Inhomogeneous broadening-dependent spectral features in a four-level atomic system

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New spectral features in an inhomogeneously broadened N-type four-level atomic system are analyzed and discussed. The atomic-level scheme uses an incoherent pumping rate in place of the incoherent recycling pump. We show that the gain profile includes an extra dip that appears only in the Doppler-broadened case. The dependence of this feature on various parameters, as well as consequences for the dispersion, are explored theoretically. For certain combinations of temperature and incoherent pump rate, this gain is shown to be independent of temperature fluctuations. © 2000 Optical Society of America [S0740-3224(00)01602-7]

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1. INTRODUCTION

Quite a lot of work has been done in the past ten years or so related to atomic coherences. Two phenomena in particular have received a substantial amount of attention. Electromagnetically induced transparency (EIT) has been observed in a number of atomic systems, including strontium and rubidium.2,3 Each of the cited papers includes the effects of Doppler broadening, which play an important role in our results. The other coherence effect that has been intensively studied recently is gain without inversion (GWI) and its companion, lasing without inversion.4–6 Experiments in lasing without inversion have been conducted in the Doppler-broadened7 and nonbroadened8 regimes, in both sodium7,8 and rubidium.9 In addition to gain considerations, the dispersion properties of a medium exhibiting GWI have also been studied.10 There are, in addition, several review papers11,12 available that cover much of the early work in this field.

Several groups have undertaken the task of constructing theoretical results that specifically model the effects of Doppler broadening on gain both in two-level systems with atomic recoil13 and in four-level GWI14–16 arrangements. We have also chosen to study the coherence effects of a four-level system; however, our theory differs from previous ones in several respects, particularly the specific level scheme and the incoherent pumping mechanism. For our system we have found the existence of novel spectral features, such as the appearance of an extra dip in the gain peak, which is caused by a coupling-field-induced saturation phenomenon that is effectively the same as spectral hole burning. The system also exhibits an independence of gain on the density of the medium for certain combinations of parameters. In this paper we theoretically investigate these phenomena by a semiclassical model using the density-matrix equations, eventually deriving expressions for the absorption coefficient and dispersion. These are then solved numerically to generate our results.

Although the gain processes discussed in this paper are not exclusively limited to inversionless systems, we have chosen to introduce them in this manner because of the contemporary relevance of steady-state GWI. When we present the results of our analysis later in the paper, our examples will primarily occur in the case of positive inversion. The reason we chose this region is because the gain is larger in this area and is therefore easier to compare visually in our plots. The same spectral features, however, also appear in the GWI region.

In Section 2 we discuss the atomic structure of our system, the effective pump model, the lasers, and the mathematical formulation of our theory. In Section 3 we present our numerical results for gain and dispersion for a variety of parameter dependencies, and Section 4 gives conclusions and a discussion of the physical nature and implications of our results.

2. PHYSICAL AND MATHEMATICAL MODELS

In this paper we analyze an N-type four-level system with inhomogeneous broadening that has some interesting spectral features that are not present, to our knowledge, in other systems. This model is also attractive owing to its experimental accessibility and its relation to certain recent publications on EIT2,3 and lasing-without-inversion4–6 systems. Inspection of Fig. 1(a) reveals that it is identical to the familiar EIT A system,4 except for the presence of the extra level |4⟩. We drive the |3⟩→|2⟩ transition (frequency ω23) with a strong coupling field of Rabi frequency Ωp = −μ23Epch and frequency ωc. Similarly, the |1⟩→|2⟩ transition (frequency ω21) is coupled by a weak probe (“weak” will be qualified in the theory section) of Rabi frequency Ωc = −μ23Ec/h and frequency ωc. Ec and Ep are the field amplitudes of the probe and coupling fields, respectively, and μ23 and μ21 are the dipole moments of the two transitions. Furthermore, we allow for detuning of the lasers from atomic resonance with the parameters Δc = −ωc + ω23 and Δp = ωp − ω21. The spontaneous decay rate from level |i⟩ to level |j⟩ is given

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by $\Gamma_{ij}$. We have added, additionally, an incoherent recycling pump source that drives the transition from $|1\rangle$ to $|4\rangle$.

