Lecture 1
Tue. 8.27.2013

Axiomatic development of classical mechanics

(Ch. 1 and Ch. 2 of Unit 1)

Geometry of momentum conservation axiom

*Totally Inelastic "ka-runch" collisions**
*Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry**

Geometry of Galilean translation symmetry

Time reversal symmetry
...of COM collisions

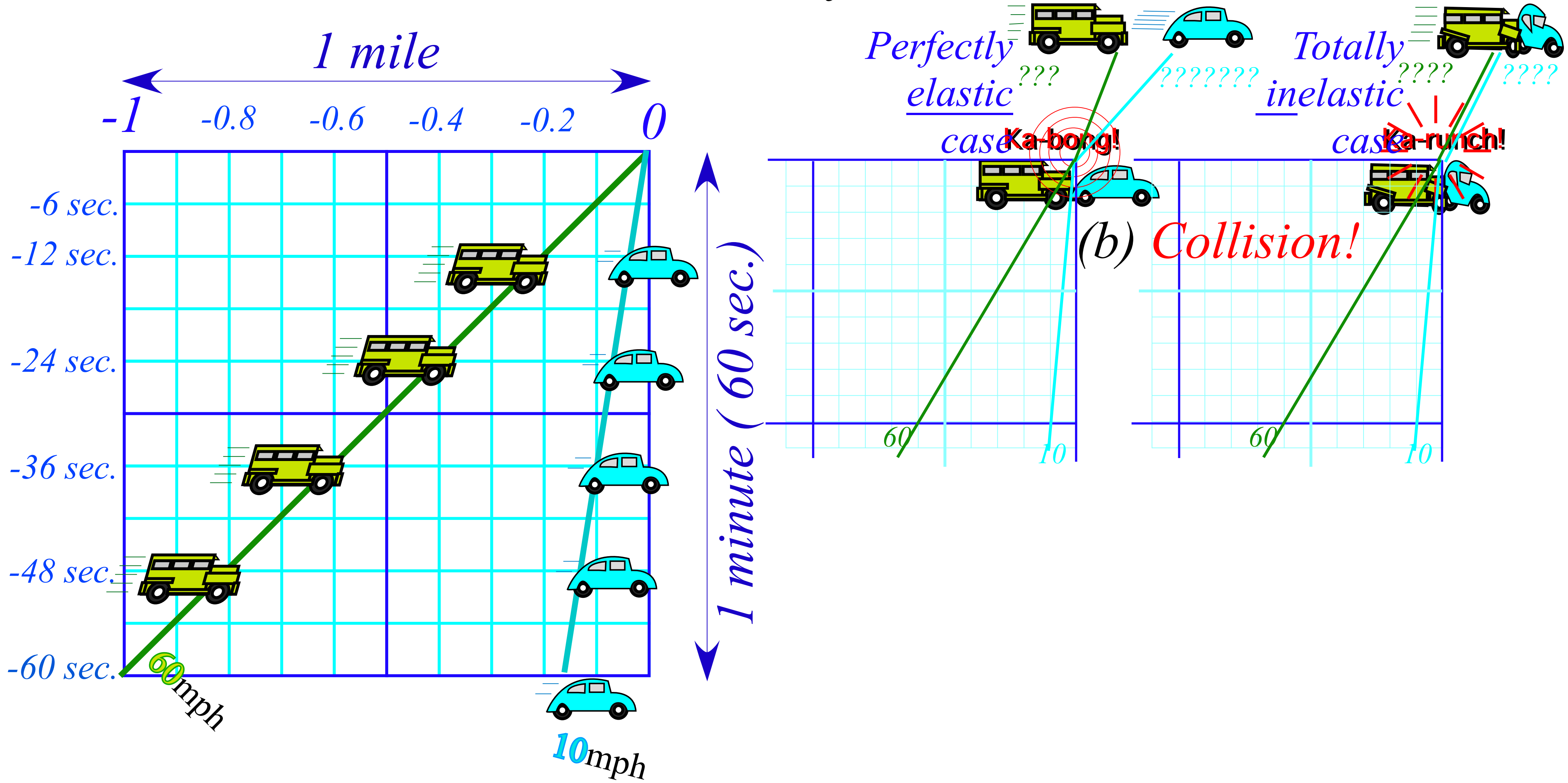
Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem

** Download Superball Collision Simulator*
http://www.uark.edu/ua/modphys/markup/BounceItWeb.html

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision..... *After collision...what velocities?*



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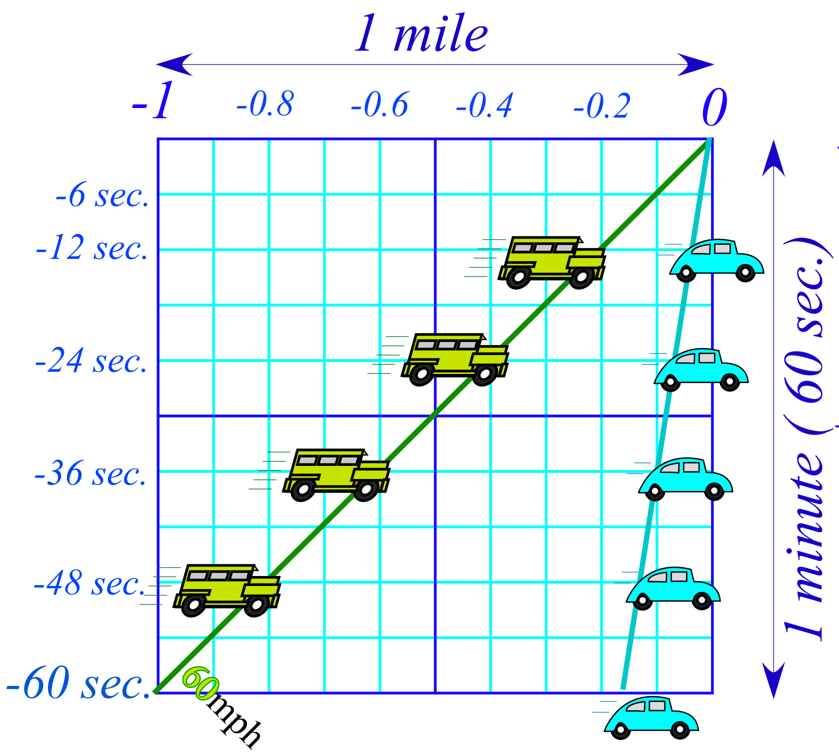
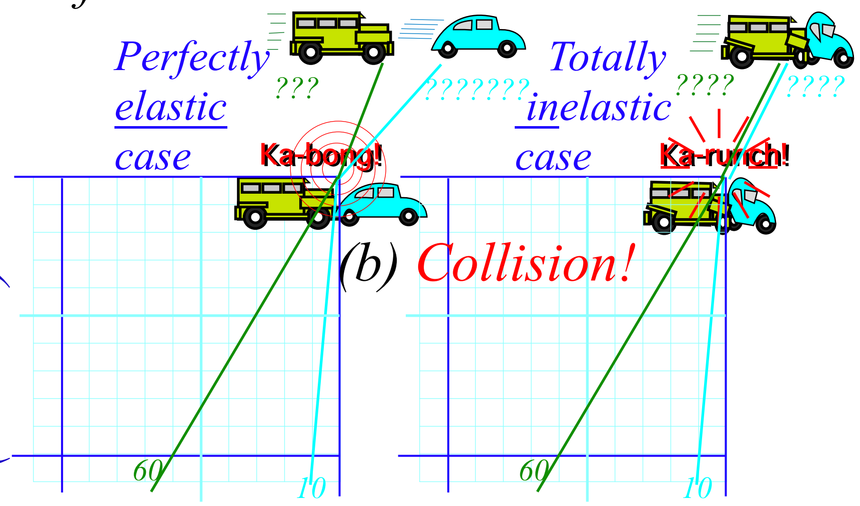


After collision...what velocities?



Conventional solution:
Get out formulas:
$\Sigma mV(\text{before}) = \Sigma mV(\text{before})$ *[momentum conservation]*
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etc.

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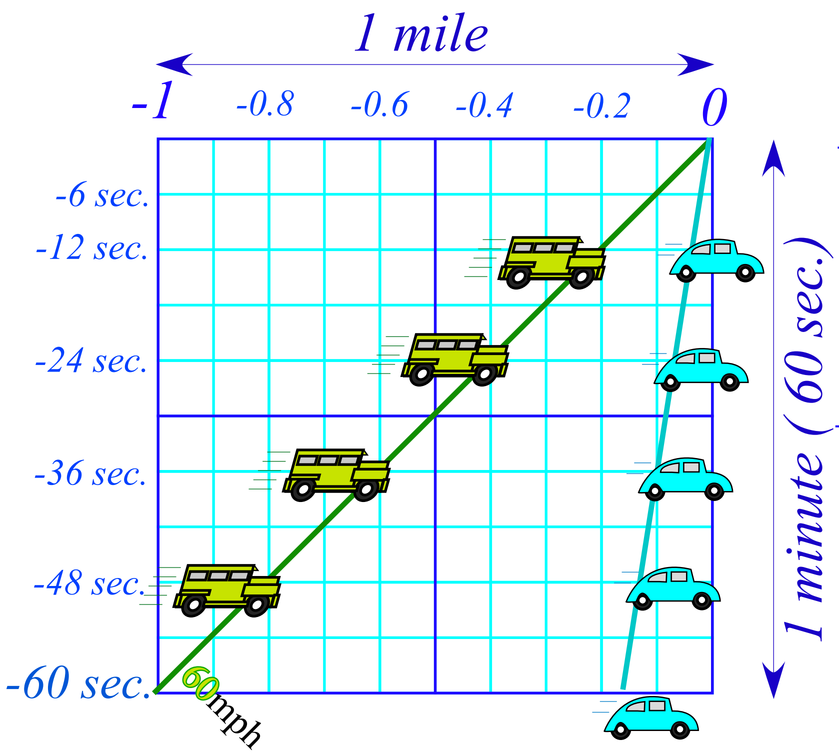

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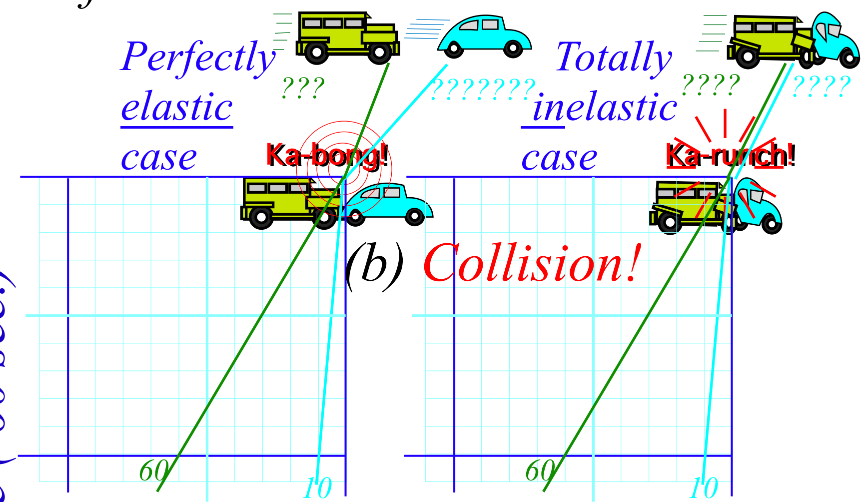

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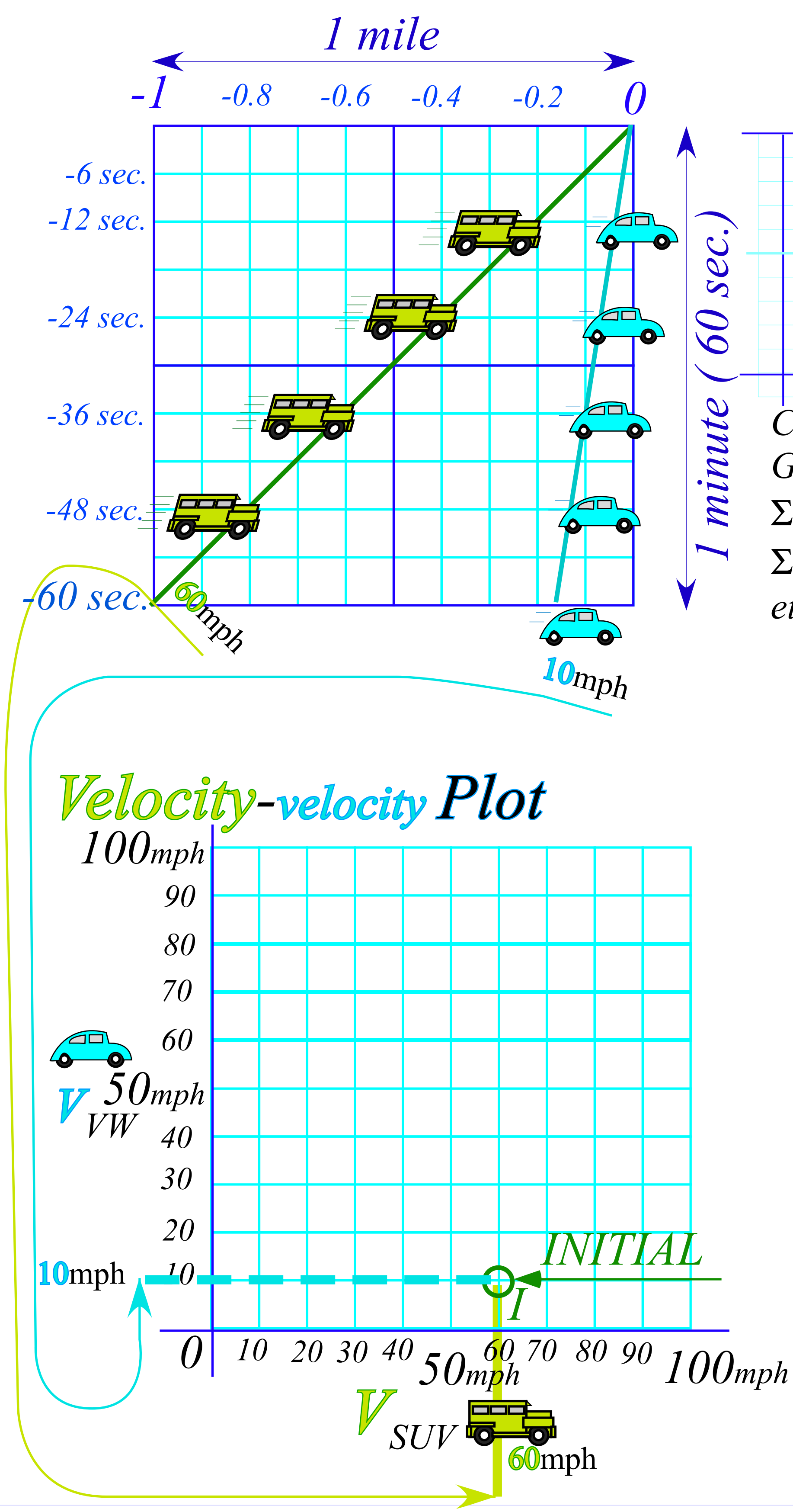
etc.

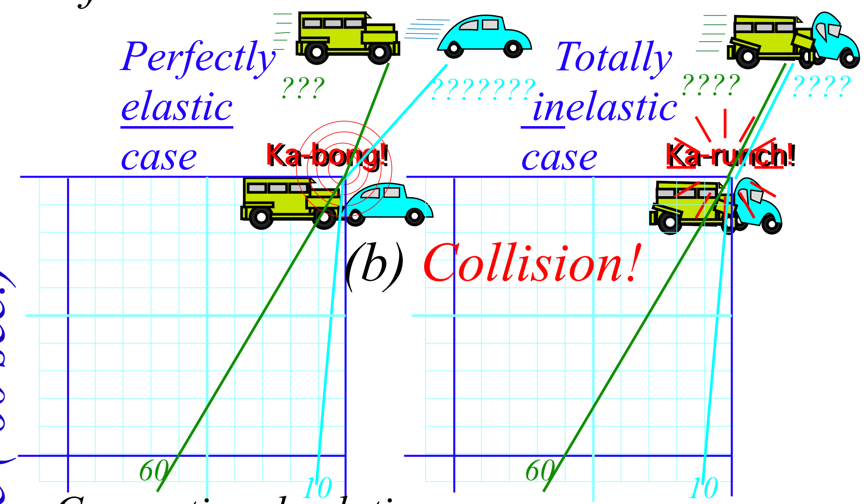
But an UNconventional way is quicker and slicker.....
..... (Just have to draw 2 lines! ... (and a circle...)

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....

After collision...what velocities?





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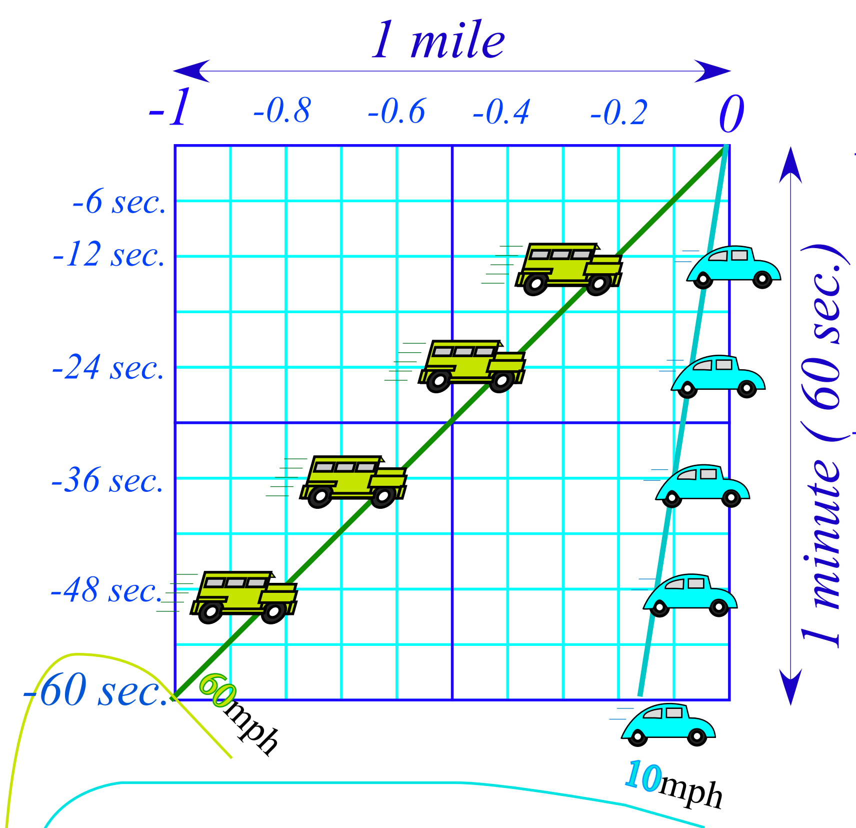
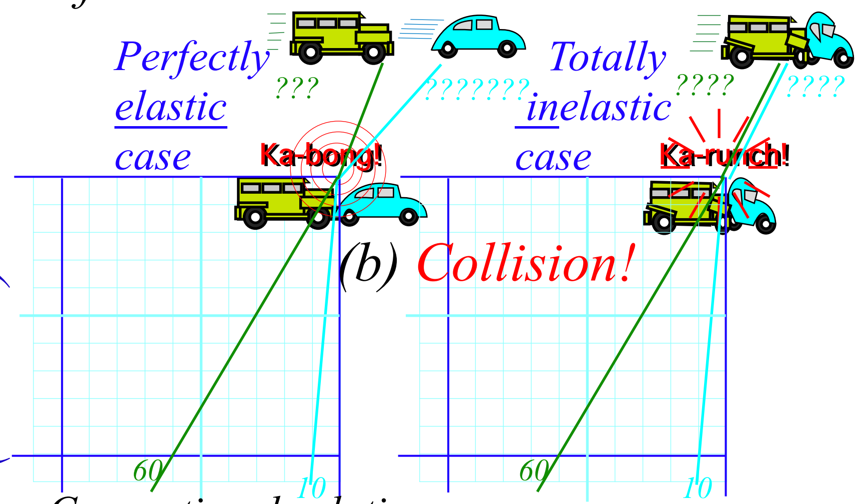


After collision...what velocities?



