Lecture 30
Tue. 12.06.2016

Relawavity and a novel introduction to relativistic mechanics I.

(Unit 8   12.06.16)

Learning about sin! *and* cos *and…Trigonometric road maps*

Hyper-Trigonometric algebra and phasors in space-time

1CW wavefunctions and phasors

Per-space-per-time vs Space-time

Wave velocity formulas

Introducing Doppler shifting

Why c is constant?!

Introducing Doppler Arithmetic and rapidity ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of 16 parameter functions of ρ and σ

Application to TE-Waveguide modes

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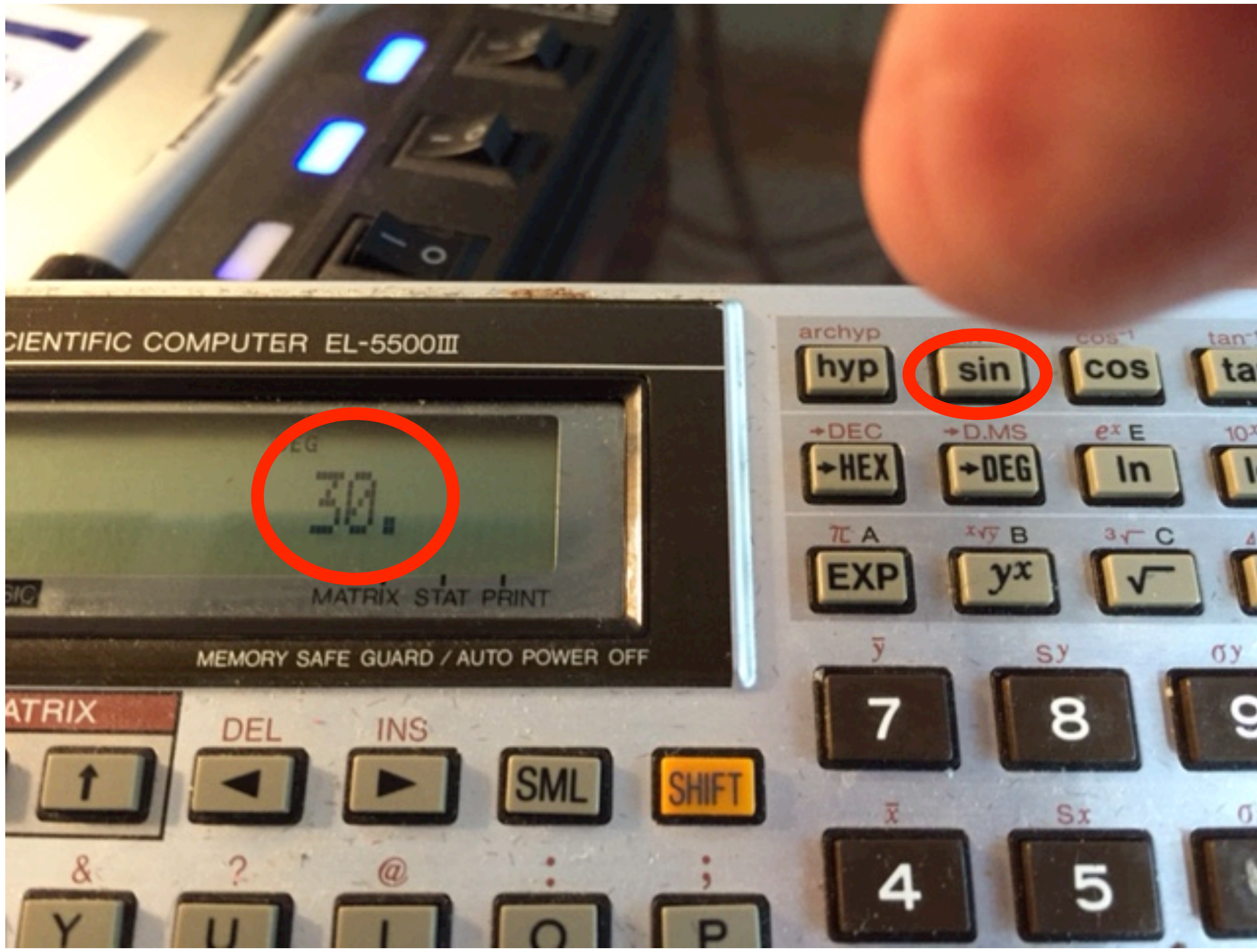
"Occams Sword" and geometry of 16 parameter functions o

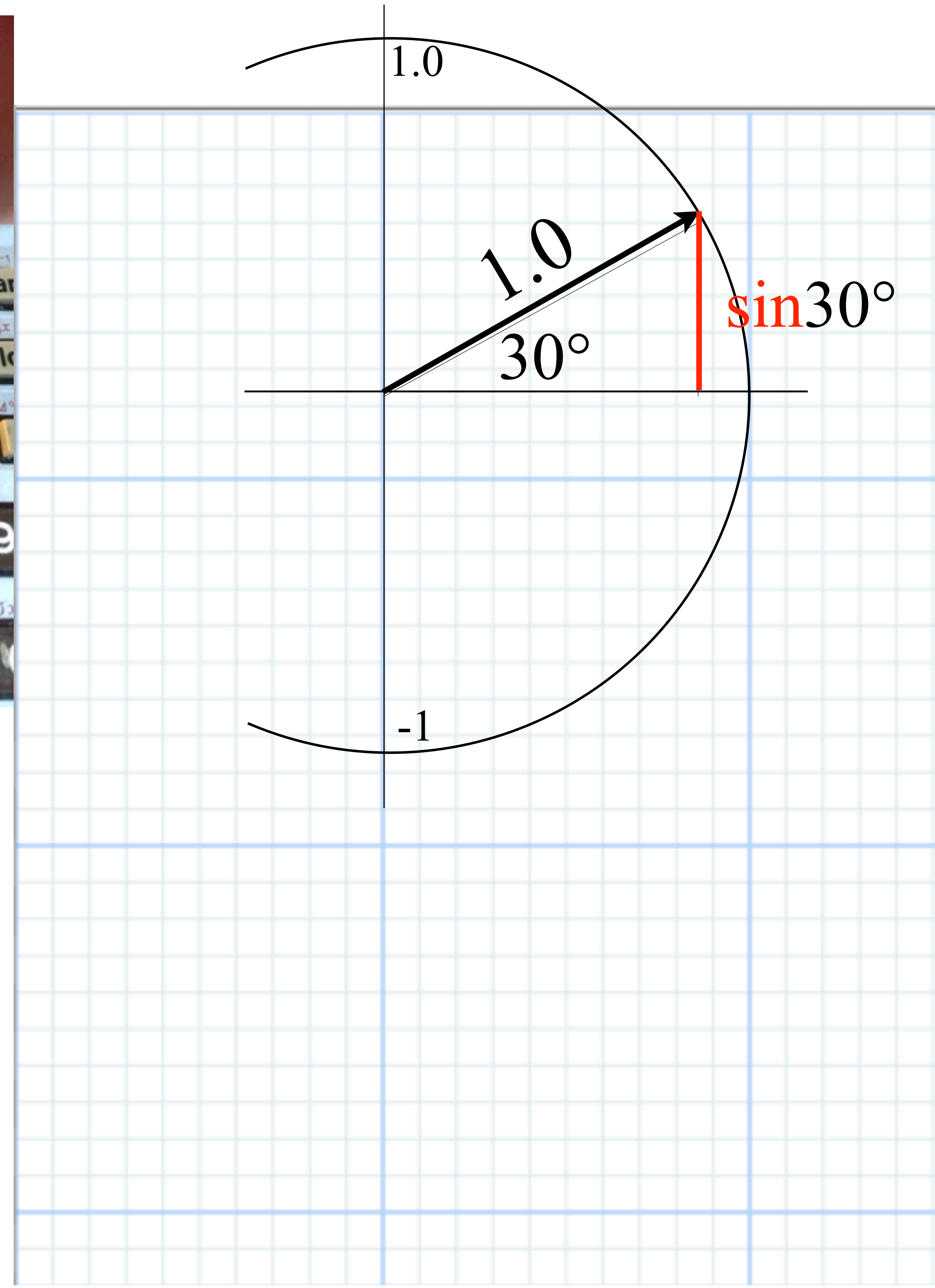
Application to TE-Waveguide modes

For an introductory, web based development of this and other concepts in special relativity see our entrant in the 2005 Pirelli Challenge*:*

A Colorful Road to Relativity
Using Occam's Razors
and
Evenson's Lasers

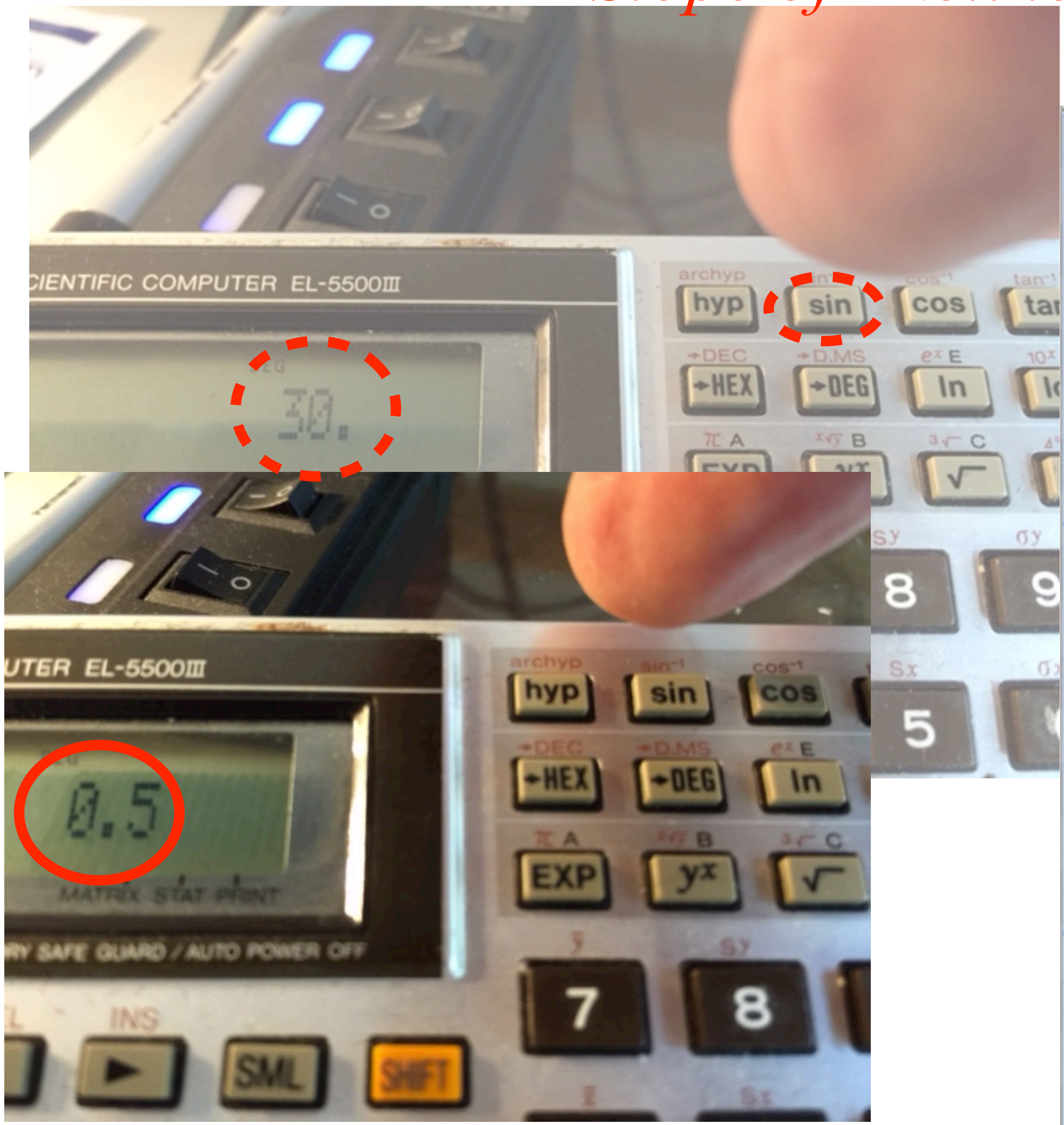
Learning about SIN

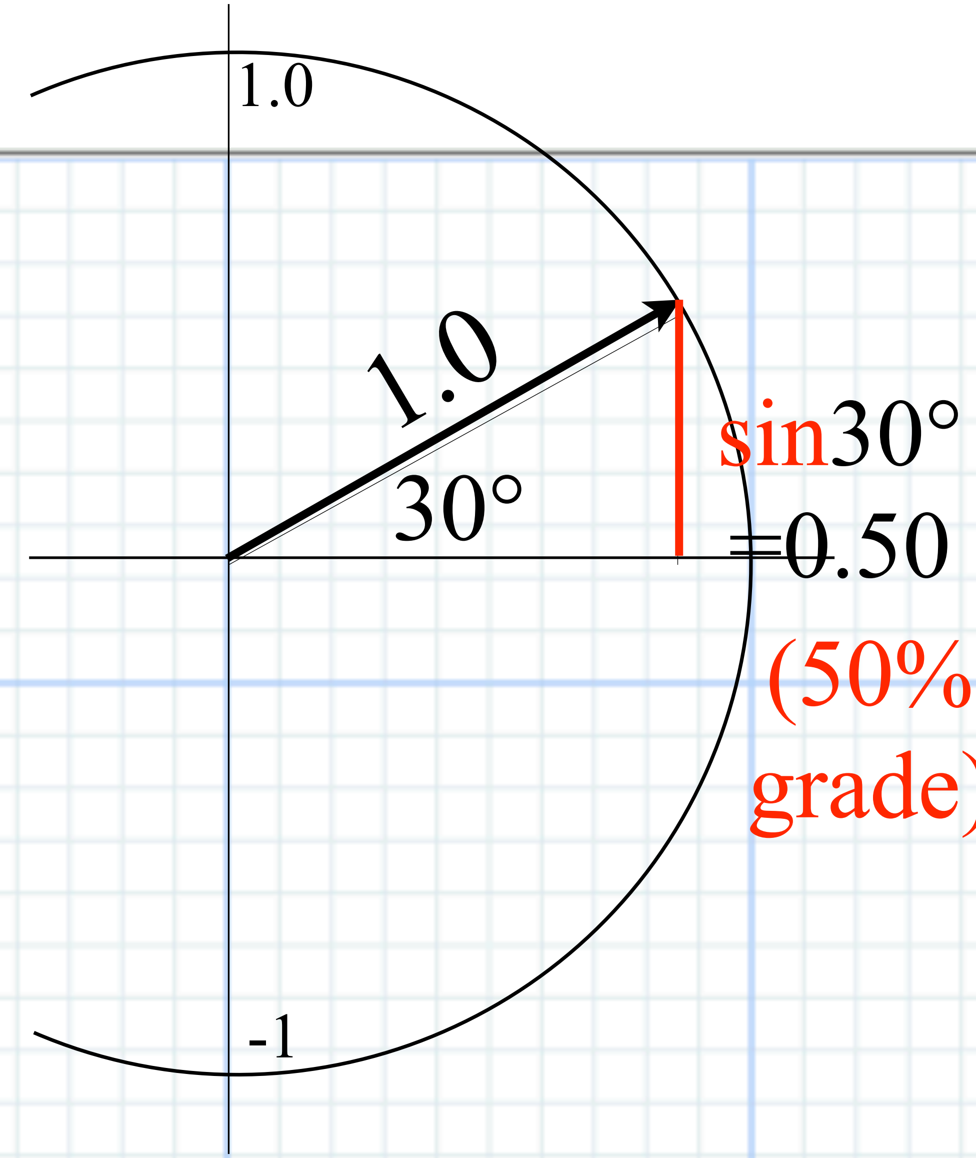




Learning about SIN

"Slope of INcline"

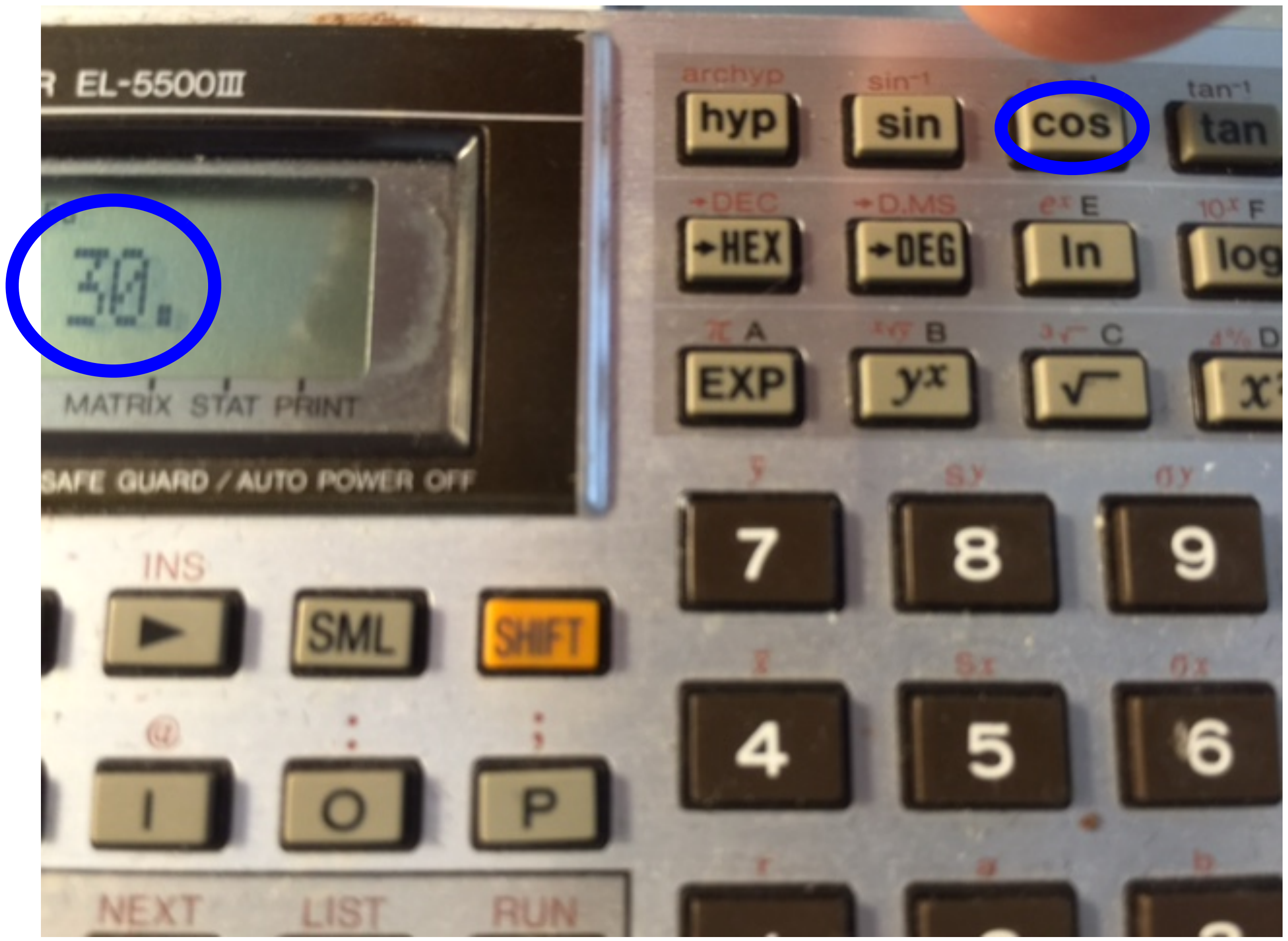


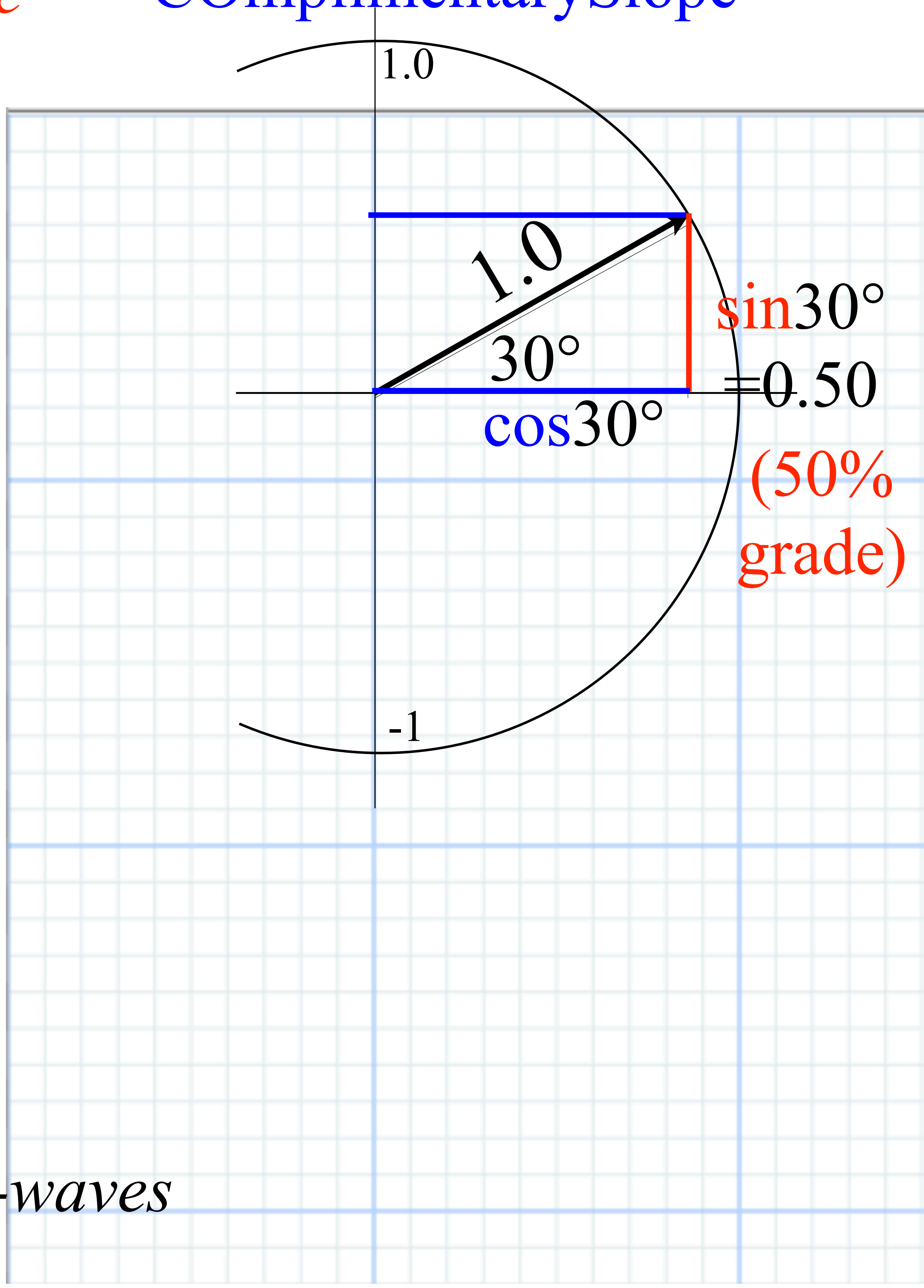


It's mostly about triangles *and sine-waves*

Learning about SIN and the COSin

"*Slope of INcline*" "COmplimentarySlope"





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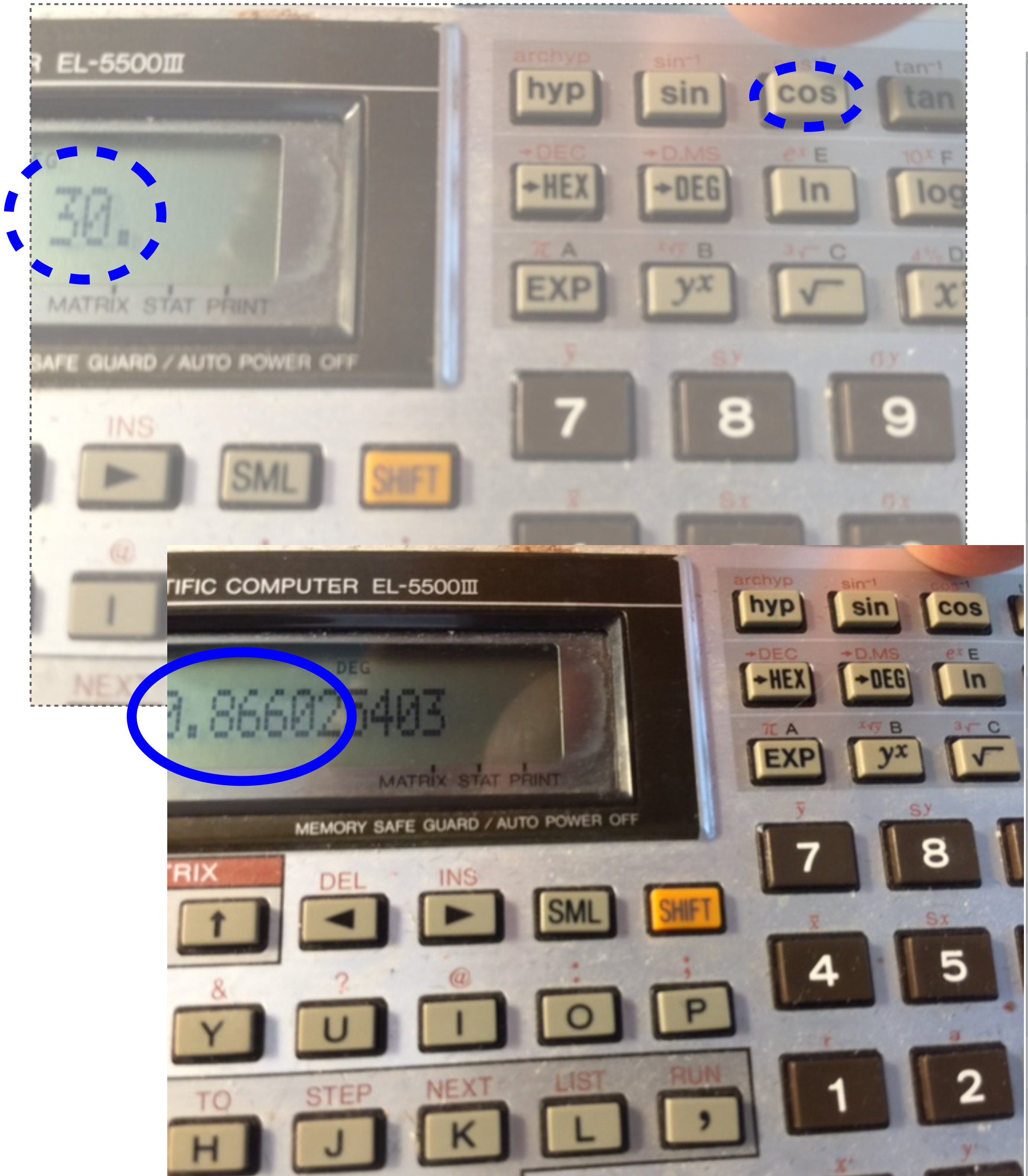
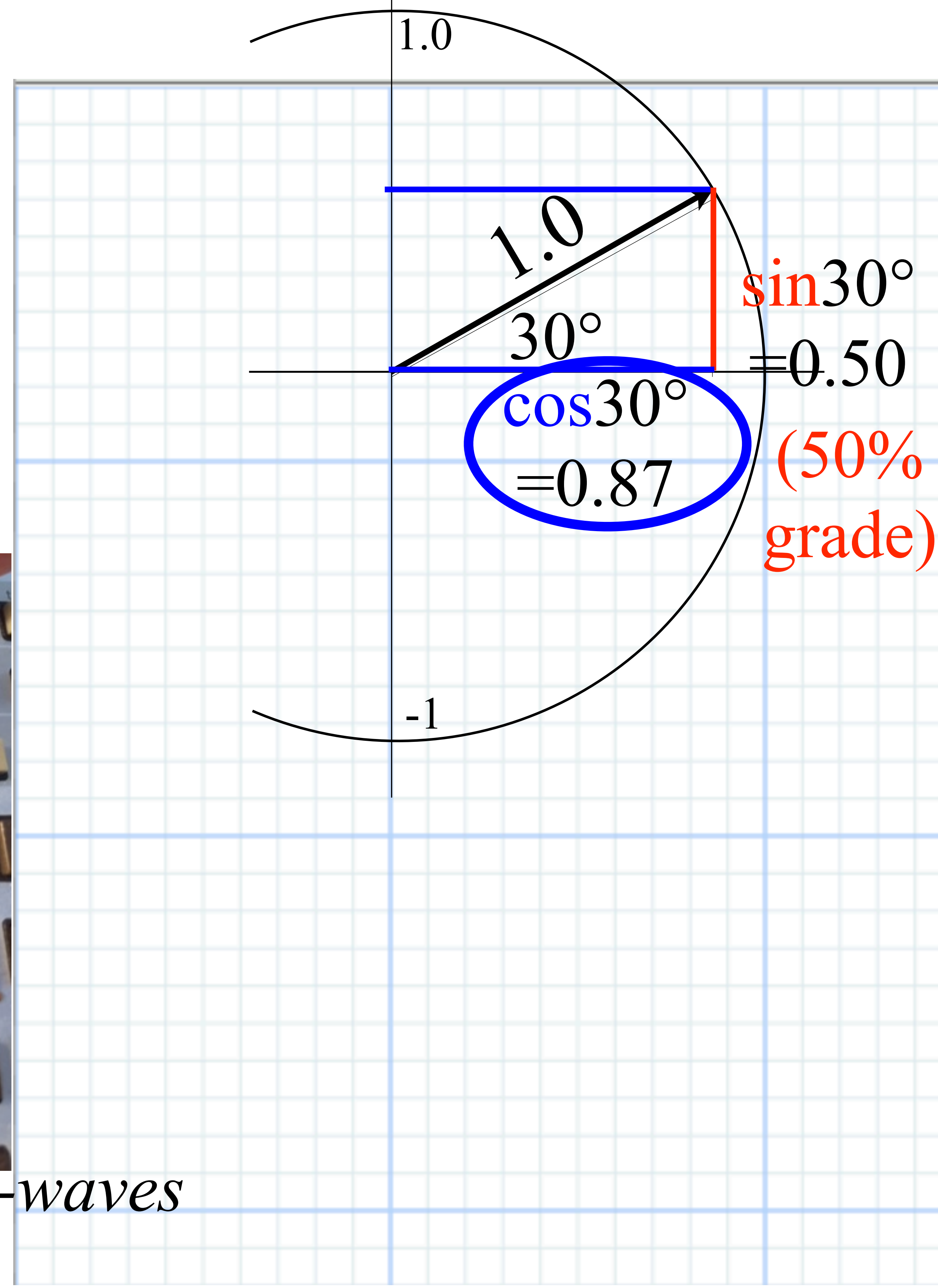
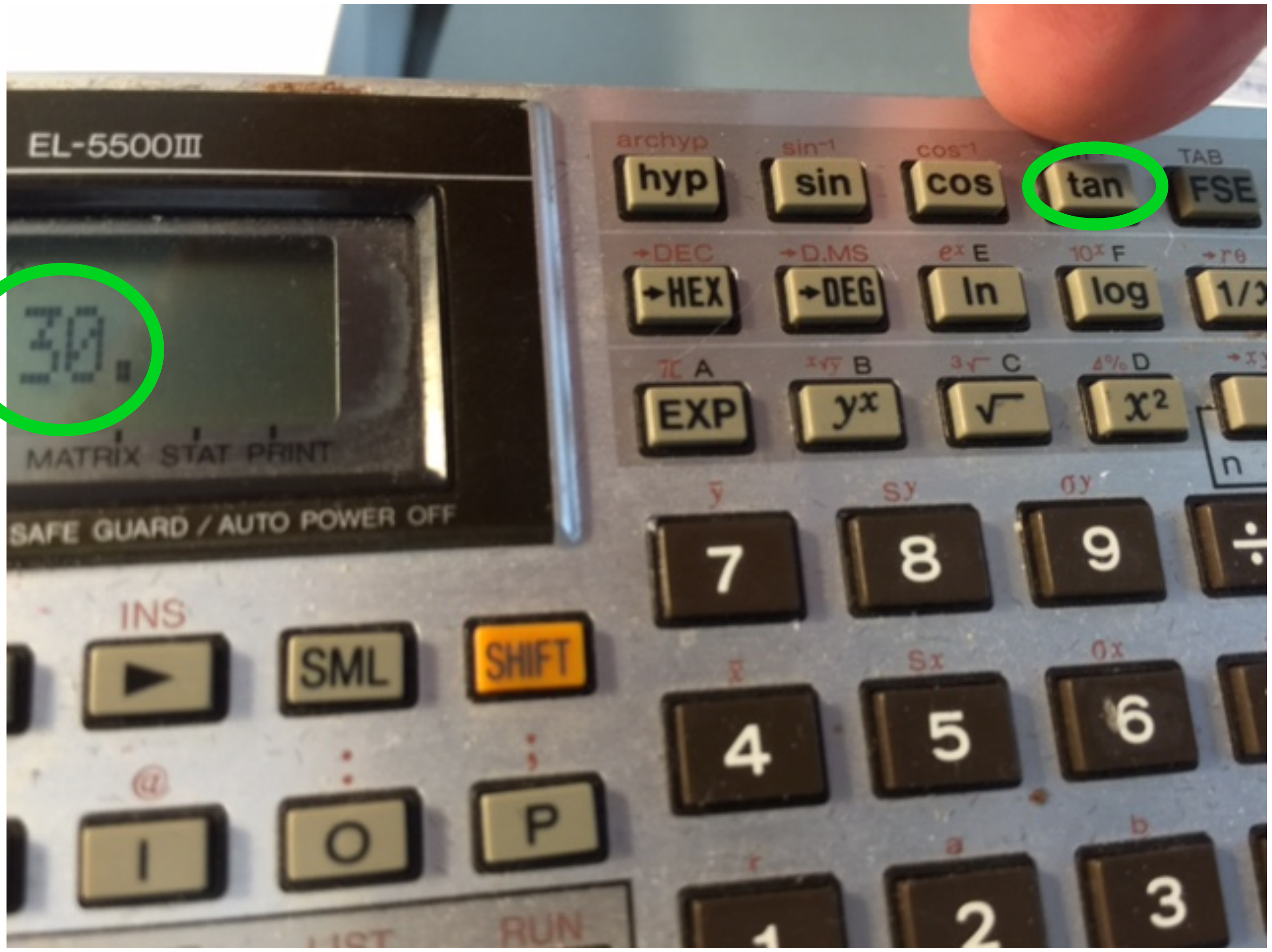


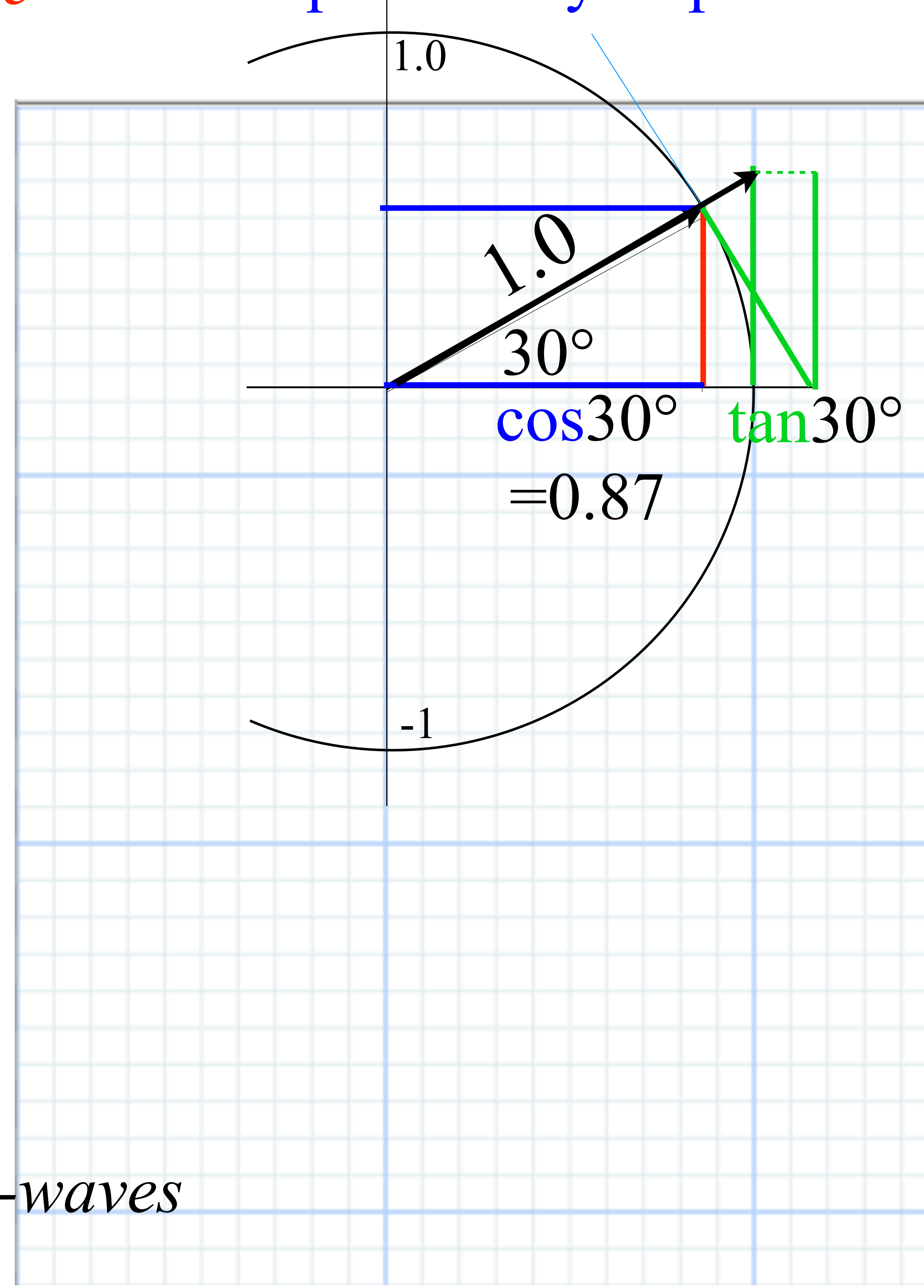


It's mostly about triangles *and sine-waves*

Learning about SIN and the COSin and TANgent

"Slope of INcline" "COmplimentarySlope"





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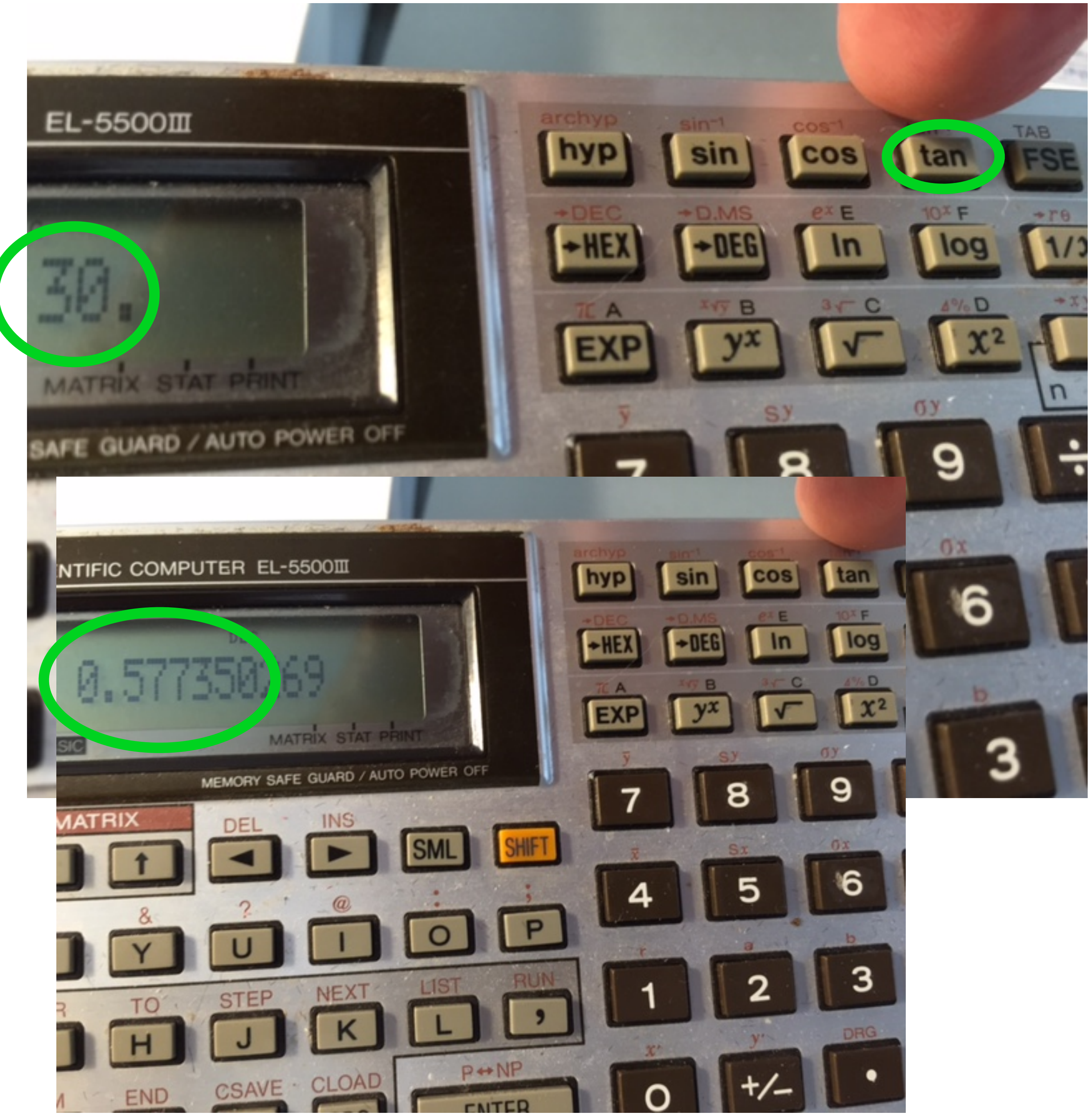
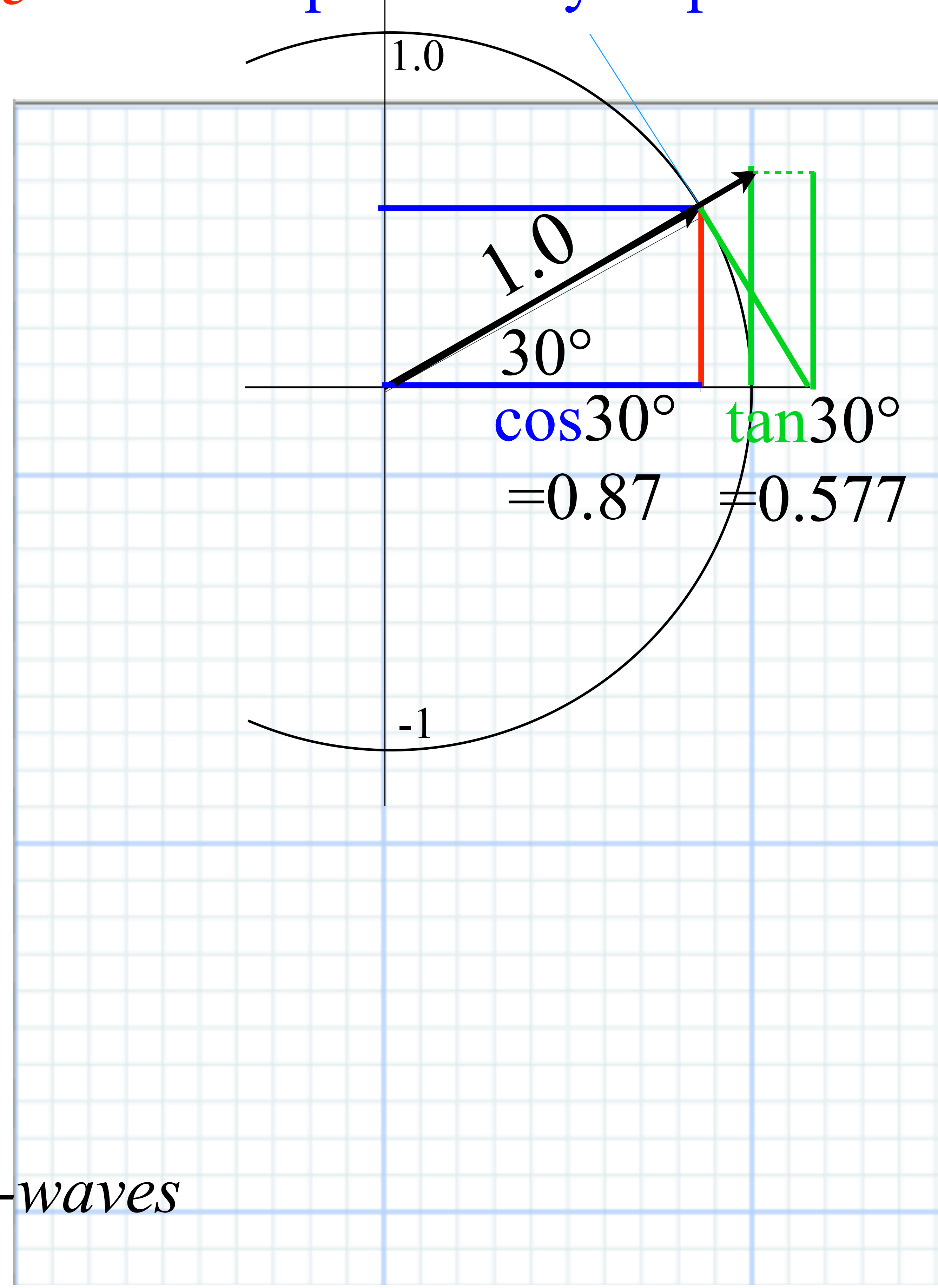
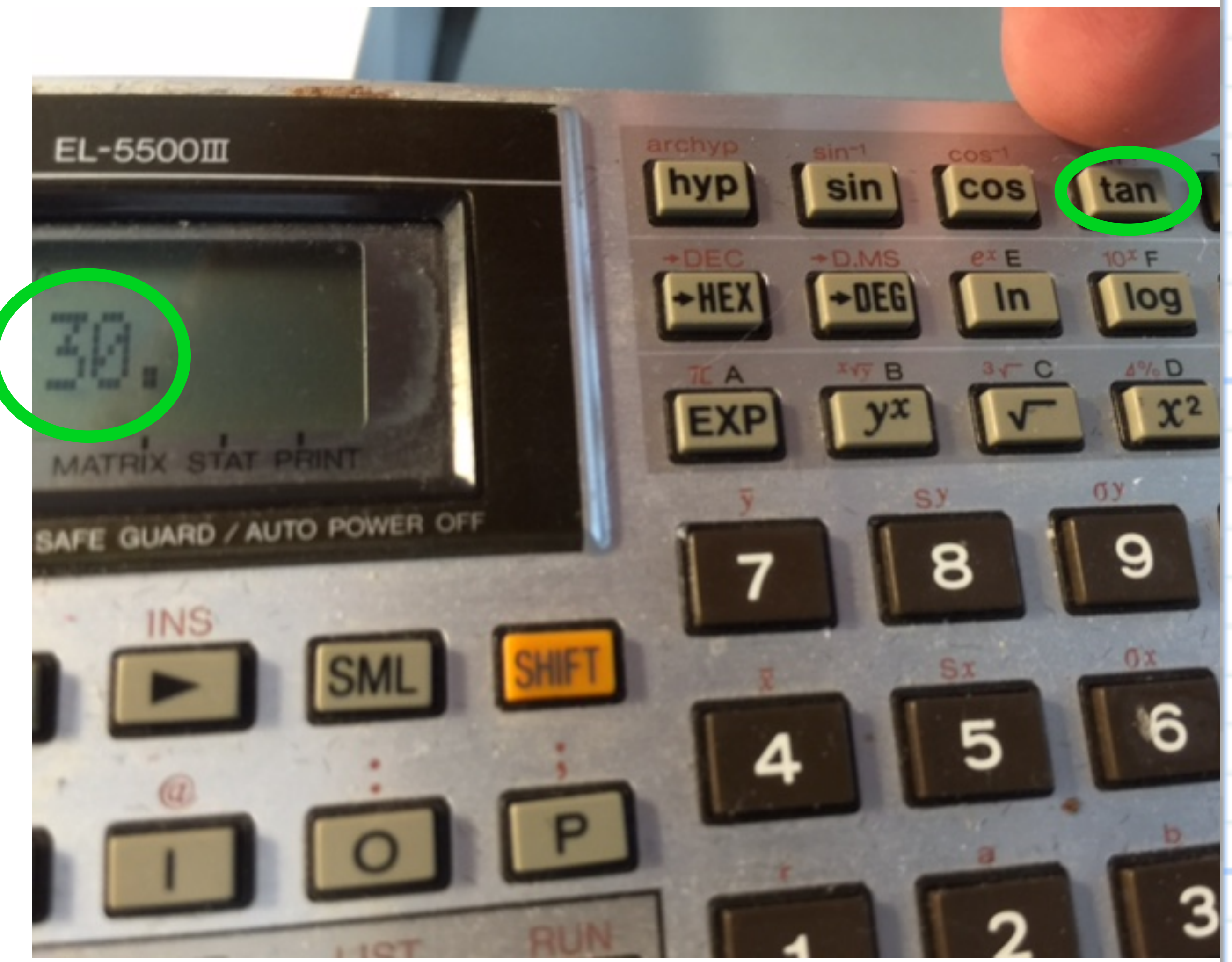


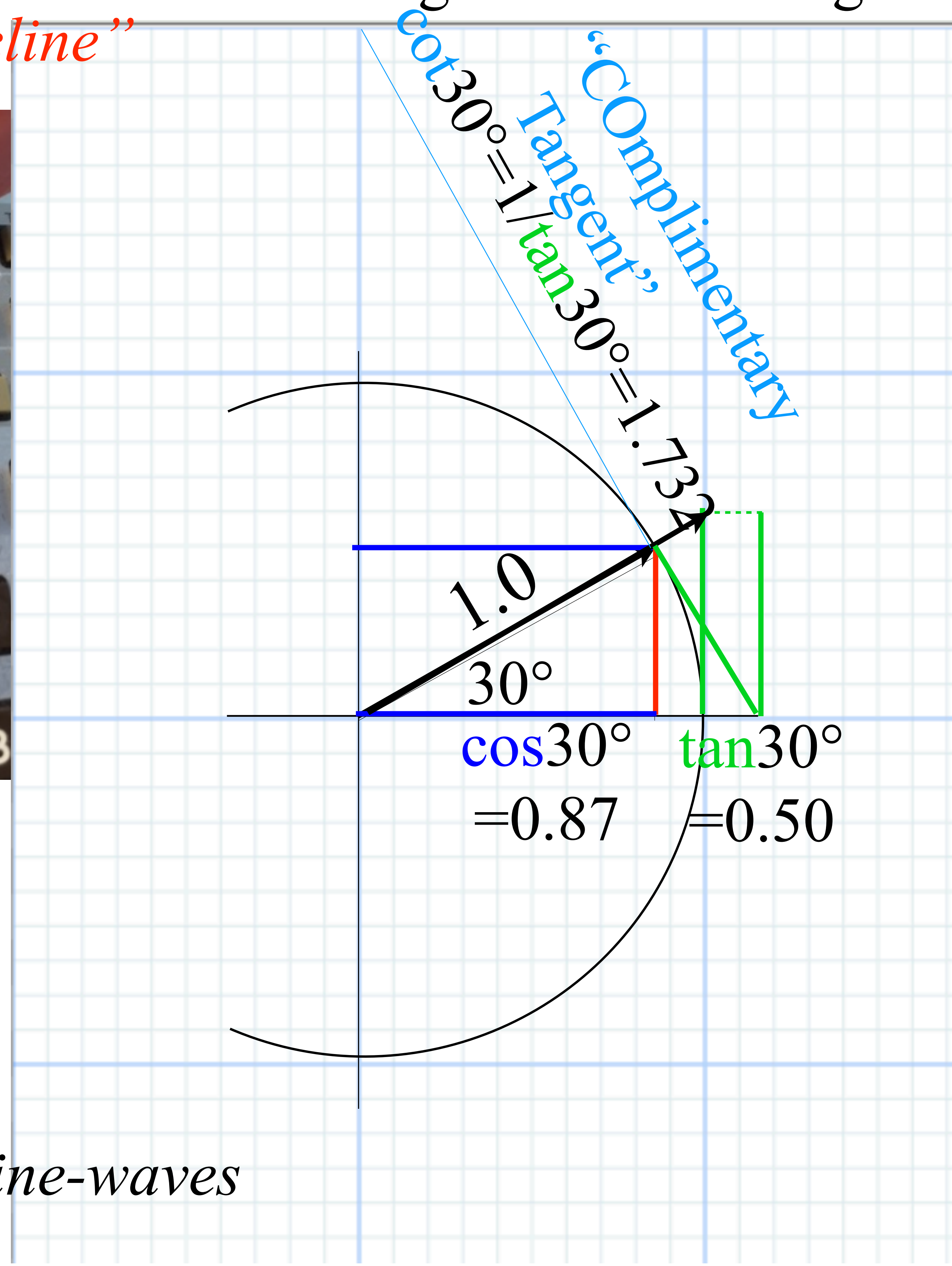


It's mostly about triangles *and sine-waves*

Learning about SIN and the COSin and TANgent and COTangent

“Slope of INcline”





