Effects related to electromagnetically induced transparency (EIT), have become very beneficial tools to investigate multilevel mixing (MWM) processes since the weak generated signals can be allowed to transmit through the resonant atomic medium with little absorption. Enhanced four-wave mixing (FWM) processes due to atomic coherence have been experimentally demonstrated in several multilevel atomic systems. Interesting phenomena, such as entangled images in the probe and signal beams in the FWM process, phase-controlled light switching at low light level, quantum destructive interference in inelastic two-wave mixing, and generation of correlated photon pairs, have all been experimentally investigated in various coherently prepared multilevel atomic systems. Recently, destructive and constructive interferences in a two-level atomic system and competition via atomic coherence in a four-level atomic system with two coexisting FWM processes were studied. In the presence of additional coupling laser fields, more FWM processes can be generated to coexist, which can be selectively suppressed or enhanced via quantum interference.

The experiment is carried out in sodium atoms (in a heat pipe oven) involving three energy levels [Fig. 1(a)]. The pulse laser beams are aligned spatially, as shown in Fig. 1(b). For the transition between $|0\rangle$ (3$S_{1/2}$) to $|1\rangle$ (3$P_{1/2}$), the coupling laser beam $E_1$ ($\omega_1, k_1$, and Rabi frequency $G_1$) together with $E'_1$ ($\omega_1', k'_1, G'_1$) having a small angle ($\sim 0.3^\circ$) propagates in the opposite direction of the weak probe field $E_3$ ($\omega_3, k_3, G_3$). These three laser beams are from the same near-transform-limited dye laser (10 Hz repetition rate, 5 ns pulse width and 0.04 cm$^{-1}$ linewidth) with the same frequency detuning $\Delta_3 = \omega_{10} - \omega_3$, where $\omega_{10}$ is the transition frequency between $|0\rangle$ and $|1\rangle$. The frequency components of $E_1$ and $E'_1$ ($\omega_1$) in beam 1 and beam 2 induce a population grating between states $|0\rangle$ to $|1\rangle$, which is probed by beam 3 ($E_2$) with the same frequency $\omega_1$. This interaction generates a degenerate FWM (DFWM) signal $E_{b1}$ [Fig. 1(a)] satisfying the phase-matching condition of $k_1 = k_3 + k_1' - k'_1$. Then, two additional coupling fields $E_2$ ($\omega_2, k_2, G_2$) and $E'_2$ ($\omega_2', k'_2, G'_2$) are applied as scanning fields connecting the transition between $|1\rangle$ and a third level $|2\rangle$ (4$D_{3/2} \rightarrow 2$) with the same frequency detuning $\Delta_2 = \omega_{21} - \omega_2$. The laser field $E_2$ is added onto beam 1 and $E'_2$ propagates in another plane ($\pi_z$) perpendicular to the $yz$ plane with a small angle from $E_1$, as shown in the inset of Fig. 1(b). $E_2$ and $E'_2$ are from another similar dye laser with its frequency set at $\omega_2$ to dress the energy level $|0\rangle$ [Fig. 1(c)]. The interaction between $E_2$, $E'_2$, and $E_1$ generates a non-DFWM (NDFWM) signal $E_{b2}$ satisfying $k_{b2} = k_1 + k_2 - k'_1$ (for the subsystem $|0\rangle \rightarrow |1\rangle \rightarrow |2\rangle$).