The idea in GWI, of course, is to pump atoms out of level $|1\rangle$ in order to supply level $|3\rangle$ (by level $|4\rangle$) with enough atoms to generate gain, but not enough to cause a population inversion. Any population in level $|4\rangle$ will decay down to $|3\rangle$ or back to $|1\rangle$ owing to spontaneous emission, but not to $|2\rangle$, which is a forbidden transition. Atoms can, however, be pumped to level $|2\rangle$ by the coupling field, where they may participate in a gain process by the probe laser. This model system is physically realizable in atomic systems such as the rubidium D 1 lines.9

Because of the mechanics of the atomic-decay processes, this system is physically equivalent to the modified $\Lambda$ system shown in Fig. 1(b). Rather than describe the recycling field as an additional incoherent pump, we have chosen to exploit the incoherent nature of the process by modeling the recycling field as an incoherent pump from level $|1\rangle$ to $|3\rangle$. The strength of the incoherent recycling field is now represented by the incoherent-pump rate $\Gamma_{1}$. Using this model for the recycling field, one may easily modify this theory to study the ladder system by simply changing the sign of the probe detuning and assuming a counterpropagating, rather than a copropagating, configuration.

The general goal is to obtain expressions for the gain coefficient and dispersion of the atomic medium, taking into account the influence of the coupling field on the dressed states, as well as the effects of the $\Gamma_{1}$ process on the various populations. To this end, we proceed in the traditional manner, calculating the expression for the density-matrix element $\rho_{21}$, finding the complex susceptibility, and finally, breaking it down into real and imaginary parts to obtain the desired results. Later in the paper we will take into account the velocity distribution of the atomic gas to find expressions with Doppler broadening.

The density-matrix equations may be calculated from the Hamiltonian for a three-level lambda system along with an added Liouvillian relaxation term$^{17}$:

$$\dot{\rho} = -\frac{i}{\hbar} [H, \rho] + \mathcal{L}(\rho), \quad (1a)$$

where

$$H = -\hbar \Delta_p |2\rangle\langle 2| - \hbar (\Delta_p - \Delta_c) |3\rangle\langle 1| + \hbar \left( \frac{\Omega_p}{c} |2\rangle\langle 1| + \frac{\Omega_c}{2} |2\rangle\langle 3| \right) + c.c., \quad (1b)$$

$$\mathcal{L}(\rho) = (\Gamma_{21}\rho_{22} + \Gamma_{31}\rho_{33}) |1\rangle\langle 1|,$$

$$- \frac{1}{2} \Gamma_{1}(|1\rangle\langle 1| \dot{\rho} + \dot{\rho}|1\rangle\langle 1|), \quad (2a)$$

$$- \frac{1}{2} (\Gamma_{21} + \Gamma_{23})(|2\rangle\langle 2| \dot{\rho} + \dot{\rho}|2\rangle\langle 2|), \quad (2b)$$

$$+ (\Gamma_{23}\rho_{22} + \Gamma_{1}\rho_{11}) |3\rangle\langle 3|,$$

$$- \frac{1}{2} \Gamma_{31}(|3\rangle\langle 3| \dot{\rho} + \dot{\rho}|3\rangle\langle 3|). \quad (2c)$$

By tracing over atomic states, the above operator equation will yield the following equations for the populations and coherences under the rotating-wave approximation:

$$\dot{\rho}_{11} = -\frac{i}{2} \Omega_{p}^{*}\rho_{21} + \frac{i}{2} \Omega_{p}\rho_{21} + \Gamma_{21}\rho_{22} + \Gamma_{31}\rho_{33} - \Gamma_{1}\rho_{11}, \quad (2d)$$

$$\dot{\rho}_{33} = -\frac{i}{2} \Omega_{c}^{*}\rho_{23} + \frac{i}{2} \Omega_{c}\rho_{23} - \Gamma_{31}\rho_{11} + \Gamma_{1}\rho_{11}, \quad (2e)$$

$$\dot{\rho}_{21} = (i\Delta_p - \gamma_{21})\rho_{21} + \frac{i}{2} \Omega_{p}(\rho_{22} - \rho_{11}) - \frac{i}{2} \Omega_{c}\rho_{31}, \quad (2f)$$

$$\dot{\rho}_{23} = (i\Delta_c - \gamma_{23})\rho_{23} + \frac{i}{2} \Omega_{c}(\rho_{22} - \rho_{33}) - \frac{i}{2} \Omega_{p}\rho_{31}, \quad (2g)$$

