Optimization of an erbium-doped fiber amplifier with radial effects
Cheng Cheng a,b,*, Min Xiao b

a Department of Applied Physics, Zhejiang University of Technology, Xiaoheshan, Hangzhou 310023, China
b Department of Physics, University of Arkansas, AR 72701, USA

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Abstract
Applying an inversing method and a genetic algorithm, two radial distributions, i.e., a core graded-index and erbium-doped concentration, are optimized for an erbium-doped fiber amplifier (EDFA) in a two-level model under single-mode condition and weakly guided approximations. There is evidence to show that the core graded-index has obvious influence on the gain bandwidth of the EDFA, and similarly, the radial distribution of the erbium concentration has effect on the bandwidth, while no effect on the gain. As an example, we provide an optimized single-fiber EDFA with the graded-index and the erbium concentration distribution, which is characterized with a 33.5 dB gain, a 30 nm bandwidth, and a noise figure of 3.55 dB. The broader bandwidth is one of the outstanding advantages over current single-fiber EDFAs with uniform radial distributions.

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Keywords: Erbium-doped fiber amplifier; Radial effect; Optimization; Gain; Bandwidth

1. Introduction
Erbium-doped fiber amplifiers (EDFAs), as key components in wavelength division multiplexing (WDM) systems in optical telecommunication, have received great attention over the past 10 years. The rapid growth and the future commercial importance of multi-wavelength optical networking create strong incentives for the development of EDFAs with higher gain and broader bandwidth. Many interesting research results were reported in recent years. For example, Chernyak and Qian [1] established modeling high-concentration L-band EDFA at high optical powers based
on inversion function. Johannes [2] reported spatial distribution effects and laser efficiency in Er/Yb-doped fibers. Wei et al. [3] utilized a genetic algorithm to optimize multistage erbium-doped fiber amplifiers systems with complex structures. A remarkable modeling was introduced by Giles and Desurvire [4], which established the propagation and rate equations for a two-level homogeneous laser medium. This approximated model is suitable for analyzing open-loop optical fiber amplifiers and also the steady-state operation of the optical fiber networks. For radial effects of the fibers on EDFA performances of the, there have been a few researches reported in recent years. One of the researches was presented by Martin [5], who studied erbium transversal distribution effects on EDFA performances of the, there have been a few researches reported in recent years. One of the researches was presented by Martin [5], who studied erbium transversal distribution influence on the effectiveness of a doped fiber by introducing a simple mathematical function, and significant gain differences were observed in active fibers. Another work was reported by Mikko and Simo [6]. They presented a spatial-mode model of EDFA gain dynamics, which includes radial distribution of dopant and modal intensity in the optical fiber. The relaxation method was used in the iterative calculation of the propagating equations in order to resolve the forward and backward traveling amplified spontaneous emission in a two-level laser system in steady-state. However, in all of the above researches on radial effects, the methods they adopted can all be ascribed as a “direct method”, i.e., solving the propagation/rate equations to obtain a direct solution by the given radial profiles or parameters. Obviously, such designed EDFAs are usually not optimal because the radial profiles or the parameters assumed in advance are not always the best for the designed EDFA gain and bandwidth. Moreover, it is impossible to attempt all radial profiles or parameters for determining the most suitable gain bandwidths.

In recent years, global optimization methods have been developed rapidly and one of these optimization technologies is called genetic algorithm. The genetic algorithm can avoid falling into a “local minimum or maximum” during global researches. In this paper, we present a method, i.e., an “inverse method” combined with the genetic algorithm, to determine the optimal radial distributions for both the erbium concentration and the core refractive index. Furthermore, an optimized EDFA with the optimal radial distributions is obtained by solving the equations for the optical power propagated in the core and for the upper-level population of the erbium ions in a two-level system under a steady-state condition and a weakly guided approximation. Finally, the radial effects on the gain bandwidths of the EDFA are discussed by comparing the graded-index with a step-index in the EDFA.

2. Equations

In general, three \(4\text{I}_{15/2}, 4\text{I}_{13/2}, 4\text{I}_{11/2}\) or more (plus \(4\text{I}_{9/2}, \ldots\)) levels of the Er\(^{3+}\) ions should be considered in a detail modeling. In some cases, however, a two-level modeling is sufficient to describe the optical power propagating in the fiber. This is because the large photon energy of the phosphate glass decayed from the upper level \(4\text{I}_{9/2}\), and \(4\text{I}_{11/2}\) is dominated by fast non-radioactive transitions, with rates of \(3.6 \times 10^{5}\) and \(3.5 \times 10^{6}\) s\(^{-1}\) [7], respectively. Due to the fast non-radioactive transitions, the populations at levels \(4\text{I}_{9/2}\) and \(4\text{I}_{11/2}\) are less than 0.2% and 0.0001% of the total Er\(^{3+}\) ions, respectively, for a high Er\(^{3+}\) concentration sample of phosphate glass with 50–100 mW pump power [8].

