Lecture 30
Tue. 12.11.2014

Geometry and Symmetry of Coulomb Orbital Dynamics II.

(Ch. 2-4 of Unit 5   12.11.14)

Eccentricity vector $\boldsymbol{\varepsilon}$ and (ε,λ)-geometry of orbital mechanics

*$\boldsymbol{\varepsilon}$-vector and Coulomb **r**-orbit geometry*

Review and connection to standard development

*$\boldsymbol{\varepsilon}$-vector and Coulomb **p**=m**v** geometry*

Example with elliptical orbit

Review of lectures 28 and 29

Analytic geometry derivation of $\boldsymbol{\varepsilon}$-construction

Algebra of $\boldsymbol{\varepsilon}$-construction geometry

Connection formulas for (a,b) and (ε,λ) with (γ,R)

Ruler & compass construction of $\boldsymbol{\varepsilon}$-vector and orbits

(R=-0.375 elliptic orbit)

(R=+0.5 hyperbolic orbit)

Properties of Coulomb trajectory families and envelopes

Graphical $\boldsymbol{\varepsilon}$-development of orbits

Launch angle fixed-Varied launch energy

Launch energy fixed-Varied launch angle

Launch optimization and orbit family envelopes

Eccentricity vector $\boldsymbol{\varepsilon}$ *and* (ε,λ)*-geometry of orbital mechanics*

➔ $\boldsymbol{\varepsilon}$*-vector and Coulomb* **r***-orbit geometry*

Review and connection to standard development

$\boldsymbol{\varepsilon}$*-vector and Coulomb* $\mathbf{p}=m\mathbf{v}$ *geometry*

$\boldsymbol{\varepsilon}$*-vector and Coulomb* $\mathbf{p}=m\mathbf{v}$ *algebra*

Example with elliptical orbit

Analytic geometry derivation of $\boldsymbol{\varepsilon}$*-construction*

Algebra of $\boldsymbol{\varepsilon}$*-construction geometry*

Connection formulas for (a,b) and (ε,λ) *with* (γ,R)

Ruler & compass construction of $\boldsymbol{\varepsilon}$*-vector and orbits*

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Eccentricity vector ε and (ε,λ) geometry of orbital mechanics

Isotropic field $V=V(r)$ guarantees conservation *angular momentum vector* **L**

(Review of Lect. 28-29) $\mathbf{L}=\mathbf{r}\times\mathbf{p}=m\,\mathbf{r}\times\dot{\mathbf{r}}$

Coulomb $V=-k/r$ also conserves *eccentricity vector* ε

$$\boldsymbol{\varepsilon}=\hat{\mathbf{r}}-\frac{\mathbf{p}\times\mathbf{L}}{km}=\frac{\mathbf{r}}{r}-\frac{\mathbf{p}\times(\mathbf{r}\times\mathbf{p})}{km}$$

(...for sake of comparison...)

IHO $V=(k/2)r^2$ also conserves *Stokes vector* ***S***

$$S_A=\tfrac{1}{2}(x_1^2+p_1^2-x_2^2-p_2^2)$$
$$S_B=x_1p_1+x_2p_2$$
$$S_C=x_1p_2-x_2p_1$$

$\mathbf{A}=km\cdot\boldsymbol{\varepsilon}$ is known as the *Laplace-Hamilton-Gibbs-Runge-Lenz vector*.

Generate symmetry groups: $U(2)\subset U(2)$

or: $R(3)\subset R(3)\times R(3)\subset O(4)$

Consider dot product of ε with a radial vector **r**:

$$\boldsymbol{\varepsilon}\bullet\mathbf{r}=\frac{\mathbf{r}\bullet\mathbf{r}}{r}-\frac{\mathbf{r}\bullet\mathbf{p}\times\mathbf{L}}{km}=r-\frac{\mathbf{r}\times\mathbf{p}\bullet\mathbf{L}}{km}=r-\frac{\mathbf{L}\bullet\mathbf{L}}{km}$$

...or of ε with momentum vector **p**:

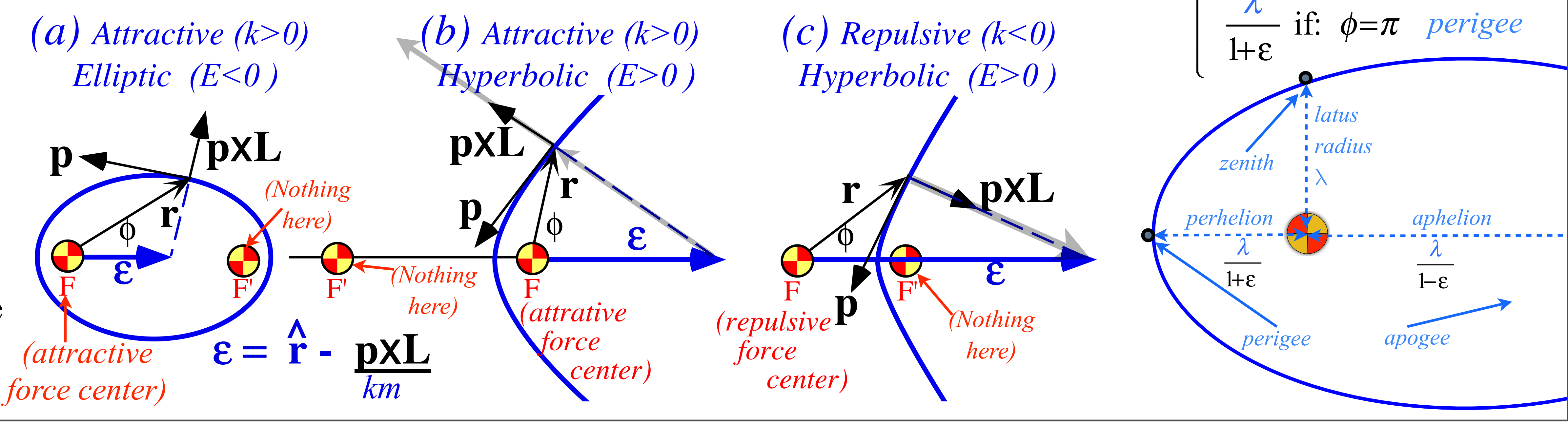
$$\boldsymbol{\varepsilon}\bullet\mathbf{p}=\frac{\mathbf{p}\bullet\mathbf{r}}{r}-\frac{\mathbf{p}\bullet\mathbf{p}\times\mathbf{L}}{km}=\mathbf{p}\bullet\hat{\mathbf{r}}=p_r$$

Let angle ϕ be angle between ε and radial vector **r**

$$\varepsilon\, r\cos\phi=r-\frac{L^2}{km}\qquad\text{or:}\qquad r=\frac{L^2/km}{1-\varepsilon\cos\phi}$$

For $\lambda=L^2/km$ that matches: $r=\dfrac{\lambda}{1-\varepsilon\cos\phi}=\begin{cases}\dfrac{\lambda}{1-\varepsilon} & \text{if: } \phi=0 \quad \textit{apogee}\\ \lambda & \text{if: } \phi=\dfrac{\pi}{2} \quad \textit{zenith}\\ \dfrac{\lambda}{1+\varepsilon} & \text{if: } \phi=\pi \quad \textit{perigee}\end{cases}$

(Rotational momentum $\mathbf{L}=\mathbf{r}\times\mathbf{p}$ is normal to the orbit plane.)



Eccentricity vector **ε** *and (ε,λ)-geometry of orbital mechanics*

ε-vector and Coulomb **r**-*orbit geometry*

➔ *Review and connection to standard development*

ε-vector and Coulomb **p**=*m***v** *geometry*

ε-vector and Coulomb **p**=*m***v** *algebra*

Example with elliptical orbit

Analytic geometry derivation of ε-construction

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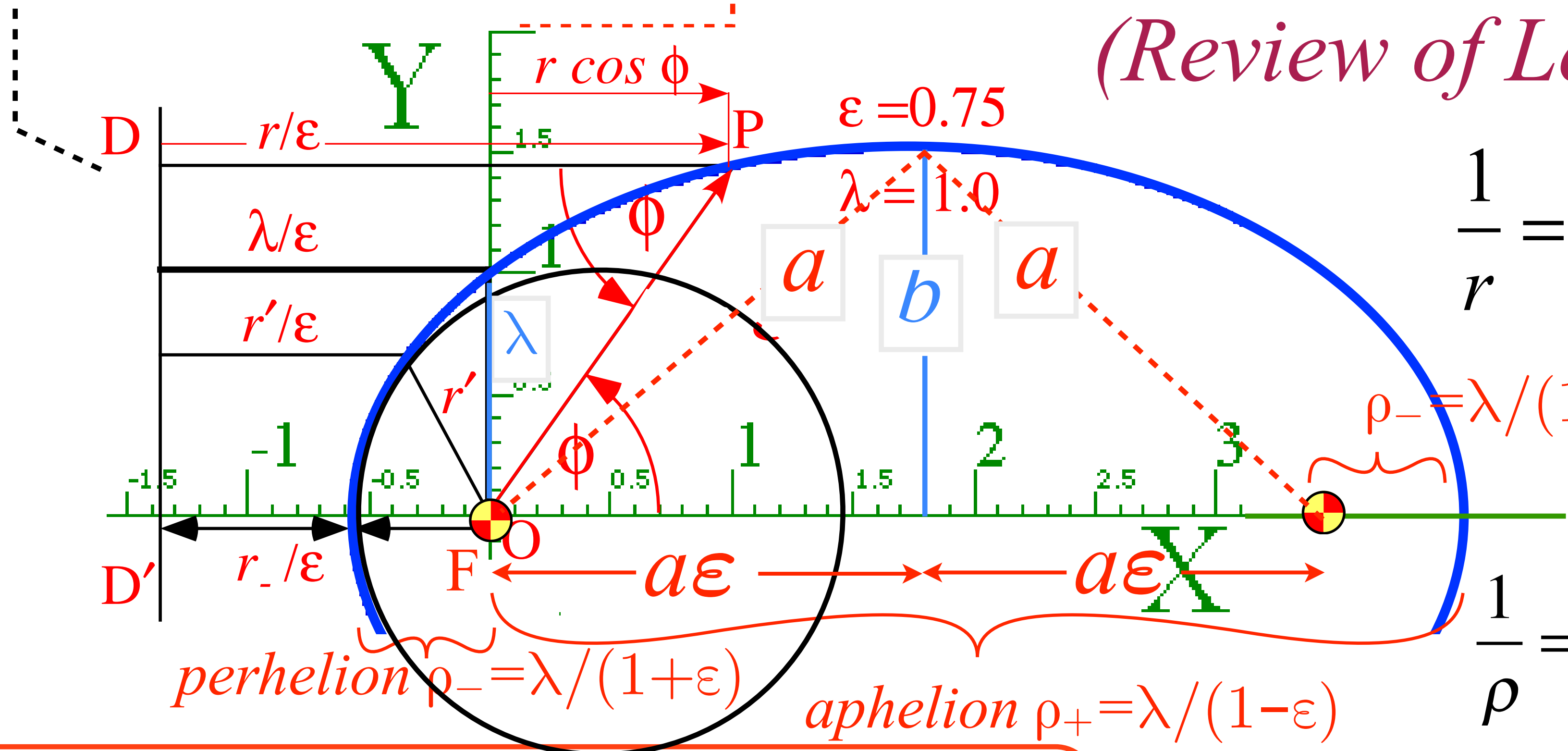
(From Lecture 28 p. 63-74) *Geometry of Coulomb orbits (Let: $r = \rho$ here)*

$$r/\varepsilon = \lambda/\varepsilon + r\cos\phi \qquad r = \lambda + r\,\varepsilon\cos\phi \qquad r = \frac{\lambda}{1-\varepsilon\cos\phi}$$

(Review of Lect. 28-29)

$$\frac{1}{r} = \frac{1-\varepsilon\cos\phi}{\lambda} = \frac{1}{\lambda} - \frac{\varepsilon}{\lambda}\cos\phi$$



$$\frac{1}{\rho} = \frac{-k}{\mu^2/m} + \frac{\sqrt{k^2 + 2E\mu^2/m}}{\mu^2/m}\cos\phi$$

All conics defined by:

*Defining **eccentricity** ε*

Distance to F_{ocal}-point $=\varepsilon\cdot$ Distance to D_{irectrix}-line

Major axis: $\rho_+ + \rho_- = 2a$

$\rho_+ + \rho_- = [\lambda(1+\varepsilon)+\lambda(1-\varepsilon)]/(1-\varepsilon^2) = 2\lambda/|1-\varepsilon^2|$

Focal axis: $\rho_+ - \rho_- = 2a\varepsilon$

$\rho_+ - \rho_- = [\lambda(1+\varepsilon)-\lambda(1-\varepsilon)]/(1-\varepsilon^2) = 2\lambda\varepsilon/|1-\varepsilon^2|$

Minor radius: $b=\sqrt{(a^2-a^2\varepsilon^2)}=\sqrt{(a\lambda)}$ *(ellipse: $\varepsilon<1$)*

Minor radius: $b=\sqrt{(a^2\varepsilon^2-a^2)}=\sqrt{(\lambda a)}$ *(hyperb: $\varepsilon>1$)*

(x,y) parameters	physical constants	(r,ϕ) parameters
$a = \dfrac{k}{2E}$	$E = \dfrac{k}{2a}$	$\varepsilon = \sqrt{\dfrac{k^2 m + 2L^2 E}{k^2 m}} = \sqrt{1 \pm \dfrac{b^2}{a^2}}$
$b = \dfrac{L}{\sqrt{2m\lvert E\rvert}}$	$L = \sqrt{km\lambda}$	$\lambda = \dfrac{L^2}{km} = \dfrac{b^2}{a}$

$\varepsilon^2 = 1 - \dfrac{b^2}{a^2}$ (ellipse: $\varepsilon<1$) $\dfrac{b^2}{a^2} = \sqrt{1-\varepsilon^2}$

$\varepsilon^2 = 1 + \dfrac{b^2}{a^2}$ (hyperbola: $\varepsilon>1$) $\dfrac{b^2}{a^2} = \sqrt{\varepsilon^2 - 1}$

$\lambda = a(1-\varepsilon^2)$ *(ellipse: $\varepsilon<1$)*

$\lambda = a(\varepsilon^2-1)$ *(hyperb: $\varepsilon>1$)*

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Dot product of ε with momentum vector **p**:

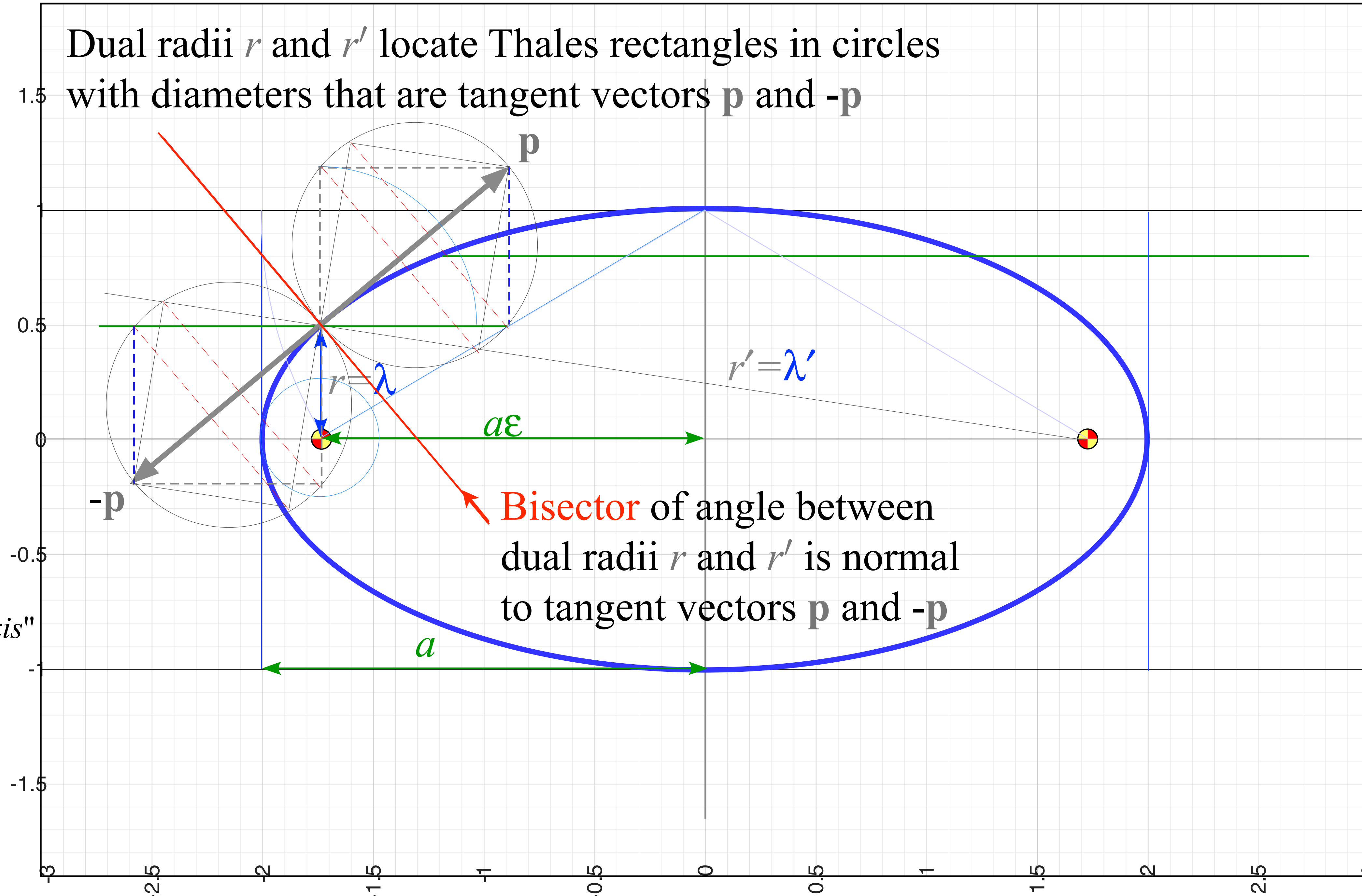
$$\boldsymbol{\varepsilon} \bullet \mathbf{p} = \frac{\mathbf{p} \bullet \mathbf{r}}{r} - \frac{\mathbf{p} \bullet \mathbf{p} \times \mathbf{L}}{km}$$

$$= \mathbf{p} \bullet \hat{\mathbf{r}} = p_r = \varepsilon p_x$$

This says:

"*Projection of* **p** *onto* **r** *is eccentricity* ε *times projection of* **p** *onto* $\hat{\mathbf{x}}$*-axis*"

($\hat{\mathbf{x}} = \hat{\boldsymbol{\varepsilon}}$)



Dot product of ε with momentum vector $\mathbf{p}$:

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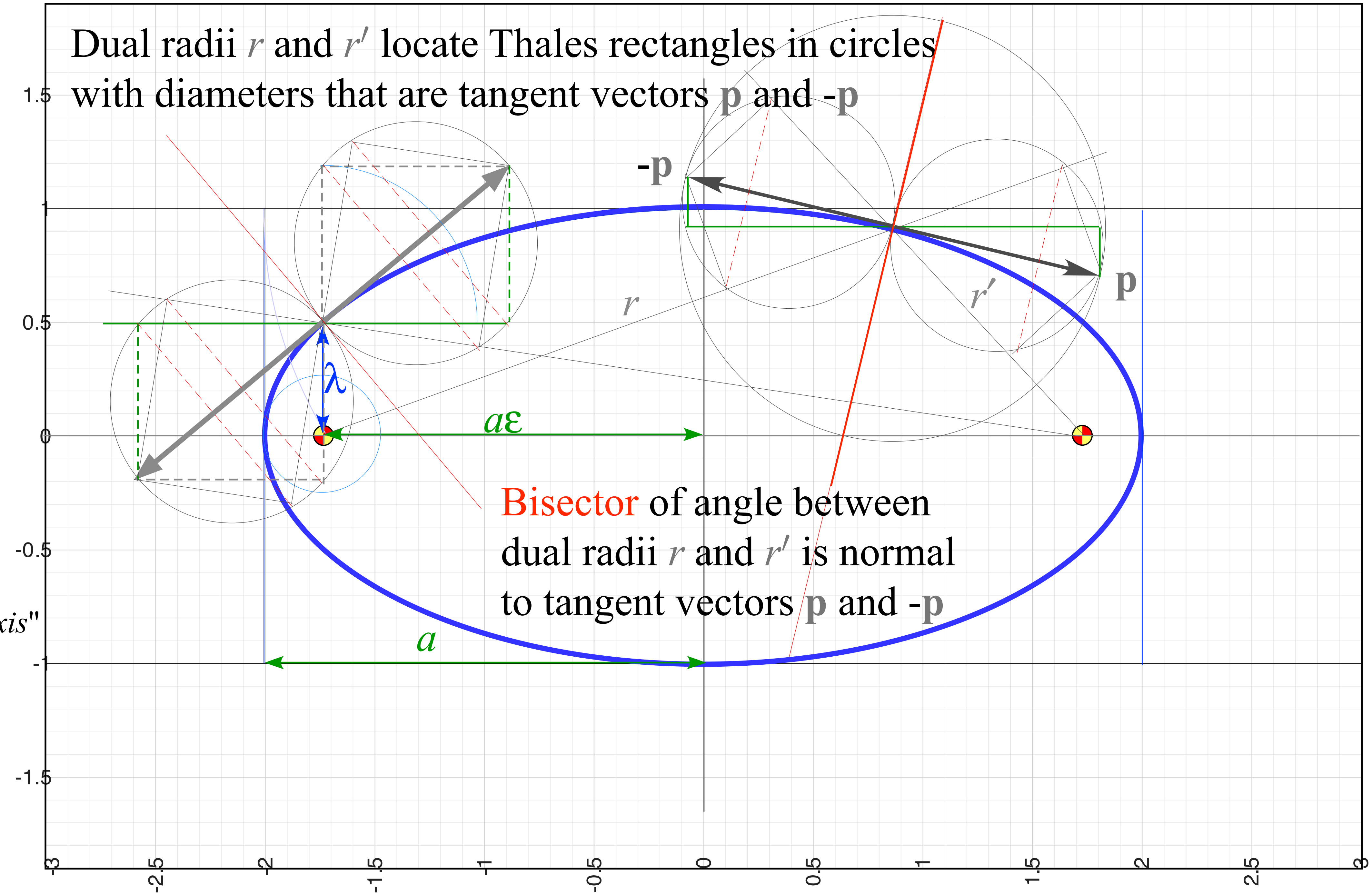

(Review of Lect. 29)

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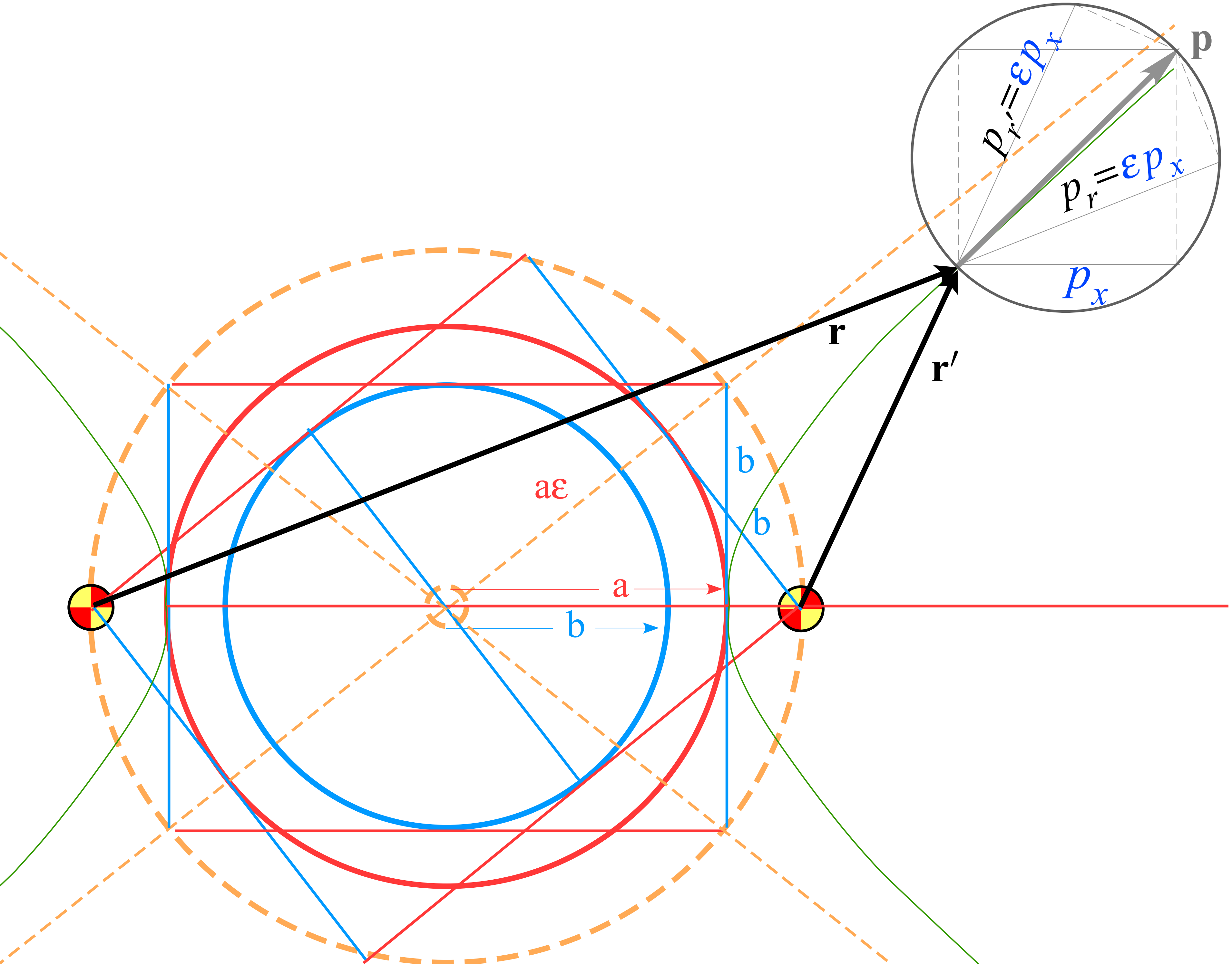

Hyperbola has eccentricity $\varepsilon > 1$

$(Here: \varepsilon = 5/4 = 1.25)$

(Review of Lect. 29)

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ε-vector and Coulomb p=mv geometry (Review of Lect. 29 p.50-62)

Finding time derivatives of orbital coordinates r, φ, x, y, and eventually velocity **v** *or momentum* **p**=m**v**

Radius r:

$$r = \frac{\lambda}{1-\varepsilon\cos\phi} = \frac{L^2/km}{1-\varepsilon\cos\phi}$$

$$\dot r = \frac{dr}{dt} = \frac{L^2}{km}\frac{-\frac{d}{dt}(-\varepsilon\cos\phi)}{(1-\varepsilon\cos\phi)^2}$$

$$\dot r = \frac{L^2}{km}\frac{-\varepsilon\sin\phi\,\dot\phi}{(1-\varepsilon\cos\phi)^2}$$

$$\dot r = -\frac{L^2}{km}\left(\frac{km}{L^2}\right)^2 r^2\dot\phi\,\varepsilon\sin\phi$$

using: $\frac{1}{(1-\varepsilon\cos\phi)^2} = \left(\frac{km}{L^2}\right)^2 r^2$

$$\dot r = -\frac{k}{L^2}mr^2\dot\phi\,\varepsilon\sin\phi = -\frac{k}{L}\varepsilon\sin\phi$$

again using: $L = mr^2\dot\phi$

Polar angle φ using: $L = mr^2\frac{d\phi}{dt} = mr^2\dot\phi$

$$\dot\phi = \frac{L}{mr^2} = \frac{L}{m}\frac{1}{r^2} = \frac{L}{m}\left(\frac{km}{L^2}\right)^2(1-\varepsilon\cos\phi)^2$$

$$r\dot\phi = \frac{L}{mr} = \frac{L}{m}\frac{1}{r} = \frac{L}{m}\left(\frac{km}{L^2}\right)(1-\varepsilon\cos\phi) = \frac{k}{L}(1-\varepsilon\cos\phi)$$

using: $\frac{1}{r^2} = \left(\frac{km}{L^2}\right)^2(1-\varepsilon\cos\phi)^2$

Cartesian $x = r\cos\phi$:

$$\dot x = \frac{dx}{dt} = \dot r\cos\phi - \sin\phi\, r\dot\phi$$

$$= -\frac{k}{L}\sin\phi$$

Cartesian $y = r\sin\phi$:

$$\dot y = \frac{dy}{dt} = \dot r\sin\phi + \cos\phi\, r\dot\phi$$

$$= \frac{k}{L}(\cos\phi - \varepsilon)$$

Velocity:

Momentum:

$$p_x = m\dot x = -\frac{mk}{L}\sin\phi$$

$$p_y = m\dot y = \frac{mk}{L}(\cos\phi - \varepsilon)$$

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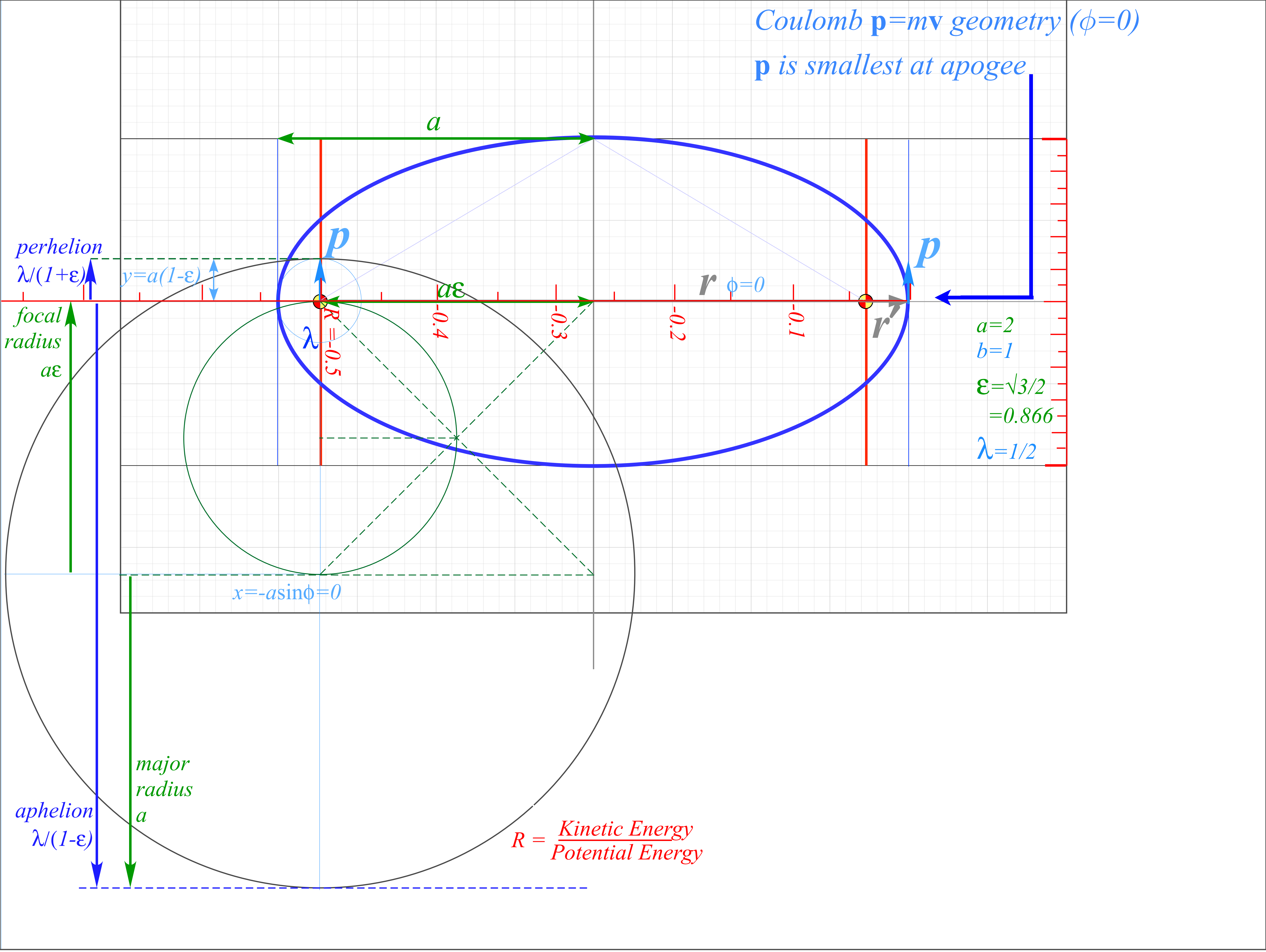
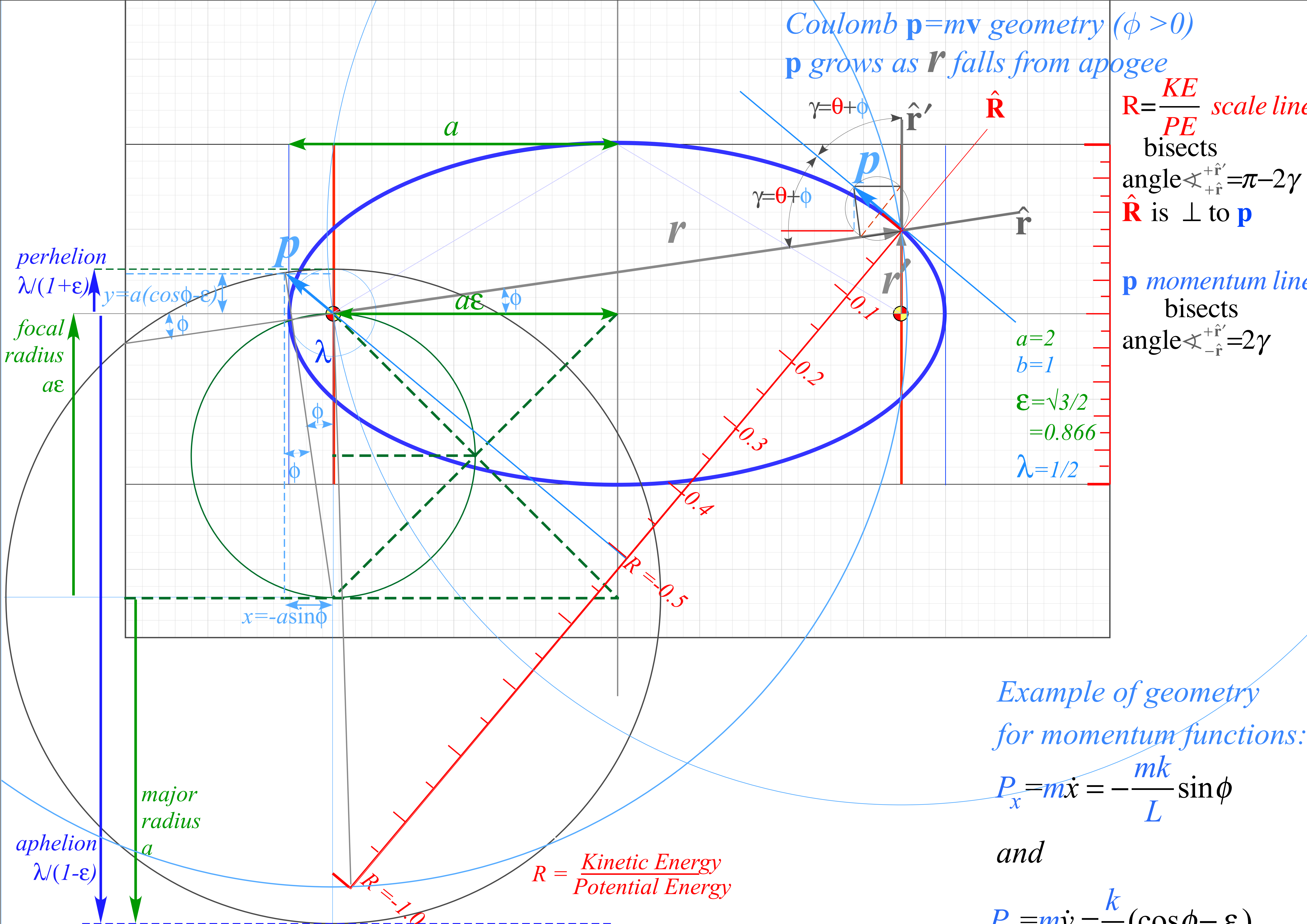


Coulomb p=mv geometry (ϕ=0)
p is smallest at apogee
a
p
p
perhelion
λ/(1+ε)
y=a(1-ε)
aε
r ϕ=0
r'
focal
radius
aε
λ
R=-0.5
-0.4
-0.3
-0.2
-0.1
a=2
b=1
ε=√3/2
=0.866
λ=1/2
x=-asinϕ=0
major
radius
a
aphelion
λ/(1-ε)
R = Kinetic Energy / Potential Energy



Example of geometry

for momentum functions:

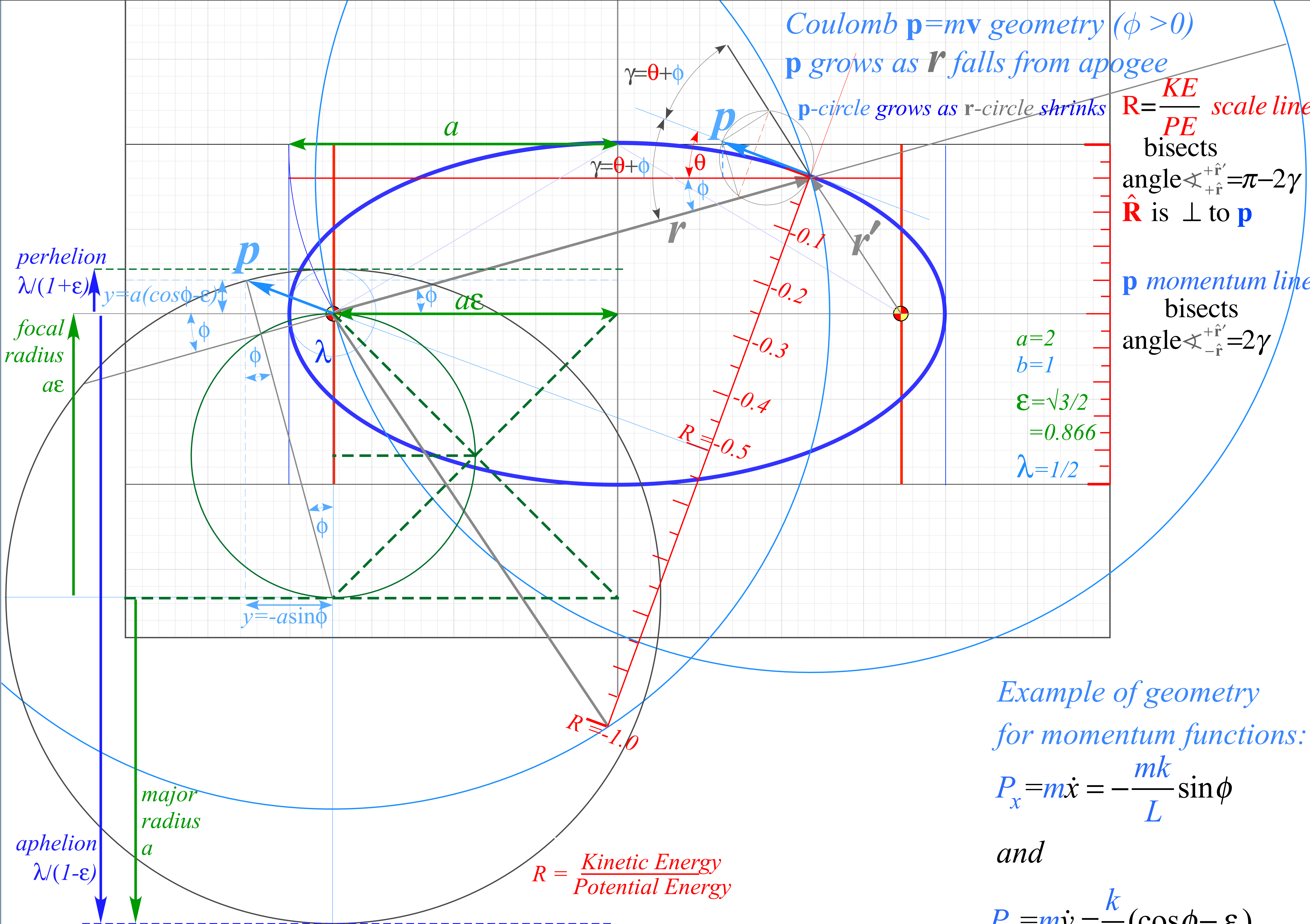
$$P_x = m\dot{x} = -\frac{mk}{L}\sin\phi$$

and

$$P_y = m\dot{y} = \frac{k}{L}(\cos\phi - \varepsilon)$$

0>R=KE/PE>-1 scale subtends angle 2γ with length 2r sinγ *as is derived several pages ahead.*

*Note similarity of (**R**,**r**)-triangle in **r**-circle of radius r to that in **p**-circle of diameter p above.*



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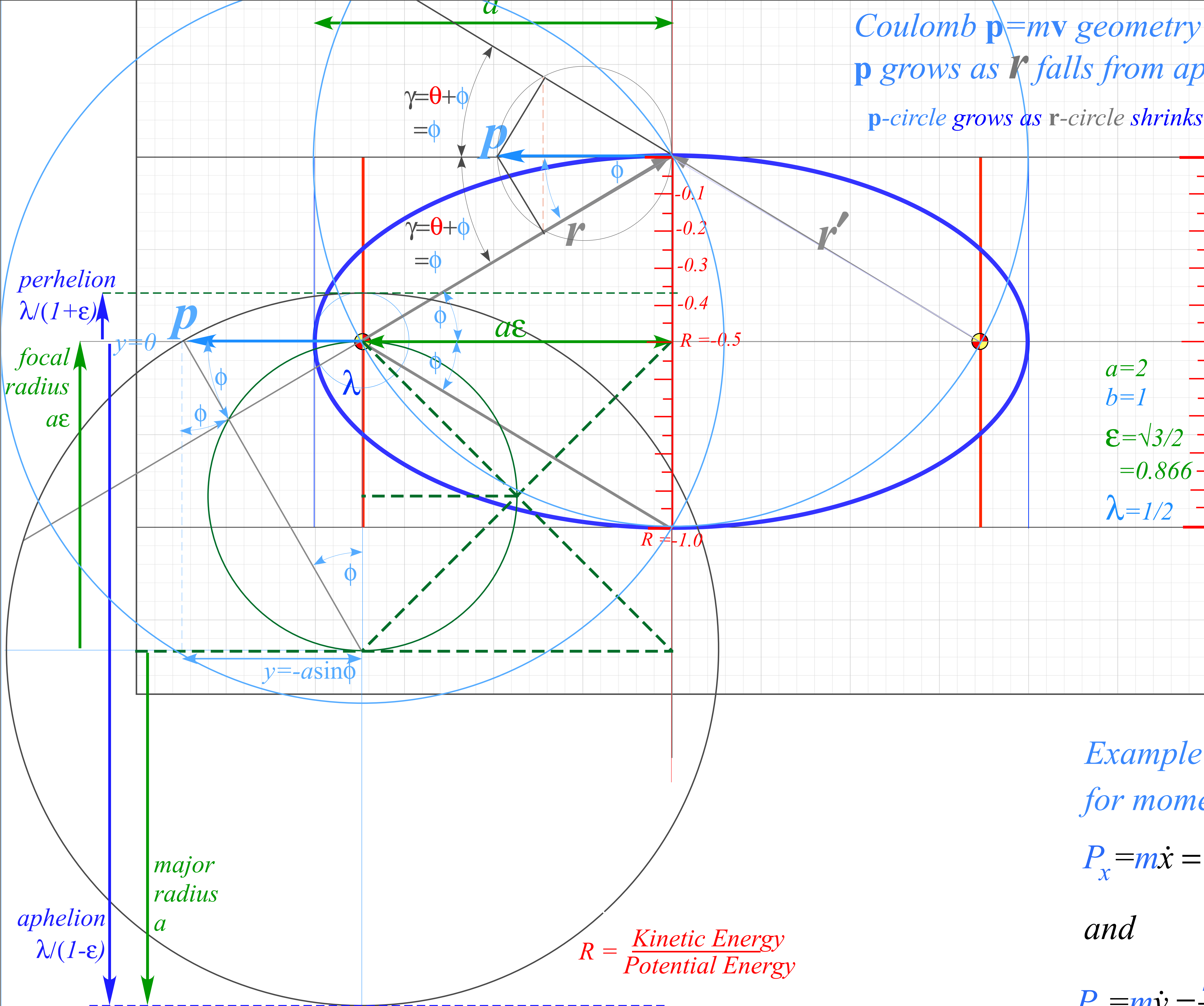
Coulomb **p**=*m***v** *geometry* $(\phi>0)$

p *grows as* **r** *falls from apogee*

p-*circle grows as* **r**-*circle shrinks*



$R=\frac{KE}{PE}$ *scale line* bisects angle $\sphericalangle^{+\hat{r}'}_{+\hat{r}}=\pi-2\gamma$

$\hat{\mathbf{R}}$ is $\perp$ to **p**

p *momentum line* bisects angle $\sphericalangle^{+\hat{r}'}_{-\hat{r}}=2\gamma$

Example of geometry for momentum functions:

