Enhancement of the cavity ringdown effect based on electromagnetically induced transparency

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We show that the unique absorption and dispersion properties of the electromagnetically induced transparency can be used effectively to relax the conditions for observing the cavity ringdown effect (CRE), which can be useful in applications of CRE in ultrasensitive detection of chemical species. A more straightforward and simple method is used to model the interesting CRE. © 2004 Optical Society of America

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When an optical cavity is excited by a monochromatic wave and the cavity length is scanned faster than the cavity round-trip time, the cavity output field profile shows an amplitude oscillation in its normal decay curve, leading to a phenomenon known as the cavity ringdown effect (CRE).\(^1,2\) Such oscillation is found to originate from the interference between the original input laser field and the intracavity circulating field, and the oscillation frequency can be controlled by the cavity scan speed.\(^1,2\) Many interesting applications of CRE have been found, including measuring ultraslow velocities of cavity mirrors,\(^3,4\) measuring the chemical kinetics and absorption bands of molecules,\(^5-7\) and measuring the concentration of minor species in flames.\(^8,9\)

Although the CRE is useful in practical applications, it suffers from serious limitations because of the stringent requirements for high cavity finesse, long cavity length, and fast cavity scan speed. For observation of the CRE, the cavity scan rate needs to be faster than the photon round-trip time inside the optical cavity, which typically demands that the cavity finesse be at least \(10^5\) for a cavity scan rate of \(1 \mu m/s\). In this Letter we propose to use an intracavity medium with electromagnetically induced transparency\(^10-12\) to relax the strong conditions mentioned above and, therefore, enhance the CRE, which will promote potential applications that use this phenomenon. An EIT medium can have a sharp dispersion change near its EIT resonance,\(^13\) which can slow down the speed of photons inside the optical cavity.\(^14-16\) Typically an intracavity medium with sharp dispersion will be accompanied by significant absorption, which damps the ringdown oscillation.\(^1\) However, an EIT medium is ideal in this regard, since it has a large dispersion change without any absorption at exact EIT resonance.

In previous works describing the CRE, the electric field inside the resonator at any instant was obtained by summing up all the wave components that underwent multiple reflections.\(^1,2\) Here we present a more straightforward method of modeling the intracavity electric field. In what follows we use \(\alpha_p\) (\(|\alpha_p|^2\) is the average photon flow, expressed in number of photons per second) to denote the intracavity field. Consider a ring cavity consisting of four mirrors, as shown in Fig. 1. \(M_1\) and \(M_3\) are input and output mirrors with intensity reflectance \(R\). \(M_2\) and \(M_4\) are perfect reflectors. \(M_2\) is mounted on a piezoelectric transducer for cavity length scanning. The change of intracavity probe field \(\alpha_p\) during a round-trip time duration \(\tau_0\) is due to the driving field \(\alpha_p^{in}\), to the cavity decay \(\gamma_c\), and to the round-trip phase shift \(\Phi_c\):

\[
\tau_0 \frac{d\alpha_p}{dt} = \sqrt{R} \alpha_p^{in} - \gamma_c \alpha_p + \Phi_c \alpha_p.
\]

For an empty cavity the total round-trip phase shift is proportional to the geometrical length of the cavity and can be expressed as \(\Phi_c = 2\pi(d_0 + v_c \tau_c)/\lambda_p\), where \(d_0\) is the initial cavity length, \(v_c\) is the cavity scan speed, and \(\lambda_p\) is the wavelength of the input field. After substitution of \(\Phi_c\) into Eq. (1), it be solved analytically to give

\[
\alpha_p(t) = \frac{\alpha_p^{in}\sqrt{\pi R}}{2i\tau_0 a} \exp \left[\frac{\gamma_c - iat}{2i\tau_0 a}\right]
\]

\[
\times \left[\text{erf}\left(\frac{\gamma_c^{2\pi a}}{2i\tau_0 a}\right) - \text{erf}\left(\frac{\gamma_c^{2\pi a} - iat}{2i\tau_0 a}\right)\right],
\]

where \(d_0\) is ignored (the cavity is in resonance with the field initially), \(a = 2\pi v_c/\lambda_p\), and erf is the error function. Under certain conditions (which will be

