Lecture 4
Thur. 9.5.2013

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations   *High mass ratio $M_1/m_2 = 49$*

Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-wall(s) crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)],[Harter, Li IMSS (2012)]

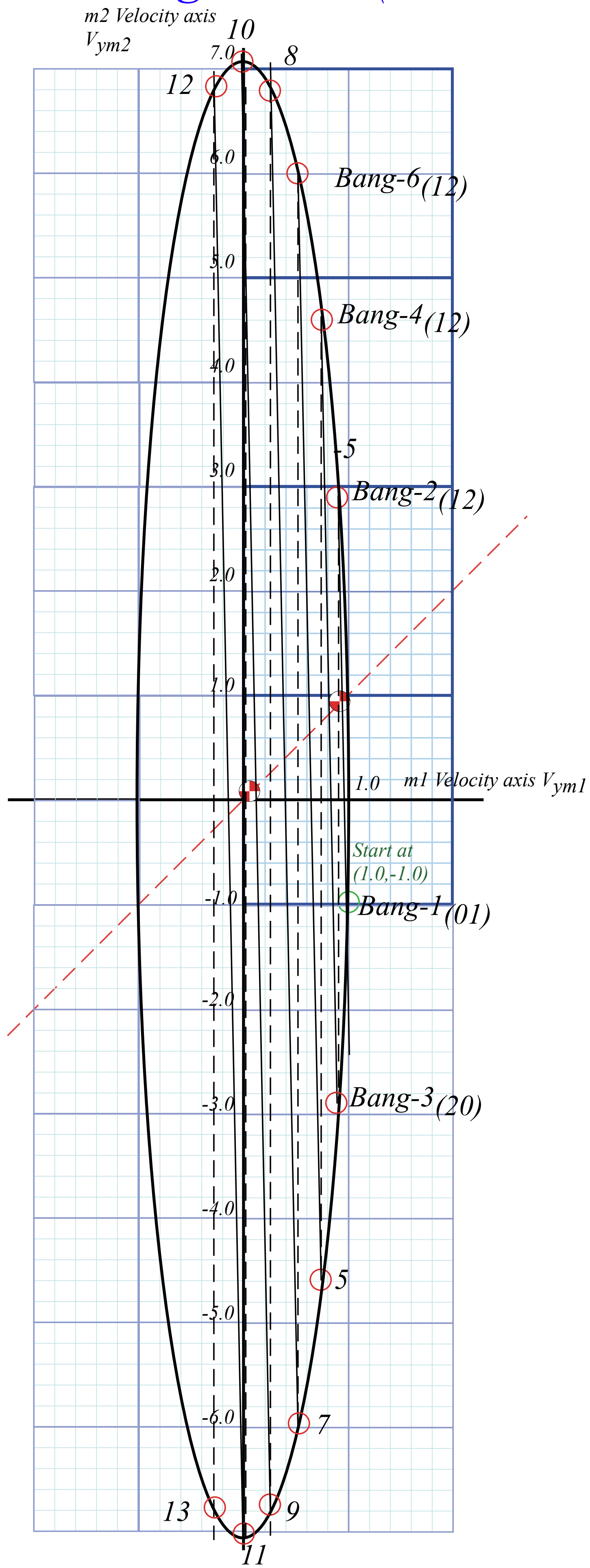
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)] [John Farey, Phil. Mag.(1816)]

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations

→ *High mass ratio $M_1/m_2 = 49$*

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1

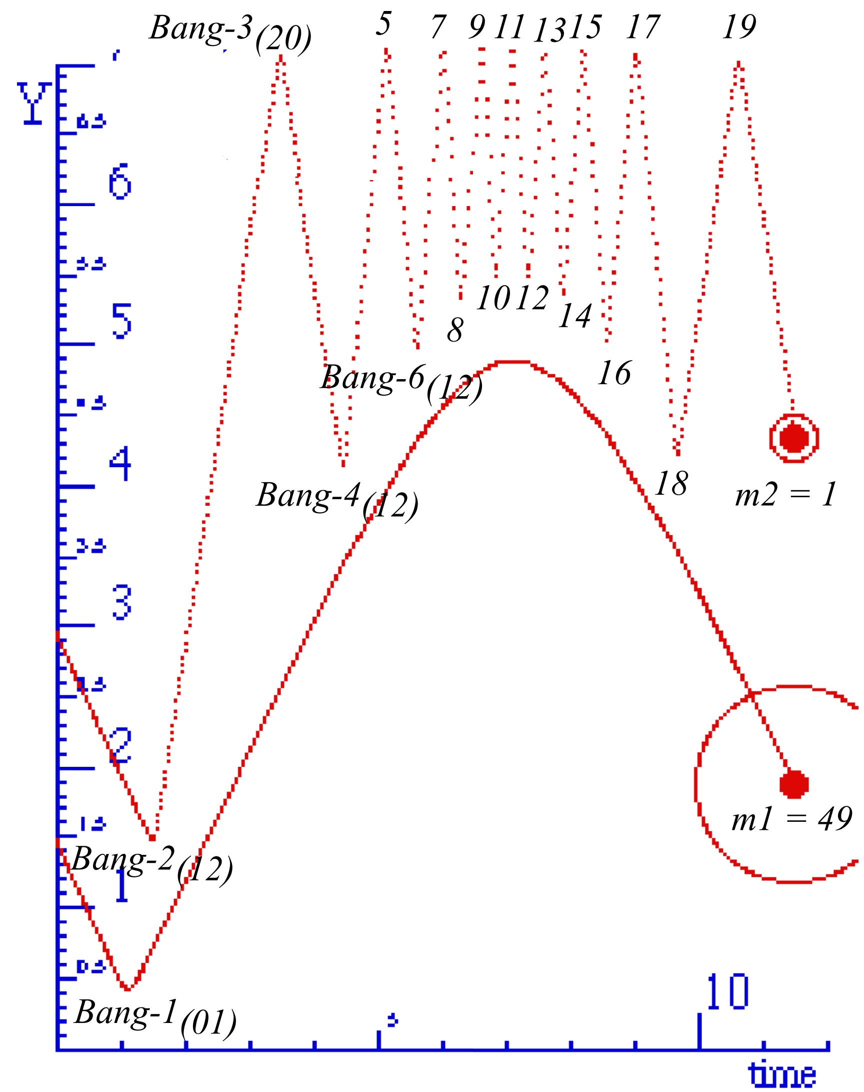


Fig. 5.1
in Unit 1

Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/m_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

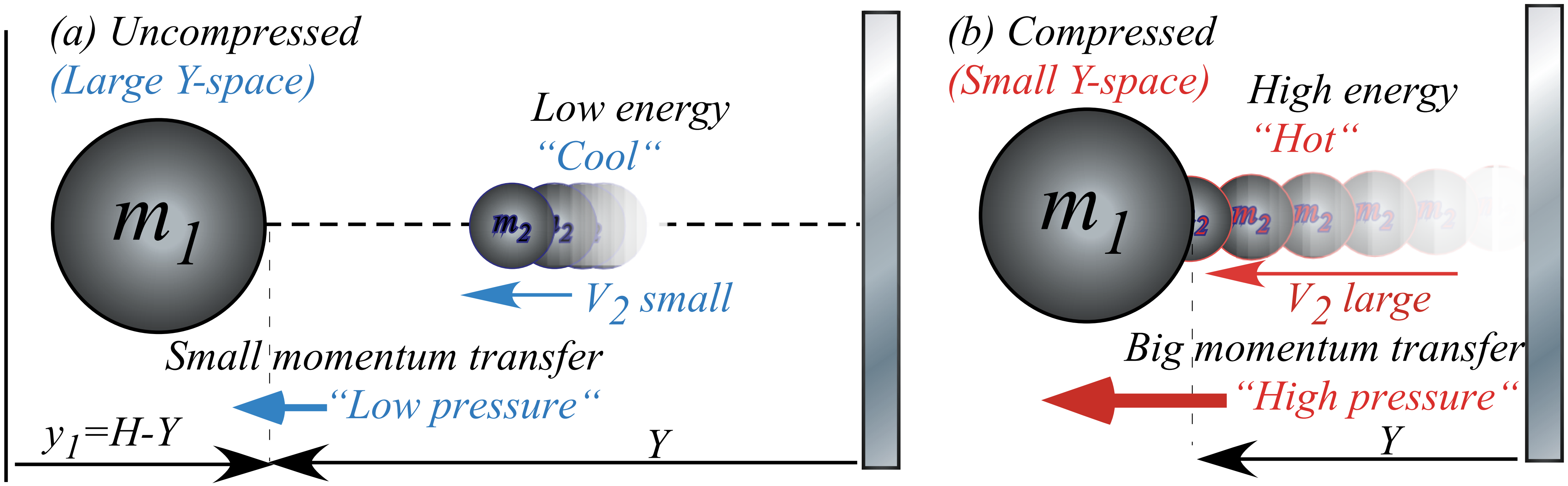
Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ *and the 1D-Adiabatic force field* $F(y)=const./y^3$

Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Unit 1
Fig. 6.1



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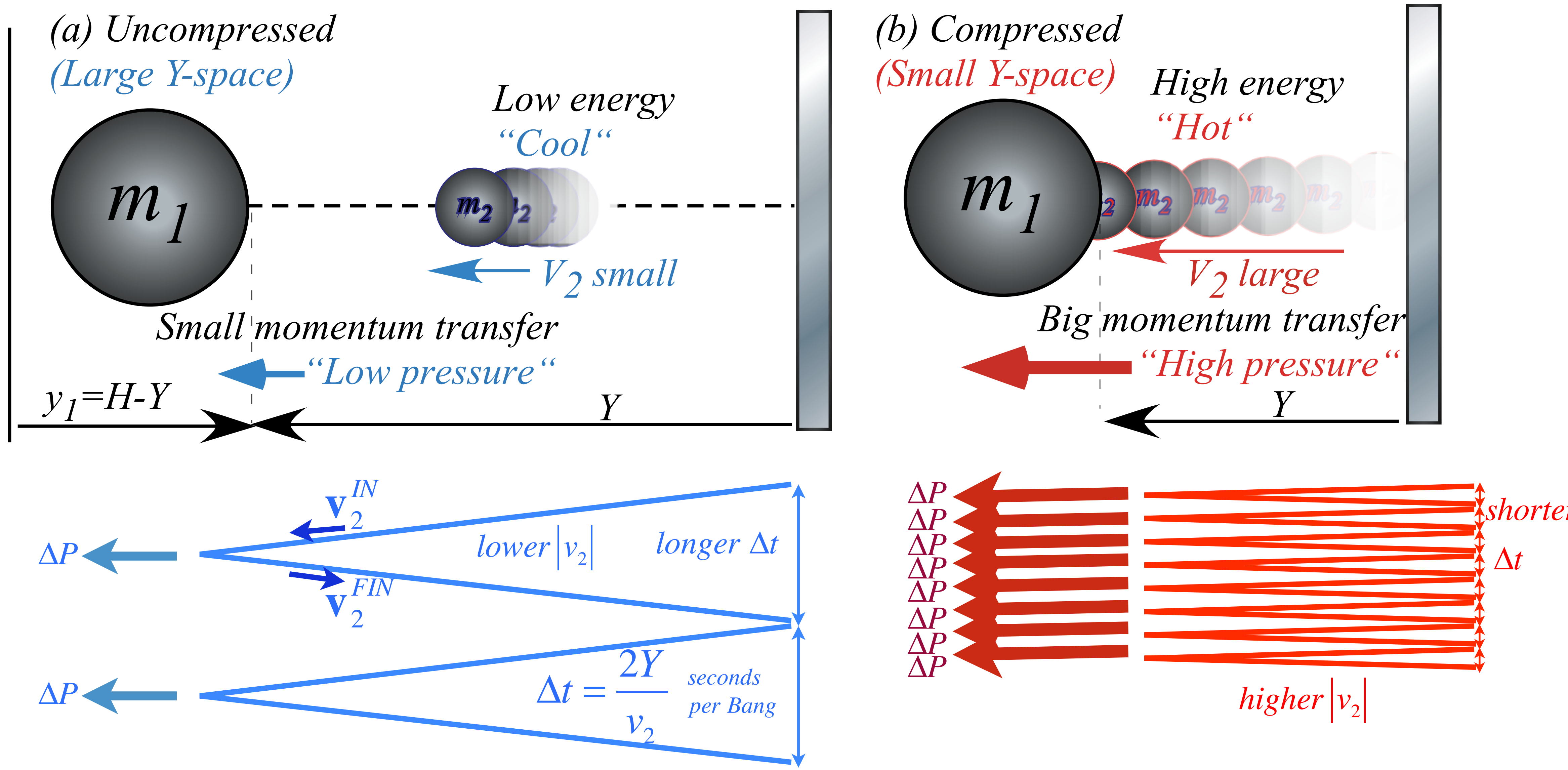

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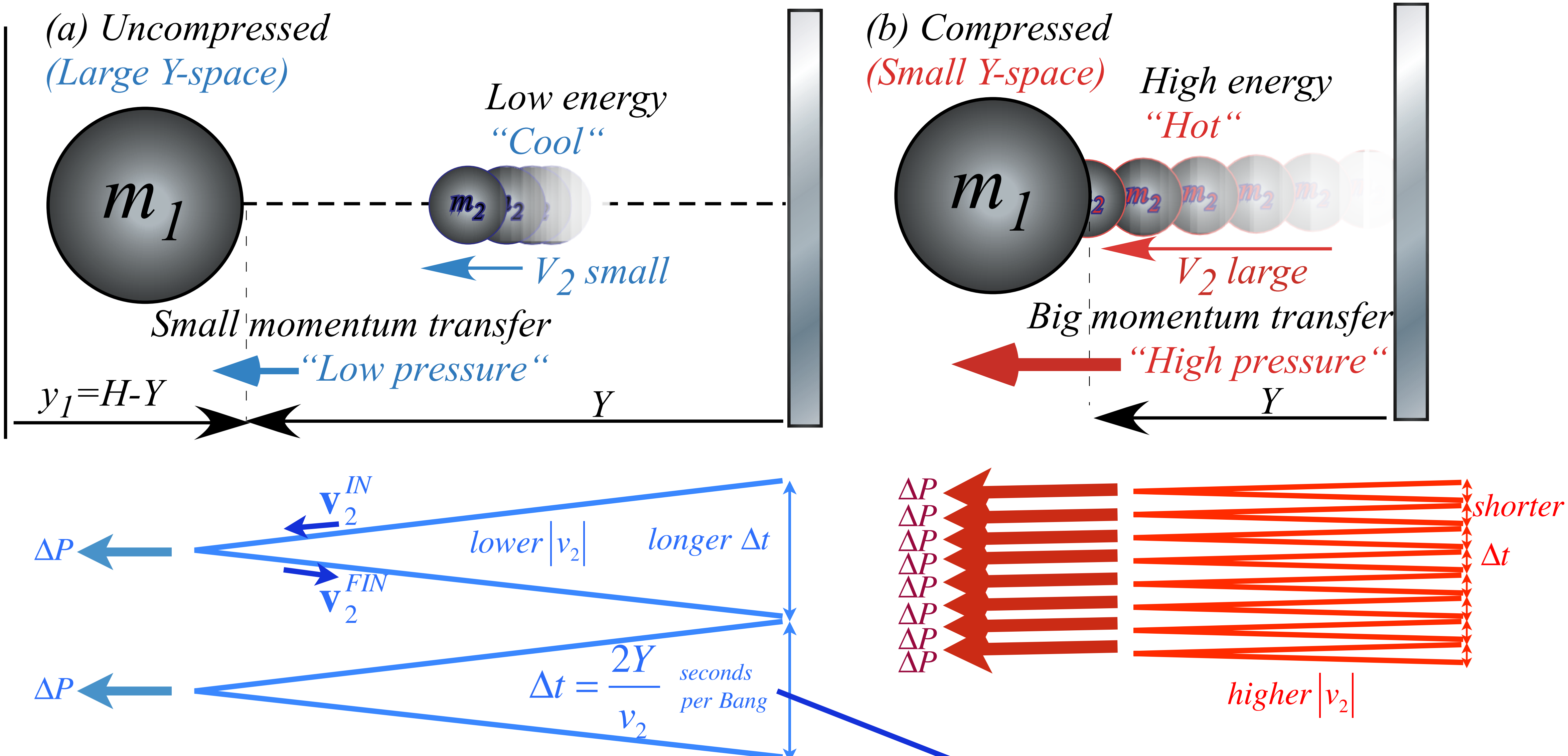
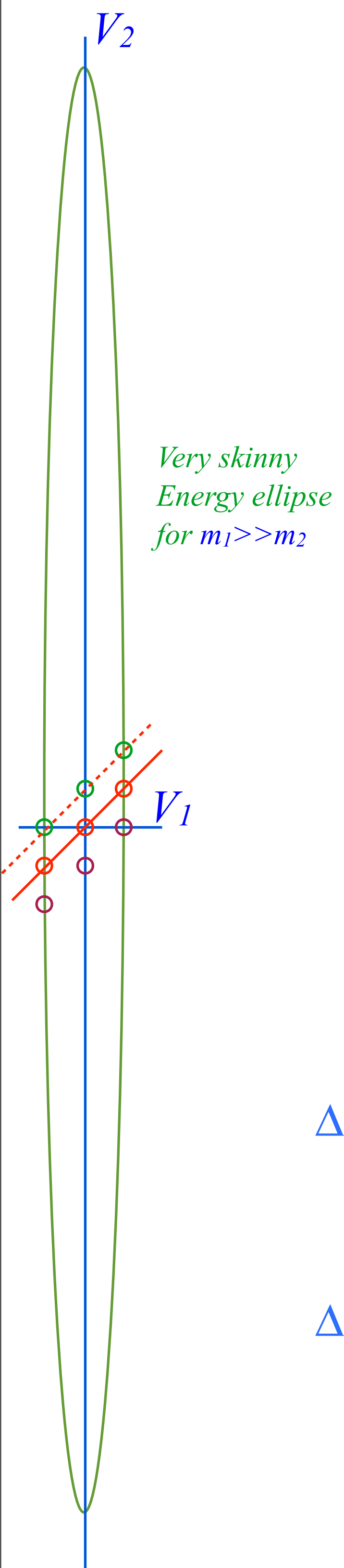


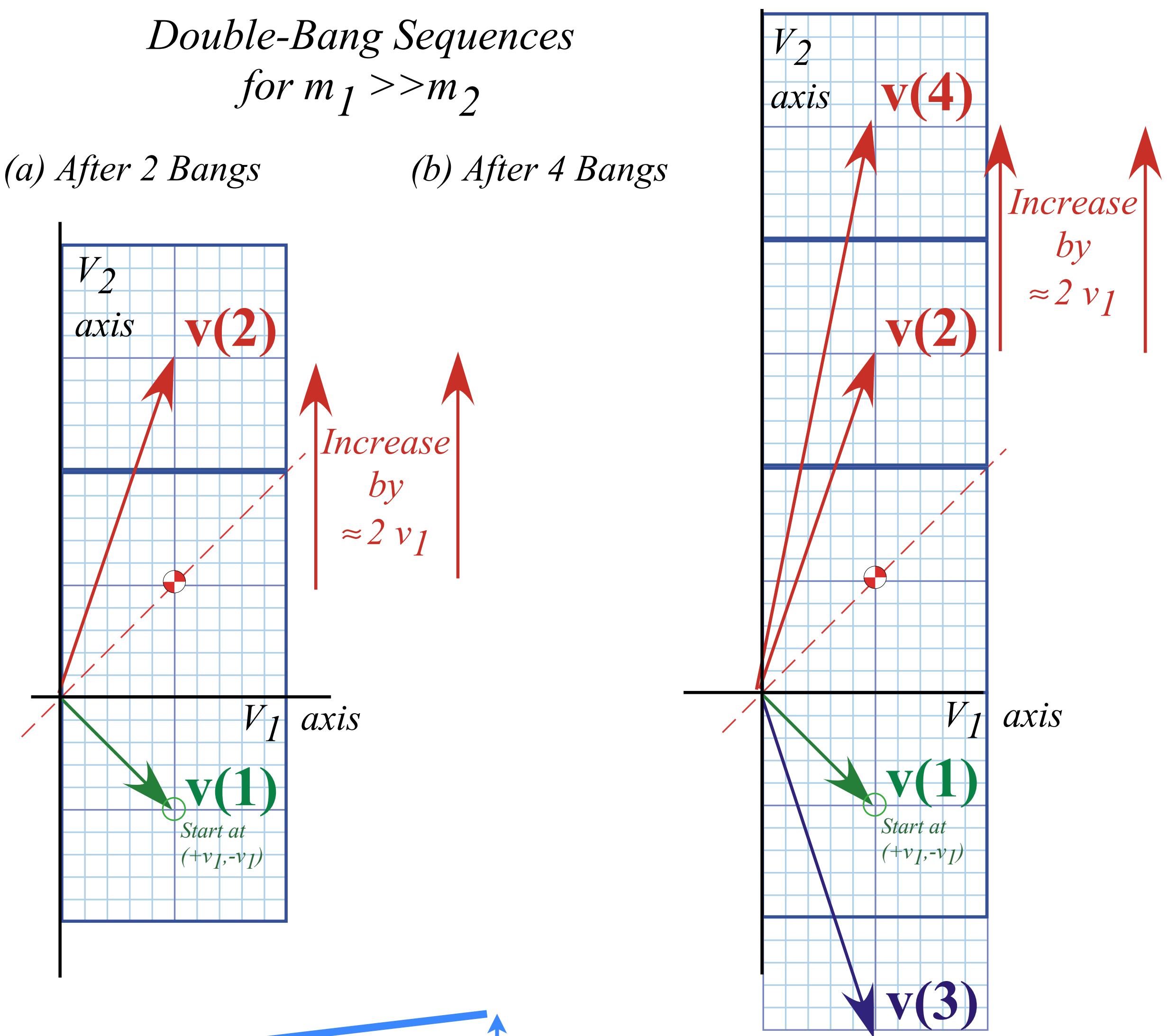
$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

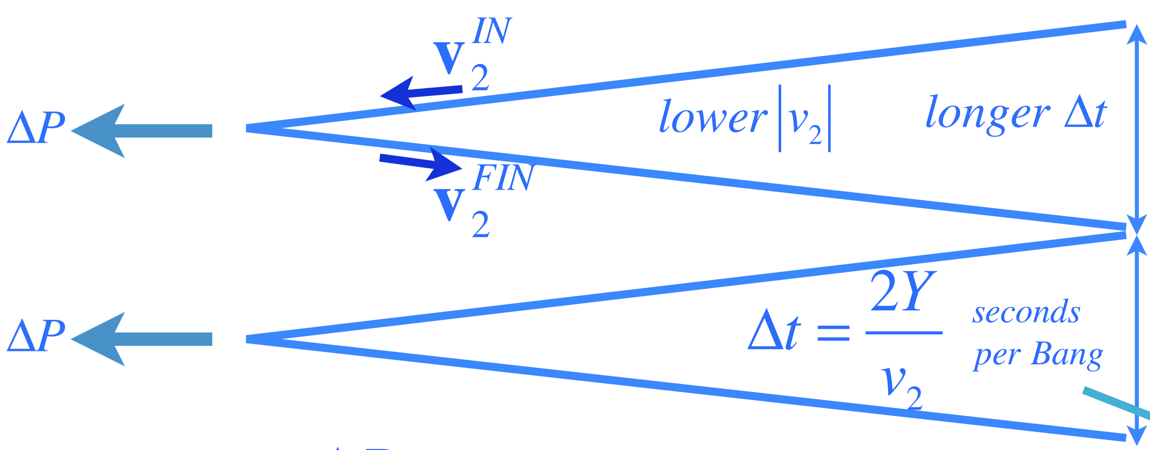


Double-Bang Sequences
for $m_1 >> m_2$

(a) After 2 Bangs (b) After 4 Bangs



Unit 1
Fig. 6.2



$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

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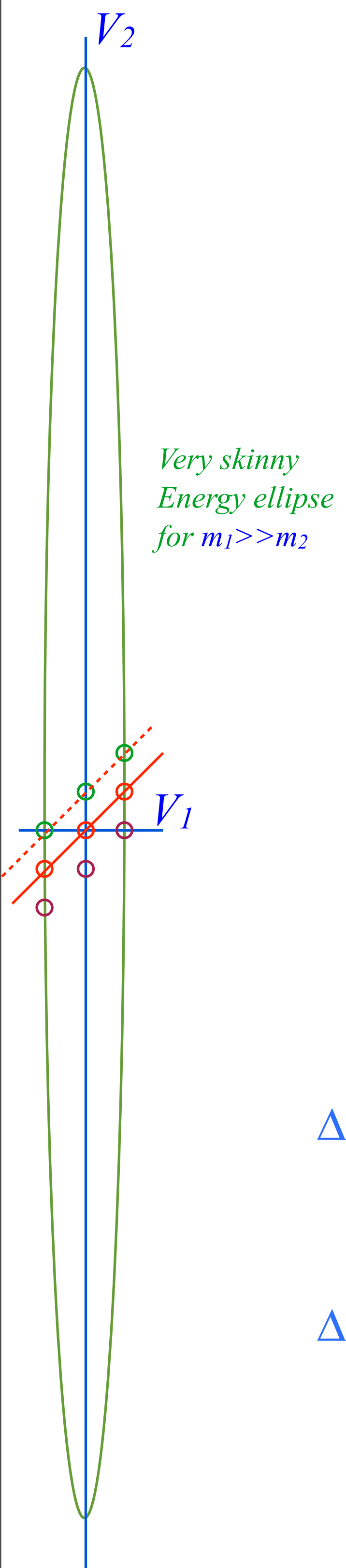