It’s mostly about triangles *and sine-waves*

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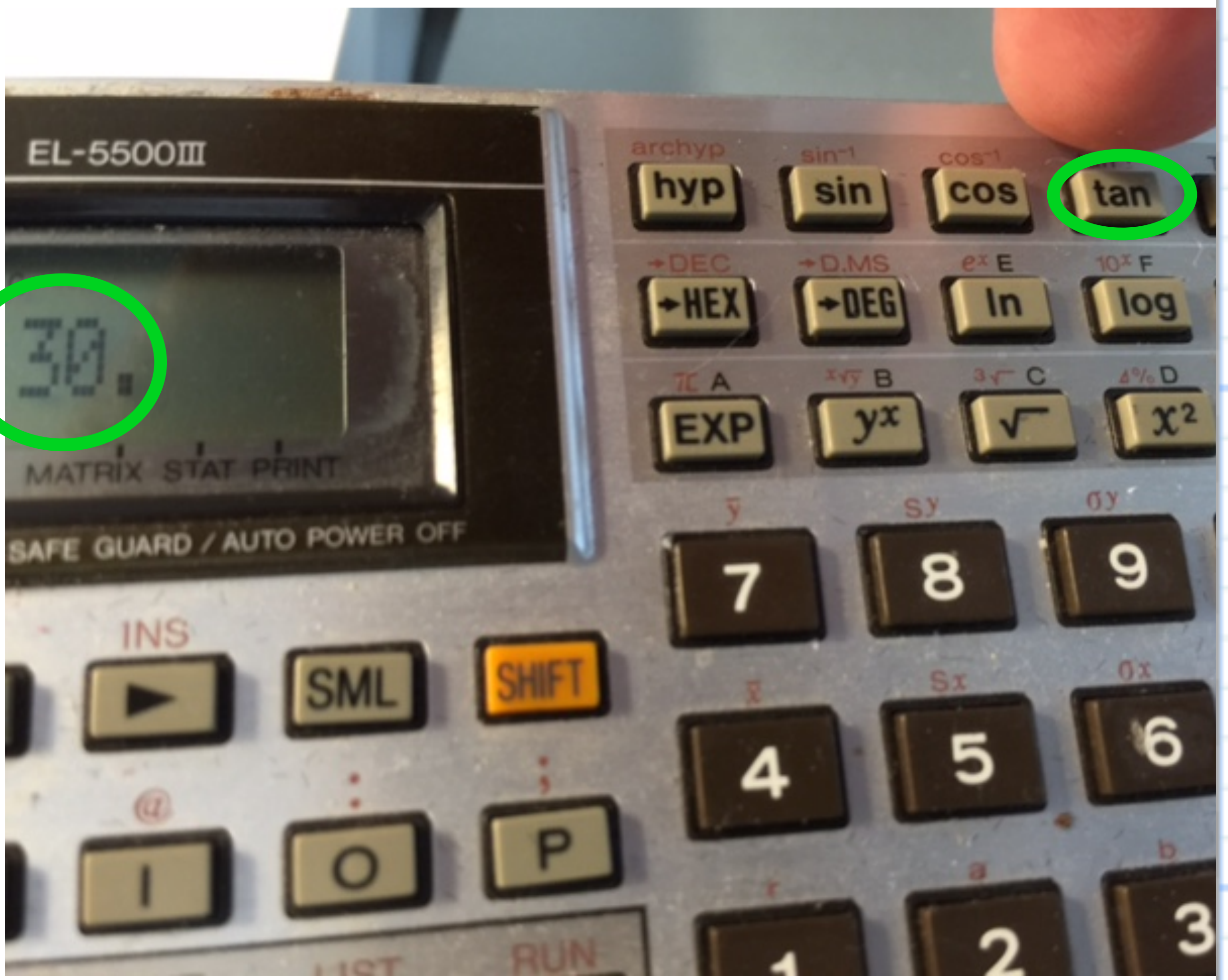

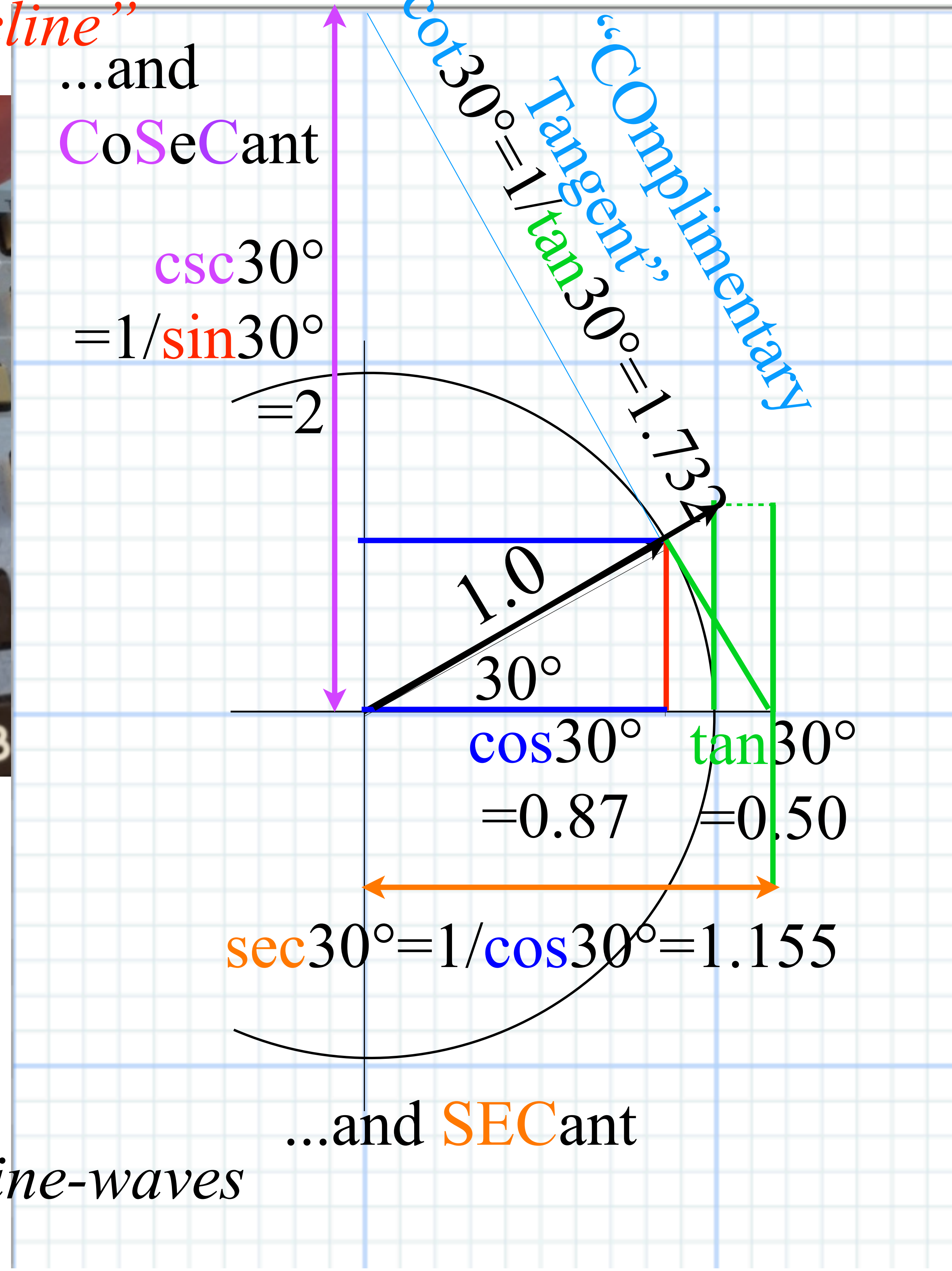

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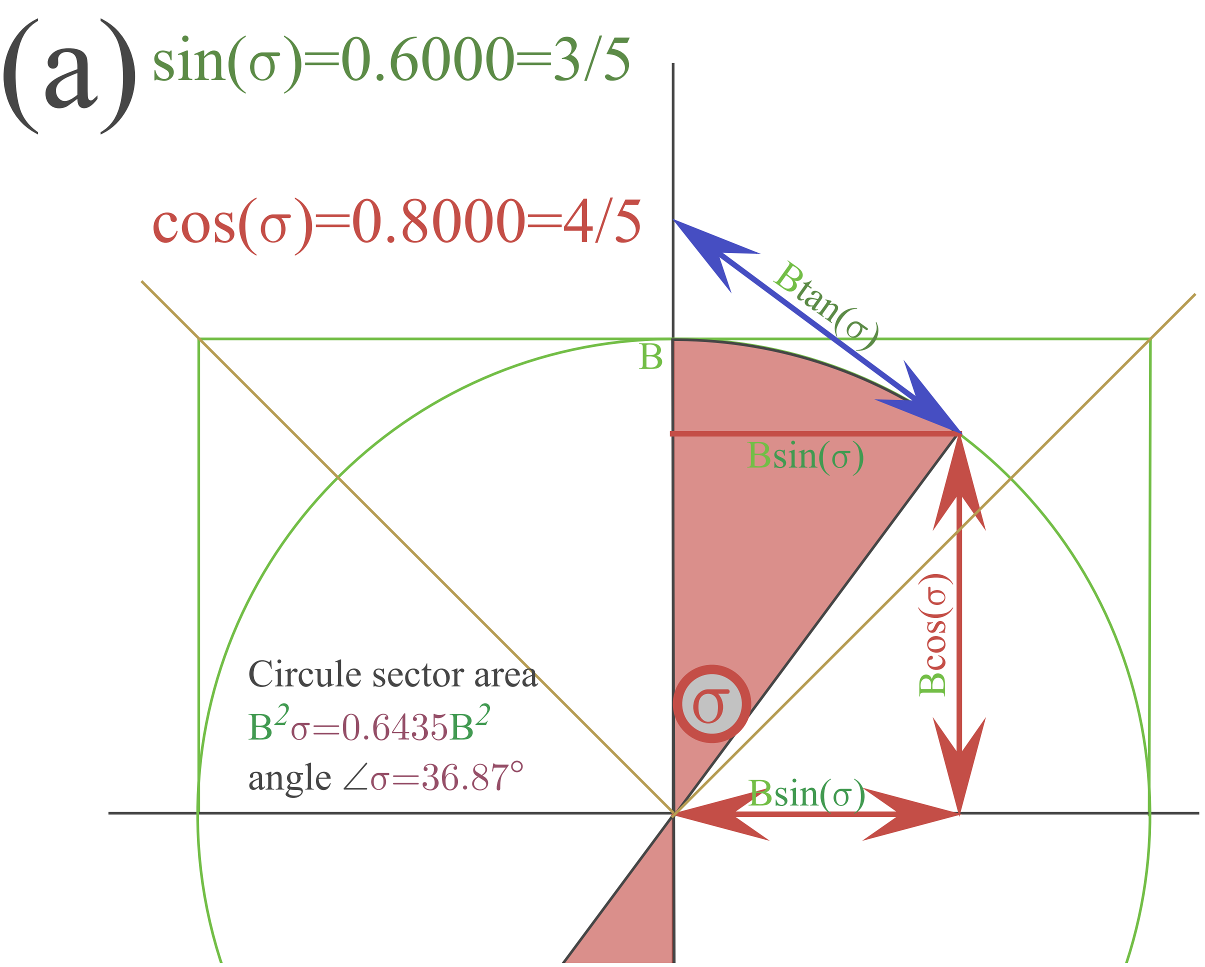
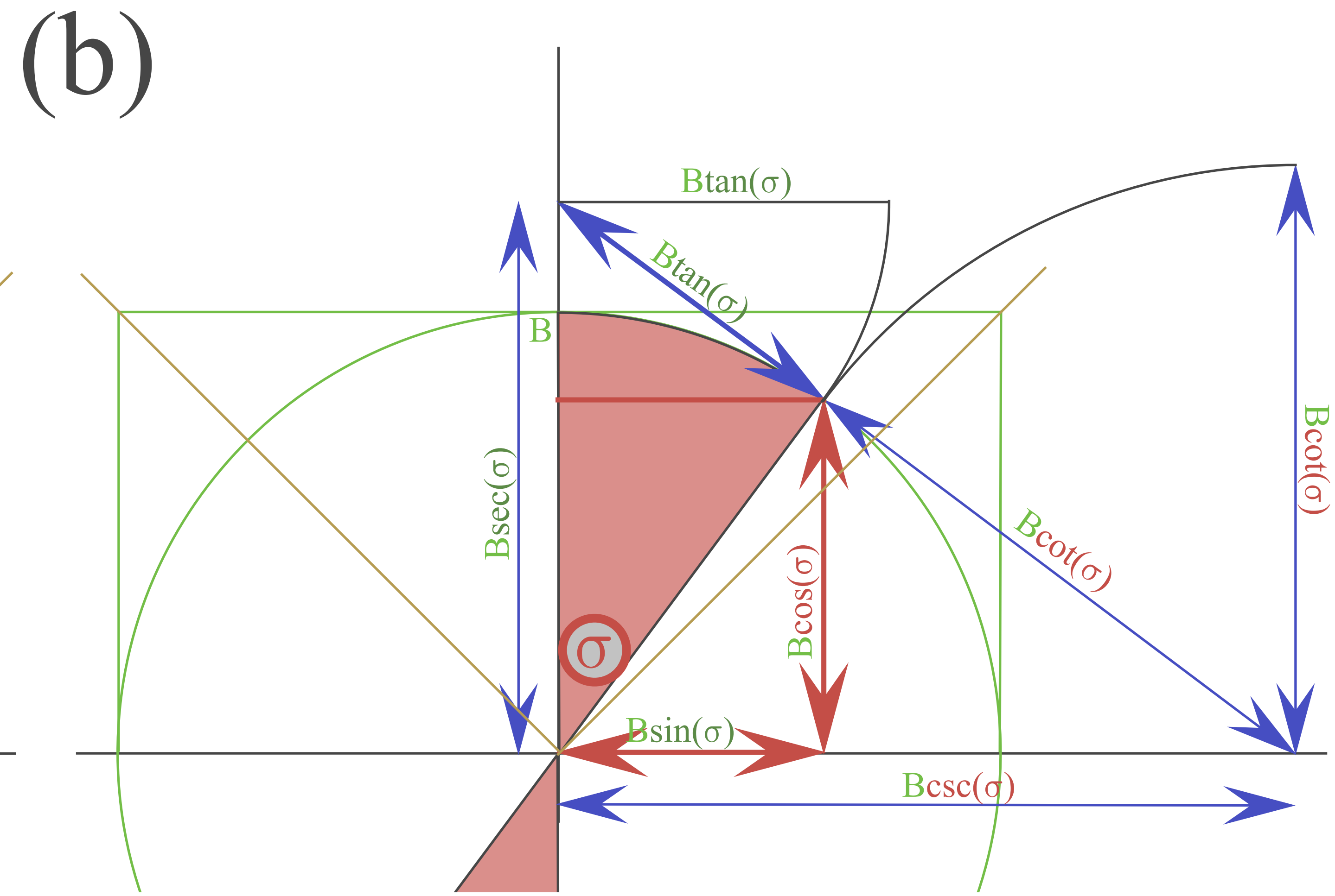
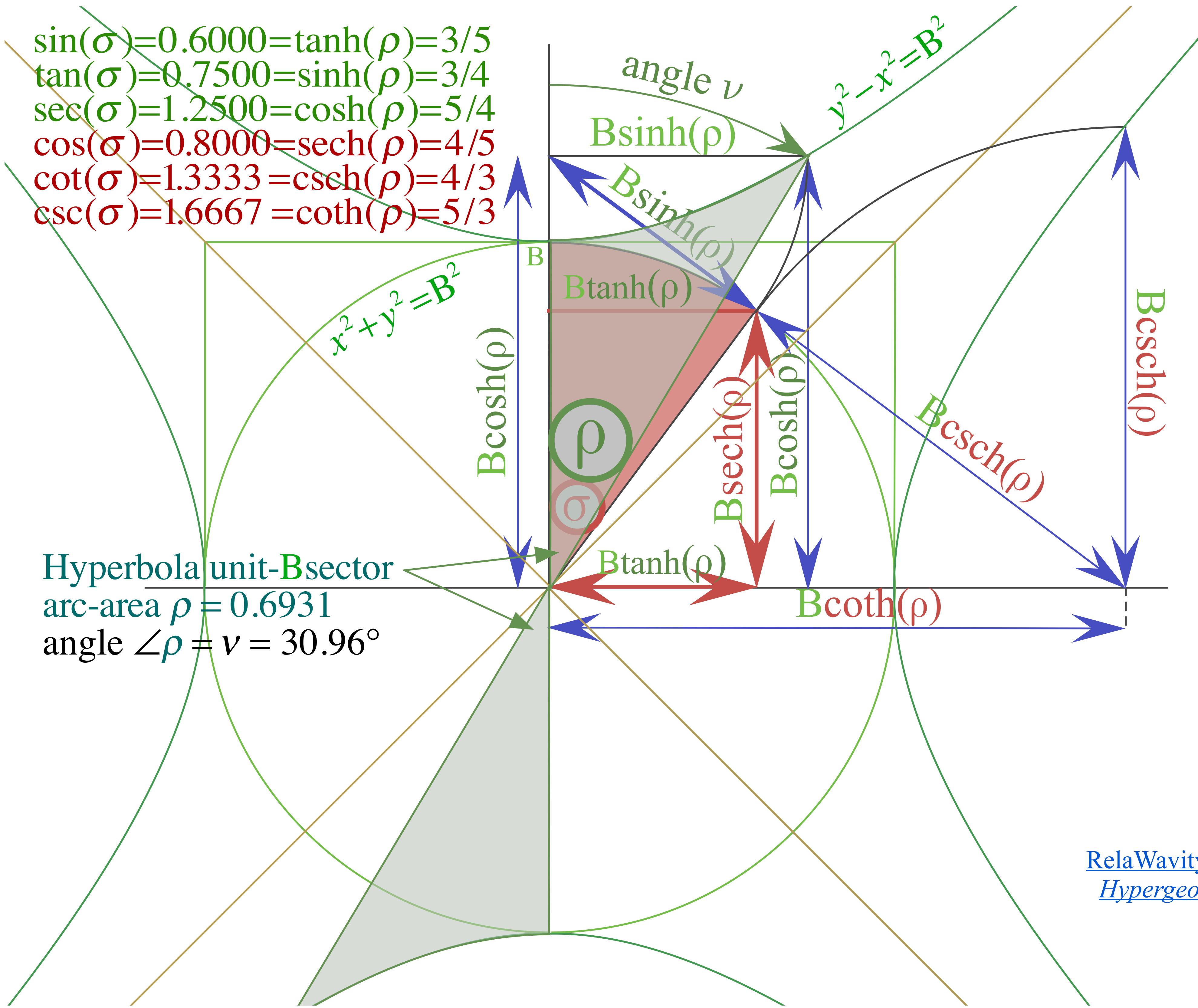
Trigonometric road maps





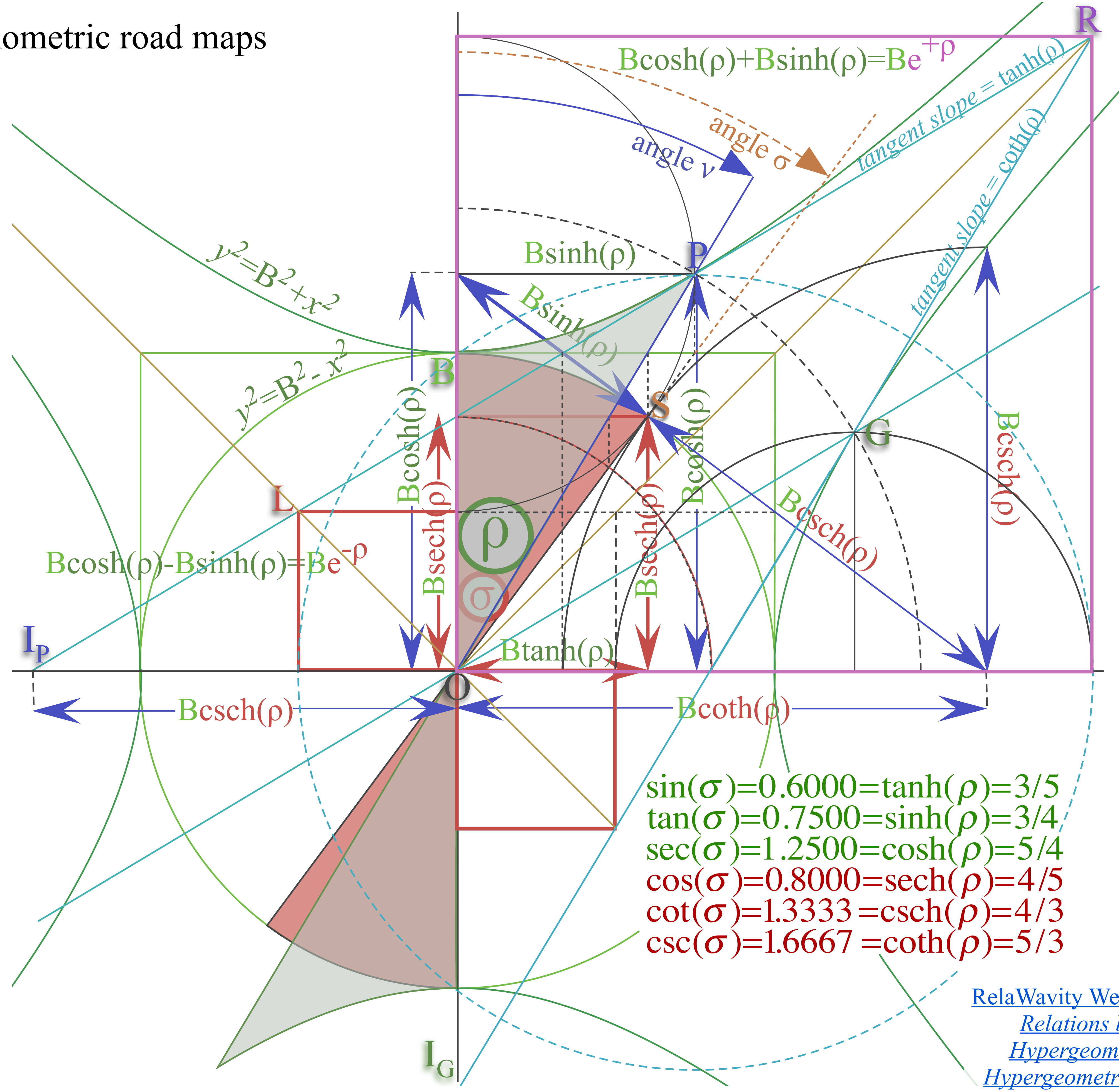
RelaWavity Web Simulation
Trigonometric functions

Hyper-Trigonometric road maps



[RelaWavity Web Simulation](#)
[*Hypergeometric functions*](#)

Hyper-Trigonometric road maps



$\sin(\sigma)=0.6000=\tanh(\rho)=3/5$
$\tan(\sigma)=0.7500=\sinh(\rho)=3/4$
$\sec(\sigma)=1.2500=\cosh(\rho)=5/4$
$\cos(\sigma)=0.8000=\mathrm{sech}(\rho)=4/5$
$\cot(\sigma)=1.3333=\mathrm{csch}(\rho)=4/3$
$\csc(\sigma)=1.6667=\coth(\rho)=5/3$

RelaWavity Web Simulation
Relations between Hypergeometric and Hypergeometric functions

Learning about sin! *and* cos *and…Trigonometric road maps*

Hyper-Trigonometric algebra and phasors in space-time

1CW wavefunctions and phasors

Per-space-per-time vs Space-time

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Application to TE-Waveguide modes

The calculus of Fig. 3 geometry uses an infinite compounding limit of the interest rate-r formula.

$$e^{rt} = \lim_{n\to\infty}\left(1+\frac{rt}{n}\right)^n \tag{3}$$

Infinite limit of binomial expansion is an exponential power series with $1/n!$ coefficients.

$$e^{rt} = 1 + rt + \frac{(rt)^2}{2} + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \ldots \tag{4a}$$

$$e^{-rt} = 1 - rt + \frac{(rt)^2}{2} - \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots \tag{4b}$$

Half-sum and half difference of $e^{\pm rt}$ series define the hyperbolic cosine (cosh(rt)) and sine (sinh(rt)).

$$\frac{e^{+rt}+e^{-rt}}{2} = 1 + \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt) \tag{5a}$$

$$\frac{e^{+rt}-e^{-rt}}{2} = rt + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \ldots = \sinh(rt) \tag{5b}$$

Sum and difference gives equation (1) and (2) consistent with figures 2 and 3. Replacing rate r with imaginary rate ir where $i=\sqrt{-1}$ gives powers $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, $i^5=i$, $i^6=-1$, $i^7=-i$,… that repeat sequence-($1,i,-1,-i$) every 4^{th}-power. Then hyper-sine-cosine becomes the circular-sine-cosine.

$$\frac{e^{+i\,rt}+e^{-i\,rt}}{2} = 1 - \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} - \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cos rt \tag{6a}$$

$$\frac{e^{+i\,rt}-e^{-i\,rt}}{2} = i\,rt - i\frac{(rt)^3}{2\cdot 3} + i\frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} - \ldots = i\sin rt \tag{6b}$$

Sum and difference of this pair gives the Euler-DeMoivre formulae.

$$e^{+i\sigma} = \cos(\sigma) + i\sin(\sigma), \qquad e^{-i\sigma} = \cos(\sigma) - i\sin(\sigma). \tag{7}$$

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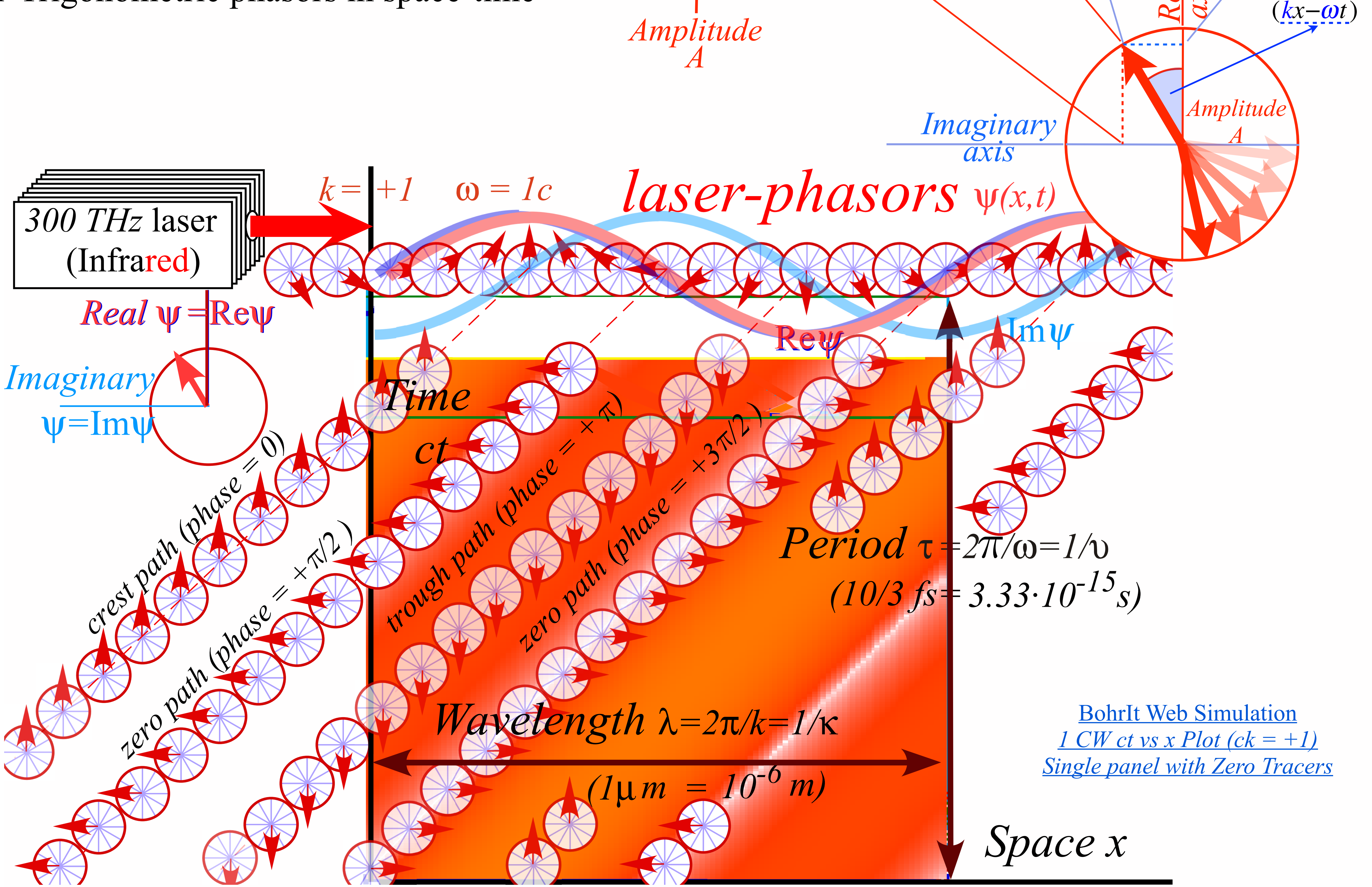
1CW Laser-phasor wave function

$$\psi = A\cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

Hyper-Trigonometric phasors in space-time



BohrIt Web Simulation
1 CW ct vs x Plot (ck = +1)
Single panel with Zero Tracers

Fig. 4(a) Single-phasor plot of wave-function at (*x*,*ct*). (b) Array of phasors at many (*x*,*ct*)-points.

1CW Laser-phasor wave function

$$\psi = A \cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$

Dimensionless Light wave-velocity c/c=1

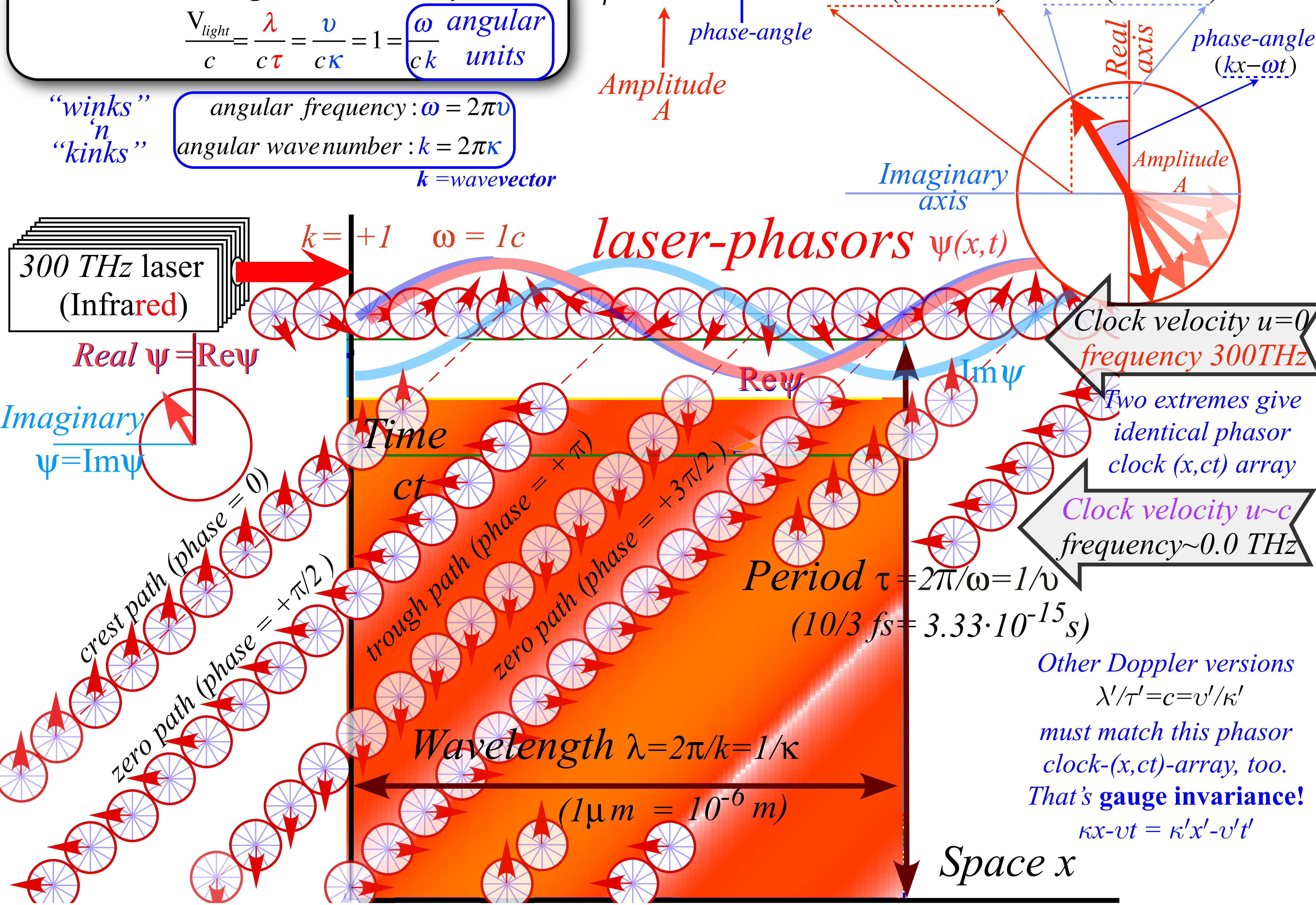
$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = 1 = \frac{\omega}{ck}$$ angular units

"winks" 'n "kinks"

angular frequency : $\omega = 2\pi\upsilon$

angular wavenumber : $k = 2\pi\kappa$

k =wave**vector**



Other Doppler versions

$\lambda'/\tau' = c = \upsilon'/\kappa'$

must match this phasor clock-(x,ct)-array, too.

That's **gauge invariance!**

$\kappa x - \upsilon t = \kappa' x' - \upsilon' t'$

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The "Keyboard of the gods" : per-space-per-time plot *versus* space-time Minkowski plot

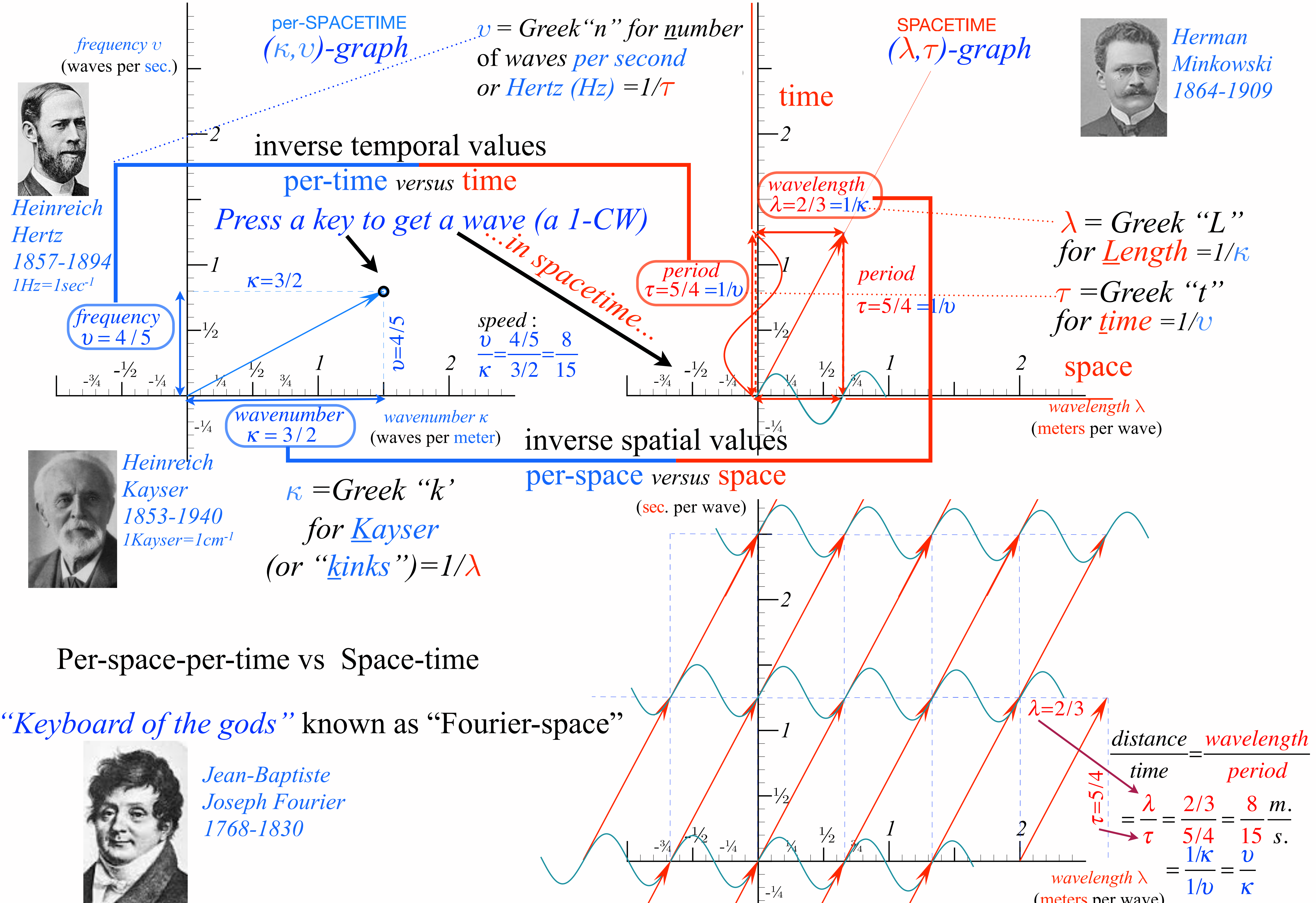
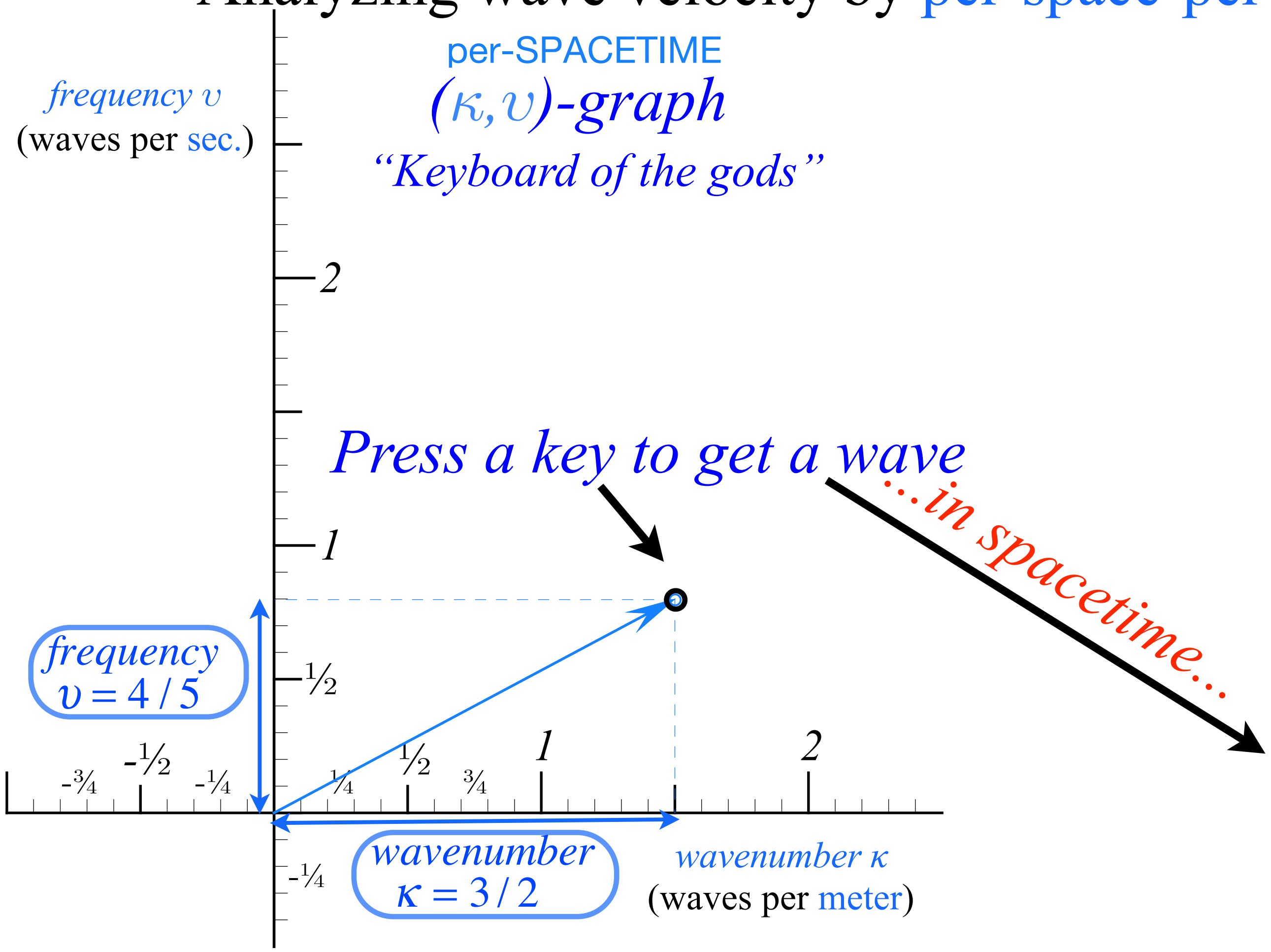
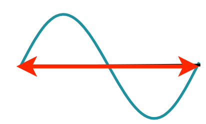


Fig. 5 Comparing a wave point in Kaiser-Hertz per-space-time to its Minkowski space-time view.

Analyzing wave velocity by per-space-per-time *and* space-time graphs



SPACETIME
(λ,τ)-graph



"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

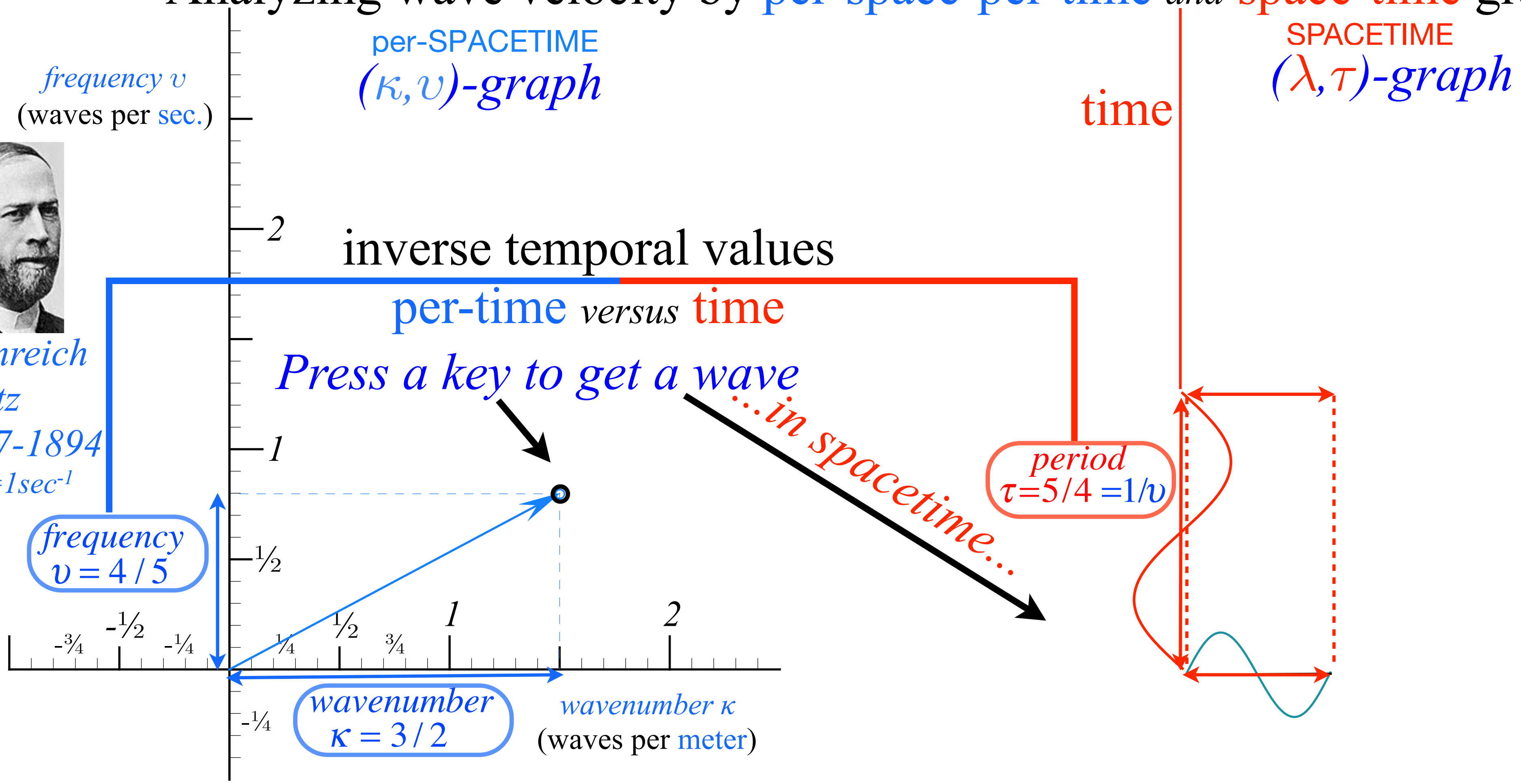
•How to understand waves
and
wave velocity V_{wave}

RelaWavity Web Simulation
Keyboard of the Gods
(per-Time *vs* per-Space)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

Heinreich Hertz 1857-1894

1Hz=1sec^{-1}



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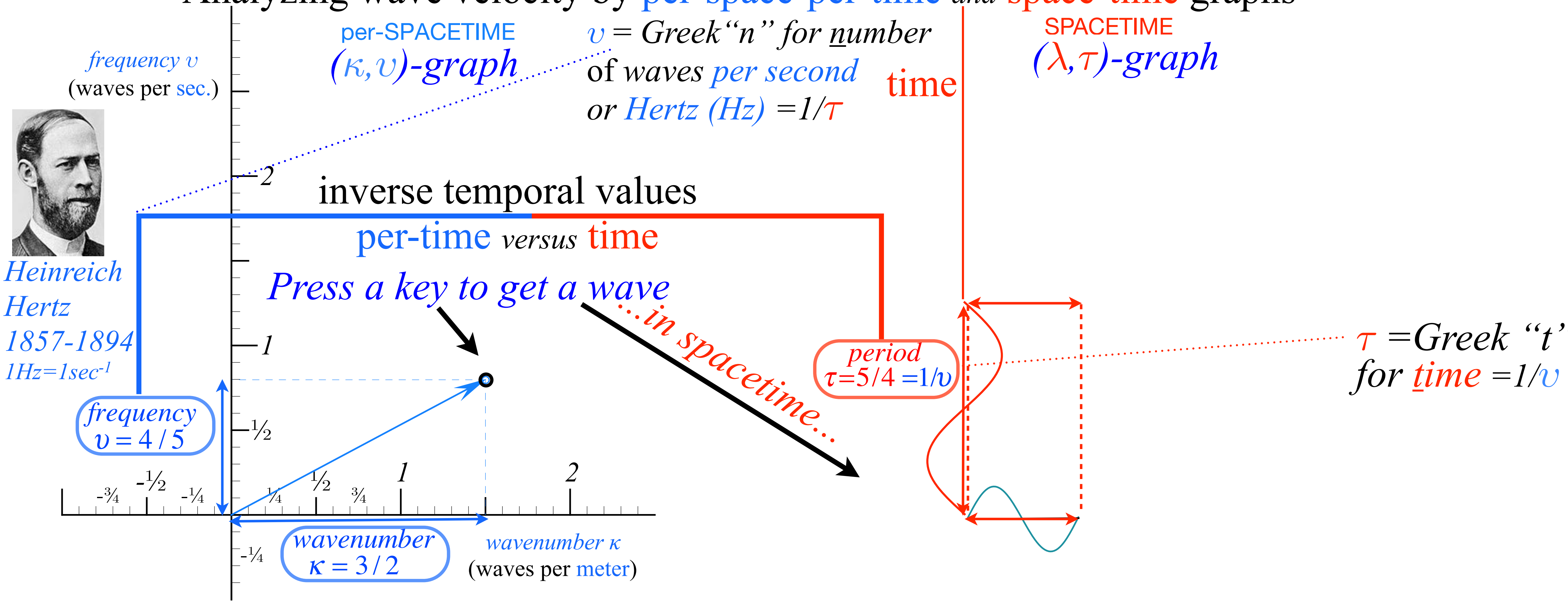
Analyzing wave velocity by per-space-per-time *and* space-time graphs



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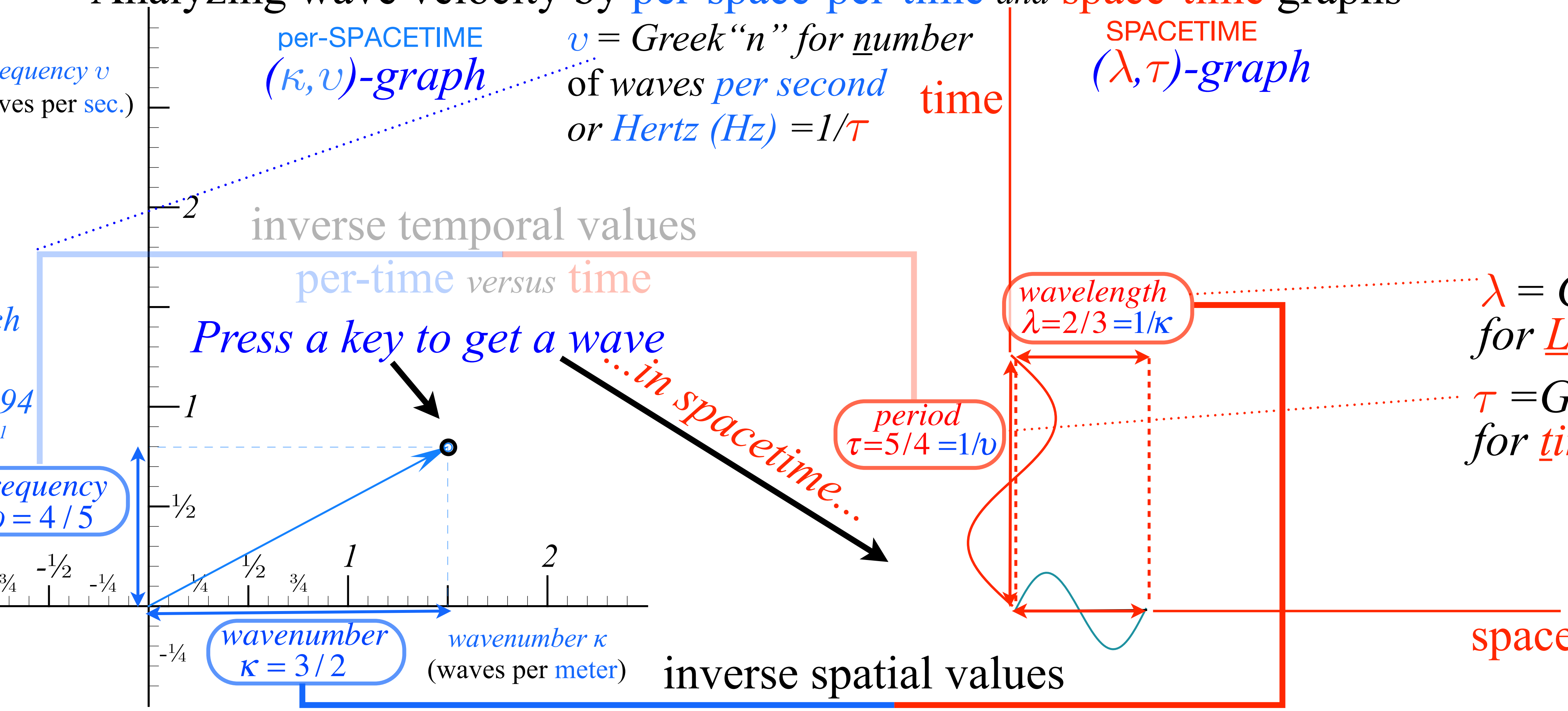
Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

