Coherent population trapping and electromagnetically induced transparency in multi-Zeeman-sublevel atoms

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We present a general formalism for studying the interaction between laser fields and degenerate-Zeeman-sublevel atoms correct up to the first order in the weak probe field. We derive from this theory the equations of motion for a $L$-type system involving $S_{1/2}$, $F = 5 \leftrightarrow P_{1/2}$, $F' = 8 \leftrightarrow S_{1/2}$ transitions in $^{87}$Rb atoms. These equations are used to numerically investigate the coherent population trapping (CPT) schemes in the $S_{1/2}$, $F = 5 \leftrightarrow P_{1/2}$, $F' = 8 \leftrightarrow S_{1/2}$ transition induced by a linearly polarized pumping field and the electromagnetically induced transparency exhibited in the weak probe spectrum in the $S_{1/2}$, $F = 5 \leftrightarrow P_{1/2}$, $F' = 8 \leftrightarrow S_{1/2}$ transition. We discuss the effects of the CPT on the probe spectrum with and without the Doppler broadenings by comparing the probe spectrum derived from the real system with that derived from an ideal $L$-type system. We show that the atoms in the CPT states are shielded from interacting with the probe field while those in the absorbent states make contributions to the weak probe by the Raman anti-Stokes process, and the effect of the real system on the probe field becomes equivalent to the ideal $L$ system only in the strong CPT parameter regime.

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I. INTRODUCTION

Electromagnetically induced transparency (EIT) and lasing without inversion (LWI) were originally predicted from theoretical studies of three- or four-level atomic systems (simple models) interacting with scalar fields [1–8]. However, the experimental demonstrations [9–17] of these phenomena were often carried out in atoms such as sodium [10,14] and rubidium [15–17], where each energy level consists of several Zeeman sublevels, interacting with polarized fields. The theoretical analysis based on simple models indicates that the foundations of EIT and LWI are the atomic coherence and the destructive quantum interference among different transition pathways. However, the atomic coherence terms and the possible interference pathways increase quadratically with the total number of levels (including the Zeeman sublevels), which can easily exceed 10 in the energy schemes of real atoms. One may question whether the main features predicted from simple models can survive in real atomic systems.

Consider an ideal $L$ system in Fig. 1(a), where the excited level 3 is coupled to level 2 by a pumping field and to level 1 by a weak probe field, and level 1 holds all the atoms. Harris, Field, and Imamoglu [4] have shown theoretically that such a medium becomes transparent to the weak probe at its resonance frequency, assuming that level 2 is metastable (or, equivalently, has a very small decay rate). This phenomenon (known as EIT) can be interpreted simply as a result of destructive interference between the $1 \leftrightarrow 3$ single-photon transition and the $1 \leftrightarrow 3 \leftrightarrow 2$ two-photon transition via the $2 \leftrightarrow 1$ atomic coherence. The experimental demonstration of EIT has been reported by Harris, Field, and Imamoglu in a heated Sr atomic vapor [9]. In their experiment, because level 2 differs from level 1 in the order of optical frequency, a very high pumping power was used to overcome the Doppler and collisional broadenings. Recently, Li and Xiao [16] have conducted an EIT experiment by employing two ground hyperfine levels in the $^{87}$Rb atom as levels 2 and 1. Since the two lower levels differ from each other in the order of microwave frequency, the two-photon-resonance condition in a copropagating configuration becomes almost Doppler free.

FIG. 1. Schematics of (a) the ideal $L$ system and (b) the $L$ system with level 2 being a degenerate doublet.
and thus this model has the advantage of the reduced requirement on the pump intensity. However, for the same reason, roughly 50% of the total populations will initially reside in level 2, making this model appear to be very much different from the ideal Λ system. The difference gives way if we treat this model as a three-level system based on the following argument: if the pump Rabi frequency is much larger than the collisional population transfer rates between levels 2 and 1, the populations in level 2 will virtually all be pumped optically to level 1, creating an ideal Λ system. However, this treatment becomes questionable as soon as we begin to apply it to real atoms involving Zeeman sublevels. As an illustration, let us consider a hypothetical atom whose energy diagram is shown in Fig. 1(b), which is the same as Fig. 1(a) except that level 2 consists of two degenerate sublevels. It is not difficult to see that the pumping field, as a result of the interference between the two transitions from levels 3 to the two sublevels in level 2, not only creates but also accumulates atoms in a coherent population trapping (CPT) state of Λ type between levels 3 and 2. Hence level 2, unlike in an ideal Λ system, possesses both population and coherence even in the presence of a strong pumping field. The difference between the two systems illustrates the need to build the Zeeman sublevels into the theoretical description in order to give an accurate account of the physics involved in real experiments.

In this paper, we present a theoretical framework for studying the laser-atom interaction correct to the first order in the weak probe, by taking into account the vector nature of laser fields and the multi-Λ-sublevel structure of real atoms. We apply this theory to a Λ-type system made up of three hyperfine levels in $^{87}$Rb atoms (Fig. 2). We investigate the properties associated with the CPT states in level 2 established by the pumping field. We mention that the CPT has been widely studied in the past both experimentally and theoretically [18–24] and has recently been utilized in such interesting applications as velocity-selective coherent population trapping [25–27] and atomic mirrors in atom optics [28,29]. What sets our work apart from the others is that our work focuses on the impact of the CPT process between levels 3 and 2 on the probe spectrum, especially the formation of EIT, between levels 3 and 1. This paper is organized as follows. In Sec. II we develop a semiclassical theory applicable to the study of EIT and LWI in degenerate multi-Zeeman-sublevel atoms. In Sec. III we illustrate how to derive the equations unique for the Λ system in Fig. 2 from the general formalism. We discuss in Sec. IV, through numerical simulations, the properties of the CPT states, the roles that the CPT plays in turning a real atomic system into an effective ideal Λ system, and the physical mechanisms underlying the transparency exhibited in the probe spectrum with and without Doppler broadenings. Section V summaries the major results.

