Electromagnetically induced transparency and its dispersion properties in a four-level inverted-Y atomic system

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Abstract

We consider a four-level atomic system in inverted-Y configuration and study the phenomenon of electromagnetically induced transparency (EIT) in this system under various parametric conditions in order to demonstrate controllability of the EIT and its dispersion properties.

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1. Introduction

In recent past a great deal of attention has been paid to observe and understand the phenomenon of electromagnetically induced transparency (EIT) and related effects in multi-level atomic systems [1–3] interacting with two or more electromagnetic fields. The closely related phenomenon with EIT is coherent population trapping (CPT) [1]. In an usual CPT situation the two electromagnetic fields interacting with the atom are of almost equal strength and the quantum interference (QI) arises from these two fields. On the other hand, in usual EIT situation, one of the fields is much stronger than the other so that the QI is mostly governed by the stronger field. In the terminology of EIT [1–3], the weaker field is called probe field and the stronger field is called the coupling field. The EIT systems are potentially useful in applications such as electro-optical devices, sharp dispersion to control speed of light, nonlinear optics, and lasing without inversion. The induced atomic coherence in multi-level atomic systems cannot only modify the linear absorption and dispersion properties, but also enhance the nonlinear optical processes such as four-wave mixing [4,5], harmonic generation [6,7], and two-photon absorption [8] and all optical switching [9]. The QI can also modify the refractive index properties of a medium. It is due to the Kramers–Kronig relation, any change in absorption of medium will be accompanied by change in the dispersion of the medium and that leads to phenomena like electromagnetically induced focusing [10] and slow light [11]. By making use of atomic coherence, the nonlinear optical properties of an atomic medium can be modified and controlled. Recently, enhanced third-
order Kerr-nonlinear index of refraction \( (n_2, \text{ in the expression of } n = n_0 + n_2|E|^2) \) was measured near the EIT condition, as well as near the more general CPT conditions in three-level \( \Lambda \)-type rubidium atoms inside an optical cavity [12]. Also, it has been observed that the EIT can lead to suppression of nonlinear response of the system. In a ladder-type four-level atomic system the suppression of two-photon absorption has been predicted theoretically and later on observed experimentally [13]. In another experiment, which also utilized a four-level EIT atomic system of cold Rb atoms, the complete suppression of both one-photon and two-photon absorption was demonstrated [14]. Also, photon switching by quantum interference inside an optical cavity [12]. Also, it has been observed experimentally [13]. These experiments have motivated us to further explore the EIT characteristics observed experimentally [15]. These experiments have motivated us to further explore the EIT characteristics in the four-level system with an inverted-Y configuration. This inverted-Y configuration is consisted of two different EIT sub-systems so it becomes interesting to look at collective behaviors of these sub-systems as well as to control the collective EIT characteristics by various parameters.

The Letter is organized as follows. In Section 2, we present our model and its approximate solution. In Section 3, we present exact numerical results and their discussion. This is followed by concluding remarks in Section 4.

2. The model

In this work, we consider a closed four-level atomic system in inverted-Y configuration as shown in Fig. 1, which has been experimentally realized in Rb atoms. Levels \(|1\rangle, |2\rangle\) and \(|3\rangle\) are in a usual three-level ladder-type configuration and level \(|0\rangle\) together with levels \(|1\rangle\) and \(|2\rangle\) forms a three-level \( \Lambda \)-type configuration. So, this composite system consists of two sub-systems, i.e., one ladder-type and other \( \Lambda \)-type. The description of the optically allowed transitions in this system is as follows. The transition \(|1\rangle \rightarrow |2\rangle\) (transition frequency \( \omega_{12} \)) interacts with a weak probe field \( E_1 \) (frequency \( \omega_1 \)) having Rabi frequency \( 2\Omega_1 = E_1 d_{12}/\hbar \). A coupling field \( E_0 \) (frequency \( \omega_0 \)) drives the transition \(|0\rangle \rightarrow |2\rangle\) (with transition frequency \( \omega_{02} \)) with the Rabi frequency \( 2\Omega_0 = E_0 d_{02}/\hbar \) while a pumping field \( E_2 \) (frequency \( \omega_2 \)) is acting on the transition \(|2\rangle \rightarrow |3\rangle\) (with transition frequency \( \omega_{23} \)) and have a Rabi frequency equals to \( 2\Omega_2 = E_2 d_{23}/\hbar \). The radiative decay constants from levels \(|3\rangle\) to \(|2\rangle\), \(|2\rangle\) to \(|0\rangle\), and \(|2\rangle\) to \(|1\rangle\) are \( \gamma_3, \gamma_2, \) and \( \gamma_1 \), respectively. The corresponding atomic detunings for these transitions are \( \Delta_2 = \omega_2 - \omega_{23}, \Delta_0 = \omega_0 - \omega_{02}, \) and \( \Delta_1 = \omega_1 - \omega_{12} \), respectively.

