Lecture 1
Tue. 8.25.2015

Axiomatic development of classical mechanics

(Ch. 1 and Ch. 2 of Unit 1)

Geometry of momentum conservation axiom
- *Totally Inelastic "ka-runch"collisions**
- *Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**
- *Comments on idealization in classical models*

Geometry of Galilean translation symmetry
- *45° shift in (V_1,V_2)-space*
- *Time reversal symmetry*
- *...of COM collisions*

Algebra,Geometry, and Physics of momentum conservation axiom
- *Vector algebra of collisions*
- *Matrix or tensor algebra of collisions*
- *Deriving Energy Conservation Theorem*

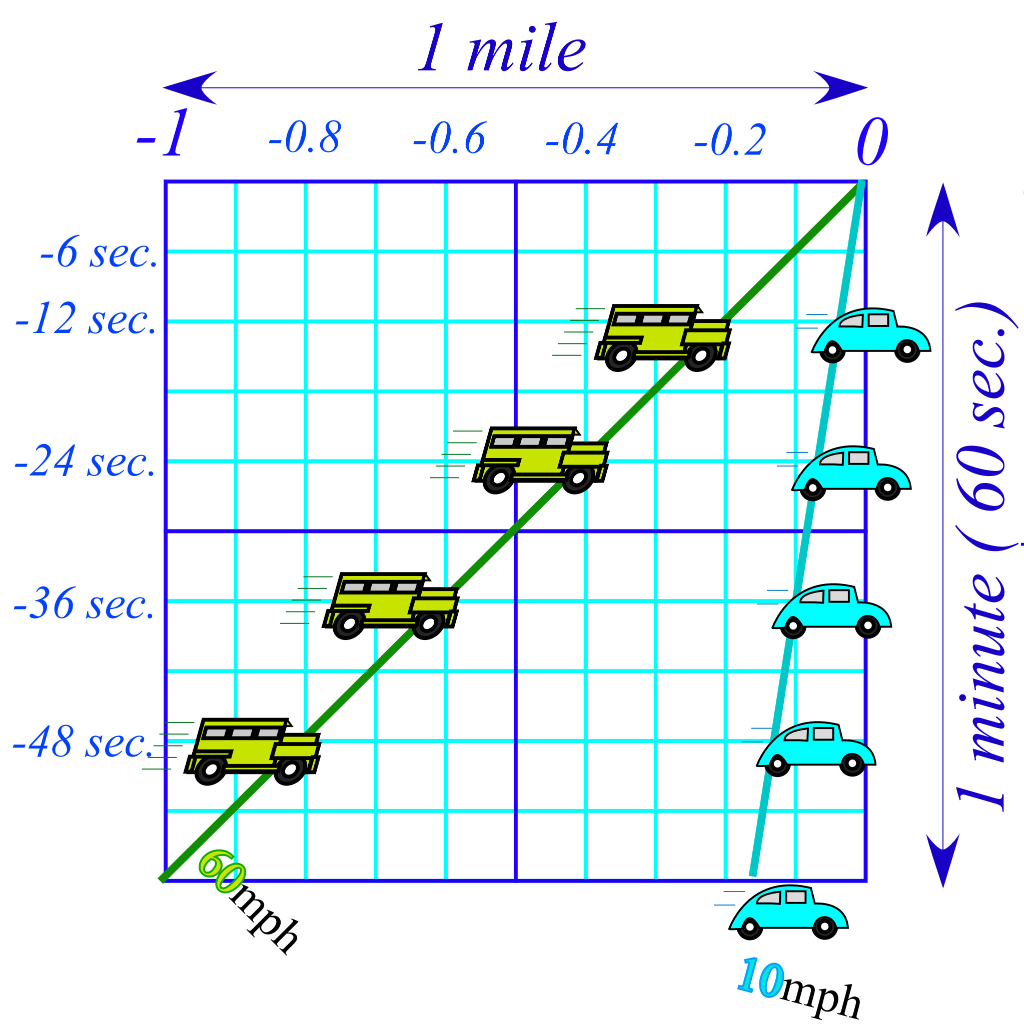
Numerical details of collision tensor algebra

*Car Collision simulator http://www.uark.edu/ua/modphys/testing/markup/CMMotionWeb.html

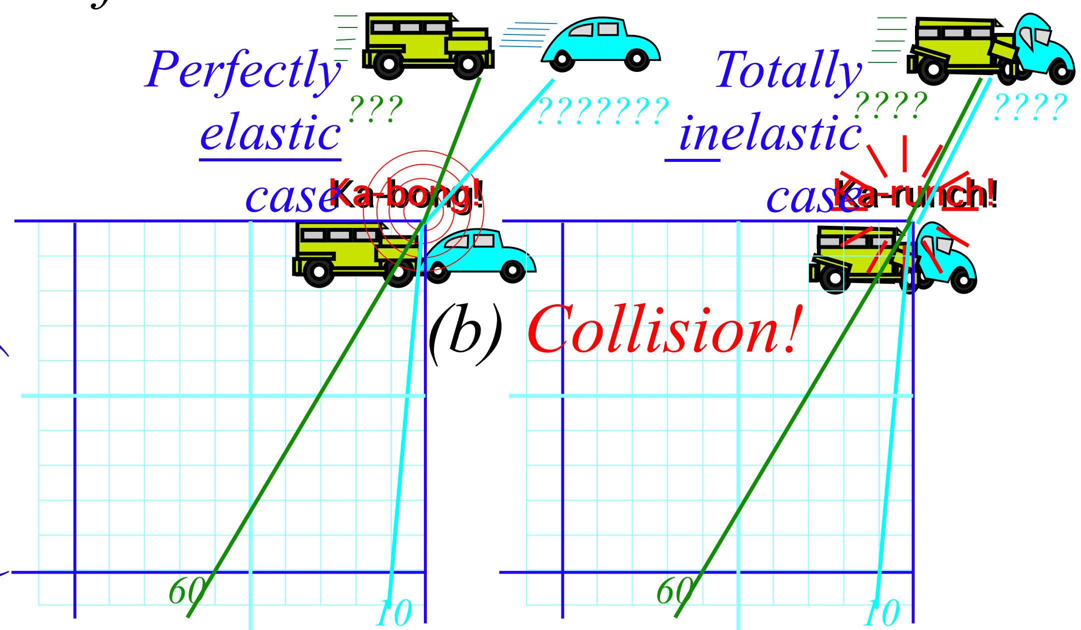
*Download Superball Collision Simulator http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....



After collision...what velocities?



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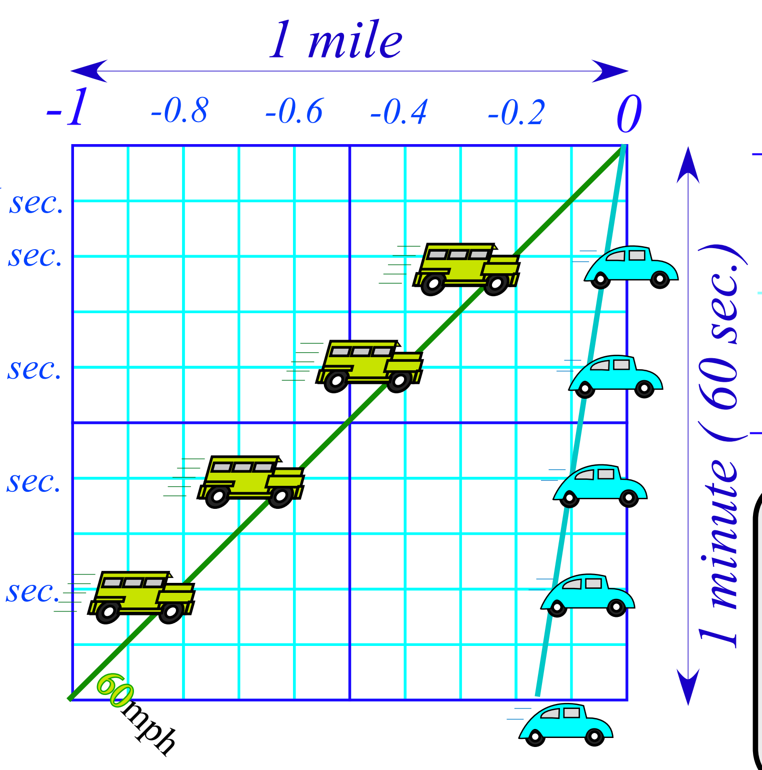

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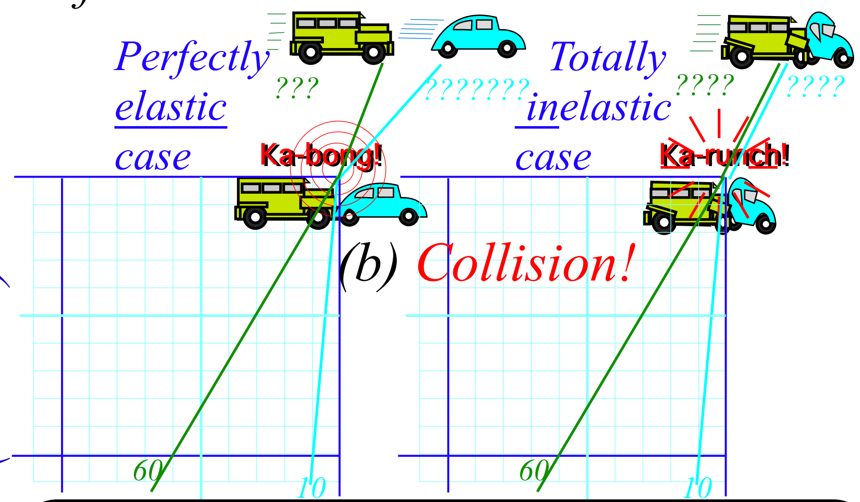

Conventional solution: Look up the usual momentum and energy formulas/axioms:

$\Sigma_i mV_i(initial) = \Sigma_i mV_i(final)$

$\Sigma_i mV^2{}_i(initial) = \Sigma_i mV^2{}_i(final)$

and solve...

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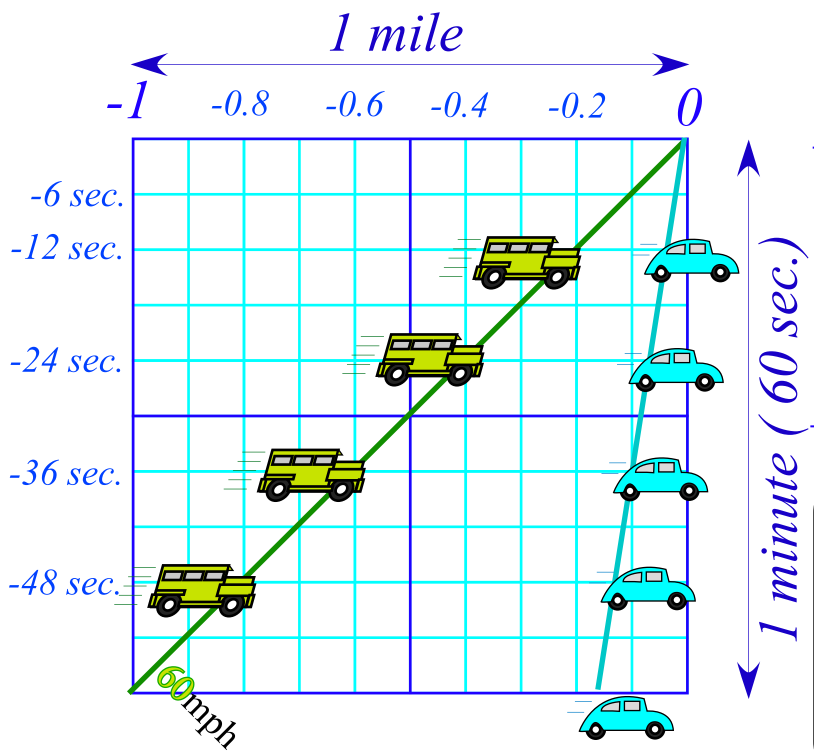

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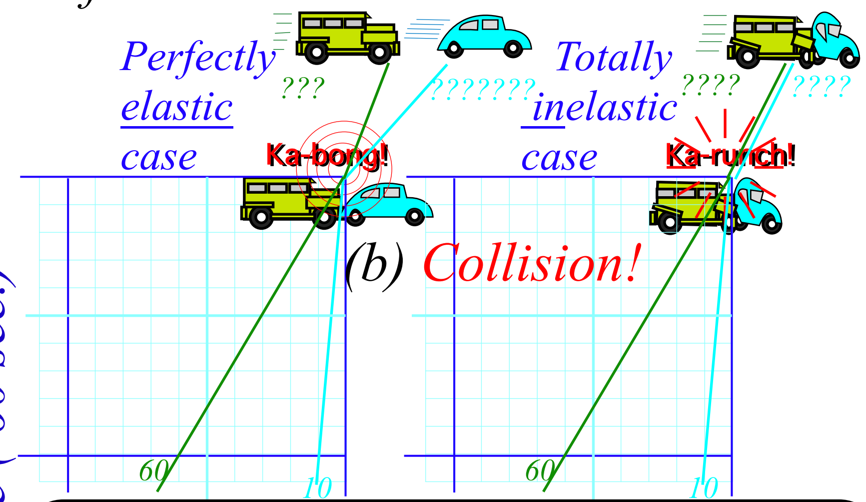

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and solve...

...But an UNconventional way is quicker and slicker.....
..... (Just have to draw 2 lines!)

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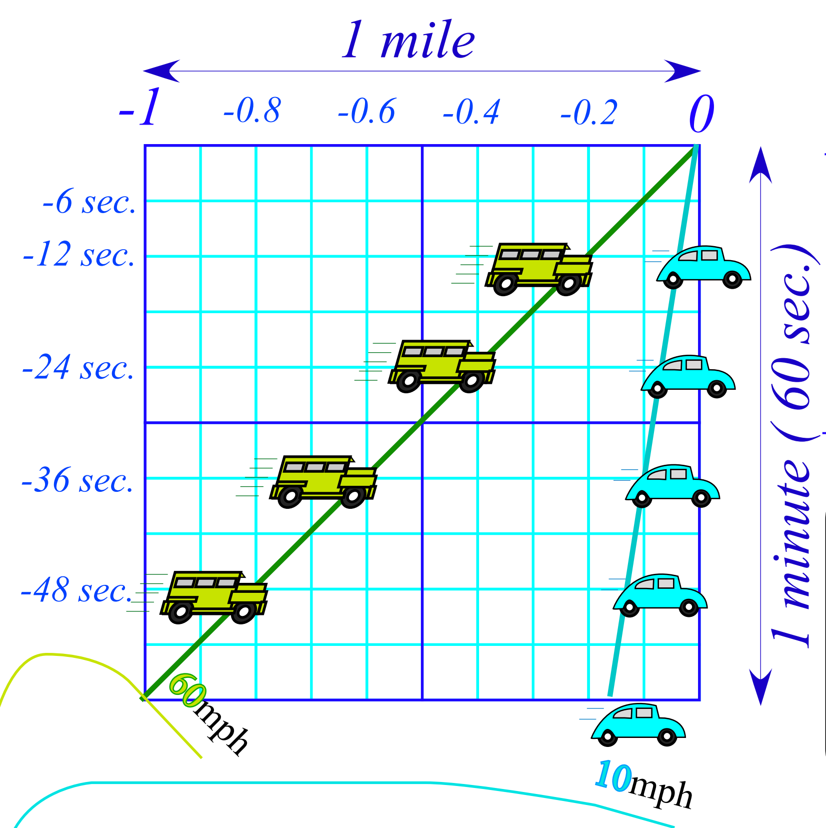

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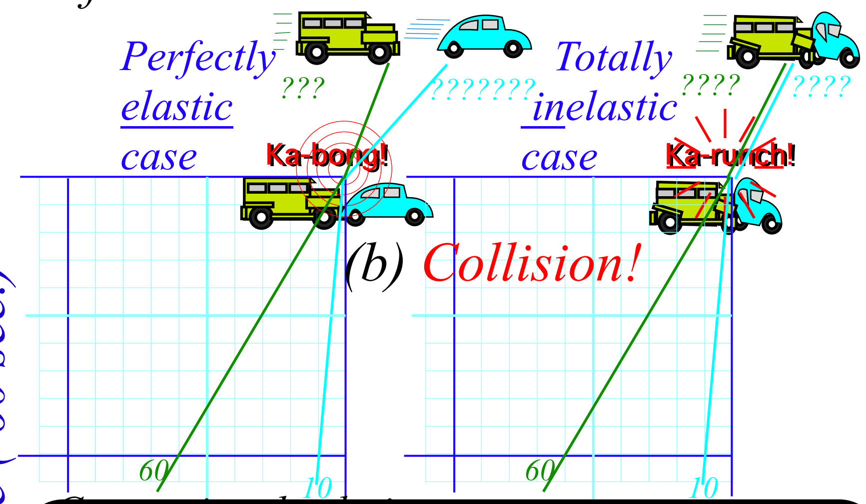

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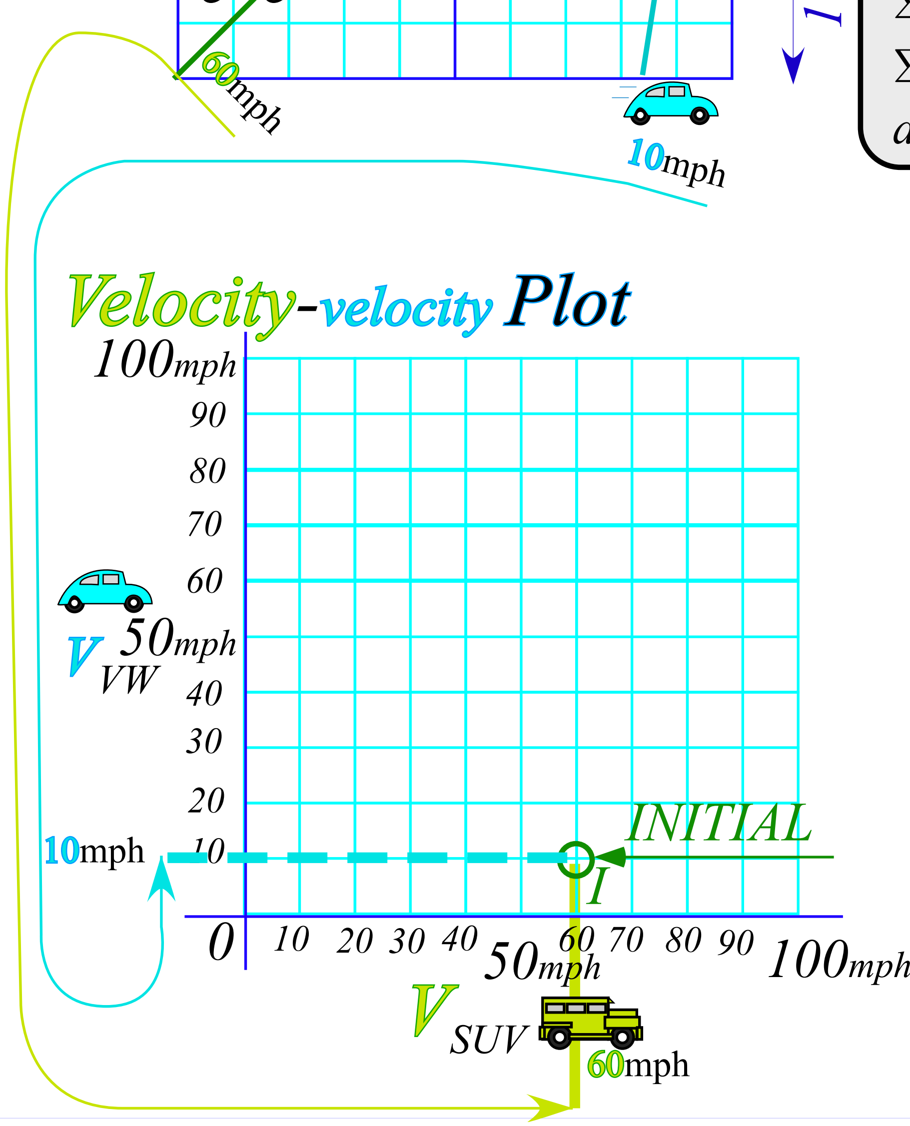

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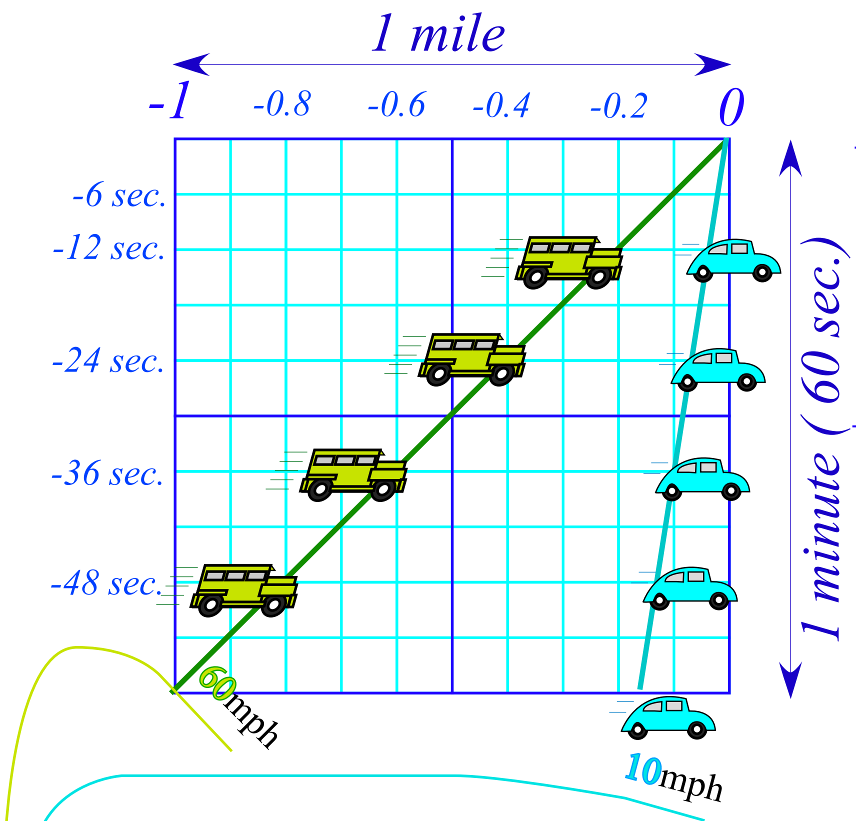

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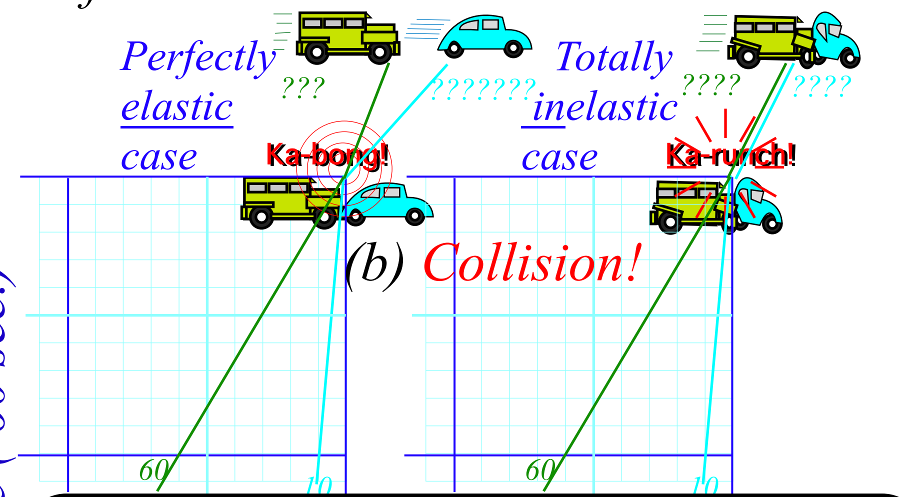

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$M_{SUV}V_{SUV}+M_{VW}V_{VW}$=constant is ***Axiom #1***

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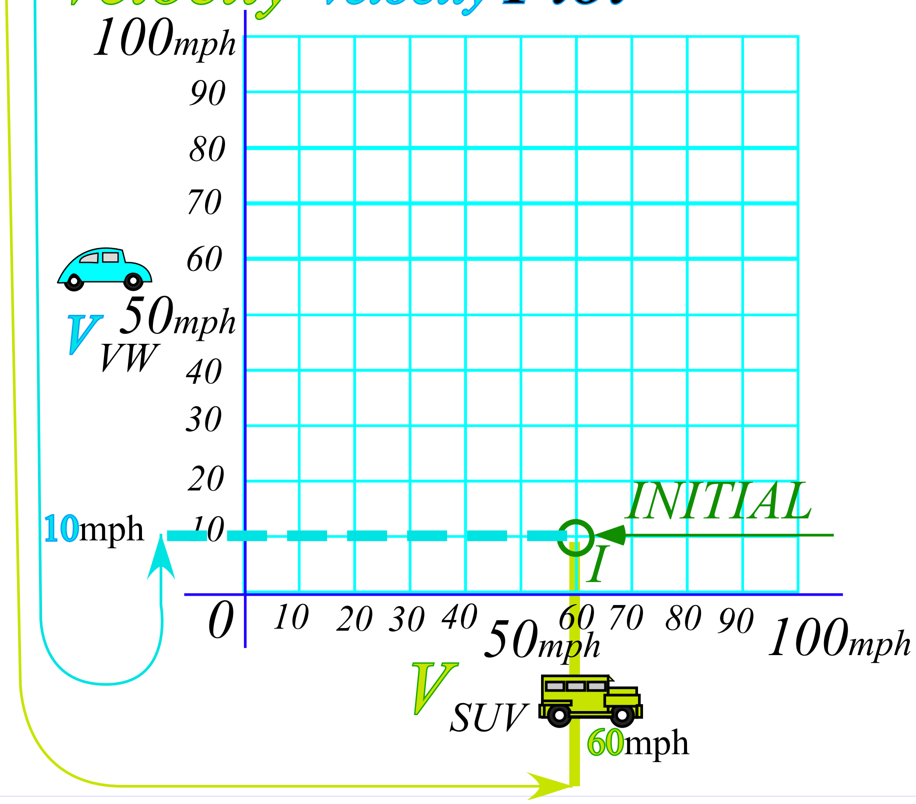
Velocity-velocity Plot