II. THEORETICAL FRAMEWORK

The general theory is based on a model in which a multi-level atom interacts resonantly with, possibly, several fields. The energy structure is the same as that of an alkali-metal atom where each hyperfine level $F$ consists of several degenerate Zeeman sublevels $m_F$. $|i,m\rangle$ is used to denote an energy level that has $\Omega_i$, atomic transition frequency (with respect to the ground level), where $i$ and $m$ represent $F$ and $m_F$, respectively. The energy levels are arranged in such an order that $\Omega_i > \Omega_j$ if $i > j$. The total field $E$ is expanded as

$$E = \sum_{i,j,q} E_{ij}^{+\delta} (\hat{\alpha}_{ij}^{\dagger})^* \exp(-i \omega_{ij} t + i k_{ij} z),$$

where

$$\hat{\alpha}_{ij}^{\dagger} = -\frac{i \hat{\gamma} + q \hat{\gamma}}{\sqrt{2}} |q| + (1 - q)(1 + q) \hat{\gamma}$$

are the spherical basis unit vectors and $E_{ij}^{+\delta}$ are the components of a field that carries a wave number $k_{ij}$ and a frequency $\omega_{ij}$ close to the $i \leftrightarrow j$ atomic transition frequency $\Omega_{ij}$ ($= \Omega_i - \Omega_j$). Note that for simplicity the letter $i$ also appears as the imaginary number by convention. The dipole moment operator in the interaction picture is written as

$$\mu = \sum_{i,j,m,n,q} \mu_{ij}^{mn} \delta_{m-q,n} \hat{\alpha}_{ij}^{\dagger} \exp(i \Omega_{ij} t) |i,m\rangle \langle j,n|,$$

where $\delta$ is a Kronecker delta function and $\mu_{ij}^{mn}$ are the components of the dipole matrix element $|i,m\rangle \mu_{ij} |j,n\rangle$. The dipole components can be further expressed as

$$\mu_{ij}^{mn} = \mu_{ij} C_{ij}^{mn},$$

where $\mu_{ij}$ is proportional to the reduced matrix element and independent of the magnetic quantum number and the coefficient $C_{ij}^{mn}$ between levels $|i,m\rangle$ and $|j,n\rangle$ is real and can be obtained from standard tables. The variables in Eqs. (2.1)–(2.3) are defined for $i > j$. Their values for $i < j$ are derived from the transformation rules

$$\omega_{ij} = -\omega_{ji}, \quad k_{ij} = -k_{ji}, \quad E_{ij}^{\delta} = (E_{ij}^{\delta})^*, \quad \hat{\alpha}_{ij}^{\dagger} = (\hat{\alpha}_{ji}^{\dagger})^*,$$

(2.4a)
\[ \mu_{m}^{ij} = (\mu_{nm}^{ji})^*, \quad \mu_{ij} = (\mu_{ji})^*, \quad C^{ij}(m,n) = C^{ji}(n,m). \] (2.4b)

With the help of the relation \( \hat{\sigma}_{ij}^* \cdot (\hat{\sigma}_{ij}^*)^* = \delta_{nn'} \), the dipole interaction Hamiltonian matrix element becomes

\[ (H')^{ij}_{mn} = \langle i,m | ( - \mathbf{\mu} \cdot \mathbf{E}) | j,n \rangle \]

\[ = - \sum_q \mu_{mn}^{ij} \delta_{m-q,n} E_{ij}^q \]

\[ \times \exp (+i \Delta_{ij} t + i k_{ij} z) \]

\[ \text{(2.5)} \]

in the rotational wave approximation, where \( \Delta_{ij} \) (\( = \Omega_{ij} - \omega_{ij} \)) is the frequency detuning. The evolution of the atomic variables in the interaction representation is governed by the master equation

\[ \frac{\partial \rho'}{\partial t} = - \frac{i}{\hbar} [H', \rho'] + \frac{(\partial \rho')}{\partial t} \text{inc}, \]

\[ \text{(2.6)} \]

where the first term results from the coherent interaction and the second term represents the dampings due to the spontaneous emission and other irreversible processes. Equation (2.6) can be changed into the one for the density-matrix element of the slowly varying function of time and space defined by

\[ \rho_{mn}^{ij} = \rho_{mn}^{ij} \exp (-i \Delta_{ij} t - i k_{ij} z), \]

\[ \text{(2.7)} \]

where \( \rho_{mn}^{ij} = \langle i,m | \rho' | j,n \rangle \). With this transformation, the first term of Eq. (2.6) becomes

\[ \frac{d \rho_{mn}^{ij}}{dt} = - i \Delta_{ij} \rho_{mn}^{ij} - \frac{i}{\hbar} \sum_{k,l} (H)^{ik}_{m} P_{kl}^{n} + \frac{i}{\hbar} \sum_{k,l} (H)^{ij}_{kl} P_{ml}^{n}, \]

\[ \text{(2.8)} \]

where

\[ (H)^{ij}_{mn} = - \sum_q \mu_{mn}^{ij} \delta_{m-q,n} E_{ij}^q \]

\[ = - \mu_{ij} \sum_q C^{ij}(m,n) \delta_{m-q,n} E_{ij}^q. \]

\[ \text{(2.9)} \]

Note, for simplicity, that the same notation \( d \rho_{mn}^{ij}/dt \) is used to denote the rate change of the density-matrix element due to the coherent as well as the incoherent interactions.