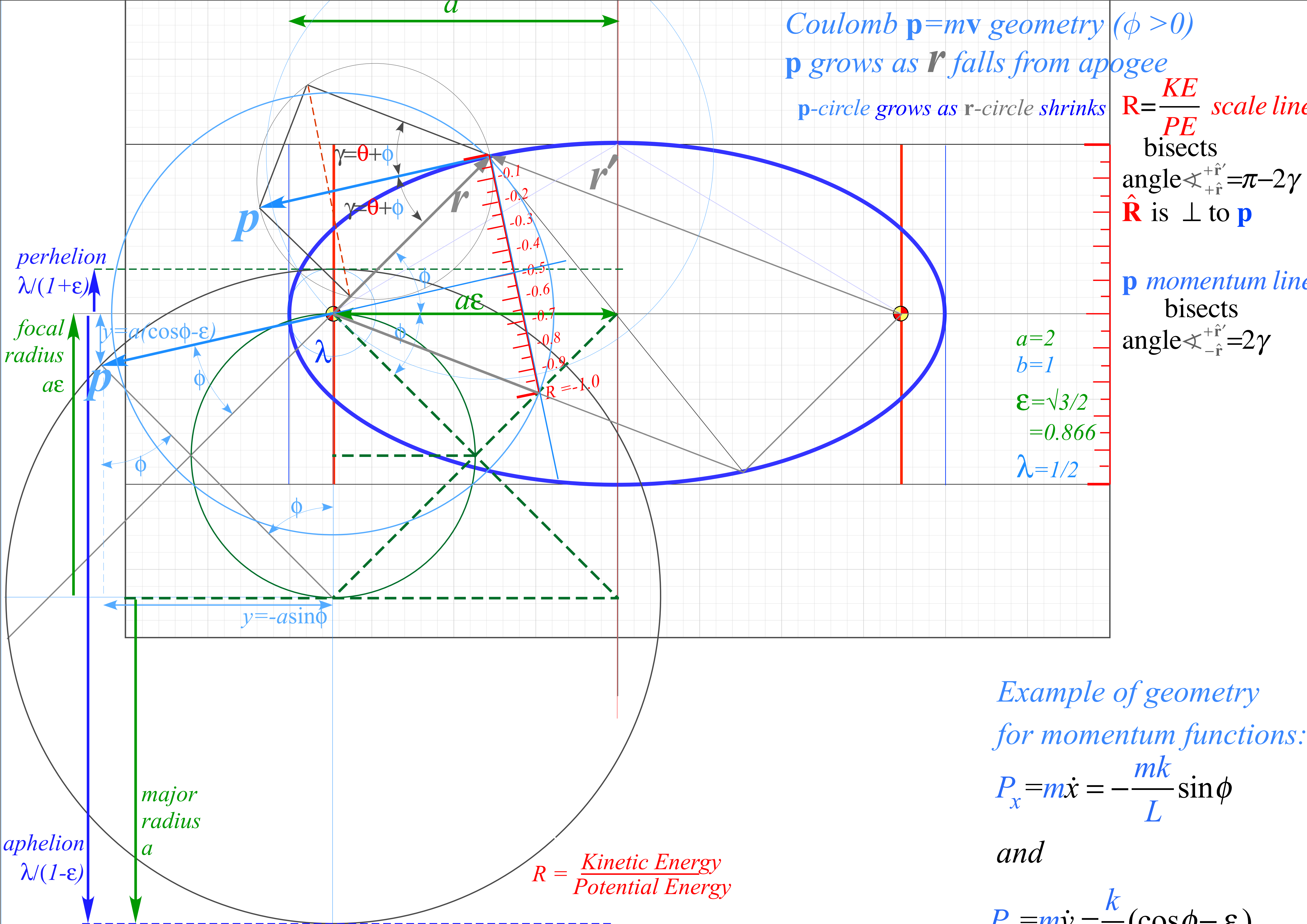
$$P_x=m\dot{x}=-\frac{mk}{L}\sin\phi$$

and

$$P_y=m\dot{y}=\frac{k}{L}(\cos\phi-\varepsilon)$$

0>R=KE/PE>-1 scale subtends angle 2γ *with length* $2r\sin\gamma$ *as is derived several pages ahead.*

*Note similarity of (***R***,***r***)-triangle in* **r**-*circle of radius r to that in* **p**-*circle of diameter p above.*



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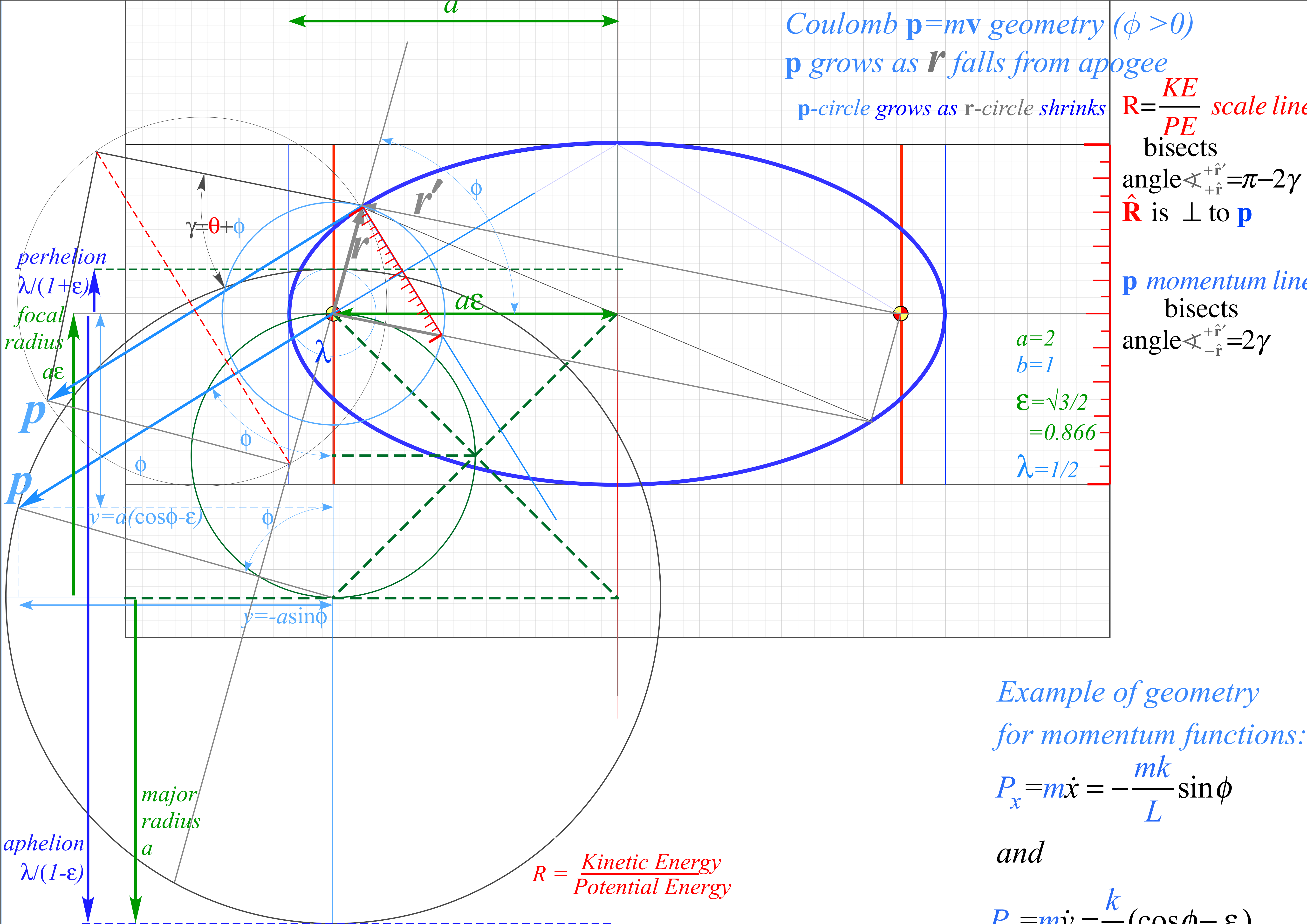

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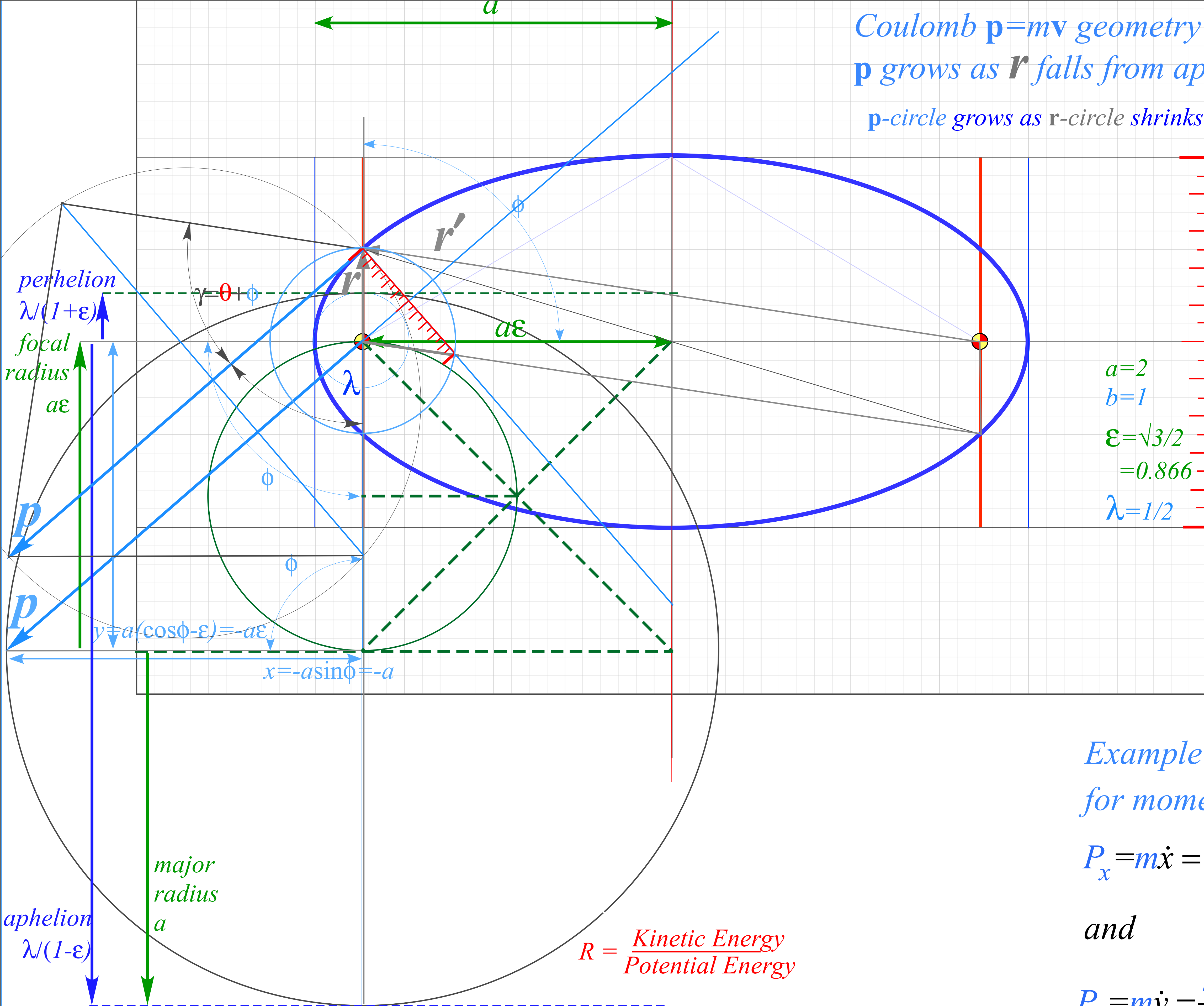
*Note similarity of (**R**,**r**)-triangle in **r**-circle of radius r to that in **p**-circle of diameter p above.*

Coulomb **p**=*m***v** geometry (ϕ >0)

p grows as **r** falls from apogee



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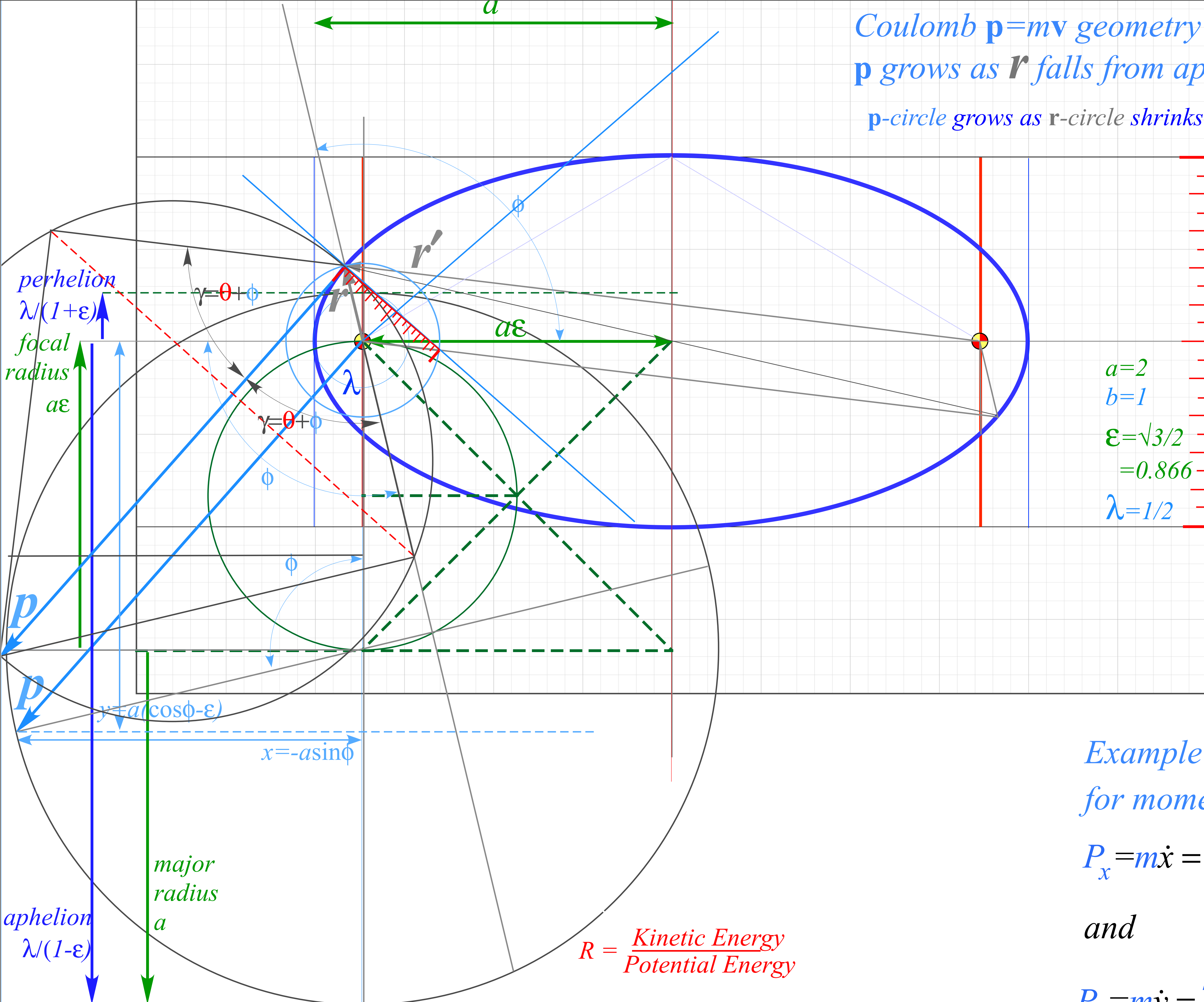

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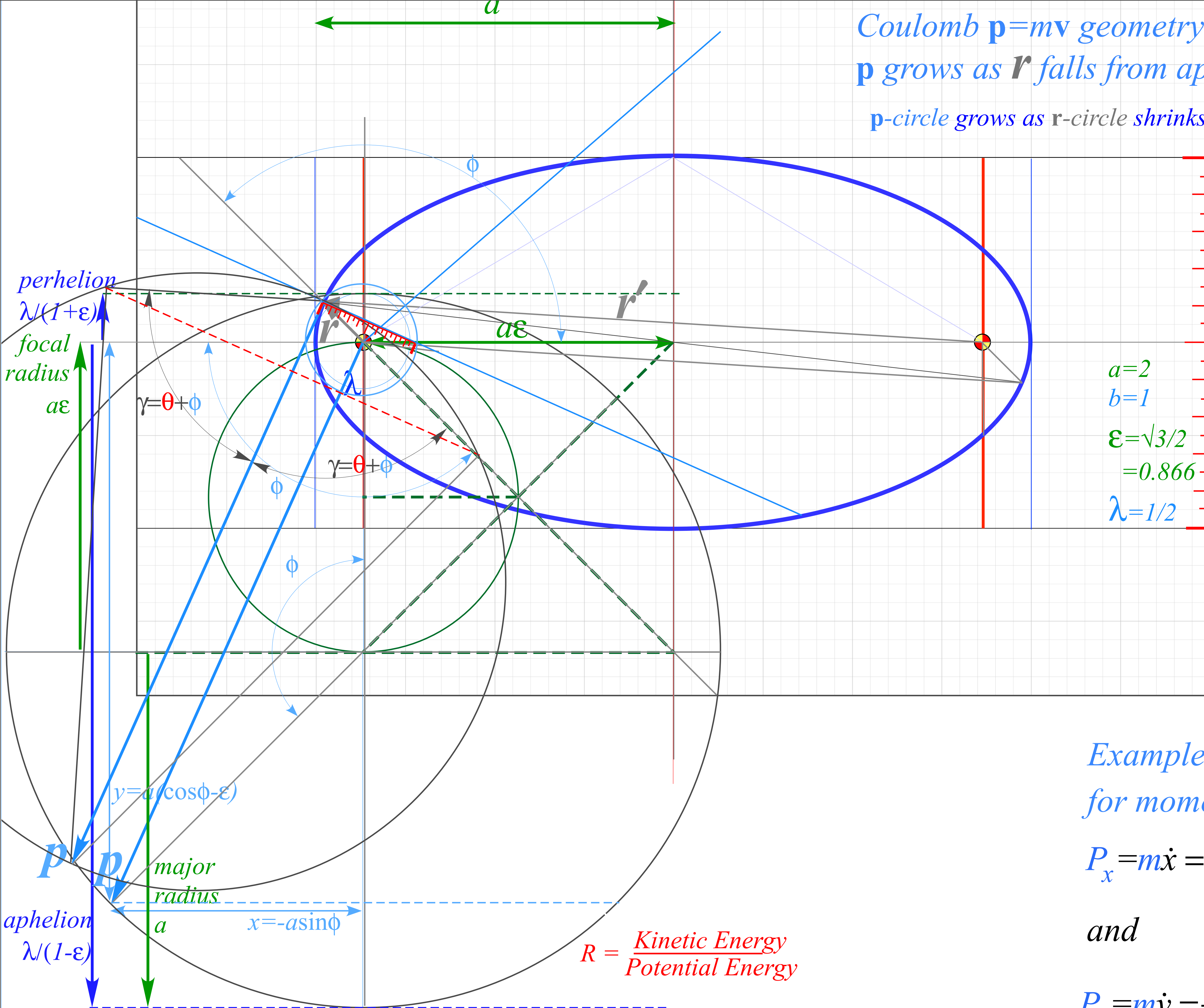

$\mathbf{R}=\dfrac{KE}{PE}$ *scale line*

bisects

angle$\sphericalangle^{+\hat{\mathbf{r}}'}_{+\hat{\mathbf{r}}}=\pi-2\gamma$

$\hat{\mathbf{R}}$ is $\perp$ to **p**

p *momentum line*

bisects

angle$\sphericalangle^{+\hat{\mathbf{r}}'}_{-\hat{\mathbf{r}}}=2\gamma$

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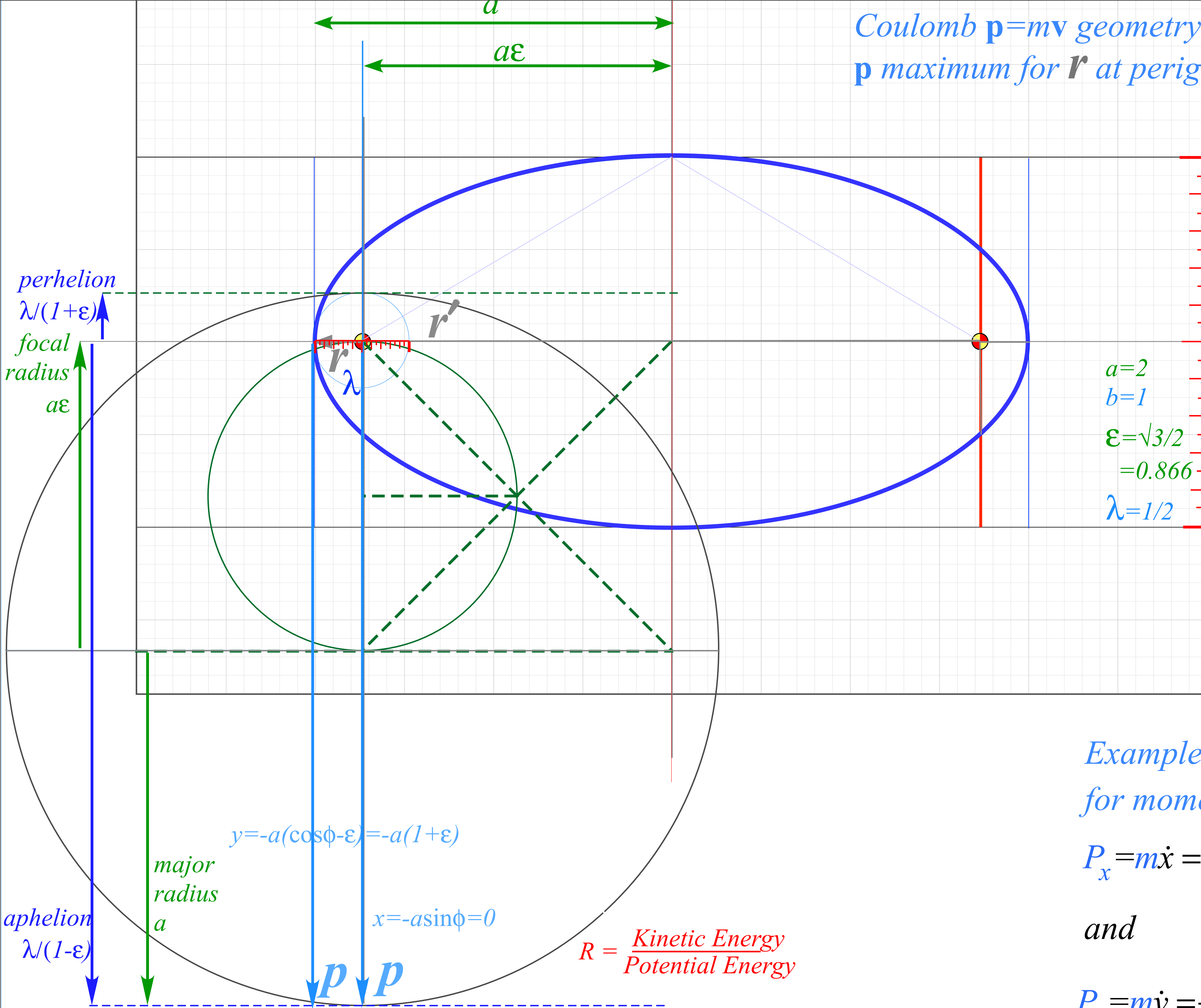
$R = \dfrac{\text{Kinetic Energy}}{\text{Potential Energy}}$

0>R=KE/PE>-1 scale subtends angle 2γ with length 2r sinγ *as is derived several pages ahead.*

*Note similarity of (***R***,***r***)-triangle in* **r**-*circle of radius r to that in* **p**-*circle of diameter p above.*

Coulomb **p**=m**v** geometry ($\phi > 0$)

p maximum for **r** at perigee



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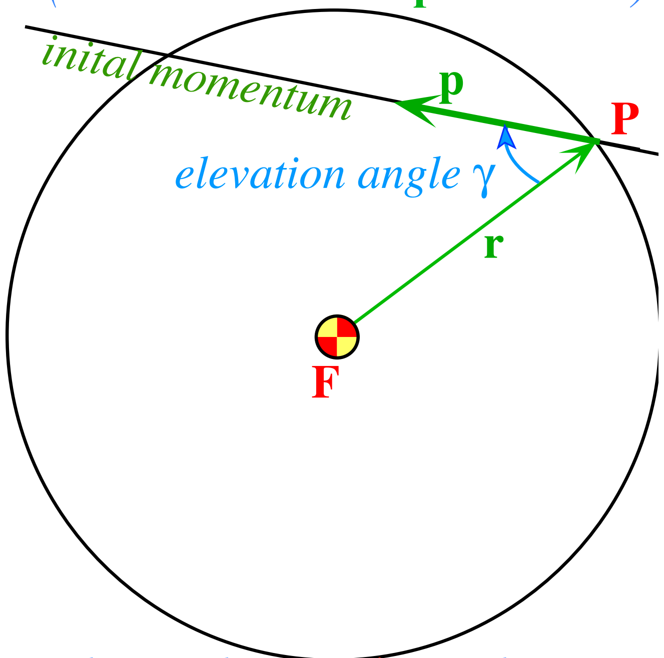
ε-vector and Coulomb orbit construction steps

Pick launch point P

(radius vector **r** *)*

and elevation angle γ *from radius*

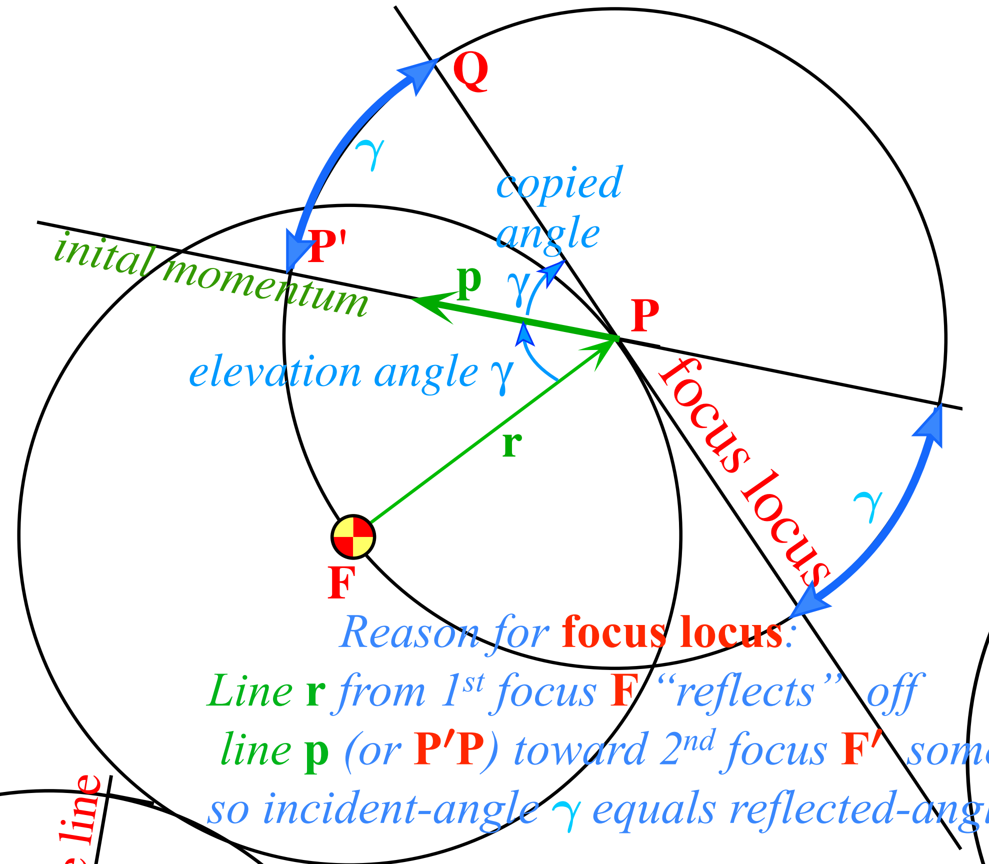
(momentum initial **p** *direction)*



Copy F-center circle around launch point P

Copy elevation angle γ *(∠FPP′) onto ∠P′PQ*

Extend resulting line QPQ′ *to make* **focus locus**



Reason for **focus locus**:

Line **r** *from 1st focus* **F** *"reflects" off*

line **p** *(or* **P′P***) toward 2nd focus* **F′** *somewhere*

so incident-angle γ *equals reflected-angle* γ

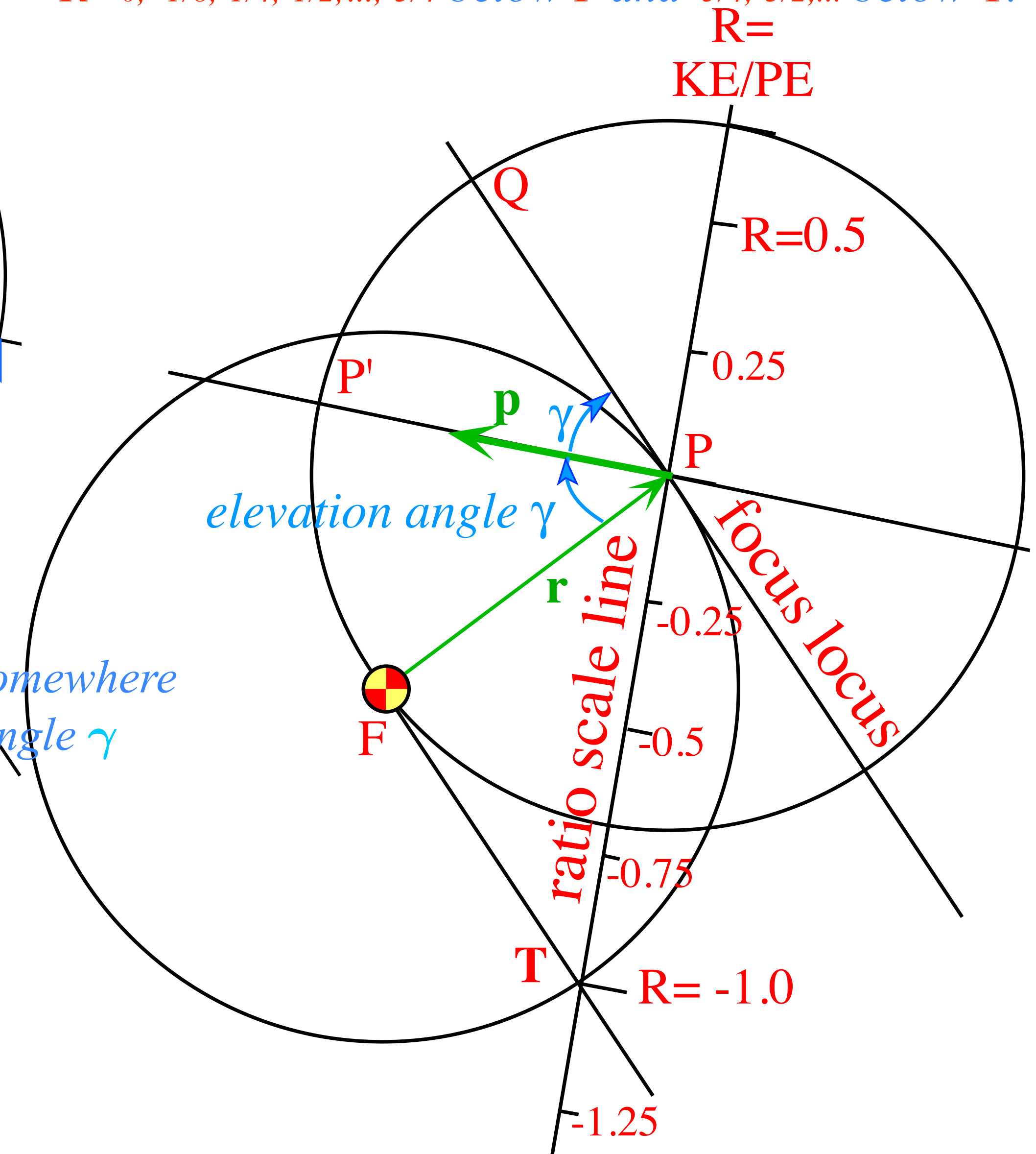
*Copy double angle 2*γ *(∠FPQ) onto ∠PFT*

Extend ∠PFT chord PT *to make R-***ratio scale line**

Label chord PT *with R=0 at* P *and R=-1.0 at* T.

*Mark R-***line** *fractions R=0, +1/4, +1/2,... above* P *and*

R=0, -1/8,-1/4,-1/2,...,-3/4 below P *and -5/4,-3/2,... below* T.

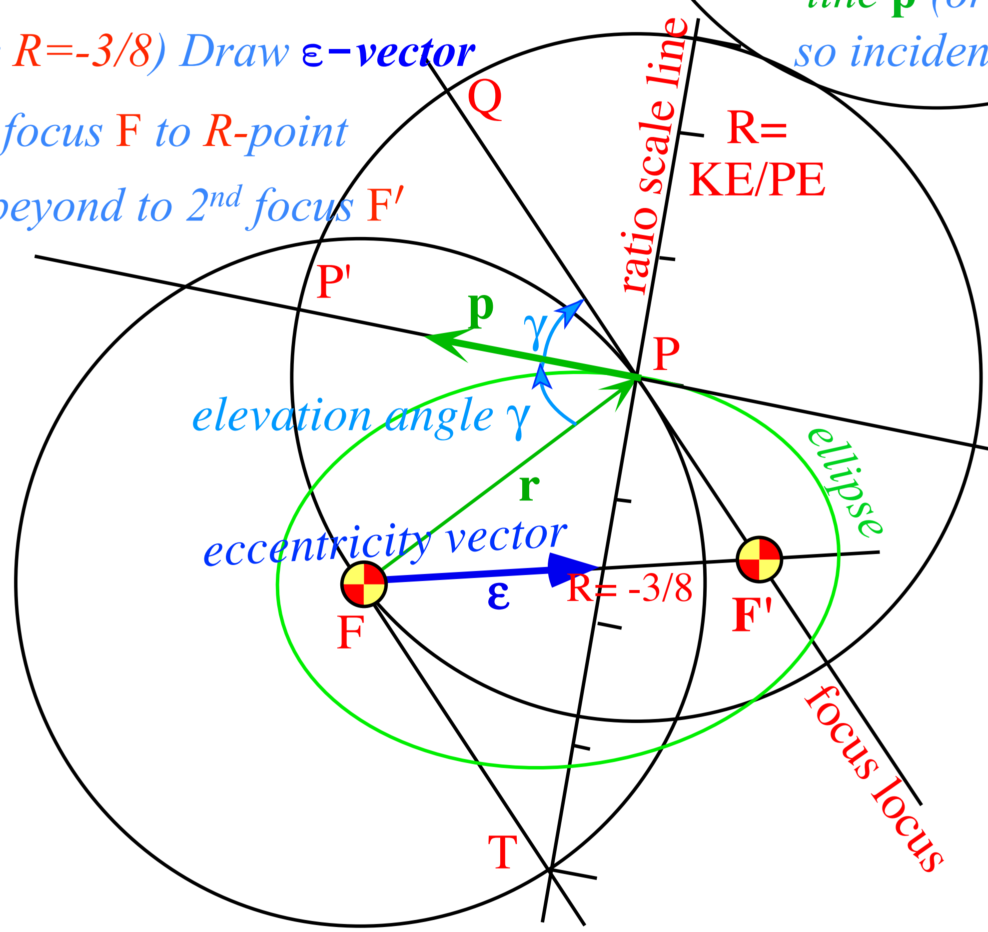


Pick initial R=KE/PE value

(here R=-3/8) Draw **ε−vector**

from focus F *to R-point*

and beyond to 2nd focus F′

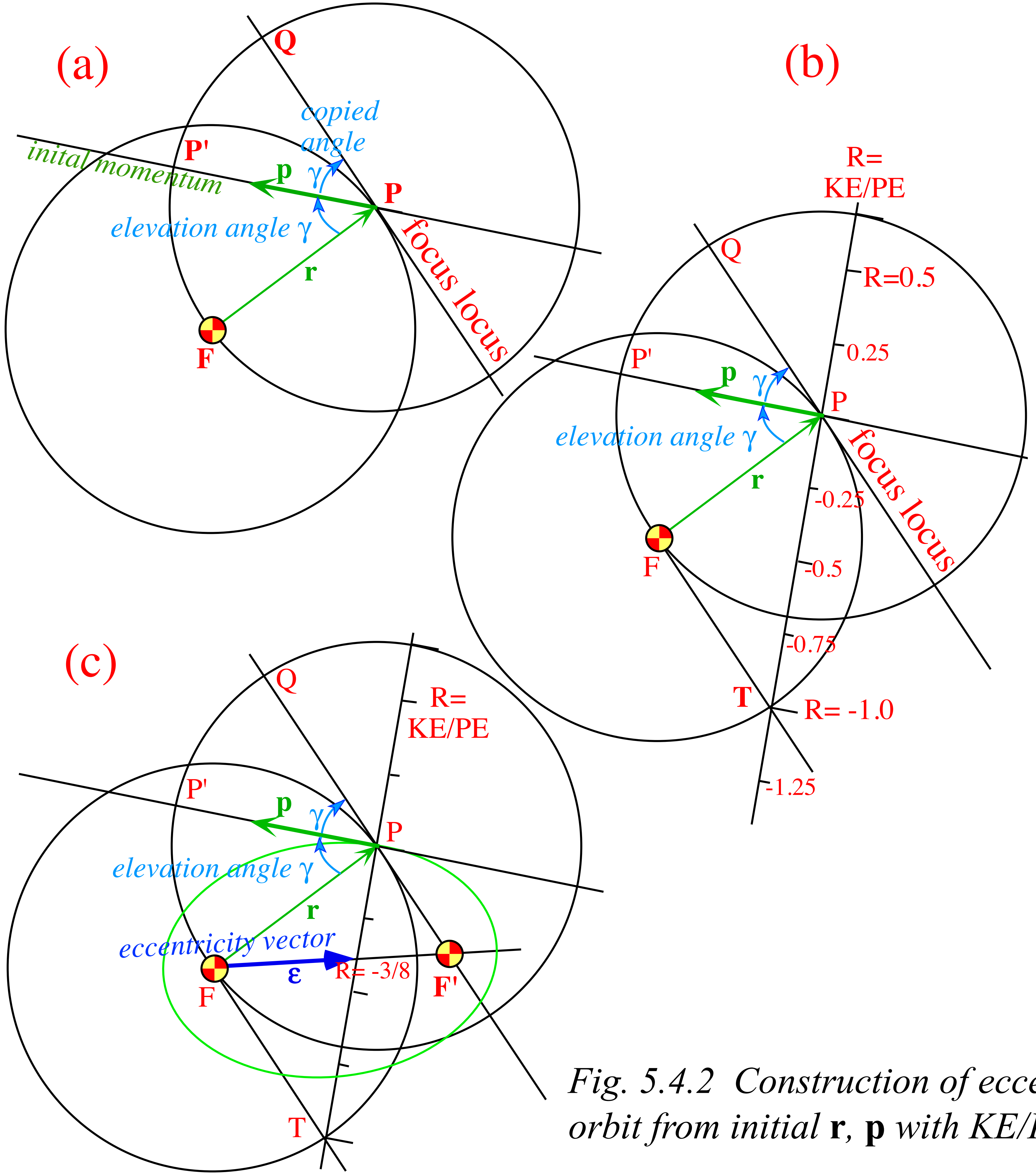


$$R = \frac{\textit{Initial KE}}{\textit{Initial PE}} = \frac{m v^2(0)/2}{-k/r(0)}$$

$$= \pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2 = \pm\frac{v^2(0)}{v^2(\infty)}$$

> *focus* F *and 2nd focus* F′ *allow final construction of orbital trajectory. Here it is an R=-3/8 ellipse.*

(Detailed Analytic geometry of **ε-vector** *follows.)*

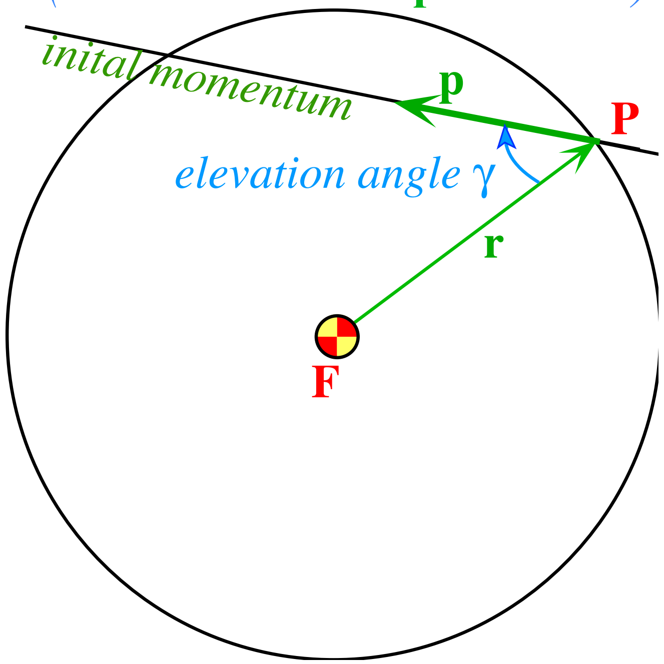


Next several pages give step-by-step constructions of ***ε-vector*** *and Coulomb orbit and trajectory physics*

Fig. 5.4.2 Construction of eccentricity vector **ε** *and orbit from initial* **r**, **p** *with KE/PE=-3/8.*

ε-vector and Coulomb orbit construction steps

Pick launch point P
(radius vector **r** *)*
and elevation angle γ *from radius*
(momentum initial **p** *direction)*



Next several pages give step-by-step constructions of ε-vector and Coulomb orbit and trajectory physics

ε-vector and Coulomb orbit construction steps

Pick launch point P
(radius vector **r** *)*
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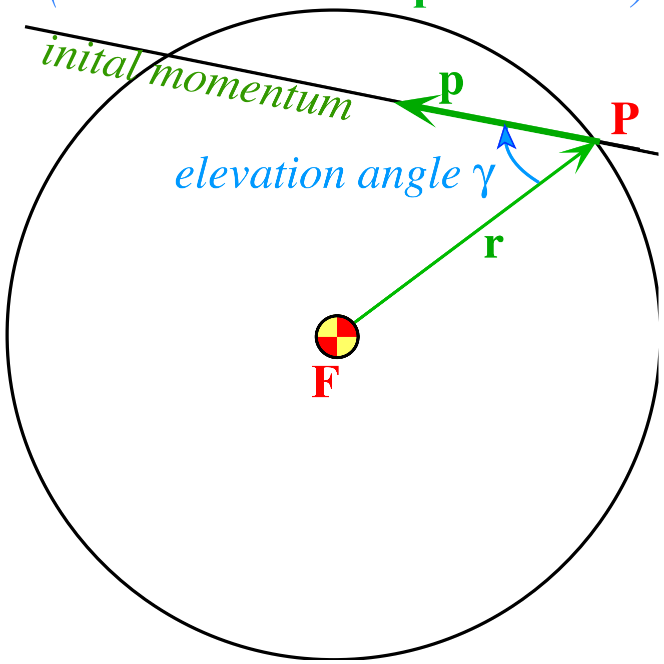
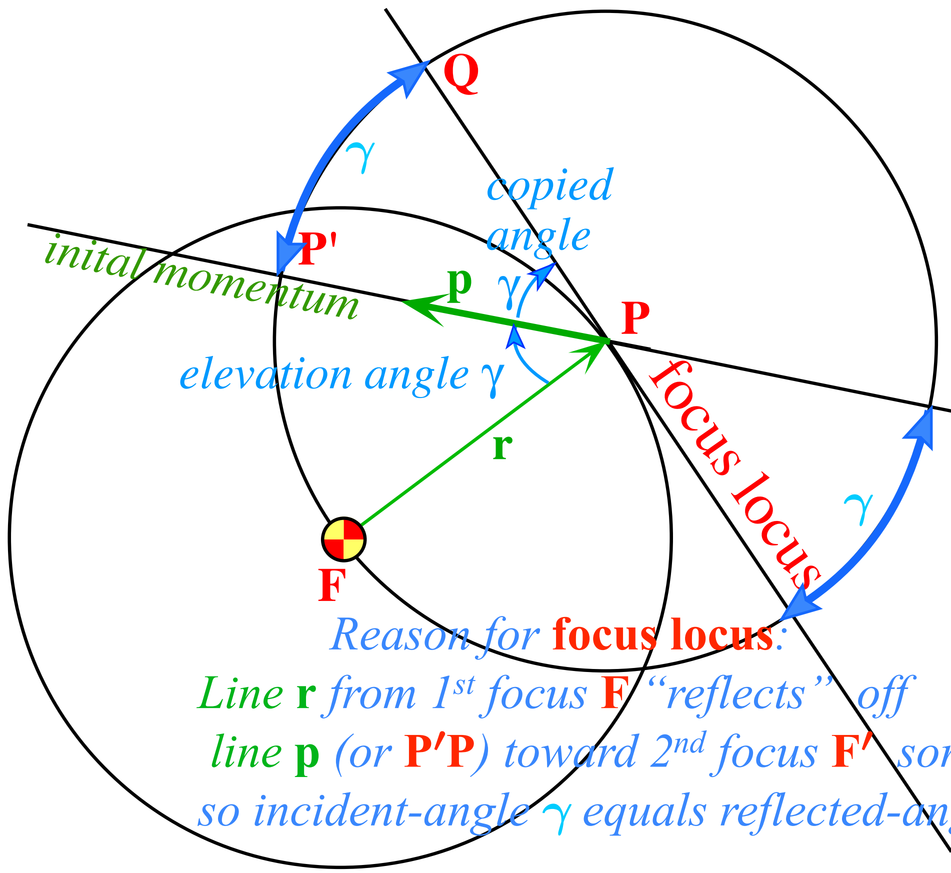


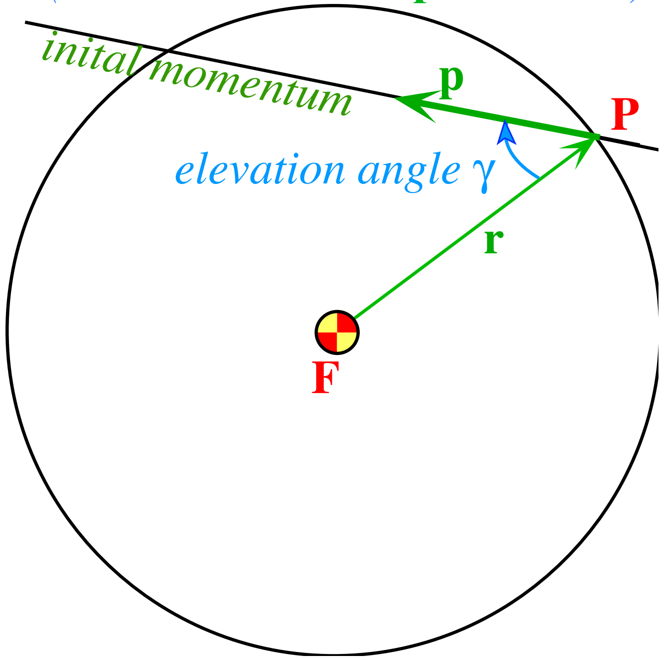
Copy F-center circle around launch point P
Copy elevation angle γ (∠FPP′) onto ∠P′PQ
Extend resulting line QPQ′ to make **focus locus**



