Interference of three multiwave mixings via electromagnetically induced transparency

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We experimentally observe the interplay among coexisting multiwave-mixing (MWM) signals via multiple electromagnetically induced transparency (EIT) windows, including two ladder-type EIT windows and one V-type EIT-like window in a five-level atomic system of 85Rb. In the presence of these EIT windows, one can control the interplay between these MWM signals easily by changing the frequency detuning. Meanwhile, we also report the spatial and temporal interferences with a femtosecond time scale among three coexisting MWM signals in two overlapped EIT windows. © 2011 Optical Society of America

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1. INTRODUCTION

When laser fields create a coherent superposition state between two ground-state sublevels of an atom, these laser beams experience reduced absorption or increased transmission, which is well known as coherent population trapping (CPT) [1–4]. Under this condition, the system falls into a dark state and the population is trapped within the two ground states. Similar to CPT, effects related to electromagnetically induced transparency (EIT) [5–7] have been very useful since the medium becomes transparent and the probe beam goes through it without attenuation. This quantum interference effect has been widely investigated and applied to many fields, such as ultraslow light [8], light storage [9–11], and quantum memory [12,13]. EIT is also used for investigating multiwave-mixing (MWM) processes since the generated signals can transmit through the resonant atomic medium with little absorption under the EIT conditions. Resorting to two ladder-type EIT windows, it has been demonstrated that the generated four-wave-mixing (FWM) and six-wave-mixing (SWM) signals can coexist in an open-cycled atomic system [14,15]. Furthermore, the competition and interference between the two coexisting FWM processes have been studied via atomic coherence in a four-level atomic system [16]. In this paper, we report an experimental demonstration of interplay and interference among two FWM and two SWM processes simultaneously in a five-level atomic system. By using atomic coherence induced by laser beams among different energy levels, the generating FWM and SWM signals can coexist with the intensities of the same order of magnitude. We can obtain a controllable phase difference between the FWM and SWM signals by carefully designing the spatial configuration of the laser beams for the phase-matching condition and introducing an appropriate optical delay in one of the coupling beams. When the phase difference changes, we can observe the temporal and spatial interferences. The temporal interference is in the femtosecond time scale, which can be used to determine the optical transition frequency. The experiment in this paper is performed with weak cw lights interacting with near-resonant atomic transitions. It is different from the works used to research liquids or solution phase dynamics [17,18]. Researching and understanding the interplay and interference among the different high-order nonlinear optical processes can find important applications in many fields, such as stable two-dimensional soliton formation [19], liquidlike surface tension [19], and nonlinear spectroscopy [17].

2. EXPERIMENT SETUP

There are five energy levels [Fig. 1(a)] from the 85Rb atom involved in the experimental schemes. Coupling beams E2 (frequency ω2, wave vector k2, and Rabi frequency G2) and E4 (ω2, k4, and G4) with the same frequency detuning Δ2 connect the transition between |1⟩ (5P3/2) and |2⟩ (5D5/2), where Δ1 = Ω1 – ω1, Ωi is the atomic resonance frequency for the corresponding transition. Pumping beams E3 (ω3, k3, and G3) and E5 (ω3, k5, and G5) with the same frequency detuning Δ3 connect the transition between |0⟩ [5S1/2 (F = 3)] and |3⟩ (5P1/2). Additional coupling beam E4 (ω4, k4, Δ4, and G4) connects the transition of |1⟩ (5P3/2) to |4⟩ (5D3/2). A weak laser beam E1 (ω1, k1, Δ1, and G1) probes the transition between |0⟩ [5S1/2 (F = 3)] and |1⟩ (5P1/2) [Fig. 1(a)]. The laser beams are aligned spatially, as shown in Fig. 1(b), with five laser beams, E1, E′1, E2, E′2, E3, E′3, E4, and E′4, propagating through the atomic medium in the same direction with small angles of ~0.3° between them. The probe beam E1 propagates in the opposite direction with a small angle.