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discussed below) the cavity transmission profile shows ringdown oscillation and the time difference between the first and second oscillation minima is found to be

\[ T_{12} = 0.5258(d_0 \rho_p/c\nu_{cav})^{1/2} \]

which agrees well with the previously reported values obtained by use of an interference method.\(^{12}\)

Next we inspect the condition for observing the CRE. By carefully examining Eq. (2) and its numerical result, we find that, to observe the CRE when \( \tau_0 \approx 1 \) ns, we should have \( \nu_{cav} \geq 50\gamma_{cav}^{-1} \); i.e., for a given cavity decay, there is a required minimum cavity scan speed. Practically, the cavity scan speed is limited by its mechanical structure and cannot exceed a certain level, typically of the order of several micrometers per second, which results in a condition for cavity decay to be smaller than \( 10^{-4} \), leading to a cavity finesse as high as \( 10^4 \).\(^{1-9}\)

The situation described above changes significantly when an EIT medium is placed inside the optical ring cavity. A typical three-level \( \Lambda \)-type system is shown in the oval at the top of Fig. 1. A strong coupling laser of frequency \( \omega_c \), near the \( \omega_{23} \) resonance couples levels |2⟩ and |3⟩, whereas a weak probe beam with frequency \( \omega_p \) near the \( \omega_{21} \) resonance couples levels |2⟩ and |1⟩. The coupling beam does not circulate inside the optical cavity and is assumed to have a different frequency than the probe beam, so it will not directly interact with the probe transition of the atomic or molecular species under detection. The absorption and the refractive index of such a system that are seen by the probe beam are related to the real and the imaginary parts of the atomic medium’s susceptibility, \( \chi = \chi' + i\chi'' \), which can be calculated analytically.\(^{10-12}\)

Figure 2 shows the absorption and dispersion coefficients given by

\[ \alpha = \alpha_0 \chi'/c \quad \text{and} \quad \beta = \omega_p \nu_0 \chi''/2c, \]

respectively, as a function of the probe frequency detuning, \( \Delta_p \). \( \nu_0 \) is the background index of refraction. The parameters used for this calculation are \( \nu_0 = 1 \), \( \lambda_p = 795 \) nm, \( \gamma_{31} \) (decay rate from level |3⟩ to |1⟩ = 0), and \( \Omega_c \) (coupling beam Rabi frequency) = 20 MHz. The transparency at resonance is accompanied by a large change of the dispersion coefficient. For a nonideal case (\( \gamma_{31} \neq 0 \)) there will be some residual absorption at the resonant frequency. Because of these EIT properties, the group velocity of light can be slowed down to a mere few meters per second.\(^{17}\) Such an effect can be used to increase greatly the photon round-trip time inside the optical cavity and thus to significantly narrow the cavity linewidth\(^{14-16}\) and effectively decrease the cavity decay rate near EIT resonance.

After introducing the EIT medium inside the optical cavity, the photon round-trip time becomes

\[ \tau = \tau_0(1 + \eta), \]

where \( \eta = \omega_{21}(d_0/\hbar)(\partial \chi'/\partial \omega_p)|_{\omega_p = \omega_{21}} \) represents the dispersion slope of the EIT medium of length \( l \). At the exact EIT resonant frequency, the non-linear index of refraction \( n_2 \) is zero,\(^{18}\) and thus the non-linear phase shift is also zero. The linear phase shift is just a constant and only horizontally shifts the overall cavity transmission profile. Without loss of generality, we can let the linear phase shift be zero also. Thus Eq. (1), which governs the evolution of the intracavity field, is still valid after the introduction of the EIT medium inside the optical cavity as long as \( \tau_0 \) is replaced with \( \tau \), or equivalently we can rewrite Eq. (1) as

\[ \frac{d\alpha_p}{dt} = \frac{\sqrt{\mathcal{R}} \alpha_p^{in}}{1 + \eta} - \gamma_{cav}^{EIT} \alpha_p + i \frac{2\pi \nu_{cav} l}{\lambda_p(1 + \eta)} \alpha_p. \]

