A degenerate optical parametric oscillator coupled with $N$ two-level atoms: effects of detuning in the optical spectra

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Abstract. We study the effects of atomic detuning in a composite system consisting of a degenerate optical parametric oscillator and a set of $N$ homogeneously broadened two-level atoms. The stability regions and the optical spectra of this composite system are investigated as a function of atomic detuning and other parameters. Near the lower turning point of the bistable curve, the unsqueezed field quadrature couples into the squeezed field quadrature through the atomic detuning.

Keywords: Light–atom interaction, squeezed states, nonlinear optics

1. Introduction

Squeezed states in a degenerate optical parametric oscillator (DOPO) and an absorptive optical bistability (AOB) system have been studied extensively both theoretically and experimentally [1–3]. A composite system combining these two systems (i.e., $N$ two-level atoms inside a DOPO cavity) has also been studied theoretically [4–6]. Steady-state behaviours (i.e., optical bistability) [4], optical spectra [5] and total intracavity squeezing [6] were calculated under the condition of resonance between the atomic transition frequency and the frequency of the sub-harmonic intracavity field. Optical bistability exists in both good- and bad-cavity limits of this composite system. An interesting feature in the bistability of this composite system is that the lower branch of the bistable curve always has a zero steady-state intensity, as in the case of a DOPO below threshold. This property allows us to use the Schwinger representation [5, 7] for the fields and atomic variables in the entire lower branch of the bistable curve and to write down a set of coupled linear operator equations for the field operators and atomic polarization operators. By transforming both the field and atomic operators into selected quadratures, we found that, at atomic resonance, these operator equations separate into two independent pairs of equations. The squeezed field quadrature of the DOPO couples to only one atomic quadrature, and the unsqueezed quadrature of the DOPO couples to the other atomic quadrature. This decoupling between the squeezed field quadrature and unsqueezed field quadrature gives us a clear picture of the underlying physics of how the squeezed and unsqueezed field quadratures of the DOPO affect the atomic dynamics. Also, the decoupling of equations at atomic resonance allows us to obtain simple analytical expressions for the optical spectra with arbitrary cavity linewidth and coupling strength between the atoms and the cavity field.

In this paper, we consider the influences of the atomic detuning on the steady-state behaviours, the stability boundaries and the optical spectra of this composite system. Due to the coupling between the squeezed and unsqueezed quadratures of the field for finite detuning, the squeezed spectra are altered by the unsqueezed field quadrature. This influence is only evident near the lower turning point of the bistable curve, where the field intensity of the unsqueezed quadrature becomes very large. At the good-cavity limit (i.e., the atomic decay rate is much larger than the cavity decay rate) the system is stable all the way up to the turning point of the lower branch, and the increase in the detuning only produces changes in the structure of the spectra. However, in the bad-cavity limit, the system becomes unstable before it reaches the lower turning point. The stability boundary remains almost constant as the detuning increases, and the instability point reaches the turning point of the lower branch only for a sufficient large detuning.

The arrangement of this paper is as follows. In section 2, we give the basic equations for this composite system with atomic detuning. In section 3, we discuss the stability regions of the system in both the good- and bad-cavity limits. In section 4, we give numerical calculations of the optical spectra with atomic detuning. Section 5 serves as a conclusion.
2. Basic equations

We study a system consisting of a nonlinear crystal and $N$ two-level atoms inside an optical cavity. The system is pumped by a strong external coherent field with a frequency close to twice that of the atomic transition frequency.

The Hamiltonian describing this system has been treated in previous papers [4, 5], and it is a standard procedure to derive a set of nonlinear stochastic differential equations from the system Hamiltonian [4, 8]. For the sake of simplicity in this paper, we will only briefly describe the procedure of deriving the steady-state equations from the system Hamiltonian. The system Hamiltonian (under the electric-dipole and rotating-wave approximations) consists of seven terms describing the free energy of the system, the atom-field interaction, the DOPO process, external driving for the DOPO, decay of the two-level atoms and decays of the cavity fields (both cavity sub-harmonic field and driving field) [4]. Using standard techniques, as described in [8], one can translate all the operators into the corresponding numbers and derive a Fokker–Planck equation. To study the steady-state solutions of this system, we obtain a set of nonlinear stochastic differential equations involving three atomic variables and four field variables (two field modes).