The experiment is carried out in hot 85Rb atoms by four external cavity diode lasers (ECDLs) with linewidths of less than or equal to 1 MHz. The probe laser beam E1 with wavelength of 780.245 nm and a horizontal polarization from an ECDL has a power of 0.7 mW. The laser beams E2 and E′2 with...
wavelength of 775.978 nm and a vertical polarization are from another ECDL split with powers of 35 and 5 mW, respectively. The beam \( E_0' \) is delayed by an amount \( \tau \) using a computer-controlled stage. The laser beams \( E_0' \) and \( E_3' \) with wavelength 780.235 nm and a vertical polarization are from the third ECDL split with equal power of 15 mW. The laser beam \( E_4' \) with wavelength of 776.157 nm and a vertical polarization from the fourth ECDL has a power of 35 mW. The cell with length of 5 cm is heated to 60 °C and the density is about 2.5 x 10^11 cm^-3. The optical depth of the atomic sample is about 43. Great care was taken in aligning the six laser beams with spatial overlaps and wave vector phase-matching conditions, as shown in Fig. 1(b). Under certain conditions, two FWM signals \( (E_{F1}, \text{phase}-\text{matching condition } k_{F1} = k_3 - k_1' + k_1) \) via the path of \( [0, E_{[3]}, E_{[1]}, E_{[1]}'] \) and one FWM signal \( (E_{S1}, \text{horizontal polarization}) \) are all in the direction of \( E_{SW} \) with the same frequency \( \omega_1 \) and horizontal polarization at the right lower corner of Fig. 1(b) and are detected by an avalanche photodiode detector. The transmitted probe beam is detected by photodiode detector.

3. BASIC THEORY
The dressed-state picture can be used to describe this composite system. We consider the investigated atomic system as a combination of one V-type three-level subsystem \((1) - [0] - [3]\), one ladder-type three-level subsystem \((0) - [1] - [2]\), and two V-type four-level subsystems \((2) - [1] - [0] - [3]\) and \((4) - [1] - [0] - [3]\) corresponding to each MWM process described in Section 2. Table 1 gives all possible phase-matched Liouville pathways for coexisting FWM and SWM processes.

First, without the beams \( E_2, E_3, \) and \( E_4, \) a pure FWM process \( E_{F1} \) will be generated via the Liouville pathway \((F1)\) \( \rho_{00} \rightarrow \rho_{01} \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' \) in the V-type three-level subsystem \((1) - [0] - [3]\), where the density matrix element \( \rho_{ij}^{(0)} \) represents the particle population at the state \([i]\) when \( i = k \) and \( k \) represents the atomic coherence between the states \([i]\) and \([j]\) when \( i \neq k \). \( \alpha \) is the order number representing the iteration process in solving the density matrix equations. The corresponding third-order nonlinear density matrix element, which gives the leading contributions to the FWM process, is \( \rho_{ij}^{(1)} = G_{F1} \exp [i(k_{F1} \cdot r - \omega_1 t)]/A_{F1} = \rho_{ij}^{(0)} \exp [i(k_{F1} \cdot r - \omega_1 t + \phi_{51})] \), where \( G_{F1} = -iG_{1}G_{3}(G_{1}'G_{3}')^* \), \( A_{F1} = \Gamma_{00}d_3d_1 + i\Delta_1, \) \( d_3 = \Gamma_{30} + i\Delta_3, \) and \( \Gamma_{00} \) is the transverse relaxation rate between states \([i]\) and \([j]\). Then, when the beams \( E_2 \) and \( E_4 \) are turned on simultaneously, they will affect the energy level \((1) \) to create two dressed states \([G_2G_1\pm]\). So the Liouville pathway of the pure FWM process \( E_{F1} \) can be modified as \((DF1)\) \( \rho_{00} \rightarrow \rho_{01}' \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' \). Furthermore, the density matrix element \( \rho_{ij}^{(3)} \) related to the FWM process \( E_{F1} \) can be modified as \( \rho_{DF1}^{(3)} = G_{DF1} \exp [i(k_{DF1} \cdot r - \omega_1 t)]/A_{DF1} = \rho_{ij}^{(0)} \exp [i(k_{DF1} \cdot r - \omega_1 t + \phi_{51})] \) via the pathway \( DF1 \), where \( A_{DF1} = \Gamma_{00}d_3d_1 + |G_2G_1|^2/2 + |G_2G_1|^2/2d_2, \) \( d_2 = \Gamma_{20} + i(\Delta_1 + \Delta_2), \) and \( d_1 = \Gamma_{00} + i(\Delta_1 + \Delta_2) \).

