Lecture 6
Thur. 9.08.2016

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

Compare mks units of Coulomb Electrostatic vs. Gravity

Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Introducing 2D IHO orbits and phasor geometry

Phasor "clock" geometry

Geometry of common power-law potentials

Geometric (Power) series

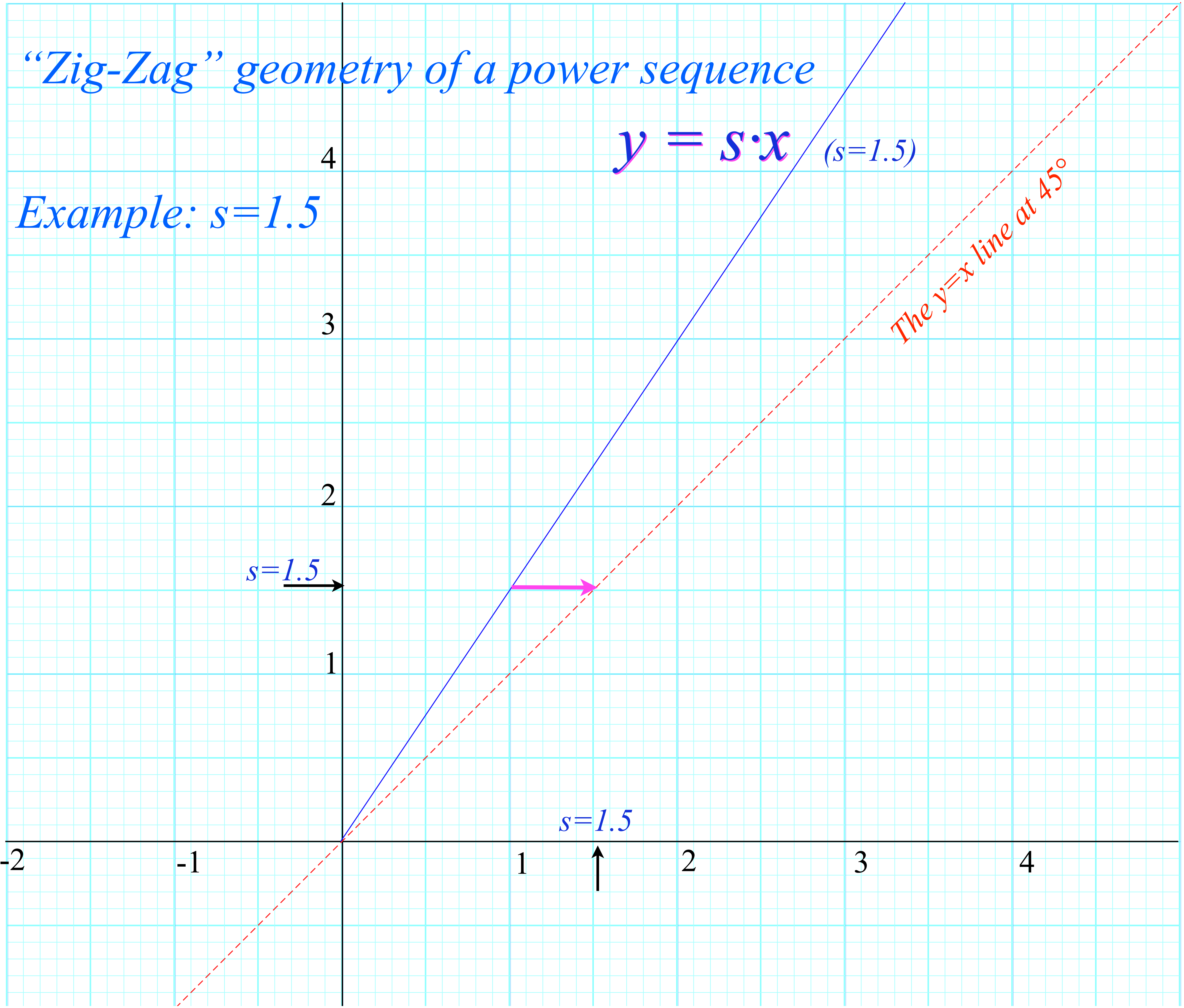
→ *"Zig-Zag" exponential geometry*

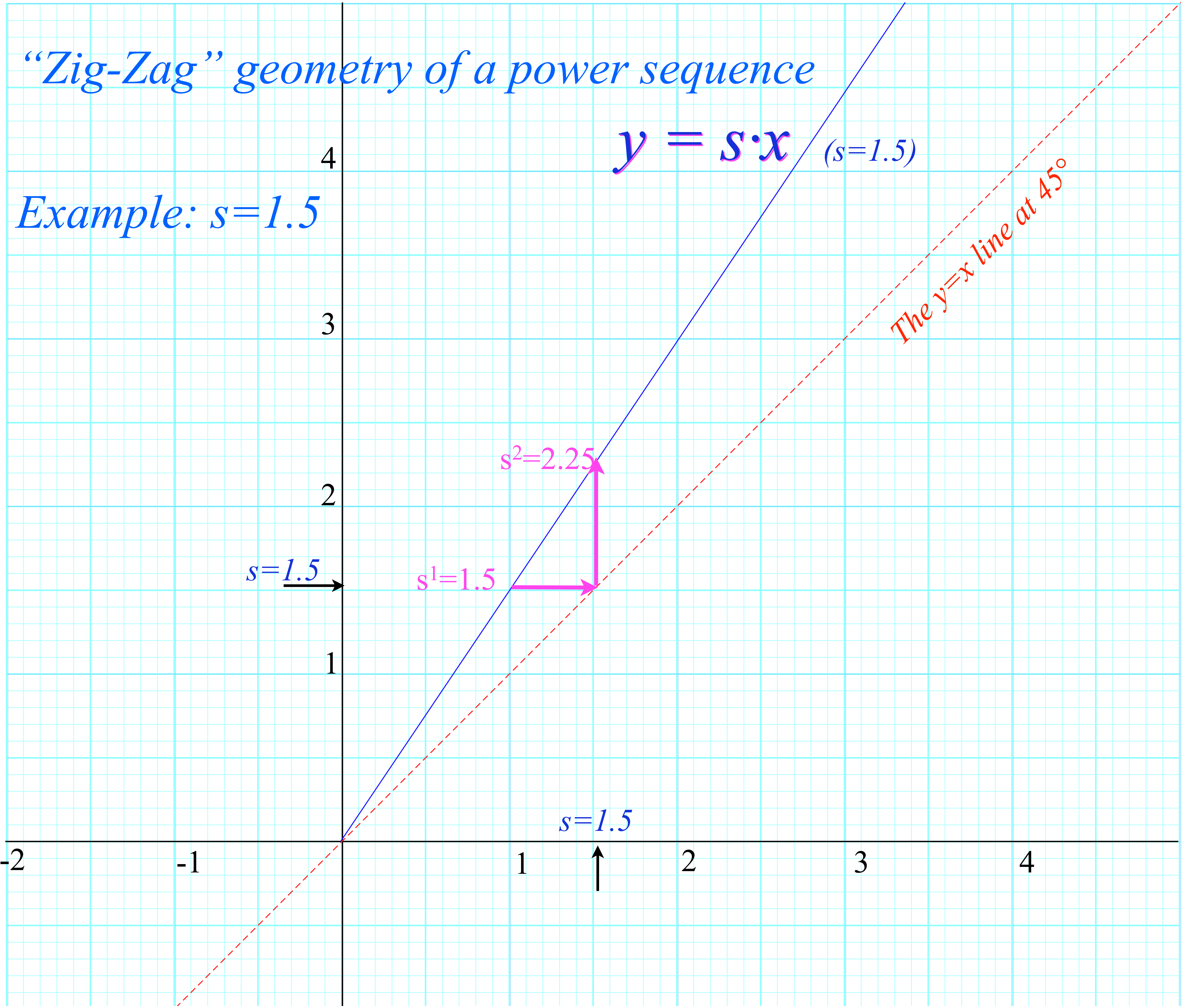
Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

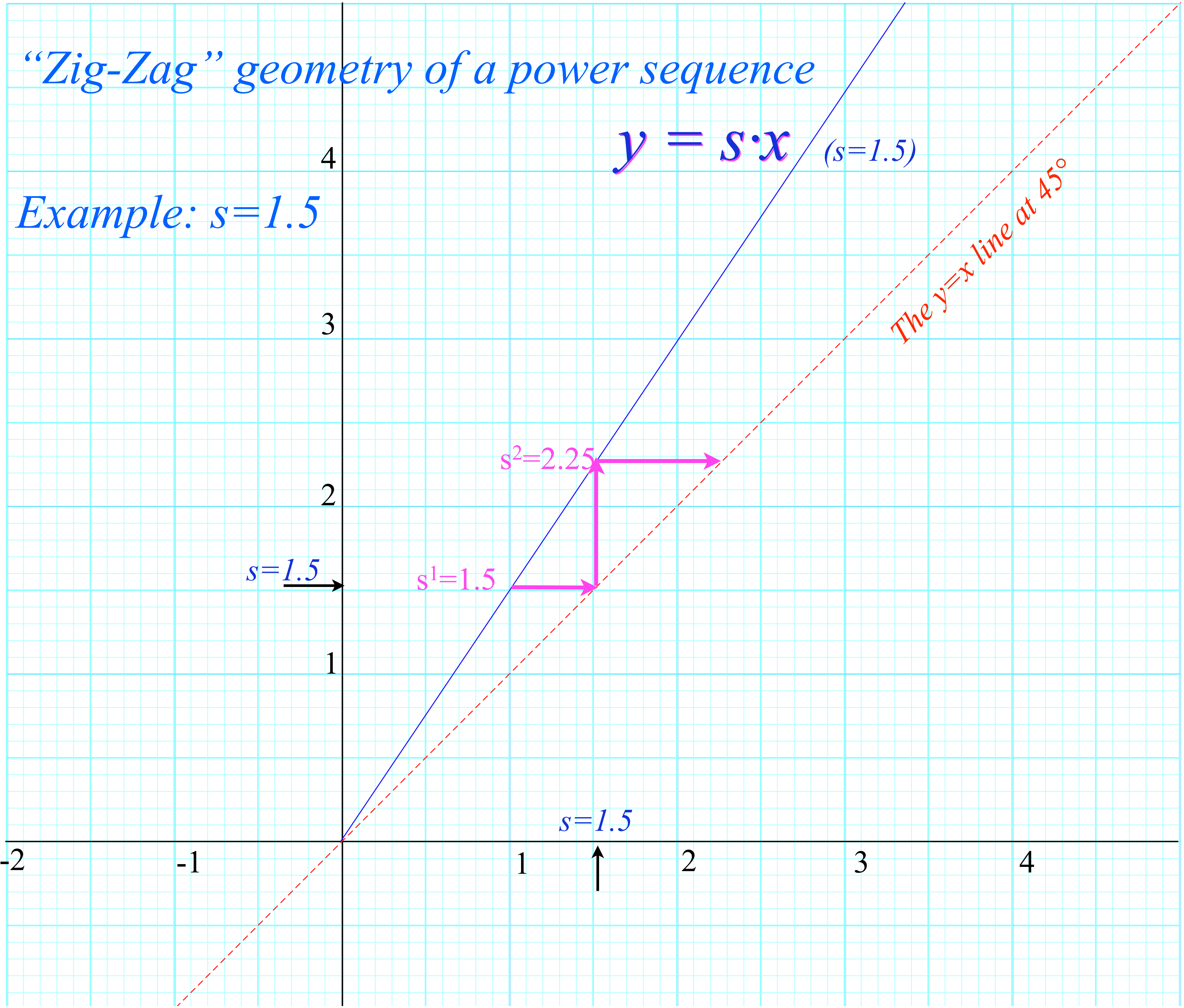
Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

Compare mks units of Coulomb Electrostatic vs. Gravity

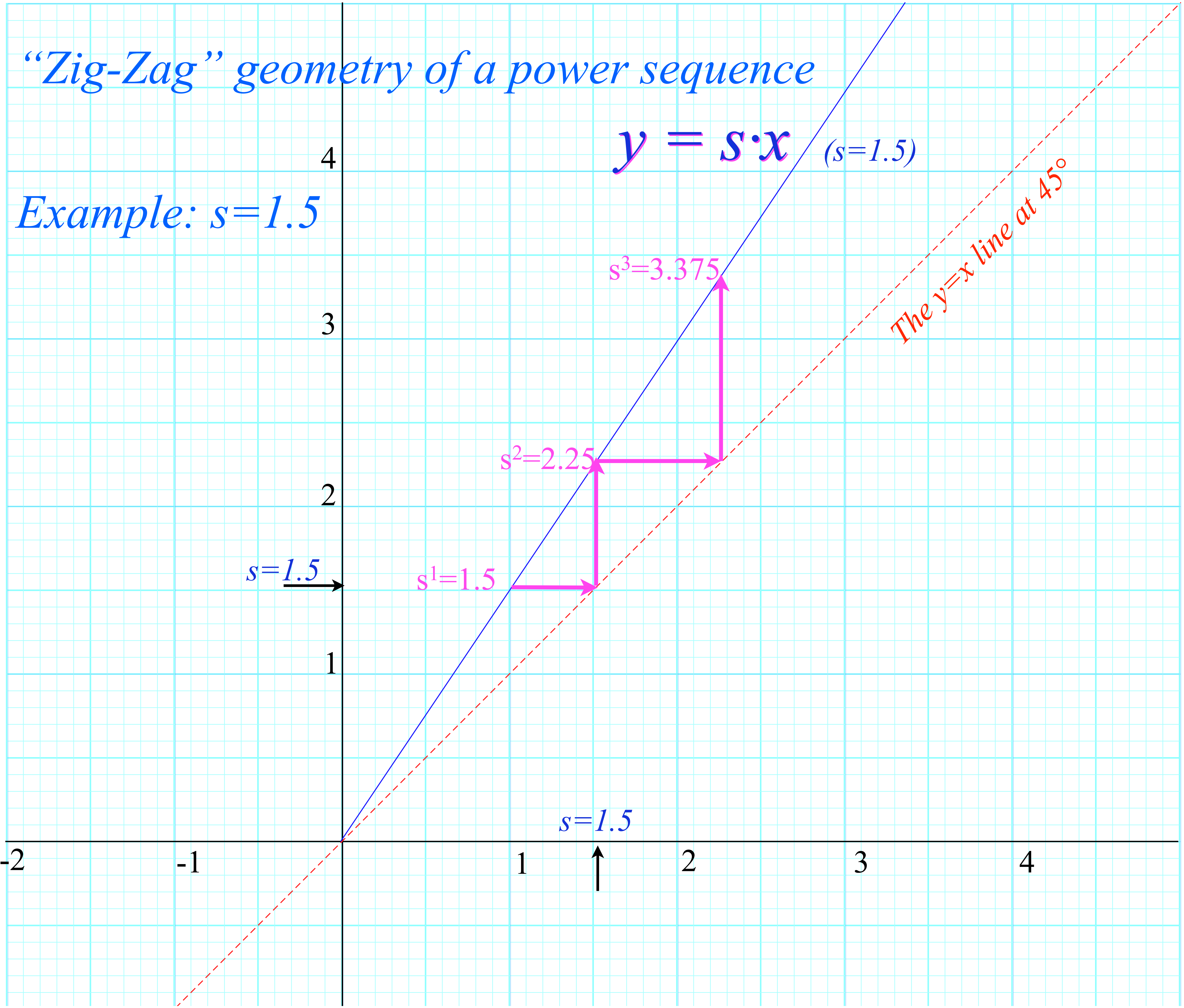

"Zig-Zag" geometry of a power sequence
$y = s \cdot x$ (s=1.5)
Example: s=1.5
The y=x line at 45°
s=1.5
s=1.5
4
3
2
1
-2
-1
1
2
3
4



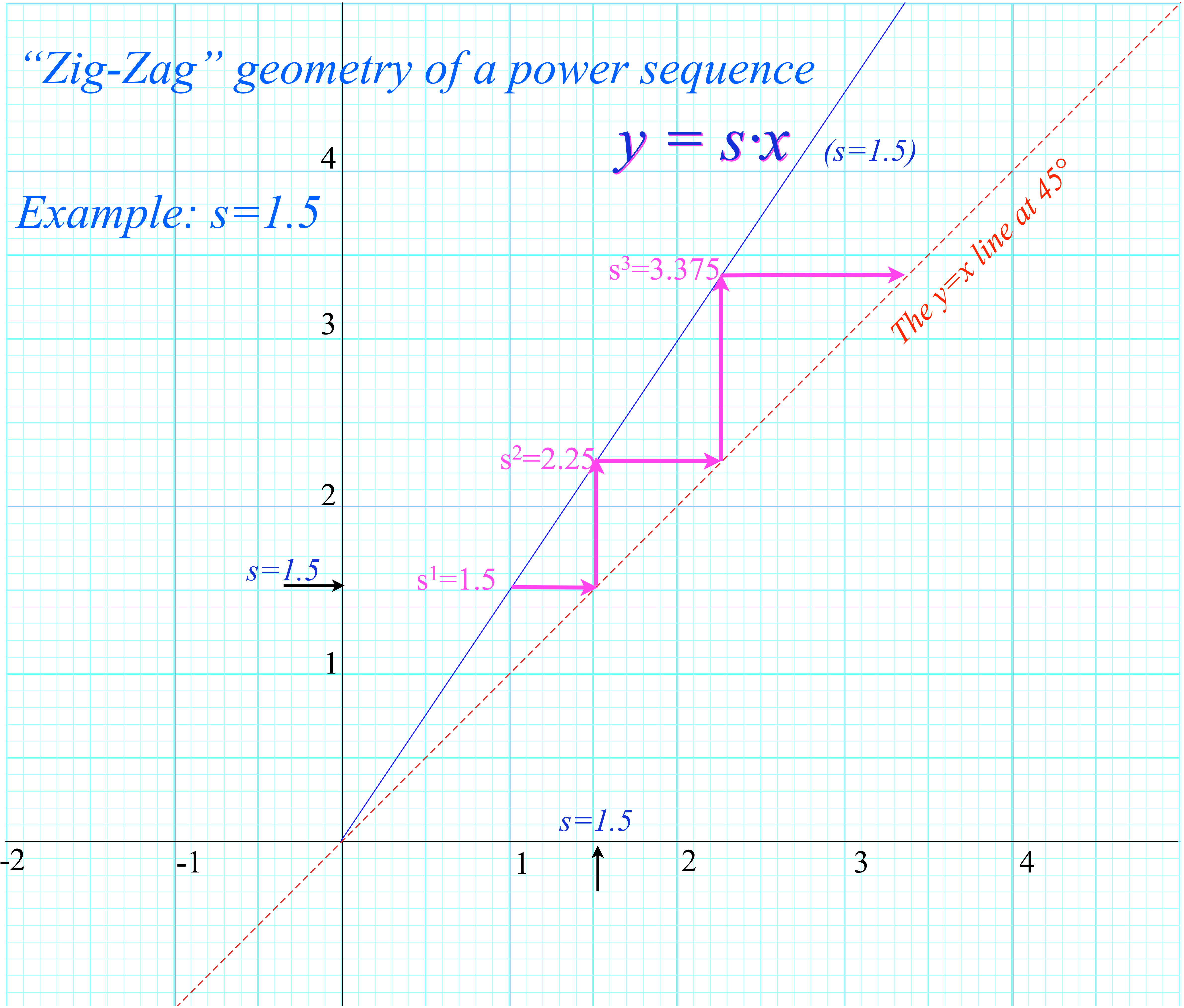
"Zig-Zag" geometry of a power sequence
$y = s \cdot x$
(s=1.5)
Example: s=1.5
The y=x line at 45°
s^2=2.25
s^1=1.5
s=1.5
s=1.5
4
3
2
1
-2
-1
1
2
3
4



"Zig-Zag" geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
The y=x line at 45°
4
3
2
1
s=1.5
s^1=1.5
s^2=2.25
s=1.5
-2
-1
1
2
3
4



"Zig-Zag" geometry of a power sequence
y = s·x
(s=1.5)
Example: s=1.5
4
3
2
1
s³=3.375
s²=2.25
s¹=1.5
s=1.5
The y=x line at 45°
s=1.5
-2
-1
1
2
3
4



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3
2
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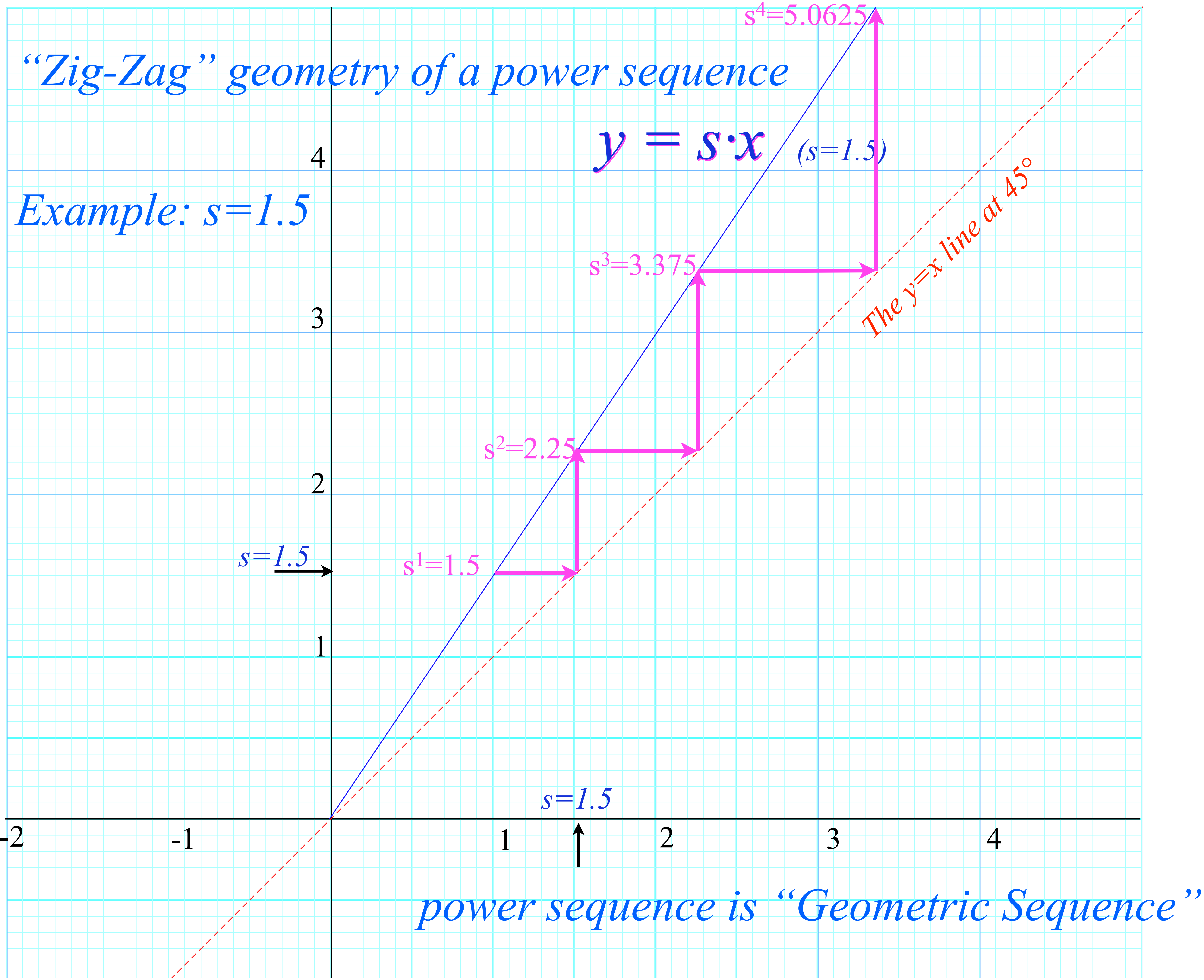
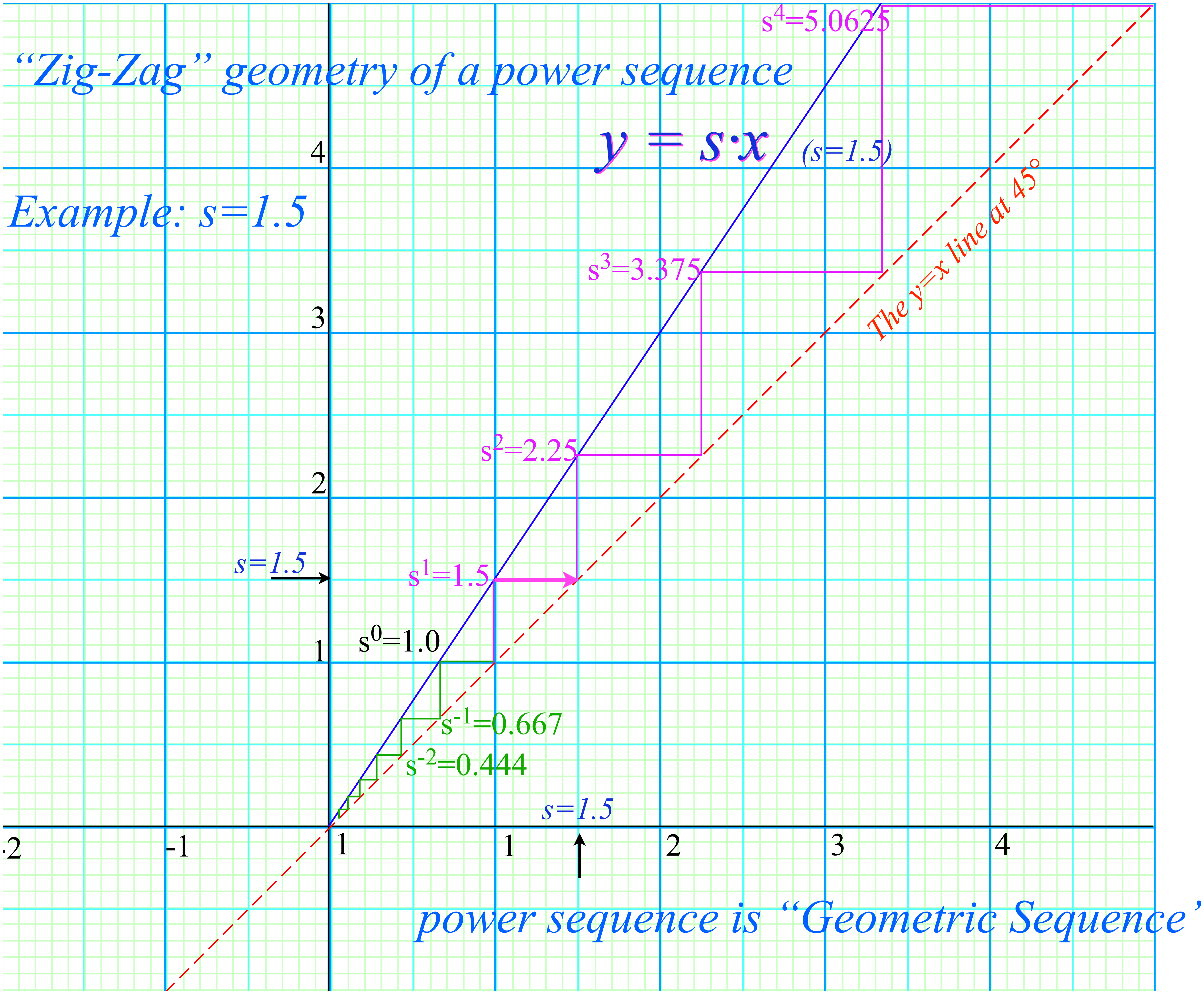
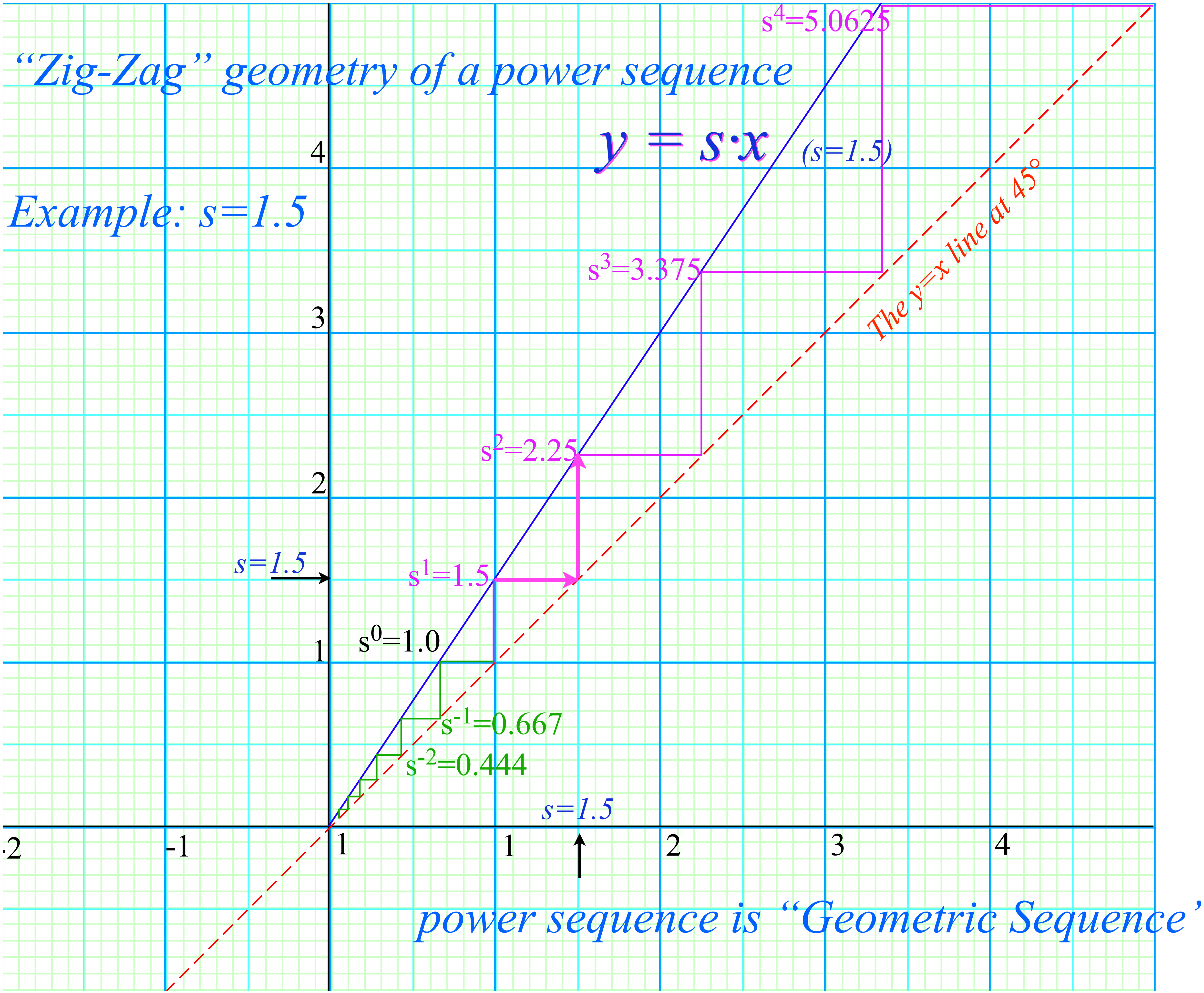
-2
-1
1
2
3
4



“Zig-Zag” geometry of a power sequence
y = s·x
(s=1.5)
Example: s=1.5
s⁴=5.0625
s³=3.375
s²=2.25
s¹=1.5
s=1.5
The y=x line at 45°
-2
-1
1
2
3
4
s=1.5
power sequence is “Geometric Sequence”



“Zig-Zag” geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
s^-1=0.667
s^-2=0.444
s=1.5
s=1.5
The y=x line at 45°
4
3
2
1
-2
-1
1
1
2
3
4
power sequence is “Geometric Sequence”



"Zig-Zag" geometry of a power sequence
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4
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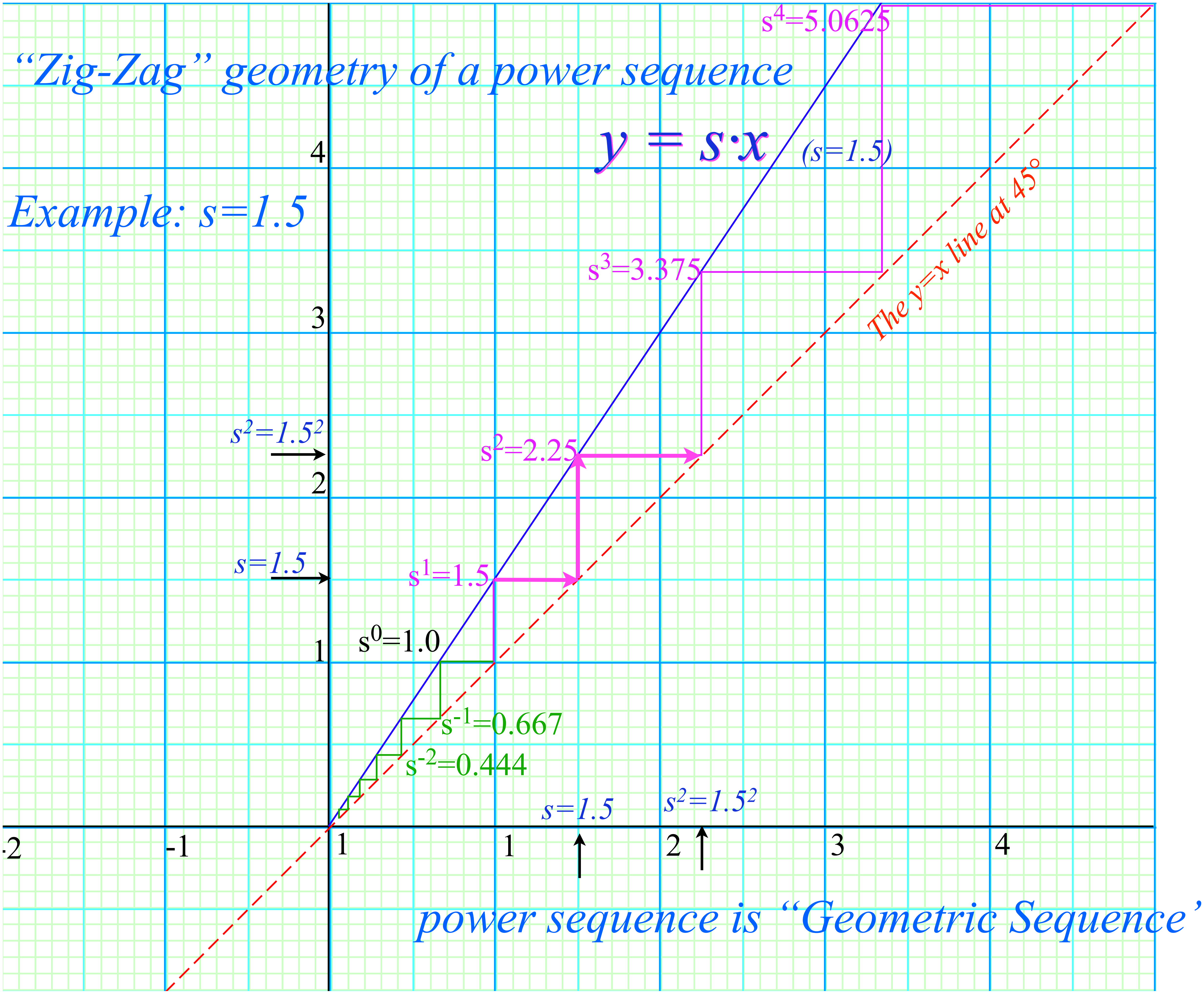
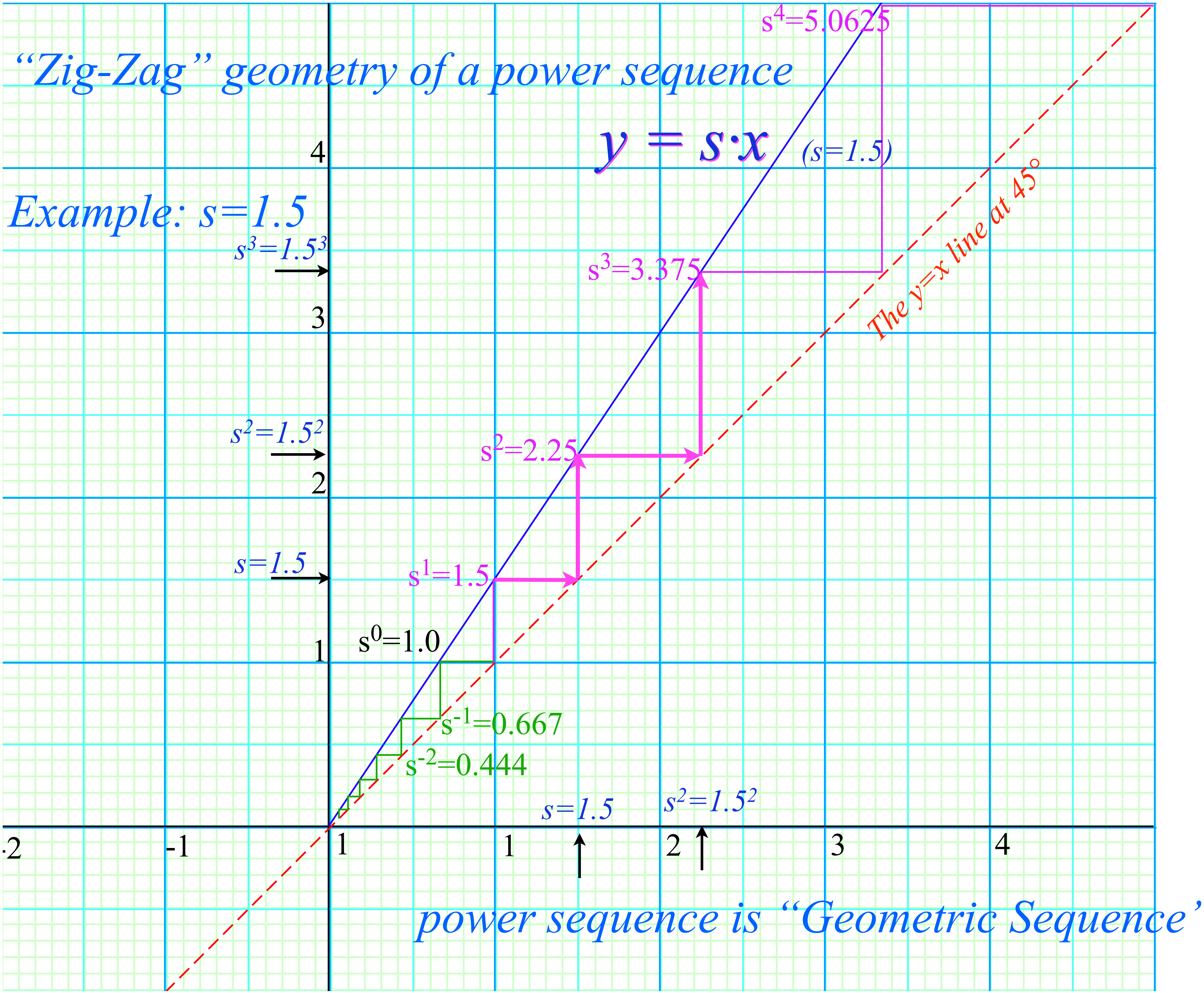
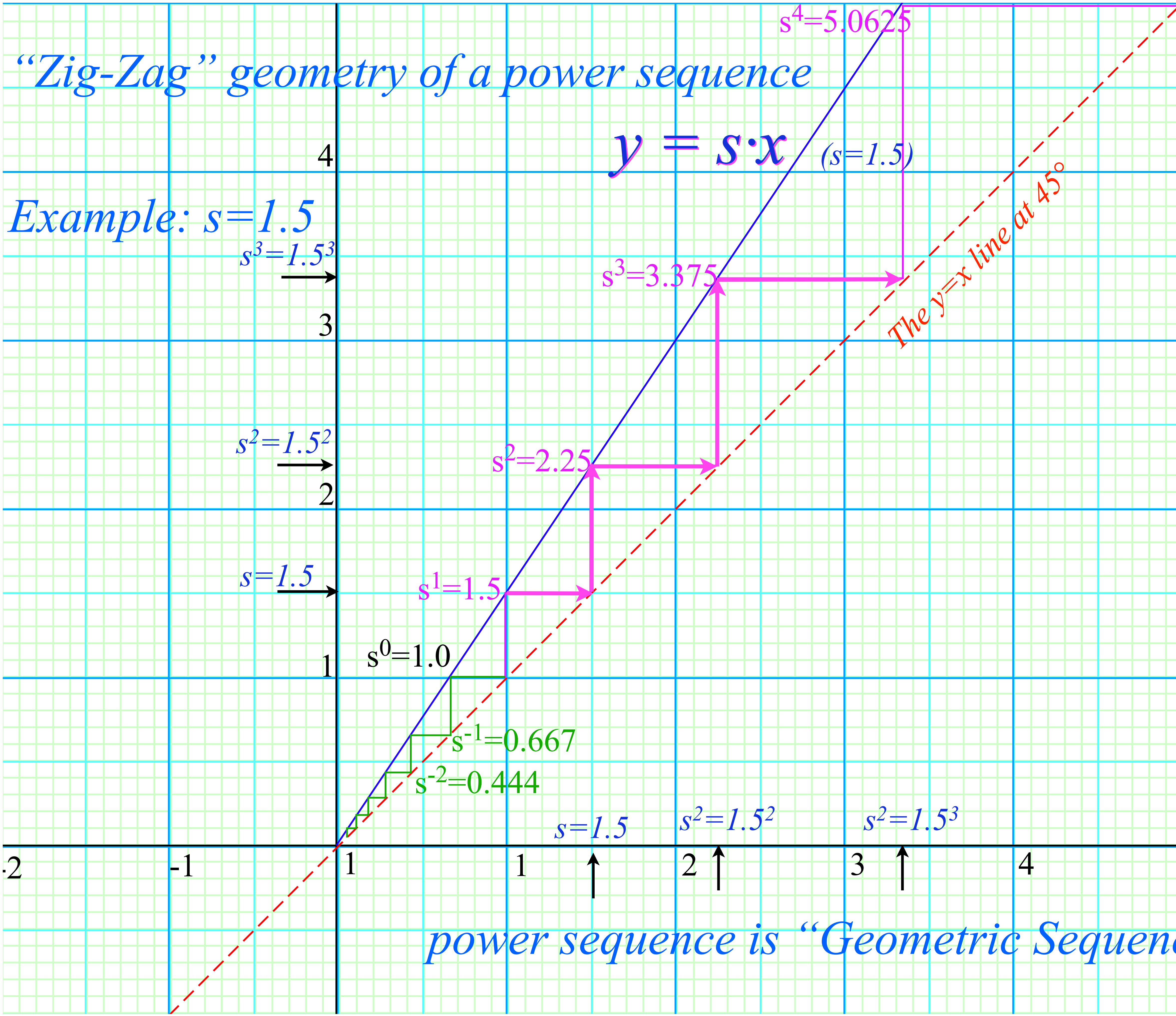