If there is no coupling field then this configuration reduces to a standard ladder-type three-level EIT system driven by the probe and the pumping fields. On the other hand if we set the pumping field \( E_2 \) to zero then this configuration along with probe and coupling fields forms a standard \( \Lambda \)-type three-level EIT system. The density-matrix equations of motion in dipole and rotating-wave approximations for this system can be written as follows:

\[
\begin{align*}
\dot{\rho}_{11} & = 2\gamma_1 \rho_{22} + 2\gamma_0 \rho_{00} + i\Omega_1 (\rho_{12} - \rho_{21}), \\
\dot{\rho}_{22} & = -2(\gamma_1 + \gamma_2) \rho_{22} + 2\gamma_3 \rho_{33} - i\Omega_1 (\rho_{12} - \rho_{21}) - i\Omega_2 (\rho_{02} - \rho_{20}) + i\Omega_2 (\rho_{23} - \rho_{32}), \\
\dot{\rho}_{33} & = -2\gamma_3 \rho_{33} - i\Omega_2 (\rho_{23} - \rho_{32}), \\
\dot{\rho}_{00} & = 2\gamma_2 \rho_{22} - 2\gamma_0 \rho_{00} + i\Omega_0 (\rho_{02} - \rho_{20}), \\
\dot{\rho}_{12} & = -(\gamma_1 + \gamma_2 - i\Delta_1) \rho_{12} + i\Omega_1 \rho_{10} + i\Omega_2 \rho_{13} + i\Omega_1 (\rho_{11} - \rho_{22}), \\
\dot{\rho}_{13} & = -(\gamma_1 - i\Delta_1 + \Delta_2) \rho_{13} + i\Omega_2 \rho_{12} - i\Omega_1 \rho_{23}, \\
\dot{\rho}_{23} & = -(\gamma_1 + \gamma_2 + \gamma_3 - i\Delta_2) \rho_{23} - i\Omega_1 \rho_{13} - i\Omega_0 \rho_{03} + i\Omega_2 (\rho_{22} - \rho_{33}).
\end{align*}
\]
\[ \rho_{10} = -\left(\gamma_0 - i(\Delta_1 - \Delta_0)\right)\rho_{10} + i\Omega_0\rho_{12} - i\Omega_1\rho_{20}, \]
\[ \rho_{20} = -\left(\gamma_1 + \gamma_2 - i\Delta_0\right)\rho_{20} + i\Omega_1\rho_{10} + i\Omega_2\rho_{02} + i\Omega_0(\rho_{00} - \rho_{22}), \]
\[ \rho_{03} = -\left(\gamma_0 + \gamma_3 - i(\Delta_0 + \Delta_2)\right)\rho_{03} + i\Omega_2\rho_{02} - i\Omega_0\rho_{23}, \]
(1)
in which \( \gamma_0 \) is related to non-radiative relaxation of state \( |0\rangle \) and the trace condition \( \sum_i \rho_{ii} = 1 \). For obtaining linear susceptibility, we need to solve the density matrix equations (1) under the steady-state condition. Under the assumption that the coupling field \( E_0 \) and the pumping field \( E_2 \) are much stronger than the probe field \( E_1 \), it can be assumed that almost all the atoms are in the ground state \( |1\rangle \). Under the weak probe field approximation, the expression of the \( \rho_{12} \) in the first-order goes as
\[ \rho_{12} = \frac{i\Omega_1(\rho_{11} - \rho_{22})}{(\gamma_1 + \gamma_2 - i\Delta_1) + \frac{\Omega_1^2}{\gamma_1-i(\Delta_1+\Delta_2)} + \frac{\Omega_2^2}{\gamma_1-i(\Delta_1+\Delta_2)}}. \]
(2)