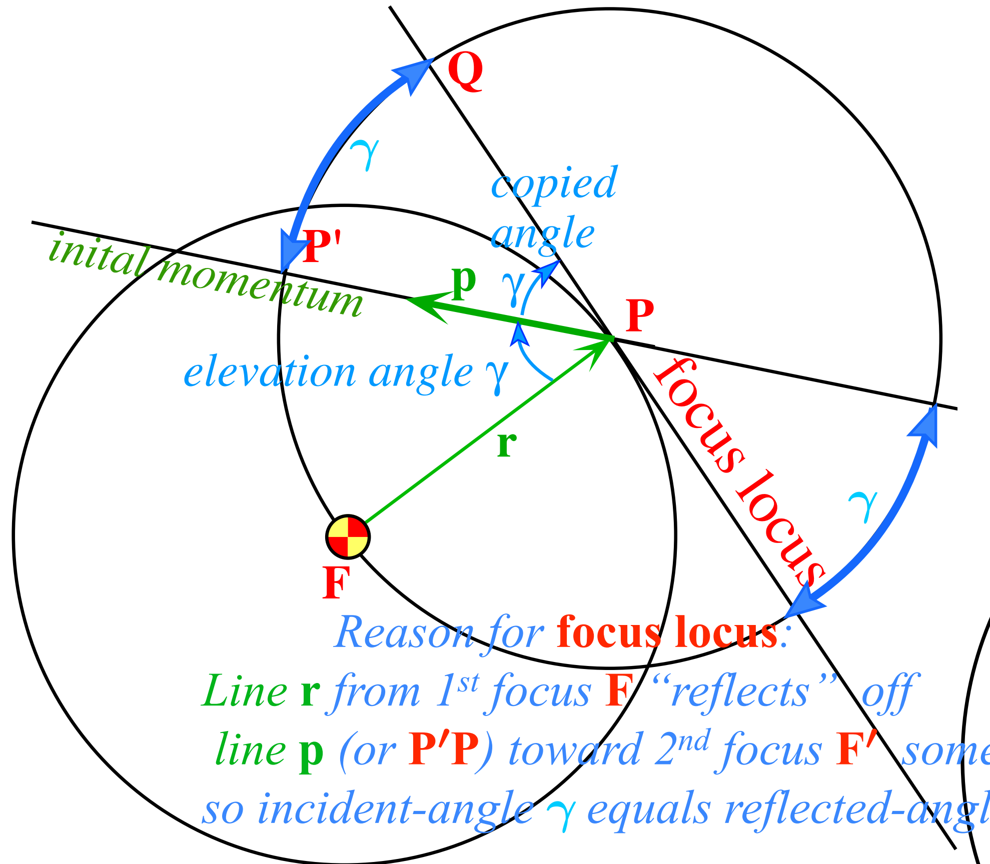
Next several pages give step-by-step constructions of ε-vector and Coulomb orbit and trajectory physics

ε-vector and Coulomb orbit construction steps

Pick launch point P
(radius vector **r** *)*
and elevation angle γ *from radius*
(momentum initial **p** *direction)*

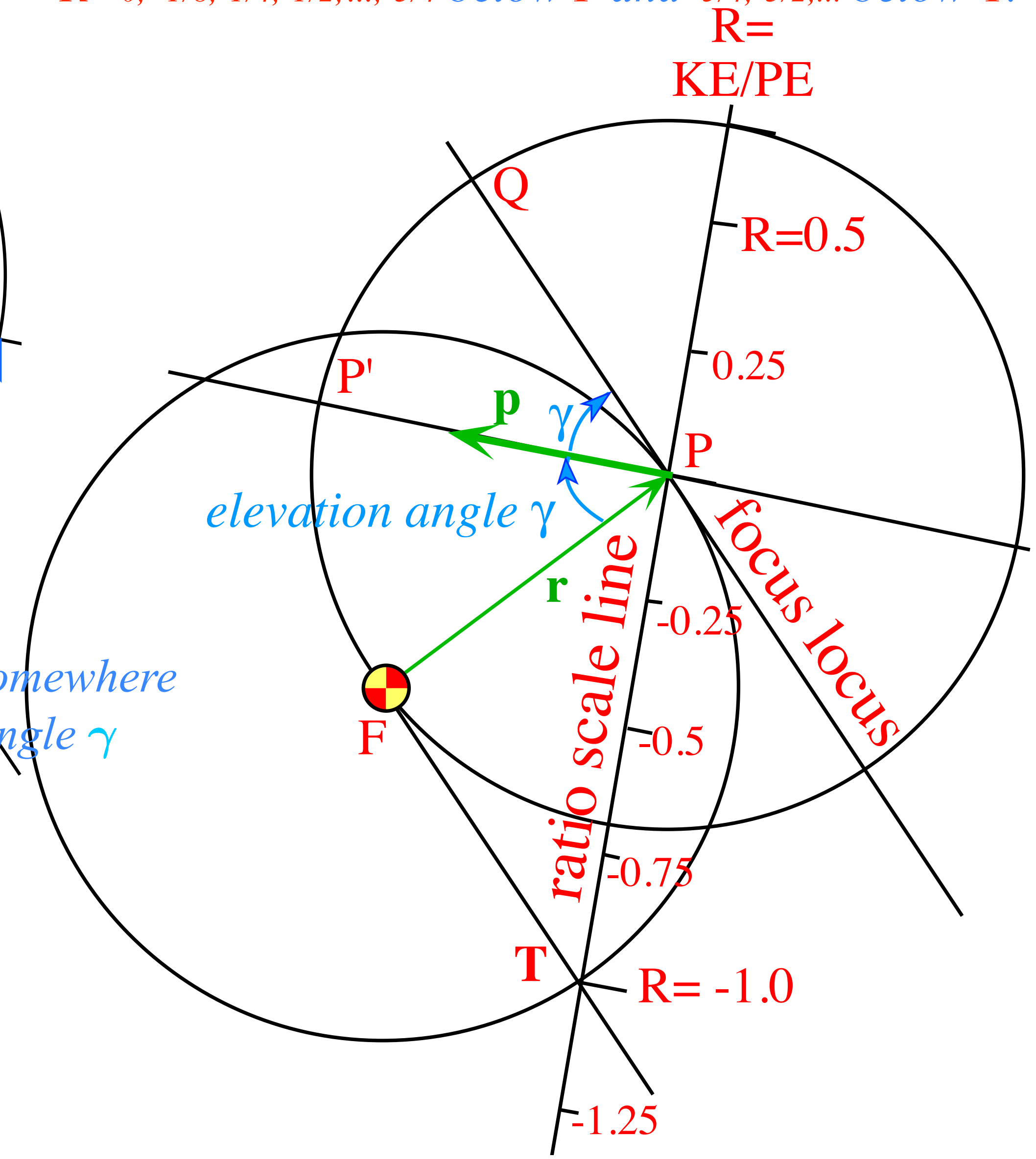


Copy F-center circle around launch point P
Copy elevation angle γ *(∠FPP′) onto ∠P′PQ*
Extend resulting line QPQ′ *to make* **focus locus**



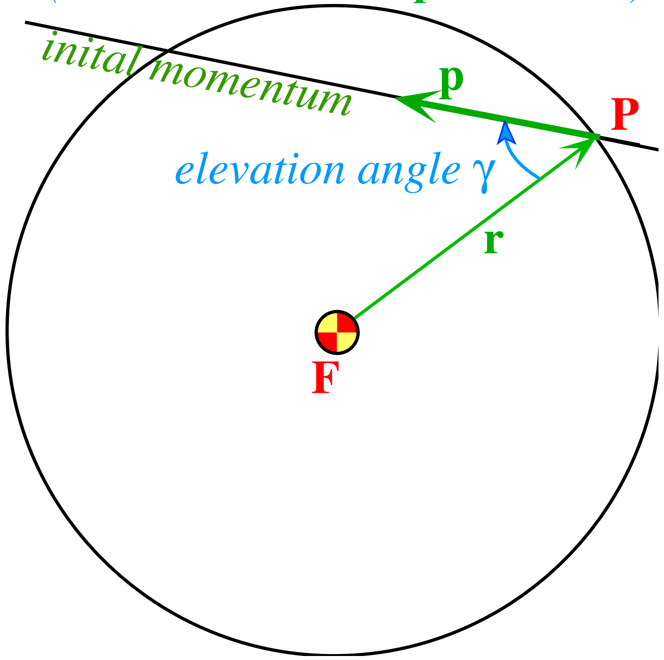
Reason for **focus locus***:*
Line **r** *from 1st focus* **F** *"reflects" off*
line **p** *(or* **P′P***) toward 2nd focus* **F′** *somewhere*
so incident-angle γ *equals reflected-angle* γ

*Copy double angle 2*γ *(∠FPQ) onto ∠PFT*
Extend ∠PFT chord PT *to make R-***ratio scale line**
Label chord PT *with R=0 at* P *and R=-1.0 at* T.
*Mark R-***line** *fractions R=0, +1/4, +1/2,... above* P *and*
R=0, -1/8,-1/4,-1/2,...,-3/4 below P *and -5/4,-3/2,... below* T.

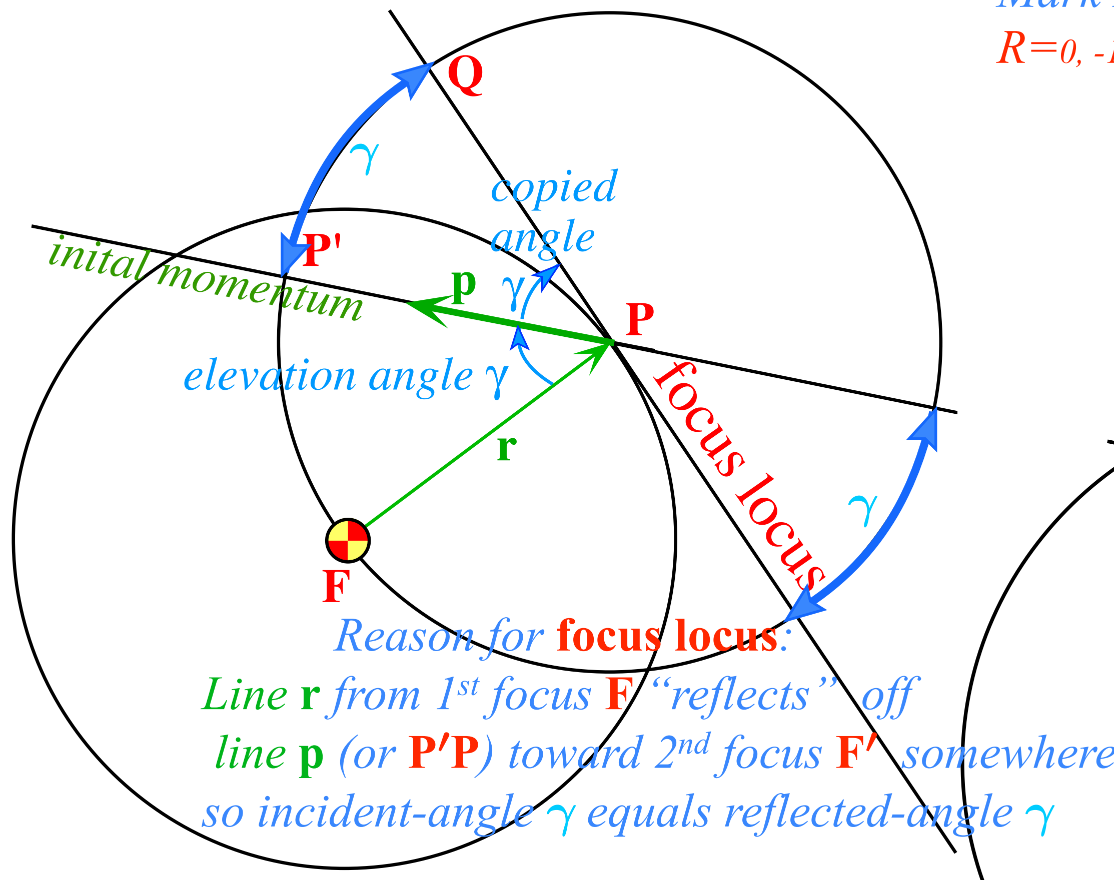


ε-vector and Coulomb orbit construction steps

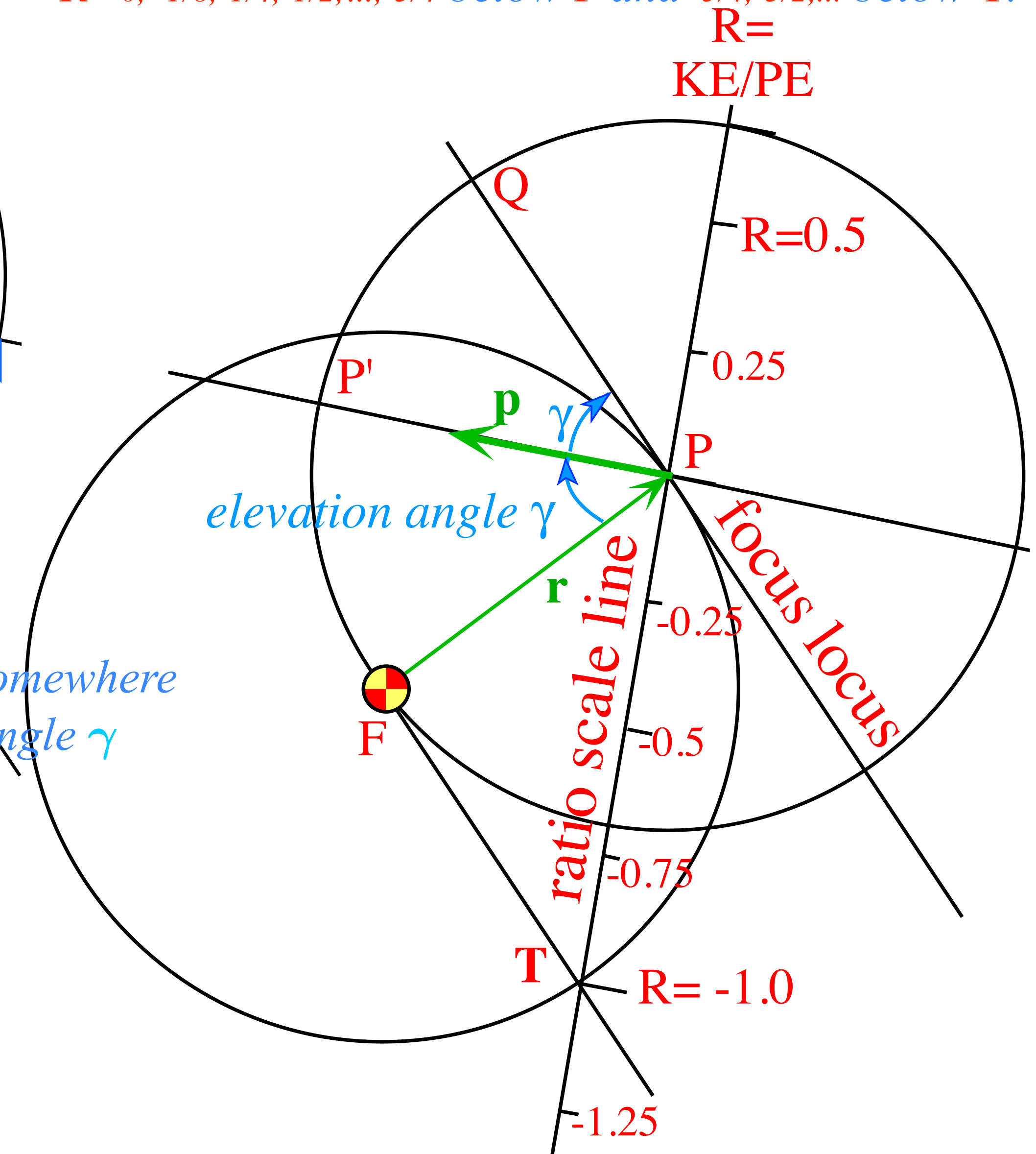
Pick launch point P
(radius vector **r** *)*
and elevation angle γ *from radius*
(momentum initial **p** *direction)*



Copy F-center circle around launch point P
Copy elevation angle γ *(∠FPP′) onto ∠P′PQ*
Extend resulting line QPQ′ *to make* **focus locus**



Copy double angle 2γ *(∠FPQ) onto ∠PFT*
Extend ∠PFT chord PT *to make R-***ratio scale line**
Label chord PT *with R=0 at* P *and R=-1.0 at* T.
*Mark R-***line** *fractions R=0, +1/4, +1/2,... above* P *and*
R=0, -1/8,-1/4,-1/2,...,-3/4 below P *and -5/4,-3/2,... below* T.

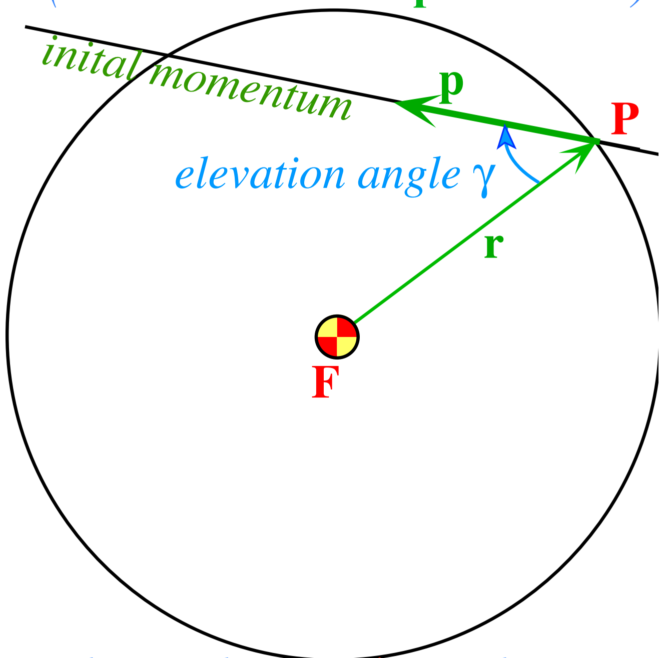


$$R = \frac{\textit{Initial KE}}{\textit{Initial PE}} = \frac{m v^2(0)/2}{-k/r(0)}$$

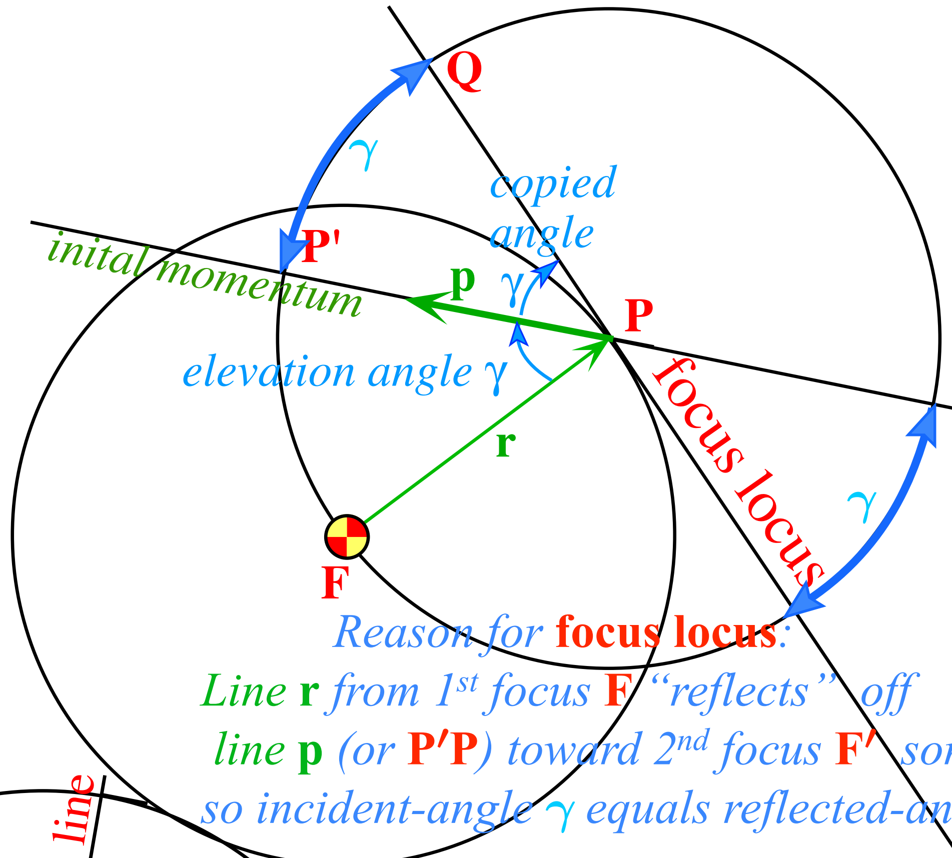
$$= \pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2 = \pm\frac{v^2(0)}{v^2(\infty)}$$

ε-vector and Coulomb orbit construction steps

Pick launch point P
(radius vector **r** *)*
and elevation angle γ from radius
(momentum initial **p** *direction)*

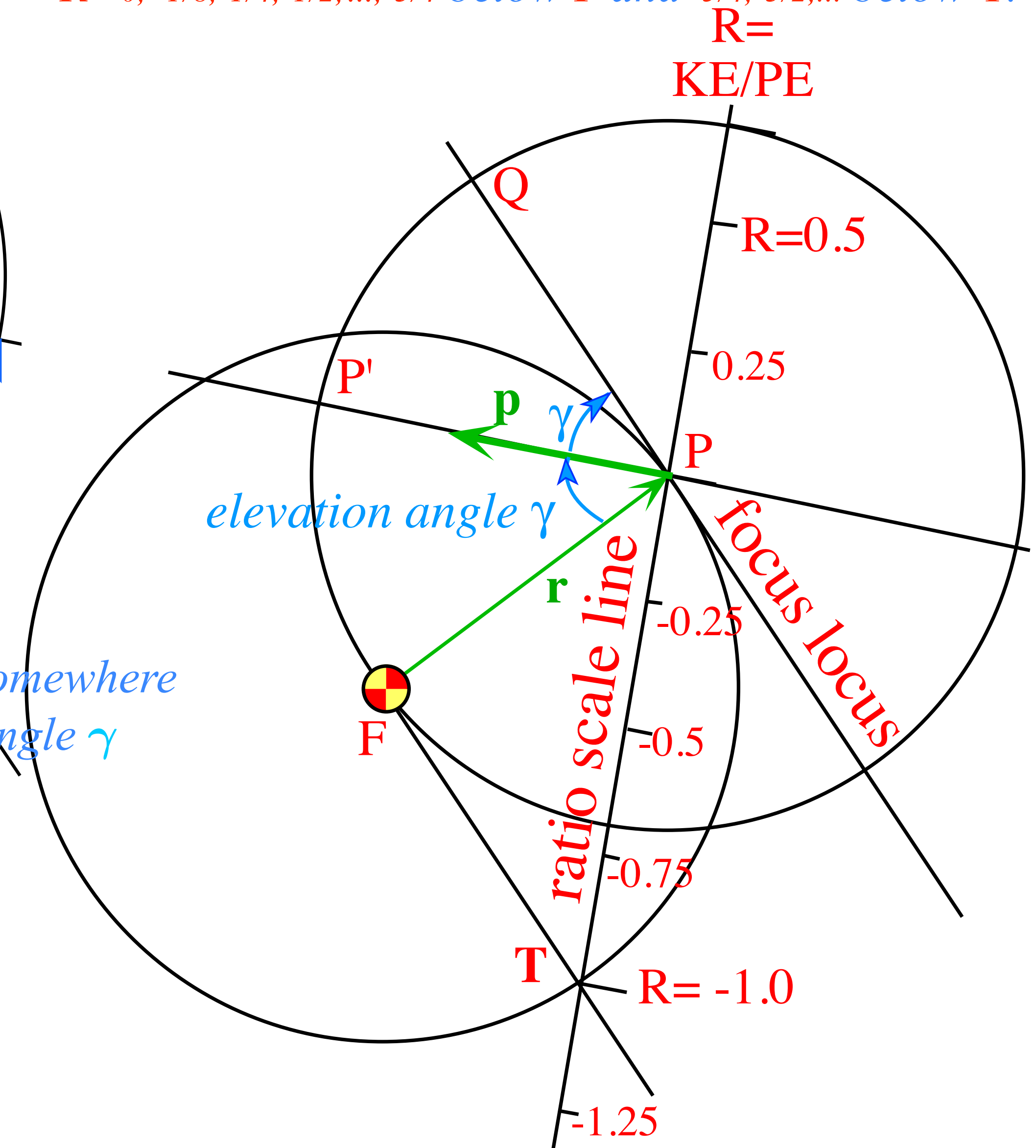


Copy F-center circle around launch point P
Copy elevation angle γ (∠FPP′) onto ∠P′PQ
Extend resulting line QPQ′ to make **focus locus**

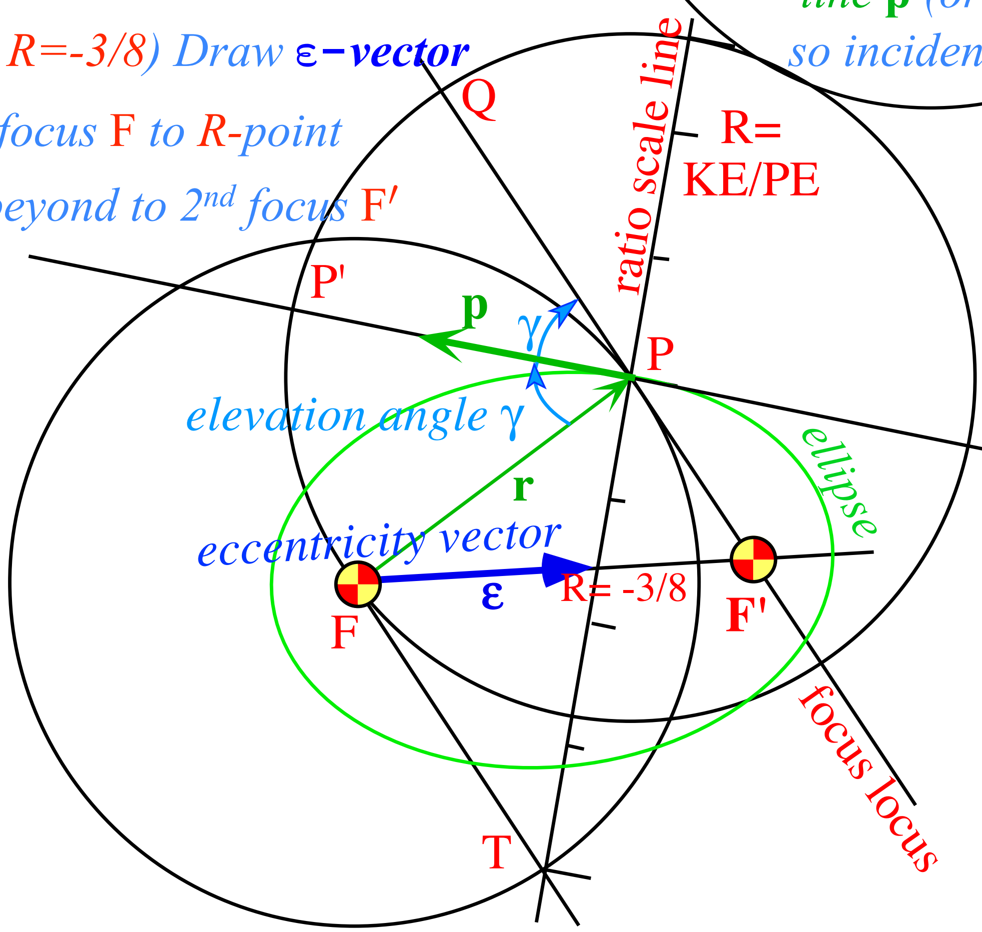


Reason for **focus locus***:*
Line **r** *from 1st focus* **F** *"reflects" off line* **p** *(or* **P′P***) toward 2nd focus* **F′** *somewhere so incident-angle γ equals reflected-angle γ*

Copy double angle 2γ (∠FPQ) onto ∠PFT
*Extend ∠PFT chord PT to make R-***ratio scale line**
Label chord PT with R=0 at P and R=-1.0 at T.
*Mark R-***line** *fractions R=0, +1/4, +1/2,... above P and R=0, -1/8,-1/4,-1/2,...,-3/4 below P and -5/4,-3/2,... below T.*



Pick initial R=KE/PE value
(here R=-3/8) Draw ***ε−vector***
from focus F to R-point
and beyond to 2nd focus F′



focus F and 2nd focus F′ allow final construction of orbital trajectory. Here it is an R=-3/8 ellipse.

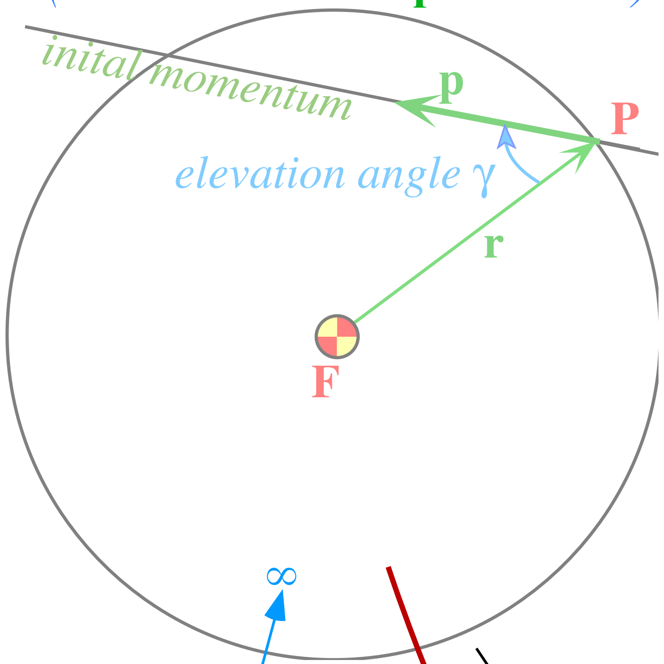
(Detailed Analytic geometry of ***ε-vector*** *follows.)*

$$R=\frac{Initial\ KE}{Initial\ PE}=\frac{mv^2(0)/2}{-k/r(0)}$$

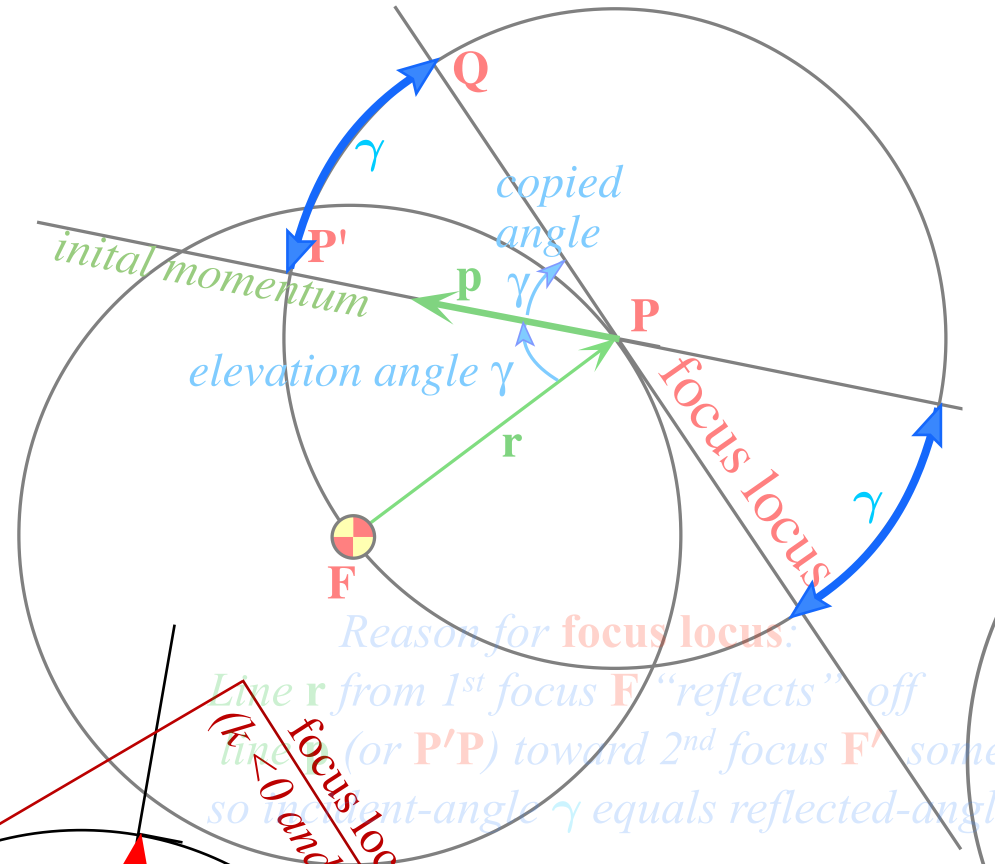
$$=\pm\left(\frac{Initial\ velocity}{Escape\ velocity}\right)^2=\pm\frac{v^2(0)}{v^2(\infty)}$$

ε-vector and Coulomb orbit construction steps

Pick launch point P
(radius vector **r** *)*
and elevation angle γ *from radius*
(momentum initial **p** *direction)*



Copy F-center circle around launch point P
Copy elevation angle γ *(∠FPP′) onto ∠P′PQ*
Extend resulting line QPQ′ *to make* **focus locus**



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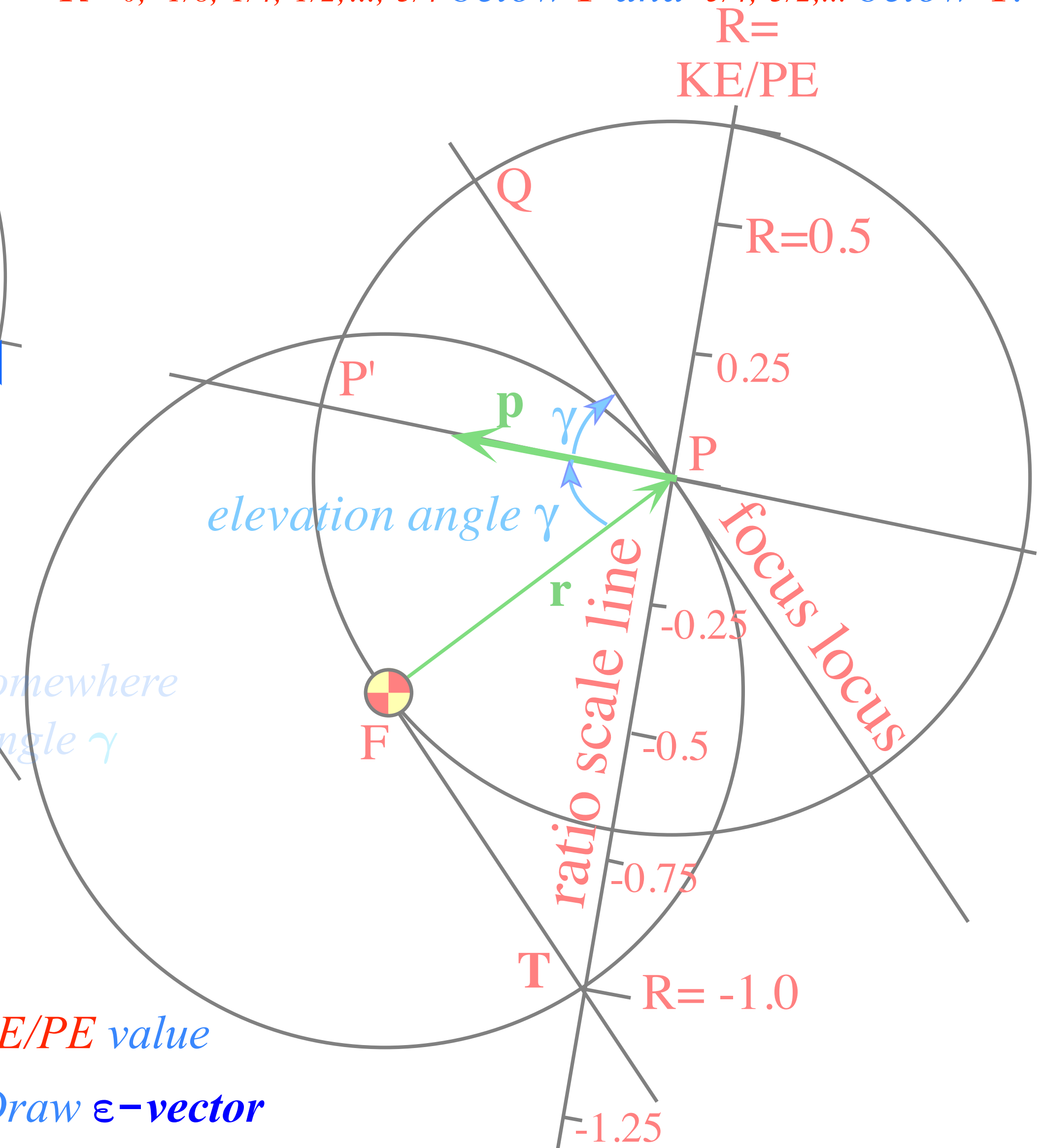
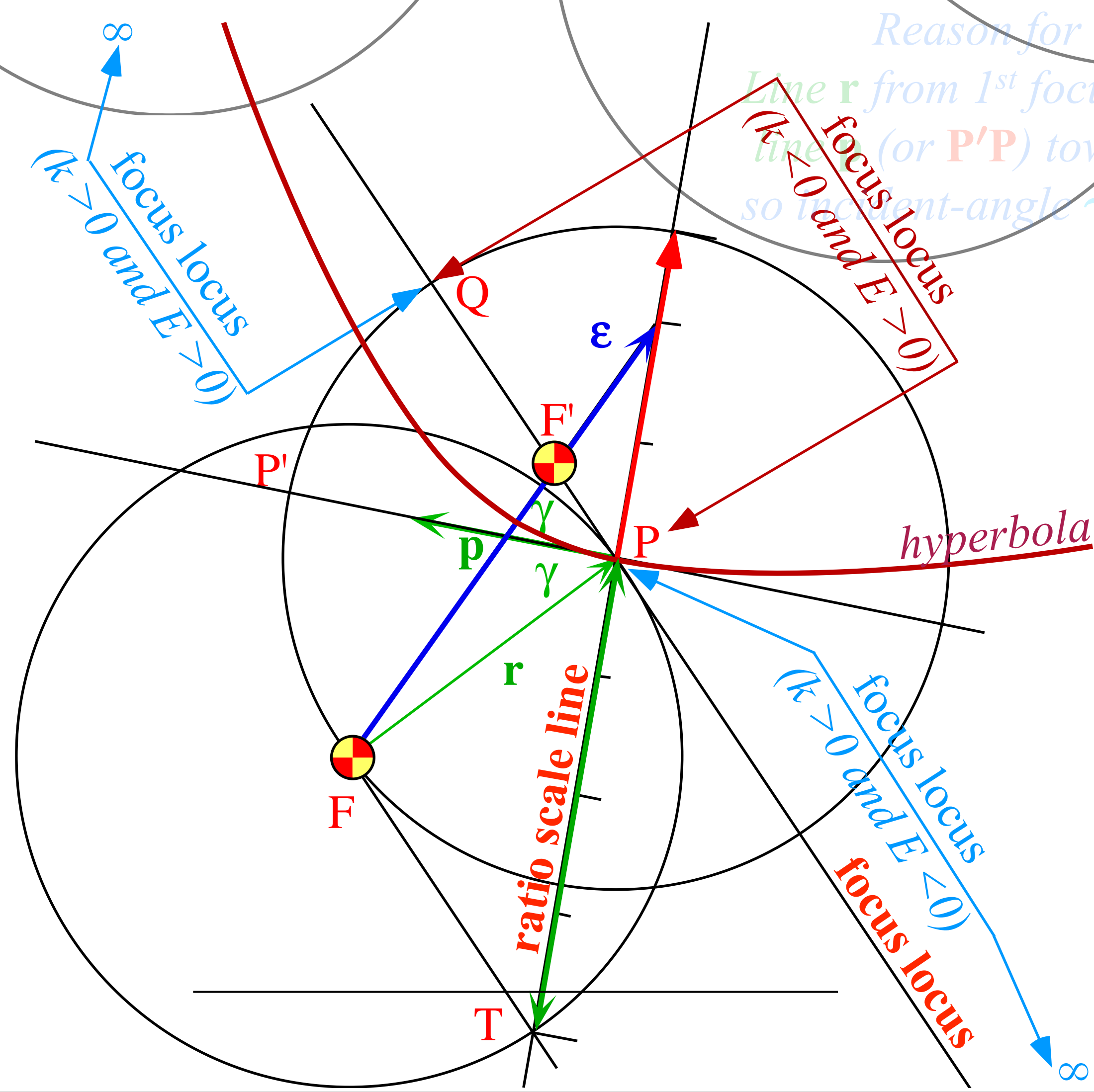




Pick initial R=KE/PE value
(here R=+1/2) Draw ***ε−vector***
from focus F *to R-point*
(Here it intersects 2nd focus F′

focus F *and 2nd focus* F′ *allow final construction of orbital trajectory. Here it is an R=+1/2 hyperbola.*

$$R=\frac{\textit{Initial KE}}{\textit{Initial PE}}=\frac{mv^2(0)/2}{-k/r(0)}$$

$$=\pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2=\pm\frac{v^2(0)}{v^2(\infty)}$$

(Detailed Analytic geometry of ***ε-vector*** *follows.)*

Eccentricity vector $\boldsymbol{\varepsilon}$ *and* (ε,λ)*-geometry of orbital mechanics*

$\boldsymbol{\varepsilon}$*-vector and Coulomb* $\mathbf{r}$*-orbit geometry*

Review and connection to standard development

$\boldsymbol{\varepsilon}$*-vector and Coulomb* $\mathbf{p}=m\mathbf{v}$ *geometry*

$\boldsymbol{\varepsilon}$*-vector and Coulomb* $\mathbf{p}=m\mathbf{v}$ *algebra*

Example with elliptical orbit

Analytic geometry derivation of $\boldsymbol{\varepsilon}$*-construction*

➔ *Algebra of* $\boldsymbol{\varepsilon}$*-construction geometry*

Connection formulas for (a,b) and (ε,λ) *with* (γ,R)

Ruler & compass construction of $\boldsymbol{\varepsilon}$*-vector and orbits*

(R=-0.375 elliptic orbit)

(R=+0.5 hyperbolic orbit)

Analytic geometry derivation of ε-constructions

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} - \frac{\mathbf{p}\times\mathbf{L}}{km} = \hat{\mathbf{r}} - \frac{(mv_0)(mv_0r_0)\sin\gamma}{km}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

where: $\mathbf{L}_{\mathbf{p}\times} \equiv \mathbf{p}\times\mathbf{L}$

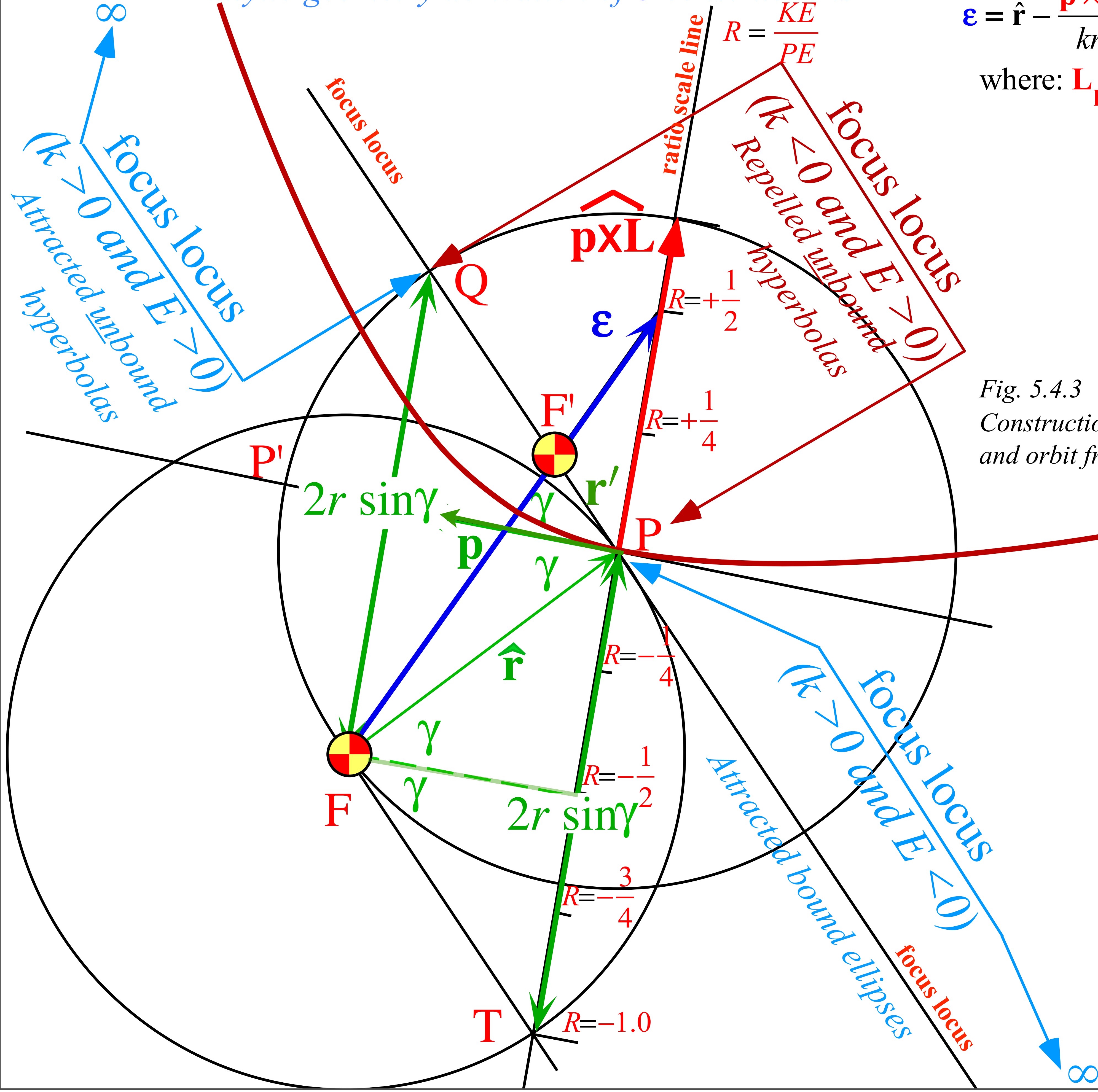


Fig. 5.4.3
Construction of eccentricity vector ε
and orbit from initial **r**, **p** *with KE/PE=+1/2.*

$$R = \frac{\textit{Initial KE}}{\textit{Initial PE}} = \frac{m\,v^2(0)/2}{-k/r(0)}$$

$$= \pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2 = \pm\frac{v^2(0)}{v^2(\infty)}$$