When $E_1$, $E_2$, $E'_2$, and $E_3$ are all turned on simultaneously, the DFWM process $E_{b1}$ and NDFWM process $E_{b2}$ are generated simultaneously and there exist interplays between these two FWM signals. In addition, both of these two generated FWM signals are in the same EIT window formed by the ladder system ($|0\rangle \rightarrow |1\rangle \rightarrow |2\rangle$) in the two-photon Doppler-free configuration. These generated FWM signals have the same frequency $\omega_1(=\omega_2)$ and propagate in two directions (i.e., strictly counter-propagating $E'_1$ and approximately counter-propagating $E'_2$, respectively) detected onto beam 1 and beam 2 induce a population grating between states $|0\rangle$ to $|1\rangle$, which is probed by beam 3 ($E_2$) with the same frequency $\omega_1$. This interaction generates a degenerate FWM (DFWM) signal $E_{b1}$ [Fig. 1(a)] satisfying the phase-matching condition of $k_1 = k_3 + k_1' - k'_1$. Then, two additional coupling fields $E_2$ ($\omega_2, k_2, G_2$) and $E'_2$ ($\omega_2', k'_2, G'_2$) are applied as scanning fields connecting the transition between $|1\rangle$ and a third level $|2\rangle$ (4$D_{3/2} \rightarrow 2$) with the same frequency detuning $\Delta_2 = \omega_{21} - \omega_2$. The laser field $E_2$ is added onto beam 1 and $E'_2$ propagates in another plane ($\pi_z$) perpendicular to the $yz$ plane with a small angle from $E_1$, as shown in the inset of Fig. 1(b). $E_2$ and $E'_2$ are from another similar dye laser with its frequency set at $\omega_2$ to dress the energy level $|0\rangle$ [Fig. 1(c)]. The interaction between $E_2$, $E'_2$, and $E_1$ generates a non-DFWM (NDFWM) signal $E_{b2}$ satisfying $k_{b2} = k_1 + k_2 - k'_1$ (for the subsystem $|0\rangle \rightarrow |1\rangle \rightarrow |2\rangle$).

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Fig. 1. (a) The diagram of relevant Na energy levels. (b) The scheme of the experiment. Inset gives the spatial alignments of the incident beams. (c) and (d) The dressed-state pictures of the suppression and enhancement of FWM $E_{b1}$ for the two-level system, respectively. (e) The dressed-state picture of the suppression of FWM $E_{b2}$ for the ladder system.
by a photomultiplier tube and a fast gated integrator (gate width of 200 ns), respectively.

In order to better understand the experimental results, we calculate the two interacting FWM processes. First, we consider the DFWM process $E_{f1}$ dressed (or perturbed) by the coupling laser beams $E_2$ and $E'_2$ [Fig. 1(d)]. There are two transition paths for generating FWM, which are described by the dressed perturbation chains

\[
\begin{align*}
(a) & \quad (E_f^1)_{(E_f^2)} \to (E_f^3), \\
(b) & \quad (E_f^1)_{(E_f^2)} \to (E_f^3), \\
(c) & \quad (E_f^1)_{(E_f^2)} \to (E_f^3), \\
(d) & \quad (E_f^1)_{(E_f^2)} \to (E_f^3).
\end{align*}
\]

The solved expressions of the corresponding FWM processes are: 

\[
\rho_3^{(3)} = \frac{G_3}{(\Gamma_0 + C_1^2)}, \quad \rho_3^{(3)} = \frac{-G_3}{(\Gamma_0 + C_1^2)}, \quad \rho_c^{(3)} = \frac{-G_3}{(C_1 C_2)}, \quad \rho_d^{(3)} = -G_3/(C_1^2 C_2),
\]

$G_3 = G_2 G_3 C_1$, $\Gamma_0 = \Gamma_0 + 2 \Delta_1 + \Delta_2$, $\Delta_1 = |G_3|^2/d_2$, $A_1 = |G_3|^2/d_2$, $A_2 = |G_3|^2/d_2$, $d_1 = i(\Delta_1 + \Gamma_0)$, and $d_2 = i(\Delta_2 + \Gamma_0)$. $G_i = -\mu E_i/\hbar (i=1,2,3)$ is the Rabi frequency; $\Gamma_{10}, \Gamma_{20}$, and $\Gamma_{00}$ are the transverse relaxation rates. The total contribution is

\[
\rho_{DFWM}^{(3)} = \rho_3^{(3)} + \rho_3^{(3)} + \rho_c^{(3)} + \rho_d^{(3)}.
\]

Similarly, for the ladder-type three-level system [Fig. 1(e)], the dressed perturbation chain is

\[
\begin{align*}
E_3 & \quad (E_f^1) \to (E_f^3), \\
E_3 & \quad (E_f^1) \to (E_f^3), \\
E_3 & \quad (E_f^1) \to (E_f^3), \\
E_3 & \quad (E_f^1) \to (E_f^3).
\end{align*}
\]

The third-order nonlinear process of the NDFWM $E_{f2}$ can be described by

\[
\rho_{NDFWM}^{(3)} = -G_2 G_3 [d_2 (d_1 + B_3)^2],
\]

where $B_3 = |G_3|^2/\Gamma_{11}$. From Eqs. (1) and (2), one can see that the two FWM processes are closely connected by dressed effects. By adjusting the frequency detuning $\Delta_1$ and scanning the dressed field detuning $\Delta_2$, many interesting phenomena can be obtained.