"Zig-Zag" geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
s⁴=5.0625
s³=3.375
s²=2.25
s¹=1.5
s⁰=1.0
s⁻¹=0.667
s⁻²=0.444
s²=1.5²
s=1.5
s=1.5
s²=1.5²
The y=x line at 45°
4
3
2
1
-2
-1
1
1
2
3
4
power sequence is "Geometric Sequence"

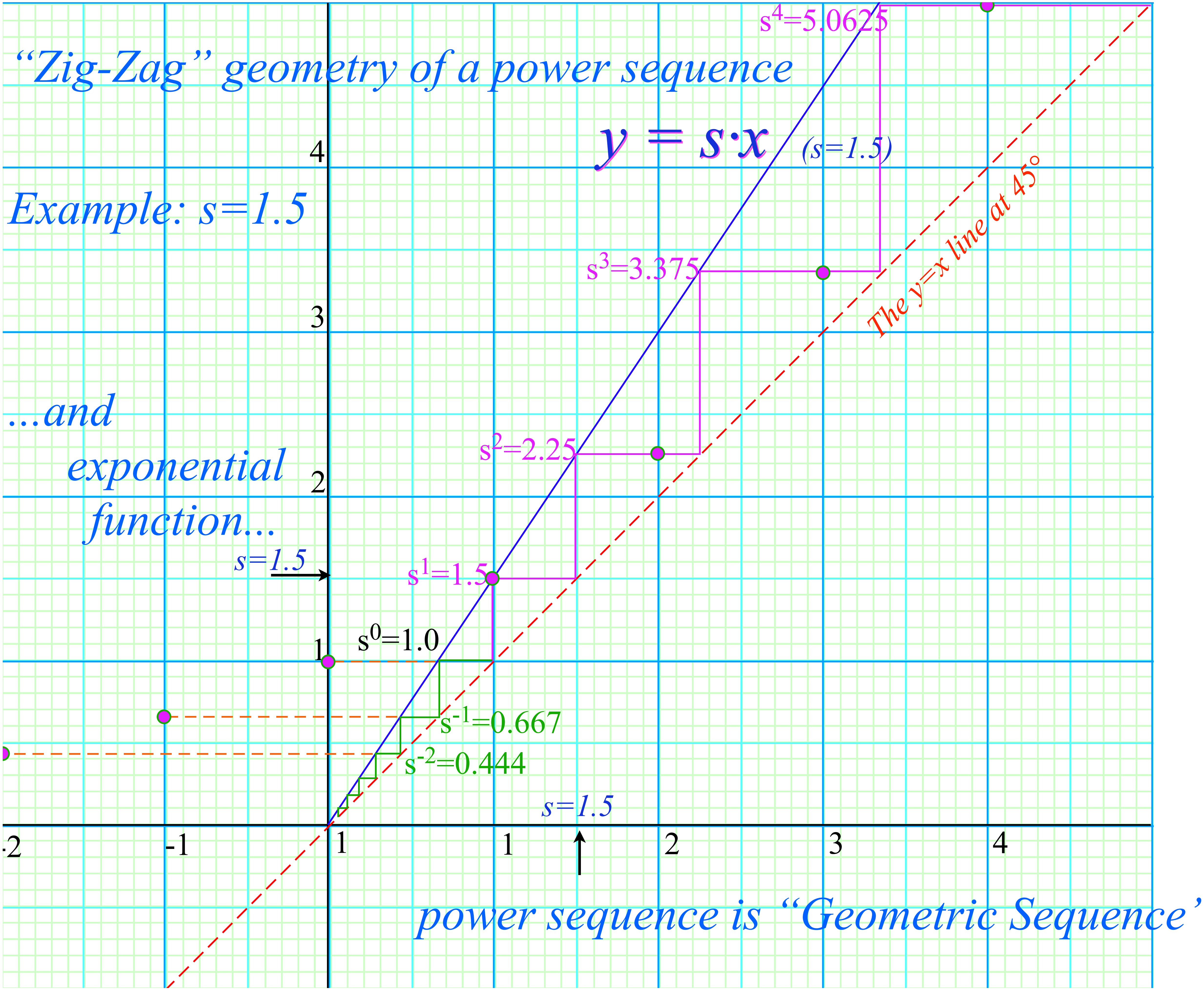


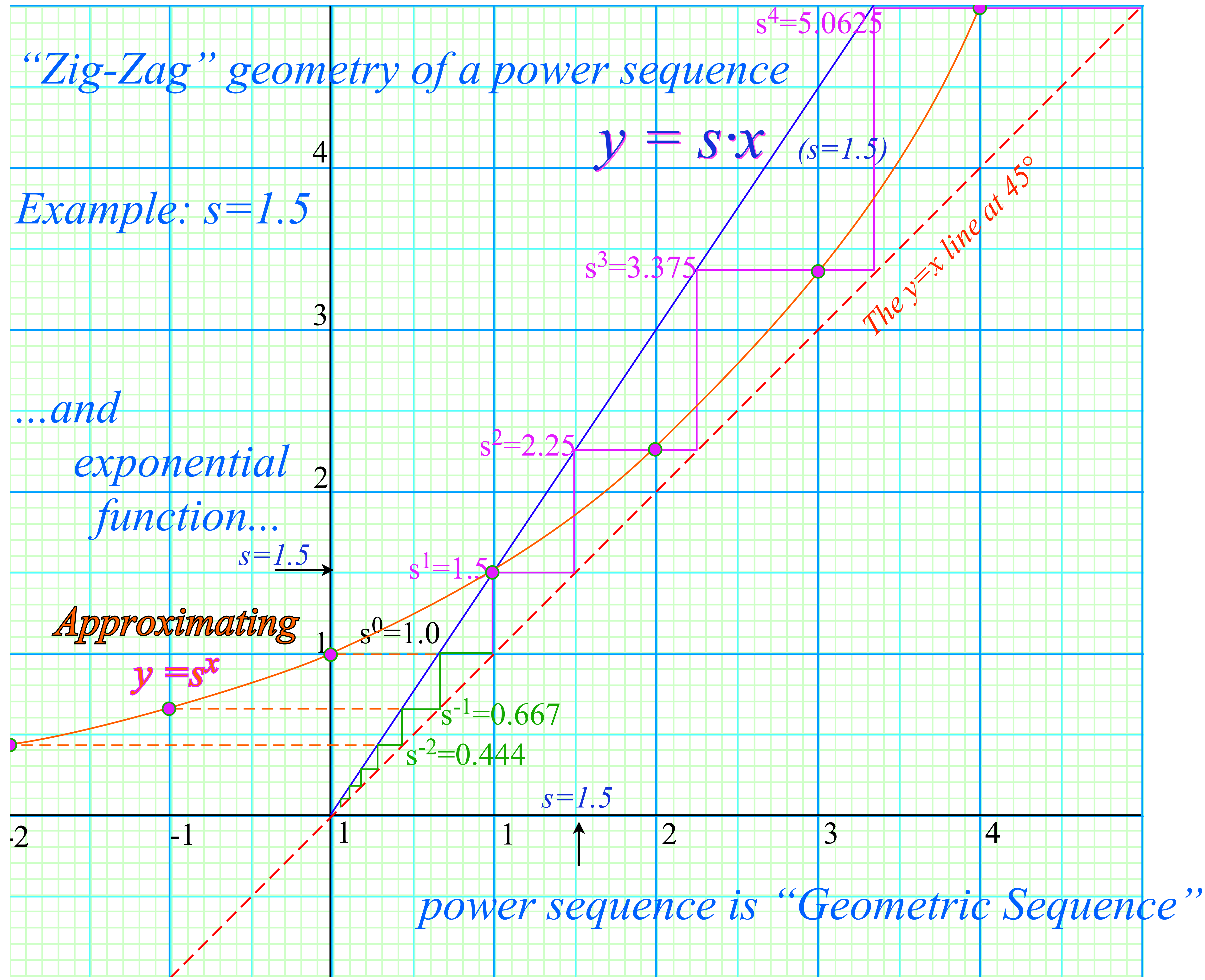
“Zig-Zag” geometry of a power sequence
y = s·x
(s=1.5)
Example: s=1.5
s⁴=5.0625
s³=1.5³
s³=3.375
s²=1.5²
s²=2.25
s=1.5
s¹=1.5
s⁰=1.0
s⁻¹=0.667
s⁻²=0.444
s=1.5
s²=1.5²
The y=x line at 45°
power sequence is “Geometric Sequence”

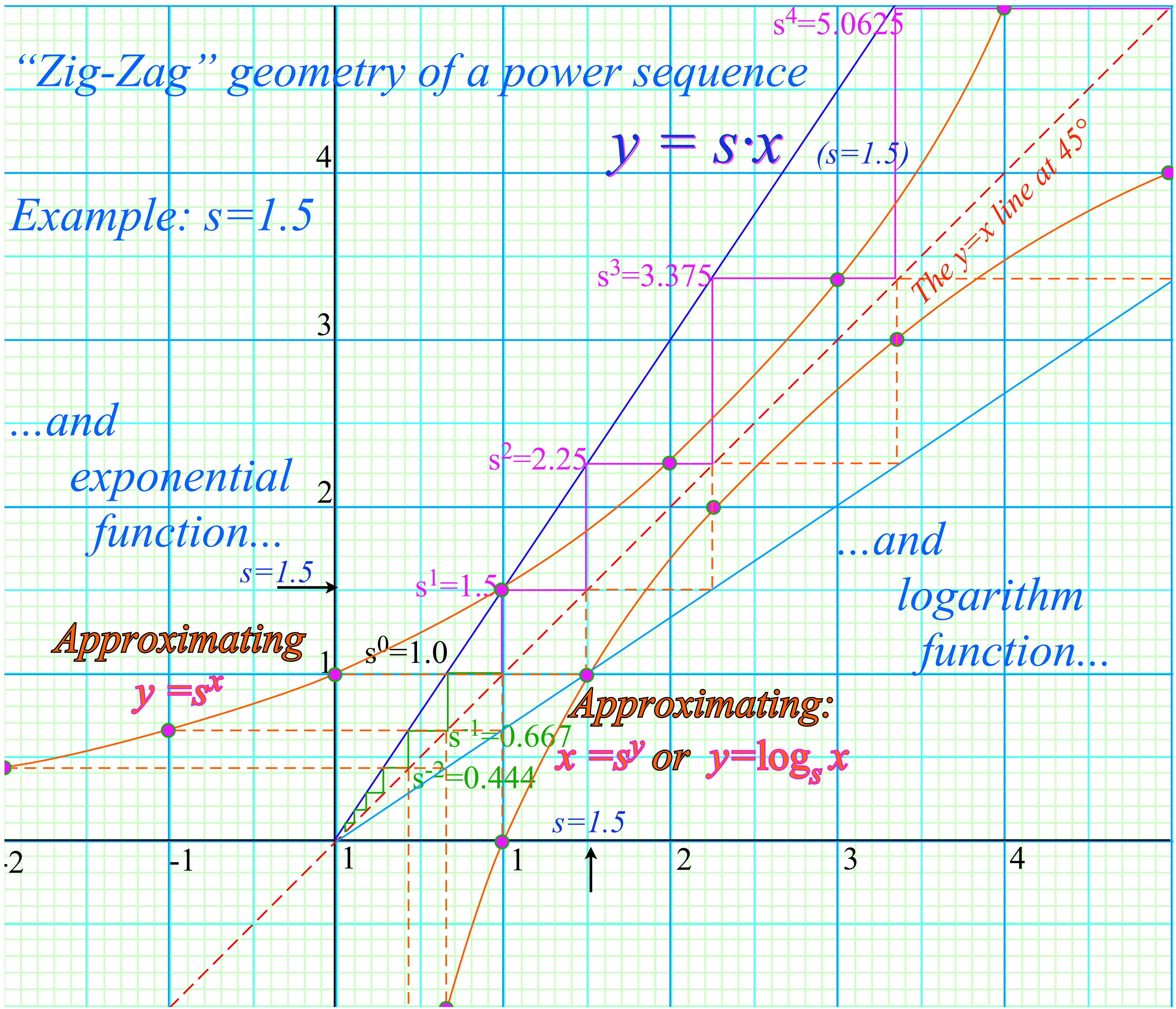
"Zig-Zag" geometry of a power sequence



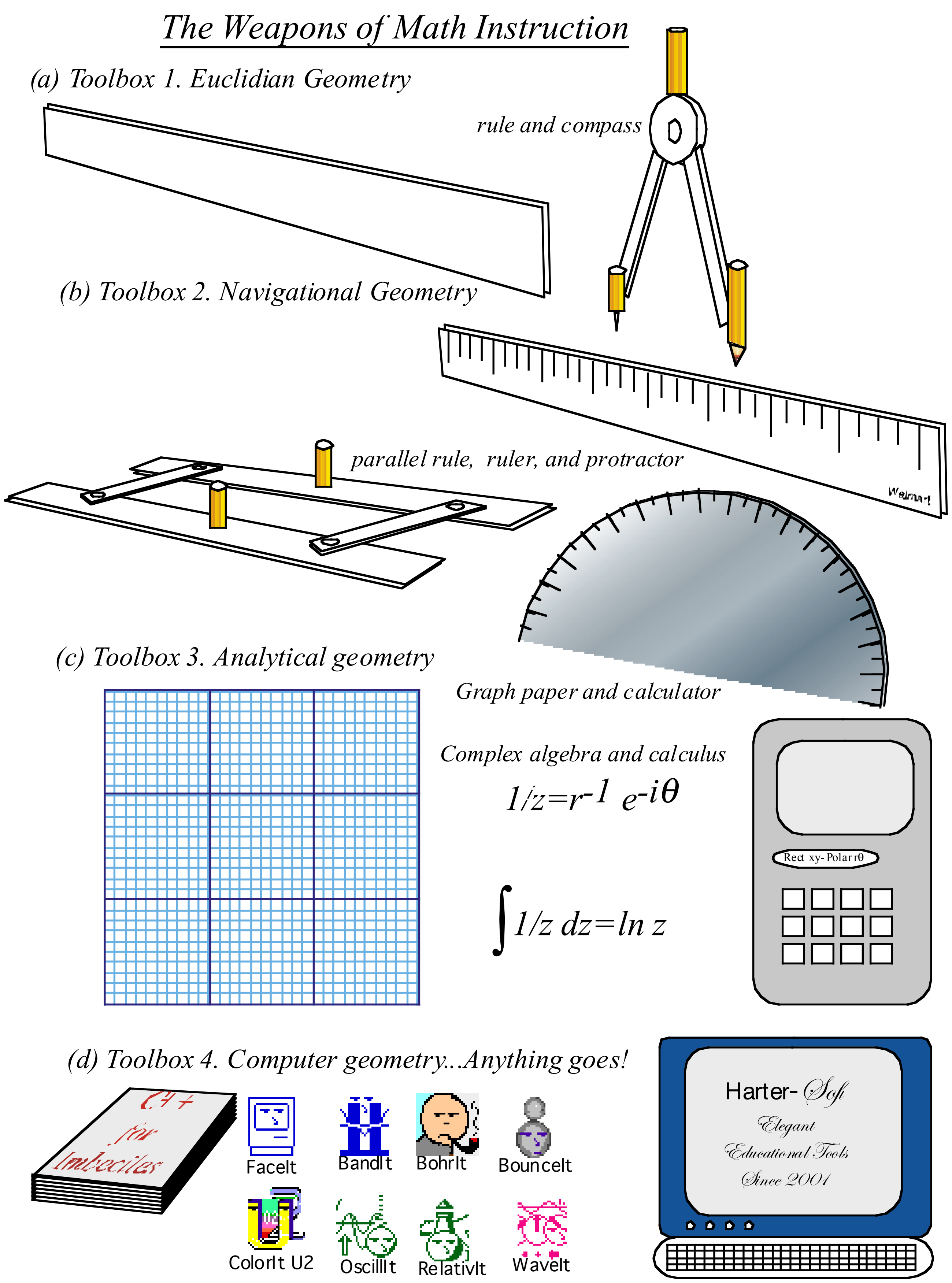
power sequence is "Geometric Sequence"


"Zig-Zag" geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
...and
exponential
function...
s=1.5
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
s^-1=0.667
s^-2=0.444
s=1.5
The y=x line at 45°
-2
-1
1
1
2
3
4
1
2
3
4
power sequence is "Geometric Sequence"


“Zig-Zag” geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
...and
exponential
function...
Approximating
y =s^x
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
s^-1=0.667
s^-2=0.444
s=1.5
s=1.5
The y=x line at 45°
4
3
2
1
-2
-1
1
1
2
3
4
power sequence is “Geometric Sequence”



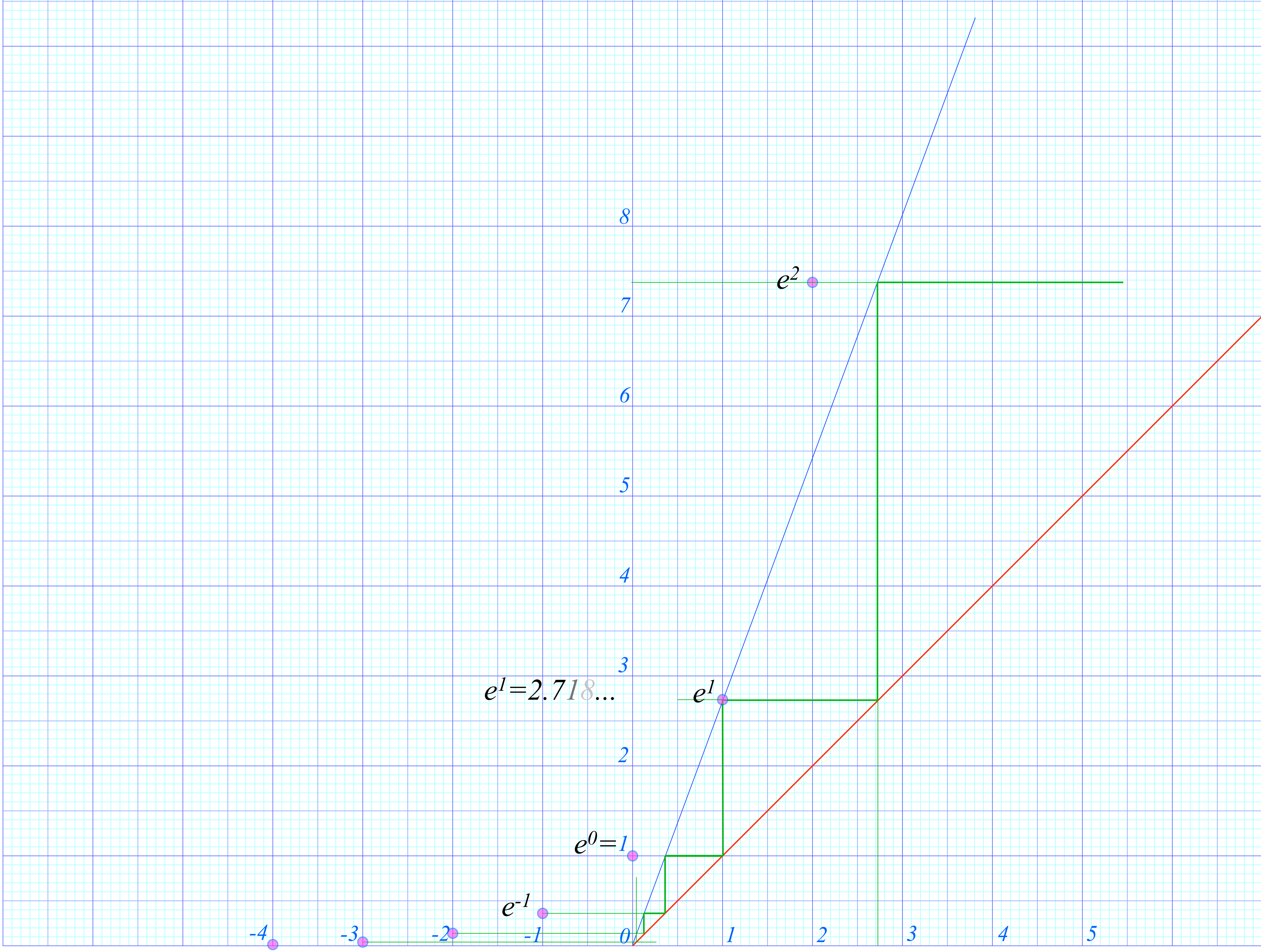
"Zig-Zag" geometry of a power sequence
Example: s=1.5
y = s·x (s=1.5)
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
s^-1=0.667
s^-2=0.444
The y=x line at 45°
...and exponential function...
...and logarithm function...
Approximating y = s^x
Approximating: x = s^y or y=log_s x
s=1.5
s=1.5
4
3
2
1
-2
-1
1
1
2
3
4

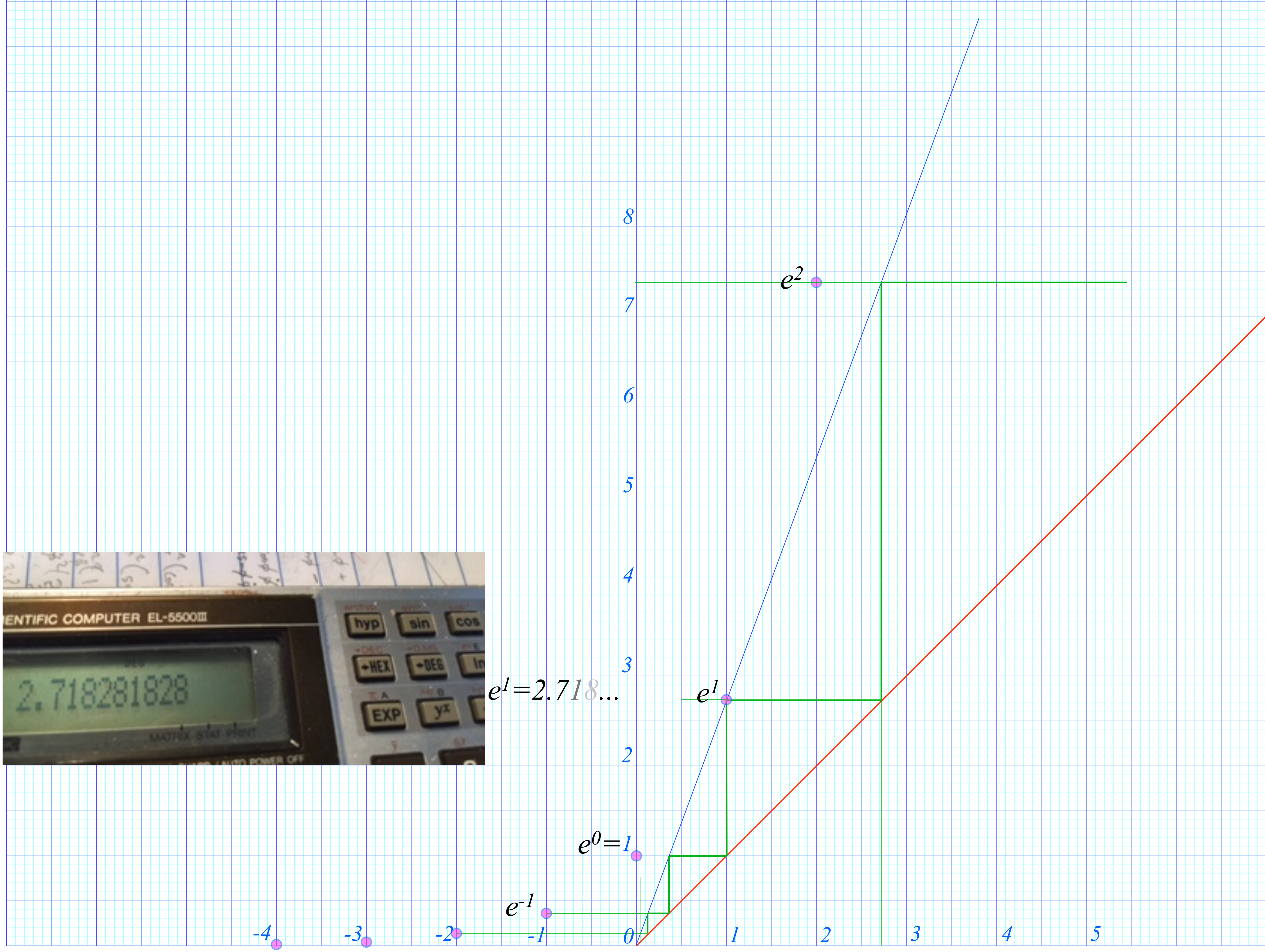


So far we mostly use Toolbox (a-b)

What follows uses Toolbox (c) ...

...and Toolbox (d)


8
7
6
5
4
3
2
e^2
e^1=2.718...
e^1
e^0=1
e^-1
-4
-3
-2
-1
0
1
2
3
4
5


e^2
e^1=2.718...
e^1
e^0=1
e^-1
-4 -3 -2 -1 0 1 2 3 4 5
1 2 3 4 5 6 7 8
ENTIFIC COMPUTER EL-5500III
2.718281828
hyp sin cos
→HEX →DEG
EXP yx
MATRIX STAT PRINT

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

→ *Projective or perspective geometry*

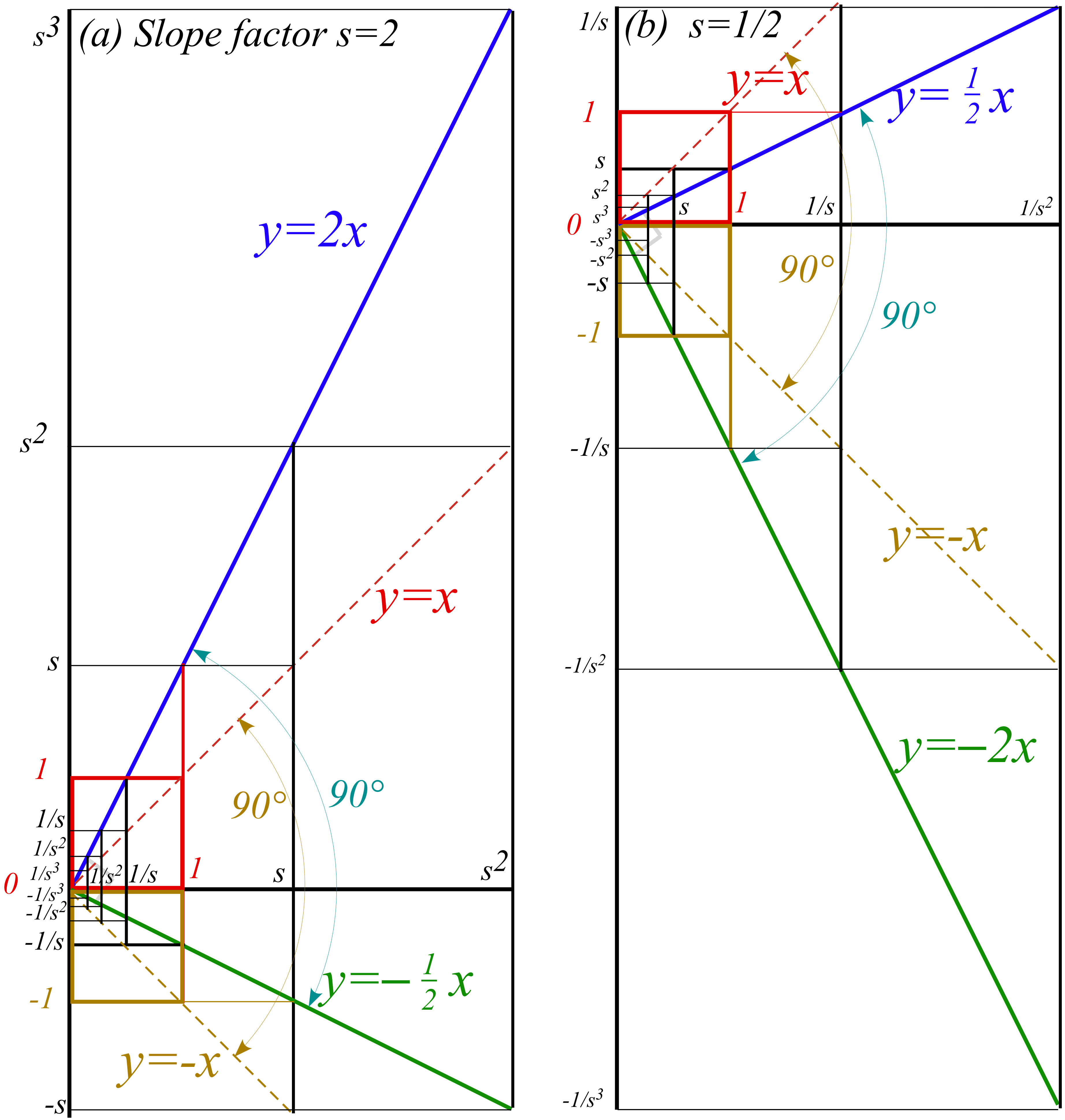
Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

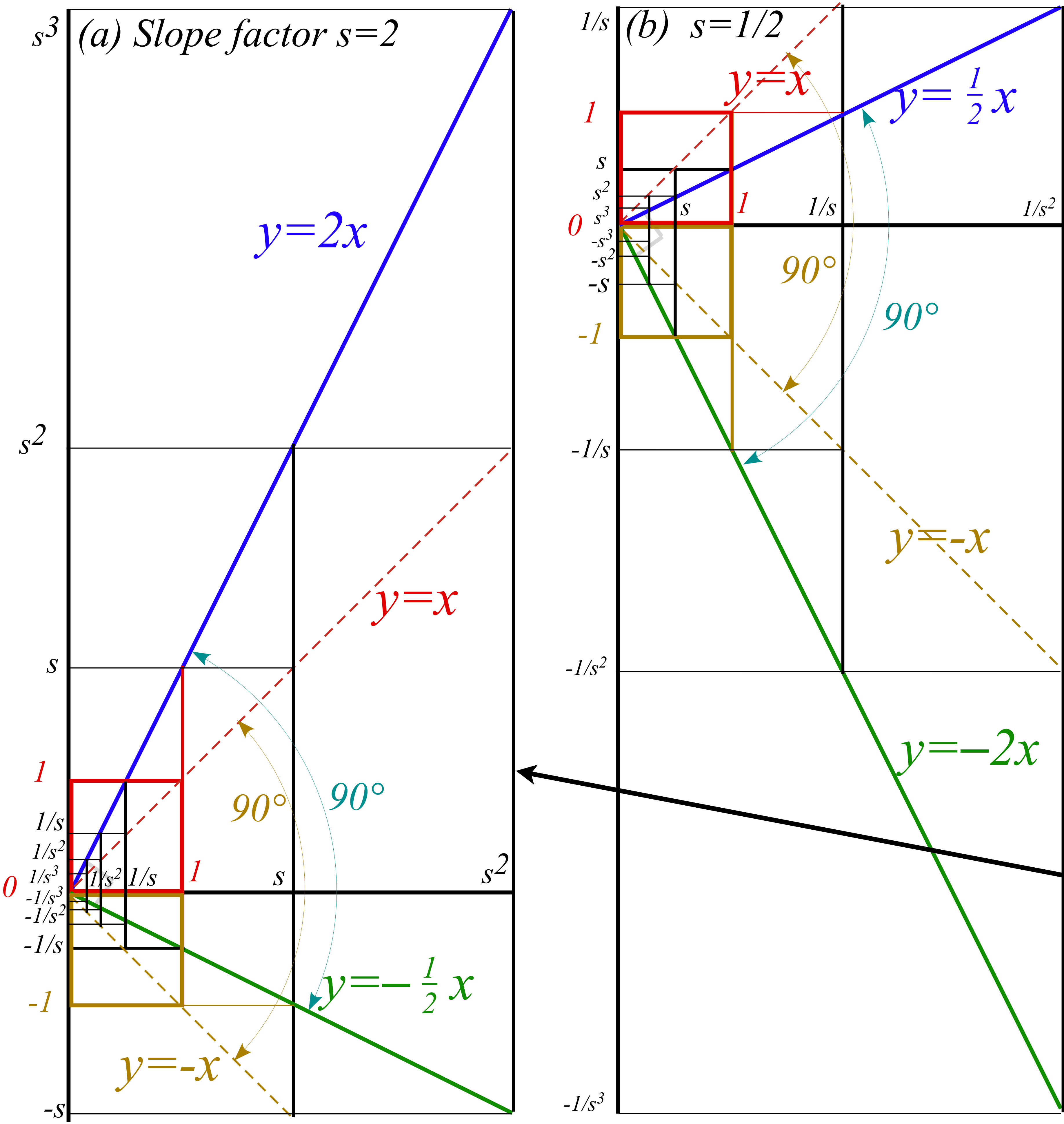
Compare mks units of Coulomb Electrostatic vs. Gravity

"Zig-Zags" give perspective geometry

(1D-vanishing point)



Unit 1
Fig. 9.2



"Zig-Zags" give perspective geometry

(1D-vanishing point)

Unit 1
Fig. 9.2

1st-day-of-school perspective of 12th-grader

1st-day-of-school perspective of 1st-grader

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

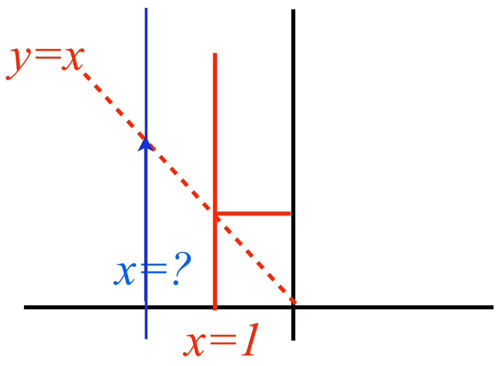
→ *Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

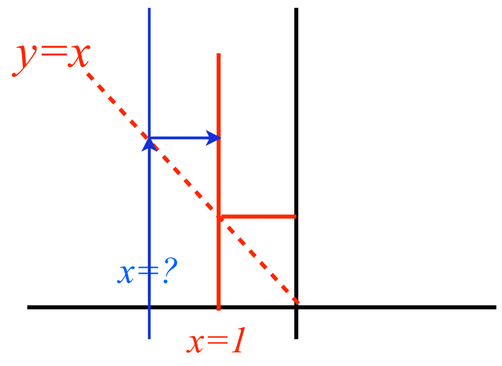
Compare mks units of Coulomb Electrostatic vs. Gravity

Each $y=x^2$ parabola point found by just one "Zig-Zag"

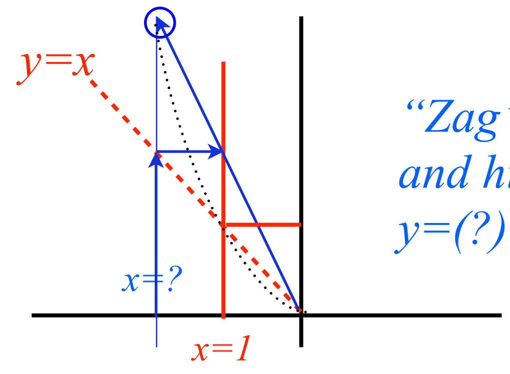
1. Pick an (x=?)-line



2. "Zig" from its y=x intersection to x=1 line



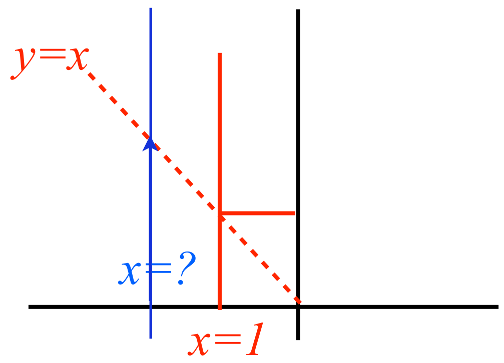
3. "Zag" from origin back to (x=?)-line



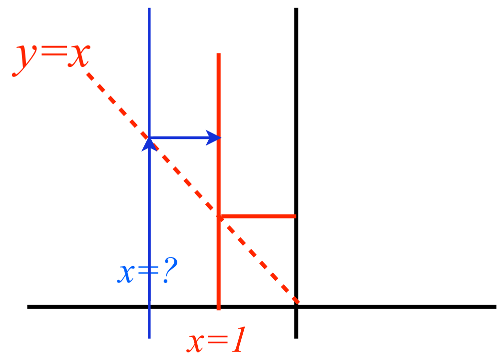
"Zag" line is y=(?)·x and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$

Each $y=x^2$ parabola point found by just one "Zig-Zag"

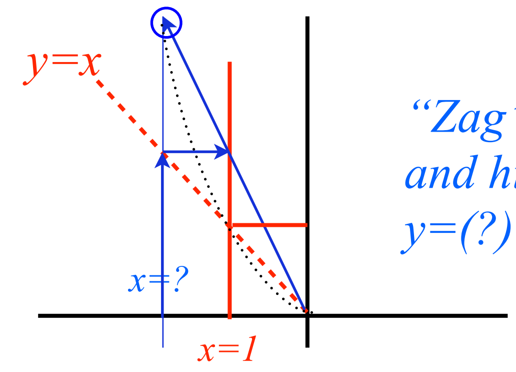
1. Pick an (x=?)-line



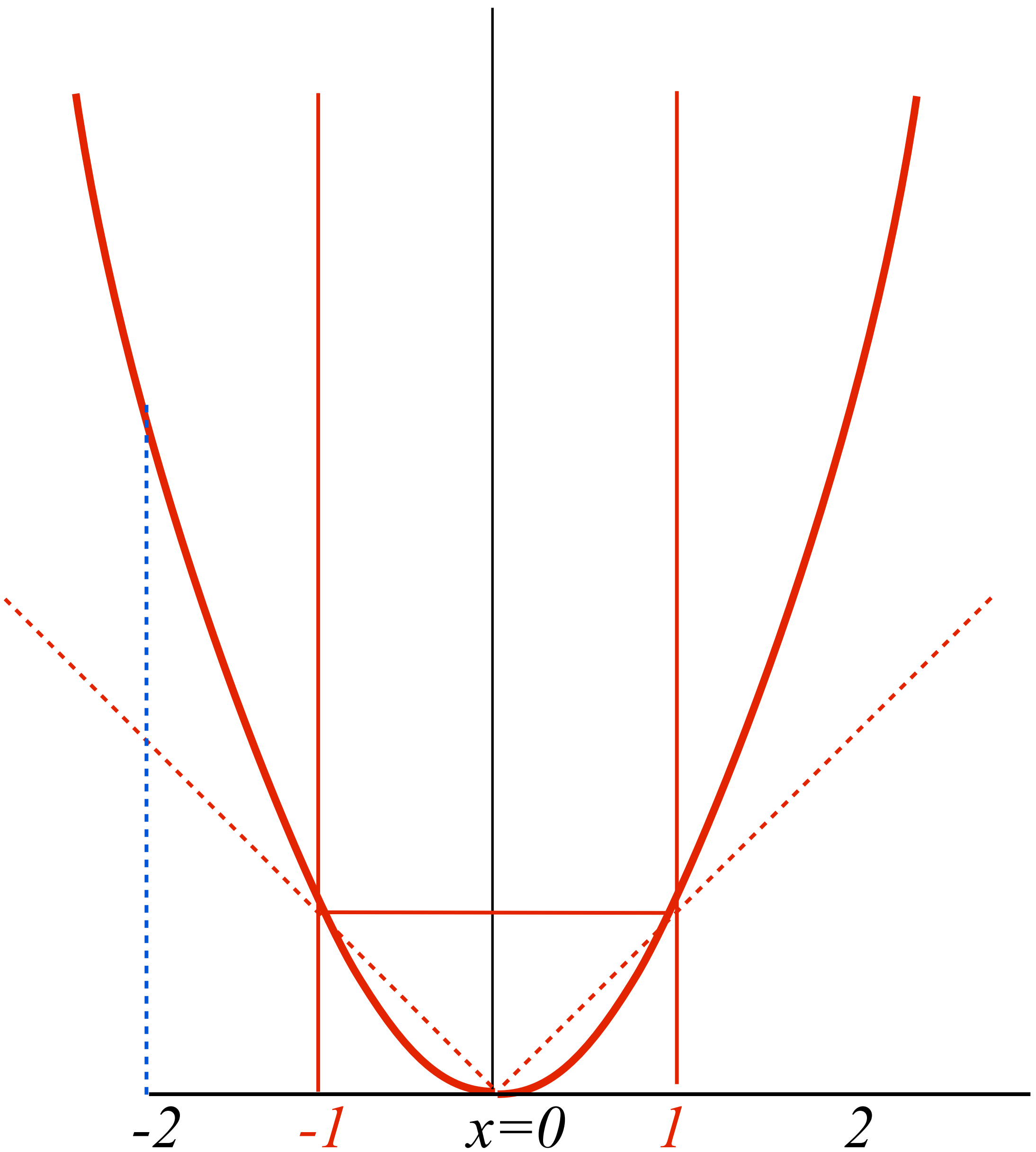
2. "Zig" from its y=x intersection to x=1 line



3. "Zag" from origin back to (x=?)-line



"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$



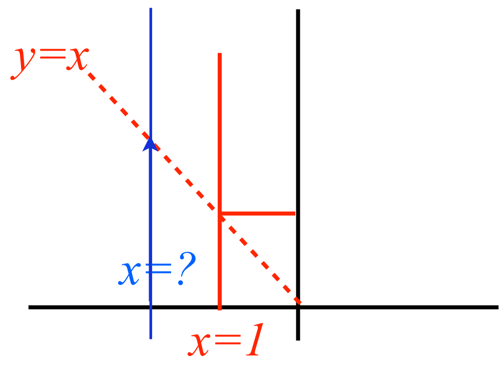
Unit 1
Fig. 9.1

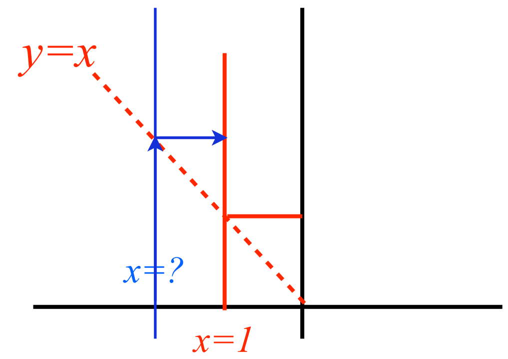
Each $y=x^2$ parabola point found by just one "Zig-Zag"