Analytic geometry derivation of ε-constructions

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} - \frac{\mathbf{p}\times\mathbf{L}}{km} = \hat{\mathbf{r}} - \frac{(mv_0)(mv_0 r_0)\sin\gamma}{km}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

where: $\mathbf{L}_{\mathbf{p}\times} \equiv \mathbf{p}\times\mathbf{L}$

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} + 2\sin\gamma\,\frac{mv_0^{\,2}/2}{-k/r_0}\hat{\mathbf{L}}_{\mathbf{p}\times} = \hat{\mathbf{r}} + 2\sin\gamma\,\frac{KE}{PE}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

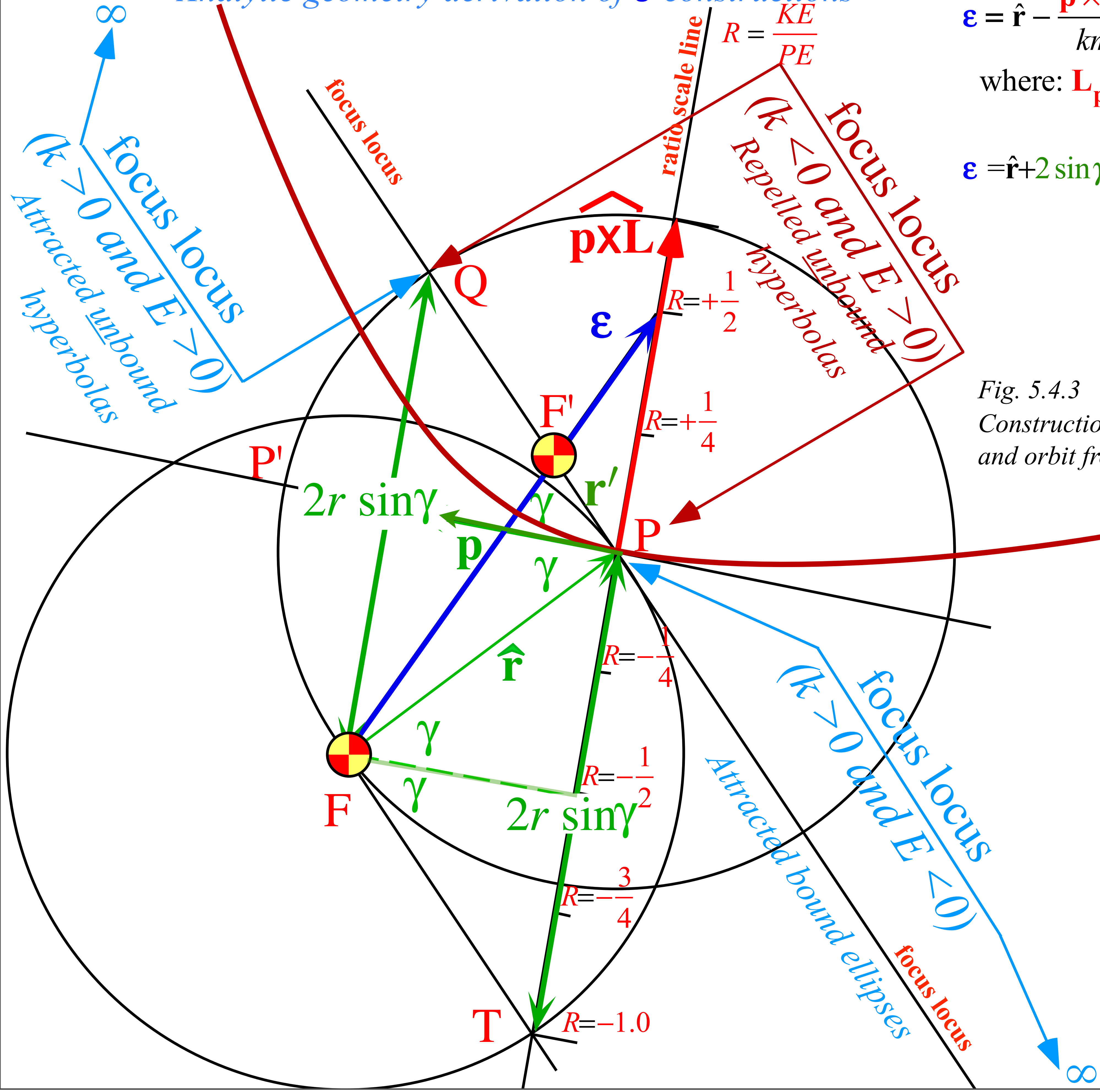
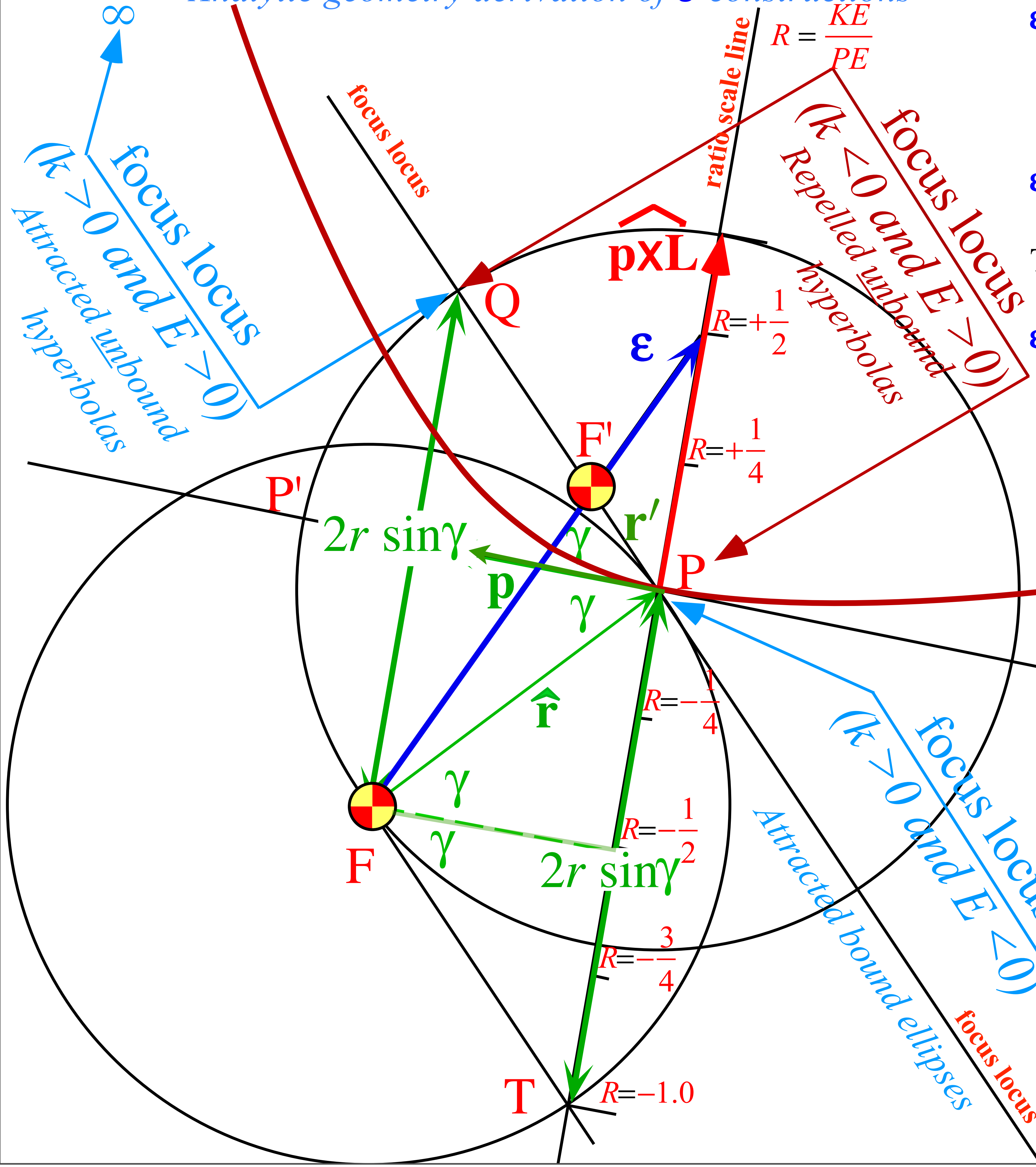


Fig. 5.4.3
Construction of eccentricity vector ε
and orbit from initial **r**, **p** *with KE/PE=+1/2.*

$$R = \frac{\textit{Initial KE}}{\textit{Initial PE}} = \frac{m\,v^2(0)/2}{-k/r(0)} = \pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2 = \pm\frac{v^2(0)}{v^2(\infty)}$$

Analytic geometry derivation of ε-constructions



$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} - \frac{\mathbf{p}\times\mathbf{L}}{km} = \hat{\mathbf{r}} - \frac{(mv_0)(mv_0 r_0)\sin\gamma}{km}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

where: $\mathbf{L}_{\mathbf{p}\times} \equiv \mathbf{p}\times\mathbf{L}$

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} + 2\sin\gamma\,\frac{mv_0^{\,2}/2}{-k/r_0}\hat{\mathbf{L}}_{\mathbf{p}\times} = \hat{\mathbf{r}} + 2\sin\gamma\,\frac{KE}{PE}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

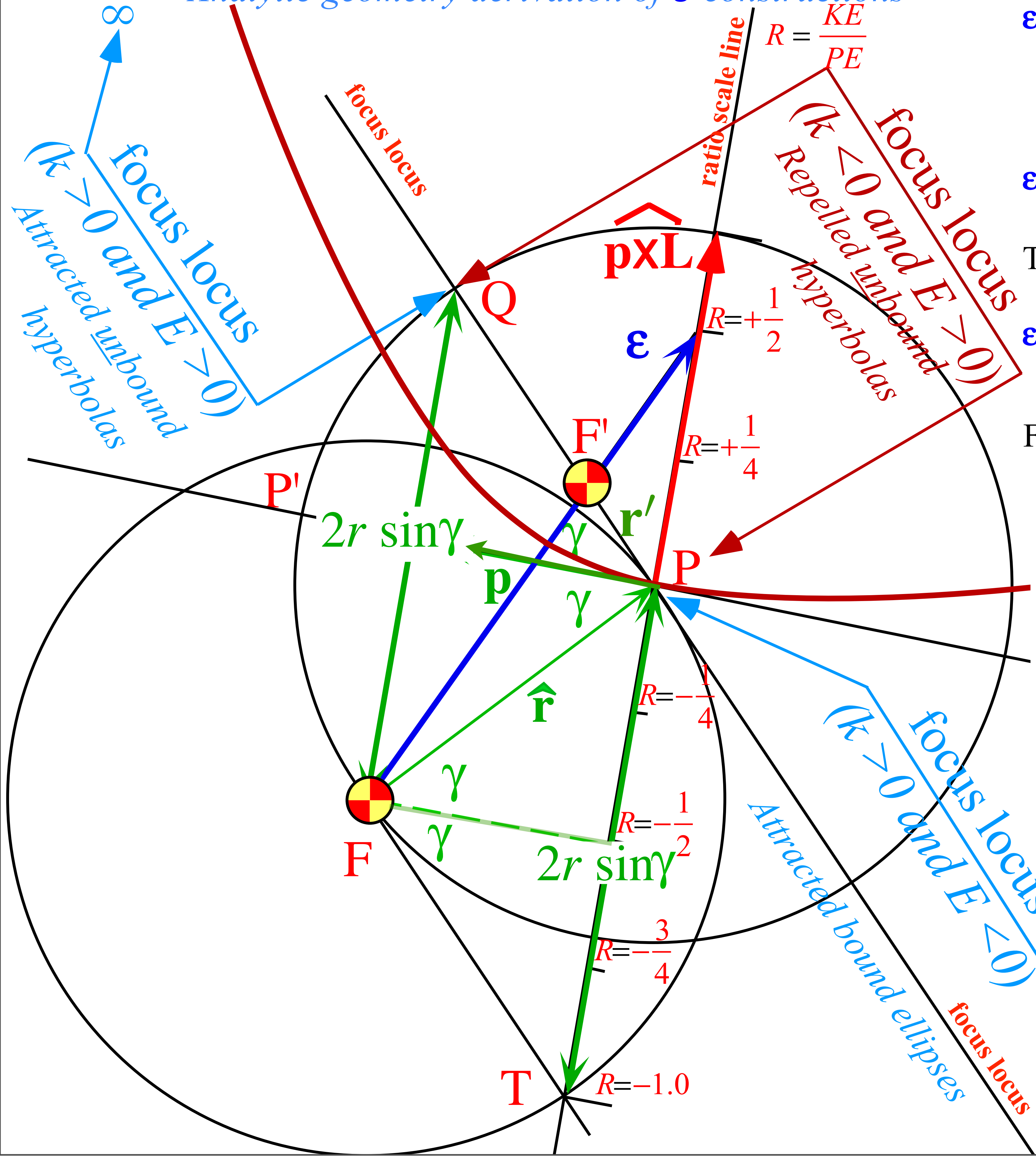
The *eccentricty* vector is:

$$\boldsymbol{\varepsilon} = \begin{pmatrix}\cos\gamma\\ \sin\gamma\end{pmatrix} + 2\sin\gamma\begin{pmatrix}0\\1\end{pmatrix}R = \begin{pmatrix}\cos\gamma\\ (2R+1)\sin\gamma\end{pmatrix}$$

$$R = \frac{\textit{Initial KE}}{\textit{Initial PE}} = \frac{m\,v^2(0)/2}{-k/r(0)}$$

$$= \pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2 = \pm\frac{v^2(0)}{v^2(\infty)}$$

Analytic geometry derivation of ε-constructions



$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} - \frac{\mathbf{p}\times\mathbf{L}}{km} = \hat{\mathbf{r}} - \frac{(mv_0)(mv_0 r_0)\sin\gamma}{km}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

where: $\mathbf{L}_{\mathbf{p}\times} \equiv \mathbf{p}\times\mathbf{L}$

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} + 2\sin\gamma\,\frac{mv_0^{\,2}/2}{-k/r_0}\hat{\mathbf{L}}_{\mathbf{p}\times} = \hat{\mathbf{r}} + 2\sin\gamma\,\frac{KE}{PE}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

The *eccentricty* vector is:

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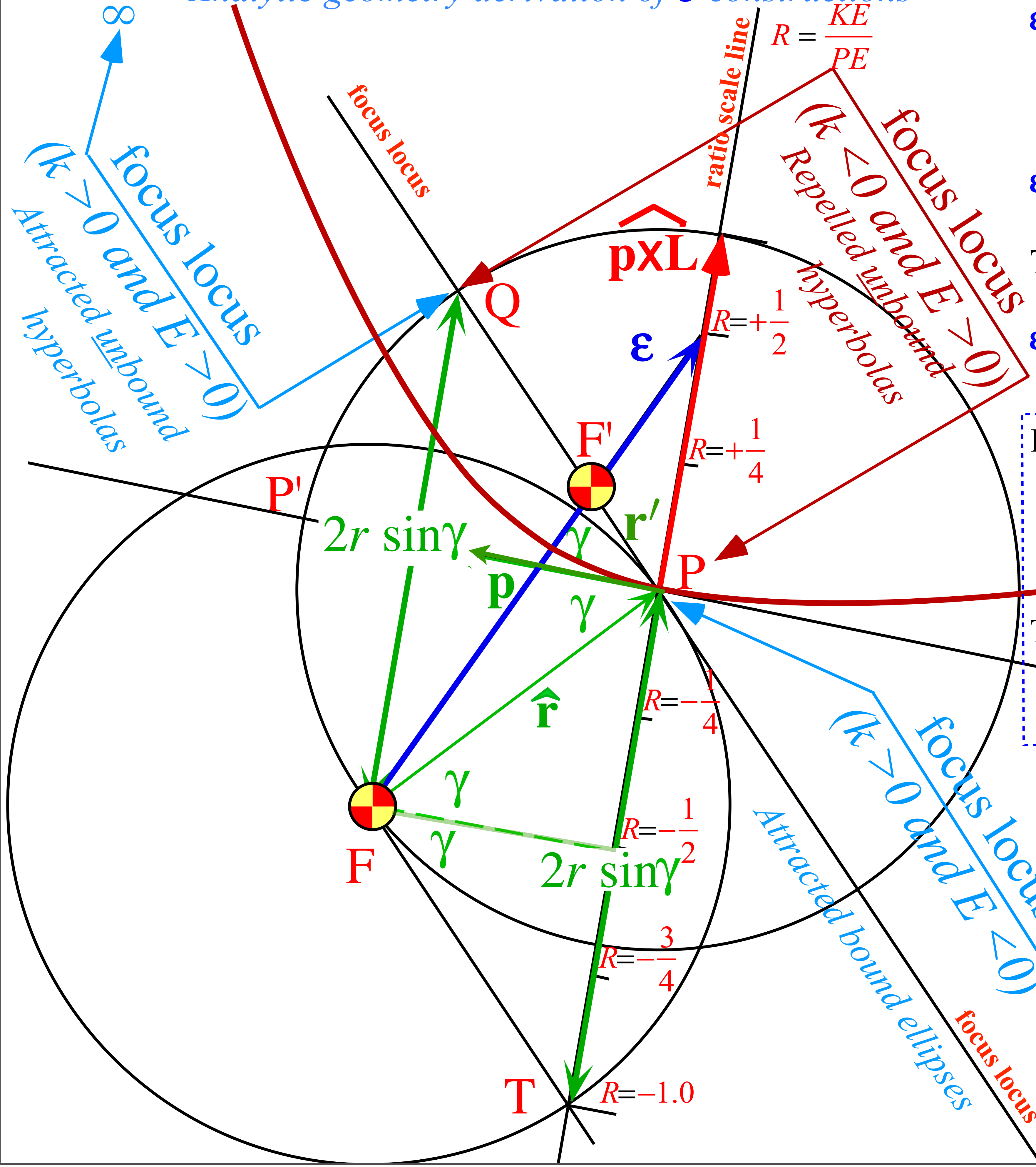
For: $\gamma = 45°$ and: $R = +\frac{1}{2}$

$$\boldsymbol{\varepsilon} = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\,(2R+1)\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ 2/\sqrt{2}\end{pmatrix},$$

$$R = \frac{\textit{Initial KE}}{\textit{Initial PE}} = \frac{m\,v^2(0)/2}{-k/r(0)}$$

$$= \pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2 = \pm\frac{v^2(0)}{v^2(\infty)}$$

Analytic geometry derivation of ε-constructions



$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} - \frac{\mathbf{p}\times\mathbf{L}}{km} = \hat{\mathbf{r}} - \frac{(mv_0)(mv_0 r_0)\sin\gamma}{km}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

where: $\mathbf{L}_{\mathbf{p}\times} \equiv \mathbf{p}\times\mathbf{L}$

$$\boldsymbol{\varepsilon} = \hat{\mathbf{r}} + 2\sin\gamma \frac{mv_0^2/2}{-k/r_0}\hat{\mathbf{L}}_{\mathbf{p}\times} = \hat{\mathbf{r}} + 2\sin\gamma \frac{KE}{PE}\hat{\mathbf{L}}_{\mathbf{p}\times}$$

The *eccentricity* vector is:

$$\boldsymbol{\varepsilon} = \begin{pmatrix}\cos\gamma\\ \sin\gamma\end{pmatrix} + 2\sin\gamma\begin{pmatrix}0\\1\end{pmatrix}R = \begin{pmatrix}\cos\gamma\\ (2R+1)\sin\gamma\end{pmatrix}$$

For: $\gamma = 45°$ and: $R = +\frac{1}{2}$

$$\boldsymbol{\varepsilon} = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}(2R+1)\end{pmatrix} = \begin{pmatrix}1/\sqrt{2}\\ 2/\sqrt{2}\end{pmatrix},$$

The *eccentricity* parameter defined by:

$$\varepsilon^2 = \cos^2\gamma + (2R+1)^2\sin^2\gamma = 1 \pm \frac{a^2}{b^2}$$

$$= 1 + 4R(R+1)\sin^2\gamma = \frac{5}{2}$$

$$R = \frac{\textit{Initial KE}}{\textit{Initial PE}} = \frac{m v^2(0)/2}{-k/r(0)}$$

$$= \pm\left(\frac{\textit{Initial velocity}}{\textit{Escape velocity}}\right)^2 = \pm\frac{v^2(0)}{v^2(\infty)}$$

Eccentricity vector $\boldsymbol{\varepsilon}$ *and* (ε,λ)*-geometry of orbital mechanics*

- $\boldsymbol{\varepsilon}$*-vector and Coulomb* **r***-orbit geometry*
  - *Review and connection to standard development*
- $\boldsymbol{\varepsilon}$*-vector and Coulomb* **p**$=m$**v** *geometry*
- $\boldsymbol{\varepsilon}$*-vector and Coulomb* **p**$=m$**v** *algebra*
  - *Example with elliptical orbit*

Analytic geometry derivation of $\boldsymbol{\varepsilon}$*-construction*

- *Algebra of* $\boldsymbol{\varepsilon}$*-construction geometry*

➔ *Connection formulas for (a,b) and* (ε,λ) *with* (γ,R)

Ruler & compass construction of $\boldsymbol{\varepsilon}$*-vector and orbits*

(R=-0.375 elliptic orbit)

(R=+0.5 hyperbolic orbit)

Algebra of ε-construction geometry

Three pairs of parameters for Coulomb orbits:

1.Cartesian (a,b), 2.Physics (E,L), 3.Polar (ε,λ)

Now we relate a 4th pair: 4.Initial (γ,R)

The *eccentricty* parameter relates ratios $R = \frac{KE}{PE}$ and $\frac{b^2}{a^2}$

$$\varepsilon^2 = 1 + 4R(R+1)\sin^2\gamma$$

$$= 1 - \frac{b^2}{a^2} \quad \text{for ellipse} \quad (\varepsilon < 1)$$

$$= 1 + \frac{b^2}{a^2} \quad \text{for hyperbola} \quad (\varepsilon > 1)$$

Algebra of ε-construction geometry

Three pairs of parameters for Coulomb orbits:
1.Cartesian (a,b), 2.Physics (E,L), 3.Polar (ε,λ)
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$$\varepsilon^2 = 1 + 4R(R+1)\sin^2\gamma$$

$$= 1 - \frac{b^2}{a^2} \quad \text{for ellipse} \quad (\varepsilon < 1) \quad \text{where:} \quad 4R(R+1)\sin^2\gamma = -\frac{b^2}{a^2} = \varepsilon^2 - 1$$

$$= 1 + \frac{b^2}{a^2} \quad \text{for hyperbola} \quad (\varepsilon > 1) \quad \text{where:} \quad 4R(R+1)\sin^2\gamma = +\frac{b^2}{a^2} = \varepsilon^2 - 1$$

Algebra of ε-construction geometry

Three pairs of parameters for Coulomb orbits:
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$$\varepsilon^2 = 1 + 4R(R+1)\sin^2\gamma$$

$= 1 - \frac{b^2}{a^2}$ for ellipse ($\varepsilon < 1$) where: $4R(R+1)\sin^2\gamma = -\frac{b^2}{a^2} = \varepsilon^2 - 1$ implying: $R(R+1) < 0$

$= 1 + \frac{b^2}{a^2}$ for hyperbola ($\varepsilon > 1$) where: $4R(R+1)\sin^2\gamma = +\frac{b^2}{a^2} = \varepsilon^2 - 1$ implying: $R(R+1) > 0$

Algebra of ε-construction geometry

Three pairs of parameters for Coulomb orbits:
1.Cartesian (a,b), 2.Physics (E,L), 3.Polar (ε,λ)
Now we relate a 4th pair: 4.Initial (γ,R)

The *eccentricty* parameter relates ratios $R=\frac{KE}{PE}$ and $\frac{b^2}{a^2}$

$$\varepsilon^2 = 1+4R(R+1)\sin^2\gamma$$

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(or: $-R^2>R$)
(or: $0>R>-1$)

$$= 1+\frac{b^2}{a^2} \quad \text{for hyperbola} \quad (\varepsilon>1) \quad \text{where:} \quad 4R(R+1)\sin^2\gamma = +\frac{b^2}{a^2} = \varepsilon^2 - 1 \quad \text{implying:} \quad R(R+1)>0$$

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Algebra of ε-construction geometry

Three pairs of parameters for Coulomb orbits:

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(or: $-R^2 > R$)
(or: $0 > R > -1$)

$$= 1 + \frac{b^2}{a^2} \quad \text{for hyperbola} \quad (\varepsilon > 1) \quad \text{where:} \quad 4R(R+1)\sin^2\gamma = +\frac{b^2}{a^2} = \varepsilon^2 - 1 \quad \text{implying:} \quad R(R+1) > 0$$

(or: $-R^2 < R$)
(or: $0 < R < -1$)

Total $\frac{-k}{2a} = E = energy = KE + PE$ relates ratio $R = \frac{KE}{PE}$ to individual radii a, b, and λ.

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Algebra of ε-construction geometry

Three pairs of parameters for Coulomb orbits:
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(or: $-R^2>R$)
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Latus radius is similarly related:

$$\lambda = \frac{b^2}{a} = \mp 2rR\sin^2\gamma$$

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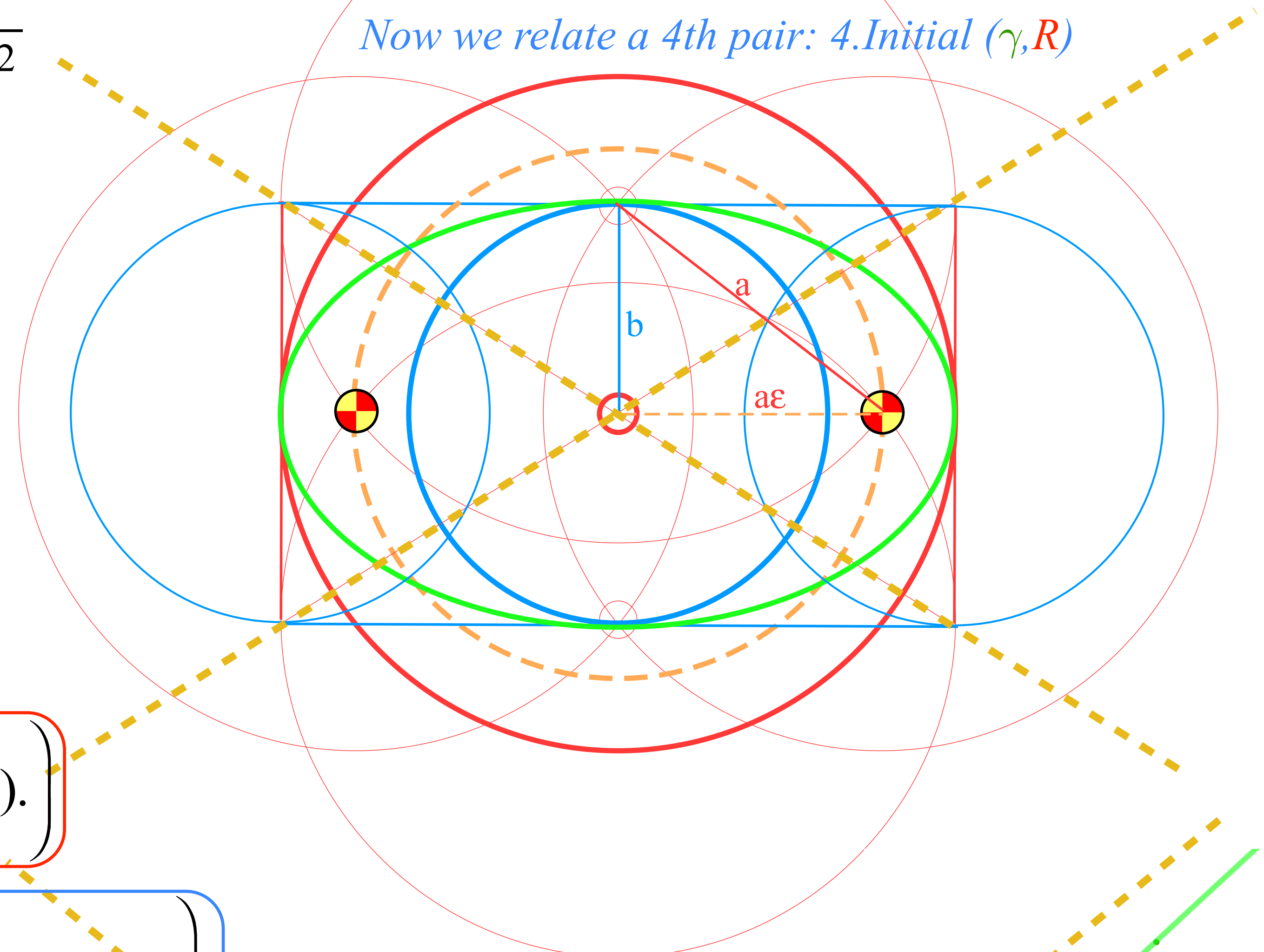

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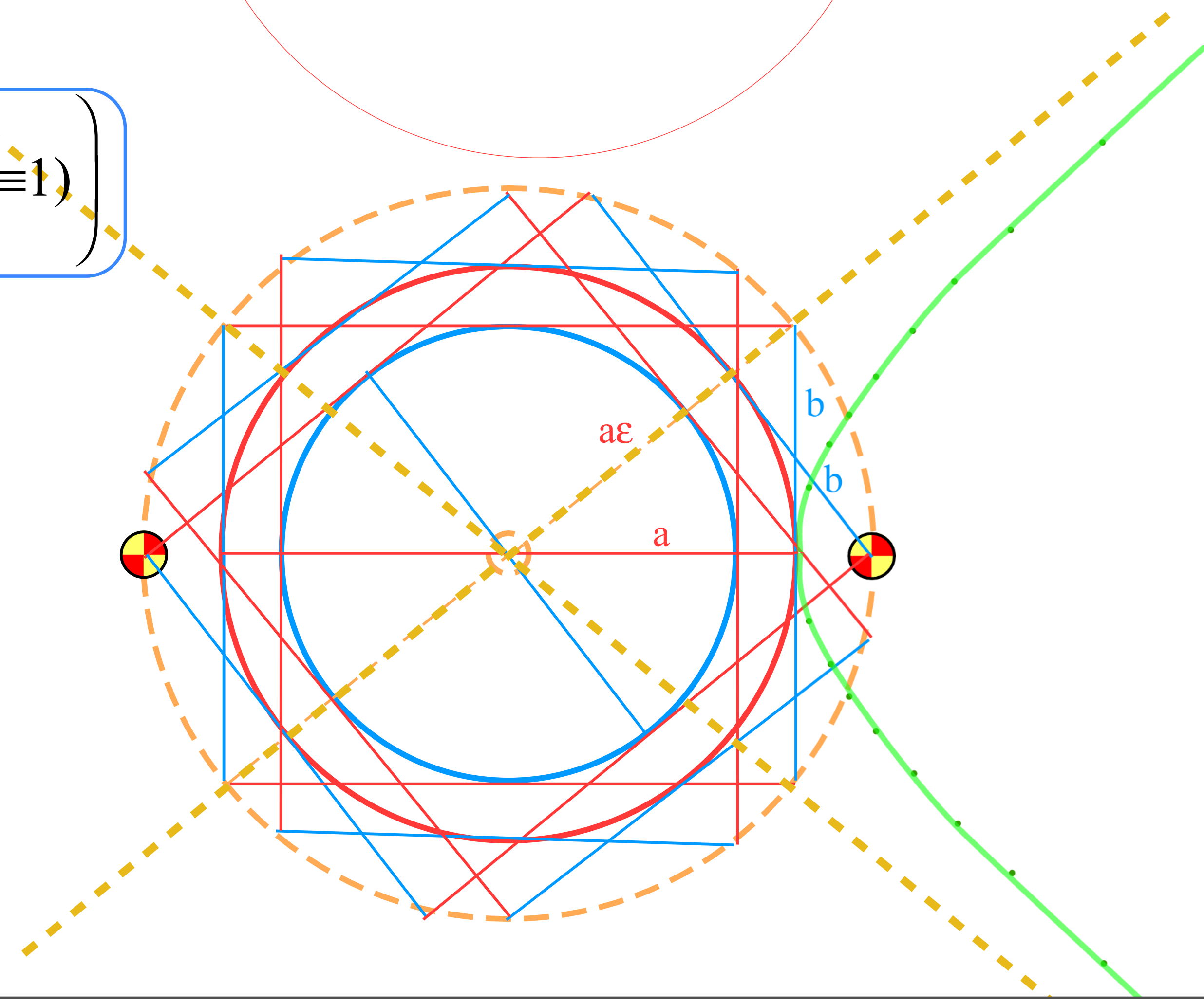
From ε^2 result (at top):

$$\frac{b}{a}=2\sqrt{\mp R(R+1)}\sin\gamma=\sqrt{\pm(1-\varepsilon^2)}$$



Eccentricity vector $\boldsymbol{\varepsilon}$ *and* (ε,λ)*-geometry of orbital mechanics*

$\boldsymbol{\varepsilon}$-vector and Coulomb **r***-orbit geometry*

Review and connection to standard development

$\boldsymbol{\varepsilon}$-vector and Coulomb $\mathbf{p}=m\mathbf{v}$ *geometry*

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Example with elliptical orbit

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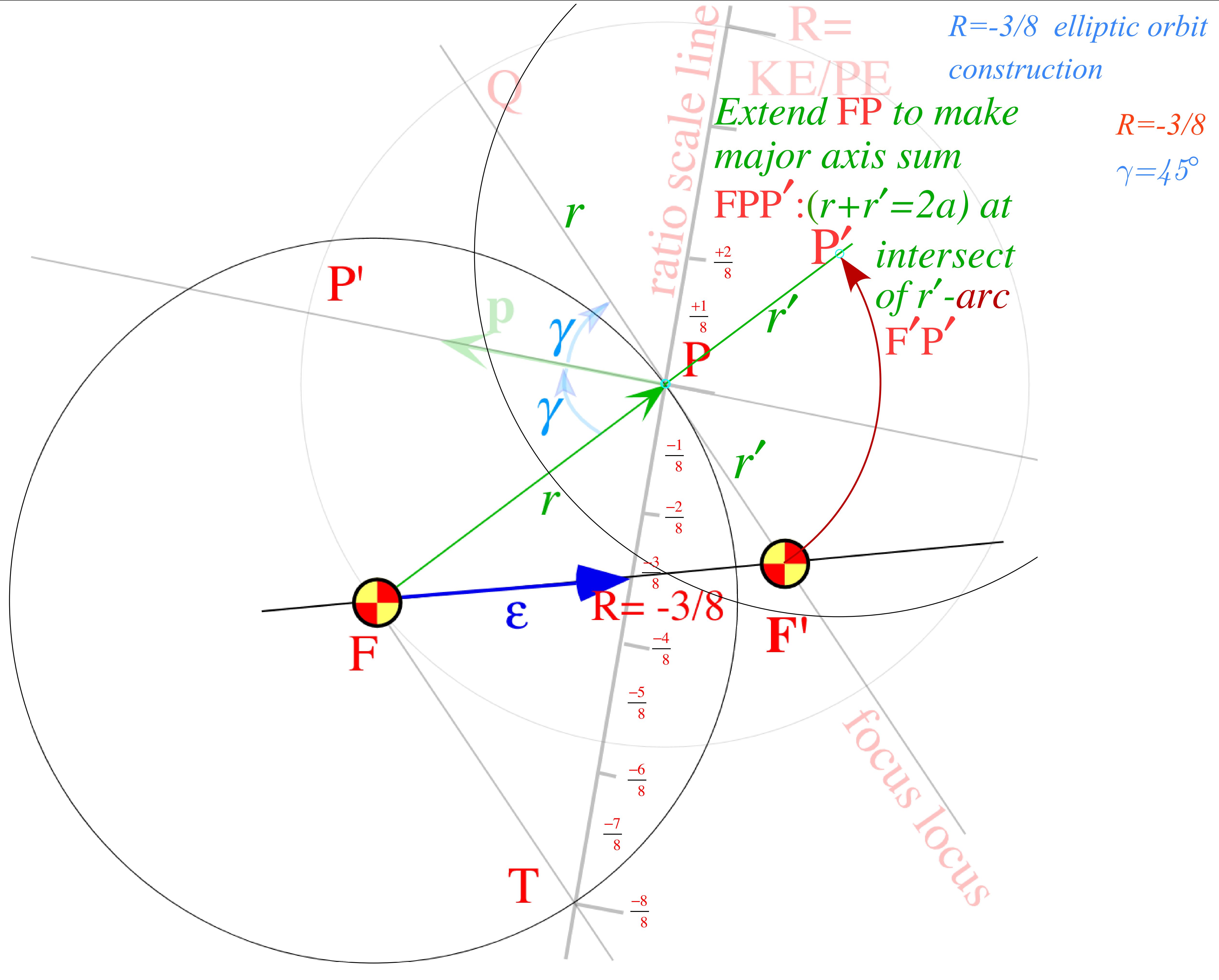
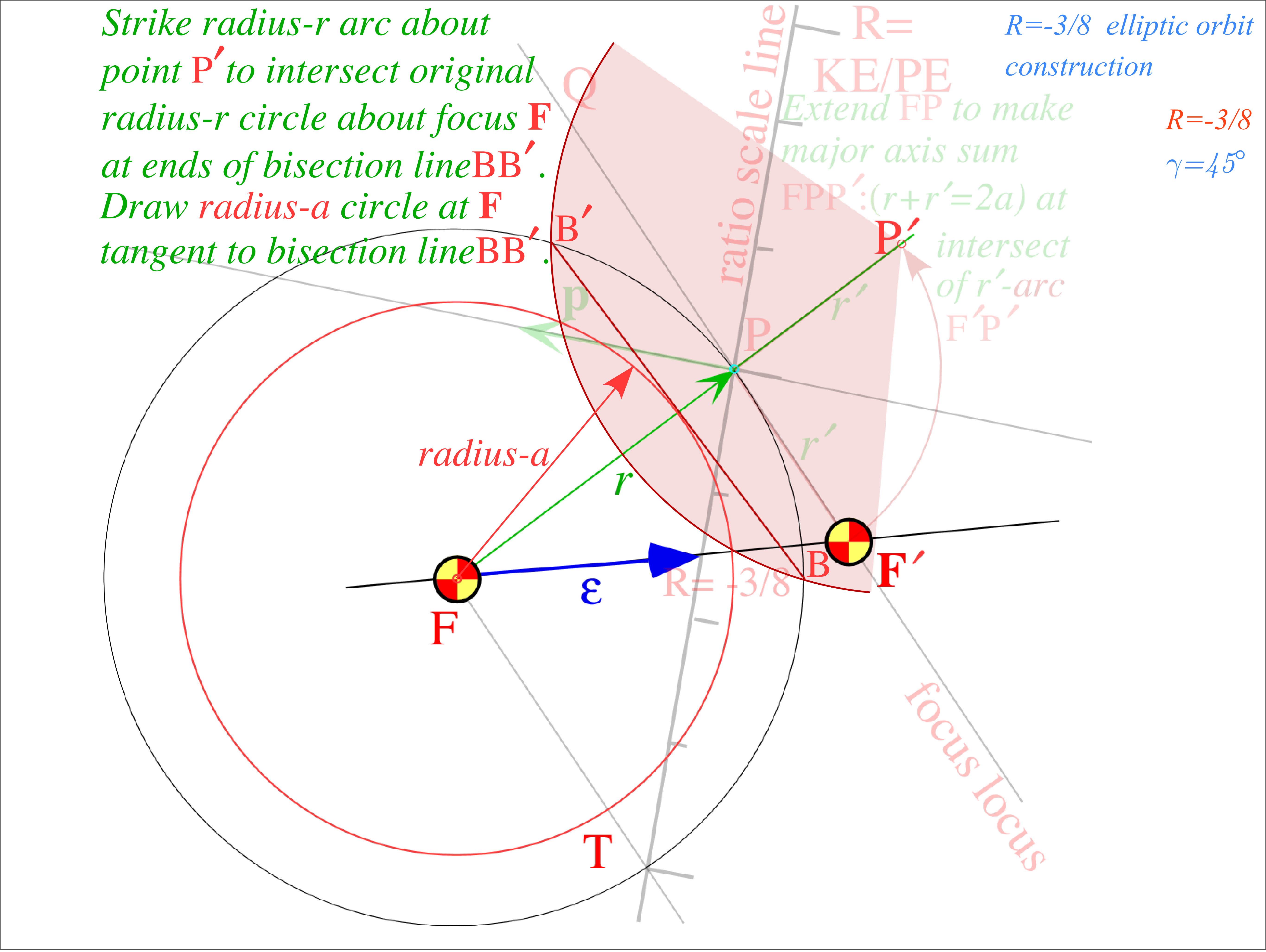
➔ *(R=-0.375 elliptic orbit)*

(R=+0.5 hyperbolic orbit)


R=-3/8 elliptic orbit
construction
R=-3/8
γ=45°
R=
KE/PE
ratio scale line
Q
Extend FP to make
major axis sum
FPP′:(r+r′=2a) at
P′
intersect
of r′-arc
F′P′
r
P'
p
γ
γ
r′
P
r′
r
+2/8
+1/8
−1/8
−2/8
−3/8
−4/8
−5/8
−6/8
−7/8
−8/8
ε
R= -3/8
F
F'
T
focus locus


Strike radius-r arc about
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at ends of bisection line BB′.
Draw radius-a circle at F
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R=-3/8 elliptic orbit
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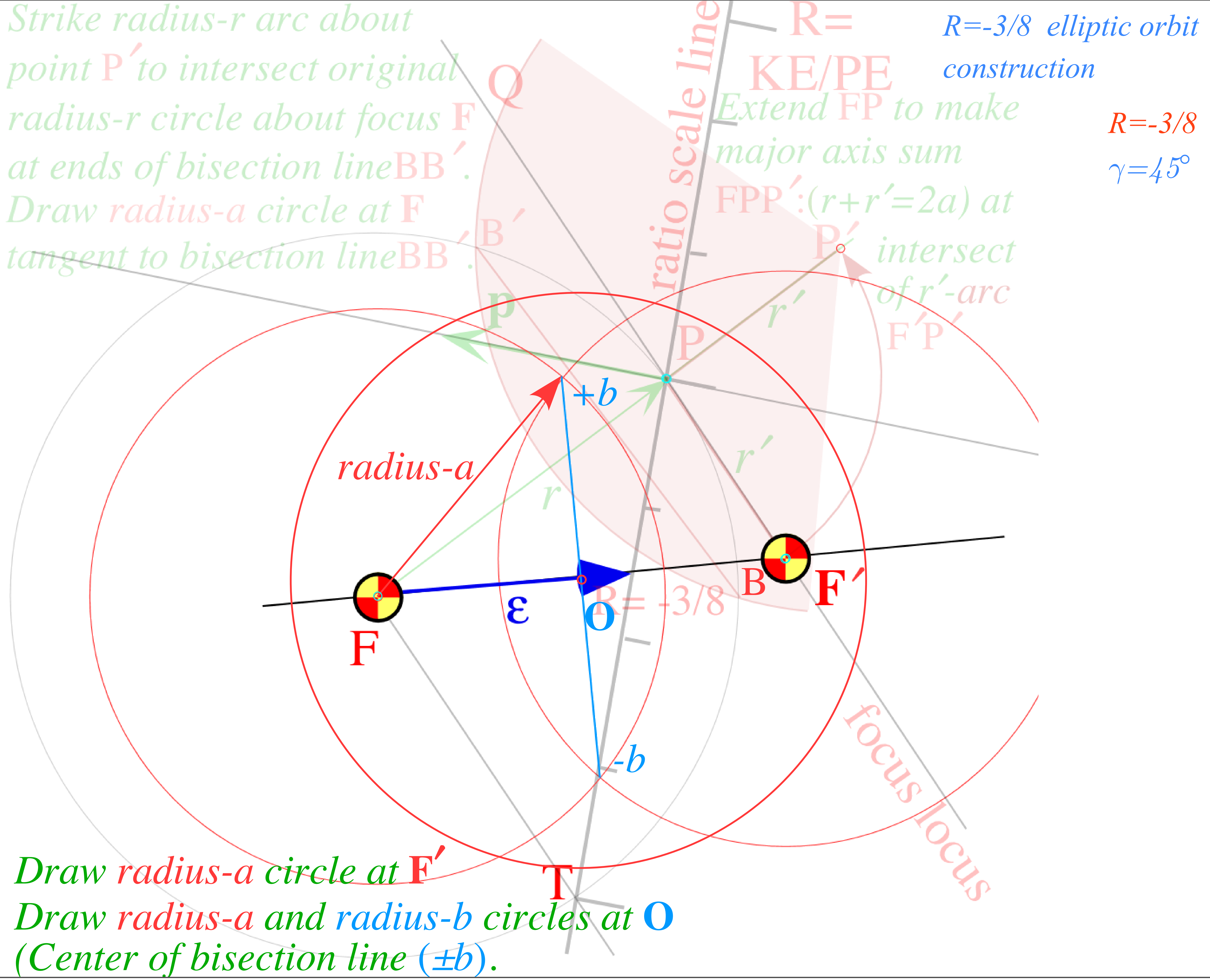
P′
p
P
r′
r′
radius-a
r
B
F′
ε
R=-3/8
F
T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R= KE/PE
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R=-3/8 elliptic orbit construction
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-b
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T
Draw radius-a circle at F′
Draw radius-a and radius-b circles at O
(Center of bisection line (±b).

