Modulating the multi-wave mixing processes via the polarizable dark states

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Abstract: We have observed multi-wave mixing (MWM) processes in reversed-Y (RY) type system in $^{87}\text{Rb}$ atoms with electromagnetically induced transparency (EIT) windows at different laser polarization configurations. Interesting rules of changing the MWM processes and EIT profiles are obtained. We have found that the degenerate Zeeman sublevels and their dressed-state effects are responsible for these observed phenomena. Polarizable dark states are used to describe the multi-level dressed states. The experimental data are in good agreement with the results from the theoretical calculation that takes into account all the 16 Zeeman sublevels in the RY system.

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References and links

16. H. Y. Ling, Y. Q Li, and M. Xiao, “Coherent population trapping and electromagnetically induced
1. Introduction

The generated multi-wave mixing (MWM) signals in multi-level systems can transmit through the resonant atomic medium with little absorption under the electromagnetically induced transparency (EIT) conditions [1-2]. Enhanced MWM processes due to laser-induced atomic coherence have been experimentally demonstrated in several multi-level atomic systems [3-5]. Interesting effects, such as quantum destructive interference in inelastic two-wave mixing [6], phase-controlled light switching at low light level [7], entangled images in the probe and signal beams in the four-wave mixing (FWM) process [8], and generation of correlated photon pairs [9-10], have been experimentally studied in various coherently-prepared multi-level atomic systems. When a strong light field interacts with an atomic transition, the light-atom system can be considered as coupled harmonic oscillators with split normal modes, called dressed states. When more than one light field interact with the same or connected atomic transitions, multi-dressed states can be formed [11]. Recently, interplay among multi-dressed FWM processes [12], competition via atomic coherence in a four-level atomic system with two co-existing FWM processes [13] and destructive/constructive interferences in a two-level atomic system [9] were studied. In recent years, many schemes have been developed to enhance higher-order nonlinear wave-mixing processes. More importantly, with induced atomic coherence and interference, the higher-order processes can become comparable or even greater in amplitude than the lower order wave-mixing processes [14].

In this paper, we report combined theoretical and experimental studies on the polarization dependence of the MWM processes and EIT profile, using rubidium atoms as the nonlinear medium. We present the results obtained for the intensities of the MWM signals as a function of the polarization of the incident laser beams. The relations between the high-order (e.g. third- and fifth-order) nonlinear susceptibilities and the polarizations of the incident beams are shown. To explain the observed EIT profiles and different rules for the MWM signal peaks with different incident beam polarizations, we take into account the degenerate Zeeman sublevels and their dressed effects. We have found that the changes in the spectrum of the MWM processes can be attributed to the modulation in the probe field variation. The experimentally measured data are in good agreements with the results from the theoretically calculated results that take into account all the 16 relevant Zeeman sublevels in the reversed-Y (RY) system in rubidium atom.