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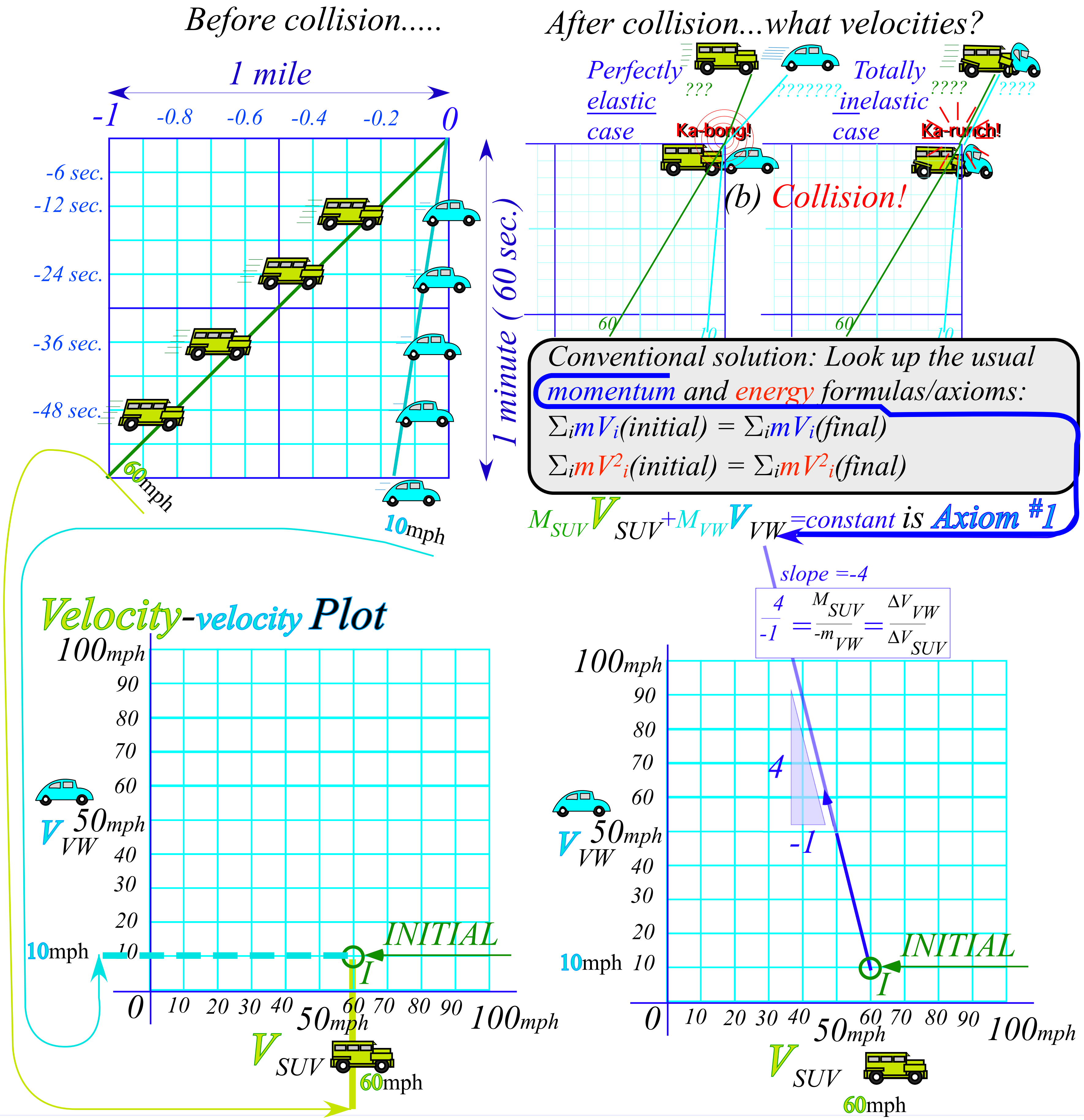

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Geometry of momentum conservation axiom

*Totally Inelastic "ka-runch" collisions**

*Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**

Comments on idealization in classical models

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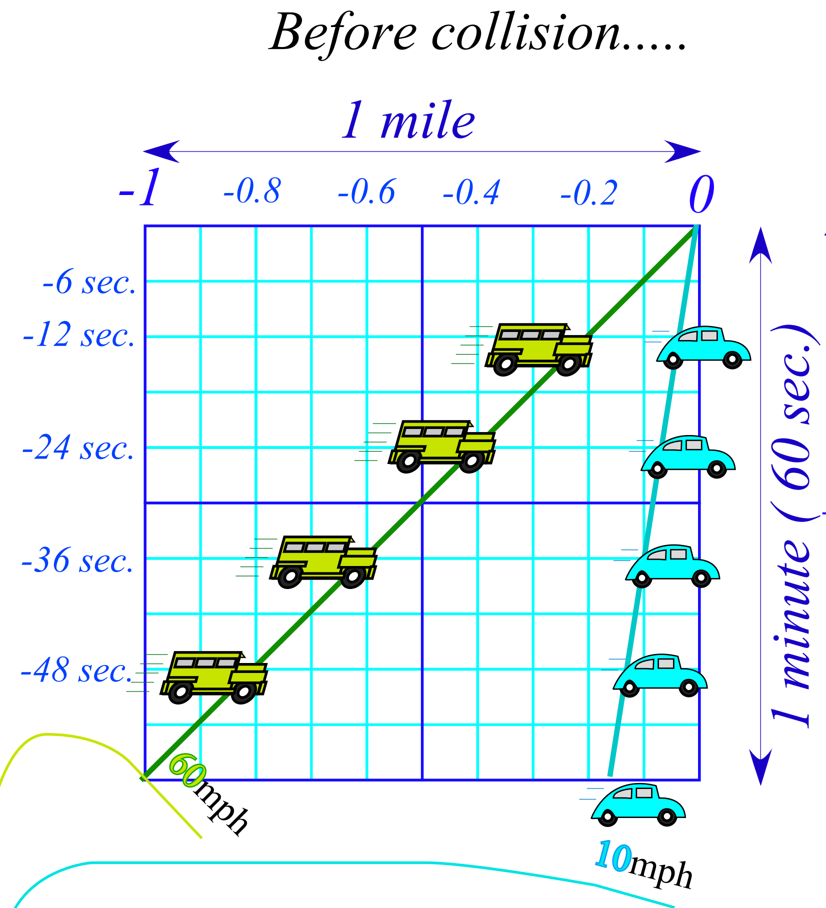

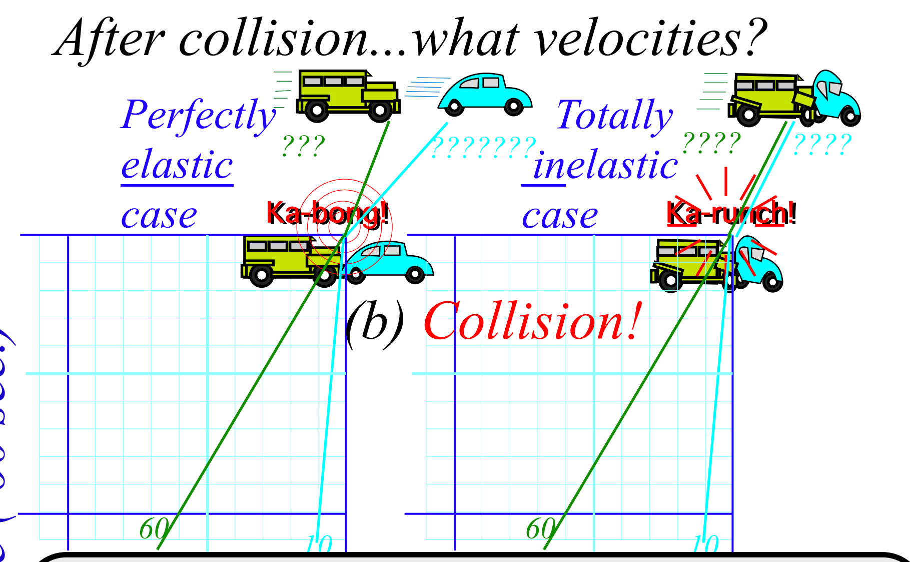

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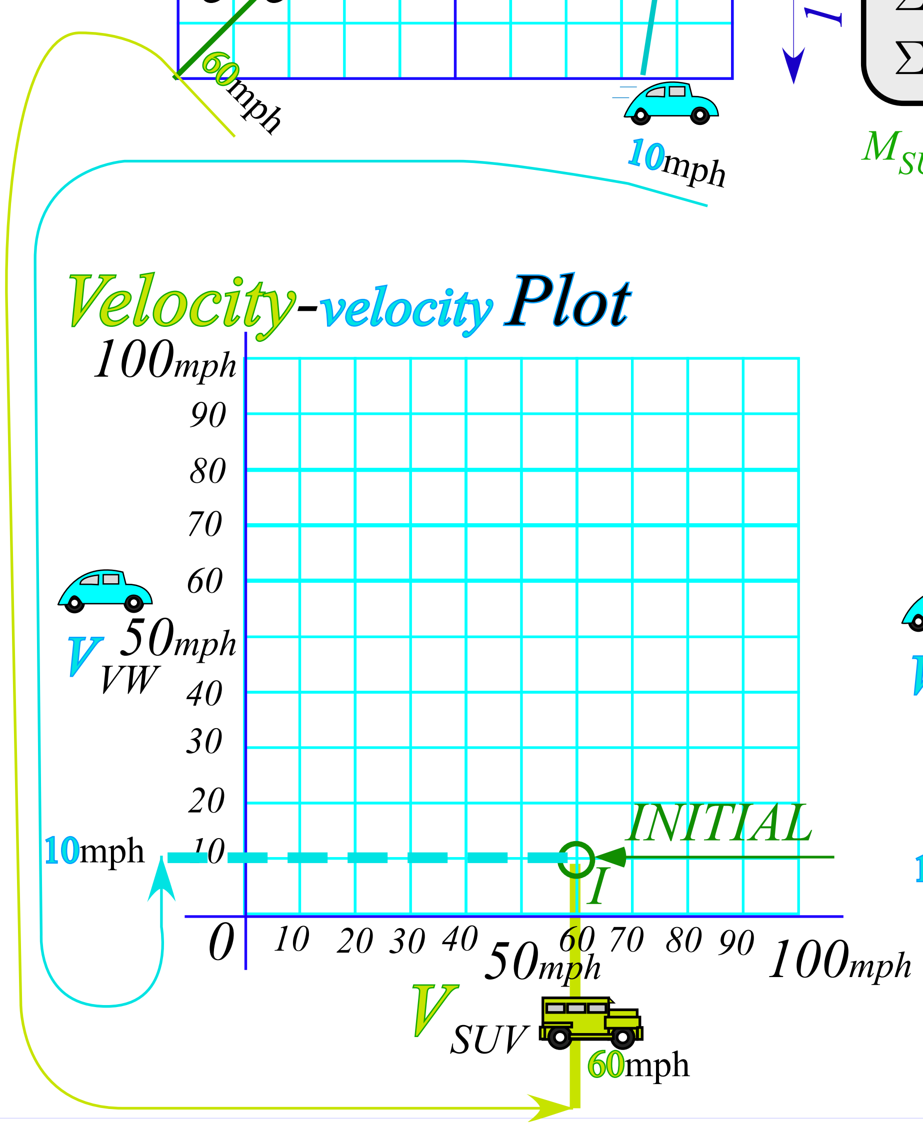

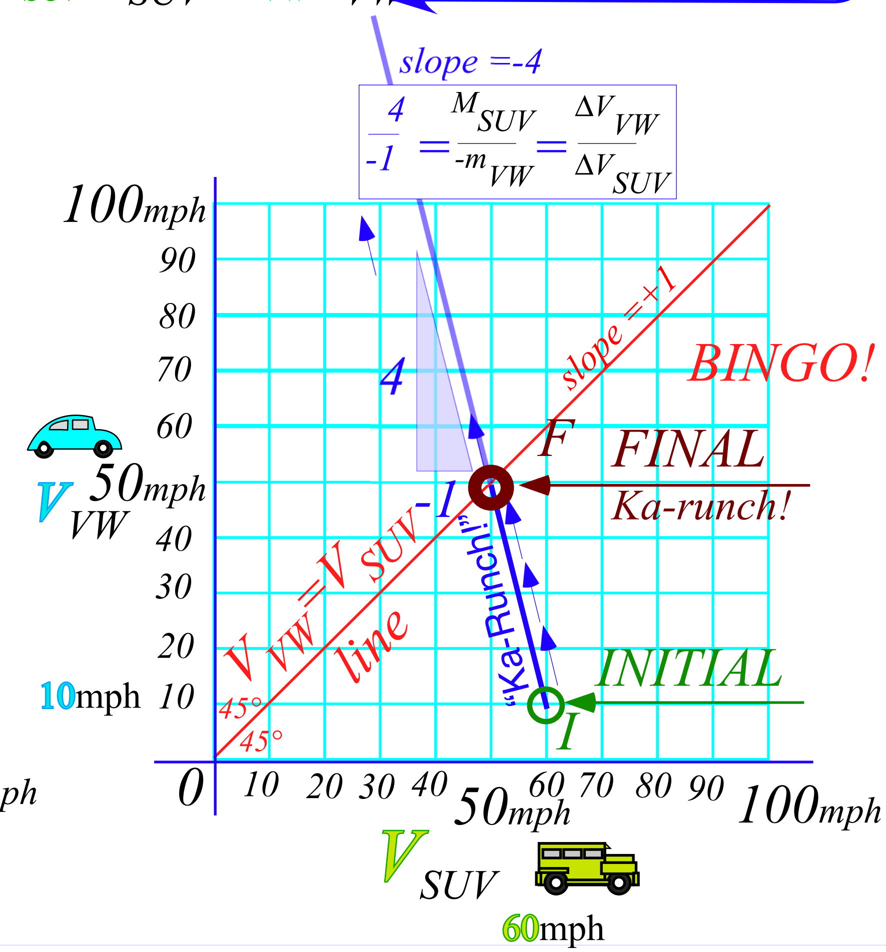

Totally Inelastic

(*Ka – Runch*!)

Geometry of momentum conservation axiom

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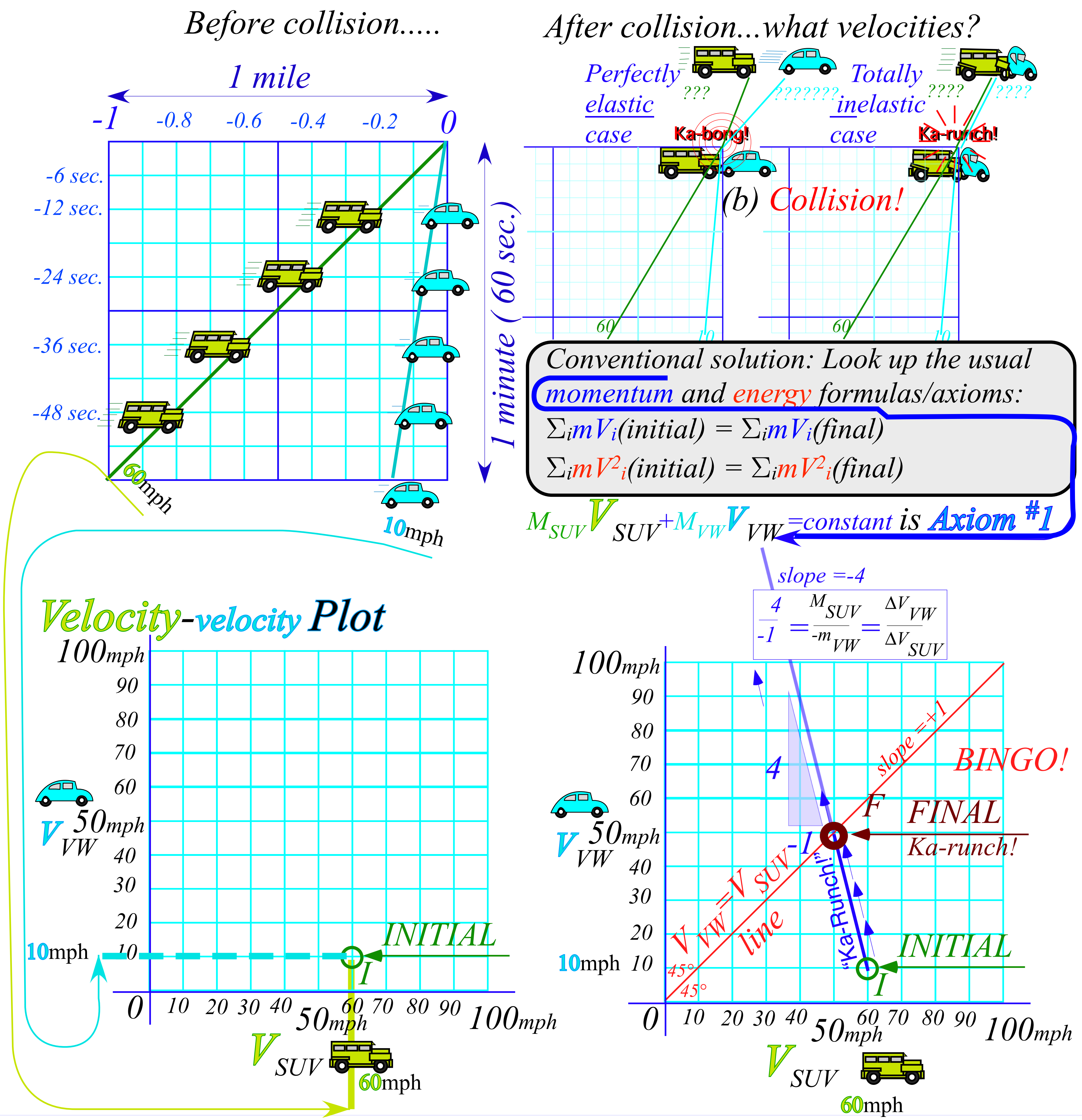

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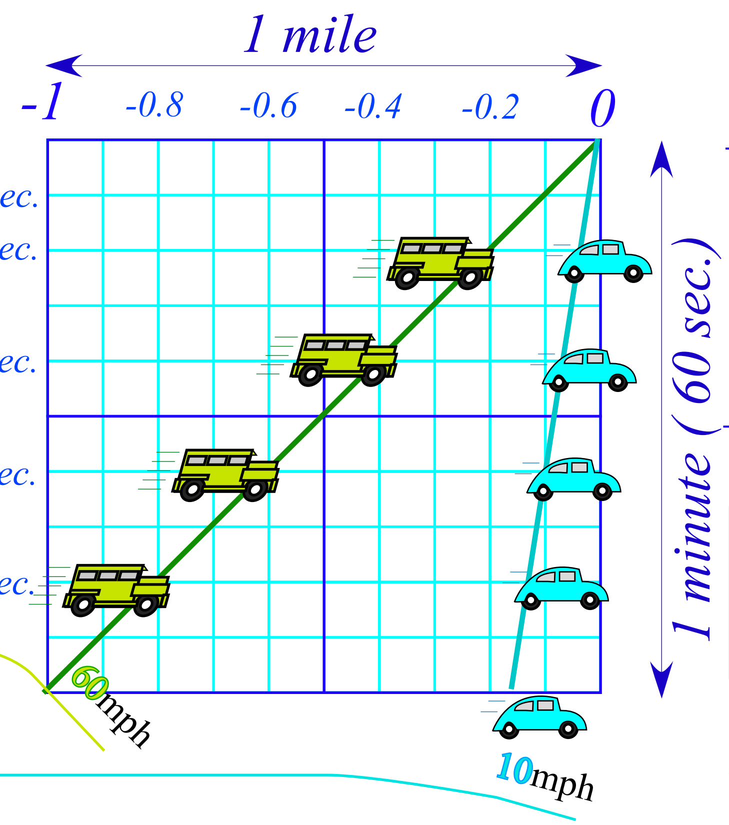
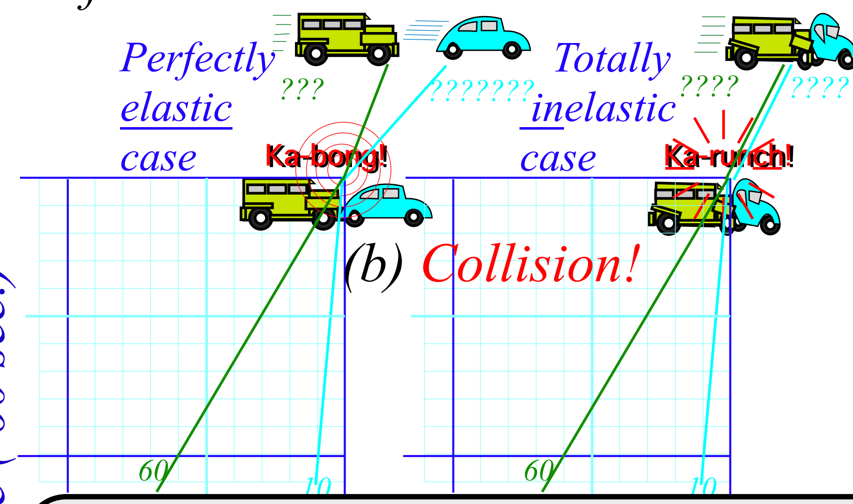


After collision...what velocities?