The decay terms due to the spontaneous emission can be obtained from the master equation for an atomic system (of multiple Zeeman sublevels) interacting with vacuum reservoir [30]. The results are summarized in the set of equations

\[ \frac{d \rho_{mn}^{ij}}{dt} = - \sum_k \Gamma_{i-k} \rho_{mn}^{ij} + \sum_{j,q} \Gamma_{jm+q;jn+q} \rho_{mn}^{ij} P_{m+q,n+q}^{j}, \]

\[ \text{(2.10a)} \]

\[ \frac{d \rho_{mn}^{ij}}{dt} = - 0.5 \left( \sum_k \Gamma_{i-k} + \sum_k \Gamma_{j-k} \right) \rho_{mn}^{ij}, \quad i \neq j, \]

\[ \text{(2.10b)} \]

where

\begin{table}[h]
\centering
\caption{\( C^{ij}(m,n) \)}
\begin{tabular}{cccc}
\hline
\( n \) & -1 & 0 & +1 \\
\hline
\( m \) & & & \\
-1 & 1 & -1 & 0 \\
0 & 1 & 0 & -1 \\
+1 & 0 & 1 & -1 \\
\hline
\end{tabular}
\end{table}

\[ \Gamma_{i-k} = \Gamma_{ik} \sum_q C^{ik}(m,m+q) C^{ik}(m,m+q), \]

\[ \text{(2.11a)} \]

\[ \Gamma_{jm+q;jn+q} = \Gamma_{ij} C^{ij}(m+q,m) C^{ij}(n+q,n), \]

\[ \text{(2.11b)} \]

and

\[ \Gamma_{ij} = \left\{ \begin{array}{ll}
| \mu_{ij} |^2 \Omega_{ij}^2 / 3 \pi \epsilon_0 \delta c & \text{if } i > j \\
0 & \text{if } i < j
\end{array} \right. \]

\[ \text{(2.12)} \]

Note that \( \Gamma_{i-k} \) is independent of \( m \) due to the isotropy of spontaneous emission. The atomic polarization \( \mathbf{P} \) is calculated from

\[ \mathbf{P} = N \text{ Tr} (\rho' \mathbf{\mu}), \]

where \( N \) is the atomic density. With the help of relations (2.2) and (2.7), we find that

\[ \mathbf{P} = \sum_{i,j,q} P_{ij}^{+q} (\hat{\sigma}_{ij}^* \cdot \mathbf{\hat{\sigma}}_{ij}^*) \exp (-i \omega_{ij} t + i k_{ij} z), \]

where

\[ P_{ij}^{+q} = \sum_{m,n} \rho_{mn}^{ij} (\mu_{mn}^{ij})^* \delta_{n-q,m} \]

\[ = \sum_{m,n} \rho_{mn}^{ij} (\mu_{ij})^* C^{ij}(m,n) \delta_{n-q,m}. \]

\[ \text{(2.13)} \]

The connection between the fields and the polarizations is closed through the Maxwell wave equations. For the slowly varying amplitudes and in the steady state, they are reduced to the coupled wave equations

\[ \frac{d E_{ij}^{+q}}{dz} = i \frac{\omega_{ij}}{2 \epsilon_0 c} P_{ij}^{+q}, \]

\[ \text{(2.14)} \]

where \( c \) and \( \epsilon_0 \) correspond, respectively, to the speed of light and the dielectric susceptibility in vacuum.

III. EQUATIONS OF MOTION FOR A \( \Lambda \)-TYPE SYSTEM IN RUBIDIUM ATOMS

We now derive, with the help of the general theory, the equations of motion for a \( \Lambda \)-type system in \(^{85}\)Rb atoms. Figure 2 shows the relevant atomic energy levels. Levels 1, 2, and 3 correspond to \( 5S_{1/2}, F=1, 5S_{1/2}, F=2, \) and \( 5P_{1/2}, F'=1 \) hyperfine levels, respectively. The coefficients \( C^{ij}(m,n) \) in matrix form are shown in Table I for transitions between levels 3 and 1 and in Table II for transitions between

\[ \text{(2.15)} \]
The atoms are subject to two linearly polarized fields. One is a strong pumping field having a wave number $k_p$ and a frequency $\omega_c$ close to the $3\leftrightarrow 2$ transition and the other is a weak probe field having a wave number $k_p$ and a frequency $\omega_p$ tuned around the $3\leftrightarrow 1$ transition. Let us select the field propagation direction to be the quantization direction. The nonzero field variables in this scheme are

$$E_{32}^+ = E_{c}^+, \quad E_{32}^- = E_{c}^-, \quad \omega_{32} = \omega_{c}, \quad k_{32} = k_{c} \quad (3.1a)$$

for the pumping field and

$$E_{31}^+ = E_{p}^+, \quad E_{31}^- = E_{p}^-, \quad \omega_{31} = \omega_{p}, \quad k_{31} = k_{p} \quad (3.1b)$$

for the weak probe field. This model has 11 atomic levels and therefore requires 121 density-matrix elements in order to fully describe its dynamics. Close inspection of the 121 equations derived from coherent contribution [Eq. (2.8)] indicates that they split up into two closed families. One family has 60 members, all of which are coherence terms, and the other, in contrast, has 61 members, which include all the population terms. Since coherence terms are initially all zero and cannot evolve to significant values by themselves, it seems legitimate that we ignore the former family and focus only on the equations for the variables in the latter family (the dominant family). The Appendix lists the relevant 61 equations. It is impractical to derive all the equations. In what follows, as an example to illustrate the main steps involved in translating the general equations in Sec. II into the ones for our model, we derive the equation for $\rho_{22}^{-1,-1}$. We begin the derivation by calculating the coherent contribution from Eq. (2.8). A straightforward substitution along with a summation over all possible $k$ yields

$$\frac{d\rho_{22}^{-1,-1}}{dt} = -i \Delta_{22} \rho_{22}^{-1,-1} + \frac{i}{\hbar} \sum_{l} (H)_{l,-1}^{23} \rho_{l,-1}^{32}$$

$$+ \frac{i}{\hbar} \sum_{l} (H)_{l,-1}^{32} \rho_{l,-1}^{23}, \quad (3.2)$$

where

$$(H)_{l,-1}^{32} = -\mu_{32} \sum_{q} C^{32}(q, -1, -1) \delta_{l,q,-1} E_{c}^{q} \quad (3.3a)$$

and

$$(H)_{l,-1}^{23} = [(H)_{l,-1}^{32}]^*$$

$$- (\mu_{32})^* \sum_{q} C^{32}(q, -1, -1) \delta_{l,q,-1} (E_{c}^{q})^*. \quad (3.3b)$$

