

THE SPECTRUM OF SOLUTION PRIMITIVES FOUND IN CONCEPTUAL
ELECTRICITY AND MAGNETISM PROBLEMS IN UNDERGRADUATE PHYSICS
TEXTBOOKS

A Masters Project submitted in partial fulfillment
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Physics

by

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ABSTRACT

In this study, all of the 1602 conceptual electricity and magnetism problems were identified in seven popular undergraduate physics textbooks, and a new generalized problem with solution was written based on each of the 1369 unique problems identified. The solutions of the new problems were broken into small pieces called solution primitives, and the spectrums of the problems, problem parts, and solution primitives were studied in the seven textbooks.

This is for John and Gay Stewart, who made this possible,
for my husband and kids, who endured six years without,
for Laura, whose conviction never wavered,
and for Boppa, who still believes.

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CHAPTER 1

Introduction

In 1985, Halloun and Hestenes studied students' misconceptions about introductory physics. They found that these misconceptions can impair the student's ability to learn the subject, and they learned that traditional instructional techniques do not significantly replace misconception with subject understanding[1]. It was found that students' preconceived notions are not always completely eradicated by physics instruction, but, instead, that a student can store both his original, incorrect notion and the correct principle or model learned in physics class without making any real connections between the two[2, 3]. Kim and Pak investigated the resistance of students' preconceptions even to "traditional textbook problem solving." They found that, even after answering an average of 1500 problems, the typical student resolves few of his previously-held misconceptions about introductory conceptual mechanics[4].

John Clement suggested in 1981 that mechanics preconceptions often take the form of a sort of theory, albeit an unorthodox one, called the 'naive impetus theory.' This theory, Clement asserted, is "a stable, alternative view" to Newtonian mechanics[5]. This theory asserts that when an agent moves a body, the agent imparts an impetus, "a certain force enabling the body to move in the direction in which the mover starts it, be it upward, downward, sideward or in a circle. It is because of this impetus that a stone moves on after the thrower has ceased moving it"[6]. In 1988, diSessa insisted that even though impetus theory appears to be "quite robust,"

it is not actually a theory. “Instead, intuitive physics is a fragmented collection of ideas, loosely connected and reinforcing, having none of the commitment or systematicity that one attributes to theories”[7].

Regardless of its status as a theory, this incorrect view is like one espoused by Aristotle and is held by many students in introductory physics courses. Caramazza, et al. found in 1981 that “students’ responses [to mechanics questions] fell into a few basic patterns.” They said that this fact “suggests that the [naive] beliefs about motion are not entirely idiosyncratic” but possibly a result of students’ observations [8]. These observations may, in reality, be flawed. McCloskey contends that students may persistently and inadvertently ignore details of an event that are counter to their preconceived naive notions [6, 9]. In 2005, Liu and MacIsaac “examined the degree to which students consistently applied the [naive] impetus theory across different items”[10], and they found that students applied the theory with a significant but incomplete consistency. Much of physics education research to this point has focused on conceptual change, or the process of trading these student misconceptions with the accepted principles of physics.

In addition, physics education research has often studied the context-dependency of problem solving for students taking introductory physics courses. The context of a problem is the physical system presented in the problem. In 1989, Gliner found that students’ characterizations of problem types are largely dependent on the contexts of the problems[11], and Gliner determined in 1991 that “the successful problem solvers” are more likely to be those students who can see past the context of a problem and into its mathematical structures[12].

In 1986, Clough and Driver bridged the gap between students’ preconceptions and the dependency of students’ problem-solving abilities on the contexts of problems. They examined not only “student’s ideas or conceptual frameworks” but also “the consistency with which the ideas were used by individuals in different contexts.” They found that students were not entirely

consistent in the application of their conceptual frameworks when solving problems in different contexts[13].

This project seeks to combine many of the subjects of past research. For this study, an expert problem solver scanned seven frequently-used undergraduate physics textbooks, identifying all of the 1602 conceptual electricity and magnetism problems and solutions located in the books and rewriting 1369 of the problems and solutions after stripping the problems of their contexts. The researchers sought to answer the following questions

1. How much conceptual coverage exists in standard textbooks?
2. How does that coverage vary among textbooks?

The problem collection produced by this project will form the basis for a national conceptual problem database and an automated concept inventory construction tool.

CHAPTER 2

The Textbooks

The researchers began by choosing two calculus-based textbooks, two algebra-based textbooks, and one famous conceptual physics textbook. Two additional textbooks were investigated, and both came from the physics education research community. The study's purpose was to investigate the quantitative differences in conceptual coverage across commonly-used undergraduate physics textbooks. As such, the researchers chose seven books that are used at many institutions and should be immediately recognizable to most physics educators. Other choices of texts were possible, but the collection chosen should be fairly representative of the physics texts most often used.

The textbooks chosen for the study were assigned identifiers as shown in Table 1: Understanding Physics (clup), Physics: Principles with Applications, Sixth Edition (gp), Conceptual

Identification	Edition	Author(s)	Nature
clup[14]	1	Cummings, Laws, Redish, and Cooney	calculus-based (PER)
gp[15]	6	Giancoli	algebra-based
hcp[16]	8	Hewitt	conceptual
hrw[17]	7	Halliday, Resnick, and Walker	calculus-based
kp[18]	1	Knight	calculus-based(PER)
yfu[19]	11	Young and Freedman	calculus-based
ygc[20]	8	Young and Geller	algebra-based

Table 1. The Books

Identification	Pages Studied	Number of Pages	Chapters Studied	Number of Chapters
hrw	561-825	265	21-30	10
clup	633-984	352	22-33	12
gp	439-614	176	16-21	6
hcp	372-450	79	21-24	4
kp	781-1083	303	25-33	9
yfu	792-1213	421	21-31	11
ygc	545-760	216	17-22	6

Table 2. The Pages and the Chapters

Physics, Eighth Edition (hcp), Fundamentals of Physics, Seventh Edition (hrw), Physics for Scientists and Engineers with Modern Physics: A Strategic Approach (kp), Sears and Zemansky's University Physics with Modern Physics, Eleventh Edition (yfu), and Sears and Zemansky's College Physics, Eighth Edition (ygc). From this point, the books will be referred to by these abbreviations.

The electricity and magnetism portions of each textbook were studied. This included topics from electric charge through Maxwell's equations and the displacement current, but optics was excluded. A summary of the chapters and pages of the texts studied is shown in Table 2.

