Lecture 27.5

Geometry and Symmetry of Orbital Dynamics

(Ch. 2-4 of Unit 5 12.04.12)

Geometry and Symmetry of Coulomb orbits

Detailed elliptic geometry

Detailed hyperbolic geometry

Rutherford scattering and differential scattering crossections

Ruler & compass construction

➔ Eccentricity vector ε and orbital phase geometry

Ruler & compass construction

➤ *Eccentricity vector ε and orbital phase geometry*

Ruler & compass construction

Isotropic field guarantees conservation of the *angular momentum vector* $\mathbf{L}$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = m\,\mathbf{r} \times \dot{\mathbf{r}}$$

Coulomb field guarantees conservation of the *eccentricity vector* $\boldsymbol{\varepsilon}$

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} - \frac{\mathbf{p} \times \mathbf{L}}{km} = \frac{\mathbf{r}}{r} - \frac{\mathbf{p} \times (\mathbf{r} \times \mathbf{p})}{km}$$

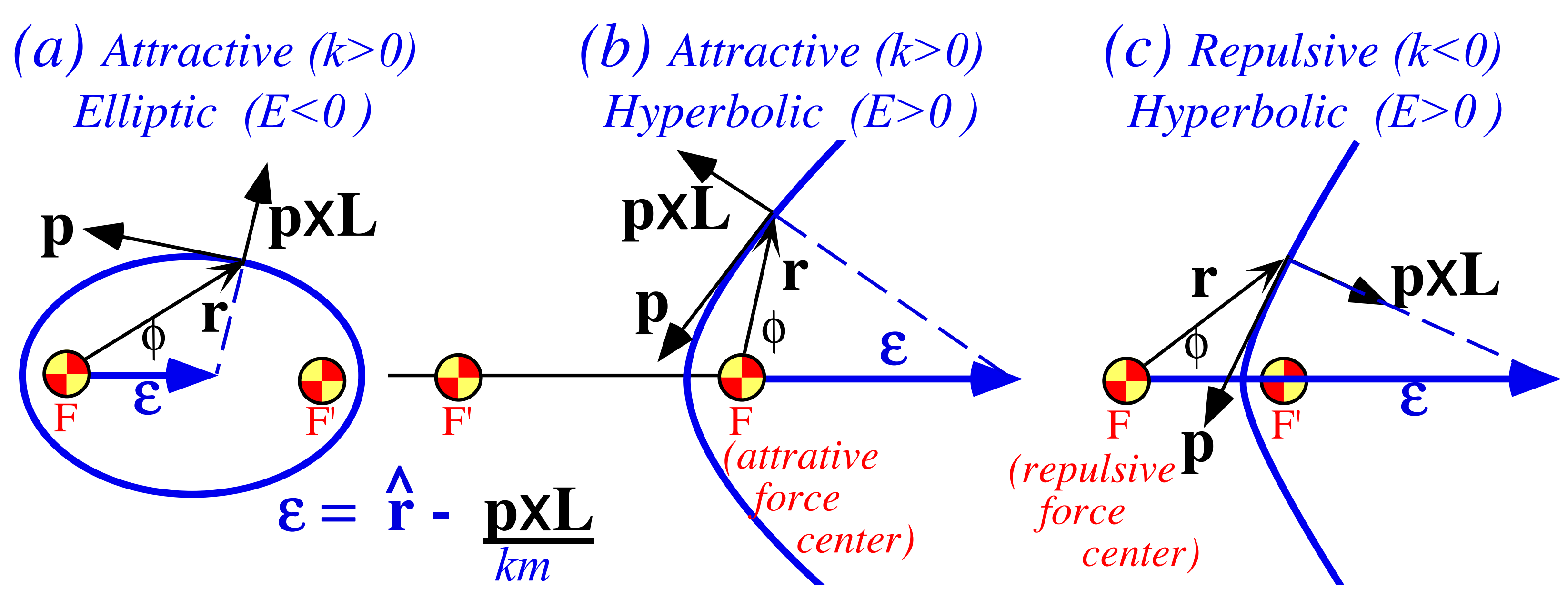
$\mathbf{A} = km\,\boldsymbol{\varepsilon}$ known as the *Laplace-Hamilton-Gibbs-Runge-Lenz vector*.

Dot product of $\boldsymbol{\varepsilon}$ with a radial vector $\mathbf{r}$.

$$\boldsymbol{\varepsilon} \bullet \mathbf{r} = \frac{\mathbf{r} \bullet \mathbf{r}}{r} - \frac{\mathbf{r} \bullet \mathbf{p} \times \mathbf{L}}{km} = r - \frac{\mathbf{r} \times \mathbf{p} \bullet \mathbf{L}}{km} = r - \frac{\mathbf{L} \bullet \mathbf{L}}{km}$$

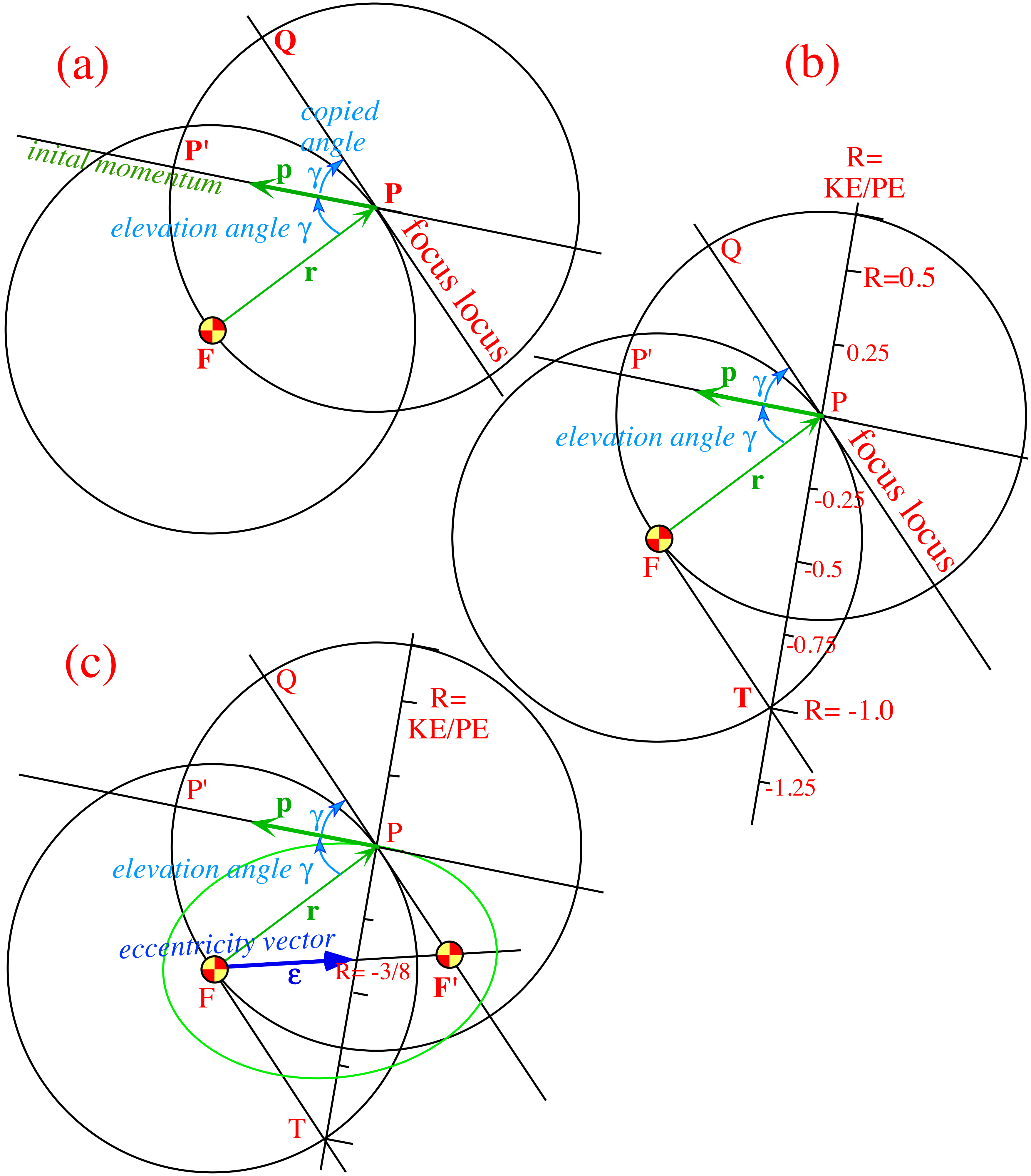
Polar angle ϕ is the angle between $\boldsymbol{\varepsilon}$ and the radial vector $\mathbf{r}$

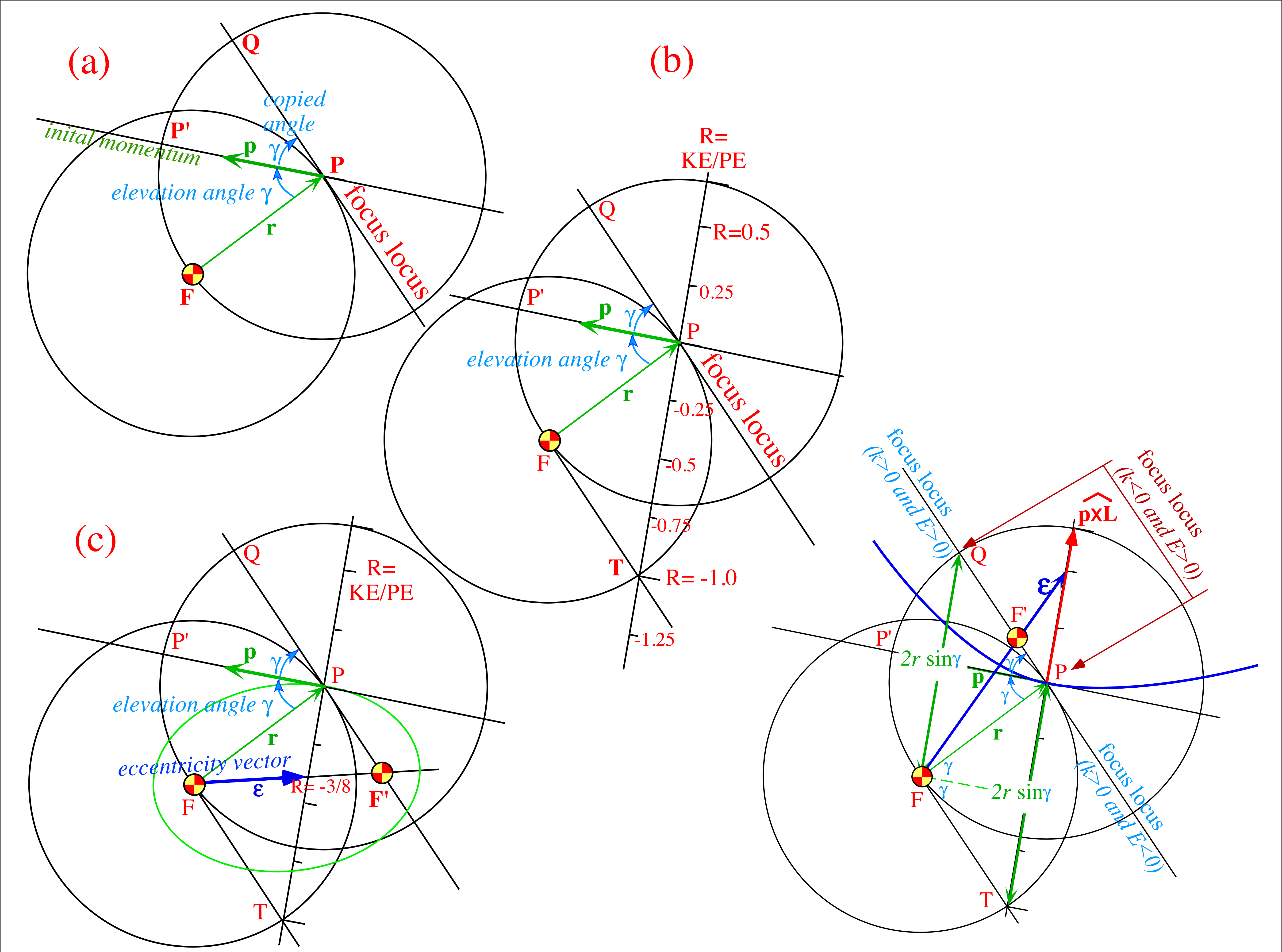
$$\varepsilon\, r \cos\phi = r - \frac{L^2}{km}, \quad \text{or:} \quad r = \frac{-L^2 / km}{1 - \varepsilon \cos\phi}$$

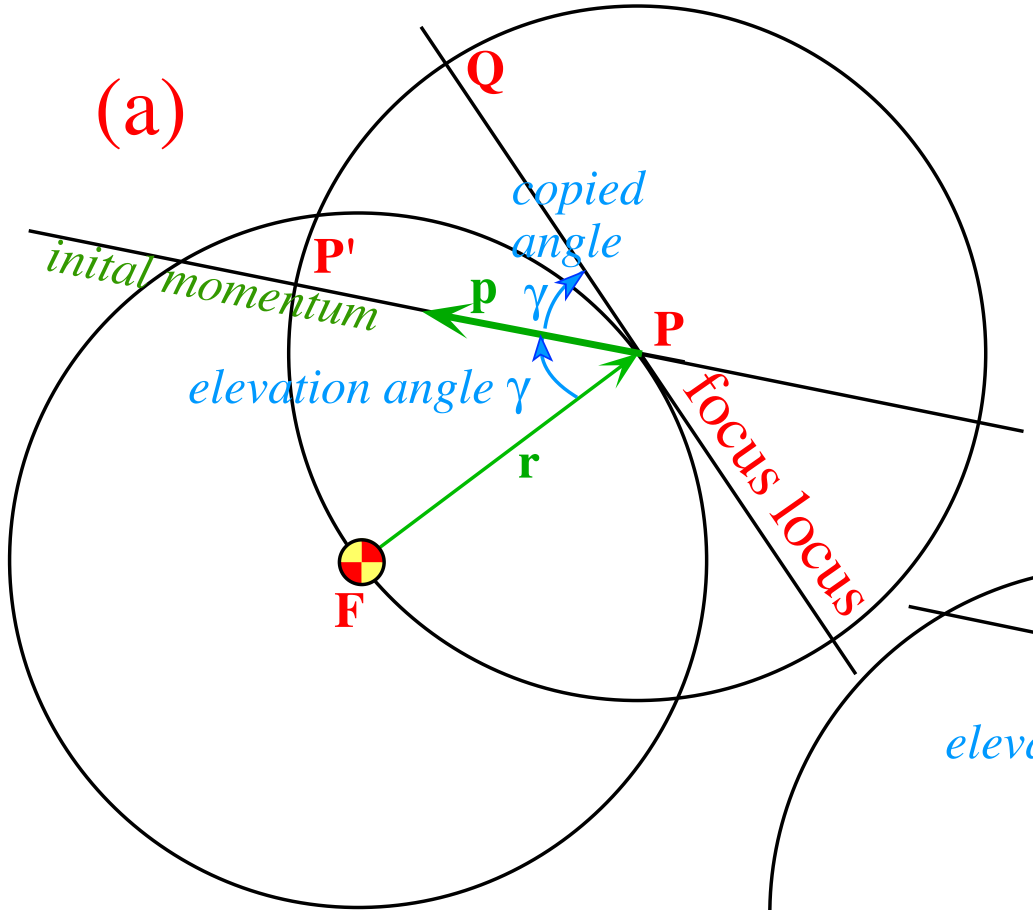


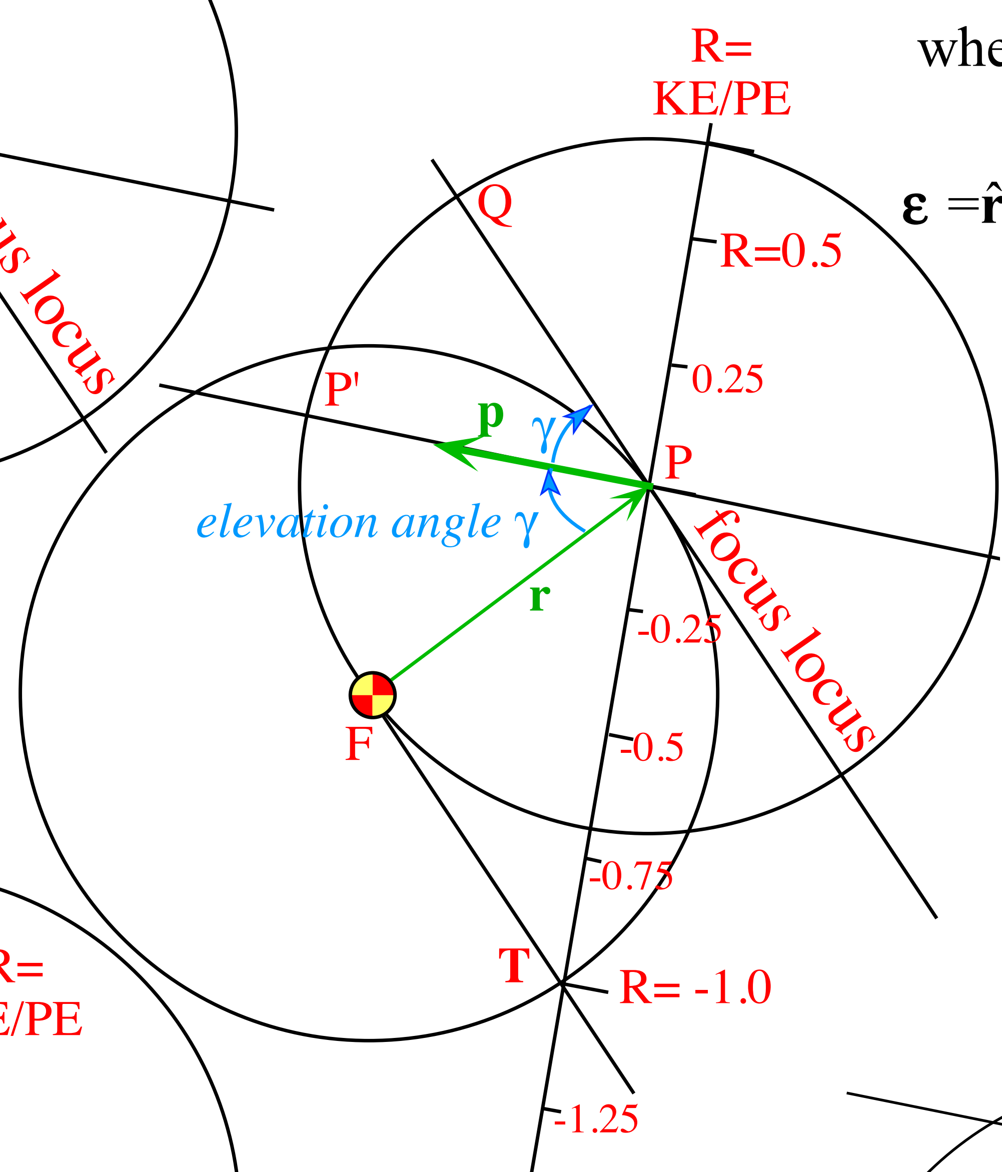
Eccentricity vector ε and orbital phase geometry

➤ *Ruler & compass construction*


(a)
Q
copied
angle
initial momentum
P'
p
γ
P
elevation angle γ
focus locus
r
F
(b)
R=
KE/PE
Q
R=0.5
0.25
P'
p
γ
P
elevation angle γ
focus locus
r
-0.25
F
-0.5
-0.75
T
R= -1.0
-1.25
(c)
Q
R=
KE/PE
P'
p
γ
P
elevation angle γ
r
eccentricity vector
ε
R= -3/8
F
F'
T


(a)
Q
copied
angle
P'
inital momentum
p
γ
P
elevation angle γ
focus locus
r
F
(b)
R=
KE/PE
Q
R=0.5
0.25
P'
p
γ
P
elevation angle γ
focus locus
r
-0.25
F
-0.5
-0.75
T
R= -1.0
-1.25
(c)
Q
R=
KE/PE
P'
p
γ
P
elevation angle γ
r
eccentricity vector
ε
R= -3/8
F
F'
T
focus locus
(k>0 and E>0)
focus locus
(k<0 and E>0)
pxL
Q
ε
F'
P'
2r sinγ
p
γ
γ
P
r
F
γ
γ
2r sinγ
focus locus
(k>0 and E<0)
T

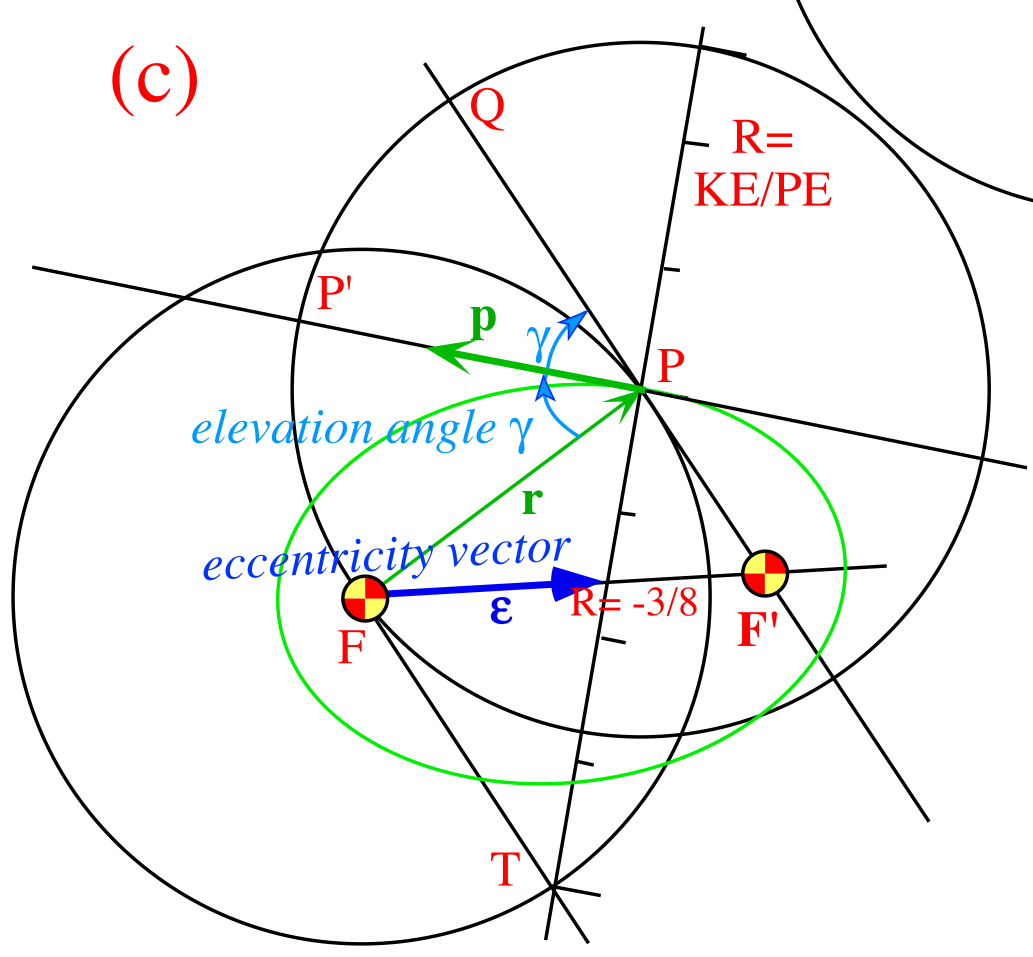


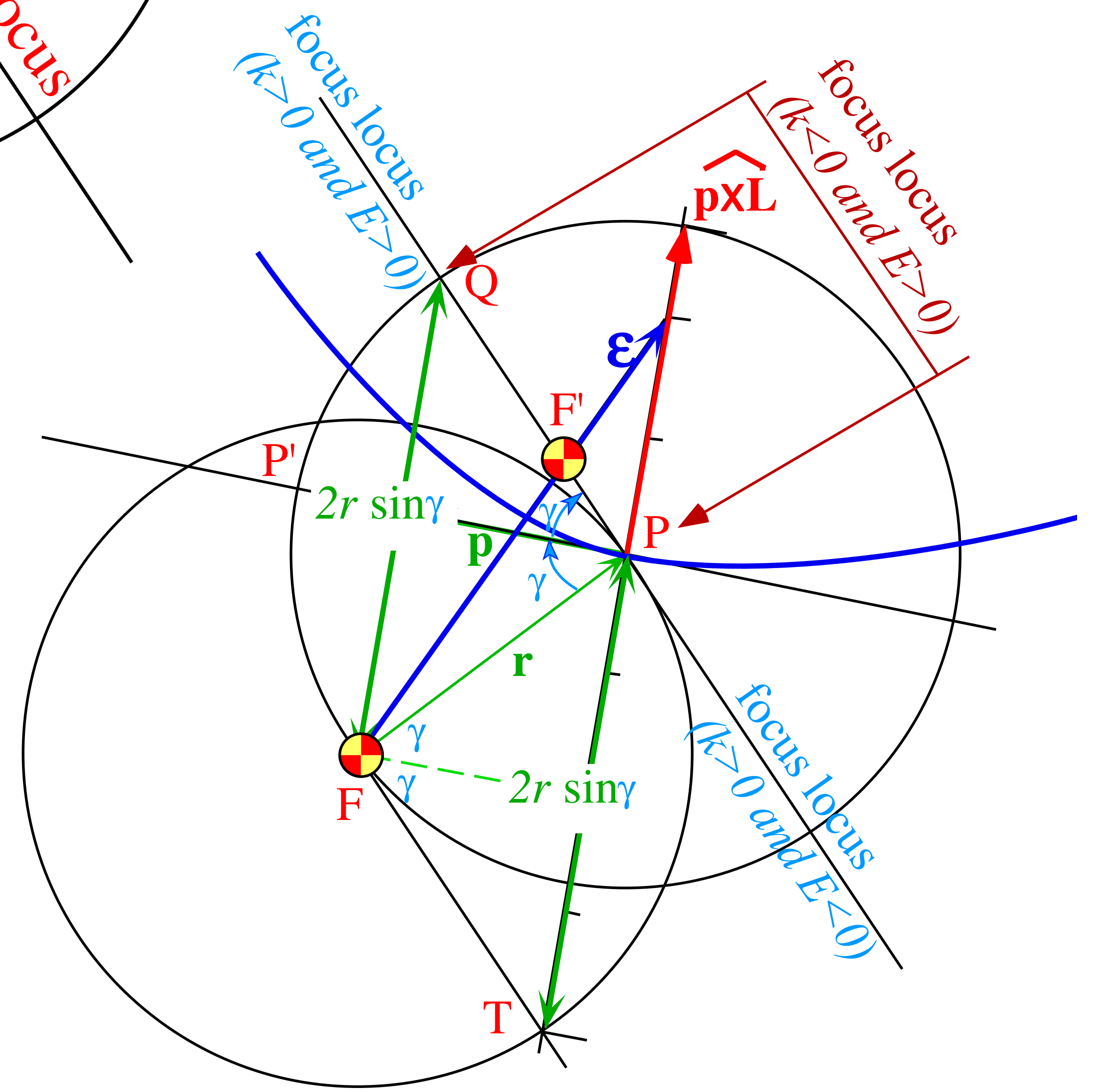


$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} - \frac{\mathbf{p}\times\mathbf{L}}{km} = \hat{\mathbf{r}} - \frac{(mv_0)(mv_0 r_0)\sin\gamma}{km}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

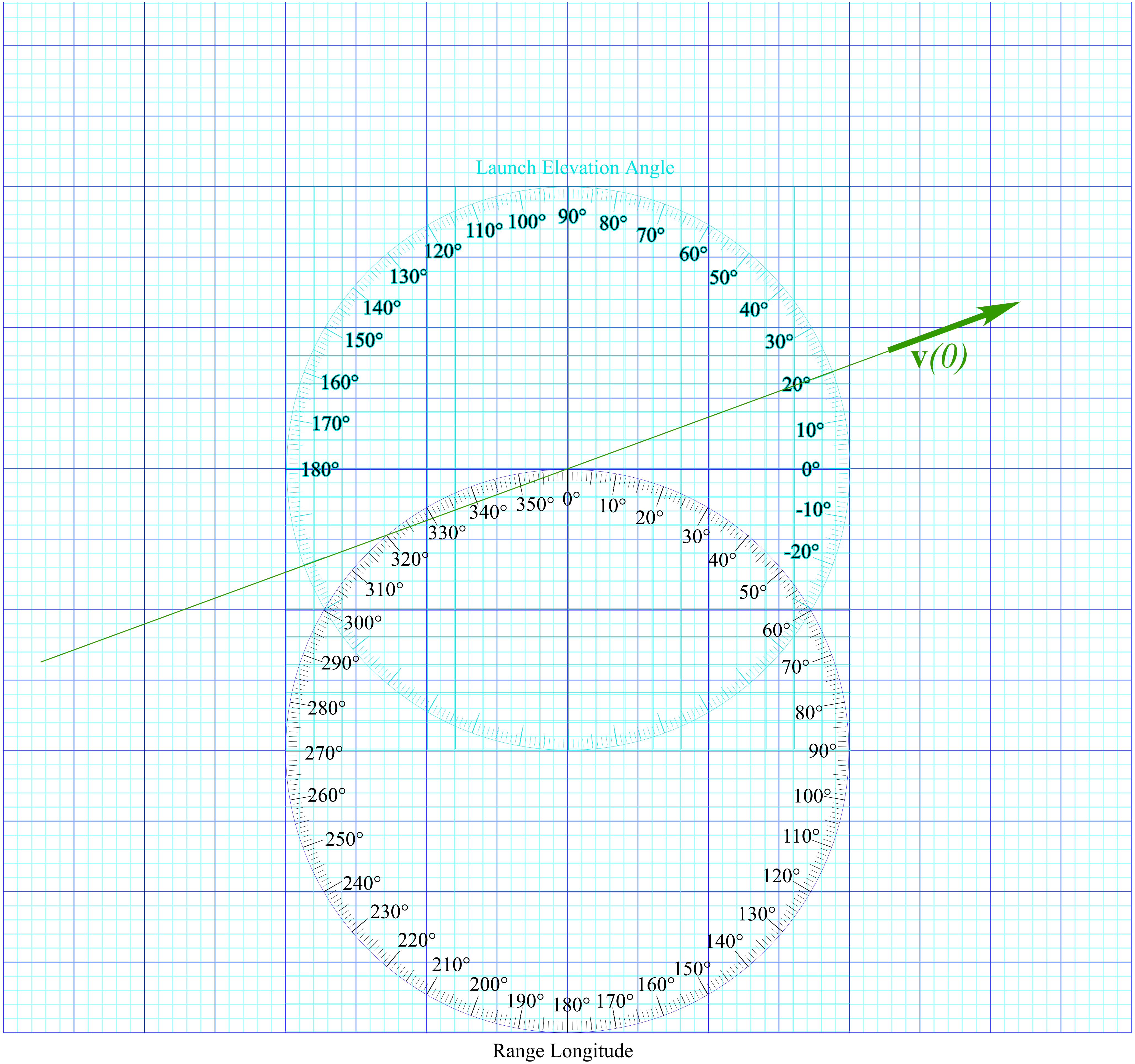
where: $\mathbf{L}_{\mathbf{p}\times} \equiv \mathbf{p}\times\mathbf{L}$

