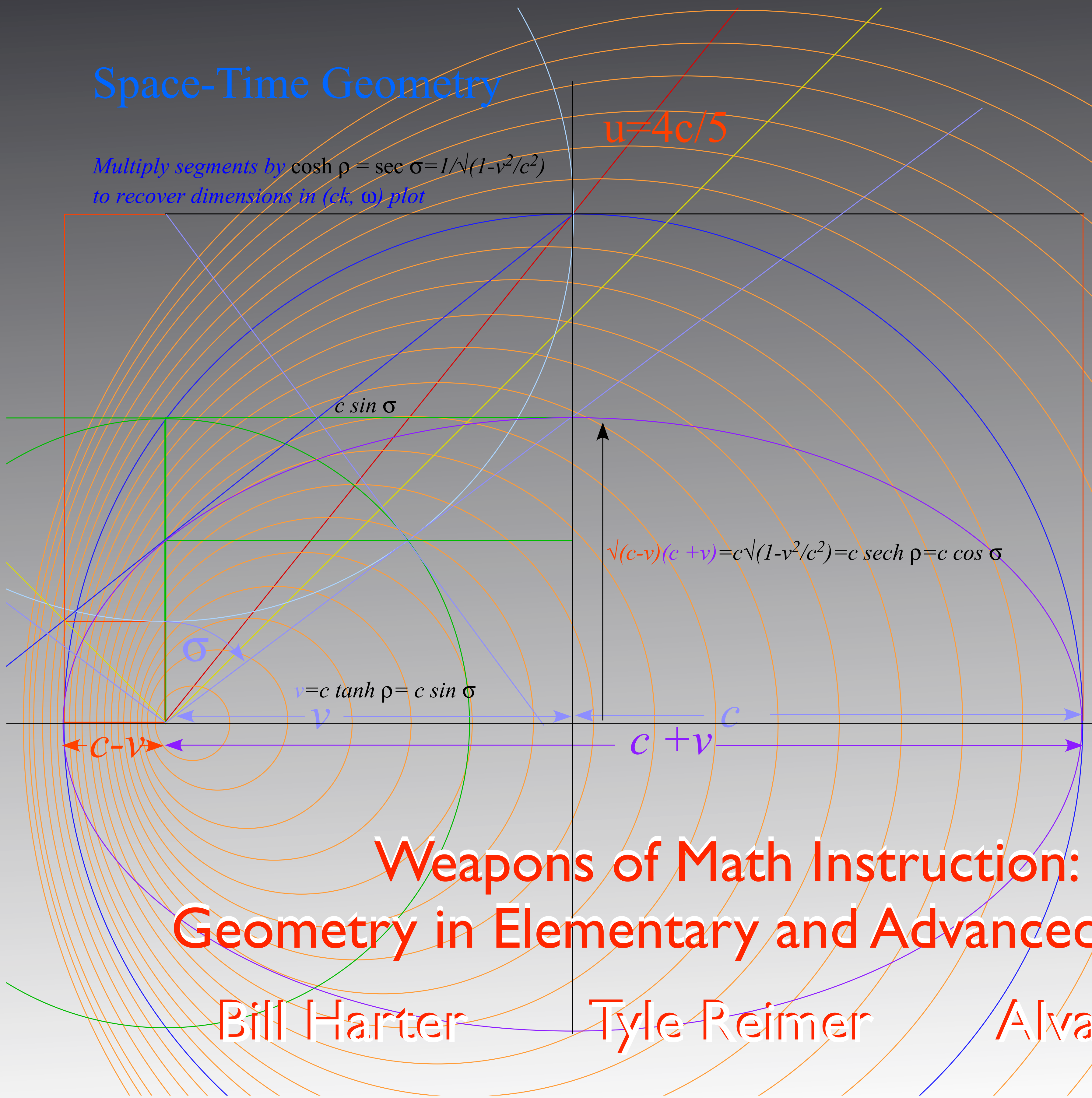
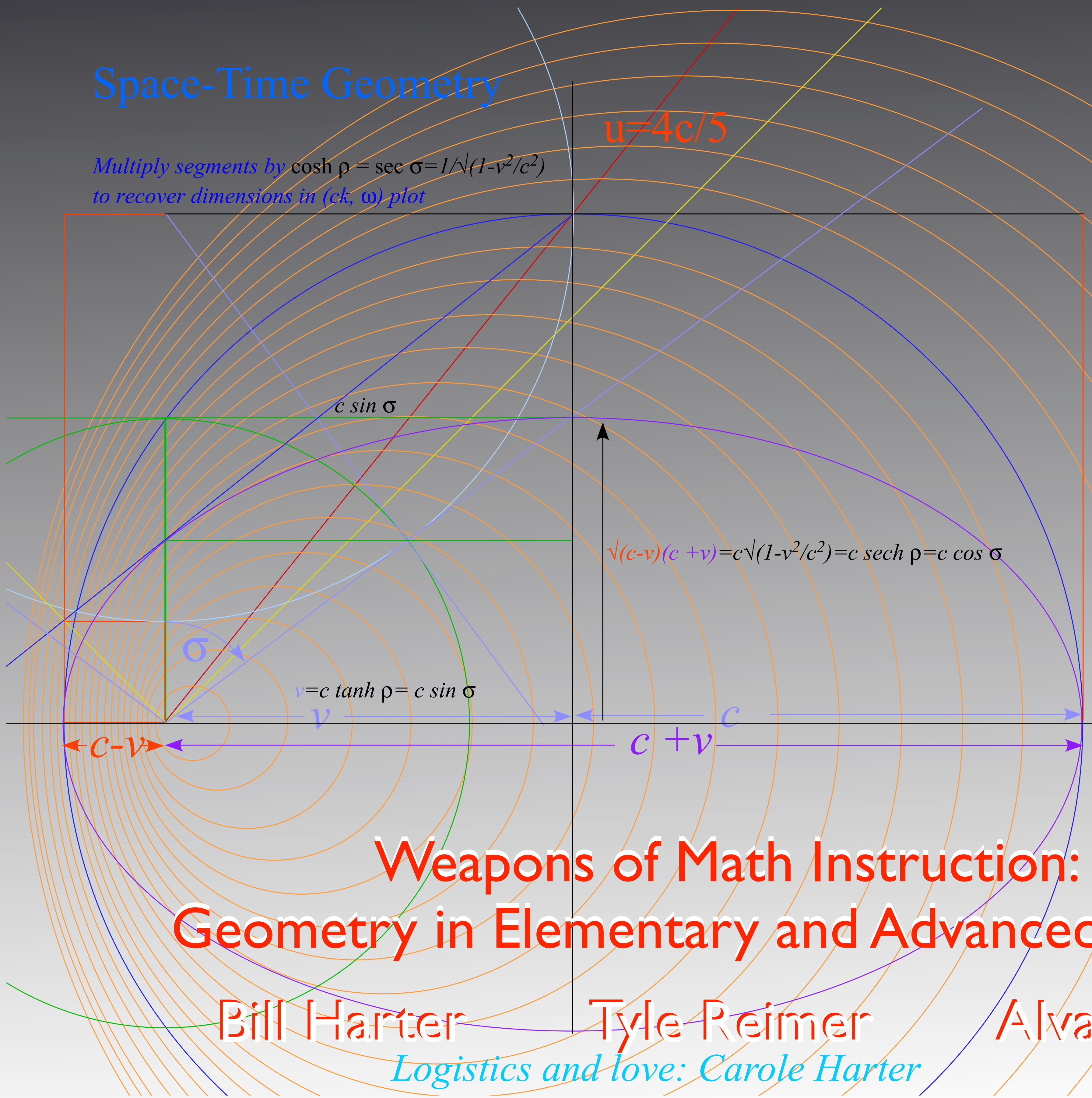
2012 INBRE workshop



Weapons of Math Instruction: Geometry in Elementary and Advanced Physics

Bill Harter   Tyle Reimer   Alvason Li

2012 INBRE workshop



Weapons of Math Instruction:
Geometry in Elementary and Advanced Physics

Bill Harter   Tyle Reimer   Alvason Li

Logistics and love: Carole Harter

SOME PHYSICS YOU CAN DO BETTER WITH GEOMETRY

- Superball collision problems (Discover momentum-energy rules and Lagrange, Hamilton, and Poincare classical mechanics)

- Rutherford-Coulomb scattering orbits and caustics
- Runga-Lenz-Lagrange scattering orbits and caustics

- Space-time wave fractal ("quantum carpet") gives a lesson in fractions that is quite appealing

- Einstein-Lorentz-Minkowskii relativity
  (Discover relativistic quantum mechanics)

- Accurate pocket sundial that tells time and predicts sunrise, sunset, civil and nautical twilight, and sunburn hazard.

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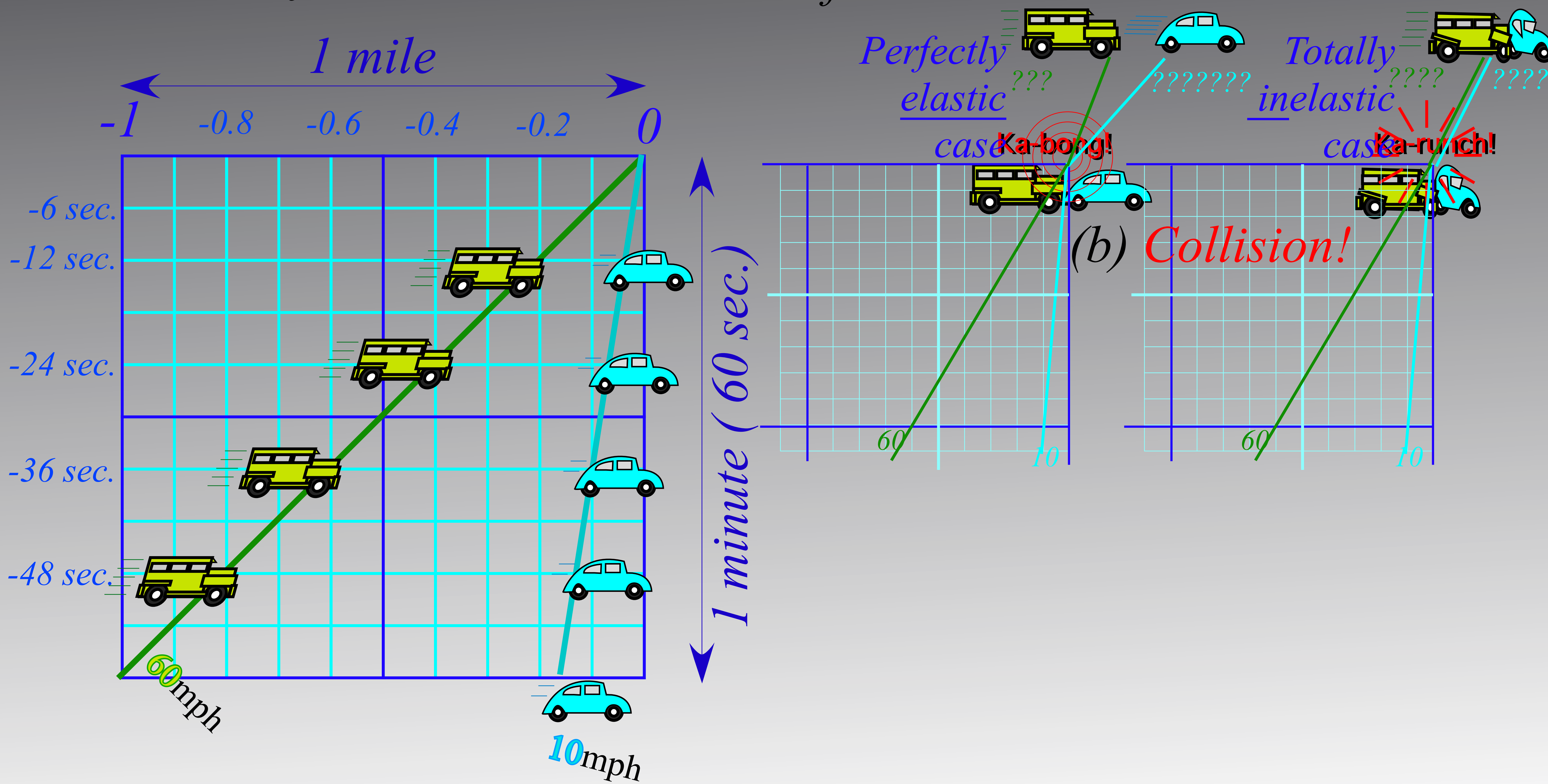
A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Before collision.....

After collision...what velocities?



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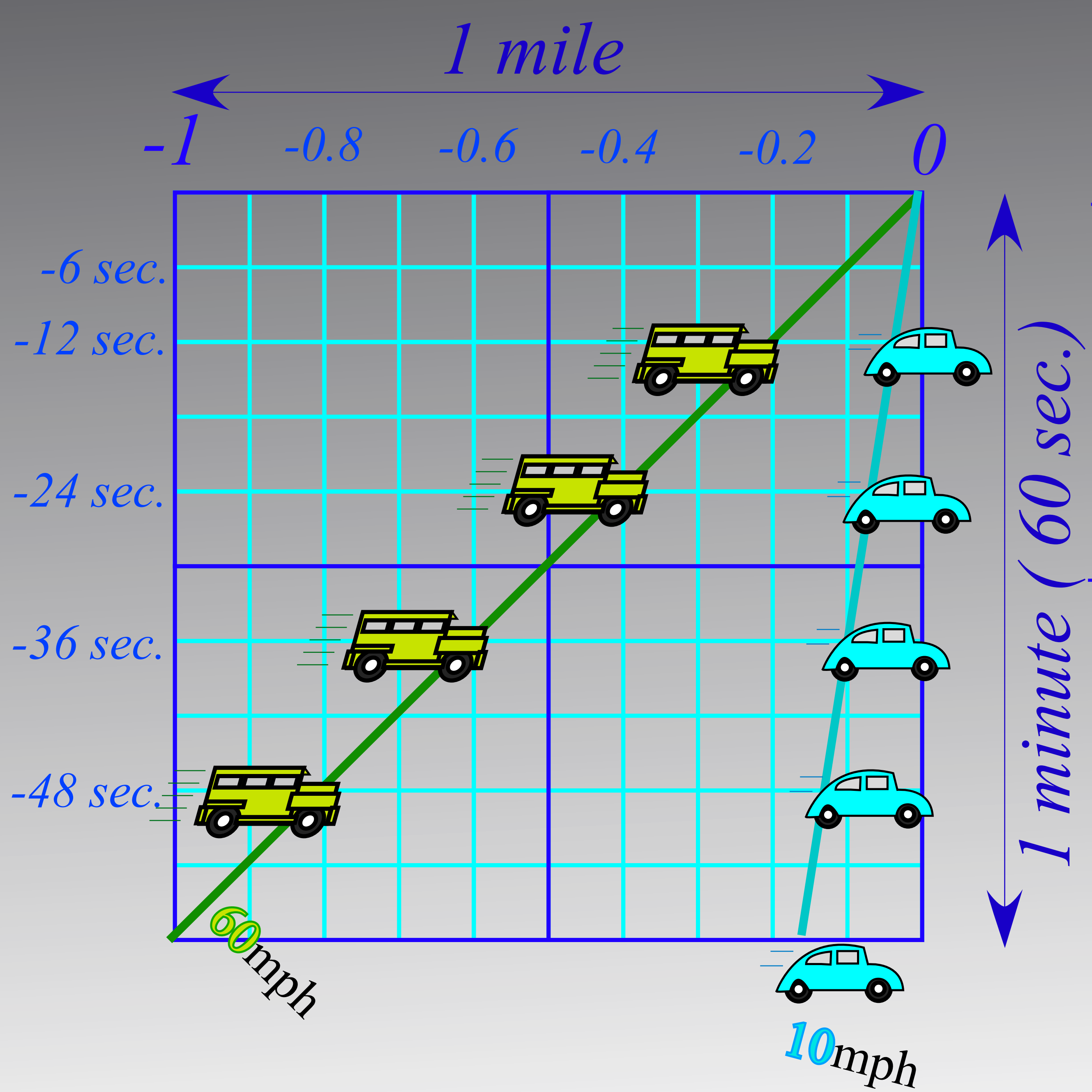

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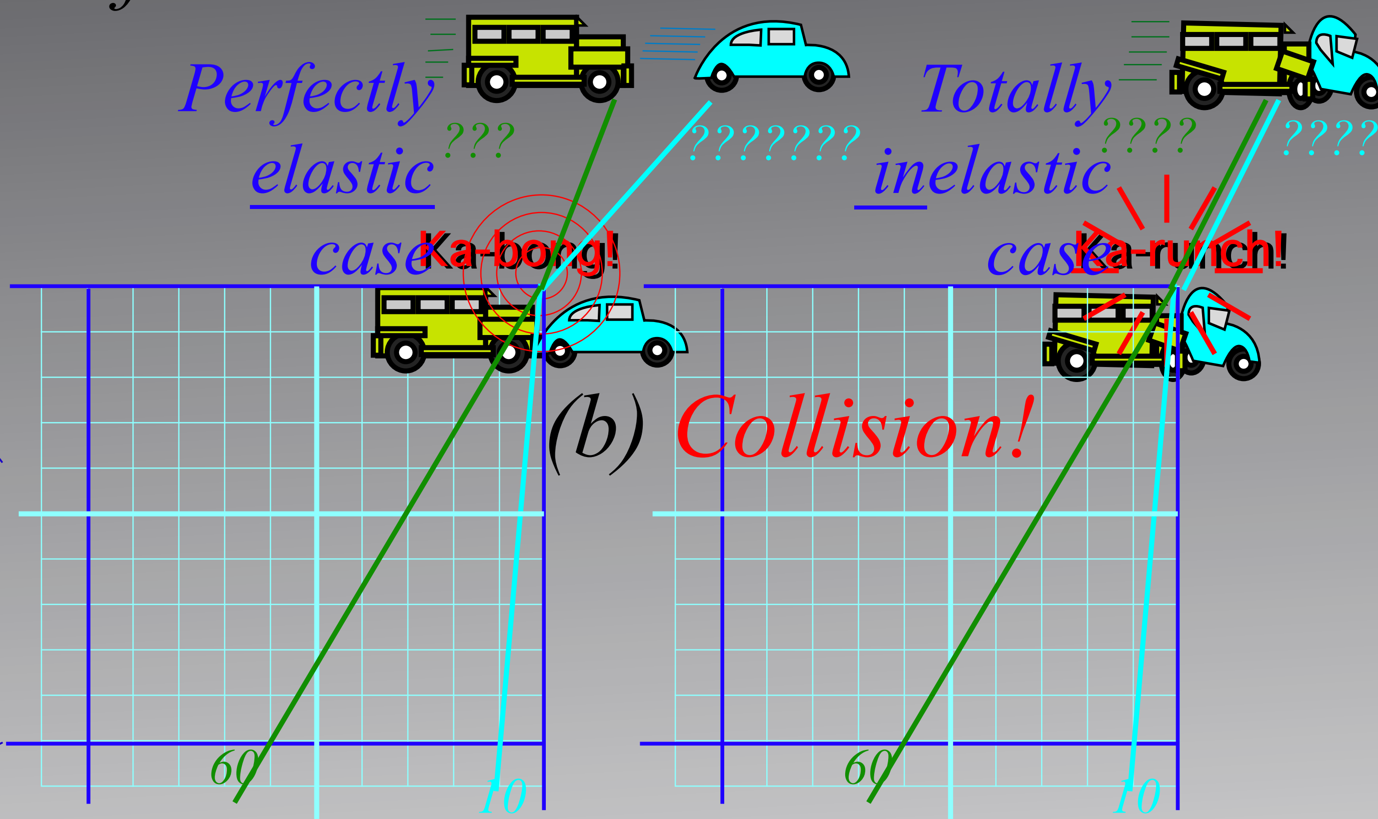

Conventional solution:
Get out formulas:

$\Sigma\; mV(before) = \Sigma\; mV(after)$ *[momentum conservation]*

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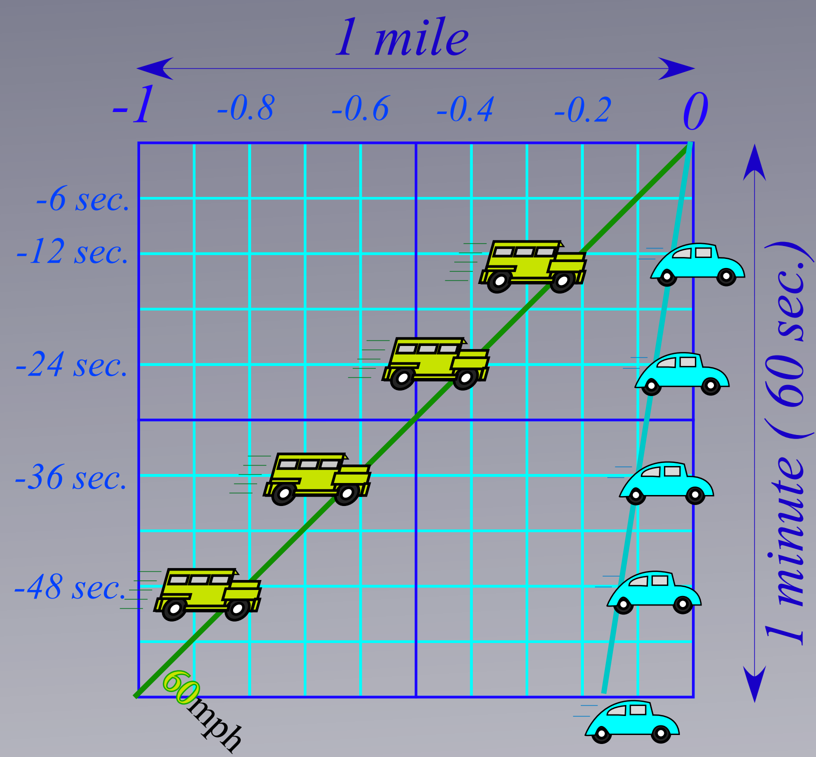

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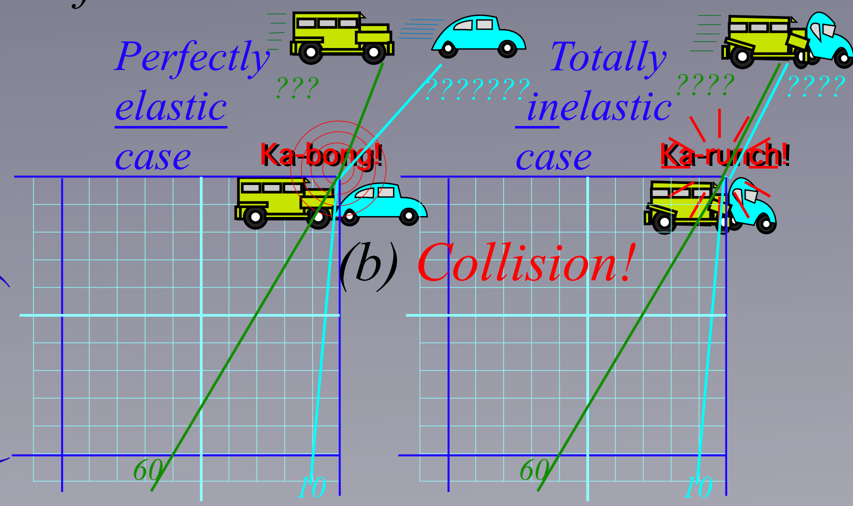

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etc.

But an UNconventional way is quicker and slicker.....
..... (Just have to draw 2 lines!   ... (and a circle...)

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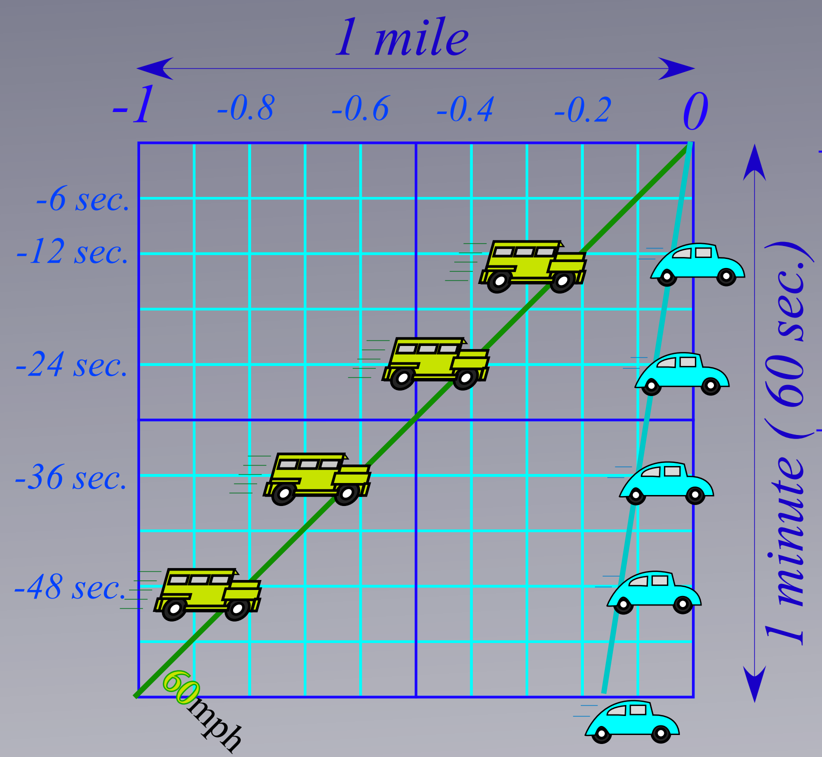

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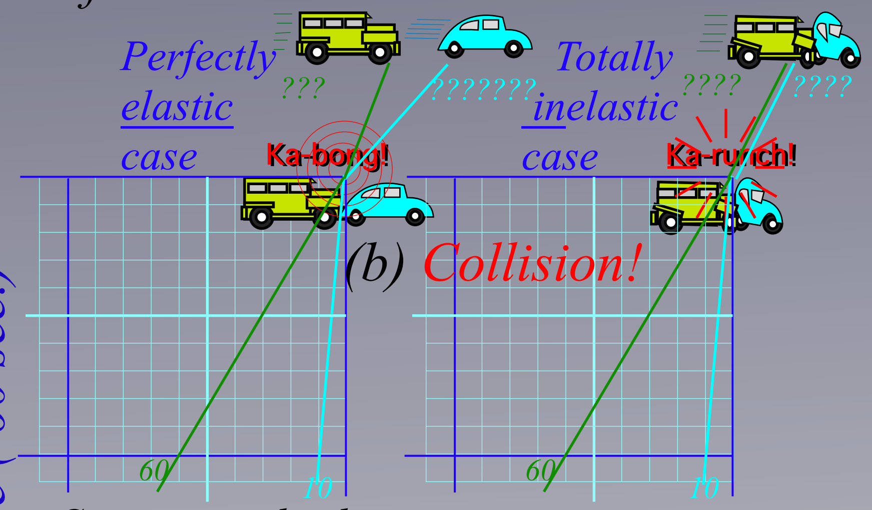

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..... and most importantly, DERIVE LOGICAL STRUCTURE!

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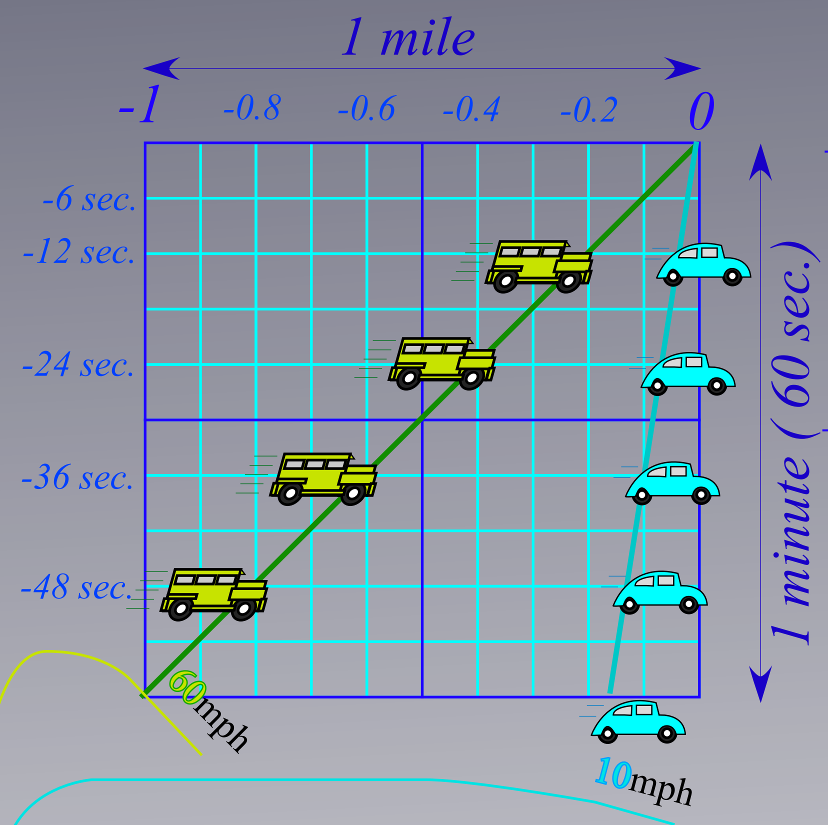

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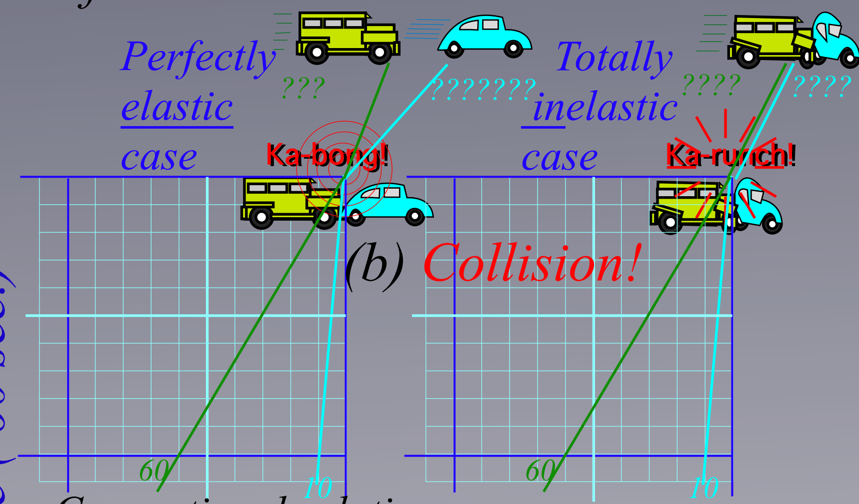

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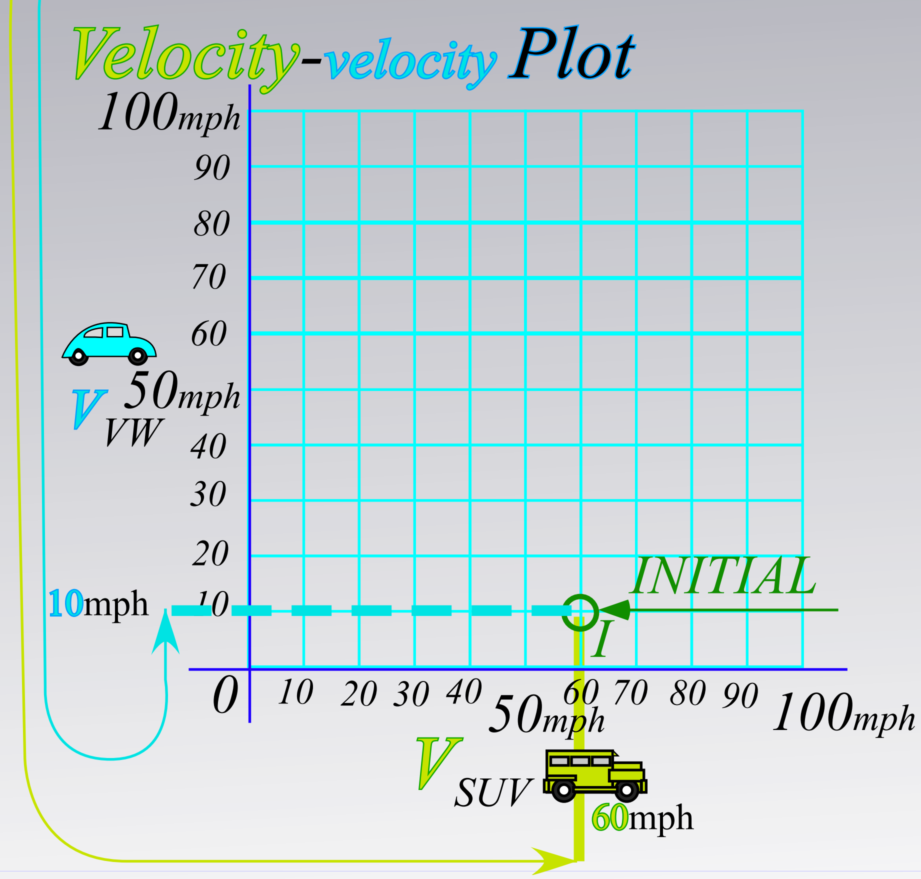

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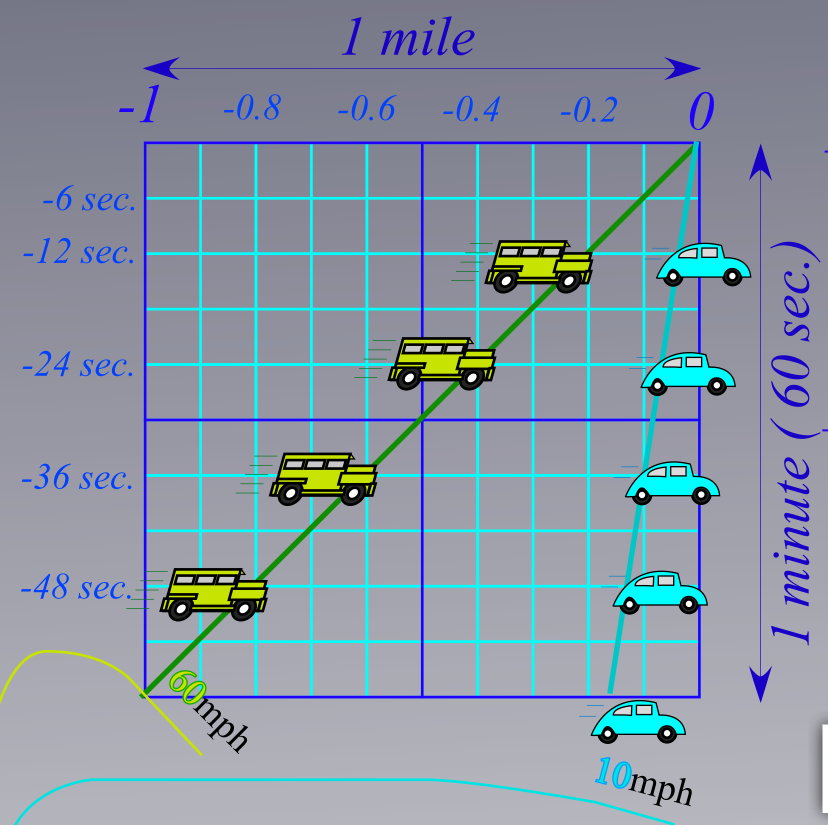

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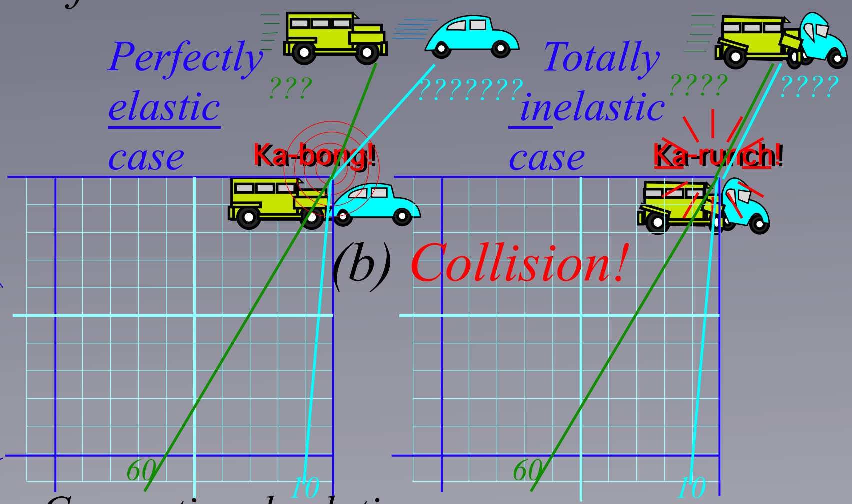