In deriving Eq. (3.2), we have used Eq. (3.1a), the fact that $\mu_{i,-1} = \mu_{31} = 0$, and the transformation rules (2.4). Next we substitute Eqs. (3.3) into Eq. (3.2) and expand explicitly the sum over $l$ and $q$ to reach

$$\frac{d\rho_{22}^{-1,-1}}{dt} = + i C^{32}(0, -1) (E_{c}^{i})^* \rho_{0,-1}^{32} + i C^{32}(-2, -1)$$

$$\times (E_{c}^{-})^* \rho_{-2,-1}^{32} - i C^{32}(0, -1) E_{c}^{i} (\rho_{0,-1}^{32})^*$$

$$- i C^{32}(-2, -1) E_{c}^{-} (\rho_{-2,-1}^{32})^*, \quad (3.4)$$

where we have introduced the Rabi frequencies of the pumping field

$$E_{c}^i = \frac{\mu_{32} E_{c}^{i+1}}{\hbar}, \quad E_{c}^{-} = \frac{\mu_{32} E_{c}^{-1}}{\hbar}. \quad (3.5)$$

Finally, we substitute the coefficients from Table II into Eq. (3.4) to obtain

$$\frac{d\rho_{22}^{-1,-1}}{dt} = - i \frac{1}{\sqrt{2}} (E_{c}^{i})^* \rho_{0,-1}^{32} + i \frac{1}{\sqrt{2}} E_{c}^{-} (\rho_{-2,-1}^{32})^*. \quad (3.6)$$

We now move to the next phase of derivation by calculating the damping terms due to spontaneous emission from Eqs. (2.10). Because level 3 is the only level that has dipole connection with level 2, it is not difficult to see that Eq. (2.10a) is reduced to

$$\frac{d\rho_{22}^{-1,-1}}{dt} = \sum_{q} \Gamma_{3,-1+q,3,-1+q-2,-1+q-2,-1+q} \rho_{3,-1+q,-1+q}^{33} \quad (3.7)$$

where the various coefficients are determined by Eq. (2.11b), for example,

$$\Gamma_{3,-1,3,-1-2,-1,2,-1} = \Gamma_{32} C^{32}(1, -1) C^{32}(-1, -1) = \frac{\Gamma_{32}}{2}.$$

Carrying out the sum over all possible $q$, we arrive at

$$\frac{d\rho_{22}^{-1,-1}}{dt} = \frac{\Gamma_{32}}{2} \left( \rho_{3,-1,1}^{33} + \rho_{0,0}^{33} \right). \quad (3.8)$$

The final phase of the derivation deals with the decays due to collisions. The decay rates due to the various collisions under the isotropic condition are most conveniently introduced via the equations for the multipole polarizations in the spherical tensor representations since each multipole relaxes with a characteristic time that may be different for different multipoles [32–34]. Here, to simplify the problem, we assume that the atomic density is low enough so that the collisional decay rates are much smaller than the spontaneous decay rates of the excited levels. Under this assumption, one only has to take into consideration the effects of the collisions on the two ground hyperfine levels. Because the energy between the ground hyperfine levels corresponds to frequencies in the microwave range, the collisions not only damp the coherence but also affect the populations of levels 1 and 2. To simulate this fact, we introduce phenomenologi-
cally the population transfer rate from $|2,m\rangle$ to $|1,n\rangle$, $\Lambda_{21}$, and the population transfer rate from $|1,m\rangle$ to $|2,n\rangle$, $\Lambda_{12}$. The contribution due to this consideration is summarized in the equation
\[
\frac{d\rho_{21}^{\text{21}}}{dt} = -3\Lambda_{21}\rho_{21}^{\text{21}} + \Lambda_{12}(\rho_{31}^{\text{11}} + \rho_{00}^{\text{11}} + \rho_{11}^{\text{11}}).
\]
(3.9)

The equation for $\rho_{21}^{\text{21}}$ in the Appendix is obtained by combining Eqs. (3.6), (3.8), and (3.9). The rest of the equations in the Appendix are derived in a similar fashion. The wave equations for the Rabi frequencies of the weak probe field
\[
E_p^+ = \frac{\mu_3 E_p^{+1}}{h}, \quad E_p^- = \frac{\mu_3 E_p^{-1}}{h}
\]
are derived from Eq. (2.14) and have the form
\[
\begin{align}
\frac{dE_p^+}{dz} &= -i \frac{\mu_3 |2| \omega_p}{2 \delta E_0 c h} (\rho_{21}^{\text{11}} + \rho_{01}^{\text{11}}), \quad (3.10a) \\
\frac{dE_p^-}{dz} &= i \frac{\mu_3 |2| \omega_p}{2 \delta E_0 c h} (\rho_{21}^{\text{11}} + \rho_{01}^{\text{11}}), \quad (3.10b)
\end{align}
\]
which will be used in the next section to derive the probe spectrum.

IV. NUMERICAL SIMULATIONS AND DISCUSSIONS

For the purpose of numerical simulation, we organize the 61 linear equations in the Appendix into the matrix form
\[
[M + E_p^+ + M_{+\,\phi}(E_p^+)^* + M_{-}(E_p^-)^* + M] \psi = I,
\]
(4.1)
where $\psi$ [Eq. (A1)] is a column vector containing the variables in the dominant family, $M_{+\,\phi}$, $M_{-\,\phi}$, $M_{-}$, and $M_{-\,\phi}$ are matrices whose elements are derived from the coefficients of the terms proportional to $E_p^+$, $(E_p^+)^*$, $E_p^-$, and $(E_p^-)^*$, respectively, $M$ is a matrix containing the rest of the terms, and $I$ is a column vector whose elements are all 0 except one element being 1, which originates from replacing one of the equations in the Appendix with the closure relation. Before performing numerical calculations, let us summarize the measures taken in simplifying the numerical simulations that follow. First, we assume, without loss of generality, the pumping field to be linearly polarized along the $\hat{y}$ direction with the form
\[
E_p(z,t) = 2\sqrt{2}E_c \hat{y} \cos(\omega_c t - k_c z),
\]
(4.2)
where $E_c$ is a real number, representing the field strength. Equation (4.2), expanded according to Eq. (2.1), gives rise to $E_c^+ = E_c = iE_c$. Second, we choose $2\to1$ population transfer rates such that $\Lambda_{21}/\Lambda_{12} = 5/3$, the ratio of the number of sublevels in level 2 to that in level 1. This is to simulate the fact that the populations in lower levels are almost equal in the absence of the pumping field as a result of $h\Delta_{21}$ being much smaller than the thermal kinetic energy. Finally, we select $\Gamma_{31}$ to be the unit for the decay rates, the frequency detunings, and the Rabi frequencies.

