Generation of a two-mode generalized coherent state in a cavity QED system

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Abstract

A collection of three-level atoms in Λ-configuration confined in a bimodal cavity and strongly driven by a classical field is considered and it is shown that under certain conditions cavity field can evolve in to a SU(2) generalized coherent state. The realization of SWAP gate operation is also demonstrated.

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1. Introduction

In recent years the investigation of nonclassical states of electromagnetic field and their generation in various physical processes has been an exciting topic in the quantum optics literature [1]. Among other states, the two-mode squeezed state generated by the nonlinear interaction [2] of type: \( H = aa^\dagger b^\dagger + \text{H.c.} \) is found to be highly entangled state under certain conditions, which can be employed for verification of Bell’s inequalities [3] and implementing quantum teleportation [4]. The two mode squeezed operator \( S(β) = e^{(βa^\dagger b^\dagger + β^* ab)} \) is the generator of SU(1,1) generalized coherent state (GCS) [5,6]. The fault tolerant phase-gate operation using such states was recently demonstrated [7]. On the other hand the two mode operator of type \( S(χ) = e^{(χa^\dagger b^\dagger + χ^* ab^\dagger)} \) is a generator of SU(2) GCS. The SU(2) GCS exhibits sub-Poissonian statistics and anti-correlations [8]. Generation of such states in parametric process and interaction of collection of two-level atoms with a single mode radiation field has been reported [5,6,8]. Our aim in this work is how to realize this generator of SU(2) GCS in a cavity quantum electrodynamics (CQED) system.

A large number of proposals for quantum state engineering, entanglement generation for quantum information processing were given [9,10] in the realm of cavity QED [11]. Here we propose a scheme of deterministic generation of SU(2) GCS for the bimodal cavity field. We consider a collection of three-level atoms which interacts with the quantum fields sustained in the bimodal cavity and also strongly driven by a classical field. We will show that under the suitable choice of initial atomic state and interaction time the cavity field can evolve in to the SU(2) state.

The rest of the Letter is organized as follows. In Section 2, we give the model to generate SU(2) GCS. The implementation of SWAP gate operation is discussed in Section 3. In Section 4, some concluding remarks are given.

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2. The model

We consider a collection of $N$ identical three-level atoms in $A$-configuration interacting simultaneously with two-mode cavity field and driven by a classical field. The modes with annihilation (creation) operators $\hat{a}$ and $\hat{b}$ interact with the $|g\rangle \leftrightarrow |e\rangle$ and $|l\rangle \leftrightarrow |e\rangle$ transitions, respectively.

The Hamiltonian under the rotating wave approximation for this system in the interaction picture is given by

$$H_{\text{int}} = H_{\text{int}}^1 + H_{\text{int}}^2,$$

where $\Delta_1 = \omega_{g1} - \omega_1$, $g_{1k}$ and $\Delta_2 = \omega_{g2} - \omega_2$, $g_{2k}$ are one-photon detunings, atom-field coupling strength of the cavity modes for respective transitions. $\Omega_k$ is the effective Rabi frequency of the classical driving field causing two-photon transition and it is assumed that the Stark shifts arising due to the virtual transitions are opposite in sign so cancel each other [12]. For simplicity we assume $\Delta_1 = \Delta_2 = \Delta_{12}$ and identical coupling strengths for each atom with the cavity fields so $g_{1k} = g_1$, $g_{2k} = g_2$, $\Omega_k = \Omega$ (Fig. 1). Under the condition that $\Delta_1 > \sqrt{N}g_1$, $\Delta_2 > \sqrt{N}g_2$, the transition to state $|e\rangle$ is negligible during the interaction and the level $|e\rangle$ can be adiabatically eliminated [13] and $H_{\text{int}}^1$ modifies to an effective Hamiltonian

$$H_{\text{eff}}^1 = \sum_{k=1}^{N} \left( \alpha_1 a^\dagger a |g_k\rangle \langle g_k| + \alpha_2 b^\dagger b |l_k\rangle \langle l_k| + G \left[ |g_k\rangle \langle l_k| a^\dagger b e^{-i\Delta_{12}t} + |l_k\rangle \langle g_k| a b^\dagger e^{i\Delta_{12}t} \right] \right),$$

in which $\alpha_1 = g_1^2/\Delta_1$, $\alpha_2 = g_2^2/\Delta_1$, $G = g_1g_2/\Delta_1$. In this effective Hamiltonian the first two terms give Stark shift contribution and the next two terms give interaction leading to two-photon transition from the initial state to the final state. Next, we define new atomic basis

$$|+\rangle_k = \frac{1}{\sqrt{2}}(|g_k\rangle + |l_k\rangle), \quad |-\rangle_k = \frac{1}{\sqrt{2}}(|g_k\rangle - |l_k\rangle),$$

