

Lecture 5
Tue. 9.9.2014

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 6, and Ch. 7 of Unit 1)

Review of $(V_1, V_2) \rightarrow (y_1, y_2)$ relations *High mass ratio $M_1/m_2 = 49$*

Force “field” or “pressure” due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y) = \text{const.}/y$ and the 1D-Adiabatic force field $F(y) = \text{const.}/y^3$

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y) = \text{const.}/y^2$

Physicist's Definition $F = -\Delta U/\Delta y$ vs. Mathematician's Definition $F = +\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y) = \text{const.} \ln(y)$

“Monster Mash” classical segue to Heisenberg action relations

Example of very very large M_1 ball-wall(s) crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The “Tiny-Big-Bang” [Harter, J. Mol. Spec. 210, 166-182 (2001)], [Harter, Li IMSS (2012)]

A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

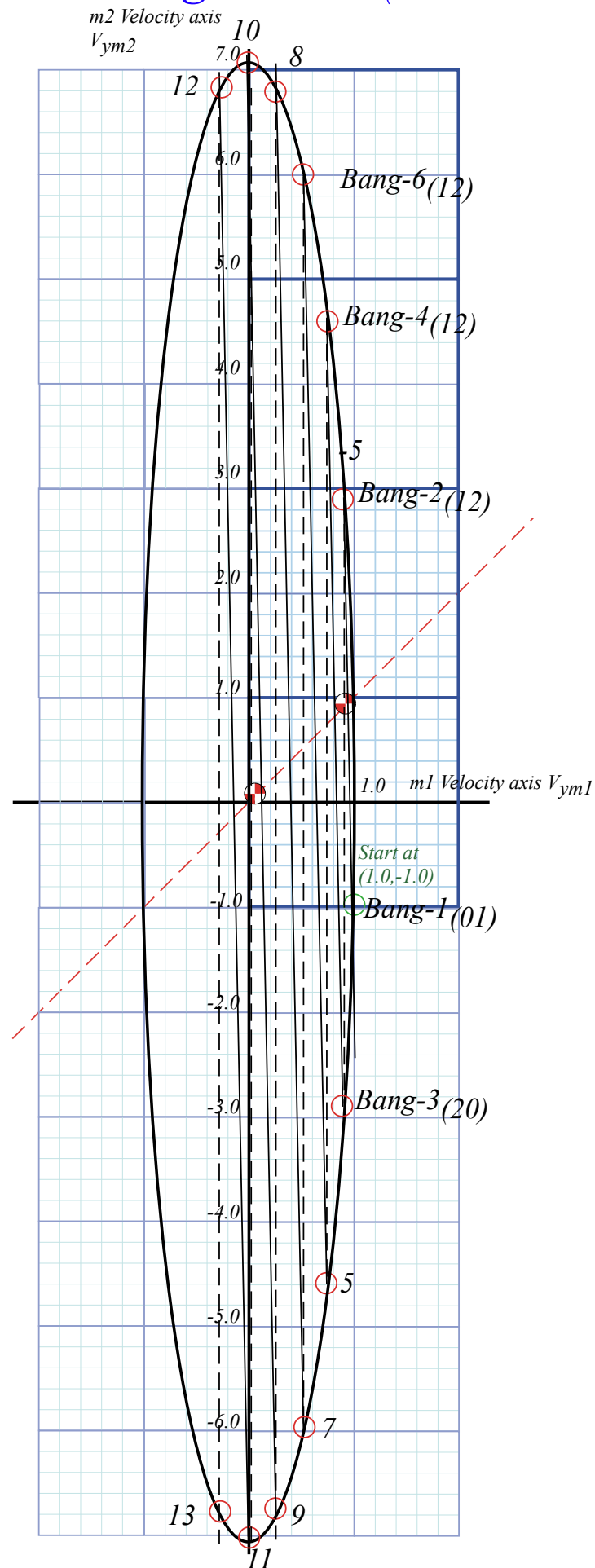
[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

[John Farey, Phil. Mag.(1816)]

Review of $(V_1, V_2) \rightarrow (y_1, y_2)$ relations

 *High mass ratio $M_1/m_2 = 49$*

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: $m_1=49$ and $m_2=1$

Kinetic Energy Ellipse

$$KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$\begin{aligned} a_1 &= \sqrt{2KE/m_1} \\ &= \sqrt{2KE/49} \\ &= \sqrt{50/49} \\ &= 1.01 \end{aligned}$$

Ellipse radius 2

$$\begin{aligned} a_2 &= \sqrt{2KE/m_2} \\ &= \sqrt{2KE/1} \\ &= \sqrt{50/1} \\ &= 7.07 \end{aligned}$$

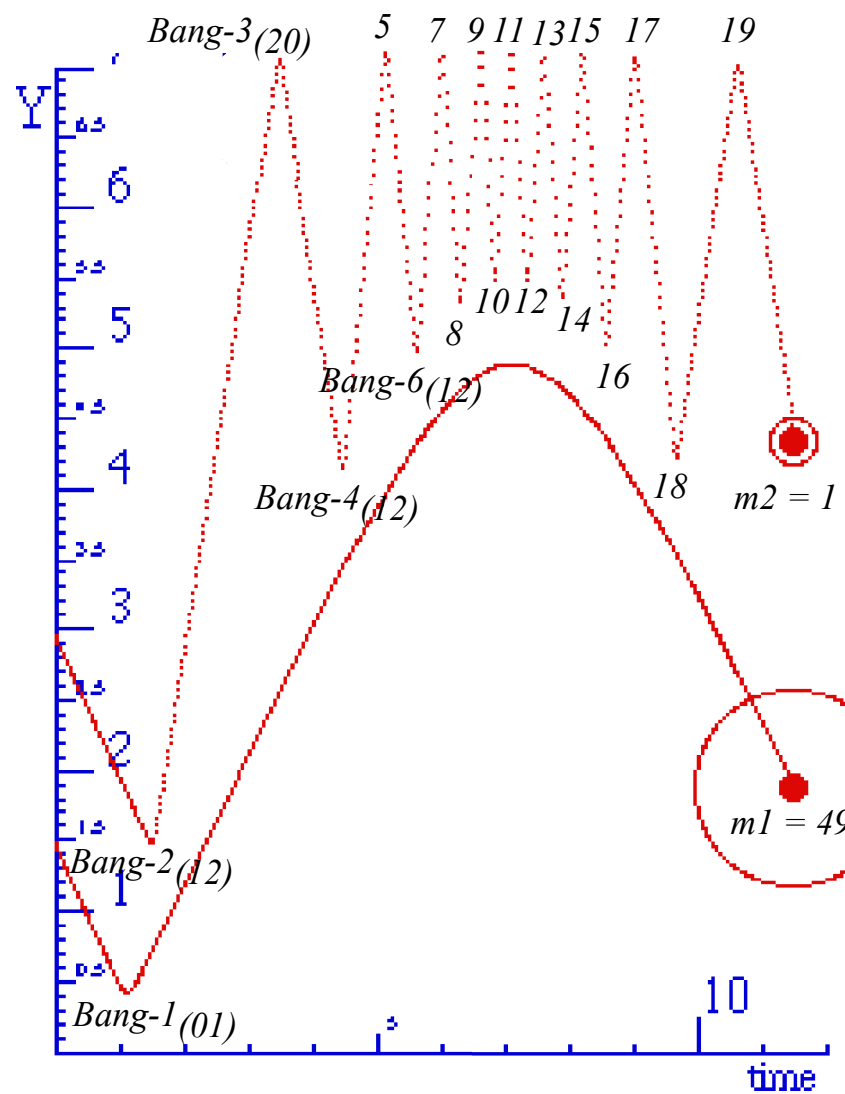


Fig. 5.1
in Unit 1

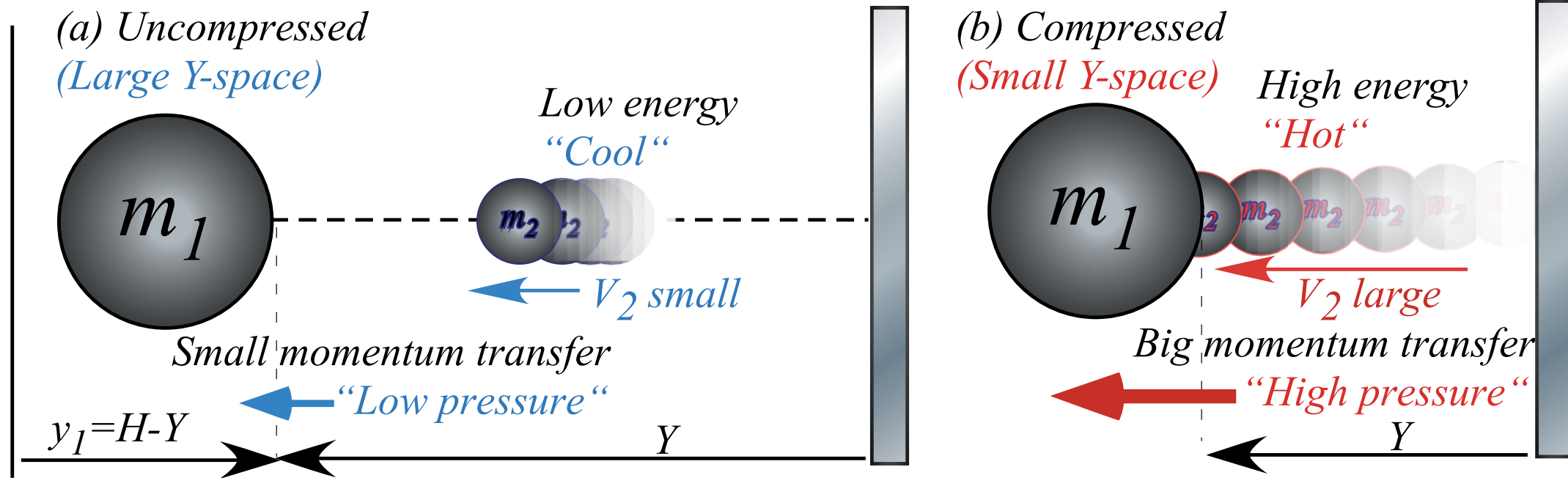
Force “field” or “pressure” due to many small bounces

 *Force defined as momentum transfer rate*

The 1D-Isothermal force field $F(y)=\text{const.}/y$ and the 1D-Adiabatic force field $F(y)=\text{const.}/y^3$

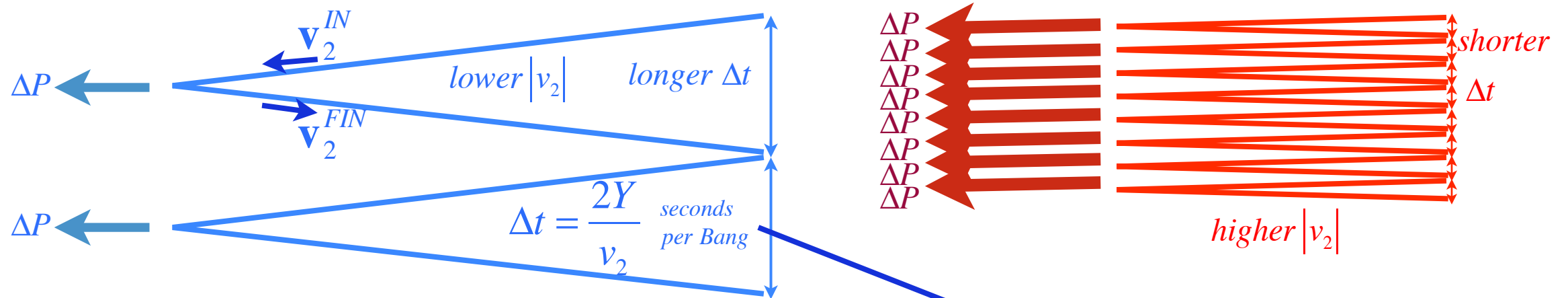
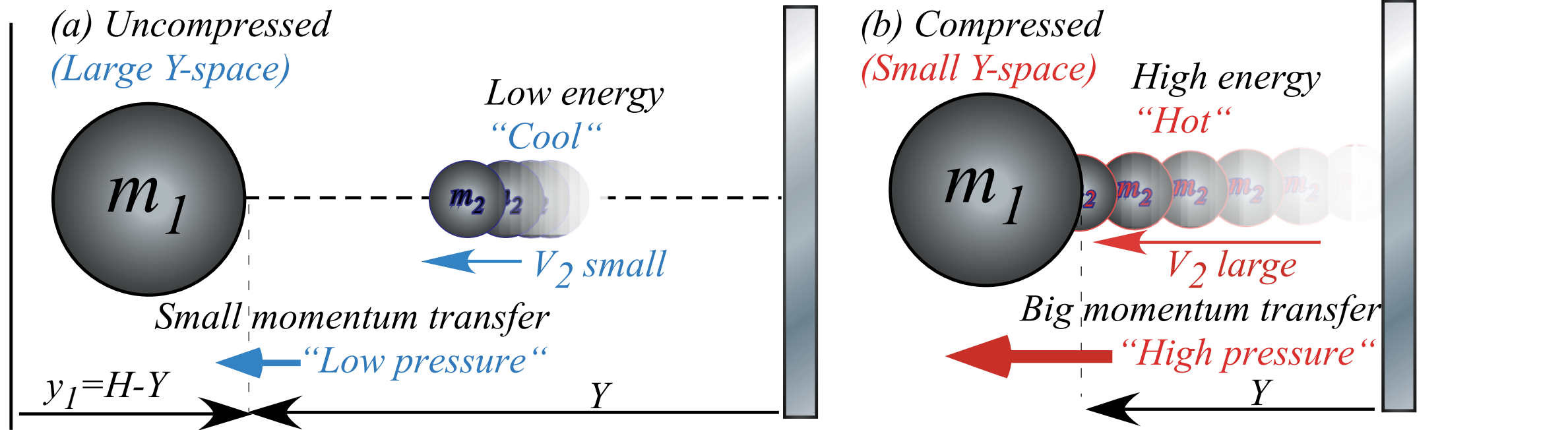
Big mass- m_1 ball feeling “force-field” or “pressure” of small ($m_2 \ll m_1$) rapidly ($v_2 \gg v_1$) bouncing ball

Unit 1
Fig. 6.1



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Unit 1
Fig. 6.1



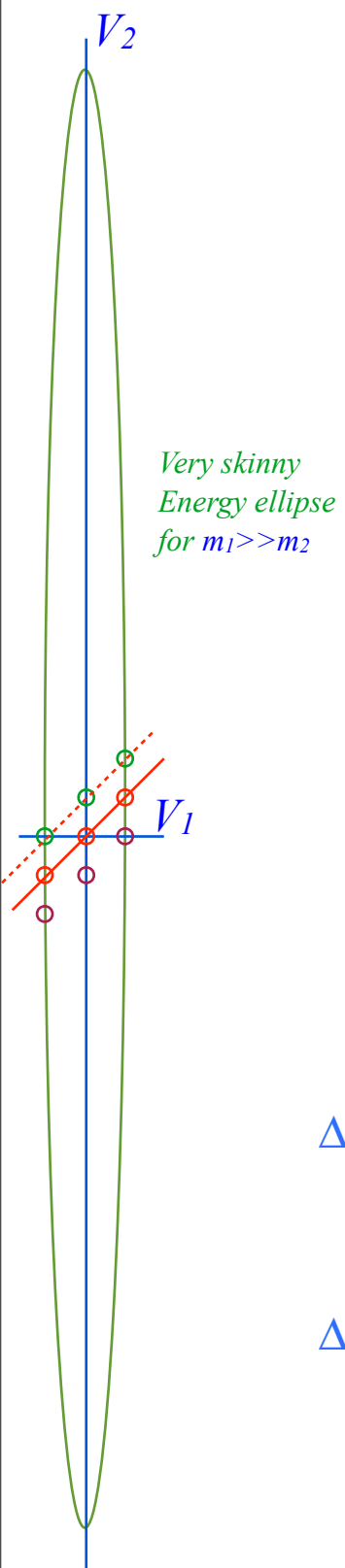
$$F = \frac{\Delta P}{\Delta t}$$

$$\Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN}$$

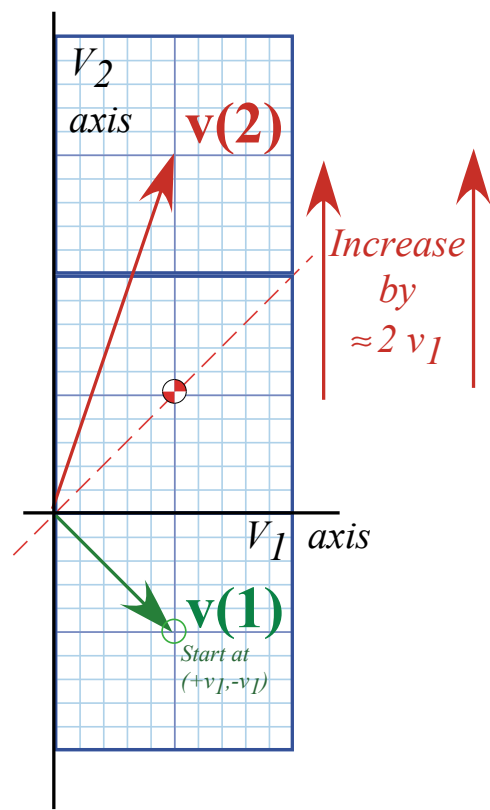
$$\frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on $m_1 = (\text{Momentum per sec.}) = (\text{Momentum per Bang}) \cdot (\text{Bangs per second})$

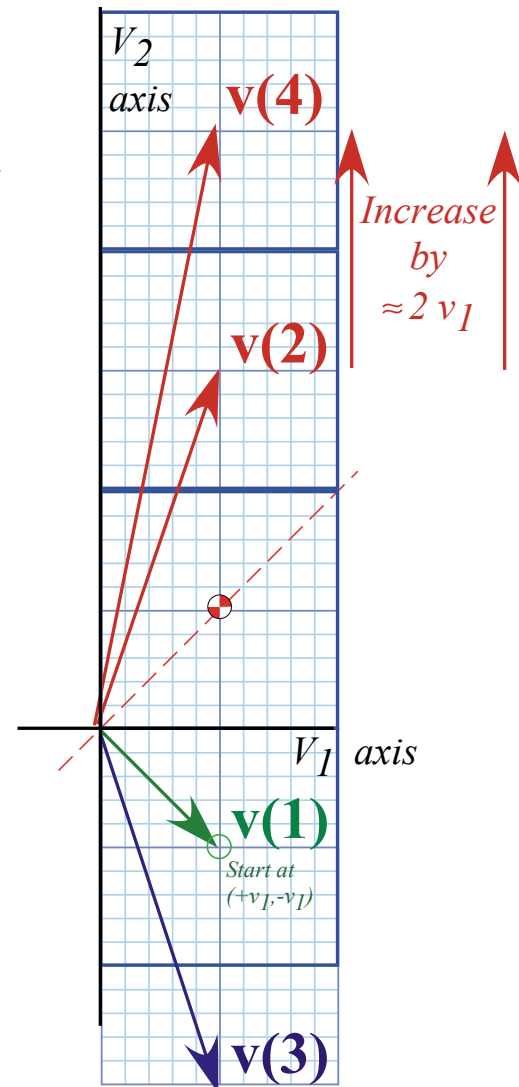
Double-Bang Sequences for $m_1 \gg m_2$



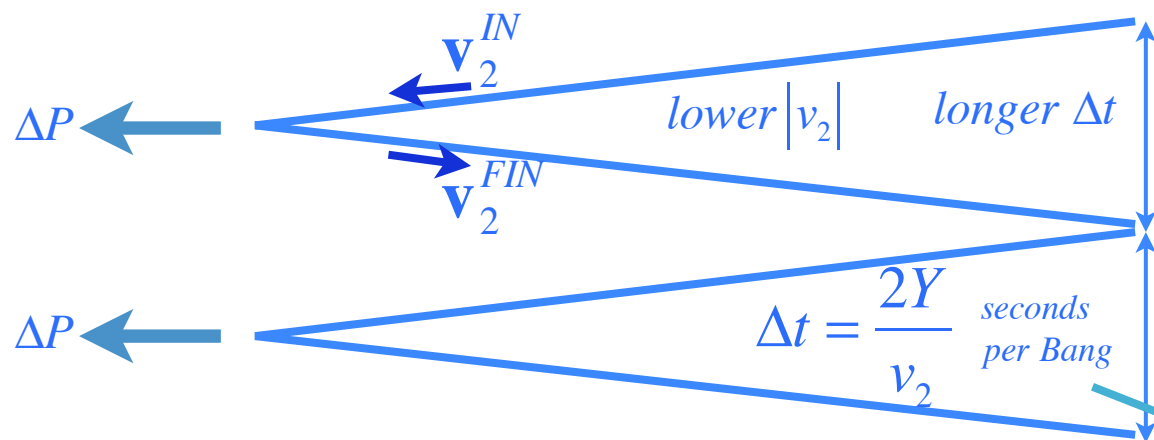
(a) After 2 Bangs



(b) After 4 Bangs



Unit 1
Fig. 6.2



$$|v_2^{FIN}| = |v_2^{IN}| + |2v_1| \quad \text{for: } m_1 \gg m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

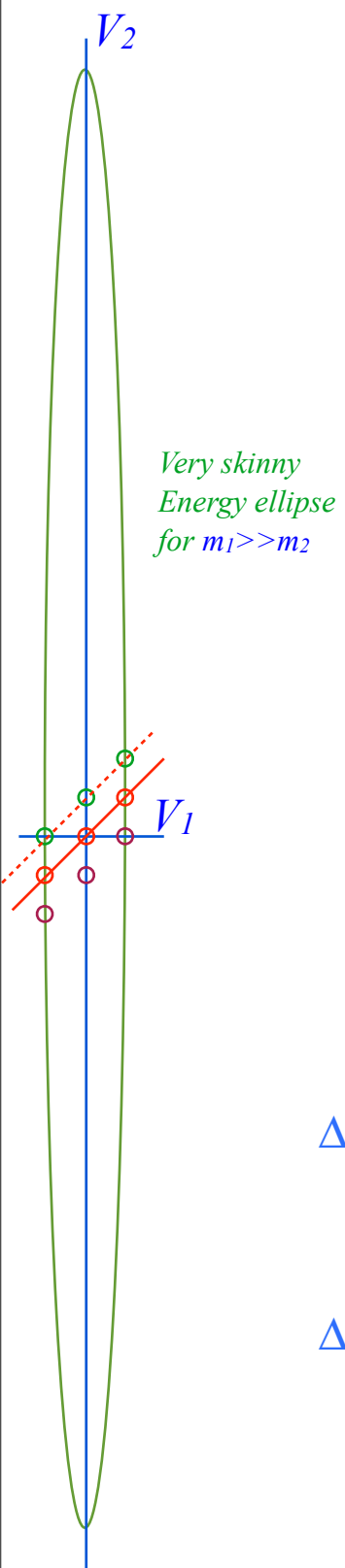
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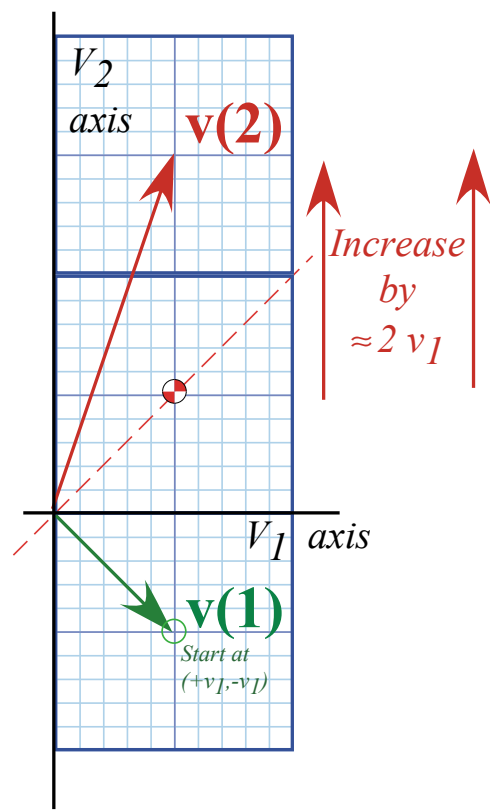
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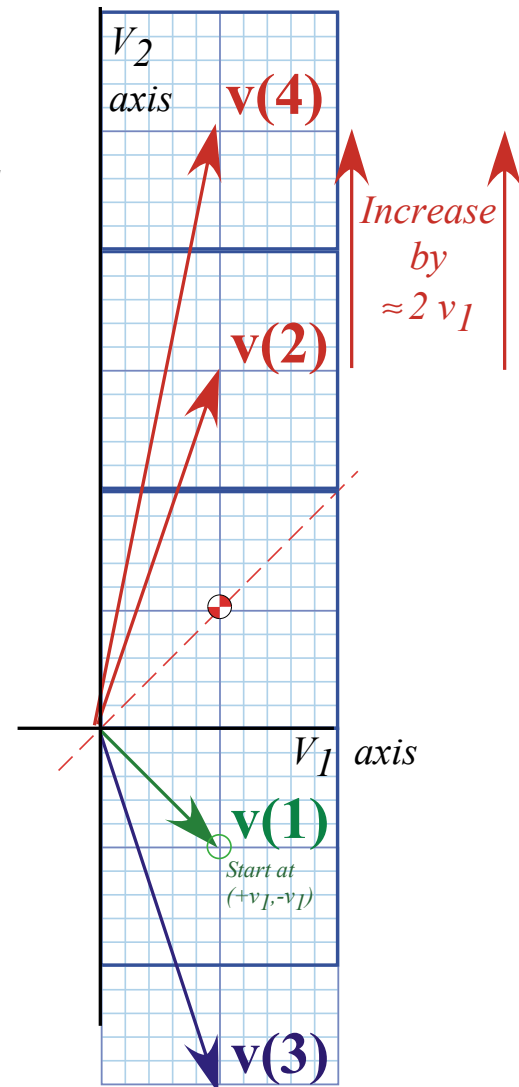
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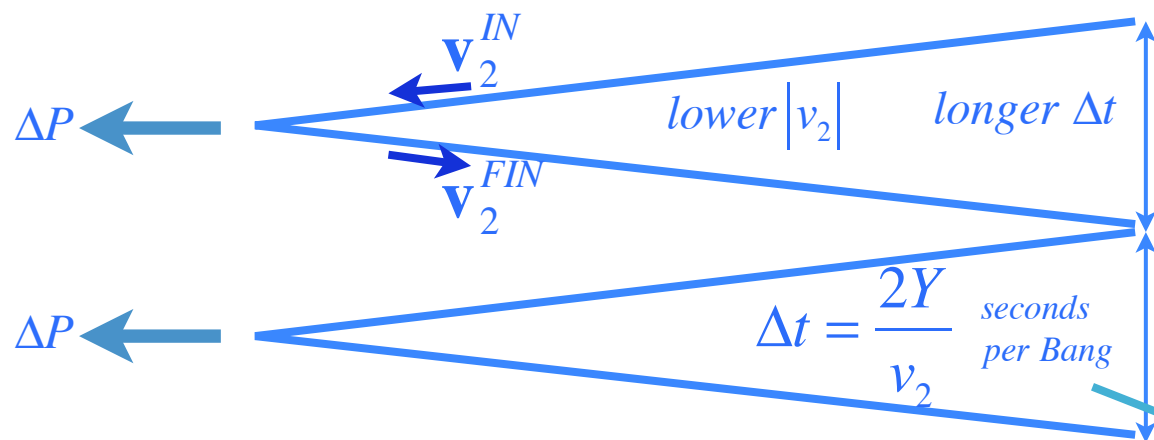
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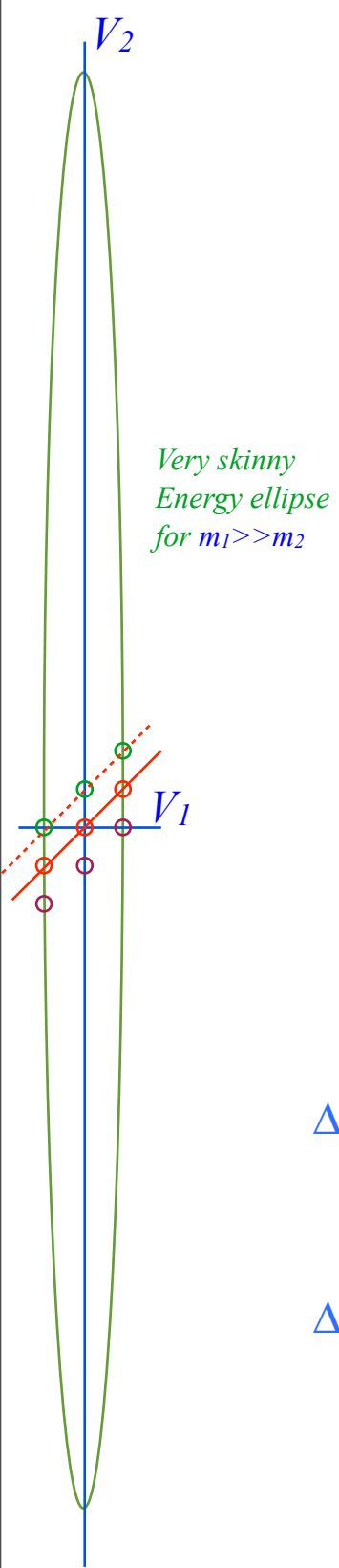
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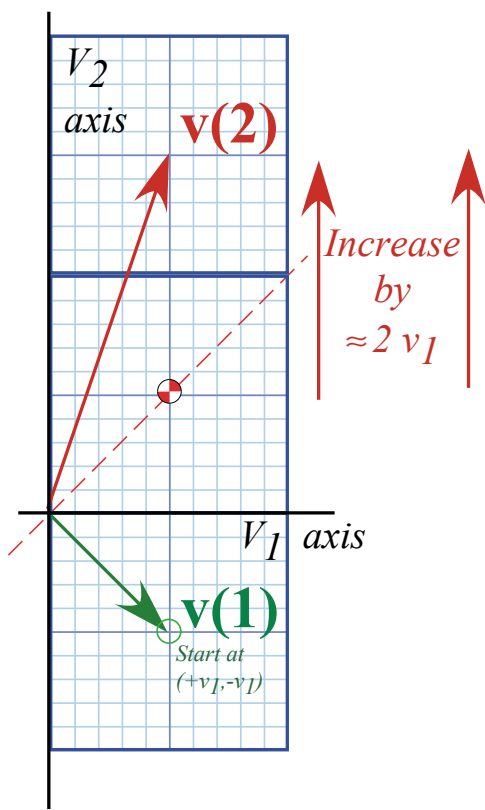
$$\Delta P = m_2 v_2^{IN} - m_2 (-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1$$