R=-3/8 elliptic orbit construction

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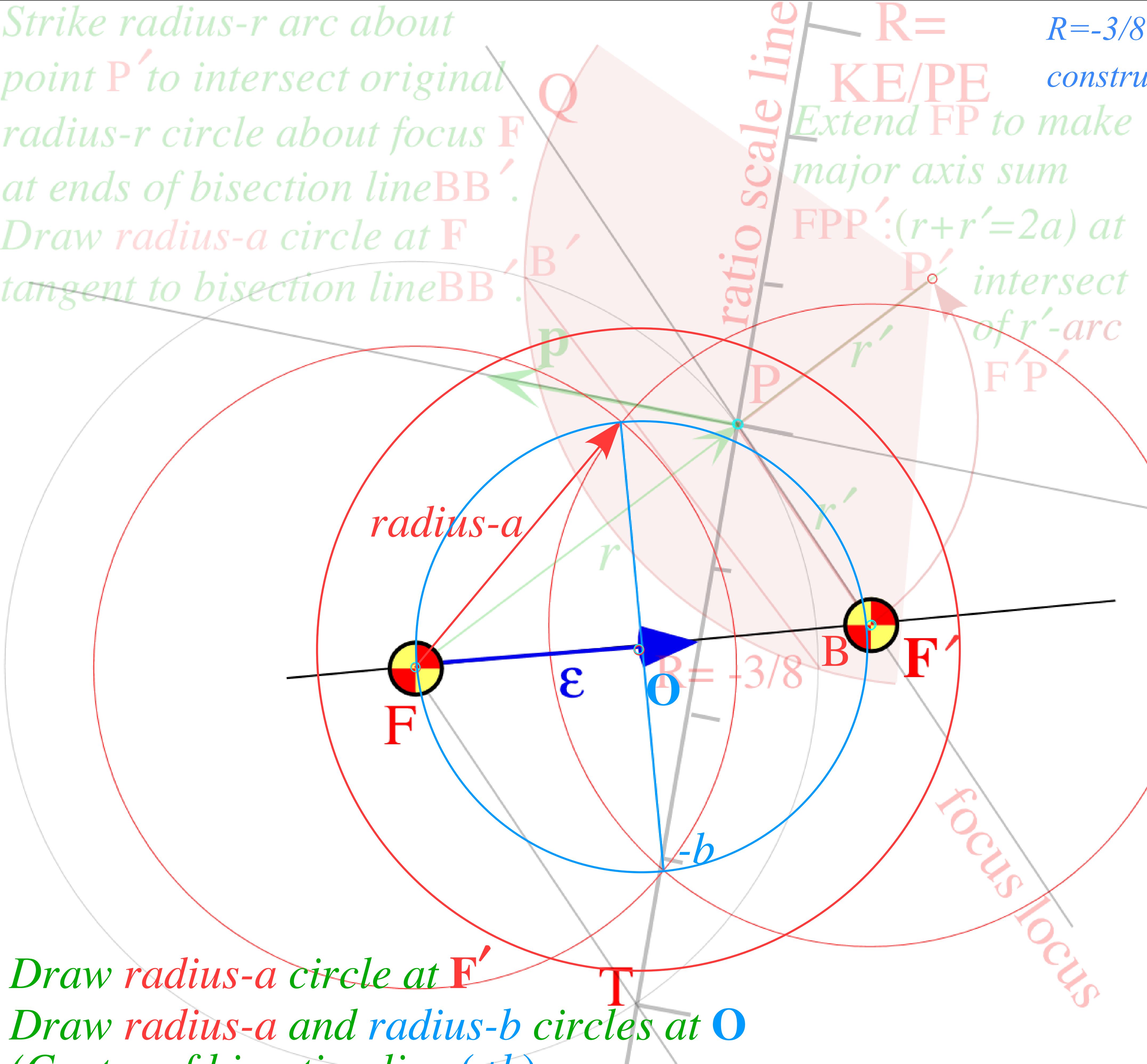


Draw radius-a circle at F′

Draw radius-a and radius-b circles at O

(Center of bisection line (±b).

R=-3/8 elliptic orbit construction

R=-3/8

$\gamma=45°$

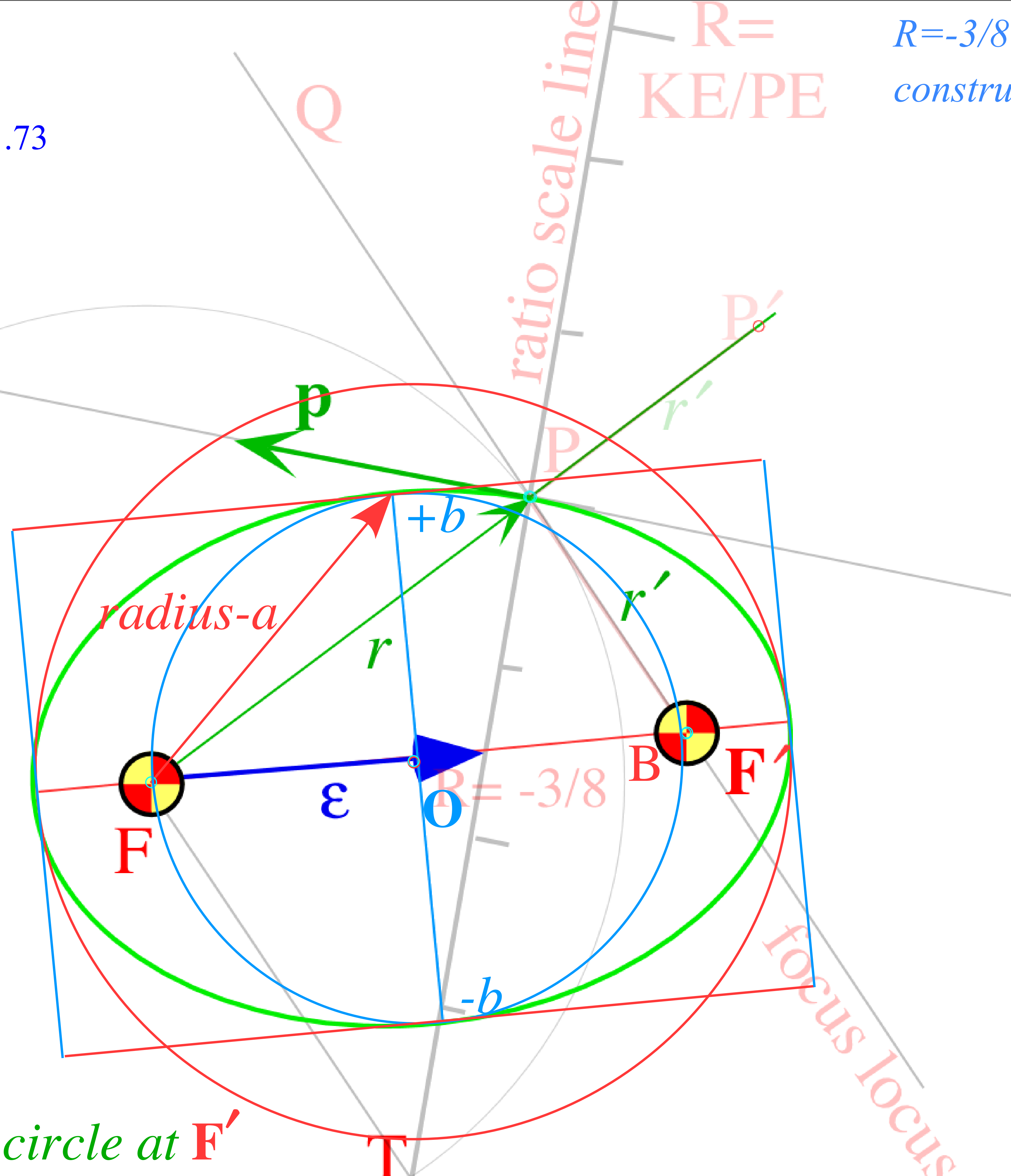
$$\varepsilon=\sqrt{1+4R(R+1)\sin^2\gamma}=\frac{\sqrt{34}}{8}=.73$$

$$a=\frac{1}{2(R+1)}=\frac{4}{5}$$

$$b=\sqrt{\frac{R}{R+1}}\sin\gamma=\sqrt{\frac{3}{10}}=.54$$

$$\lambda=\frac{b^2}{a}=2R\sin^2\gamma=\frac{3}{8}=.375$$

$$\frac{b}{a}=2\sqrt{R(R+1)}\sin\gamma=\tan 34°$$



*Draw radius-a circle at **F′***

*Draw radius-a and radius-b circles at **O***

(Center of bisection line (±b).) Do (a,b)-ellipse construction.

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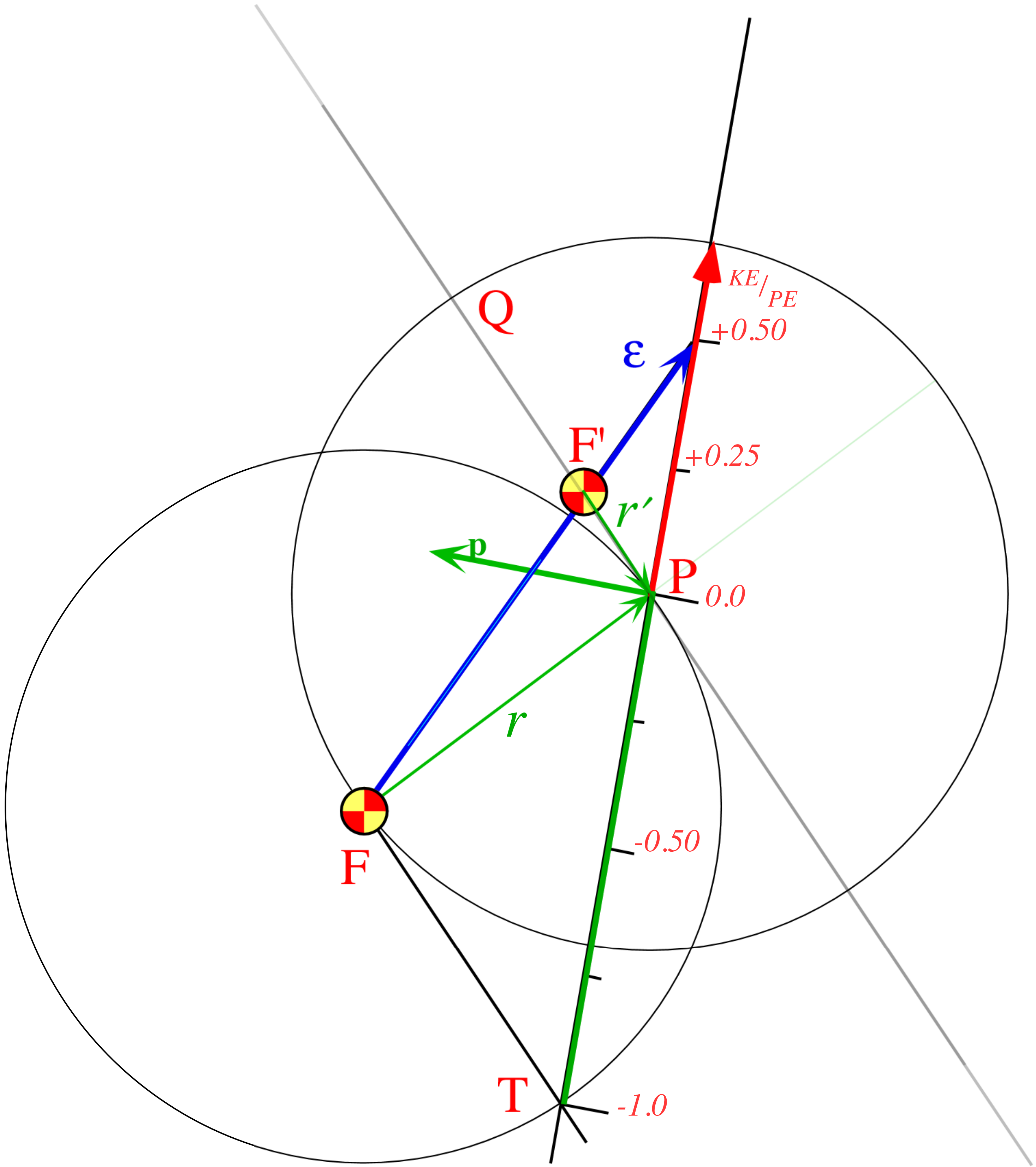
Major diameter 2a is difference (r-r′=2a).
Major radius a is half of difference (r-r′)/2=a
Major diameter 2a needs to be centered on F-F′ focal axis

R=+1/2 hyperbolic orbit construction

R=+1/2

$\gamma=45°$



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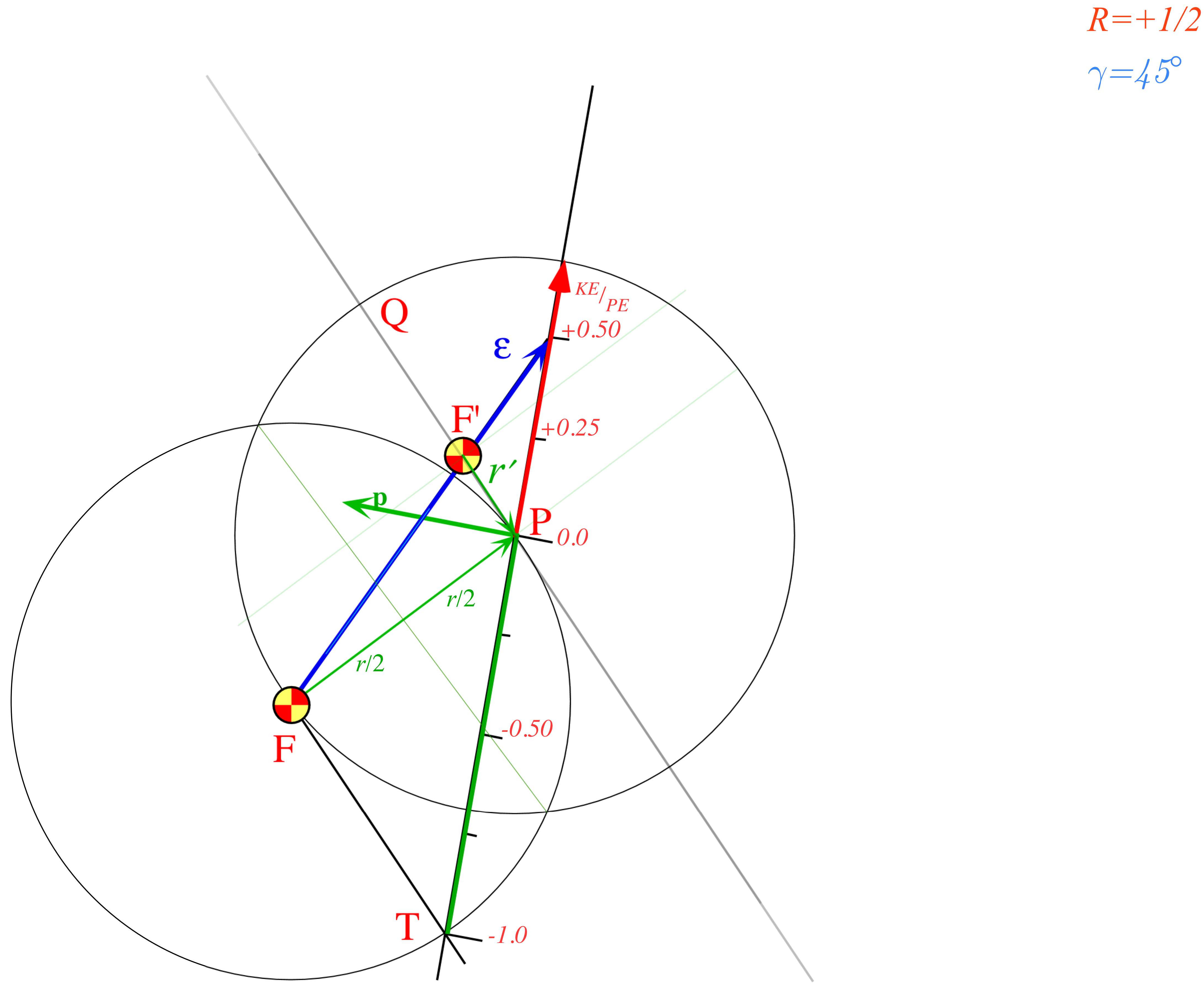
1. Bisect F-P radius r using F-P circle intersections to define r/2 sections.

R=+1/2 hyperbolic orbit construction



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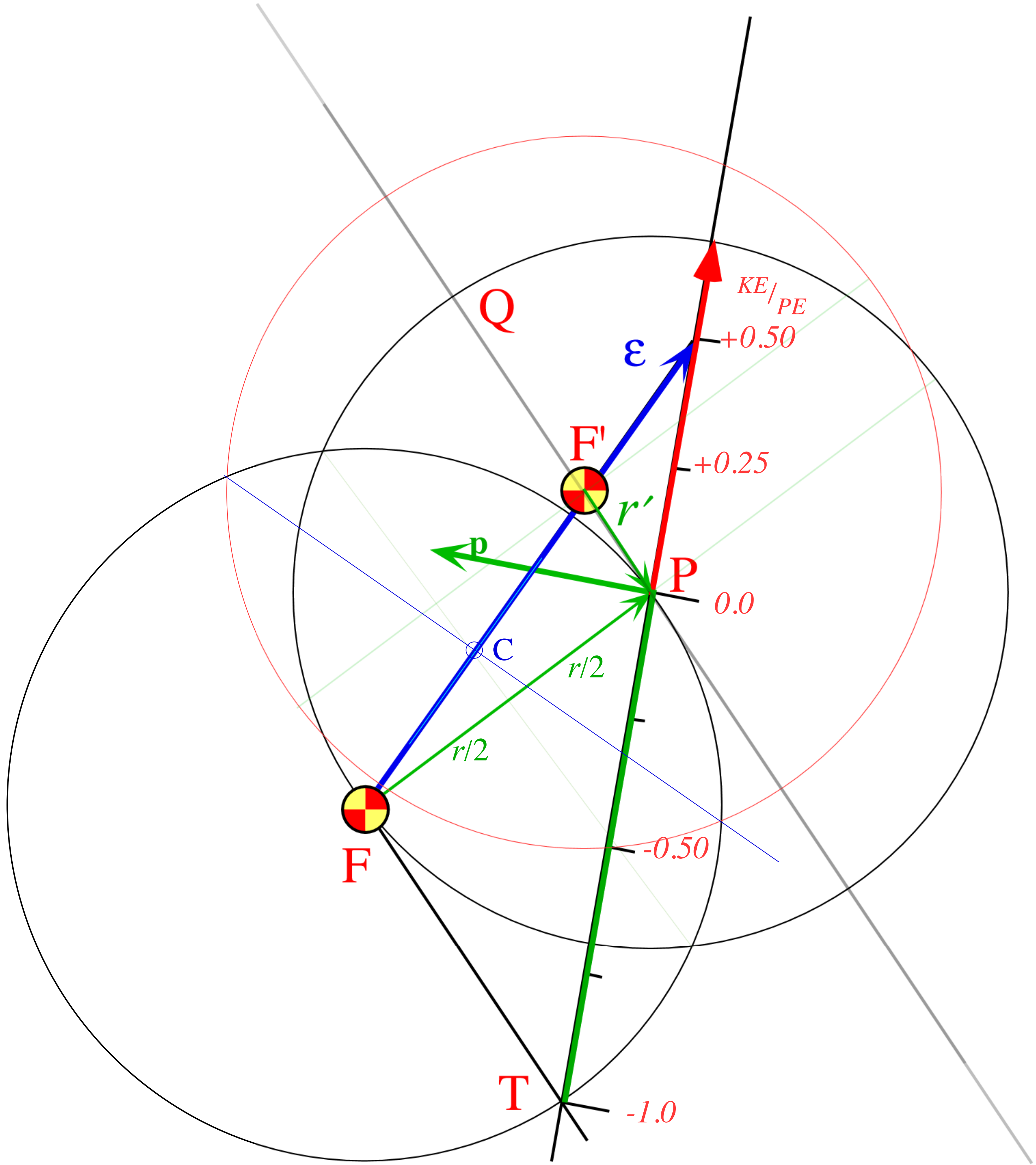
R=+1/2 hyperbolic orbit construction

R=+1/2

γ=45°



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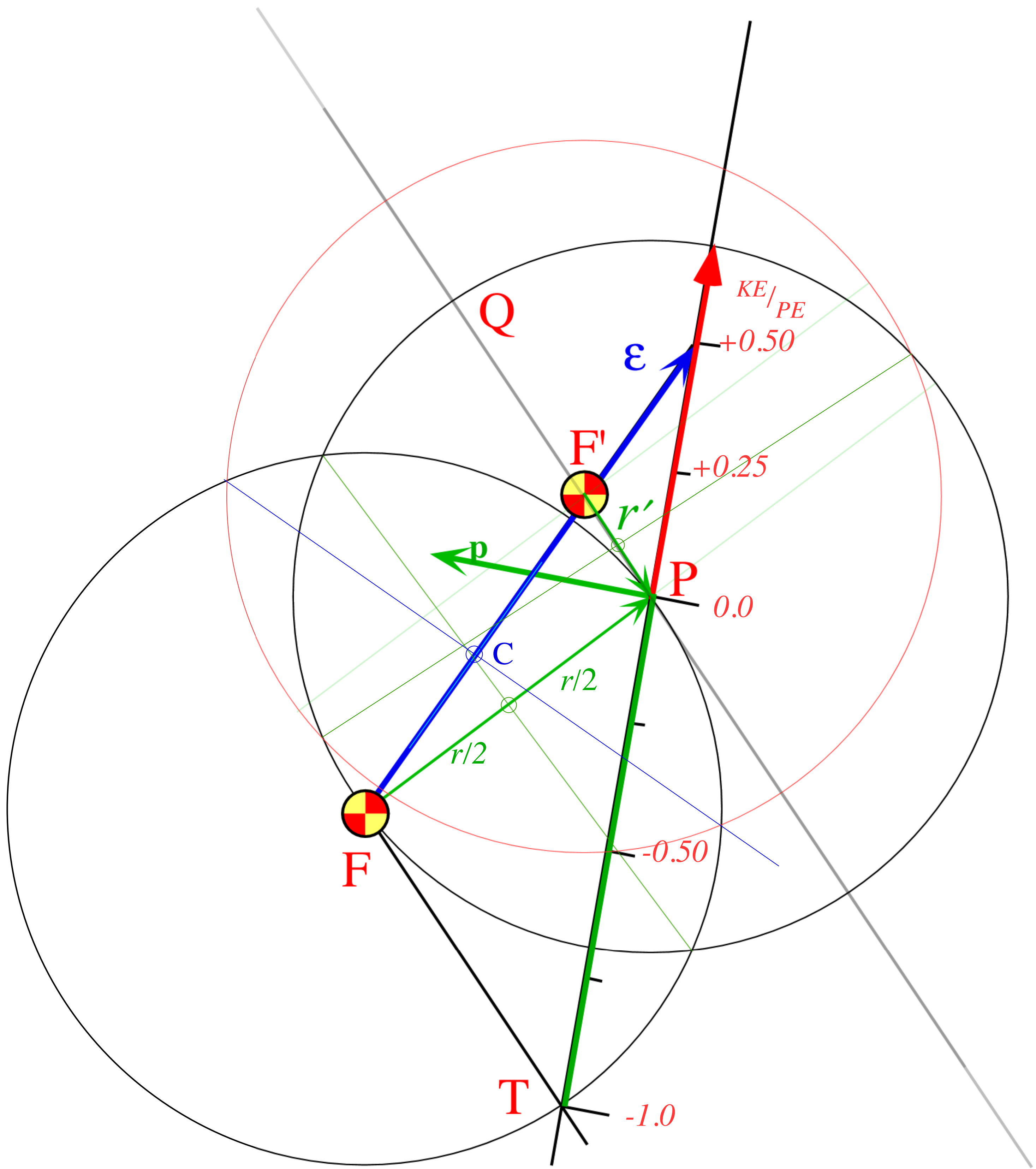

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4. Swing radius r′/2 onto r/2 section to make major radius a=(r-r′)/2.

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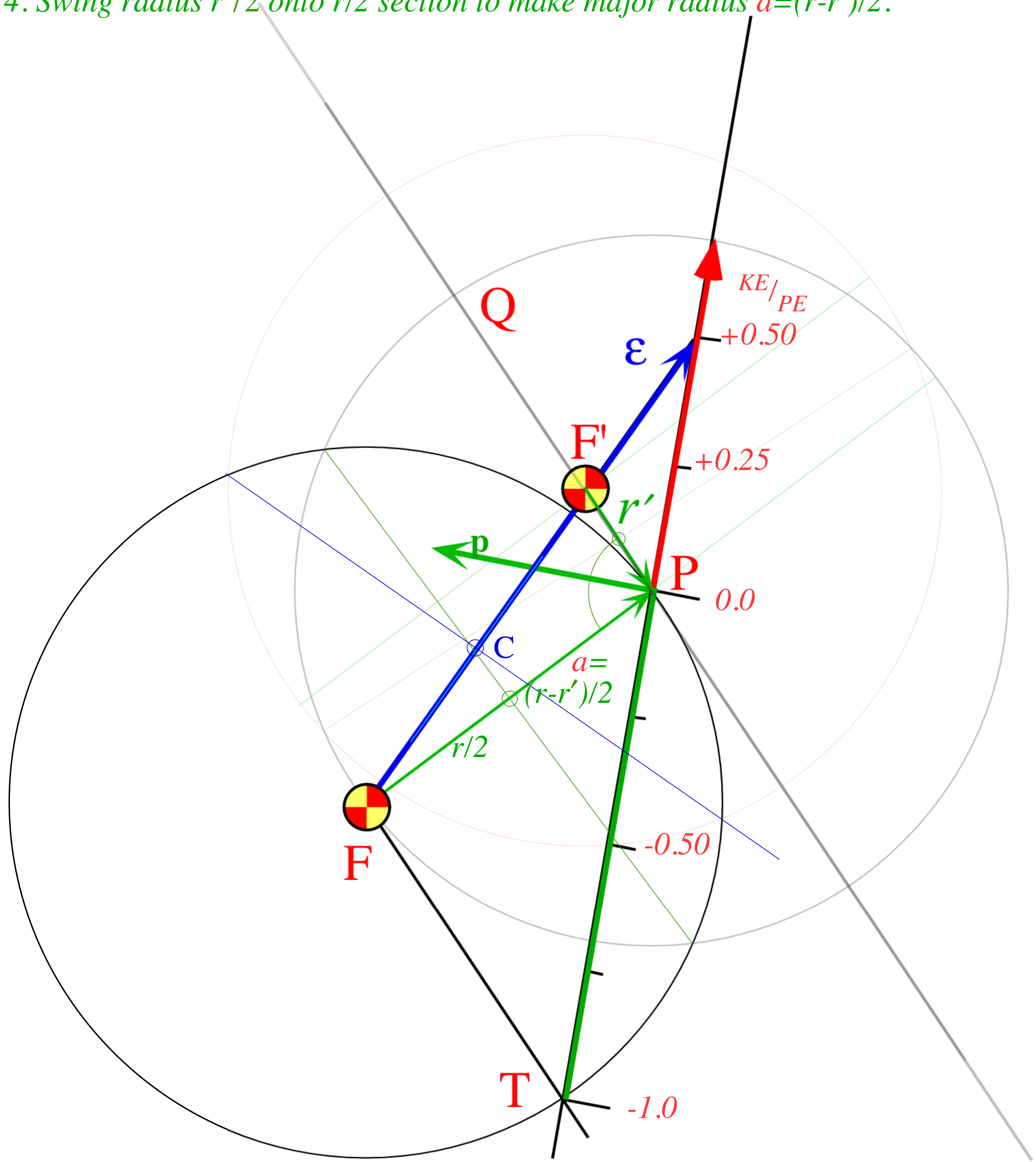

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5. *Copy circle of major radius a=(r-r′)/2 about orbit center C .*

R=+1/2 hyperbolic orbit construction

R=+1/2

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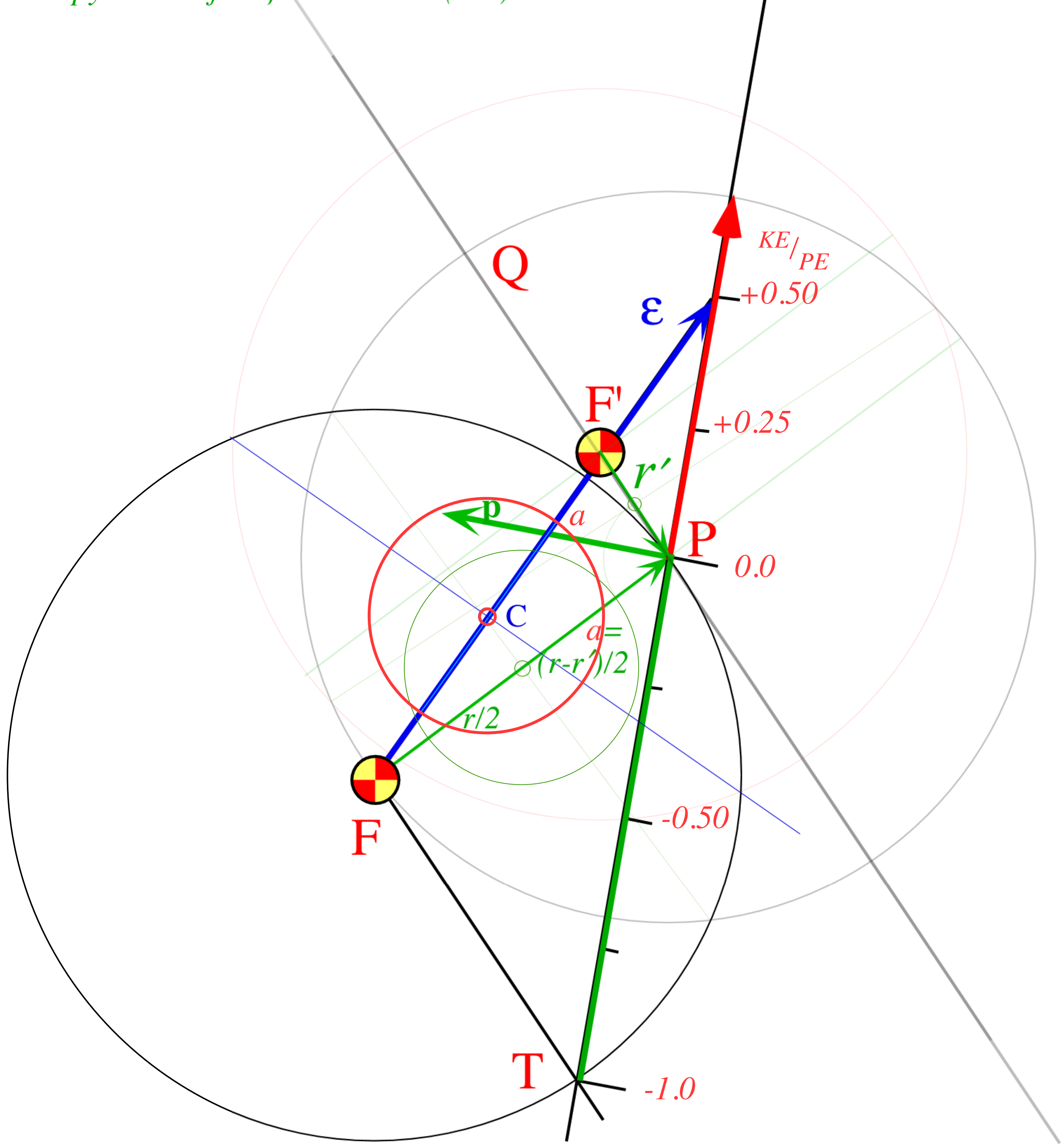

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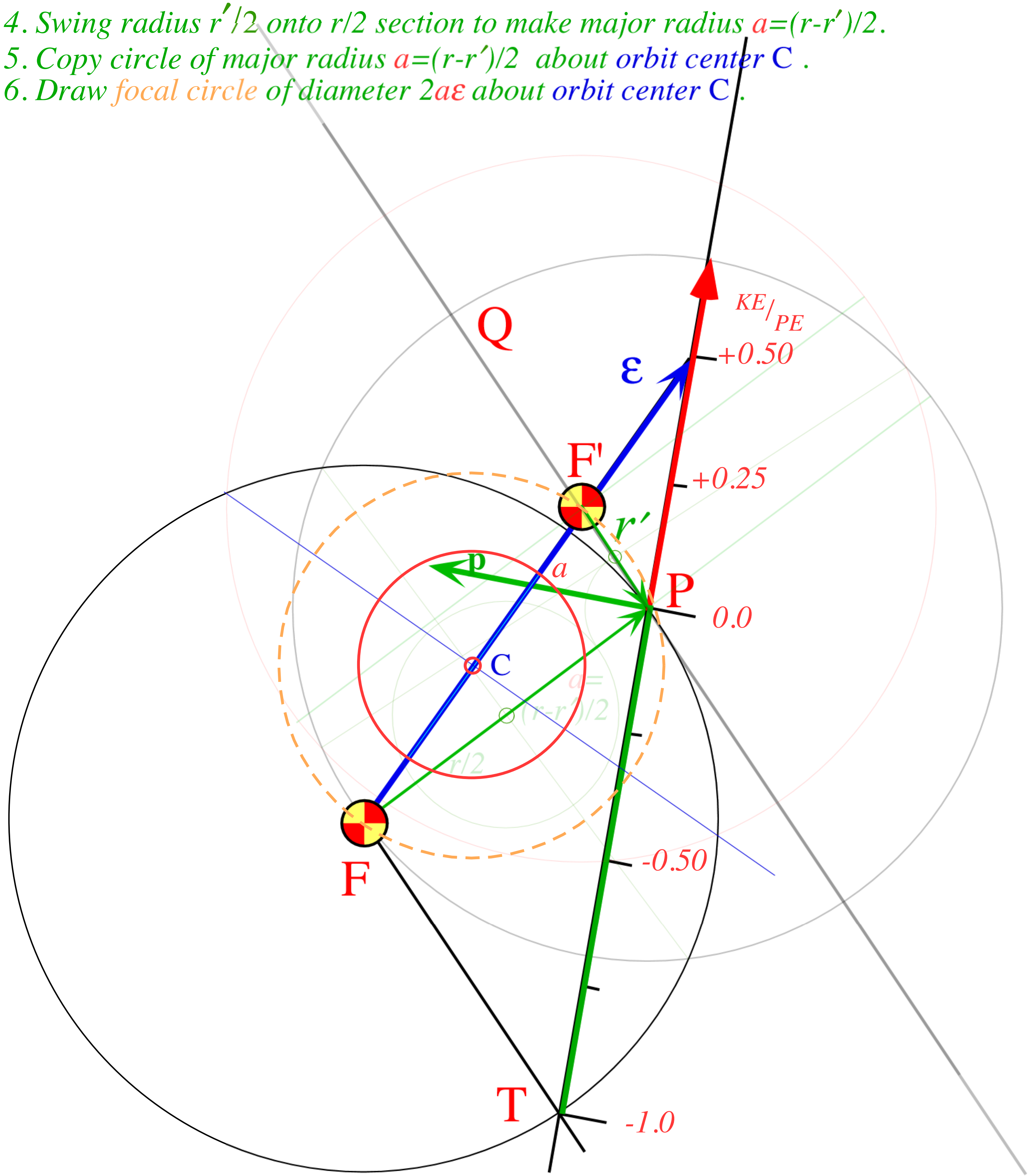
5. *Copy circle of major radius a=(r-r′)/2 about orbit center C .*
6. *Draw focal circle of diameter 2aε about orbit center C .*

R=+1/2 hyperbolic orbit construction

R=+1/2
γ=45°



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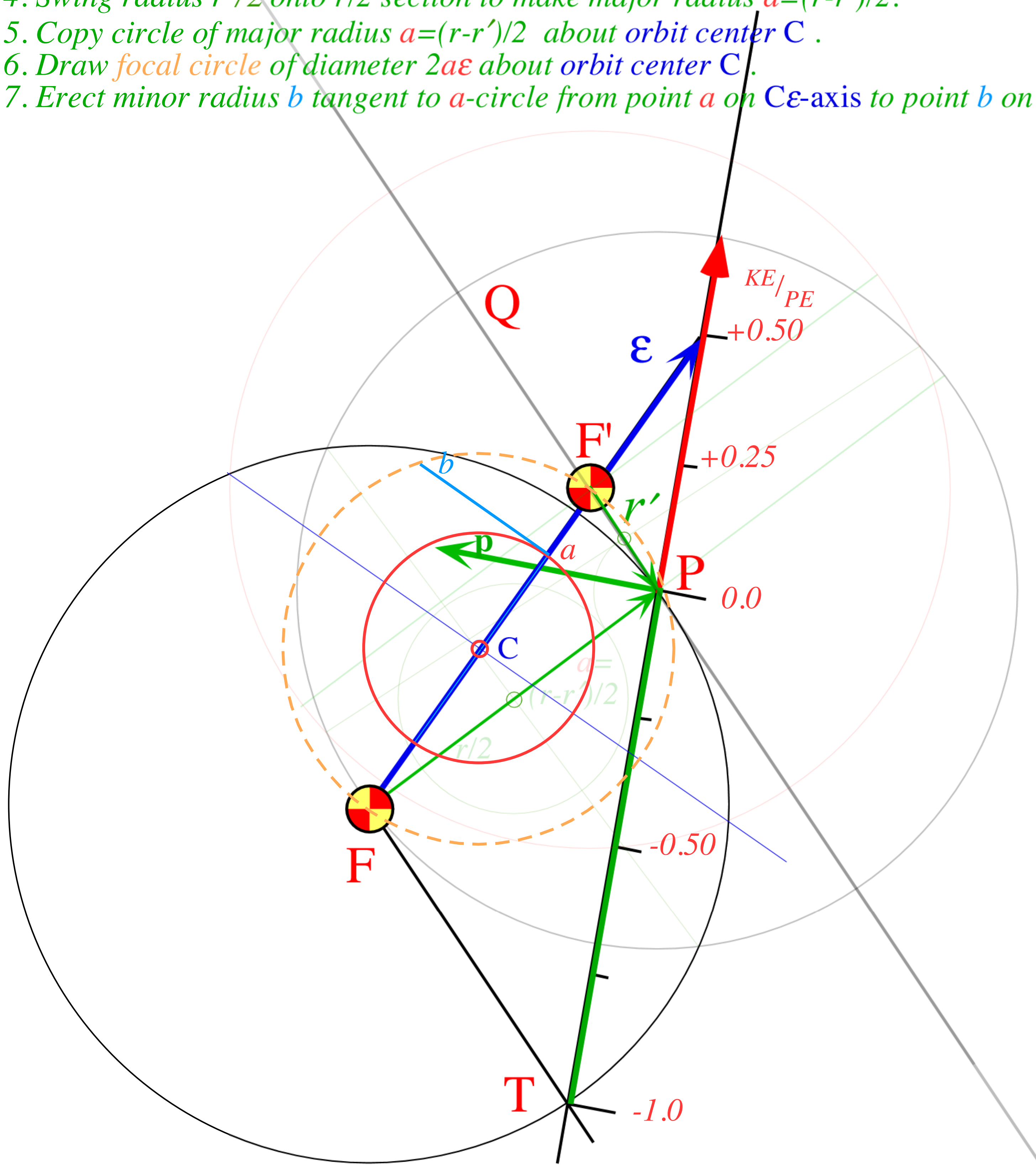
7. *Erect minor radius b tangent to a-circle from point a on Cε-axis to point b on focal circle.*

R=+1/2 hyperbolic orbit construction

R=+1/2

$\gamma=45°$



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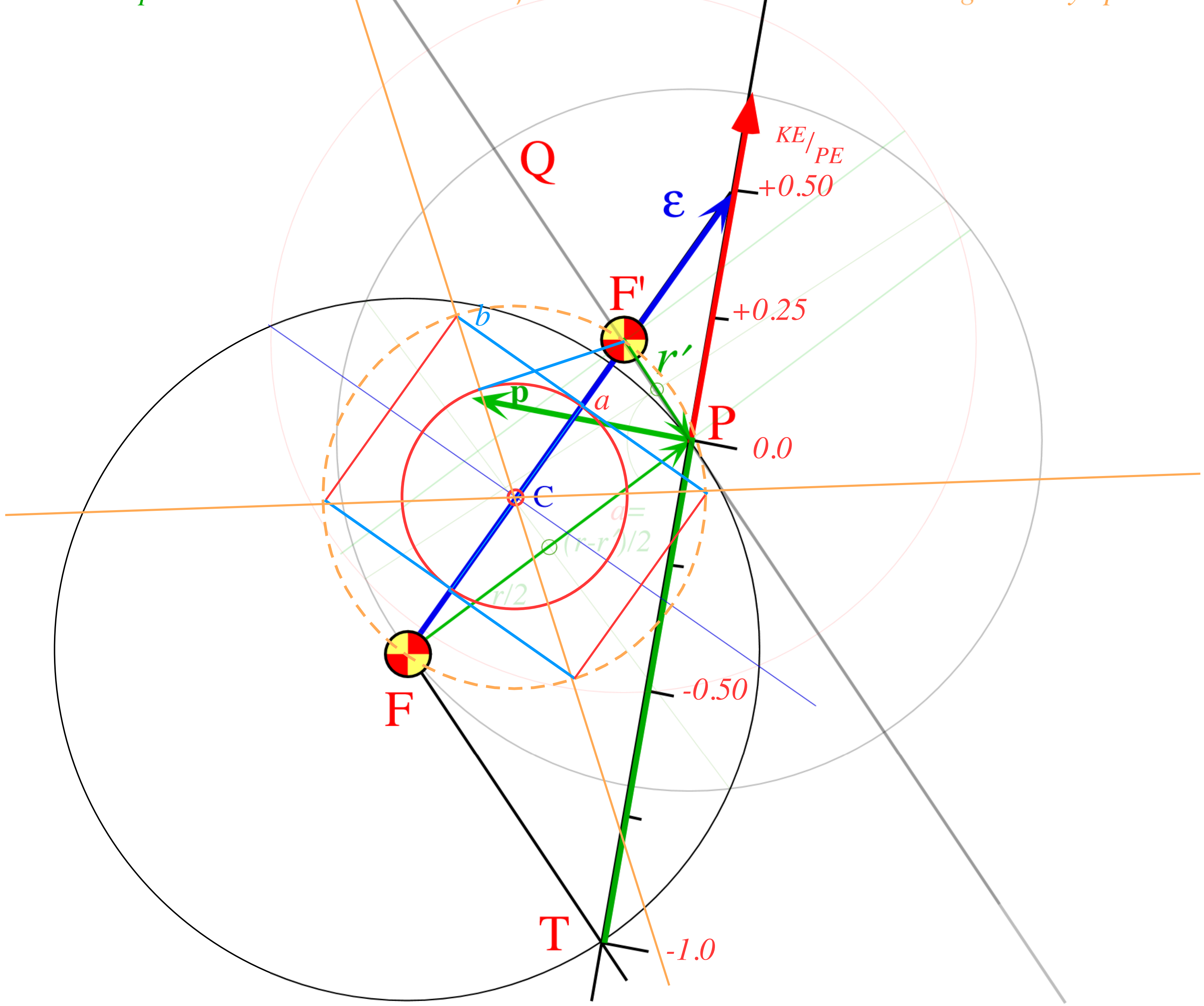
8. *Complete orbit a-x-b box between focal circle and a-circle and its diagonal asymptotes.*