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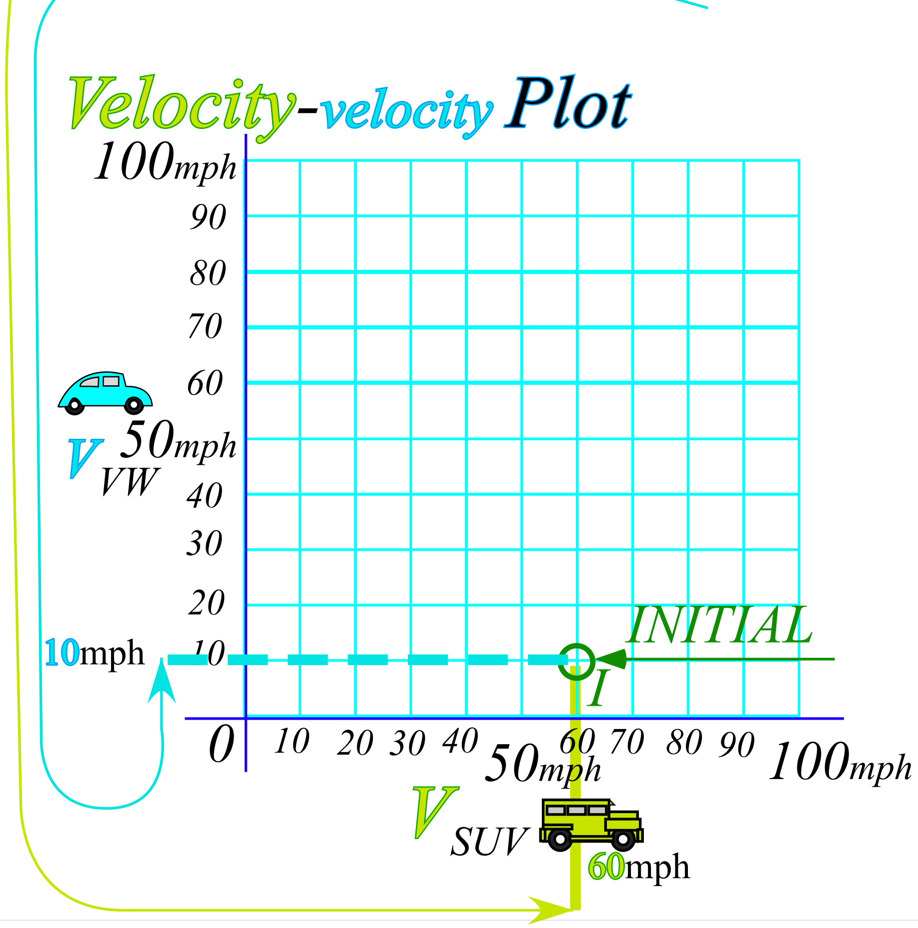
$M_{SUV}V_{SUV} + M_{VW}V_{VW} = constant$ is ***Axiom #1***



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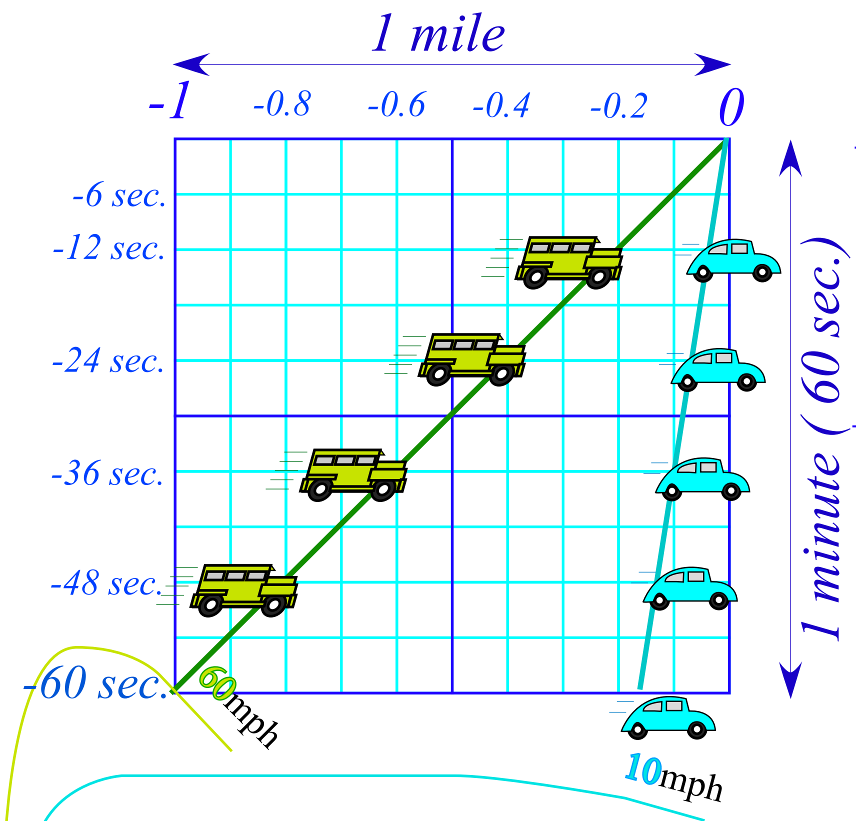

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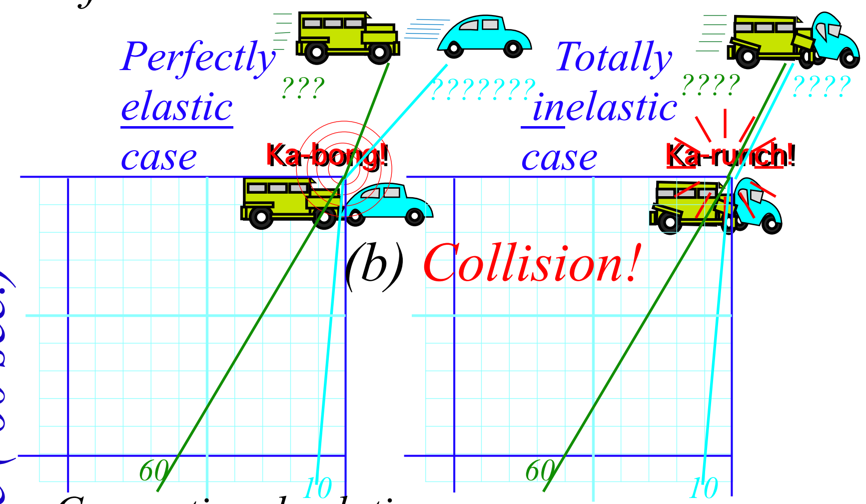

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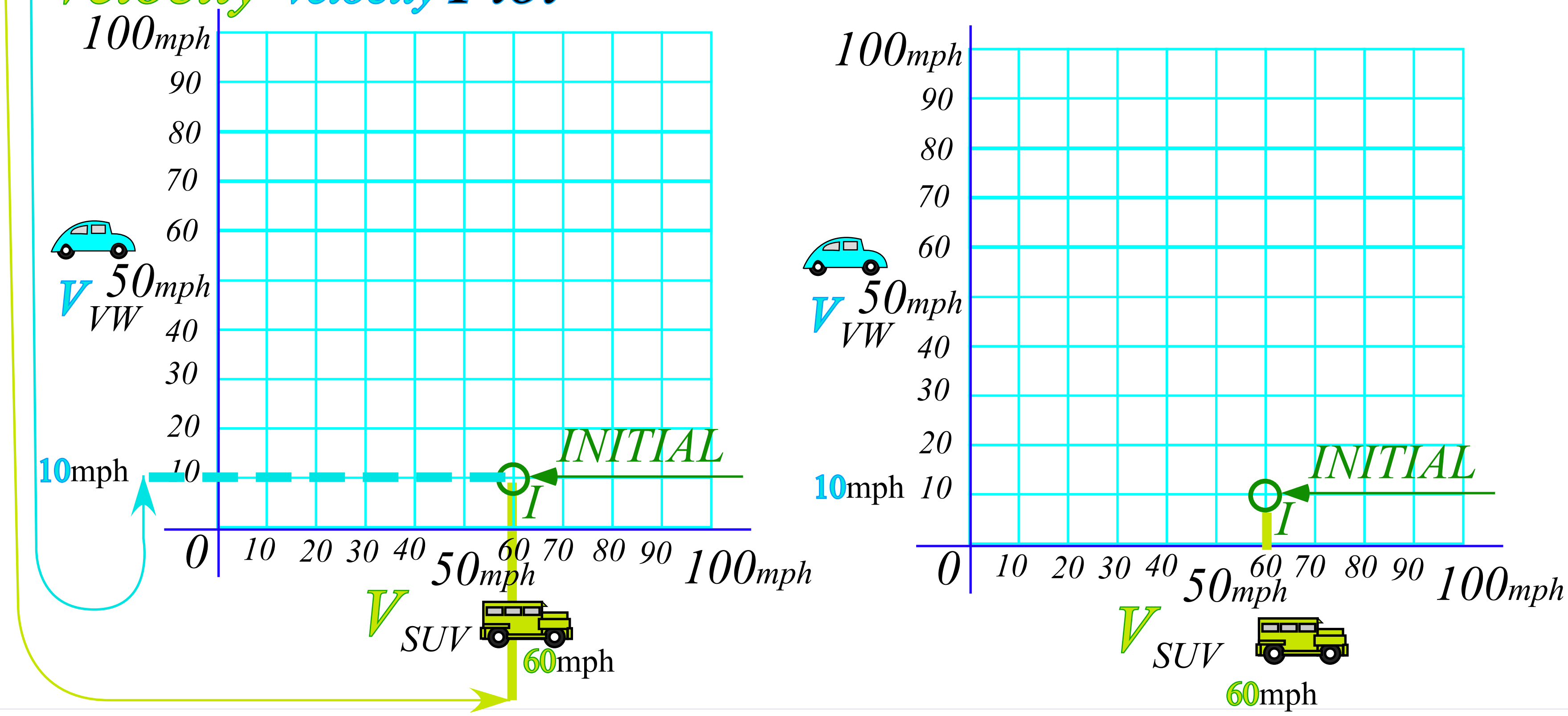
Velocity-velocity Plot



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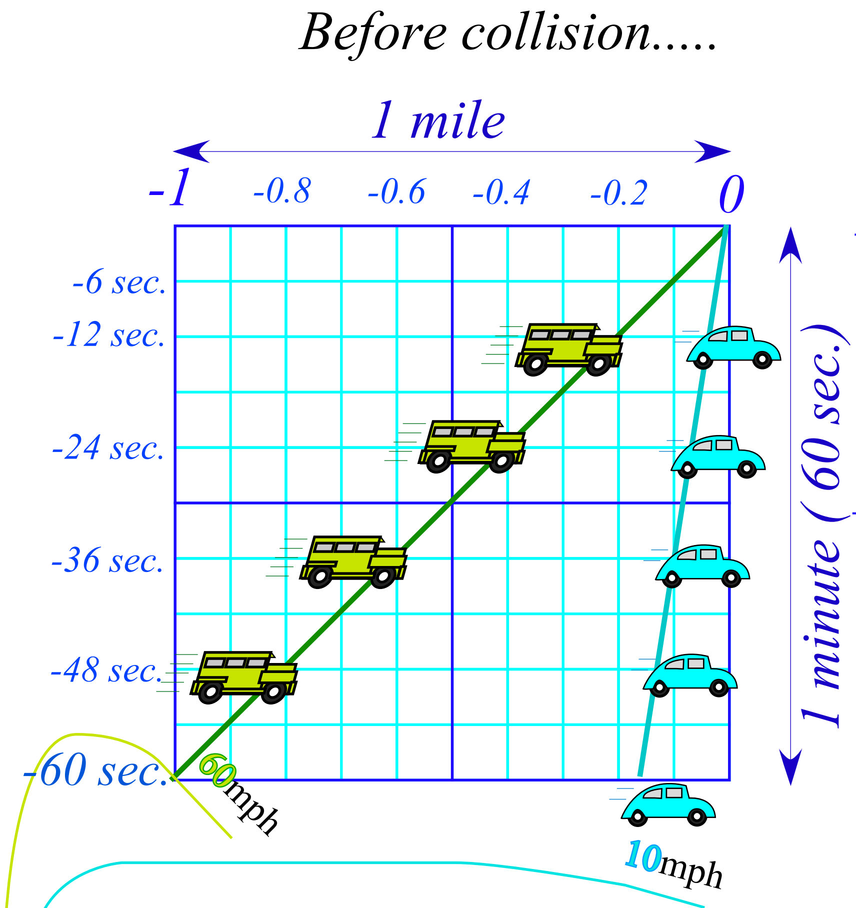

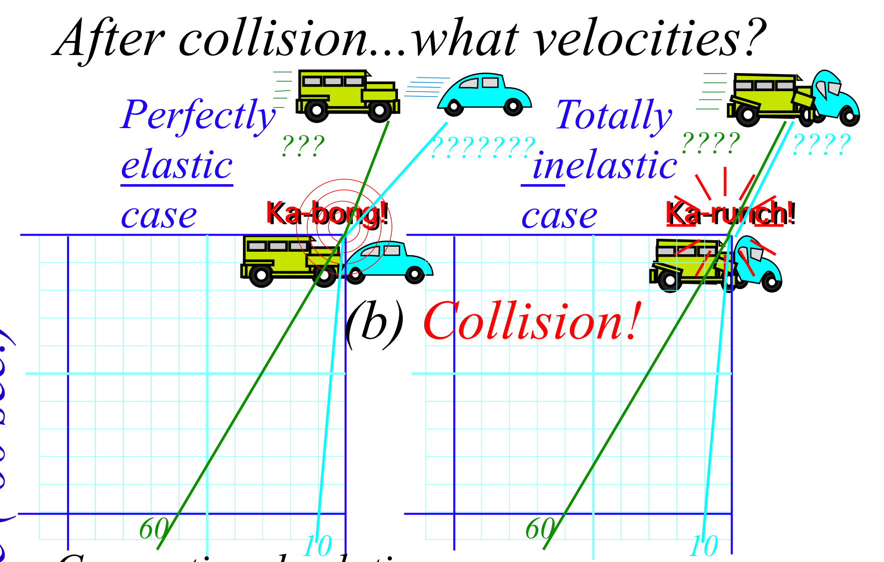

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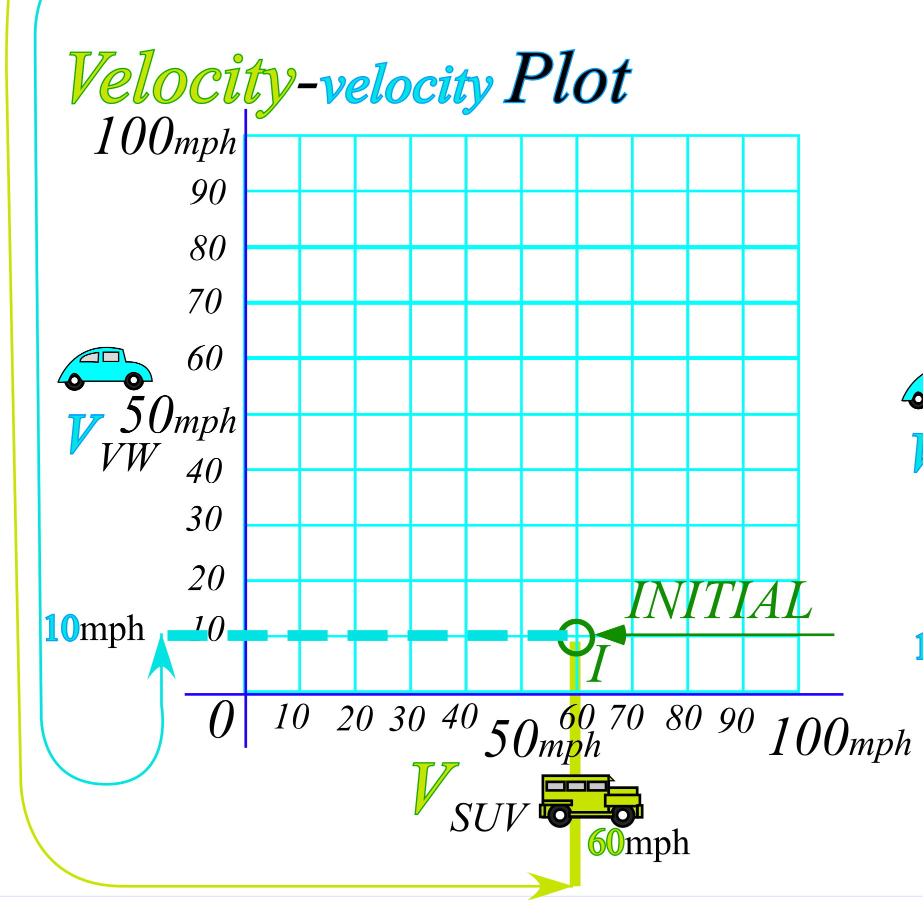

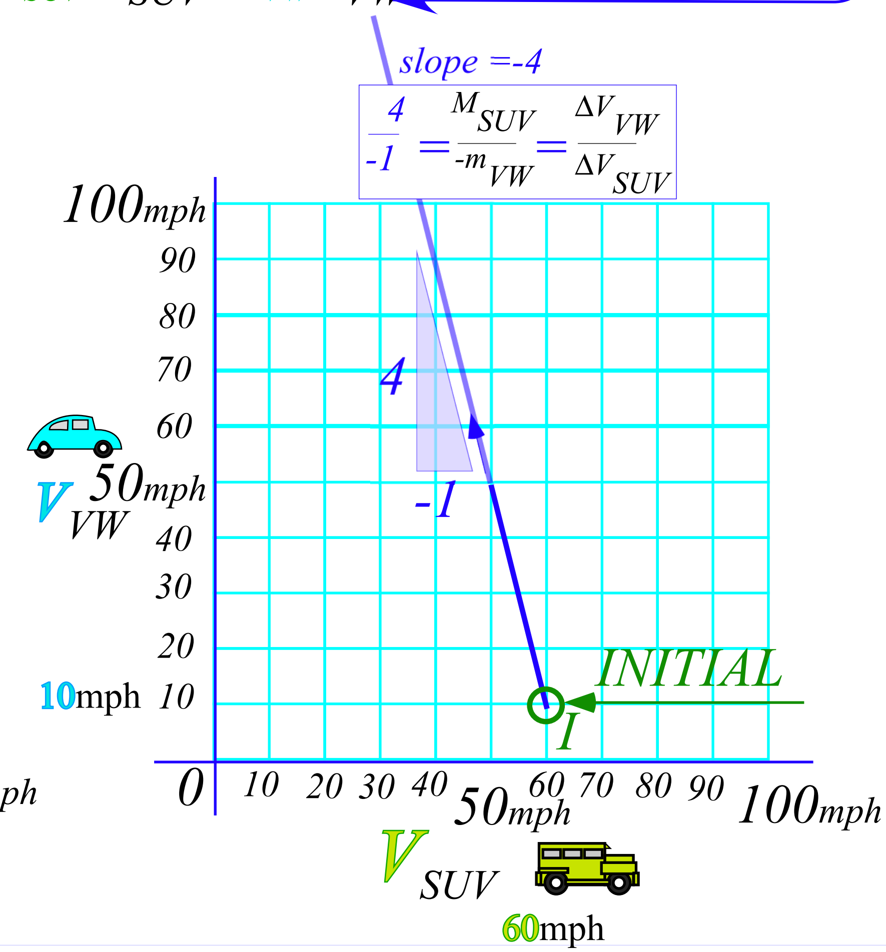

Geometry of momentum conservation axiom

- *Totally Inelastic "ka-runch" collisions*
- *Perfectly Elastic "ka-bong" and Center Of Momentum (COM) symmetry*

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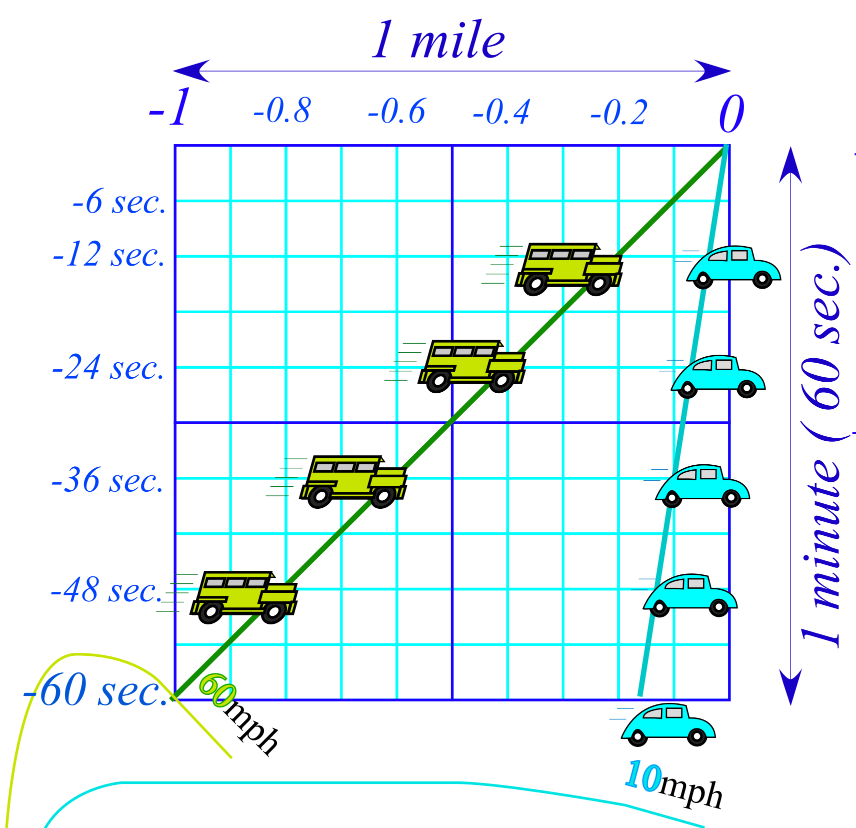
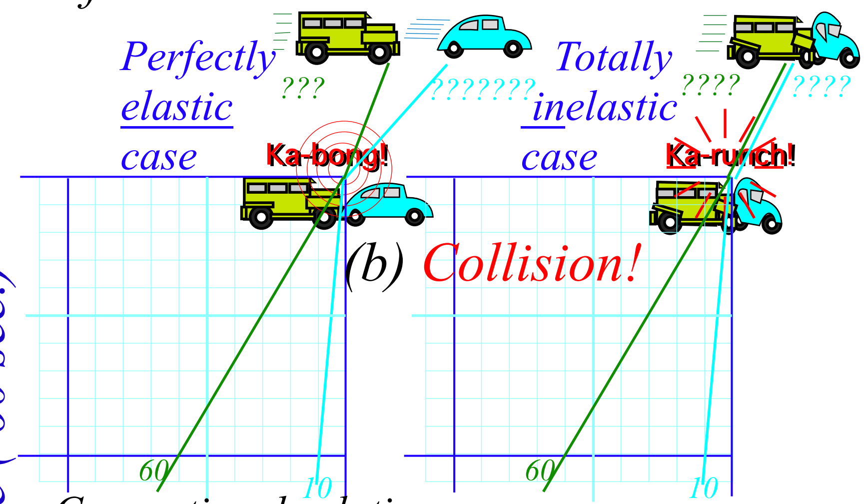


After collision...what velocities?



