Lecture 29 part II
Mon. 12.03.2018

Relawavity : a novel introduction to relativistic mechanics II.

(CMwBang! Unit 8 , AMOP Ch.0,)

Why ***Men in Black*** *shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra
1CW wavefunctions and phasors
Per-space-per-time vs Space-time (How to understand wave parameters)
Wave velocity formulas
Introducing Doppler shifting
Why is c *so* constant?!
Introducing Doppler Arithmetic and *Rapidity* ρ
Optical interference "baseball-diamond" displays *phase* and *group* velocity
Details of 2CW wavefunctions in rest frame
Pulse waves (PW) versus Continuous Waves (CW)
Doppler shifted "baseball-diamond" displays Lorentz frame transformation
Analyzing wave velocity by per-space-per-time *and* space-time graphs
16 coefficients of relativistic 2CW interference
Two "famous-name" coefficients and the Lorentz transformation
Thales mean geometry of Lorentz transformation
Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and geometry of functions of ρ and σ Minkowski animations
Application to TE-Waveguide modes. synchrotron beam relativity

part II ↓

A running collection of links to course-relevant sites and articles

Physics Web Resources

Comprehensive Harter-Soft Resource Listing

UAF Physics YouTube channel

LearnIt Physics Web Applications

"Texts"

Classical Mechanics with a Bang!

Quantum Theory for the Computer Age

Principles of Symmetry, Dynamics, and Spectroscopy

Modern Physics and its Classical Foundations

Classes

2014 AMOP

2017 Group Theory for QM

2018 AMOP

2018 Adv Mechanics

Neat external material to start the class:
- AIP publications
- AJP article on superball dynamics
- AAPT summer reading

AMOP Detailed Development of Relawavity
AMOP Ch 0 Space-Time Symmetry - 2019
Seminar at Rochester Institute of Optics, Auxiliary slides, June 19, 2018
Springer AMO Handbook - Ch 32 - Harter-Reimer-2019

These *are* hot off the presses:
- Sorting ultracold atoms in a 3D optical lattice in a realization of Maxwell's demon - Kumar-Nature-Letters-2018
- Synthetic three-dimensional atomic structures assembled atom by atom - Berredo-Nature-Letters-2018

Slightly Older ones:
- Wave–particle duality of C60 molecules
- Optical vortex knots – One Photon at a Time

"*Relawavity*" and quantum basis of *Lagrangian* & *Hamiltonian* mechanics:
- 2-CW laser wave - BohrIt Web App
- Lagrangian vs Hamiltonian - RelaWavity Web App

Older Links from Lectures 14-20

http://thearmchaircritic.blogspot.com/2011/11/punkin-chunkin.html
http://www.sussexcountyonline.com/news/photos/punkinchunkin.html
Shooting-range-for-medieval-siege-weapons-Anybody-knows
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html?scenario=MontezumasRevenge
https://modphys.hosted.uark.edu/markup/TrebuchetWeb.html?scenario=SeigeOfKenilworth
The trebuchet, Chevedden, Sci Am 1995
'*Simple*' *Pendulum* Sim: https://modphys.hosted.uark.edu/markup/PendulumWeb.html
'*Cycloid*' *Pendulum*: https://modphys.hosted.uark.edu/markup/CycloidulumWeb.html
Google search on: "Satelite view of Patricia" (Images)
Physics Girl Channel - Fun with Vortex Rings in the Pool
iBall demo - Quasi-periodicity: https://youtu.be/_jntDtULxDc
https://modphys.hosted.uark.edu/markup/CoulItWeb.html?scenario=SynchrotronMotion
https://modphys.hosted.uark.edu/markup/CoulItWeb.html?scenario=SynchrotronMotion2
Mechanical Analog to EM Motion (YouTube video) - https://youtu.be/hTd5FTJ-vRk
CoulIt Web Simulation: Bound-state motion in parabolic coordinates
CoulIt Web Simulation: Bound-state motion in hyperbolic coordinates
OscillIt Web App: Simulations of various types of resonance: 18, 27, 31, 35, 38, 39
Smith Chart
http://nobelprize.org/

AnalyIt Web Application, posted 10/22/2018 in our *testing* area:
https://modphys.hosted.uark.edu/testing/markup/AnalyItBJS.html

Older Links from Lectures 21-23

Advanced Atomic and Molecular Optical Physics 2018 Class #9, pages: 5, 61
BoxIt Web Simulations
- Pure A-Type w/Cosine
- Pure B-Type w/Cosine
- Pure B-Type w/Freq ratios
- Mixed AB-Type 2:1 Freq ratio
- Pure A-Type A=4.9, B=0 ,C=0, & D=4.0
- Pure B-Type: A=4.0, B=-0.2, C=0, & D=4.0
- Pure C-Type A,D=4.055, B=0, C=0.1
- Mixed AB-Type w/Cosine
- Mixed AB Type A=4.0, BU2=0.866…, CU2=0, & D=1.0 w/Stokes & Freq rats
- Mixed AB Type A=5.086 B=-0.27 C=0 D=2.024 w/Stokes plot
- Mixed ABC Type A=4.833 B=0.2403 C=0.4162 D=4.277 w/Stokes plot
- Recent mixed ABC Type A=0.325 B=0.375 C=0.825 D=0.05 w/Stokes plot

Classical Mechanics with a Bang! 2018
- Lectures 8, 9, 23 page 93
- Text Unit 6, page=27

ColorU2 for the Web - in development
Group Theory for Quantum Mechanics - 2017 Lectures: 6, 7, 8, and the combined 9-10
Quantum Theory for the Computer Age Unit 3 Ch.7-10, page=90
Web based 3D & XR (x∈{A,M,V}, R=Reality) https://www.babylonjs.com/
Web based 3D graphics WebGL API (Graphics Layer modeled after OpenGL)
Wiki on Pafnuty Chebyshev

continued ↘

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Repeated from previous page

Older Links from Lectures 24-27

JerkIt Web App: 2-, 2+, Amp50Omega147-, Amp50Omega296, Amp50Omega602, Gap(1)
MolVibes Web App: C3vN3
WaveIt Web App:
Dim = 3 w/Wave Components;
Static Char Table: 6, 12, 12(b), 16, 36, 256
Quantum Carpet with N=20: Gaussian, Boxcar
Quantum Revivals of Morse Oscillators and Farey-Ford Geometry - Li-Harter-CPL-2015
QTCA Unit_5 Ch14 2013
Lester. R. Ford, Am. Math. Monthly 45,586(1938)
John Farey, Phil. Mag.(1816) Wolfram
Harter, J. Mol. Spec. 210, 166-182 (2001)
Harter, Li IMSS (2013)
Li, Harter, Chem.Phys.Letters (2015)
OscillatorPE Web App: IHO Scenario 2, Coulomb Scenario 3
RelaWavity Web App/Simulator/Calculator: Elliptical - IHO orbits
CoulIt Web App Simulations: p19, p32, p72, p73, p92, R=-0.375, R=+0.5, Rutherford

Older Links from Lectures 28

CMwBang Text 2012 Unit 6 page=5
BounceIt Web App/Scenarios:
5002, 5003
CoulIt Web App/Scenarios:
TwoParticleCollision_LToR, TwoParticleCollision_LToR_CM, TwoParticleOrbit_Coulomb,
TwoParticleOrbit_Coulomb_CM, TwoParticleOrbit_Hooke, TwoParticleOrbit_Hooke_CM
Singular Motion of Asymetric Rotators AJP 44, 11 p1080 Harter-Kim-1976
Molecular Eigensolution Symmetry Analysis and Fine Structure - Int.J.MolSci1.4.13 Harter-Mitchell-IJMS-2013
Lenz Vector and Orbital Analog Computers - AJP 44 p348 1976
Some Geometric Aspects of Classical Coulomb Scattering AJP 40 4 p1852 1972
How Molecules do Self-NMR - Harter-Mitchell-Columbus-2009
Classical Mechanics with a Bang! - Asymmetric Top Demo
Allbookstores.com - Compare for Heller's SemiClassical Way - 0691163731
"My Bomerang Won't Come Back" (YouTube: Playlist)
Rotating Solid Bodies in Microgravity (YouTube)
Dancing T-handle in zero-g (YouTube)

Links to supplement Lecture 29: Parts I & II

AMOP Chapter 0: Space-Time Symmetry
AMOP Detailed Development of RelaWavity
2018 Rochester Talk (Auxilary Slides)
Special Relativity and Quantum Theory by Ruler and Compass - Earlier, expanded draft
Ruler & Compass - Relawavity Exercise
2018 RelaWavity Portal Page
Pirelli Relativity Challenge Web Site:
Title Page, Clocks_12_hr, Clocks_24_hr_QT, Phasors Addition
BohrIt Web App/Simulations: -130022, -30001, -30104, 30004, 30022
GuideIt Web App/Scenarios: 230, 260
RelativIt Web App/Scenarios: 22, 24
RelaWavity Web App/Scenarios: 0,9, 3,6, 3,6 NoMink, 4,8, 6,1, 6,3a, 6,3b, 6.3c, 7,1, 7,2,1, 7,2,2, 7,2,3, 7,2,7, 8,3, 8,5, 8,7, 8,8

➛ *Why* ***Men in Black*** *shot little Suzie…Learning about* sin!, cos *and…Trigonometric road maps*

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Longitudinal hyperbolic *ρ*-geometry connects to transverse cir…

"Occams Sword" and geometry of 16 parameter functions o…

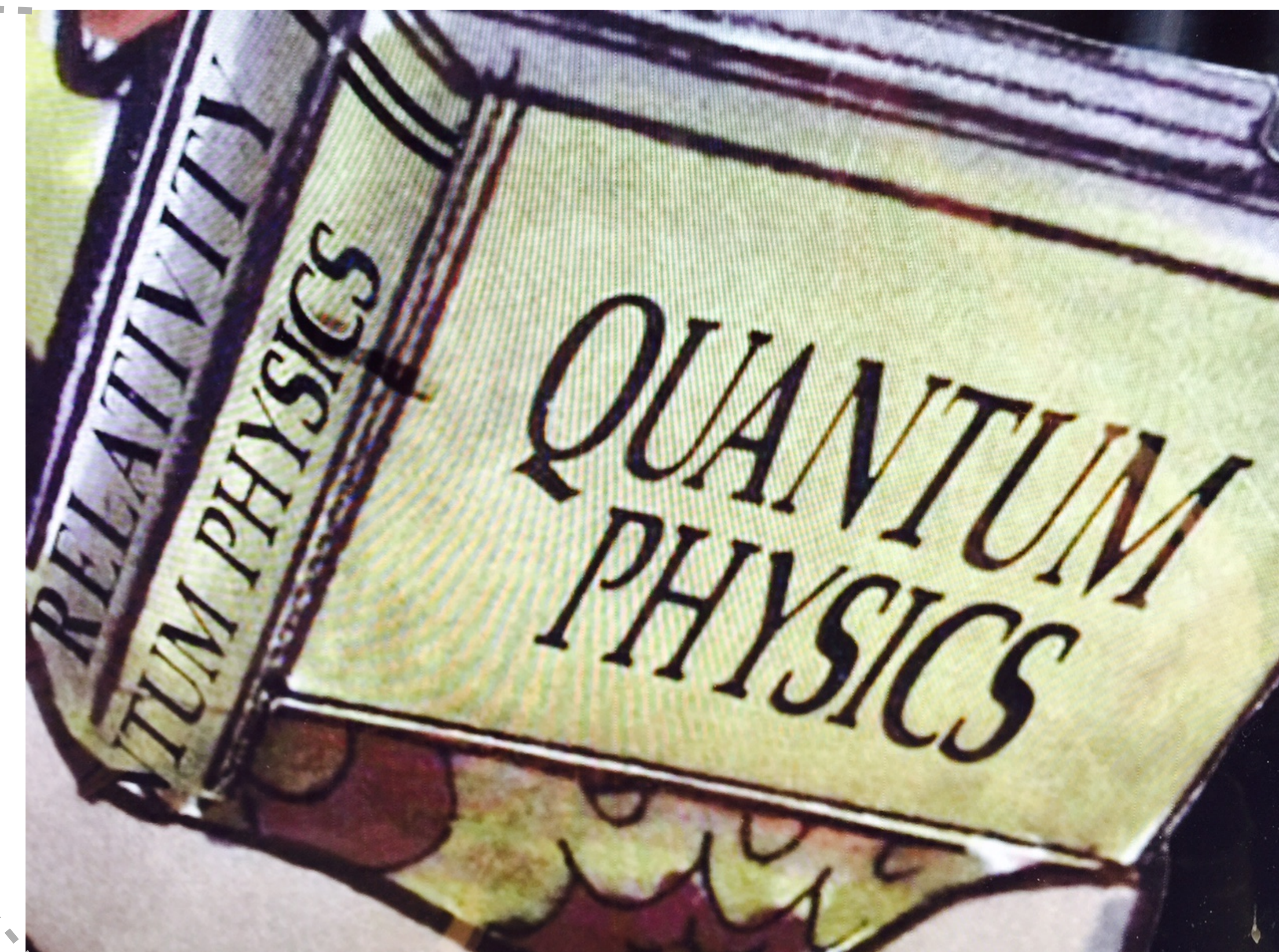
Application to TE-Waveguide modes and synchrotron be…

For an introductory, web based development of this and other concepts in special relativity see our entrant in the 2005 Pirelli Challenge*:*

A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers

From AMOP Ch.0 article.

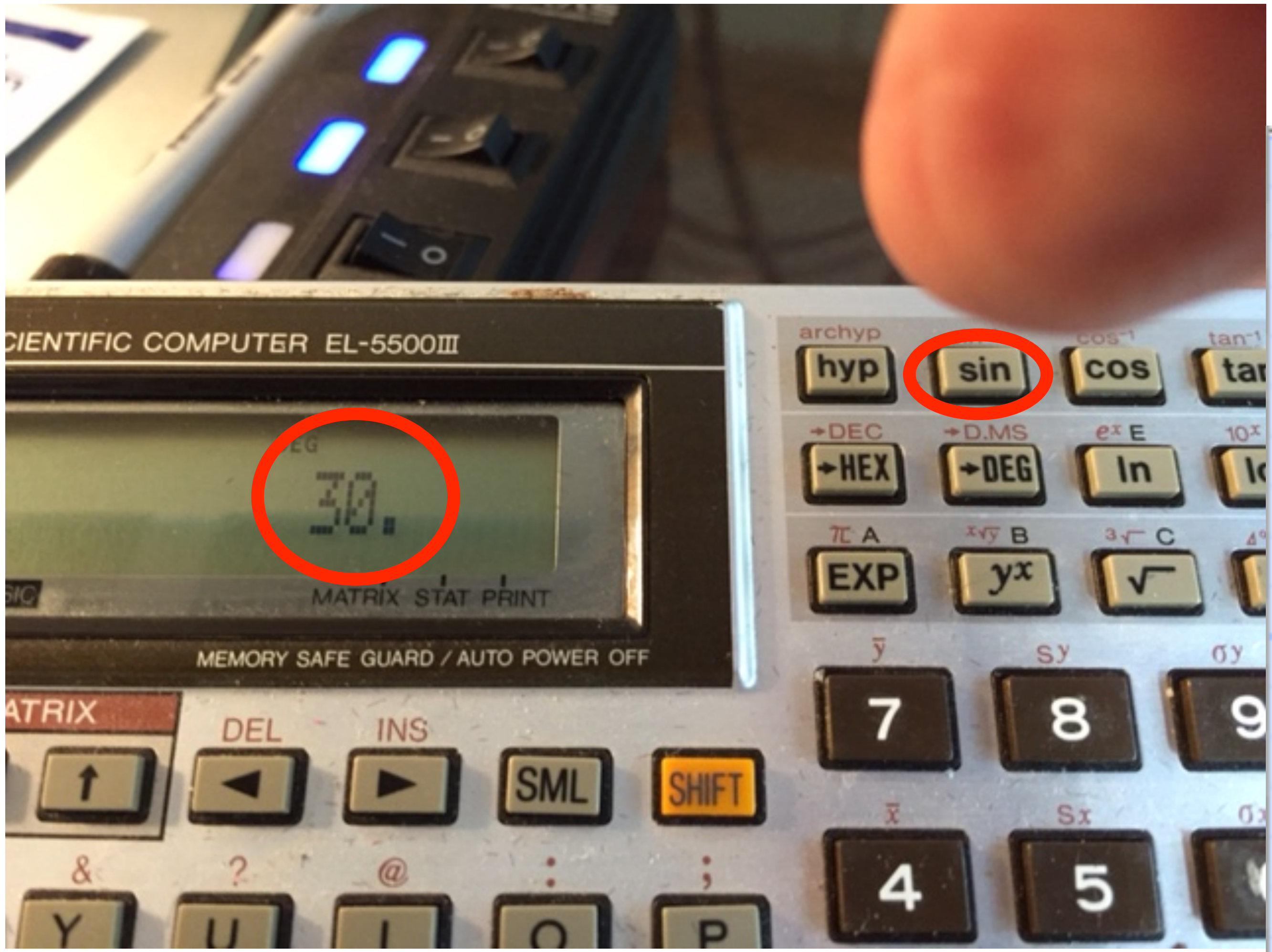
Why did a ***Men In Black*** candidate shoot little Suzy?

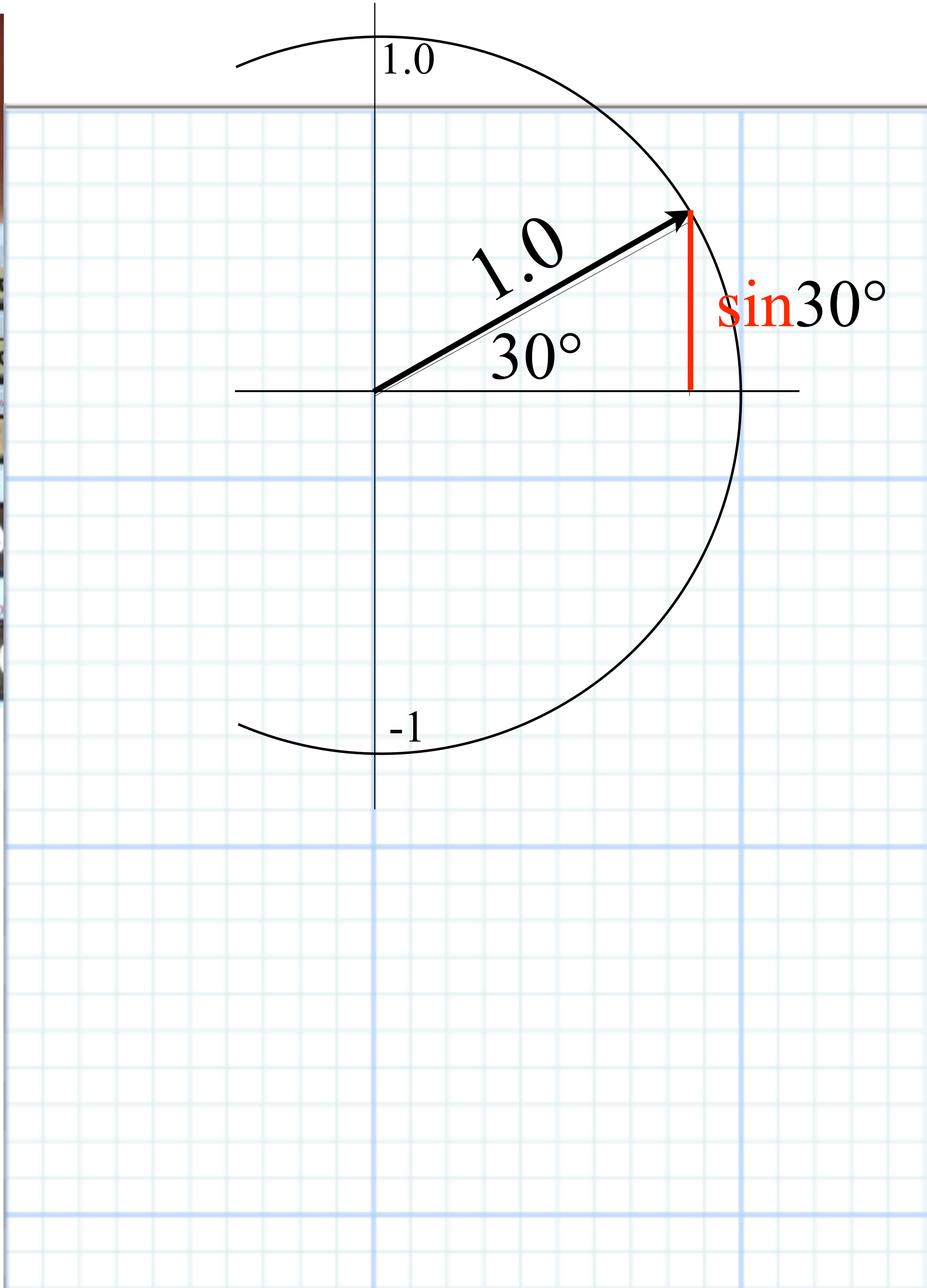


Bad Suzy!
Relativity and Quantum Theory need to be unified in *one* book *half* the size of those old tomes!

We call that a ***Relawavity*** *book.*
(It's a *lot* lighter!)

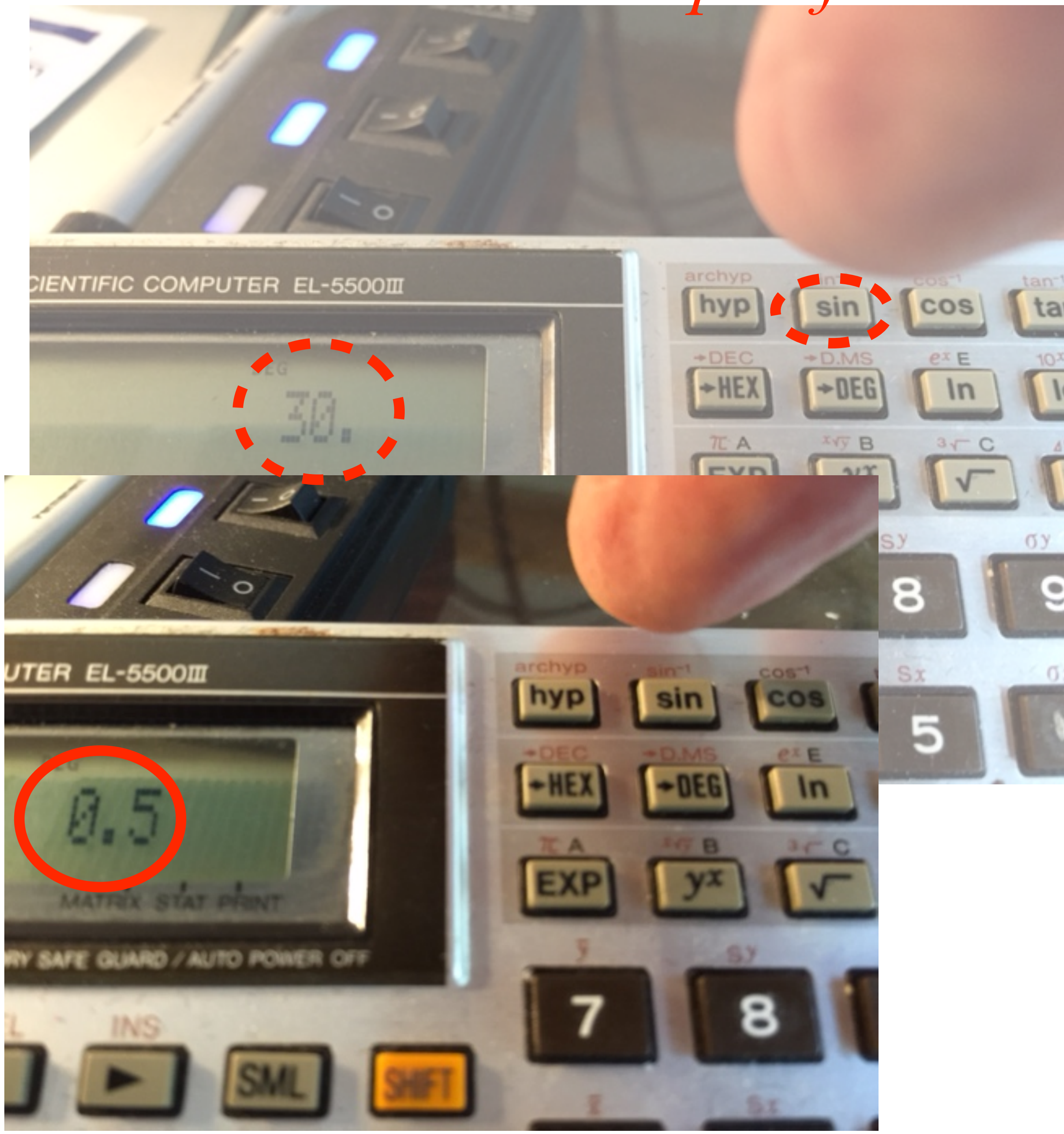
Learning about SIN

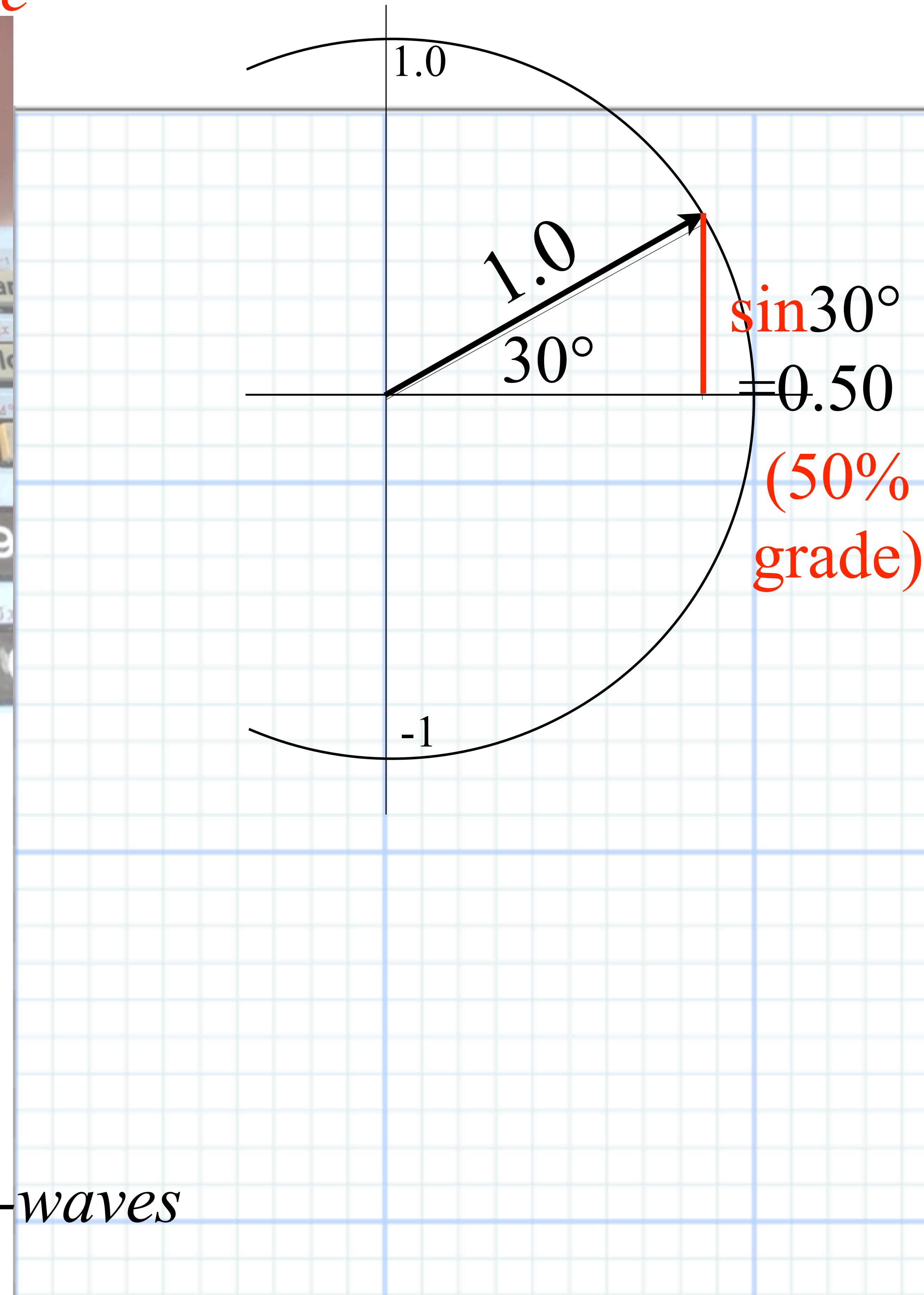




Learning about SIN

"Slope of INcline"

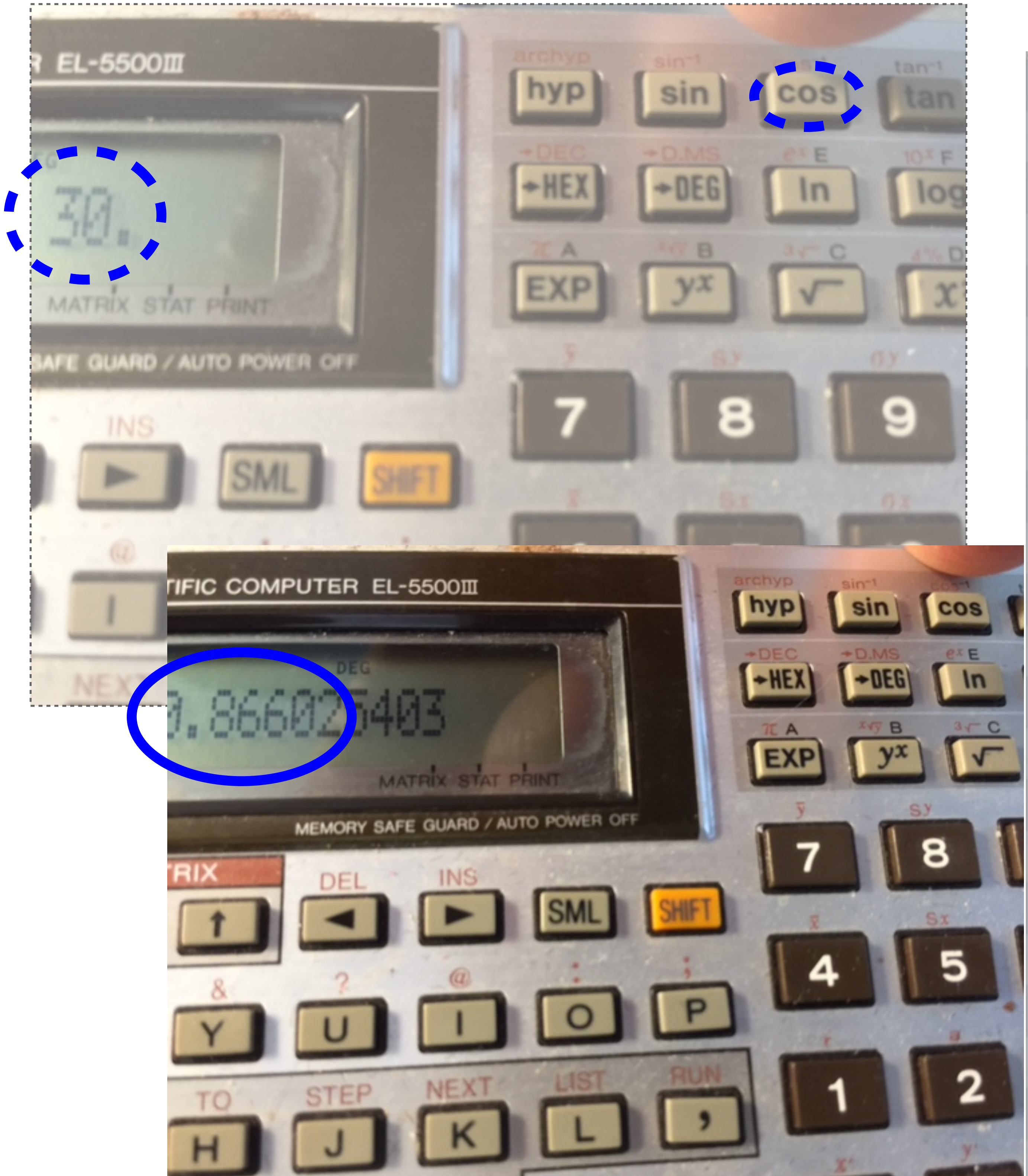


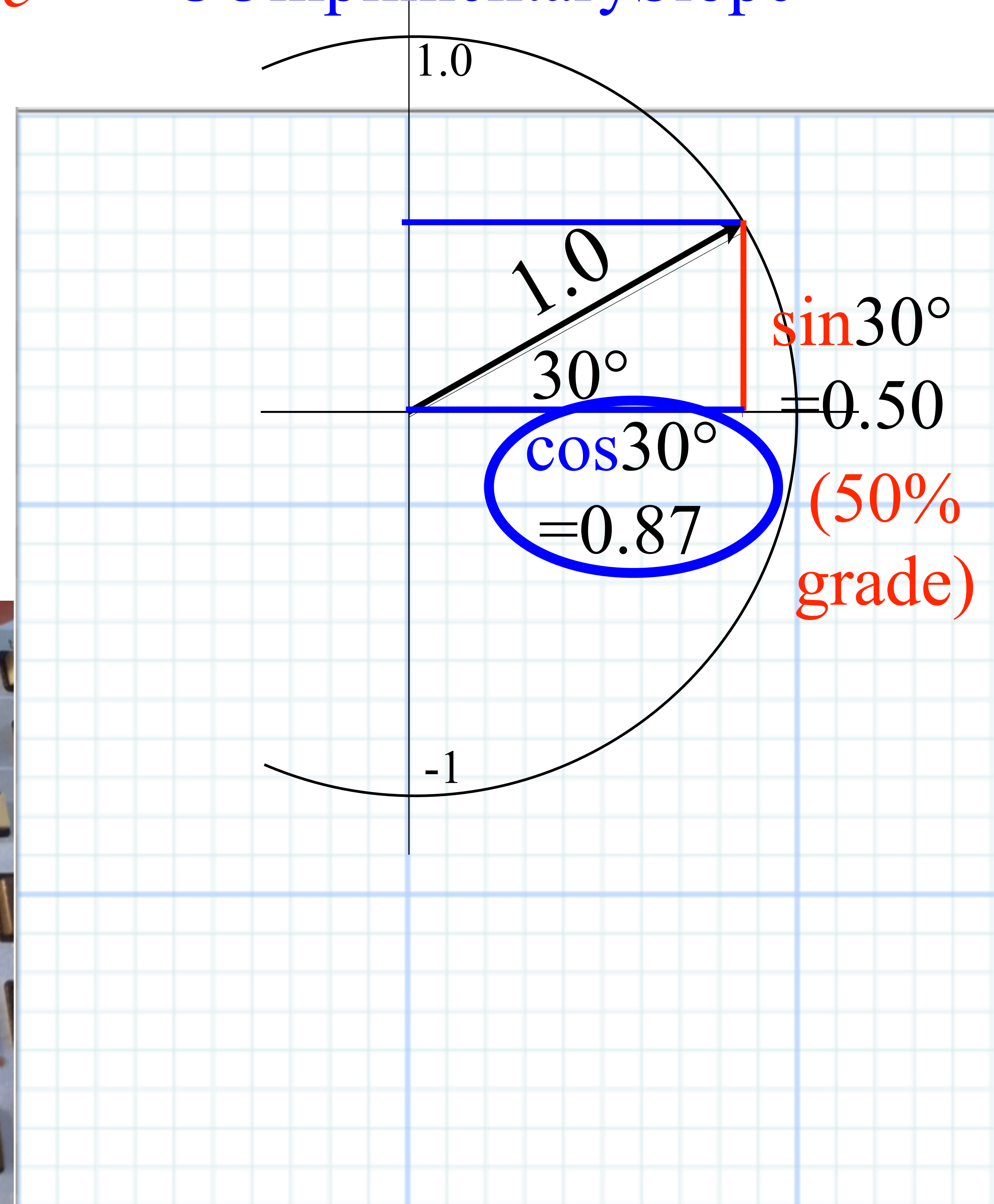


It's mostly about triangles *and sine-waves*

Learning about SIN and the COSin

"*Slope of INcline*" "COmplimentarySlope"

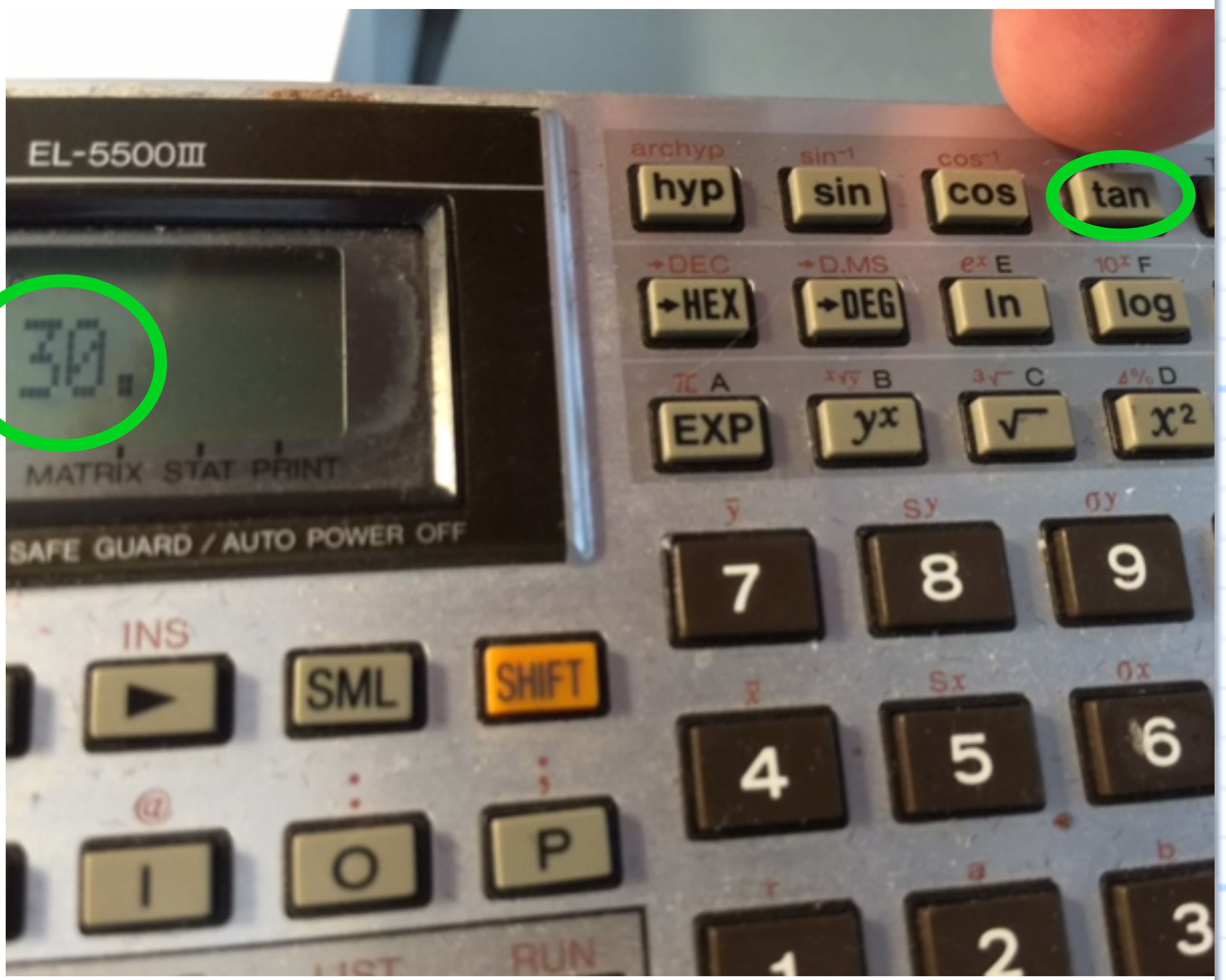


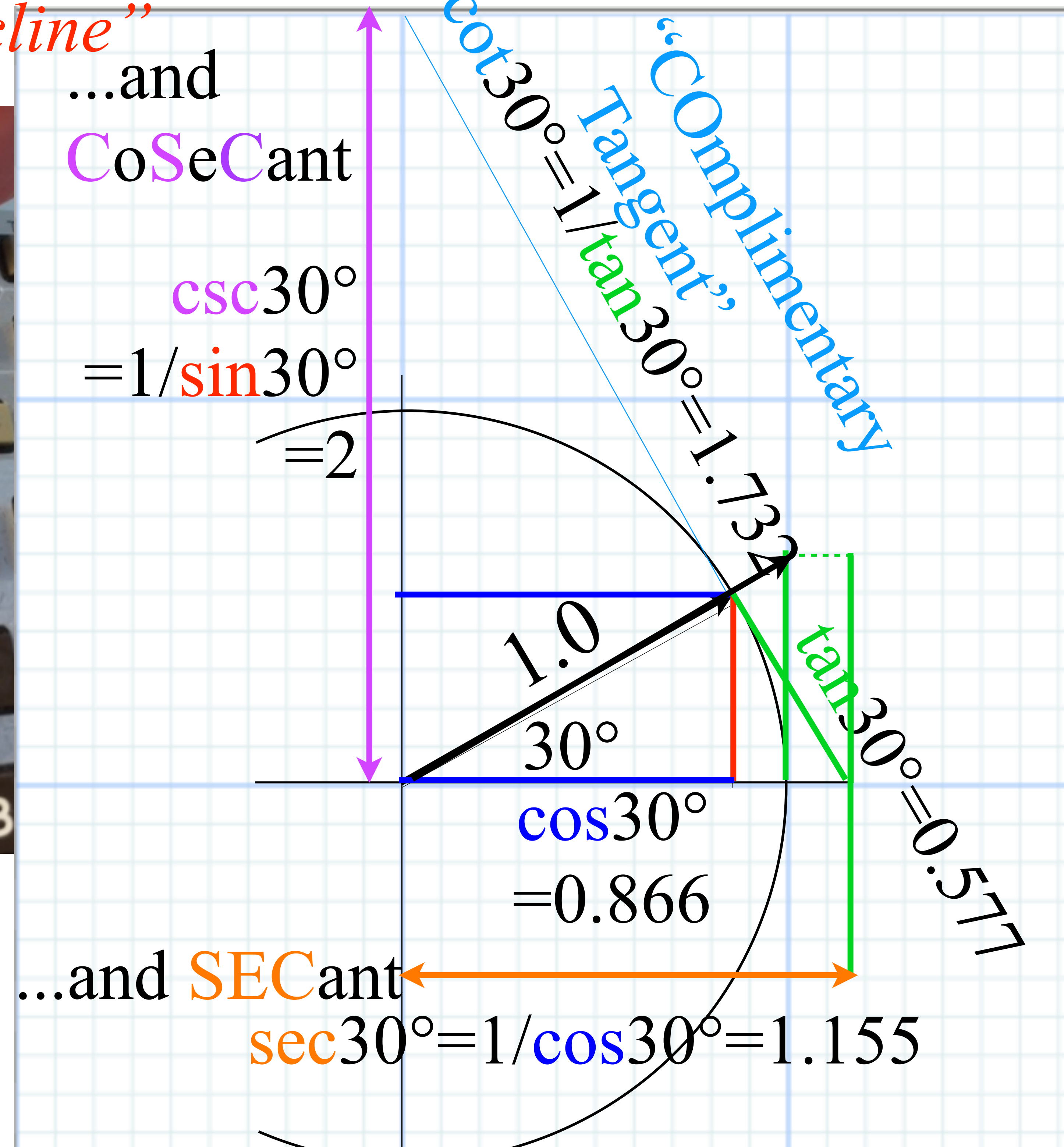


It's mostly about triangles *and sine-waves and cosine-waves*

Learning about SIN and the COSin and TANgent and COTangent

"Slope of INcline"





Fundamental relativity and quantum wave mechanics is mostly about triangles *and sine-waves and cosine-waves*

*Why **Men in Black** shot little Suzie…Learning about* sin!,cos *and…Trigonometric road maps*

➨ Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra
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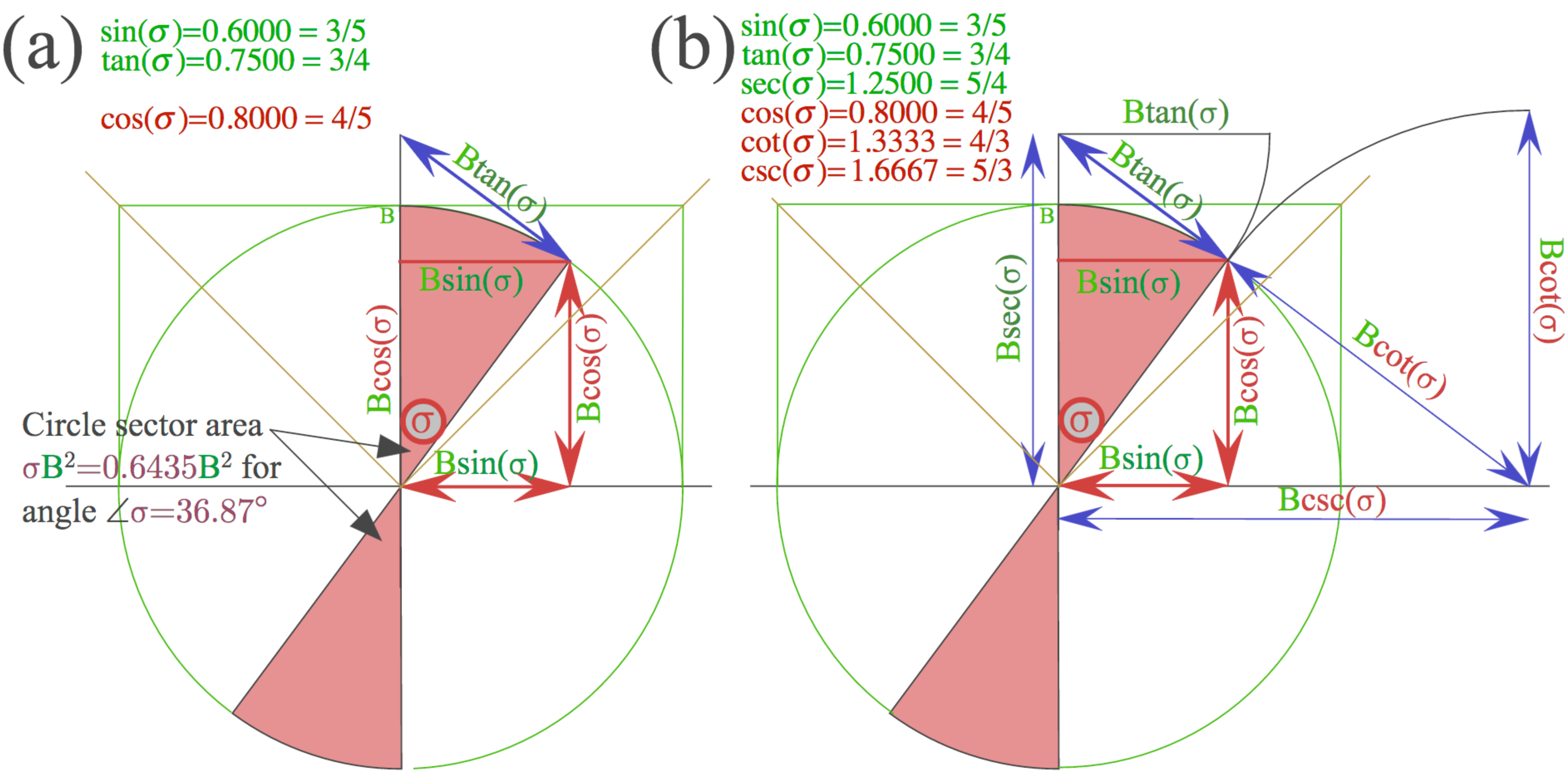
Trigonometric road maps



All this physics of relativity
is mostly simple trigonometry
of optical wave interference!

And, it derives fundamentals
of quantum theory, too!

RelaWavity Web Simulation
Relations between
Hypergeometric and
Hypergeometric functions

Trigonometric road maps become hyperbolic trig maps…

RelaWavity Web Simulation
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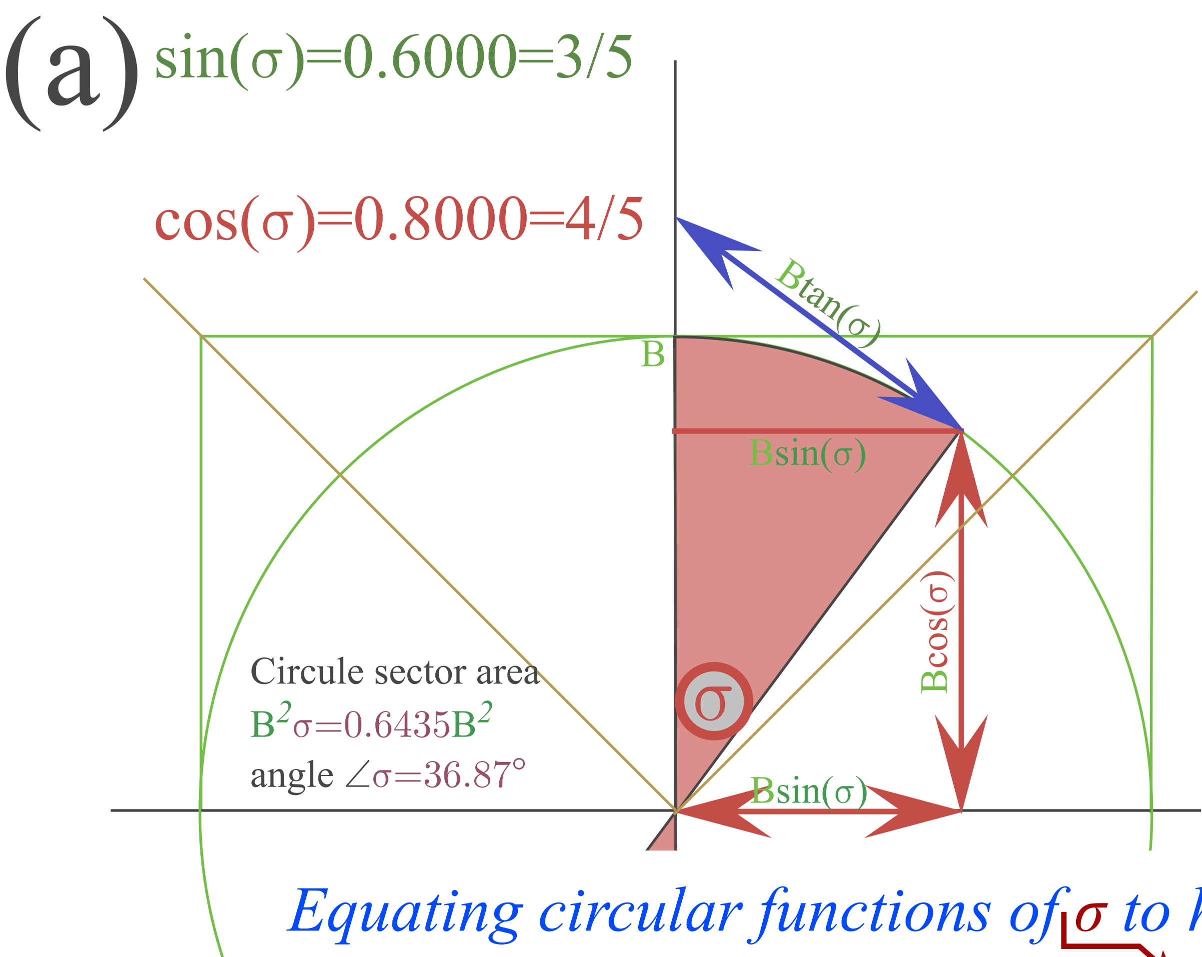
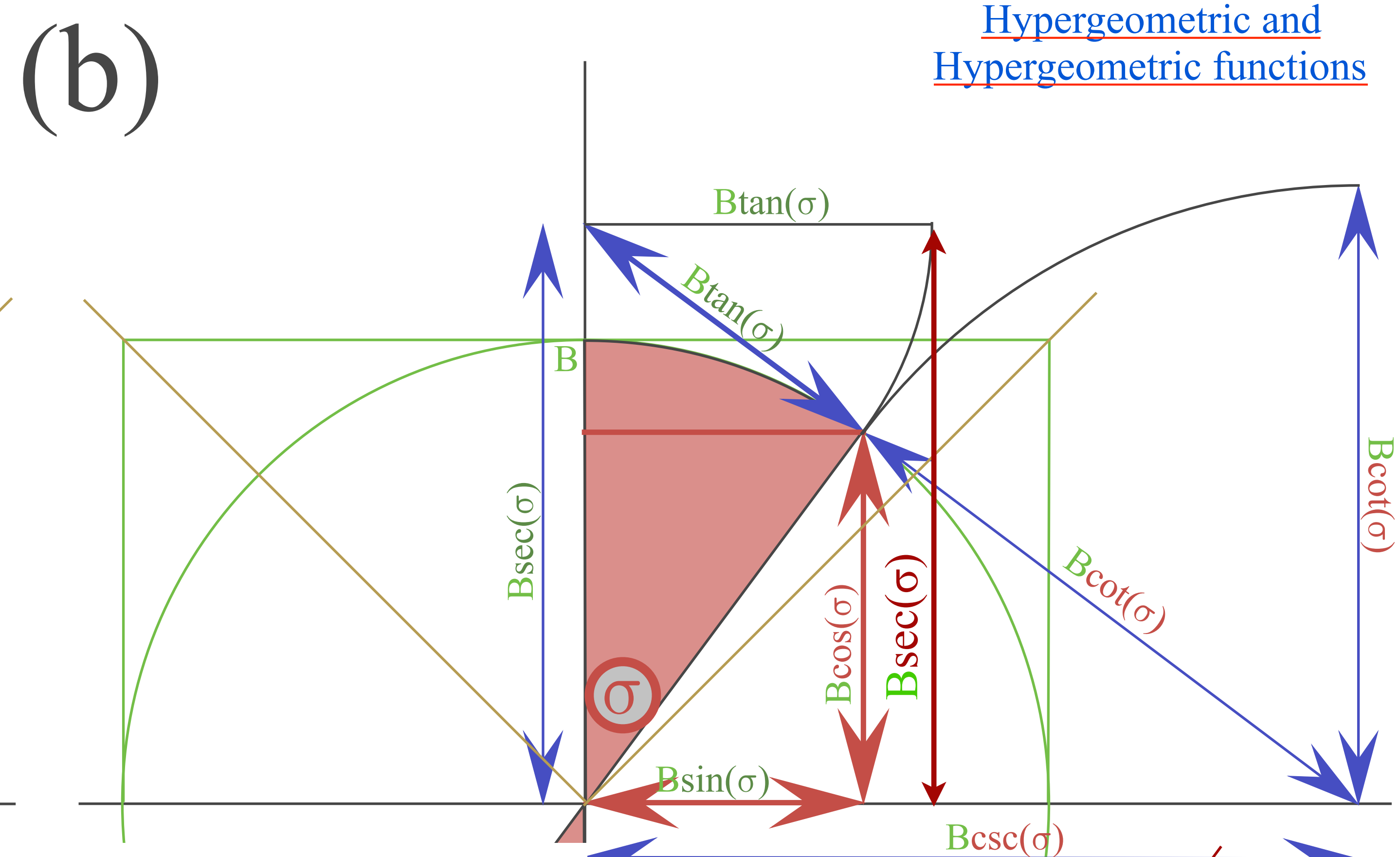
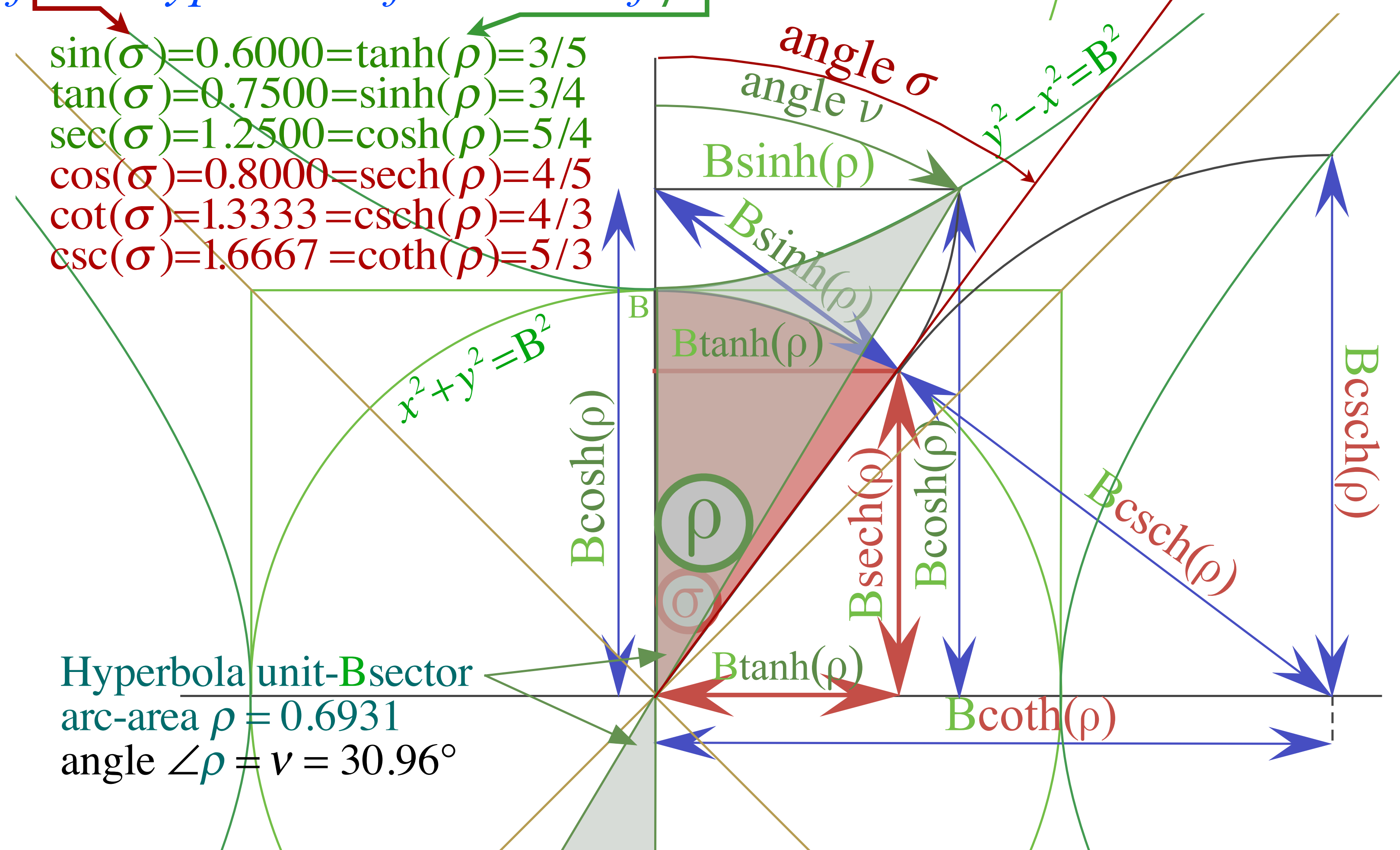




Equating circular functions of σ to hyperbolic functions of ρ

sin(σ)=0.6000=tanh(ρ)=3/5
tan(σ)=0.7500=sinh(ρ)=3/4
sec(σ)=1.2500=cosh(ρ)=5/4
cos(σ)=0.8000=sech(ρ)=4/5
cot(σ)=1.3333=csch(ρ)=4/3
csc(σ)=1.6667=coth(ρ)=5/3



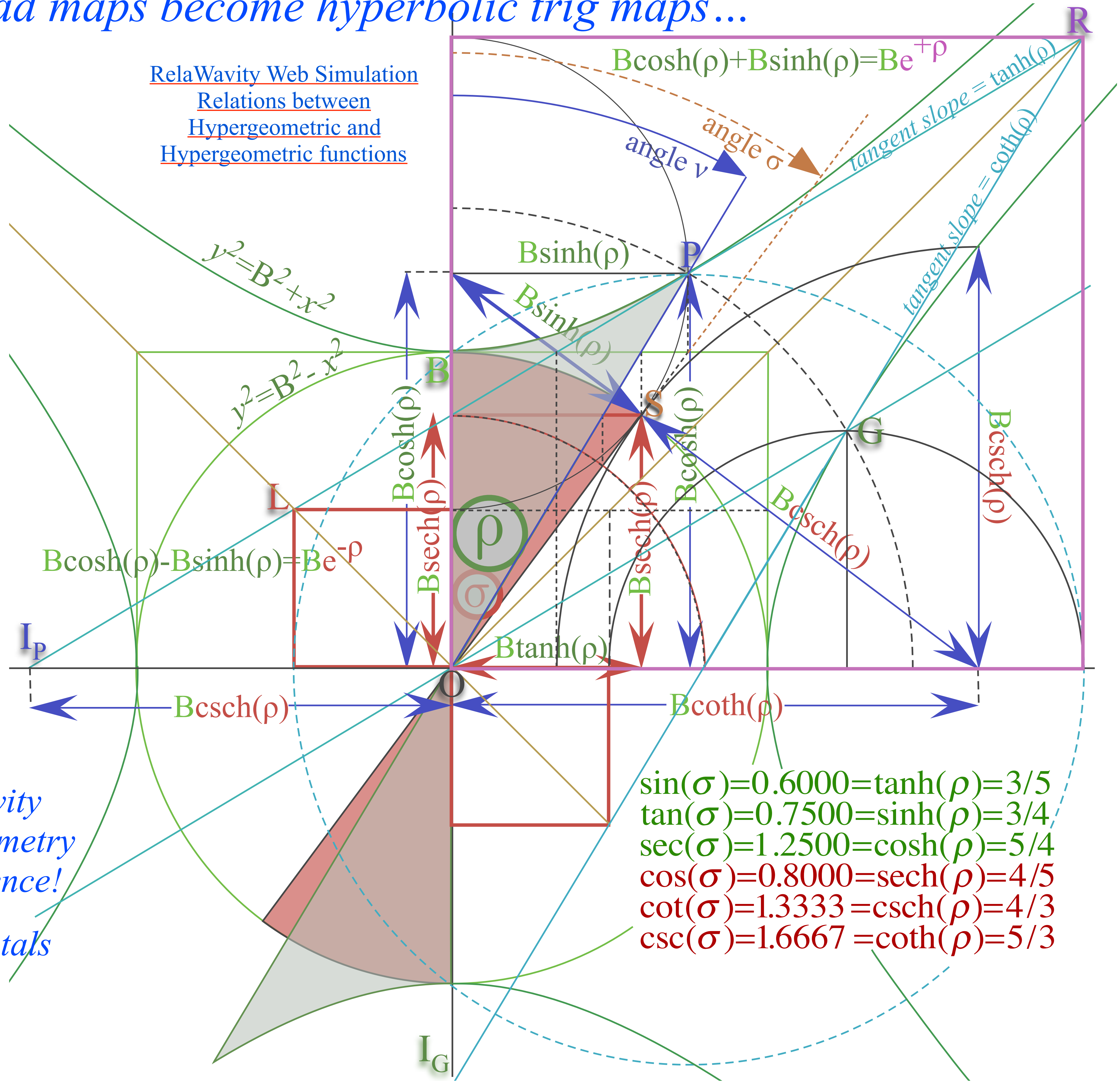
AMOP Ch.0 article p.9.

All this physics of relativity is mostly simple trigonometry of optical wave interference!

