1. INTRODUCTION
Noise can significantly enhance the synchronization of a periodic modulation signal in a bistable system. This is manifested in stochastic resonance, where the response of a nonlinear system with a periodically modulated signal and added noise could lead to an improved signal-to-noise ratio, a counterintuitive scenario [1,2]. Noise-induced switching has been experimentally studied in many dynamical nonlinear systems, e.g., parametrically driven electrons in a Penning trap [3], radio frequency driven Josephson junction [4], magneto-optically trapped atoms [5], and micromechanical and nanomechanical oscillators [6–8]. The noise enables these systems to overcome the activation barrier resulting in transition between the dynamical states. Noise can induce an undamped signal transmission in an array of one-way coupled bistable elements [9] and can form a regular output signal gradually along the chain of a monovibrator [10]. These noise-induced phenomena are equally important in many other physical, chemical, and biological systems [1,2].

The deterministic descriptions of optical bistable systems have been well established [11–13]. However, the responses of these systems to fluctuations are still intriguing, and the presence of noise can significantly influence the switching dynamics between its two dynamical states. The noise could induce switching when the system is below its deterministic threshold [14]. Interesting works of this kind on lasers [11,12,15], hybrid electro-optical systems [16], and passive all-optical double-cavity membrane systems [17] were reported in the past.

This paper reports on the studies of noise-induced switching between the two states in the three-level atomic optical bistability (AOB) system as they have advantages of controllability [13]. The intracavity medium consists of vapor of an $^87$Rb atom in a \( \Lambda \)-configuration of its levels, which is an electromagnetically induced transparency (EIT) medium with a greatly enhanced Kerr-type nonlinearity due to induced atomic coherence [18]. Any fluctuations in the interacting field’s intensity/frequency, or fluctuations in the optical cavity length, would cause changes in the induced atomic coherence in such an EIT medium and consequently in the refractive index of the medium, which leads to a change in the round-trip cavity phase and therefore induces the switching. This phenomenon is physically different from the periodic nature of instability [19,20] explained using deterministic equations.

2. THE MODEL
To understand the physical mechanism behind such optical switching we resort to a simple model, where the coupled cavity-atom system considered in an optical ring cavity is displaying dispersive AOB, so the internal medium of cavity is a Kerr-medium whose refractive index depends linearly on the cavity field intensity. The cavity response function \( S(\omega_p) = \frac{I_{\text{out}}}{I_{\text{in}}} \) of the optical ring cavity with length \( L \) with an atomic vapor cell of length \( l \) placed in one arm of the cavity is given by [21]

\[
S(\omega_p) = \frac{T^2}{(1 - R \kappa)^2 + 4R \kappa \sin^2(\theta/2)},
\]

where \( T \) and \( R \) are the transmission and the reflection coefficients [12], respectively, of cavity mirrors such that \( R = 1 - T \) and the empty cavity finesses \( Y = \sqrt{2R/(1 - R)} \). The effect of medium absorption is \( \kappa = \exp(-\alpha l/2) \) with \( \alpha \) being the single-pass absorption. The phase shift of the cavity field upon completion of a round trip through the cavity is

\[
\theta = \frac{\omega_p L}{c} + (n_0 - 1)l \frac{\omega_p}{c} + l \frac{\omega_p}{c} n_2 I_p,
\]

in which \( \omega_p \) is the frequency of the probe field, \( n_0 \) is the linear index of refraction of the medium, and \( n_2 \) is the nonlinear Kerr-index of refraction, \( I_p \) is the intracavity field intensity.
Here we are interested in dispersive optical bistable states controlled by the nonlinear refractive index of the intracavity medium. The relaxation of the intracavity round-trip phase (due to the nonlinear refractive index) is described by the following Debye equation (Refs. [12,14]) in the small-cavity limit (cavity round-trip time $t_R$ is short compared to medium relaxation time $\tau$), and in the absence of any fluctuations of the field

$$\theta_{nl} + (1/\tau)\theta_{nl} = \frac{Y_{nl}P}{(1 - R\kappa)^2 + 4R\kappa \sin^2(\theta_{nl} + \omega \xi R/2)}, \quad (3)$$