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} + 2\sin\gamma\,\frac{mv_0^2/2}{-k/r_0}\hat{\mathbf{L}}_{\mathbf{p}\times} = \hat{\mathbf{r}} - 2\sin\gamma\,\frac{KE}{PE}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

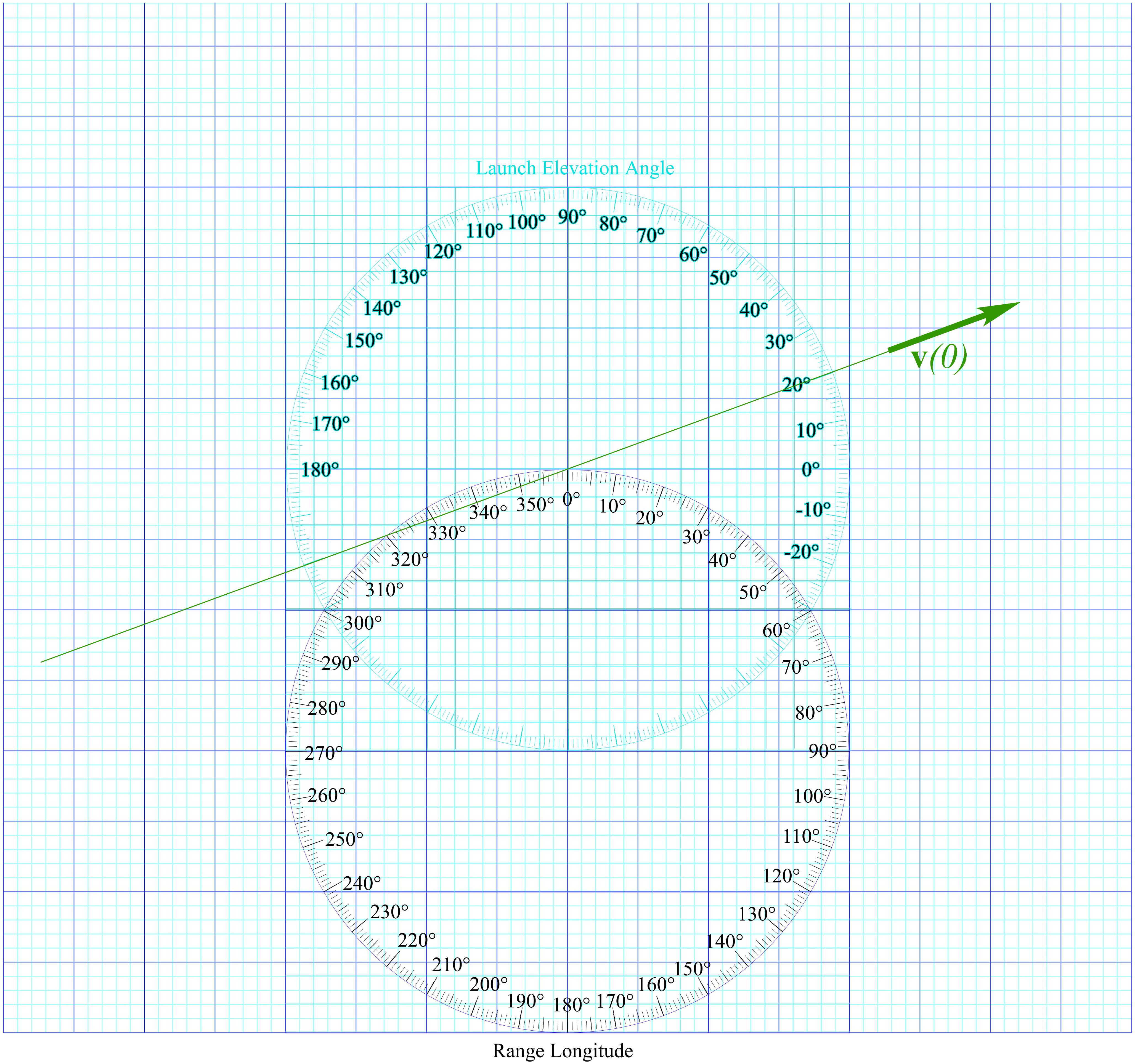




Start with initial velocity **v***(0)*



Start with initial velocity **v***(0)* *or* **-v***(0)*

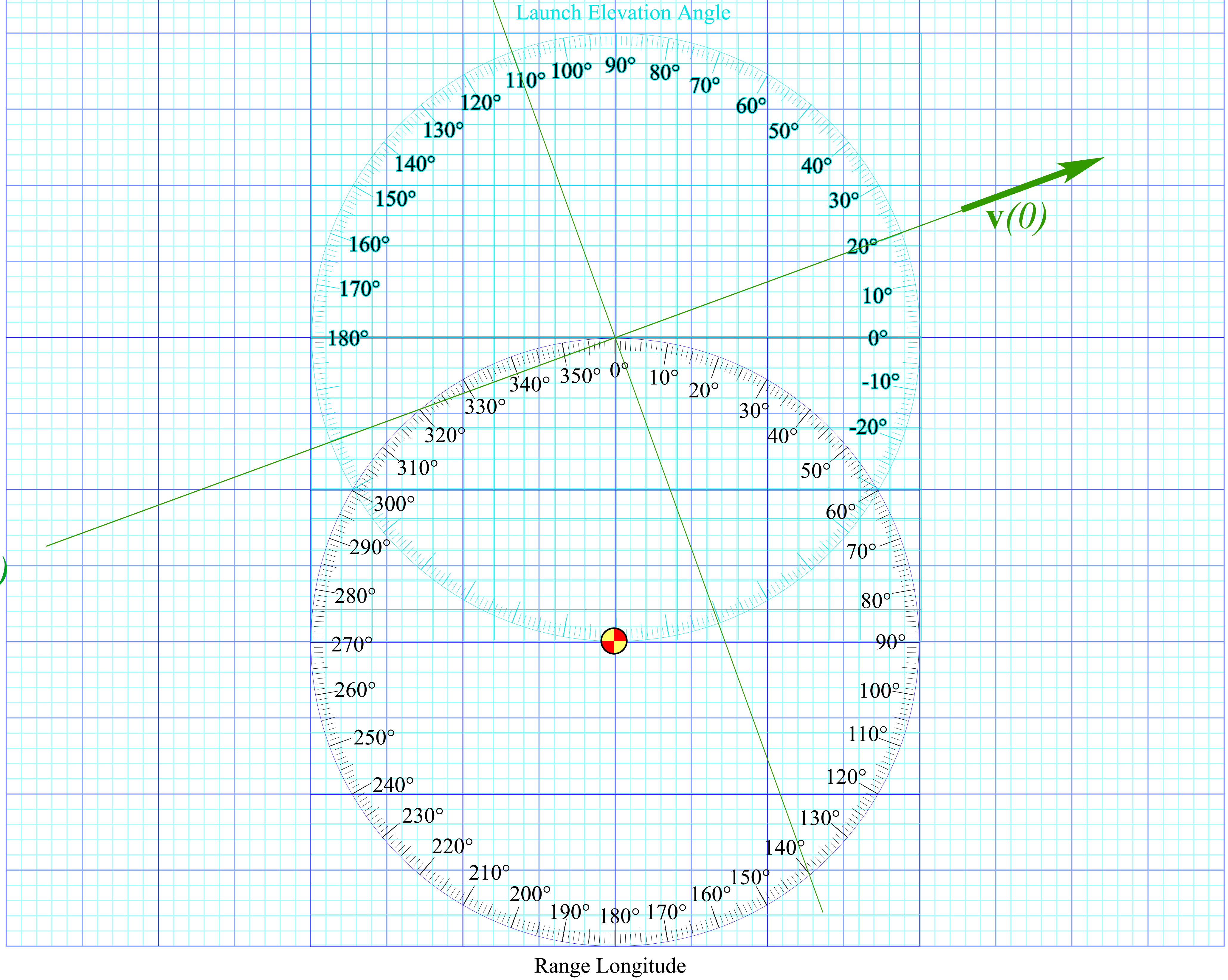


or **-v***(0)*

Start with initial velocity **v***(0)* *or* **-v***(0)*

Label Main Focus F

Construct *R-line normal* to initial velocity **v***(0)* line



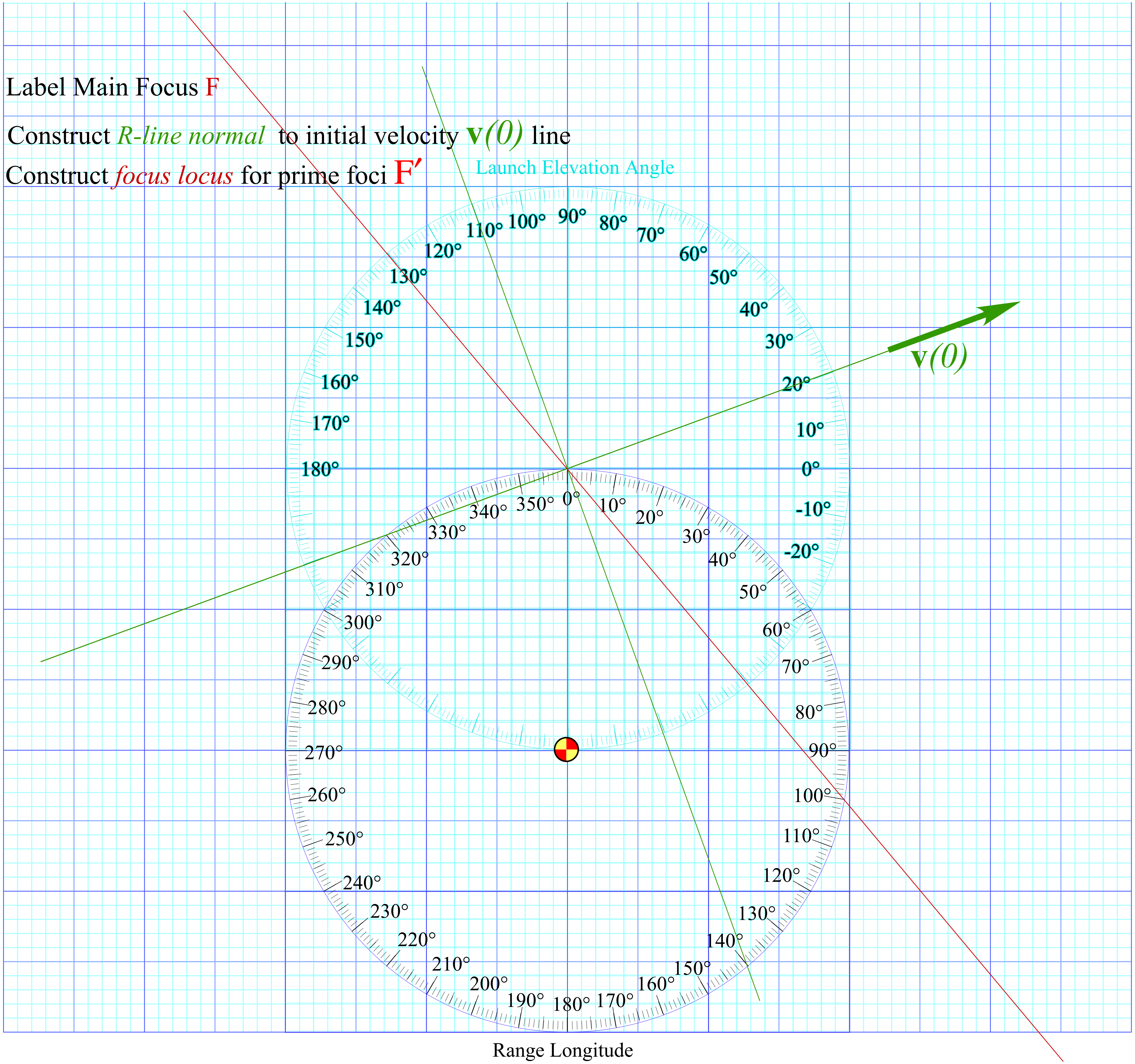
or **-v***(0)*

Start with initial velocity **v***(0)* *or* **-v***(0)*

Label Main Focus F

Construct *R-line normal* to initial velocity **v***(0)* line

Construct *focus locus* for prime foci **F′**



or **-v***(0)*

Start with initial velocity **v***(0)* *or* **-v***(0)*

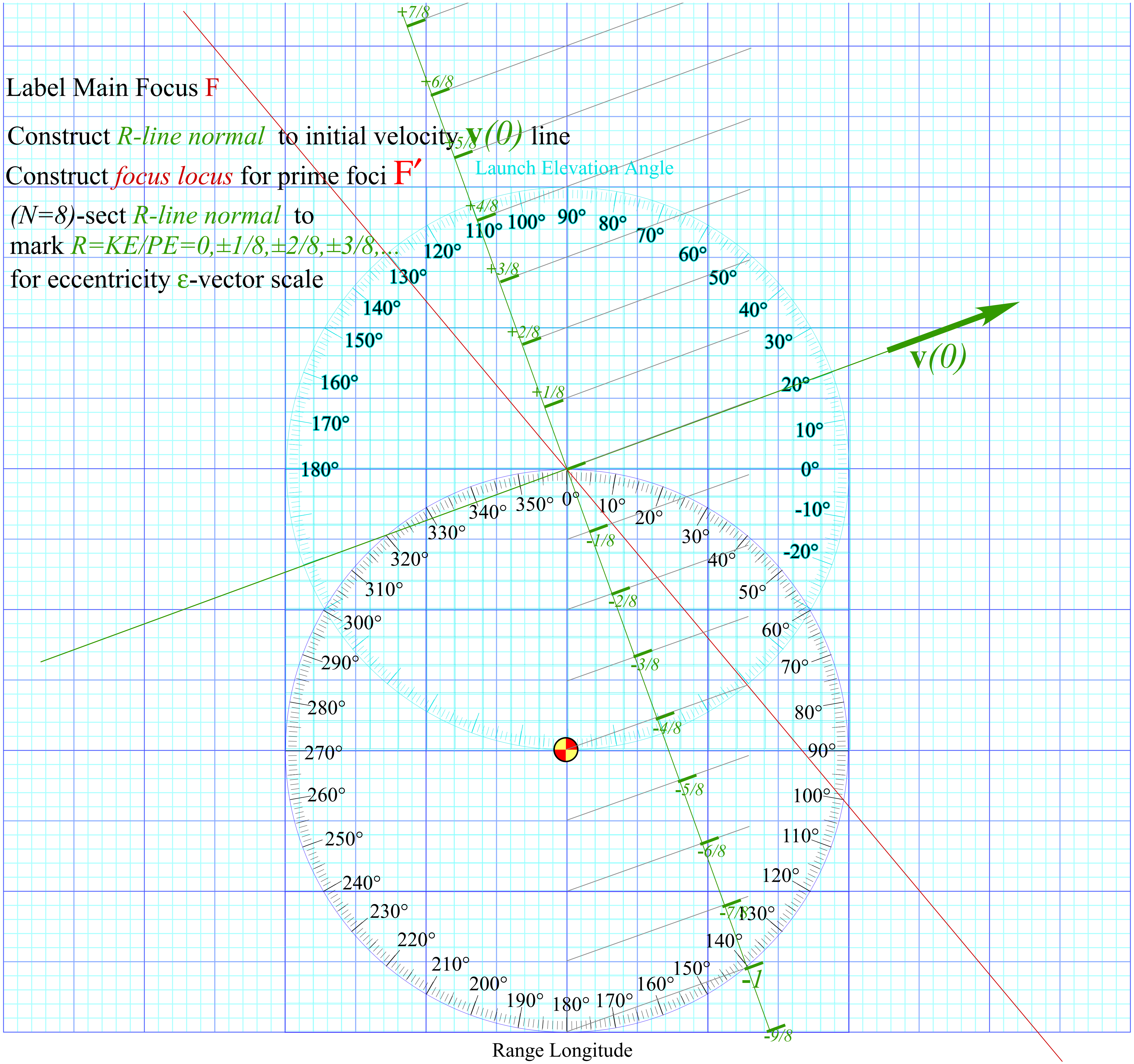
Label Main Focus F

Construct *R-line normal* to initial velocity **v***(0)* line

Construct *focus locus* for prime foci **F′**

(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...*

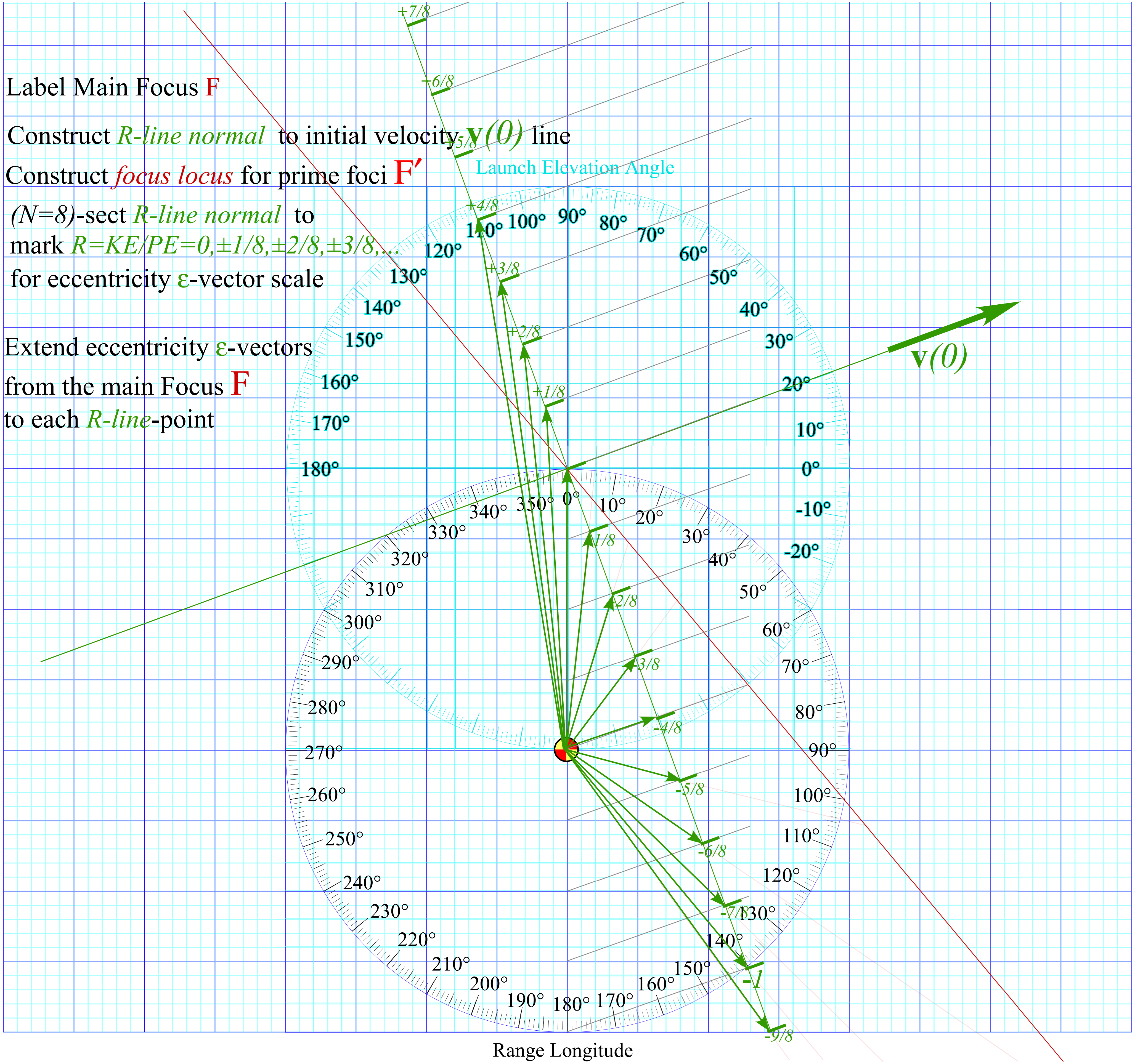
for eccentricity ε-vector scale



or **-v***(0)*

Start with initial velocity **v**(0)

or **-v**(0)



or **-v**(0)

Start with initial velocity **v***(0)*

or **-v***(0)*

Label Main Focus F

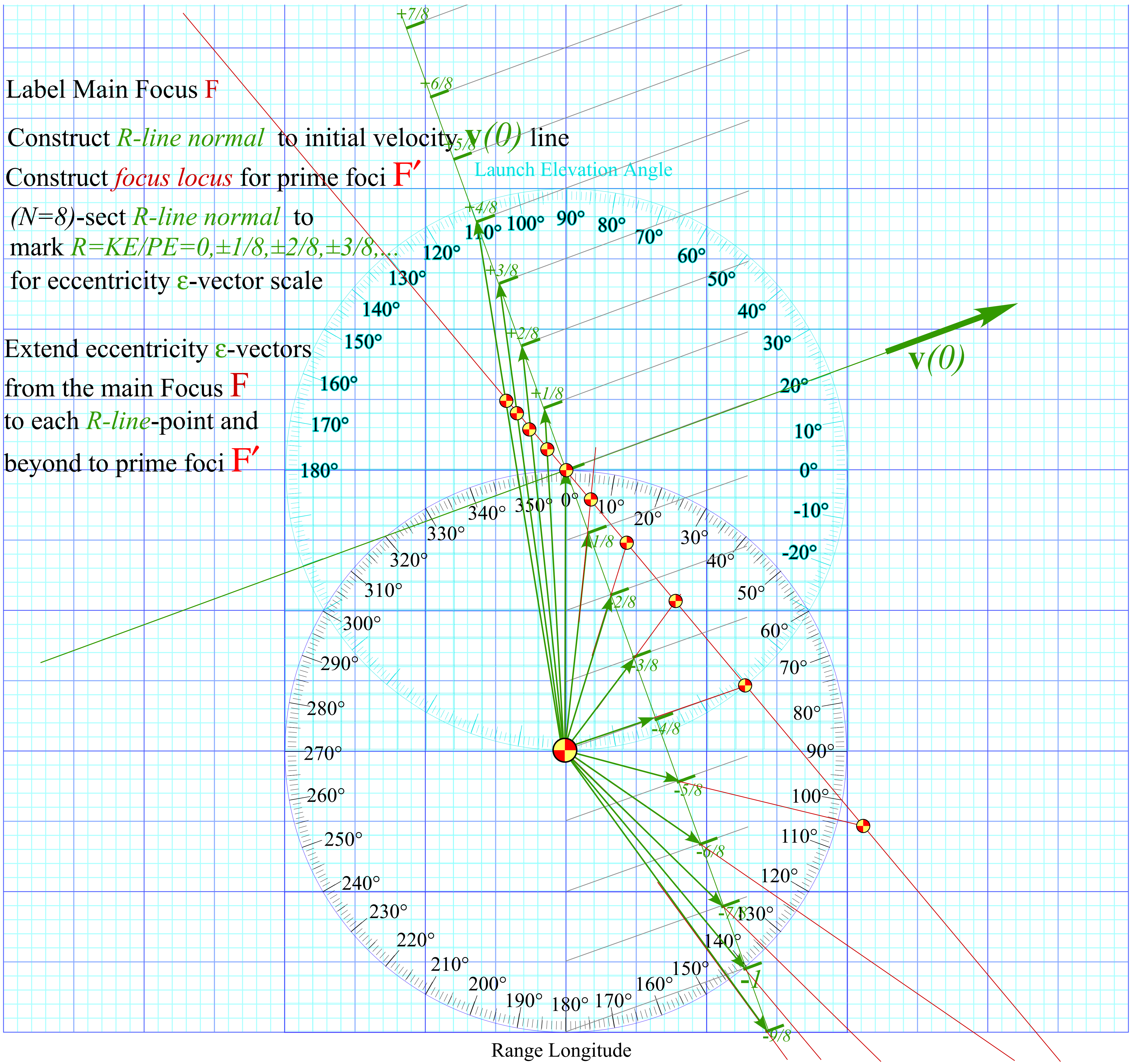
Construct *R-line normal* to initial velocity **v***(0)* line

Construct *focus locus* for prime foci F′

(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

or **-v***(0)*



Start with initial velocity **v**(0) *or* **-v**(0)

Label Main Focus F

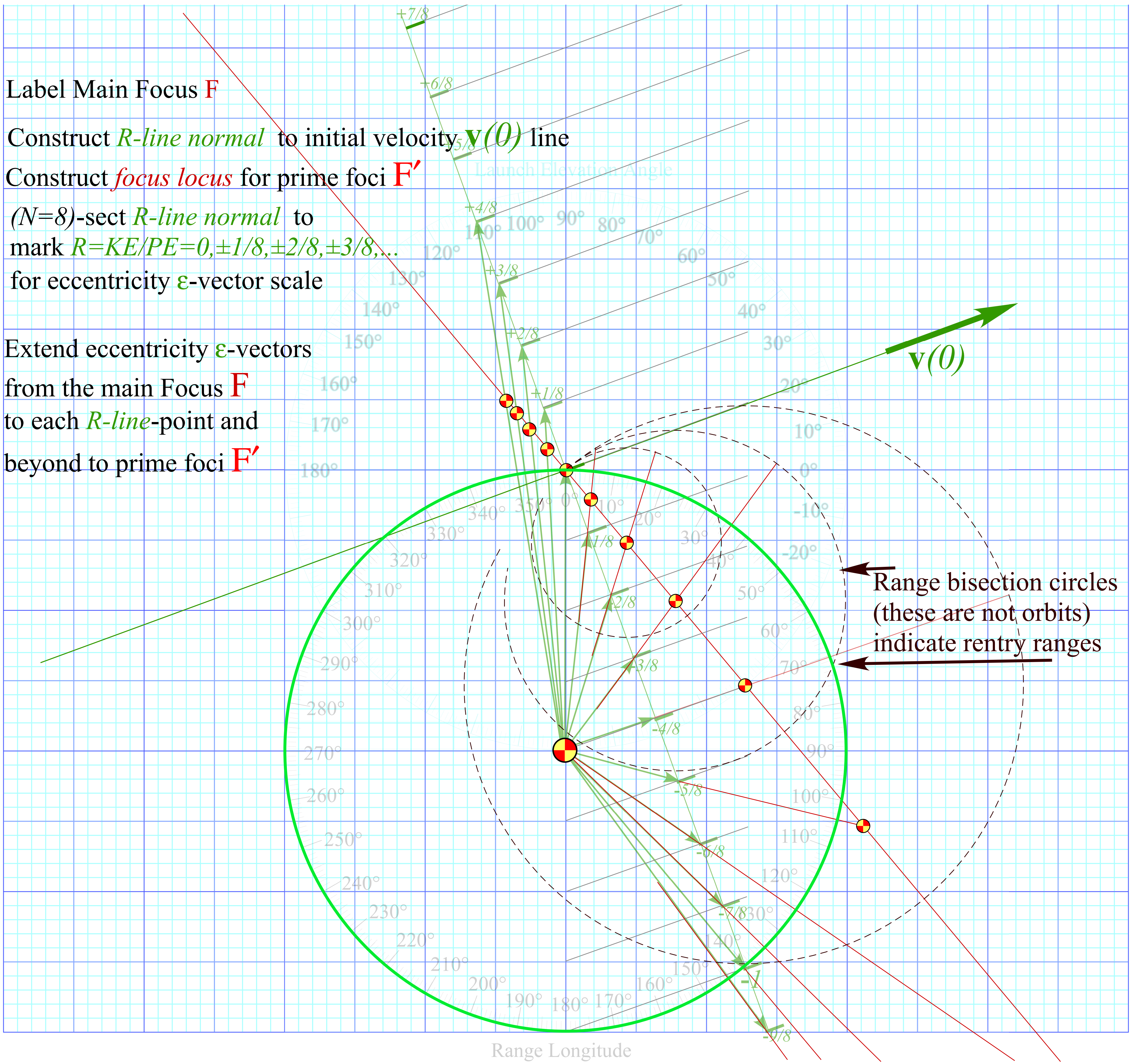
Construct *R-line normal* to initial velocity **v**(0) line