[RelaWavity Web Simulation *Keyboard of the Gods* (Dual Plot)]

Analyzing wave velocity by per-space-per-time *and* space-time graphs



υ = Greek "n" for number of *waves per second* *or Hertz (Hz) =1/τ*

Heinreich Hertz 1857-1894

1Hz=1sec⁻¹

λ = Greek "L" for Length =1/κ

τ =Greek "t" for time =1/υ

κ =Greek "k" for Kayser (or "kinks")=1/λ

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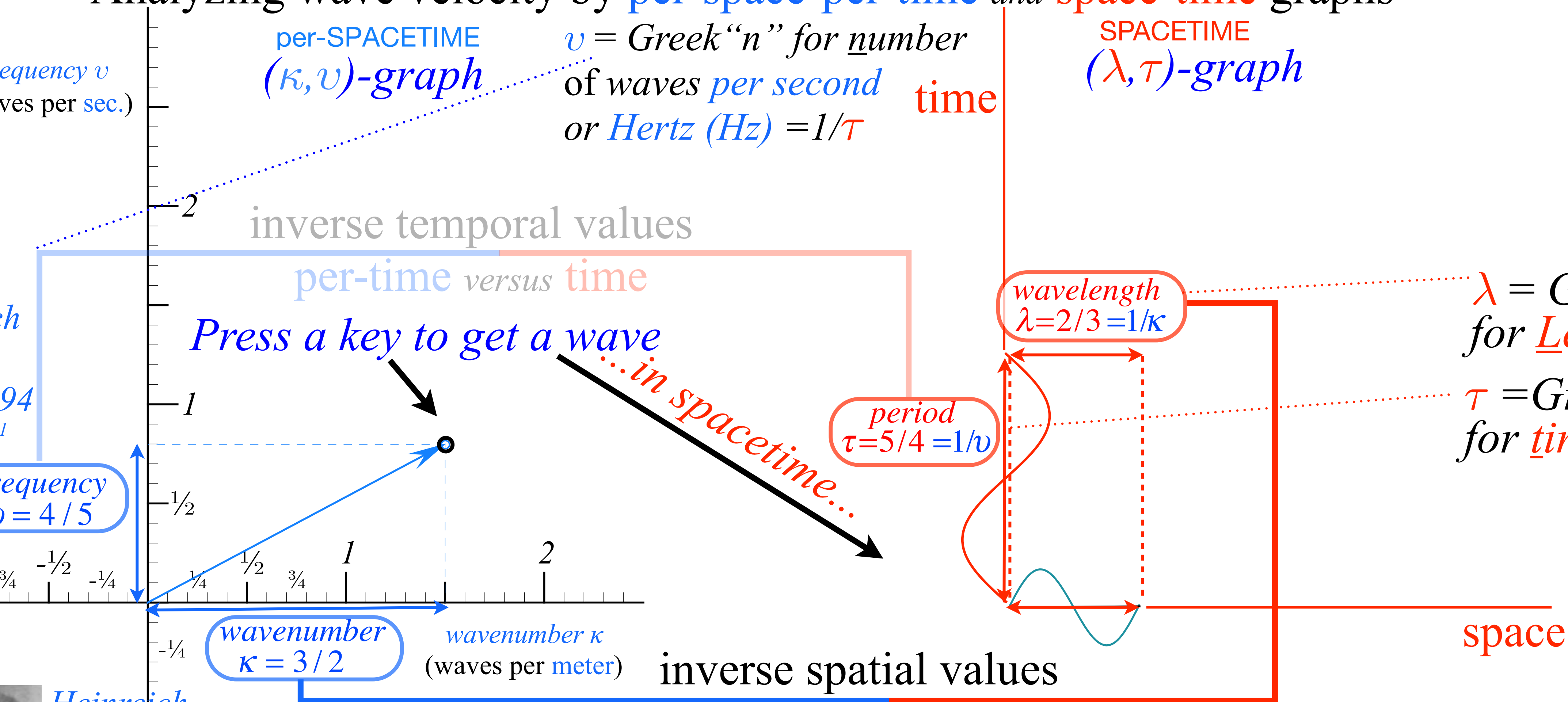
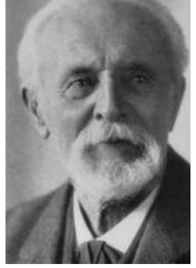
Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

Analyzing wave velocity by per-space-per-time *and* space-time graphs



Heinreich Hertz 1857-1894
1Hz=1sec⁻¹



Heinreich Kayser 1853-1940
1Kayser=1cm⁻¹

κ =Greek "k" for Kayser (or "kinks")=1/λ

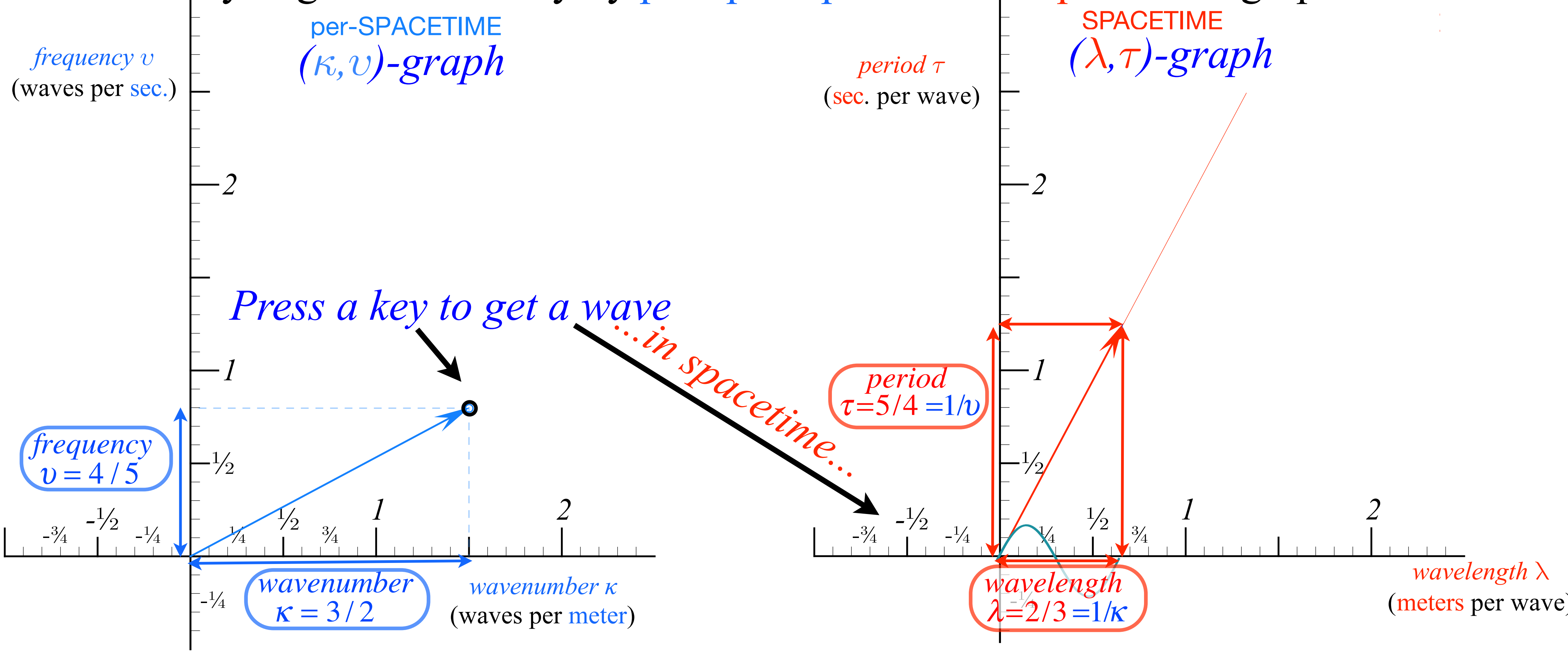
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

•How to understand waves and wave velocity V_{wave}

RelaWavity Web Simulation
Keyboard of the Gods
(Dual Plot 1)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



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Jean-Baptiste
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•*How to understand waves*
and
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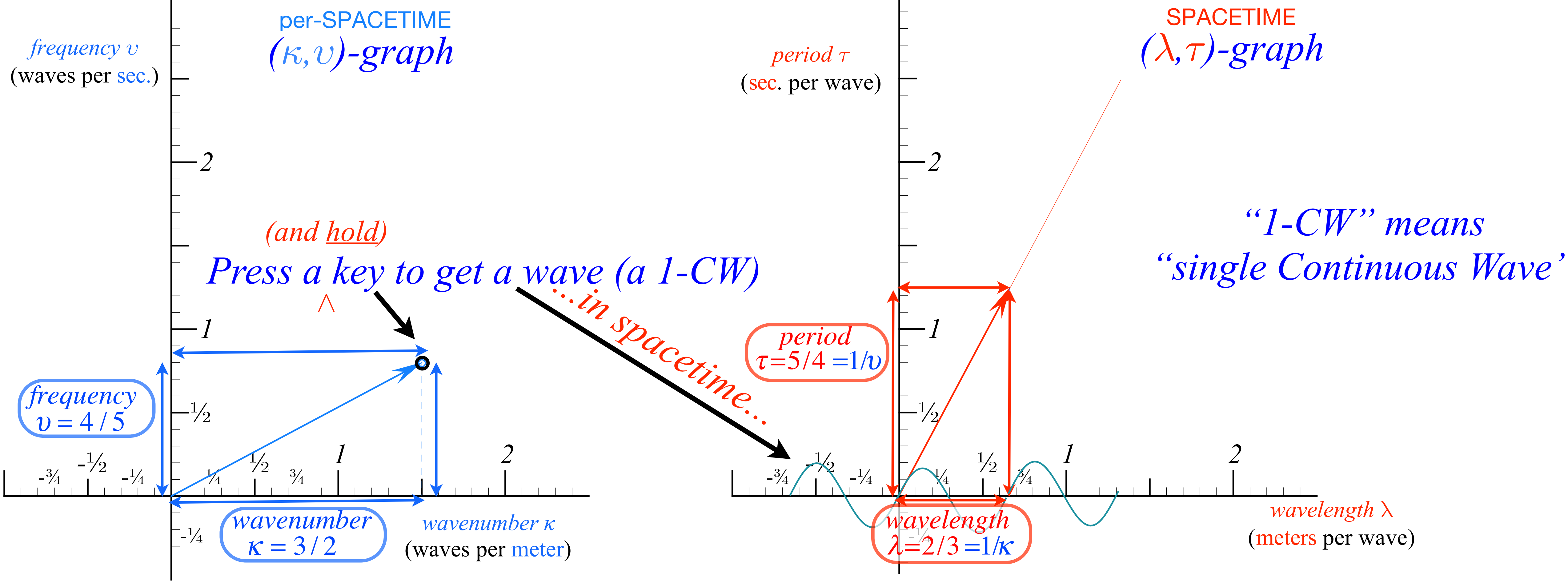
Analyzing wave velocity by per-space-per-time *and* space-time graphs



"*Keyboard of the gods*" is known as "Fourier-space"

Jean-Baptiste
Joseph Fourier
1768-1830

•How to understand waves
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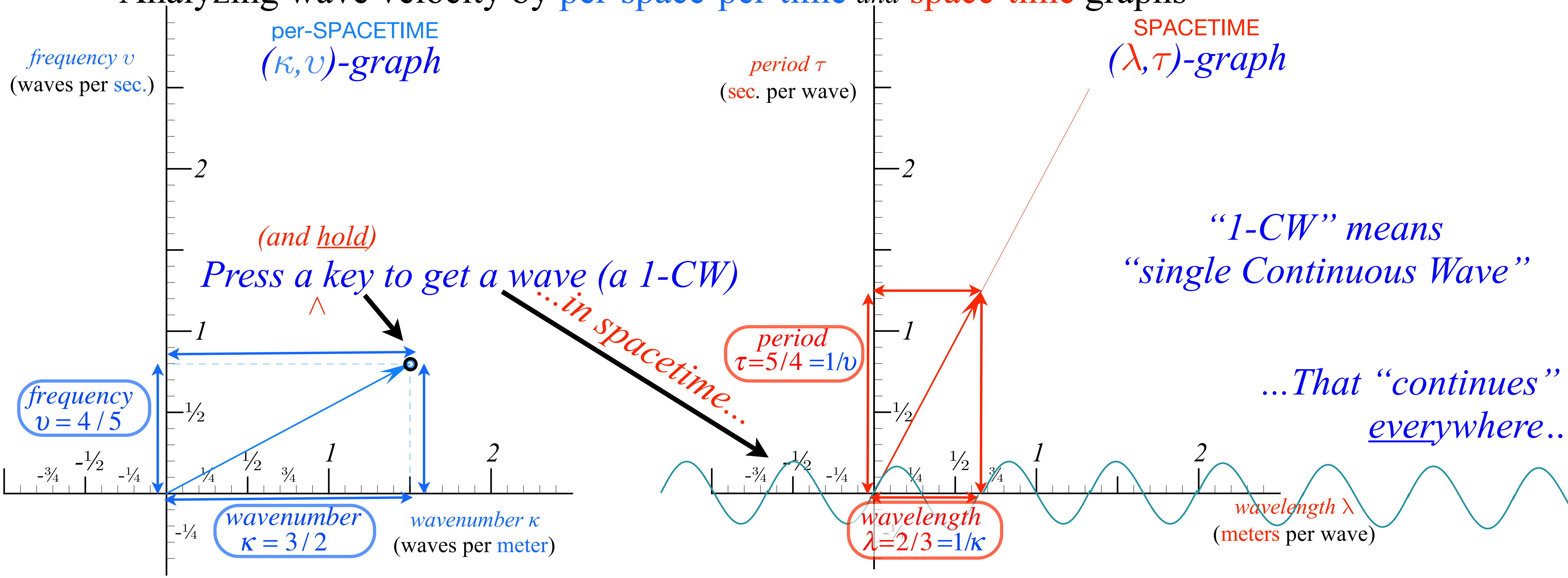
wave velocity V_{wave}

RelaWavity Web Simulation
Keyboard of the Gods
(Dual Plot 2)

Analyzing wave velocity by per-space-per-time *and* space-time graphs



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Jean-Baptiste
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•How to understand waves
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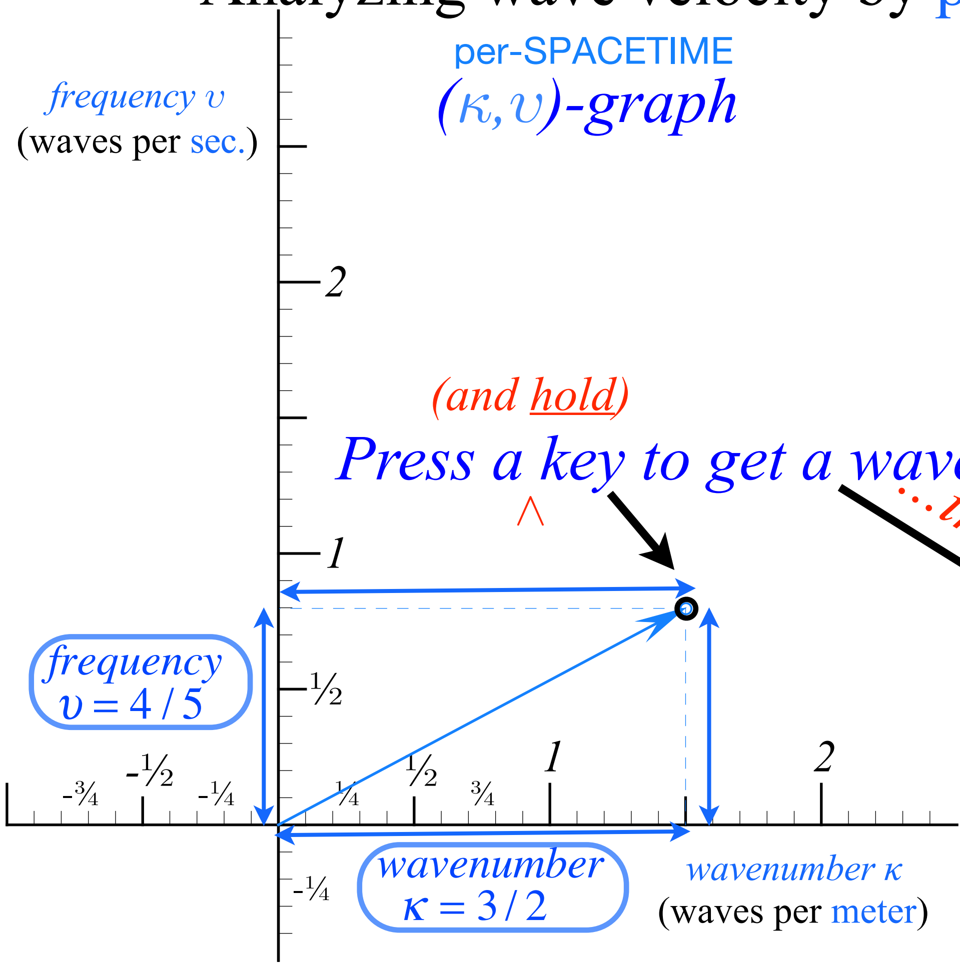
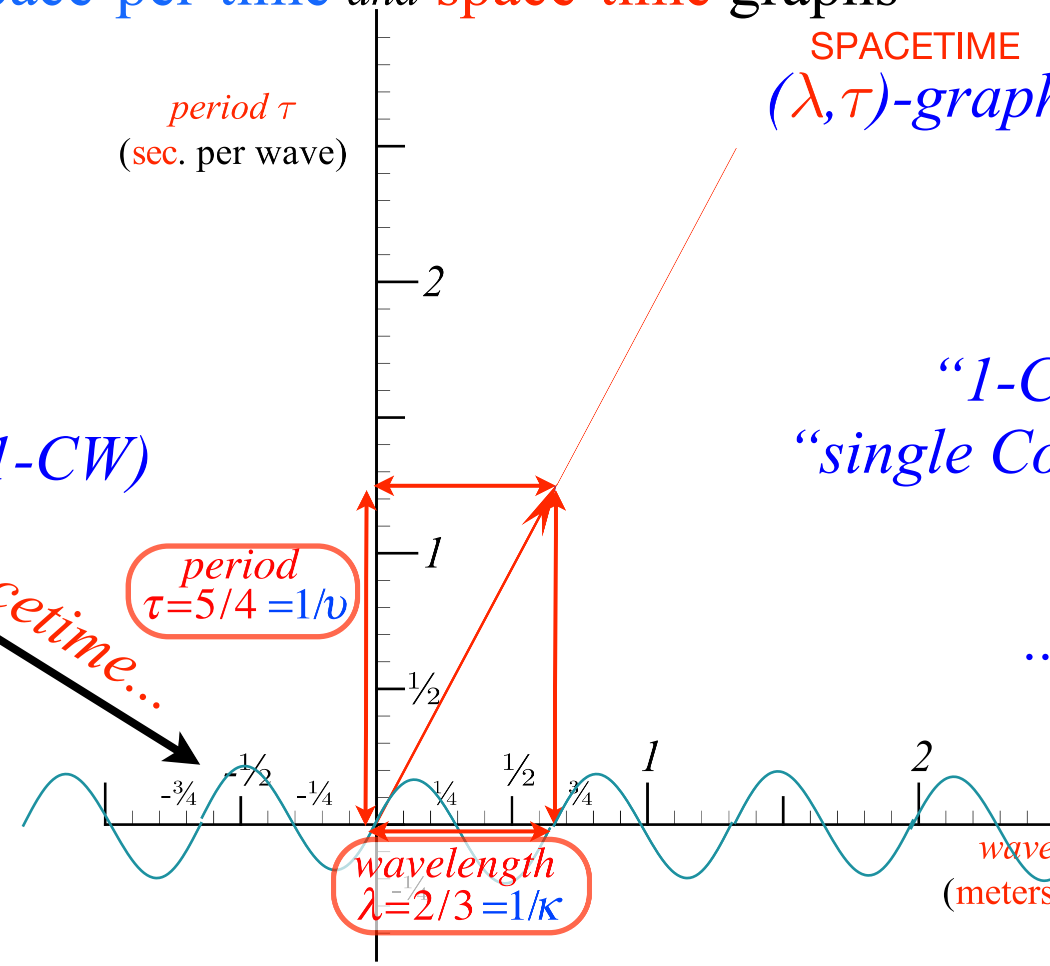
[RelaWavity Web Simulation](#)
[*Keyboard of the Gods*](#)
[(Dual Plot 3)](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs

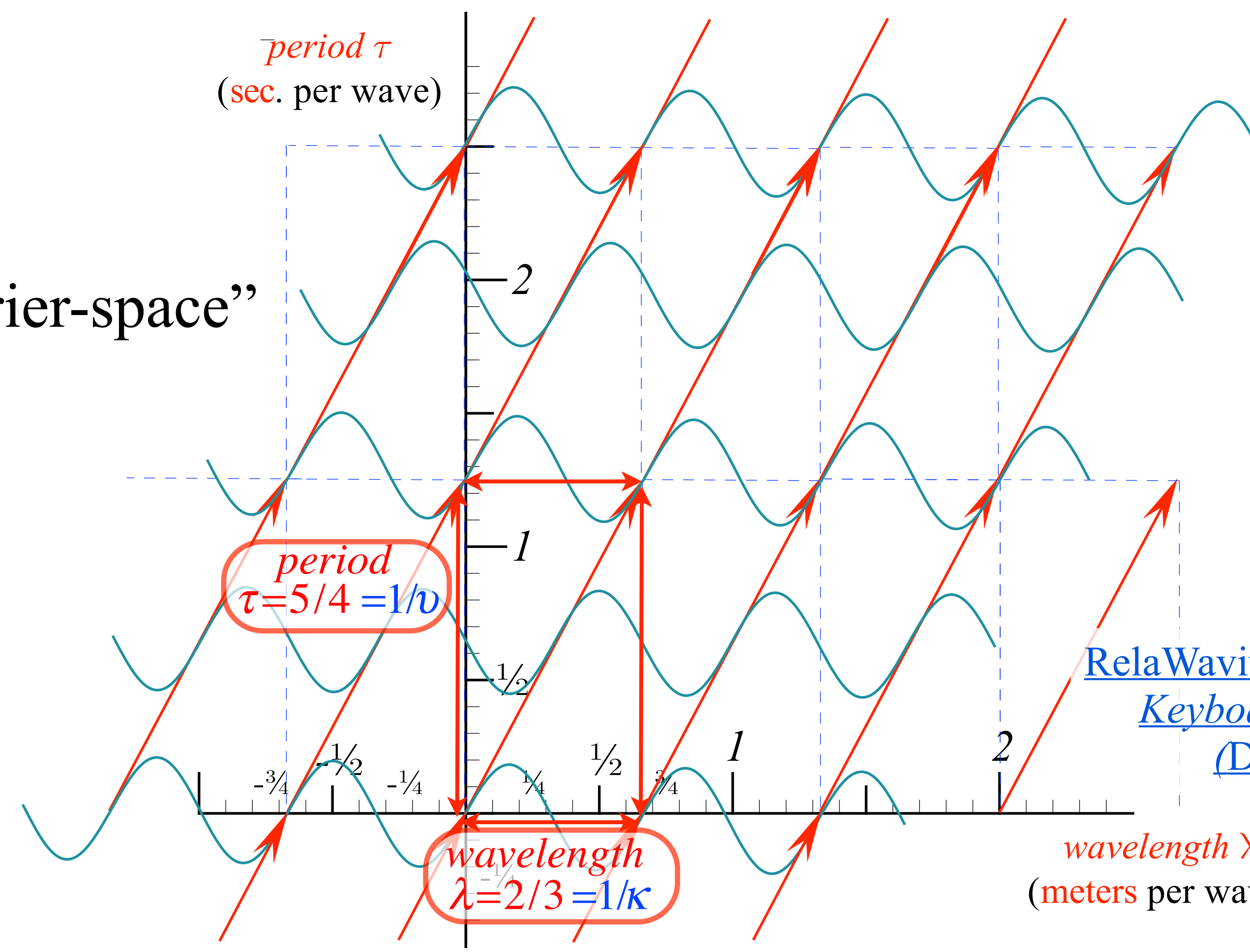




"1-CW" means "single Continuous Wave"

...That "continues" everywhere..

...for all time...



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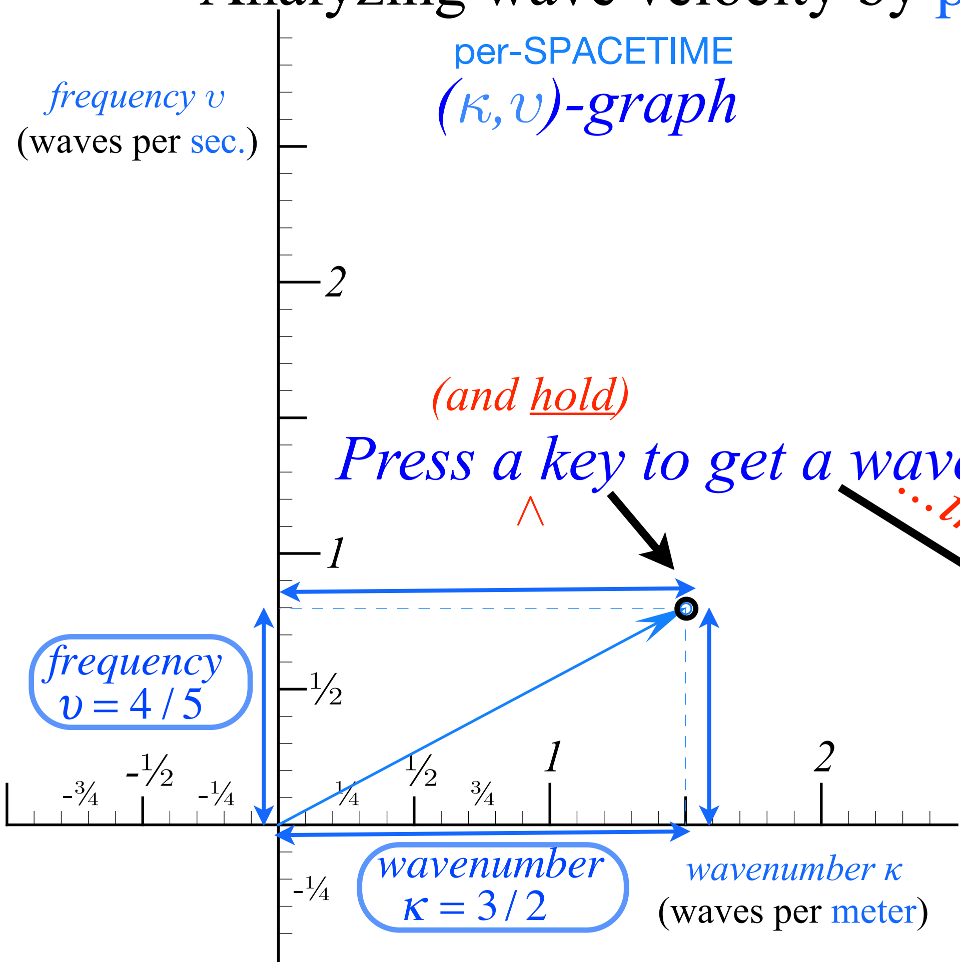
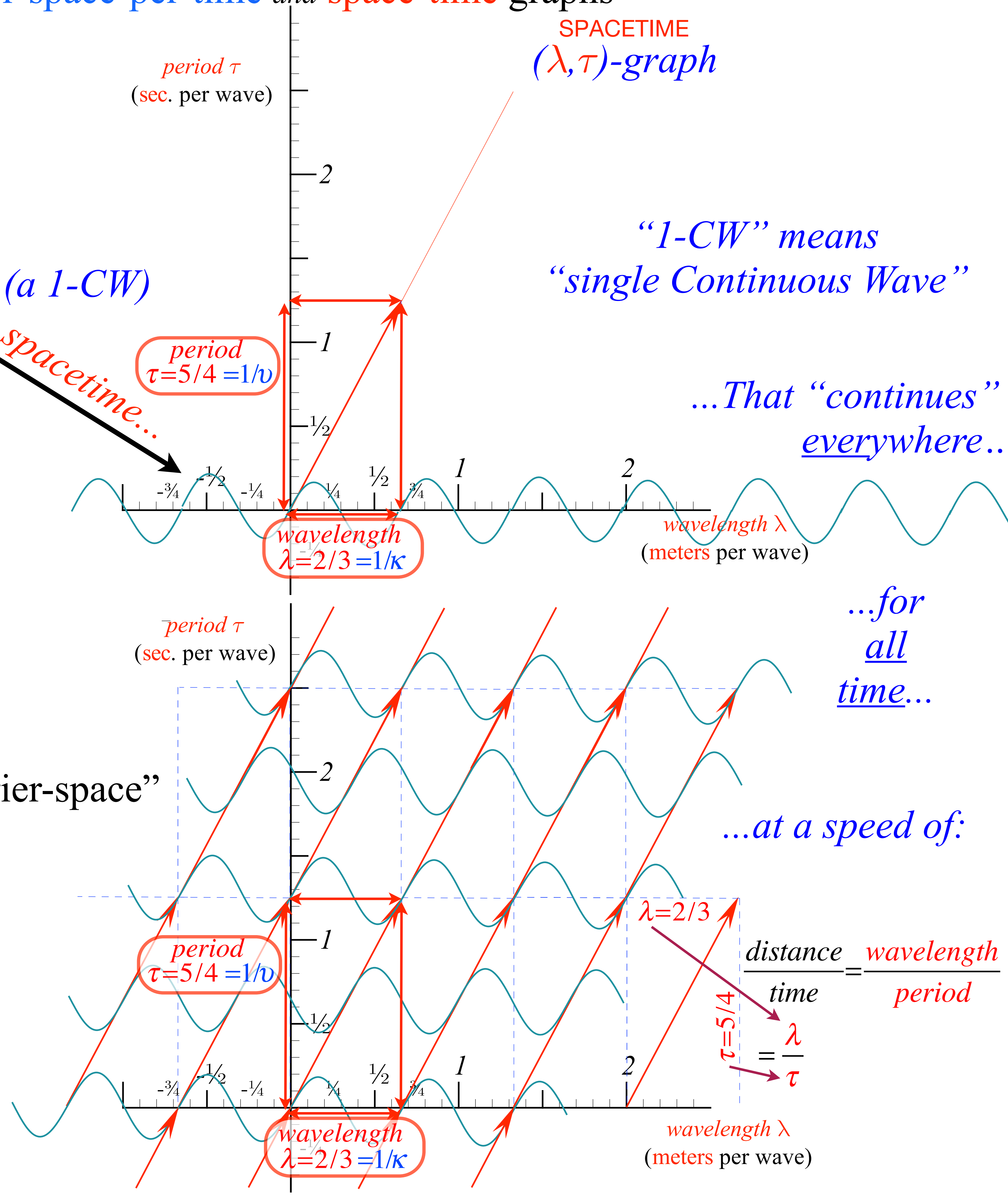
[RelaWavity Web Simulation *Keyboard of the Gods* *(Dual Plot 7)*]

- *How to understand waves and wave velocity V_{wave}*

Analyzing wave velocity by per-space-per-time *and* space-time graphs



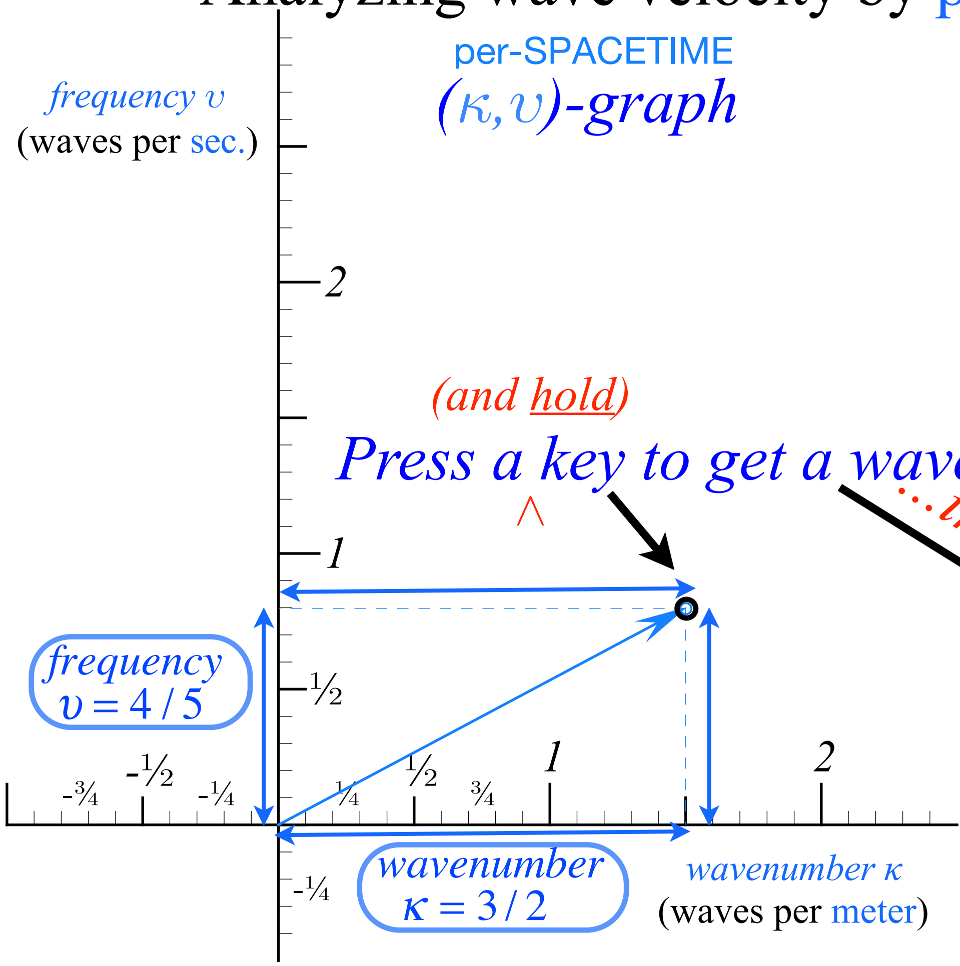


"*Keyboard of the gods*" is known as "Fourier-space"

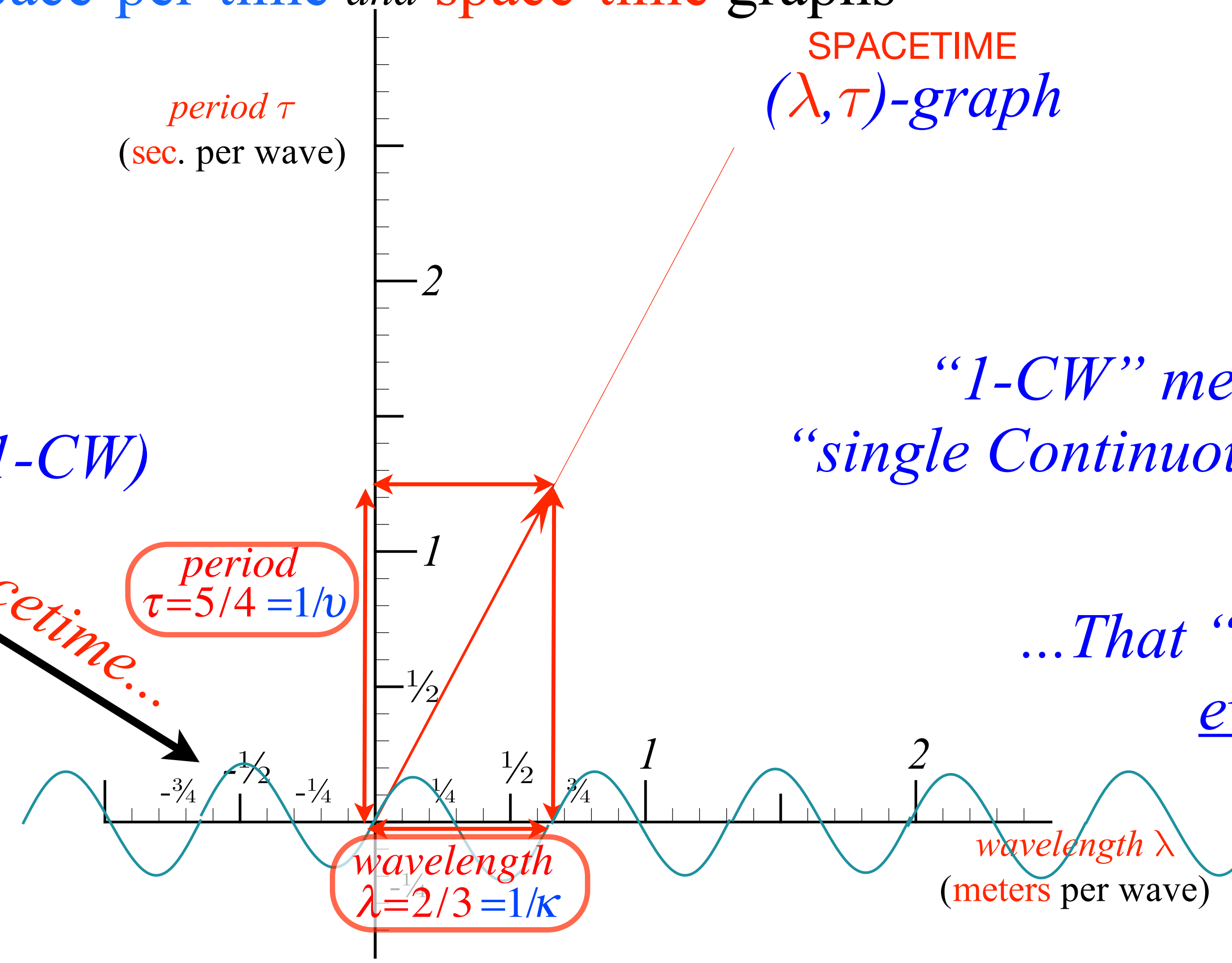
Jean-Baptiste
Joseph Fourier
1768-1830

•How to understand waves
and
wave velocity V_{wave}

Analyzing wave velocity by per-space-per-time *and* space-time graphs



...in spacetime...



"1-CW" means "single Continuous Wave"

...That "continues" everywhere..

...for all time...

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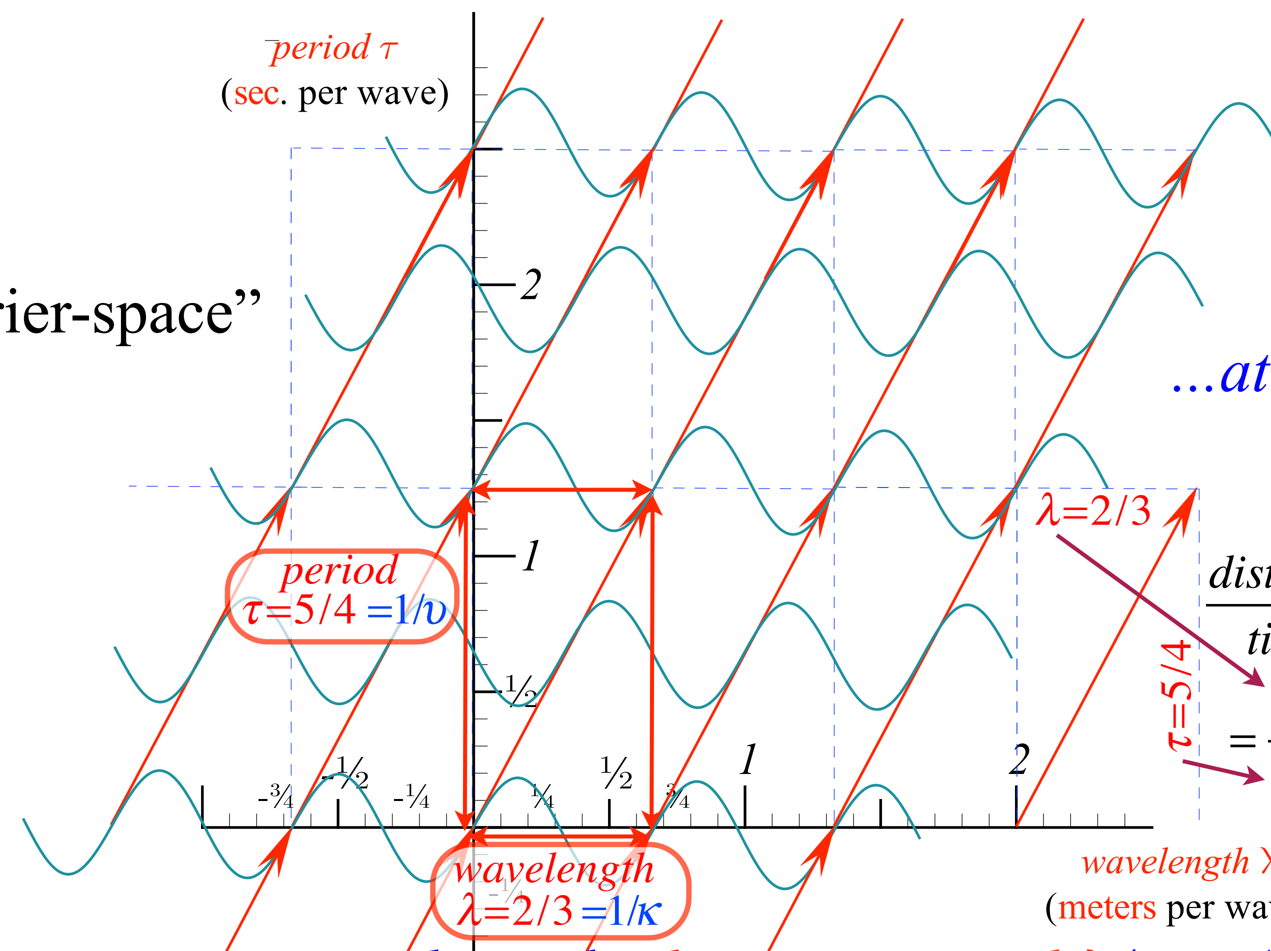


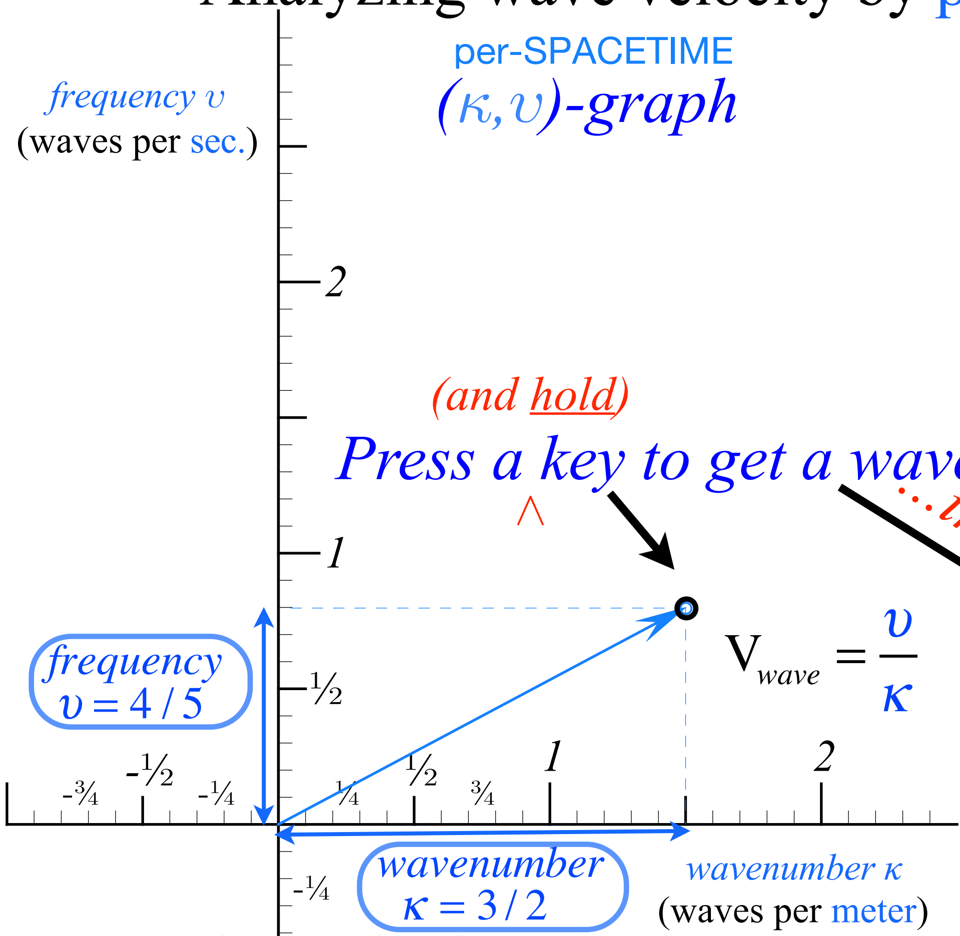
...at a speed of:

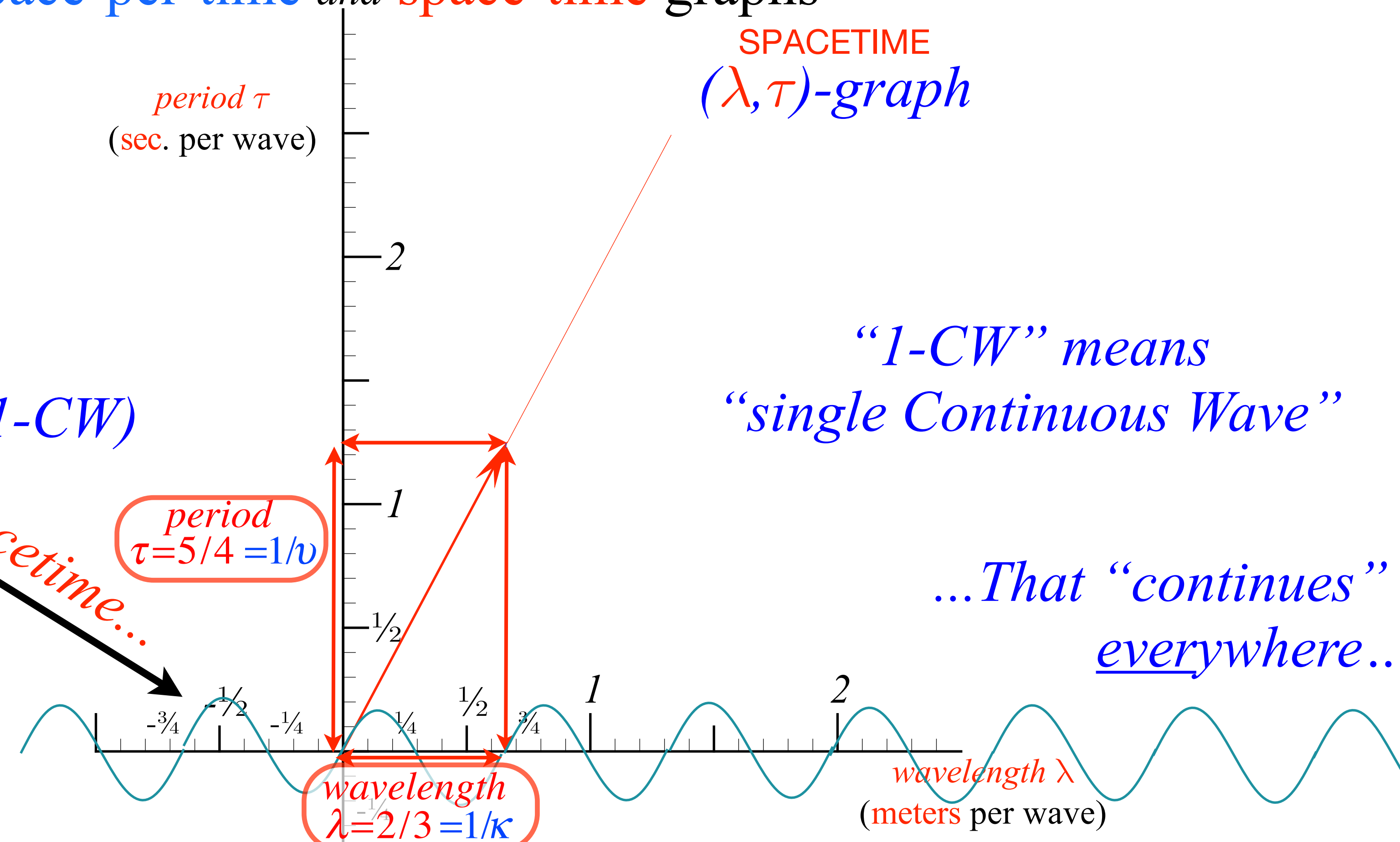
$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

•How to understand waves and wave velocity V_{wave}

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

Analyzing wave velocity by per-space-per-time *and* space-time graphs





"1-CW" means "single Continuous Wave"

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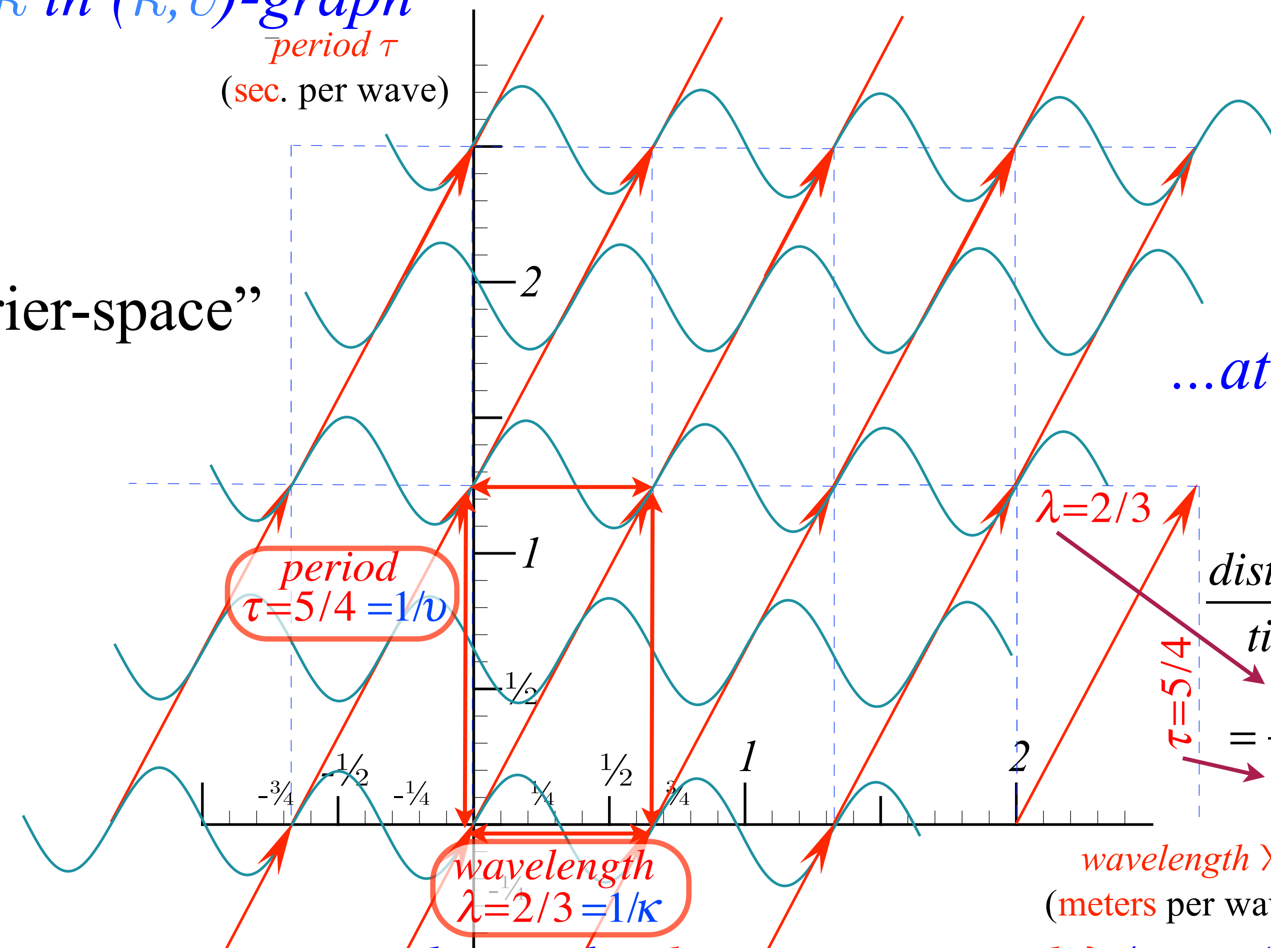
wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph

...for all time...