where $\theta_{nl}$ is the nonlinear phase shift experienced by the cavity field and defined as $(\omega p/c) n_{nl} P$ and $Y = 2\pi n_g LT^2/c$. When the cavity field frequency and intensity have fluctuations ($\delta \omega p$ and $\delta P$) about some mean values ($\omega p$ and $P$), Eq. (3) can be modified to

$$\theta_{nl} + (1/\tau)\theta_{nl} = \frac{Y(\omega p + \delta \omega p)(P + \delta P)}{(1 - R)^2 + 4R\kappa \sin^2(\theta_{nl} + \omega p + \delta \omega p)t_R/2)}, \quad (4)$$

We can write $(\omega p + \delta \omega p)t_R = -D + \omega p + \delta \omega p/t_R$, in which $D = (\omega_{nl} - \omega p)t_R$ and $\omega_{nl}$ is the cavity resonance frequency. Furthermore, we have $\omega_{nl}t_R = 2\pi n_i (n_i$ is an integer) because of cavity conditions. Thus we can rewrite Eq. (4) as

$$\theta_{nl} + (1/\tau)\theta_{nl} = \frac{Q(1 + \delta \omega p/\omega p)}{(1 - R)^2 + 4R\kappa \sin^2(\theta_{nl} + \omega p + \delta \omega p / \omega p - 1/2)t_R/2)}, \quad (5)$$

in which $\tau$ is absorbed in redefining the time variable, $Q = \omega p/\omega_{nl}$, $\omega_{nl} = c/(2\pi n_g LT^2)$ such that $P$ is the intracavity field intensity (the mean value); $\eta(t) = \delta \omega p/\omega p$ and $\delta \omega p/\omega p$ are stochastic variables of zero mean describing intensity and frequency fluctuations, respectively. Next, we define a new phase variable $\phi = \theta_{nl} - \theta_0 + \delta \omega p/t_R$, in which $\theta_0$ is related to a steady-state solution of Eq. (5) in the absence of any fluctuations obtained from

$$\theta_0 = K(Q, \theta_0). \quad (6)$$

The function $K(Q, \theta_0)$ is defined by the right hand side of Eq. (5) in the absence of any fluctuations as

$$K(Q, \theta_0) = \frac{Q}{(1 - R\kappa)^2 + 4R\kappa \sin^2(\theta_{nl} - D)/2)} \quad (7)$$

The dynamical equation for the new phase variable including fluctuations [14] is given by

$$\frac{d\phi}{dt} = M_\phi(\phi) + M_1, \quad (8)$$

in which $M_\phi(\phi) = -\phi/W_0 + \phi^2/2K^2(2)$, $M_1 = (\theta_0 + \omega p t_R)(\xi + \dot{\xi}) + \theta_0\eta$, $W_0 = 1/(1 - K^2(2))$. $K^2(2)$ is a second derivative of $K$ with respect to $\theta_{nl}$ and $\xi = \delta \omega p/\omega p$ describes the frequency fluctuations. Note that in the expression of $M_1$, the terms depending on frequency fluctuation are multiplied by $\theta_0 + \omega p t_R$, but the term depending on amplitude fluctuation is multiplied by $\theta_0$ only. At optical frequencies and a ring cavity of length 37 cm, $\omega p t_R \gg \theta_0$ (about a factor of $\sim 10^5$). Hence the frequency fluctuation dominates over the amplitude fluctuation. The correlation times of fluctuations are much smaller than the deterministic relaxation time for the nonlinear phase shift. The Fokker–Planck equation associated with Langevin Eq. (8) is [11]