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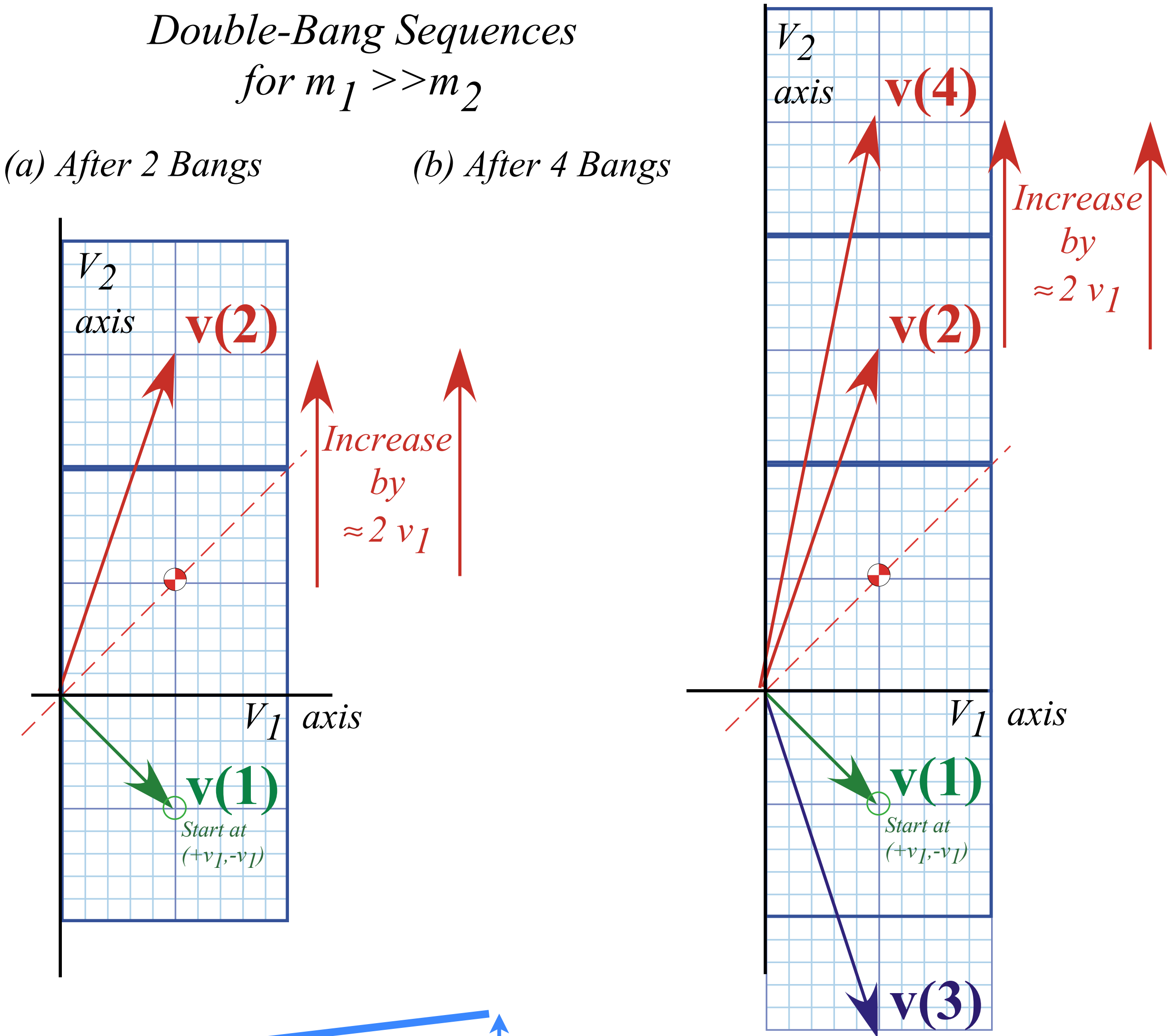
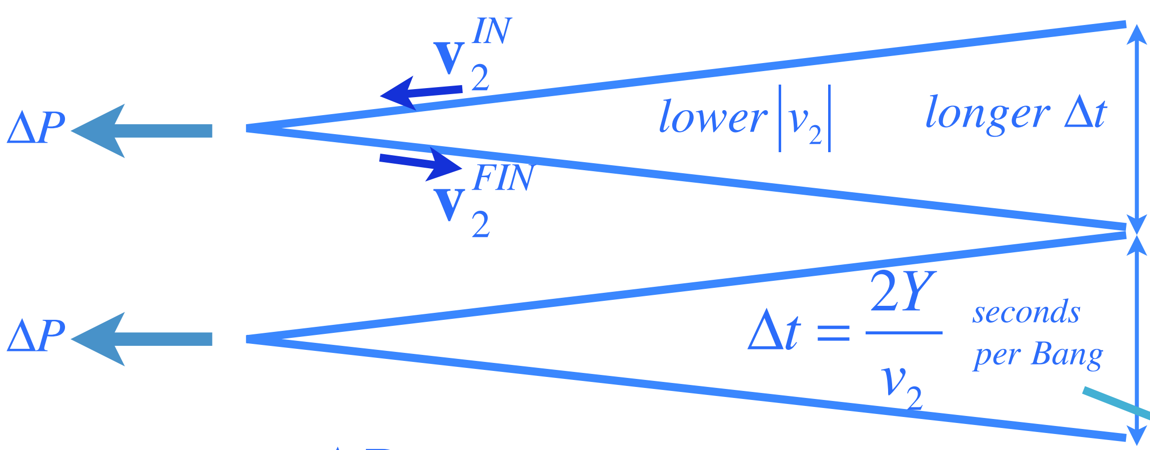


Unit 1
Fig. 6.2



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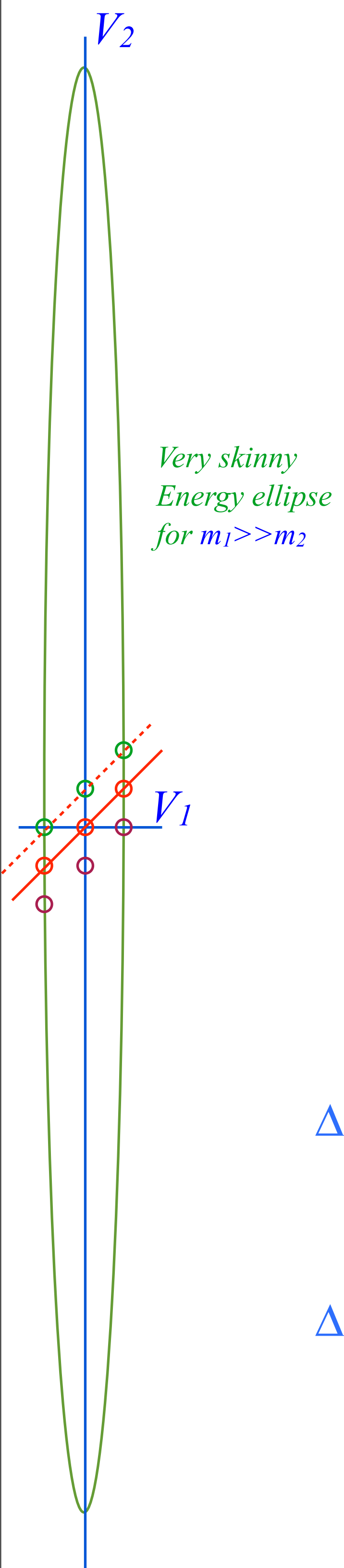

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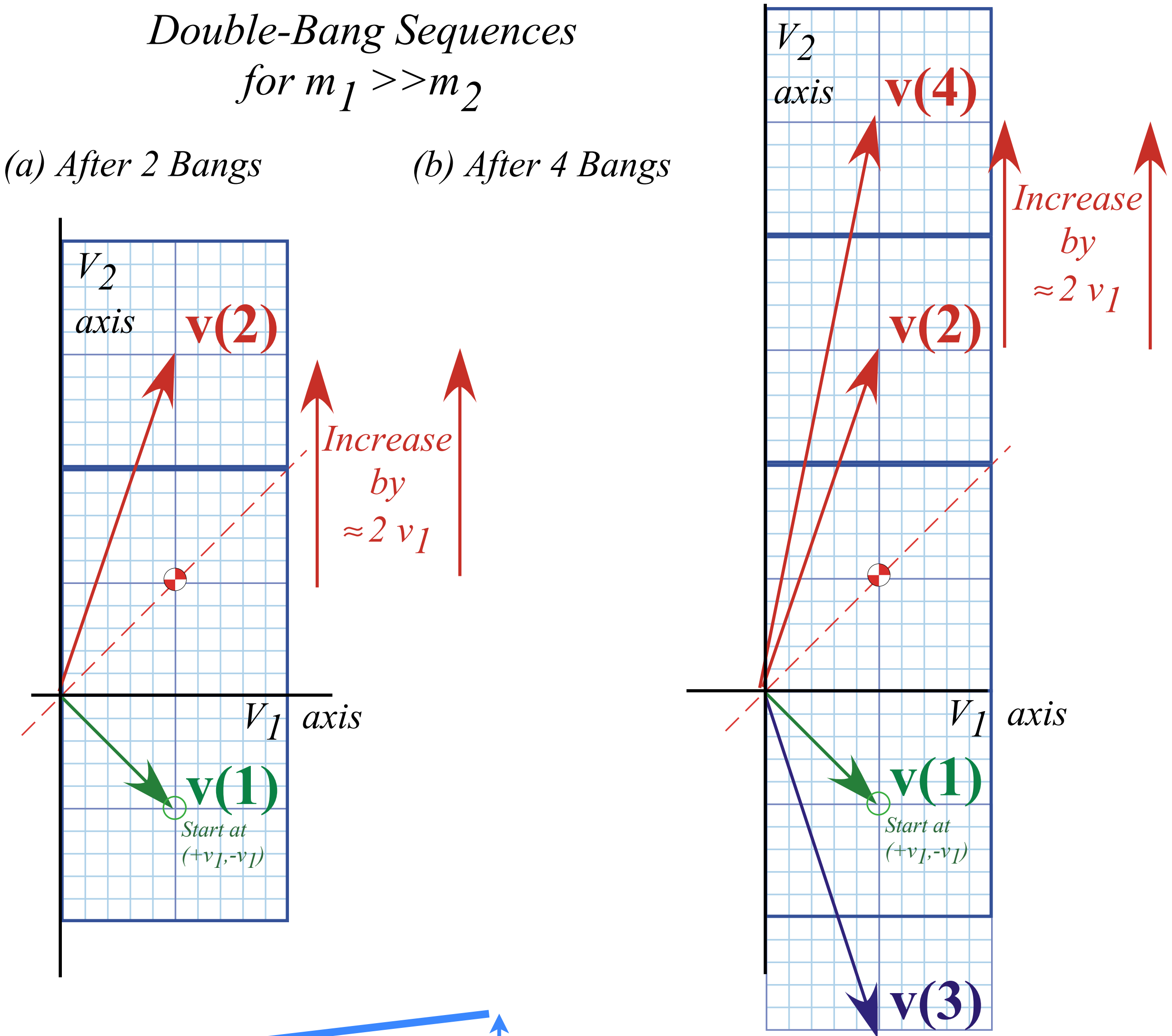
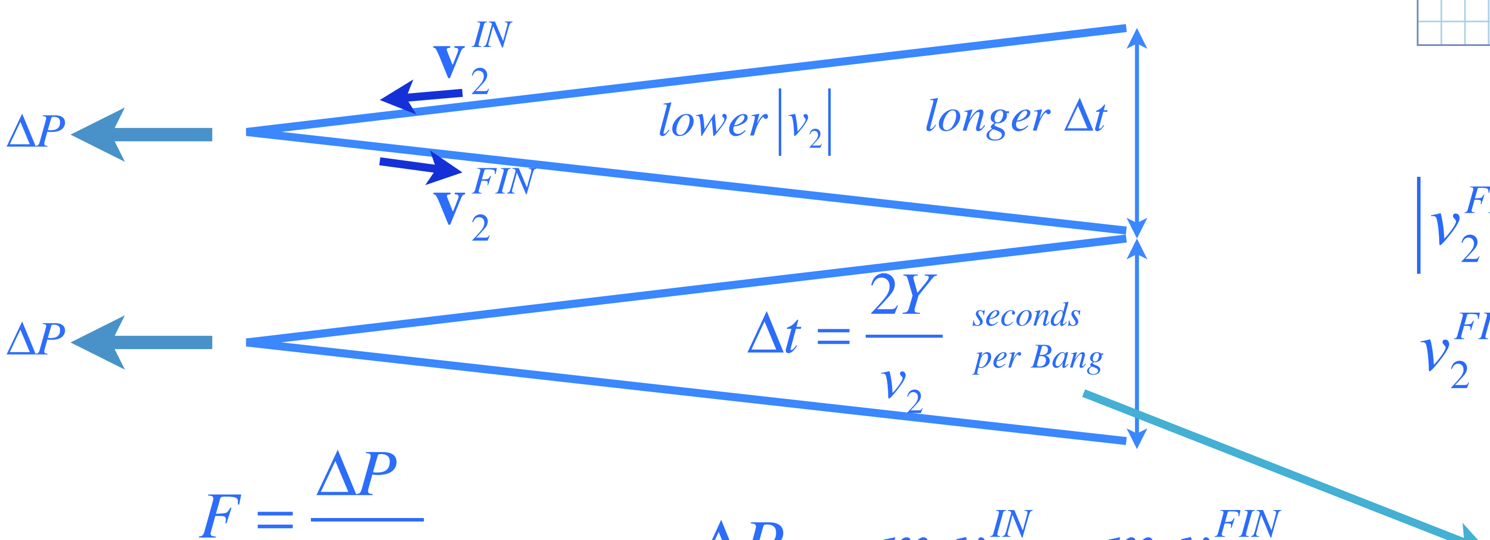


Unit 1
Fig. 6.2



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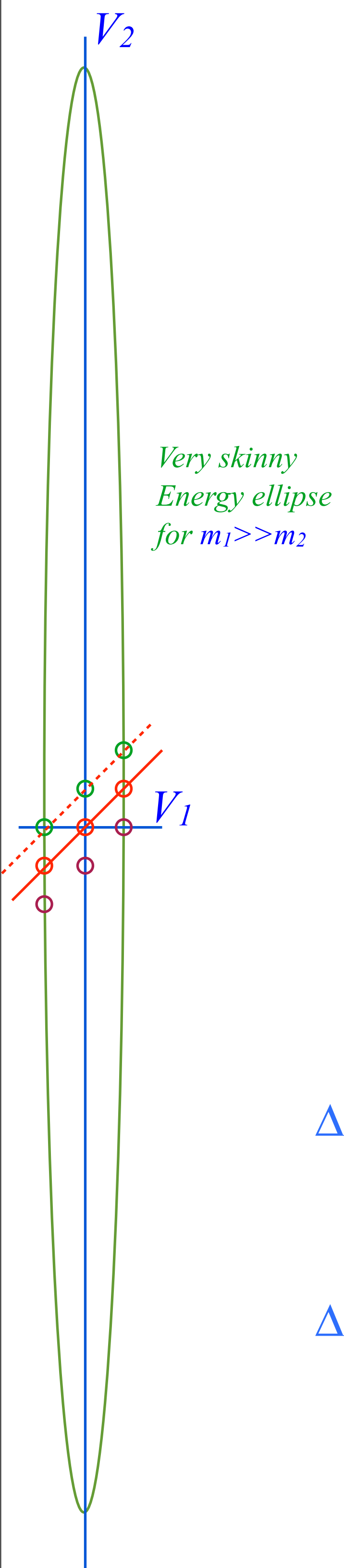
Assuming slow m_1 : $v_1 \ll v_2$



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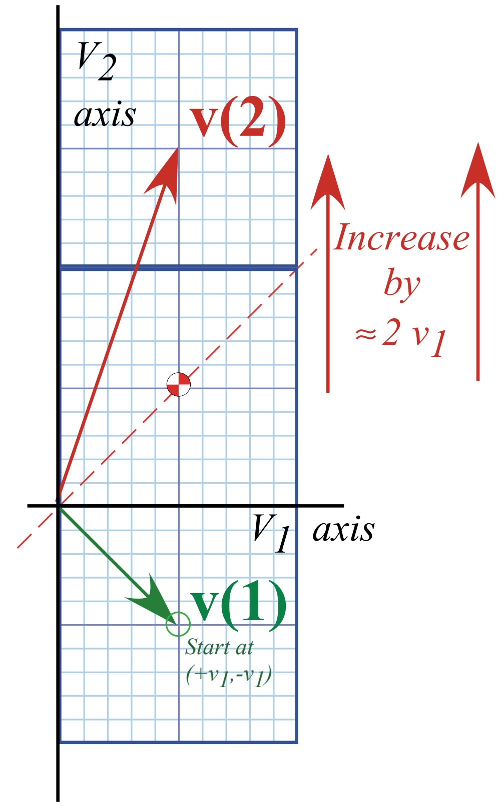
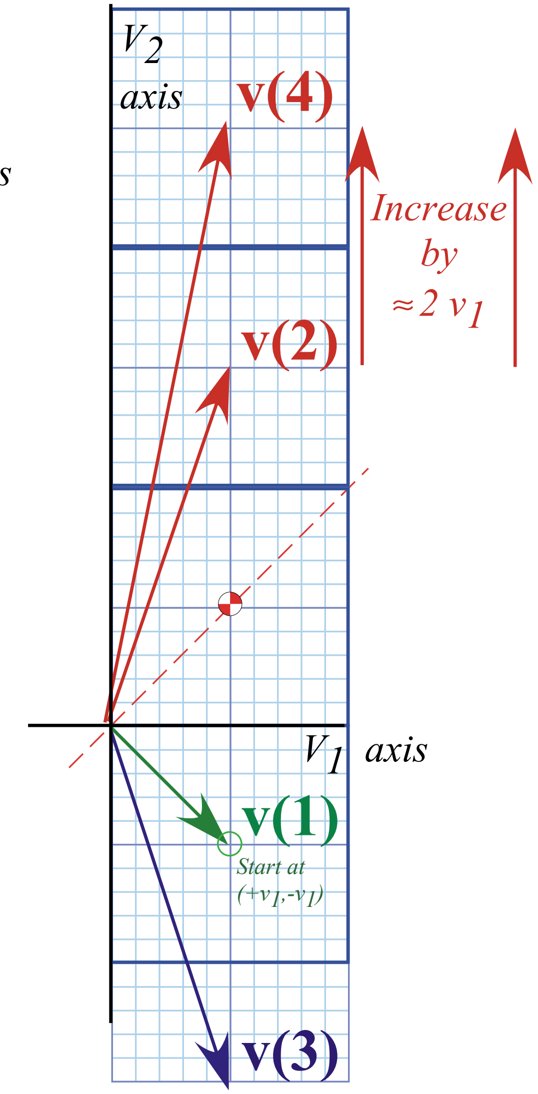
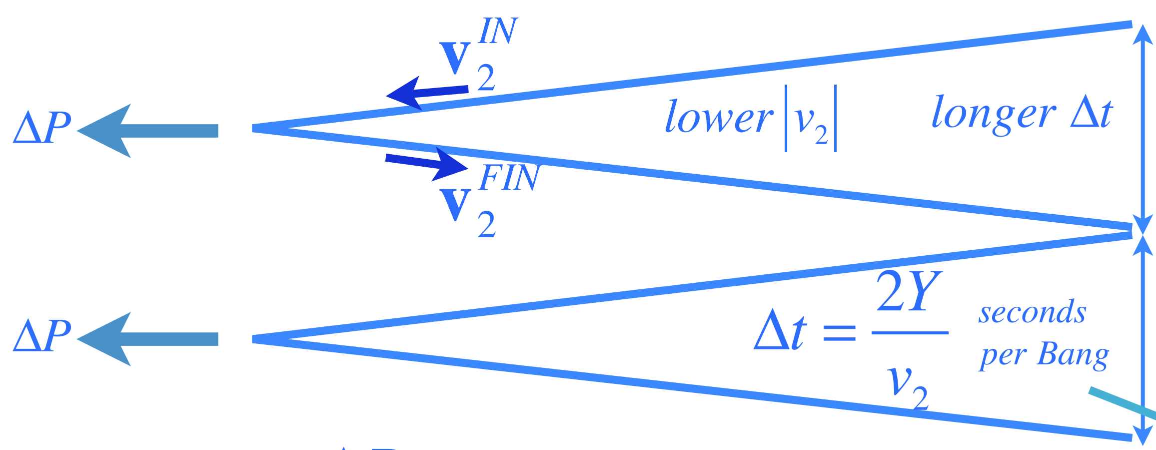


(b) After 4 Bangs



Unit 1
Fig. 6.2



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$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

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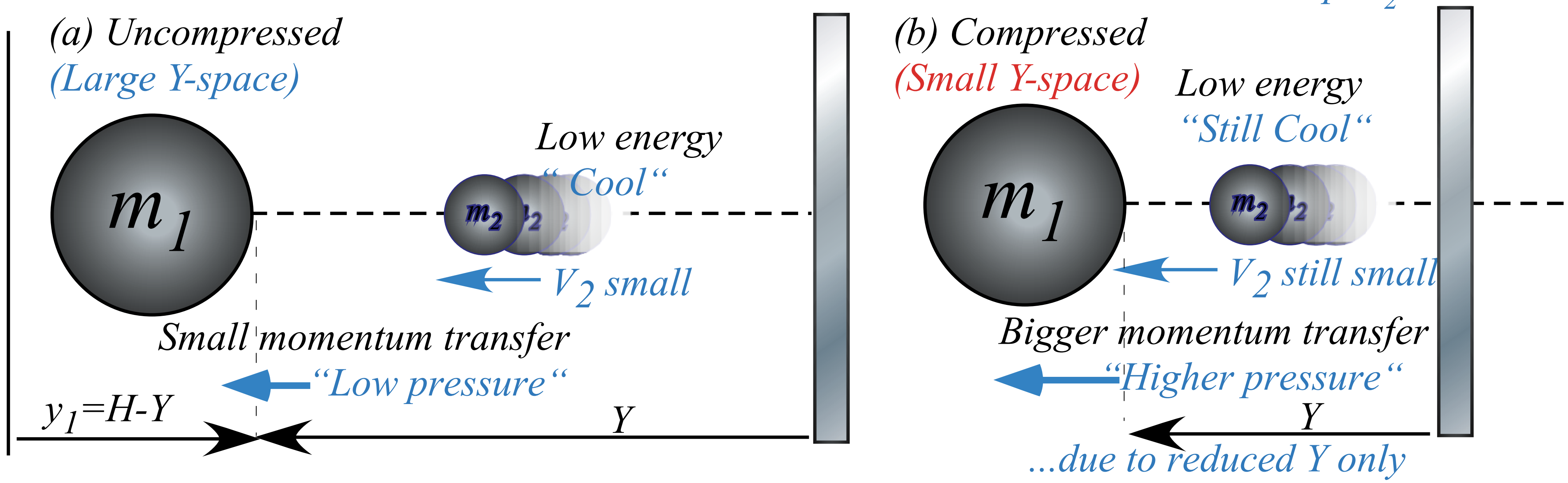
1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

Isothermal expansion or contraction: *Wall serves as thermal bath to keep m_2 cool*



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1D-Isothermal Force Law (assume v_2 is constant for all Y): $F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B = v_2/2Y$.

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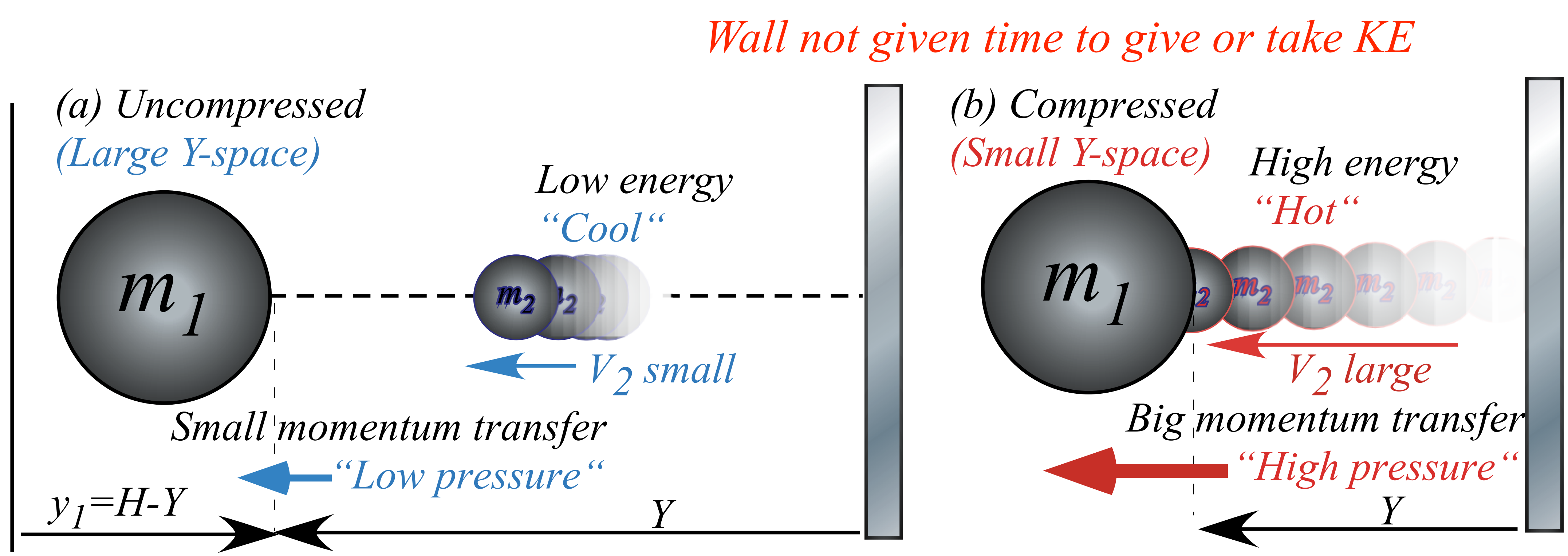

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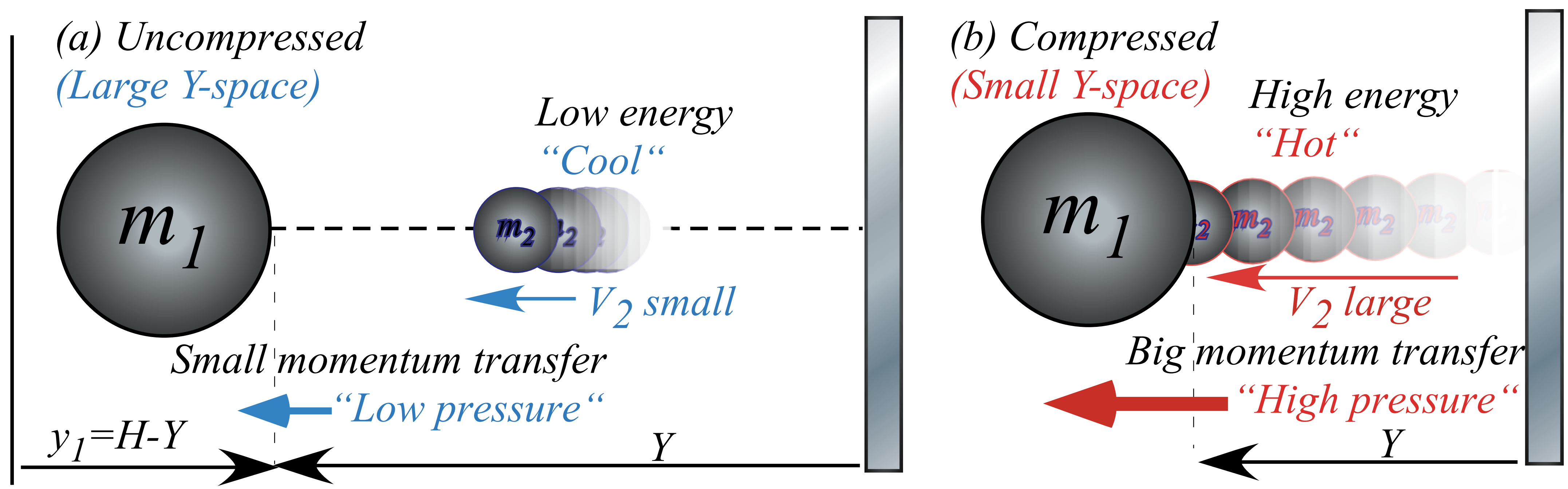
Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e(e^C x)$

$\frac{dv_2}{v_2} = -\frac{dY}{Y}$ integrates to: $\ln v_2 = -\ln Y + C$ or: $\ln v_2 = \ln \frac{const.}{Y}$ or: $v_2 = \frac{const.}{Y}$

Wall not given time to give or take KE



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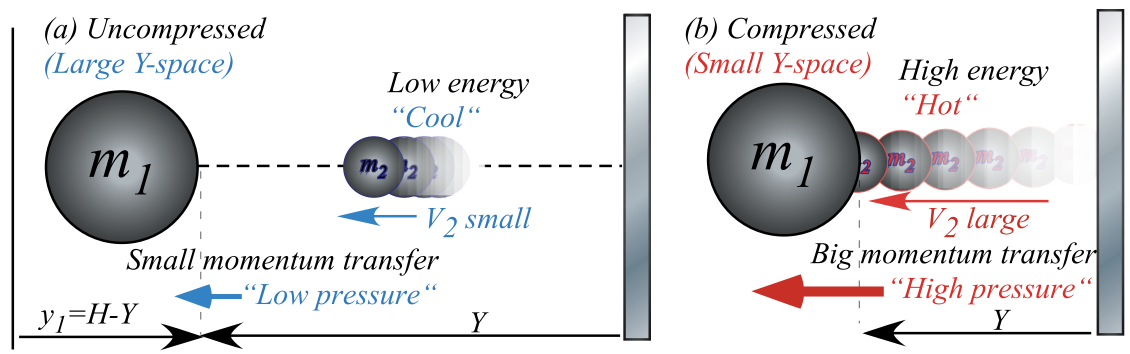
$\frac{dv_2}{v_2}=-\frac{dY}{Y}$ integrates to: $\ln v_2=-\ln Y+C$ or: $\ln v_2=\ln\frac{const.}{Y}$ or: $v_2=\frac{const.}{Y}$

Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2=\frac{const.}{Y}=\frac{v_2^{IN}Y(t=0)}{Y}$*):* $F=\frac{m_2\left(v_2^{IN}Y(t=0)\right)^2}{Y^3}=\frac{const.}{Y^3}$



Potential field due to many small bounces

→ *Example of 1D-Adiabatic potential $U(y)=const./y^2$*

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const. \ln(y)$

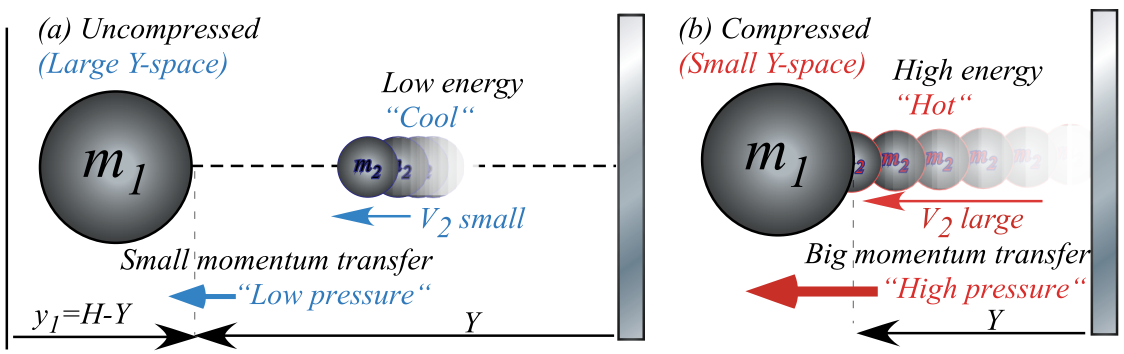
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy KE(v_1) vs Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



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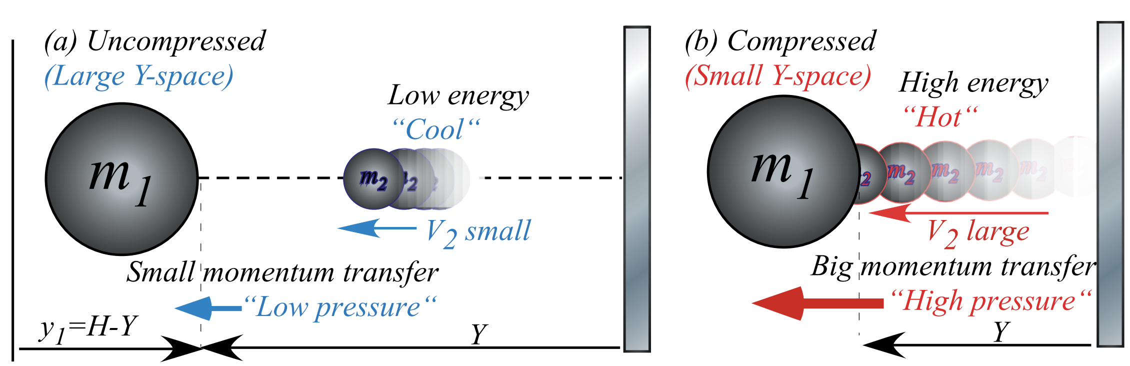
Potential energy PE(Y)=U(Y)$=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F \cdot dY = \pm dU$

Q*?Another axiom?* A: No.



