Improving spatial resolution in quantum imaging beyond the Rayleigh diffraction limit using multiphoton W entangled states

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ABSTRACT

Using multiphoton entangled states, we demonstrate improving spatial imaging resolution beyond the Rayleigh diffraction limit in the quantum imaging process. In particular, we examine resolution enhancement using triphoton W state and a factor of 2 is achievable as with the use of the Greenberger–Horne–Zeilinger state, compared to using a classical-light source.

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1. Introduction

It is well known that in classical optics the spatial resolving power of optical devices is limited by diffraction. According to the Rayleigh criterion\textsuperscript{1} \cite{1}, the ability to resolve two point sources is limited by the wavelength of the light. Recently, there is a renewed interest in improve resolution beyond the Rayleigh diffraction limit. Number of proposals have been presented in the literature and most of them are based upon the use of nonclassical light state and/or new developed measurement techniques. The motivation of improving the resolving power of optics is stimulated not only from the side of the fundamental research interest, but also by developing next-generation technologies, such as quantum imaging, quantum lithography, and quantum sensors.

Among those proposals, promising schemes can be grouped into the following three categories. In category one, nonclassical light sources, especially entangled photon pairs, are applied to beat the Rayleigh diffraction limit. In this approach, quantum ghost imaging \cite{2–7} has attracted a considerable attention over the past ten years. In quantum ghost imaging, two-photon entanglement has been used for joint detection measurement. The resolution of this system has been discussed in Refs. \cite{8,9} and no obvious improvement is achievable with biphotons compared to classical optics. In category two, entangled photon-number states (especially the famous N00N state) \cite{10} and squeezed state \cite{11} have been chosen to bypass the diffraction limit. For entangled photon-number states, an interferometer is usually built to obtain sub-wavelength imaging. For squeezed states, the image is constructed through the homodyne detection. However, both of these techniques are severely affected by the loss of photons. In category three, people proposed to use classical light sources to enhance the spatial resolution. Here, two configurations have been extensively studied in recent years. One configuration is to apply high-order correlation measurements similar to quantum ghost imaging and Hanbury–Brown and Twiss experiment \cite{12,13}. The other configuration is to design an interferometric lithography with the use of classical coherent state \cite{14,15}.

In this Letter we are interested in forming sub-wavelength imaging in the frame of quantum ghost imaging with the use of multiparticle entanglement. Specifically, we shall discuss how to beat the Rayleigh diffraction limit using two inequivalent classes of triphoton entangled states, namely, the Greenberger–Horne–Zeilinger (GHZ) class and the W class. In entangled three-qubit states it has been shown that these two classes are inequivalent under stochastic local operations and classical communications \cite{16}. Related work with emphasis on the entanglement properties of continuous-variable three-partite Gaussian GHZ and W states has been presented in \cite{17}. Only recently, tripartite entangled GHZ and W states in time (or energy) and space have been reported in \cite{18}. It is this spatial correlation exhibited in these states that is useful for multiphoton quantum imaging and lithography \cite{19–21}. Quantum ghost imaging with multiphoton W states has been presented in Refs. \cite{19,20}. However, the issue of the resolution is not touched in that work. Using the GHZ states to achieve sub-Rayleigh imaging has been discussed in \cite{21} where the spatial resolving power enhancement by a factor of \(N\) is possible with input \(N+1\) entangled photon-number state, compared to classical optics. Few
drawbacks affect the realization of such an imaging process. One comes from the lack of generation mechanism for such a state. Second, the requirement of $N$-photon number detectors is beyond the current labs capability. Although the research of designing photon number detectors has achieved impressive progress, their functionality is not ready for commercial applications. In addition, the diffraction will severely cause the decoherence of the state. Therefore, it would be interesting to see whether the W state can be used for sub-Rayleigh imaging. This is the driving force for this Letter.

Here, through theoretical calculations we show that it is possible to improve the spatial resolution of the imaging beyond the diffraction limit with the use of the W state. To be specific, we use three-photon W state to illustrate the principle. Our results tell that under certain conditions, using a W state can reach the same spatial resolution enhancement by using a GHZ state, despite they belong to different classes. But the underlying physics is different. It has been shown in [18] that the GHZ state is fragile to photon loss while the W state is robust against the loss. Hence, the results presented here are more close to practical applications when considering to improve sensitivities of classical sensors and spatial resolution of medical images.

