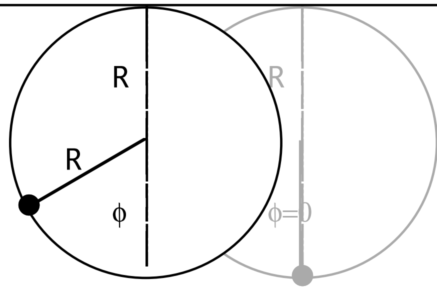
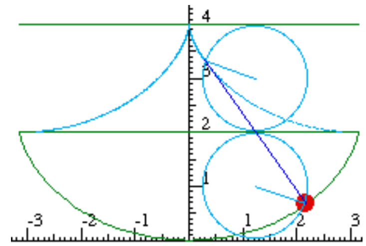
Assignment 9 - **Classical Mechanics 5103** 10/17/18 Due Wed. Oct. 24

Main Reading: In new text (*Classical Mechanics with a BANG!*) Unit 2 thru 2.8 and Unit 3 thru 3.8.

Huygen's problem. For 40 years Christian Huygens worked to improve harmonicity of pendulums and only solved the problem you are about to do, just before he died. Let's hope it doesn't take <u>*you*</u> *as long!*





A really scary Halloween roller coaster (cackle! cackle!)

Ex.1. A mass m slides frictionlessly along a cycloid of radius $R=\frac{3}{\pi}\,cm.$ in gravity $g\sim 10m\cdot s^{-2}$. Or else, a very light hoop is held up against a magnetized ceiling so it rolls with no slip carrying mass m on its rim. (See sketch).

(a_1) First construct *24pt* normal cycloids using ruler-compass on graph paper. (Attached is a 6*cm.* graph whose *x-axis* has 24 intervals that "roll" circles of radii $R=3/\pi cm.$ by angles $n\pi/12$ above and below and π out of phase. Check geometry with algebra.))

(a_2) Write parametric equations $x(\phi)$, $y(\phi)$ for a circumference point mass on a wheel rolling on the ceiling where ϕ is the wheel-center-relative angle that mass has rotated from its lowest point at lab origin $(x=0,y=0)$.

(b) Derive Lagrangian $L(d\phi/dt,\phi)$ and find canonical momentum p_ϕ and equation of motion.

(c) Derive total energy and Hamiltonian function H. Are any of these *(L, p_ϕ,E, or H) ever* constants of motion?

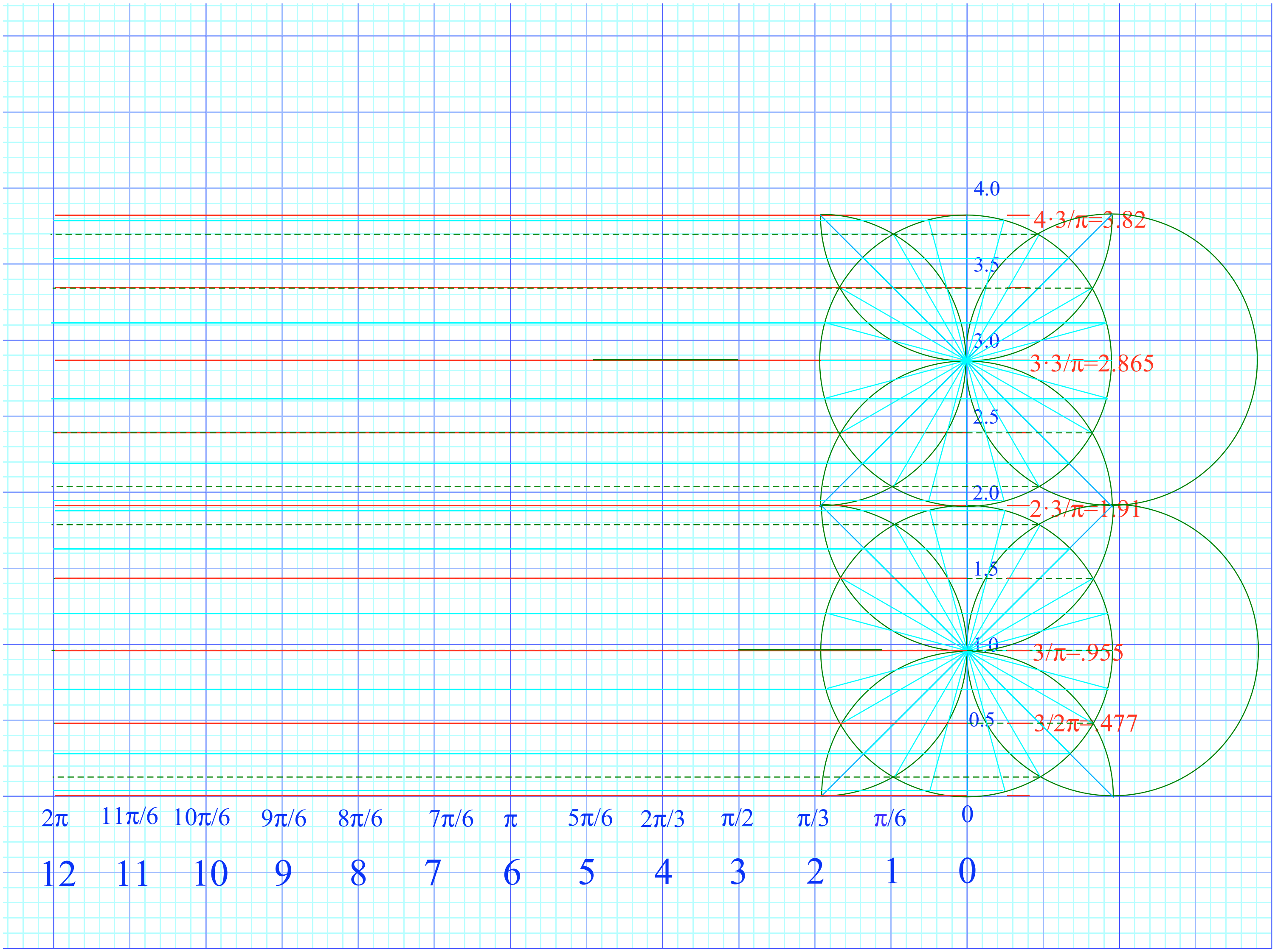
(d) Derive an expression for the arc length $s(\phi)$ that m travels to ϕ from the lowest point $\phi=0$. How long is a string wrapped around a cycloid from $\phi=0$ to $\phi=\pi$.