Construct *focus locus* for prime foci F′

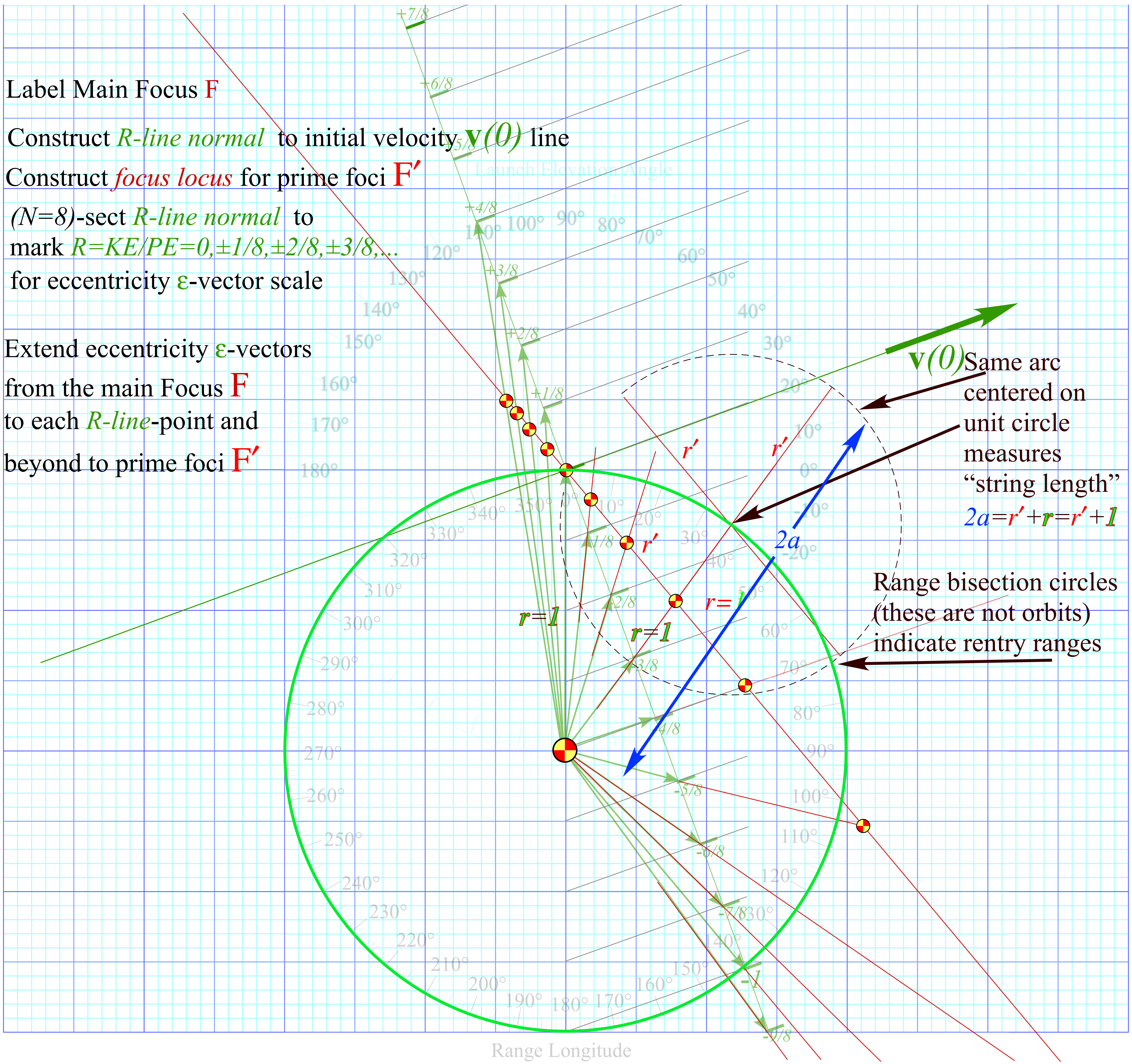
(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

or **-v**(0)

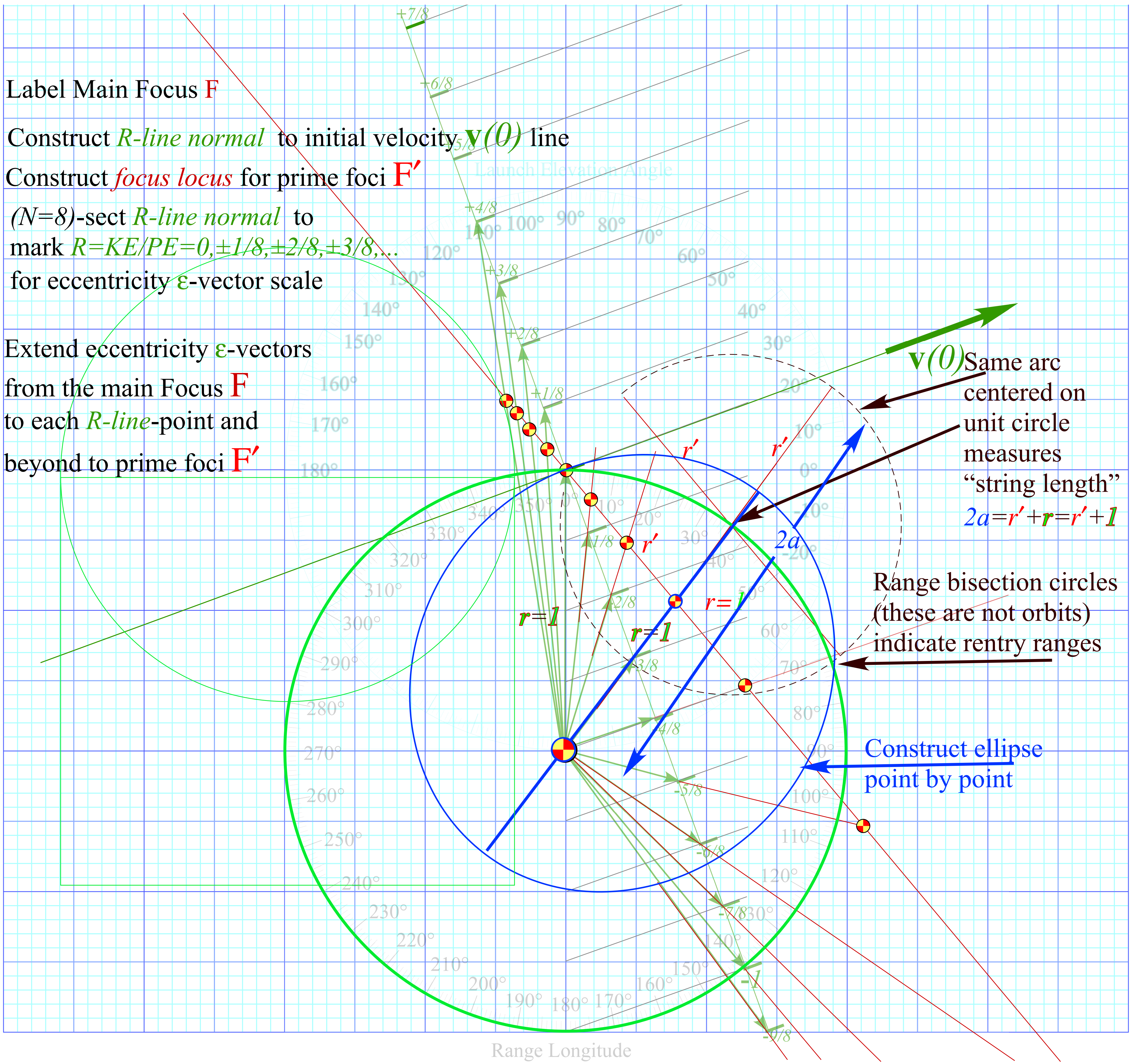


Start with initial velocity **v***(0)* *or* **-v***(0)*

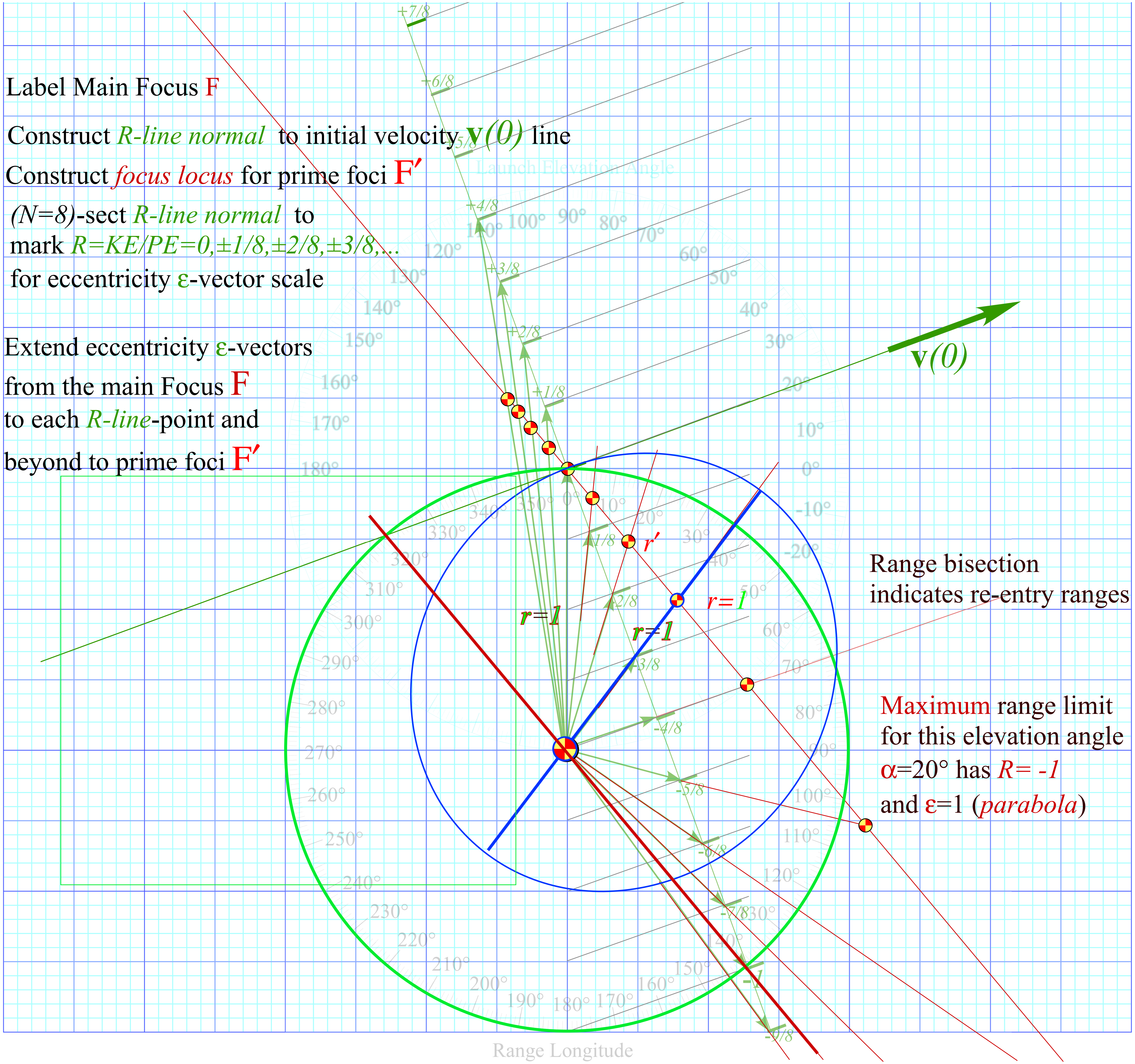


or **-v***(0)*

Start with initial velocity **v***(0)* *or* **-v***(0)*



or **-v***(0)*


Label Main Focus F
Construct *R-line normal* to initial velocity **v**(0) line
Construct *focus locus* for prime foci F′
(N=8)-sect *R-line normal* to
mark *R=KE/PE=0,±1/8,±2/8,±3/8,...*
for eccentricity ε-vector scale
Extend eccentricity ε-vectors
from the main Focus F
to each *R-line*-point and
beyond to prime foci F′
Range bisection
indicates re-entry ranges
Maximum range limit
for this elevation angle
α=20° has *R= -1*
and ε=1 (*parabola*)
v(0)
Launch Elevation Angle
Range Longitude
r=1
r=1
r=1
r′
+7/8
+6/8
+5/8
+4/8
+3/8
+2/8
+1/8
-1/8
-2/8
-3/8
-4/8
-5/8
-6/8
-7/8
-1
-9/8

Label Main Focus F

Construct *R-line normal* to initial velocity **v**(0) line

Construct *focus locus* for prime foci F′

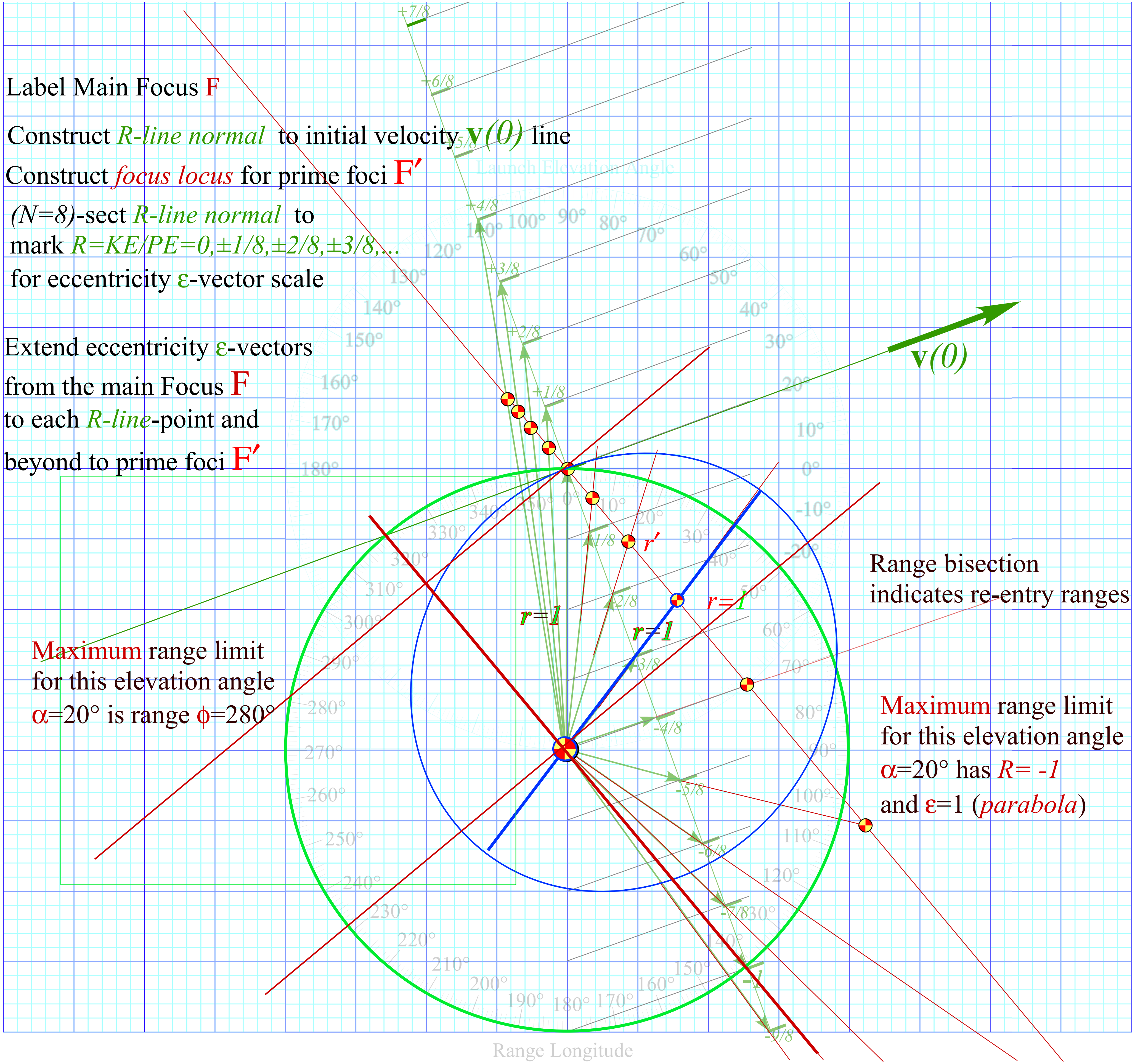
(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

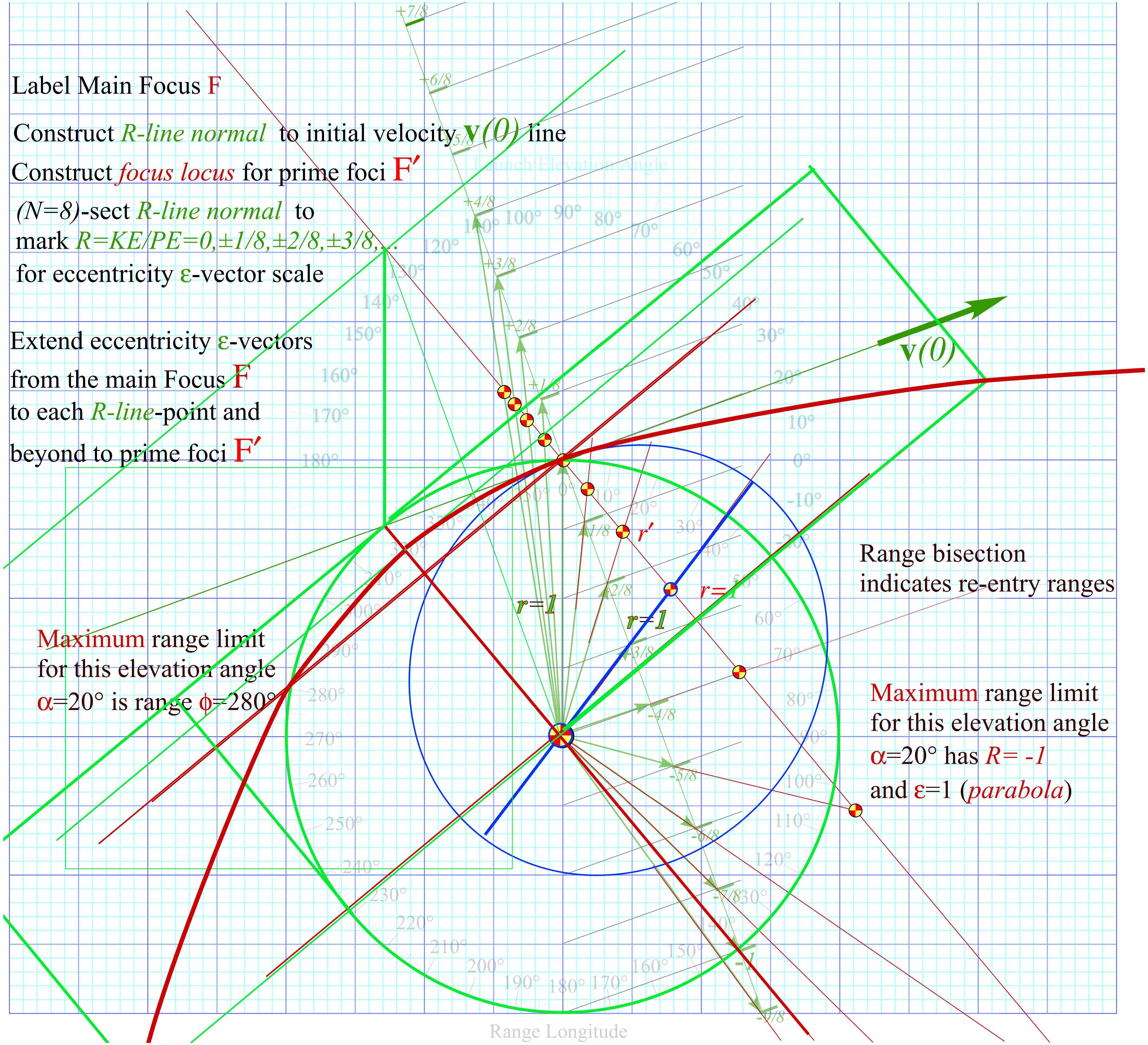
Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

Range bisection indicates re-entry ranges

Maximum range limit for this elevation angle α=20° is range ϕ=280°

Maximum range limit for this elevation angle α=20° has *R= -1* and ε=1 (*parabola*)




Label Main Focus F
Construct R-line normal to initial velocity v(0) line
Construct focus locus for prime foci F′
(N=8)-sect R-line normal to
mark R=KE/PE=0,±1/8,±2/8,±3/8,...
for eccentricity ε-vector scale
Extend eccentricity ε-vectors
from the main Focus F
to each R-line-point and
beyond to prime foci F′
v(0)
Range bisection
indicates re-entry ranges
Maximum range limit
for this elevation angle
α=20° is range ϕ=280°
Maximum range limit
for this elevation angle
α=20° has R= -1
and ε=1 (parabola)
r=1
r=1
r=1
r′
Launch Elevation Angle
Range Longitude

Start with initial velocity **v***(0)*

or **-v***(0)*

Label Main Focus F

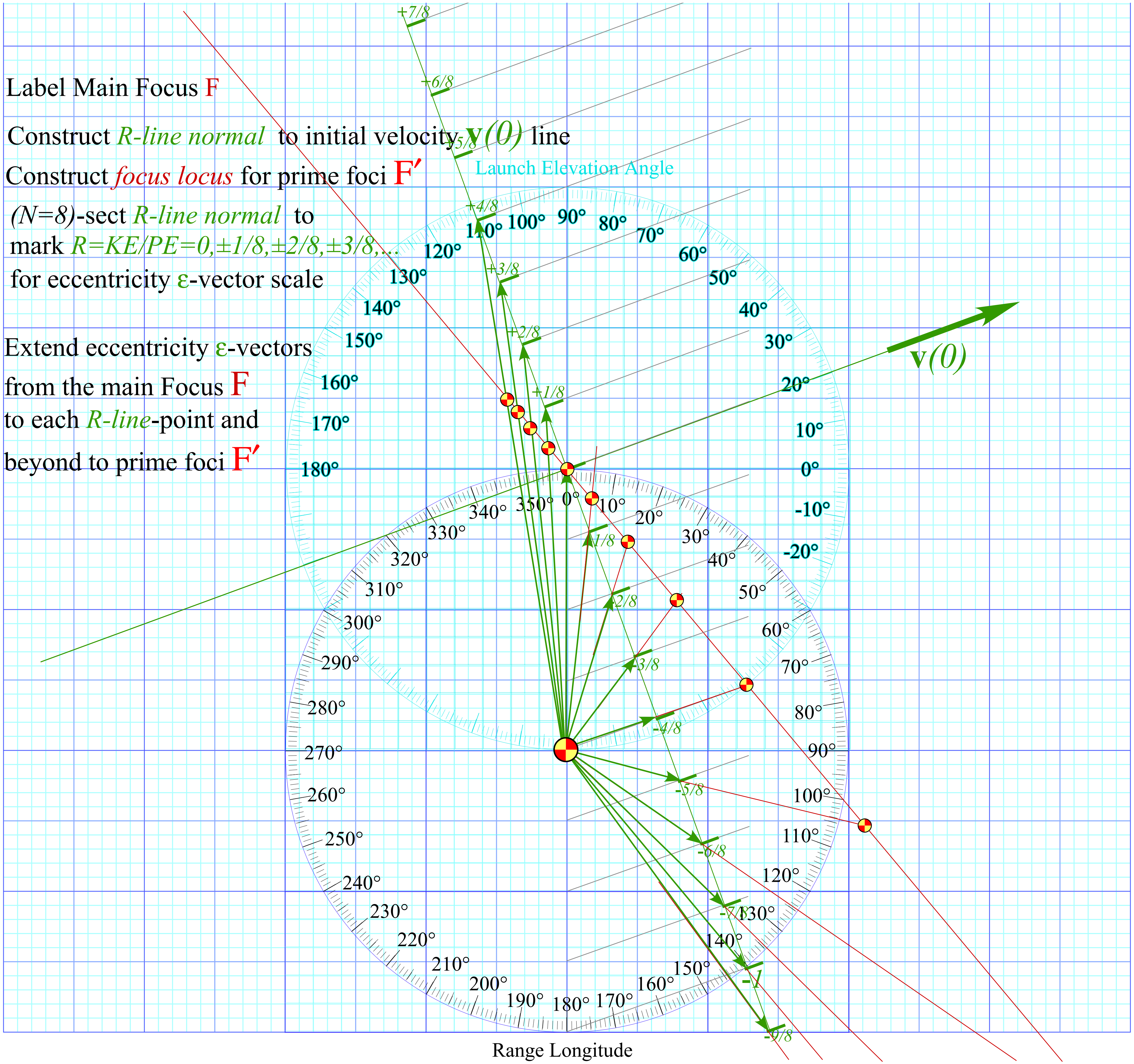
Construct *R-line normal* to initial velocity **v***(0)* line

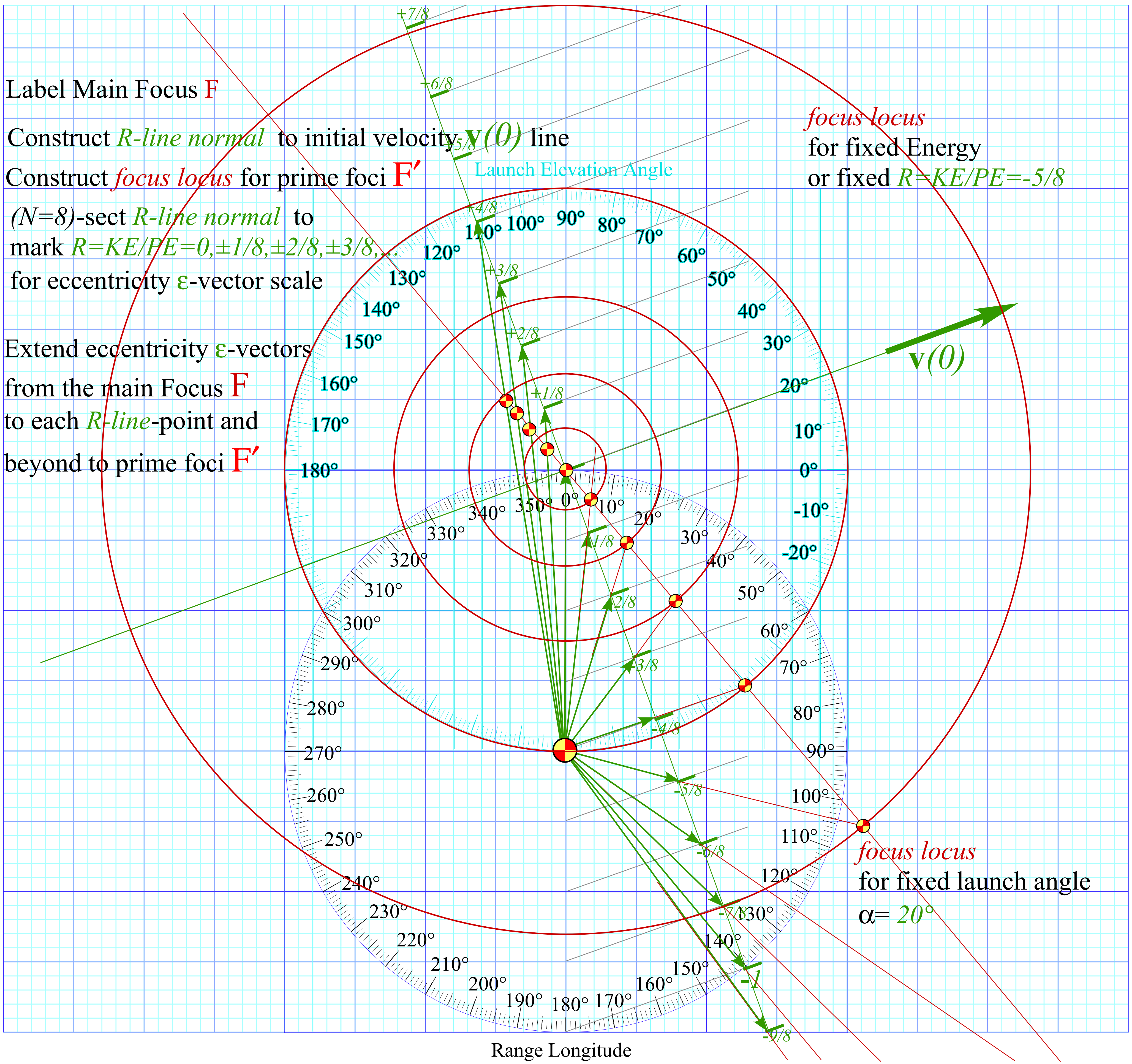
Construct *focus locus* for prime foci F′

(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

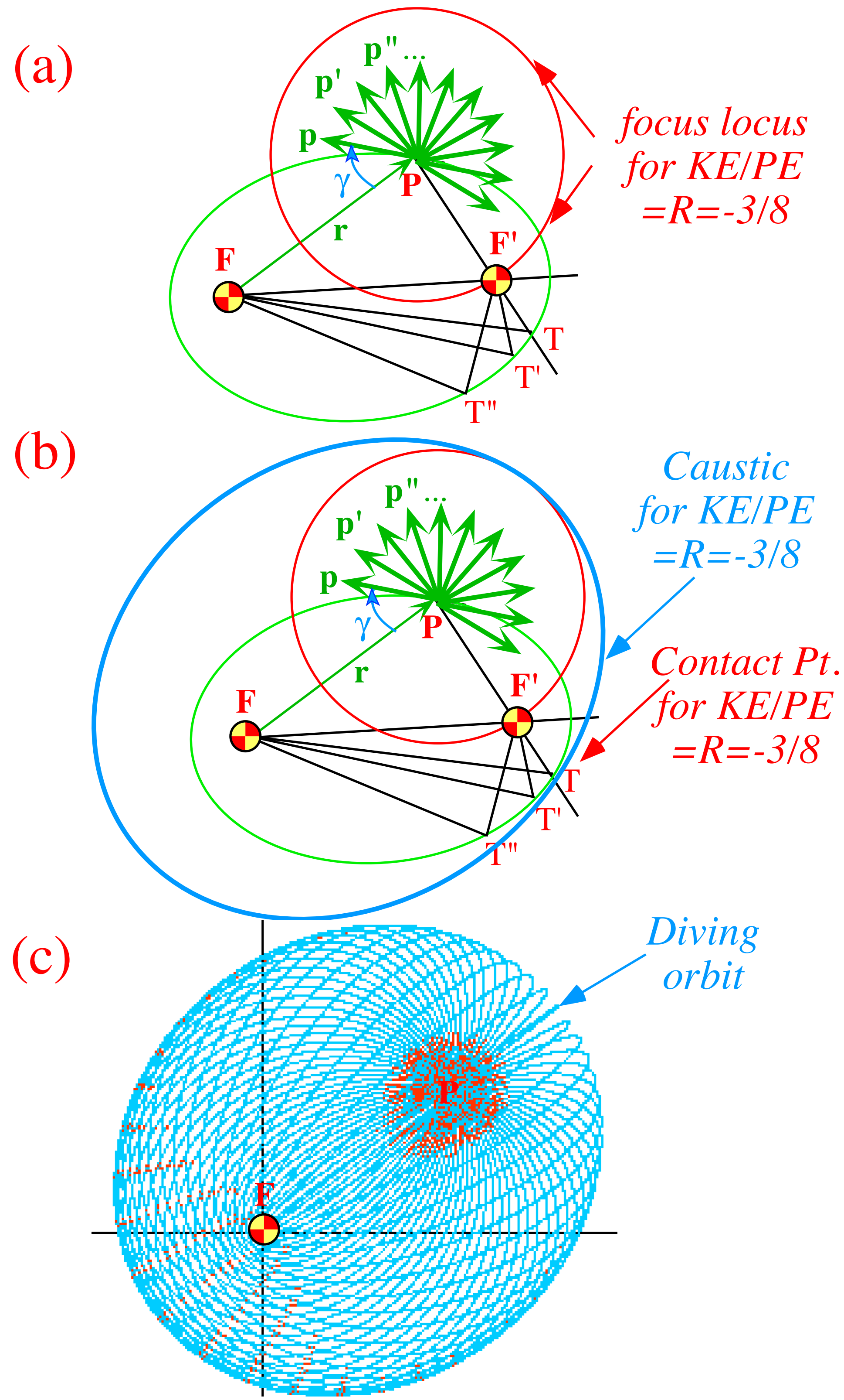
Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