With a circularly symmetric fiber, considering radial effects, the power propagation equation in a two-level system is given by the following [4]:

\[
\frac{dP_k(z)}{dz} = u_k \sigma_{ok} \int_0^\infty i_k(r,t)n_2(r,z,t) \left[P_k(z) + m h v_k A v_k \right] 2\pi r \, dr - u_k \sigma_{ok} \int_0^\infty i_k(r,t)n_1(r,z,t) P_k(z) 2\pi r \, dr \\
- u_k i_k P_k(z),
\]

where \(P_k\) is the power propagated in the fiber with the frequency \(v_k\); each beam is traveling either in the forward \((u_k = +1)\) or backward \((u_k = -1)\) direction; \(\sigma_{ok}\) is the emission cross-section (absorption cross-section); \(i_k\) is the normalized transverse mode intensity; \(n_{1,2}\) are the populations of the ground level \(4\text{I}_{15/2}\) and the upper-level \(4\text{I}_{13/2}\).
respectively; \( l_k \) is excess fiber loss per length; and \( \Delta v_k \) is an effective noise bandwidth. \( m h v \Delta v_k \) is the contribution of spontaneous emission from the local \( n_2 \) population, here \( m = 2 \) since amplifier spontaneous emissions (ASEs) have both the forward and backward components. In Eq. (1), the first term is the contributions from the emission and spontaneous decay, the second term is from the absorption loss, and the last one represents propagation loss in the fiber. Note that Eq. (1) has a modification from [4] by adding the radial distribution, i.e., \( n_2 = n_2(r,z,t) \).

In order to investigate the radial effects on amplifier characteristics, the erbium concentration is allowed to search in a range as high as \( 1 \times 10^{20} \text{cm}^{-3} \). Therefore, ion–ion interactions have to be included in this modeling. The most important ion–ion interaction is cooperative up-conversion, in which two excited erbium ions interact with each other, and one erbium ion transfers its energy to the opponent, leaving itself in the ground state and the other ion in a higher excited state. The acceptor may then relax to the ground state via radiation decay or to the original excited state via radiation /non-radiation decay. When the pumping power increases and more ions remain in the excited states, this cooperative up-conversion interaction will be prominent, limiting the performance of erbium-doped amplifiers. Including the cooperative up-conversion, the rate equation for the population of the erbium upper-level \( ^4 \! I_{13/2} \) can be written as follows:

\[
\frac{\partial n_2(r,t)}{\partial t} = \sum_k \frac{P_k i_k \sigma_{ak}}{h \nu_k} n_1 - \sum_k \frac{P_k i_k \sigma_{ek}}{h \nu_k} n_2 - \frac{n_2}{\tau} - C_{22} n_2^2 \\
\equiv S_{ak} n_1 - S_{ek} n_2 - \frac{n_2}{\tau} - C_{22} n_2^2, \tag{2}
\]

where \( \tau \) is the lifetime of the metastable level, \( C_{22} \) is the coefficient of the cooperative up-conversion, \( i_k = i_k(r,t), n_{1,2} = n_{1,2}(r,z,t) \). With steady-state approximation to the upper-level population, we have

\[
n_2(r) = \frac{1}{2C_{22}} \left\{ \left( [S_{ak} + S_{ek} + 1/\tau] \right)^2 + 4C_{22} E_r(r) S_{ak} \right\}^{1/2} - (S_{ak} + S_{ek} + 1/\tau), \tag{3}
\]

where the total concentration \( E_r(r) = n_1 + n_2 \). There are several functional forms that can be chosen to describe the radial distribution of the erbium ions. We use an exponential function as follows:

\[
E_r(r) = E_{r0} \exp \left( -\frac{r}{\beta} \right)^{\delta} (\beta, \delta > 0), \tag{4}
\]

where \( E_{r0} \) is the center concentration, \( \beta \) and \( \delta \) are the parameters required to be optimized in the genetic algorithm. Function (4) can show various radial profiles with different values of \( \beta \) and \( \delta \). At \( r = 0 \), \( E_r \) reaches a maximum, which is coincident with actual erbium-doped concentration.