Next, without the beams \( E_2', E_3, \) and \( E_4, \) the second pure FWM process \( E_{F2} \) from the ladder-type three-level subsystem \((0) - [1] - [2]) \) will be generated via the Liouville pathway \((F2)\) \( \rho_{00} \rightarrow \rho_{01} \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{10}' \), which, via the pathway \( F2 \), gives \( \rho_{DF2}^{(3)} = G_{DF2} \exp [i(k_{DF2} \cdot r - \omega_1 t + \omega_2 t)]/A_{DF2} = \rho_{ij}^{(0)} \exp [i(k_{DF2} \cdot r - \omega_1 t + \omega_2 t + \phi_{52})] \). The transmitted probe beam is detected by photodiode detector. The transmitted probe beam is detected by photodiode detector.

Table 1. Phase-Matching Conditions and Liouville Pathways of Coexisting FWM and SWM Processes

<table>
<thead>
<tr>
<th>FWM processes</th>
<th>( k_{F1} = k_{DF1} = k_3 - k_1' + k_1 )</th>
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<tbody>
<tr>
<td>( \rho_{00} \rightarrow \rho_{00}' \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' ) (F1)</td>
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<tr>
<td>( \rho_{00} \rightarrow \rho_{01}' \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' ) (DF1)</td>
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<td>( k_{F2} = k_{DF2} = k_1 + k_2 - k_1' )</td>
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<tr>
<td>( \rho_{00} \rightarrow \rho_{01} \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{10}' ) (F2)</td>
<td></td>
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<tr>
<td>( k_{DF2} = k_{S1} = k_3 - k_1' + k_1 + k_2 - k_1 )</td>
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<tr>
<td>( \rho_{00} \rightarrow \rho_{01}' \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' ) (S1)</td>
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<tr>
<td>( \rho_{00} \rightarrow \rho_{01}' \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' ) (DF2)</td>
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<table>
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<tr>
<th>SWM processes</th>
<th>( k_{S1} = k_{S2} = k_3 - k_1' + k_1 + k_2 - k_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{00} \rightarrow \rho_{01}' \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' ) (S1)</td>
<td></td>
</tr>
<tr>
<td>( \rho_{00} \rightarrow \rho_{01}' \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{00}' ) (S2)</td>
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Now, we focus on the two SWM processes generated from the two V-type four-level subsystems \((2) - [1] - [0] - [3] \) and \((4) - [1] - [0] - [3] \). On the one hand, when blocking the beams \( E_2 \) and \( E_4 \) and ignoring the dressing effect of the beam \( E_2 \), we can obtain the pure SWM process \( E_{S1} \) from the V-type four-level subsystem \((2) - [1] - [0] - [3] \) via the Liouville pathway \((S1)\) \( \rho_{00} \rightarrow \rho_{01} \rightarrow \rho_{20} \rightarrow \rho_{20}' \rightarrow \rho_{10}' \). This pure SWM via the pathway \( S1 \) has the corresponding fifth-order nonlinear density matrix element \( \rho_{ij}^{(5)} = G_{S1} \exp [i(k_{S1} \cdot r - \omega_1 t)]/A_{S1} = \rho_{ij}^{(0)} \exp [i(k_{S1} \cdot r - \omega_1 t + \phi_{51})] \), where \( G_{S1} = -iG_{1}G_{3}(G_{1}'G_{3}')^* \) and \( A_{S1} = \Gamma_{00}d_3d_5. \) On the other hand, when blocking the beams \( E_2 \) and \( E_4 \) and ignoring the dressing effect of \( E_4 \), we can obtain the other pure SWM process \( E_{S2} \) from the second V-type four-level subsystem \((4) - [1] - [0] - [3] \). By the Liouville pathway \((S2)\)