(e) Prove period of oscillation of mass m is independent of initial velocity $\dot{\phi}(0)$ at $\phi=0$ for $\dot{\phi}(0)$ less than________?

What is that max $v_{\max}=\dot{x}_{\max}=$_____? and max $\omega_{\max}=\dot{\phi}_{\max}=$_____? (Discuss and give both algebraic and numerical results.)

(f) Derive $\phi(t)$ for free oscillation. Can the hoop roll across the ceiling with constant speed $v_{const}=\dot{x}_{const}$ for some initial conditions ? Discuss. Does such a uniform velocity state exist if instead the hoop rolls on the floor?

(g) Show how a cycloid path is generated by a string unwrapping (*evolute*) from another cycloid and that a normal cycloid is the locus of the center of curvature (*involute*) of a π-out-of-phase cycloid like itself. (Use (a) to discuss.)



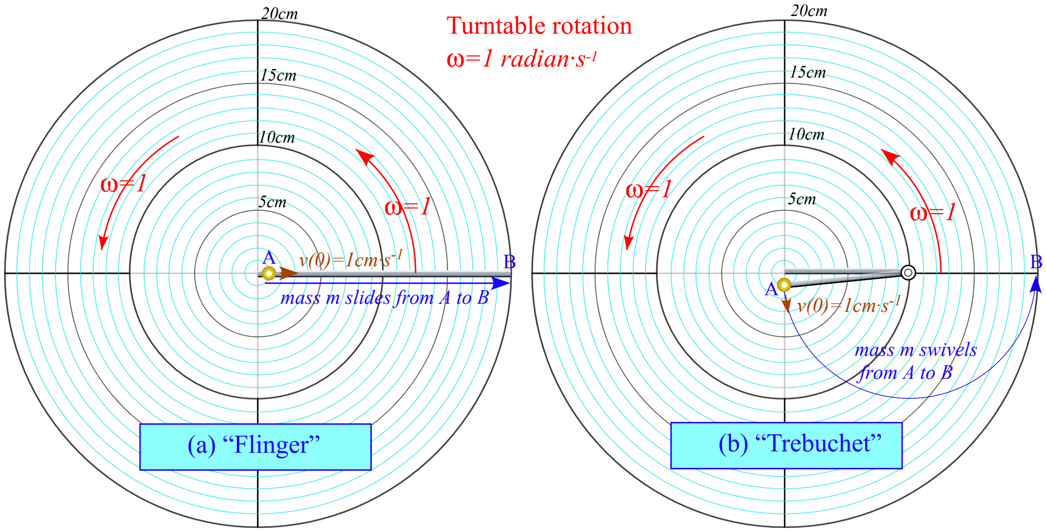
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
4·3/π=3.82
3·3/π=2.865
2·3/π=1.91
3/π=.955
3/2π=.477
2π
11π/6
10π/6
9π/6
8π/6
7π/6
π
5π/6
2π/3
π/2
π/3
π/6
0
12
11
10
9
8
7
6
5
4
3
2
1
0

Flinger vs. Trebuchet on turntable (geometric version)

Ex.2. Compare dynamics of mass m on a "Flinger" (Fig. (a)) to what it does on a "Trebuchet" (Fig. (b)).
Both begin at point A of radius $r(0)=1cm.$ from the center of a turntable rotating at $\omega=1(radian)s^{-1}$. Both have an initial speed of $v(0)=1cm\cdot s^{-1}$ and move from that point A to a final point B relative to turntable having radius $r(t_r)=20cm$ where we assume m is then released into the laboratory.

In Fig. (a) m slides $19cm$ along a rod of length $\ell=20cm$. (The $20cm$ rod is fixed to turntable.)

In Fig. (b) m swivels on a rod of length $\ell=10cm$ around a point fixed to turntable at $r=10cm$ radius.



Relative to turntable…

Find m release speed for "Flinger." _________

Relative to laboratory…

Find m release speed for "Flinger."_________

To scale†, sketch lab $\mathbf{v}(t_r)$ assuming release at B.

† Let $1cm$ be $1cm\cdot s^{-1}$.

Relative to turntable…

Find m release speed for "Trebuchet."__________

Relative to laboratory…

Find m release speed for "Trebuchet."__________

To scale†, sketch lab $\mathbf{v}(t_r)$ assuming release at B.

† Let $1cm$ be $1cm\cdot s^{-1}$.

How long does m take to go from A to release point B? _____sec.
Plot or (preferably) construct its orbit on a polar graph like Fig. (a) but in the lab-relative frame.