The steady-state solutions of this system can be found to be [4]

$$\left[1 + \frac{2C}{1 + \Delta^2 + X} + rX\right]^2 + \left[\frac{2C\Delta}{1 + \Delta^2 + X}\right]^2 = Y \quad (1)$$

and

$$X = 0, \quad (2)$$

where the normalized dimensionless parameters are defined as

$$C = \frac{N g^2}{2\gamma_1}, \quad \Delta = \frac{\Delta_d}{\gamma_1} = \frac{w_c - w_\perp}{\gamma_1}, \quad \Delta_c = \frac{\Delta_c}{\gamma_1},$$

$$x = xx^*, \quad y = yy^*, \quad \gamma = \frac{\epsilon_2}{\epsilon_0}, \quad r = \frac{N_g}{N_0}.$$ 

$x$ is the amplitude of the intracavity field at the subharmonic frequency. $C$ is the atomic cooperative parameter used as a measure of atomic absorption relative to cavity losses. $\gamma_1$ is the atomic polarization decay and $\gamma_1$ is the cavity decay rate of the subharmonic field. $g$ is the coupling strength between a single atom and the intracavity field. $w_c$ is the frequency of the cavity subharmonic field and $w_\perp$ is the atomic transition frequency. $\epsilon_2$ is the complex classical amplitude of the external driving field and $\epsilon_0$ is the normal DOPO field strength. $r$ is a measure of the relative strength of the atomic system over the DOPO.

Because of the presence of the set of two-level atoms, the threshold value of the pumping amplitude is increased in comparison to the DOPO process to the value

$$|\tilde{c}^e| = |\epsilon_0| \left[\left(\frac{2C\Delta}{1 + \Delta^2}\right)^2 + \left(1 + \frac{2C\Delta}{1 + \Delta^2}\right)^2\right]^{1/2} \quad (4)$$

An example of different intracavity intensities as a function of the coherent driving field intensity is shown in figure 1.

Due to the dependence of the bistability on the coupling between the atoms and the intracavity subharmonic field, the bistable region decreases when the atomic detuning $\Delta$ is increased. In addition, the turning point decreases as $\Delta$ increases, and when $\Delta \rightarrow \infty$ the system is identical to a normal DOPO system.

The lower branch ($X = 0$) is the one to be addressed in this study, so we will neglect pumping de

![Figure 1. Intracavity subharmonic intensity $X$ versus pumping intensity $Y$ for different detuning $\Delta$. Parameters are $\Delta = 0, 0.5, 1, 1.5, 2$.](image)

Applying these atomic operators to the system Hamiltonian, we obtain a set of new linear operator equations [5]:

$$\dot{\hat{a}} = -\gamma_1 \hat{a} + \sqrt{\gamma_2 N} \hat{g} \hat{b} + \frac{\kappa}{\gamma_2} \hat{a}^\dagger \epsilon_2 + \hat{\Gamma}_a(t), \quad (7a)$$

$$\dot{\hat{a}}^\dagger = -\gamma_1 \hat{a}^\dagger + \sqrt{\gamma_2 N} \hat{g} \hat{b}^\dagger + \frac{\kappa}{\gamma_2} \hat{a} \epsilon_2^\ast + \hat{\Gamma}_a^\dagger(t), \quad (7b)$$

$$\dot{\hat{b}} = -(\gamma_1 + i\Delta_a) \hat{b} - \sqrt{\gamma_2 N} \hat{g} \hat{a} + \hat{\Gamma}_b(t), \quad (7c)$$

$$\dot{\hat{b}}^\dagger = -(\gamma_1 - i\Delta_a) \hat{b}^\dagger - \sqrt{\gamma_2 N} \hat{g} \hat{a}^\dagger + \hat{\Gamma}_b^\dagger(t), \quad (7d)$$

with

$$\left[\hat{\Gamma}_a(t)\hat{\Gamma}_a(t')\right] = \frac{\kappa}{\gamma_2} \epsilon_2 \delta(t - t'), \quad (8a)$$