Fig. 1. (a) Experimental atomic system. (b) Square box pattern beam geometry used in the experiment. (c) Dressed-state picture for the experimental atomic system.
and $d$ and $\rho$ are not satisfied by two-photon absorption and gives rise to the FWM signal $A_{ij}\exp[\ph{\Delta \omega I}I]$ for $E_{S2}$, where $G_{S2} = -\Gamma G_1G_2^*G_4^*G_4^*$ and $A_{S2} = \Gamma_0d_2^*d_1d_4$. If only the beam $E_{s}'$ is clocked and the dressing effects of both beams $E_{s}$ and $E_{s}'$ is considered, the above-mentioned two pure SWM processes will coexist and dress each other. These processes can be described by the modified Liouville pathways $\rho_{S0}\rightarrow E_{s} \rightarrow \rho_{S0}\rightarrow P_{G_{0}G_{0}}\rightarrow \rho_{S0}\rightarrow P_{G_{0}G_{0}}$ (DS1) and $\rho_{S0} \rightarrow E_{s}' \rightarrow \rho_{S0}\rightarrow P_{G_{0}G_{0}}\rightarrow \rho_{S0}\rightarrow P_{G_{0}G_{0}}$ (DS2).

By the pathways DS1 and DS2, we can obtain $\rho_{S1}^E = G_{S1}\exp[\ph{\Delta \omega I}I]/A_{S1} = |\rho_{S1}^E|^2 \exp[\ph{\Delta \omega I}I]$ for $E_{S1}$, where $A_{S1} = \Gamma_0d_4^*d_1 + |G_2|^2/d_1 + |G_4|^2/d_4^2$ and $\rho_{S2}^E = G_{S2}\exp[\ph{\Delta \omega I}I]/A_{S2} = |\rho_{S2}^E|^2 \exp[\ph{\Delta \omega I}I]$ for $E_{S2}$, where $A_{S2} = \Gamma_0d_3^*d_1 + |G_4|^2/d_1 + |G_4|^2/d_4^2$, $d_1' = \Gamma_0 + i\Delta_1$, $d_2' = \Gamma_0 + i\Delta_2$, and $d_4 = \Gamma_0 + i\Delta_1 + \Delta_2$.

4. RESULTS AND DISCUSSION

When we investigate the interplay among generated MWM signals, the beam $E_{s}'$ is always turned off. Thus, there are two SWM ($E_{s}'$ and $E_{s}$) and one FWM ($E_{F1}$) signals propagating in the same direction simultaneously in the experiment. There exist several interesting physical processes in this composite system that describe the interplay among these MWM processes. First, when one SWM process $E_{S1}$ (or $E_{S2}$) and one FWM process $E_{F1}$ are overlapped in frequency, the interplay between them is dominated by the contributions of the optical pumping effect of the beams $E_{s}$ and $E_{s}'$ with large frequency detuning. The second one is the mutual dressing effects between two SWM processes $E_{S1}$ and $E_{S2}$ in the two V-type four-level subsystems, which perturb two SWM processes and modify the total amplitude of the SWM processes, especially when the two SWM signals are tuned together in frequency by adjusting frequency detunings. On the other hand, when the beam $E_{s}'$ is turned on and generated MWM signals (two SWM, $E_{S1}$ and $E_{S2}$, and one FWM, $E_{F2}$) overlap in frequency, we investigate temporal and spatial interferences among these three nonlinear optical processes.