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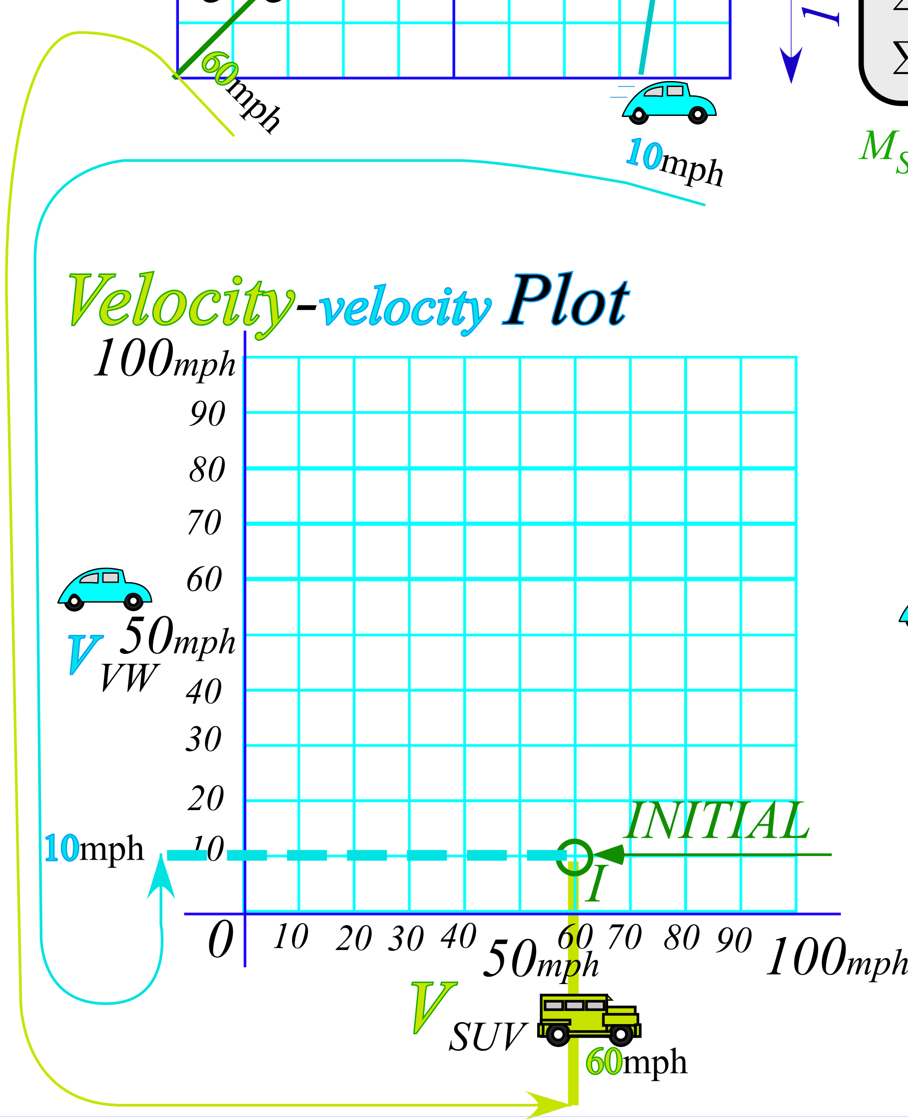
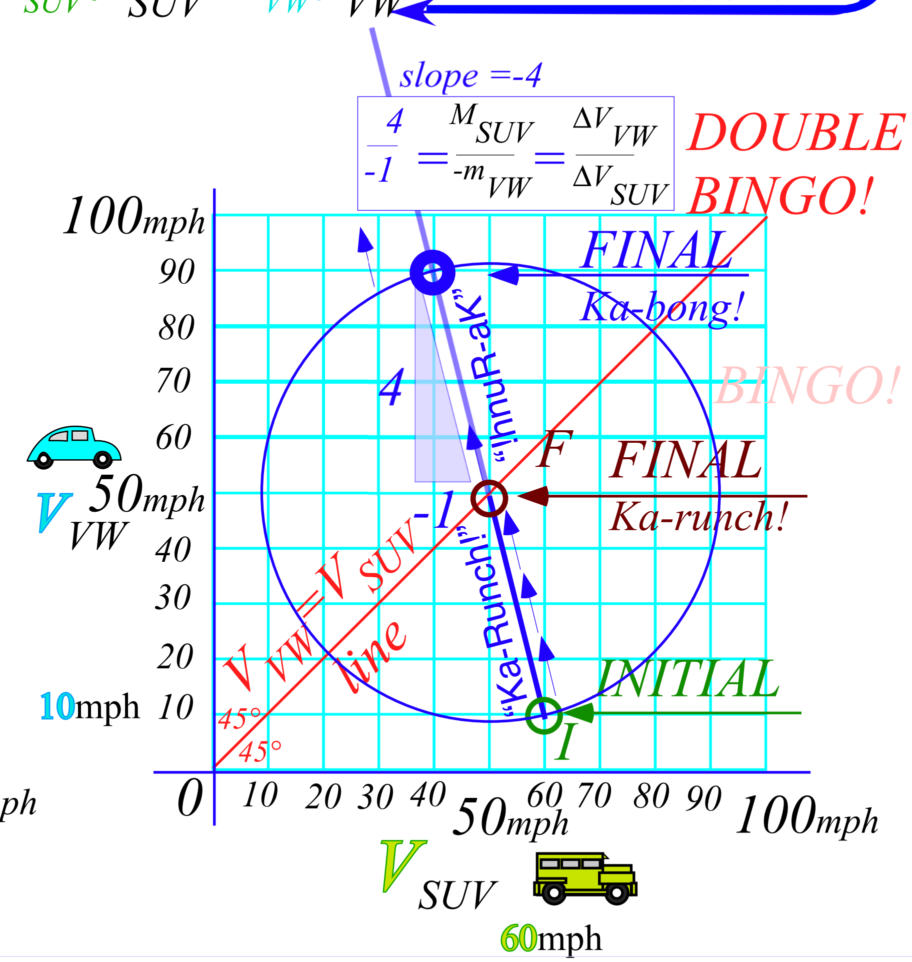
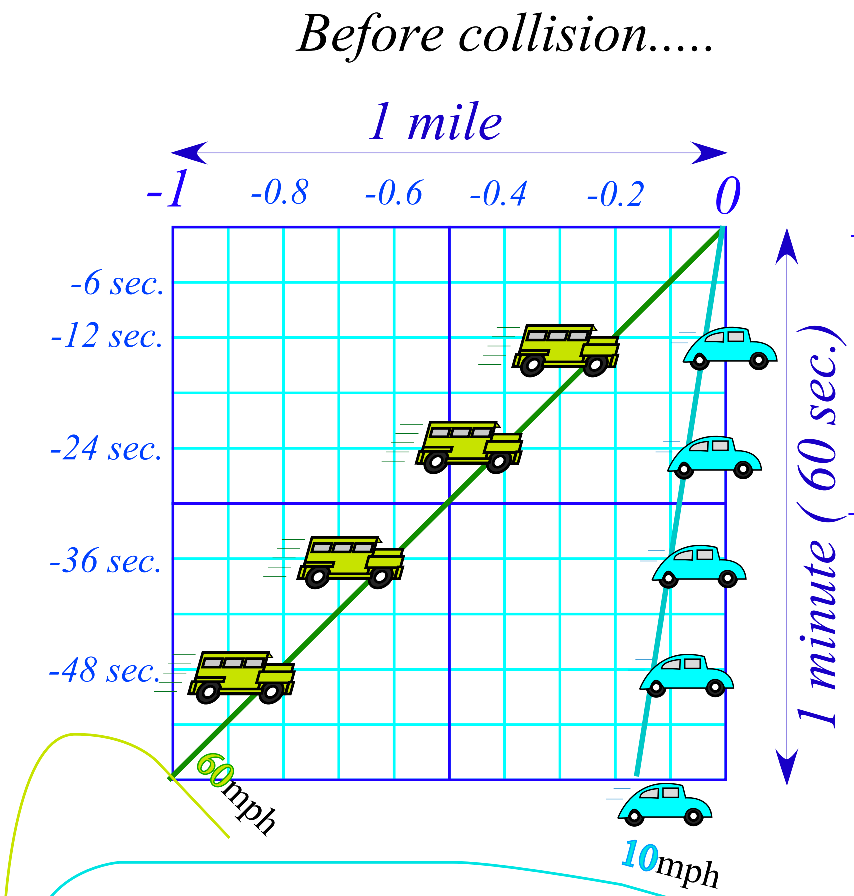
...But an UNconventional way is quicker and slicker.....
..... (Just have to draw 2 lines!)
... (and a circle...)

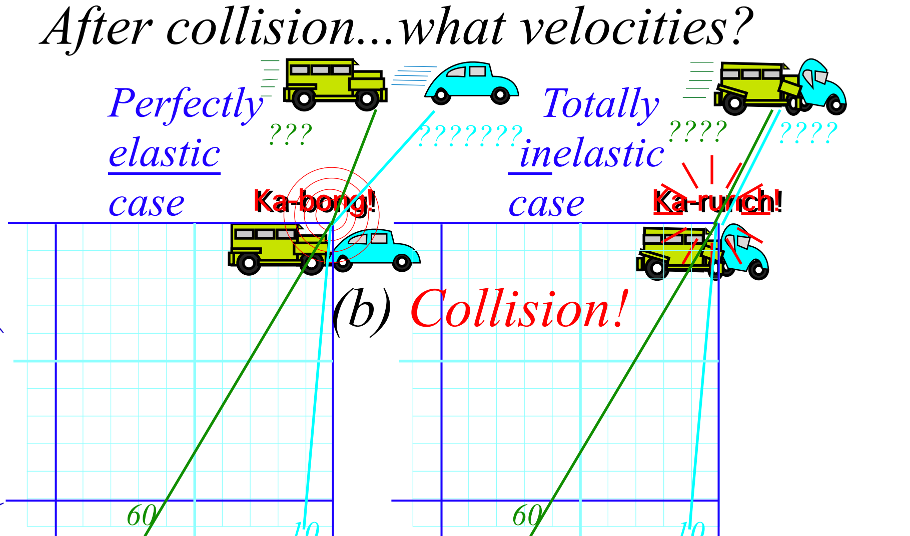




Totally Elastic
(*Ka – Bong*!)

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)





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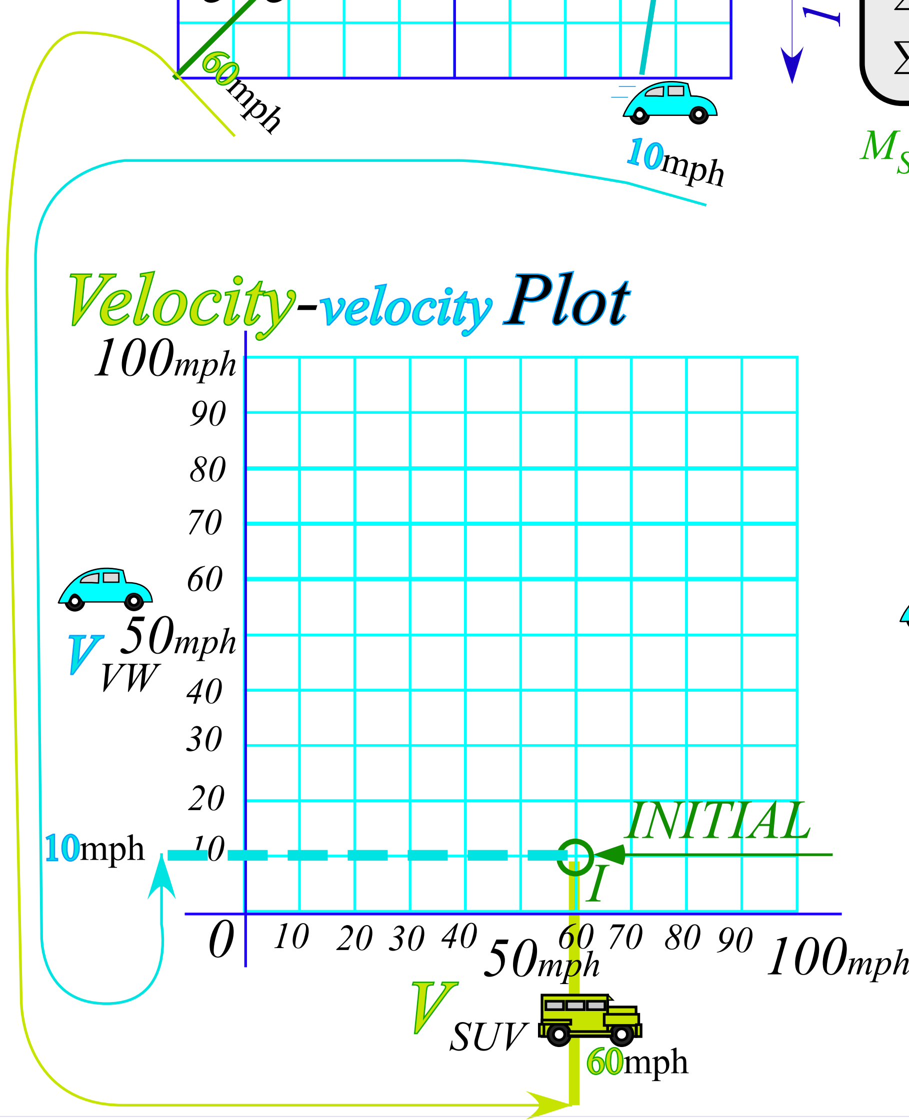

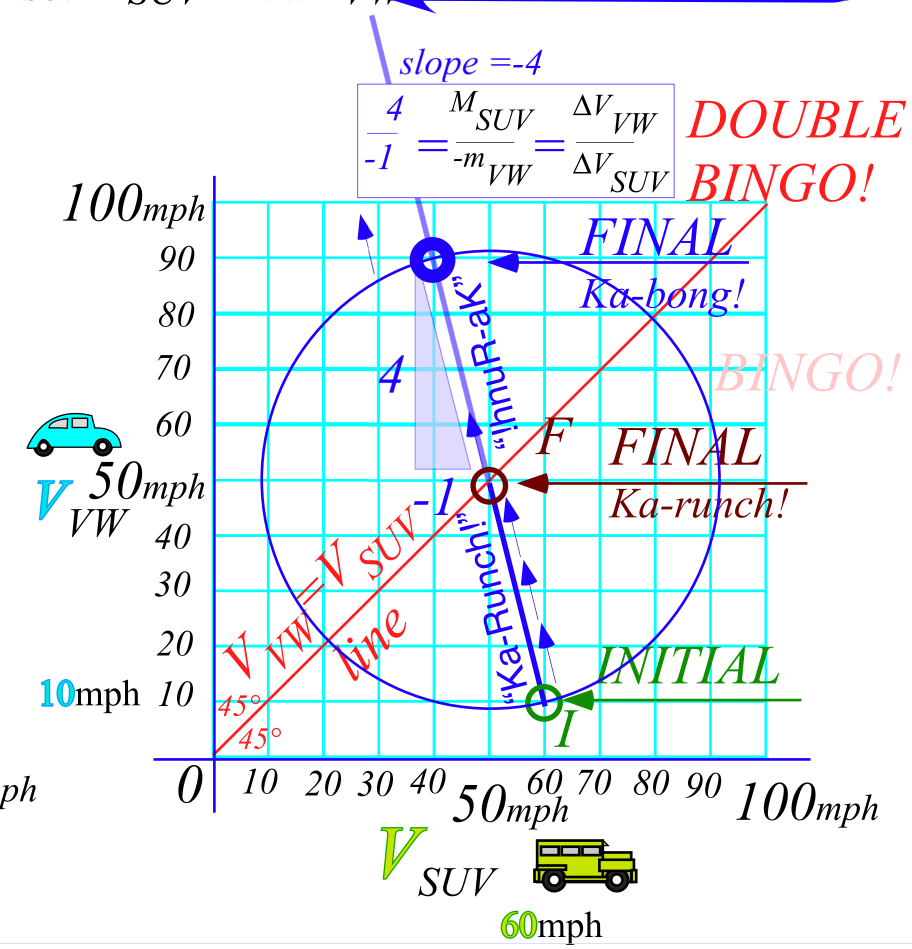

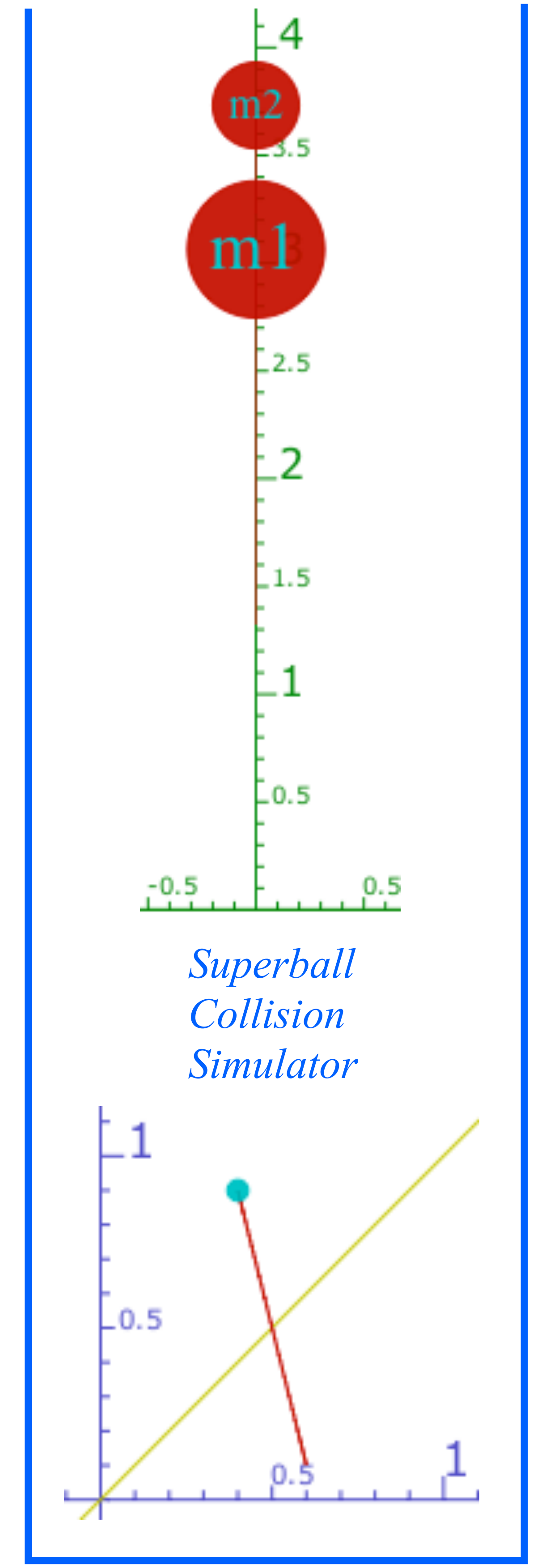

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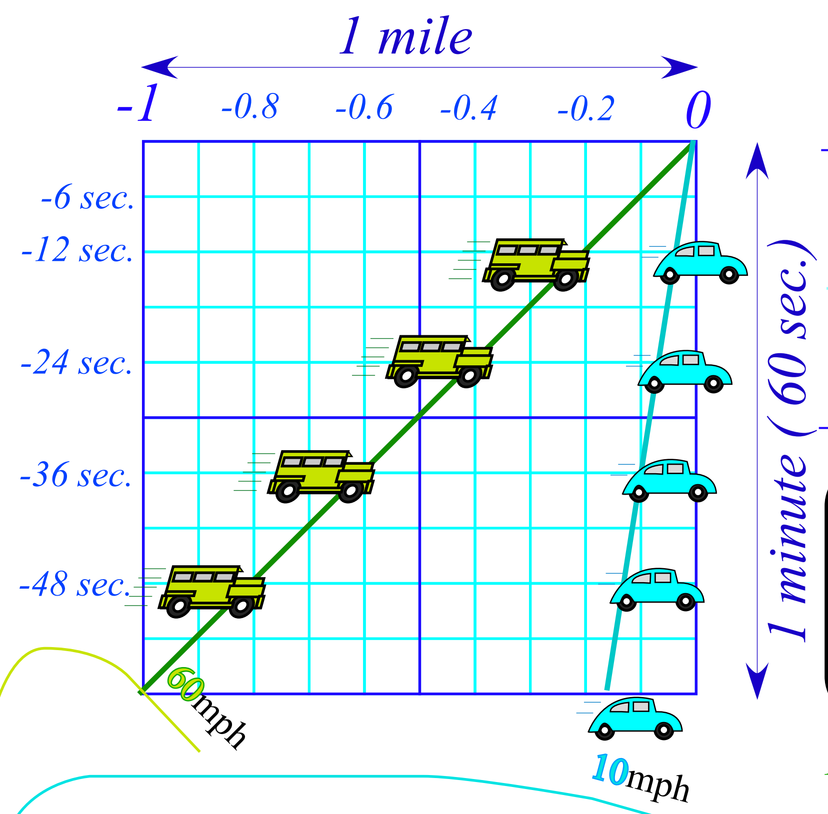

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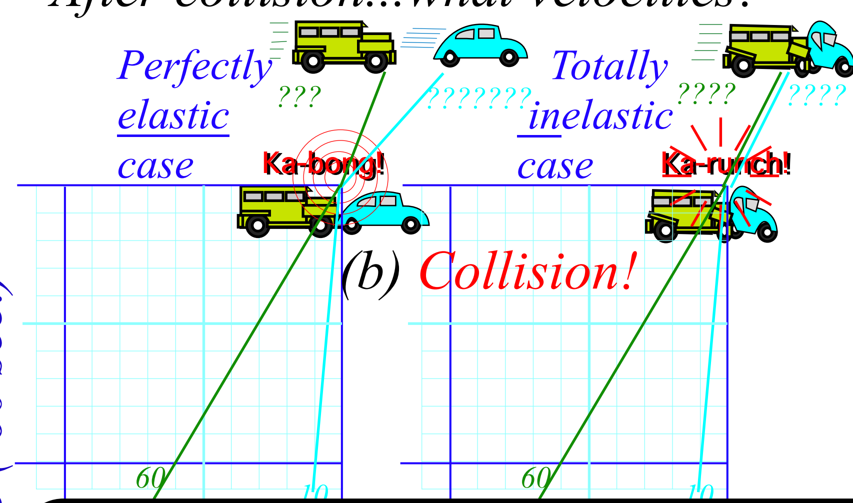

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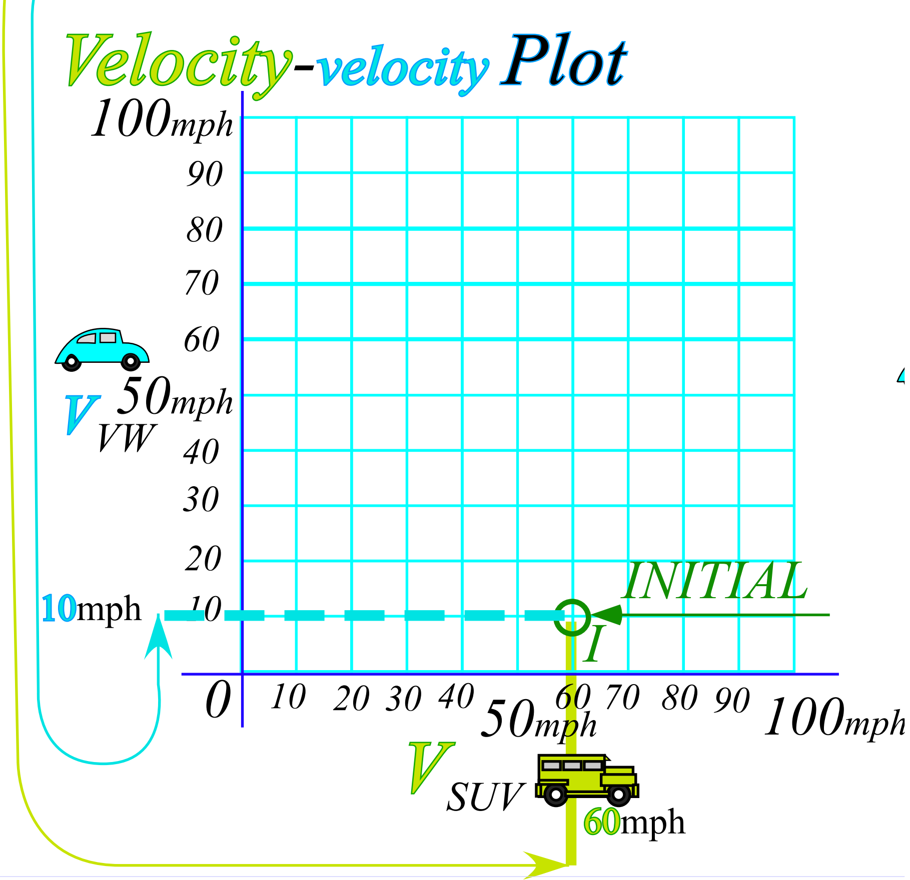
$M_{SUV} V_{SUV} + M_{VW} V_{VW} = \text{constant}$ is ***Axiom #1***

slope =-4

...But an UNconventional way is quicker and slicker.....
..... (Just have to draw 2 lines!)