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi} \left[ \frac{\partial U}{\partial \phi} P \right] + D \frac{\partial^2 P}{\partial \phi^2}, \quad (9)$$

where $D$ is the probability distribution of the process $[11] U(\phi) = \phi^2/2W_0 - \phi^3/6K^2(2)$. So, the situation is diffusion of a particle trapped in a potential well $U(\phi)$ subjected to a stochastic process characterized by diffusion constant $D$. The minimum (maximum) of $U(\phi)$ is at $\phi = 0$ ($\phi = 2(1/K^2(2))$. To make correspondence with the original problem we consider the solution of deterministic steady-state Eq. (6) in the vicinity of the upswitching point. The typical hysteresis curve ABCD for dispersive AOB obtained between $\theta$ and $Q$ by solving Eq. (6) is as shown in Fig. 1. The hysteresis curve has stable branches AB and CD and an unstable branch BC. The conditions involved in upswitching can be obtained by solving $\theta = K(Q, \theta)$, $d\theta/dQ = \infty$, where $K(Q, \theta)$ is defined (we have dropped the subscript on $\theta$ for brevity) in Eq. (7) providing the upper threshold or switching value $Q_-.\theta$ marked as point B on the hysteresis curve in Fig. 1. Similarly, we can have conditions for downswitching or lower threshold, which is marked as point C on the hysteresis curve in Fig. 1, and together with the upswitching value it gives the width of the hysteresis cycle of the AOB (i.e., the separation between points C and B on $Q$-axis of Fig. 1). One can determine the solution of Eq. (6) around the switching point by the Taylor series expansion of $K(Q, \theta)$ to obtain $\theta_+ = \theta_0 \pm \sqrt{(2\theta_0 \Delta Q/K^2(2))}$ (in which negative (positive) sign is for $\theta_+ (\theta_-)$) and $\Delta Q = (Q_s - Q)$ represents (Fig. 1) a deviation of $Q$ value from the upswitching value, i.e., upper threshold for AOB given by $Q_s$. Clearly, the distance between maximum and minimum of potential is $\theta_+ - \theta_0 = \sqrt{(2\theta_0 \Delta Q/K^2(2))}$, which is the separation of the unstable and stable branches of the hysteresis cycle of the AOB (see Fig. 1). The particle starting from minimum arrives at maximum because of a noise spike, where it can either fall back to original position or escape. In the optical bistable system (Fig. 1), the state is initially at one stable point (phase $\theta_0$) and then migrates to the unstable

![Fig. 1. Plot of $\theta$ (dimensionless) as a function of $Q$ (arbitrary units) using Eq. (6) for $\Delta R = 4.5$ $(N = 1 - R_0)/(2 - R_0)$ showing a hysteresis cycle ABCD having stable branches AB and CD and an unstable branch BC; $\theta_0$ and $\Delta Q$ are defined in the text.](image-url)
point (phase $\theta_1$) situated on the unstable branch of the hysteresis curve due to noise surge. At this point the system will either return to $\theta_0$ or switch to the next stable branch [14] (which is the upper branch) and thus completes the noise-induced upswitching. In this study, the average escape time for a particle initially at $\theta_0$ is given by

$$T^* = \frac{c^{(1)}}{\Delta Q} \exp\left[\frac{(\Delta Q)^{3/2}}{3}c^{(2)}Q\right],$$

where $c^{(1)}$ and $c^{(2)}$ are positive constants related to the experimental parameters. For small $\Delta Q$ there is a reasonably large switching rate caused by noise spikes. With increasing $\Delta Q$, which can be controlled by moving the operating point on the hysteresis curve width, the average time for noise-induced switching increases very rapidly. Thus the particle escape time serves as an estimate of the average time between noise-induced switching. The above model provides a very good physical understanding of the experimental results. The experimental result as we will see in the following are fairly matching with the theory within the limitation of the experimental control of parameters.

