15. Suppose one mole of an ideal gas undergoes the reversible cycle ABCA shown in the P-V diagram above, where AB is an isotherm. The molar heat capacities are \( C_p \) at constant pressure and \( C_V \) at constant volume. The net heat added to the gas during the cycle is equal to

(A) \( RT_h \frac{V_2}{V_1} \)
(B) \( -C_p(T_h - T_c) \)
(C) \( C_V(T_h - T_c) \)
(D) \( RT_h \ln \frac{V_2}{V_1} - C_p(T_h - T_c) \)
(E) \( RT_h \ln \frac{V_2}{V_1} - R(T_h - T_c) \)

16. The mean free path for the molecules of a gas is approximately given by \( \frac{1}{\eta \sigma} \), where \( \eta \) is the number density and \( \sigma \) is the collision cross section. The mean free path for air molecules at room conditions is approximately

(A) \( 10^{-4} \) m
(B) \( 10^{-7} \) m
(C) \( 10^{-10} \) m
(D) \( 10^{-13} \) m
(E) \( 10^{-16} \) m

17. The wave function for a particle constrained to move in one dimension is shown in the graph above \((\Psi = 0 \text{ for } x \leq 0 \text{ and } x \geq 5)\). What is the probability that the particle would be found between \( x = 2 \) and \( x = 4 \)?

(A) \( \frac{17}{64} \)
(B) \( \frac{25}{64} \)
(C) \( \frac{5}{8} \)
(D) \( \sqrt{\frac{5}{8}} \)
(E) \( \frac{13}{16} \)
18. Consider a potential of the form

\[ V(x) = \begin{cases} 
0, & x \leq a \\
V_0, & a < x < b \\
0, & x \geq b 
\end{cases} \]

as shown in the figure above. Which of the following wave functions is possible for a particle incident from the left with energy \( E < V_0 \)?

(A) \( \Psi(x) \)

(B) \( \Psi(x) \)

(C) \( \Psi(x) \)

(D) \( \Psi(x) \)

(E) \( \Psi(x) \)

19. When alpha particles are directed onto atoms in a thin metal foil, some make very close collisions with the nuclei of the atoms and are scattered at large angles. If an alpha particle with an initial kinetic energy of 5 MeV happens to be scattered through an angle of 180°, which of the following must have been its distance of closest approach to the scattering nucleus? (Assume that the metal foil is made of silver, with \( Z = 50 \).)

(A) \( 1.22 \times 5^{10} \text{ fm} \)

(B) \( 2.9 \times 10^{-15} \text{ m} \)

(C) \( 1.0 \times 10^{-12} \text{ m} \)

(D) \( 3.0 \times 10^{-4} \text{ m} \)

(E) \( 1.7 \times 10^{-7} \text{ m} \)

20. A helium atom, mass 4u, travels with nonrelativistic speed \( v \) normal to the surface of a certain material, makes an elastic collision with an (essentially free) surface atom, and leaves in the opposite direction with speed 0.6\( v \). The atom on the surface must be an atom of

(A) hydrogen, mass 1u

(B) helium, mass 4u

(C) carbon, mass 12u

(D) oxygen, mass 16u

(E) silicon, mass 28u

GO ON TO THE NEXT PAGE.
21. The period of a physical pendulum is \( T = 2\pi\sqrt{\frac{I}{mgd}} \), where \( I \) is the moment of inertia about the pivot point and \( d \) is the distance from the pivot to the center of mass. A circular hoop hangs from a nail on a barn wall. The mass of the hoop is 3 kilograms and its radius is 20 centimeters. If it is displaced slightly by a passing breeze, what is the period of the resulting oscillations?

(A) 0.63 s  
(B) 1.0 s  
(C) 1.3 s  
(D) 1.8 s  
(E) 2.1 s

22. The curvature of Mars is such that its surface drops a vertical distance of 2.0 meters for every 3600 meters tangent to the surface. In addition, the gravitational acceleration near its surface is 0.4 times that near the surface of Earth. What is the speed a golf ball would need to orbit Mars near the surface, ignoring the effects of air resistance?

(A) 0.9 km/s  
(B) 1.8 km/s  
(C) 3.6 km/s  
(D) 4.5 km/s  
(E) 5.4 km/s

23. Suppose that the gravitational force law between two massive objects were \( F_{12} = \frac{Gm_1m_2}{r_{12}^{3+\epsilon}} \), where \( \epsilon \) is a small positive number. Which of the following statements would be FALSE?