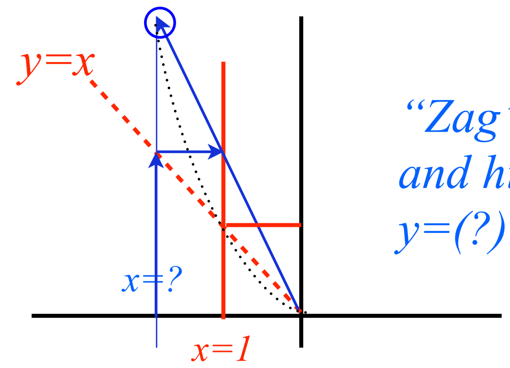
1. Pick an (x=?)-line

2. "Zig" from its y=x intersection to x=1 line

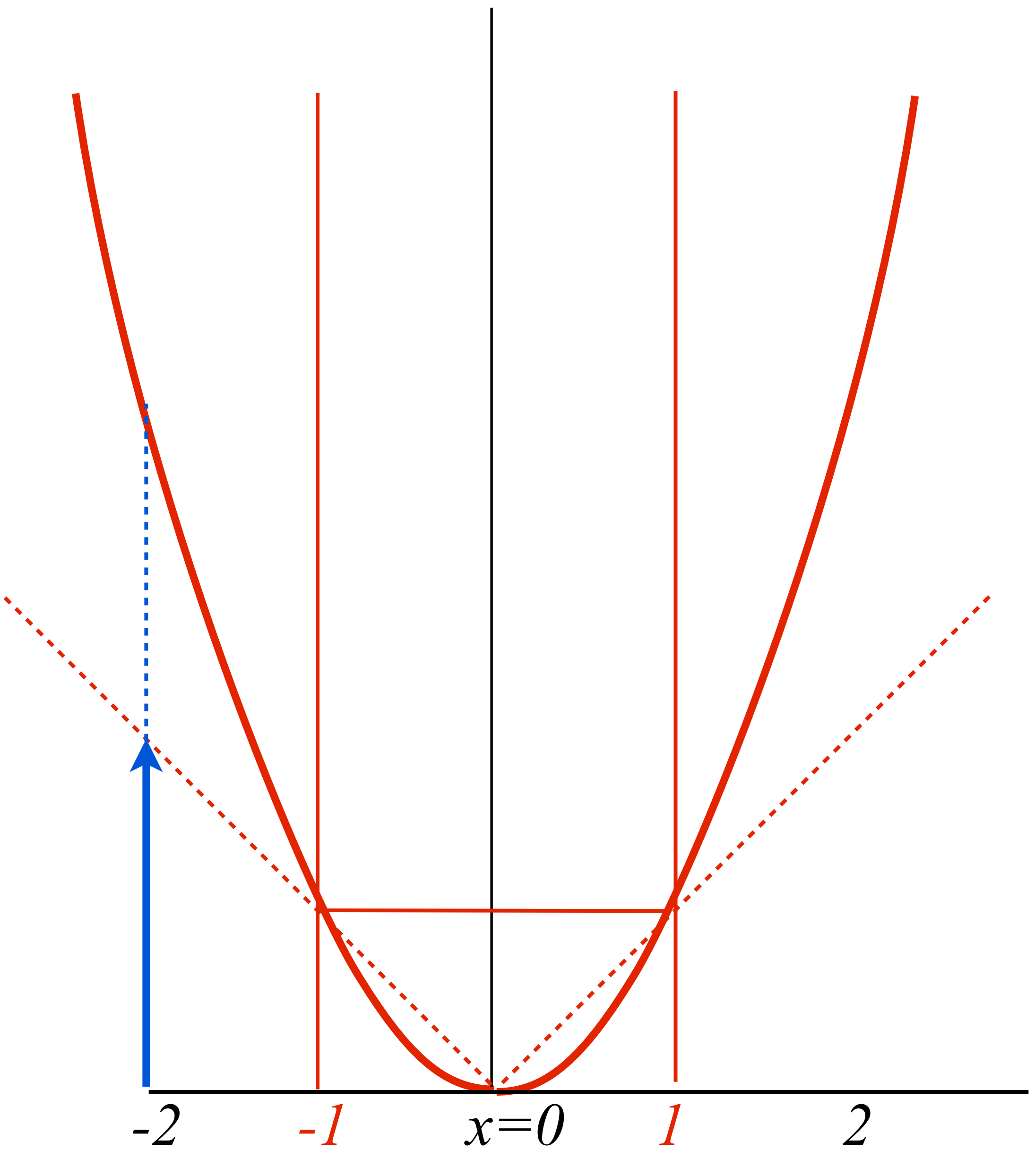
3. "Zag" from origin back to (x=?)-line







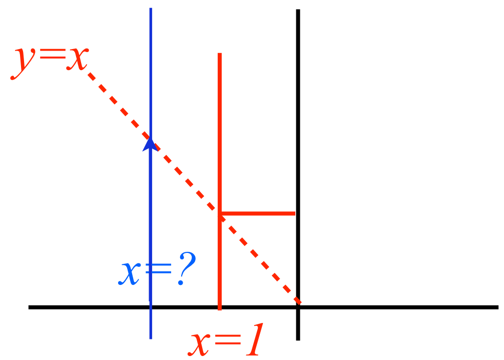
"Zag" line is y=(?)·x and hits (x=?)-line at y=(?)·(?)=(?)²



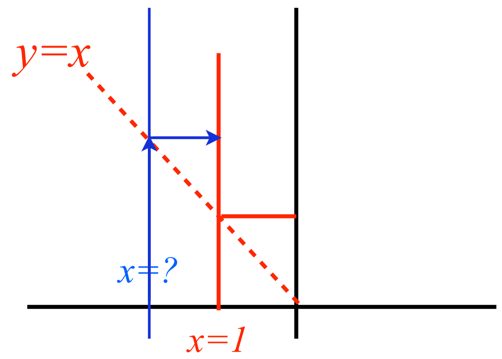
Unit 1
Fig. 9.1

Each $y=x^2$ parabola point found by just one "Zig-Zag"

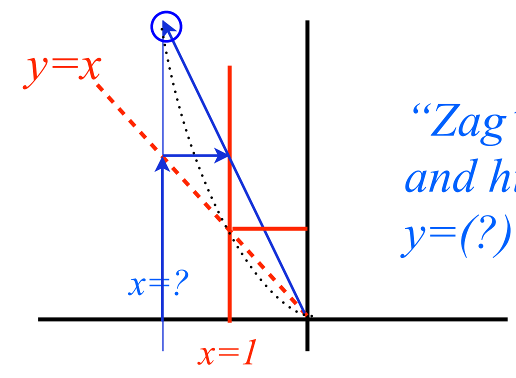
1. Pick an (x=?)-line



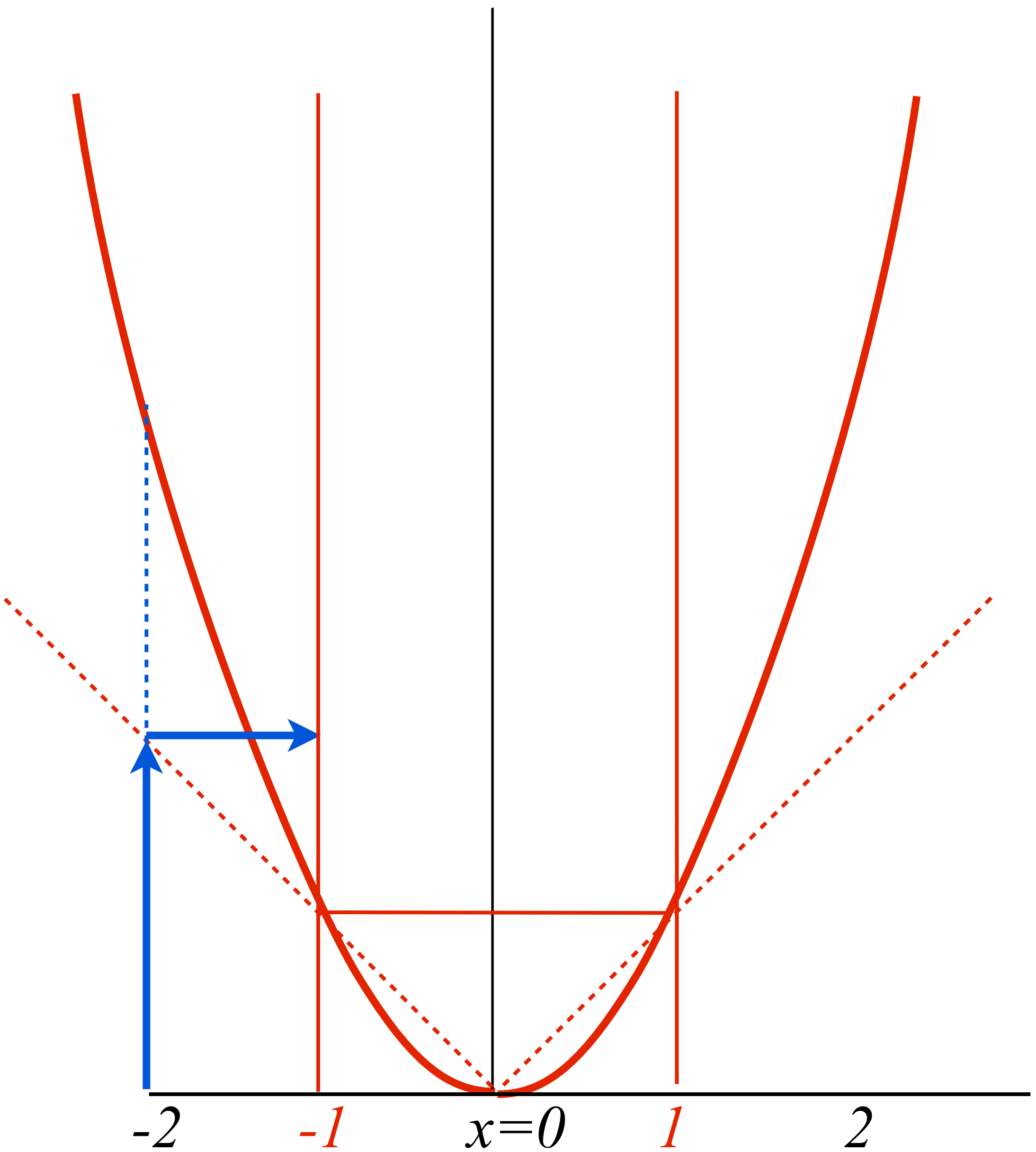
2. "Zig" from its y=x intersection to x=1 line



3. "Zag" from origin back to (x=?)-line



"Zag" line is y=(?)·x and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$



Unit 1
Fig. 9.1

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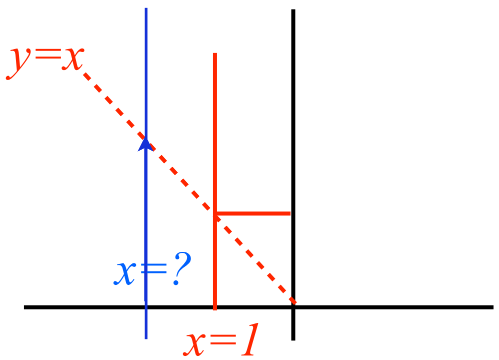

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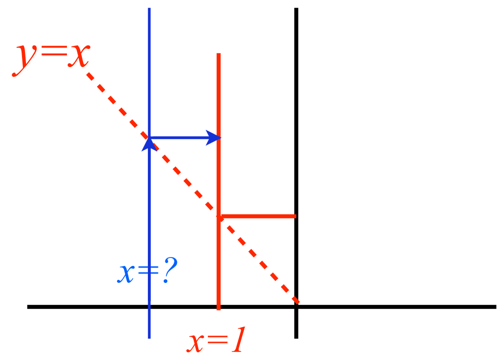

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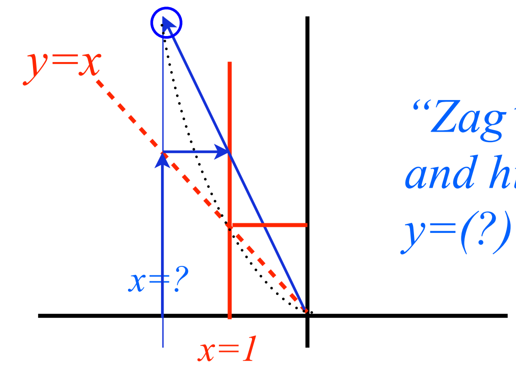
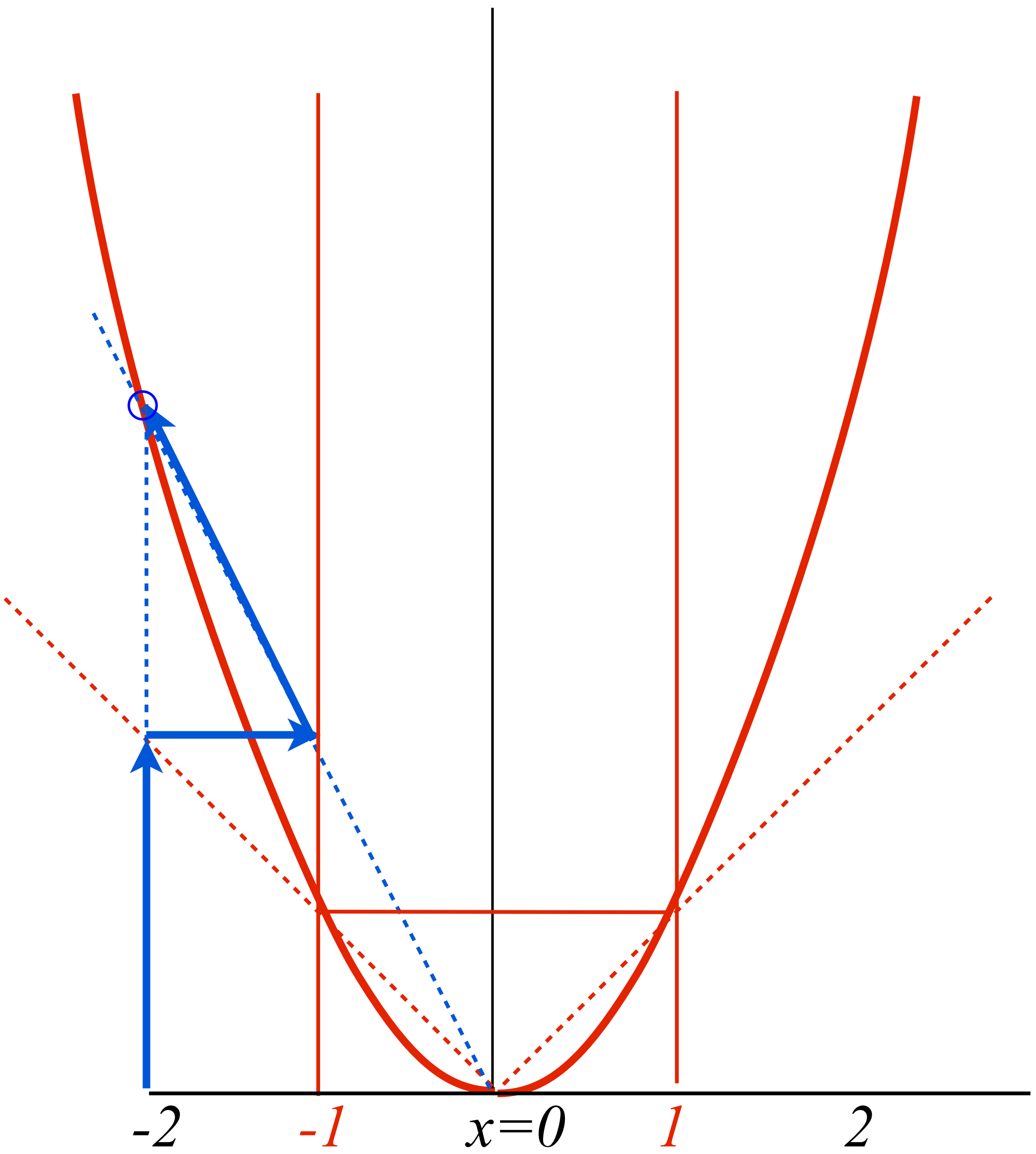


"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$



Unit 1
Fig. 9.1

Each $y=x^2$ parabola point found by just one "Zig-Zag"

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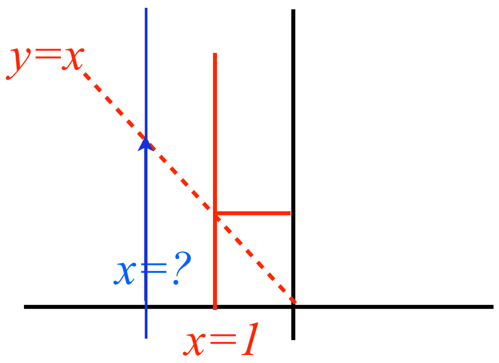
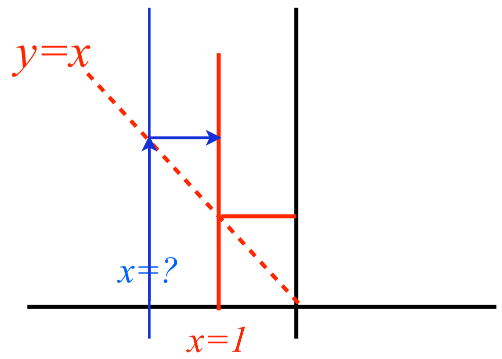
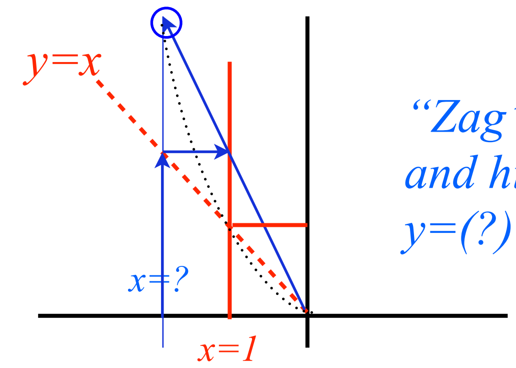


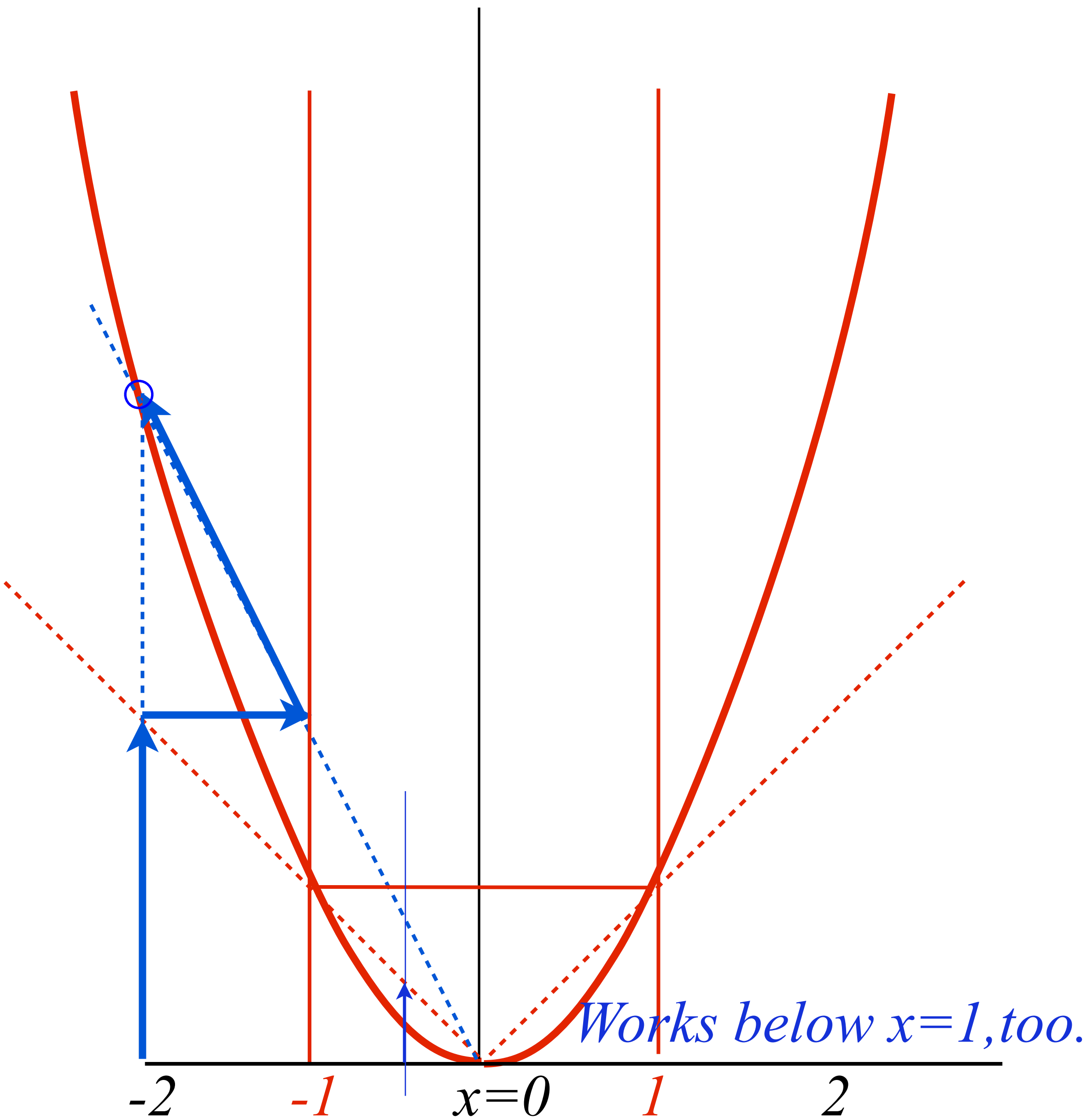
2. "Zig" from its y=x intersection to x=1 line



3. "Zag" from origin back to (x=?)-line

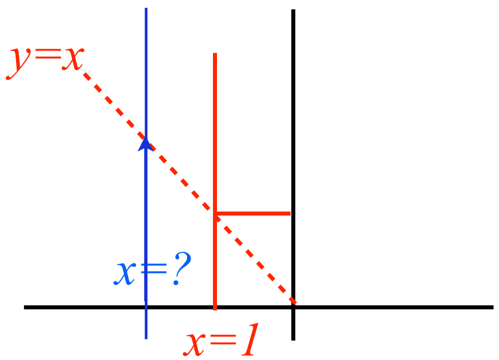


"Zag" line is y=(?)·x and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$

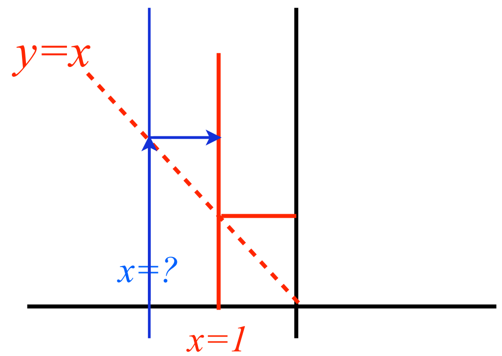


Each $y=x^2$ parabola point found by just one "Zig-Zag"

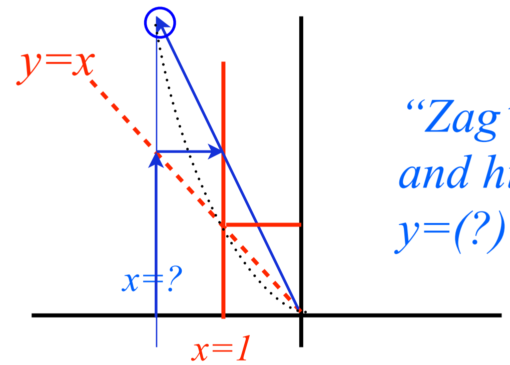
1. Pick an (x=?)-line



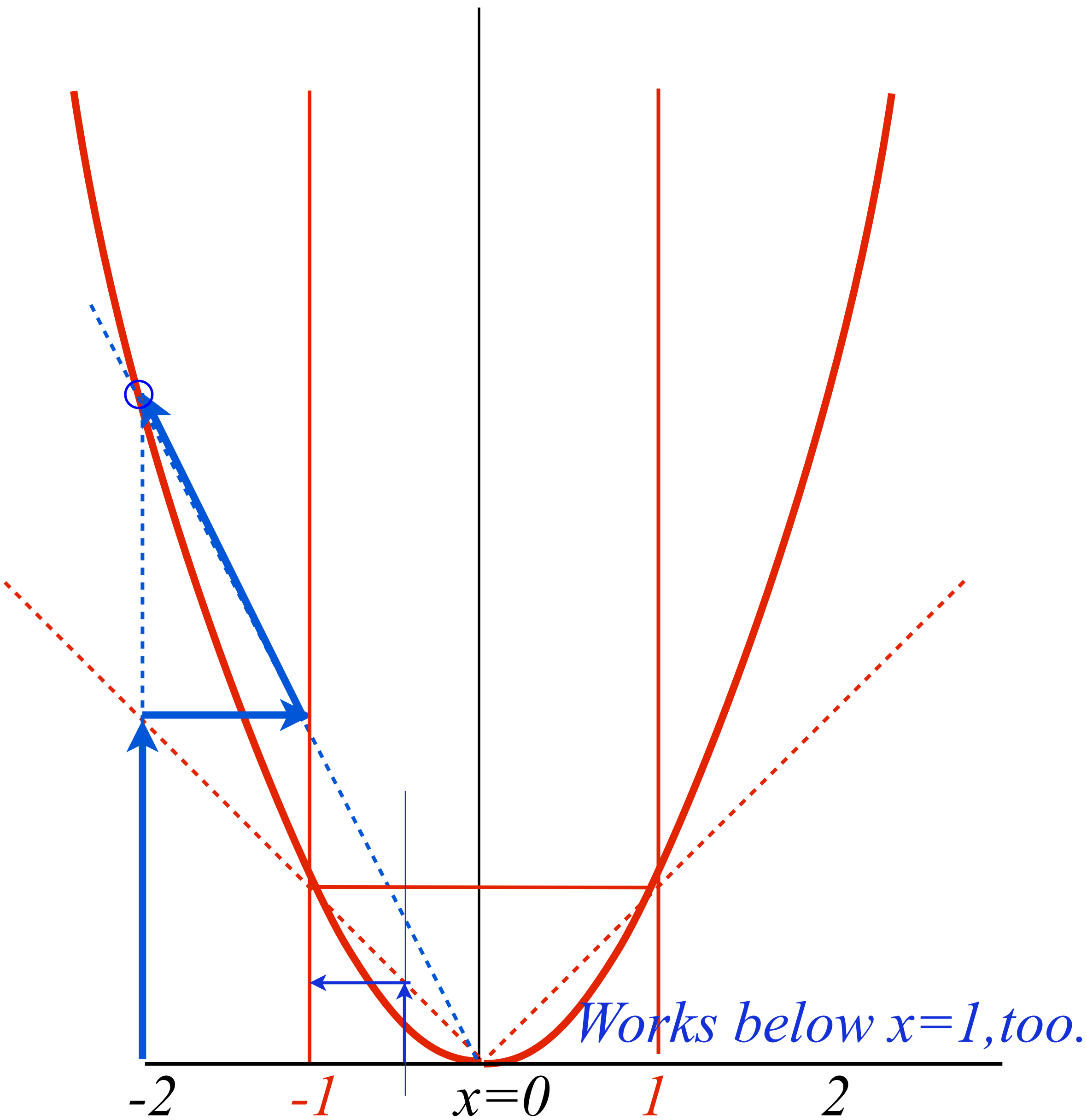
2. "Zig" from its y=x intersection to x=1 line



3. "Zag" from origin back to (x=?)-line

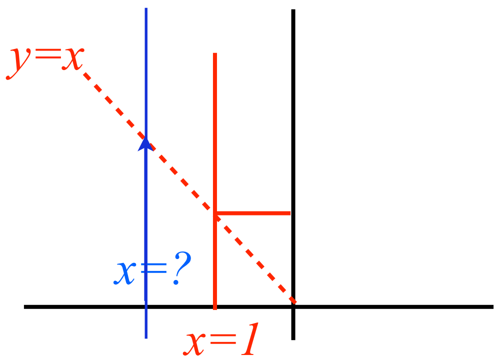


"Zag" line is y=(?)·x and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$

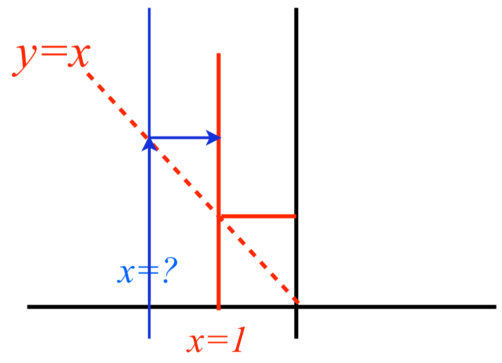


Each $y=x^2$ parabola point found by just one "Zig-Zag"

1. Pick an (x=?)-line



2. "Zig" from its y=x intersection to x=1 line



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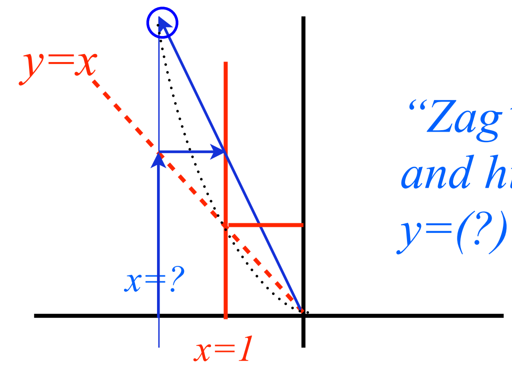

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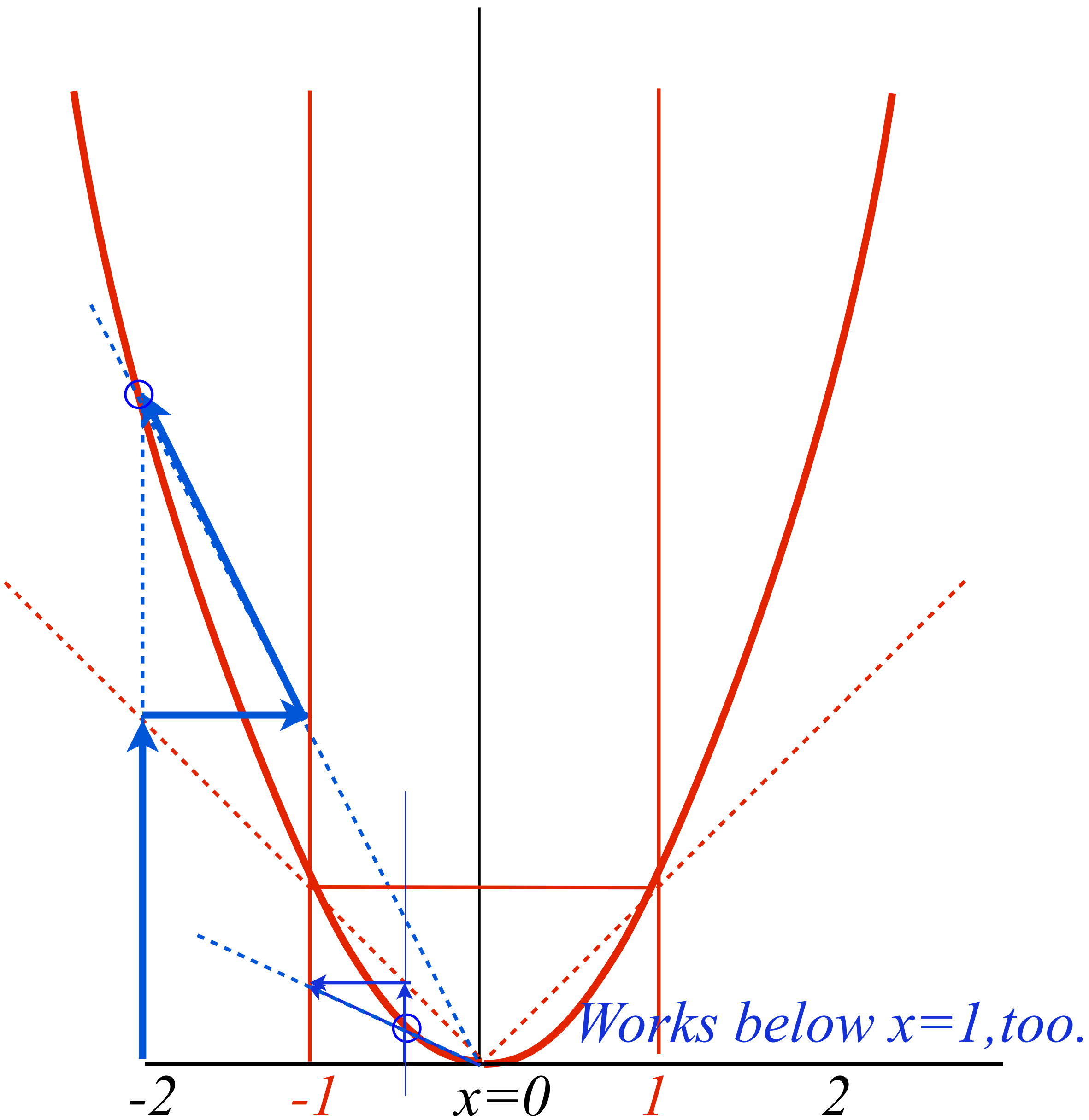
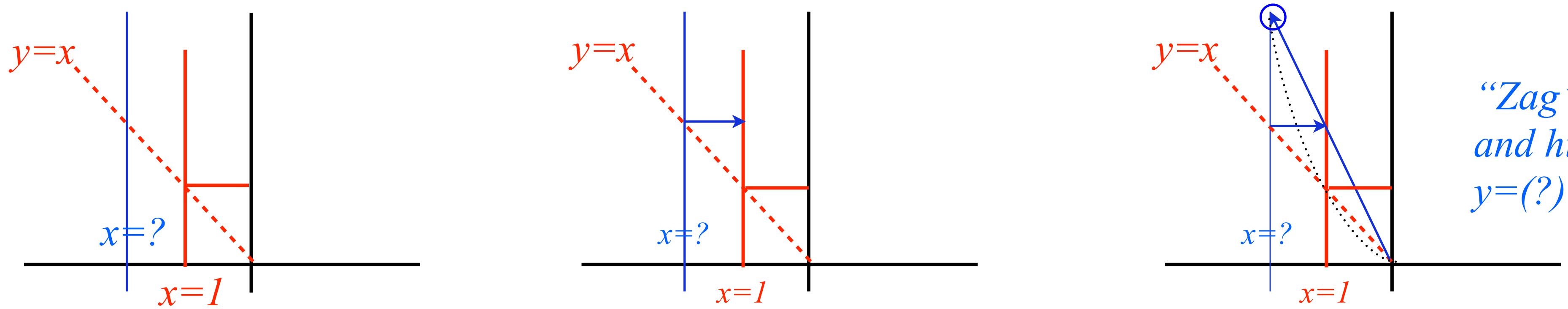


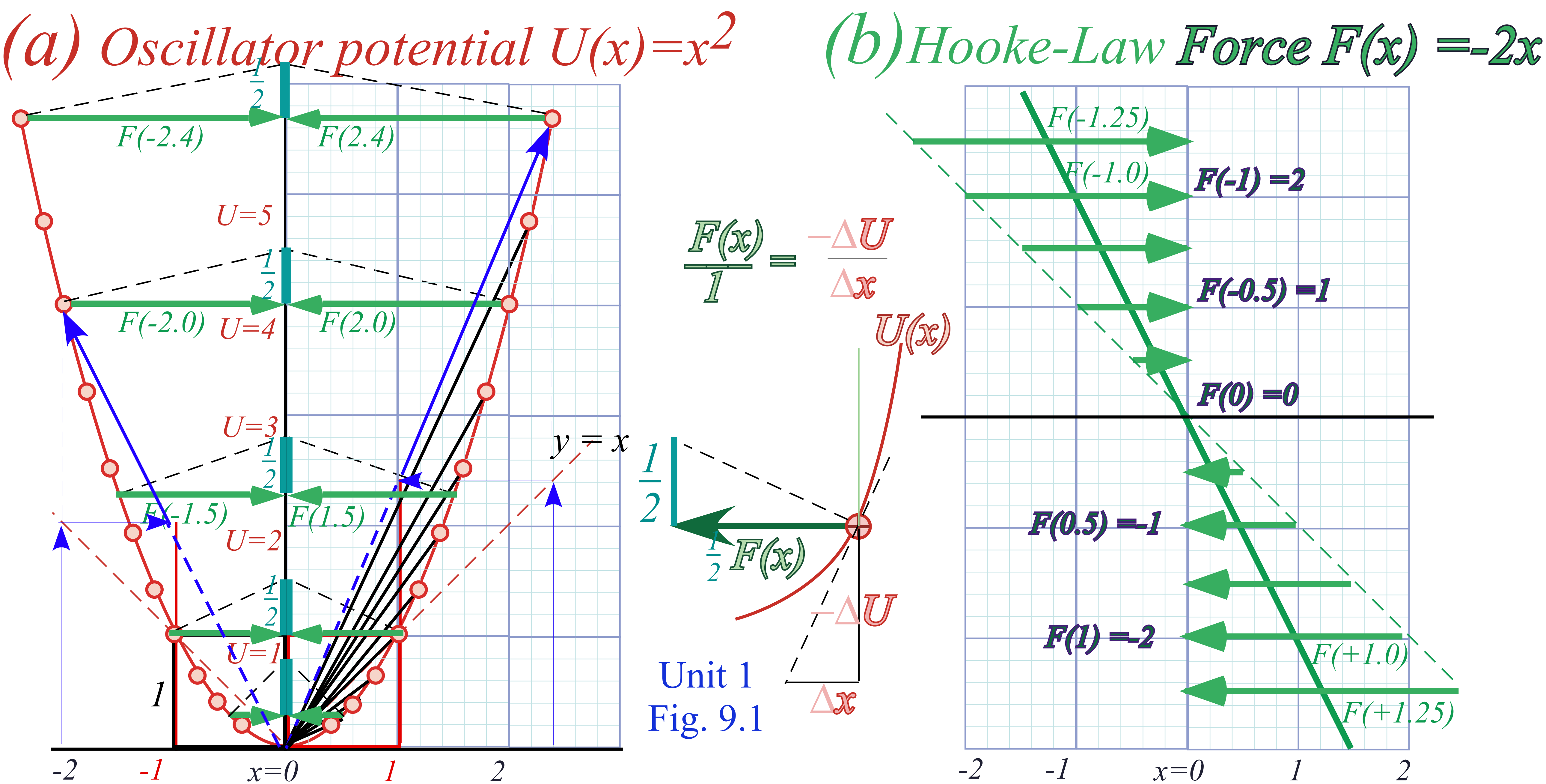
Unit 1
Fig. 9.1

Each $y=x^2$ parabola point found by just one "Zig-Zag"

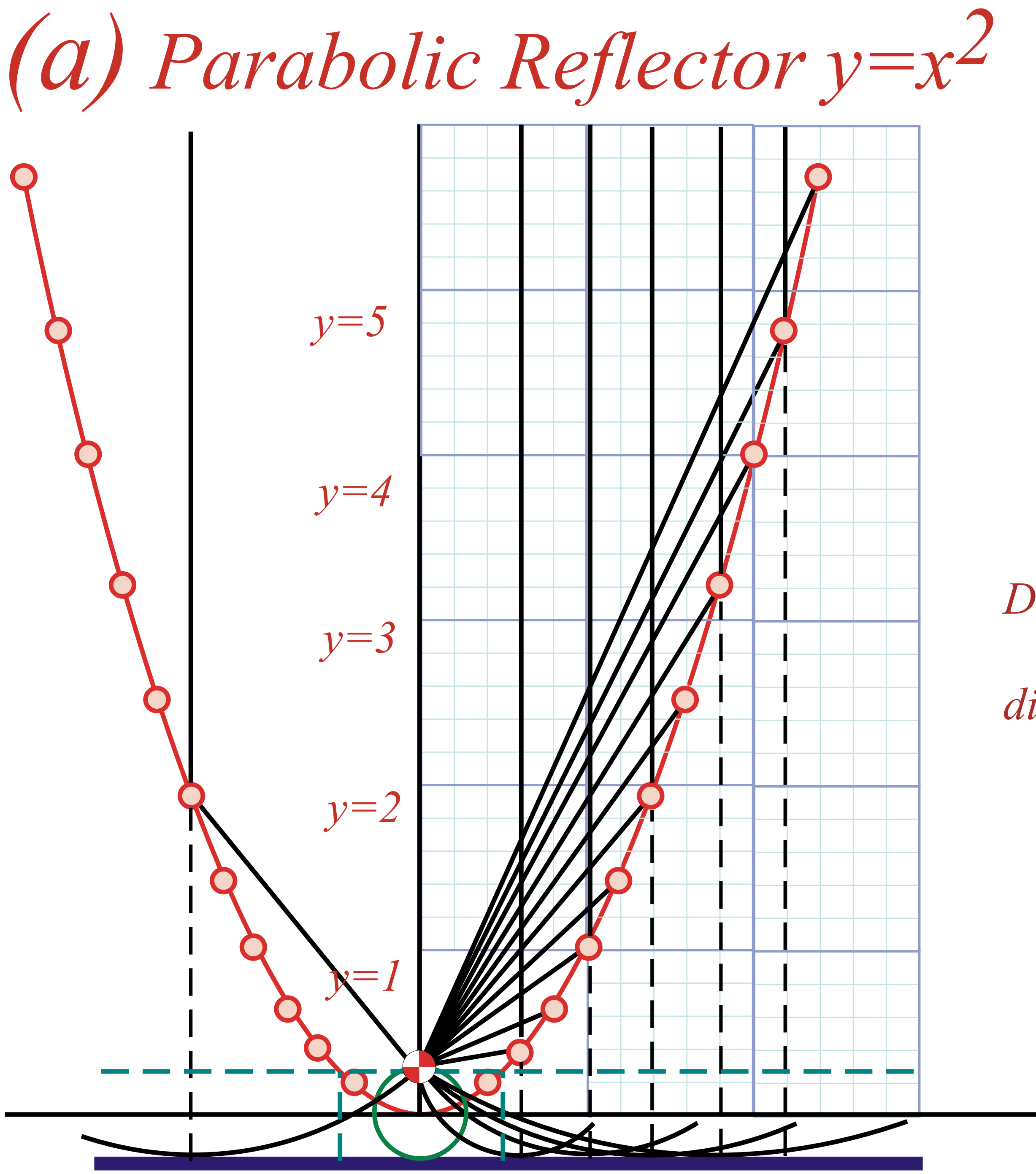
1. Pick an (x=?)-line
2. "Zig" from its y=x intersection to x=1 line
3. "Zag" from origin back to (x=?)-line