2. Theoretical model and analysis

The relevant experimental energy-level diagram is shown in Fig. 1(a). Four energy levels from the $^{87}\text{Rb}$ atoms are involved in the experimental schemes used in this work. The laser beams are spatially aligned as shown in Fig. 1(b). In Fig. 1(a), energy levels $|0\rangle$ ($5S_{1/2}$, $F = 2$), $|1\rangle$ ($5P_{3/2}$, $F' = 2$), $|2\rangle$ ($5D_{5/2}$, $F'' = 1$) and $|3\rangle$ ($5S_{5/2}$, $F = 1$) form the RY-type four-level atomic system. Strong coupling laser beam $E_1$ ($\omega_1$, $k_1$, and Rabi frequency $G_1$) together with $E'_1$ ($\omega_1$, $k'_1$, and Rabi frequency $G'_1$) with small angle (0.5°) and the same frequency detuning $\Delta_1$ ($= \omega_0 - \omega_1$), connecting the transition between $|1\rangle$ to $|2\rangle$, propagate in the opposite direction of the weak probe beam $E_2$ ($\omega_2$, $k_2$, and Rabi frequency $G_2$), which has the frequency detuning $\Delta_2$ ($= \omega_0 - \omega_2$) and connects the transition between $|0\rangle$ to $|1\rangle$, as shown in the inset of Fig. 1(b). Two additional coupling beams $E_3$ ($\omega_3$, $k_3$, and Rabi frequency $G_3$) and $E'_3$ ($\omega_3$, $k'_3$, and Rabi frequency $G'_3$), with the same frequency detuning $\Delta_3$ ($= \omega_0 - \omega_3$) connecting the transition between $|3\rangle$ to $|1\rangle$, also propagate in the opposite direction of
the weak probe beam $E_p$ [Fig. 1(b)] with small angles. When all five laser beams ($E_1$, $E_2$, $E_3$, $E_4$, and $E_5'$) are turned on simultaneously, only one ladder-type EIT window opens, whose position in frequency domain depends on the frequency detuning of the $\omega_m$ laser beams. Meanwhile, there will be co-existing MWM processes that generate signal beams at frequency $\omega_s$ (where $\omega_s = \omega_m$) in such multi-level system. First, without the coupling beams $E_1$ and $E_4'$, one pure FWM $E_2'$ process will be generated in the ladder system ($\ket{0} \rightarrow \ket{1} \rightarrow \ket{2}$) satisfying the phase-matching condition of $k_s = k_1 + k_2 - k_3'$. The signal emerging from the “P” polarization direction is detected by an avalanche photodiode detector (APD). Second, when all beams (except with $E_2'$ blocked) are turned on, the strong coupling beam $E_3$ will dress the energy level $\ket{1}$ to create the dressed states $|+\rangle$ and $|-\rangle$. In such case, there exist a dressed FWM process $k_s' = k_1 + k_2 - k_3'$ and a coexisting six-wave mixing (SWM) process, in which two photons from $E_3$ and one photon each from $E_1$, $E_4$ and $E_5'$ participate in the SWM process to generate $E_s$ with phase-matching condition of $k_s = k_1 + k_2 - k_3 + k_3'$. 

![Energy level diagram](image)

Fig. 1. (a) Energy level diagram of relevant $^8\text{Rb}$ energy levels. (b) The schematic diagram of the experiment. D denotes the photodiode and APD denotes the avalanche diode detector; PBS denotes the polarized beam splitter and WP denotes the wave plate. Inset: the spatial alignment of the laser beams.

Although only four energy levels are explicitly shown in Fig. 1(a), there are Zeeman sublevels for each of these energy levels. There are multiple quantum paths for each step of the nonlinear optical processes. The probability amplitudes for these paths depend on the Clebsch-Gordan coefficients, which are intimately related to the polarizations of the input laser beams. Therefore, one can easily manipulate the contributions of the interference terms to the MWM signals by controlling the polarizations of the input laser beams.

To quantitatively investigate the polarization dependence of the MWM signals, we develop a theoretical model to treat the polarization-dependent nonlinear processes. The observed MWM signal intensities in the experiments are proportional to the square of the polarizations induced in Rb vapor at frequency $\omega_m$. First, for the FWM signal $E_{2'}$ (by blocking laser beams $E_1$ and $E_4'$), the nonlinear atomic polarization $P^{(3)}(\omega_s)$ along the $i$ ($i=x,y$) direction, from first-order perturbation theory, is given by [15]

$$P_i^{(3)}(\omega_s) = \varepsilon_0 \sum_{jk} x_{ij}^{(3)}(\omega_s, \omega_1, \omega_2, \omega_3),$$

where the third-order susceptibility contains the microscopic information about the atomic system. The susceptibility of the nonlinear tensor $\chi^{(3)}_{ij} (\omega_s, \omega_1, \omega_2, \omega_3)$ is also related to the polarization components of the incident and generated fields. For an isotropic medium, as in the rubidium vapor, only four elements are not zero and they are related to each other by $\chi_{x'z'x} = \chi_{zx'z} + \chi_{x'z'z}$. According to the experimental arrangement, we assume that the coupling beams $E_1$, $E_2'$, $E_3$, and $E_5'$ have fixed polarization along y-axis (S polarization), while the probe beam $E_p$ can have polarization in any direction in the $xy$ plane. In this case, the polarization of the generated FWM signal beam $E_{2'}$ will have two components, i.e. P and S polarizations.
where \( \chi_s \) and \( \chi_y \) are the effective susceptibilities. If there is a half-wave plate placed on the path of the probe beam \( E_s \), then

\[
\chi_s^{(3)} = \chi_{xxyy} \cos 2\theta, \\
\chi_y^{(3)} = \chi_{yxxx} \sin 2\theta.
\]

For a quarter-wave plate in the probe beam,

\[
\chi_s^{(3)} = \chi_{xxxx} \sqrt{\sin^4 \phi + \cos^4 \phi}, \\
\chi_y^{(3)} = \chi_{yyyy} \sqrt{2 \sin \phi \cos \phi}.
\]

\( \theta, \phi \) are the rotated angles of the half-wave plate and quarter-wave plate, respectively, relative to the x-axis.