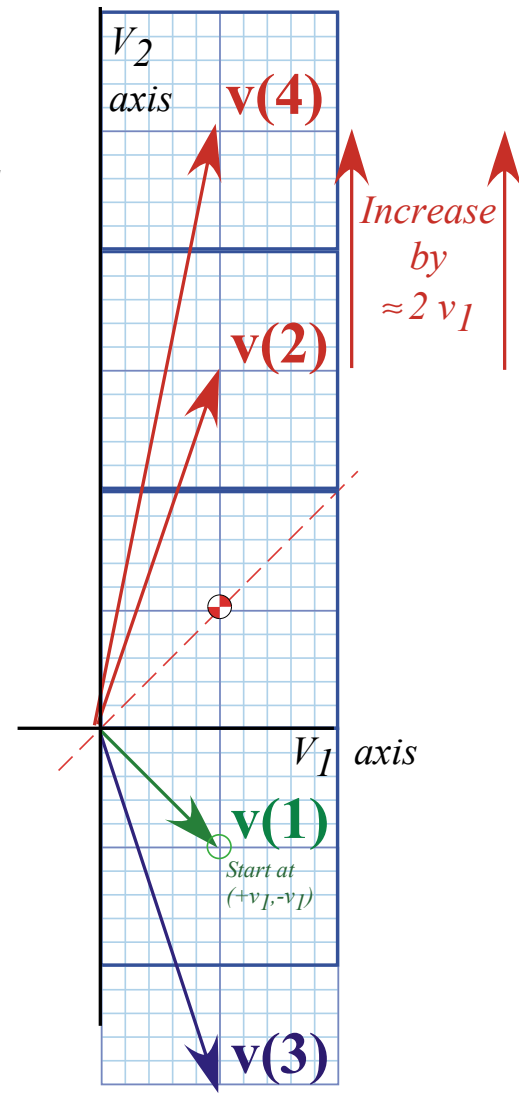
Very skinny
Energy ellipse
for $m_1 \gg m_2$

Double-Bang Sequences for $m_1 \gg m_2$

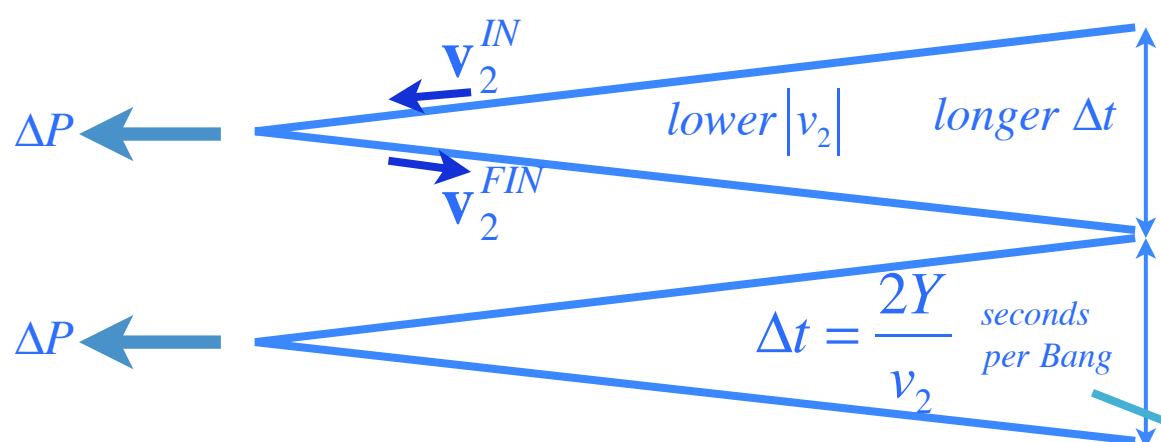
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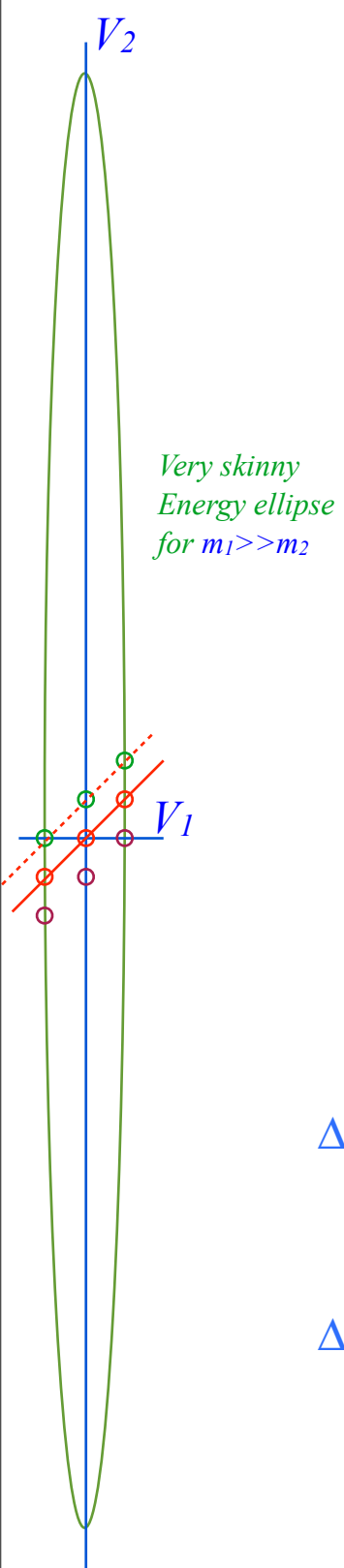
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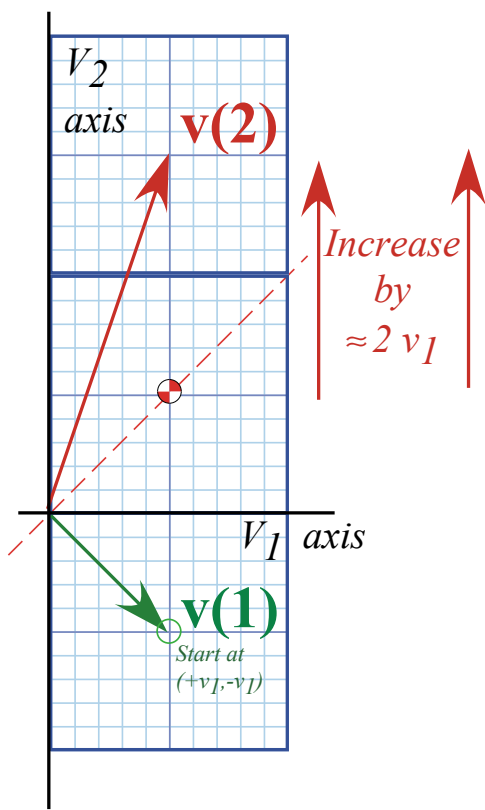
Assuming slow $m_1 : v_1 \ll v_2$



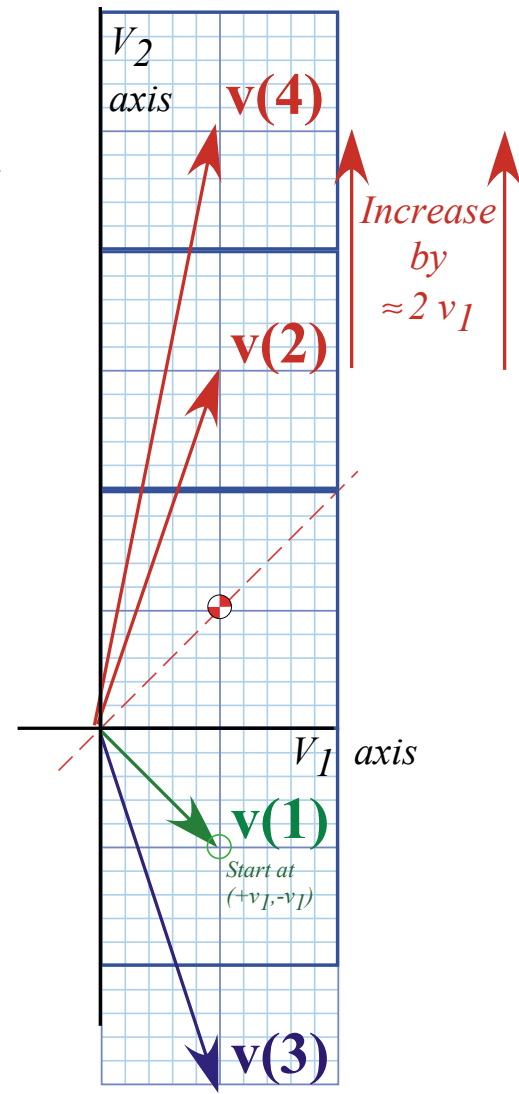
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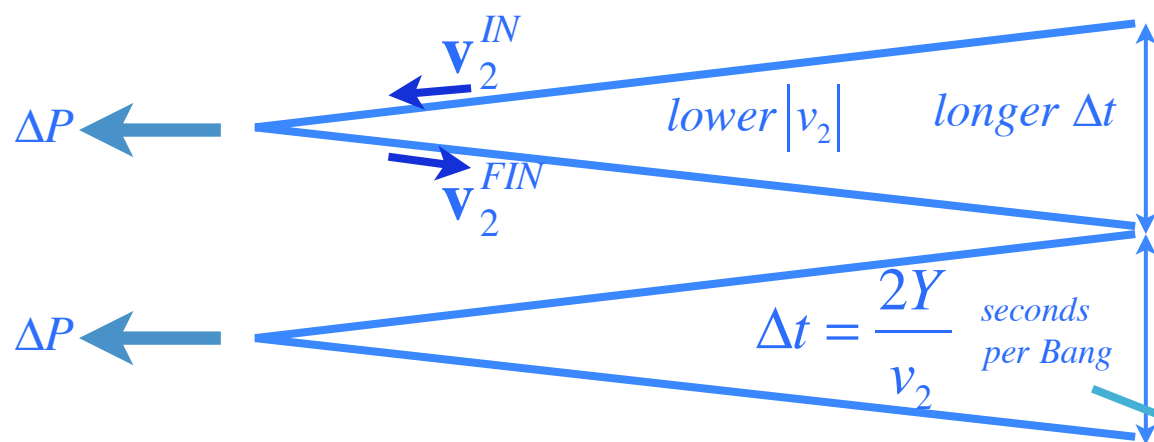
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$$F = \frac{\Delta P}{\Delta t} = (\Delta P \approx 2m_2 v_2) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y} \right) \approx \frac{m_2 v_2^2}{Y}$$

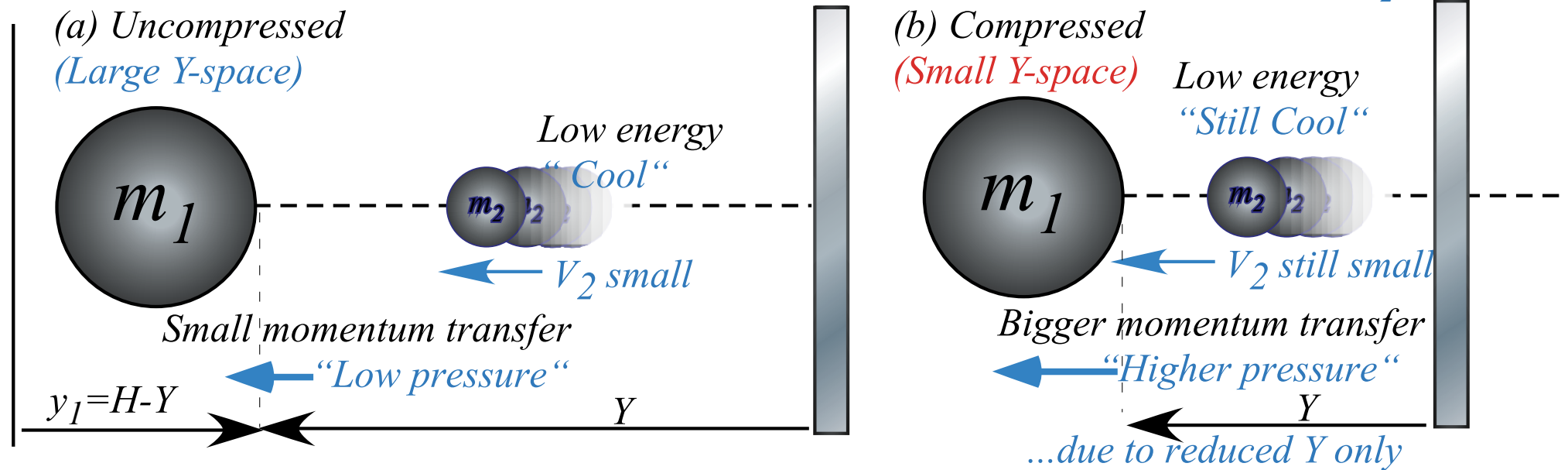
Assuming slow $m_1 : v_1 \ll v_2$

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1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2v_2^2}{Y} = \frac{\text{const.}}{Y}$$

Isothermal expansion or contraction: Wall serves as thermal bath to keep m_2 cool



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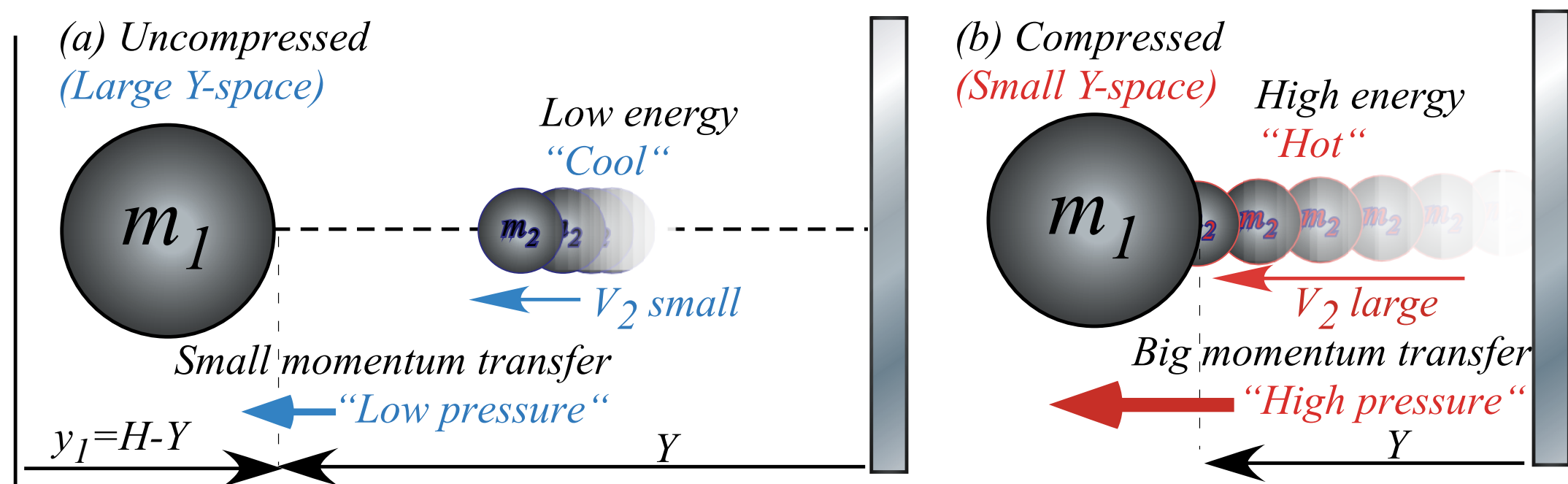
$$F = \frac{m_2v_2^2}{Y} = \frac{\text{const.}}{Y}$$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y : $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2 . This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1B = 2v_1 \frac{v_2}{2Y} = -2 \frac{dY}{dt} \frac{v_2}{2Y}$$

Wall not given time to give or take KE



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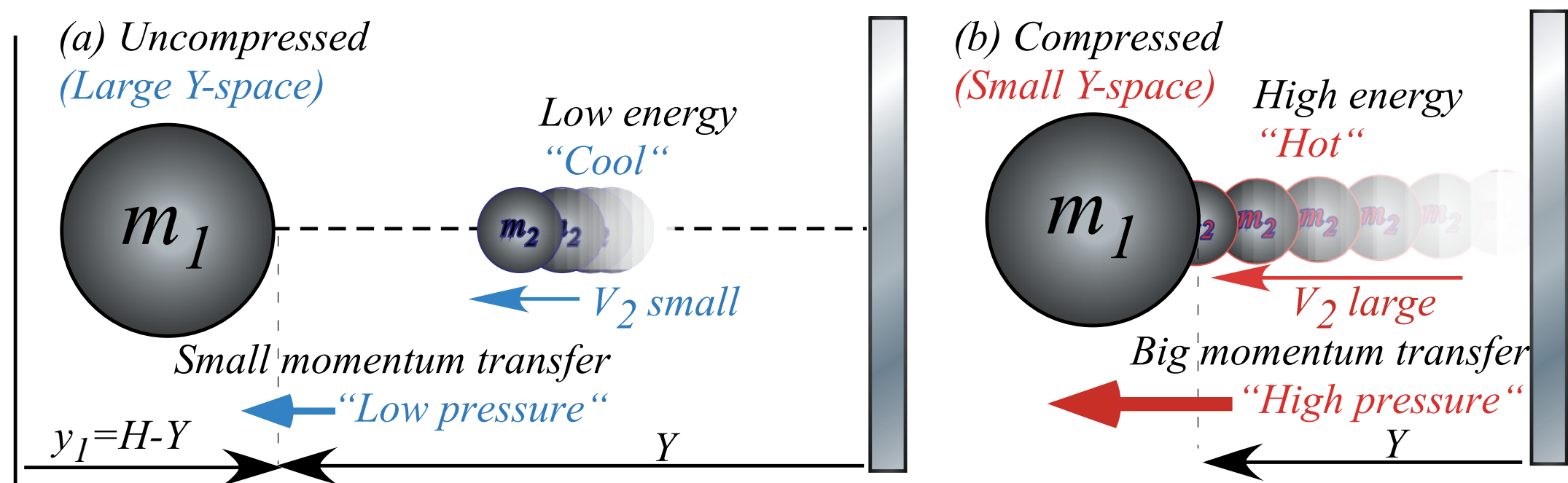
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Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e (e^C x)$

$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \text{ integrates to: } \ln v_2 = -\ln Y + C \text{ or: } \ln v_2 = \ln \frac{\text{const.}}{Y} \text{ or: } v_2 = \frac{\text{const.}}{Y}$$

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$$F = \frac{\Delta P}{\Delta t} = (\Delta P \approx 2m_2v_2) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y} \right) \approx \frac{m_2v_2^2}{Y}$$

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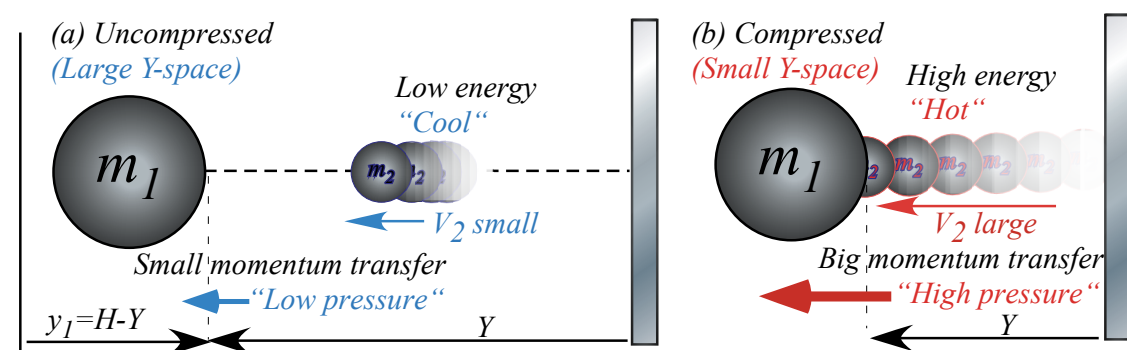
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Force law with this variable v_2 is called *adiabatic* or *not-diabatic* or *not-gradual*.

1D-Adiabatic Force Law (assume v_2 varies: $v_2 = \frac{\text{const.}}{Y} = \frac{v_2^{IN} Y(t=0)}{Y}$): $F = \frac{m_2 (v_2^{IN} Y(t=0))^2}{Y^3} = \frac{\text{const.}}{Y^3}$



Potential field due to many small bounces



Example of 1D-Adiabatic potential $U(y)=\text{const.}/y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=\text{const.} \ln(y)$

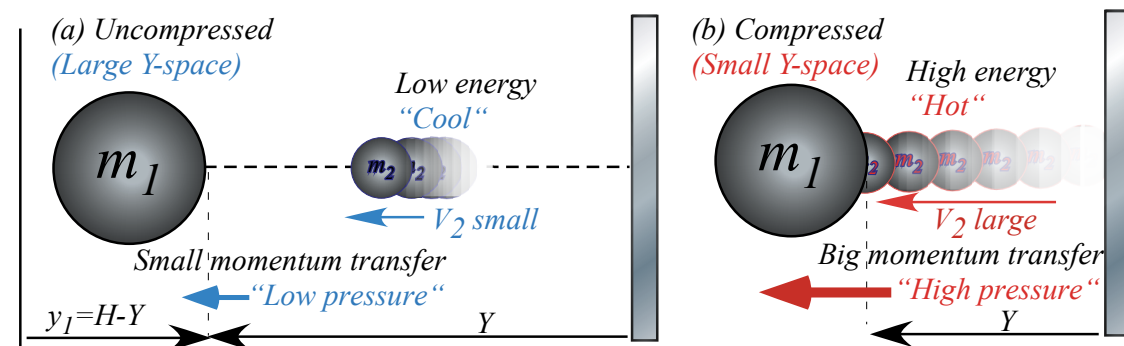
Big mass- m_1 ball feeling “potential-field” or “gradient” due to small ($m_2 \ll m_1$) rapidly ($v_2 \gg v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{\text{const.}}{Y}$ the total energy E is strictly conserved.

$$\text{const.} = E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{\text{const.}}{Y} \right)^2$$

Define for big mass m_1 : Kinetic energy $KE(v_1)$ vs Potential energy $PE(Y) = U(Y)$

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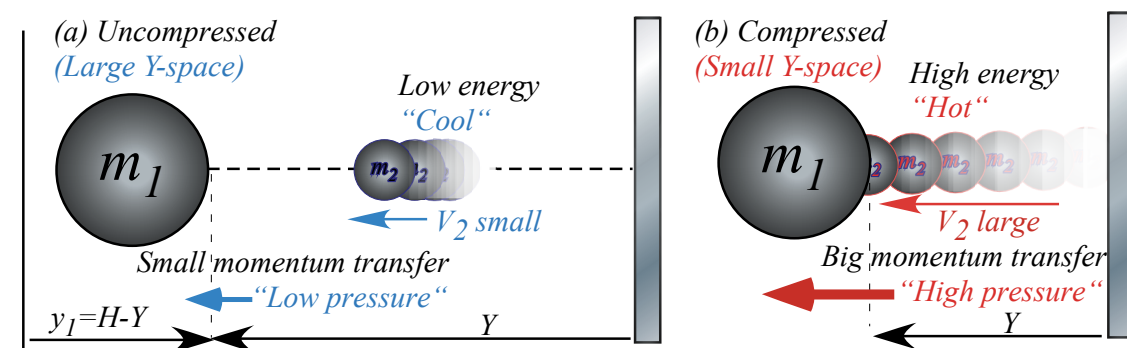
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Potential energy $PE(Y) = U(Y) = \frac{1}{2} m_2 \left(\frac{\text{const.}}{Y} \right)^2$ relates to Force $F(Y)$ thru Work relations $F \cdot dY = \pm dU$

Q? Another axiom? A: No.



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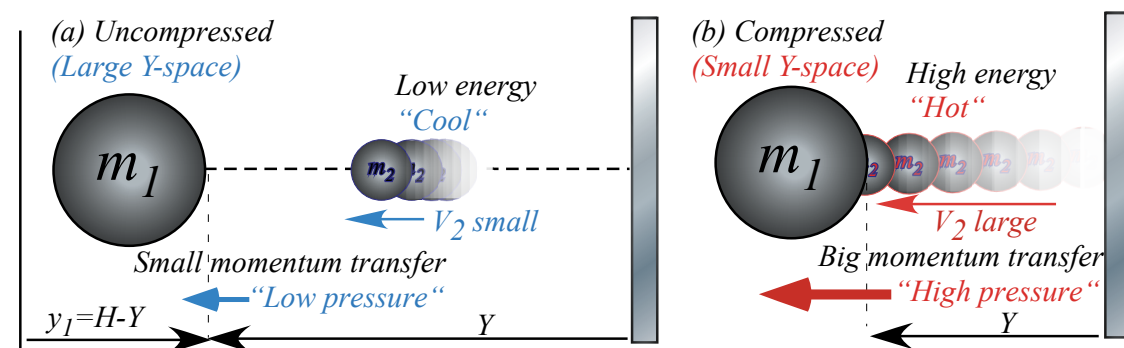
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Q? Another axiom? A: No. $\int F \cdot dY = \int \frac{dp}{dt} \cdot dY = \int \frac{dY}{dt} \cdot dp = \int V \cdot dp = \int V \cdot d(mV) = m \frac{V^2}{2} + \text{const} = U$



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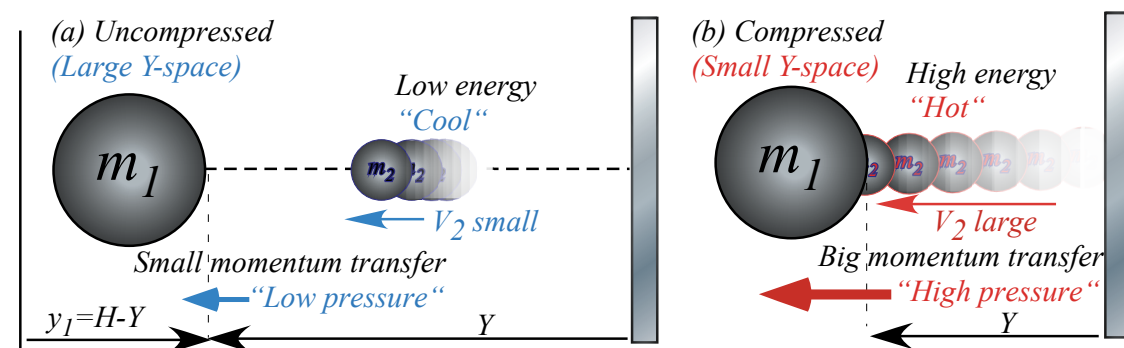
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or else : $F \cdot \frac{dY}{dt} = \frac{dp}{dt} \cdot V = \frac{d(mV)}{dt} \cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt}$



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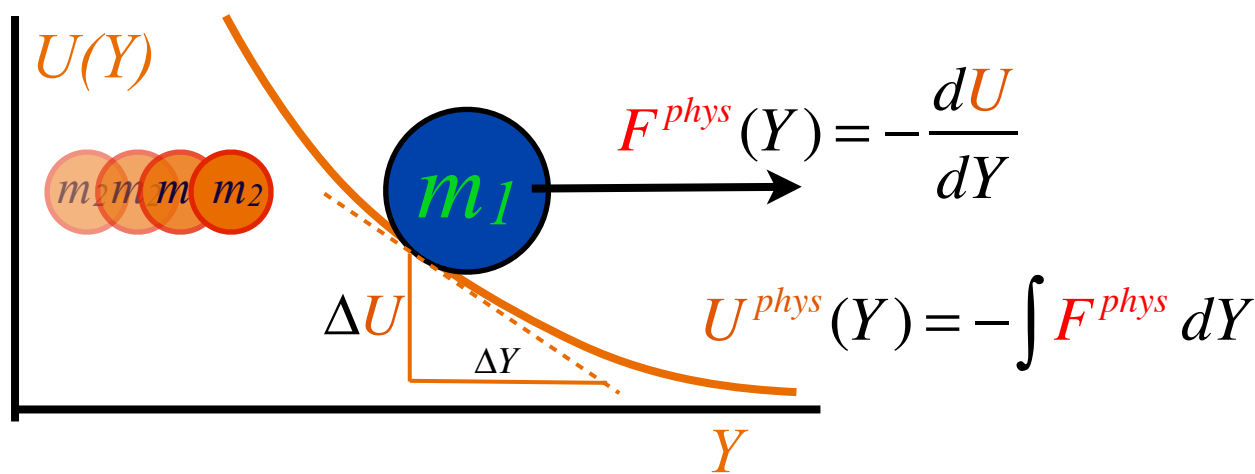
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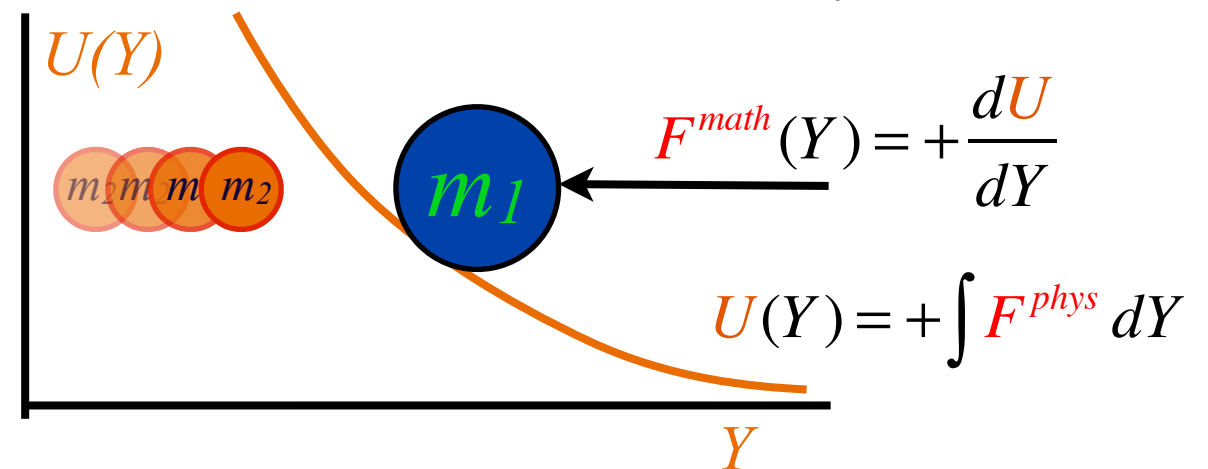
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The “Physicist” View of Force



The “Mathematician” View of Force



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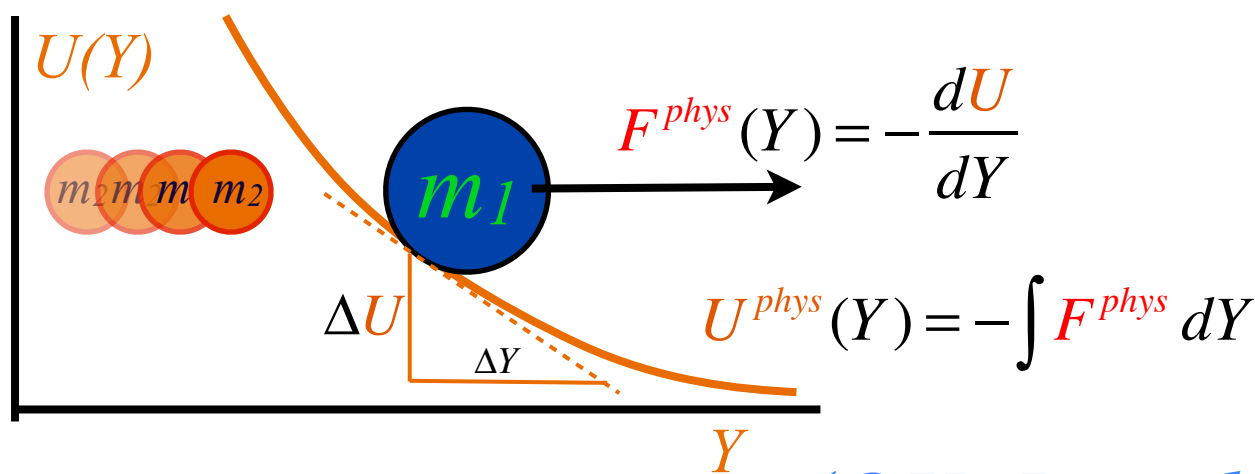
In adiabatic case where $v_2 = \frac{\text{const.}}{Y}$ the total energy E is strictly conserved.

$$\text{const.} = E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 \left(\frac{\text{const.}}{Y} \right)^2$$

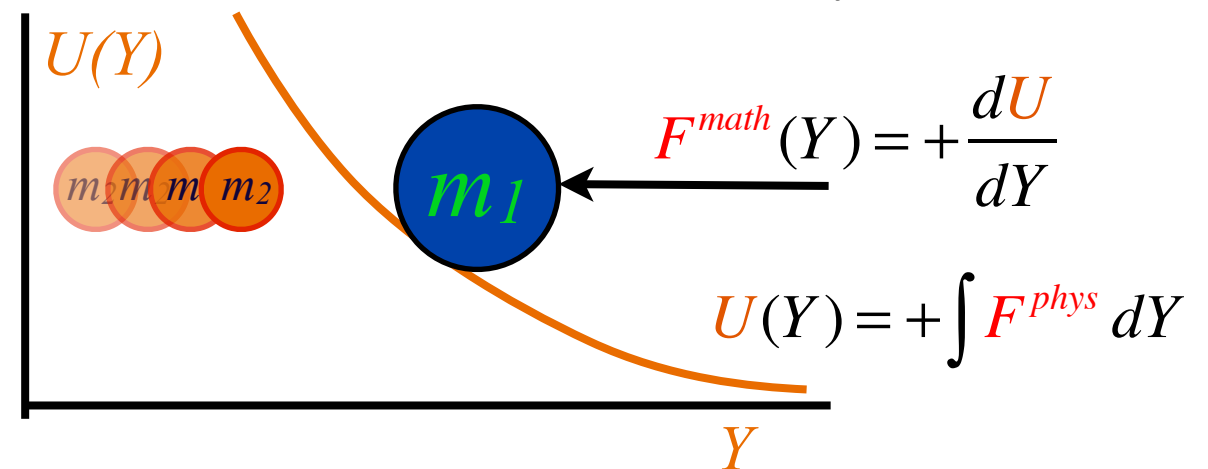
Define for big mass m_1 : Kinetic energy $KE(v_1)$ vs Potential energy $PE(Y) = U(Y)$

Potential energy $PE(Y) = U(Y) = \frac{1}{2} m_2 \left(\frac{\text{const.}}{Y} \right)^2$ relates to Force $F(Y)$ thru Work relations $F \cdot dY = \pm dU$

The “Physicist” View of Force



The “Mathematician” View of Force



(OK, But, does it work?)