$$\left[\hat{\Gamma}_a^\dagger(t)\hat{\Gamma}_a^\dagger(t')\right] = \frac{\kappa}{\gamma_2} \epsilon_2^\ast \delta(t - t'), \quad (8b)$$
In equations (13) that determine the dynamic quadratures where
\[ θ = \frac{\kappa}{\gamma} \]
is the cavity decay rate for this field. These equations can be rewritten using two sets of quadratures describing the intracavity field and the atomic polarization,
\[ \hat{X}_+ = \hat{a} + \hat{a}^\dagger, \quad \hat{X}_- = \frac{\hat{a} - \hat{a}^\dagger}{i} \]
and
\[ \hat{Z}_+ = \hat{b} + \hat{b}^\dagger, \quad \hat{Z}_- = \frac{\hat{b} - \hat{b}^\dagger}{i} \]
which correspond to the unsqueezed and squeezed quadratures \((\hat{X}_+, \hat{X}_-)\) respectively, with relative phase angle \(θ = 0\) for the DOPO system and the corresponding atomic quadratures \((\hat{Z}_+, \hat{Z}_-)\). The new set of equations is
\[
\begin{align*}
\frac{d\hat{X}_+}{dt} &= -\lambda_- \hat{X}_+ + \sqrt{N} g \hat{Z}_+ + \hat{\Gamma}_+, \\
\frac{d\hat{X}_-}{dt} &= -\lambda_- \hat{X}_- + \sqrt{N} g \hat{Z}_- + \hat{\Gamma}_-, \\
\frac{d\hat{Z}_+}{dt} &= -\gamma_\perp \hat{Z}_+ - \sqrt{N} g \hat{X}_+ + \Delta_a \hat{Z}_-, \\
\frac{d\hat{Z}_-}{dt} &= -\gamma_\perp \hat{Z}_- - \sqrt{N} g \hat{X}_- - \Delta_a \hat{Z}_-.
\end{align*}
\]
These linear differential equations can be written in the standard form
\[ \frac{d\hat{u}}{dt} = A\hat{u} + D^{1/2} \hat{\eta}(t), \]
where
\[
A = \begin{pmatrix}
-\lambda_- & g\sqrt{N} & 0 & 0 \\
-g\sqrt{N} & -\gamma_\perp & \Delta_a & g\sqrt{N} \\
0 & 0 & -\lambda_- & -g\sqrt{N} \\
0 & -\Delta_a & -g\sqrt{N} & -\gamma_\perp
\end{pmatrix}
\]
and
\[
D = \begin{pmatrix}
\frac{2\kappa}{\gamma} \epsilon_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \frac{2\kappa}{\gamma} \epsilon_2 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]
In addition, the white noise operators \(\hat{\eta}(t)\) satisfy \(\langle \hat{\eta}(t) \rangle = 0\) and \(\langle \hat{\eta}(t) \hat{\eta}^\dagger(t') \rangle = \delta_{ij} \delta(t - t')\). The modified effective decay rates are
\[
\lambda_- \equiv \gamma_1 - \frac{\kappa}{\gamma} \epsilon_2 \quad \text{and} \quad \lambda_+ \equiv \gamma_1 + \frac{\kappa}{\gamma} \epsilon_2.
\]

3. Stability regions

In order to determine stability regions, we plot the pumping field as a function of atomic detuning for different parameters \(\mu\) and \(C\). The new parameter \(\mu \equiv \gamma_1 / \gamma_2\) represents cavity quality relative to the atomic polarization decay rate. For each particular set of parameters \((\mu, C)\), there are four eigenvalues for matrix \(A\) in equations (13) that determine the dynamic behaviours of the system. Since the steady state depends on the pumping field amplitude \(y\), the signs of the eigenvalues will depend on parameters \(y, \mu, C\) and detuning \(\Delta\). If one of the eigenvalues is positive, the system will become unstable. Even in the stable region the system will behave differently depending on whether the eigenvalues are real or complex.

Figure 2 shows the characteristics of the eigenvalues in the bad-cavity limit \((\mu = 100)\) and \(C = 1\). The upper limit is the threshold value of the pumping intensity given by equation (4). The first eigenvalue presents a negative value even for high detuning up to the turning point. If \(\Delta\) is fixed and \(y\) is increased, the second eigenvalue, in contrast, changes from a real negative value to a complex value before the turning point. Furthermore, the value of \(y\) at which the change takes place is almost constant when \(\Delta\) is varied. With further increase in \(y\), the complex region finally changes to an unstable region that extends to the end of the lower branch. The third eigenvalue exhibits a similar behaviour as the second eigenvalue, but once the detuning is included, the real negative region disappears. It is also interesting to notice that with a small detuning \((\Delta < 1.8)\), an increase in the pumping amplitude in the unstable region converts the eigenvalue to a stable value again. The last eigenvalue is unstable in the region of high pumping field amplitude and small detuning, in the same location in which the third eigenvalue is stable. Keeping \(\Delta\) fixed and increasing \(y\), the eigenvalue (except for the case \(\Delta = 0\)) changes from a stable complex value to a negative real value until the lower turning point is reached. The unstable region is determined by the second and third eigenvalues, that show an almost constant lower limit even for a high detuning. In this limit the effect of the detuning is basically to change the eigenvalues from real values to complex values or vice versa.