And, it derives fundamentals of quantum theory, too!

Trigonometric road maps become hyperbolic trig maps…

Need to see how trig road map matches the physical map on following page.



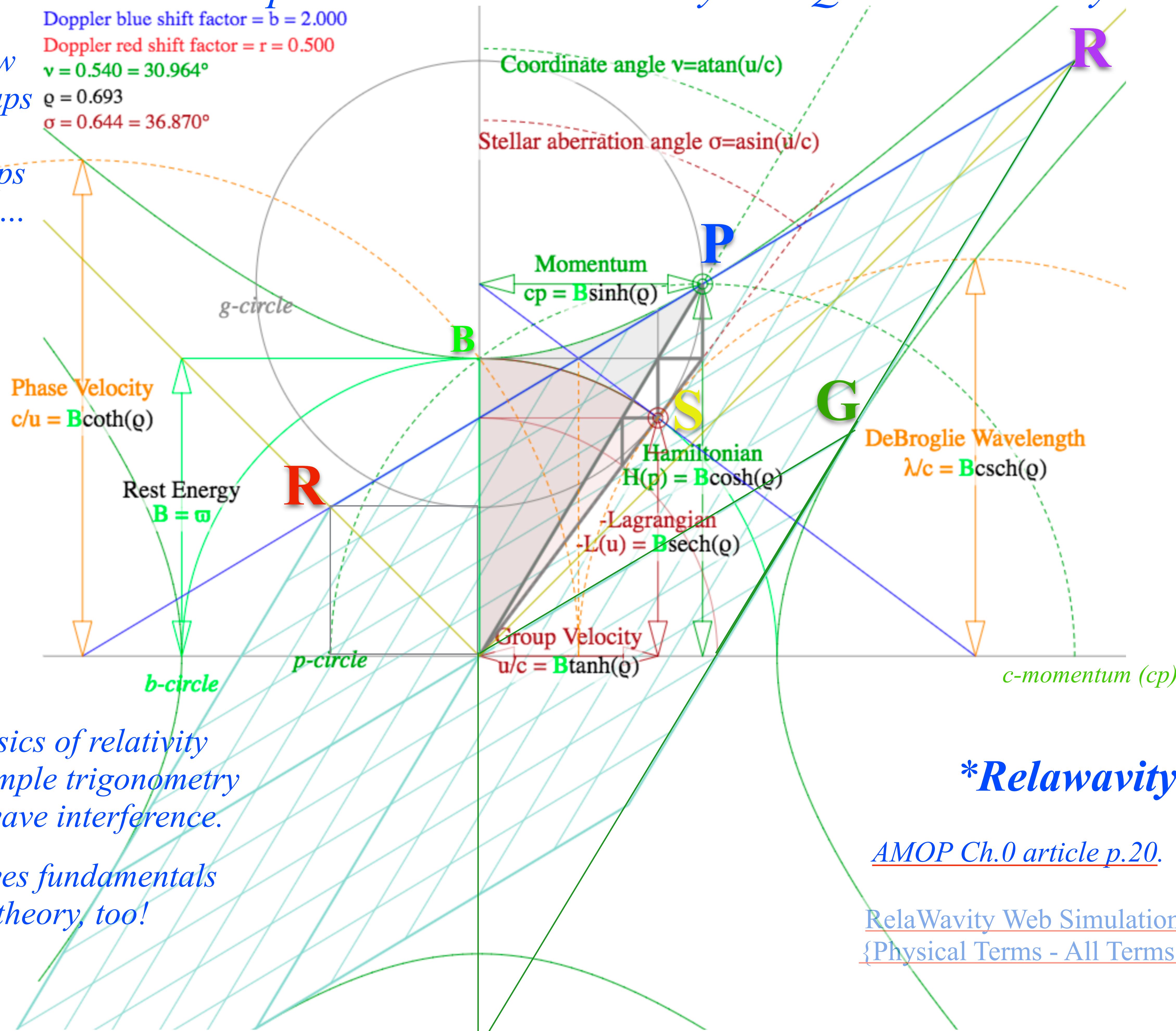
AMOP Ch.0 article p.10.

All this physics of relativity is mostly simple trigonometry of optical wave interference!

And, it derives fundamentals of quantum theory, too!

*Trigonometric road maps…. to Relativity and Quantum Theory**

Need to show trig road maps can match physical maps like this one…



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***Relawavity**

AMOP Ch.0 article p.20.

RelaWavity Web Simulation
{Physical Terms - All Terms}

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Hyper-Trigonometric *Relawavity* geometry and Euler exponential algebra ←

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Hyper-Trigonometric algebra easily derives *Circular*-Trigonometric-algebra

Exponential derived by infinite-n-compounding limit of the interest rate-r formula.

$$e^{rt} = \lim_{n\to\infty}\left(1+\frac{rt}{n}\right)^n$$

Infinite-n limit of binomial series is an exponential power-p series of $(rt)^p$ with $1/p!$ coefficients.

$$e^{rt} = 1 + rt + \frac{(rt)^2}{2} + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \ldots$$

$$e^{-rt} = 1 - rt + \frac{(rt)^2}{2} - \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots$$

Half-sum and half difference of $e^{\pm rt}$ series define the hyperbolic cosine (cosh(rt)) and sine (sinh(rt)).

$$\frac{e^{+rt}+e^{-rt}}{2} = 1 + \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt)$$

$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt)$$

Hyper-Trig

coshρ and sinhρ

Hyper-Trigonometric algebra easily derives *Circular*-Trigonometric-algebra

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$$e^{-rt} = 1 - rt + \frac{(rt)^2}{2} - \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt) - \sinh(rt)$$

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$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt)$$

Hyper-Trig

coshρ and sinhρ

Replace rate *r* with imaginary rate *ir* and $i=\sqrt{-1}$ powers $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, $i^5=i$, $i^6=-1$, $i^7=-i$,…

Then *hyper*-sine-cosine becomes the *circular*-sine-cosine.

$$\frac{e^{+i\,rt}+e^{-i\,rt}}{2} = 1 - \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cos rt$$

$$\frac{e^{+i\,rt}-e^{-i\,rt}}{2} = i\,rt - i\frac{(rt)^3}{2\cdot 3} + i\frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} - \ldots = i\sin rt$$

Circular-Trig

cosσ and sinσ

Sum and difference of this pair gives the Euler-DeMoivre relations of exponentials vs trig-functions.

$$e^{+i\sigma} = \cos\sigma + i\sin\sigma\ , \qquad e^{-i\sigma} = \cos\sigma - i\sin\sigma .$$

$$e^{+\rho} = \cosh\rho + \sinh\rho\ , \qquad e^{-\sigma} = \cosh\rho - \sinh\rho .$$

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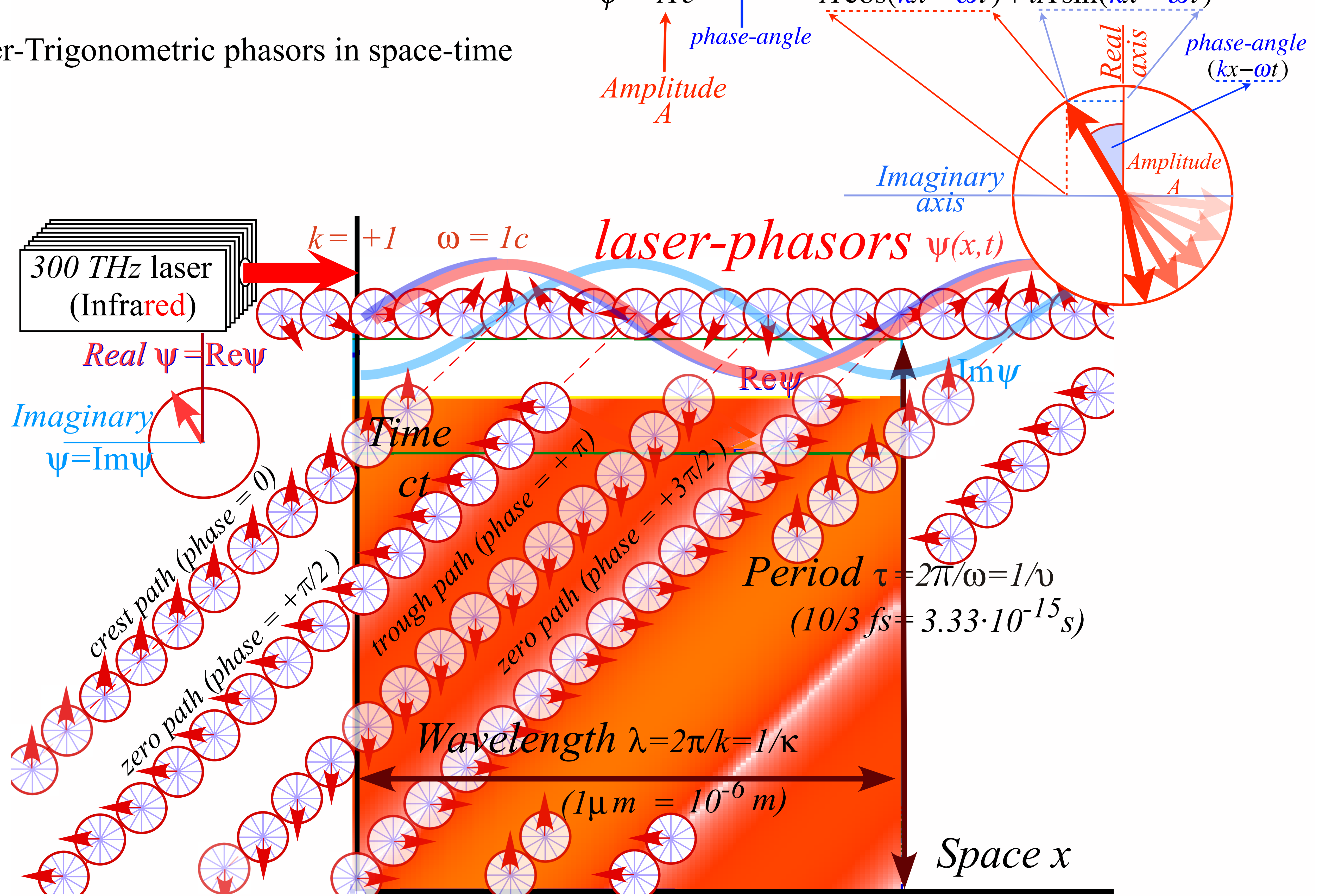
1CW Laser-phasor wave function

$$\psi = A \cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

Hyper-Trigonometric phasors in space-time



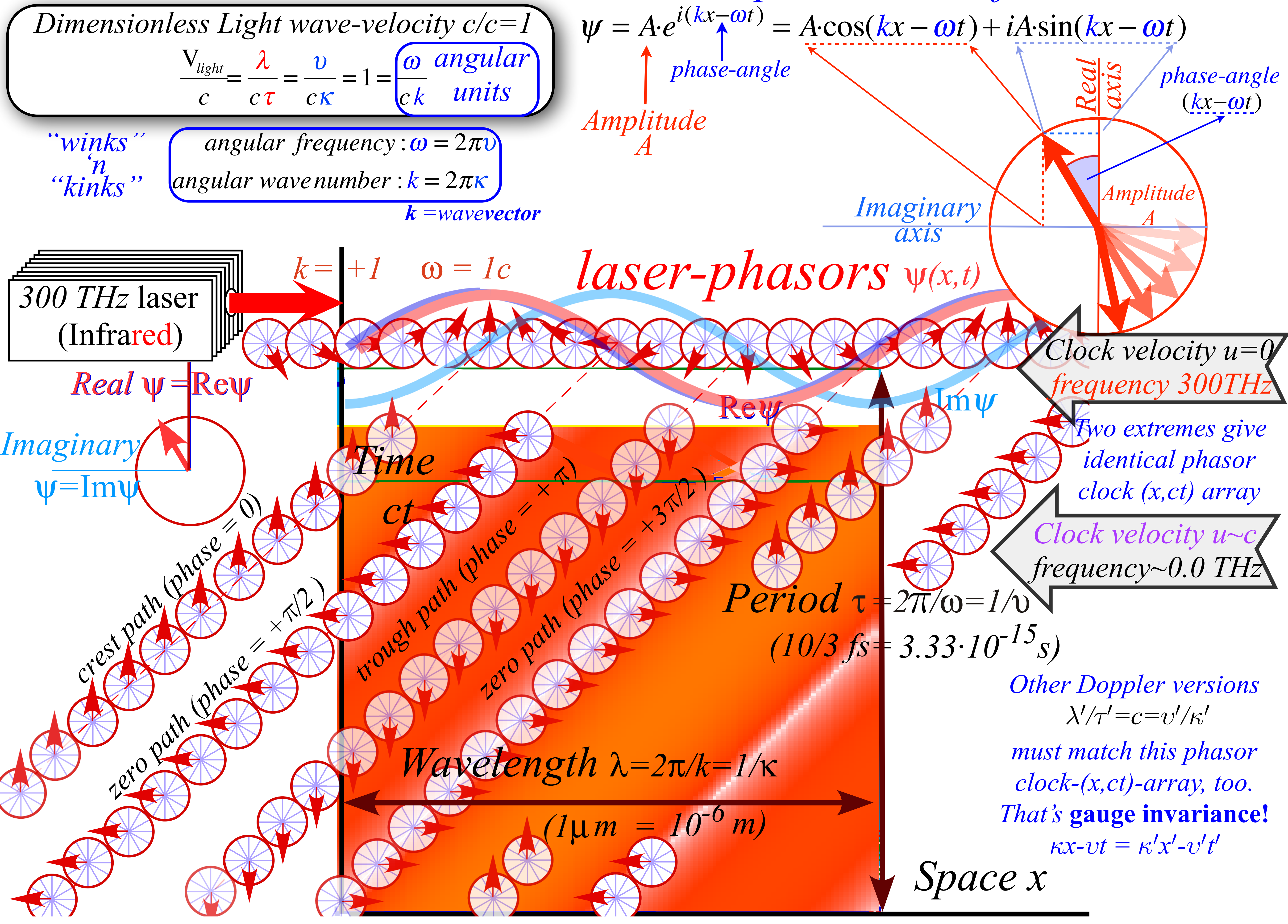
(a) Single-phasor plot of wave-function at (x,ct). (b) Array of phasors at many (x,ct)-points.

Pirelli Site *World of Clocks* Animations

24 hr. clock

12 hr. clock

1CW Laser-phasor wave function



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The "Keyboard of the gods" : per-space-per-time plot *versus* space-time Minkowski plot

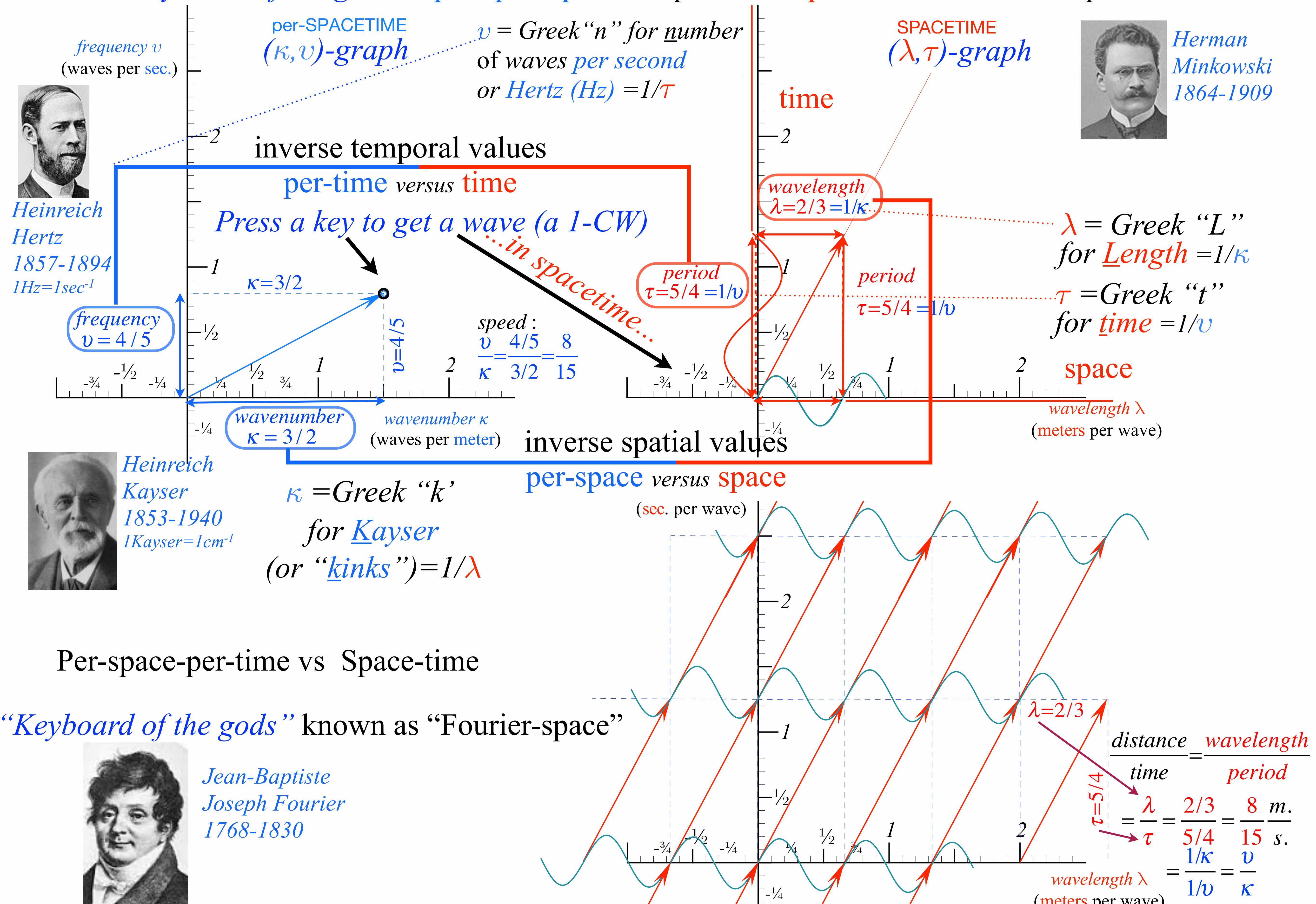
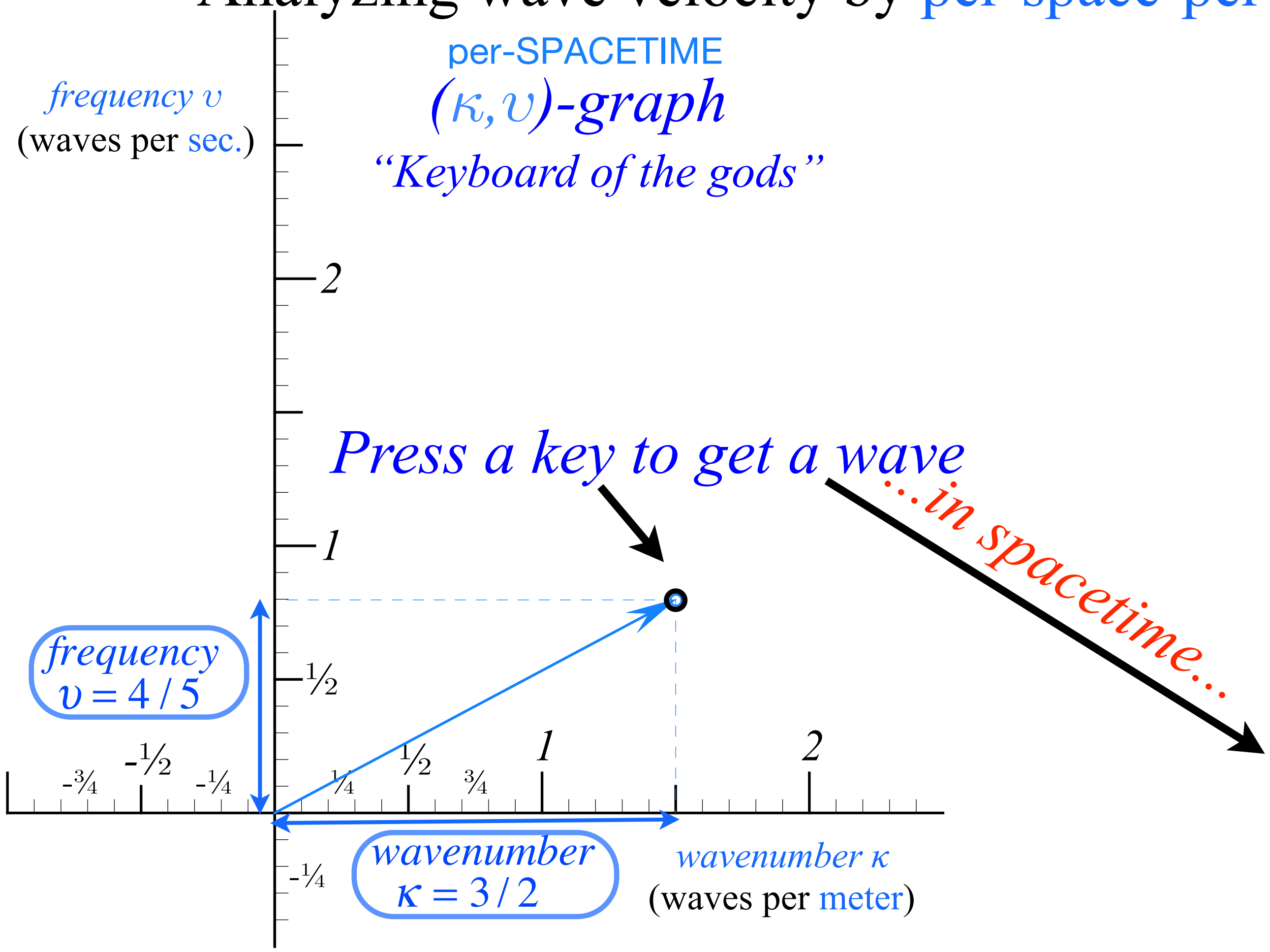


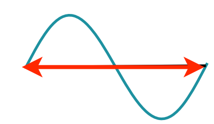
Fig. 5 Comparing a wave point in Kaiser-Hertz per-space-time to its Minkowski space-time view.

Analyzing wave velocity by per-space-per-time *and* space-time graphs



SPACETIME
(λ,τ)-graph

Herman Minkowski 1864-1909



"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

- *How to understand waves and wave parameters*

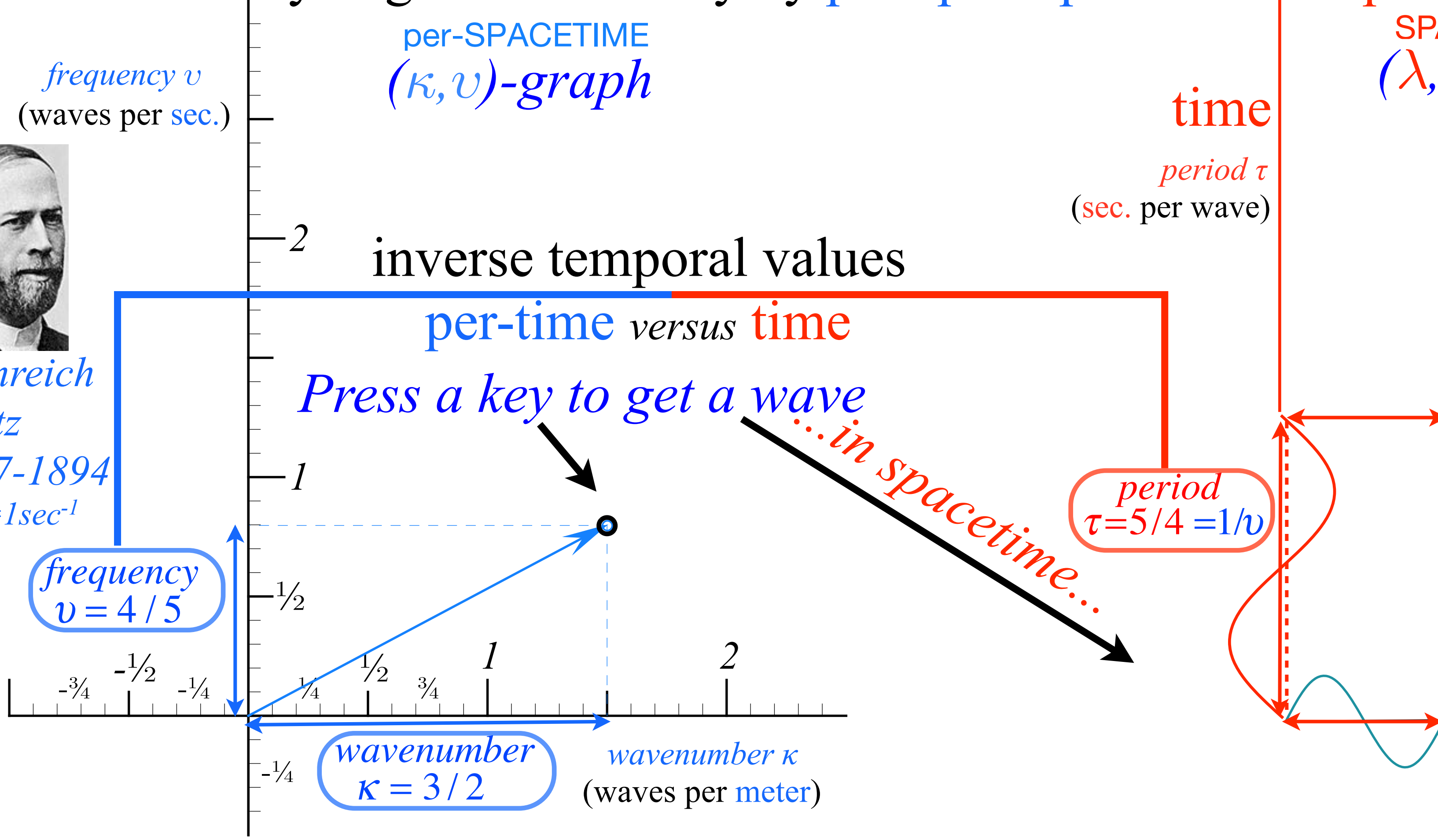
wave frequency υ	*wave period* τ
wavenumber κ	*wavelength* λ

RelaWavity Web Simulation
Keyboard of the Gods (per-Time vs per-Space)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Heinreich Hertz 1857-1894
1Hz=1sec^{-1}

Herman Minkowski 1864-1909



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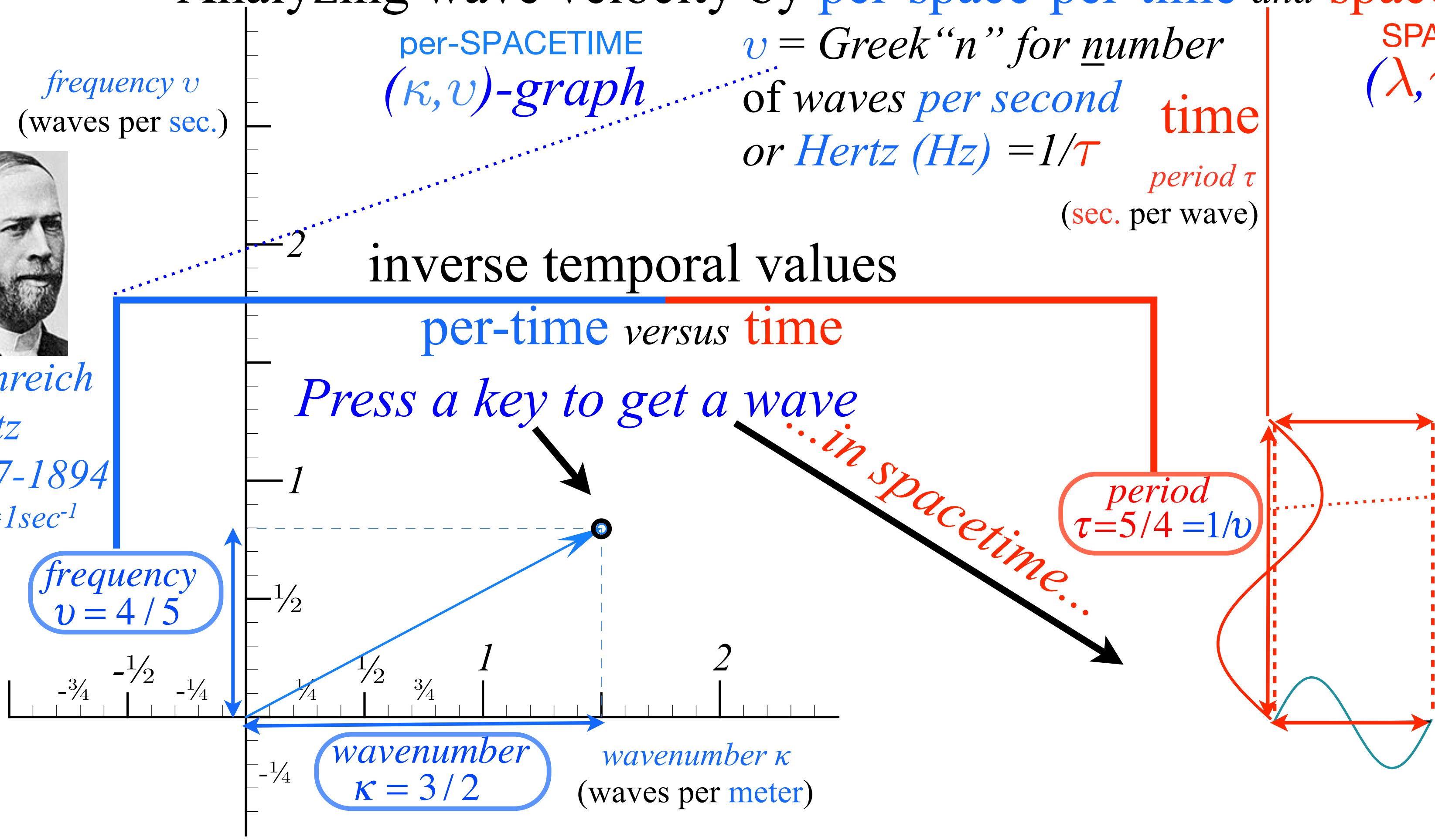
Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909

Heinreich Hertz 1857-1894
1Hz=1sec⁻¹



τ =Greek "t"
for time =1/υ

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wave frequency υ	*wave period τ*
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[RelaWavity Web Simulation](#)
[Keyboard of the Gods (Dual Plot #1)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Herman Minkowski 1864-1909

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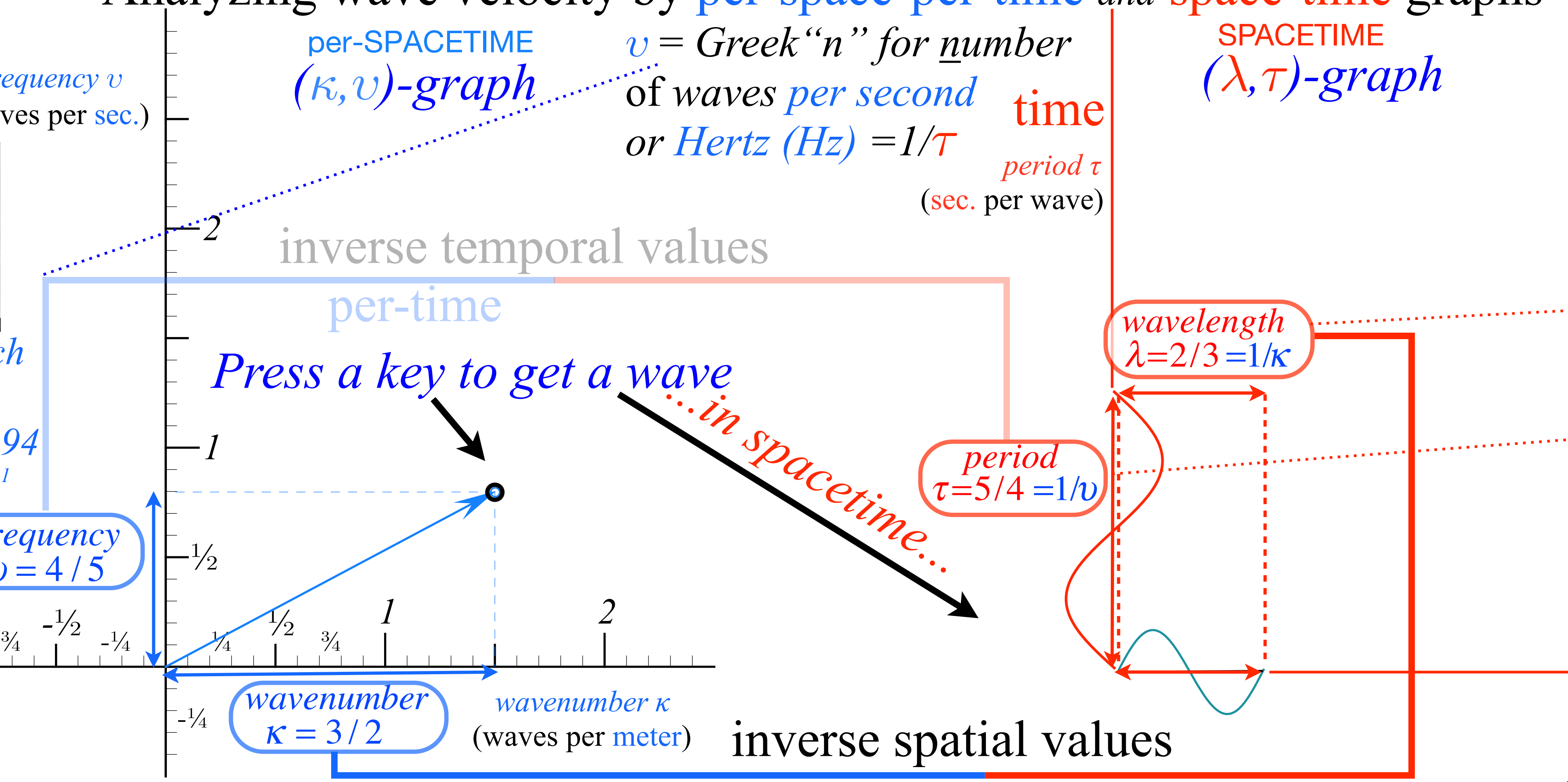

κ =Greek "k"
for Kayser
(or "kinks")=1/λ

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•*How to understand waves and wave parameters*

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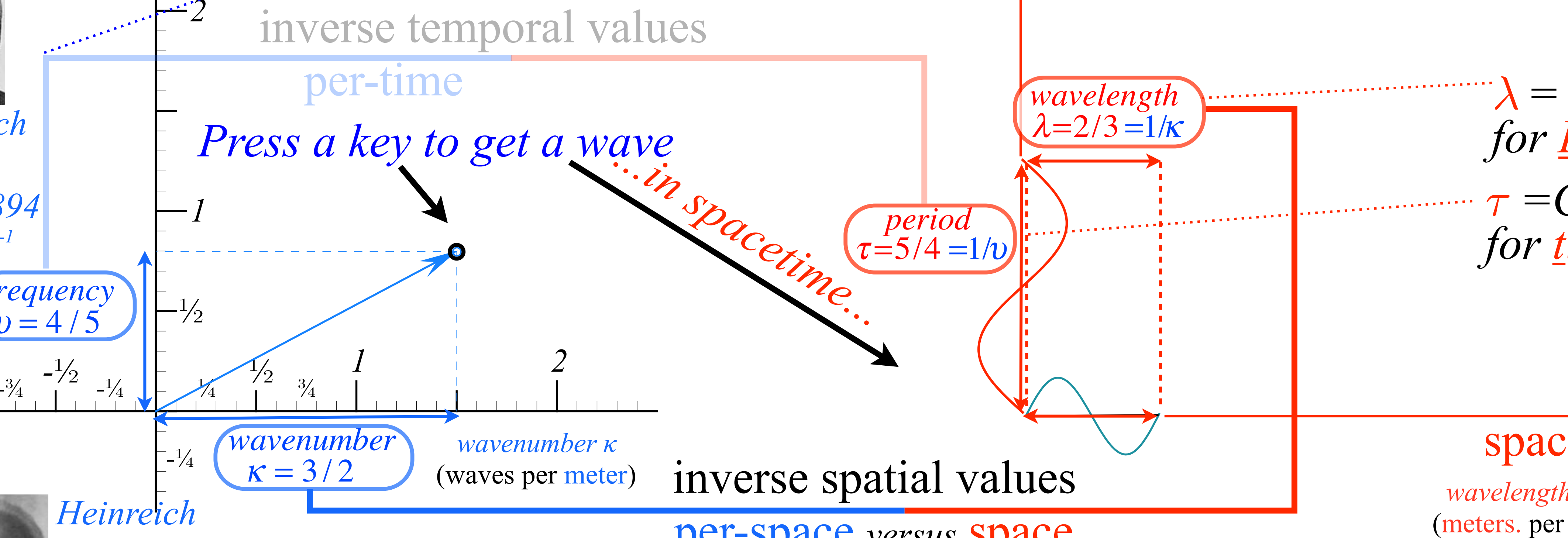

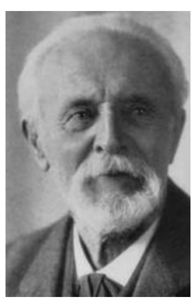

Heinreich Kayser 1853-1940

1Kayser=1cm^{-1}

κ =Greek "k"

for Kayser

(or "kinks")=1/λ

"Keyboard of the gods" is known as "Fourier-space"

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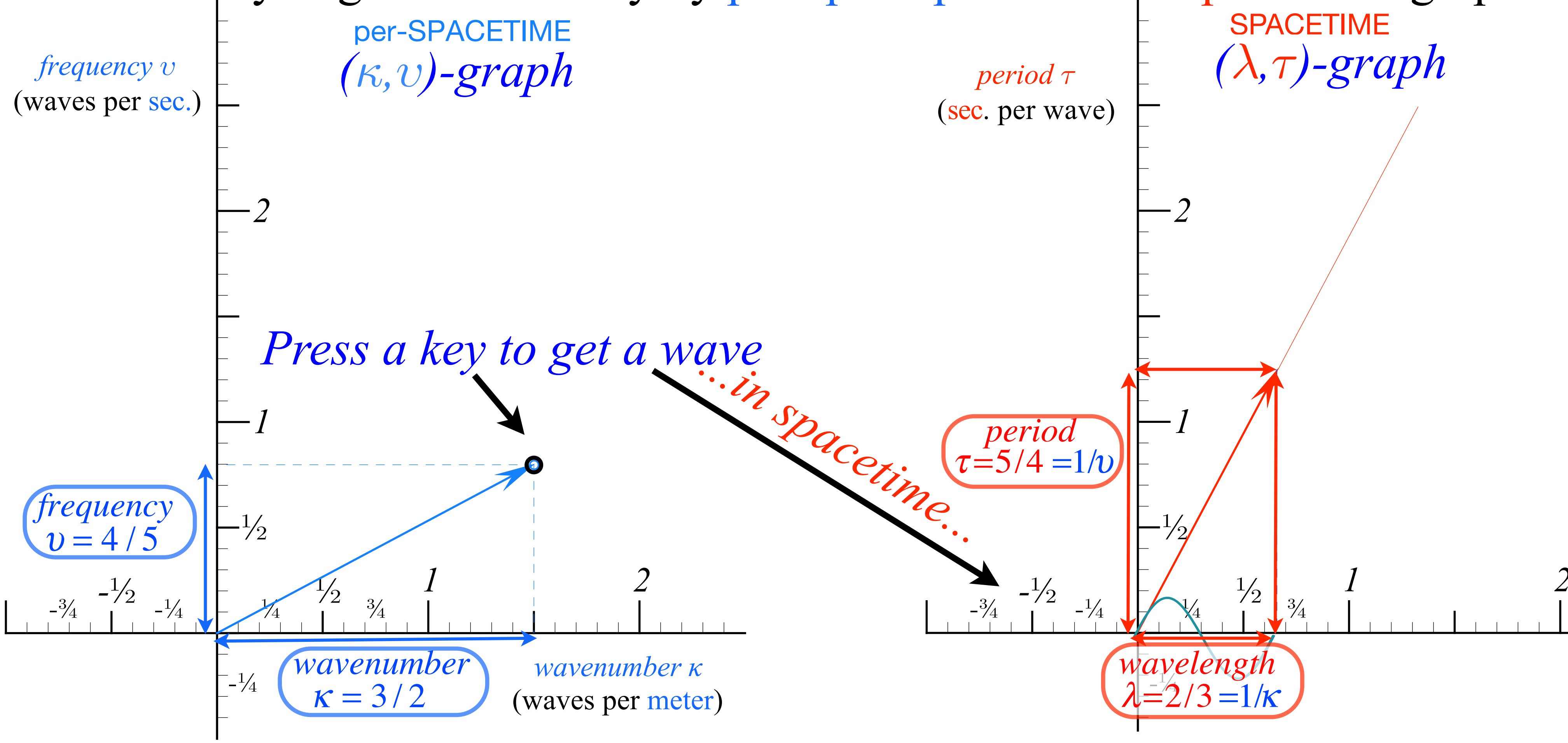
•How to understand waves and wave parameters

wave frequency υ	*wave period τ*
wavenumber κ	*wavelength λ*

RelaWavity Web Simulation

Keyboard of the Gods (Dual Plot #1)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

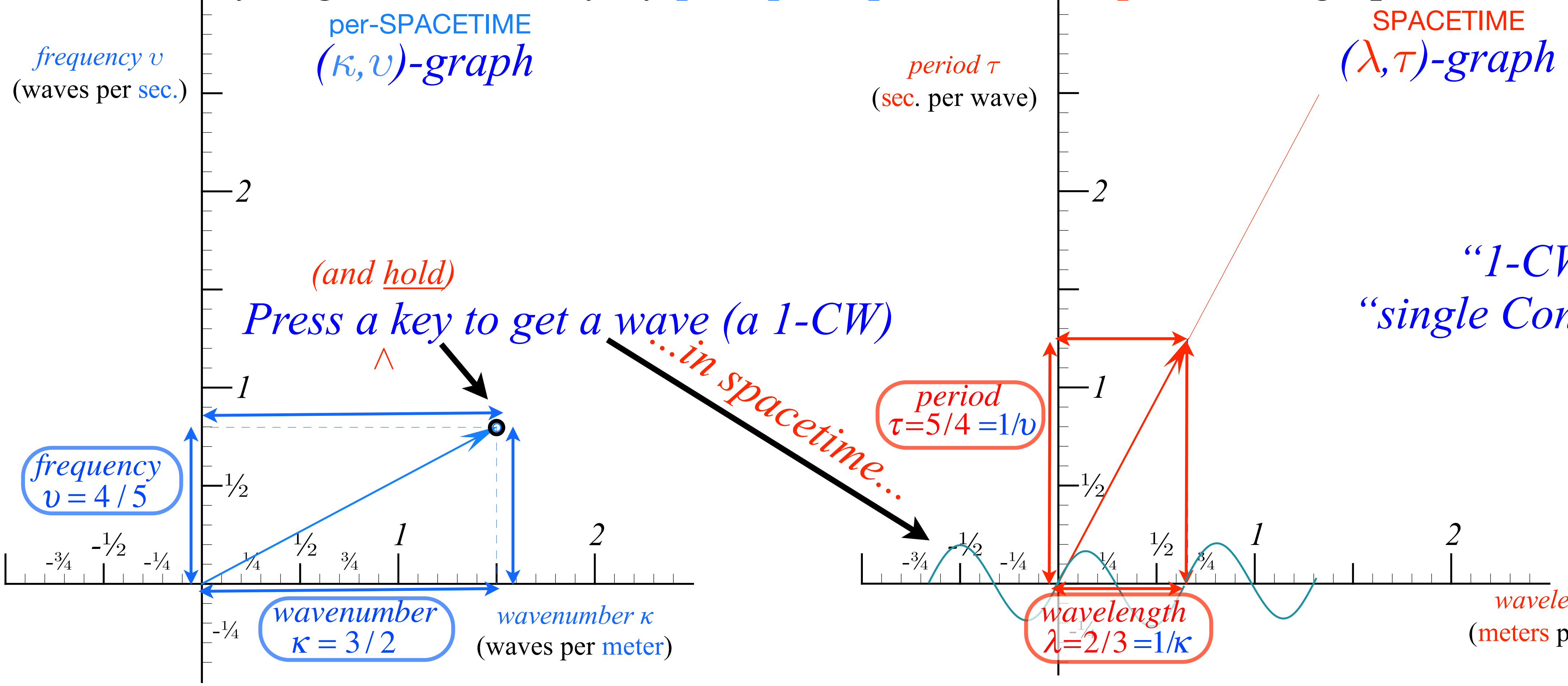


"Keyboard of the gods" is known as "Fourier-space"

•How to understand waves and wave parameters

wave frequency υ	*wave period τ*
wavenumber κ	*wavelength λ*

Analyzing wave velocity by per-space-per-time *and* space-time graphs



Herman Minkowski 1864-1909

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•*How to understand waves and wave parameters*

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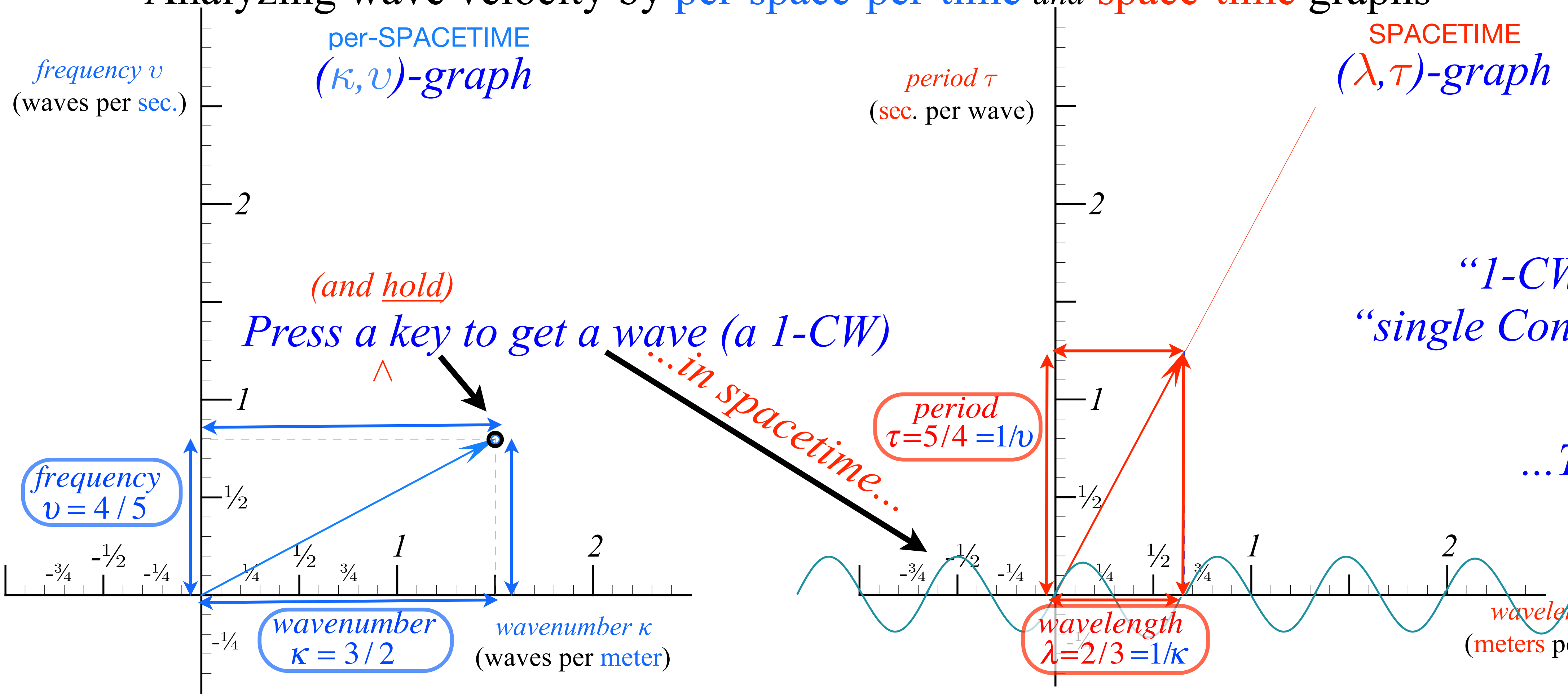
RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #2)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



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Jean-Baptiste
Joseph Fourier
1768-1830

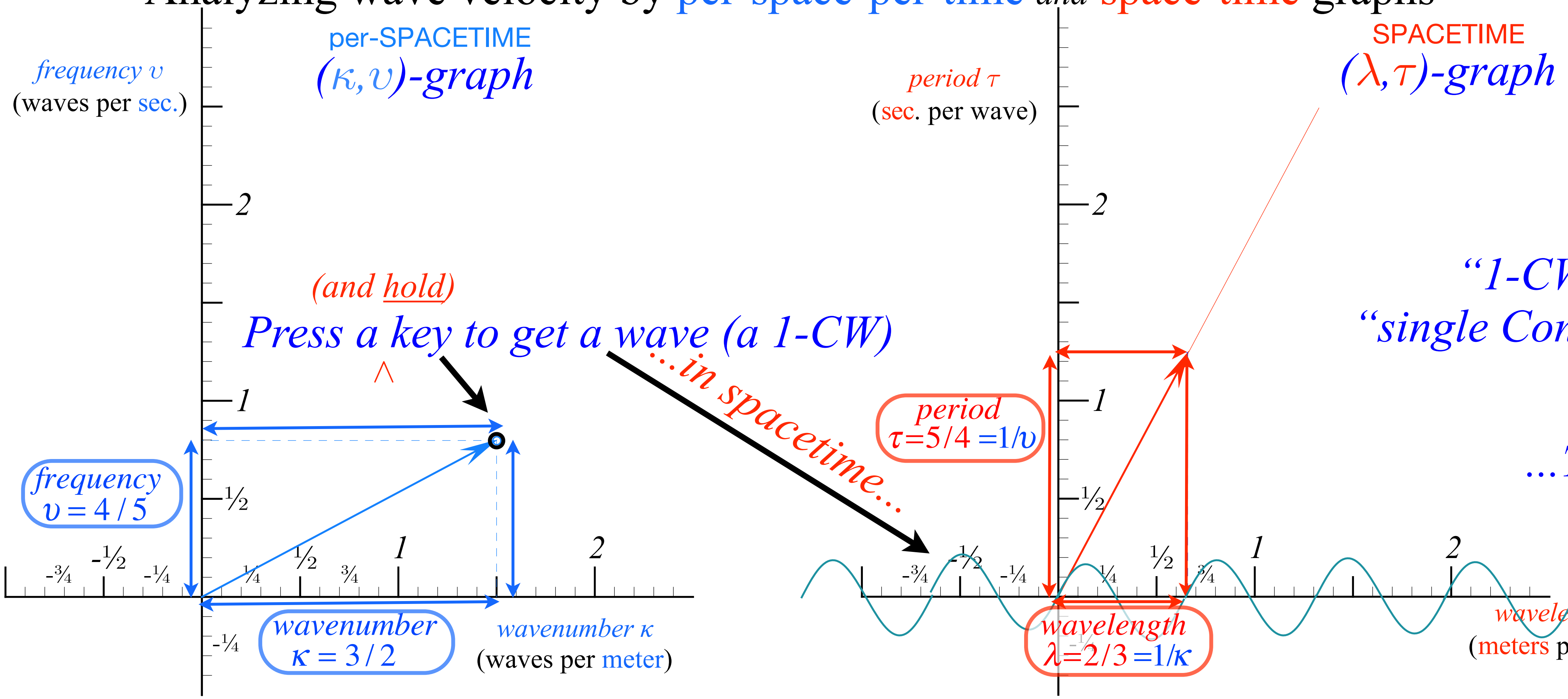
•How to understand waves and wave parameters

wave frequency υ — *wave period τ*
wavenumber κ — *wavelength λ*

[RelaWavity Web Simulation](#)
[Keyboard of the Gods (Dual Plot #3)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

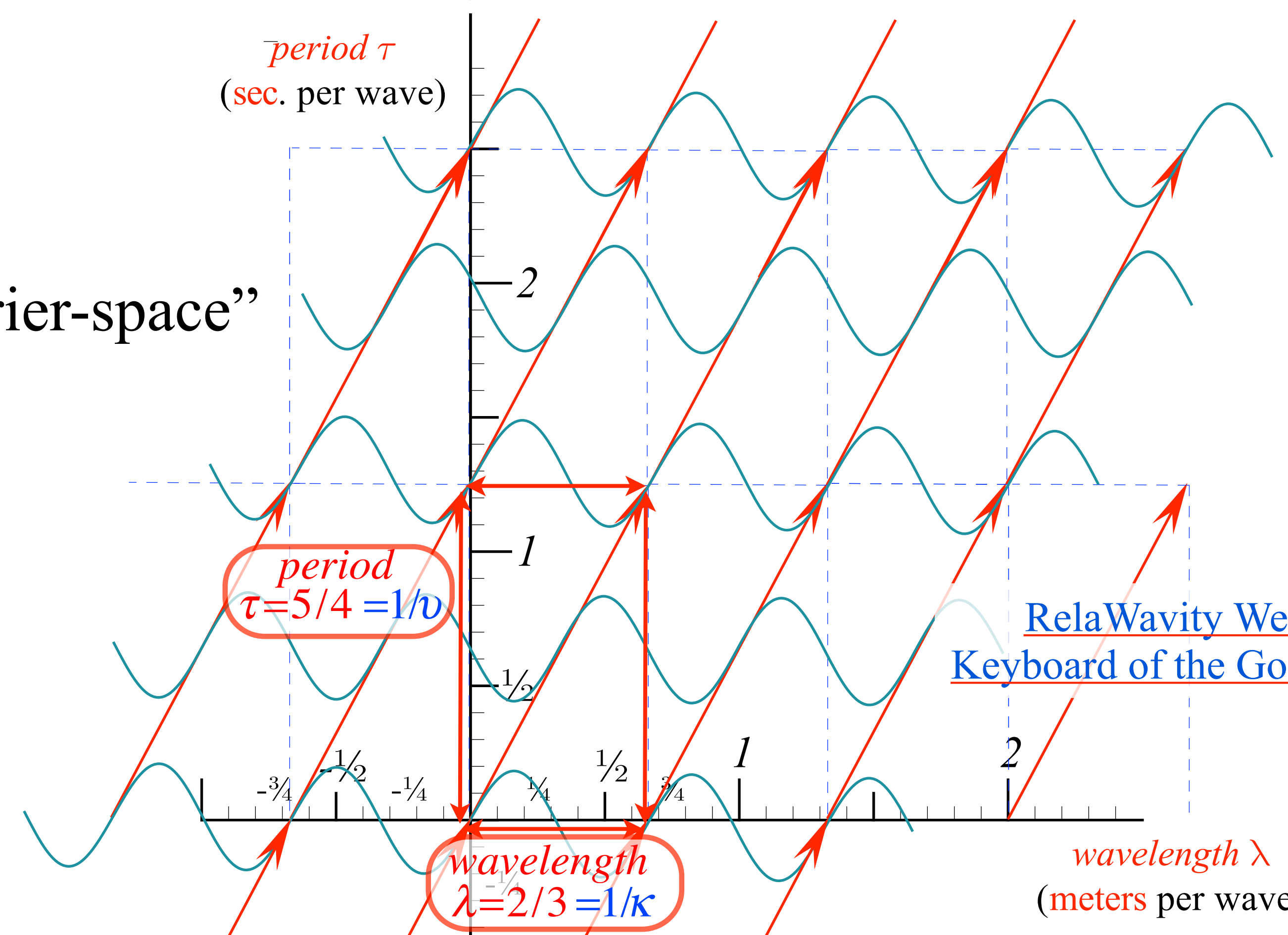
Herman Minkowski 1864-1909



"1-CW" means "single Continuous Wave"

...That "continues" everywhere...

...for all time...



RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot #7)

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Jean-Baptiste Joseph Fourier 1768-1830

- *How to understand waves and wave parameters*

wave frequency υ	*wave period τ*
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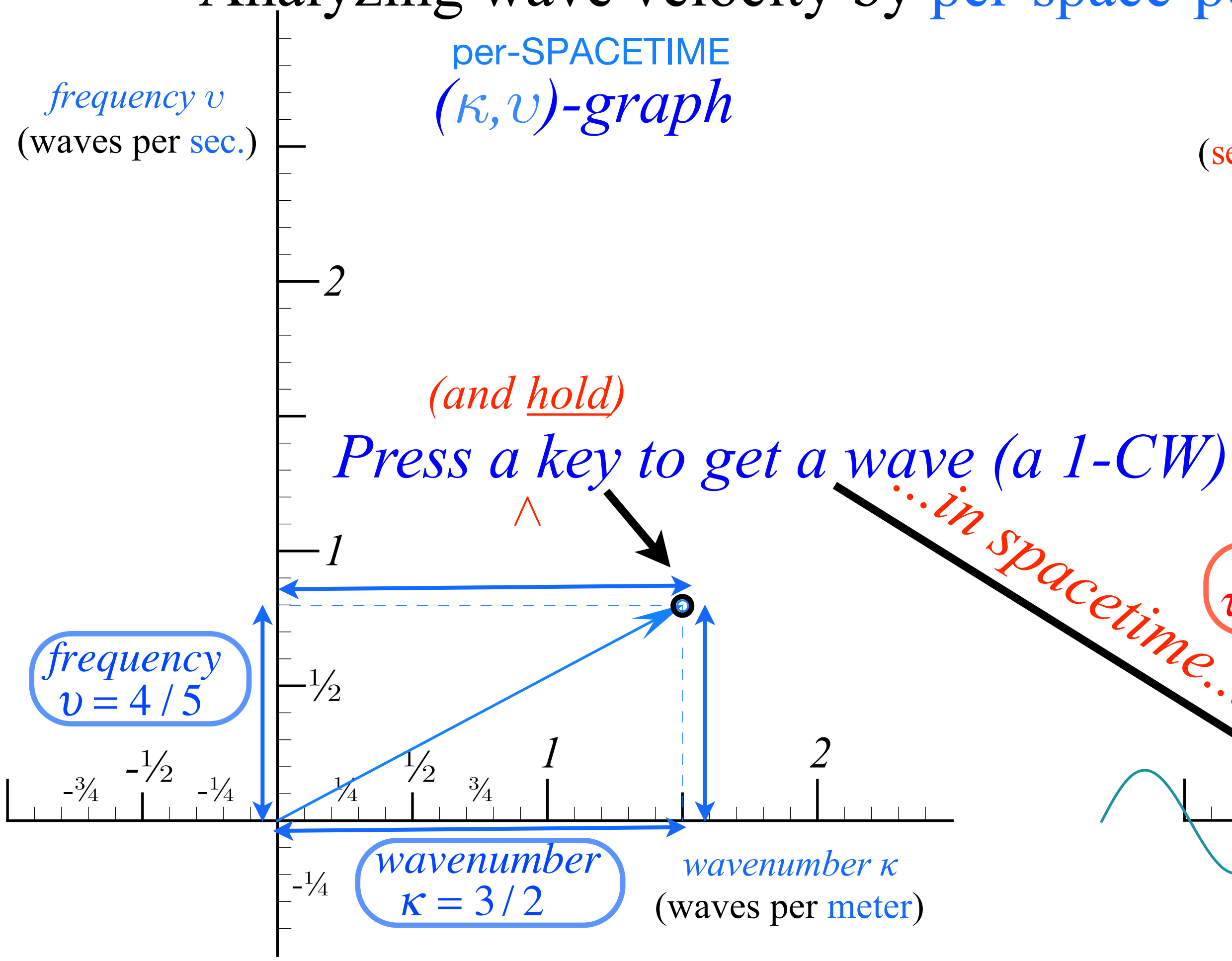

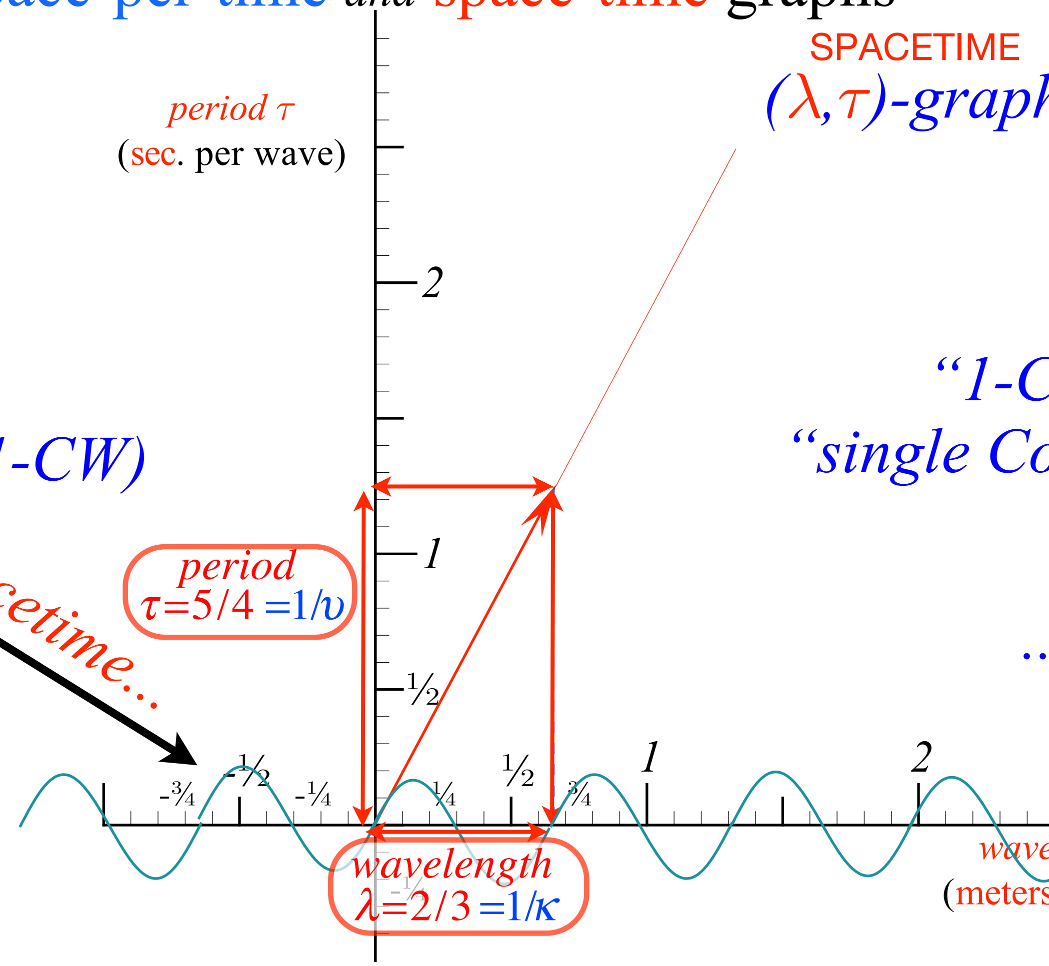

Herman Minkowski 1864-1909

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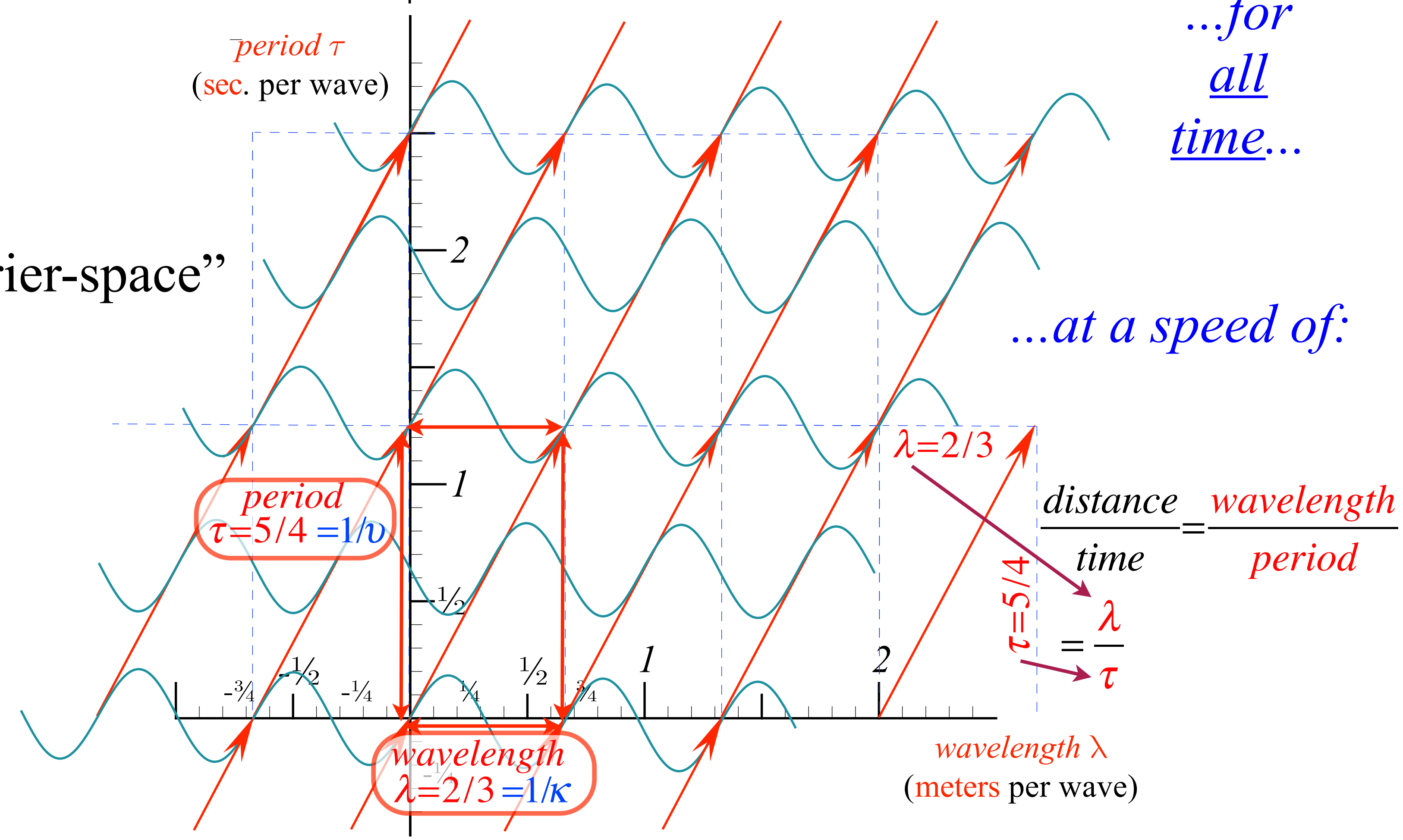
...at a speed of:



$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau}$$

"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

Analyzing wave velocity by per-space-per-time *and* space-time graphs

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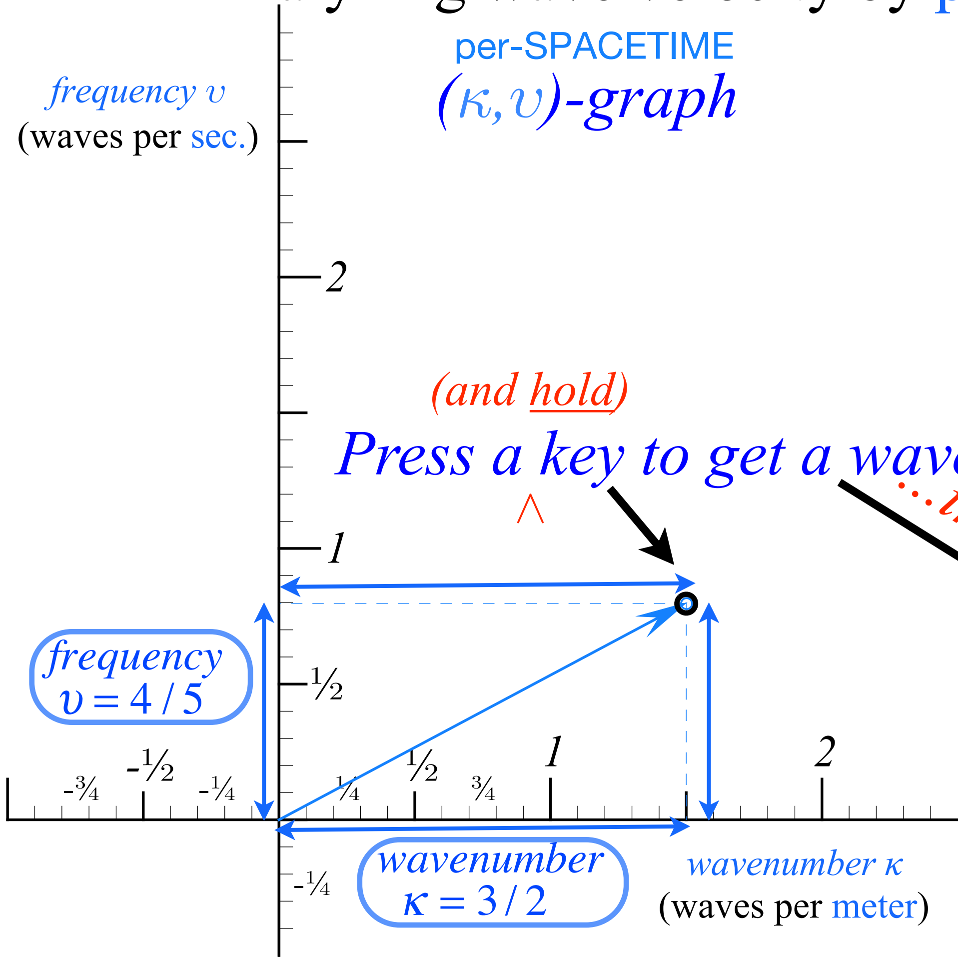

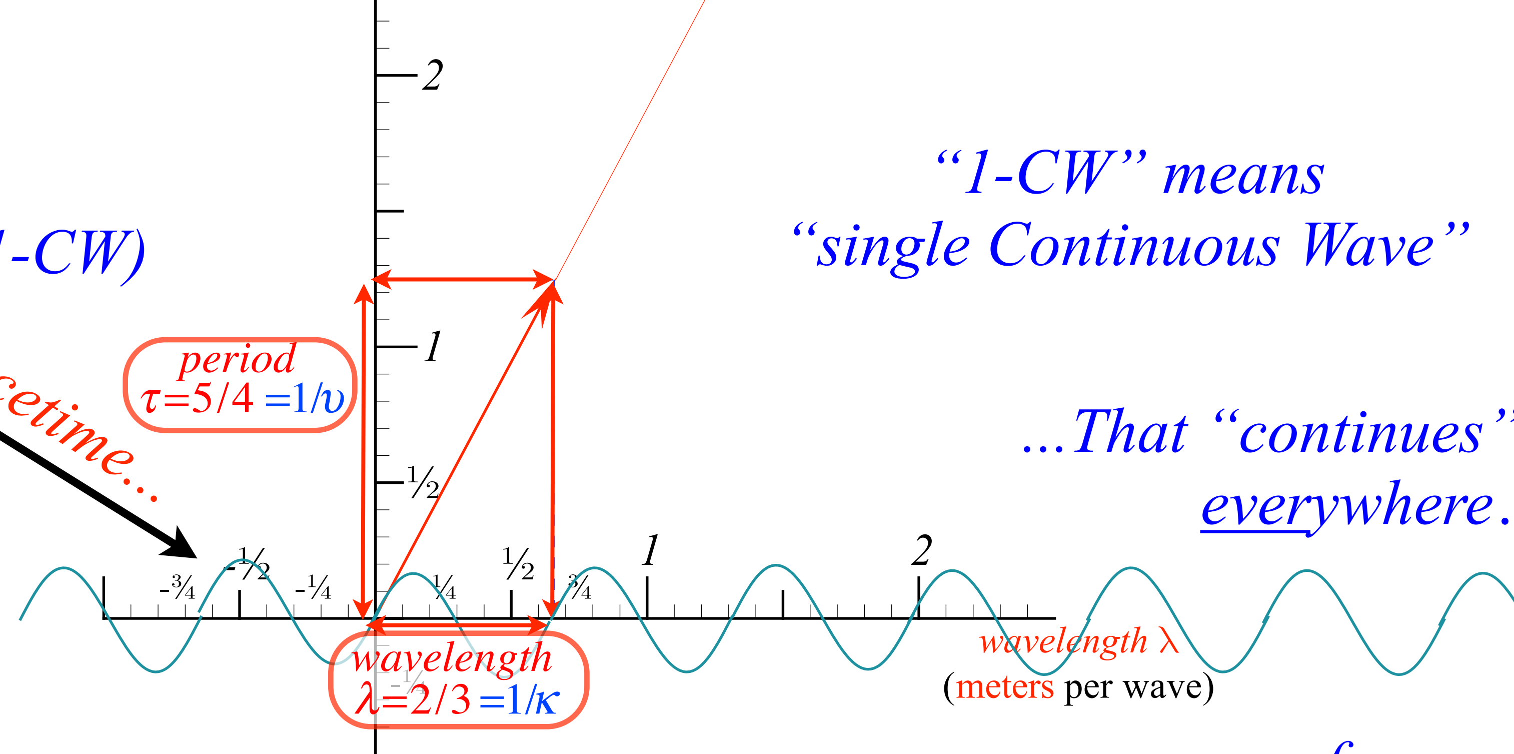

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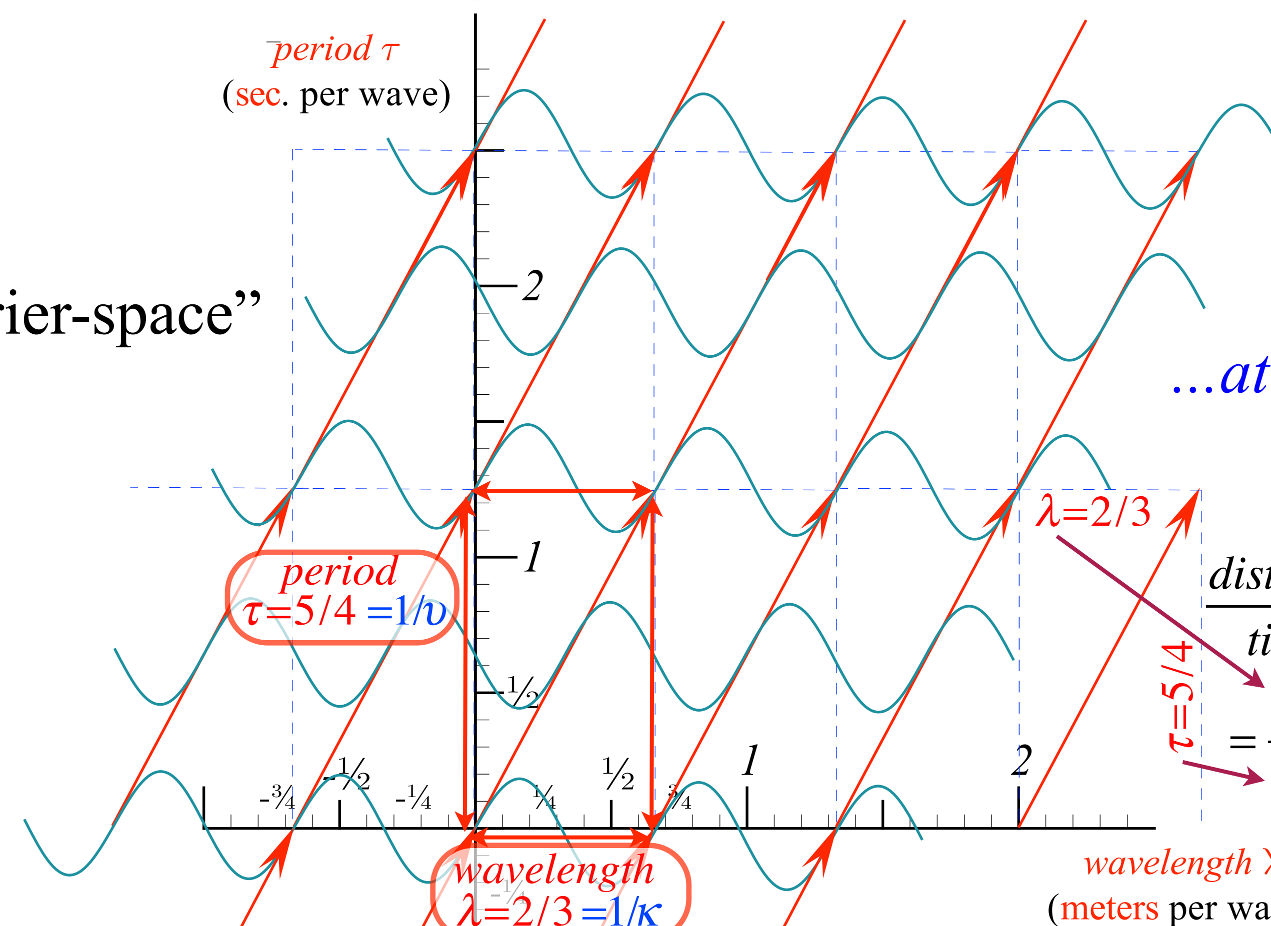

…for
all
time…

…at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\,\frac{m.}{s.}$$

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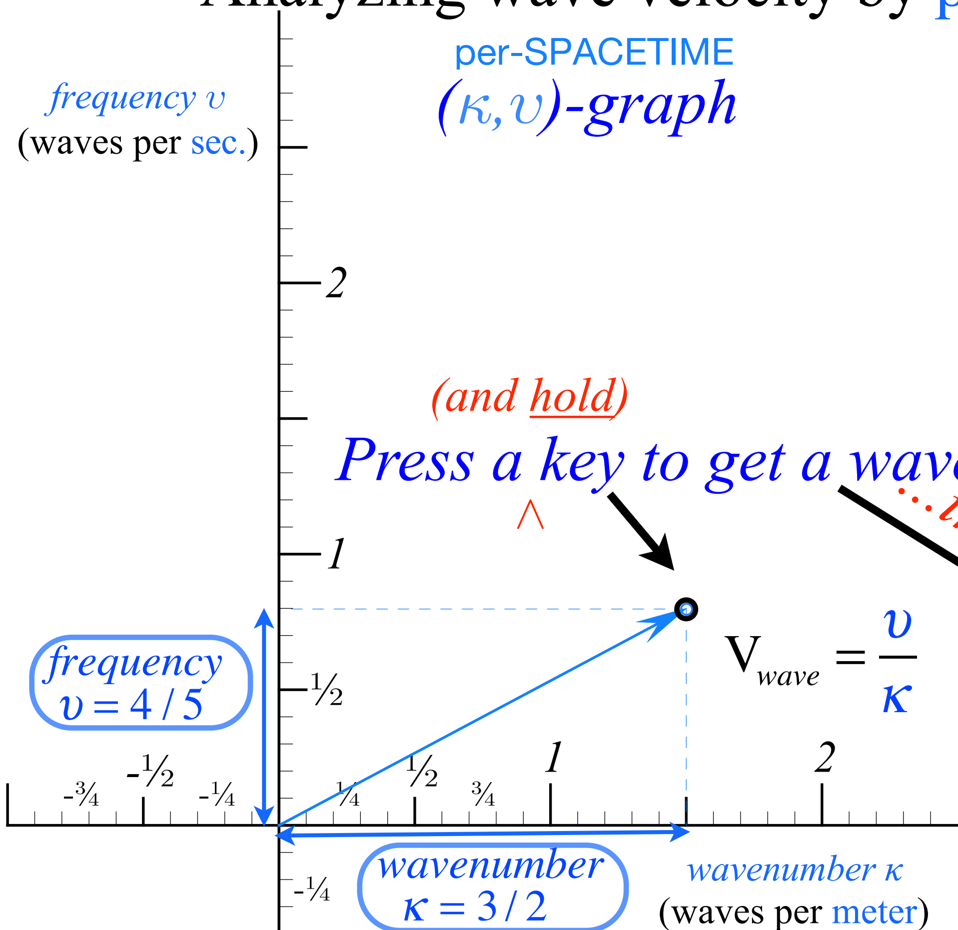
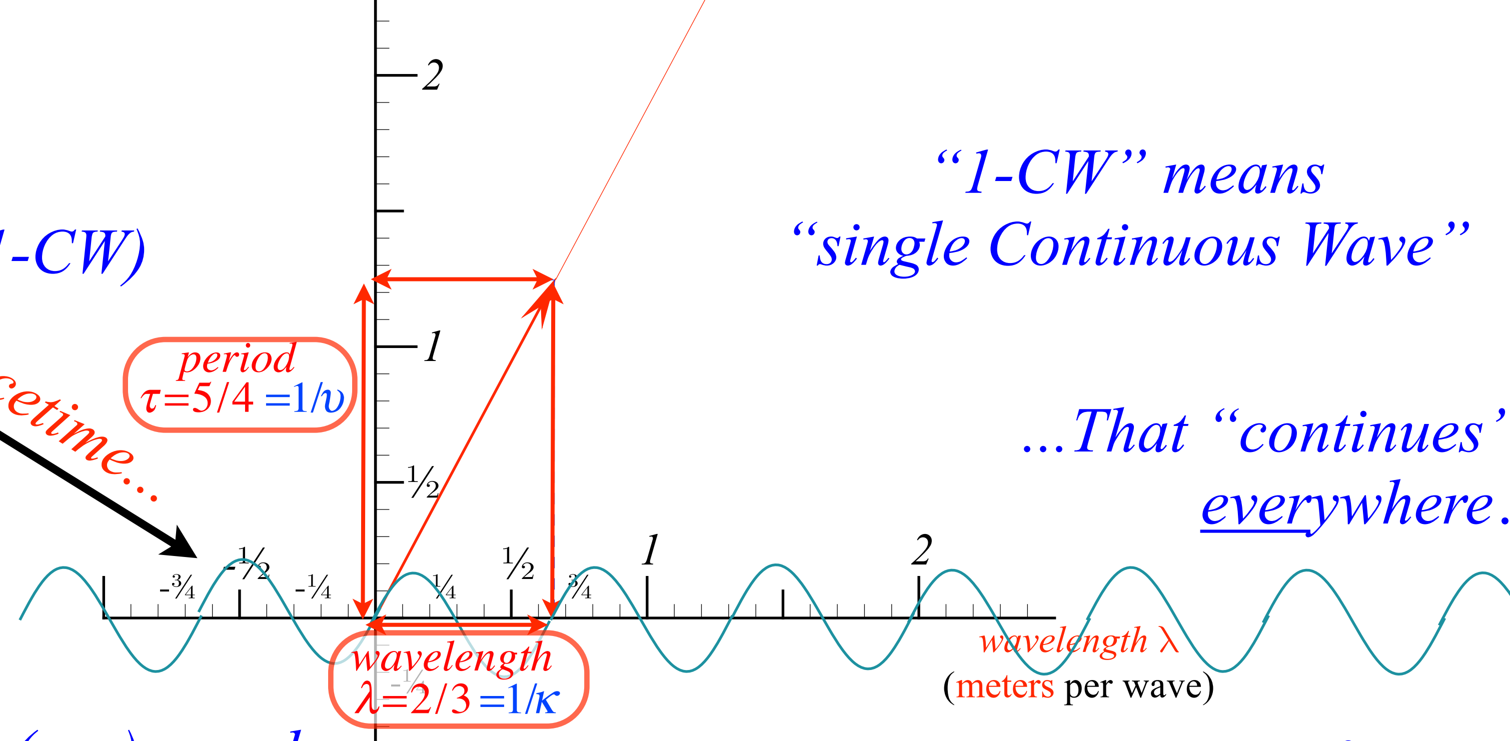
•How to understand waves
and
wave velocity V_{wave}

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

Analyzing wave velocity by per-space-per-time *and* space-time graphs



wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph



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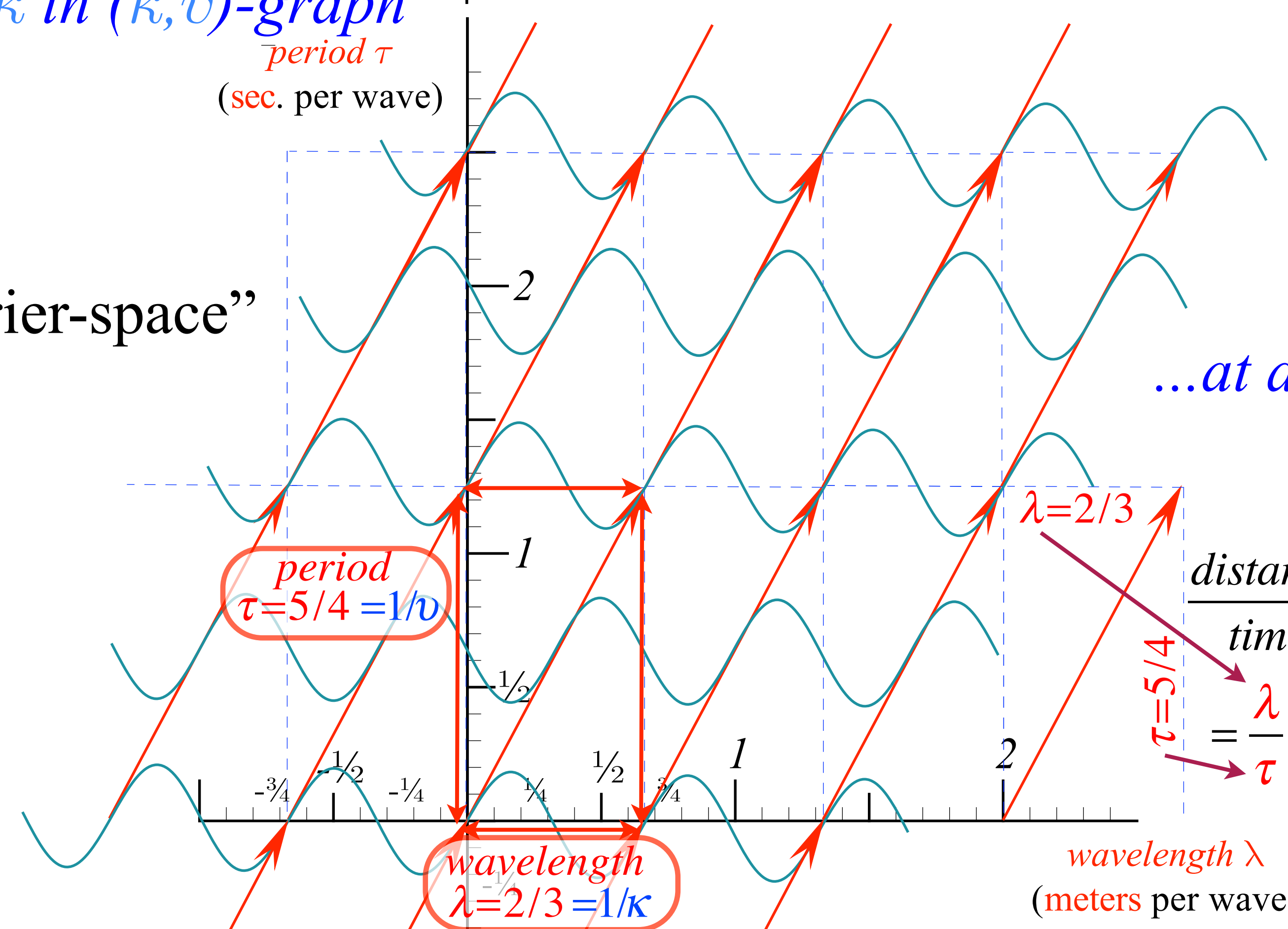

...at a speed of:

$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa}$$

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

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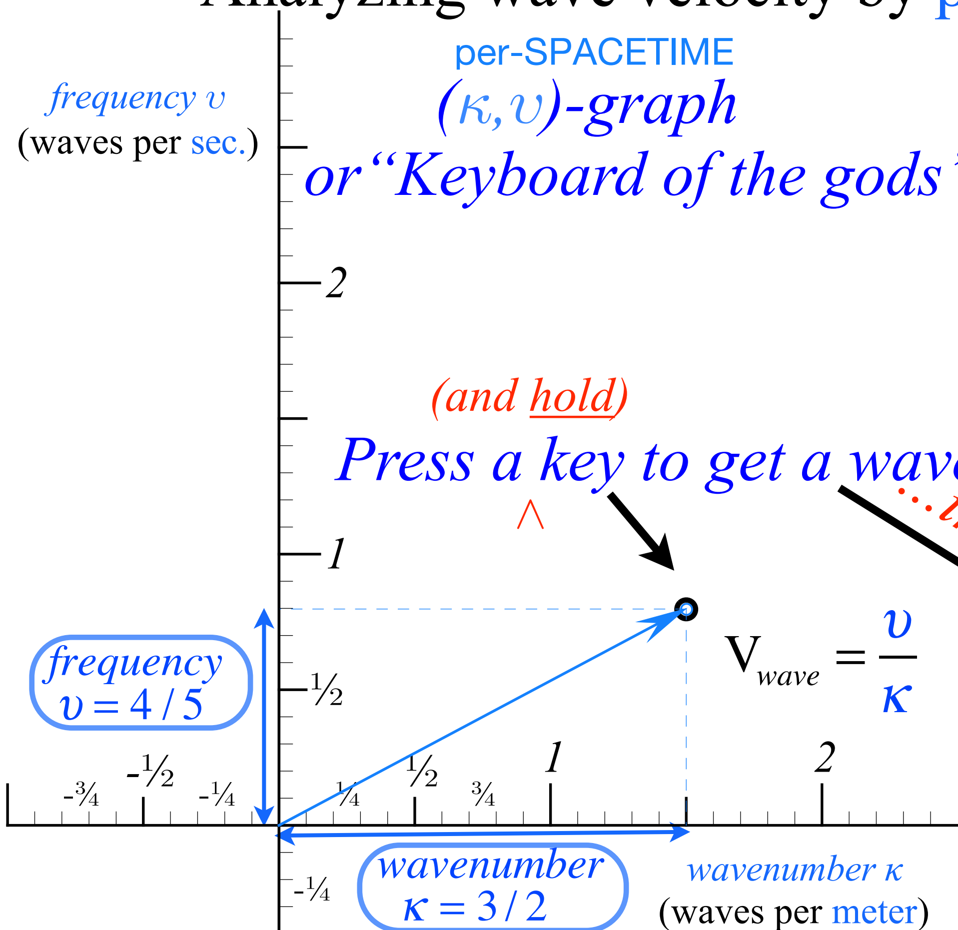
- *How to understand waves and wave velocity V_{wave}*

Analyzing wave velocity by per-space-per-time *and* space-time graphs



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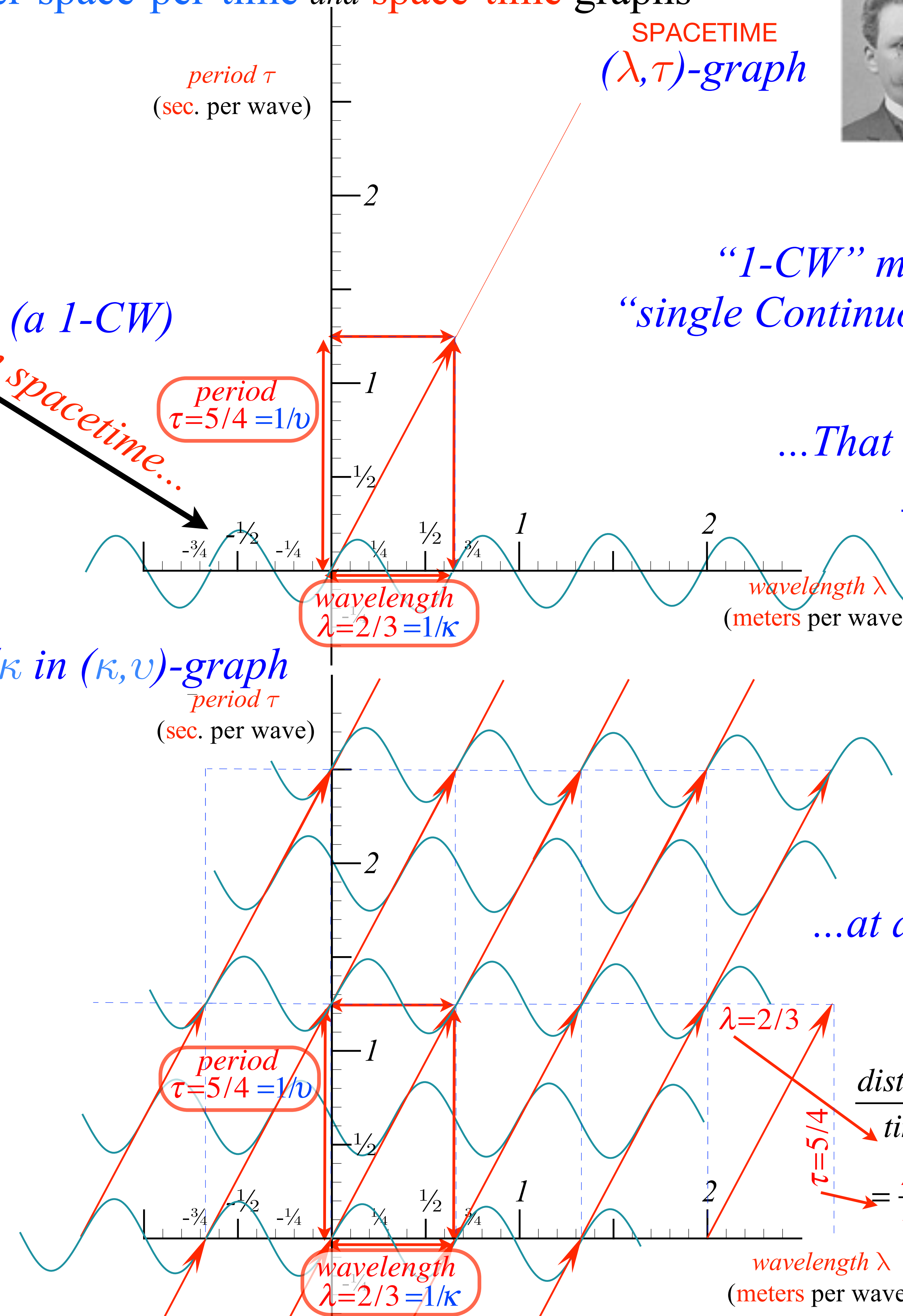

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wave-velocity formulas

$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\text{frequency}}{\text{wavenumber}}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions

•How to understand waves
and
wave velocity V_{wave}

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

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➔ Introducing Doppler shifting

Why is c *so* constant?!

Introducing Doppler Arithmetic and *Rapidity* ρ

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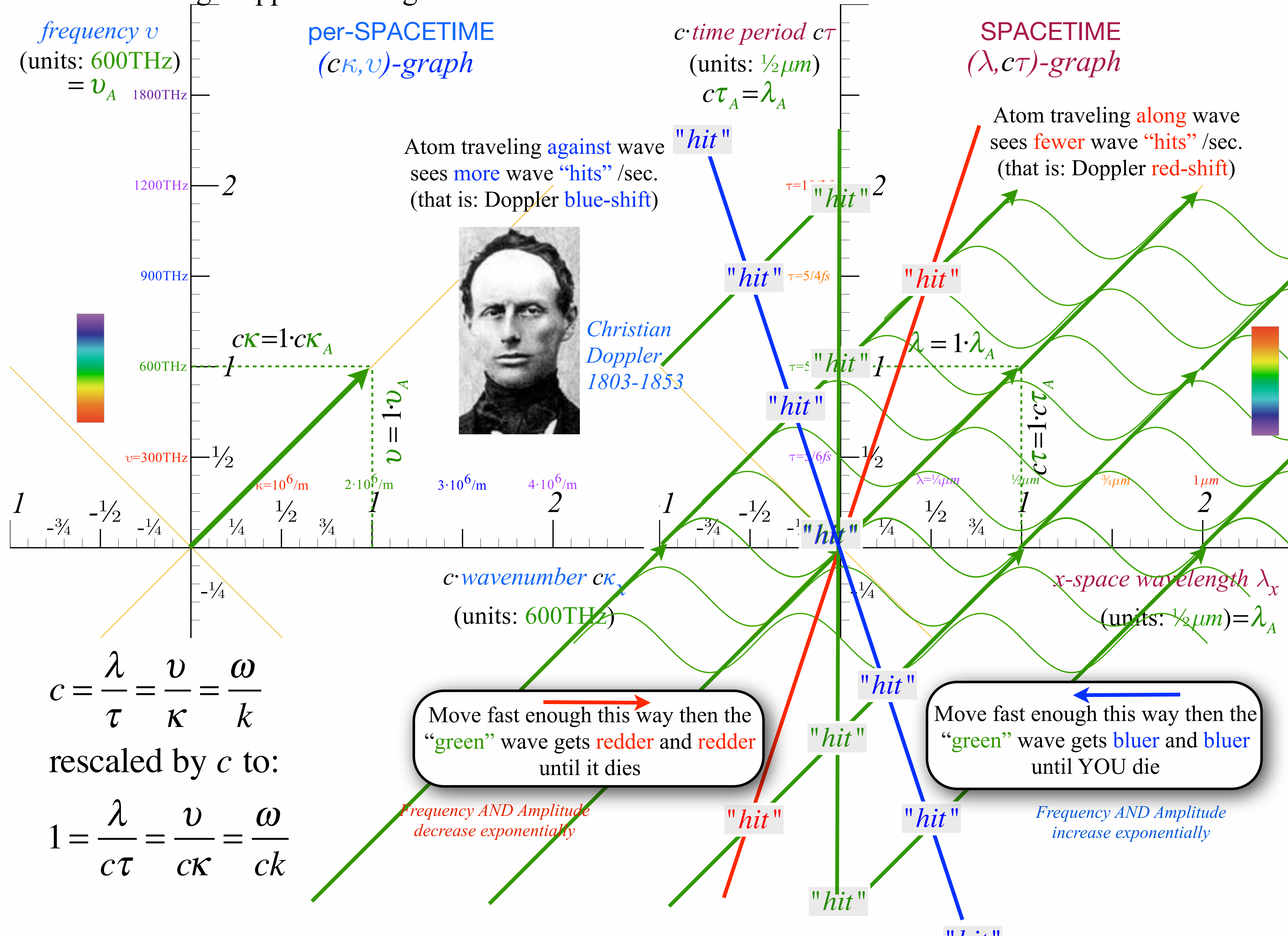

$$c = \frac{\lambda}{\tau} = \frac{\upsilon}{\kappa} = \frac{\omega}{k}$$

rescaled by c to:

$$1 = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = \frac{\omega}{ck}$$

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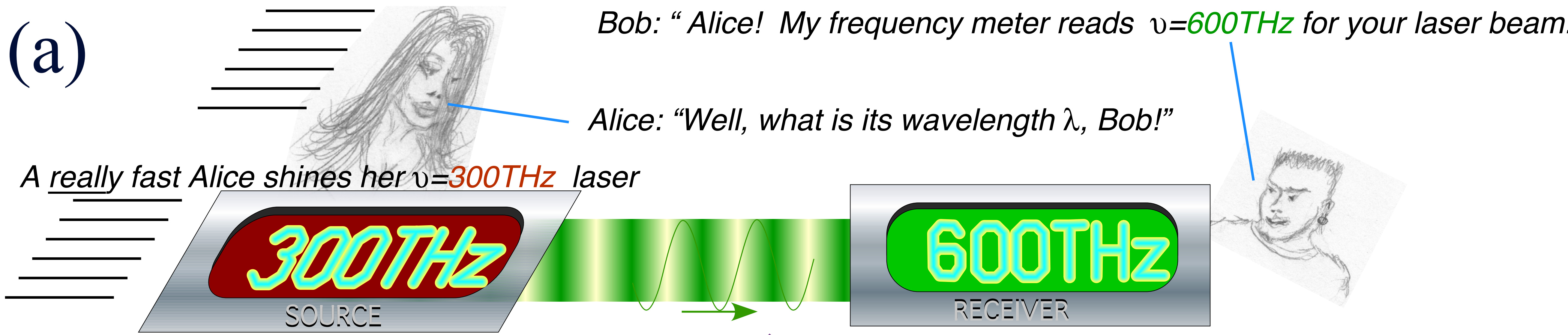
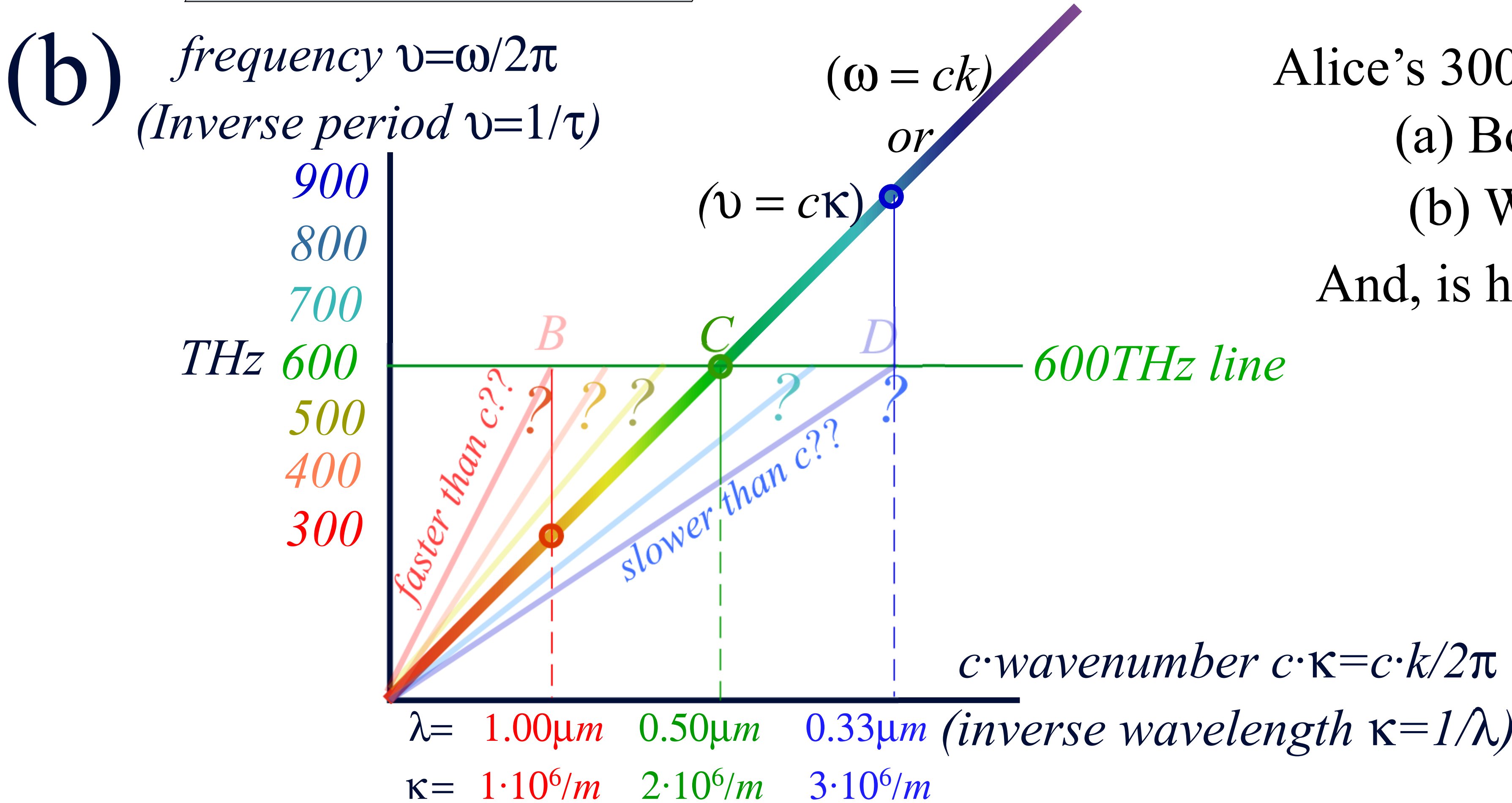
Introducing Doppler shifting and why *c* is *so* constant (and *so slow*)





Alice's 300THz laser approaches Bob.

(a) Bob sees *υ*=600THz.

(b) What *λ=1/κ* does Bob measure?

And, is he seeing a *'phony' green*?

Introducing Doppler shifting and why c is constant

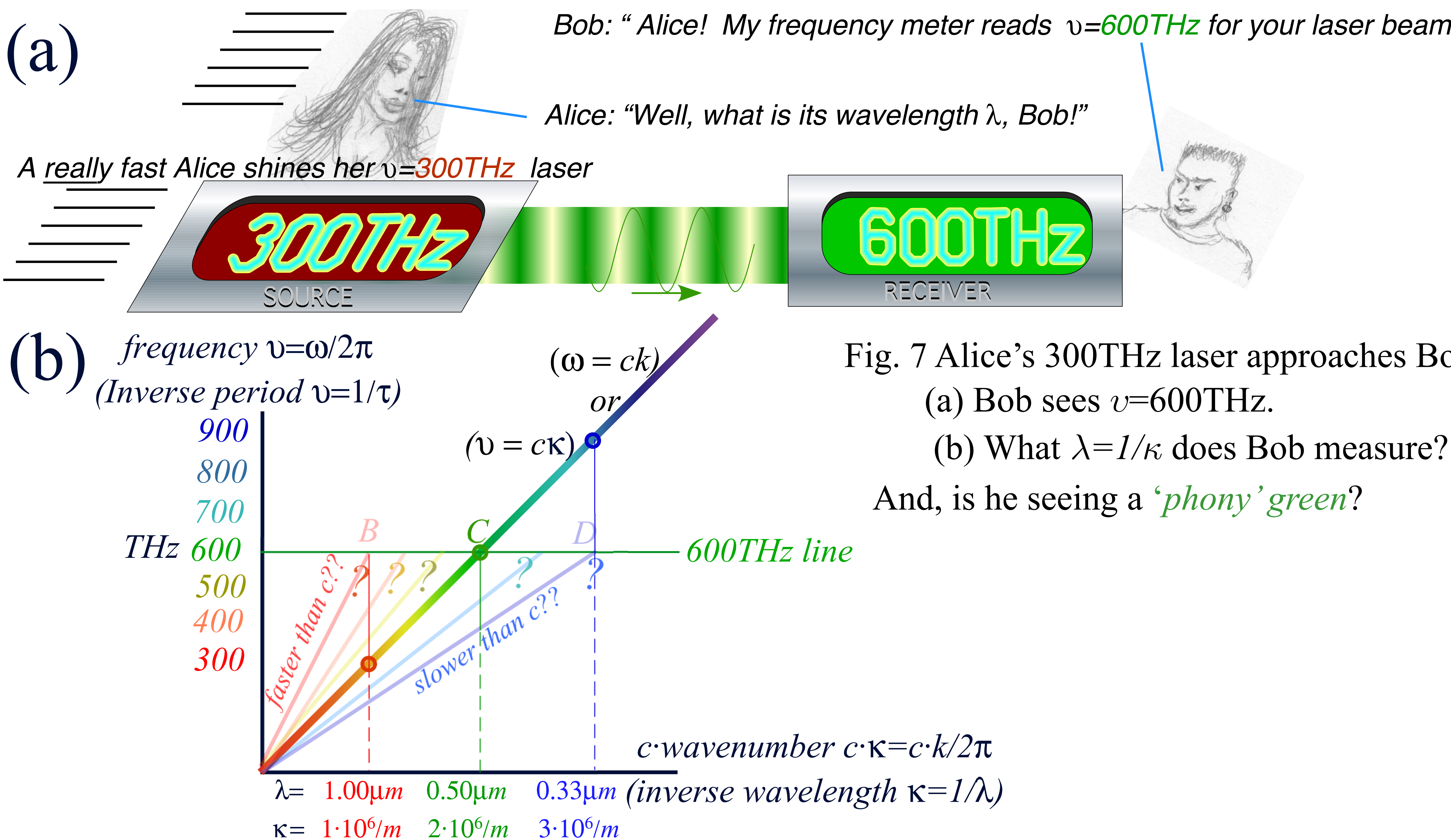
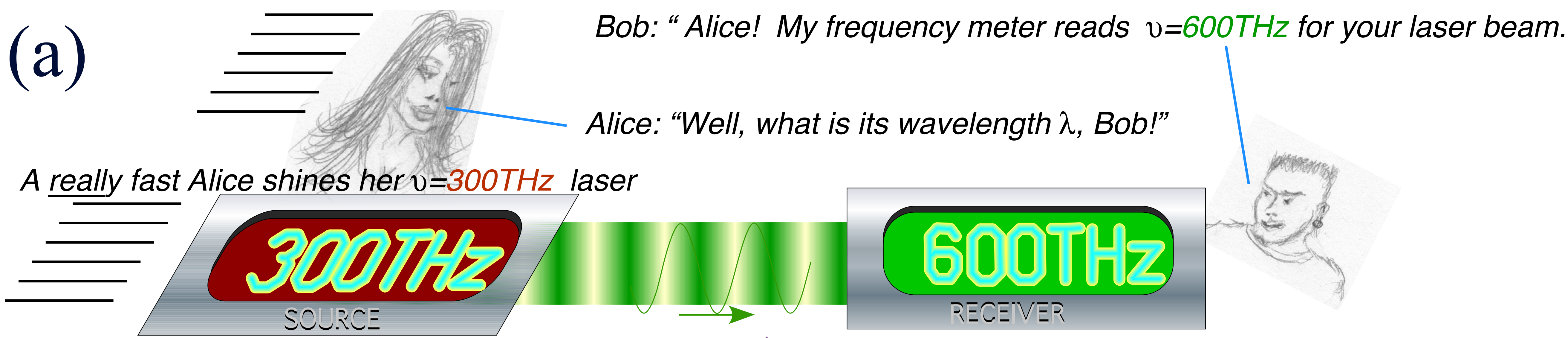


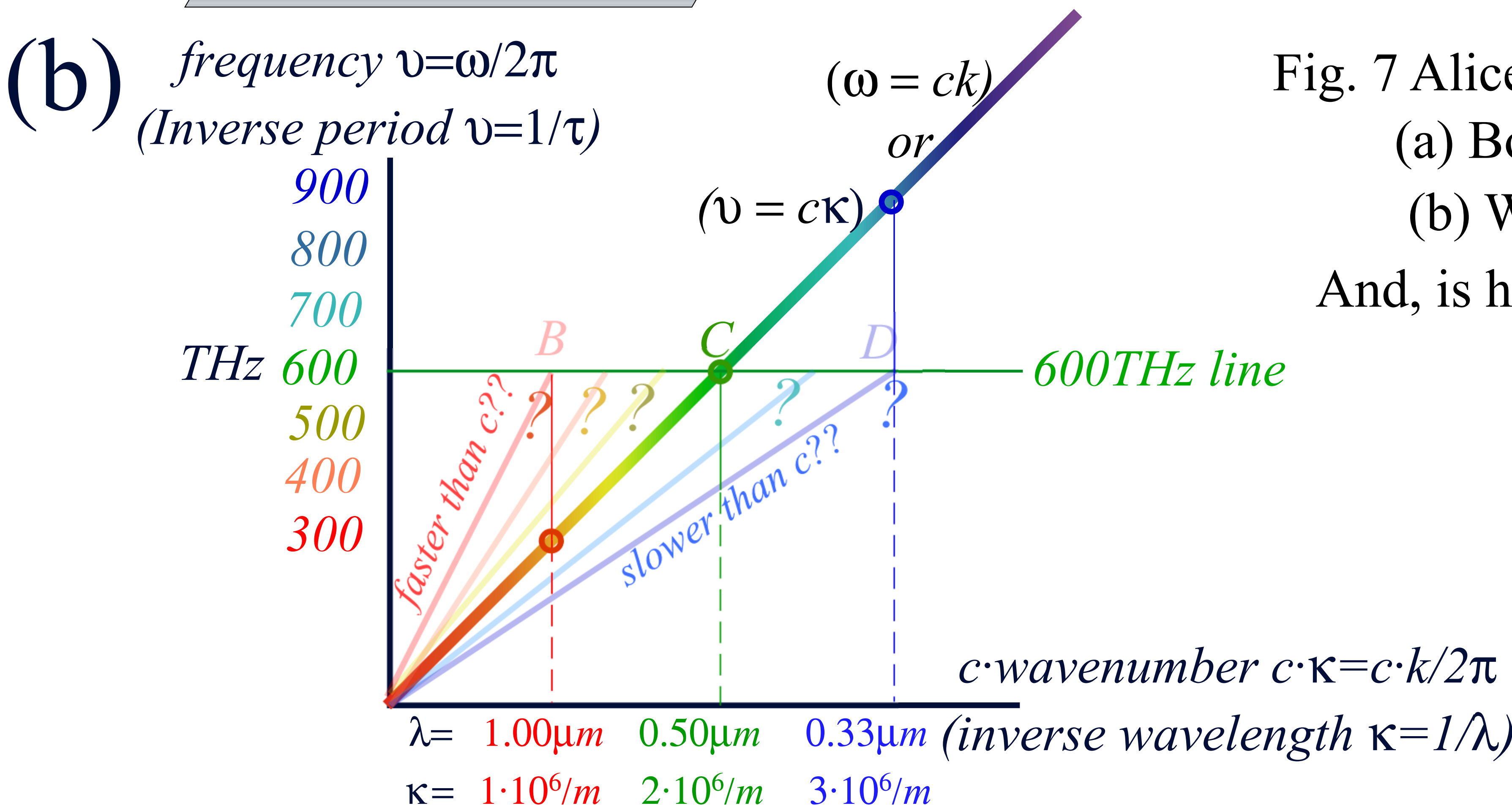
Fig. 7 Alice's 300THz laser approaches Bob.
(a) Bob sees υ=600THz.
(b) What $\lambda=1/\kappa$ does Bob measure?
And, is he seeing a *'phony' green*?

Years of spectroscopy rule out 'phony' 600THz blue-green that do not have wavelength λ=0.5*micron*.

The only choice is C.

Introducing Doppler shifting and why c is constant





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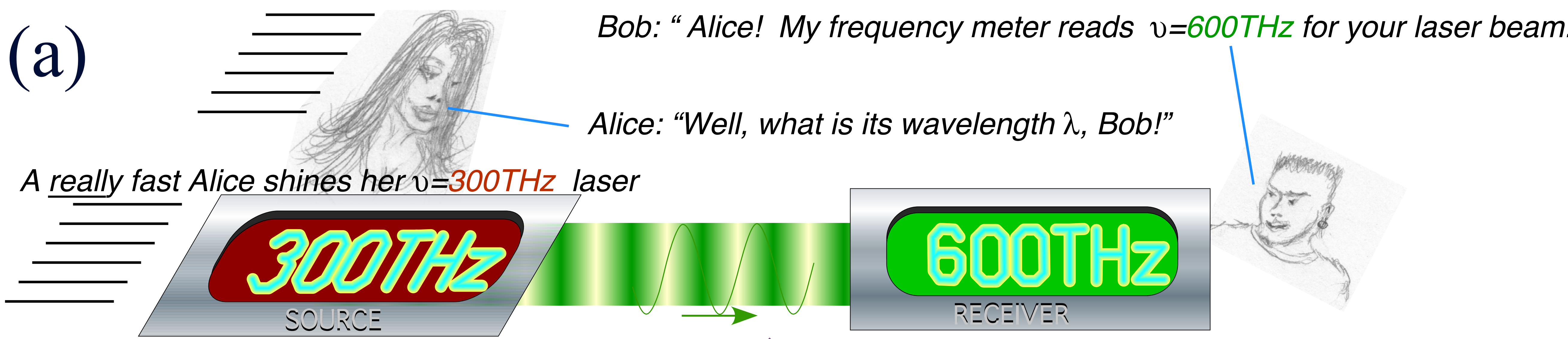
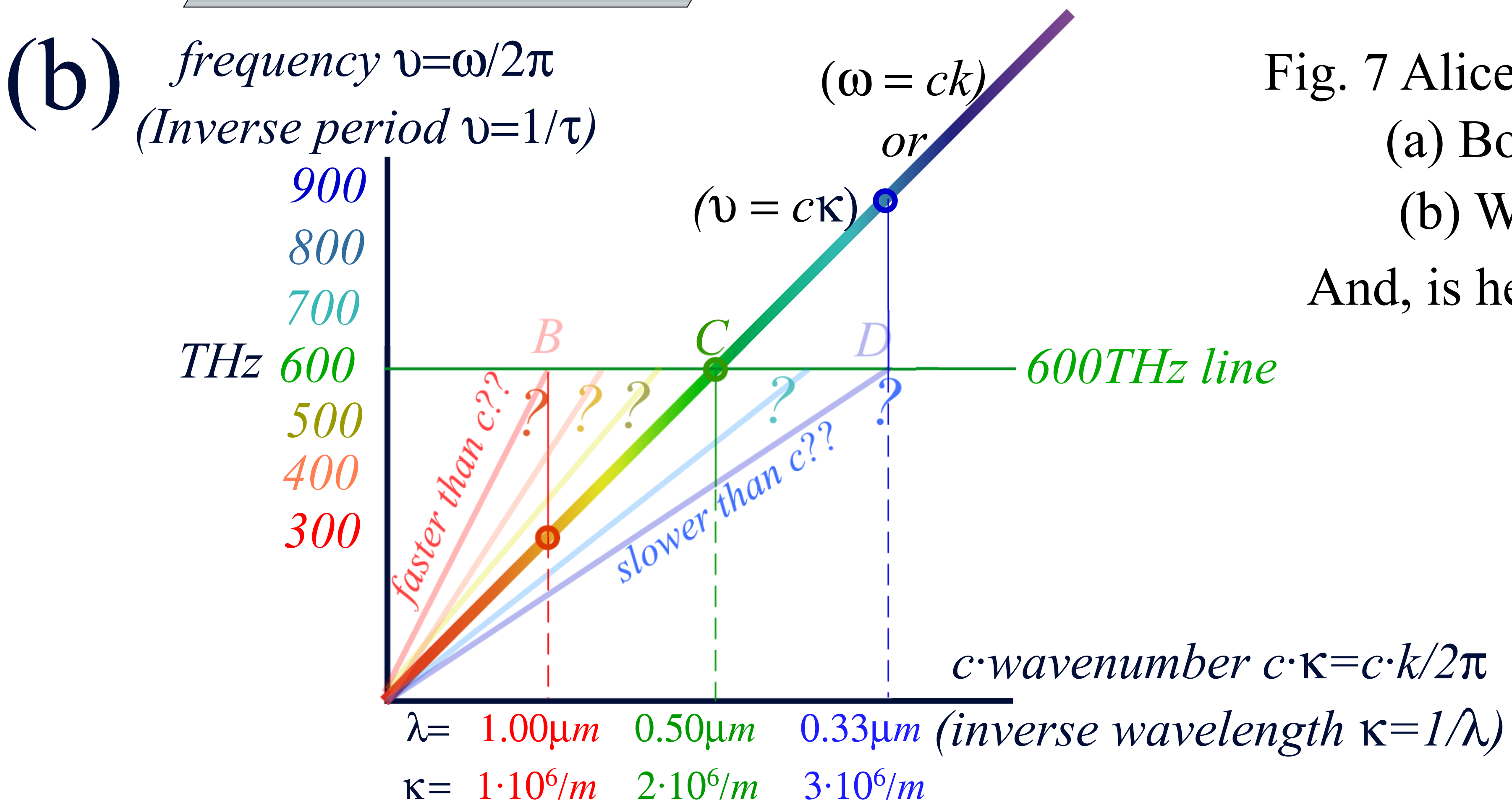
Introducing Doppler shifting and why c is constant





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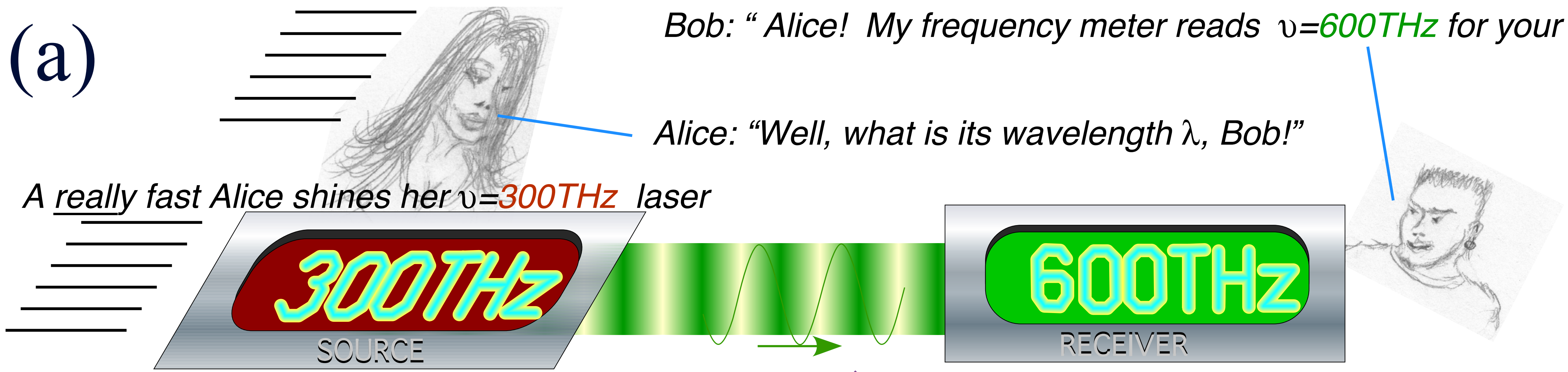
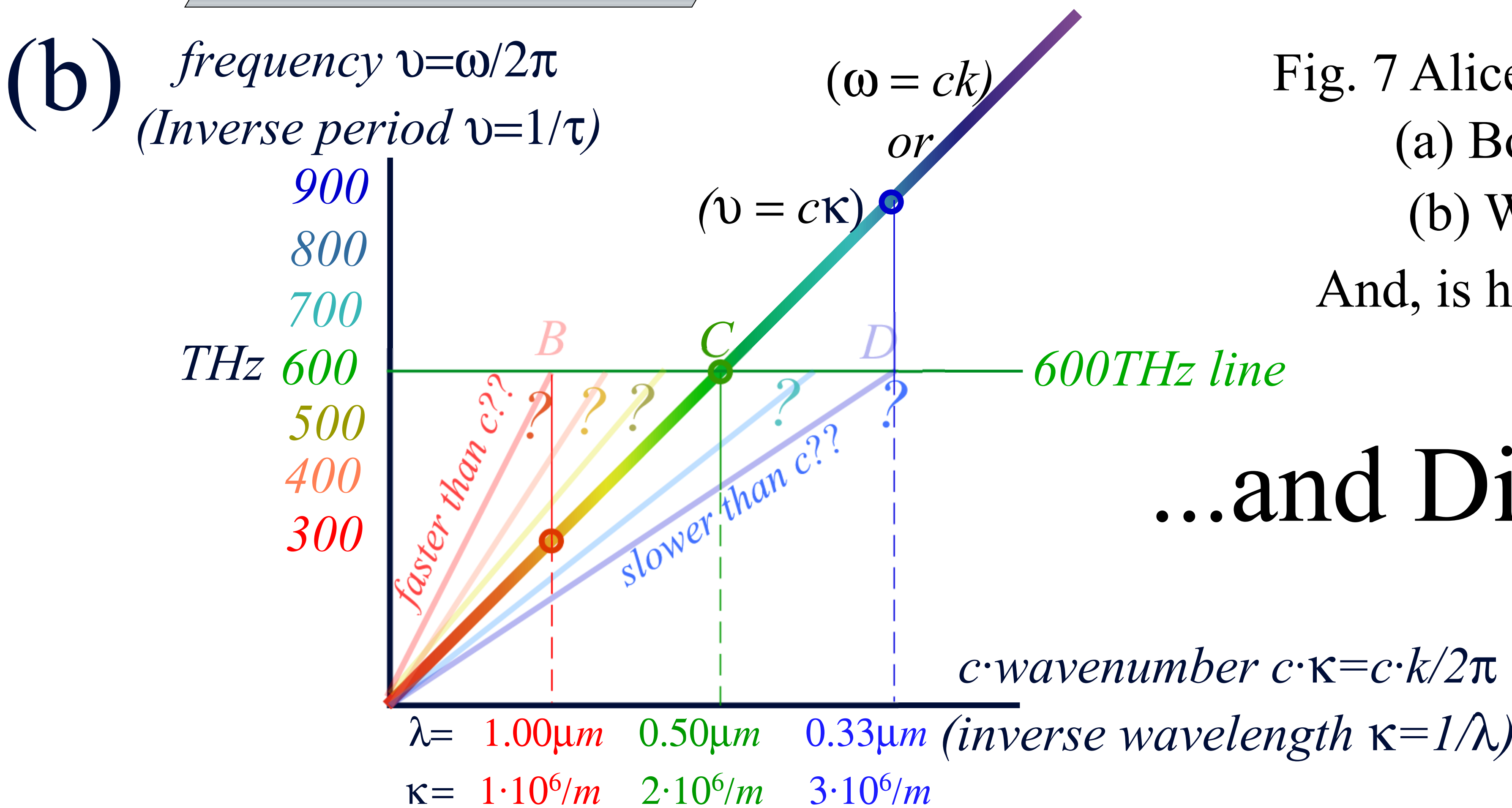
Actually: $2.99792458\cdot 10^{8}$m·s^{-1}

Introducing Doppler shifting and why c is constant





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...and Dispersion-Free!