While writing down Eq. (2), we assumed that initially the values of \( \rho_{20} \) and \( \rho_{23} \) are approximately zero. Eq. (2) gives some ideas of how coupling field \( E_0 \) and the pumping field \( E_2 \) affect the absorption and dispersion properties of the probe field \( E_1 \), which give rise to the EIT conditions. When \( E_2 = 0 \), we get the EIT equation (from Eq. (2)) for a three-level system in a \( \Lambda \) configuration. On the other hand when \( E_0 = 0 \), then Eq. (2) reduces to EIT equation for a three-level system in a ladder configuration. Note that Eq. (2) is strictly valid only under the weak-probe field approximation so we will concentrate on general results using numerical solutions of Eq. (1) in the steady-state limit without invoking the weak-probe field approximation.

3. Numerical results and discussion

We now discuss the results for this composite EIT system under consideration by numerical integration of Eq. (1) under the steady-state condition. For this purpose we will examine the coherence term \( \rho_{12} \) for the probe transition in terms of its real and imaginary parts as a function of \( \Delta_1/\gamma_1 \). The imaginary (real) part of \( \rho_{12} \) versus \( \Delta_1/\gamma_1 \) represents the probe absorption (dispersion) spectrum up to all orders.

We first concentrate on the situation when the EIT is most prominent, i.e., under the condition of \( \Delta_0/\gamma_1 = \Delta_2/\gamma_1 = 0 \). In Fig. 2, the plots of \( \text{Im}(\rho_{12}) \) as a function of \( \Delta_1/\gamma_1 \) are shown with \( \gamma_1 = \gamma_2 = \gamma_3 = 1.0, \gamma_0 = 0 \) and \( \Omega_1/\gamma_1 = 0.01 \). Curve (a) is for \( \Omega_0/\gamma_1 = 0.5, \Omega_2/\gamma_1 = 0.0 \), which reminds us the ideal EIT for a three-level system in \( \Lambda \) configuration.

