67. A black hole is an object whose gravitational field is so strong that even light cannot escape. To what approximate radius would Earth (mass = \(5.98 \times 10^{24}\) kilograms) have to be compressed in order to become a black hole?

(A) 1 nm
(B) 1 \(\mu\)m
(C) 1 cm
(D) 100 m
(E) 10 km

68. A bead is constrained to slide on a frictionless rod that is fixed at an angle \(\theta\) with a vertical axis and is rotating with angular frequency \(\omega\) about the axis, as shown above. Taking the distance \(s\) along the rod as the variable, the Lagrangian for the bead is equal to

(A) \(\frac{1}{2}ms^2 - mgs \cos \theta\)
(B) \(\frac{1}{2}ms^2 + \frac{1}{2}m(\omega s)^2 - mgs\)
(C) \(\frac{1}{2}ms^2 + \frac{1}{2}m(\omega s \cos \theta)^2 + mgs \cos \theta\)
(D) \(\frac{1}{2}m(s \sin \theta)^2 - mgs \cos \theta\)
(E) \(\frac{1}{2}ms^2 + \frac{1}{2}m(\omega s \sin \theta)^2 - mgs \cos \theta\)

69. Two long conductors are arranged as shown above to form overlapping cylinders, each of radius \(r\), whose centers are separated by a distance \(d\). Current of density \(J\) flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown. What are the magnitude and direction of the magnetic field at point \(A\)?

(A) \((\mu_0/2\pi)\pi d J\), in the +y-direction
(B) \((\mu_0/2\pi)d^2 J/r\), in the +y-direction
(C) \((\mu_0/2\pi)d^2 J/r\), in the -y-direction
(D) \((\mu_0/2\pi)J^2/d\), in the -y-direction
(E) There is no magnetic field at \(A\).

70. A charged particle, \(A\), moving at a speed much less than \(c\), decelerates uniformly. A second particle, \(B\), has one-half the mass, twice the charge, three times the velocity, and four times the acceleration of particle \(A\). According to classical electrodynamics, the ratio \(P_B/P_A\) of the powers radiated is

(A) 16
(B) 32
(C) 48
(D) 64
(E) 72
71. The figure above shows the trajectory of a particle that is deflected as it moves through the uniform electric field between parallel plates. There is potential difference $V$ and distance $d$ between the plates, and they have length $L$. The particle (mass $m$, charge $q$) has nonrelativistic speed $v$ before it enters the field, and its direction at this time is perpendicular to the field. For small deflections, which of the following expressions is the best approximation to the deflection angle $\theta$?

(A) $\arctan\left(\frac{(L/d)(Vq/mv^2)}{v}ight)$
(B) $\arctan\left(\frac{(L/d)(Vq/mv^2)^2}{v}ight)$
(C) $\arctan\left(\frac{(L/d)^2(Vq/mv^2)}{v}ight)$
(D) $\arctan\left(\frac{(L/d)(2Vq/mv^2)^{1/2}}{v}ight)$
(E) $\arctan\left(\frac{(L/d)^{1/2}(2Vq/mv^2)}{v}\right)$

72. In a voltage amplifier, which of the following is NOT usually a result of introducing negative feedback?

(A) Increased amplification
(B) Increased bandwidth
(C) Increased stability
(D) Decreased distortion
(E) Decreased voltage gain

73. The adiabatic expansion of an ideal gas is described by the equation $PV^\gamma = C$, where $\gamma$ and $C$ are constants. The work done by the gas in expanding adiabatically from the state $(V_i, P_i)$ to $(V_f, P_f)$ is equal to

(A) $P_fV_f$
(B) $\frac{(P_i + P_f)}{2}(V_f - V_i)$
(C) $\frac{P_fV_f - P_iV_i}{1 - \gamma}$
(D) $\frac{P_i(V_f^{1+\gamma} - V_i^{1+\gamma})}{1 + \gamma}$
(E) $\frac{P_f(V_f^{1-\gamma} - V_i^{1-\gamma})}{1 - \gamma}$