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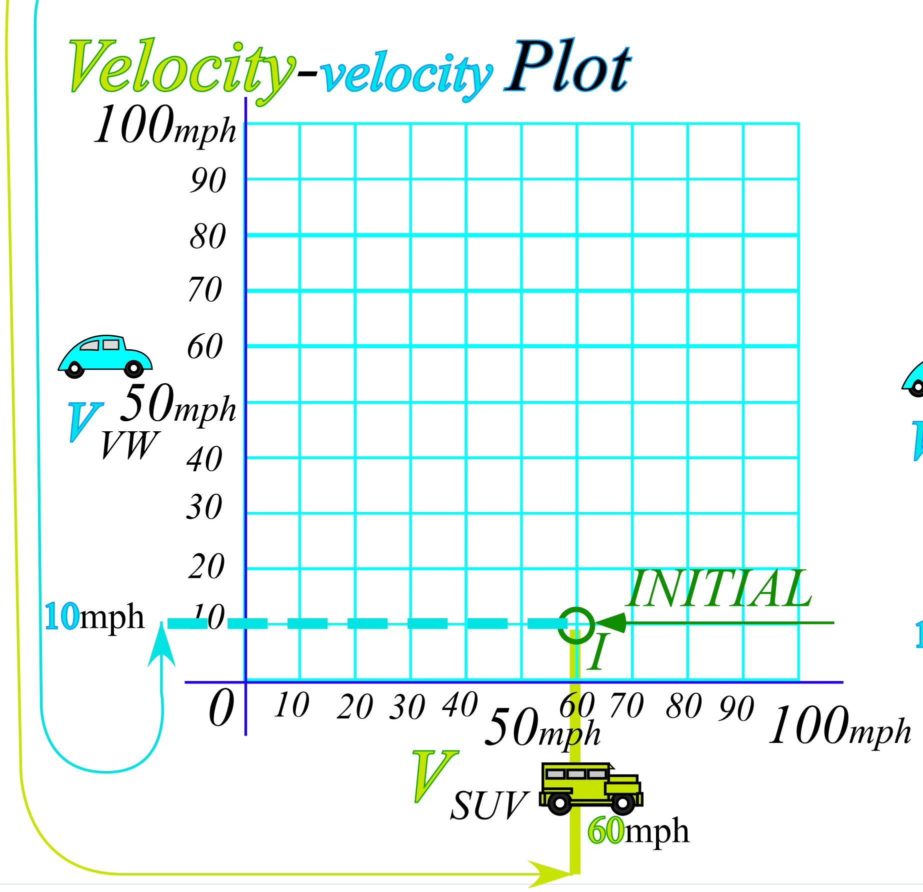

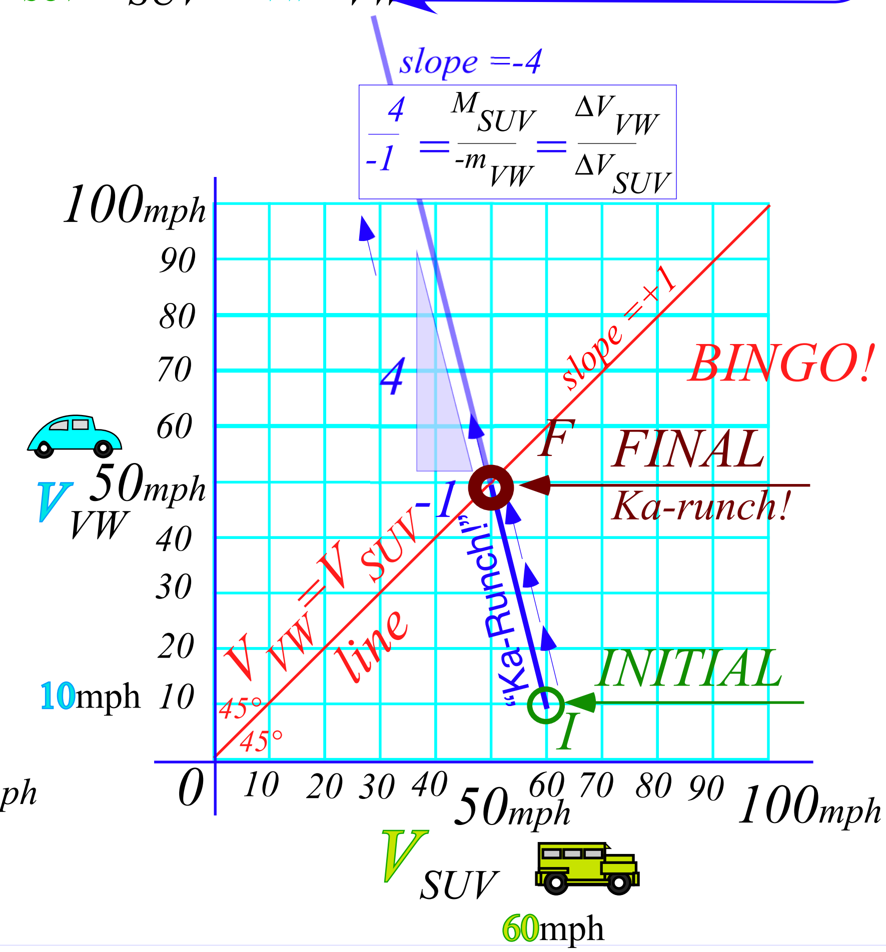

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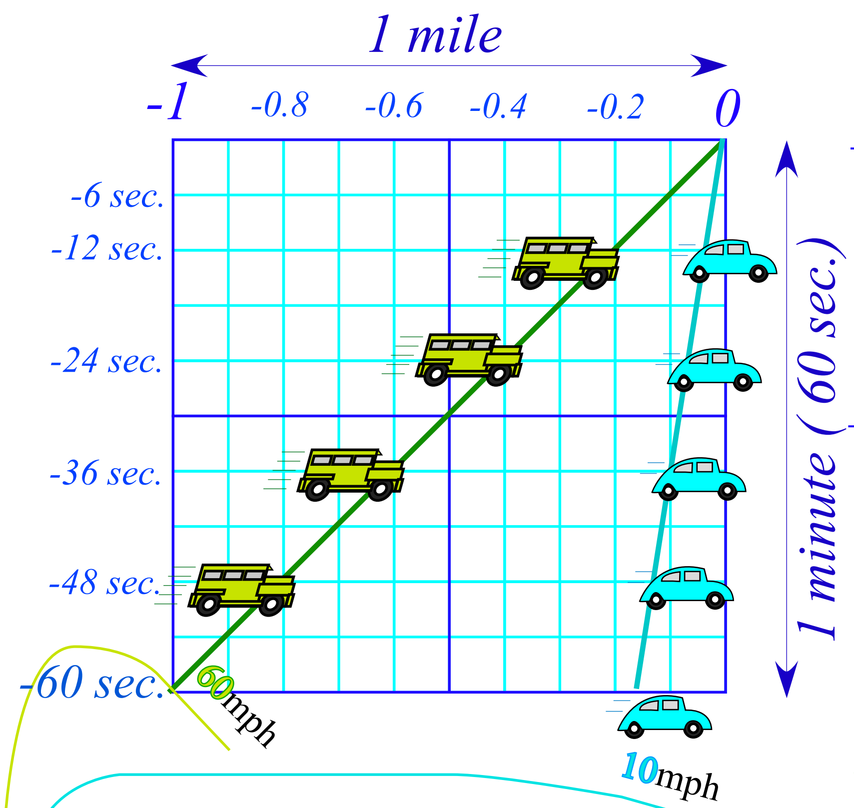

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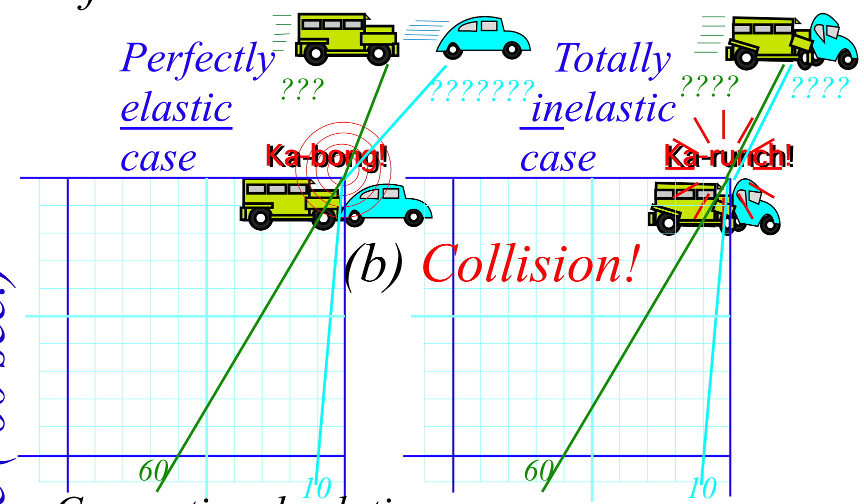

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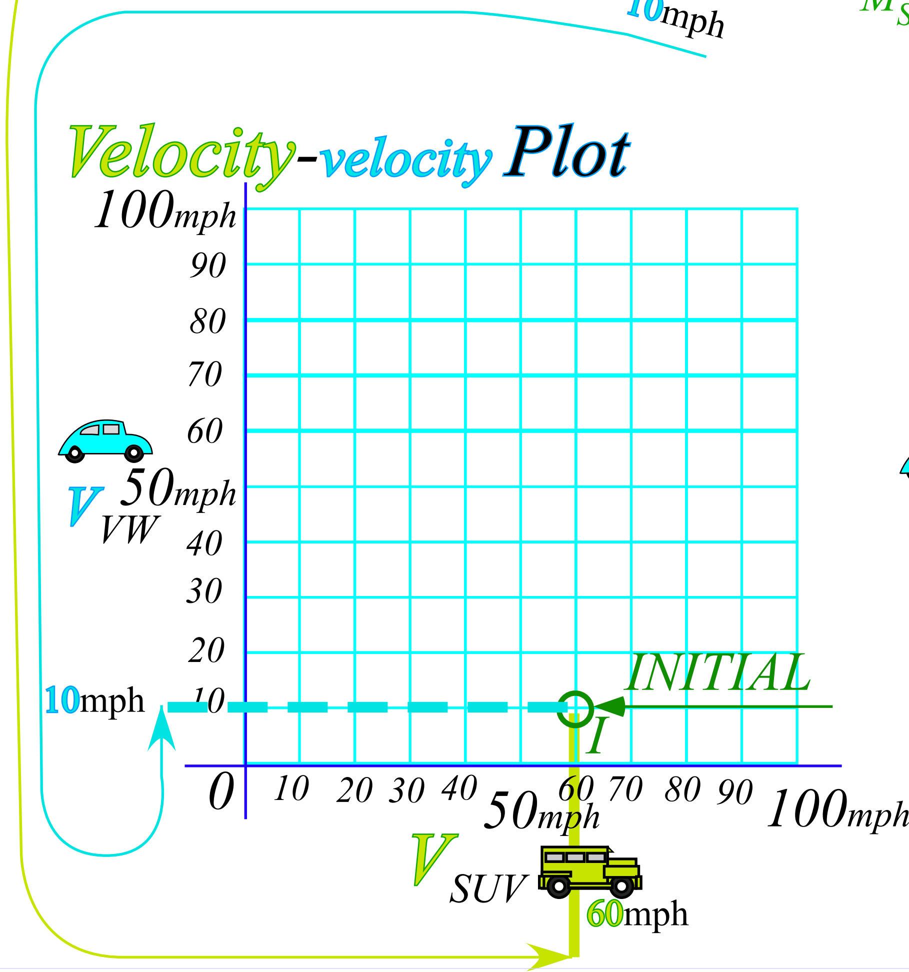

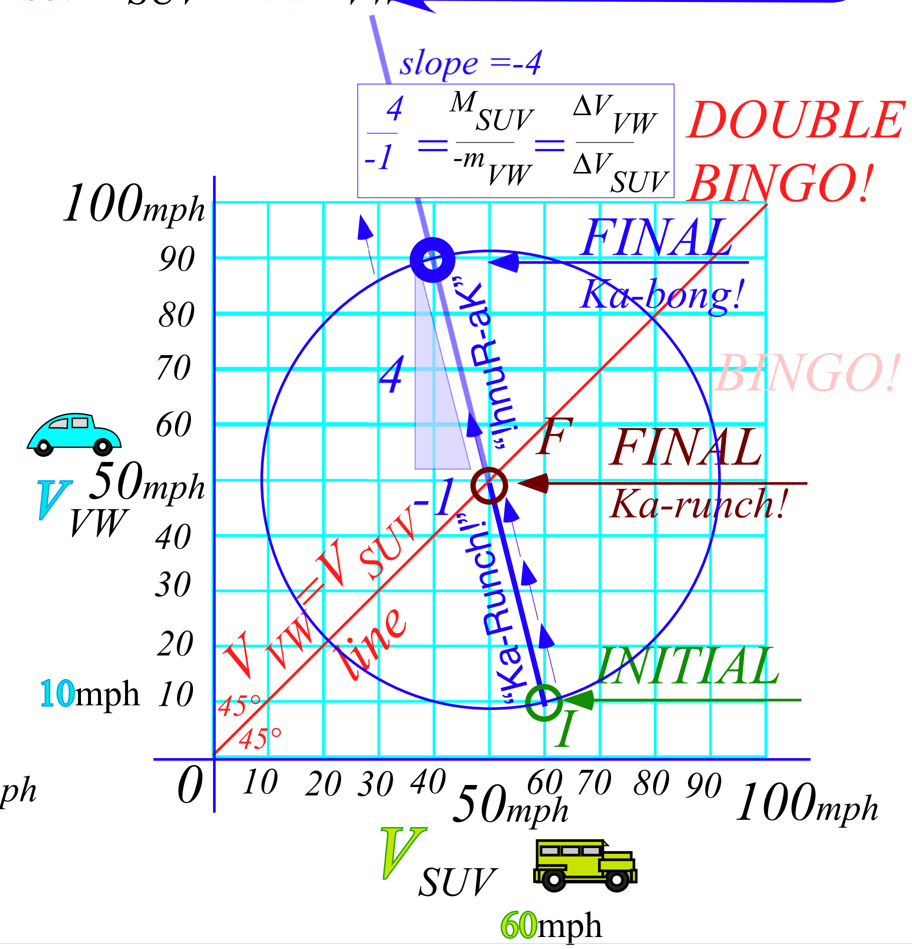

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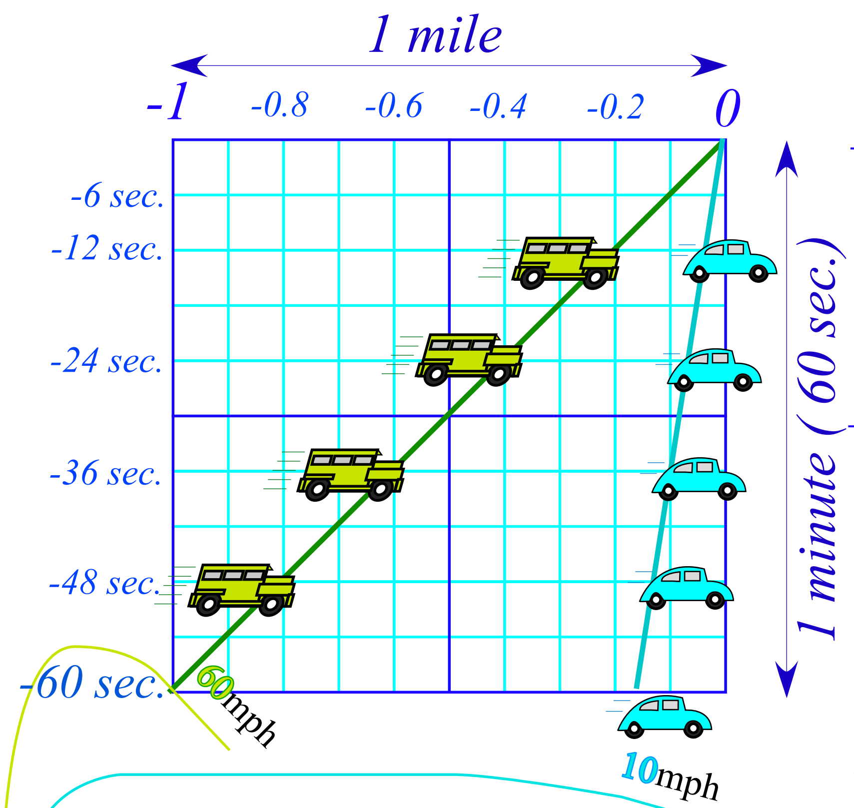

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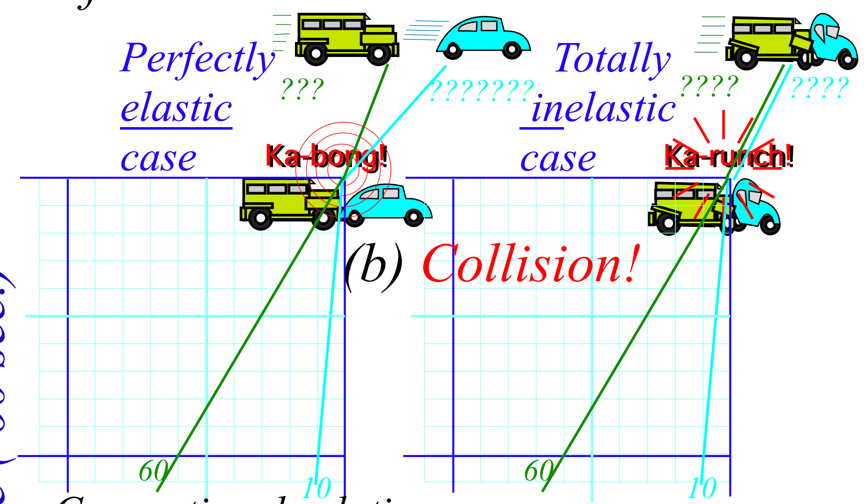

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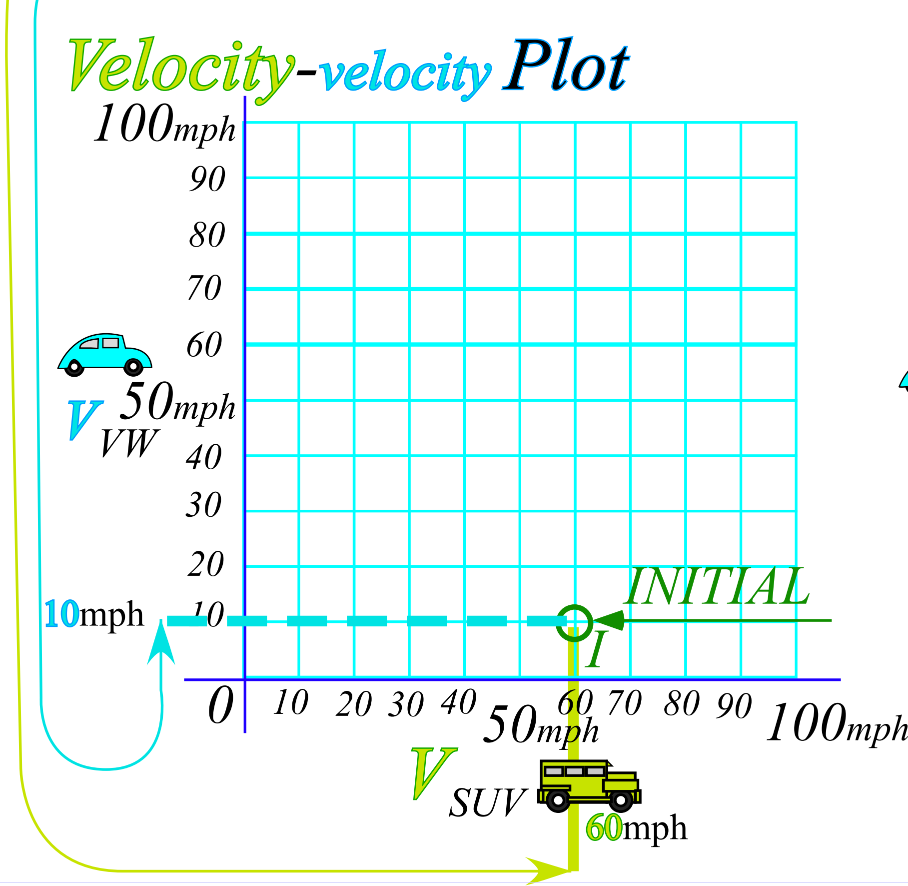