The QI and the dark state play the role for observing EIT in this sub-system. Curve (b) is for \( \Omega_0/\gamma_1 = 0.0, \Omega_2/\gamma_1 = 1.5 \), which depicts the EIT in a three-level system in ladder configuration. In a three-level atomic system in ladder configuration with probe field applied to the lower transition, the value of the radiative decay constant \( (\gamma_1) \) of the upper most level can lead to a significant degradation in the depth of EIT in \( \text{Im}(\rho_{12}) \) [3]. If \( \gamma_1 \) is smaller than \( \gamma \) we observe deeper EIT in comparison to the situation when \( \gamma \geq \gamma_1 \) [3]. The combined effects of these two sub-systems (the \( \Lambda \)-sub-system and the ladder-sub-system) can be seen in curve (c) where we keep \( \Omega_0/\gamma_1 = 0.5 \) and \( \Omega_2/\gamma_1 = 1.5 \), which shows a double EIT effect having features of both Fig. 2(a) and (b). In this situation, in the steady-state limit, both coherences terms \( \rho_{10} \) and \( \rho_{32} \) get developed which is reflected in the probe absorption spectrum in terms of double EIT. This system can be analyzed in terms of the dressed states created by the coupling field and the pumping field. It is the destructive interference between two excitation pathways to these dressed states from the ground state \( |1\rangle \) that gives rise to the EIT. We can see this in a more transparent manner by a simple analysis. In the resonant condition, the interaction Hamiltonian is \( H = \hbar\Omega_1|1\rangle\langle 2| + \hbar\Omega_0|0\rangle\langle 2| + \hbar\Omega_2|2\rangle\langle 3| + \text{h.c.} \), where \( |1\rangle, |0\rangle, |2\rangle, \) and \( |3\rangle \) are the discrete atomic states (Fig. 1). The eigenvalues of this Hamiltonian are \( 0, 0, \pm\sqrt{\Omega_1^2 + \Omega_0^2 + \Omega_2^2} \), respectively. The corresponding eigenstates to the zero eigenvalues are linear combination of the states \( |1\rangle, |0\rangle, \) and \( |3\rangle \). Thus, we have two dark states or the degenerated dark states in this case. If the atoms are in any or both of the dark states then EIT is exhibited. The interesting aspect of this EIT situation is its controllability by appropriately selecting the field amplitudes for the coupling field and the pumping field. This has been shown in curve (d) where we keep \( \Omega_0/\gamma_1 = 0.5, \Omega_2/\gamma_1 = 3.0, \) and the EIT dip-width and amplitude are controlled as desired. The real part of \( \rho_{12} \) or \( \text{Re}(\rho_{12}) \) is plot-
Fig. 2. \textit{Im}(\rho_{12}) as a function of $\Delta_1/\gamma_1$ for the parametric conditions: $\gamma_1 = \gamma_2 = \gamma_3 = 1.0$; $\Delta_0/\gamma_1 = \Delta_2/\gamma_1 = 0.0$; $\Omega_1/\gamma_1 = 0.01$. Curves (a), (b), (c), and (d) are for $\Omega_0/\gamma_1 = 0.5$, $\Omega_2/\gamma_1 = 0.0$, $\Omega_0/\gamma_1 = 0.0$, $\Omega_2/\gamma_1 = 1.5$; $\Omega_0/\gamma_1 = 0.5$, $\Omega_2/\gamma_1 = 1.5$; and $\Omega_0/\gamma_1 = 0.5$, $\Omega_2/\gamma_1 = 3.0$, respectively.

Fig. 3. \textit{Re}(\rho_{12}) as a function of $\Delta_1/\gamma_1$ with all other conditions same as in Fig. 2.
Fig. 4. $\text{Im}(\rho_{12})$ as a function of $\Delta_1/\gamma_1$ for the parametric conditions: $\gamma_1 = \gamma_2 = \gamma_3 = 1.0; \, \Omega_1/\gamma_1 = 0.01, \, \Omega_0/\gamma_1 = 0.5, \, \Omega_2/\gamma_1 = 1.5$. Curves (a), (b), (c), and (d) are for $\Delta_0/\gamma_1 = -1.0, \Delta_2/\gamma_1 = 0.0; \, \Delta_0/\gamma_1 = -2.5, \Delta_2/\gamma_1 = 0.0; \, \Delta_0/\gamma_1 = 0.0, \Delta_2/\gamma_1 = 2.0; \, \text{and} \, \Delta_0/\gamma_1 = -2.0, \, \Delta_2/\gamma_1 = 2.0$, respectively.

ted in Fig. 3(a), (b), (c), (d) for the same parametric conditions of Fig. 2(a), (b), (c), (d). Fig. 3(a) shows the typical EIT dispersion curve for the three-level system in $\Lambda$ configuration and Fig. 3(b), on the contrary, represents EIT dispersion for the three-level system in a ladder configuration [3]. Fig. 3(c) accounts for the combined EIT dispersion due to the presence of $\Lambda$ and ladder sub-systems in this four-level system. The most prominent effect can be seen near the $\Delta_1/\gamma_1 = 0$ region where most of the changes occur due to the double EIT condition. The controllability of this dispersion by manipulating the coupling and the pumping laser fields is clearly seen in Fig. 3(d).