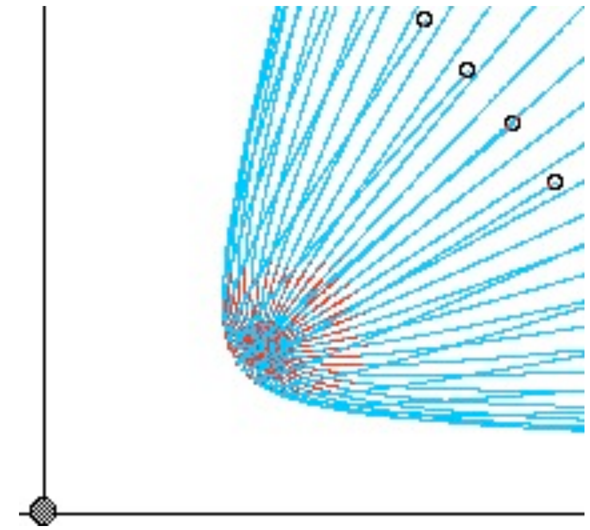
or **-v***(0)*

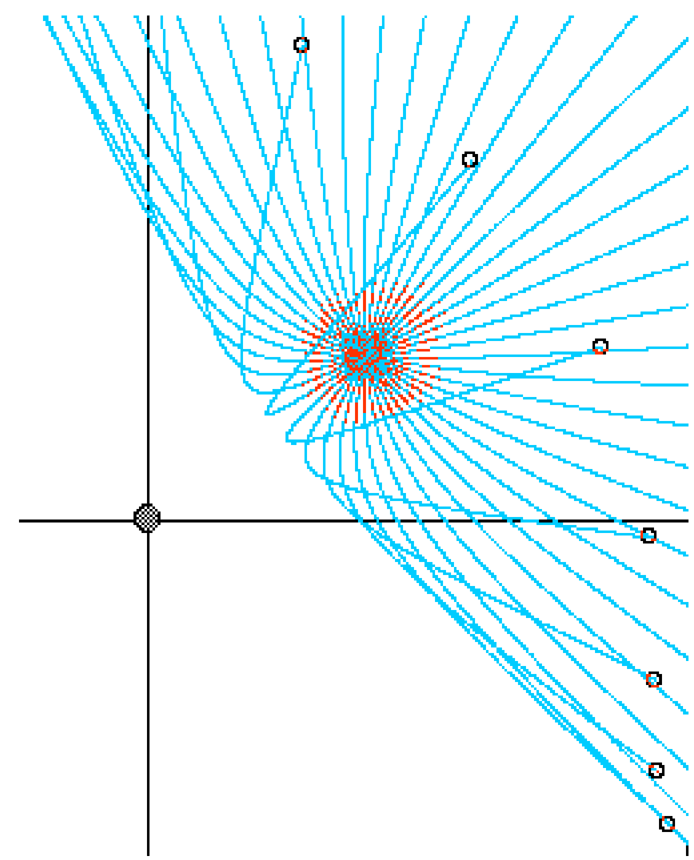




Label Main Focus F
Construct R-line normal to initial velocity v(0) line
Construct focus locus for prime foci F′
(N=8)-sect R-line normal to
mark R=KE/PE=0,±1/8,±2/8,±3/8,...
for eccentricity ε-vector scale
Extend eccentricity ε-vectors
from the main Focus F
to each R-line-point and
beyond to prime foci F′
focus locus
for fixed Energy
or fixed R=KE/PE=-5/8
Launch Elevation Angle
v(0)
focus locus
for fixed launch angle
α= 20°
Range Longitude


(a)
p"...
p'
p
γ
P
r
F
F'
T
T'
T"
focus locus
for KE/PE
=R=-3/8
(b)
Caustic
for KE/PE
=R=-3/8
Contact Pt.
for KE/PE
=R=-3/8
(c)
Diving
orbit
F





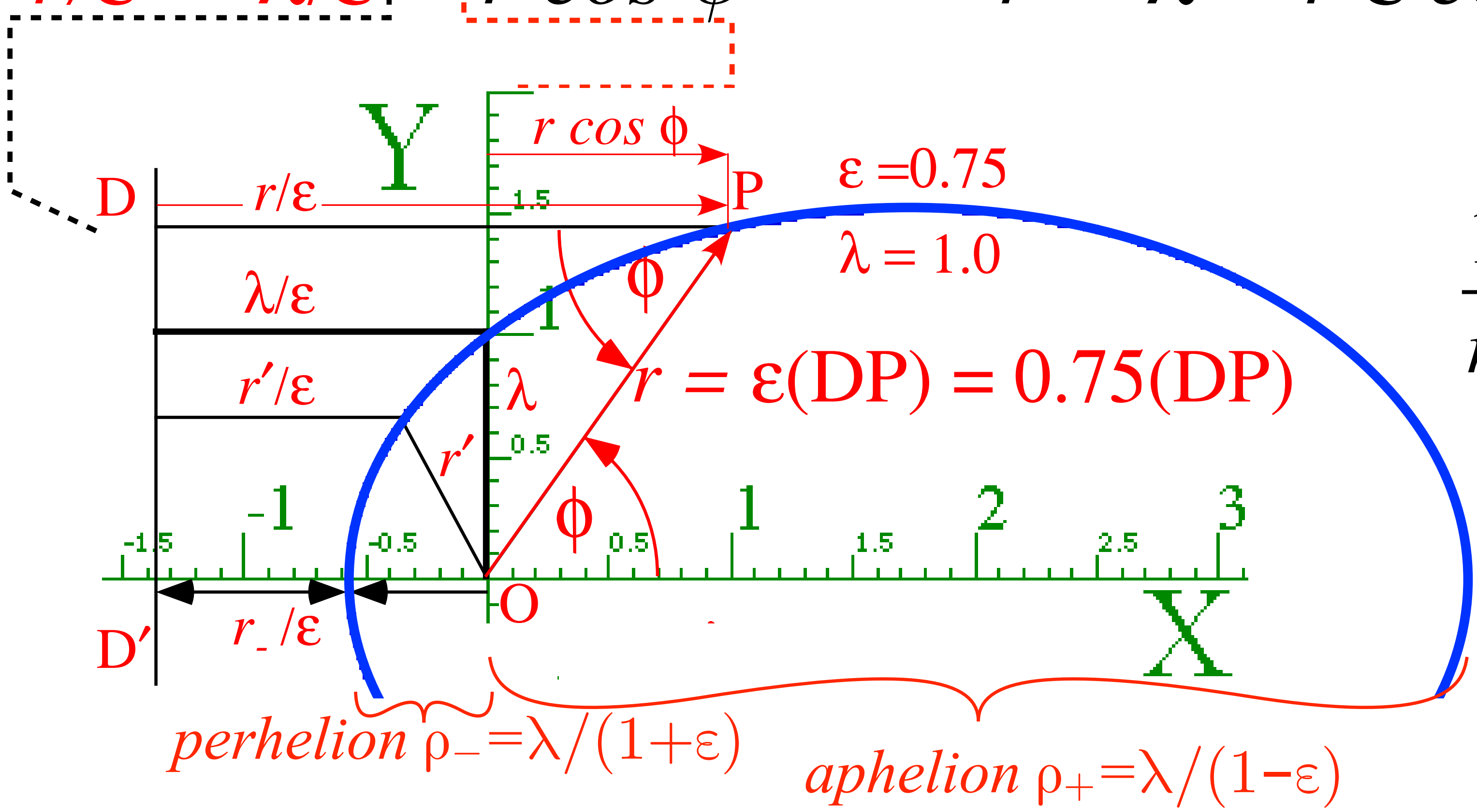
➤ *Geometry and Symmetry of Coulomb orbits*

Detailed elliptic geometry

Detailed hyperbolic geometry

Geometry of Coulomb conic section orbits (Let: $r = \rho$ here)

$$r/\varepsilon = \lambda/\varepsilon + r\cos\phi \qquad r = \lambda + r\,\varepsilon\cos\phi \qquad r = \frac{\lambda}{1-\varepsilon\cos\phi}$$

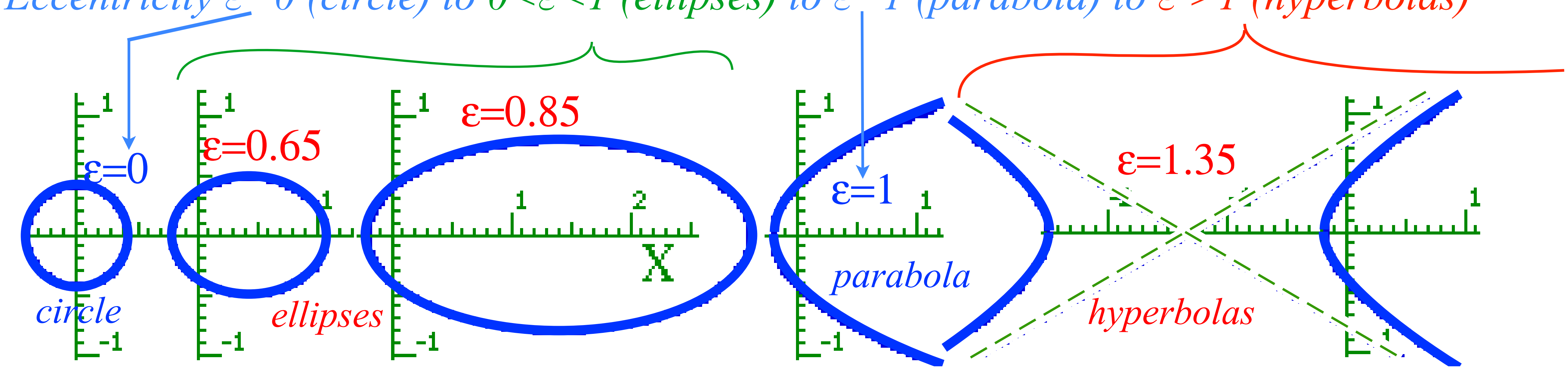


$$\frac{1}{r} = \frac{1-\varepsilon\cos\phi}{\lambda} = \frac{1}{\lambda} - \frac{\varepsilon}{\lambda}\cos\phi$$

$$\frac{1}{\rho} = \frac{-k}{\mu^2/m} + \frac{\sqrt{k^2 + 2E\mu^2/m}}{\mu^2/m}\cos\phi$$

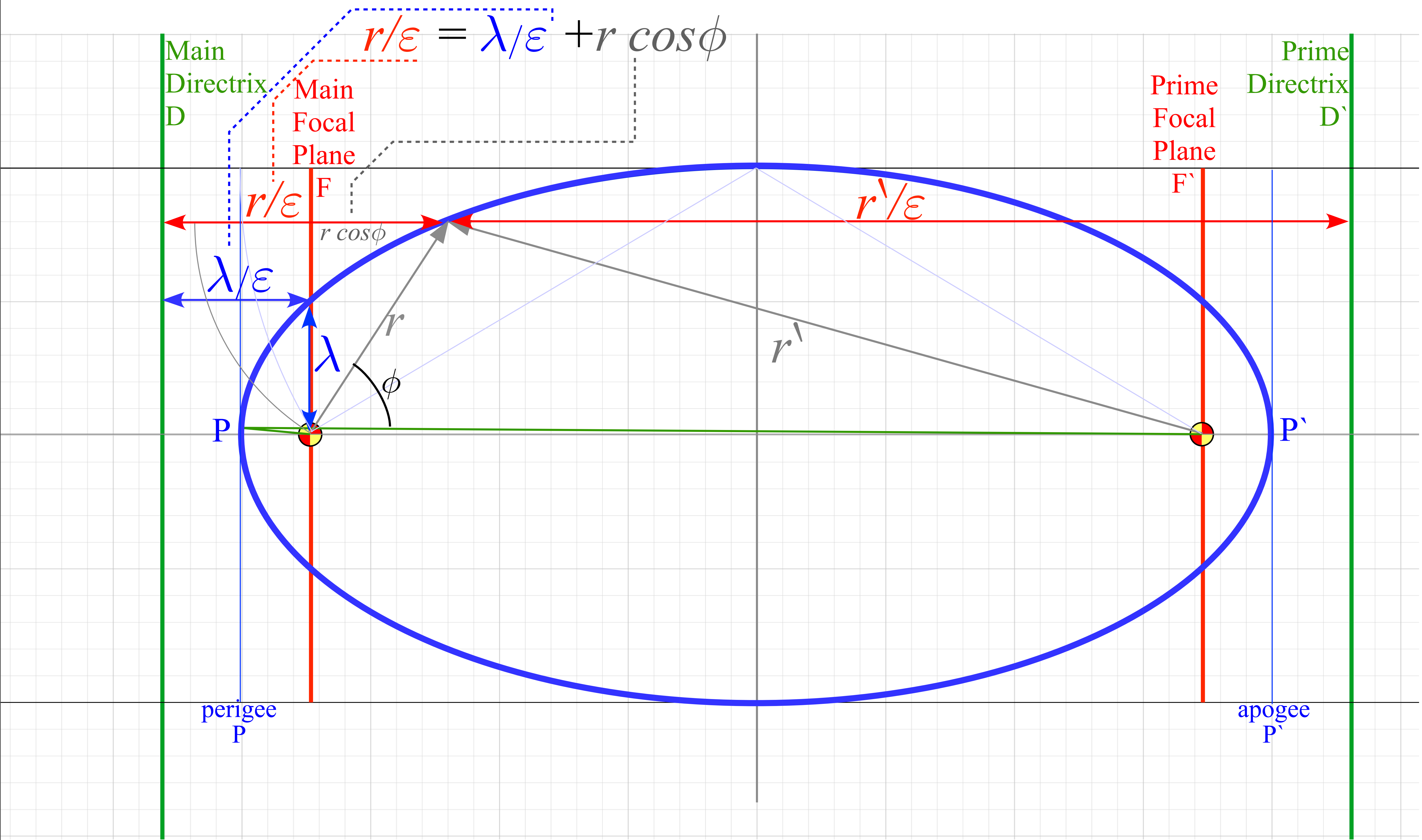
Becoming more and more eccentric...

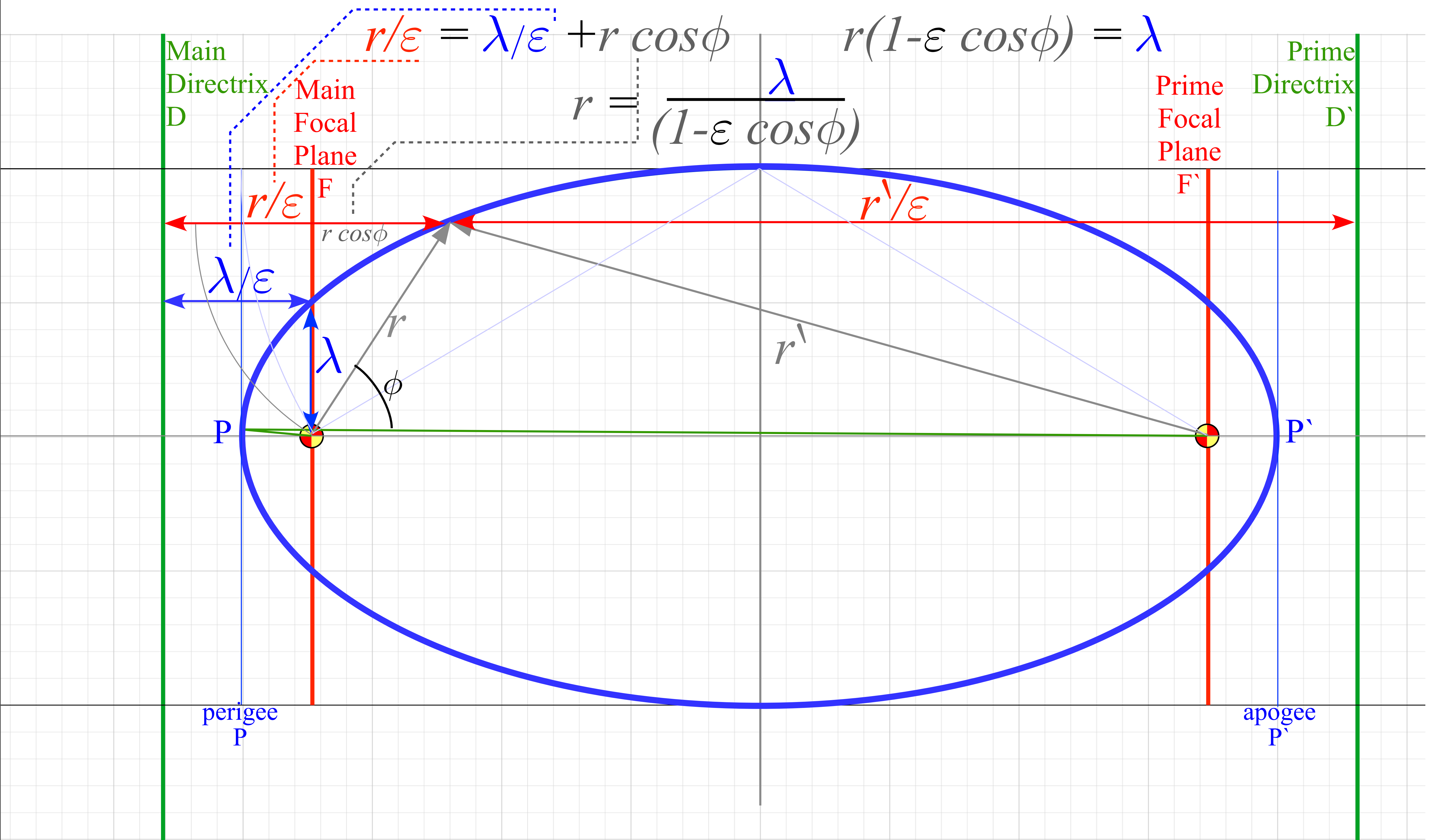
Eccentricity $\varepsilon=0$ (circle) to $0<\varepsilon<1$ (ellipses) to $\varepsilon=1$ (parabola) to $\varepsilon>1$ (hyperbolas)

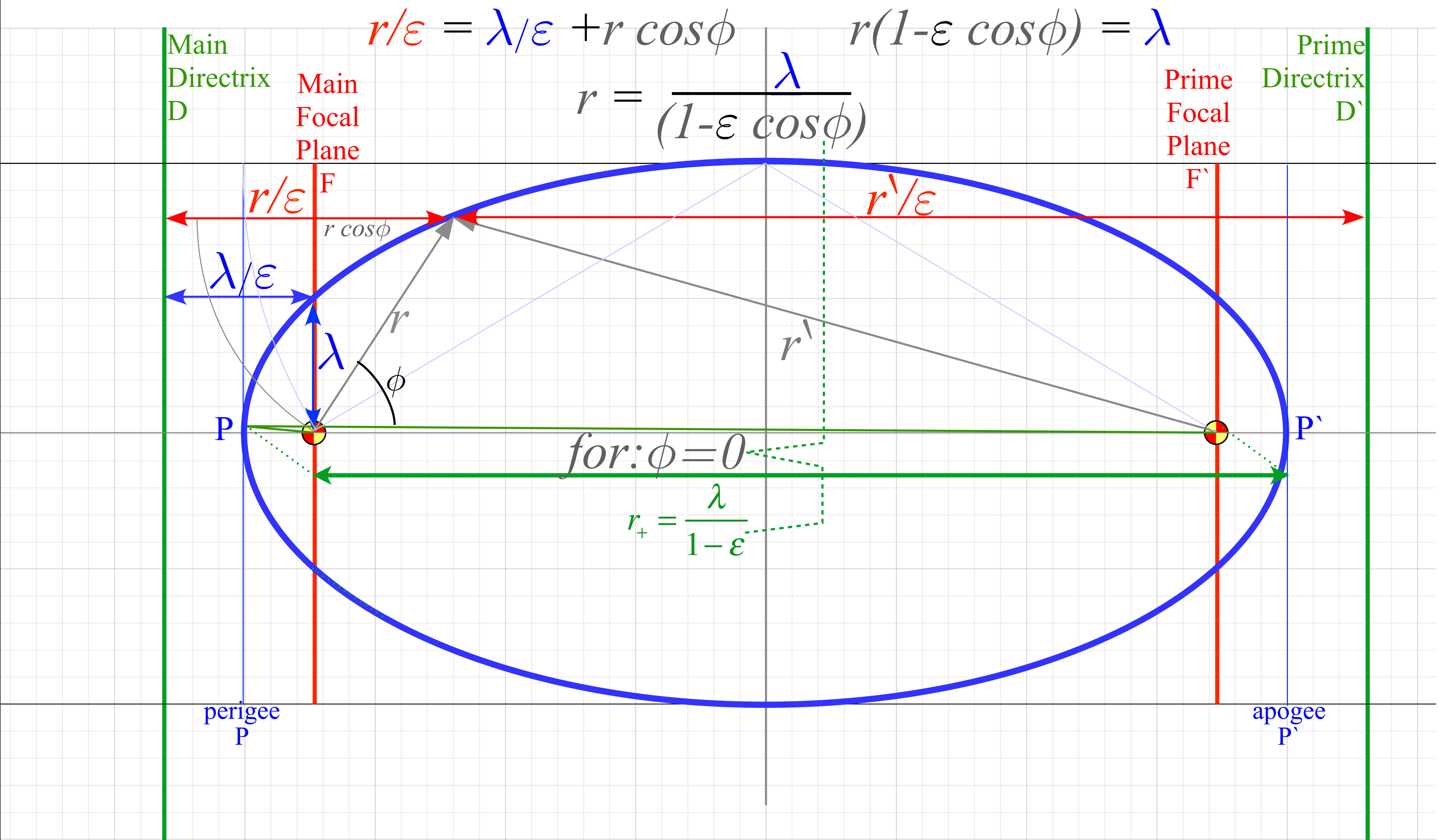


Geometry and Symmetry of Coulomb orbits

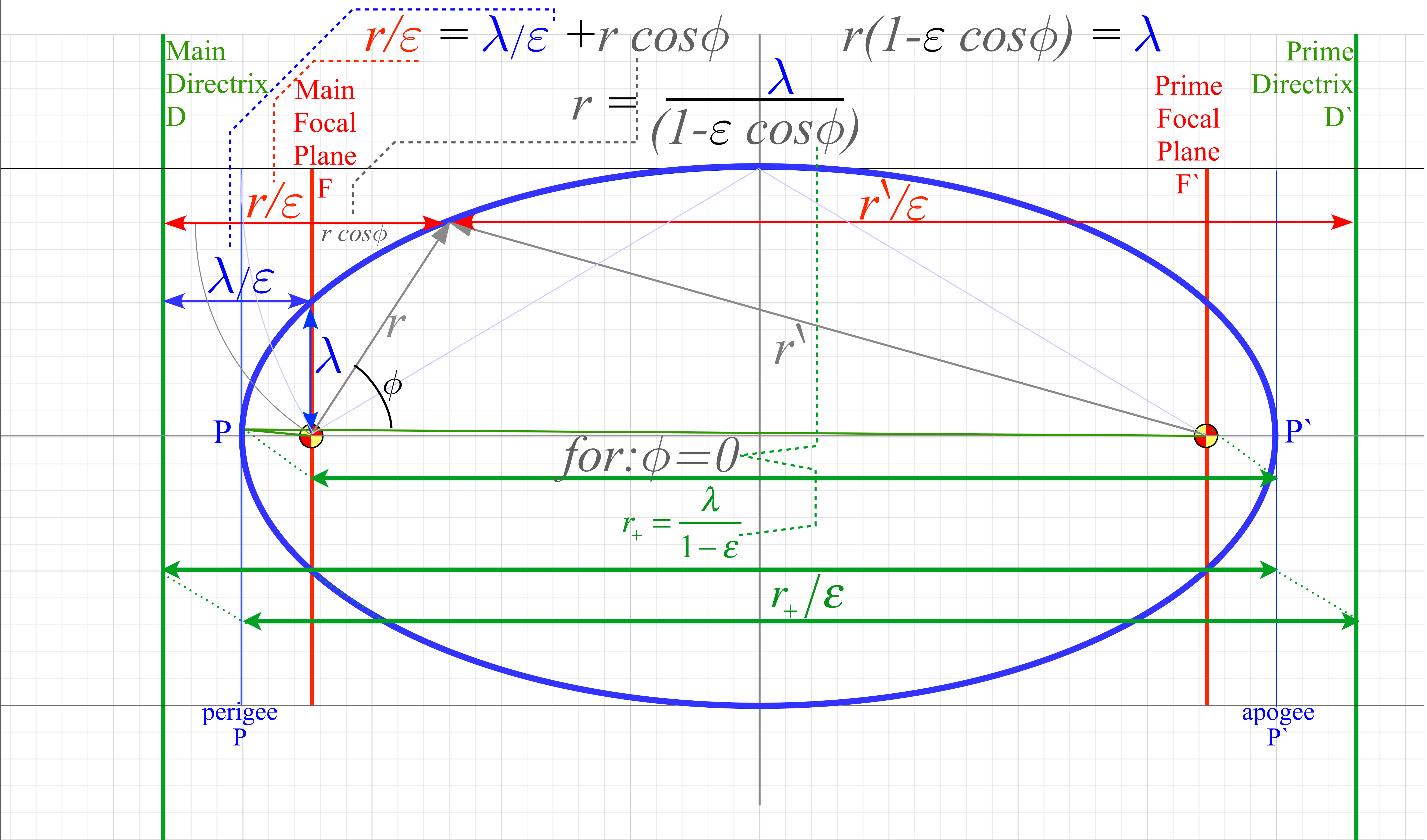
- ➤ *Detailed elliptic geometry*
- *Detailed hyperbolic geometry*

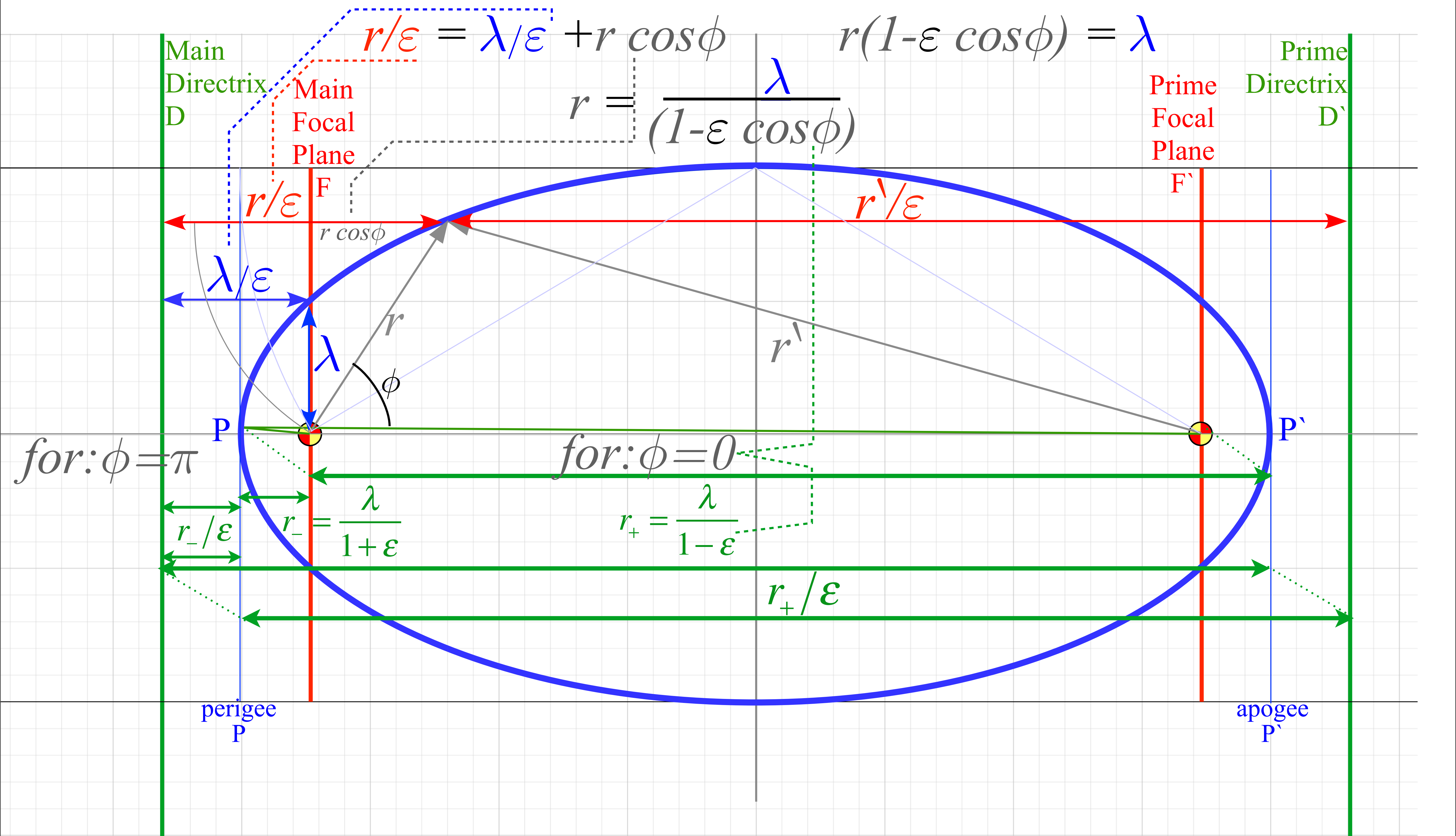

$r/\varepsilon = \lambda/\varepsilon + r\cos\phi$
Main Directrix D
Main Focal Plane
F
Prime Focal Plane
F`
Prime Directrix D`
r/ε
$r\cos\phi$
r'/ε
λ/ε
λ
r
r'
ϕ
P
P`
perigee P
apogee P`


r/ε = λ/ε + r cosφ
r(1-ε cosφ) = λ
r = λ / (1-ε cosφ)
Main Directrix D
Main Focal Plane F
Prime Focal Plane F`
Prime Directrix D`
r/ε
r cosφ
r`/ε
λ/ε
λ
r
r`
φ
P
P`
perigee P
apogee P`