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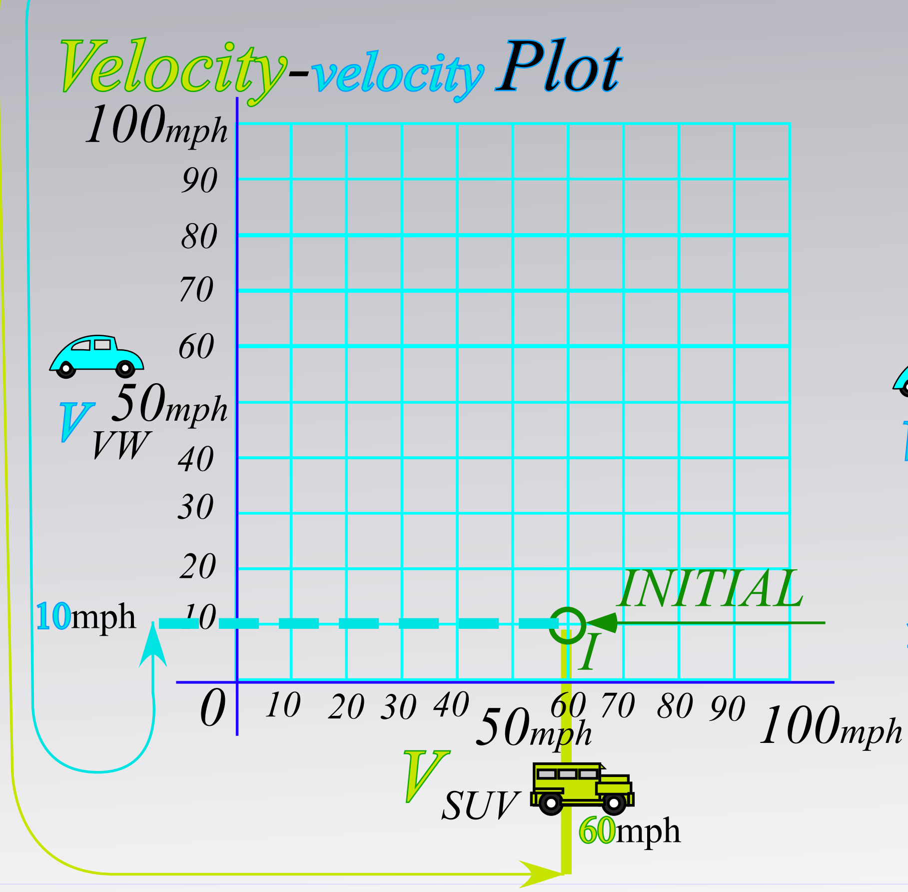
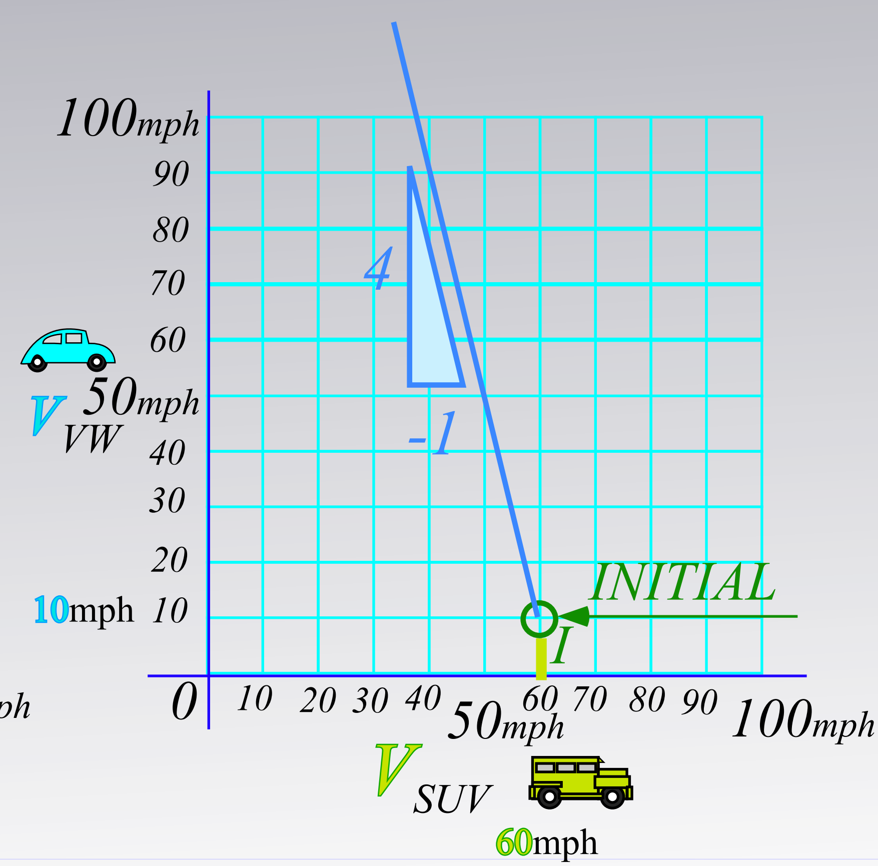
$M_{SUV} V_{SUV} + M_{VW} V_{VW}$ =constant is **Axiom #1**





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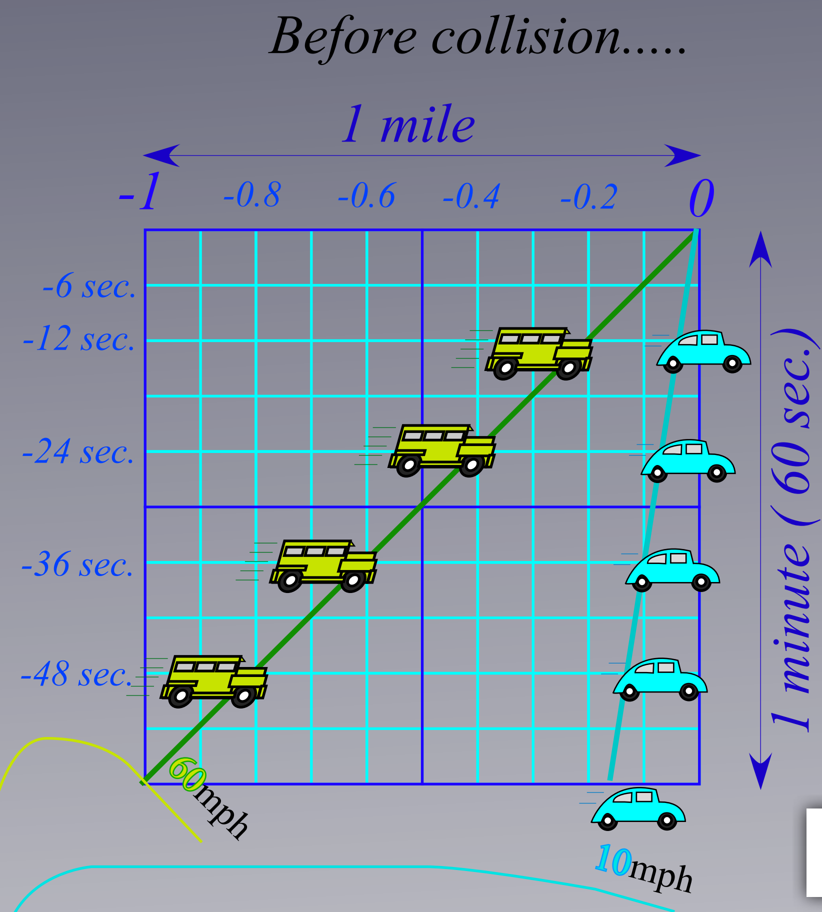

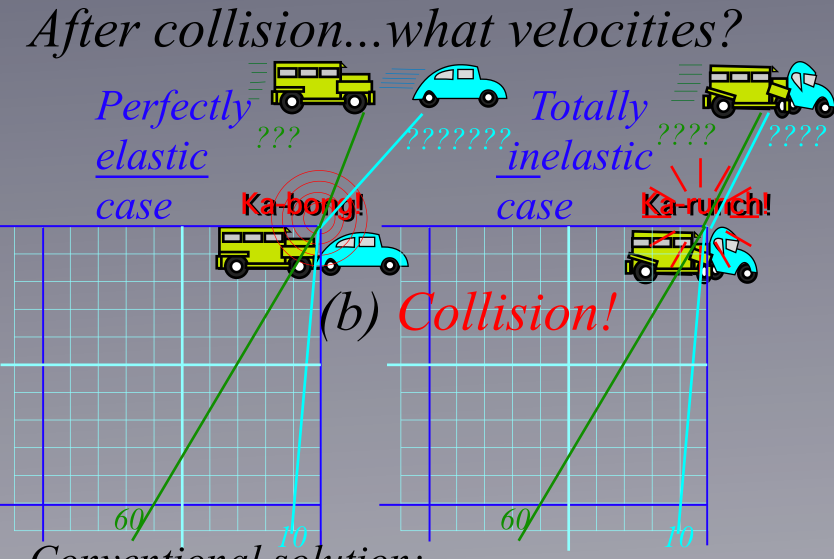

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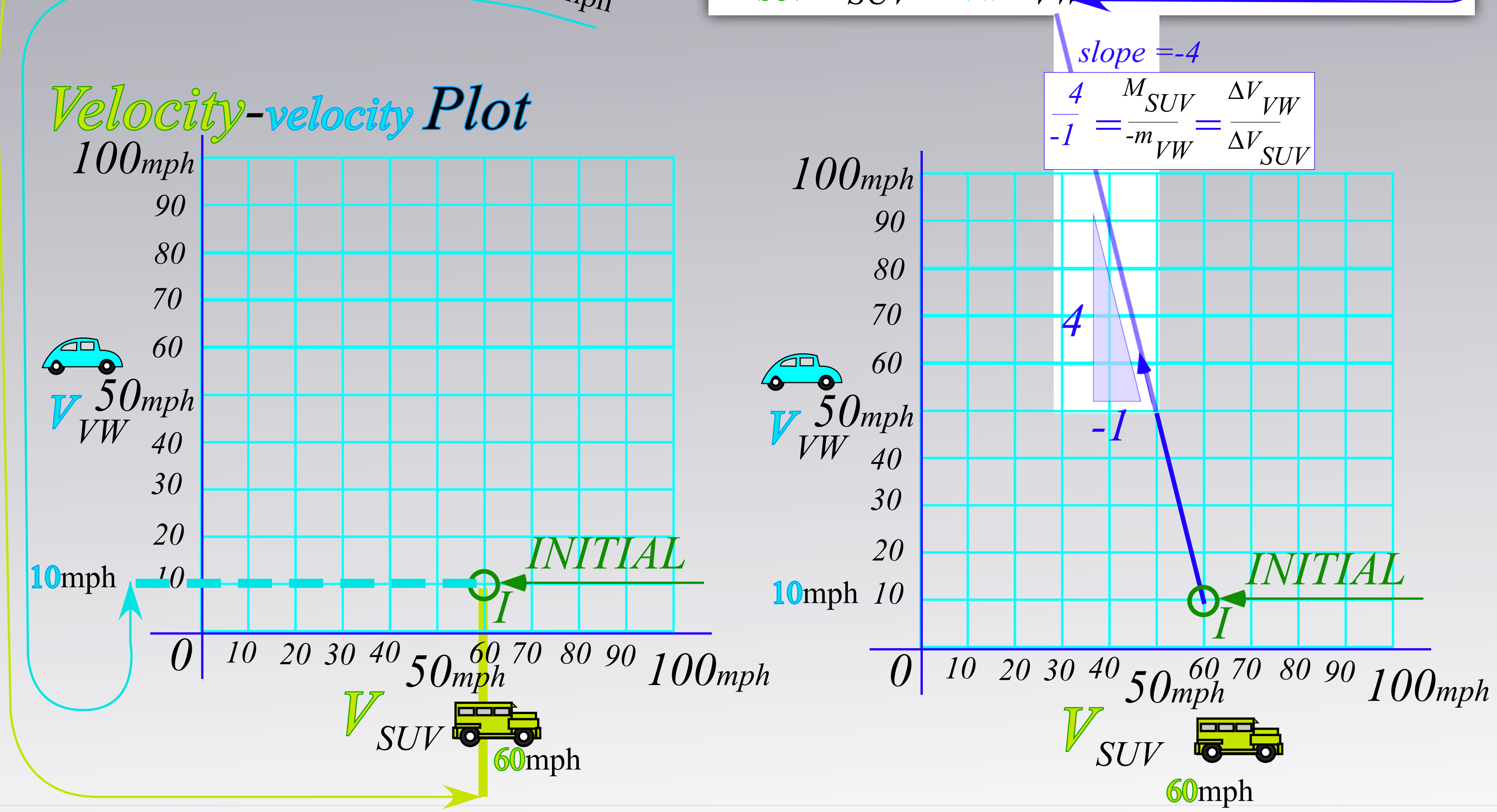

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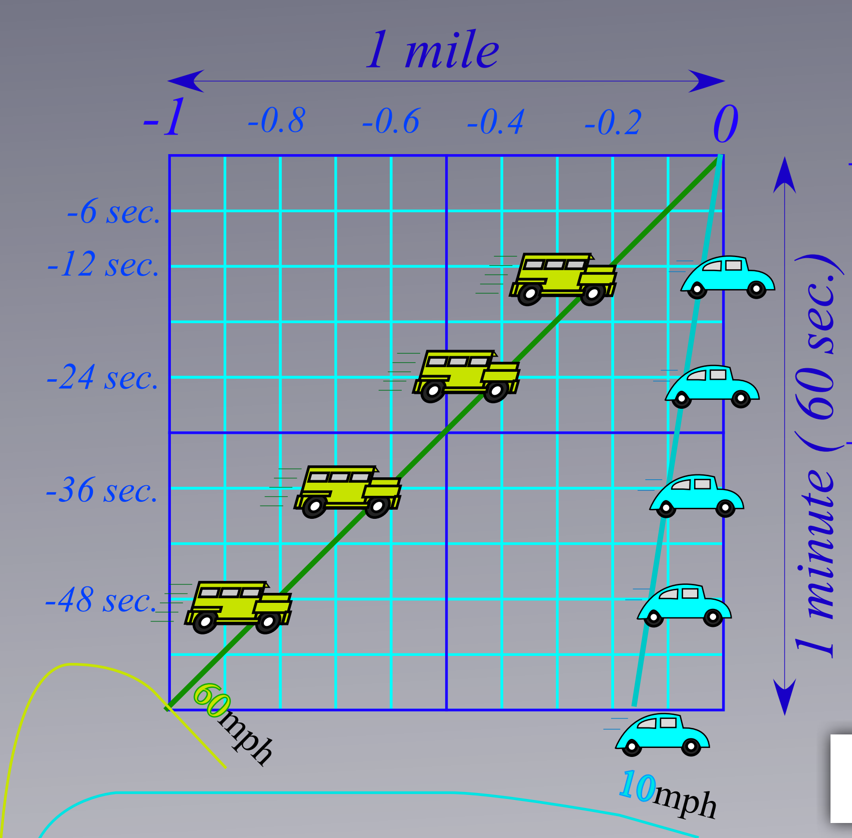

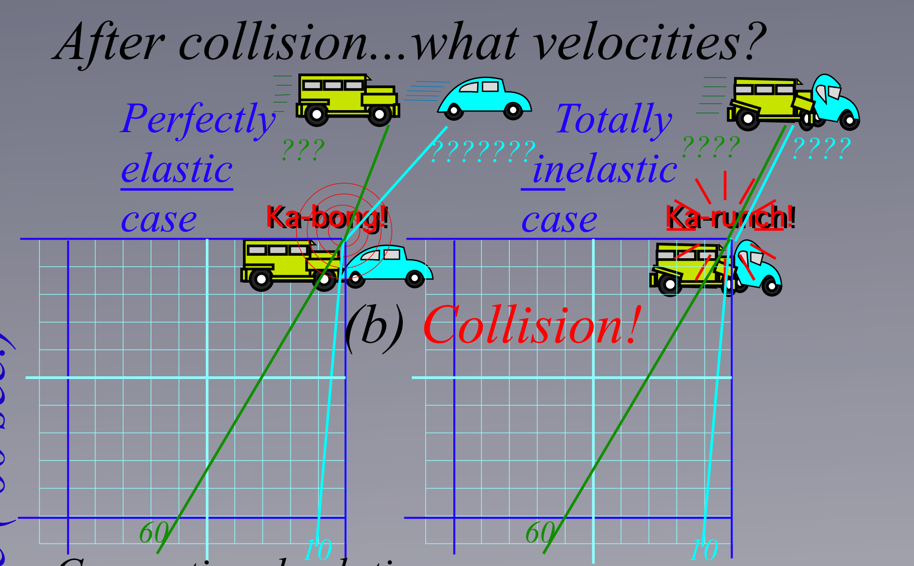

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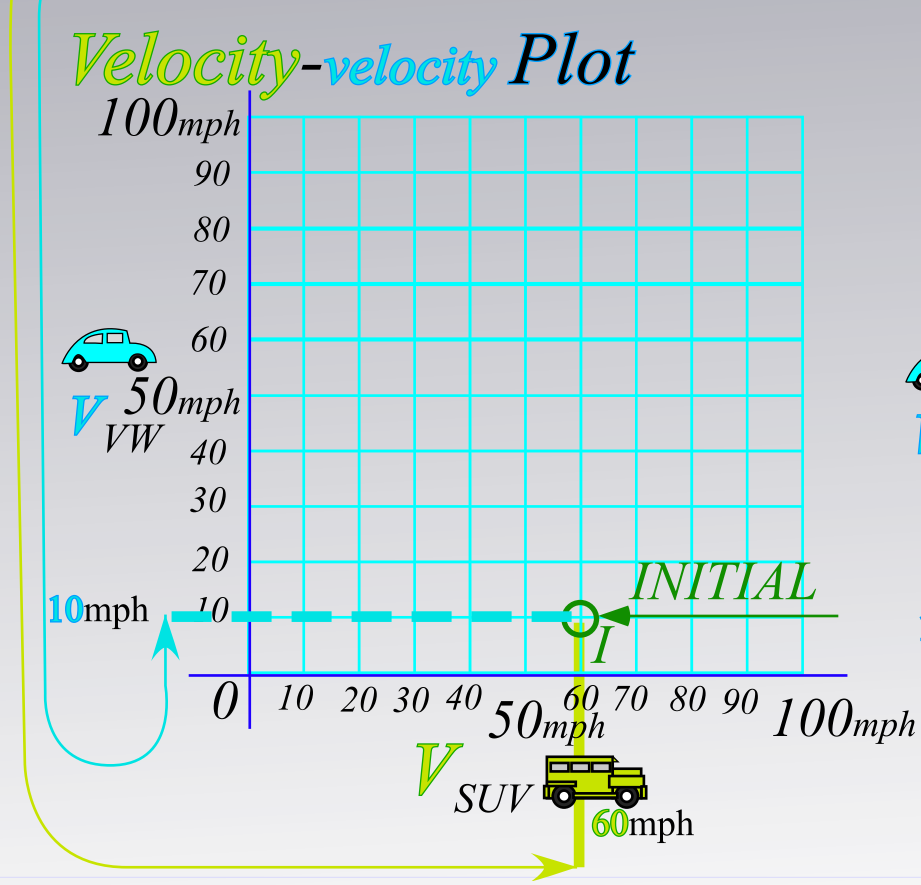

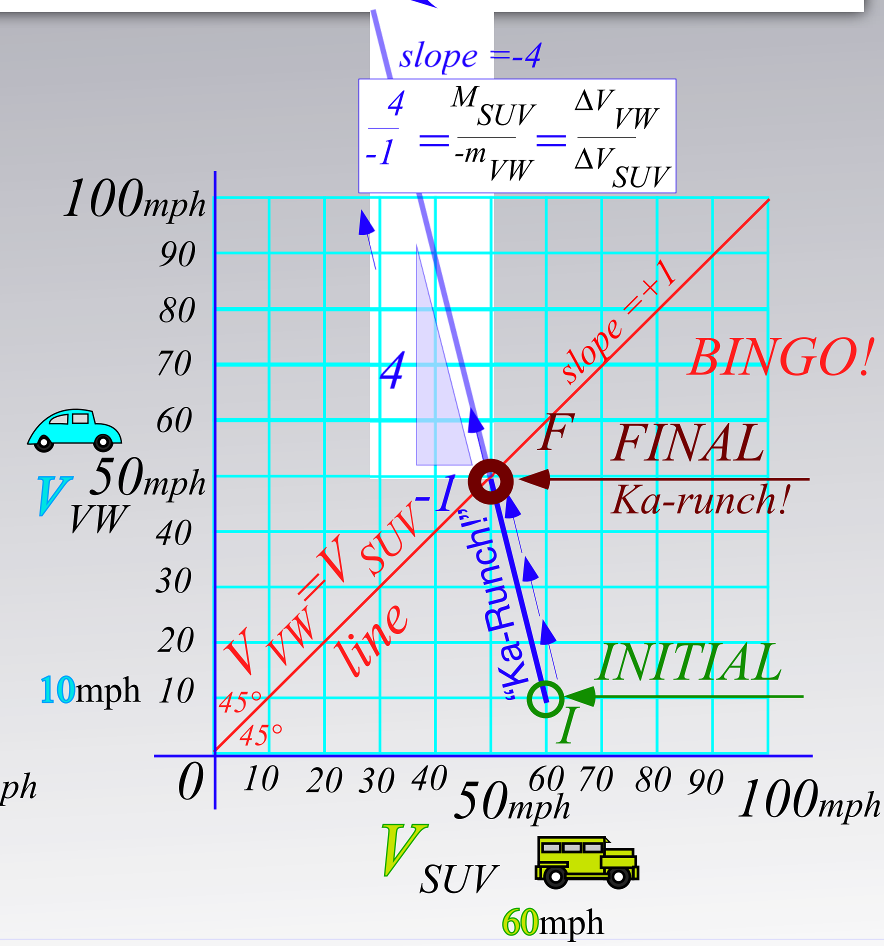

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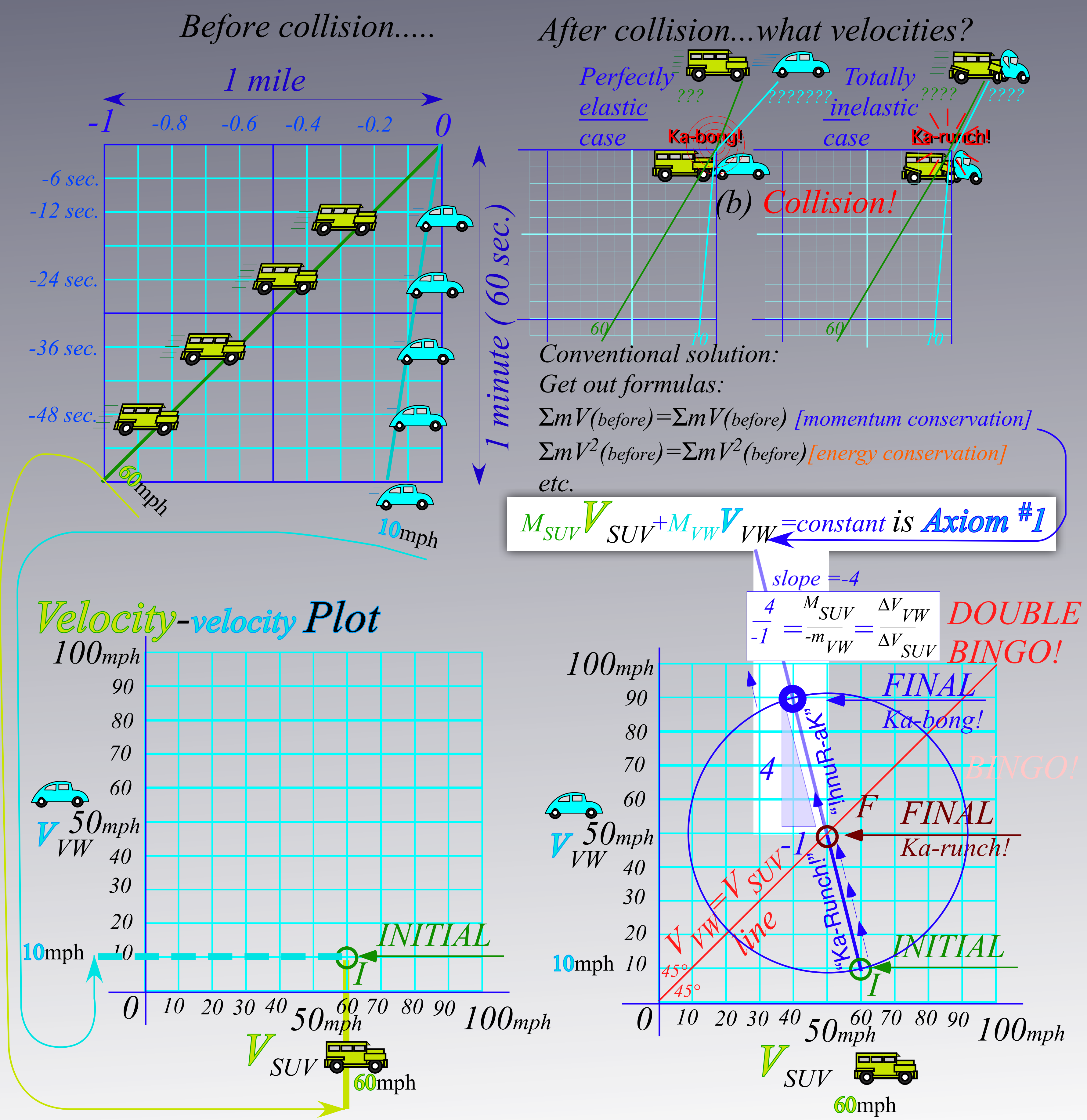

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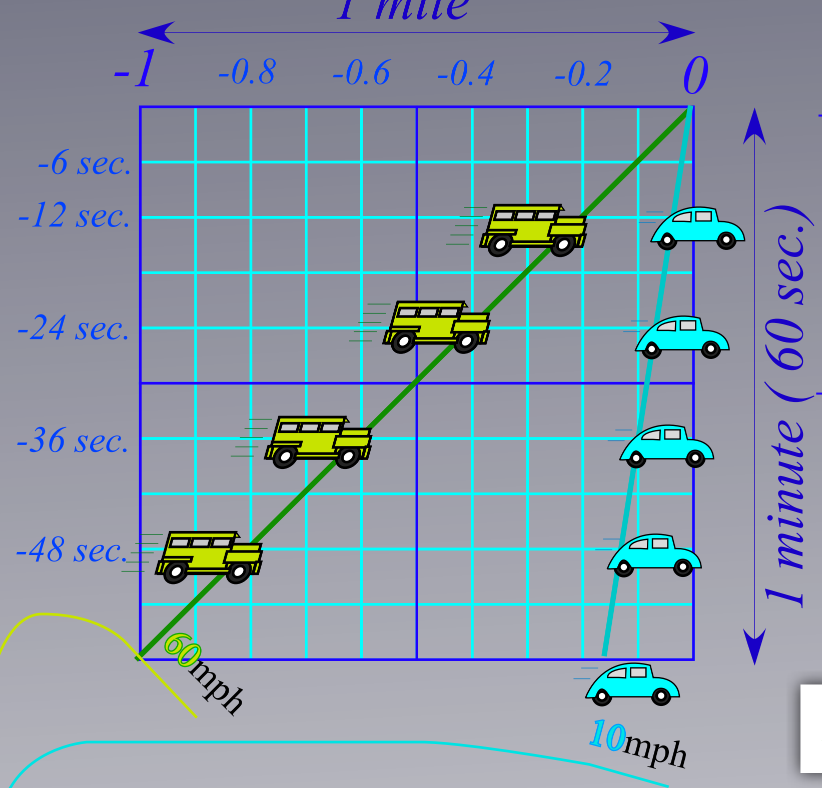

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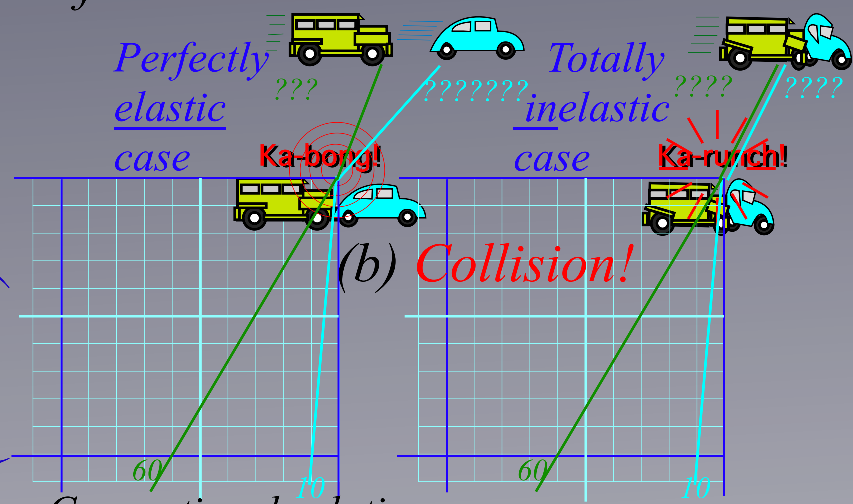

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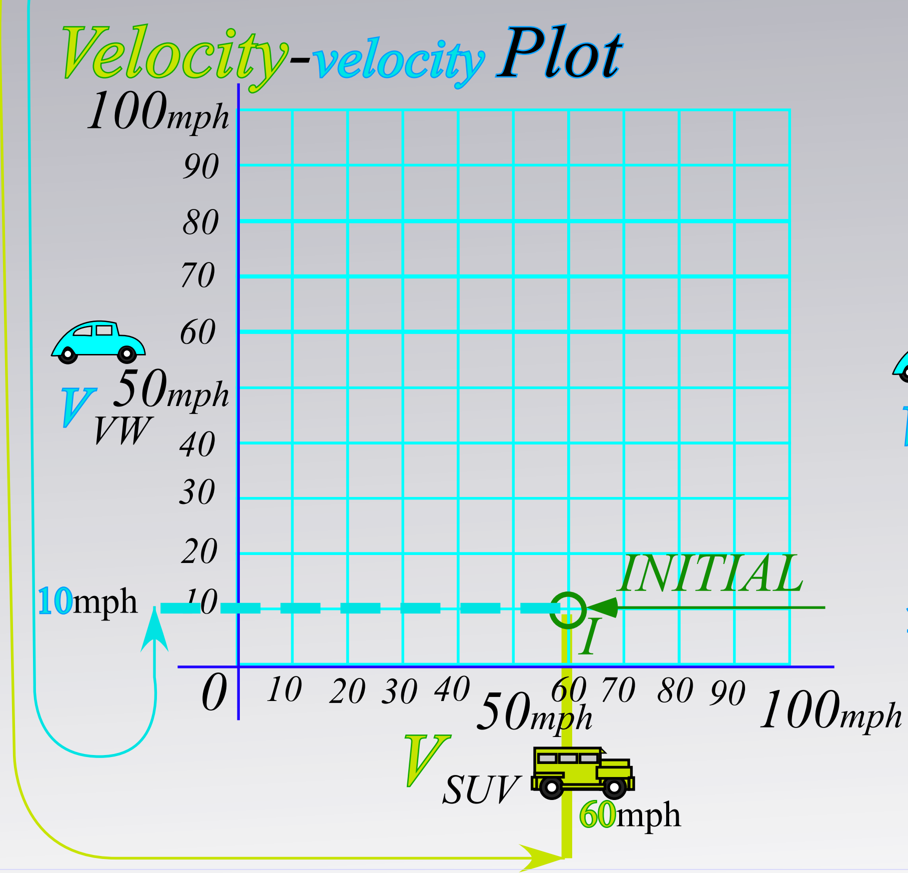
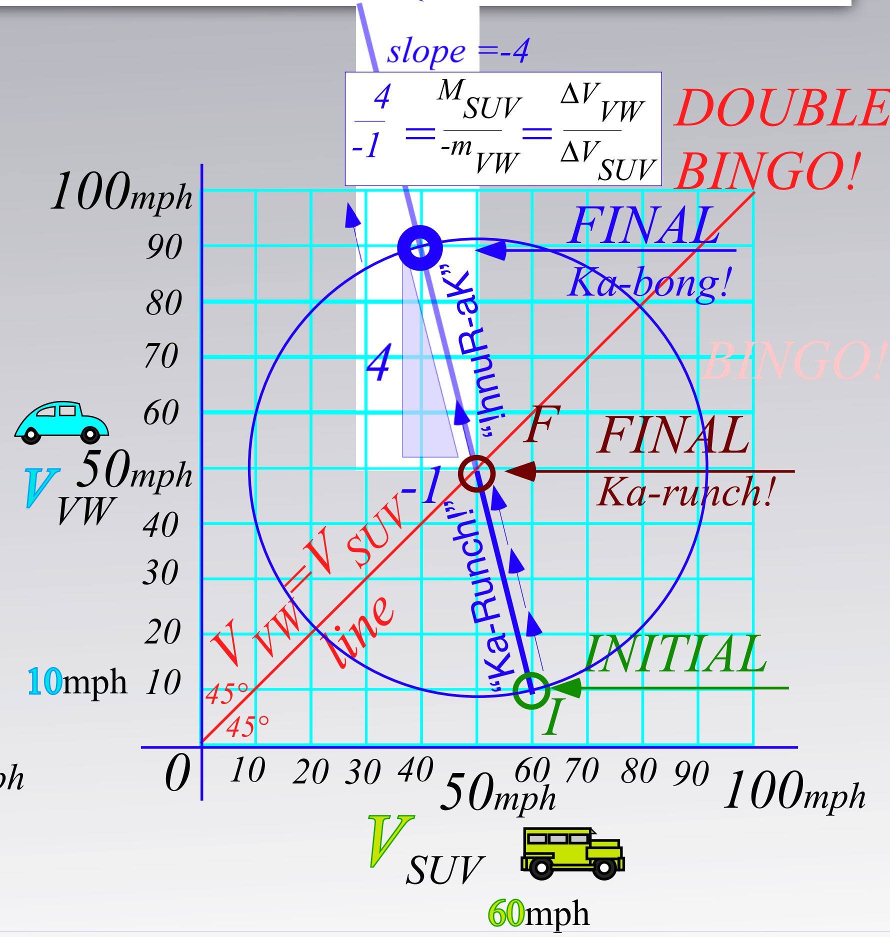
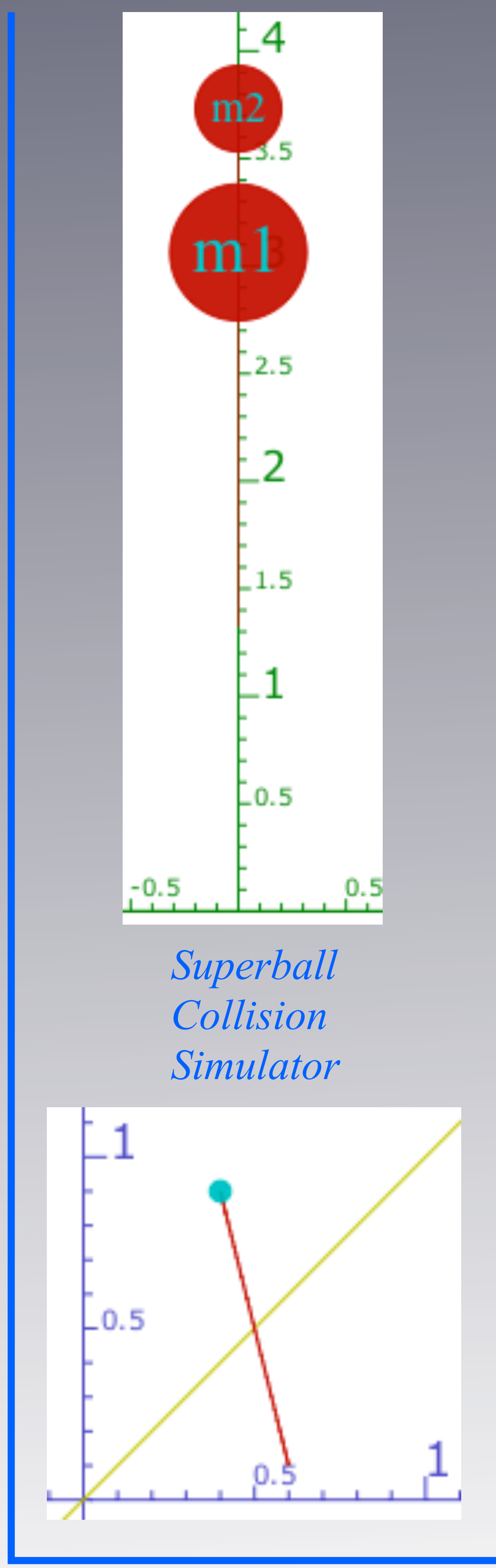






A problem in *space-time* : (60mph Cell-faxing 4ton SUV rear-ends 10mph 1ton VW)

Geometry of Galilean translation (A symmetry transformation)

If you increase your velocity by 50 mph,...