Figure 3 shows two eigenvalues for the good cavity limit \((\mu = 0.01)\) and \(C = 1\). In this case, there are two pairs of eigenvalues that behave identically up to the turning point. It is interesting to notice that the first two eigenvalues are complex except in a very small region near the resonant case \(\Delta = 0\); therefore, a very small detuning changes these eigenvalues from real to complex. The third and fourth eigenvalues change from complex to real values as \(y\) increases. Changes in the detuning shift the value of \(y\) at which the change takes place. In this limit, the system is stable in the entire lower branch, and changing the detuning has no effect on the stability. In both limits, special features in the field transmission spectra will occur which depend on \(Y\) and \(\Delta\).

There is a clear and more physical way to understand the instability in this system. In the good-cavity limit the atomic decay rate is larger in comparison to the cavity decay rate; therefore, photons in the subharmonic field will dissipate mainly by atomic fluorescence. For the intracavity field to start to oscillate, the atoms need to be saturated, and this
occurs only at the turning point of the lower branch. However, in the bad-cavity limit the situation is different: since the atomic decay rate is not large enough, the atoms cannot dump the photons fast enough through fluorescence. In this case, the number of photons in the cavity increases rapidly up to the point where the intracavity field starts to oscillate (becoming unstable) before reaching the turning point. A reasonable amount of detuning does not change how fast the photons are being brought out by fluorescence if the atomic decay rate is small enough. However, in the good-cavity limit, atomic detuning will have a significant impact on the atom-field interaction because the ‘effective’ number of atoms interacting with the intracavity field decreases with detuning.

4. Optical spectra

We can calculate the optical spectra of this composite system using [9]

\[ S(w) = (i w I + A)^{-1} D (-i w I + A^T)^{-1}, \]

where

\[ S(w) = \begin{pmatrix}
\langle X_+ X_+ \rangle & \langle X_+ Z_+ \rangle & \langle X_+ X_- \rangle & \langle X_+ Z_- \rangle \\
\langle Z_+ X_+ \rangle & \langle Z_+ Z_+ \rangle & \langle Z_+ X_- \rangle & \langle Z_+ Z_- \rangle \\
\langle X_- X_+ \rangle & \langle X_- Z_+ \rangle & \langle X_- X_- \rangle & \langle X_- Z_- \rangle \\
\langle Z_- X_+ \rangle & \langle Z_- Z_+ \rangle & \langle Z_- X_- \rangle & \langle Z_- Z_- \rangle
\end{pmatrix} \]

are the correlations between the intracavity field and the atomic polarization quadratures. \( D \) and \( A \) are given by equations (13). \( I \) is the unit matrix.

In the case of \( \Delta_1 = 0 \), the 4 × 4 matrix can be separated into two independent 2 × 2 matrices because the off-diagonal 2 × 2 matrices become zero. In this situation, the squeezed quadratures are independent of the unsqueezed quadratures, and even in the case near the instability point where \( \langle X_+ X_+ \rangle \) is very large, \( \langle X_- X_- \rangle \) remains small and negative.

Typical spectral shapes for the squeezed field quadrature are given in figure 4 for the parameters \( \mu = 100 \) and \( C = 1 \). For the case \( \Delta = 0 \), the spectrum is negative with a peak at \( \Omega = 0 \) due to atomic resonance absorption. An increase in the pumping intensity towards the turning point produces more squeezing at frequencies outside the atomic resonance peak.

Both quadratures will be affected once the detuning is increased, because the off-diagonal terms in \( S \) will serve as a ‘bridge’ between the squeezed quadrature and the unsqueezed quadrature. Since the ‘noisy’ (or unsqueezed) quadrature generates two peaks centred at frequencies \( \pm \Omega_0 \), parts of them will be fed into the squeezed quadrature. Figures 4(b) and (c) also show the squeezed quadrature when detuning is included. Even a small amount of detuning affects the spectra and produces two large peaks. As the pumping amplitude approaches the instability point, the amount of
DOPO coupled with \( N \) two-level atoms

Figure 3. Eigenvalues in the case \( C = 1, \mu = 0.01 \). I: stable (negative), II: stable (complex).

Fluctuation increases very quickly, and the size of the two peaks changes by several orders of magnitude.