### 3. EXPERIMENT AND ANALYSIS

The experimental setup is outlined in Fig. 2. The energy levels of the $D_1$ line of $^{87}$Rb atom are employed to form the required three-level system in a $\Lambda$-type configuration [13]. The optical ring cavity consists of three mirrors, two of which ($M_1$ and $M_3$) have about 1% and 3% transmissivities, respectively, while the third one ($M_3$) is almost a perfect reflector and is mounted onto a piezoelectric transducer (PZT) in order to tune the cavity length. The rubidium atomic vapor is contained in a 5 cm glass cell with Brewster windows. Magnetic shielding of the cell is provided by a $\mu$-metal sheet wrapped around it, and the temperature of the cell is controlled by a heating-tape outside the $\mu$-metal. The two Hitachi HL7851G tunable diode lasers are extended-cavity diode lasers that are temperature and current stabilized. Frequencies of both the probe and the coupling beams are controlled precisely by locking to Fabry–Perot cavities FP1 and FP2. A feedback loop is used for each of these lasers for further stabilization [22]. The probe laser beam (driving the transition $5S_{1/2}(F=1) \rightarrow 5P_{1/2}(F'=2)$) circulates inside the ring cavity as the cavity field while the coupling beam (driving the transition $5S_{1/2}(F=2) \rightarrow 5P_{1/2}(F'=2)$) is misaligned from the cavity axis slightly (about a 2° angle) so it does not

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**Fig. 2.** Experimental setup. LD1 and LD2, diode lasers; EOM, electro-optic modulator; PBS1–PBS5, polarizing cubic beam splitters; $M_1$–$M_3$, cavity mirrors; APD, avalanche photodiode detectors; PZT, piezoelectric transducer; MM, mode matching lens; PH, pinhole; BS, beam splitter; $/\lambda/2$, half-wave plates; FR, Faraday rotator; FP1 and FP2, Fabry–Perot cavities; D1 and 2, photodiode detectors; $M_4$ $M_5$, extended cavity mirrors of LD1–LD2. The lock-in loops attached with LD1 and LD2 are for stabilization of diode lasers and cavity lock-in loop is for stabilization of optical ring cavity. The direction of arrows after each lock-in loop indicates the correcting signal provided to PZT of extended cavity mirrors (for LD1–LD2) or PZT of $M_3$ (for optical cavity). The bubble shows a three-level system in the $\Lambda$-configuration.
circulate in the cavity. The beam waist of the coupling (probe) field at the center of the vapor cell is 700 (80) μm. By propagating the coupling and the probe (cavity) laser beams collinearly, the first-order Doppler effect is eliminated [21]. A third diode laser has been used as a reference beam to lock the frequency of the optical cavity to a Fabry–Perot cavity (similar kind of setup as mentioned above for diode lasers) with the stabilization provided by a cavity lock-in loop as depicted in Fig. 2.

In the experiment, intensities and frequency detunings of the coupling/probe beams, Rb cell temperature, and the cavity frequency detuning \( \Delta_\phi \) can be used to control AOB. The electro-optic modulator (EOM) scanned the intensity of the cavity input field back and forth (like an intensity modulator). To see the typical bistable curve, the EOM is switched on with triangular shaped driving pulses. After establishing the proper refractive optical bistability, the EOM scanning is then switched off. EOM is also used to set the input intensity level of the probe laser entering the optical cavity. The cavity input power is then set in such a way that it lies almost in the middle of the observed AOB hysteresis loop. The typical linewidth of the diode laser is several megahertz. Though the lasers used here are an extended cavity type, they do possess residual phase (frequency) and intensity noise responsible for fluctuations in the nonlinear refractive index causing the switching phenomenon. As discussed above [after Eq. (8)] the frequency fluctuation dominates over the amplitude fluctuation. Also, our measurements when laser is far-off resonant from the atomic absorption line suggest that the magnitude of intensity fluctuation is about 10–100 times smaller than the frequency fluctuation responsible for such switching. Any frequency jitter in the ring cavity is essentially associated with the linewidth of the third laser, which is of the same order of magnitude as that of the probe laser (i.e., several megahertz). Hence such jitter is taken into account along with the frequency fluctuation of the probe laser circulating in the cavity. The cavity output is monitored by an avalanche photodiode detector and stored digitally in the oscilloscope for observing switching phenomenon due to fluctuating refractive index of the medium. To obtain an averaged response of the optical cavity output, the above process has been repeated say about 50 times for a fixed time interval. A typical switching phenomenon of the output cavity field observed experimentally is shown in Fig. 3(a) for the experimental conditions of cavity temperature \( T = 68°C \), coupling laser power \( P_C = 12 \text{ mW} \), probe laser frequency detuning \( \Delta_P = 50 \text{ MHz} \), and cavity detuning \( \Delta_\phi = 50 \text{ MHz} \). In Fig. 3, curves (a), (b), and (c), are for three different AOB hysteresis curves with decreased widths by selecting coupling laser frequency detuning \( \Delta_C = 55, 35, \text{ and } 25 \text{ MHz} \), respectively, but keeping other parameters unchanged. From these curves one can see that if the operating point of the input intensity is moved on the hysteresis curve toward the upper threshold of the AOB curves (this is done by reducing the width of AOB) the occurrence of switching to the upper metastable state in the same time interval increases provided it is still far from the upper threshold so that critical slowing down does not dominate the dynamical process. At each impulse from the noise peak there is a sharp spike up to the upper state, which immediately decays down to the lower state owing to asymmetric potential \( U(\phi) \) [after Eq. (9)]. The intracavity EIT medium causes cavity linewidth narrowing [23] thus a small change in refractive index will easily cause the cavity mode to shift by a linewidth leading to switching from no cavity transmission (lower branch) to cavity transmission (upper branch).