We organize the Letter as follows. In Section 2 we will study spatial resolution enhancement in quantum imaging using multipartite entanglement. We first briefly review our imaging scheme with the use of the tripartite GHZ state. We then show that it is possible to achieve the same spatial resolution improvement by a factor of $2$ with the use of a W triphoton state. The required conditions are also explicitly given. Finally, we will draw our conclusions in Section 3.

2. Quantum imaging with multipartite entanglement

Conventional imaging usually involves only the first-order correlation measurement. Ghost imaging, however, takes advantage of high-order correlation measurement through multi-party joint detections. One advantage is that more properties of the light can be observed in such a measurement and cannot be found simply through the first-order correlation measurement. Quantum imaging with entangled paired photons has been extensively studied in the last decade. However, extension to multipartite entanglement has been explored only very recently, partially because of the experimental difficulty of creating such entangled states. In multipartite systems, there exist many different classes of entanglement. It has been shown that even in tripartite systems, there exist two inequivalent classes of entanglement, namely, the GHZ class and the W class. In this section, we consider quantum imaging with multipartite entanglement by taking advantage of its spatial properties. In particular, we concentrate on beating the Rayleigh diffraction limit using a triphoton W state entangled in time and space. Our motivation comes from practical applications. That is, it is more easier to produce a triphoton W state entangled in time and space from the nonlinear optical process in the current labs.

2.1. Summary of quantum imaging with a GHZ triphoton state

In [21] quantum imaging with multipartite GHZ entangled states including the triphoton case has been discussed, where it was shown that the resolution of images illuminated by sources composed of $N$ photons in which one nondegenerate photon is entangled with $N$ degenerate photons can be improved by a factor of $N$ compared to using a classical source. As shown in [21], by sending the degenerate photons to an $N$-photon detector while sending the nondegenerate one to a single-photon detector, the spatial resolution improvement by a factor of $N$ is possible no matter whether the $N$-photon detector is a point detector or a bucket one. The difference between these two imaging schemes is that using the point $N$-photon number detector yields a coherent image while the use of a bucket detector (i.e., $N$ single-photon detectors) leads to an incoherent image. The advantage of applying the bucket detector is that it is easier to construct than a point $N$-photon number detector. However, as mentioned in the introductory, no feasible generation mechanism was yet found to produce such a state entangled in time and space. Hence, it would be interesting to look for another alternative way to achieve the sub-Rayleigh ghost imaging.

2.2. Quantum imaging with multipartite W entangled states

In Section 2.1, we briefly reviewed that with the entangled photon-number state $|1,2\rangle$, the ability to resolving two point sources in the object can be improved by a factor of 2 by sending two degenerate photons to the object with coincidence with the nondegenerate photon and imaging lens in the other arm. Due to the lack of generating such a state, in this section we will look at the spatial resolution enhancement by replacing the GHZ state with a triphoton W state as shown in Fig. 1. We assume that the two nearly degenerate photons (photon 1 and photon 2) with same central frequency $\Omega_1$ and wave number $K_1$ are sent to the object, while the nondegenerate photon with central frequency $\Omega_2$ and wave number $K_2$ passes through the imaging lens and then fires a single-photon point detector $D_3$. In [19,20] ghost imaging with multipartite W entangled states has been studied. However, the issue of spatial resolution in imaging was not explicitly discussed in those works [19,20]. It is therefore a natural question of whether one can beat the Rayleigh diffraction limit with entangled multiphoton W states. Here we answer this question by considering the triphoton W state and show that under certain conditions, indeed one can enhance the spatial resolution in imaging beyond the diffraction limit with such a state.

Following the procedure done in [21], the three-photon amplitude for detectors $D_1$, $D_2$, and $D_3$, located at $(z_1, \bar{\rho}_1), (z_1, \bar{\rho}_2)$, and $(z_3, \bar{\rho}_3)$, respectively, takes the form

$$
\Psi_{1,2,3} = \langle 0 | E_3^{(+)}(\bar{\rho}_3, z_3, t_3) E_2^{(+)}(\bar{\rho}_2, z_2, t_2) \times E_1^{(+)}(\bar{\rho}_1, z_1, t_1) | \Psi_{1,2,3} \rangle,
$$