Figure 5(a) shows \( \langle X^-X^- \rangle \) in the case of \( \Delta = 0.5 \). The spectrum is basically the same as the one without detuning, but with two large and very narrow peaks near the lower turning point at \( \Delta = \pm \Omega_0 \). Their locations are the same as those of the two peaks shown in figure 5(b), which correspond to the ‘noisy’ quadrature as a function of \( \Omega \) and \( y \) with \( C = 1 \) and \( \Delta = 0.5 \). This shows how even a small detuning has a large influence on the squeezed quadrature near threshold.

With further increase in \( \Delta \), the two peaks become higher until they reach a maximum value, and then their locations start to change. At the same time, their heights start to decrease until they eventually disappear. Figure 6 shows the behaviour of the squeezed quadrature as a function of \( \Omega \) and \( \Delta \). In this case \( y = 1 \), which means a point not as near to the instability point as in figure 5(a), so the peaks are smaller and broader. The resonance peak also increases as a function of \( \Delta \) for a while, and then it starts to decrease around \( \Delta = 30 \). In the limit \( \Delta \to \infty \), all the peaks disappear and the squeezing approaches its maximum value, as in a DOPO.

When detuning (\( \Delta \)) between the frequency of the intracavity subharmonic field and the atomic transition is introduced, the central peak in \( \langle X^-X^- \rangle \) evolves since the atomic resonance absorption changes its size. It increases until it reaches a maximum value and then it decreases until

Figure 4. Different spectra for \( X^- \) quadrature in the bad-cavity limit, with \( \mu = 100, y = 1.009 \) and \( C = 1 \). (a) \( \Delta = 0 \), (b) \( \Delta = 0.11 \), (c) \( \Delta = 0.5 \).

The system behaves like a normal DOPO system with a very large \( \Delta \). Figure 7 shows the values of the detuning that maximize the central peak for each given \( y \). It is interesting to notice that even when the system is in its unstable region, the peak reaches its maximum value at the turning point of the lower branch.
Figure 5. (a) Squeezing spectrum \langle X, X_\perp \rangle and (b) unsqueezed spectrum \langle X, X_\parallel \rangle versus \Omega and y in the bad-cavity limit (\mu = 100) with C = 1 and \Delta = 0.5.

Figure 6. The squeezing spectrum \langle X, X_\perp \rangle versus \Omega and \Delta in the bad-cavity limit (\mu = 100) with C = 1 and y = 1.

The ‘noisy’ field quadrature, in contrast, exhibits two peaks that become larger as the pumping intensity increases towards the instability point. The nature of the eigenvalues plays an important role in the structure of the spectrum. Figure 5(b) shows the unsqueezed (or ‘noisy’) field quadrature for y = 1, which corresponds to a point near the region where the real parts of two of the eigenvalues change from negative to positive and become unstable. The instability point is below the turning point given by equation (4). If y is increased from zero, the gap between the peaks becomes larger and larger. In addition, each peak increases in size and becomes narrower. Once the pumping field crosses the point y = 0.71, which corresponds to the change from real to complex of two of the eigenvalues, the amount of fluctuation starts to increase by orders of magnitude until the instability point is reached.

The ‘noisy’ quadrature \langle X, X_\parallel \rangle also exhibits some characteristics similar to the spectra for X_\perp. However, that spectral feature is more obscured due to the fact that the ‘noisy’ quadrature near the instability point is four orders of magnitude higher than the maximum value of \langle X, X_\perp \rangle.

In the good-cavity limit (\mu = 0.01), in contrast, the amount of squeezing is smaller than that of the ideal DOPO since the atomic decay rate \gamma_1 is much larger than the cavity output rate \gamma_1. In this case, the atoms do not exhibit an absorption peak in \langle X, X_\perp \rangle because the atomic absorption spectrum is much broader than the cavity linewidth. This broad atomic spectrum limits the maximum squeezing for the intracavity field. Typical shapes of the squeezing spectra are given in figure 8. Since the eigenvalues are stable up to the turning point of the lower branch, the maximum pumping amplitude is y = 3 in the case \Delta = 0. The ‘noisy’ quadrature has only one peak at the centre of the spectra, which increases with the pumping intensity.