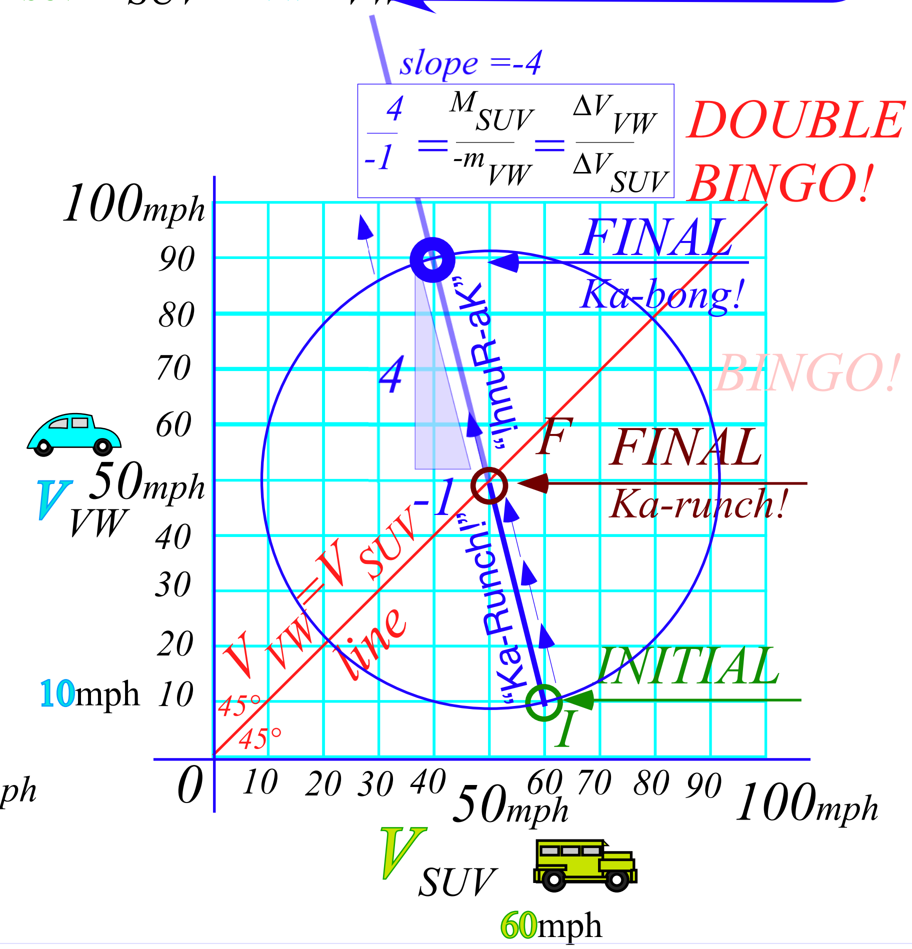

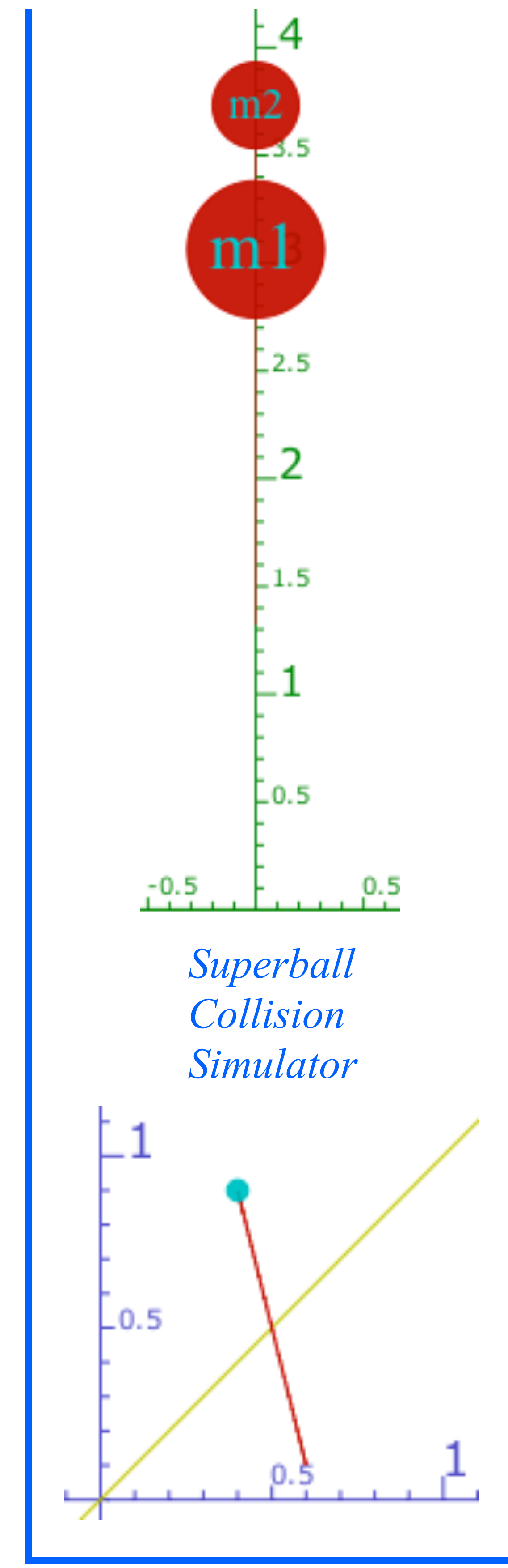

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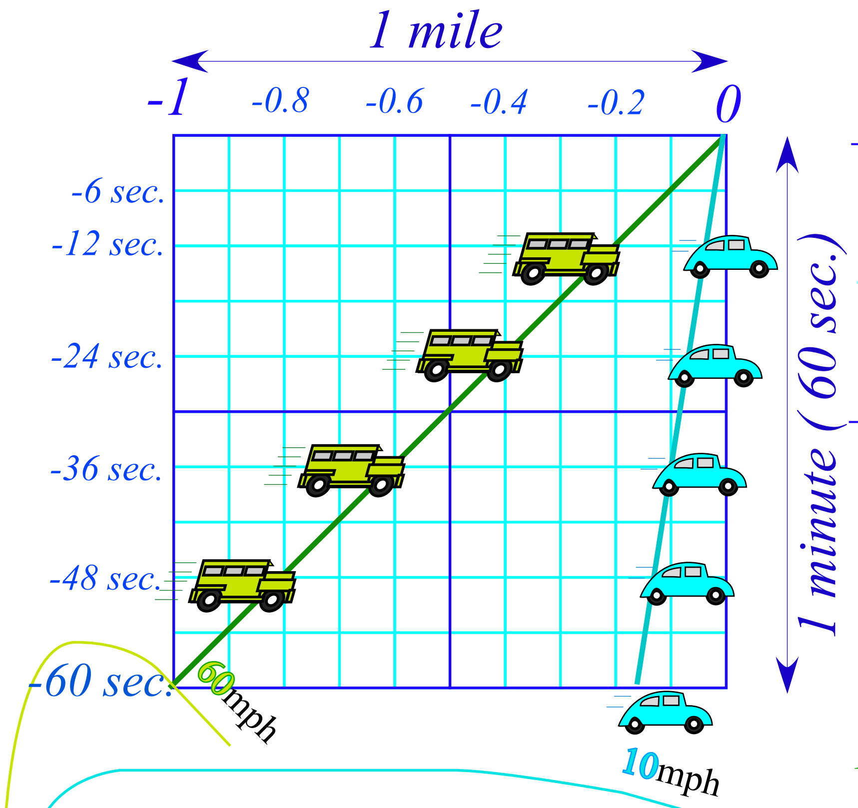

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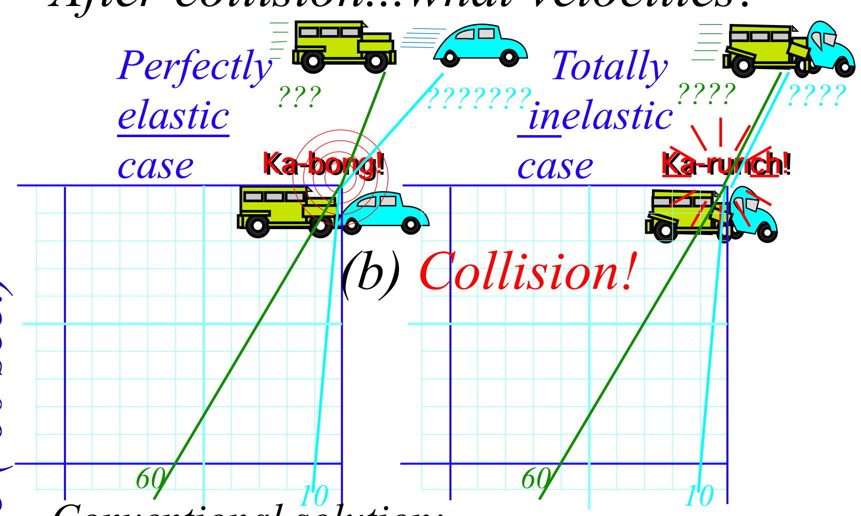

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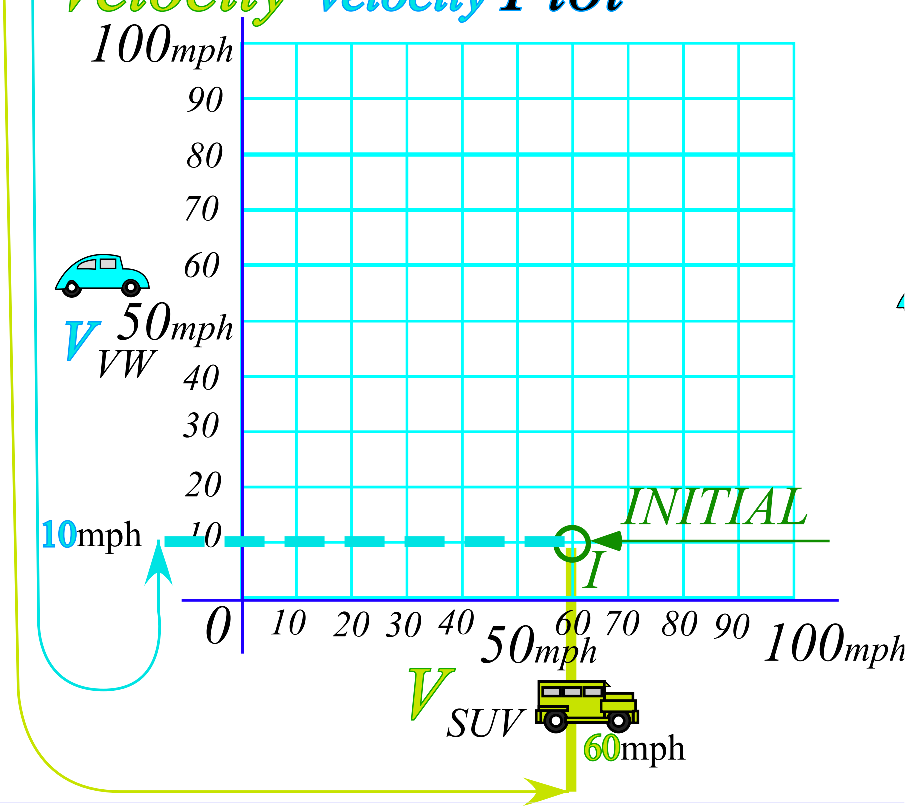
slope =-4

Velocity-velocity Plot



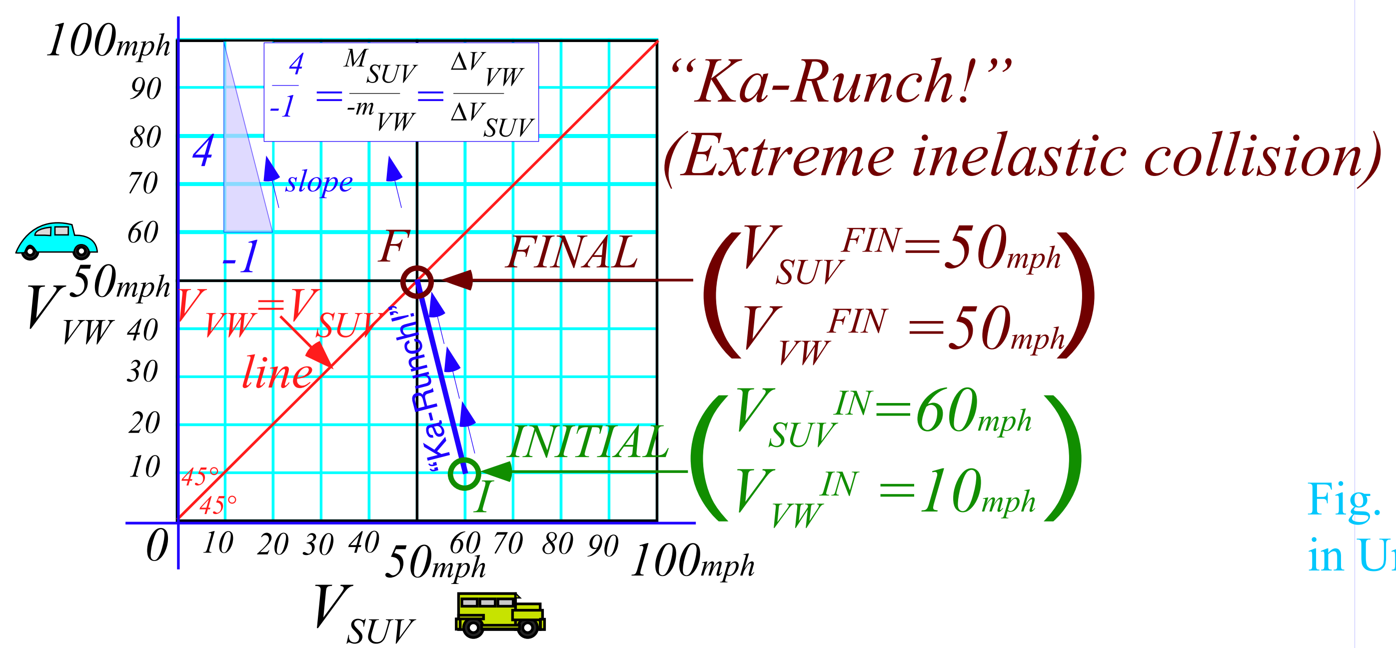
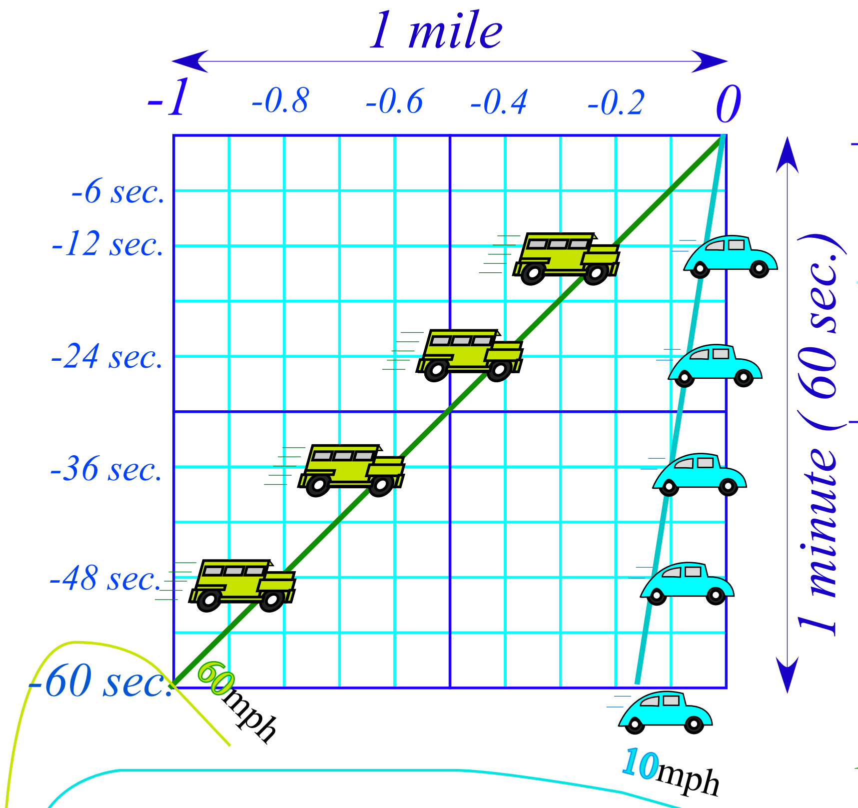


Fig. 2.1
in Unit 1

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Notice "Ka-Bong" Figure 2.2 scaling
(ft./min. is more realistic)

Before collision.....



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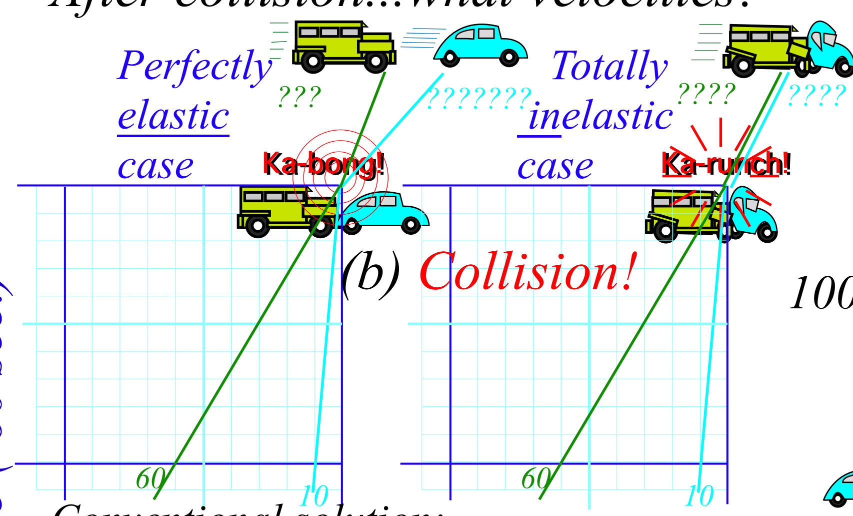

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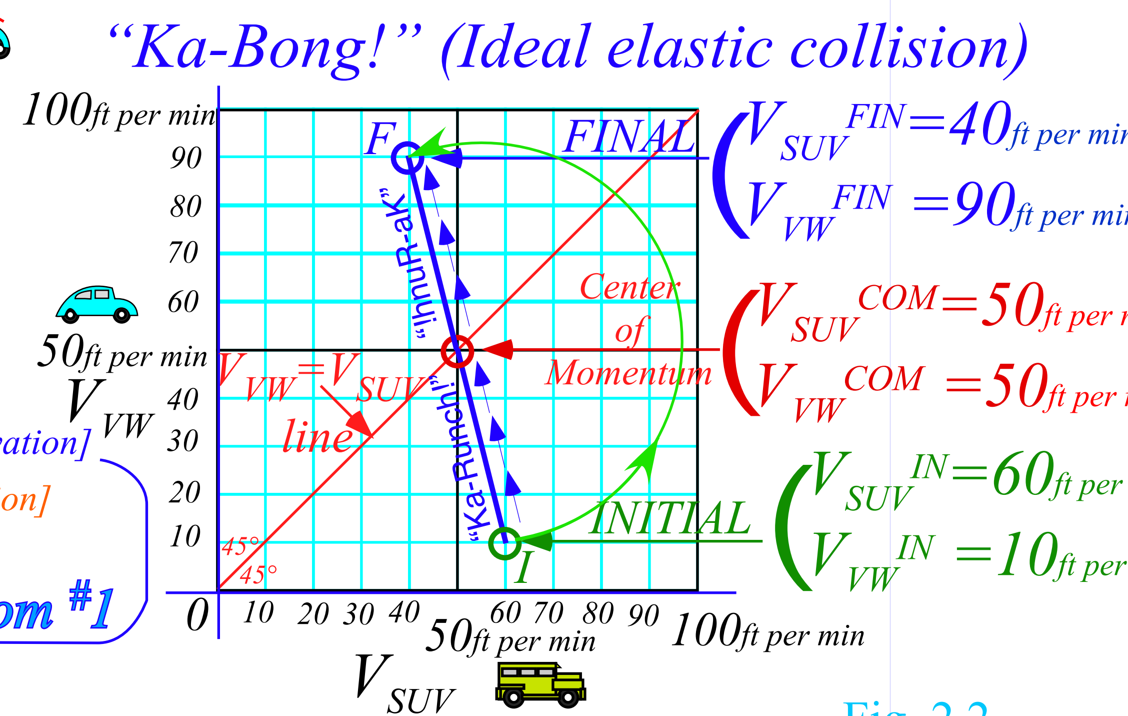
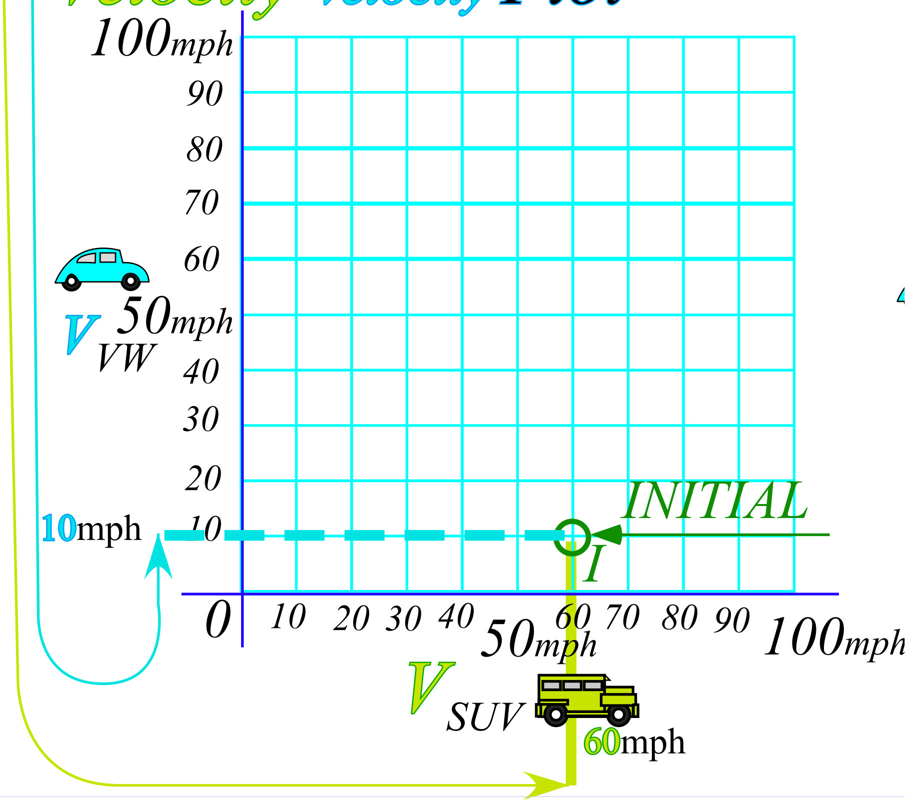