CHAPTER 3

The Problems

1. Defining and Identifying Conceptual Problems

concept *n* 1. something that somebody has thought up, or that somebody might be able to imagine. Also called *conception* 2. a broad abstract idea or a guiding general principle, such as one that determines how a person or culture behaves, or how nature, reality, or events are perceived 3. the most basic understanding of something 4. a method, plan, or type of product or design

In this work, the word “concept” is used as in definition 2 above, but the word has also achieved an additional but not fully specific meaning in the physics community. In fact, the definition of concept often varies from person to person. When presented with a problem, a physicist can categorize the problem as conceptual or non-conceptual, but numerous attempts by this group to construct a rigorous definition of the term “conceptual problem” have failed; a counter-example to any potential restriction could be found. One of the future results of this research will be a classification of the collection of problems into various conceptual levels.

Conceptual problems were collected by two researchers, the author and Richard Campbell. To ensure that no conceptual problems were missed, the first researcher (this author) scanned the textbooks acquiring new problems for the database and excluding only those prob-

lems she considered to be categorically non-conceptual. In general, questions with decimals or scientific notation in the problem statement were excluded. Problems with solutions requiring more than simple arithmetic beyond addition and subtraction of whole numbers or simple fractions were also rejected. Solutions with multiplication, division, or exponents were also excluded unless the results were simple whole numbers. Simple symbolic algebraic manipulation was accepted, but any complex proof was excluded from the problem database. No hard-and-fast rules were applied in the acquisition of problems for the database. If the problem *might* be considered conceptual, it was collected. To understand the difficulty of conceptual identification, consider the following problem:

Problem: An electron with charge $-1.6 \times 10^{-19}\text{C}$ is near the earth with a net charge of $-5 \times 10^5\text{C}$. Is the magnitude of the electric force due to the electron on the earth larger than, smaller than, or equal to the magnitude of the electric force due to the earth on the electron?

The problem statement contains scientific notation, but most physicists would consider it conceptual.

Figure 1 shows a purely qualitative problem that was identified as conceptual and included in the problem database. Figure 2 shows a problem that was rejected because its solution required too many algebraic steps, and Figure 3 shows a problem that was rejected because its solution contains decimal calculation. One should note, that in the problems in Figures 2 and 3, one has to examine both the problem and solution to determine if the problem is conceptual or non-conceptual.

2. Defining Unique Conceptual Problem Types

Most introductory physics textbooks cover the same basic set of topics. As one might expect, some very similar problems were identified among the books studied. Each problem

Figure 1. Conceptual and Included in the Database

Example: Do Two Pieces of Tape Attract or Repel Each Other?

Problem: Two pieces of tape cut from the same roll are stuck to a surface. The pieces of tape are then pulled off of the surface. If the pieces of tape are brought close to each other, will they attract each other, repel each other, or have no effect on each other at all? Why?

(a) When the sticky pieces of tape are peeled off of the table, the pieces of tape acquire a net charge either by losing or by gaining electrons.

(b) The two pieces of tape are made of the same material and are stuck to the same surface, so the pieces of tape will have the same sign of charge when pulled from the surface.

(c) Since like charges repel, the pieces of tape repel each other when brought close together.

Figure 2. Not Conceptual and Excluded from the Database: Complicated Algebra

Example: Finding Electric Field Immediately Outside of an Infinite Conducting Wire

Problem:

Using Gauss's law and a cylindrical gaussian surface of radius r , prove that the electric field immediately outside an infinite straight conducting wire with uniform linear charge density of Q/L is equal to $\frac{Q}{2\pi\epsilon_0 r l}$. Hint: there is no flux through either end of the cylinder.

The electric flux Φ through a surface is equal to $E A \cos\theta$ where θ is the angle between the electric field E and the normal of the surface with area A . Since there is no flux through the two ends of the cylinder, there can only be flux through the remaining surface. This surface is essentially a rectangle "wrapped" around the perimeters of the two circular ends. So, the rectangle has dimensions of l and $2\pi r$; the area A of this surface is then equal to $l2\pi r$. The electric field is everywhere parallel to the normal of this surface, so $\theta = 0$. Thus, $E l 2\pi r \cos\theta$ becomes $E l 2\pi r$. Gauss's law says that the total flux through a closed gaussian surface is also equal to the charge enclosed by that surface divided by ϵ_0 , so $E l 2\pi r = \frac{Q_{enc}}{\epsilon_0}$. The linear charge density is given in the problem as Q/L . The charge enclosed by a gaussian cylinder of length l must then be $(Q/l)l = Q$. Plugging Q in for Q_{enc} gives $E l 2\pi r = \frac{Q}{\epsilon_0}$. Finally, $E = \frac{Q}{2\pi\epsilon_0 r l}$.

Figure 3. Not Conceptual and Excluded from the Database: Too Many Numbers

Example: Electric Forces Between Point Charges

Problem: The electric force F between two point charges of equal magnitude located a distance of $r = 6\text{cm}$ apart has a magnitude of $F = 20\text{N}$. The two charges have equal magnitude; what is this magnitude?

The magnitude of the electric force F between two point charges is $F = k\frac{q_1q_2}{r^2}$. F is given in the problem as 20N , and r is given as 6cm . $q_1q_2 = \frac{20\text{N}}{k}(6\text{cm})^2 = \frac{.072\text{Nm}}{k}$. $q_1 = q_2$, so $2q_1 = \frac{.072\text{Nm}}{k}$. $q_1 = q_2 = \frac{.036\text{Nm}}{k}$.

acquired for the database was stripped of its context making the problem abstract, if possible. Once made abstract, two problems were considered identical if both their problem statements and their solutions were the same. When a problem was found that was sufficiently similar to a problem that had already been acquired, a new problem was not written. However, the new problem was associated with the existing problem in the database and was treated as if a new problem had been entered for it during analysis.

3. Writing the Problems

Problems were extracted from chapters covering topics from electric charge through Maxwell's equations. Problems were found both within the bodies of the textbooks' chapters and at the chapters' ends, and the new problems were assigned unique identifiers and placed into a conceptual-problem database. Often, a problem was divided into its component parts by the textbook's author or authors. Each problem entered into the database maintained the original author's division into parts.

4. Stripping the Context

Each unique problem found in the textbooks was stripped of its context and rewritten in general form, along with its solution. As a rule, this meant removing the physical system from the problem, leaving the problem as abstract as possible. For example, a charged pith ball in the textbook would become a charged particle in the database. When a problem was identified that could not be generalized, it was put into the database exactly as found in the textbook. Each of these problems was labeled “identical to” the original problem found in the textbook.

5. Selecting the Solutions

This project studies the solutions written and entered into the database. Because the answer to almost any physics problem can be found by using more than one method, two or more solutions could reasonably be used to solve a single physics problem. Different authors could write different solutions to the same problem that vary in their similarity to the solution of the problem in the database. Characterizing this variability is an interesting avenue for future research, but it is not considered here. For this study, each problem was answered using the physics introduced in the chapter containing it. Hopefully, this ensures that the solutions entered into the database are as close as possible to those intended by the textbooks’ authors.