A. Zeroth-order solution and coherent population trapping

By setting the probe field components to zero in Eq. (4.1), we turn it into $M \psi^{(0)} = I$, from which we find the zeroth-order solution $\psi^{(0)} = M^{-1}I$. Let us consider the $\Delta_{12} = 0$ situation (where the pumping field is on resonance with the $3\to2$ transition) under the decay rates $\Gamma_{31} = 6$, $\Gamma_{12} = 0.001$, and $\Lambda_{21} = 0.001$. Here the choice of $\Gamma_{31}/\Gamma_{12} = 6$ is based on the fact that in $^87$Rb the $5P_{3/2}, F = 1 \leftrightarrow \Sigma_{3/2}$, $F = 2$ transition is five times stronger than the $5P_{3/2}, F = 1 \leftrightarrow \Sigma_{3/2}$, $F = 1$ transition. Figure 3 shows how the population of each Zeeman sublevel of level 2 changes with $E_c$. As expected, when $E_c = 0$, the population at each Zeeman sublevel is 0.1 because 0.5 populations in level 2 are equally divided among its five sublevels. As the pumping field increases, in contrast to the three-level $\Lambda$ model (without Zeeman-sublevel structure) where all the populations in level 2 are optically pumped to level 1, the populations in level 2 mainly undergo a redistribution among its own Zeeman sublevels even at large $E_c$. For example, when $E_c = 1.5$, of the initial 0.5 populations in level 2, only 0.055 are optically pumped to level 1 and the rest are distributed according to $\rho_{22}^{32} = 0.0288$, $\rho_{21}^{31} = 0.0011$, and $\rho_{02}^{32} = 0.1656$. To illustrate how the coherences $|\rho_{mn}^{32}|$ and $|\rho_{mn}^{22}|$ ($m \neq n$) change with $E_c$, we consider, as an example, $|\rho_{32}^{31}|$ and $|\rho_{22}^{22}|$ in Fig. 4. As expected, both coherences are zeros in the absence of $E_c$ and increase with $E_c$ initially. Surprisingly, as $E_c$ further increases, $|\rho_{32}^{31}|$ reduces to an extremely small value while $|\rho_{22}^{22}|$ grows to a value comparable to the populations in level 2. For example, when $E_c = 1.5$, $|\rho_{32}^{31}| = 0.0001$ and $|\rho_{22}^{22}| = 0.1094$. These results imply that the atoms in level 2 are, on the one hand, decoupled from level 3 and, on the other hand, have a strong correlation among themselves. To understand these features, we first replace the five Zeeman sublevels of level 2 with five orthonormal superposition states (derived under the condition $E_c^+ = E_c^-$)
Among them, \(|\text{NAS 1}\rangle = \frac{1}{\sqrt{2}} |2,-1\rangle - \frac{1}{\sqrt{2}} |2,+1\rangle,\)

|NAS 2\rangle = \frac{1}{\sqrt{8}} |2,-2\rangle - \sqrt{3/4} |2,0\rangle + \frac{1}{\sqrt{8}} |2,+2\rangle,

|\text{AS 1}\rangle = \frac{1}{\sqrt{2}} |2,-1\rangle + \frac{1}{\sqrt{2}} |2,+1\rangle,

|\text{AS 2}\rangle = \frac{1}{\sqrt{2}} |2,-2\rangle - \frac{1}{\sqrt{2}} |2,+2\rangle,

|\text{AS 3}\rangle = \sqrt{3/8} |2,-2\rangle + \frac{1}{2} |2,0\rangle + \sqrt{3/8} |2,+2\rangle.

Among them, \(|\text{NAS 1}\rangle\) is a CPT (or nonabsorbent) state of \(\Lambda\) type made up of \(m_F=0\) of level 3 and \(m_F=-1\) and +1 of level 2, \(|\text{NAS 2}\rangle\) is also a CPT state but of inverted \(W\) type made up of \(m_F=-1\) and +1 of level 3 and \(m_F=-2, 0,\) and +2 of level 2, and the rest are the absorbent states [26]. We next plot in Fig. 5 their populations as a function of \(E_c\). As the pumping field increases, the populations at absorbent states quickly deplete while those at CPT states grow significantly. Here the pumping field plays dual roles, with one as a catalyst to the formation of the coherent superposition states and the other as an optical pump for the atoms at absorbent states. The features shown in Figs. 3 and 4 are nothing but those displayed by the atoms coherently trapped in level 2. The CPT scheme in our system has two unique characteristics. First, the CPT is not highly sensitive to the pump detuning \(\Delta_{32}\) as depicted in Fig. 6. This can be understood as follows. In a CPT scheme, such as ours, where all the atomic transitions are degenerate, the CPT condition (or, equivalently, the two-photon-resonance condition) requires the frequency of the field component that couples each transition to be the same. This is always satisfied in our model irrespec-

\[\text{FIG. 4. Coherences as functions of the pump field strength } E_p \text{ under the same parameters as in Fig. 3. The solid curve represents } |\rho_{11}|_{J=4} \text{ and the dashed curve } |\rho_{11}|_{J=2}.\]

\[\text{FIG. 5. Populations of the coherent superposition states of level } 2 \text{ as functions of the pump field strength under the same parameters as in Fig. 3. The solid curve represents the population in one of the equally populated CPT states and the dashed curves represent the populations in three absorbent states (two of them are equally populated).}\]