74. A body of mass $m$ with specific heat $C$ at temperature 500 K is brought into contact with an identical body at temperature 100 K, and the two are isolated from their surroundings. The change in entropy of the system is equal to

(A) $\frac{4m}{3}C$ ln $9/5$
(B) $mC$ ln $9/5$
(C) $mC$ ln $3$
(D) $-mC$ ln $5/3$
(E) 0

GO ON TO THE NEXT PAGE.
75. Window A is a pane of glass 4 millimeters thick, as shown above. Window B is a sandwich consisting of two extremely thin layers of glass separated by an air gap 2 millimeters thick, as shown above. If the thermal conductivities of glass and air are 0.8 watt/meter °C and 0.025 watt/meter °C, respectively, then the ratio of the heat flow through window A to the heat flow through window B is

(A) 2  
(B) 4  
(C) 8  
(D) 16  
(E) 32

76. A Gaussian wave packet travels through free space. Which of the following statements about the wave packet are correct for all such wave packets?

I. The average momentum of the wave packet is zero.
II. The width of the wave packet increases with time, as \( t \to \infty \).
III. The amplitude of the wave packet remains constant with time.
IV. The narrower the wave packet is in momentum space, the wider it is in coordinate space.

(A) I and III only  
(B) II and IV only  
(C) I, II, and IV only  
(D) II, III, and IV only  
(E) I, II, III, and IV

GO ON TO THE NEXT PAGE.
Two ions, 1 and 2, at fixed separation, with spin angular momentum operators $S_1$ and $S_2$, have the interaction Hamiltonian $H = -J \cdot S_1 \cdot S_2$, where $J > 0$. The values of $S_1^2$ and $S_2^2$ are fixed at $S_1(S_1 + 1)$ and $S_2(S_2 + 1)$, respectively. Which of the following is the energy of the ground state of the system?

(A) 0

(B) $-JS_1S_2$

(C) $-J[S_1(S_1 + 1) - S_2(S_2 + 1)]$

(D) $-\frac{J/2}{[(S_1 + S_2)(S_1 + S_2 + 1) - S_1(S_1 + 1) - S_2(S_2 + 1)]}$

(E) $\frac{-J}{2} \left[ S_1(S_1 + 1) + S_2(S_2 + 1) \right]$

In an n-type semiconductor, which of the following is true of impurity atoms?

(A) They accept electrons from the filled valence band into empty energy levels just above the valence band.

(B) They accept electrons from the filled valence band into empty energy levels just below the valence band.

(C) They accept electrons from the conduction band into empty energy levels just above the conduction band.

(D) They donate electrons to the filled valence band from donor levels just above the valence band.

(E) They donate electrons to the conduction band from filled donor levels just below the conduction band.

For an ideal diatomic gas in thermal equilibrium, the ratio of the molar heat capacity at constant volume at very high temperatures to that at very low temperatures is equal to

(A) 1

(B) $5/3$

(C) 2

(D) 7/3

(E) 3

A string consists of two parts attached at $x = 0$. The right part of the string ($x > 0$) has mass $\mu_r$ per unit length and the left part of the string ($x < 0$) has mass $\mu_l$ per unit length. The string tension is $T$. If a wave of unit amplitude travels along the left part of the string, as shown in the figure above, what is the amplitude of the wave that is transmitted to the right part of the string?

(A) 0

(B) $\frac{2}{1 + \sqrt{\mu_l/\mu_r}}$

(C) $\frac{2\sqrt{\mu_l/\mu_r}}{1 + \sqrt{\mu_l/\mu_r}}$

(D) $\frac{\sqrt{\mu_l/\mu_r} - 1}{\sqrt{\mu_l/\mu_r} + 1}$

(E) 0

A piano tuner who wishes to tune the note $D_2$ corresponding to a frequency of 73.416 hertz has tuned $A_4$ to a frequency of 440.000 hertz. Which harmonic of $D_2$ (counting the fundamental as the first harmonic) will give the lowest number of beats per second, and approximately how many beats will this be when the two notes are tuned properly?