6. Verifying the Solutions

The solutions in the problem database are primarily the work of this author. They were written to be good solutions to present to college physics students. Each solution was verified by a second researcher both for its accuracy and for its usefulness as instructional material.

CHAPTER 4

Verifying the Problem-Collection Process

The problem-collection process was lengthy and presented ample opportunity for error. To estimate and correct for error, a second researcher accumulated a complete list of conceptual problems from the same set of textbooks. In this way, he established his own list of conceptual problems from the set of textbooks investigated by the first researcher. He included only those problems he considered to be unquestionably conceptual. Unlike the original problem solver, he did not write new problems of the same type as those he identified in the source textbooks, and it was found that many problems could not be positively characterized as either conceptual or non-conceptual until the solution was actually written. The more restrictive criteria was used to ensure no patently conceptual problems were missed and to approximate the number of problems that are only somewhat conceptual. The second researcher's set was then compared with that of the initial researcher.

The second researcher used more restrictive criteria for identifying the problems than that used by the initial researcher. As a result, it was found that the first problem solver identified 303 more conceptual problems in the textbooks than the second researcher. The problems representing the difference between these two identifications represents the gray area between problems that are definitely conceptual and problems that might or might not be considered conceptual.

CHAPTER 5

Solution Primitives

1. Counting Concepts

The goal of this project was to characterize the amount of and the variability of conceptual problems and examples in a collection of popular textbooks. The issue immediately arose of how sizeable a single problem is conceptually. One can simply count problems, but should a problem with multiple parts not count more than a problem with just a single part? What if the single-part problem is difficult while the multiple-part problem is easy? But, then, how does one characterize “easy?” To quantify the conceptual content of a problem, the researchers examined the expert solutions written for the project and designed to educate a college student. These solutions were decomposed into fundamental pieces of communication, termed “solution primitives.” A solution with more solution primitives contains more conceptual ideas and should be counted as conceptually more sizeable than a solution containing fewer solution primitives.

2. Defining Solution Primitives

A solution primitive was defined to be the smallest separable part of a solution. That is, a solution primitive is the smallest part of the larger solution to any given problem that can

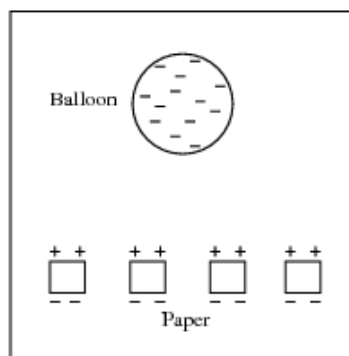
Figure 4. Solution Before Decomposition

Example: Picking Up Paper with a Balloon

Problem: After rubbing a balloon in your hair, you can use it to lift small pieces of paper off a table. Explain.

Solution

When I rub a balloon in my hair, charge is transferred to/from the balloon from/to my hair and it acquires a net charge. The net charge on the balloon causes the charges in the atoms of the insulating paper pieces to polarize slightly leaving (assuming a negatively charged balloon) some excess negative charge farther from the balloon and some excess positive charge nearer to—as shown to the right. The electric force falls off with distance so that the positive charges nearer the balloon feel a larger attractive force than the repulsive force felt by the negative charges on the paper farther from the balloon, giving a net attractive force between the balloon and the pieces of paper.



IsContainedIn: ProblemType:Explaining the Attraction of Charged and Uncharged Objects

act alone as the answer to another, smaller problem. Figure 4 shows a solution in the database, and Figure 5 is its decomposition into solution primitives.

After some experimentation, a problem decomposition protocol was developed as shown in Figure 6.

3. Testing Solution Primitive Identification

To test the reproducibility of solution primitive identification using the protocol, thirty problems, both conceptual and non-conceptual, were chosen at random from textbooks not included in the database (Serway and Jewett [21], Tipler [22], and Ohanian [23]). The problems' original authors provided the solutions and the division of the problems into parts, but the

Figure 5. Solution After Decomposition with Solution Primitives Bulleted

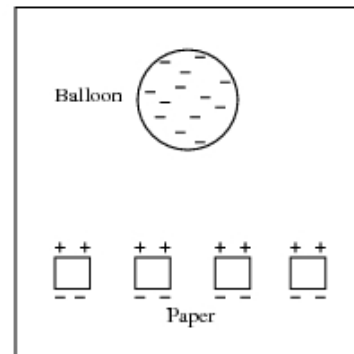
Example: Picking Up Paper with a Balloon

Problem: After rubbing a balloon in your hair, you can use it to lift small pieces of paper off a table. Explain.

Solution

(Composite) Attraction of Charged Object to Uncharged Object

- Charging By Rubbing or Friction:** When I rub a balloon in my hair, charge is transferred to/from the balloon from/to my hair and it acquires a net charge.
- Charged Object Causes Charge Separation or Polarization in An Uncharged Object:** The net charge on the balloon causes the charges in the atoms of the insulating paper pieces to polarize slightly leaving (assuming a negatively charged balloon) some excess negative charge farther from the balloon and some excess positive charge nearer to—as shown to the right.
- Electric Force Falls Off with Increasing Distance:** The electric force falls off with distance so that the positive charges nearer the balloon feel a larger attractive force than the repulsive force felt by the negative charges on the paper farther from the balloon,
- Net Force of Uncharged Object on Charged Object is Attractive:** giving a net attractive force between the balloon and the pieces of paper.



IsContainedIn: ProblemType:Explaining the Attraction of Charged and Uncharged Objects

Figure 6. Problem Decomposition Protocol; SP = Solution Primitive

1. Separate the solution into individual clauses. This naturally divides the problem by higher subdivisions; paragraphs and sections with headings or numbering.
2. Denoting Decomposition - Decomposition is denoted by placing a vertical line between solution primitives.
3. Separated Clauses - If two segments that contribute to the same piece of information are separated, arrows should be drawn connecting the related segments.
4. Repetition - If a segment conveys multiple pieces of the same information, like a repeated calculation, the repetition should be denoted by rep(5) for 5 repeats. This is a shortcut and will be counted exactly as if 5 SPs had been identified.
5. Group clauses into units which add individual new pieces of information to the solution. These units are the solution primitives (SP).
6. If in doubt about whether a set of clauses is a SP, apply the test, "Could a physics question be framed such that a subset of the clauses was the answer to the question?"
7. When two groups give a general statement, then the specific application to the solution of the problem, then the two groups should be combined into a single SP.
8. Mathematical phrases must also be examined and subdivided. Each equals sign in the solution represents a new clause.
9. Mathematical phrases should be examined for reasoning that is not supported by the text, such as a variable being set equal to zero or the substitution of a formula that has not already been introduced in the text. These actions add new information to the solution and should be identified as SPs.
10. Mathematical manipulations that do not introduce new information are not separated into individual SPs, except as follows:
11. A purely algebraic manipulation should only be identified as an SP if it has been textually singled out by the author. For example, 1. Solve for v , $v=qBr/m$ would be an SP.
12. Clauses that restate givens are separated but do not represent SPs and will not be used in further analysis.
13. Sub-clause SPs - It is possible to pack multiple SPs in the same clause. This usually happens when either the subject or predicate contains a conjunction. In this case, the clause could be re-written as multiple clauses by removing the conjunctions. The number of SPs marked should equal this number of re-written clauses.

division into solution primitives was done by the researchers.