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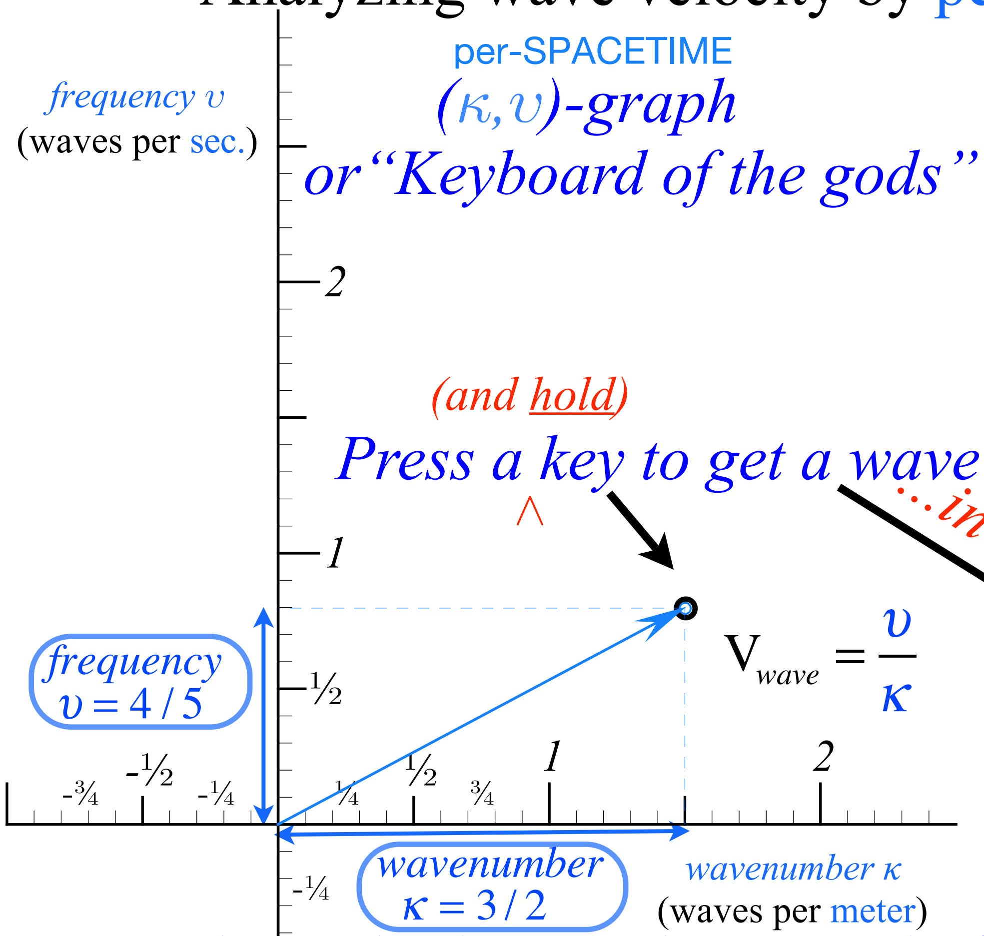
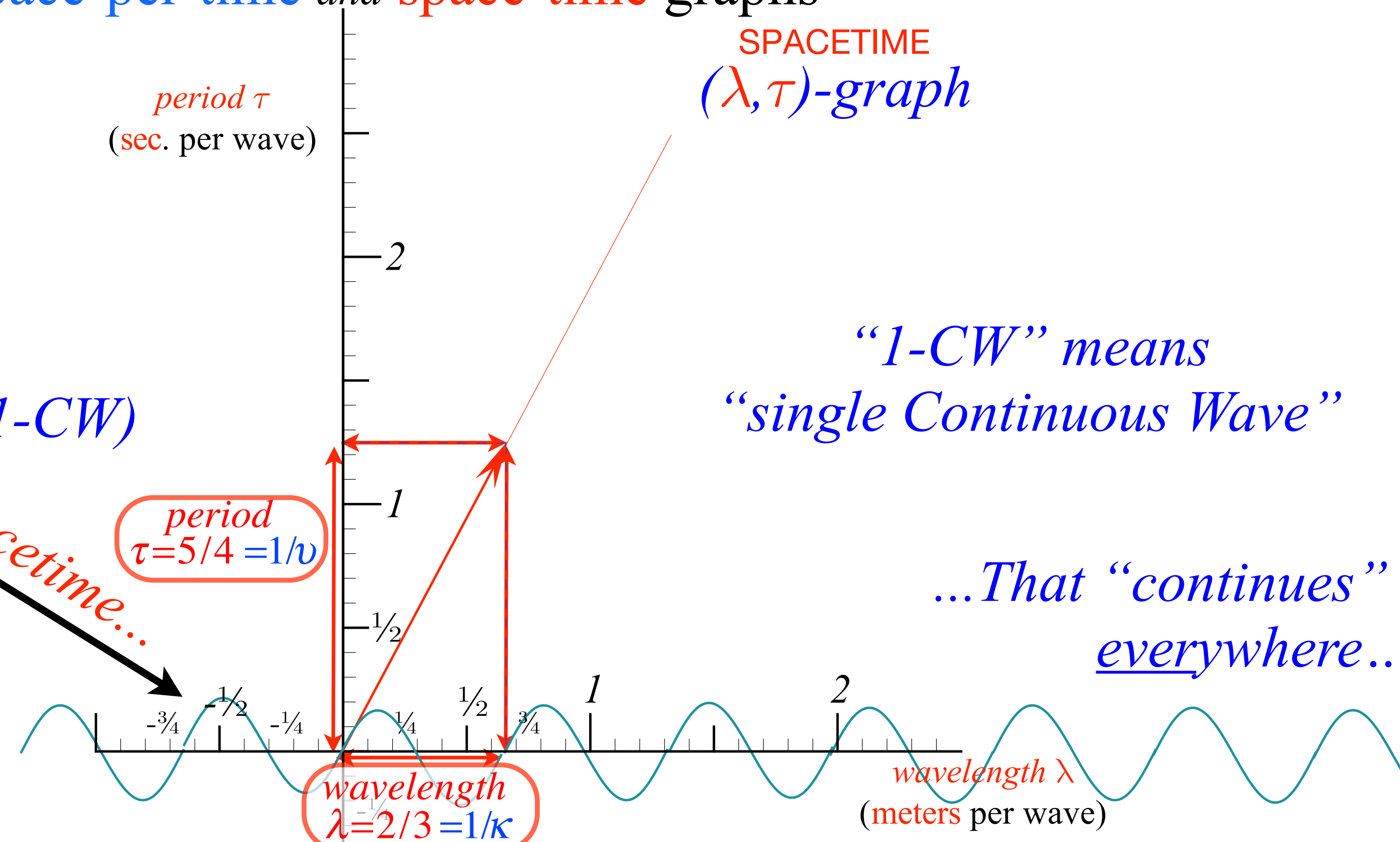




wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph

wave-velocity formulas

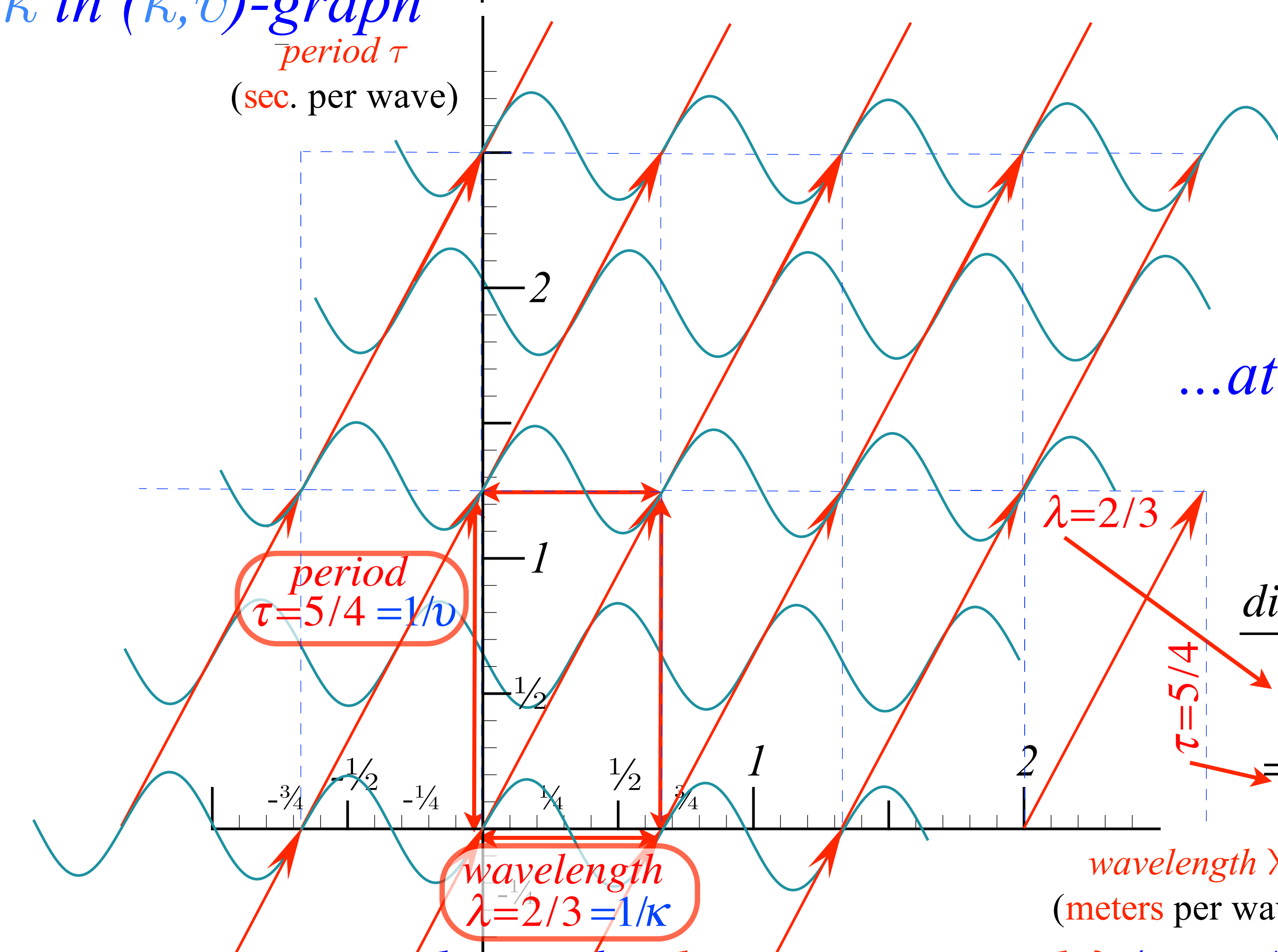
$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{frequency}{wavenumber}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15}\frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions

- *How to understand waves and "1st quantization"*



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➔ Introducing Doppler shifting

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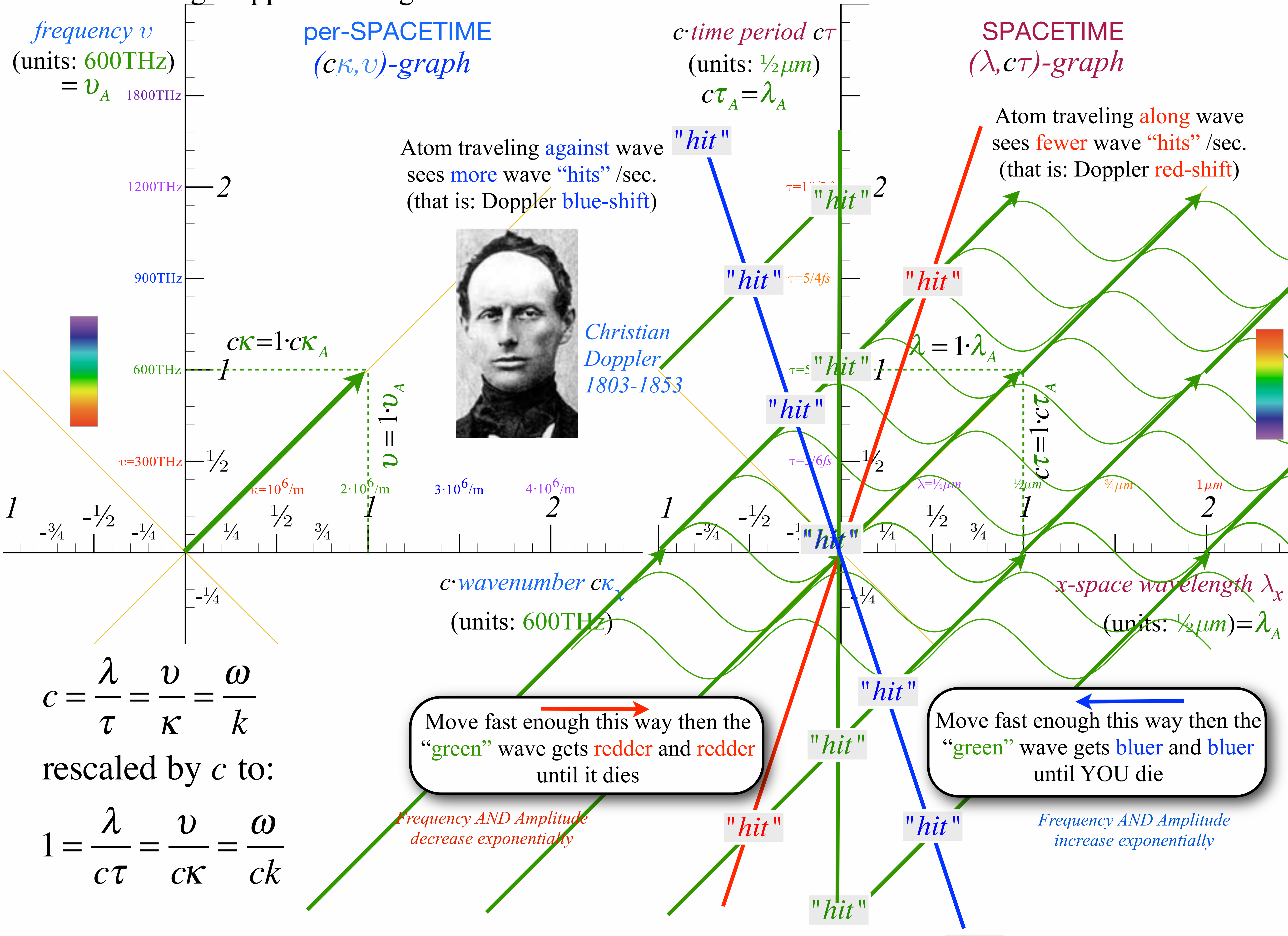
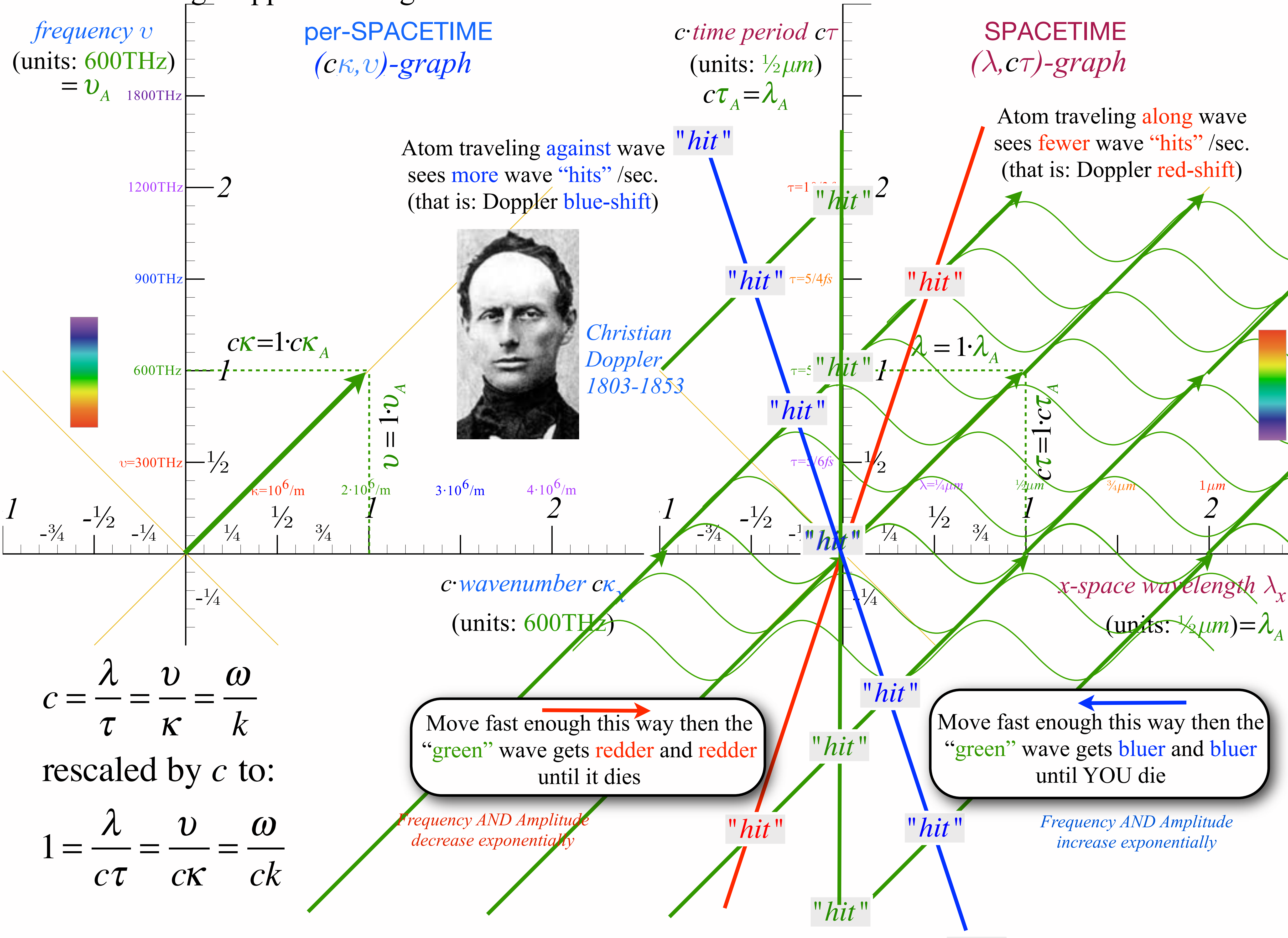


$$c = \frac{\lambda}{\tau} = \frac{\upsilon}{\kappa} = \frac{\omega}{k}$$

rescaled by c to:

$$1 = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = \frac{\omega}{ck}$$

Introducing Doppler shifting



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Introducing Doppler shifting and why c is constant

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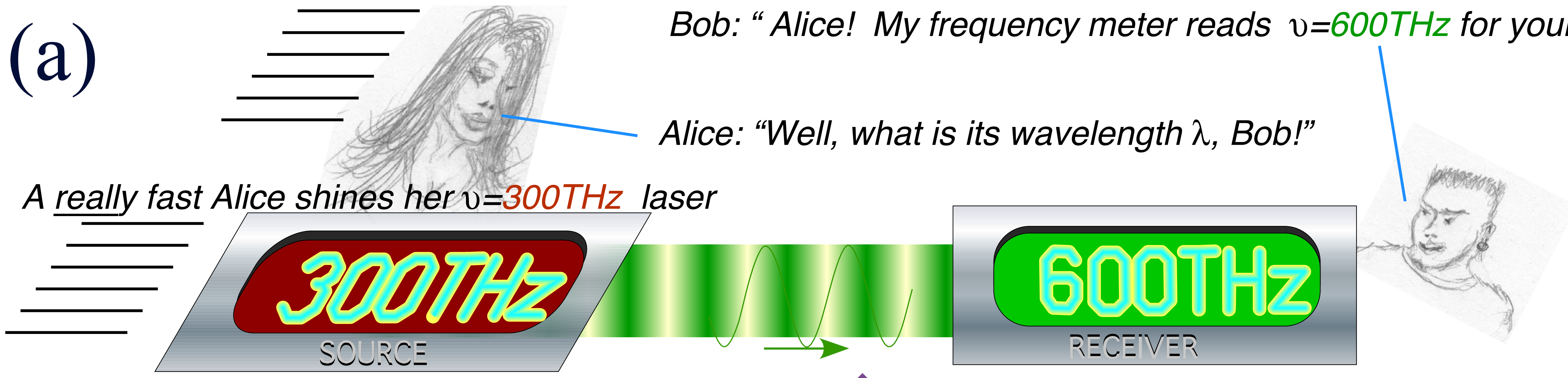

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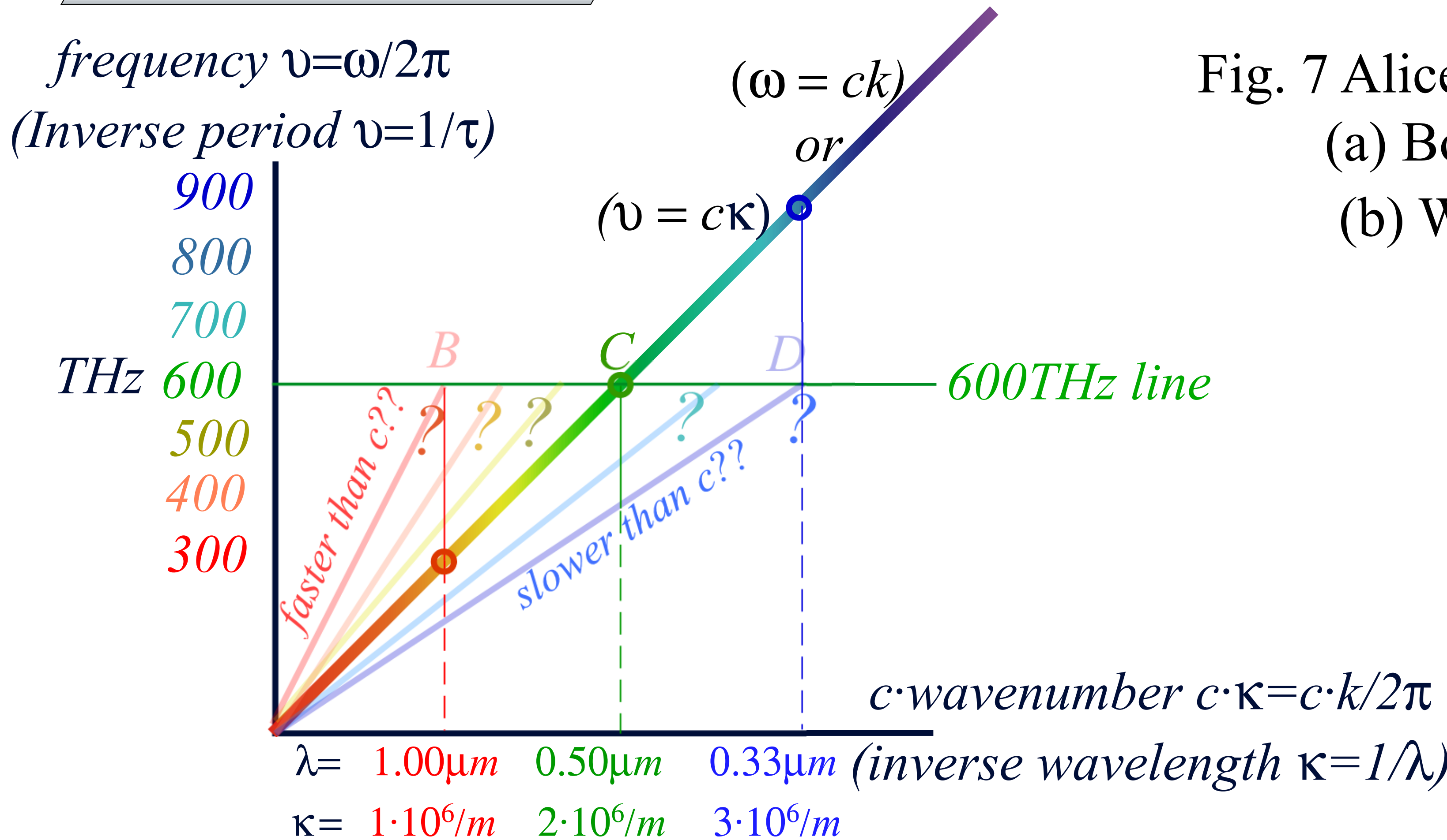
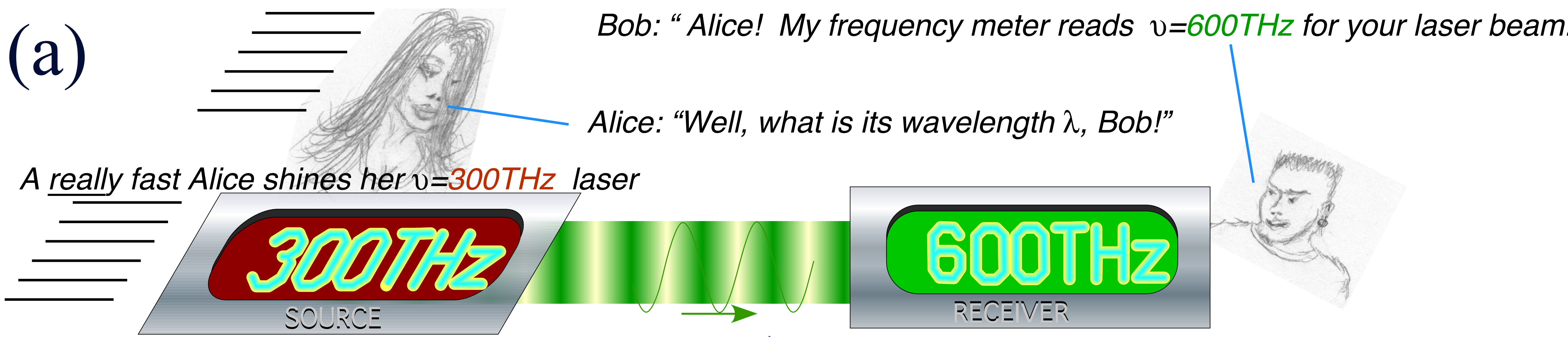
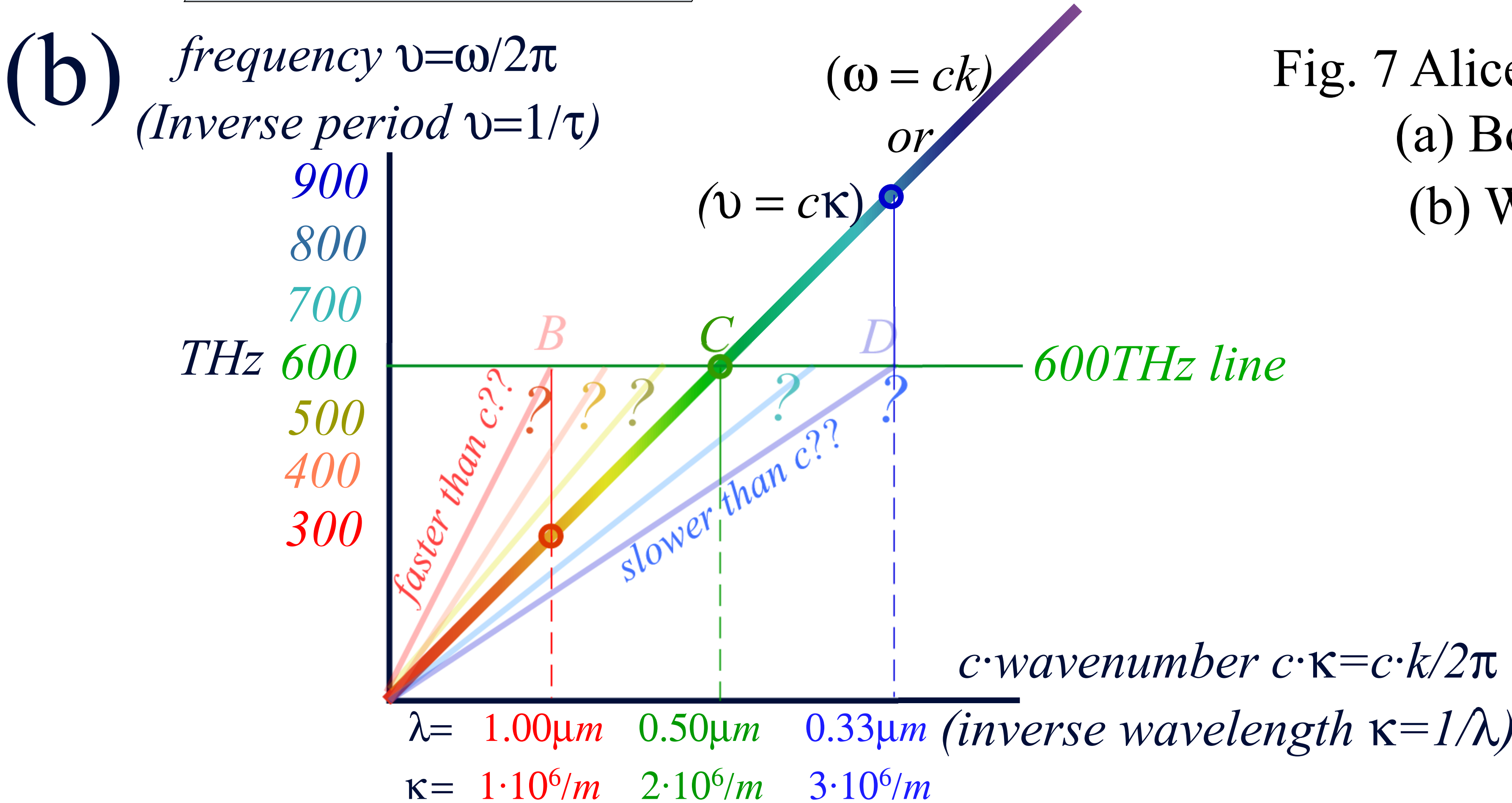


Fig. 7 Alice's 300THz laser approaches Bob.
(a) Bob sees υ=600THz.
(b) What $\lambda=1/\kappa$ does Bob measure?

Introducing Doppler shifting and why c is constant





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Years of spectroscopy rule out 'phony' 600THz blue-green that do not have wavelength λ=0.5*micron.*

The only choice is C.

Introducing Doppler shifting and why c is constant

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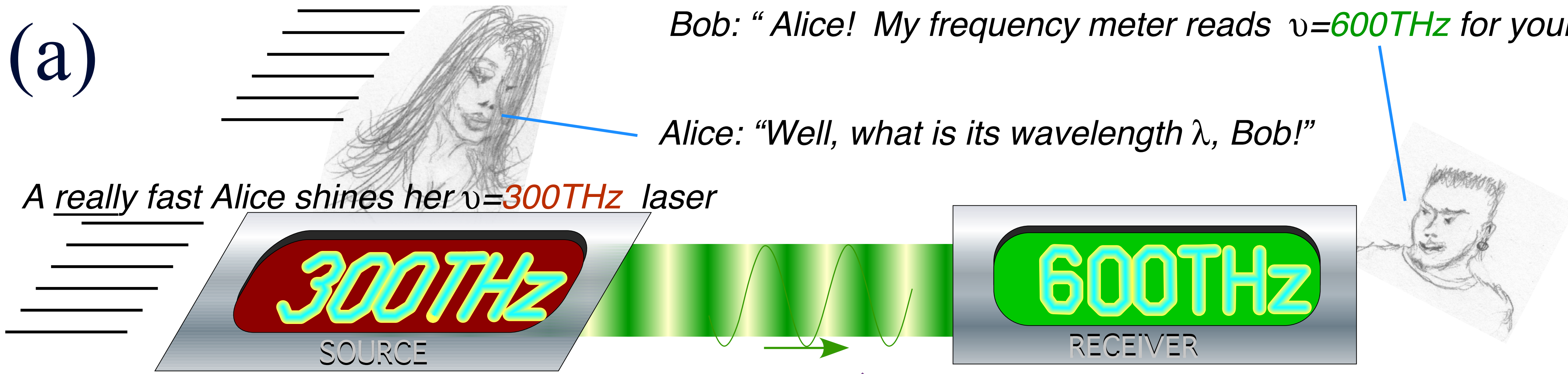

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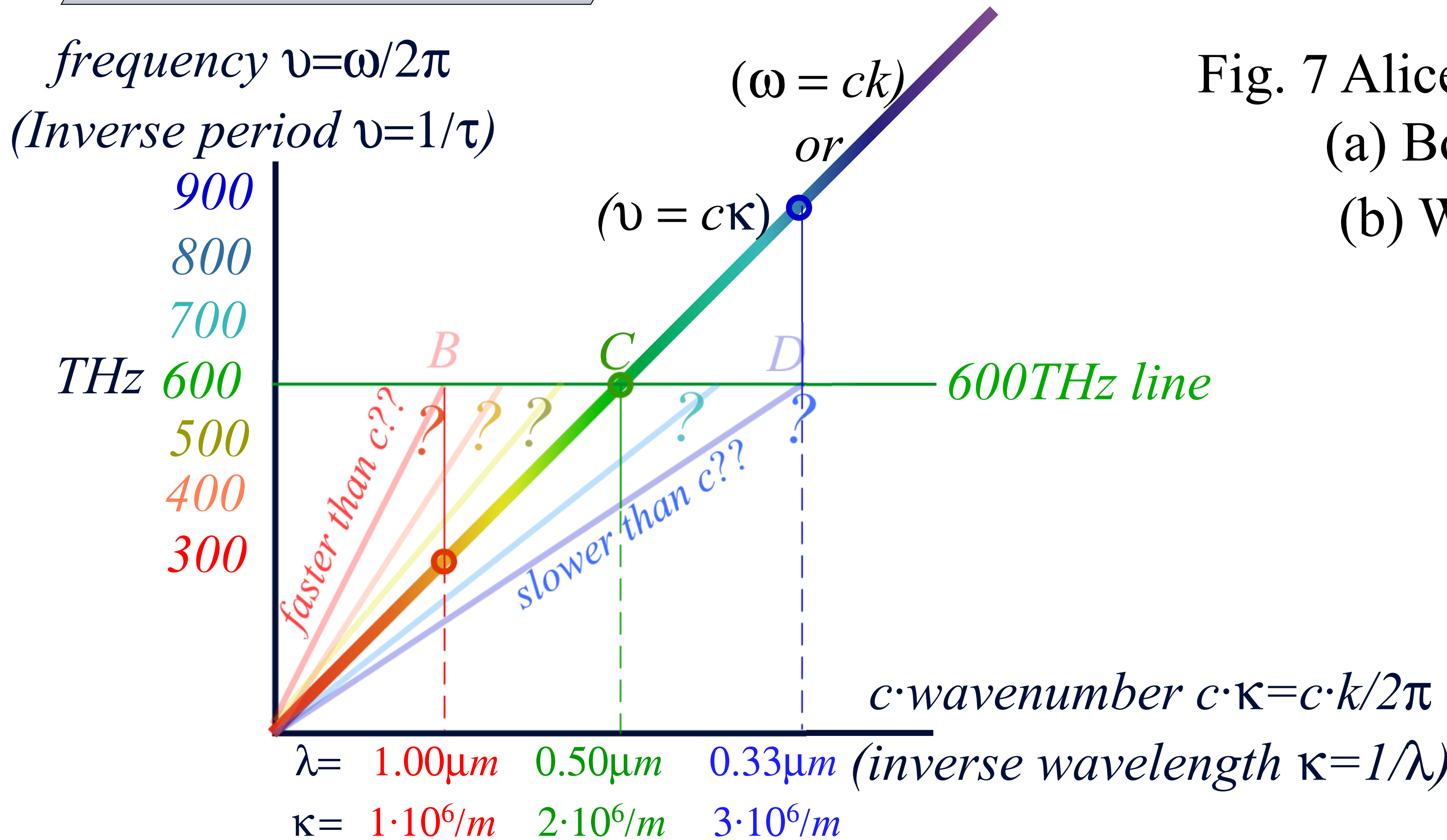

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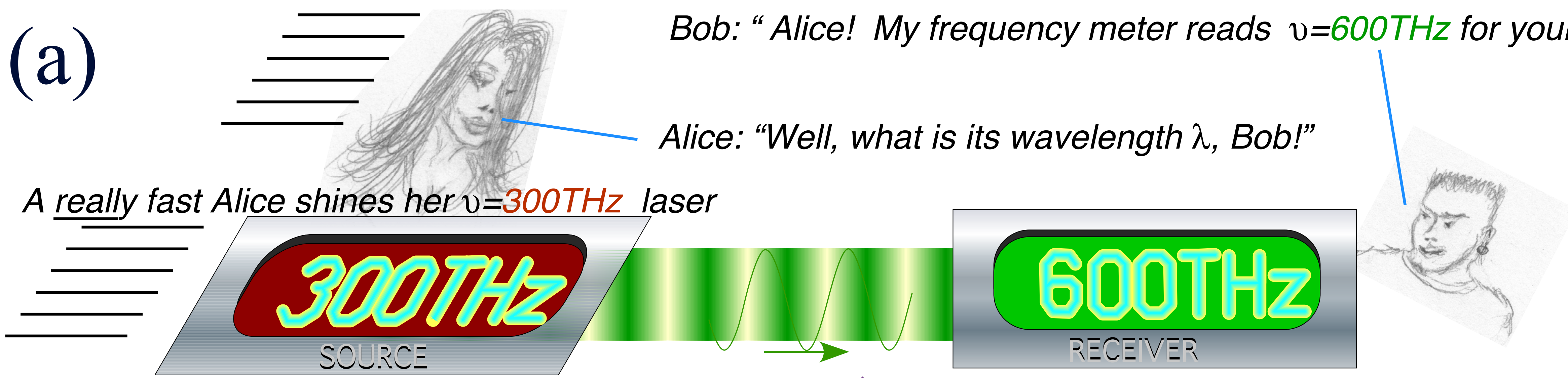
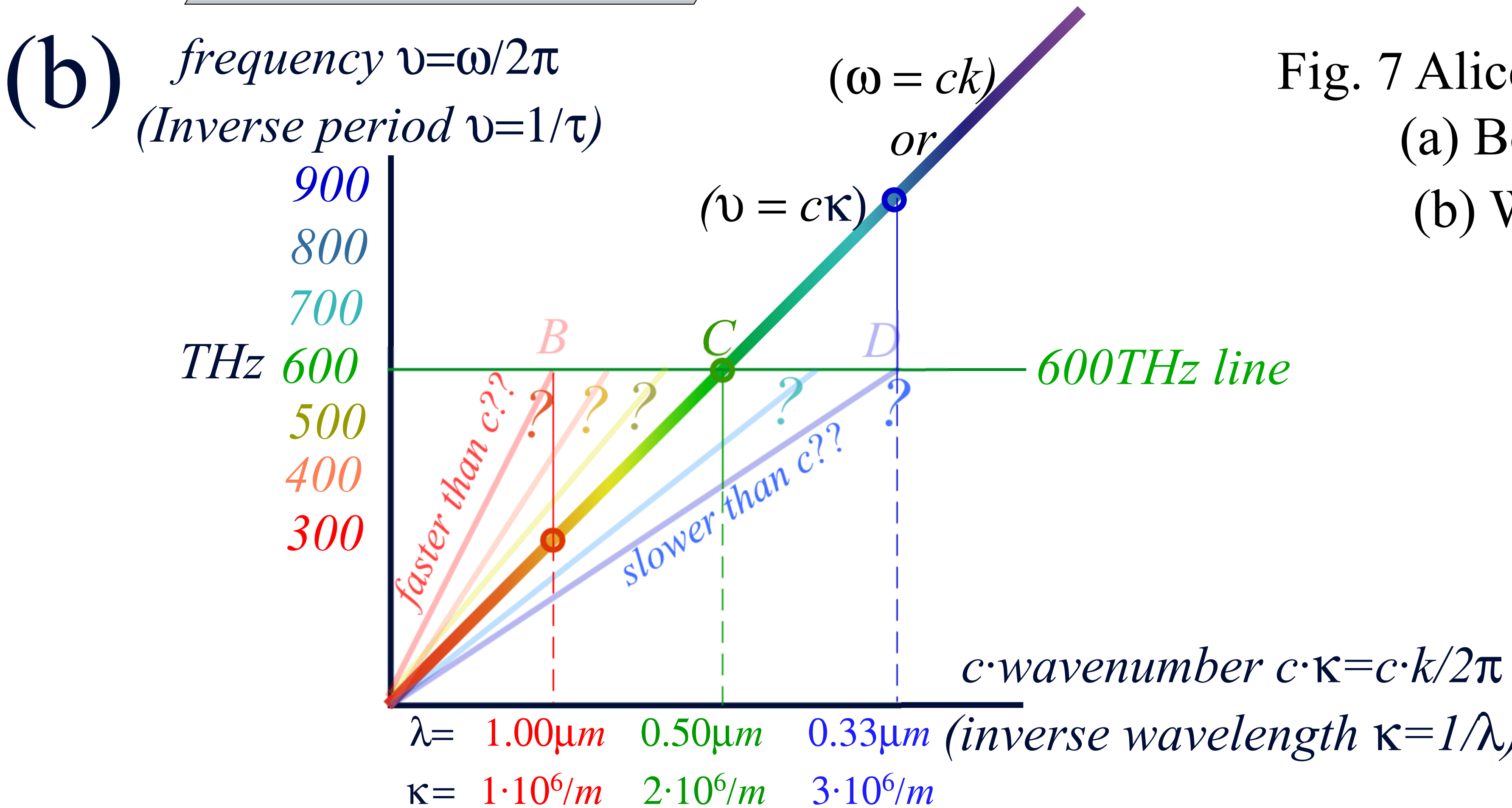
Introducing Doppler shifting and why c is constant





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Actually: $2.99792458\cdot 10^{8}$m·s-1

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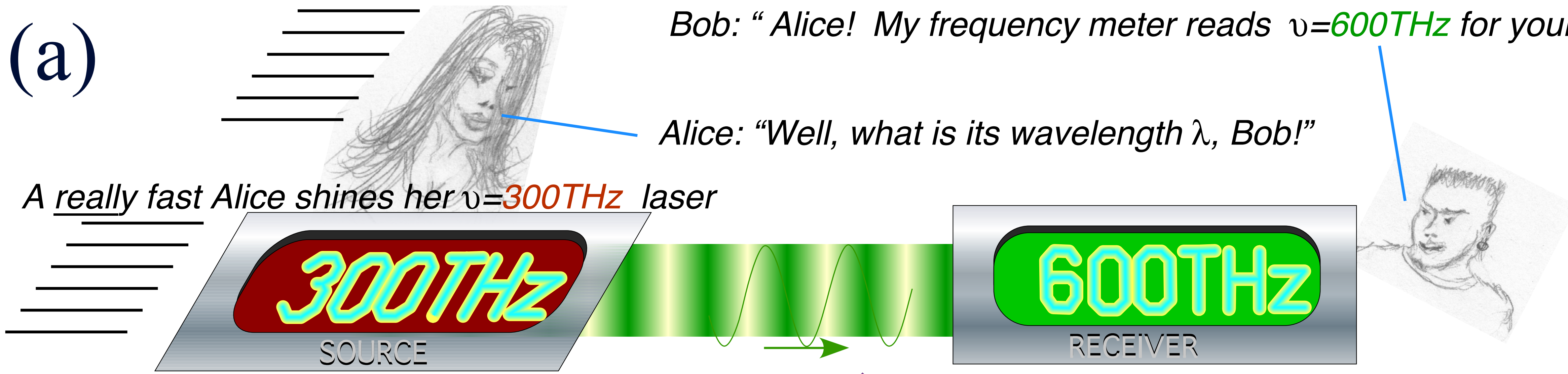

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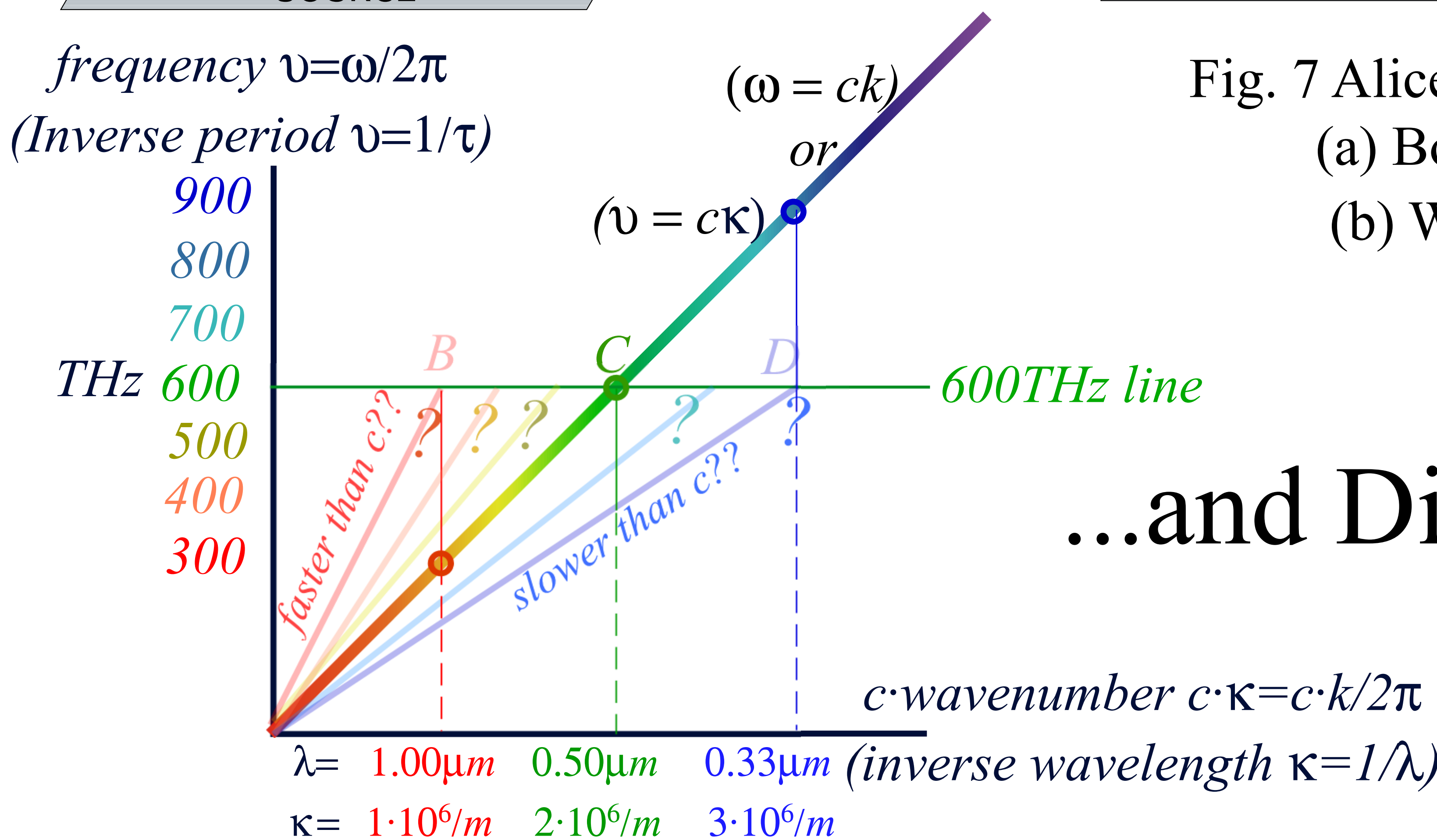

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...and Dispersion-Free!