In the ladder-type three-level system, the coupling fields $E_1$ and $E'_1$ (with diameter of 0.8 mm and power of 3 $\mu W$) and the probe field $E_3$ (with diameter of 0.8 mm and power of 5 $\mu W$) are tuned to the line center (589.0 nm) of the $|0\rangle$ to $|1\rangle$ transition, which generate the DFWM signal $E_{f1}$ at frequency $\omega_1$. The coupling fields $E_2$ and $E'_2$ (with diameter of 1.1 mm and power of 100 $\mu W$) are scanned simultaneously crossing the $|1\rangle$ to $|2\rangle$ transition to dress the DFWM process $E_{f1}$ and generate a NDFWM signal $E_{f2}$.

We first set $\Delta_1$ at one point and scan $\Delta_2$. Evolution from suppression to enhancement is observed as shown in Fig. 2. The probe field is changed from high to low frequency side. As frequency detuning goes from $\Delta_1 < 0$ to zero, the DFWM signal $E_{f1}$ is enhanced gradually to the maximum value [right side of Fig. 2(a)], which is an enhanced process. Then, it undergoes a partial enhancement/suppression [Fig. 2(b)], until the FWM signal is purely suppressed at the resonant point [Fig. 2(c) and the upper curves of Fig. 4(a)]. When $\Delta_1$ changes to be positive, it shows a symmetric process [i.e., a partial suppression/enhancement in Fig. 2(c)], and a pure enhanced process in the left side of Fig. 2(a)].

One way to explain these observed effects is by using the dressed-state picture. Let us first consider the case of large $G_2$ (e.g., 15.7 GHz). The dressing field couples the transition $|2\rangle$ to $|1\rangle$ and creates the dressed states $|G_2\pm\rangle$ [Fig. 1(d)]. Therefore, the DFWM signal $E_{f1}$ for a large one-photon detuning is extremely small when $G_2=0$. The strong dressing field can cause resonant excitation for one of the dressed states if the condition $\omega_1 + \omega_2 = \omega_{10} + (\omega_{21} \pm \Delta_{G2})$ (i.e., $\Delta_1 + \Delta_2 = \Delta_{G2} = 0$) is satisfied, where $\Delta_{G2}$ is the splitting level relative to the original position of the state $|1\rangle$ by the dressing field $E_2$ or $E'_2$, so that $\Delta_2 = (G_2^2 - \Delta_1^2)/\Delta_1$. For example, the DFWM signal $E_{f1}$ is strongly enhanced in the

![Fig. 2. (Color online) The evolution of the dressed effects for different $\Delta_1$ values: (a) $\Delta_1 = -101$ GHz (squares), -84.3 GHz (circles), -67 GHz (triangles), 71.3 GHz (reverse triangles), 88.6 GHz (pentagons), and 105 GHz (hexagons) from right to left. Inset: theoretical plots using experimental parameters. (b) $\Delta_1 = -30.3$ GHz (squares), -2.6 GHz (triangles) and -13 GHz (circles). (c) $\Delta_1 = 29.3$ GHz (squares), 38 GHz (triangles) and 42.2 GHz (circles).](http://apl.aip.org/aps/abstract/ApplPhysLett/v95/i6/p041103_d/fig2.png)

![Fig. 3. (Color online) (a) Measured enhanced DFWM signal spectra for different coupling field ($E_{f2}$) intensities, 17.4 $\mu W$ (squares), 34.2 $\mu W$ (circles) and 64.3 $\mu W$ (triangles). Inset: the experimental data and theoretical curve of the power dependence of the enhanced FWM signal, $\Delta_1 = -55.8$ GHz. (b) Measured suppressed DFWM signal spectra for different coupling field ($E_{f2}$) intensities, 81.8 $\mu W$ (squares), 72.7 $\mu W$ (circles) and 54.6 $\mu W$ (triangles). Inset: the experimental data and theoretical curve of the power dependence of the suppressed FWM signal, $\Delta_1 = 0$ GHz.](http://apl.aip.org/aps/abstract/ApplPhysLett/v95/i6/p041103_d/fig3.png)
presence of dressing field when $\Delta_1 = -67$ GHz [Fig. 2(a)], which is mainly due to the one-photon $(|\psi\rangle \rightarrow |G_{2}\rangle)$ resonance. Thus, initially, $\Delta_1$ for $E_{f1}$ is very large, so the dressed effect only gives the enhancement. As $\Delta_1$ goes toward zero, the suppression effect gets into play gradually due to the dressed states $|G_{2}\rangle$ [Fig. 1(c)]. In this case, when the frequency changes from high to low values it results in a suppression for the DFWM signal first and then an enhancement. At the $\Delta_1 = 0$ point (resonant case), only suppression effect exists [Fig. 1(c)]. For the $\Delta_1 > 0$ part, it has a symmetric evolution between the enhancement and suppression effects, as shown in Fig. 2(c).