Three researchers read the problems and solutions, marking the solution primitives according to the problem decomposition protocol. The solution primitive identifications of the three researchers were then compared as shown in Table 3.

It was found that when the researchers worked separately, they completely agreed on the number of the solution primitives and on the exact solution primitive marking for each problem 60 percent of the time, but when two or more researchers worked together marking a problem, they virtually always came to agreement on the number of solution primitives and on the exact solution primitive marking of the problem. That is, the disagreement represented correctable errors in marking, not fundamental problems with the protocol.

For each problem, the average number of solution primitives identified by each of the three researchers was found, as well as the number of solution primitives completely agreed upon by the researchers. The ratio of the number of solution primitives completely agreed upon by the researchers to the average number of solution primitives identified was then found for each problem, and this ratio was defined to be the agreement among the researchers. The average of this ratio among the 30 problems studied was found to be .839, or an 83.9 percent agreement. For the kind of experiment being conducted, this agreement is exceptional. Further, the total number of solution primitives identified in all 30 problems was found. The sum of the average number of solution primitives identified by the three researchers was also found. The average value of the difference between the two sums was found and divided by the sum of the average number of solution primitives identified by the three researchers. The result was defined to be the error in the total number of solution primitives identified per researcher, and in this experiment was found to be 5.8 percent.

Although the verification was done with published textbook material, much of the error

Problem	Researcher 1	Researcher 2	Researcher 3	Average	All Agree	Agreement
22-7-cex	24	26	24	24.67	23	.93
32-4-ex	6	6	5	6	6	1
19-1-ex	2	2	2	2	2	1
28-7-ex	3	3	3	3	3	1
28-8-ex	9	9	8	8.67	8	0.92
4-p940-ex	13	13	13	13	13	1
28-3-ex	4	4	4	4	4	1
36-9-ex	5	6	3	4.67	3	0.64
8-6-ex	8	8	10	8.67	7	0.81
2-9-cex	6	6	6	6	6	1
24-2-ex	5	7	4	5.33	2	0.375
22-3-ex	3	4	3	3.33	1	0.3
34-8-ex	4	4	4	4	4	1
24-14-ex	4	4	4	4	4	1
31-5-ex	6	5	6	5.67	4	0.71
8-5-cex	3	5	6	4.67	2	0.43
26-17-ex	4	4	4	4	4	1
25-14-ex	1	1	1	1	1	1
4-2-ex	2	2	2	2	2	1
25-2-ex	2	2	2	2	2	1
31-2-ex	1	1	1	1	1	1
31-9-ex	6	6	5	5.67	4	0.71
9-5-ex	4	5	4	4.33	2	0.46
29-10-ex	2	2	2	2	2	1
4-3-ex	9	13	8	10	7	0.70
29-3-ex	4	4	4	4	4	1
22-5-ex	9	17	7	11	2	0.18
23-4-ex	2	2	2	2	2	1
4-2-ex	2	2	2	2	2	1
total	159	179	156	164.7		

Table 3. Solution Primitive Verification

seemed to have resulted from what were considered to be poorly written solutions. A poorly-written solution was defined to be a solution with assumed steps, multiple pieces of reasoning buried in mathematics strung on the same line, or substantial information packed into multiple phrases in the same sentence. When these problems were removed from the problem set, an agreement of 93.7 percent was found.

4. Decomposing the Problems

One researcher decomposed the problems based on the protocol through the topic of capacitance, covering all of the electrostatics. From the above analysis, one would expect an error rate of 5.8 percent in the decomposition.

CHAPTER 6

Analysis

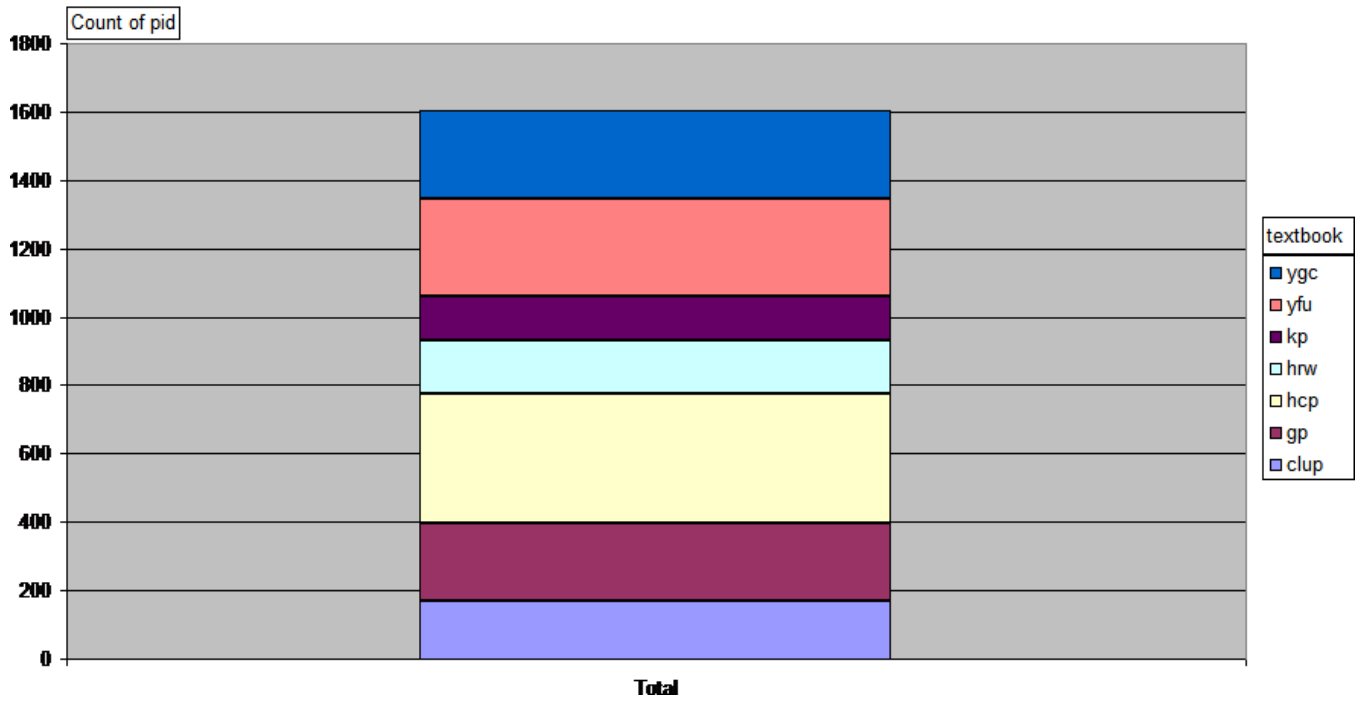
1. Problems Identified

In total, 1602 conceptual problems were identified in the seven source textbooks, and 1369 total problems were written based on those problems. An additional 233 problems were identified which were considered to be identical to problems existing in the database. The problems were noted in the database and paired with the problems of the same type; however, new problems were not included in the database. Table 4 shows the number of problems written, number of problems identified as identical to some other problem, and number of total problems identified per textbook. Figure 7 shows this information in graphical form.