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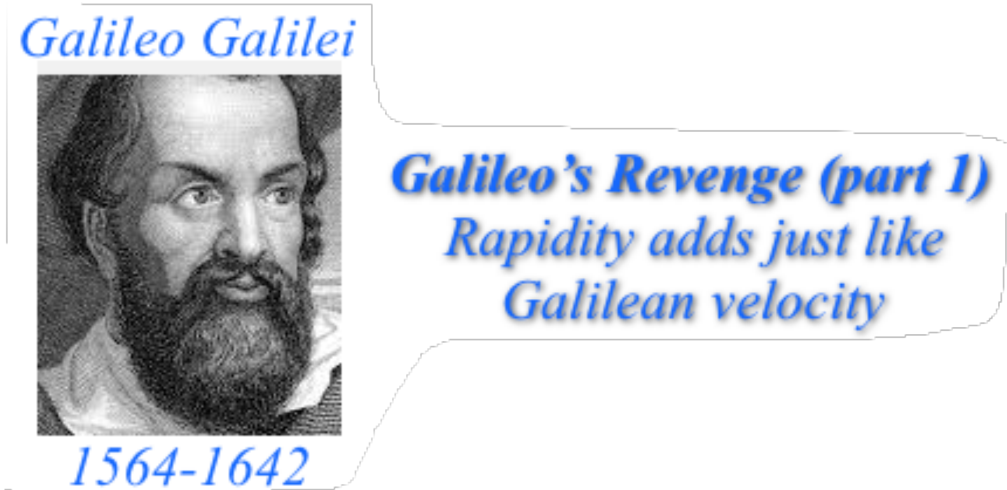

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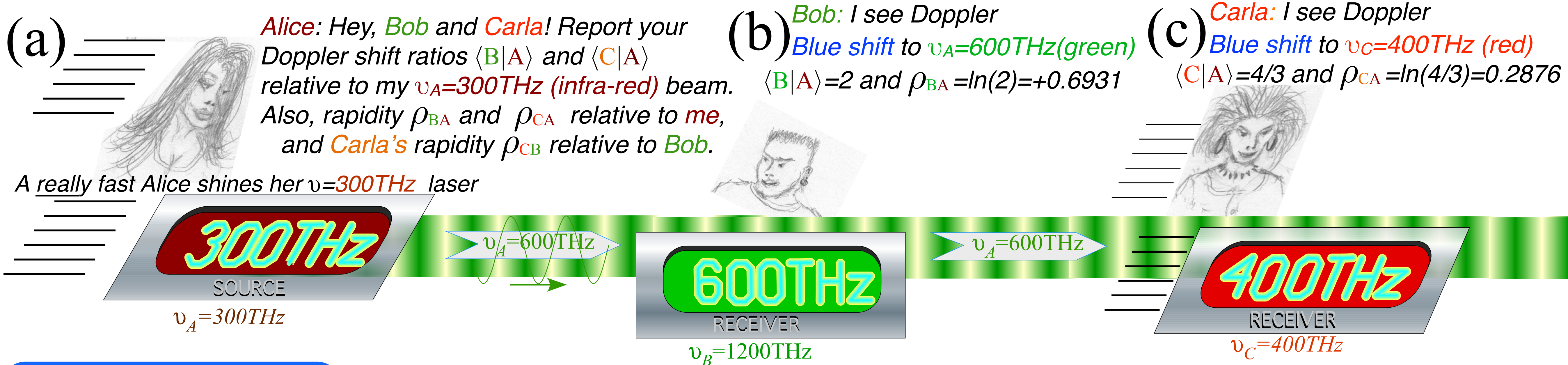


Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

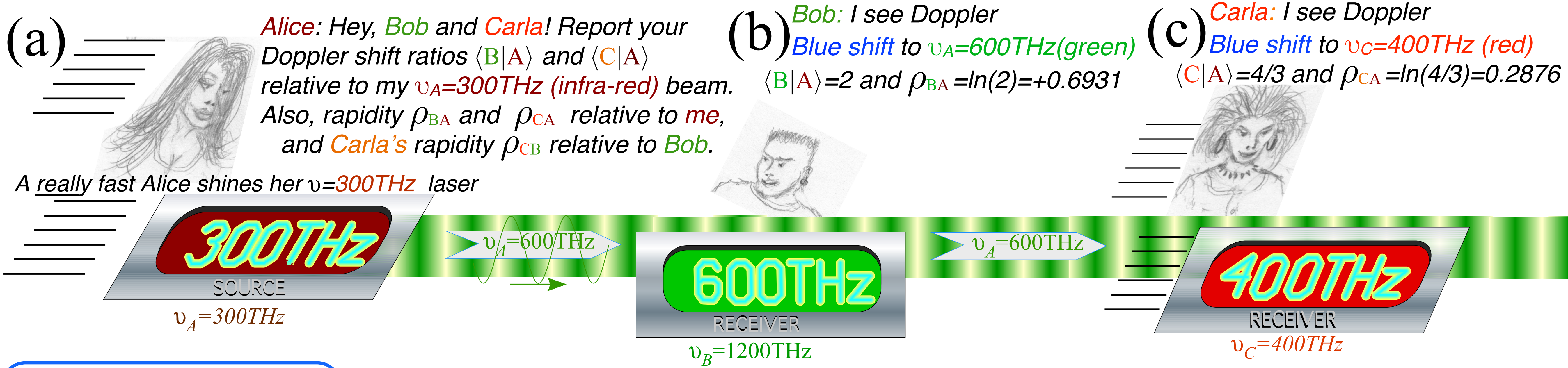
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Introducing Doppler Arithmetic and rapidity ρ



Definition of Rapidity ρ_{RS}

Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

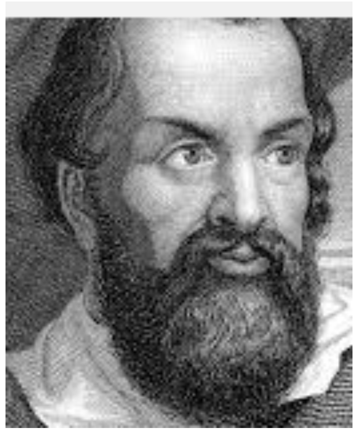
Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle = \frac{4}{3}\frac{1}{2} = \frac{2}{3}$$

Galileo Galilei

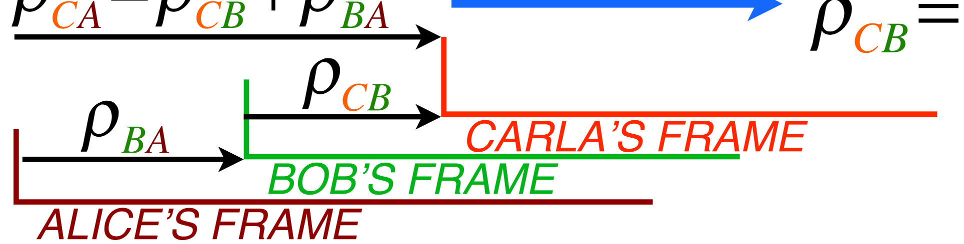


1564-1642

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CA} = \rho_{CB} + \rho_{BA}$$

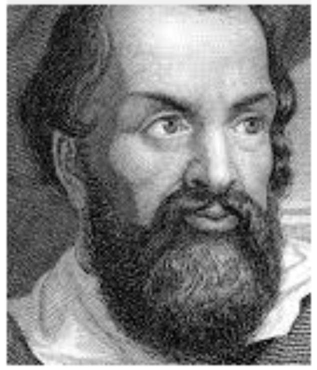


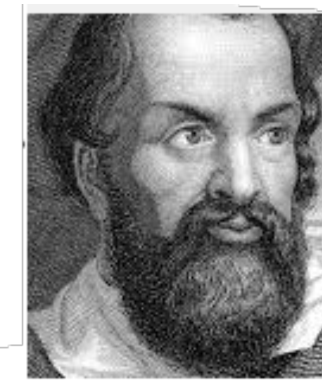
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = 0.2876 - 0.6931 = -0.4055$$

$$= \ln\frac{4}{3} + \ln\frac{1}{2} = \ln\frac{2}{3}$$





➔ Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation

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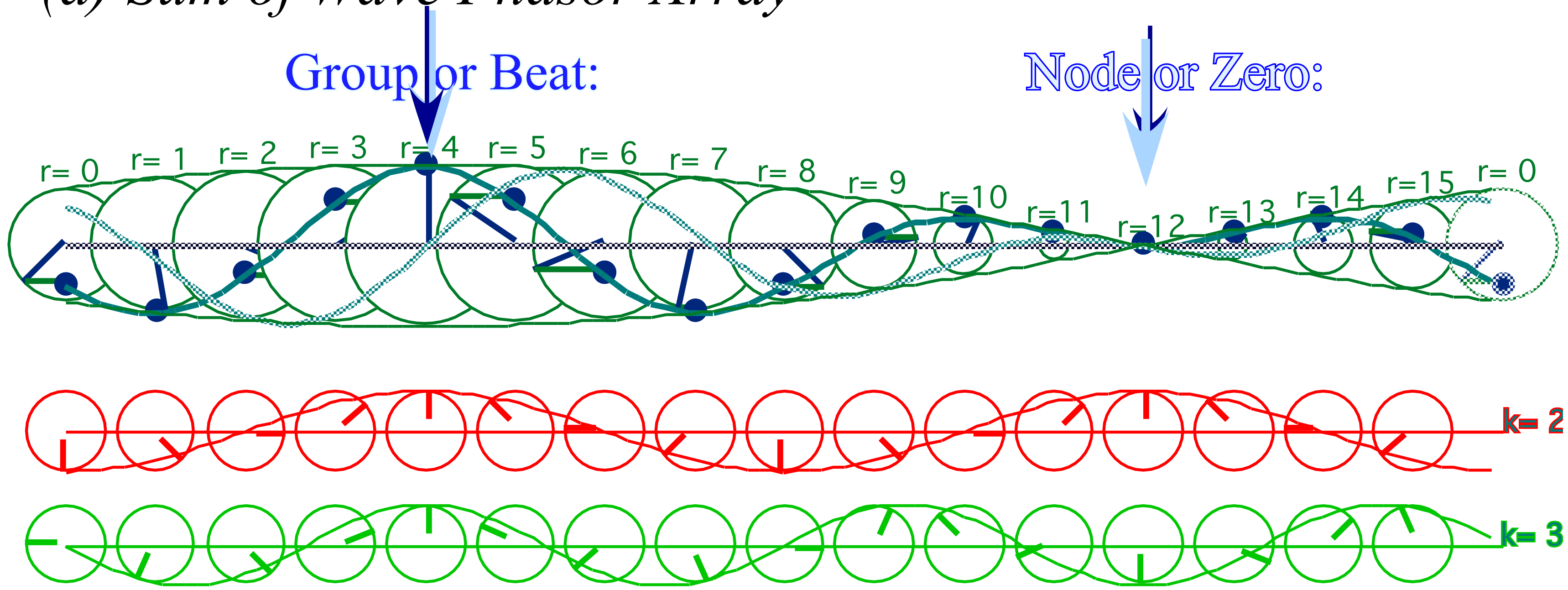
"Occams Sword" and geometry of 16 parameter functions of ρ and σ

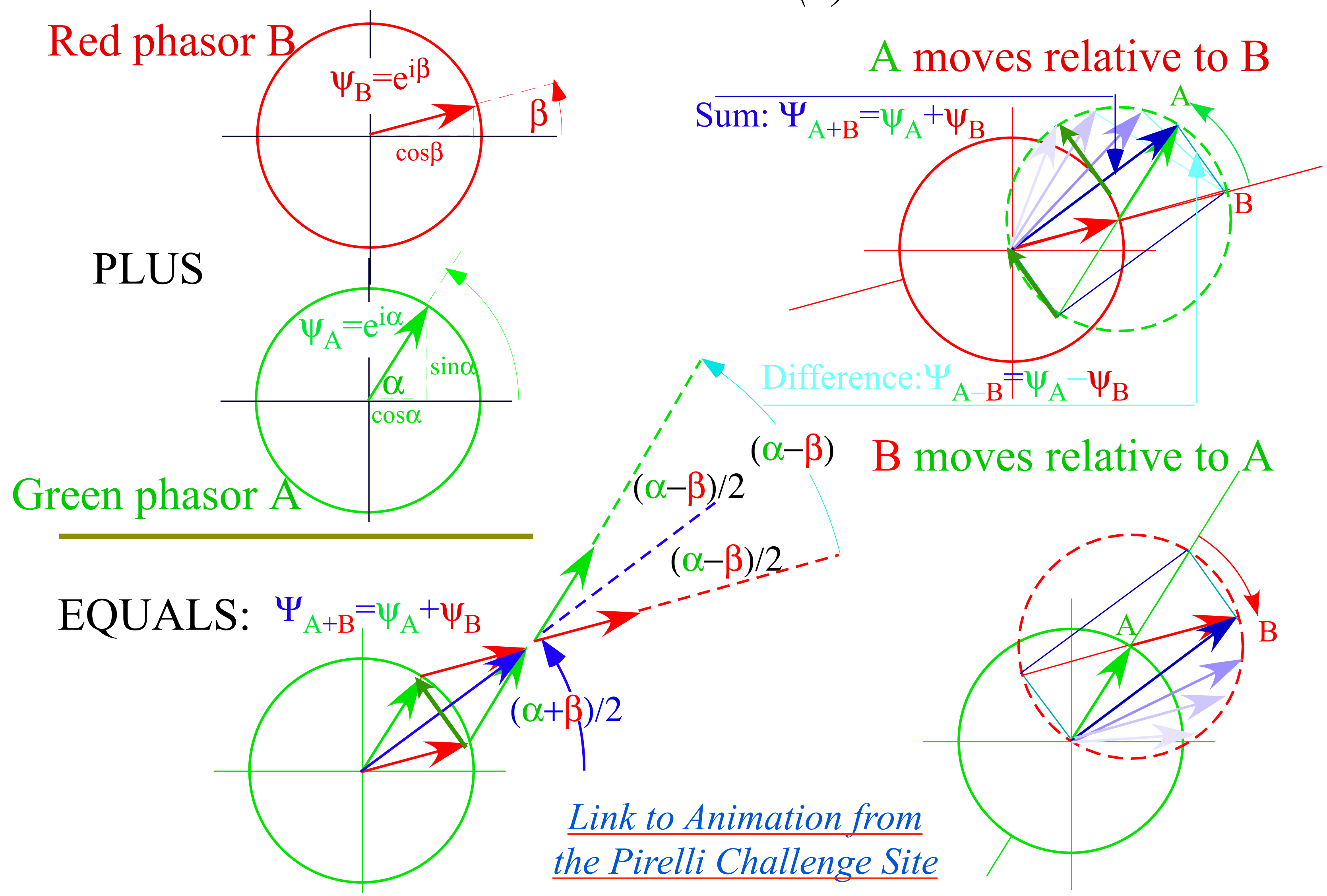
Application to TE-Waveguide modes and synchrotron beam relativity

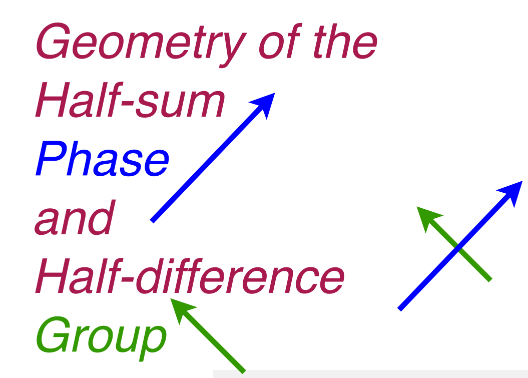
(a) Sum of Wave Phasor Array



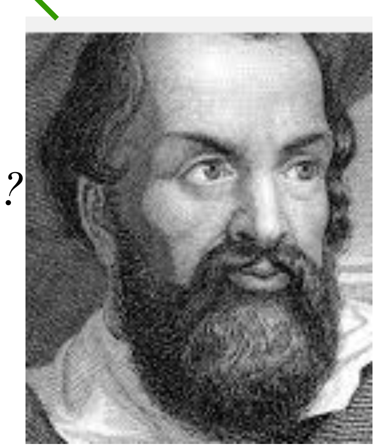
(b) Typical Phasor Sum:

(c) Phasor-relative views



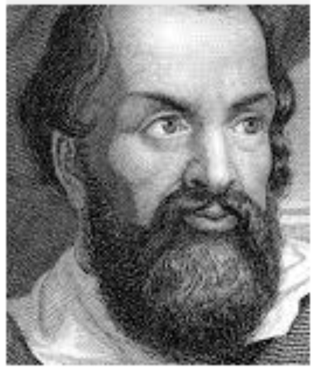


Happy now?



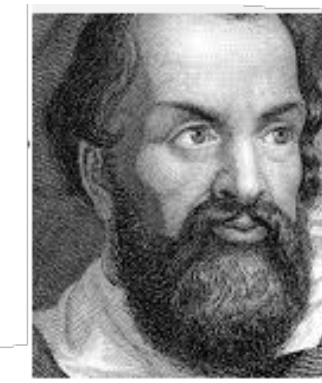
Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

➔ Details of 2CW wavefunctions in rest frame

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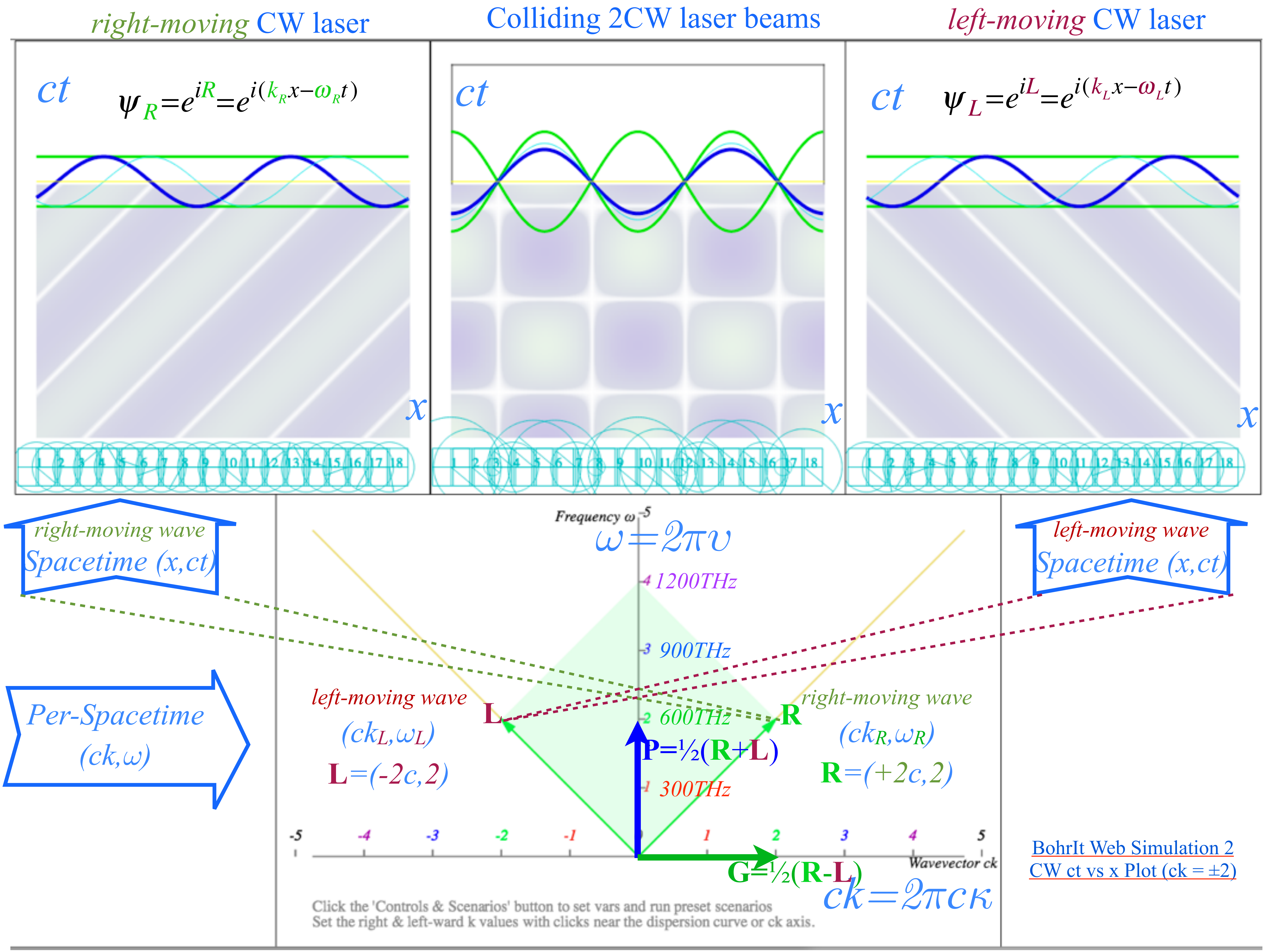
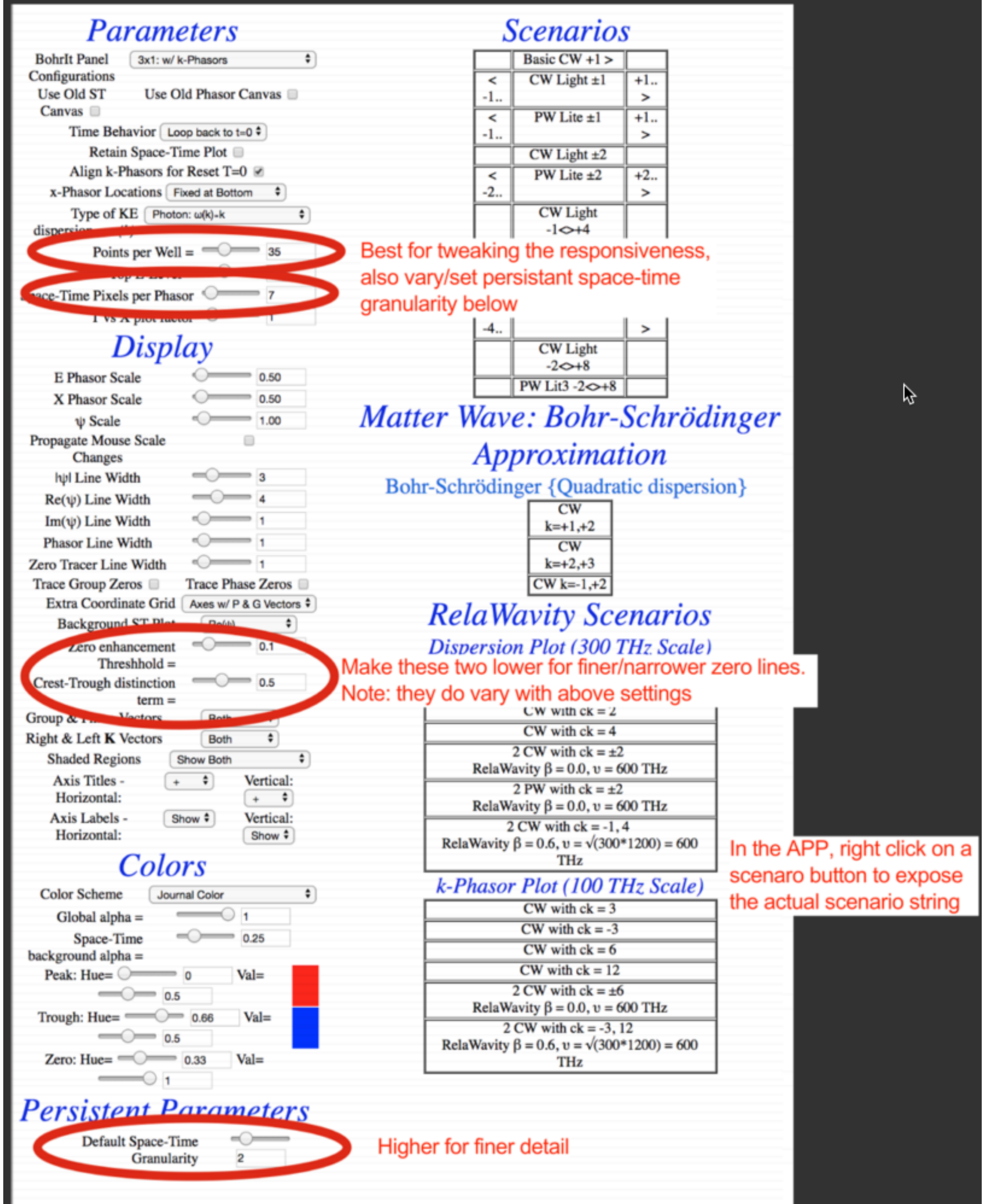
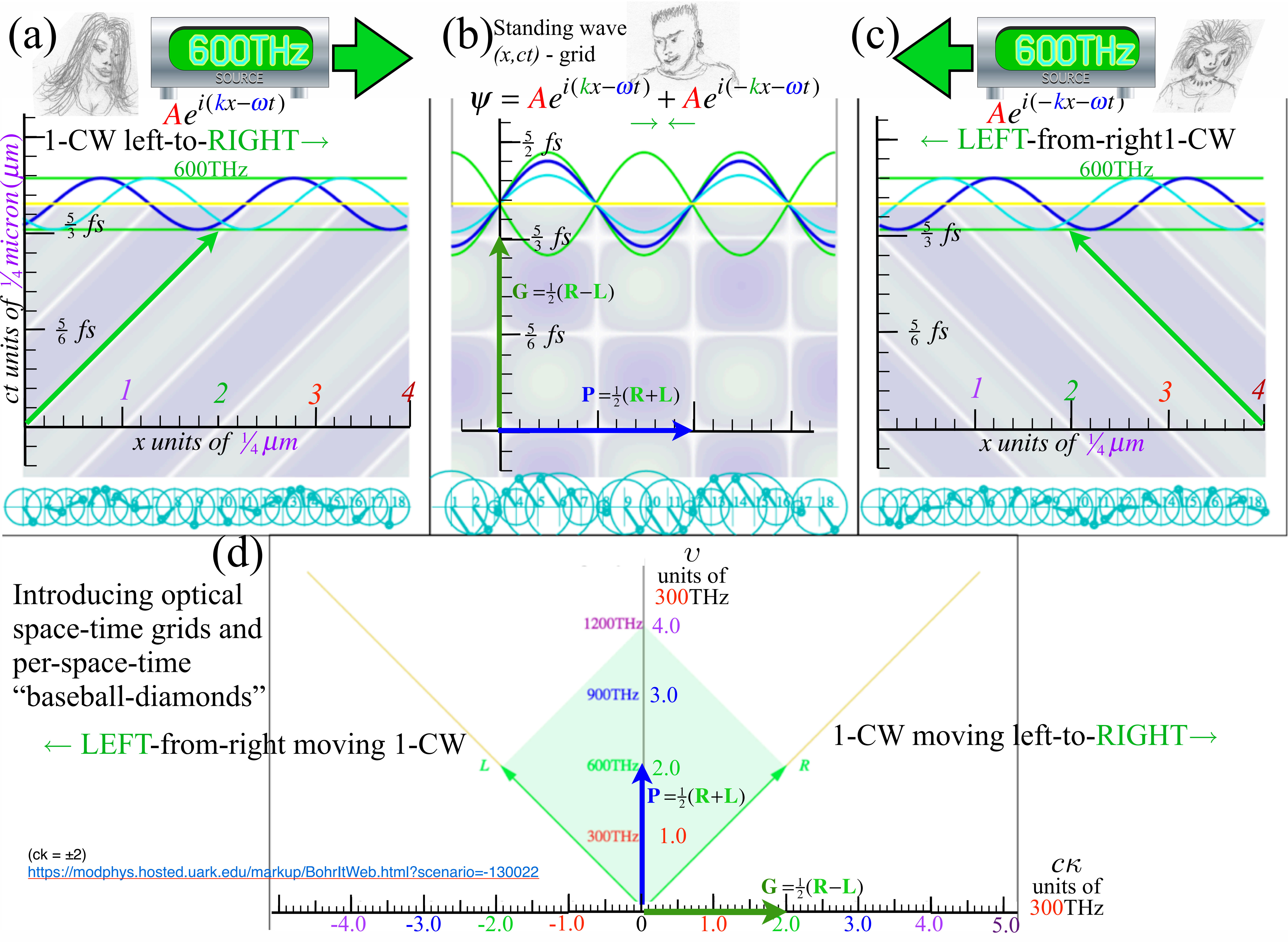
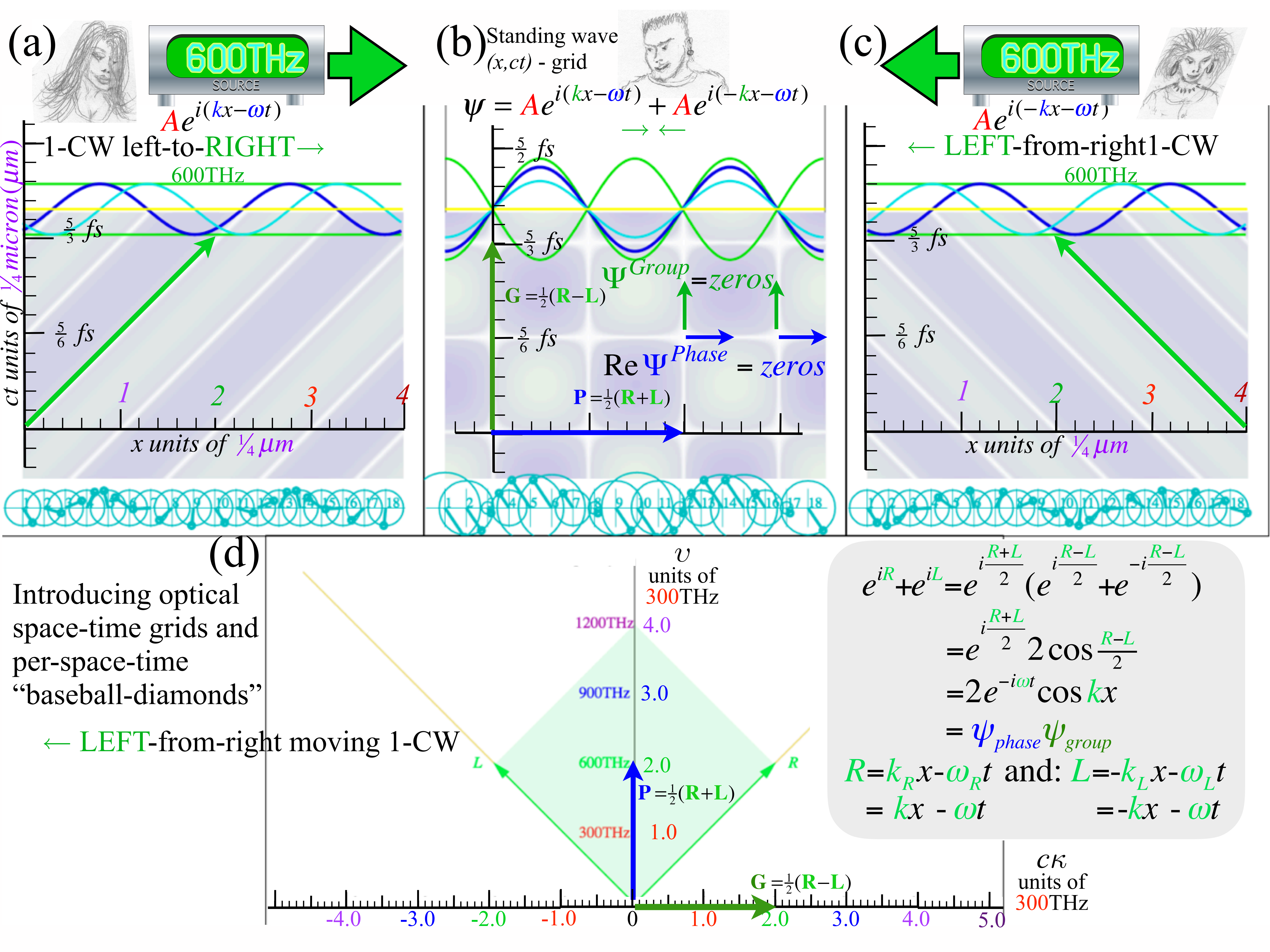
Application to TE-Waveguide modes. — synchrotron beam relativity


right-moving CW laser
Colliding 2CW laser beams
left-moving CW laser
ct
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
x
ct
x
ct
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
$\omega = 2\pi\upsilon$
1200THz
900THz
600THz
300THz
left-moving wave
(ck_L, ω_L)
L=(-2c,2)
right-moving wave
(ck_R, ω_R)
R=(+2c,2)
P=½(R+L)
G=½(R-L)
Wavevector ck
$ck = 2\pi c\kappa$
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation 2
CW ct vs x Plot (ck = ±2)


Parameters
BohrIt Panel Configurations
3x1: w/ k-Phasors
Use Old ST Canvas
Use Old Phasor Canvas
Time Behavior
Loop back to t=0
Retain Space-Time Plot
Align k-Phasors for Reset T=0
x-Phasor Locations
Fixed at Bottom
Type of KE
Photon: ω(k)-k
Points per Well =
35
ace-Time Pixels per Phasor
7
Best for tweaking the responsiveness, also vary/set persistant space-time granularity below
Display
E Phasor Scale
0.50
X Phasor Scale
0.50
ψ Scale
1.00
Propagate Mouse Scale Changes
|ψ| Line Width
3
Re(ψ) Line Width
4
Im(ψ) Line Width
1
Phasor Line Width
1
Zero Tracer Line Width
1
Trace Group Zeros
Trace Phase Zeros
Extra Coordinate Grid
Axes w/ P & G Vectors
Zero enhancement Threshhold =
0.1
Crest-Trough distinction term =
0.5
Make these two lower for finer/narrower zero lines.
Note: they do vary with above settings
Right & Left K Vectors
Both
Shaded Regions
Show Both
Axis Titles - Horizontal:
Vertical:
Axis Labels - Horizontal:
Show
Vertical:
Show
Colors
Color Scheme
Journal Color
Global alpha =
1
Space-Time background alpha =
0.25
Peak: Hue=
0
Val=
0.5
Trough: Hue=
0.66
Val=
0.5
Zero: Hue=
0.33
Val=
1
Persistent Parameters
Default Space-Time Granularity
2
Higher for finer detail
Scenarios
Basic CW +1 >
< -1..
CW Light ±1
+1.. >
< -1..
PW Lite ±1
+1.. >
CW Light ±2
< -2..
PW Lite ±2
+2.. >
CW Light -1<>+4
-4..
>
CW Light -2<>+8
PW Lit3 -2<>+8
Matter Wave: Bohr-Schrödinger Approximation
Bohr-Schrödinger {Quadratic dispersion}
CW k=+1,+2
CW k=+2,+3
CW k=-1,+2
RelaWavity Scenarios
Dispersion Plot (300 THz Scale)
CW with ck = 4
2 CW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 PW with ck = ±2 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -1, 4 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz
In the APP, right click on a scenaro button to expose the actual scenario string
k-Phasor Plot (100 THz Scale)
CW with ck = 3
CW with ck = -3
CW with ck = 6
CW with ck = 12
2 CW with ck = ±6 RelaWavity β = 0.0, υ = 600 THz
2 CW with ck = -3, 12 RelaWavity β = 0.6, υ = √(300*1200) = 600 THz

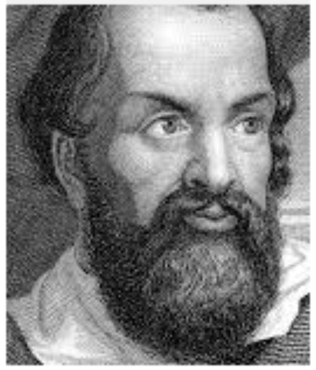


(a)
600THz
SOURCE
$Ae^{i(kx-\omega t)}$
1-CW left-to-RIGHT→
600THz
ct units of ¼ micron (μm)
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1 2 3 4
x units of ¼ μm
(b) Standing wave
(x,ct) - grid
$\psi = Ae^{i(kx-\omega t)} + Ae^{i(-kx-\omega t)}$
→ ←
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
$\frac{5}{6}$ fs
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
(c)
600THz
SOURCE
$Ae^{i(-kx-\omega t)}$
← LEFT-from-right1-CW
600THz
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1 2 3 4
x units of ¼ μm
(d)
Introducing optical space-time grids and per-space-time "baseball-diamonds"
← LEFT-from-right moving 1-CW
1-CW moving left-to-RIGHT→
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
L
R
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
cκ units of 300THz
-4.0 -3.0 -2.0 -1.0 0 1.0 2.0 3.0 4.0 5.0
(ck = ±2)
https://modphys.hosted.uark.edu/markup/BohrItWeb.html?scenario=-130022



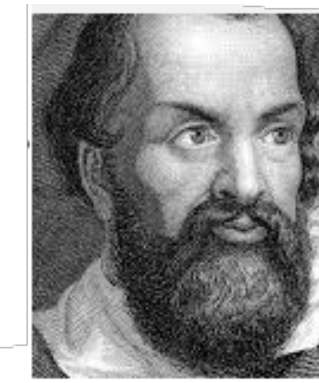
(a)
600THz
SOURCE
$Ae^{i(kx-\omega t)}$
1-CW left-to-RIGHT→
600THz
ct units of ¼ micron (μm)
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(b)
Standing wave
(x,ct) - grid
$\psi = Ae^{i(kx-\omega t)} + Ae^{i(-kx-\omega t)}$
→ ←
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
Ψ^{Group} = zeros
G = ½(R−L)
Re Ψ^{Phase} = zeros
P = ½(R+L)
(c)
600THz
SOURCE
$Ae^{i(-kx-\omega t)}$
← LEFT-from-right 1-CW
600THz
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(d)
Introducing optical space-time grids and per-space-time "baseball-diamonds"
← LEFT-from-right moving 1-CW
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
L
R
P = ½(R+L)
G = ½(R−L)
cκ units of 300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0
$e^{iR} + e^{iL} = e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}} + e^{-i\frac{R-L}{2}}\right)$
$= e^{i\frac{R+L}{2}}\, 2\cos\frac{R-L}{2}$
$= 2e^{-i\omega t}\cos kx$
$= \psi_{phase}\psi_{group}$
$R = k_R x - \omega_R t$ and: $L = -k_L x - \omega_L t$
$= kx - \omega t$
$= -kx - \omega t$

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

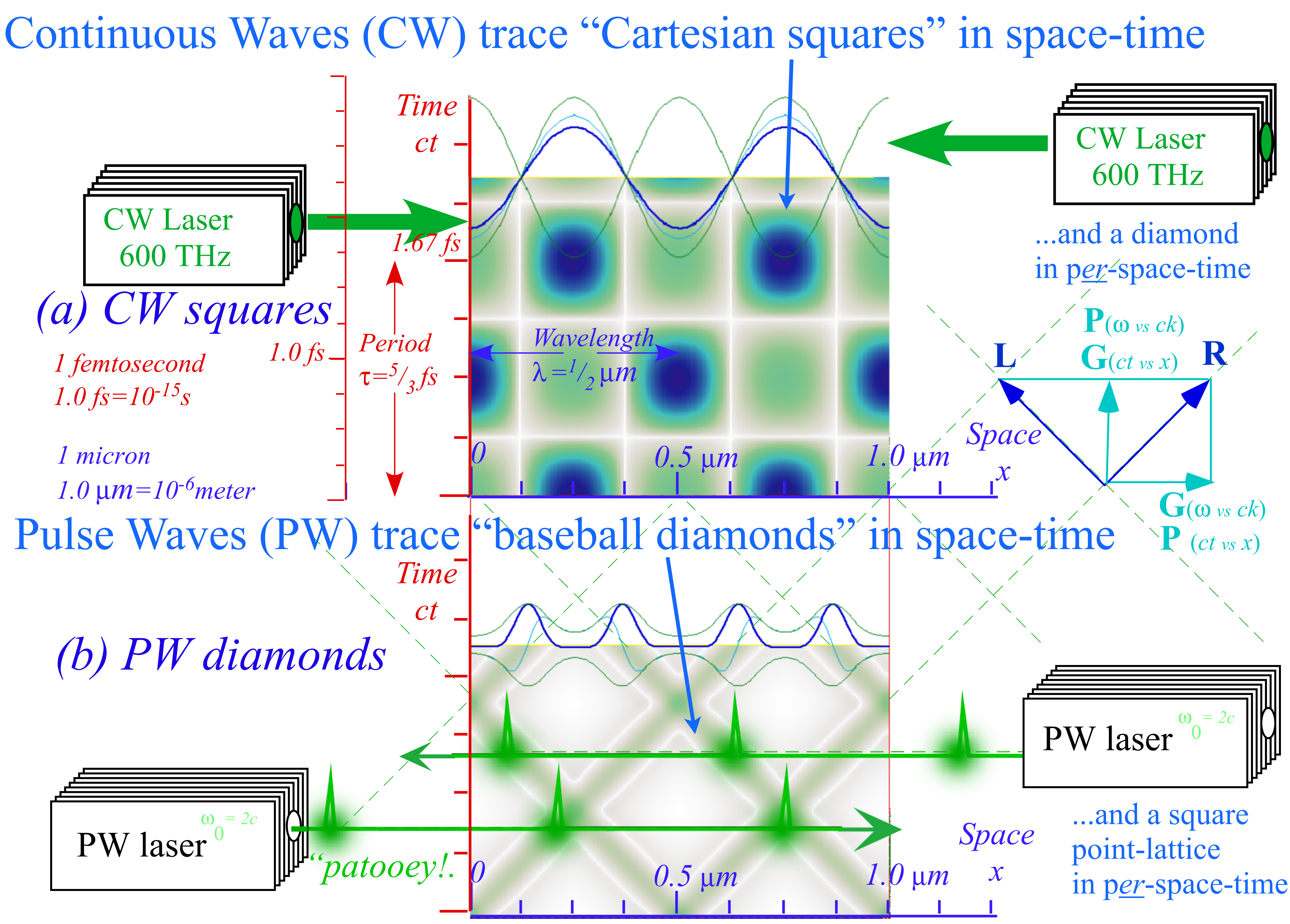
- Details of 2CW wavefunctions in rest frame
- ➔ Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

- Analyzing wave velocity by per-space-per-time *and* space-time graphs
  - 16 coefficients of relativistic 2CW interference
  - Two "famous-name" coefficients and the Lorentz transformation
  - Thales geometry of Lorentz transformation

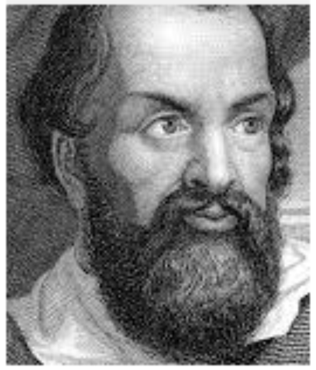
Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and geometry of functions of ρ and σ — Minkowski animations
  - Application to TE-Waveguide modes. — synchrotron beam relativity



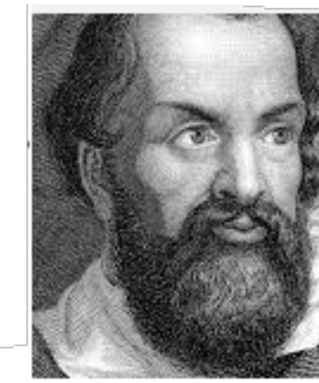
BohrIt Web Simulation: *Multi-Panel 2 PW ct vs x Plot*

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
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adds just like
Galilean velocity

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  - "Occams Sword" and geometry of functions of ρ and σ Minkowski animations
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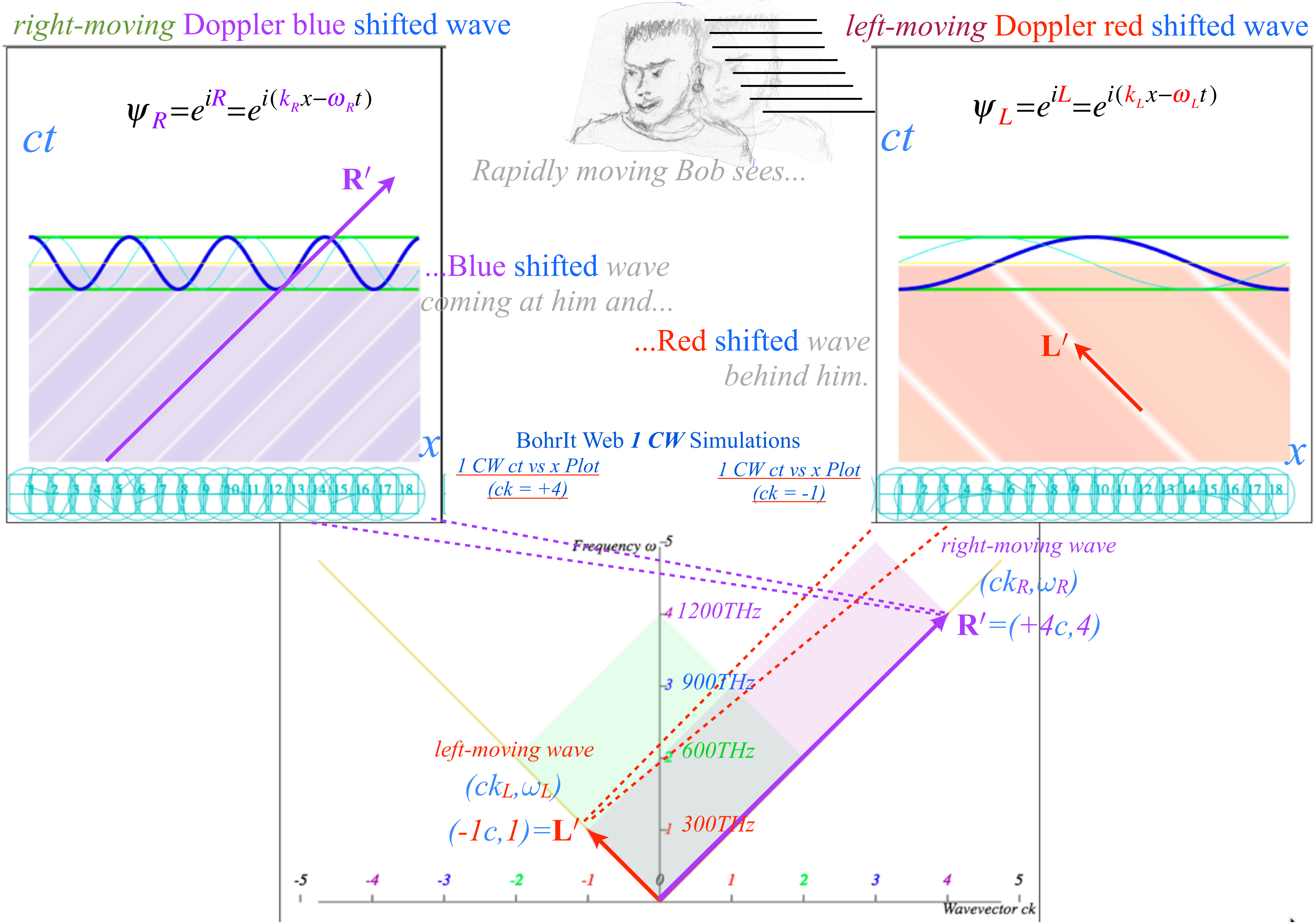
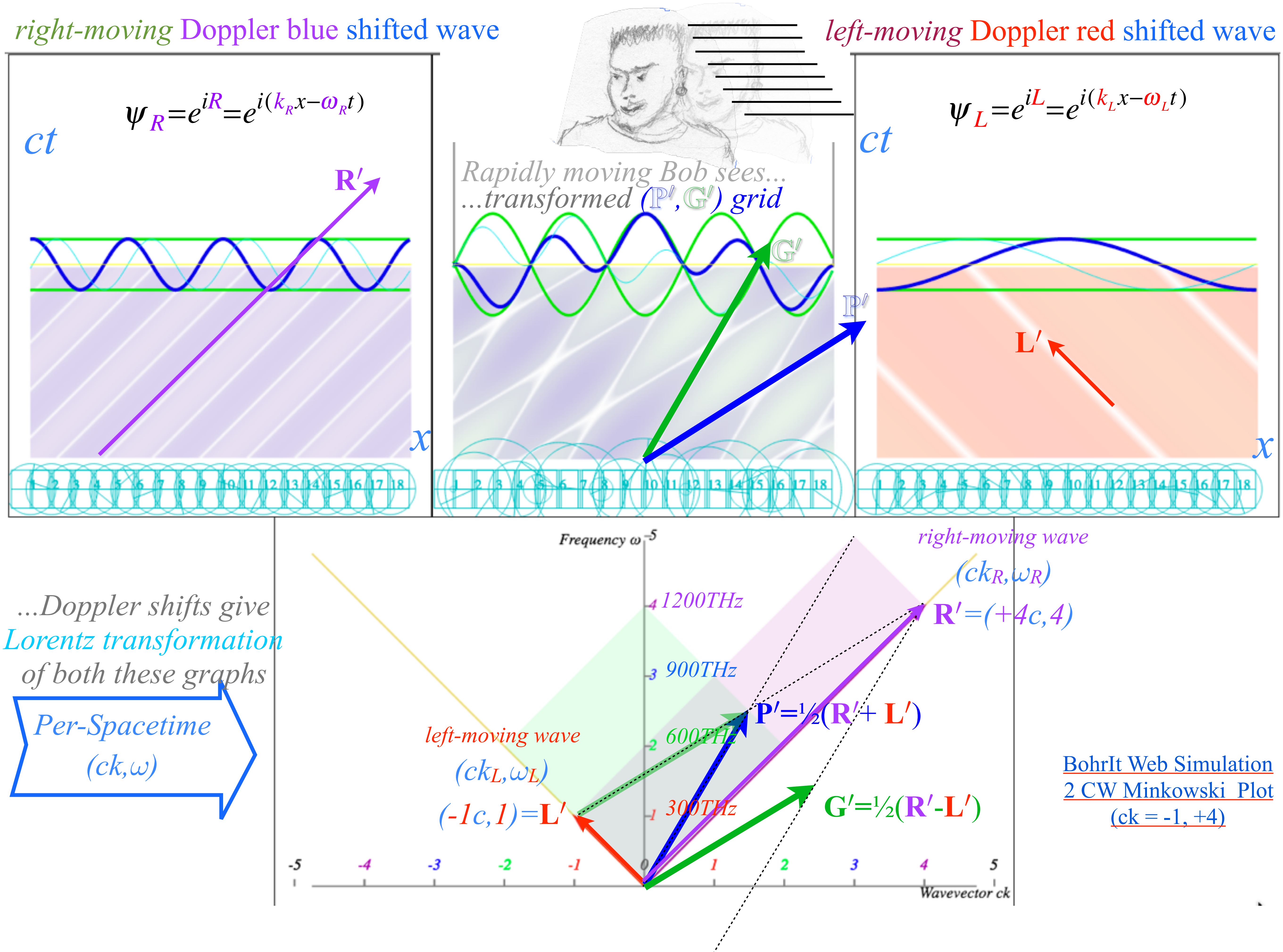
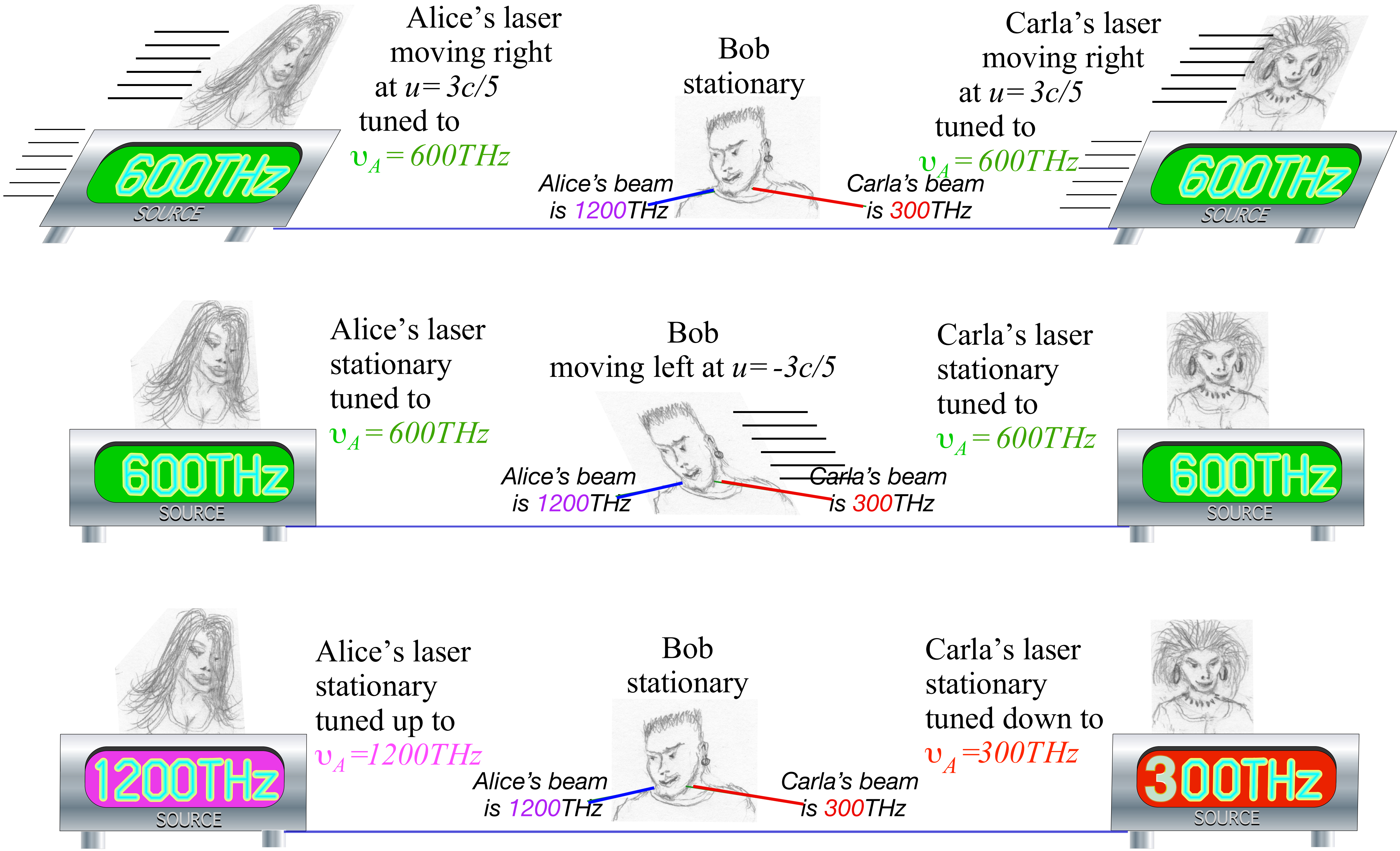

right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
ct
R′
x
Rapidly moving Bob sees...
...Blue shifted wave coming at him and...
...Red shifted wave behind him.
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
L′
x
BohrIt Web 1 CW Simulations
1 CW ct vs x Plot (ck = +4)
1 CW ct vs x Plot (ck = -1)
Frequency ω
1200THz
900THz
600THz
300THz
right-moving wave
(ck_R, ω_R)
R′=(+4c,4)
left-moving wave
(ck_L, ω_L)
(-1c,1)=L′
Wavevector ck

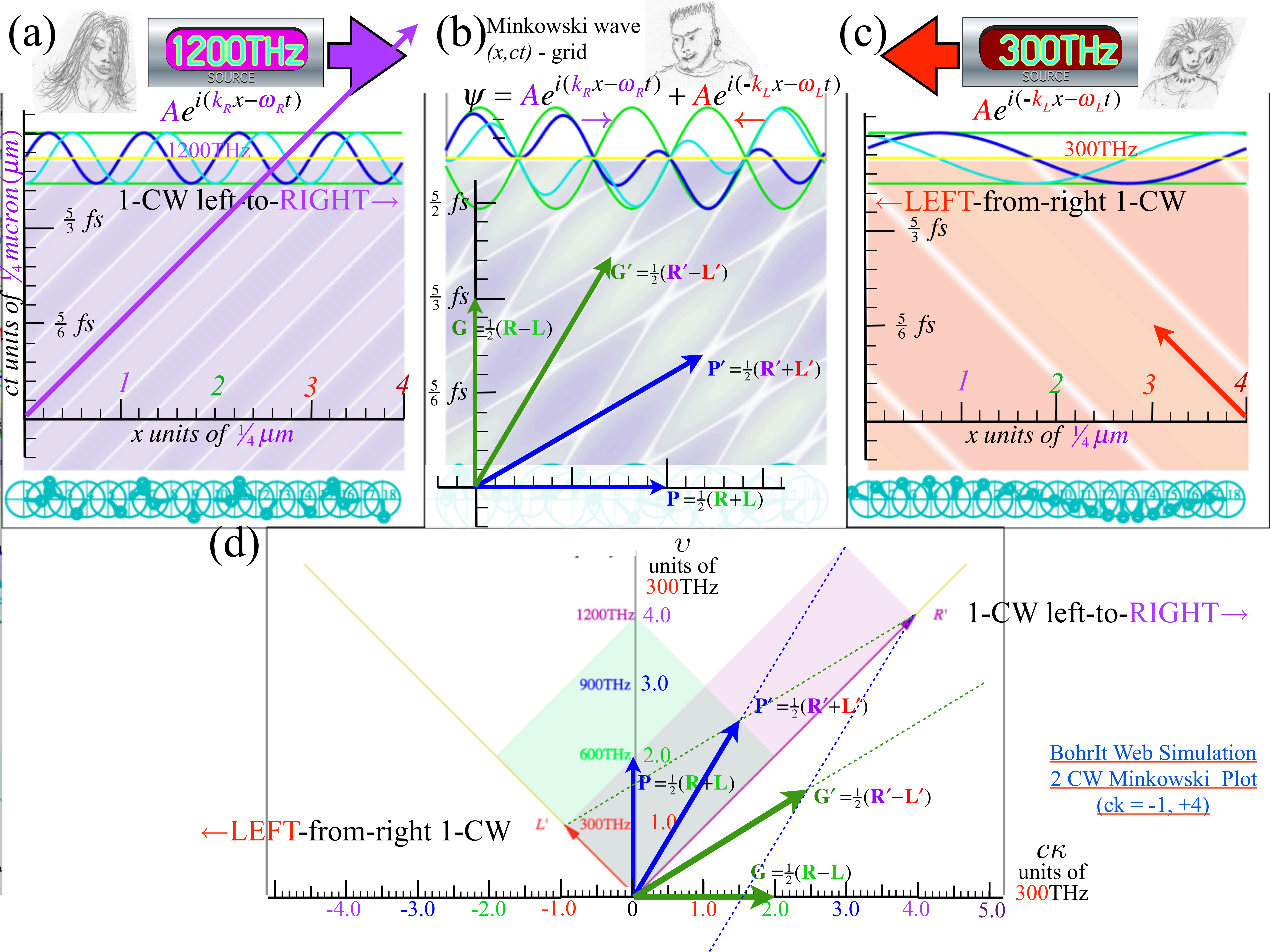


right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
x
R′
L′
Rapidly moving Bob sees...
...transformed (P′,G′) grid
G′
P′
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
...Doppler shifts give
Lorentz transformation
of both these graphs
Per-Spacetime
(ck,ω)
Frequency ω
1200THz
900THz
600THz
300THz
right-moving wave
(ck_R, ω_R)
R′=(+4c,4)
P′=½(R′+ L′)
G′=½(R′-L′)
left-moving wave
(ck_L, ω_L)
(-1c,1)=L′
-5 -4 -3 -2 -1 0 1 2 3 4 5
Wavevector ck
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

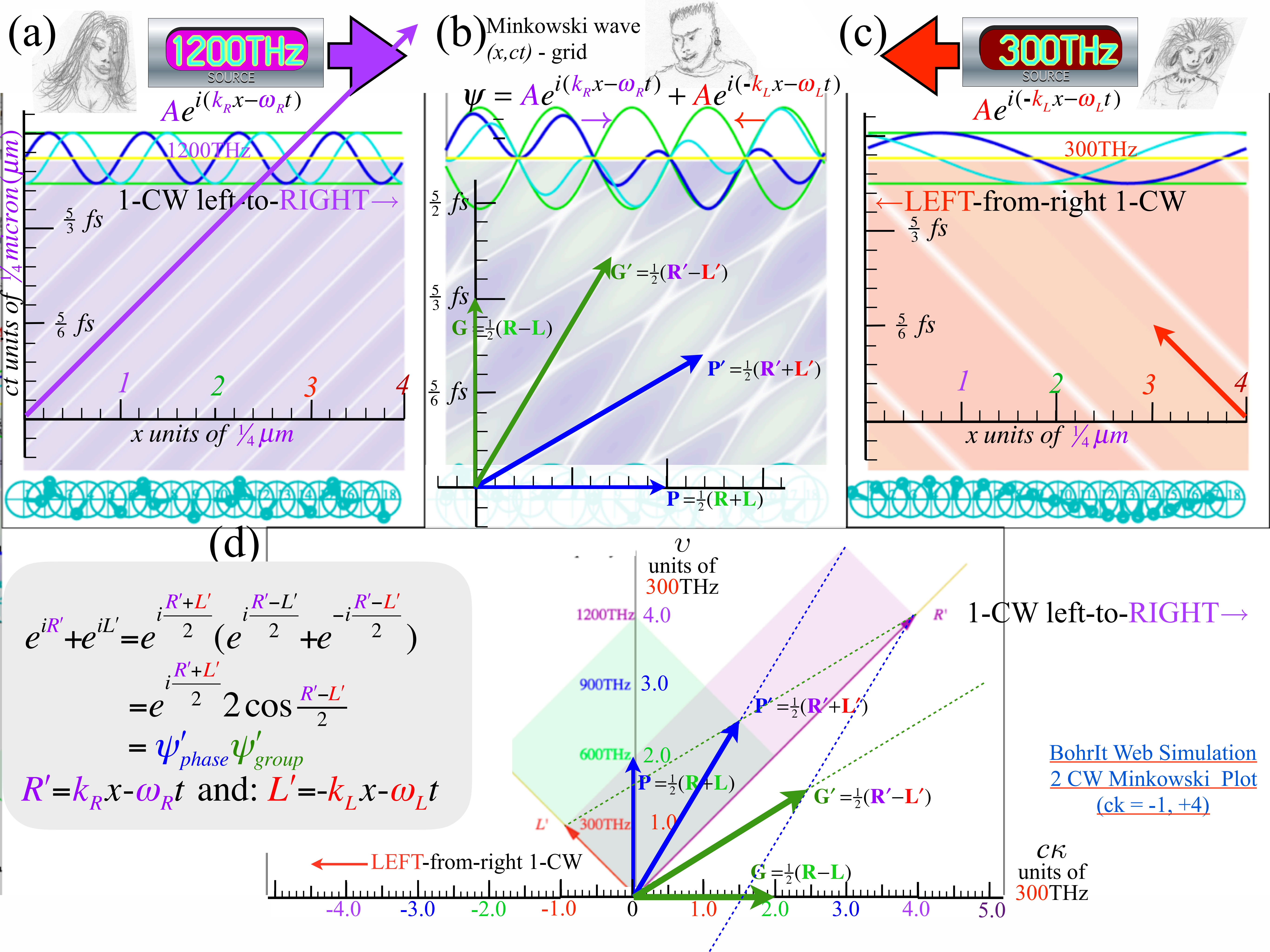
Three scenarios that look the same to Bob



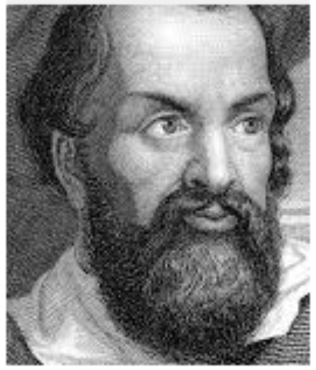
Much cheaper (and safer) to do the 3rd scenario!$!