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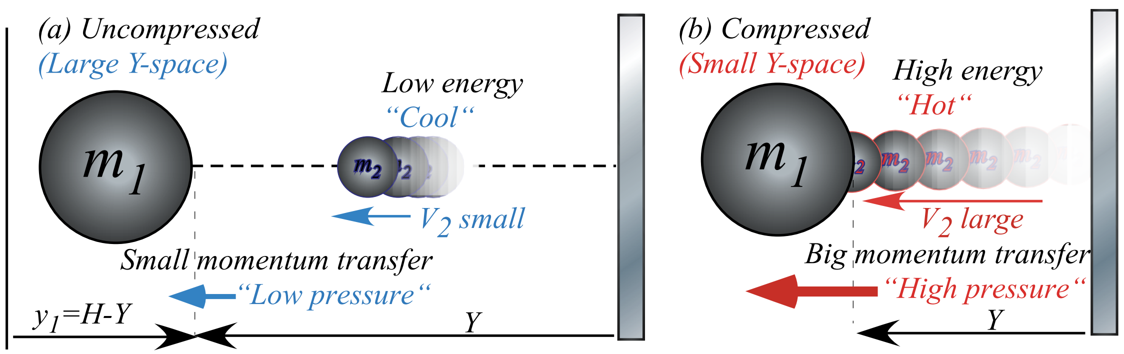

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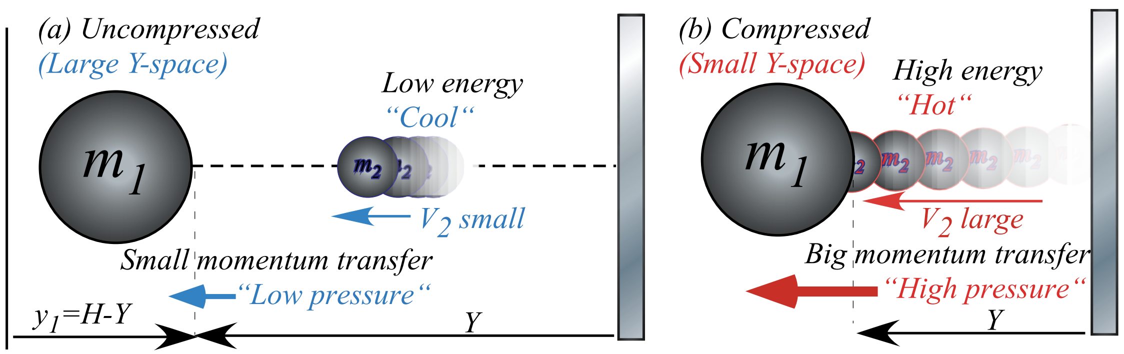
or else :

$$F \cdot \frac{dY}{dt} = \frac{dp}{dt} \cdot V = \frac{d(mV)}{dt} \cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt}$$



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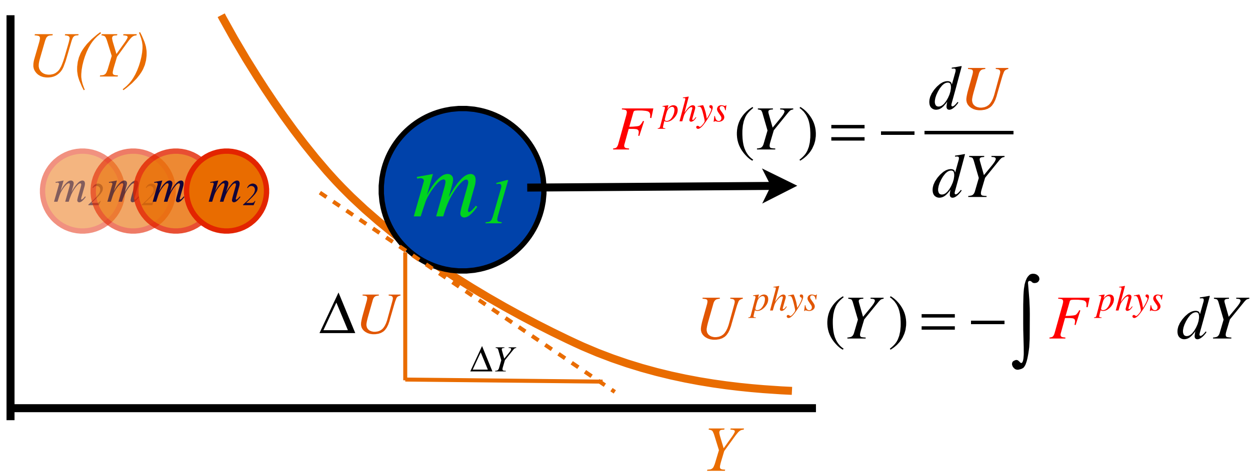
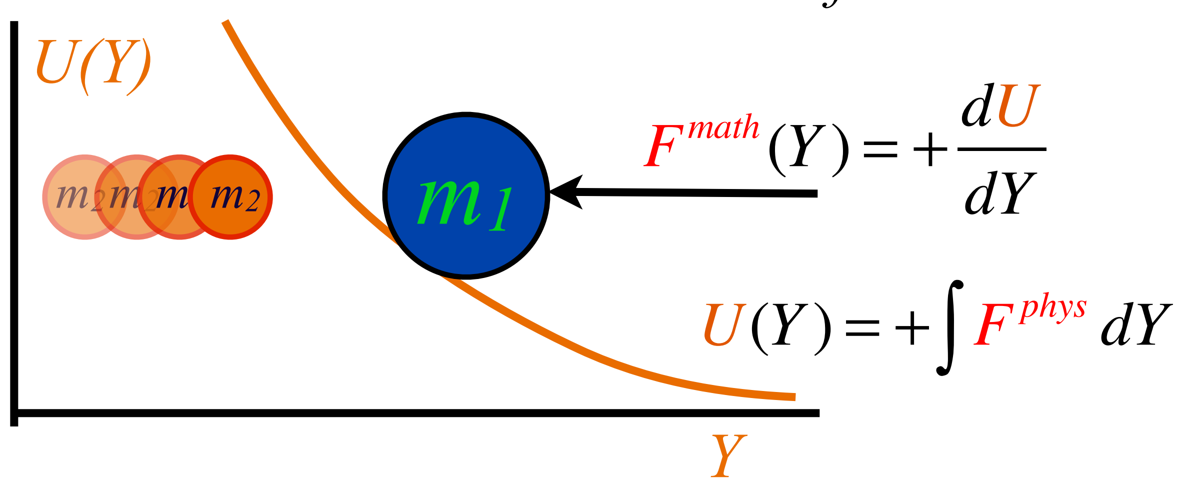
The "Physicist" View of Force



The "Mathematician" View of Force



Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

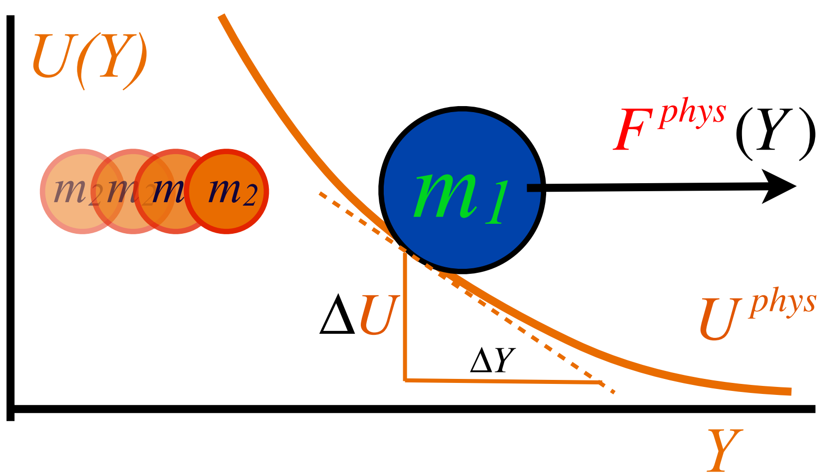
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

Potential energy PE(Y)=U(Y)=$\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F \cdot dY = \pm dU$

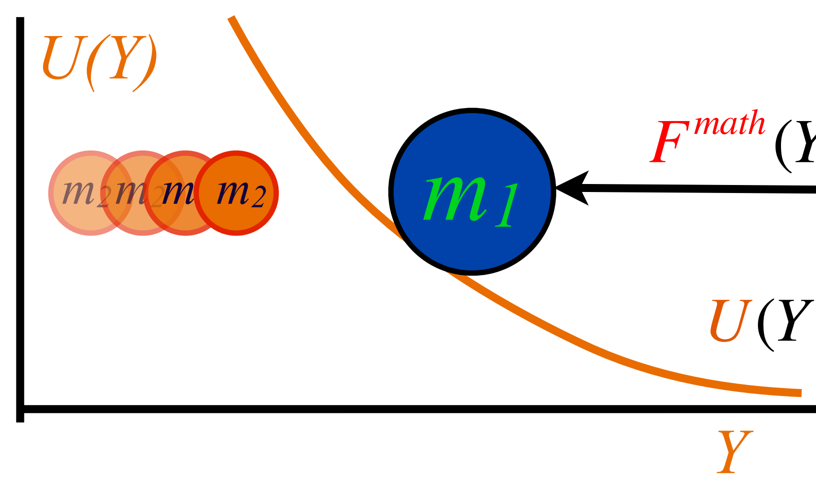
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

(OK, But, does it work?)

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

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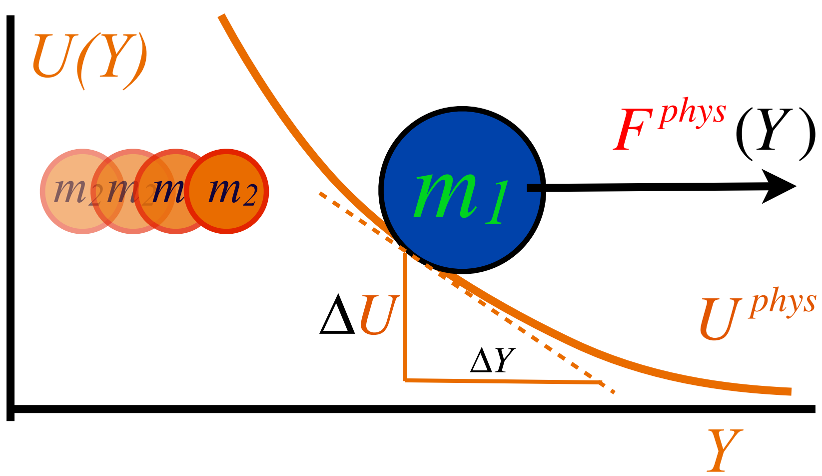

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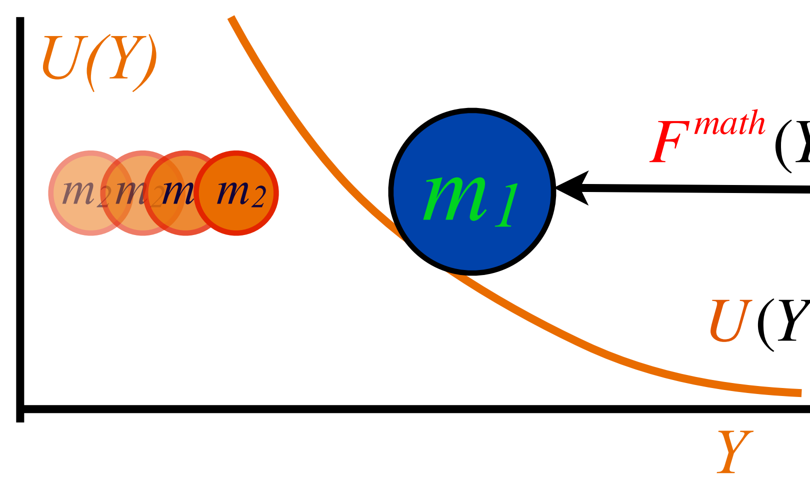
The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

(OK, But, does it work?)

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with :

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

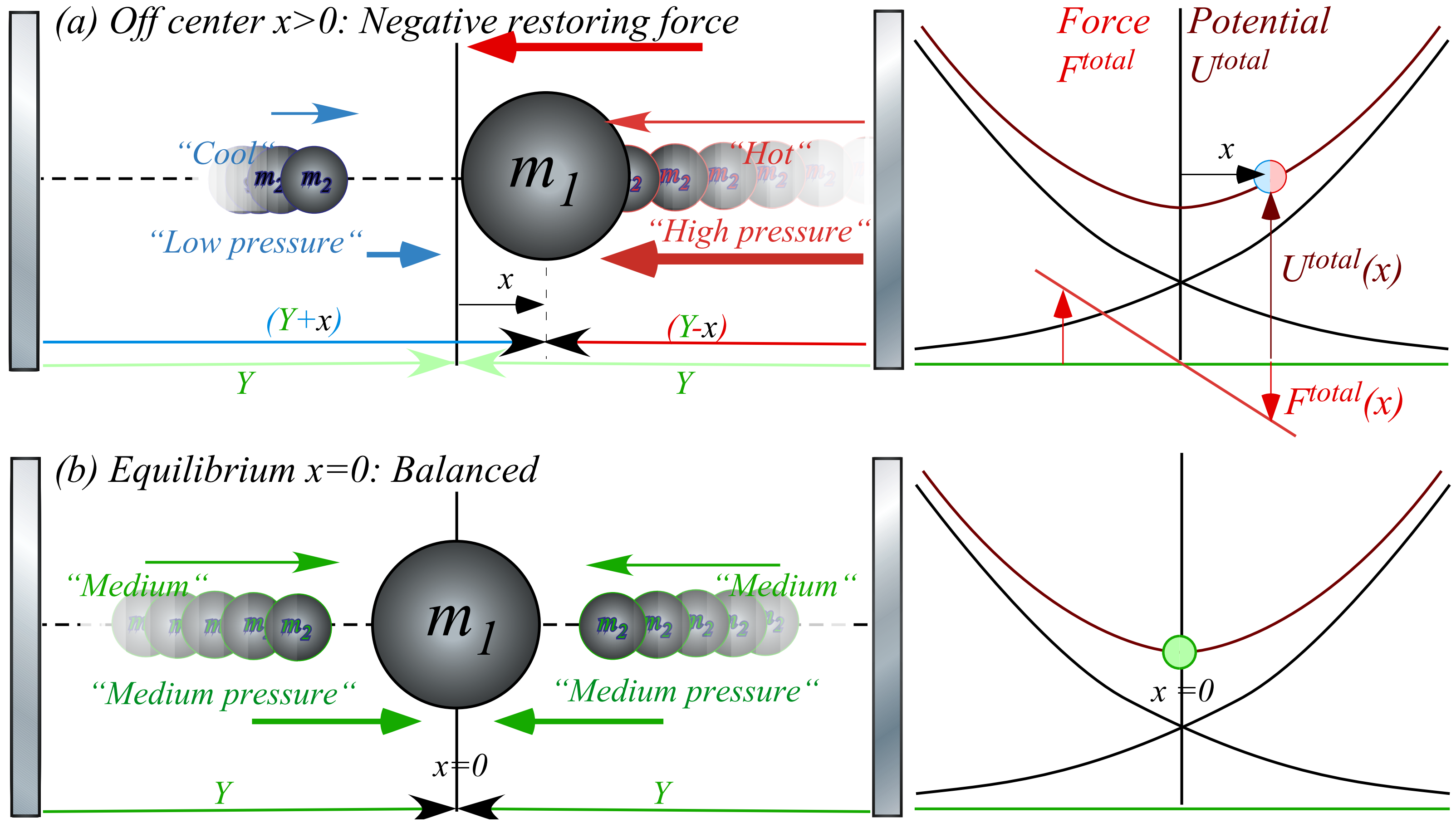
Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

→ *Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$*

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U=-m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

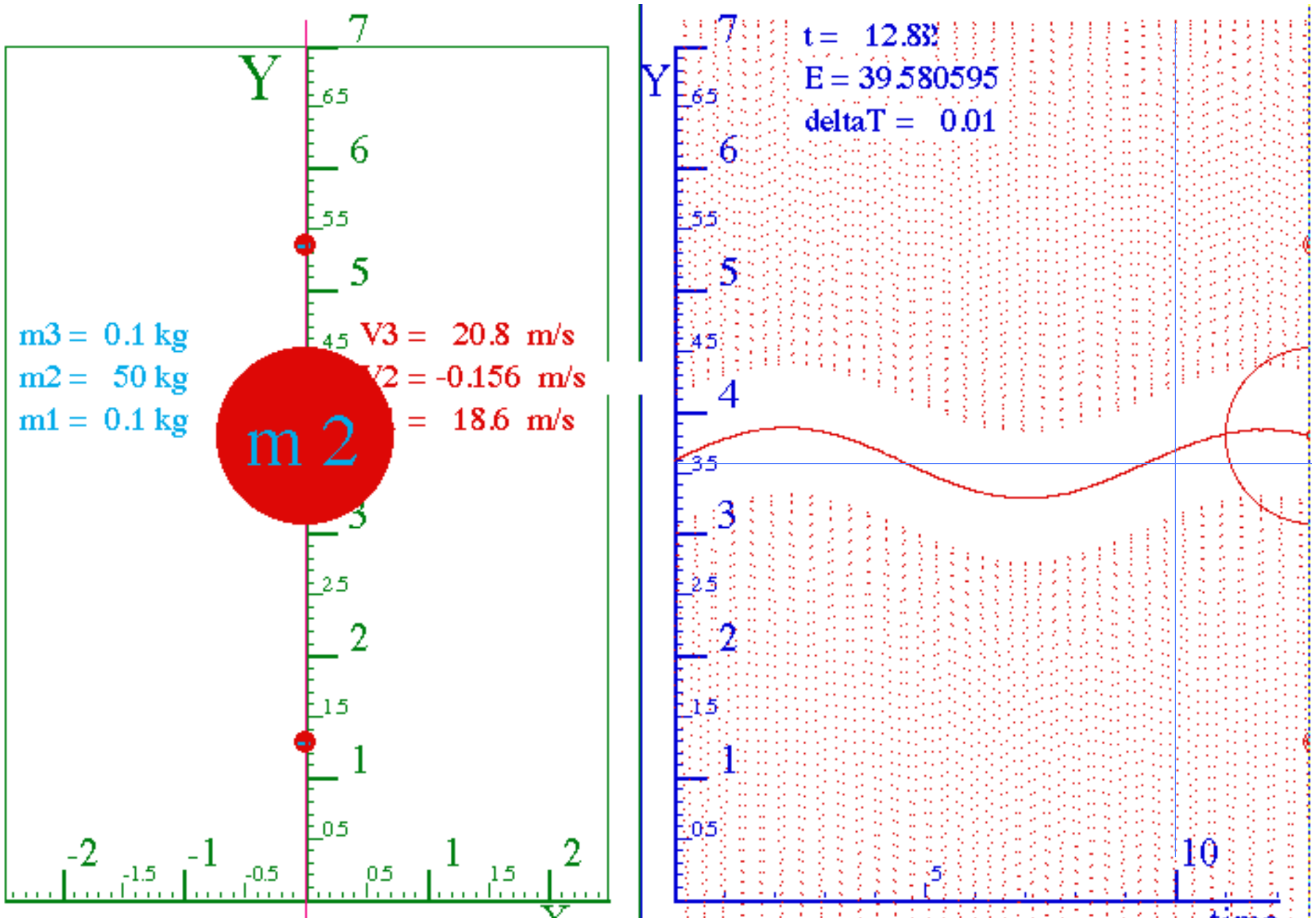
Anharmonic oscillator terms...

Harmonic oscillator term

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{1+x}-\frac{f}{1-x}=f\left[1-x+x^2-x^3...\right]-f\left[1+x+x^2+x^3...\right]=-2f\cdot x-2f\cdot x^3-...$$

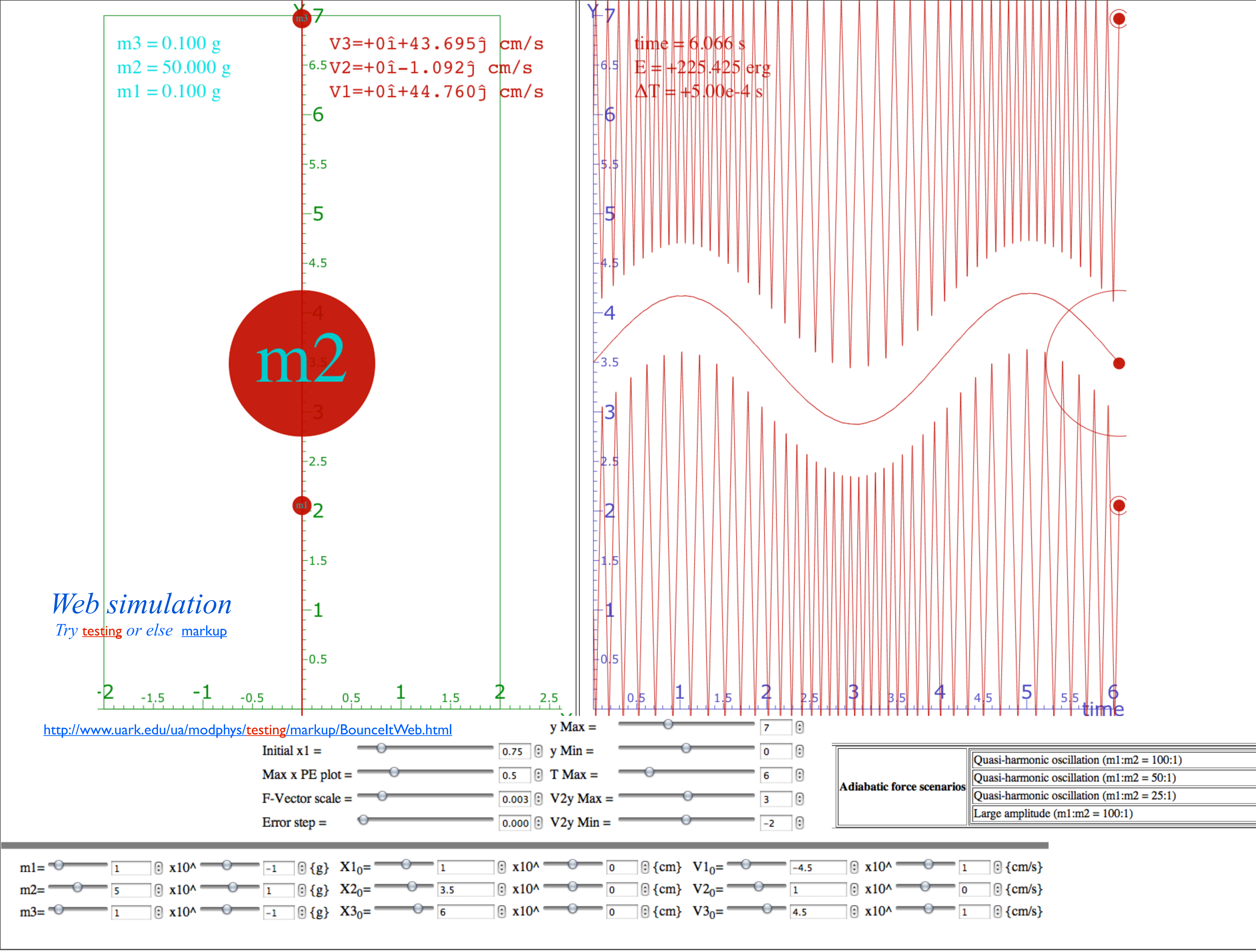
$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x} \qquad U^{HO}=\frac{1}{2}k\cdot x^2=-\int F^{HO}\,dx \qquad \text{HO}\ \measuredangle\text{frequency: } \omega=\sqrt{\frac{k}{m_1}}=2\pi\upsilon$$



Unit 1
Fig. 6.3

Simulation of
the adiabatic case

See Homework problem 1.6.1: *Compute frequency and/or period for both isoT and adiabatic cases*


m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g
V3=+0î+43.695ĵ cm/s
V2=+0î-1.092ĵ cm/s
V1=+0î+44.760ĵ cm/s
m2
m3
m1
time = 6.066 s
E = +225.425 erg
ΔT = +5.00e-4 s
time
Web simulation
Try testing or else markup
http://www.uark.edu/ua/modphys/testing/markup/BounceItWeb.html
Initial x1 = 0.75
Max x PE plot = 0.5
F-Vector scale = 0.003
Error step = 0.000
y Max = 7
y Min = 0
T Max = 6
V2y Max = 3
V2y Min = -2
Adiabatic force scenarios
Quasi-harmonic oscillation (m1:m2 = 100:1)
Quasi-harmonic oscillation (m1:m2 = 50:1)
Quasi-harmonic oscillation (m1:m2 = 25:1)
Large amplitude (m1:m2 = 100:1)
m1= 1 x10^ -1 {g} X1₀= 1 x10^ 0 {cm} V1₀= -4.5 x10^ 1 {cm/s}
m2= 5 x10^ 1 {g} X2₀= 3.5 x10^ 0 {cm} V2₀= 1 x10^ 0 {cm/s}
m3= 1 x10^ -1 {g} X3₀= 6 x10^ 0 {cm} V3₀= 4.5 x10^ 1 {cm/s}

