48. The half-life of a π⁺ meson at rest is $2.5 \times 10^{-8}$ second. A beam of π⁺ mesons is generated at a point 15 meters from a detector. Only $\frac{1}{2}$ of the π⁺ mesons live to reach the detector. The speed of the π⁺ mesons is

(A) $\frac{1}{2}c$
(B) $\sqrt{\frac{2}{5}}c$
(C) $\frac{2}{\sqrt{5}}c$
(D) c
(E) $2c$

49. The infinite xy-plane is a nonconducting surface, with surface charge density $\sigma$, as measured by an observer at rest on the surface. A second observer moves with velocity $\vec{v} \times \hat{z}$ relative to the surface, at height $h$ above it. Which of the following expressions gives the electric field measured by this second observer?

(A) $\frac{\sigma}{2\varepsilon_0} \hat{z}$
(B) $\frac{\sigma}{2\varepsilon_0} \sqrt{1-\frac{v^2}{c^2}} \hat{z}$
(C) $\frac{\sigma}{2\varepsilon_0} \frac{\sqrt{1-\frac{v^2}{c^2}}}{\hat{z} + \frac{v}{c} \hat{z}}$
(D) $\frac{\sigma}{2\varepsilon_0} \frac{\sqrt{1-\frac{v^2}{c^2}}}{\hat{z} - \frac{v}{c} \hat{z}}$
(E) $\frac{\sigma}{2\varepsilon_0} \frac{\sqrt{1-\frac{v^2}{c^2}}}{\hat{z} + \frac{v}{c} \hat{f}}$

50. In inertial frame $S$, two events occur at the same instant in time and 3c - minutes apart in space. In inertial frame $S'$, the same events occur at 5c - minutes apart. What is the time interval between the events in $S'$?

(A) 0 min
(B) 2 min
(C) 4 min
(D) 8 min
(E) 16 min

51. The solution to the Schrödinger equation for a particle bound in a one-dimensional, infinitely deep potential well, indexed by quantum number $n$, indicates that in the middle of the well the probability density vanishes for

(A) the ground state ($n = 1$) only
(B) states of even $n$ ($n = 2, 4, \ldots$)
(C) states of odd $n$ ($n = 1, 3, \ldots$)
(D) all states ($n = 1, 2, 3, \ldots$)
(E) all states except the ground state

52. At a given instant of time, a rigid rotator is in the state $\psi(\theta, \phi) = \sqrt{3/4\pi} \sin\theta \sin\phi$, where $\theta$ is the polar angle relative to the z-axis and $\phi$ is the azimuthal angle. Measurement will find which of the following possible values of the $z$-component of the angular momentum, $L_z$?

(A) 0
(B) $\hbar/2$, $-\hbar/2$
(C) $\hbar$, $-\hbar$
(D) $2\hbar$, $-2\hbar$
(E) $\hbar$, 0, $-\hbar$

53. Positronium is the bound state of an electron and a positron. Consider only the states of zero orbital angular momentum ($\ell = 0$). The most probable decay product of any such state of positronium with spin zero (singlet) is

(A) 0 photons
(B) 1 photon
(C) 2 photons
(D) 3 photons
(E) 4 photons

GO ON TO THE NEXT PAGE.
Questions 54-55 concern a plane electromagnetic wave that is a superposition of two independent orthogonal plane waves and can be written as the real part of
\[ E = \hat{x}E_1 \exp(i(kz - \omega t)) + \hat{y}E_2 \exp(i(kz - \omega t + \pi)), \]
where \( k \), \( \omega \), \( E_1 \), and \( E_2 \) are real.

54. If \( E_2 = E_1 \), the tip of the electric field vector will
describe a trajectory that, as viewed along the \( z \)-axis
from positive \( z \) and looking toward the origin, is a
(A) line at \( 45^\circ \) to the \( +z \)-axis
(B) line at \( 335^\circ \) to the \( +z \)-axis
(C) clockwise circle
(D) counterclockwise circle
(E) random path

55. If the plane wave is split and recombined on a
screen after the two portions, which are polarized
in the \( x \)- and \( y \)-directions, have traveled an optical
path difference of \( 2\pi/\lambda \), the observed average
intensity will be proportional to
(A) \( E_1^2 + E_2^2 \)
(B) \( E_1^2 - E_2^2 \)
(C) \( (E_1 + E_2)^2 \)
(D) \( (E_1 - E_2)^2 \)
(E) 0

56. A light source is at the bottom of a pool of water
(the index of refraction of water is 1.33). At what
minimum angle of incidence will a ray be totally
reflected at the surface?
(A) \( 0^\circ \)
(B) \( 25^\circ \)
(C) \( 50^\circ \)
(D) \( 75^\circ \)
(E) \( 90^\circ \)