$$\dot{\rho}_{31} = [i(\Delta_p - \Delta_c) - \gamma_{31}]\rho_{31} - \frac{i}{2} \Omega_{c}\rho_{21} + \frac{i}{2} \Omega_{p}\rho_{23},$$

where

$$\gamma_{21} = \frac{1}{2}(\Gamma_{21} + \Gamma_{23} + \Gamma_{1}), \quad \gamma_{23} = \frac{1}{2}(\Gamma_{21} + \Gamma_{23} + \Gamma_{31}), \quad \gamma_{31} = \frac{1}{2}(\Gamma_{1} + \Gamma_{31}).$$

We can include the influence of the laser linewidths by making the following substitutions to the detunings$^{2}$ (assuming the laser line shapes are Lorentzian):

$$\Delta_p \rightarrow (\omega_p + i\gamma_p) - \omega_{21}, \quad (3a)$$
which are phenomenologically equivalent to the substitutions

\[ \gamma_{21} \to \gamma_{21} + \gamma_p, \]

\[ \gamma_{23} \to \gamma_{23} + \gamma_c, \]

\[ \gamma_{31} \to \gamma_{31} + \gamma_p + \gamma_c. \]

The rates \(\gamma_p\) and \(\gamma_c\) are the HWHM linewidths of the probe and the coupling lasers, respectively.

The important quantity for calculating the absorption coefficient and dispersion is \(\rho_{21}\), which can be solved in the steady state by use of the density-matrix equations given above. From Eq. (2d) and Eq. (2e) we have

\[ \rho_{23}^* = \frac{i}{2} \frac{\Omega_p^* \rho_{31} - \Omega_c^* (\rho_{22} - \rho_{33})}{i \Delta_c + \gamma_{23}}, \]

\[ \rho_{31} = \frac{i}{2} \frac{\Omega_c^* \rho_{21} - \Omega_p \rho_{23}^*}{i (\Delta_p - \Delta_c) - \gamma_{32}^2}. \]

Using these equations, we can find a solution for \(\rho_{31}\) in terms of the populations,

\[ \rho_{31} = \frac{(i \Delta_c + \gamma_{23}) \Omega_p^* \Omega_c (\rho_{22} - \rho_{31}) - (i \Delta_p - \Delta_c) \Omega_c^* (\rho_{22} - \rho_{33})}{4 \left( i (\Delta_p - \Delta_c) - \gamma_{32}^2 \right) (i \Delta_p - \gamma_{21}) (i \Delta_c + \gamma_{23}) + (i \Delta_p - \Delta_c) \Omega_c^* (i \Delta_c + \gamma_{23}) (i \Omega_c^2 - (i \Delta_p - \Delta_c) \Omega_p^2).} \]

When the time comes to use this expression to find \(\rho_{21}\) and ultimately the gain and the refractive index, the mathematics will go much smoother if we can justify ignoring the \(\|\Omega_p^2\|^2\) term in \(\rho_{31}\) in favor of a first-order expression. In order to show that this term is negligible, we will require that, from the denominator of Eq. (6),

\[ \frac{\gamma_{22}^2 \gamma_{21} \gamma_{31} - 4 \Delta_p^2 + \|\Omega_c^2\|^2 + 4 \gamma_{22}^2 \Delta_p^2 (2 \gamma_{21} - \gamma_{31})^2}{(\Delta_p^2 + \gamma_{21})^2 |\Omega_p|^4} \geq 1. \]

For reasonable values of the Rabi frequencies, one might suppose from an inspection of Eq. (6) that the second-order term in \(\Omega_p\) becomes significant for certain probe detunings. Fortunately, this is not a problem. Given values of \(\Omega_p = 0.2\) MHz (which is generous) and \(\Omega_c = 5.0\) MHz, all four roots of \(R(\Delta_p) = 1\) come out complex when rubidium D1 line decay parameters are used (see Section 3), and it turns out that the smallest value that \(R(\Delta_p)\) achieves is \(\sim 2 \times 10^2\). Therefore we can neglect the probe-field Rabi frequency in the second order as long as the coupling field remains on resonance. The effects of the second-order probe term on the gain profile are also negligible. Actually, we will soon add Doppler broadening to the equations, and under these circumstances, any resonances corresponding to poles in Eq. (6) that might otherwise have appeared at specific values of the detunings will be washed out.