(A) The total mechanical energy of the planet-Sun system would be conserved.  
(B) The angular momentum of a single planet moving about the Sun would be conserved.  
(C) The periods of planets in circular orbits would be proportional to the \((3 + \epsilon)/2\) power of their respective orbital radii.  
(D) A single planet could move in a stationary noncircular elliptical orbit about the Sun.  
(E) A single planet could move in a stationary circular orbit about the Sun.

24. Two identical conducting spheres, \( A \) and \( B \), carry equal charge. They are initially separated by a distance much larger than their diameters, and the force between them is \( F \). A third identical conducting sphere, \( C \), is uncharged. Sphere \( C \) is first touched to \( A \), then to \( B \), and then removed. As a result, the force between \( A \) and \( B \) is equal to

(A) 0  
(B) \( F/16 \)  
(C) \( F/4 \)  
(D) \( 3F/8 \)  
(E) \( F/2 \)

25. Two real capacitors of equal capacitance \((C_1 = C_2)\) are shown in the figure above. Initially, while the switch \( S \) is open, one of the capacitors is uncharged and the other carries charge \( Q_0 \). The energy stored in the charged capacitor is \( U_0 \). Sometime after the switch is closed, the capacitors \( C_1 \) and \( C_2 \) carry charges \( Q_1 \) and \( Q_2 \), respectively; the voltages across the capacitors are \( V_1 \) and \( V_2 \); and the energies stored in the capacitors are \( U_1 \) and \( U_2 \). Which of the following statements is INCORRECT?

(A) \( Q_0 = \frac{1}{2}(Q_1 + Q_2) \)  
(B) \( Q_1 = Q_2 \)  
(C) \( V_1 = V_2 \)  
(D) \( U_1 = U_2 \)  
(E) \( U_0 = U_1 + U_2 \)

26. A series \( RLC \) circuit is used in a radio to tune to an FM station broadcasting at 103.7 MHz. The resistance in the circuit is 10 ohms and the inductance is 2.0 microhenries. What is the best estimate of the capacitance that should be used?

(A) 200 pF  
(B) 50 pF  
(C) 1 pF  
(D) 0.2 pF  
(E) 0.02 pF

GO ON TO THE NEXT PAGE.
27. In laboratory experiments, graphs are employed to determine how one measured variable depends on another. These graphs generally fall into three categories: linear, semilog (logarithmic versus linear), and log-log. Which type of graph listed in the third column below would NOT be the best for plotting data to test the relationship given in the first and second columns?

<table>
<thead>
<tr>
<th>Relation</th>
<th>Variables Plotted</th>
<th>Type of Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) ( \frac{dN}{dt} \approx e^{-2t} )</td>
<td>Activity vs. time for a radioactive isotope</td>
<td>Semilog</td>
</tr>
<tr>
<td>(B) ( eV_s = hf - W )</td>
<td>Stopping potential vs. frequency for the photoelectric effect</td>
<td>Linear</td>
</tr>
<tr>
<td>(C) ( s \propto t^2 )</td>
<td>Distance vs. time for an object undergoing constant acceleration</td>
<td>Log-log</td>
</tr>
<tr>
<td>(D) ( V_{out}/V_{in} \approx 1/\omega )</td>
<td>Gain vs. frequency for a low-pass filter</td>
<td>Linear</td>
</tr>
<tr>
<td>(E) ( P \approx T^4 )</td>
<td>Power radiated vs. temperature for blackbody radiation</td>
<td>Log-log</td>
</tr>
</tbody>
</table>

28. The figure above represents the trace on the screen of a cathode ray oscilloscope. The screen is graduated in centimeters. The spot on the screen moves horizontally with a constant speed of 0.5 centimeter/millisecond, and the vertical scale is 2 volts/centimeter. The signal is a superposition of two oscillations. Which of the following are most nearly the observed amplitude and frequency of these two oscillations?