(a)
1200THz
SOURCE
$Ae^{i(k_R x-\omega_R t)}$
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron (μm)
x units of ¼ μm
(b) Minkowski wave (x,ct) - grid
$\psi = Ae^{i(k_R x-\omega_R t)} + Ae^{i(-k_L x-\omega_L t)}$
$G' = \frac{1}{2}(R'-L')$
$G = \frac{1}{2}(R-L)$
$P' = \frac{1}{2}(R'+L')$
$P = \frac{1}{2}(R+L)$
(c)
300THz
SOURCE
$Ae^{i(-k_L x-\omega_L t)}$
300THz
←LEFT-from-right 1-CW
x units of ¼ μm
(d)
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
1-CW left-to-RIGHT→
←LEFT-from-right 1-CW
cκ units of 300THz
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

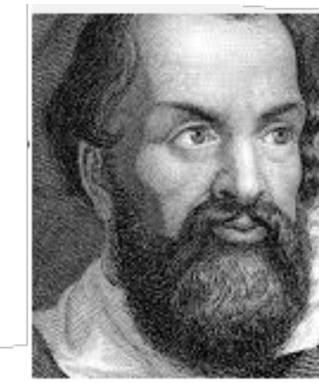

(a)
1200THz
SOURCE
$Ae^{i(k_R x-\omega_R t)}$
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron (μm)
5/3 fs
5/6 fs
1
2
3
4
x units of ¼ μm
(b)
Minkowski wave (x,ct) - grid
$\psi = Ae^{i(k_R x-\omega_R t)} + Ae^{i(-k_L x-\omega_L t)}$
5/2 fs
5/3 fs
5/6 fs
$\mathbf{G}' = \frac{1}{2}(\mathbf{R}'-\mathbf{L}')$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
$\mathbf{P}' = \frac{1}{2}(\mathbf{R}'+\mathbf{L}')$
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
(c)
300THz
SOURCE
$Ae^{i(-k_L x-\omega_L t)}$
300THz
←LEFT-from-right 1-CW
5/3 fs
5/6 fs
1
2
3
4
x units of ¼ μm
(d)
$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$
$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$
$$=\psi'_{phase}\psi'_{group}$$
$R'=k_R x-\omega_R t$ and: $L'=-k_L x-\omega_L t$
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R'
L'
1-CW left-to-RIGHT→
$\mathbf{P}' = \frac{1}{2}(\mathbf{R}'+\mathbf{L}')$
$\mathbf{P} = \frac{1}{2}(\mathbf{R}+\mathbf{L})$
$\mathbf{G}' = \frac{1}{2}(\mathbf{R}'-\mathbf{L}')$
$\mathbf{G} = \frac{1}{2}(\mathbf{R}-\mathbf{L})$
←LEFT-from-right 1-CW
cκ units of 300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

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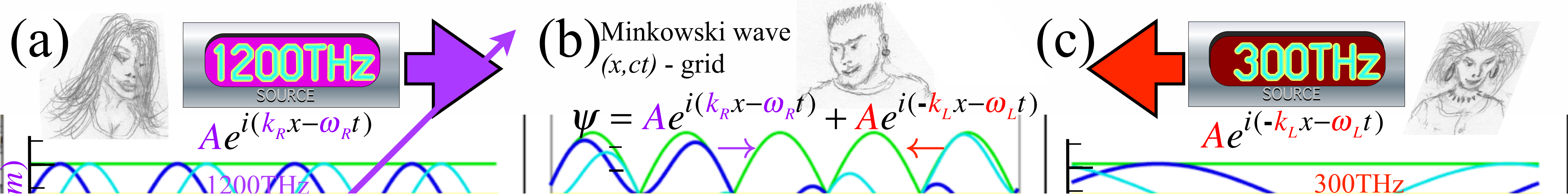

$$\mathbf{P}'=\begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'+\mathbf{L}')=\upsilon_A\begin{pmatrix} \frac{1}{2}(e^{\rho}+e^{-\rho}) \\ \frac{1}{2}(e^{\rho}-e^{-\rho}) \end{pmatrix}=\upsilon_A\begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}_{\text{Bob's View}} \text{ or: } \upsilon_A\begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\text{Alice's View}}$$

$$\mathbf{G}'=\begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'-\mathbf{L}')=\upsilon_A\begin{pmatrix} \frac{1}{2}(e^{\rho}-e^{-\rho}) \\ \frac{1}{2}(e^{\rho}+e^{-\rho}) \end{pmatrix}=\upsilon_A\begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}_{\text{Bob's View}} \text{ or: } \upsilon_A\begin{pmatrix} 0 \\ 1 \end{pmatrix}_{\text{Alice's View}}$$

$$e^{-\rho}=\frac{1}{2} \qquad e^{\rho}=2$$

(d)

$$\mathbf{L}'=\upsilon_A\begin{pmatrix} -e^{-\rho} \\ e^{-\rho} \end{pmatrix}=\upsilon_A\begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$
$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$
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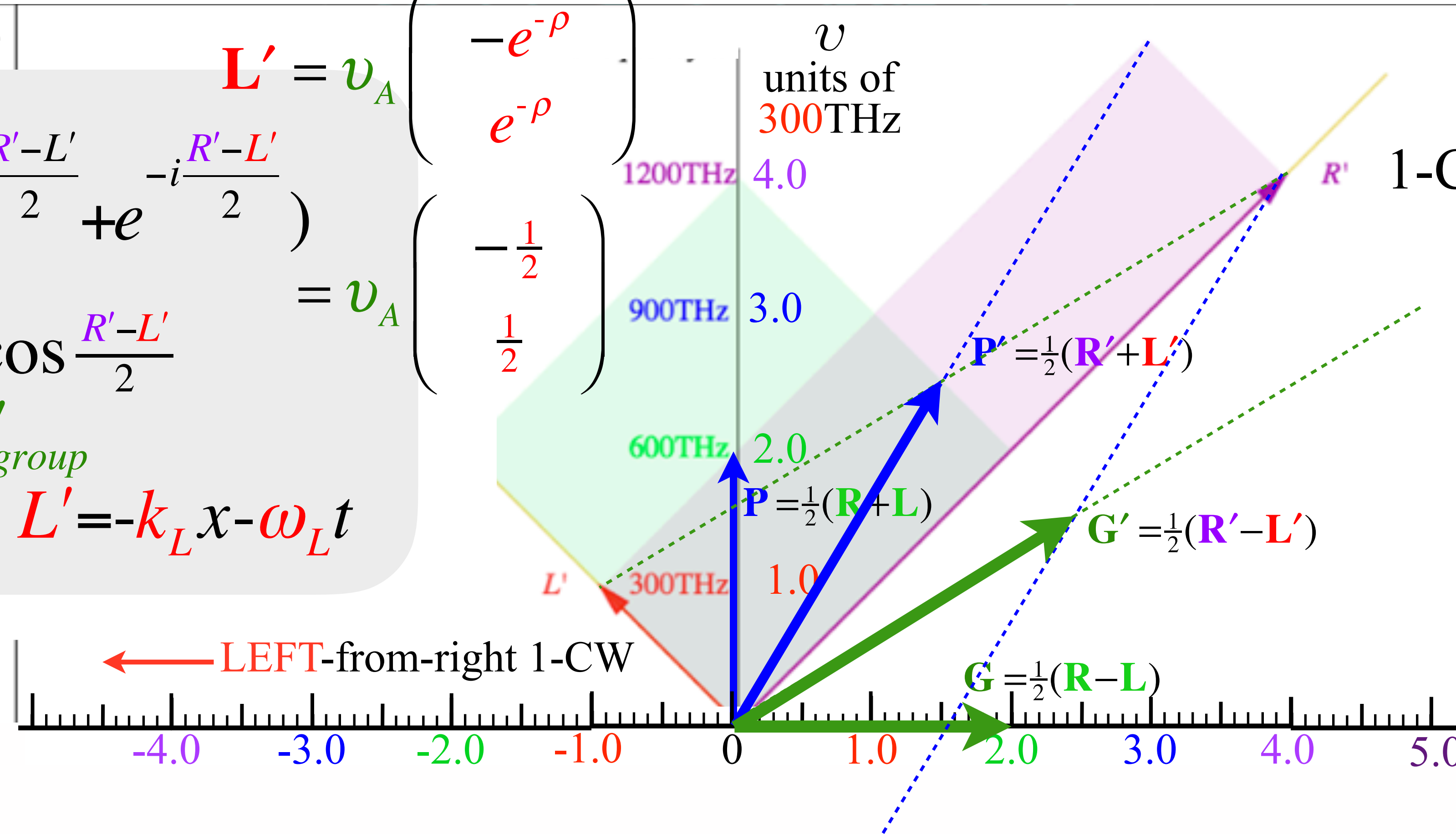
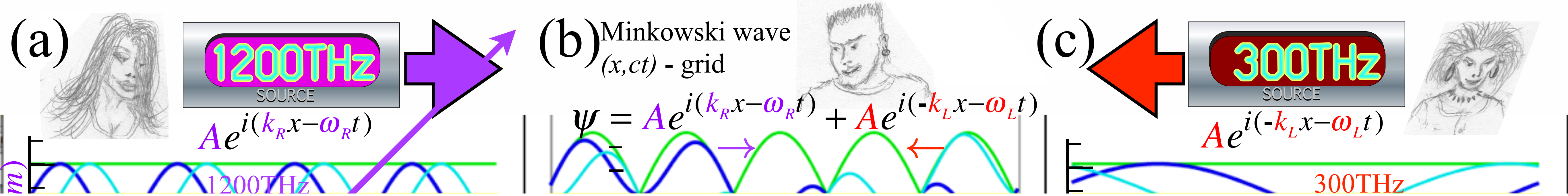
$R'=k_R x-\omega_R t$ and: $L'=-k_L x-\omega_L t$



1-CW left-to-RIGHT→

$$\mathbf{R}'=\upsilon_A\begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix}=\upsilon_A\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$



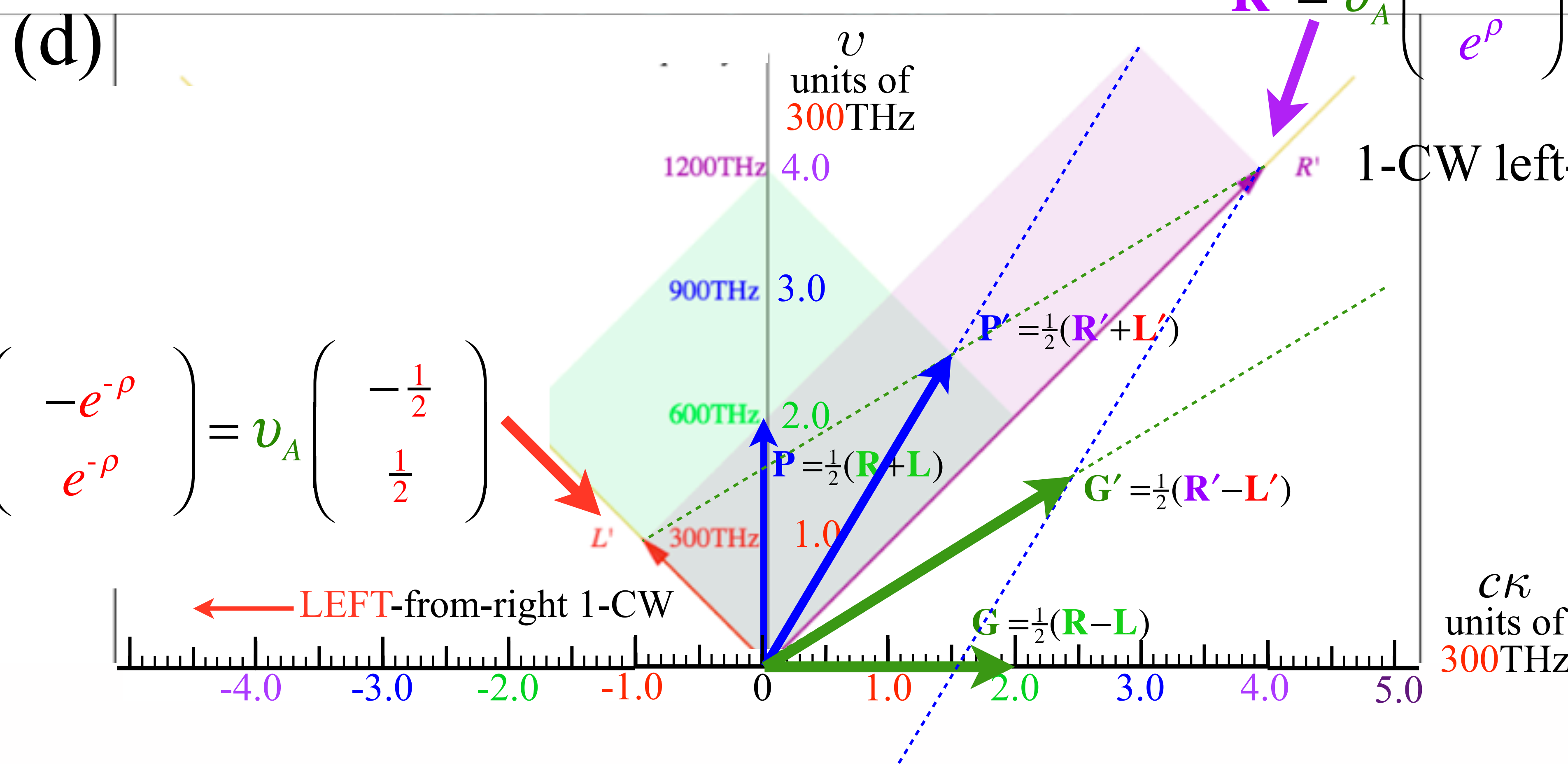
$$\mathbf{P}'=\begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'+\mathbf{L}')=\upsilon_A\begin{pmatrix} \tfrac{1}{2}(e^{\rho}+e^{-\rho}) \\ \tfrac{1}{2}(e^{\rho}-e^{-\rho}) \end{pmatrix}=\upsilon_A\begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} \tfrac{5}{4} \\ \tfrac{3}{4} \end{pmatrix}_{\substack{Bob's\\View}} \quad \text{or: } \upsilon_A\begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\substack{Alice's\\View}}$$

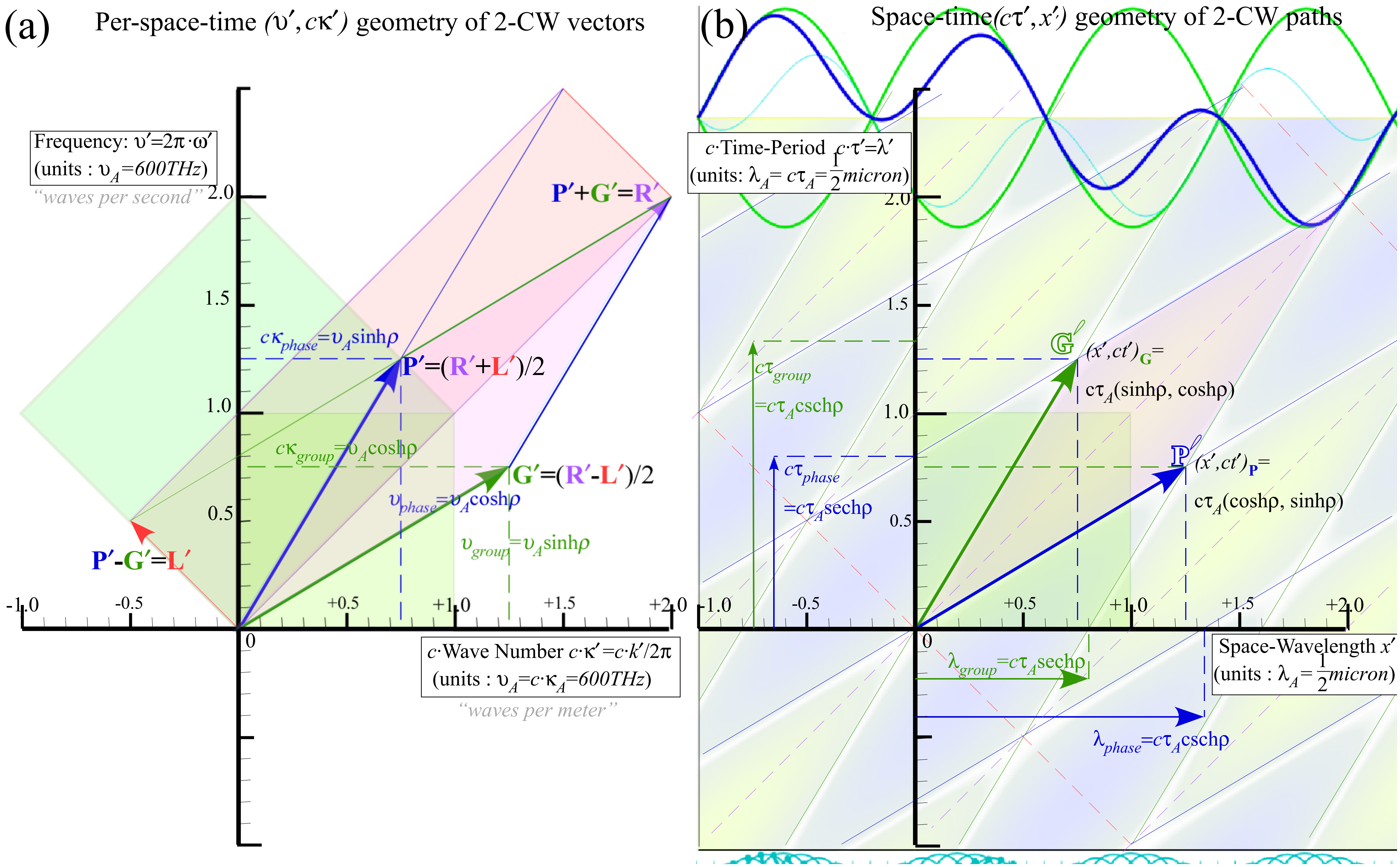
$$\mathbf{G}'=\begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'-\mathbf{L}')=\upsilon_A\begin{pmatrix} \tfrac{1}{2}(e^{\rho}-e^{-\rho}) \\ \tfrac{1}{2}(e^{\rho}+e^{-\rho}) \end{pmatrix}=\upsilon_A\begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} \tfrac{3}{4} \\ \tfrac{5}{4} \end{pmatrix}_{\substack{Bob's\\View}} \quad \text{or: } \upsilon_A\begin{pmatrix} 0 \\ 1 \end{pmatrix}_{\substack{Alice's\\View}}$$

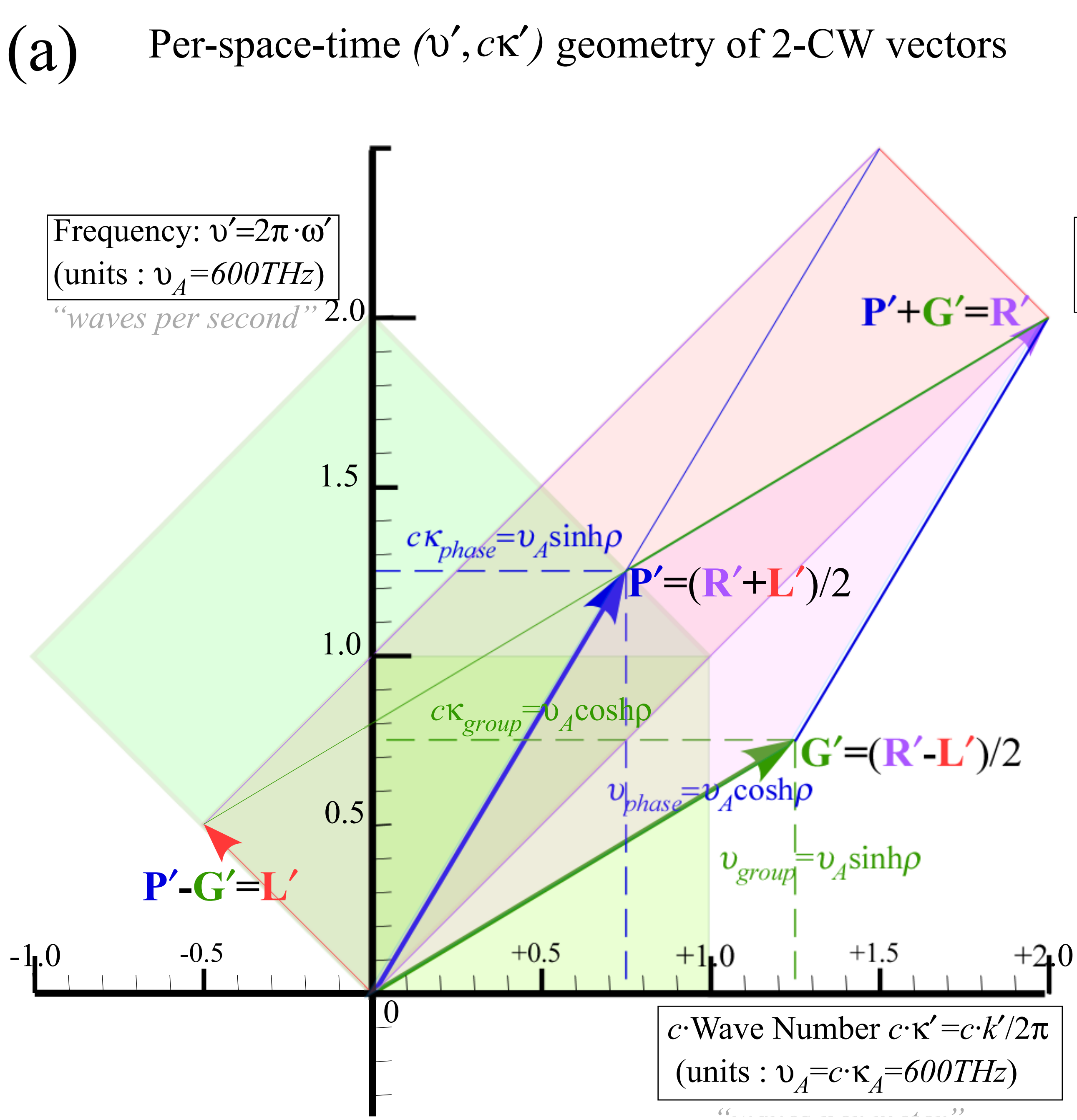
$$e^{-\rho}=\frac{1}{2} \qquad e^{\rho}=2$$

$$\mathbf{R}'=\upsilon_A\begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix}=\upsilon_A\begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\mathbf{L}'=\upsilon_A\begin{pmatrix} -e^{-\rho} \\ e^{-\rho} \end{pmatrix}=\upsilon_A\begin{pmatrix} -\tfrac{1}{2} \\ \tfrac{1}{2} \end{pmatrix}$$

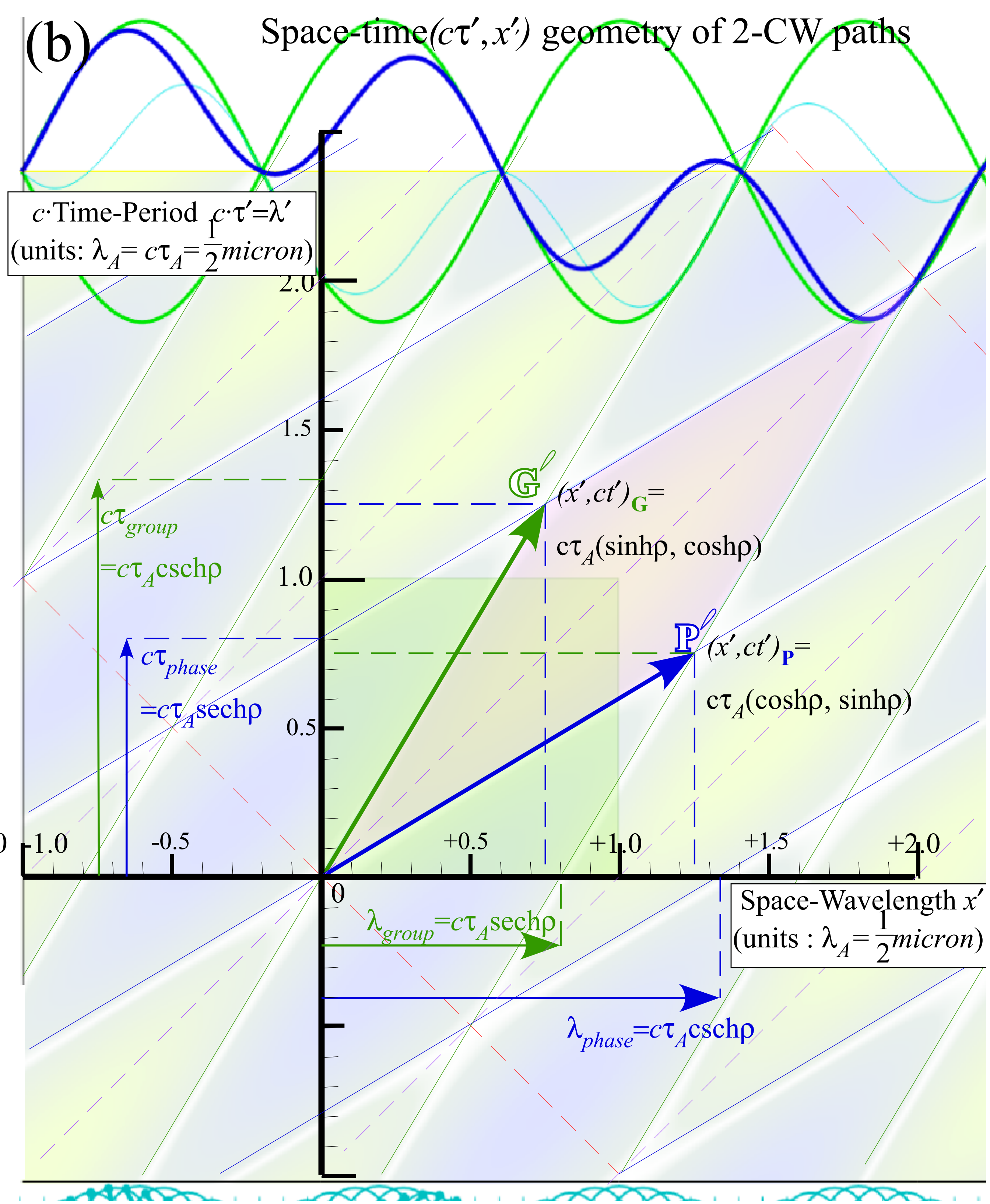



(a) Per-space-time (υ′, cκ′) geometry of 2-CW vectors
(b) Space-time (cτ′, x′) geometry of 2-CW paths
Frequency: υ′=2π·ω′
(units : υA=600THz)
"waves per second"
P′+G′=R′
P′=(R′+L′)/2
G′=(R′-L′)/2
P′-G′=L′
cκphase=υAsinhρ
cκgroup=υAcoshρ
υphase=υAcoshρ
υgroup=υAsinhρ
c·Wave Number c·κ′=c·k′/2π
(units : υA=c·κA=600THz)
"waves per meter"
c·Time-Period c·τ′=λ′
(units: λA= cτA= 1/2 micron)
cτgroup
=cτAcschρ
cτphase
=cτAsechρ
G′ (x′,ct′)G=
cτA(sinhρ, coshρ)
P′ (x′,ct′)P=
cτA(coshρ, sinhρ)
λgroup=cτAsechρ
λphase=cτAcschρ
Space-Wavelength x′
(units : λA= 1/2 micron)



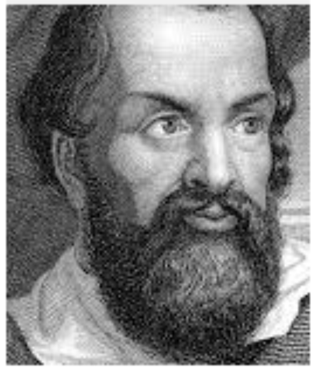
The slope of Bob's group vector **G′** in $(c\kappa,\upsilon)$-plot is actual group wave velocity in c-units.

$$\frac{V^{group}}{c} = \frac{\upsilon'_{group}}{c\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \tanh\rho = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5} \equiv \frac{u}{c} \equiv \beta$$



Group vector G′ in (x,ct)-plot has 3/5 slope relative to time axis

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
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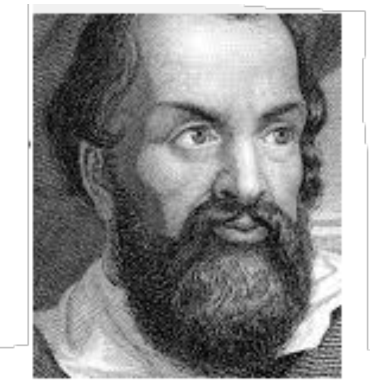

Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

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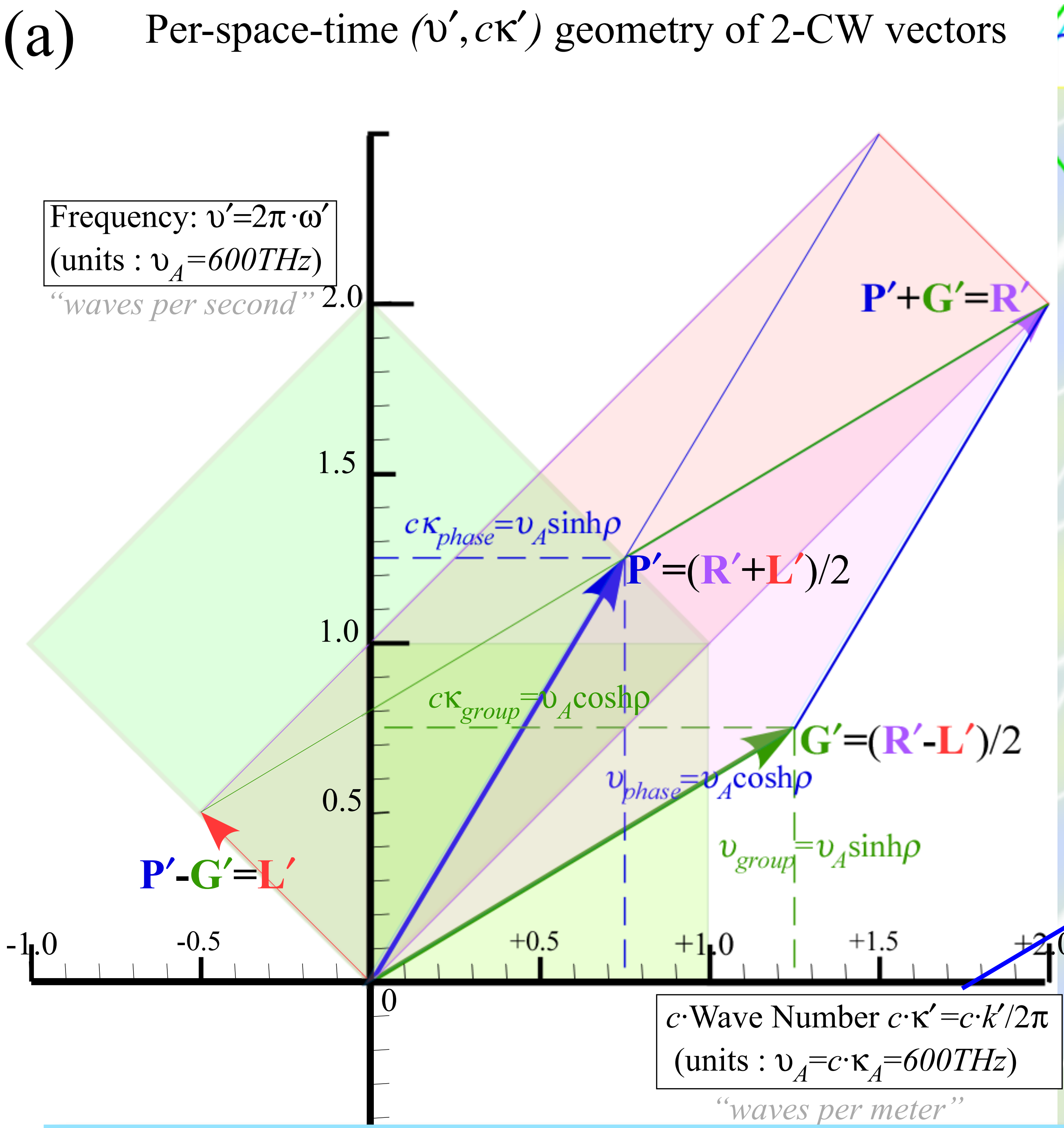
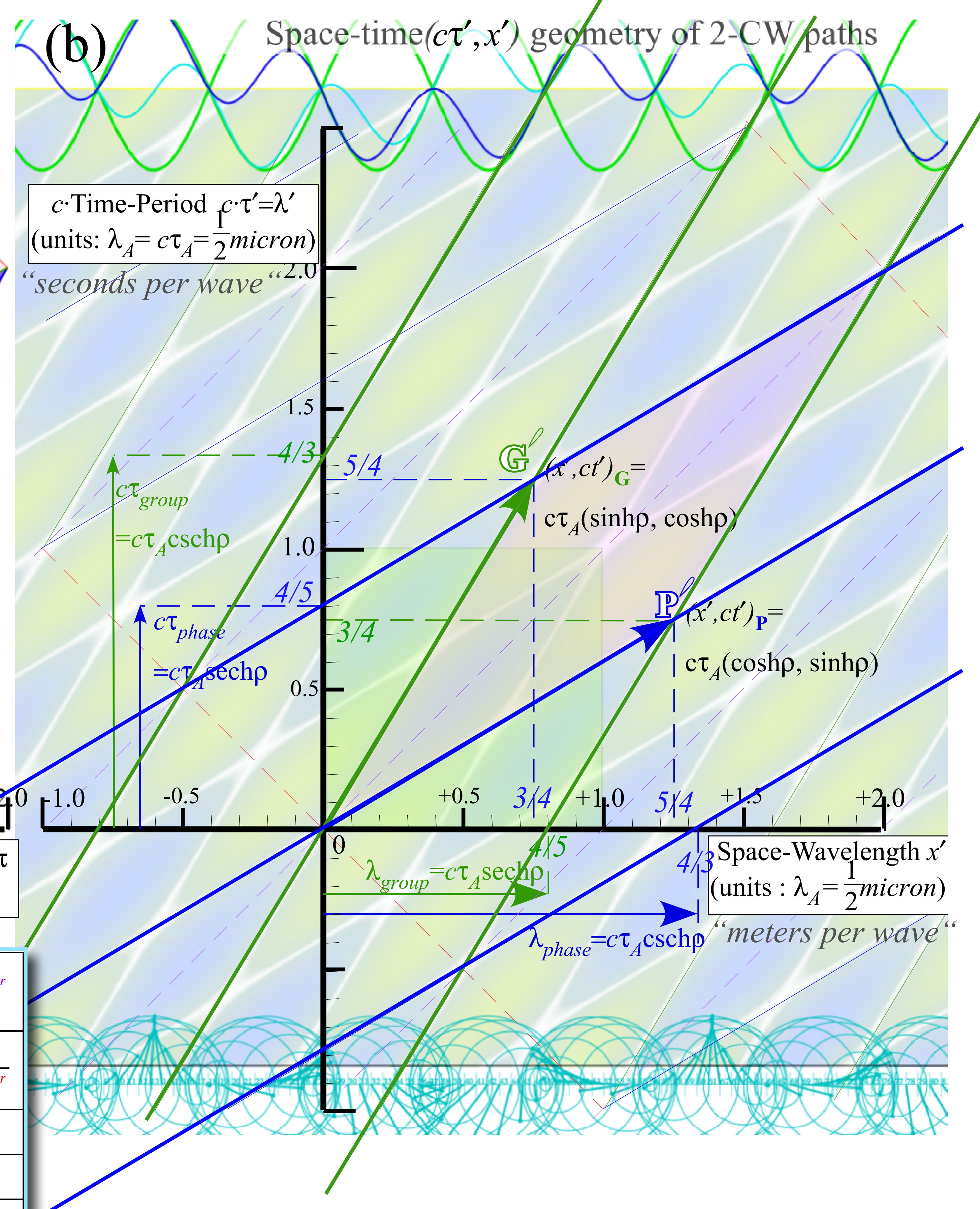




group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Fig. 11 in text Relawavity...

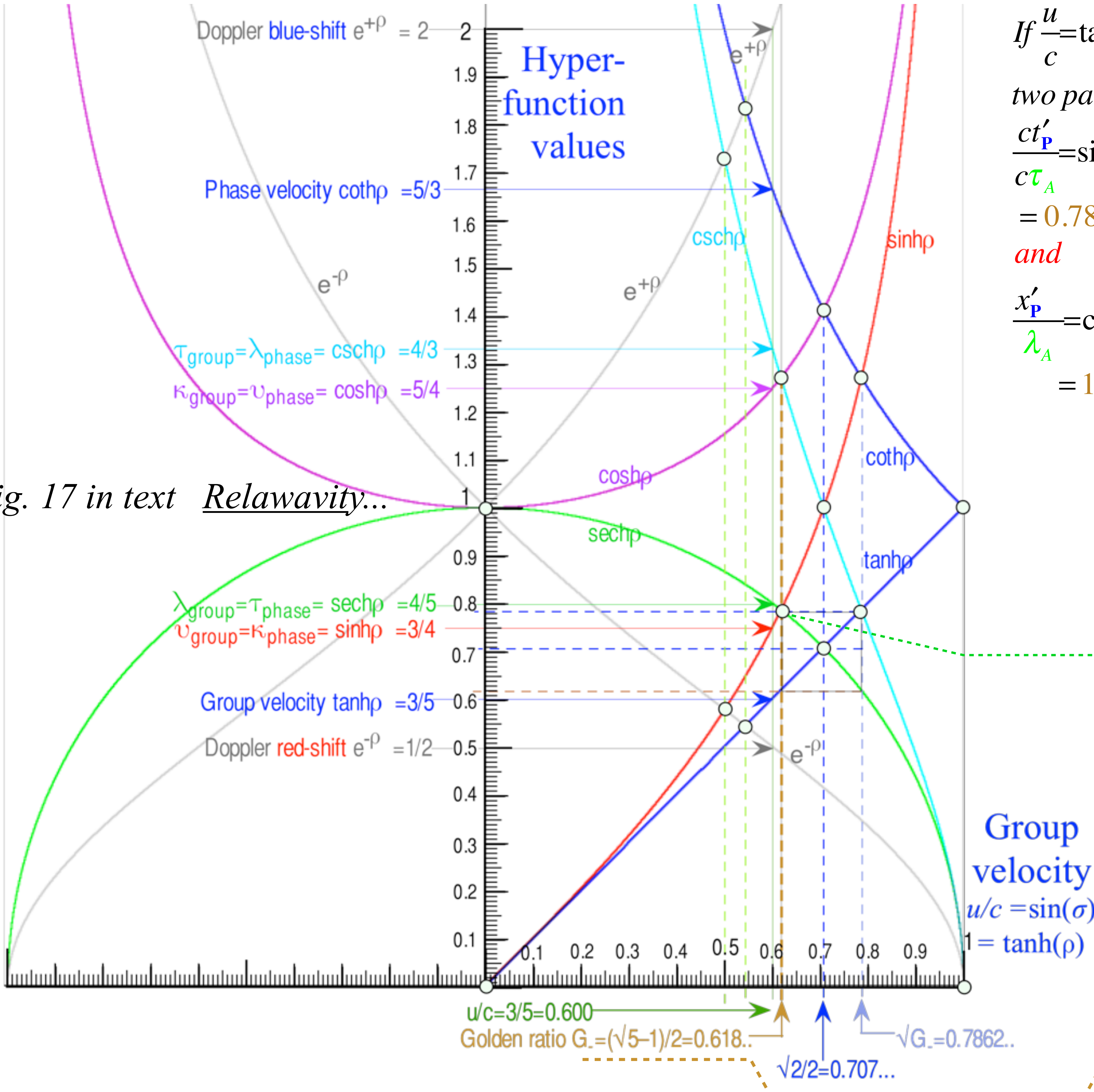


Fig. 17 in text *Relawavity...*

$$\text{If } \frac{u}{c} = \tanh\rho = 0.618\ldots(\text{Golden-Mean } G\_)$$

two parameters become exactly equal :

$$\frac{ct'_P}{c\tau_A} = \sinh\rho = \frac{\lambda_{group}}{\lambda_A} = \frac{\tau_{phase}}{\tau_A} = \operatorname{sech}\rho$$

$$= 0.786.. = \sqrt{G\_} \qquad = 0.786..$$

and

$$\frac{x'_P}{\lambda_A} = \cosh\rho = \frac{\lambda_{phase}}{\lambda_A} = \frac{\tau_{group}}{\tau_A} = \operatorname{csch}\rho$$

$$= 1.272.. = 1/\sqrt{G\_} \qquad = 1.272..$$

Solve :

$\operatorname{sech}\rho = \sinh\rho$

or:

$\sinh\rho\cosh\rho = 1$

or:

$\sinh 2\rho = 2$

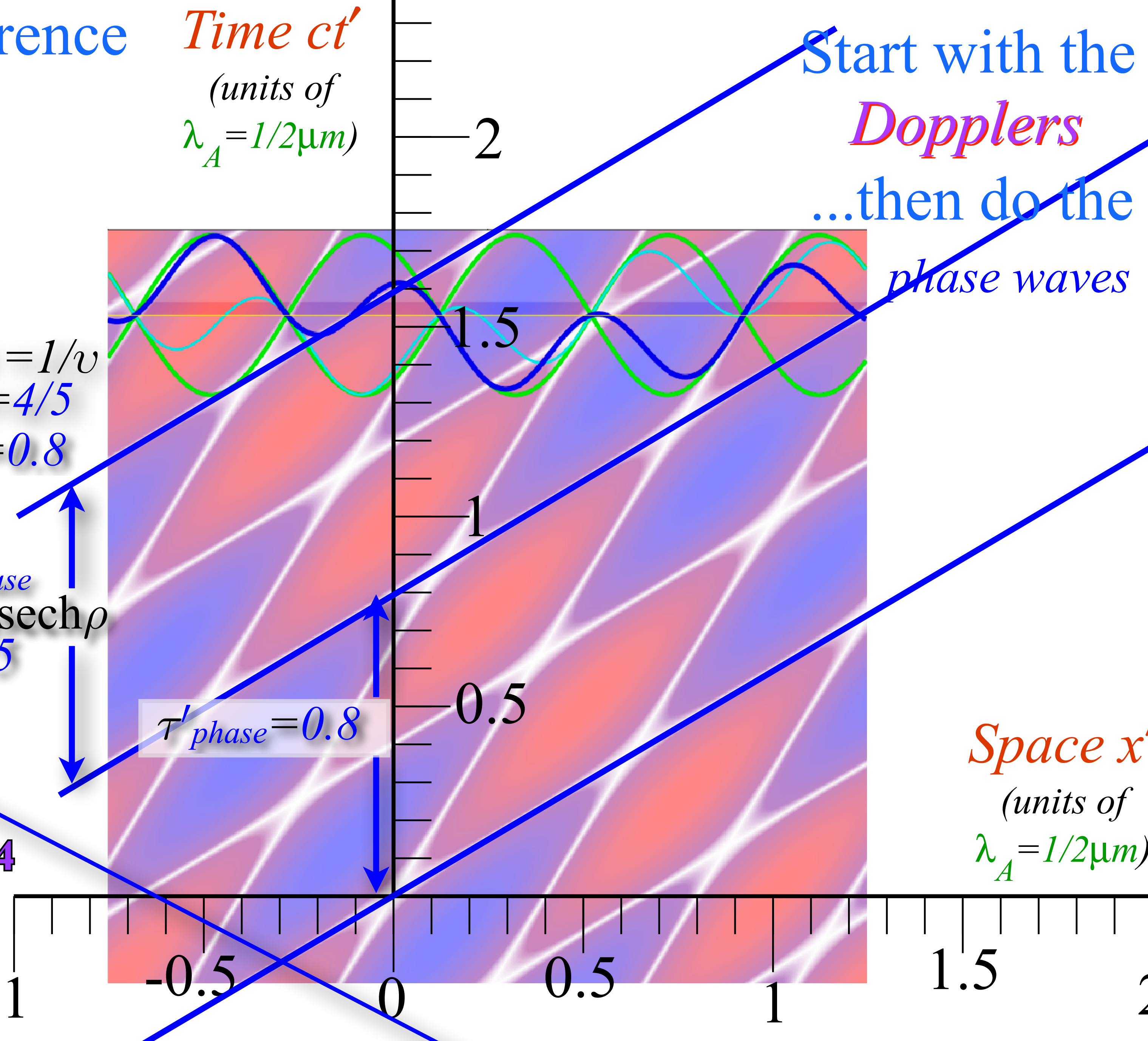
$$\rho = \frac{1}{2}\sinh^{-1}2 = 0.7218\ldots$$

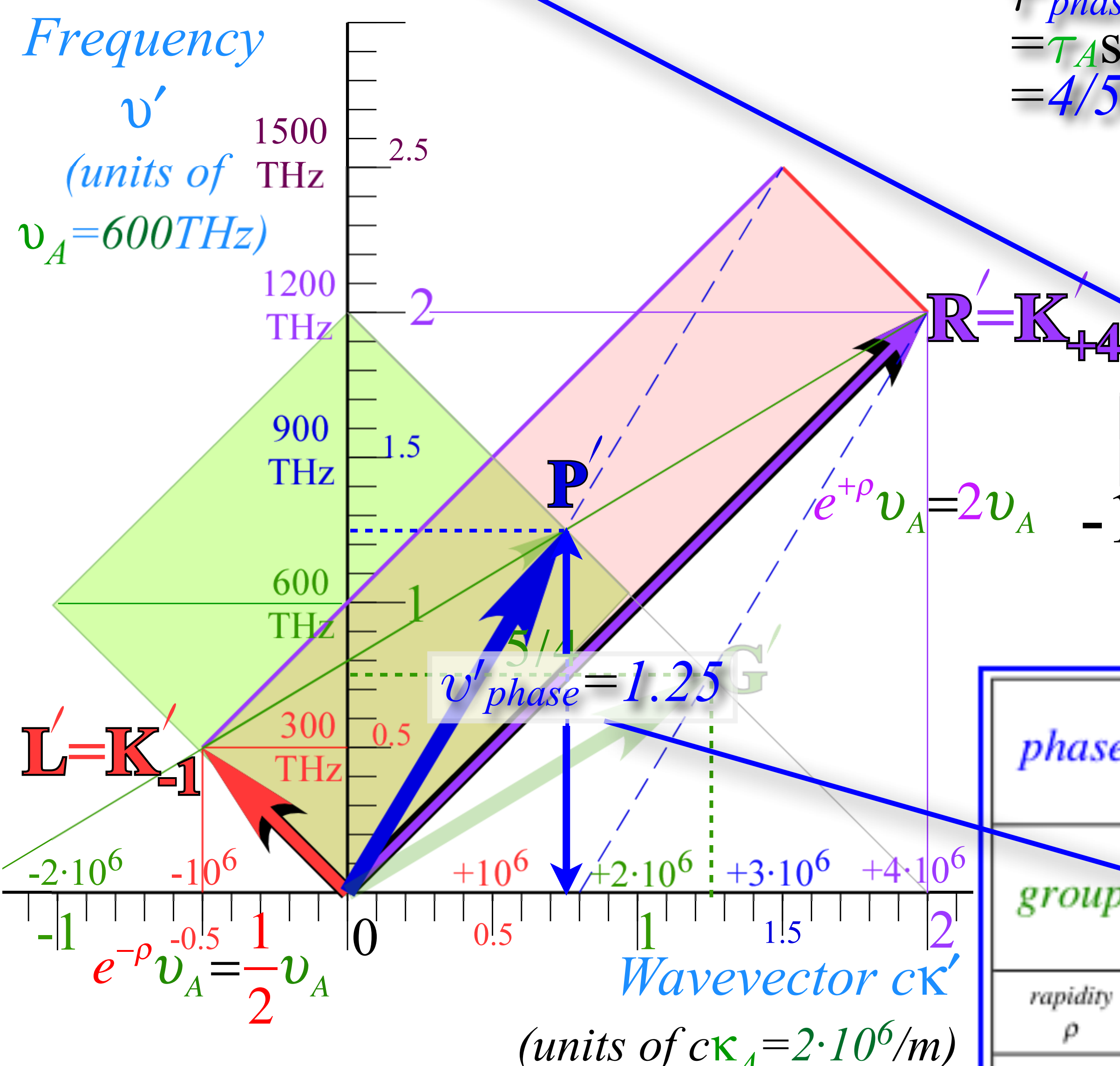
$$\tanh\rho = 0.618\ldots = \frac{\sqrt{5}-1}{2}$$

The 16 dimensions of 2CW interference

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$ flips to *Phase period* $\tau = 1/\upsilon$: $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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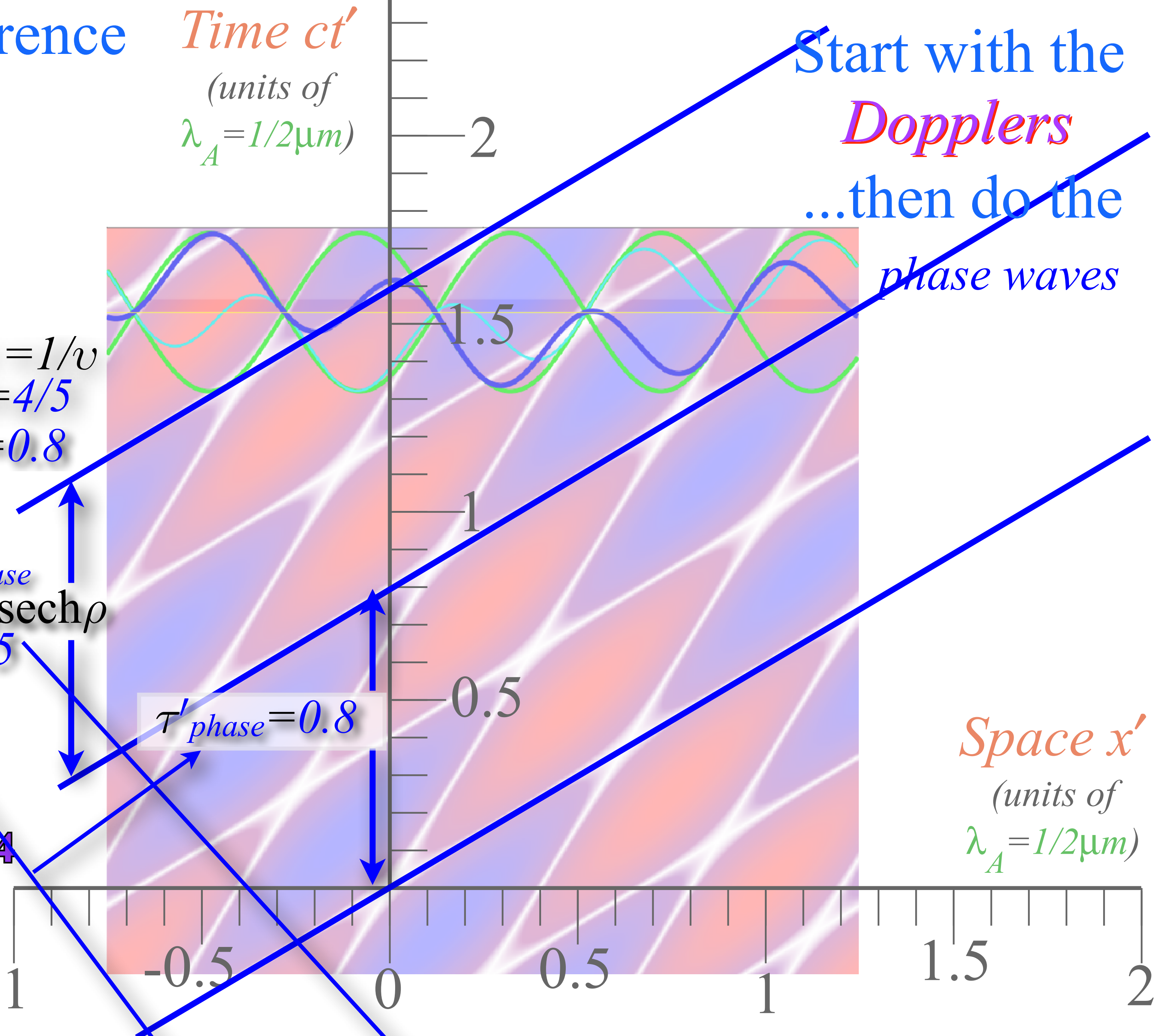
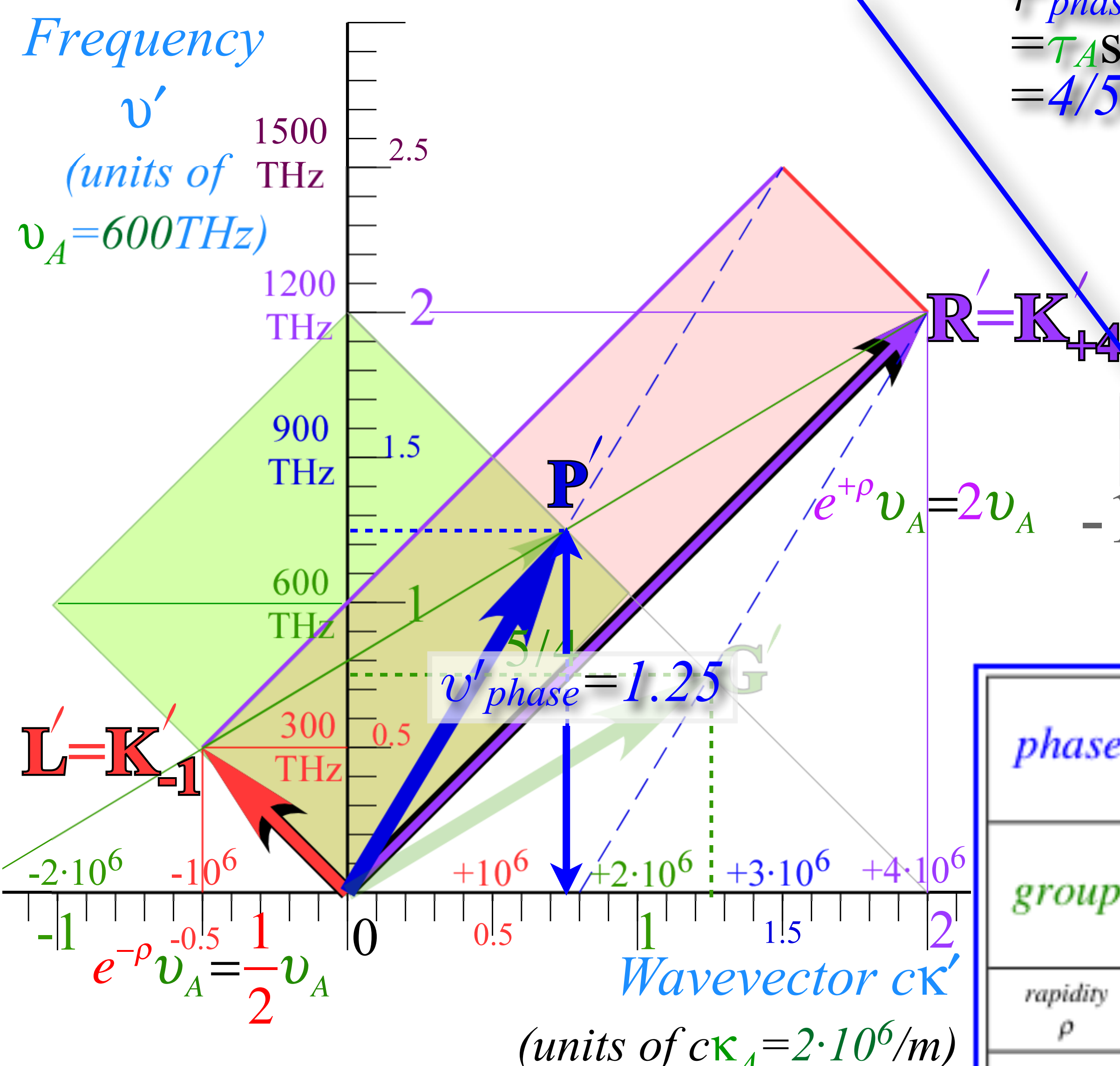
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

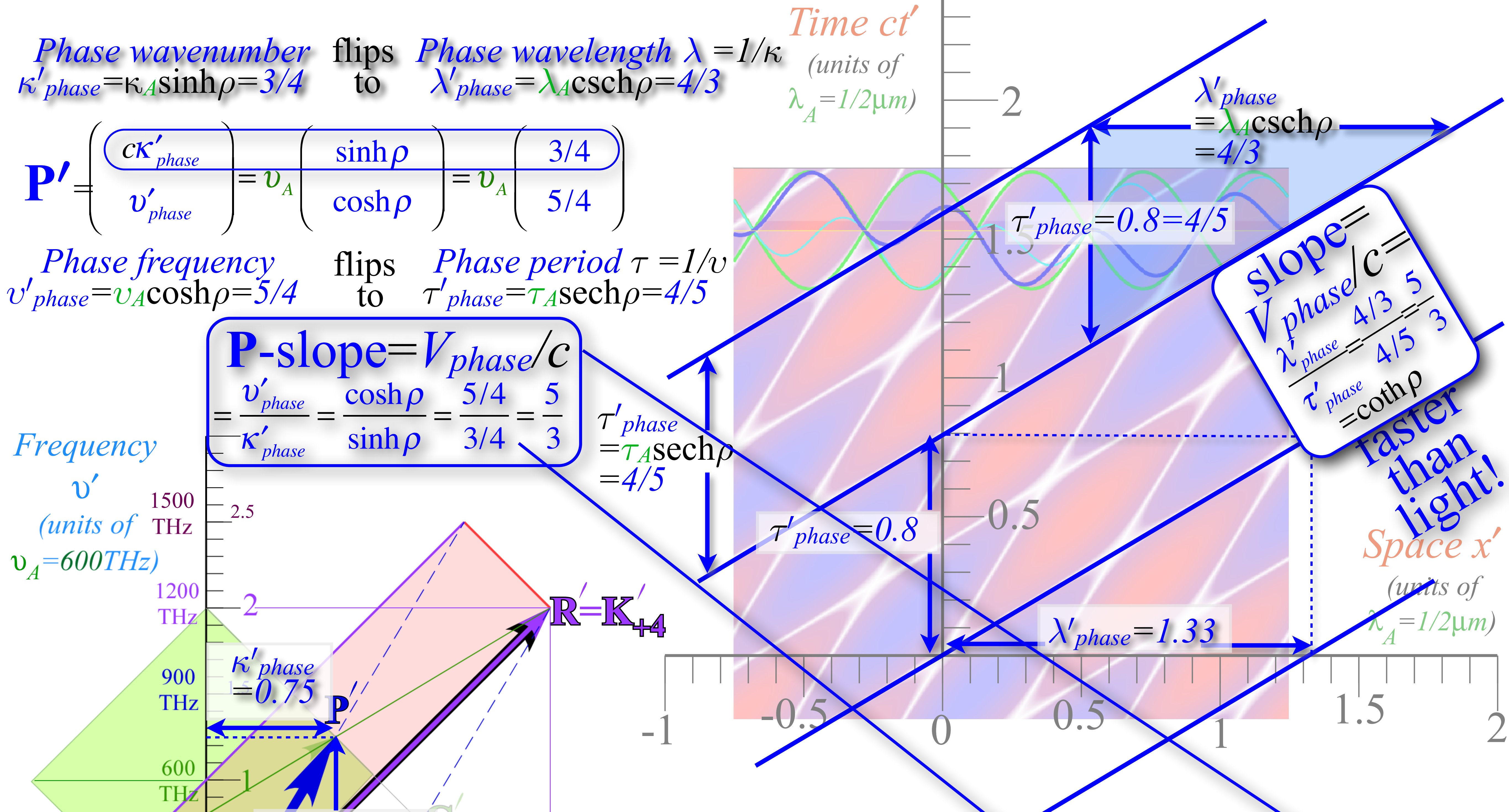
flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

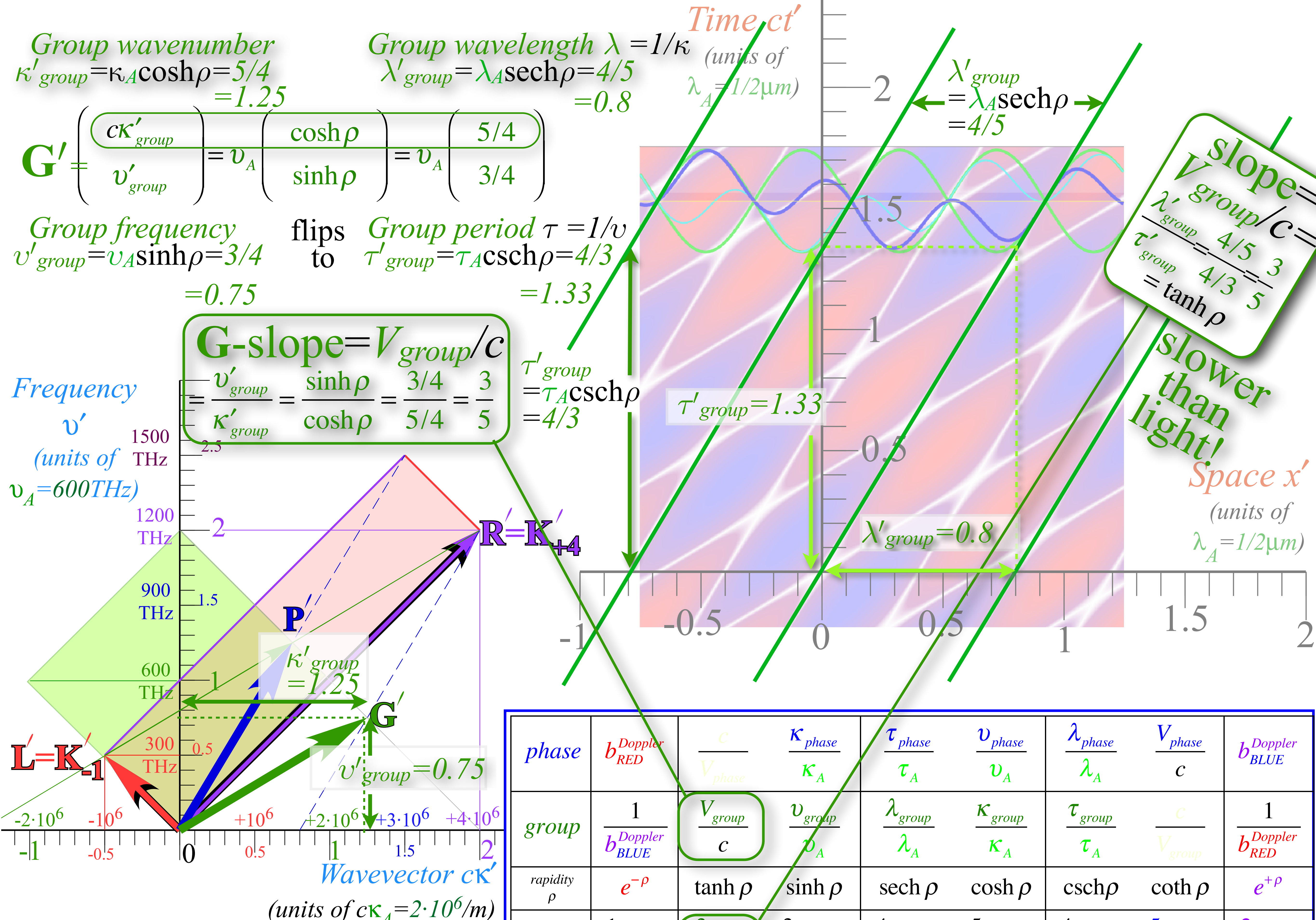




phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$



phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	tanh ρ	sinh ρ	sech ρ	cosh ρ	csch ρ	coth ρ	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0



phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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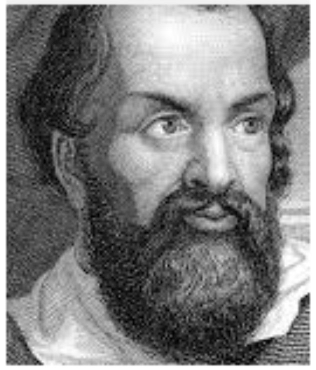
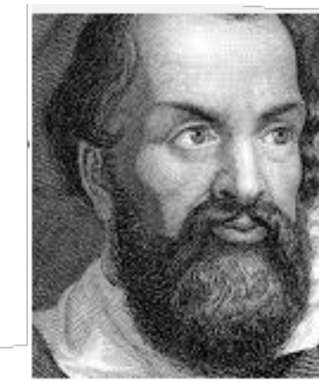
Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
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Optical interference "baseball-diamond" displays *phase* and *group* velocity
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Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