Similarly, for the generated SWM signal \( E_g \) (when blocking the coupling laser beam \( E_c' \)), the fifth-order nonlinear polarization \( P^{(5)}(\omega) \) along the \( i (i=x,y) \) direction is then given by

\[
P^{(5)}(\omega) = e_0 \sum_{ijklmn} \chi_{ijklmn}^{(5)} E_i^* E_j E_k^* E_l E_m^* E_n (\omega_k) E_{\omega l} (\omega_m),
\]

where \( \chi_{ijklmn}^{(5)} \) is the fifth-order nonlinear susceptibility. For an isotropic medium, as in the Rb vapor, there are sixteen nonzero components and only fifteen of them are independent because they are related to each other by

\[
\chi_{xxxxx} = \chi_{yxxxx} + \chi_{xyxxx} + \chi_{yyxxx} + \chi_{yyxyy} + \chi_{yyyyy} + \chi_{yxyyx} + \chi_{xyxyx} + \chi_{xyxyy} + \chi_{xyxxy} + \chi_{xxyxy} + \chi_{xxyyx} + \chi_{xxxyy} + \chi_{xxxyx} + \chi_{xxxyy} + \chi_{xxxxx}.
\]

Under the same condition as mentioned above for the FWM case, the generated SWM polarization will have two components, parallel and perpendicular to the beam’s polarization,

\[
P_s^{(5)}(\omega) = e_0 \chi_s, E_x^* E_y^* E_z, \]

\[
P_y^{(5)}(\omega) = e_0 \chi_y, E_x^* E_y^* E_z.
\]

According to the experimental arrangement, we assume that the probe field can have linear polarization in any direction in the xy plane determined by the wave-plate, while the other coupling fields have fixed polarization along the same y-axis (S polarization). In this case, the polarization of the generated SWM signal beam will have two components, i.e. parallel and perpendicular to the coupling beams’ polarization. For a half-wave plate placed in front of the probe beam, the polarizations of the SWM signal beam become

\[
\chi_s = \chi_{xyyy} \cos 2\theta, \\
\chi_y = \chi_{yyyy} \sin 2\theta.
\]

In addition, for a quarter-wave plate, these fifth-order nonlinear susceptibility components are

\[
\chi_s = \chi_{yyyy} \sqrt{\sin^4 \phi + \cos^4 \phi}, \\
\chi_y = \chi_{yyyy} \sqrt{2 \sin \phi \cos \phi}.
\]

In our experimental [Fig 1(b)], we first use a half-wave plate to horizontally
polarize the probe beam (P polarization). Then the beam passes through a half-wave plate (or a quarter-wave plate) that is rotated by an angle \( \theta \) (or \( \phi \)) before going into the \(^{87}\text{Rb} \) atomic vapor cell. By blocking different incident beams, the probe beam and the generated FWM signal (or SWM signal) [13] both pass through a polarized beam splitter (PBS) before being detected. The detector D receives the P polarization component of the transmitted probe beam, \( E_\omega (t_\omega) \), whereas the APD receives the horizontally polarized component of the generated MWM signal \( E_\omega (t_\omega) \).

3. Experimental results

The experiment is done with hot \(^{87}\text{Rb} \) atoms by three external cavity diode lasers (ECDL) and linewidths of less than or equal to 1 MHz. Each output power is as follow: 0.7 mw of probe field \( E_1 \); 65 mw of coupling beams \( E_2, E_2' \) and 15 mw of coupling laser beams \( E_3, E_3' \). The cell whose length is 5cm is heat up to 60°C and the density is \( 2.5 \times 10^{11} \) / cm\(^3\).

Figure 2 shows the EIT spectra (lower dark curves) and the SWM spectra (upper red curves) of the \(^{87}\text{Rb} \) atoms, by blocking laser beam \( E_2' \), for probe field with different polarizations rotated by a half-wave plate. The coupling beams are linearly polarized in the S polarizations direction and the probe beam is linearly polarized in the P polarization direction initially (defined as \( \theta = 0^\circ \)).