57. Consider a single-slit diffraction pattern for a slit of
width \( d \). It is observed that for light of wavelength
400 nanometers, the angle between the first minimum
and the central maximum is \( 4 \times 10^{-3} \) radians.
The value of \( d \) is
(A) \( 1 \times 10^{-5} \) m
(B) \( 5 \times 10^{-5} \) m
(C) \( 1 \times 10^{-4} \) m
(D) \( 2 \times 10^{-4} \) m
(E) \( 1 \times 10^{-3} \) m

58. A collimated laser beam emerging from a commercial HeNe laser has a diameter of about 1 millimeter. In order to convert this beam into a well-collimated beam of diameter 10 millimeters, two convex lenses are to be used. The first lens is of focal length 1.5 centimeters and is to be mounted at the output of the laser. What is the focal length, \( f \), of the second lens and how far from the first lens should it be placed?

\[
\begin{array}{|c|c|}
\hline
f & \text{Distance} \\
\hline
(A) & 4.5 \text{ cm} \quad 6.0 \text{ cm} \\
(B) & 10 \text{ cm} \quad 10 \text{ cm} \\
(C) & 10 \text{ cm} \quad 11.5 \text{ cm} \\
(D) & 15 \text{ cm} \quad 15 \text{ cm} \\
(E) & 15 \text{ cm} \quad 16.5 \text{ cm} \\
\hline
\end{array}
\]

59. The approximate number of photons in a femto-
second \( (10^{-15} \text{s}) \) pulse of 600 nanometers wave-
length light from a 10-kilowatt peak-power dye laser is
(A) \( 10^3 \)
(B) \( 10^7 \)
(C) \( 10^{11} \)
(D) \( 10^{15} \)
(E) \( 10^{18} \)

60. The Lyman alpha spectral line of hydrogen
(\( \lambda = 122 \) nanometers) differs by \( 1.8 \times 10^{-12} \) meter
in spectra taken at opposite ends of the Sun’s
equator. What is the speed of a particle on the
equator due to the Sun’s rotation, in kilometers per
second?
(A) \( 0.22 \)
(B) \( 2.2 \)
(C) \( 22 \)
(D) \( 220 \)
(E) \( 2200 \)

GO ON TO THE NEXT PAGE.
61. A sphere of radius $R$ carries charge density proportional to the square of the distance from the center: $\rho = Ar^2$, where $A$ is a positive constant. At a distance of $R/2$ from the center, the magnitude of the electric field is

(A) $A/4\pi\epsilon_0$

(B) $AR^2/40\epsilon_0$

(C) $AR^2/24\epsilon_0$

(D) $AR^2/5\epsilon_0$

(E) $AR^2/3\epsilon_0$

62. Two capacitors of capacitances 1.0 microfarad and 2.0 microfarads are each charged by being connected across a 5.0-volt battery. They are disconnected from the battery and then connected to each other with resistive wires so that plates of opposite charge are connected together. What will be the magnitude of the final voltage across the 2.0-microfarad capacitor?

(A) 0 V

(B) 0.6 V

(C) 1.7 V

(D) 3.3 V

(E) 5.0 V

63. According to the Standard Model of elementary particles, which of the following is NOT a composite object?

(A) Muon

(B) Pi-meson

(C) Neutron

(D) Deuteron

(E) Alpha particle

64. The binding energy of a heavy nucleus is about 7 million electron volts per nucleon, whereas the binding energy of a medium-weight nucleus is about 8 million electron volts per nucleon. Therefore, the total kinetic energy liberated when a heavy nucleus undergoes symmetric fission is most nearly

(A) 1876 MeV

(B) 938 MeV

(C) 7 MeV

(D) 8 MeV

(E) 7 MeV

65. A man of mass $m$ on an initially stationary boat gets off the boat by leaping to the left in an exactly horizontal direction. Immediately after the leap, the boat, of mass $M$, is observed to be moving to the right at speed $v$. How much work did the man do during the leap (both on his own body and on the boat)?

(A) $1/2Mv^2$

(B) $1/2mv^2$

(C) $1/2(M + m)v^2$

(D) $1/2\left(\frac{M^2}{m}+M\right)v^2$

(E) $1/2\left(M\frac{m}{M+m}\right)v^2$

66. When it is about the same distance from the Sun as is Jupiter, a spacecraft on a mission to the outer planets has a speed that is 1.5 times the speed of Jupiter in its orbit. Which of the following describes the orbit of the spacecraft about the Sun?