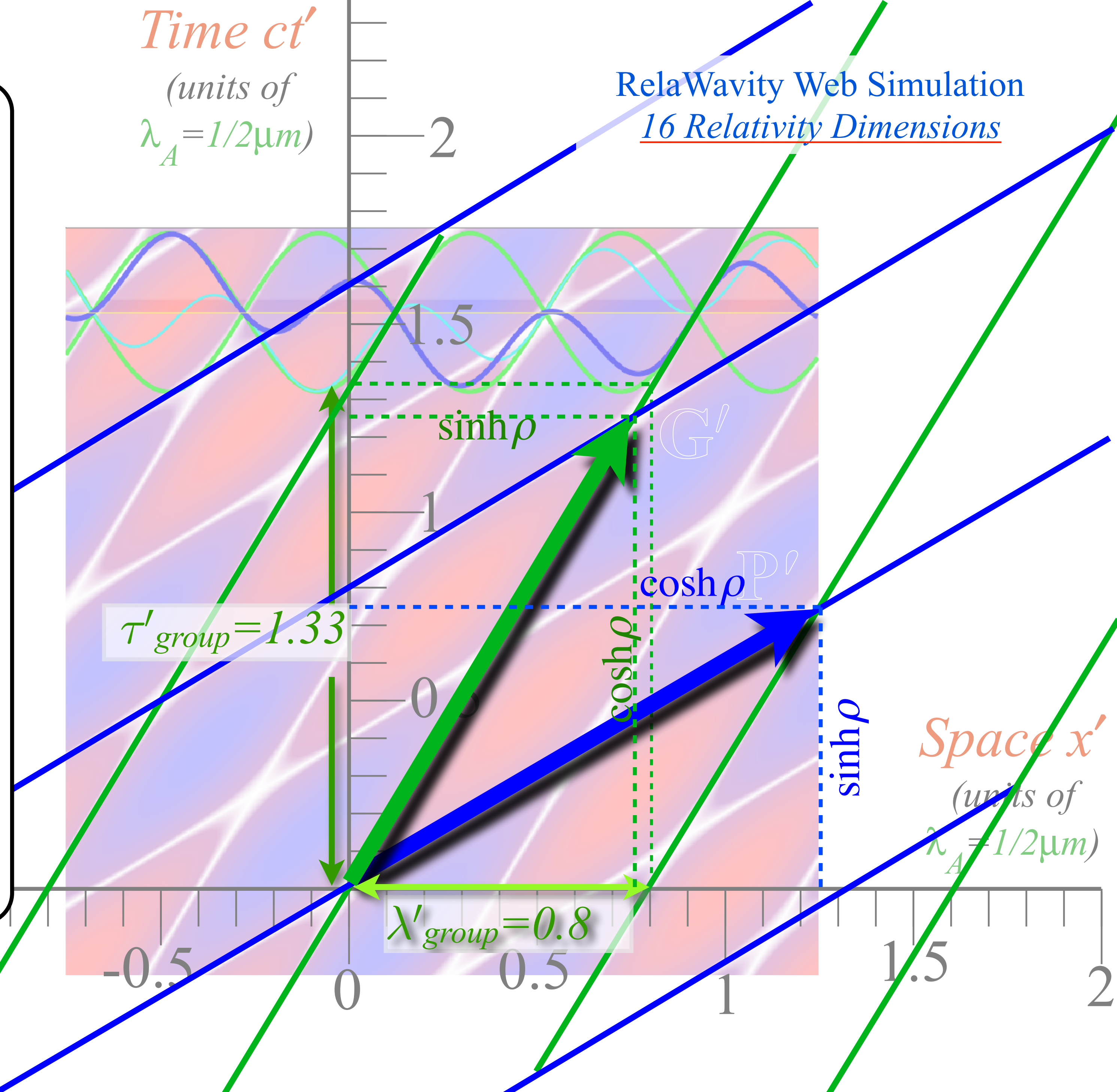
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

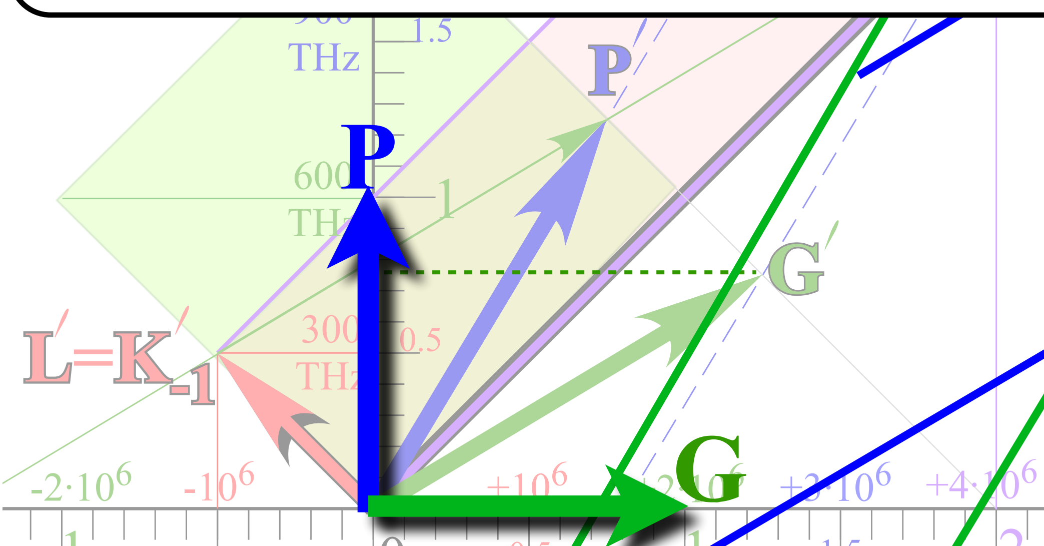
$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G} \sinh\rho + \mathbf{P} \cosh\rho$$



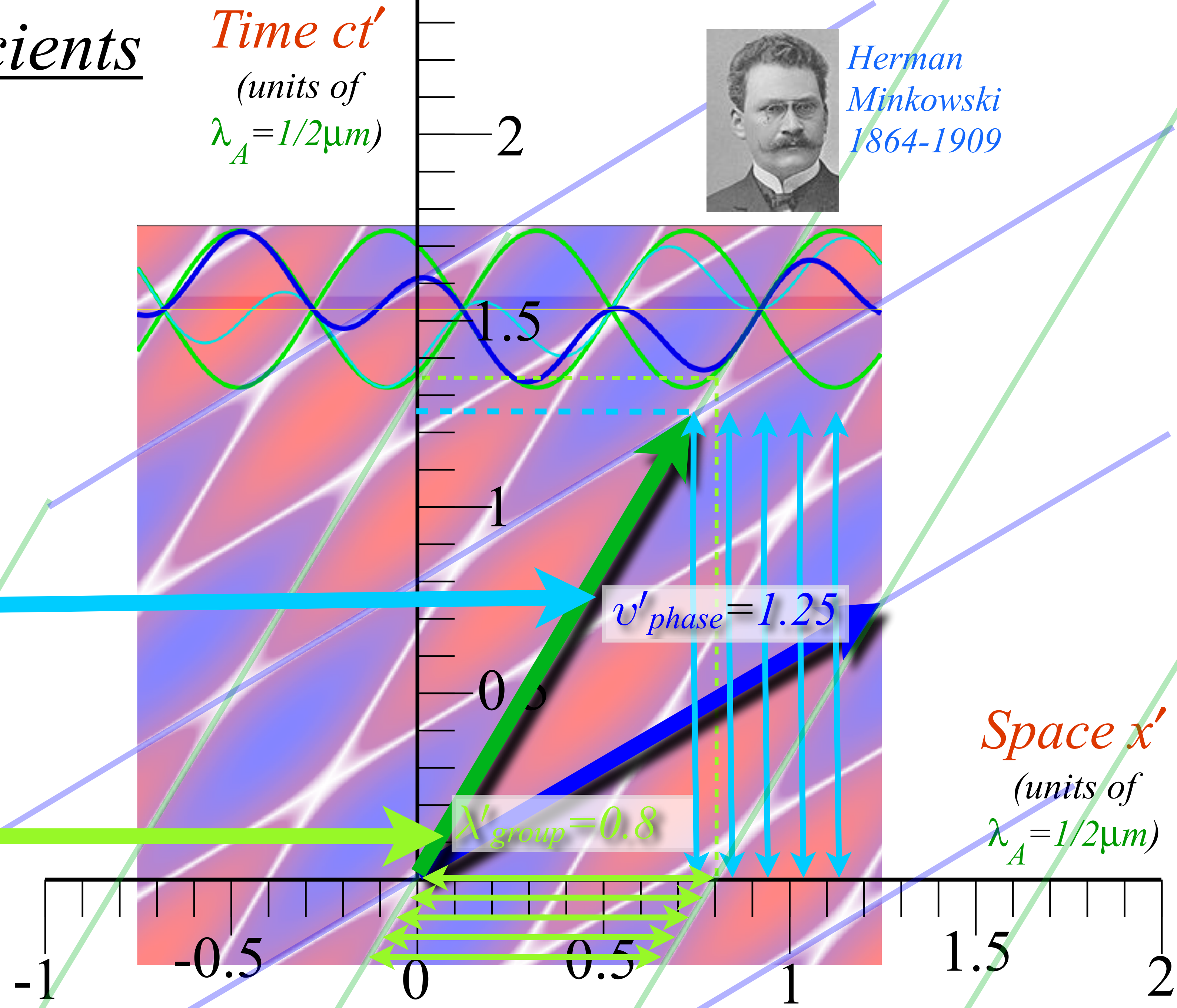


$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$

Lorentz transform matrix

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

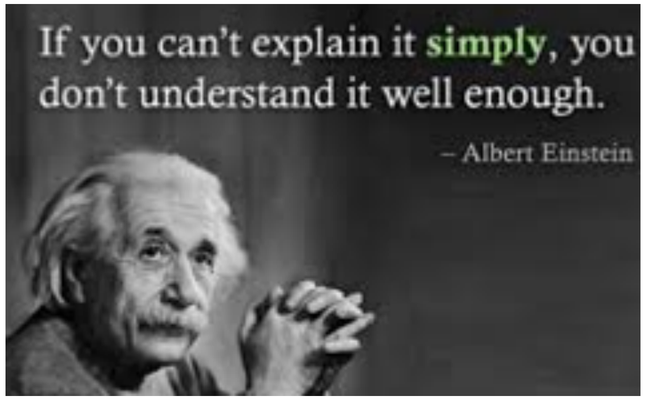
Two Famous-Name Coefficients



This number
is called an: Einstein
time-dilation
(dilated by 25% here)

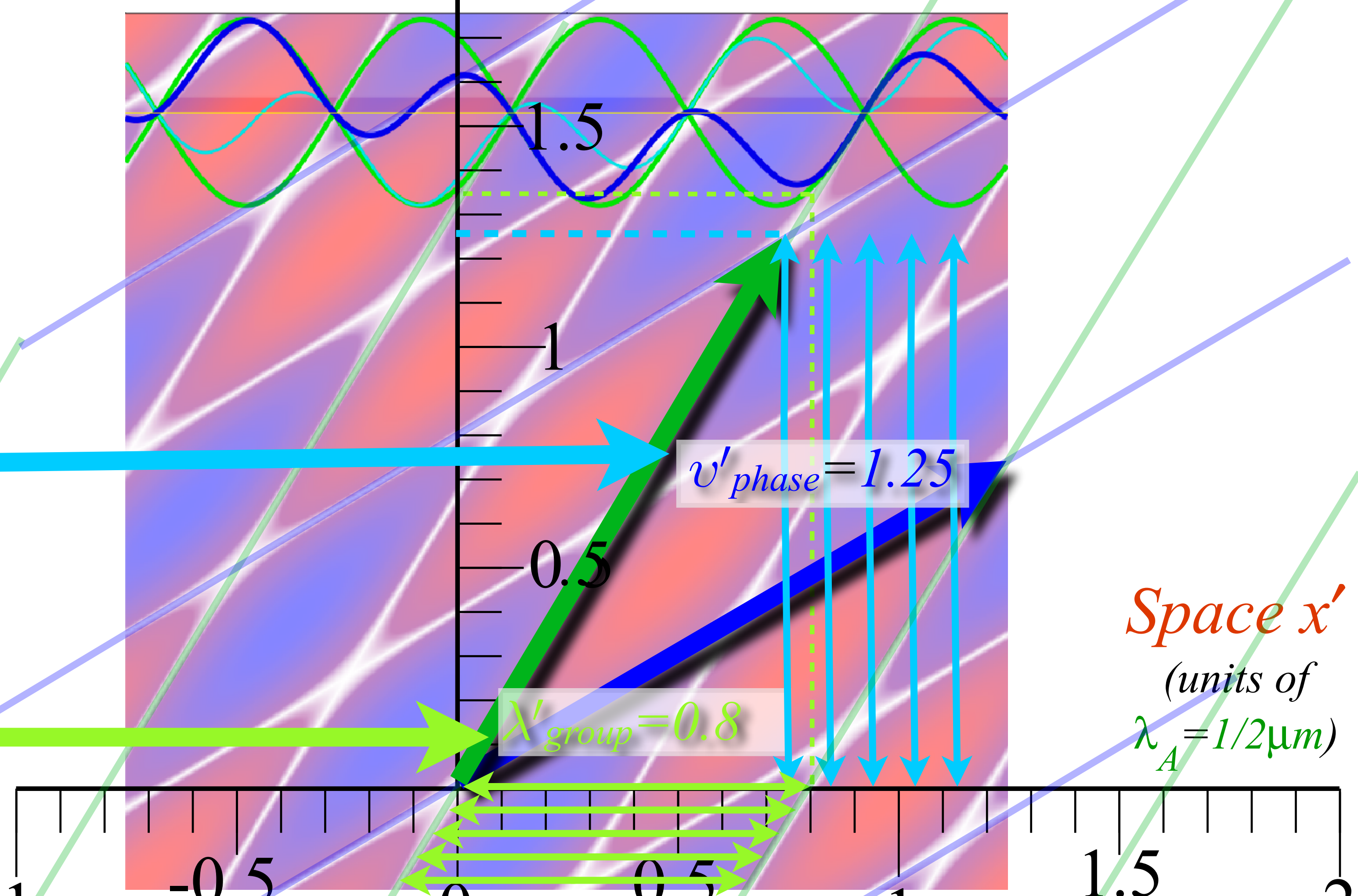
This number
is called a: Lorentz
length-contraction
(contracted by 20% here)

Two Famous-Name Coefficients



Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation**
(dilated by 25% here)

This number is called a: **Lorentz length-contraction**
(contracted by 20% here)

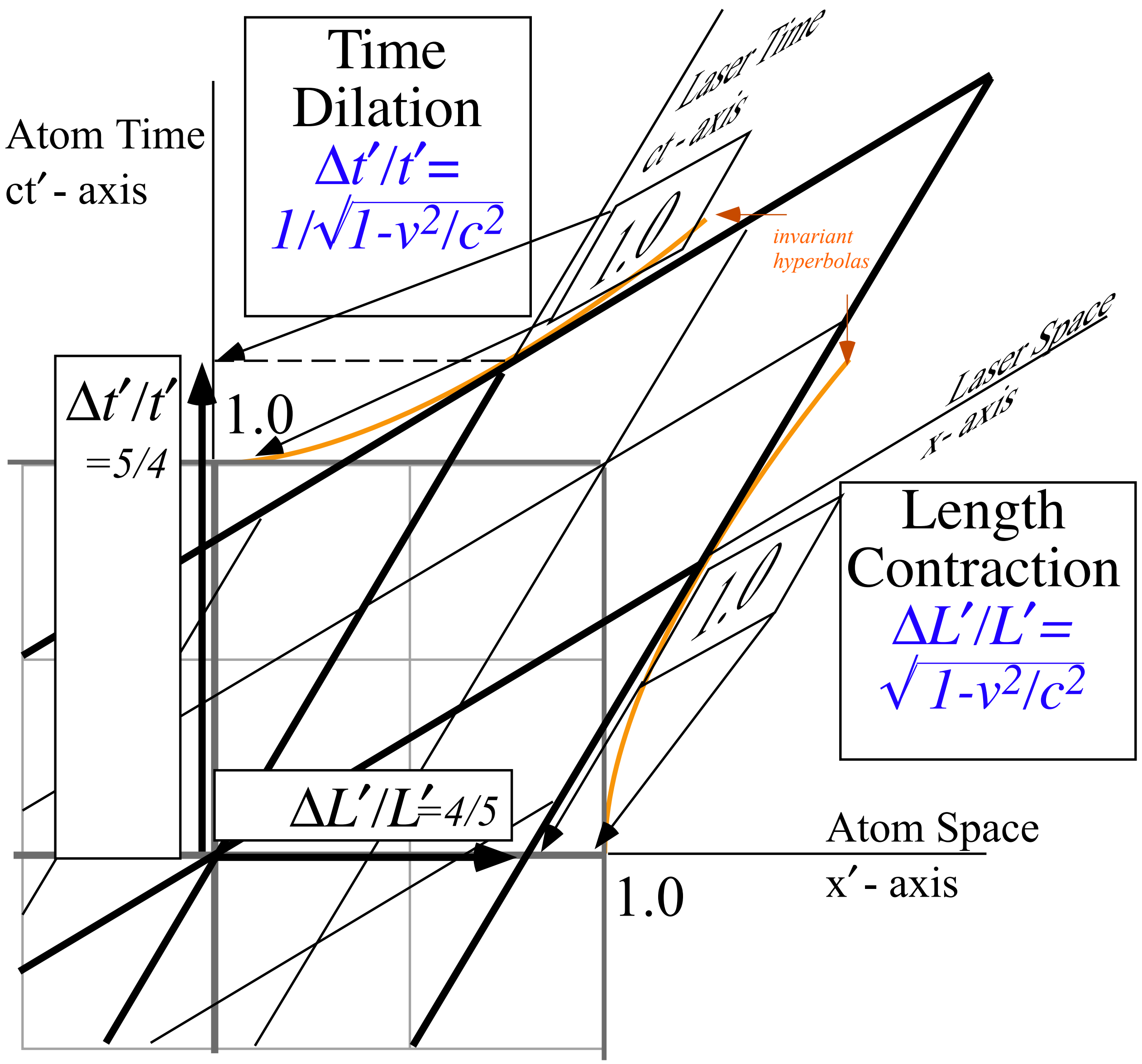
Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation
Relativistic Terms (Expanded Table)

Reading Minkowski graph plots for $e^{\rho}=2.0$ or $\beta=u/c=3/5$



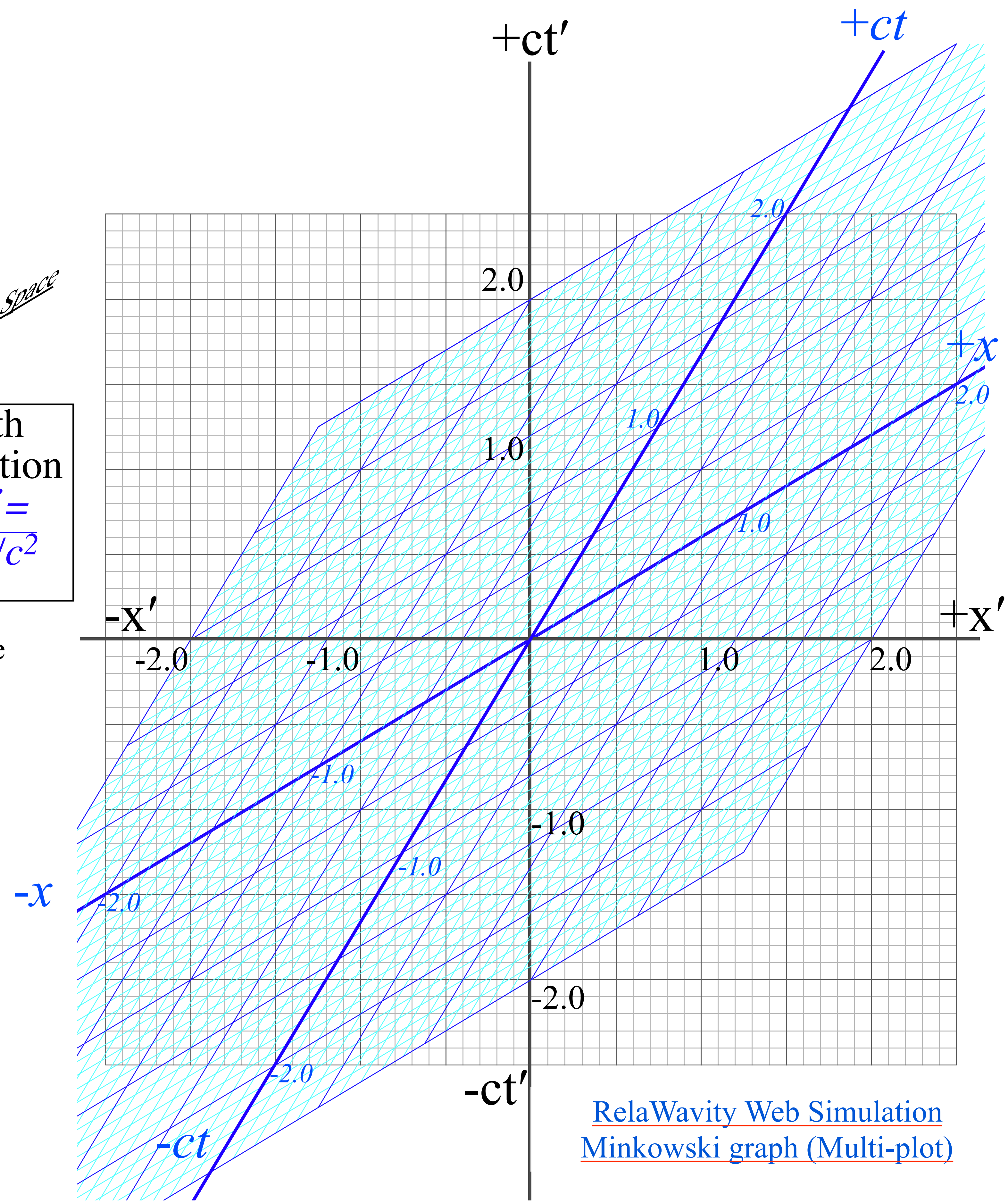
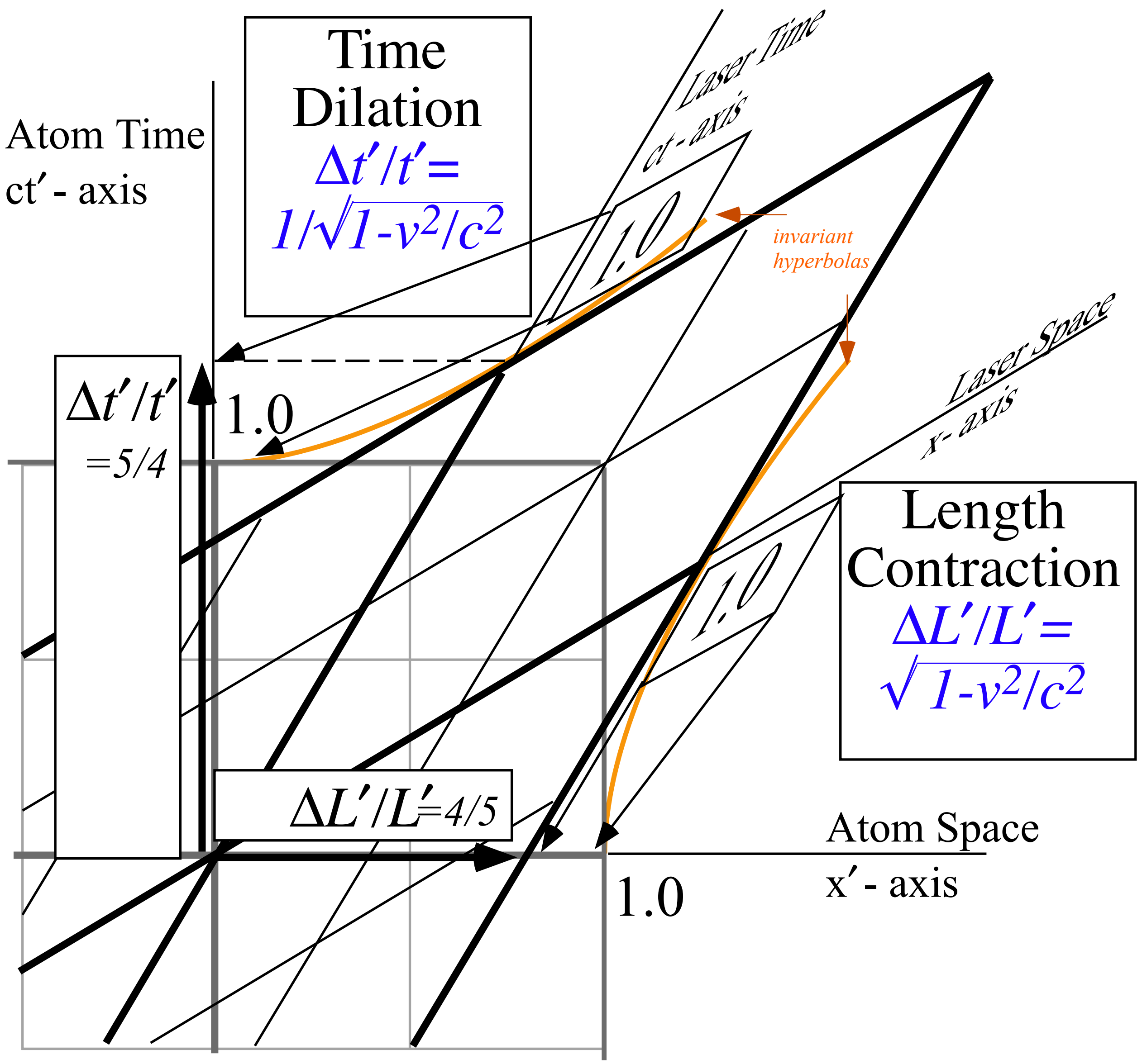


Fig. 8.3.6
CMwBang! Unit 8

RelaWavity Web Simulation
Minkowski graph (Multi-plot)

Reading Minkowski graph plots for $e^{\rho}=2.0$ or $\beta=u/c=3/5$



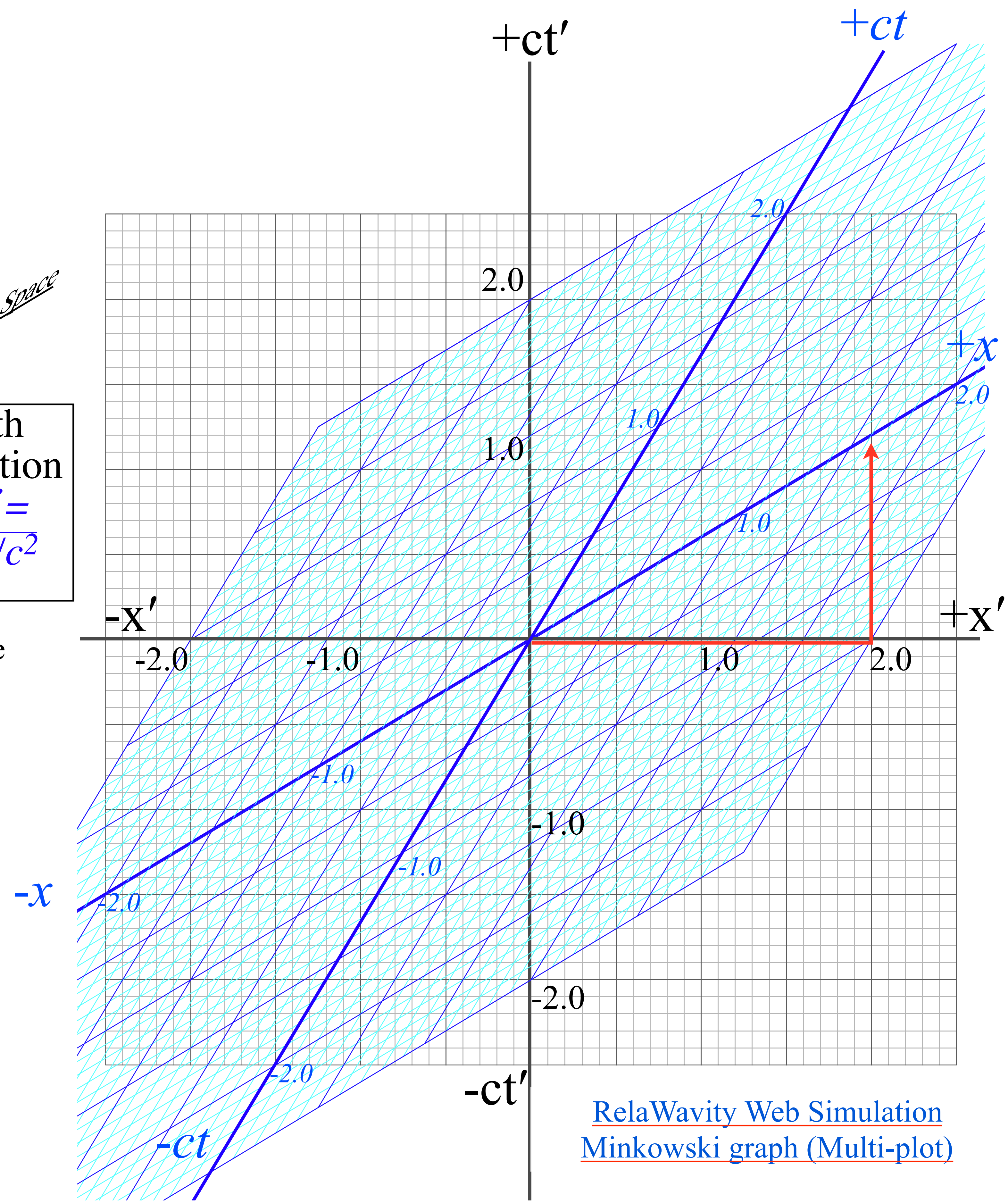
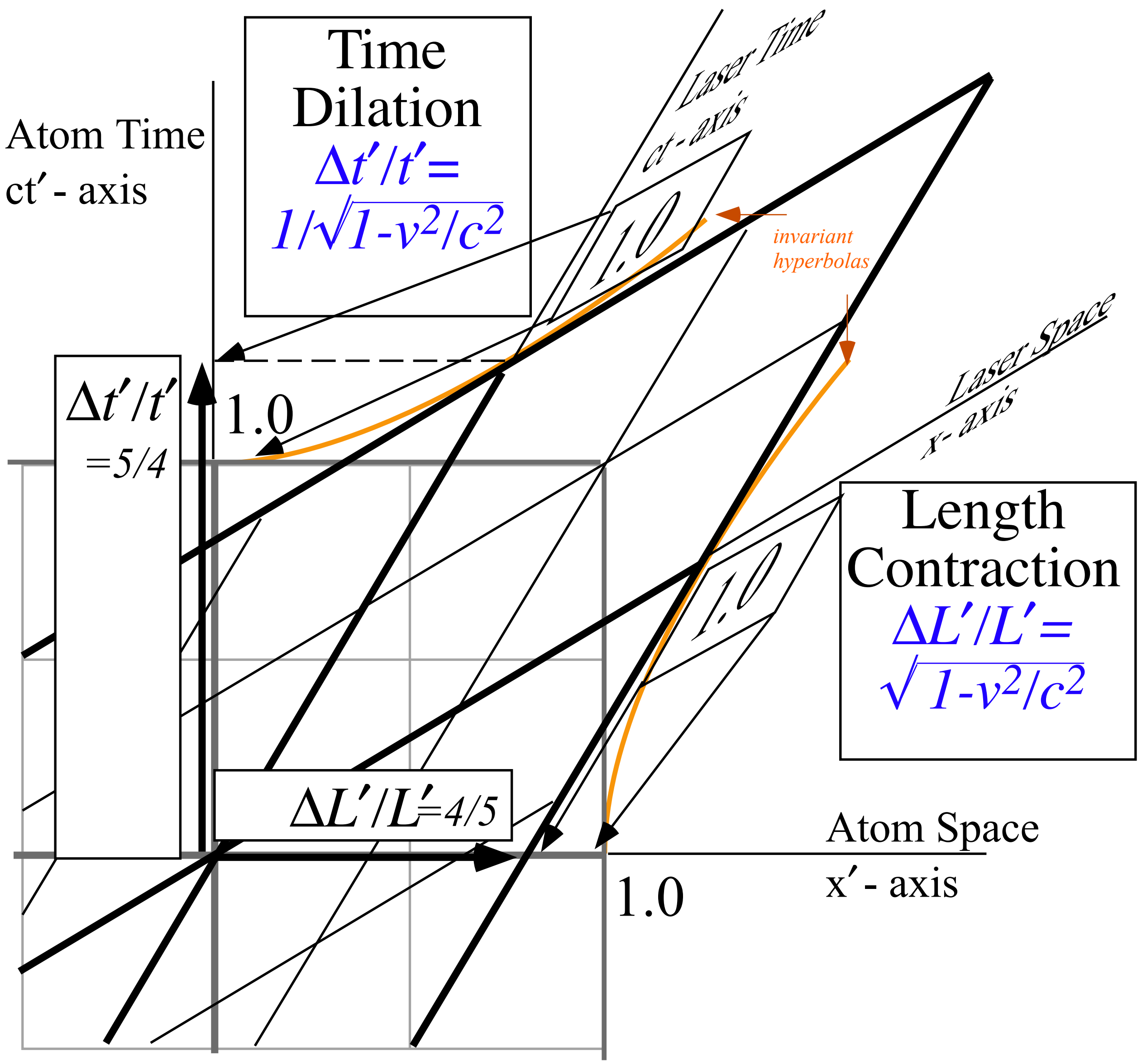


Fig. 8.3.6
CMwBang! Unit 8

Reading Minkowski graph plots for $e^{\rho}=2.0$ or $\beta=u/c=3/5$



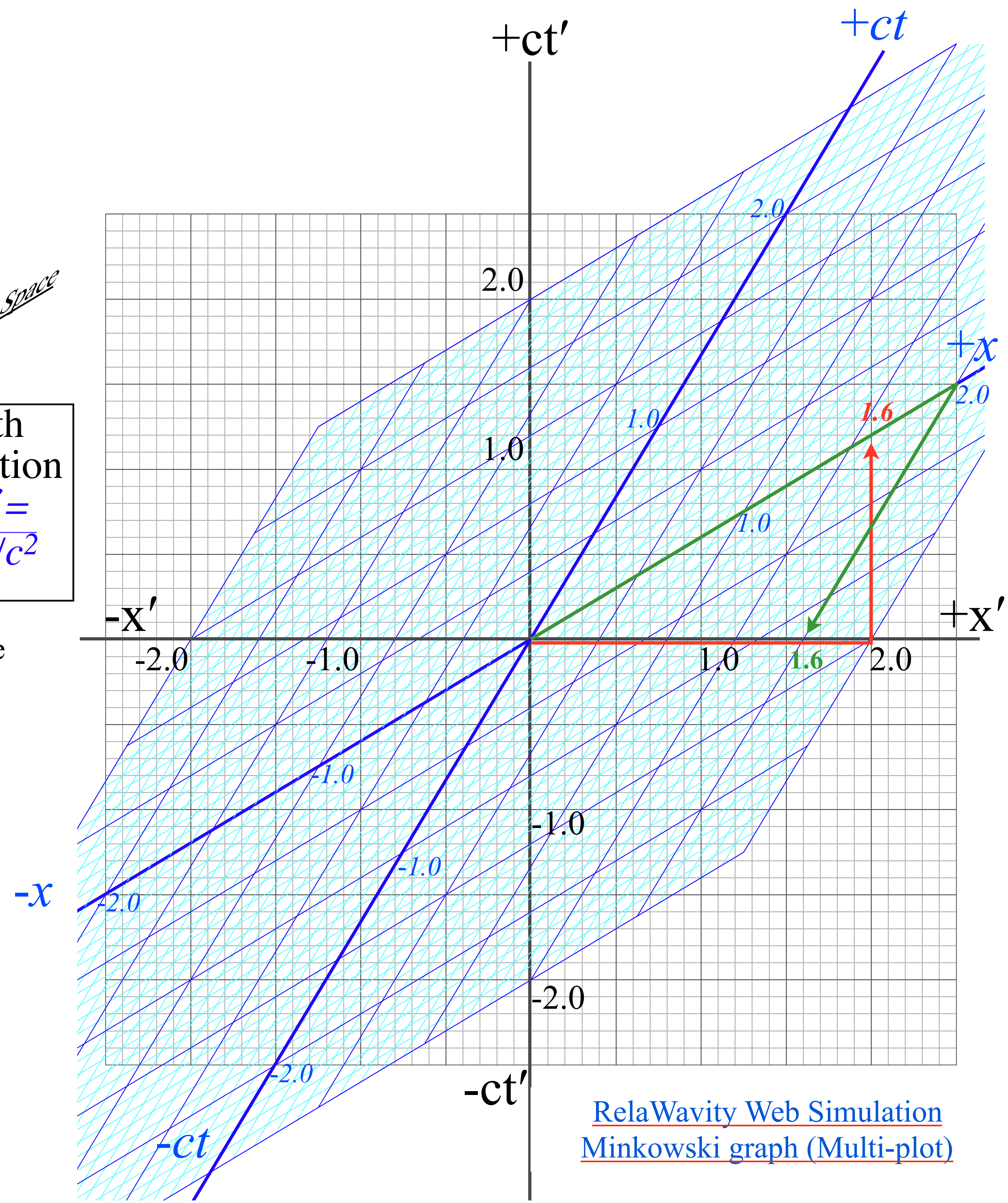
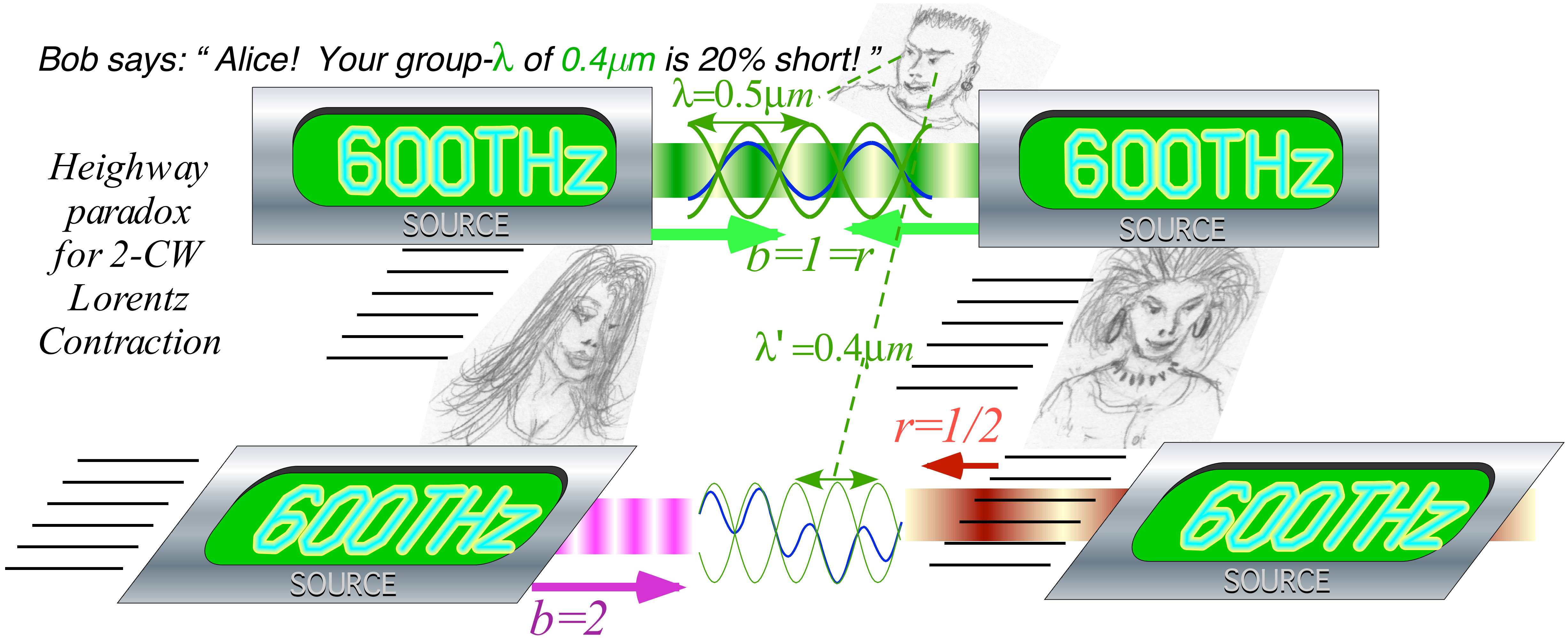
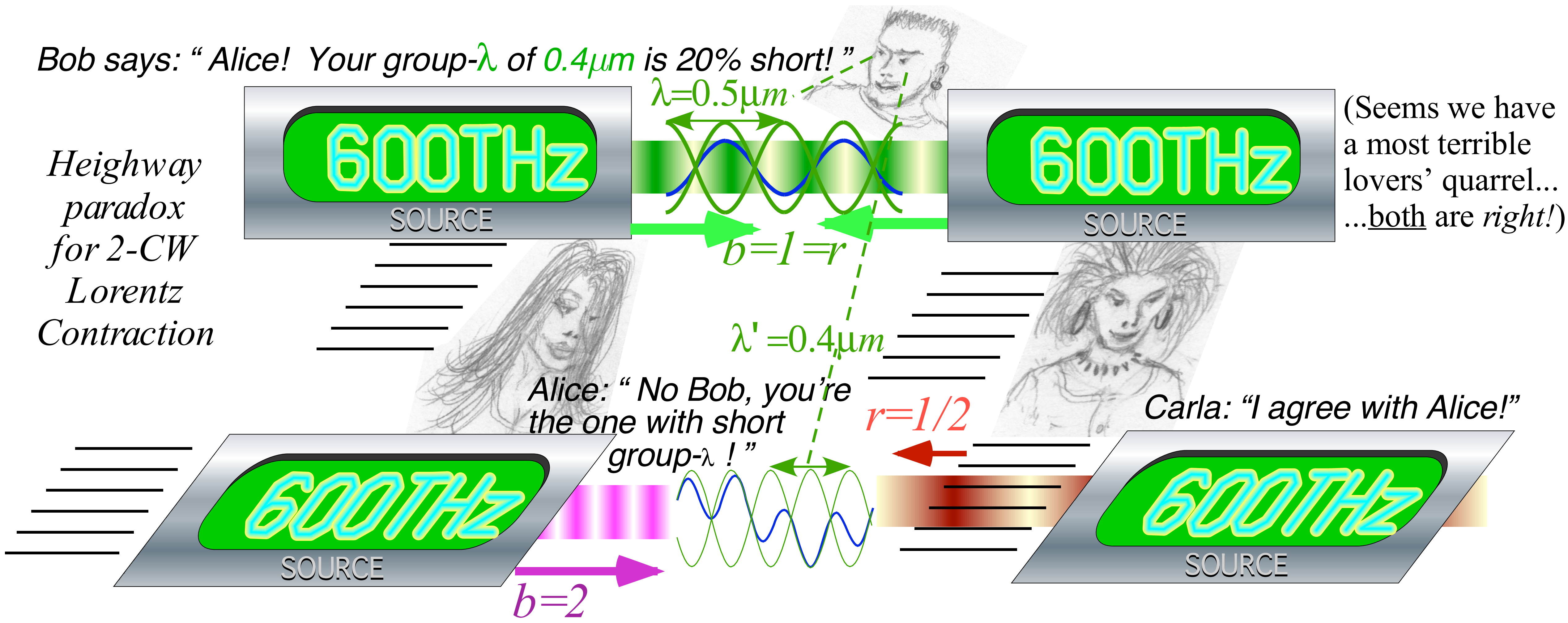


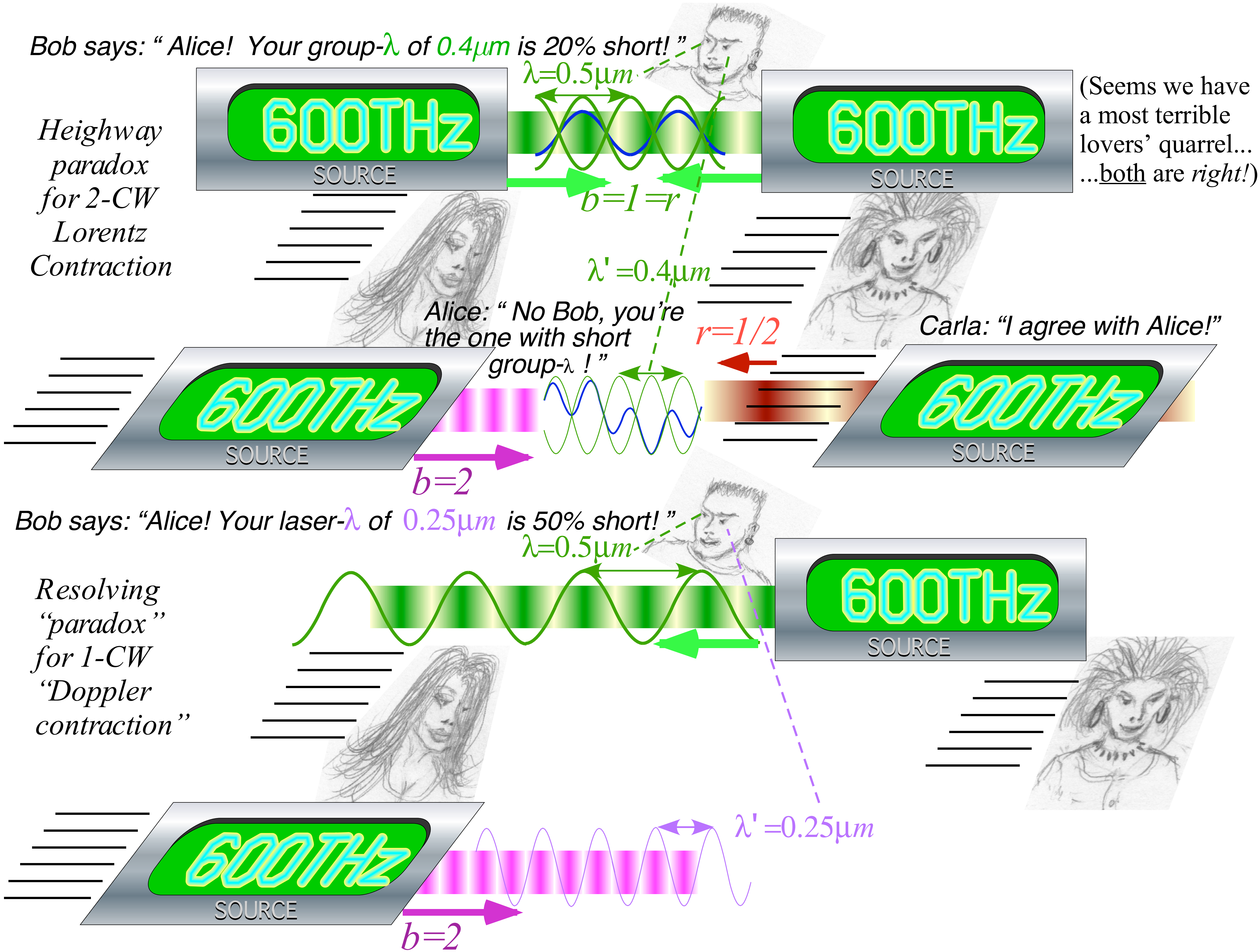
Fig. 8.3.6
CMwBang! Unit 8

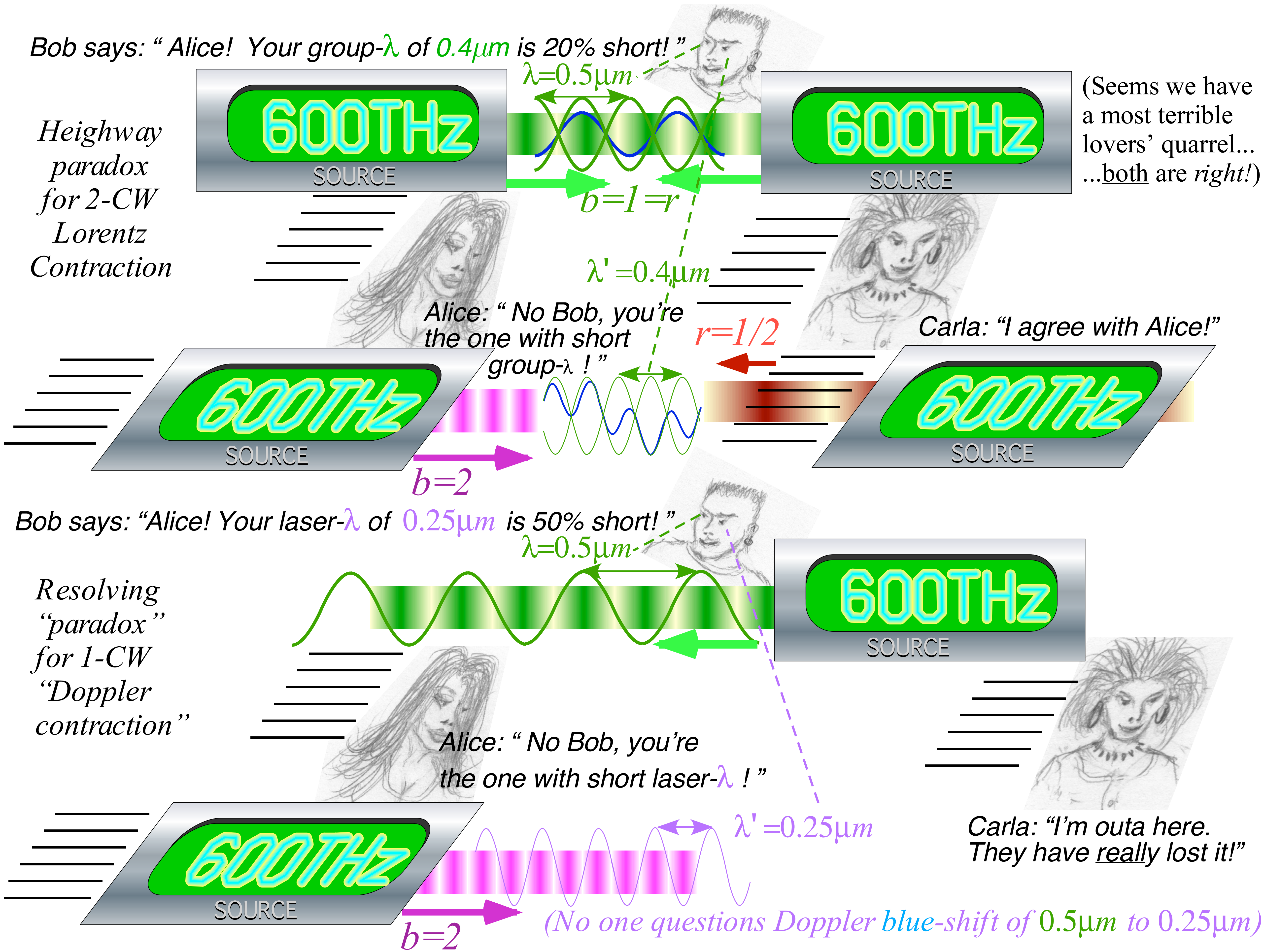
[RelaWavity Web Simulation
Minkowski graph (Multi-plot)]



Bob says: " Alice! Your group-λ of 0.4μm is 20% short! "
Heighway paradox for 2-CW Lorentz Contraction
λ=0.5μm
600THz
SOURCE
600THz
SOURCE
b=1=r
λ' =0.4μm
r=1/2
600THz
SOURCE
600THz
SOURCE
b=2


Bob says: “ Alice! Your group-λ of 0.4μm is 20% short! ”
λ=0.5μm
Heighway paradox for 2-CW Lorentz Contraction
600THz
SOURCE
600THz
SOURCE
(Seems we have a most terrible lovers’ quarrel... ...both are right!)
b=1=r
λ' =0.4μm
Alice: “ No Bob, you’re the one with short group-λ ! ”
r=1/2
Carla: “I agree with Alice!”
600THz
SOURCE
600THz
SOURCE
b=2

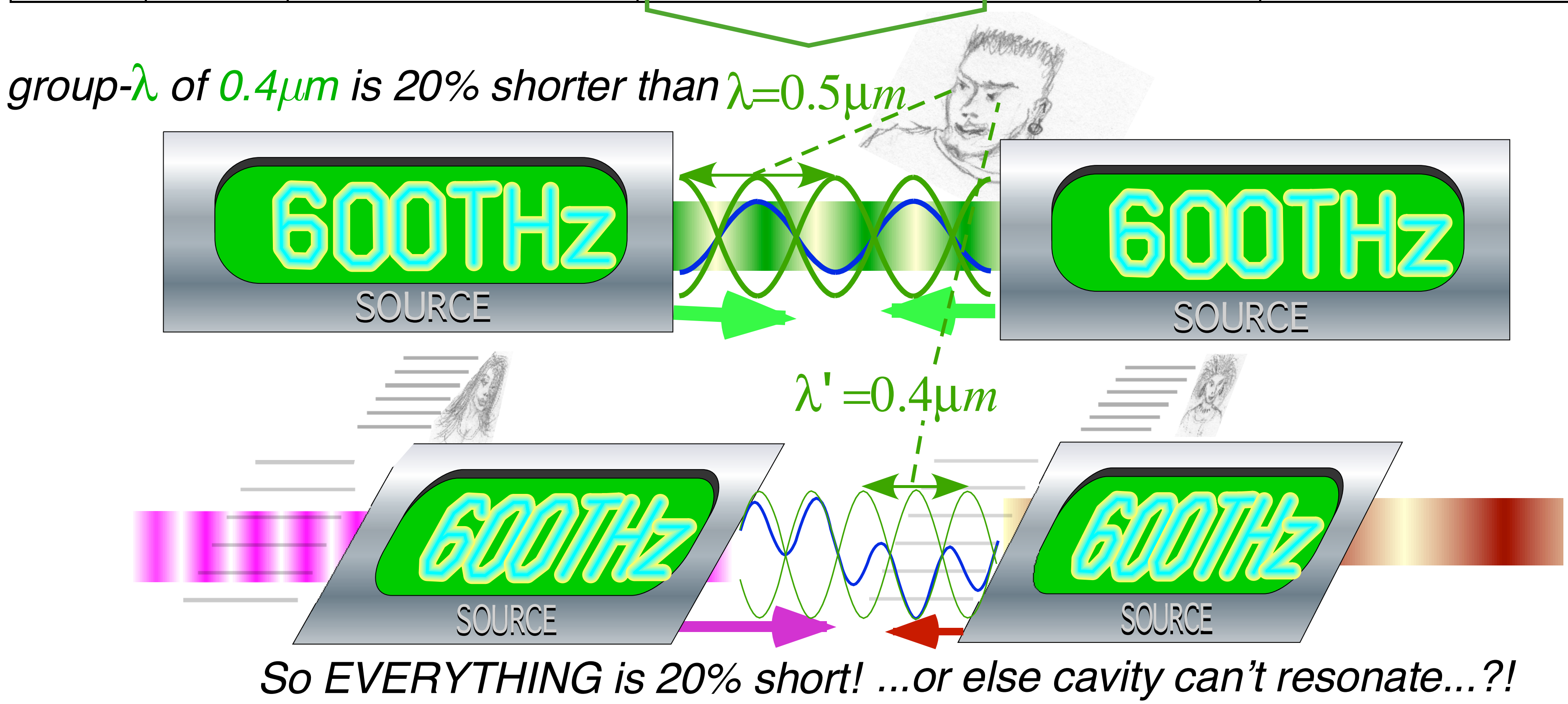

Bob says: " Alice! Your group-λ of 0.4μm is 20% short! "
λ=0.5μm
600THz
SOURCE
600THz
SOURCE
(Seems we have a most terrible lovers' quarrel... ...both are right!)
Heighway paradox for 2-CW Lorentz Contraction
b=1=r
λ'=0.4μm
Alice: " No Bob, you're the one with short group-λ ! "
r=1/2
Carla: "I agree with Alice!"
600THz
SOURCE
600THz
SOURCE
b=2
Bob says: "Alice! Your laser-λ of 0.25μm is 50% short! "
λ=0.5μm
600THz
SOURCE
Resolving "paradox" for 1-CW "Doppler contraction"
600THz
SOURCE
λ'=0.25μm
b=2



Bob says: " Alice! Your group-λ of 0.4μm is 20% short! "
λ=0.5μm
600THz
SOURCE
600THz
SOURCE
(Seems we have a most terrible lovers' quarrel... ...both are right!)
Heighway paradox for 2-CW Lorentz Contraction
b=1=r
λ'=0.4μm
Alice: " No Bob, you're the one with short group-λ ! "
r=1/2
Carla: "I agree with Alice!"
600THz
SOURCE
600THz
SOURCE
b=2
Bob says: "Alice! Your laser-λ of 0.25μm is 50% short! "
λ=0.5μm
600THz
SOURCE
Resolving "paradox" for 1-CW "Doppler contraction"
Alice: " No Bob, you're the one with short laser-λ ! "
λ'=0.25μm
Carla: "I'm outa here. They have really lost it!"
600THz
SOURCE
b=2
(No one questions Doppler blue-shift of 0.5μm to 0.25μm)

Lorentz contraction is a quantum matter-wave effect

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b_{RED}^{Doppler}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*$^{(Lorentz)}$ τ_{phase}*-contraction*	*t-dilation*$^{(Einstein)}$ υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b_{BLUE}^{Doppler}$



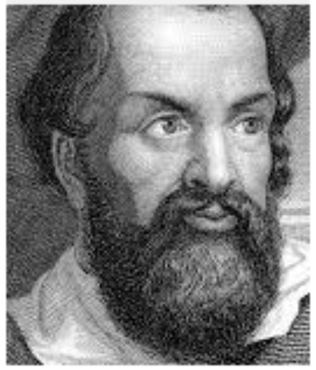
More generally:

Quantum mechanics is a relativistic effect,

and

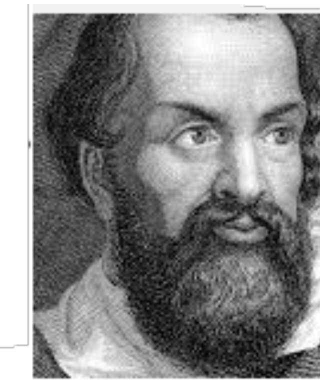
Relativity is a quantum mechanical effect.

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Optical interference "baseball-diamond" displays *phase* and *group* velocity
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➔ Doppler Jeopardy & Thales geometric mean υ geometry of Lorentz transformation
Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
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Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\,\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity? $\frac{300}{900}$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity? $\frac{300}{900}$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

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Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$\frac{300}{900}$

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$\sqrt{6\cdot 3} = 3\sqrt{2} = 4.24$

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

Geometric mean

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$\frac{300}{900}$

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} \qquad V_{group} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L} = c\frac{600 - 300}{600 + 300} = \frac{1}{3}c$$

Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R?