First, when the beam $E_{s}'$ is blocked, the interplay between the coexistent MWM signals has been considered (Figs. 2 and 3). By two-photon Doppler-free configurations, there exist two ladder-type EIT windows in the five-level atomic system [Fig. 1(a)]. With the beam $E_{s}$ propagating in the opposite direction of the weak probe field $E_{1}$ in the $|0\rangle - |1\rangle - |2\rangle$ ladder-type subsystem, the two-photon Doppler-free condition is satisfied [20]. Thus, one EIT window (named EIT1, satisfying $\Delta_1 + \Delta_2 = 0$, the right peak in the lower curve of Fig. 2(a2)) forms, which is induced by the beam $E_{s}$. Similarly, the other EIT window (named EIT2, satisfying $\Delta_1 - \Delta_2 = 0$, the right peak in the lower curve of Fig. 2(a3)) appears in the $|0\rangle - |1\rangle - |4\rangle$ ladder-type subsystem induced by the beam $E_{s}$. In contrast, in the $|1\rangle - |0\rangle - |3\rangle$ V-type three-level subsystem, since the beam $E_{s}$ propagates in the opposite direction to $E_{s}$ and $E_{s}'$, the two-photon Doppler-free condition is not satisfied [20]. But similar saturated absorption will arise, which leads to the reduced absorption sharp peak in the probe transmission signal [the lower curve at $F = 3$ in Fig. 2(b1)]. Such a reduced absorption sharp peak can enhance FWM signals so that we refer to it as an EIT-like phenomenon [5,7].

On the other hand, there will be coexisting MWM processes in the above-mentioned EIT windows that generate signal beams at frequency $\omega_1$. First, when beams $E_{s}$, $E_{s}'$, and $E_{s}$ are turned on, an FWM signal $E_{F1}$ [Fig. 2(c1)] is generated with the phase-matching condition of $\kappa_{F1} = \kappa_{s} - \kappa_{s}' + \kappa_{s}$ [Fig. 2(d1)], as shown by the upper curve in Fig. 2(a1), which falls into the V-type EIT-like window [the lower curve in Fig. 2(a1)]. Physically, the interference between the beams $E_{s}'$ and $E_{s}$ produces a static population grating in the medium. Such a static grating diffracts $E_{s}$ and gives rise to the FWM signal $E_{F1}$. Since the beams $E_{s}$ and $E_{s}'$ are not satisfied by two-photon Doppler-free configurations, the FWM signal $E_{F1}$ is not Doppler free. Next, when the beam $E_{s}'$ is used at one upper transition of $|1\rangle - |2\rangle$, an SWM signal $E_{S1}$ [Fig. 2(c2)] satisfying the phase-matching condition of $\kappa_{S1} = \kappa_{s} - \kappa_{s}' + \kappa_{s} + \kappa_{s} - \kappa_{s}$ [Fig. 2(d2)] forms, as shown by the right peak of the upper curve in Fig. 2(a2), which falls into the $|0\rangle - |1\rangle - |2\rangle$.
ladder-type EIT window [the peak of the lower curve in Fig. 2(a2)]. The SWM signal $E_{S1}$ is Doppler free since it is in the EIT window. Finally, when the beam $E_1$ is turned on and the beam $E_2$ is blocked, another SWM signal $E_{S2}$ [Fig. 2(c3)] satisfying the phase-matching condition of $k_{S2} = k_1 - k_2$ corresponds to $\Delta k = k_1 + k_2 - k_3$ [Fig. 2(d3)] is generated, as shown by the right peak of the upper curve in Fig. 2(a3), which falls into the $|0\rangle - |1\rangle - |4\rangle$ ladder-type EIT windows [the right peak of the lower curve in Fig. 2(a3)]. Also, the SWM signal $E_{S2}$ is Doppler free since it is in the EIT window. Figure 2(a4) shows the generated MWM signals and the corresponding EIT windows when five laser beams mentioned above are all turned on.