The first term simply changes the overall scale of the output field and \( \gamma_{cav}^{EIT} = \gamma_{cav}/(1 + \eta) \). Thus, we can clearly see that EIT effectively reduces the cavity decay by a factor of \( (1 + \eta) \). If EIT is non-ideal, the effective decay is then determined by

\[ \gamma_{cav}^{EIT} = \gamma_{cav}/F, \]

where the cavity enhancement factor is given by

\[ F = (1 + \eta)\sqrt{k(1 - R)/(1 - Rk)} \]

and

\[ k = \exp(-\alpha l/2) \]

represents the residual absorption of the EIT medium.\(^{14-16}\) Because of the reduction of the cavity decay, the required cavity scan speed also decreases and is given by

\[ \nu_{cav}^{EIT} = \nu_{cav}(1 + \eta)/F^2 \]

Thus the required cavity scan speed for a cavity with an intracavity EIT medium can be much slower than that for an empty cavity as long as \( F^2 > 1 + \eta \). Under the ideal EIT condition, \( k = 1 \) and \( \nu_{cav}^{EIT} = \nu_{cav}/(1 + \eta) \).

Figure 3 shows cavity enhancement factor \( F \) as a function of \( \Omega_c \). Here \( \gamma_{31} \) is taken to be 0.1 MHz, which is a realistic value for rubidium atoms inside a vapor cell at room temperature. One can easily see that the cavity enhancement is enhanced quickly as \( \Omega_c \) increases initially. Beyond a certain level, as \( \Omega_c \) increases further, the cavity enhancement factor begins to decrease because of the power broadening of the EIT spike. The cavity enhancement factor also depends sensitively on the value of \( \gamma_{31} \). The inset of Fig. 3 shows the relationship between maximum achievable \( F \) and \( \gamma_{31} \).

Figure 4 shows two cavity transmission profiles with respect to the normalized time \( t/\tau \). The dashed curve is the cavity transmission of an empty cavity. The parameters used are \( \gamma_{cav} = 0.02 \) (equivalent to a cavity finesse of 150) and \( \nu_{cav} = 2 \times 10^4 \) μm/s. The CRE can barely be seen since the cavity finesse is low

![Fig. 2. Theoretical calculations of the absorption (dotted curve) and dispersion (solid curve) coefficients of the three-level EIT medium.](image-url)
Fig. 3. Cavity enhancement factor versus coupling power for $\gamma_{31} = 0.1$ MHz. The inset shows the maximum values of the cavity enhancement factor as a function of $\gamma_{31}$. The other parameters are $R = 0.98$, $l = 5$ cm, and $d_0 = 36$ cm.

Fig. 4. Two examples of cavity transmission profile. The inset shows the required minimum cavity scan speed as a function of cavity decay. In both cases, the dashed curve is for an empty cavity and the solid curve is for the cavity with an intracavity EIT medium.

and the cavity scan speed is at the lower limit of the requirement. Even at this lower limit the scan speed is still huge compared with the typical cavity scan speed achievable in practice. The CRE under such conditions will not be useful in applications. The solid curve is the cavity transmission with an intracavity EIT medium for $\gamma_{\text{cav}} = 0.02$ and $v_{\text{cav}} = 4$ $\mu$m/s (the CRE would not be seen if the cavity were empty under these conditions). The parameters used in the EIT calculation are $\gamma_{31} = 0$ and $\Omega_c = 5$ MHz, resulting in $\eta = 24,000$. Under these conditions, the cavity transmission field clearly shows the ringdown oscillation. Thus, with the help of EIT, the CRE can be observed for cavities with much lower finesse and with much lower cavity scan speeds. The inset of Fig. 4 shows a comparison of the minimum cavity scan speed required for observation of the CRE with and without EIT as a function of cavity decay. The EIT parameters are the same as in Fig. 4. One can see that the cavity scan speed can be significantly reduced (it is 4 orders smaller) with the intracavity EIT medium.

The methodology discussed here can be used to enhance the usual cavity ringdown spectroscopy for the quantitative detection of atomic and molecular species with high sensitivity. The species to be detected can itself act as an EIT system in some cases, or one can have a background EIT medium to create the condition mentioned above, and then the species of interest can be detected in the EIT window with ultrasensitivity.

In summary, we have shown that an intracavity EIT medium can greatly enhance the CRE by significantly reducing the cavity decay rate without introducing additional absorption. In such a new system, the CRE can still be observed even in a short optical cavity with a much lower finesse and slower scan speed for the cavity. Such improvements in the CRE will make applications of this interesting effect much easier in sensitive measurements and detections of chemical species. A new model has been presented to describe the CRE that gives an analytical solution and a more transparent physical picture.

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