Velocity-velocity Plot



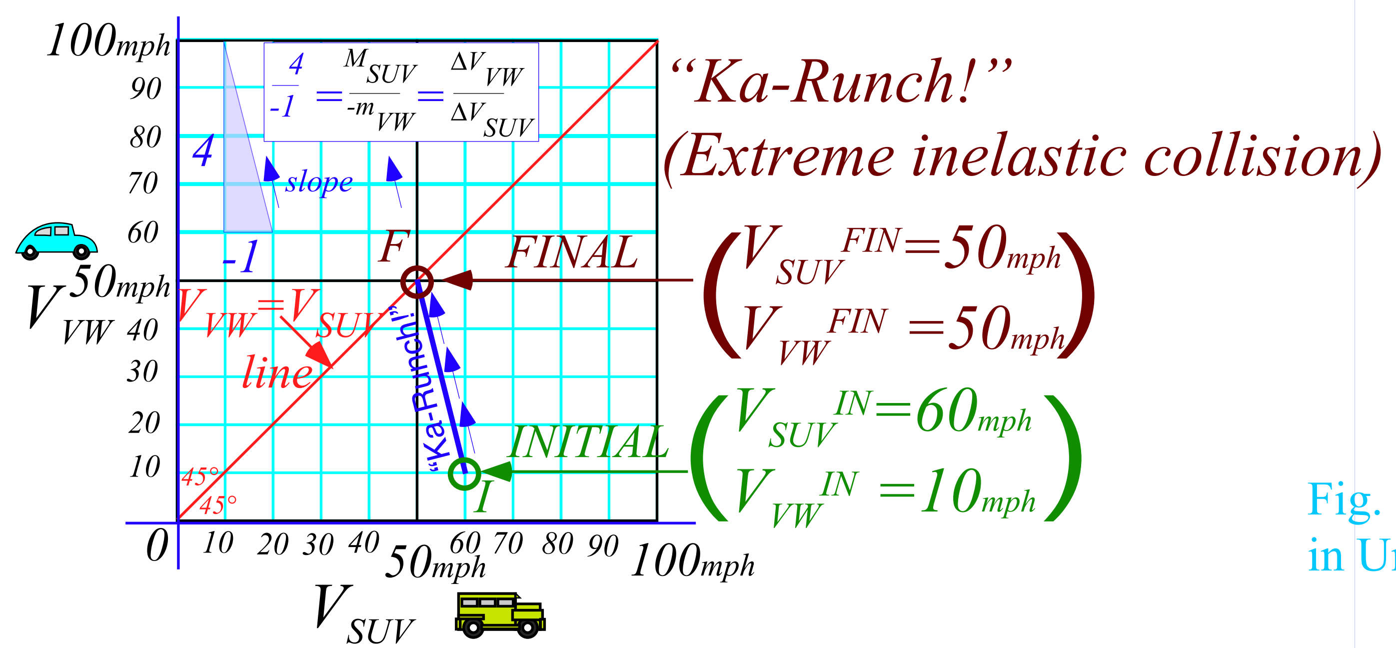
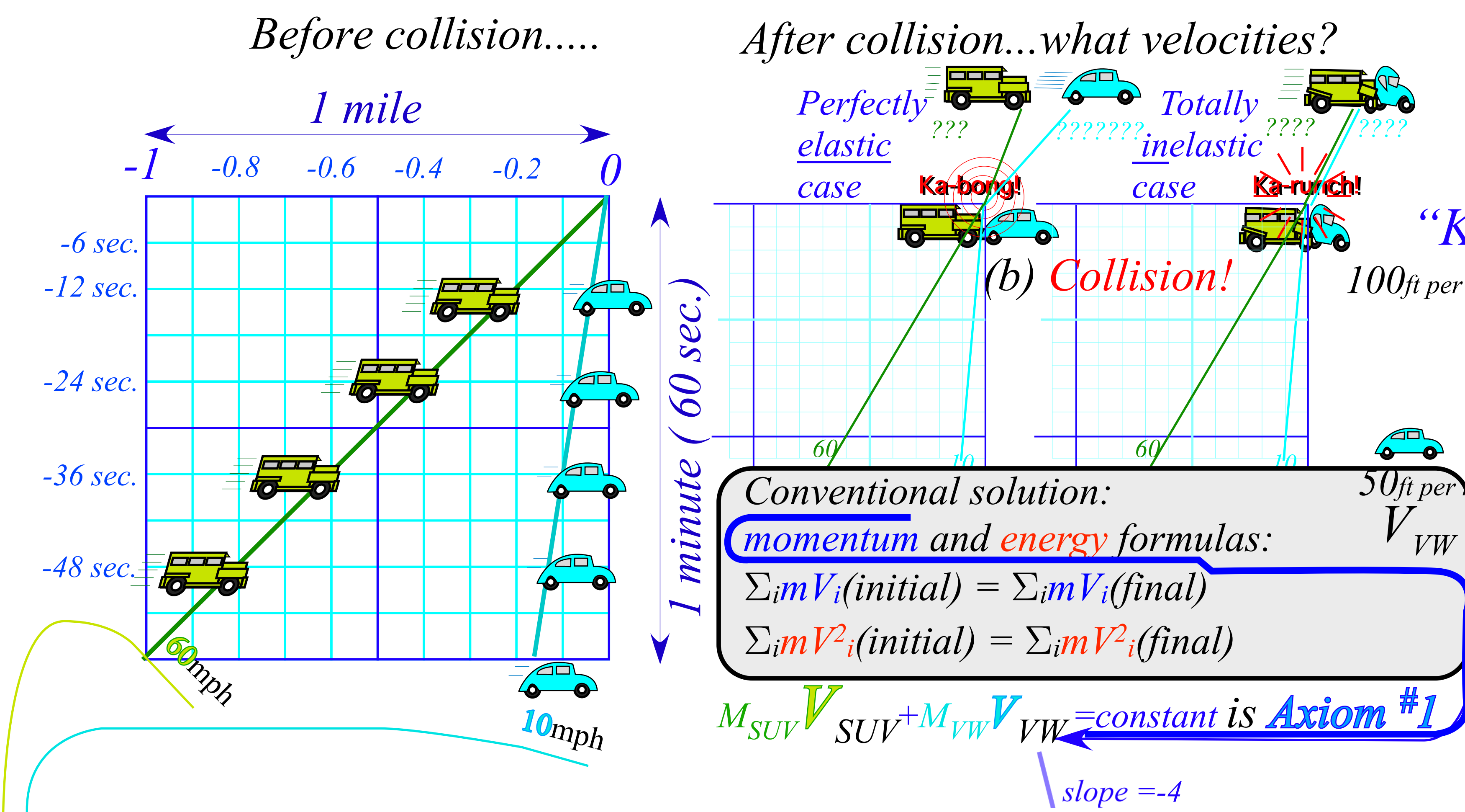


Fig. 2.1
in Unit 1

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Notice "Ka-Bong"
Figure 2.2 scaling
(ft./min. is more realistic)



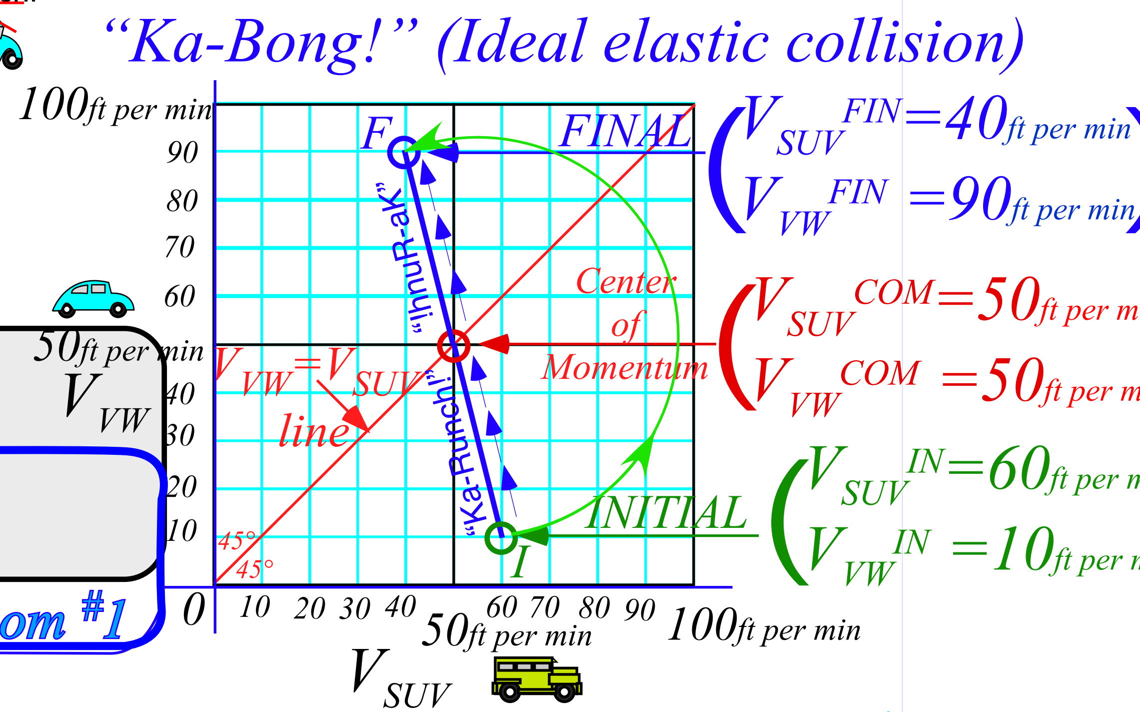
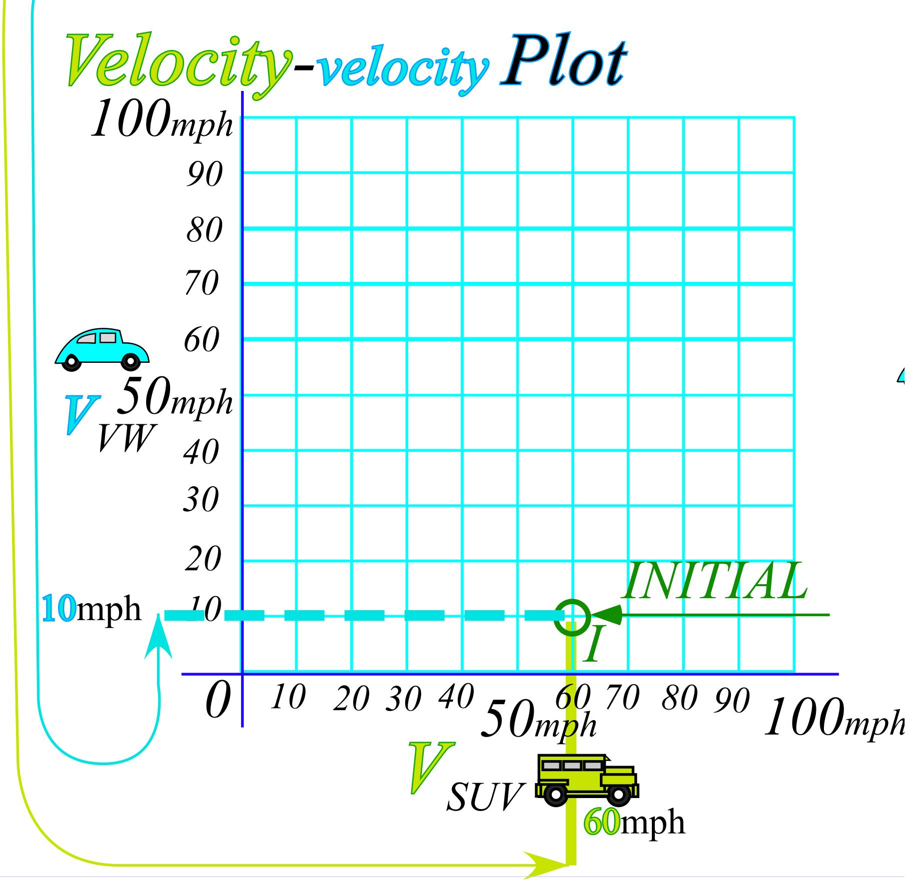


Fig. 2.2 in Unit 1



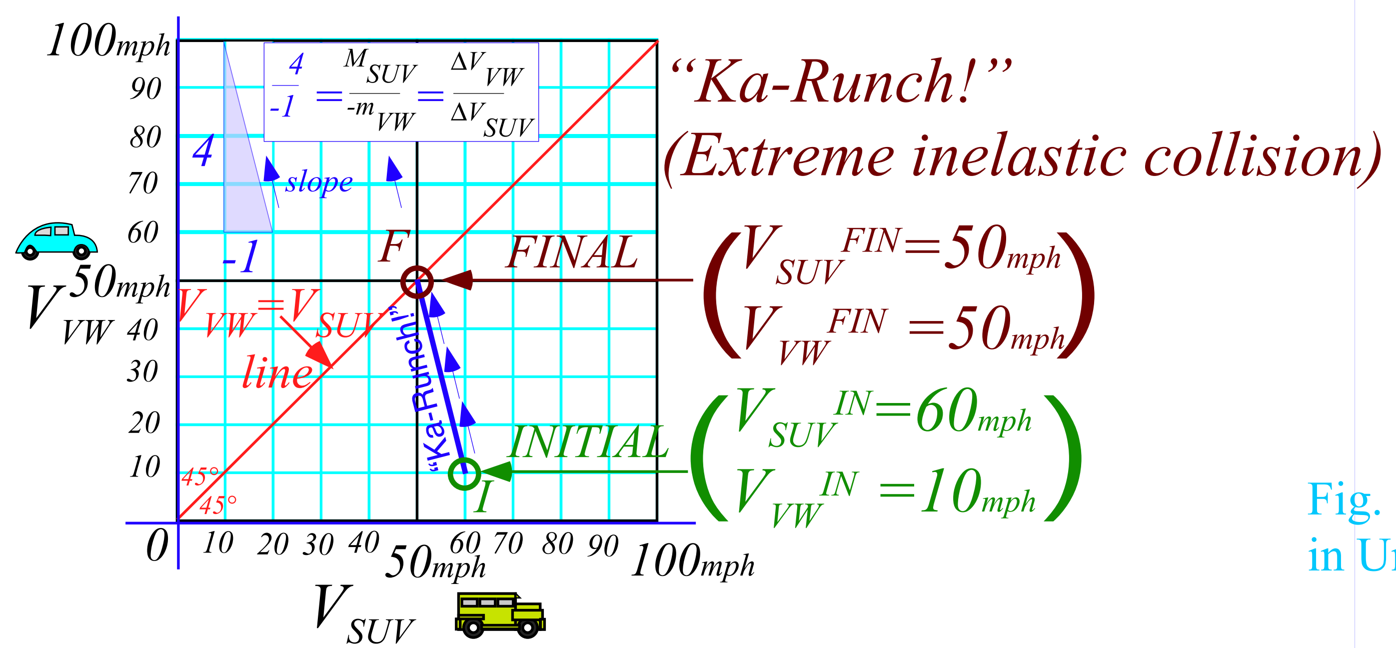


Fig. 2.1 in Unit 1

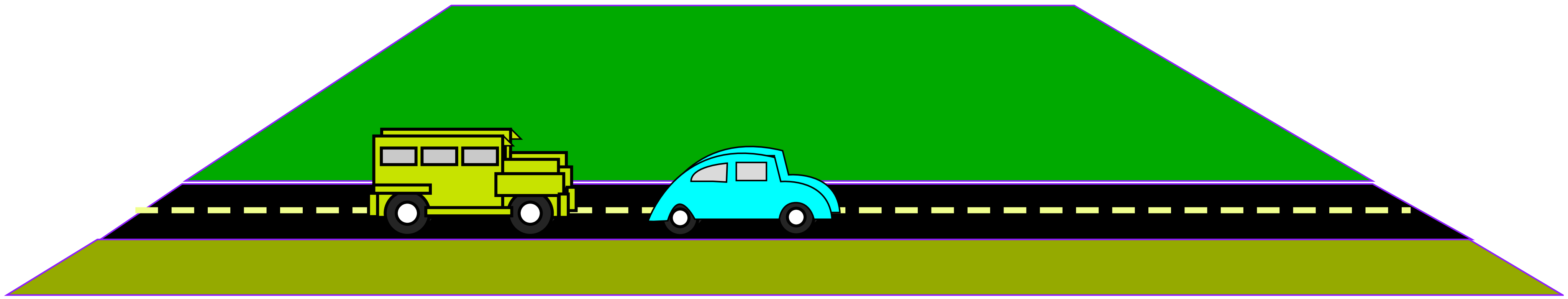
Geometry of momentum conservation axiom

*Totally Inelastic “ka-runch”collisions**

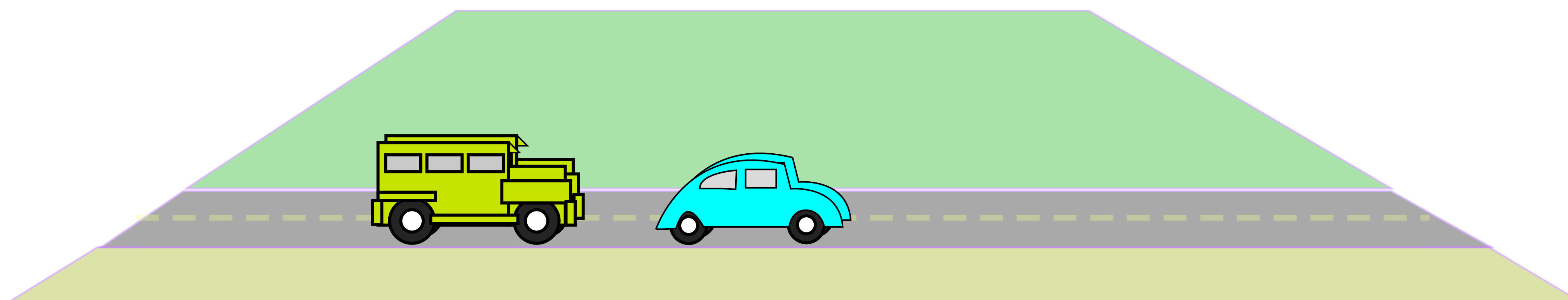
*Perfectly Elastic “ka-bong” and Center Of Momentum (COM) symmetry**

→ *Comments on idealization in classical models*

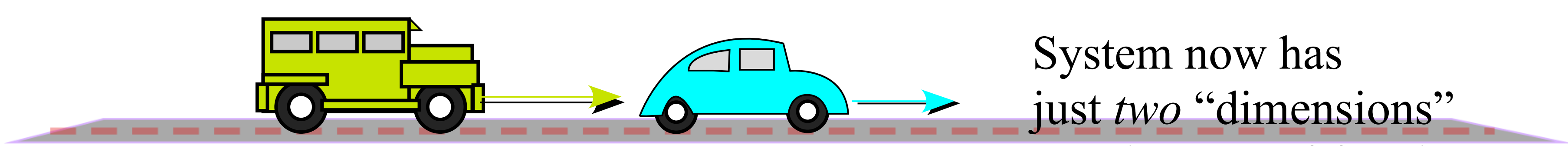
The SUV and VW *Idealized* thought experiments



Idealization 1. Ignore background.
(No rolling friction, air resistance, etc.)



Idealization 2. Make each 1-dimensional.
(Cars "constrained" to ride on frictionless rail)

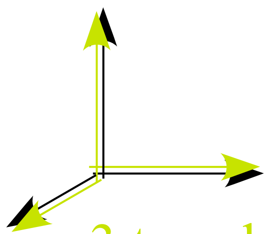


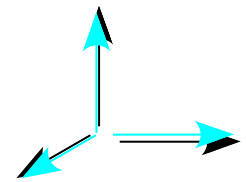
System now has
just *two* "dimensions"
or "*degrees-of-freedom*"

Landscape 1.1 Idealized model for collision model and thought experiments

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)





6 translational degrees of freedom for SUV and VW.

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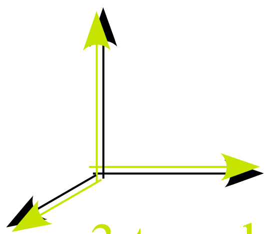
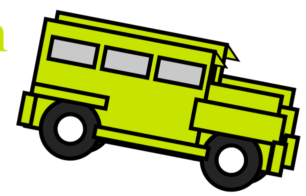
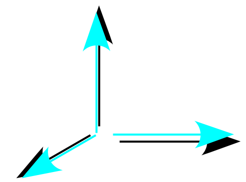


3 translation dimensions





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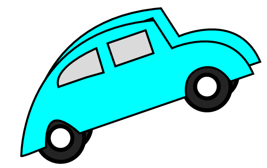
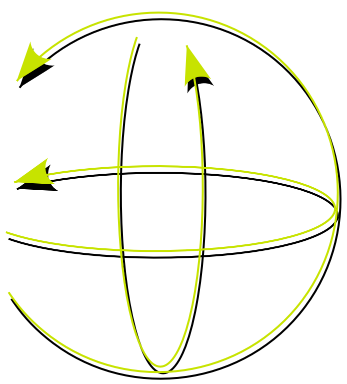


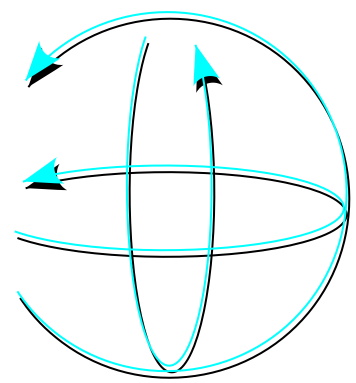
6 translational degrees of freedom for SUV and VW.

Rotation (Each body has 3 rotationaldegrees of freedom) (Intoduced in Units 3 and 7)



3 rotational dimensions

yaw-pitch-roll Euler angles



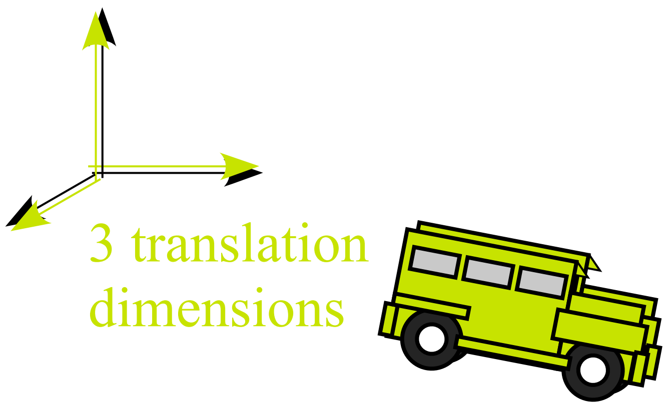
3 rotational dimensions

yaw-pitch-roll Euler angles

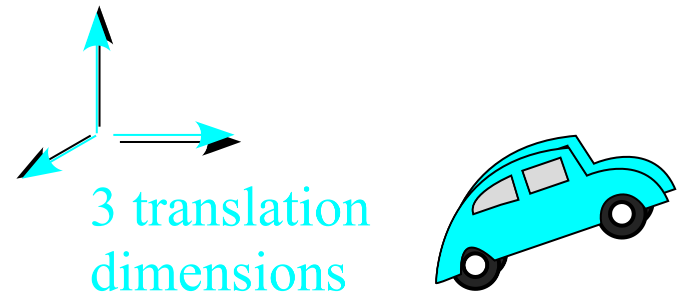
6 rotational degrees of freedom for SUV and VW.

Summary of Classical Mechanical Degrees of Freedom

Translation (Each body has 3 translational degrees of freedom) (Intoduced in Units 1 and 2)



3 translation dimensions



3 translation dimensions

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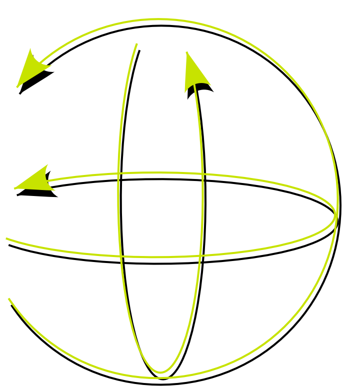
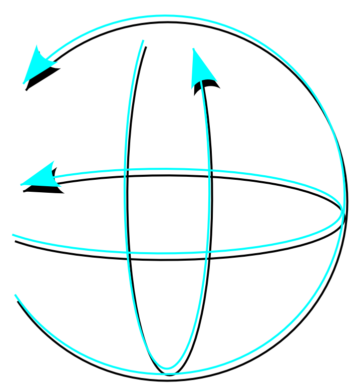


3 rotational dimensions

yaw-pitch-roll Euler angles



3 rotational dimensions

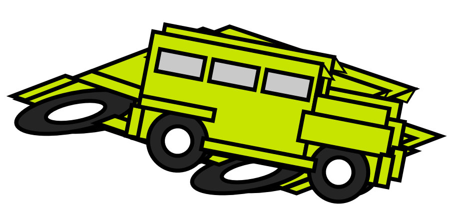
yaw-pitch-roll Euler angles

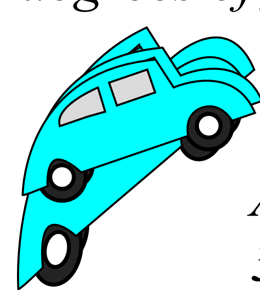
6 rotational degrees of freedom for SUV and VW.

SUV and VW system involves
12 rigid-body degrees of freedom

Vibration (Each body has many vibrational degrees of freedom) (Intoduced in Units 3-8)

Generalized Curvilinear Coordinates (GCC) introduced in Unit 1 Chapters 10 -12





An N-atom molecule has 3N-6 vibrational degrees of freedom

Landscape 1.2 Some idealized classical model degrees of freedom

Geometry of Galilean translation symmetry

→ *45° shift in (V_1,V_2)-space*

Time reversal symmetry

...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,... *(In some direction x,y, or z…)*

...the rest of the world appears to be 50 mph slower *(In that direction…)*

(a) Galileo transforms to COM frame

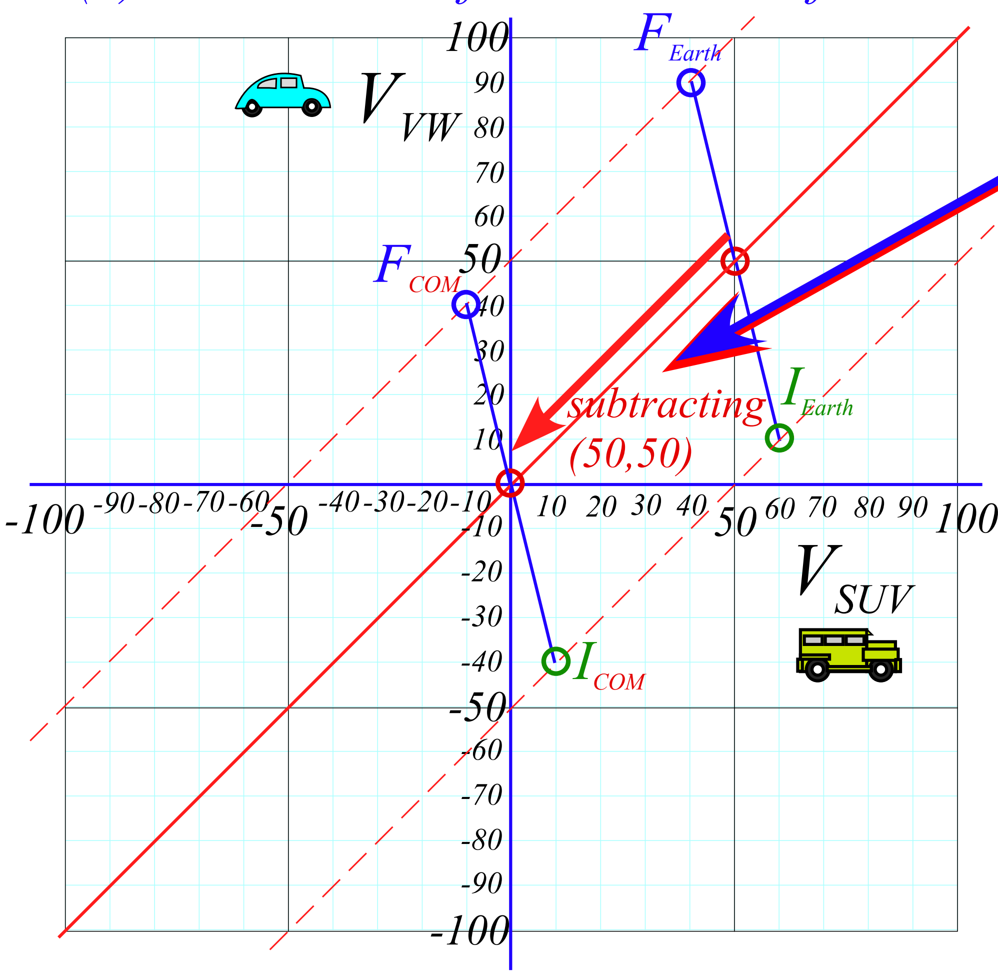


Fig. 2.5a in Unit 1

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,... *(In some direction x,y, or z...)*

...the rest of the world appears to be 50 mph slower *(In that direction...)*

(a) Galileo transforms to COM frame *(b) ... and to five or six other reference frames*