Figure 2(b1) shows two groups of FWM signals (upper curve) with the same phase-matching condition $k_1 = k_3 - k_4 + k_4$, corresponding to the scanning of the probe beam $E_1$ (lower curve) around the transitions from $|S\rangle_{1/2}, F = 2\rangle$ to $|S\rangle_{1/2}, F = 1, 2, 3\rangle$ and from $|S\rangle_{1/2}, F = 3\rangle$ to $|S\rangle_{3/2}, F = 2, 3, 4\rangle$, respectively. The probe transmission spectrum shows the increased absorption in the left $|S\rangle_{1/2}, F = 2\rangle$ dip [the lower curve in Fig. 2(b1)] due to the optical pumping effect of the beams $E_1$ and $E_2^\prime$ from the ground state $|S\rangle_{1/2}, F = 3\rangle$. The increased absorption of the probe transmission spectrum is regarded as being like electromagnetically induced absorption, which is beneficial to the generation of the FWM signal. Another effect of optical pumping is to generate a multipeak FWM signal corresponding to the hyperfine-level transition from $|S\rangle_{1/2}, F = 2\rangle$ to $|S\rangle_{3/2}, F = 1, 2, 3\rangle$ [the upper curve in Fig. 2(b1)]. Those peaks of the FWM signal from left to right correspond to $|S\rangle_{3/2}, F = 3\rangle$ and $|S\rangle_{3/2}, F = 2\rangle$ with a separation of about $63\text{MHz}$. More attractively, although without satisfying the Doppler-free condition for the FWM process $E_{F1}$ due to opposite propagation of beams $E_1$ and $E_3 + E_3^\prime$ in the V-type subsystem, two sharp FWM signal peaks form at $|S\rangle_{1/2}, F = 3\rangle$ because of enhanced atomic coherence in the EIT-like windows. Those two peaks correspond to $|S\rangle_{3/2}, F = 2\rangle$ and $|S\rangle_{3/2}, F = 3\rangle$ and their separation of about $362\text{MHz}$ approximately equals the energy spacing between $|S\rangle_{1/2}, F = 3\rangle$ and $|S\rangle_{3/2}, F = 2\rangle$. Figure 2(b2) [or 2(b3)] shows the SWM signal $E_{S1}$ (or $E_{S2}$) coexisting with the FWM signal $E_{F1}$ at $|S\rangle_{1/2}, F = 3\rangle$ when the $E_2$ (or $E_4$) beam is turned on.

In the following, we show the interplay and competition among EIT windows as well as generated MWM signals by changing different frequency detuning. In the five-level atomic system shown in Fig. 1(a), the two EIT windows (EIT1 and EIT2) in the two-ladder-type subsystems and the EIT-like window in the V-type subsystem will simultaneously form. Since the EIT-like window is much weaker than the two EIT windows, as shown by the lower curves in Fig. 3, we investigate only the interplay between the two EIT windows. As shown in Fig. 3, by changing the frequency detuning $\Delta_4$ with fixed frequency detuning $\Delta_5$ and scanning the probe frequency detuning $\Delta_5$, the two EIT windows can be controlled to overlap or separate. The lower curves in Figs. 3(a1)–(a7) show two modified EIT windows at different positions of the probe transmission profile determined by the EIT condition $\Delta_4 = -\Delta_5$ and $\Delta_4 = -\Delta_5 - \Delta_4$, respectively. When the two EIT windows overlap (i.e., $\Delta_4 = \Delta_4'$) and dress each other, as shown in Fig. 3(a2), the area of the EIT windows is slightly larger than that when the two EIT windows are far away [Fig. 3(a3)].