...the rest of the world appears to be 50 mph slower

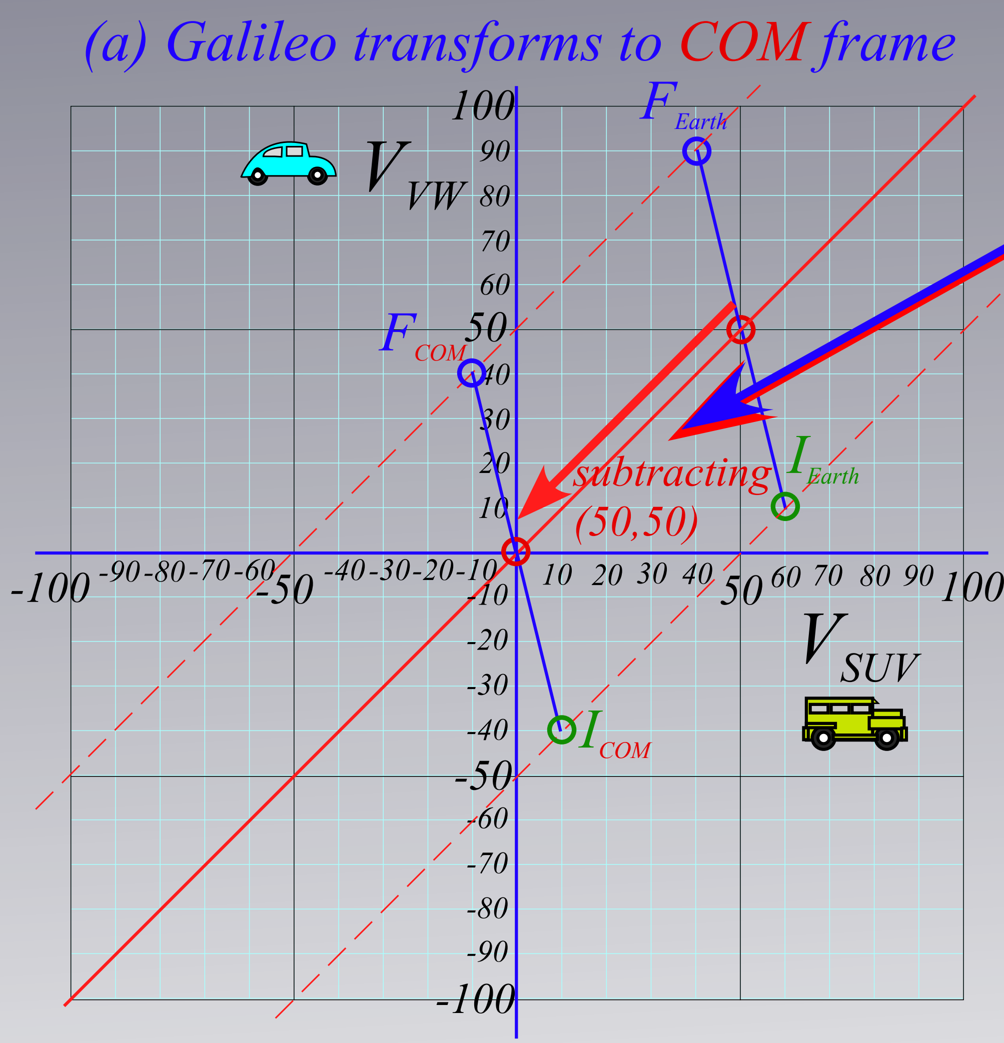
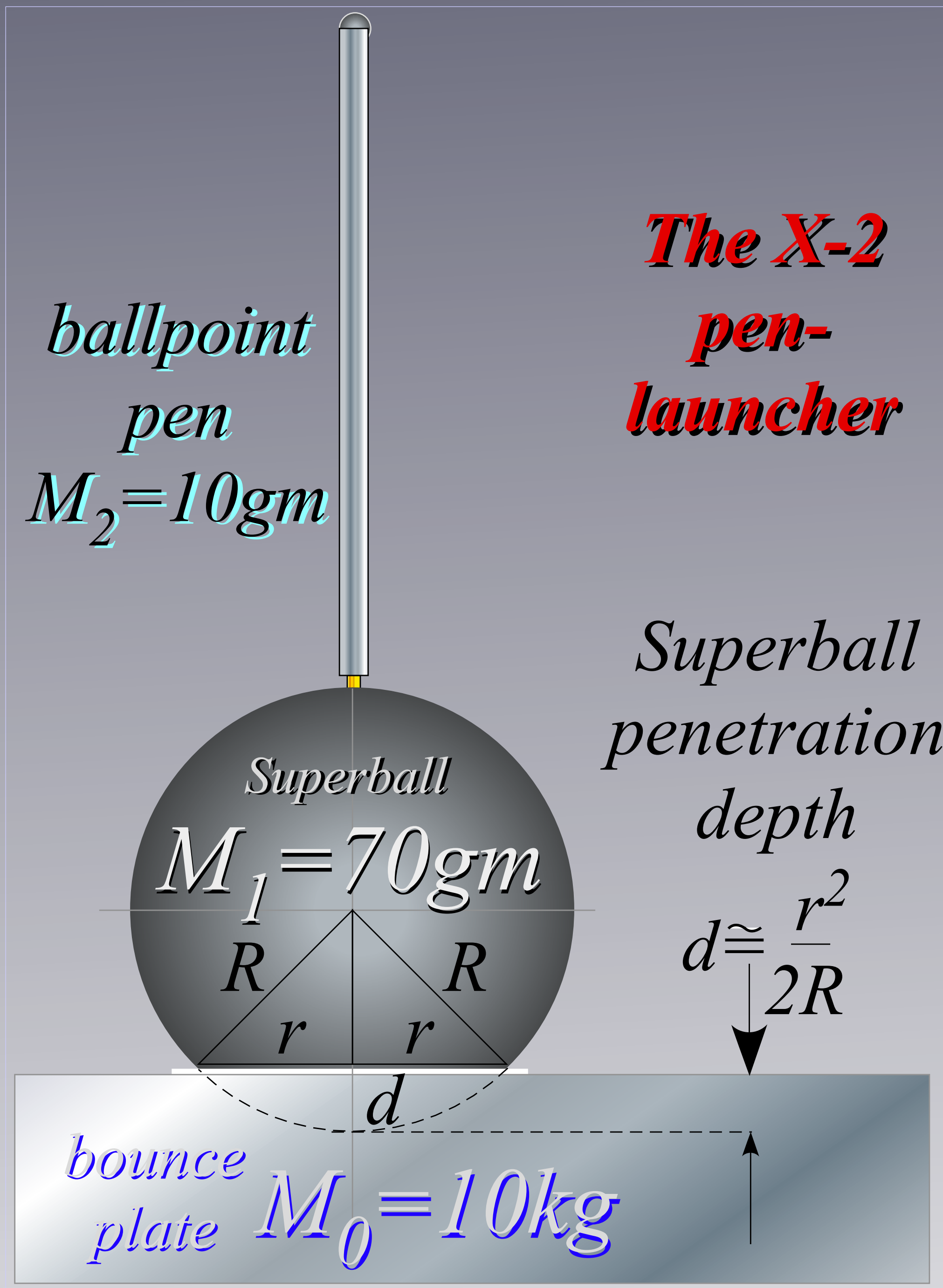
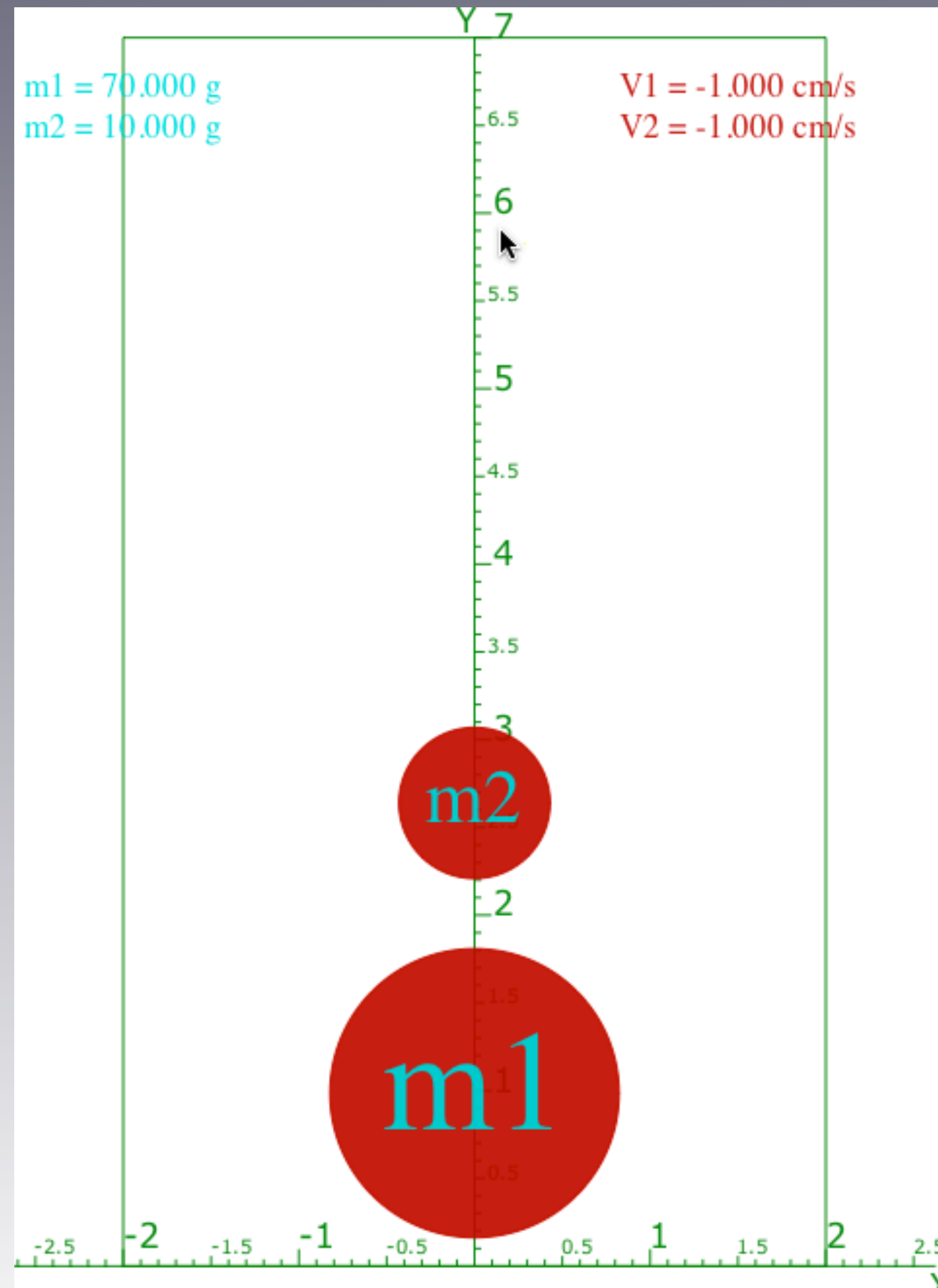


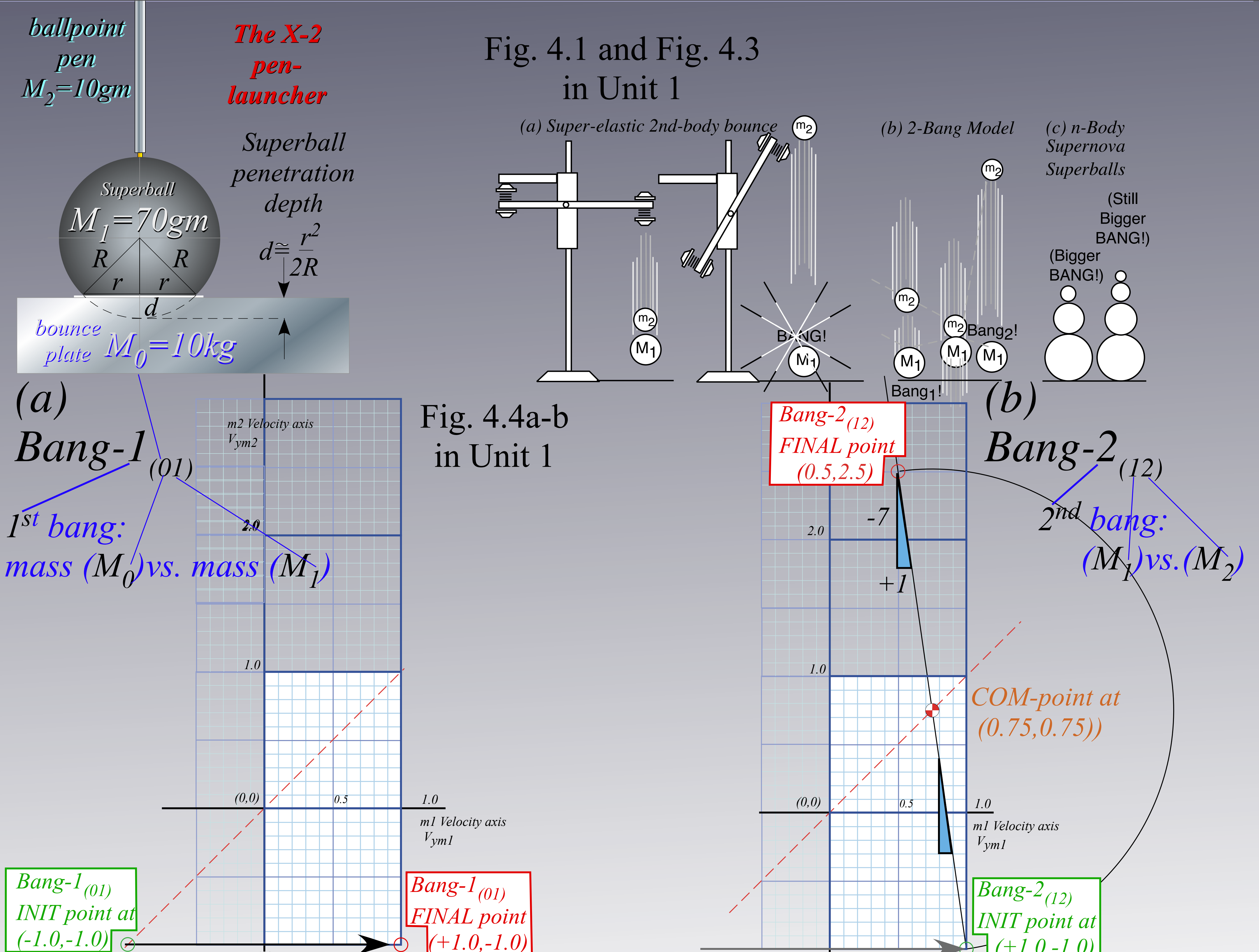
Fig. 2.5a
in Unit 1

The X-2 Pen launcher and *Superball Collision Simulator**





**Simulator Website:* http://www.uark.edu/rso/modphys/animations/BounceItWeb.html



This 1st bang is a floor-bounce of M_1 off very massive plate/Earth M_0

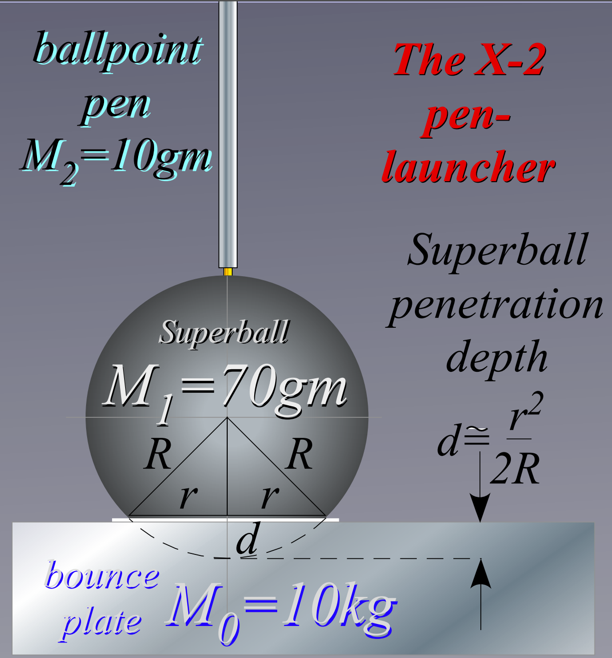
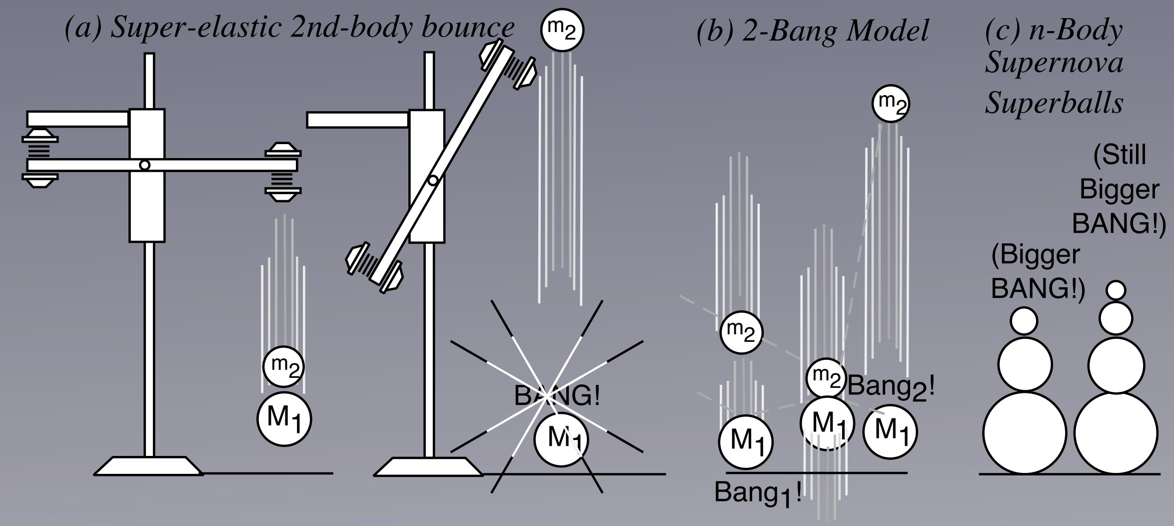


Fig. 4.1 and Fig. 4.3
in Unit 1



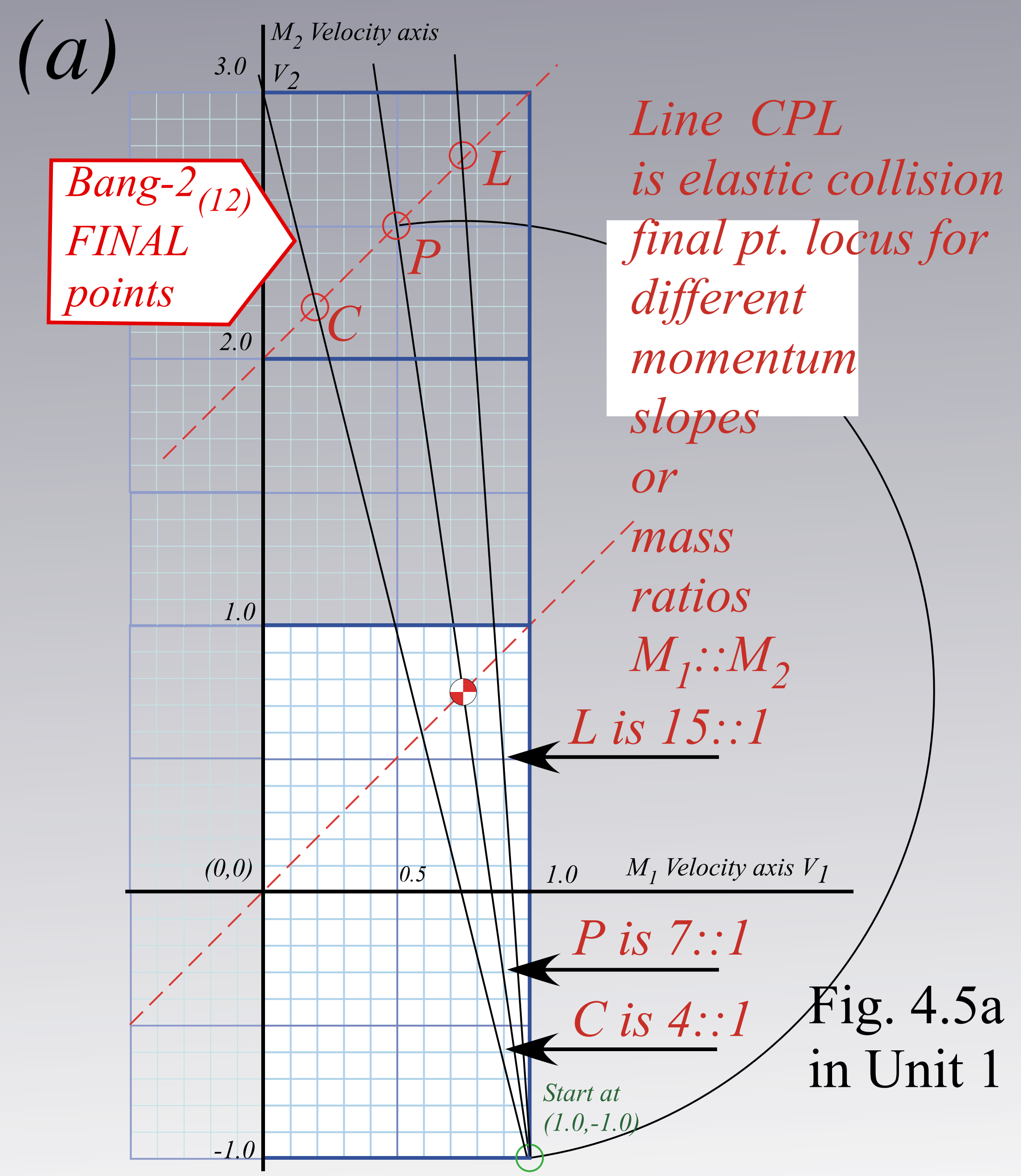
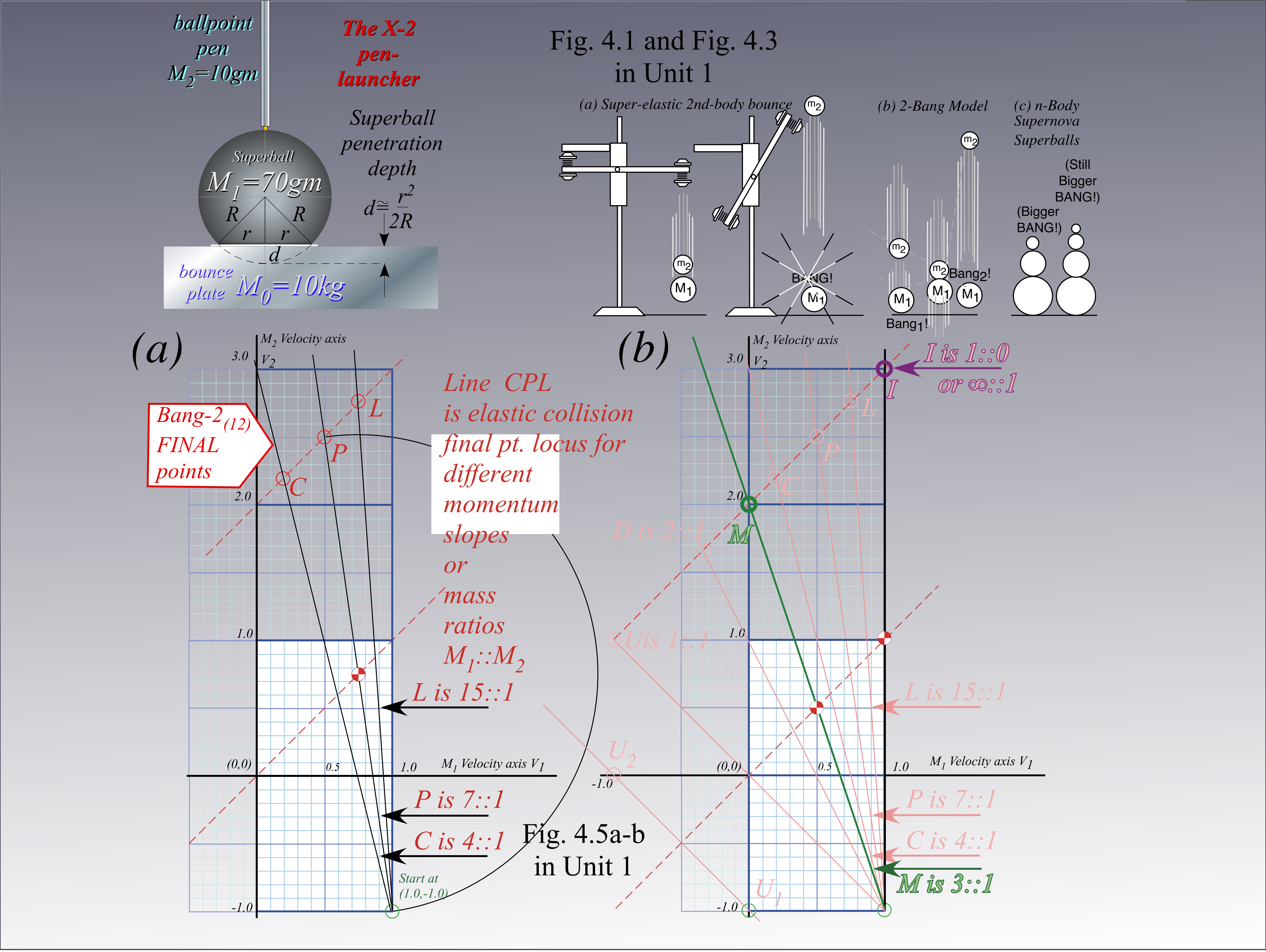
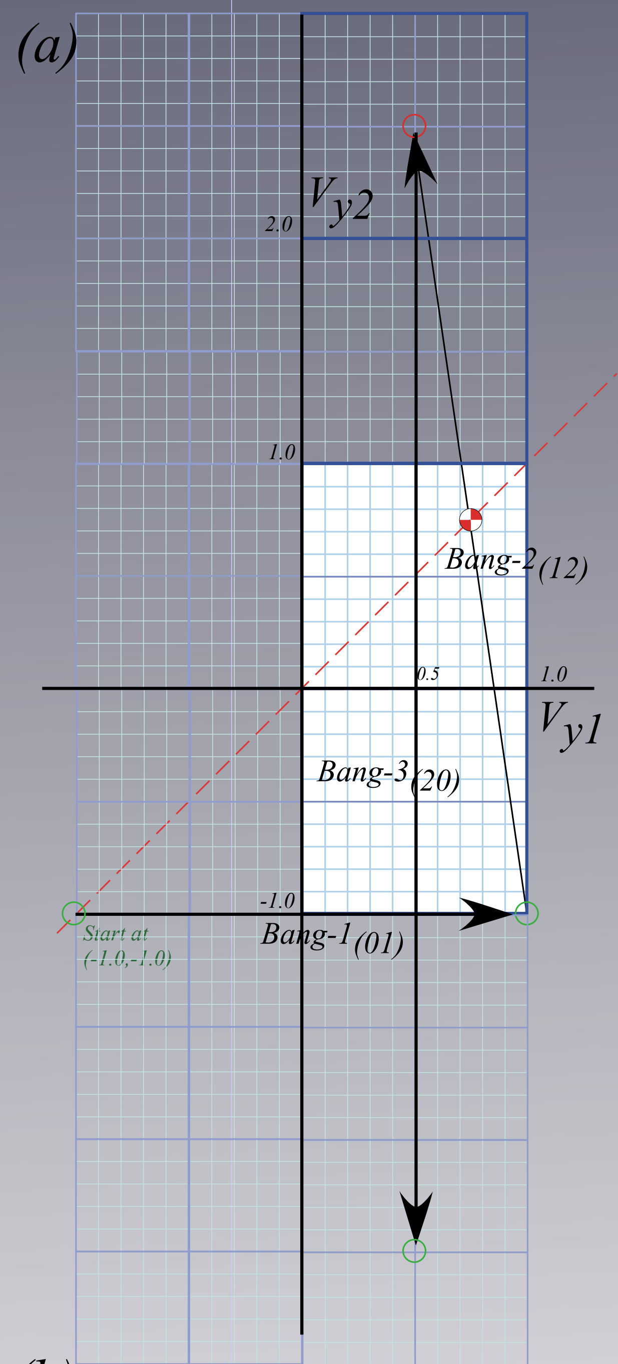
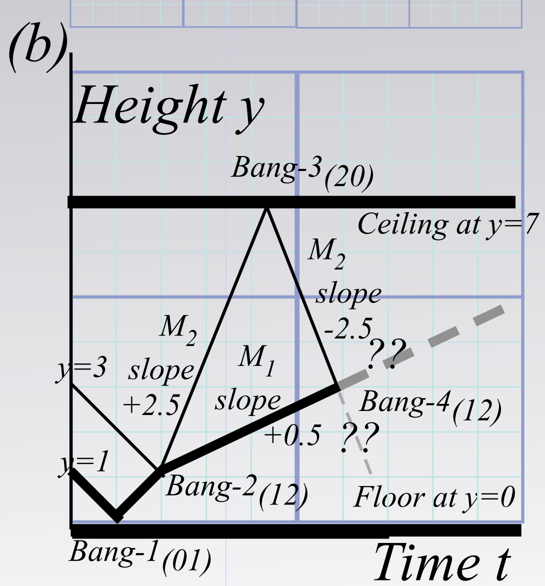


Fig. 4.5a
in Unit 1


ballpoint pen M_2=10gm
The X-2 pen-launcher
Superball M_1=70gm
R R r r d
Superball penetration depth
$d \cong \frac{r^2}{2R}$
bounce plate M_0=10kg
Fig. 4.1 and Fig. 4.3 in Unit 1
(a) Super-elastic 2nd-body bounce
(b) 2-Bang Model
(c) n-Body Supernova Superballs
(Still Bigger BANG!)
(Bigger BANG!)
BANG!
Bang$_1$!
Bang$_2$!
(a)
(b)
M_2 Velocity axis V_2
M_1 Velocity axis V_1
3.0
2.0
1.0
(0,0)
0.5
-1.0
Bang-2$_{(12)}$ FINAL points
Line CPL is elastic collision final pt. locus for different momentum slopes or mass ratios M_1::M_2
L is 15::1
P is 7::1
C is 4::1
Start at (1.0,-1.0)
I is 1::0 or ∞::1
D is 2::1
U is 1::1
M is 3::1
U_1
U_2
Fig. 4.5a-b in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)





Geometric "Integration" (Converting Velocity data to Spacetime)

Kinetic Energy Ellipse

$$KE = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2 = \frac{1}{2} + \frac{7}{2} = 4$$

$$1 = \frac{V_1^2}{2KE / M_1} + \frac{V_2^2}{2KE / M_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

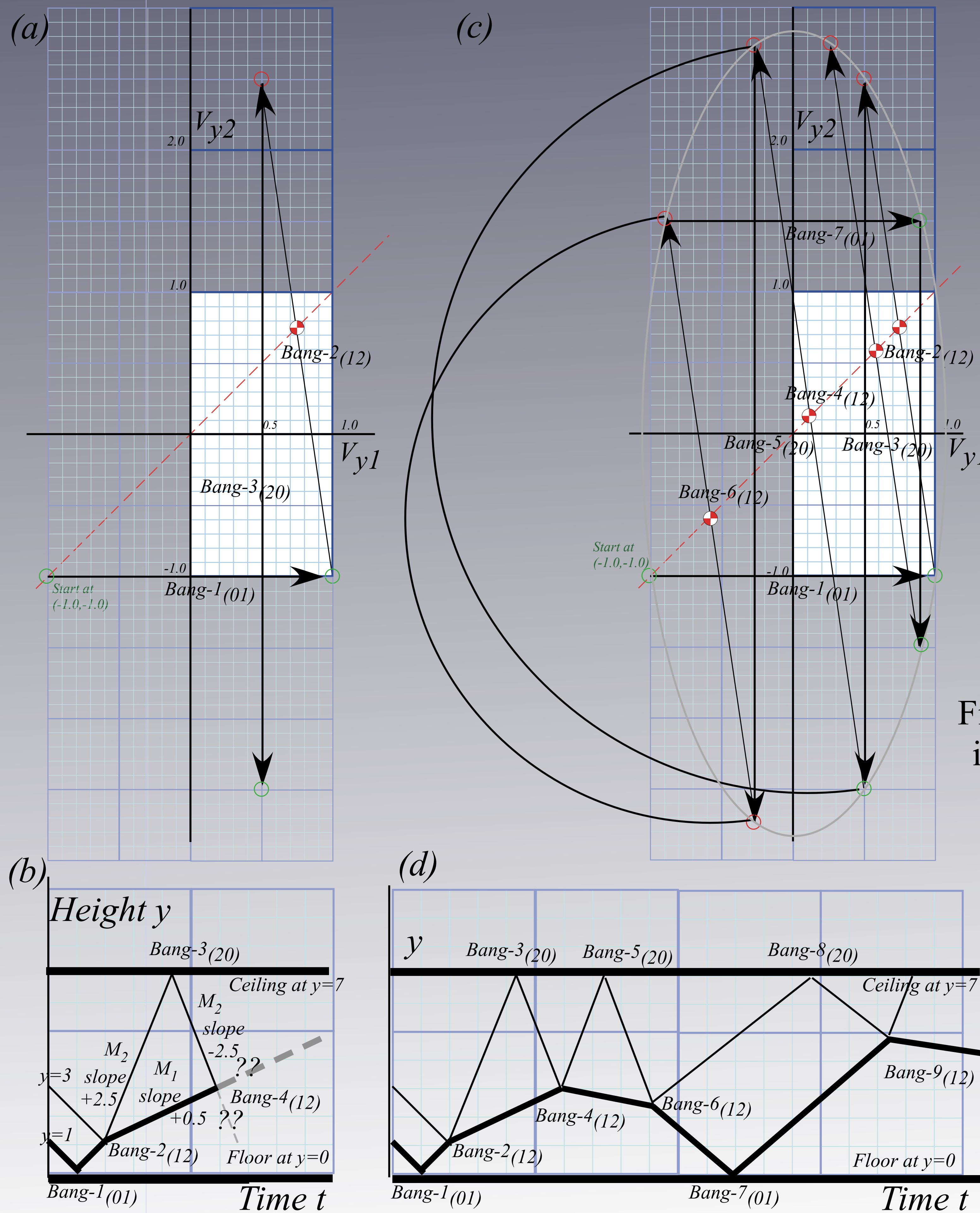
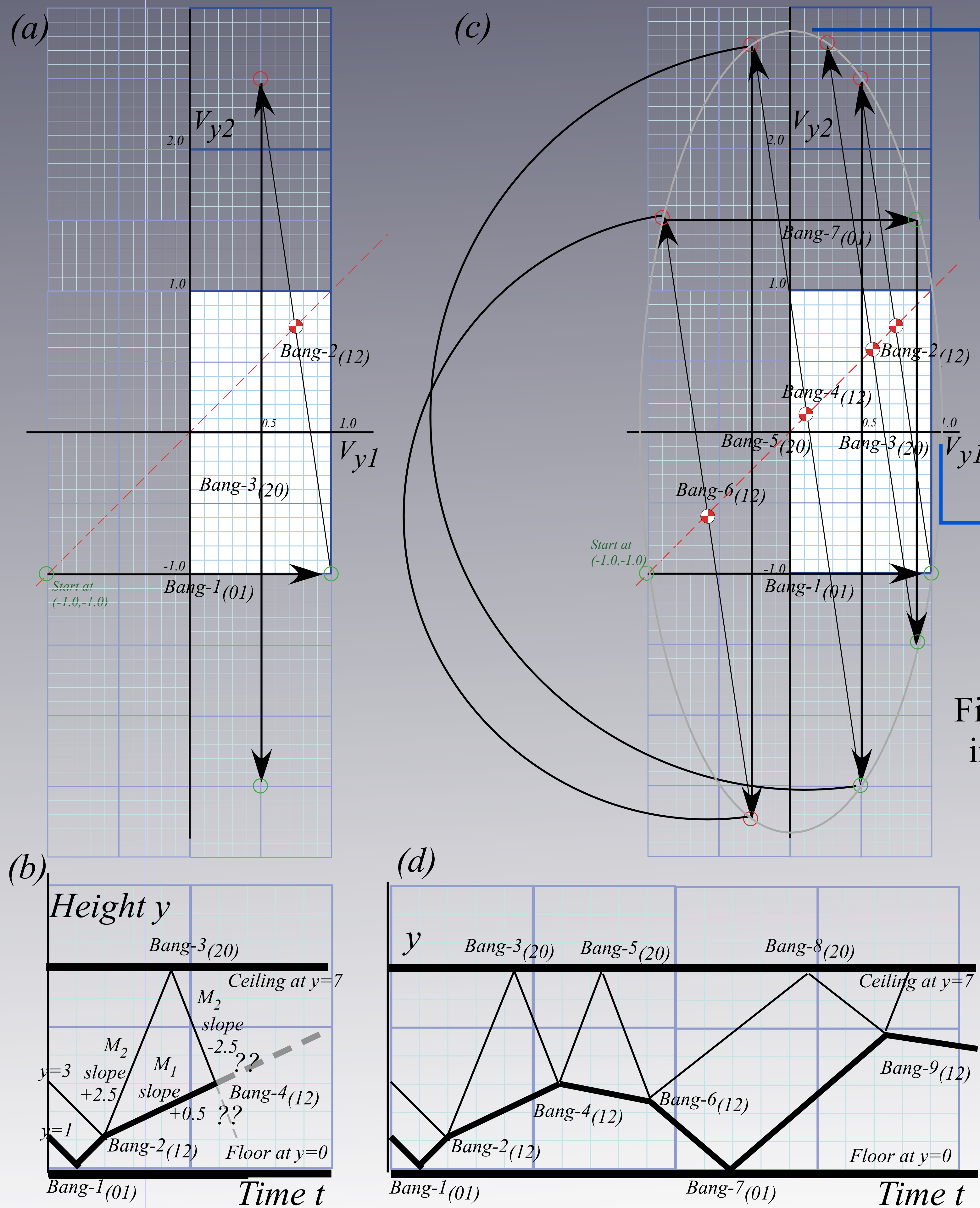


Fig. 4.7a-d in Unit 1

Geometric "Integration" (Converting Velocity data to Spacetime)



Kinetic Energy Ellipse

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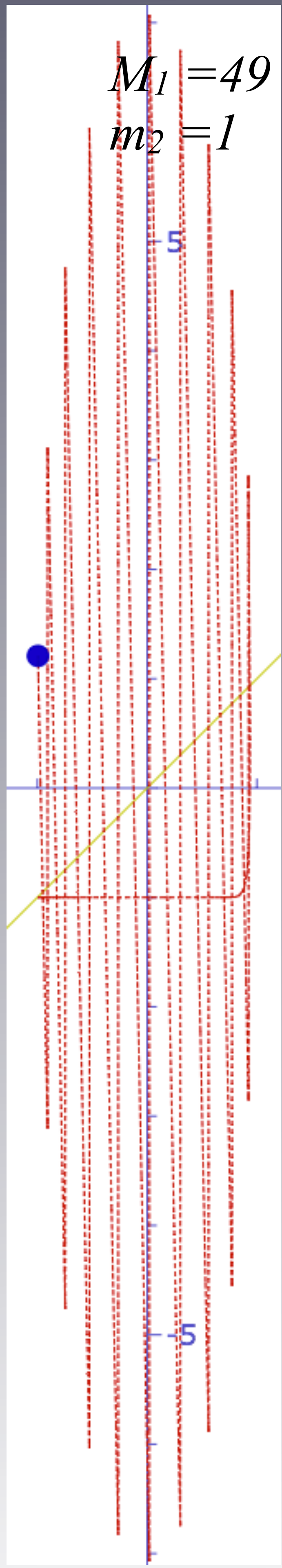
Ellipse radius 1