Textbook	Problems Written	Identical Problems	Total Problems
clup	123	46	169
gp	215	13	228
hcp	341	36	377
hrw	115	43	158
kp	104	26	130
yfu	246	37	283
ygc	225	32	257
Total	1369	233	1602

Table 4. Problems Identified by Source

Figure 7. Number of Problems Per Book
Number of Problems Per Book



Textbook	Problems Within Chapters	Problems at the Chapters' Ends	Total
clup	99	70	169
gp	47	181	228
hcp	46	331	377
hrw	76	82	158
kp	61	69	130
yfu	56	227	283
ygc	45	212	257
Total	430	1172	1602

Table 5. Problem Environments

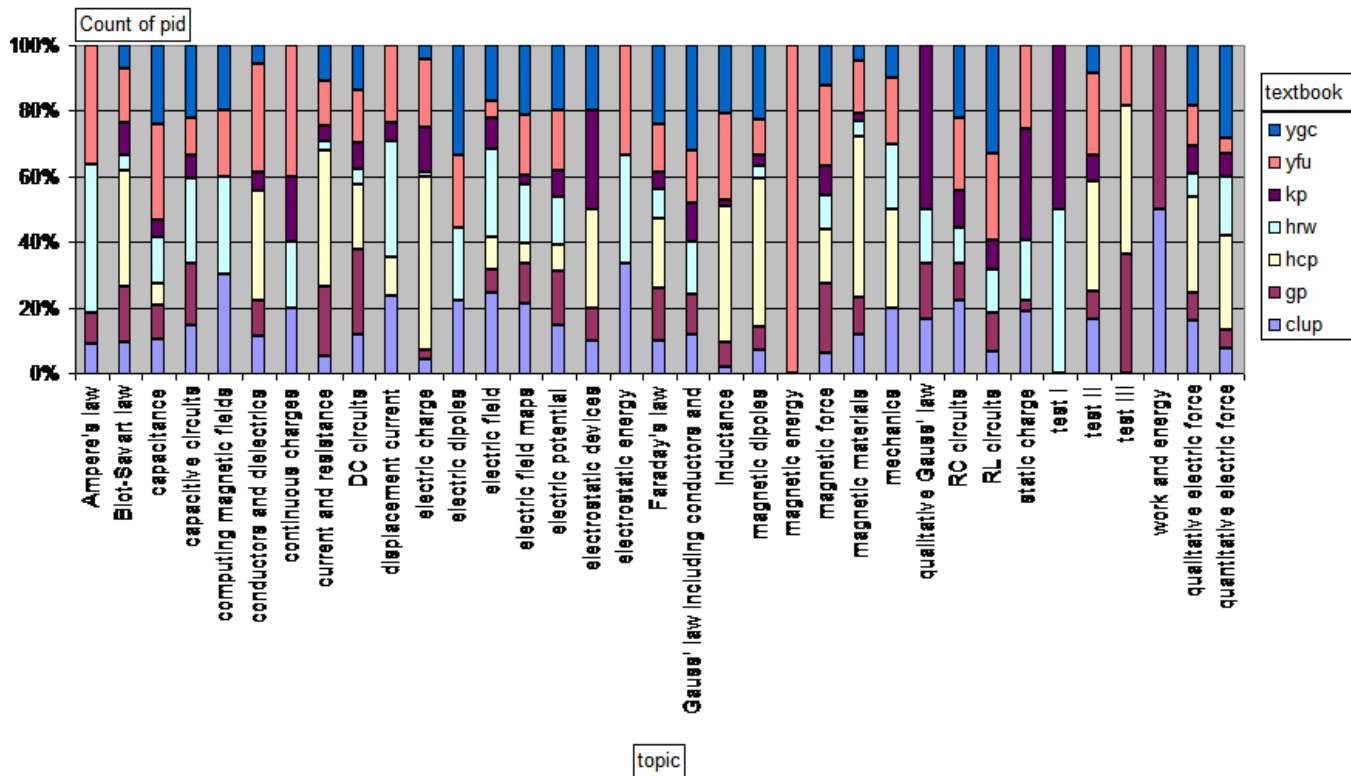
2. Problem Environments

For this study, the environment for a given problem was defined to be the location of the problem in the textbook chapter. Two environments were identified: “within the chapter,” meaning within the body of the chapter itself, and “at the chapter’s end,” meaning at the end of the chapter, normally a section dedicated to questions and problems. Of the 1602 total problems identified, 73 percent were found at the textbooks’ chapters’ ends, with the remaining 27 percent found within the chapters. This distinction is interesting because the problems within the chapter are the problems that should be encountered by all students. Table 5 shows the problem environment breakdown for each textbook.

3. Problem Topics

The problems identified in the textbooks were mapped onto a single sequence of topics using the topical sequence presented at the University of Arkansas at Fayetteville. Problems from 35 electricity-and-magnetism topics were studied: Ampere’s law, Biot-Savart law, capacitance, capacitive circuits, computing magnetic fields, conductors and dielectrics, continuous charges, current and resistance, DC circuits, displacement current, electric charge, electric

Figure 8. Problems by Topic
Problems by Topic



dipoles, electric field, electric field maps, qualitative electric force, quantitative electric force, electric potential, electrostatic devices, electrostatic energy, Faraday's law, static charge, Gauss' law including conductors and dielectrics, inductance, magnetic dipoles, magnetic energy, magnetic force, magnetic materials, mechanics, RC circuits, RL circuits and RLC circuits, qualitative Gauss' law, final topics for test I, final topics for test II, final topics for test III, and work and energy. Table 6 and Figure 8 show the distribution of problems within topics. Note the strong variation of level of coverage by textbook and the disproportionate number of problems relating to current and DC circuits.