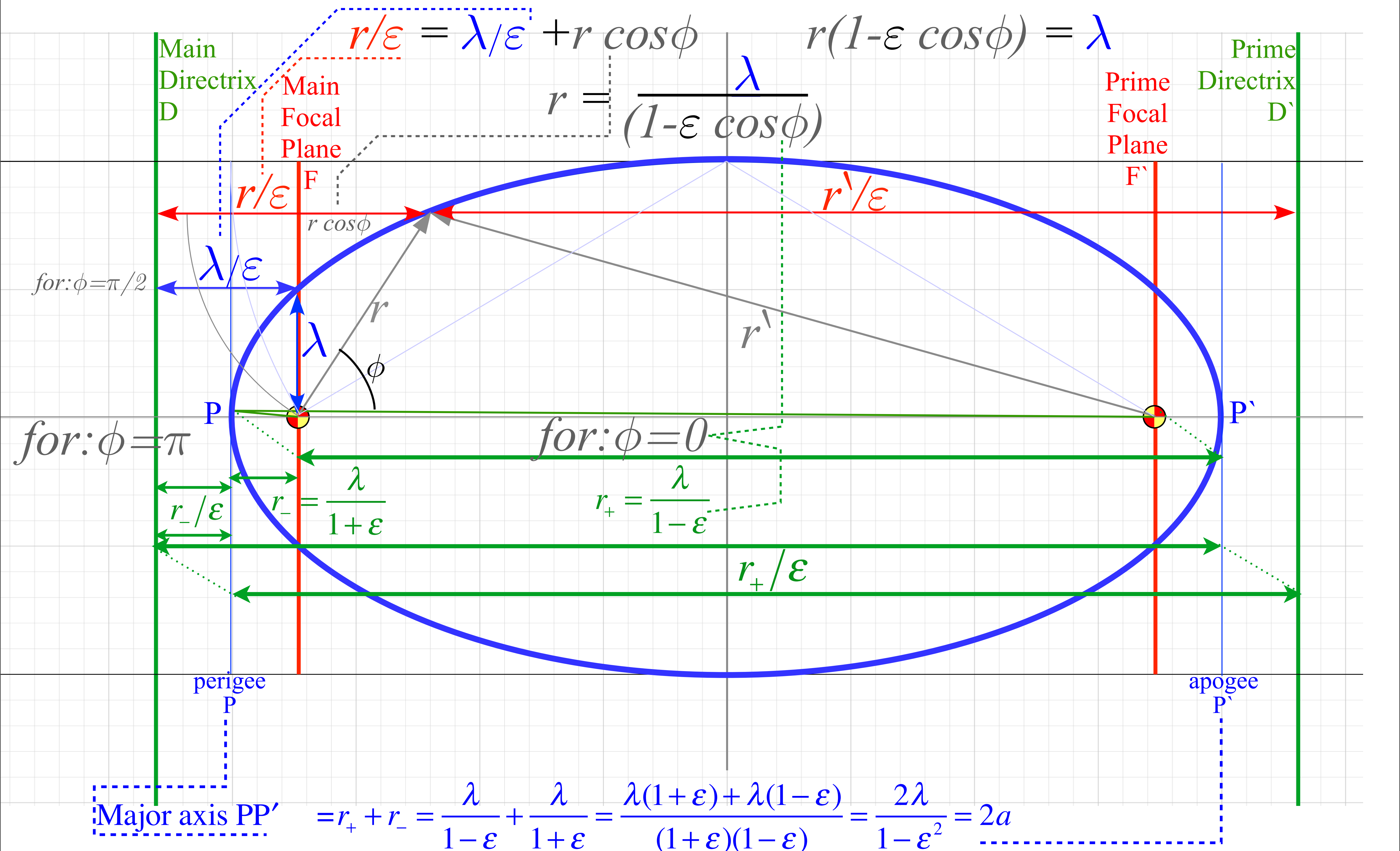


$r/\varepsilon = \lambda/\varepsilon + r\cos\phi$
$r(1-\varepsilon\cos\phi) = \lambda$
$r = \frac{\lambda}{(1-\varepsilon\cos\phi)}$
Main Directrix D
Main Focal Plane
F
Prime Focal Plane
F`
Prime Directrix D`
r/ε
$r\cos\phi$
r'/ε
λ/ε
λ
r
r'
ϕ
P
P`
$for: \phi = 0$
$r_+ = \frac{\lambda}{1-\varepsilon}$
perigee P
apogee P`

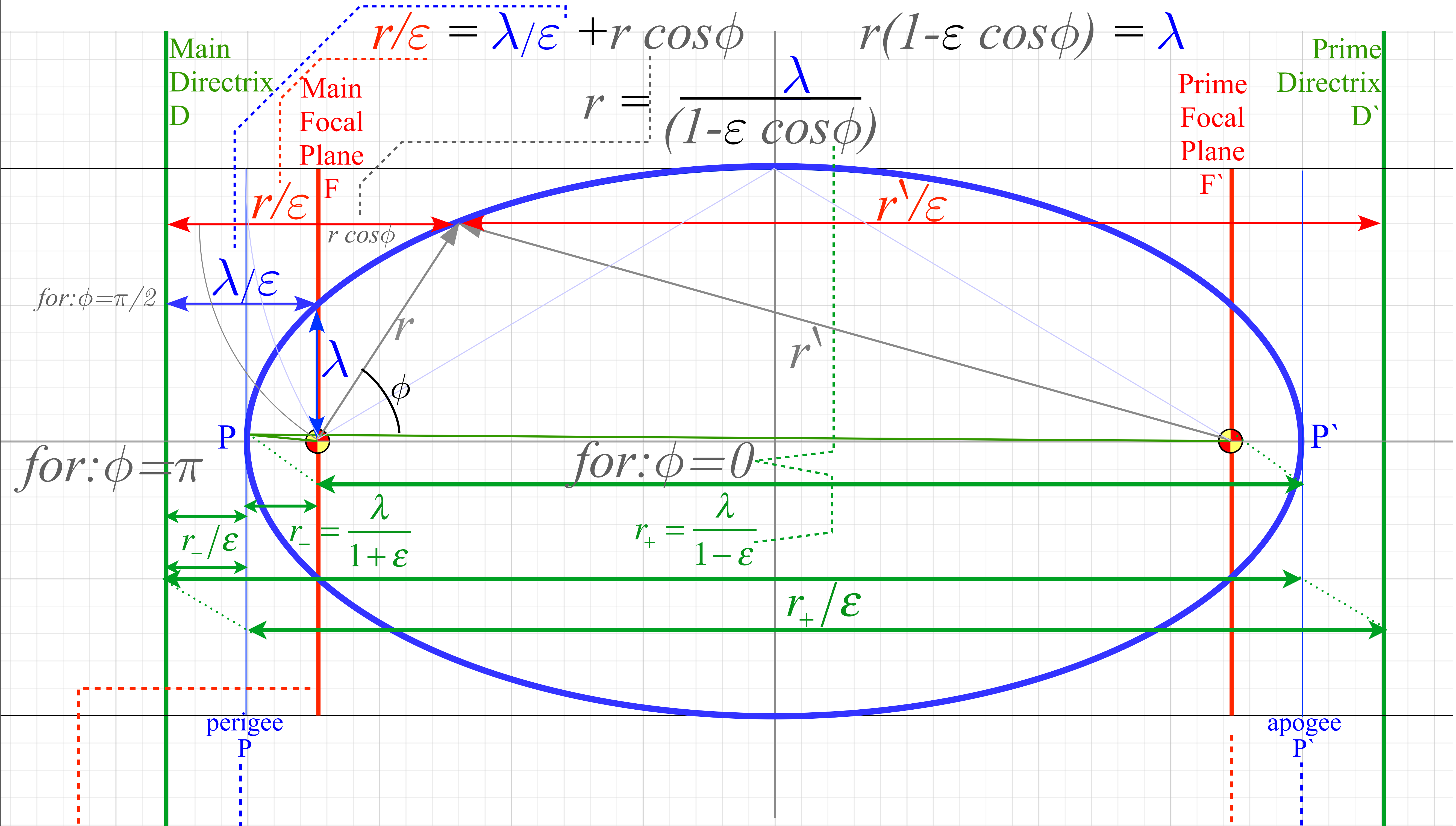

$r/\varepsilon = \lambda/\varepsilon + r\cos\phi$
$r(1-\varepsilon\cos\phi) = \lambda$
$r = \dfrac{\lambda}{(1-\varepsilon\cos\phi)}$
Main Directrix D
Main Focal Plane F
Prime Focal Plane F`
Prime Directrix D`
r/ε
$r\cos\phi$
r'/ε
λ/ε
λ
r
r'
ϕ
P
P`
$for{:}\phi=0$
$r_+ = \dfrac{\lambda}{1-\varepsilon}$
r_+/ε
perigee P
apogee P`



r/ε = λ/ε + r cosφ
r(1-ε cosφ) = λ
r = λ / (1-ε cosφ)
Main Directrix D
Main Focal Plane F
Prime Focal Plane F`
Prime Directrix D`
r/ε
r cosφ
r`/ε
λ/ε
λ
r
r`
φ
P
P`
for:φ=π
for:φ=0
r_ = λ / (1+ε)
r+ = λ / (1−ε)
r_/ε
r+/ε
perigee P
apogee P`

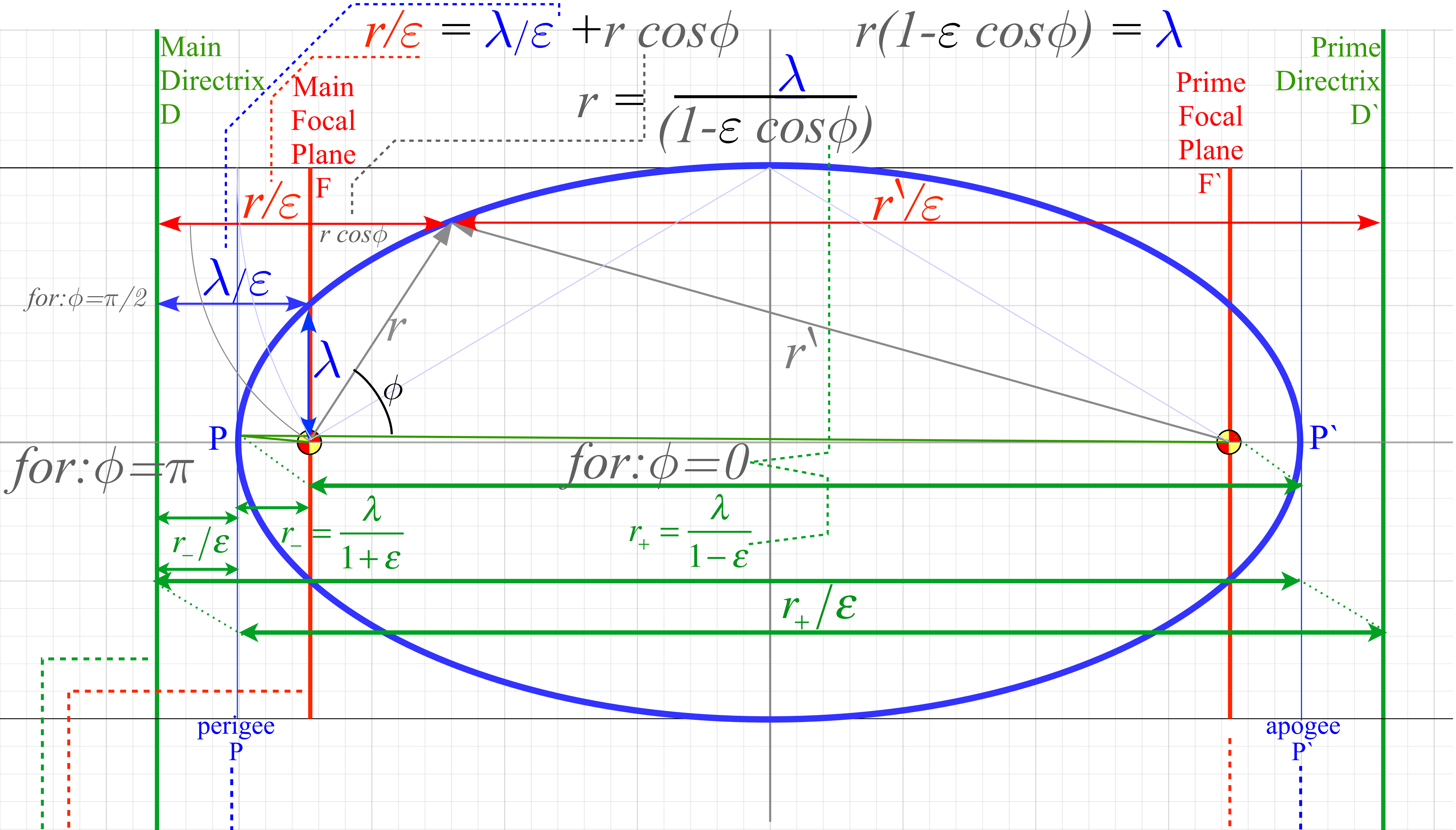


$r/\varepsilon = \lambda/\varepsilon + r\cos\phi$
$r(1-\varepsilon\cos\phi) = \lambda$
$r = \frac{\lambda}{(1-\varepsilon\cos\phi)}$
Main Directrix D
Main Focal Plane F
Prime Focal Plane F`
Prime Directrix D`
r/ε
$r\cos\phi$
$r`/\varepsilon$
$for:\phi=\pi/2$
λ/ε
λ
r
$r`$
ϕ
P
P`
$for:\phi=\pi$
$for:\phi=0$
r_-/ε
$r_- = \frac{\lambda}{1+\varepsilon}$
$r_+ = \frac{\lambda}{1-\varepsilon}$
r_+/ε
perigee P
apogee P`
Major axis PP′
$= r_+ + r_- = \frac{\lambda}{1-\varepsilon} + \frac{\lambda}{1+\varepsilon} = \frac{\lambda(1+\varepsilon)+\lambda(1-\varepsilon)}{(1+\varepsilon)(1-\varepsilon)} = \frac{2\lambda}{1-\varepsilon^2} = 2a$



Major axis PP′ $= r_+ + r_- = \dfrac{\lambda}{1-\varepsilon} + \dfrac{\lambda}{1+\varepsilon} = \dfrac{\lambda(1+\varepsilon)+\lambda(1-\varepsilon)}{(1+\varepsilon)(1-\varepsilon)} = \dfrac{2\lambda}{1-\varepsilon^2} = 2a$

Focal axis FF′ $= r_+ - r_- = \dfrac{\lambda}{1-\varepsilon} - \dfrac{\lambda}{1+\varepsilon} = \dfrac{2\lambda\varepsilon}{1-\varepsilon^2} = 2a\varepsilon$

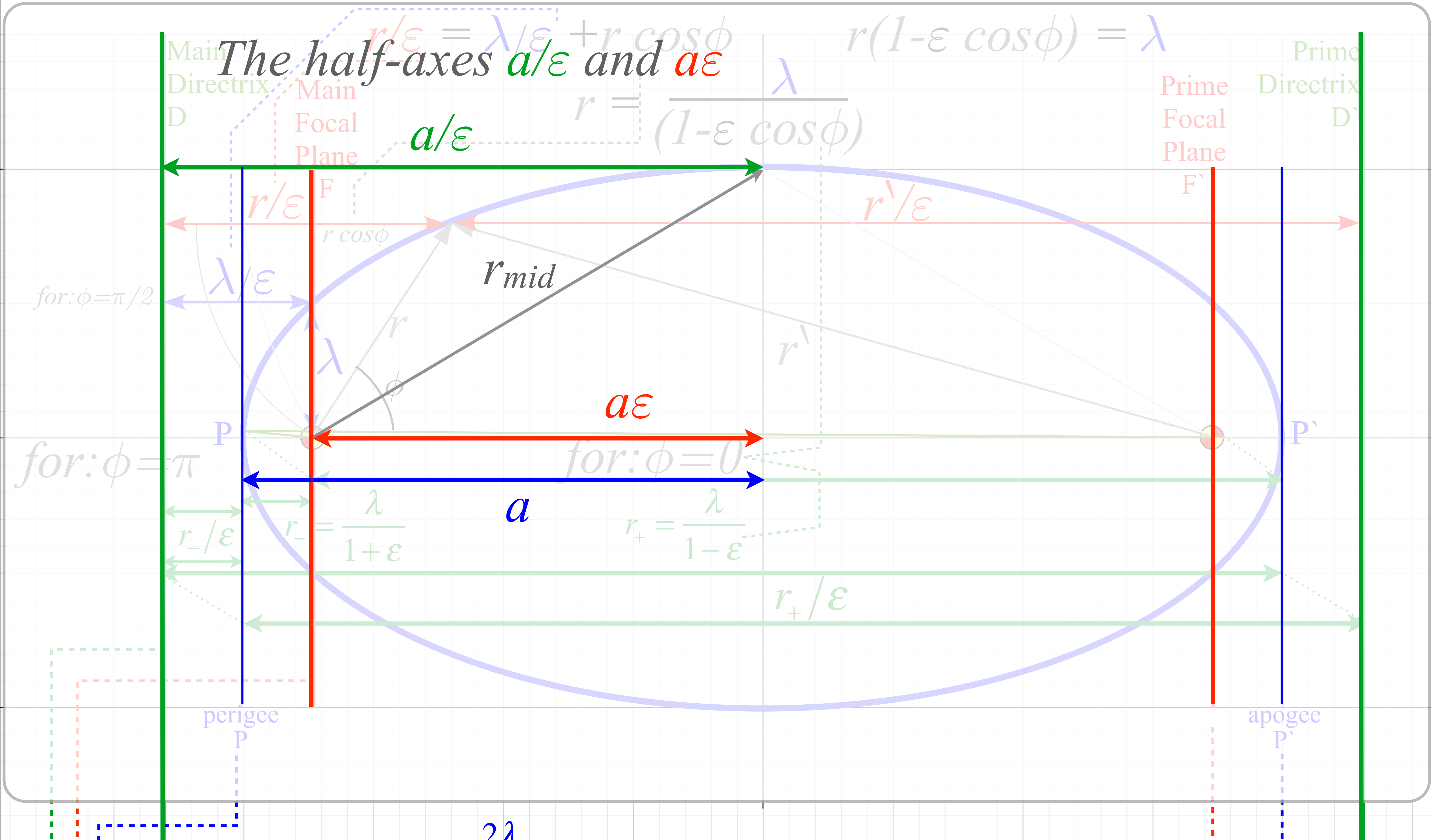


Major axis PP′ $= r_+ + r_- = \dfrac{\lambda}{1-\varepsilon} + \dfrac{\lambda}{1+\varepsilon} = \dfrac{\lambda(1+\varepsilon)+\lambda(1-\varepsilon)}{(1+\varepsilon)(1-\varepsilon)} = \dfrac{2\lambda}{1-\varepsilon^2} = 2a$

Focal axis FF′ $= r_+ - r_- = \dfrac{\lambda}{1-\varepsilon} - \dfrac{\lambda}{1+\varepsilon} = \dfrac{2\lambda\varepsilon}{1-\varepsilon^2} = 2a\varepsilon$

Directrix axis DD′ $= \dfrac{r_+}{\varepsilon} + \dfrac{r_-}{\varepsilon} = \dfrac{2a}{\varepsilon}$

The half-axes a/ε and $a\varepsilon$

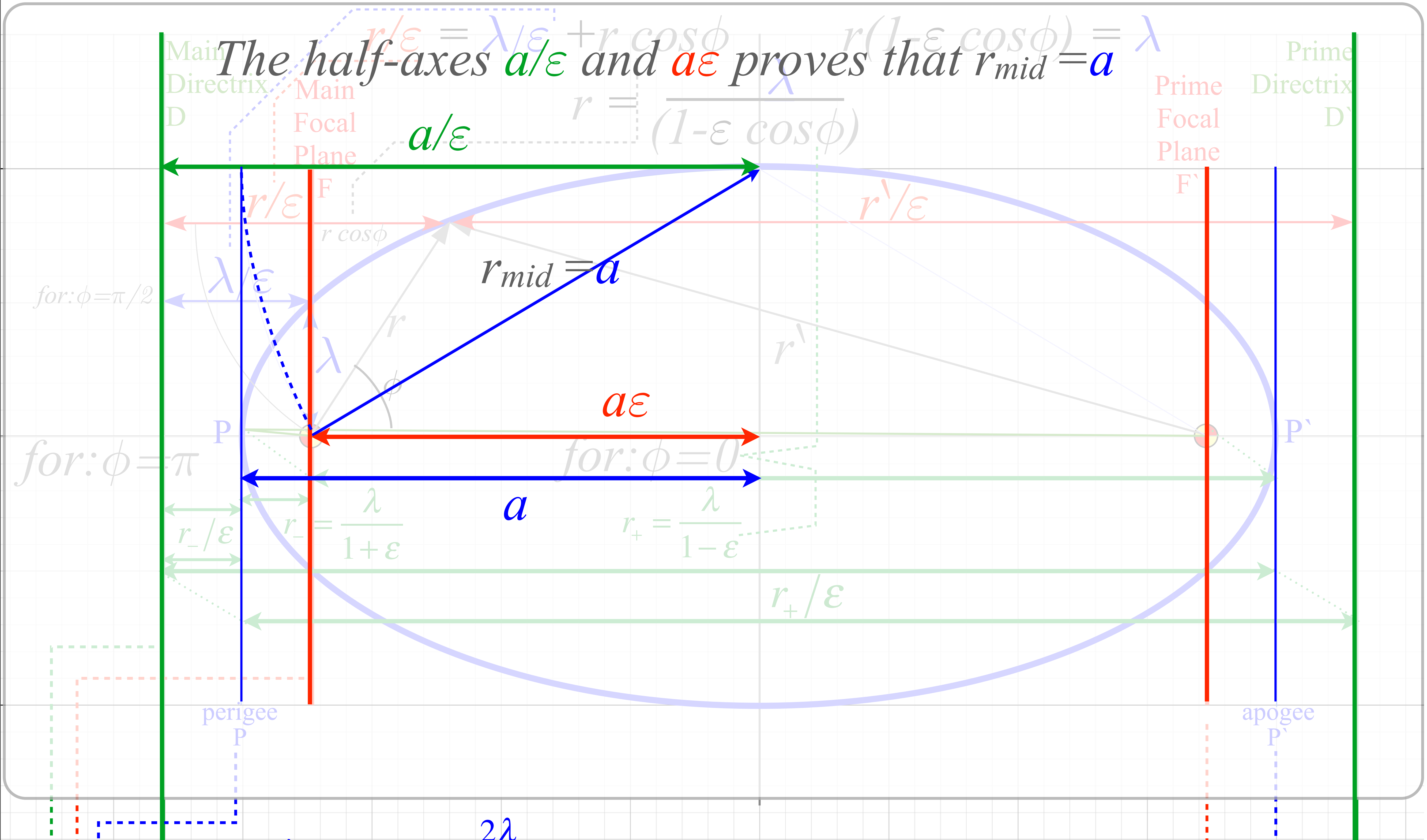


Major axis PP′ $= r_+ + r_- = \frac{2\lambda}{1-\varepsilon^2} = 2a$

Focal axis FF′ $= r_+ - r_- = \frac{2\lambda\varepsilon}{1-\varepsilon^2} = 2a\varepsilon$

Directrix axis DD′ $= \frac{r_+}{\varepsilon} + \frac{r_-}{\varepsilon} = \frac{2a}{\varepsilon}$

The half-axes a/ε and $a\varepsilon$ proves that $r_{mid} = a$

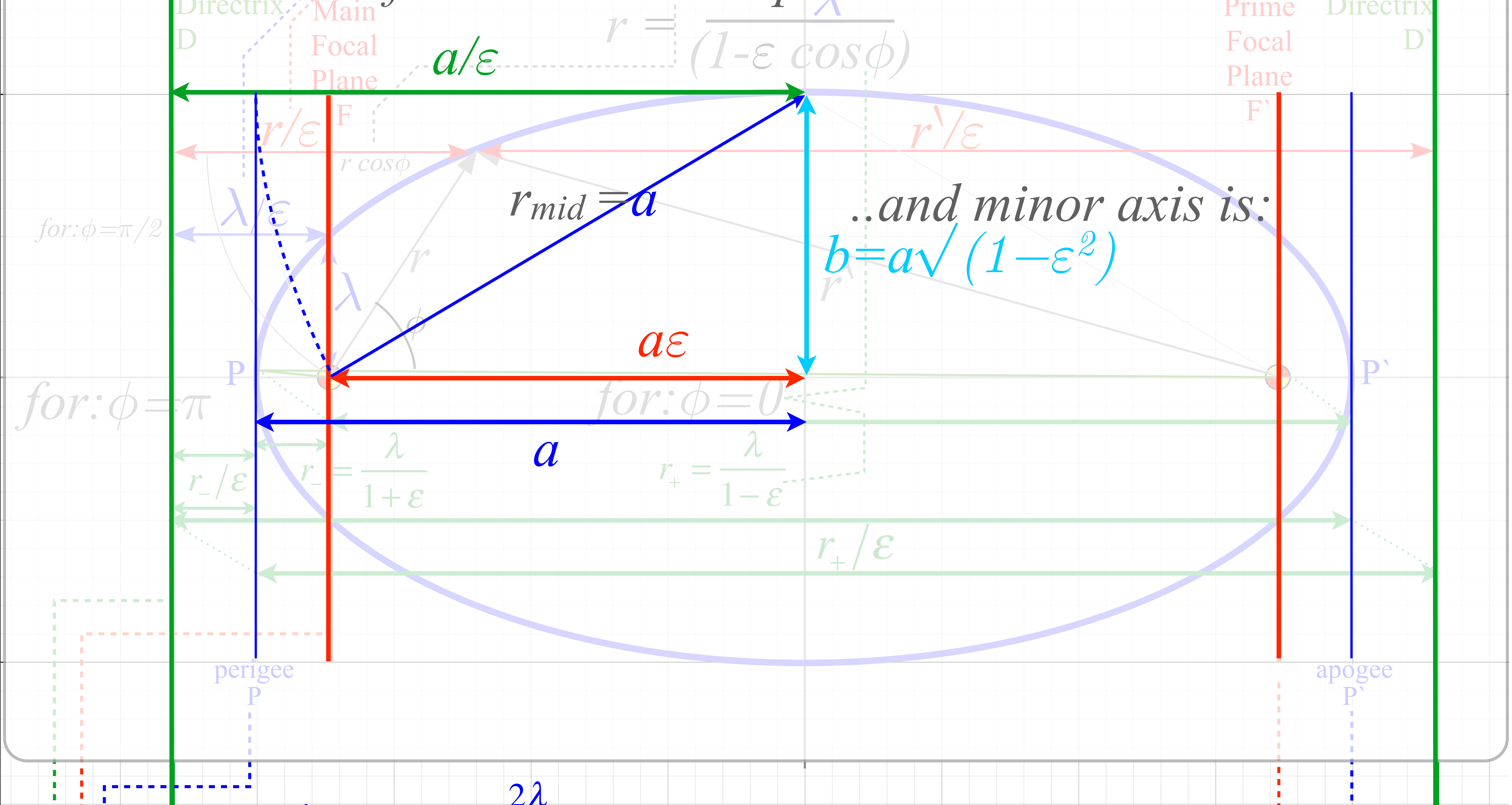


Major axis PP′ $= r_+ + r_- = \frac{2\lambda}{1-\varepsilon^2} = 2a$

Focal axis FF′ $= r_+ - r_- = \frac{2\lambda\varepsilon}{1-\varepsilon^2} = 2a\varepsilon$

Directrix axis DD′ $= \frac{r_+}{\varepsilon} + \frac{r_-}{\varepsilon} = \frac{2a}{\varepsilon}$

The half-axes a/ε and $a\varepsilon$ proves that $r_{mid} = a$



$$\text{Major axis } PP' = r_{+} + r_{-} = \frac{2\lambda}{1-\varepsilon^2} = 2a$$