The upper curves in Fig. 3 show the interplay and competition between the generated MWM signals. We can identify and control them by selectively detuning different laser frequencies and blocking different laser beams. First, when we turn on five laser beams, except for $E_3$, and only change the frequency detuning $\Delta_4$, the SWM signals $E_{S2}$ will shift from right to left through the SWM signals $E_{S1}$ and the FWM signals $E_{F1}$, as shown in Figs. 3(a1)–(a7). Here, we discuss only the interplay between two SWM signals. In the expression $\rho_{S1}$ of the dressed SWM process $E_{DS1}$, the beam $E_2$ is both the dressing field of the $E_{DS1}$ signal and the field to generate it, so we refer to the dressing effect of the beam $E_2$ as self-dressing. Whereas, the beam $E_1$ is only the dressing field, so we refer to the dressing effect of $E_1$ as external dressing. Similarly, for the second dressed SWM process $E_{DS2}$, $E_1$ and $E_2$ are the self-dressing field and the external-dressing field, respectively. The enhancement or suppression of the wave-mixing signal is induced by the external-dressing field. For the external dressing by the beam $E_4$ in $E_{DS1}$, according to $\rho_{S1}$, we can obtain the suppression condition as $\Delta_4 = -\Delta_4$. Similarly, for the external dressing by the beam $E_2$ in $E_{DS2}$, the suppression condition is $\Delta_4 = -\Delta_4$ according to $\rho_{S2}$. In particular, comparing Fig. 3(a2) with Fig. 3(a3), when the two SWM signals $E_{S1}$ and $E_{S2}$ overlap with each other [the upper curve in Fig. 3(a2)], i.e., satisfying $\Delta_4 = \Delta_4' = -\Delta_4 = -\Delta_2$, there exists a maximum suppression for both SWM signals. Physically, when $\Delta_2 = \Delta_4 = -\Delta_4 = -\Delta_5$, since the beam $E_1$ touches the virtual energy level created by the dressing fields $E_2$ and $E_4$ as shown in Fig. 1(c), the mutual suppression of the two SWM signals is strongest. Here, two EIT windows corresponding to the two ladder subsystems $|0\rangle - |1\rangle - |4\rangle$ and $|0\rangle - |1\rangle - |4\rangle$ also overlap each other since two EIT conditions are simultaneously satisfied by $\Delta_4 = -\Delta_4' = -\Delta_4 = -\Delta_2$ [the lower curve in Fig. 3(a2)].
Next, when the beam \( E_1 \) is blocked (or the beam \( E_2 \) is blocked), we observe the interplay between the FWM signal \( E_{F1} \) and the SWM signal \( E_{S1} \) (or \( E_{S2} \)) as shown in the upper curves of Figs. 3(b1)–(b7) [or Figs. 3(c1)–(c7)]. In the upper curves of Figs. 3(b1)–(b7) [or Figs. 3(c1)–(c7)], the fixed peaks correspond to the FWM signals \( E_{F1} \) and the shifting peaks correspond to the SWM signal \( E_{S1} \) for different frequency detuning \( \Delta_2 \) with the beam \( E_4 \) blocked (or the SWM signal \( E_{S2} \) for different frequency detuning \( \Delta_4 \) with the beam \( E_2 \) blocked). By comparing Fig. 3(b5) with Fig. 3(b6) [or by comparing Fig. 3(c4) with Fig. 3(c5)], one can see that the total intensity of the FWM signal \( E_{F1} \) and the SWM signal \( E_{S1} \) (or the SWM signal \( E_{S2} \)) is enhanced when they overlap each other. This is because the optical pumping effect of beams \( E_3 \) and \( E_4 \) with large frequency detuning destroys the suppression condition mentioned above. In detail, first, due to the optical pumping effect of the beams \( E_3 \) and \( E_4 \) with large frequency detuning, the FWM signal \( E_{F1} \) is largely enhanced. Second, when the SWM signal \( E_{S1} \) (or the SWM signal \( E_{S2} \)) is tuned to overlap with \( E_{F1}, E_{S1} \) (or \( E_{S2} \)) is also maximally enhanced. Thus, the overlapped \( E_{F1} \) and \( E_{S1} \) (or \( E_{S2} \)) show the enhancement, since the increased intensity of \( E_{F1} \) and \( E_{S1} \) (or \( E_{S2} \)) caused by the optical pumping effect is larger than the reduced intensity of them caused by the mutual suppression.