When detuning is involved, part of the ‘noisy’ quadrature penetrates (X, X_\perp) (the off-diagonal terms in matrix A in equations (13)) through the coupling between them, and one peak at \Omega = 0 appears. A graph of the squeezed field quadrature as a function of \Delta and \Omega for y = 0.72 is shown in figure 9. The central peak increases with \Delta for a while until it reaches a maximum, and then it decreases to zero as \Delta becomes very large. Figure 7 shows that, for each y, \Delta maximizes the peak. Notice that the same graph (figure 7) applies to both the good-cavity limit and the bad-cavity limit. In the good-cavity limit the detuning has a greater effect in the field quadratures than in the bad-cavity limit, due to the fact that the atomic decay rate is predominant. A detuning of around \Delta = 10 causes the system to behave like a normal DOPO system. Figure 10 shows the squeezed (X, X_\perp) and unsqueezed (X, X_\parallel) spectra, respectively, as a function of \Omega and y for the good-cavity limit in the case \Delta = 2. A small detuning produces a large peak at \Omega = 0 in (X, X_\perp) near the threshold.

5. Conclusions

We have studied the influences of the atomic detuning on the composite system consisting of a DOPO and a set of
$N$ two-level atoms. As the atomic detuning increases, the bistability region reduces. For a large atomic detuning, the DOPO decouples from the atomic system and the bistability disappears. The stability boundaries of the system strongly depend on the detuning $\Delta_1$ and $\mu$. In the good-cavity limit ($\mu = 0.01$), the subharmonic field photons are absorbed by the atoms and dissipate through atomic fluorescence, and it is only when the atoms are saturated at the turning point that the intracavity field can start to oscillate. The effect of the detuning is to change the interaction between the intracavity subharmonic field and the atoms. Therefore, it changes the turning point, and in the limit $\Delta_1 \to \infty$ the system reduces to a simple DOPO system. In the bad-cavity limit ($\mu = 100$), the atoms cannot absorb the photons fast enough, and the cavity field starts to oscillate before the turning point, which causes the system to become unstable. The atomic detuning does not affect that process, and it only produces changes in the optical spectra by changing the structures of the eigenvalues.

For a reasonable atomic detuning, the coupling between the squeezed quadrature and the unsqueezed quadrature is weak. Therefore, for a system far from ‘threshold’ (lower $\Delta_1$).

Figure 8. Different spectra for $X_-$ quadrature in the good-cavity limit, with $\mu = 0.01$ and $C = 1$. (a) $\Delta = 0$ and $y = 2.8$. The instability point is in this case $y_{in} = 3$, (b) $\Delta = 0.1$ and $y = 2.8$, $y_{in} = 2.99$, (c) $\Delta = 1$ and $y = 2.1$, $y_{in} = 2.23$.

Figure 9. The squeezing spectrum $\langle X_-X_- \rangle$ versus $\Omega$ and $\Delta$ in the good-cavity limit ($\mu = 0.01$) with $C = 1$ and $y = 1$.

Figure 10. (a) The squeezing spectrum $\langle X_-X_- \rangle$ and (b) the unsqueezed spectrum $\langle X_+X_+ \rangle$ versus $\Omega$ and $y$ in the good-cavity limit ($\mu = 0.01$) with $C = 1$ and $\Delta = 2$.
turning point), the squeezed field quadrature will not be greatly influenced. However, near the ‘threshold’ point, the unsqueezed quadrature (or ‘noisy’ quadrature) becomes extremely large (it diverges at ‘threshold’). In such a case, even a weak coupling to the unsqueezed quadrature will have a large influence on the squeezed quadrature, which is small (the best squeezing will have a value of minus one). We have investigated the influences of the unsqueezed quadratures on the squeezed quadrature for different parameters such as $\mu$ (cavity decay over atomic decay), $y$ (pumping strength), $C$ (atomic cooperativity parameter or atomic loss over cavity loss) and atomic detuning.

The problem of coupling between a squeezed quadrature and unsqueezed quadrature also exists in other systems, such as two-level atoms inside a squeezed vacuum [10], atoms inside an optical cavity with squeezed inputs [11, 12] and squeezing in an atomic optical bistability [3]. The results from the current study could provide some understanding of this problem since the system used here is simple and physically clear. A better degree of squeezing is always accompanied by a large degree of fluctuation in the conjugate quadrature, and coupling between these two quadratures through detuning or some other mechanism could limit the potential applications of the squeezed states of light.

Since small detuning always exists in an experimental situation due to the cavity stability and frequency stability of the pumping laser, the issue of how the atomic detuning influences the optical spectra of interest is an important one. We found that, for reasonable experimental parameters, the unsqueezed quadrature is narrow compared to the squeezing bandwidth, and, therefore, can be avoided in an experiment by detecting the squeezed spectra at a different frequency away from the centre peak due to atomic absorption (in the bad-cavity limit) and side peaks due to the coupling of the ‘noisy’ quadrature.

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