Big mass- m_1 ball feeling “potential-field” or “gradient” due to small ($m_2 \ll m_1$) rapidly ($v_2 \gg v_1$) bouncing ball

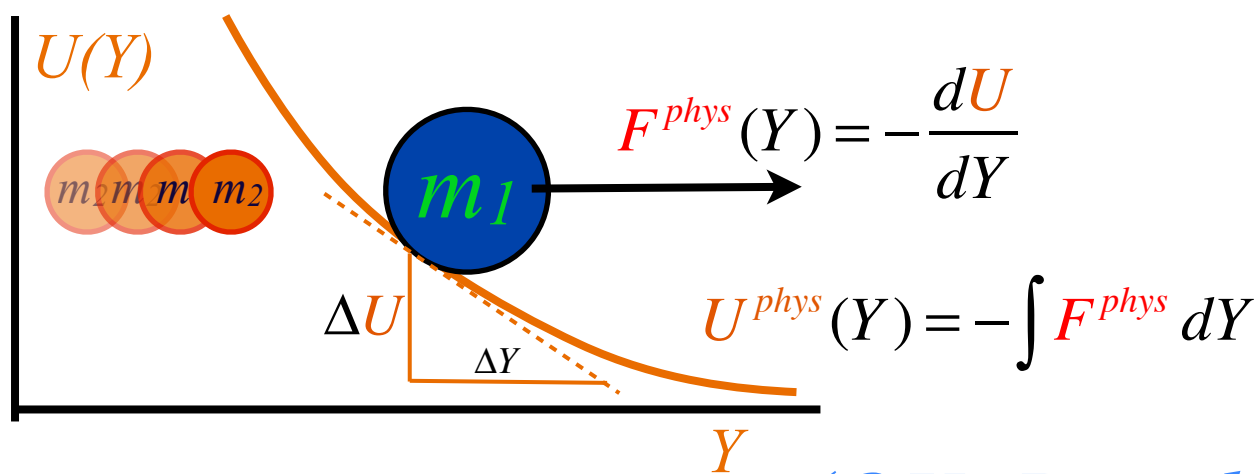
In adiabatic case where $v_2 = \frac{\text{const.}}{Y}$ the total energy E is strictly conserved.

$$\text{const.} = E = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \boxed{\frac{1}{2} m_1 v_1^2} + \frac{1}{2} m_2 \left(\frac{\text{const.}}{Y} \right)^2$$

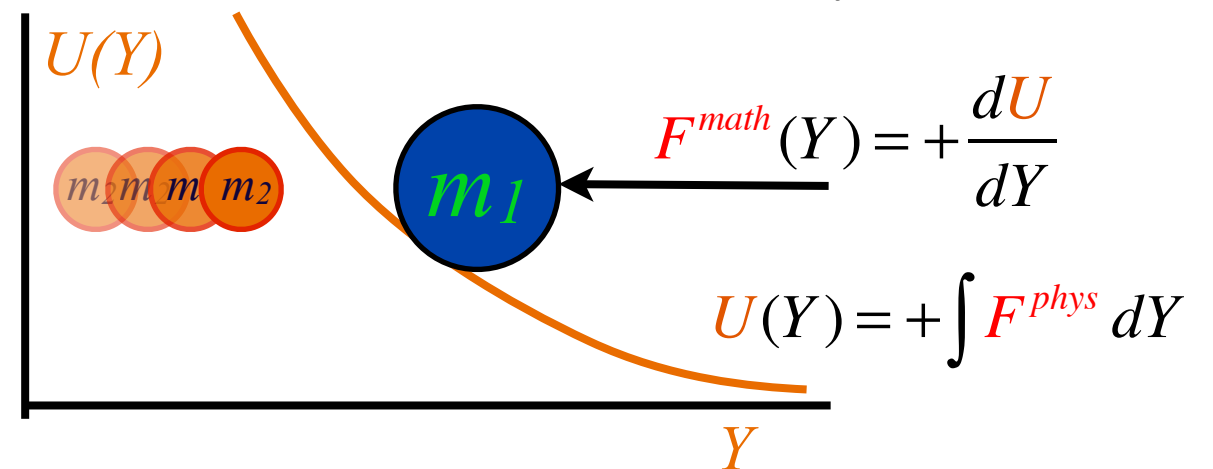
Define for big mass m_1 : Kinetic energy $KE(v_1)$ vs Potential energy $PE(Y) = U(Y)$

Potential energy $PE(Y) = U(Y) = \frac{1}{2} m_2 \left(\frac{\text{const.}}{Y} \right)^2$ relates to Force $F(Y)$ thru Work relations $F \cdot dY = \pm dU$

The “Physicist” View of Force



The “Mathematician” View of Force



(OK, But, does it work?)

$$F^{phys} = m_2 \frac{(\text{const.})^2}{Y^3} \quad \text{consistent with :} \quad F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY} \frac{1}{2} m_2 \left(\frac{\text{const.}}{Y} \right)^2 = m_2 \frac{(\text{const.})^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=\text{const.}/y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

 *Example of 1D-Isothermal potential $U(y)=\text{const.} \ln(y)$*

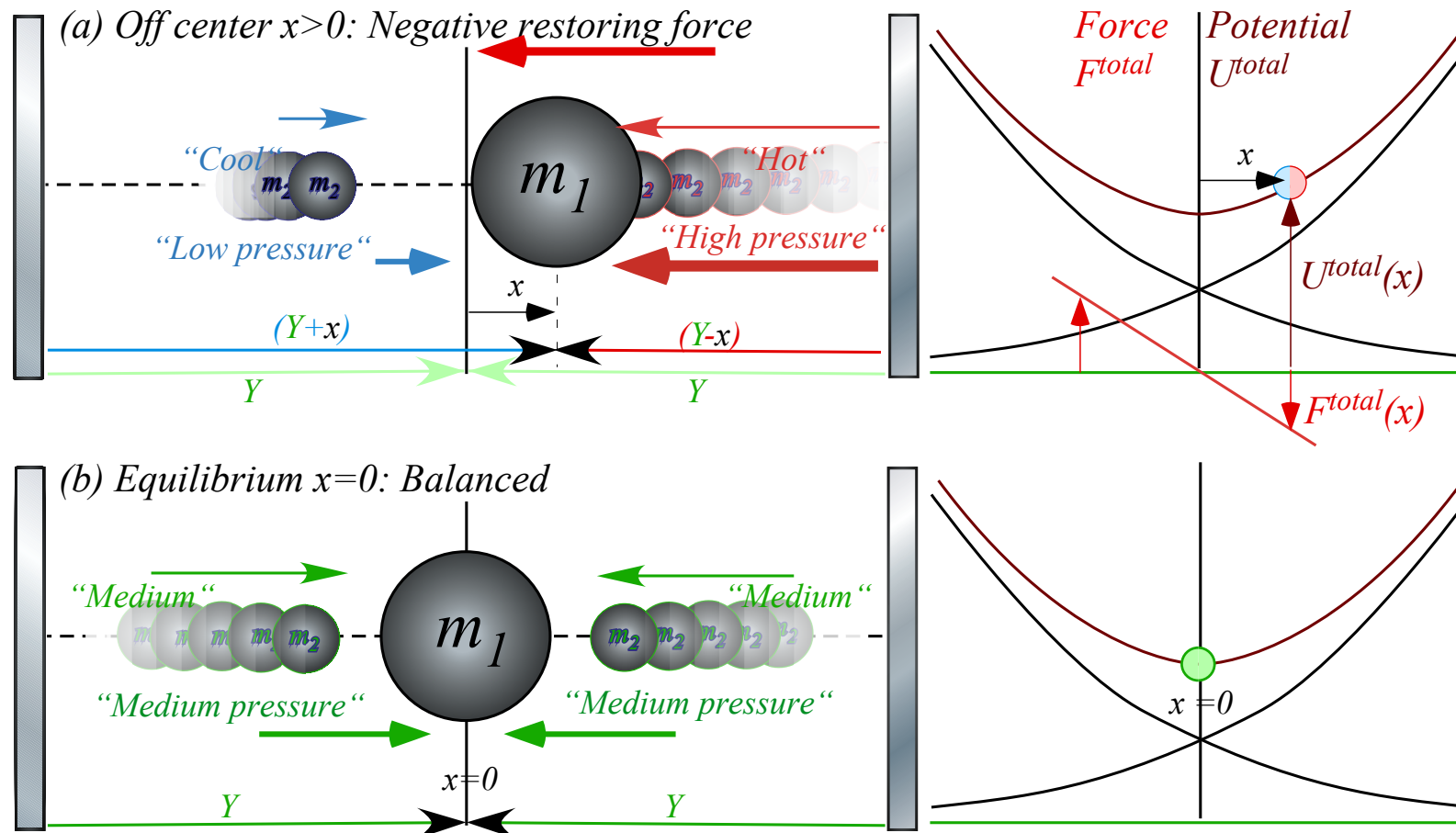
1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{\text{const.}}{Y}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y}$$

implies :

$$U = -m_2 v_2^2 \ln(Y)$$



Unit 1
Fig. 6.2

Anharmonic
oscillator
terms...

Harmonic
oscillator
term

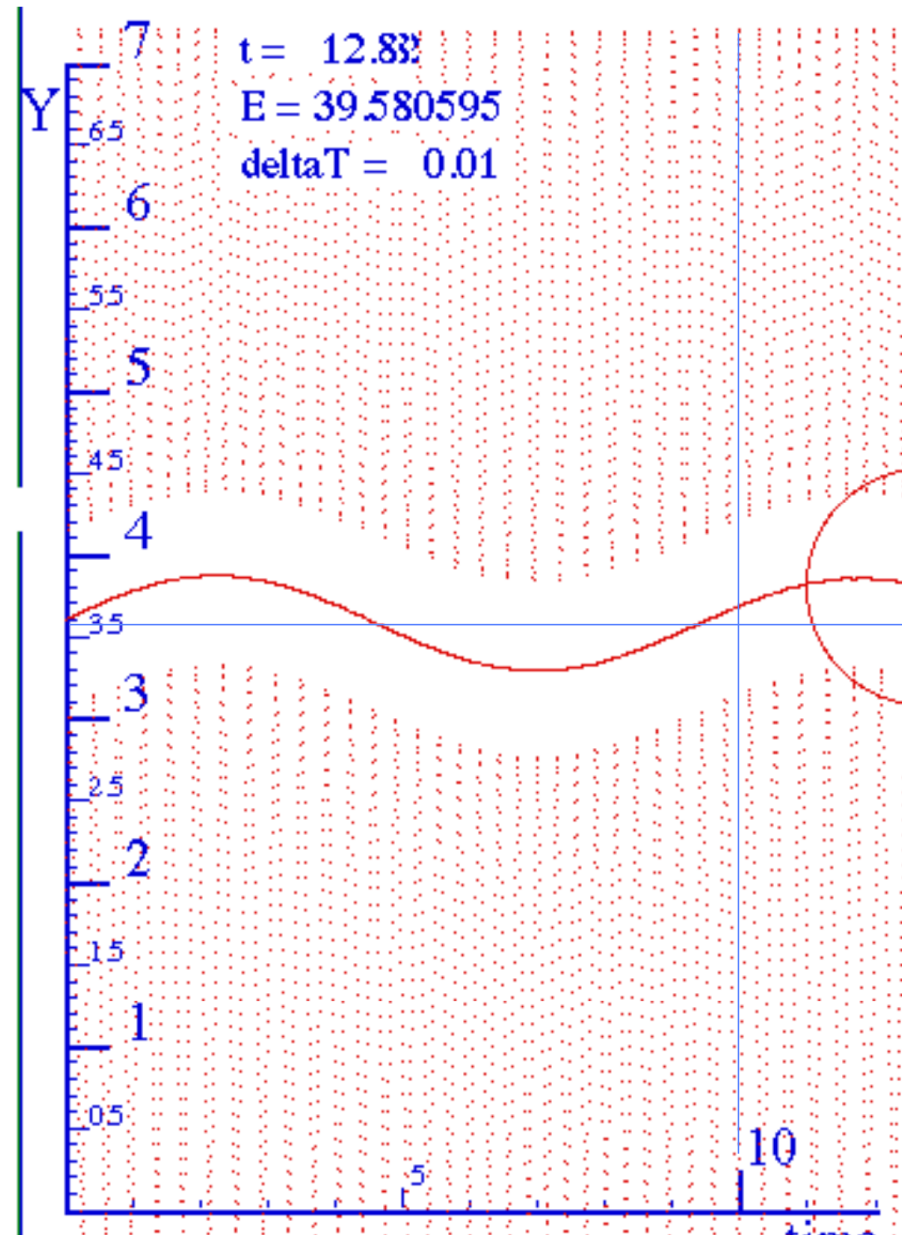
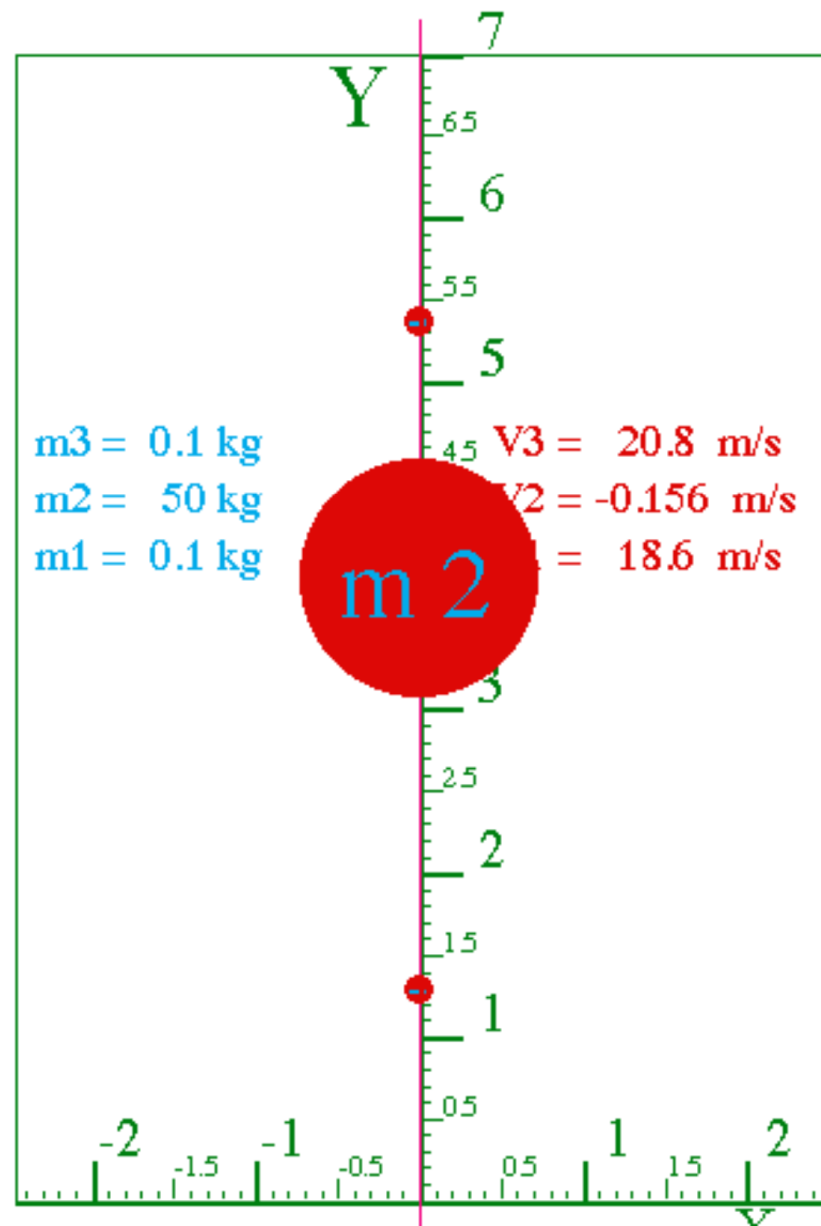
Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{1+x} - \frac{f}{1-x} = f [1 - x + x^2 - x^3 \dots] - f [1 + x + x^2 + x^3 \dots] = -2f \cdot x - 2f \cdot x^3 - \dots$$

$$F^{HO} = -k \cdot x = -\frac{\partial U^{HO}}{\partial x}$$

$$U^{HO} = \frac{1}{2} k \cdot x^2 = -\int F^{HO} dx$$

$$\text{HO } \angle \text{frequency: } \omega = \sqrt{\frac{k}{m_1}} = 2\pi\nu$$



Unit 1
Fig. 6.3

Simulation of
the **adiabatic case**

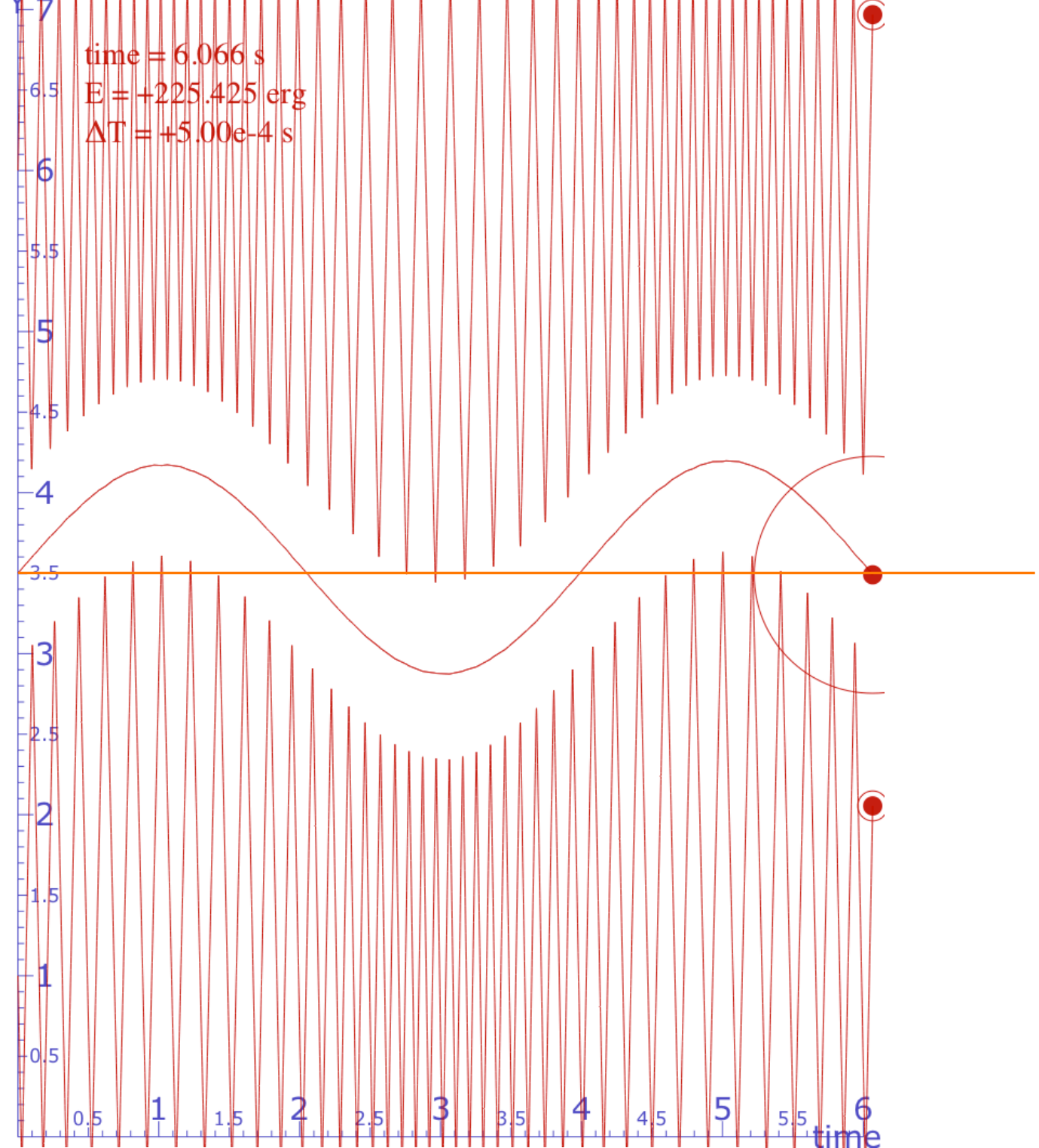
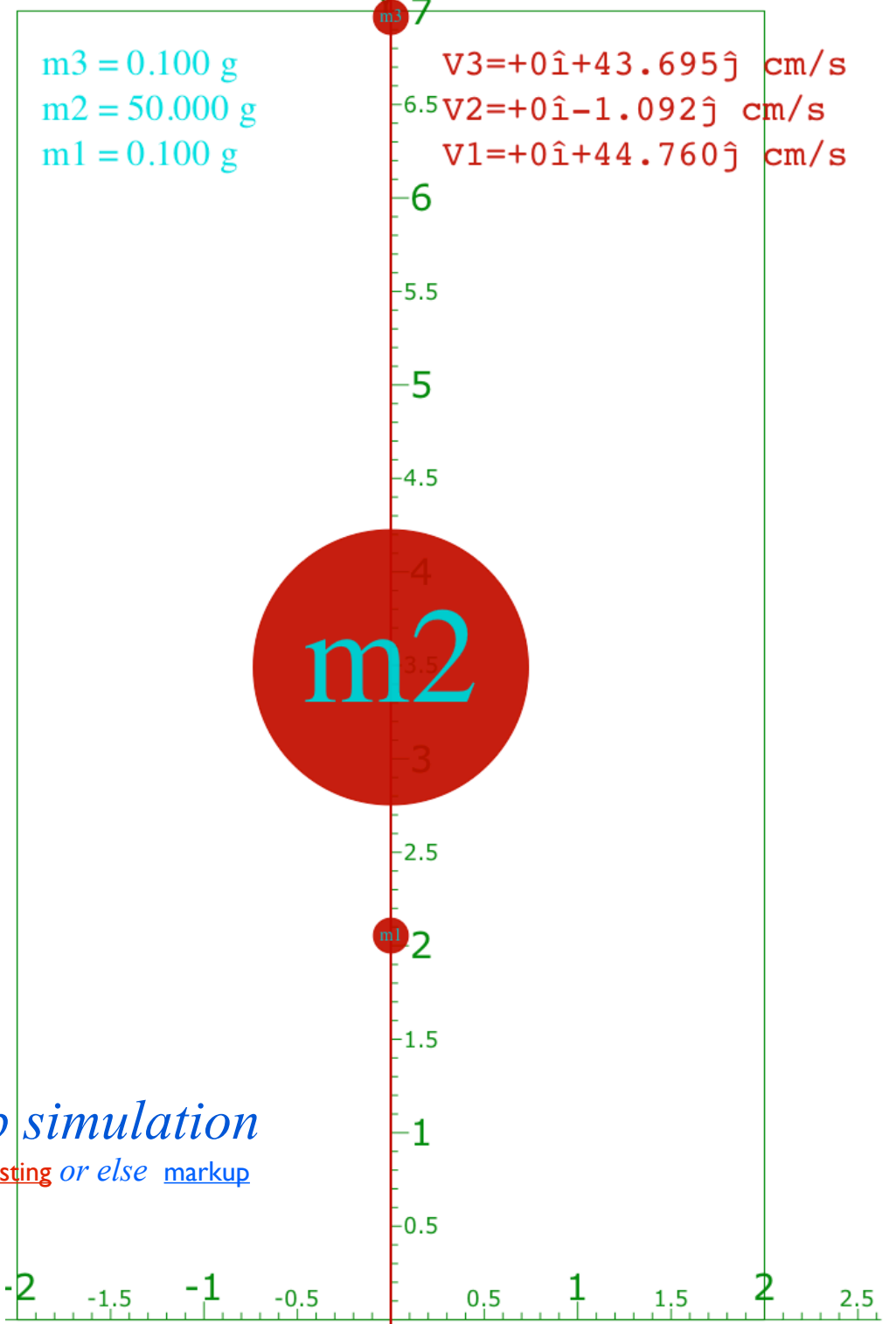
See Homework problem 1.6.1: *Compute frequency and/or period for both isoT and adiabatic cases*

m3 = 0.100 g
 m2 = 50.000 g
 m1 = 0.100 g

V3 = +0i + 43.695j cm/s
 V2 = +0i - 1.092j cm/s
 V1 = +0i + 44.760j cm/s



Web simulation
 Try [testing](#) or else [markup](#)



<http://www.uark.edu/ua/modphys/testing/markup/BounceltWeb.html>

Initial x1 = y Max =
 Max x PE plot = y Min =
 F-Vector scale = T Max =
 Error step = V2y Max =
 V2y Min =

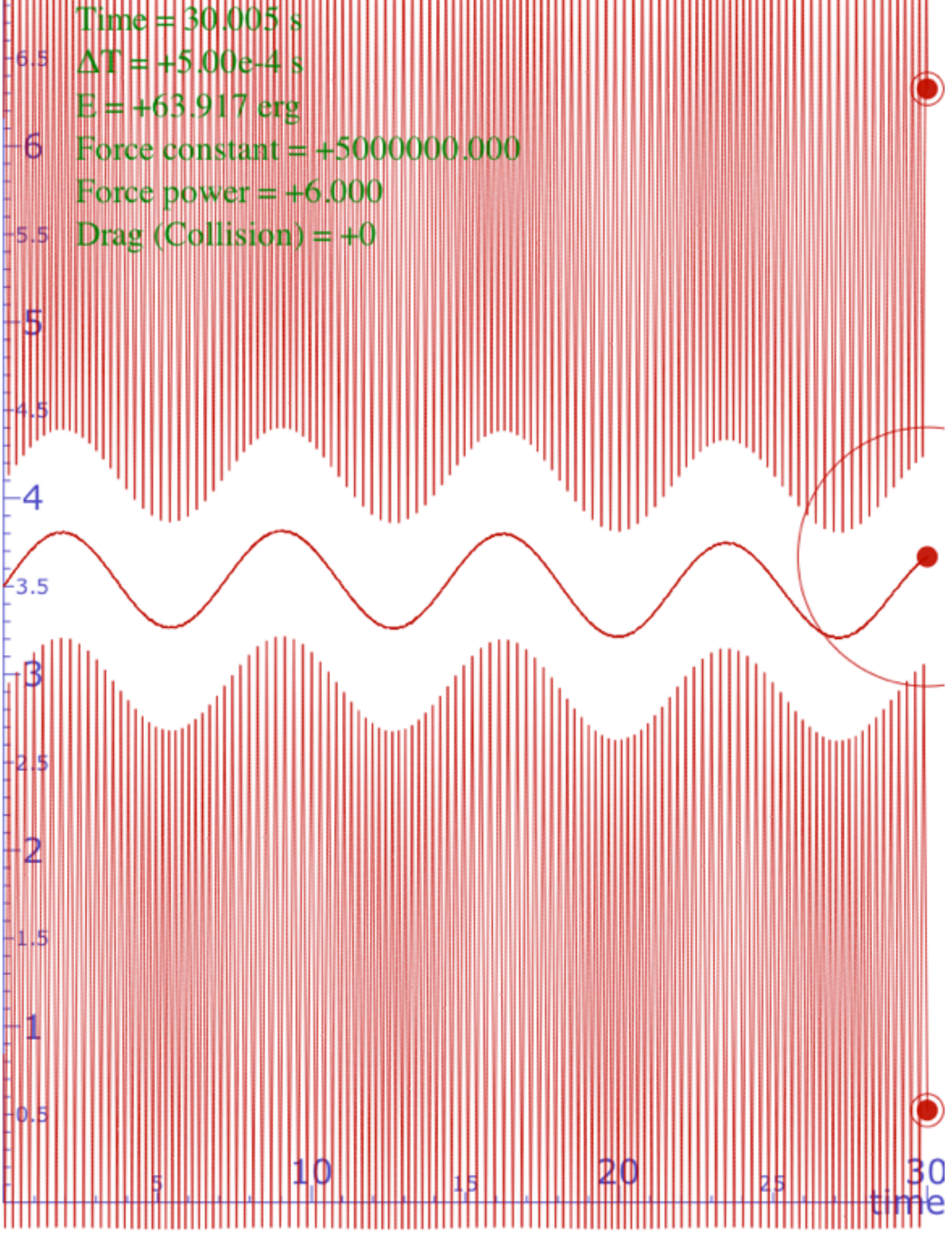
Adiabatic force scenarios

- Quasi-harmonic oscillation (m1:m2 = 100:1)
- Quasi-harmonic oscillation (m1:m2 = 50:1)
- Quasi-harmonic oscillation (m1:m2 = 25:1)
- Large amplitude (m1:m2 = 100:1)

m1 = x10^ {g} X1_0 = x10^ {cm} V1_0 = x10^ {cm/s}
 m2 = x10^ {g} X2_0 = x10^ {cm} V2_0 = x10^ {cm/s}
 m3 = x10^ {g} X3_0 = x10^ {cm} V3_0 = x10^ {cm/s}

m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g

V3 = +0i - 27.079j cm/s
V2 = +0i + 0.143j cm/s
V1 = +0i - 23.127j cm/s



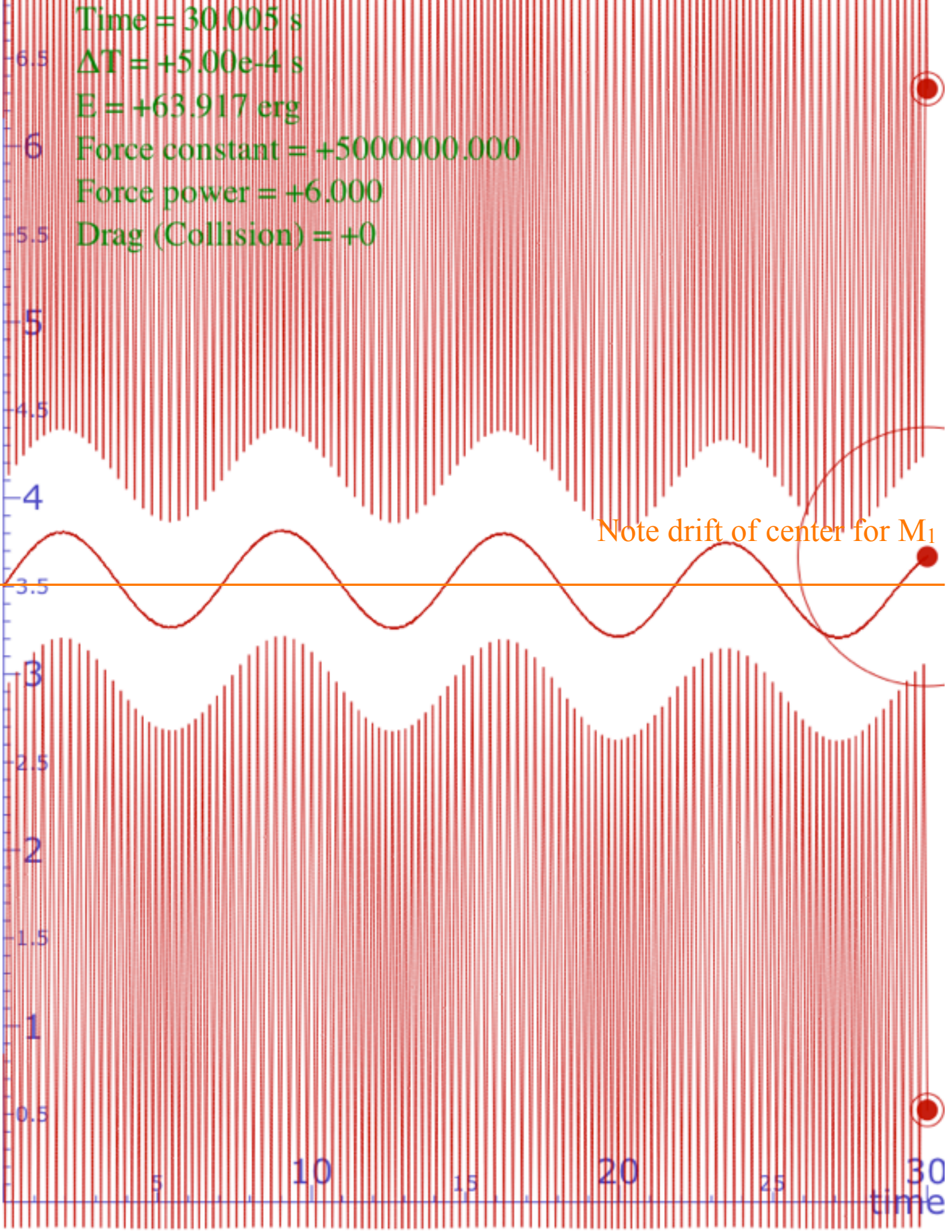
<http://www.uark.edu/ua/modphys/markup/BounceltWeb.html?scenario=2081>