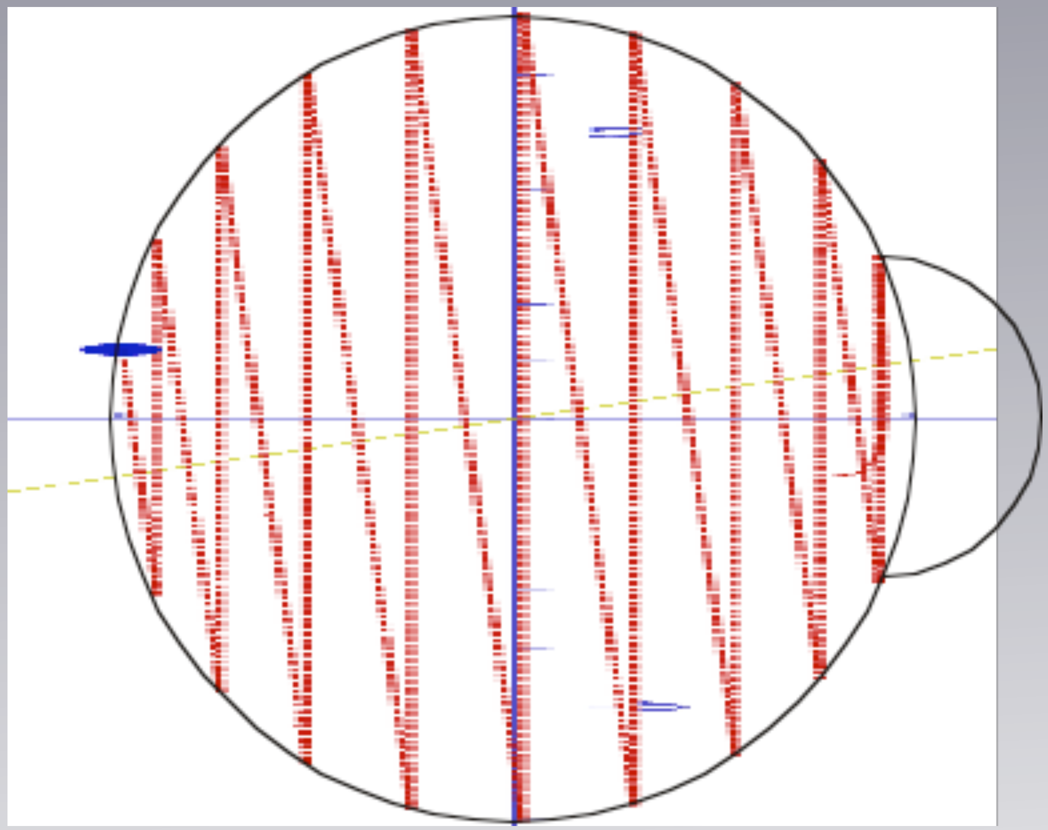
$$a_1 = \sqrt{2KE/M_1} = \sqrt{2KE/7} = \sqrt{8/7} = 1.07$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/M_1} = \sqrt{2KE/1} = \sqrt{8/1} = 2.83$$

Fig. 4.7a-d in Unit 1





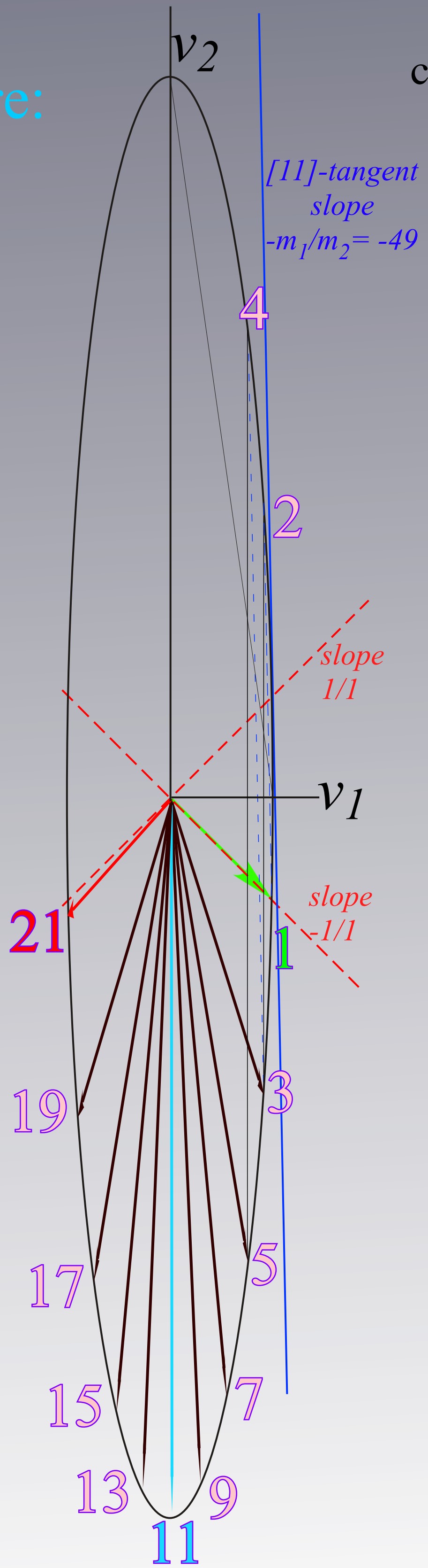
Difficult to see high mass-ratio-skinny-ellipse improved by
Scale transformation $M_1 v_1 \to \sqrt{M_1}\, v_1$

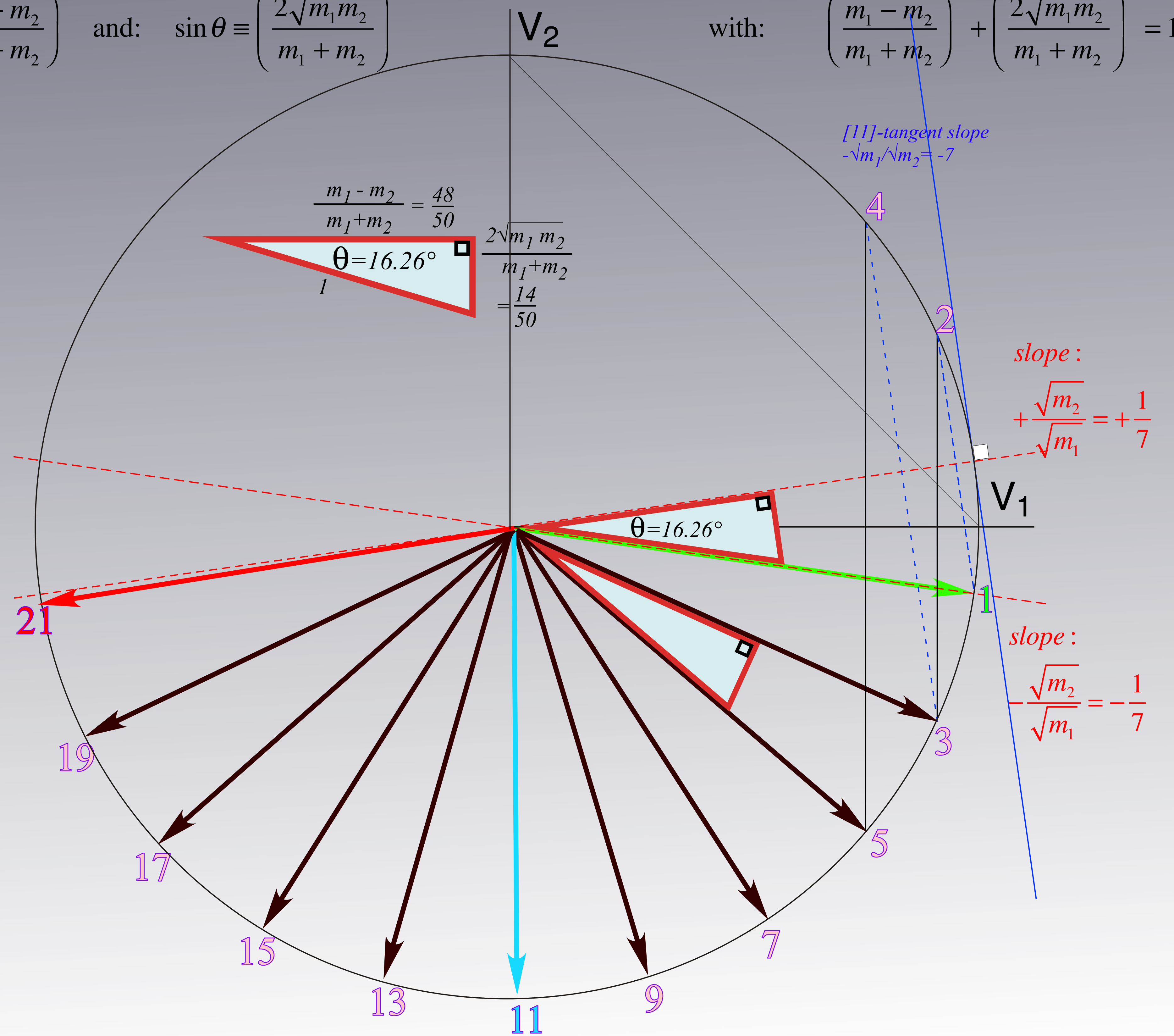
Ellipse rescaling geometry and reflection symmetry analysis

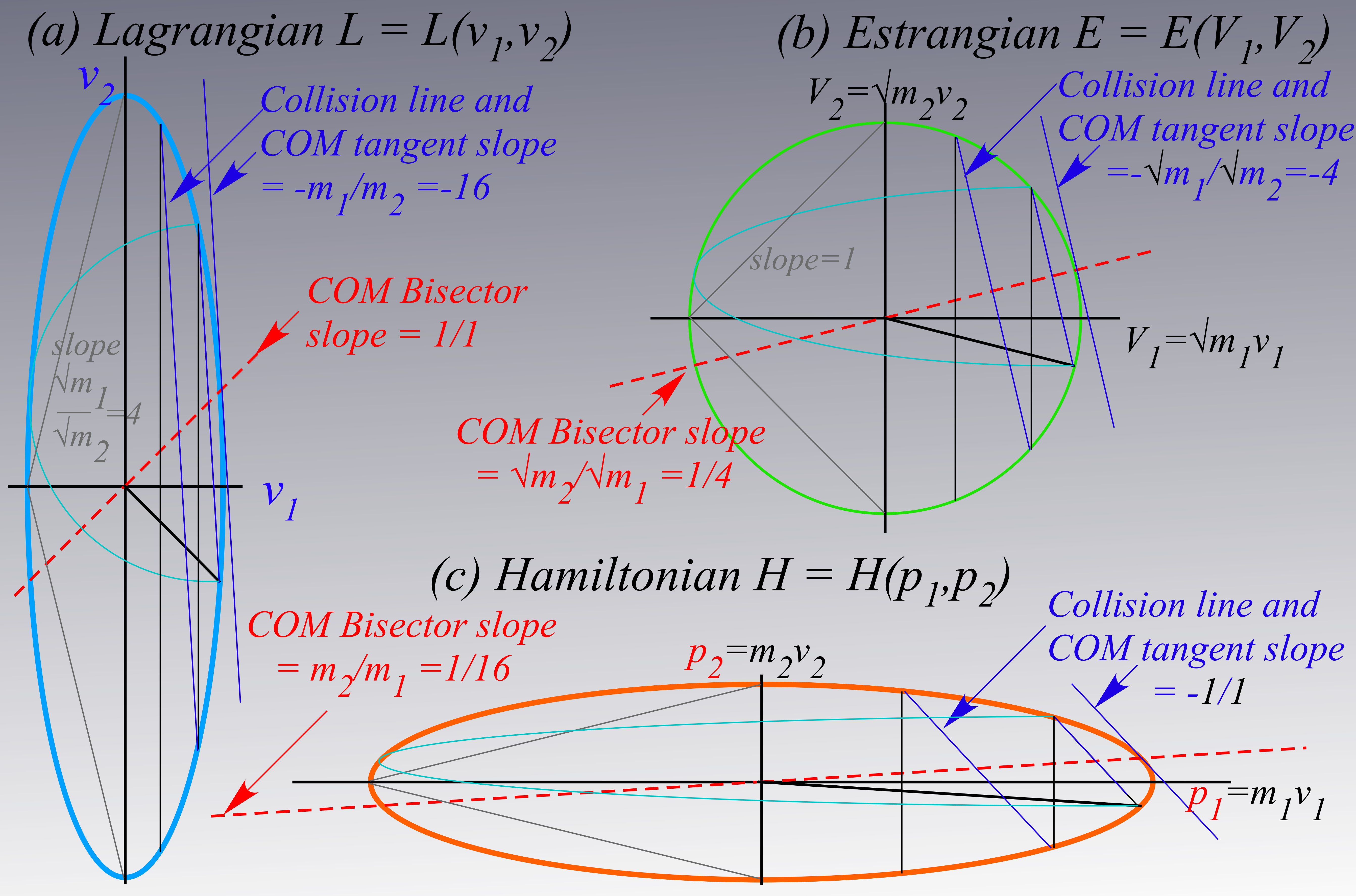
Convert to rescaled velocity: $V_1 = v_1 \cdot \sqrt{m_1}$, $V_2 = v_2 \cdot \sqrt{m_1}$, symmetrize: $KE = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} V_1^2 + \frac{1}{2} V_2^2$

Then collisions become *reflections* $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$ and double-collisions become *rotations*

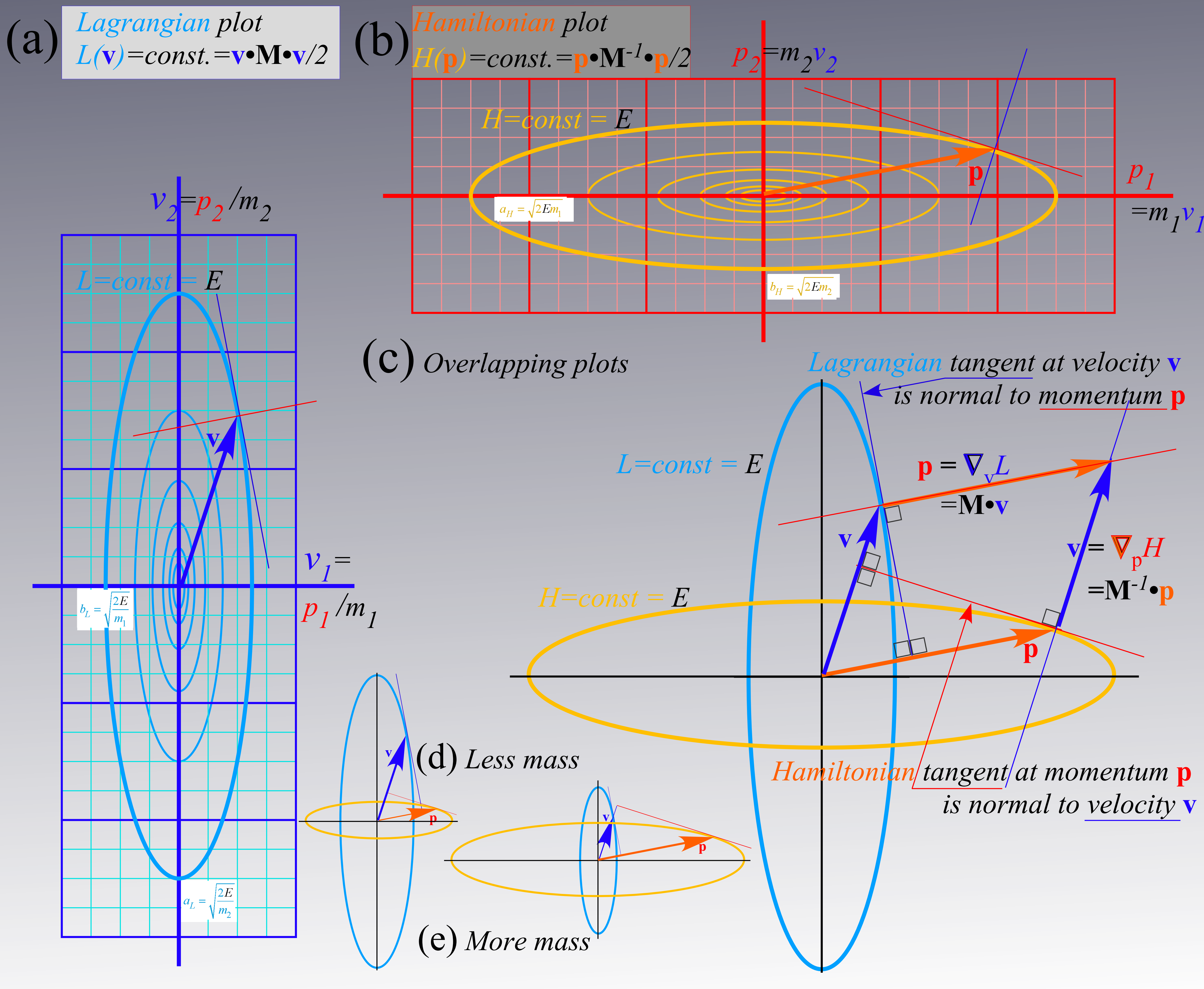
where: $\cos\theta \equiv \left(\frac{m_1 - m_2}{m_1 + m_2}\right)$ and: $\sin\theta \equiv \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)$ with: $\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 + \left(\frac{2\sqrt{m_1 m_2}}{m_1 + m_2}\right)^2 = 1$



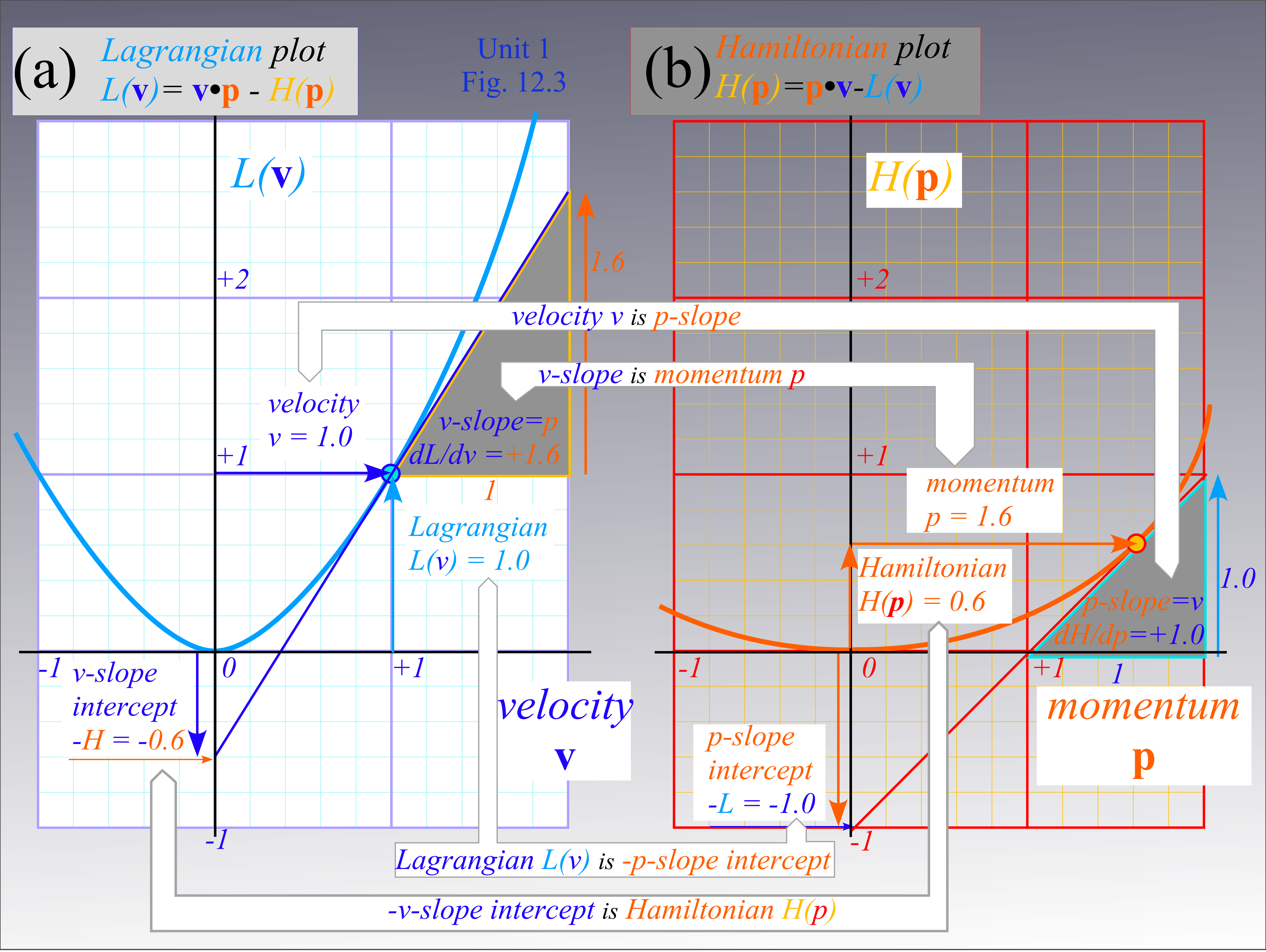




(a) Lagrangian L = L(v1,v2)
v2
v1
Collision line and
COM tangent slope
= -m1/m2 =-16
slope
√m1/√m2 =4
COM Bisector
slope = 1/1
COM Bisector slope
= m2/m1 =1/16
(b) Estrangian E = E(V1,V2)
V2=√m2v2
V1=√m1v1
Collision line and
COM tangent slope
=-√m1/√m2=-4
slope=1
COM Bisector slope
= √m2/√m1 =1/4
(c) Hamiltonian H = H(p1,p2)
p2=m2v2
p1=m1v1
Collision line and
COM tangent slope
= -1/1



(a) Lagrangian plot
$L(\mathbf{v})$=const.=$\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}/2$
(b) Hamiltonian plot
$H(\mathbf{p})$=const.=$\mathbf{p}\bullet\mathbf{M}^{-1}\bullet\mathbf{p}/2$
$p_2=m_2v_2$
H=const = E
p_1
$=m_1v_1$
$a_H=\sqrt{2Em_1}$
$b_H=\sqrt{2Em_2}$
$v_2=p_2/m_2$
L=const = E
$v_1=$
p_1/m_1
$b_L=\sqrt{\frac{2E}{m_1}}$
$a_L=\sqrt{\frac{2E}{m_2}}$
(c) Overlapping plots
Lagrangian tangent at velocity v
is normal to momentum p
L=const = E
$\mathbf{p}=\nabla_{\mathbf{v}}L$
$=\mathbf{M}\bullet\mathbf{v}$
$\mathbf{v}=\nabla_{\mathbf{p}}H$
$=\mathbf{M}^{-1}\bullet\mathbf{p}$
H=const = E
Hamiltonian tangent at momentum p
is normal to velocity v
(d) Less mass
(e) More mass



(a) Lagrangian plot
L(v)= v•p - H(p)
Unit 1
Fig. 12.3
(b) Hamiltonian plot
H(p)=p•v-L(v)
L(v)
H(p)
+2
+1
-1
0
+1
1.6
velocity v is p-slope
v-slope is momentum p
velocity
v = 1.0
v-slope=p
dL/dv =+1.6
1
Lagrangian
L(v) = 1.0
momentum
p = 1.6
Hamiltonian
H(p) = 0.6
p-slope=v
dH/dp=+1.0
1.0
1
v-slope
intercept
-H = -0.6
velocity
v
p-slope
intercept
-L = -1.0
momentum
p
Lagrangian L(v) is -p-slope intercept
-v-slope intercept is Hamiltonian H(p)

"Monster Mash" classical segue to Heisenberg action relations

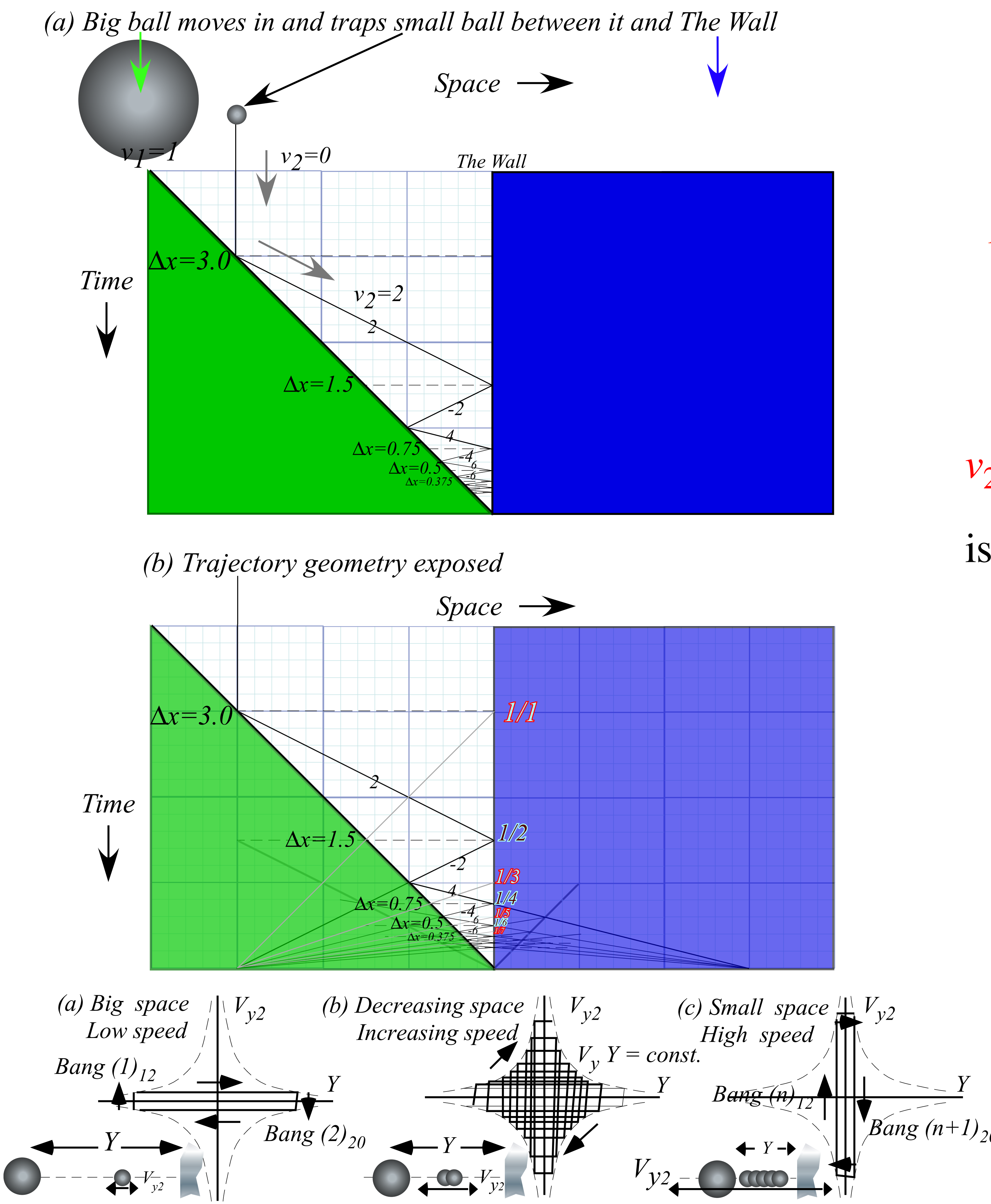
Example of very very large M_1 ball-walls crushing a poor little m_2

How m_2 keeps its action

An interesting wave analogy: The "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*],[*Harter, Li IMSS (2012)*]

A lesson in geometry of fractions: Ford Circles and Farey Sums

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*] [*John Farey, Phil. Mag.(1816)*]



The Classical "Monster Mash"

Classical introduction to

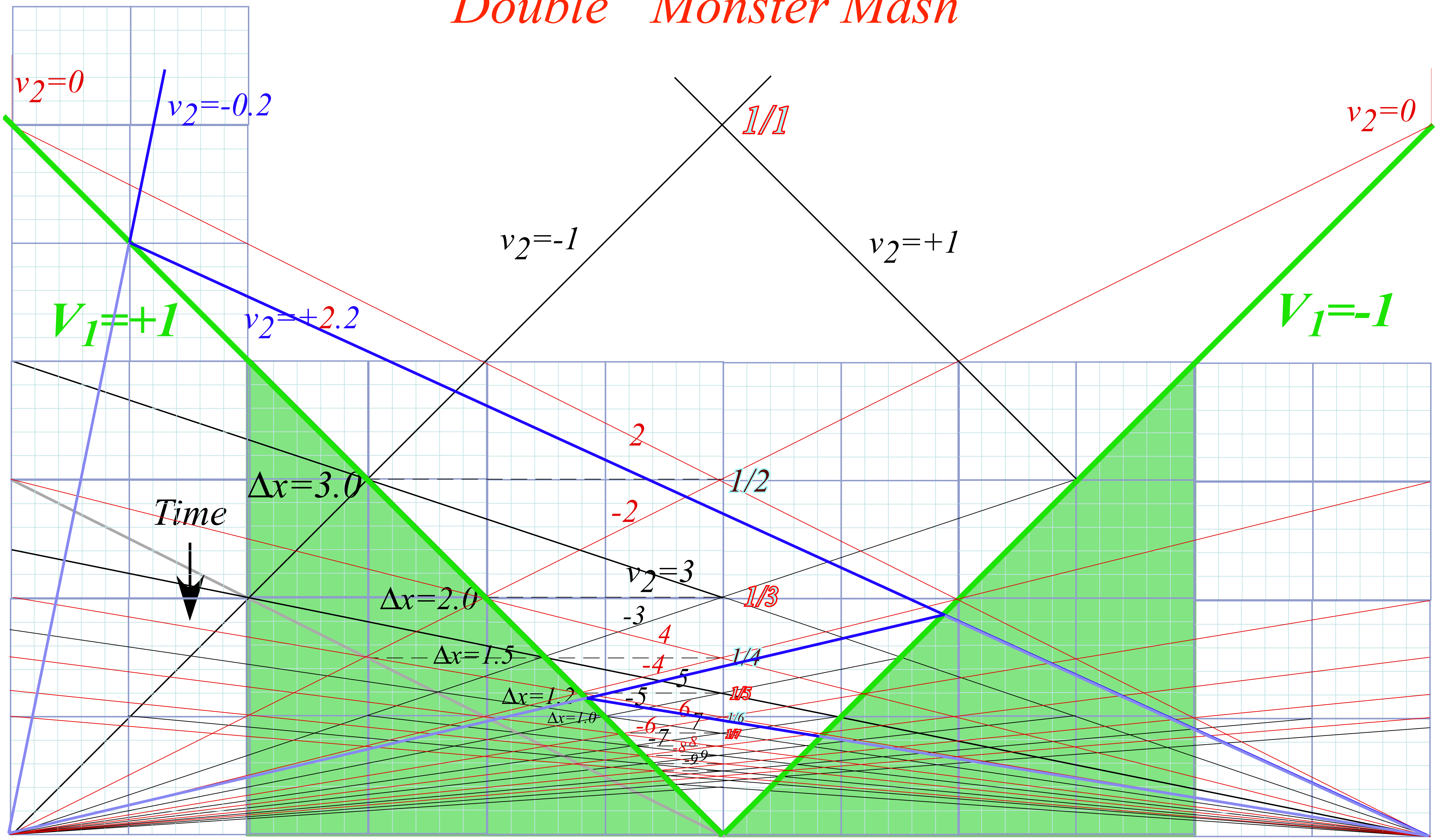
Heisenberg "Uncertainty" Relations

$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

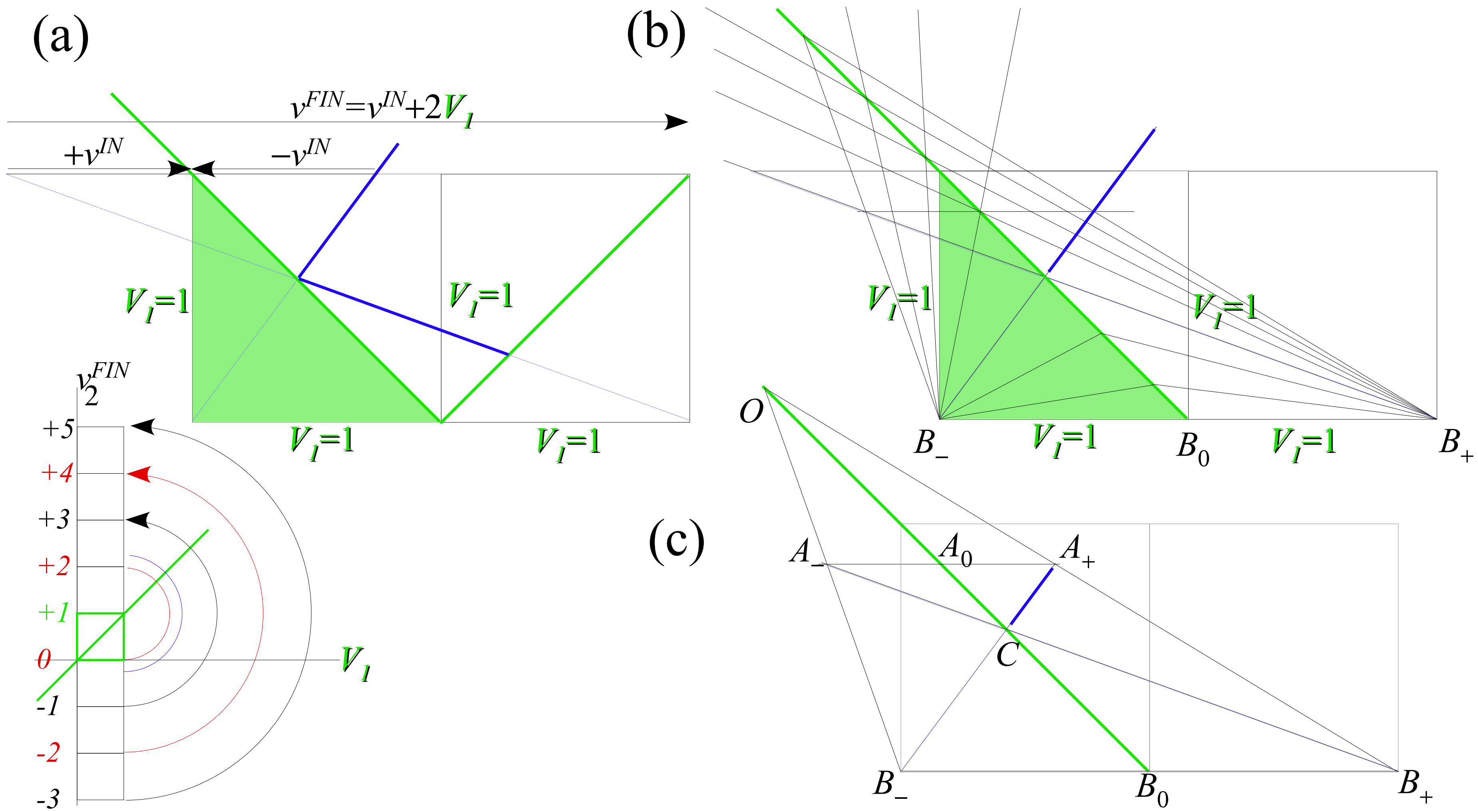
is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