\[\text{FIG. 6. Populations of the coherent superposition states of level } 2 \text{ as functions of the pump field detuning } \Delta_{32} \text{ with the parameters } E_c=5, \Gamma_{33}=6, \Gamma_{31}=1, \Delta_{21}=0.0167, \text{ and } \Delta_{12}=0.01. \text{ The curve with a circle legend represents the population in one of the equally populated CPT states, the rest the populations in the three absorbent states (two of them are equally populated).}\]
from which we find $c^*$ are the four column vectors having the same dimensions as $c$. Replacing the density-matrix elements in Eq. (3.10) with the first-order solutions, we are able to reach

$$
\frac{dE_p^+}{d\eta} = \alpha_+^+ E_p^+ + \alpha_- E_p^- , \tag{4.5a}
$$

$$
\frac{dE_p^-}{d\eta} = \alpha_+ E_p^- + \alpha_-^+ E_p^+ , \tag{4.5b}
$$

where $\eta$ is the distance $z$ scaled to $2e_0c\hbar/(|\mu_3|^2\omega_p)$ and

$$
\alpha_+ = i\psi_+[37] + i\psi_+[41] , \quad \alpha_- = i\psi_-[37] + i\psi_-[41] ,
$$

with 37, 39, 41, and 43 being, respectively, the locations of the elements $\rho_{31}^{31}, \rho_{01}^{01}, \rho_{11}^{01}$, and $\rho_{11}^{10}$ in the column vector $\psi$. Equation (4.5) indicates that a linearly polarized weak probe will, in general, change its polarization state during its propagation as a result of the anisotropy ($\alpha_+ \neq \alpha_- \neq 0$) and the cross couplings ($\alpha_+ \neq \alpha_-$) between its two orthogonal components. However, in our model, $\alpha_+ \neq \alpha_- \neq 0$ due to the lack of a direct coupling between the probe field and the atoms in level 2 and hence they will be ignored. Furthermore, the model is symmetric with respect to $\sigma^+$ and $\sigma^-$ probe transitions and $\alpha_+ = \alpha_- = \alpha$. Thus it is justifiable to consider $\text{Re}(\alpha)$ and $\text{Im}(\alpha)$ versus the probe frequency as the absorption and the dispersion spectra of the weak probe. Figure 8 depicts typical absorption and dispersion spectra, indicating EIT around the two photon resonance.

We now turn our attention to the effects of the CPT on the probe spectrum. One obvious effect is that the absorption profile undergoes a uniform reduction because the CPT effect traps the populations in level 2 that are otherwise available to level 1 for absorption. A less intuitive effect is that, in spite of the presence of atoms in level 2, the probe spectrum, in the parameter regime where the CPT is strong, is equivalent to that of a simple system in which levels 2 and 3 are both empty and level 1 inherits the population of level 1 in the real atomic system.

To demonstrate this effect, consider, again, the spectra in Fig. 8. This represents a case where atoms in level 2 are virtually all in CPT states, as can be seen from the population distributions in the zeroth-order solutions: 55.5% are in level 1, 44.2% in nonabsorbent states, 0.25% in the absorbent states, and 0.027% in level 3. The behavior of the simple system is simulated by the same C computer program for the
FIG. 9. Comparison of the absorption probe spectra between the real and the simple models. The solid curve is for the real system and the dashed curve is for the simple system. The parameters for the real system are $\Delta_{32}=0$, $E_e=2$, $\Gamma_{32}=6$, $\Gamma_{31}=1$, $A_{21}=0.167$, and $A_{12}=0.1$.

real system with the exception of keeping, in the zeroth-order solution, only the population in level 1. Our numerical simulation yields probe spectra (not shown) almost in perfect agreement with those in Fig. 8.

We caution that the probe spectrum derived from the simple system is not always in agreement with that derived from the real system, especially in the parameter regime where the CPT is weak while the pumping field is not. Consider a situation the same as Fig. 8 except with the increased collisional decay rates $A_{12}=0.1$ and $A_{31}=0.1667$. This is the case in which a sizable amount of population in level 2 is left in absorbent states as reflected from the following population weights: 52.2% are in level 1, 32.3% in the CPT states, 14.0% in the absorbent states, and 1.5% in level 3. The increased populations in the absorbent states are, of course, due to the increased collisional decay rates, which shorten the lifetime of the CPT states and demand a higher pumping power to keep the atoms in the CPT states (Fig. 7). The solid and the dashed curves in Fig. 9 represent the absorption spectra obtained from the real and the simple systems, respectively. They clearly disagree with each other; the transparency based on the real system is deeper than that based on the simple system. To account for the difference, one realizes that the increased populations in the absorbent states coupled with the pumping field create favorable conditions for the Raman anti-Stokes process while the population accumulation in level 3 accelerates the stimulated emission process. Both processes are in favor of the production, not the absorption, of the probe photons, but are absent from the simple system as a result of the elimination of the populations in levels 3 and 2. That explains the difference between the solid and the dashed curves in Fig. 9.

The equivalence of the two models in a strong CPT regime demonstrates a remarkable feature of the CPT process: its ability to spontaneously filter out from the real system the processes that do not appear in an ideal $\Lambda$ system. Physically, CPT accomplishes this filtration by the very foundation of the CPT states: they are decoupled from level 3. It follows that the populations in the excited level 3 can no longer accumulate and the coherences between levels 3 and 2 diminish. This simple argument explains how the CPT process destroys the stimulated emission and the Raman anti-Stokes processes. Note that the atoms in level 2 make their presence known to the probe field through the $3\leftrightarrow2$ coherences. The extremely small values in the $3\leftrightarrow2$ coherences also imply that the atoms in the CPT states are “invisible” to the weak probe field between levels 3 and 1. From the experimental point of view, this feature of the CPT process is extremely beneficial. A system as complex as Fig. 2, after the filtration of the CPT process, becomes a system as simple as Fig. 1(a) so far as the probe spectrum between levels 3 and 1 is concerned. This makes it possible to perform a detailed, quantitative comparison of the experimental results against a simple theory based on an ideal $\Lambda$ system (but only in the strong CPT regime).