![Fig. 3](image1.png)  
**Fig. 3.** Typical noise-induced switching observed in cavity output intensity as a function of time in the AOB system with experimental parameters as mentioned in the text. Curves (a), (b), and (c) are for three different coupling laser detunings, respectively, with all other parameters fixed.

![Fig. 4](image2.png)  
**Fig. 4.** Occurrence of switching events in cavity output intensity as a function of width of the AOB hysteresis curve. The experimental parameters are \( T = 68°C, P_C = 12 \text{ mW}, \Delta_P = 50 \text{ MHz} \), and cavity detuning \( \Delta_\phi = 50 \text{ MHz} \). AOB widths are controlled by using \( \Delta_C \) in the range from 25 to 65 MHz.
However the frequency stabilization factor \[\proportional \frac{\partial^2}{\partial \omega^2} \ln(n) + \frac{1}{2} \Delta^2 \]
pulls the system back to its original state, leading to the observed behaviors as shown in Fig. 3.

In Fig. 4, measured occurrences of the cavity output intensity switching to the upper branch of the AOB hysteresis curve as a function of AOB widths are displayed. The experimental parameters are as shown in the caption. The operating point of the input power level is kept at the middle of each AOB curve. Clearly, with an increase of the AOB width, the occurrence of the switching phenomenon diminishes. This is because the noise spikes are not strong enough compared to the increase in half width of the AOB curve to bring an adequate change in the refractive index required for switching. Hence average time required for switching increases, which is in good agreement with Eq. (10).

To quantitatively analyze such switching phenomenon further we select three different AOB hysteresis curves of different widths in Fig. 5. The experimental parameters are shown in the caption. We sit at different operating points on each of the hysteresis curves (shown in the insets of Fig. 5) and observe the occurrences of switching phenomenon and plot them as a function of relative position of operating point within the AOB curve. When the width of hysteresis curve is small [Fig. 5(a)] the occurrence of switching events at low operating points (far from threshold) is large in comparison to wider width hysteresis curves in Figs. 5(b) and 5(c). The noise spikes are strong enough to bring a required phase change for switching the cavity field to upper state for a narrower width AOB curve. As the operating points get near to the upper threshold point the switching occurrences increase in all the curves and show similar kinds of trends.

4. CONCLUSION

In summary, we studied the phenomenon of noise-induced switching in the passive optical bistable system consisting of three-level atoms inside an optical cavity. This system was driven by a cavity field with intrinsic fluctuations in its frequency and intensity, causing atomic coherence and thus enhanced nonlinearity to fluctuate, which led to the phenomenon of random switching between bistable states. This system could be potentially useful in generating pseudorandom numbers (in which statistics could show correlations or random behavior [24, 25]), a crucial requirement in quantum communication and cryptology. The results discussed here can have broad relevance to other fields of science.

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