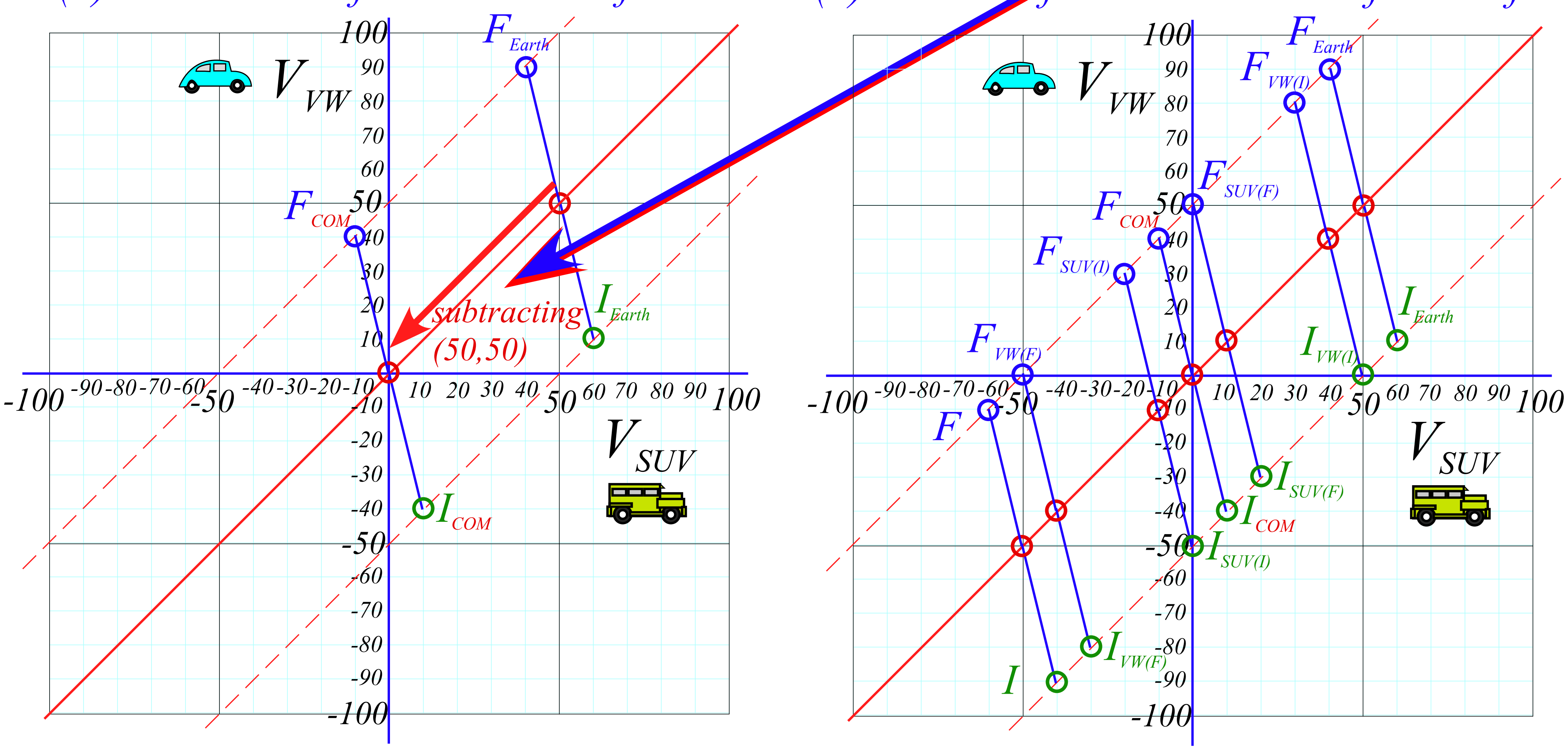


Fig. 2.5a in Unit 1

Fig. 2.5b in Unit 1

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

→ *Time reversal symmetry*

...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

*...the rest of the world appears to be 50 mph **slower***

Final F and Initial I trade places...

Time-reversal (F-I) symmetry pairs (Four examples)

(a) Galileo transforms to COM frame

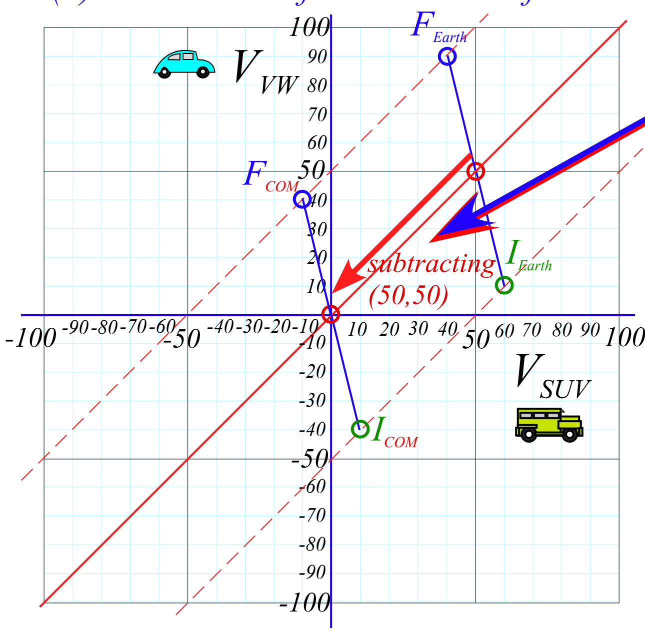


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

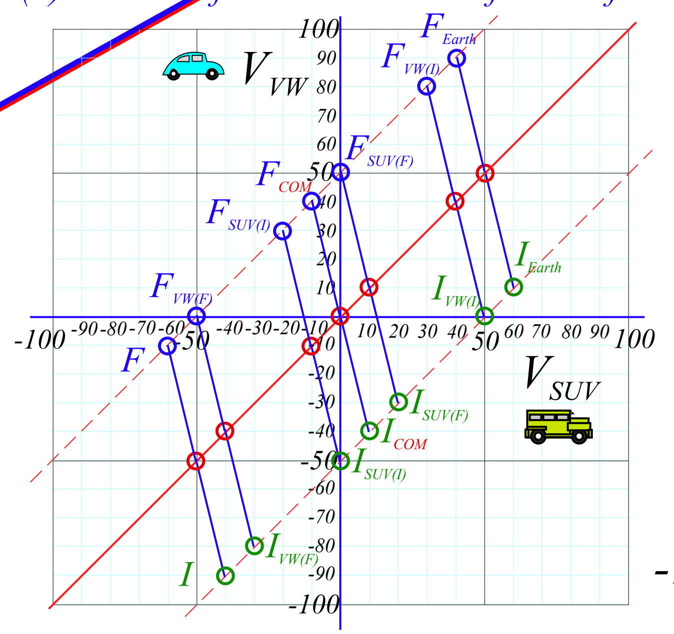
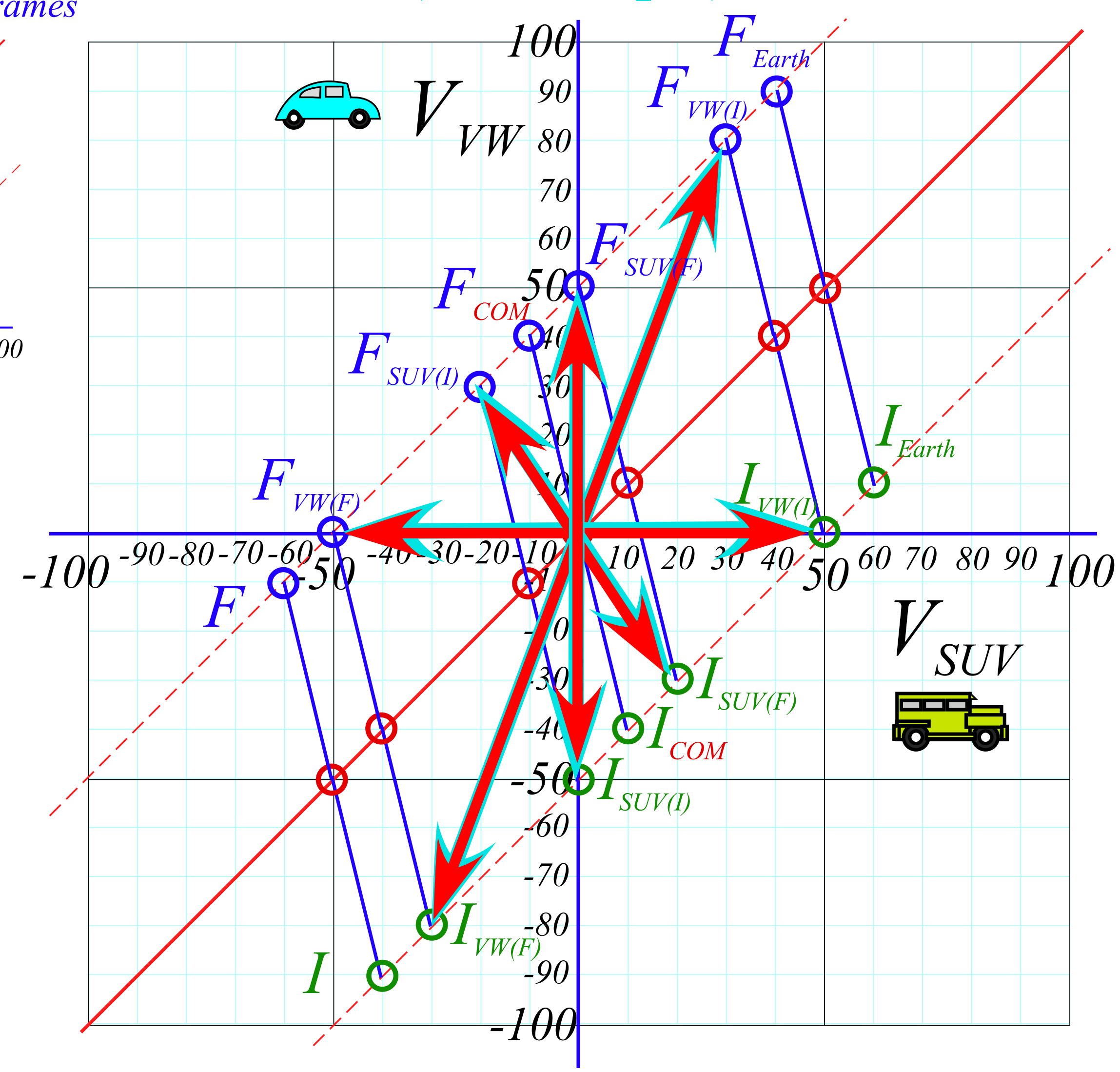


Fig. 2.5b
in Unit 1



Time-reversal means flip t to -t...
(Run a movie backwards)

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

(a) Galileo transforms to COM frame

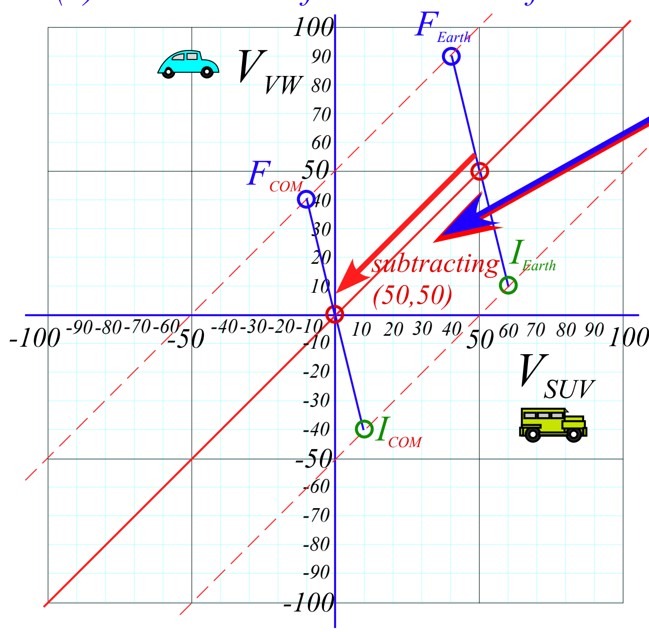


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

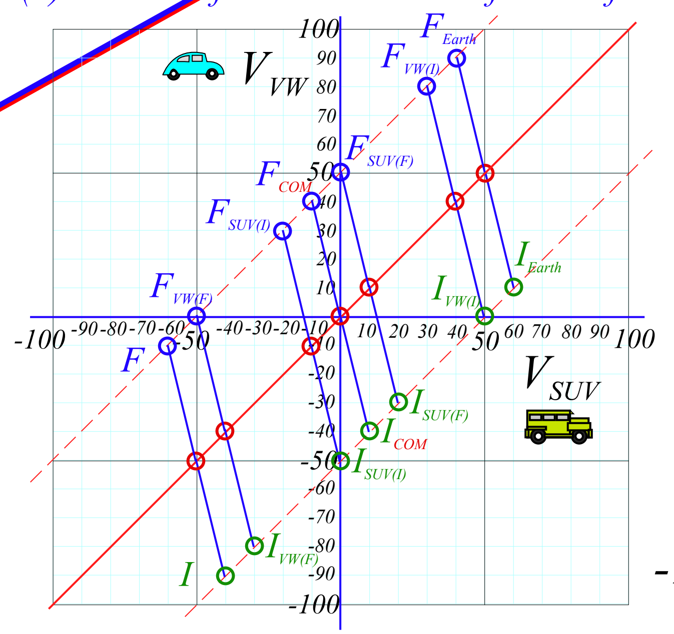
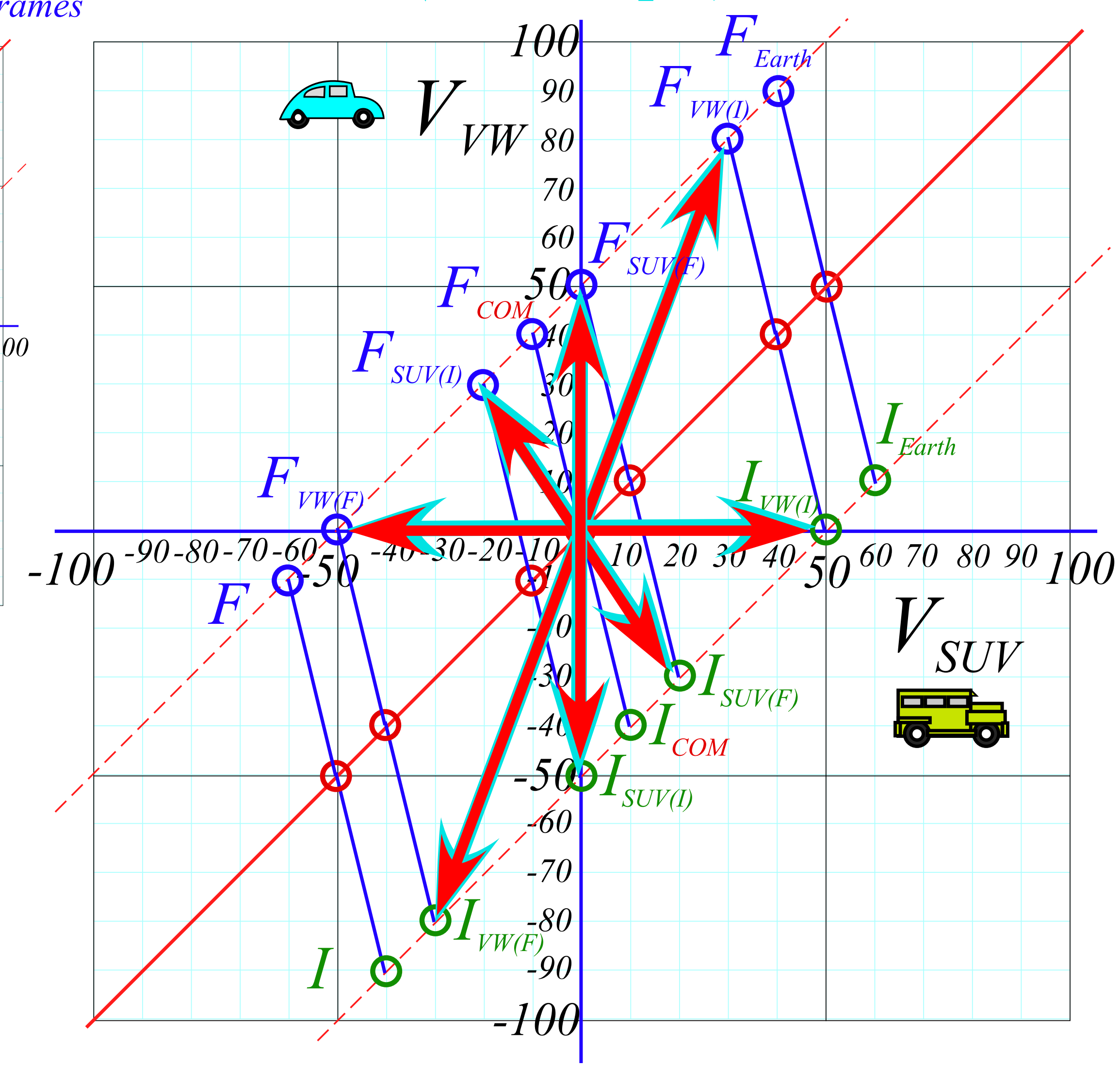


Fig. 2.5b
in Unit 1

Final F and Initial I trade places...

Time-reversal (F-I) symmetry pairs (Four examples)



Time-reversal means flip t to -t...
(Run a movie backwards)

That means you flip Velocity V to -V...
(Everything goes backwards)

Geometry of Galilean translation symmetry

45° shift in (V_1,V_2)-space

Time reversal symmetry

→ *...of COM collisions*

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

*...the rest of the world appears to be 50 mph **slower***

THE
COM Time-reversal symmetry pair (Just 1 case)

(a) Galileo transforms to COM frame

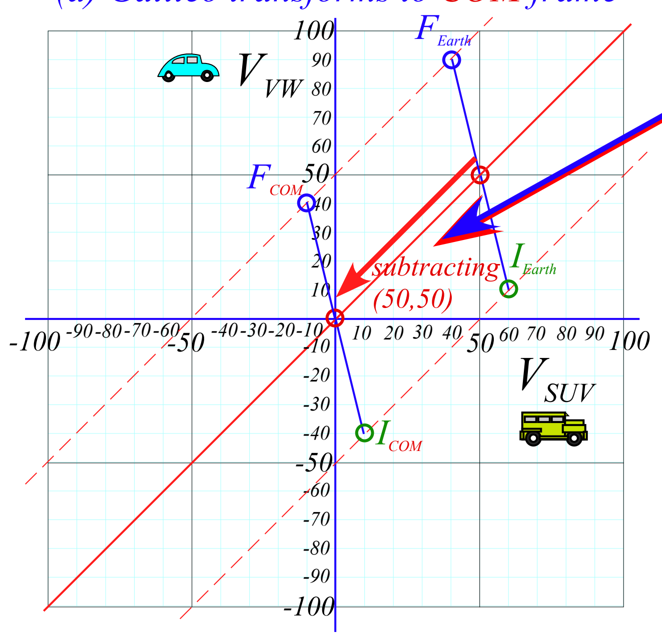


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

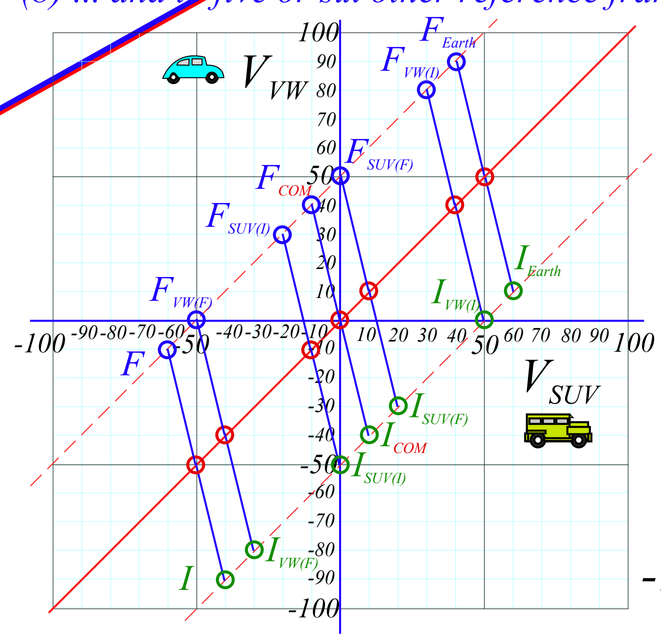
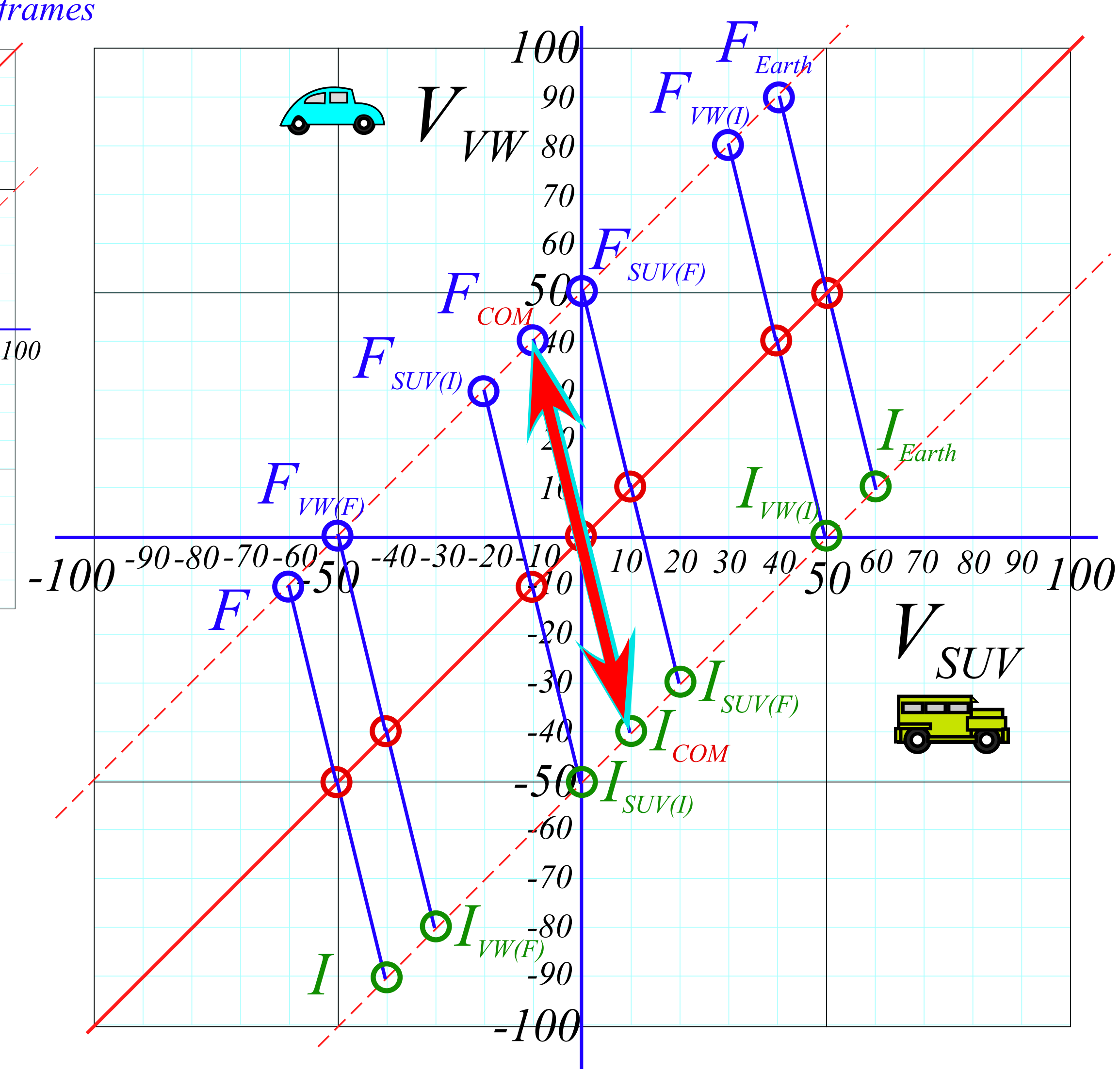


Fig. 2.5b
in Unit 1



There is just one velocity frame in which the time-reversed collision looks just like the original collision

That is the
Center-of-Momentum
***(COM)**-frame*

Time-reversal means flip t to -t...
(Run a movie backwards)

That means you flip Velocity V to -V...
(Everything goes backwards)

Algebra,Geometry, and Physics of momentum conservation axiom

→ *Vector algebra of collisions*
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
Energy Ellipse geometry