As shown in Fig. 2, various polarized configurations of the coupling and probe beams, by rotating the half-wave plate, result in quite different spectra. The absorption curve of the probe beam changes from EIT to a dispersion-like curve, and then back to EIT again while the polarization of the input probe beam changes one period. Meanwhile, the rules of the two SWM signal peaks change quite differently. These observed experimental results reveal that the degenerate Zeeman sublevels [16] and the dressed-state effects play crucial roles in the EIT and SWM spectra.

![Fig. 2. (color online) Polarization dependence of the transmitted probe and SWM signal beams versus the rotating angle of the half-wave plate. (a1-a6) EIT (lower curves) and SWM (upper curves) spectra versus the rotating angle. The experimental parameters are \( G_1 = 2 \pi \times 5 \text{MHz} \), \( G_2 = G_2' = 2 \pi \times 80 \text{MHz} \), \( G_3 = G_3' = 2 \pi \times 35 \text{MHz} \), and \( \Delta_1 = \Delta_2 = 0 \).](image)

To understand the above experimental results, we show the theoretical calculation of the probe absorption spectrum that considers all the 16 relevant Zeeman sublevels [17] in the RY system. Our calculations are limited to the cases that the coupling fields are linearly polarized and the magnetic field is absent. Three examples of the excitations of the coupling and the probe beams are illustrated in Fig. 3. Figure 3(a) shows the case with the probe beam polarization to be orthogonal to the coupling beams and Fig.3(c) corresponds to the case with parallel polarization. Figure 3(b) gives the results with probe input polarization in between those two cases. For simplicity, we consider the cascade three-level system (by blocking coupling fields \( E_1 \) and \( E_3' \)), which forms an EIT configuration. For the case as given in Fig. 3(a), the density-matrix \( \rho \) considering all the Zeeman levels is solved with the following equations:

\[
\rho = (1/\hbar) [H_\text{atom} + H_\text{coupling} + H_\text{probe} \cdot \rho] + [d\rho/dt],
\]
\[ H_{\text{inter}} = \hbar \omega_{i1} \sum_{i=1}^{10} |i><i| + \hbar \omega_{i2} \sum_{i=1}^{13} |i><i|, \]  
\[ H_{\text{coupling}} = -\exp[i\omega_{p1} t]|(\hbar \Omega_{p1}/2)(17><11|+ 19><13|) + (\hbar \Omega_{p2}/2)(18><12|) + \text{c.c.}, \]  
\[ H_{\text{probe}} = -\exp[i\omega_{p2} t]|(\hbar \Omega_{p3}/2)(11><71| + 15><91|) + (\hbar \Omega_{p4}/2)(12><81| + 14><82|) + (\hbar \Omega_{p5}/2)(13><91| + 13><71|) + (\hbar \Omega_{p6}/2)(12><61| + 14><10|) + \text{c.c.}, \]  
\[ \frac{d\rho}{dt} = -\Gamma_1 \sum_{i=1}^{10} \rho_{ii} |i><i| - \Gamma_2 \sum_{i=1}^{13} \rho_{ii} |i><i| \]  
\[ + \sum_{i=1}^{10} \sum_{j=1}^{13} \Gamma_{ij} \rho_{ij} |i><j| + \sum_{i=1}^{10} \sum_{j=1}^{13} \Gamma_{ji} \rho_{ji} |j><i| \]  
\[ - \frac{1}{2} [\Gamma_1 \sum_{i=1}^{10} \sum_{j=1}^{13} \rho_{ij} |i><j| + \Gamma_2 \sum_{i=1}^{10} \sum_{j=1}^{13} \rho_{ij} |j><i| + \text{c.c.}] \]  
\[ - [\Gamma_1 \sum_{i=1}^{10} \sum_{j=1}^{13} \rho_{ij} |i><j| + \Gamma_2 \sum_{i=1}^{10} \sum_{j=1}^{13} \rho_{ij} |j><i| + \text{c.c.}], \]  

where \( \omega_{i1} \) is the \( F = 2 \rightarrow F' = 2 \) transition frequency and \( \omega_{i2} \) is the \( F' = 2 \rightarrow F'' = 1 \) transition frequency. \( \omega_{p1} \) is the frequency of the coupling field and \( \omega_{p2} \) is frequency of the probe laser. \( \Omega_{p1} \) is the Rabi frequency for the transition indicated by its subscripts. The expression of \( \{d\rho/dt\} \) describes all the relaxation processes in the system. \( \Gamma_1 \) and \( \Gamma_2 \) are the spontaneous decay rates of the \( 5D_{5/2}, \quad F'' = 1 \) and \( 5P_{3/2}, \quad F' = 2 \) excited states, respectively. \( \Gamma_{ij} \) is the spontaneous emission rate from an excited state \( |j> \) to a ground state \( |i> \). \( \Gamma' \), \( \Gamma'' \) and \( \Gamma''' \) are the decoherence rates for the relaxation processes other than the spontaneous decay. To obtain linear susceptibility, we need to solve the density-matrix equations (11) under the steady-state condition. Under the weak probe field approximation [2], the expressions of the first-order matrix elements can be easily calculated.