"Monster Mash" classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

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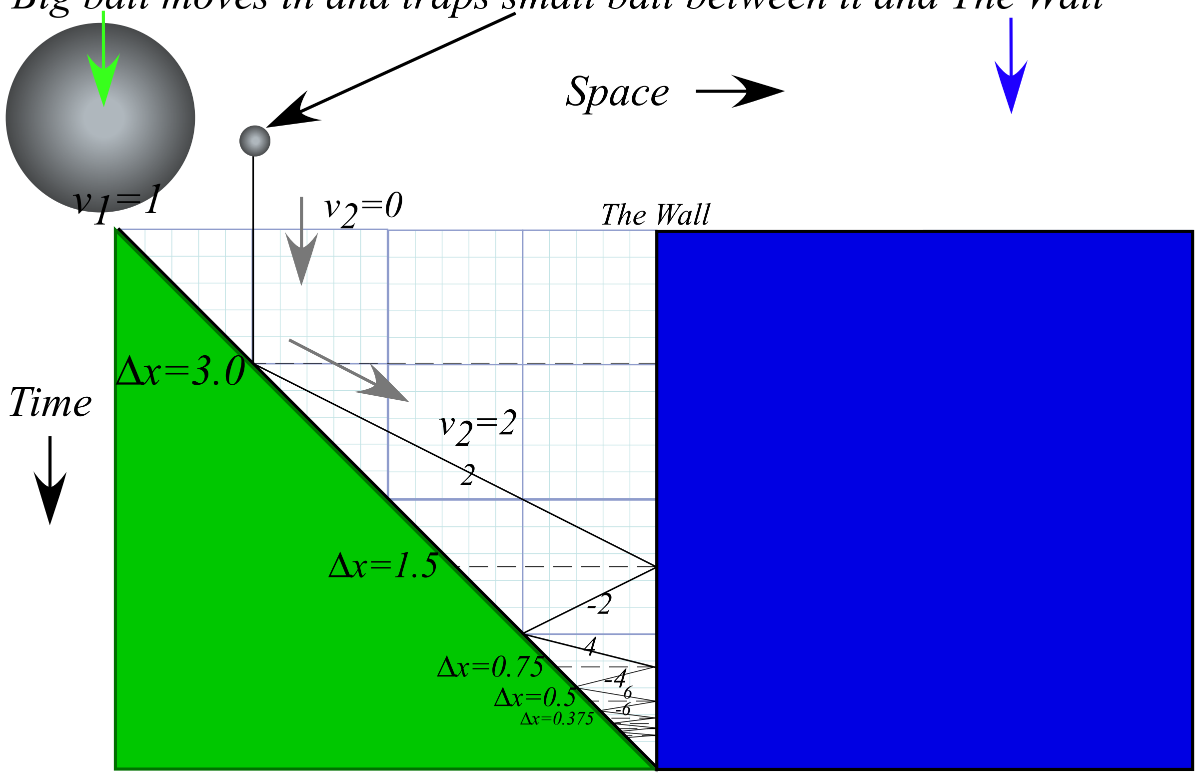
[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]

The Classical "Monster Mash"

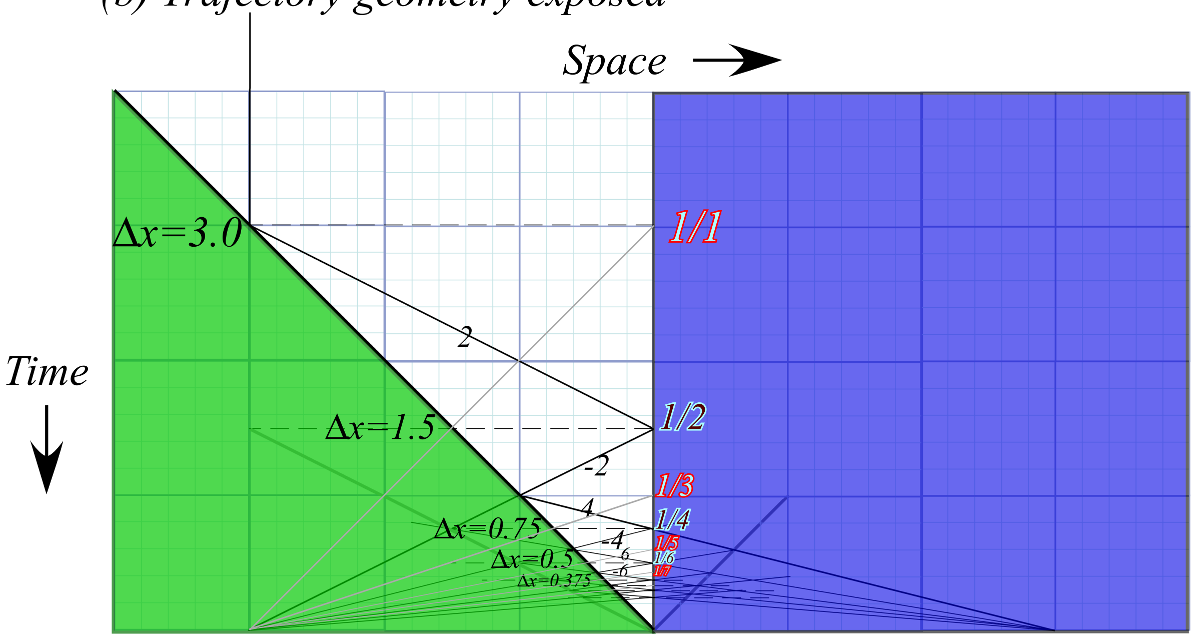
Classical introduction to

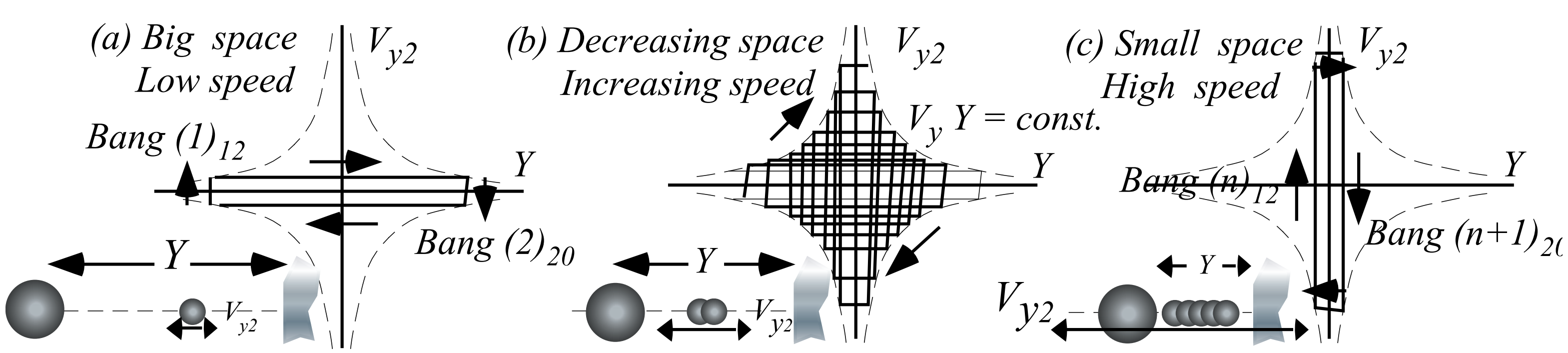
Heisenberg "Uncertainty" Relations

(a) Big ball moves in and traps small ball between it and The Wall



(b) Trajectory geometry exposed



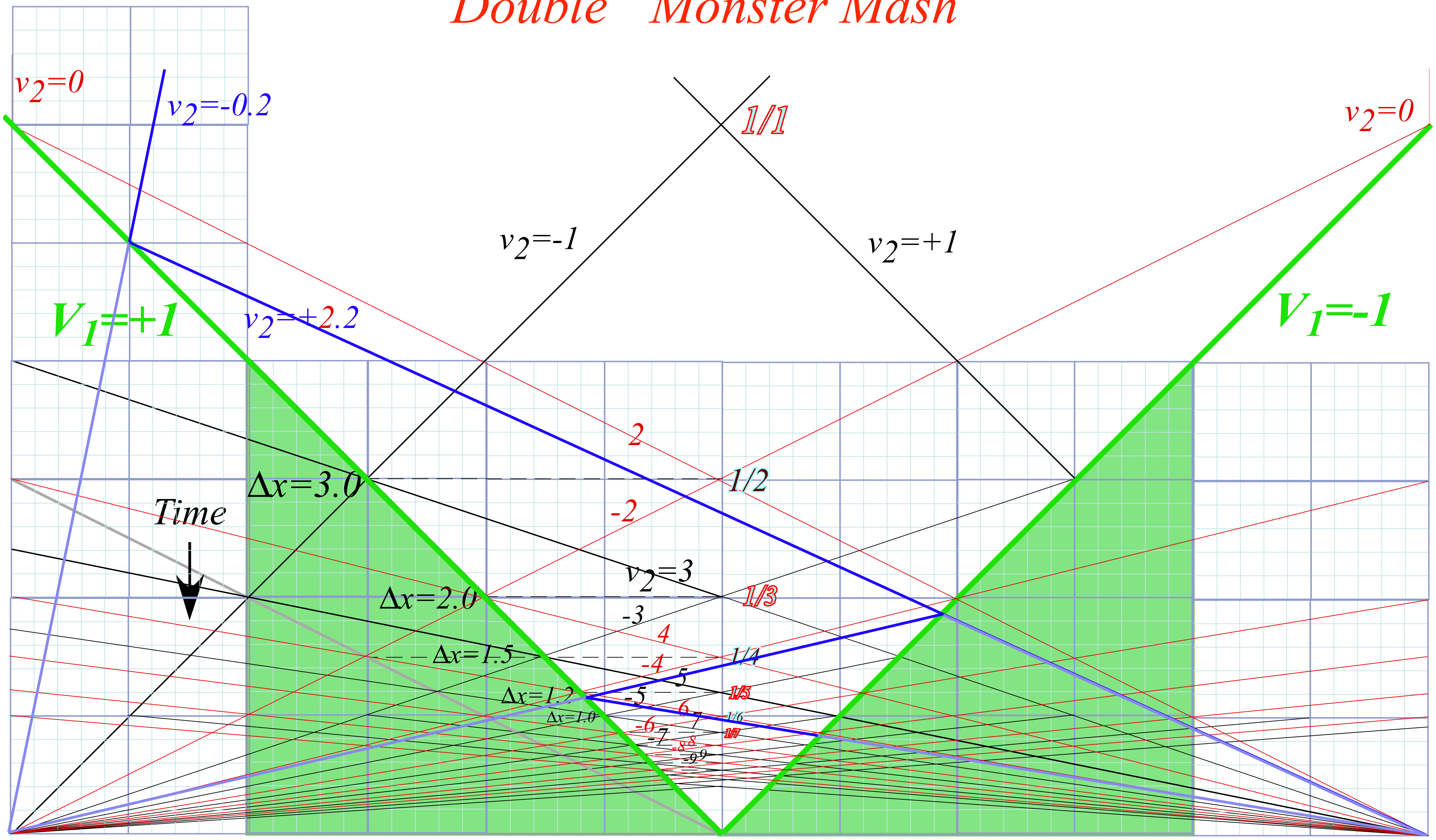


$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

Unit 1
Fig. 6.4

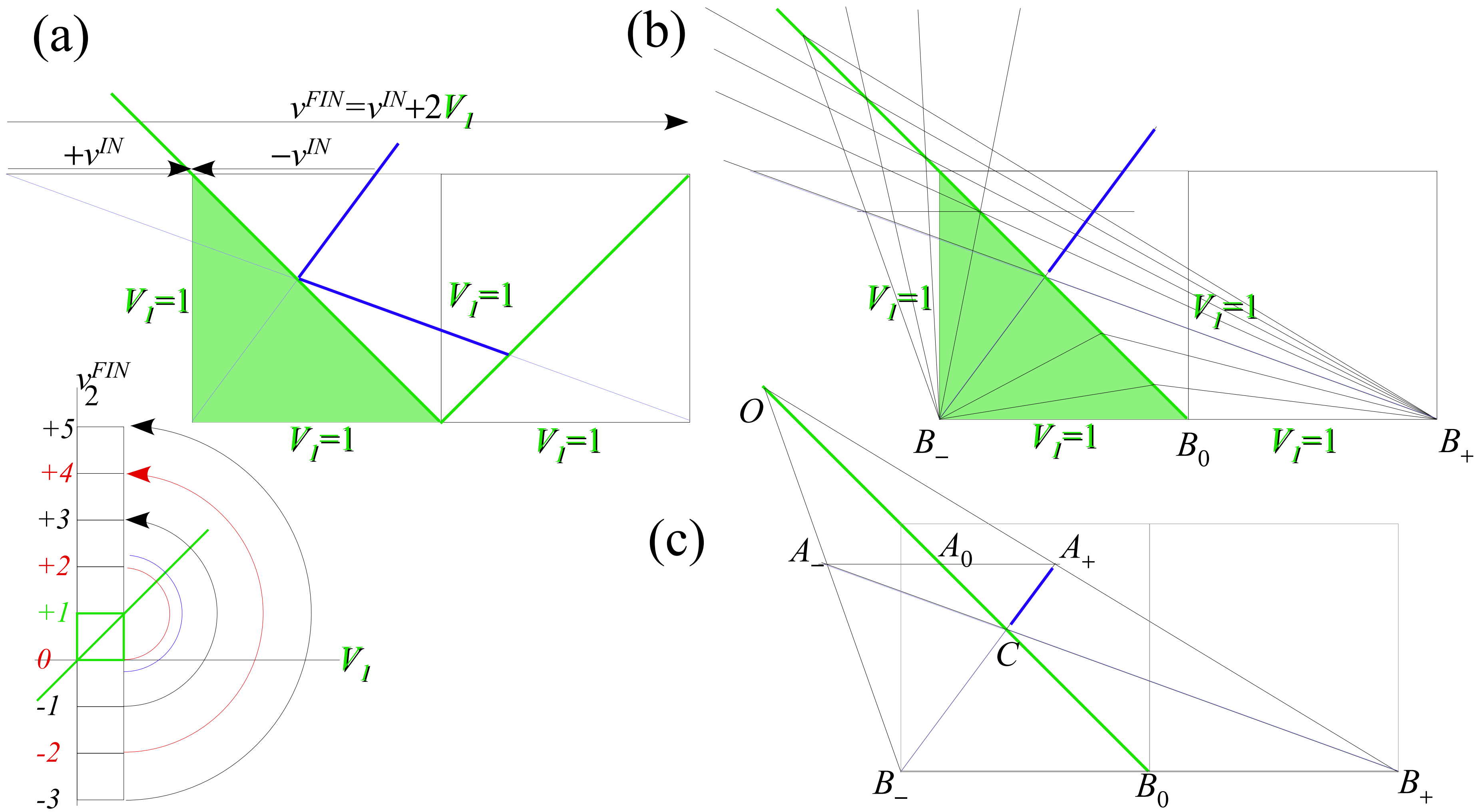
Double "Monster Mash"

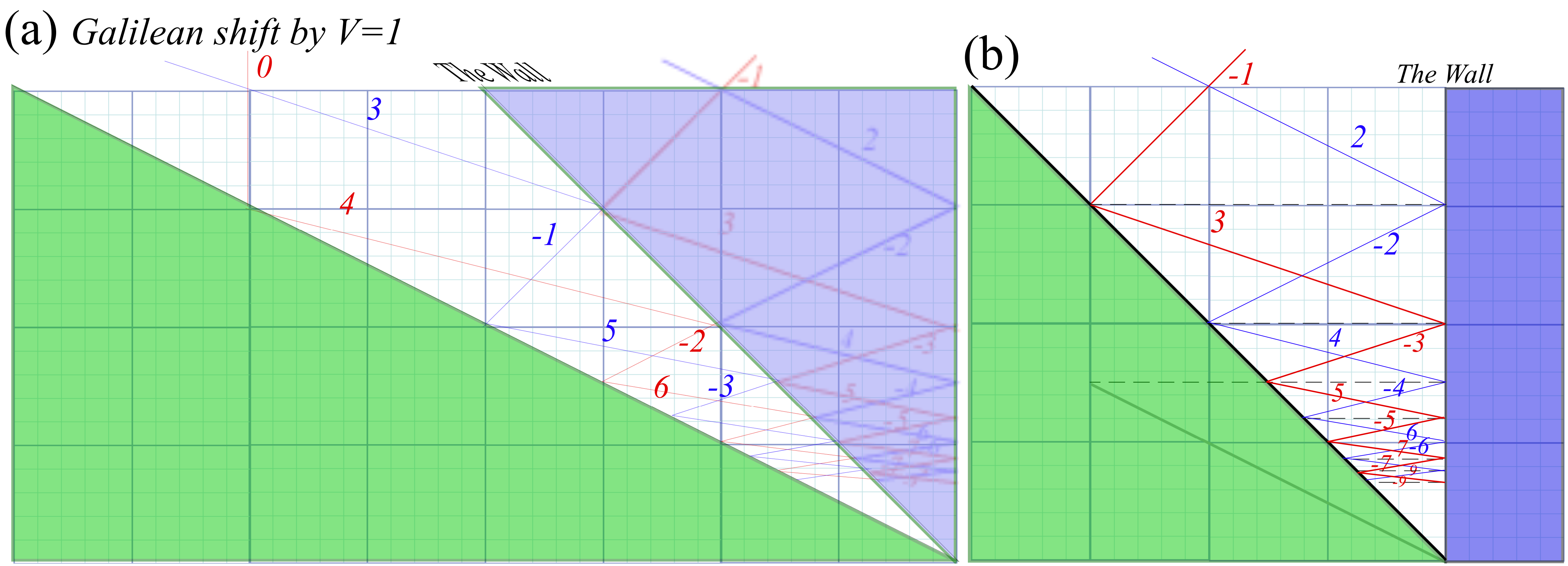


Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7





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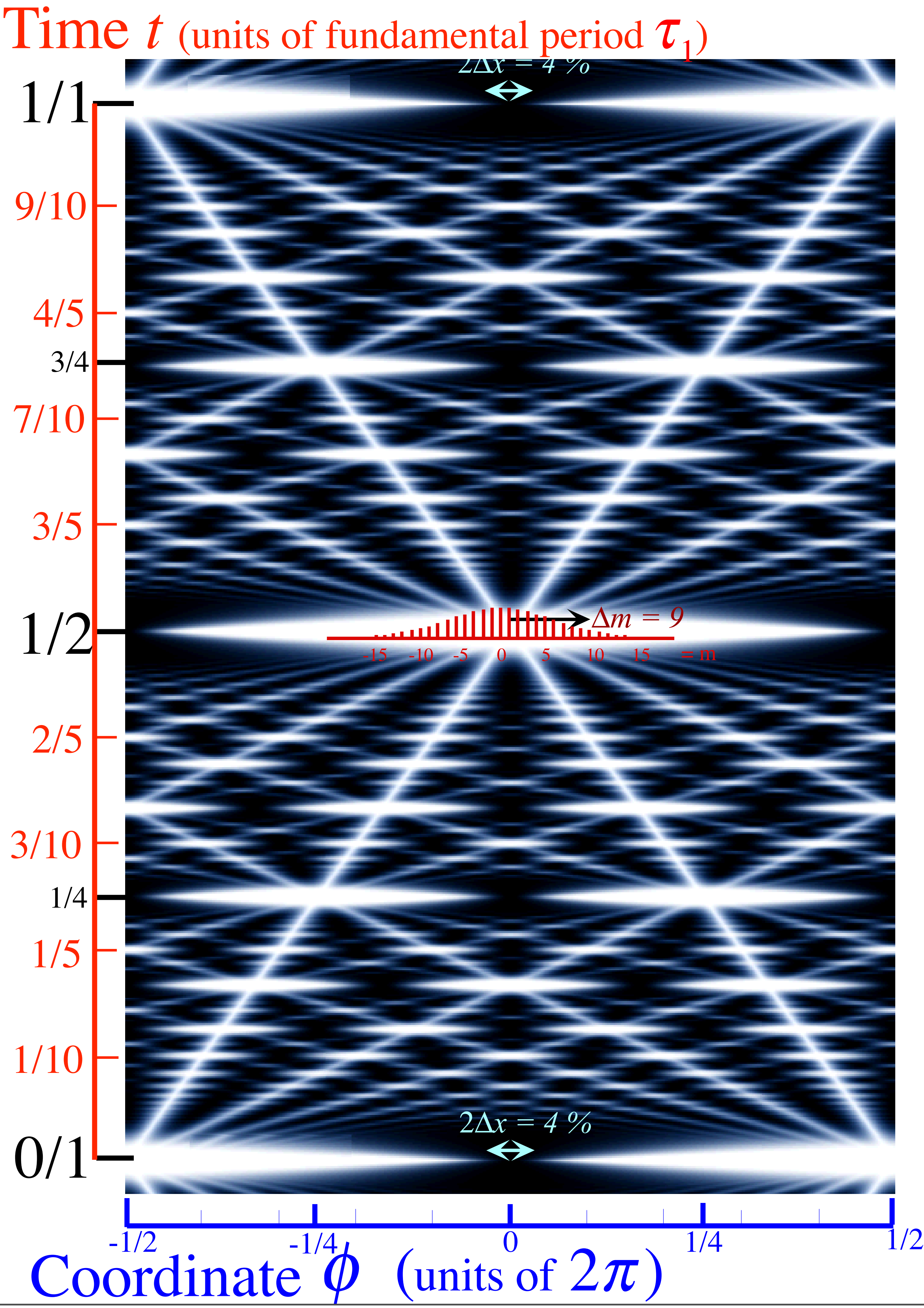
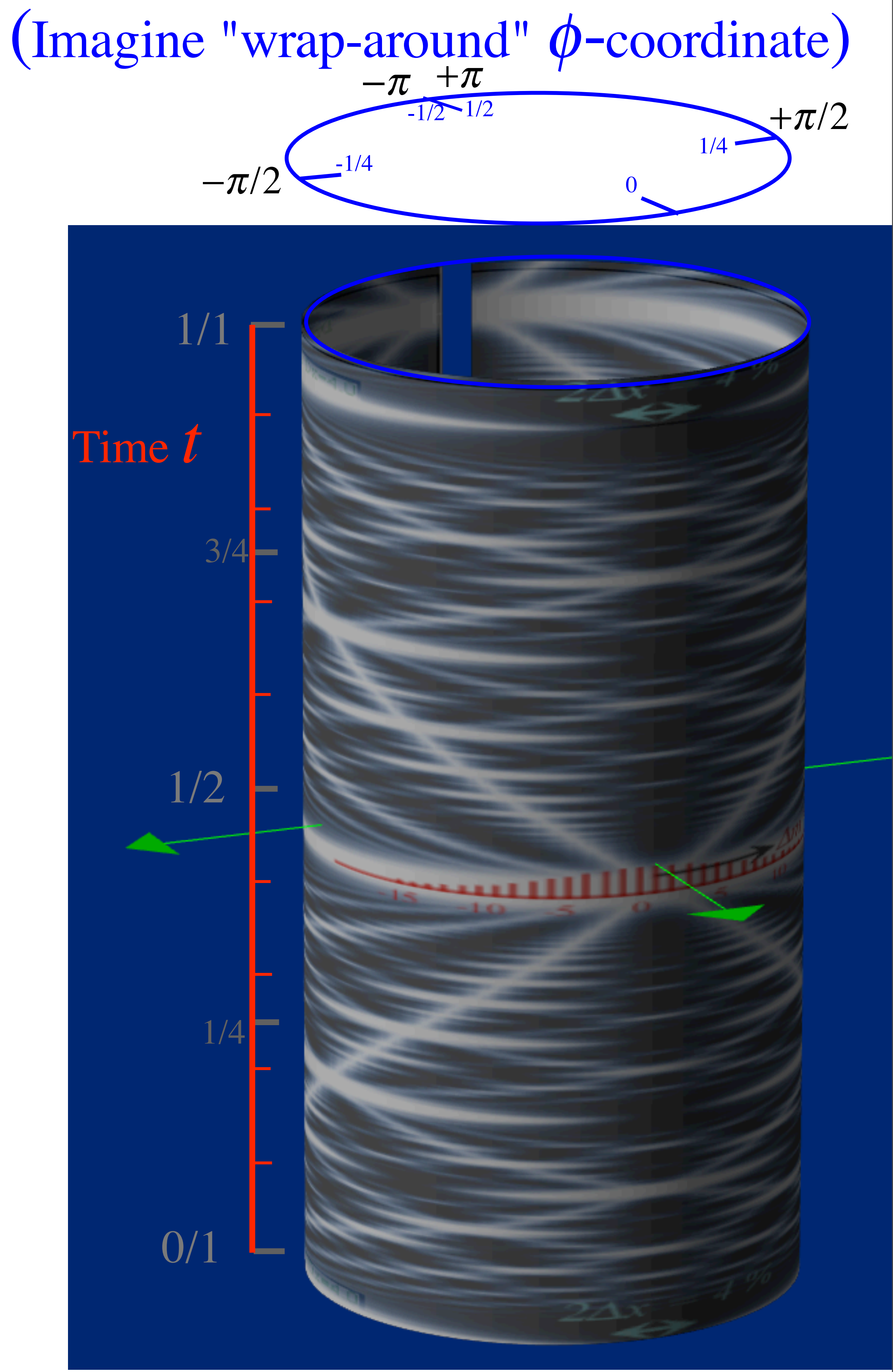


Time t (units of fundamental period τ_1)
1/1
9/10
4/5
3/4
7/10
3/5
1/2
2/5
3/10
1/4
1/5
1/10
0/1
$2\Delta x = 4\ \%$
$\Delta m = 9$
-15 -10 -5 0 5 10 15 = m
-1/2 -1/4 0 1/4 1/2
Coordinate ϕ (units of 2π)



(Imagine "wrap-around" ϕ-coordinate)
$-\pi$ $+\pi$
-1/2 1/2
$+\pi/2$
1/4
-1/4
$-\pi/2$
0
Time t
1/1
3/4
1/2
1/4
0/1

Web simulation http://www.uark.edu/ua/modphys/markup/WaveItWeb.html

Try testing *or else* markup

Click here….

Launch | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

..then here….