$m_3 = 0.100 \text{ g}$
 $m_2 = 50.000 \text{ g}$
 $m_1 = 0.100 \text{ g}$

$V_3 = +0\hat{i} - 27.079\hat{j} \text{ cm/s}$
 $V_2 = +0\hat{i} + 0.143\hat{j} \text{ cm/s}$
 $V_1 = +0\hat{i} - 23.127\hat{j} \text{ cm/s}$



Note drift of total E
from 64.052
to 63.917

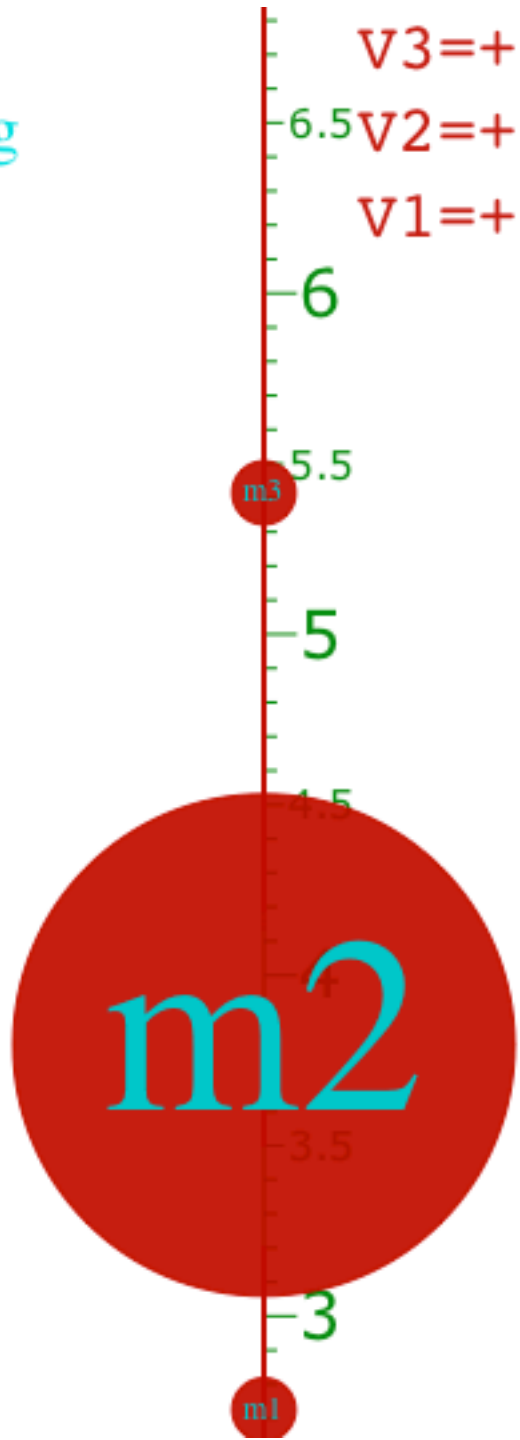


Time = 30.005 s
 $\Delta T = +5.00e-4 \text{ s}$
 $E = +63.917 \text{ erg}$
 Force constant = +5000000.000
 Force power = +6.000
 Drag (Collision) = +0

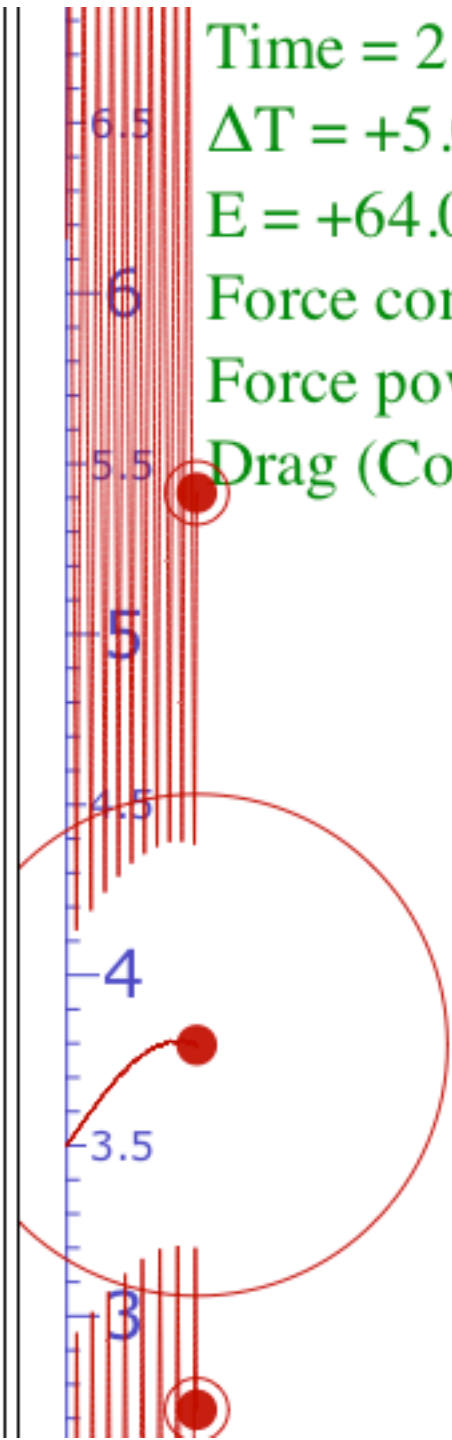
Note drift of center for M_1

<http://www.uark.edu/ua/modphys/markup/BounceltWeb.html?scenario=2081>

m3 = 0.100 g
m2 = 50.000 g
m1 = 0.100 g



$V3 = +0\hat{i} + 27.212\hat{j}$ cm/s
 $V2 = +0\hat{i} - 0.058\hat{j}$ cm/s
 $V1 = +0\hat{i} - 23.212\hat{j}$ cm/s



Time = 2.181 s
 $\Delta T = +5.00e-4$ s
E = +64.052 erg
Force constant = +5000000.000
Force power = +6.000
Drag (Collision) = +0

“Monster Mash” classical segue to Heisenberg action relations

 *Example of very very large M_1 ball-walls crushing a poor little m_2*

How m_2 keeps its action

An interesting wave analogy: The “Tiny-Big-Bang” [Harter, *J. Mol. Spec.* 210, 166-182 (2001)], [Harter, *Li IMSS* (2012)]

A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, *Am. Math. Monthly* 45,586(1938)]

[John Farey, *Phil. Mag.*(1816)]

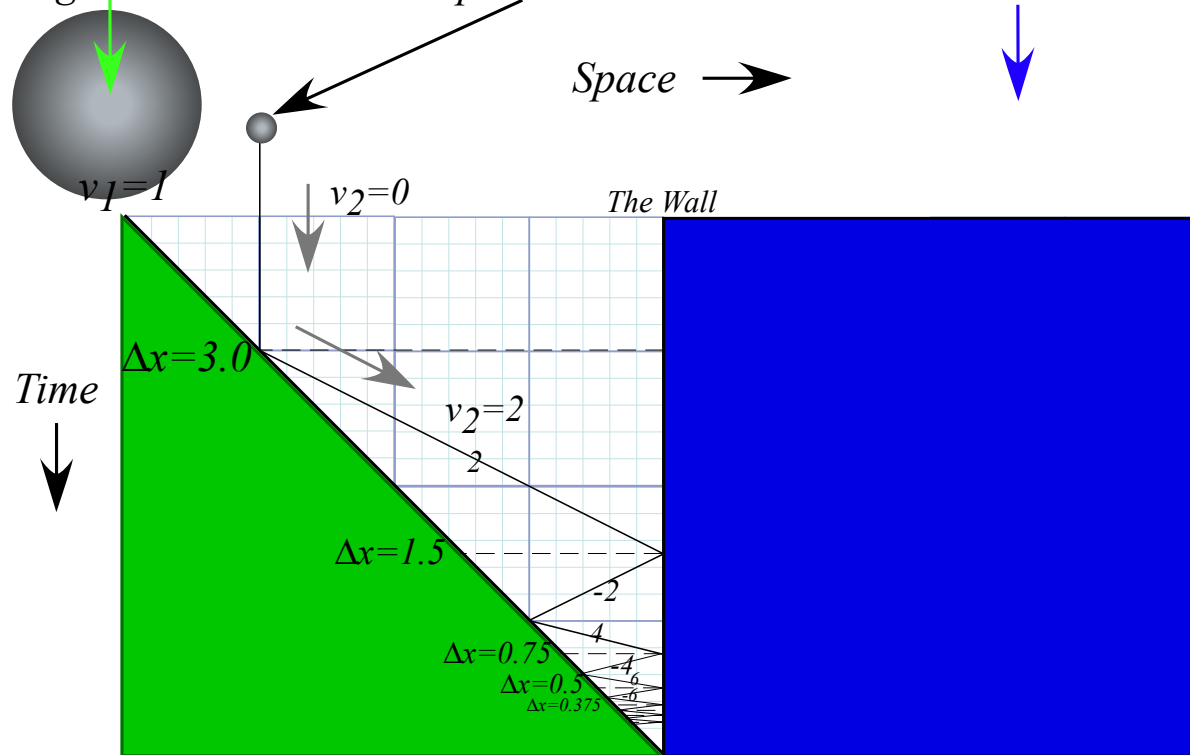
The Classical "Monster Mash"

Classical introduction to
Heisenberg "Uncertainty" Relations

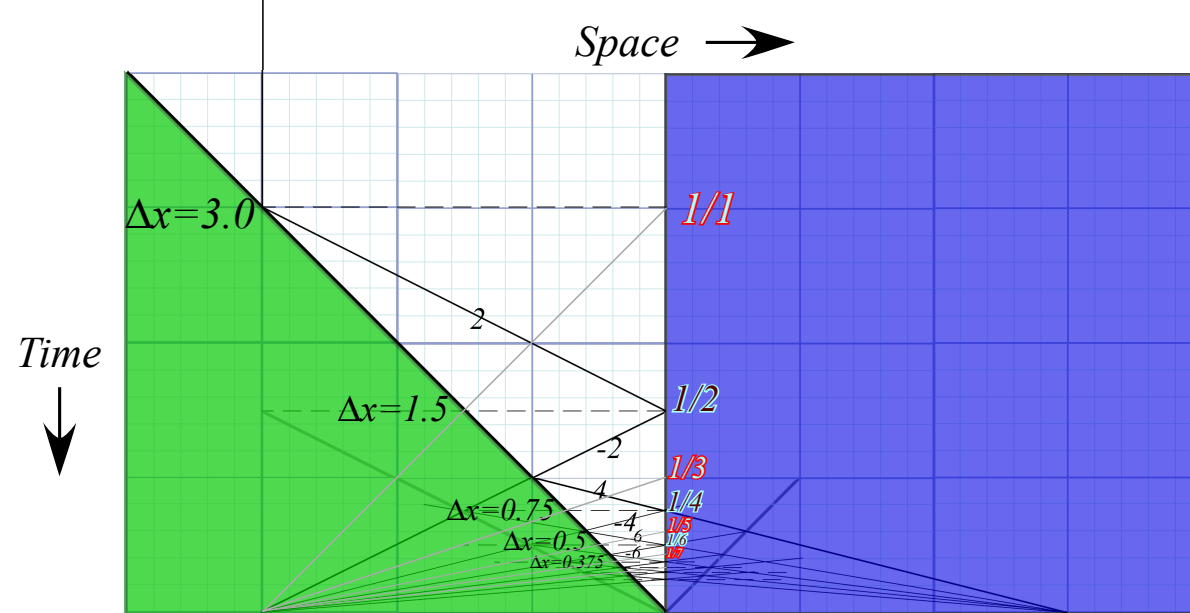
$$v_2 = \frac{\text{const.}}{Y} \quad \text{or:} \quad Y \cdot v_2 = \text{const.}$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

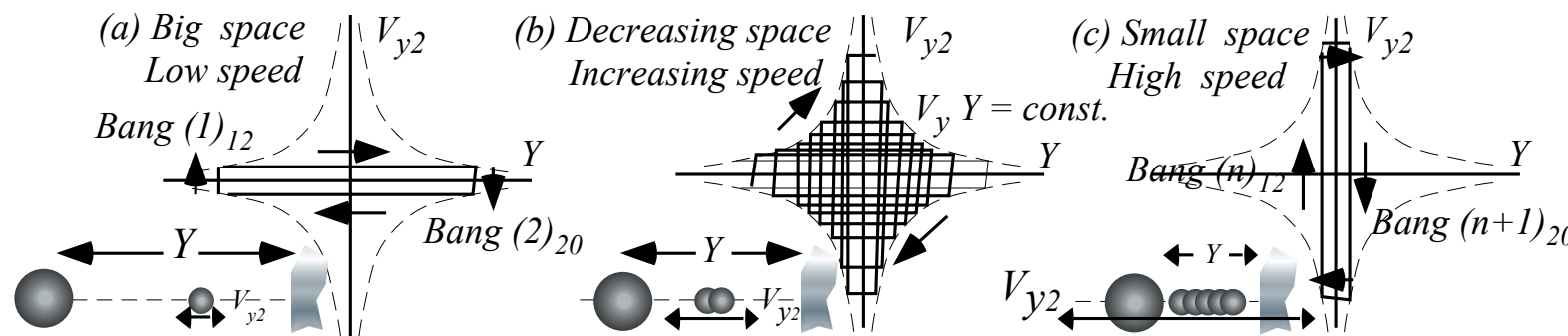
(a) Big ball moves in and traps small ball between it and The Wall



(b) Trajectory geometry exposed

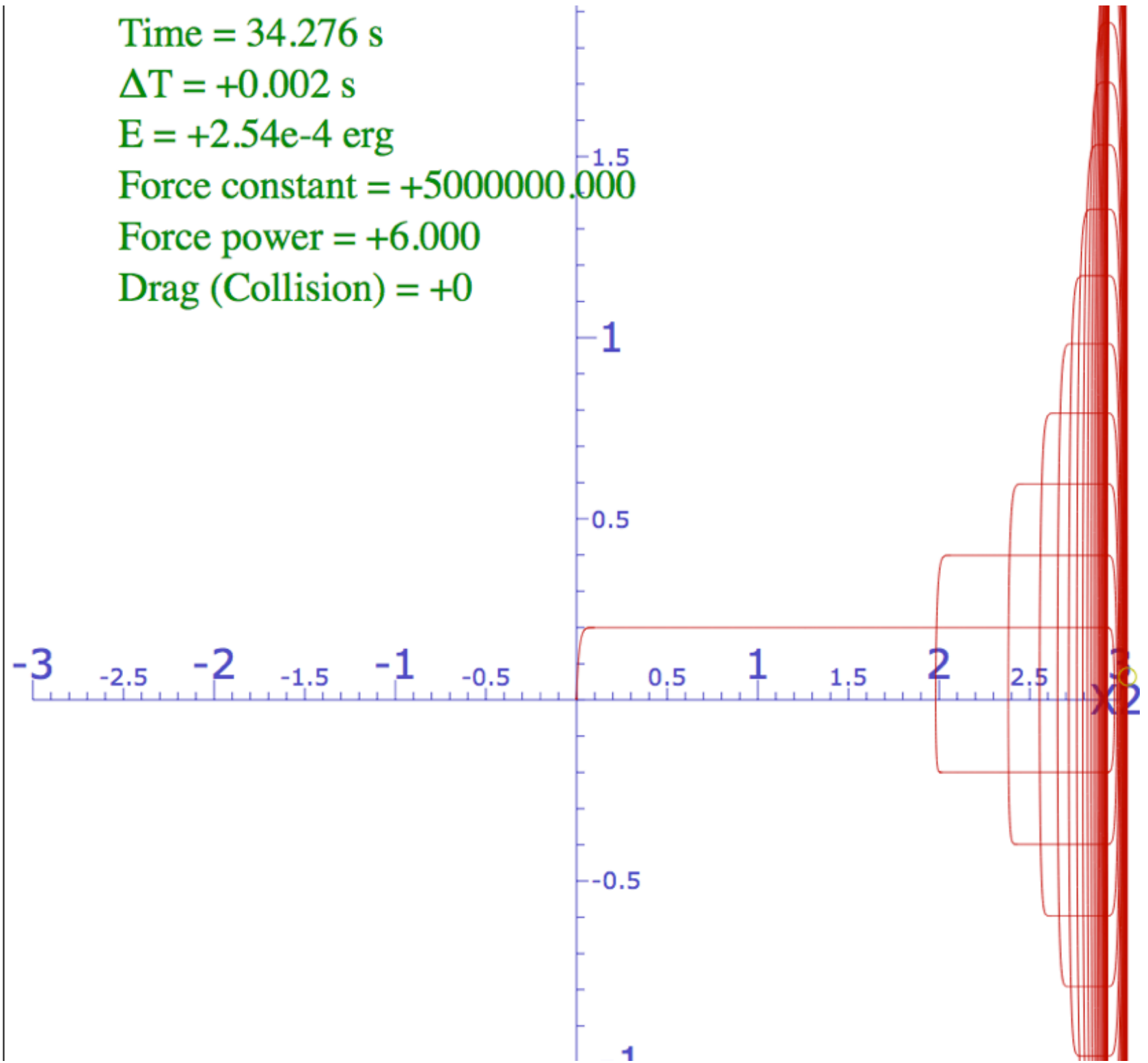
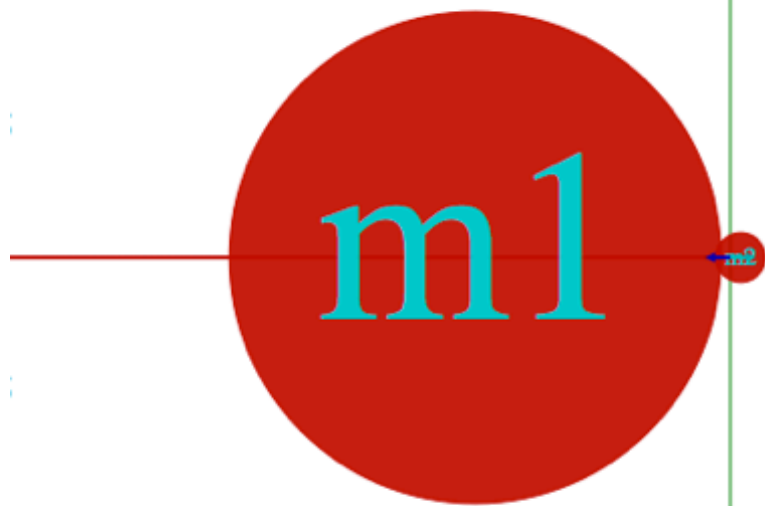


Unit 1
Fig. 6.4



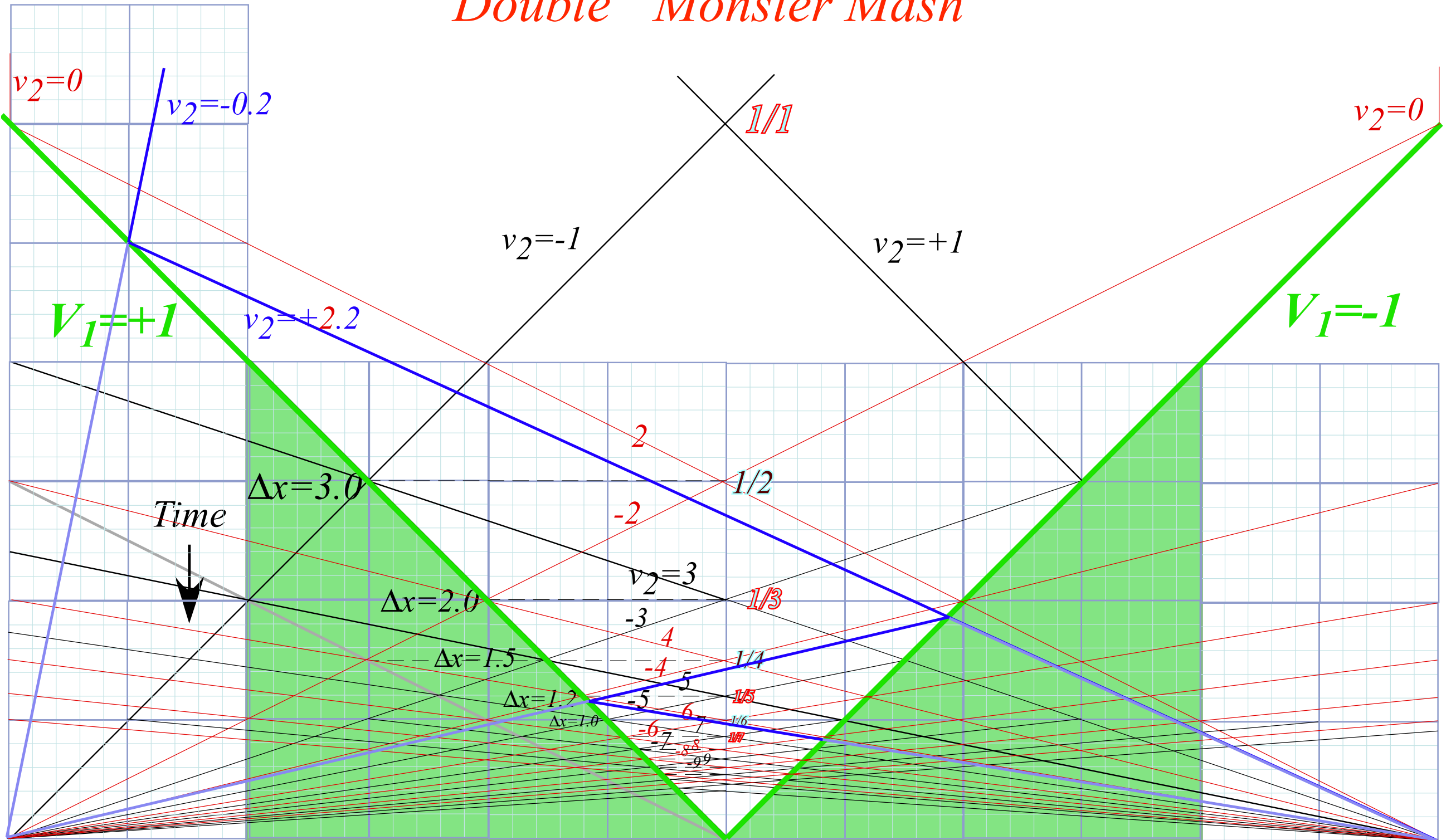
$V_2 = +0.064\hat{i} + 0\hat{j}$ cm/s
 $V_1 = -9.98e-4\hat{i} + 0\hat{j}$ cm/s

Time = 34.276 s
 $\Delta T = +0.002$ s
E = $+2.54e-4$ erg
Force constant = $+5000000.000$
Force power = $+6.000$
Drag (Collision) = $+0$



<http://www.uark.edu/ua/modphys/markup/BounceltWeb.html?scenario=3000>

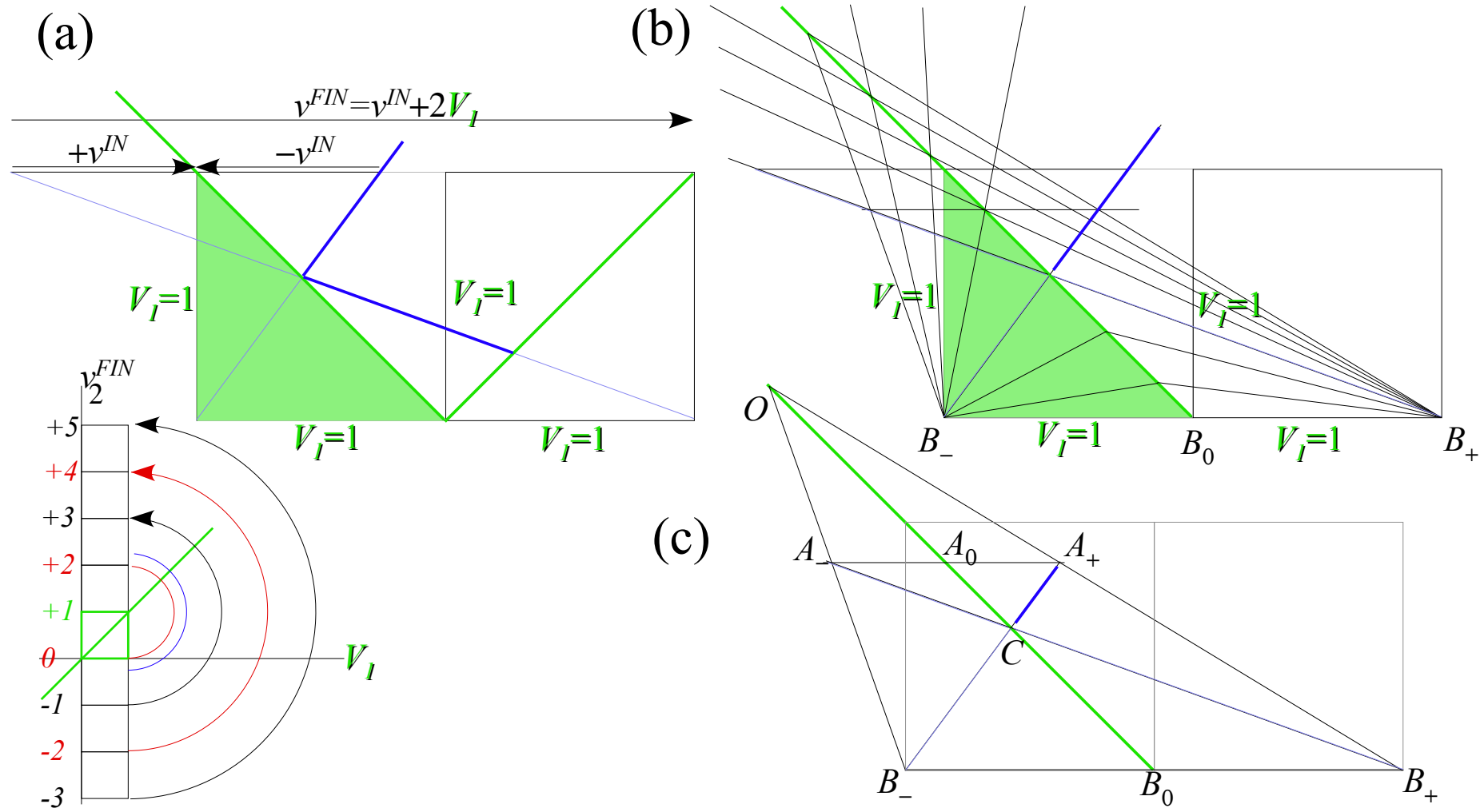
Double "Monster Mash"