(1) For the right-hand-circularly (RHC) polarized sub-ladder systems:

\[ \rho_{1,7} = \frac{i \Omega_{p1}}{i \Delta_{p1} + \Gamma_{1,7} + \frac{\Omega_{p1}^2}{i (\Delta_{p1} + \Delta_{1,7} + \Gamma_{1,1})}}, \]
\[ \rho_{2,8} = \frac{i \Omega_{p2}}{i \Delta_{p2} + \Gamma_{2,8} + \frac{\Omega_{p2}^2}{i (\Delta_{p2} + \Delta_{2,8} + \Gamma_{1,2})}}, \]
\[ \rho_{3,9} = \frac{i \Omega_{p3}}{i \Delta_{p3} + \Gamma_{3,9} + \frac{\Omega_{p3}^2}{i (\Delta_{p3} + \Delta_{3,9} + \Gamma_{1,3})}}, \]
\[ \rho_{4,10} = \frac{i \Omega_{p4}}{i \Delta_{p4} + \Gamma_{4,10} + \frac{\Omega_{p4}^2}{i (\Delta_{p4} + \Delta_{4,10} + \Gamma_{1,4})}}. \]
are the differences between the laser and the remaining calculations are similar. We can then get the expressions of the matrix

\[
\rho_{2,6} = \frac{i\Omega_{p2}}{i\Delta_{p4} + \Gamma_{2,6}} \rho_{2,6}^{(0)},
\]

\[
\rho_{3,7} = \frac{i\Omega_{p3}}{i\Delta_{p3} + \Gamma_{3,7}} \rho_{3,7}^{(0)},
\]

\[
\rho_{4,8} = \frac{i\Omega_{p4}}{i\Delta_{p4} + \Gamma_{4,8}} \rho_{4,8}^{(0)},
\]

\[
\rho_{5,9} = \frac{i\Omega_{p5}}{i\Delta_{p5} + \Gamma_{5,9}} \rho_{5,9}^{(0)},
\]

\[
\text{(16)}
\]

where frequency detuning parameters \(\Delta_j\) are the differences between the laser frequency and the corresponding atomic transition frequency.

For the level configuration shown in Fig. 5(e), the probe Hamiltonian is changed to

\[
H_{\text{probe}} = -\exp[i\omega_{p}t](i\hbar\Omega_{p1} / 2)(11 \leftrightarrow 61 \leftrightarrow 15) + \exp[i\omega_{p}t](i\hbar\Omega_{p2} / 2)(12 \leftrightarrow 71 \leftrightarrow 4 \leftrightarrow 9) + \text{c.c.},
\]

\[
\text{(17)}
\]

and the remaining calculations are similar. We can then get the expressions of the matrix elements for the linearly polarized sub-ladder systems as:
The case in Fig. 3(b) is an integration of the results from Figs. 3(a) and 3(c). All the theoretical results of the different polarization configurations are obtained in the same way.

When we take into account all the Zeeman sublevels in the RY system, the unexpected profiles of the EIT spectra and different changing rules of the two SWM signal peaks can then be easily explained. For instance, initially, the polarizations of the probe beam and coupling beams are perpendicular [Fig. 3(a)]. There exist three RHC (blue arrows) and three LHC (green arrows) EIT subsystems with one RHC and one LHC probe transitions without the interference of the coupling fields. The observed transmission spectrum is the combination of the Lorentzian absorption profile and the EIT profile for the probe beam. However, in the Fig. 3(a), the intensity of the Lorentzian absorption profile is much smaller than that of the EIT profile, which is the result of different dipole momentums between the Zeeman sublevels [18]. In the case of Fig. 3(b), as the half-wave plate rotating, there are two linearly polarized EIT subsystems (red arrows) with the RHC and LHC EIT subsystems decreasing. Also, there are two additional linear probe transitions. The intensity of the Lorentzian absorption profile is comparable to that of the EIT subspectrums. Therefore, a narrower Lorentzian shape peak can be seen clearly on top of the broader EIT profile as shown in Fig. 2(a3). Due to the detuning of the coupling field, the curve appears to be dispersion-like. When all the laser beams have the same polarization [Fig. 3(c)], there are only two linearly-polarized EIT subsystems and two linear probe transitions. Because of the different dipole momentums among different Zeeman sublevels, the Lorentzian absorption profile is much larger than that of the EIT profile, so the EIT profile has an absorption-type shape with subnatural linewidth [19]. This effect doesn’t present in Fig. 2 due to our current experimental configuration, i.e., the signal is detected only in the P polarization.