Topic	clup	gp	hcp	hrw	kp	yfu	ygc	Total
Ampere's law	1	1	0	5	0	4	0	11
Biot-Savart law	4	7	15	2	4	7	3	42
capacitance	6	6	4	8	3	17	14	58
capacitive circuits	4	5	0	7	2	3	6	27
computing magnetic fields	3	0	0	3	0	2	2	10
conductors and dielectrics	2	2	6	0	1	6	1	18
continuous charges	1	0	0	1	1	2	0	5
current and resistance	11	45	88	6	10	29	23	212
DC circuits	18	39	30	7	12	24	21	151
displacement current	4	0	2	6	1	4	0	17
electric charge	3	2	38	1	10	14	4	70
electric dipoles	2	0	0	2	0	2	3	9
electric field	10	3	4	11	4	2	7	41
electric field maps	7	4	2	6	1	6	7	33
qualitative electric force	13	7	24	6	7	10	15	82
quantitative electric force	5	4	19	12	5	3	19	67
electric potential	16	18	9	16	9	20	22	110
electrostatic devices	1	1	3	0	3	0	2	10
electrostatic energy	1	0	0	1	0	1	0	3
Faraday's law	13	21	28	12	7	19	32	132
static charge	11	2	0	11	20	15	0	59
Gauss' law including conductors and dielectrics	3	3	0	4	3	4	8	25
inductance	1	4	22	0	1	14	11	53
magnetic dipoles	4	4	26	2	2	6	13	57
magnetic energy	0	0	0	0	0	2	0	2
magnetic force	7	24	19	12	10	28	14	114
magnetic materials	5	5	21	2	1	7	2	43
mechanics	2	0	3	2	0	2	1	10
RC circuits	2	1	0	1	1	2	2	9
RL circuits and RLC circuits	5	9	0	10	7	20	25	76
qualitative Gauss' law	1	1	0	1	3	0	0	6
test I	0	0	0	1	1	0	0	2
test II	2	1	4	0	1	3	1	12
test III	0	8	10	0	0	4	0	22
work and energy	1	1	0	0	0	0	0	2
Total	169	228	377	158	130	282	258	1602

Table 6. Problems by Topic

Textbook	Identified	Problems Identified	Parts Per Problem
clup	335	169	2.0
gp	345	228	1.5
hcp	457	377	1.2
hrw	329	158	2.1
kp	182	130	1.4
yfu	453	283	1.6
ygc	343	257	1.3
Total	2444	1602	1.5

Table 7. Parts Per Problem by Source

4. Parts Identified

The problems were divided into parts, each part asking a single question. A total of 2444 parts were identified in the seven textbooks, for an average of 349.1 parts identified per book. The average parts per problem varied from 1.2 in hcp, which had a tendency of asking simple problems to 2.1 in hrw, which often asked exceptionally complicated problems as shown in Table 7.

5. Part Environments

Table 8 shows that of the 2444 total parts identified, 1745 were found at the chapters' ends, and only 699 were found within the chapters' bodies. Note the strong variation among the texts.

6. Part Topics

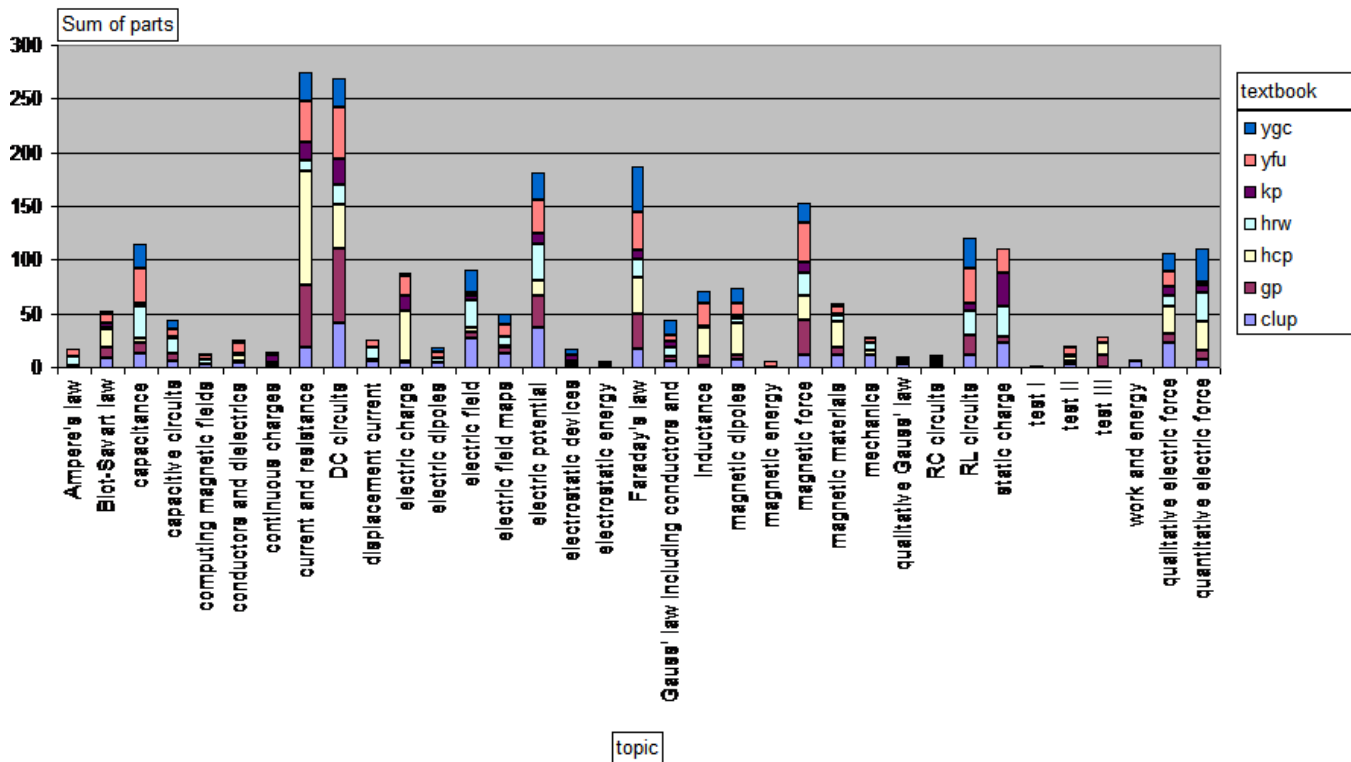
Table 9 and Figure 9 show the distribution of parts by topic. Note the imbalance of parts in current and DC circuits.