R=+1/2 hyperbolic orbit construction

R=+1/2

γ=45°

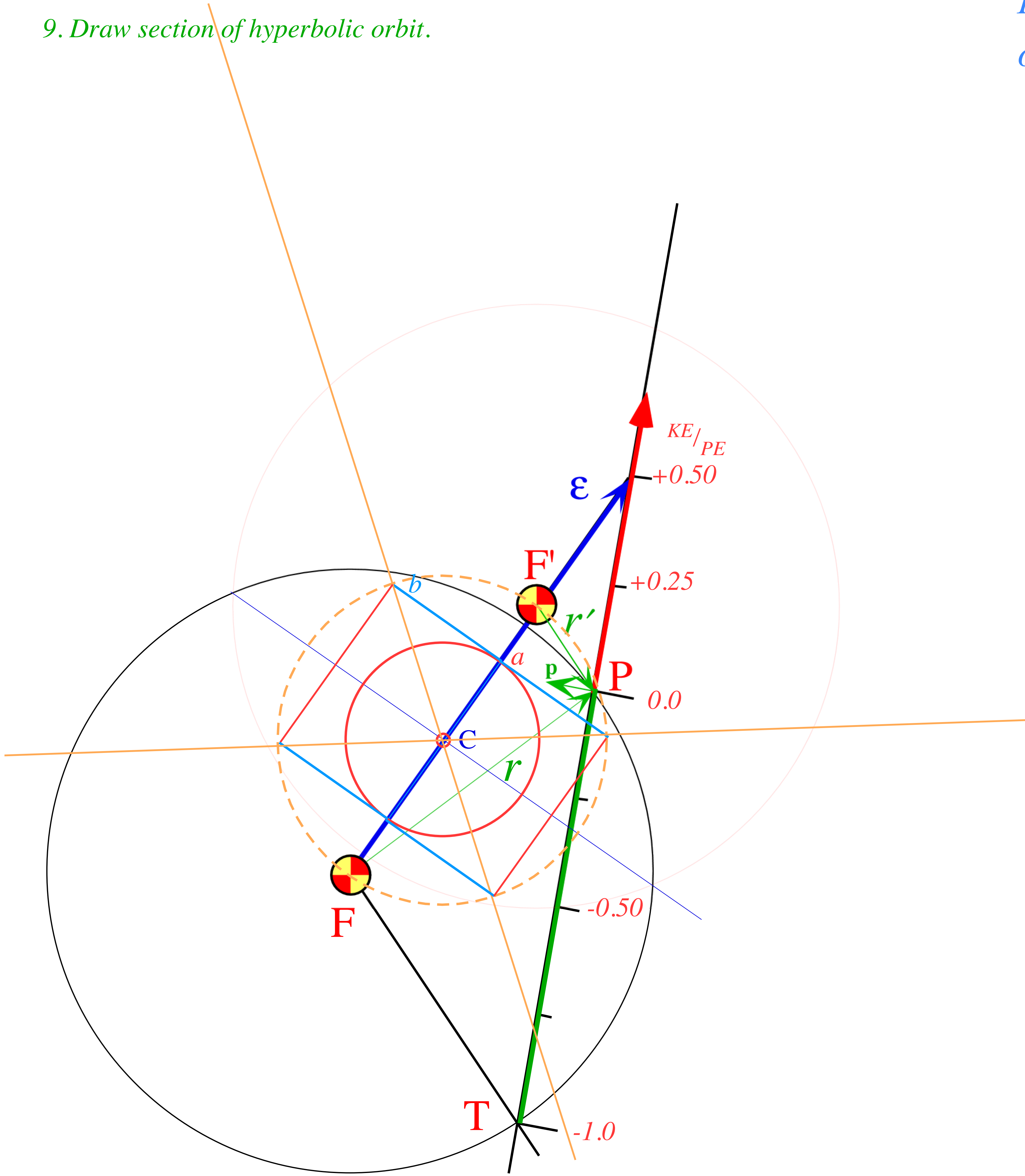


9. *Draw section of hyperbolic orbit.*

R=+1/2 hyperbolic orbit construction

R=+1/2

$\gamma=45°$



R=+1/2 hyperbolic orbit construction

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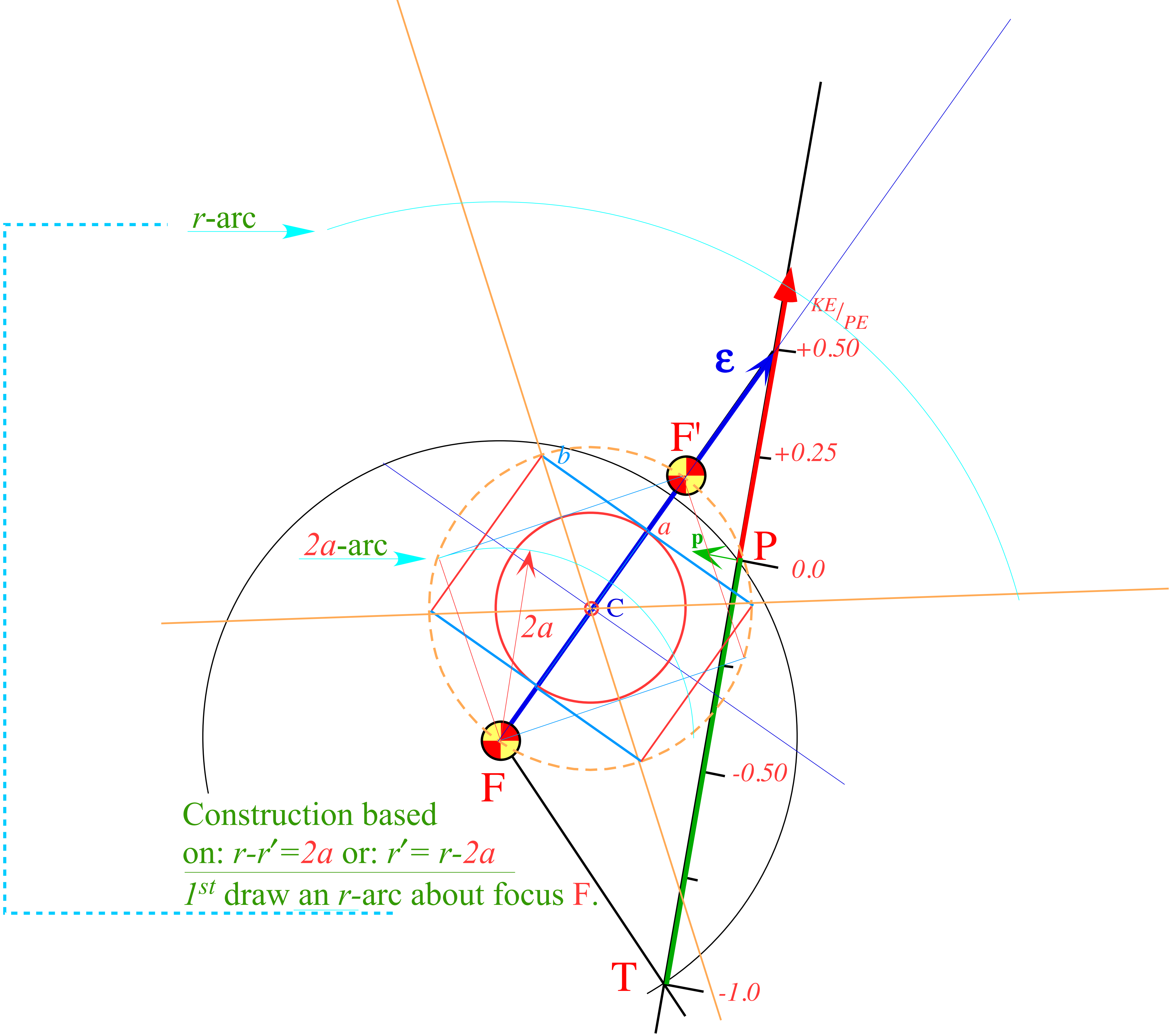


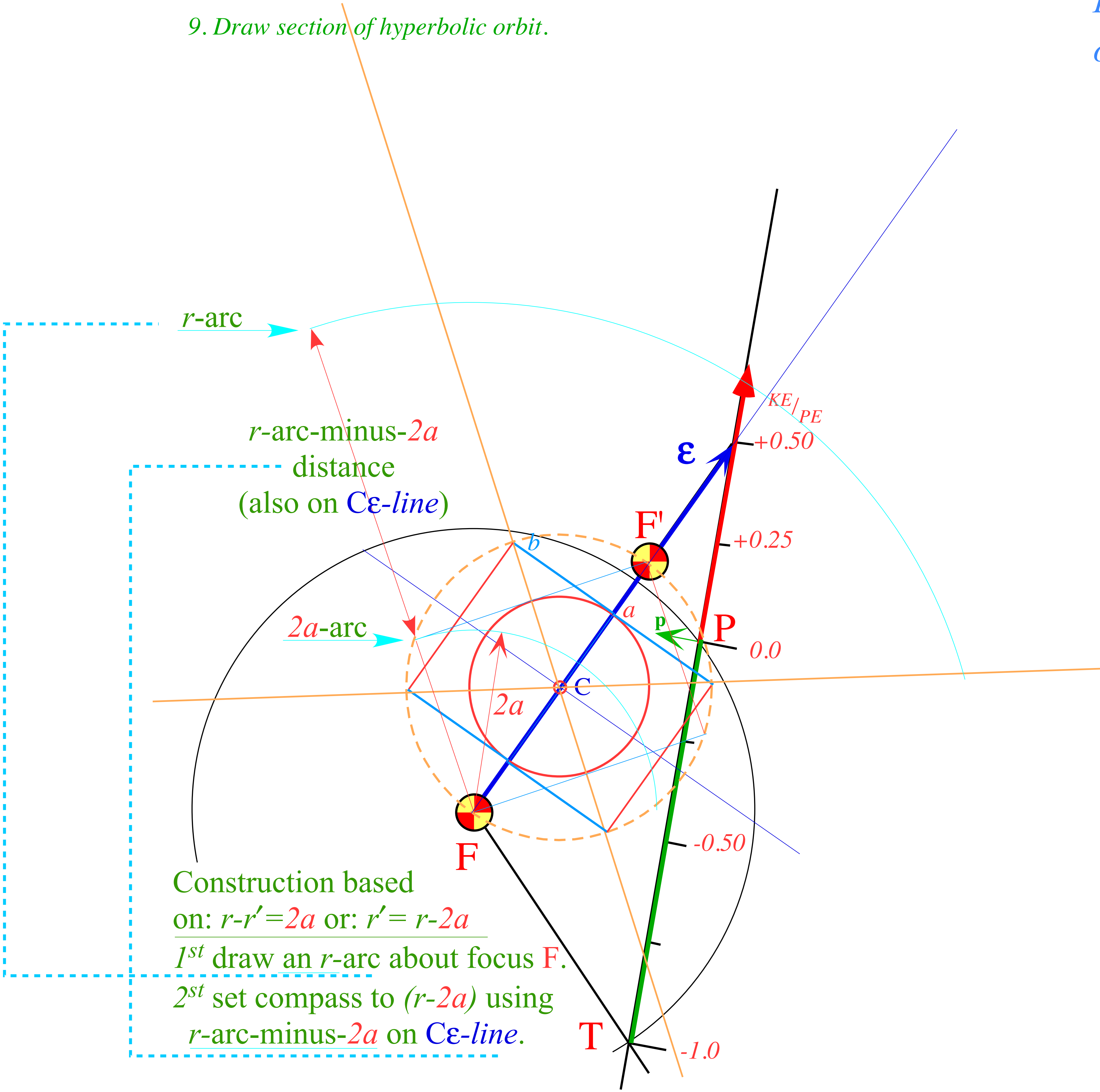
Construction based
on: $r\text{-}r'=2a$ or: $r' = r\text{-}2a$

1^{st} draw an *r*-arc about focus F.

R=+1/2 hyperbolic orbit construction

$R=+1/2$

$\gamma=45°$



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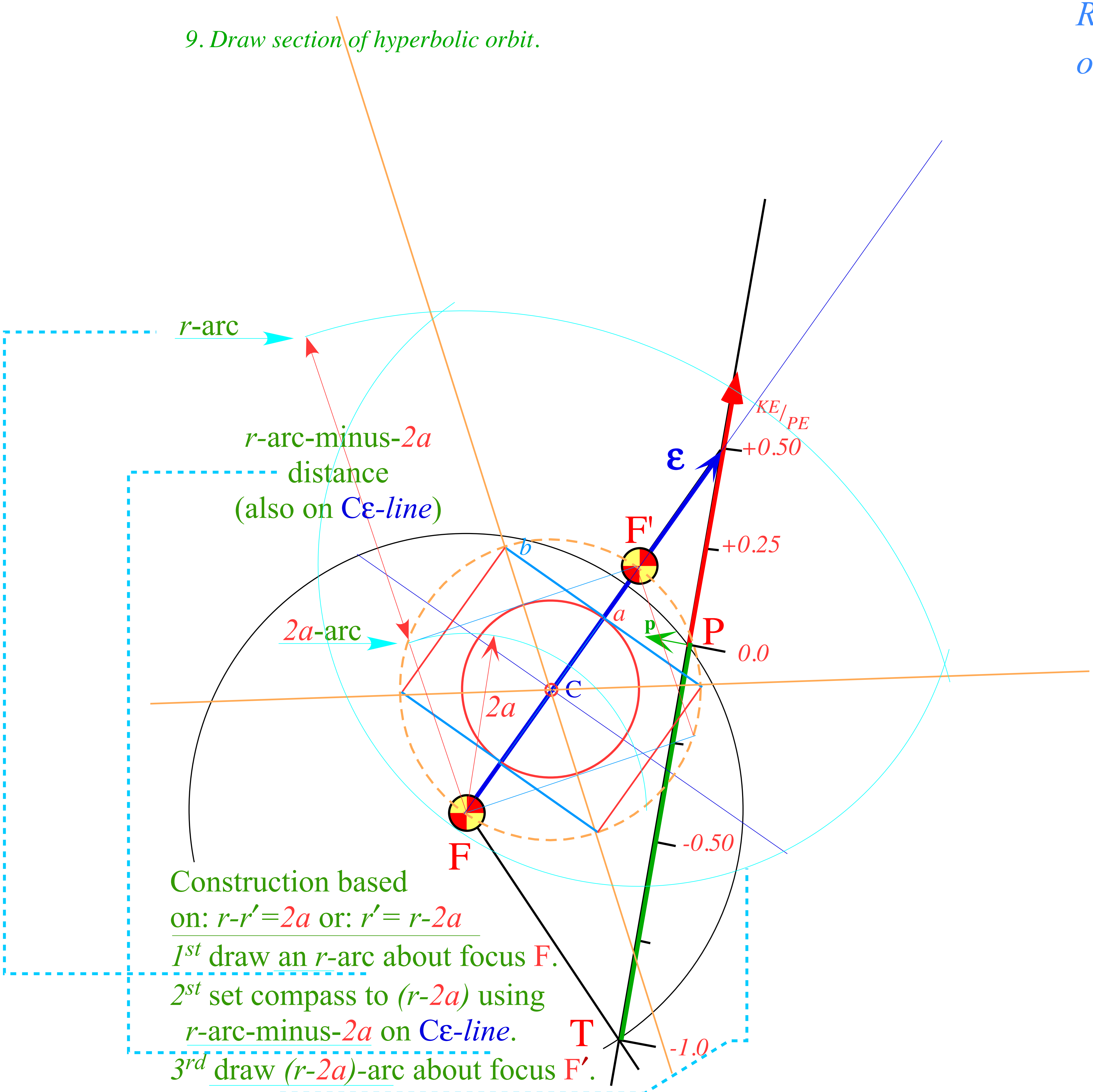

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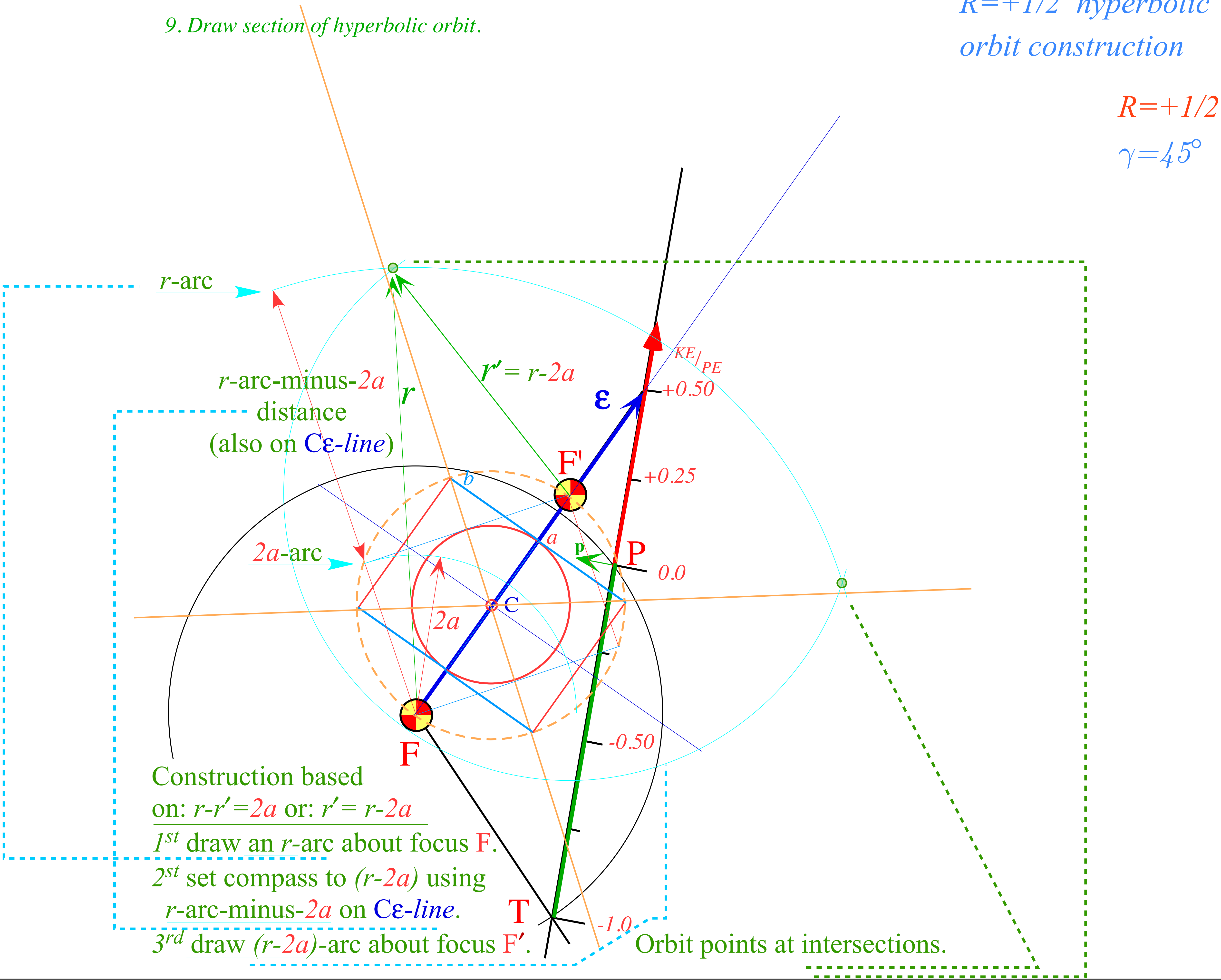

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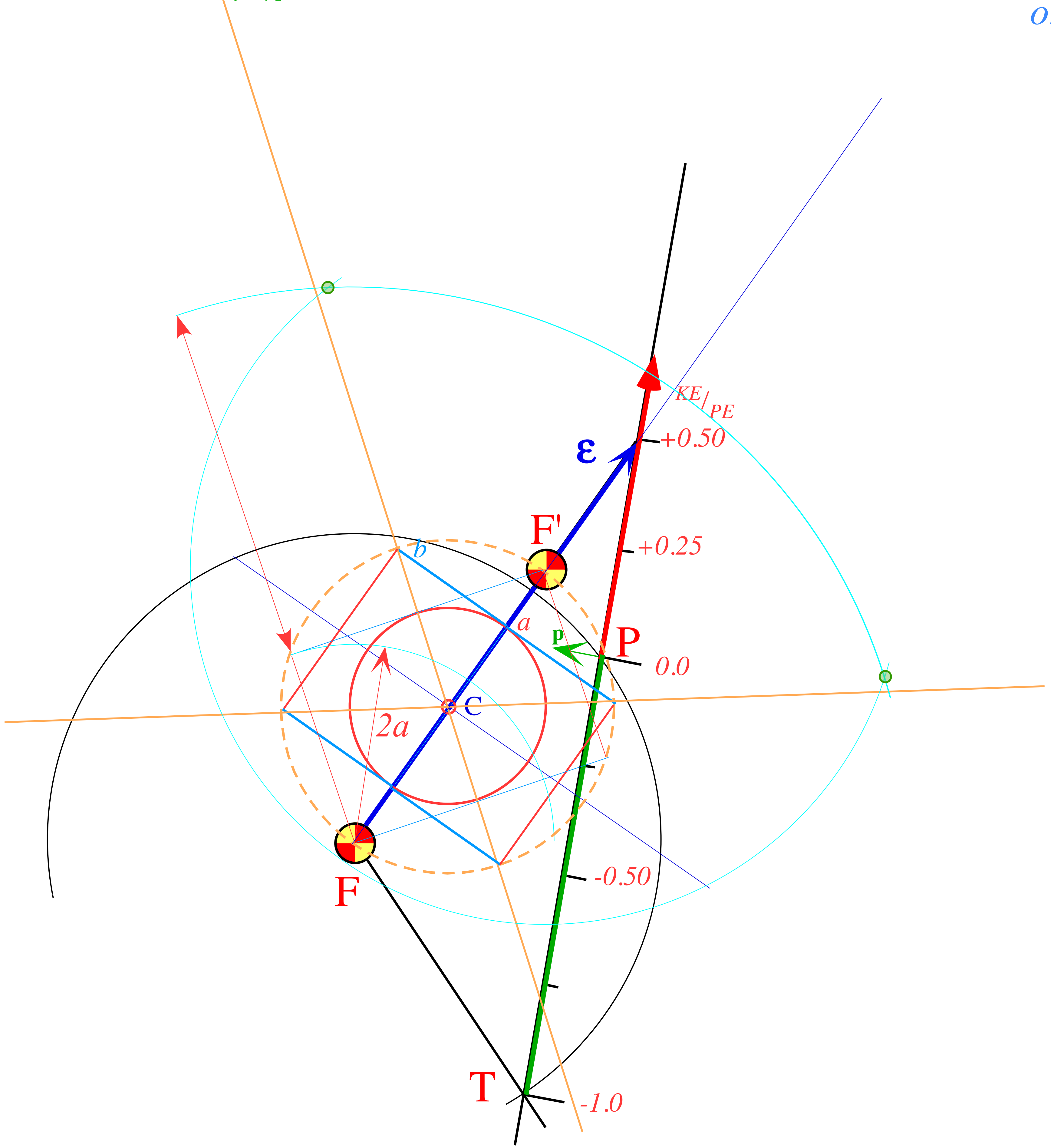

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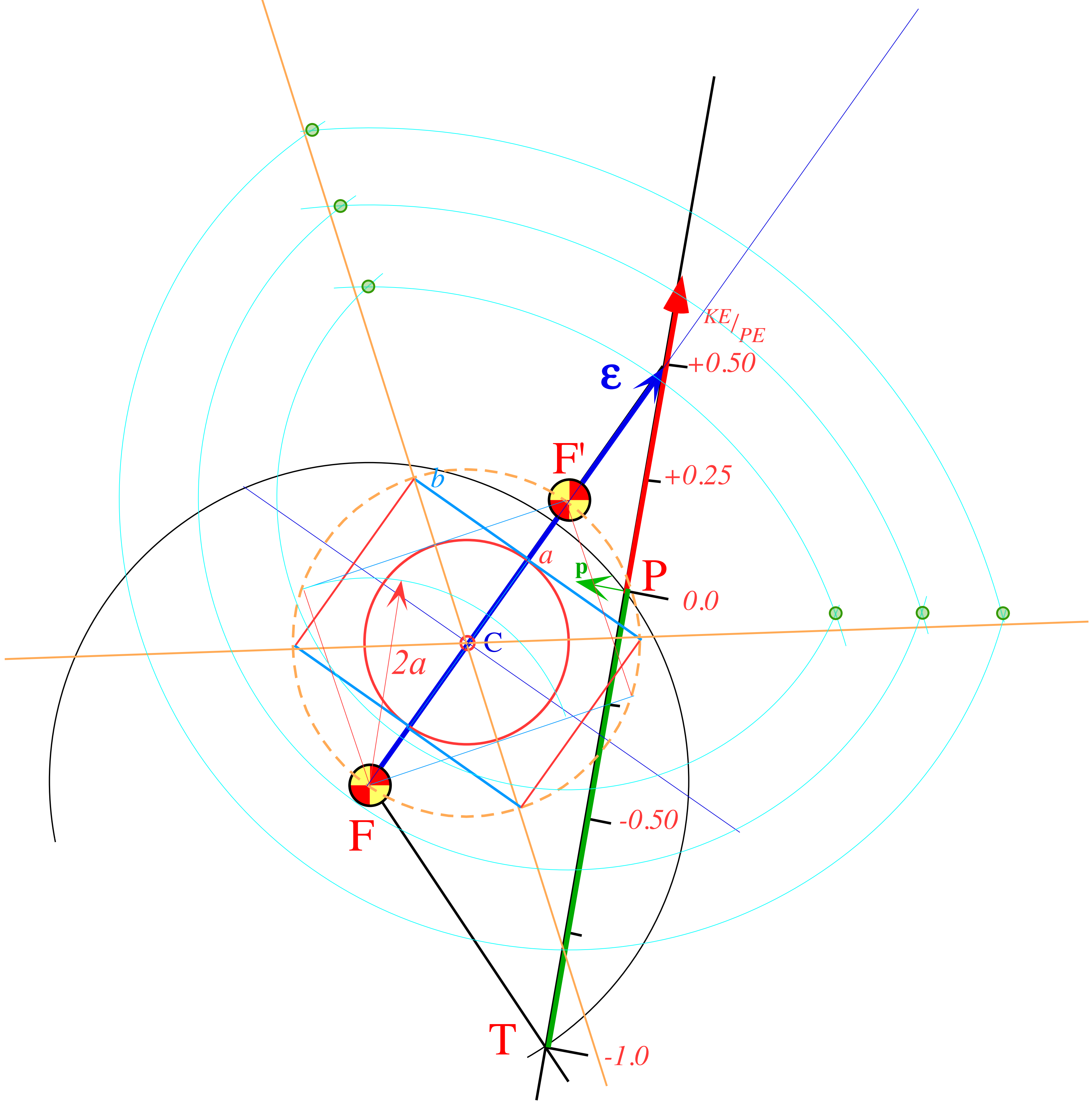

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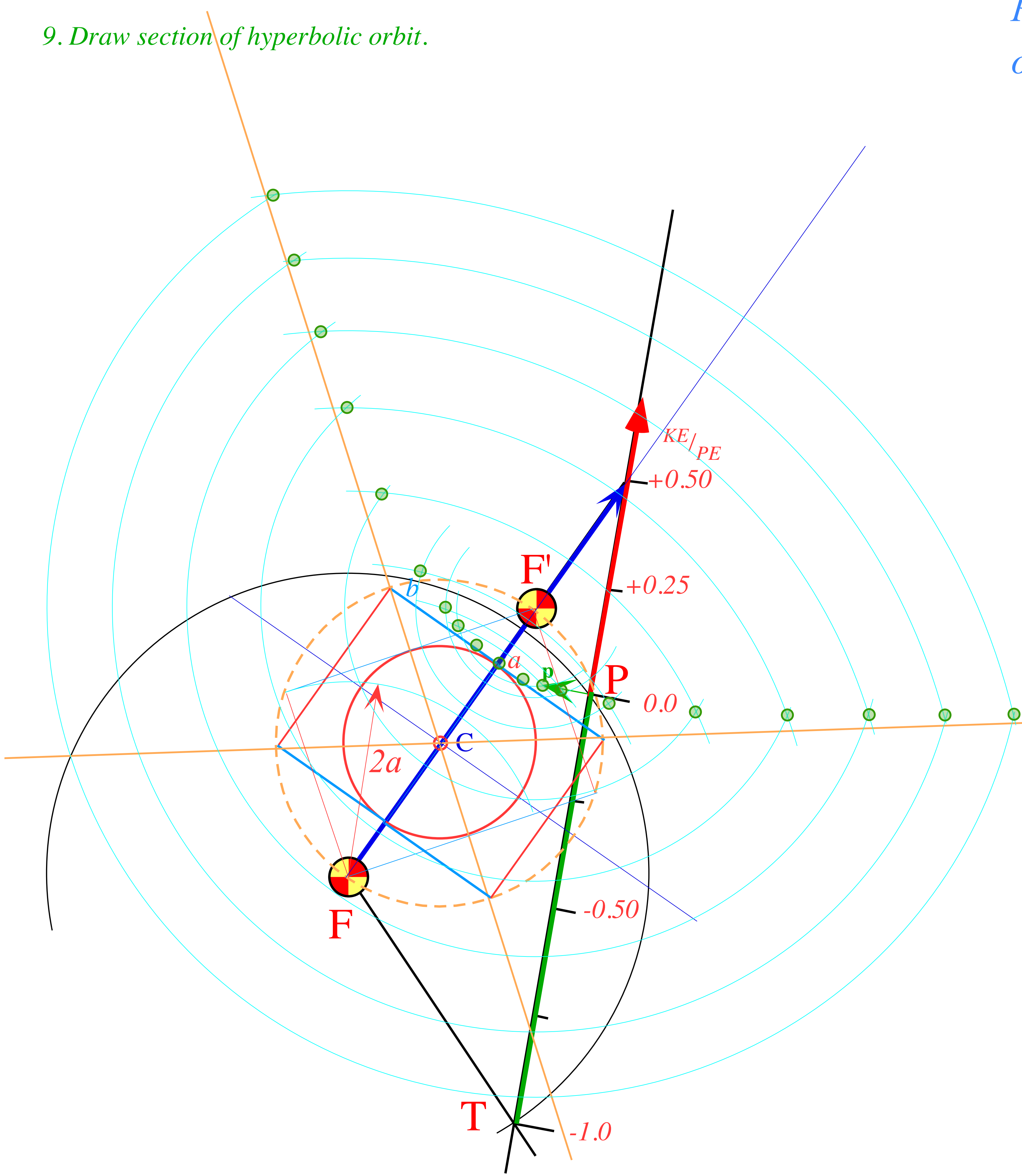
R=+1/2 hyperbolic orbit construction



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γ=45°

9. Draw section of hyperbolic orbit.

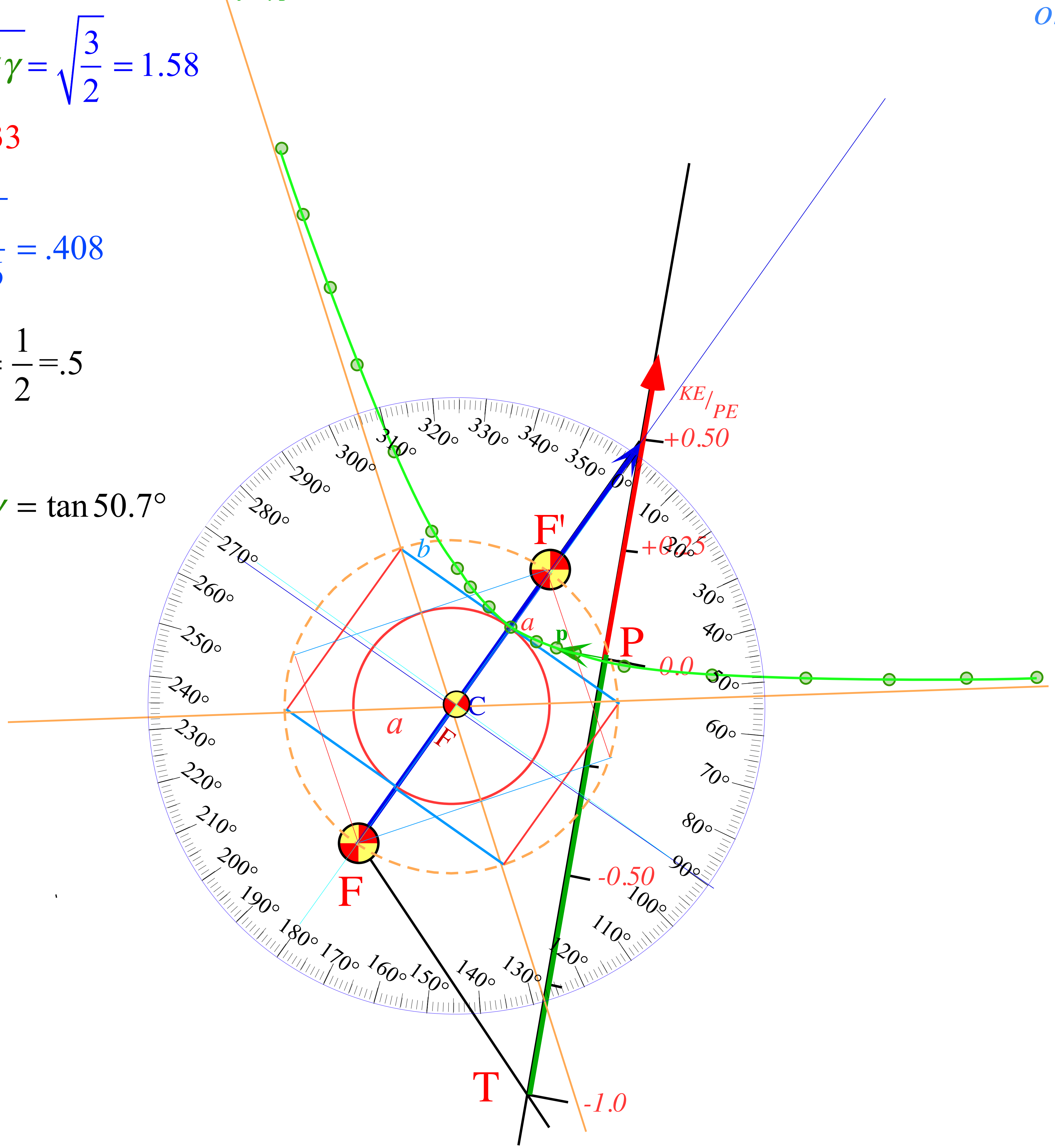
$$\varepsilon=\sqrt{1+4R(R+1)\sin^2\gamma}=\sqrt{\frac{3}{2}}=1.58$$

$$a=\frac{1}{2(R+1)}=\frac{1}{3}=.33$$

$$b=\sqrt{\frac{R}{R+1}}\sin\gamma=\sqrt{\frac{1}{6}}=.408$$

$$\lambda=\frac{b^2}{a}=2R\sin^2\gamma=\frac{1}{2}=.5$$

$$\frac{b}{a}=2\sqrt{R(R+1)}\sin\gamma=\tan 50.7°$$



Properties of Coulomb trajectory families and envelopes

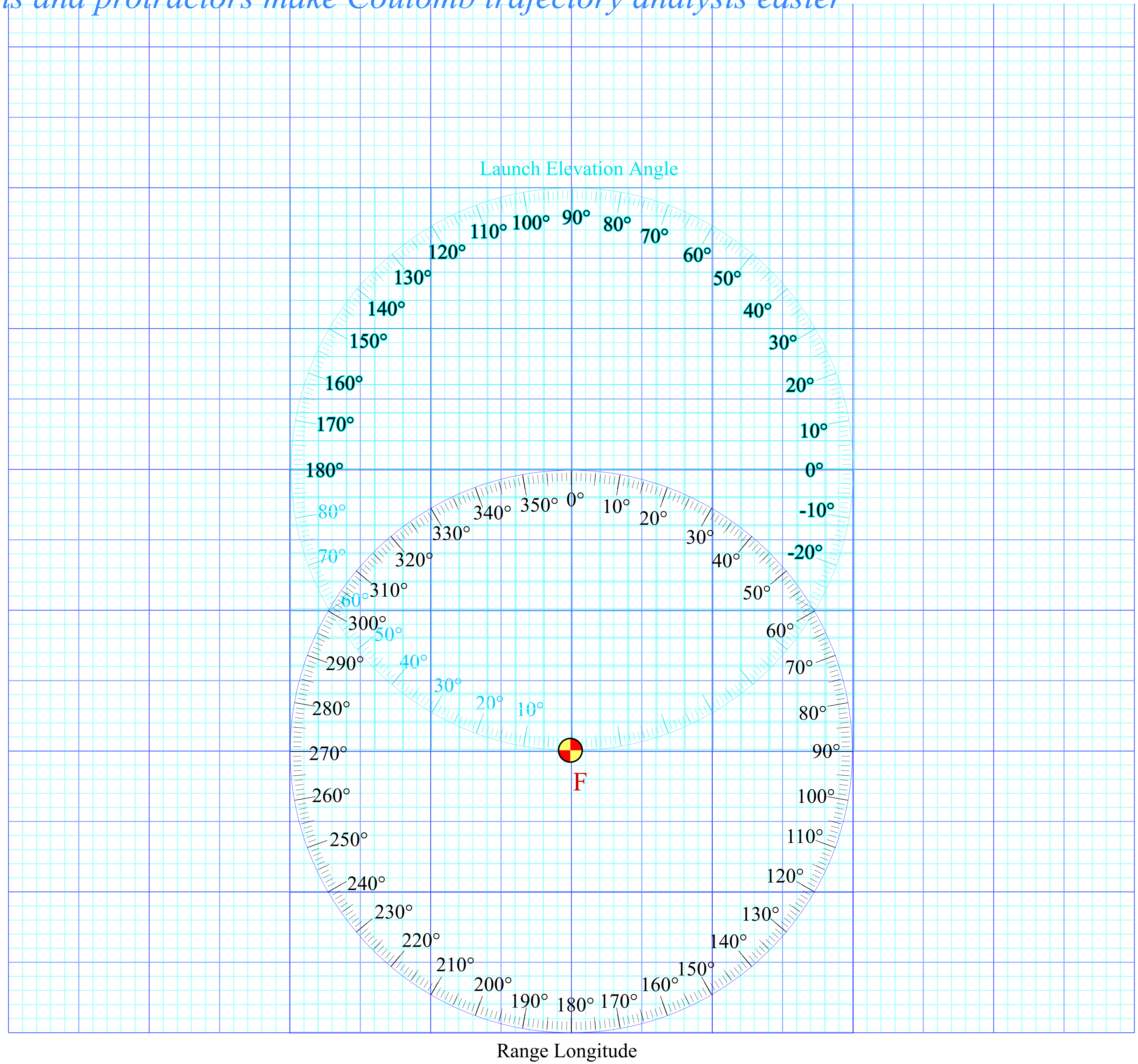
Graphical ε-development of orbits

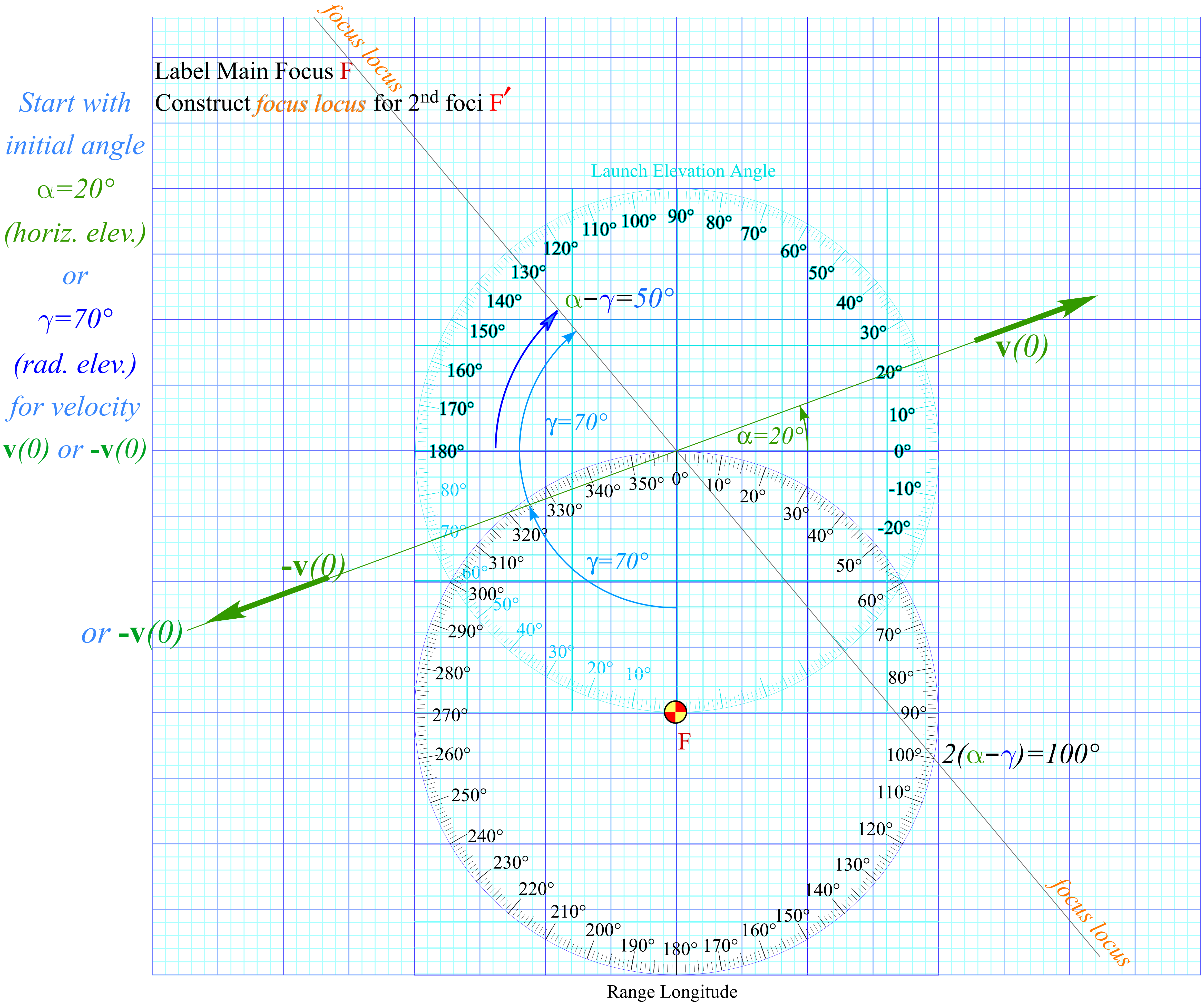
➔ *Launch angle fixed-Varied launch energy*

Launch energy fixed-Varied launch angle

Launch optimization and orbit family envelopes

Graphs and protractors make Coulomb trajectory analysis easier



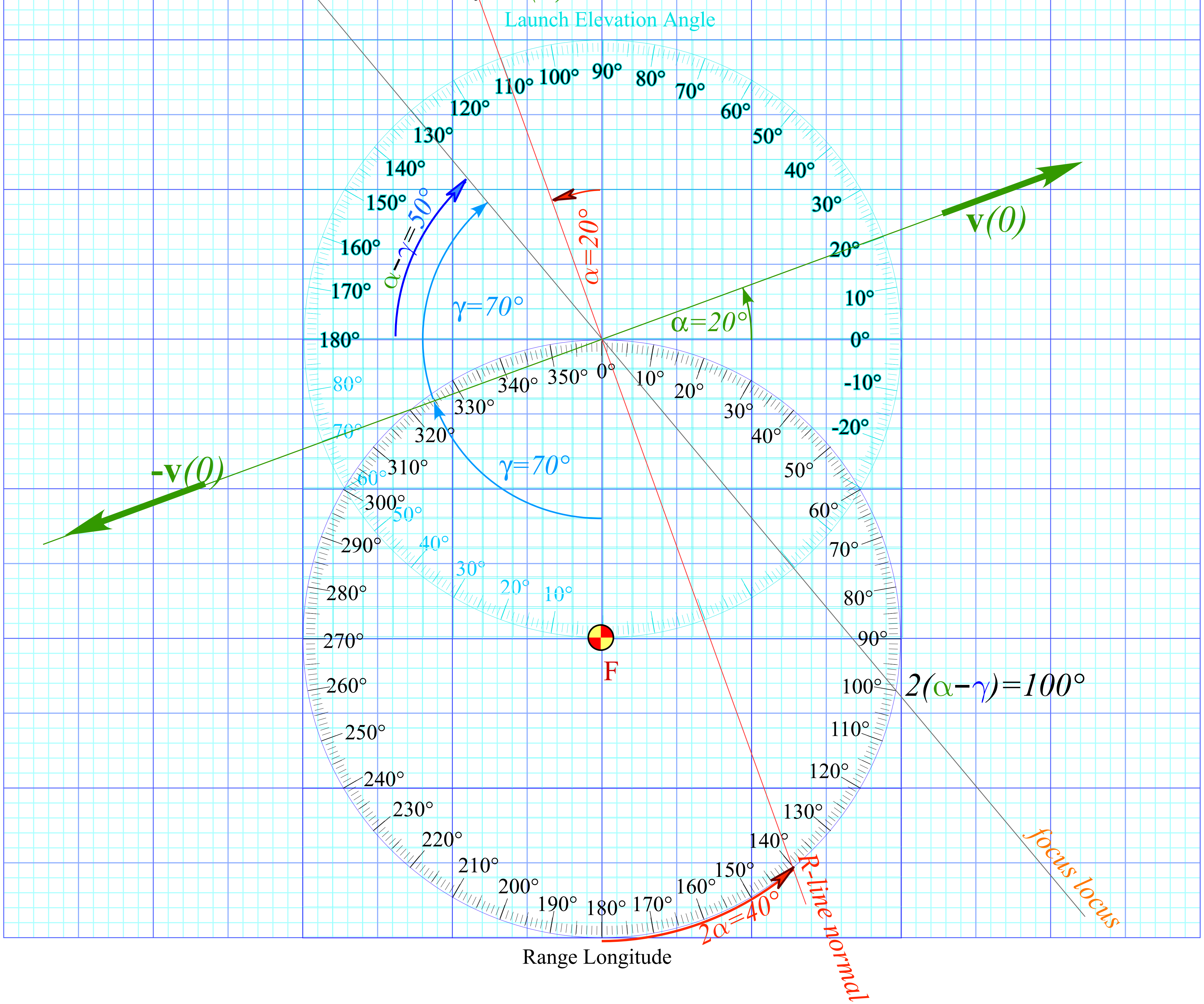

Label Main Focus F
Construct focus locus for 2nd foci F′
Start with
initial angle
α=20°
(horiz. elev.)
or
γ=70°
(rad. elev.)
for velocity
v(0) or -v(0)
focus locus
Launch Elevation Angle
α−γ=50°
γ=70°
α=20°
v(0)
-v(0)
or -v(0)
γ=70°
F
2(α−γ)=100°
focus locus
Range Longitude

Start with initial angle
$\alpha=20°$
(horiz. elev.)
or
$\gamma=70°$
(rad. elev.)
for velocity
$\mathbf{v}(0)$ *or* $-\mathbf{v}(0)$