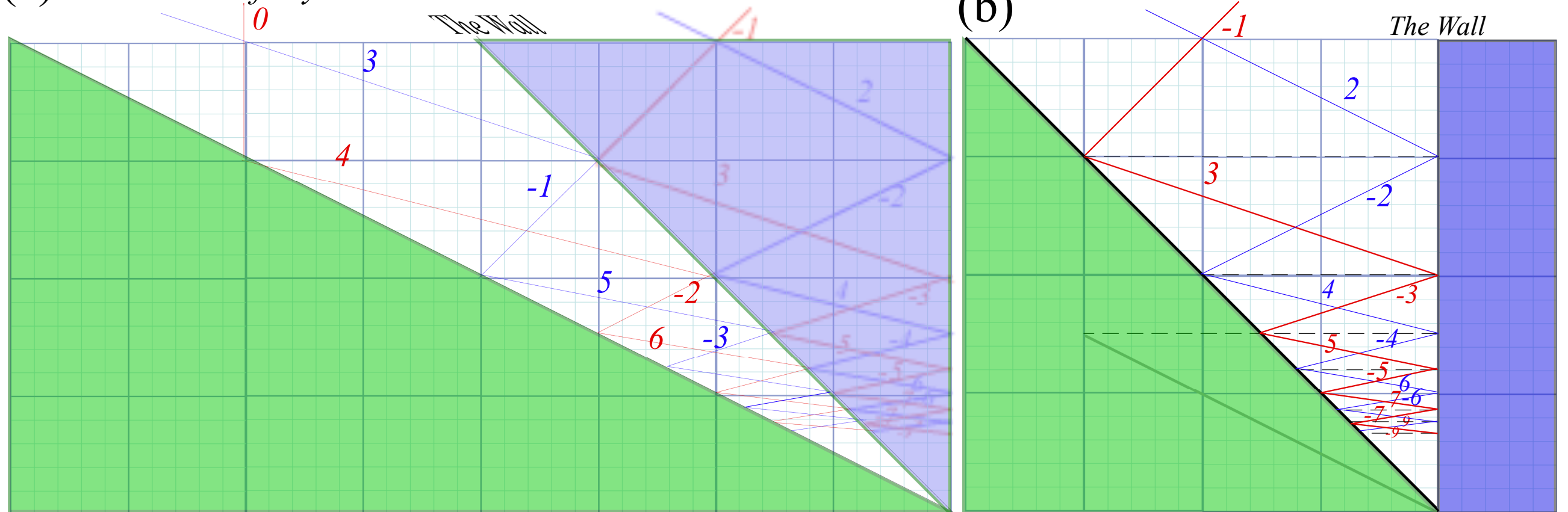
Unit 1
Fig. 6.5

See Homework problem 1.6.2: *Construct related spacetime case*

Unit 1
Fig. 6.6
and
Fig. 6.7



(a) Galilean shift by $V=1$



“Monster Mash” classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

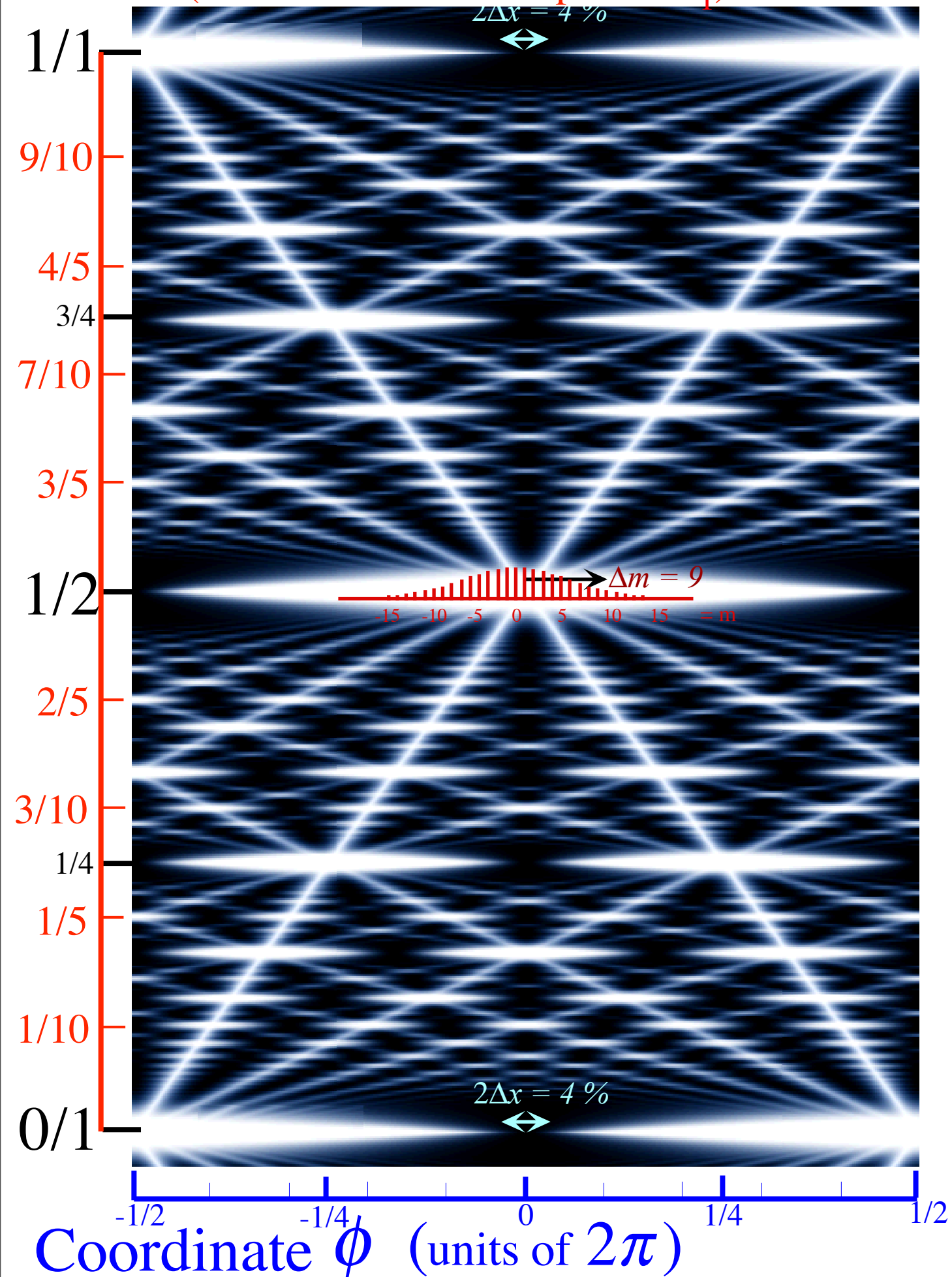
 *An interesting wave analogy: The “Tiny-Big-Bang” [Harter, J. Mol. Spec. 210, 166-182 (2001)], [Harter, Li IMSS (2012)]*

A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

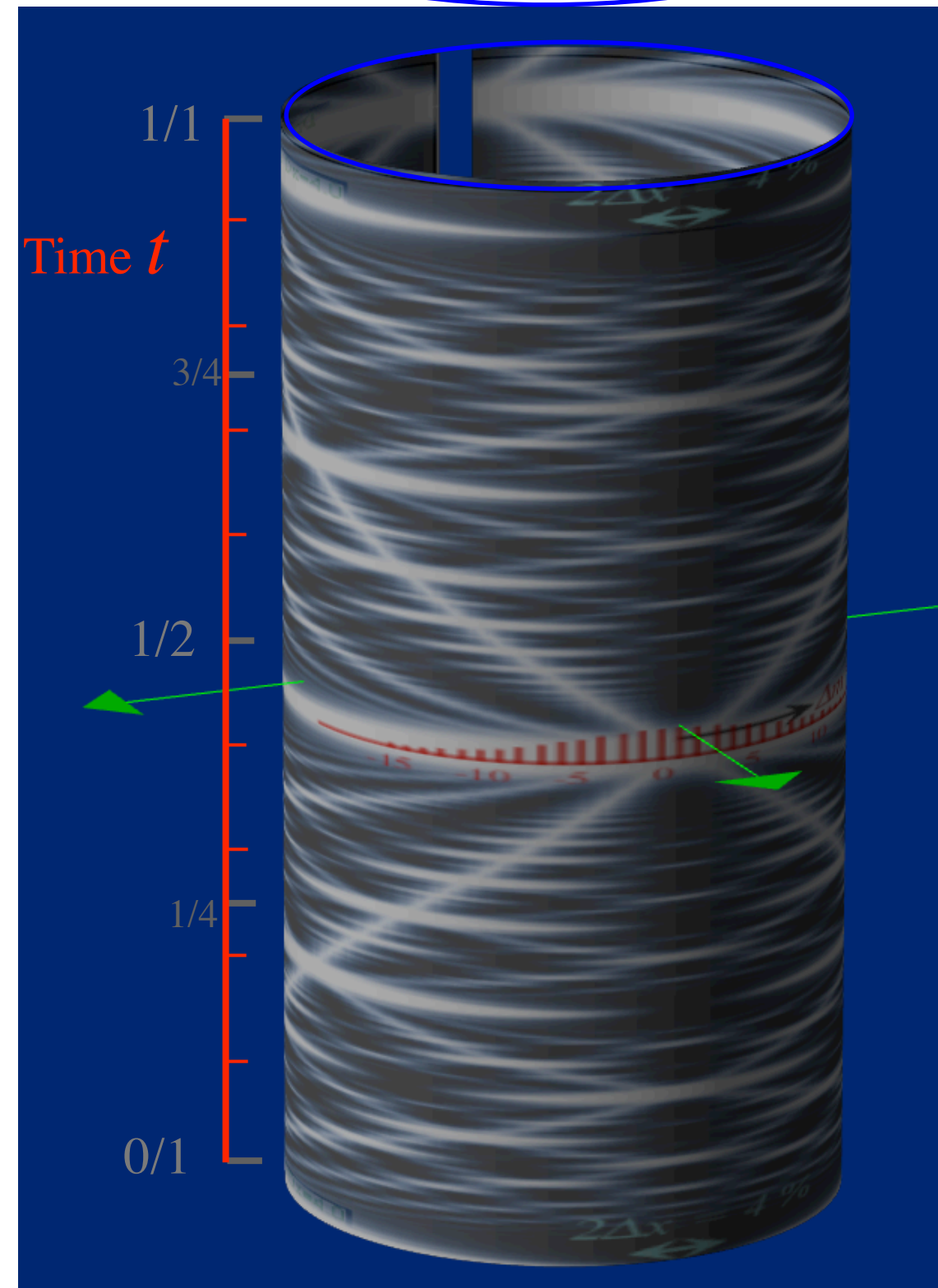
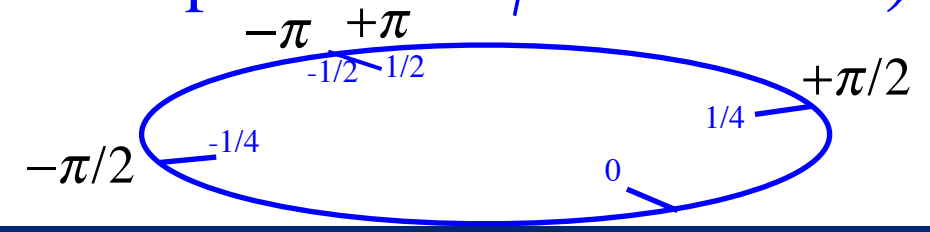
[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

[John Farey, Phil. Mag.(1816)]

Time t (units of fundamental period τ_1)



(Imagine "wrap-around" ϕ -coordinate)



Web simulation

or:

<http://www.uark.edu/ua/modphys/markup/WaveletWeb.html>

<http://www.uark.edu/ua/modphys/markup/WaveletWeb.html?scenario=Quantum%20Carpet>

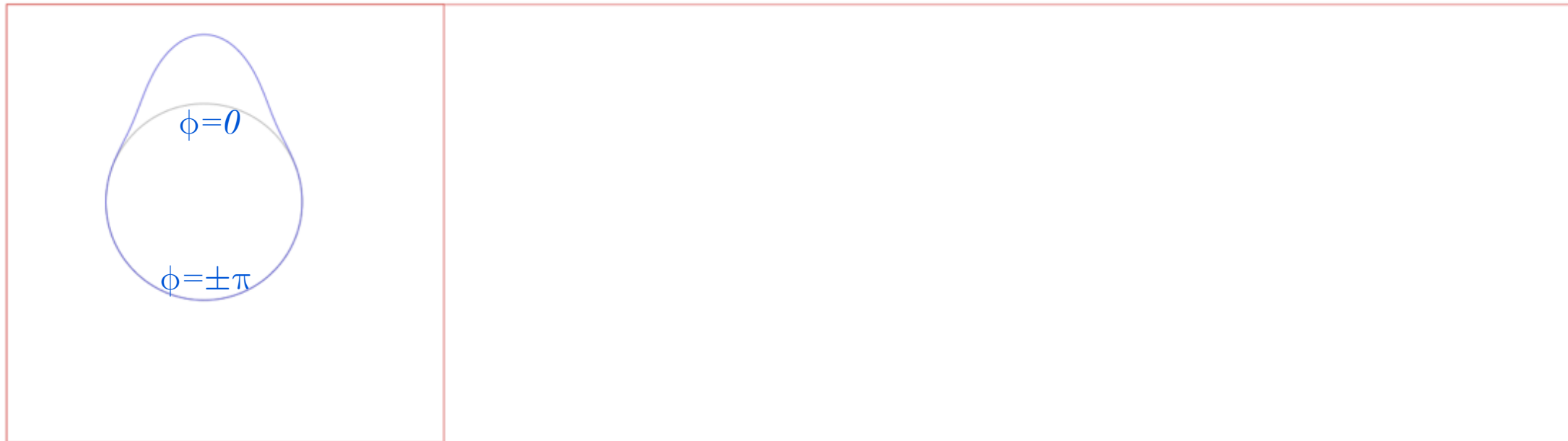
Also, try [testing](#) or else [markup](#)

Click here....

T-Scale=

..then here....

$\phi = -\pi$ $\phi = 0$ $\phi = +\pi$



Starts with Gaussian $\Psi(\phi, t)$ at $\phi = 0$ on Bohr wave ring that expands and "beats"



Web simulation

or:

<http://www.uark.edu/ua/modphys/markup/WaveltWeb.html>

<http://www.uark.edu/ua/modphys/markup/WaveltWeb.html?scenario=Quantum%20Carpet>

Also, try [testing](#) or else [markup](#)

Click here....

Launch

Fourier Control

Scenarios

Pause

Set T=0

Zero Amps

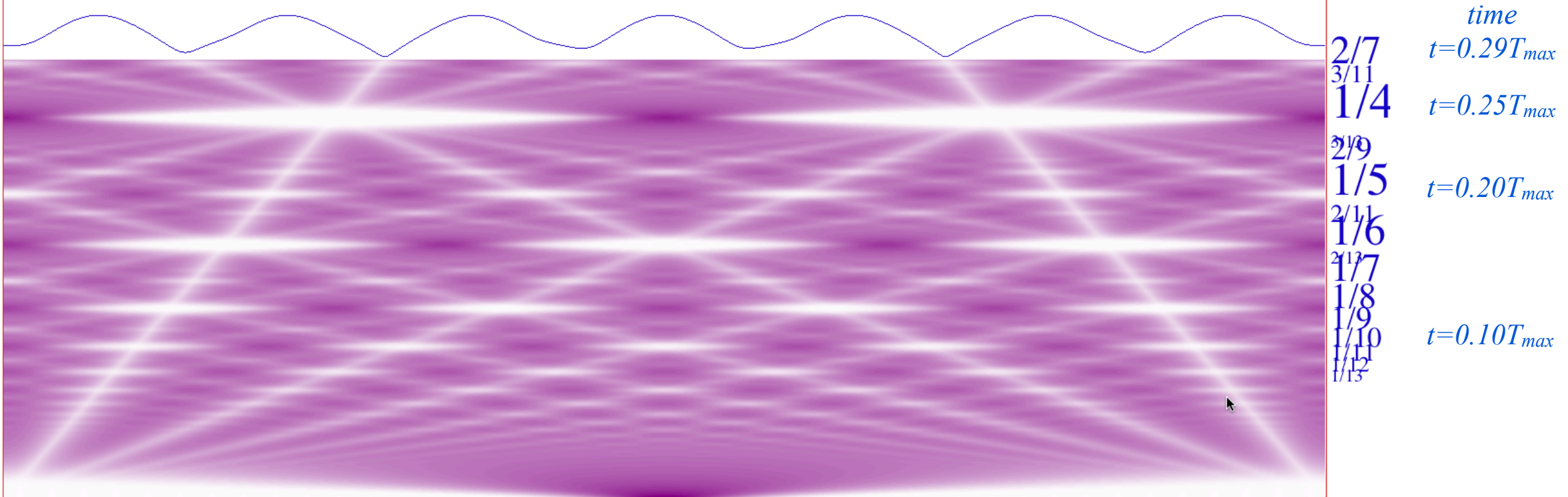
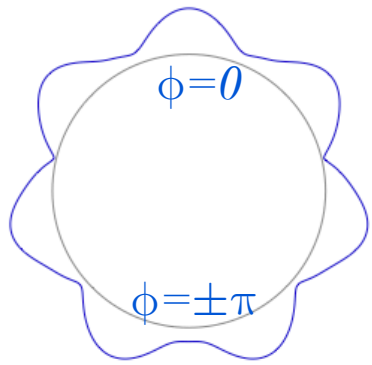
T-Scale= 1



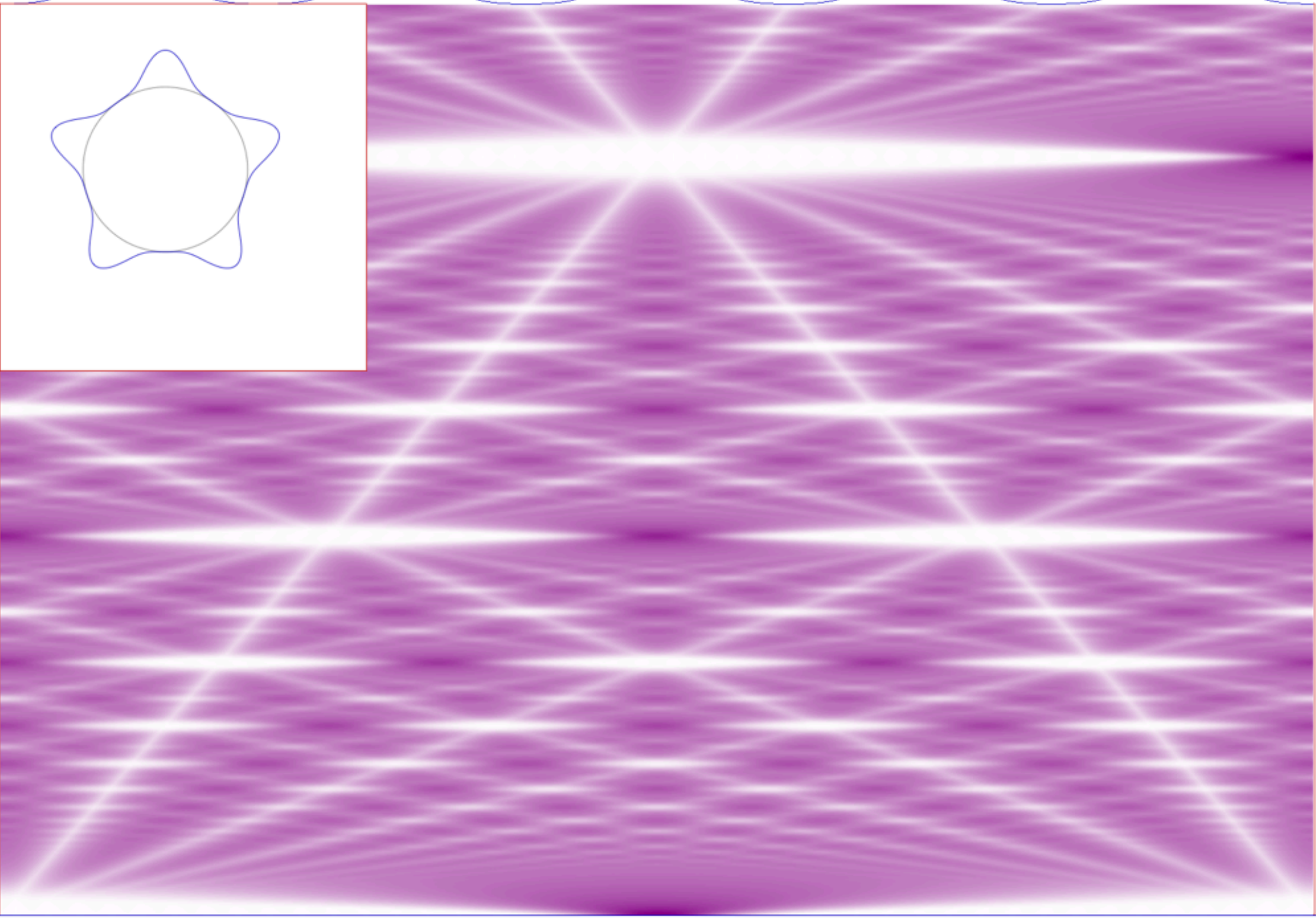
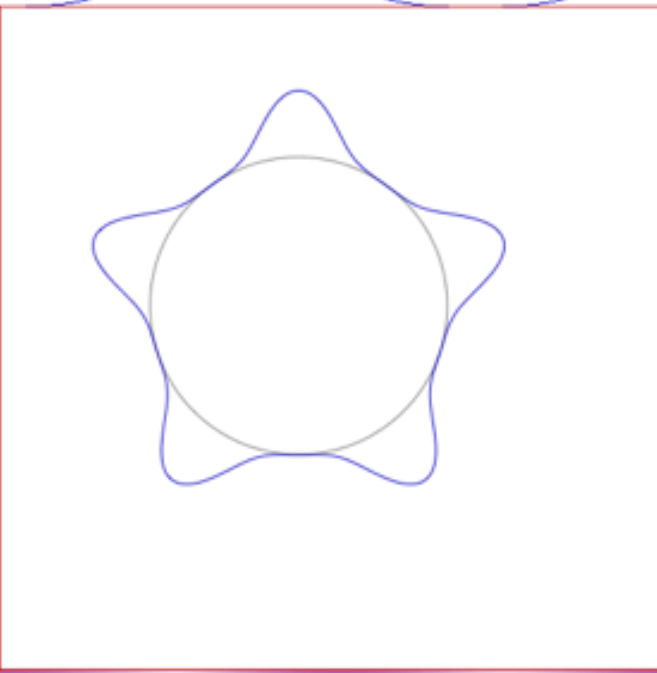
..then here....

Quantum Carpet

$\phi = -\pi$ $\phi = 0$ $\phi = +\pi$



time = 0.60T



- 3/5
- 7/12
- 4/7
- 5/9
- 6/11
- 7/13
- 1/2
- 6/13
- 5/11
- 4/9
- 3/7
- 5/12
- 2/5
- 5/12
- 3/8
- 4/11
- 1/3
- 4/13
- 3/10
- 2/7
- 3/11
- 1/4
- 2/9
- 1/5
- 2/11
- 1/6
- 2/12
- 1/7
- 1/8
- 1/9
- 1/10
- 1/11
- 1/12
- 1/13

Launch

Fourier Control

Scenarios

Pause

Set T=0

Zero Amps

T-Scale= 1

Set this and then click here....

Type Quantum Carpet

Time Behavior Pause at End

Time Start (% Period) = 0

Time End (% Period) = 60

Del-x Width (% L) = 4

Excitation (Max n) = 20

Left (% L) = 0

Right (% L) = 100

n-Mean (% Max n) = 0

Peak1 Mean (% L) = 50

OverAll Scale = 1

Peak2 Mean (% L) = 0

Peak2 Amp (% Peak1) = 0

Draw Ring m/n Labels

m-Boxcar

Draw m-Bars m-Bars Max = 30

Aspect Ratio {W/H} = 1.5

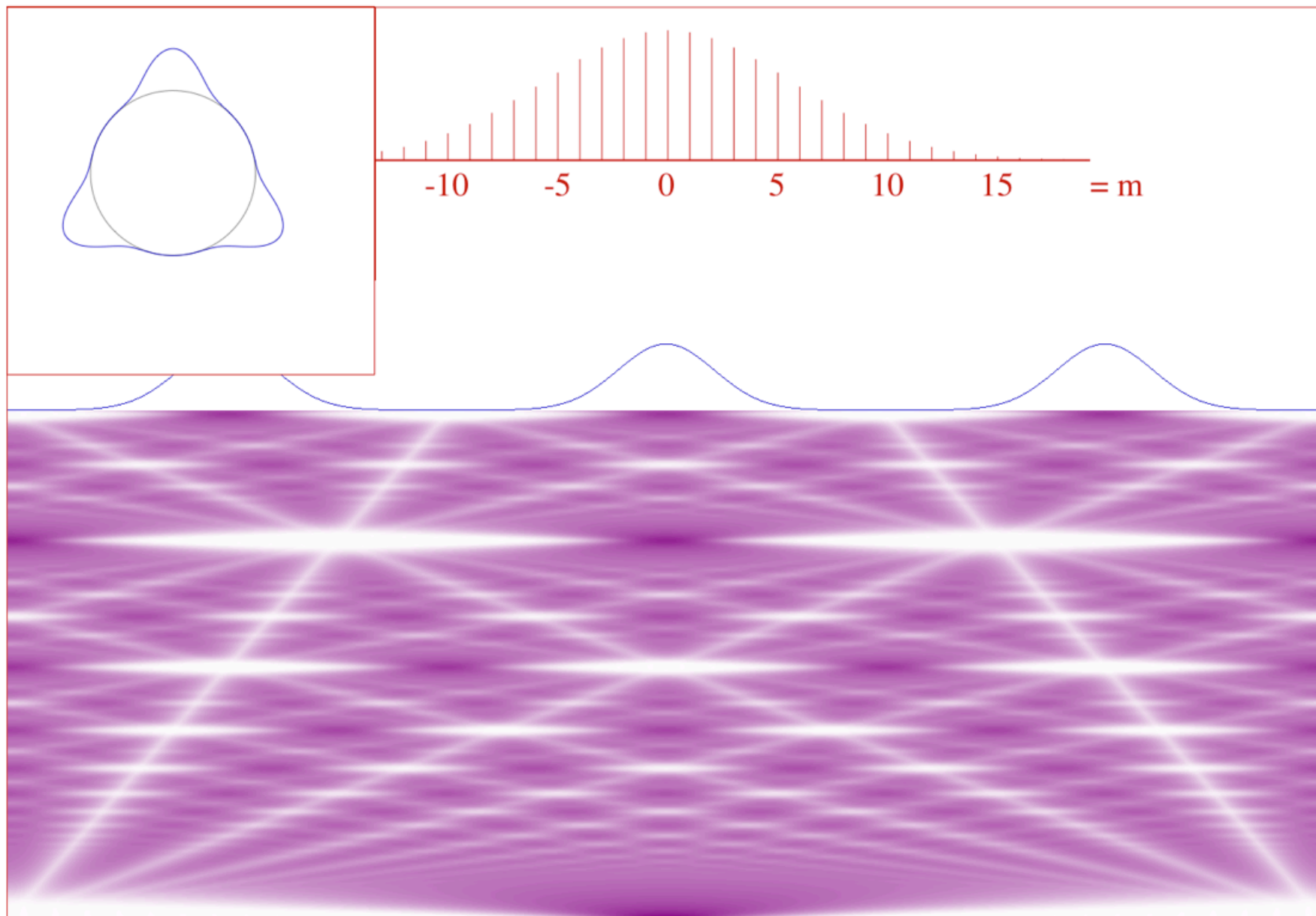
Red Level = 128

Green Level = 0

Blue Level = 128

Alpha Level = 1

Definition Level = 0.5

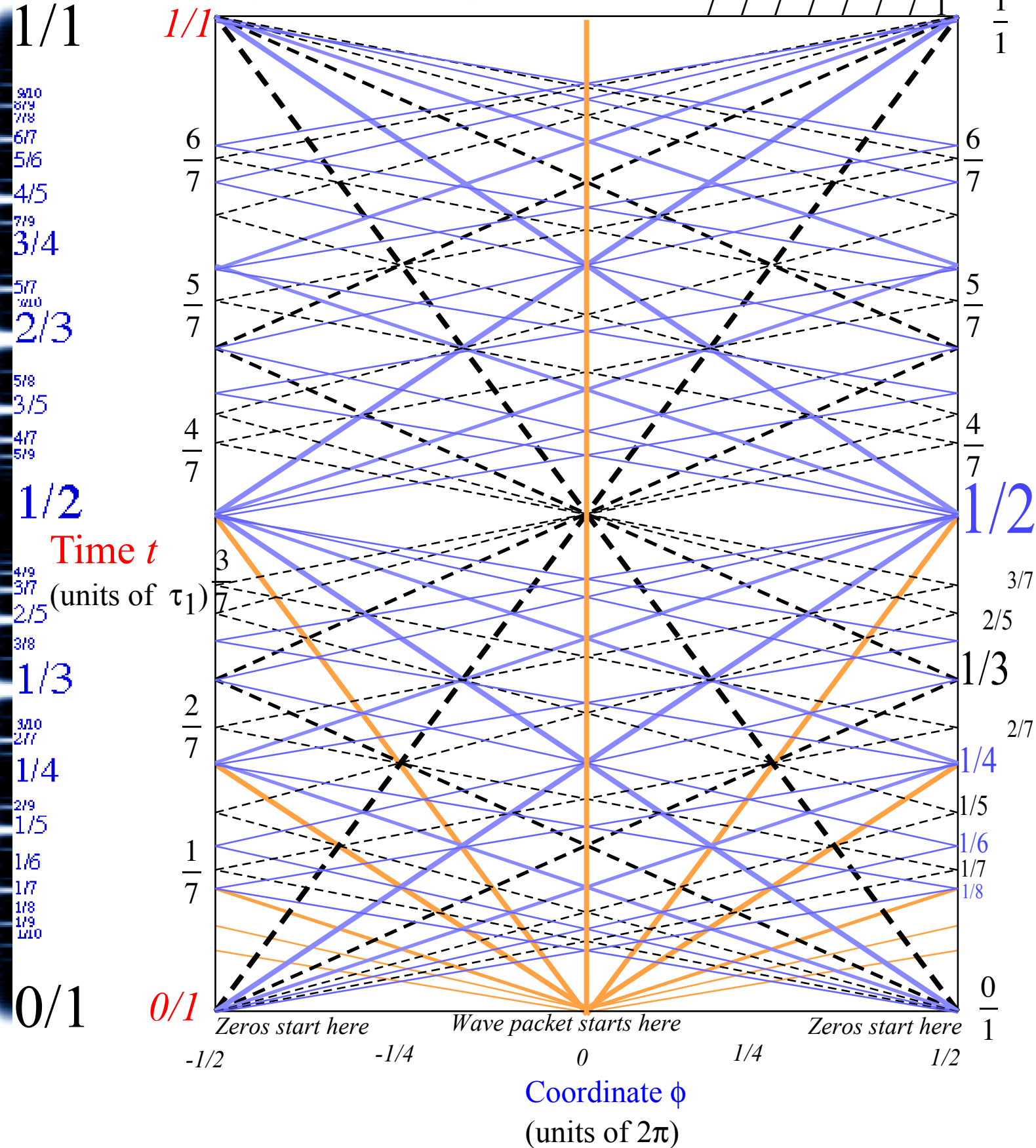
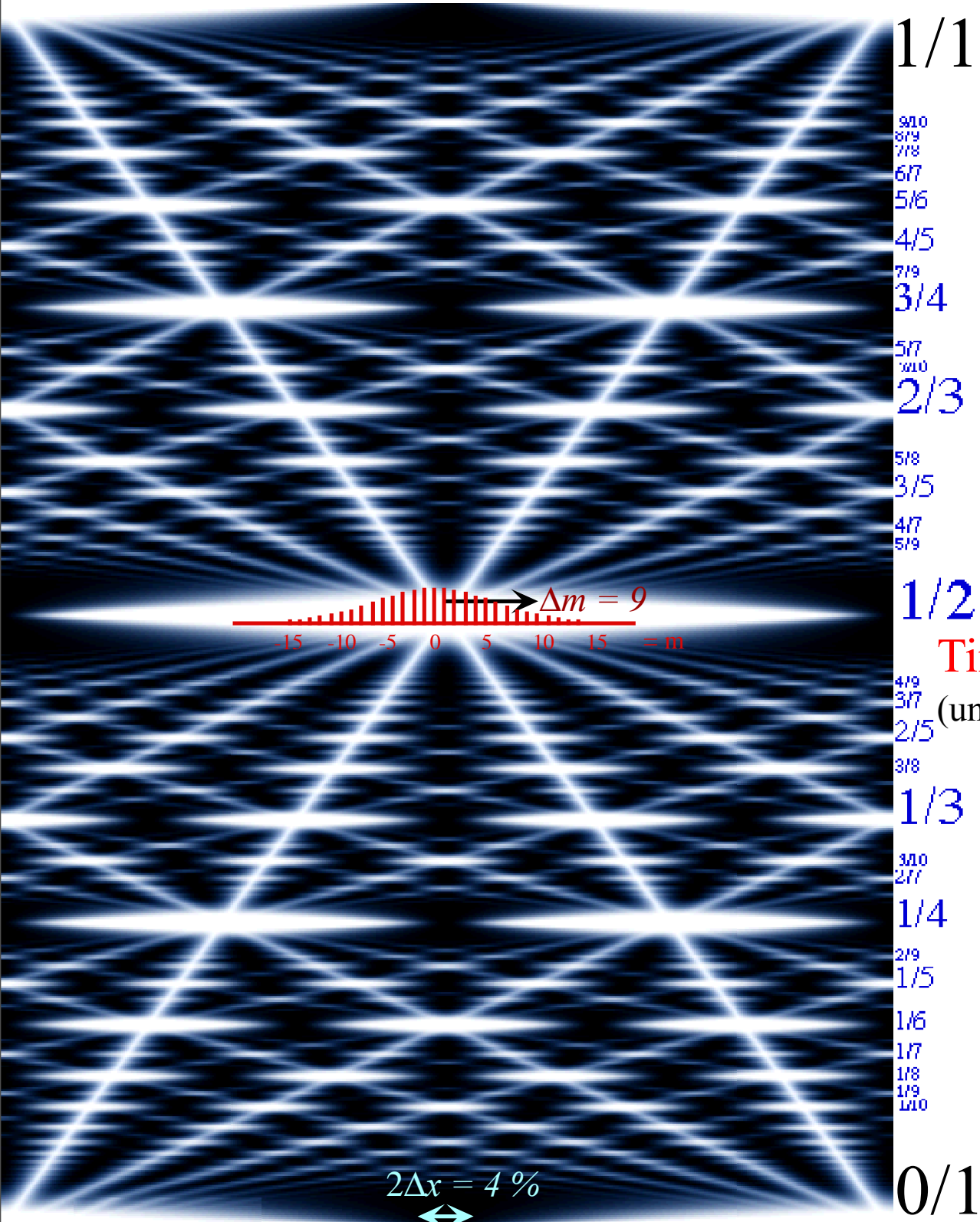