Textbook	Parts Within Chapters	Parts at the Chapters' Ends	Total
clup	192	143	335
gp	73	272	345
hcp	52	405	457
hrw	167	162	329
kp	72	110	182
yfu	90	363	453
ygc	53	290	343
Total	699	1745	2444

Table 8. Part Environments

Figure 9. Parts Per Topic

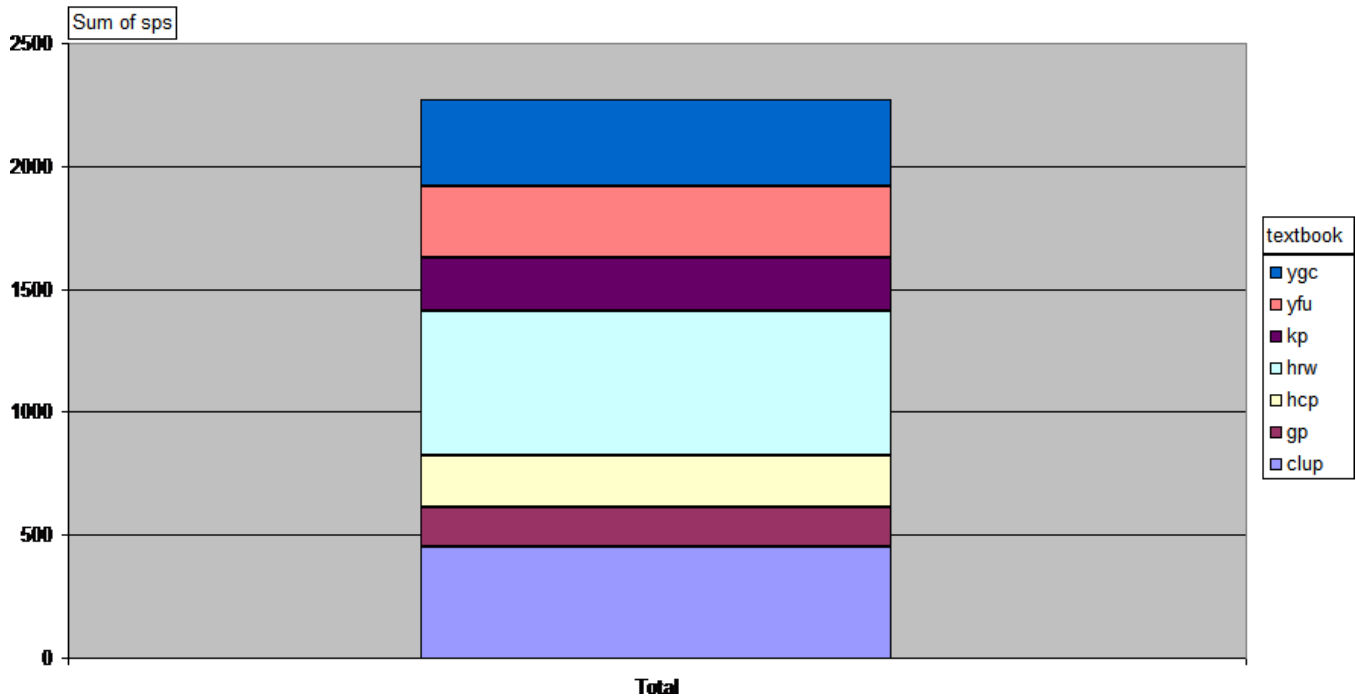
Parts Per Topic



Topic	clup	gp	hcp	hrw	kp	yfu	ygc	Total
Ampere's law	1	1	0	8	0	7	0	17
Biot-Savart law	8	10	17	2	4	9	3	53
capacitance	13	10	4	30	3	32	22	114
capacitive circuits	6	7	0	14	2	6	9	44
computing magnetic fields	3	0	0	4	0	4	2	13
conductors and dielectrics	4	2	6	0	1	10	3	26
continuous charges	1	0	0	3	8	2	0	14
current and resistance	19	58	105	10	17	38	27	274
DC circuits	41	69	42	18	24	48	27	269
displacement current	5	0	2	11	1	6	0	25
electric charge	4	2	46	1	13	19	3	88
electric dipoles	4	0	0	4	0	6	5	19
electric field	27	5	5	25	4	4	20	90
electric field maps	13	5	2	8	1	11	9	49
qualitative electric force	23	8	26	10	8	14	17	106
quantitative electric force	7	8	28	26	7	3	31	110
electric potential	37	30	13	35	9	32	25	181
electrostatic devices	2	1	3	0	5	0	6	17
electrostatic energy	2	0	0	2	0	1	0	5
Faraday's law	17	32	34	17	9	36	42	187
static charge	23	5	0	28	32	23	0	111
Gauss' law II	6	4	0	9	5	6	14	44
inductance	2	8	27	0	1	22	11	71
magnetic dipoles	7	4	30	4	3	11	15	74
magnetic energy	0	0	0	0	0	5	0	5
magnetic force	12	32	23	21	10	37	18	153
magnetic materials	11	8	23	6	1	8	2	59
mechanics	11	0	5	7	0	4	1	28
RC circuits	2	2	0	1	1	3	2	11
RL circuits and RLC circuits	12	18	0	22	7	33	28	120
qualitative Gauss' law	3	2	0	2	3	0	0	10
test I	0	0	0	1	1	0	0	2
test II	3	2	5	0	2	7	1	20
test III	0	11	11	0	0	6	0	28
work and energy	6	1	0	0	0	0	0	7
Total	335	345	457	329	182	453	343	2444

Table 9. Parts by Topic

Figure 10. SPs Per Book
SPs Per Book



7. Solution Primitives Identified

The solution primitive spectrum was studied for 18 topics: capacitance, conductors and dielectrics, continuous charges, electric charge, electric dipoles, electric field, electric field maps, qualitative electric force, quantitative electric force, electric potential, electrostatic devices, electrostatic energy, static charge, Gauss' law including conductors and dielectrics, mechanics, qualitative Gauss' law, final topics for test I, and work and energy. Within these 18 topics, 2192 total solution primitives were identified. The distribution of solution primitives by text is shown in Figure 10.

Topic	clup	gp	hcp	hrw	kp	yfu	ygc	Total
capacitance	24	17	4	51	5	57	50	208
conductors and dielectrics	14	5	5	0	5	12	5	46
continuous charges	6	0	0	12	8	15	0	41
electric charge	7	5	54	1	20	27	6	120
electric dipoles	11	0	0	14	0	9	8	42
electric field	54	15	5	138	10	5	26	253
electric field maps	23	14	5	22	4	15	31	114
qualitative electric force	55	14	56	29	21	33	46	254
quantitative electric force	21	12	31	70	19	3	59	215
electric potential	81	64	19	77	26	59	59	385
electrostatic devices	4	1	5	0	13	0	13	36
electrostatic energy	2	0	0	2	0	1	0	5
static charge	81	5	0	108	61	30	0	285
Gauss' law including conductors and dielectrics	11	4	0	18	14	8	29	84
mechanics	20	0	6	16	0	9	5	56
qualitative Gauss' law	13	2	0	10	9	0	0	34
test I	0	0	0	4	1	0	0	5
work and energy	7	2	0	0	0	0	0	9
Total	434	160	190	572	216	283	337	2192

Table 10. Solution Primitives by Topic

8. Solution Primitive Topics

Table 6 shows the distribution of solution primitives by topic.

