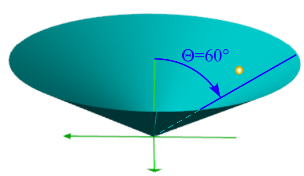
Assignment 10 - **Classical Mechanics 5103** 10/24/18 Due Wed. Oct. 31

Main Reading: In new text (*Classical Mechanics with a BANG!*) Unit 2 thru 2.8 and Unit 3 thru 3.8.

Ex.1. Derivatives of spherical coordinate metric g_{mn} (Ex.1 Set 9) give Christoffel coefficients $\Gamma_{ij;k}$ (1st kind) and $\Gamma_{ij;}{}^{k}$ (2nd kind) by eq. (3.6.10) or Lect 17 p.24. It is easier to use Lagrange derivative equations (2.4.1) or covariant force Lagrange equations (3.5.10) as in Sec. 3.7 to derive both kinds of Christoffel Γ coefficients. (Also: Lect. 17 p.51.) Discuss what "ficticious" forces or accelerations the spherical coordinate $\Gamma_{ij;k}$ or $\Gamma_{ij;}{}^{k}$ give. (It might help to redo cylindrical $\Gamma_{ij;k}$ without looking at 3.7.)

Write out the covariant and the contravariant Riemann equations while solving for coefficients. (Lect. 17 p.51.)



Ex.**2.** *Funny funnel orbits*

(a) Analyze orbits for mass sliding on funnel cone of polar angle Θ in gravity $g_z \sim 10m \cdot s^{-2}$. (Recall "I-Ball" in Lecture 17.)
What closed and periodic orbit (if any) listed in Fig. 3.8.1 is closest to being achieved for Θ=60°? ω_r/ω_ϕ=______?

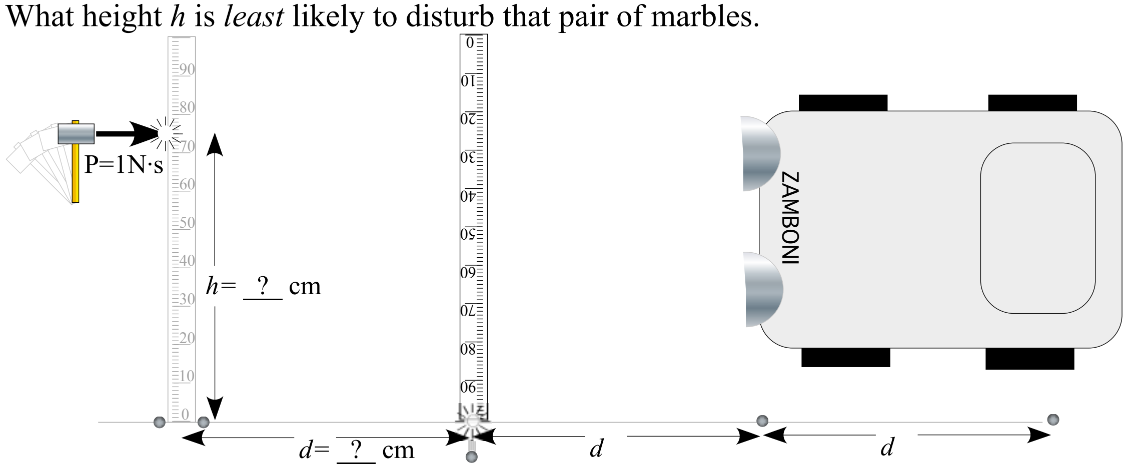
(b) Find Θ-cones that do give closed orbits with frequency ratios given below and (if orbit is possible) sketch its path.

$\omega_r/\omega_\phi=1/1$: $\Theta_{1/1}$=___; $\omega_r/\omega_\phi=2/1$: $\Theta_{2/1}$=___; $\omega_r/\omega_\phi=3/1$: $\Theta_{3/1}$=___;

$\omega_r/\omega_\phi=1/2$: $\Theta_{1/2}$=___; $\omega_r/\omega_\phi=3/2$: $\Theta_{3/2}$=___; $\omega_r/\omega_\phi=1/3$: $\Theta_{1/3}$=___; $\omega_r/\omega_\phi=2/3$: $\Theta_{2/3}$=___.

An icy cycloid problem

Ex.3 (a) A *1kg*. meter stick lies on a smooth icy hockey rink surface with two marbles sitting at its end on either side of the 0.0cm mark. (See figure) A hammer give impulse $\mathbf{P}=(1\text{N}\cdot\text{s})\mathbf{e}_x$ to the stick at the *h*-cm. mark.
What height *h* is *least* likely to disturb that pair of marbles.



(b) Now assume *h*-value from (**a**) and friction-free "icy" surface. At what distances *d, 2d, 3d,* … along *x*-axis should the *3rd, 4th, 5th,…*marbles be placed so they are most likely to be knocked below the axis. Draw 12 equal time Δt interval snapshots of the stick as it flips thru 180°. (**c**) What is Δt for a 1kg stick?

Electromagnetic cycloids

Ex.4 A unit mass $m=1\ kg$ and charge $Q=1\ Coul.$ (Dangerous!) starts at $(x=0=y)$ on a frictionless (x,y)-surface in vertical Earth gravity (Say $g_y=-10m/s^2$) and in a strong z-axial magnetic field $\mathbf{B}_z=(0,0,B_z)$ normal to surface.

(a) What field B_z (in *Tesla*) causes the mass with zero initial velocity $(v_x(0),v_y(0))=(0,0)$ to follow a cycloid of 0.5 meter radius along $-x$ axis? What x-axis points does it hit? Are these hit points different for different $\mathbf{v}(0)$?

(b) What initial $\mathbf{v}(0)$ would cause the mass to fly a straight line along the $-x$-axis? … along the $+x$-axis?

(c) Describe and plot the resulting trajectory if instead the mass is thrown down with $(v_x(0),v_y(0))=(0,-2m/s)$.