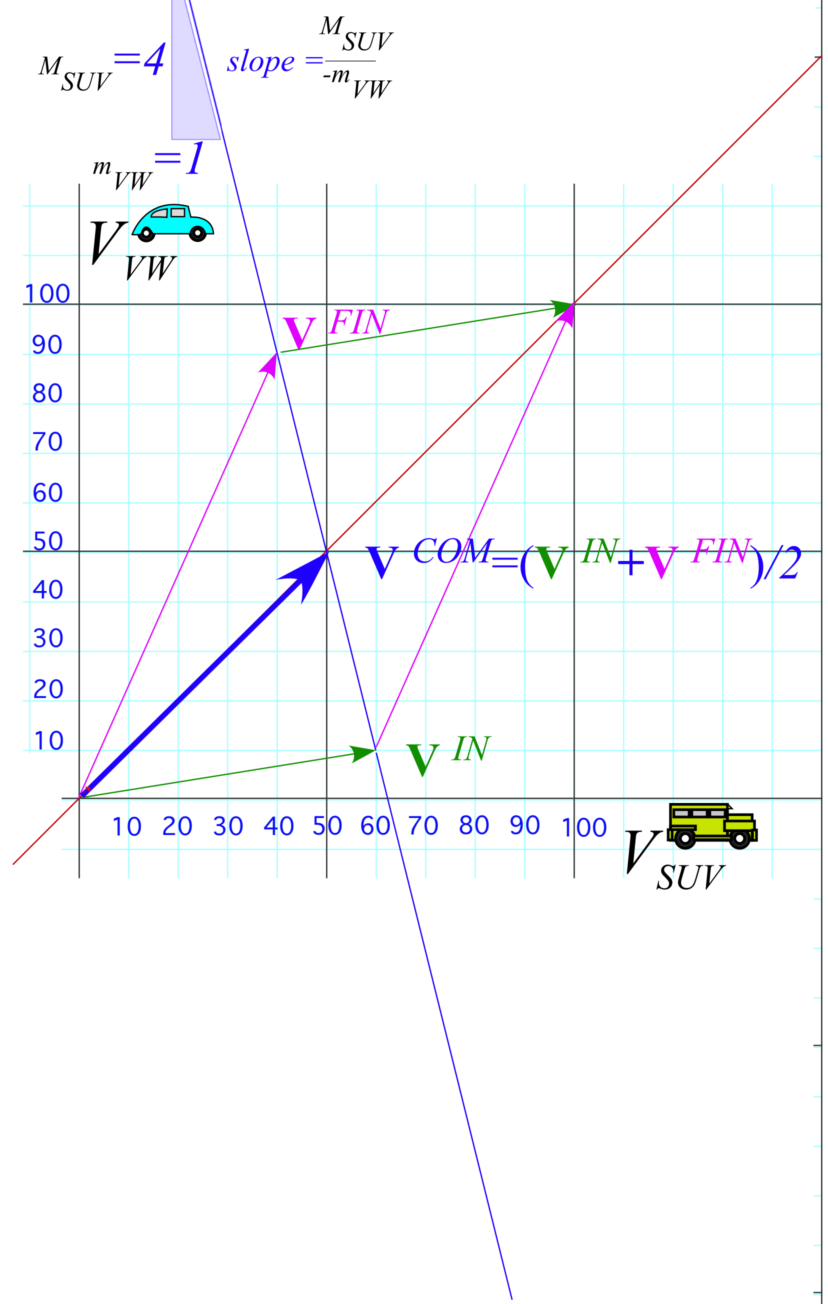
Developing
Conservation-of-Momentum
The key axiom of mechanics

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$

Wow! This is constant!
(Says the axiom)



Developing
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The key axiom of mechanics

Algebra, Geometry, and Physics of Momentum Conservation Axiom

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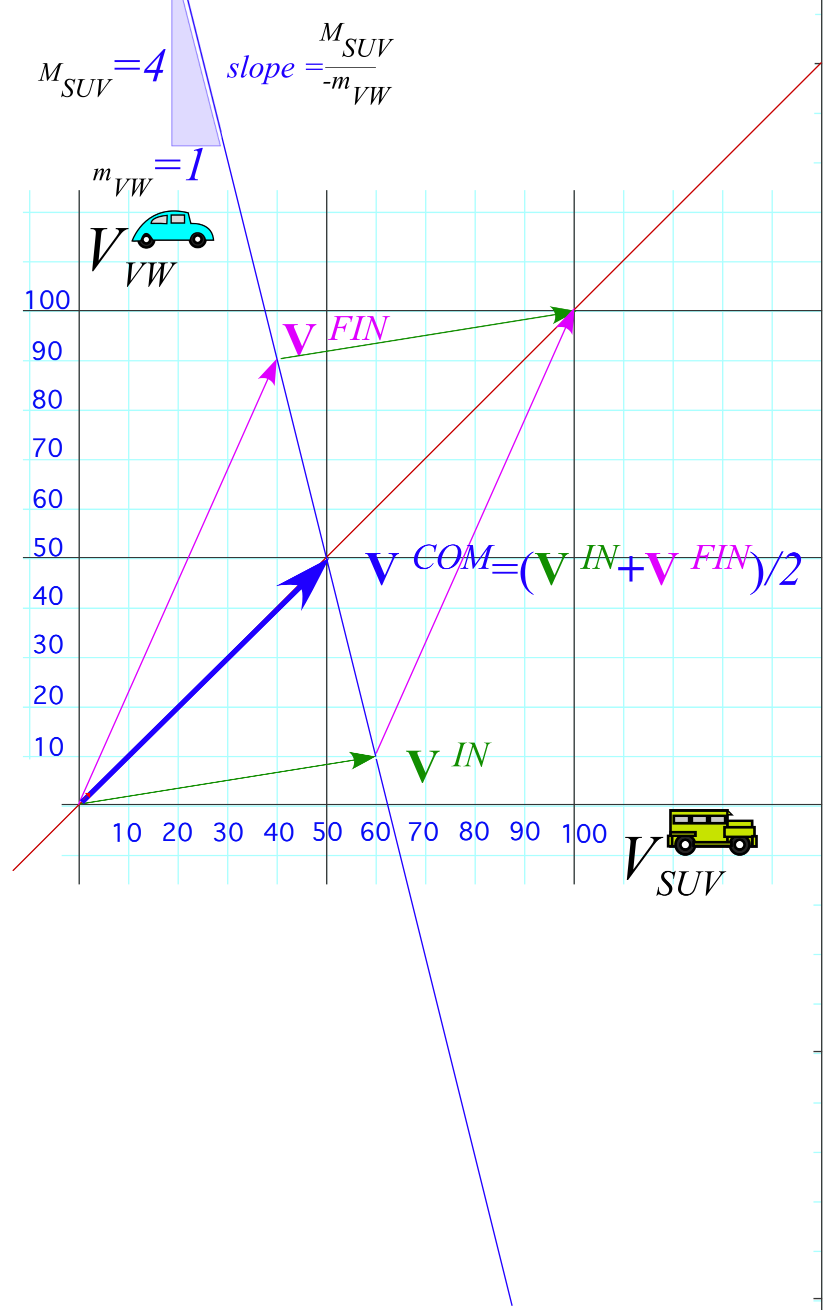

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}, \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix}, \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

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Conservation of momentum line:

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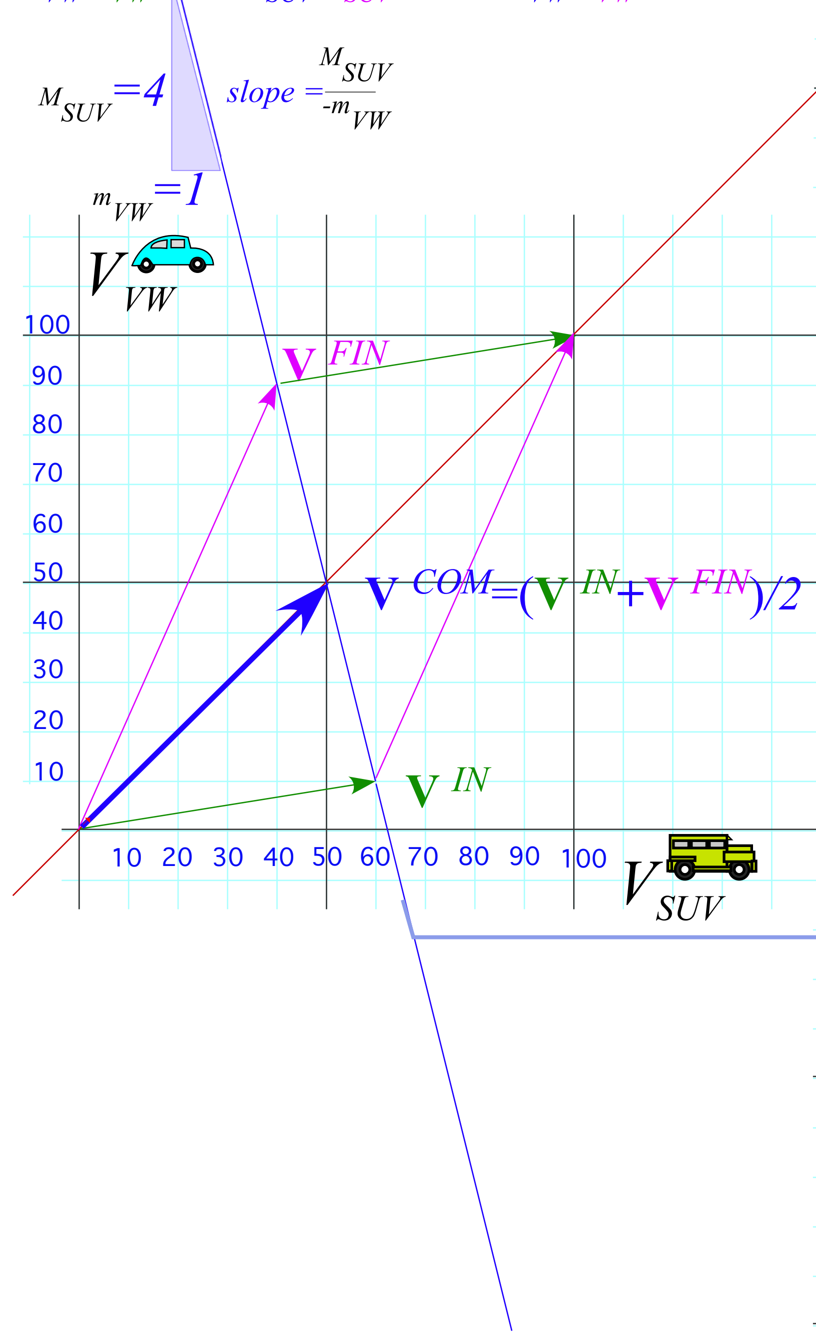


Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}, \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix}, \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Algebra, Geometry, and Physics of Momentum Conservation Axiom

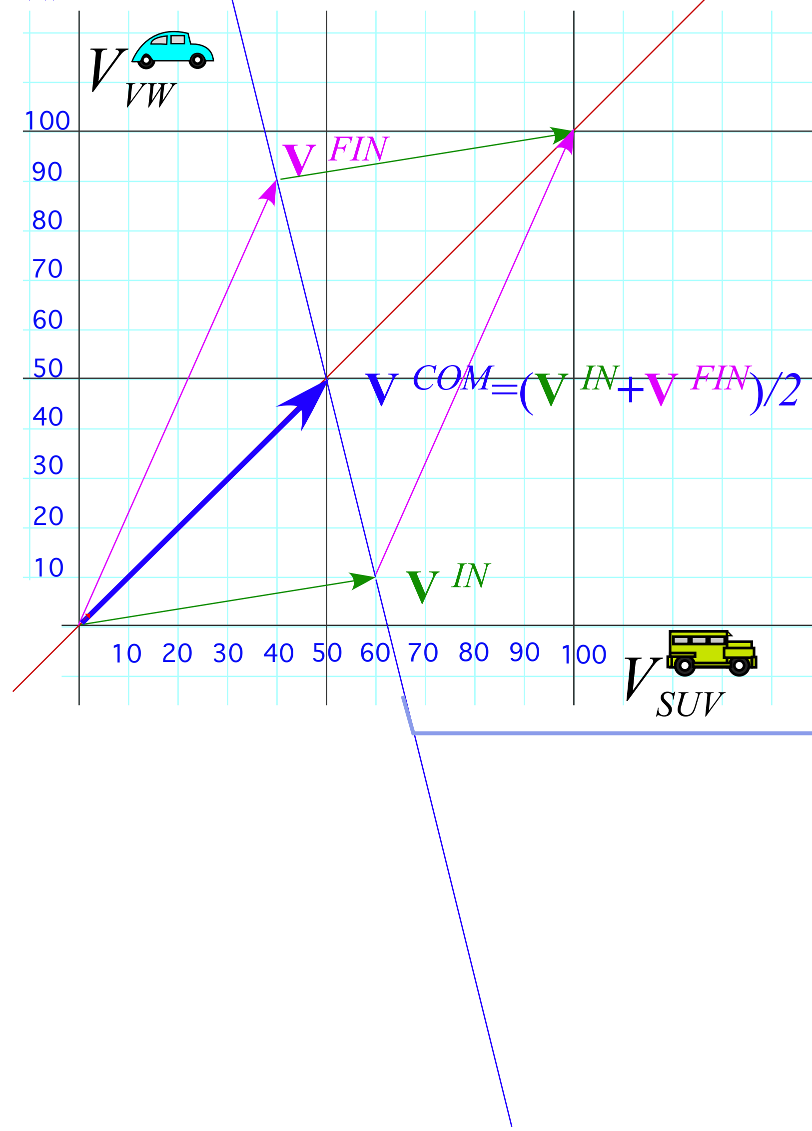
Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Wow! This is constant!
(Says the axiom)



Developing
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Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}, \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix}, \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

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Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

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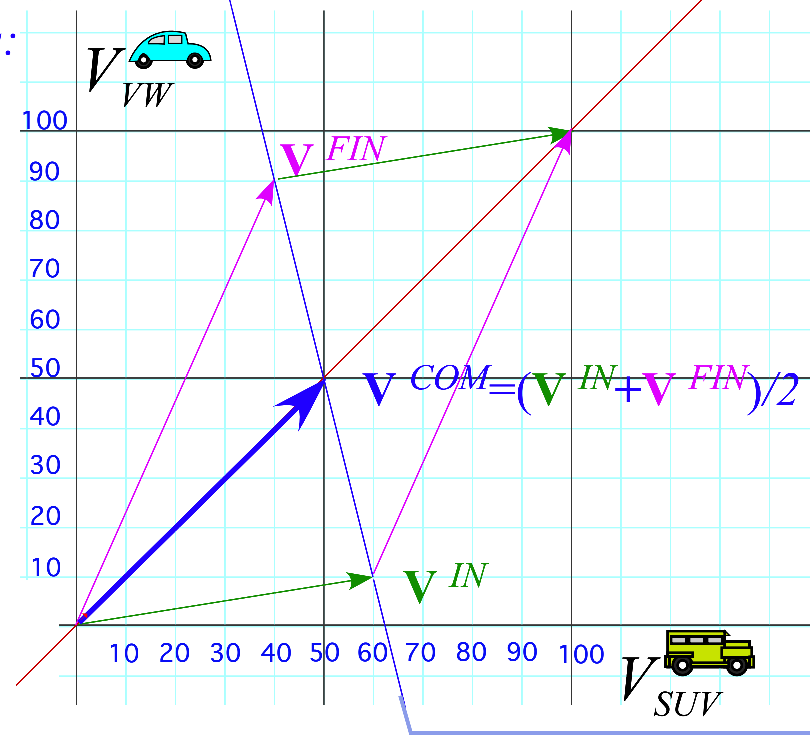
Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$



Developing
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The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

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Conservation of momentum line:

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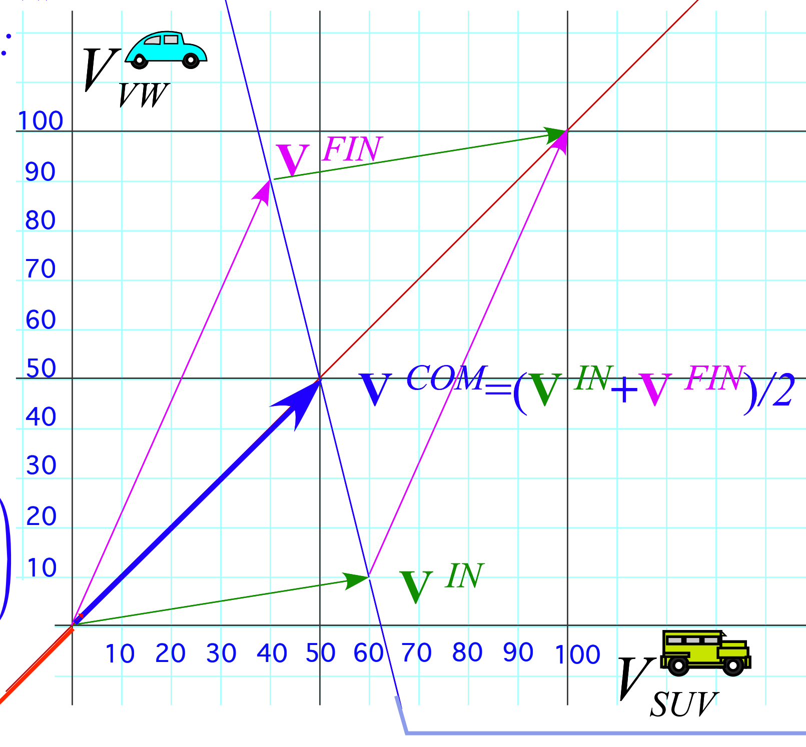
$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$=V^{COM}\mathbf{u}$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$



Define funny-unit vector : $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

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Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

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Vector $\mathbf{V}^{COM}$ is along *45° line*

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Vector algebra of collisions

→ *Matrix or tensor algebra of collisions*

Deriving Energy Conservation Theorem

Energy Ellipse geometry

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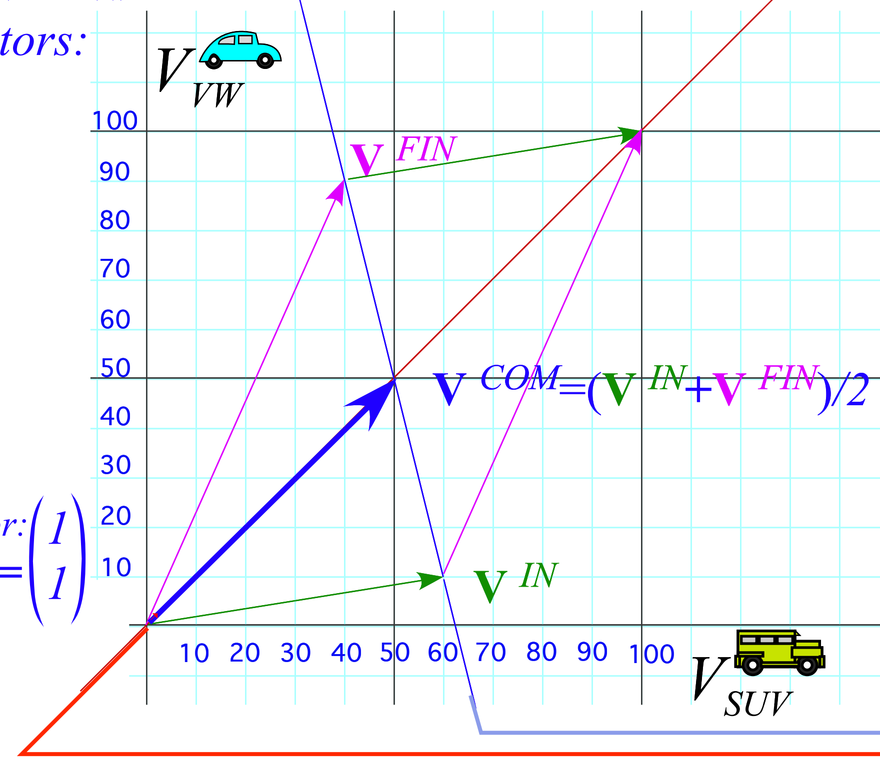
Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}=\begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}=\begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$=V^{COM}\mathbf{u}$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$



Define funny-unit vector : $\mathbf{u}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

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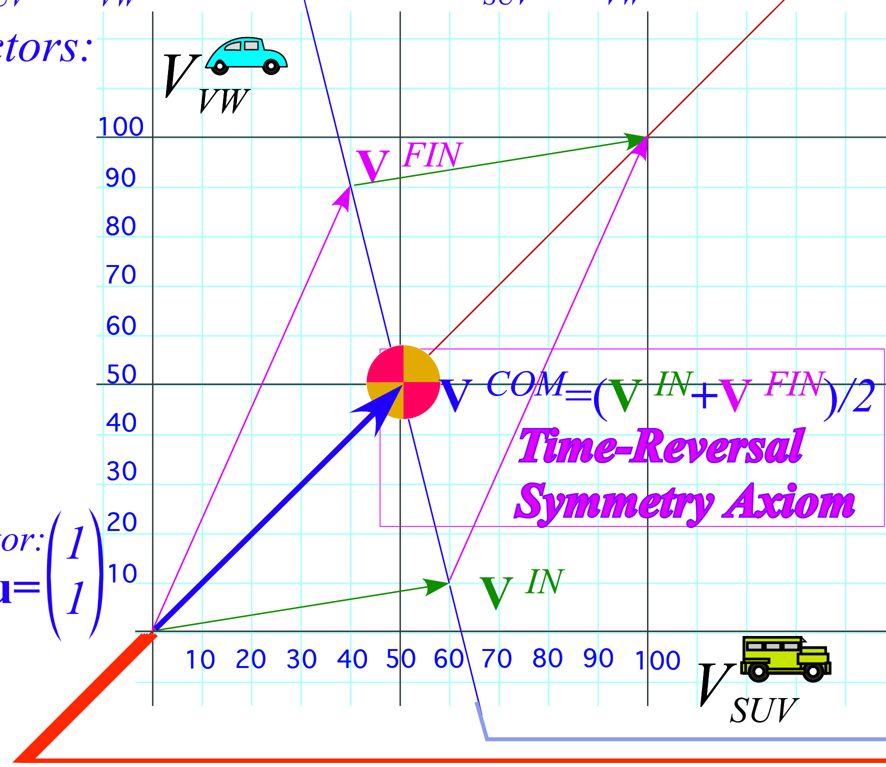
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}$$



...that give momentum vector:

$$\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$$

Define funny-unit vector : $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

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$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}=\begin{pmatrix} 60 \\ 10 \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$=V^{COM}\mathbf{u}$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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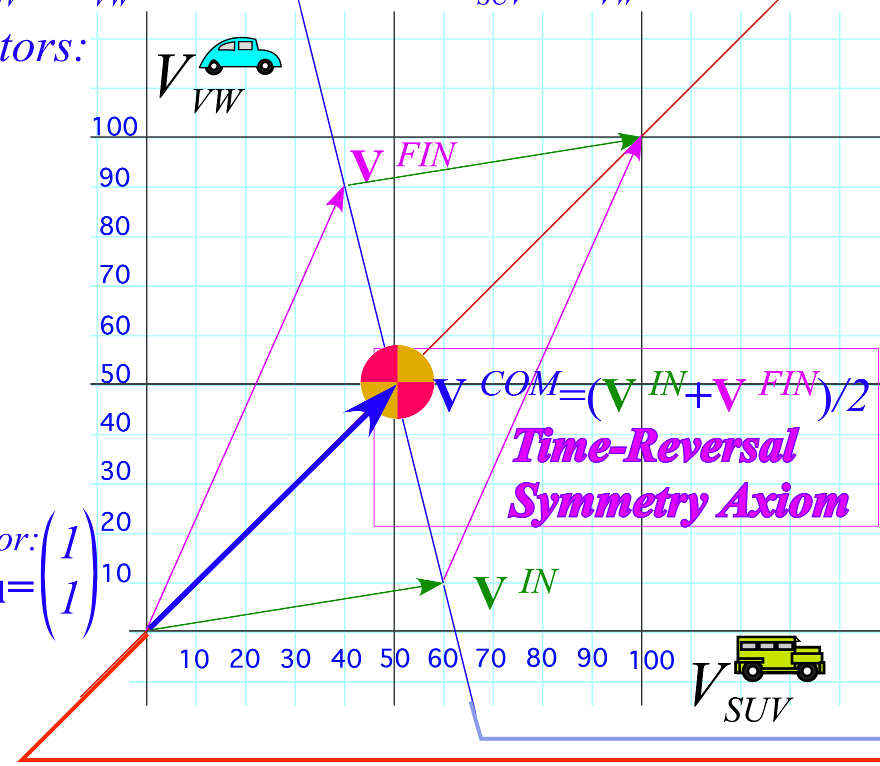

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whose sum of components is constant.

(by $\mathbf{u}\bullet$ product)

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Define funny-unit vector : $\mathbf{u}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

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$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

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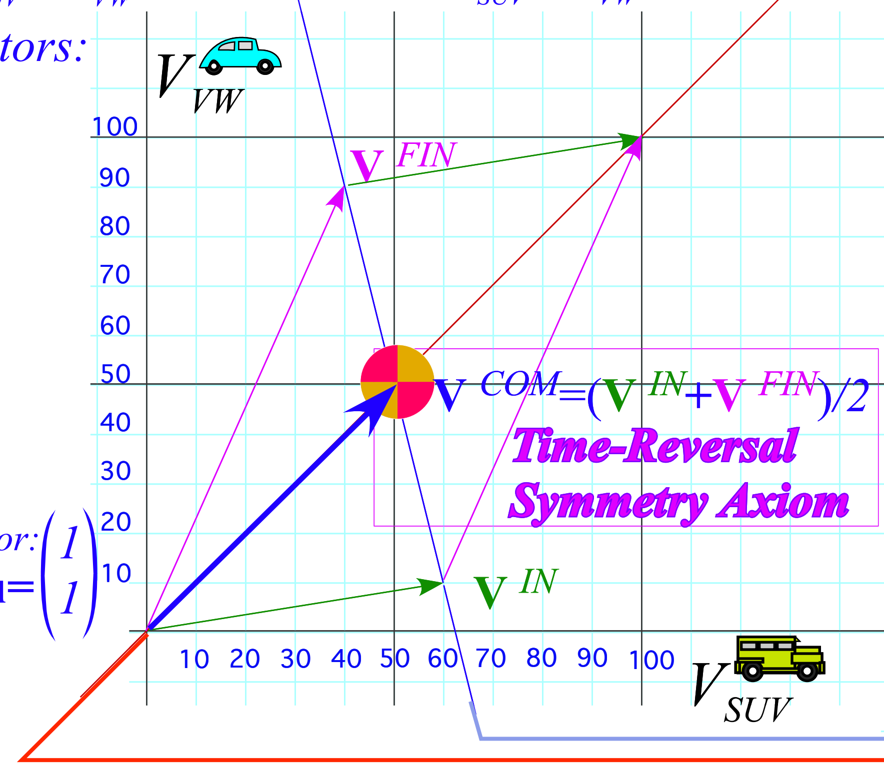

Developing
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whose sum of components is constant.