Fig. 2.2
in Unit 1

Velocity-velocity Plot



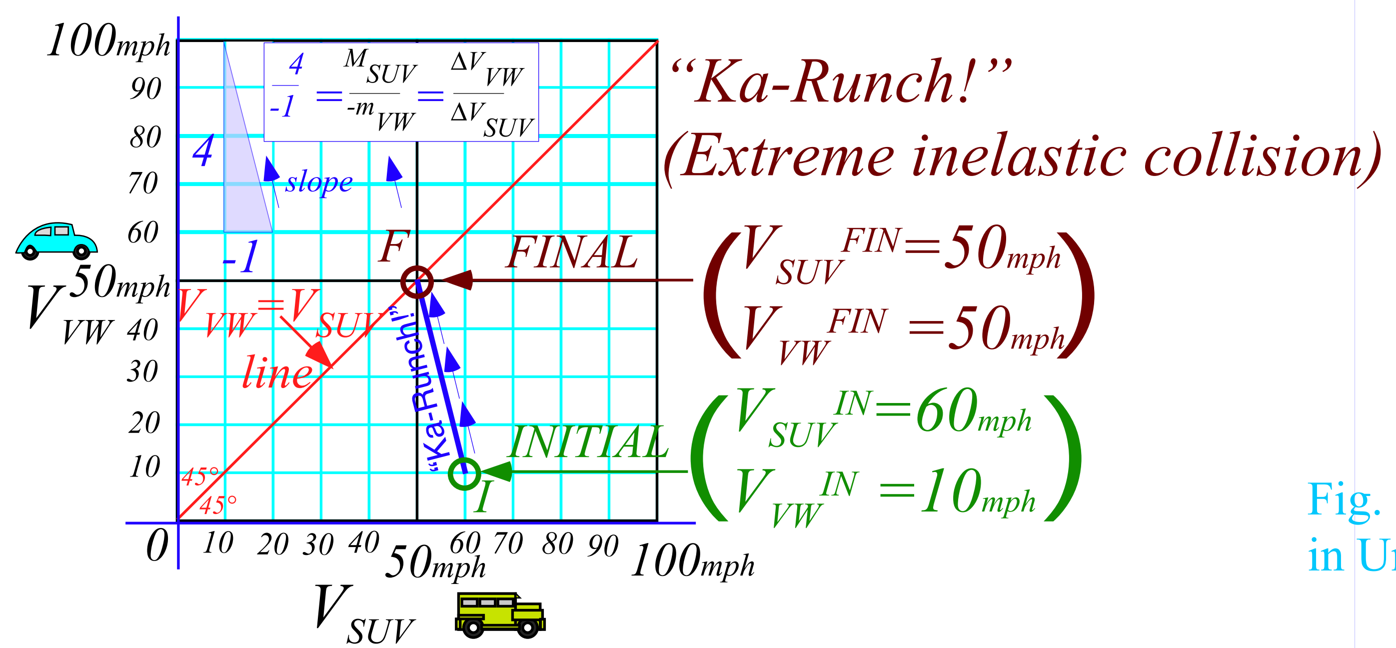


Fig. 2.1
in Unit 1

Geometry of Galilean translation symmetry

→ *45° shift in (V_1,V_2)-space*

Time reversal symmetry

...of COM collisions

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph ***slower***

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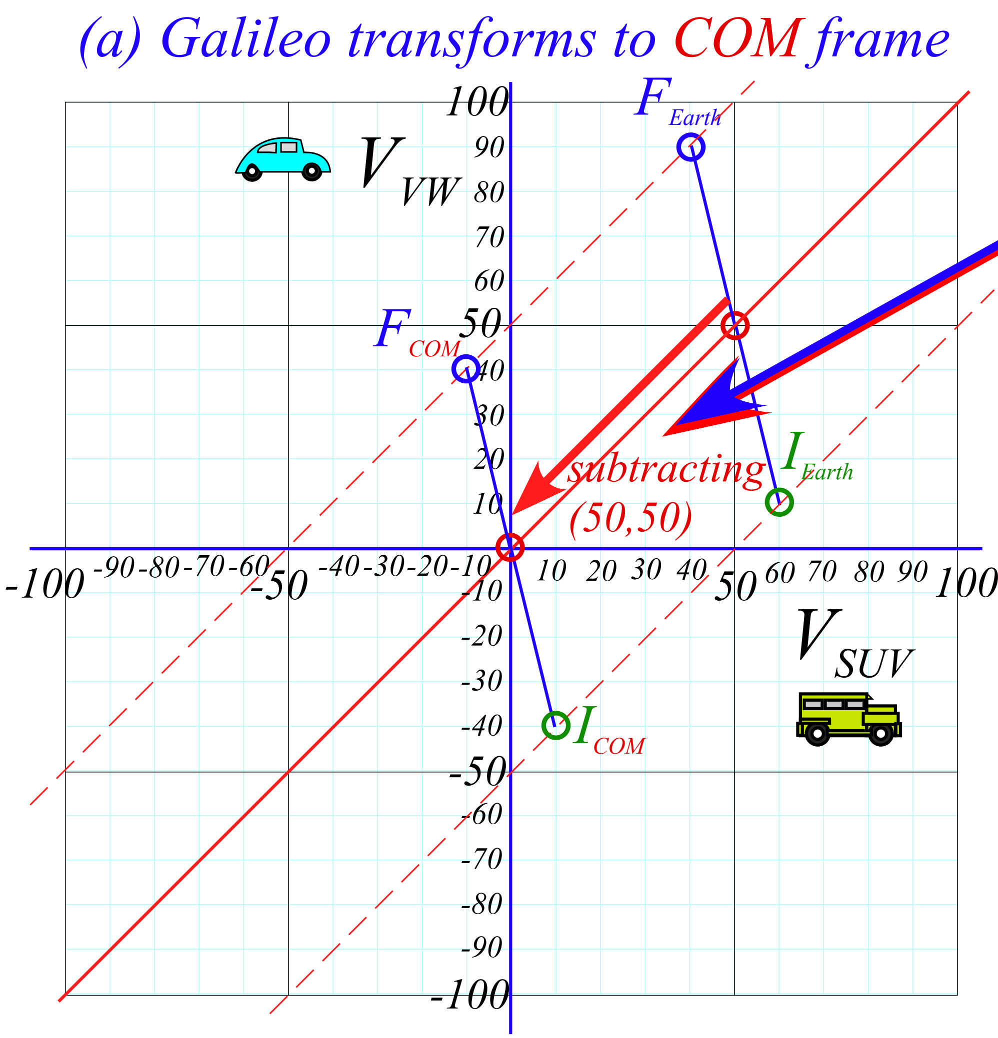

Fig. 2.5a
in Unit 1

A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A *symmetry transformation*)

If you increase your velocity by 50 mph,...

*...the rest of the world appears to be 50 mph **slower***

(a) Galileo transforms to COM frame *(b) ... and to five or six other reference frames*

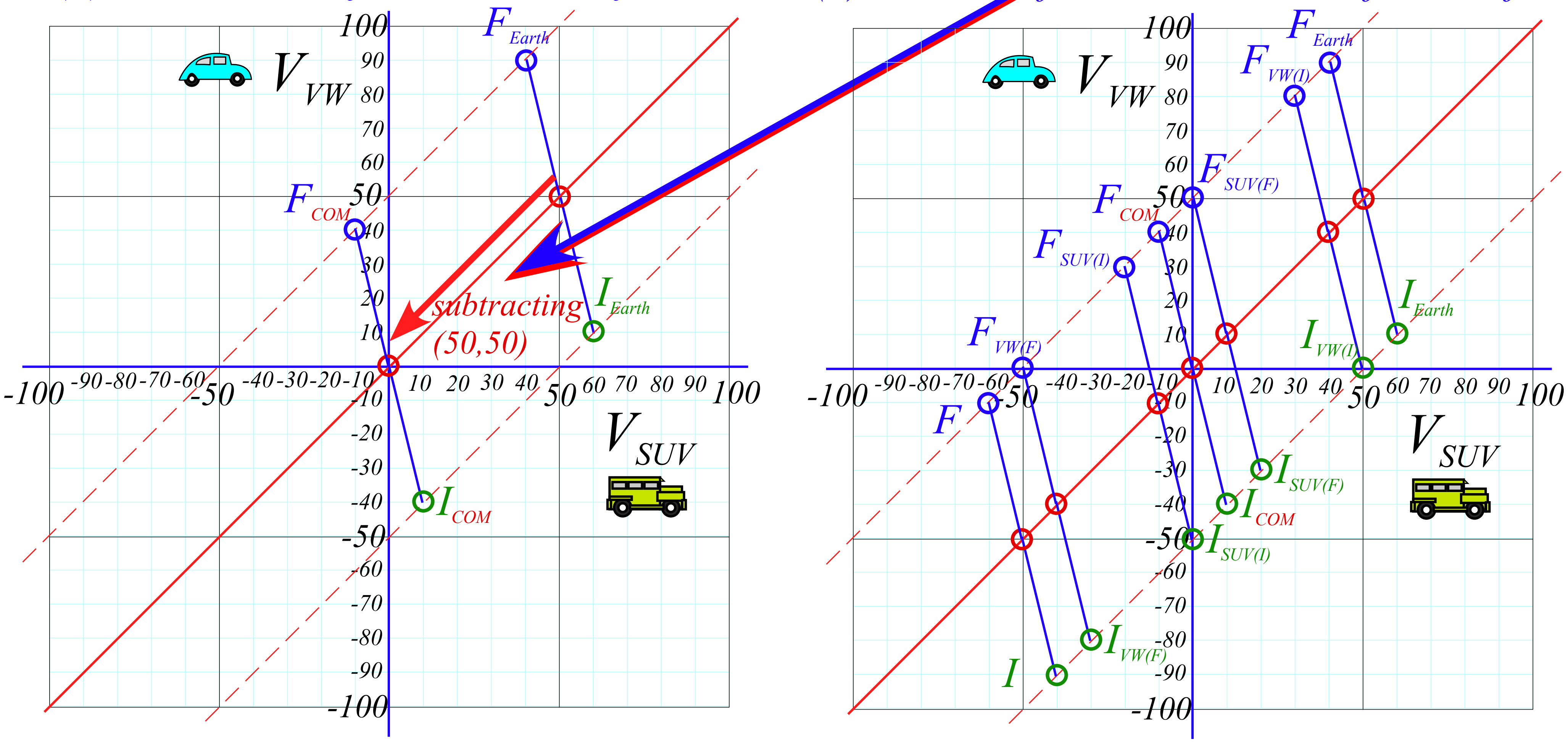

Fig. 2.5a in Unit 1

Fig. 2.5b in Unit 1

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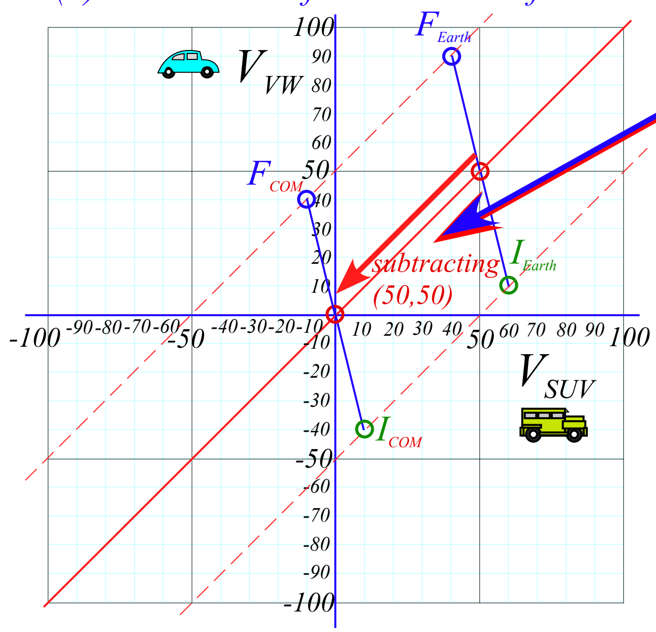


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

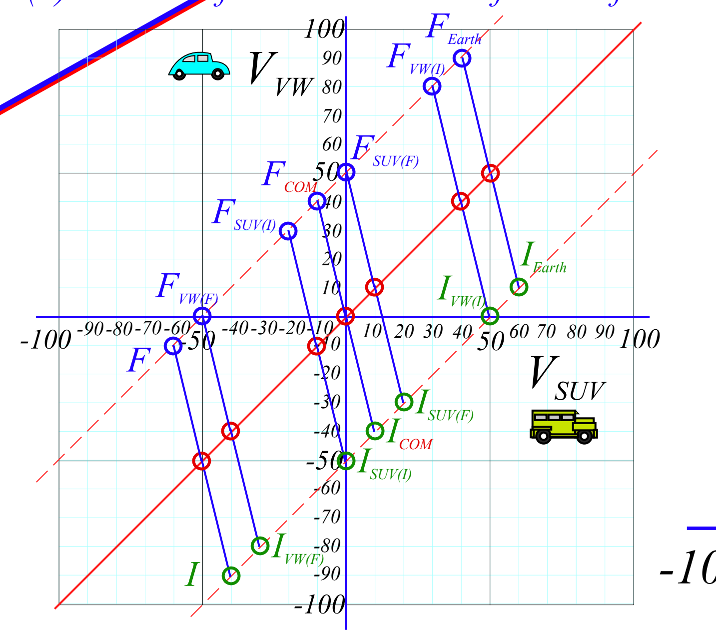
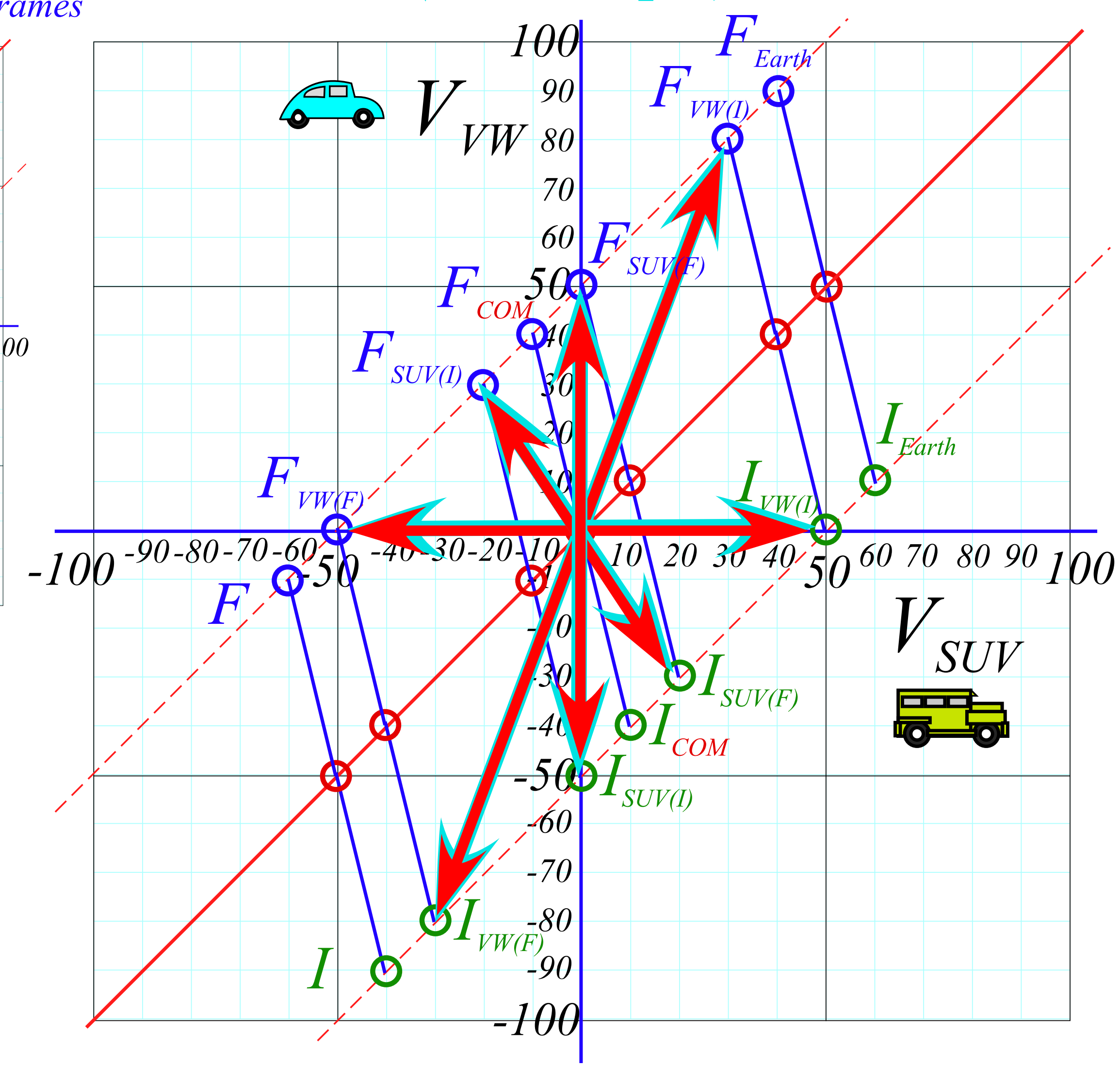


Fig. 2.5b
in Unit 1

Time-reversal (F-I)
symmetry pairs
(Four examples)



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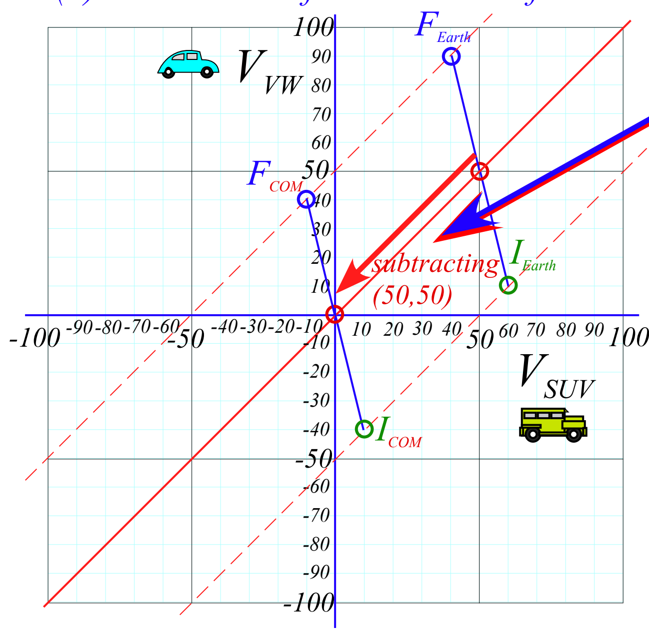