where the positive-frequency part of the free-space electromagnetic field is

$$
E_j^{(+)} = \int d\omega_j \int d^2 \alpha_j E_j f(\omega_j) e^{-i \omega_j t_j} \times \mathbf{g}_j(\bar{\alpha}_j, \omega_j; \bar{\beta}_j, \bar{\gamma}_j) \mathbf{a}(\bar{\alpha}_j, \omega_j),
$$

with $E_j = \sqrt{\hbar \omega_j/2\epsilon_0}$, $\bar{\alpha}_j$ is the transverse wave vector, $\omega_j$ is the angular frequency, and $\mathbf{a}(\bar{\alpha}_j, \omega_j)$ is a photon annihilation operator at the output surface of the source,

$$
[a(\bar{\alpha}, \omega), a^\dagger(\bar{\alpha}', \omega')] = \delta(\bar{\alpha} - \bar{\alpha}') \delta(\omega - \omega').
$$

Fig. 1. (Color online.) Schematic of quantum imaging with a triphoton W entangled state. $d_1$ is the distance from the output surface of the source to the object, $l_1$ is the distance from the object to a two-photon point detector, $D_1$, $d_2$ is the distance from the output surface of the source to the imaging lens with focal length $f$ and $l_2$ is the length from the imaging lens to a single-photon detector $D_3$, which scans coming signal photons in its transverse plane. "C.C." represents the three-fold joint-detection measurement.
The function of \( f_j(\omega) \) is a narrow bandwidth filter function which is assumed to be peaked at central frequency \( \Omega_j \). The function \( g_j \) is the Green's function that describes the propagation of each mode from the output surface of the source to the \( j \)th detector at the transverse coordinate \( \vec{r}_j \), at the distance from the output surface through the medium to the plane of the detector, \( z_j \). Following the treatments in [4,19,21], we evaluate the Green's functions \( g_1(\vec{r}_1, \omega_1; \vec{r}_1, \omega_1) \), \( g_2(\vec{r}_2, \omega_2; \vec{r}_2, \omega_2) \), and \( g_3(\vec{r}_3, \omega_3; \vec{r}_3, \omega_3) \) for the experimental setup of Fig. 1 by assuming that the narrow bandwidth filters allow us to make the assumption of \( \omega_1 = \Omega_j + \nu_j \) where \( |\nu_j| \ll \Delta_2 \) and \( 2\Omega_1 + \Omega_2 = \Omega \). In the paraxial approximation, we obtain

\[
g_1(\vec{a}_1, \Omega_1; \vec{p}_1, z_1) \propto e^{-\frac{d_1(\nu_j^2)}{\Delta_1}} \int d^2 \rho_0 A(\vec{\rho}_0) e^{i K_1(\vec{\rho}_0)} \times e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_1}} \delta(\omega_1 - \Omega_1) \delta(\omega - \Omega_1) \delta(\vec{a}_1 - \vec{a}_2 + \vec{a}_3) \times a^\dagger(\vec{a}_2, \omega_2) a(\vec{a}_1, \omega_1) \cdot \langle 0 |.
\]

(4)

The \( \delta \)-functions describe that three photons in such a state are frequency and transverse momentum correlated. The distinction between this W state and the GHZ state can be characterized by looking at the bipartite and tripartite correlation measurements, and in the W state the former has more degrees of freedom than the GHZ state as shown in [18]. One major difference between this state and the GHZ state can be characterized by looking at the bipartite and tripartite correlation measurements, and in the W state the former has more degrees of freedom than the latter.

By applying the Gaussian thin lens imaging condition in Eq. (12)

\[
1 \frac{1}{f} = 1 + \frac{1}{L_3 + d_1 + d_1(\lambda_1/2\lambda_3)},
\]

(13)

the transverse part of the three-photon amplitude becomes

\[
B_{1,2,3} = B_1 \int d^2 \nu_0 A(\vec{\rho}_0) e^{i K(\vec{\rho}_0)} e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_1}} \times e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_2}{\Delta_1}} \times e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_1}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_3}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_2}{\Delta_3}} \\
\times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_3}}.
\]

(14)

where all the slowly varying constants have been absorbed into \( B_1 \). In the following we will only examine the spatial resolution with the use of a two-photon point detector. The bucket detector case can be analyzed using the same procedure as done for the GHZ case [21].