Twelve (n=12) oscillator

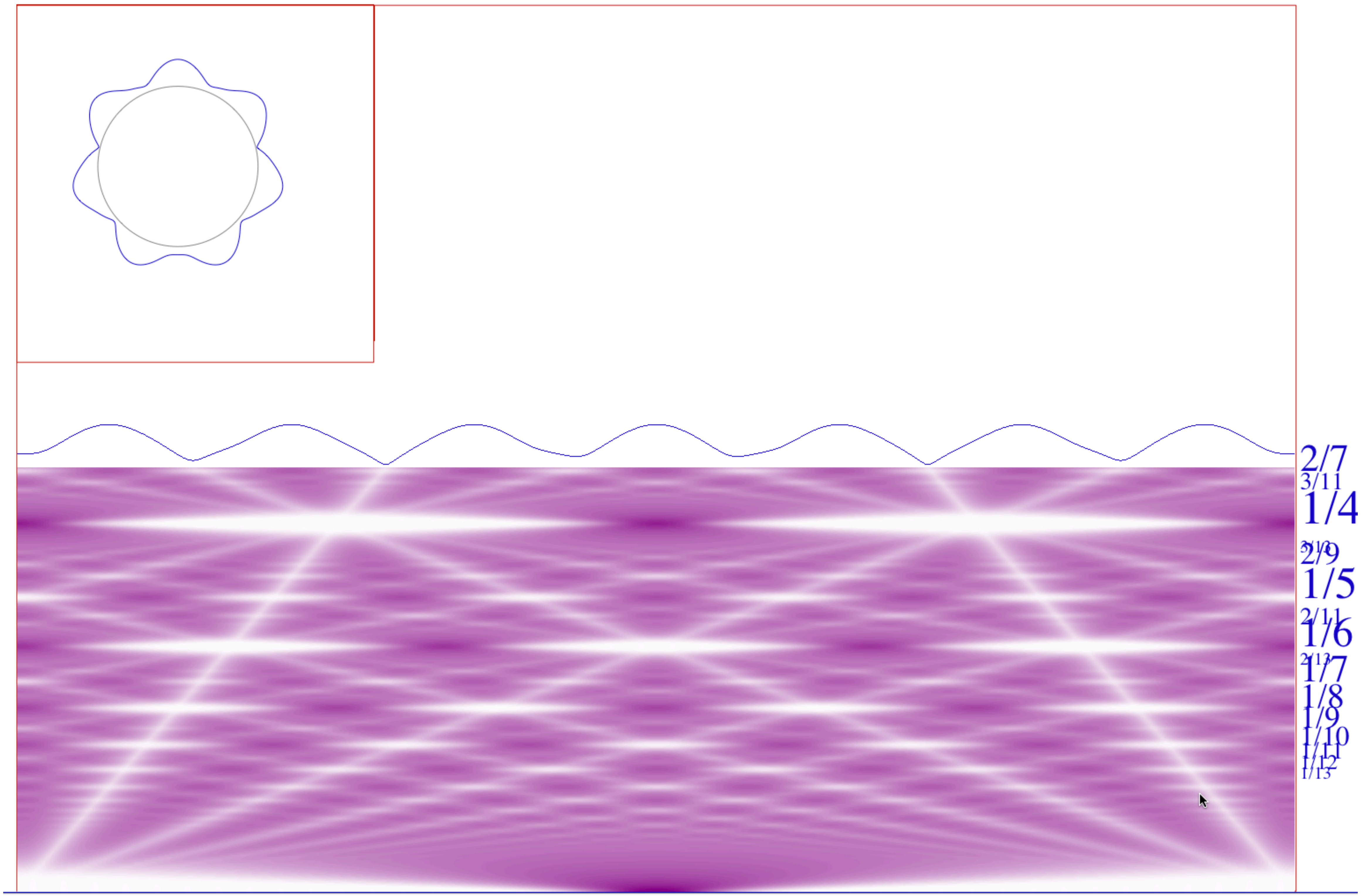
Twelve (n=12) oscillator

Twelve (n=12) oscillator

C(n) Character Table

Quantum Carpet

time = 0.29T



Web simulation http://www.uark.edu/ua/modphys/markup/WaveItWeb.html

Try testing or else markup

Click here....

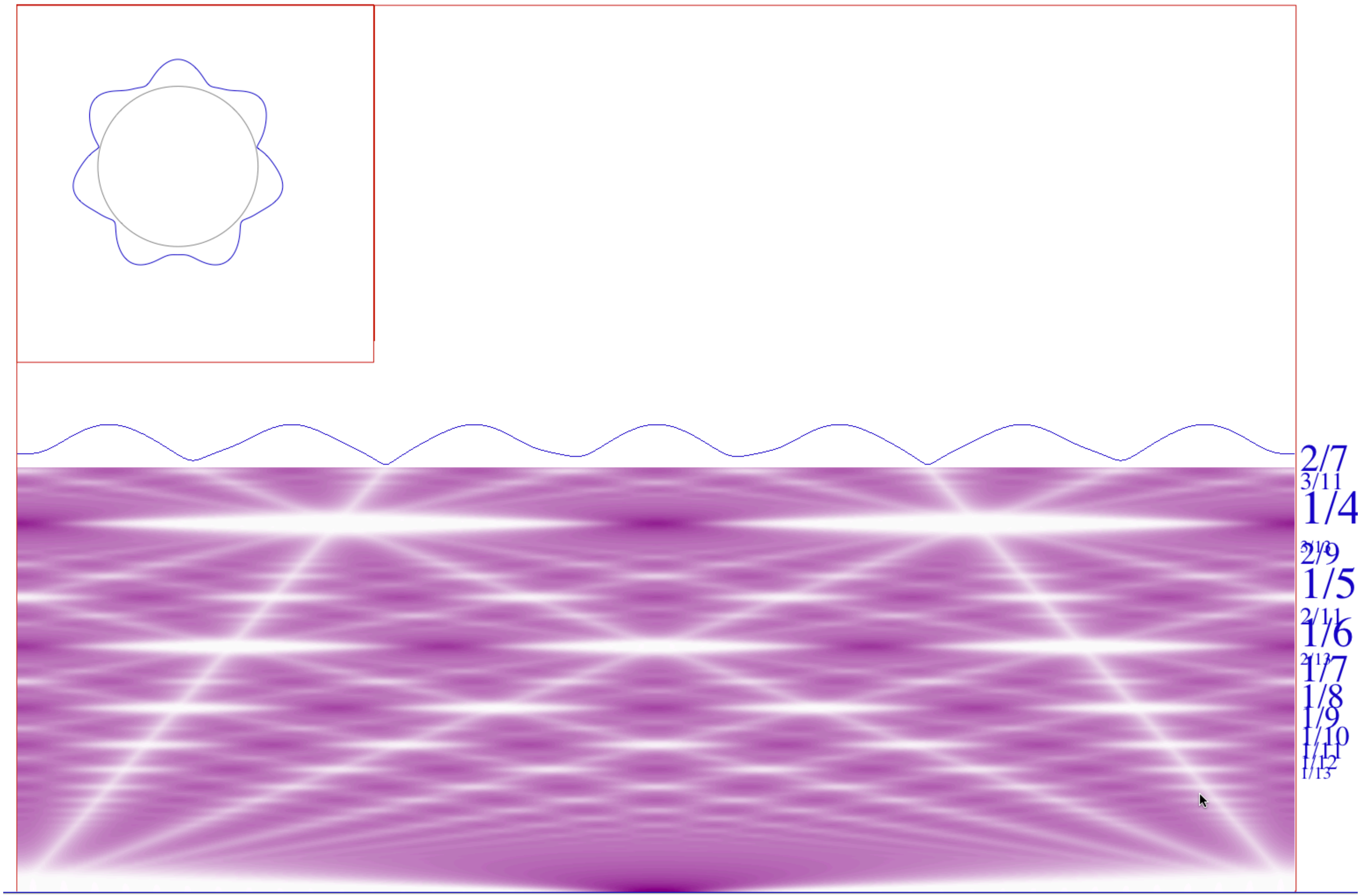
Local Control | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

Twelve (n=12) oscillator
Twelve (n=12) oscillator
Twelve (n=12) oscillator
C(n) Character Table
Quantum Carpet

..then here....

Click here to reset controls

time = 0.29T



Launch | Fourier Control | Scenarios | Pause | Set T=0 | Zero Amps | T-Scale= 1

Set this and then click here….

Type: Quantum Carpet

Time Behavior: Pause at End

Time Start (% Period) = 0

Time End (% Period)= 60

Del-x Width (% L) = 4

Excitation (Max n) = 20

Left (% L) = 0

Right (% L)= 100

n-Mean (% Max n)= 0

Peak1 Mean (% L)= 50

OverAll Scale = 1

Peak2 Mean (% L)= 0

Peak2 Amp (% Peak1)= 0

Draw Ring ☑ m/n Labels ☑

m-Boxcar ☐

Draw m-Bars ☑ m-Bars Max = 30

Aspect Ratio {W/H} = 1.5

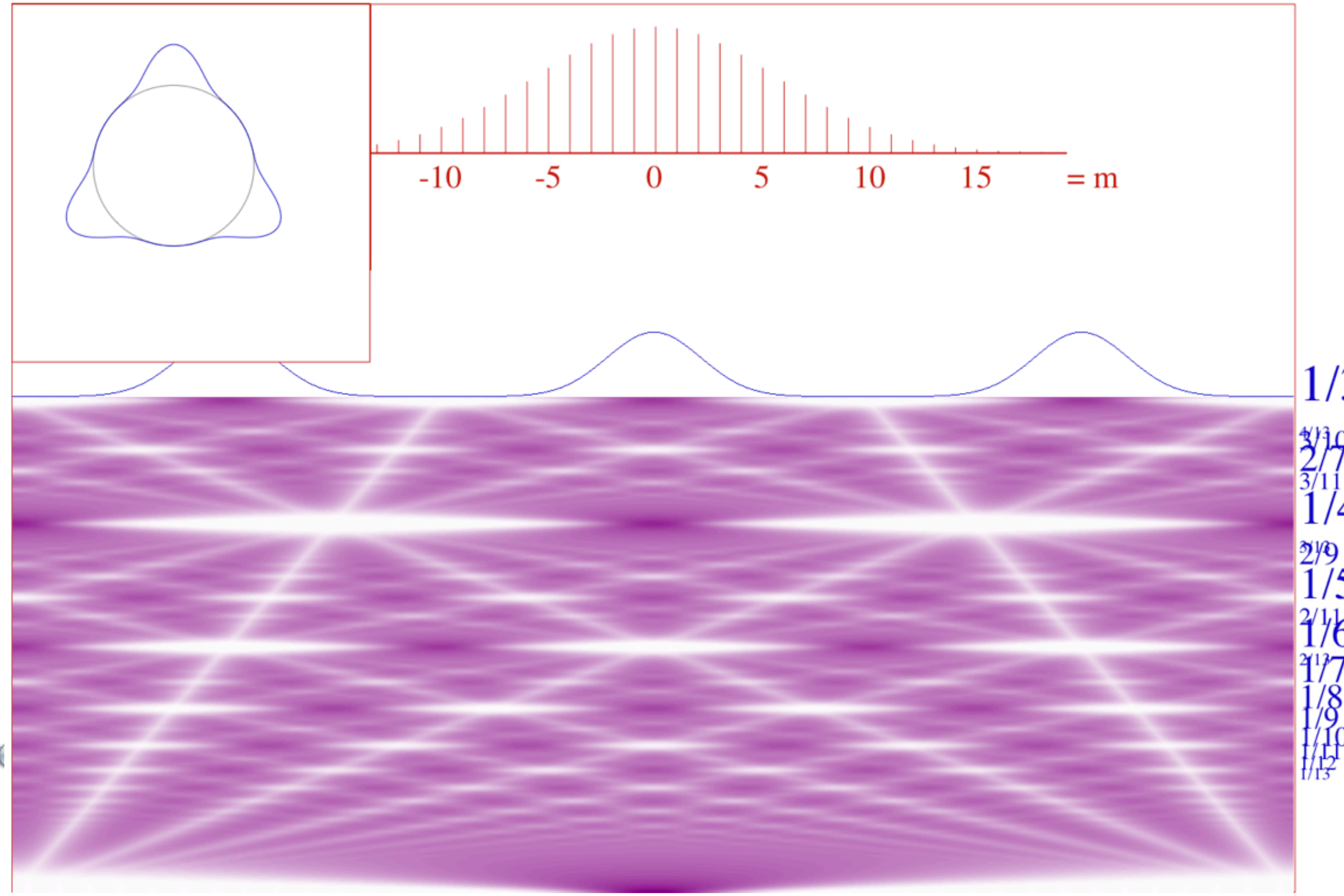
Red Level = 128

Green Level = 0

Blue Level = 128

Alpha Level = 1

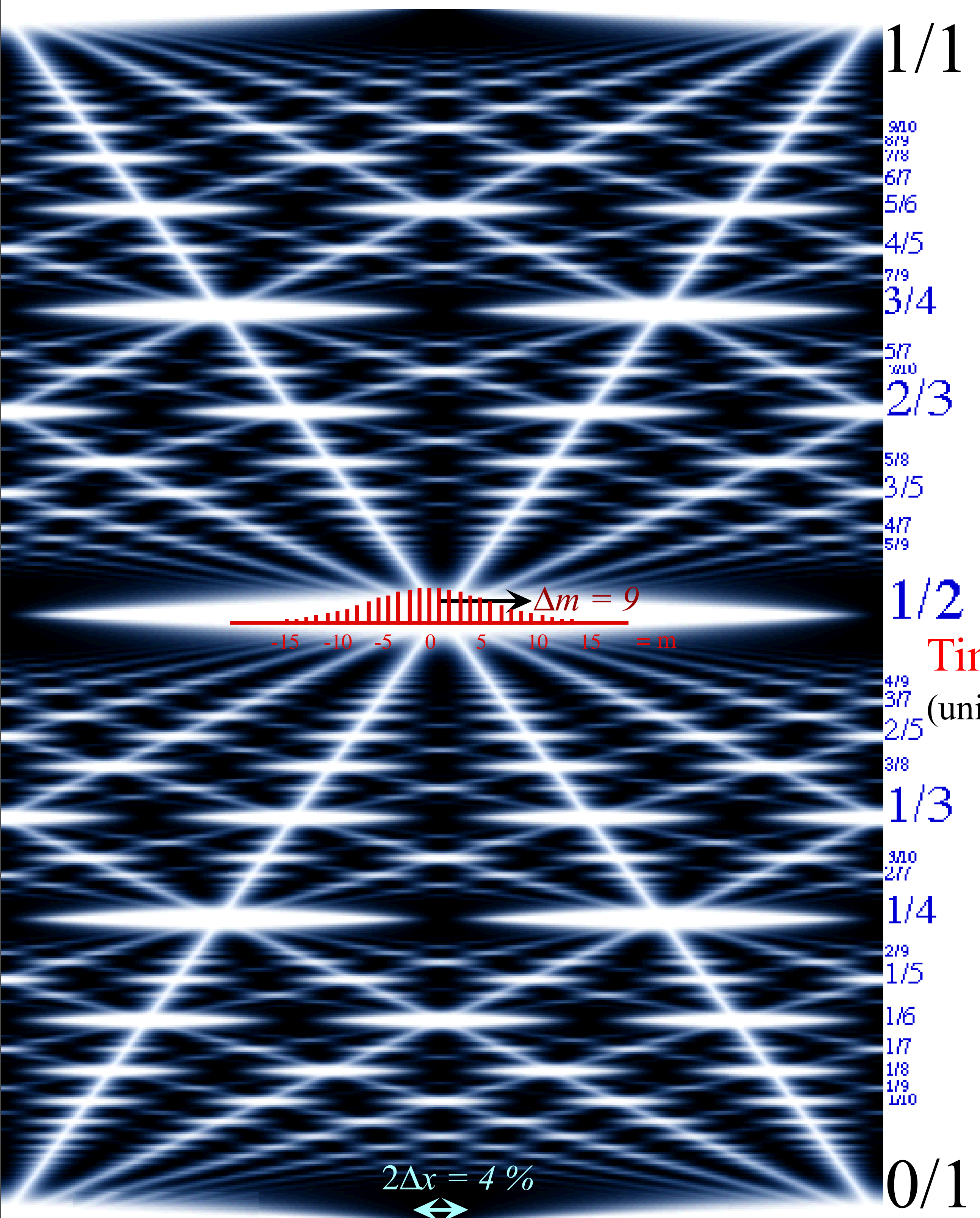
Definition Level = 0.5

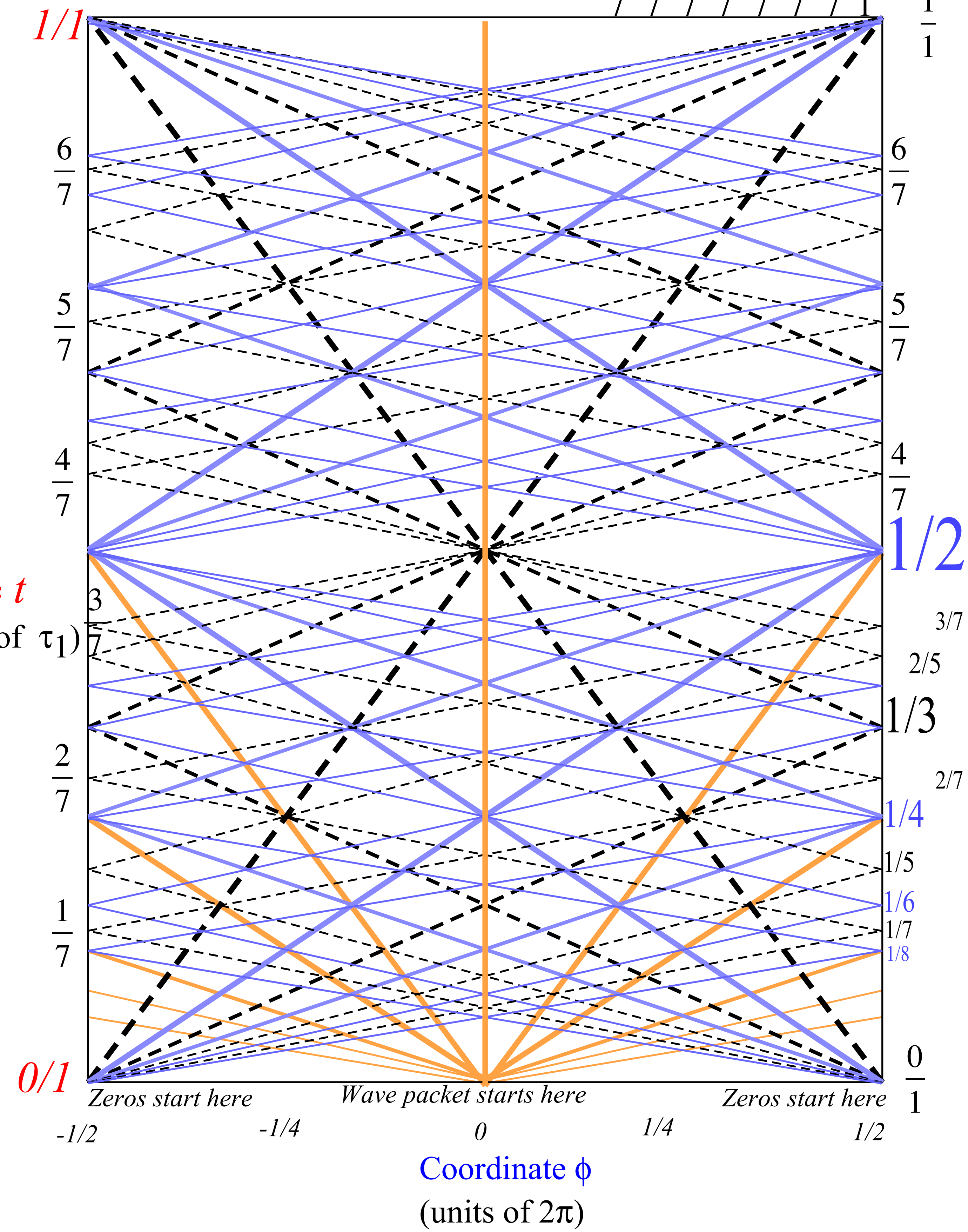


N-level-system and revival-beat wave dynamics

(9 or10-levels $(0, \pm1, \pm2, \pm3, \pm4, ..., \pm9, \pm10, \pm11...)$ excited)

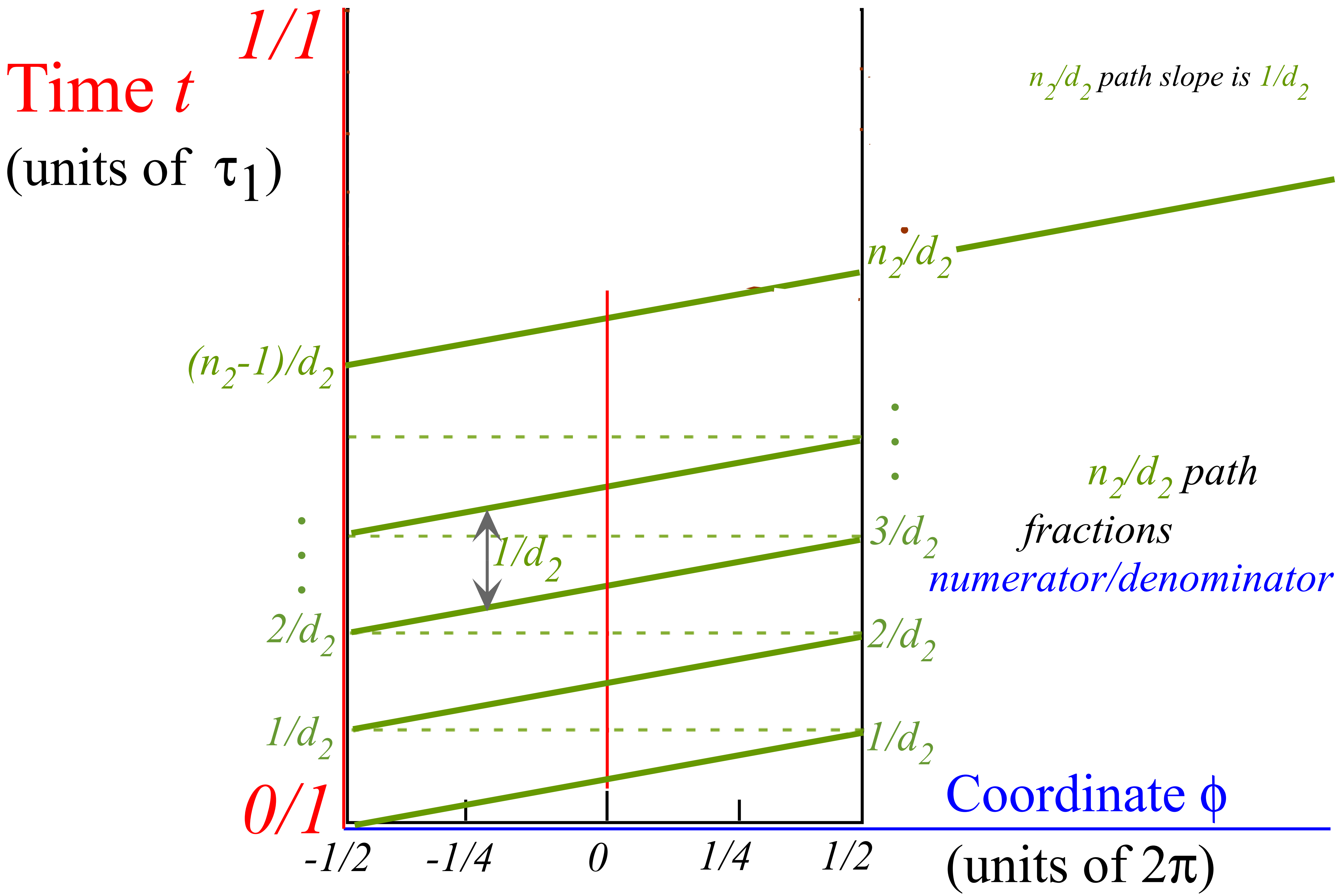
Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$





Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



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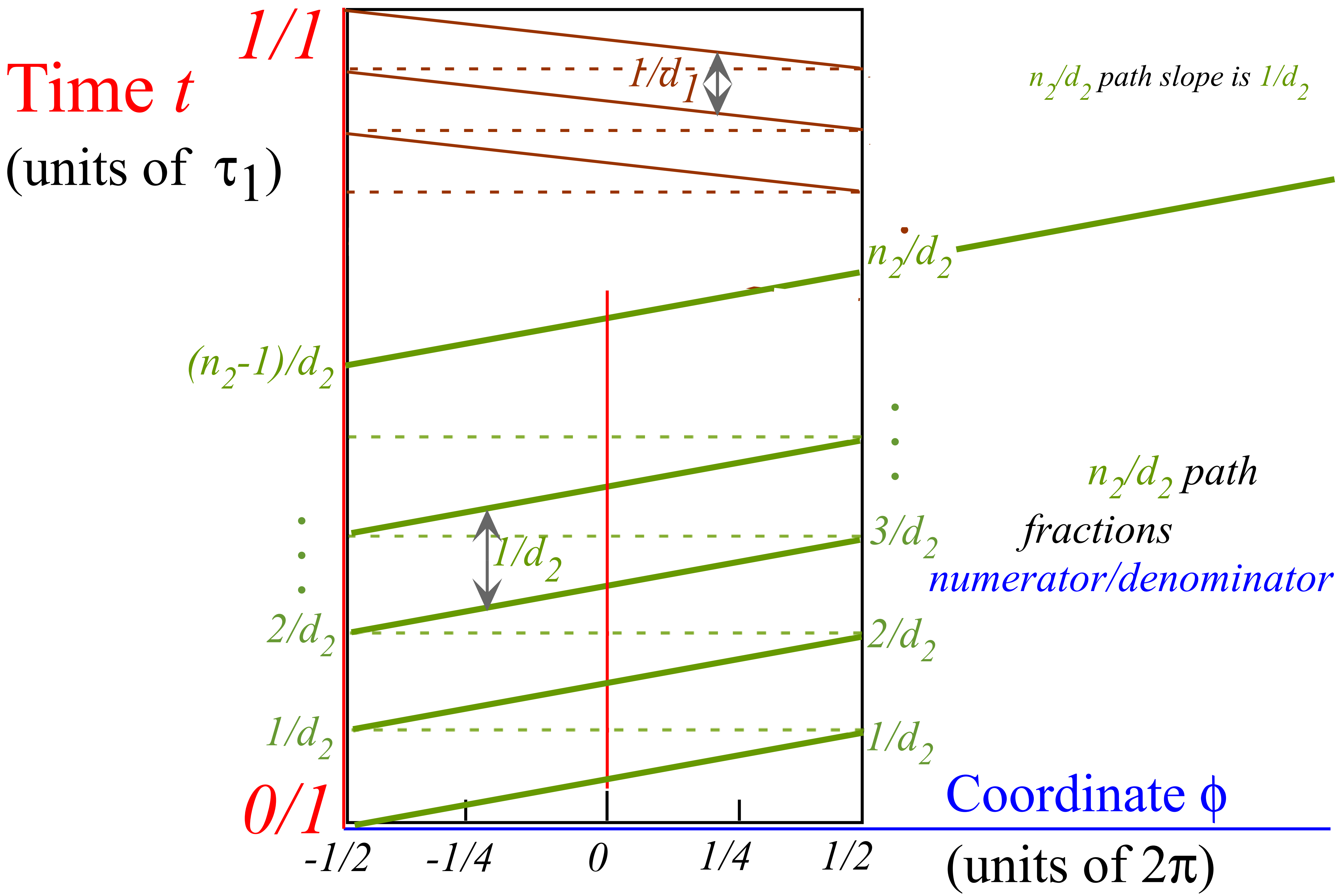

