Appendix C: Equations Underlying Tables and Figures

Table 3 and Figure 7
The revenue limit calculations of Table 3 are based on these equations:\(^1\)

\[
\begin{align*}
\Delta R_M &= -p_M \Delta E_M < 0, \\
\Delta R_x &= p_x \Delta E_x > 0.
\end{align*}
\]

This frees up \(p_M \Delta E_M - p_x \Delta E_x\) in aid, which is split among property taxpayers in all districts:\(^2\)

\[
\Delta L_i = -(p_M \Delta E_M - p_x \Delta E_x) \cdot (V_i / \sum V_i),
\]
and each district’s aid is adjusted upward accordingly:

\[
\Delta A_i = \Delta R_i + (p_M \Delta E_M - p_x \Delta E_x) \cdot (V_i / \sum V_i).
\]

In the bottom half of Table 3, the net benefits are decomposed as follows:

\[
(p_M \Delta E_M - p_x \Delta E_x) = (p_M - p_x) \Delta E_M - p_x (\Delta E_x - \Delta E_M).
\]

The first term on the right-hand side is the reduction in spending on those students who switch, and the second term is the spending on vouchers for students who would not have attended MPS.

Table 4 and Figure 8
Given \(\Delta R_M\) from (7), the 2/3 rule implies

\[
\Delta L_i = (\Delta R_M / 3) \cdot (V_i / \sum V_i),
\]
and each district’s aid is adjusted upward accordingly:

\[
\Delta A_i = \Delta R_i - (\Delta R_M / 3) \cdot (V_i / \sum V_i).
\]

Table 5 and FY94-99 in Figure 9

---

\(^1\) Note that the changes we are considering are relative to having no vouchers. Thus \(\Delta E\) is actually \(E\). Also, the enrollment change for Milwaukee \(\Delta E_M\), is defined in absolute value, rather than algebraic.

\(^2\) There is a slight complication due to the reduction in Wisconsin’s total public enrollment, which will slightly raise the average valuation per pupil, \(v\). Strictly speaking, the analyses in this paper hold \(v\) constant and only let \(v\) vary. The impact of this simplification is a very slight distortion of the distribution of levies between Milwaukee and the other districts.
During this period, Milwaukee's revenue limits include the voucher students, so 
\[ R_M = \rho_M (E_M + E_x), \]
and
\[ \Delta R_M = \rho_M \cdot (\Delta E_x - \Delta E_M) > 0. \]

The expressions for changes in local levies and aid reflect the MPCP aid deduction and levy substitution, as well as the 2/3 rule:

\[ \Delta L_i = \frac{\Delta R_M}{3} \cdot (V_i / \sum V_i) \]
\[ \Delta A_i = \frac{\Delta R_M}{3} \cdot (V_i / \sum V_i) \]

Table 6 and FY00-01 in Figure 9
Voucher students were removed from revenue limits, so we have (7) again, for \( \Delta R_M \).

The expressions for changes in local levies and aid reflect the split of the MPCP aid deduction between Milwaukee and other districts (denoted as \( \sim M \)). The expressions also reflect the 2/3 rule, including the addition of aid for 2/3 of the MPCP levy hike:

\[ \Delta L_M = \frac{p_x \Delta E_x}{2} + \left[ \frac{(\Delta R_M / 3) - (2/3)(p_x \Delta E_x)}{\sum V_i} \right] \cdot (V_M / \sum V_i) \]
\[ \Delta A_M = \frac{\Delta R_M}{3} \cdot (V_M / \sum V_i) \]

Tables 7-8 and FY02-07 in Figure 9
The revenue limit calculation is unchanged from (7\( ^{\text{II}} \)). The expressions for changes in local levies and aid reflect the elimination of the MPCP deduction from non-Milwaukee districts and the adjustment of Milwaukee's deduction to 45%. The expressions also reflect the removal of the MPCP levy hike from the 2/3 rule.

\[ \Delta L_M = 0.45 \cdot p_x \Delta E_x + \left[ \frac{(\Delta R_M / 3)}{\sum V_i} \right] \cdot (V_M / \sum V_i) \]
\[ \Delta A_M = -0.45 \cdot p_x \Delta E_x - \frac{\Delta R_M}{3} \cdot (V_M / \sum V_i) \]