<table>
<thead>
<tr>
<th>Oscillation 1</th>
<th>Oscillation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) 5V, 250Hz</td>
<td>2.5V, 1000Hz</td>
</tr>
<tr>
<td>(B) 1.5V, 250Hz</td>
<td>3V, 1500Hz</td>
</tr>
<tr>
<td>(C) 5V, 6Hz</td>
<td>2V, 2Hz</td>
</tr>
<tr>
<td>(D) 2.5V, 83Hz</td>
<td>1.25V, 500Hz</td>
</tr>
<tr>
<td>(E) 6.14V, 98Hz</td>
<td>1.35V, 257Hz</td>
</tr>
</tbody>
</table>

GO ON TO THE NEXT PAGE.
Let \( Q \) positive if added, work positive if done by the system.

\[
\Delta U = Q - W
\]

Full cycle, \( U = nRT \), \( \Delta U = 0 \)

\( W = Q \) \ This is also true for an iso-therm, since \( \Delta T = 0 \)

**Path AB**

\[
\text{Work} = \text{Area under PV curve.}
\]

\[
W_{AB} = \int P\,dV = \int_{V_1}^{V_2} \frac{nRT}{V} \,dV
\]

\[
= nRT_h \ln \left( \frac{V_2}{V_1} \right) = Q_{AB}
\]
Path BC (constant pressure)

\[ Q_{BC} = n \, Cp \, (T_g - T_h) \]

Path CA (constant volume)

\[ Q_{CA} = n \, Cv \, (T_h - T_o) \]

\[ Q = Q_{AB} + Q_{BC} + Q_{CA} \]

\[ = n \, RT_h \ln \frac{V_2}{V_1} + \frac{n \, Cv \, (T_h - T_o) + n \, Cp \, (T_2 - T_n)}{n \, Cv - n \, Cp} \]

\[ \Downarrow \]

\[ (n \, Cv - n \, Cp) \, (T_h - T_o) \]

\[ Cp - Cv = R \]

\[ Q = n \, RT_h \ln \left( \frac{V_2}{V_1} \right) - n \, R \, (T_h - T_o) \]

\[ n = 1 \]
Mean free path \( = \frac{1}{N_0} \)

Cross-section \( \approx (1A^o)^2 = 10^{-20} m^2 \approx 10^{-19} m^2 \)

Too big

Mass density of air \( = 1.29 Kg/m^3 \)

Molar weight \( (M^o) \approx 30 gm/mole = 3 \times 10^{-2} Kg/mole \)

Molar density of air \( = \frac{1.3 Kg/m^3}{3 \times 10^{-2} Kg/mole} \)

\( \approx 40 \text{ mol/m}^3 \)

Number Density

\[ \text{40 mole/m}^3 \cdot 6 \times 10^{23} \text{ atoms/mole} \]

\[ N = 2.4 \times 10^{25} \text{ atoms/m}^3 \]

Mean Free Path

\[ \frac{1}{N_0} = \frac{1}{2.4 \times 10^{25} \text{ atoms/m}^3 \cdot 10^{-19} m^2} \]

\( \approx 10^{-6} m \)
\( \psi \psi = 1 + 1 + 4 + 9 + 1 + 0 \)

\[ = 16 \]

\[ \psi \rightarrow \psi/4 \text{ to normalize} \]

\[ P_{23} + P_{34} = \left( \frac{1}{2} \right)^2 + \left( \frac{3}{4} \right)^2 \]

\[ = \frac{1}{4} + \frac{9}{16} = \frac{13}{16} \]
Choice (c) is correct because the wave function must oscillate when $E > V_0$ and decay when $E < V_0$. 
\[ T = 5 \text{ MeV} = \frac{k q_1 q_2}{d^4} \]

\[ = \text{Potential energy} \]

\[ d = \frac{k q_1 q_2}{5 \text{ MeV}} = \frac{(9 \times 10^9 \text{ N m}^2/\text{c}^2)(2e)(50e)}{5 \times 10^6 \text{ eV}} \]

\[ q_1 = 2e \quad q_2 = 50e \]

\[ 1 \text{ eV} = (e)(1\text{V}) = e \left(\frac{\text{N} \cdot \text{m}}{\text{C}}\right) \]

\[ d = \frac{9 \times 10^{-11} \text{ m}}{5 \times 10^6 \text{ e}} \]

\[ = \frac{(9 \times 10^{-11} \text{ m/c}) (1.6 \times 10^{-19} \text{ C})}{5 \times 10^6} = 3 \times 10^{-24} \text{ m} \]

\[ = 30 \text{ fm} \]
\[ v_i = v \quad v_f = -\frac{3}{5}v \]

**Momentum**

\[ m v_i = m v_f + M v_1 \]

**Energy (elastic)**

\[ \frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2 + \frac{1}{2} M v_1^2 \]

Let \( \gamma = \frac{M}{m} \quad M = m \gamma \)

**Momentum**

\[ v_i = v_f + \gamma v_1 \]

**Energy**

\[ v_i^2 = v_f^2 + \gamma v_1^2 \]