<table>
<thead>
<tr>
<th>Harmonic</th>
<th>Number of Beats</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) 6</td>
<td>5</td>
</tr>
<tr>
<td>(B) 6</td>
<td>0.5</td>
</tr>
<tr>
<td>(C) 5</td>
<td>0.1</td>
</tr>
<tr>
<td>(D) 3</td>
<td>0.372</td>
</tr>
<tr>
<td>(E) 2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Consider two horizontal glass plates with a thin film of air between them. For what values of the thickness of the film of air will the film, as seen by reflected light, appear bright if it is illuminated normally from above by blue light of wavelength 488 nanometers?

(A) 0, 122 nm, 244 nm

(B) 0, 122 nm, 366 nm

(C) 0, 244 nm, 488 nm

(D) 122 nm, 244 nm, 366 nm

(E) 122 nm, 366 nm, 610 nm
67 Number of the Day 1cm

\[
KE = \frac{1}{2} mc^2 = PE = \frac{m\, M\, L}{R_{\text{earth}}}
\]

\[
R = \frac{Z\, M\, L}{c^2}
\]

\[
= Z \left(6 \times 10^{24}\right) \left(6.67 \times 10^{-11}\right) \frac{1}{\left(3 \times 10^8\right)^2}
\]

\[
= 8 \times 10^{24} \, \text{m} \approx 1\, \text{cm}
\]
\[ T = \frac{1}{2} m \dot{s} + \frac{1}{2} m (\omega s \cos \theta)^2 = \frac{1}{2} m (\omega s \sin \theta)^2 \]

\[ V = mg \sin \theta \]

\[ L = T - V = \frac{1}{2} m \dot{s} + \frac{1}{2} m (\omega s \sin \theta)^2 - mg \cos \theta \]
\[ I_{\text{enc}} = \pi \rho^2 \]

\[ B = \frac{\mu_0 I_{\text{enc}}}{2 \pi \rho} = \frac{\mu_0 \pi \rho^2}{2 \pi \rho} = \frac{\mu_0 \rho}{2} \]

\[ = \frac{\mu_0 \pi \rho}{2 \pi} \]

\[ = \frac{\mu_0}{2 \pi} (\pi \rho) \]

The field is twice this.

\[ B = \frac{\mu_0}{2 \pi} (2 \pi \rho) \]

The field is evaluated at \( d/2 \)

\[ B = \frac{\mu_0}{2 \pi} (\pi J_0) \]
Larmor Formula

\[ P = \frac{q^2 a^2}{6 \pi \varepsilon_0 c^2} \]

\[ \frac{P_B}{P_A} = \left( \frac{2q}{2q + 4a} \right)^2 \frac{(2q)^2 (4a)^2}{q^2 a^2} = 64 \]
\[ (7) \quad E = \frac{V}{d} \]

\[ \Delta t = \frac{L}{v_0} \]

\[ h = \frac{1}{2} a (\Delta t)^2 = \frac{1}{2} \frac{gE}{m} (\Delta t)^2 \]

\[ = \frac{1}{2} \frac{gE}{m} \left( \frac{L}{v_0} \right)^2 \]

\[ \Theta \sim \frac{h}{L} = \frac{1}{2} \frac{g \Delta L}{md v_0^2} \]
Negative feedback usually reduces amplification.
\[ W = \int_{v_i}^{v_f} P dV \]

\[ = \int_{v_i}^{v_f} \frac{C}{\nu^{\gamma+1}} dV \]

\[ = \left. \frac{CV^{-\gamma+1}}{1-\gamma} \right|_{v_i}^{v_f} \]