Finally, when all laser beams, including beam \( E_2' \), are turned on and two EIT windows in the five-level optical system overlap each other in frequency, temporal and spatial interferences between nonlinear optical processes are also investigated. On the one hand, the FWM signal \( E_{F2} \) is also Doppler free and highly efficient since it is in the \([0] \rightarrow [1] \rightarrow [2] \) EIT window. On the other hand, due to \( G_2 \gg G_1 \), the SWM signal \( E_{S1} \) can coexist with the FWM signal \( E_{F2} \) in the \([0] \rightarrow [1] \rightarrow [2] \) EIT window. By changing the frequency detuning \( \Delta_4 \), the \([0] \rightarrow [1] \rightarrow [4] \) EIT window can be shifted toward the \([0] \rightarrow [1] \rightarrow [2] \) EIT window. When the two EIT windows overlap with each other, temporal and spatial interferences of two SWM (\( E_{S1}, E_{S2} \)) and one FWM (\( E_{F2} \)) can be observed. If we neglect the MWM processes that are either weak or propagating in other directions, the coexisting one FWM and two SWM signals give the total detected intensity as \( I(\tau, r) \propto |\rho_{DF2}^{(3)} + \rho_{DS1}^{(5)} + \rho_{DS2}^{(5)}|^2 = A_1 + A_2 \cos(\varphi_{32} - \varphi_{31} + \varphi) + A_3 \cos(\varphi_{32} - \varphi_{31} + \varphi), \) where \( A_1 = |\rho_{DF2}^{(3)} + \rho_{DS1}^{(5)}|^2 + |\rho_{DS2}^{(5)}|^2 + 2|\rho_{DS1}^{(5)}||\rho_{DS2}^{(5)}| \cos(\varphi_{31} - \varphi_{32}), \) \( A_2 = 2|\rho_{DF2}^{(3)}||\rho_{DS1}^{(5)}|, \) \( A_3 = 2|\rho_{DF2}^{(3)}||\rho_{DS2}^{(5)}|, \) \( \varphi = \Delta k \cdot r - \omega_s \tau, \) and \( \Delta k = (\mathbf{k}_2 - \mathbf{k}_1) - (\mathbf{k}_4 - \mathbf{k}_3). \) Since the generated SWM signals \( E_{S1} \) and \( E_{S2} \) have strong correlation characteristics, the equation about \( I(\tau, r) \) can be rewritten as \( I(\tau, r) \propto A_1 + (A_2 + A_3) \cos(\varphi_{32} - \varphi_{31} + \varphi), \) where \( \varphi_5 = \varphi_{31} \neq \varphi_{32}. \)

From the equation for \( I(\tau, r) \), one can see that the total signal has an ultrafast time oscillation with a period of \( 2\pi/\omega_2 \) and spatial interference with a period of \( 2\pi/\Delta k \), which form a spatiotemporal interferogram. Figure 4 shows a two-dimensional interferogram pattern [Fig. 4(a)] and its projections on time [Fig. 4(b)]. Figure 4(b) depicts a typical temporal interferogram with the temporal oscillation period of \( 2\pi/\omega_2 = 2.587 \) fs corresponding to the \( 5P_{3/2} \) to \( 5D_{3/2} \) transition frequency of \( \Omega_2 = 2.427 \) fs\(^{-1}\) in \( ^{85}\text{Rb} \). This gives a technique for precision measurement of the atomic transition frequency in the optical wavelength range. The spatial interference is determined by \( \Delta k \). Thus, 1.2 interference fringes form, as shown by the structure along the \( r \) direction in Fig. 4(a) since we have \( 2\pi/\Delta k = 3 \) mm in our experiment. The spatial interference pattern can change from constructive to destructive at the center of the beam profile (\( r = 0 \)) when the phase delay on the \( E_r' \) beam is varied. The solid curve in Fig. 4(b) is the theoretical calculation from the full density matrix equations. It can be seen that the theoretical results are in good agreement with the experimental results.

5. CONCLUSION
In summary, several generated MWM processes can coexist in a five-level atomic system with carefully arranged laser beams. Two SWM signals (\( E_{S1} \) and \( E_{S2} \)) falling into each EIT window and one FWM signal (\( E_{F1} \)) in an EIT-like window can be tuned to overlap or separate by varying the corresponding frequency detuning. The experimental observation has clearly shown the interplay among these MWM signals, including the enhancement between the FWM process \( E_{F1} \) and any of the SWM processes due to the optical pumping effect and the mutual suppression of two SWM signals caused by the mutual dressing. On the other hand, when two EIT windows merge in frequency, the temporal and spatial interferences among the two coexistent SWM signals and the FWM signal \( E_{F2} \) are also reported. The generated spatiotemporal interferogram with a femtosecond time scale can be used to determine
the optical transition frequency. In this paper, different higher-order nonlinear optical processes coexist and compose correlative and coherent multichannel signals together with the probe beam. Since such multichannel signals can be used in the optical switch, multichannel optical router, and optical logical calculation, it is very important to understand and optimize higher-order nonlinear optical processes by research on their interplay and interference. In addition, the work in this paper also opens the door for other fields, such as optical imaging storage and quantum information processing.

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