1/3
 2/7
 3/11
 1/4
 2/9
 1/5
 1/6
 1/7
 1/8
 1/9
 1/10
 1/13

N -level-system and revival-beat wave dynamics

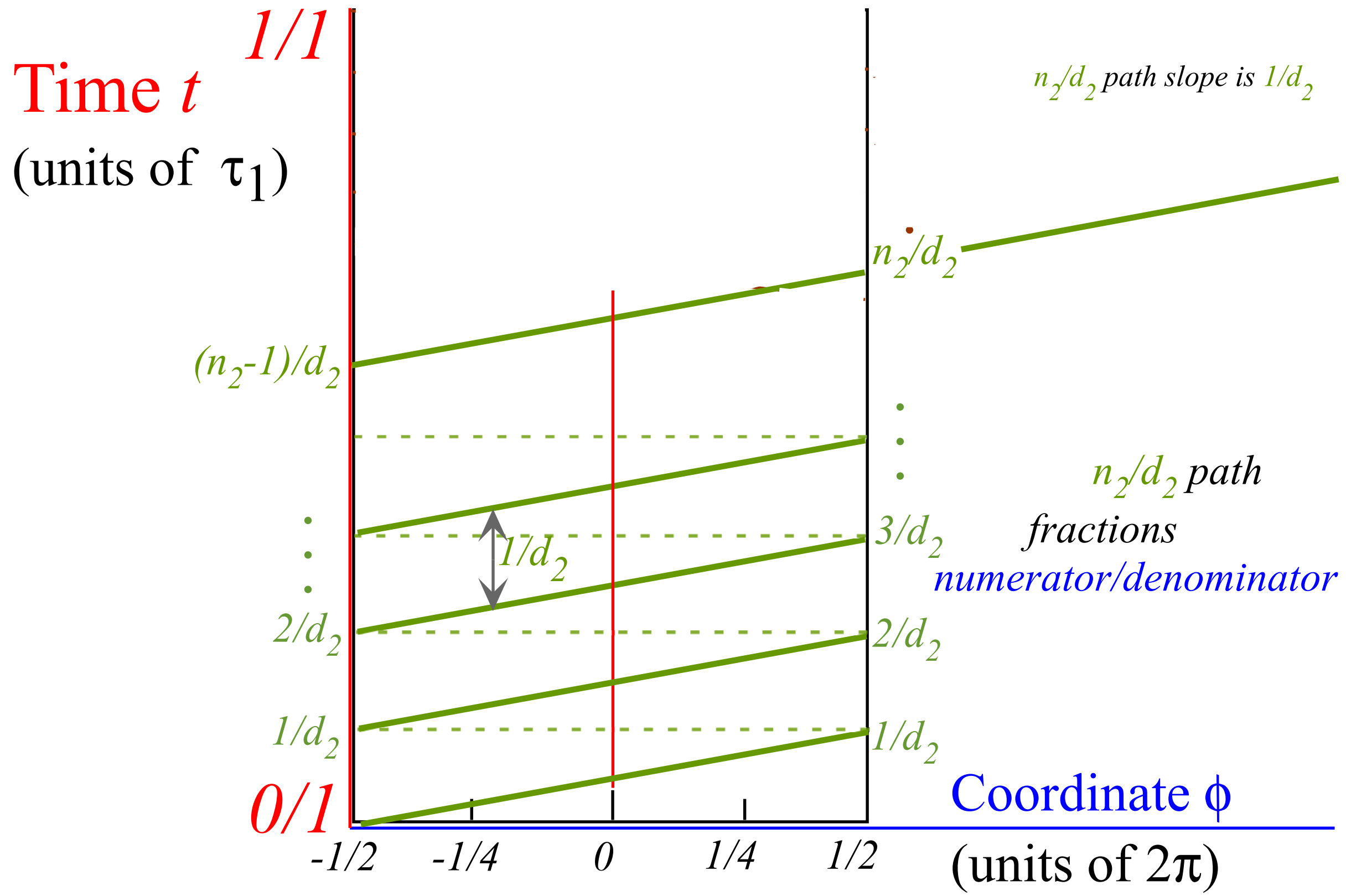
(9 or 10-levels $(0, \pm 1, \pm 2, \pm 3, \pm 4, \dots, \pm 9, \pm 10, \pm 11, \dots)$ excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$



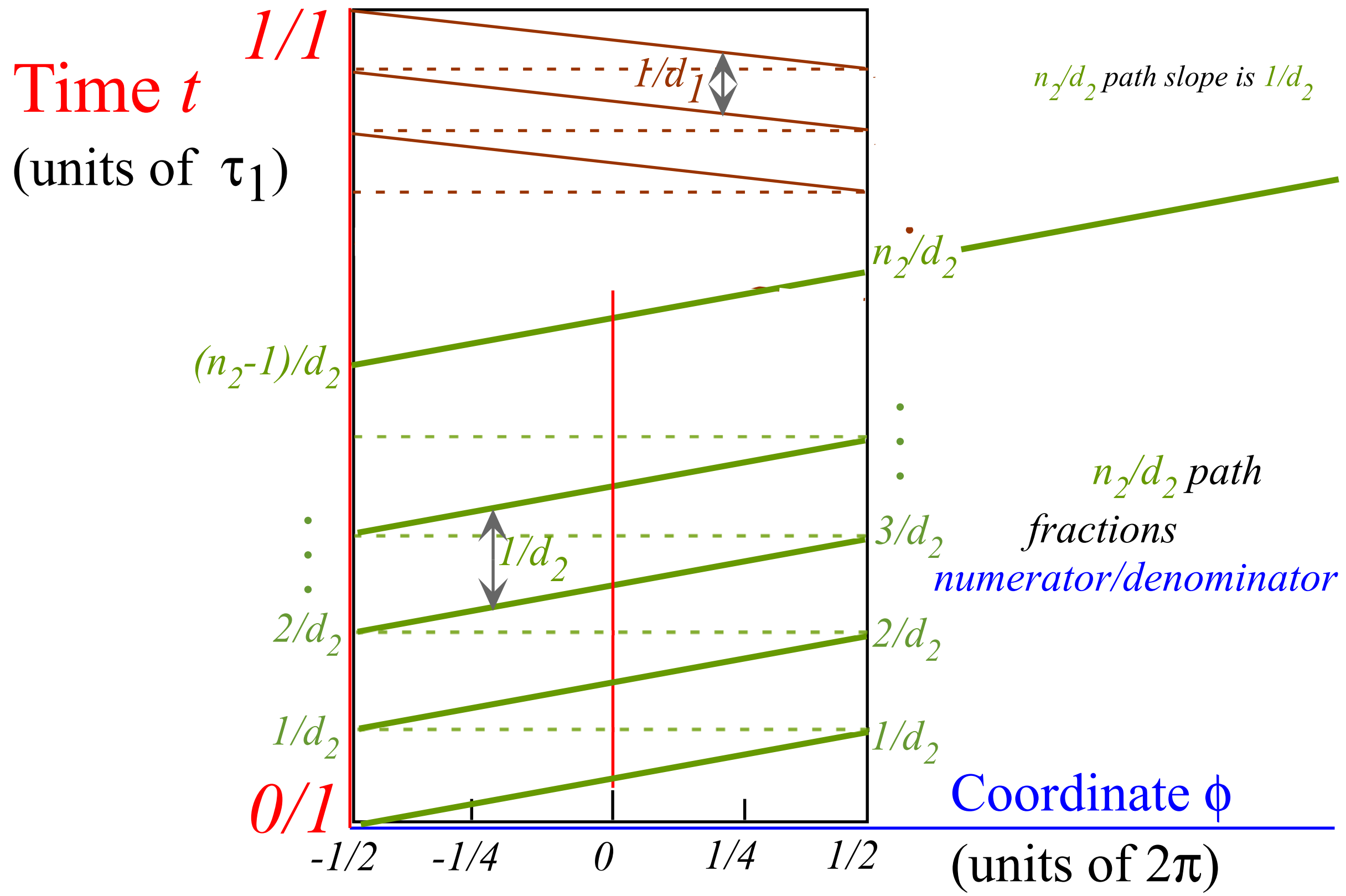
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators* N and *denominators* D of rational fractions N/D



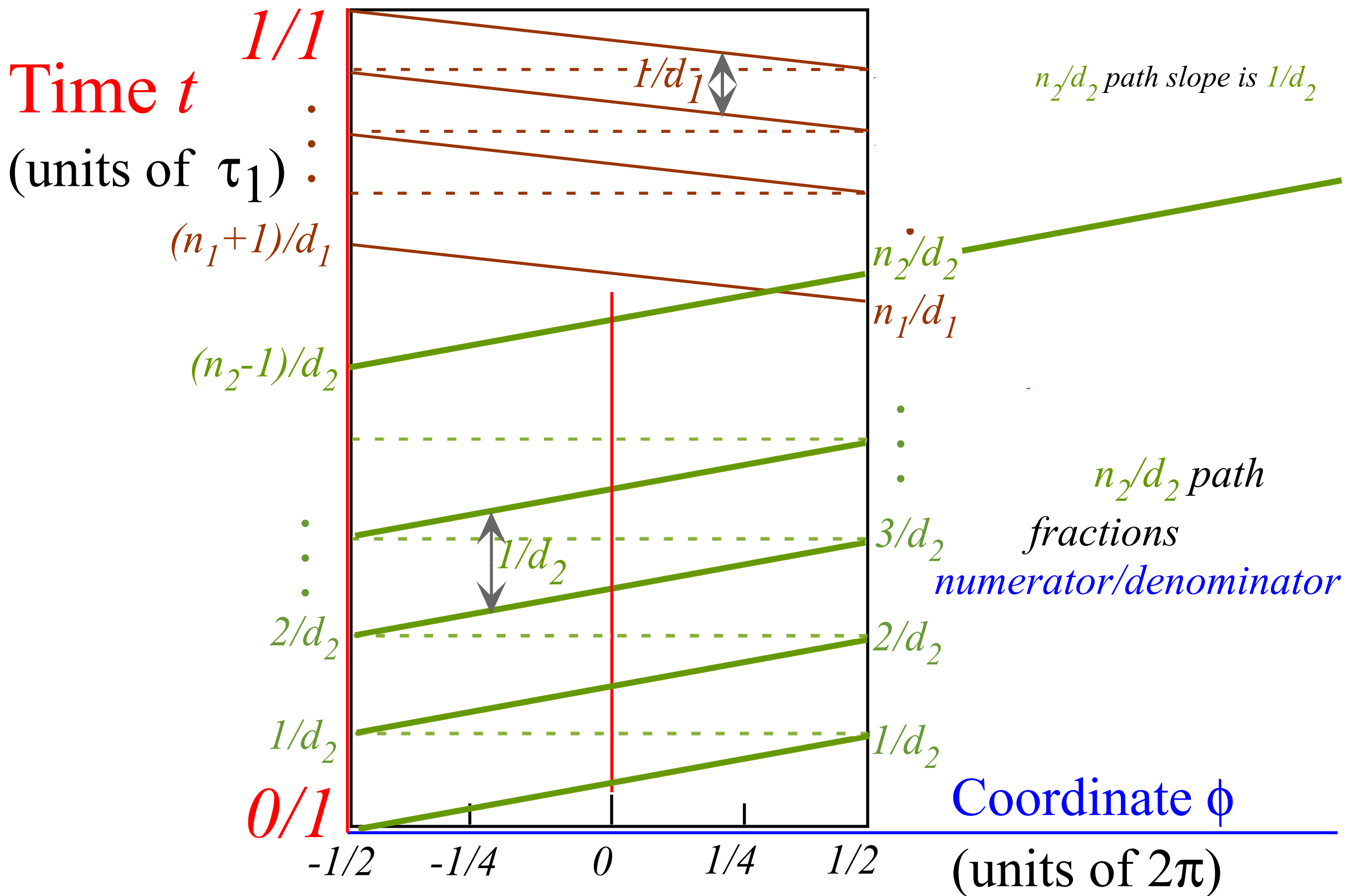
Farey Sum algebra of revival-beat wave dynamics

Label by numerators N and denominators D of rational fractions N/D



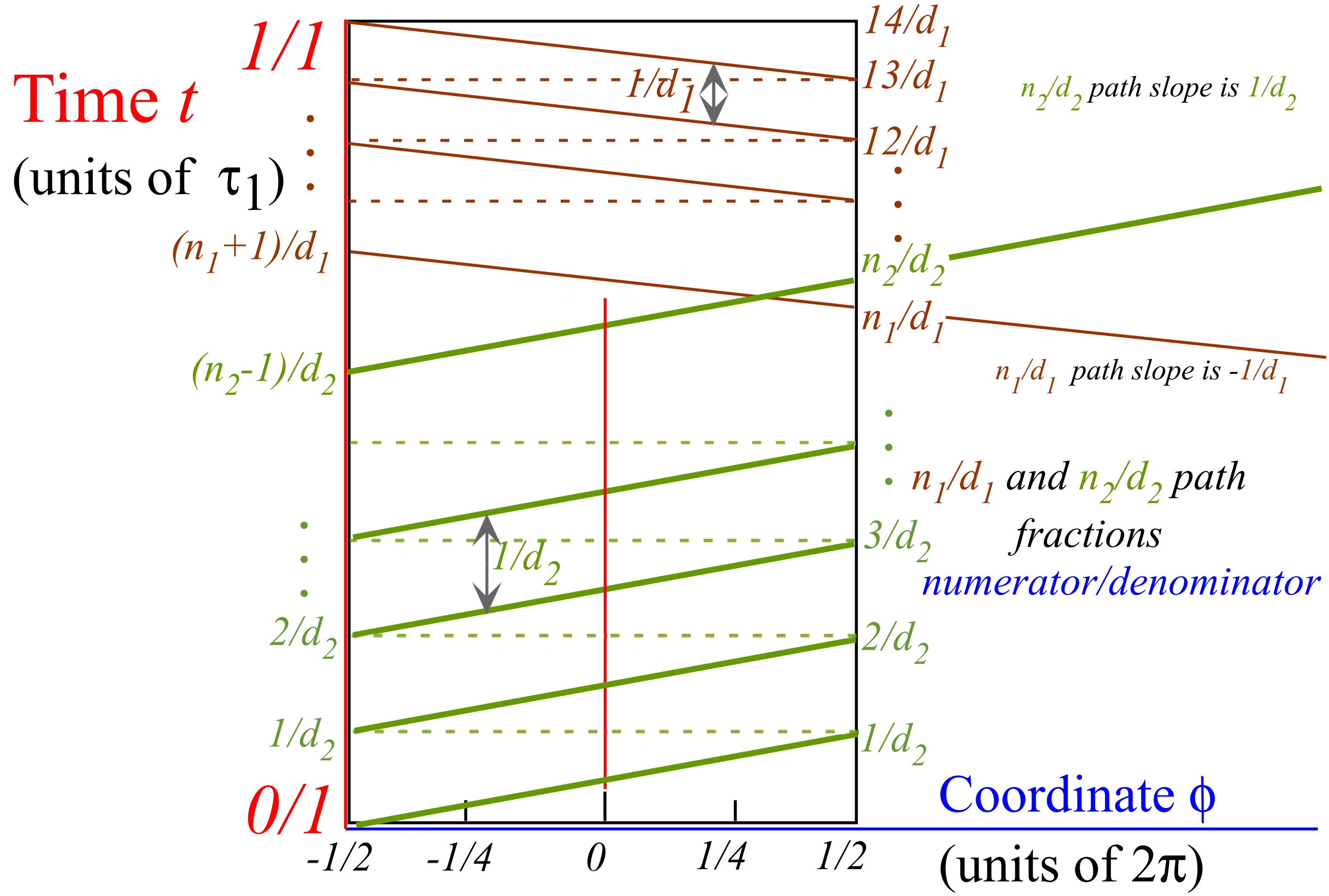
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators* N and *denominators* D of rational fractions N/D



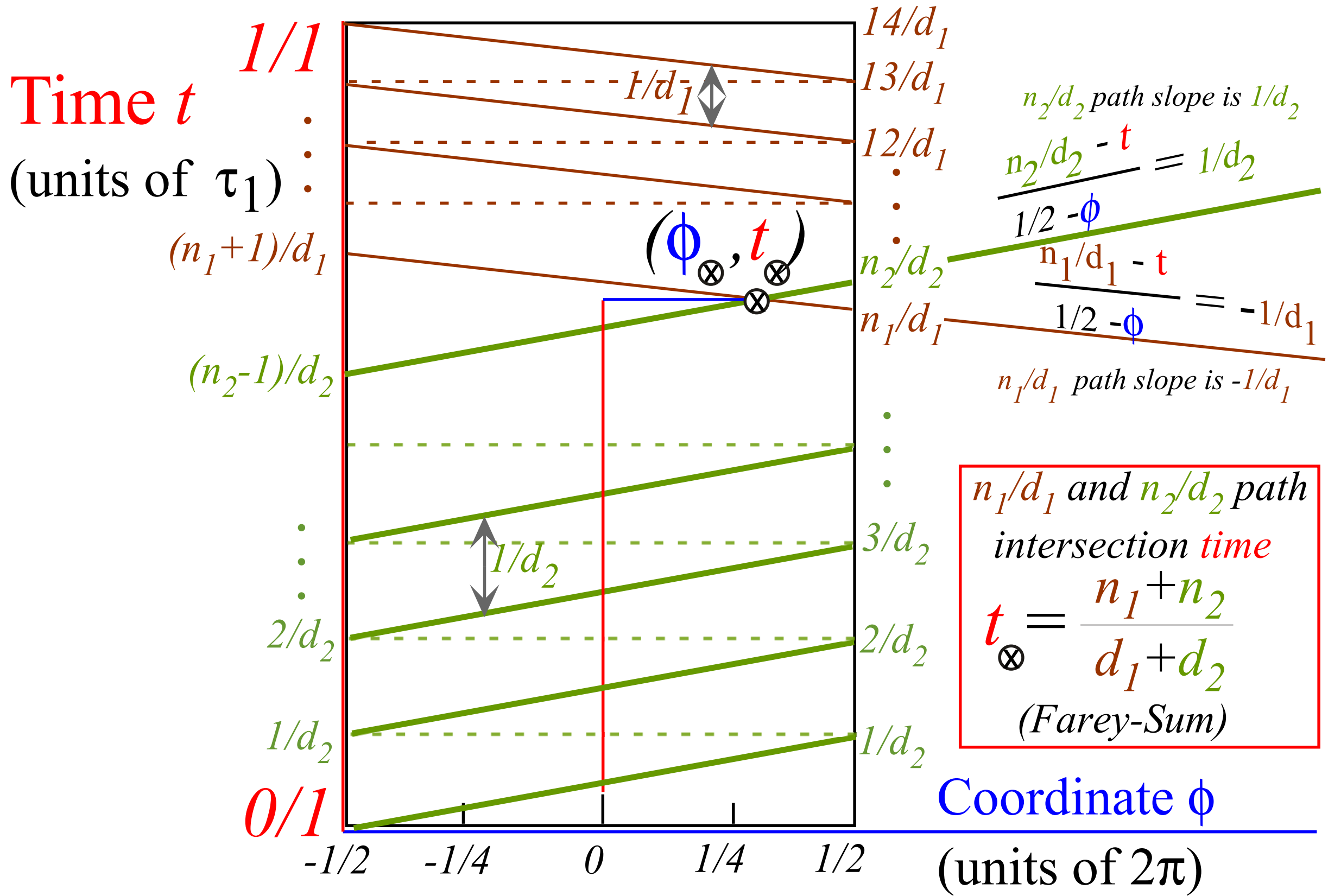
Farey Sum algebra of revival-beat wave dynamics

Label by numerators N and denominators D of rational fractions N/D



Farey Sum algebra of revival-beat wave dynamics

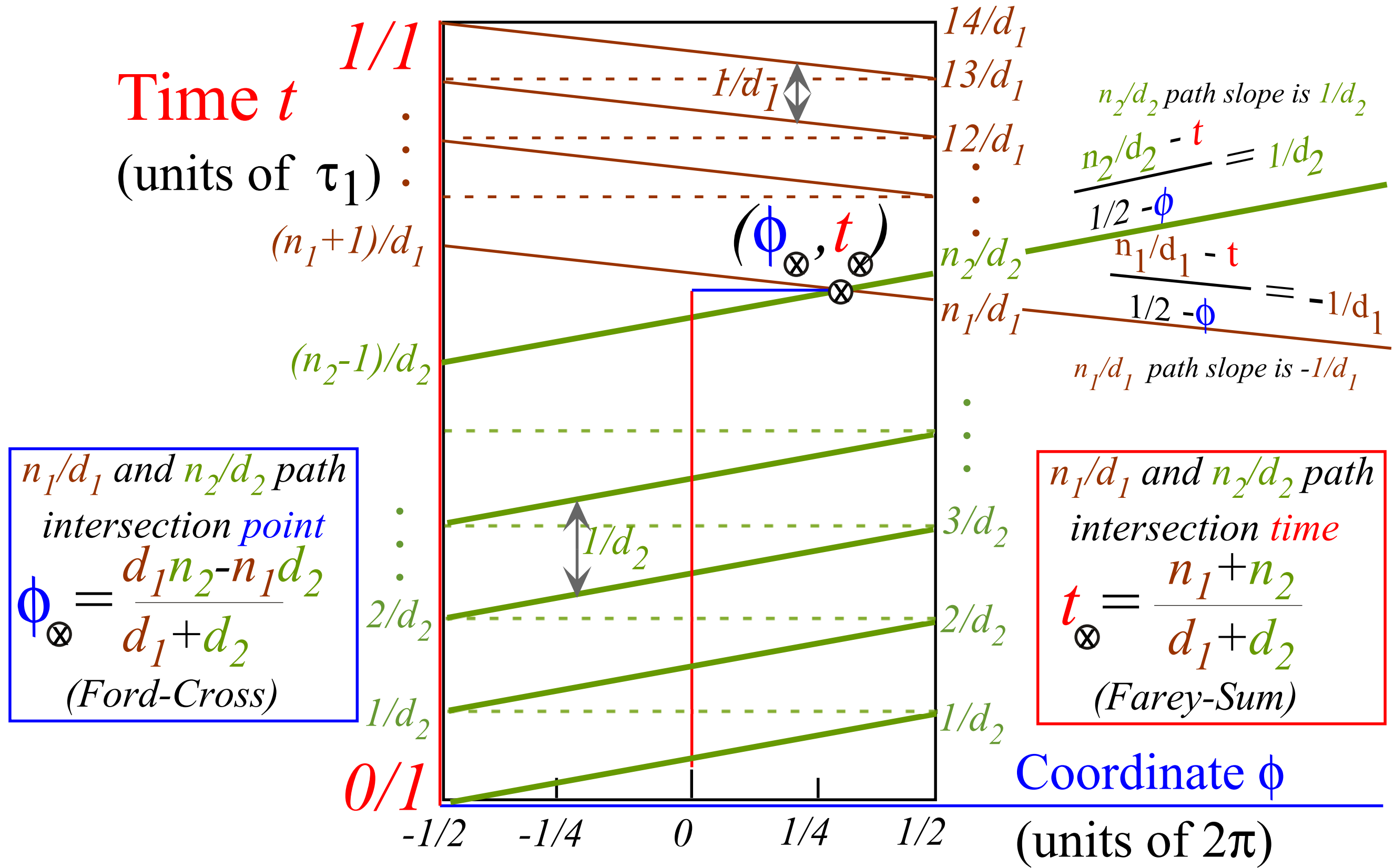
Label by numerators N and denominators D of rational fractions N/D



[John Farey, Phil. Mag.(1816)]

Farey Sum algebra of revival-beat wave dynamics

Label by numerators N and denominators D of rational fractions N/D



[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

[John Farey, Phil. Mag.(1816)]

“Monster Mash” classical segue to Heisenberg action relations

Example of very very large M_1 ball-walls crushing a poor little m_2

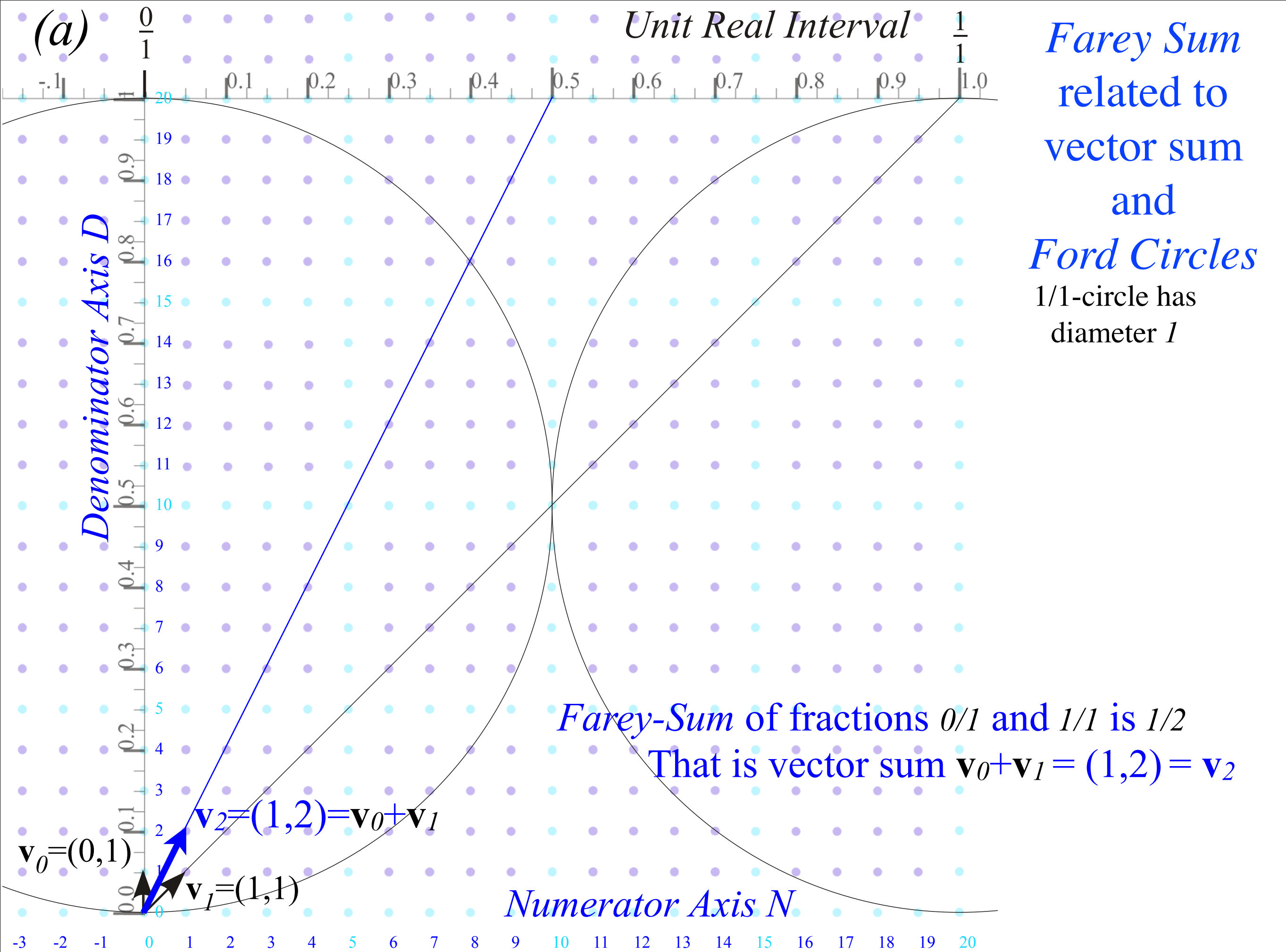
How m_2 keeps its action

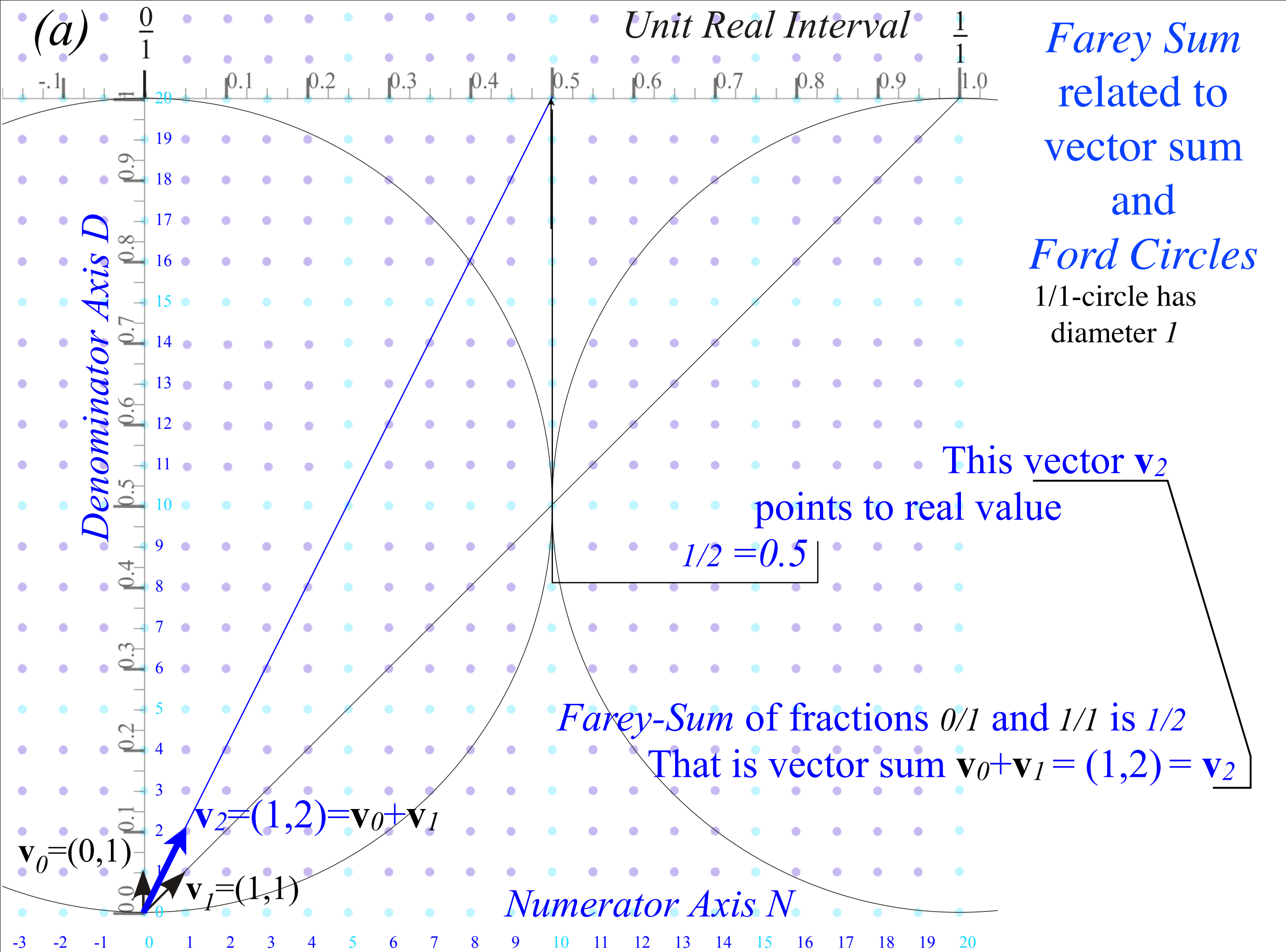
An interesting wave analogy: The “Tiny-Big-Bang” [Harter, J. Mol. Spec. 210, 166-182 (2001)], [Harter, Li IMSS (2012)]