Label Main Focus F

Construct *focus locus* for 2nd foci F′

Construct *R-line normal* to initial velocity $\pm\mathbf{v}(0)$ line

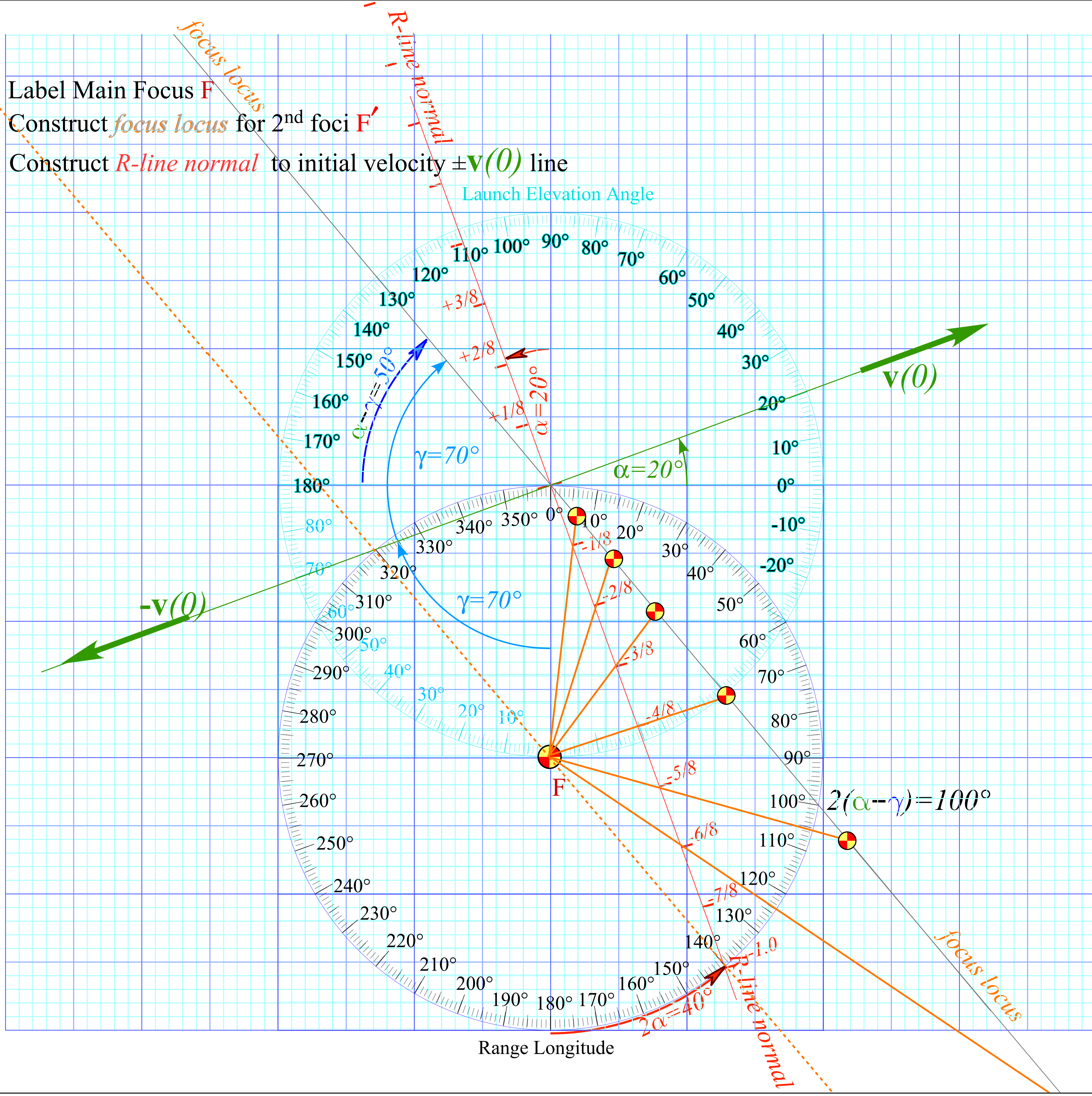


Start with initial angle $\alpha=20°$ *(horiz. elev.)* *or* $\gamma=70°$ *(rad. elev.)* *for velocity* $\mathbf{v}(0)$ *or* $-\mathbf{v}(0)$

Label Main Focus F

Construct *focus locus* for 2nd foci F′

Construct *R-line normal* to initial velocity $\pm\mathbf{v}(0)$ line

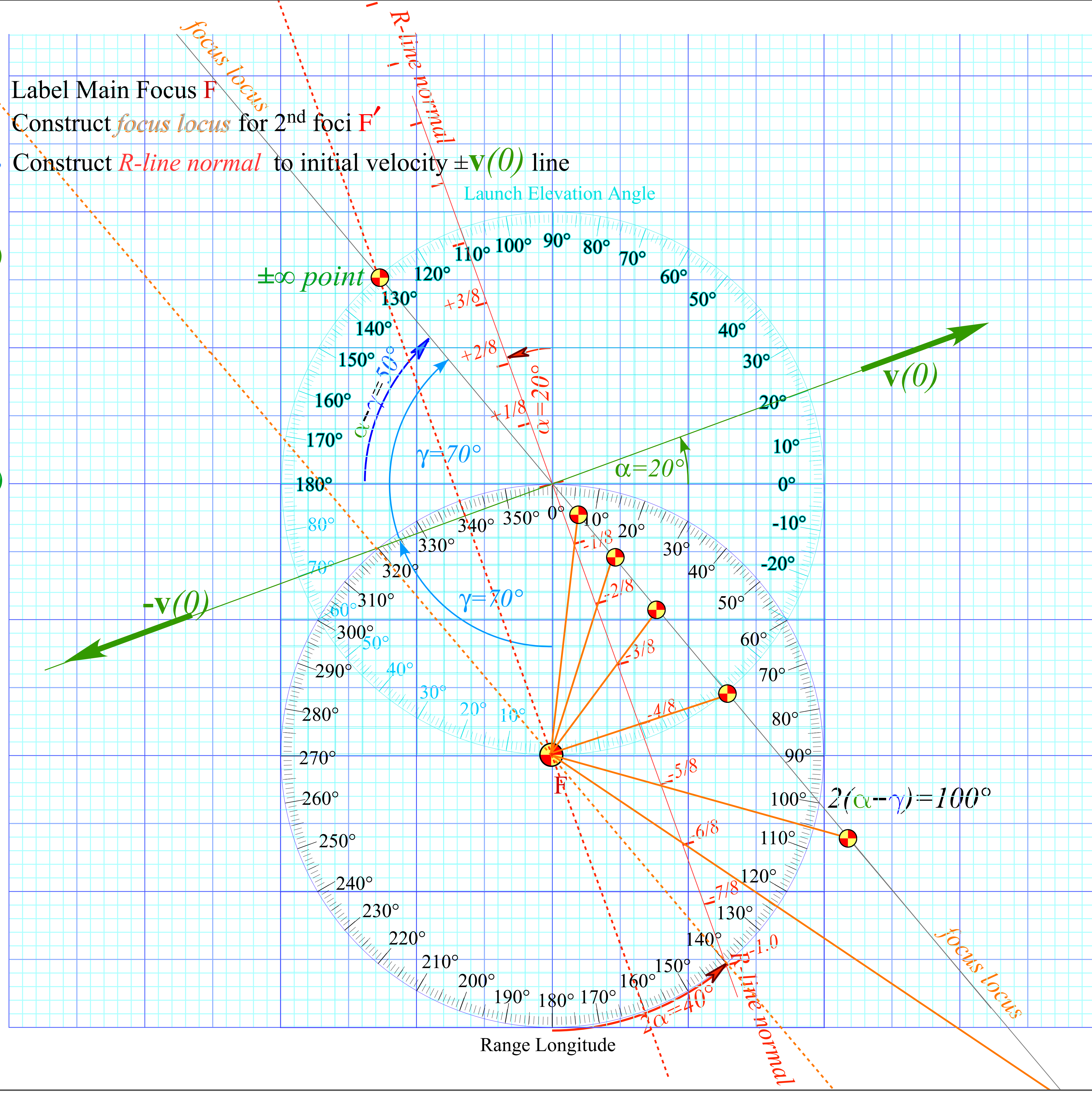


Start with initial angle $\alpha=20°$ *(horiz. elev.)* *or* $\gamma=70°$ *(rad. elev.)* *for velocity* **v**(0) *or* -**v**(0)

Label Main Focus **F**

Construct *focus locus* for 2^{nd} foci **F′**

Construct *R-line normal* to initial velocity ±**v**(0) line



Start with initial angle

$\alpha=20°$

(horiz. elev.)

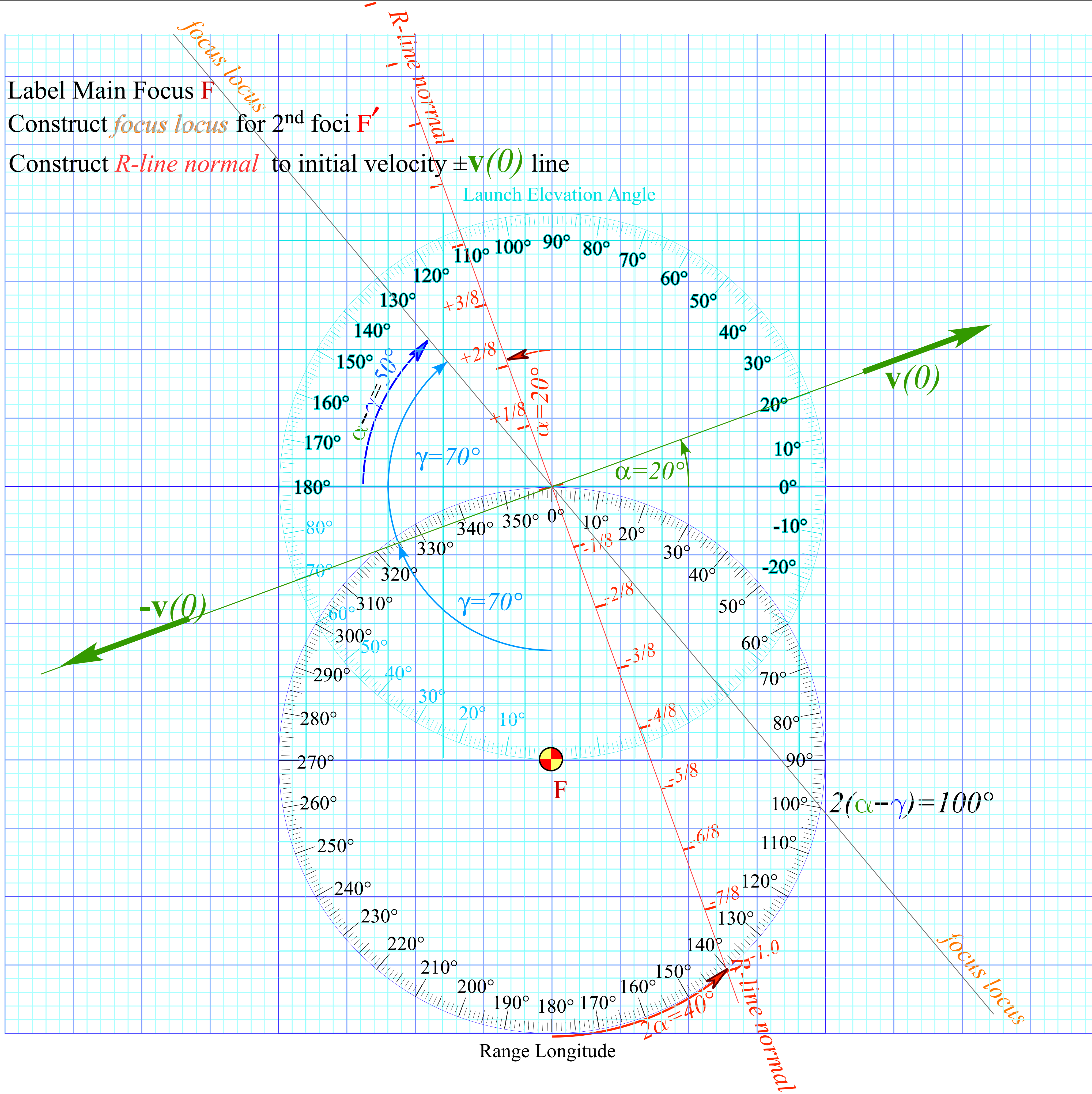
or

$\gamma=70°$

(rad. elev.)

for velocity

$\mathbf{v}(0)$ *or* $-\mathbf{v}(0)$



Start with initial angle

α=20° (horiz. elev.)

or

γ=70° (rad. elev.)

for velocity **v**(0) *or* -**v**(0)

or -**v**(0)

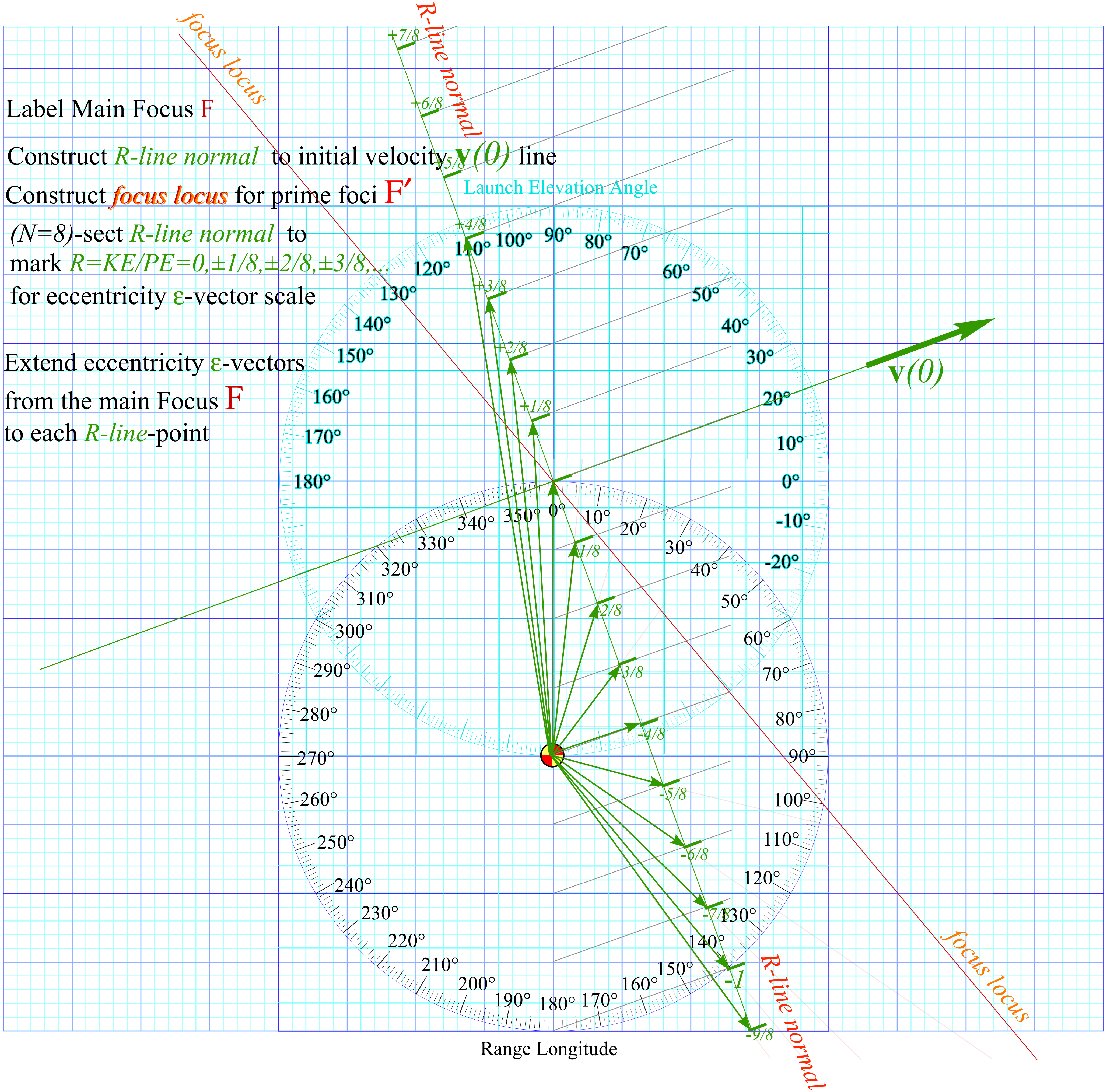
Label Main Focus F

Construct *R-line normal* to initial velocity **v**(0) line

Construct *focus locus* for prime foci F′

(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point



Start with initial angle

$\alpha = 20°$

(horiz. elev.)

or

$\gamma = 70°$

(rad. elev.)

for velocity

$\mathbf{v}(0)$ *or* $-\mathbf{v}(0)$

Label Main Focus F

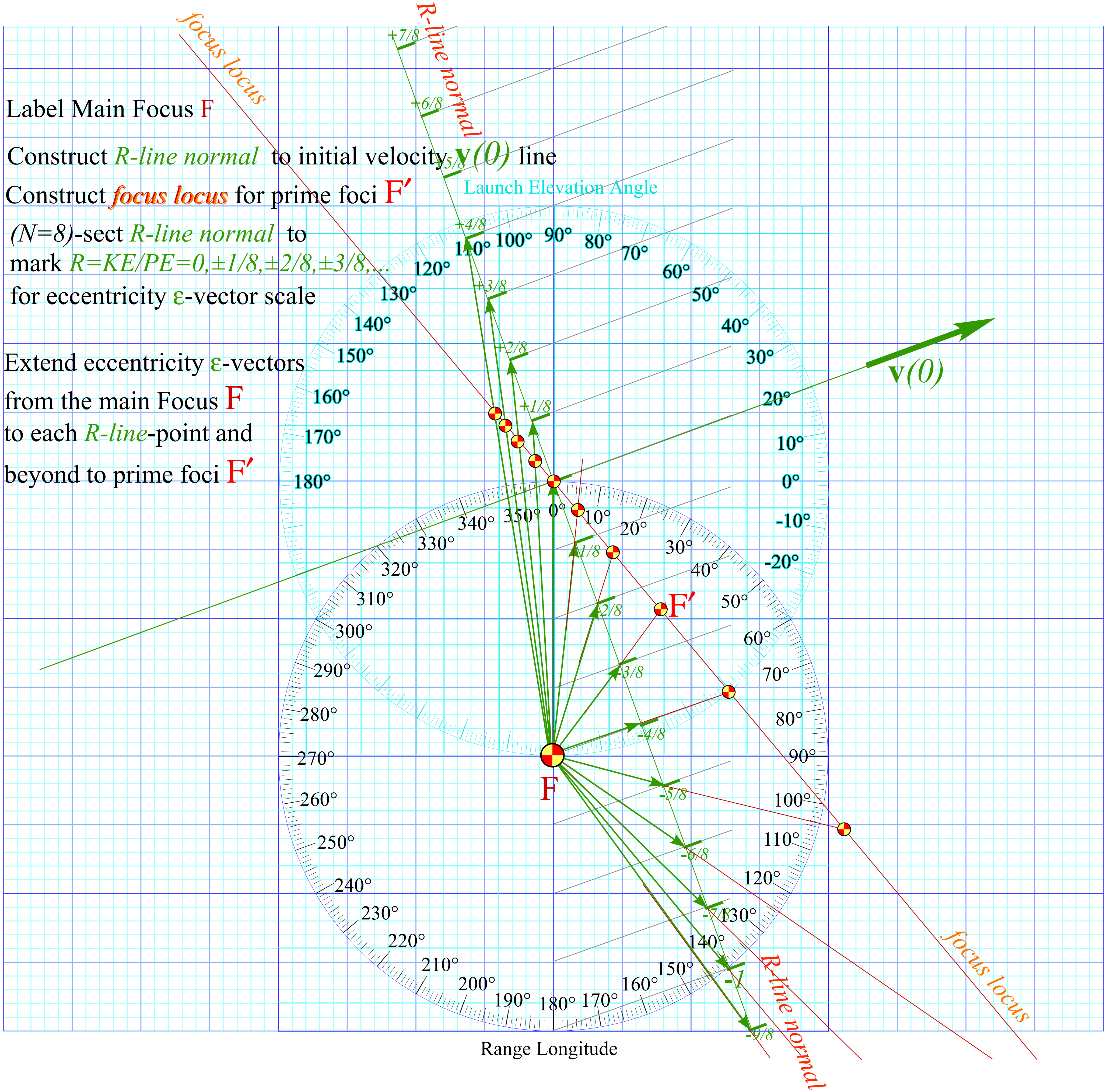
Construct *R-line normal* to initial velocity $\mathbf{v}(0)$ line

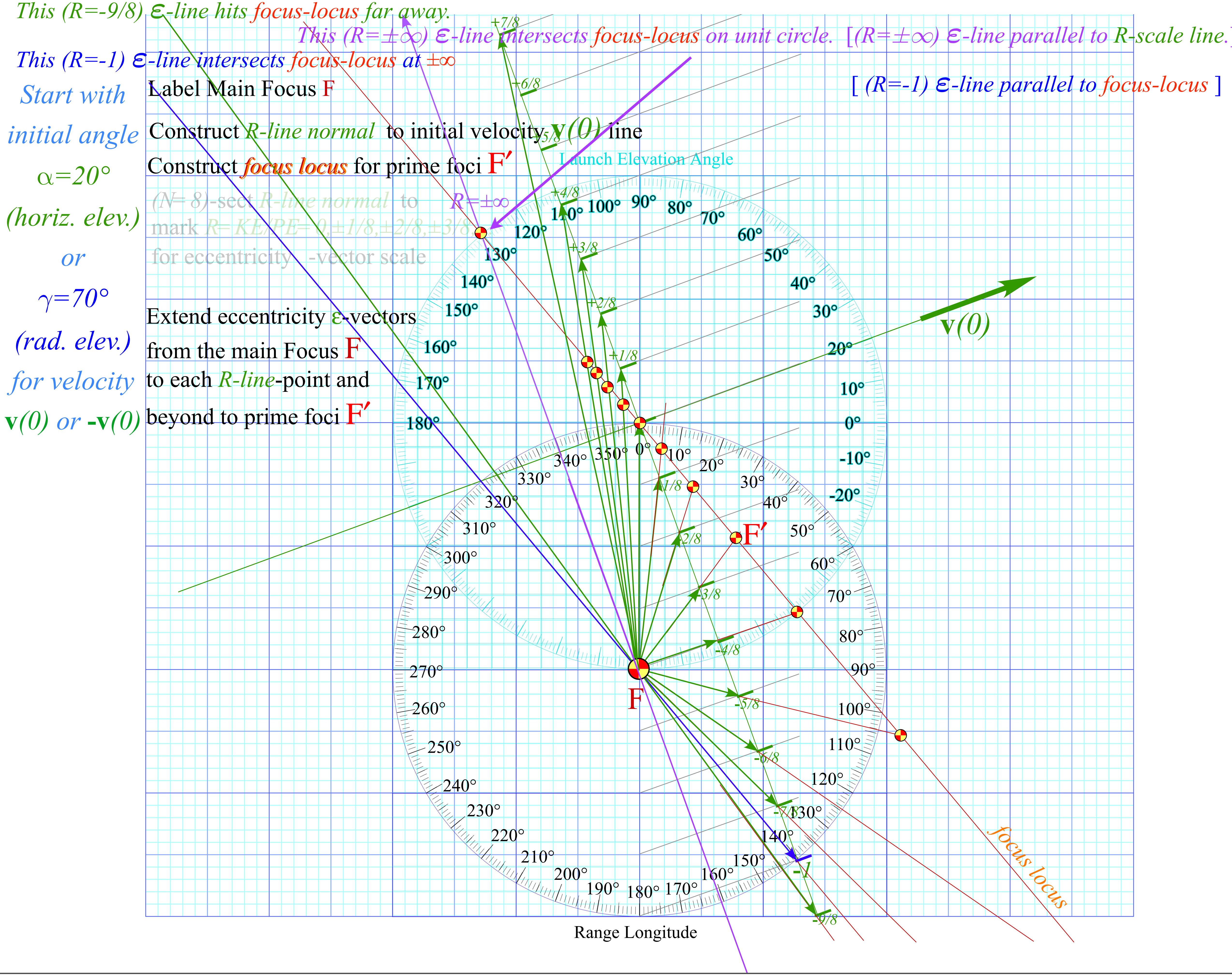
Construct *focus locus* for prime foci F′

(N=8)-sect *R-line normal* to mark $R=KE/PE=0,\pm1/8,\pm2/8,\pm3/8,...$ for eccentricity ε-vector scale

Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

or $-\mathbf{v}(0)$





This (R=-9/8) ε-line hits focus-locus far away.
This (R=±∞) ε-line intersects focus-locus on unit circle. [(R=±∞) ε-line parallel to R-scale line.]
This (R=-1) ε-line intersects focus-locus at ±∞
[(R=-1) ε-line parallel to focus-locus]
Start with
initial angle
α=20°
(horiz. elev.)
or
γ=70°
(rad. elev.)
for velocity
v(0) or -v(0)
Label Main Focus F
Construct R-line normal to initial velocity v(0) line
Construct focus locus for prime foci F′
(N= 8)-sect R-line normal to
mark R= KE/PE= 0,±1/8,±2/8,±3/8
for eccentricity -vector scale
R=±∞
Extend eccentricity ε-vectors
from the main Focus F
to each R-line-point and
beyond to prime foci F′
Launch Elevation Angle
v(0)
F
F′
focus locus
Range Longitude
+7/8
+6/8
+5/8
+4/8
+3/8
+2/8
+1/8
-1/8
-2/8
-3/8
-4/8
-5/8
-6/8
-7/8
-1
-9/8

Properties of Coulomb trajectory families and envelopes

Graphical ε-development of orbits

Launch angle fixed-Varied launch energy

➤ *Launch energy fixed-Varied launch angle*

Launch optimization and orbit family envelopes

Start with initial velocity **v***(0)* *or* **-v***(0)*

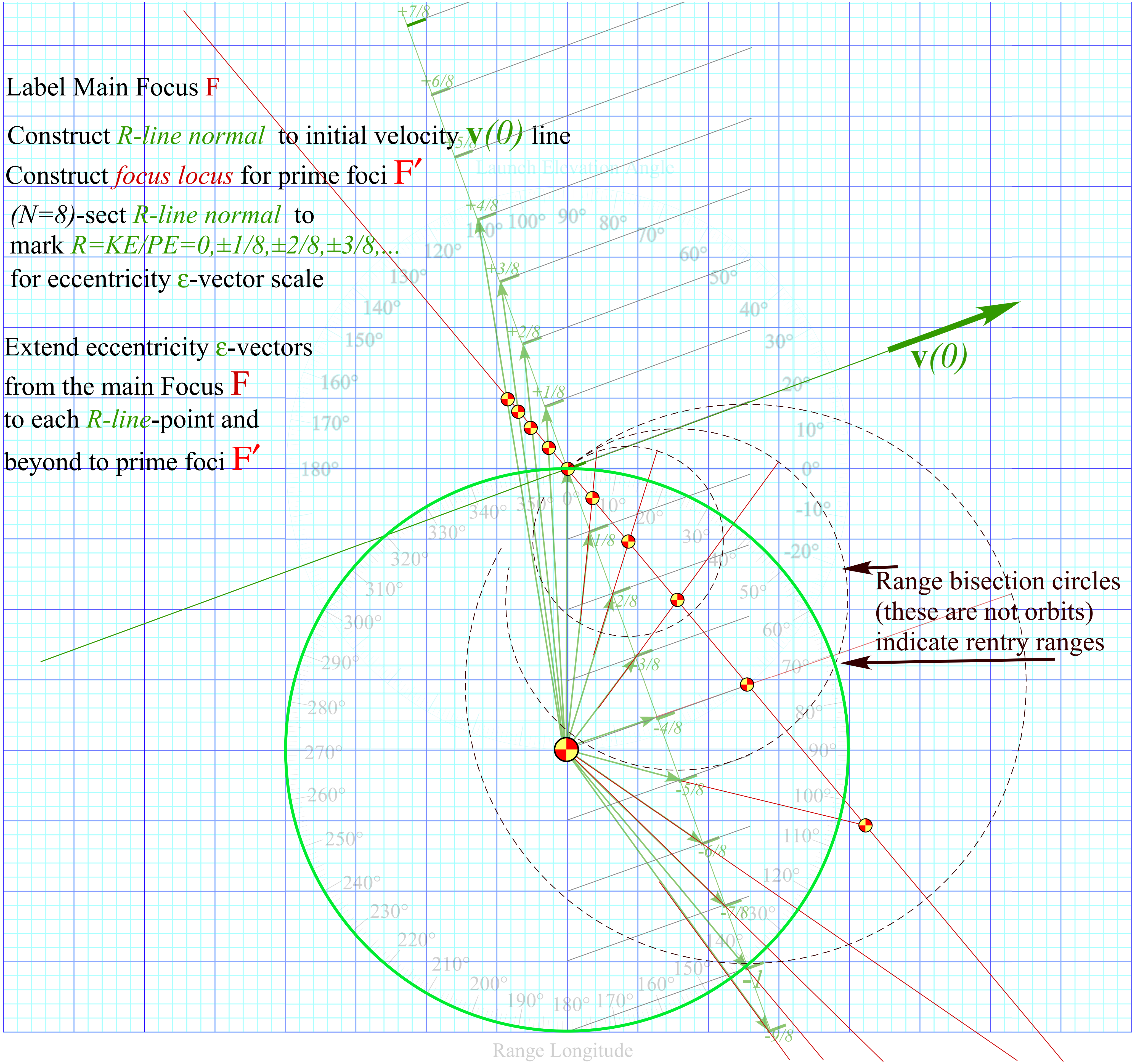
Label Main Focus F

Construct *R-line normal* to initial velocity **v***(0)* line

Construct *focus locus* for prime foci F′

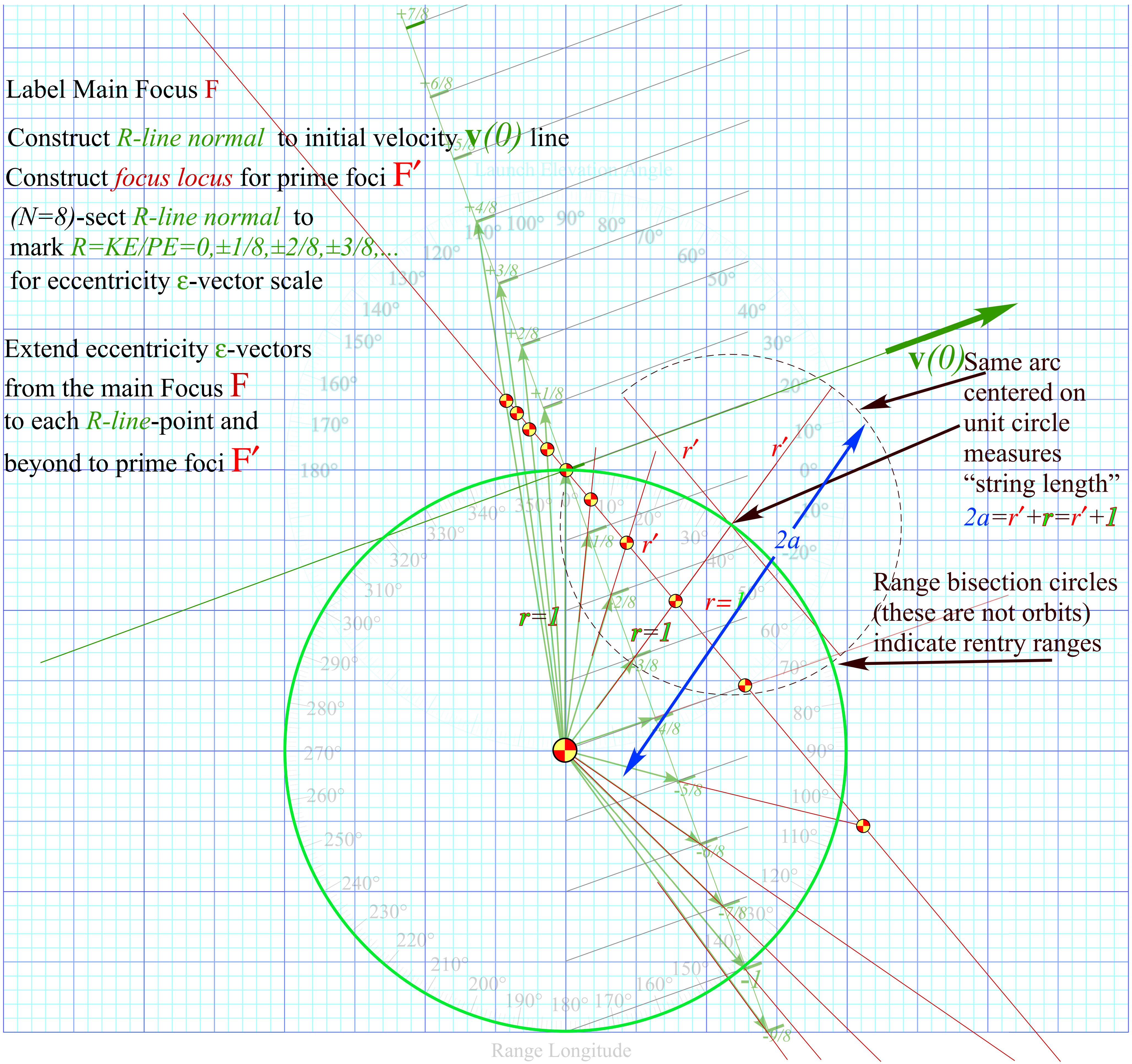
(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′



or **-v***(0)*

Start with initial velocity **v***(0)* *or* **-v***(0)*



Label Main Focus F

Construct *R-line normal* to initial velocity **v***(0)* line

Construct *focus locus* for prime foci F′

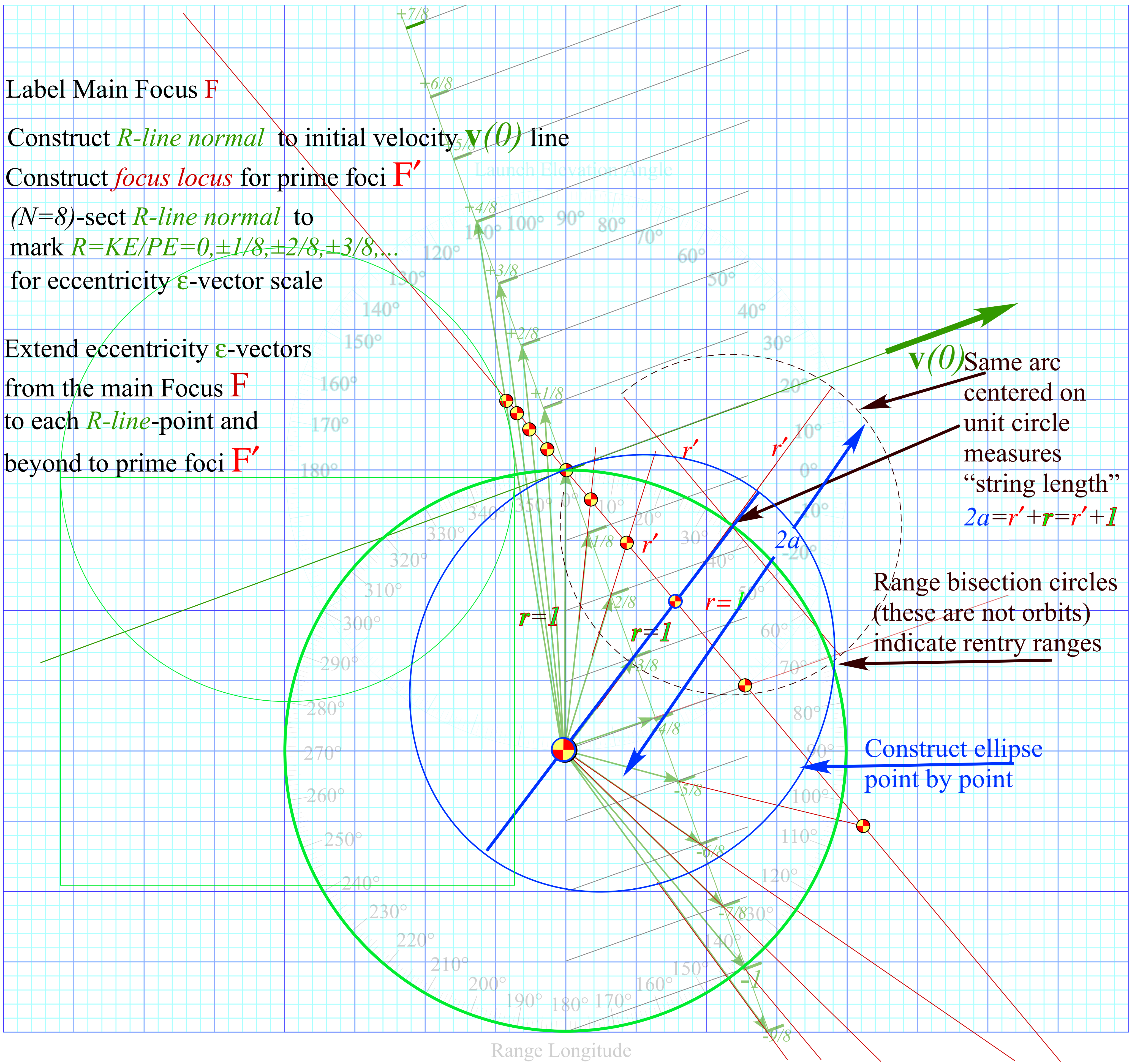
(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

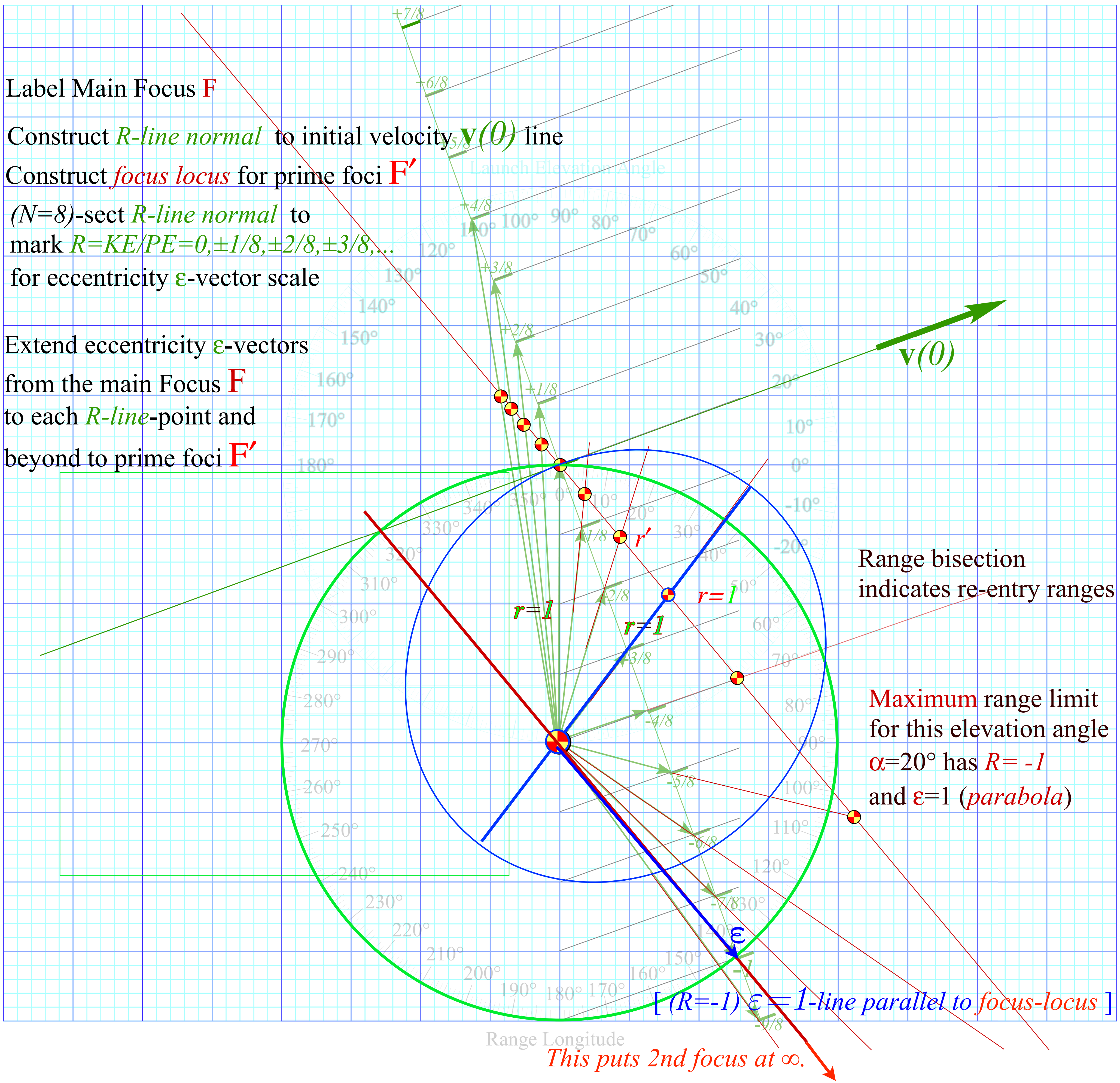
or **-v***(0)*

Start with initial velocity **v***(0)*

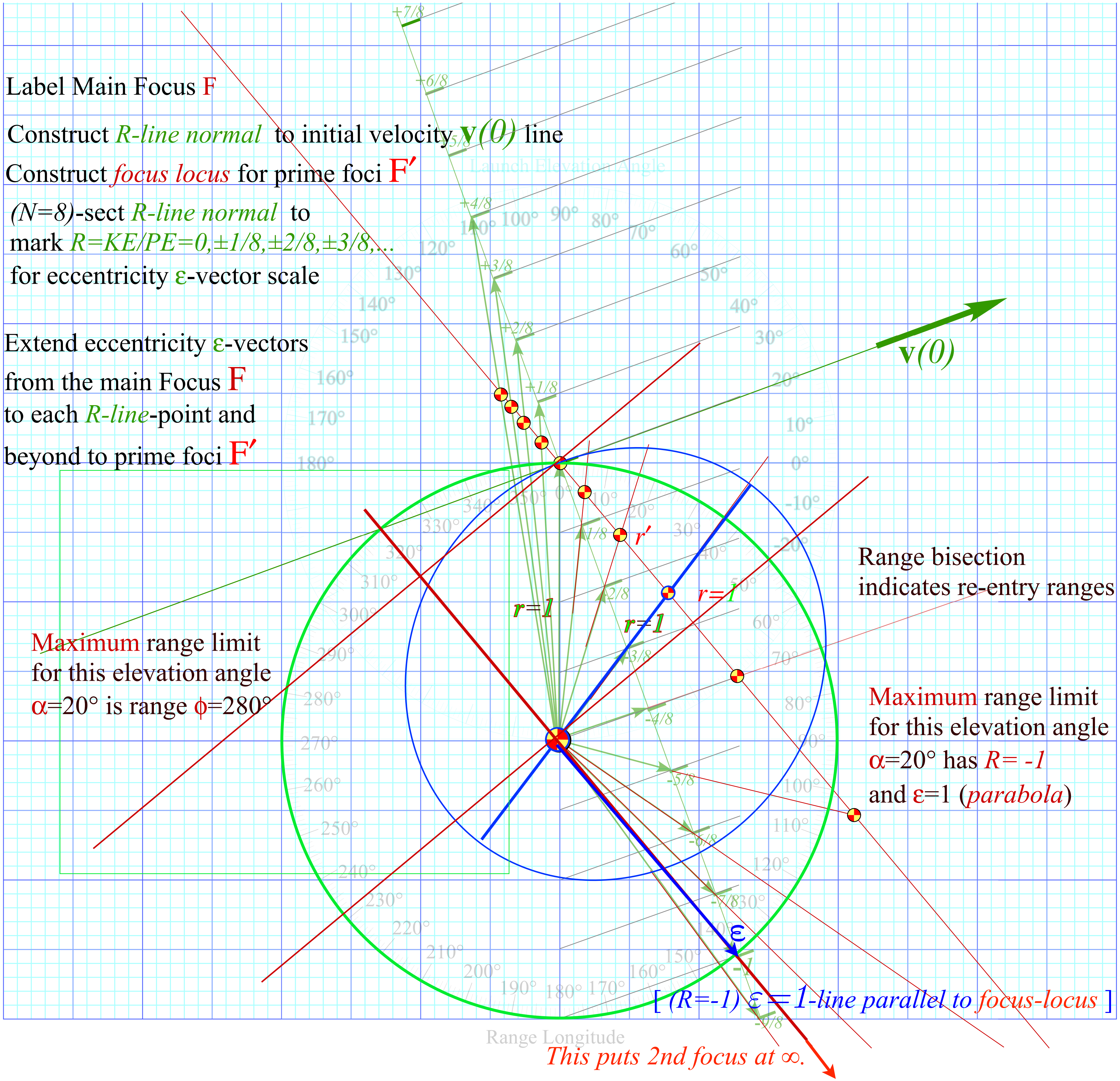
or **-v***(0)*

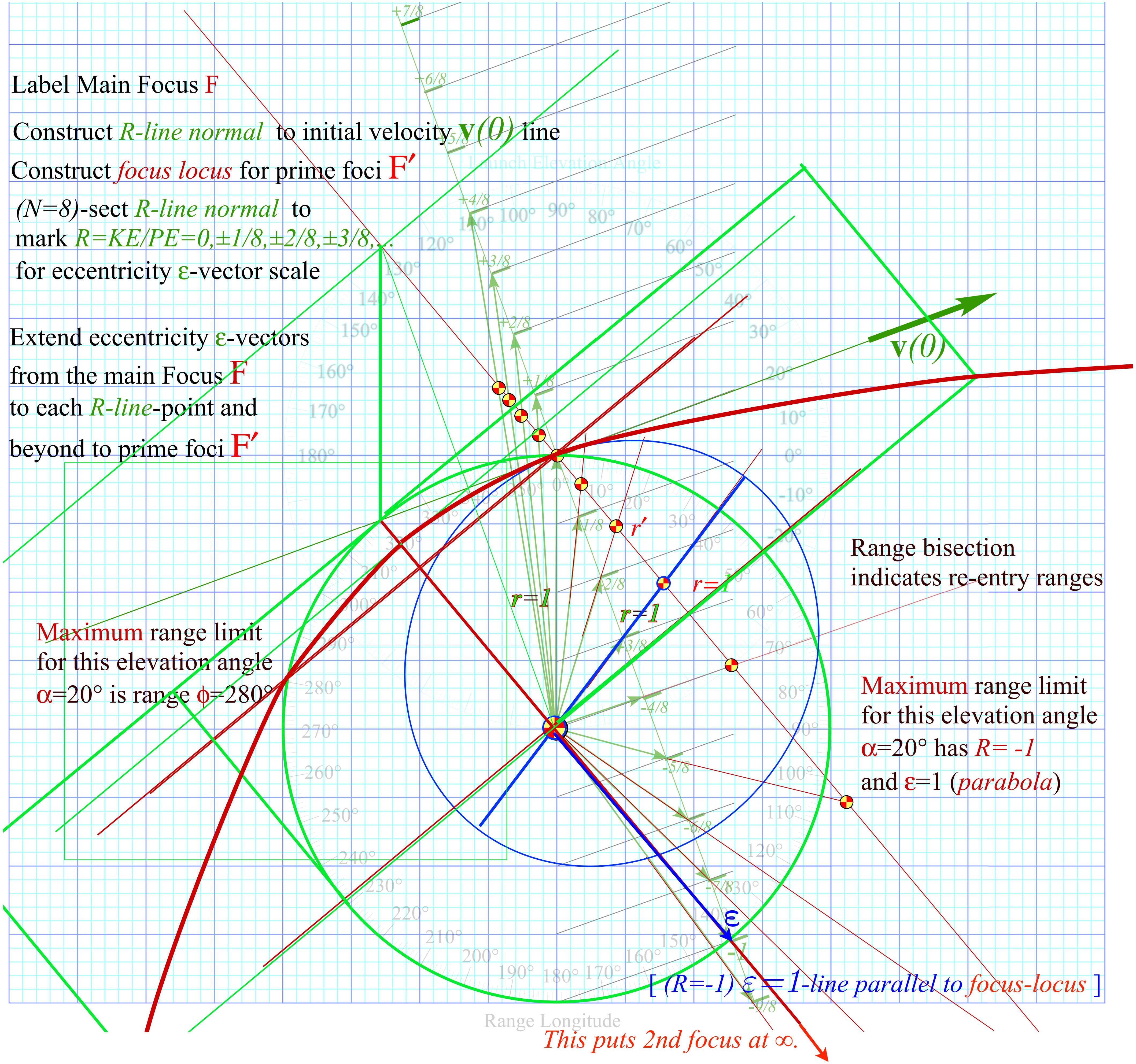


or **-v***(0)*



Label Main Focus F
Construct R-line normal to initial velocity v(0) line
Construct focus locus for prime foci F′
(N=8)-sect R-line normal to
mark R=KE/PE=0,±1/8,±2/8,±3/8,...
for eccentricity ε-vector scale
Extend eccentricity ε-vectors
from the main Focus F
to each R-line-point and
beyond to prime foci F′
Launch Elevation Angle
v(0)
r′
r=1
r=1
r=1
ε
Range bisection
indicates re-entry ranges
Maximum range limit
for this elevation angle
α=20° has R= -1
and ε=1 (parabola)
[(R=-1) ε=1-line parallel to focus-locus]
Range Longitude
This puts 2nd focus at ∞.