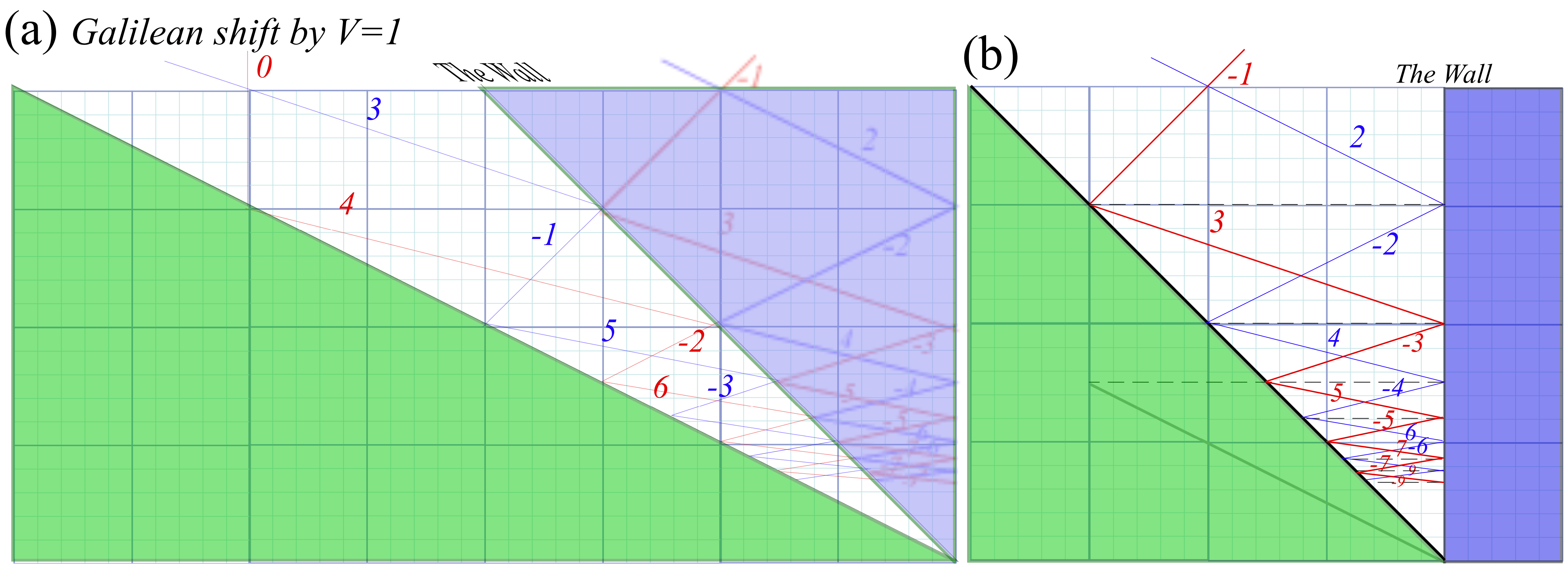
Unit 1
Fig. 6.4

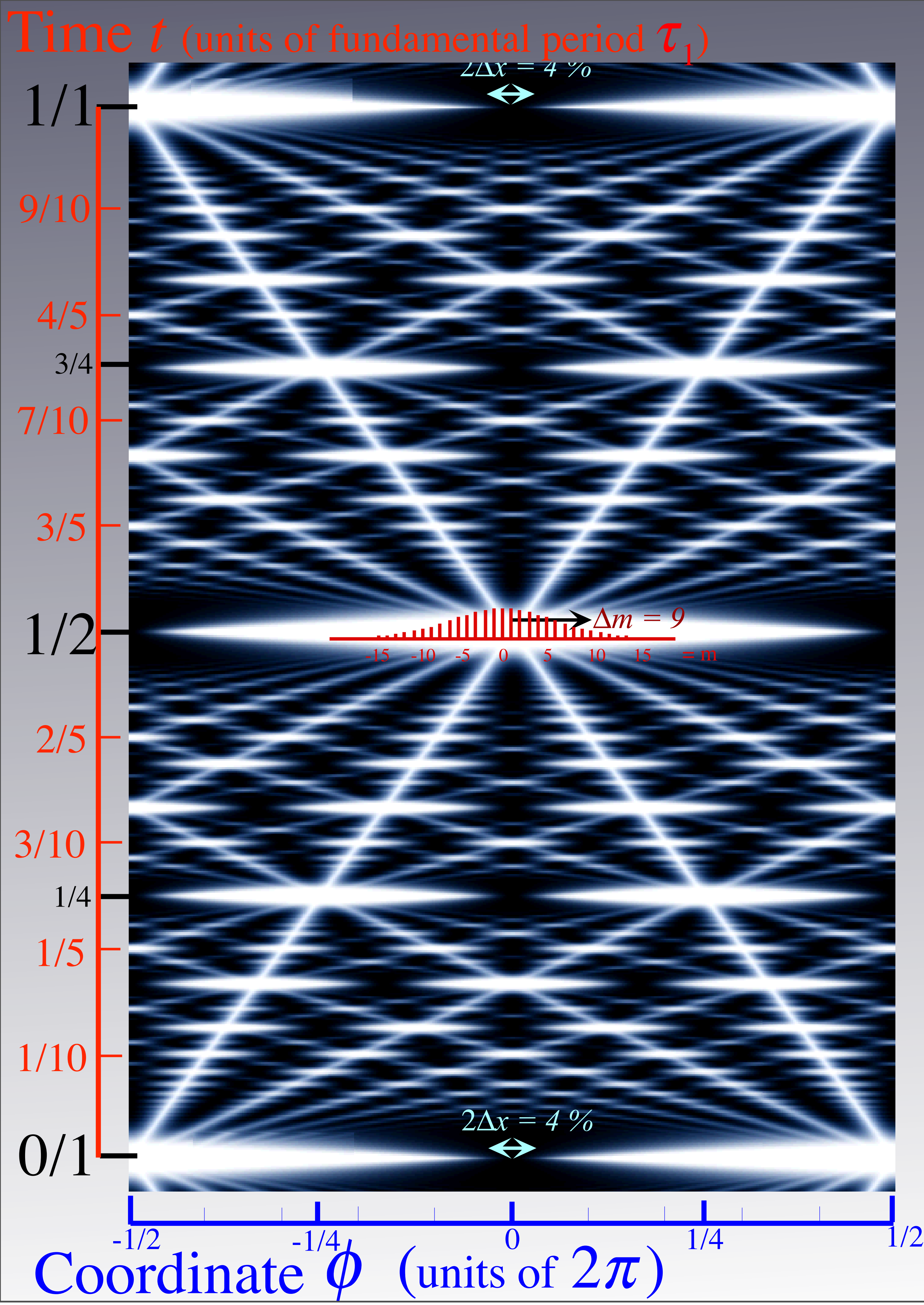
Double "Monster Mash"

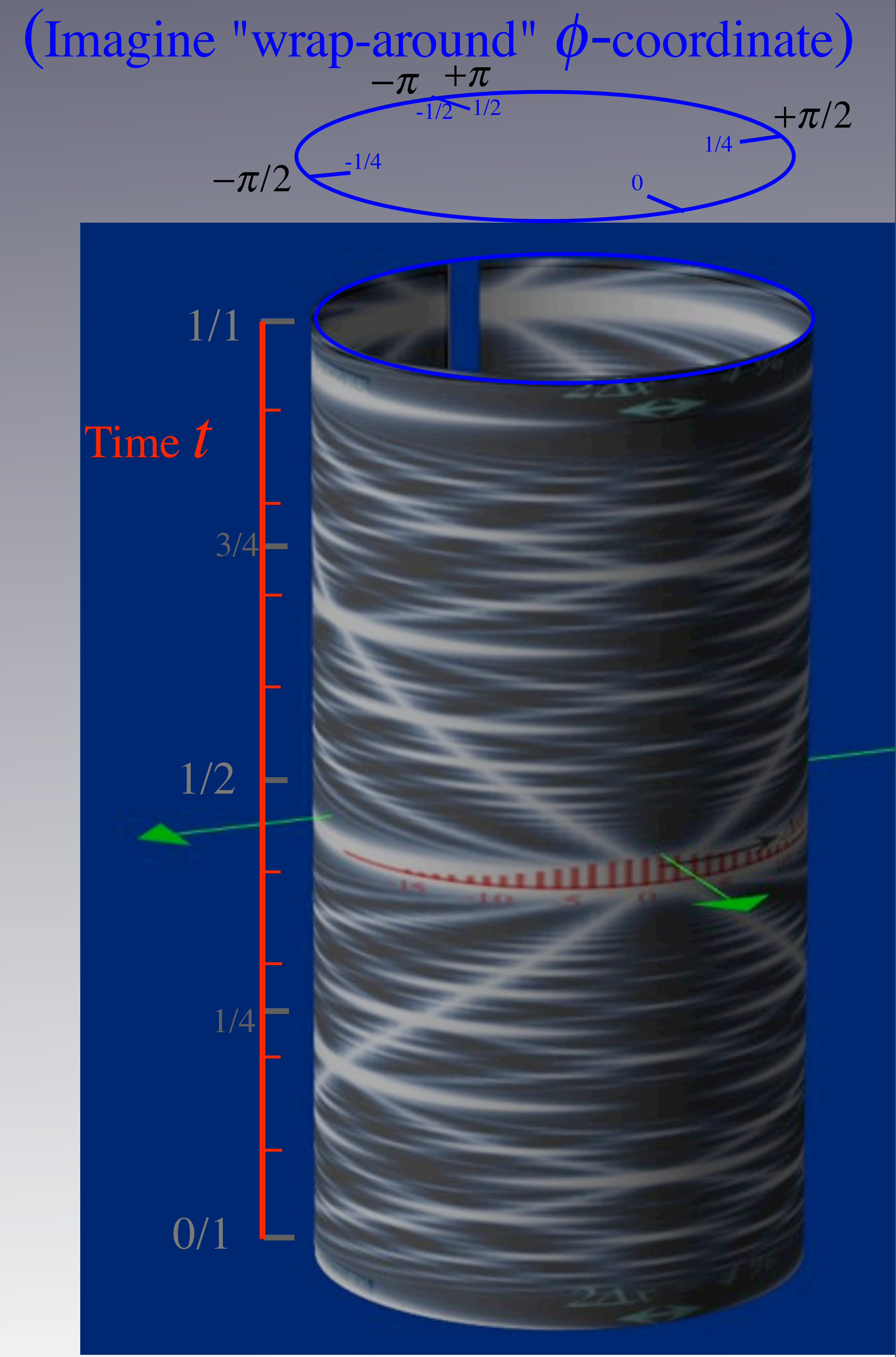


Unit 1
Fig. 6.6
and
Fig. 6.7







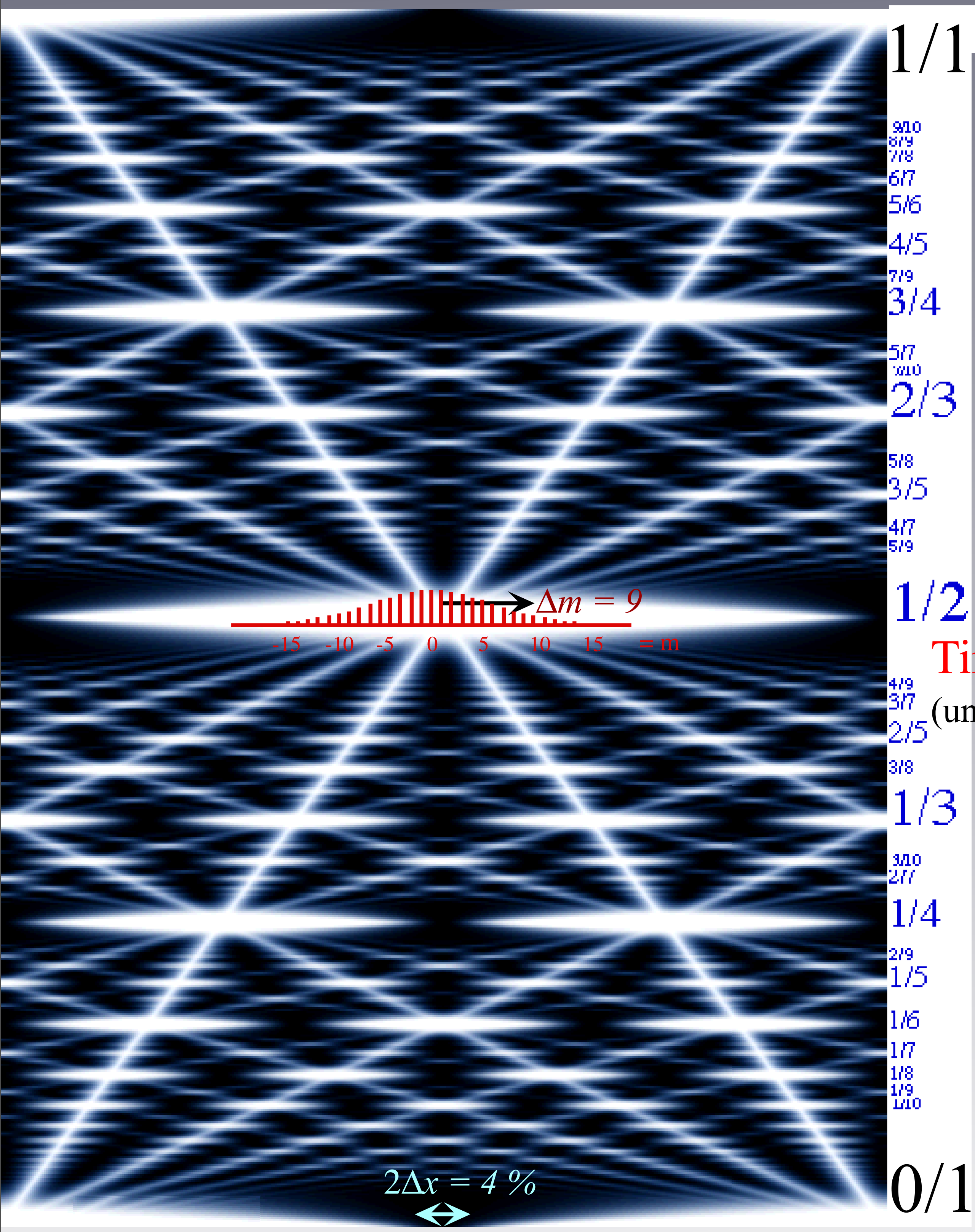


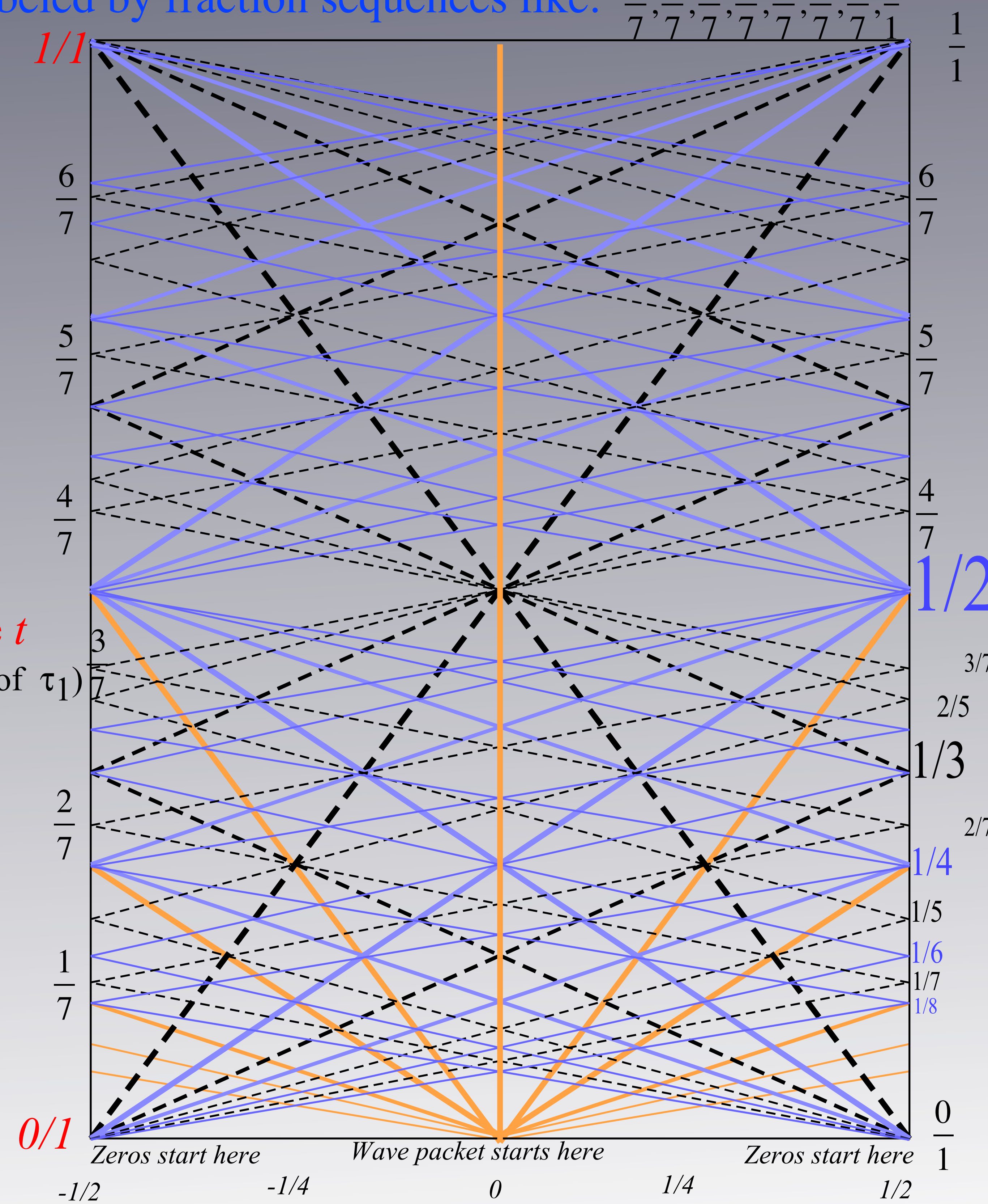
[*Harter, J. Mol. Spec. 210, 166-182 (2001)*]

N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$



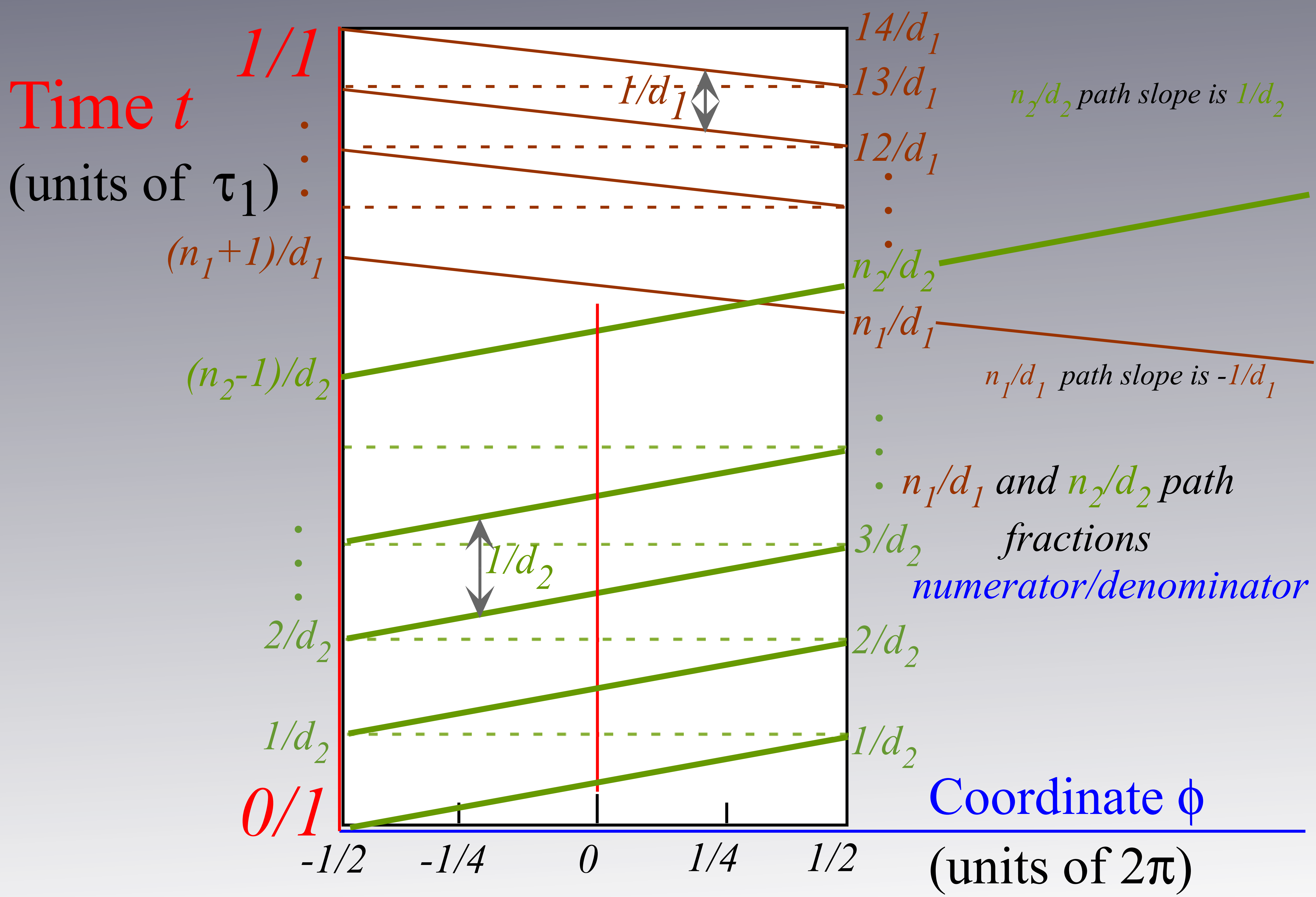


Coordinate ϕ
(units of 2π)

[Harter, J. Mol. Spec. 210, 166-182 (2001)]

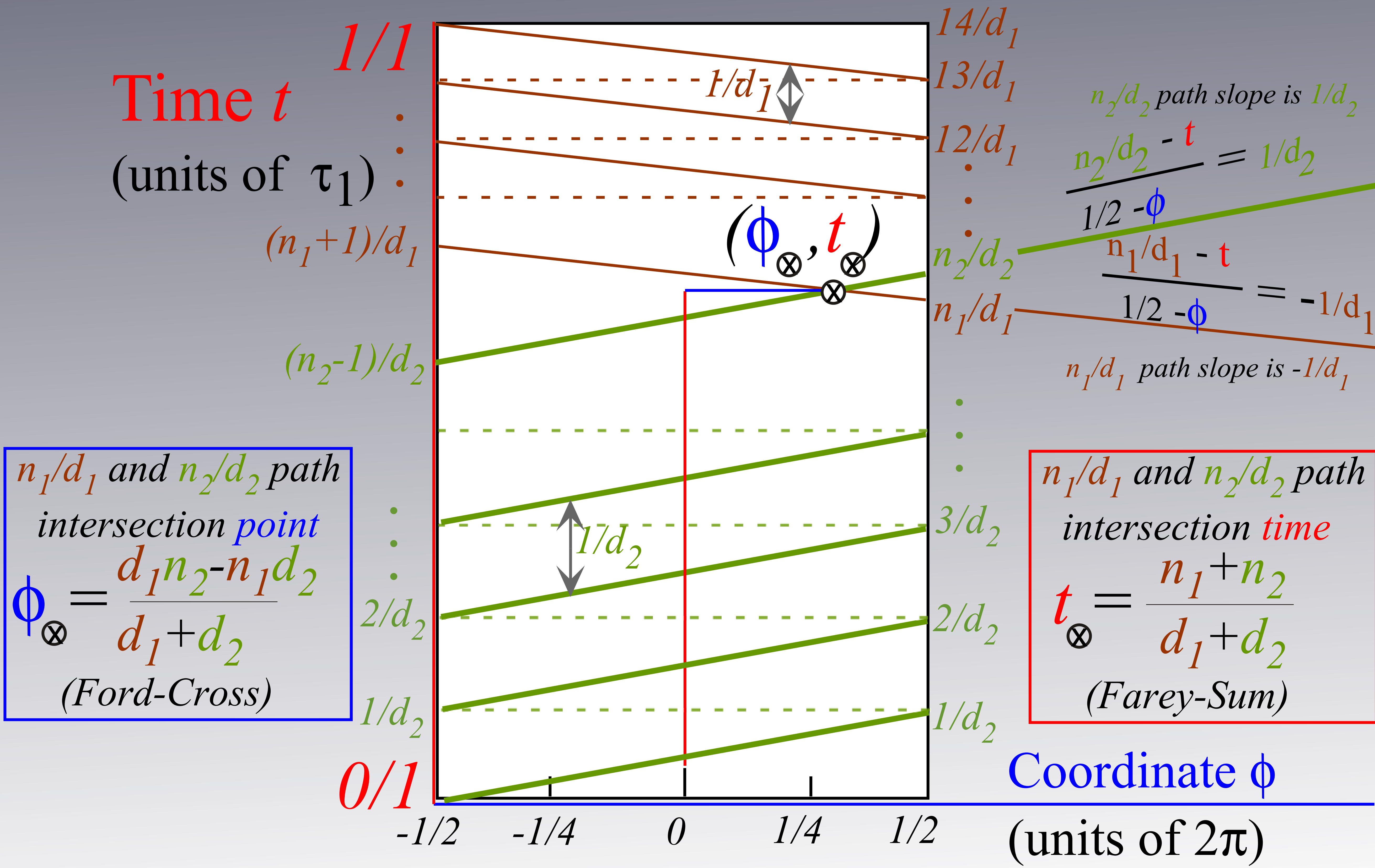
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



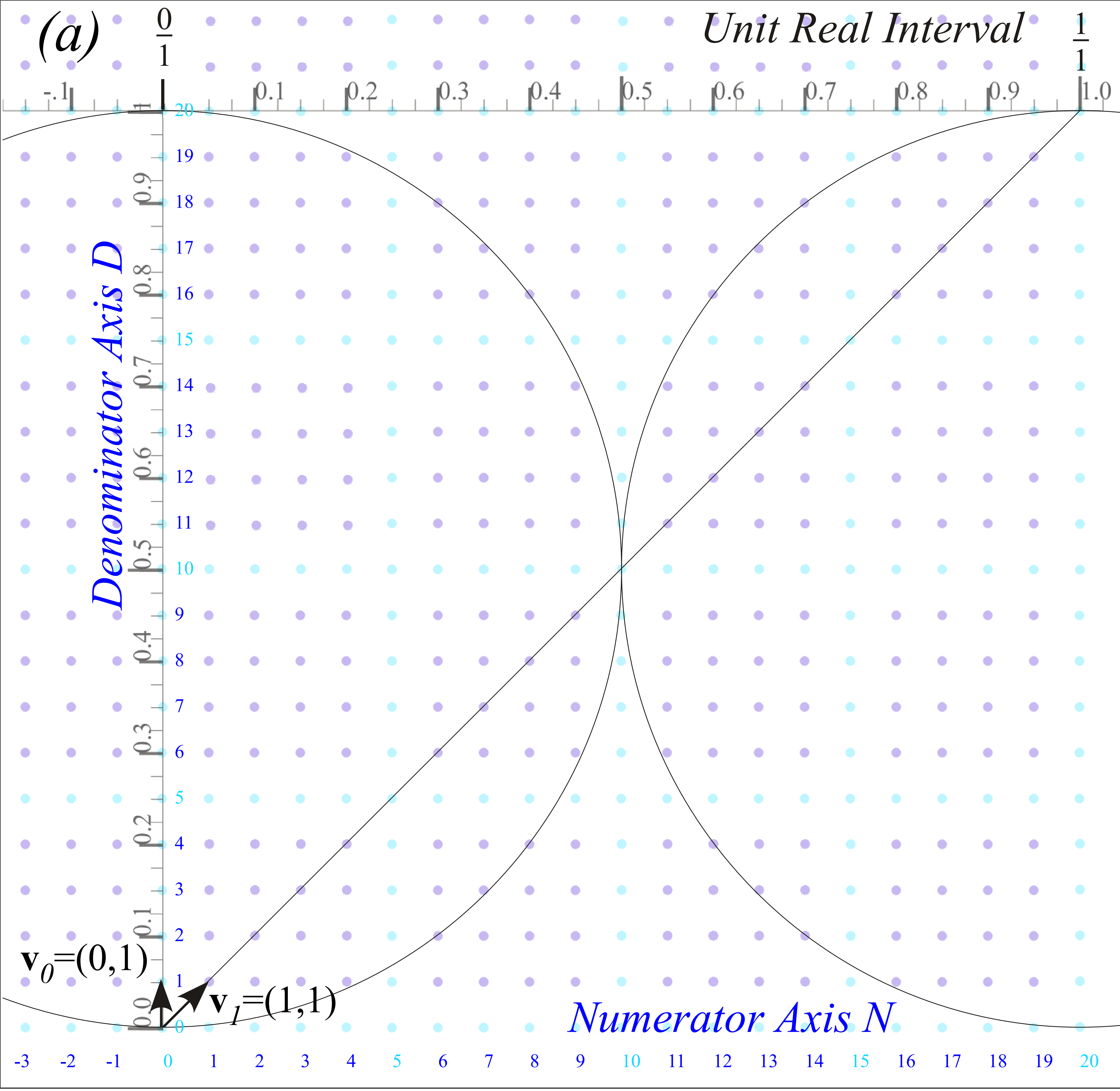
Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



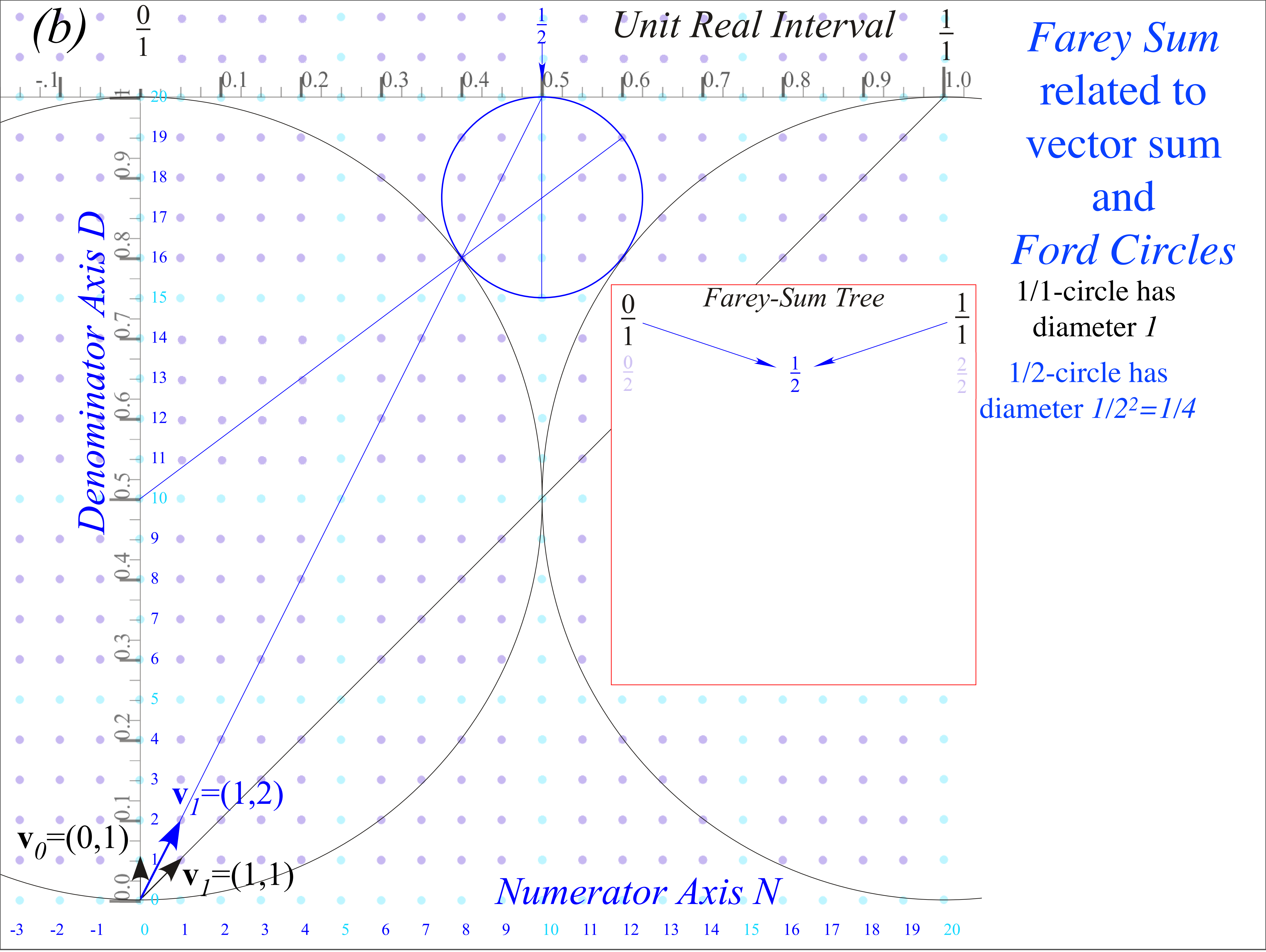
[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]

[John Farey, Phil. Mag.(1816)]

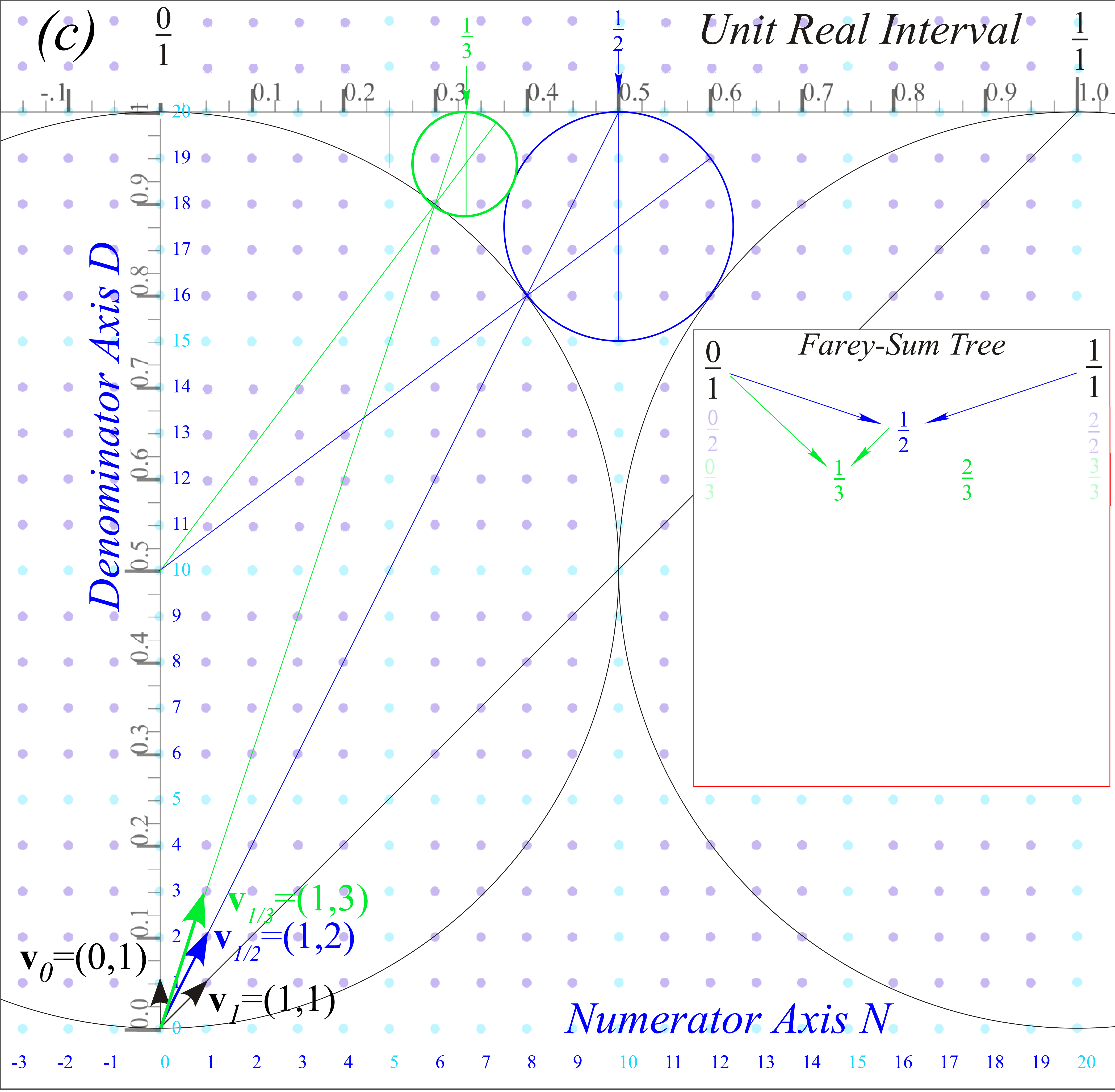


Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



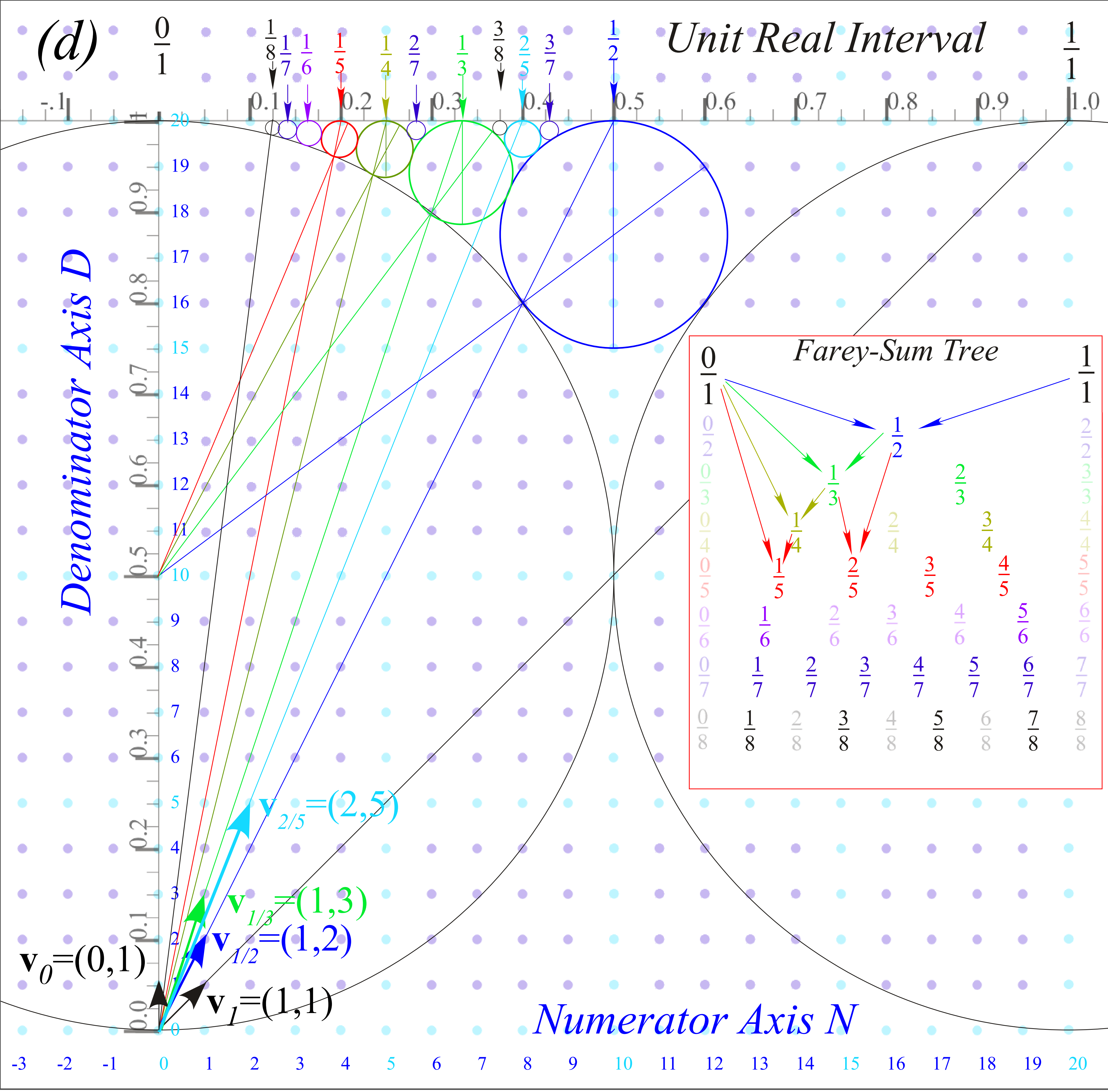
(b)
Unit Real Interval
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
1/2-circle has diameter 1/2²=1/4
Farey-Sum Tree
Denominator Axis D
Numerator Axis N
v0=(0,1)
v1=(1,1)
v1=(1,2)



Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$



Farey Sum related to vector sum and *Ford Circles*

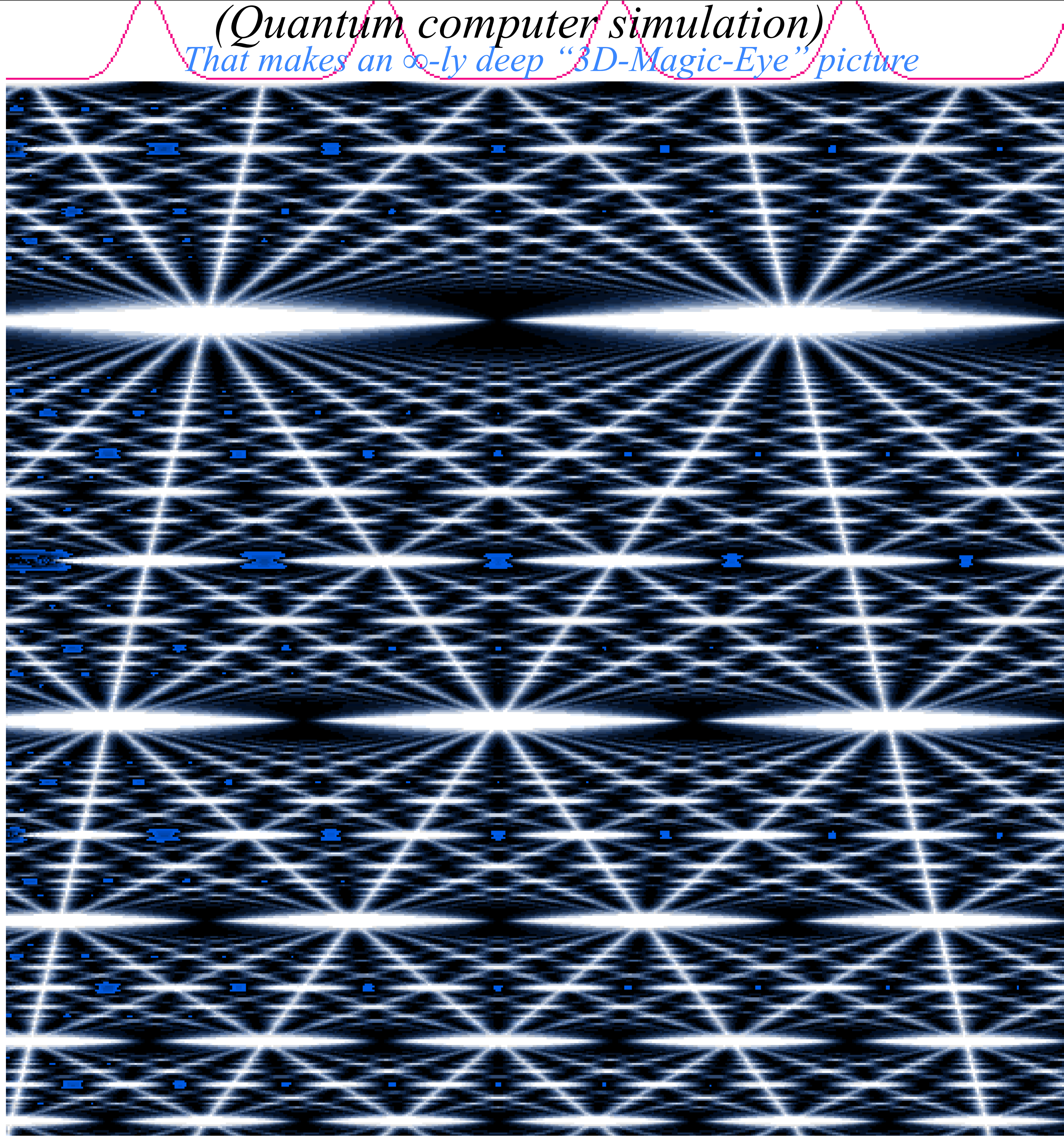
1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

(Quantum computer simulation)

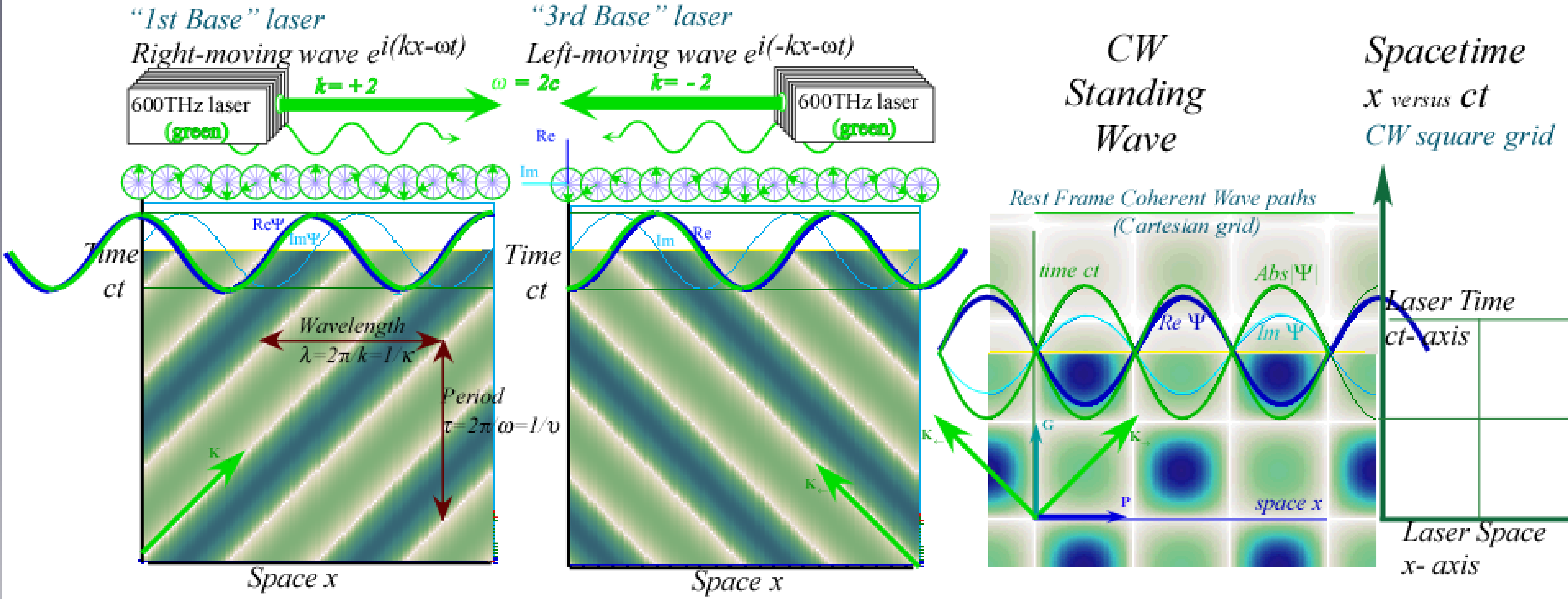
That makes an ∞-ly deep "3D-Magic-Eye" picture



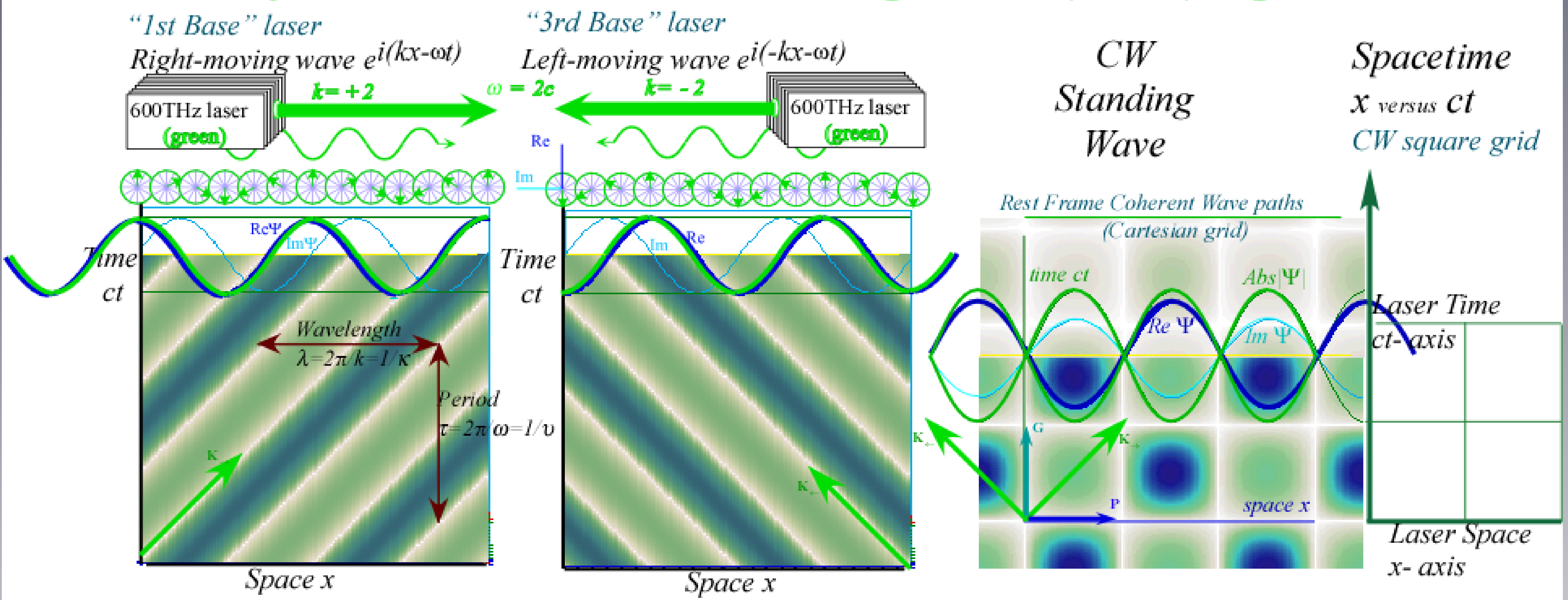
- Einstein-Lorentz-Minkowskii relativity

(Discover relativistic quantum mechanics)

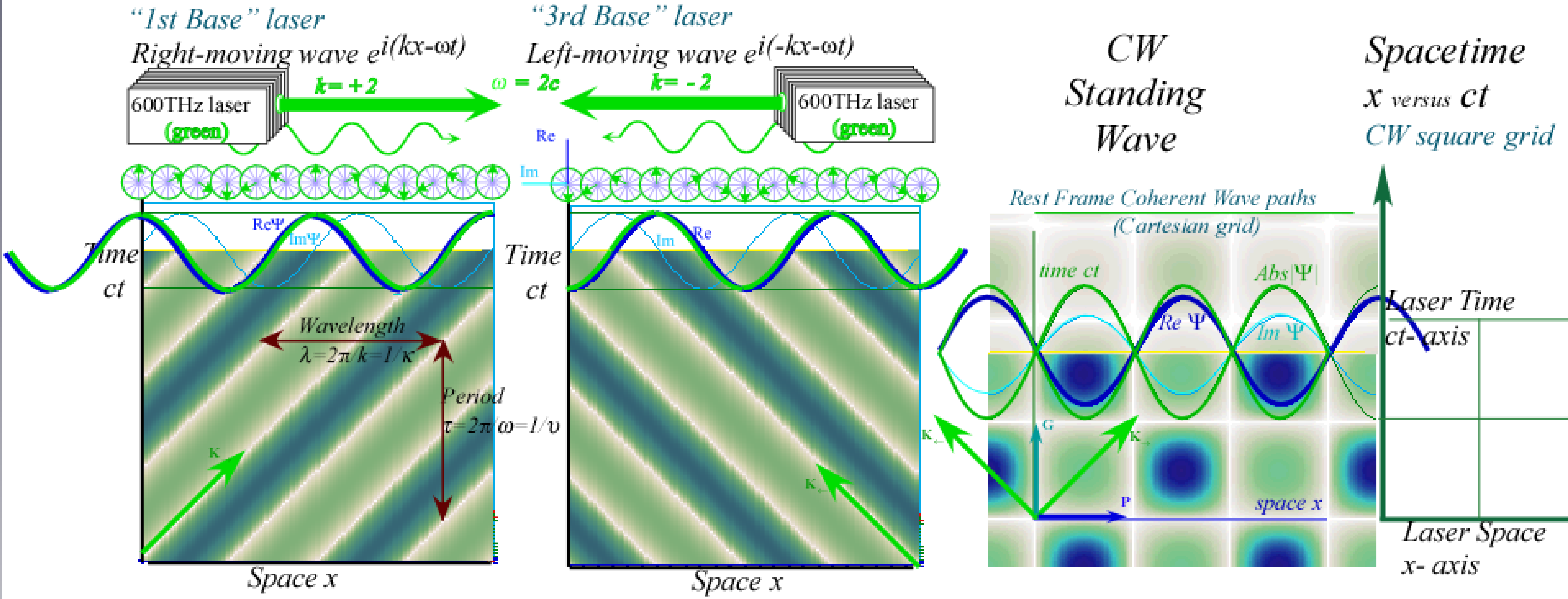
Zeros of head-on CW sum gives (x,ct)-grid



Zeros of head-on CW sum gives (x,ct)-grid



Zeros of head-on CW sum gives (x,ct)-grid



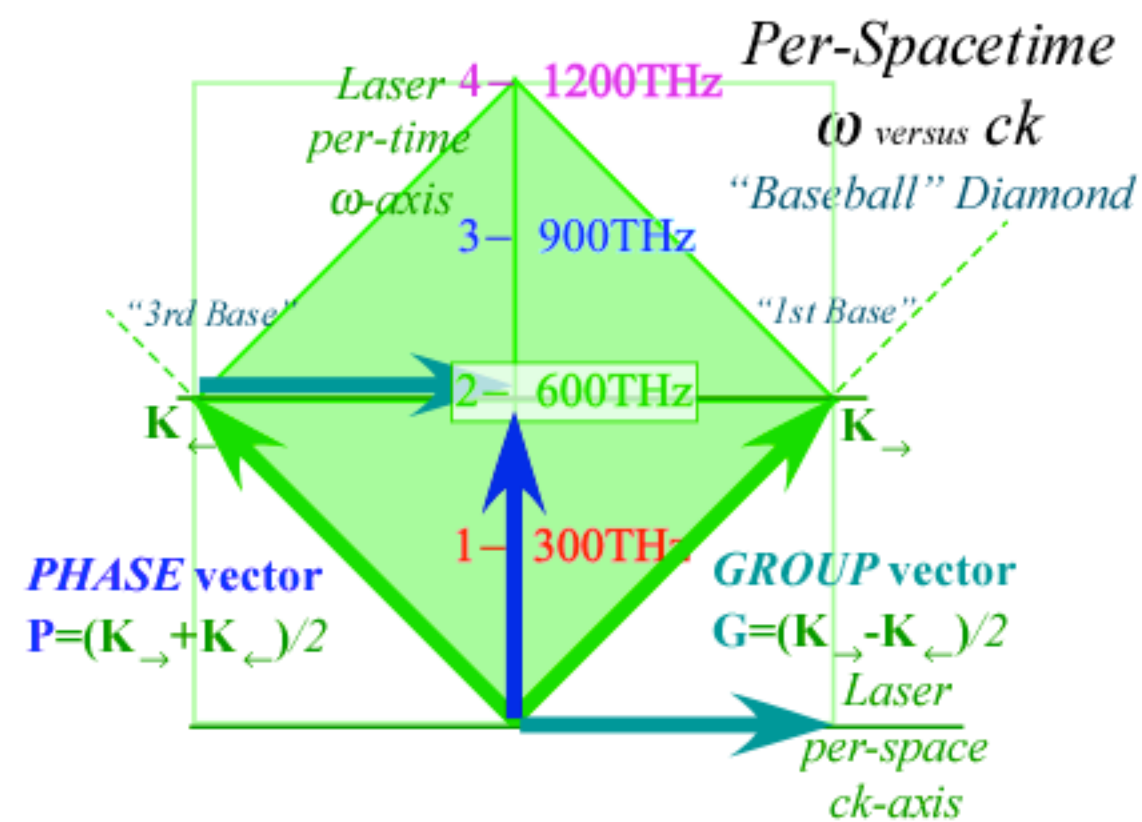
Find zeros by factoring sum:

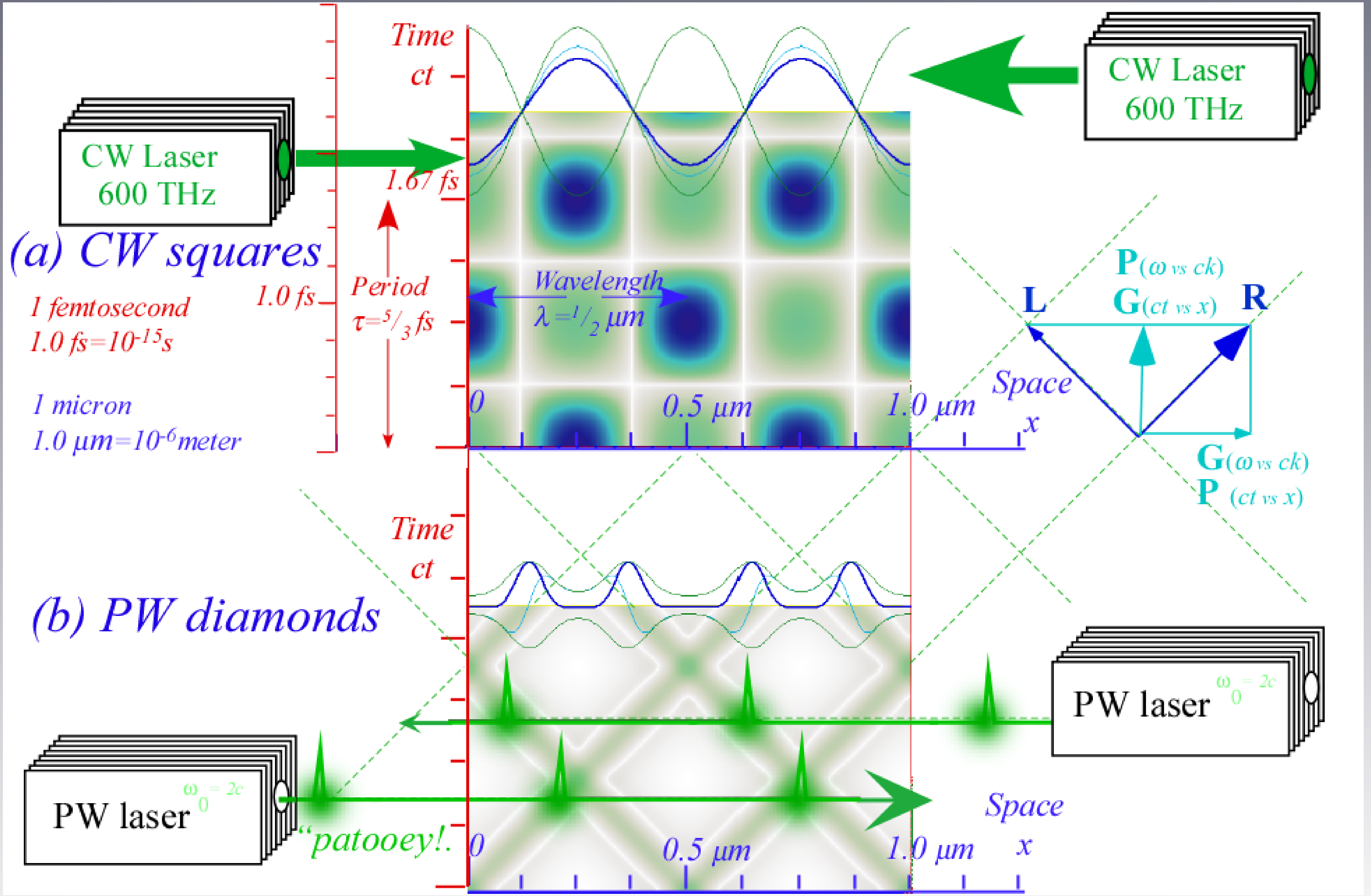
$$\Psi = \overset{(kx-\omega t)}{e^{ia}} + \overset{(-kx-\omega t)}{e^{ib}}$$

$$= \underbrace{e^{i(a+b)/2}}_{\text{Phase factor:}} \underbrace{\left(e^{i(a-b)/2} + e^{-i(a-b)/2}\right)}_{\text{Group factor:}}$$

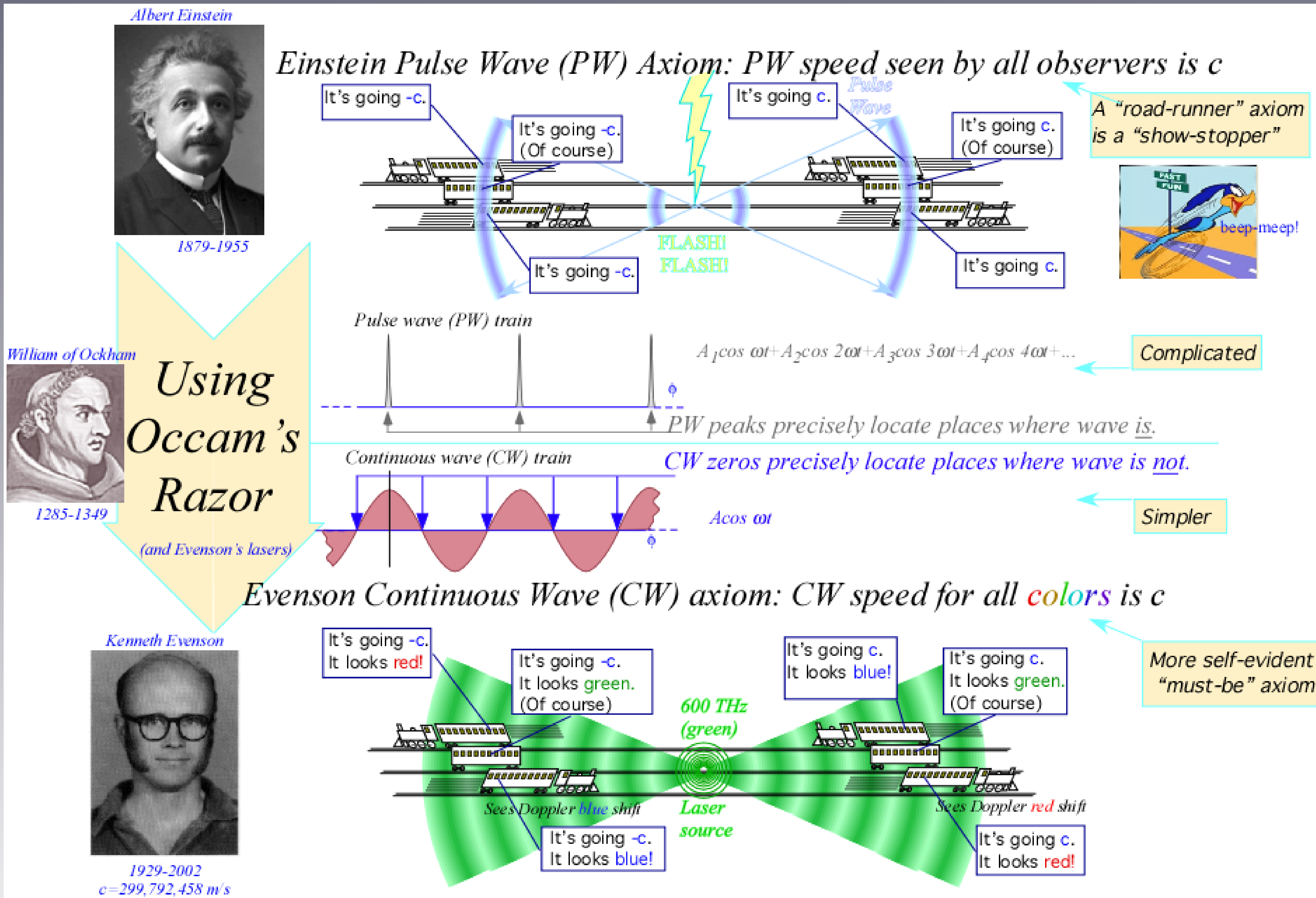
Phase factor: $\exp i\left(\frac{a+b}{2}\right) = e^{-i\omega t}$

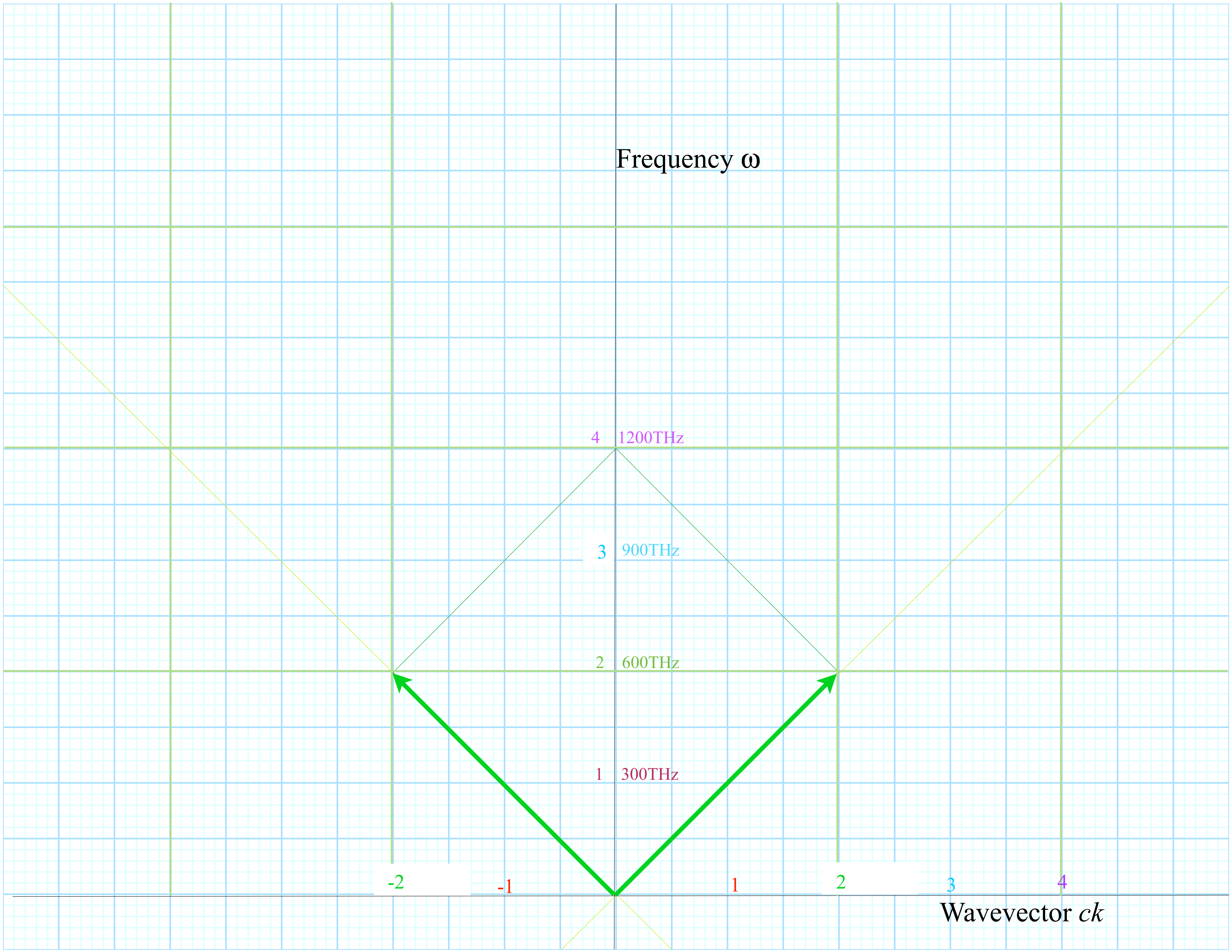
Group factor: $2\cos\left(\frac{a-b}{2}\right) = 2\cos(kx)$

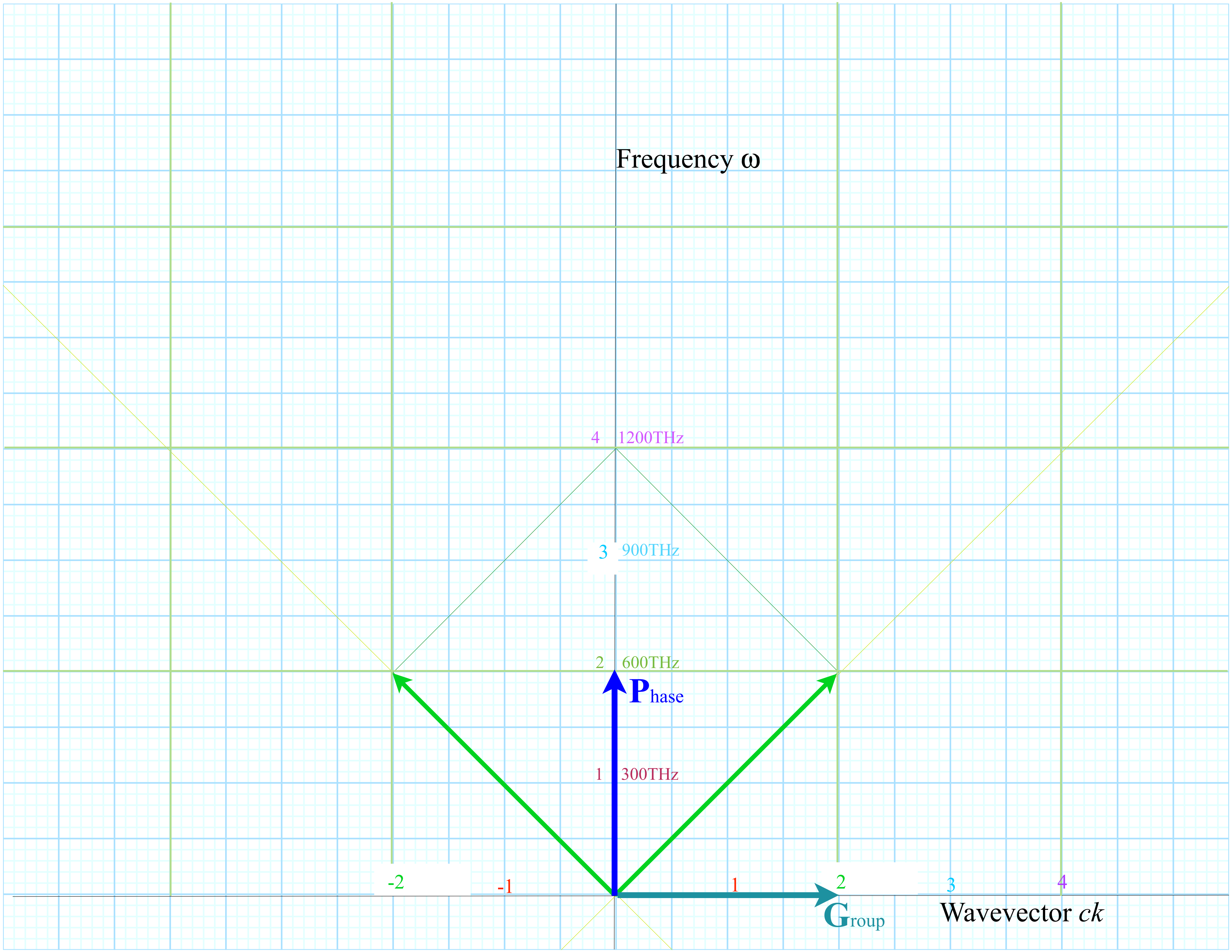


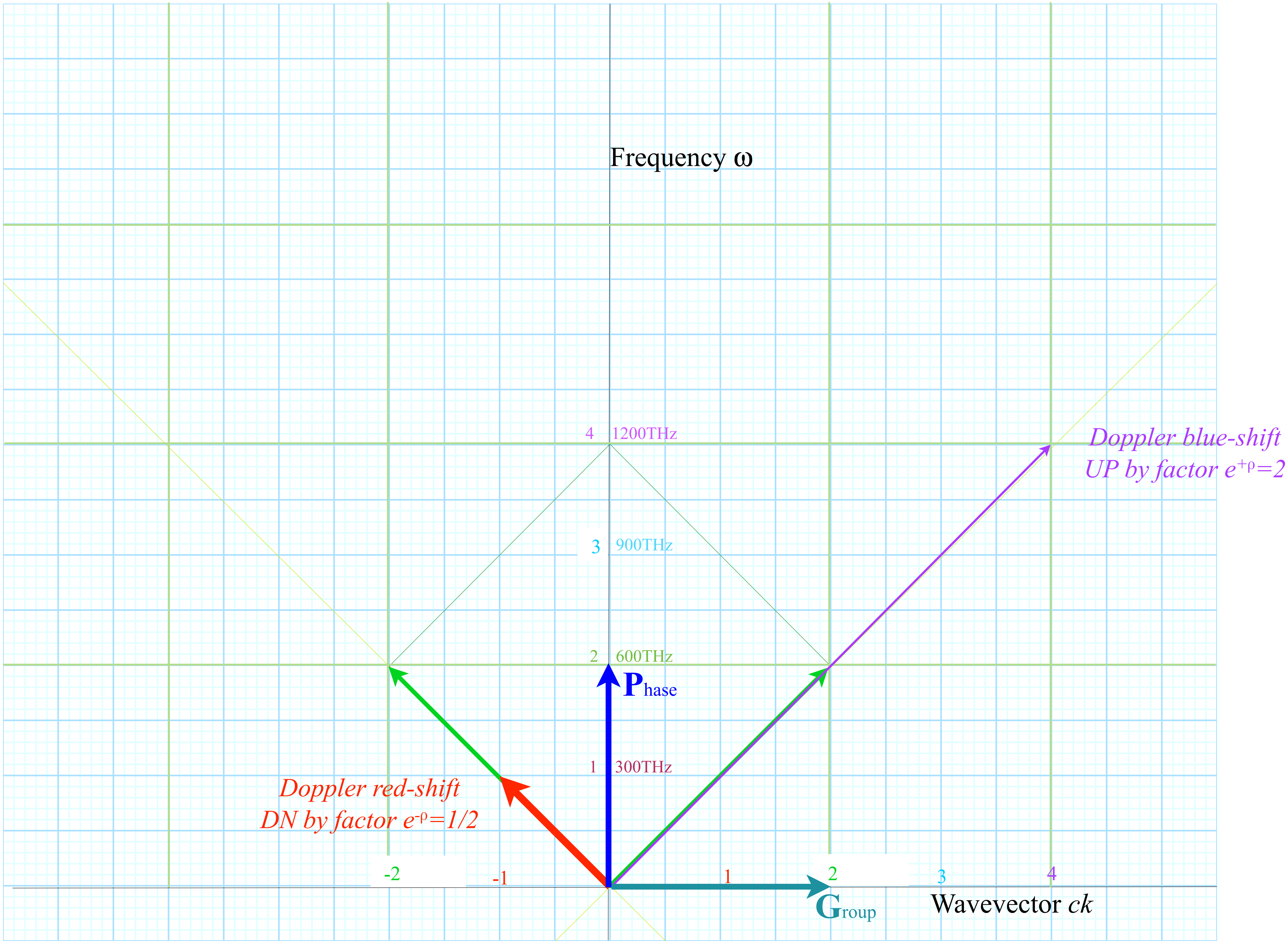


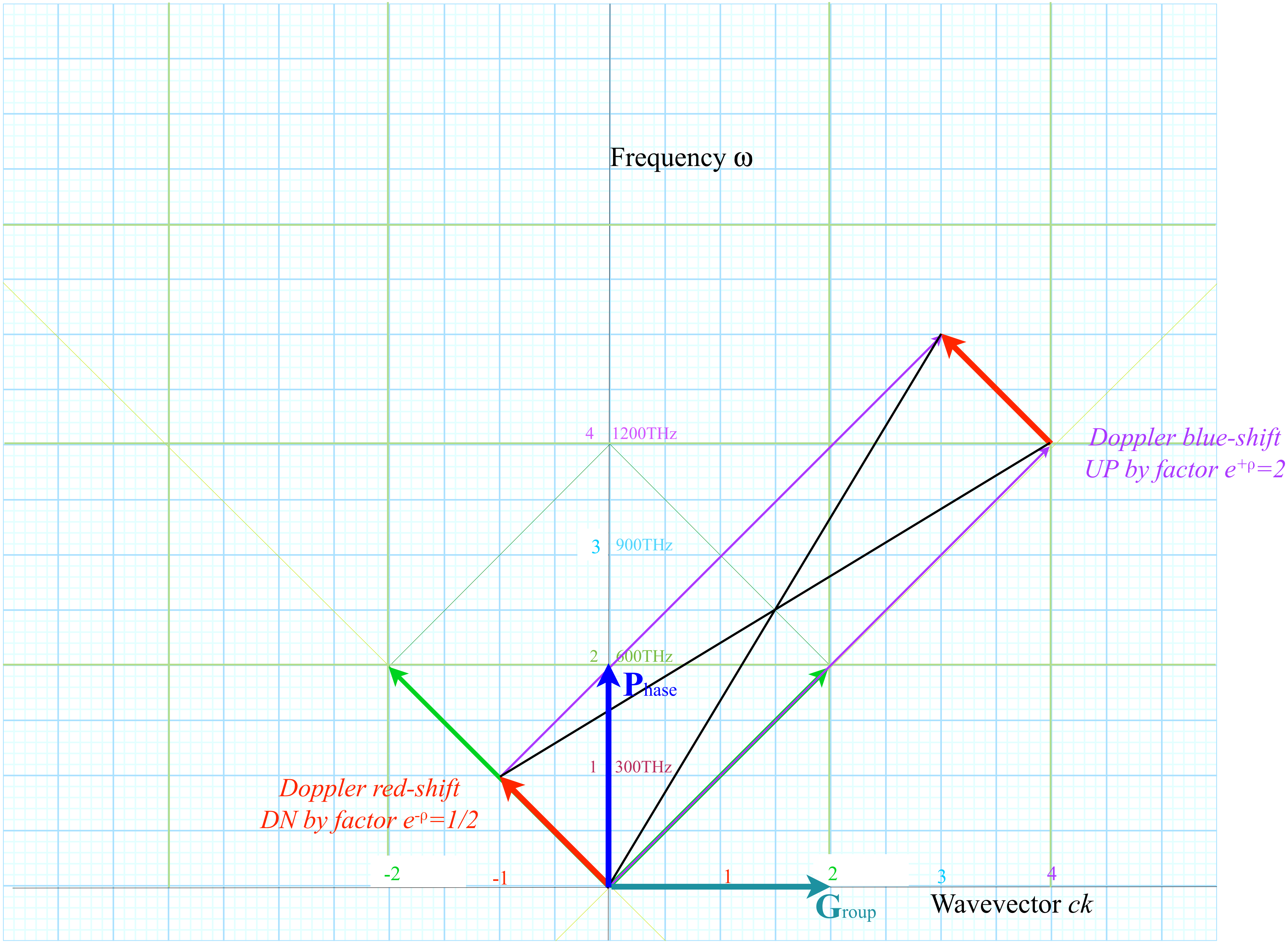
CW Laser
600 THz
CW Laser
600 THz
Time
ct
1.67 fs
(a) CW squares
1 femtosecond
1.0 fs=10-15s
1 micron
1.0 μm=10-6meter
1.0 fs
Period
τ=5/3 fs
Wavelength
λ=1/2 μm
0
0.5 μm
1.0 μm
Space
x
L
P(ω vs ck)
G(ct vs x)
R
G(ω vs ck)
P (ct vs x)
Time
ct
(b) PW diamonds
PW laser
PW laser
"patooey!.
0
0.5 μm
1.0 μm
Space
x