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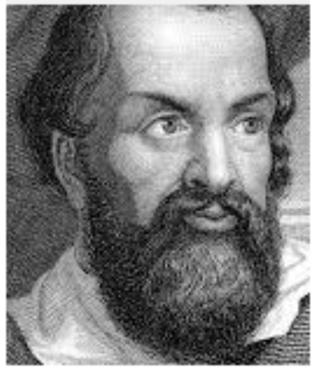
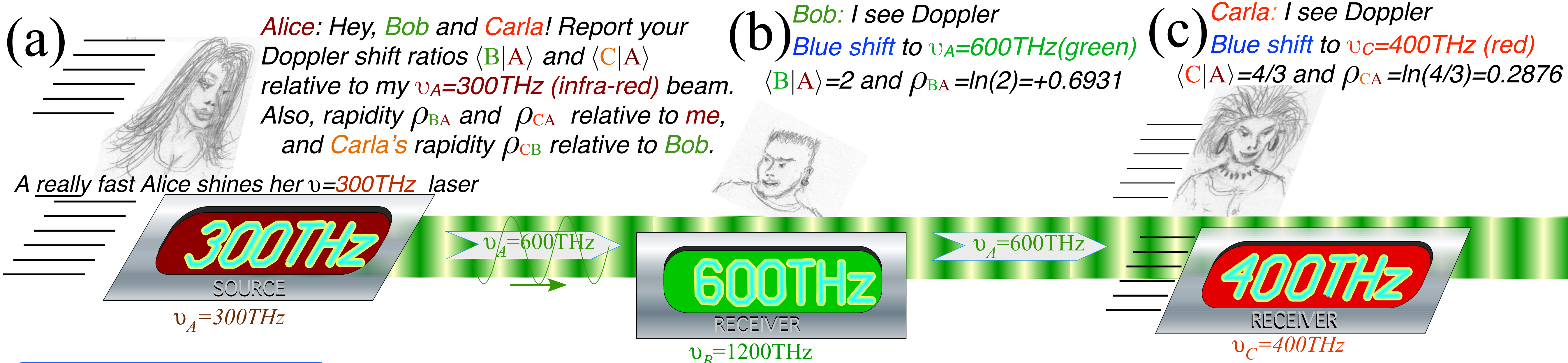
Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like Galilean velocity



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

Bob-Alice rapidity:

$$\rho_{BA} = \ln\langle B|A\rangle = \ln\frac{2}{1} = 0.6931$$

$$\rho_{AB} = \ln\langle A|B\rangle = \ln\frac{1}{2} = -0.6931 = -\rho_{BA}$$

Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$

Introducing Doppler Arithmetic and rapidity ρ

(a) *Alice: Hey, Bob and Carla! Report your Doppler shift ratios ⟨B|A⟩ and ⟨C|A⟩ relative to my υ_A=300THz (infra-red) beam. Also, rapidity ρ_{BA} and ρ_{CA} relative to me, and Carla's rapidity ρ_{CB} relative to Bob.*

(b) *Bob: I see Doppler Blue shift to υ_A=600THz(green)*
⟨B|A⟩=2 and ρ_{BA}=ln(2)=+0.6931

(c) *Carla: I see Doppler Blue shift to υ_C=400THz (red)*
⟨C|A⟩=4/3 and ρ_{CA}=ln(4/3)=0.2876

A really fast Alice shines her υ=300THz laser

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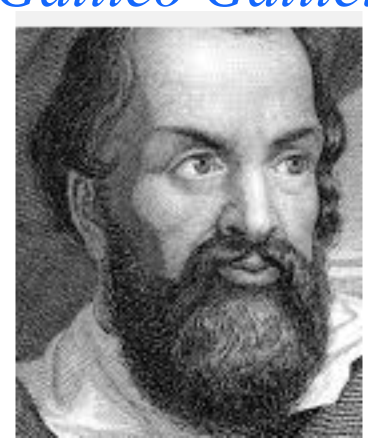
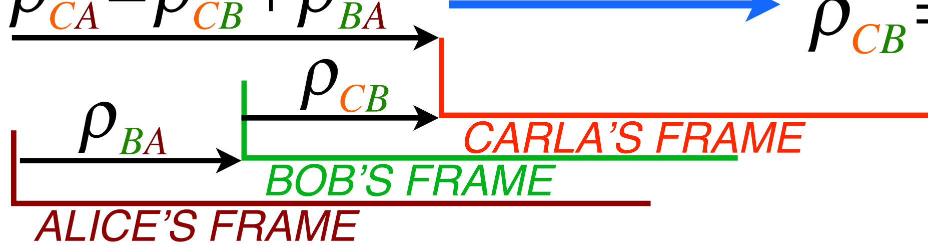


1564-1642

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CA} = \rho_{CB} + \rho_{BA}$$



Carla-Bob Doppler ratio:

$$\langle C|B\rangle = \frac{\upsilon_C}{\upsilon_B} = \frac{\upsilon_C}{\upsilon_A}\frac{\upsilon_A}{\upsilon_B} = \langle C|A\rangle\langle A|B\rangle = \frac{4}{3}\frac{1}{2} = \frac{2}{3}$$

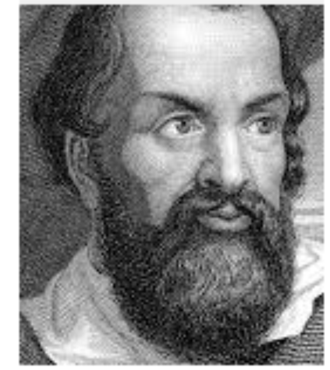
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}}e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = 0.2876 - 0.6931 = -0.4055$$

$$= \ln\frac{4}{3} + \ln\frac{1}{2} = \ln\frac{2}{3}$$

Galileo Galilei



1564-1642

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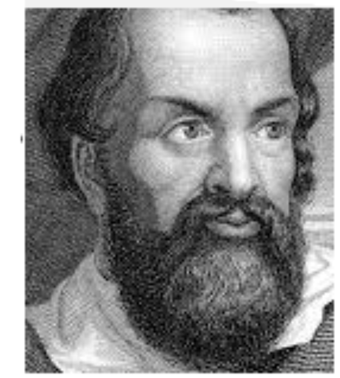

Galileo's Revenge (part 2)
Phasor angular velocity *adds just like Galilean velocity*

Learning about sin! *and* cos *and…Trigonometric road maps*
- Hyper-Trigonometric algebra and phasors in space-time
- 1CW wavefunctions and phasors

Per-space-per-time vs Space-time
- Wave velocity formulas

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- Why c is constant?!
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- Details of 2CW wavefunctions in rest frame
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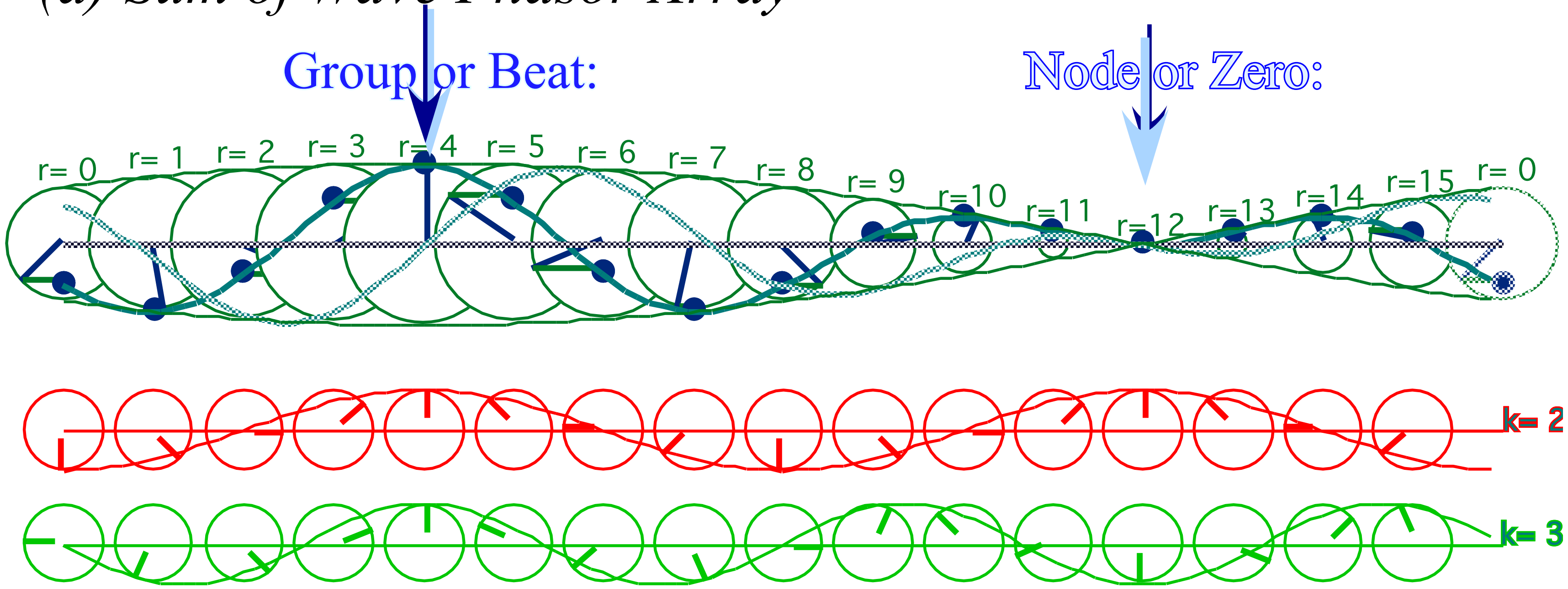
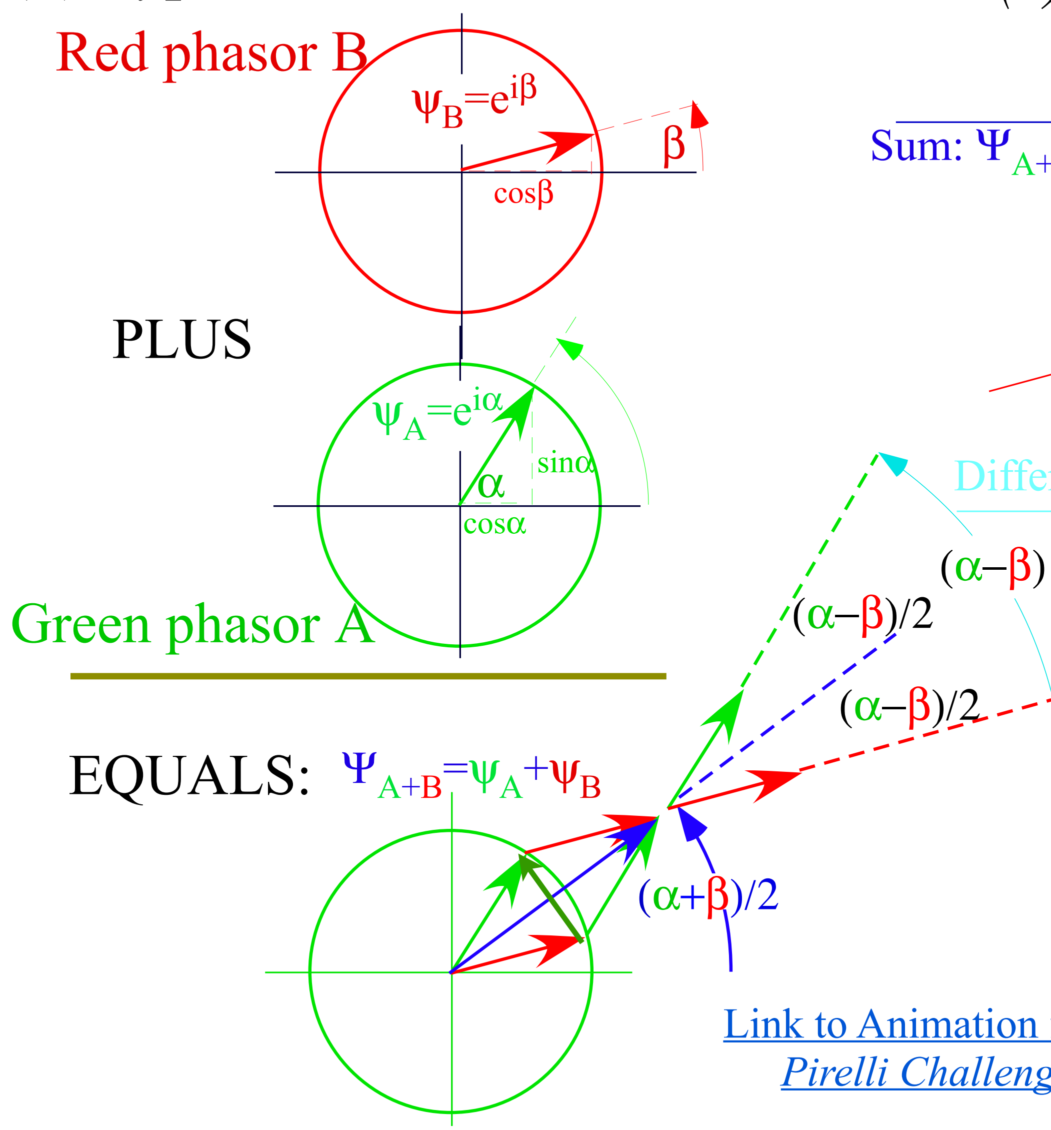
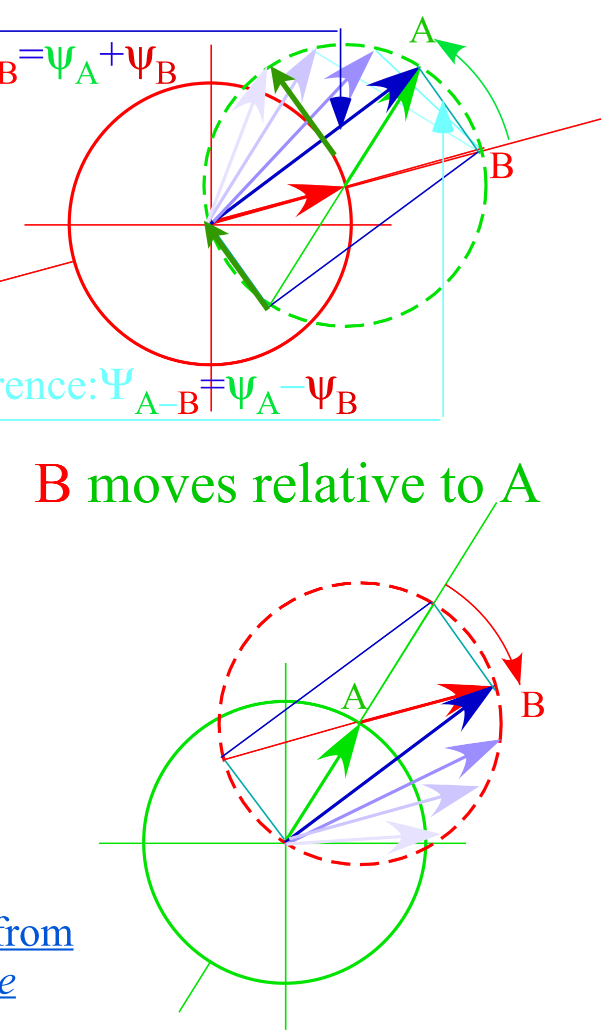
(a) Sum of Wave Phasor Array



(b) Typical Phasor Sum:

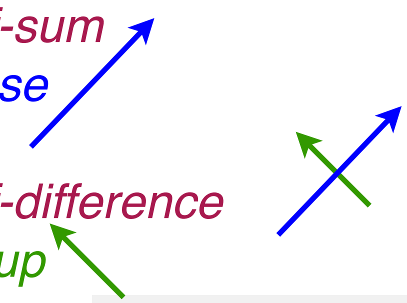


(c) Phasor-relative views

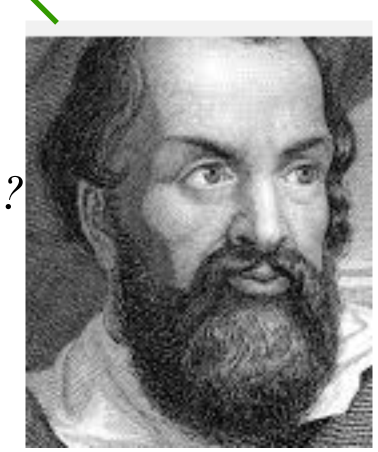


[Link to Animation from *Pirelli Challenge*]

Geometry of the Half-sum Phase and Half-difference Group



Happy now?



Galileo's Revenge (part 2)
Phasor angular velocity
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Galilean velocity

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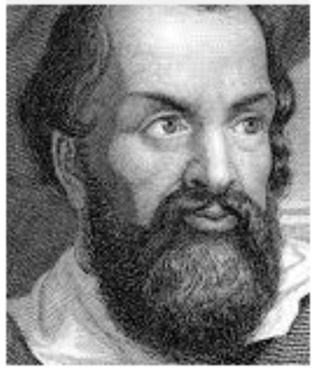

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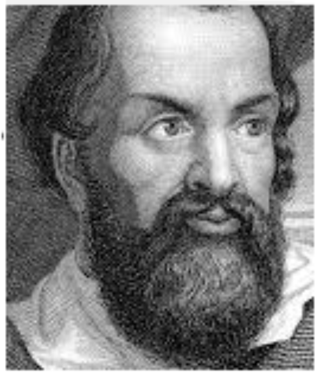

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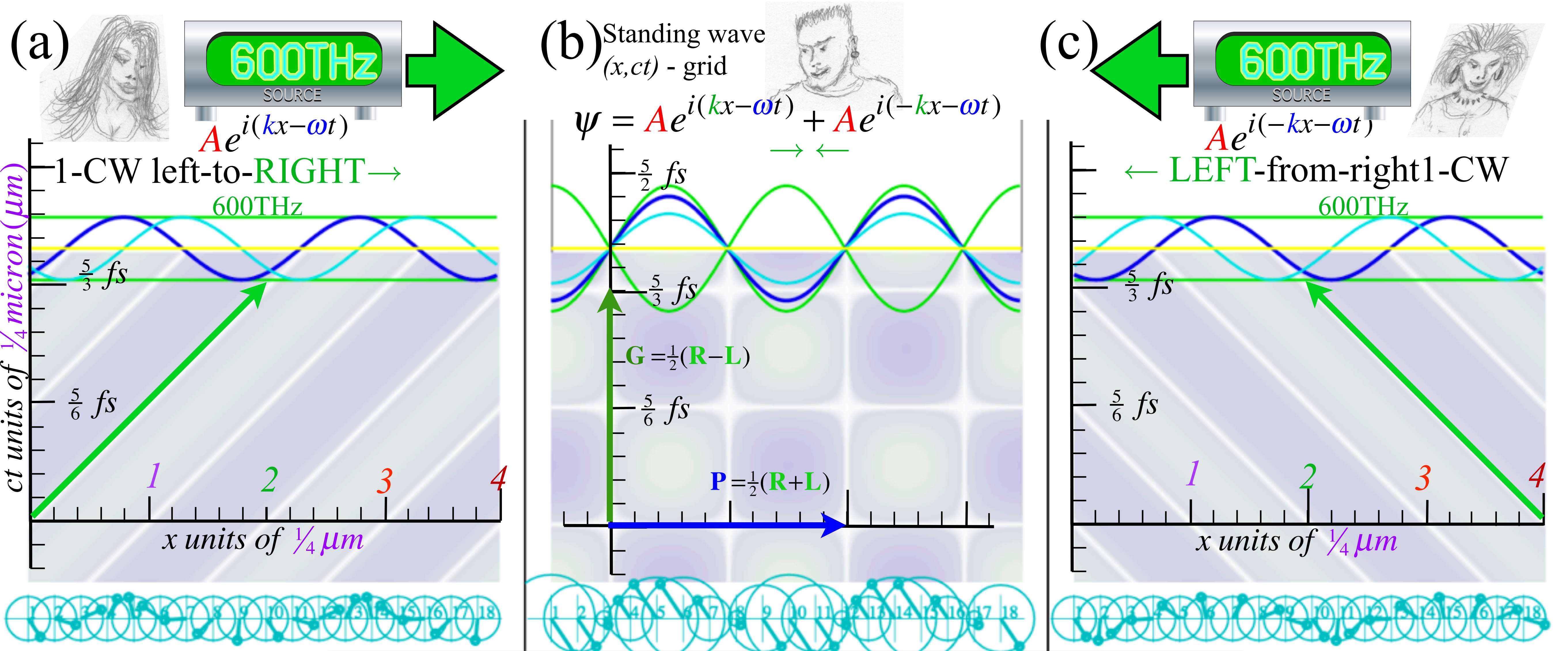
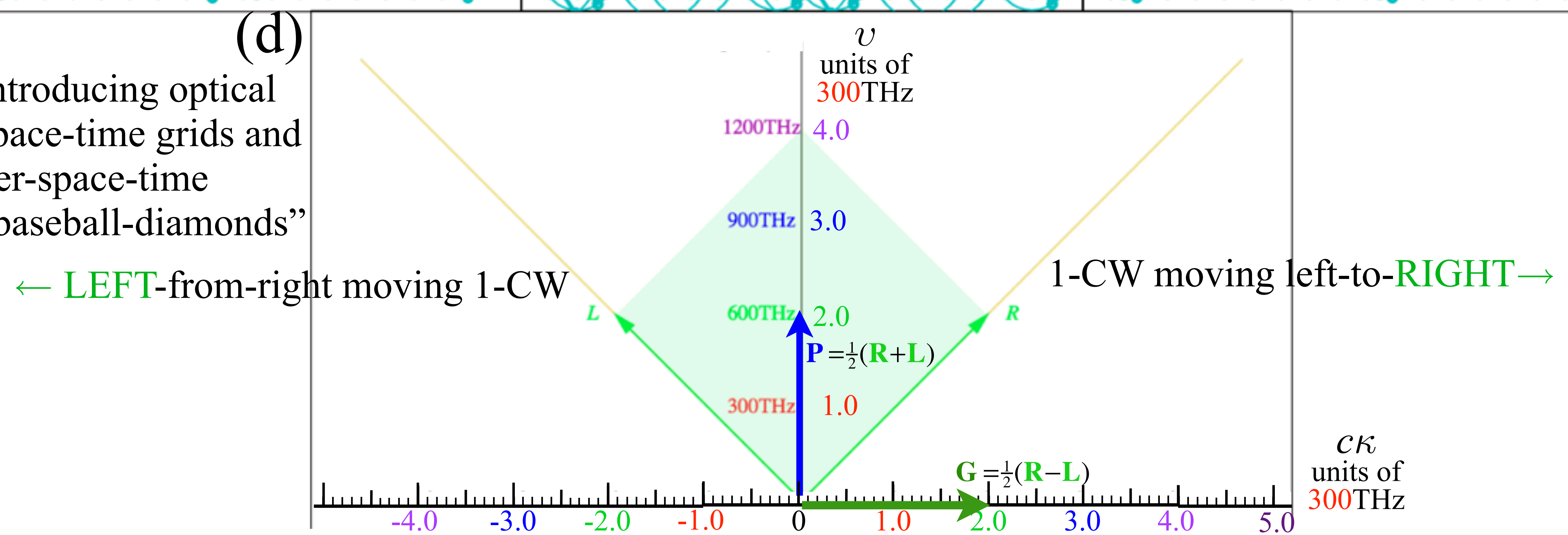
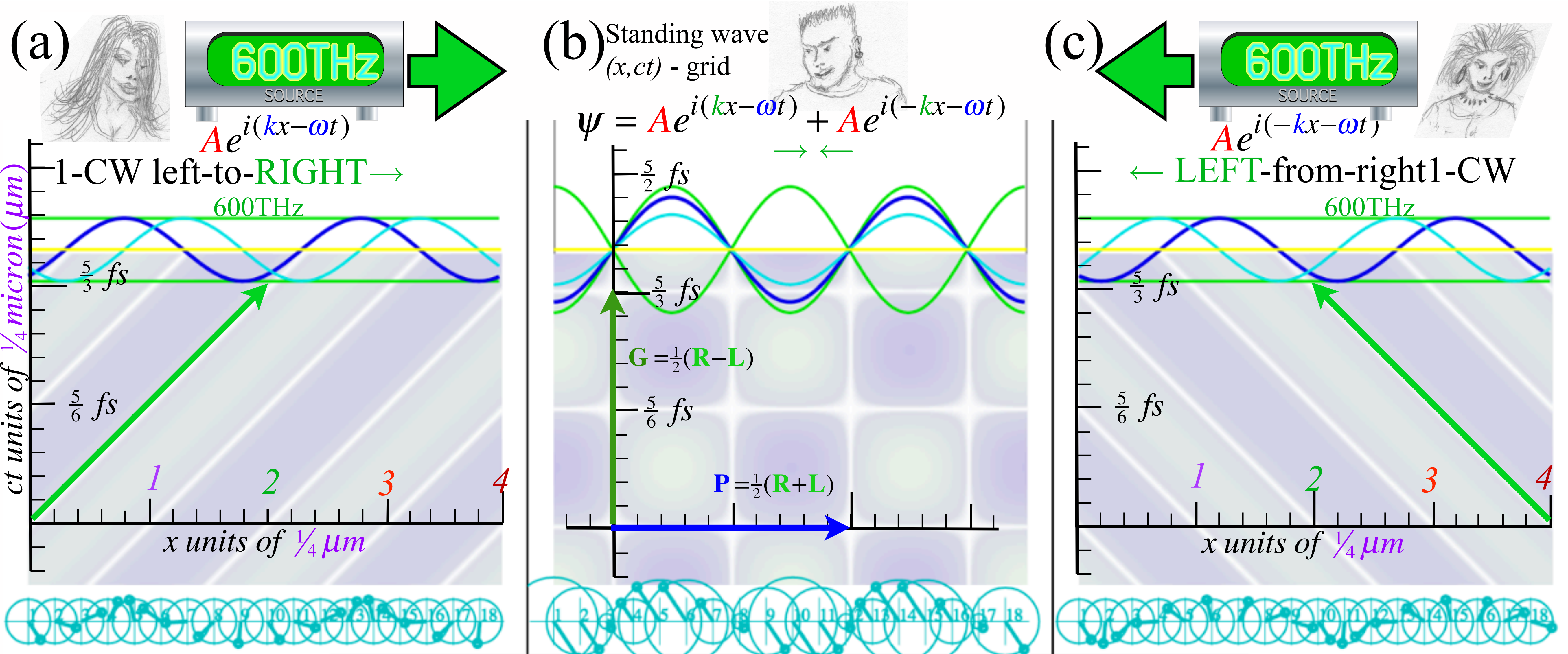


Introducing optical space-time grids and per-space-time "baseball-diamonds"

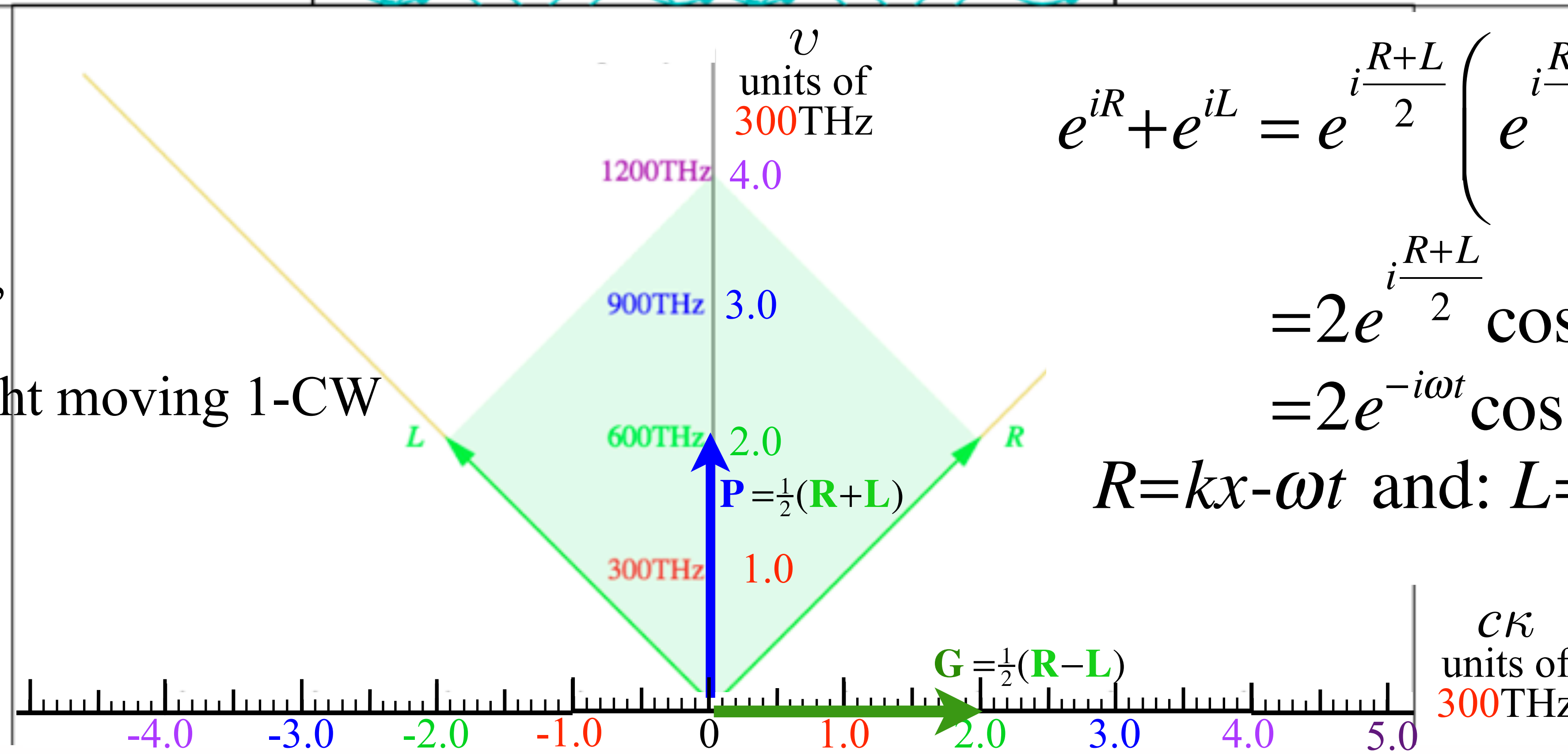




(d)

Introducing optical space-time grids and per-space-time “baseball-diamonds”

← LEFT-from-right moving 1-CW

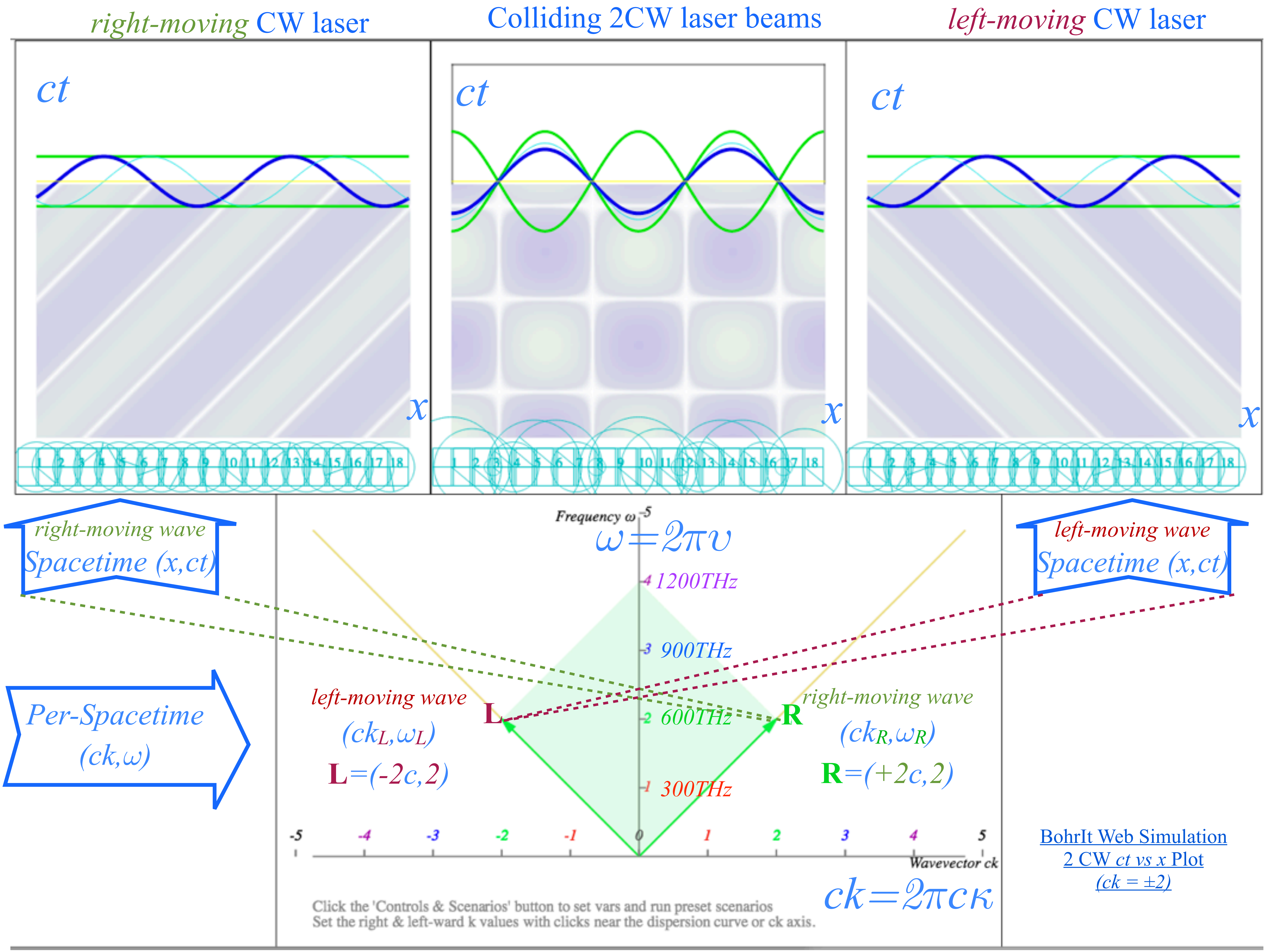


$$e^{iR}+e^{iL} = e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$

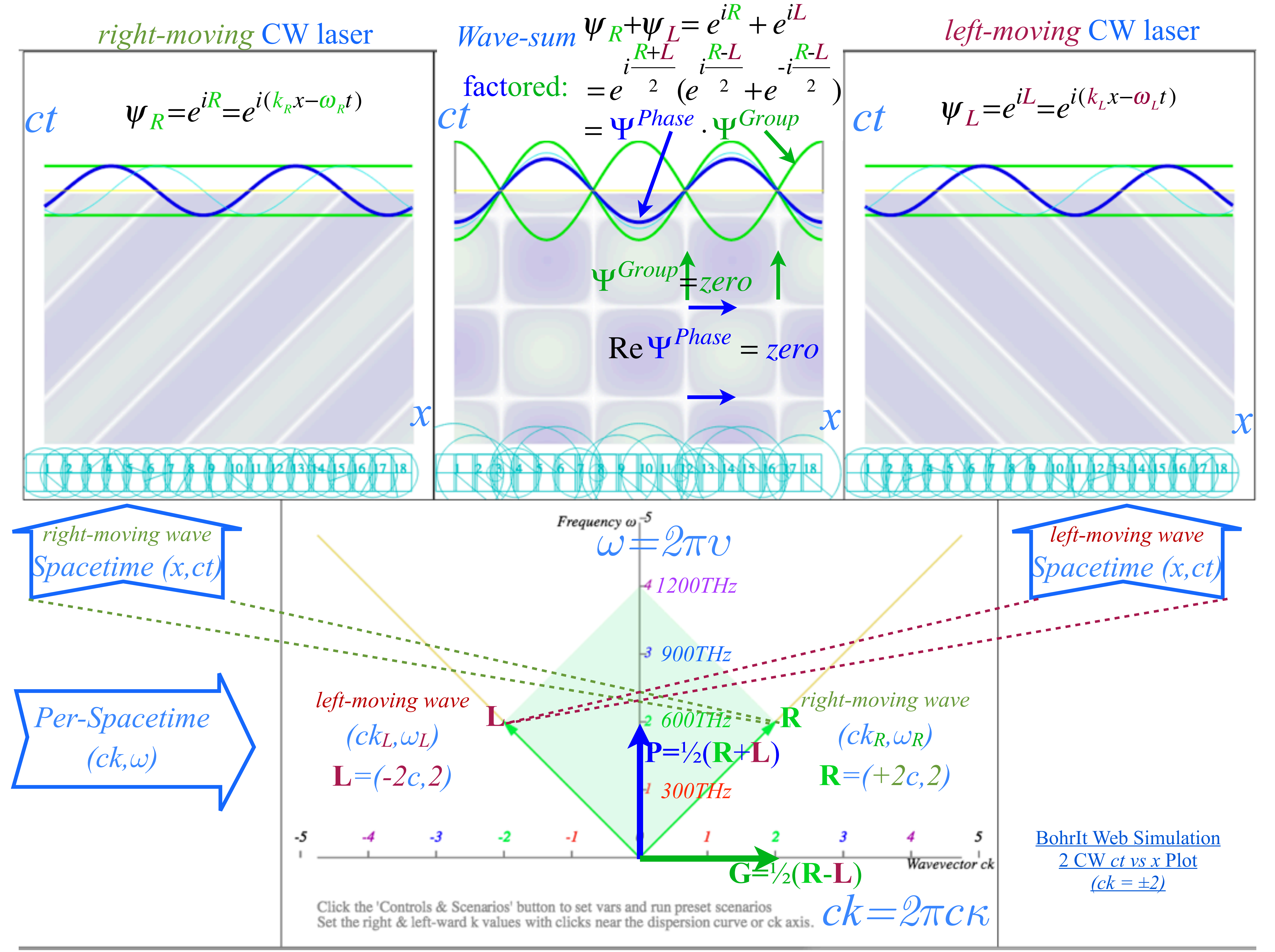
$$=2e^{i\frac{R+L}{2}}\cos\frac{R-L}{2}$$

$$=2e^{-i\omega t}\cos kx$$

$R=kx\text{-}\omega t$ and: $L=\text{-}kx\text{-}\omega t$



right-moving CW laser
Colliding 2CW laser beams
left-moving CW laser
ct
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
ω=2πυ
1200THz
900THz
600THz
300THz
left-moving wave
(ckL,ωL)
L=(-2c,2)
right-moving wave
(ckR,ωR)
R=(+2c,2)
Wavevector ck
ck=2πcκ
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation
2 CW ct vs x Plot
(ck = ±2)


right-moving CW laser
Wave-sum $\psi_R+\psi_L = e^{iR} + e^{iL}$
factored: $= e^{i\frac{R+L}{2}} (e^{i\frac{R-L}{2}} + e^{-i\frac{R-L}{2}})$
$= \Psi^{Phase} \cdot \Psi^{Group}$
left-moving CW laser
ct
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
$\Psi^{Group} = zero$
$\mathrm{Re}\,\Psi^{Phase} = zero$
x
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
$\omega = 2\pi\upsilon$
1200THz
900THz
600THz
300THz
left-moving wave
(ck_L, ω_L)
L=(-2c,2)
right-moving wave
(ck_R, ω_R)
R=(+2c,2)
L
R
P=½(R+L)
G=½(R-L)
Wavevector ck
$ck = 2\pi c\kappa$
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
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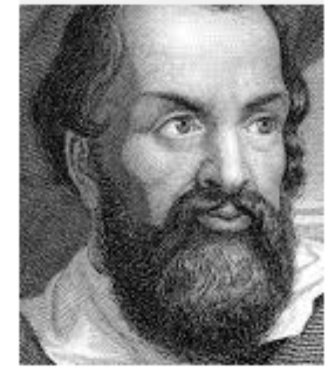
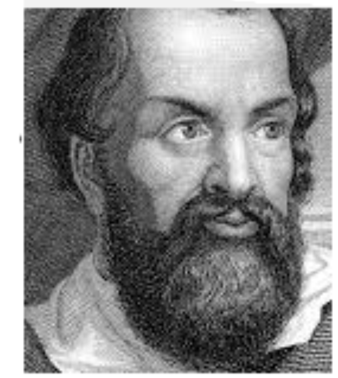
2 CW ct vs x Plot
(ck = ±2)

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like Galilean velocity

Learning about sin! *and* cos *and…Trigonometric road maps*
Hyper-Trigonometric algebra and phasors in space-time
1CW wavefunctions and phasors
Per-space-per-time vs Space-time
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Introducing Doppler shifting
Why c is constant?!
Introducing Doppler Arithmetic and rapidity ρ
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➔ Pulse waves (PW) versus Continuous Waves (CW)
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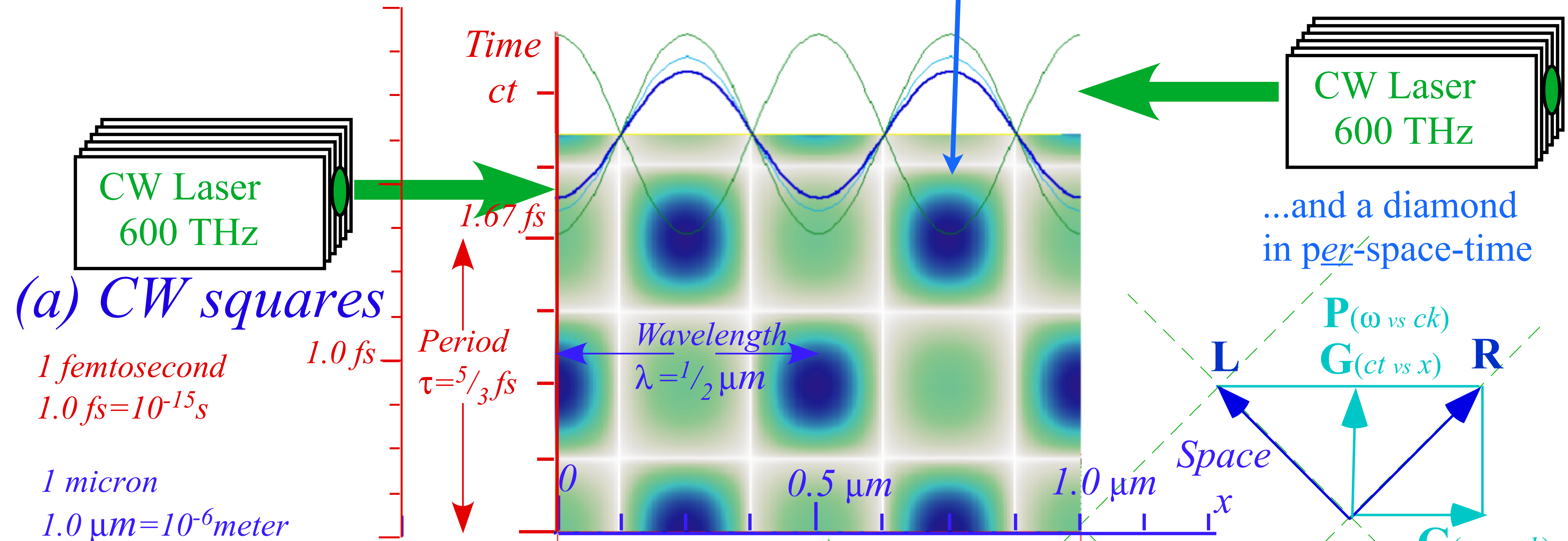
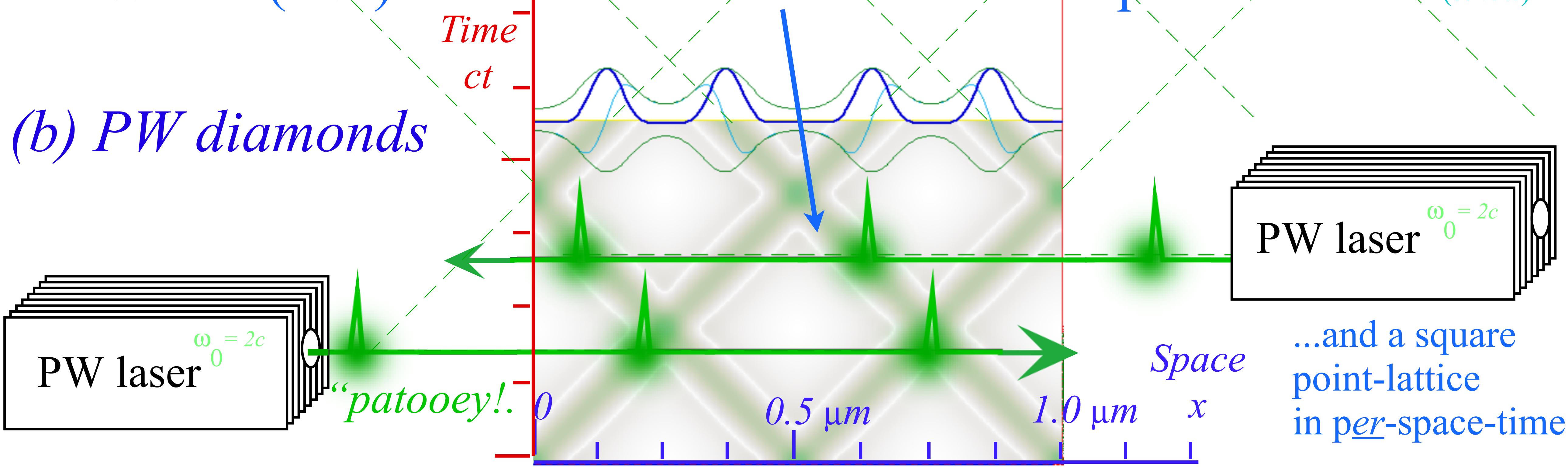
Continuous Waves (CW) trace "Cartesian squares" in space-time

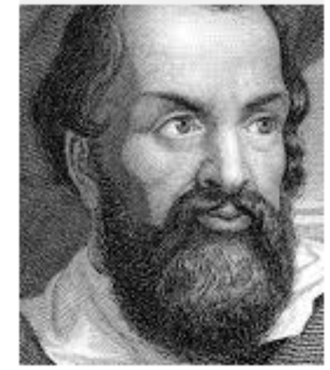


Pulse Waves (PW) trace "baseball diamonds" in space-time



BohrIt Web Simulation: 2 PW *ct vs x* Plot

Galileo Galilei



1564-1642

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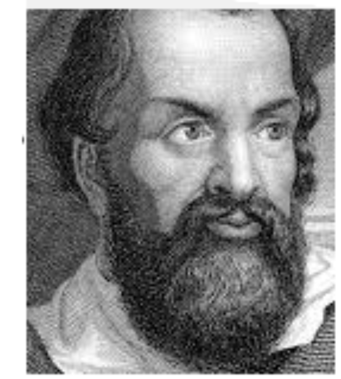
Galilean velocity



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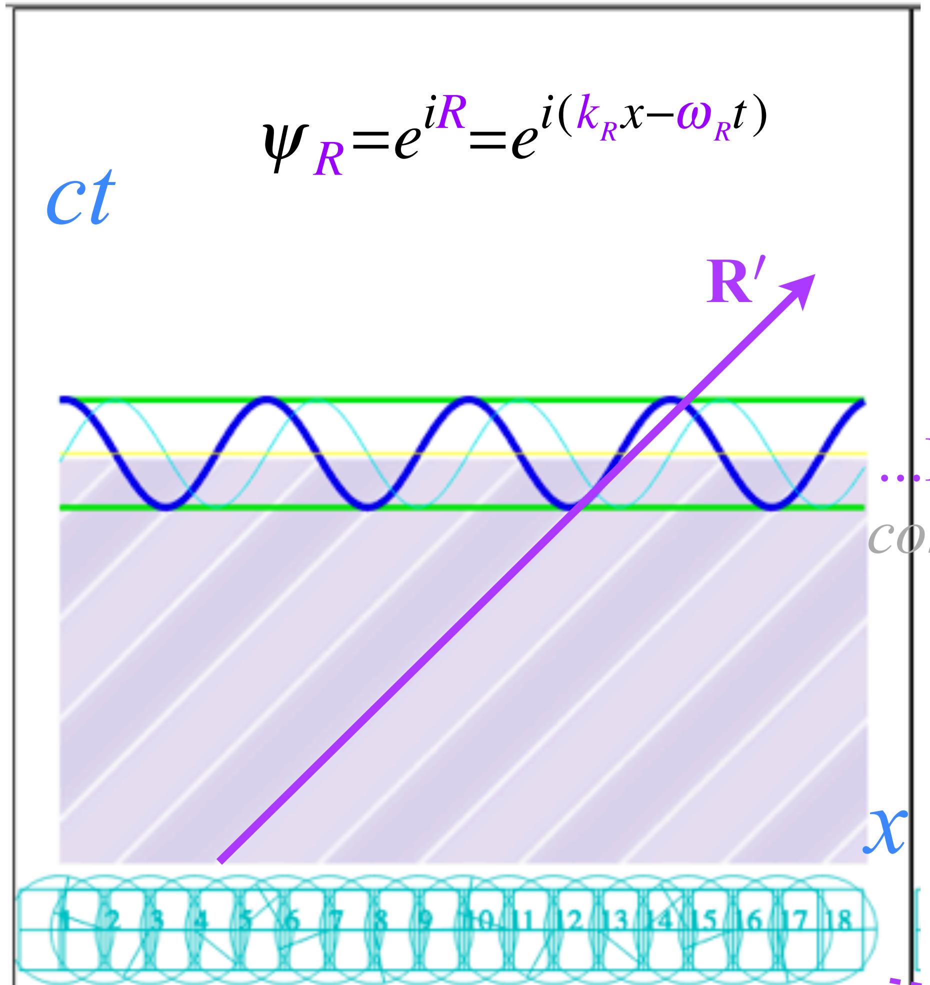
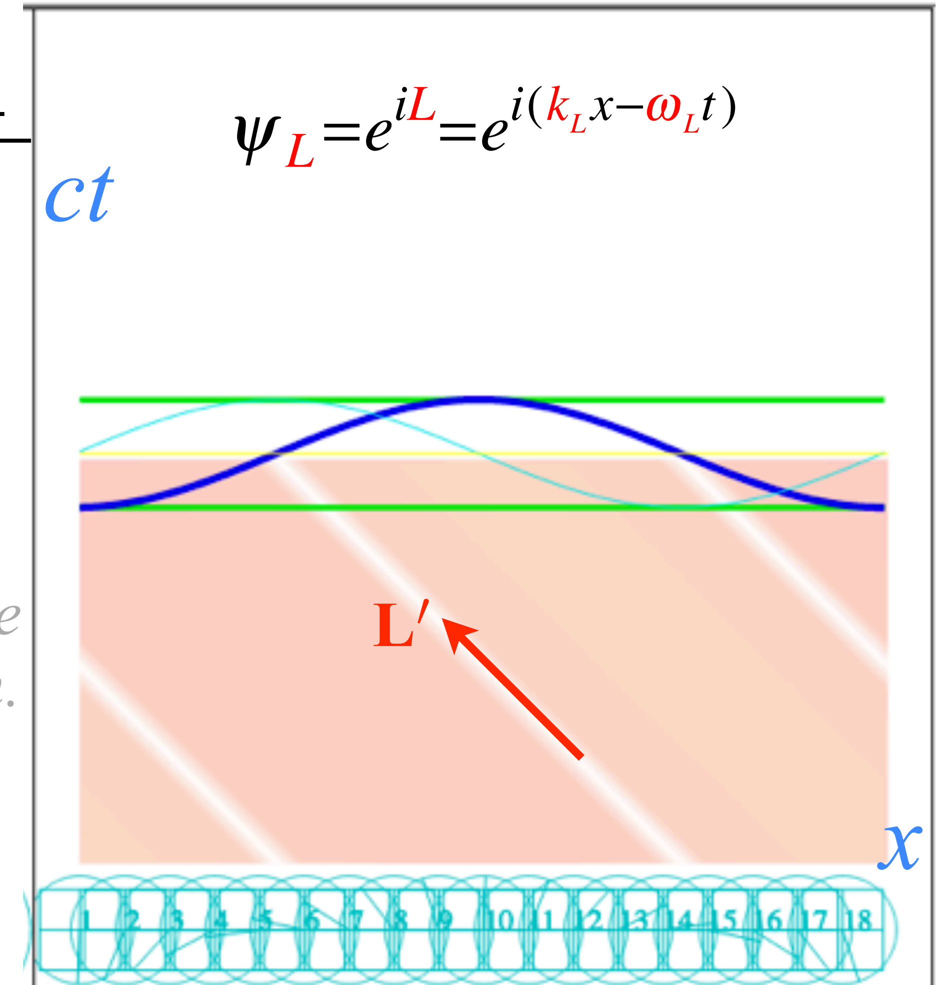
right-moving Doppler blue shifted wave

left-moving Doppler red shifted wave





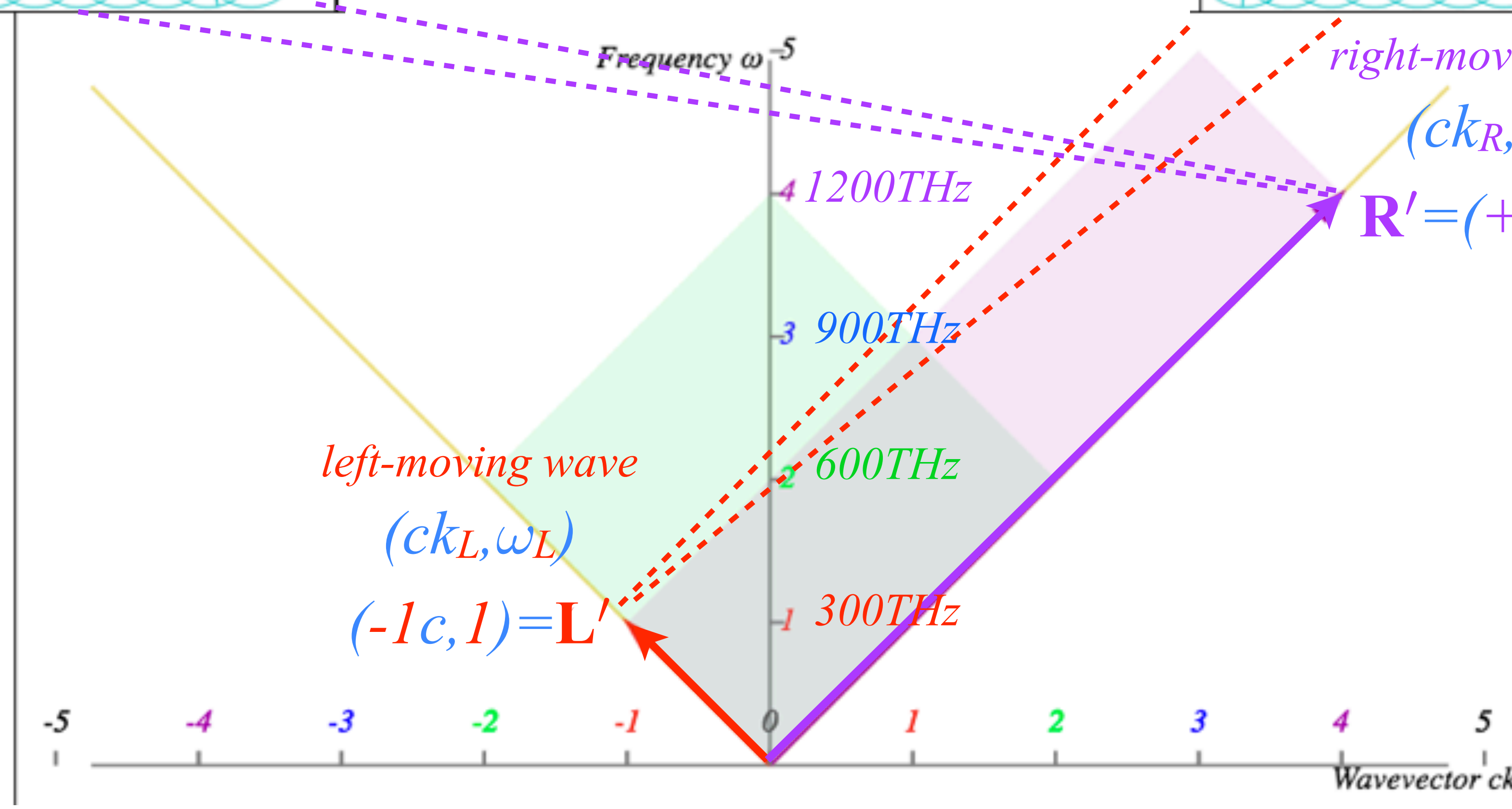
Rapidly moving Bob sees...