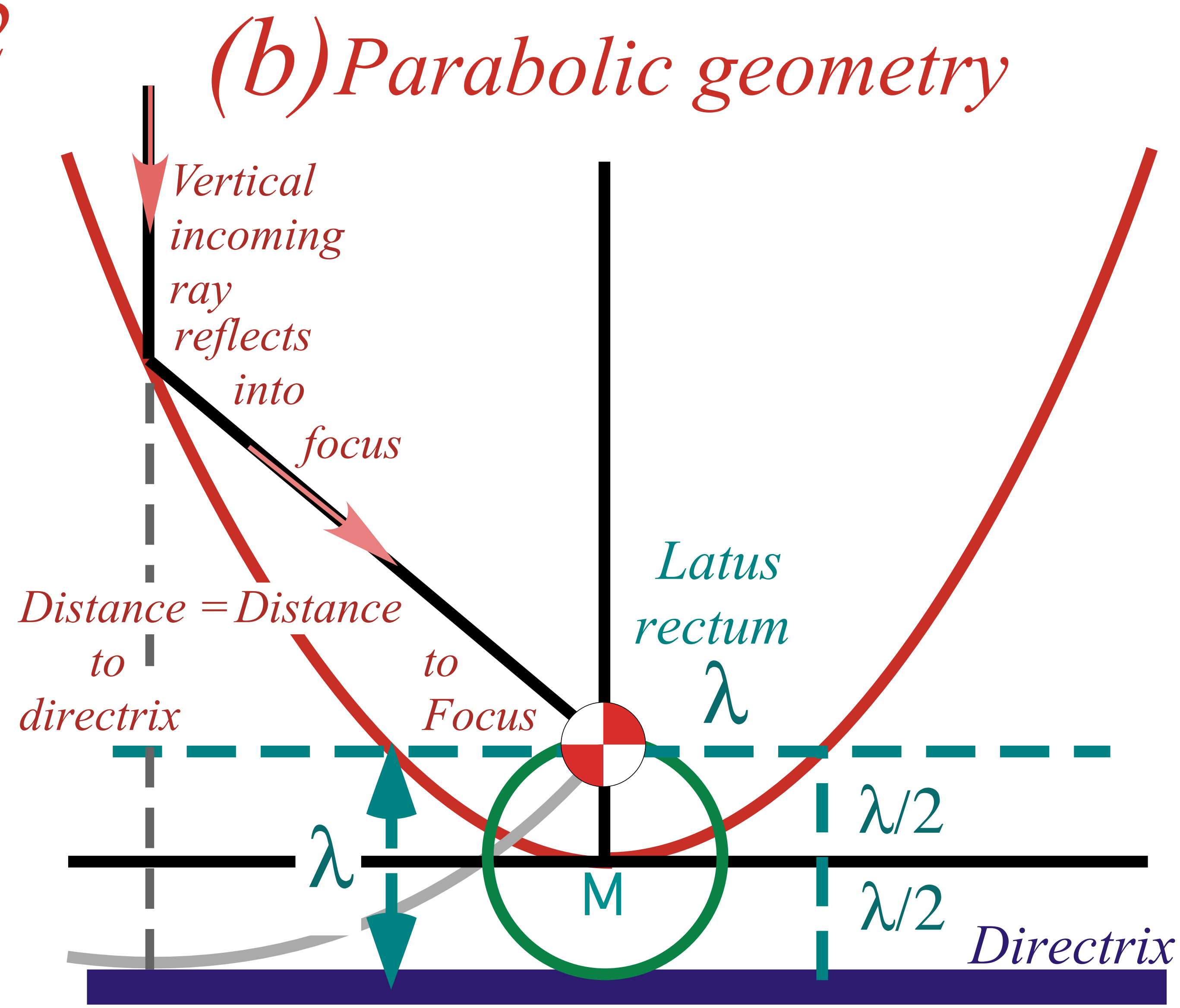


"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$



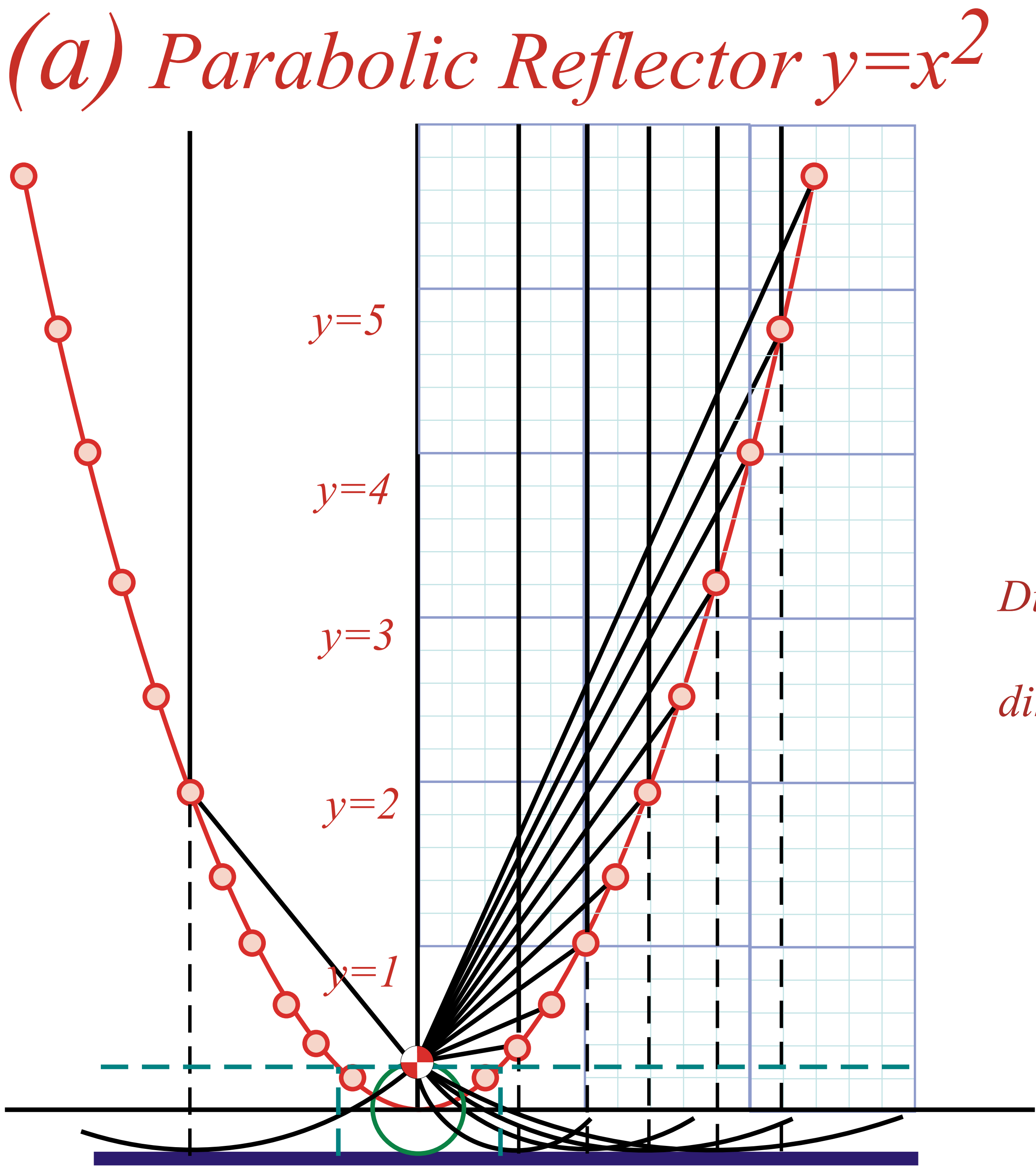
A more conventional parabolic geometry…(uses focal point)

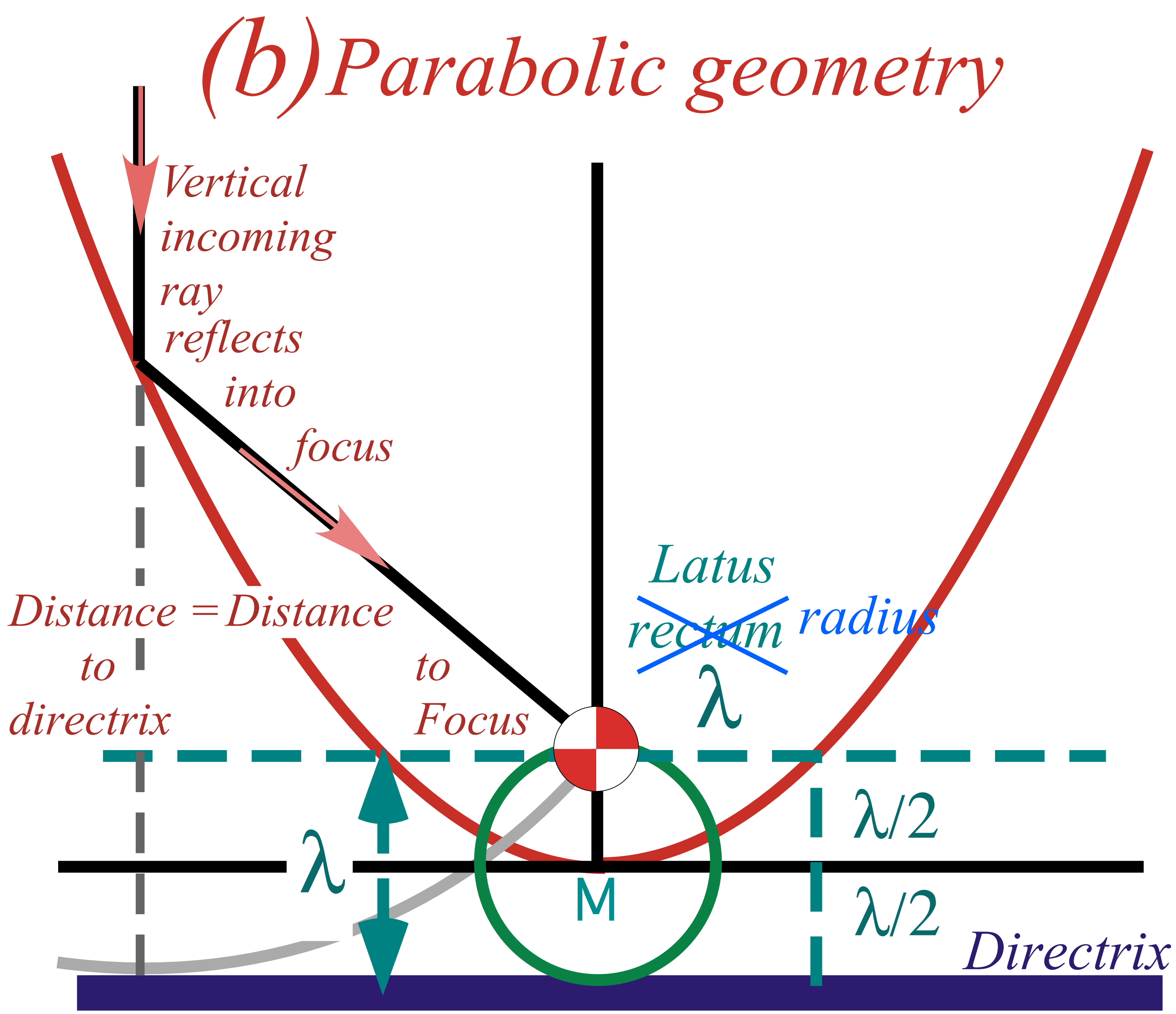




Unit 1
Fig. 9.3

A more conventional parabolic geometry...



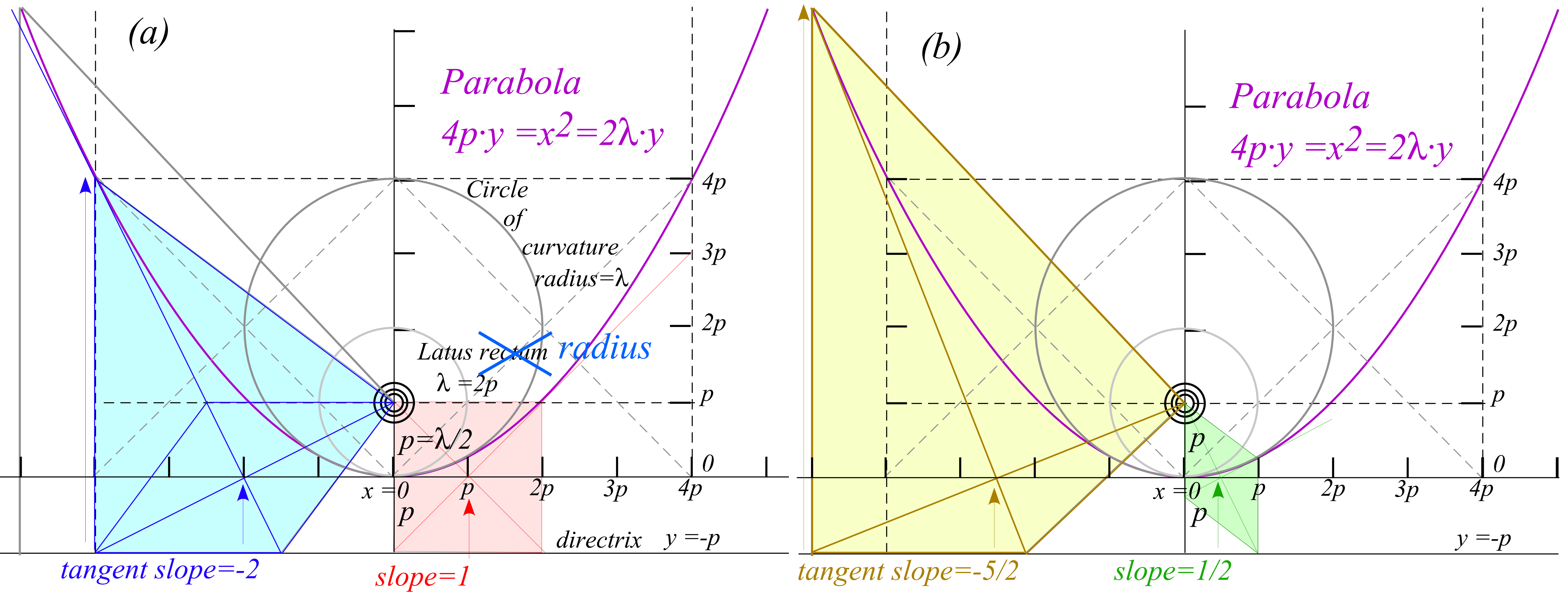


Better name† for λ : *latus radius*

†Old term *latus rectum* is exclusive copyright of
X-Treme Roidrage Gyms
Venice Beach, CA 90017

Unit 1
Fig. 9.3

...conventional parabolic geometry...carried to extremes...



Unit 1
Fig. 9.4

Geometry of common power-law potentials

Geometric (Power) series

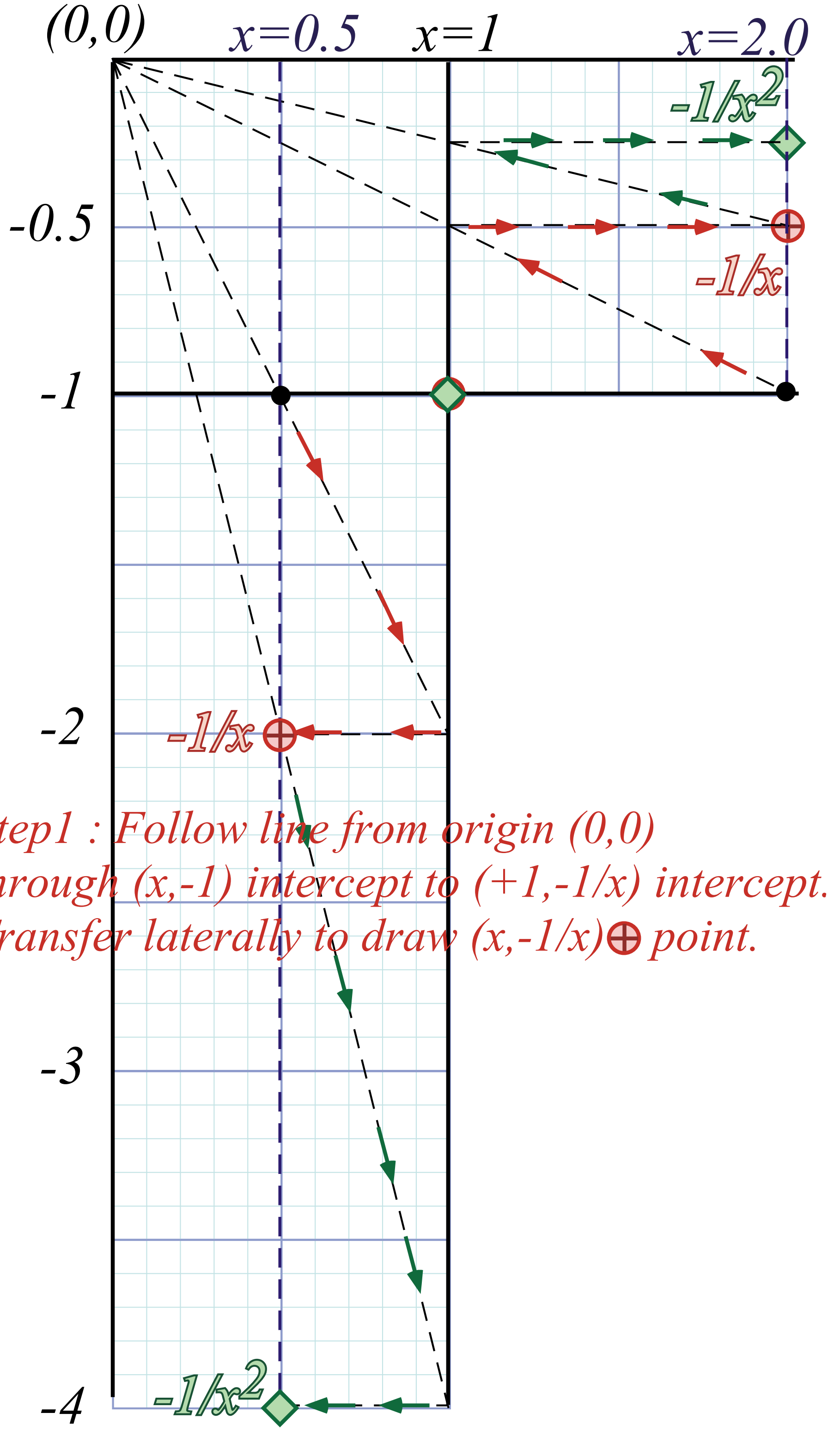
"Zig-Zag" exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

→ *Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields*

Compare mks units of Coulomb Electrostatic vs. Gravity

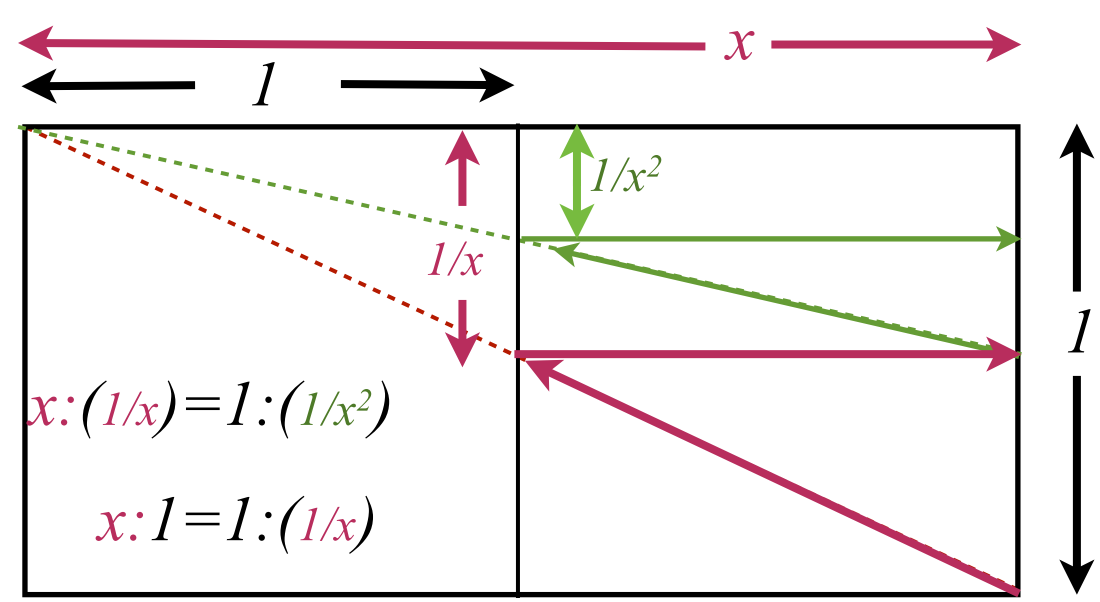


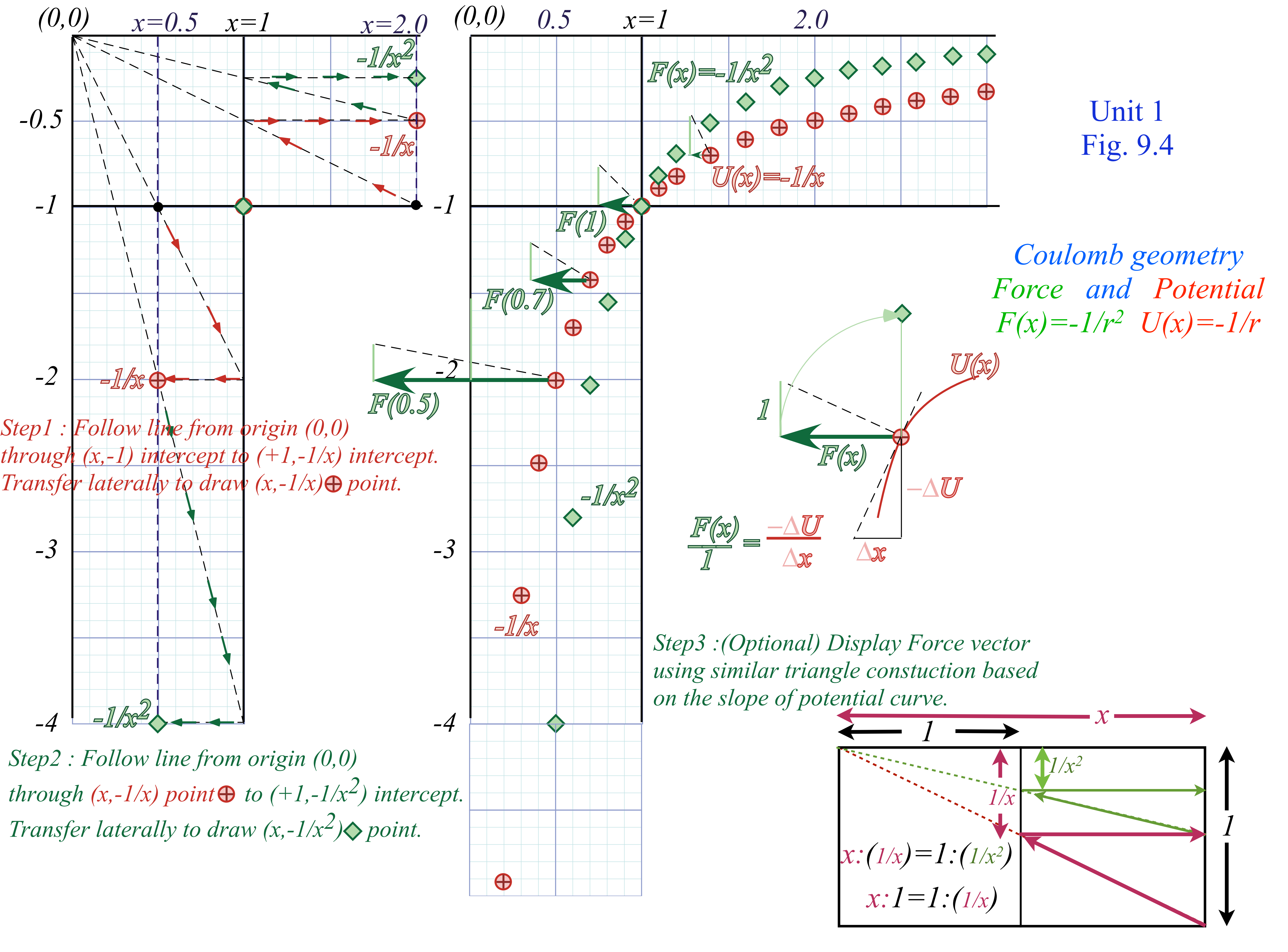
Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.

Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to $(+1,-1/x^2)$ intercept. Transfer laterally to draw $(x,-1/x^2)$◇ point.

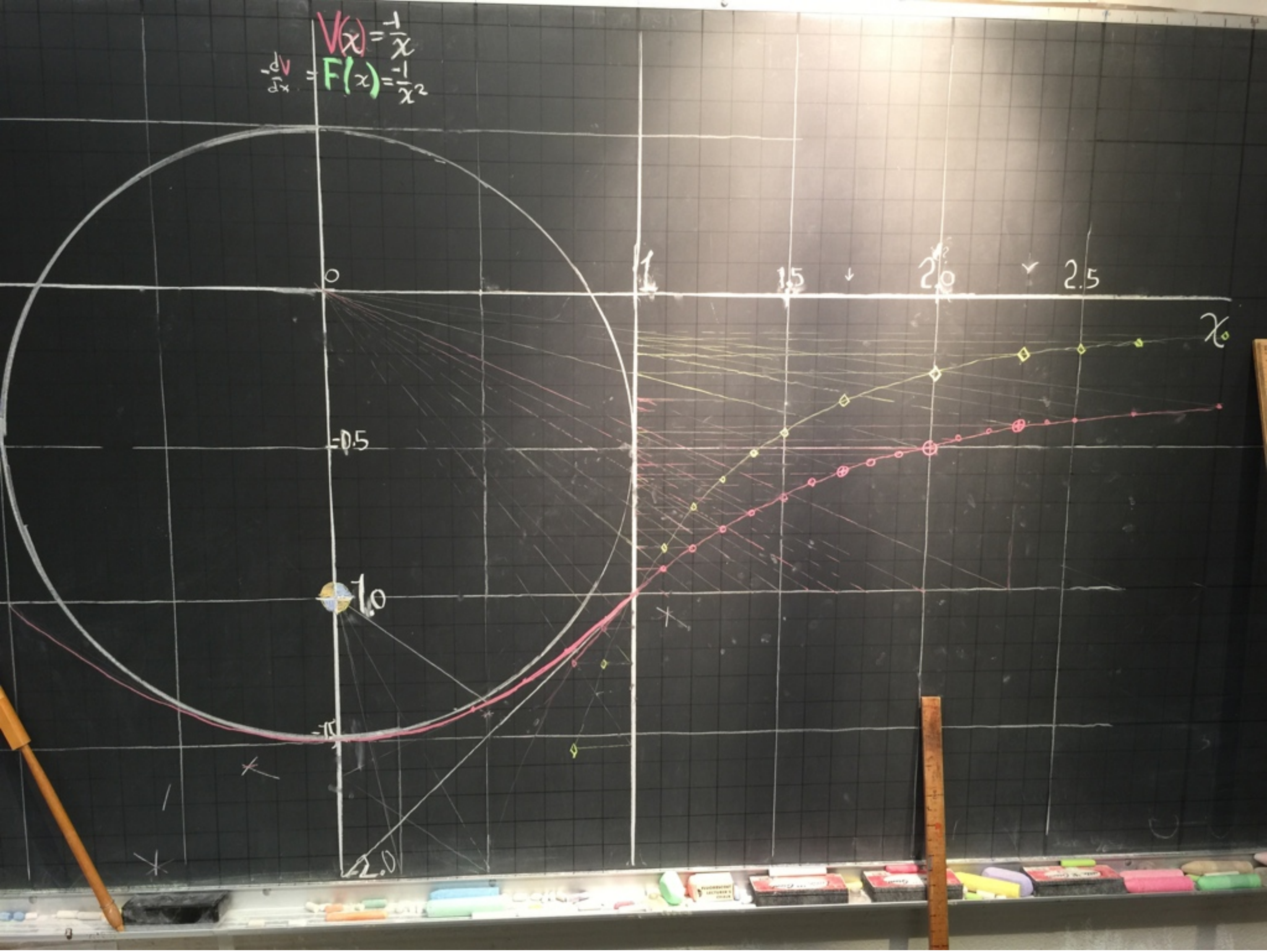
Unit 1
Fig. 9.4

Coulomb geometry
Force and Potential
$F(x)=-1/r^2$ $U(x)=-1/r$





Unit 1
Fig. 9.4
Coulomb geometry
Force and Potential
F(x)=-1/r² U(x)=-1/r
(0,0) x=0.5 x=1 x=2.0
-0.5
-1
-2
-3
-4
-1/x²
-1/x
(0,0) 0.5 x=1 2.0
F(x)=-1/x²
U(x)=-1/x
F(1)
F(0.7)
F(0.5)
U(x)
F(x)
−ΔU
Δx
F(x)/1 = −ΔU/Δx
Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.
Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x²) intercept. Transfer laterally to draw (x,-1/x²)◇ point.
Step3 :(Optional) Display Force vector using similar triangle constuction based on the slope of potential curve.
x
1
1/x²
1/x
x:(1/x)=1:(1/x²)
x:1=1:(1/x)



$V(x) = \frac{-1}{x}$
$-\frac{dV}{dx} = F(x) = \frac{-1}{x^2}$
0
1
1.5
2.0
2.5
x
-0.5
-1.0
-2.0

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

→ *Compare mks units of Coulomb Electrostatic vs. Gravity*

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong ?.?\cdot 10^{?} \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

~9E9~10^{10}

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

Compare mks units for Coulomb fields

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More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551\cdot 10^9 Nm^2/C^2$ $\sim 9\cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022\cdot 10^{-19}$ Coulomb

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

...but 1 Ampere = 1 Coulomb/sec.

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

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↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

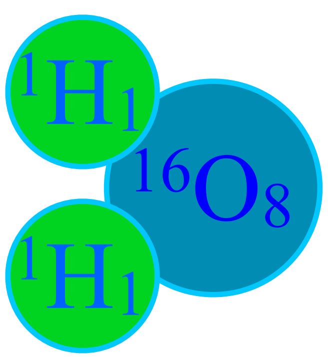
...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 8 notes that 1 $(cm)^3$ =1gm of water (1/18 Mole) has (1/18) $6\cdot 10^{23}$ molecules

$\sim 0.3 \cdot 10^{23}$

Avogodro's Number

H_2O Molecular weight~18



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quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb

↑

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Attractive (+)(-) or (-)(+)

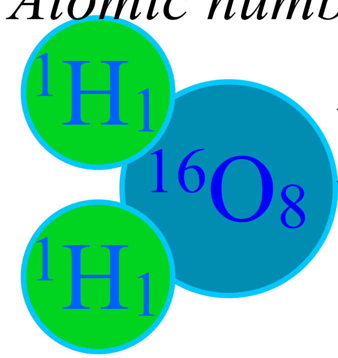
...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 9 notes that 1 $(cm)^3$ =1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules or ~ $3 \cdot 10^{23}$ electrons

$\sim 0.3 \cdot 10^{23}$ and ~ $3 \cdot 10^{23}$ protrons.

H_2O Molecular weight~18

Atomic number = 10



10 electrons
10 protons

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

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quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb

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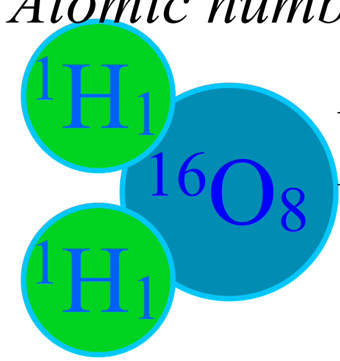

10 electrons *That is ~ − $3 \cdot 10^{23} \, 1.6022 \cdot 10^{-19}$ Coulomb or about $-0.5 \cdot 10^{+5}$ C or − 50,000 Coulomb*

10 protons *plus ~ $+3 \cdot 10^{23} \, 1.6022 \cdot 10^{-19}$ Coulomb or about $+0.5 \cdot 10^{+5}$ C or +50,000 Coulomb*

Equals zero total charge

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

$\sim 9E9 \sim 10^{10}$

More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2$ $\sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)
vs
Always Attractive (so far)

↓

↕ COMPARE! ↕

BIG
vs
small

2. Gravitational force between m(kilograms) and M(kg.)

$$F^{grav.}(r) = -G\frac{mM}{r^2} \quad where: G = \boxed{?.? \cdot 10^{?}} \ \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

$\sim 9E9 \sim 10^{10}$

More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ *Coulomb*

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)
vs
Always Attractive (so far)

↓

↕ COMPARE! ↕

BIG
vs
small

2. Gravitational force between m(kilograms) and M(kg.) !!!!

$$F^{grav.}(r) = -G\frac{mM}{r^2} \quad where: G = 0.000{,}000{,}000{,}067 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

$\sim {}^2/_3 10^{-10} \sim 10^{-10}$

More precise value for gravitational constant : $G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

quantum of charge: $|e|=1.6022\cdot 10^{-19}$ Coulomb

↑
Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q(Coulombs) and Q(C.)

~9E9~10^{10} !!!!

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Joule}{per\ square\ Coulomb}$$

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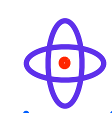
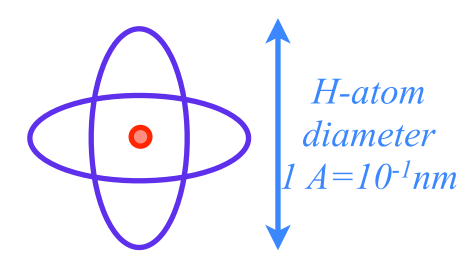


Nuclear size ~ 10^{-15} m = 1 femtometer =1fm

Atomic size ~ 1 Angstrom = 10^{-10} m



Compare mks units for Coulomb fields

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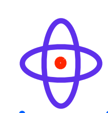
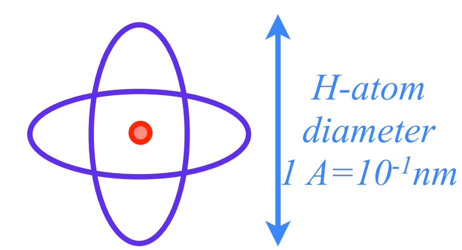


Nuclear size ~ 10^{-15} m = 1 femtometer =1fm

Atomic size ~ 1 Angstrom = 10^{-10} m

Big molecule ~ 10 Angstrom = 10^{-9} m = 1nanometer=1nm



Compare mks units for Coulomb fields

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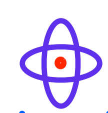
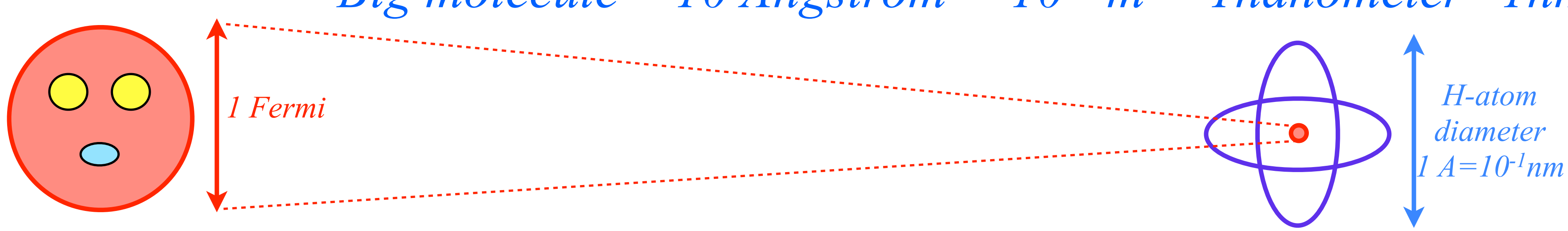


Nuclear size ~ 10^{-15} m = 1 femtometer =1fm

Atomic size ~ 1 Angstrom = 10^{-10} m

Big molecule ~ 10 Angstrom = 10^{-9} m = 1nanometer=1nm

also:1fm = 10^{-13} cm =1Fermi
=1Fm



Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

quantum of charge: $|e|=1.6022\cdot10^{-19}$ Coulomb

Discussion of repulsive force and PE in Ch. 9...

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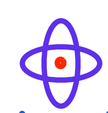
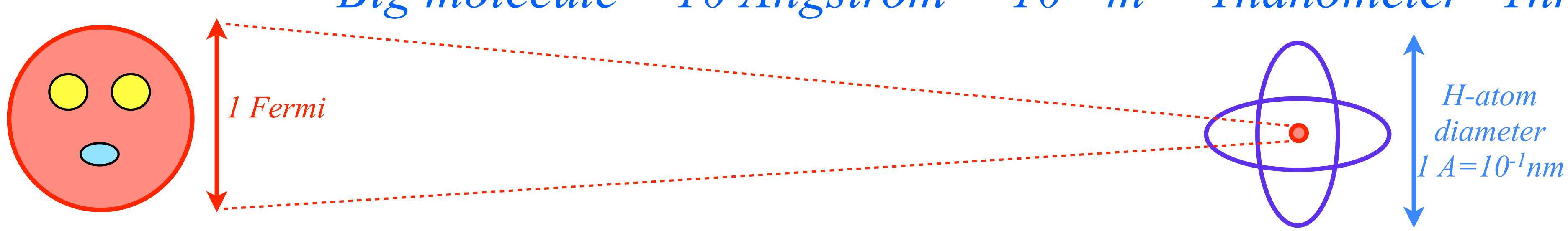


Nuclear size ~ 10^{-15} m = 1 femtometer =1fm

Atomic size ~ 1 Angstrom = 10^{-10} m

Big molecule ~ 10 Angstrom = 10^{-9} m = 1nanometer=1nm

also:1fm = 10^{-13} cm =1Fermi =1Fm



nuclear radii are 100,000 to 1,000,000 times smaller than atomic/chemical radii

*...so nuclear qQ/r energy 100,000 to 1,000,000 times **bigger** that of atomic/chemical...*

Geometry of idealized "Sophomore-physics Earth"

→ *Coulomb field outside*　　　*Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

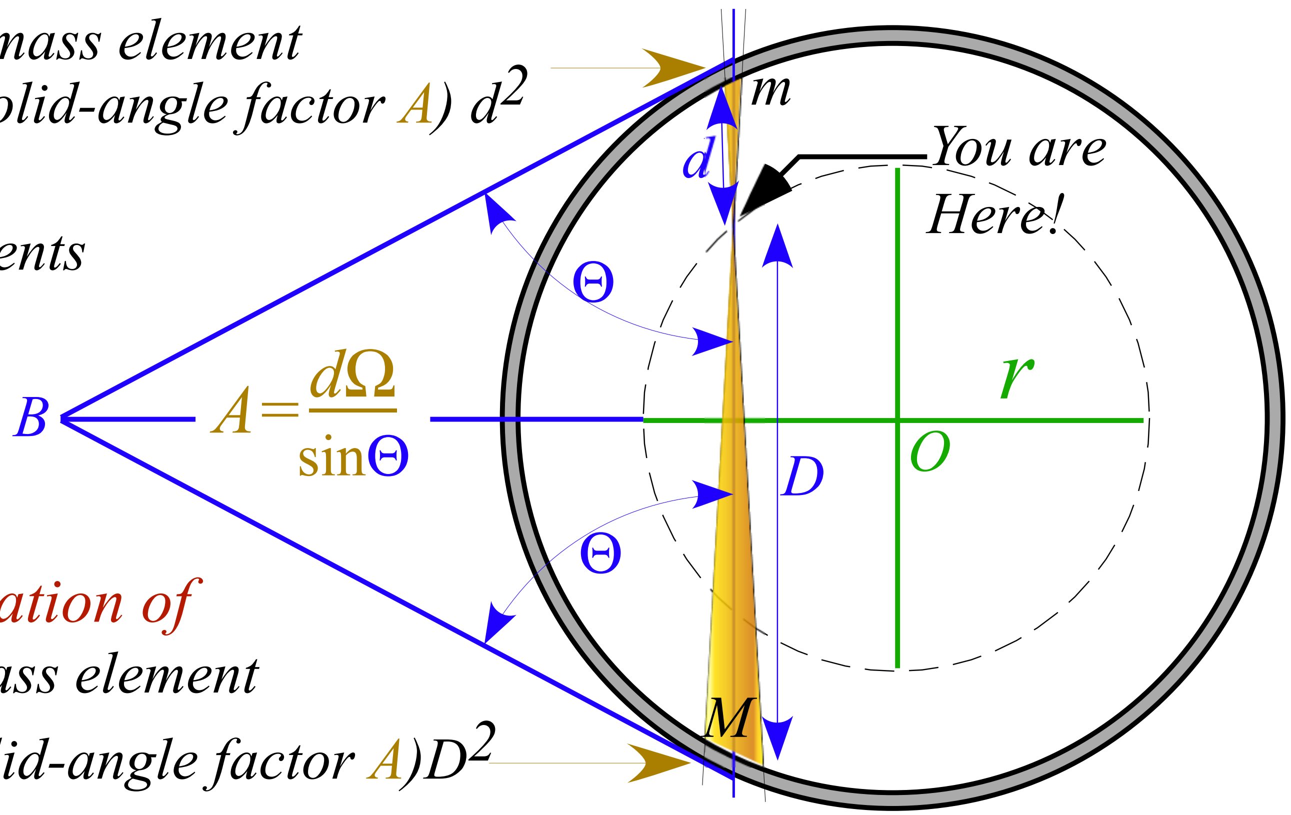
Shell mass element
m =(solid-angle factor A) d^2