(A) Spiral

(B) Circle

(C) Ellipse

(D) Parabola

(E) Hyperbola
\( t_{1/2} = 2.5 \times 10^{-8} \text{s} \)

Since only half the particles survive to the target,

\( t_{\text{lab}} = \frac{1}{\nu} \) is the half-life in the lab.

\[
\frac{T_{\text{lab}}}{T_{\text{rest}}} = \frac{1}{\sqrt{v_0}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} > 1
\]

\[
1 - \frac{v^2}{c^2} = \frac{v^2 t_0^2}{\lambda^2}
\]

\[
1 = \left( 1 + \frac{t_0^2 c^2}{\lambda^2} \right)^2
\]

\[
B = \frac{v}{c}
\]

\[
\left( 1 + \frac{t_0^2 c^2}{\lambda^2} \right) = \left( 1 + \left( \frac{2.5 \times 10^{-8} \text{s}}{15 \text{ m}} \right)^2 \right)
\]

\[
= (1 + \left( \frac{1}{2} \right)^2) = \frac{5}{4}
\]

\[
B^2 = \frac{4}{5} \quad \nu = \frac{2}{\sqrt{5}} < c
\]
The length contracts in the direction of motion increasing the charge density.

\[ \sigma' = \frac{\sigma}{\sqrt{1-v^2/c^2}} \quad > \quad \sigma \]

\[ E = \frac{\sigma'}{\varepsilon_0} = \frac{\sigma}{2\varepsilon_0 \sqrt{1-v^2/c^2}} \]
\[ \Delta t = 0 \quad \Delta x = 3 \text{ minutes} \]

\[ \Delta t' = 7 \quad \Delta x' = 5 \text{ minutes} \]

\[ t' = \gamma \left( t - \frac{ux}{c^2} \right) \]

\[ x' = \gamma \left( x - ct \right) \]

\[ \Delta x' = \gamma (\Delta x - c\Delta t) = \gamma \left( 3 \text{ minutes} - c \cdot 0 \right) = 5 \text{ minutes} \]

\[ \gamma = \frac{5}{3} \]

\[ \Delta t' = \gamma \left( \Delta t - \frac{u}{c^2} \Delta x \right) \]

\[ = \gamma \left( 0 - \frac{u}{c} \cdot 3 \right) \quad (\text{time measure}) \]

\[ \gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \]

\[ 1 - \frac{u^2}{c^2} = \frac{1}{\gamma^2} \]
\[
\frac{\gamma^2}{c^2} = 1 - \frac{1}{\gamma^2} = 1 - \frac{9}{25} = \frac{16}{25}
\]

\[
\frac{\gamma}{c} = \frac{4}{5}
\]

\[
\Delta t' = 8 \left( -\frac{\gamma}{c} \cdot 3 \right)
\]

\[
= \frac{5}{3} \left( -\frac{4}{5} \cdot 3 \right)
\]

\[
= 4 \text{ minutes}
\]
All even states
\[ \Psi = \sqrt{\frac{3}{4}} \pi \sin \theta \sin \phi \]

\[ = \sqrt{\frac{3}{4}} \pi \sin \theta \left[ \frac{e^{i\phi} - e^{-i\phi}}{2i} \right] \]

\[ m = \pm 1 \quad L_z = \pm \hbar \]
0 photons does not conserve energy

1 photon does not conserve momentum

2 photons can also conserve angular momentum.
\[ \vec{E} = x E_1 e^{i(kz-\omega t)} + y E_2 e^{i(kx-\omega t + \pi)} \]

Waves, \( e^{i\pi} = -1 \) out of phase

\[ \vec{E} = (x E_1 - y E_2) e^{i(kz-\omega t)} \]

\[ \text{I.a } \vec{E} \cdot \vec{E} = E_1^2 + E_2^2 \]

Nothing changes if one wave travels, \( \frac{2\pi}{k} = \lambda \) farther.
\[ n_w \sin \Theta_c = n_{air} \sin 90^\circ \]

\[ \sin \Theta_c = \frac{1}{n_w} = \frac{3}{4} \]

\[ \Theta_c = 50^\circ \quad (48.6^\circ) \]
\[ \Theta = 4 \times 10^{-3} \text{ radians} \]

\[ d \sin \theta = m \lambda \quad m = 1 \]

\[ d = \frac{\lambda}{\sin \theta} = \frac{\lambda}{\Theta} = \frac{4 \times 10^{-7} \text{m}}{4 \times 10^{-3}} \]

\[ = 1 \times 10^{-4} \text{m} \]
similar triangles

Distance $f_e + f_o = 16.5\text{cm}$
\[ E = P \Delta t \]
\[ = \left(10^4 \text{ J/s} \right) \left(10^{-15} \text{s} \right) \]
\[ = 10^{-11} \text{ J} \]

\[ E_{\text{photon}} = \hbar \nu = h \nu = \frac{hc}{\lambda} \]
\[ = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s} \cdot 3 \times 10^8 \text{ m/s}}{6 \times 10^{-7} \text{ m}} \]
\[ = 3.3 \times 10^{-19} \text{ J} \]

\[ N = \frac{E}{E_{\text{photon}}} = \frac{10^{-11} \text{ J}}{3.3 \times 10^{-19} \text{ J}} = \frac{1}{3} 10^{8} = 3 \times 10^7 \]
Doppler Shift

\[ \chi' = \sqrt{\frac{1-u/c}{1+u/c}} \lambda \]

\[ \frac{\Delta \lambda}{\lambda} = 1 - \sqrt{\frac{1+u/c}{1-u/c}} \]

