Appendix C: Equations Underlying Tables and Figures

Table 3 and Figure 7

The revenue limit calculations of Table 3 are based on these equations:[1]

$$(7)\qquad \begin{aligned} \Delta R_M &= -p_M \Delta E_M < 0, \\ \Delta R_x &= p_x \Delta E_x > 0. \end{aligned}$$

This frees up $p_M \Delta E_M$ - $p_x \Delta E_x$ in aid, which is split among property taxpayers in all districts:[2]

$$(8)\qquad \Delta L_i = -\left(p_M \Delta E_M - p_x \Delta E_x\right) \cdot (V_i / \sum V_i),$$

and each district's aid is adjusted upward accordingly:

$$(9)\qquad \Delta A_i = \Delta R_i + \left(p_M \Delta E_M - p_x \Delta E_x\right) \cdot (V_i / \sum V_i).$$

In the bottom half of Table 3, the net benefits are decomposed as follows:

$$(p_M \Delta E_M - p_x \Delta E_x) = (p_M - p_x) \Delta E_M - p_x (\Delta E_x - \Delta E_M).$$

The first term on the right-hand side is the reduction in spending on those students who switch, and the second term is the spending on vouchers for students who would not have attended MPS.

Table 4 and Figure 8

Given ΔR_M from (7), the 2/3 rule implies

$$(8')\qquad \Delta L_i = \left(\Delta R_M / 3\right) \cdot (V_i / \sum V_i),$$

and each district's aid is adjusted upward accordingly:

$$(9')\qquad \Delta A_i = \Delta R_i - \left(\Delta R_M / 3\right) \cdot (V_i / \sum V_i).$$

Table 5 and FY94-99 in Figure 9

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[1] Note that the changes we are considering are relative to having no vouchers. Thus ΔE_x is actually E_x. Also, the enrollment change for Milwaukee ΔE_M, is defined in absolute value, rather than algebraic.

[2] There is a slight complication due to the reduction in Wisconsin's total public enrollment, which will slightly raise the average valuation per pupil, v^3. Strictly speaking, the analyses in this paper hold v^3 constant and only let v^2 vary. The impact of this simplification is a very slight distortion of the distribution of levies between Milwaukee and the other districts.

During this period, Milwaukee's revenue limits include the voucher students, so $R_M = p_M \cdot (E_M + E_x)$, and

(7[I]) $$\Delta R_M = p_M \cdot (\Delta E_x - \Delta E_M) > 0.$$

The expressions for changes in local levies and aid reflect the MPCP aid deduction and levy substitution, as well as the 2/3 rule:

(8[I]) $$\begin{aligned} \Delta L_M &= p_x \Delta E_x + (\Delta R_M / 3) \cdot (V_M / \sum V_i) \\ \Delta L_i &= (\Delta R_M / 3) \cdot (V_i / \sum V_i) \end{aligned}$$

(9[I]) $$\begin{aligned} \Delta A_M &= (\Delta R_M - p_x \Delta E_x) - (\Delta R_M / 3) \cdot (V_M / \sum V_i) \\ \Delta A_i &= -(\Delta R_M / 3) \cdot (V_i / \sum V_i) \end{aligned}$$

<u>Table 6 and FY00-01 in Figure 9</u>

Voucher students were removed from revenue limits, so we have (7) again, for ΔR_M.
The expressions for changes in local levies and aid reflect the split of the MPCP aid deduction between Milwaukee and other districts (denoted as $\sim M$). The expressions also reflect the 2/3 rule, including the addition of aid for 2/3 of the MPCP levy hike:

(8[II]) $$\begin{aligned} \Delta L_M &= (p_x \Delta E_x)/2 + [(\Delta R_M / 3) - (2/3)(p_x \Delta E_x)] \cdot (V_M / \sum V_i) \\ \Delta L_{\sim M} &= (p_x \Delta E_x)/2 + [(\Delta R_M / 3) - (2/3)(p_x \Delta E_x)] \cdot (V_{\sim M} / \sum V_i) \end{aligned}$$

(9[II]) $$\begin{aligned} \Delta A_M &= \Delta R_M - (p_x \Delta E_x)/2 + [(2/3)(p_x \Delta E_x) - (\Delta R_M / 3)] \cdot (V_M / \sum V_i) \\ \Delta A_{\sim M} &= -(p_x \Delta E_x)/2 + [(2/3)(p_x \Delta E_x) - (\Delta R_M / 3)] \cdot (V_{\sim M} / \sum V_i) \end{aligned}$$

<u>Tables 7-8 and FY02-07 in Figure 9</u>

The revenue limit calculation is unchanged from (7[II]). The expressions for changes in local levies and aid reflect the elimination of the MPCP deduction from non-Milwaukee districts and the adjustment of Milwaukee's deduction to 45%. The expressions also reflect the removal of the MPCP levy hike from the 2/3 rule.

(8[III]) $$\begin{aligned} \Delta L_M &= 0.45 \cdot p_x \Delta E_x + (\Delta R_M / 3) \cdot (V_M / \sum V_i) \\ \Delta L_i &= (\Delta R_M / 3) \cdot (V_i / \sum V_i) \end{aligned}$$

(9[III]) $$\begin{aligned} \Delta A_M &= \Delta R_M - 0.45 \cdot p_x \Delta E_x - (\Delta R_M / 3) \cdot (V_M / \sum V_i) \\ \Delta A_i &= -(\Delta R_M / 3) \cdot (V_i / \sum V_i) \end{aligned}$$