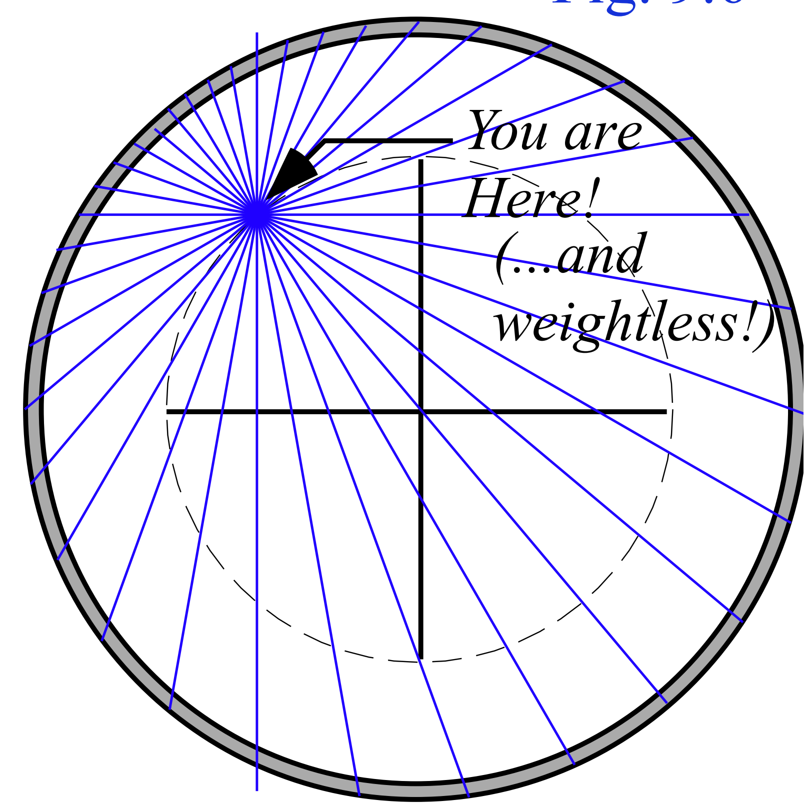
Gravity at r
due to shell mass elements

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

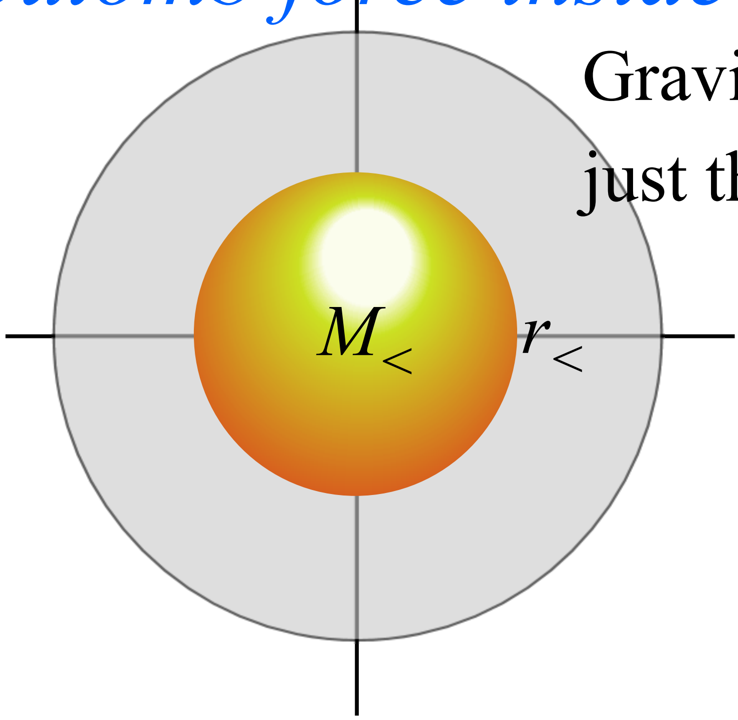
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

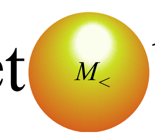
Cancellation of
Shell mass element
M =(solid-angle factor A)D^2

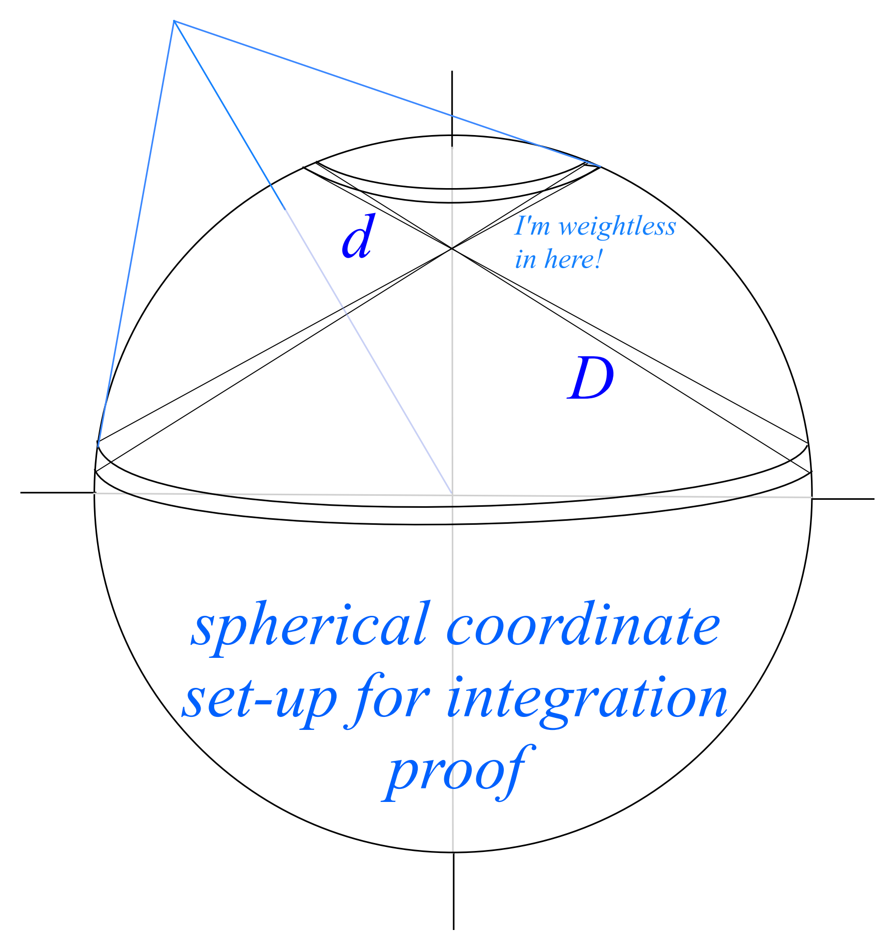




Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is just that of planet  below $r_<$



Coulomb force vanishes inside-spherical shell *(Gauss-law)*

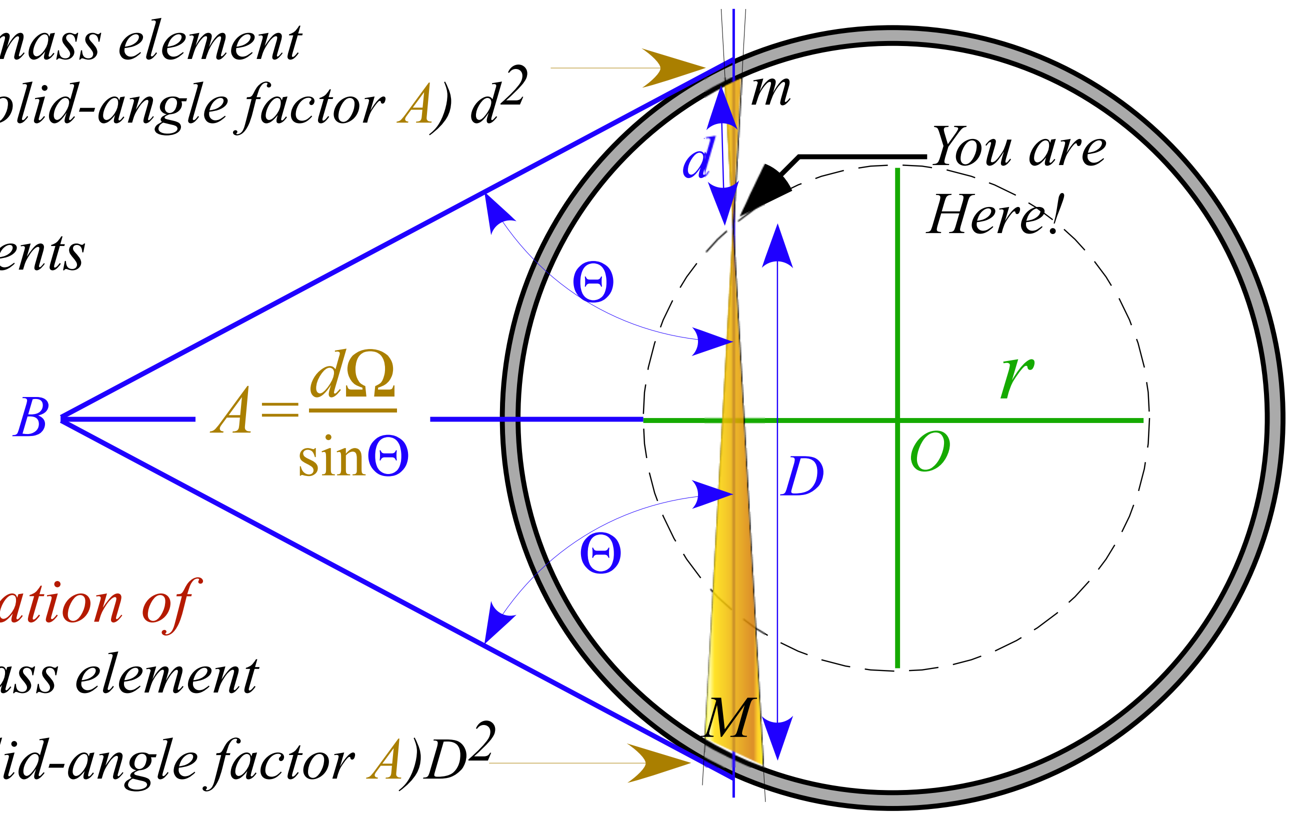
Unit 1
Fig. 9.6

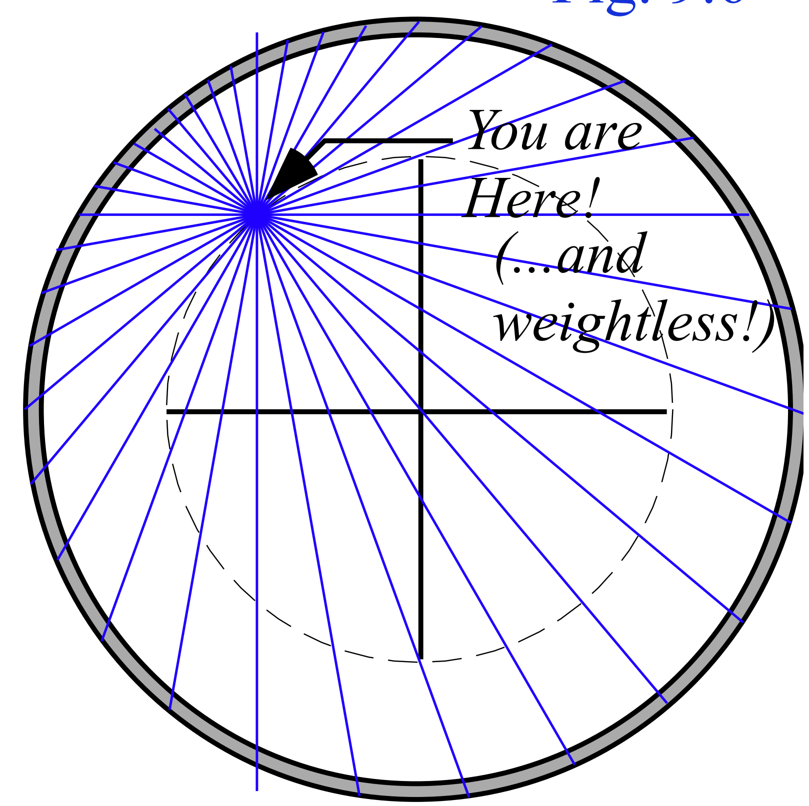
Gravity at r due to shell mass elements

$$\frac{GM}{D^2} - \frac{Gm}{d^2} = $$

$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

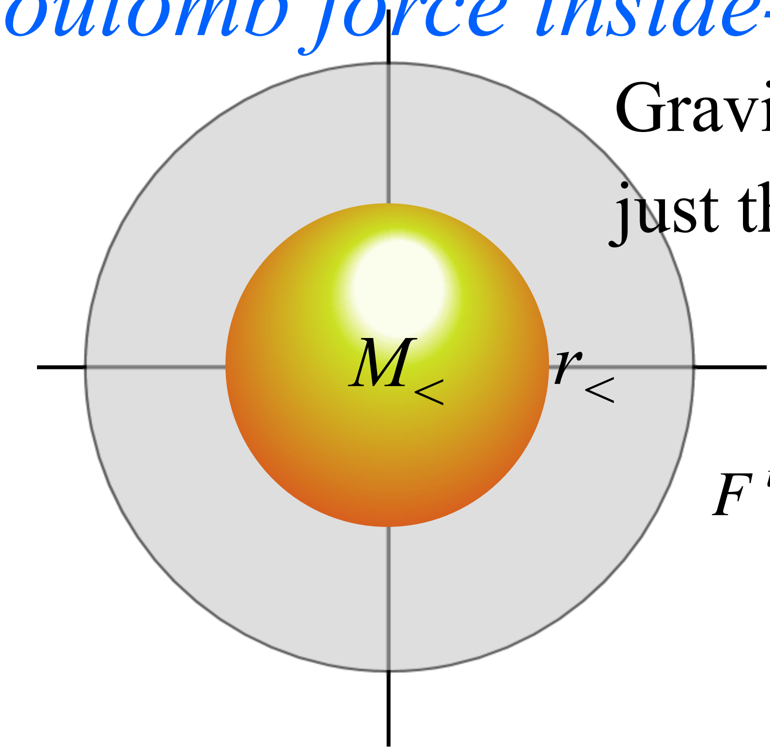
Shell mass element
m = (solid-angle factor A) d^2

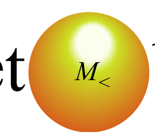




Cancellation of Shell mass element
M = (solid-angle factor A) D^2

Coulomb force inside-spherical body due to stuff <u>below</u> you, <u>only</u>.



Gravitational force at $r_<$ is just that of planet  below $r_<$

Note: Hooke's (linear) force law for $r_<$ inside uniform body

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg \cdot x$$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

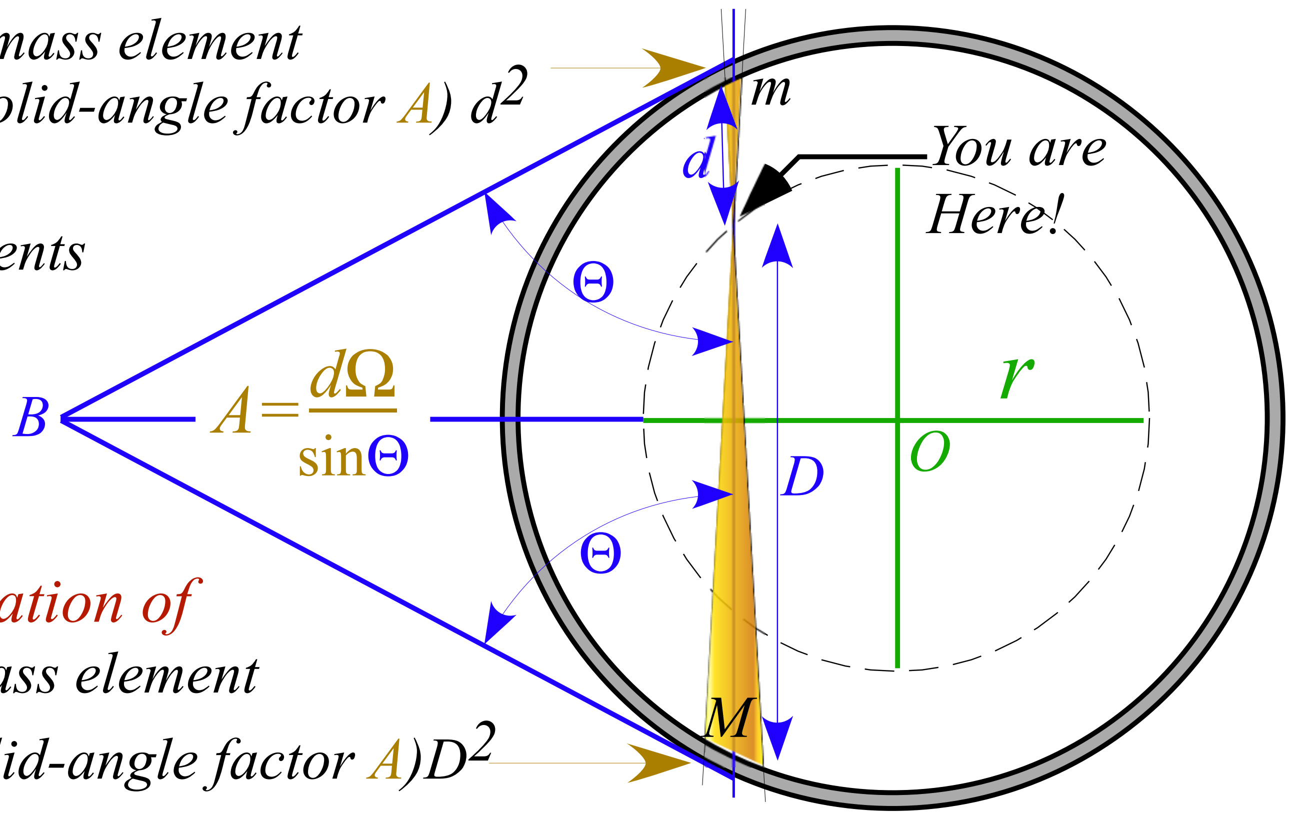
Shell mass element
m =(solid-angle factor A) d²

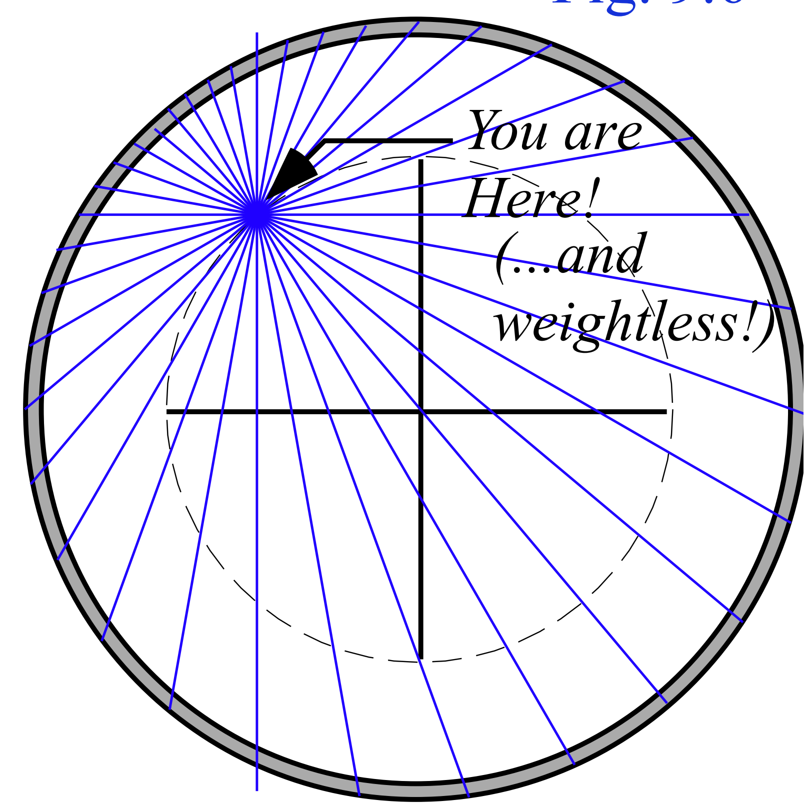
Gravity at r
due to shell mass elements

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

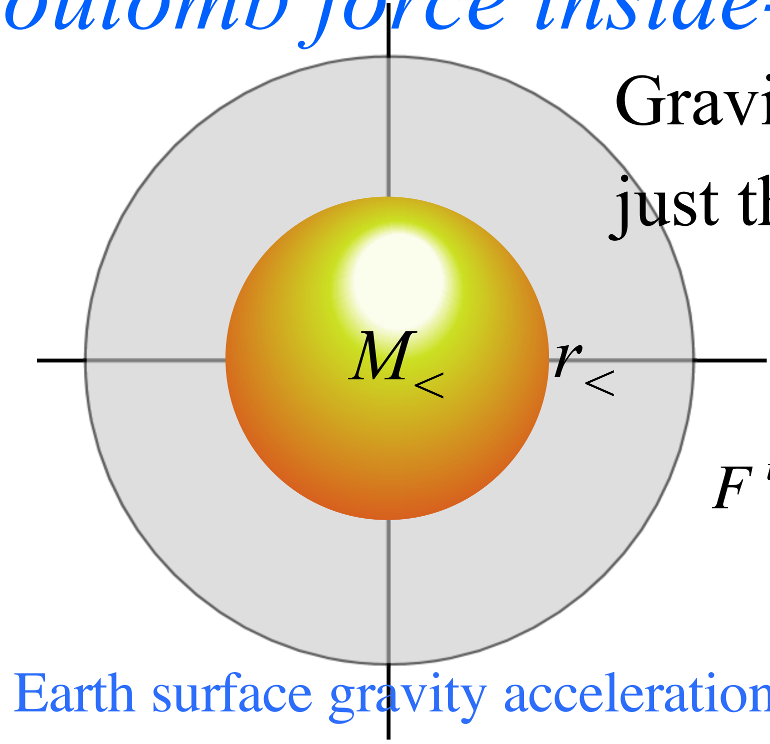
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

Cancellation of
Shell mass element
M =(solid-angle factor A)D²





Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is just that of planet ($M_<$) below $r_<$

Note:
Hooke's (linear) force law
for $r_<$ inside uniform body

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

Unit 1
Fig. 9.6

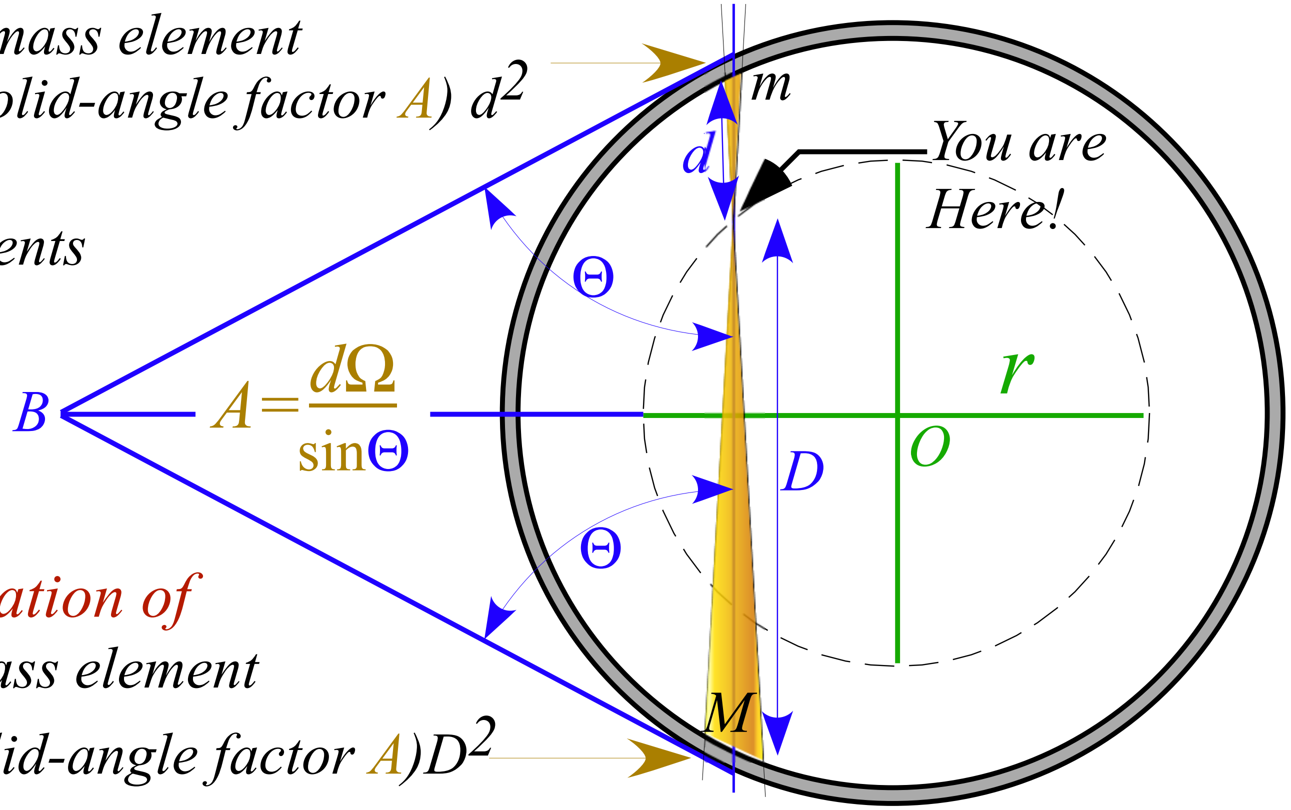
Shell mass element
m =(solid-angle factor A) d^2

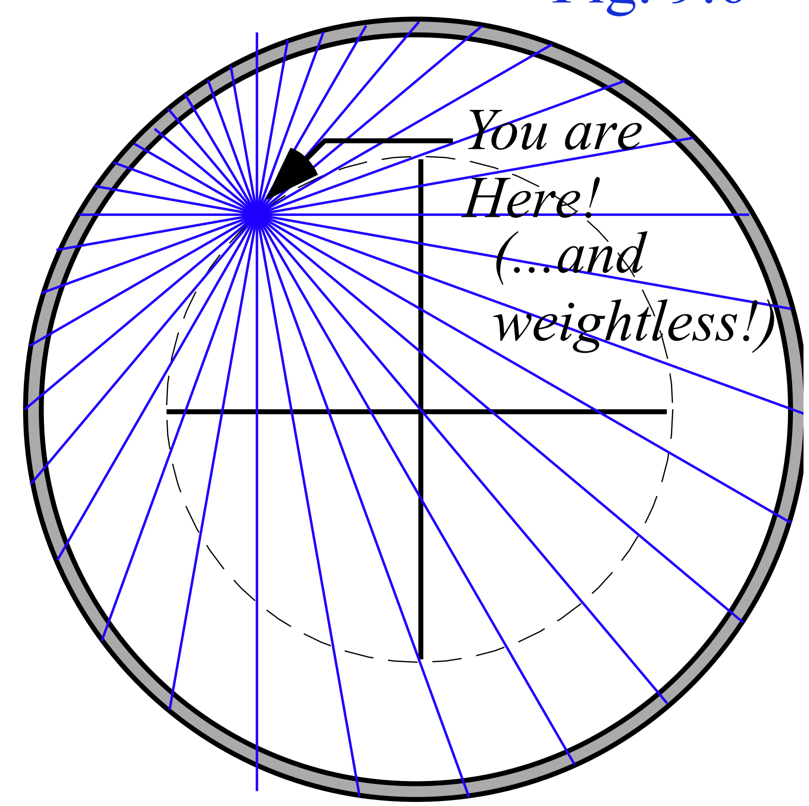
Gravity at r
due to shell mass elements

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

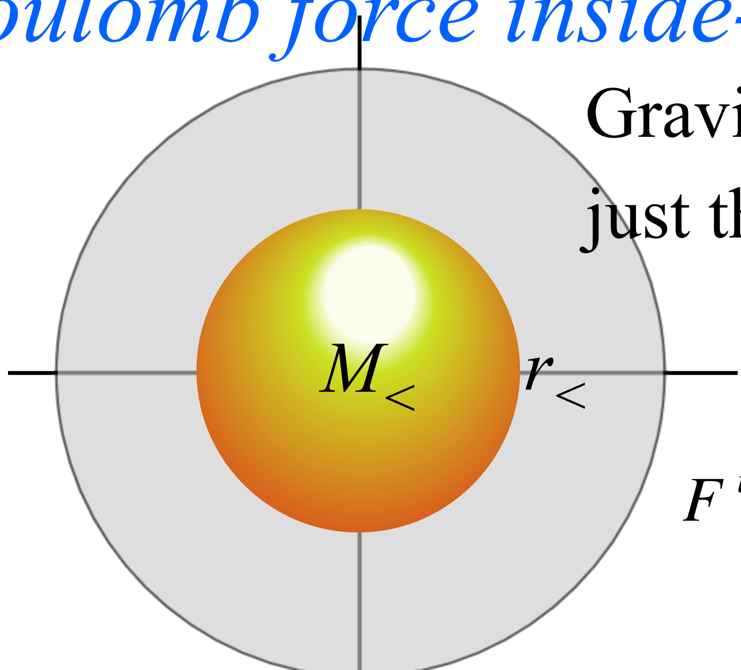
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

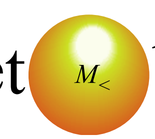
Cancellation of
Shell mass element
M =(solid-angle factor A)D^2





Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet  below $r_<$

Note:
Hooke's (linear) force law
for $r_<$ inside uniform body

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Earth radius : $R_\oplus = 6.371\cdot 10^6\,m \simeq 6.4\cdot 10^6\,m$

Earth mass : $M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0\cdot 10^{24}\,kg.$

Solar radius : $R_\odot = 6.955 \times 10^8\,m. \simeq 7.0\cdot 10^8\,m.$

Solar mass : $M_\odot = 1.9889 \times 10^{30}\,kg. \simeq 2.0\cdot 10^{30}\,kg.$

Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*

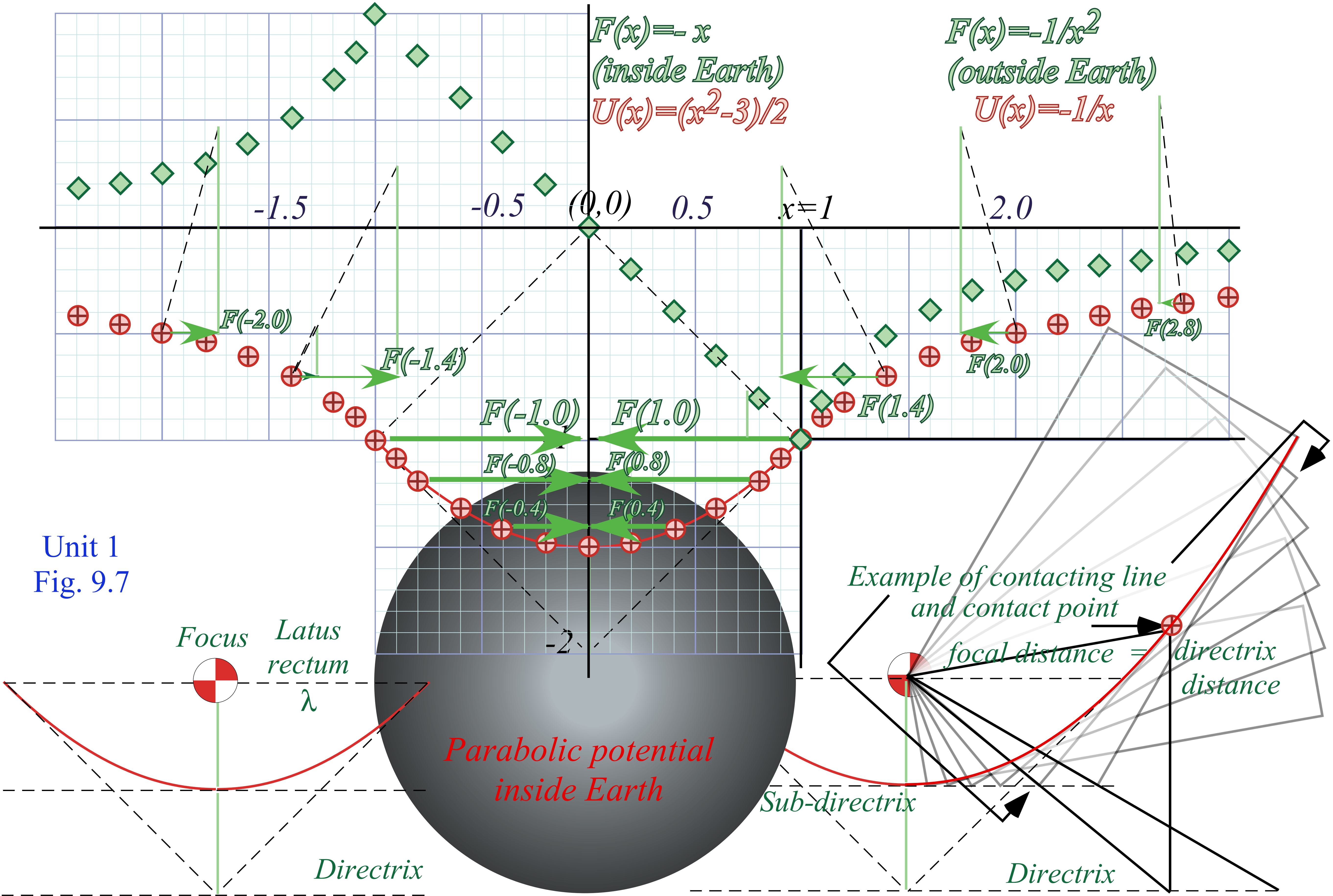
→ *Contact-geometry of potential curve(s)*

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

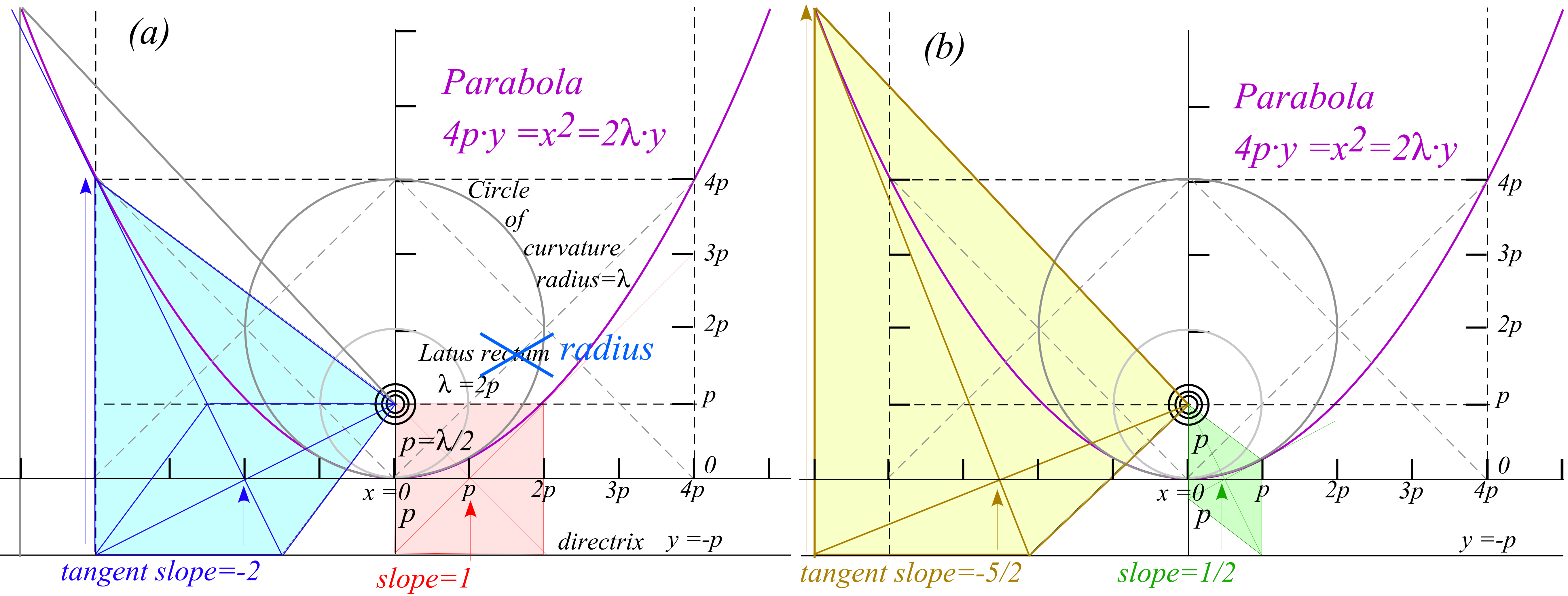
*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

The ideal "Sophomore-Physics-Earth" model of geo-gravity



...conventional parabolic geometry...carried to extremes...

(From p.18)



Unit 1
Fig. 9.4

Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*

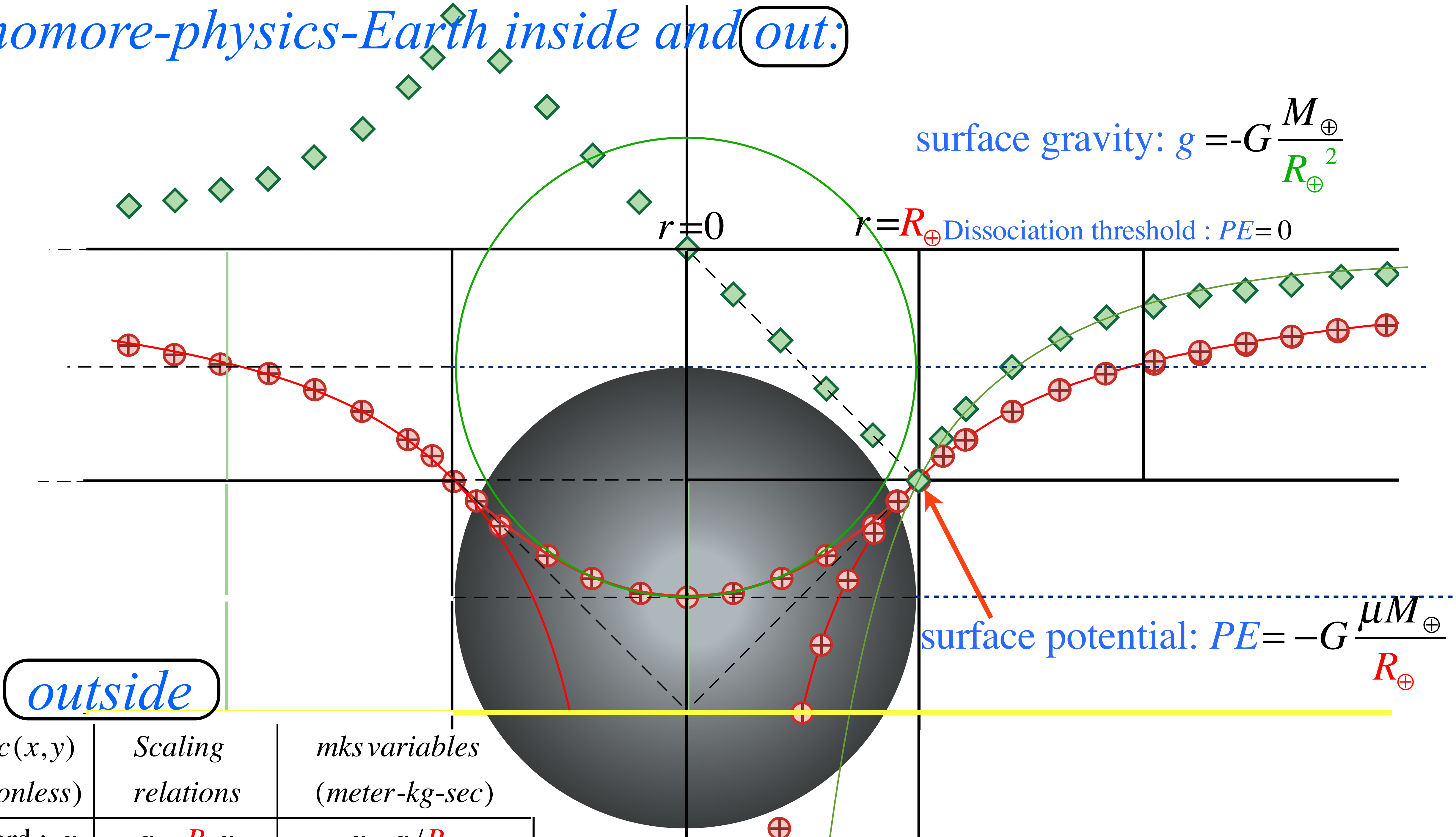
Contact-geometry of potential curve(s)

→ *"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

Earth matter vs nuclear matter:

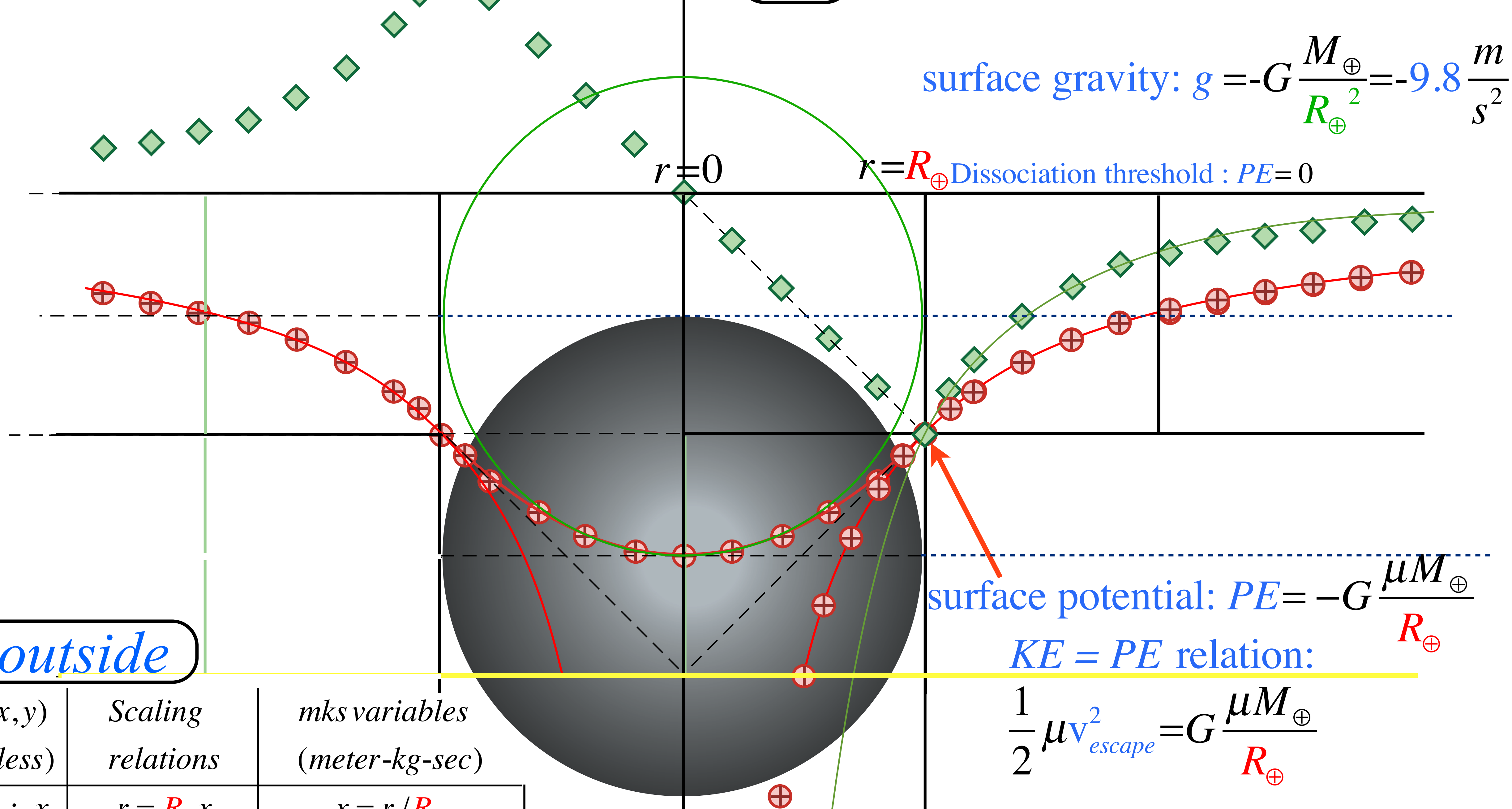
*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Sophomore-physics-Earth inside and out:



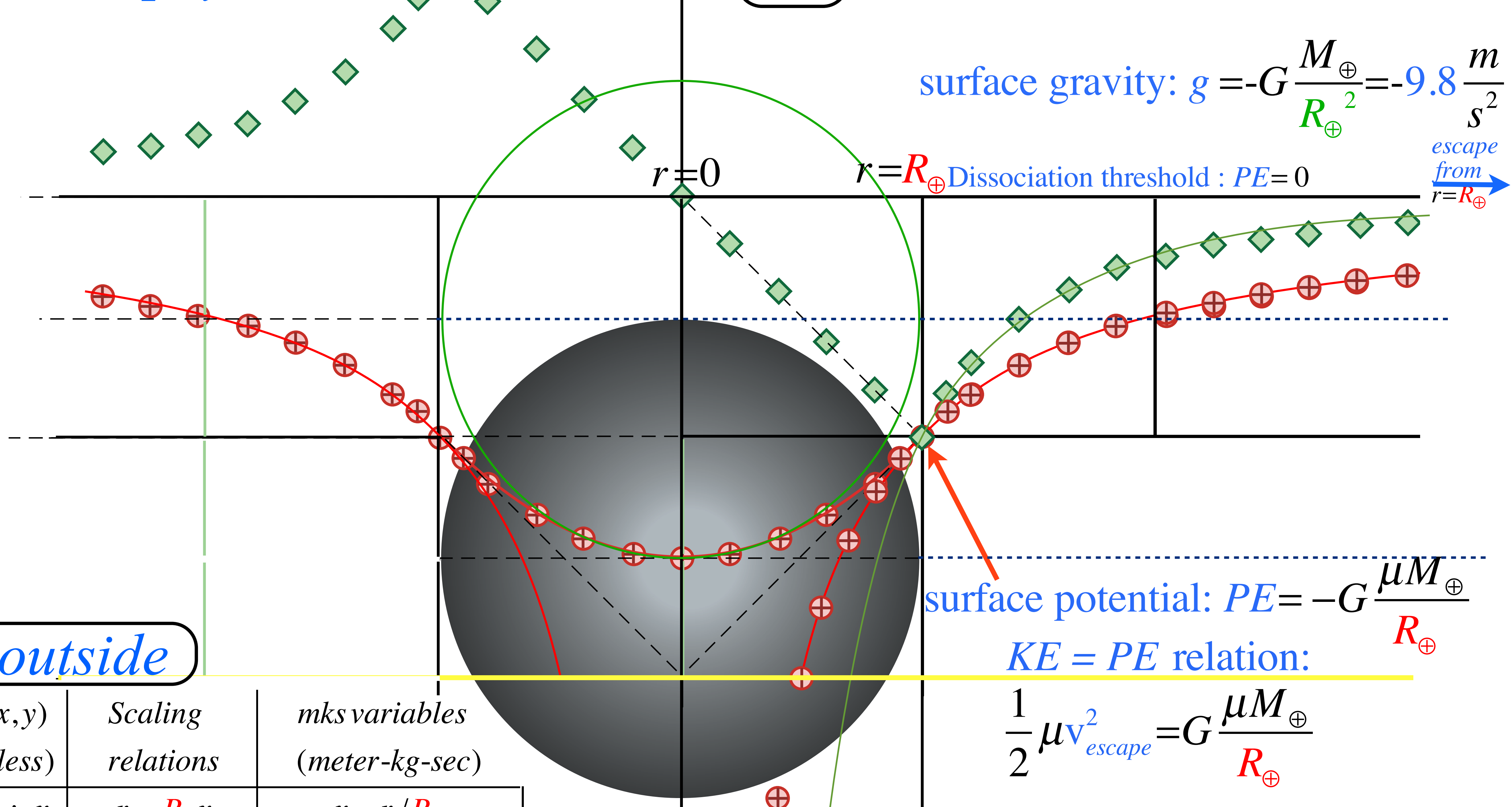
Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$

Sophomore-physics-Earth inside and out:



Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x \rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$

Sophomore-physics-Earth inside and out:



surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^{\;2}} = -9.8\dfrac{m}{s^2}$

surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

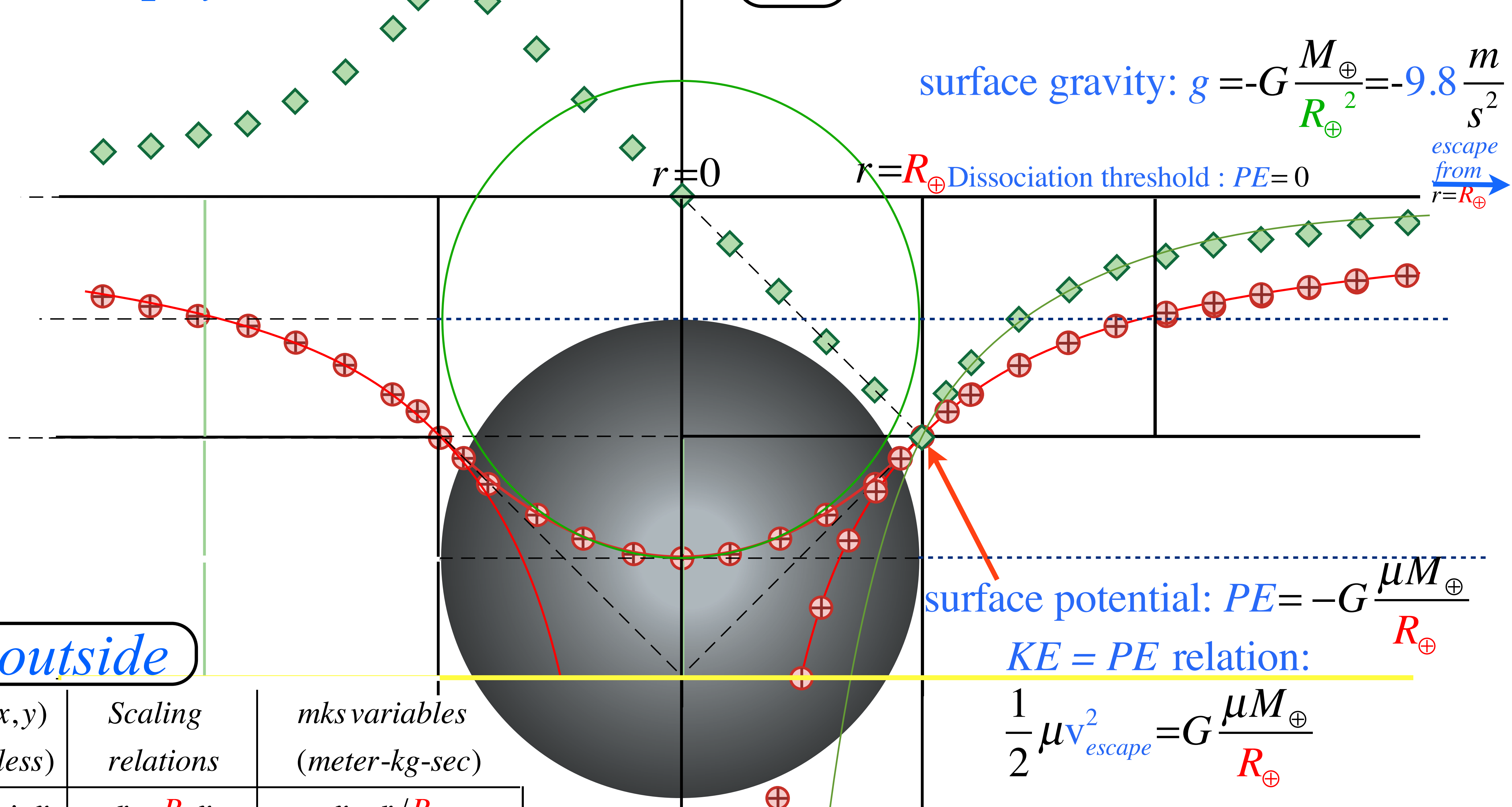
$$\frac{1}{2}\mu v_{escape}^2 = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

Geometric (x,y) *(Dimensionless)*	*Scaling relations*	*mks variables (meter-kg-sec)*
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\dfrac{GM\mu}{r} = -\dfrac{GM\mu}{R_\oplus}\dfrac{1}{x}$

Sophomore-physics-Earth inside and out:



surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^{\,2}} = -9.8\,\frac{m}{s^2}$

surface potential: $PE = -G\frac{\mu M_\oplus}{R_\oplus}$

$KE = PE$ relation:

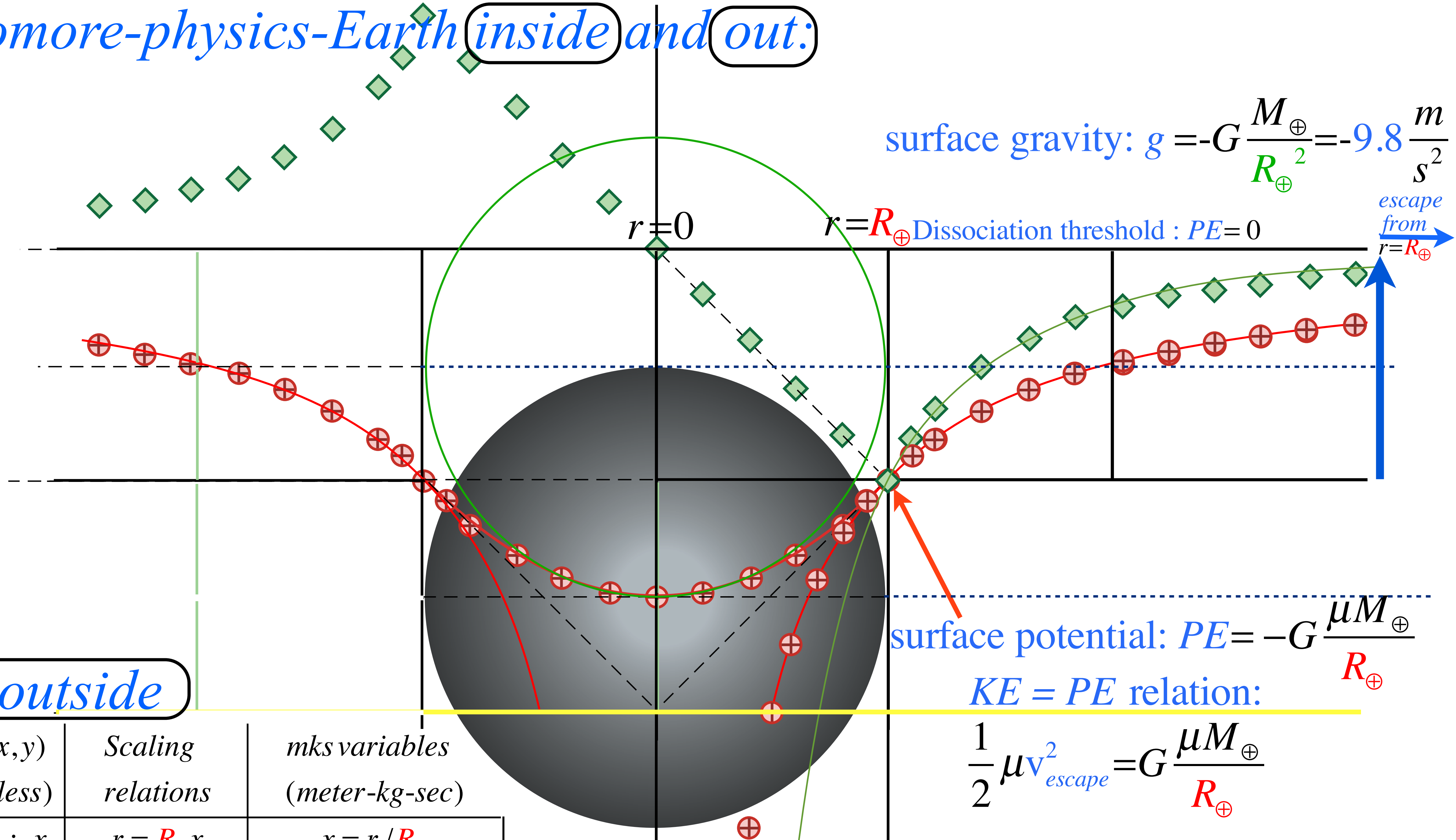
$$\frac{1}{2}\mu v_{escape}^2 = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$
$Force$ for $\lvert x\rvert \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_\oplus^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_\oplus^2}\frac{1}{x^2}$

Sophomore-physics-Earth inside and out:



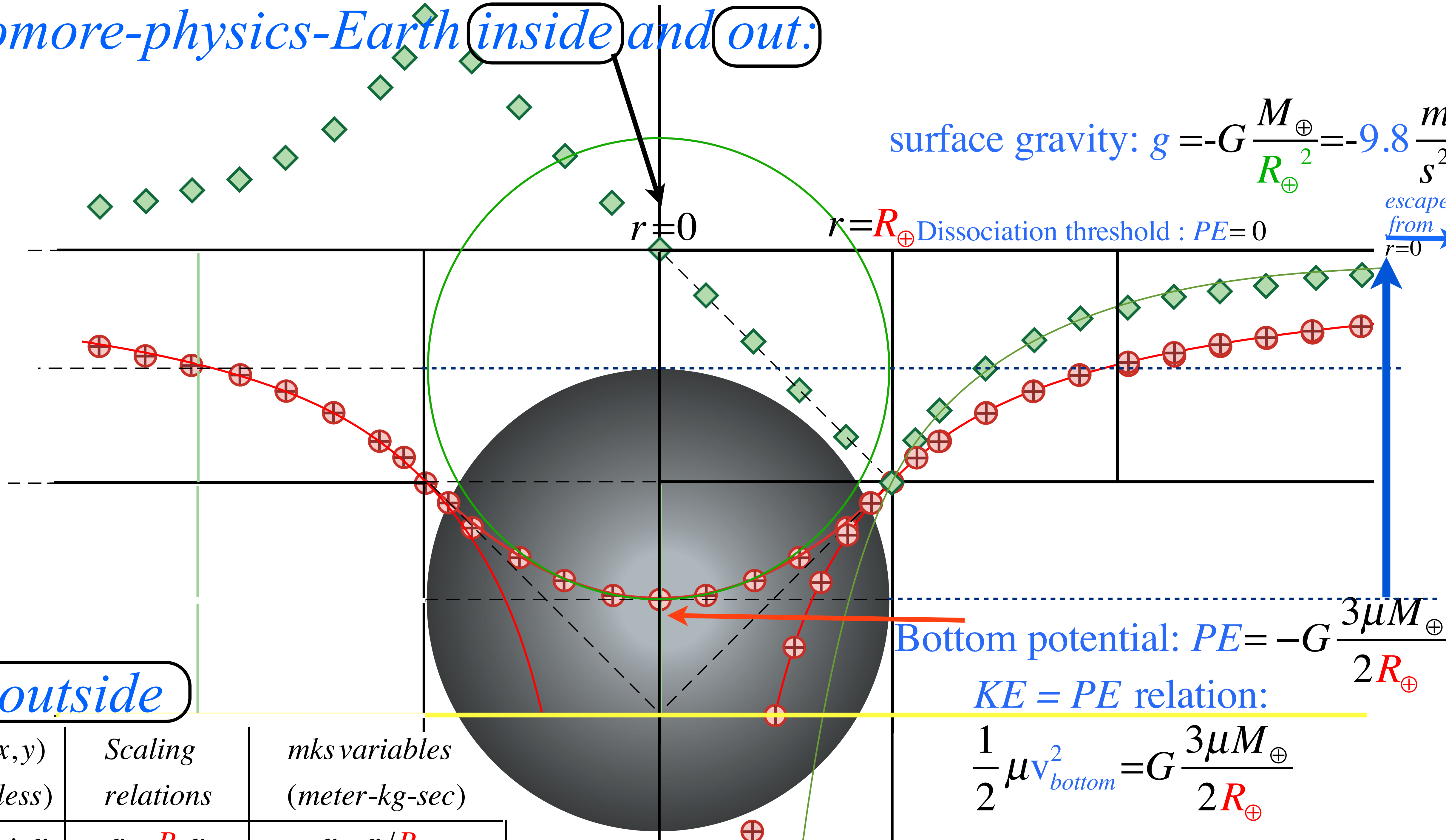
Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)	inside	
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$		
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$	PE for $\lvert x\rvert < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$
$Force$ for $\lvert x\rvert \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_\oplus^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_\oplus^2}\frac{1}{x^2}$	$Force$ for $\lvert x\rvert < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_\oplus^3} r$

$R_\oplus$-escape-velocity

$$v_{escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Sophomore-physics-Earth inside and out:



surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2} = -9.8\frac{m}{s^2}$

Bottom potential: $PE = -G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

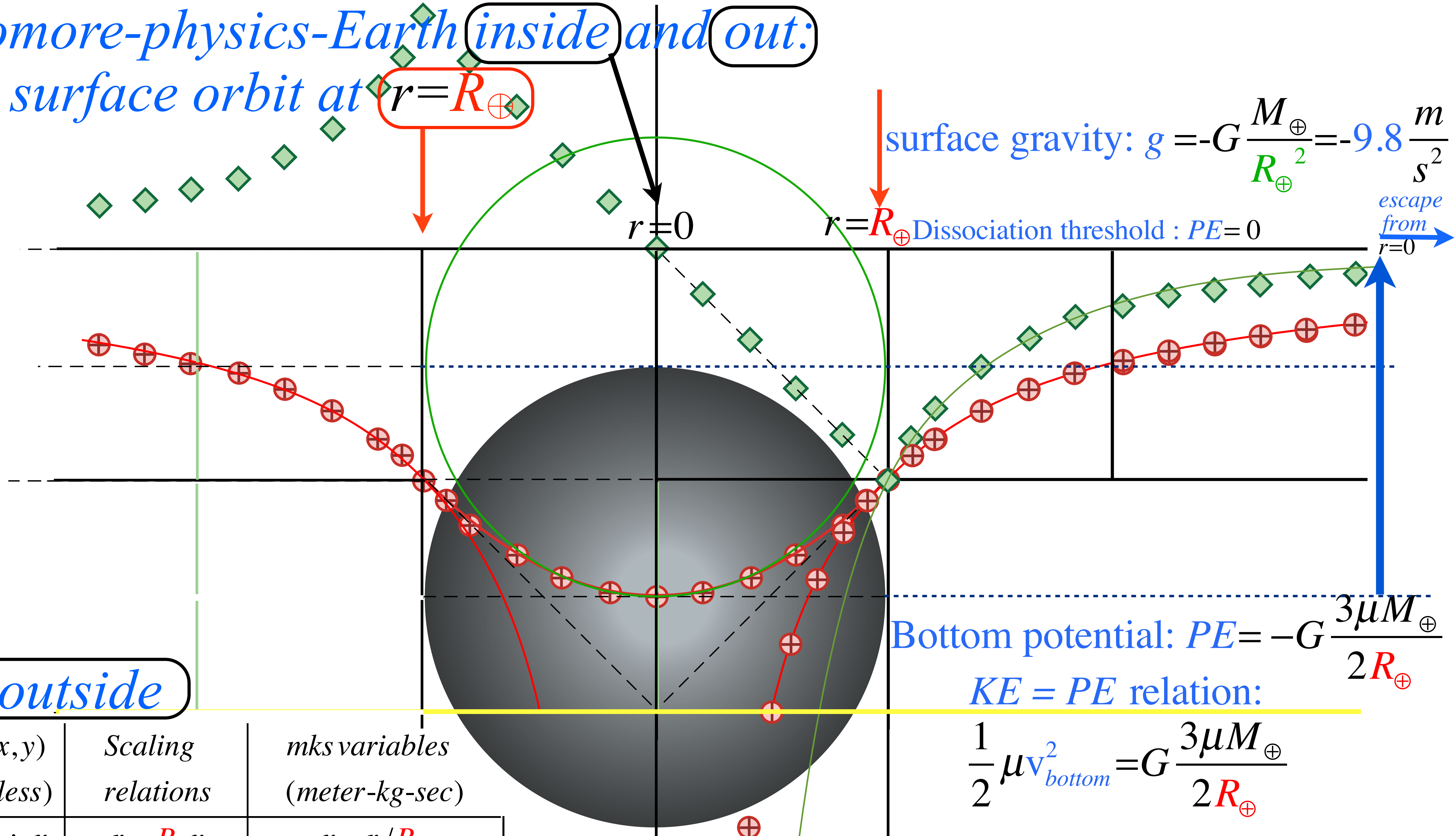
$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Geometric (x,y) (Dimensionless)	*Scaling relations*	*mks variables (meter-kg-sec)*		*inside*
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$		
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$	PE for $\lvert x\rvert < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$
$Force$ for $\lvert x\rvert \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_\oplus^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_\oplus^2}\frac{1}{x^2}$	$Force$ for $\lvert x\rvert < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_\oplus^3} r$

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$



surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^{\,2}} = -9.8\frac{m}{s^2}$

Bottom potential: $PE = -G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

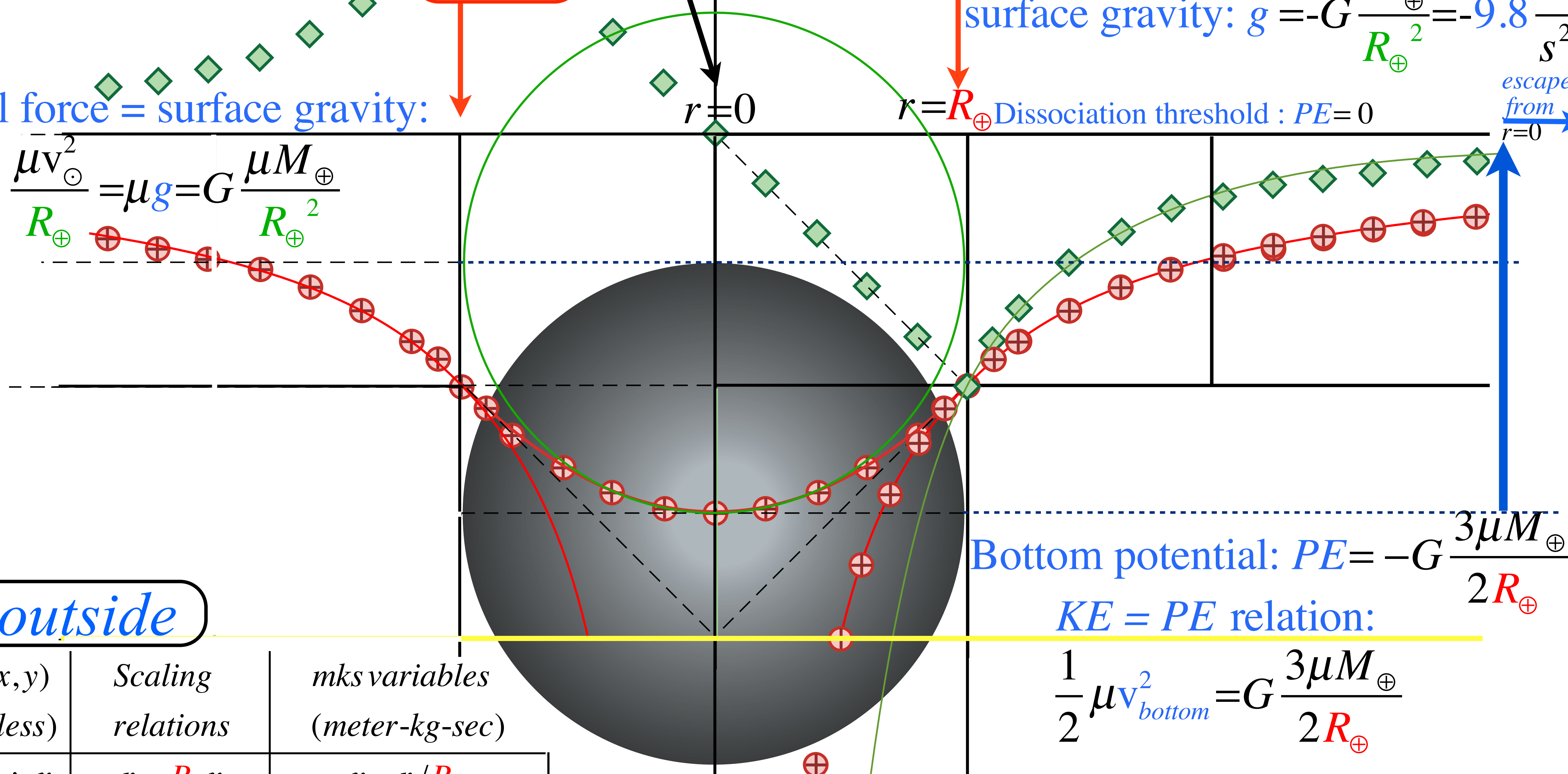
$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Geometric (x,y) (Dimensionless)	*Scaling relations*	*mks variables (meter-kg-sec)*		
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$		
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$	PE for $\lvert x\rvert < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$
$Force$ for $\lvert x\rvert \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_\oplus^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_\oplus^2}\frac{1}{x^2}$	$Force$ for $\lvert x\rvert < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_\oplus^3} r$

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$



Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^2}$$

surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2} = -9.8\frac{m}{s^2}$

Bottom potential: $PE = -G\frac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

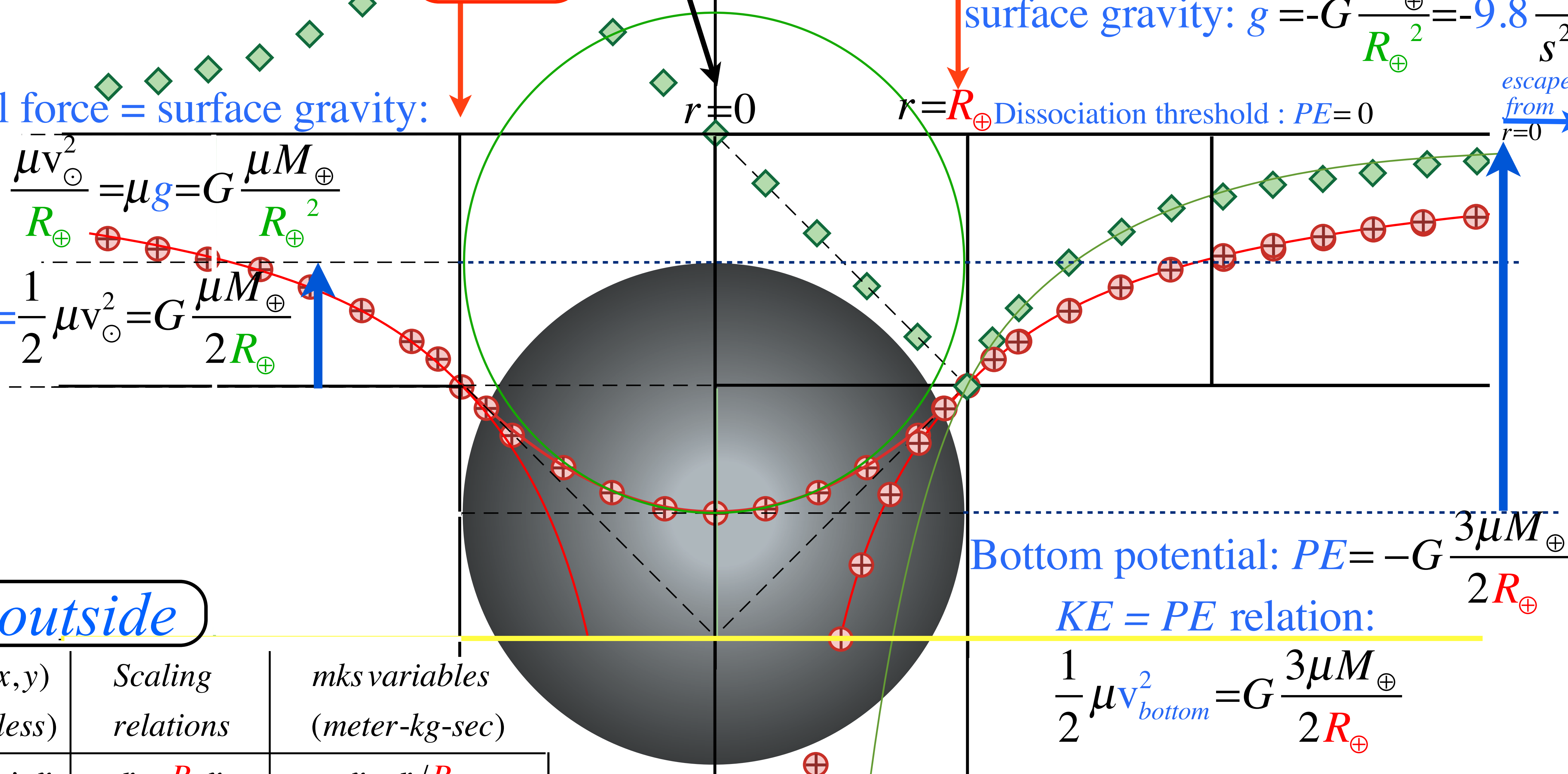
Geometric (x,y) (Dimensionless)	*Scaling relations*	*mks variables (meter-kg-sec)*		*inside*
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$		
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$	*PE* for $\lvert x\rvert < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$
Force for $\lvert x\rvert \geq 1$: $y^{Force} = \frac{-1}{x^2}$	$F^{mks}(r) = \frac{GM\mu}{R_\oplus^2} y^{Force}$	$F^{mks}(r) = -\frac{GM\mu}{r^2} = -\frac{GM\mu}{R_\oplus^2}\frac{1}{x^2}$	*Force* for $\lvert x\rvert < 1$: $y^{Force} = -x$	$F^{mks}(r) = -\frac{GM\mu}{R_\oplus^3} r$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$



surface gravity: $g=-G\frac{M_\oplus}{R_\oplus^{\ 2}}=-9.8\frac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus^{\ 2}}$$

Orbit KE$=\frac{1}{2}\mu v_\odot^2=G\frac{\mu M_\oplus}{2R_\oplus}$

Bottom potential: $PE=-G\frac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v_{bottom}^2=G\frac{3\mu M_\oplus}{2R_\oplus}$$

Geometric (x,y) *(Dimensionless)*	*Scaling relations*	*mks variables (meter-kg-sec)*		
space coord.: x	$r=R_\oplus x$	$x=r/R_\oplus$		**inside**
PE for $\lvert x\rvert\geq 1$: $y^{PE}=\frac{-1}{x}$	$PE^{mks}(r)=\frac{GM\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r)=-\frac{GM\mu}{r}=-\frac{GM\mu}{R_\oplus}\frac{1}{x}$	PE for $\lvert x\rvert<1$: $y^{PE}=\frac{x^2}{2}-\frac{3}{2}$	$PE^{mks}(r)=\frac{GM\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2}-\frac{3}{2}\right)$
$Force$ for $\lvert x\rvert\geq 1$: $y^{Force}=\frac{-1}{x^2}$	$F^{mks}(r)=\frac{GM\mu}{R_\oplus^2}y^{Force}$	$F^{mks}(r)=-\frac{GM\mu}{r^2}=-\frac{GM\mu}{R_\oplus^2}\frac{1}{x^2}$	$Force$ for $\lvert x\rvert<1$: $y^{Force}=-x$	$F^{mks}(r)=-\frac{GM\mu}{R_\oplus^3}r$

$(r=0)$-escape-velocity

$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$

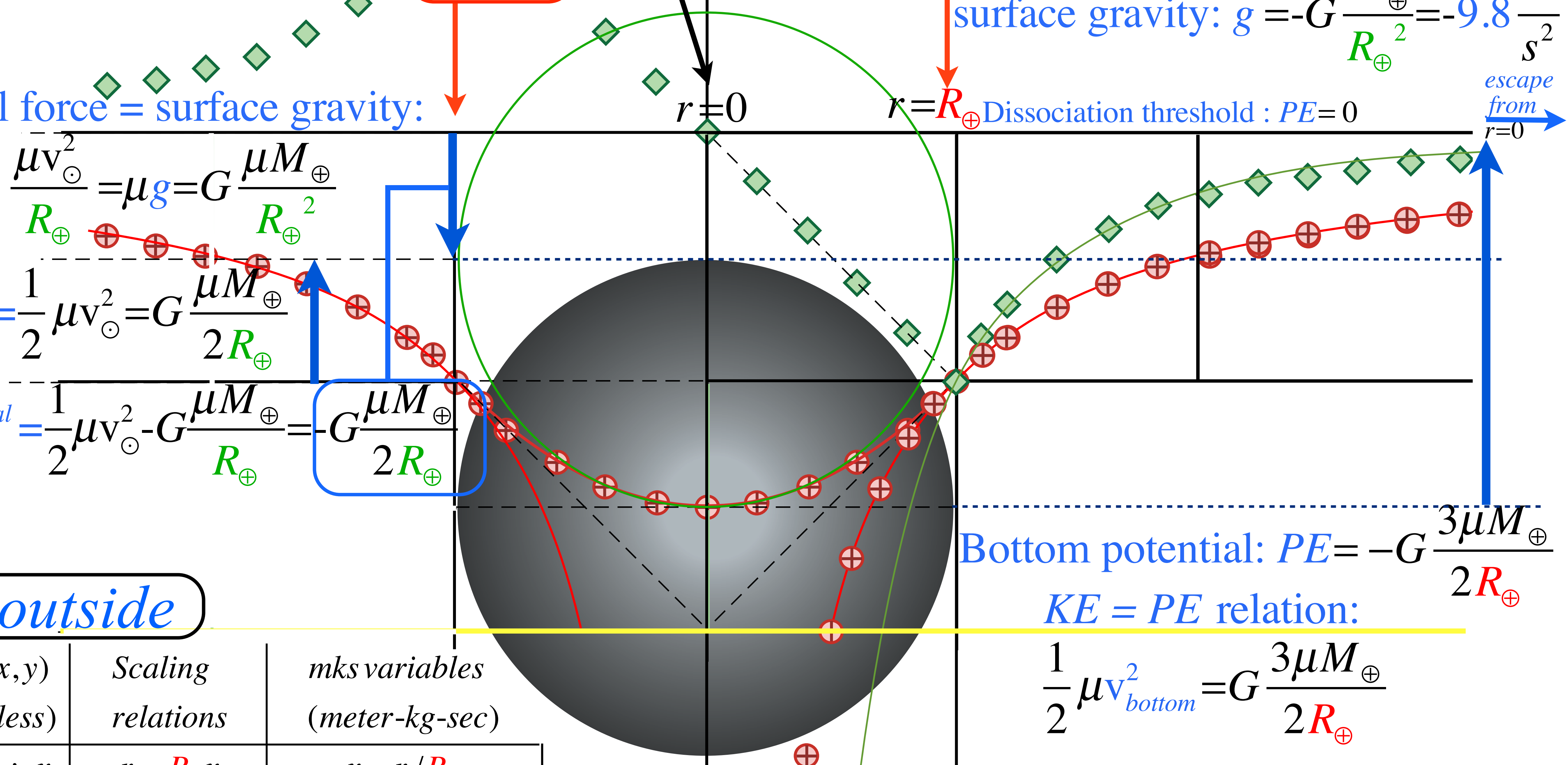
surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2} = -9.8\dfrac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^2}$$

Orbit KE $= \dfrac{1}{2}\mu v_\odot^2 = G\dfrac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total} = \dfrac{1}{2}\mu v_\odot^2 - G\dfrac{\mu M_\oplus}{R_\oplus} = -G\dfrac{\mu M_\oplus}{2R_\oplus}$



Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

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Geometric (x,y) (Dimensionless)	*Scaling relations*	*mks variables (meter-kg-sec)*		*inside*
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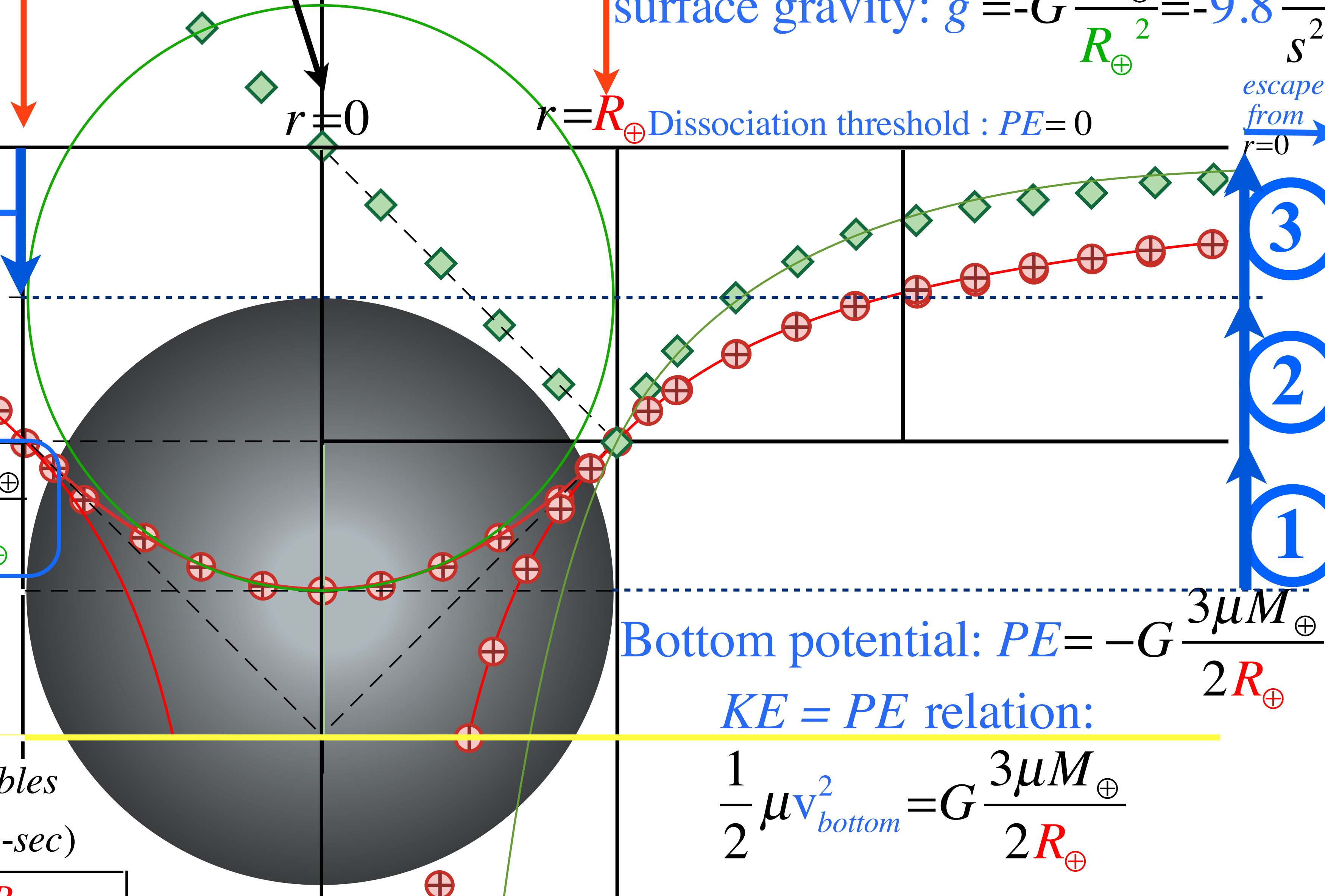
Sophomore-physics-Earth inside and out: "3-steps out of (or into) Hell"
...and surface orbit at $r=R_\oplus$



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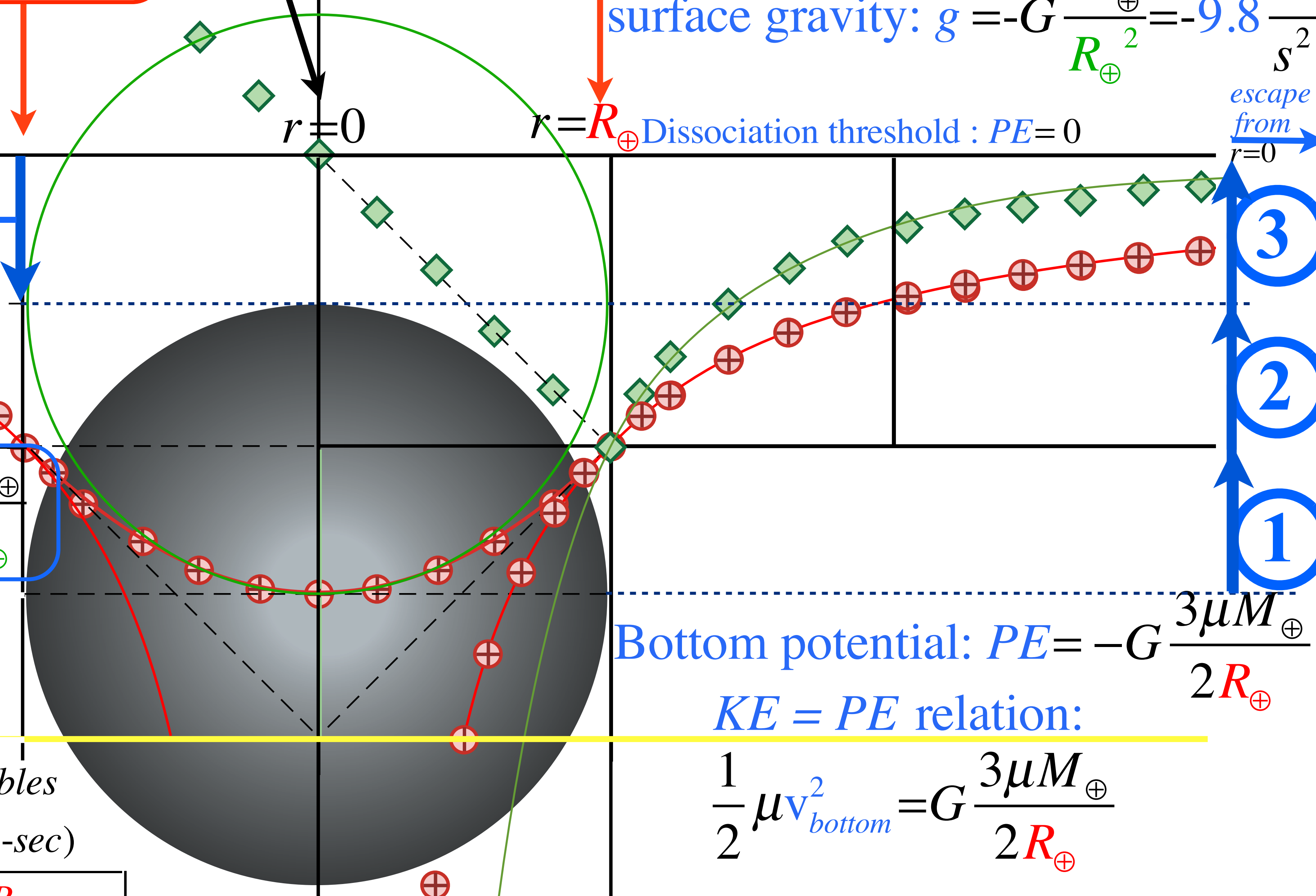