Let us look at the two-photon point detector case. In this case, the two-photon point detector is helpful to retrieve the information of degenerate photons scattered off the same spatial point on the target. That is, we apply the same assumption that the detector \( D_1 \) is only sensitive to the signal fields from the same point in the object, i.e., \( \delta(\vec{\rho}_0 - \vec{\rho}_0) \). With this assumption Eq. (10) reduces to

\[
B_{1,2,3} = B_1 \int d^2 \rho_0 A(\vec{\rho}_0) e^{i K(\vec{\rho}_0)} e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_1}} \times e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_2}{\Delta_1}} \times e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_1}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_3}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_2}{\Delta_3}} \\
\times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_3}}.
\]

(11)

Completing the integrations on the transverse modes \( \vec{a}_1 \) and \( \vec{a}_2 \) in Eq. (11) leads to

\[
B_{1,2,3} = B_1 \int d^2 \rho_0 A(\vec{\rho}_0) e^{i K(\vec{\rho}_0)} e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_2}{\Delta_1}} \times e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_1}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_3}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_3}}.
\]

(12)

By applying the Gaussian thin lens imaging condition in Eq. (12)

\[
1 \frac{1}{f} = 1 + \frac{1}{L_3 + d_1 + d_1(\lambda_1/2\lambda_3)}
\]

the transverse part of the three-photon amplitude becomes

\[
B_{1,2,3} = B_1 \int d^2 \rho_0 A(\vec{\rho}_0) e^{i K(\vec{\rho}_0)} e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_2}{\Delta_1}} \times e^{-\frac{i K_1 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_1}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_3}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_3}{\Delta_3}} \times e^{-\frac{i K_3 \vec{\rho}_0 \cdot \vec{r}_1}{\Delta_3}}.
\]

(14)

where the magnification factor is \( m_f = L_3/[d_3 + (\lambda_1/2\lambda_3)d_1] \). It is now clear that Eqs. (13) and (14) take the same form as Eqs. (15) and (16) obtained in [21], respectively. This indicates that under such a situation both GHZ and W states can be used to enhance the spatial resolving power in images. From the point of spatial resolution in imaging, there is no difference between these two states although they belong to two inequivalent classes. But this does not indicate that these two states are fully equivalent. It is easy to show that in the bipartite correlation measurement, the GHZ state reduces to a product state while the W state still maintains some entanglement [18]. Eq. (13) can be further verified from Eqs. (35) and (36) in [19] by taking account of the degeneracy.

The calculations on resolving two spatially separated point scatterers are the same as in the GHZ case for both the point two-photon detector scheme and two-photon bucket detector scheme. A resolution improvement by a factor of 2 can be achieved in both schemes except that the point two-photon detector scheme offers
a coherent image while the bucket detector scheme gives an incoherent image. We will not repeat those calculations here and interested reader may follow the procedure done in [21] to verify that.

Before ending this section, we would like to briefly discuss how to experimentally generate such a state. As a matter of fact, there are many ways to create a triphoton W state entangled in time (or energy) and space. One possible way is to directly use the third-order atomic cascade transition [22]. That is, one input photon is down converted into three daughter photons, similar as the SPDC process. The difficulty of this method is that the third-order nonlinearity is so weak that no appreciable counts could be observed in the lab. The second way is to use two down conversions followed by one up-conversion, as the scheme proposed by Keller et al. [23]. One problem for this scheme is that two down conversions may dominate the process and the real entangled triphotons will be hard to be detected in coincidences. There is also another way to generate a triphoton W state using cascade four-wave mixings (FWMs), as recently proposed by some of us [24]. The challenge in this mechanism is that one needs to confine the interaction volume for the second FWM process. However, it is nontrivial to generate a three-photon GHZ state entangled in time and space.

3. Conclusions

In summary, we have studied to improve the spatial resolution in the object beyond the Rayleigh diffraction limit using a triphoton entangled W state in time (or energy) and space. We have shown that by sending two degenerate photons to the target, propagating the nondegenerate photon through the imaging lens, and using a two-photon resolving detector or a bucket detector, a factor of 2 is achievable in spatial resolution enhancement using the Rayleigh criterion. The enhancement coincides with the GHZ triphoton case as discussed in [21]. The image is nonlocal and the quantum nature of the states leads to the sub-Rayleigh imaging resolution with high contrast. We emphasize that although both GHZ and W states can be chosen to improve the spatial resolving power, these two states are inequivalent and belong to different classes. The generalization to multiphoton entangled W states is straightforward by assuming all the observed degenerate photons are scattered off the same spatial point in the object. Our imaging protocol may be of importance in practical applications such as imaging and telescope.

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