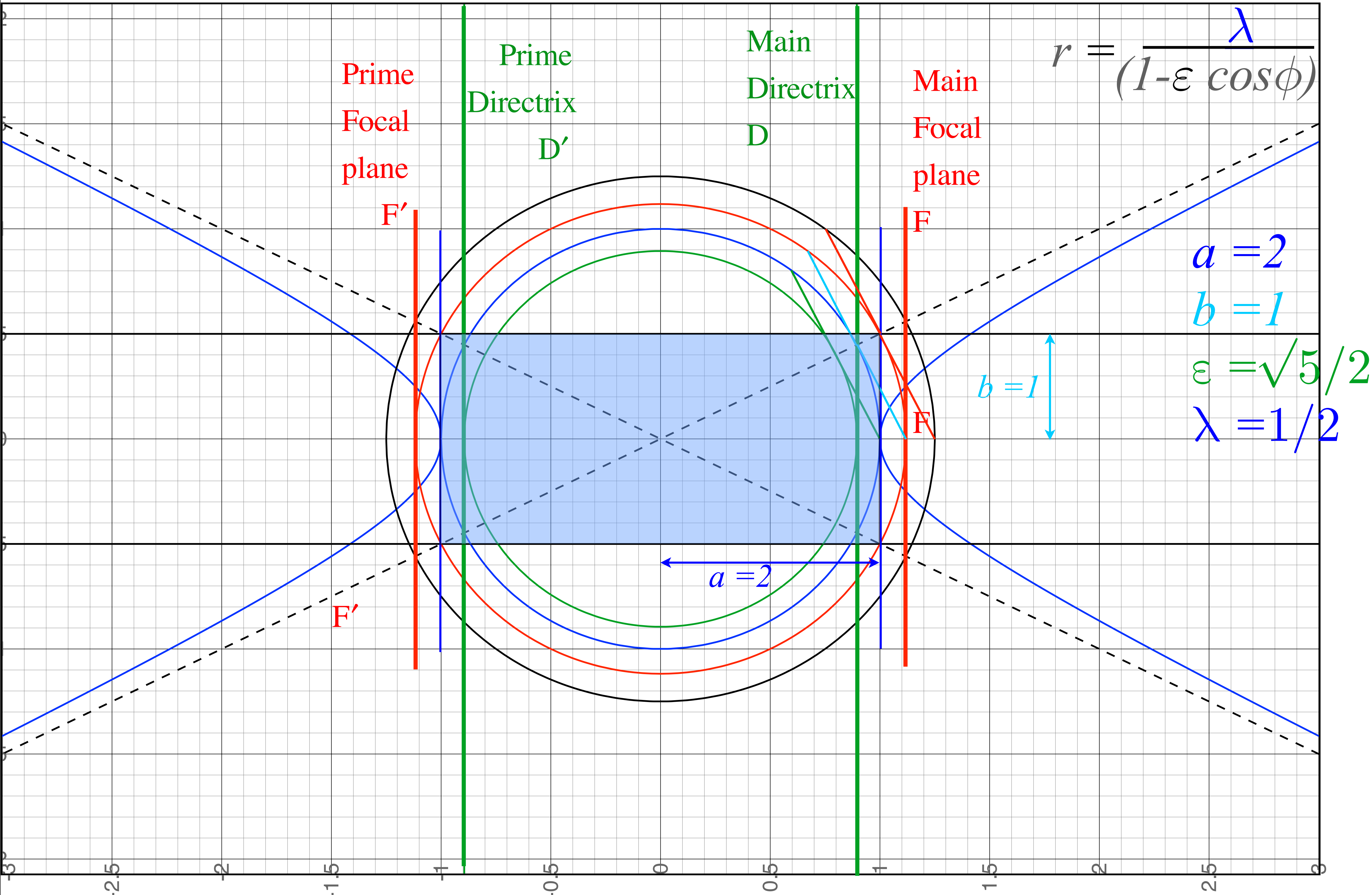
$$\text{Focal axis } FF' = r_{+} - r_{-} = \frac{2\lambda\varepsilon}{1-\varepsilon^2} = 2a\varepsilon$$

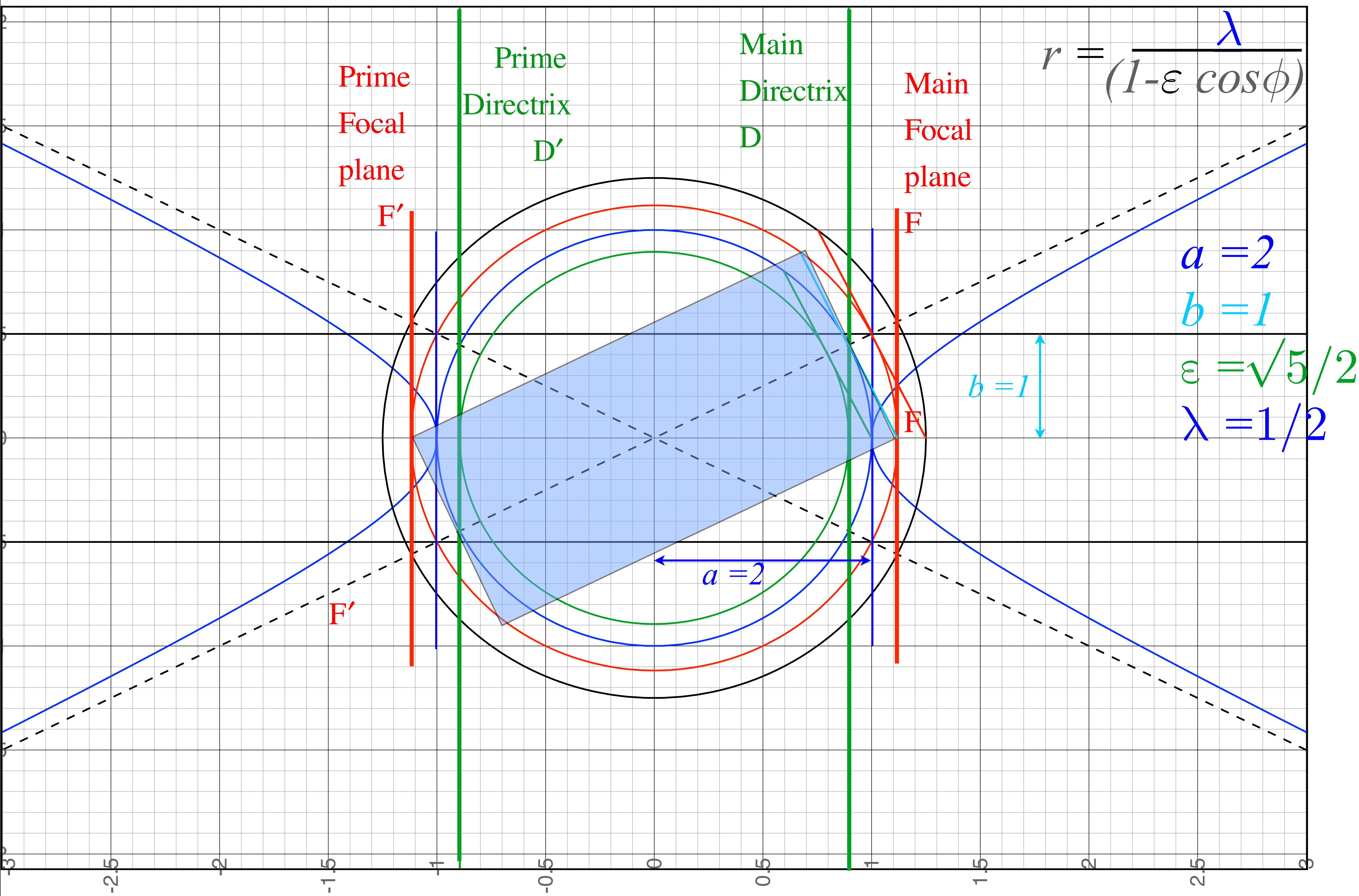
$$\text{Directrix axis } DD' = \frac{r_{+}}{\varepsilon} + \frac{r_{-}}{\varepsilon} = \frac{2a}{\varepsilon}$$

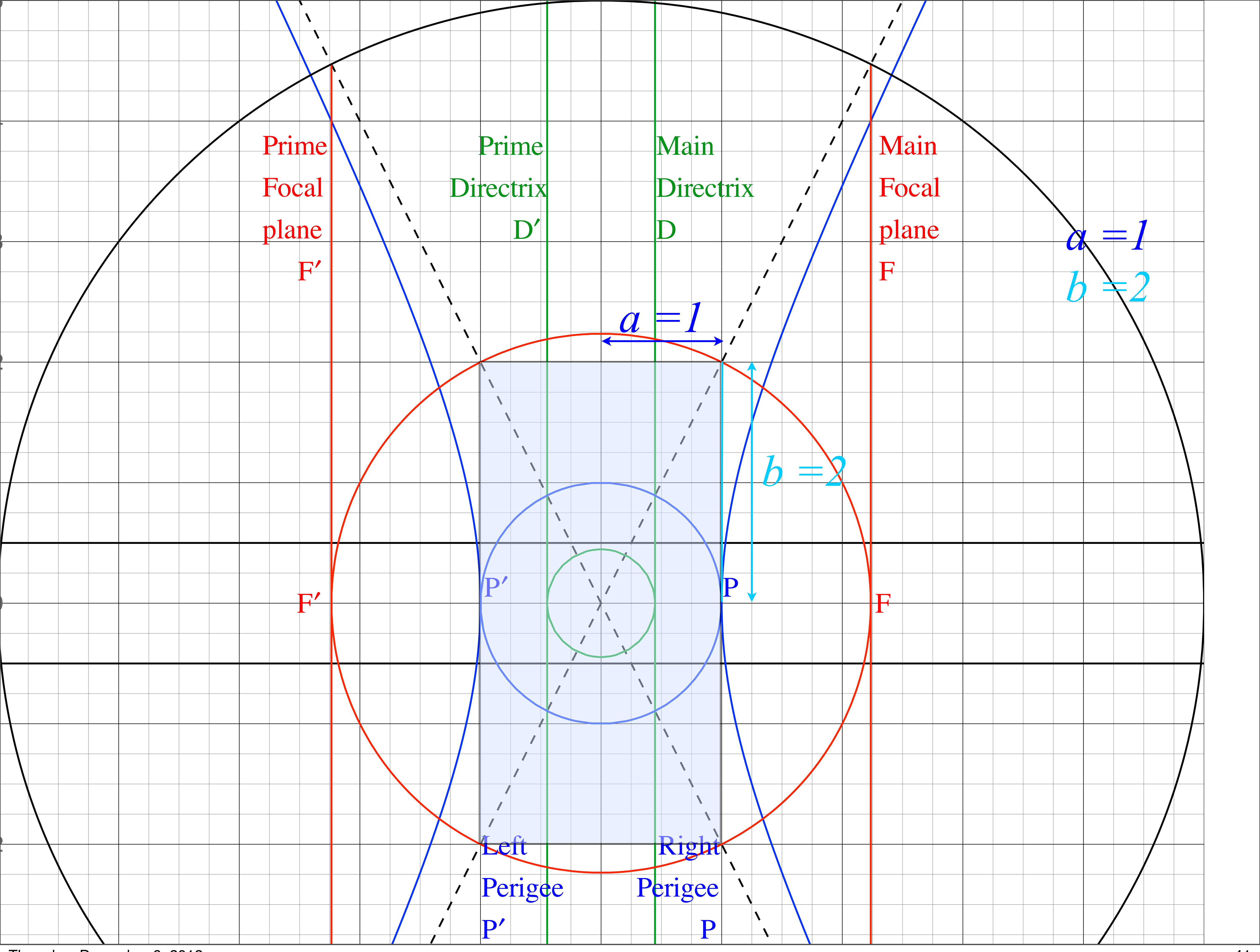
Geometry and Symmetry of Coulomb orbits

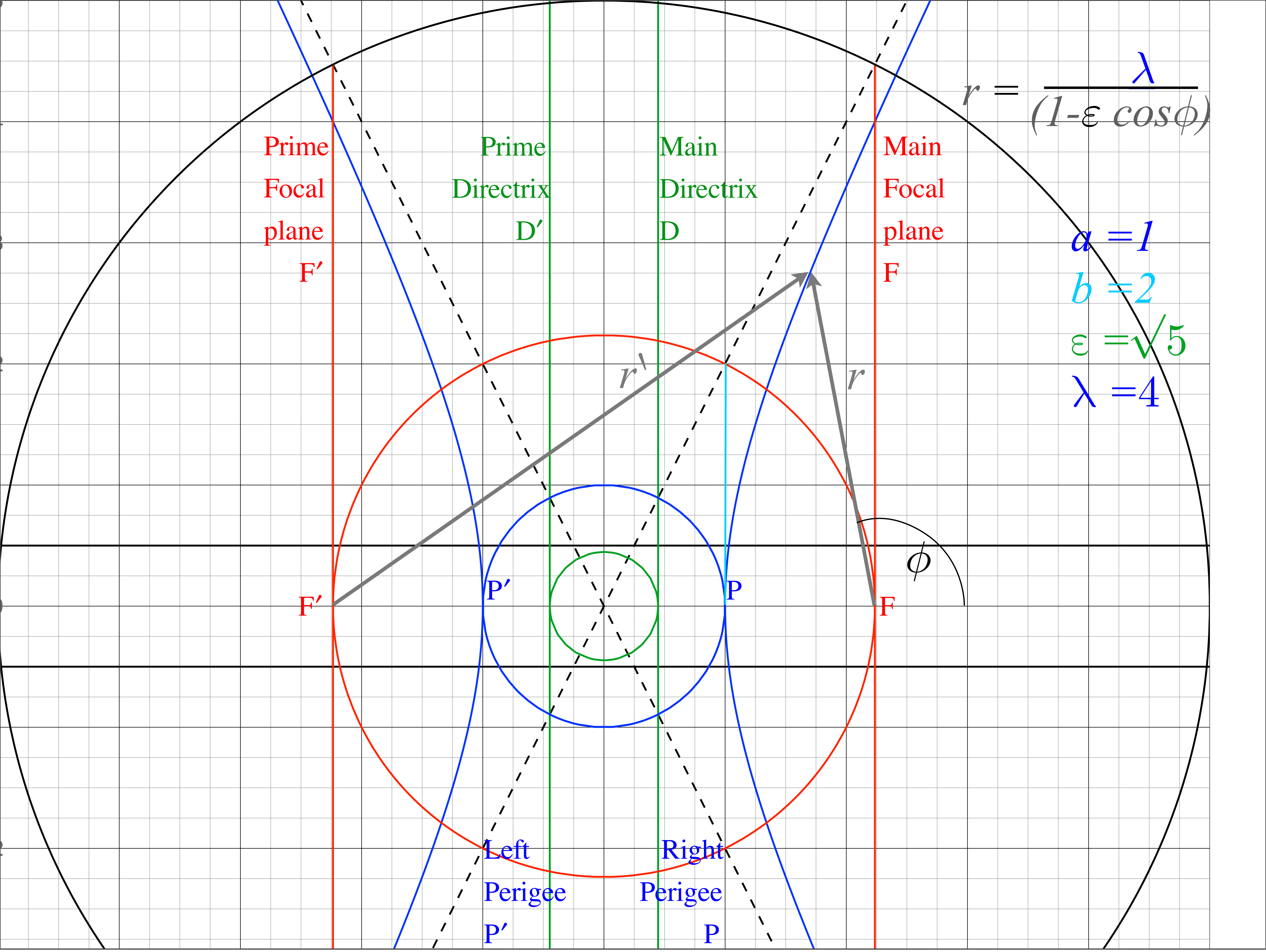
Detailed elliptic geometry

➤ *Detailed hyperbolic geometry*

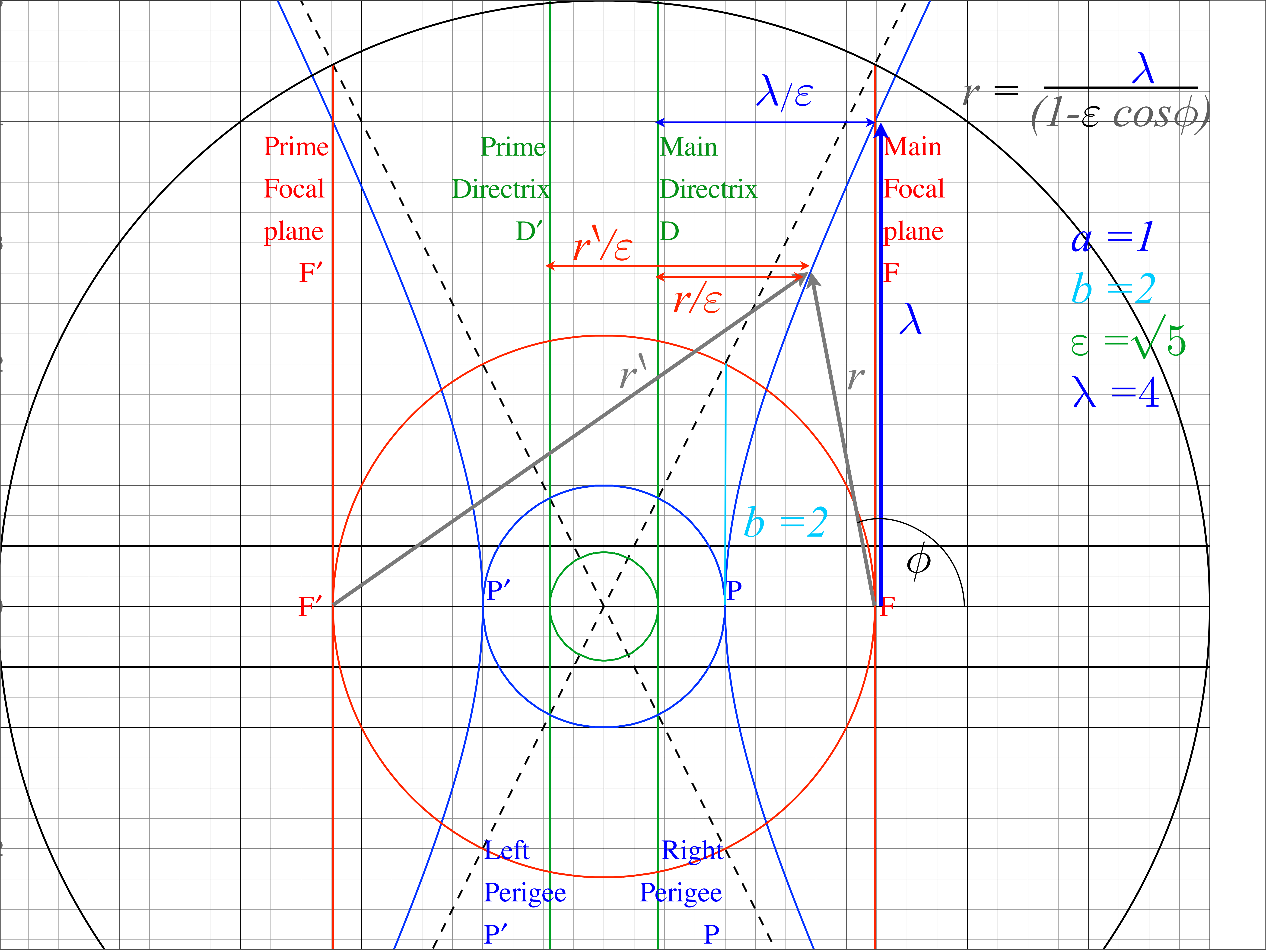

Prime
Focal
plane
F′
Prime
Directrix
D′
Main
Directrix
D
Main
Focal
plane
F
r = λ/(1-ε cosφ)
a =2
b =1
ε =√5/2
λ =1/2
b =1
F
a =2
F′
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3

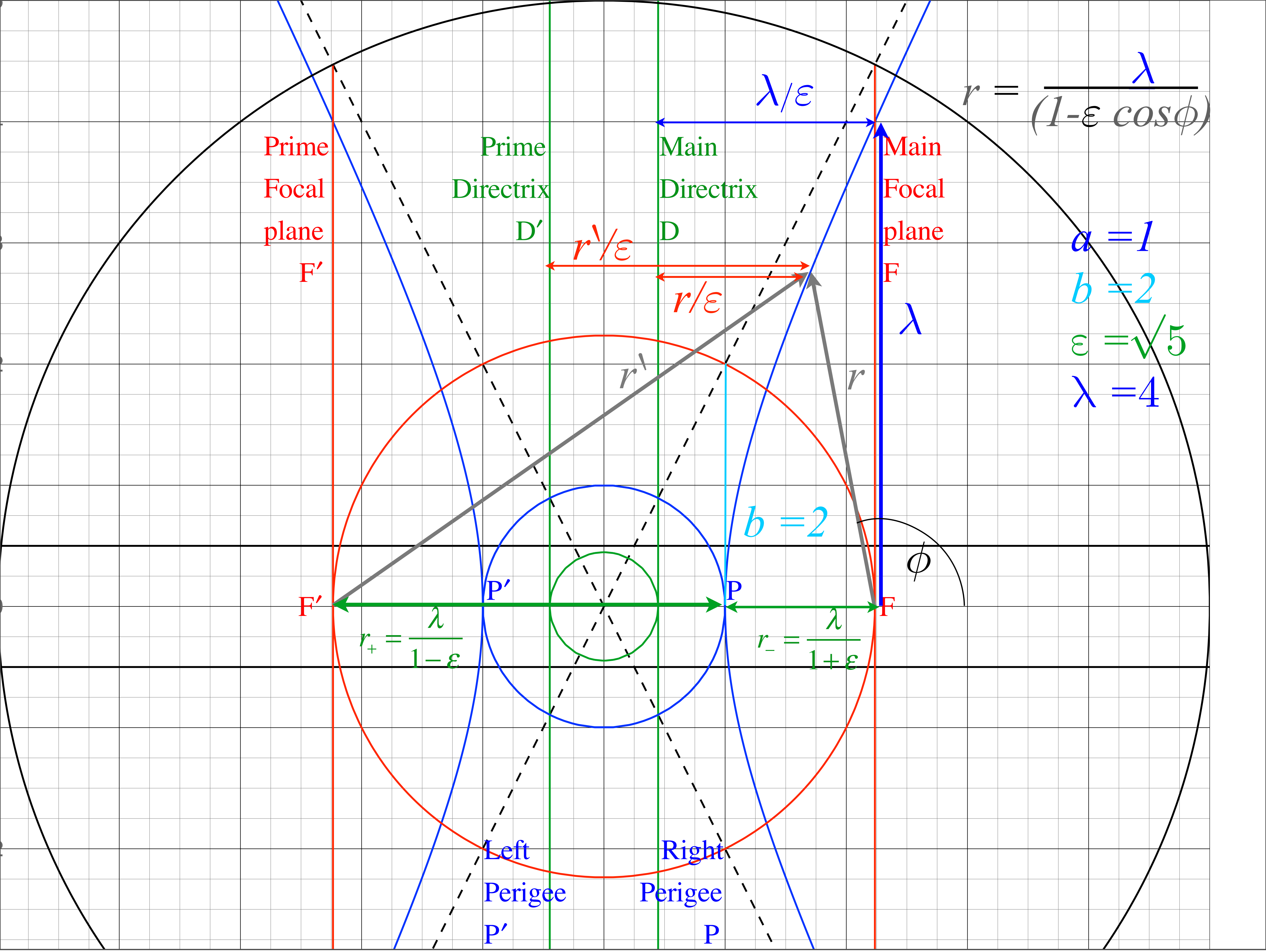

Prime
Focal
plane
F′
Prime
Directrix
D′
Main
Directrix
D
Main
Focal
plane
F
$r = \frac{\lambda}{(1-\varepsilon \cos\phi)}$
$a = 2$
$b = 1$
$\varepsilon = \sqrt{5}/2$
$\lambda = 1/2$
$b = 1$
F
$a = 2$
F′
-3
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
3


Prime Focal plane F′
Prime Directrix D′
Main Directrix D
Main Focal plane F
$a = 1$
$b = 2$
$a = 1$
$b = 2$
P′
P
F′
F
Left Perigee P′
Right Perigee P

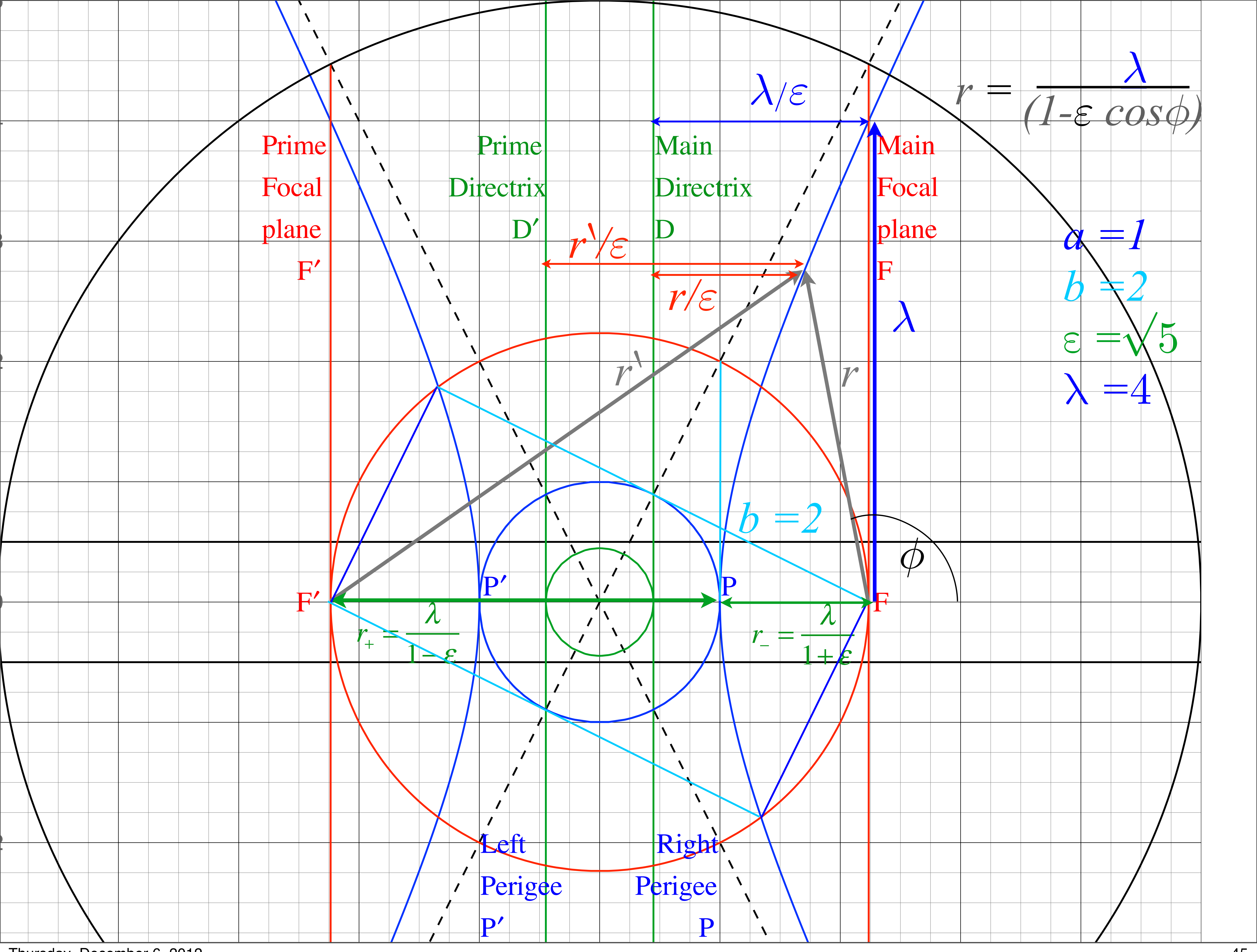


$r = \frac{\lambda}{(1-\varepsilon \cos\phi)}$
Prime
Focal
plane
F′
Prime
Directrix
D′
Main
Directrix
D
Main
Focal
plane
F
a =1
b =2
ε =√5
λ =4
r'
r
φ
F′
P′
P
F
Left
Perigee
P′
Right
Perigee
P

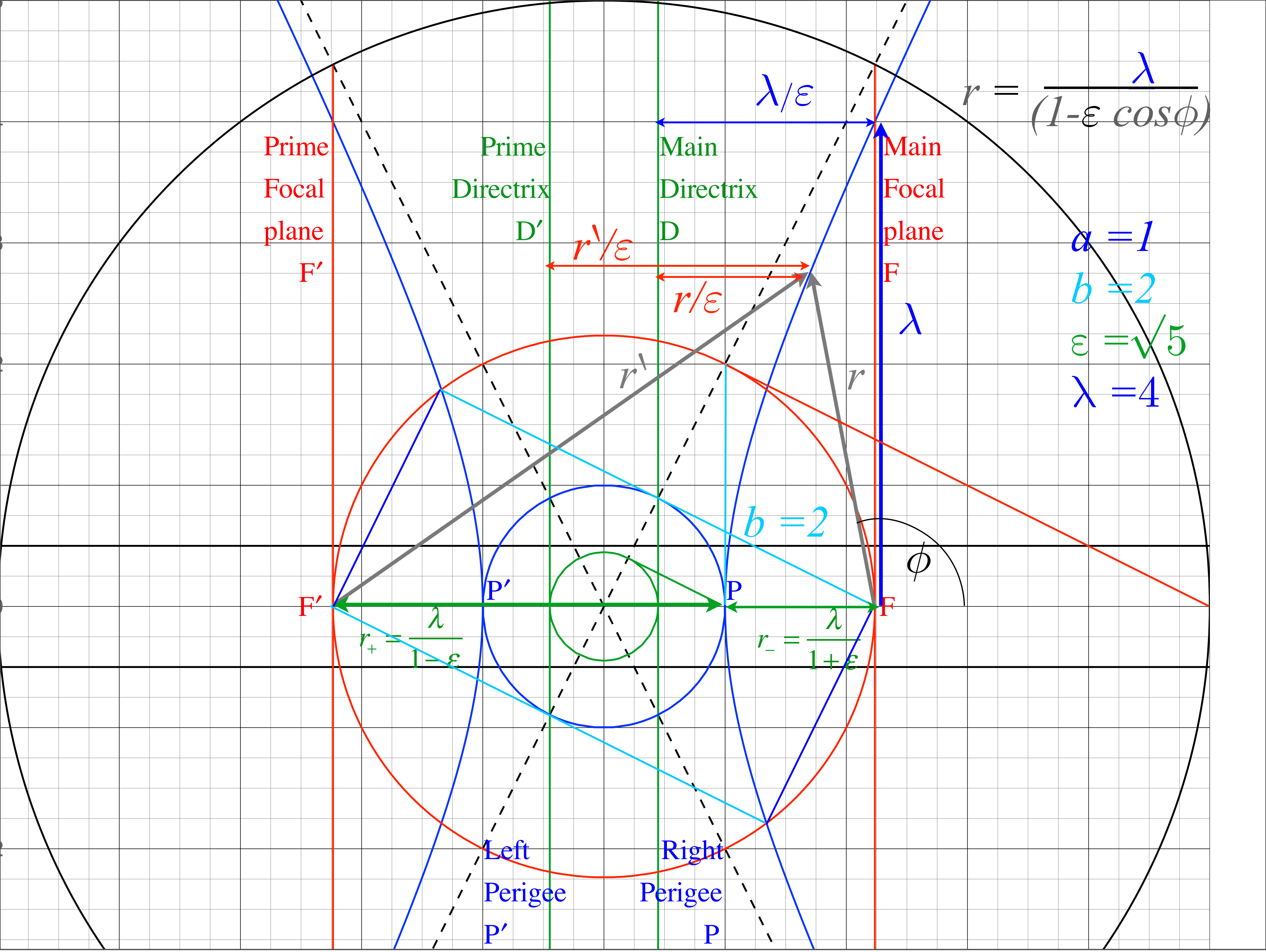

λ/ε
$r = \frac{\lambda}{(1-\varepsilon \cos\phi)}$
Prime Focal plane F′
Prime Directrix D′
Main Directrix D
Main Focal plane F
r'/ε
r/ε
λ
a =1
b =2
ε =√5
λ =4
r'
r
b =2
ϕ
F′
P′
P
F
Left Perigee P′
Right Perigee P