and transform the effective Hamiltonian (Eq. (2)) in this basis to get

$$H_{\text{eff}}^1 = \frac{1}{2} \sum_{k=1}^{N} \left( \alpha_1 a^\dagger a (l_k + \sigma_k^+ + \sigma_k^-) + \alpha_2 b^\dagger b (l_k - \sigma_k^+ - \sigma_k^-) \right)$$

$$+ G \left[ (\sigma_k^+ - \sigma_k^-) a^\dagger b e^{-i\Delta_{12}t} + (\sigma_k^+ + \sigma_k^-) a b^\dagger e^{i\Delta_{12}t} \right]$$

and the second part of Hamiltonian changes to

$$H_{\text{int}}^2 = \sum_{k=1}^{N} \Omega_k \sigma_k^z,$$

where $I_k$ and $\sigma_k^\pm$, $\sigma_k^z$ are identity and Pauli’s matrices in the new basis, respectively.

Fig. 1. Schematics of strongly driven (by classical field $\Omega$) three-level atom coupled to cavity fields ($\hat{a}$, $\hat{b}$). $\Delta_1$ ($\Delta_2$) and $g_1$ ($g_2$) are field frequency detuning and coupling strength of respective transitions.
This system has wave function \( \psi(t) \), and the time evolution of the system is governed by the Schrödinger equation

\[
\frac{i}{\hbar} \frac{\partial \psi(t)}{\partial t} = (H_{\text{eff}} + H_{\text{int}}^2)\psi(t).
\]  
(6)

If we transform wave function according to the following transformation

\[
\psi(t) = e^{-i H_{\text{int}} t} \psi'(t),
\]  
(7)

then the effective Hamiltonian is also transformed to

\[
H_{\text{eff}}^1 = \frac{1}{2} \sum_{k=1}^{N} \left( \alpha_1 a^\dagger a \left( I_k + e^{-i \Omega t} \sigma_k^+ + e^{i \Omega t} \sigma_k^- \right) + \alpha_2 b^\dagger b \left( I_k - e^{-i \Omega t} \sigma_k^+ - e^{i \Omega t} \sigma_k^- \right) \right.
\]
\[
+ G \left[ (\sigma_k^+ - e^{-i \Omega t} \sigma_k^+ + e^{i \Omega t} \sigma_k^-) a^\dagger b e^{-i \Delta_{12} t} + (\sigma_k^- + e^{-i \Omega t} \sigma_k^+ - e^{i \Omega t} \sigma_k^-) a b^\dagger e^{i \Delta_{12} t} \right].
\]  
(8)

Next we assume the strong driving condition for the classical driving field which implies \( \Omega > \alpha_1, \alpha_2, G, \Delta_{12} \) so that the rapidly oscillating terms containing the terms like \( e^{\pm i \Omega t} \) can be neglected in Eq. (8) and the Hamiltonian \( H_{\text{eff}}^1 \) further simplifies to

\[
H_{\text{eff}}^1 = \frac{1}{2} \sum_{k=1}^{N} \left( \alpha_1 a^\dagger a + \alpha_2 b^\dagger b \right) I_k + G \left( a^\dagger b e^{-i \Delta_{12} t} + a b^\dagger e^{i \Delta_{12} t} \right) \sigma_k^-. 
\]  
(9)

At this stage we make a choice for the initial atomic state by considering all the atoms to be in a coherent superposition of their lower states so the combined state of the atoms is a product state of the type

\[
|\psi(0)\rangle_{\text{atom}} = \prod_{k=1}^{N} \frac{1}{\sqrt{2}} \left( |g_k\rangle + |l_k\rangle \right) = \prod_{k=1}^{N} |\mp\rangle_k.
\]  
(10)

This state \( |\psi(0)\rangle_{\text{atom}} \) is an eigenstate of the operator \( \sum_{k=1}^{N} \sigma_k^- \) such that \( \sum_{k=1}^{N} \sigma_k^- |\psi(0)\rangle_{\text{atom}} = N |\psi(0)\rangle_{\text{atom}} \). If atoms are initially in this state then they will remain in this state throughout the interaction with fields thus implying that the cavity fields are decoupled with the atomic state. This can be expressed mathematically as follows. If

\[
|\psi(0)\rangle_{\text{total}} = |\psi(0)\rangle_{\text{atom}} |\psi(0)\rangle_{\text{field}},
\]  
(11)

then

\[
|\psi(t)\rangle_{\text{total}} = |\psi(0)\rangle_{\text{atom}} e^{H_{\text{field}}^\text{red}} |\psi(0)\rangle_{\text{field}}.
\]  
(12)

Under such condition we can separate out the field Hamiltonian part \( H_{\text{field}}^\text{red} \) from Eq. (9), which governs the evolution of the cavity field only as

\[
H_{\text{field}}^\text{red} = \frac{1}{2} \sum_{k=1}^{N} \left[ (\alpha_1 a^\dagger a + \alpha_2 b^\dagger b) I_k + G (a^\dagger b e^{-i \Delta_{12} t} + a b^\dagger e^{i \Delta_{12} t}) \right].
\]  
(13)