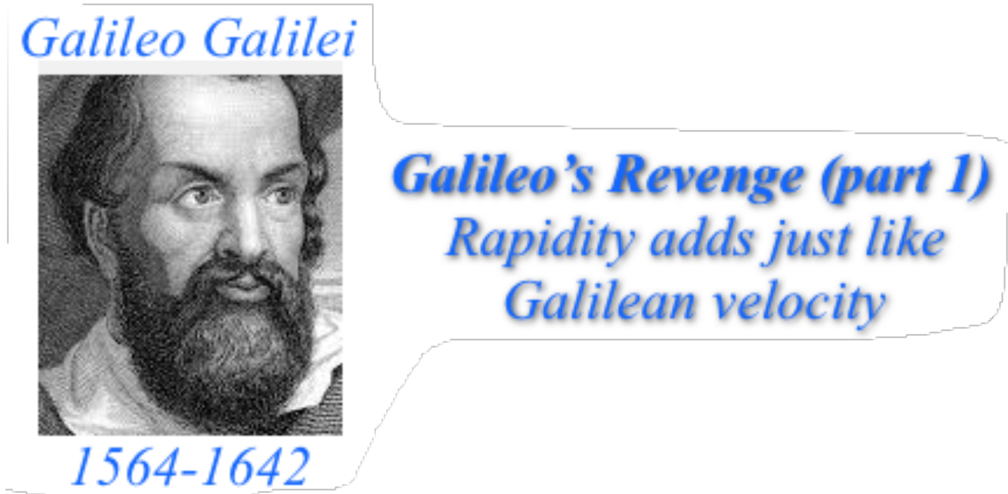
$\sqrt{6\cdot 3} = 3\sqrt{2} = 4.24$

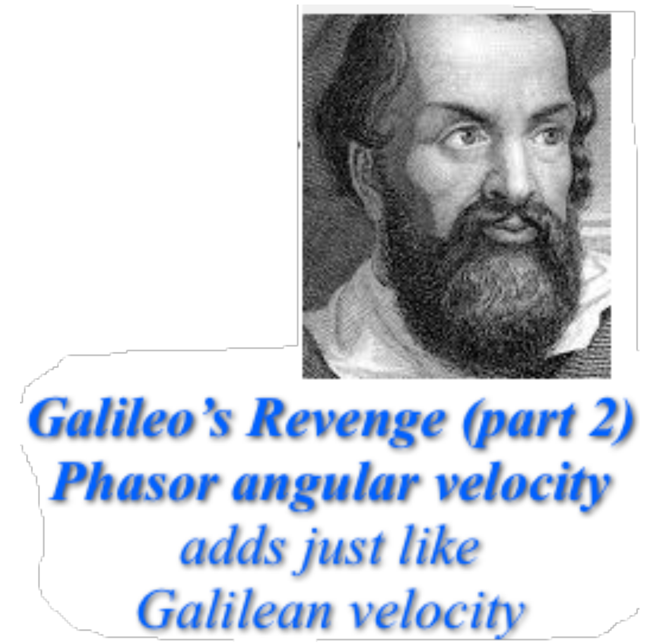
$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R/\omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

$$\omega_E = \sqrt{\omega_R \cdot \omega_L} = \sqrt{180000} = 424$$

V_{group}/c is ratio of difference mean $\omega_{group} = \frac{\omega_R - \omega_L}{2}$ to arithmetic mean $\omega_{phase} = \frac{\omega_R + \omega_L}{2}$. Frequency $\omega_E = B$ is the geometric mean $\sqrt{\omega_R \cdot \omega_L}$ of left and right-moving frequencies defining the geometry

Geometric mean





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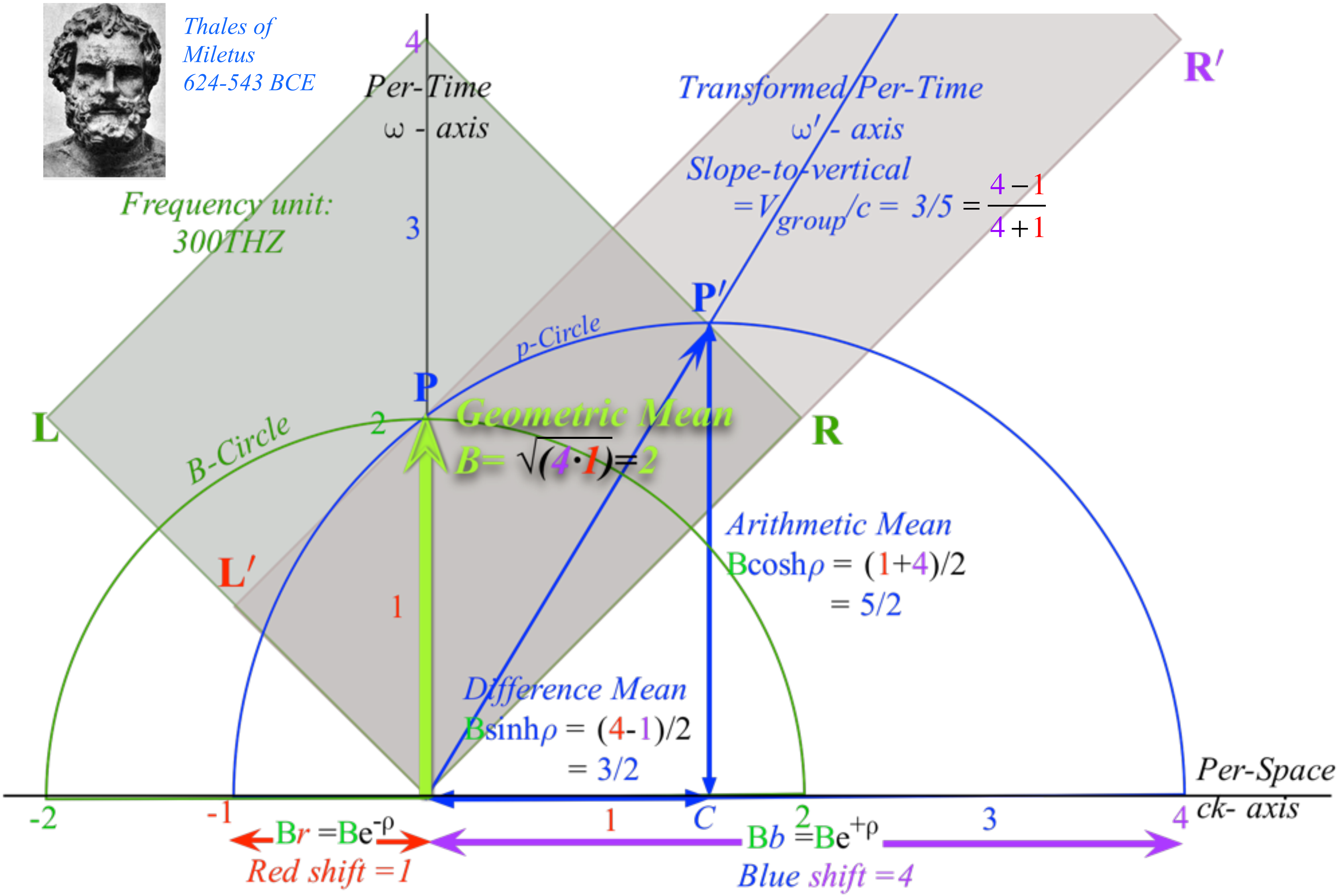
"Occams Sword" and geometry of functions of *ρ* and *σ*   Minkowski animations

Application to TE-Waveguide modes.   synchrotron beam relativity

Thales Mean Geometry (600BCE)

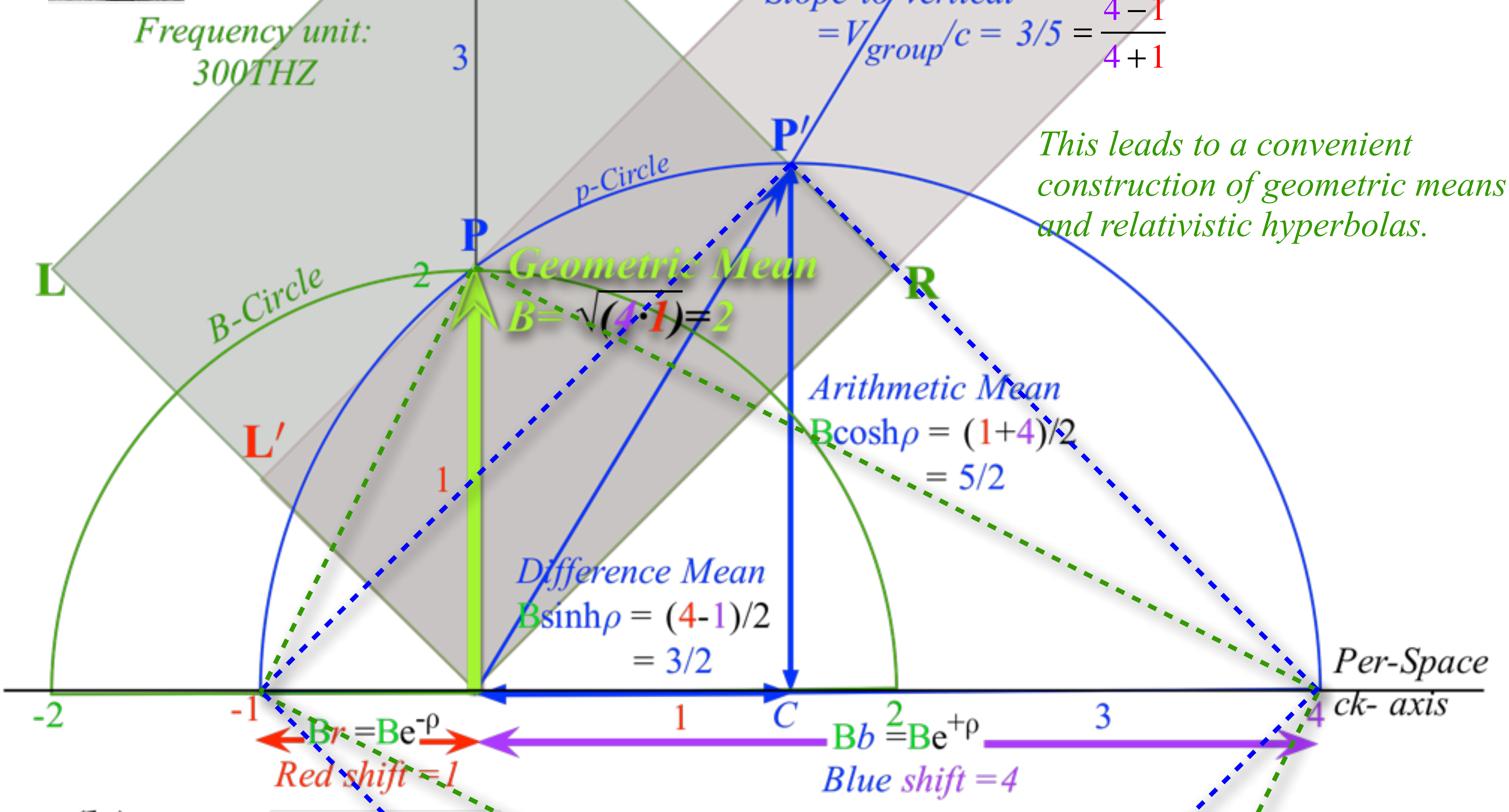
helps "Relawavity"



Thales Mean Geometry (600BCE)

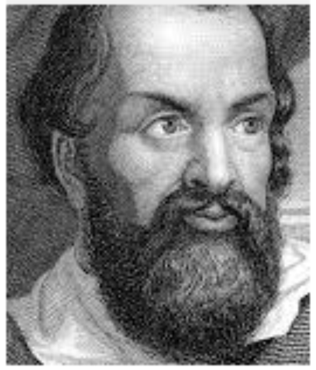
helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*

Thales of Miletus 624-543 BCE



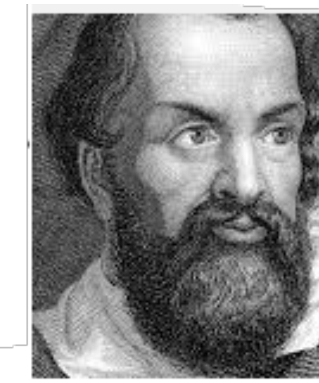
This leads to a convenient construction of geometric means and relativistic hyperbolas.

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
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Galilean velocity

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16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation ➔...and invariant hyperbolas ⬅

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of functions of ρ and σ Minkowski animations

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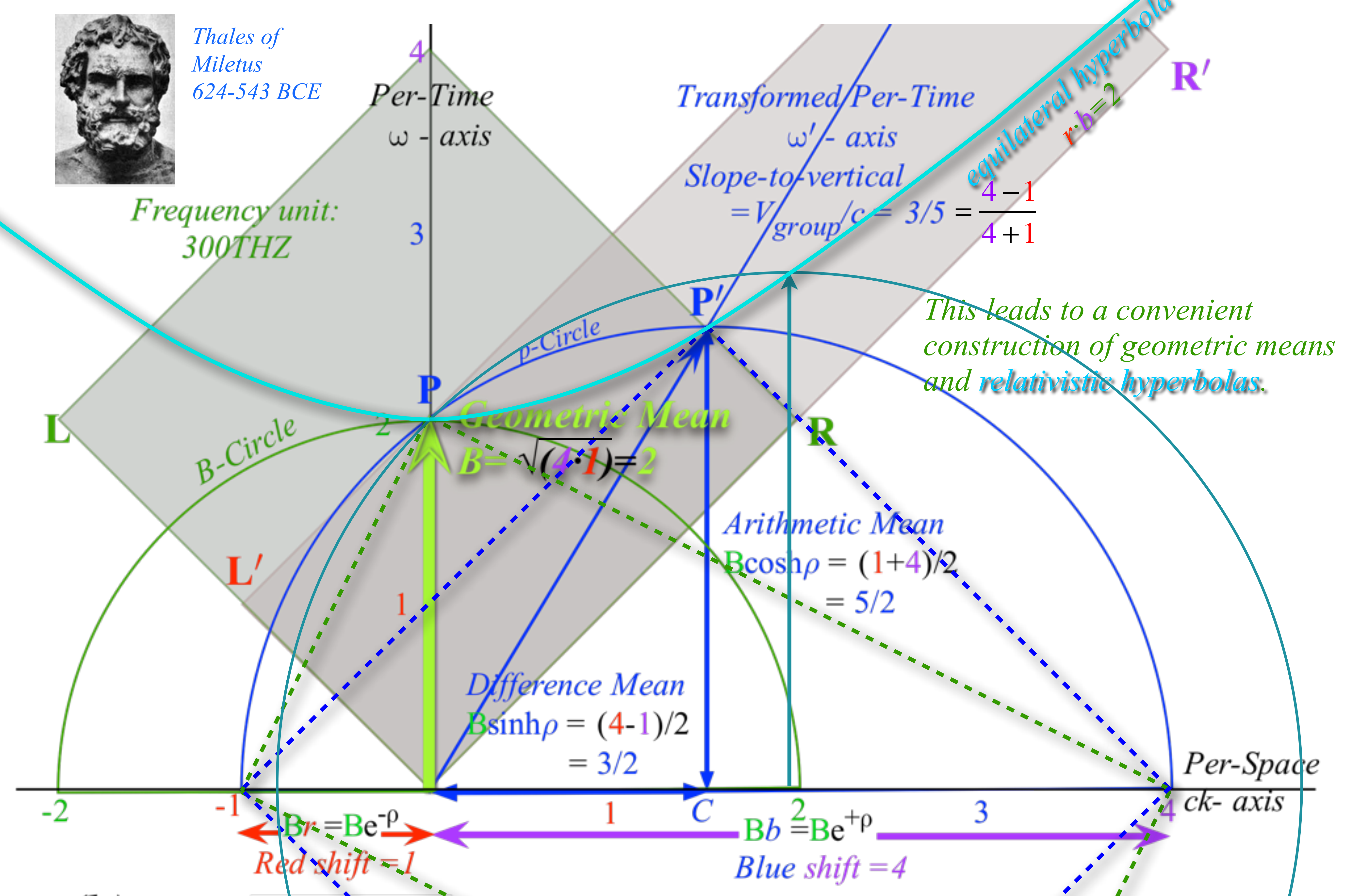
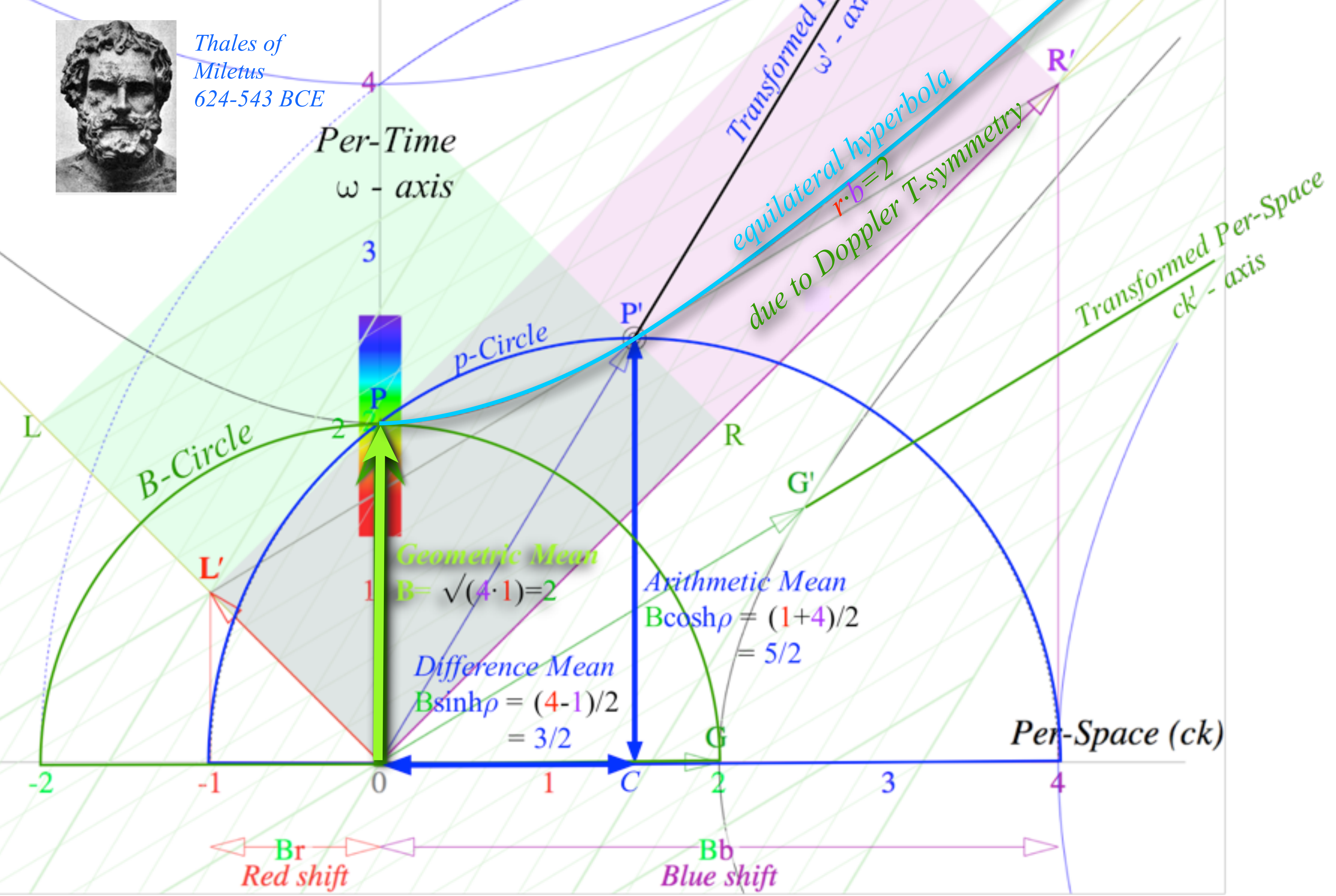
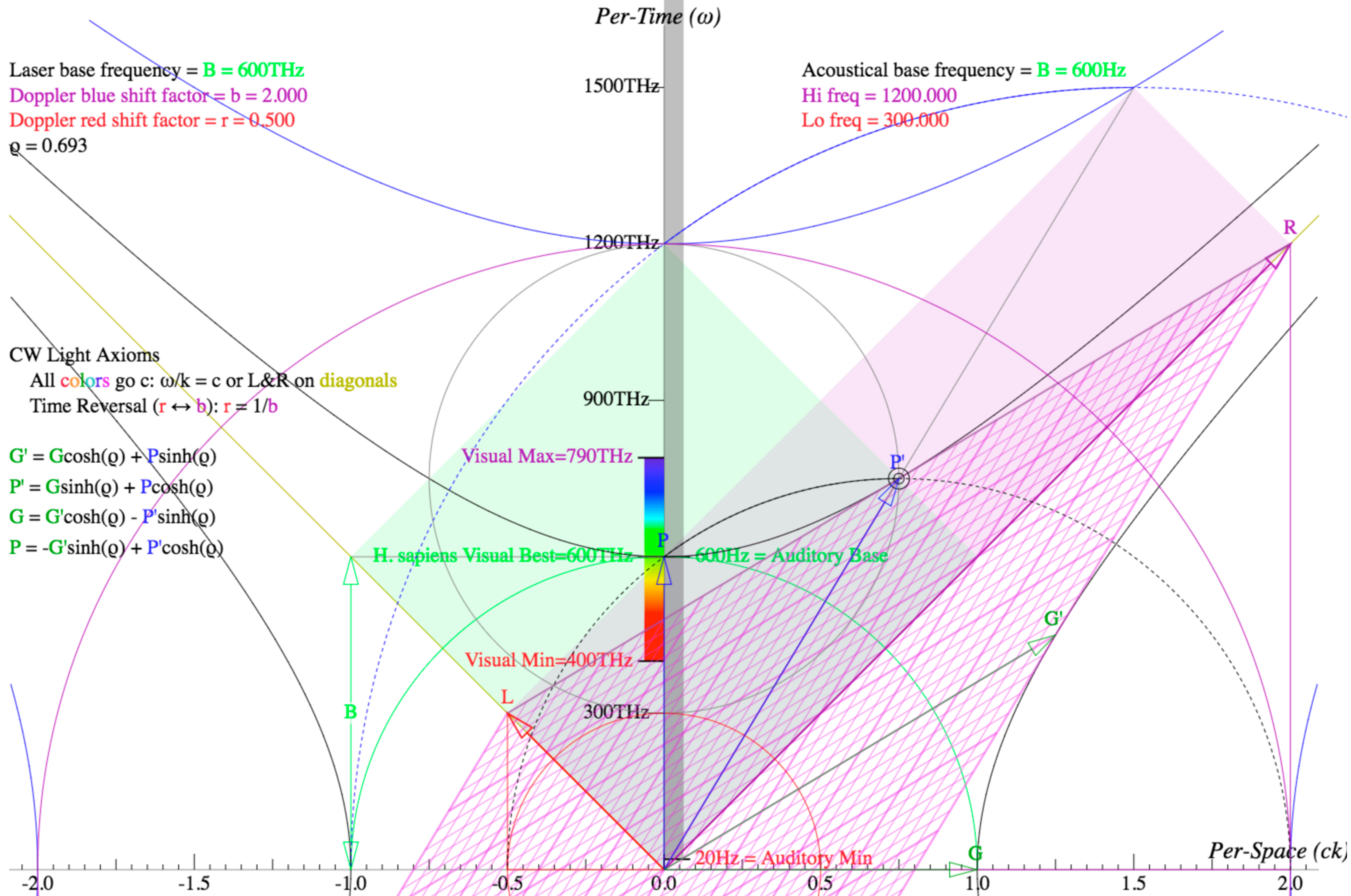


Thales Mean Geometry (600BCE)

helps "Relawavity"





RelaWavity Web Simulation

Detailed Thales Geometry

Select from the top menus to choose the view type and sub-type.
Click the 'Controls' button to set shared model & display vars.

Set parameters with click (& drag) near the ck axis: r,b; the green semi-circle: σ; the hyperbolae: v
Right (or CTRL+) click figure to set plot specific vars.

A view of a conical intersection: Any vertical cross-section is hyperbolic avoided-crossing

Recall ABD U(2) system in Lect. 23 p.93

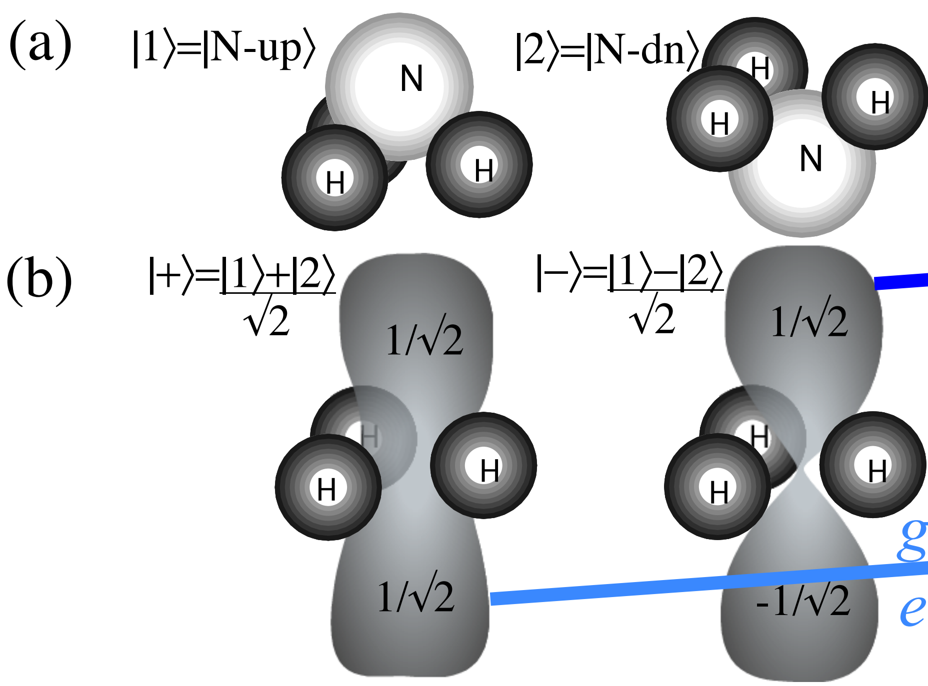
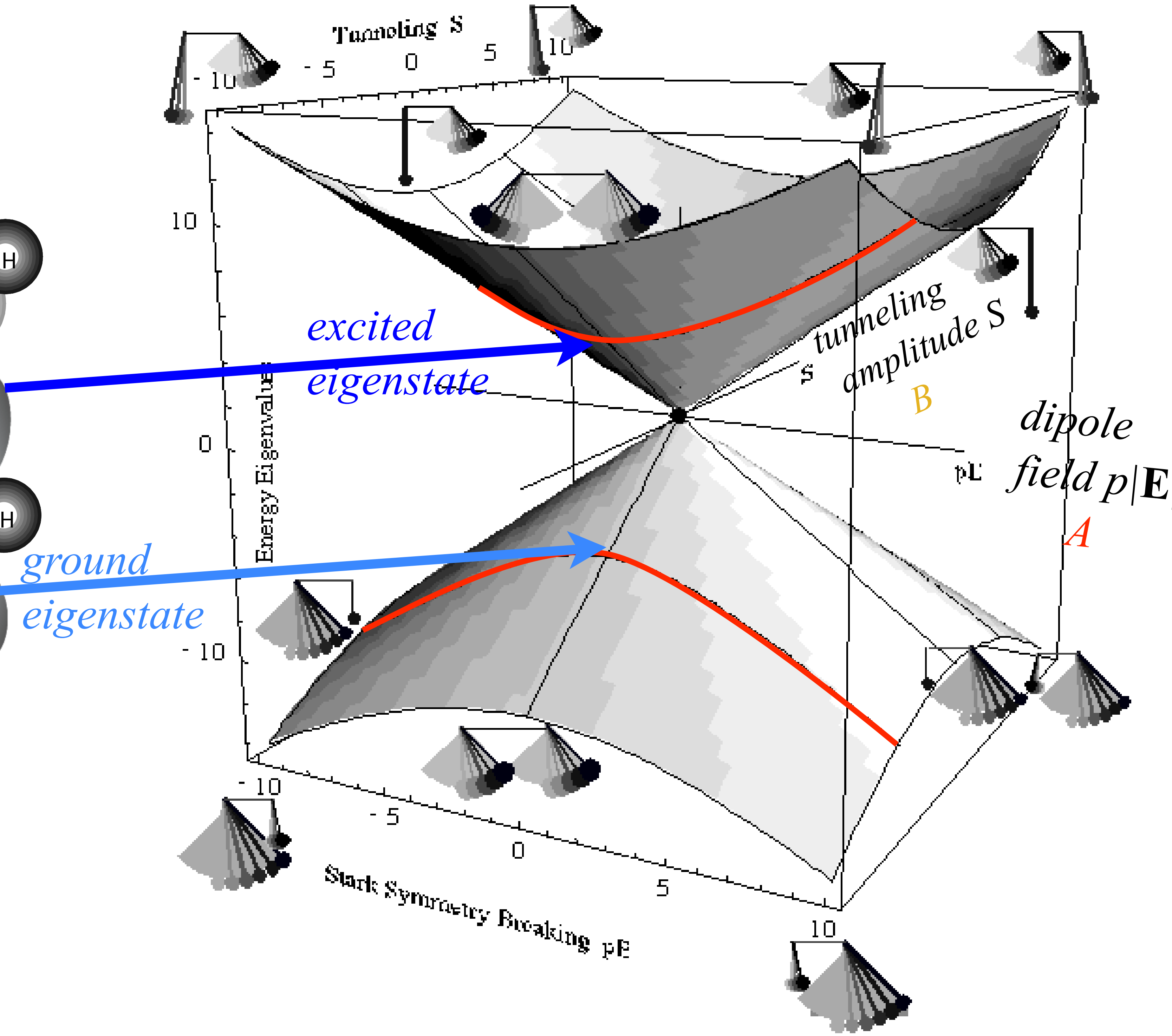


Fig. 10.3.2 Ammonia (NH$_3$) inversion states
(a) Base states (b) C$_2$-Eigenstates



10.3.1 (a) Two state eigenvalue "diablo" surfaces and conical intersection and pendulum eigenstates.

(Also known as a “Dirac-point”)

A to B to A Symmetry breaking described by hyperbolic eigenvalues of $A\boldsymbol{\sigma}_A + B\boldsymbol{\sigma}_B = \mathbf{H} = \begin{pmatrix} +A & B \\ B & -A \end{pmatrix}$

$\mathbf{H} = \begin{pmatrix} +A & B \\ B & -A \end{pmatrix}$ Secular equation: $\varepsilon^2 - 0 \cdot \varepsilon - (A^2 + B^2)$ gives *hyperbolic* energy levels: $\varepsilon = \pm\sqrt{A^2 + B^2}$

Recall ABD U(2) system in Lect. 23 p.90

$\mathbf{H}(B-basis)$ $\quad$ $\mathbf{H}(A-basis)$

$$\begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} = \frac{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}{\sqrt{2}} \begin{pmatrix} +A & B \\ B & -A \end{pmatrix} \frac{\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}}{\sqrt{2}}$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} +A & B \\ B & -A \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} +A+B & B-A \\ +A-B & B+A \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2} \begin{pmatrix} 2B & 2A \\ 2A & -2B \end{pmatrix}$$

$$= \begin{pmatrix} +B & A \\ A & -B \end{pmatrix}$$

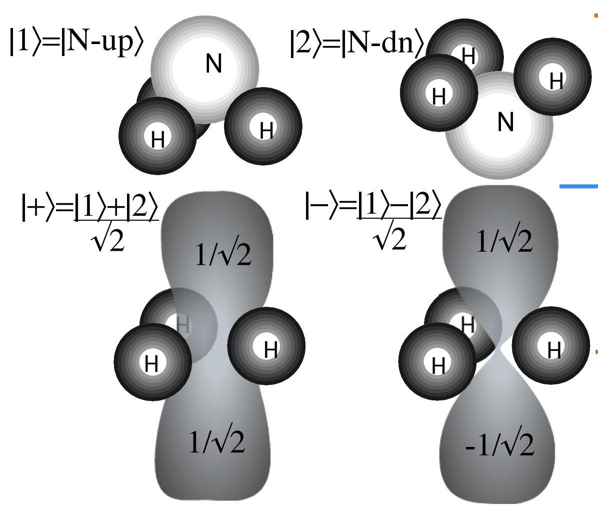


Fig. 10.3.2 Ammonia (NH_3) inversion states
(a) Base states (b) C_2-Eigenstates

Here we display eigenvalues and eigenvectors while holding B constant and varying A. Obviously it can be done vice-versa and with varying C, too.

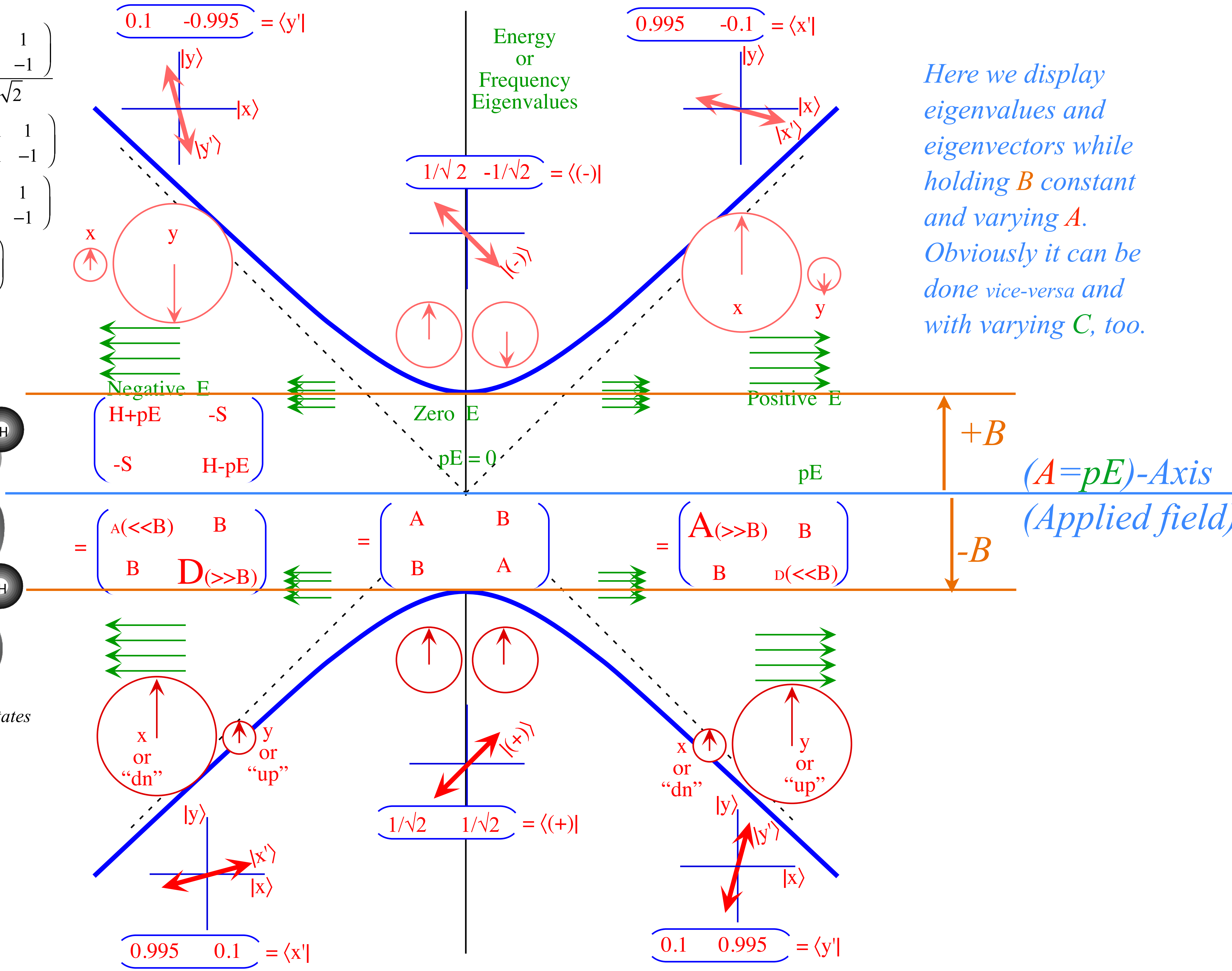
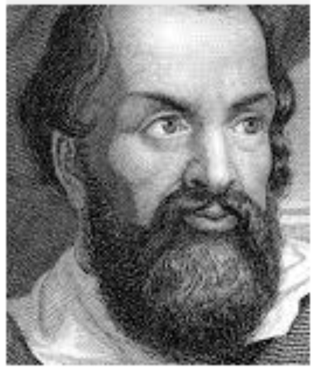


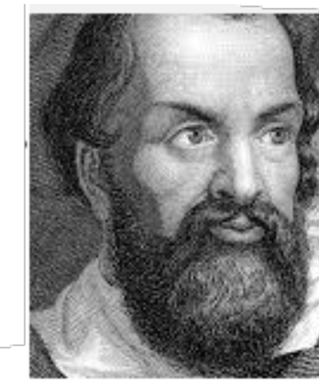
Fig. 10.3.1 (b) Wigner avoided level crossing. (Fixed tunneling B=-S and variable A-D=pE field.)

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
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Optical interference "baseball-diamond" displays *phase* and *group* velocity

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➤ *Rapidity* ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity ◄

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of functions of ρ and σ Minkowski animations

Application to TE-Waveguide modes. synchrotron beam relativity

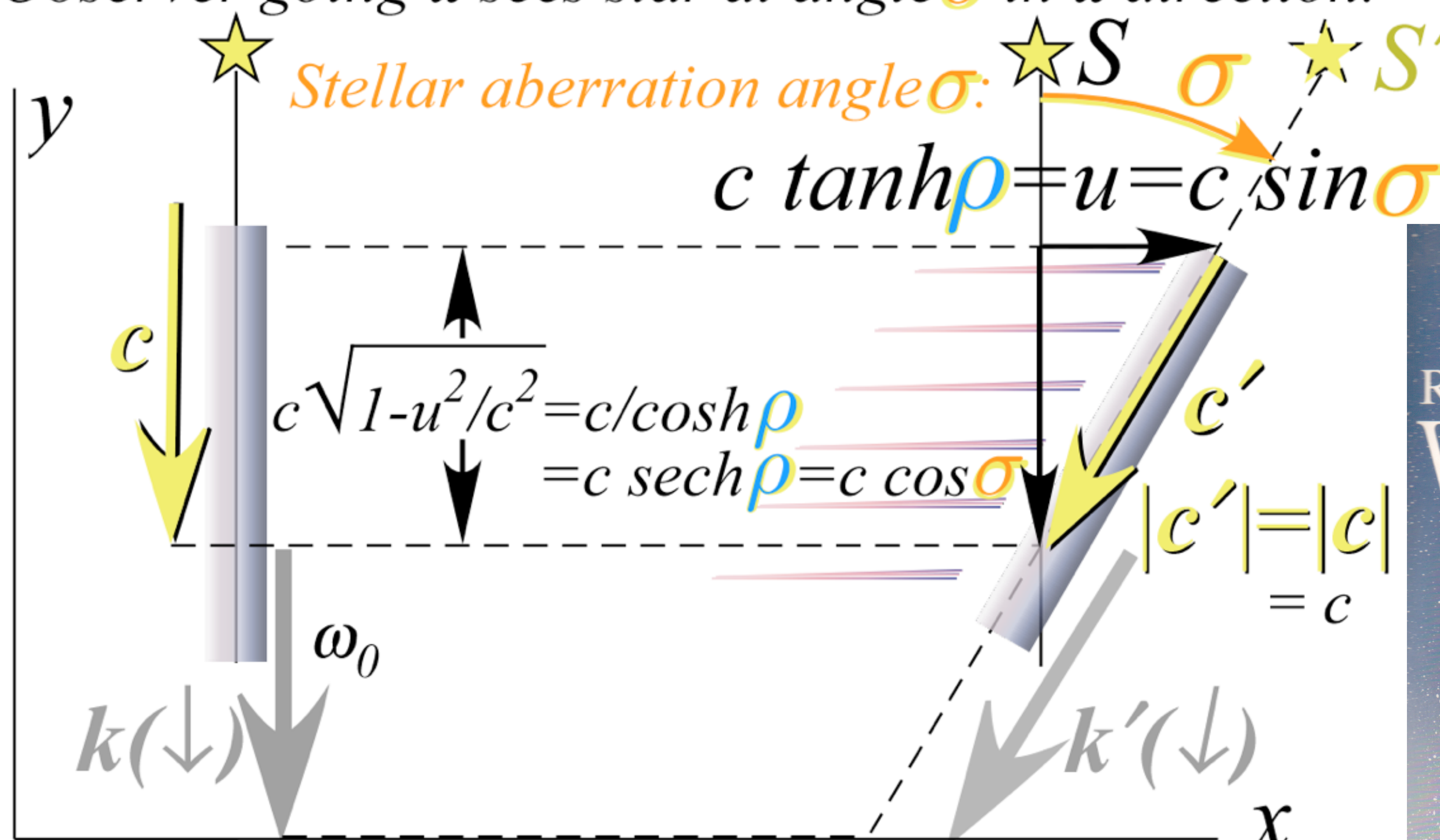
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

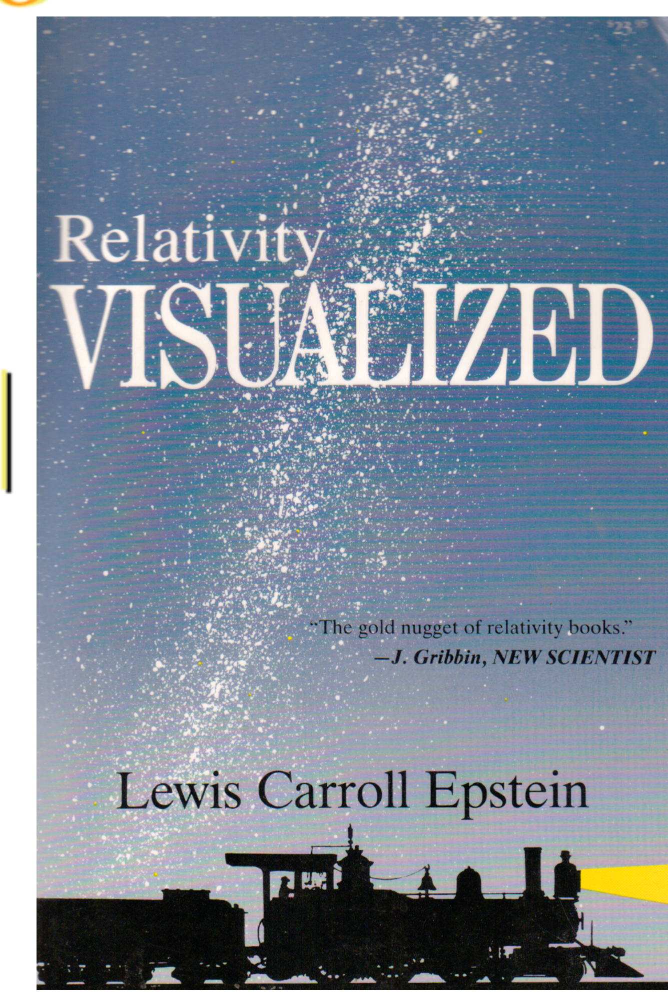
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



We used notation σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



Purchase at: AllBookstores.com amazon

Details of stellar aberration angle σ of K-vexctor rotation

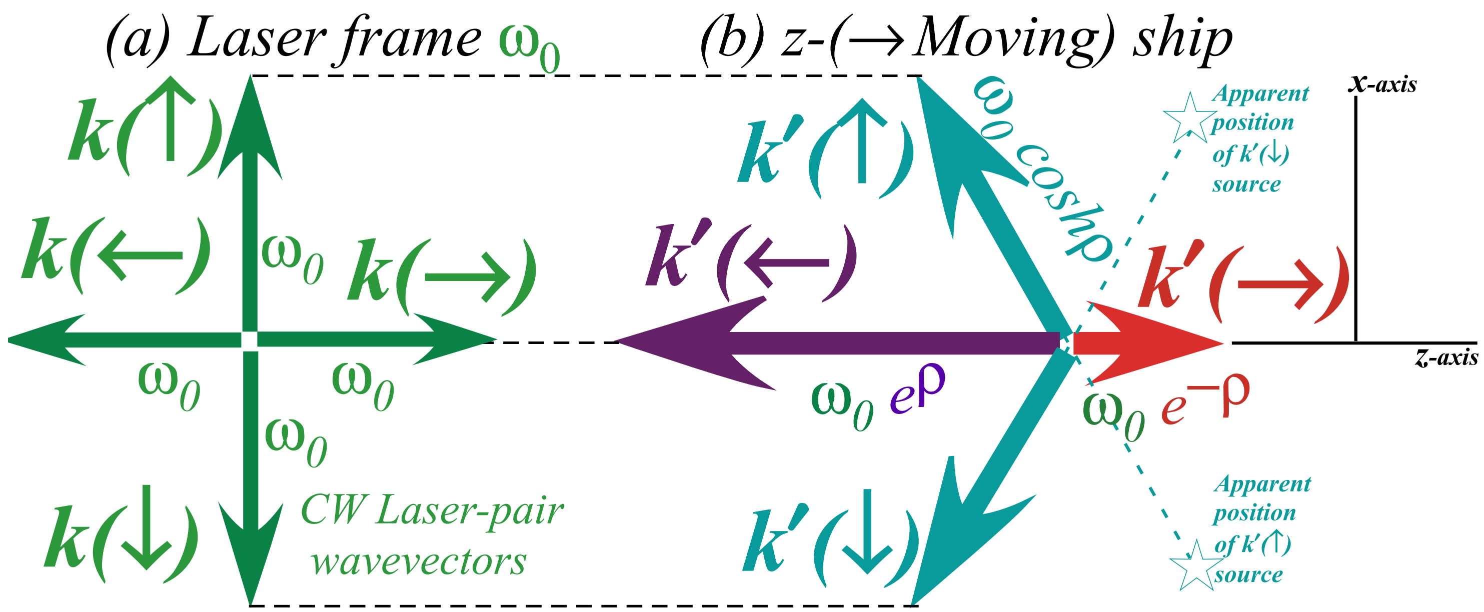


Fig. 8.5.7
Unit 8

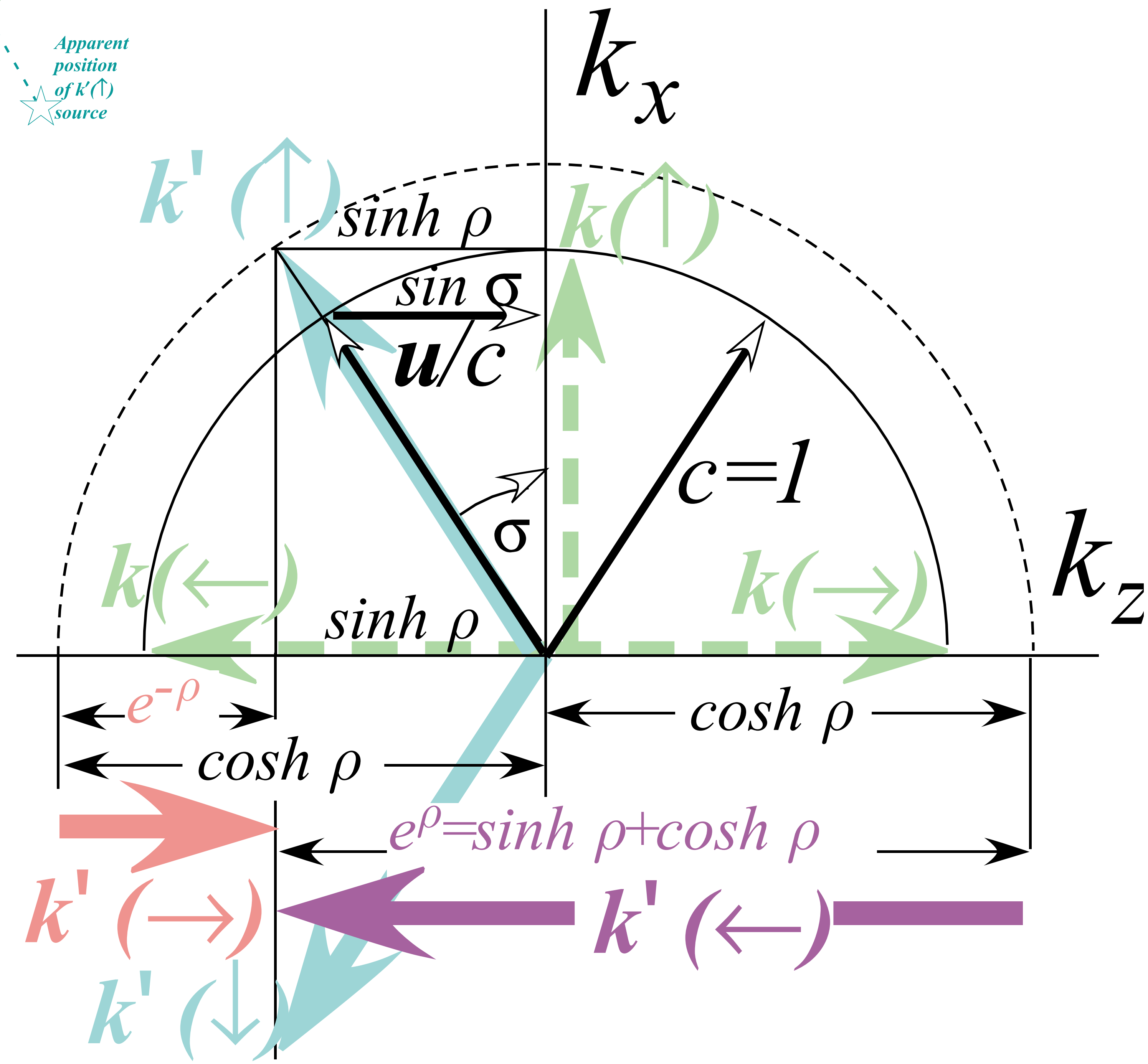
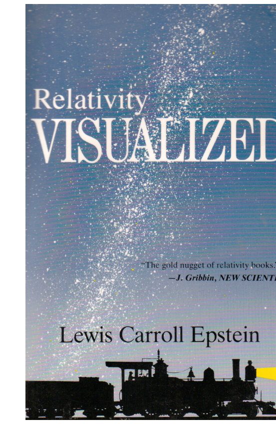


Fig. 8.5.10
Unit 8

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

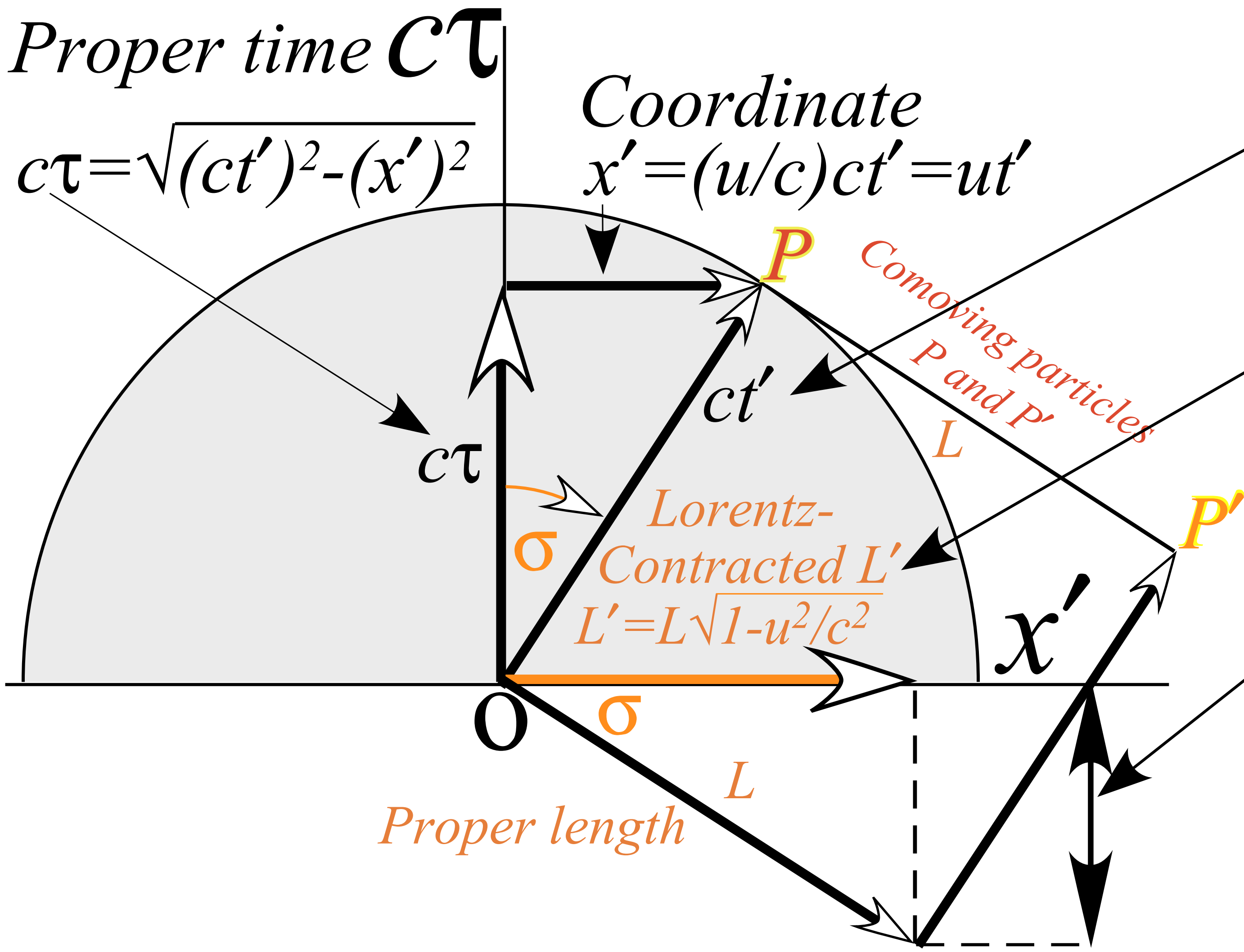
to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame

RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

RelativIt Web Simulation *Epstein plot*

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to a Transverse*relativity parameter: Stellar aberration angle σ

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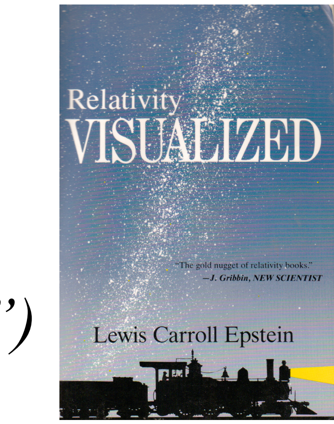

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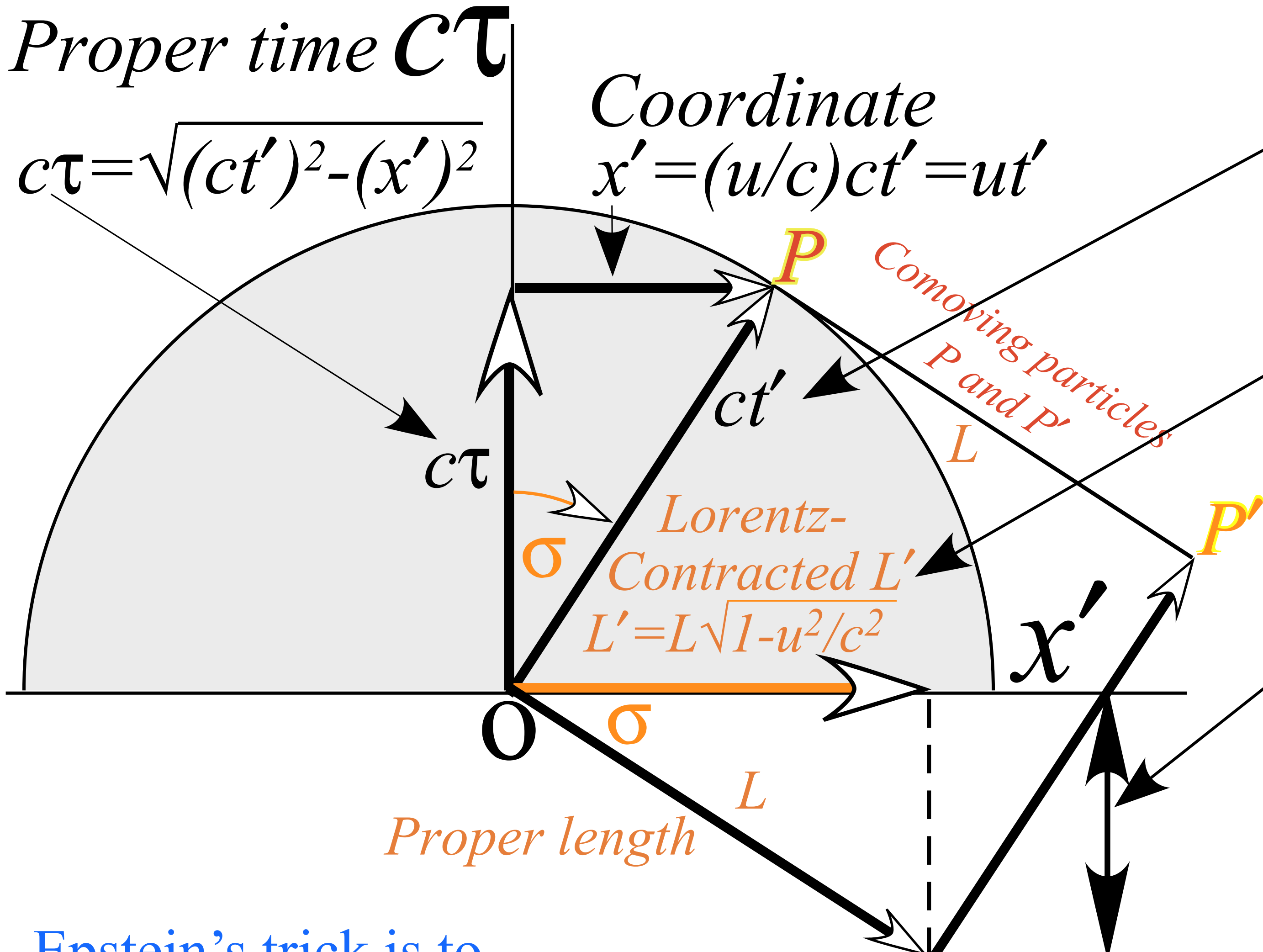

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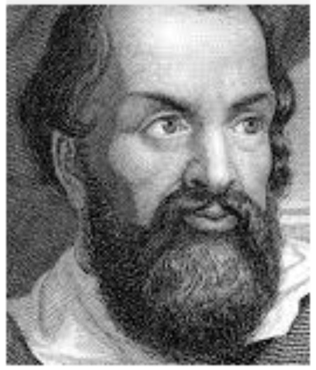
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2 + (x')^2} = (ct')$$

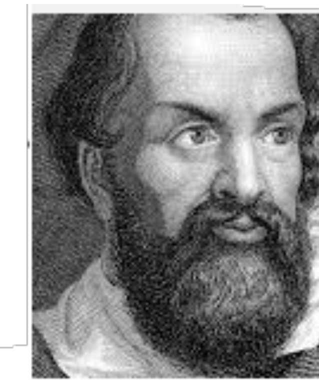
Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



Galileo's Revenge (part 2)
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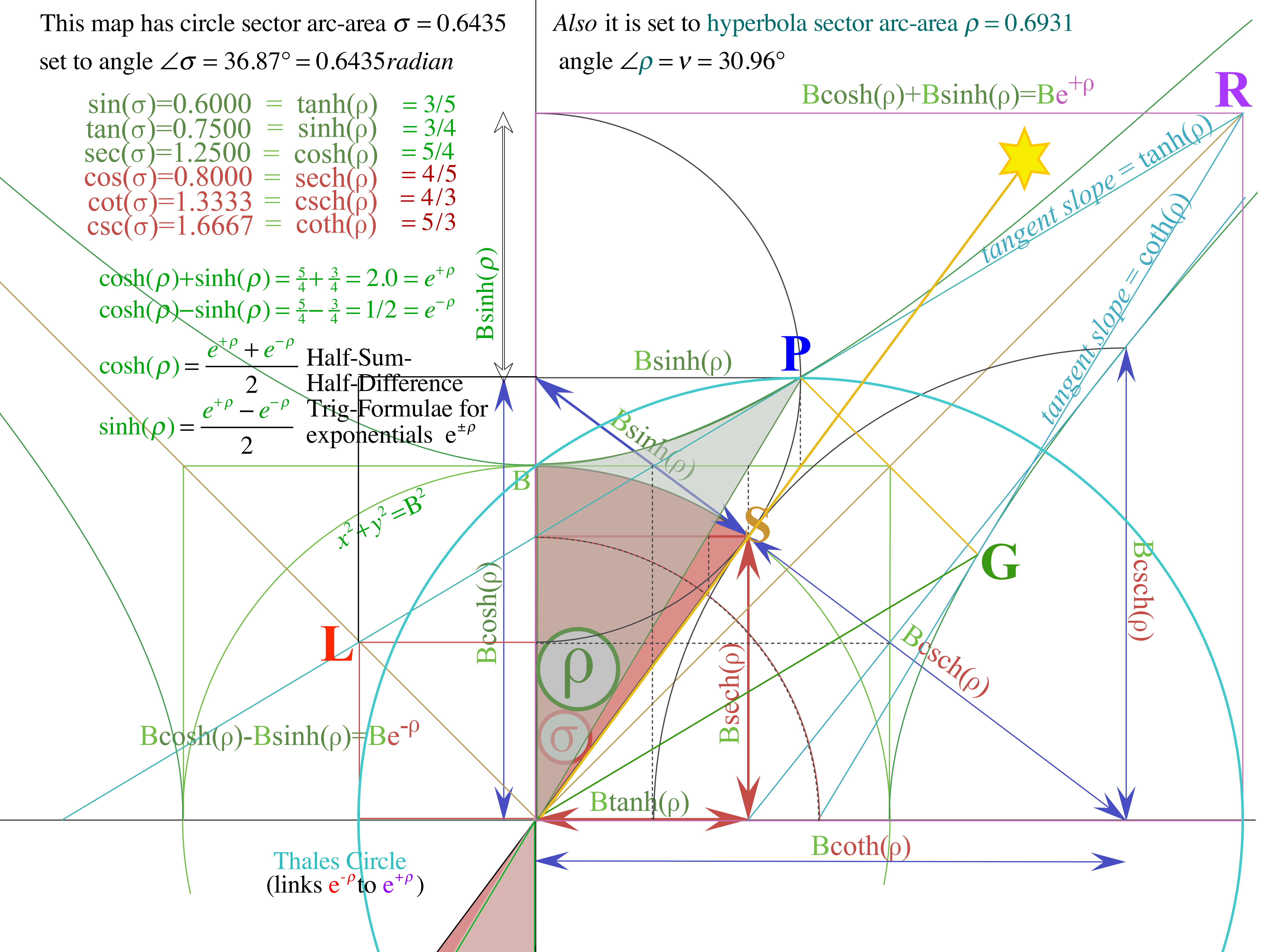
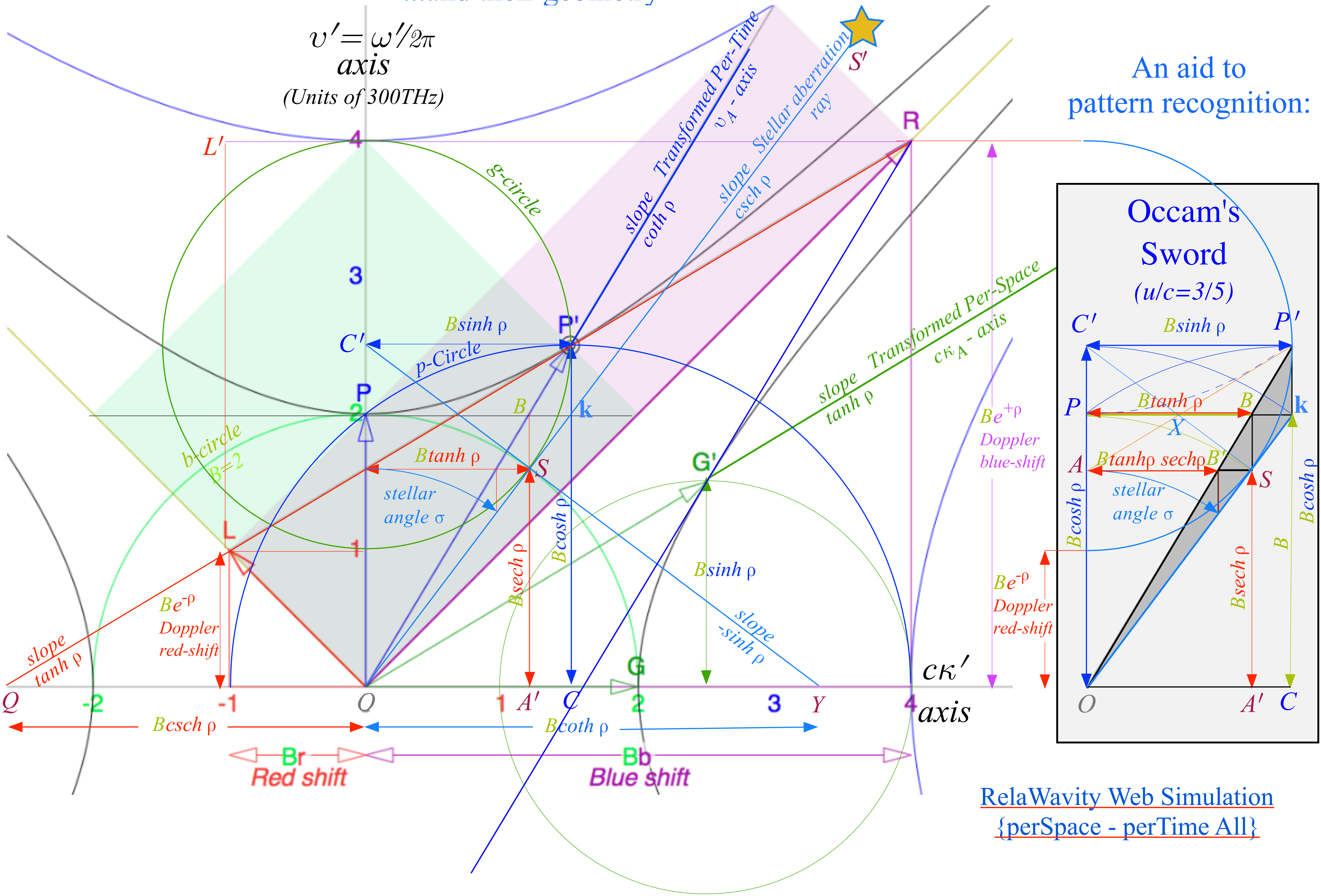
→ "Occams Sword" and geometry of functions of ρ and σ Minkowski animations
Application to TE-Waveguide modes and synchrotron beam relativity