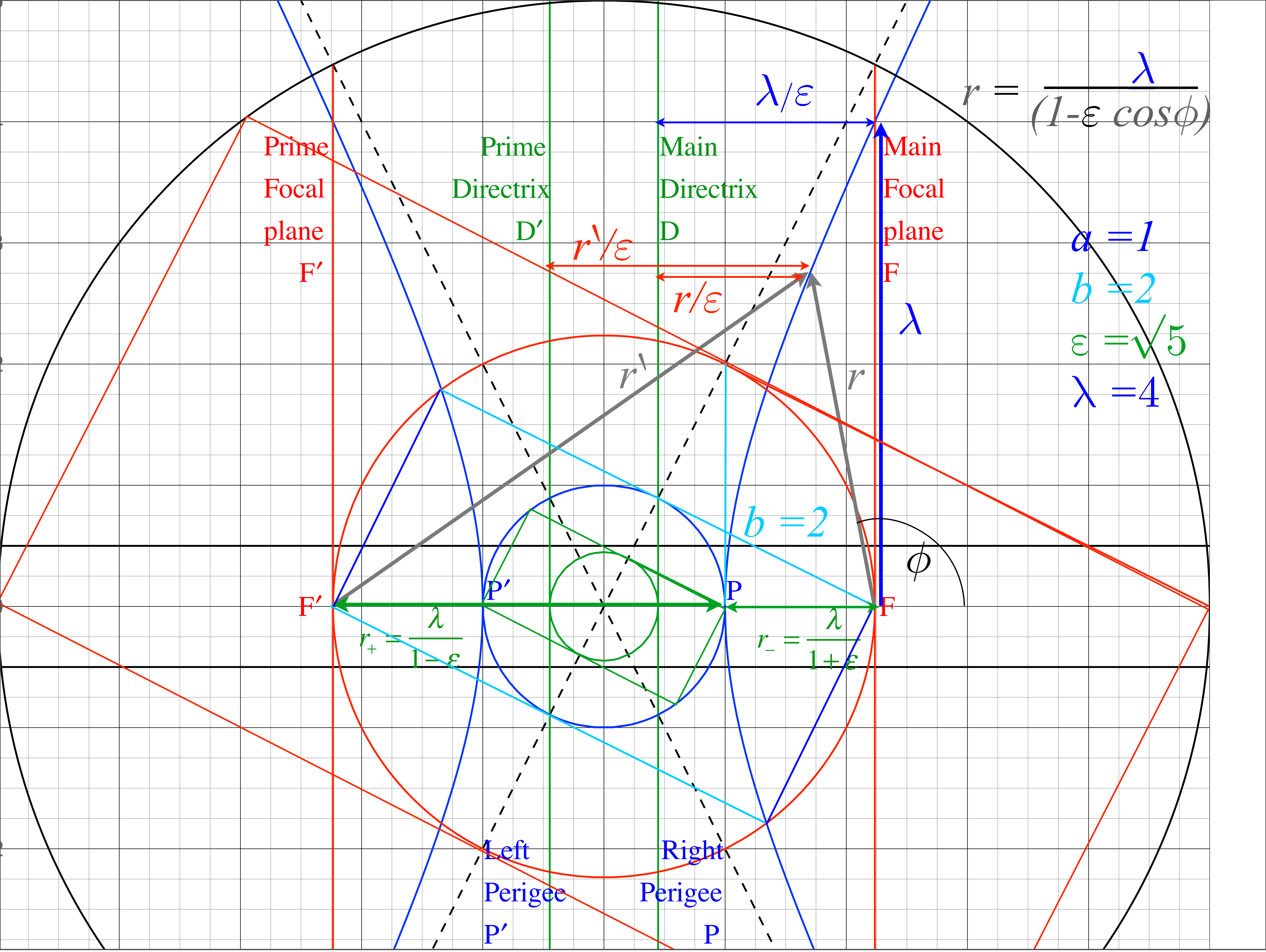
$r = \frac{\lambda}{(1-\varepsilon \cos\phi)}$
λ/ε
Prime Focal plane F′
Prime Directrix D′
Main Directrix D
Main Focal plane F
r'/ε
r/ε
λ
$a = 1$
$b = 2$
$\varepsilon = \sqrt{5}$
$\lambda = 4$
r'
r
$b = 2$
ϕ
F′
P′
P
F
$r_+ = \frac{\lambda}{1-\varepsilon}$
$r_- = \frac{\lambda}{1+\varepsilon}$
Left Perigee P′
Right Perigee P

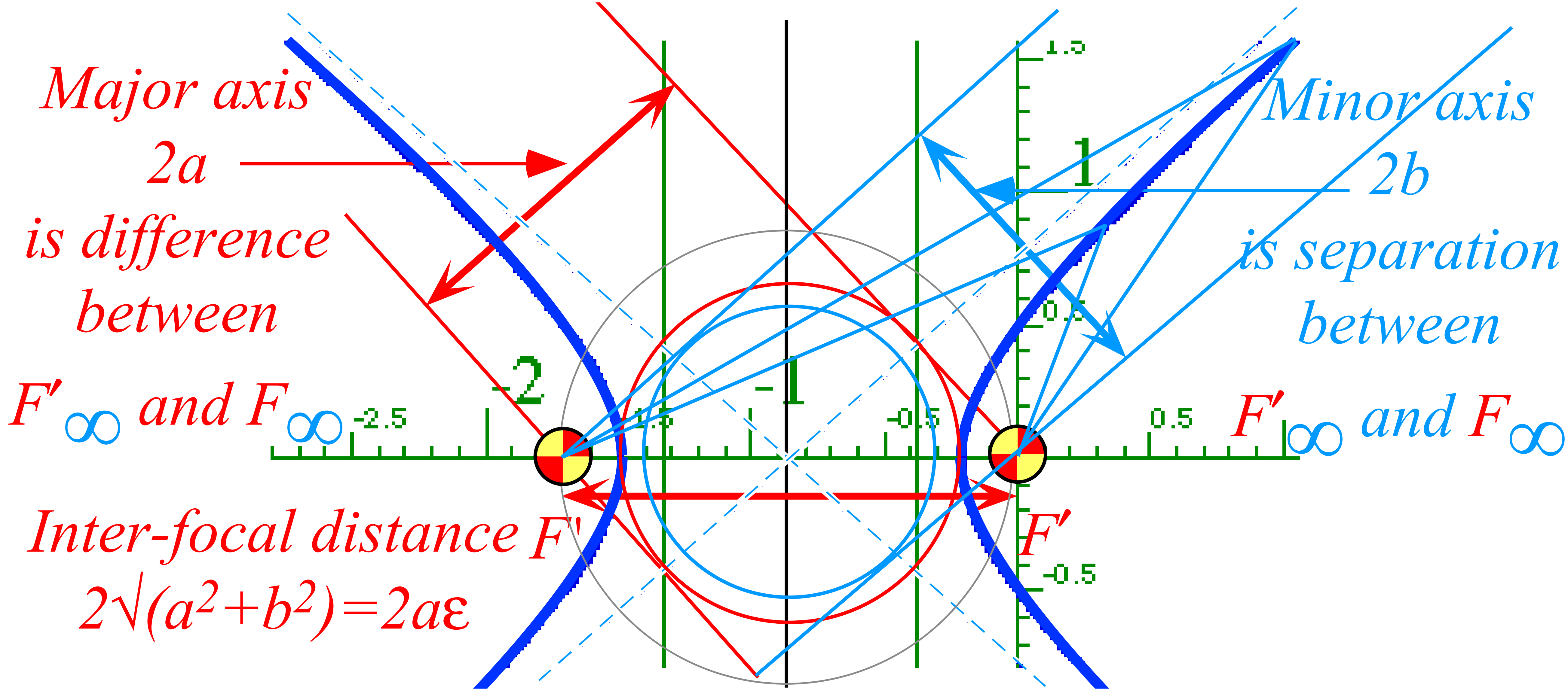


λ/ε
r = λ / (1-ε cosφ)
Prime
Focal
plane
F′
Prime
Directrix
D′
Main
Directrix
D
Main
Focal
plane
F
r′/ε
r/ε
λ
a =1
b =2
ε =√5
λ =4
r′
r
b =2
φ
F′
P′
P
F
r₊ = λ / 1−ε
r₋ = λ / 1+ε
Left
Perigee
P′
Right
Perigee
P

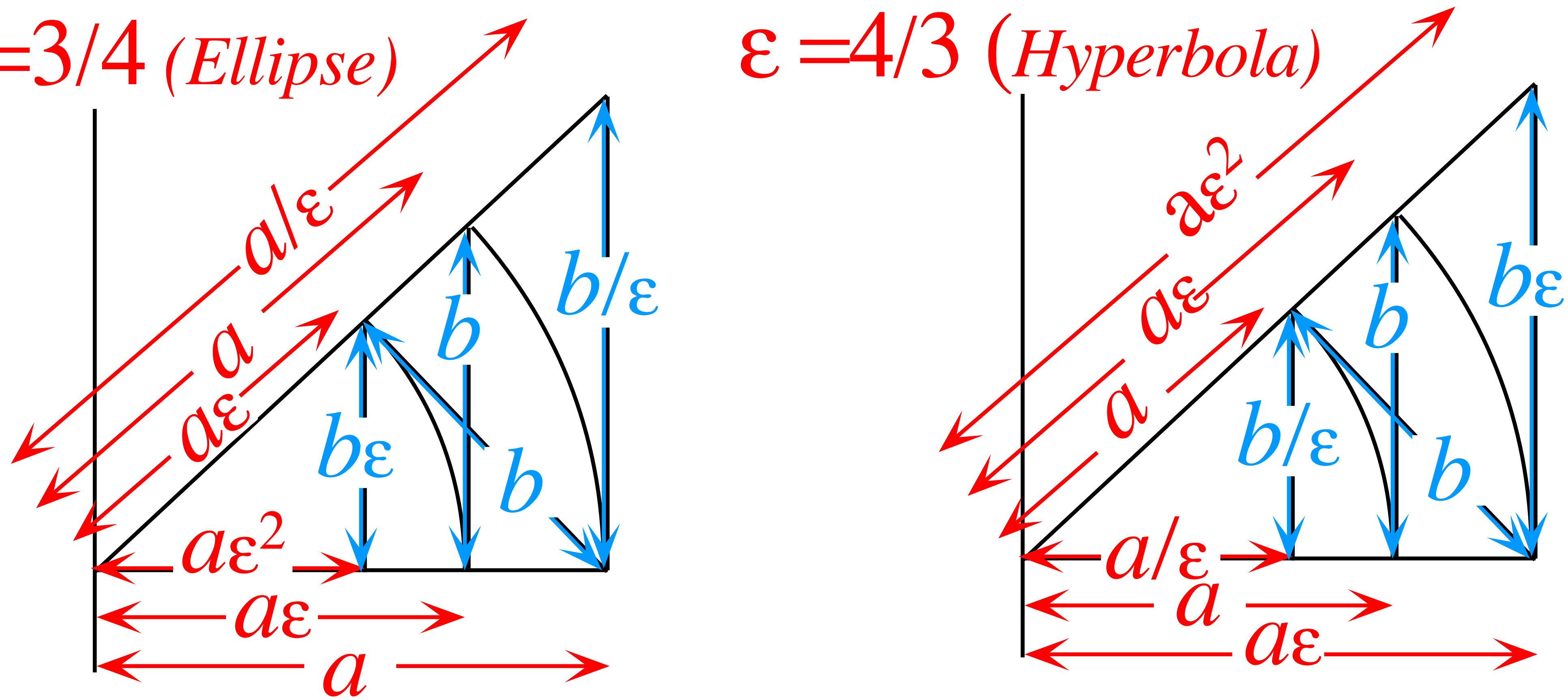


$r = \frac{\lambda}{(1-\varepsilon\cos\phi)}$
λ/ε
Prime Focal plane F′
Prime Directrix D′
Main Directrix D
Main Focal plane F
r'/ε
r/ε
λ
a =1
b =2
ε =√5
λ =4
r'
r
b =2
φ
F′
P′
P
F
$r_+ = \frac{\lambda}{1-\varepsilon}$
$r_- = \frac{\lambda}{1+\varepsilon}$
Left Perigee P′
Right Perigee P


$r = \frac{\lambda}{(1-\varepsilon \cos\phi)}$
λ/ε
Prime Focal plane F′
Prime Directrix D′
Main Directrix D
Main Focal plane F
r'/ε
r/ε
λ
r'
r
a =1
b =2
ε =√5
λ =4
b =2
φ
F′
P′
P
F
$r_+ = \frac{\lambda}{1-\varepsilon}$
$r_- = \frac{\lambda}{1+\varepsilon}$
Left Perigee P′
Right Perigee P



Major axis
2a
is difference
between
F′∞ and F∞
Minor axis
2b
is separation
between
F′∞ and F∞
Inter-focal distance F′F
2√(a²+b²)=2aε
-2
-1
1
-2.5
-1.5
-0.5
0.5
1.5
0.5
-0.5



For the elliptic geometry *(ε < 1)*:

$b^2 = a^2 - a^2\varepsilon^2 = a\lambda$,

$b = a\sqrt{(1-\varepsilon^2)} = \sqrt{(a\lambda)}$,

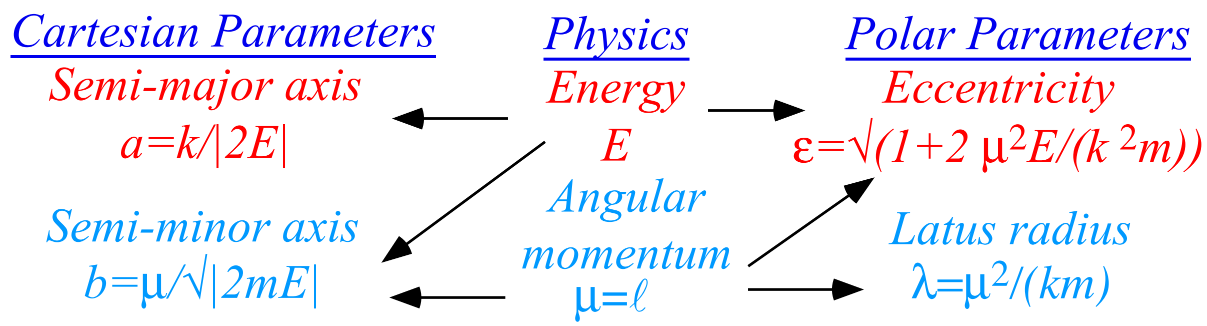
For hyperbolic geometry *(ε > 1)*:

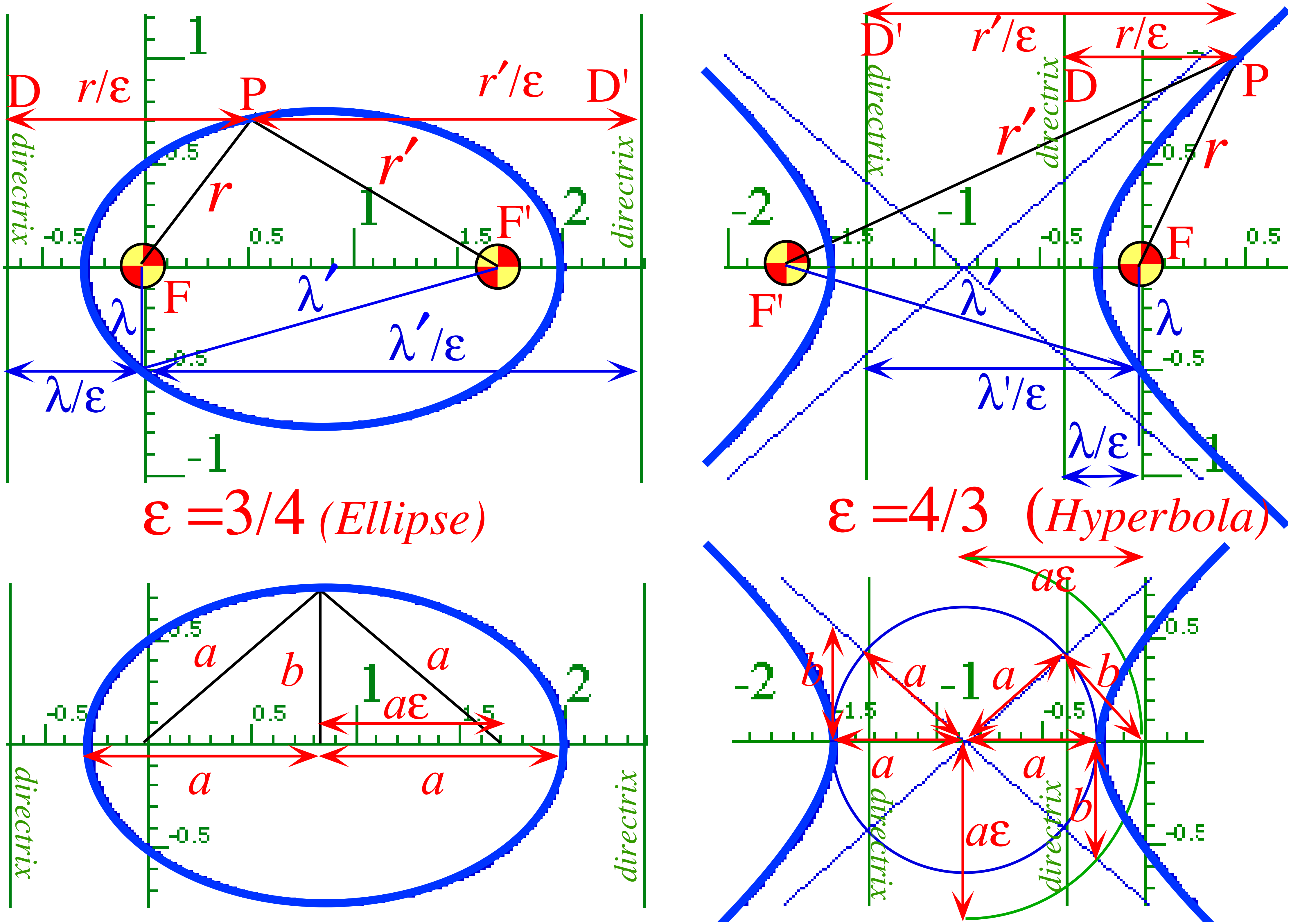
$b^2 = a^2\varepsilon^2 - a^2 = a\lambda$,

$b = a\sqrt{(\varepsilon^2-1)} = \sqrt{(a\lambda)}$.

(λ,ε)-*(a,b)* expressions and their inverses follow.

$a = \lambda/(1-\varepsilon^2)$	$a = \lambda/(\varepsilon^2-1)$
$b^2 = \lambda^2/(1-\varepsilon^2)$	$b^2 = \lambda^2/(\varepsilon^2-1)$
$\lambda = a(1-\varepsilon^2) = b^2/a$	$\lambda = a(\varepsilon^2-1) = b^2/a$
$\varepsilon^2 = 1 - b^2/a^2$	$\varepsilon^2 = 1 + b^2/a^2$



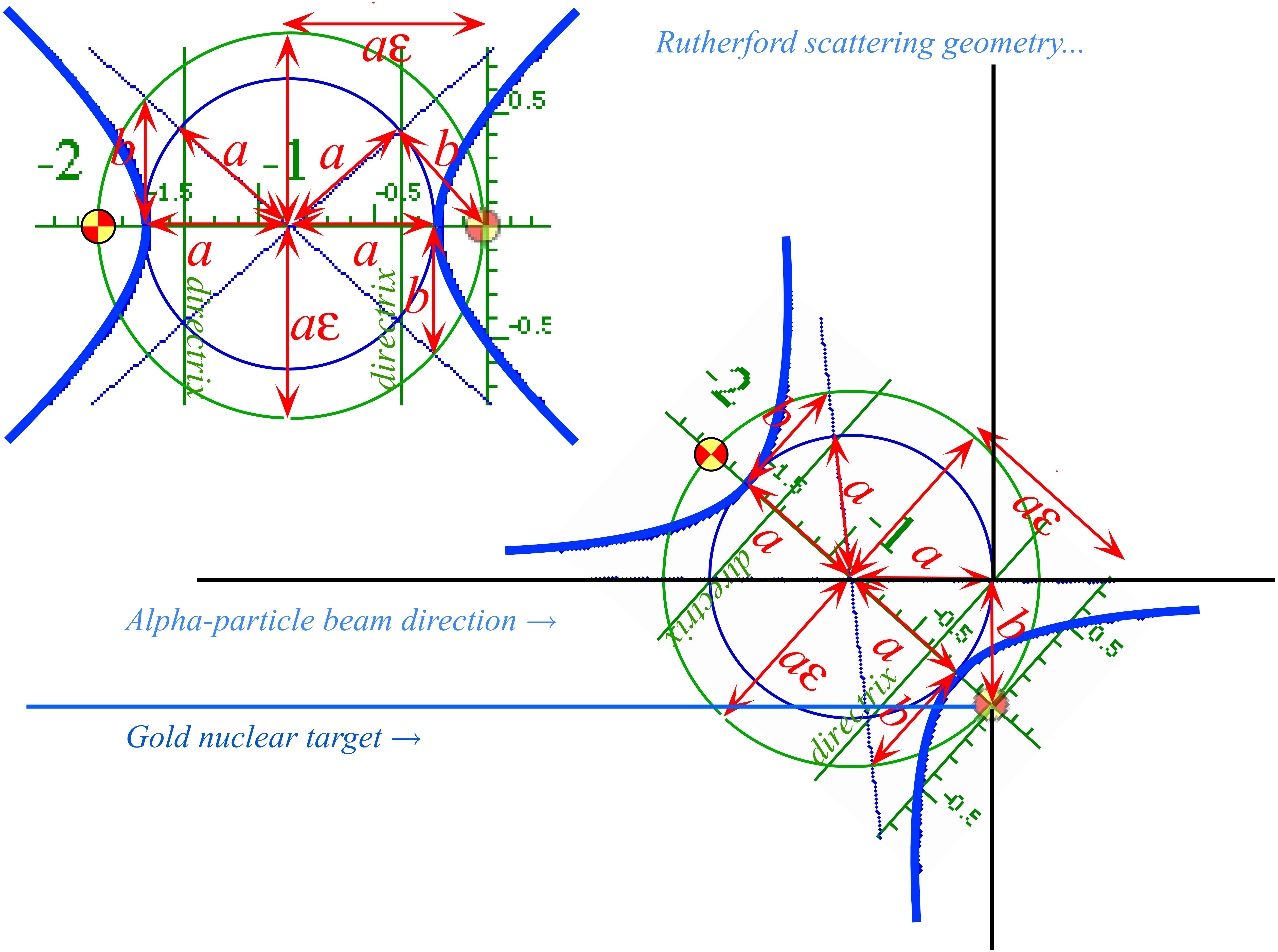


D
r/ε
P
r′/ε
D'
r
r′
F
F'
λ
λ′
λ/ε
λ′/ε
directrix
directrix
ε =3/4 (Ellipse)
D'
r′/ε
r/ε
D
P
r′
r
F
F'
λ′
λ
λ'/ε
λ/ε
directrix
directrix
ε =4/3 (Hyperbola)
a
b
a
aε
a
a
directrix
directrix
aε
b
a
a
b
a
a
b
aε
directrix
directrix

- *Rutherford scattering and differential scattering crossections*

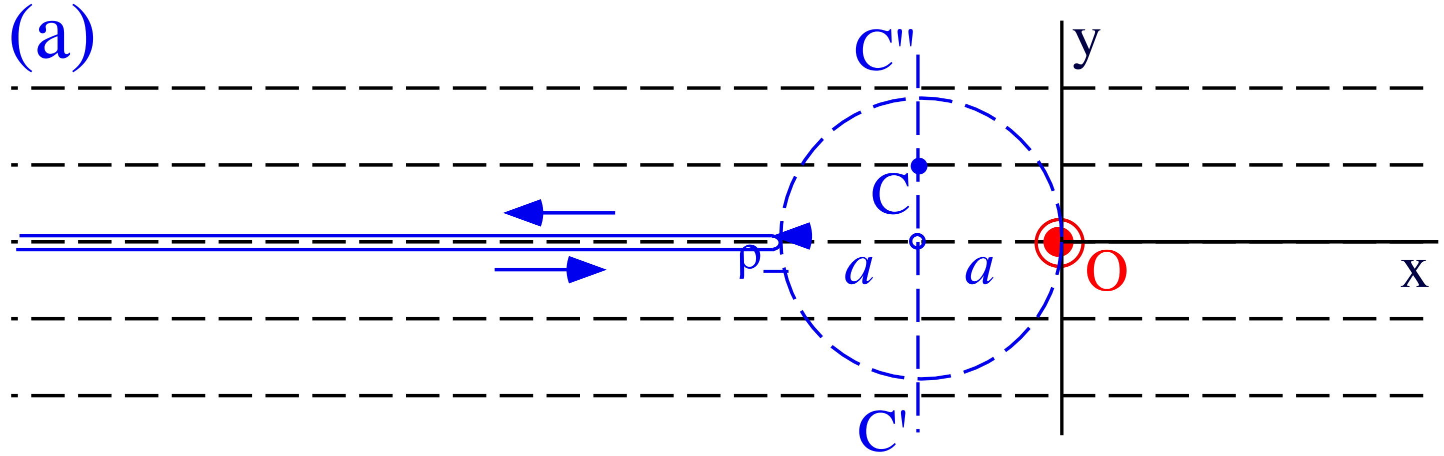
  Ruler & compass construction

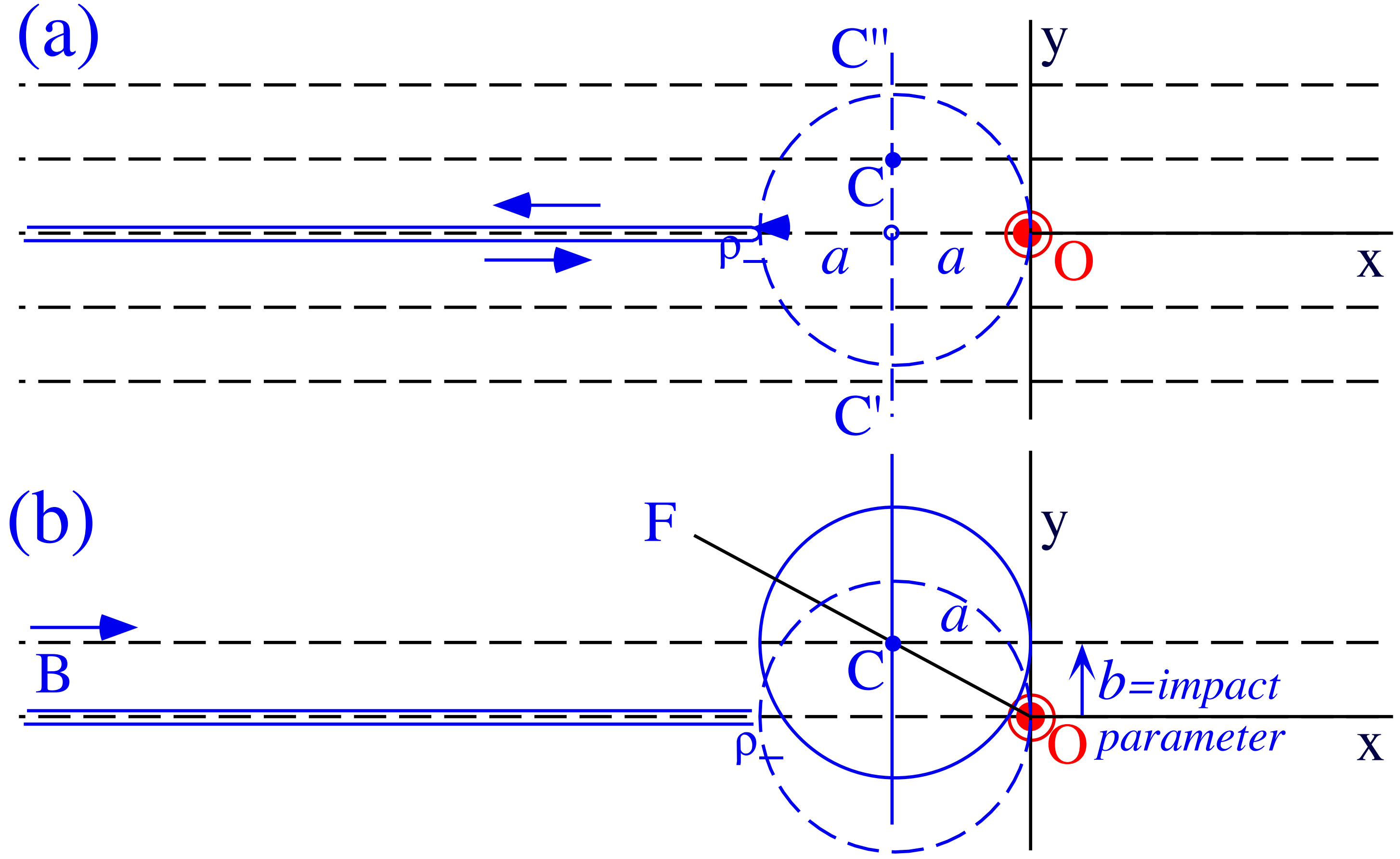
Rutherford scattering geometry...