In addition, if the coupling fields \( E_1 \) and \( E_3 \) are open, there are doubly-dressed effects for the EIT curve. The analytical solutions will be changed as following.

1. For the RHC-polarized sub-RY systems:

\[
\rho_{1,7}^{(0)} = \frac{i \Omega_{\rho_1}^1}{i \Delta_{\rho_1} + \Gamma_{1,7}} \rho_{1,7}^{(0)},
\]
\[
\rho_{2,7}^{(0)} = \frac{i \Omega_{\rho_2}^2}{i \Delta_{\rho_2} + \Gamma_{2,7}} \rho_{2,7}^{(0)},
\]
\[
\rho_{3,0}^{(0)} = \frac{i \Omega_{\rho_3}^3}{i \Delta_{\rho_3} + \Gamma_{3,0}} \rho_{3,0}^{(0)},
\]
\[
\rho_{4,9}^{(0)} = \frac{i \Omega_{\rho_4}^4}{i \Delta_{\rho_4} + \Gamma_{4,9}} \rho_{4,9}^{(0)}.
\]
\[ \rho_{3,9} = \frac{i\Omega_{p,3}}{i\Delta_{p,3} + \Gamma_{3,9} + \frac{|\Omega_{s,1}|^2}{i(\Delta_{p,3} + \Delta_{s,1}) + \Gamma_{13,3}} + \frac{|\Omega_{s,4}|^2}{i(\Delta_{p,3} - \Delta_{s,4}) + \Gamma_{16,3}}}\rho_{3,9}^{(0)}. \]

(2) For the LHC-polarized sub-RY systems:

\[ \rho_{3,7} = \frac{i\Omega_{p,3}}{i\Delta_{p,3} + \Gamma_{3,7} + \frac{|\Omega_{s,1}|^2}{i(\Delta_{p,3} + \Delta_{s,1}) + \Gamma_{11,3}} + \frac{|\Omega_{s,4}|^2}{i(\Delta_{p,3} - \Delta_{s,4}) + \Gamma_{14,3}}}\rho_{3,7}^{(0)}, \]

\[ \rho_{4,8} = \frac{i\Omega_{p,2}}{i\Delta_{p,2} + \Gamma_{4,8} + \frac{|\Omega_{s,1}|^2}{i(\Delta_{p,2} + \Delta_{s,2}) + \Gamma_{12,4}} + \frac{|\Omega_{s,4}|^2}{i(\Delta_{p,2} - \Delta_{s,4}) + \Gamma_{14,4}}}\rho_{4,8}^{(0)}, \]

\[ \rho_{5,9} = \frac{i\Omega_{p,3}}{i\Delta_{p,5} + \Gamma_{5,9} + \frac{|\Omega_{s,1}|^2}{i(\Delta_{p,5} + \Delta_{s,1}) + \Gamma_{13,5}} + \frac{|\Omega_{s,4}|^2}{i(\Delta_{p,5} - \Delta_{s,4}) + \Gamma_{16,5}}}\rho_{5,9}^{(0)}. \]

(3) For the linearly-polarized sub-RY systems:

\[ \rho_{2,7} = \frac{i\Omega_{p,3}}{i\Delta_{p,2} + \Gamma_{2,7} + \frac{|\Omega_{s,2}|^2}{i(\Delta_{p,2} + \Delta_{s,2}) + \Gamma_{11,2}} + \frac{|\Omega_{s,4}|^2}{i(\Delta_{p,2} - \Delta_{s,4}) + \Gamma_{14,2}}}\rho_{2,7}^{(0)}, \]

\[ \rho_{4,9} = \frac{i\Omega_{p,2}}{i\Delta_{p,4} + \Gamma_{4,9} + \frac{|\Omega_{s,2}|^2}{i(\Delta_{p,4} + \Delta_{s,2}) + \Gamma_{13,4}} + \frac{|\Omega_{s,4}|^2}{i(\Delta_{p,4} - \Delta_{s,4}) + \Gamma_{16,4}}}\rho_{4,9}^{(0)}. \]

These expressions indicate that the additional coupling fields will modulate the EIT profiles. Different frequency detuning configurations of the three stronger coupling fields will affect the EIT spectral shape. By controlling the frequency detuning of the coupling fields, either EIT or electromagnetic induced absorption (EIA) spectrum will appear.