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($r=0$)-escape-velocity

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13.7km/sec

($r=R_\oplus$)-orbit speed:

$$v_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus}} = \sqrt{gR_\oplus}$$

7.9km/sec

Sophomore-physics-Earth inside and out: "3-steps out of (or into) Hell"
...and surface orbit at $r=R_\oplus$

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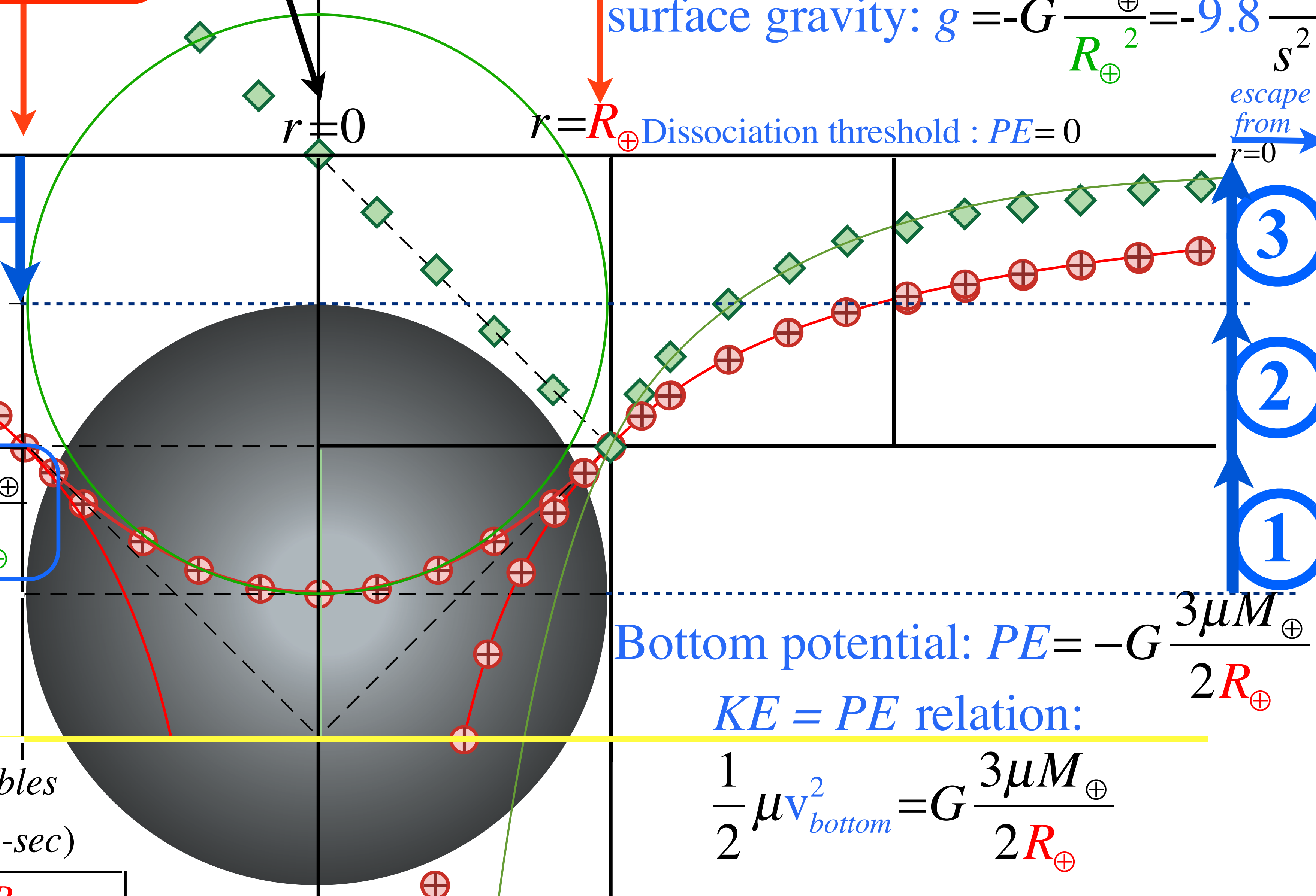
Orbit $E_\odot^{Total} = \dfrac{1}{2}\mu v_\odot^2 - G\dfrac{\mu M_\oplus}{R_\oplus} = -G\dfrac{\mu M_\oplus}{2R_\oplus}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$



Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

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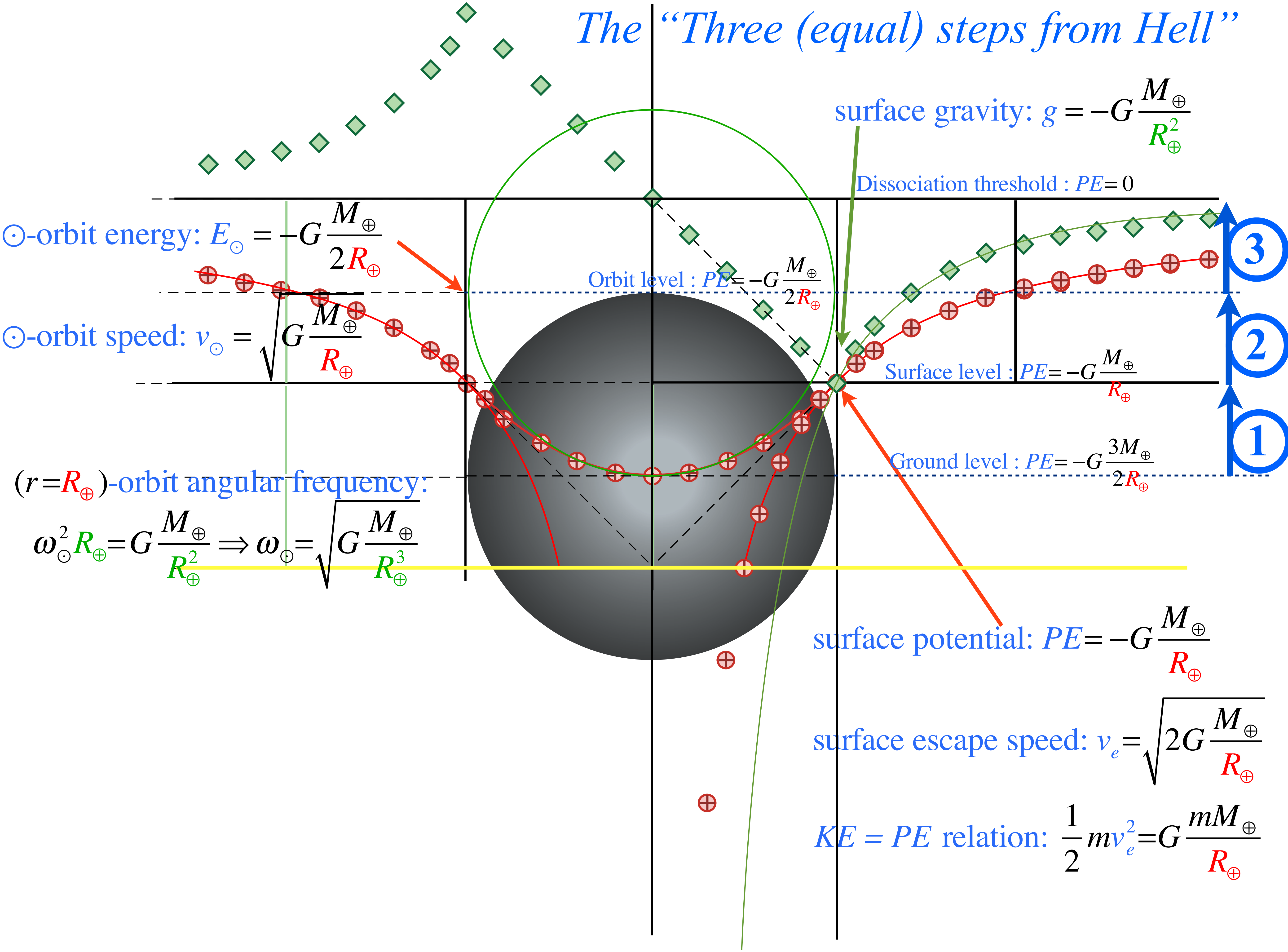
$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

$(r=R_\oplus)$-escape velocity:

$$v_{escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec



The "Three (equal) steps from Hell"
surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2}$
Dissociation threshold : $PE = 0$
⊙-orbit energy: $E_\odot = -G\frac{M_\oplus}{2R_\oplus}$
Orbit level : $PE = -G\frac{M_\oplus}{2R_\oplus}$
⊙-orbit speed: $v_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus}}$
Surface level : $PE = -G\frac{M_\oplus}{R_\oplus}$
Ground level : $PE = -G\frac{3M_\oplus}{2R_\oplus}$
3
2
1
$(r=R_\oplus)$-orbit angular frequency:
$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$
surface potential: $PE = -G\frac{M_\oplus}{R_\oplus}$
surface escape speed: $v_e = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$
$KE = PE$ relation: $\frac{1}{2}mv_e^2 = G\frac{mM_\oplus}{R_\oplus}$

The "Three (equal) steps from Hell"

surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2}$

$\odot$-orbit energy: $E_\odot = -G\frac{M_\oplus}{2R_\oplus}$

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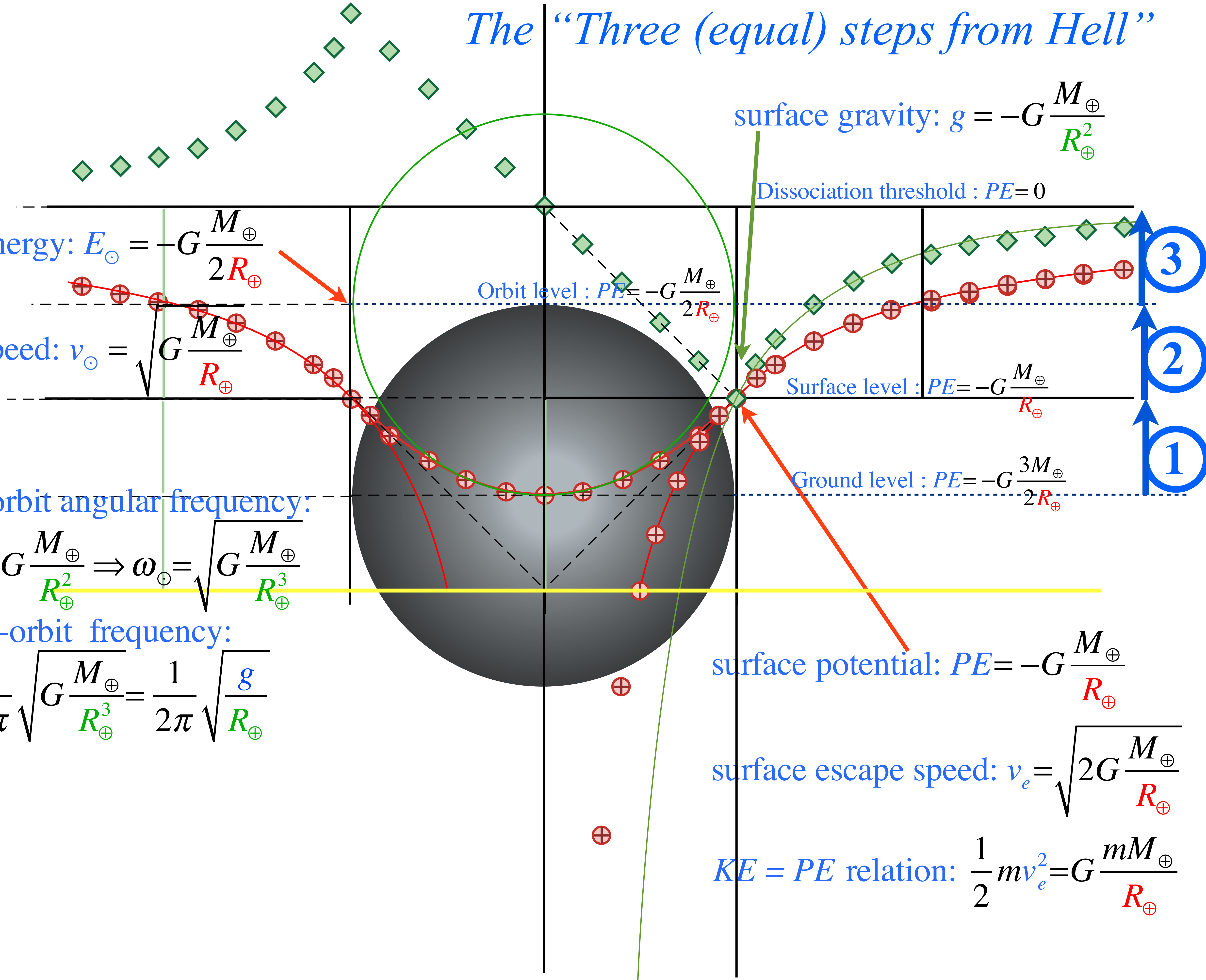

$(r = R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

$(r = R_\oplus)$-orbit frequency:

$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

surface potential: $PE = -G\frac{M_\oplus}{R_\oplus}$

surface escape speed: $v_e = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$

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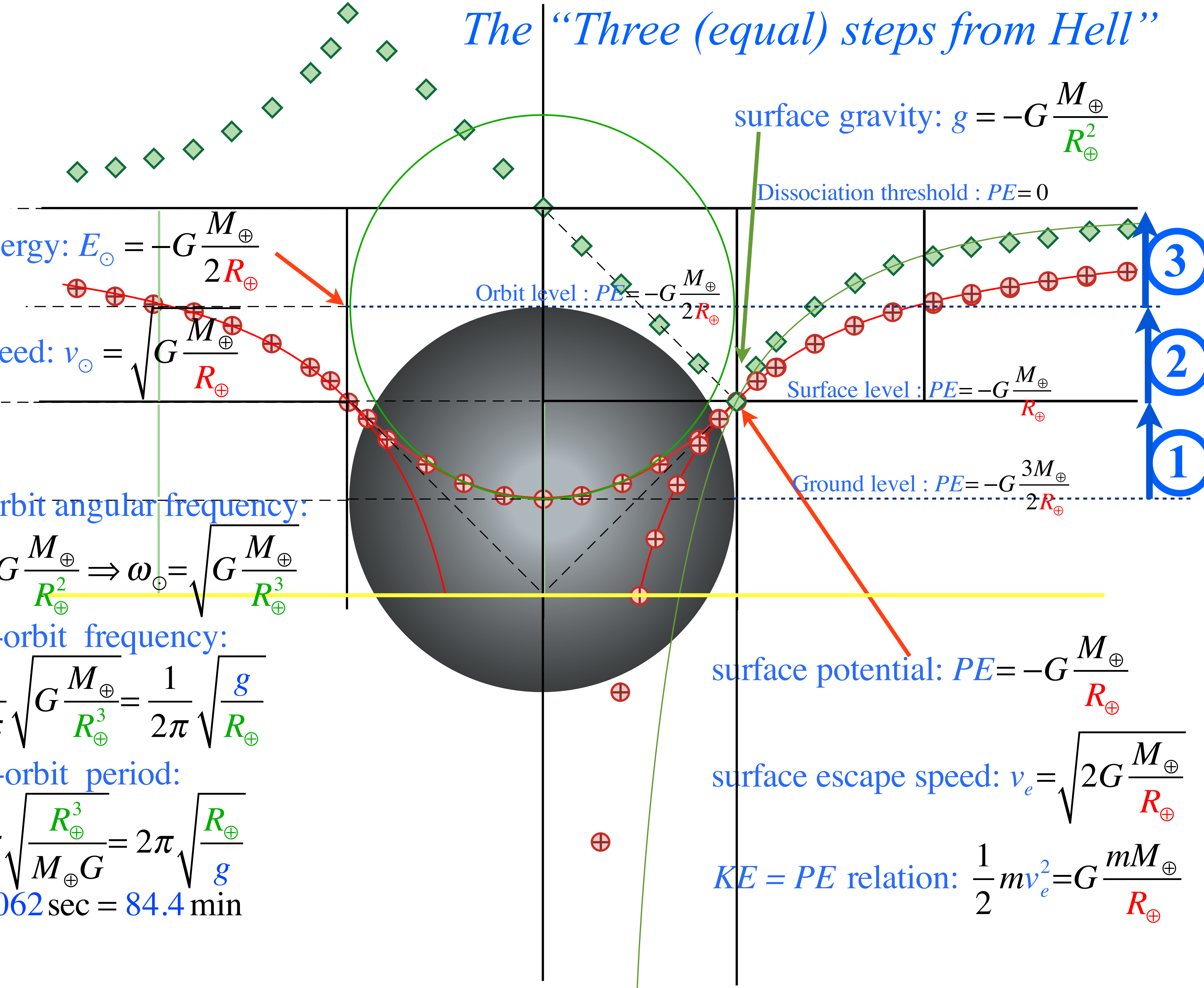
The "Three (equal) steps from Hell"



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$(r=R_\oplus)$-orbit frequency:

$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

$(r=R_\oplus)$-orbit period:

$$\upsilon_\odot = 2\pi\sqrt{\frac{R_\oplus^3}{M_\oplus G}} = 2\pi\sqrt{\frac{R_\oplus}{g}}$$

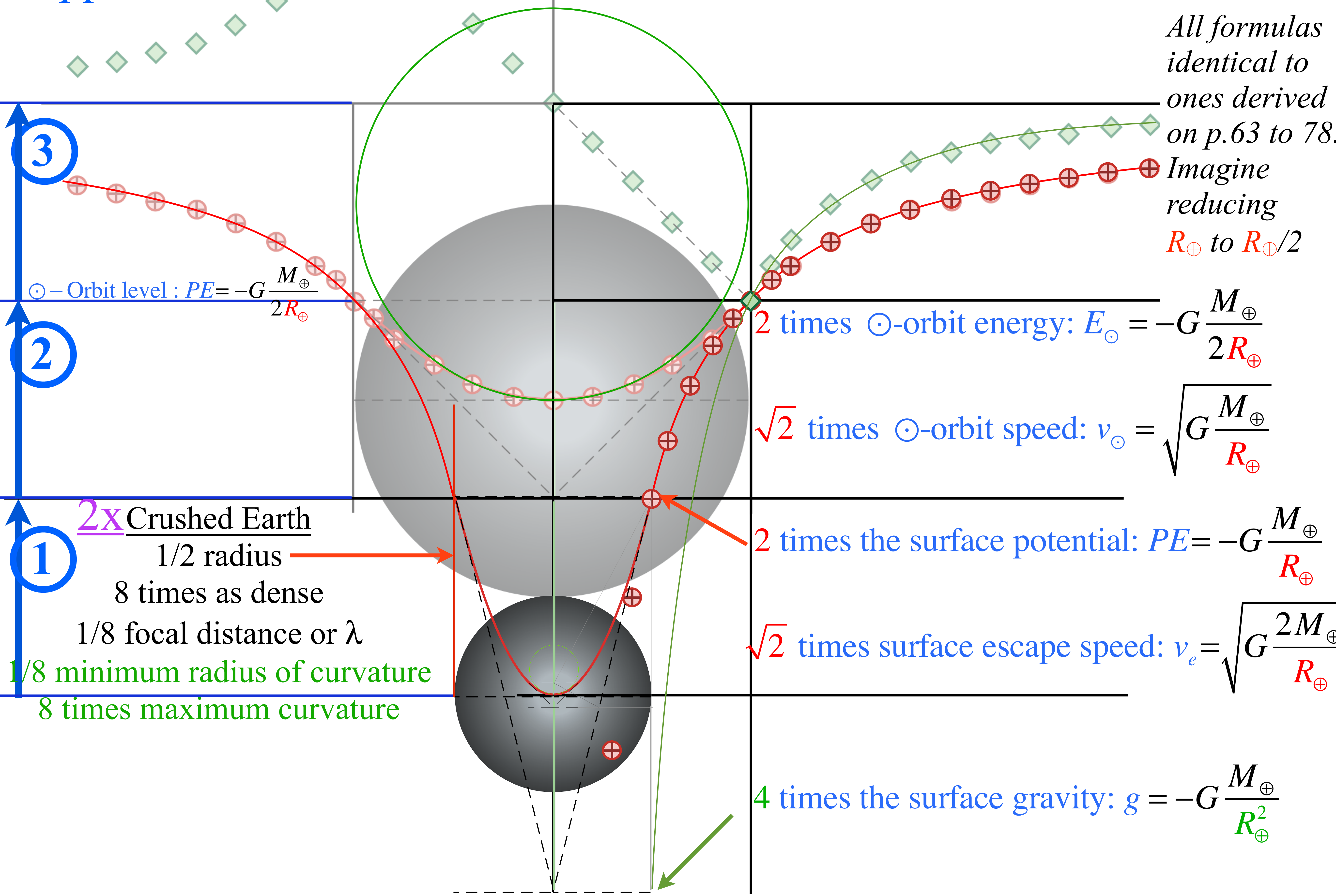
$= 5062\,\text{sec} = 84.4\,\text{min}$

surface potential: $PE = -G\frac{M_\oplus}{R_\oplus}$

surface escape speed: $v_e = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$

$KE = PE$ relation: $\frac{1}{2}mv_e^2 = G\frac{mM_\oplus}{R_\oplus}$

Suppose Earth radius crushed to 1/2: *($R_\oplus$=6.4·10^{6}m crushed to $R_\oplus$/2=3.2·10^{6}m)*



Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

→ *Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Examples of "crushed" matter

Earth matter

$Earth\,mass: M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ Density $\rho_{\oplus} = ??$

$Earth\,radius: R_{\oplus} = 6.371 \cdot 10^{6}\,m \simeq 6.4 \cdot 10^{6}\,m \quad Earth\,volume: (4\pi/3)R_{\oplus}{}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^{3}$

$(6.4)^3 \sim 262$ *and* $(4\pi/3)260 = 1098 \sim 10^3$

Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus^{\ 3}}$$

Earth matter

$Earth\,mass: M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ Density $\rho_\oplus \sim 6.0 \cdot 10^{24\text{-}21} \sim 6 \cdot 10^3 kg/m^3$

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$(6.4)^{3} \sim 262$ and $(4\pi/3)260 = 1089 \sim 10^{3}$

Density of solid Fe $= 7.9 \cdot 10^{3} kg/m^{3}$

Density of liquid Fe $= 6.9 \cdot 10^{3} kg/m^{3}$

Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus{}^3}$$

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Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27}kg. \sim 2 \cdot 10^{-27}kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27}kg$.

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$^{4\pi}/_3 3^3 = 36\pi = 113 \sim 10^2$

That's $100 \cdot 10^{-27} = 10^{-25}\ kg$ packed into a volume of $^{4\pi}/_3 r^3 = {}^{4\pi}/_3\ (3 \cdot 10^{-15})^3\ m^3$ or about $10^{-43}\ m^3$.

$36\pi = 113 \sim 10^2$

Examples of “crushed” matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus{}^3}$$

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Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in a fingertip$(1\text{cm})^3$.

$(1cm)^3 = (10^{-2}m)^3 = 10^{-6}m^3$

Examples of "crushed" matter

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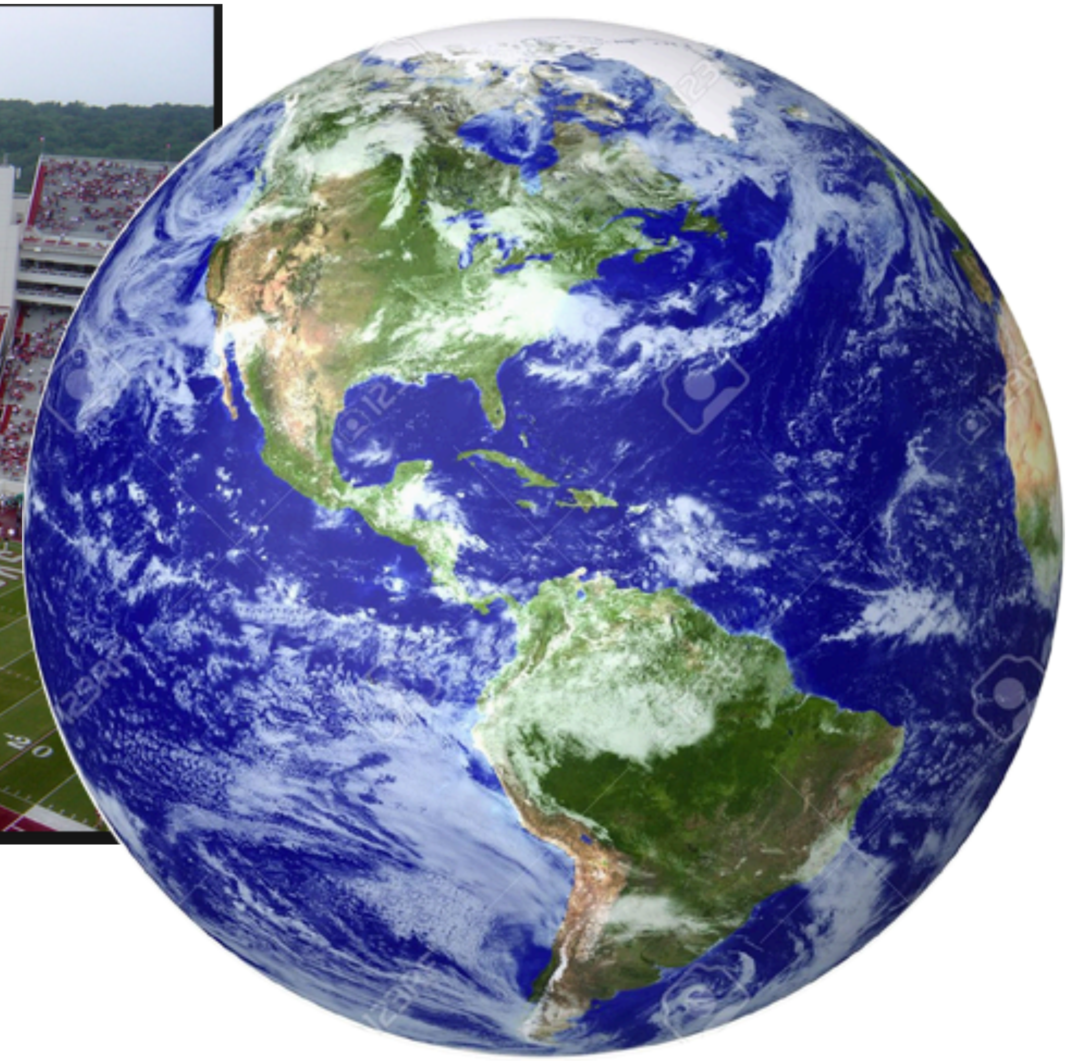
Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.



Geometry and algebra of idealized "Sophomore-physics Earth" fields

Coulomb field *<u>outside</u>* *Isotropic Harmonic Oscillator (IHO) field* *<u>inside</u>*

Contact-geometry of potential curve(s) and "kite" geometry

"Ordinary-Earth" models: 3 key energy "steps" and 4 key energy "levels"

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

→ *Introducing the "neutron starlet"*

*Fantasizing a "***Black-Hole-Earth***"*

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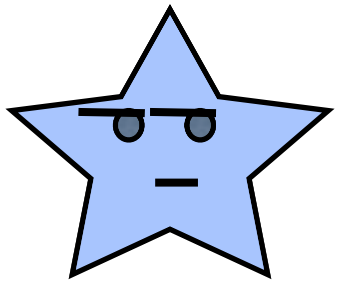
Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300\,m$ would approach neutron-star density.

Introducing the "Neutron starlet"

1 cm^3 *of nuclear matter: mass* $= 10^{12}$ *kg.*



Geometry and algebra of idealized "Sophomore-physics Earth" fields

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

Contact-geometry of potential curve(s) and "kite" geometry

"Ordinary-Earth" models: 3 key energy "steps" and 4 key energy "levels"

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

Introducing the "neutron starlet"

→ *Fantasizing a "***Black-Hole-Earth***"*

Examples of "crushed" matter

Earth matter

Earth mass : $M_{\oplus} = 5.9722 \times 10^{24}\, kg. \simeq 6.0 \cdot 10^{24}\, kg$. Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^{3} kg/m^{3}$

Earth radius : $R_{\oplus} = 6.371 \cdot 10^{6}\, m \simeq 6.4 \cdot 10^{6}\, m$ *Earth volume* : $(4\pi/3) R_{\oplus}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\, m^{3}$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

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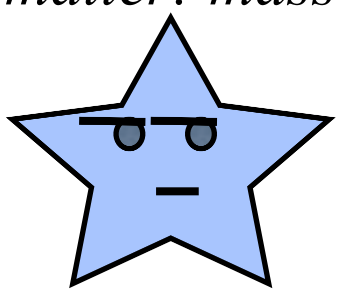

Fantasizing the "**Black Hole Earth**"

Suppose Earth is crushed so that its surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^{8} m/s$.

$c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$V_{escape} = \sqrt{(2GM/R_{\otimes})}$

(from p. 65, 66,..,75)

$G=6.67384(80) \cdot 10^{-11} Nm^{2}/C^{2} \sim (2/3) 10^{-10}$

Examples of "crushed" matter

$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus^{\ 3}}$

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Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

$^{4\pi}/_3 3^3 = 36\pi = 113 \sim 10^2$

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Introducing the "Neutron starlet"

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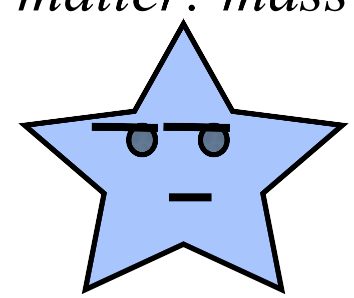

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(from p. 65, 66,..,75)

$c = \sqrt{(2GM/R_\otimes)}$

$R_\otimes = 2GM/c^2 = 8.9mm \sim 1cm$

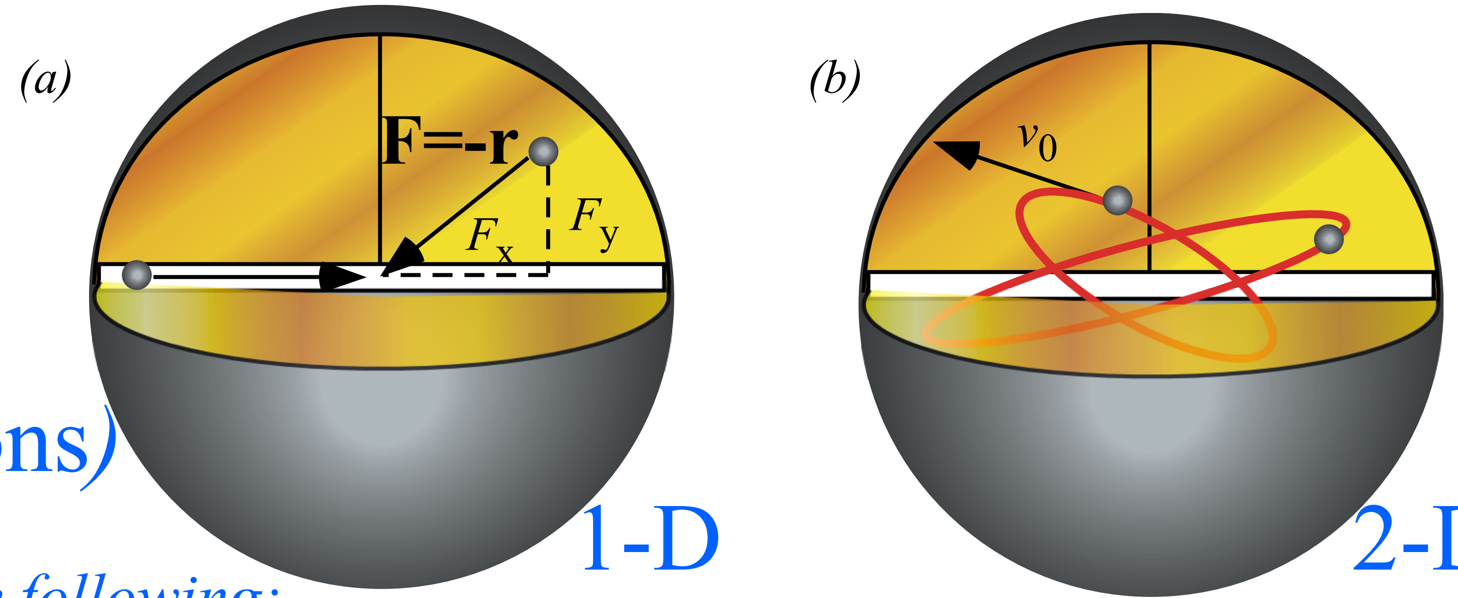
(fingertip size!)

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

→ *Introducing 2D IHO orbits and phasor geometry*

Phasor "clock" geometry

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

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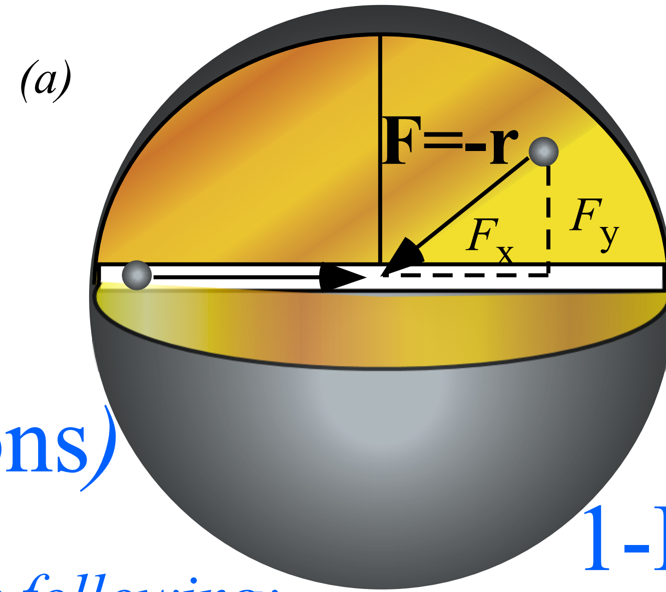
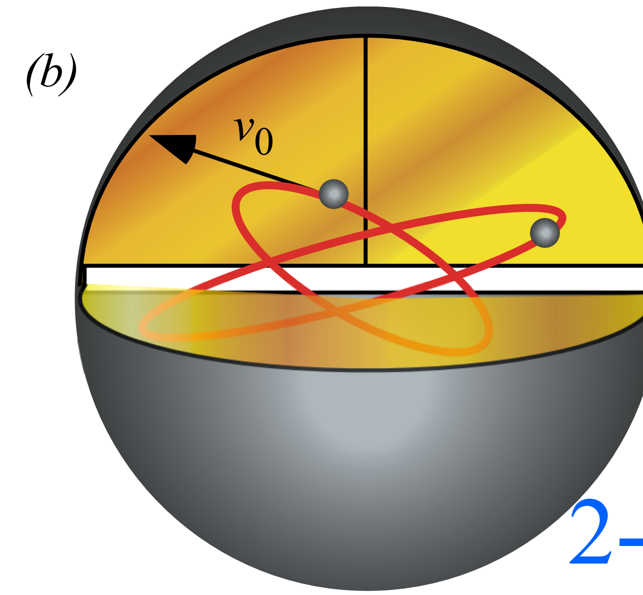
$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D



Unit 1
Fig. 9.10

2-D or 3-D

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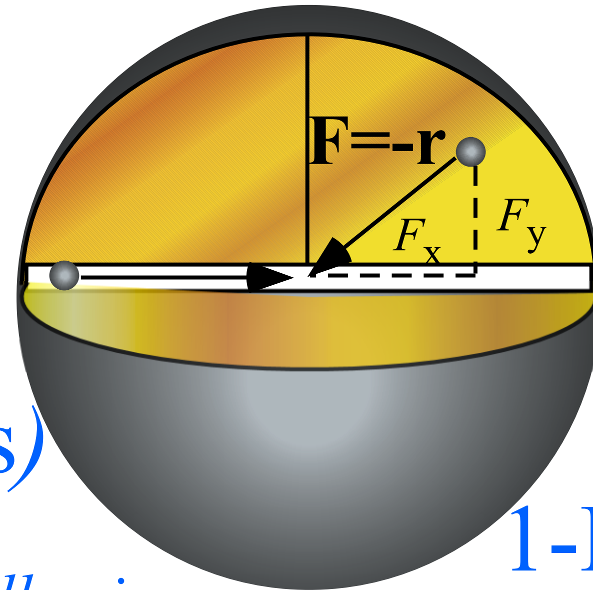
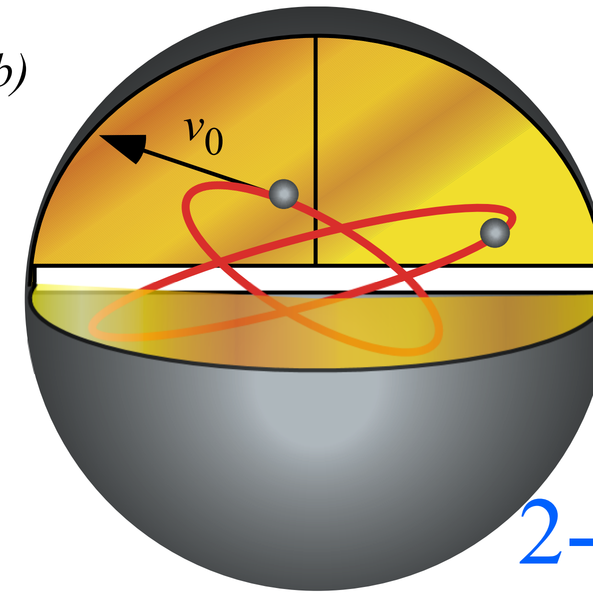
Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 9.10

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1-D

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$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

velocity: *position:*

$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

F =-x (1-Dimension)

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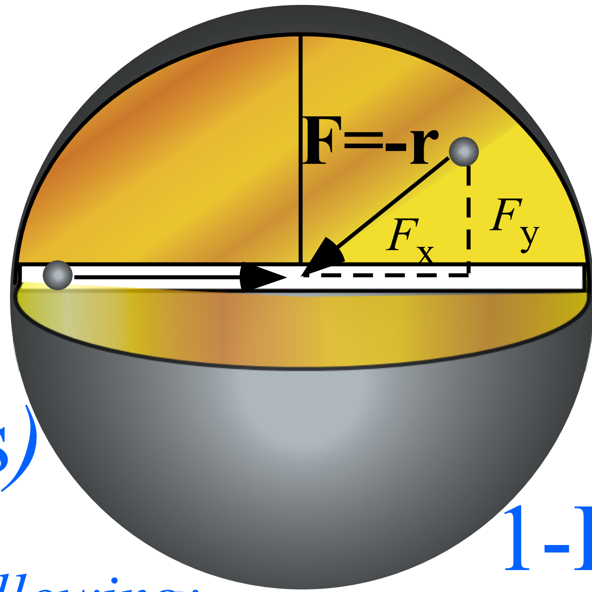
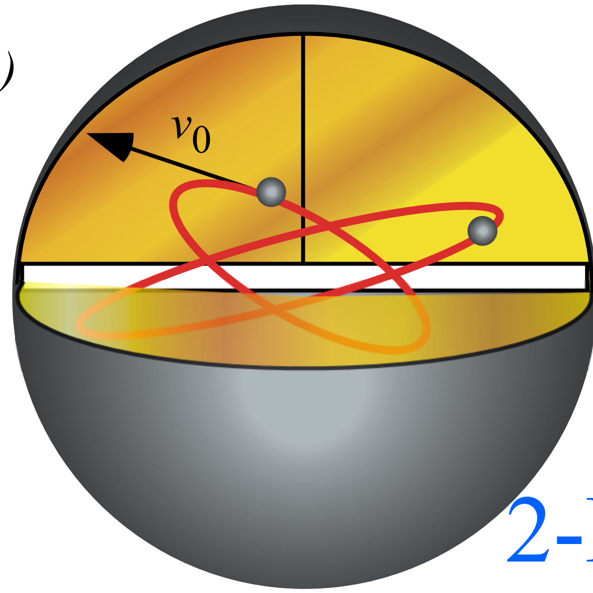
(a)



1-D

(b)



Unit 1
Fig. 9.10

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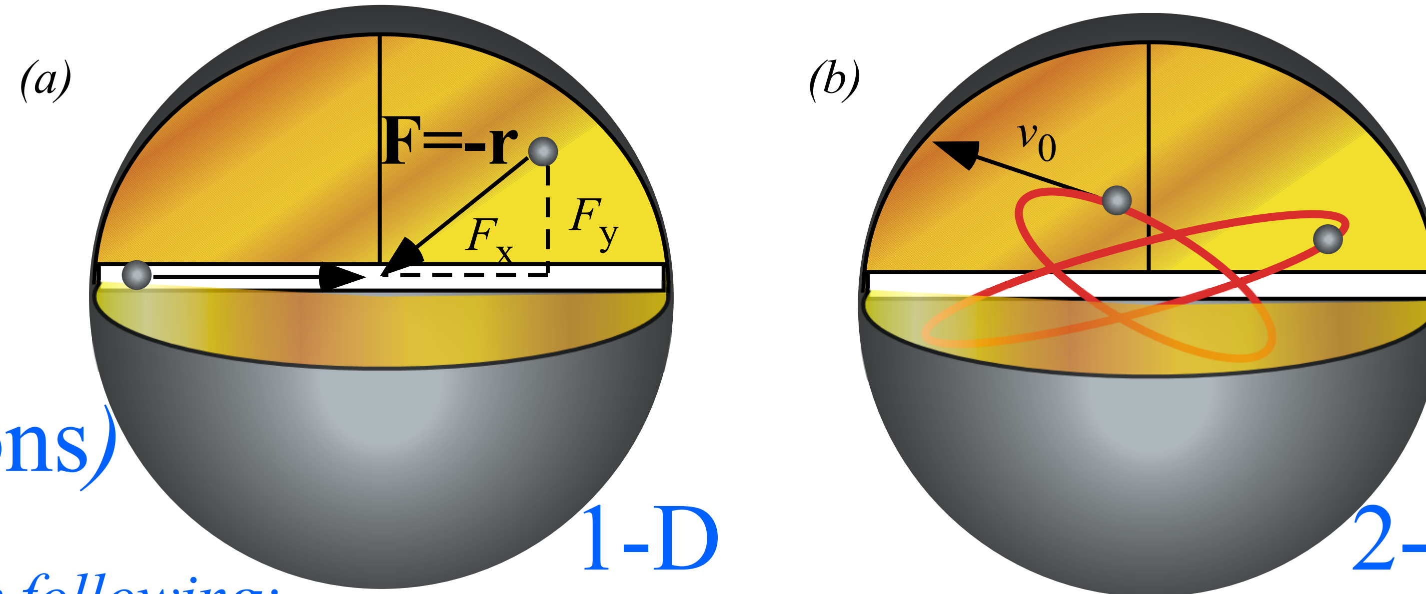
Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 9.10