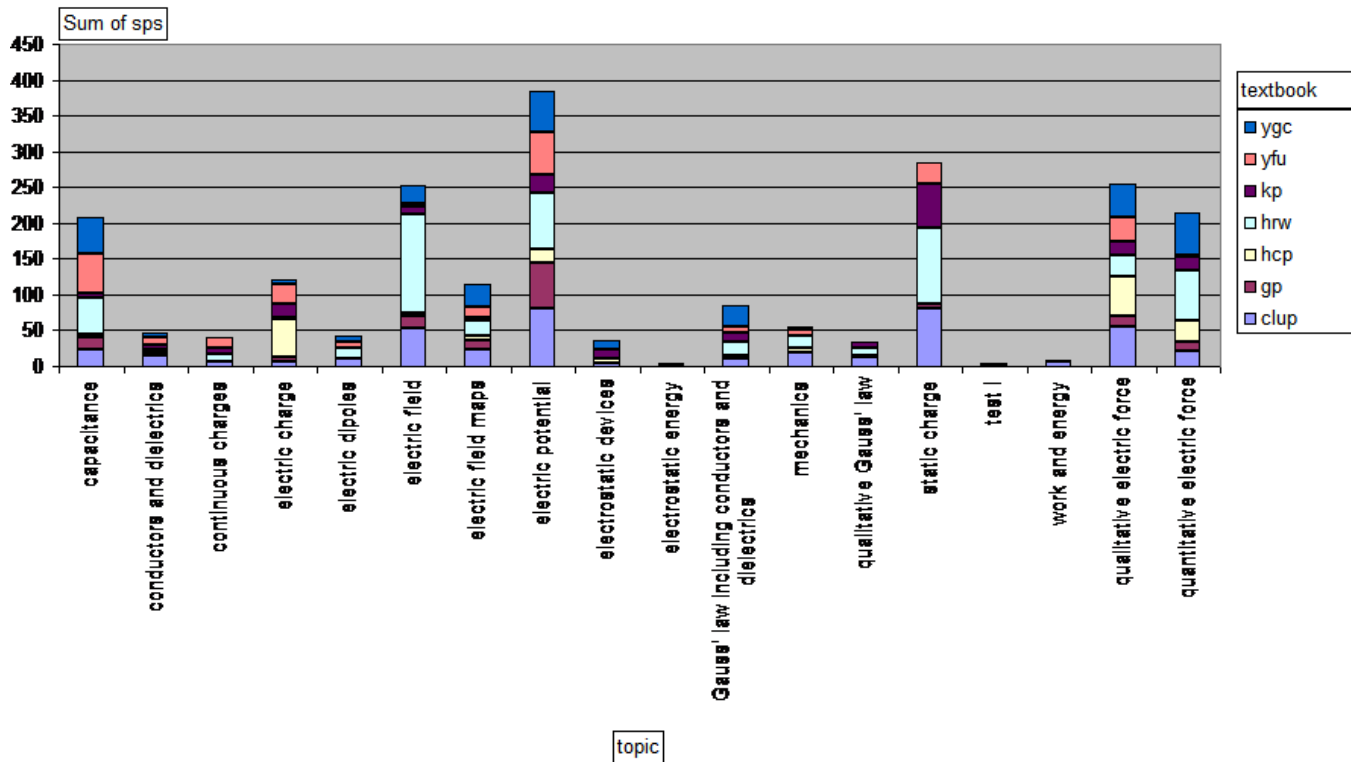
9. Solution Primitive Environments

Solution primitives were marked for topics from electric charge through capacitance. Of the 2192 solution primitives identified, about 66 percent were found at the chapters' ends, and the remaining 34 percent were found within the chapters as shown in Table 11. Note the strong variation of the number of solution primitives per text and their locations.

Textbook	SPs Found Within Chapters	SPs Found at the Chapters' Ends	Total
clup	256	195	451
gp	23	138	161
hcp	30	182	212
hrw	259	326	585
kp	87	129	216
yfu	44	247	291
ygc	60	293	353
Total	759	1510	2269

Table 11. Solution Primitive Environment

Figure 11. SPs Per Topic (Restricted)

SPs Per Topic (Restricted)

CHAPTER 7

Discussion

In the end, many more problems were identified in hcp than in any of the other textbooks. In fact, the problems found in hcp comprise almost one-fourth of the total problems identified. However, hcp's problems were not considered to be nearly as complex as the problems identified in most of the other textbooks because it has the fewest parts per problem of the books studied.

Moreover, it was found that hcp shows the largest disparity between the number of problems and parts identified within the bodies of its chapters and the number found at its chapters' ends. On the other hand, clup actually contains more conceptual problems, parts, and solution primitives in the bodies of its chapters than at its chapters' ends, and hrw has an almost equal number of problems in its chapters and at their ends.

Because more solution primitives and parts per problem were identified in hrw and clup than in the others, these textbooks seemed to have the highest levels of complexity. Further, hrw and clup seemed to cover the topics with more uniformity than most of the other textbooks studied. The books showed little bias in their coverage of the topics studied.

However, some books did show a good deal of bias towards the topics of current and DC circuits. About one-third of the problems in gp and hcp were devoted to the two topics. In fact, the bias seems to pervade the set of books studied. Almost one-fourth of the conceptual problems studied involved the topics of current and DC circuits. Out of the seven books studied,

only hrw does not appear to show a bias towards the two topics.

The quantitative analysis of seven undergraduate physics textbooks was performed by assessing and categorizing the conceptual problems, parts, and solution primitives identified in the books. It was determined that a sizeable variation in conceptual coverage (from overly-simplistic to exceedingly-complex problems, to certain topics of problems given more space within the textbooks' chapters or afforded more conceptual coverage) exists in these textbooks.

CHAPTER 8

Conclusion

Ultimately, the conceptual presentations of textbooks used for the study varied widely. They were dissimilar in the number of conceptual problems identified; in the uniformity of their problems, parts, and solution primitives across the topics studied; and in the complexity of their problems and solutions.

CHAPTER 9

Future Research

In the future, conceptual-problem identification will be further studied to establish a precise definition of the term conceptual problem. In addition, the differences in the identification of conceptual problems by individual researchers will be used to study the spectrum of problems that can be categorized as belonging to the gray area between quantitative and purely conceptual problems. Question-primitive marking will also be verified, and the question primitives in the seven textbooks will be studied in detail.

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