**Eliminate** \( v_1 \)

\[ v_1 = \frac{v_i - v_f}{\gamma} \]
\[ \begin{align*}
V_i^2 & = V_f^2 + \gamma \left( \frac{V_i - V_f}{\delta} \right)^2 \\
& = V_f^2 + \frac{1}{\delta} (V_i - V_f)^2 \\
V_i^2 - V_f^2 & = (V_i - V_f)(V_i + V_f) = \frac{1}{\delta} (V_i - V_f)(V_i - V_f) \\
\gamma & = \frac{V_i - V_f}{V_i + V_f} = \frac{V - (-3/5)V}{V + (-3/5)V} = 4 \\
\Rightarrow \quad \mathcal{N} = 8 \quad \Rightarrow \quad \text{oxygen.}
\end{align*} \]
\( I_{cm} = MR^2 \)

Parallel axis thm: \( I = I_{cm} + Md^2 \)

\[
I = MR^2 + MR^2 = 2MR^2
\]

\[
T = 2\pi \sqrt{\frac{I}{MgR}} = 2\pi \sqrt{\frac{2MR^2}{MgR}}
\]

\[
= 2\pi \sqrt{\frac{2R}{g}} = 2\pi \sqrt{\frac{2}{5.10}}
\]

\[
= 2\pi \sqrt{0.04s^2} = 2\pi \cdot 0.2s
\]

\( T = 1.2s \)
22

\[ 3600 \text{ m} = d \]

\[ 2m = h \]

\[ g_m = \frac{2}{5} g \]

**orb is drop is less than 2m**

\[ d = v \Delta t \]

\[ h = \frac{1}{2} g_m \Delta t^2 = \frac{1}{2} g_m \left( \frac{d}{v} \right)^2 \]

\[ v^2 = \sqrt{\frac{g_m d^2}{2h}} \]

\[ v = \sqrt{\frac{g_m}{2h}} \]

\[ = \sqrt{\frac{10.2}{10.2}} \cdot d = 3600 \text{ m/s} \]
(23) (D) The shape of the orbit depends on the details of the force.

Circular orbit still works. Angular momentum only depends on central force. Conservation of energy is true because central forces are conservative.
\[ F = \frac{kQ_1Q_2}{d^2} = \frac{\pi \left( \frac{Q}{2} \right) \left( \frac{3Q}{4} \right)}{d^2} = \frac{3}{8} F_0 \]
Parallel Circuit, $\Delta V$

$$U_0 = \frac{1}{2} C (\Delta V)^2$$

Each capacitor now has same $V, Q, U$ as the lone capacitor.

$$V_0 = V_1 = V_2 \quad Q_0 = Q_1 = Q_2$$

$$U_0 = U_1 = U_2$$

So (E) is incorrect.
\( f = 1 \times 10^9 \text{Hz} \)

\( R = 10 \Omega \)

\( L = 2 \times 10^{-6} \text{H} \)

Resonant Frequency LRC Circuit

\[
f = \frac{1}{2\pi} \cdot \frac{1}{\sqrt{LC}}
\]

\[
C = \frac{1}{(2\pi f)^2 L}
\]

\[
= \frac{1}{(2\pi \cdot 1 \times 10^9 \text{s}^{-1})^2} \cdot \frac{1}{2 \times 10^{-6} \text{H}}
\]

\[
= \frac{1}{(2\pi)^2} \cdot \frac{1}{2 \times 10^{10} \text{F}^{-1}}
\]

\[
= \frac{1}{8\pi^2} \cdot 10^{-10} \text{F}
\]

\[
= 10^{-12} \text{F} = 1 \text{pF}
\]

\[8\pi^2 \approx 100\]
27

Exponentials  semi-log

power laws  log-log

\[ \frac{V_{out}}{V_{in}} = \omega^{-1} \quad \text{power law} \]

\[ \Rightarrow \text{log-log} \]
Oscillation 1

Amplitude 2 cm = 4 V

Period 6 cm

\[ 6 \text{ cm} \cdot \frac{10^{-3} \text{s}}{0.5 \text{ cm}} = 1.2 \times 10^{-2} \text{s} \]

\[ f = \frac{1}{T} = 83 \text{ Hz} \]

Oscillation 2

Period 1 cm

Frequency

\[ = \frac{1}{1 \text{ cm}} \cdot \frac{0.5 \text{ cm}}{10^{-3} \text{s}} \]

\[ = 500 \text{ Hz} \]