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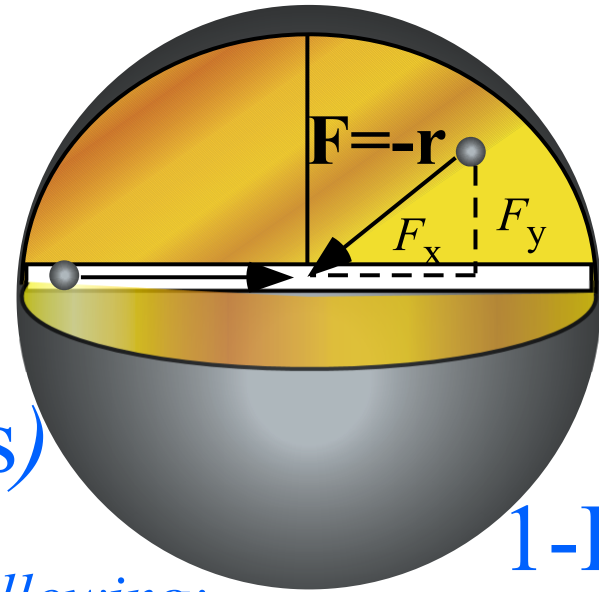
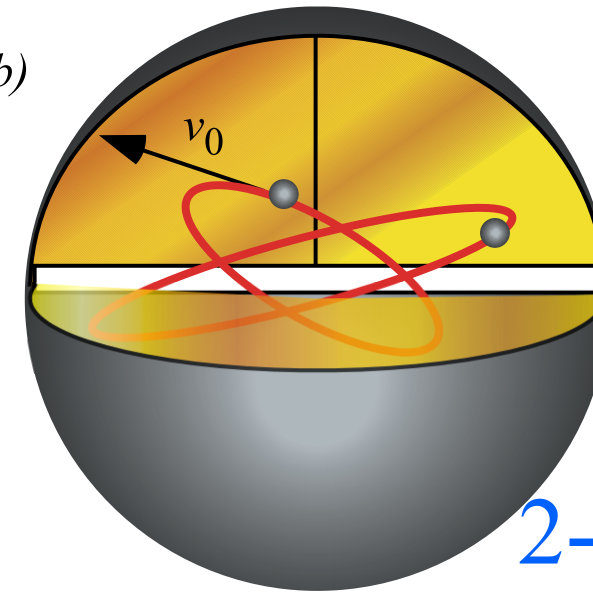
angular velocity: def. **(3)** $\omega = \frac{d\theta}{dt}$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt}$$

by **(1)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 9.10

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$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta}$$

by **(1)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

$F = -x$ (1-Dimension)

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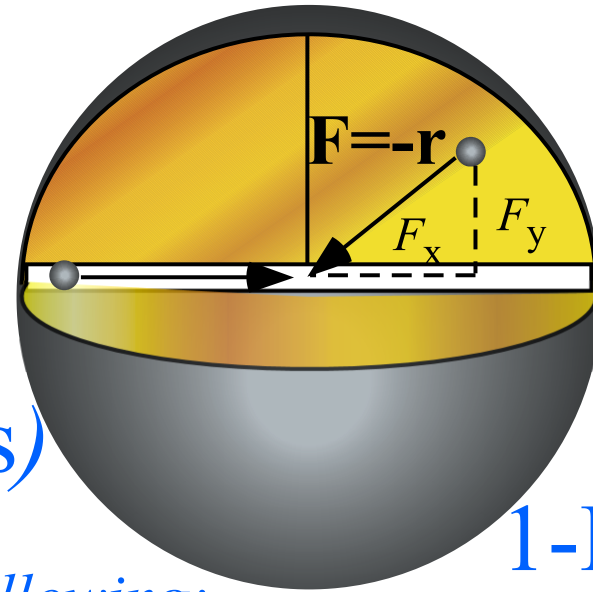
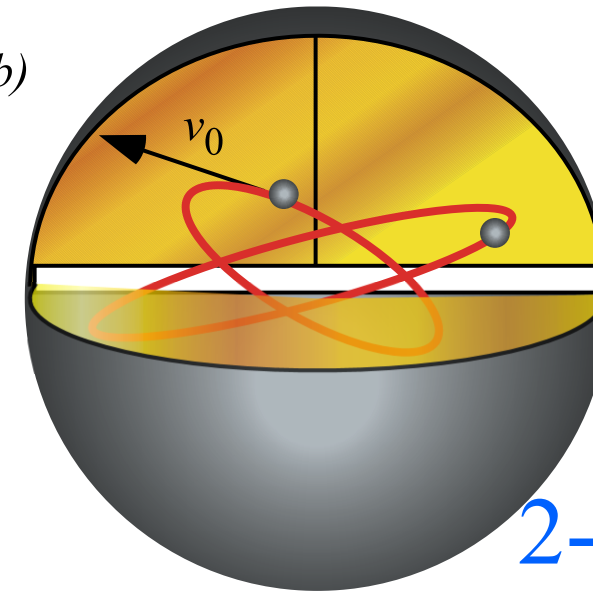
(a)



1-D

(b)



Unit 1
Fig. 9.10

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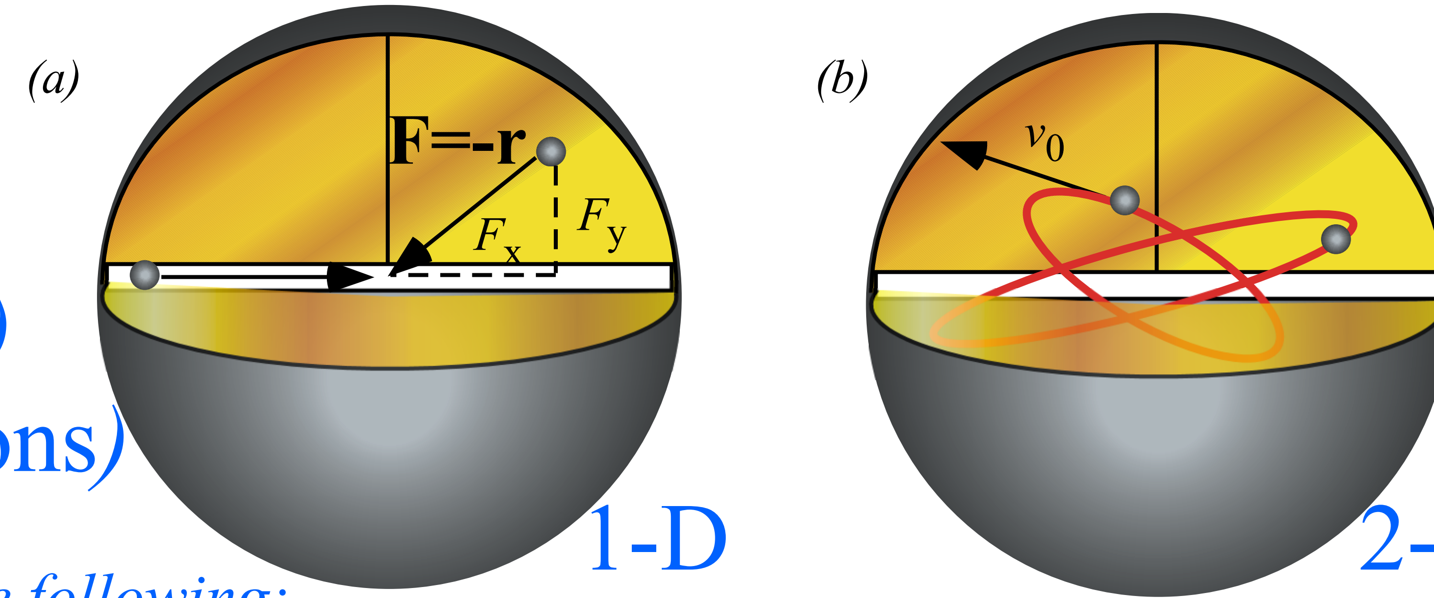
angular velocity: *def.* **(3)** $\omega = \frac{d\theta}{dt}$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta}$$

by **(1)** *by def.* **(3)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
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Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

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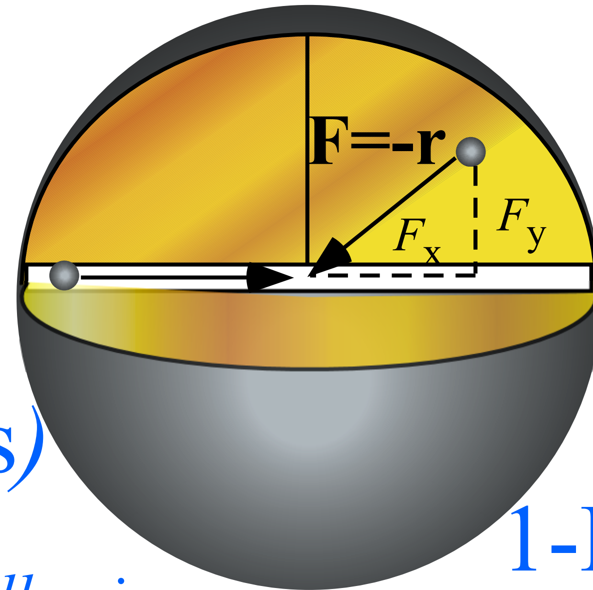
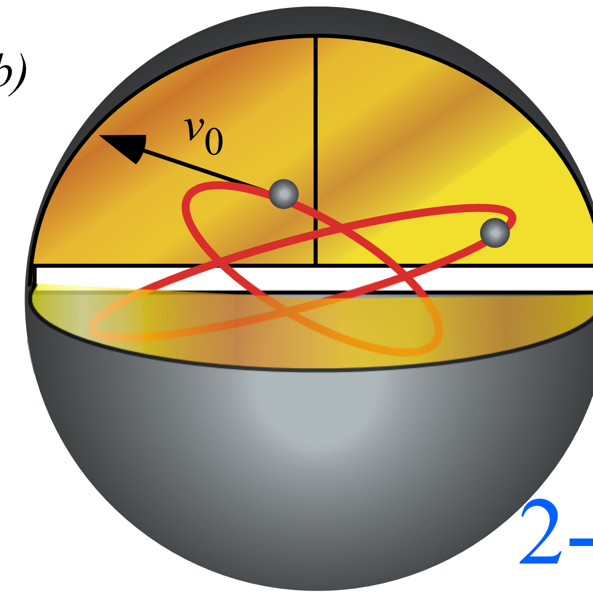


1-D

(b)



Unit 1
Fig. 9.10

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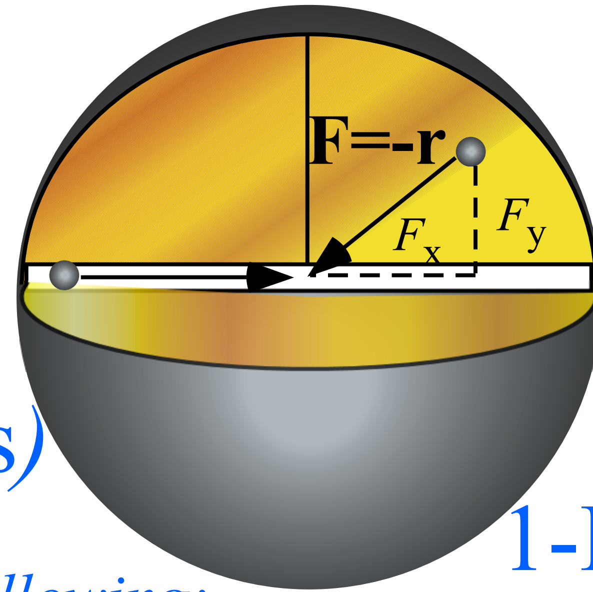
by (1) | by def. (3) | by (2)

by def. (3)

$$\omega = \frac{d\theta}{dt}$$

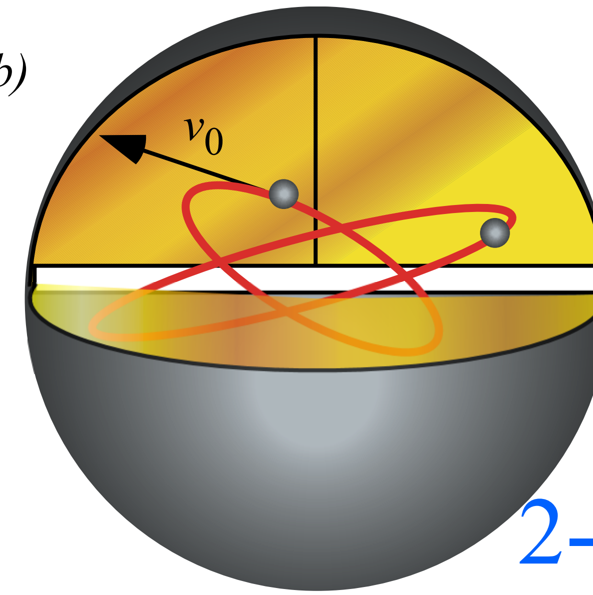
Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



1-D

(b)



Unit 1
Fig. 9.10

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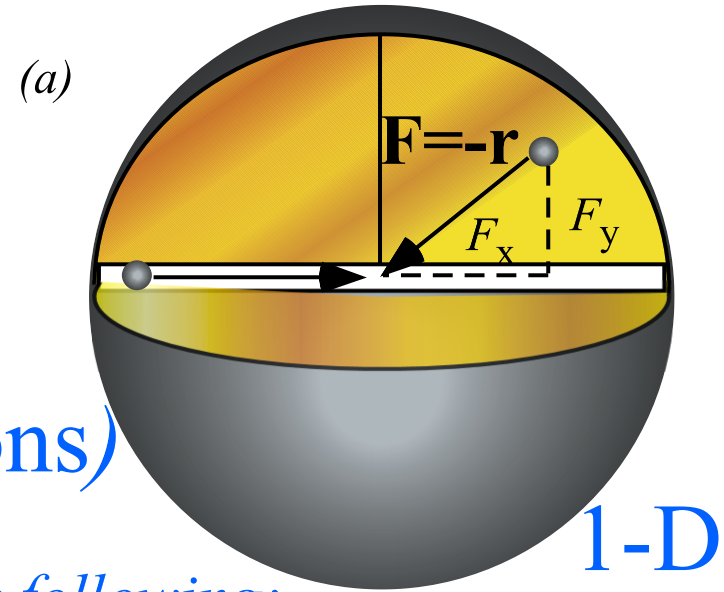
by **(1)** — *by def.* **(3)** — *by* **(2)**

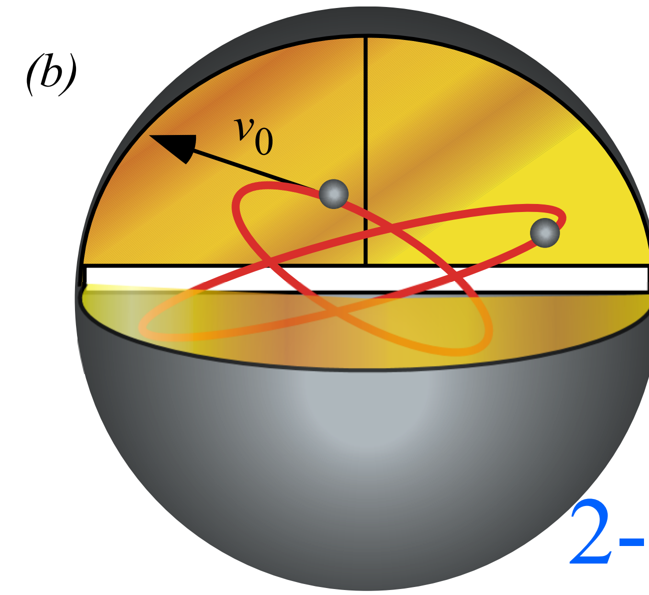
by def. **(3)**

$$\omega = \frac{d\theta}{dt} = \frac{\sqrt{\frac{2E}{m}}\cos\theta}{\sqrt{\frac{2E}{k}}\cos\theta}$$

divide **(1)** — *by* **(2)** *derivative*

Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

F =-*x* (1-Dimension)

F =-**r** (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let: \ \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and: \ \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \mathbf{(3)}\ \ \omega = \frac{d\theta}{dt}$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by **(1)**   *by def.* **(3)**   *by* **(2)**

by def. **(3)**

$$\omega = \frac{d\theta}{dt} = \frac{\sqrt{\frac{2E}{m}}\cos\theta}{\sqrt{\frac{2E}{k}}\cos\theta} = \sqrt{\frac{k}{m}}$$

divide **(1)**   *by* **(2)** *derivative*

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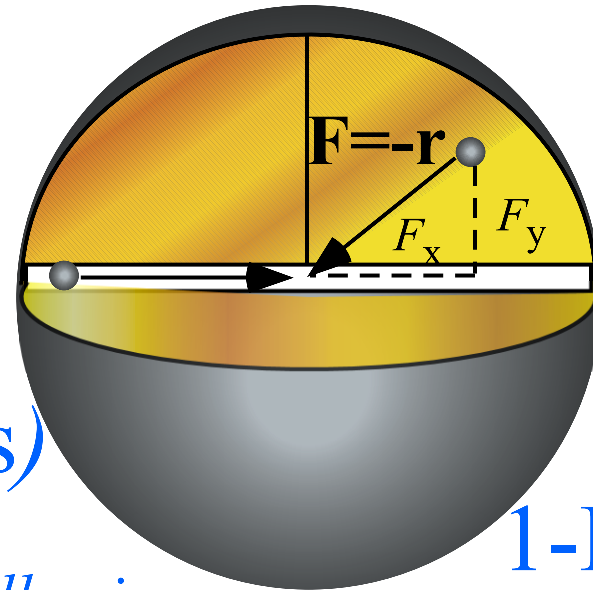
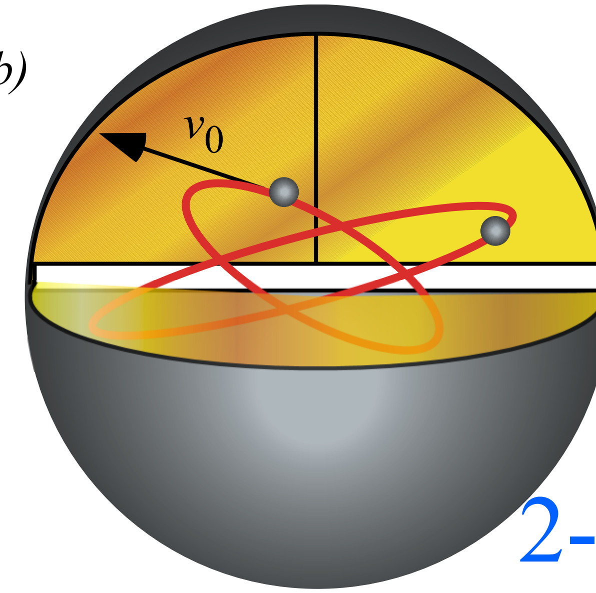
(a)



1-D

(b)



Unit 1
Fig. 9.10

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

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by **(1)** *by def.* **(3)** *by* **(2)**

by def. **(3)**

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

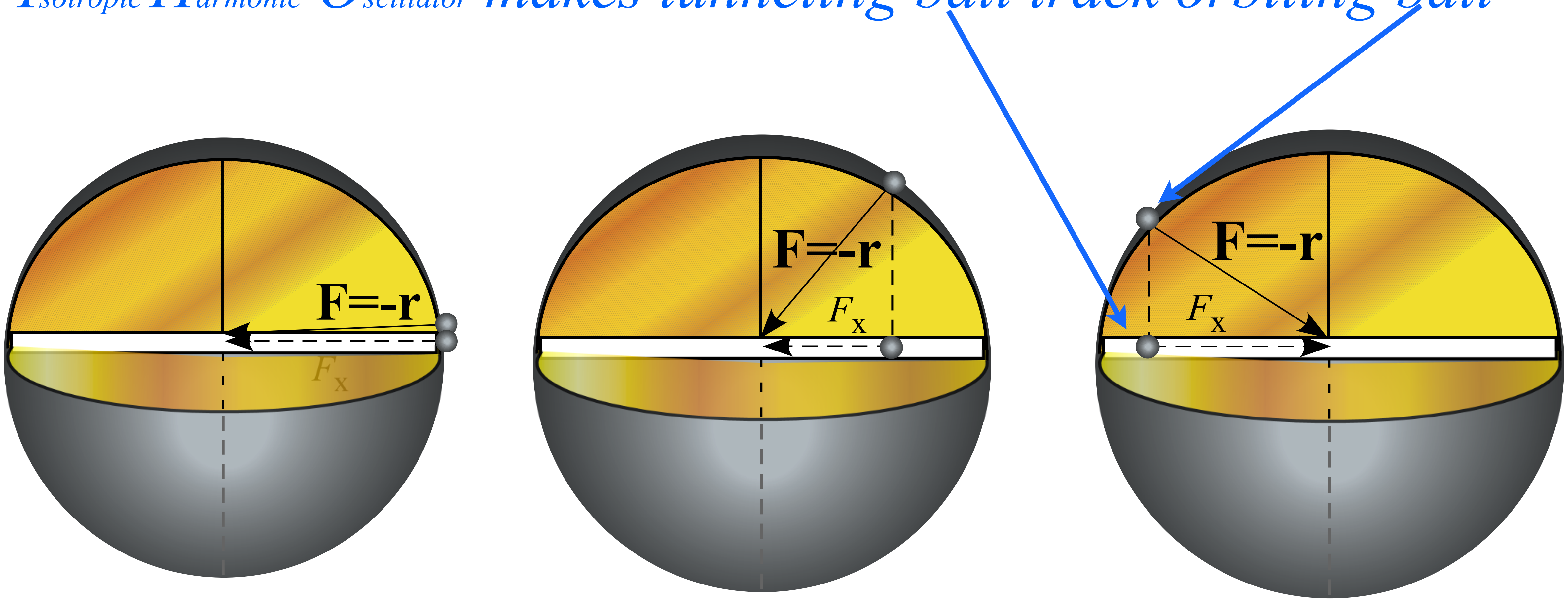
by integration given constant ω:

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

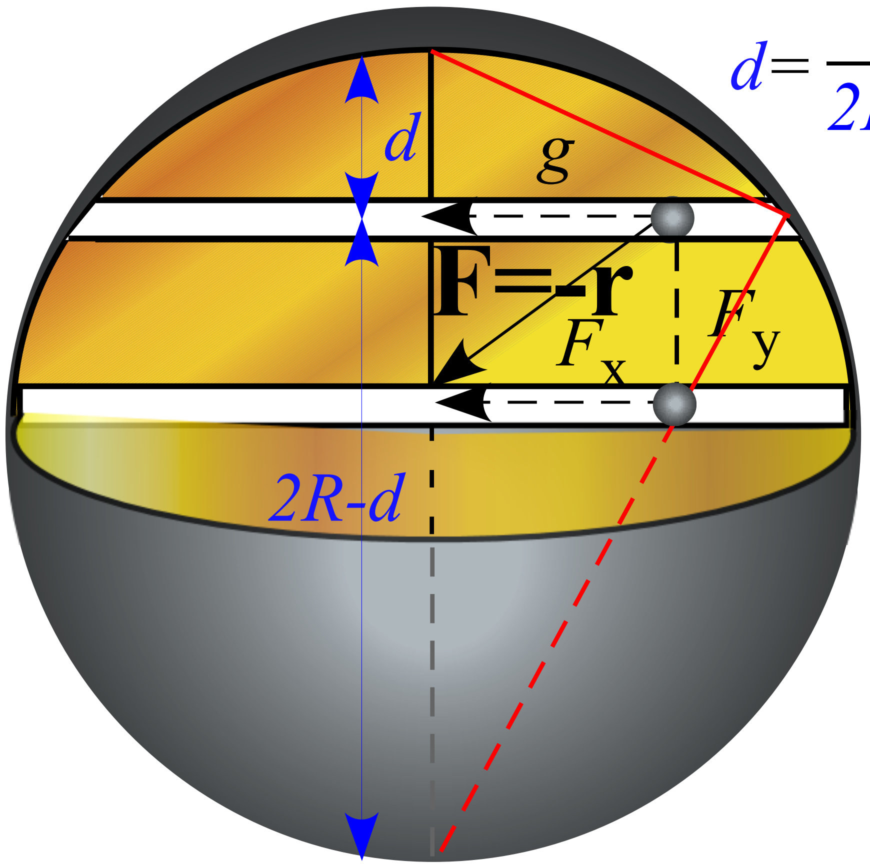
→ *Introducing 2D IHO orbits and phasor geometry*

Phasor "clock" geometry

Isotropic Harmonic Oscillator makes tunneling ball track orbiting ball

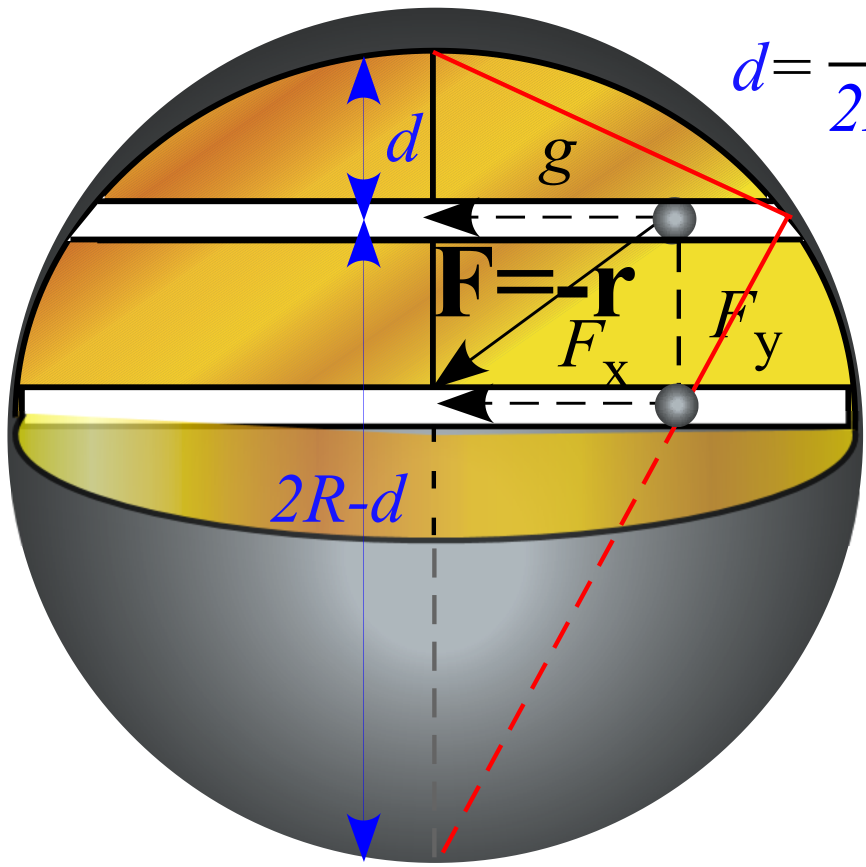


Isotropic Harmonic Oscillator makes balls in parallel tunnel track each other



$$d = \frac{g^2}{2R-d} \sim \frac{g^2}{2R}$$

Isotropic Harmonic Oscillator makes balls in parallel tunnels track each other...



$$d = \frac{g^2}{2R-d} \sim \frac{g^2}{2R}$$

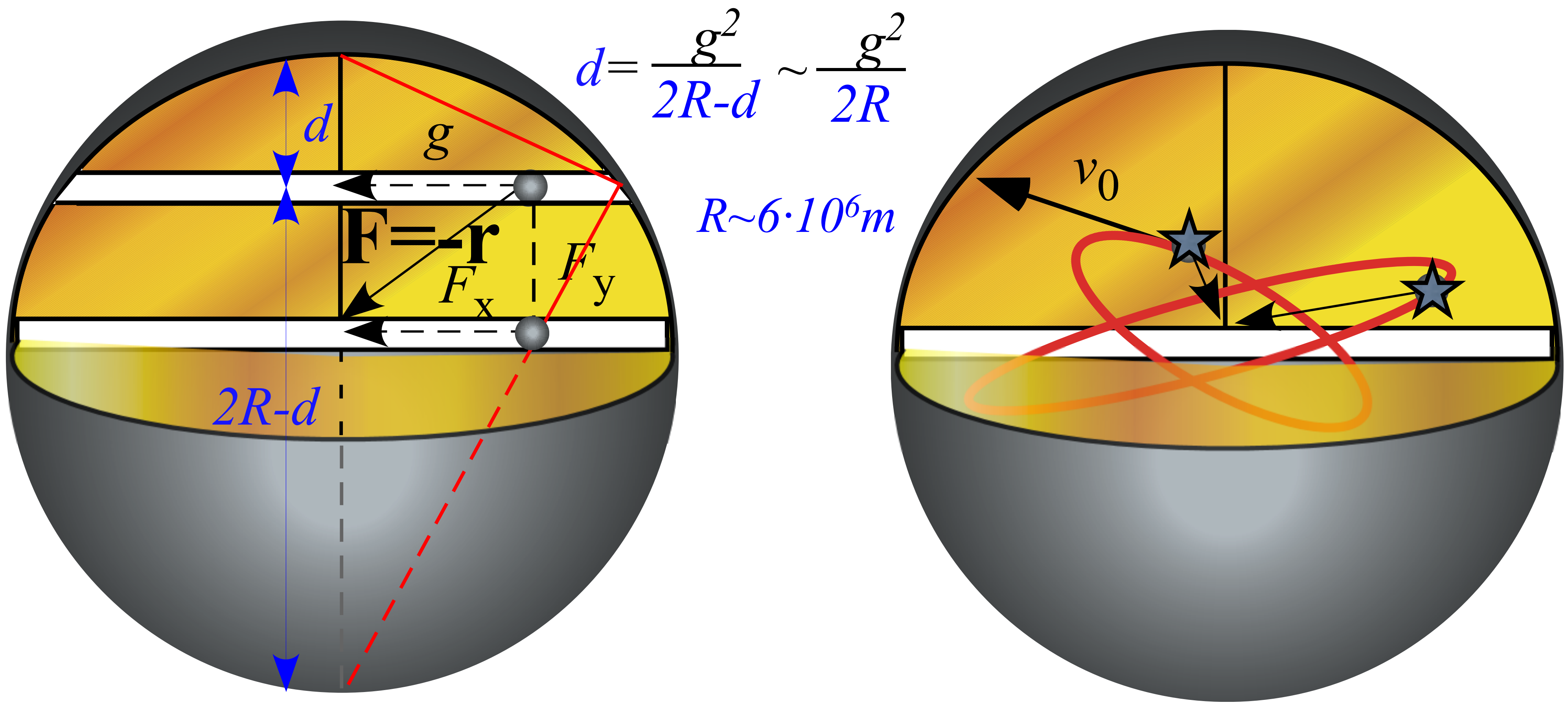
$R \sim 6 \cdot 10^6 m$

$$d \sim \frac{1}{2R}$$

...even if track length is just $g = 1m$ *so* $d \sim (1/12)$*micron*

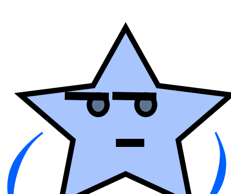
They all take about 84 minutes to go from right to left and back, again.

Isotropic Harmonic Oscillator makes balls in parallel tunnels track each other...



...even if track length is just $g = 1m$ *so* $d = (1/12)$*micron*

The all take about 84 minutes to go from right to left and back, again.

Most neutron starlet ( *)orbits are centered ellipses*

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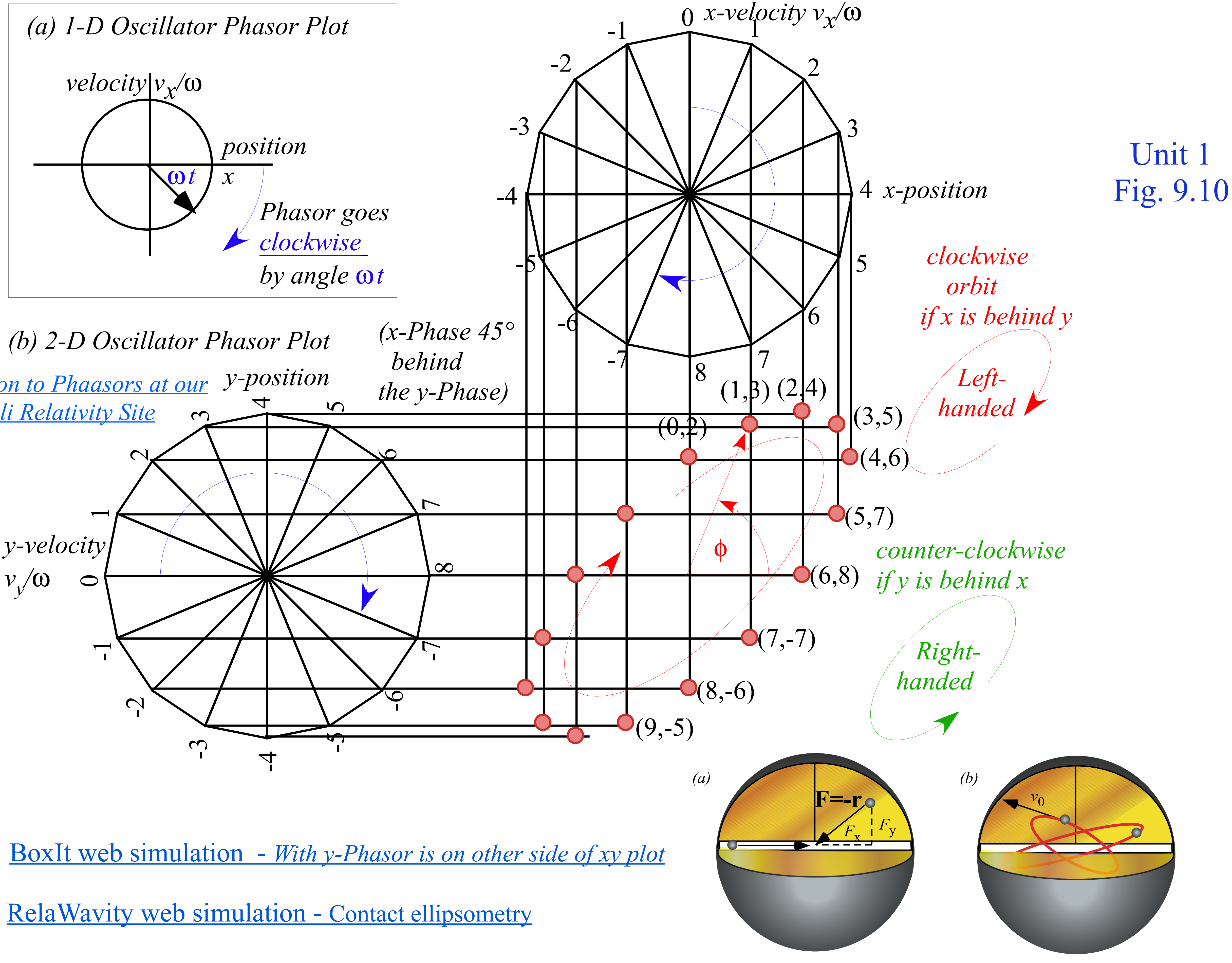
Unit 1
Fig. 9.10



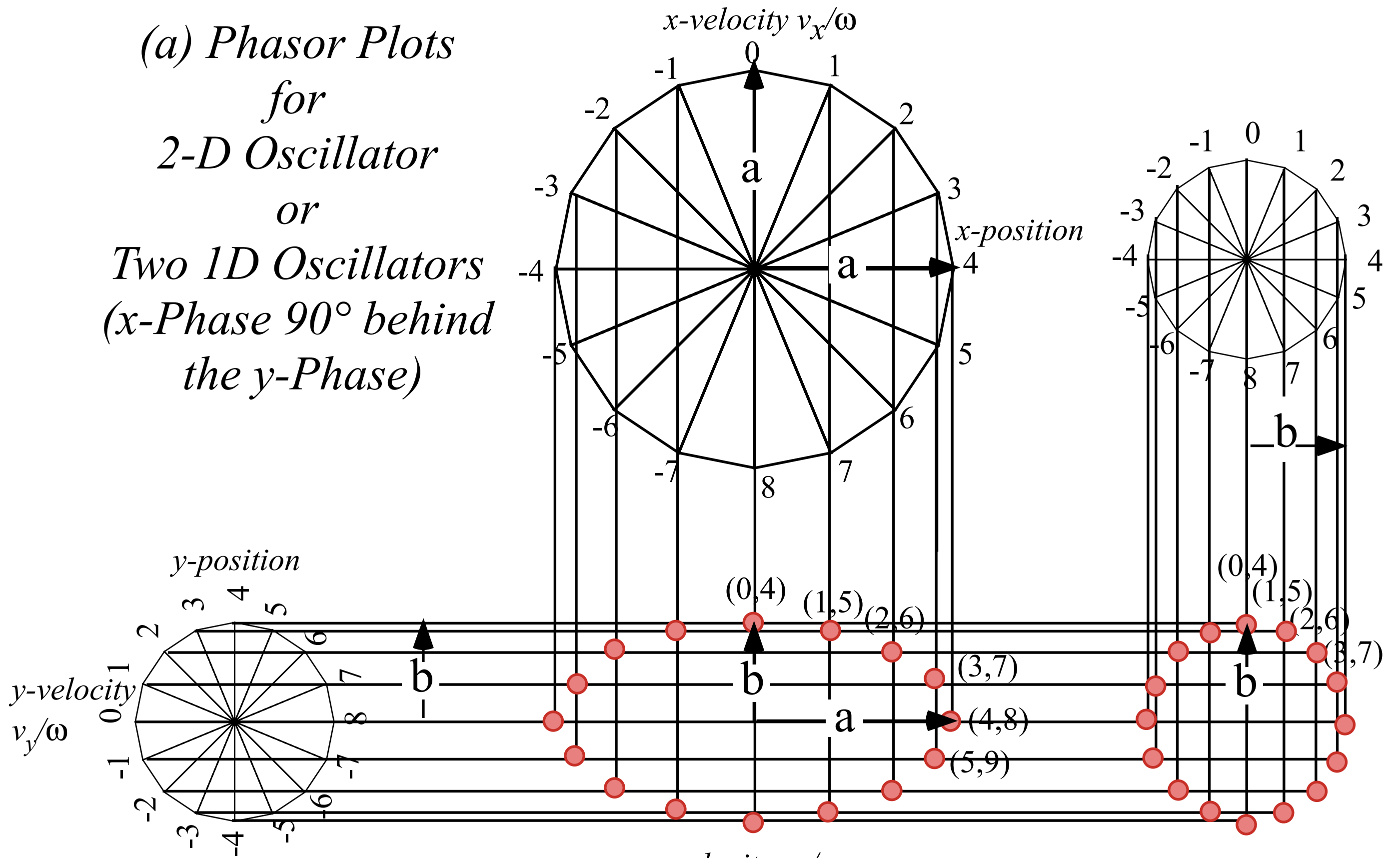
[Introduction to Phaasors at our Pirelli Relativity Site]()

[BoxIt web simulation - *With y-Phasor is on other side of xy plot*]()

[RelaWavity web simulation - Contact ellipsometry]()

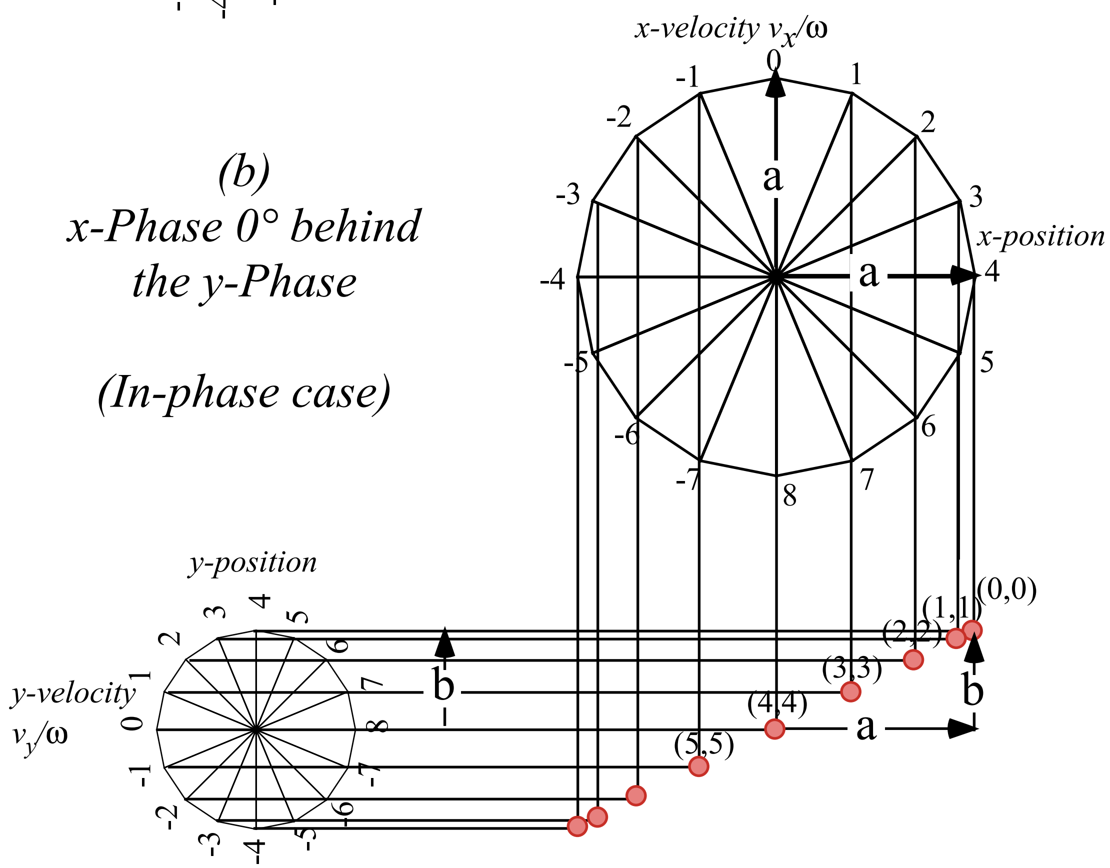
Unit 1
Fig. 9.12

(a) Phasor Plots for 2-D Oscillator or Two 1D Oscillators (x-Phase 90° behind the y-Phase)



(b) x-Phase 0° behind the y-Phase

(In-phase case)



These are more generic examples with radius of x-phasor differing from that of the y-phasor.

RelaWavity web simulation - Contact ellipsometry *(User Mouse Input allowed for setting phasor values)*