In order to get any further with our analysis, we need expressions for the populations. Since we assume that the probe field is weak enough that it is nonperturbative for the populations, we can solve the density-matrix equations in the steady state to zeroth order in \(\Omega_p\) to obtain

\[ \rho_{33}^{(0)} = \frac{\Gamma_1}{\Gamma_{21} + \Gamma_1}, \]

where

\[ \rho_{22}^{(0)} = \frac{\Gamma_i}{\Gamma_{21} + \Gamma_1}, \]

\[ \rho_{13}^{(0)} = \frac{W_c + \Gamma_{21} + \Gamma_1}{W_c(1 + e) + (\Gamma_{23} - \Gamma_1)e + (\Gamma_{31} + \Gamma_1) \Gamma_1 + \Gamma_1}, \]

\[ \rho_{12}^{(0)} = \frac{\Gamma_1}{\Gamma_{21} + \Gamma_1}. \]

We also assume our system is closed, that is, \(\rho_{11}^{(0)} = 1 - \rho_{22}^{(0)} - \rho_{33}^{(0)}\). These are the expressions for the populations that we will use from this point forward. We also define the population inversion of the system as \(P = \rho_{22}^{(0)} - \rho_{33}^{(0)}\). Using Eq. (6) in the steady-state solution to Eq. (2c) yields (after some algebra)

\[ \rho_{21} = \frac{\Omega_p (\rho_{22}^{(0)} - \rho_{11}^{(0)})(\Delta_p - i \gamma_{21}) - \Omega_c \rho_{23}^{(0)}}{2 \Delta_p^2 + \gamma_{21}^2} + \frac{\Omega_p}{2} \times \frac{(AD + BC)(i \Delta_p + \gamma_{21}) + (AC - BD)(\Delta_p - i \gamma_{21})}{(C^2 + D^2)(\Delta_p^2 + \gamma_{21}^2)}, \]

where

\[ A = |\Omega_c|^2 \gamma_{22}^{(0)} - \rho_{22}^{(0)} - \rho_{21}^{(0)} - \rho_{33}^{(0)}, \]

\[ B = |\Omega_c|^2 \gamma_{23}^{(0)} - \rho_{22}^{(0)} - \rho_{21}^{(0)} - \rho_{33}^{(0)}, \]

\[ C = 2 \gamma_{31}(\Delta_p \Delta_c + \gamma_{21} \gamma_{23}) \]

\[ - 4(\Delta_p - \Delta_c)(\Delta_p \gamma_{21} - \Delta_c \gamma_{23}) + \gamma_{22} |\Omega_c|^2, \]

\[ D = 2 \gamma_{31}(\Delta_p \gamma_{23} - \Delta_c \gamma_{21}) \]

\[ + 4(\Delta_p - \Delta_c)(\Delta_p \Delta_c + \gamma_{21} \gamma_{23}) - \Delta_c |\Omega_c|^2. \]

We can find the complex susceptibility in the traditional way, by using

\[ \chi = \frac{2N \mu_{23}^2}{\hbar \Omega_p \rho_{21}} \]

and separating this into imaginary and real parts to find (up to a multiplicative constant) the gain coefficient and the refractive index, respectively.

In order to include the effects of Doppler broadening, we assume the coupling and the probe beams are copropagating, and we make the substitutions

\[ \Delta_c \to \Delta_c + \frac{\omega_p V}{c}, \]

\[ \Delta_c \to \Delta_c + \frac{\omega_c V}{c}. \]
then integrate the susceptibility over the Maxwellian atomic-velocity distribution

\[ N(v)dv = \left( \frac{N_0}{u} \right) \frac{1}{\sqrt{\pi}} \exp\left( \frac{-v^2}{u^2} \right) dv, \tag{13} \]

with an rms atomic velocity of \( u \), where \( u^2 = 2kT/m \). The final result for the normalized gain (or absorption) coefficient is

\[ \alpha = \frac{\alpha_0}{\alpha_{0,\text{max}}} = \int_0^\infty \left( \frac{\gamma_{21}(\rho_{22}^{(0)} - \rho_{11}^{(0)})}{(\Delta_p + \omega_p V/c)^2 + \gamma_{21}^2} + \frac{(\Delta_p + \omega_p V/c)(A D + B C) - \gamma_{21}(A C - B D)}{[(\Delta_p + \omega_p V/c)^2 + \gamma_{21}^2](C^2 + D^2)} \right) \times \exp\left( \frac{-v^2}{u^2} \right) dv, \tag{14} \]