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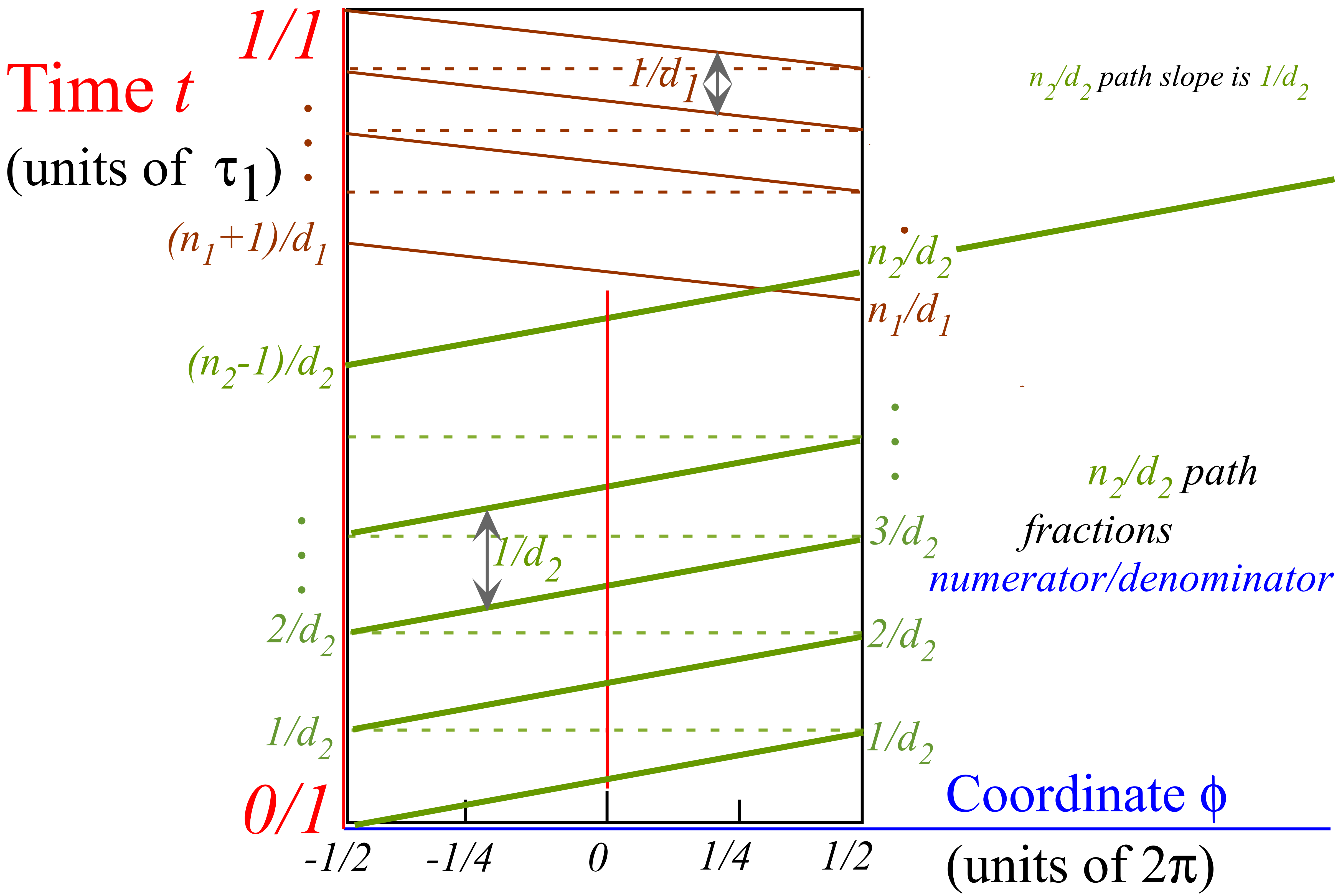

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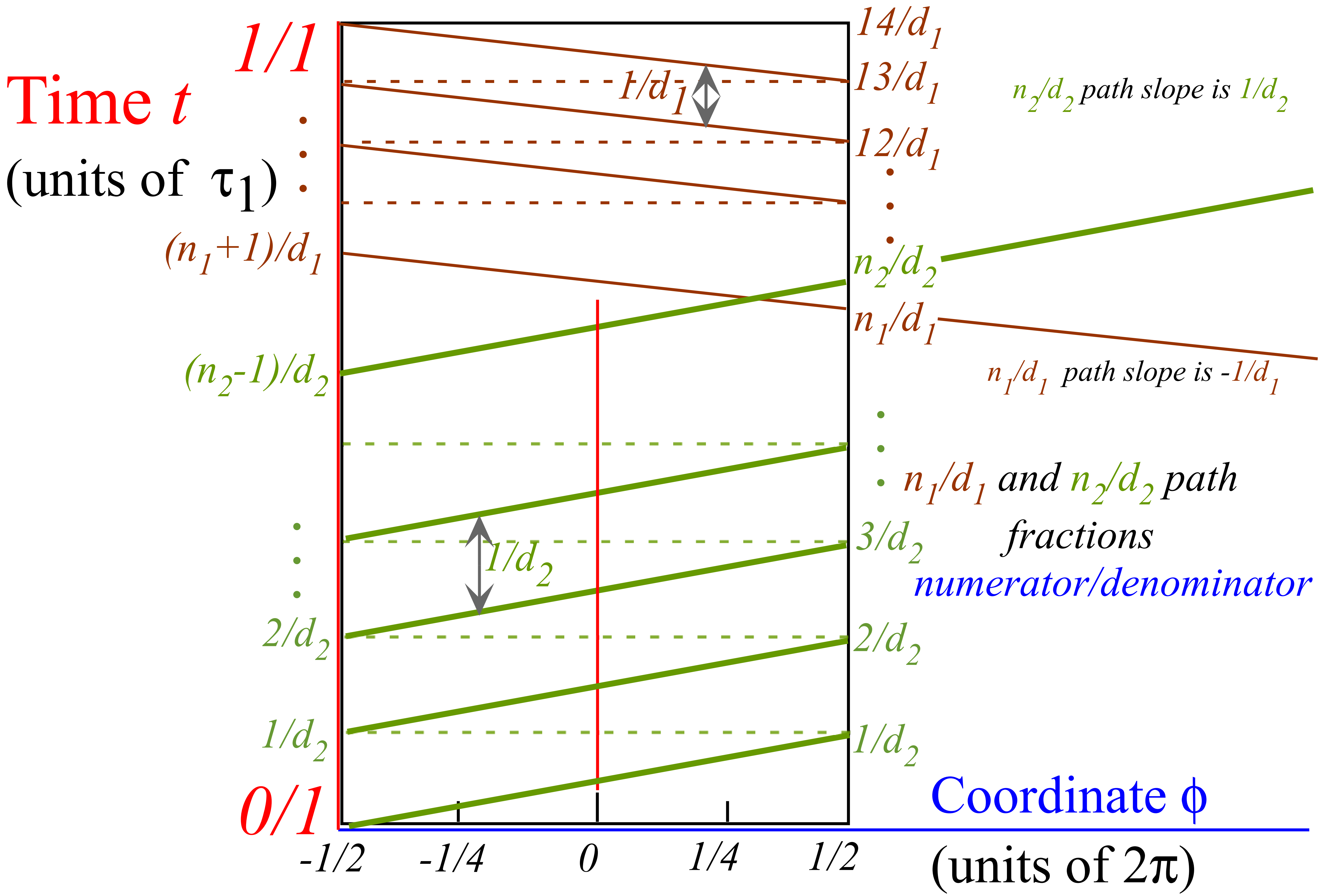

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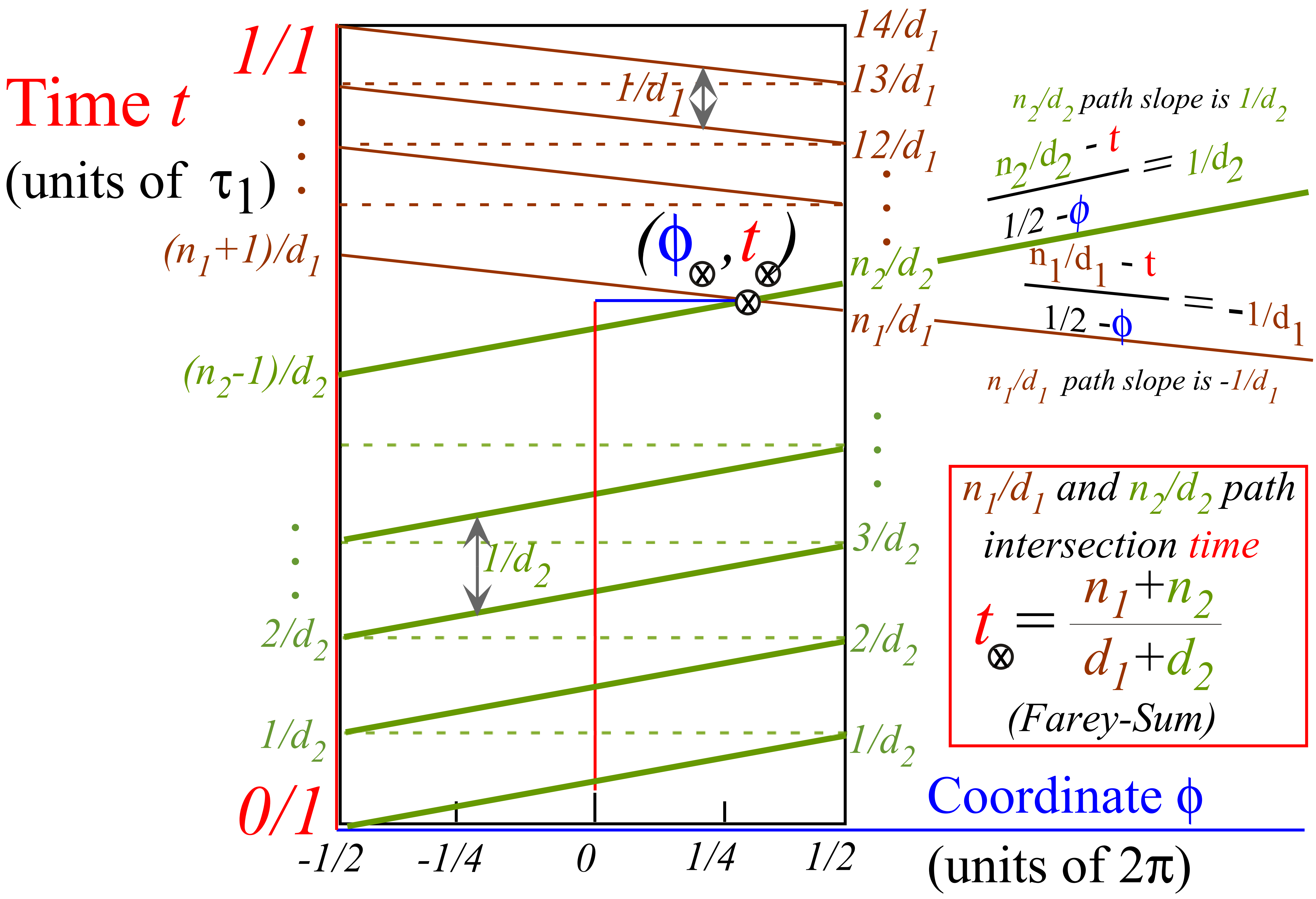


n_1/d_1 and n_2/d_2 path intersection time

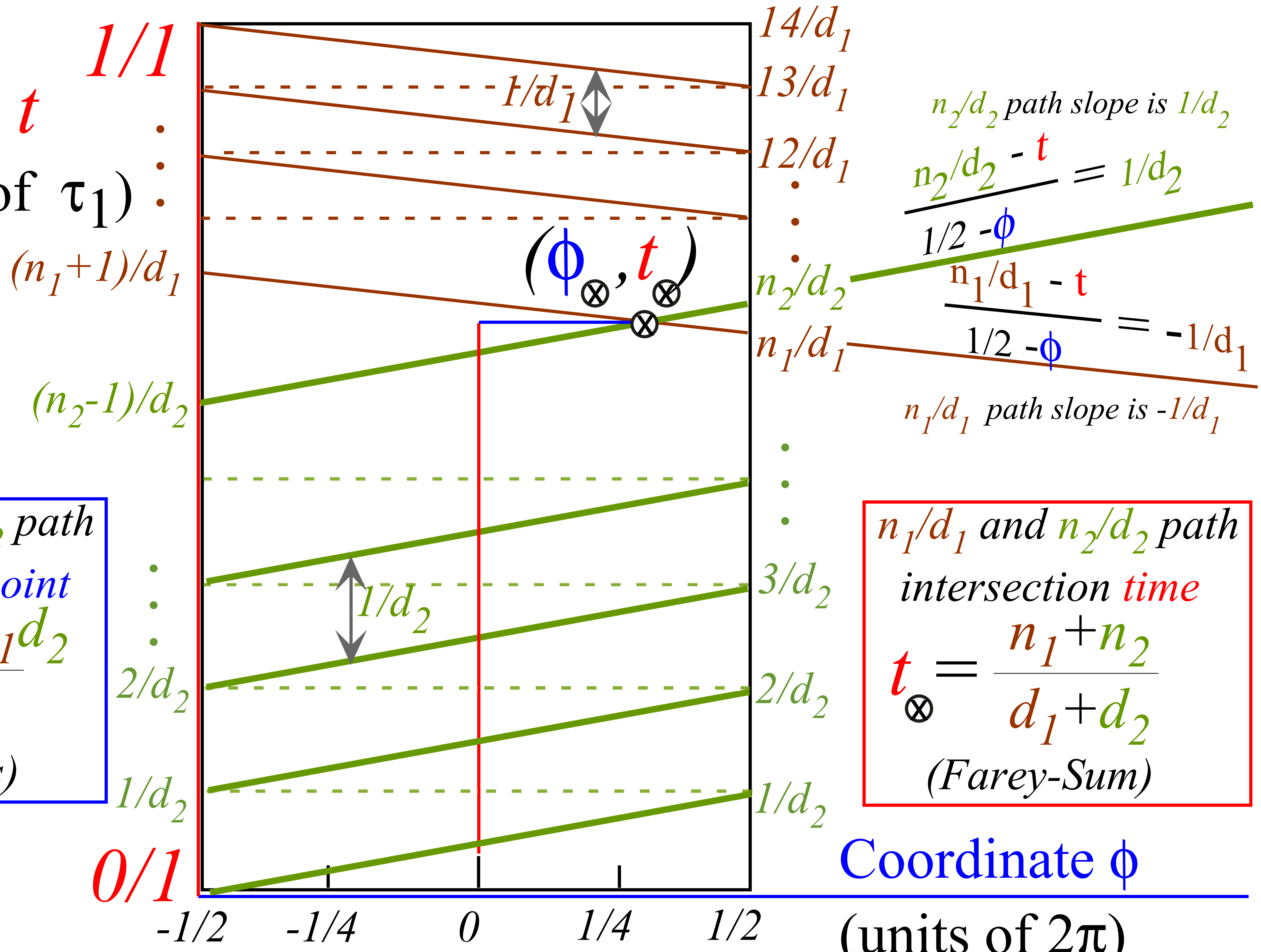
$$t_\otimes = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[*John Farey, Phil. Mag.(1816)*]

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



$$\frac{n_2/d_2 - t}{1/2 - \phi} = 1/d_2$$

$$\frac{n_1/d_1 - t}{1/2 - \phi} = -1/d_1$$

n_1/d_1 and n_2/d_2 path intersection point

$$\phi_\otimes = \frac{d_1 n_2 - n_1 d_2}{d_1 + d_2}$$

(Ford-Cross)

n_1/d_1 and n_2/d_2 path intersection time

$$t_\otimes = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

[John Farey, Phil. Mag.(1816)]

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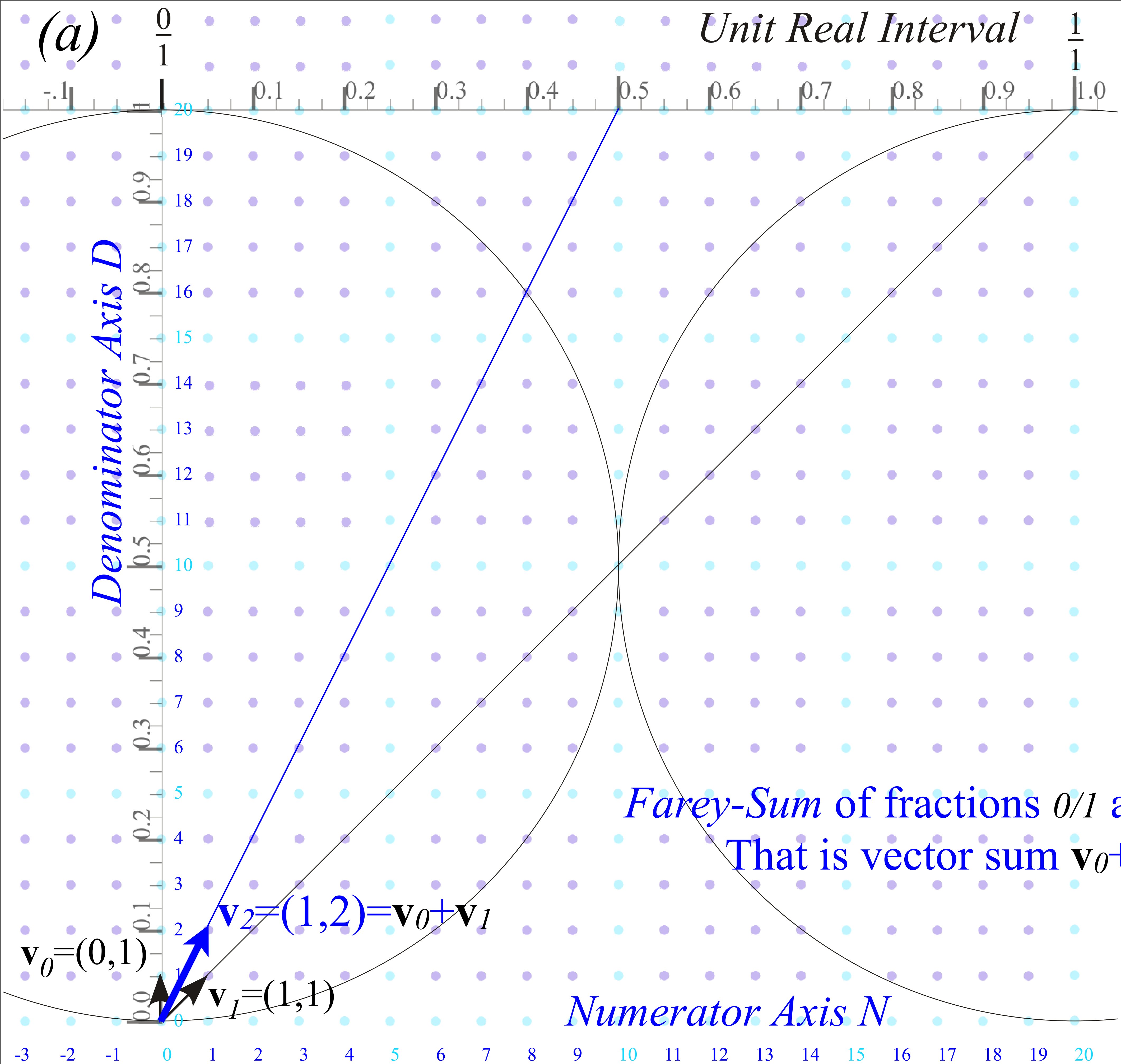
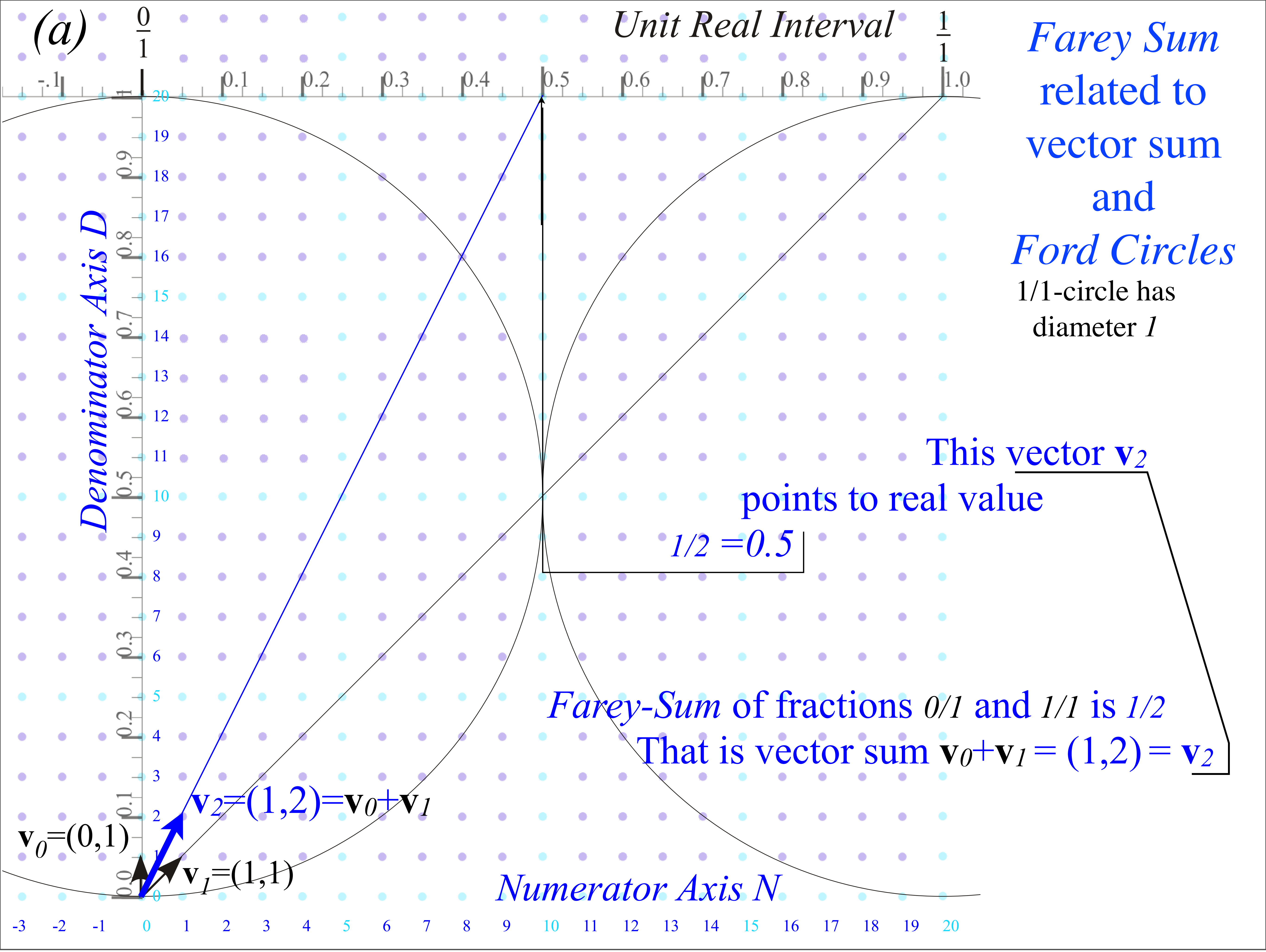
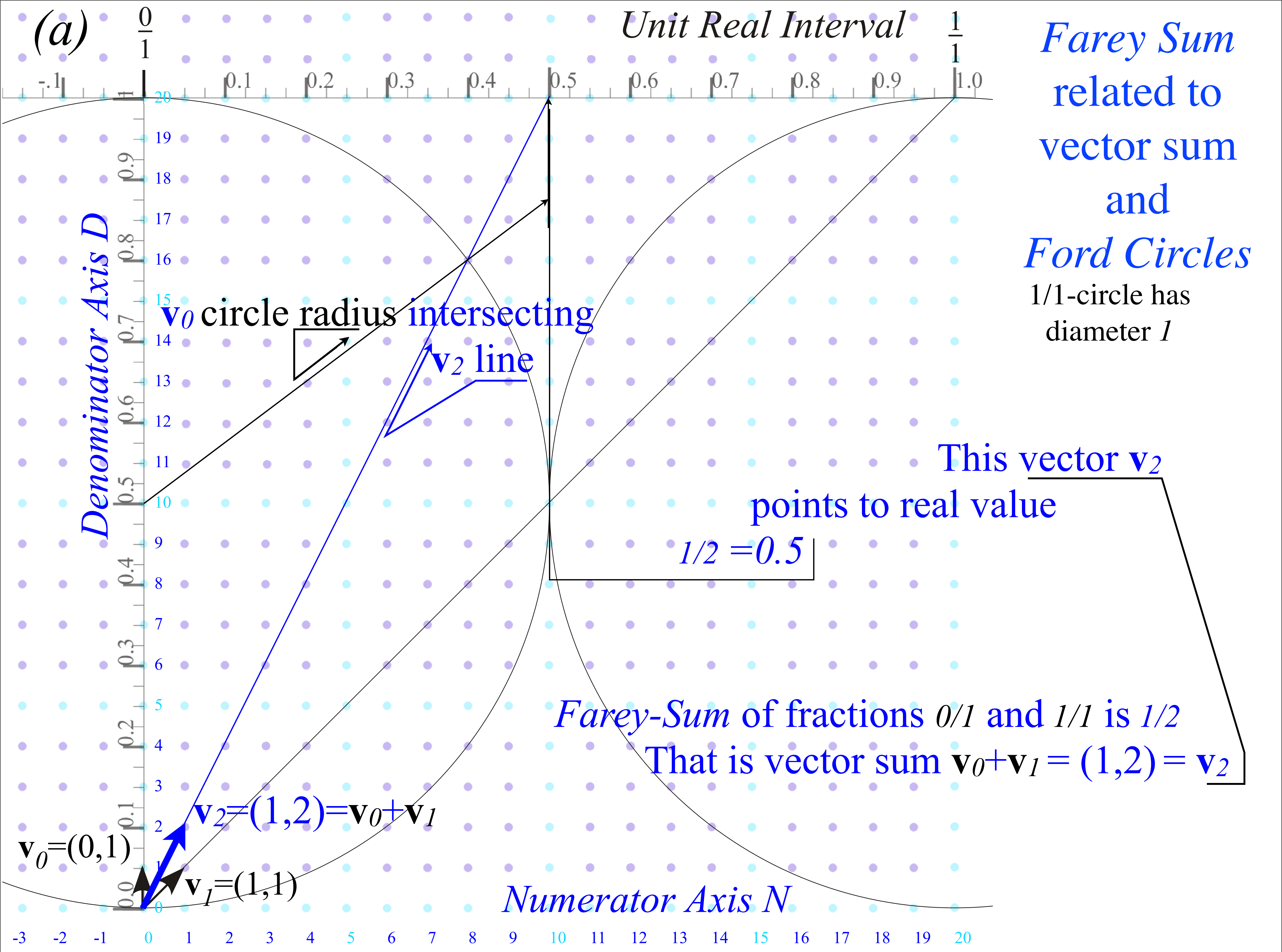
Farey Sum related to vector sum and *Ford Circles*