Fig. 2.5a
in Unit 1

(b) ... and to five or six other reference frames

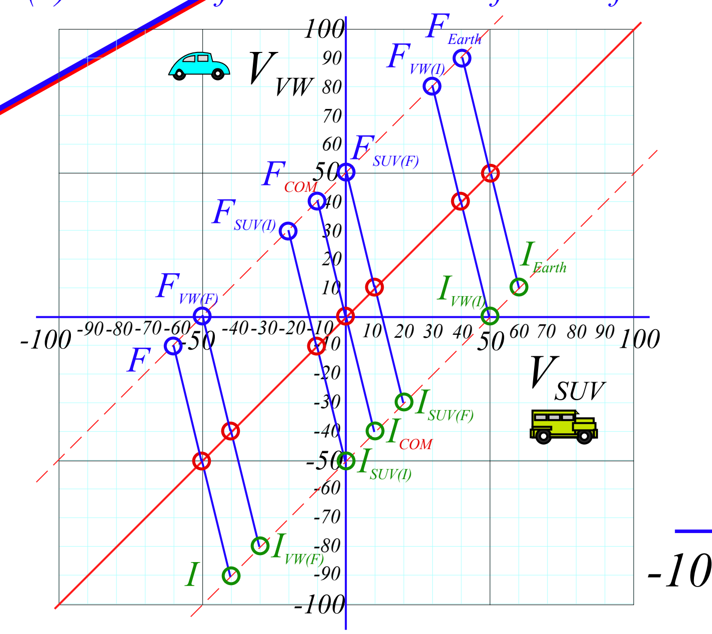
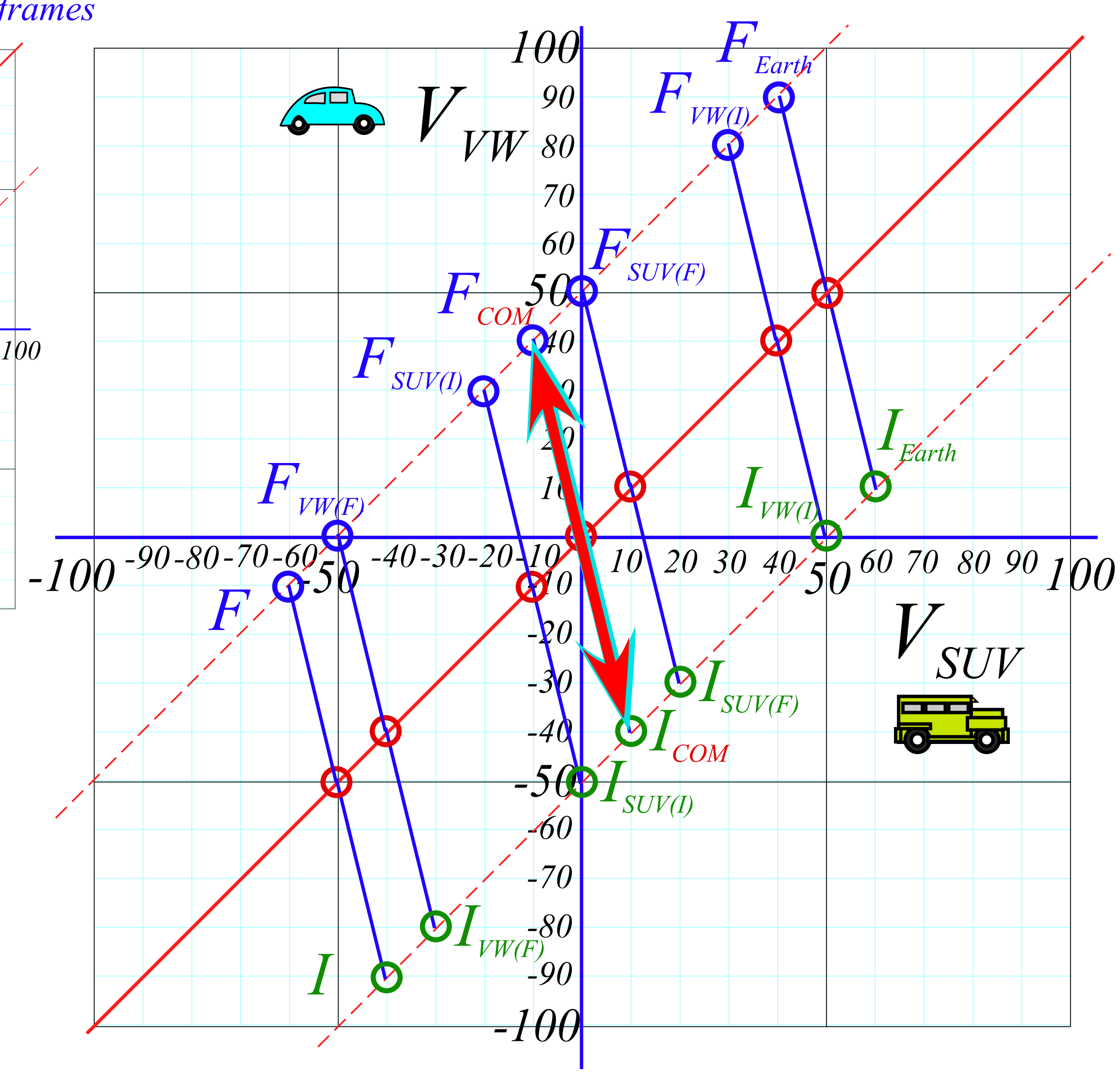


Fig. 2.5b
in Unit 1

THE
COM Time-reversal
symmetry pair
(Just 1 case)



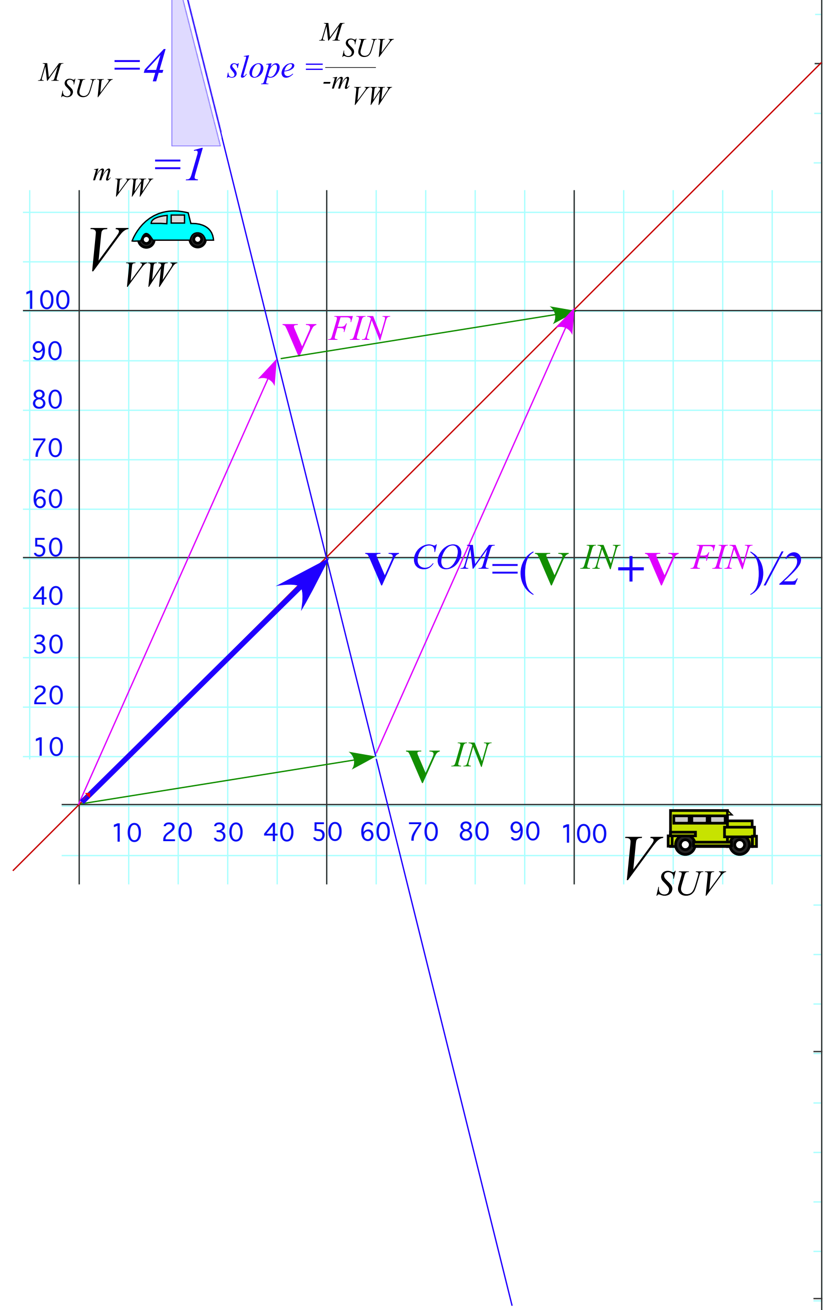
Algebra,Geometry, and Physics of momentum conservation axiom

→ *Vector algebra of collisions*
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$



Algebra, Geometry, and Physics of Momentum Conservation Axiom

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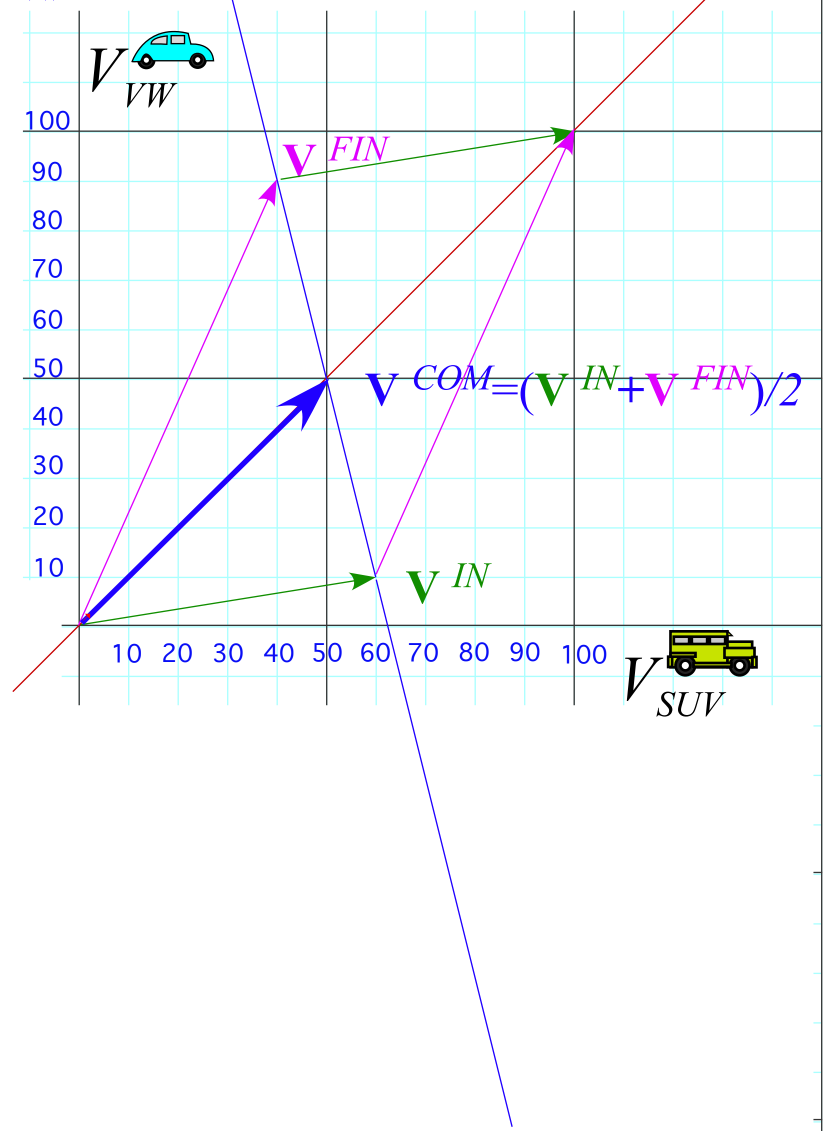
Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$



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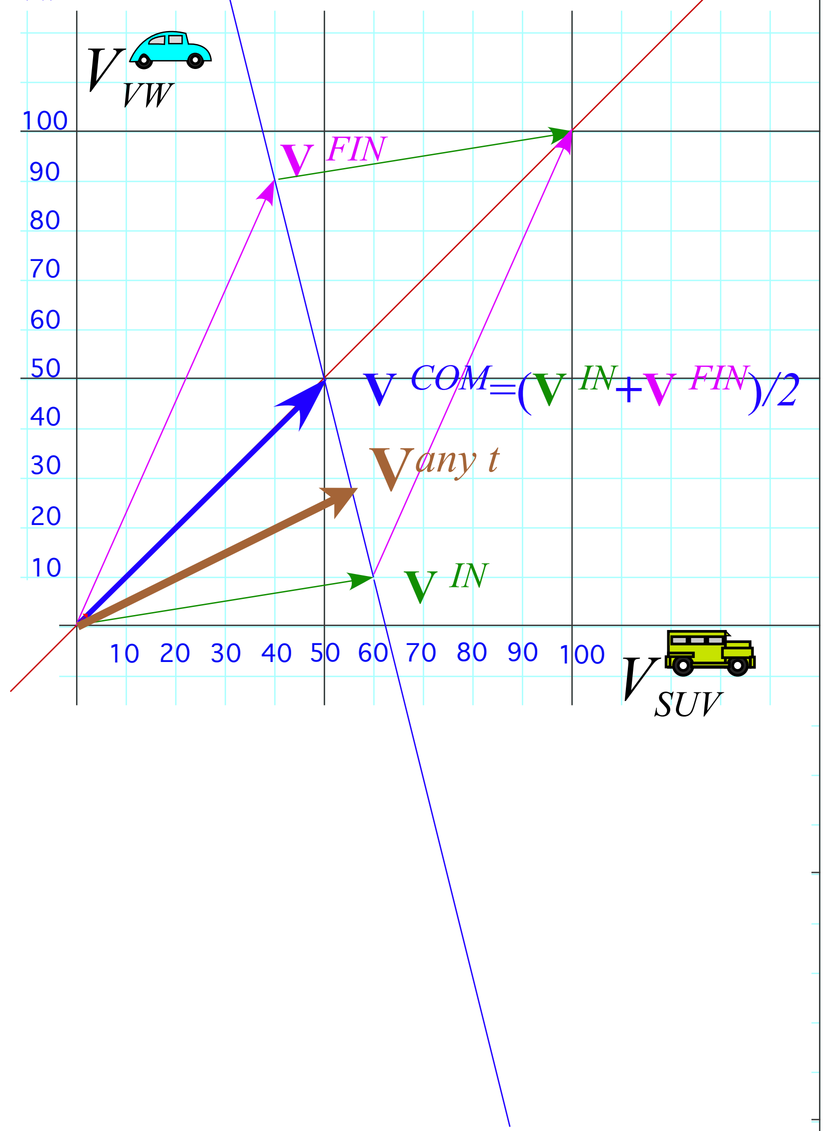
Mass weighted average velocity at anytime is Center of Mass velocity V^{COM}:

$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$



Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

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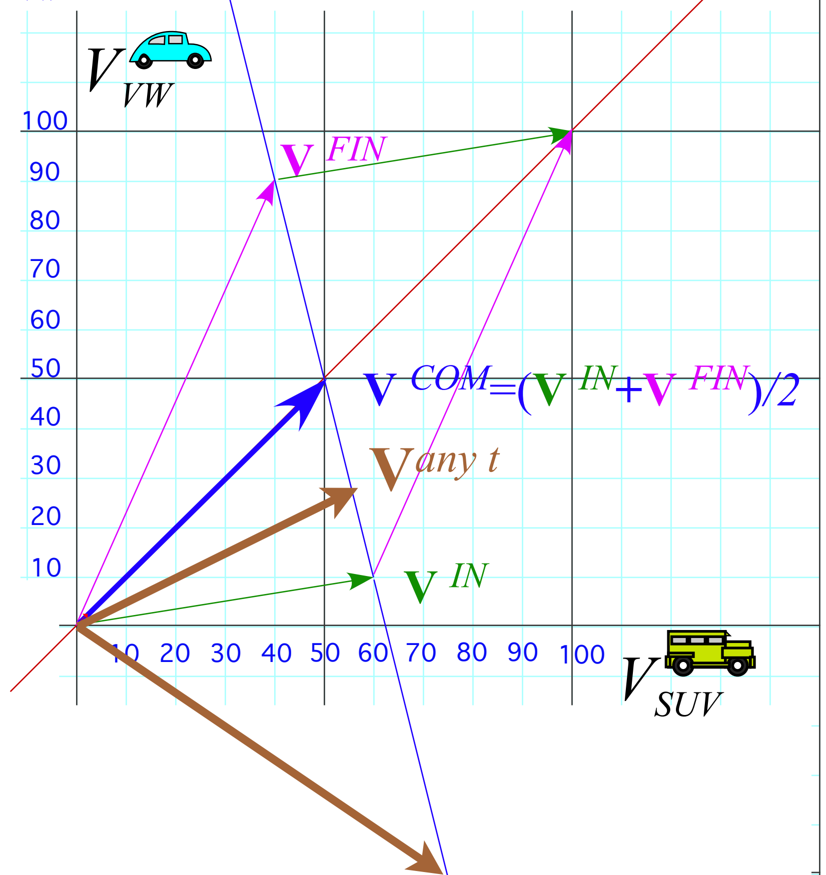

The weighted average applies to any old $\mathbf{V}^{any\ t}$ on momentum line

$$V^{COM}=\frac{M_{SUV}V_{SUV}^{anyt}+M_{VW}V_{VW}^{anyt}}{M_{SUV}+M_{VW}}$$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

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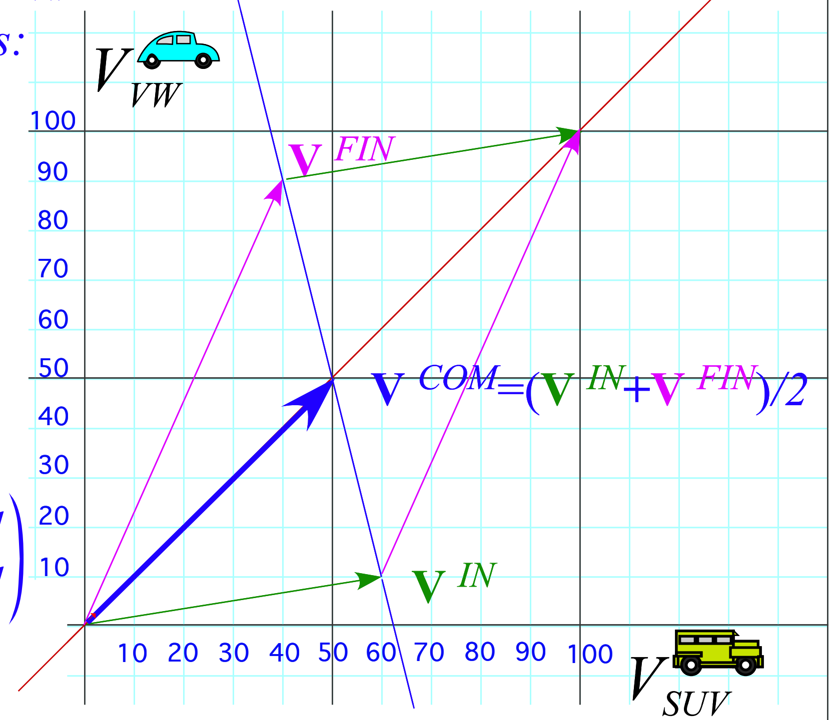
Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}$$

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\ V_{VW}^{FIN}\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\ V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$ Define funny-unit vector: $\mathbf{u}=\begin{pmatrix}1\\ 1\end{pmatrix}$



Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

→ *Matrix or tensor algebra of collisions*

Deriving Energy Conservation Theorem

Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

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Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

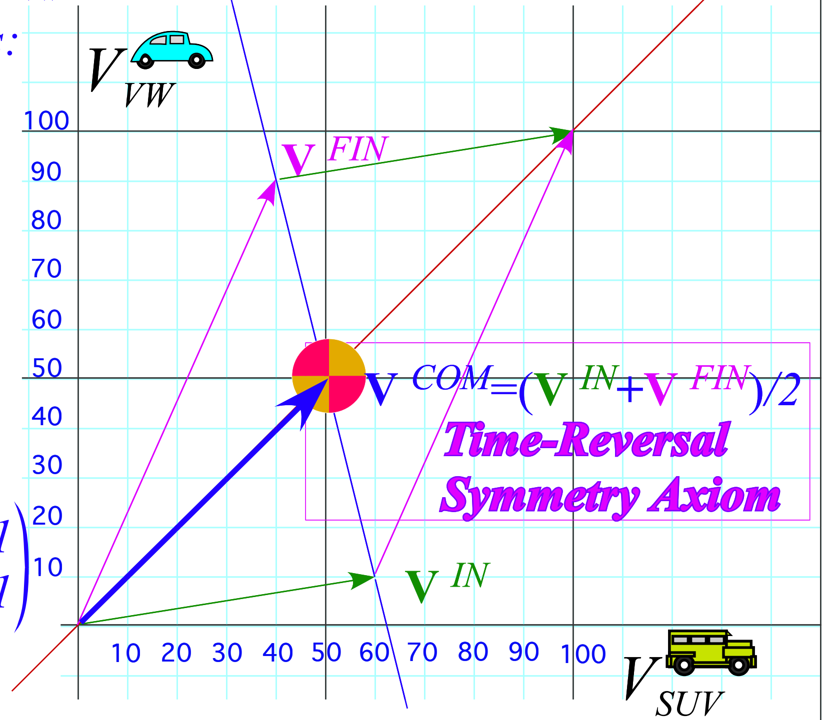
$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$=V^{COM}\mathbf{u}$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix} M_{SUV} & 0 \\ 0 & M_{VW} \end{pmatrix}$$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix} M_{SUV} & 0 \\ 0 & M_{VW} \end{pmatrix}\begin{pmatrix} V_{SUV} \\ V_{VW} \end{pmatrix}=\begin{pmatrix} P_{SUV} \\ P_{VW} \end{pmatrix}=\begin{pmatrix} M_{SUV}V_{SUV} \\ M_{VW}V_{VW} \end{pmatrix}$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