For the fibers with radial distributions of the core refractive index (i.e., graded-index fibers), some parameters (e.g., cut-off frequency) describing light propagation in a step-index are no longer available since the graded-index alters with the core radius. Some detail analyses on this aspect were already developed, and one of these is a usual variational method [9], in which a graded-index is reduced into an equivalent step-index. Here, a useful formula for the core refractive index is given by

\[
n^2_{\text{core}} = n^2_{\text{clad}} \left[ 1 + 2\Delta \! H \left( \frac{r}{a} \right) \right] (r \leq a), \tag{5}
\]

where \( a \) is the fiber core radius and \( \Delta n \) is the relative refractive-index difference. The function \( H \) has the following form:

\[
H \left( \frac{r}{a} \right) = 1 - \left( \frac{r}{a} \right)^{\alpha}, \quad 0 < \alpha < \infty. \tag{6}
\]

In Eq. (6), the \( H \) function converges to one when \( \alpha \to \infty \), then a graded-index reduces to a step-index. Accordingly, an equivalent core radius is \( a_e = a \left( \frac{\alpha+2}{1+\alpha} \right)^{1/2} \), an equivalent normalized frequency is \( V_e = \frac{r}{(1+\alpha/2)^{1/2}} = \frac{2na_e\Delta n_e}{\lambda} \), where \( V \) is a normalized frequency, and \( N A_e \) is an equivalent numerical aperture. Under these notations, a graded-index can be treated as an equivalent step-index.

The modeling subjects to two premises: (1) a weakly guided approximation; (2) a signal single-mode condition, by choosing the refractive difference \( \Delta n = 0.0063 \) and the core radius \( a = 4.1 \mu m \), to be consistent with the current fiber data of the Lucent. Only the positive pumping direction, i.e.,
$u_k = +1$ in Eq. (1), is implemented in the current work.

Under the condition of the constant pumping power and signal power, the optical power $P_k$ in Eq. (1) depends on six parameters ($E_{r0}$, $\beta$, $\delta$, $\alpha$, $L$ and $\lambda_p$), in which $L$ is the length of the fiber, and $\lambda_p$ is the pumping wavelength. In programming, the genetic algorithm is taken as a main program and the propagation/rate equations as a sub-program. The main program is used to implement an inversing process, i.e., from the objective function values (e.g., gain, bandwidth) output by the sub-program to inversely obtain the optimal parameters that determine the objective function values. The random values of the six parameters ($E_{r0}$, $\beta$, $\delta$, $\alpha$, $L$, $\lambda_p$) in the main program are distributed in the searching ranges (Table 1). In the genetic algorithm, each optimizing parameter is randomly encoded into binary sequences called a gene, and a set of genes form a chromosome. Then, these decimal chromosomes are transferred into binary forms and digital number of genes (or byte number) is defined according to the required precision. These genes randomly group into hundreds of chromosomes. The computation procedure then calls the subprogram and transfers the binary chromosomes into the decimal system, and finally calculates corresponding objective functions, which are then ranked according to their functional values. The chromosomes with large objective functions are kept and those with small objective functions are discarded. The program chooses parents by the “binary tournament selection rule” to form offspring chromosomes. Through gene mutations and after $\sim$30 generations, one can reach the global maximum of the objective function. In this way, the chromosome with the best gene combination can be found, corresponding to the six optimized parameters. More detailed description about the genetic algorithm can be found in [10–12]. Table 2 lists the data used in the genetic algorithm.

In the genetic algorithm, the objective function is defined as follows:

$$f_{\text{obj}} = \Delta(E_{r0}, \beta, \delta, \alpha, L, \lambda_p) + \gamma G_s(E_{r0}, \beta, \delta, \alpha, L, \lambda_p) \quad (G_s > 30 \text{ dB})$$

where $\Delta$ is the $-3$ dB bandwidth, $G_s$ is the signal gain and $\gamma$ is introduced to balance the desired gain and bandwidth for the amplifier. Choosing different $\gamma$ values meet different requirements for reaching a high gain and/or a broadband, here we choose $\gamma = 0.15 \text{ nm/dB}$.