...Blue shifted *wave coming at him and...*

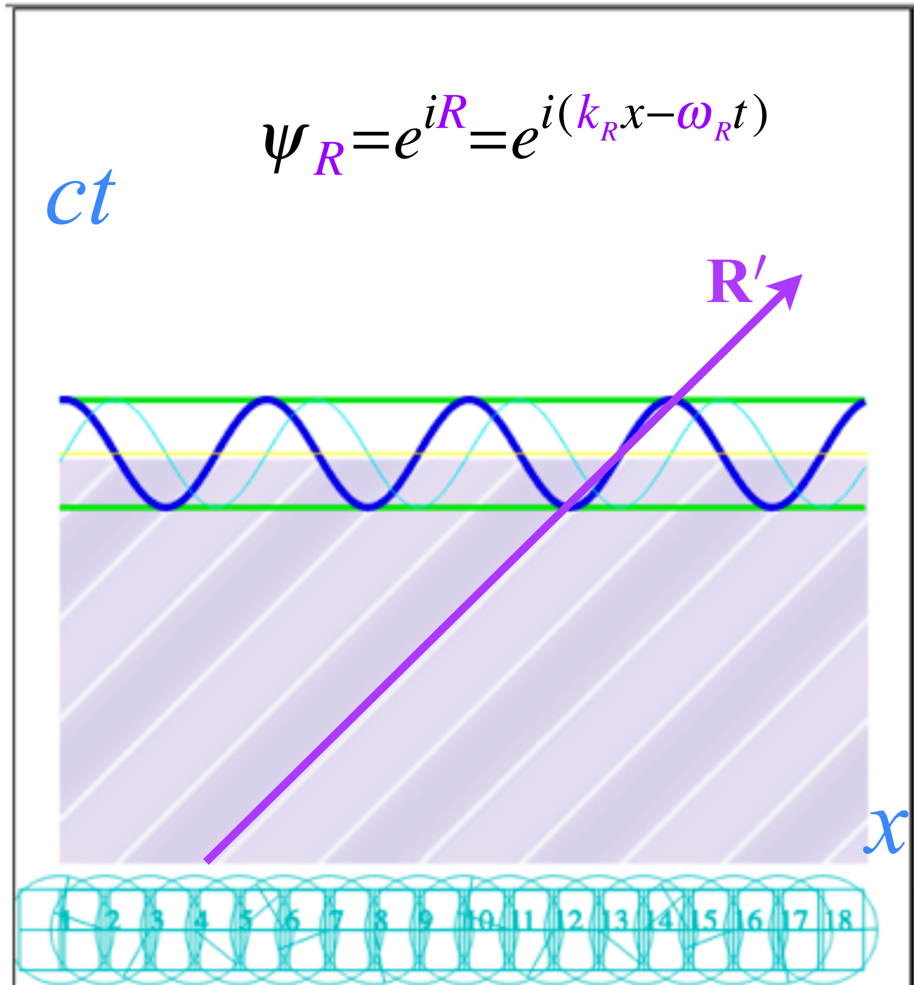
...Red shifted *wave behind him.*

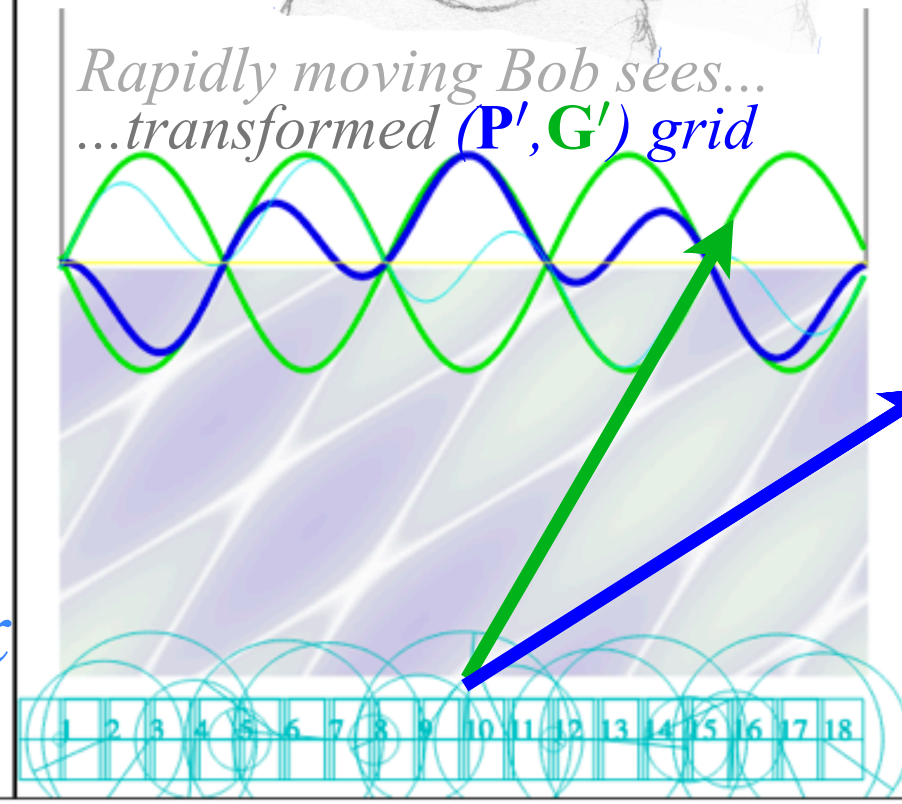
Web Simulation
1 CW *ct vs x* Plot
(ck = +4)

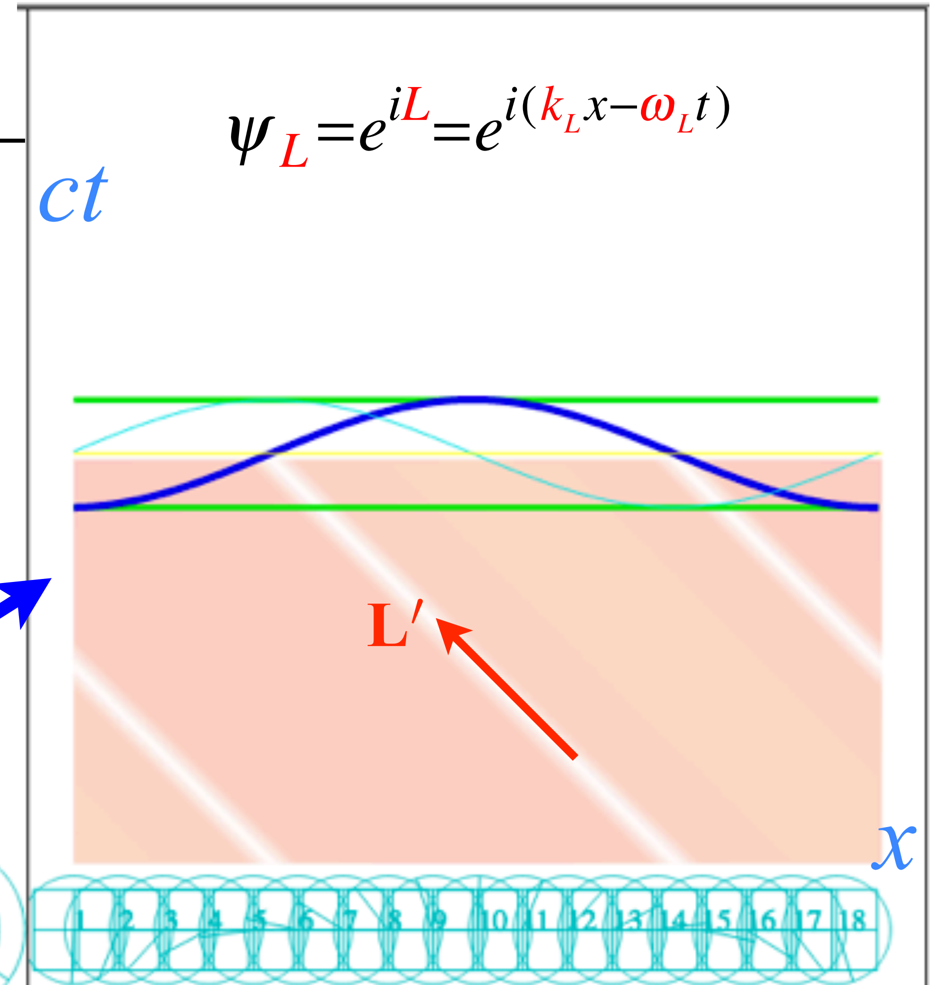
Web Simulation
1 CW *ct vs x* Plot
(ck = -1)



right-moving Doppler blue shifted wave — *left-moving* Doppler red shifted wave

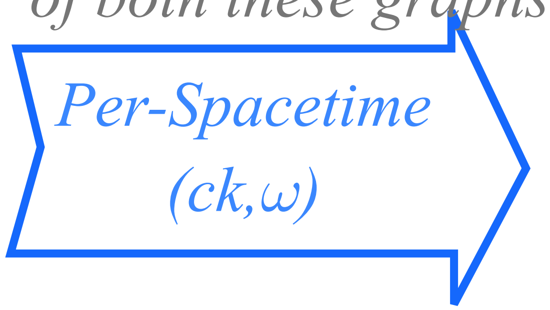


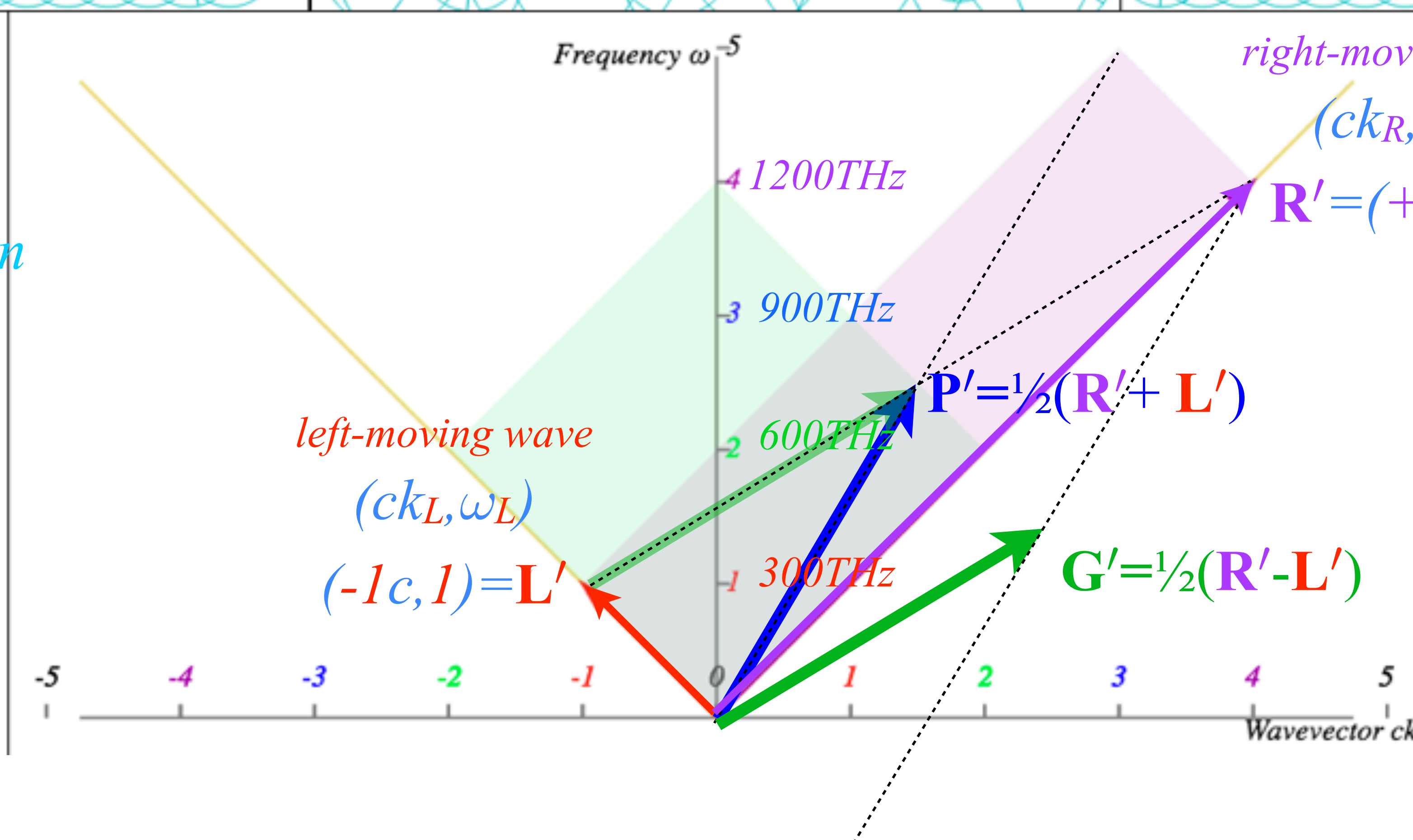




Web Simulation
1 CW *ct vs x* Plot
(ck = +4)

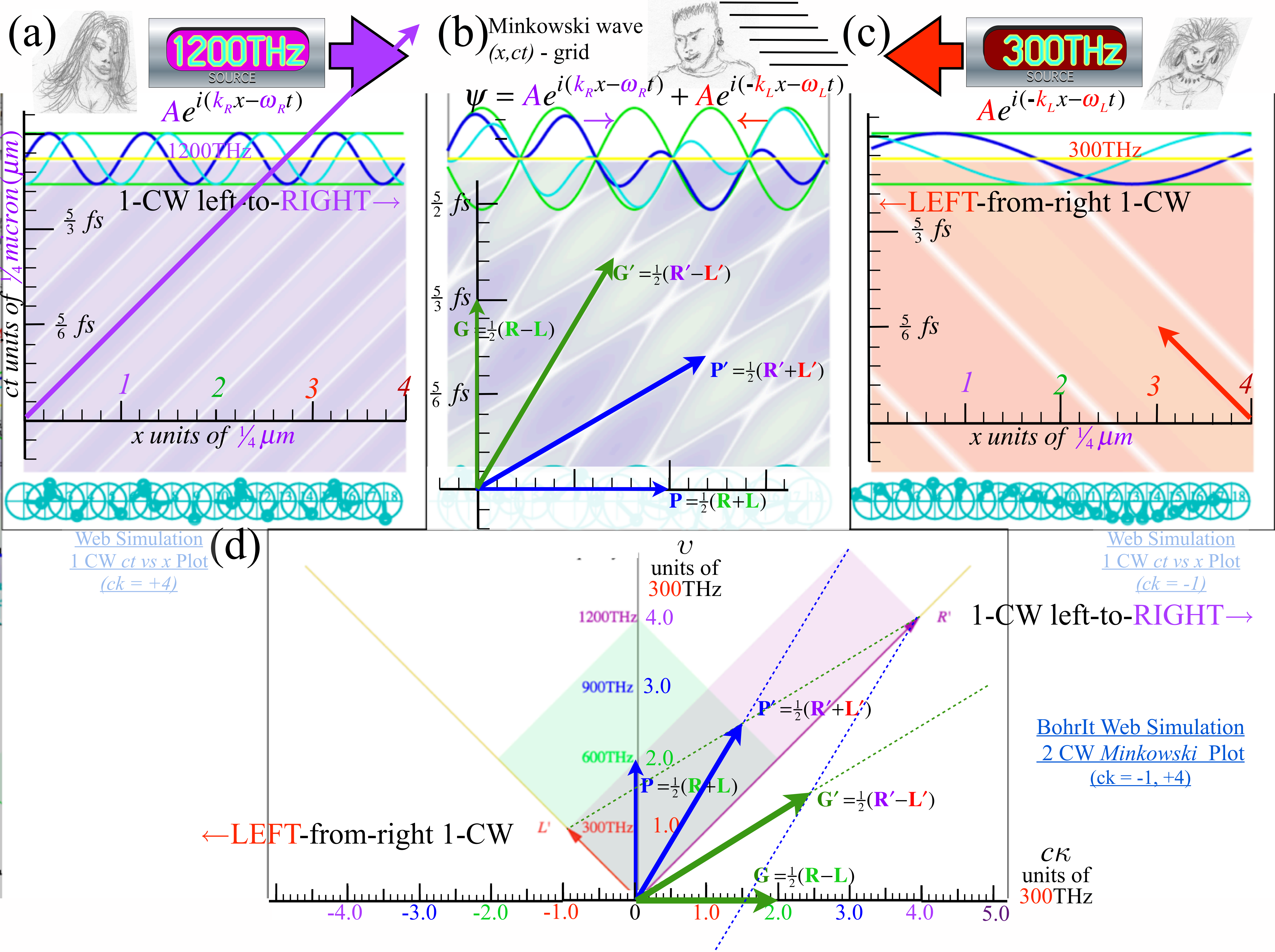
...Doppler shifts give
Lorentz transformation
of both these graphs

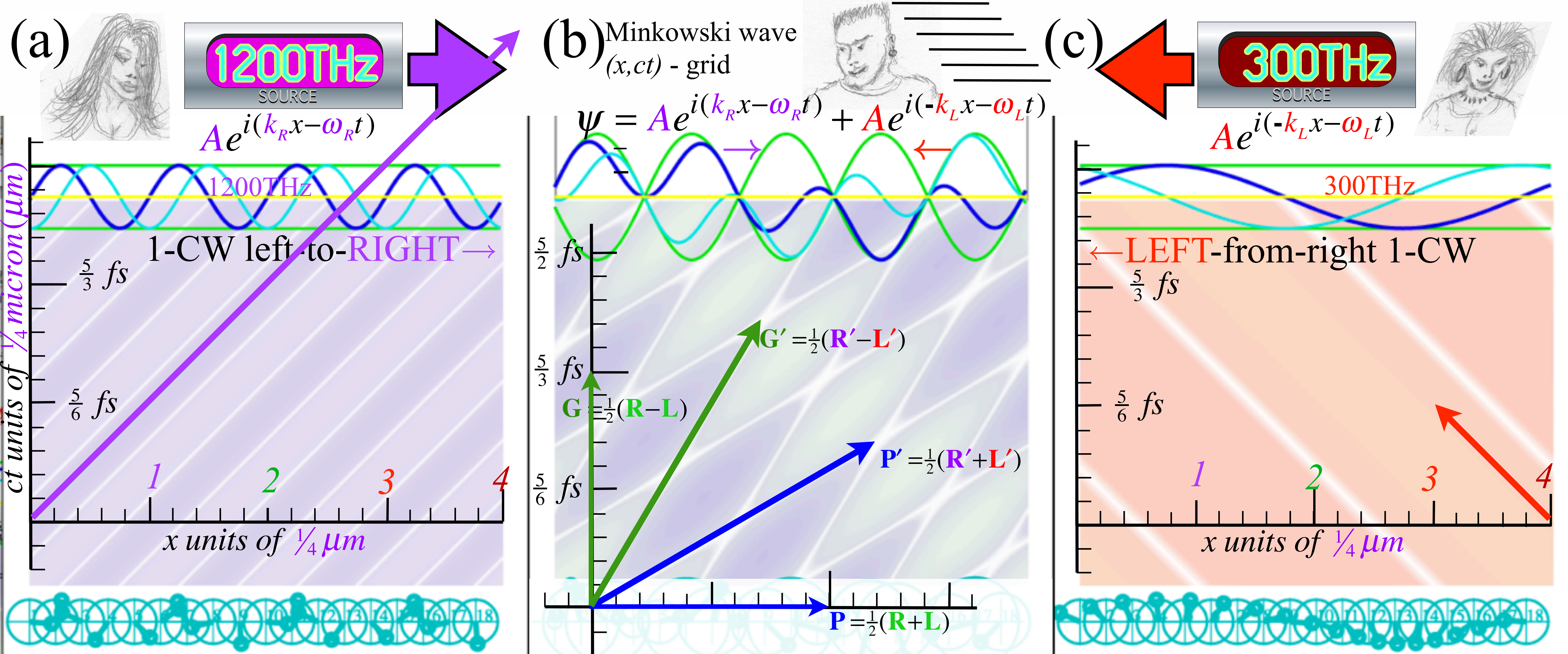




Web Simulation
1 CW *ct vs x* Plot
(ck = -1)

BohrIt Web Simulation
2 CW *Minkowski* Plot
(ck = -1, +4)


(a)
1200THz
SOURCE
$Ae^{i(k_R x-\omega_R t)}$
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron(μm)
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1 2 3 4
x units of ¼ μm
(b) Minkowski wave (x,ct) - grid
$\psi = Ae^{i(k_R x-\omega_R t)} + Ae^{i(-k_L x-\omega_L t)}$
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
$G' = \frac{1}{2}(R'-L')$
$G = \frac{1}{2}(R-L)$
$P' = \frac{1}{2}(R'+L')$
$P = \frac{1}{2}(R+L)$
(c)
300THz
SOURCE
$Ae^{i(-k_L x-\omega_L t)}$
300THz
←LEFT-from-right 1-CW
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1 2 3 4
x units of ¼ μm
Web Simulation
1 CW ct vs x Plot
(ck = +4)
(d)
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R'
L'
1-CW left-to-RIGHT→
$P' = \frac{1}{2}(R'+L')$
$P = \frac{1}{2}(R+L)$
$G' = \frac{1}{2}(R'-L')$
$G = \frac{1}{2}(R-L)$
←LEFT-from-right 1-CW
cκ units of 300THz
-4.0 -3.0 -2.0 -1.0 0 1.0 2.0 3.0 4.0 5.0
Web Simulation
1 CW ct vs x Plot
(ck = -1)
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)



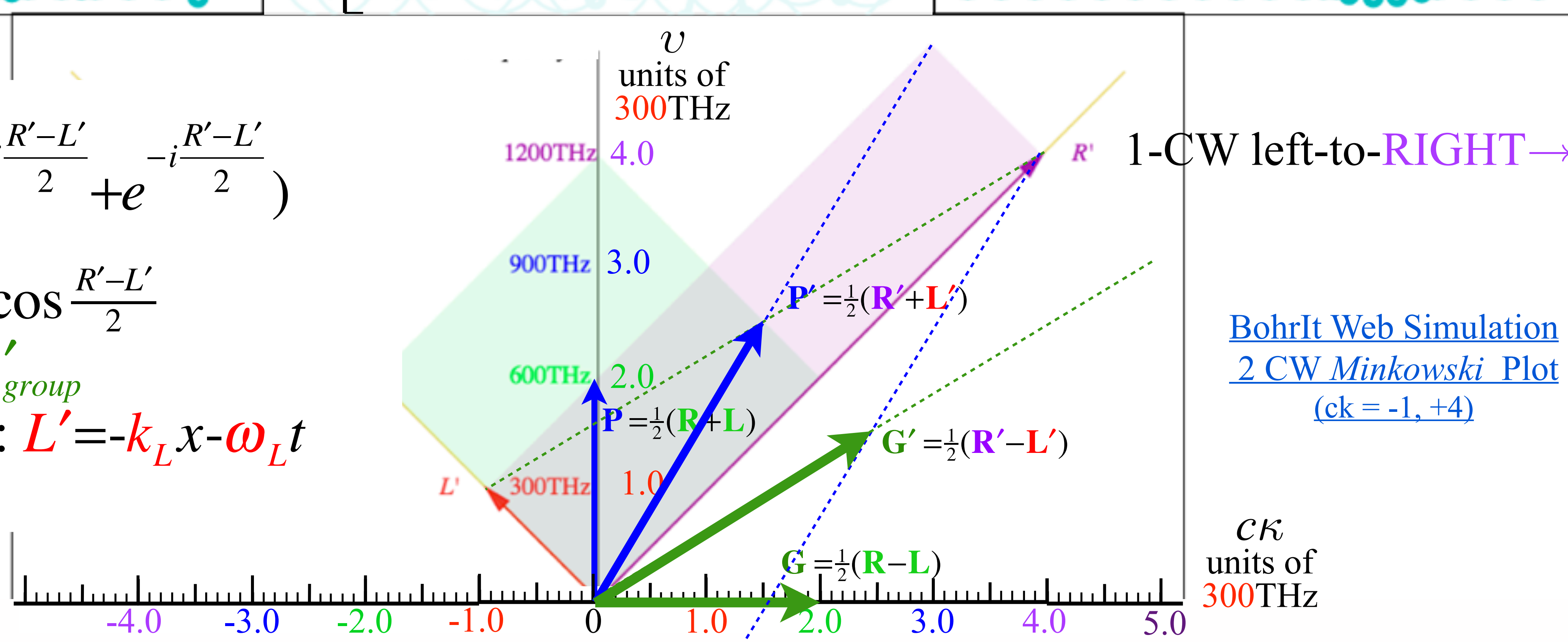
(d)

$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$

$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$

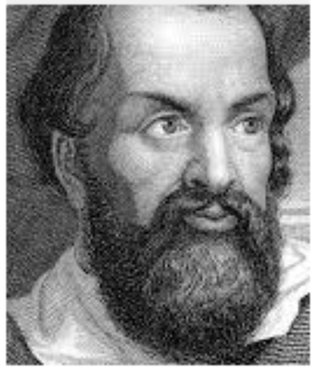
$$=\psi'_{phase}\psi'_{group}$$

$R'=k_R x-\omega_R t$ and: $L'=-k_L x-\omega_L t$



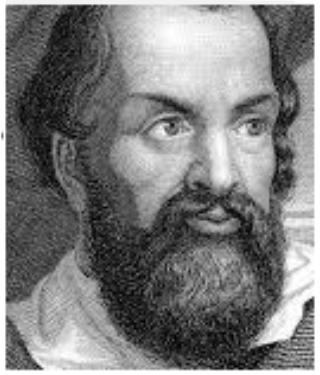
BohrIt Web Simulation
2 CW *Minkowski* Plot
(ck = -1, +4)

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
Galilean velocity



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adds just like
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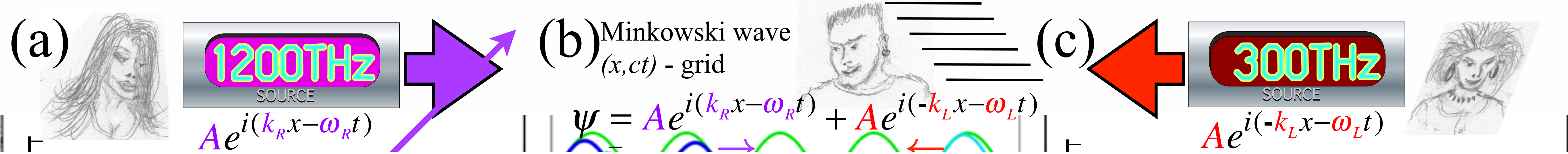


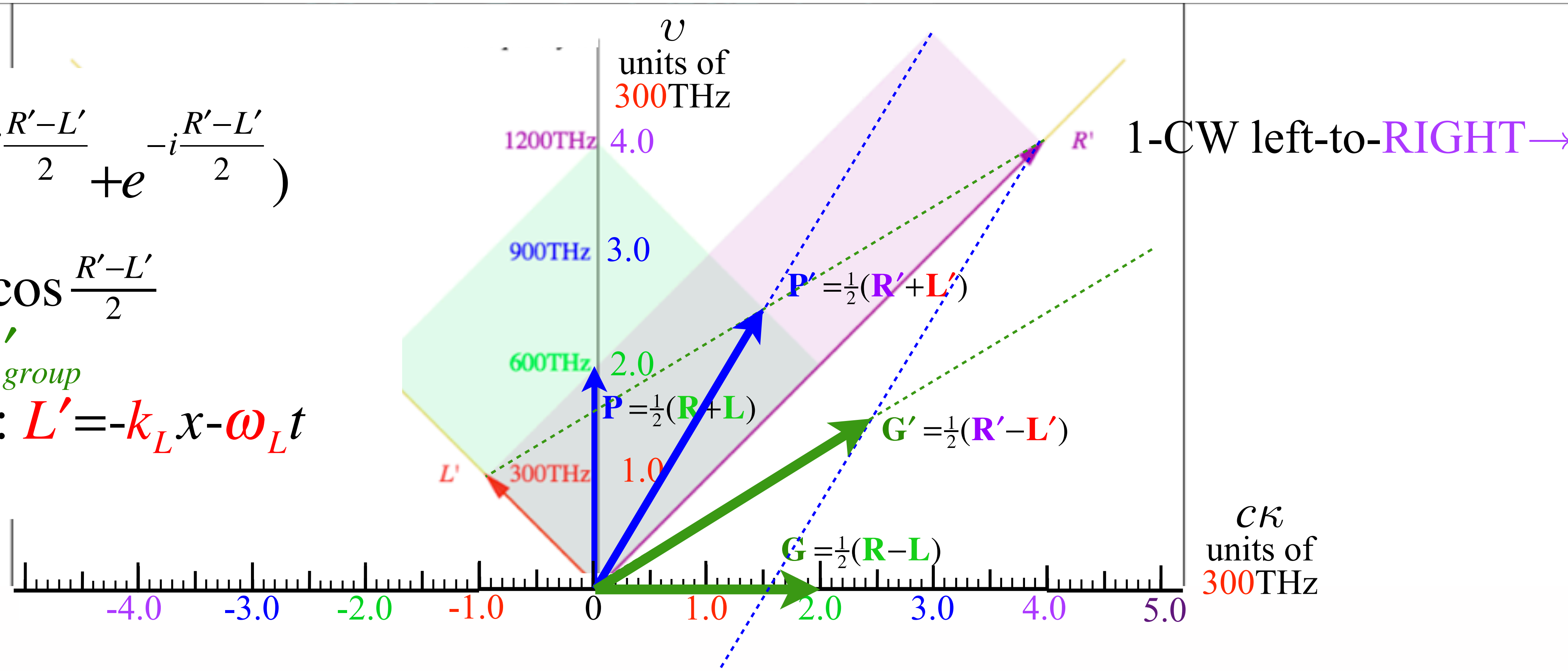
$$\mathbf{P}'=\begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase}\end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'+\mathbf{L}')=\upsilon_A\begin{pmatrix}\frac{1}{2}(e^{\rho}+e^{-\rho})\\ \frac{1}{2}(e^{\rho}-e^{-\rho})\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{5}{2}\\ \frac{3}{2}\end{pmatrix}_{Bob's\ View} \text{ or: } \upsilon_A\begin{pmatrix}1\\ 0\end{pmatrix}_{Alice's\ View}$$

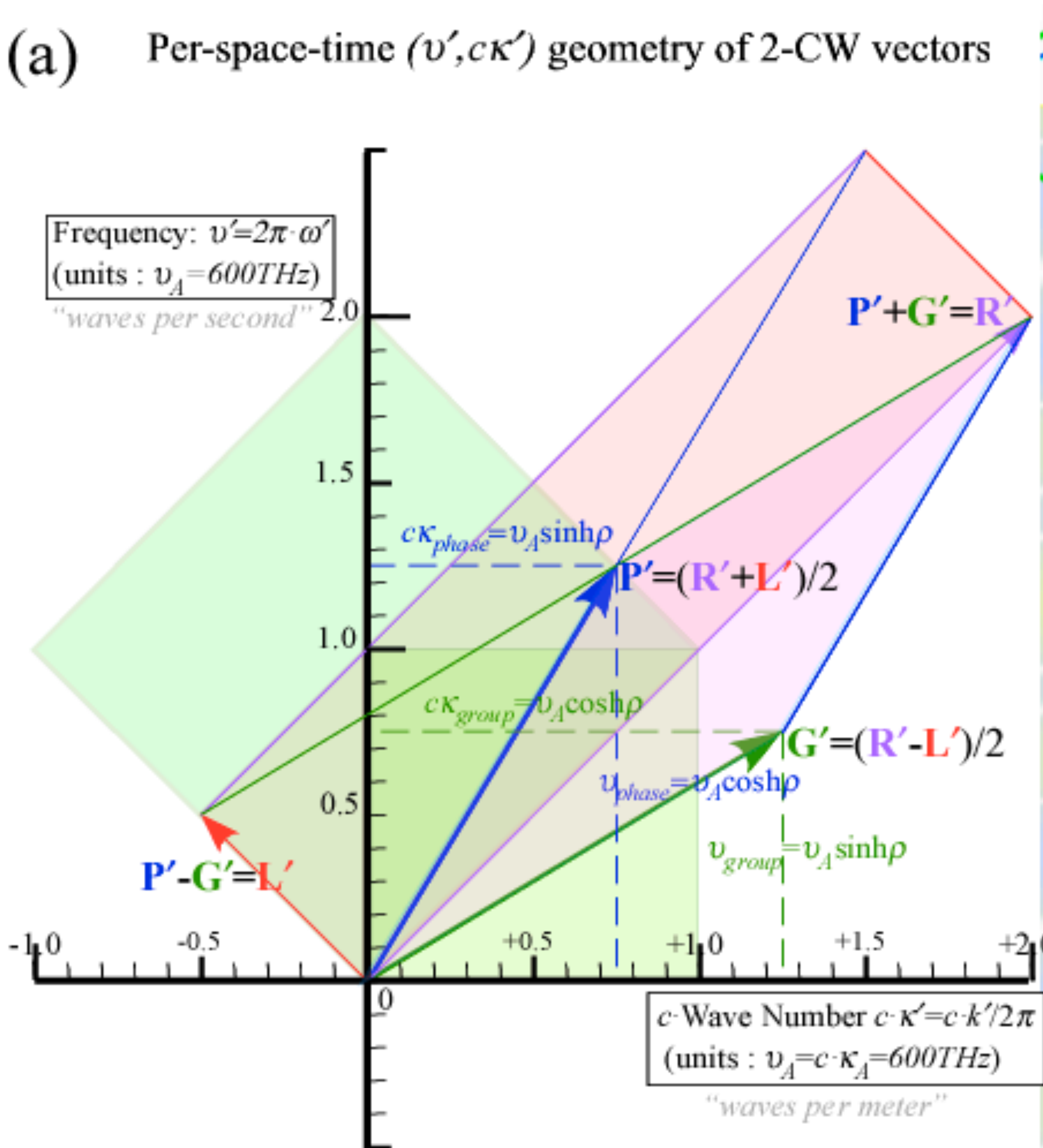
$$\mathbf{G}'=\begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group}\end{pmatrix}=\tfrac{1}{2}(\mathbf{R}'-\mathbf{L}')=\upsilon_A\begin{pmatrix}\frac{1}{2}(e^{\rho}-e^{-\rho})\\ \frac{1}{2}(e^{\rho}+e^{-\rho})\end{pmatrix}=\upsilon_A\begin{pmatrix}\sinh\rho\\ \cosh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{3}{2}\\ \frac{5}{2}\end{pmatrix}_{Bob's\ View} \text{ or: } \upsilon_A\begin{pmatrix}0\\ 1\end{pmatrix}_{Alice's\ View}$$

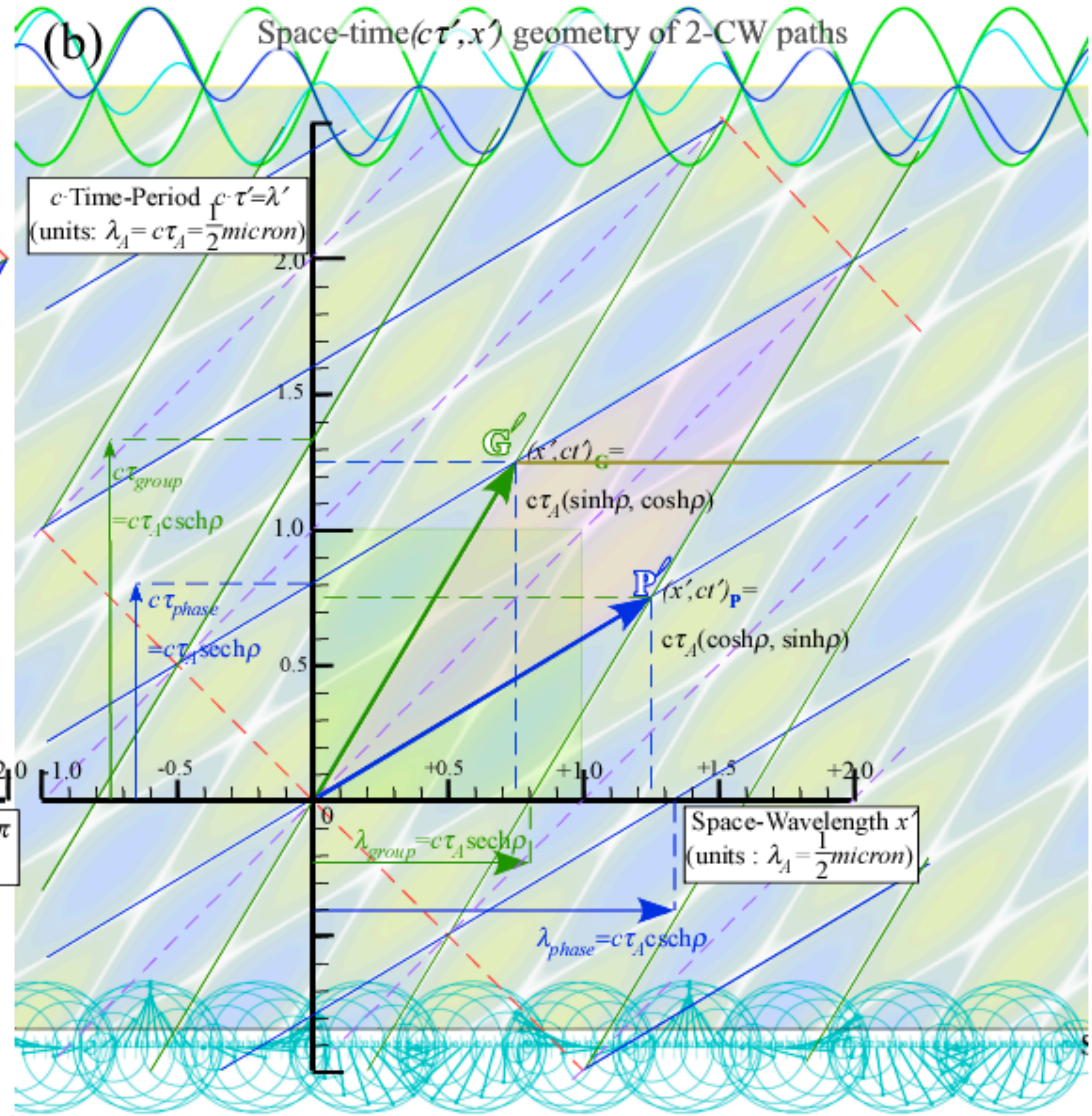
(d)

$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$
$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$
$$=\psi'_{phase}\psi'_{group}$$

$R'=k_R x-\omega_R t$ and: $L'=-k_L x-\omega_L t$

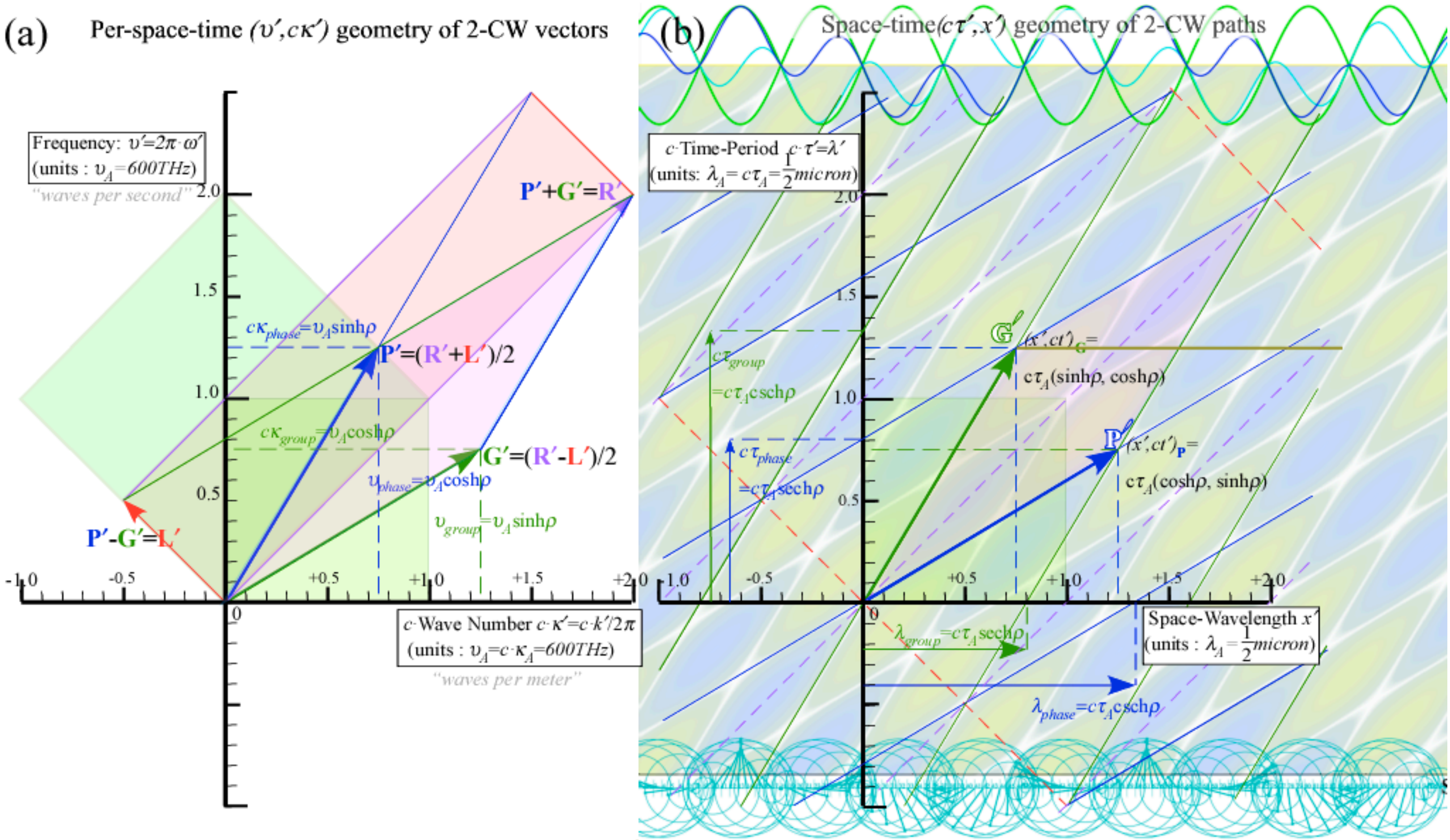






[RelaWavity Web Simulation](#)
[Relating *Per-space-time* and *Space-time*](#)

[BohrIt Web Simulation](#)
[2 CW Minkowski Plot (ck = -1, +4)](#)
[with Phase and Group relations](#)

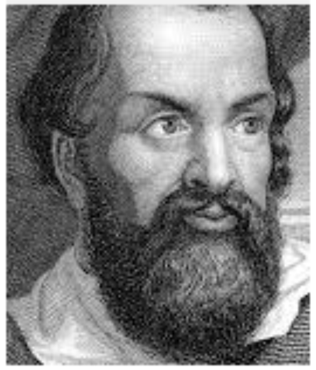


$$\frac{V^{group}}{c}=\frac{\upsilon'_{group}}{c\kappa'_{group}}=\frac{\sinh\rho}{\cosh\rho}=\tanh\rho=\frac{\frac{3}{2}}{\frac{5}{2}}=\frac{3}{5}\equiv\frac{u}{c}\equiv\beta$$

BohrIt Web Simulation
2 CW Minkowski Plot (ck = -1, +4)
with Phase and Group relations

RelaWavity Web Simulation
Relating *Per-space-time* and *Space-time*

Galileo Galilei



1564-1642

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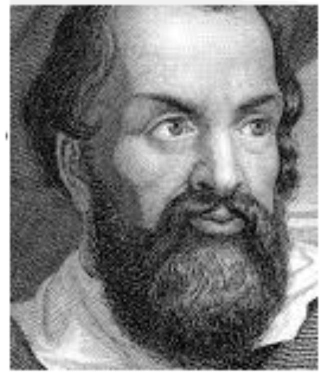

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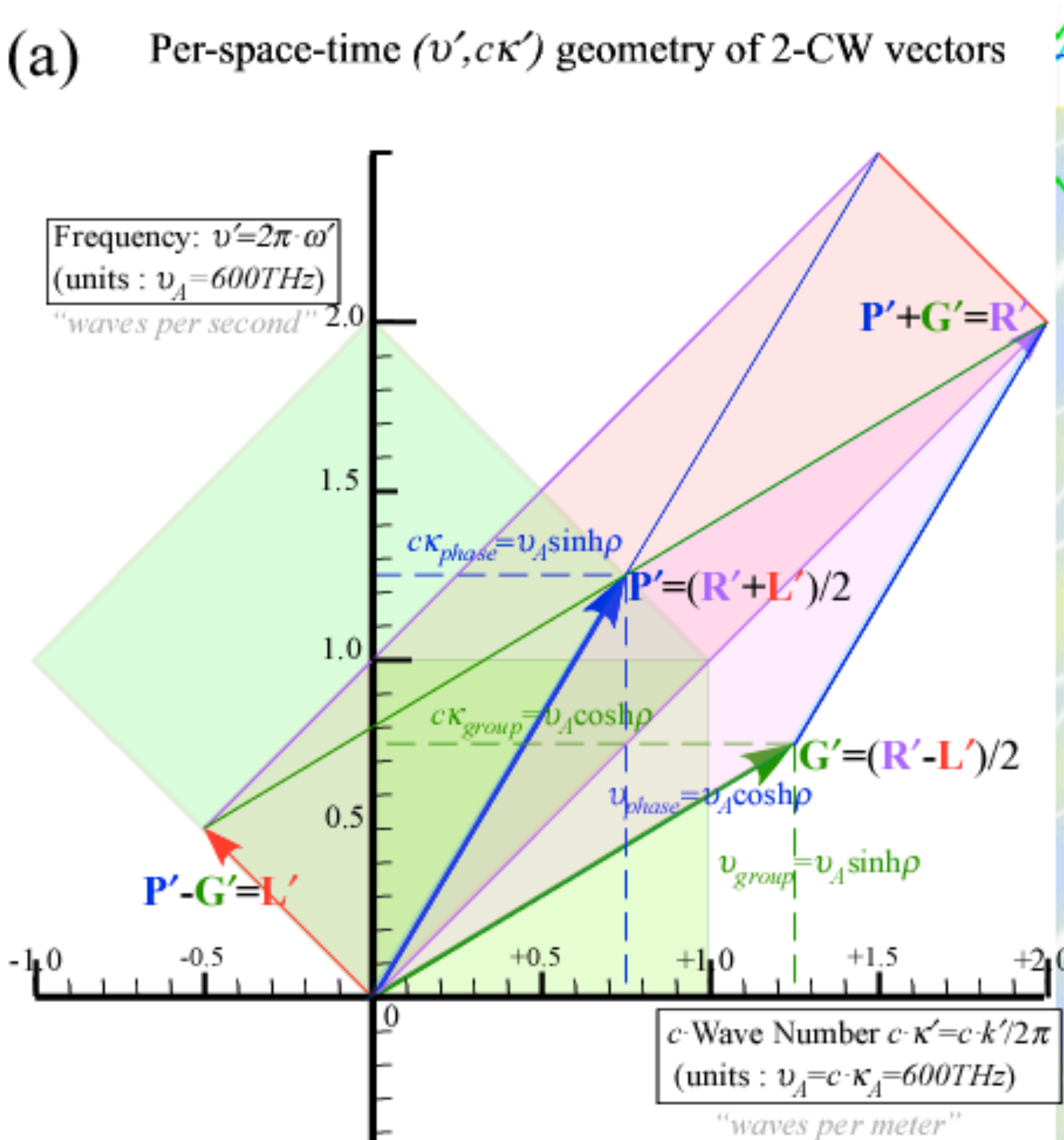
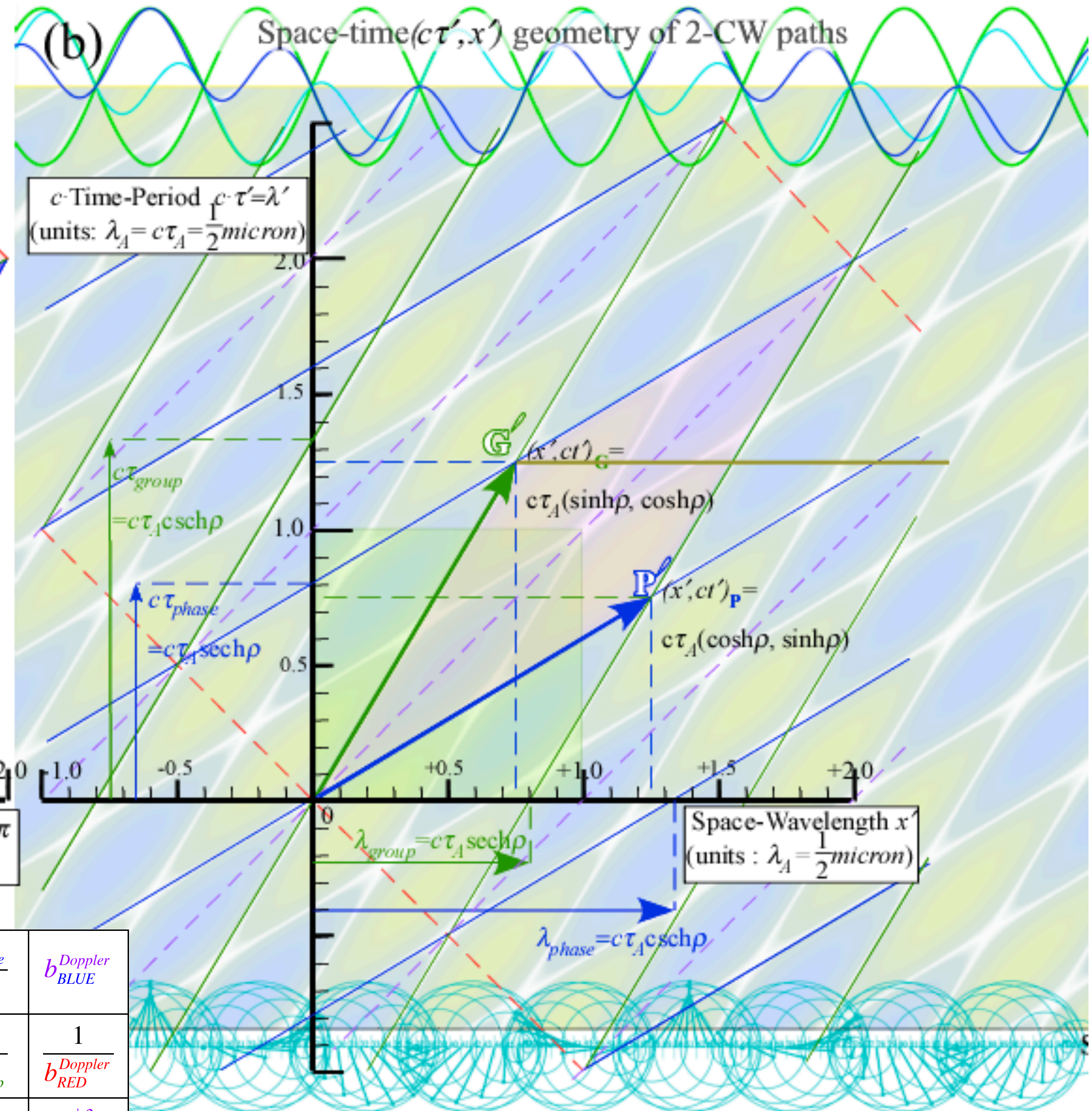




phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

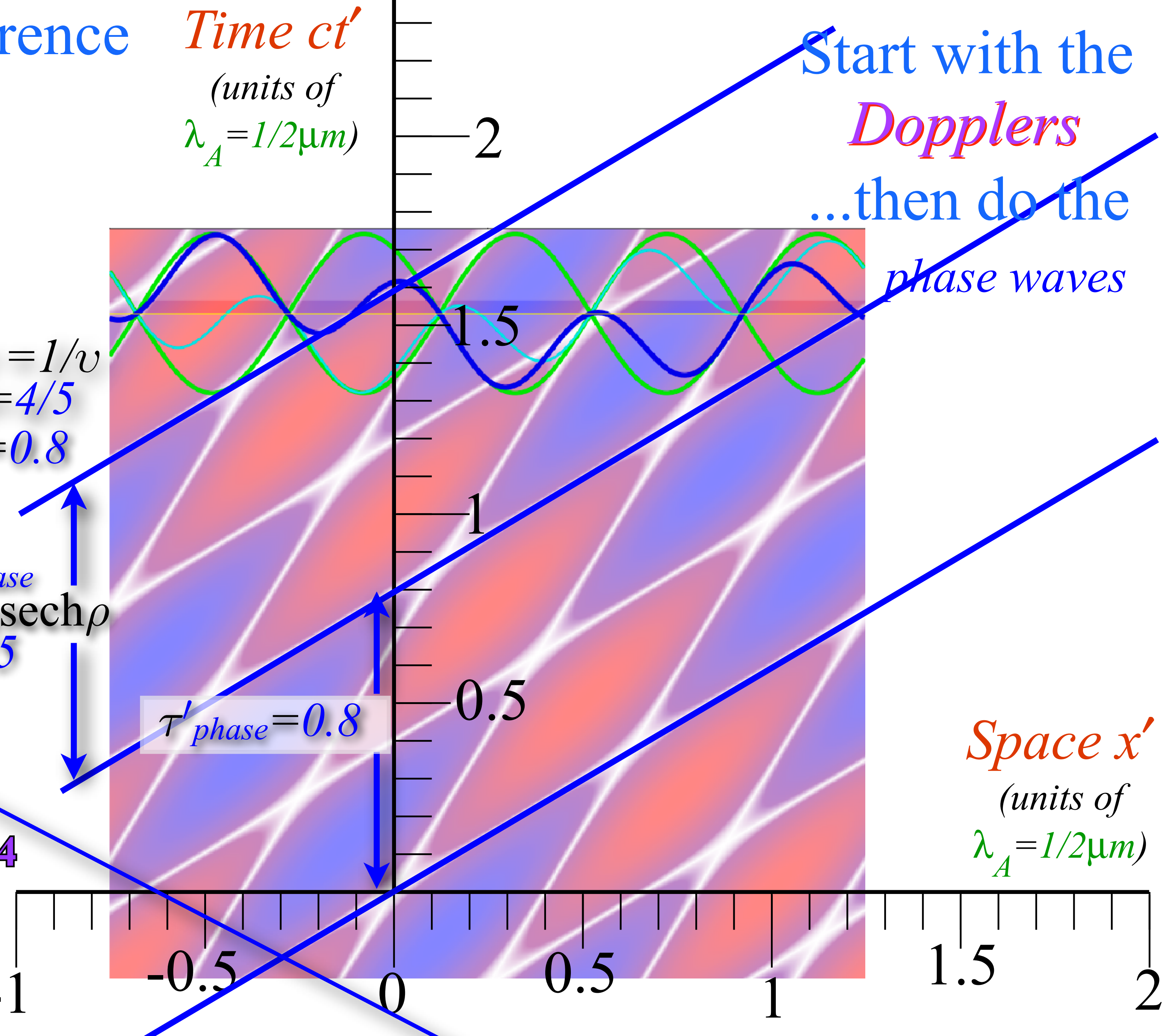
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