C. Effect of Doppler broadening

The theory developed so far describes the behavior of a group of atoms that move at a well-defined velocity and applies directly to the experimental arrangement involving three mutually orthogonal and well-collimated beams: an atomic beam, a pump beam, and a probe laser beam. In the case of a gas cell where the atoms moving at different velocities can all interact with the fields, the correct theory requires the superposition of the contributions from the atoms in various velocity groups. Let $\omega_p$ and $\omega_c$ be the probe and pump frequencies in the rest atomic frame and $\Delta_{310}$ and $\Delta_{320}$ be their corresponding frequency detunings. Then the probe and the pump frequency detunings are Doppler shifted to

$$\Delta_{31}=\Delta_{310}+\frac{\omega_c0v}{c},$$

$$\Delta_{32}=\Delta_{320}+\frac{\omega_p0v}{c},$$
in the perspective of an atom moving with a velocity $v$. The complex absorption coefficient in the presence of Doppler broadening $\alpha(\Delta_{310},\Delta_{320},\Delta\omega)$ can thus be obtained by the integral

$$\alpha(\Delta_{310},\Delta_{320},\Delta\omega) = \frac{1}{\sqrt{\pi}\Delta\omega} \int_{-\infty}^{\infty} \alpha(\Delta_{31},\Delta_{32}) \times \exp\left(-\frac{(\Delta_{31}-\Delta_{310})^2}{\Delta\omega^2}\right) d\Delta_{31}$$

under the Maxwellian-distribution assumption, where $\Delta\omega$ is proportional to the Doppler width and $\alpha(\Delta_{31},\Delta_{32})$ is the same $\alpha$ defined in Sec. IV B, but evaluated at $\Delta_{31}$ and $\Delta_{32}$. Figure 10 compares the probe spectrum in the presence of the pumping field (solid curve) with that in the absence of the pumping field (dashed curve), revealing, besides the EIT, the heavy presence of the populations in level 2 because the
peak value of the solid curve, compared to that of the dashed curve, is far less than doubled.

One noticeable effect of the Doppler broadening on the transparency is the increased contributions from the Raman anti-Stokes and the stimulated emission processes. To explain this effect, let us divide the atoms in level 2 into groups according to the velocities of the atoms or equivalently the pump frequencies seen by the atoms. This division enables us to regard Fig. 6 as the distribution of the atoms between the CPT states and the absorbent states as a function of the group. Figure 6 indicates that the share of the atoms in the absorbent states increases steadily from the groups near to those far away from the pump resonance frequency. Note that each point in the Doppler-broadened spectrum is the superposition of the contributions from the atoms in every group inside the Gaussian distribution curve. It follows that, as the Gaussian distribution curve widens, more atoms in the absorbent states will contribute to the probe spectrum, resulting in a transparency deeper than that derived from the simple system. Figure 11 compares the probe spectrum in the real system to that in the simple system, clearly indicating the increased contributions from the Raman anti-Stokes and the stimulated emission processes that govern the dynamics of this model.

We found that the pumping field applied between levels 2 and 3 causes a significant number of atoms to be coherently trapped among the Zeeman sublevels of level 2 as a result of the coexistence of two CPT states in level 2: the coherent superposition of \( m_F = -1 \) and \( +1 \) and that of \( m_F = +2, 0, \) and \( +2 \). We have studied numerically the influence of the pumping field and the various decay rates on the CPT states and found that the CPT states are not highly sensitive to the pump detuning and can be formed by a relatively weak pumping field.

We have calculated numerically the probe spectrum between levels 3 and 1 and explored the effects of the CPT on the probe spectrum and the physical mechanisms behind the transparency in the real system with several important results.

(i) The atoms in the absorbent states of level 2 make contributions to the probe spectrum by the Raman anti-Stokes process while those in the CPT states are shielded from interacting with the probe field.

(ii) The CPT process has the ability to filter out some processes from the real system. The real system approaches the effective ideal \( \Lambda \) system only when level 2 is prepared in a mixture of (almost) pure CPT states. In this case, the transparency is equivalently to EIT, a phenomenon due solely to the atomic coherence between levels 2 and 1 established by the pumping field.

V. SUMMARY

In this paper, we have established a theoretical framework for studying the interaction between electromagnetic fields and degenerate multi-Zeeman-sublevel atoms. We have applied this theory to a \( \Lambda \)-type system made up of three hyperfine levels in \(^{87}\text{Rb} \) atoms and derived a complete set of equations that govern the dynamics of this model.

We have found that the pumping field applied between levels 2 and 3 causes a significant number of atoms to be coherently trapped among the Zeeman sublevels of level 2 as a result of the coexistence of two CPT states in level 2: the coherent superposition of \( m_F = -1 \) and \( +1 \) and that of \( m_F = +2, 0, \) and \( +2 \). We have studied numerically the influence of the pumping field and the various decay rates on the CPT states and found that the CPT states are not highly sensitive to the pump detuning and can be formed by a relatively weak pumping field.

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(iii) In the parameter regime where a significant number of atoms are in the absorbent states, the effective ideal $\Lambda$ system fails. The weak probe obtained from the real system exhibits a transparency deeper than that obtained from the effective ideal $\Lambda$ model. In this case, the transparency is due to, besides the atomic coherence between levels 2 and 1 (EIT), the Raman anti-Stokes and the stimulated emission processes.

We have also performed numerical calculations of the probe spectrum in the presence of the Doppler broadening. The results indicate that the essential features, the CPT and the EIT, of the homogeneous broadened model are well preserved at a moderate pumping power. The results also show that the Doppler broadening increases the contributions of the Raman anti-Stokes and the stimulated emission processes to the transparency. We attribute this phenomenon to the interaction between the pumping field and the increased atomic populations in the absorbent states.