In order to quantitatively compare with the observed changes of the SWM signal peaks as probe polarization changes, we choose to calculate the area of the spectrum under the peak [see Fig. 4(b1)] and the height of each peak [see Figs. 4(b2) and 4(b3)]. As a comparison, the area curve of the EIT [Fig. 4(a1)] and the absorption height of each
Fig 4. (a) Measured (dots) and calculated (solid curves) areas and heights of the two transmitted EIT peaks. (b) Measured (dots) and calculated (solid curves) areas and heights of the SWM signal peaks.

peak [Figs. 4(a2) and 4(a3)] are also presented. From Figs. 4(a1) and 4(a1), the changes of the SWM spectrum, as well as for the EIT peaks, are well described by the function \( \cos^2 2\theta \), as calculated above. It is interesting to see the polarization dependence of the SWM emission signal following the polarization of the probe field. The theoretical curves (the solid lines) are in good agreement with the experimental data. The different change rules of the two SWM signal peaks can be attributed to the modulation of the modified EIT spectral profile. The dotted lines of Figs. 4(a2) and 4(a3) represent the absorption peak heights of the probe field at different polarization states. The dotted lines of Figs. 5(a1) and 5(a2) depict the EIT depths which show different coupling processes of the total EIT effect via the different polarization dark states. As can be seen from the figures, the rule of change in the positive part of the EIT dispersion-like curve [Fig 5(a1)] is dominated by the polarization property. However, in the negative part [Fig. 5(a2)], it gradually changes from EIT to absorption in a half period. After the absorption is larger than the EIT dip, the increase of absorption is faster than the polarized attenuation, and then the roles of them are switched. Figures 4(b2) and 4(b3) illustrate the measured dependence of the relative SWM signal intensity on the rotation angle \( \theta \) (half-wave plate) when the diode laser (probe) is tuned to the 5S_{1/2}, \( F = 2 \rightarrow 5P_{3/2}, \ F' = 2 \) one-photon transition. Besides its direct dependence on the polarization of the probe beam, the SWM signal spectrum is also modulated by the EIT effect since it transmits through the medium in the EIT window. Figures 5(b1) and 5(b2) show the corresponding differences of the experiment data and theoretically calculated curves which represent different coupling paths of the total SWM processes via different polarization configurations. The left peak is always in the positive part of the dispersion-like EIT curve which gets dramatically modulated effect due to EIT. This means that the fifth-order susceptibility \( \chi^{(5)}_{\text{photon}} \) must be taken into account due to dressed effect, which affects the evolution of the peak together with the polarization dependence of \( \cos^2 2\theta \). The right peak is in the negative region of the dispersion-like EIT shape, where it is dominated by the polarization property due to the increase in absorption. We can conclude that the evolution of the SWM spectrum is modulated by the modified EIT spectrum.
Next, we concentrate on the experimentally measured and theoretically calculated probe transmission and FWM spectra by rotating the quarter-wave plate (in front of the input probe beam) whose period is 180 degree as can be seen in Fig. 6. Here, the MWM process is the FWM by blocking laser beams $E_1$ and $E_1'$ while other beams are turned on. The theoretical results can be obtained from the same procedure as above by eliminating the absent items, that is, LHC subsystems in the RY system. The EIT spectrum also shows the same profile, including the dispersion-like curve. However, the positive peak changes from a single peak into two peaks. Although the way of polarization with a quarter-wave plate is different from with a half-wave plate, its rules of evolution can be explained by the same method as for the half-wave plate. Schemes (d), (e) and (f) in Fig. 3 show the ways of coupling in different polarization configurations. Different from using a half-wave plate whose period is 90 degrees, the right-hand elliptically polarized beam is present during 0~45 and 45~90 degrees [Fig. 3(e)], and a pure RHC-polarized beam at 45 degrees [Fig. 3(f)]. However, within 0~45 degrees, the RHC component increases gradually while the linear component decreases. In the region of 45~90 degrees the opposite process is true. Taking the case of 0~45 degrees as an example, we can take the right-hand elliptically-polarized beam being composed of a vertical, linearly-polarized beam and a RHC-polarized beam. Therefore, in our experiment, the original symmetric EIT configurations [Fig. 3(d)] are replaced by two linear EIT and three RHC EIT subsystems [Fig. 3(e)] that are asymmetric due to the difference in the dipole moments among different Zeeman sublevels. The destruction of this symmetry results in different polarizable dark states leading to the modified EIT spectrum. It is different from the case with half-wave plate, which does not destroy the symmetry in the EIT spectral shape. The situation in the rest of the period (45~90 degree) can be discussed in the same way. So, the spectrum by rotating a quarter-wave plate is different from the case of using a half-wave plate.
Fig. 6. (color online) Polarization dependence of the transmitted probe and FWM signal beams versus the rotation angle of the quarter-wave plate. (a1-a7) The EIT (lower curves) and FWM (upper curves) spectra versus the rotation angle. The experimental parameters are \( G_1 = 2\pi \times 5\text{MHz} \), \( G_2 = G'_2 = 2\pi \times 80\text{MHz} \), and \( \Delta = 0 \).