where the normalization constant is

\[ \alpha_0 = \int_\infty^\infty \frac{\gamma_{21}(\Gamma_i = 0)}{\gamma_{21}(\Gamma_i = 0)} \frac{1}{\gamma_{21}} \exp\left( \frac{-v^2}{u^2} \right) dv, \tag{15} \]

which is the maximum of the absorption peak for a probe beam passing through a gas of bare atoms (\( \Omega_c = \Gamma_i = 0 \)). The dispersion coefficient can be found similarly,

\[ \beta = \frac{\beta_{\text{max}}}{\beta_{\text{max}}} = \int_\infty^\infty \left( \frac{\gamma_{21}(\rho_{22}^{(0)} - \rho_{11}^{(0)})}{(\Delta_p + \omega_p V/c)^2 + \gamma_{21}^2} + \frac{(\Delta_p + \omega_p V/c)(A D + B C) - \gamma_{21}(A C - B D)}{[(\Delta_p + \omega_p V/c)^2 + \gamma_{21}^2](C^2 + D^2)} \right) \times \exp\left( \frac{-v^2}{u^2} \right) dv, \tag{16} \]

and in this case the normalization constant \( \beta_{\text{max}} \) is defined by our taking the expression for the dispersion for the case \( \Omega_c = \Gamma_i = 0 \), solving

\[ \frac{d}{d\Delta_p} \int_{-\infty}^\infty \frac{\Delta_p + \omega_p V/c}{(\Delta_p + \omega_p V/c)^2 + [\gamma_{21}(\Gamma_i = 0)]^2} \times \exp\left( \frac{-v^2}{u^2} \right) dv = 0 \tag{17} \]

numerically for the global extrema, and then finding the absolute value of the integral at either the maximum or minimum. (As long as the pump detuning is zero, the curve will be symmetric. Otherwise, there will only be one extremum, the sign of which will depend on that of \( \Delta_c \).

3. NUMERICAL CALCULATIONS

There is an interesting feature that arises in the gain as a function of probe detuning. One would expect that in the case of \( \Gamma_i > 0 \) the absorption dip that appears in the spectral peak in the case of Doppler-broadened EIT would continue to rise past zero absorption, indicating a positive gain. Of course this happens, but what is not intuitive is that another dip appears in the center of the peak (when

Table 1. Default Atomic and Laser Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
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<tr>
<td>( \Gamma_{21} )</td>
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<tr>
<td>( \Gamma_{23} )</td>
<td>5.0 MHz</td>
</tr>
<tr>
<td>( \Gamma_{31} )</td>
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<td>( \Omega_c )</td>
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<td>( \Gamma_i )</td>
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<tr>
<td>Temp.</td>
<td>300 K</td>
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<tr>
<td>( \gamma_{p, 21} )</td>
<td>0.5 MHz</td>
</tr>
</tbody>
</table>

*These are the values used in the numerical calculations, unless otherwise specified.*

Fig. 2. Gain for different values of \( \Gamma_i \). The normalized gain is plotted versus the probe detuning for \( \Gamma_i = 0.4\Gamma_{21} \) (dashed curve), with a population inversion of 0.29, and for \( \Gamma_i = 0.3\Gamma_{21} \) (solid curve), with a population inversion of 0.17. The other parameters are given in Table 1.

Fig. 3. Dispersion for different values of \( \Gamma_i \). The normalized gain is plotted versus the probe detuning for \( \Gamma_i = 0.4\Gamma_{21} \) (dashed curve), with a population inversion of 0.29, and for \( \Gamma_i = 0.3\Gamma_{21} \) (solid curve), with a population inversion of 0.17. The other parameters are given in Table 1.

the coupling field is on resonance), and we have only a small amount of gain in a place where it should be greatest—where the probe field is on resonance.

We have chosen the rubidium D1 lines for our numerical calculations. In this case, level 1 is the F = 1 hyperfine level of \( 5S_{1/2} \), level 2 is the F = 2 hyperfine level...
of 5S_{1/2}, and level |3⟩ is the F' = 2 hyperfine level of 5P_{1/2}. For all of the results in this section we use the default parameters in Table 1 unless otherwise specified.