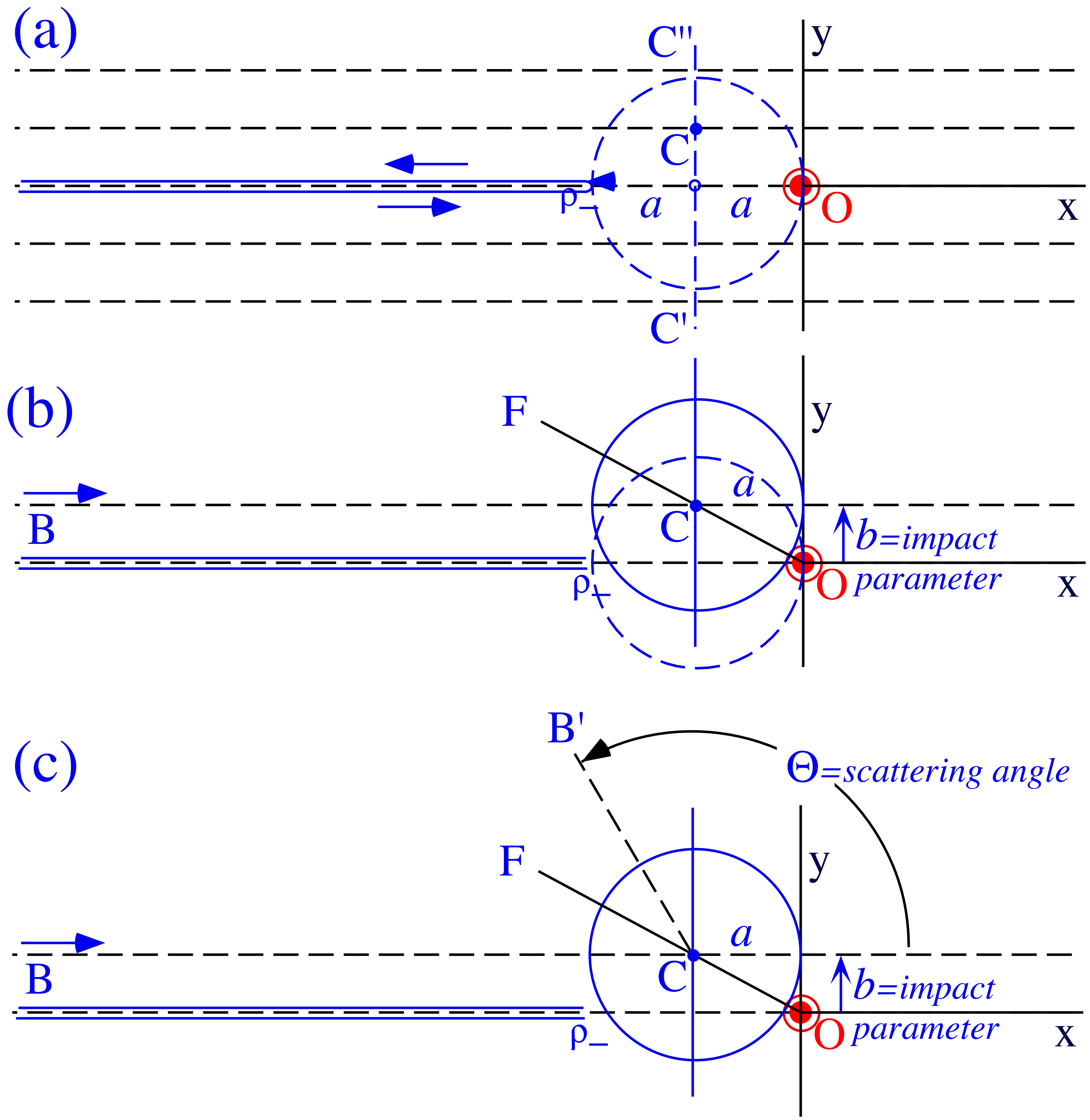
Rutherford scattering and differential scattering crossections

➡ *Ruler & compass construction*

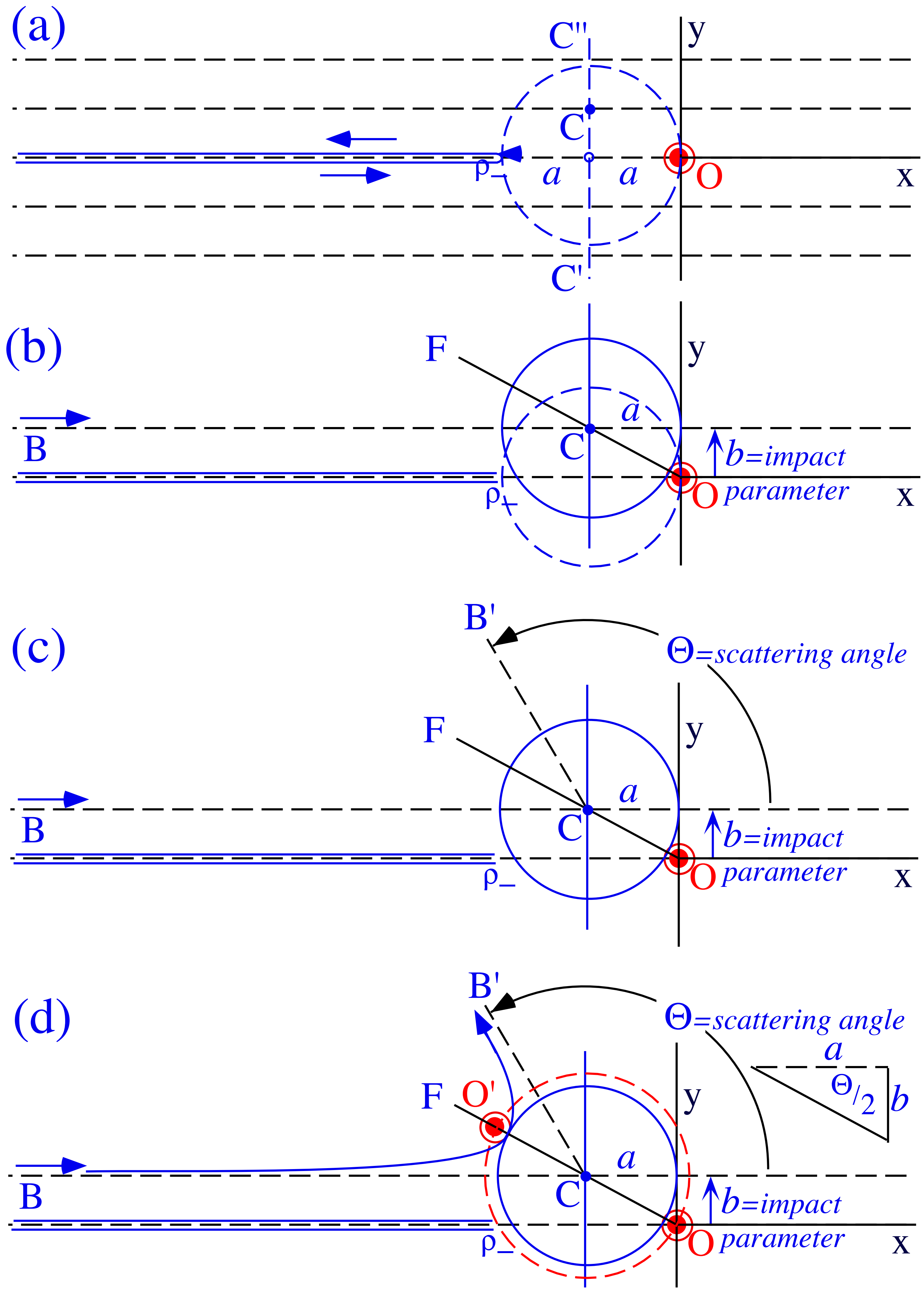

(a)
C''
y
C
ρ
a
a
O
x
C'

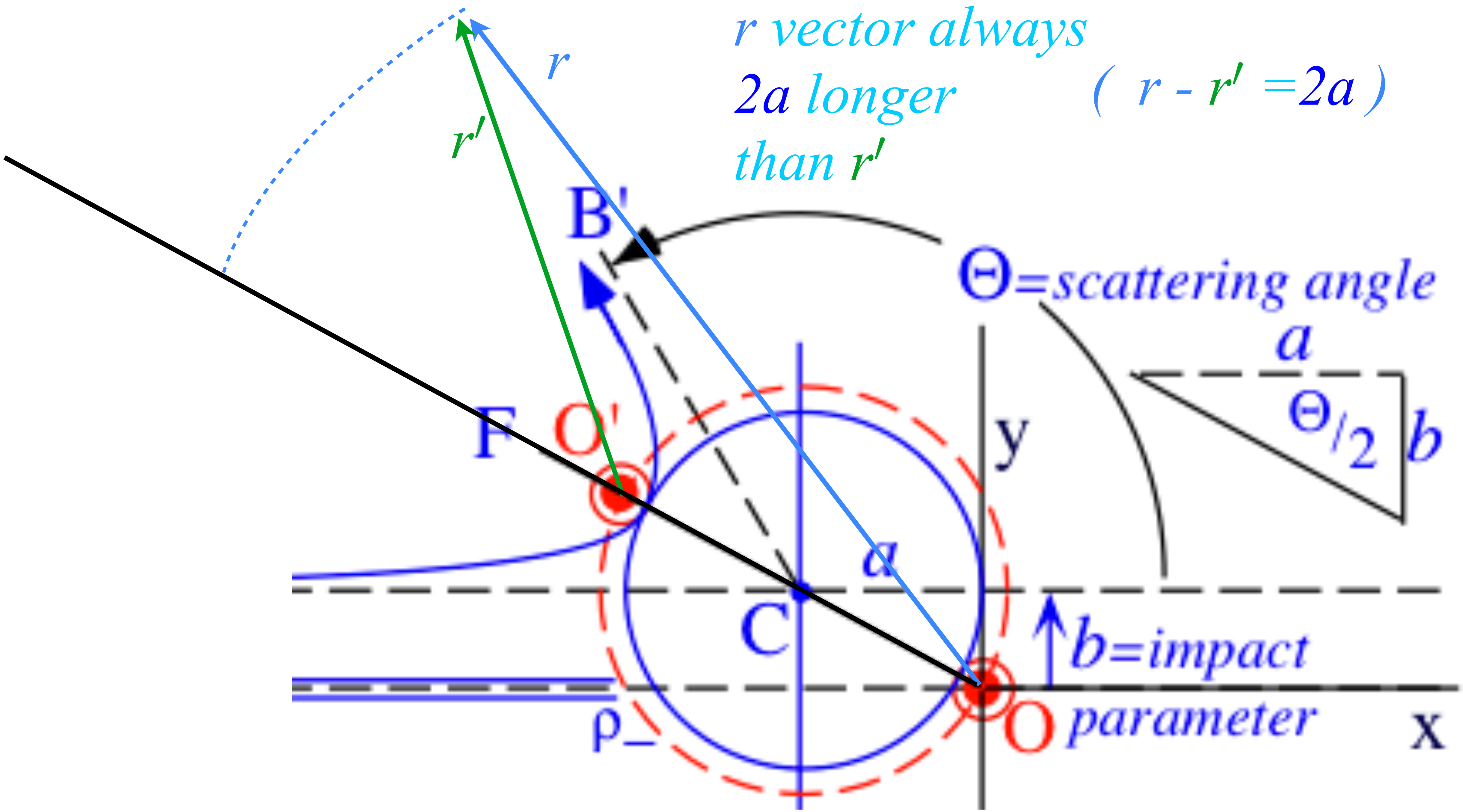


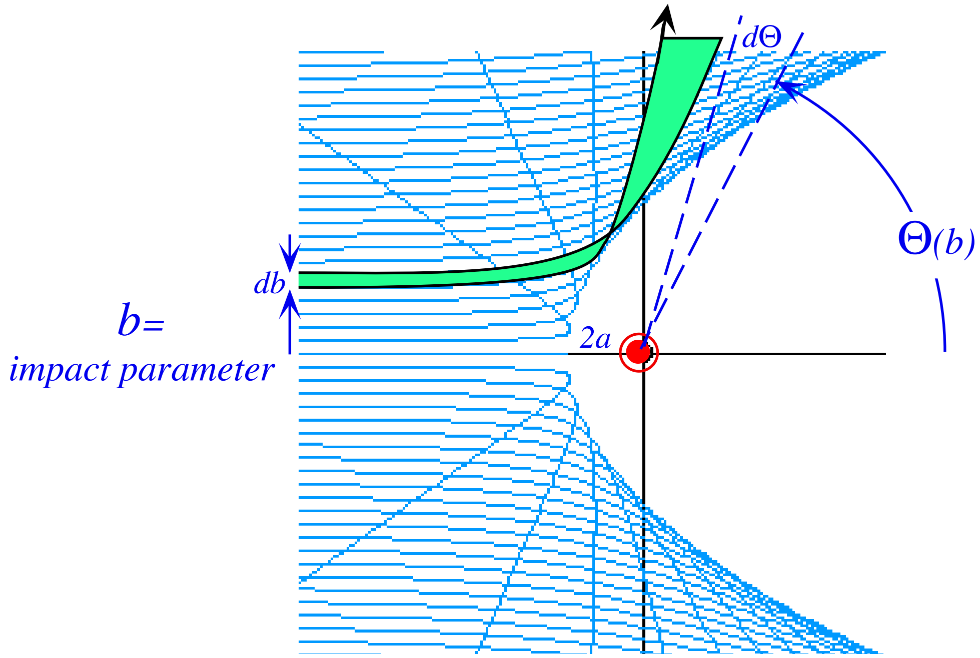
(a)
C''
y
C
ρ
a
a
O
x
C'
(b)
F
y
a
B
C
b=impact
parameter
ρ
O
x

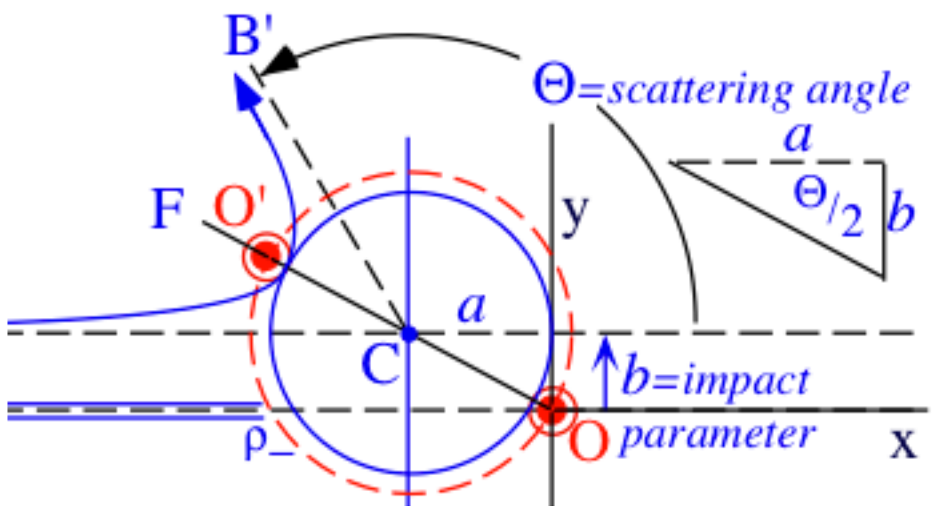


(a)
C''
y
C
ρ_
a
a
O
x
C'
(b)
F
y
a
B
C
b=impact
ρ_
O
parameter
x
(c)
B'
Θ=scattering angle
F
y
a
B
C
b=impact
ρ_
O
parameter
x


(a)
C''
y
C
ρ_
a
a
O
x
C'
(b)
F
y
a
B
C
b=impact
parameter
ρ+
O
x
(c)
B'
Θ=scattering angle
F
y
a
B
C
b=impact
parameter
ρ_
O
x
(d)
B'
Θ=scattering angle
a
Θ/2
b
F
O'
y
a
B
C
b=impact
parameter
ρ_
O
x


r vector always
2a longer
than r′
(r - r′ =2a)
r
r′
B'
Θ=scattering angle
a
Θ/2
b
F
O'
y
a
C
b=impact
parameter
ρ_
O
x



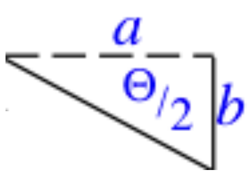


Particle going in incremental window $d\sigma = b\, db$ normal to beam at x=-∞ ends up in an area

$$dA = R^2 \sin\Theta d\Theta d\varphi = R^2 d\Omega$$

on a sphere at R=+∞ where $d\Omega = \sin\Theta d\Theta d\varphi$ is called the *incremental solid angle dΩ*.

The ratio $\frac{d\sigma}{d\Omega} = \frac{b\, db\, d\varphi}{\sin\Theta d\Theta d\varphi} = \frac{b}{\sin\Theta}\frac{db}{d\Theta}$ is called the *differential scattering crossection (DSC)*.



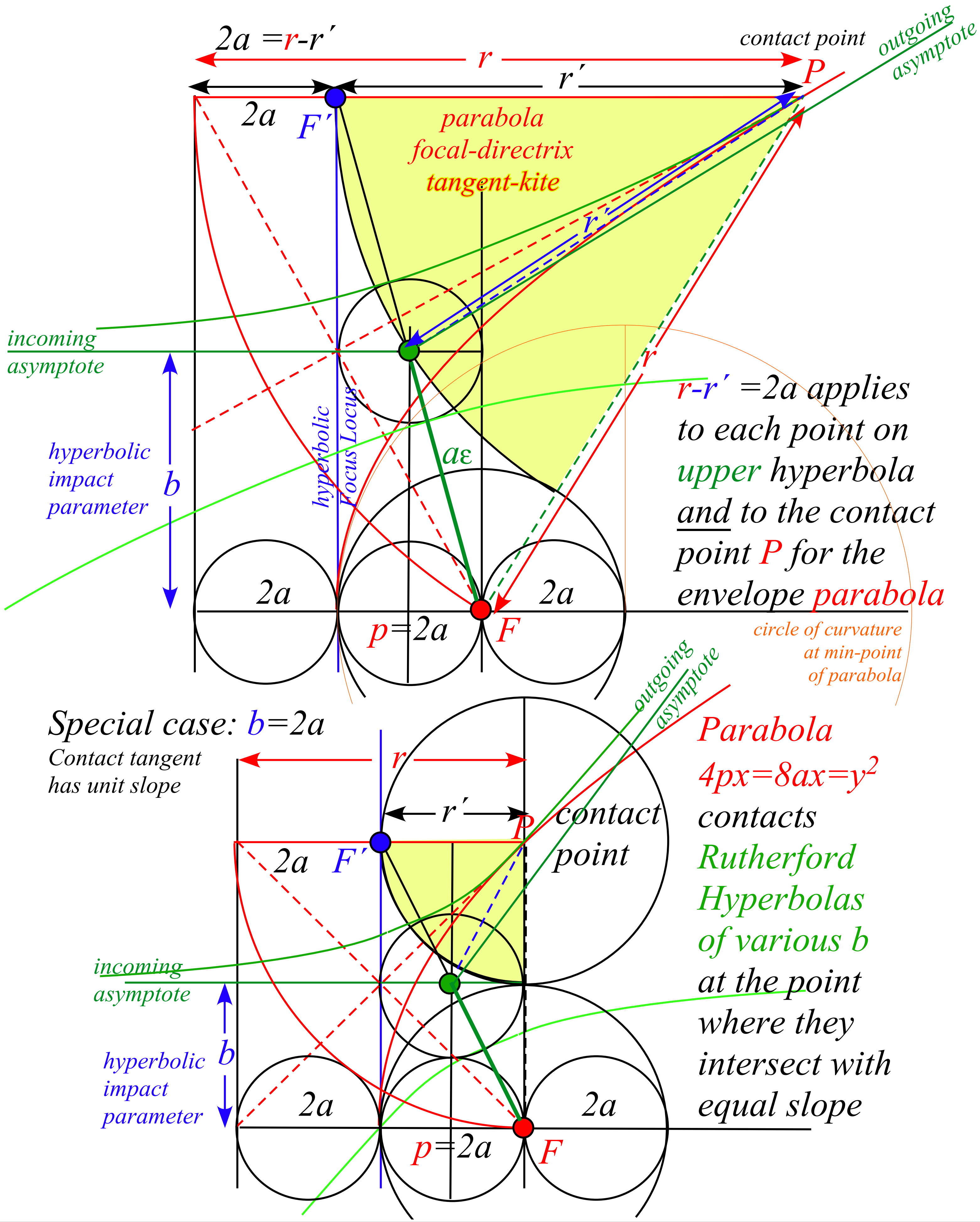
$b = a\ \cot\frac{\Theta}{2} = \frac{k}{2E}\cot\frac{\Theta}{2}$ gives: *Rutherford DSC*: $\frac{d\sigma}{d\Omega} = \frac{k^4}{16E^2}\sin^{-4}\frac{\Theta}{2}$

Partial scattering crossection.

$$\sigma\Big|_{(\varphi_0,\Theta_0)}^{(\varphi_1,\Theta_1)} = \int_{(\varphi_0,\Theta_0)}^{(\varphi_1,\Theta_1)} d\Omega \frac{d\sigma}{d\Omega} = \int_{\varphi_0}^{\varphi_1} d\varphi \int_{\Theta_0}^{\Theta_1} d\Theta \frac{k^4}{16E^2}\sin^{-4}\frac{\Theta}{2}$$

(Blows up)

Rutherford scattering geometry for beam path contact points

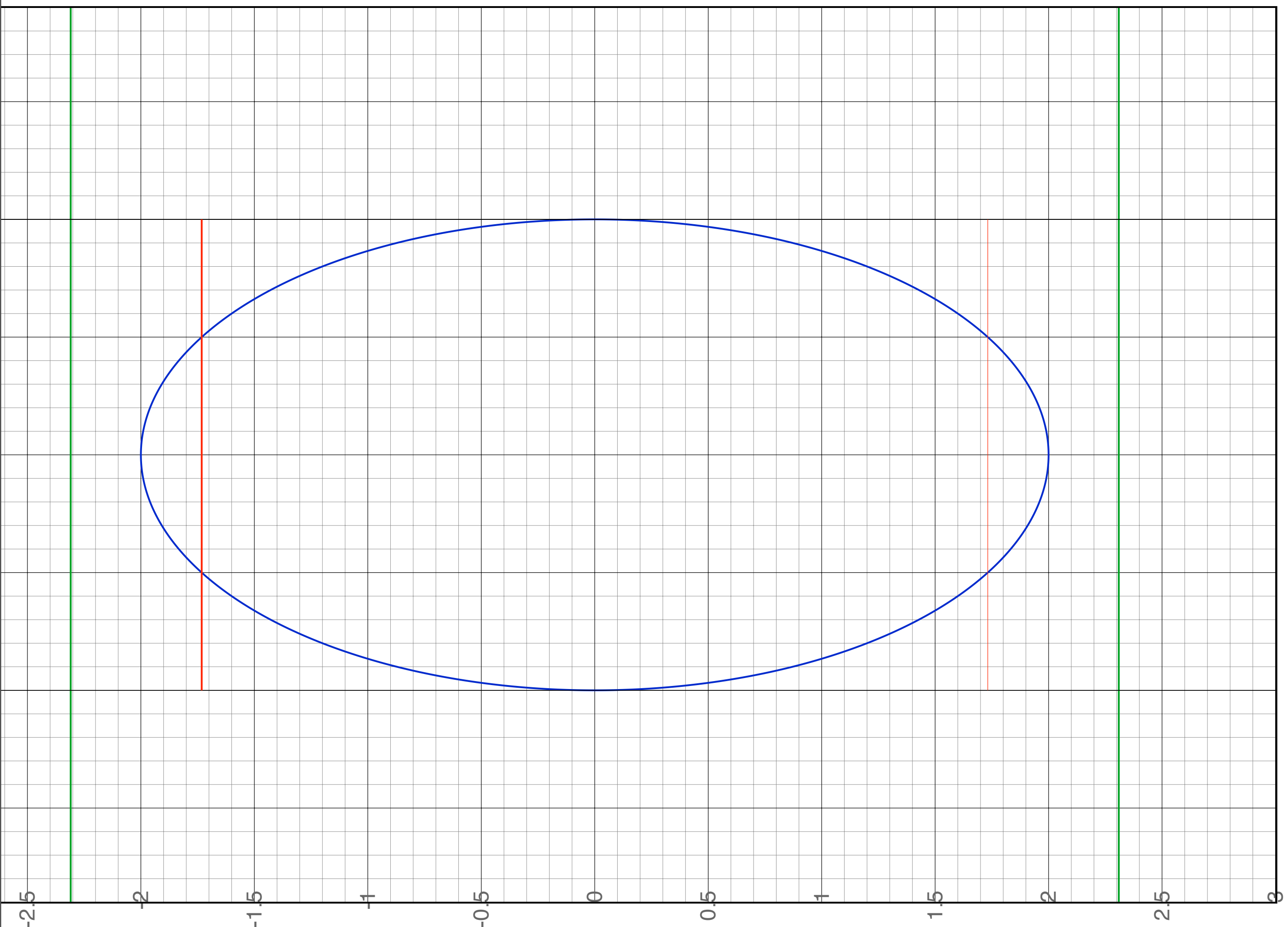


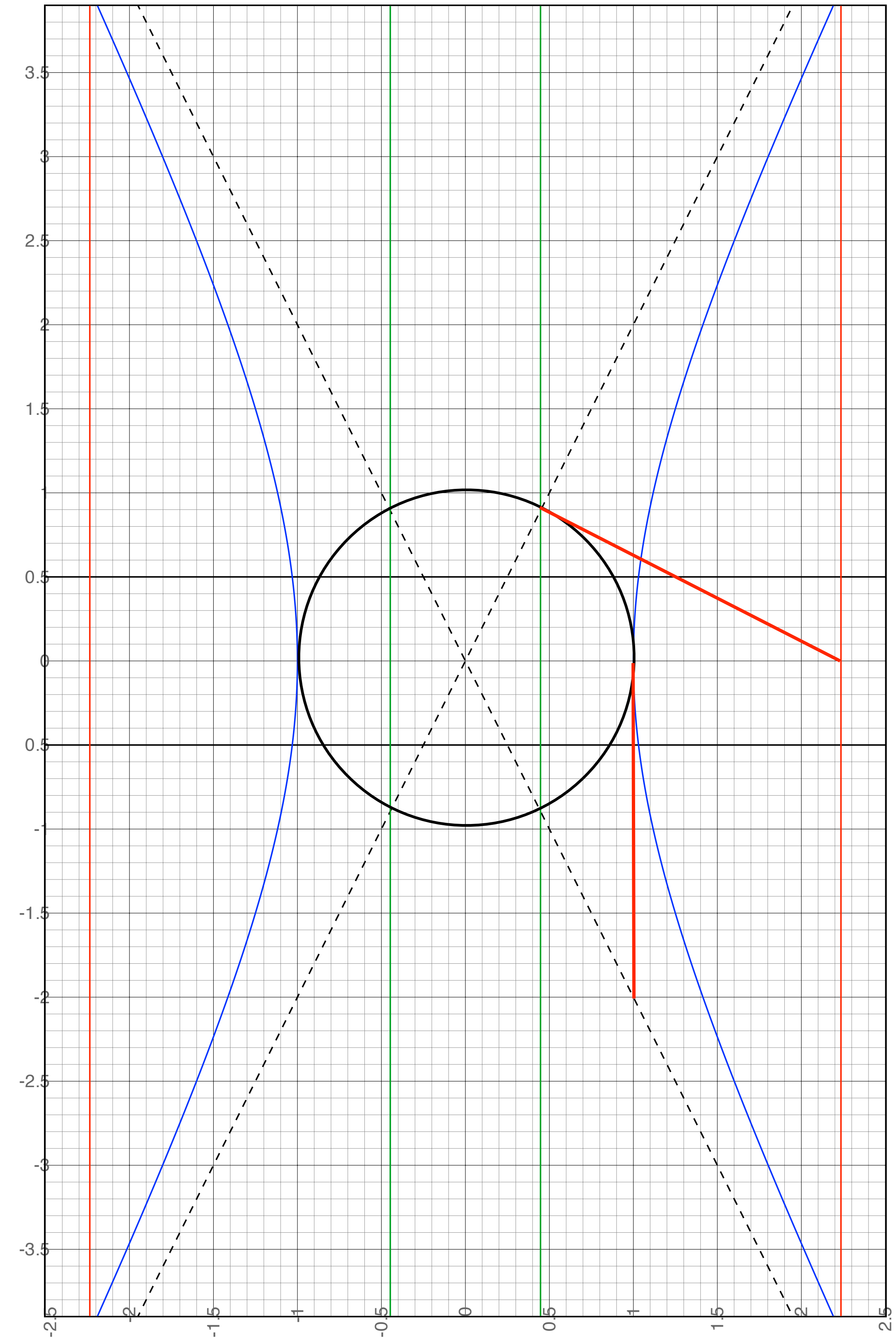
r-r′ =2a applies to each point on upper hyperbola and to the contact point P for the envelope parabola

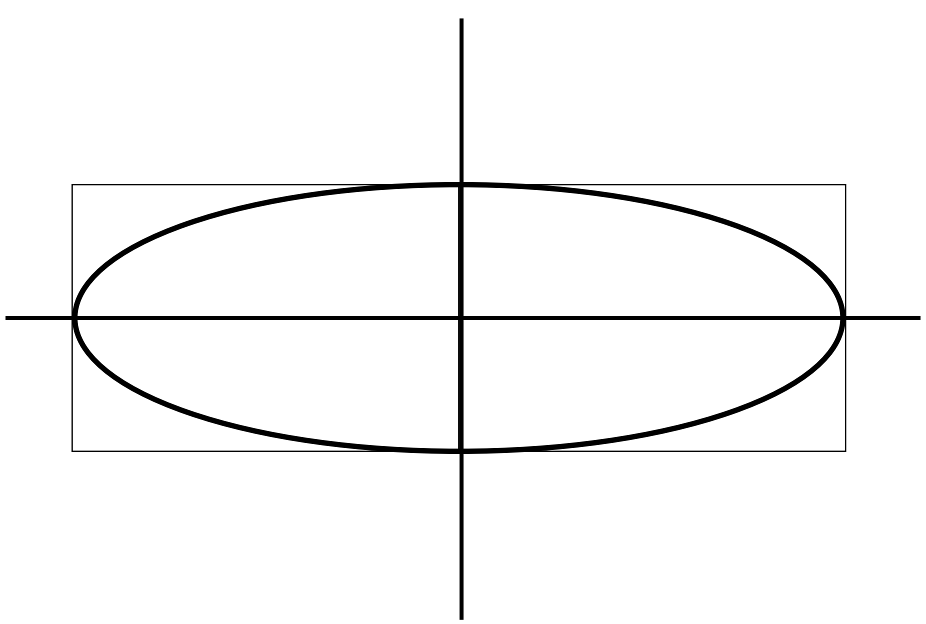
Special case: b=2a

Contact tangent has unit slope

Parabola $4px=8ax=y^2$ contacts Rutherford Hyperbolas of various b at the point where they intersect with equal slope


-2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5 3


3.5
3
2.5
2
1.5
0.5
0
0.5
-1.5
-2
-2.5
-3
-3.5
-2.5 -2 -1.5 -1 -0.5 0 0.5 1 1.5 2 2.5



918 Tanglebriar Ln

is SW corner of Tanglewood and Applebury
(Applebury ends there)

Path shown from Physics