Let us close our work by briefly extending our analysis to the same model except level 3 being replaced by $5P_{1/2}$, $F'=2$ in $^{87}$Rb atoms as shown in Fig. 12. The energy structure between levels 3 and 2 can be divided into a $W$ scheme made up of $m_F=-2$, 0, and +2 of level 3 and $m_F=-1$ and +1 of level 2 and an inverted $W$ scheme the same as that in Fig. 2. Note that the atoms cannot be coherently trapped in $m_F=-1$ and +1 of level 2 because they are coupled to three excited Zeeman sublevels. There is only one CPT state: the coherent superposition of $m_F=-2, 0,$ and +2 of level 2. We thus anticipate that more atoms (more than 75% of the total populations, an estimation based on the fact that $\Gamma_{31}=\Gamma_{32}$) will be optically pumped to level 1. Apart from that, the conclusions drawn from this paper can all be applied to this model.

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APPENDIX

The variables in the dominant family are summarized in $\psi$, a column vector defined

$$\psi =\text{Transpose of } \begin{pmatrix} \rho_{-1-1}^{33}, \rho_{00}^{33}, \rho_{1+1}^{33}, \rho_{-1+1}^{33}, (\rho_{-1-1}^{33})^*, \\
\rho_{-2-2}^{22}, \rho_{-1-1}^{22}, \rho_{00}^{22}, \rho_{1+1}^{22}, \rho_{-1+1}^{22}, (\rho_{-1-1}^{22})^*, \\
\rho_{-1+1}^{22}, (\rho_{-1+1}^{22})^*, (\rho_{00}^{22})^*, (\rho_{00}^{22})^*, (\rho_{00}^{22})^*, (\rho_{00}^{22})^*, \\
\rho_{-1-1}^{11}, (\rho_{00}^{11})^*, (\rho_{00}^{11})^*, (\rho_{00}^{11})^*, (\rho_{00}^{11})^*, (\rho_{00}^{11})^*, \\
\rho_{-1-1}^{11}, (\rho_{-1+1}^{11})^*, (\rho_{-1+1}^{11})^*, (\rho_{-1+1}^{11})^*, \\
\rho_{00}^{21}, (\rho_{00}^{21})^*, (\rho_{00}^{21})^*, (\rho_{00}^{21})^*, (\rho_{00}^{21})^*, (\rho_{00}^{21})^*, \\
\rho_{00}^{00}, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*, \\
(\rho_{00}^{00})^*, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*, (\rho_{00}^{00})^*. \end{pmatrix}$$

(A1)

To simplify the expressions for the equations of motion, let us introduce

$$\Gamma_3 = \frac{2}{3} \Gamma_{32} + 2 \Gamma_{31}, \quad \gamma_{32} = \frac{1}{2} (\Gamma_3 + 3 \Lambda_{21}),$$

$$\gamma_{31} = \frac{1}{2} (\Gamma_3 + 5 \Lambda_{12}), \quad \gamma_{21} = \frac{1}{2} (3 \Lambda_{21} + 5 \Lambda_{12}),$$

$$\rho_{11}^{10} = \rho_{-1-1}^{10} + \rho_{00}^{10} + \rho_{1+1}^{10},$$

$$\rho_{22}^{20} = \rho_{-2-2}^{20} + \rho_{-1-1}^{20} + \rho_{00}^{20} + \rho_{1+1}^{20} + \rho_{-1+1}^{20} + \rho_{-1+1}^{20},$$

$$\Delta_{32} = \Omega_{32} - \omega_c, \quad \Delta_{31} = \Omega_{31} - \omega_p, \quad \Delta_{21} = \Delta_{31} - \Delta_{32}.$$

Below is a list of the equations for the variables in the dominant family:

$$\frac{d \rho_{33}^{33}}{dt} = -\Gamma_3 \rho_{33}^{33} - i \left[ (E^+_{c})^* \rho_{-1-2}^{33} + \frac{1}{\sqrt{6}} (E^-_{c})^* \rho_{-10}^{33} \right]$$

$$+ (E^+_{c})^* \rho_{-10}^{33} \text{ c.c.},$$

$$\frac{d \rho_{00}^{33}}{dt} = -\Gamma_3 \rho_{00}^{33} + i \left[ \frac{1}{\sqrt{2}} (E^+_{c})^* \rho_{00-1}^{33} + \frac{1}{\sqrt{2}} (E^-_{c})^* \rho_{00+1}^{33} \right]$$

$$- (E^+_{c})^* \rho_{00-1}^{33} + (E^-_{c})^* \rho_{00+1}^{33} \text{ c.c.},$$
\[
\frac{d\rho_{p+1}^{32}}{dt} = -\Gamma_3 \rho_{p+1}^{32} + \frac{i}{\sqrt{6}} (E_c^+) \rho_{p+1}^{32} + i E_c^- (\rho_{p+1}^{32})^* + i E_c^- (\rho_{p+1}^{32})^*
\]
\[
\frac{d\rho_{12}}{dt} = -(\gamma_{32} + i\Delta_{32})\rho_{12} + iE^+_{e}\rho_{33} - iE^-_{e}\rho_{22} - \frac{i}{\sqrt{6}} E^r_{e}\rho_{02} \\
+ iE^+_{p}(\rho_{20})^*,
\]
\[
\frac{d\rho_{32}}{dt} = -(\gamma_{32} + i\Delta_{32})\rho_{32} + iE^-_{e}\rho_{33} - \frac{i}{\sqrt{6}} E^-_{e}\rho_{02} \\
- iE^+_{p}\rho_{22} - iE^-_{p}(\rho_{20})^*,
\]
\[
\frac{d\rho_{1-1}}{dt} = -(\gamma_{12} + i\Delta_{12})\rho_{1-1} - iE^+_{p}(\rho_{20})^* \\
- i \frac{1}{\sqrt{2}} (E^r_{e})\rho_{01},
\]
\[
\frac{d\rho_{20}}{dt} = -(\gamma_{20} + i\Delta_{20})\rho_{20} + iE^-_{p}(\rho_{10})^* - i \frac{1}{\sqrt{2}} (E^-_{e})\rho_{10},
\]
and the equations for the rest variables (the ones with a superscript *) in \( \psi \) are obtained from the equations of their complex conjugate variables.