After studying EIT at exact resonant conditions of the coupling and the pumping laser fields with atomic transition frequencies (e.g., $\Delta_0/\gamma_1 = 0, \, \Delta_2/\gamma_1 = 0$), we next study how these detunings bring changes in the EIT characteristics of the system. In Fig. 4 we plot $\text{Im}(\rho_{12})$ as a function of $\Delta_1/\gamma_1$ with $\gamma_1 = \gamma_2 = \gamma_3 = 1.0$ and $\Omega_1/\gamma_1 = 0.01, \, \Omega_0/\gamma_1 = 0.5, \, \Omega_2/\gamma_1 = 1.5$. Curves (a) and (b) are for non-zero coupling detuning but pumping field detuning zero, e.g., $\Delta_0/\gamma_1 = -1.0, \, \Delta_2/\gamma_1 = 0.0$ and $\Delta_0/\gamma_1 = -2.5, \, \Delta_2/\gamma_1 = 0.0$, respectively. We again obtain the double EIT but EIT-dips shifted with each other or in other words situated at different values of $\Delta_1/\gamma_1$ in Figs. 4(a) and (b). As expected the EIT due to $\Lambda$-sub-system changing its location because only the coupling field detuning is non-zero in these two curves. If the coupling field detuning is kept zero ($\Delta_0/\gamma_1 = 0, \, 0$) and the pumping field detuning is kept zero ($\Delta_0/\gamma_1 = -2.0, \, \Delta_2/\gamma_1 = 2.0$), then there is a shift in the EIT resonance of the ladder-sub-system, as shown in Fig. 4(c). Under the two-photon resonant condition, e.g., $\Delta_0/\gamma_1 = -2.0, \, \Delta_2/\gamma_1 = 2.0$, there is a merging of the two EIT peaks into a single peak and there is a loss of symmetry in the resulting single EIT peak (Fig. 4(d)). One side of this single EIT peak is very sharply changing but the other side is slowly changing. This implies that controlling detunings of the coupling laser and the pumping laser fields can lead to controlling of double EIT width, shape, and location very effectively—something difficult to achieve with one of the sub-systems alone. The real part of $\rho_{12}$ ($\text{Re} \, \rho_{12}$) is shown in Fig. 5(a), (b), (c), (d) for the identical conditions of the parameters as in Fig. 4(a), (b),
Fig. 5. Re$(\rho_{12})$ as a function of $\Delta_1/\gamma_1$ with all other conditions same as in Fig. 4.

(c), (d). The dispersion properties represented by these curves are again the combined response of the two three-level sub-systems. The locations of peaks and dips are sensitive functions of the values of $\Delta_0/\gamma_1$ and $\Delta_2/\gamma_1$, which can be clearly observed in Fig. 5(a), (b). At the two-photon resonant condition (Fig. 5(d)) there is merge of two dips which are related to the two sub-systems and there is an asymmetry in this dip also. This means that we can also control the dispersive part very easily by changing the detuning parameters.