Next, we choose an appropriate $\Delta_1$ value at the position of either enhanced or suppressed DFWM process $E_{f1}$ (i.e., the peak or dip in Fig. 3). The dressing field $E_2$ is applied to create the dressed states $|G_{2}\rangle$. First, we set $E_{13}$ at a large detuning $\Delta_1 = -55.8$ GHz, which leads to the one-photon $(|\psi\rangle \rightarrow |G_{2}\rangle)$ resonance [Fig. 1(d)] resulting in an enhanced DFWM $E_{f1}$ signal. Under this condition, we measure the power dependence of the enhanced DFWM process by scanning $\Delta_2$ [Fig. 3(a)]. The enhancement effect gets bigger and bigger as the dressing field power increases, until it saturates at certain point, as shown in the inset of Fig. 3(a). Figure 3(b) shows that there is a suppressed dip at the line center, which indicates that at exact one-photon resonance ($\Delta_1 = 0$), the DFWM signal intensity is greatly suppressed when scanning the dressing field $E_2$ across its resonance ($\Delta_2 = 0$). As expected, the DFWM signal intensity decreases gradually as the coupling intensity increases due to the dressed effect [the inset plot in Fig. 3(b)], which is well described by the theoretical curve (solid line).

Finally, we concentrate on the interactions between these two FWM processes. Figure 4(a) shows mutual suppression, where the upper curves are the DFWM signal $E_{f1}$, while the lower curves are the NDFWM signal $E_{f2}$. As shown in Fig. 4(a), different suppressed FWM signals can be seen by blocking different laser beams. The square points are pure DFWM signal case. When the additional laser beam $E_2$ or $E_2'$ is tuned on, suppressed DFWM signals are obtained (triangle points in upper curves). Using the same procedure, we obtain the suppressed NDFWM signals (triangle points in lower curves) comparing to the pure NDFWM signal (square points in lower group). When all five laser beams are turned on, the two FWM processes couple to each other with mutual suppressions, as shown by the circle points both in upper and lower groups of curves in Fig. 4(a). Then, we set $\Delta_1$ at a point with enhanced DFWM and obtain a group of curves showing interplays between these two FWM processes, where the DFWM signal $E_{f1}$ is enhanced while the NDFWM signal $E_{f2}$ is suppressed by scanning $\Delta_2$. Similarly, comparing to the pure DFWM signal $E_{f1}$ [square points in Fig. 4(b) and 1], the dressed DFWM signal $E_{f1}$ is enhanced by blocking different laser beams [triangle points and circle points in Fig. 4(b) and 1]. Other than the DFWM process, the NDFWM process [square points in Fig. 4(b) and 2] is suppressed by the coupling fields $E_1$, $E_1'$ [triangle points and circle points in Fig. 4(b) and 2]. For DFWM signal $E_{f1}$, the one-photon $(|\psi\rangle \rightarrow |G_{2}\rangle)$ resonant condition is satisfied in enhancing $E_{f1}$ in Fig. 1(d). At the same time, for the NDFWM signal $E_{f2}$ in Fig. 1(e), the state $|1\rangle$ is dressed by the coupling fields $E_1$ and $E_1'$, and separated into the dressed states $|G_{1}\rangle$, which always suppress $E_{f2}$ due to destructive interference.

In summary, we report our experimental results with theoretical analysis on the evolutions of the dressed effects of the DFWM and NDFWM processes. The experimental data show that the FWM signal can be enhanced by adding coupling fields. In addition, we have measured the power dependences of enhanced and suppressed FWM signals. The experimental data are in good agreements with our theoretically calculated results. These studies provide detail physical mechanisms to control and optimize the efficiencies of the MWM processes in multilevel systems.

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