Label Main Focus F
Construct R-line normal to initial velocity v(0) line
Construct focus locus for prime foci F′
(N=8)-sect R-line normal to
mark R=KE/PE=0,±1/8,±2/8,±3/8,...
for eccentricity ε-vector scale
Extend eccentricity ε-vectors
from the main Focus F
to each R-line-point and
beyond to prime foci F′
Maximum range limit
for this elevation angle
α=20° is range ϕ=280°
v(0)
Launch Elevation Angle
r=1
r′
Range bisection
indicates re-entry ranges
Maximum range limit
for this elevation angle
α=20° has R= -1
and ε=1 (parabola)
ε
[(R=-1) ε=1-line parallel to focus-locus]
Range Longitude
This puts 2nd focus at ∞.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(N=8)-sect R-line normal to
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Maximum range limit
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α=20° is range ϕ=280°
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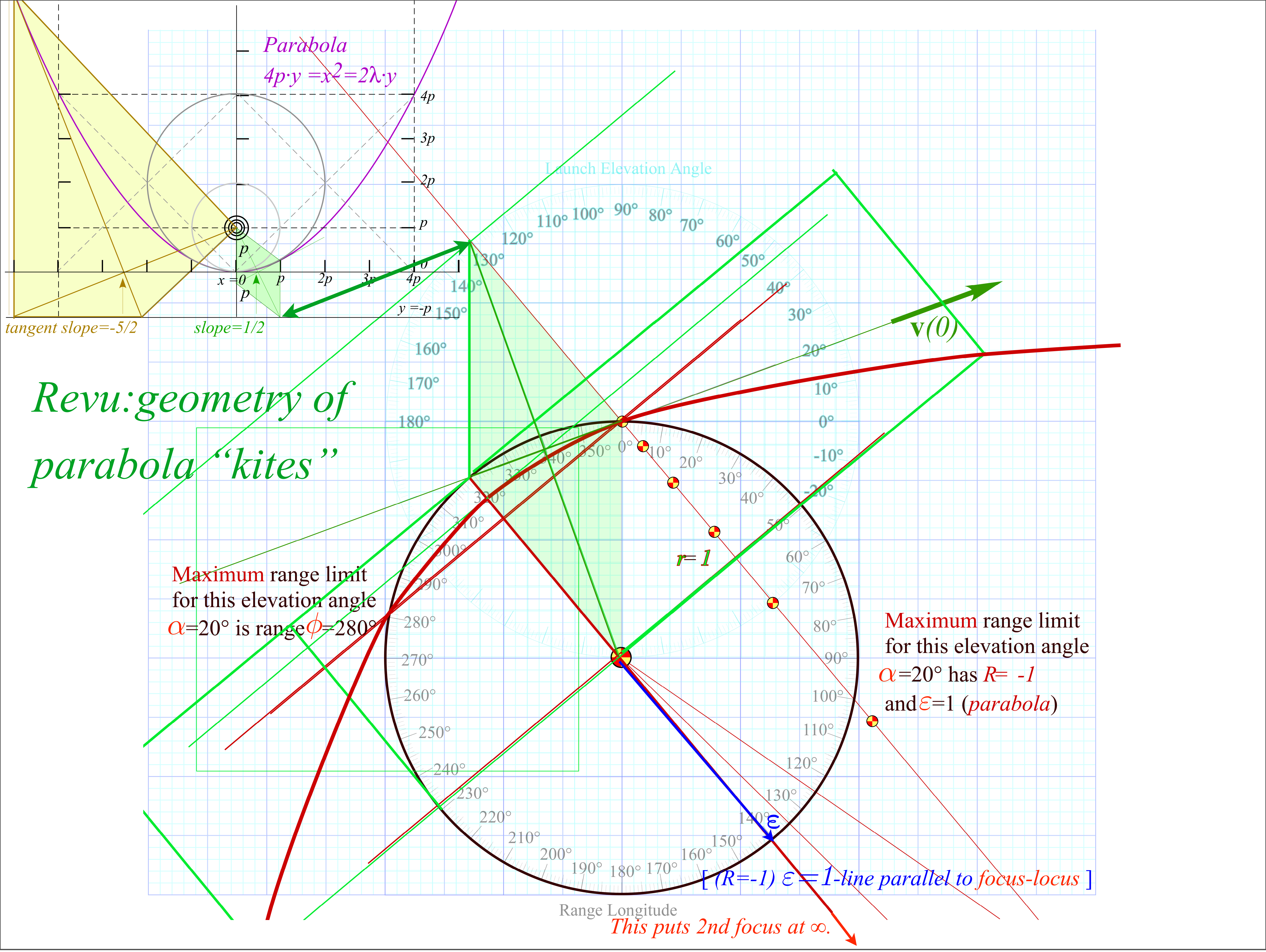
indicates re-entry ranges
Maximum range limit
for this elevation angle
α=20° has R= -1
and ε=1 (parabola)
r=1
r=1
r=1
r′
ε
[(R=-1) ε=1-line parallel to focus-locus]
Launch Elevation Angle
Range Longitude
This puts 2nd focus at ∞.

Revu:geometry of parabola "kites"



Properties of Coulomb trajectory families and envelopes

Graphical ε-development of orbits

Launch angle fixed-Varied launch energy

➤ *Launch energy fixed-Varied launch angle*

Launch optimization and orbit family envelopes

Start with initial velocity $\mathbf{v}(0)$ *or* $-\mathbf{v}(0)$

Label Main Focus F

Construct *R-line normal* to initial velocity $\mathbf{v}(0)$ line

Construct *focus locus* for prime foci F′

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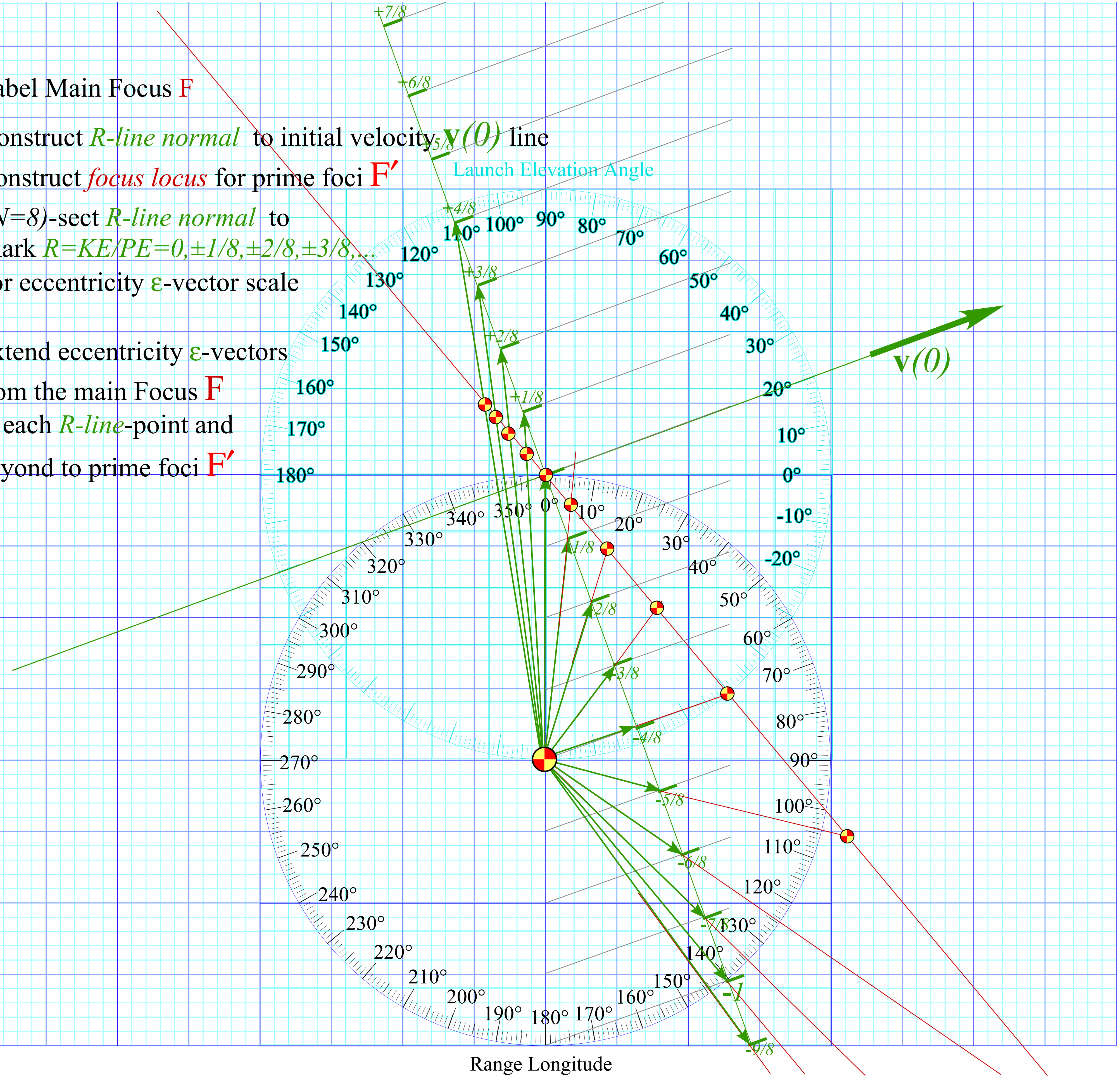
Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

or $-\mathbf{v}(0)$



Label Main Focus F

Construct *R-line normal* to initial velocity **v***(0)* line

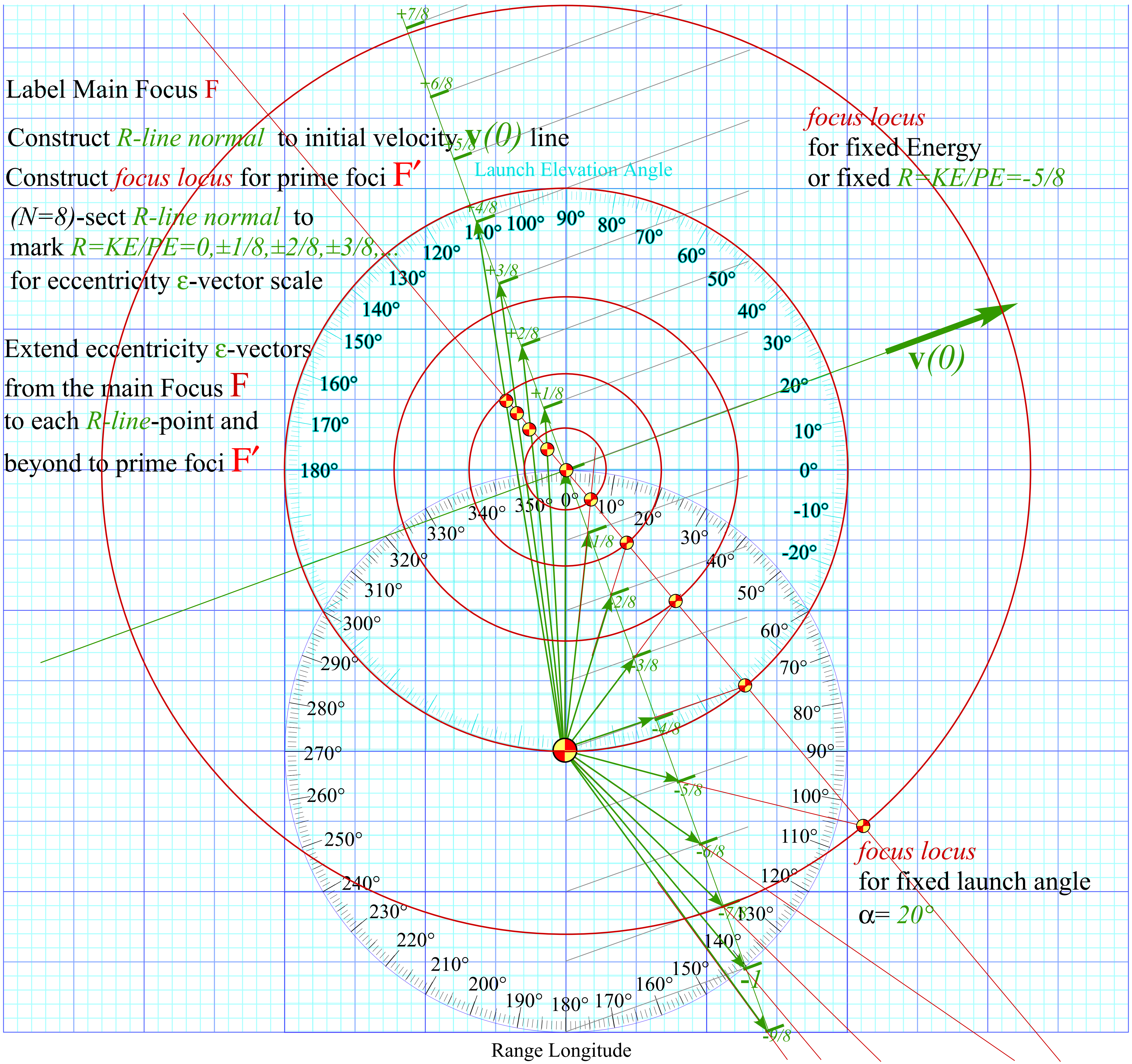
Construct *focus locus* for prime foci F′

(N=8)-sect *R-line normal* to mark *R=KE/PE=0,±1/8,±2/8,±3/8,...* for eccentricity ε-vector scale

Extend eccentricity ε-vectors from the main Focus F to each *R-line*-point and beyond to prime foci F′

focus locus for fixed Energy or fixed *R=KE/PE=-5/8*

focus locus for fixed launch angle α= *20°*



Properties of Coulomb trajectory families and envelopes

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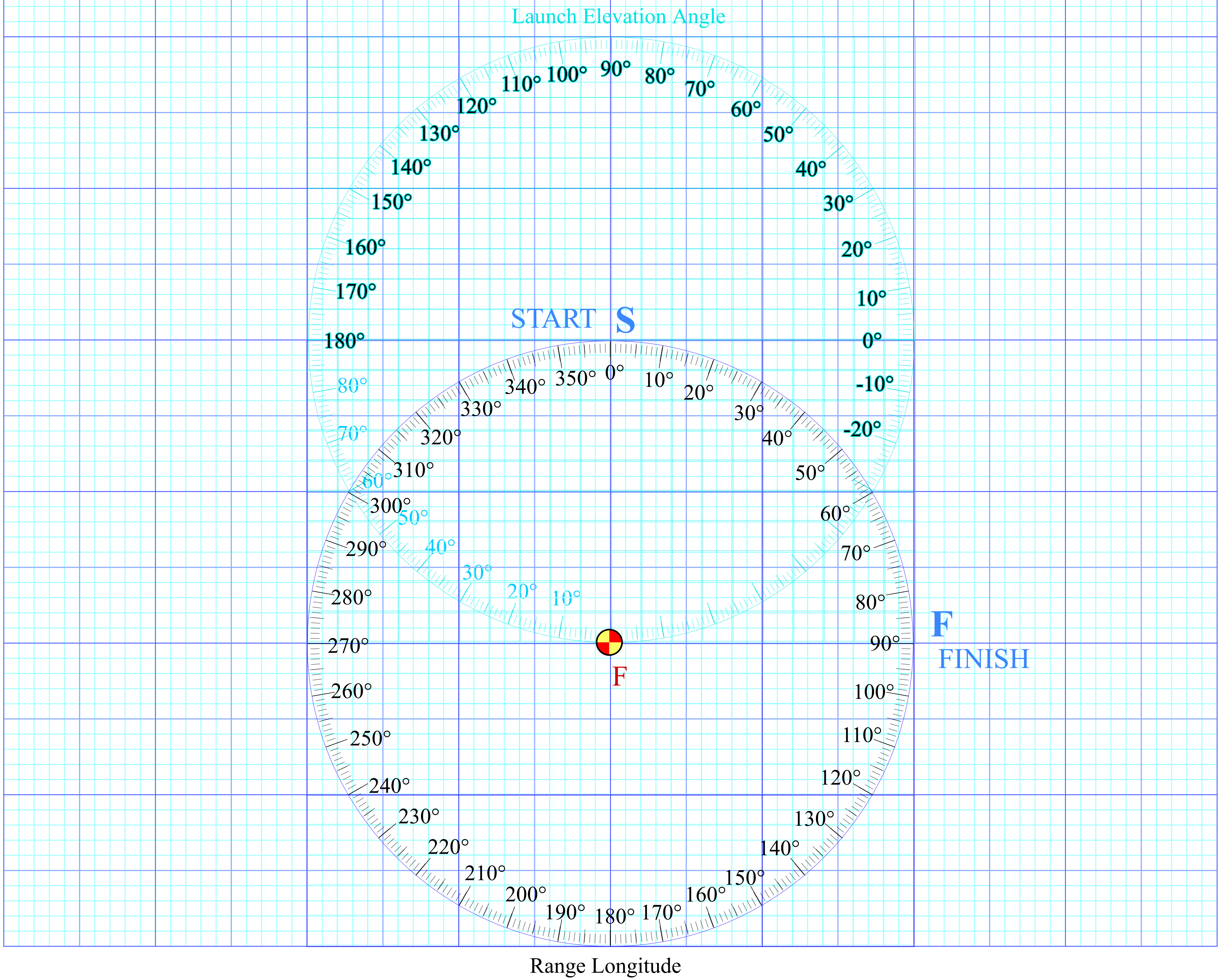
Launch energy fixed-Varied launch angle

➤ *Launch optimization and orbit family envelopes*

Graphs and protractors help Coulomb trajectory launch optimization and orbit family envelopes

Problem:

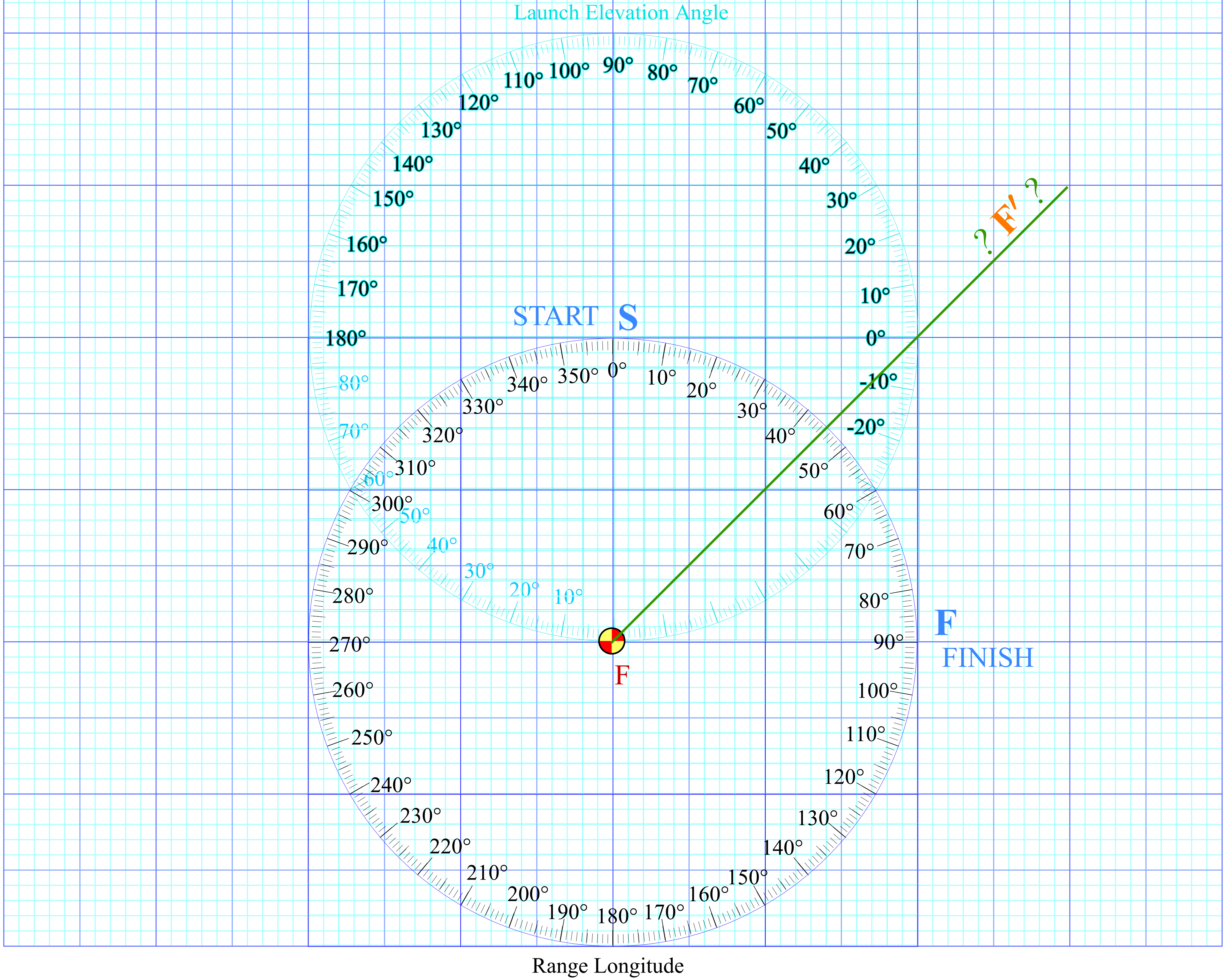
Find trajectory angle of minimum energy to fly 90° of arc (1/4 around planet)



Problem:

Find trajectory angle of minimum energy to fly 90° of longitude (1/4 around planet)

Solution: Prime focus **F′** *lies on radial line that bisects longitude angle*

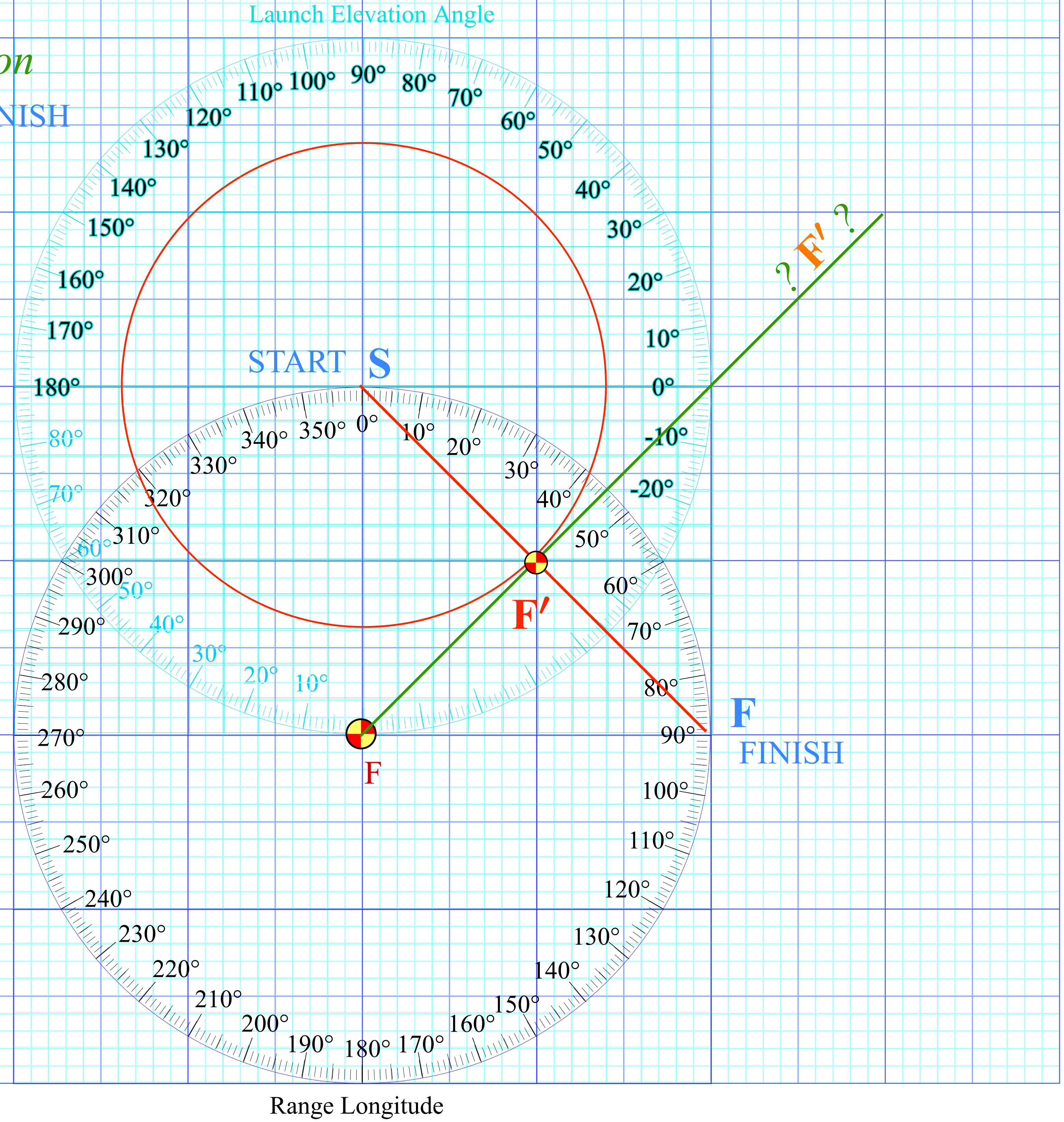


Problem:

Find trajectory angle of minimum energy to fly 90° of longitude (1/4 around planet)

Solution: Prime focus **F′** *lies on radial line that bisects longitude angle*

Optimal prime focus **F′** *lies on line connecting* START *and* FINISH *at tangent point of minimal energy circle* **SF′**.



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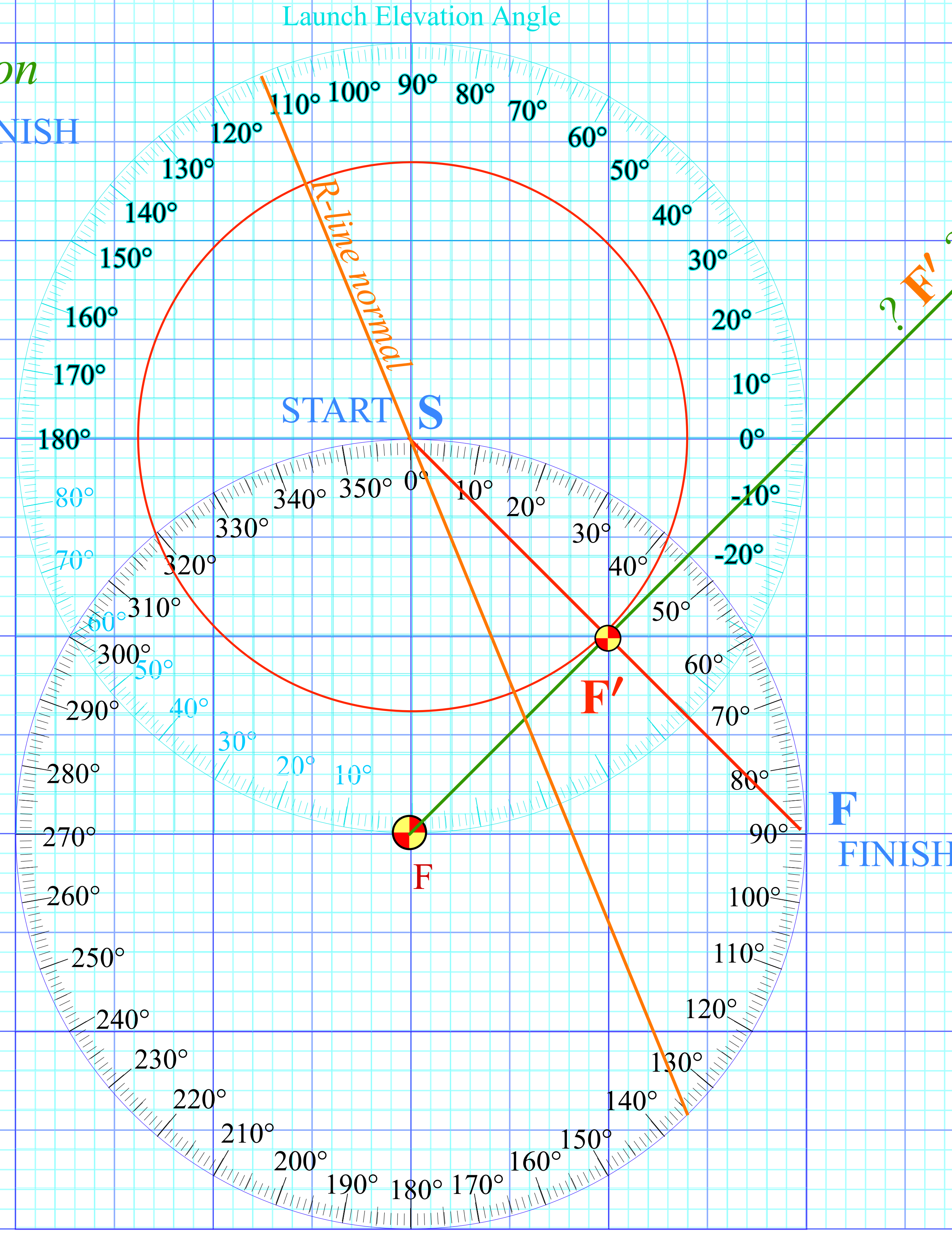
R-line normal must bisect angle **FSF′** *connecting foci* **F** *and* **F′** *and is normal to initial launch vector* $\mathbf{v}_0$



Graphs and protractors help Coulomb trajectory launch optimization and orbit family envelopes

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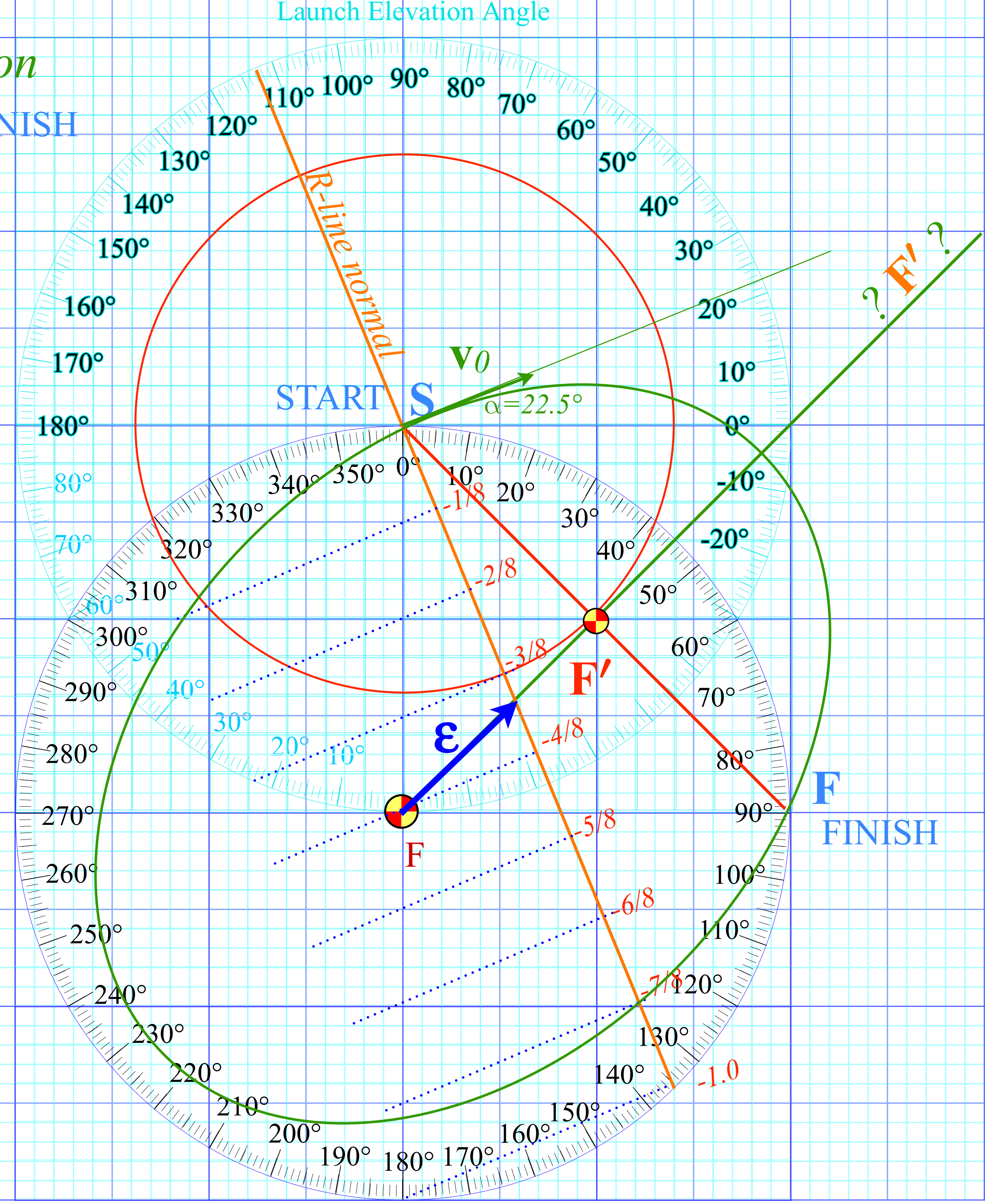
R-line normal must bisect angle **FSF′** *connecting foci* **F** *and* **F′** *and is normal to initial launch vector* $\mathbf{v}_0$ *with launch angle* $\alpha=22.5°$

The ε*-vector and R-value:*



Range Longitude

Problem:

Find trajectory angle of minimum energy to fly 90° of longitude (1/4 around planet)

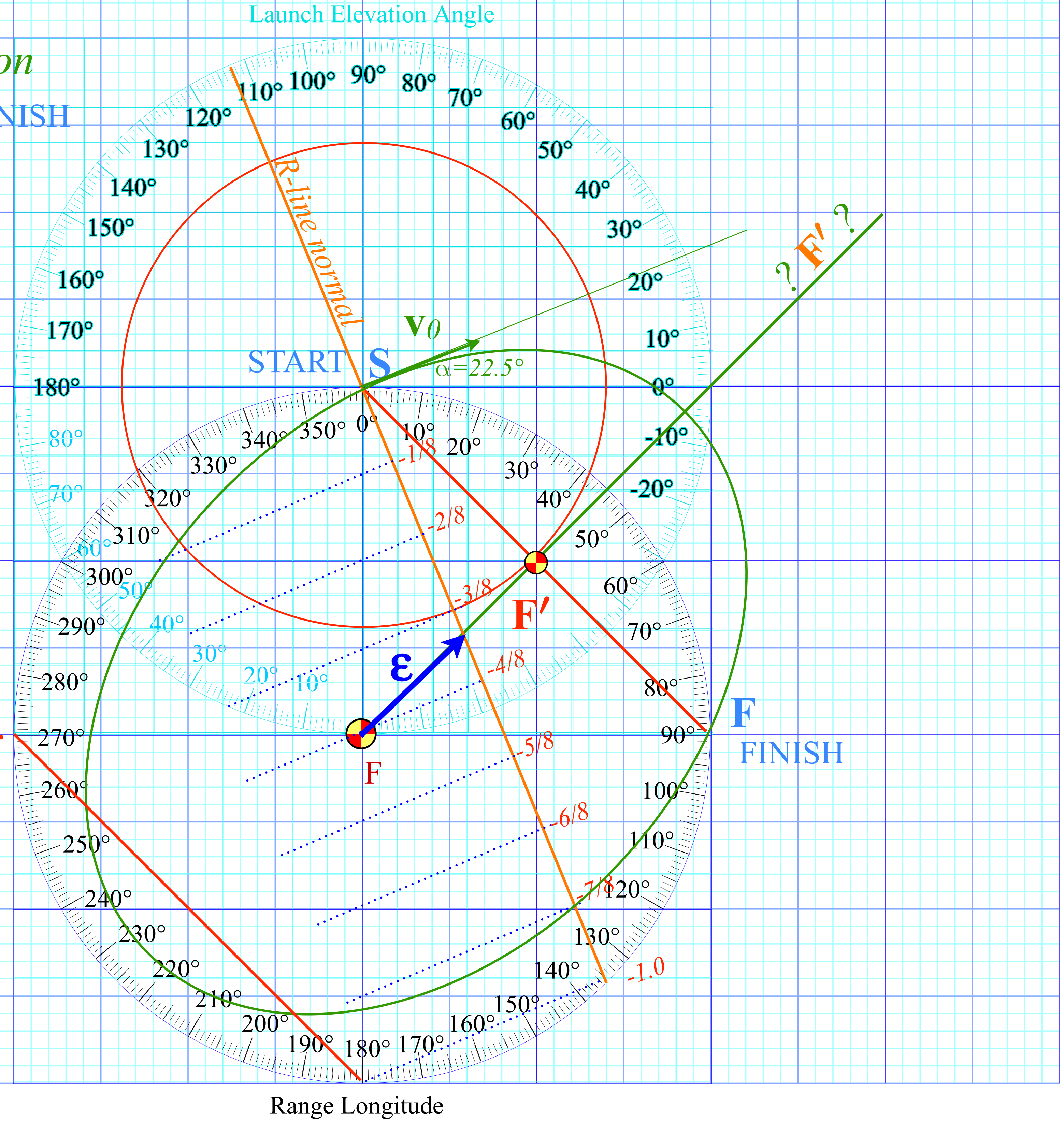
*Solution: Prime focus **F′** lies on radial line that bisects longitude angle*

*Optimal prime focus **F′** lies on line connecting START and FINISH at tangent point of minimal energy circle **SF′**.*

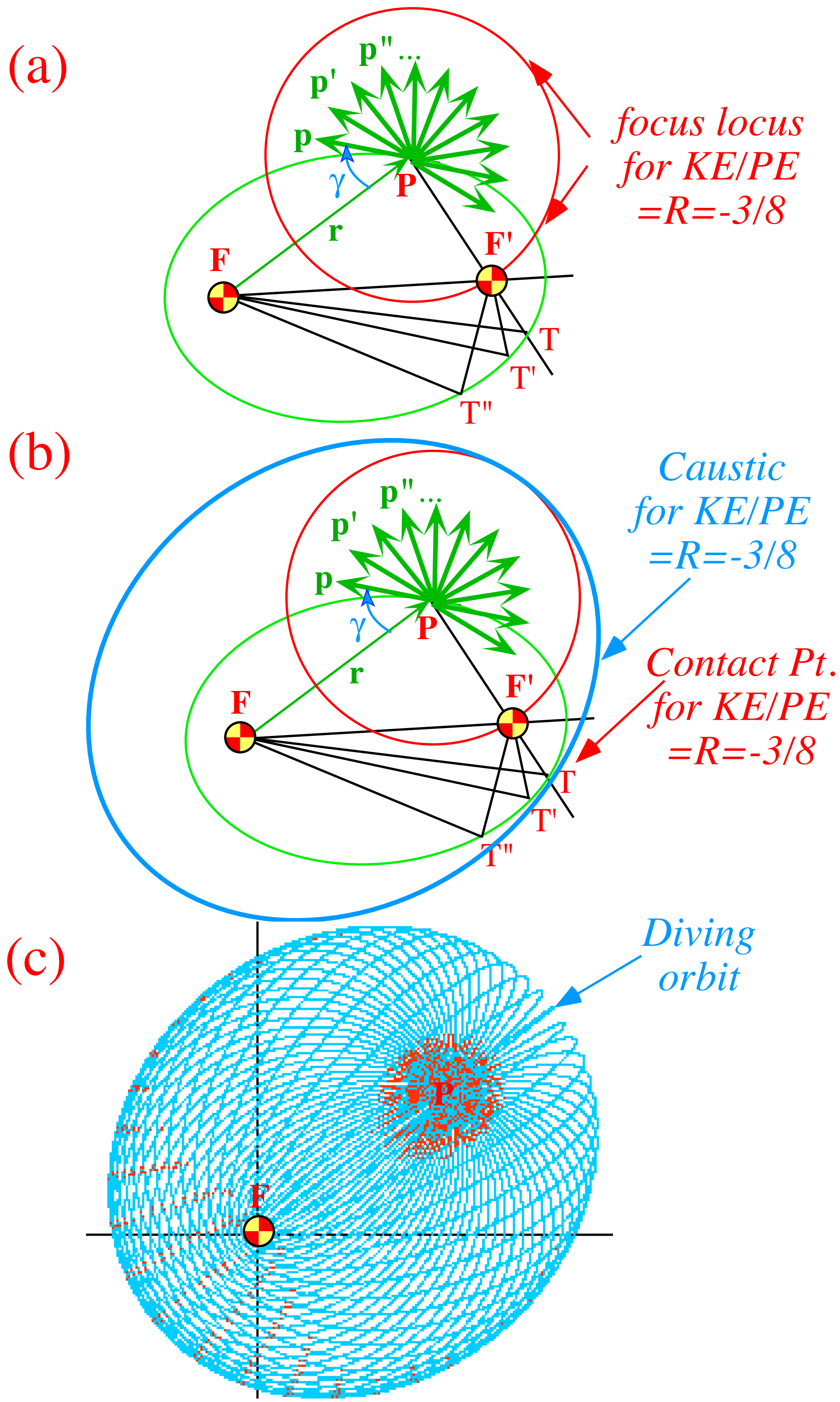
*R-line normal must bisect angle **FSF′** connecting foci **F** and **F′** and is normal to initial launch vector $\mathbf{v}_0$ with launch angle $\alpha=22.5°$*

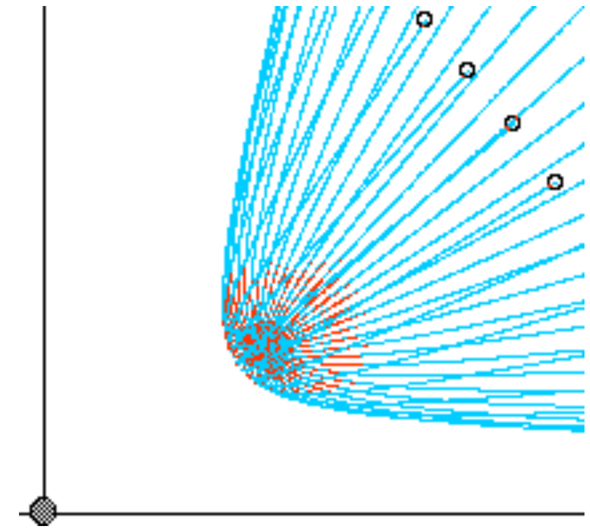
The ε-vector and R-value:

Maximum range 269.999°:

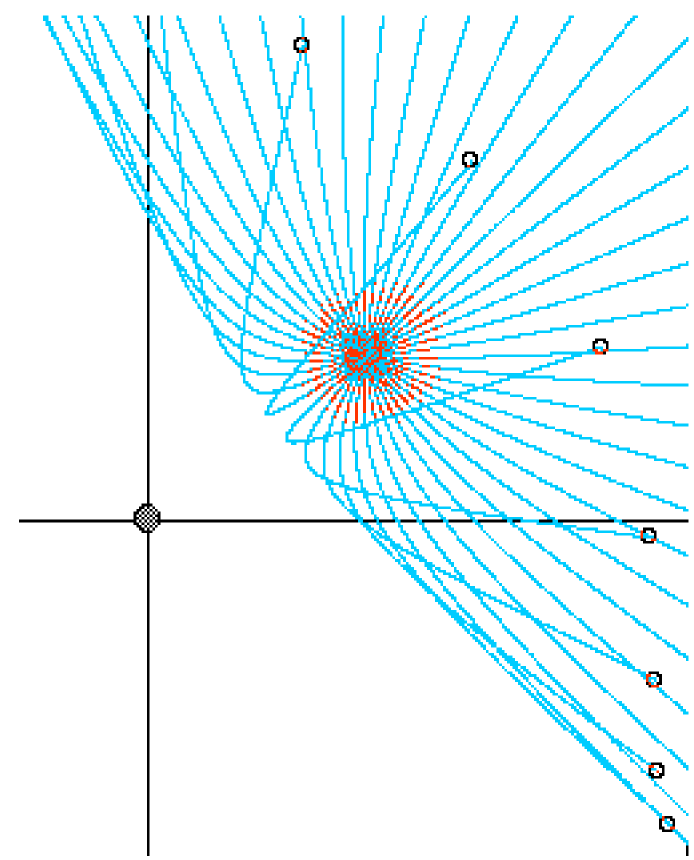


Coulomb envelope geometry





Ideal comet “heads” or “tails” in solar wind



Launch optimization