We further study our this EIT system under some different parametric conditions. In Fig. 6, we plot Im$(\rho_{12})$ as a function of $\Delta_1/\gamma_1$ with $\gamma_1 = \gamma_2 = \gamma_3 = 1.0$. In curve (a) we keep $\Delta_0/\gamma_1 = -2.0$, $\Delta_2/\gamma_1 = 0.0$, $\Omega_0/\gamma_1 = 0.5$, $\Omega_2/\gamma_1 = 3.0$, $\Omega_1/\gamma_1 = 0.01$. Double EIT appears as expected, however, the EIT dip due to $\Lambda$-sub-system situated near $\Delta_1/\gamma_1 = -2$ is showing dispersion like features. Interestingly, the Re$(\rho_{12})$ under the same parametric conditions (as shown in Fig. 7(a)) has a peak-like structure. When we set the two-photon resonant condition for the ladder-sub-system, e.g., $\Delta_0/\gamma_1 = -3.0$, $\Delta_2/\gamma_1 = 3.0$, with $\Omega_0/\gamma_1 = 1.0$, $\Omega_2/\gamma_1 = 1.0$, $\Omega_1/\gamma_1 = 0.1$, the two EIT peaks are not only merging into one but producing a true dispersion-like profile for Im$(\rho_{12})$ in Fig. 6(b) and a perfect dip-like feature in Re$(\rho_{12})$ in Fig. 7(b) situated near the region of $\Delta_1/\gamma_1 = -3.0$. This dispersion-like EIT profile in Im$(\rho_{12})$ (or the dip-like feature in Re$(\rho_{12})$) can easily be controlled in width, amplitude, and position by suitably selecting the system parameters. For example, we can obtain the dispersion-like EIT feature exactly at $\Delta_1/\gamma_1 = 0$, which is not easy to obtain with the any of the single sub-systems. By selecting $\Delta_0/\gamma_1 = 0.0$, $\Delta_2/\gamma_1 = 2.0$, with $\Omega_0/\gamma_1 = 1.0$, $\Omega_2/\gamma_1 = 4.0$, $\Omega_1/\gamma_1 = 0.1$, we have shown how to control the location as well as width/height/shape of EIT profile for Im$(\rho_{12})$ and Re$(\rho_{12})$ in Fig. 6(c) and Fig. 7(c), respectively. On the other hand we can completely suppress the EIT due to one of the sub-system by suitably adjusting the magnitude of control field and/or the pumping field. To demonstrate this we keep $\Delta_0/\gamma_1 = 0.0$, $\Delta_2/\gamma_1 = 0.0$, with $\Omega_0/\gamma_1 = 2.5$, $\Omega_2/\gamma_1 = 6.0$ and $\Omega_1/\gamma_1 = 0.01$, and plot Im$(\rho_{12})$ [Re$(\rho_{12})$] in Fig. 6(d) [Fig. 7(d)]. Since the magnitude of the pumping field is larger compared to the coupling field, we get a considerable suppression of the EIT due to the $\Lambda$-sub-system in this situation. However, converse will be true, i.e., the EIT due to ladder-sub-system can also be suppressed if the magnitude of the coupling field is larger with respect to the pumping field.
Fig. 6. $\text{Im}(\rho_{12})$ as a function of $\Delta_1/\gamma_1$ for the parametric conditions: $\gamma_1 = \gamma_2 = \gamma_3 = 1.0$. Curves (a), (b), (c), and (d) are for $\Delta_0/\gamma_1 = -2.0, \Delta_2/\gamma_1 = 0, \Omega_1/\gamma_1 = 0.01, \Omega_0/\gamma_1 = 0.5, \Omega_2/\gamma_1 = 3.0; \Delta_0/\gamma_1 = -3.0, \Delta_2/\gamma_1 = 3.0, \Omega_1/\gamma_1 = 0.1, \Omega_0/\gamma_1 = 1.0, \Omega_2/\gamma_1 = 1.0; \Delta_0/\gamma_1 = 0.0, \Delta_2/\gamma_1 = 2.0, \Omega_1/\gamma_1 = 0.1, \Omega_0/\gamma_1 = 1.0, \Omega_2/\gamma_1 = 4.0; \Delta_0/\gamma_1 = 0.0, \Delta_2/\gamma_1 = 0.0, \Omega_1/\gamma_1 = 0.01, \Omega_0/\gamma_1 = 2.5, \Omega_2/\gamma_1 = 6.0$, respectively.

Fig. 7. $\text{Re}(\rho_{12})$ as a function of $\Delta_1/\gamma_1$ with all other conditions same as in Fig. 6.
4. Conclusions

We have studied the phenomenon of EIT and its dispersion properties in a four-level atomic system having inverted-Y configuration. Such a system can be considered to be composed of two three-level sub-systems in $\Lambda$ and ladder configurations. The EIT characteristics of both sub-systems persist in this four-level system as the coherences generated by the coupling field and the pumping field do not mutually destroy each other. By manipulating the system parameters it is possible to control the EIT characteristics of the system from a double EIT to a single EIT situation as well as from a absorption dip-like EIT feature to dispersion-like EIT features (situated at $\Delta_1/\gamma_1 = 0$) in both real and imaginary parts of the susceptibilities. Also, we can suppress the EIT due to one of the subsystems as desired by changing parameters. This kind of system is certainly useful in any optical switching devices which are based on the phenomenon of EIT because control and variation of EIT are the main features of this system. The four-level system discussed here has been easily realized in a recent experiment [14]. Also, dispersion control can find important applications in manipulating group velocity of light pulses and optical beam shaping.

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