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Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}$$

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$$=V^{COM}\mathbf{u}$$ *Define funny-unit vector:* $\mathbf{u}=\begin{pmatrix}1\\1\end{pmatrix}$

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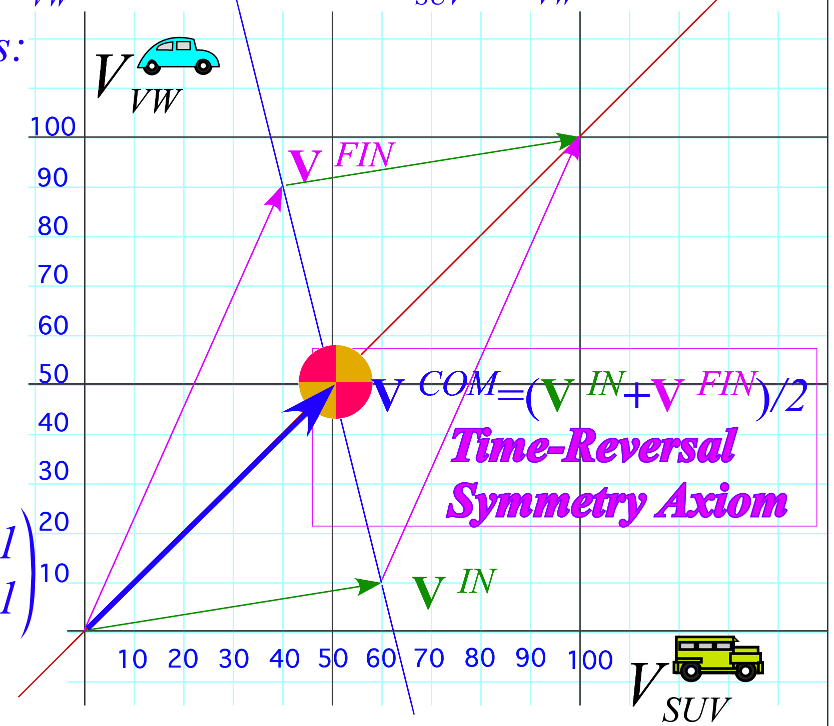

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u•** *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

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Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix} V_{SUV}^{IN} \\ V_{VW}^{IN} \end{pmatrix}$$

$$\mathbf{V}^{FIN}=\begin{pmatrix} V_{SUV}^{FIN} \\ V_{VW}^{FIN} \end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix} V^{COM} \\ V^{COM} \end{pmatrix}=V^{COM}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

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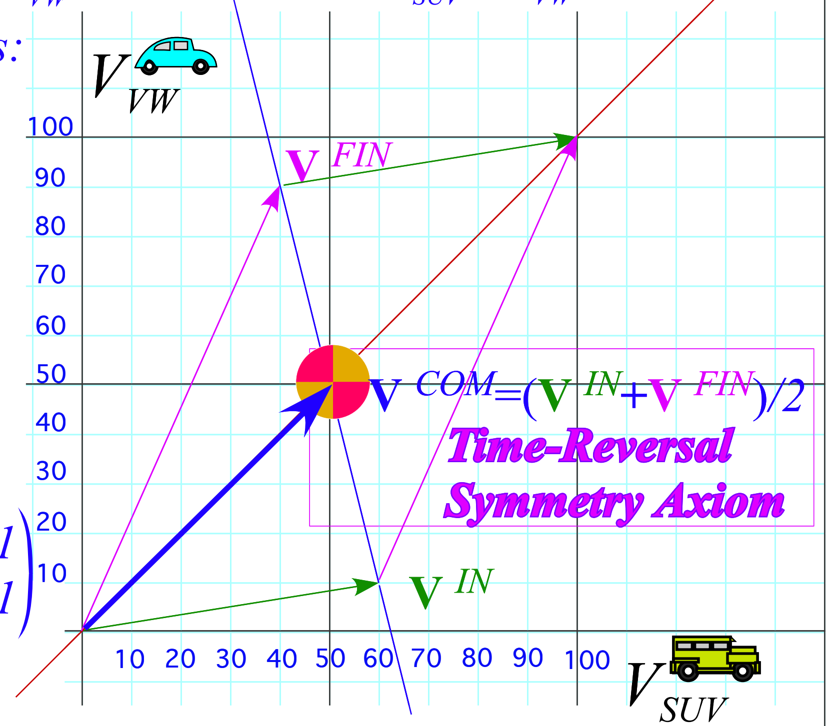


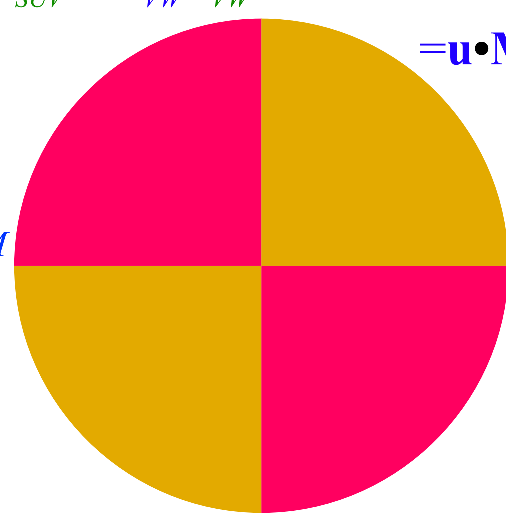
...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix} M_{SUV} & 0 \\ 0 & M_{VW} \end{pmatrix}\begin{pmatrix} V_{SUV} \\ V_{VW} \end{pmatrix}=\begin{pmatrix} P_{SUV} \\ P_{VW} \end{pmatrix}=\begin{pmatrix} M_{SUV}V_{SUV} \\ M_{VW}V_{VW} \end{pmatrix}$

whose sum of components is constant.

(by **u•** *product)*

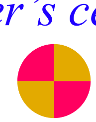
$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$



Denote Center of Momentum $\mathbf{V}^{COM}$

with engineer's centering symbol



Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

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Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}$$

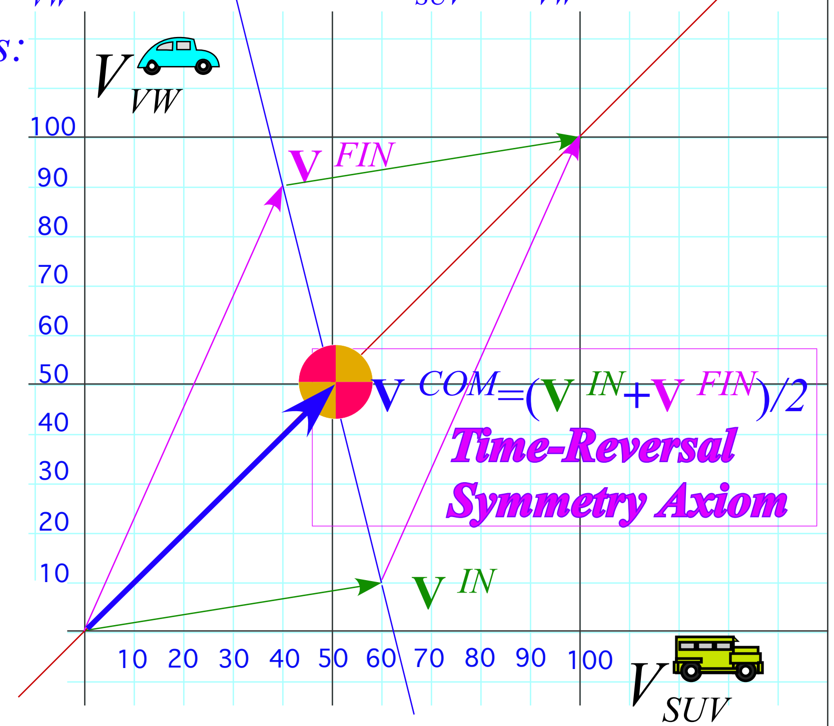
$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\ V_{VW}^{FIN}\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\ V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}$$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\ V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\ P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\ M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by $\mathbf{u}\bullet$ *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

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Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

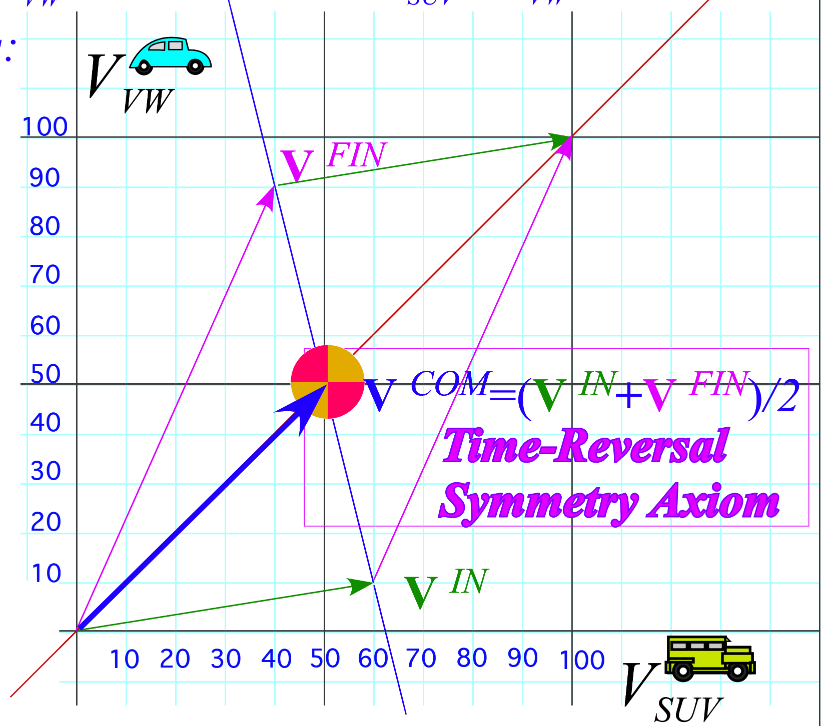
$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\V_{VW}^{FIN}\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}$$



...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant. *(by* **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

Matrix or tensor algebra of collisions

→ *Deriving Energy Conservation Theorem*

Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

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$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}$$

$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\ V_{VW}^{FIN}\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\ V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

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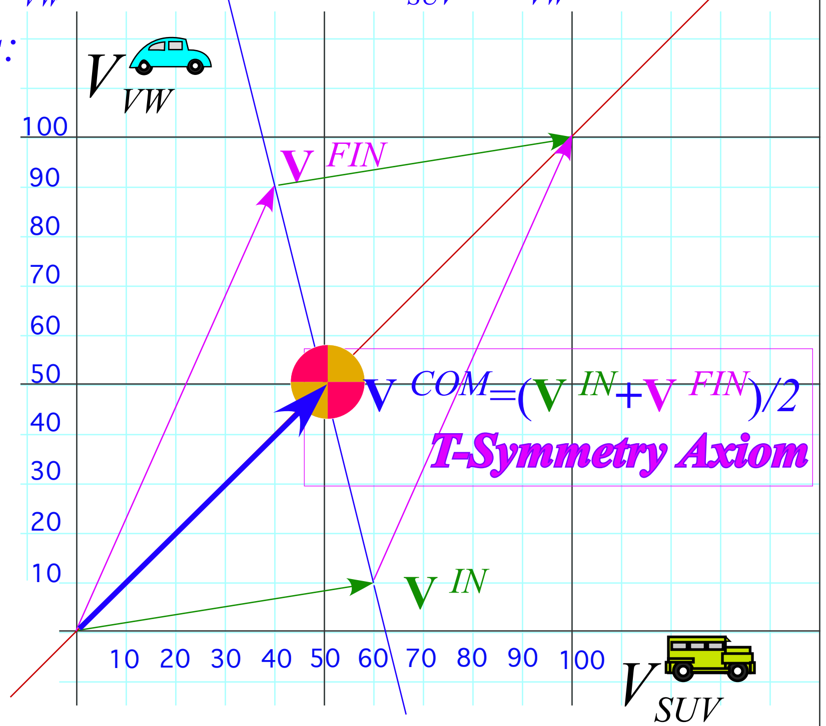

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\ V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\ P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\ M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

*(by **u**• product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

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Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

*By **T-Symmetry Axiom**:* $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$*. Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

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$$const.=V^{COM}=\frac{M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}}{(M_{SUV}+M_{VW})}=\frac{M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}}{(M_{SUV}+M_{VW})}$$

Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\V_{VW}^{IN}\end{pmatrix}$$

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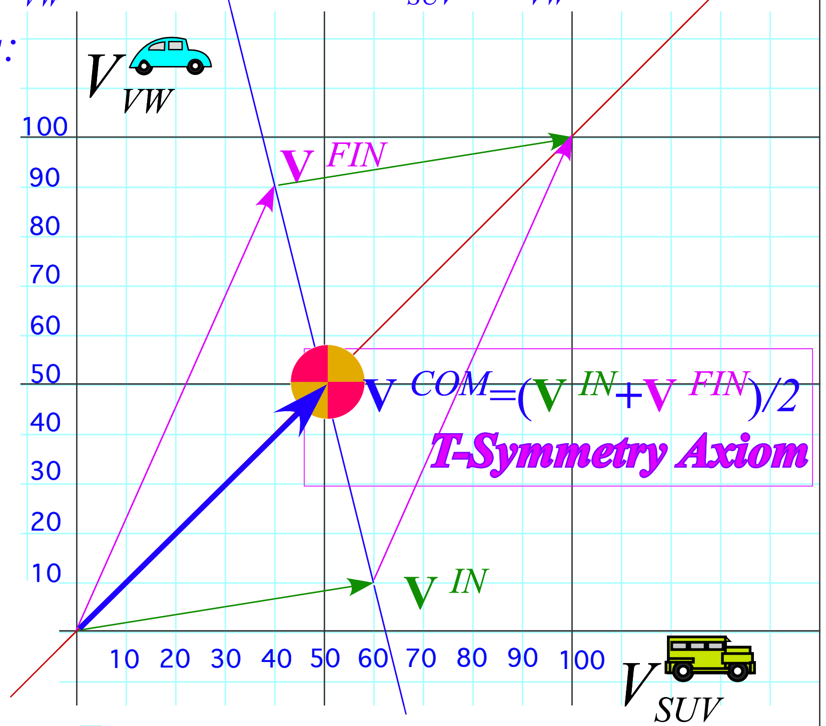
$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV}&0\\0&M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by $\mathbf{u}\bullet$ *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

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Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}$$

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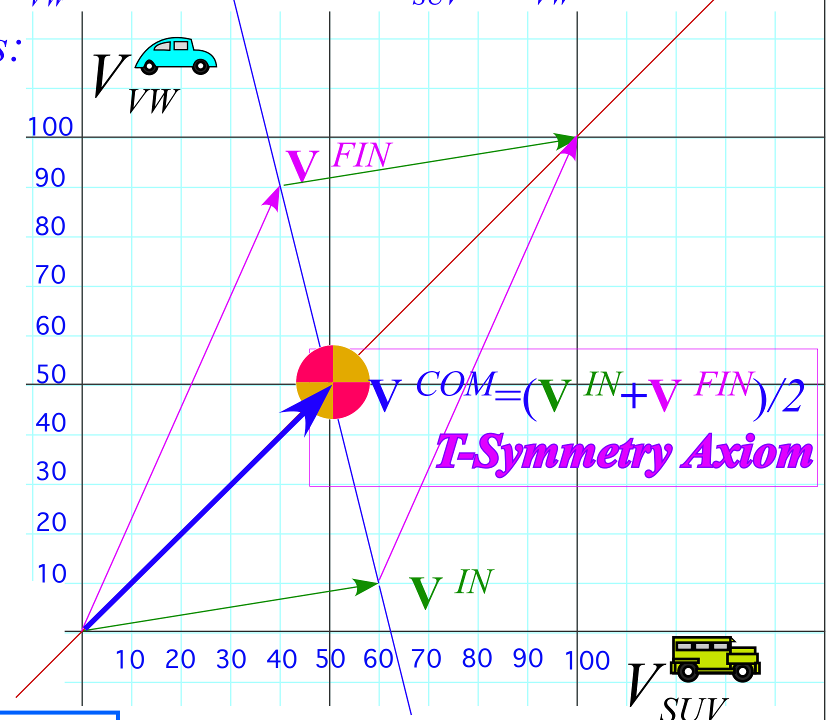
$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\ V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\ V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\ P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\ M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***M-symmetry*** $\mathbf{M}=\mathbf{M}^T$: $\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

this becomes:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}-1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$
$$=1/2\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions

Matrix or tensor algebra of collisions

→ *Completing derivation of Energy Conservation Theorem*

Energy Ellipse geometry

Algebra, Geometry, and Physics of Momentum Conservation Axiom

Conservation of momentum line:

$$(M_{SUV}+M_{VW})V^{COM}=M_{SUV}V_{SUV}^{IN}+M_{VW}V_{VW}^{IN}=M_{SUV}V_{SUV}^{FIN}+M_{VW}V_{VW}^{FIN}$$

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Express this using velocity vectors:

$$\mathbf{V}^{IN}=\begin{pmatrix}V_{SUV}^{IN}\\ V_{VW}^{IN}\end{pmatrix}$$

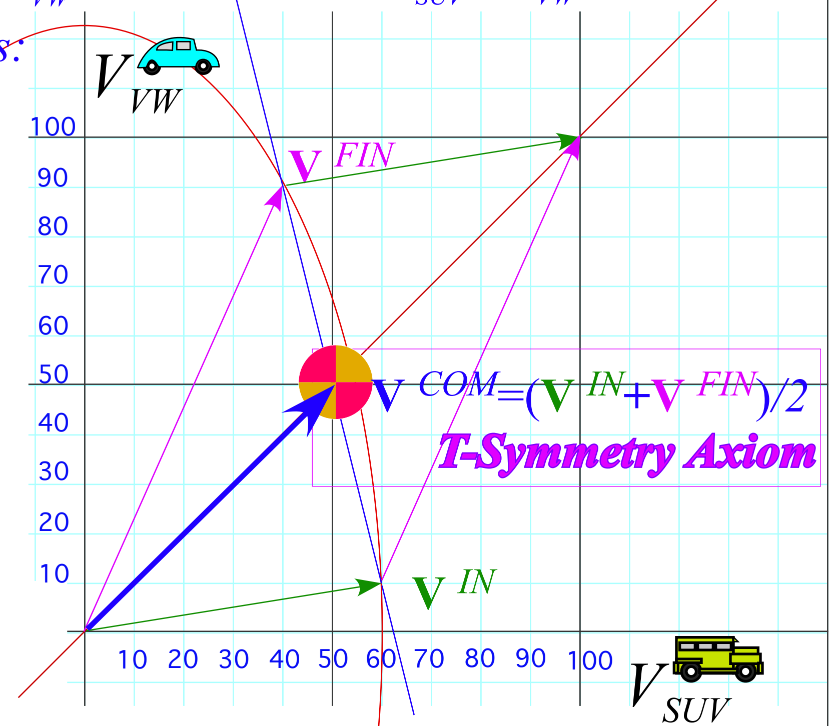
$$\mathbf{V}^{FIN}=\begin{pmatrix}V_{SUV}^{FIN}\\ V_{VW}^{FIN}\end{pmatrix}$$

$$\mathbf{V}^{COM}=\begin{pmatrix}V^{COM}\\ V^{COM}\end{pmatrix}=V^{COM}\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$=V^{COM}\mathbf{u}$$

...and matrix operators:

$$\mathbf{M}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}=\mathbf{M}^{Transpose}$$



M-symmetry $\mathbf{M}=\mathbf{M}^T$

...that give momentum vector: $\mathbf{P}=\mathbf{M}\bullet\mathbf{V}=\begin{pmatrix}M_{SUV} & 0\\ 0 & M_{VW}\end{pmatrix}\begin{pmatrix}V_{SUV}\\ V_{VW}\end{pmatrix}=\begin{pmatrix}P_{SUV}\\ P_{VW}\end{pmatrix}=\begin{pmatrix}M_{SUV}V_{SUV}\\ M_{VW}V_{VW}\end{pmatrix}$

whose sum of components is constant.