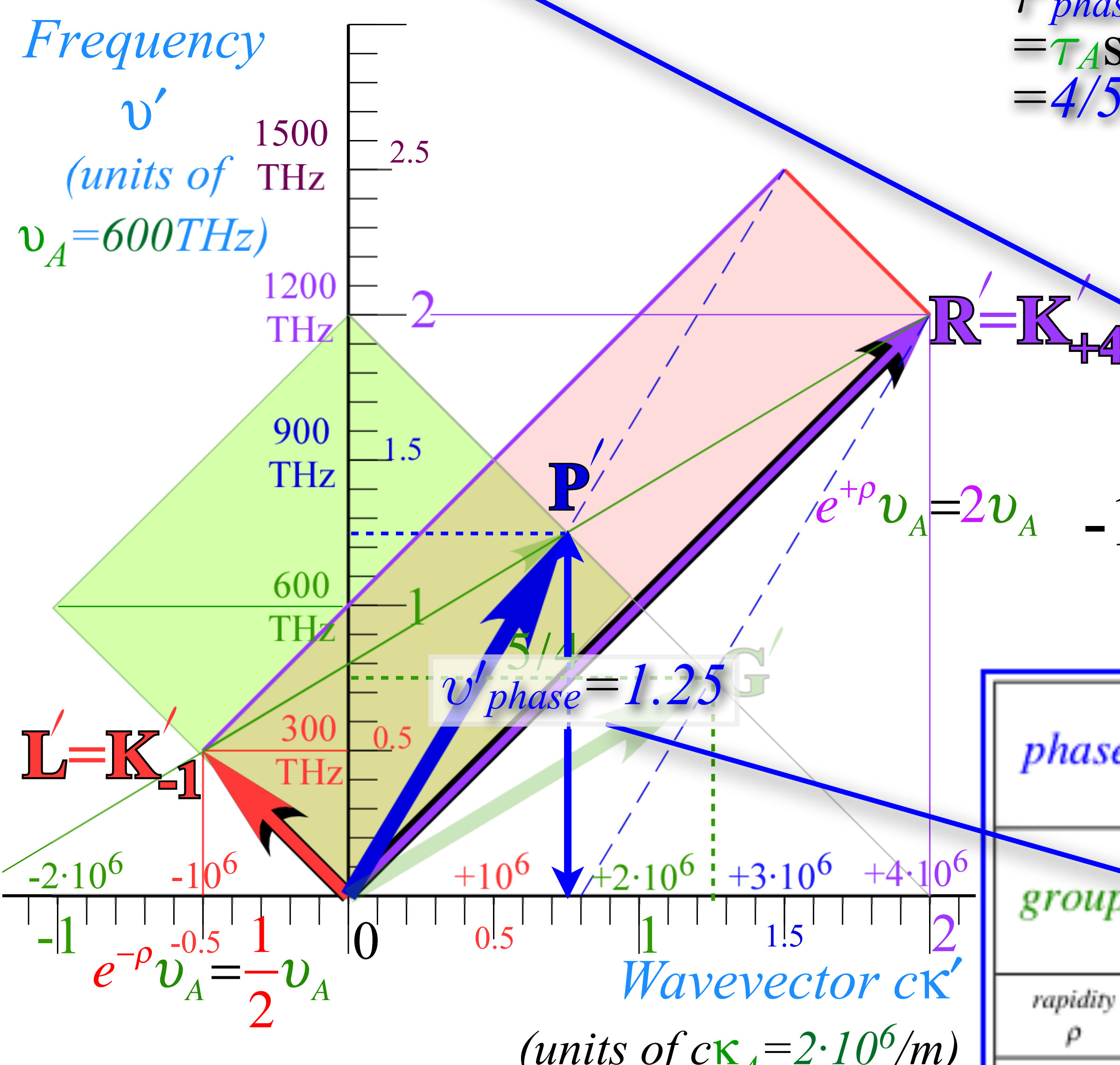
Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$: $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* ...then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

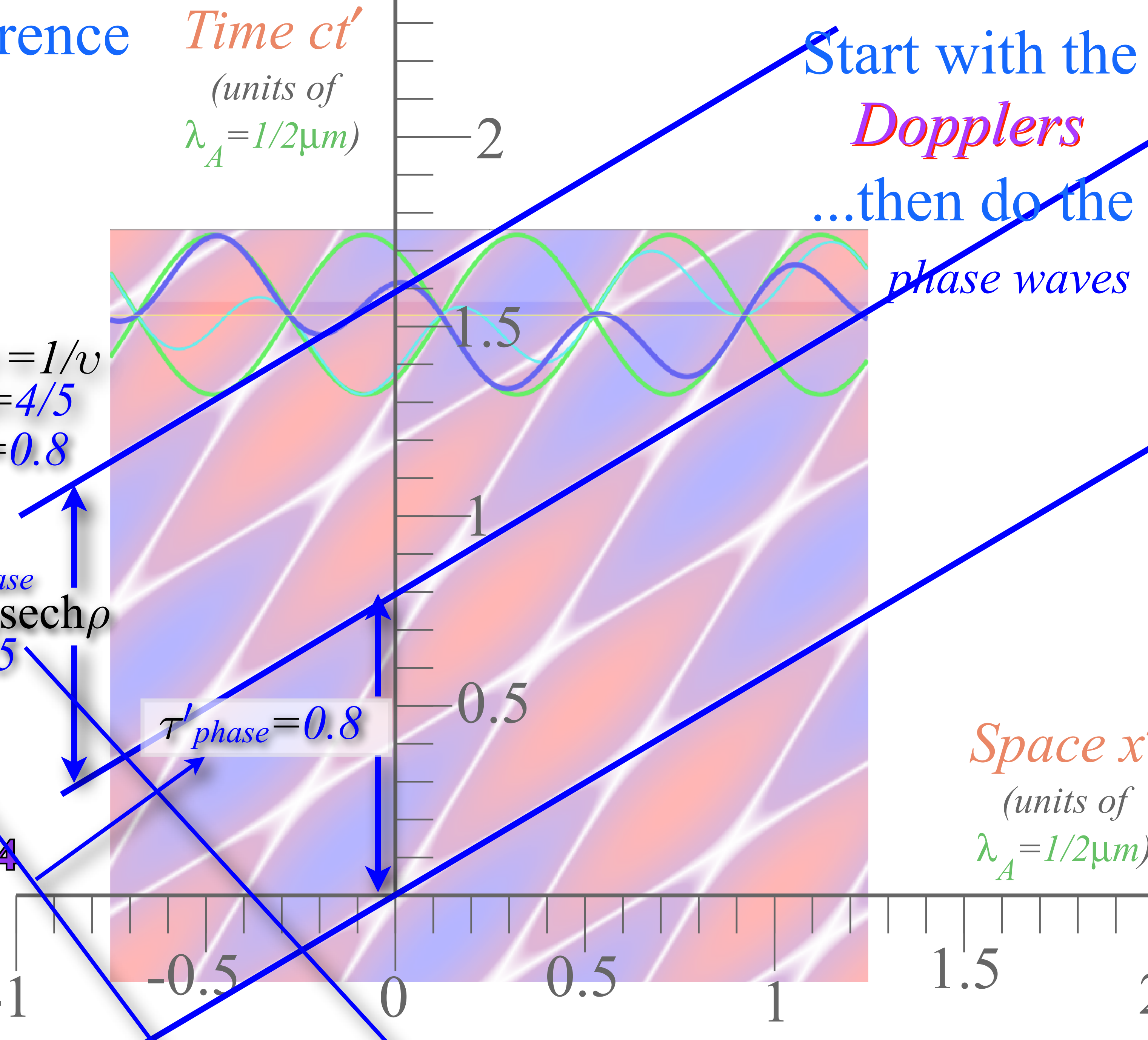
The 16 dimensions of 2CW interference

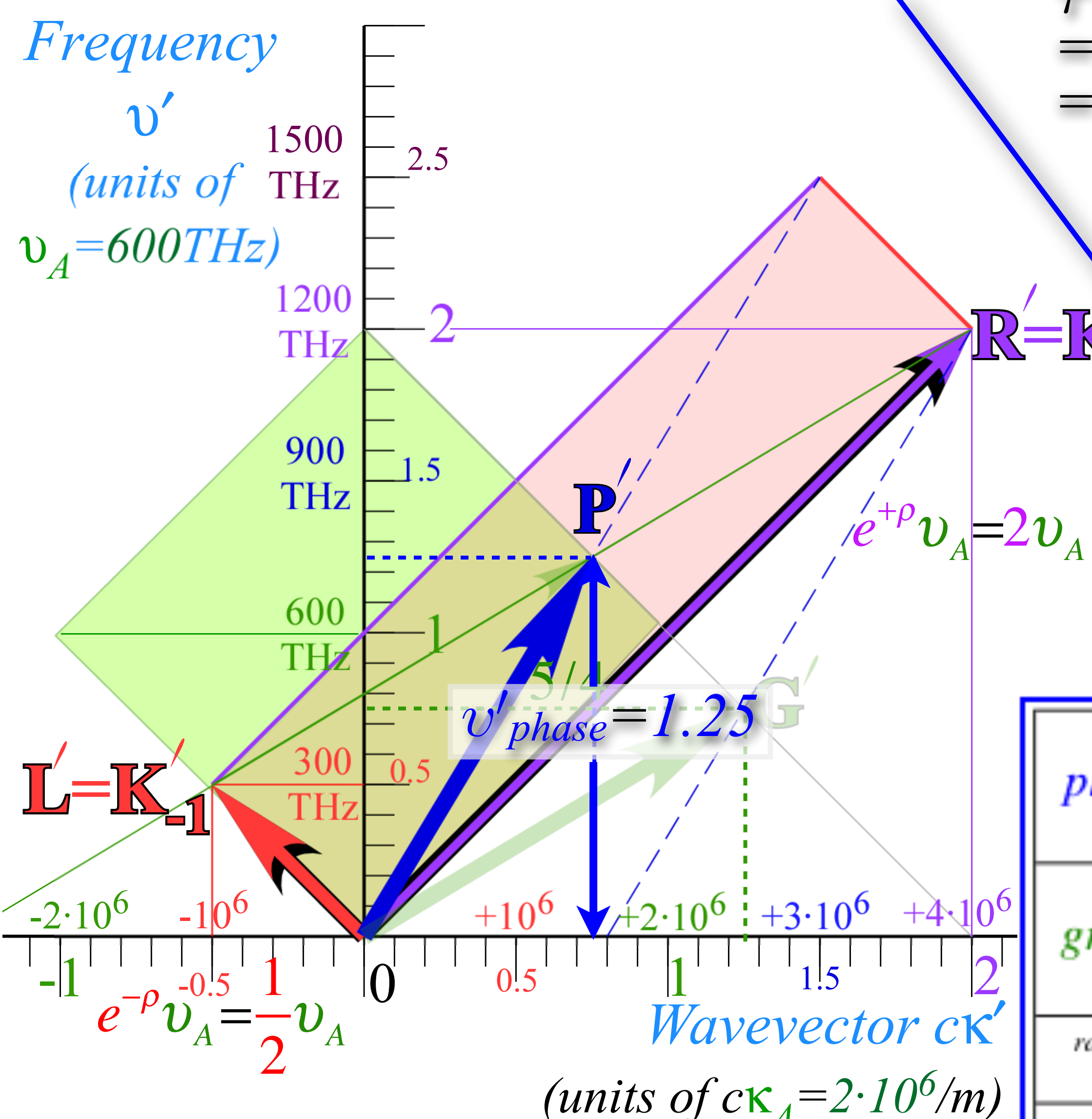
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$
$\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

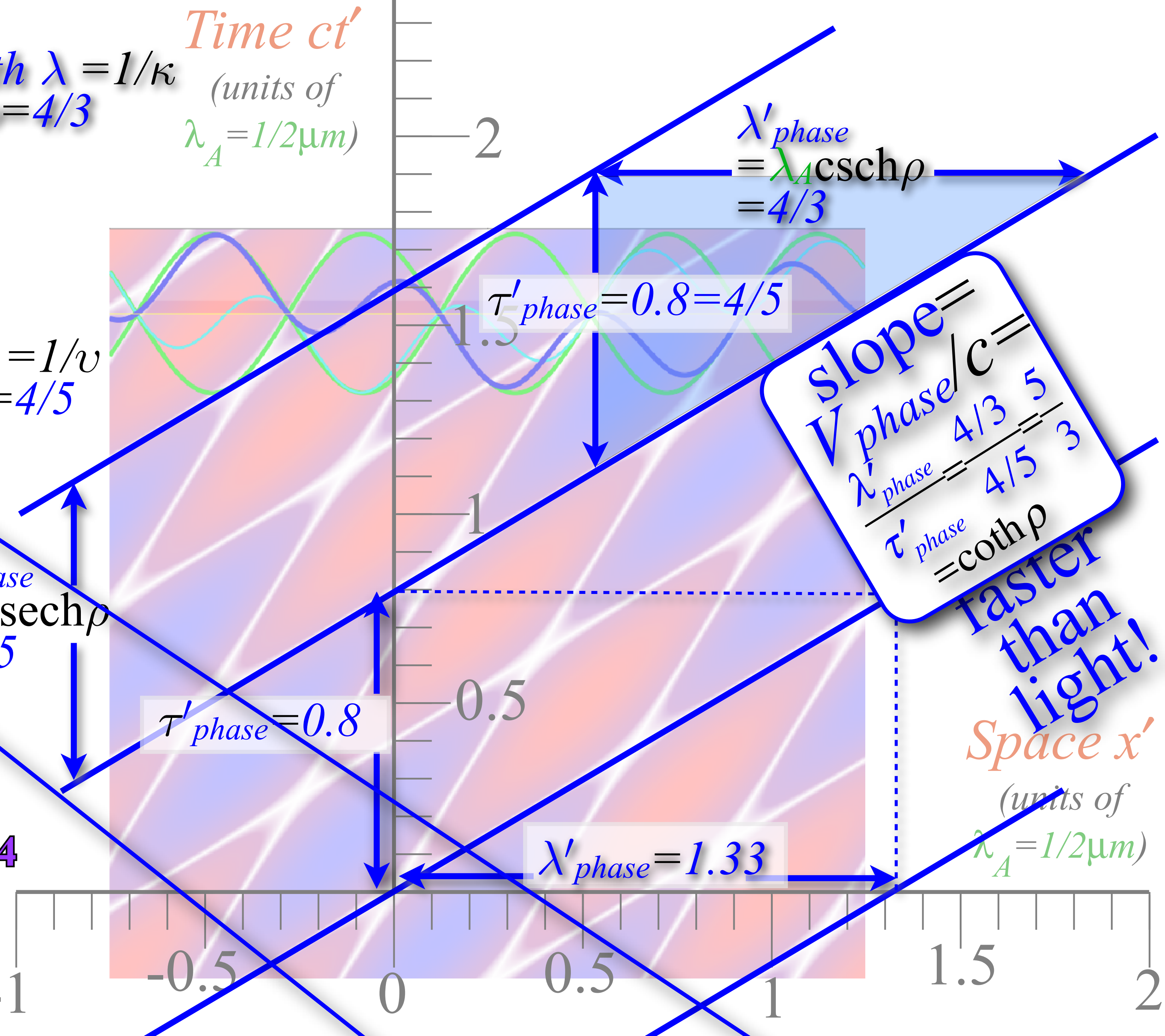
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \mathrm{csch}\rho = 4/3$

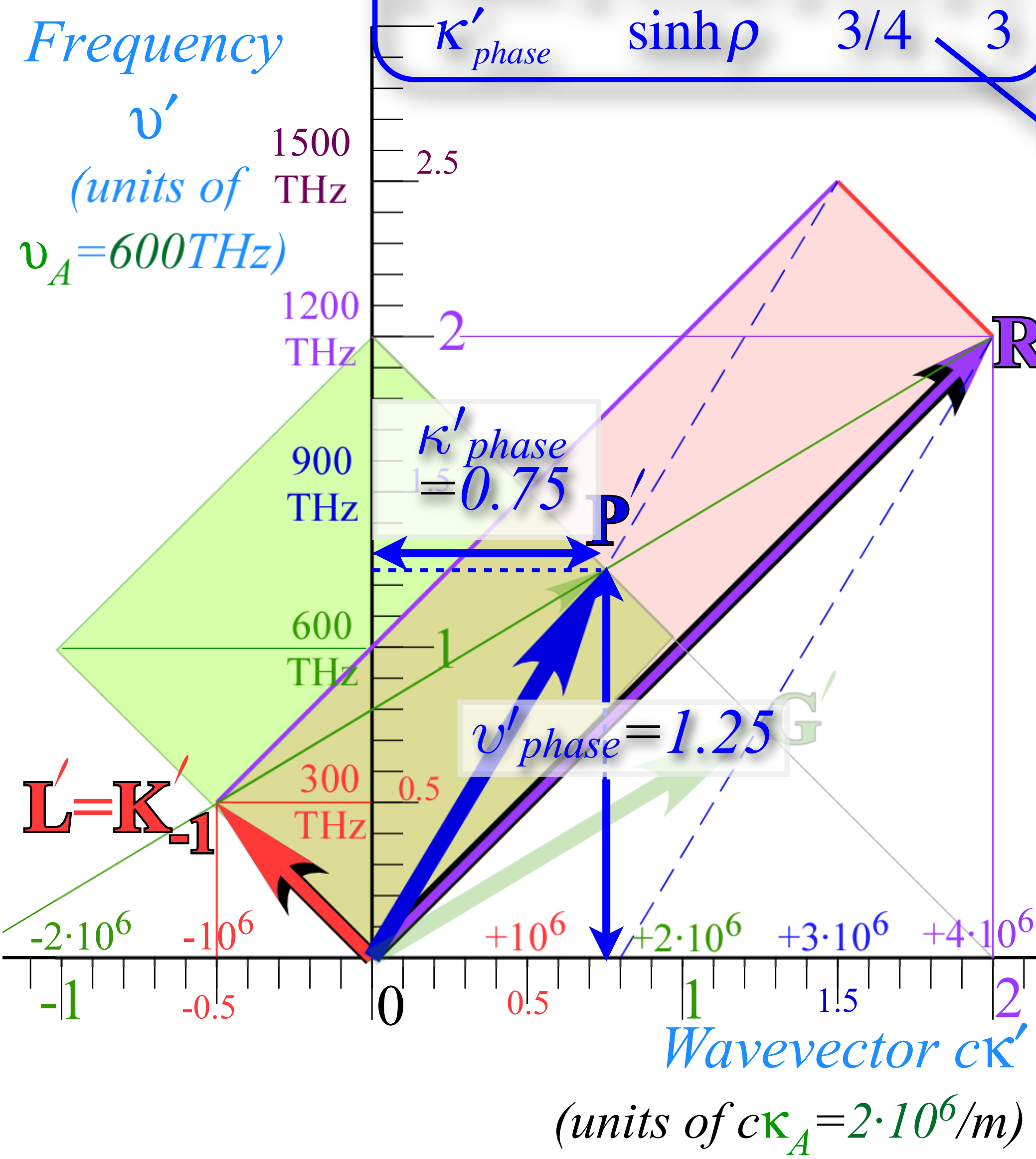
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \mathrm{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \text{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

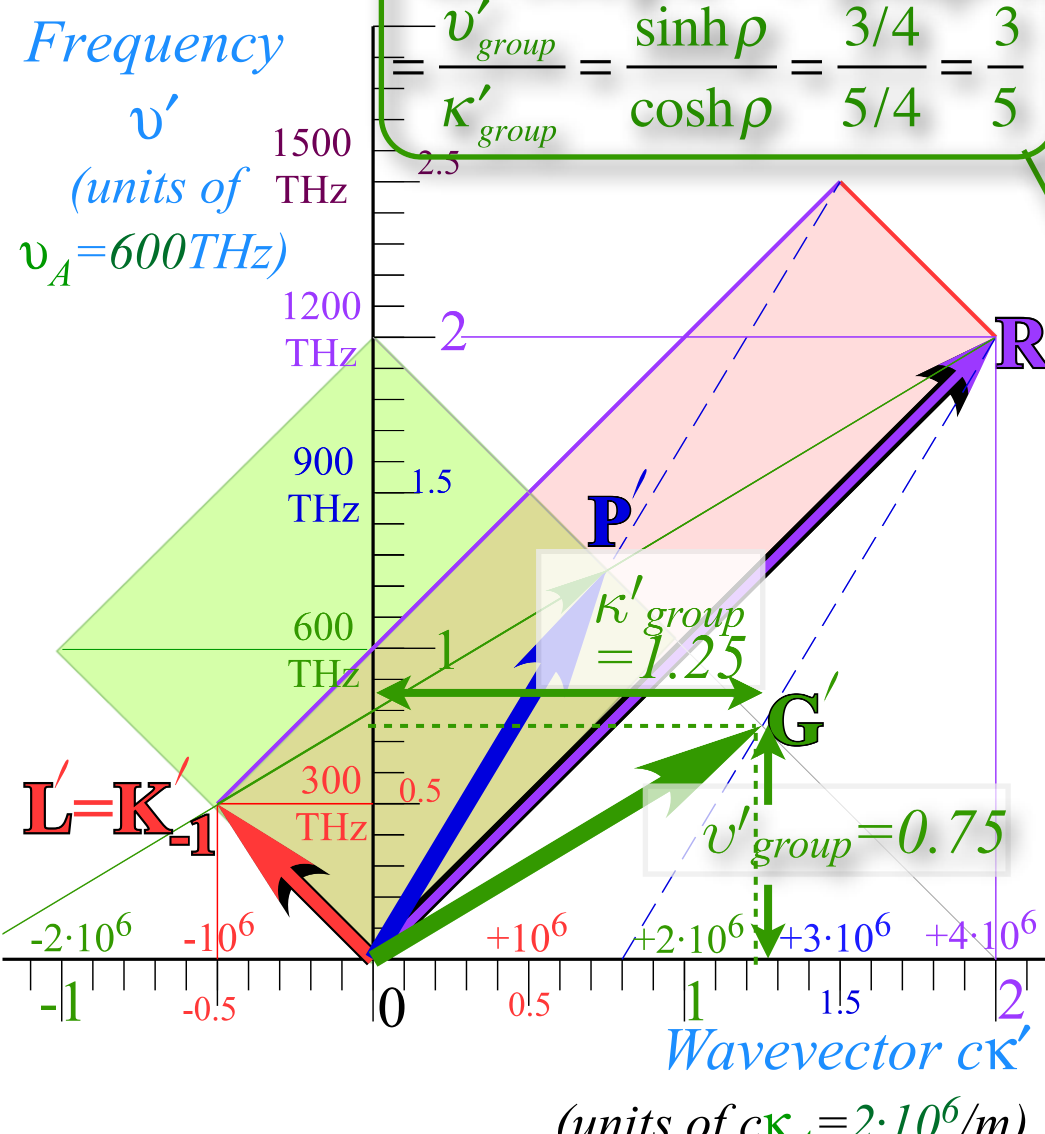
flips to

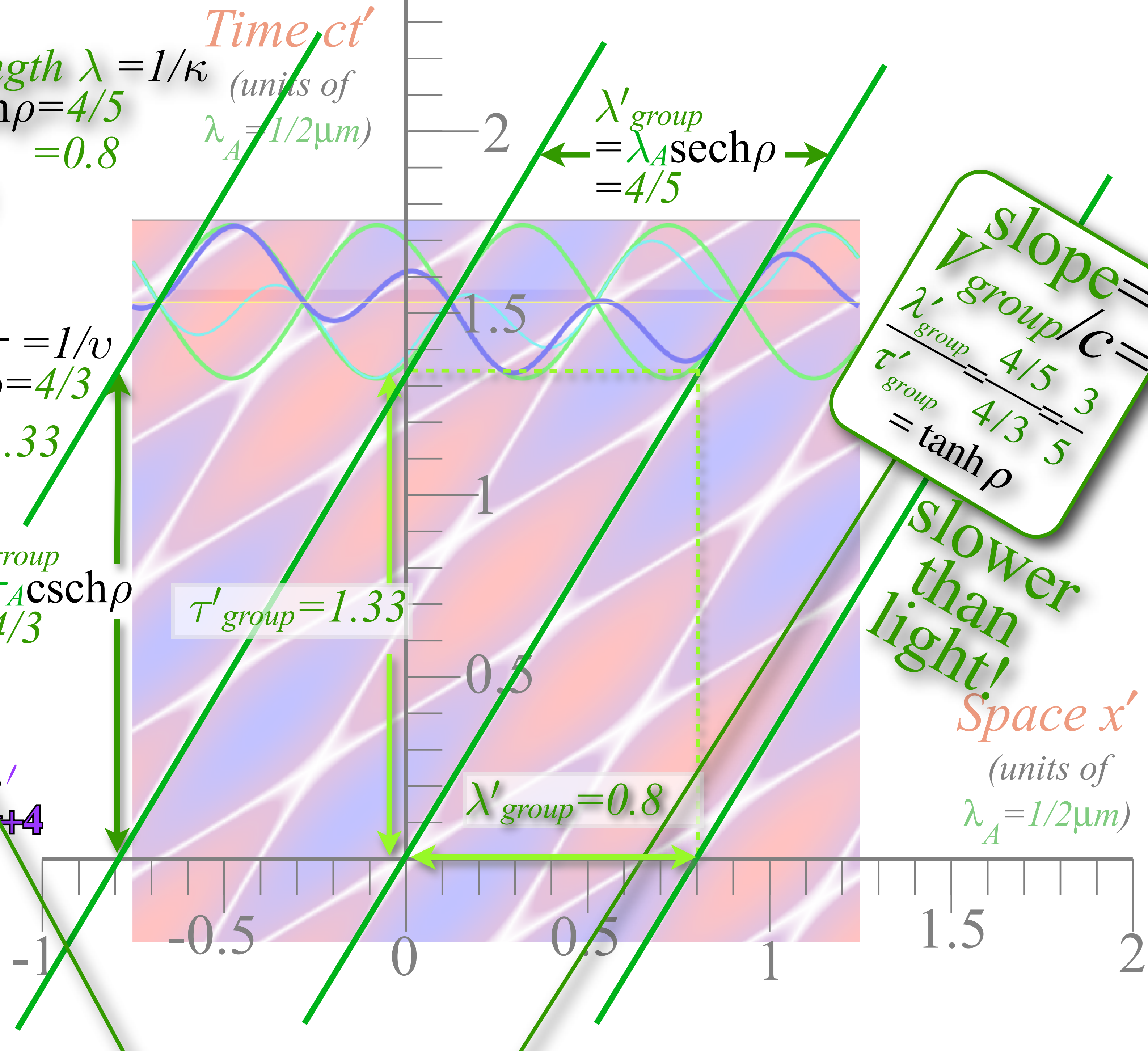
Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \text{csch}\rho = 4/3 = 1.33$

G-slope $= V_{group}/c$

$$= \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$

$\tau'_{group} = \tau_A \text{csch}\rho = 4/3$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	tanh ρ	sinh ρ	sech ρ	cosh ρ	cschρ	coth ρ	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Lorentz transformations...

write **G′** *and* **P′** *in terms of* **G** *and* **P** *using* coshρ *and* sinhρ

$$\mathbf{G}'=\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix}=\upsilon_A\begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

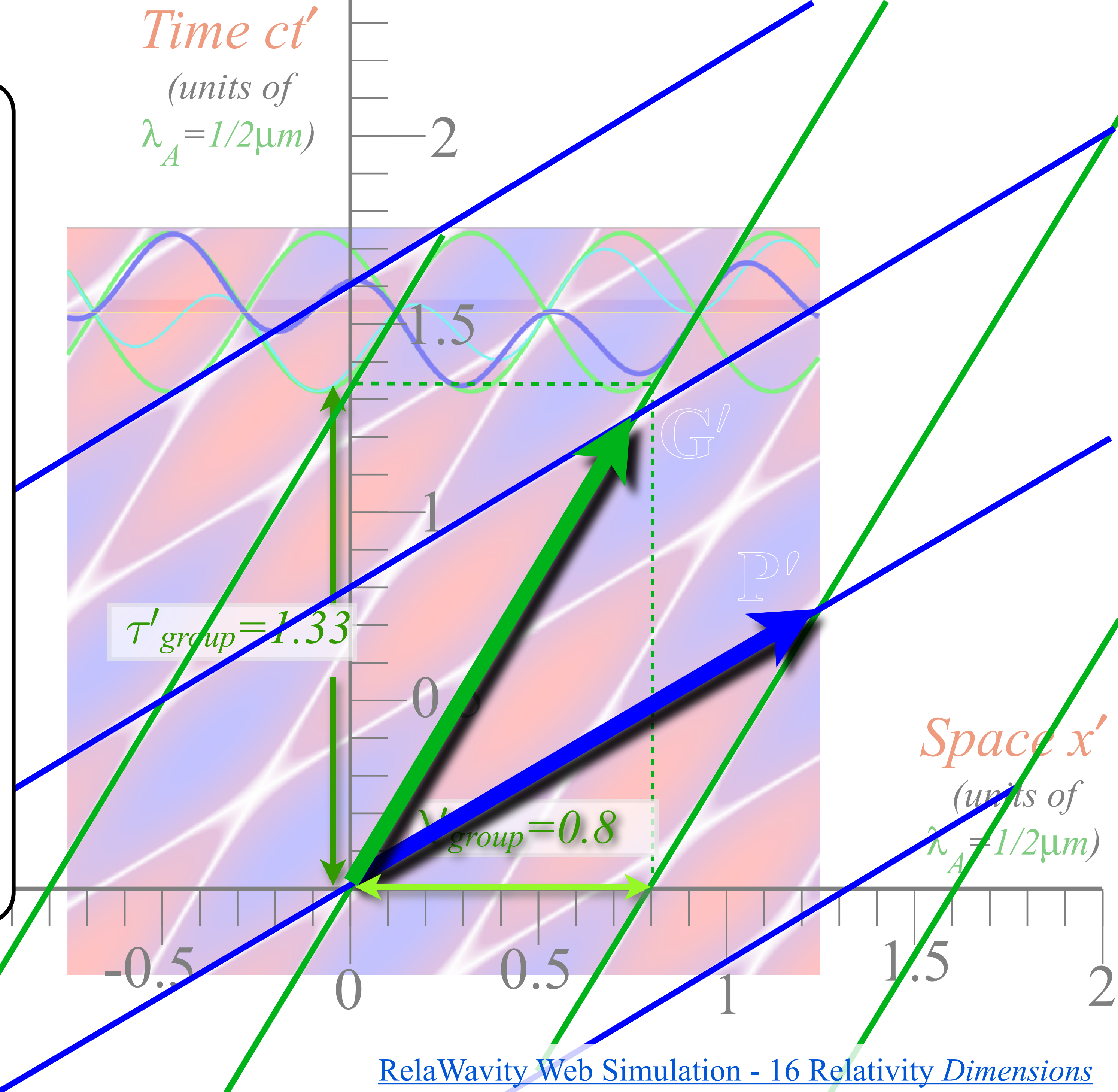
$$=\upsilon_A\begin{pmatrix} 1 \\ 0 \end{pmatrix}\cosh\rho+\upsilon_A\begin{pmatrix} 0 \\ 1 \end{pmatrix}\sinh\rho$$

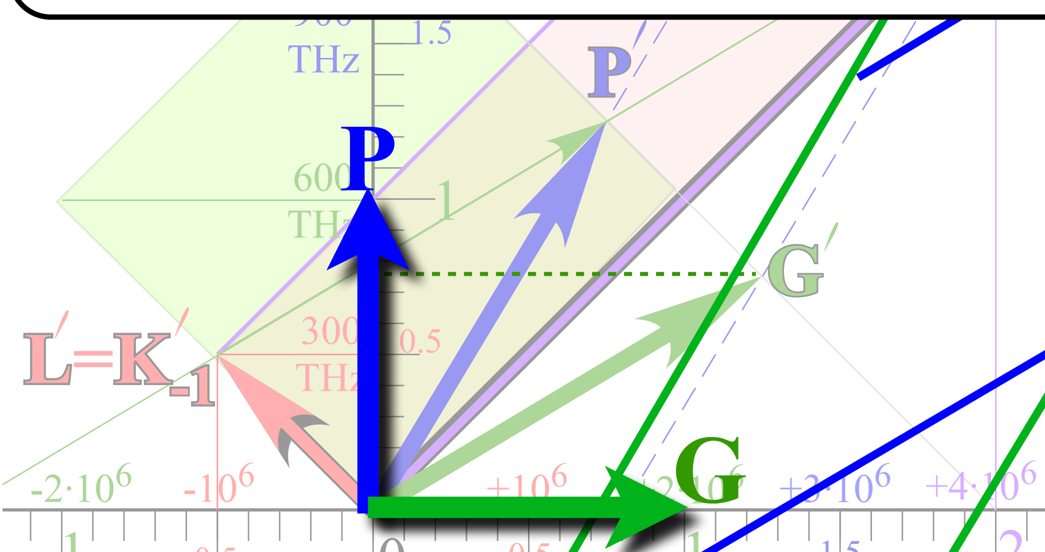
$$\mathbf{G}'=\mathbf{G}\cosh\rho+\mathbf{P}\sinh\rho$$

$$\mathbf{P}'=\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix}=\upsilon_A\begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}=\upsilon_A\begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$=\upsilon_A\begin{pmatrix} 1 \\ 0 \end{pmatrix}\sinh\rho+\upsilon_A\begin{pmatrix} 0 \\ 1 \end{pmatrix}\cosh\rho$$

$$\mathbf{P}'=\mathbf{G}\sinh\rho+\mathbf{P}\cosh\rho$$

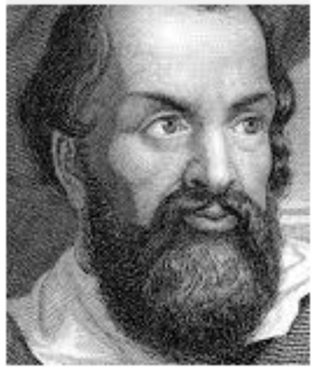




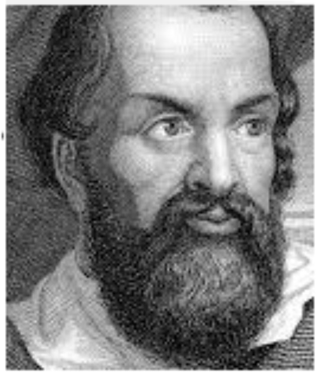
$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$ Lorentz transform matrix

RelaWavity Web Simulation - 16 Relativity *Dimensions*

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0


1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like Galilean velocity

Learning about sin! *and* cos *and…Trigonometric road maps*
- Hyper-Trigonometric algebra and phasors in space-time
- 1CW wavefunctions and phasors

Per-space-per-time vs Space-time
- Wave velocity formulas

Introducing Doppler shifting
- Why c is constant?!
- Introducing Doppler Arithmetic and rapidity ρ

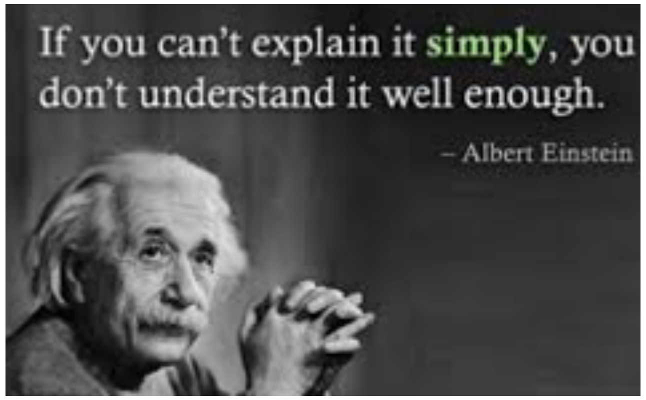
Optical interference "baseball-diamond" displays *phase* and *group* velocity
- Details of 2CW wavefunctions in rest frame
  - Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation
- Analyzing wave velocity by per-space-per-time *and* space-time graphs
  - 16 coefficients of relativistic 2CW interference
  - ➔ Two "famous-name" coefficients and the Lorentz transformation
  - Thales geometry of Lorentz transformation

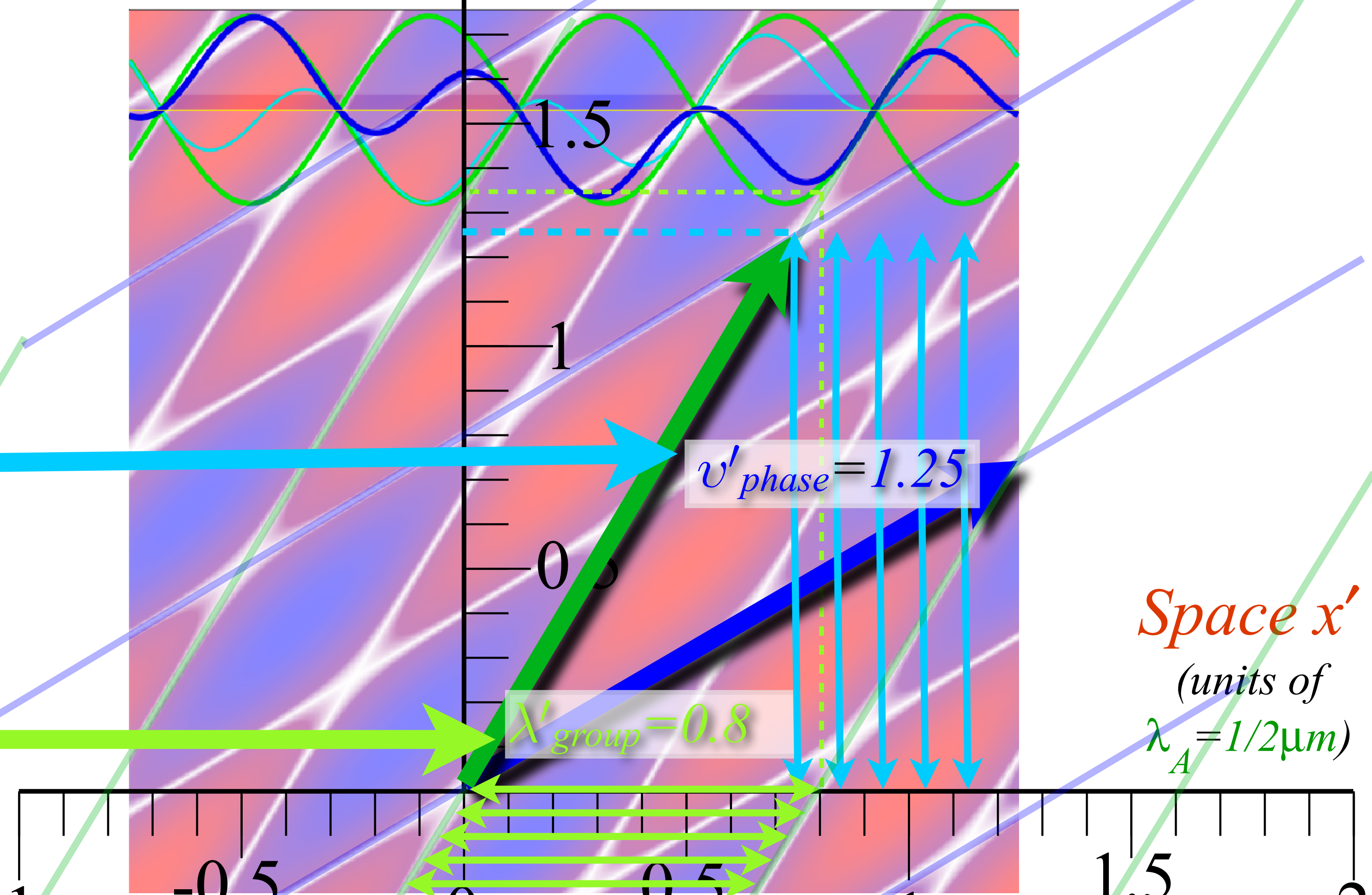
Rapidity ρ related to *stellar aberration angle σ* and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic *ρ*-geometry connects to transverse circular *σ*-geometry
  - "Occams Sword" and geometry of 16 parameter functions of *ρ* and *σ*
    - Application to TE-Waveguide modes

Two Famous-Name Coefficients

Albert Einstein 1859-1955

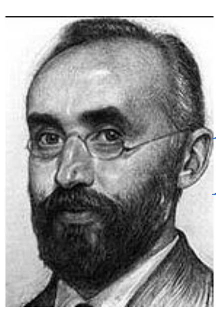


Herman Minkowski 1864-1909



This number is called an: ***Einstein time-dilation*** *(dilated by 25% here)*

This number is called a: ***Lorentz length-contraction*** *(contracted by 20% here)*

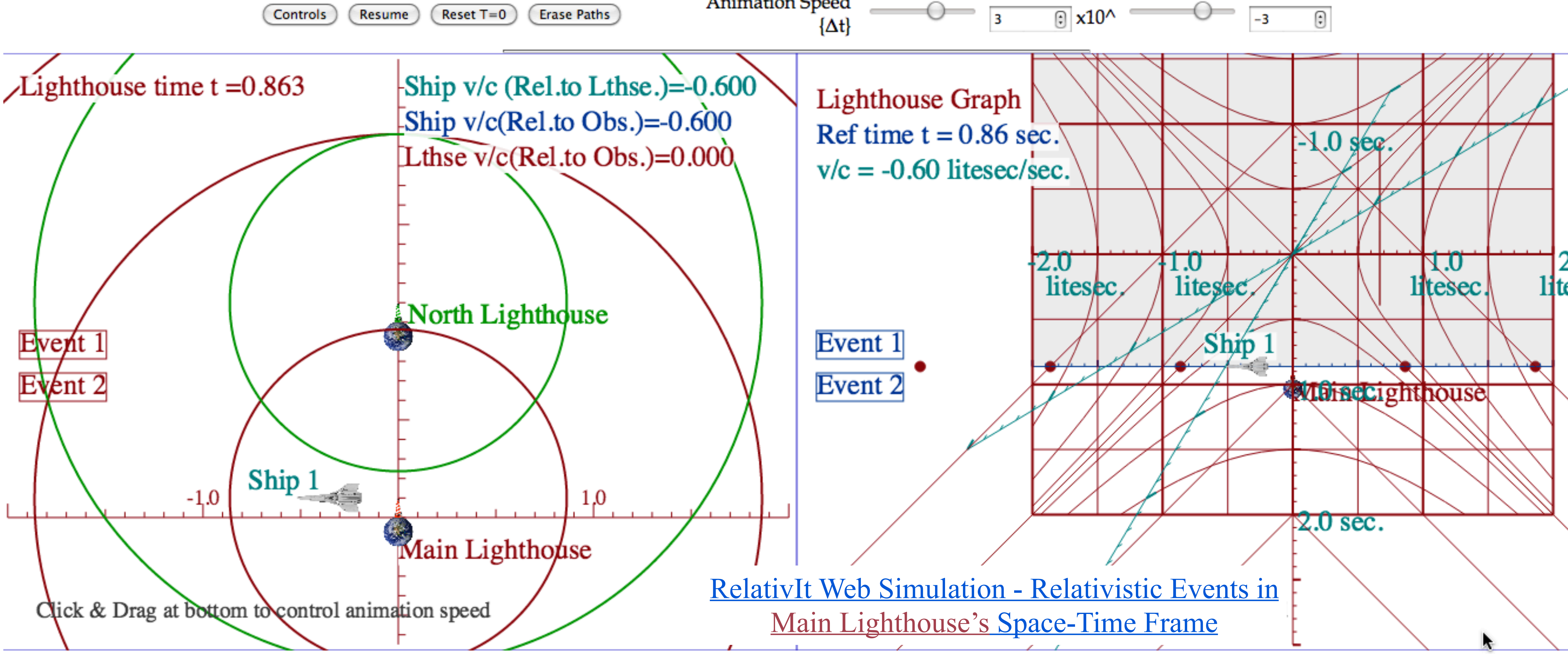


Hendrik A. Lorentz 1853-1928

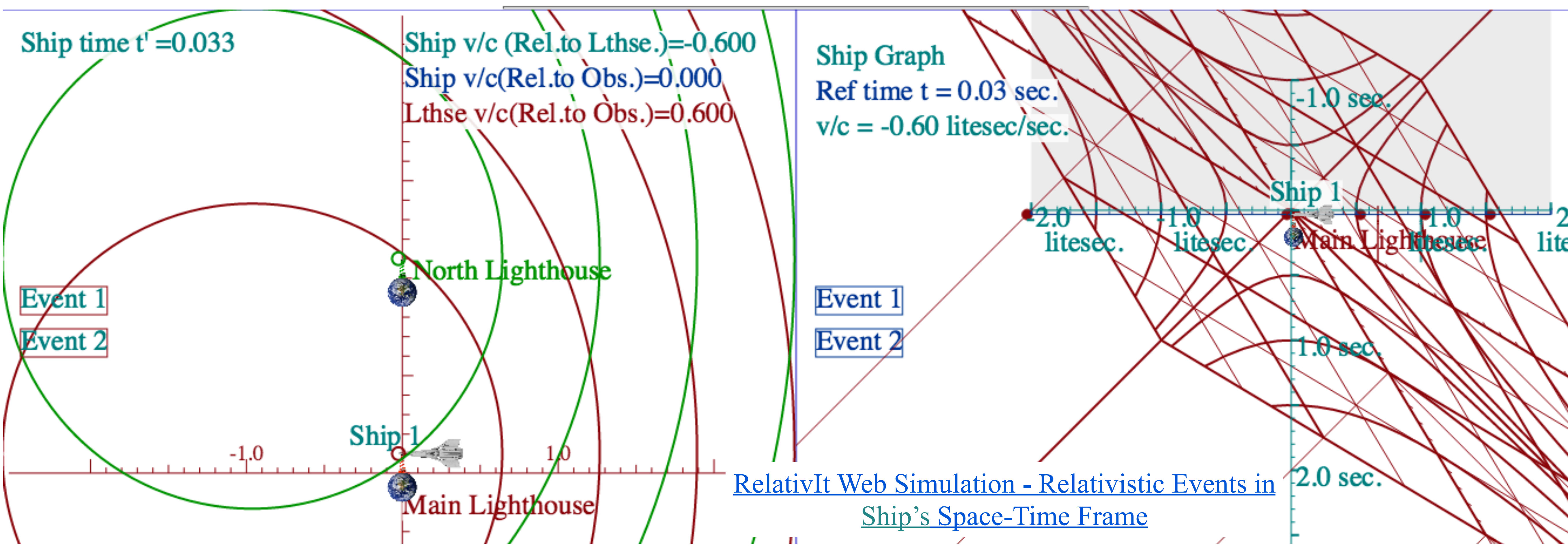
Old-Fashioned Notation

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$=0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation - Relativistic *Terms*
(Expanded Table)

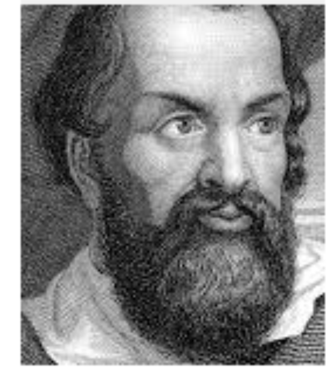


[RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame]



[RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame]

Galileo Galilei



1564-1642

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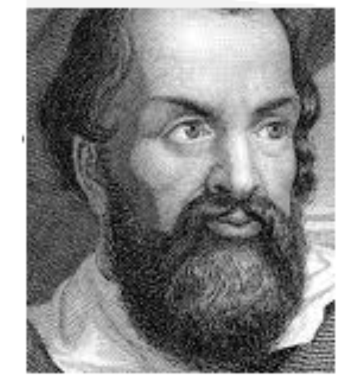
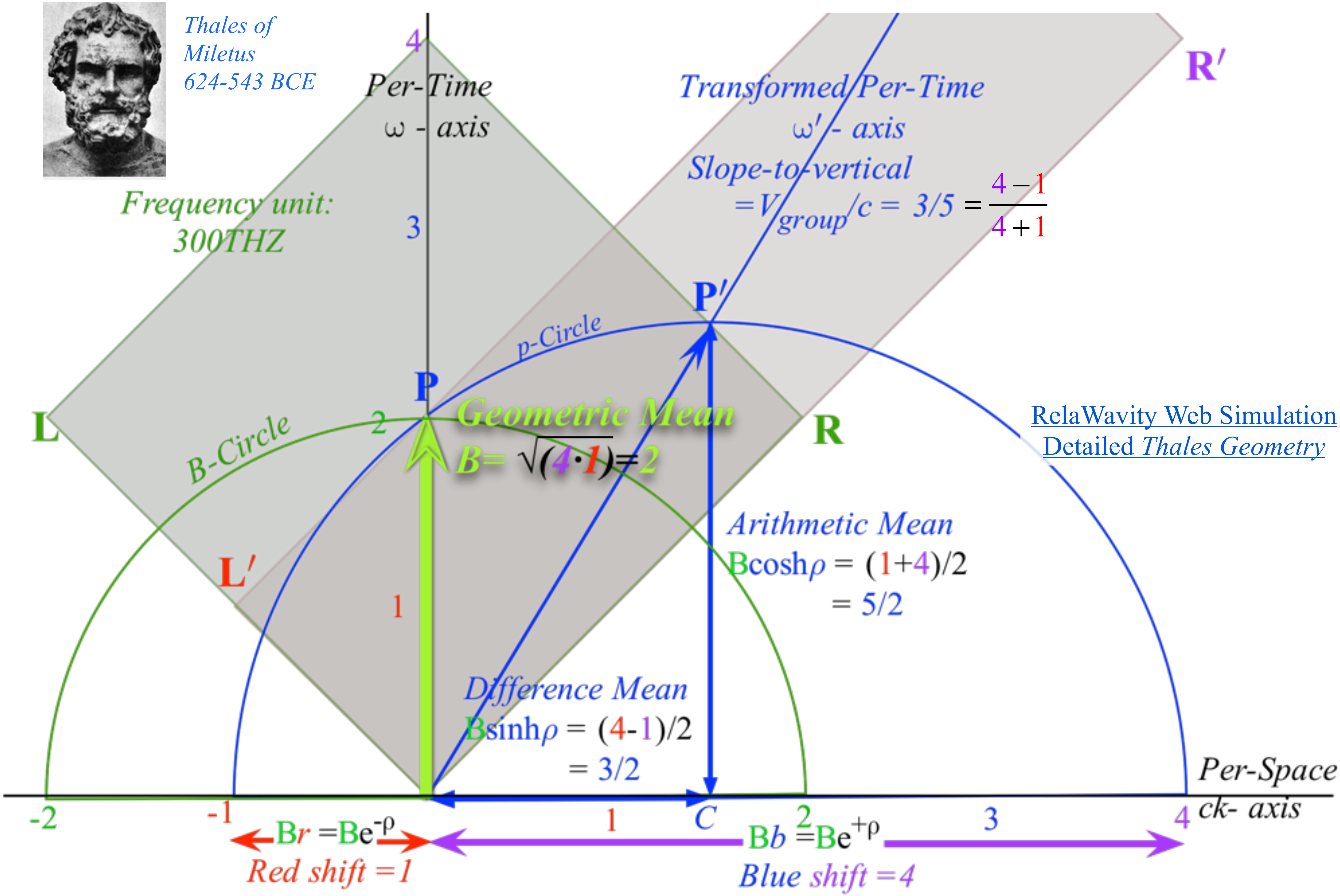


Galileo's Revenge (part 2)
Phasor angular velocity *adds just like Galilean velocity*

Thales Mean Geometry (600BCE)

helps “Relawavity”

Thales of Miletus 624-543 BCE



(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?
(2.) What is that frequency ω_E ?