In the sub-program, the pumping wavelength $\lambda_p$ is searched from 1450 to 1600 nm (Table 1), always spacing 20 nm shorter than the signal wavelength. Here, the lowest scanning wavelength is chosen at 1450 nm but not at usual 980 nm or lower wavelengths. If it is chosen at the 980 nm or lower ones, a three-level system, instead of the two-level system used in our modeling, is required due to 980 nm with a higher level, which is beyond our investigation. Using emission/absorption cross-sections as functions of wavelengths are adopted from [13] for Al/P-silica fiber. Due to the randomness of the produced genes, some bad chromosomes will induce the implementation into an infinite recycle. As a result, the temporal cost on the computation by a 3 GHz Pentium IV is estimated to be over 10 h per point in Figs. 1–3. The working parameters of the EDFA used in the modeling are listed in Table 3.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>The searching ranges of the six optimizing parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Erbium concentration, $E_{r0}$ (cm$^{-3}$)</td>
<td>(0.5–10) $\times 10^{19}$</td>
</tr>
<tr>
<td>$\beta$ (pm)</td>
<td>0.2–20</td>
</tr>
<tr>
<td>$\delta$ (m)</td>
<td>0–8</td>
</tr>
<tr>
<td>$\alpha$ (m)</td>
<td>0–64</td>
</tr>
<tr>
<td>Fiber length, $L$ (m)</td>
<td>4–80</td>
</tr>
<tr>
<td>Pumping wavelength, $\lambda_p$ (nm)</td>
<td>1450–1600</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Data used in the genetic algorithm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of chromosomes</td>
<td>1600</td>
</tr>
<tr>
<td>Single crossover point</td>
<td>Random</td>
</tr>
<tr>
<td>Comparing crossover point</td>
<td>0.9</td>
</tr>
<tr>
<td>Mutation probability (%)</td>
<td>2</td>
</tr>
<tr>
<td>Number of generations</td>
<td>30</td>
</tr>
<tr>
<td>Number of genes</td>
<td>6</td>
</tr>
<tr>
<td>Number of bytes per gene</td>
<td>10</td>
</tr>
</tbody>
</table>
Fig. 1. Radial profiles.

Fig. 2. Effects of $\beta$ on the gain and the bandwidth.
3. Results and discussion

Table 4 lists some optimized results of the EDFA and the required radial distribution of the erbium concentration is shown in Fig. 1. Note that these data are obtained under the two premises discussed above and based on the working conditions listed in Table 3. The profile of the transversal modal intensity \( i_k \) with a fundamental mode \( \mathrm{LP}_{01} \) is also illustrated in Fig. 1, where \( i_k \) is a normalized function proportional to the zeroth-order Bessel function \( J_0 \). To compare an overlapping degree between the transversal mode and the erbium concentration distribution, numerically, the overlapping factor is \( \Gamma_k = 0.5404 \) (\( \Gamma_k(v) = \frac{1}{E_{\mathrm{T}}} \int_0^a i_k(r)E_r(r)2\pi r \, dr \)), where \( E_{\mathrm{T}} \) is a total erbium concentration.

Table 3
The working parameters of the EDFA used in the modeling

<table>
<thead>
<tr>
<th>Pumping power, ( P_p ) (mW)</th>
<th>Signal power, ( P_s ) (dBm)</th>
<th>Upper-level lifetime, ( \tau ) (ms)</th>
<th>Rate coefficient, ( C_{22} ) (cm(^3)/s)</th>
<th>Fiber loss, ( l_k ) (dB/m)</th>
<th>Refractive-index difference, ( \Delta n )</th>
<th>Core radius, ( a ) (( \mu )m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>-30</td>
<td>10</td>
<td>( (0.5-2) \times 10^{-18} ) [14]</td>
<td>0.03</td>
<td>0.0063</td>
<td>4.1</td>
</tr>
</tbody>
</table>

\[ a \] Here for \( E_{\mathrm{T}} = (0.7-4) \times 10^{20} \) cm\(^{-3}\). A linear interpolation or extrapolation is used when \( E_{\mathrm{T}} \) is in or out of \( (0.7-4) \times 10^{20} \) cm\(^{-3}\).

Table 4
The optimized results of the EDFAs

<table>
<thead>
<tr>
<th>Central concentration, ( E_{\mathrm{T}} ) (cm(^{-3}))</th>
<th>( \beta ) (( \mu )m)</th>
<th>( \delta )</th>
<th>( \chi )</th>
<th>Pump wavelength, ( \lambda_p ) (nm)</th>
<th>Gain, ( G_s ) (dB)</th>
<th>Band-width, ( \Delta ) (nm)</th>
<th>Noise figure, ( NF ) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 6.43 \times 10^{19} )</td>
<td>1.846</td>
<td>1.820</td>
<td>0.108</td>
<td>1503.4</td>
<td>33.5</td>
<td>30.0</td>
<td>3.55</td>
</tr>
</tbody>
</table>
Fig. 2 shows the effects of the parameter $\beta$ (describing erbium radial distribution, as defined in Eq. (4)) on the gain bandwidths. The solid line with $\beta = 1.846$ corresponds to the optimal erbium distribution with the gain of $G_e = 33.5$ dB and the bandwidth of $\Delta = 30$ nm. With various $\beta$ values, the radial distributions of the erbium concentrations show different profiles, consequently, the gain and the bandwidth also increase or decrease. Furthermore, the radial factor $\delta$ in Eq. (4) is also altered to verify the gain bandwidths. However, it is shown that the gain is almost unchanged with varied $\delta$, although the radial profile of the erbium concentration is changed. Note that the fact that total erbium concentrations, i.e., the areas under the profiles, are also altered with various $\beta$, $\delta$ values. As a comparison, we adopt a uniform erbium concentration while keeping the total concentration the same. It is shown that the gain remains almost unchanged (to 33.4 dB from the optimal 33.5 dB), while the bandwidth decreases to 20 nm from the optimal 30 nm. Different total erbium concentrations possess different gain bandwidths. On the basis of these data, one can conclude that the total erbium concentration is a key to determine gain bandwidths in the EDFA. The radial distributions of the erbium concentration have effect on the bandwidth, but with very little effect on the gain.