The FWM spectra are modulated by the EIT spectra as shown in Fig. 6. As the quarter-wave plate rotates, the FWM spectrum changes from a two-peak structure to a three-peak one. The rule of evolution for each peak includes not only the polarized property, but also effects of dark-state modulation due to the asymmetric coupling structure in EIT spectrum. Similar to the analysis used for the half-wave plate, the rule of evolution in the height of each absorption peak in the EIT profile follows the polarization effect of the probe beam. Although each peak [Figs. 7(a1)-(a3)] is significantly modulated, the general trend in its evolution is not destroyed. Moreover, in studying the detail dark-state coupling configuration for each peak, we have investigated the peak depths of the EIT curves [Figs. 8(a1)-(a3)]. All of them undergo the combined interactions of the selectively-polarized probe beam and the polarized dark-state modulation. The two peaks in the positive part of the dispersion-like curve can be attributed to the asymmetric EIT configuration, RHC polarization and vertical linearly-polarized EIT subsystems. Due to the increase in absorption [Fig. 3(e)], the right peak in the probe transmission spectrum changes from EIT to EIA [Fig. 8(a3)]. Therefore, based on the above analysis of the EIT spectrum, we can understand the rule of change in FWM spectrum easily. The left FWM peak [Figs. 7(b1) and 8(b1)], corresponding to the left EIT peak [Fig. 8(a1)] dominated by the polarized dark-state component, is dramatically modulated and the contribution from polarization change of the probe beam is suppressed. As mentioned above, there could be combined contributions from the third-order nonlinear susceptibility \( \chi^{(3)}(\omega_1; \omega_2, -\omega_3, \omega_2) \) and the polarization effect of the probe field in the form of \( \sqrt{\sin^2 \phi + \cos^2 \phi} \). Although it is affected by the polarized dark-state modulation, the trend of change for the middle peak [Fig. 8(a2)] follows the polarization change of the probe beam. As a result, the polarization property of the middle FWM peak survives [Figs. 7(b2) and 8(b2)]. The right FWM peak is modulated and its height increases gradually due to the increased absorption of the right peak in the probe transmission spectrum. Meanwhile, the middle FWM peak decreases gradually. These two peaks have a competitive relationship, which can explain their opposite phase evolutions [Figs. 7(b3) and 8(b3)].
4. Conclusion

In summary, we have experimentally and theoretically demonstrated that the modified EIT spectra resulting from degenerate Zeeman sublevels can significantly modulate the MWM processes. The different couple ways of permitted transitions of degenerate Zeeman sublevels due to polarization configuration of incident beam which destroy the symmetric can modulate the spectra [Figs. 2 and 6]. Each peak of the MWM process will be controlled by polarization property [Figs. 4(a3) and 4(b3)], dressed effect [Figs. 4(a2) and 4(b2)], or both of them [Figs. 8(a2) and 8(b2)]. The experimental data are in good agreement with the results from theoretical calculation involving all relevant Zeeman sublevels. Our study indicates that Zeeman sublevels should be taken into account in the analysis of the MWM, such as FWM and SWM, processes involving laser-induced atomic coherence in multi-level atomic systems.

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