(by **u**• *product)*

$$const.=\mathbf{u}\bullet\mathbf{P}=P_{SUV}+P_{VW}=M_{SUV}V_{SUV}+M_{VW}V_{VW}=\mathbf{u}\bullet\mathbf{M}\bullet\mathbf{V}$$

Then: $\mathbf{V}^{COM}=V^{COM}\mathbf{u}$ *gives:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***T-Symmetry Axiom***: $\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})$. *Substituting:*

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2(\mathbf{V}^{IN}+\mathbf{V}^{FIN})\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

By ***M-symmetry*** $\mathbf{M}=\mathbf{M}^T$: $\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

this becomes:

$$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM}-1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$$

$$=1/2\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}=1/2\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$$

These are equations for energy conservation ellipse:

$$const.=\tfrac{1}{2}M_{SUV}V_{SUV}^{2}+\tfrac{1}{2}M_{VW}V_{VW}^{2}$$

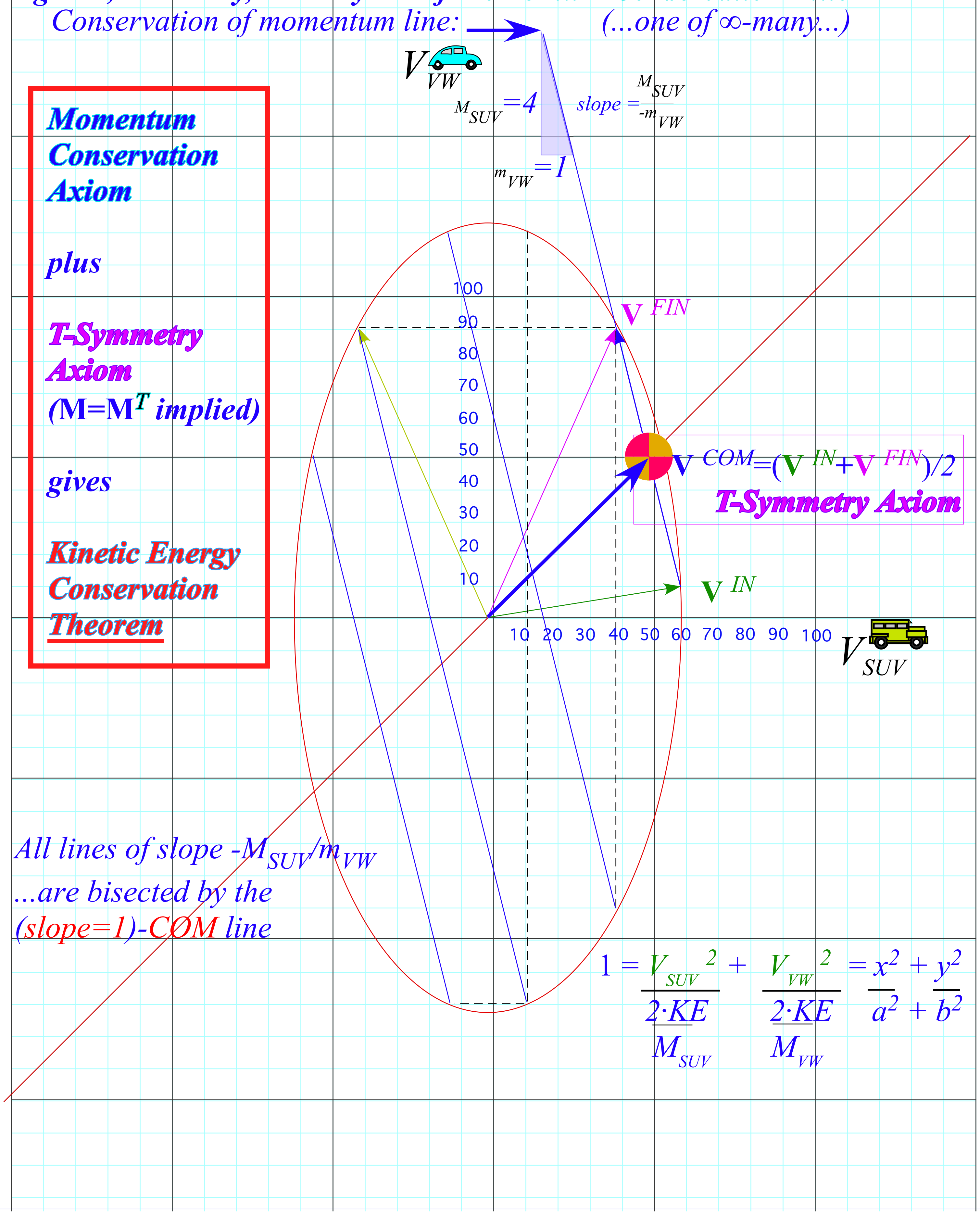
$$=\tfrac{1}{2}M_{SUV}V_{SUV}^{2}+\tfrac{1}{2}M_{VW}V_{VW}^{2}$$

=Kinetic Energy=KE is now defined
and proved a constant under T-Symmetry

Algebra,Geometry, and Physics of momentum conservation axiom

Vector algebra of collisions
Matrix or tensor algebra of collisions
Deriving Energy Conservation Theorem
→ *Energy Ellipse geometry*

Algebra, Geometry, and Physics of Momentum Conservation Axiom



$\mathbf{V}^{COM}\bullet\mathbf{M}\bullet\mathbf{V}^{COM} - 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN}$
$= 1/2\,\mathbf{V}^{IN}\bullet\mathbf{M}\bullet\mathbf{V}^{IN} = 1/2\,\mathbf{V}^{FIN}\bullet\mathbf{M}\bullet\mathbf{V}^{FIN}$

These are equations for energy conservation ellipse:

$$KE = {}^1\!/_2 M_{SUV} V_{SUV}^{\ 2} + {}^1\!/_2 M_{VW} V_{VW}^{\ 2}$$

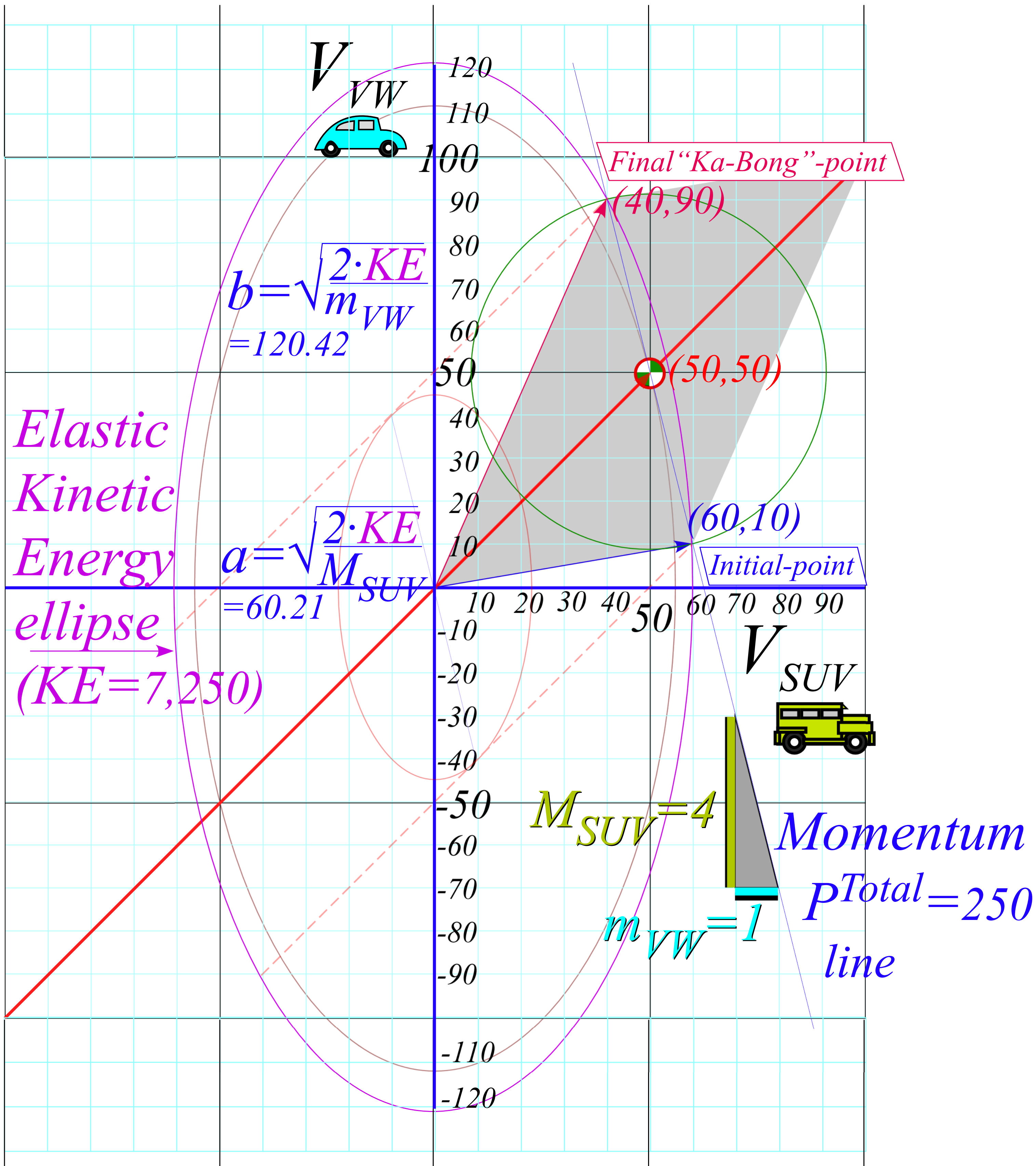


Fig. 3.1 a
in Unit 1

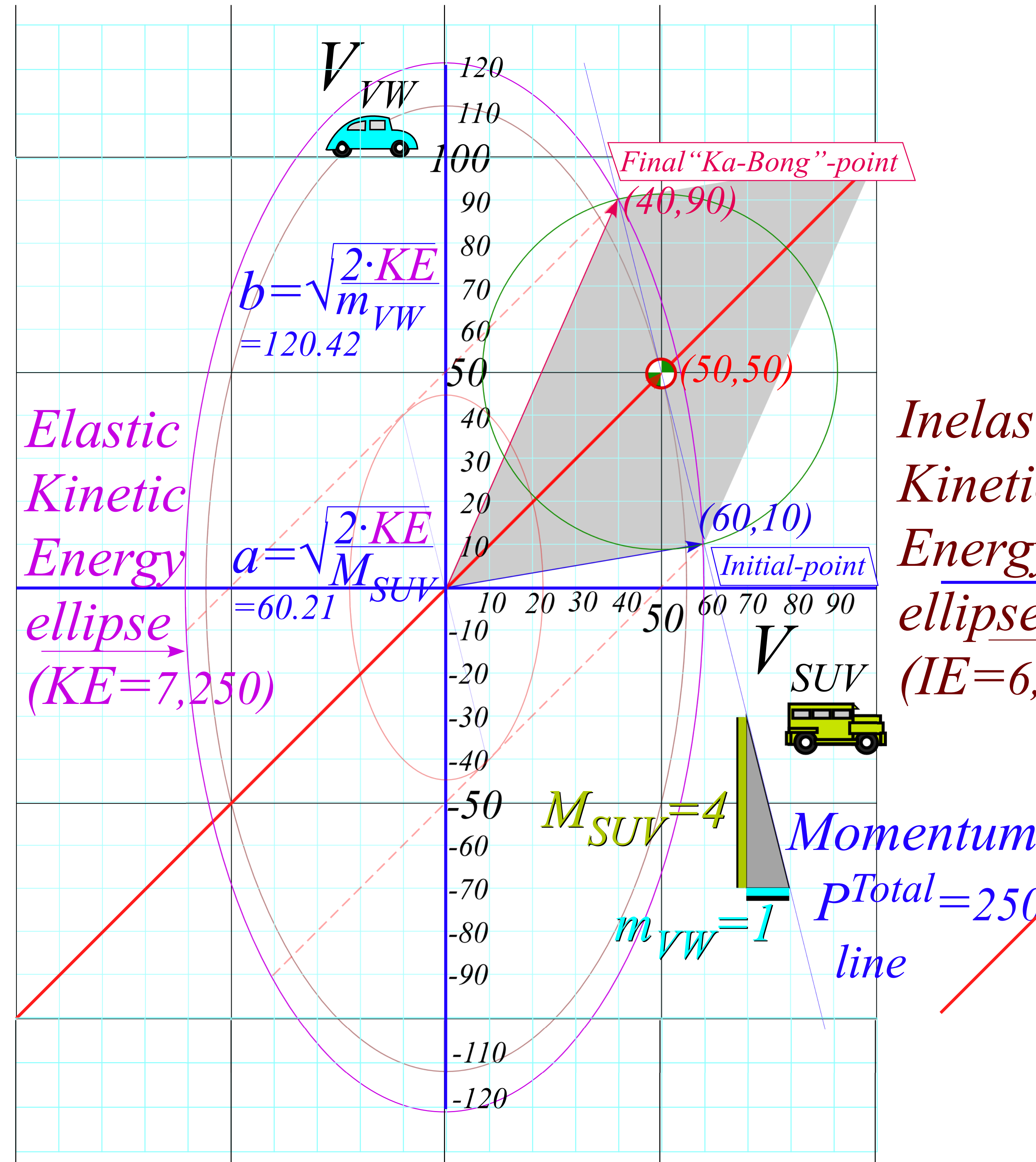


Fig. 3.1 a
in Unit 1

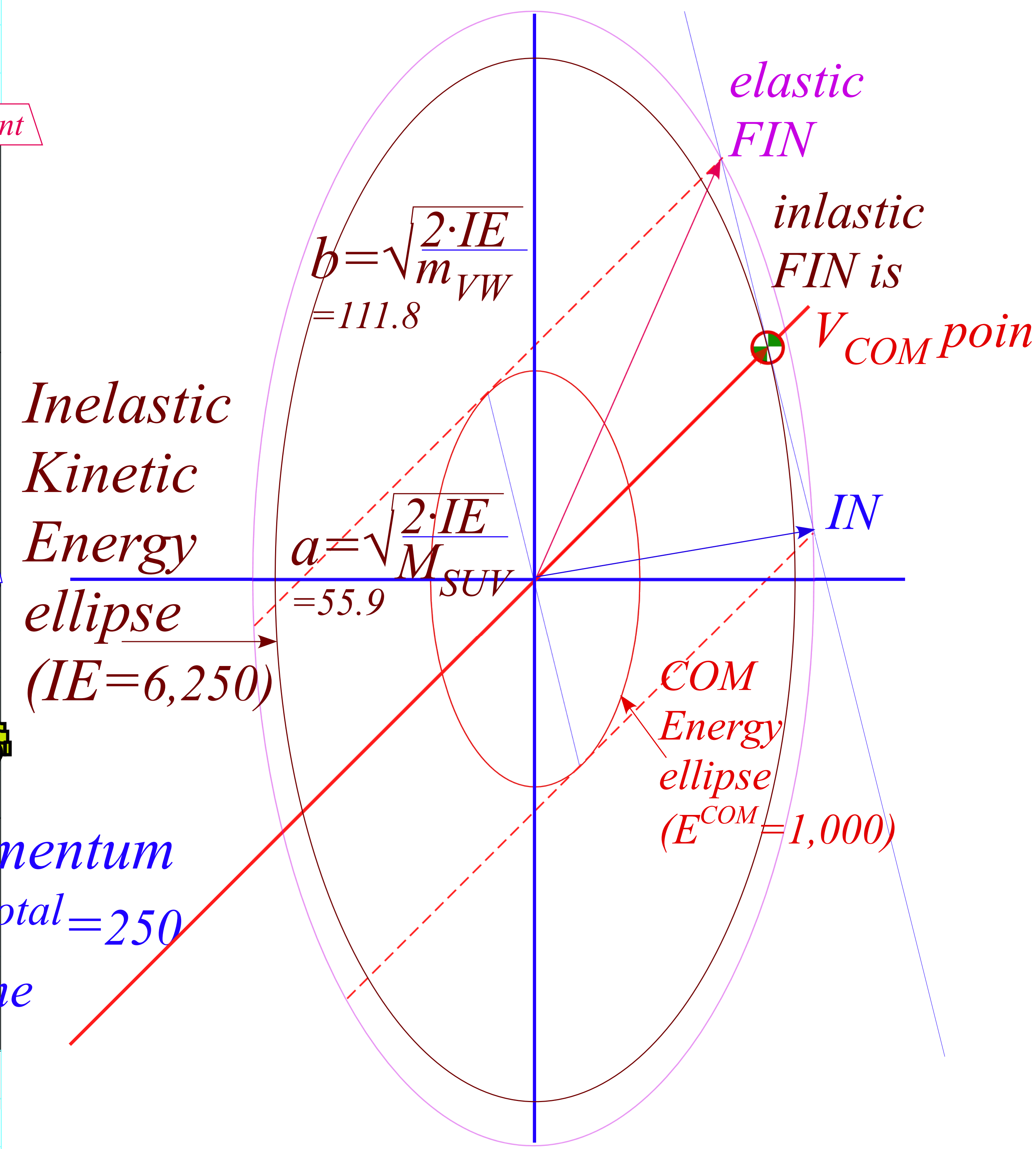


Fig. 3.1 b
in Unit 1

As usual in physics, opposite extremes are easier to analyze than the generic "real(er) world" in between!

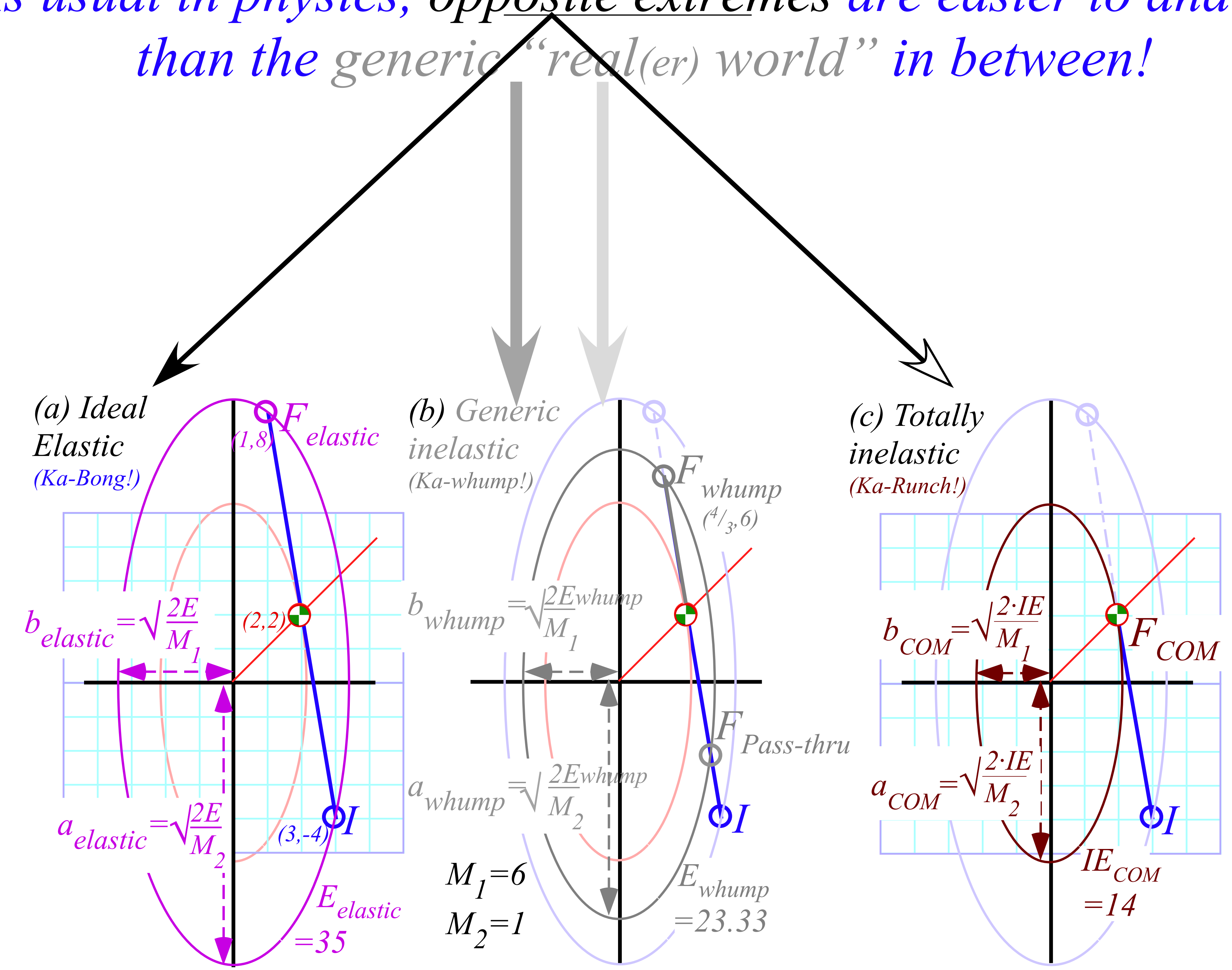


Fig. 3.2 in Unit 1

(This case has Bush era requisite SUV mass of the 6 ton "Hummer")

Next: **The X-2 pen-launcher**