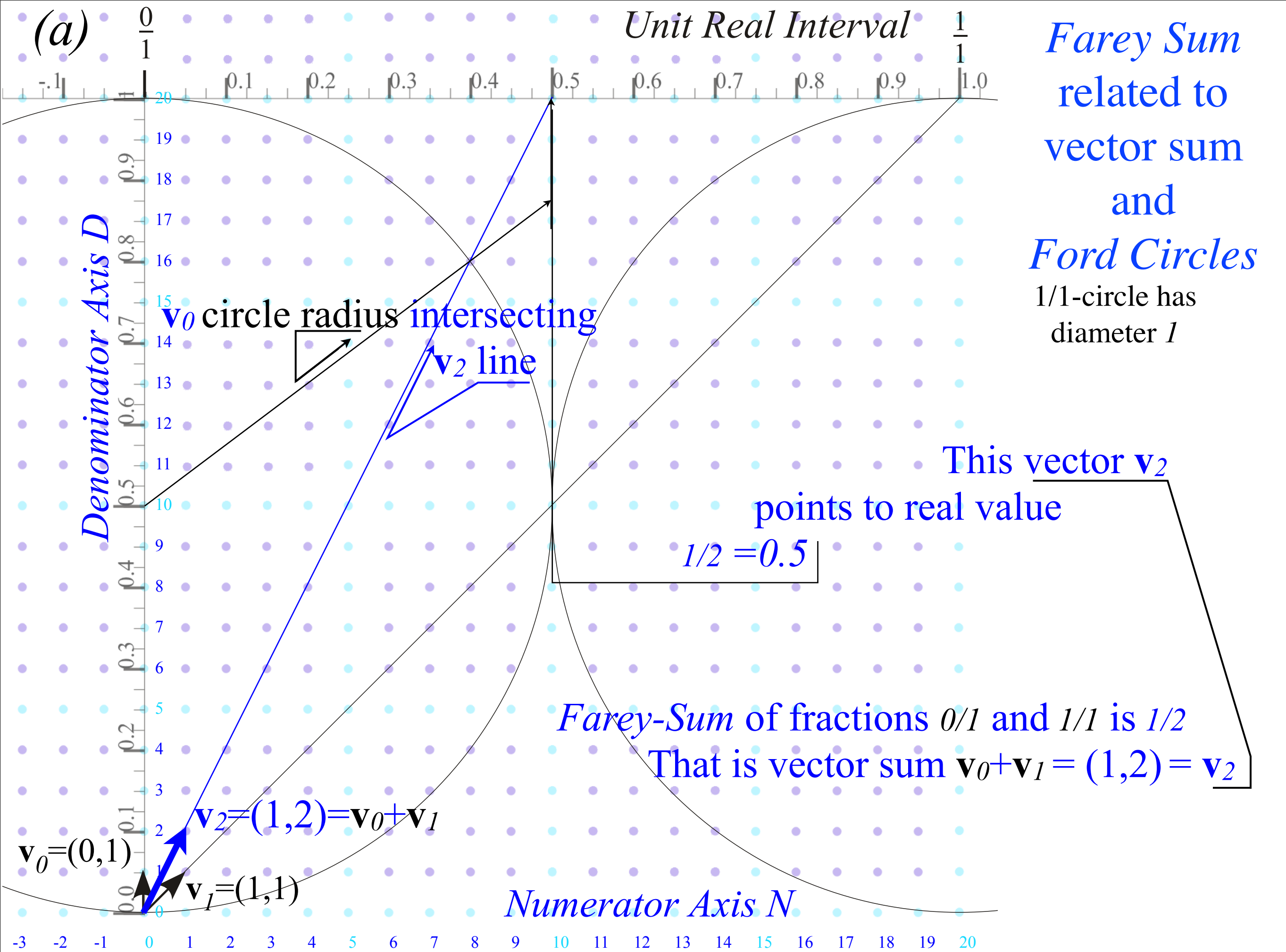
 *A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums*

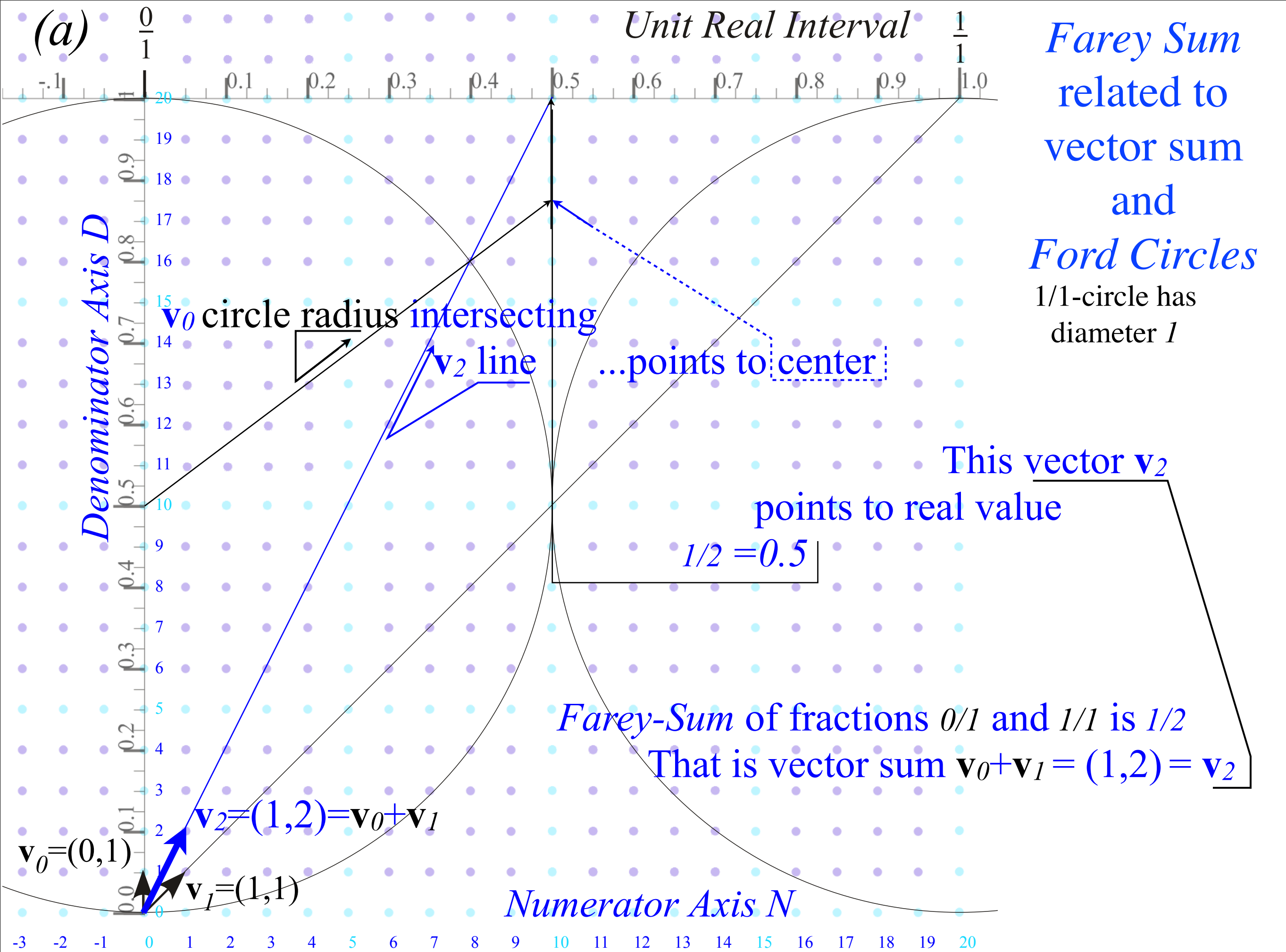
[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

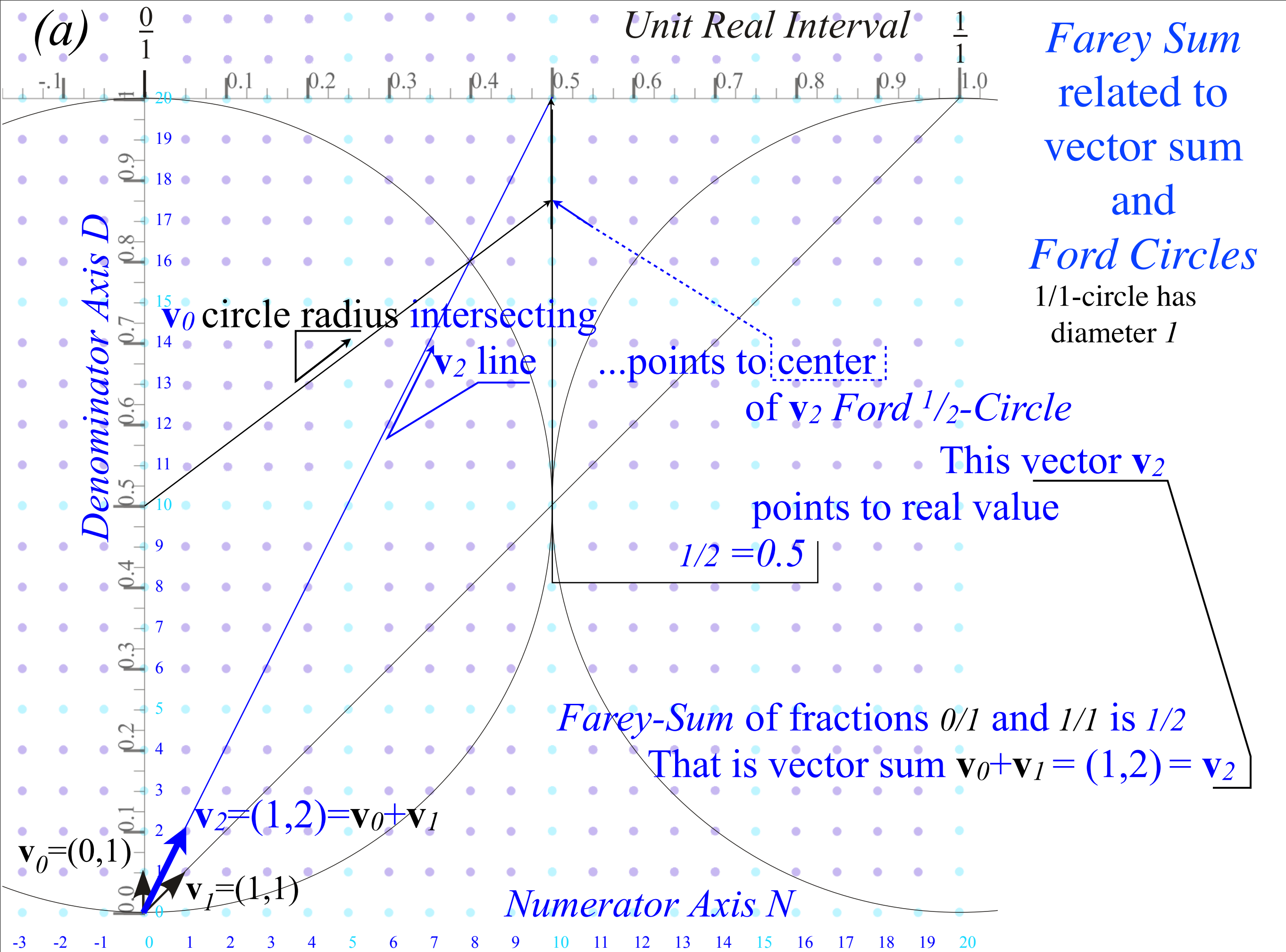
[John Farey, Phil. Mag.(1816)]

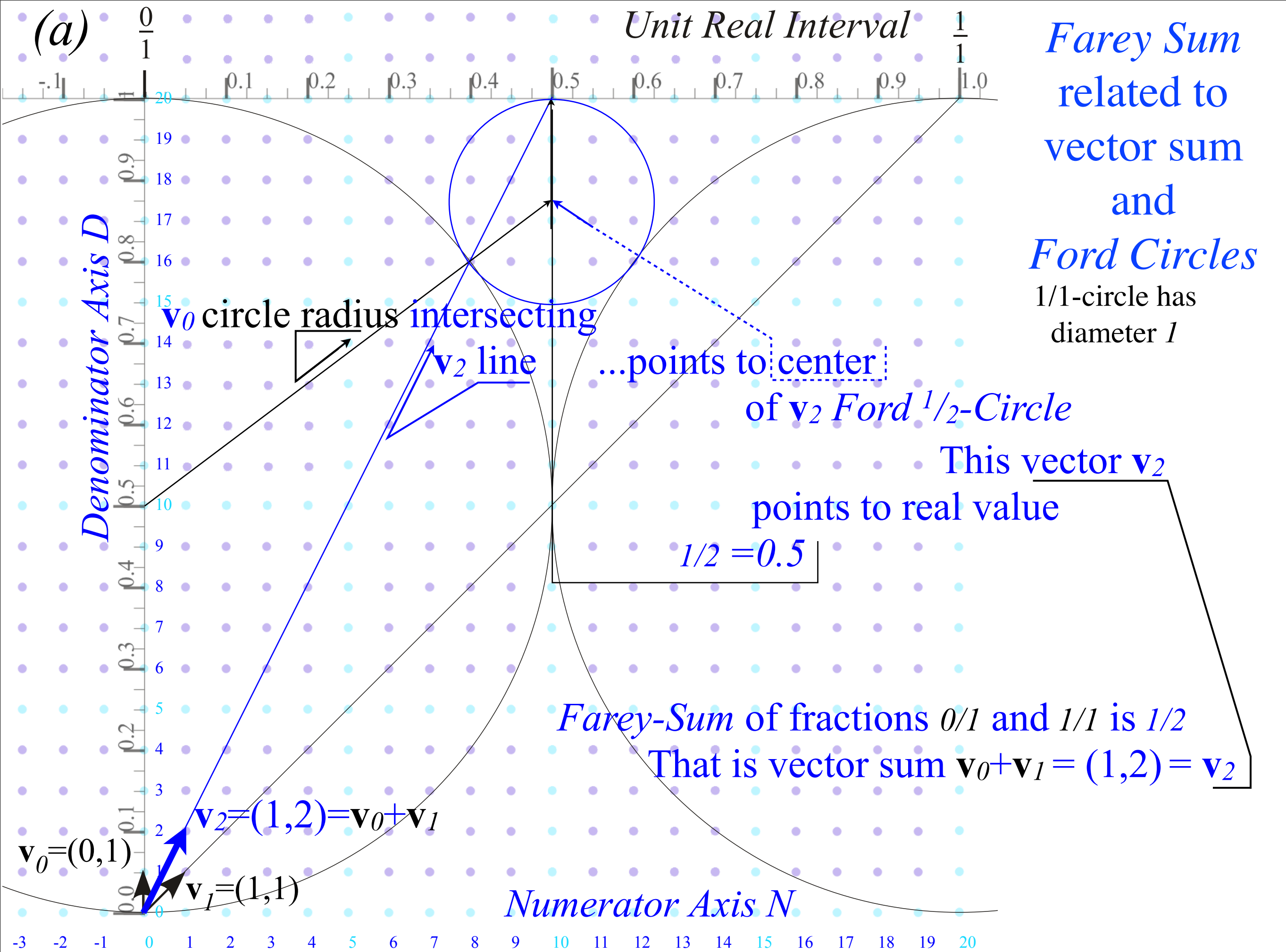


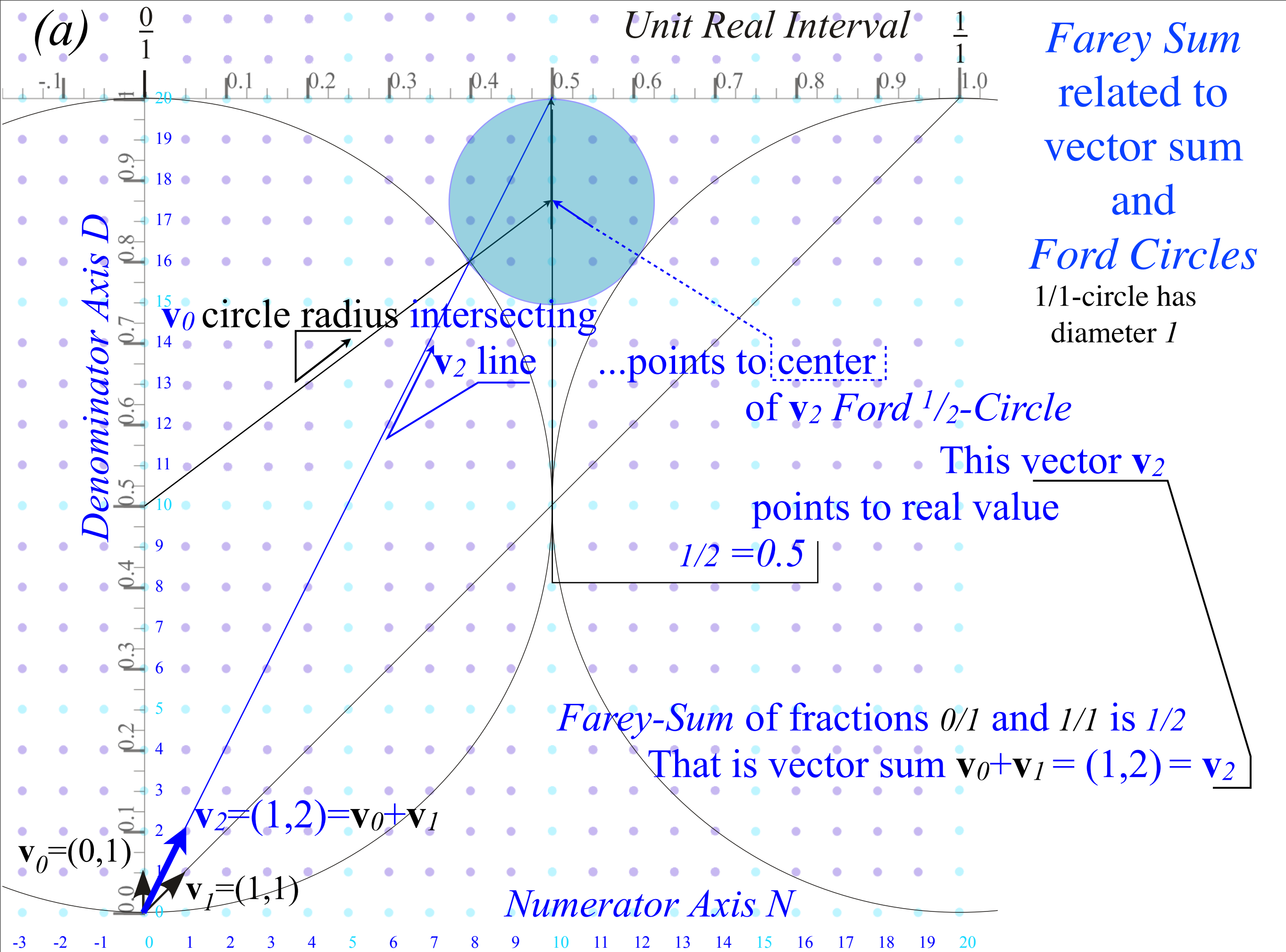


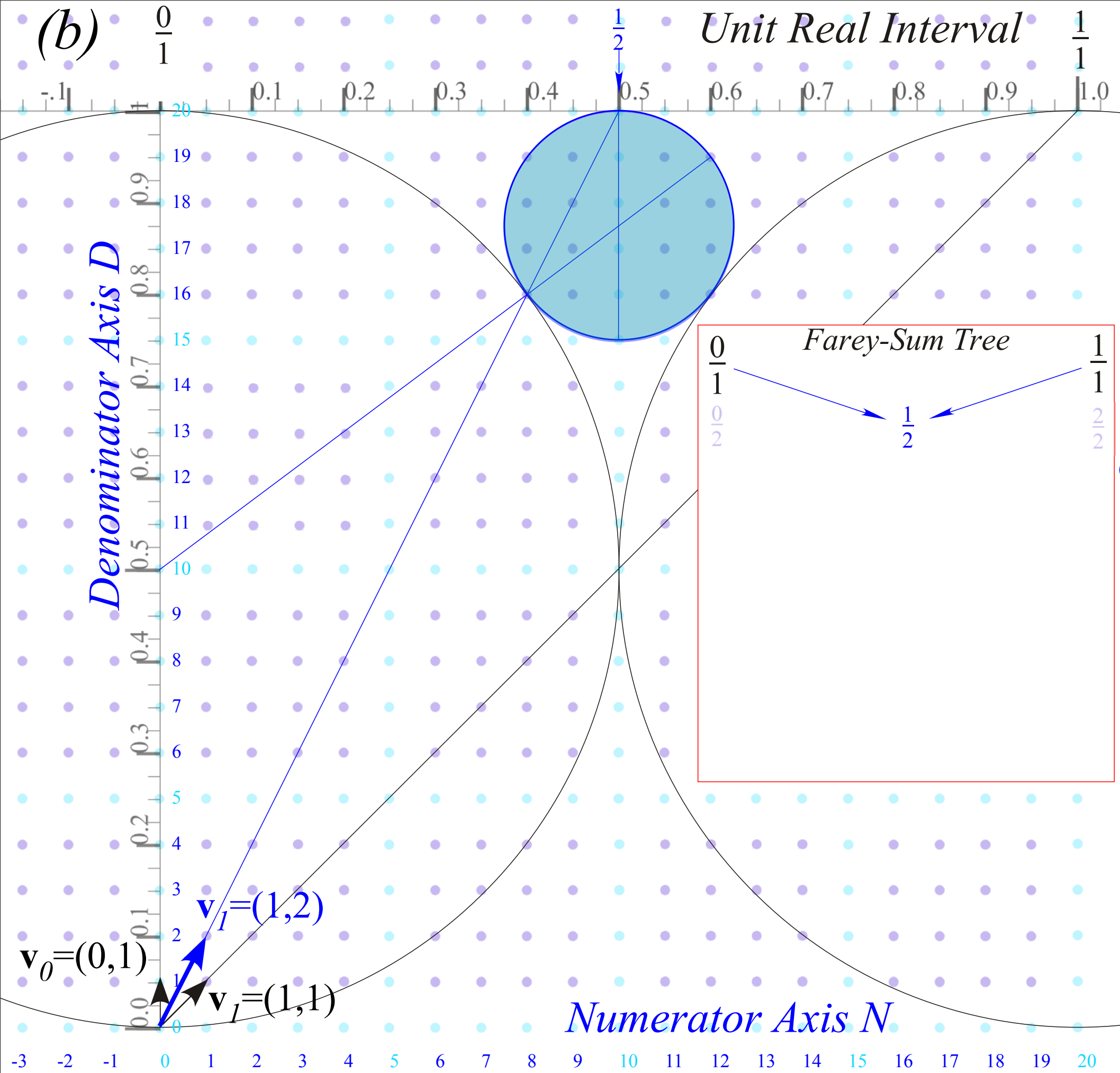




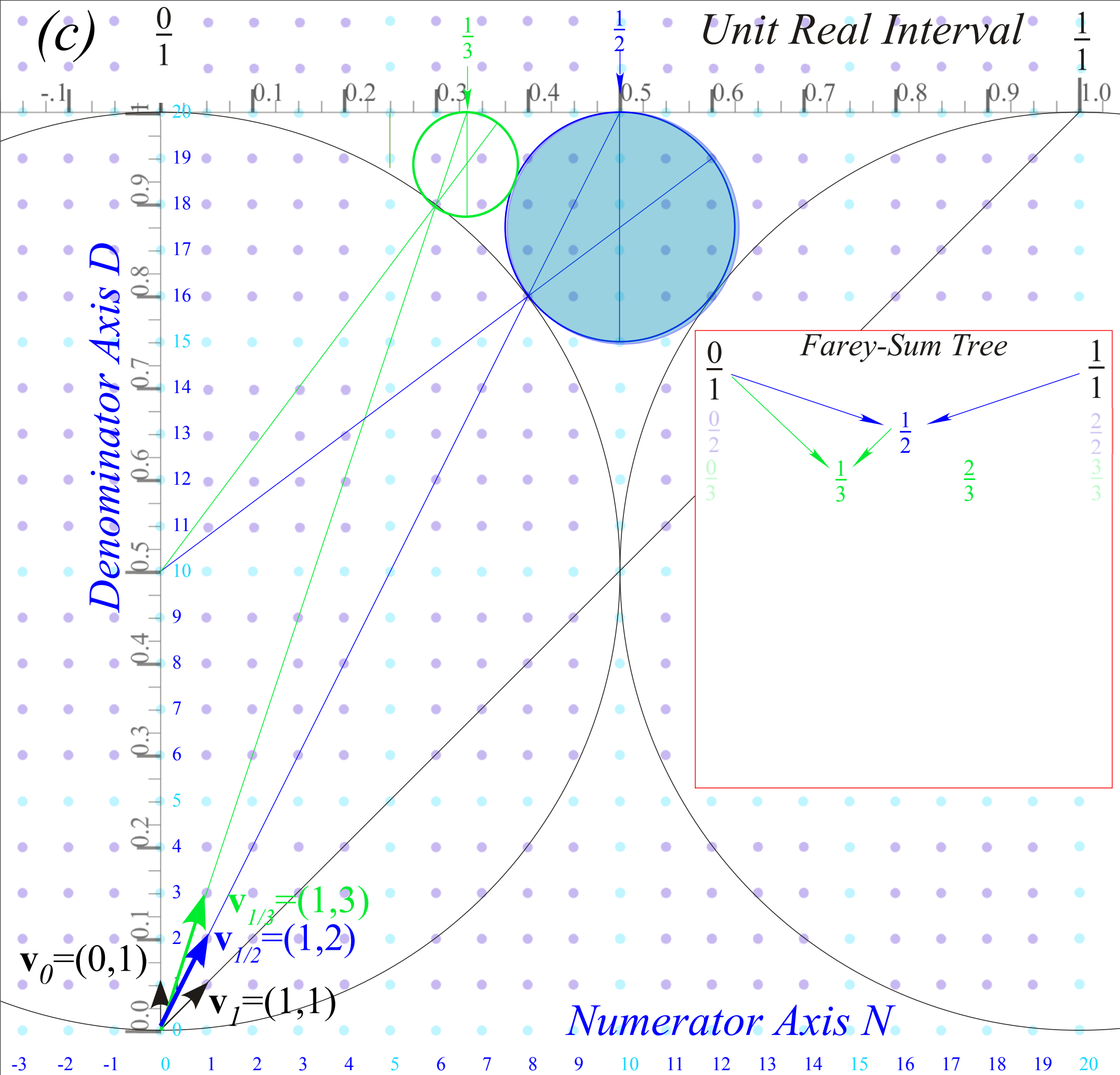






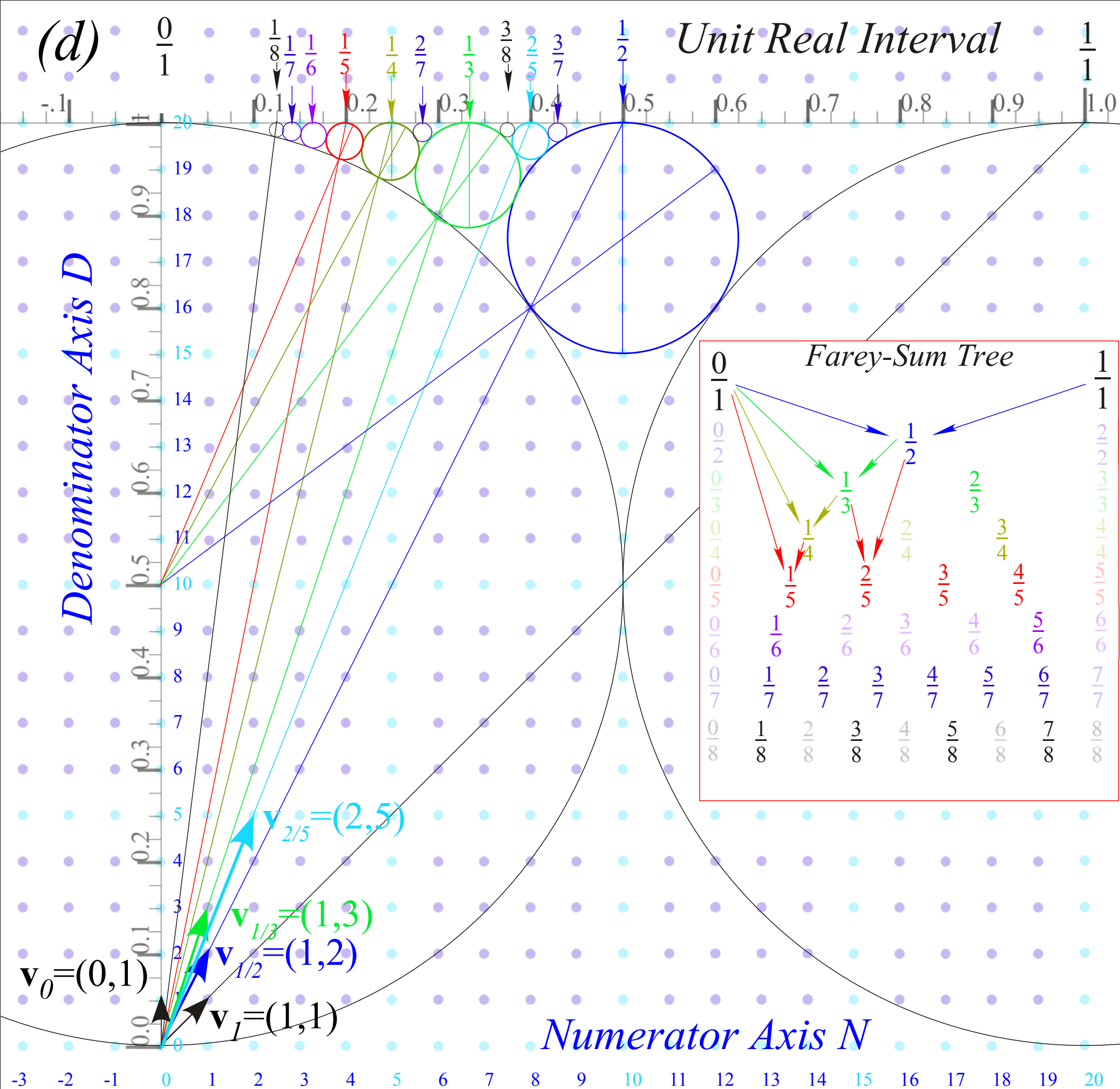


Farey Sum
 related to
 vector sum
 and
Ford Circles
 1/1-circle has
 diameter 1
 1/2-circle has
 diameter $1/2^2 = 1/4$