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\omega_{group}}{k_{group}} = \frac{\omega_R - \omega_L}{k_R - k_L} = c\frac{\omega_R - \omega_L}{\omega_R + \omega_L}$$

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?
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Query (2.) similarly: What ω_E is blue-shift $b\omega_L$ of ω_L *and* red-shift ω_R/b of ω_R ?

$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?
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Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

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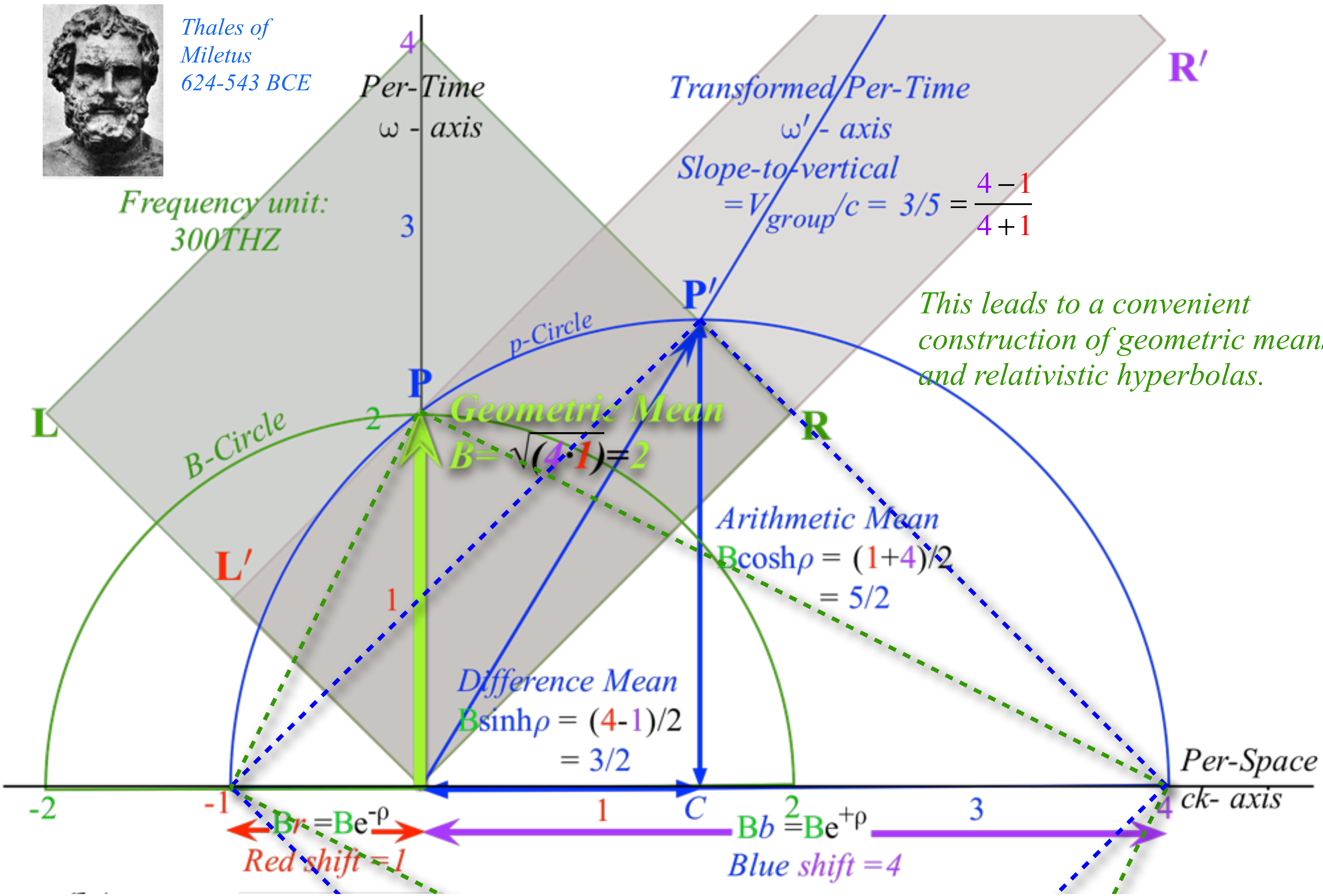
$$\omega_E = b\,\omega_L = \omega_R/b \quad \Rightarrow \quad b = \sqrt{\omega_R / \omega_L} \quad \Rightarrow \quad \omega_E = \sqrt{\omega_R \cdot \omega_L}$$

V_{group}/c is ratio of difference mean $\omega_{group} = \frac{\omega_R - \omega_L}{2}$ to arithmetic mean $\omega_{phase} = \frac{\omega_R + \omega_L}{2}$. Frequency $\omega_E = B$ is the geometric mean $\sqrt{\omega_R \cdot \omega_L}$ of left and right-moving frequencies defining the geometry

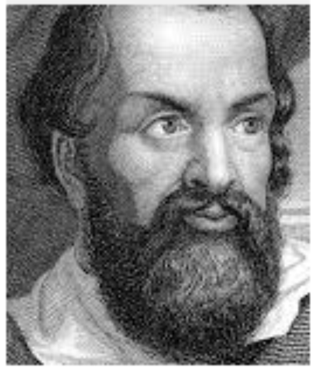
Thales Mean Geometry (600BCE)

helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*



This leads to a convenient construction of geometric means and relativistic hyperbolas.

Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like
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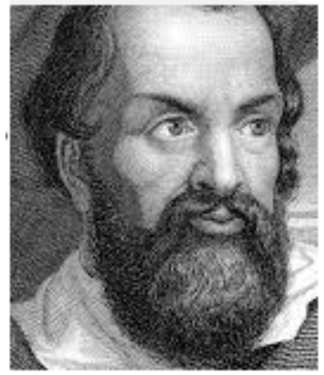

Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

Learning about sin! *and* cos *and…Trigonometric road maps*

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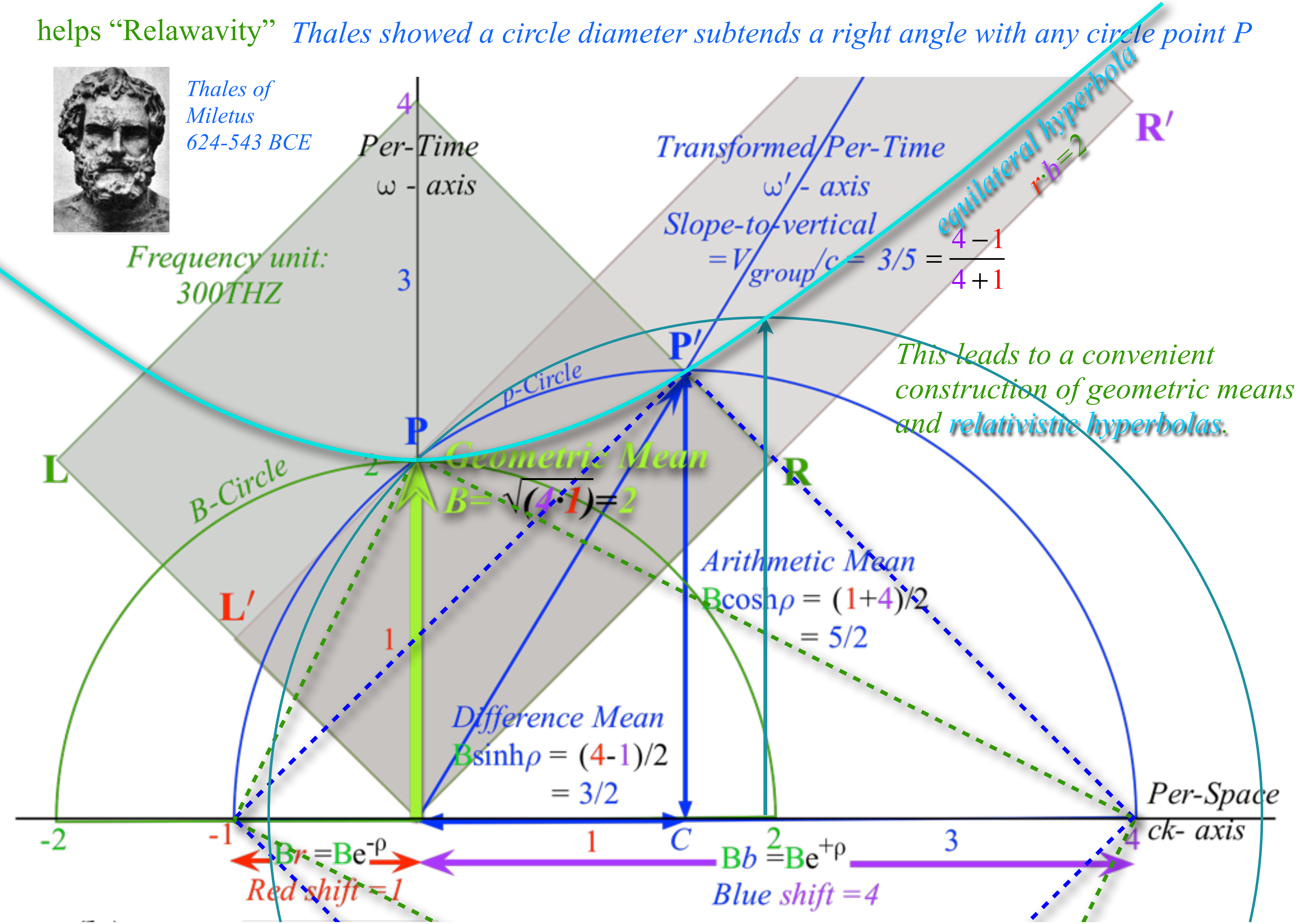

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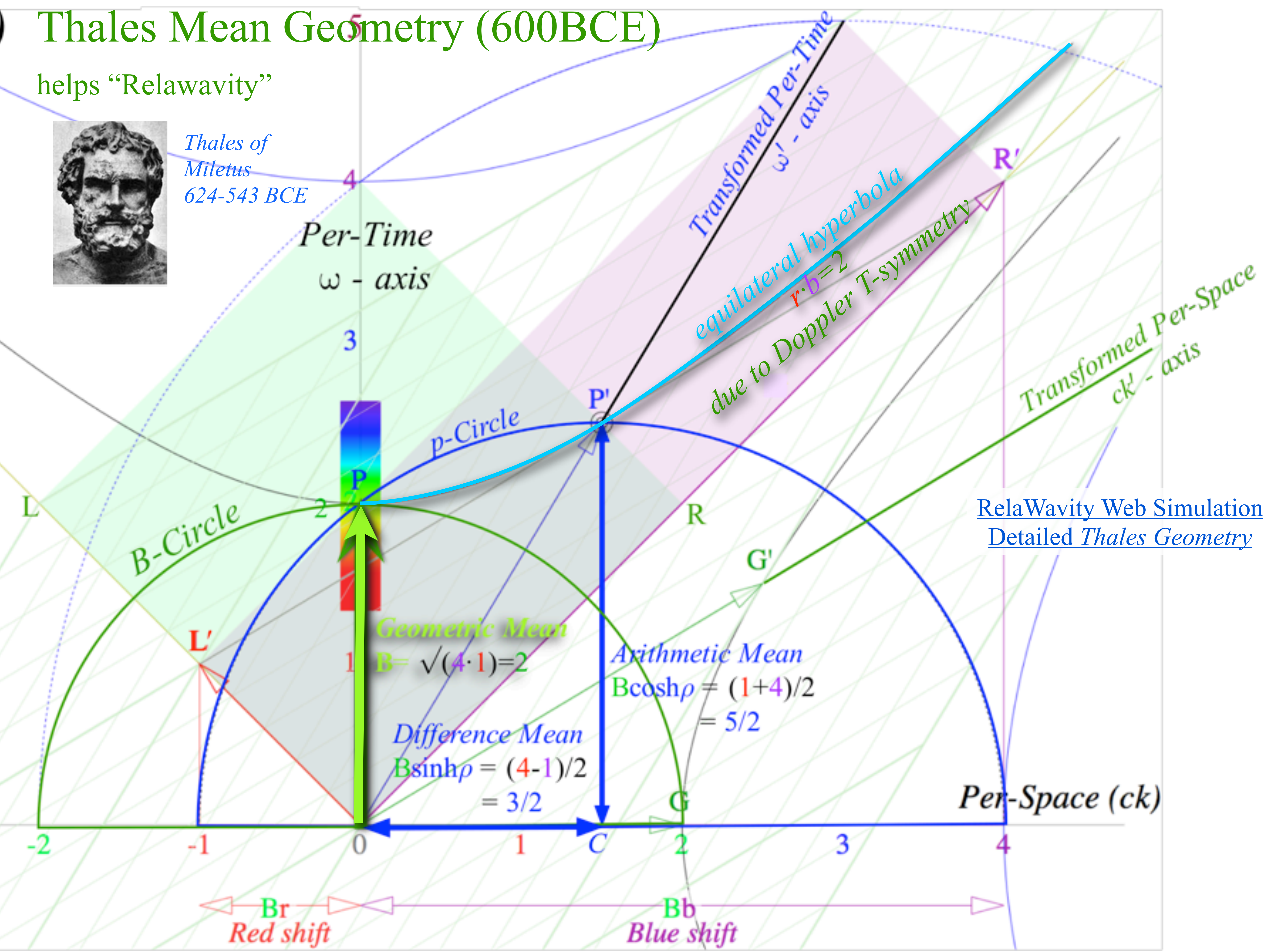
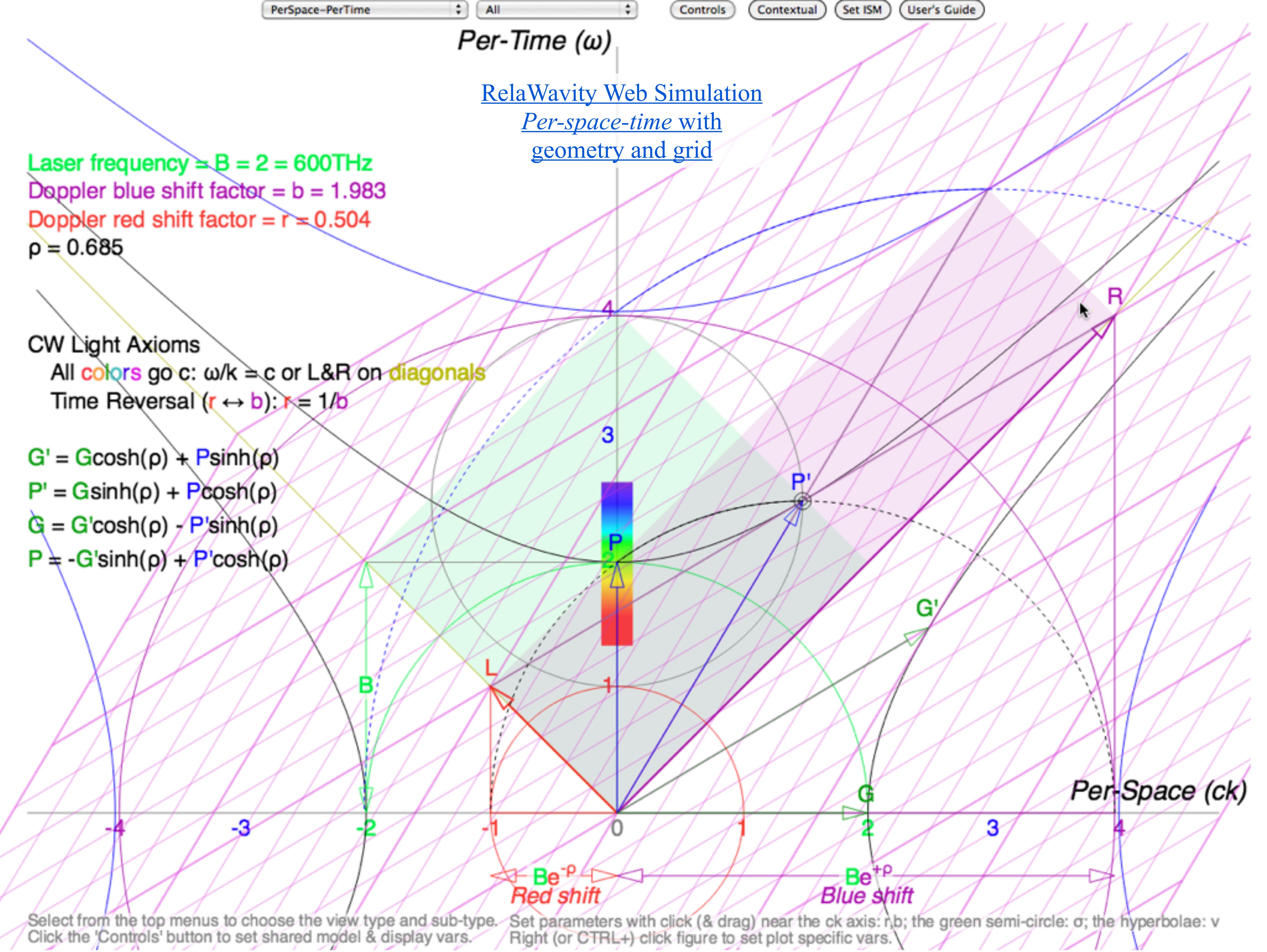
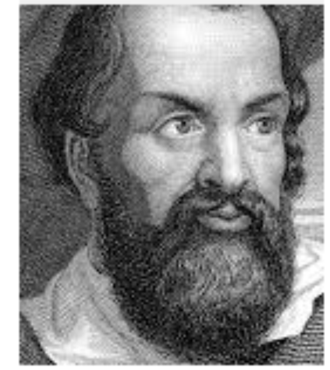


RelaWavity Web Simulation
Detailed *Thales Geometry*

RelaWavity Web Simulation
Per-space-time with geometry and grid

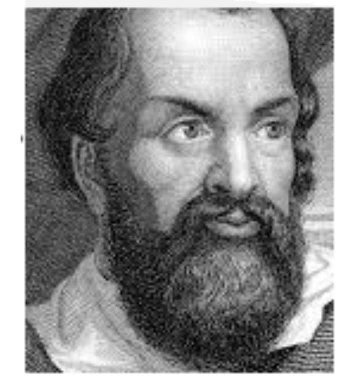


Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like Galilean velocity



Galileo's Revenge (part 2)
Phasor angular velocity *adds just like Galilean velocity*

Learning about sin! *and* cos *and…Trigonometric road maps*

Hyper-Trigonometric algebra and phasors in space-time

1CW wavefunctions and phasors

Per-space-per-time vs Space-time

Wave velocity formulas

Introducing Doppler shifting

Why c is constant?!

Introducing Doppler Arithmetic and rapidity ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity

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16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

Thales geometry of Lorentz transformation ...and hyperbolas

➜ *Rapidity* ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity

➜ Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of 16 parameter functions of ρ and σ

Application to TE-Waveguide modes

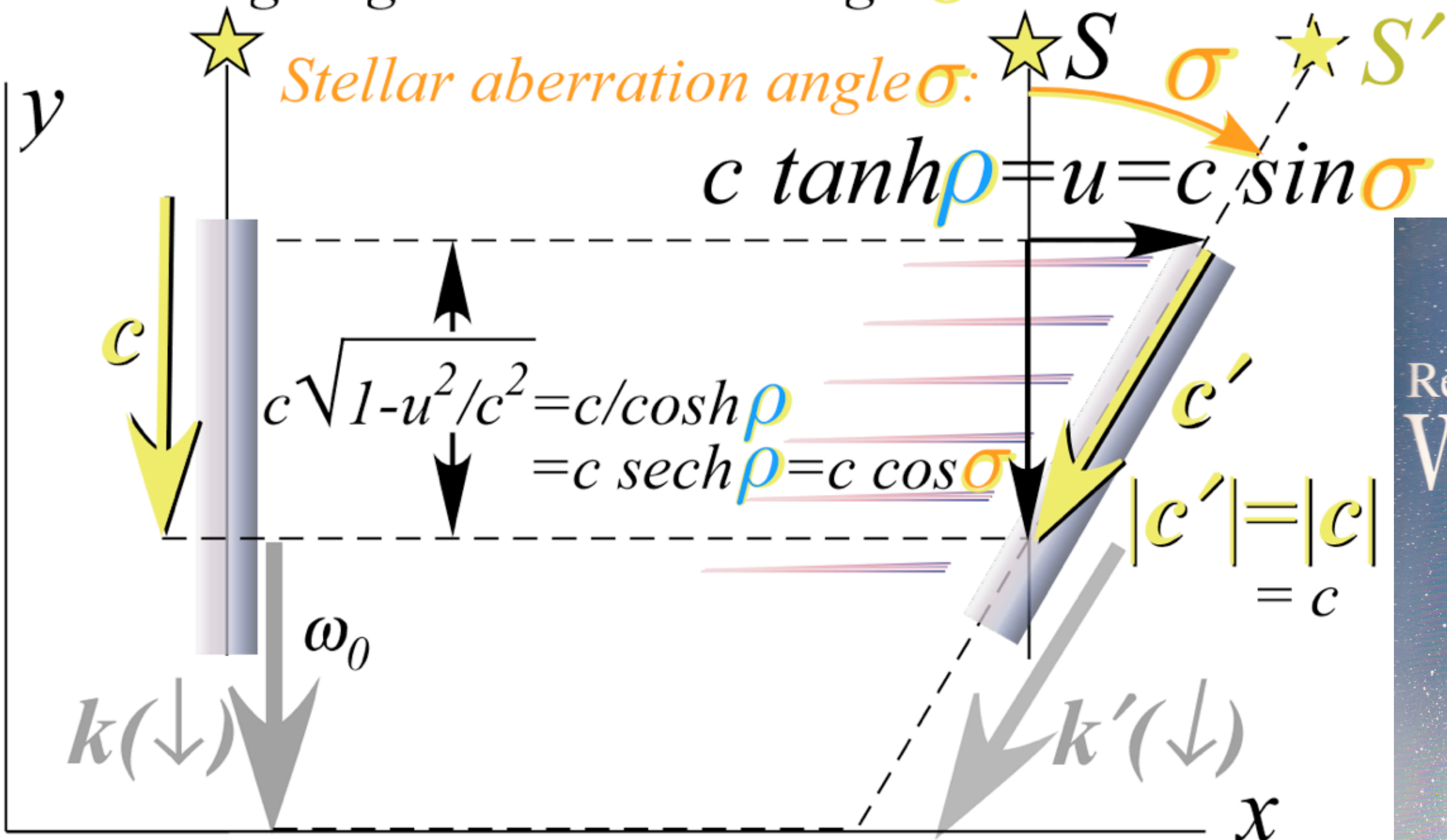
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

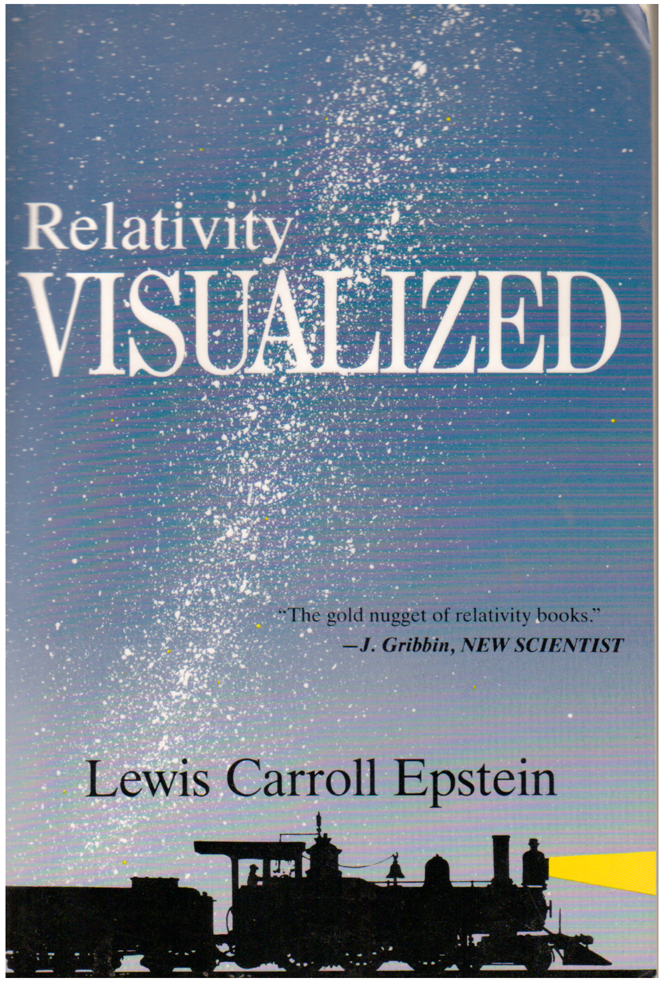
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



Purchase at: AllBookstores.com, amazon

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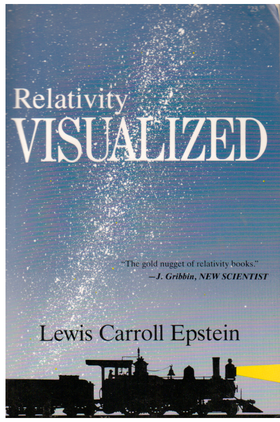
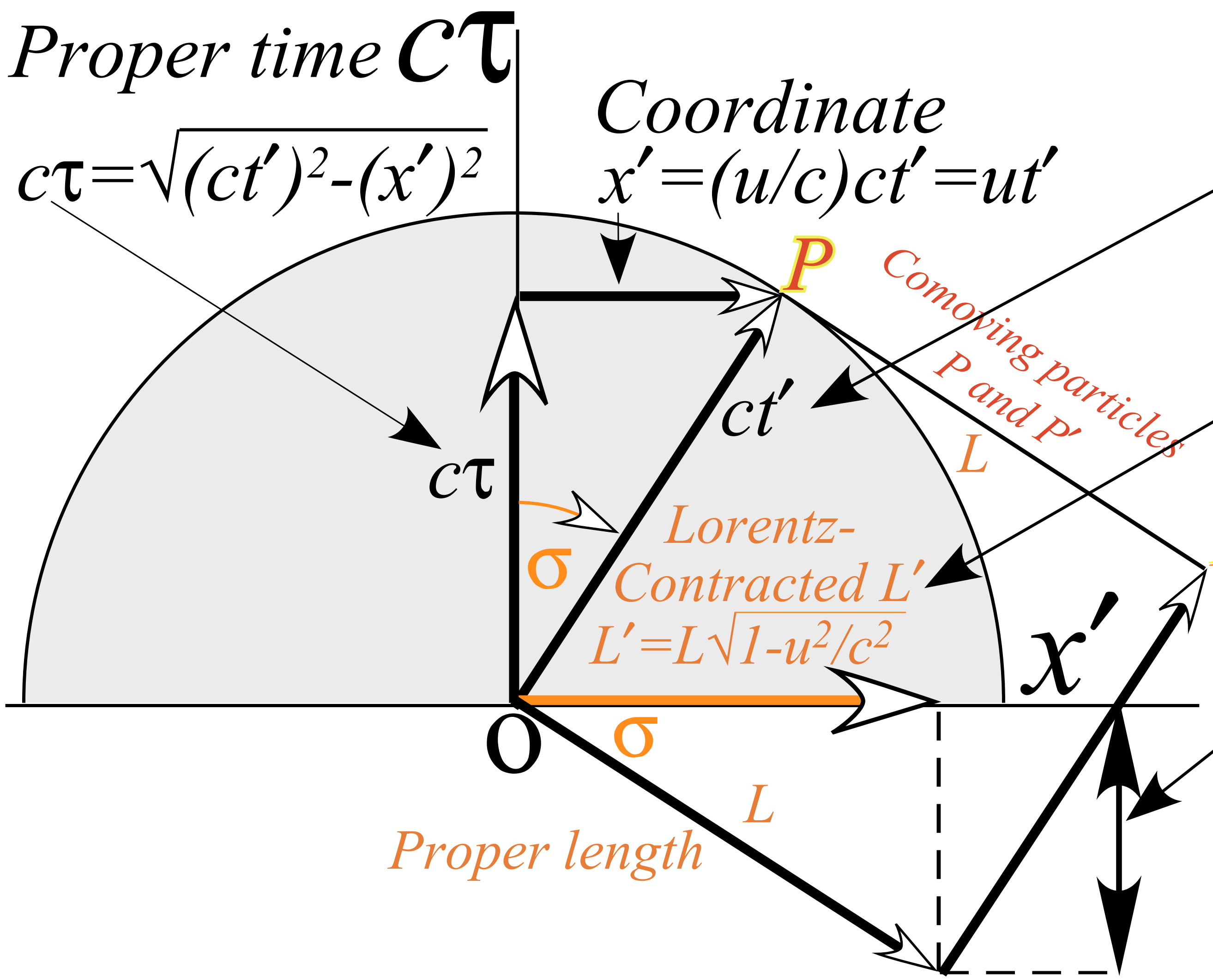


Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P' have speed u in (x',ct') and speed c in $(x, c\tau)$



Einstein time dilation:

$$ct'=c\tau\sec\sigma=c\tau\cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L'= L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift) to a Transverse*relativity parameter: Stellar aberration angle σ

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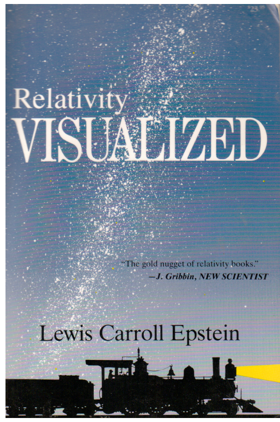
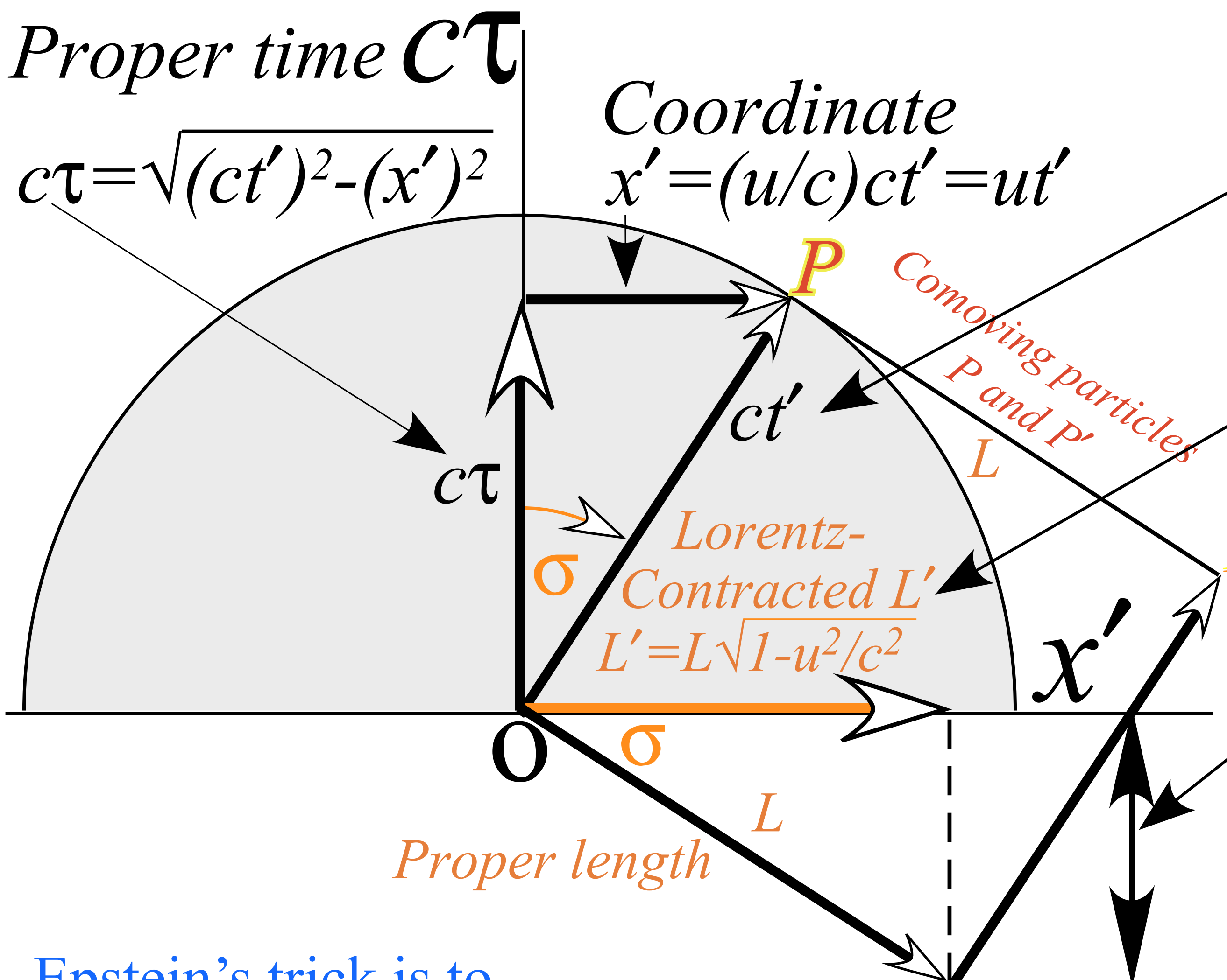


Proper time cτ vs. coordinate space x - (*L. C. Epstein's "Cosmic Speedometer"*)

Particles P and P′ have speed u in (x',ct') *and speed c in* $(x, c\tau)$



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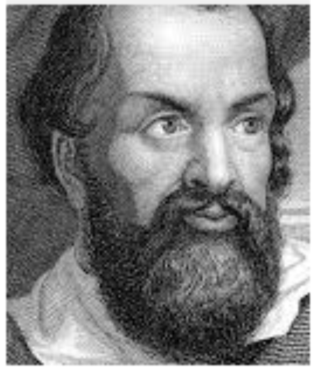
Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form:

$$\sqrt{(c\tau)^2 + (x')^2} = (ct')$$

Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

Galileo Galilei



1564-1642

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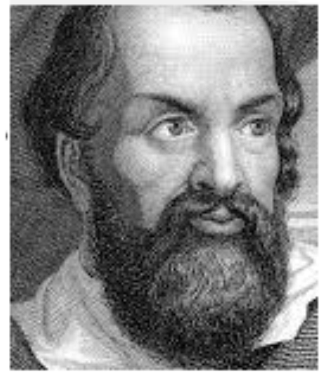

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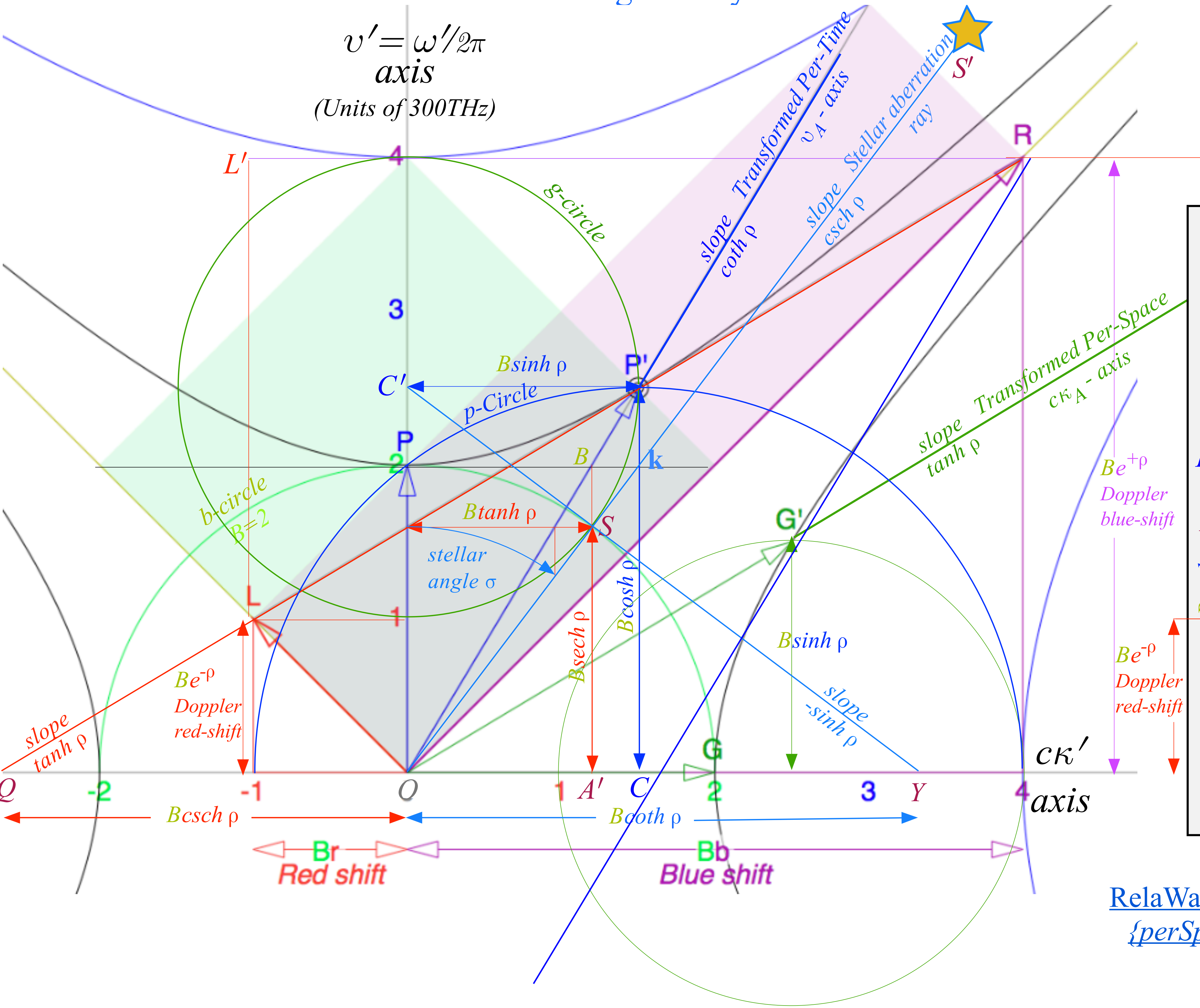
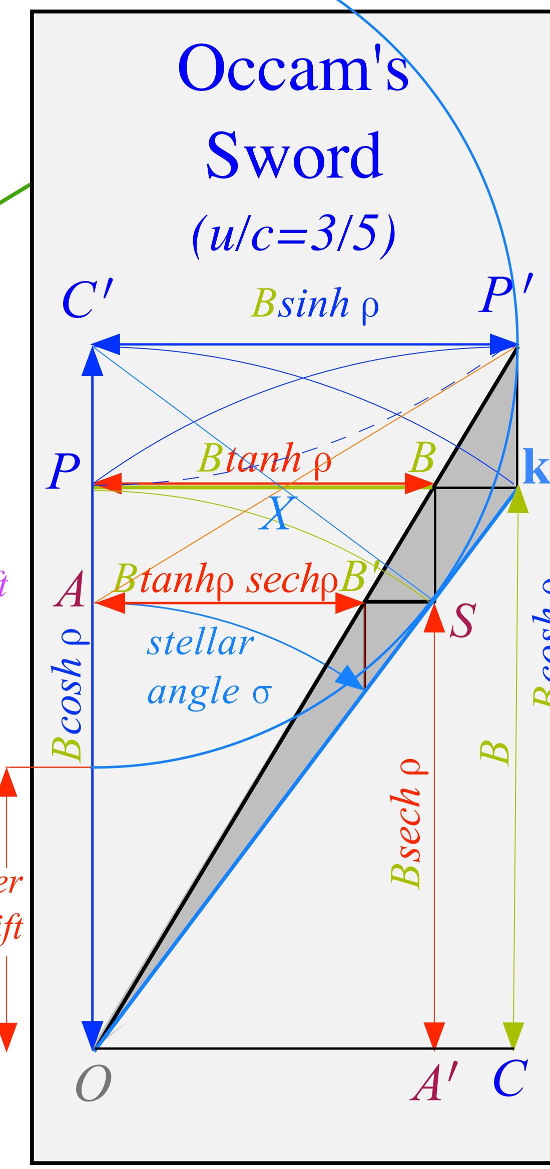
Summary of optical wave parameters for relativity and QM

...and their geometry

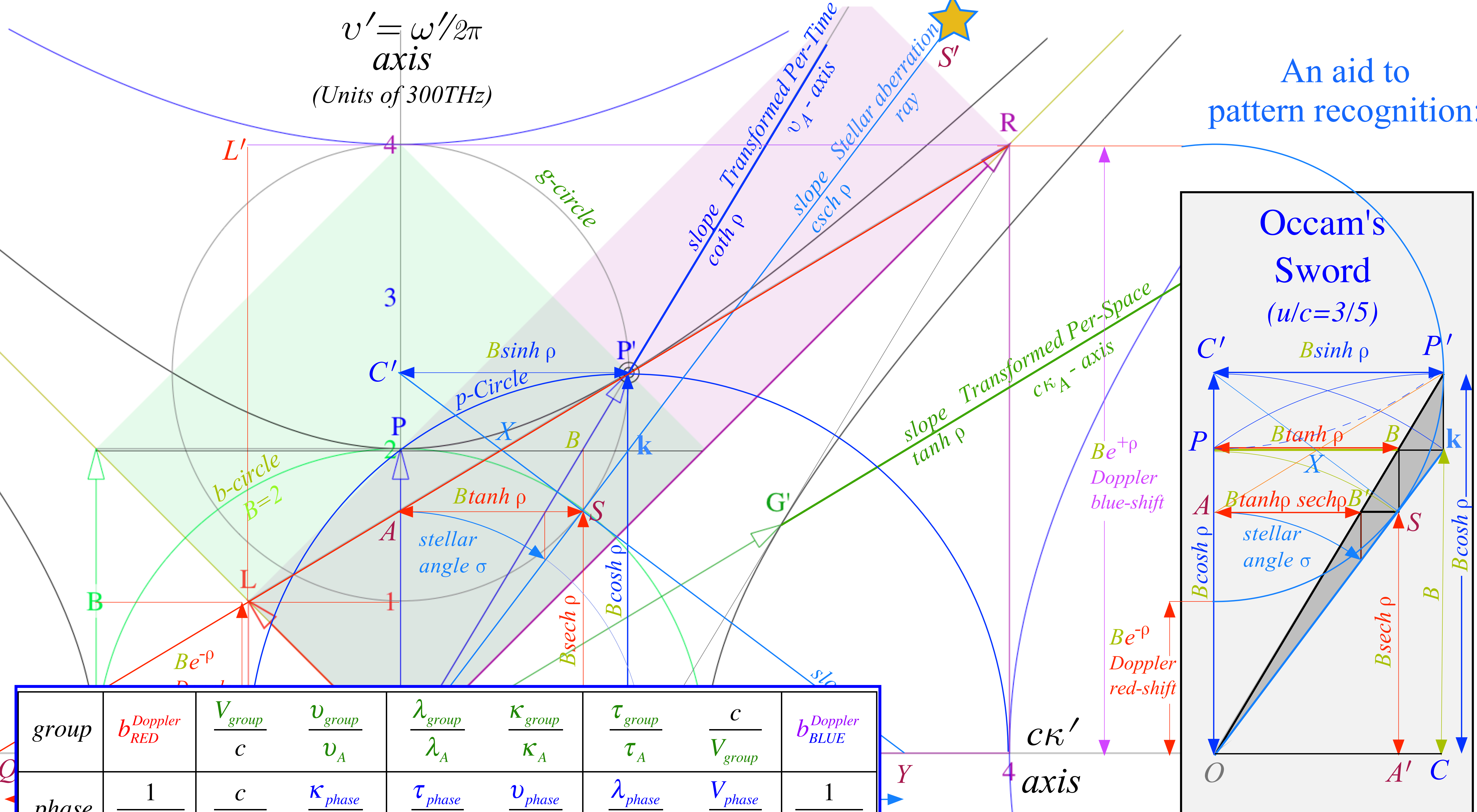


An aid to pattern recognition:



RelaWavity Web Simulation
{perSpace - perTime All}

An aid to pattern recognition:



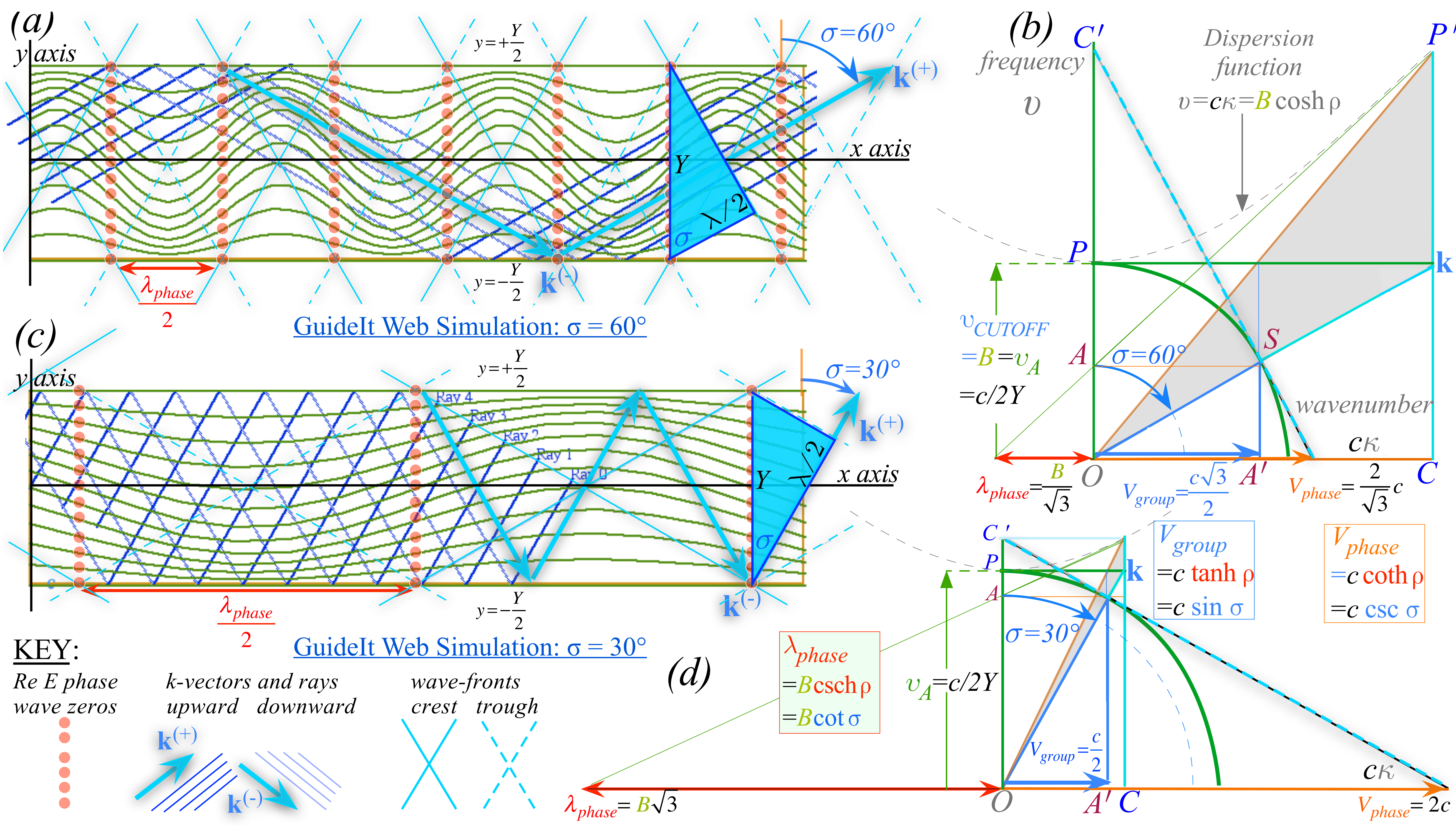
group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

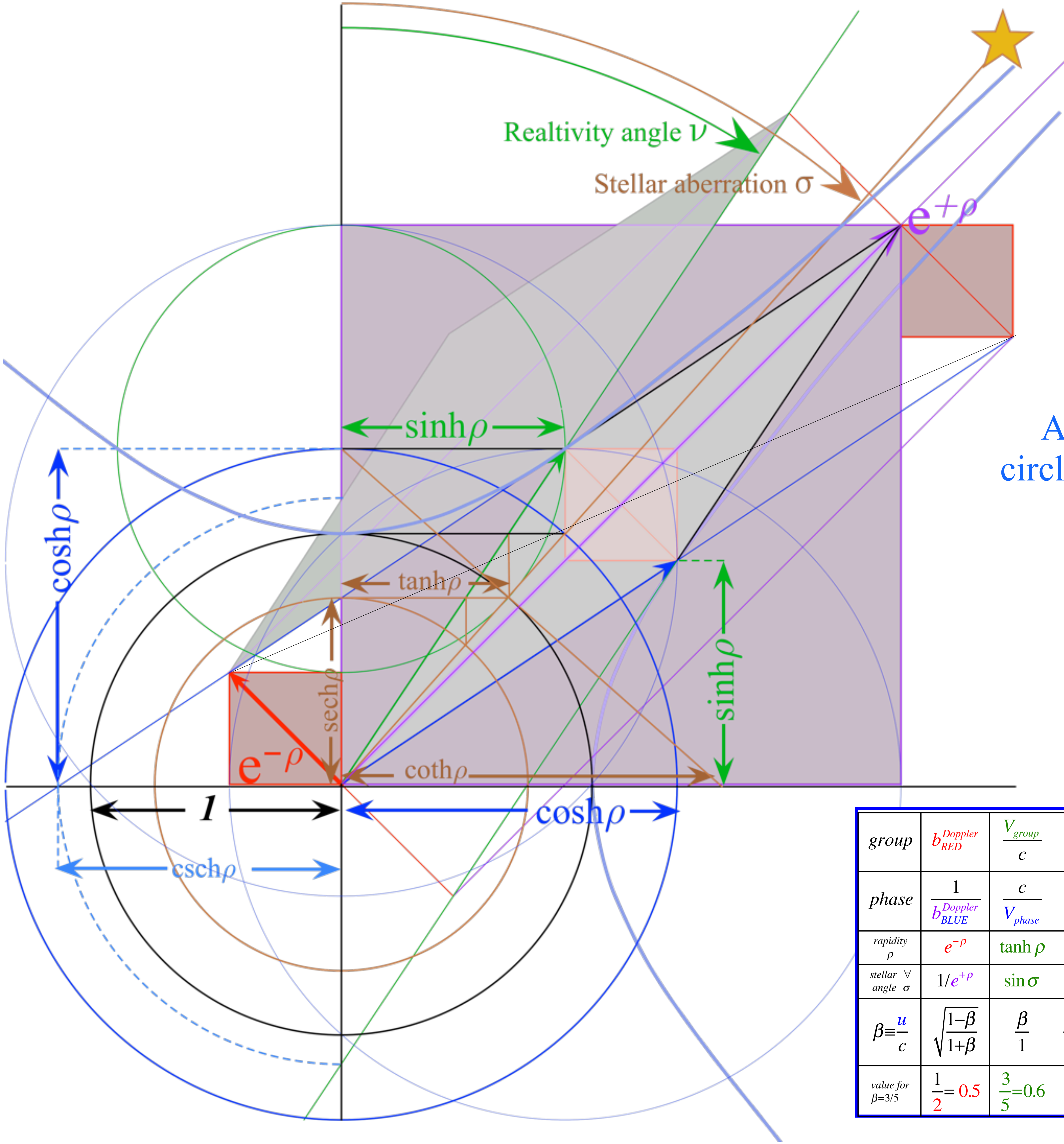
Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c*=3/5

RelaWavity Web Simulation

Expanded *Relativistic* Relations


(a)
y axis
x axis
y=+Y/2
y=−Y/2
σ=60°
k(+)
k(−)
Y
λ/2
σ
λphase/2
GuideIt Web Simulation: σ = 60°
(b)
C′
P′
frequency
υ
Dispersion function
υ=cκ=B cosh ρ
P
k
υCUTOFF
=B=υA
=c/2Y
A
σ=60°
S
wavenumber
cκ
λphase=B/√3
O
Vgroup=c√3/2
A′
Vphase=2/√3 c
C
(c)
y axis
y=+Y/2
σ=30°
Ray 4
Ray 3
Ray 2
Ray 1
Ray 0
k(+)
x axis
Y
λ/2
σ
y=−Y/2
k(−)
λphase/2
GuideIt Web Simulation: σ = 30°
KEY:
Re E phase wave zeros
k-vectors and rays
upward downward
k(+)
k(−)
wave-fronts
crest trough
(d)
C′
P
A
k
Vgroup
=c tanh ρ
=c sin σ
Vphase
=c coth ρ
=c csc σ
σ=30°
λphase
=Bcsch ρ
=Bcot σ
υA=c/2Y
Vgroup=c/2
cκ
λphase= B√3
O
A′
C
Vphase=2c



A more compact circle-based geometry

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
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