We can transform the field wave function \( |\psi(t)\rangle_{\text{field}} \) to \( |\psi'(t)\rangle_{\text{field}} \) by the following transformation

\[
|\psi(t)\rangle_{\text{field}} = e^{-i N (\alpha_1 - \alpha_2) t / 2} e^{-i \Delta_{12} t} a^\dagger b + e^{-i N (\alpha_1 - \alpha_2) t / 2} e^{i \Delta_{12} t} a b^\dagger.
\]  
(14)

which eventually transforms Hamiltonian \( H_{\text{field}}^\text{red} \) to \( H_{\text{field}}^\text{red} \) and gives

\[
H_{\text{field}}^\text{red} = \frac{N G}{2} \left[ e^{i N (\alpha_1 - \alpha_2) t / 2} e^{-i \Delta_{12} t} a^\dagger b + e^{-i N (\alpha_1 - \alpha_2) t / 2} e^{i \Delta_{12} t} a b^\dagger \right].
\]  
(15)

With the choice of parameter \( N (\alpha_1 - \alpha_2) = \Delta_{12} \) we have the evolution of the cavity field given by the Hamiltonian

\[
H_{\text{field}}^\text{red} = \frac{N G}{2} [a^\dagger b + a b^\dagger].
\]  
(16)

Such a filed Hamiltonian is a generator of the SU(2) coherent state of type \( |p, q\rangle = e^{i K_+ - i K_-} |q, -q\rangle \), where we have coefficient \( \xi = -i N G t / 2 \).
3. The SWAP gate operation

Using such a Hamiltonian (Eq. (16)) we can realize swap gate operation very conveniently [14]. For this purpose we take our initial field of bimodal cavity to be in Fock states

$$|\psi(0)\rangle_{\text{field}} = |u\rangle_a |v\rangle_b,$$

which will evolve in time as

$$|\psi(t)\rangle_{\text{field}} = e^{-iGt} (a^{\dagger} + a)^{n} |\psi(0)\rangle_{\text{field}} = Q(t) |\psi(0)\rangle_{\text{field}}.$$

For simplicity we define $\xi = NG/2$. The effect of operator $Q(t)$ on the two-mode Fock state field of the cavity can be written as [14]

$$Q(t) |u\rangle_a |v\rangle_b = \frac{1}{\sqrt{uv!}} (a^{\dagger} \cos \xi t - ib^{\dagger} \sin \xi t)^u (b^{\dagger} \cos \xi t - ia^{\dagger} \sin \xi t)^v |0\rangle_a |0\rangle_b.$$

If we choose interaction time $t = t_{int}$ such that $\xi t_{int} = \pi/2$ then we can realize

$$Q(\pi/2\xi) |u\rangle_a |v\rangle_b = (-i)^{u+v} |v\rangle_a |u\rangle_b,$$

which implies swapping operation is possible of two-mode Fock states by $Q(\pi/2\xi)$ operation. To illustrate it further we consider that $u, v = 0$ or $1$ initially, so by the operation of $Q(\pi/2\xi)$ we get transformation of different initial two-mode Fock states as

$$Q(\pi/2\xi) |0\rangle_a |0\rangle_b = |0\rangle_a |0\rangle_b,$$
$$Q(\pi/2\xi) |0\rangle_a |1\rangle_b = -i |1\rangle_a |0\rangle_b,$$
$$Q(\pi/2\xi) |1\rangle_a |0\rangle_b = -i |0\rangle_a |1\rangle_b,$$
$$Q(\pi/2\xi) |1\rangle_a |1\rangle_b = - |1\rangle_a |1\rangle_b.$$

The above Eq. (21) clearly demonstrate swap gate operation [14] apart from a phase factor that can be taken care of.

4. Summary and concluding remarks

We have proposed a cavity QED based scheme to generate SU(2) GCS without involving any conditional measurement with a suitable choice of initial atomic states. The scheme involves interaction of a collection of classically driven atoms with bimodal cavity fields and under a suitable choice of atomic and interaction parameters the cavity field gets decoupled from the atomic state and thus acts as a generator of SU(2) GCS. This scheme can be realized in Rubidium or any suitable Rydberg system of atoms. One can use the suitable Rydberg states [15] and thus acts as a generator of SU(2) GCS. This scheme can be realized in Rubidium or any suitable Rydberg system of atoms. We have proposed a cavity QED based scheme to generate SU(2) GCS without involving any conditional measurement with a suitable choice of initial atomic states. The scheme involves interaction of a collection of classically driven atoms with bimodal cavity fields and under a suitable choice of atomic and interaction parameters the cavity field gets decoupled from the atomic state and thus acts as a generator of SU(2) GCS. This scheme can be realized in Rubidium or any suitable Rydberg system of atoms. One can use the suitable Rydberg states [15] and thus acts as a generator of SU(2) GCS. This scheme can be realized in Rubidium or any suitable Rydberg system of atoms.

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