$1/2$ -circle has diameter $1/2^2 = 1/4$

$1/3$ -circles have diameter $1/3^2 = 1/9$

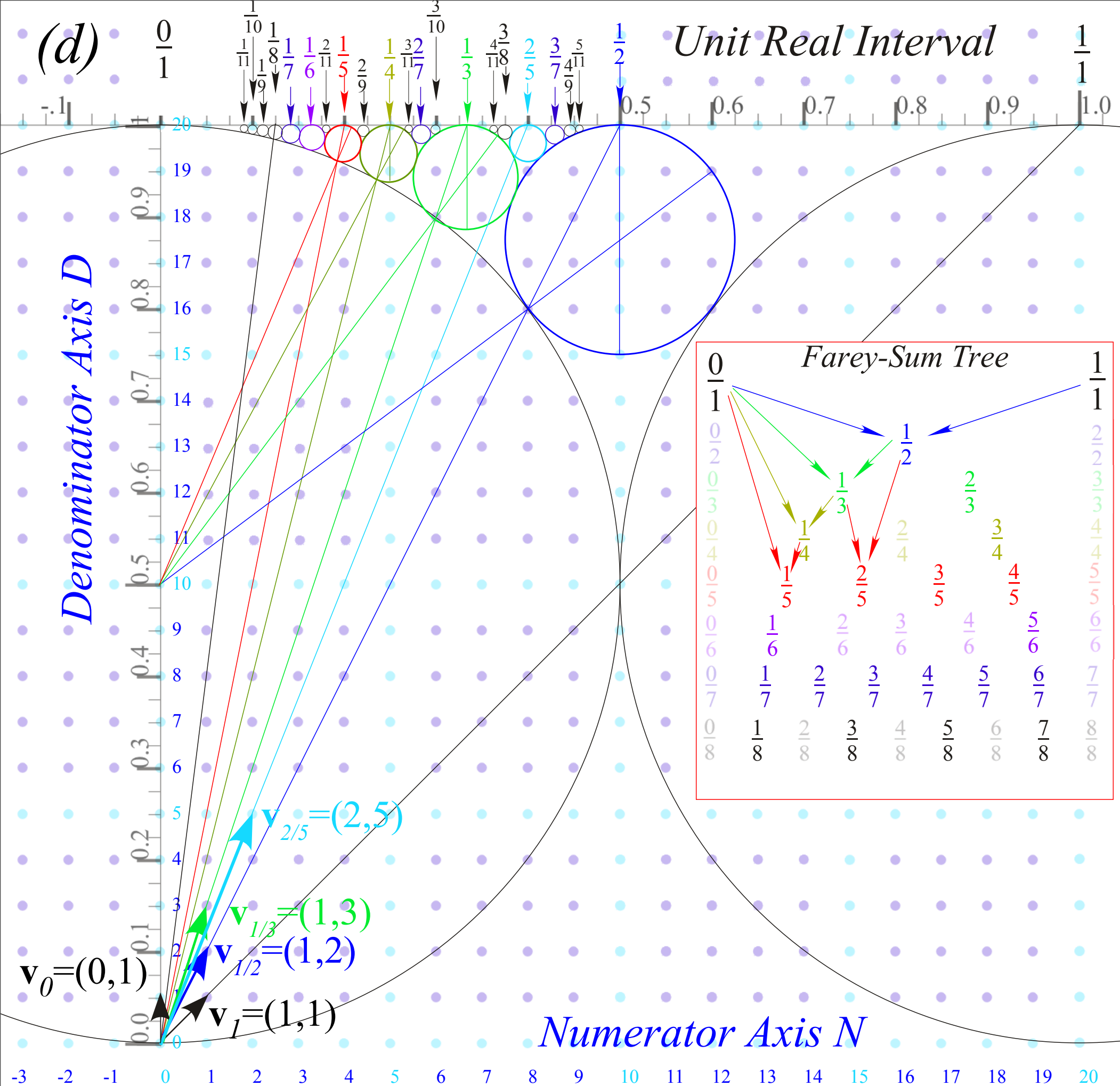


Farey Sum related to vector sum and Ford Circles

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$



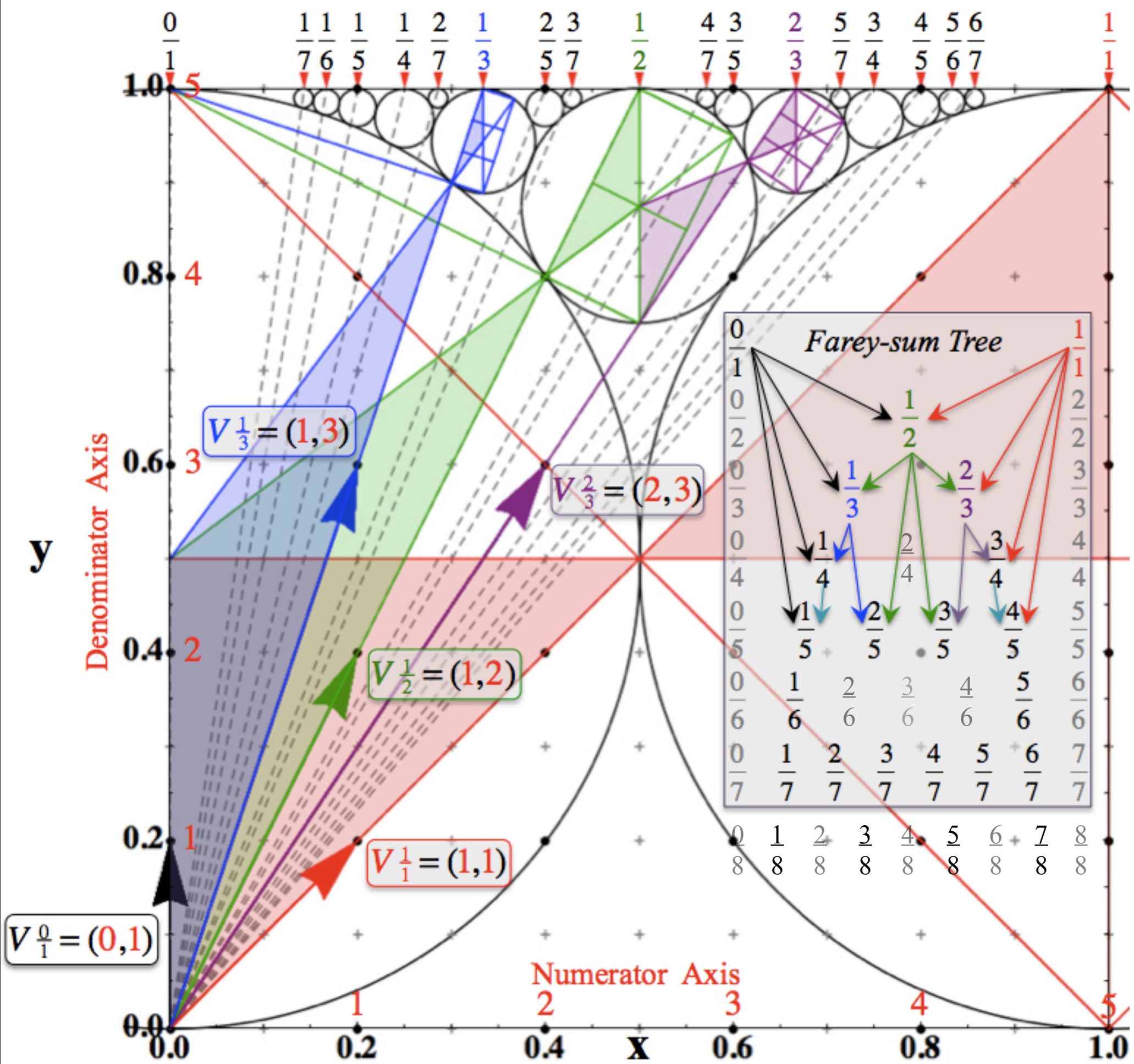
*Farey Sum
related to
vector sum
and
Ford Circles*

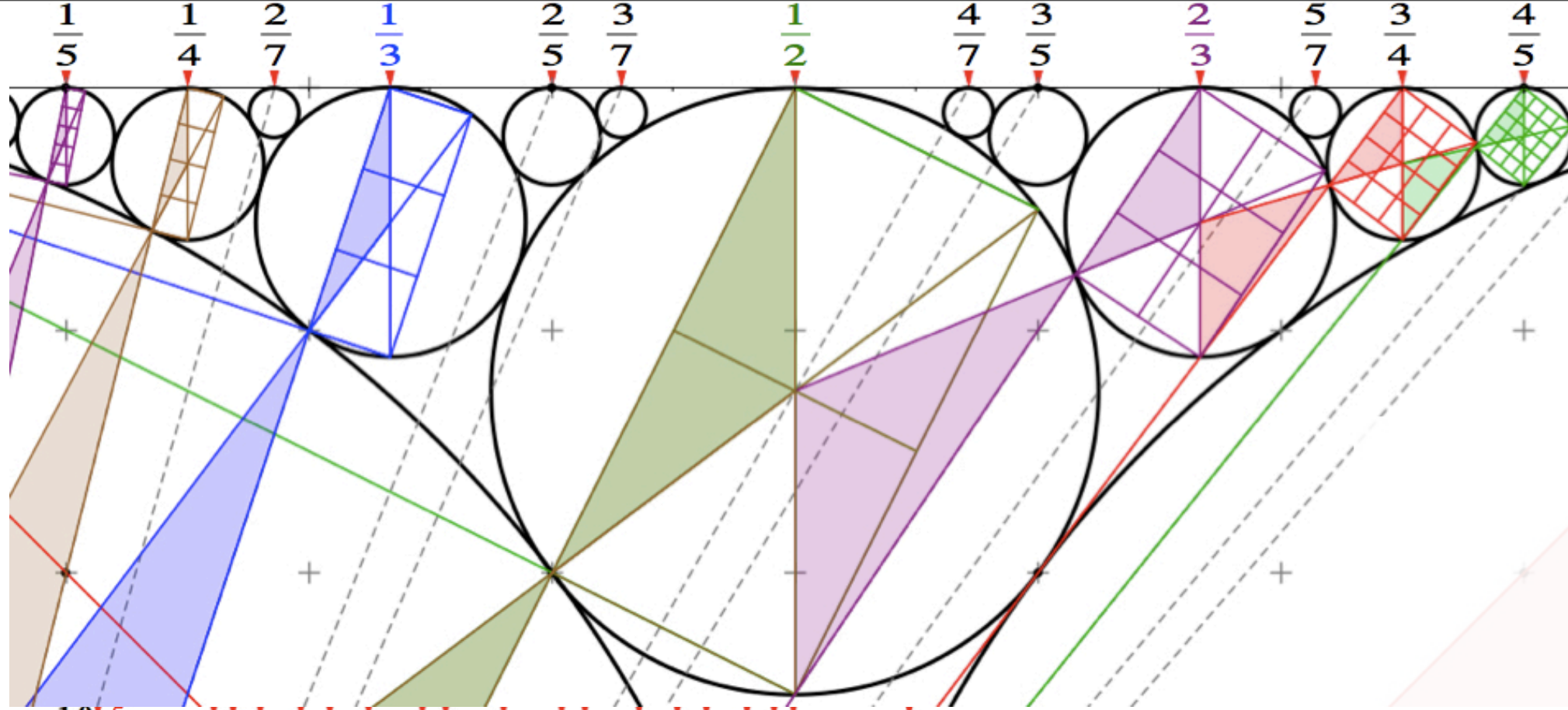
*1/2-circle has
diameter $1/2^2=1/4$*

*1/3-circles have
diameter $1/3^2=1/9$*

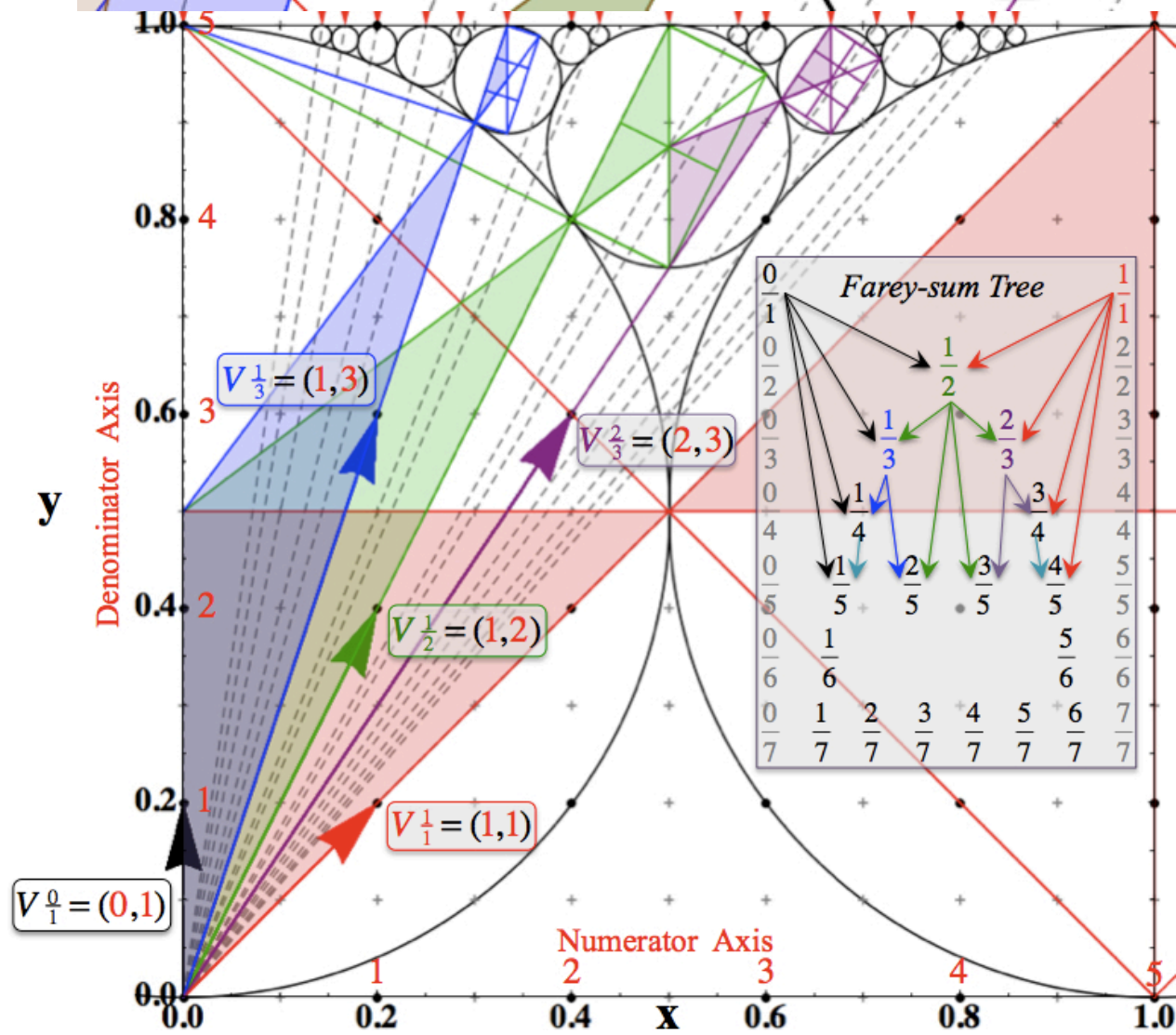
*n/d-circles have
diameter $1/d^2$*

Thales
Rectangles
provide
analytic geometry
of
fractal structure

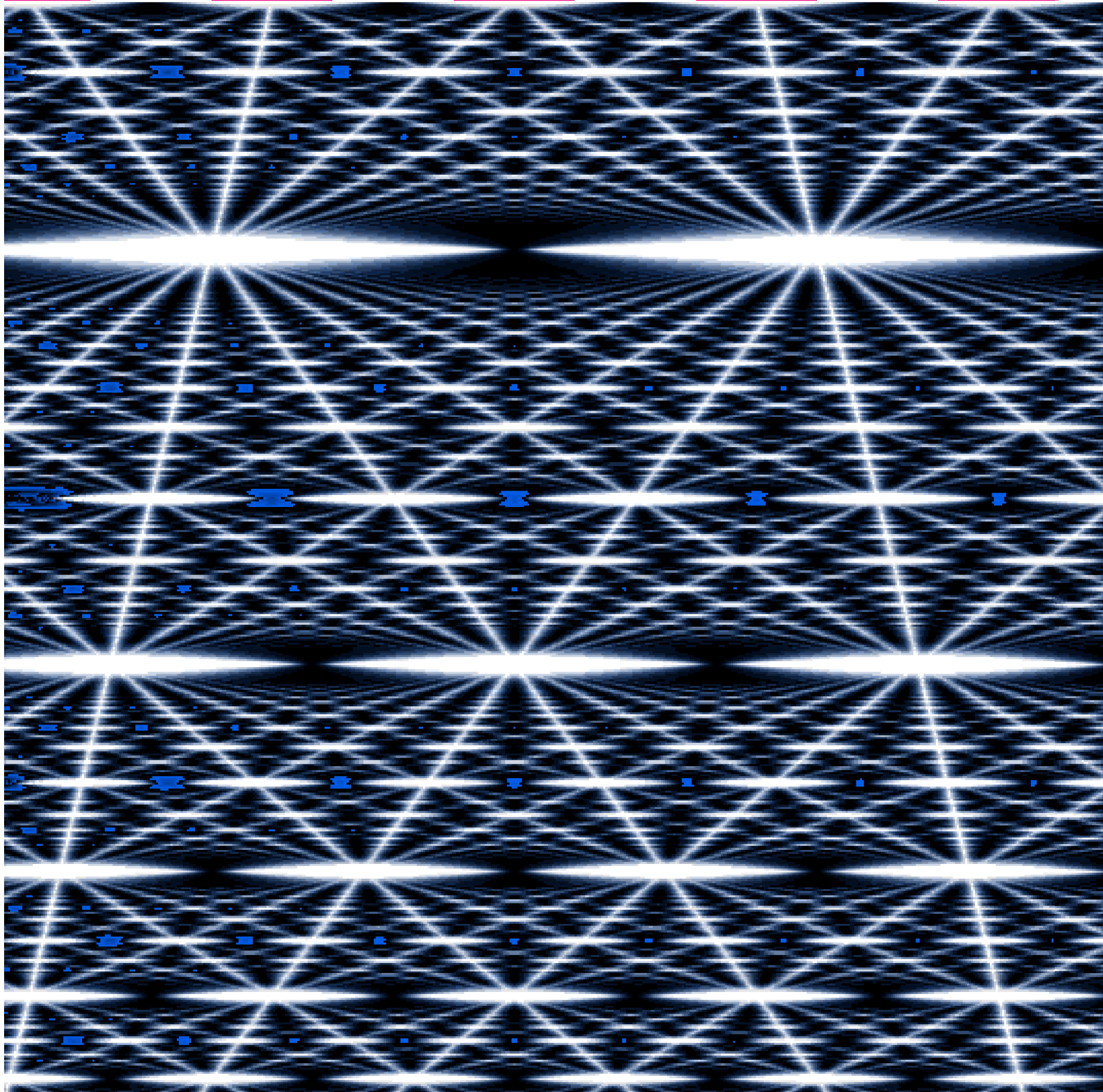




“Quantized”
Thales
Rectangles
provide
analytic geometry
of
fractal structure



*(Quantum computer simulation)
That makes an ∞ -ly deep "3D-Magic-Eye" picture*



Geometric "Integration" (Converting Velocity data to Spacetime)

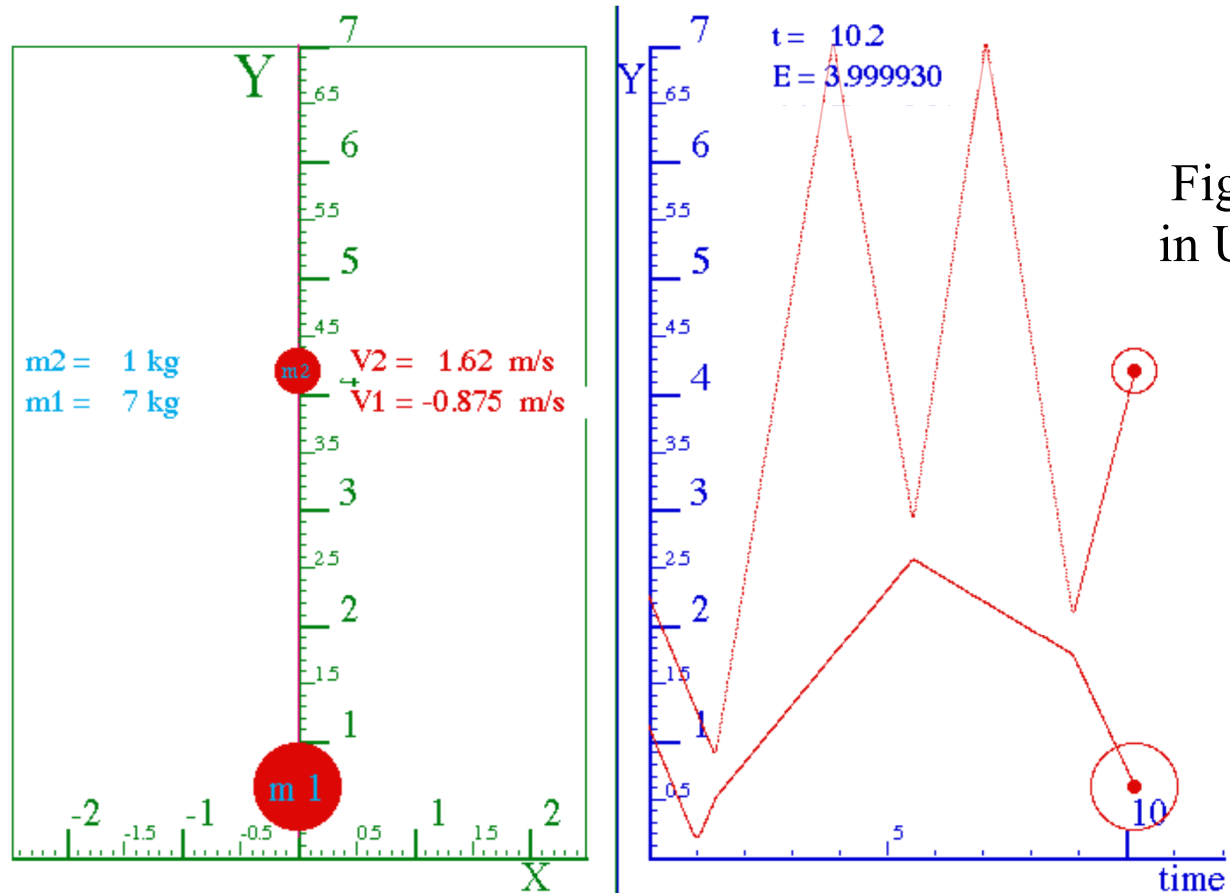


Fig. 4.8
in Unit 1

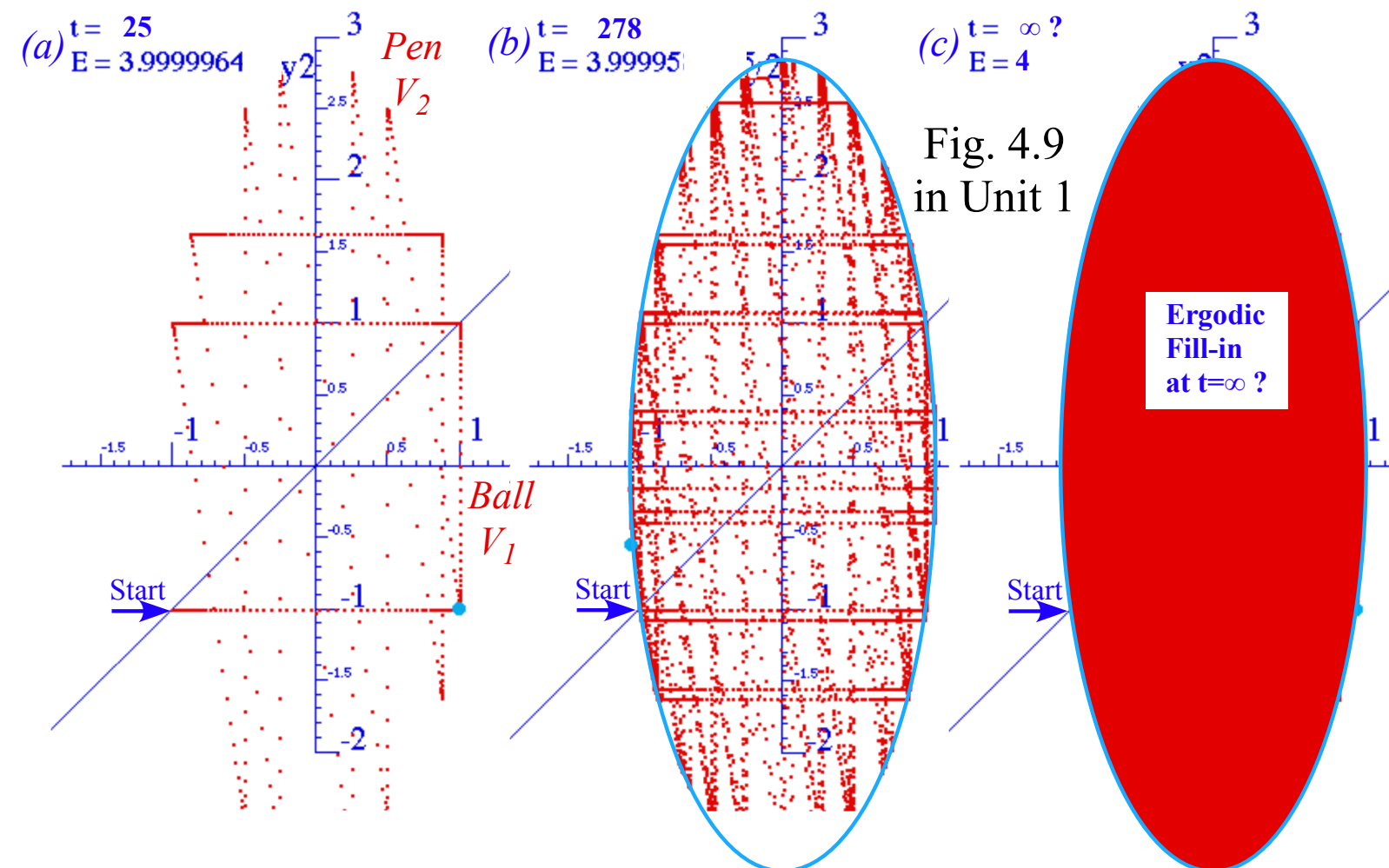
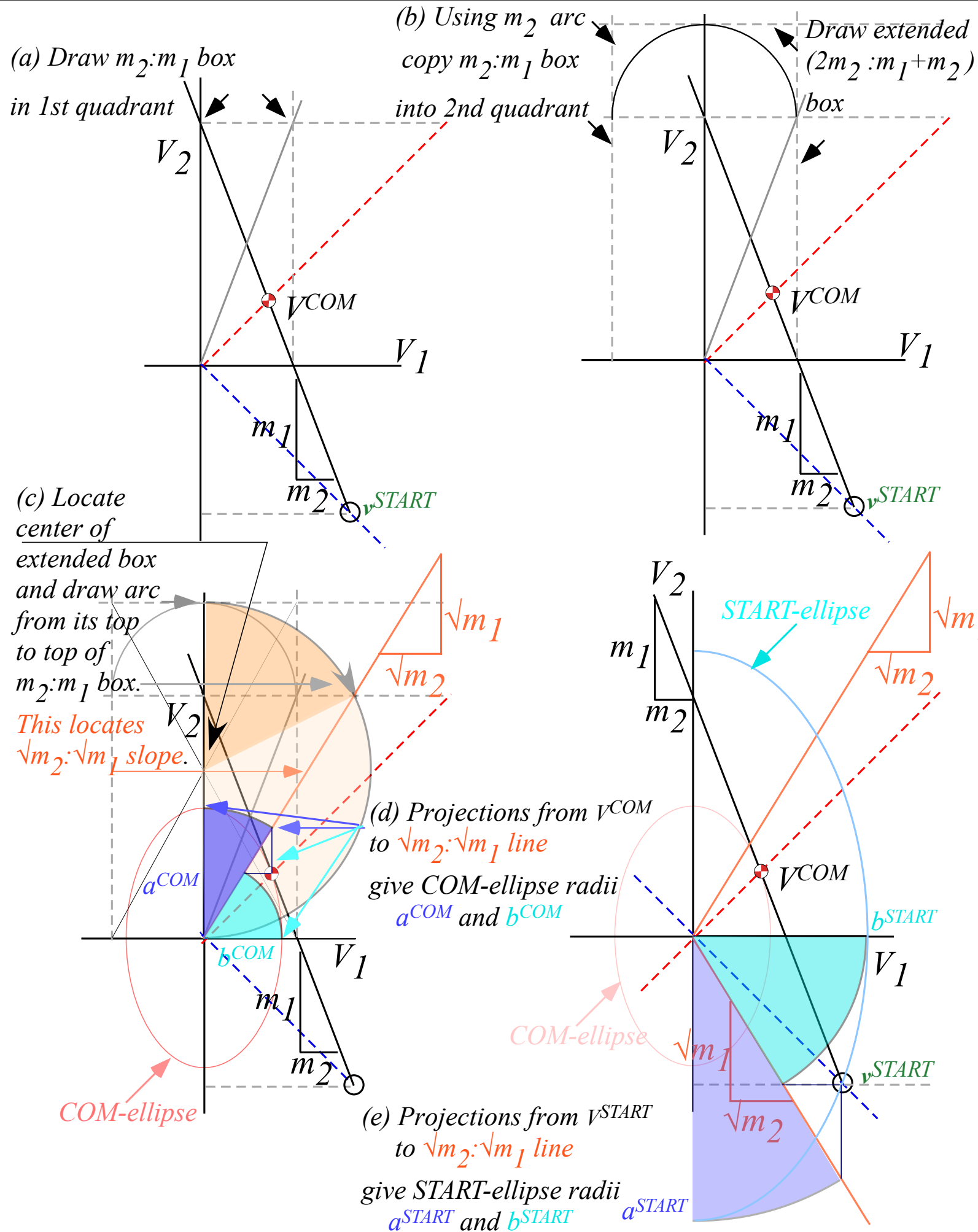


Fig. 4.9
in Unit 1



Unit 1
Fig. 8.4a-d

This is a construction of the energy ellipse in a Largangian (v_1, v_2) plot given the initial (v_1, v_2) .

The Estrangian (V_1, V_2) plot makes the (v_1, v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1, V_2) .)