Fig. 3 shows the radial effects of the core refractive index on the gain bandwidth, where the solid line corresponds to the optimal profile with a small $\alpha = 0.108$. Notice that here $\alpha$ is nearly zero considering the range of $0 \sim 64$. However, $\alpha = 0$ will result into $n_{\text{core}}^2 = n_{\text{clad}}^2$ according to definition Eqs. (5) and (6), which is not allowed in fiber propagations. It is even unexpected that the optimal profile shows a spire formation. In contrast, other two profiles seem to be more reasonable. However, analyzing the equivalent normalized frequency $V_e = \frac{r}{(1+2/n)^{1/2}} = \frac{2n a N A_e}{\lambda_0}$, one can obtain the fact that a smaller $\alpha$ will result in a smaller $V_e$. Therefore, it is easier to satisfy the requirements for single mode $V_e < 2.4048$ under the condition that the equivalent numerical aperture $N A_e$ is kept constant due to $\Delta n = 0.0063$. As a result, it will make the signals move towards a longer wavelength, where the ratio of the emission cross-section to the absorption cross-section possesses bigger values than that in the shorter wavelength region. In our computation, it is shown that the signal wavelength with a peak gain stretches to 1605 nm, which belongs to L-band. This is one of the advantages over usual step-index EDFAs. On the contrary, when $\alpha \rightarrow \infty$ (which is equivalent to a step-index), the gain $G_e$ decreases to 30.0 dB from the optimal 33.5 dB and the bandwidth $\Delta$ to 5.0 nm from the optimal 30 nm. In general, the limiting differences between the graded-index and the step-index reach to $(G_{\text{gra}} - G_{\text{ste}})_{\text{max}} \approx 4$ dB and $(\Delta_{\text{gra}} - \Delta_{\text{ste}})_{\text{max}} \approx 25$ nm. In other words, for the optimized step-index fibers, the gain decreases to $\sim 85\%$ and the bandwidth to $\sim 20\%$ compared to the optimized graded-index fibers under the same objective function.

Sometimes, e.g., in [5], the overlapping factor $\Gamma_k$ was utilized as a criterion to estimate the gains or bandwidths of the amplifiers. In fact, $\Gamma_k$ is inversely proportional to the wavelength. If one redefines an overlapping factor as $\Gamma'_k(v) = \frac{1}{\sqrt{n_e}} \int_0^\infty i_k(r) n_2(r) 2 \pi r \ dr$, it is more significant to investigate the relations between the $\Gamma'_k$ and the gain bandwidth.

4. Conclusions

Radial effects of an erbium-doped fiber on the characteristics of the EDFAs were investigated by solving optical propagation equations and rate equations in a two-level model. Applying an inverting method and a genetic algorithm, we determined two optimal radial distributions (a core graded-index and an erbium-doping concentration) under the single-mode condition and the weakly guided approximation. We have shown that the core graded-index has obvious influences on the gain bandwidths of the EDFA, similarly, the radial distribution of the erbium concentration has effect on the bandwidth while no effect on the gain. As an example, we demonstrated an optimized single-fiber EDFA with specially designed graded-index and the erbium concentration...
distributions, which is characterized with the 33.5 dB gain, the 30 nm bandwidth, and the noise figure of 3.55 dB. Comparing to a similar EDFA with the step-index, the increased gain and bandwidth reach to 3.5 dB and 25 nm, respectively. The broader bandwidth is one of the outstanding advantages over current single-fiber EDFAs with uniform distributions. Such global optimization procedure has many advantages over the traditional ones by optimizing one parameter at a time and can have great impacts in designing better EDFAs to achieve desired gain or bandwidth.

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References