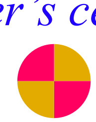
(by **u**• *product)*

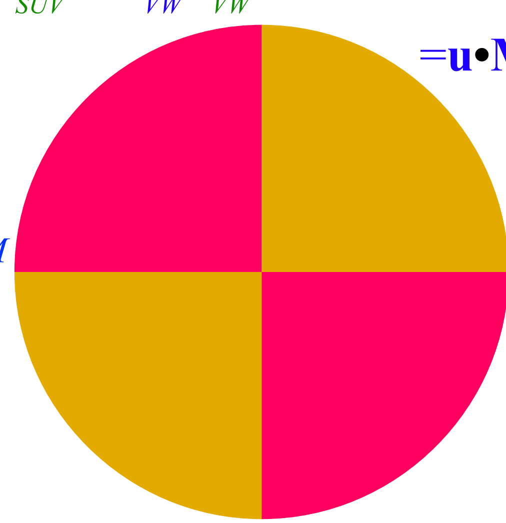
$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Denote Center of Momentum $\mathbf{V}^{COM}$

with engineer's centering symbol





Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

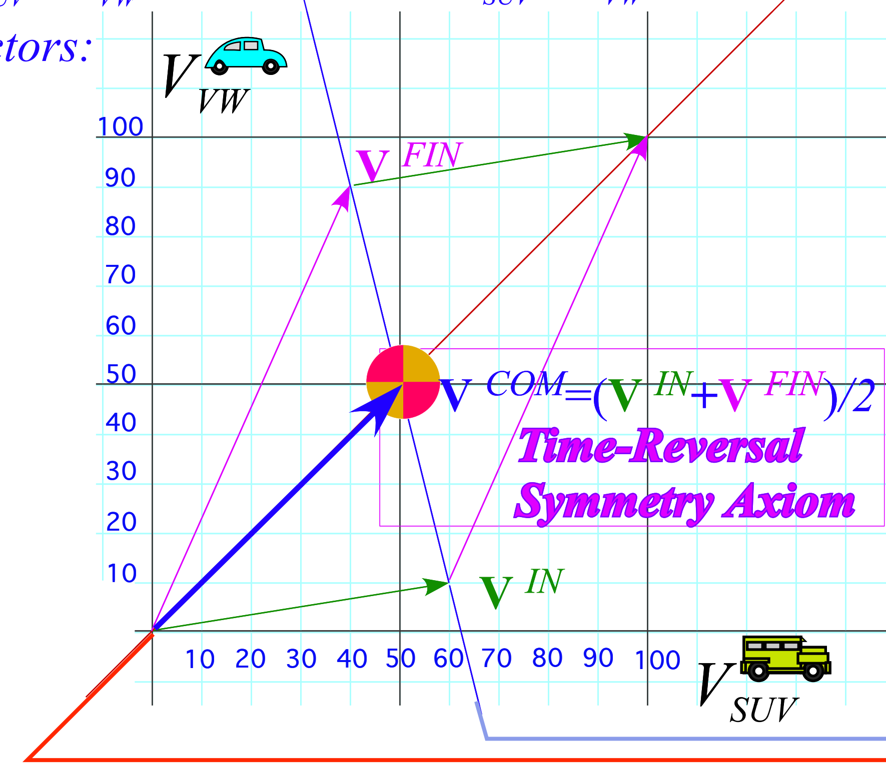
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}$$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Developing ***Conservation-of-Momentum*** *The key axiom of mechanics*

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

Matrix or tensor algebra of collisions

→ *Deriving Energy Conservation Theorem*

Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

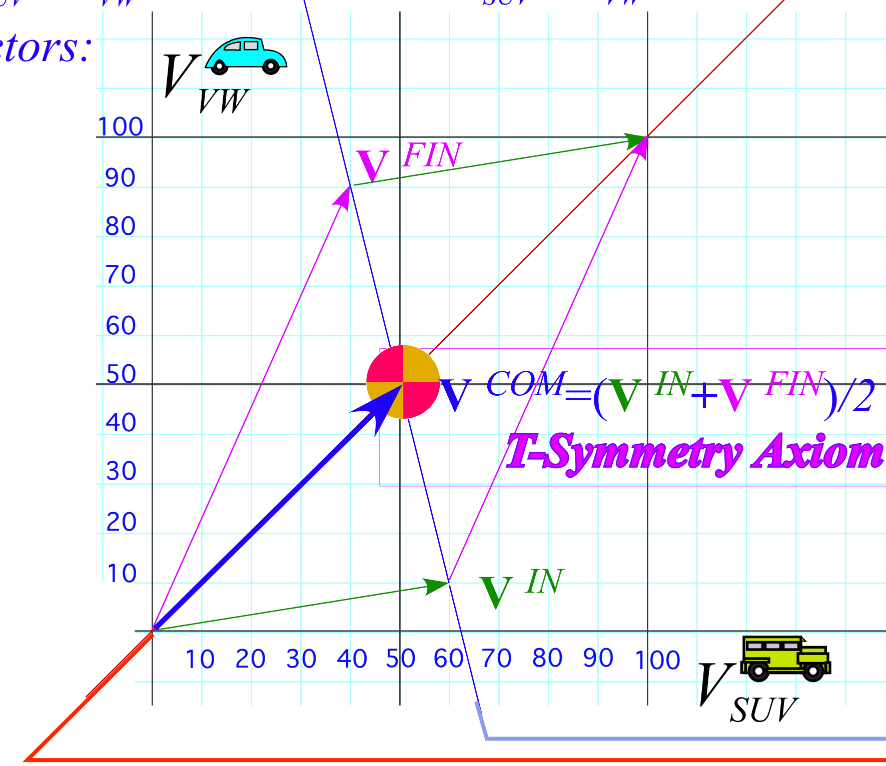
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}$$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.
(by **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$.

Developing ***Conservation-of-Momentum*** *The key axiom of mechanics*

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

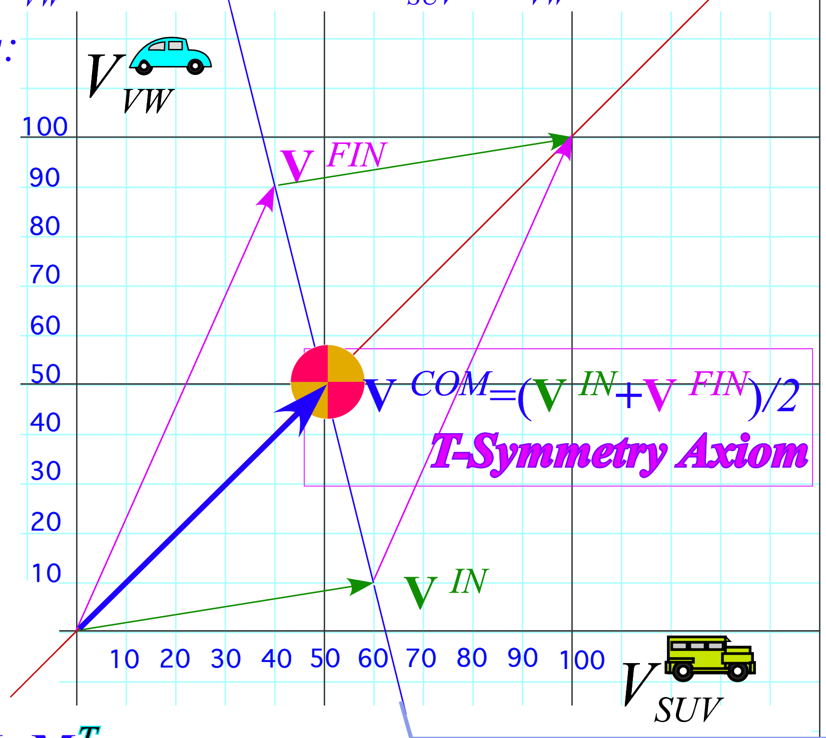
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$

M-symmetry $\mathbf{M}=\mathbf{M}^T$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant. (by $\mathbf{u}\bullet$ product)

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ gives:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$.

Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}=\begin{pmatrix}40\\90\end{pmatrix}$$

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}=\begin{pmatrix}60\\10\end{pmatrix}$$

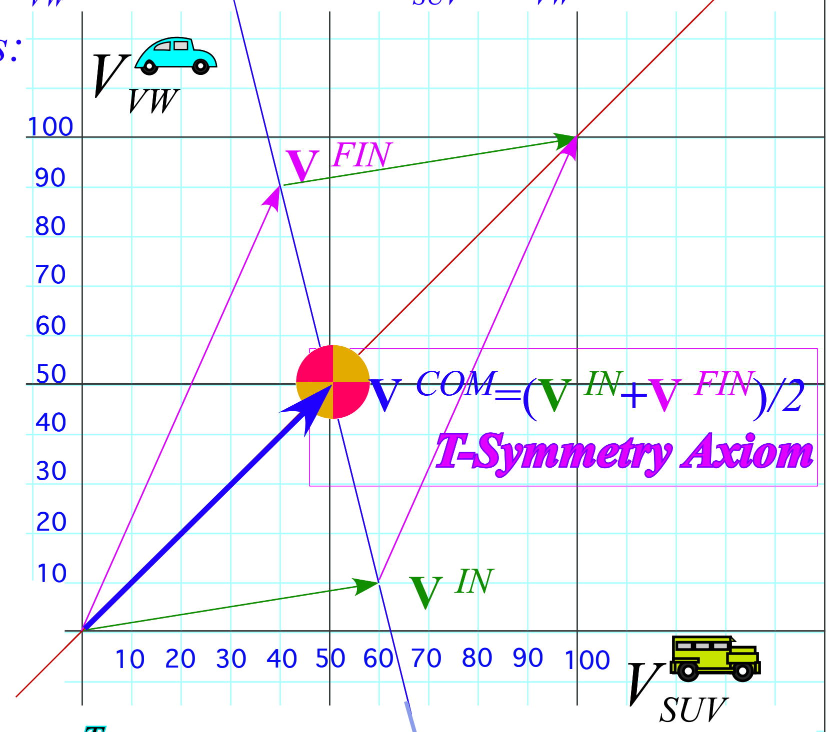
$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$

M-symmetry $\mathbf{M}=\mathbf{M}^T$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant. *(by* **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

Points of vectors

$\mathbf{V}^{FIN}$ and $\mathbf{V}^{COM}$ and $\mathbf{V}^{IN}$

all lie on

momentum - conservation line

Vector $\mathbf{V}^{COM}$ is along *45° line*

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

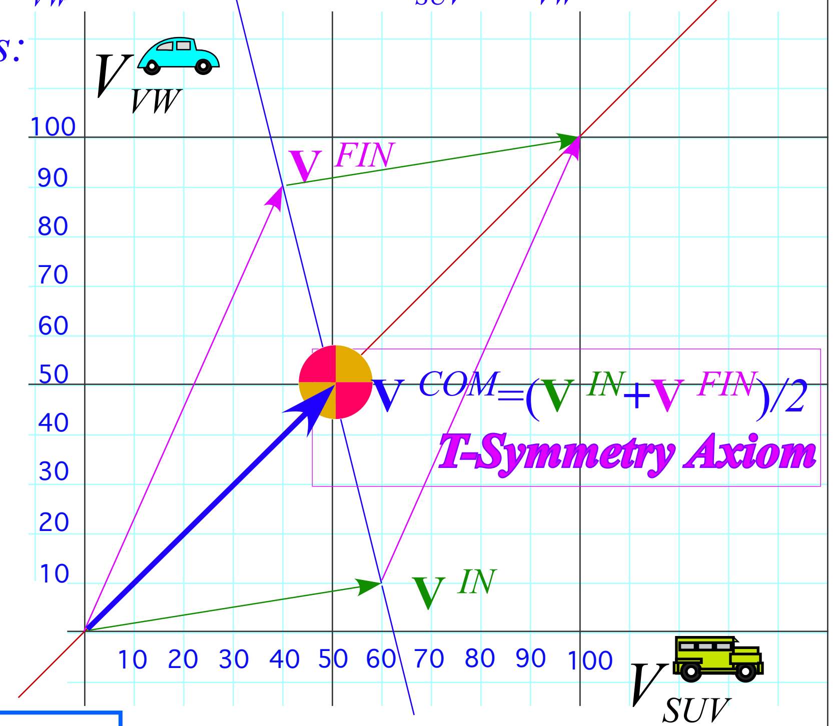
$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix} M_{SUV} & 0 \\ 0 & M_{VW} \end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix} M_{SUV} & 0 \\ 0 & M_{VW} \end{pmatrix}\begin{pmatrix} V_{SUV} \\ V_{VW} \end{pmatrix}=\begin{pmatrix} P_{SUV} \\ P_{VW} \end{pmatrix}=\begin{pmatrix} M_{SUV}V_{SUV} \\ M_{VW}V_{VW} \end{pmatrix}$

whose sum of components is constant.

*(by **u**• product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Developing
Conservation-of-Momentum
The key axiom of mechanics

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix} V_{SUV}^{COM} \\ V_{VW}^{COM} \end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

By **M-symmetry** $\mathbf{M}=\mathbf{M}^T$: $\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

this becomes:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}-1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=1/2\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
→ *Completing derivation of Energy Conservation* ***Theorem***
Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$

Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

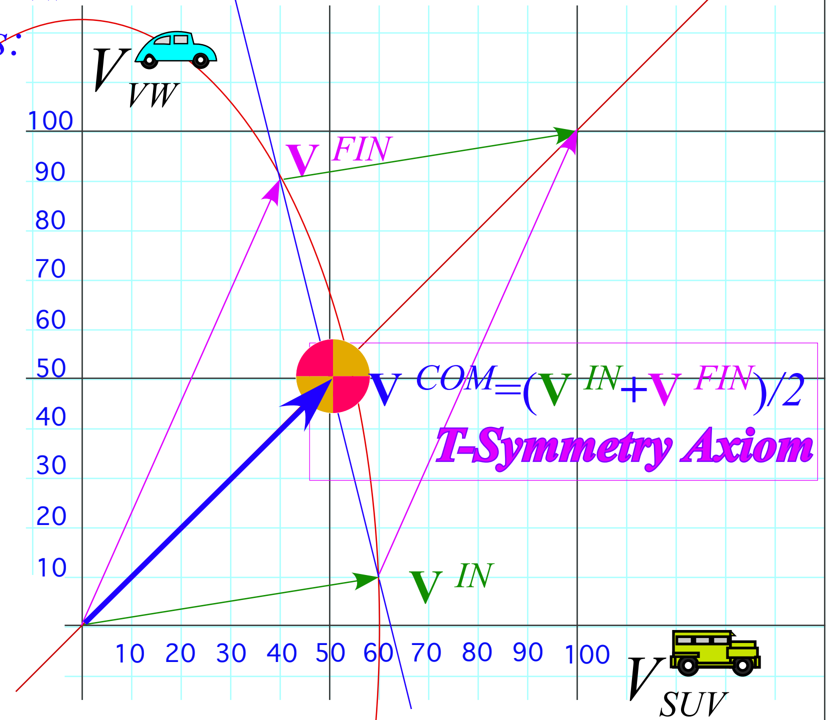
$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by $\mathbf{u}\bullet$ *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Developing
Conservation-of-Momentum
The key axiom of mechanics
leading to
Conservation-of-Energy Theorem

Define velocity vector points

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix},\ \mathbf{V}^{COM}=\begin{pmatrix}V_{SUV}^{COM}\\V_{VW}^{COM}\end{pmatrix},\ \mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

By ***M-symmetry*** $\mathbf{M}=\mathbf{M}^T$: $\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

this becomes:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}-1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=1/2\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

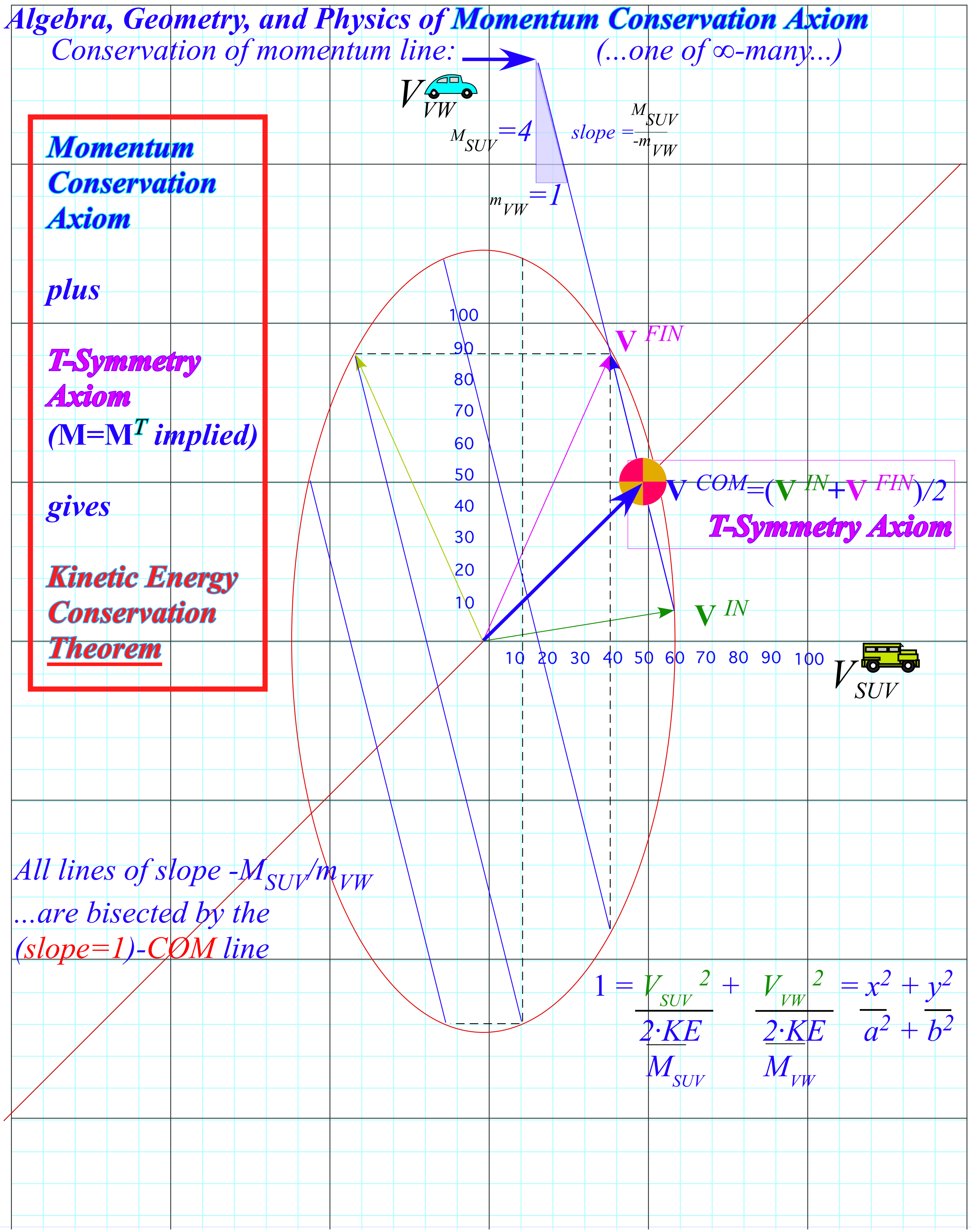
These are equations for energy conservation ellipse:

$$const.={}^1\!/_2M_{SUV}V_{SUV}^{\ 2}+{}^1\!/_2M_{VW}V_{VW}^{\ 2}$$
$$={}^1\!/_2M_{SUV}V_{SUV}^{\ 2}+{}^1\!/_2M_{VW}V_{VW}^{\ 2}$$

=Kinetic Energy=KE is now defined and proved a constant under T-Symmetry

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation <u>Theorem</u>
→ *Energy Ellipse geometry*



Developing

Conservation-of-Momentum

The key ***axiom*** *of mechanics*

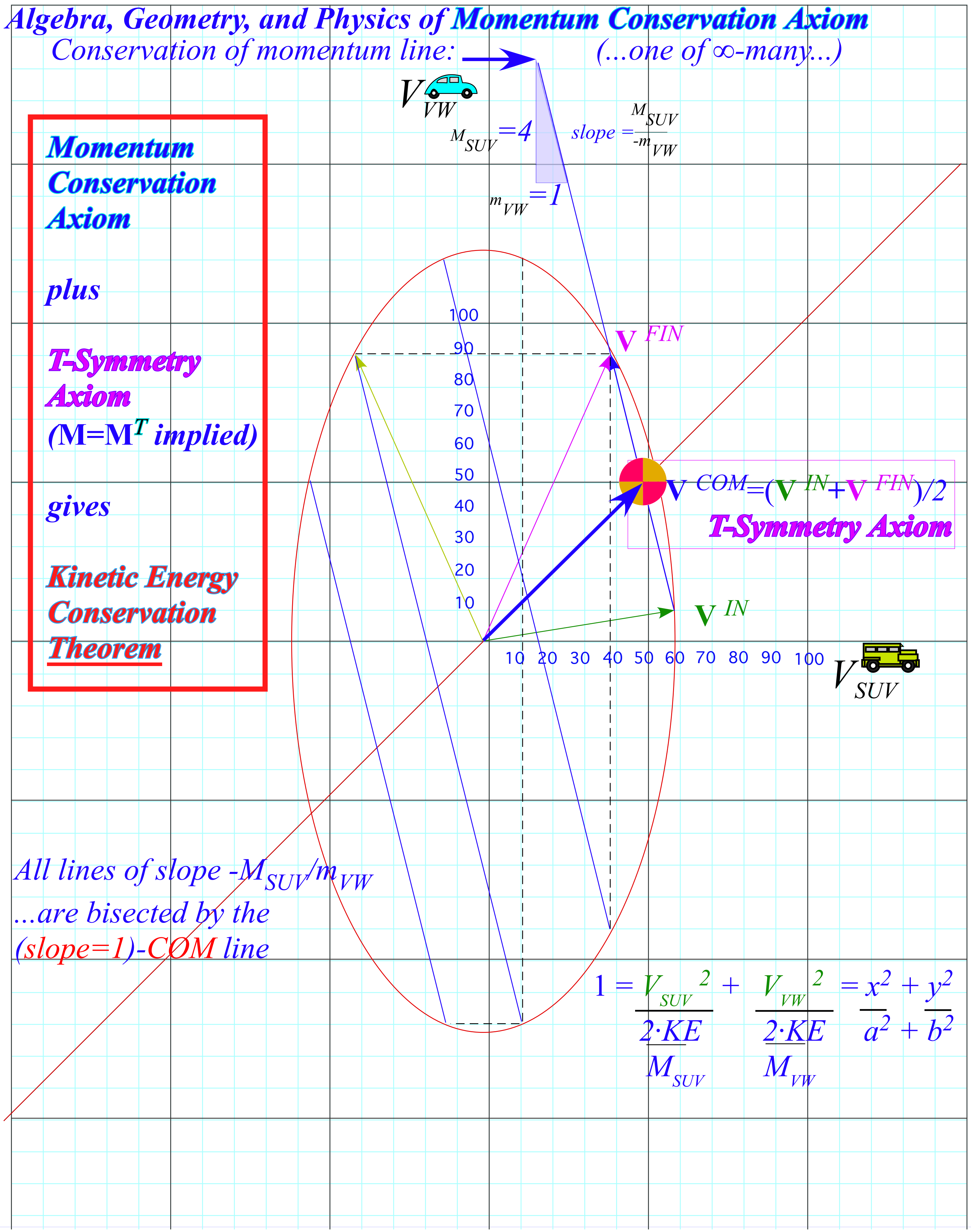
leading to

Conservation-of-Energy Theorem

$V^{COM} \bullet M \bullet V^{COM} - 1/2 V^{FIN} \bullet M \bullet V^{IN}$
$= 1/2 V^{IN} \bullet M \bullet V^{IN} = 1/2 V^{FIN} \bullet M \bullet V^{FIN}$

These are equations for energy conservation ellipse:

$$KE = {}^1/_2 M_{SUV} V_{SUV}{}^2 + {}^1/_2 M_{VW} V_{VW}{}^2$$



Developing

Conservation-of-Momentum

The key axiom of mechanics

leading to

Conservation-of-Energy Theorem

If and only if...

there is T-Symmetry

$$V^{COM}\bullet M\bullet V^{COM} - \tfrac{1}{2}V^{FIN}\bullet M\bullet V^{IN}$$
$$= \tfrac{1}{2}V^{IN}\bullet M\bullet V^{IN} = \tfrac{1}{2}V^{FIN}\bullet M\bullet V^{FIN}$$