\[ = \frac{C V_f^{-\gamma+1} - C V_i^{-\gamma+1}}{1-\gamma} \]

\[ = \frac{P_f V_f - P_i V_i}{1-\gamma} \]

since \[ C V_f^{-\gamma+1} = C V_f^{-\gamma} V_f = P_f V_f \]
\[
\Delta S = \frac{\Delta Q}{T}
\]

\[
\Delta S_1 = \int_{T_0}^{T_f} \frac{\Delta Q}{T} = \int_{T_0}^{T_f} \frac{mC \, dT}{T} = mC \ln \left( \frac{T_f}{T_0} \right)
\]

\[
\Delta S = \Delta S_1 + \Delta S_2 = mC \left[ \ln \left( \frac{T_f}{T_0} \right) + \ln \left( \frac{T_f}{T_0} \right) \right]
\]

\[
= mC \left[ \ln \left( \frac{300}{100} \right) + \ln \left( \frac{300}{500} \right) \right]
\]

\[
= mC \ln \frac{9}{5}
\]
\[ I_h = \frac{\Delta T}{R_h} \]

\[ R_h \approx \frac{l}{A\sigma_h} \]

\[ \frac{I_A}{I_B} = \frac{R_B}{R_A} = \frac{\lambda_B \sigma_A}{\lambda_A \sigma_B} \]

\[ = \frac{2\text{ mm}}{4\text{ mm}} \cdot \frac{0.8}{0.025} \]

\[ = \frac{1.6}{0.1} = 16 \]
二，四
\[ (S_1 + S_2)^2 = S_1^2 + 2S_1 \cdot S_2 + S_2^2 = S^2 \]
\[ S_1 \cdot S_2 = \frac{1}{2} \left( (S_1 + S_2)^2 - S_1^2 - S_2^2 \right) \]
\[ S^2 |\phi_0\rangle = S(s+1)\hbar^2 = S(s+1) \quad h = 1 \]

\[
H = -\frac{\hbar}{2} \left( (S_1 + S_2)(S_1 + S_2 + 1) - S_1 (s_1 + 1) - S_2 (s_2 + 1) \right)
\]
\[ C_v = \frac{d}{2} \, R \]

\( d = \text{degrees of freedom} \)

Degrees of Freedom

\begin{align*}
\text{3 translation} \\
\text{2 rotation} \\
\text{2 vibration (potential + kinetic)} \\
\hline \\
7 \\
\end{align*}

High temperature \( C_v = \frac{7}{2} \, R \)

Low temperature \( C_v = \frac{3}{2} \, R \)

\[ \frac{C_v_{\text{high}}}{C_v_{\text{low}}} = \frac{7}{3} \]
\[ T = \frac{2}{1 - \sqrt{\mu_2/\mu_1}} \]

\[ = \frac{2 \sqrt{\mu_1/\mu_2}}{1 + \sqrt{\mu_1/\mu_2}} \]
\[ f_A = 440 \text{ Hz} \]
\[ f_B = 734 \text{ Hz} \]

Beats \[ \Delta f = | f_B - f_A | = \# \text{ beats/sec} \]

\[
\begin{array}{c}
73.4 \\
\hline 6 \\
\hline 440.4
\end{array}
\]

\[ \Delta f = 0.4 \text{ beats/sec} \]
For reflection

\[ n_i > n_t \quad \text{No change} \]
\[ n_i < n_t \quad \pi \text{ change} \]

Consider only light reflected from glass surfaces that form two sides of a cavity.

The is \( \pi \) phase difference to start with.

The additional travel of the light must produce an additional \( \pi \) for the fringe to appear bright.

\[ 2kd + \pi = 2n\pi \]
\[ 2kd = (2n-1)\pi \]
\[
\frac{4\pi d}{\lambda} = \pi (2n-1)
\]

\[
d = \lambda \left( \frac{2n-1}{4} \right)
\]

\[
= \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}
\]