The Doppler shift at the edges is twice the shift from zero

\[ \frac{\Delta \lambda}{\lambda} = \frac{1.8 \times 10^{-12} \text{m}^2}{122 \times 10^{-9} \text{m}} = \frac{3}{4} \times 10^{-5} \]

Use binomial expansion \((1+x)^n = 1+nx+\ldots\)

\[ \sqrt{\frac{1+u/c}{1-u/c}} = (1+\frac{1}{2} \frac{u}{c})(1-\frac{1}{2}(\frac{u}{c})) \]

\[ = 1 + \frac{u}{c} + \ldots \]

\[ \frac{\Delta \lambda}{\lambda} = 1 - \sqrt{\frac{1+u/c}{1-u/c}} \approx 1 - (1 + \frac{u}{c}) = -\frac{u}{c} \]

\[ u = \frac{\Delta \lambda}{\lambda} c = \frac{3}{4} \times 10^{-5} \times 3 \times 10^8 \text{m/s} = \frac{9}{4} \times 10^3 \text{m/s} \]

\[ = 2.2 \text{ km/s} \]
\[ p = Ar^2 \]

\[ Q_{en} = \int_0^r 4\pi r^2 \rho \, dr = 4\pi A \int_0^r r^4 \, dr \]

\[ = \frac{4\pi Ar^5}{5} \]

\[ E = \frac{Q_{en}}{4\pi \varepsilon_0 r^2} \hat{r} \]

\[ = \frac{4\pi Ar^5/5}{4\pi \varepsilon_0 r^2} \]

\[ = \frac{Ar^3}{5\varepsilon_0} \]

\[ A + r = R/2 \]

\[ \vec{E} = \frac{AR^3}{40 \varepsilon_0} \]
\[ Q_1 = 5C_1 = 5\mu C \quad Q_2 = 5C_2 = 10\mu C \]

\[
\begin{array}{c}
\underline{5\mu C} \\
\underline{-10\mu C}
\end{array}
\]

\[
\begin{array}{c}
\underline{-5\mu C} \\
\underline{10\mu C}
\end{array}
\]

\[ Q_i = 5\mu C \quad C_{eq} = 1\mu F + 2\mu F = 3\mu C \]

\[ \Delta V = \frac{Q}{C} = \frac{5}{3} \quad V = \]

\[ \approx 1.7V \]
muon - heavier partner of electron.
\[ B_H = 7 \text{MeV} \]
\[ B_m = 8 \text{MeV} \]

Let \( A \) be the number of nucleons.

For light nuclei:
\[ A_0 \quad m_{A_0}c^2 = A_0 m_0 c^2 + A_0 (7 \text{MeV}) \]

For heavy nuclei:
\[ A_{1,2} \quad m_{A_1}c^2 + m_{A_2}c^2 = A_0 m_c c^2 + A_0 (8 \text{MeV}) \]

\[ \Delta E = A_0 \, 1 \text{MeV} \]

For heavy nuclei, \( A_0 \propto 200 \)

\[ \Delta E = 200 \text{MeV} \]
Work is total kinetic energy of system since the jump is horizontal.

**Momentum**

\[ mV = Mv \]
\[ V = \frac{Mv}{m} \]

**Energy**

\[ U = \frac{1}{2} mV^2 + \frac{1}{2} Mv^2 \]
\[ = \frac{1}{2} m \left( \frac{Mv}{m} \right)^2 + \frac{1}{2} Mv^2 \]
\[ = \frac{1}{2} \left( M + \frac{M^2}{m} \right) v^2 \]
66 \quad \mathbf{V} = 1.5 \mathbf{V}_J

What is escape velocity at Jupiter?

\begin{align*}
PE &= KE \\
\frac{m M_J G}{R_J} &= \frac{1}{2} m v^2 \\
v^2 &= \frac{2 M_J G}{R_J}
\end{align*}

How fast is Jupiter going - assume circular?

\begin{align*}
\mathbf{m}_c &= \frac{m_J M_J G}{R_J^2} \\
&= \frac{m_J v_J^2}{R_J} \\
v_J^2 &= \frac{M_J G}{R_J}
\end{align*}

Escape velocity squared = \( Z v_J^2 = V_c^2 \)
\[ v = \frac{3}{2} v_f \]

\[ v^2 = \frac{9}{4} \quad v_f^2 = 2.2 \cdot v_f^2 > \text{escape velocity} \]

hyperbola.