These are equations for energy conservation ellipse:

$$KE = \tfrac{1}{2}M_{SUV}V_{SUV}^{\ 2} + \tfrac{1}{2}M_{VW}V_{VW}^{\ 2}$$

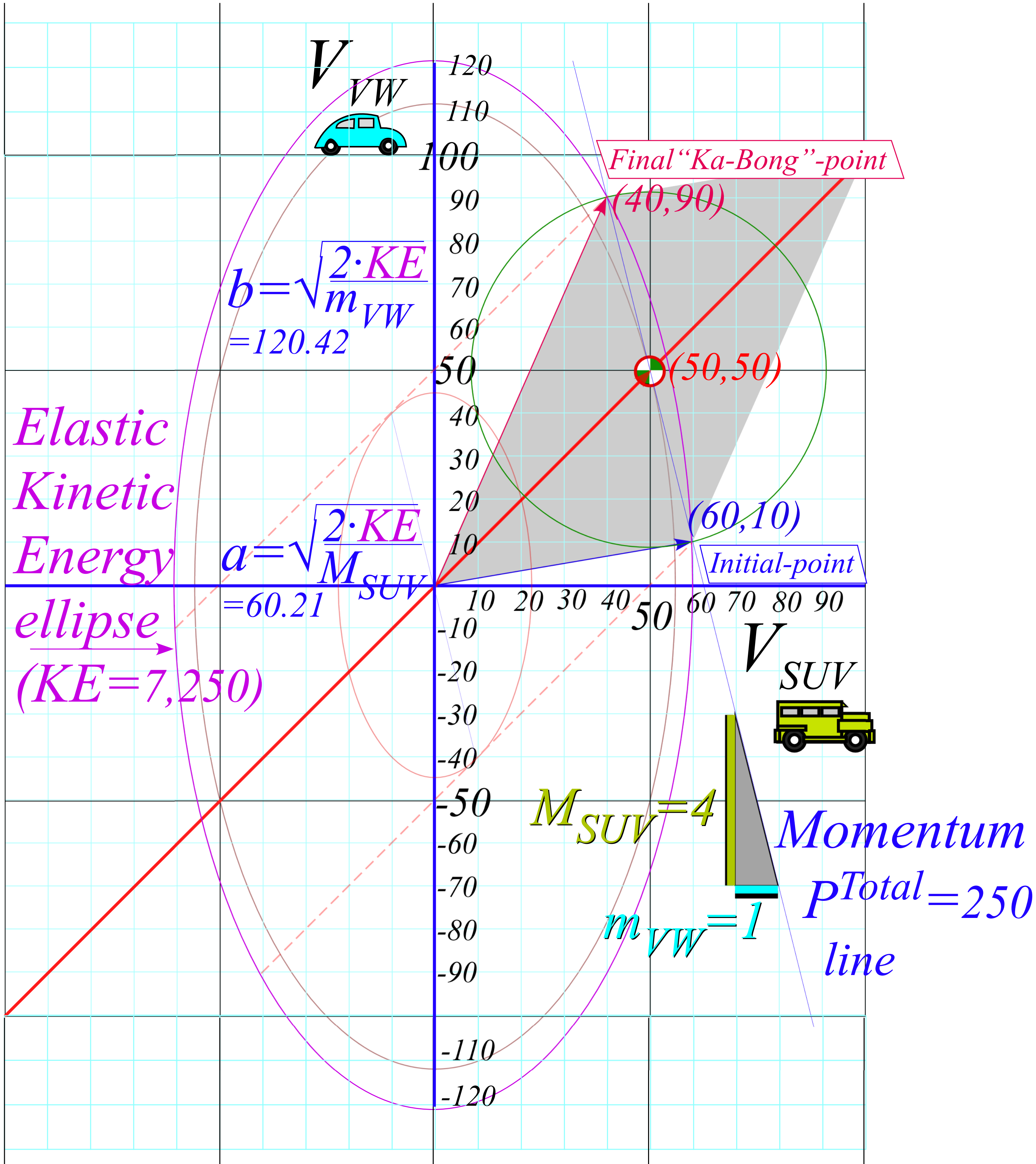


Fig. 3.1 a
in Unit 1

Developing

Conservation-of-Momentum

The key axiom of mechanics

leading to

Conservation-of-Energy Theorem

If and only if…

*there is **T-Symmetry***

Fig. 3.1

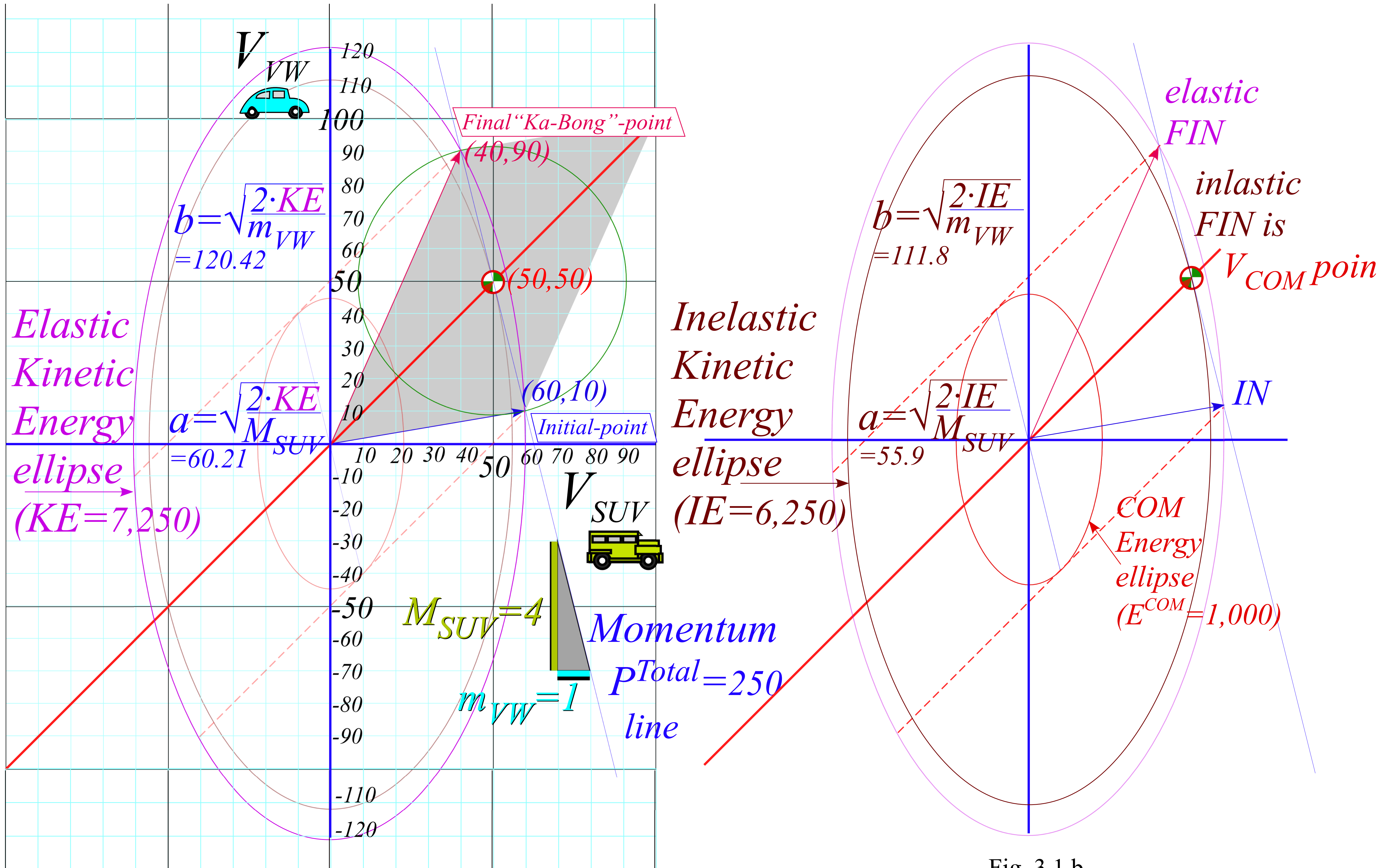


Fig. 3.1 a
in Unit 1

Fig. 3.1 b
in Unit 1

Fig. 3.1

As usual in physics, opposite extremes are easier to analyze than the generic "real(er) world" in between!

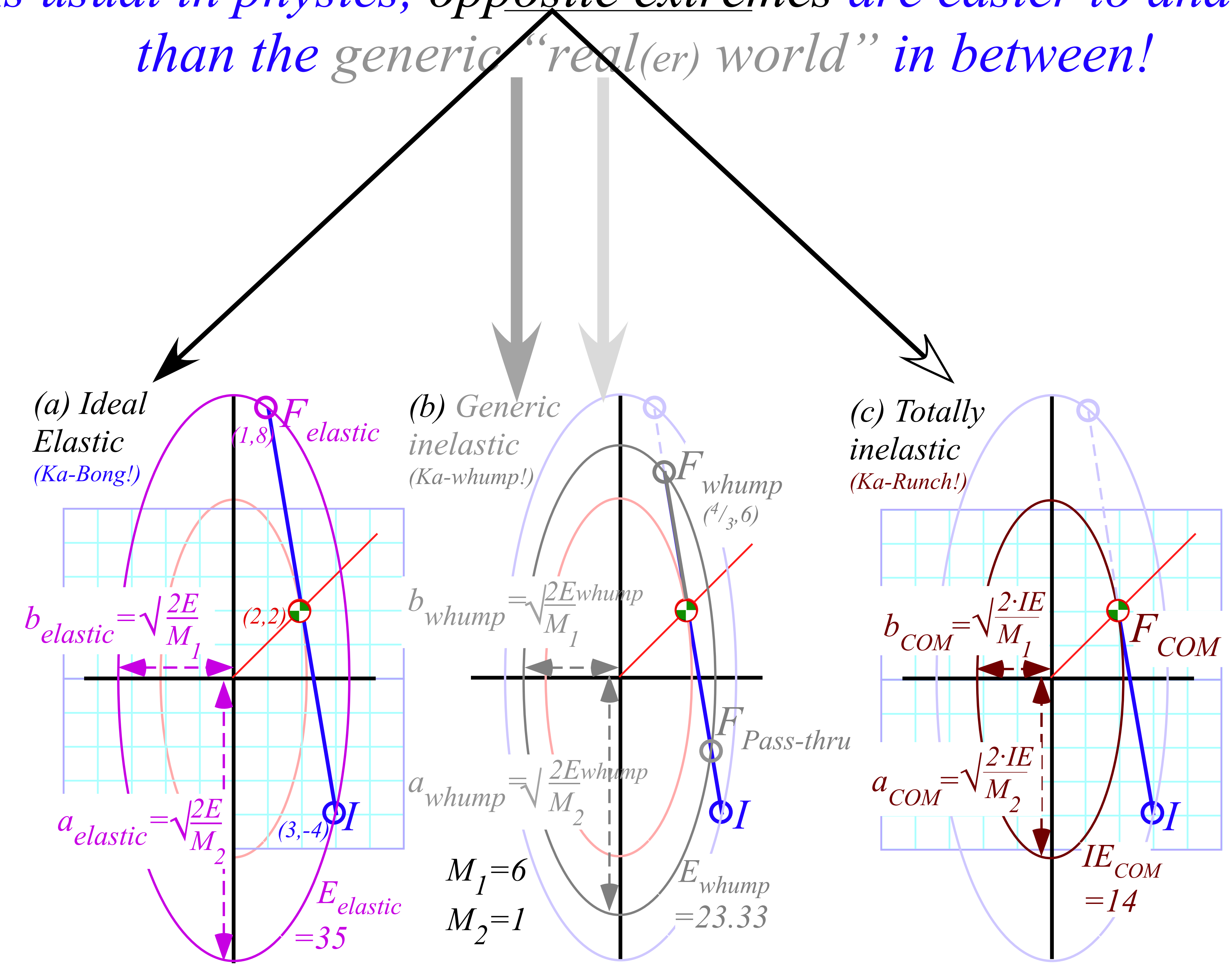


Fig. 3.2 in Unit 1

(This case has Bush era requisite SUV mass of the 6 ton "Hummer")

Next: **The X-2 pen-launcher**

Here ***T-Symmetry*** *is best*

Here ***T-Symmetry*** *is less*

Here ***T-Symmetry*** *is least*

Numerical details of collision tensor algebra

General Inertia Tensor M or inertia matrix of n^2 coefficients M_{jk} for dimension $n=2, 3,....$

$$\left.\begin{aligned} P_1 &= M_{11}V_1 + M_{12}V_2 \\ P_2 &= M_{21}V_1 + M_{22}V_2 \end{aligned}\right\} \quad denoted : \vec{\mathbf{P}} = \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}} \quad or : \begin{pmatrix} P_1 \\ P_2 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

With 45° diagonal $\mathbf{V}^{COM}$ so: . $\quad V_1^{COM} = V_2^{COM} \equiv V^{COM}$

$$P_{Total} = M_1 V_1^{IN} + M_2 V_2^{IN} = M_1 V_1^{FIN} + M_2 V_2^{FIN} = M_1 V^{COM} + M_2 V^{COM} = M_{Total} V^{COM}$$

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A product of total momentum P_{Total} and V^{COM} is expressed by *tensor quadratic forms* **v•M•u**

$$V^{COM} P_{Total} = \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN} = \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = V^{COM} M_{Total} V^{COM}$$

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Write this out with the numbers used in Fig. 3.1 where $V^{COM} = 50$. (2 pages back)

$$50 P_{Total} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix} = 50 M_{Total} 50 = 12{,}500$$

$$= 100 \cdot 125 = 100 \cdot 125 = 50 \cdot 250$$

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P_{Total} =*250* is the same at IN, FIN, and COM. Now use *T*-symmetry: $\vec{\mathbf{V}}^{COM} = (\vec{\mathbf{V}}^{FIN} + \vec{\mathbf{V}}^{IN})/2$

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Subtracting:

$$\frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2}$$

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$$V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{4}$$

Subtracting: $\dfrac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \dfrac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2}$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

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With ($M_{12} = 0 = M_{21}$) kinetic energy $KE_{Elastic} = \frac{1}{2} \overleftarrow{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

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$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE_{Elastic}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

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$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy $KE_{inelastic} = IE$ when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE_{Inelastic} = V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE_{Elastic} + \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = IE$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

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$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE_{Elastic}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy $KE_{inelastic} = IE$ when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE_{Inelastic} = V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE_{Elastic} + \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

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$$KE_{Inelastic} = \tfrac{1}{2} \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = \tfrac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 50 \\ 50 \end{pmatrix} = 6{,}250$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies:

$$\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$$

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With ($M_{12} = 0 = M_{21}$) kinetic energy $KE_{Elastic} = \frac{1}{2} \vec{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

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$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy $KE_{inelastic} = IE$ when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE_{Inelastic} = V^{COM} P_{Total} - \frac{\vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\vec{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE_{Elastic} + \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = IE$$

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The difference is inelastic "crunch" energy *KE-IE* or, for elastic cases, potential energy of compression.

$$KE_{Elastic} - KE_{Inelastic} = \frac{\vec{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} - \frac{\vec{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$1{,}000 = 3{,}625 - 2{,}625 = KE - IE$$

Transpose symmetry ($M_{jk} = M_{kj}$) of the **M**-matrix implies: $\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN} = \overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}$

$$\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix} = \begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}$$

$$= 100 \cdot 105 = 100 \cdot 105 = 10{,}500$$

With ($M_{12} = 0 = M_{21}$) kinetic energy $KE_{Elastic} = \frac{1}{2} \overleftarrow{\mathbf{V}} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}$ is the same at $\mathbf{V} = \mathbf{V}^{IN}$ and $\mathbf{V} = \mathbf{V}^{FIN}$.

$$V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{2} = \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{FIN}}{2} = KE_{Elastic}$$

$$12{,}500 - \frac{10{,}500}{2} = \frac{\begin{pmatrix} 60 & 10 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 60 \\ 10 \end{pmatrix}}{2} = \frac{\begin{pmatrix} 40 & 90 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 40 \\ 90 \end{pmatrix}}{2} = KE_{Elastic}$$

$$12{,}500 - 5{,}250 = 7{,}250 = 7{,}250$$

Consider kinetic energy $KE_{inelastic} = IE$ when $\mathbf{V} = \mathbf{V}^{COM}$. It is reduced by *1,000* from 7,250 to 6,250.

$$KE_{Inelastic} = V^{COM} P_{Total} - \frac{\overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{1}{2} V^{COM} P_{Total} = \frac{\overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM}}{2} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} + \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} = \frac{1}{2} KE_{Elastic} + \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$12{,}500 - \frac{12{,}500}{2} = 6{,}250 = 6{,}250 = 3{,}625 + 2{,}625 = IE$$

$$KE_{Inelastic} = \frac{1}{2} \overleftarrow{\mathbf{V}}^{COM} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{COM} = \frac{1}{2} \begin{pmatrix} 50 & 50 \end{pmatrix} \bullet \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \bullet \begin{pmatrix} 50 \\ 50 \end{pmatrix} = 6{,}250$$

The difference is inelastic "crunch" energy *KE*-*IE* or, for elastic cases, potential energy of compression.

$$KE_{Elastic} - KE_{Inelastic} = \frac{\overleftarrow{\mathbf{V}}^{IN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4} - \frac{\overleftarrow{\mathbf{V}}^{FIN} \bullet \overleftrightarrow{\mathbf{M}} \bullet \vec{\mathbf{V}}^{IN}}{4}$$

$$1{,}000 = 3{,}625 - 2{,}625 = KE - IE$$

This difference is the same in all reference frames including COM frame where $KE_{inelastic} = IE$ is zero.

Note "crunch" energy ElasticKE-inelasticIE = 0.21
is the same in all frames including COM-frame.

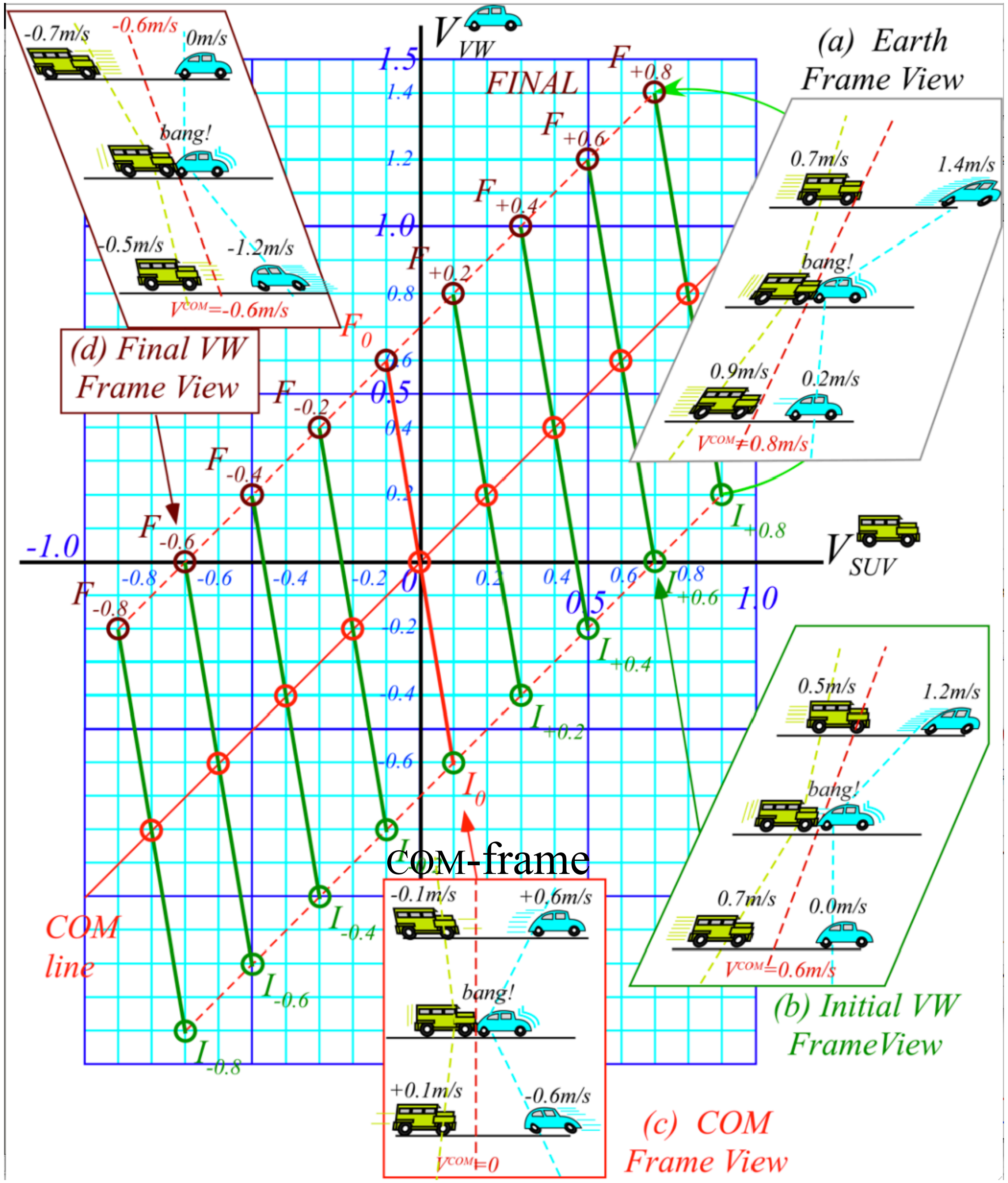

Fig. 3.4 Galilean Frame Views of collision like Fig. 2.5 or Fig. 3.1 with Bush (6:1) SUV.
(a) Earth frame view (b) Initial VW frame (VW initially fixed)
(c) COM frame view (d) Final VW frame (VW ends up fixed)

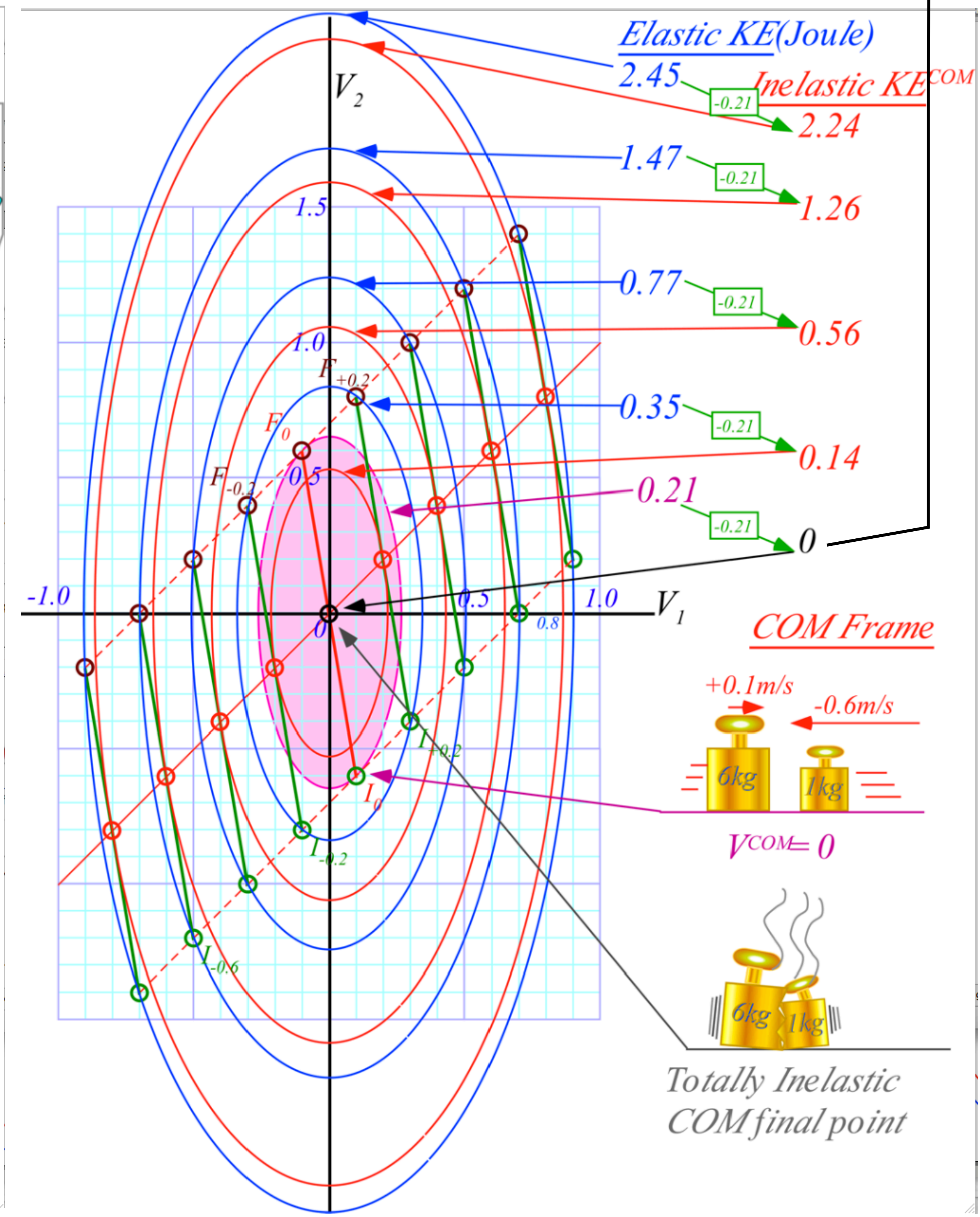

Fig. 3.5 Momentum (P=const.)-lines and energy (KE=const.)-ellipses appropriate for Fig. 3.4.