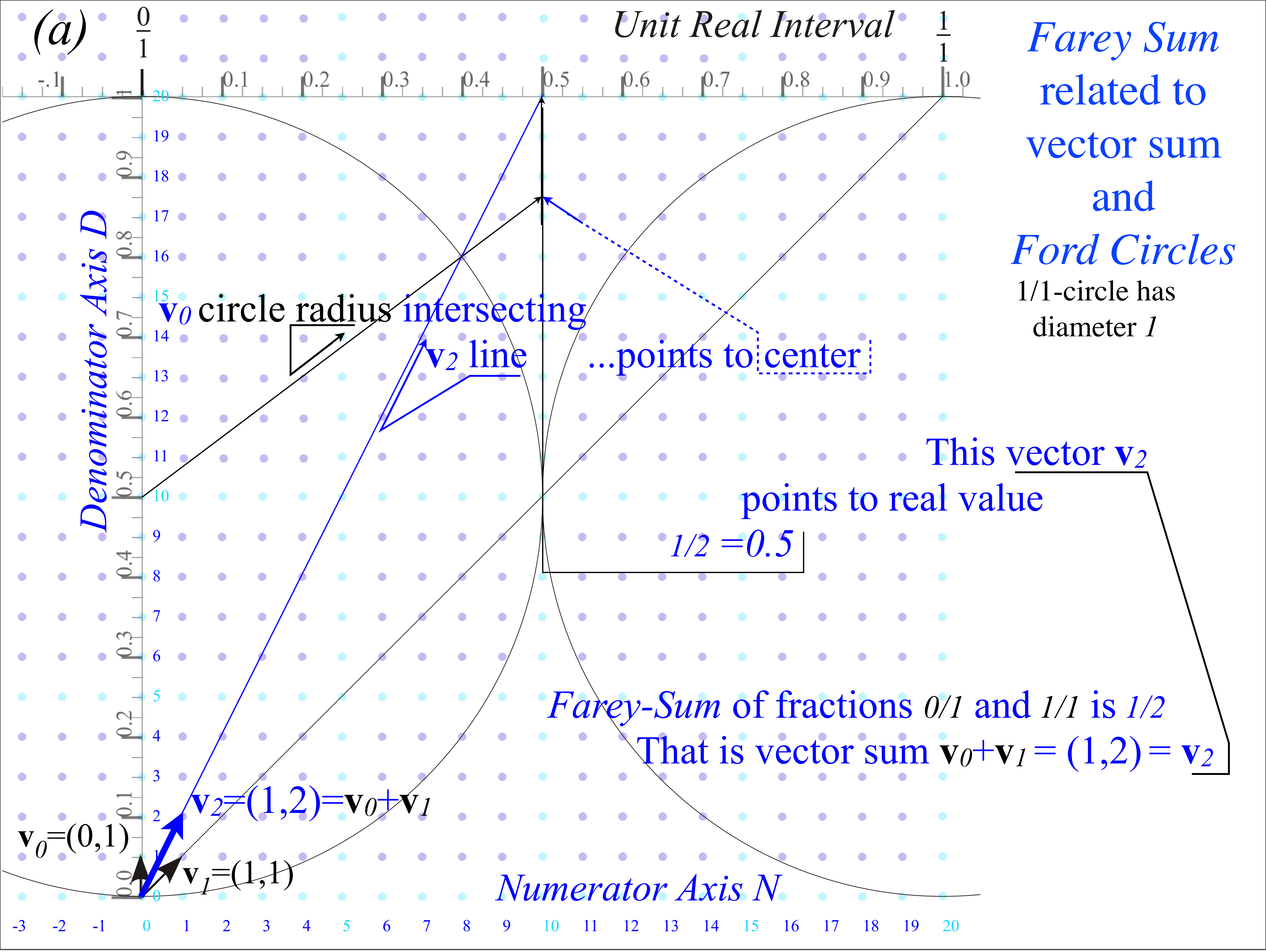
1/1-circle has diameter *1*



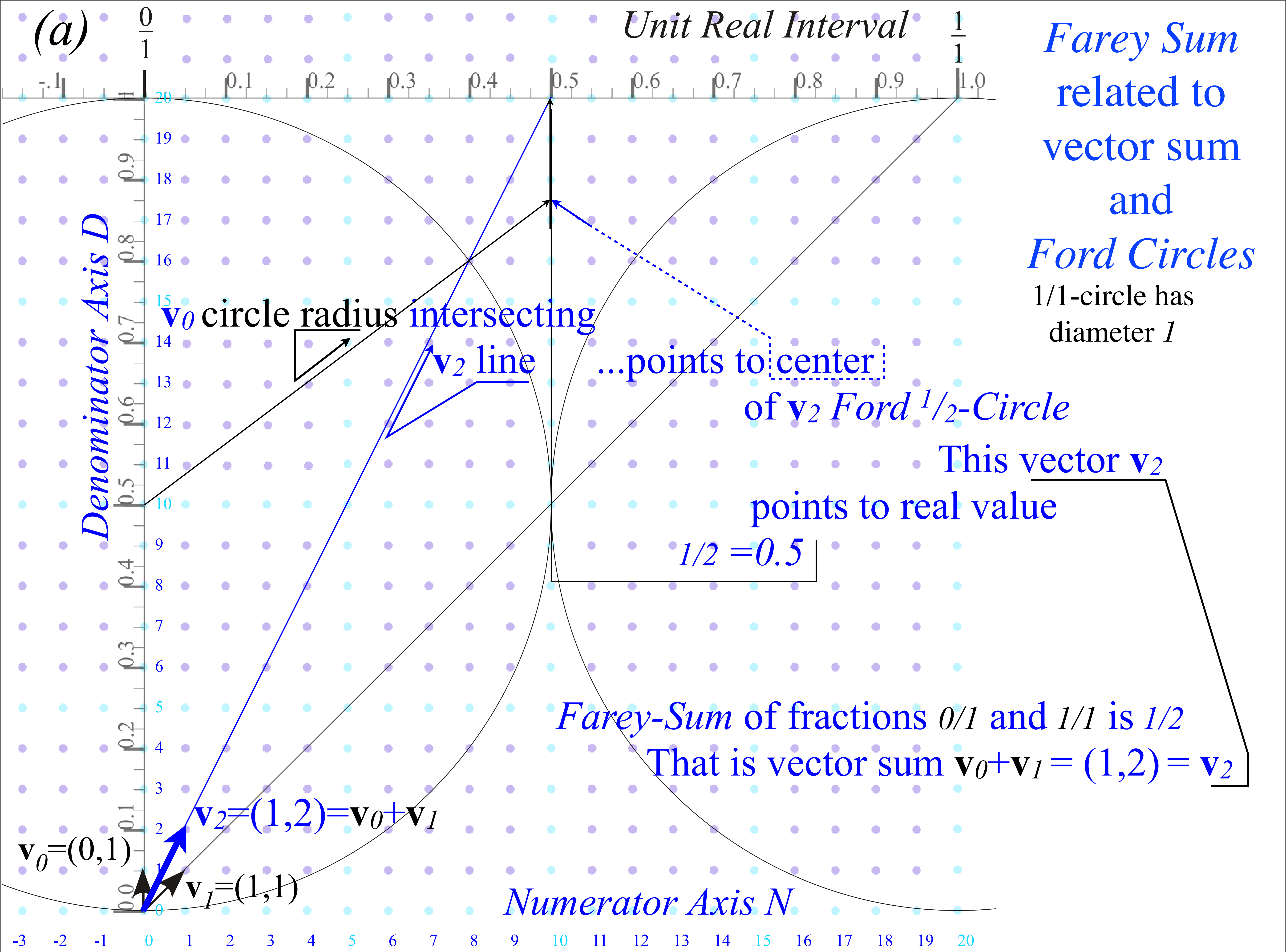


(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
This vector $\mathbf{v}_2$ points to real value $1/2 = 0.5$
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$
$\mathbf{v}_2=(1,2)=\mathbf{v}_0+\mathbf{v}_1$
$\mathbf{v}_0=(0,1)$
$\mathbf{v}_1=(1,1)$


(a)
0/1
Unit Real Interval
1/1
Farey Sum
related to
vector sum
and
Ford Circles
1/1-circle has
diameter 1
Denominator Axis D
$\mathbf{v}_0$ circle radius intersecting
$\mathbf{v}_2$ line
This vector $\mathbf{v}_2$
points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$
$\mathbf{v}_2=(1,2)=\mathbf{v}_0+\mathbf{v}_1$
$\mathbf{v}_0=(0,1)$
$\mathbf{v}_1=(1,1)$
Numerator Axis N



(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting
v2 line
...points to center
This vector v2
points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N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Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
v0 circle radius intersecting v2 line
...points to center of v2 Ford 1/2-Circle
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
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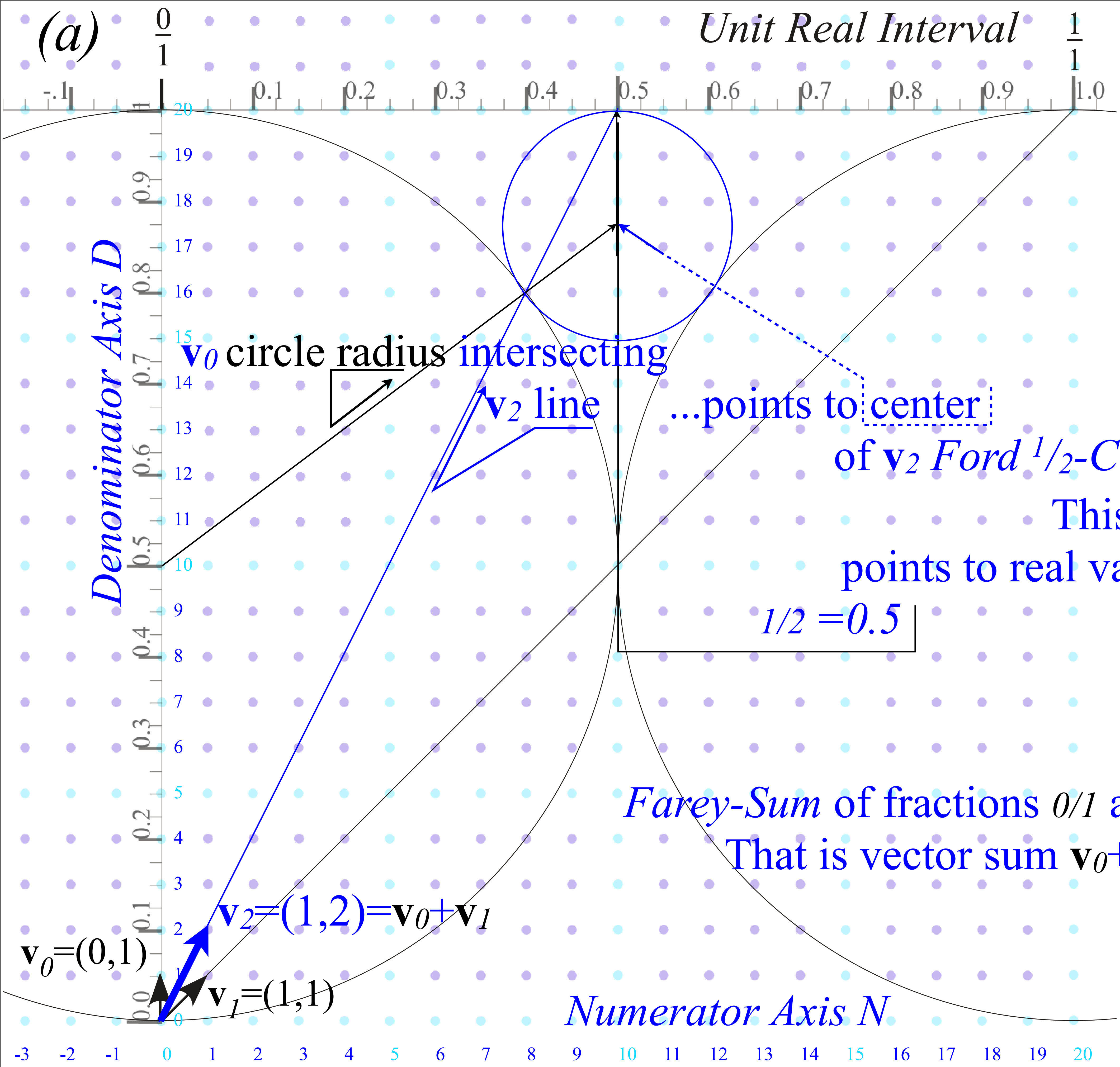
v1=(1,1)



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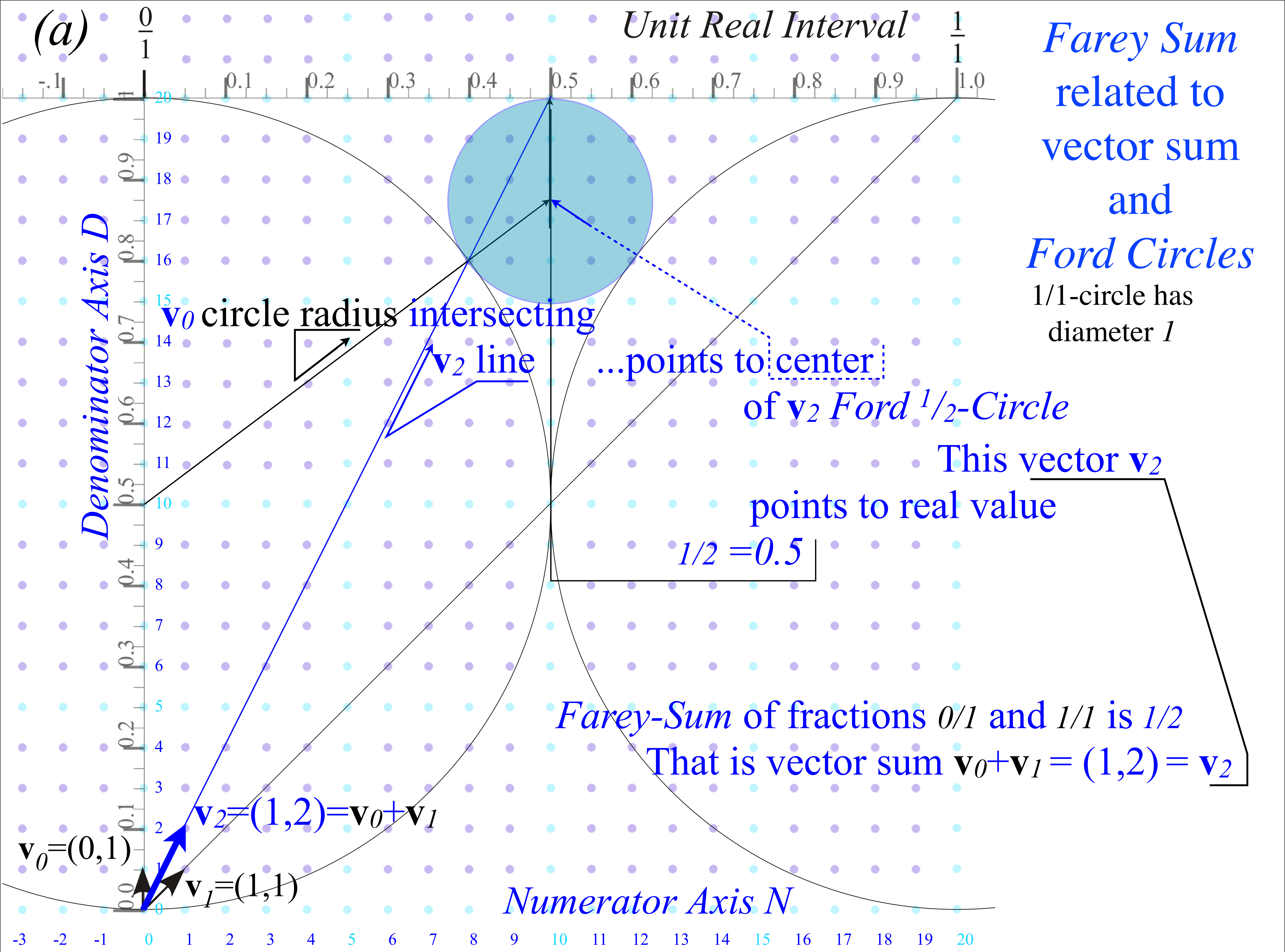
1/1-circle has diameter *1*


(a)
Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
v0 circle radius intersecting
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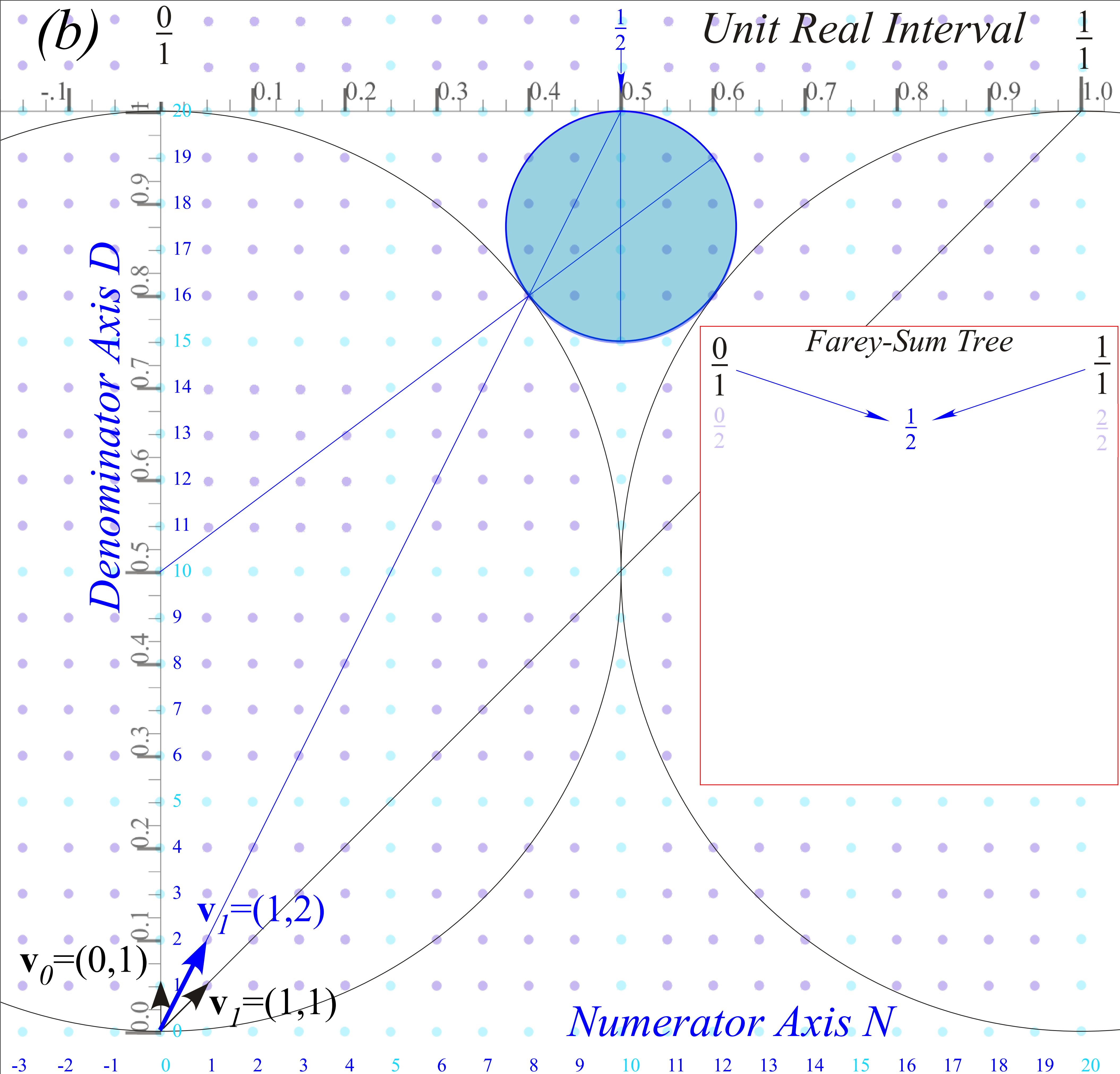
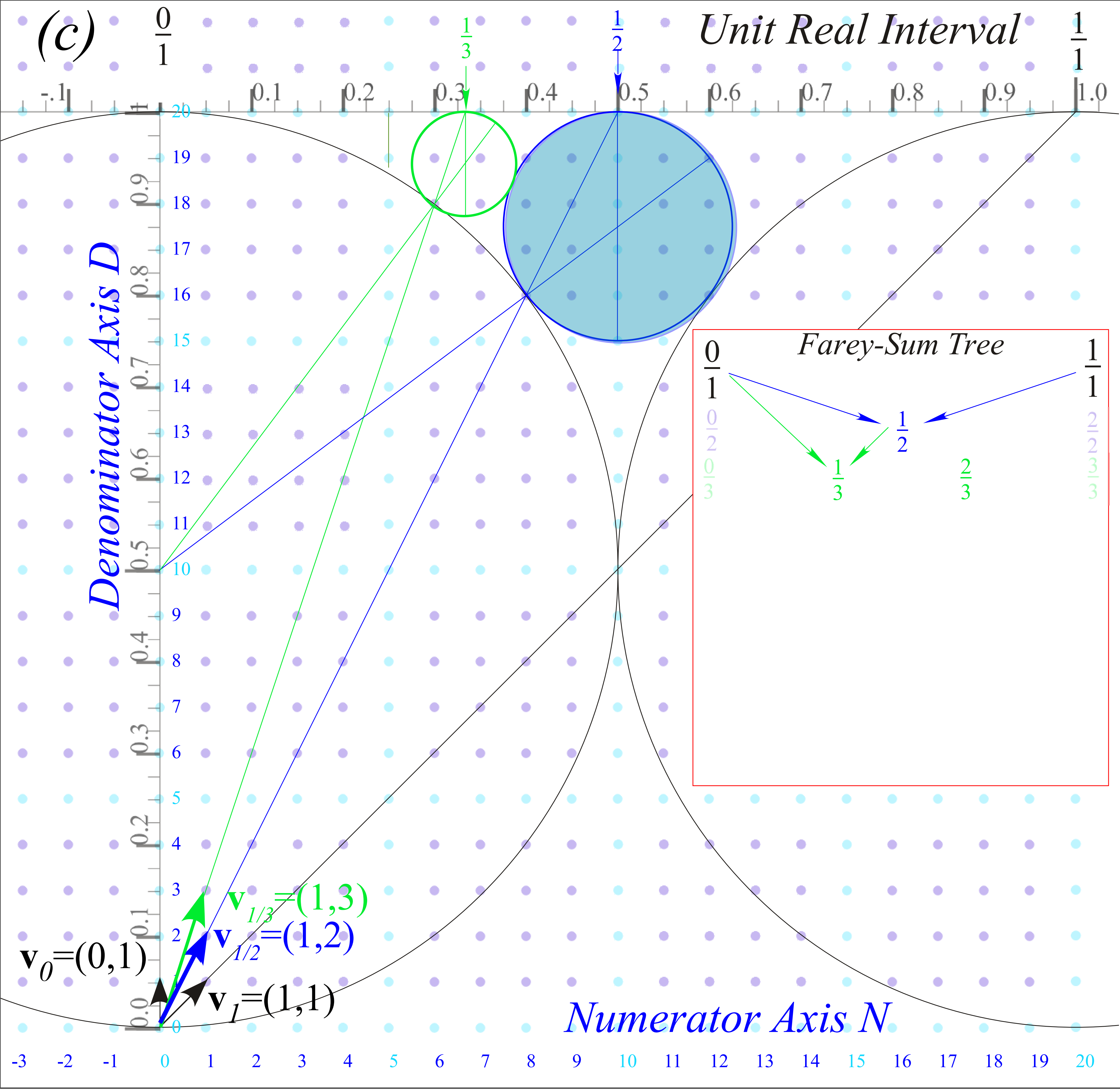
v0=(0,1)
v1=(1,1)



Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

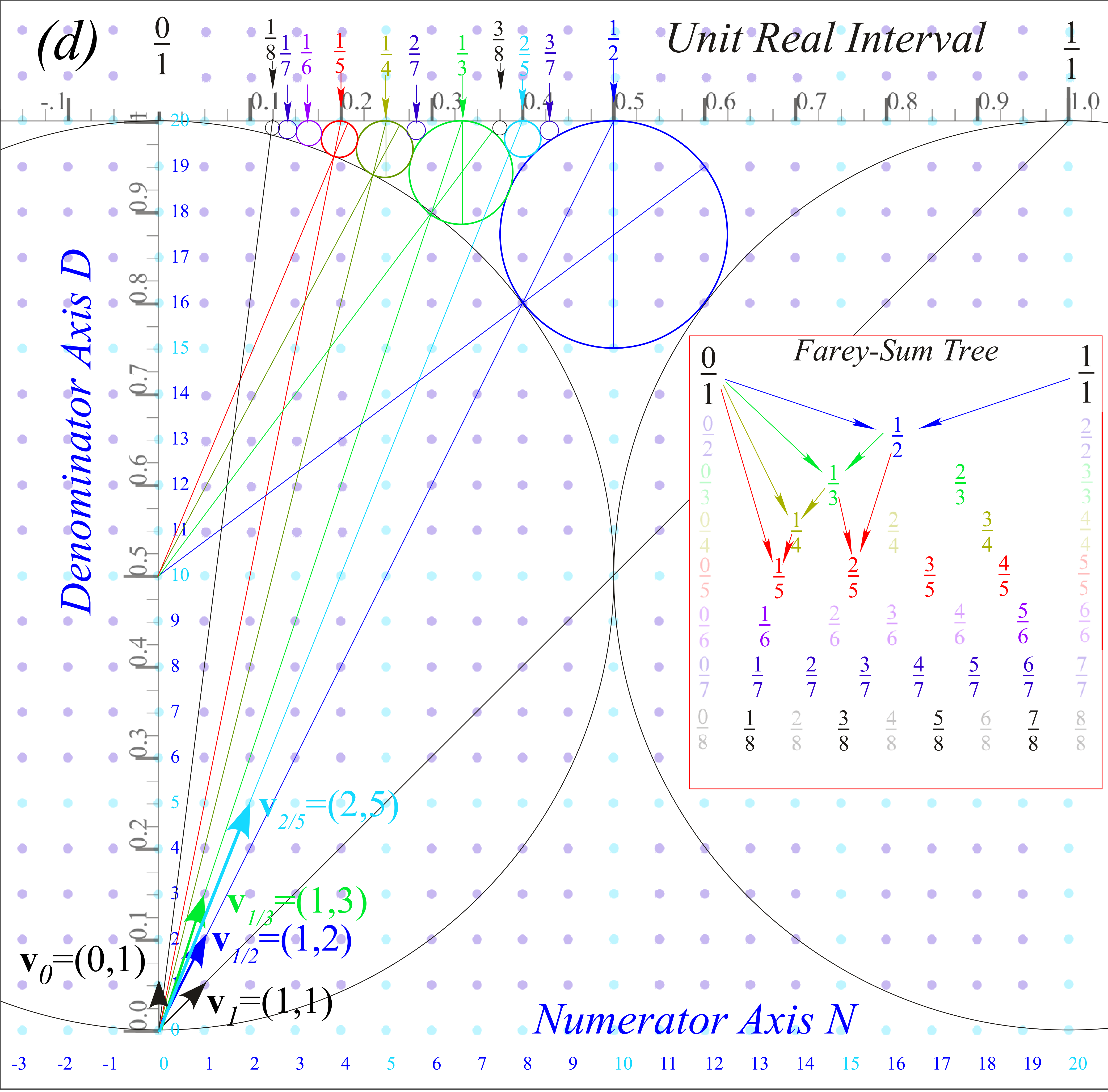
1/2-circle has diameter $1/2^2=1/4$



Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

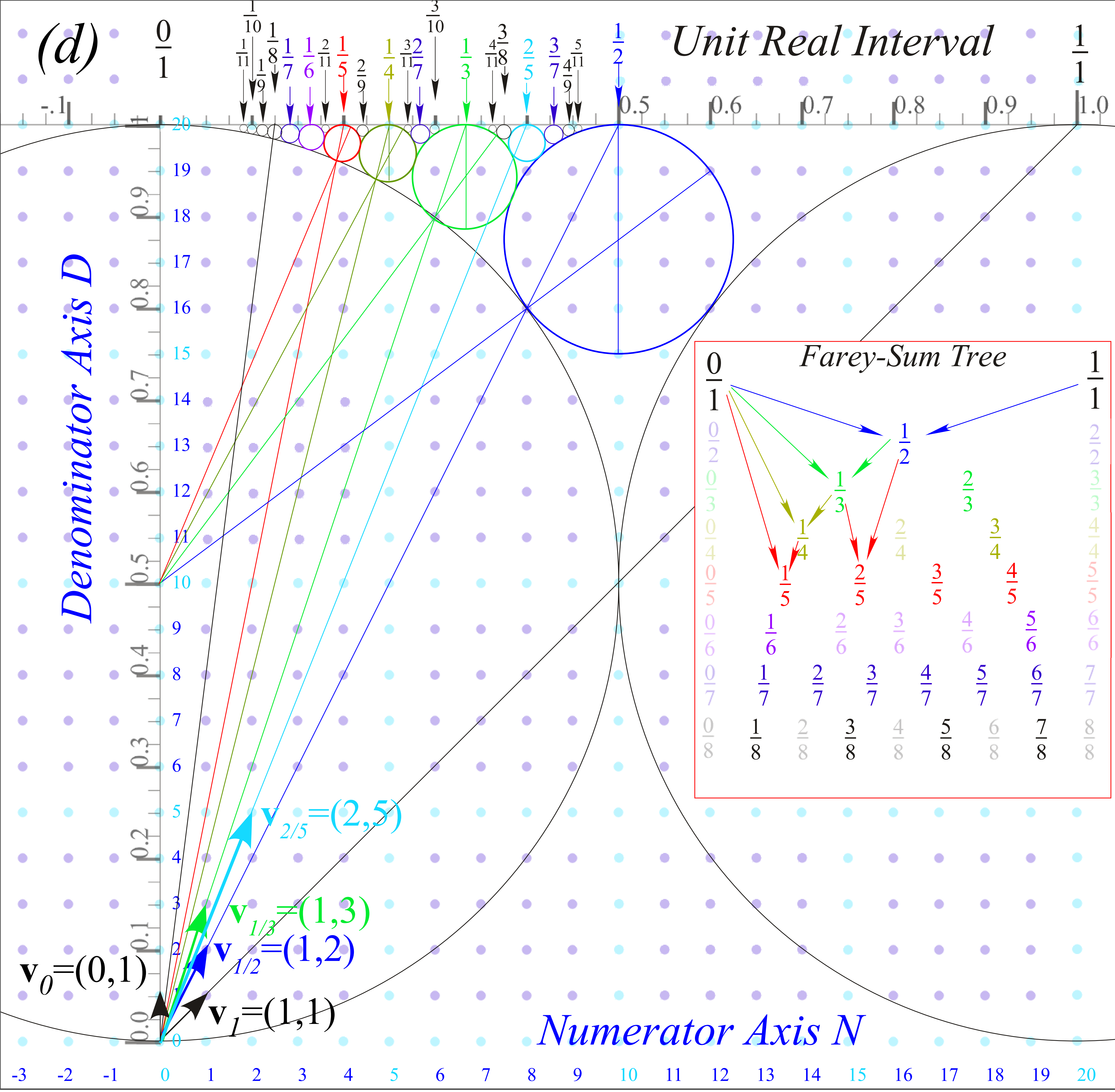


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

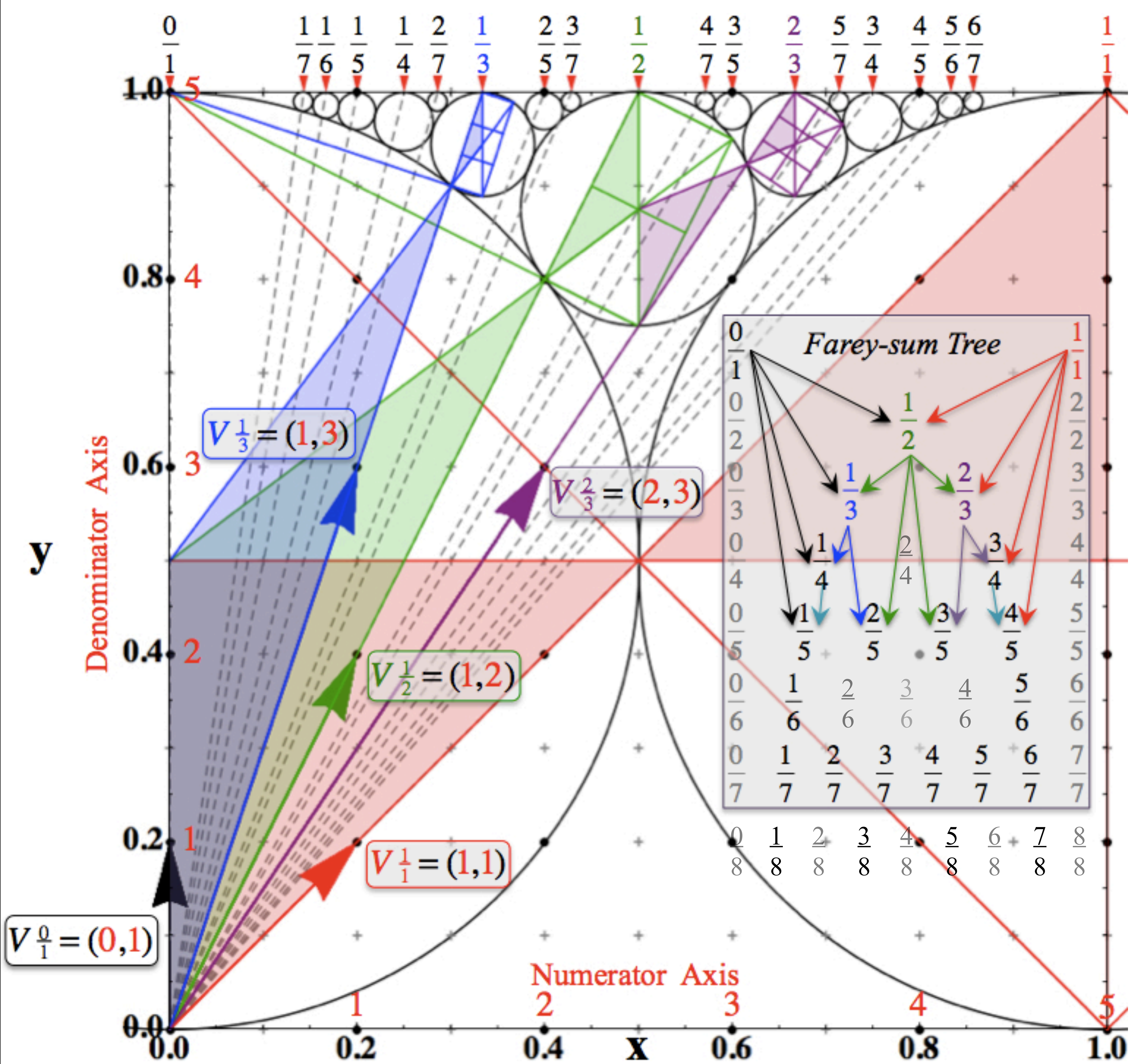


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

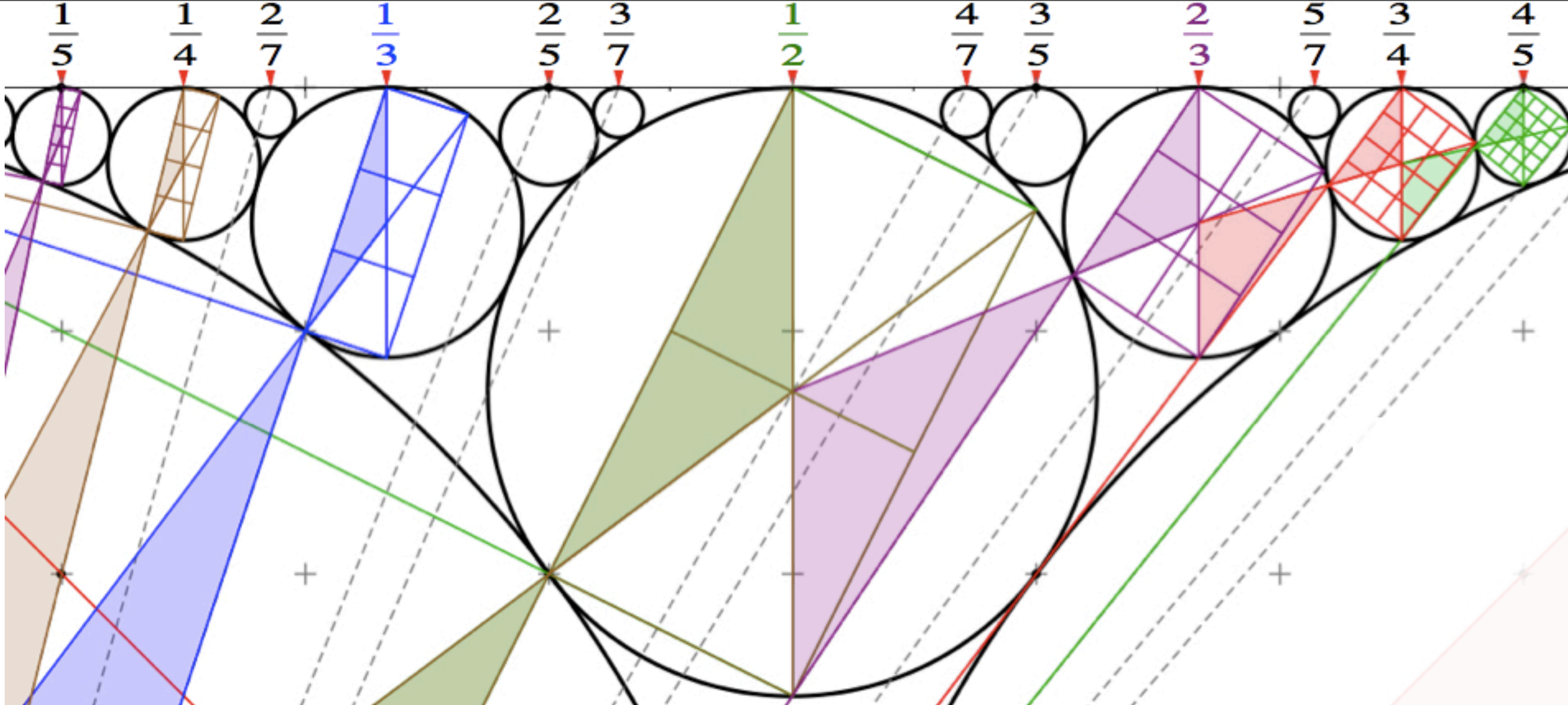
1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$



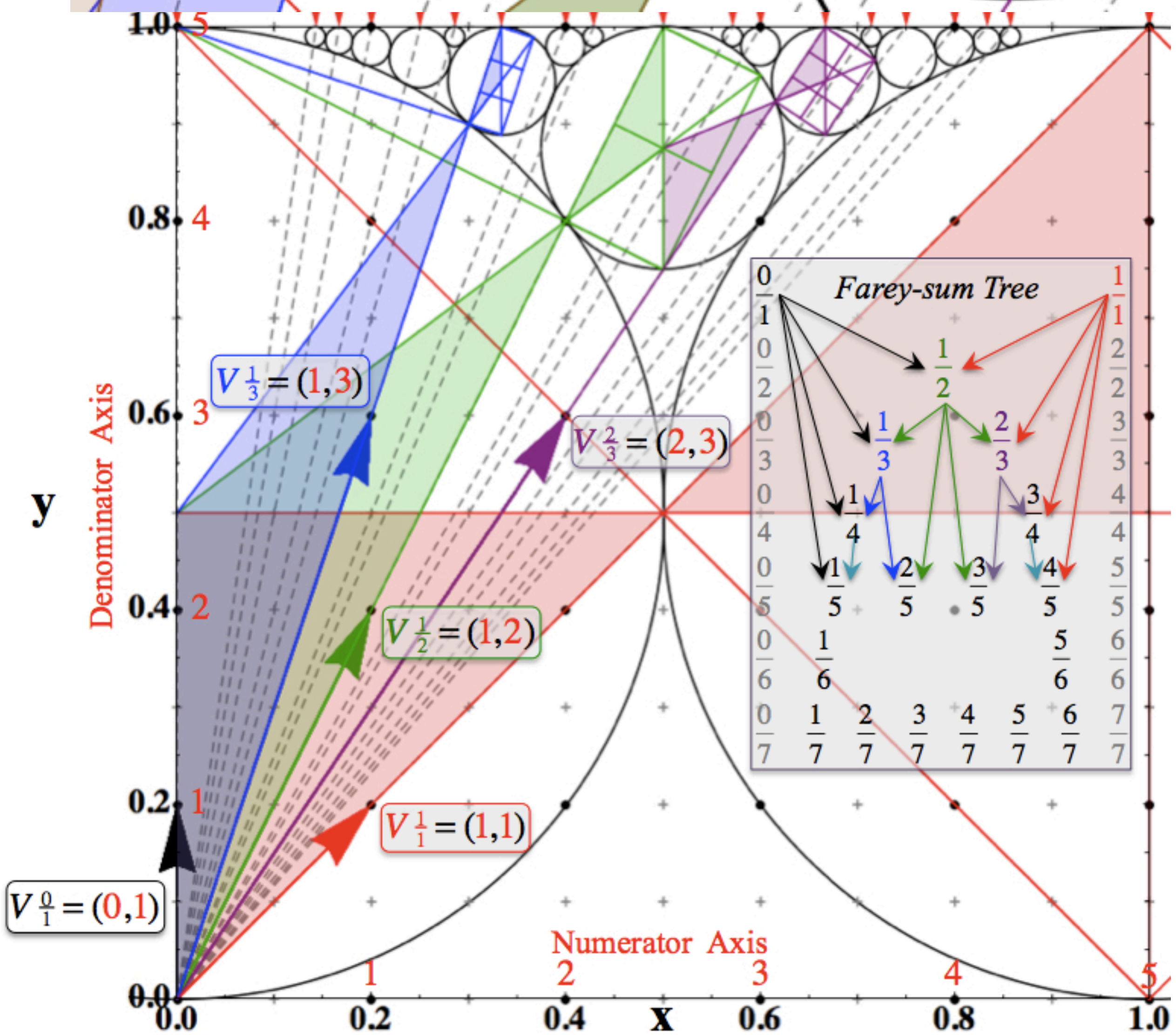
Thales Rectangles

provide
analytic geometry
of
fractal structure



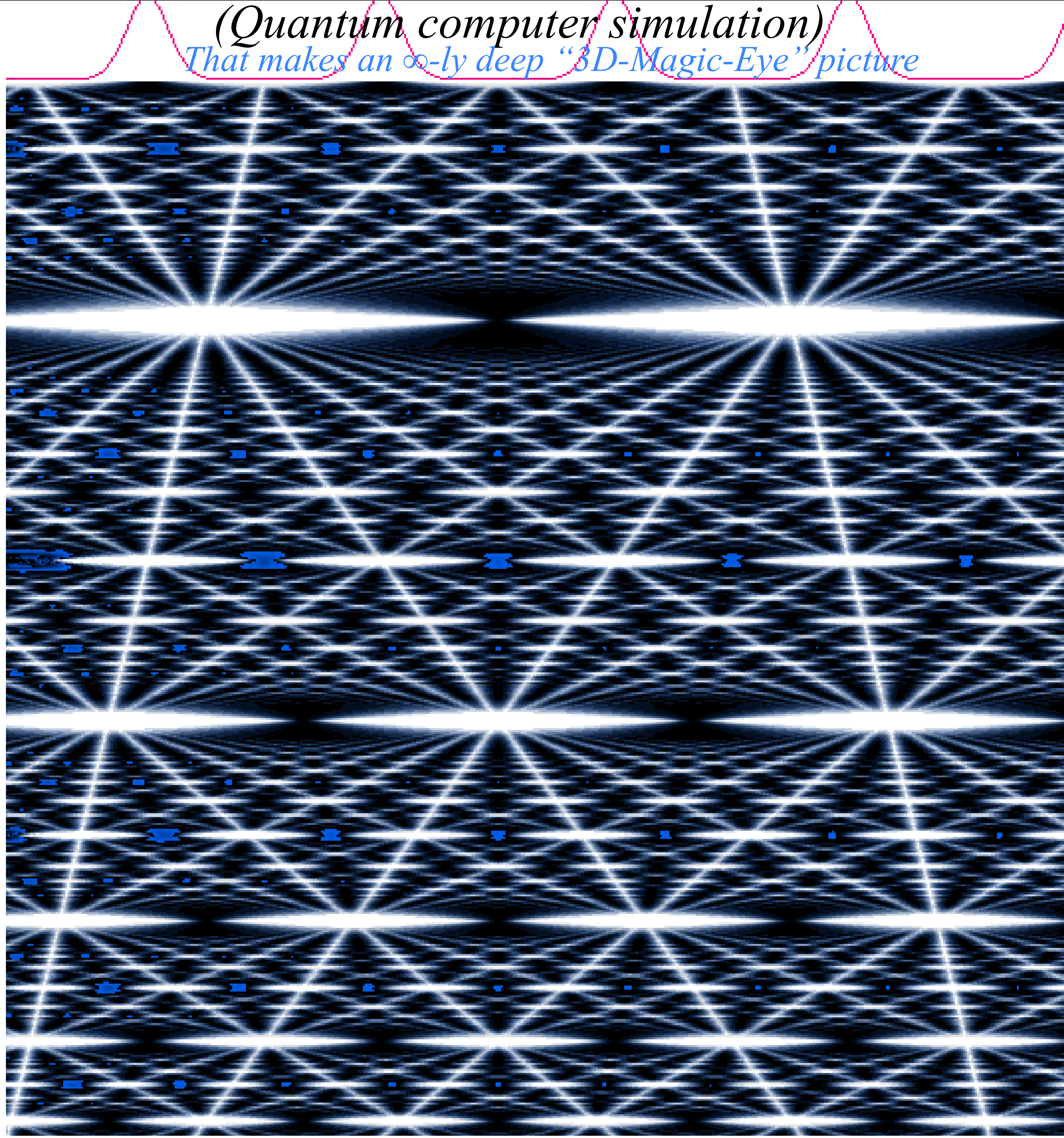
"Quantized" Thales Rectangles

provide
analytic geometry
of
fractal structure



(Quantum computer simulation)

That makes an ∞-ly deep "3D-Magic-Eye" picture



Geometric "Integration" (Converting Velocity data to Spacetime)

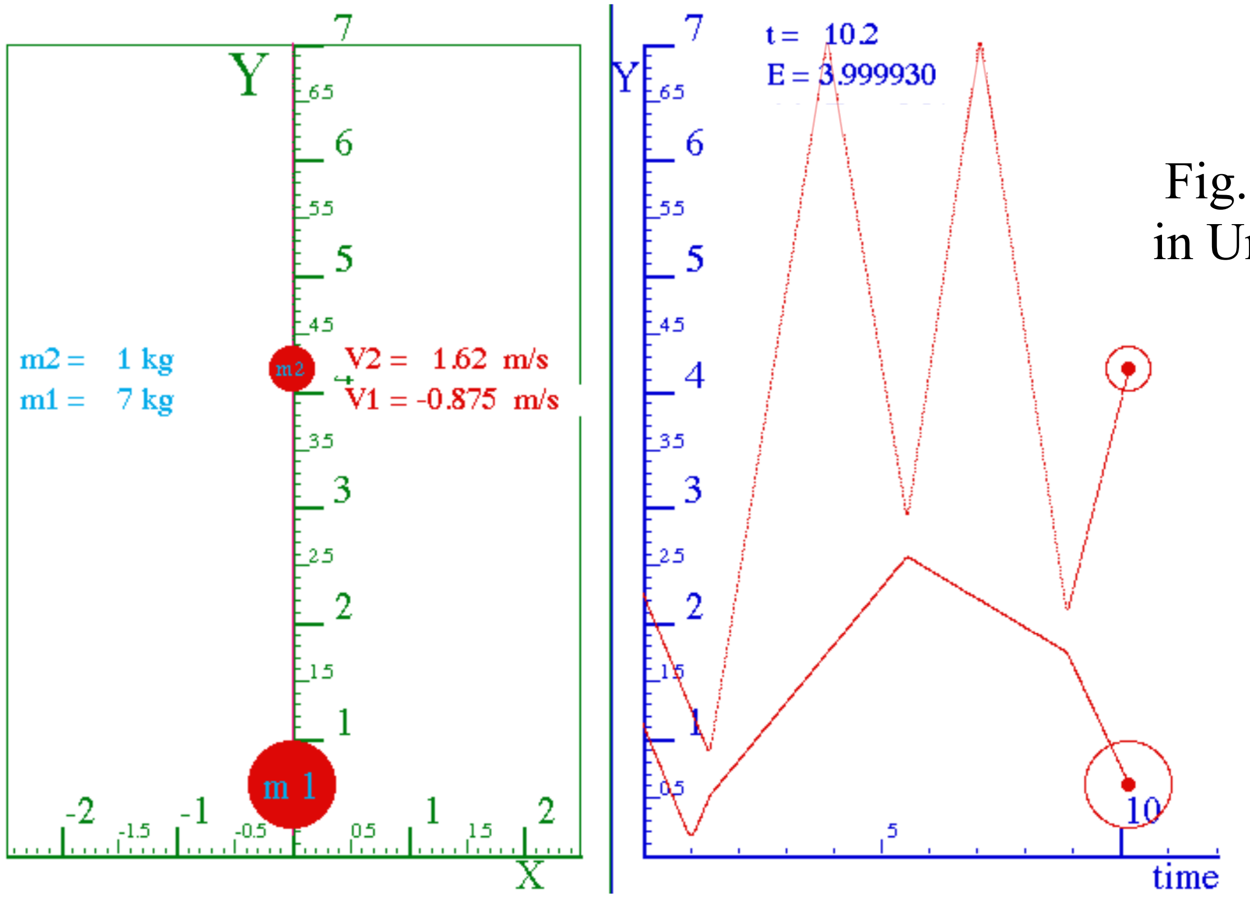


Fig. 4.8
in Unit 1

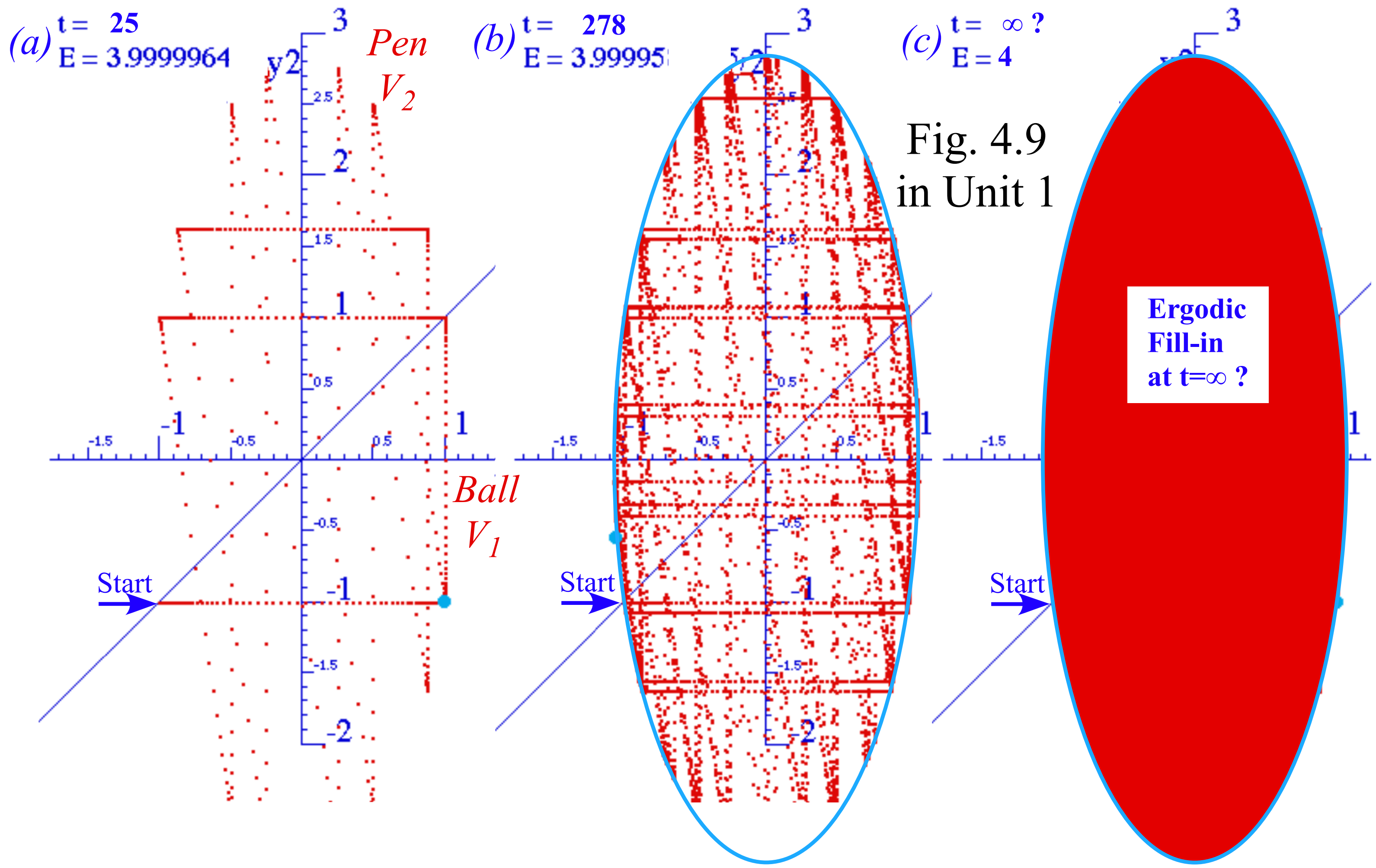
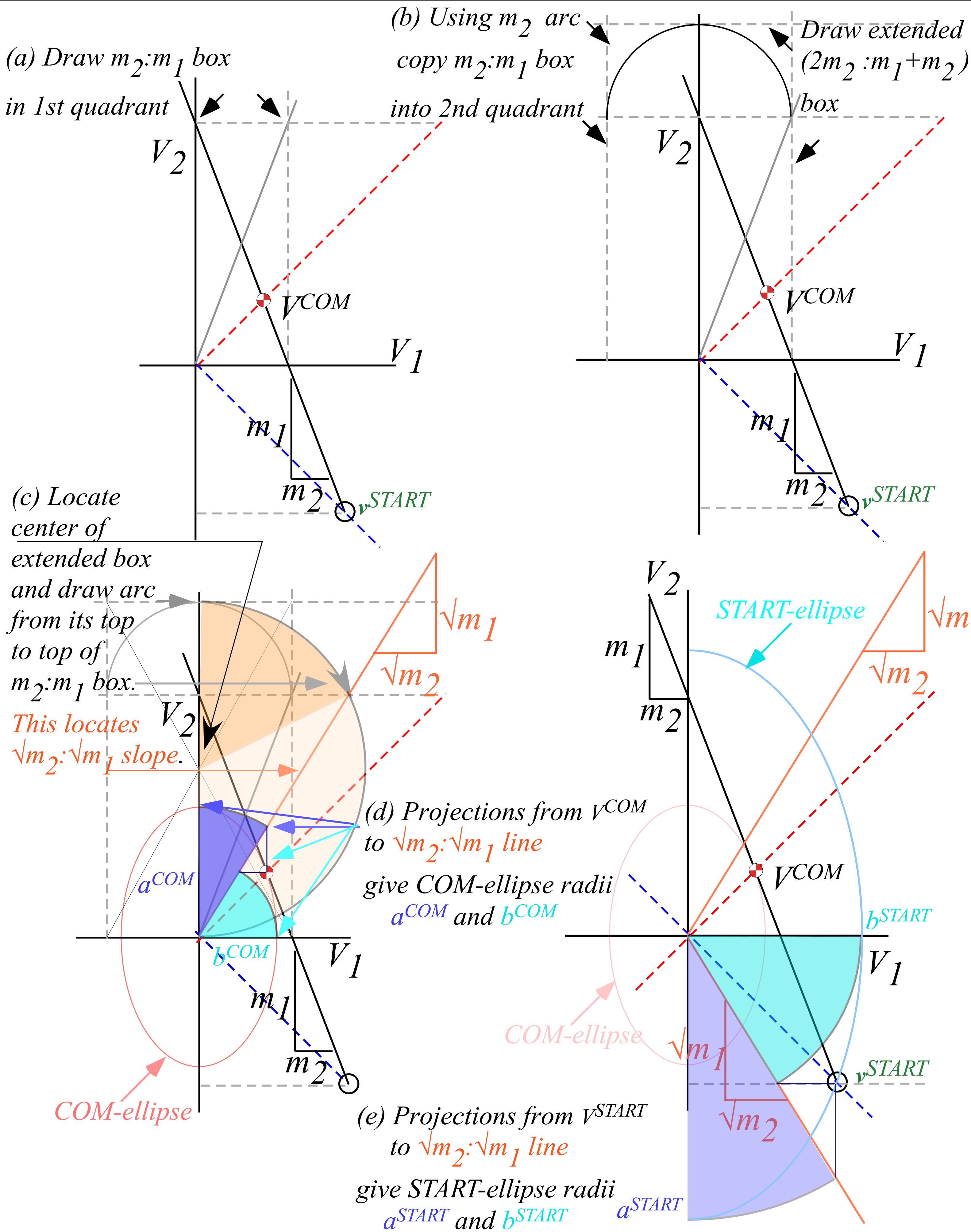


Fig. 4.9
in Unit 1



Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)