In Fig. 2 the gain is plotted for several different values of Γ, and in Fig. 3 we show the corresponding results for the dispersion, which follows the gain slope as predicted by the Kramers–Kronig relations. Obviously, the gain is quite large, even with a 1-MHz FWHM linewidth in both lasers. The dip in the middle of the gain peak is a new feature, not found in Doppler-free media.

Actually, the potential to see this dip exists even under low-temperature conditions such as a laser-cooled gas, but the dip only appears for uncommonly large values of the incoherent pumping field, such as Γ = 10 MHz at ω_c = 10 MHz and T = 1.0 mK, at which point over 90% of the population is shelved on the upper two states. As the temperature increases, the dichotomy between the values of ω_c and Γ increases sufficiently such that the peaks can be seen at even negative inversions, making this phenomenon an important consideration in inversionless gain systems. This is why the influence of Doppler broadening is critical to GWI systems.

This influence is quantified in Fig. 4, where the gain is plotted for two values of T. The normalized gain is plotted versus the probe detuning for T = 100 K (dashed curve), with a population inversion of 0.10, and for T = 300 K (solid curve), with a population inversion of 0.23. The other parameters are given in Table 1.

In Fig. 6 we have plotted the gain versus Γ for several different temperatures: (a) T = 300 K, (b) T = 125 K, and (c) T = 50 K. The probe detuning is Δ_p = 10.2 MHz, and the other parameters are given in Table 1.

This influence is quantified in Fig. 4, where the gain is plotted for two values of T. The gain increases with the temperature. Of course, this cannot continue indefinitely, for eventually the spreading of the levels owing to the motion of the atoms takes its toll on the stimulated emission, and the increases in gain with T level off. In Fig. 5 we plot the gain curve at 300 K along with that at 125 K to demonstrate that a certain amount of broadening is necessary not only to see much of the new dip, but to obtain a reasonable amount of gain in the first place.

Another spectral feature of this system involves the direct relationship of the temperature (or equivalently, the density) and Γ. In Fig. 6 we have plotted the gain versus the recycling pump rate for several different values of the temperature. For the probe detuning we chose Δ_p = 10.2Γ_{21}, which is the location of the right peak by use of the parameters from Table 1. We display a related set

Fig. 4. Gain for different values of T. The normalized gain is plotted versus the probe detuning for T = 100 K (dashed curve), with a population inversion of 0.10, and for T = 300 K (solid curve), with a population inversion of 0.23. The other parameters are given in Table 1.

Fig. 5. Gain for different values of T, one being very small. The normalized gain is plotted versus the probe detuning for T = 20 K (dashed curve), with a population inversion of −0.08, and for T = 300 K (solid curve), with a population inversion of 0.23. The other parameters are given in Table 1.

Fig. 6. Gain versus Γ for different temperatures: (a) T = 300 K, (b) T = 125 K, and (c) T = 50 K. The probe detuning is Δ_p = 10.2 MHz, and the other parameters are given in Table 1.

Fig. 7. Closeup plot of the gain versus detuning for different temperatures: (a) T = 300 K, (b) T = 125 K, and (c) T = 50 K. Notice that at detunings of ±10.2Γ_{21}, the gain is the same, regardless of T. The value of Γ is 0.1575Γ_{21}, and the other parameters are given in Table 1.
of curves in Fig. 7, where we have plotted the gain versus probe detuning for the same three values of temperature. Notice that, for a particular value of $\Gamma_1$ (in this case, 0.12$\Gamma_{21}$), all of the curves intersect. The closeup of the region of intersection reveals that this point is very close to, but slightly below, the region of zero absorption. What all of this indicates is that there exists a set of parameters for which the absorption is ~1% of the natural value, and at this point, fluctuations in the temperature of the gas have no effect on the degree of absorption. Of course, for other detuning/recycling-rate combinations in general, the temperature has a large influence, and acts as a weighting factor that determines the degree to which $\Gamma_1$ affects the gain profile.