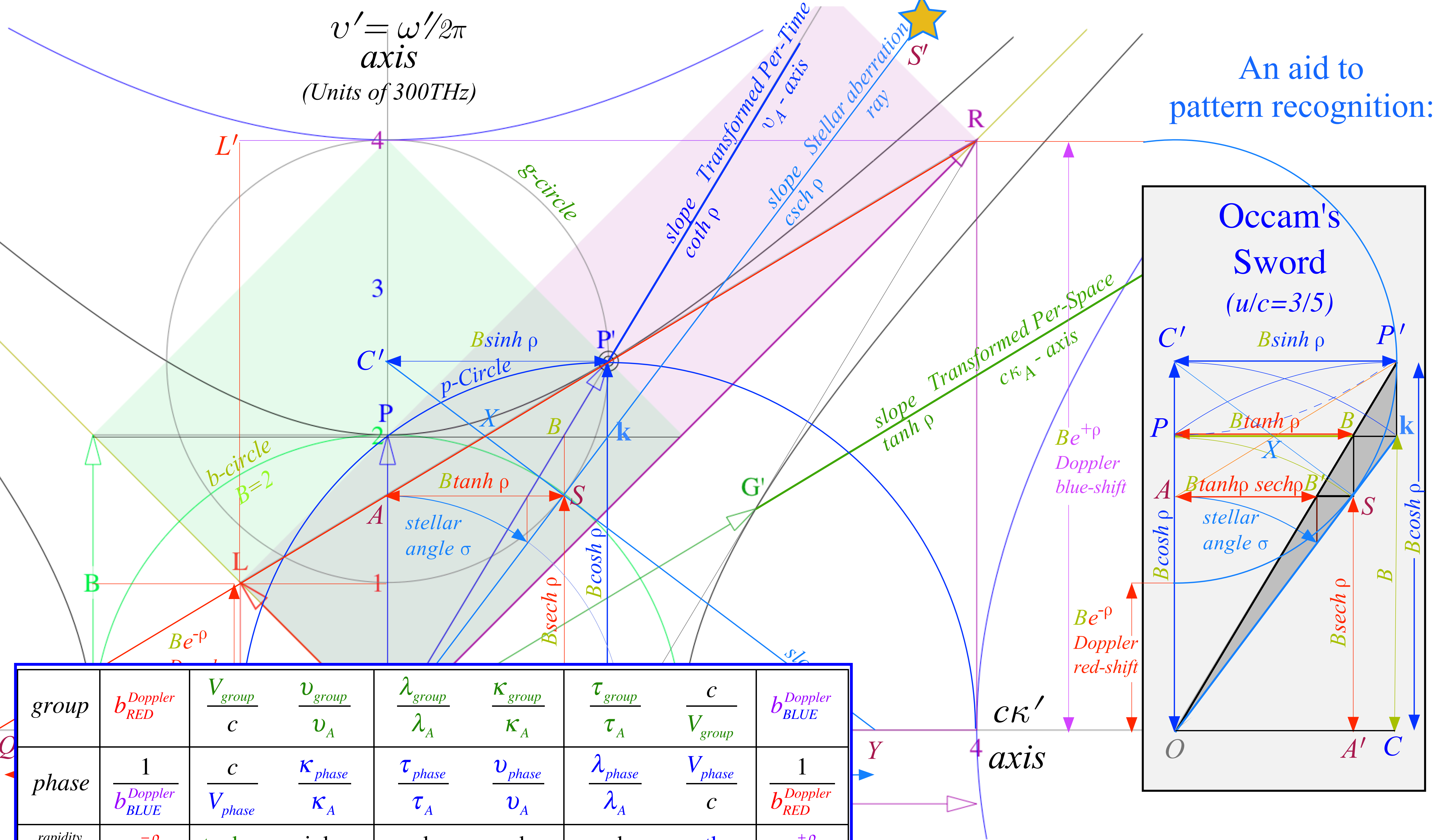

This map has circle sector arc-area $\sigma = 0.6435$
set to angle $\angle\sigma = 36.87° = 0.6435 radian$
Also it is set to hyperbola sector arc-area $\rho = 0.6931$
angle $\angle\rho = \nu = 30.96°$
sin(σ)=0.6000 = tanh(ρ) = 3/5
tan(σ)=0.7500 = sinh(ρ) = 3/4
sec(σ)=1.2500 = cosh(ρ) = 5/4
cos(σ)=0.8000 = sech(ρ) = 4/5
cot(σ)=1.3333 = csch(ρ) = 4/3
csc(σ)=1.6667 = coth(ρ) = 5/3
$\cosh(\rho)+\sinh(\rho) = \frac{5}{4} + \frac{3}{4} = 2.0 = e^{+\rho}$
$\cosh(\rho)-\sinh(\rho) = \frac{5}{4} - \frac{3}{4} = 1/2 = e^{-\rho}$
$\cosh(\rho) = \frac{e^{+\rho} + e^{-\rho}}{2}$
$\sinh(\rho) = \frac{e^{+\rho} - e^{-\rho}}{2}$
Half-Sum-Half-Difference Trig-Formulae for exponentials $e^{\pm\rho}$
Bcosh(ρ)+Bsinh(ρ)=Be$^{+\rho}$
R
Bsinh(ρ)
P
tangent slope = tanh(ρ)
tangent slope = coth(ρ)
Bsinh(ρ)
B
$x^2+y^2=B^2$
S
G
Bcsch(ρ)
L
Bcosh(ρ)
ρ
σ
Bsech(ρ)
Bcsch(ρ)
Bcosh(ρ)-Bsinh(ρ)=Be$^{-\rho}$
Btanh(ρ)
Bcoth(ρ)
Thales Circle
(links e$^{-\rho}$ to e$^{+\rho}$)

Summary of optical wave parameters for relativity and QM

...and their geometry



[RelaWavity Web Simulation {perSpace - perTime All}]



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

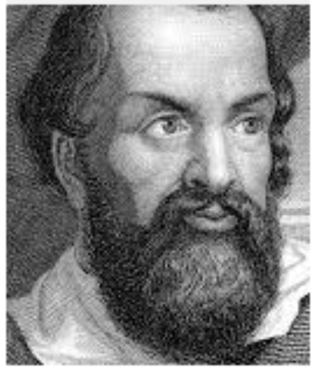
Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

RelaWavity Web Simulation

Expanded Relativistic Relations

Galileo Galilei



1564-1642

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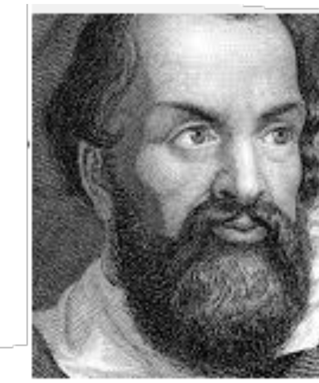

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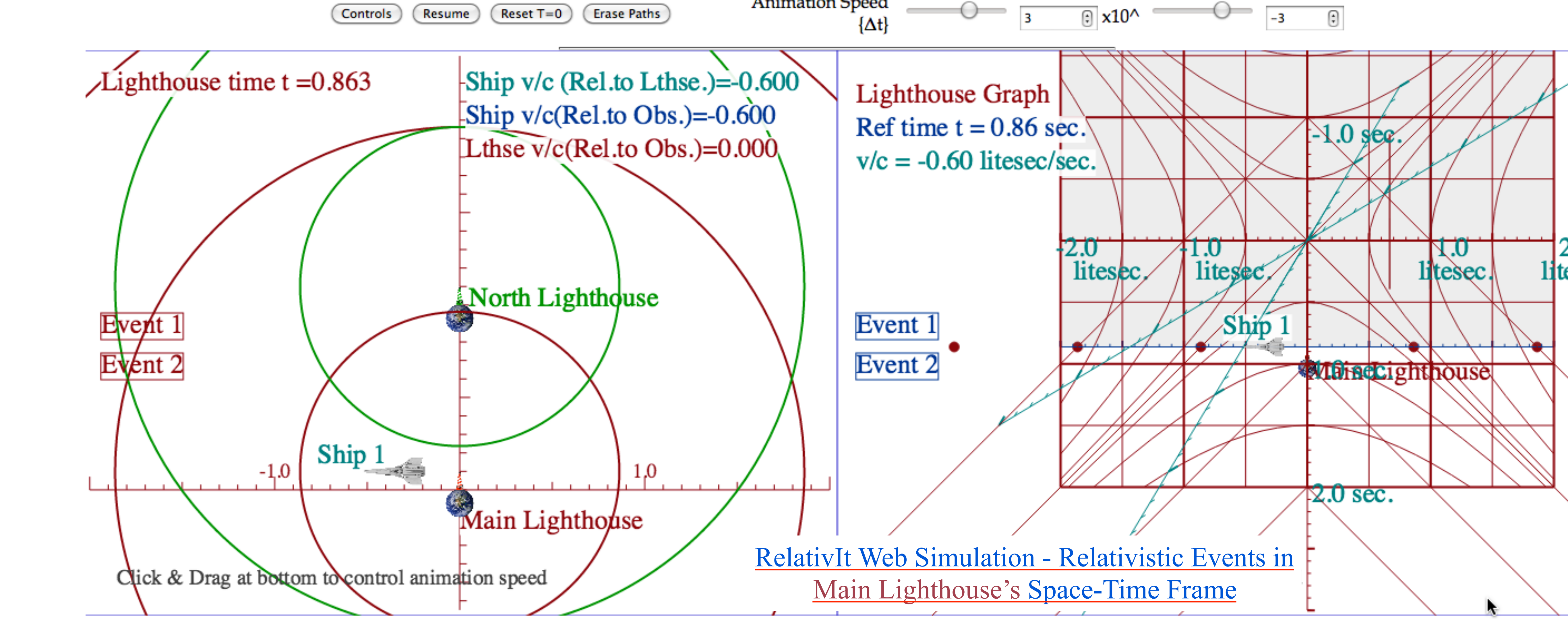
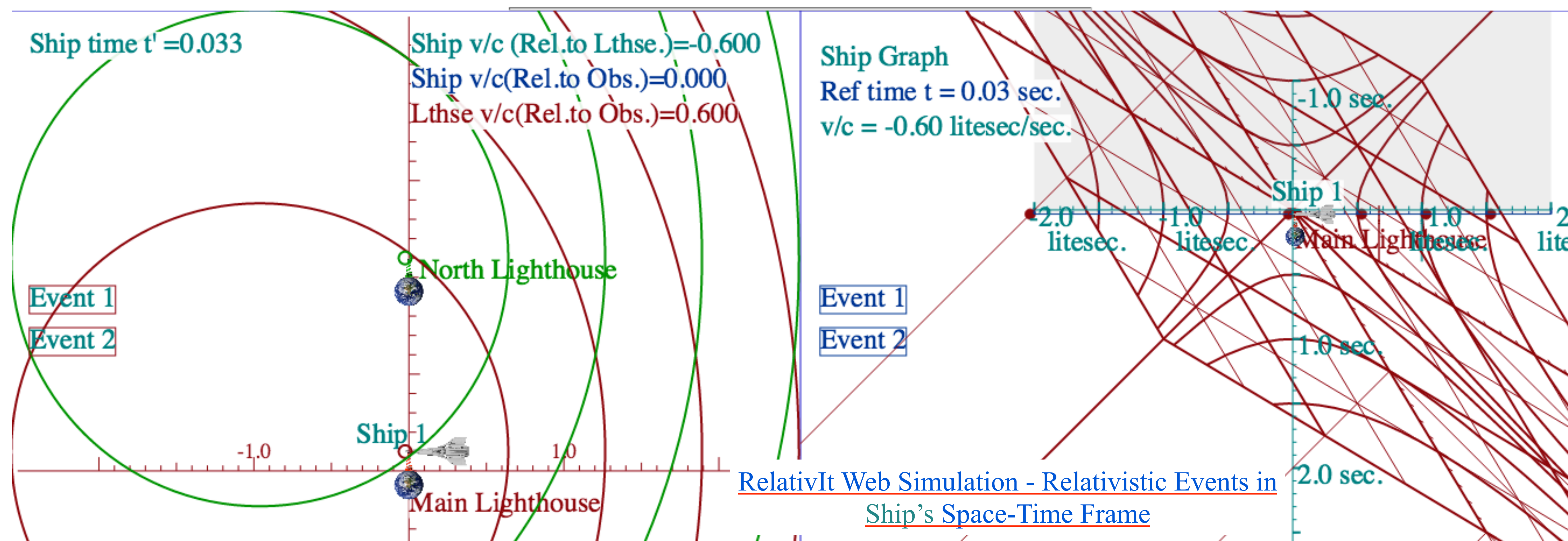
    - Application to TE-Waveguide modes synchrotron beam relativity

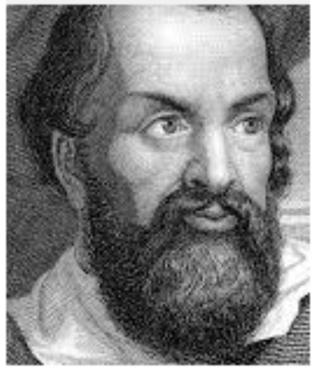


RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame



RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

Galileo Galilei



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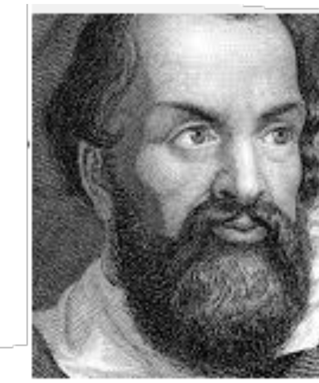

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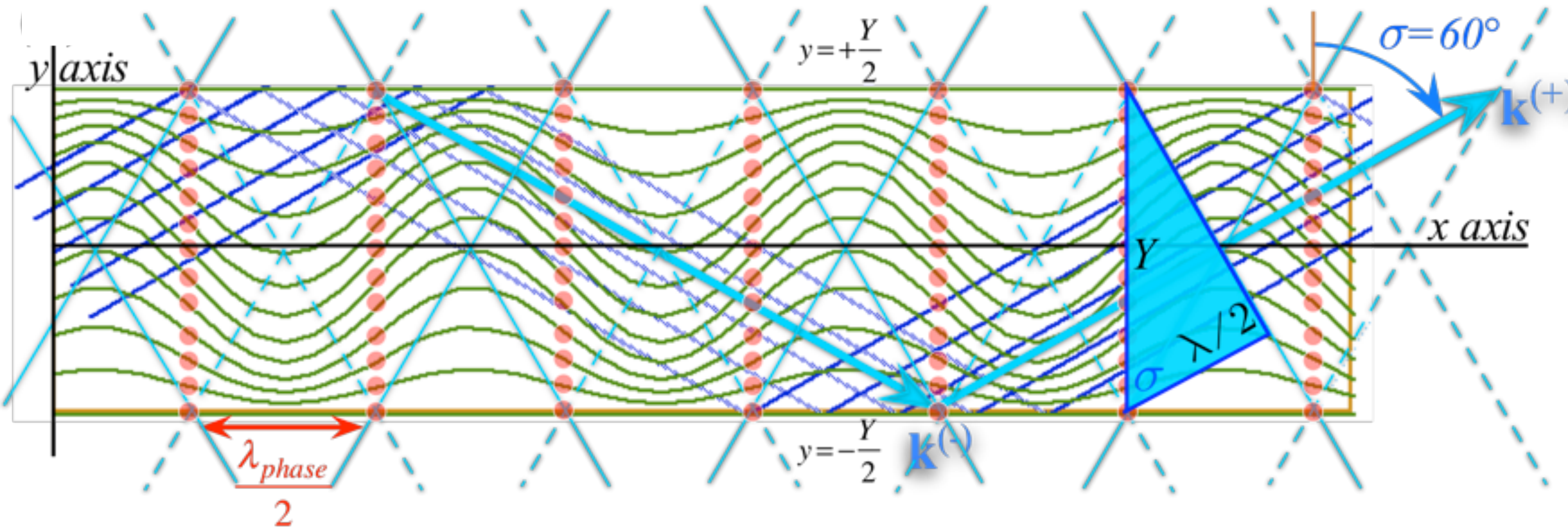
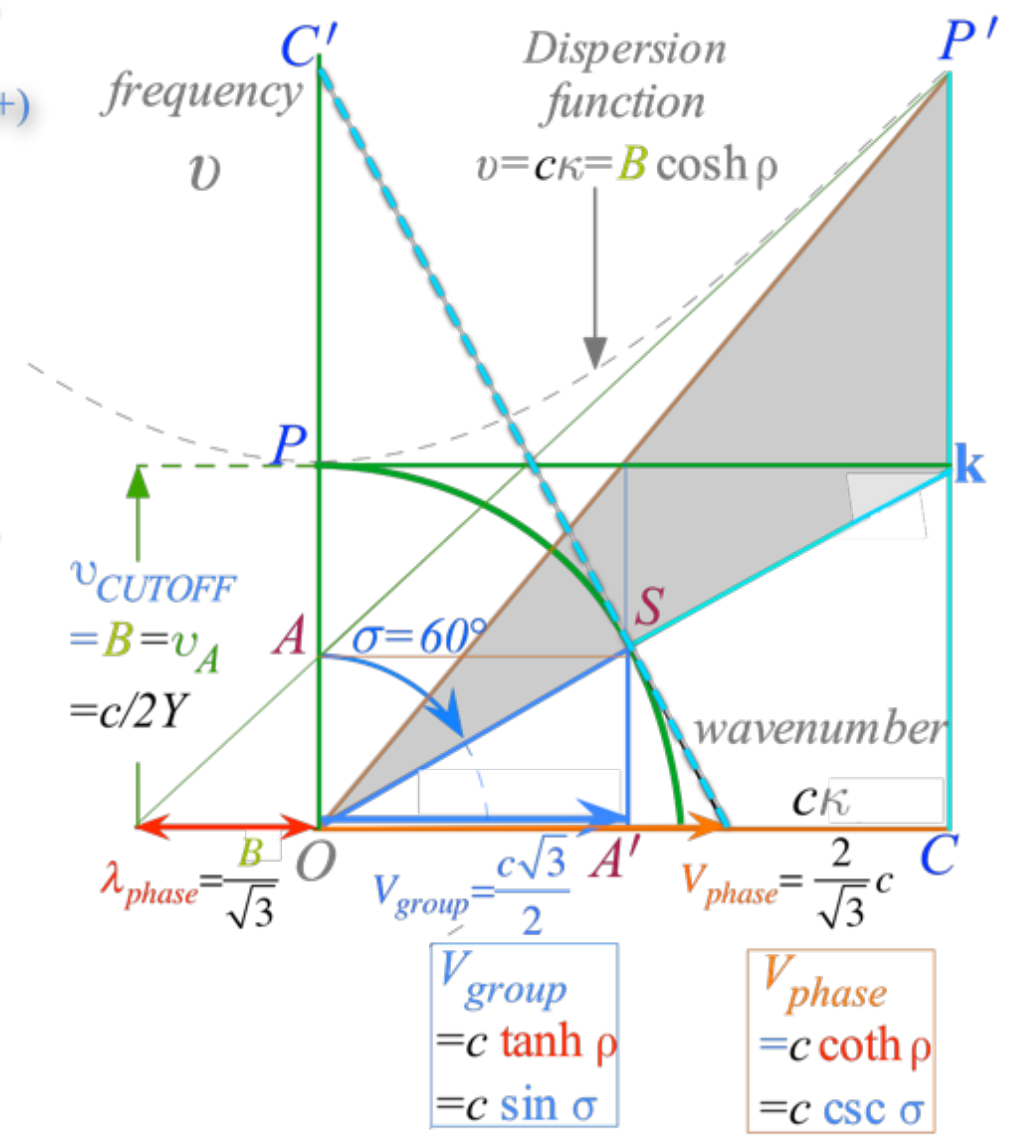
Application to TE-Waveguide modes synchrotron beam relativity

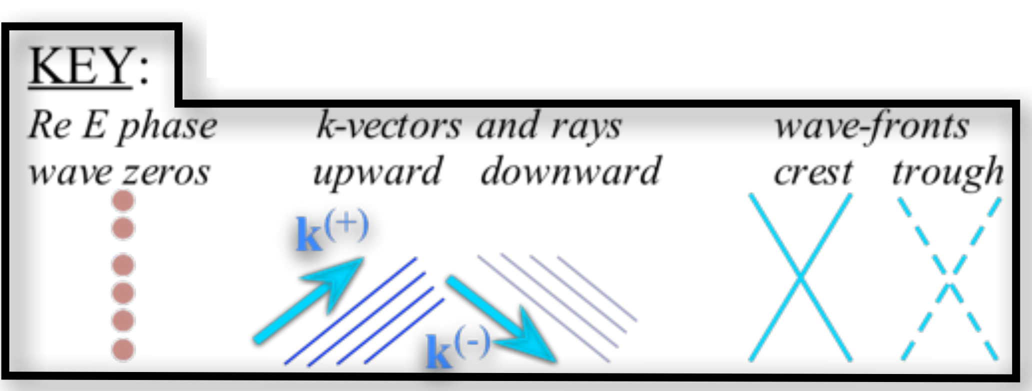
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)







Optical wave guide relativistic geometry aided by Occam's Sword

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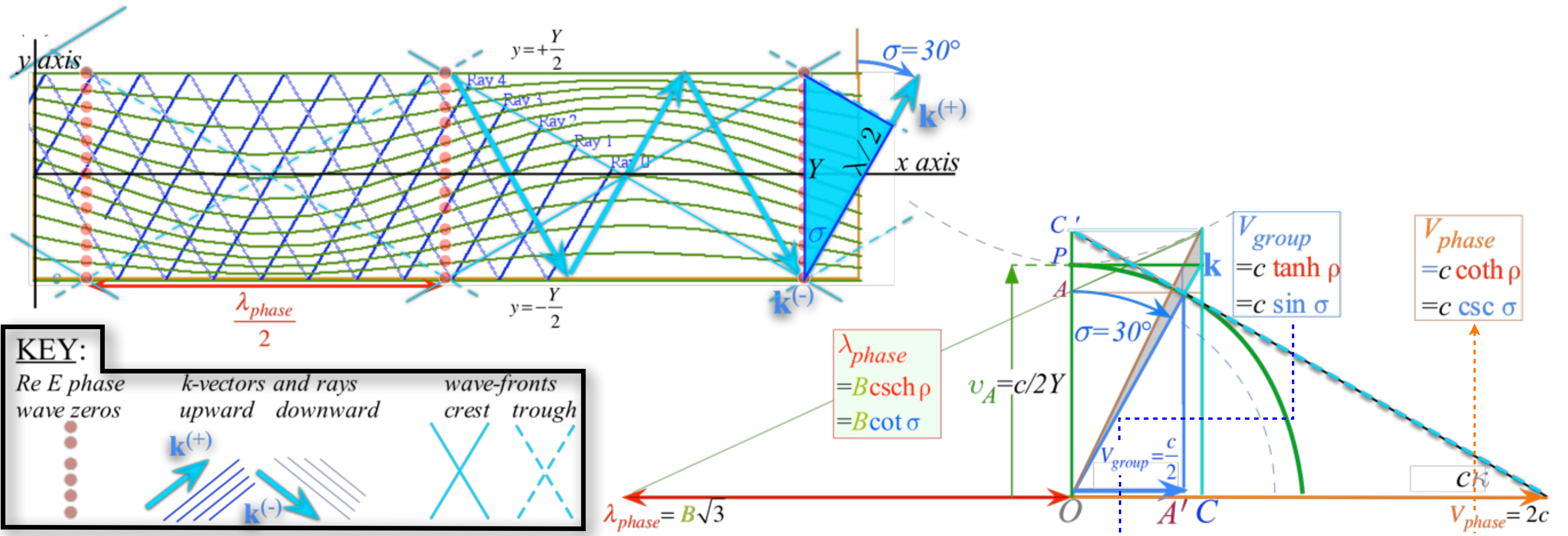

Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Optical wave guide relativistic geometry aided by Occam's Sword

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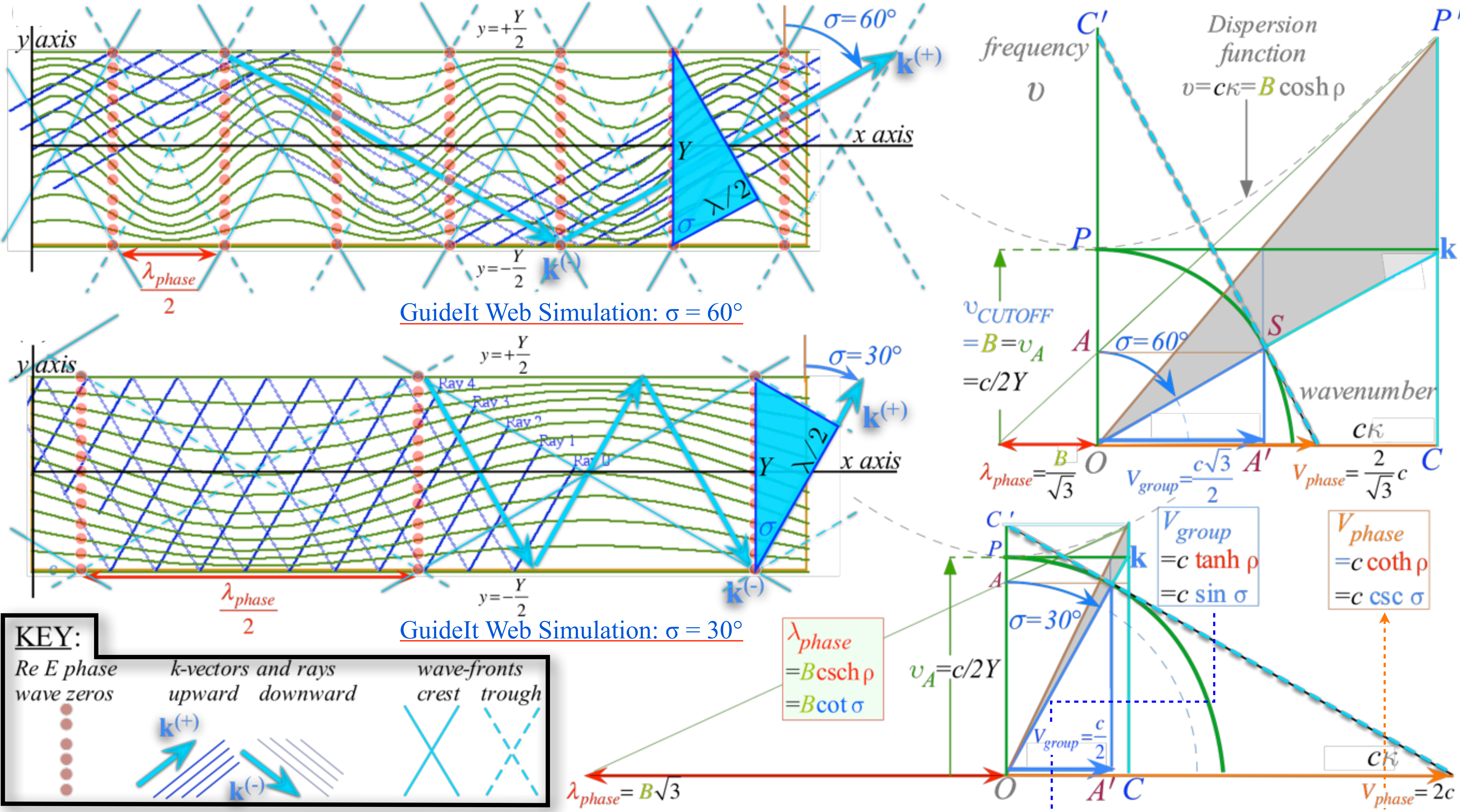

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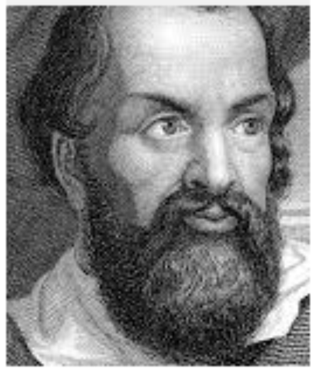
Galileo Galilei



1564-1642

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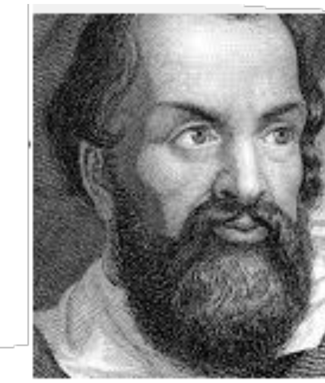

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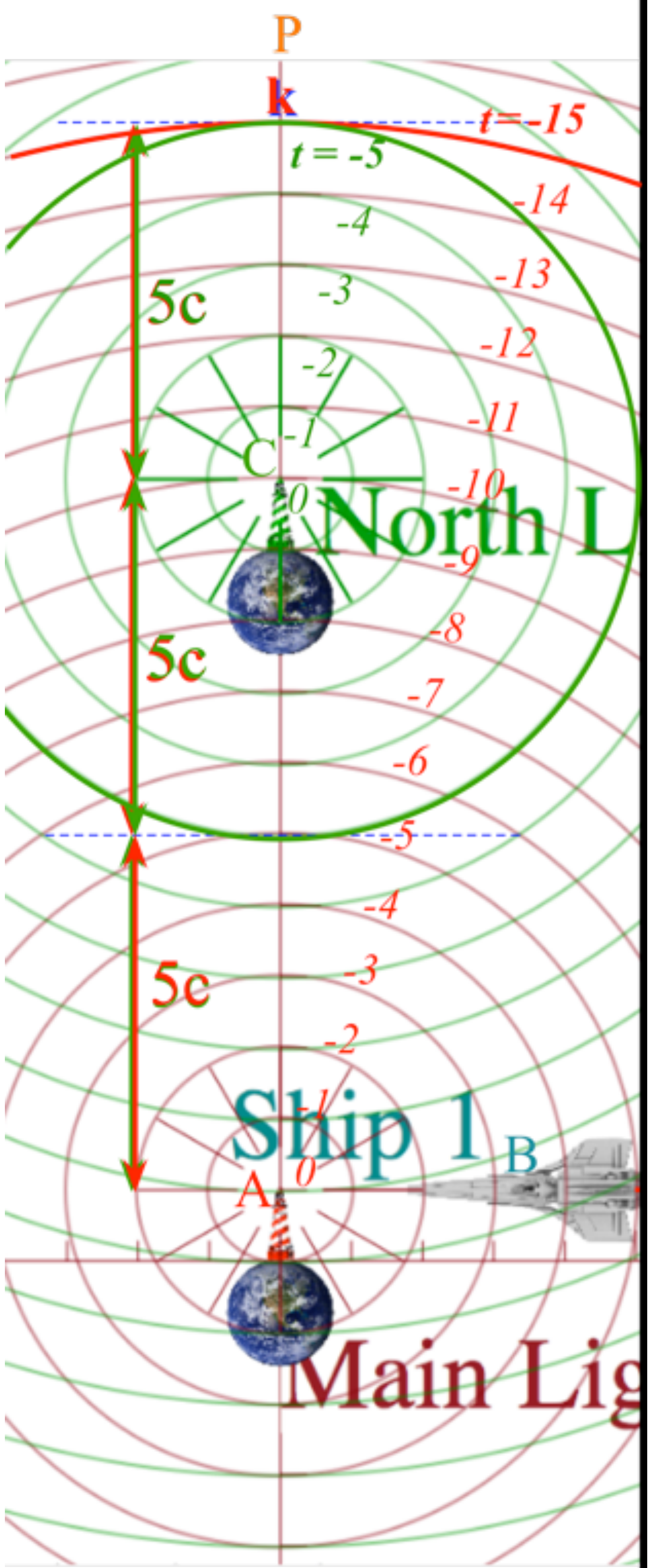
Application to TE-Waveguide modes. ➔ synchrotron beam relativity ⬅

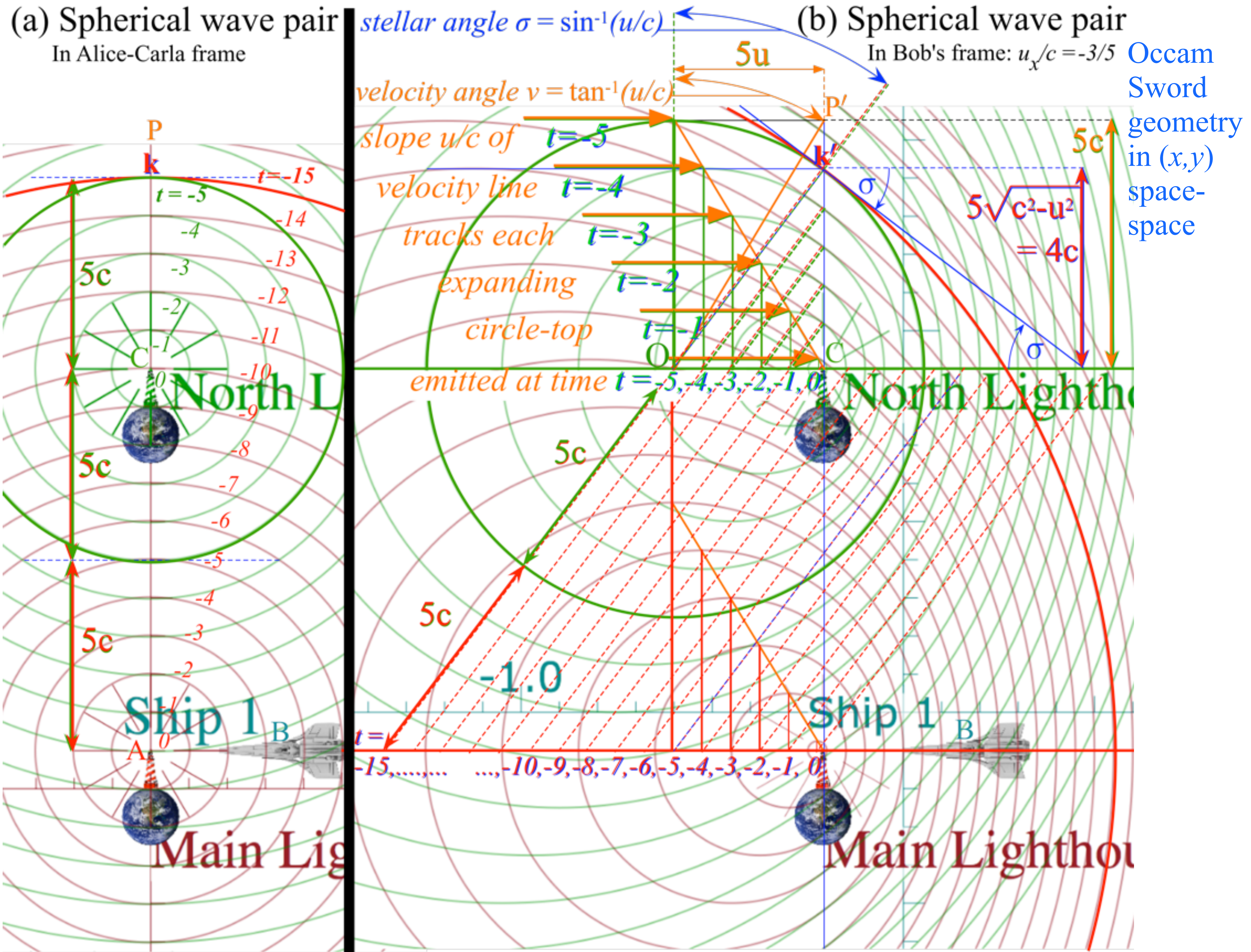
Spherical wave relativistic geometry

Also, aided by Occam's Sword

(a) Spherical wave pair

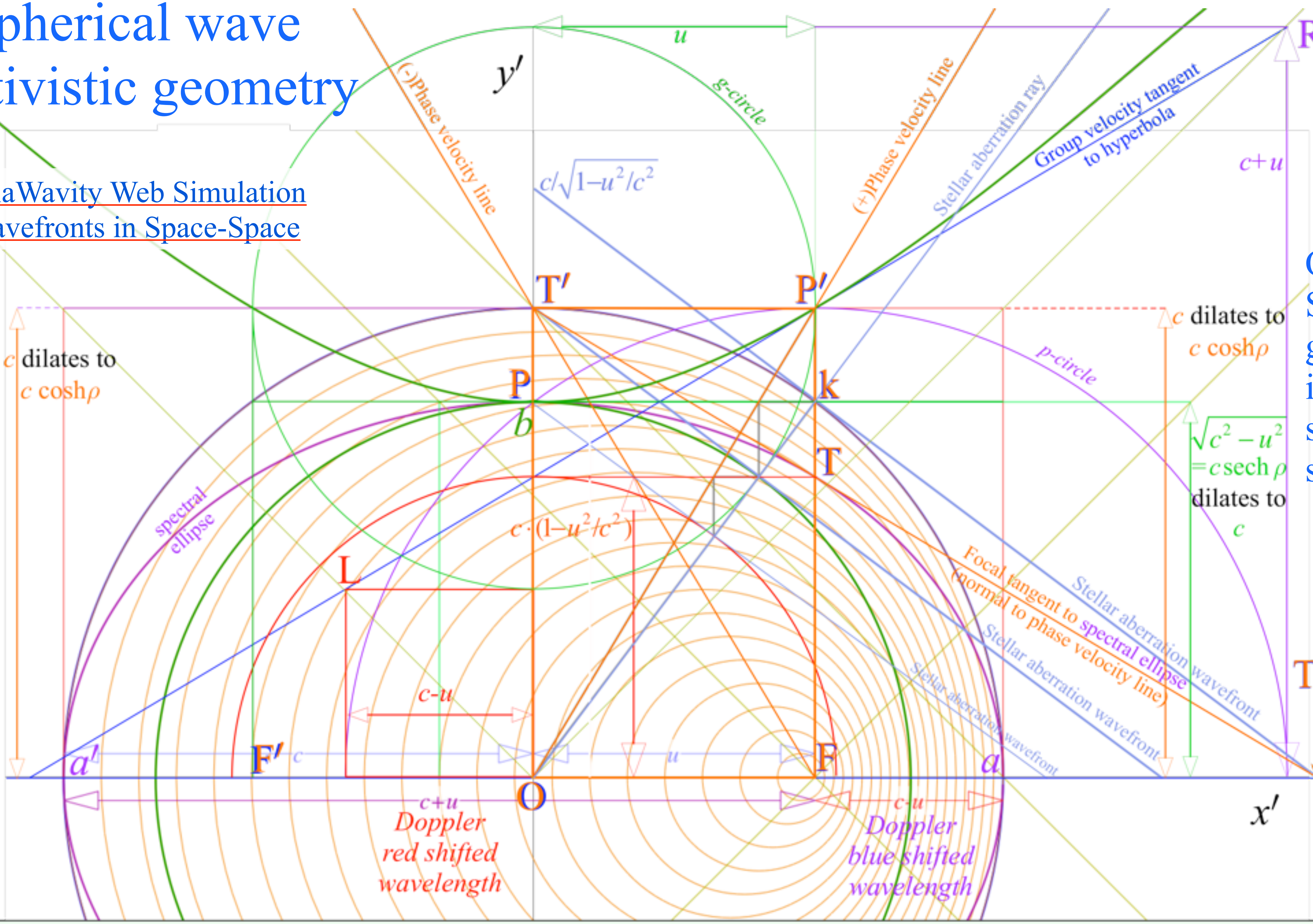
In Alice-Carla frame




(a) Spherical wave pair
In Alice-Carla frame
(b) Spherical wave pair
In Bob's frame: $u_x/c = -3/5$
stellar angle $\sigma = \sin^{-1}(u/c)$
velocity angle $\nu = \tan^{-1}(u/c)$
slope u/c of
velocity line
tracks each
expanding
circle-top
emitted at time t = -5,-4,-3,-2,-1,0
5u
5c
$5\sqrt{c^2-u^2} = 4c$
-1.0
t = -15,........ ....,-10,-9,-8,-7,-6,-5,-4,-3,-2,-1, 0
North Lighthouse
Main Lighthouse
Ship 1
Occam Sword geometry in (x,y) space-space

Spherical wave relativistic geometry

RelaWavity Web Simulation
Wavefronts in Space-Space



Occam Sword geometry in (x,y) space-space

Doppler Red $\lambda = c+u$
dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse major radius a=**OF**a=c
dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

Applications of Einstein dilation factor:
$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse focal length **FO**= $u = c\tanh\rho$
dilates to: $u\cosh\rho = c\sinh\rho$

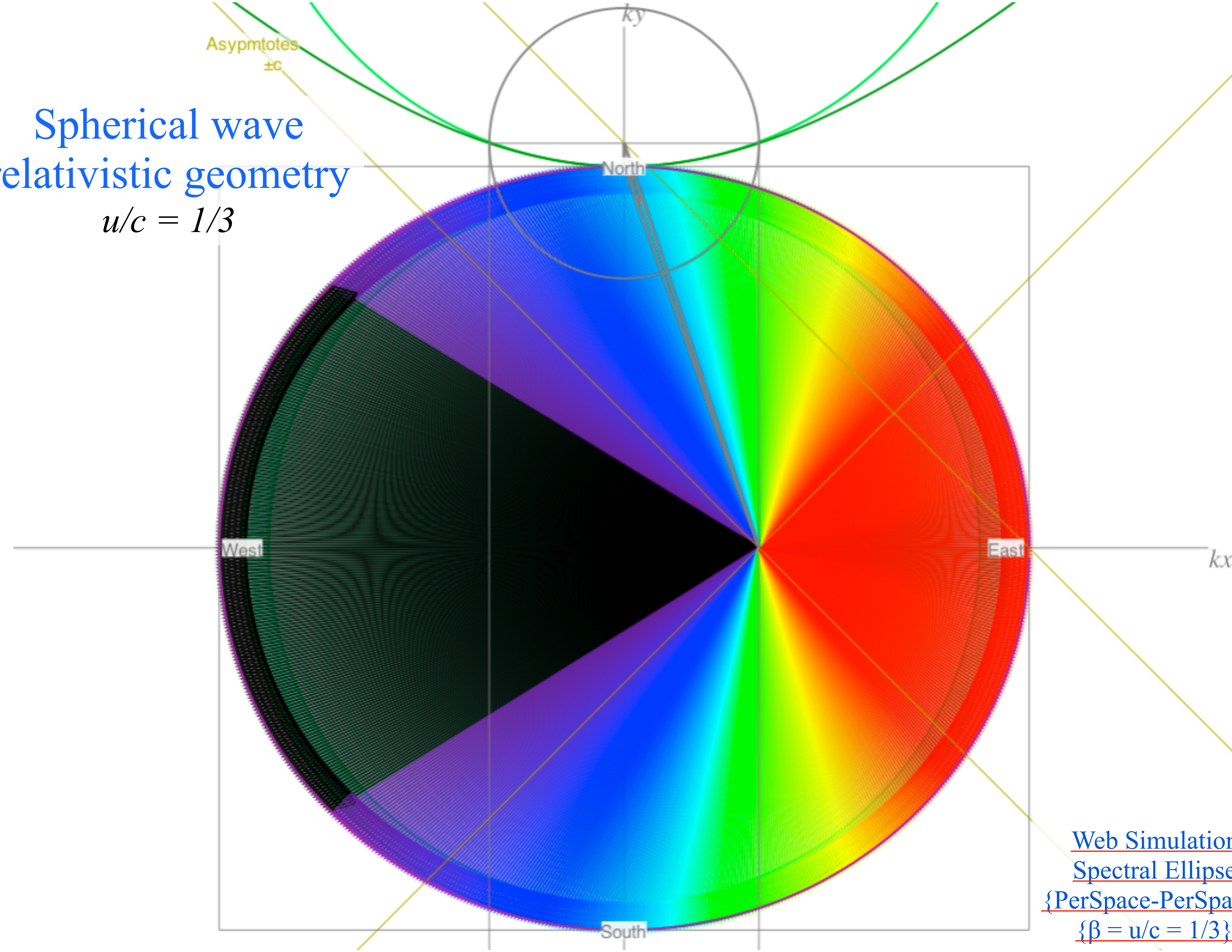
ellipse latus radius **FT**=$c(1-u^2/c^2)$
dilates to: $c(1-u^2/c^2)\cosh\rho$
$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

Doppler Blue $\lambda = c-u$
dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

Base height **FTk**=$\sqrt{c^2-u^2}$
dilates to: $\sqrt{c^2-u^2}\cosh\rho = c$
(equal to ellipse minor radius b)

Spherical wave relativistic geometry

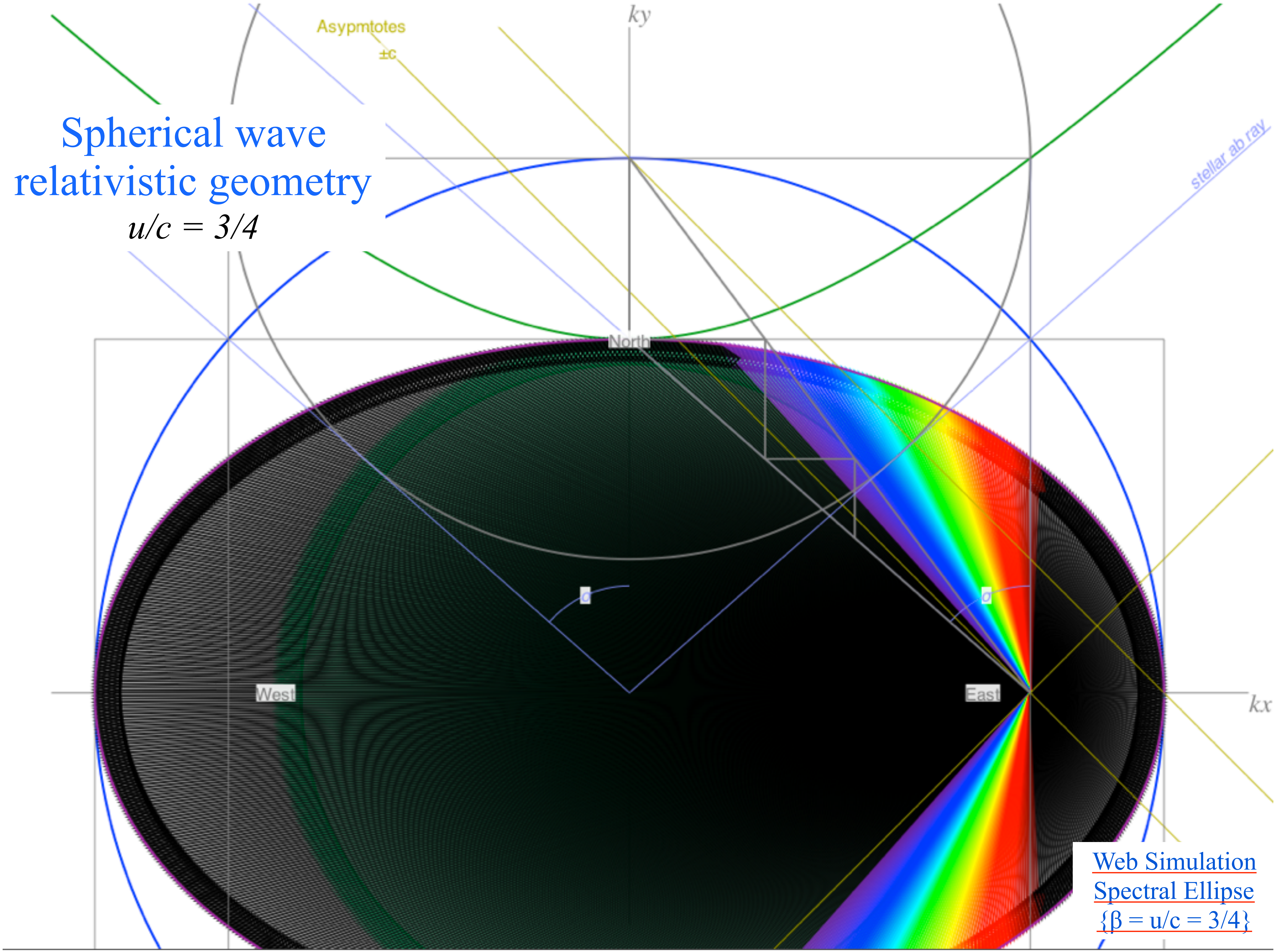
u/c = 1/3



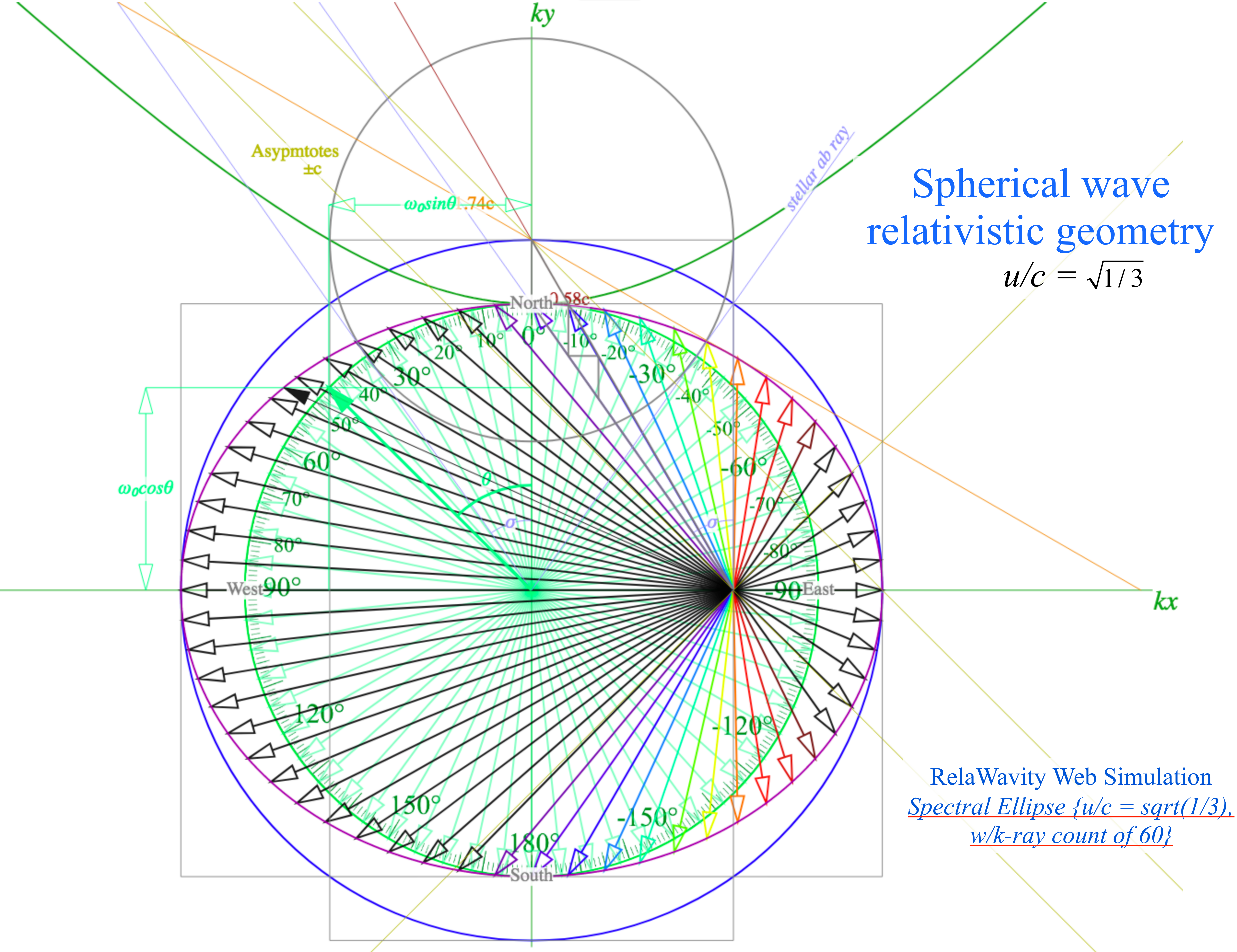
Web Simulation
Spectral Ellipse
{PerSpace-PerSpace}
{β = u/c = 1/3}

Spherical wave relativistic geometry

u/c = 3/4



[Web Simulation Spectral Ellipse {β = u/c = 3/4}]



Spherical wave relativistic geometry

$u/c = \sqrt{1/3}$

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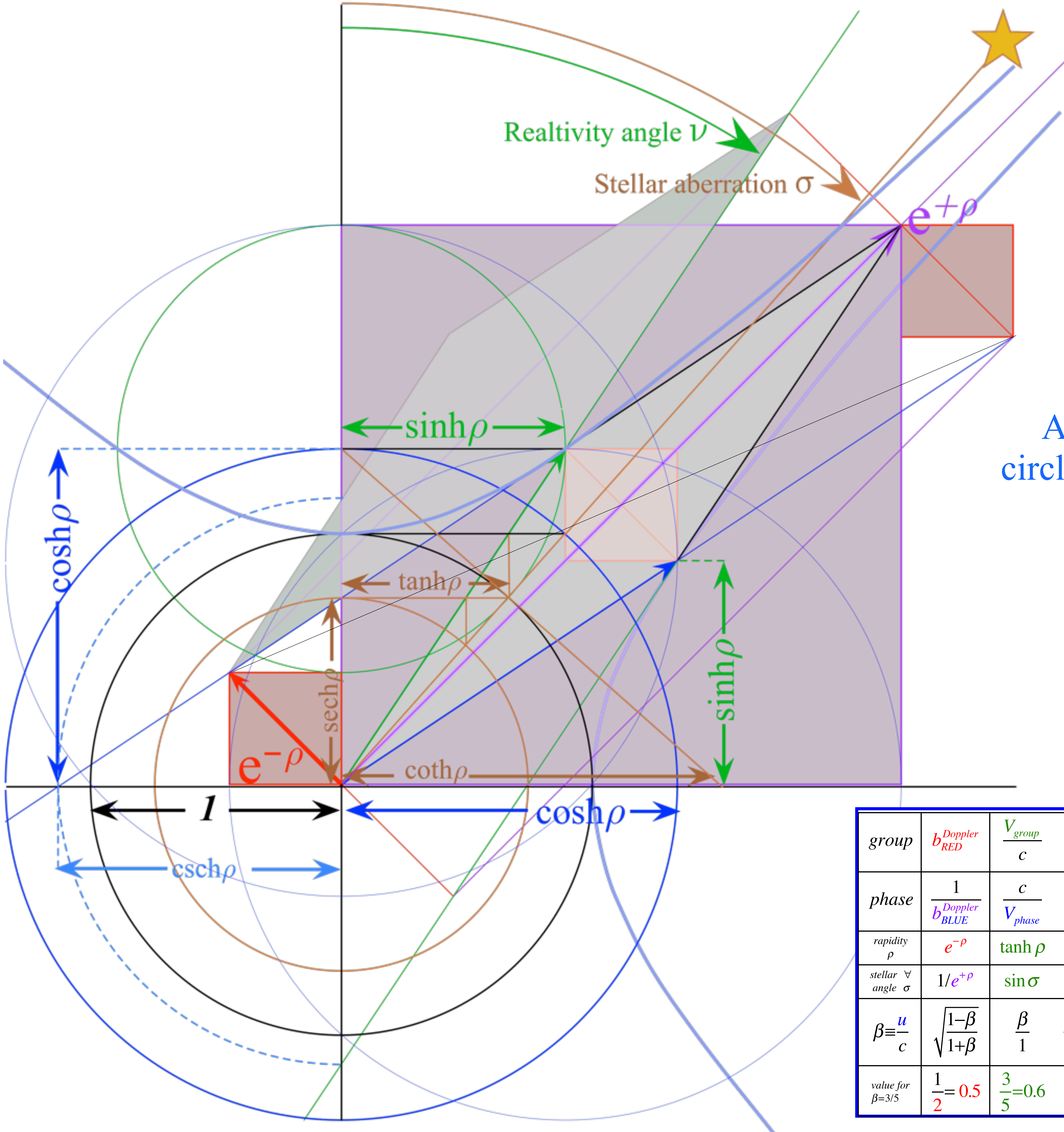
Spectral Ellipse {u/c = sqrt(1/3), w/k-ray count of 60}

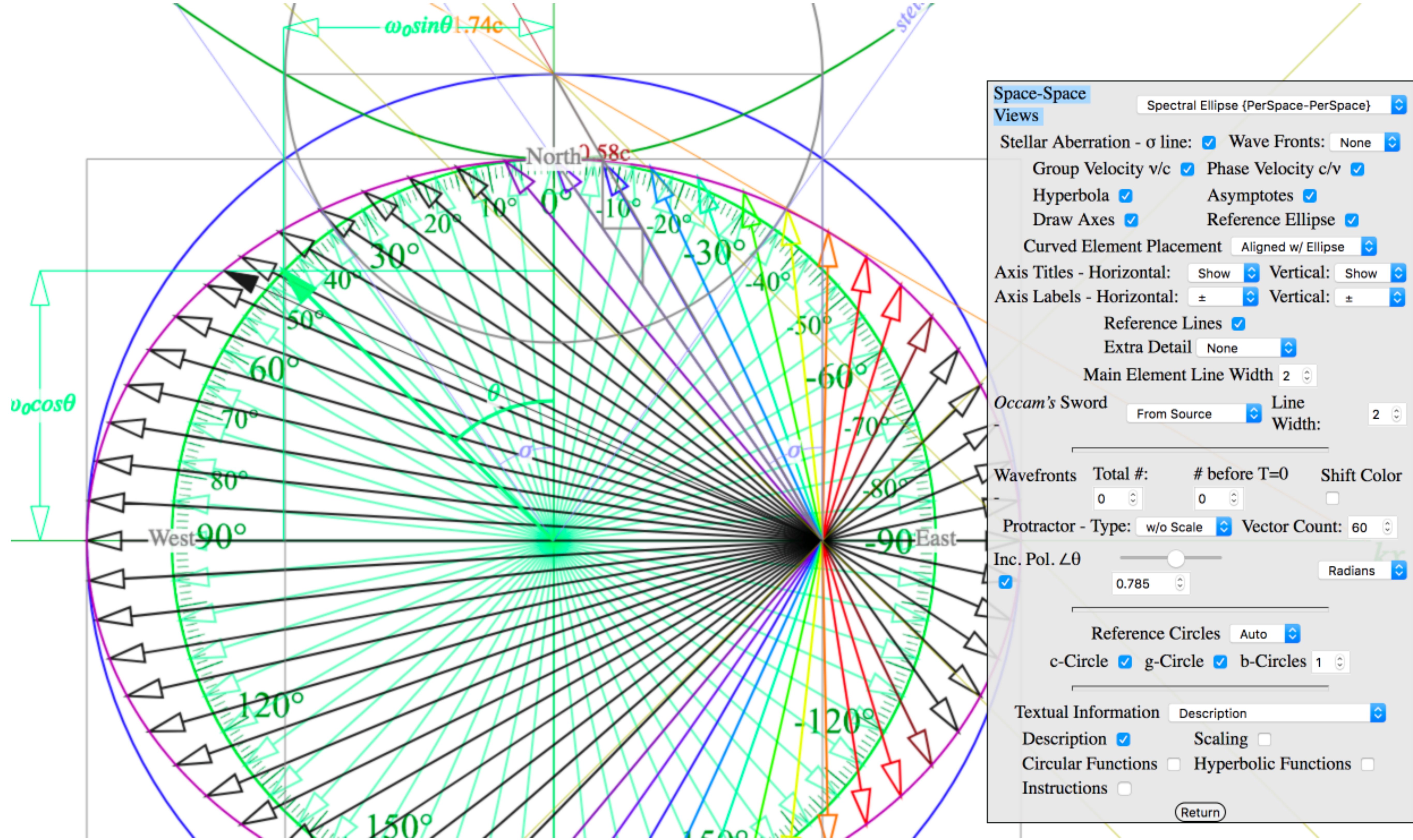
A more compact circle-based geometry



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Intentionally blank

Alternate slides to follow



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Spectral Ellipse {u/c = sqrt(1/3), w/ray count of 60}

Spherical wave relativistic geometry

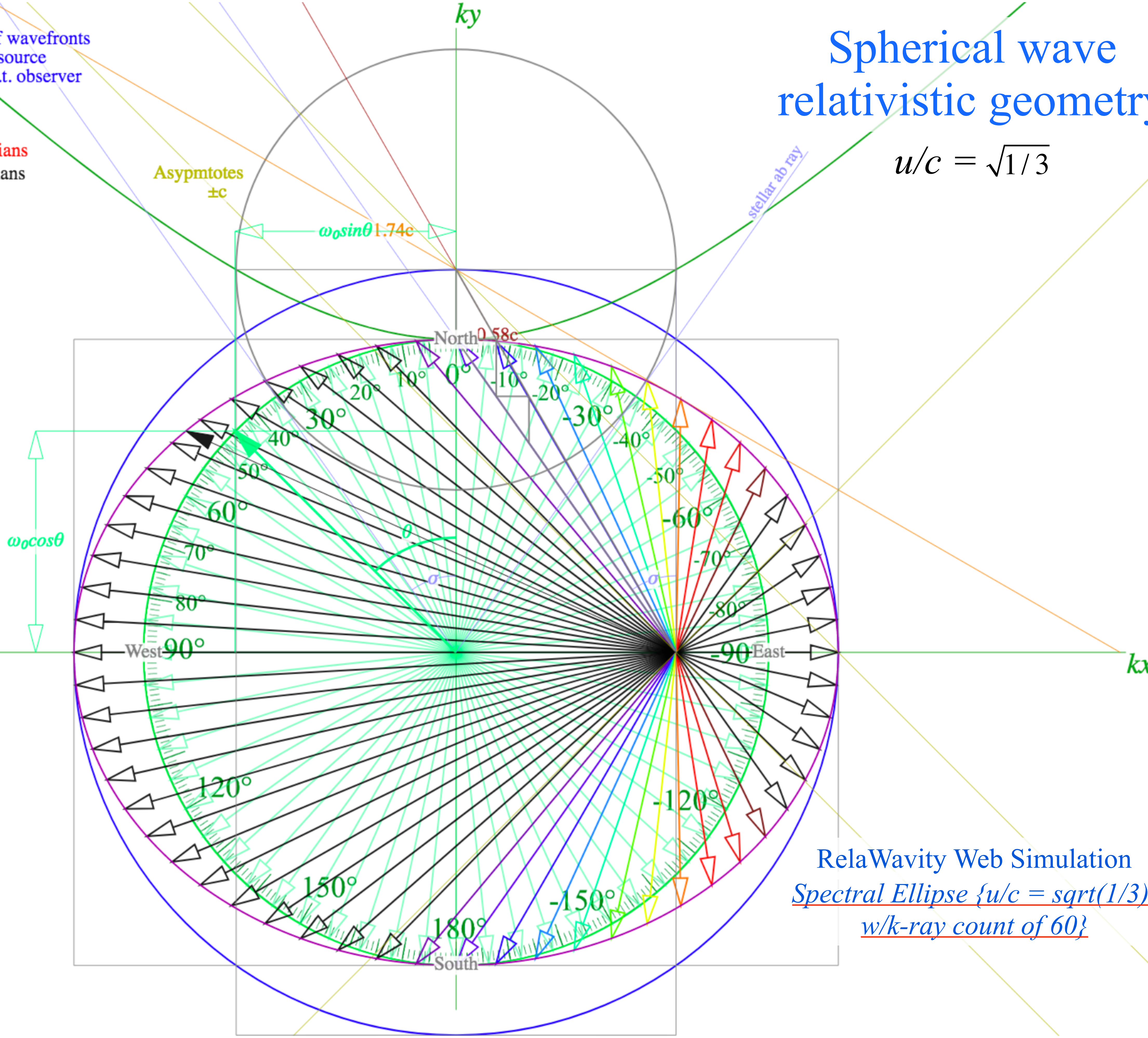
$u/c = \sqrt{1/3}$

x-Space-y-Space Plot of wavefronts dropped by CW or PW source moving at u = 0.58c w.r.t. observer

$V_{group}/c = u/c = 0.58$
$V_{phase}/c = c/u = 1.74$
Inc. Pol. ∠θ = 0.79 Radians
Shifted ∠θ´ = 0.61 Radians

Circular Functions
- m∠(σ) = 0.6132
- Length(σ) = 0.6132
- Area(σ) = 0.6132
- sin(σ) = 0.5755
- tan(σ) = 0.7037
- sec(σ) = 1.2228
- cos(σ) = 0.8178
- cot(σ) = 1.4210
- csc(σ) = 1.7376

Hyperbolic Functions
- ρ = 0.6557
- Area(ρ) = 0.6557
- tanh(ρ) = 0.5755
- sinh(ρ) = 0.7037
- cosh(ρ) = 1.2228
- sech(ρ) = 0.8178
- csch(ρ) = 1.4210
- coth(ρ) = 1.7376



RelaWavity Web Simulation
Spectral Ellipse {u/c = sqrt(1/3), w/k-ray count of 60}