4. DISCUSSION AND CONCLUSIONS

To summarize, we have theoretically and numerically analyzed an incoherent-pump-rate model of atomic coherence and gain in a four-level N-type medium, specifically for the rubidium D1 lines. We have discovered that, when Doppler-broadening effects are included in the model, one can expect a new spectral feature, the appearance of an extra dip in the center of the gain peak (although we have not presented an example here, a slight coupling-field detuning will shift the peaks to one side). We are aware that some other systems exist that have spectra with a dip at resonance, but in those cases they are due to split ground-state levels and are not caused by any combination of saturation and atomic-coherence effects, as is the case here. The gain in this system is rather large, and actually increases with the temperature. Since any references to the density of the medium are normalized out of the expressions for the absorption coefficient and dispersion, the only place where the temperature shows up is in the rms atomic velocity that appears in the Maxwellian weight factor in Eq. (13). This effect is therefore influenced by the distribution of atomic velocities.

The significance of Doppler broadening is that it requires us to use much higher coupling-field Rabi frequencies than the 3 MHz or so needed for a non-Doppler-broadened medium. With a very high coupling-field intensity the population is driven so rapidly between levels $|2\rangle$ and $|3\rangle$ that the relatively weak probe field has little chance of causing a stimulated emission by the transition from $|2\rangle$ to $|1\rangle$. Because of this, a substantial reduction in the gain profile appears in the spectra region defined by the coupling field (we have set $\Delta_p = 0$, so this reduction appears at the center of the gain peak). This effect is phenomenologically similar to spectral hole burning in conventional lasers. This explains a number of issues regarding the effects of various parameters on the gain dip. Since the population increases with $\Gamma_1$, we note that the gain peaks on either side of the dip continue to increase as well, since there are more atoms available in level $|2\rangle$ to participate in stimulated emission. The width of the dip will increase slightly with an increase in the incoherent-pump rate, but the dip depth does not change significantly with $\Gamma_1$, since the saturation does not depend on small changes in the populations of the dressed state levels. On the other hand, the dip depth will depend on $\Omega_c$. As we increase the coupling-field Rabi frequency, the EIT peaks (spaced at $\Delta_p = \pm \Omega_c$ for a resonant coupling field) move out, and the entire gain "peak" expands, as well. Furthermore, the peaks on either side of the gain dip are reduced in height, since we are increasing the saturation of the medium. This has the effect of increasing the dip width as $\Omega_c$ becomes larger. Another consequence is that the bottom of the dip will eventually flatten out as the degree of saturation increases and will eventually take on an appearance similar to that of the plots in Fig. 7. These dependencies suggest that the gain could be maximized by detuning the coupling field off resonance so that one of the side peaks of the gain profile is shifted to exact resonance. A typical value for the HWHM of the dip is 5.44 MHz, which was calculated numerically for $\Omega_c = 90\Gamma_{21}$, $\Gamma_1 = 0.3\Gamma_{21}$, and $T = 300$ K. Changing the natural linewidth of the $|2\rangle$–$|1\rangle$ transition causes the gain to decrease and the peaks on either side of the dip to decrease in magnitude, but since the natural linewidth has little to do with the saturation phenomenon other than providing a guideline for the lower limit of $\Omega_c$, it has only a marginal effect on the saturation dip width.

The temperature of the rubidium vapor is relevant for two reasons. First, as mentioned above, it broadens the atomic linewidth substantially; thus a large Rabi frequency is needed in order to observe the dip in the gain peak. Second, it controls the distribution of atomic velocities. As the temperature is increased, more atoms are Doppler shifted away from exact resonance, where the coupling field saturates the atomic interaction with the probe field. When this shifting is considerably large, the hole burning is reduced and the medium exhibits more gain just off resonance. If the atoms are Doppler shifted further, of course, the atoms move out of range of the spectral region in which the two-photon transition is active, and the gain quickly decreases. If the temperature is increased to very large values (>1000 K), the increase in gain with temperature slows considerably, since the velocity distribution flattens out near resonance. This is the reason why the flat regions in Fig. 7 expand when the temperature increases.

This spectral effect has significance for those conducting experiments on gain by atomic coherences in four-level systems. If the probe laser is tuned on resonance, the output may not be indicative of the peak gain, and a slight detuning would then be necessary.

We have also found a combination of temperature and incoherent recycling field rate that predicts a very low absorption coefficient, together with an independence of atomic temperature or density. This may be of interest in an analysis of the dispersion properties of atomic systems where temperature fluctuations preclude an accurate measurement of the desired parameters.

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