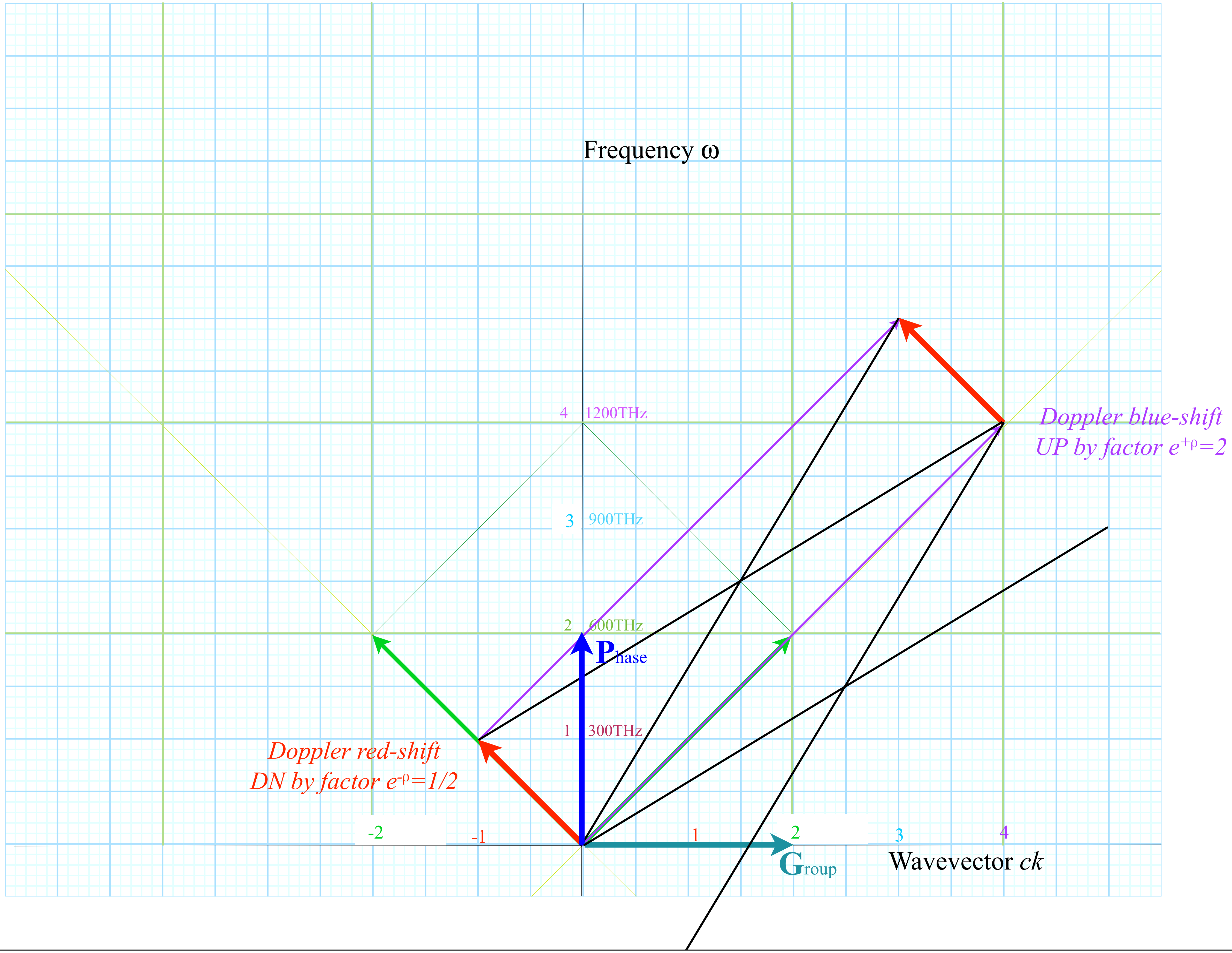

Albert Einstein
1879-1955
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c
It's going -c.
It's going -c.
(Of course)
It's going c.
Pulse
Wave
It's going c.
(Of course)
It's going -c.
FLASH!
FLASH!
It's going c.
A "road-runner" axiom
is a "show-stopper"
FAST
FUN
beep-meep!
William of Ockham
1285-1349
Using
Occam's
Razor
(and Evenson's lasers)
Pulse wave (PW) train
$A_1\cos\omega t + A_2\cos 2\omega t + A_3\cos 3\omega t + A_4\cos 4\omega t + \ldots$
Complicated
ϕ
PW peaks precisely locate places where wave is.
Continuous wave (CW) train
CW zeros precisely locate places where wave is not.
$A\cos\omega t$
Simpler
ϕ
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c
More self-evident
"must-be" axiom
Kenneth Evenson
1929-2002
c=299,792,458 m/s
It's going -c.
It looks red!
It's going -c.
It looks green.
(Of course)
600 THz
(green)
It's going c.
It looks blue!
It's going c.
It looks green.
(Of course)
Sees Doppler blue shift
Laser
source
Sees Doppler red shift
It's going -c.
It looks blue!
It's going c.
It looks red!

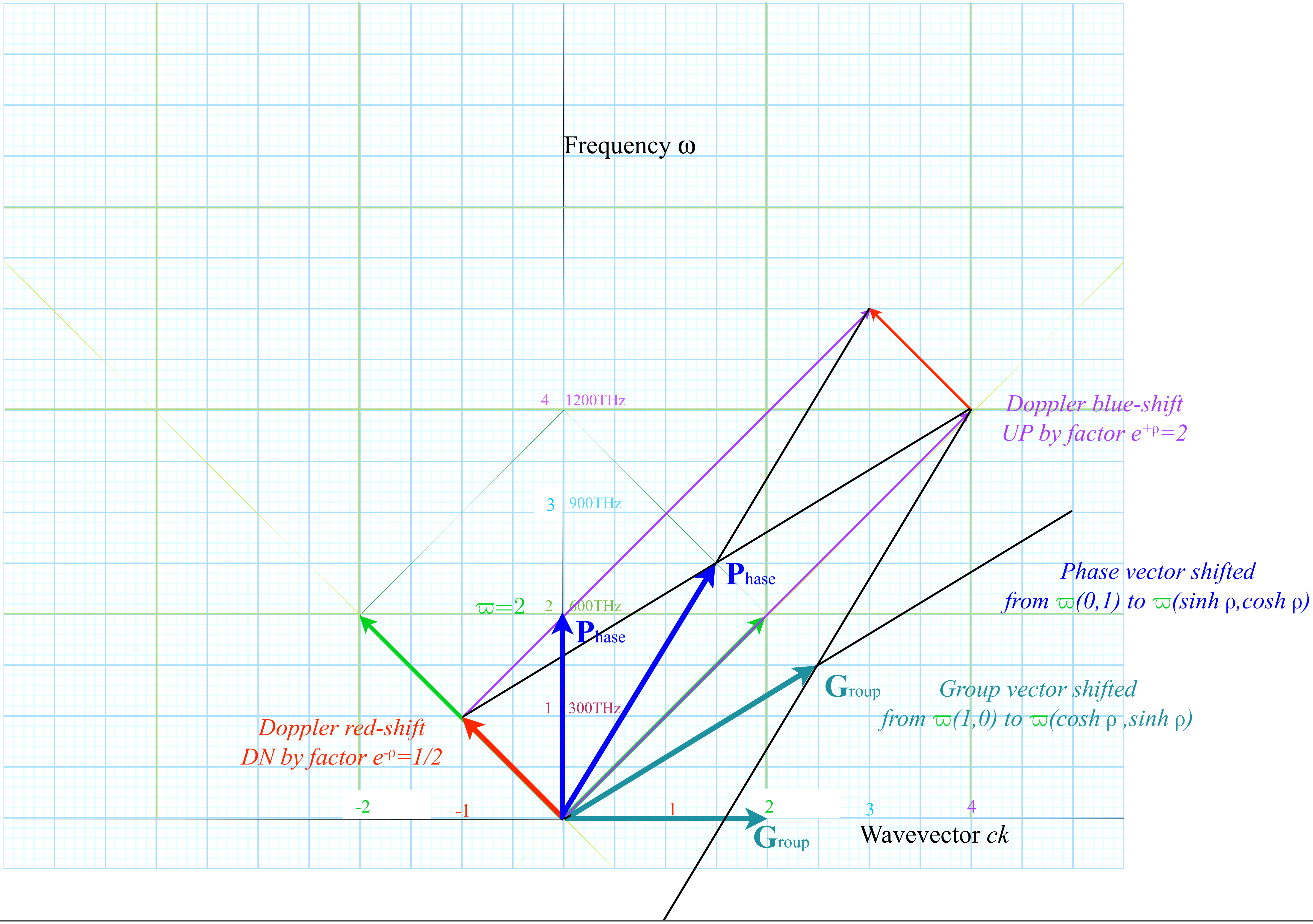

Frequency ω
4 1200THz
3 900THz
2 600THz
1 300THz
-2
-1
1
2
3
4
Wavevector ck


Frequency ω
4
1200THz
3
900THz
2
600THz
Phase
1
300THz
-2
-1
1
2
3
4
Group
Wavevector ck

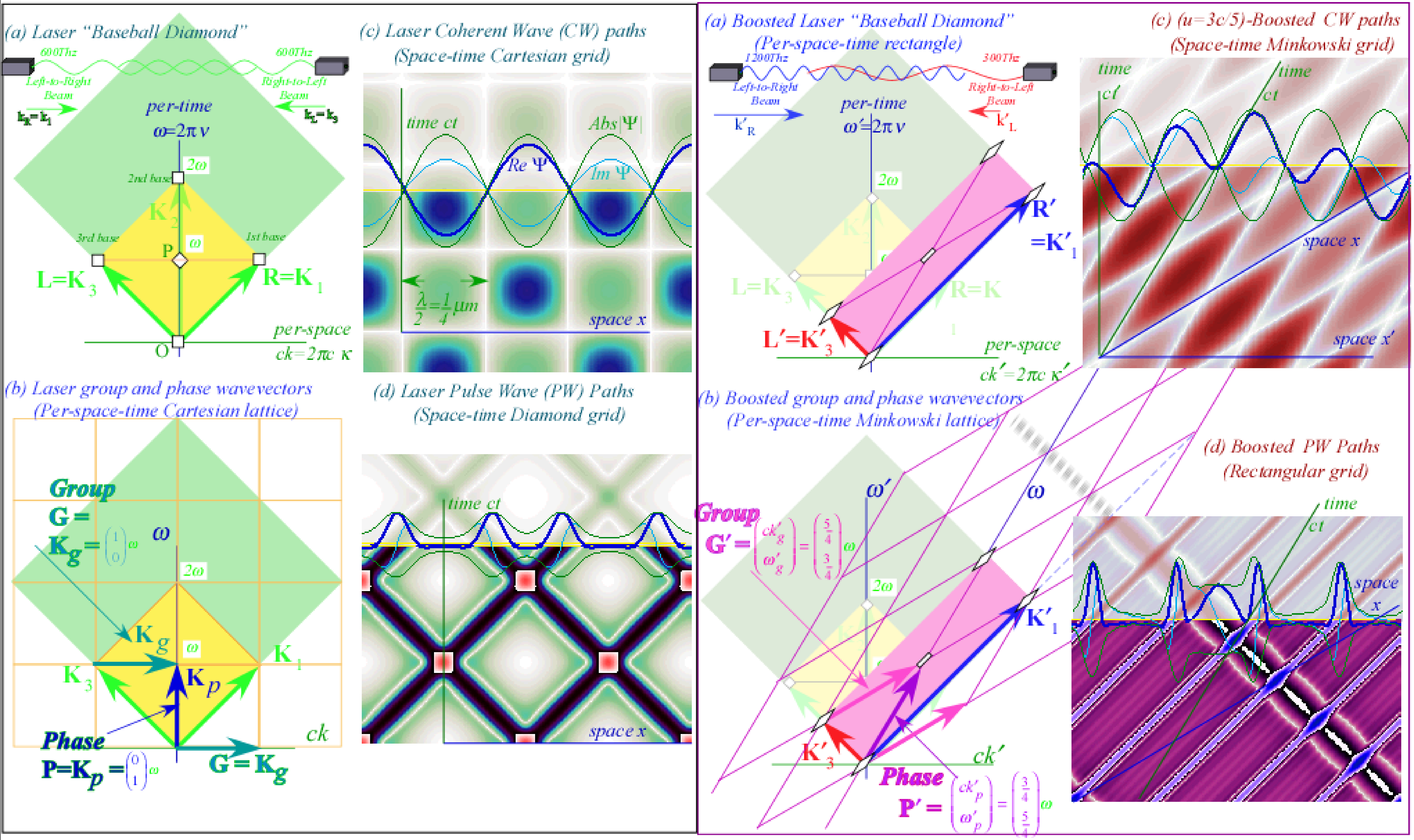

Frequency ω
4 1200THz
Doppler blue-shift
UP by factor e^{+ρ}=2
3 900THz
2 600THz
Phase
1 300THz
Doppler red-shift
DN by factor e^{-ρ}=1/2
-2
-1
1
2
3
4
Group
Wavevector ck


Frequency ω
4 1200THz
3 900THz
2 600THz
1 300THz
Phase
Doppler blue-shift
UP by factor $e^{+\rho}=2$
Doppler red-shift
DN by factor $e^{-\rho}=1/2$
-2
-1
1
2
3
4
Group
Wavevector ck


Frequency ω
4 1200THz
3 900THz
2 600THz
1 300THz
Doppler blue-shift
UP by factor e+ρ=2
Doppler red-shift
DN by factor e-ρ=1/2
Phase
Group
-2
-1
1
2
3
4
Wavevector ck

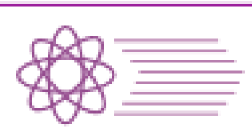


Frequency ω
4 1200THz
3 900THz
ϖ=2 2 600THz
1 300THz
Doppler blue-shift
UP by factor e^{+ρ}=2
Phase
Phase
Phase vector shifted
from ϖ(0,1) to ϖ(sinh ρ,cosh ρ)
Group
Group vector shifted
from ϖ(1,0) to ϖ(cosh ρ ,sinh ρ)
Doppler red-shift
DN by factor e^{-ρ}=1/2
-2
-1
1
2
3
4
Group
Wavevector ck

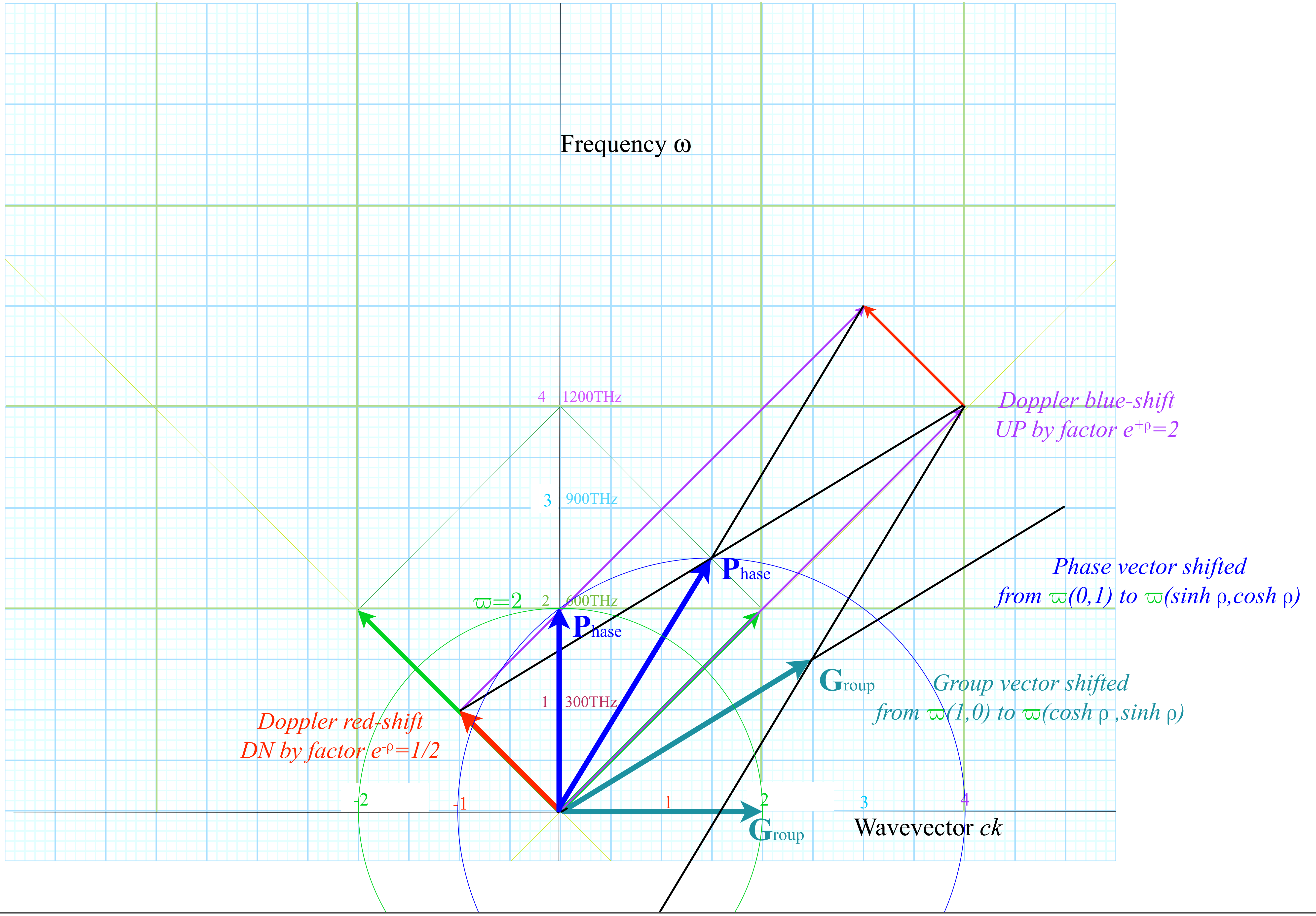


Laser lab views

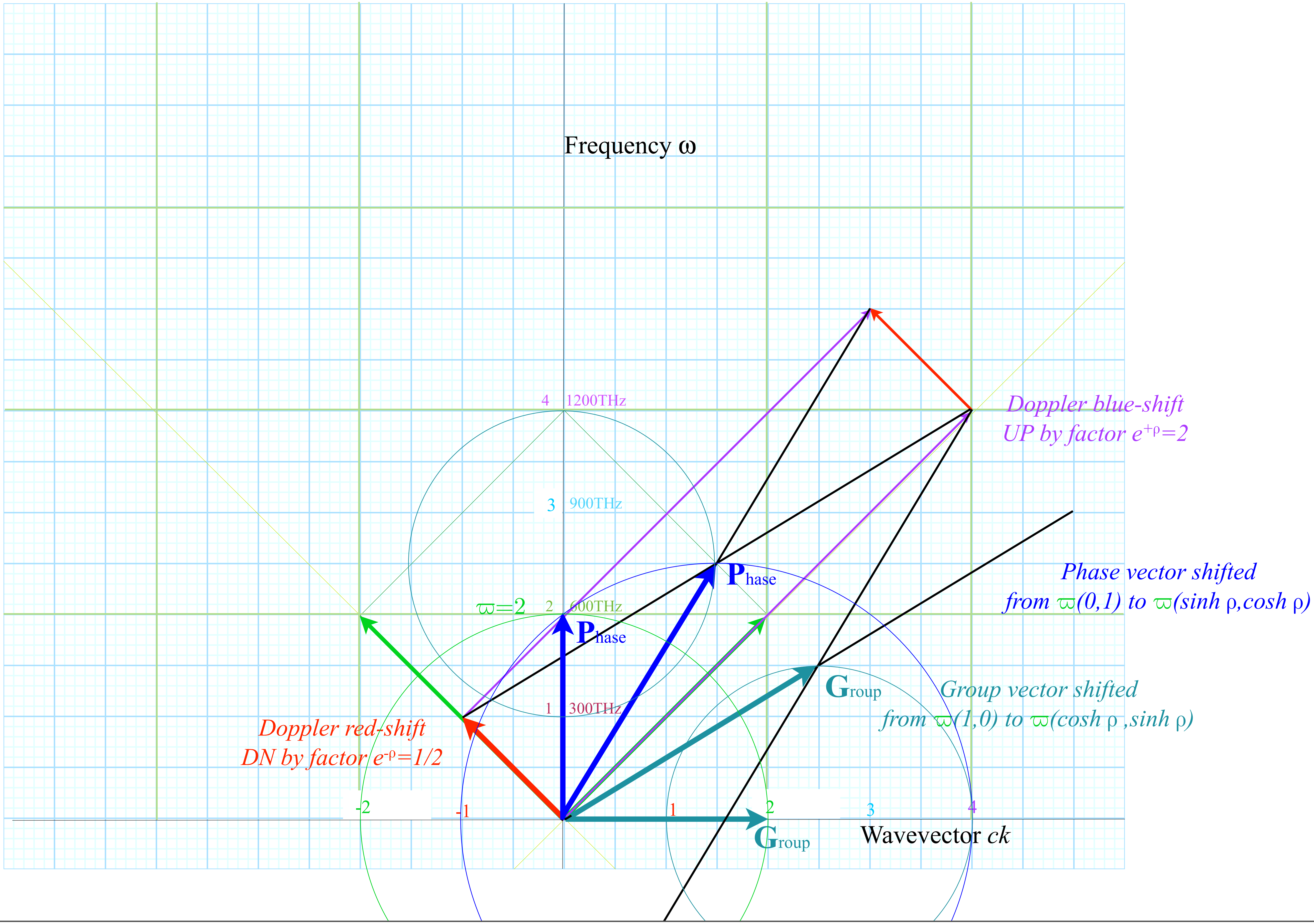
atom speed $-u=-\frac{3}{5}c$

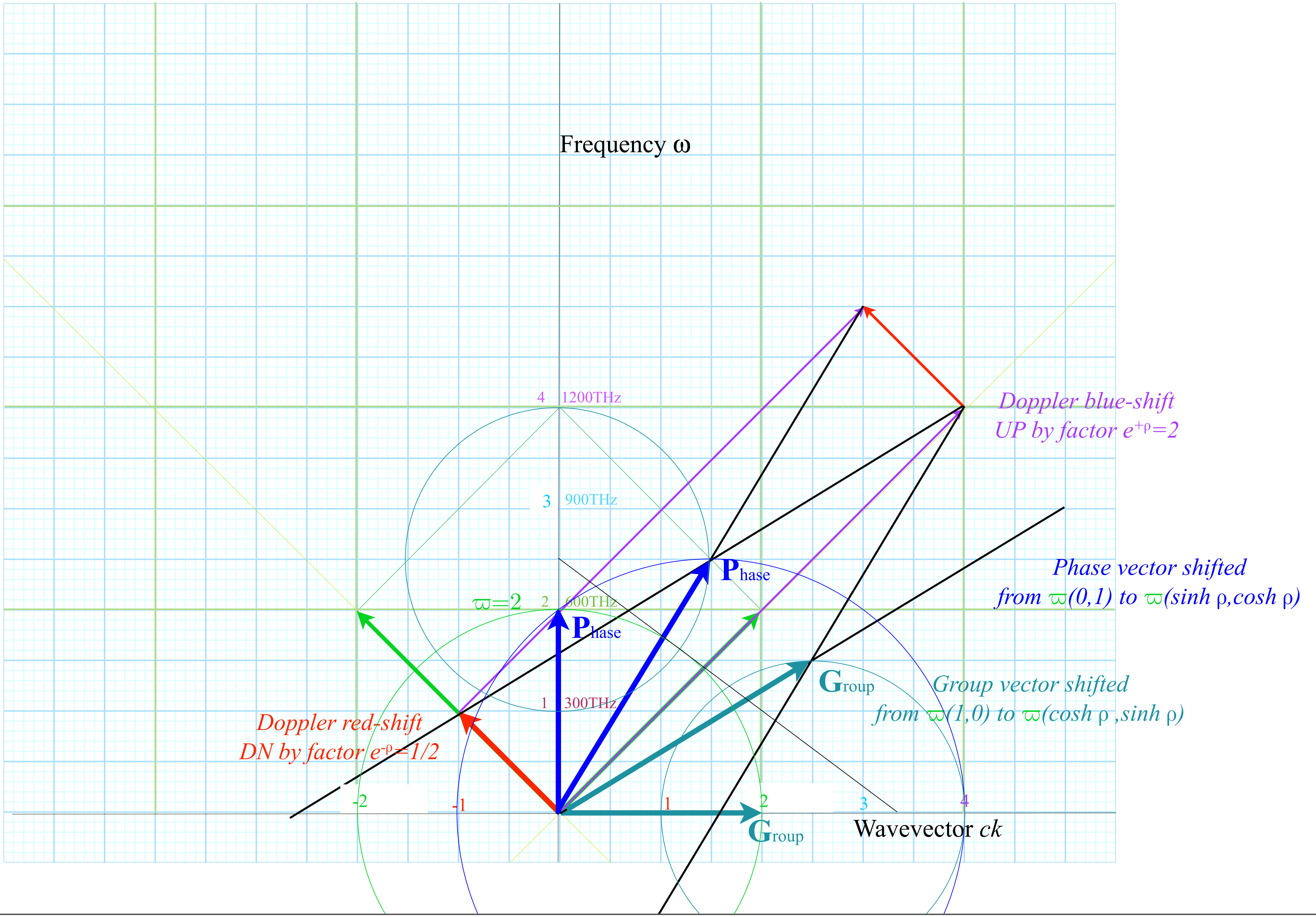


Atom views (sees lab going $+u=\frac{3}{5}c$)

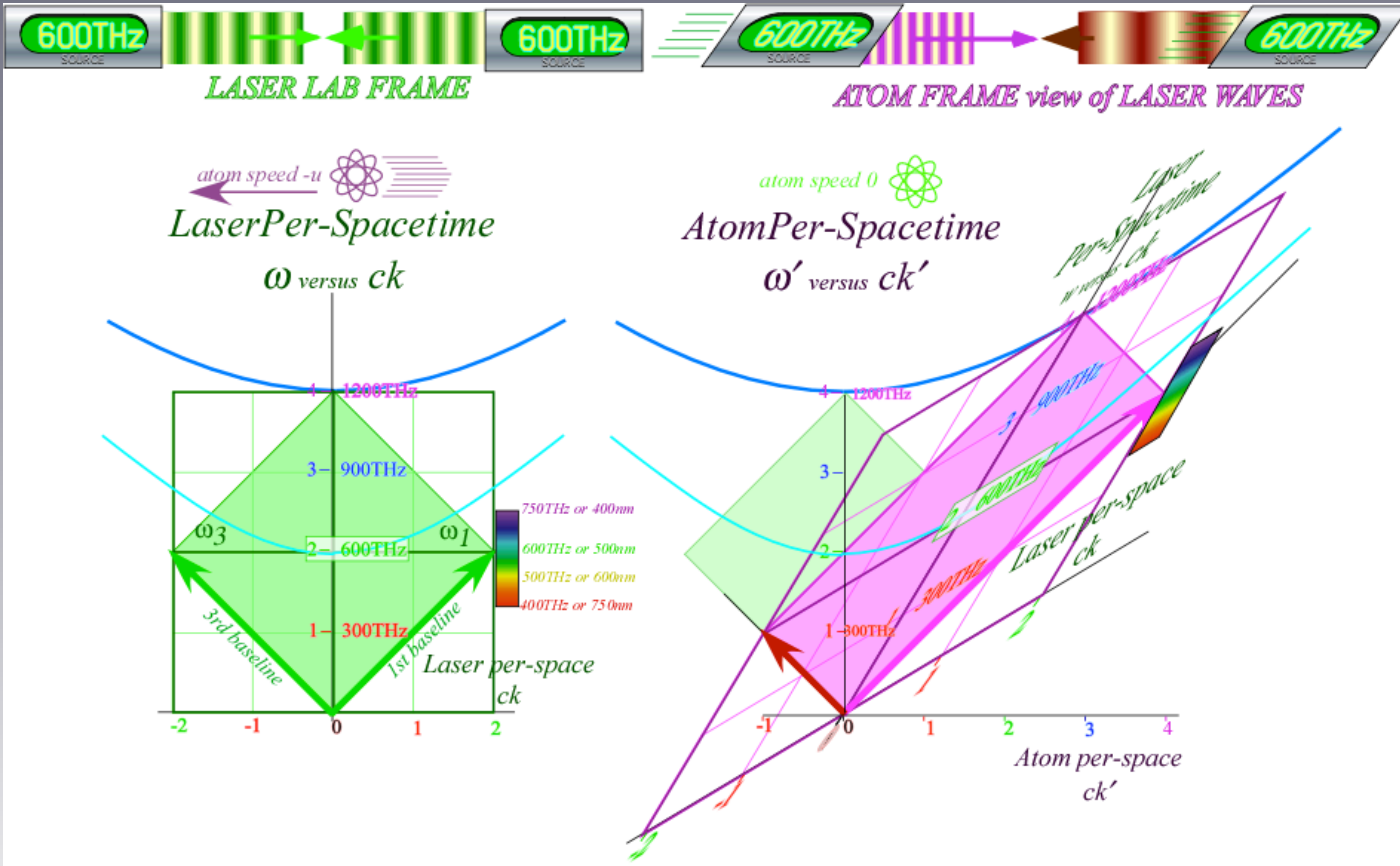


Frequency ω
4 1200THz
3 900THz
2 600THz
1 300THz
ϖ=2
Doppler blue-shift
UP by factor e+ρ=2
Phase
Phase
Phase vector shifted
from ϖ(0,1) to ϖ(sinh ρ,cosh ρ)
Group
Group vector shifted
from ϖ(1,0) to ϖ(cosh ρ ,sinh ρ)
Doppler red-shift
DN by factor e-ρ=1/2
-2
-1
1
2
3
4
Group
Wavevector ck


Frequency ω
4 1200THz
3 900THz
ϖ=2
2 600THz
1 300THz
Doppler blue-shift
UP by factor e+ρ=2
Phase
Phase
Phase vector shifted
from ϖ(0,1) to ϖ(sinh ρ,cosh ρ)
Group
Group vector shifted
from ϖ(1,0) to ϖ(cosh ρ ,sinh ρ)
Doppler red-shift
DN by factor e-ρ=1/2
-2
-1
1
2
3
4
Group
Wavevector ck



Frequency ω
4 1200THz
3 900THz
ϖ=2
2 600THz
1 300THz
Doppler blue-shift
UP by factor e^+ρ=2
Phase
Phase
Phase vector shifted
from ϖ(0,1) to ϖ(sinh ρ,cosh ρ)
Group
Group vector shifted
from ϖ(1,0) to ϖ(cosh ρ ,sinh ρ)
Doppler red-shift
DN by factor e^-ρ=1/2
-2
-1
1
2
3
4
Group
Wavevector ck



600THz
SOURCE
600THz
SOURCE
LASER LAB FRAME
600THz
SOURCE
600THz
SOURCE
ATOM FRAME view of LASER WAVES
atom speed -u
LaserPer-Spacetime
ω versus ck
atom speed 0
AtomPer-Spacetime
ω′ versus ck′
1200THz
3– 900THz
2– 600THz
1– 300THz
ω3
ω1
3rd baseline
1st baseline
Laser per-space
ck
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
-2
-1
0
1
2
Laser
Per-Spacetime
ω′ versus ck
1200THz
900THz
600THz
300THz
Laser per-space
ck
Atom per-space
ck′

Euclidian Geometry for Per-spacetime Relativity

relative speed~slope

$u/c = \sinh\rho / \cosh\rho = \tanh\rho$

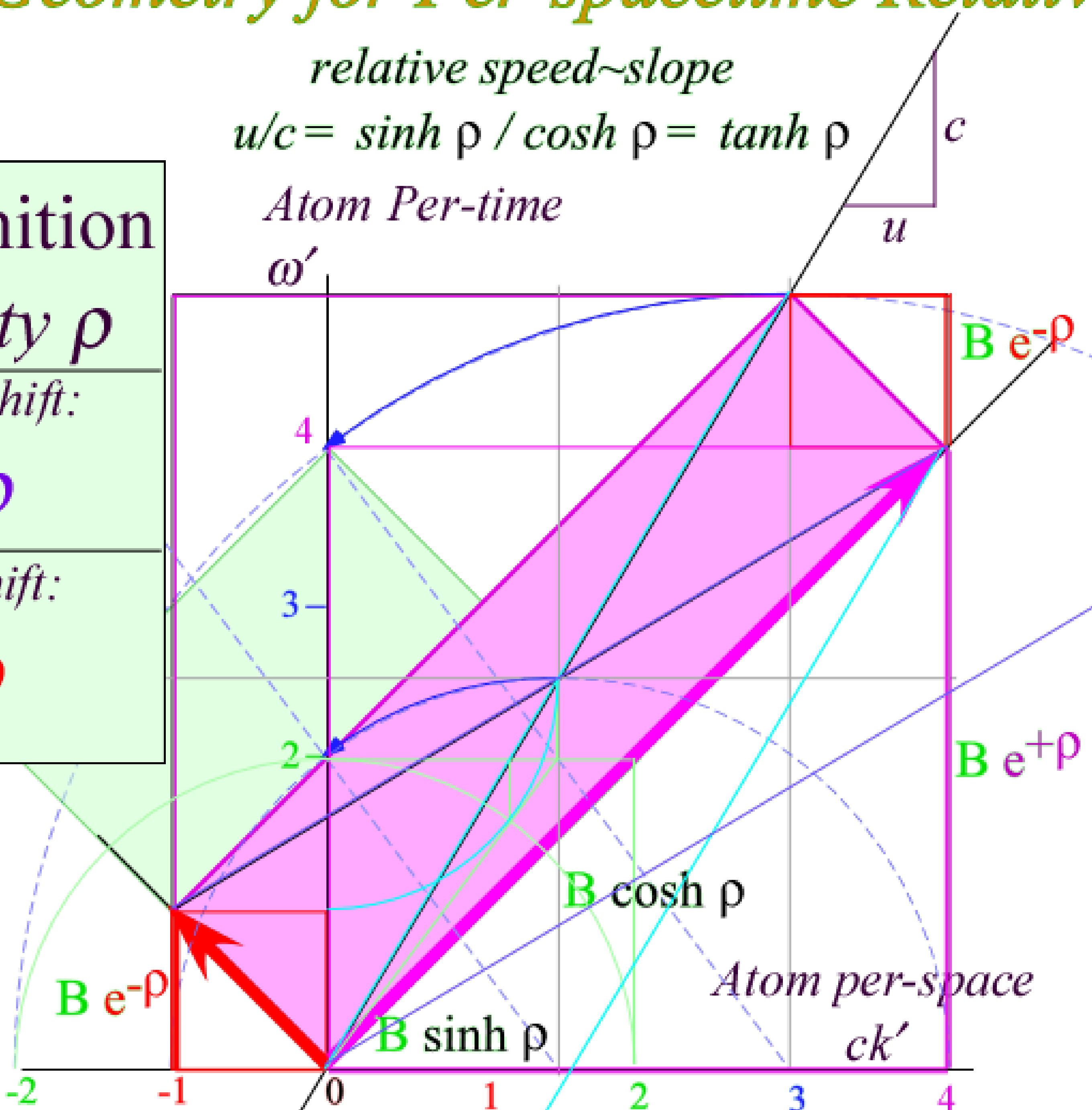
Key Definition of *Rapidity ρ*

Doppler blue shift:

$Bb = Be^{+\rho}$

Doppler red shift:

$Br = Be^{-\rho}$



Key Results:

ω vs. ck

"winks" vs. "kinks"

$\omega = B\cosh\rho$

$ck = B\sinh\rho$

group velocity:

$$\frac{\omega}{ck} = \frac{u}{c} = \tanh\rho$$

phase velocity:

$$\frac{ck}{\omega} = \frac{c}{u} = \coth\rho$$

$B\sinh\rho = (Be^{+\rho} - Be^{-\rho})/2$

$B\cosh\rho = (Be^{+\rho} + Be^{-\rho})/2$

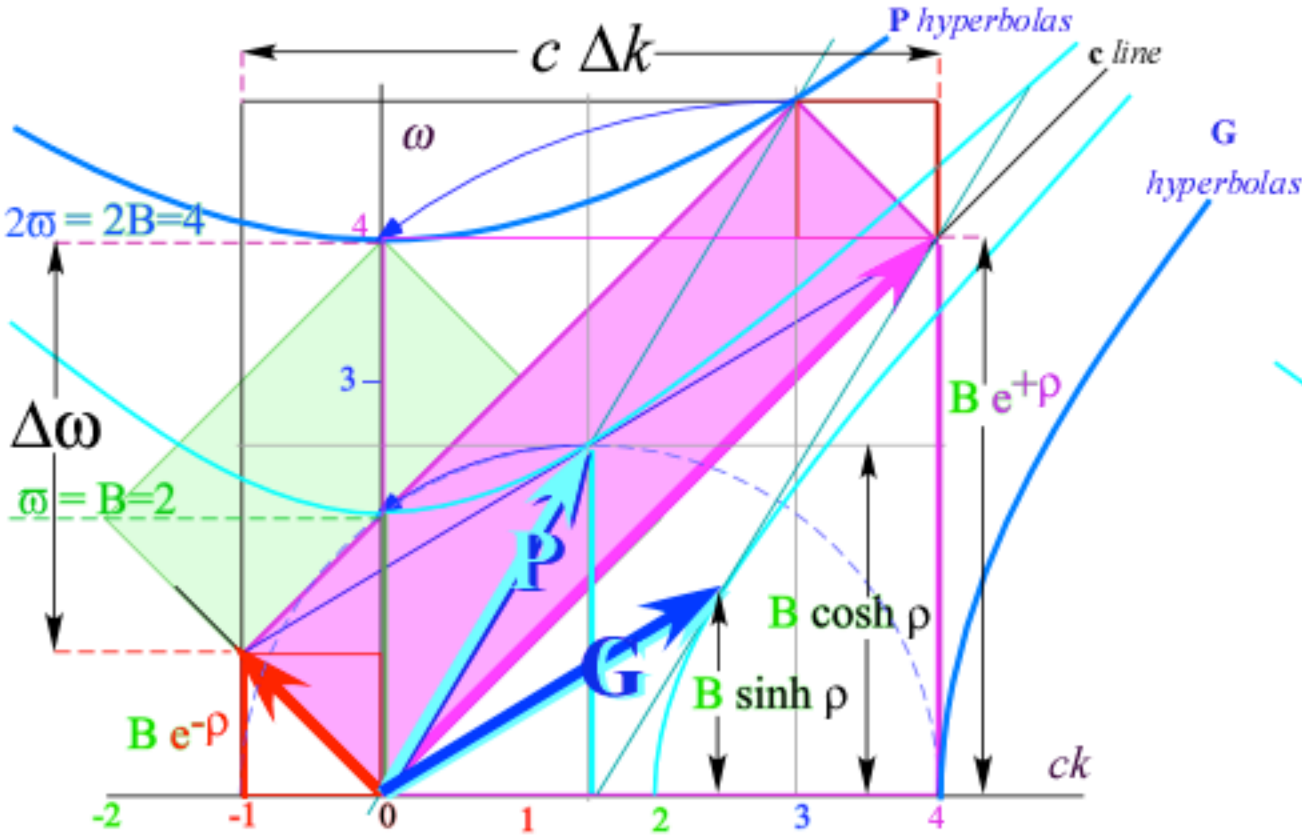
Key Quantities

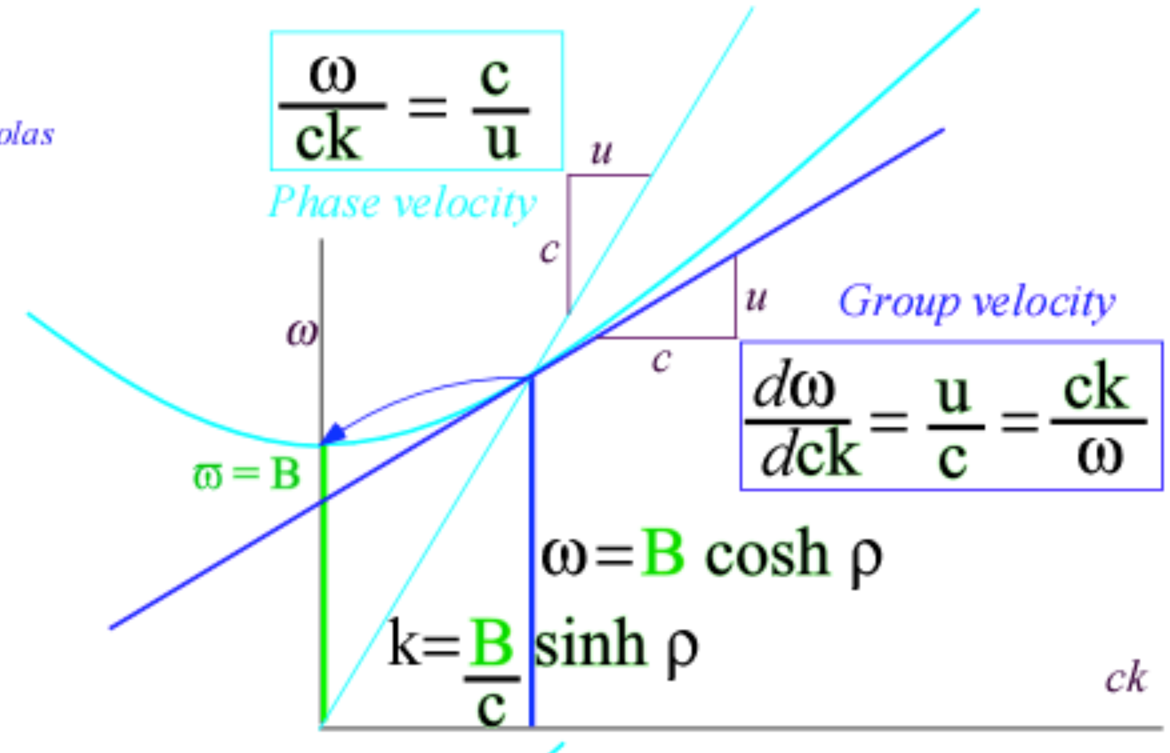
Lorentz-Einstein factors

$$\sinh\rho = \frac{\frac{u}{c}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\cosh\rho = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Group velocity u *and phase velocity* c^2/u *are hyperbolic tangent slopes*





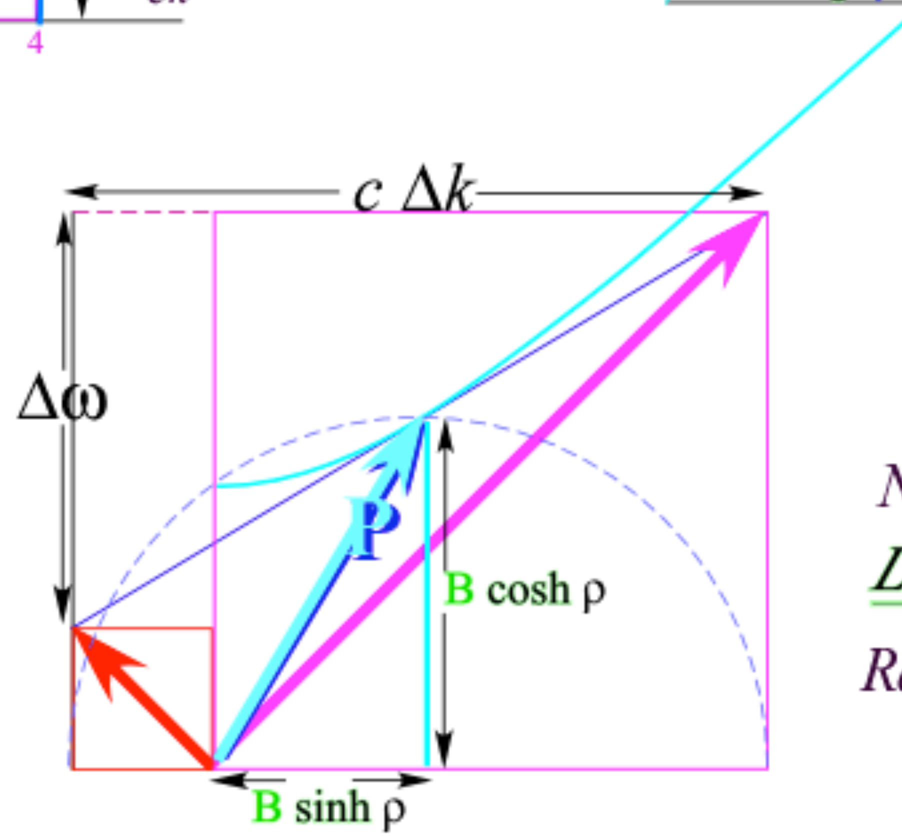
Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with LARGE Δk

(not infinitesimal)

Relativistic group wave speed $u=c\tanh\rho$

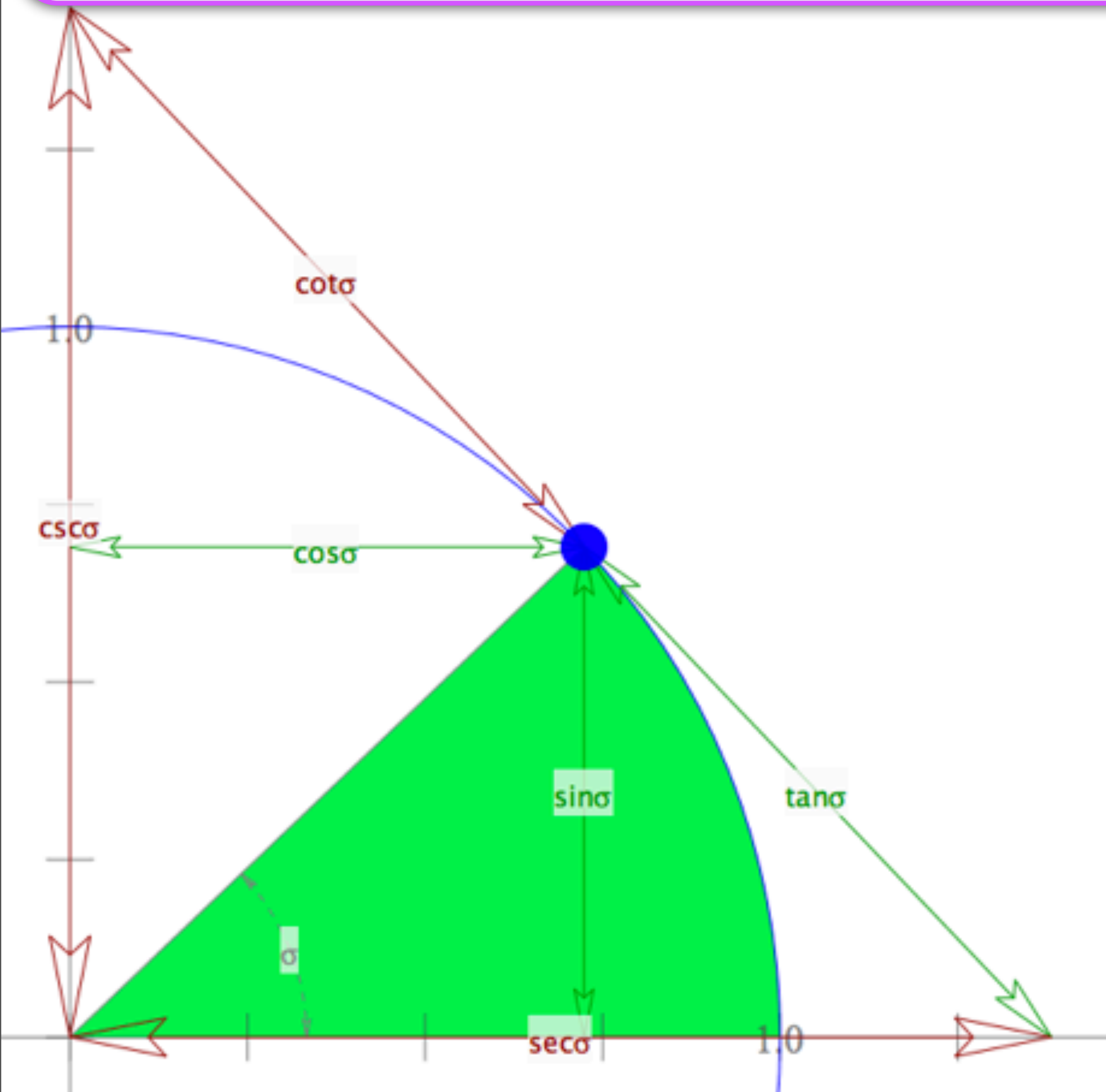


Newtonian speed $u \sim c\rho$

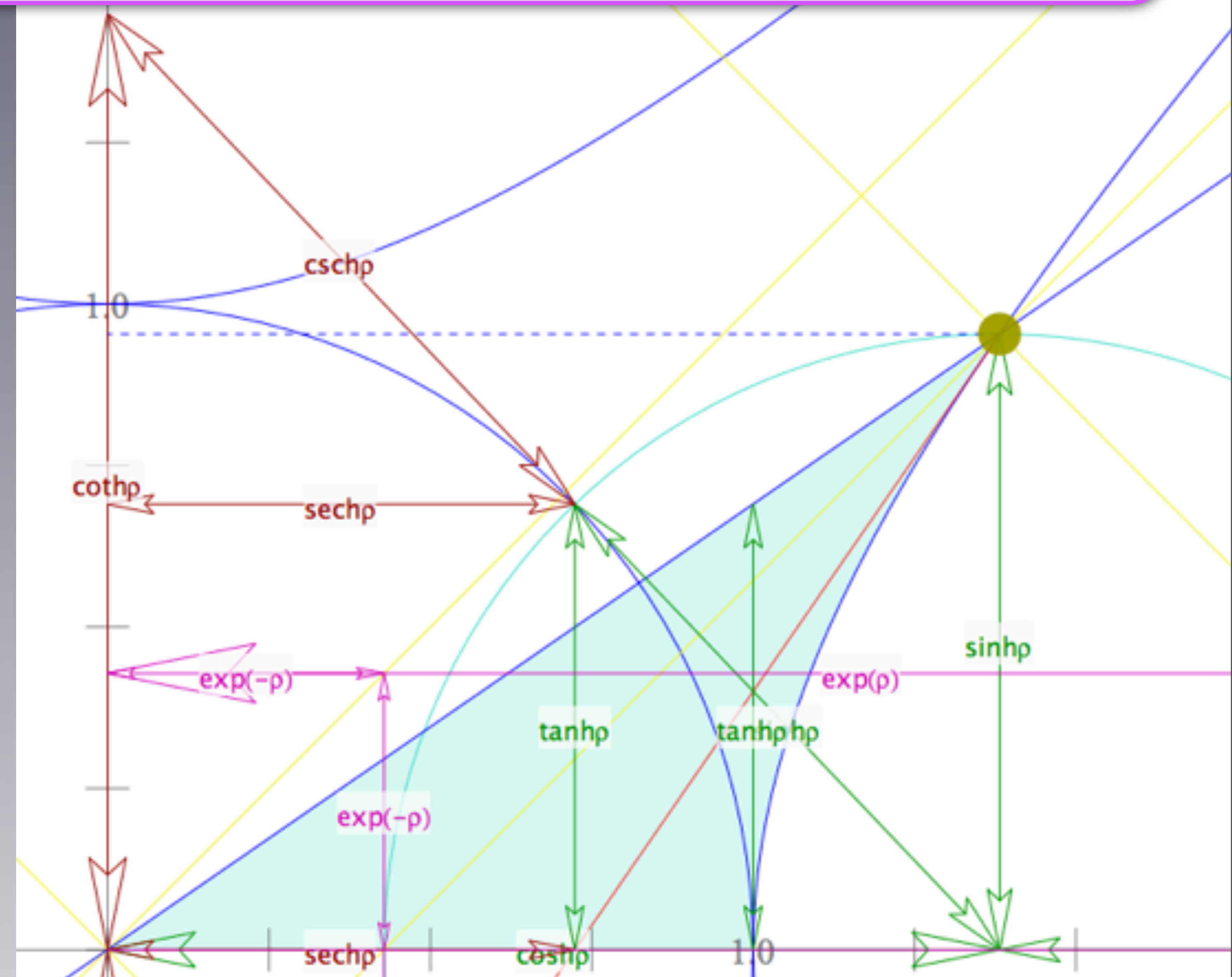
Low speed approximation

Rapidity ρ *approaches* u/c

Circular Geometry of Lagrangian Functions *versus* *Hyperbolic Geometry of Hamiltonian Functions*



The Circular Functions "Urban elite"

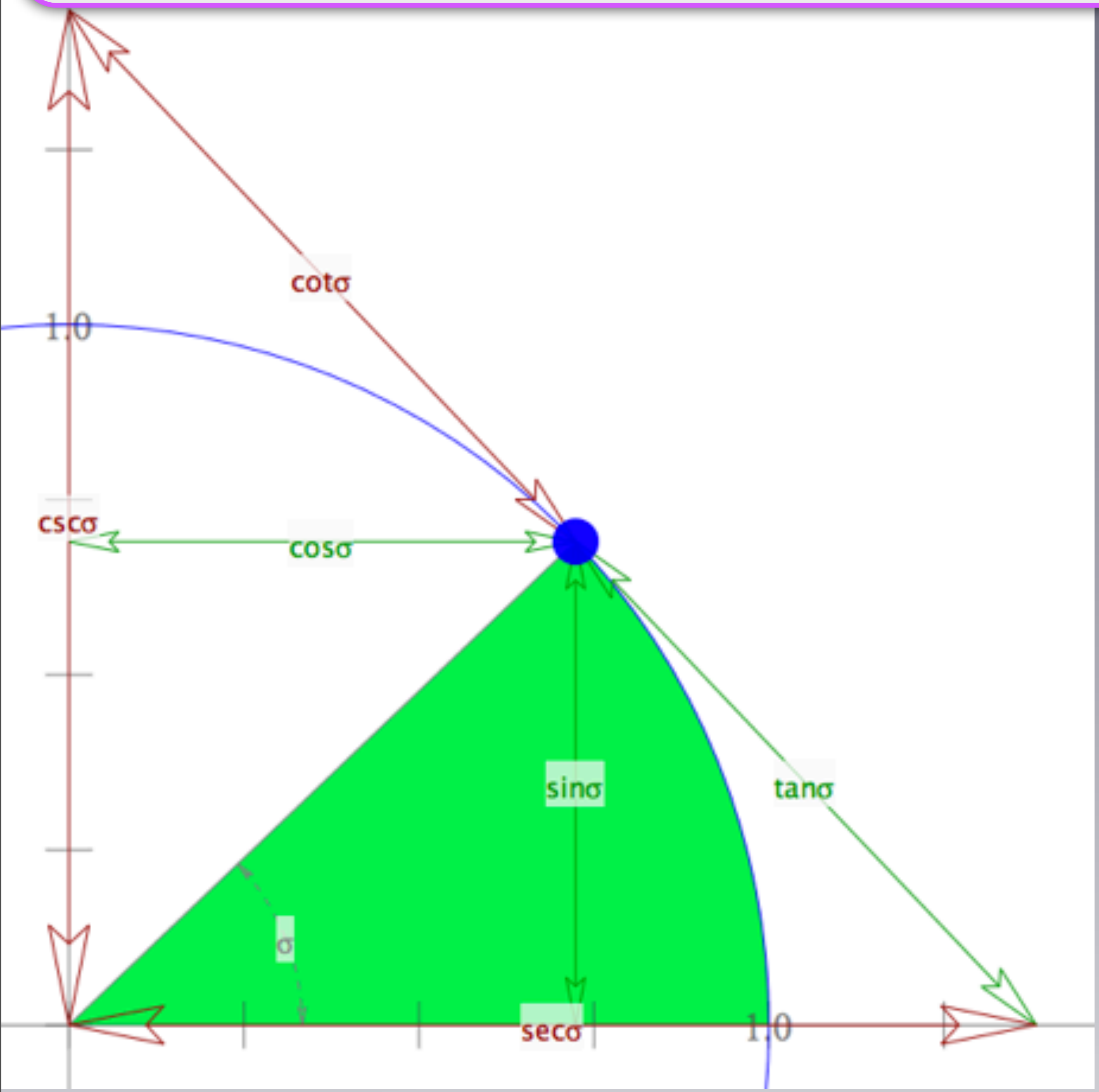


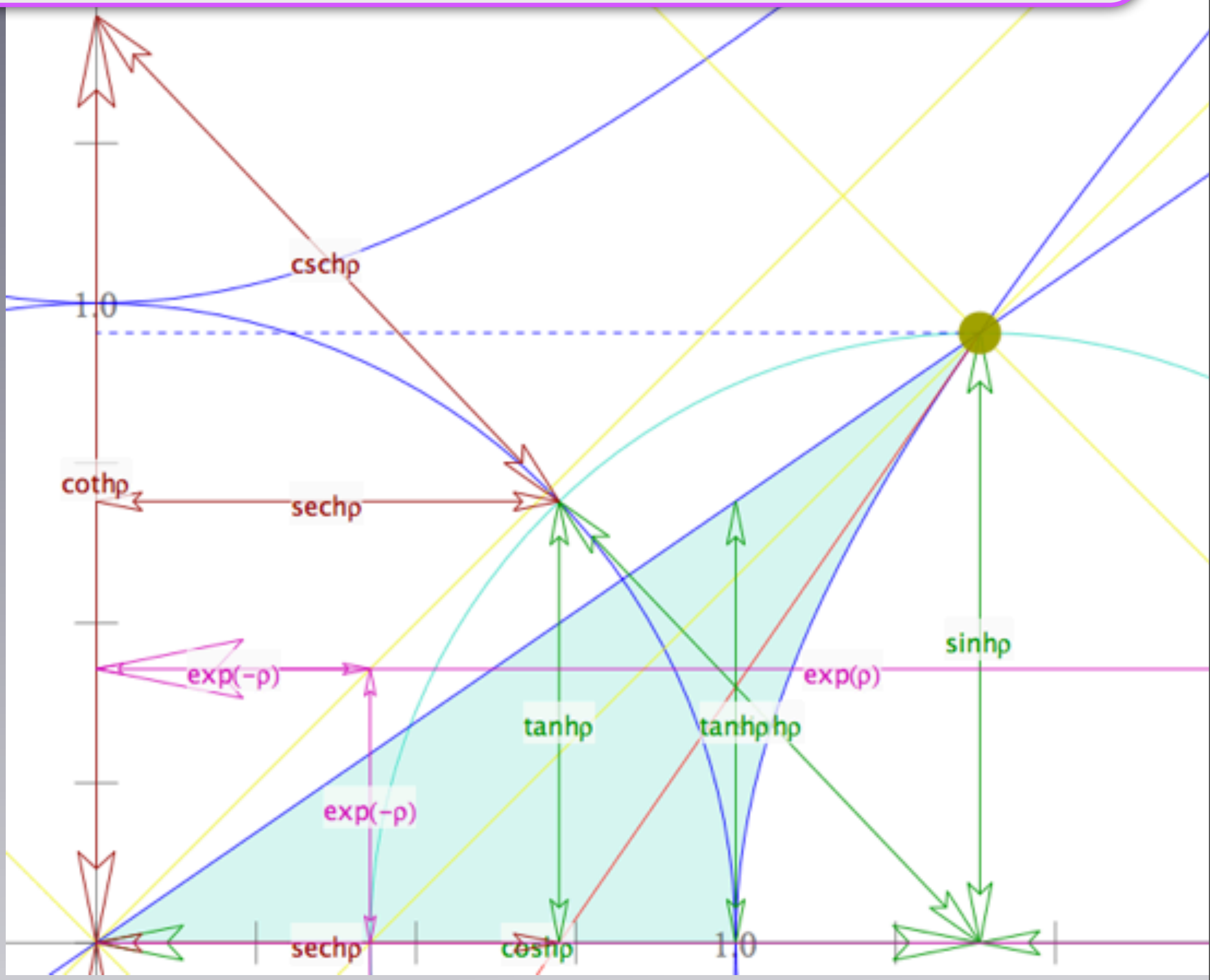
The Hyperbolic Functions "Country-cousins"

They're related by Legendre contact transformation $L = p\bullet v - H$

tan σ = *sinh* ρ

Circular Geometry of Lagrangian Functions versus Hyperbolic Geometry of Hamiltonian Functions





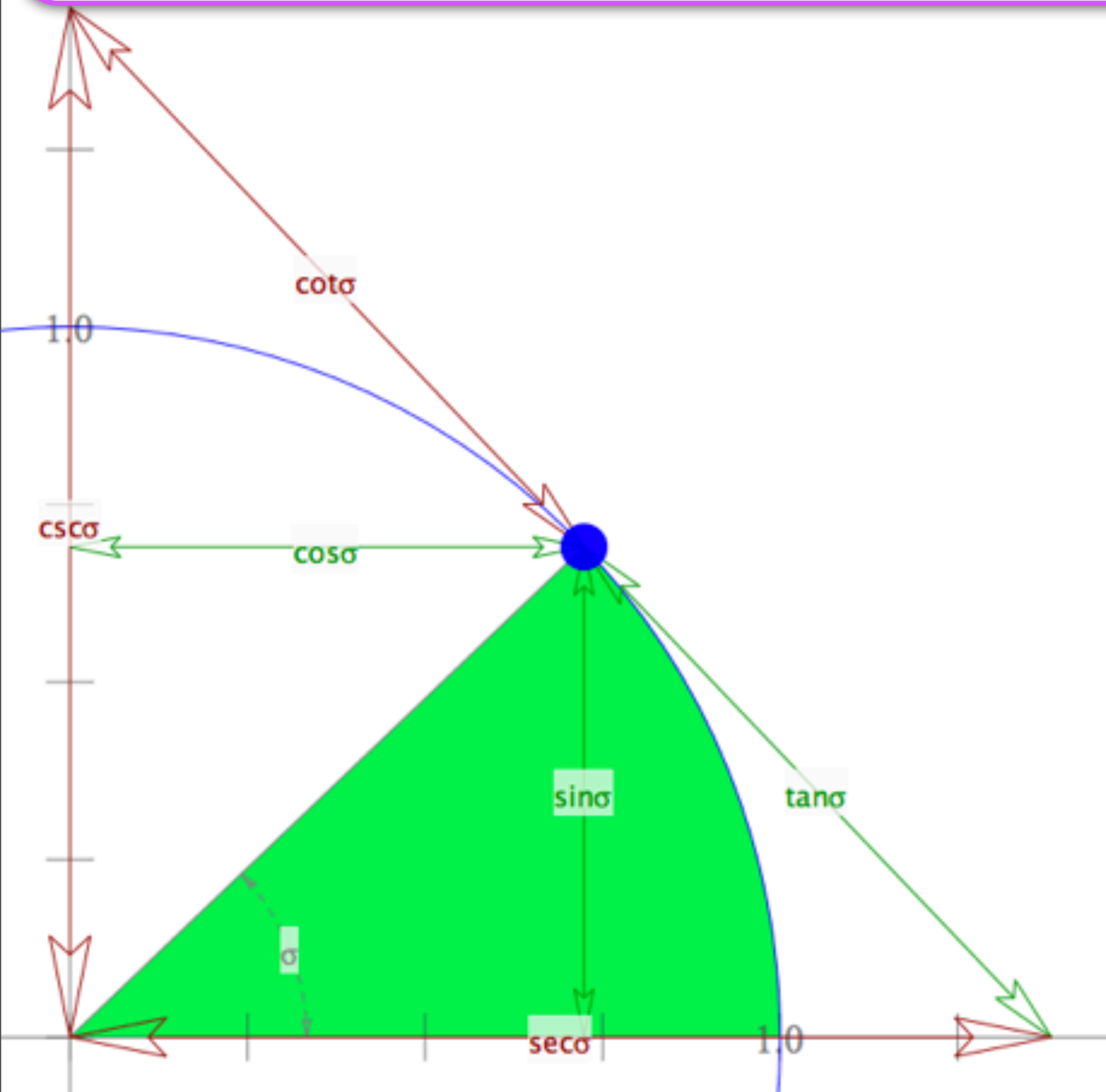
The Circular Functions "Urban elite" *The Hyperbolic Functions "Country-cousins"*

They're related by Legendre contact transformation $L = p \cdot v - H$

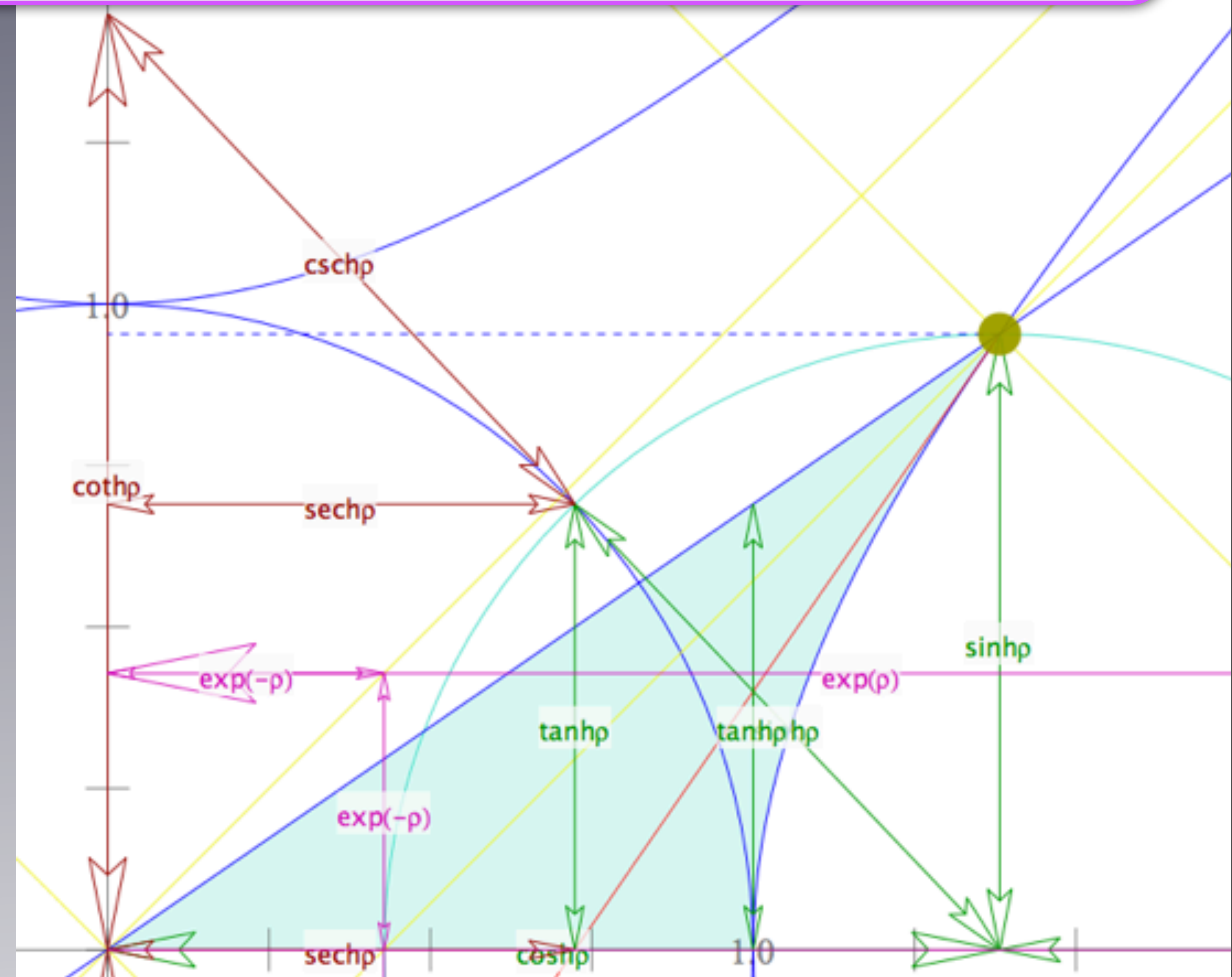
$$\tan\sigma = \sinh\rho$$

$$\sin\sigma = \tanh\rho$$

Circular Geometry of Lagrangian Functions versus Hyperbolic Geometry of Hamiltonian Functions



The Circular Functions "Urban elite"



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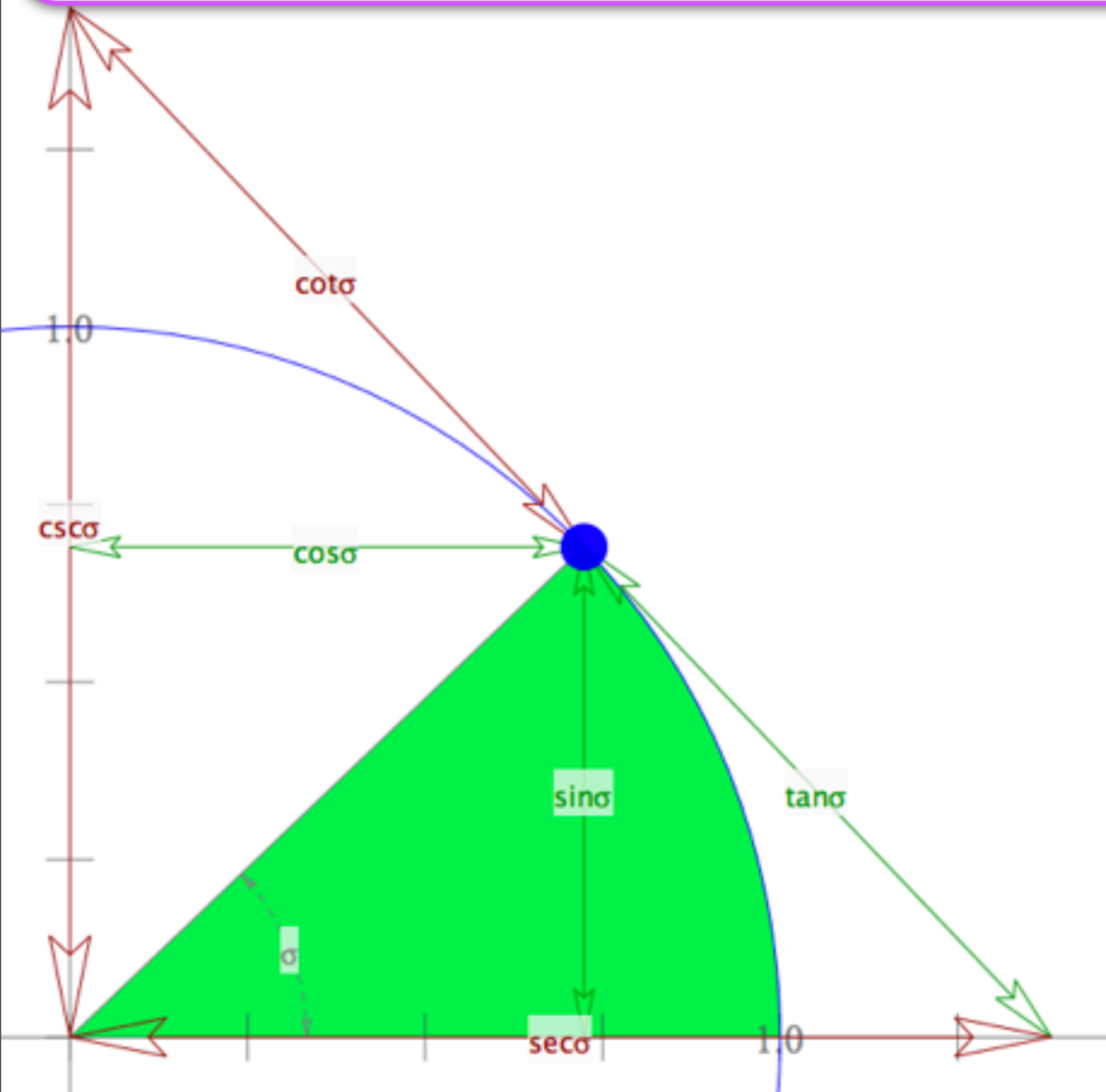
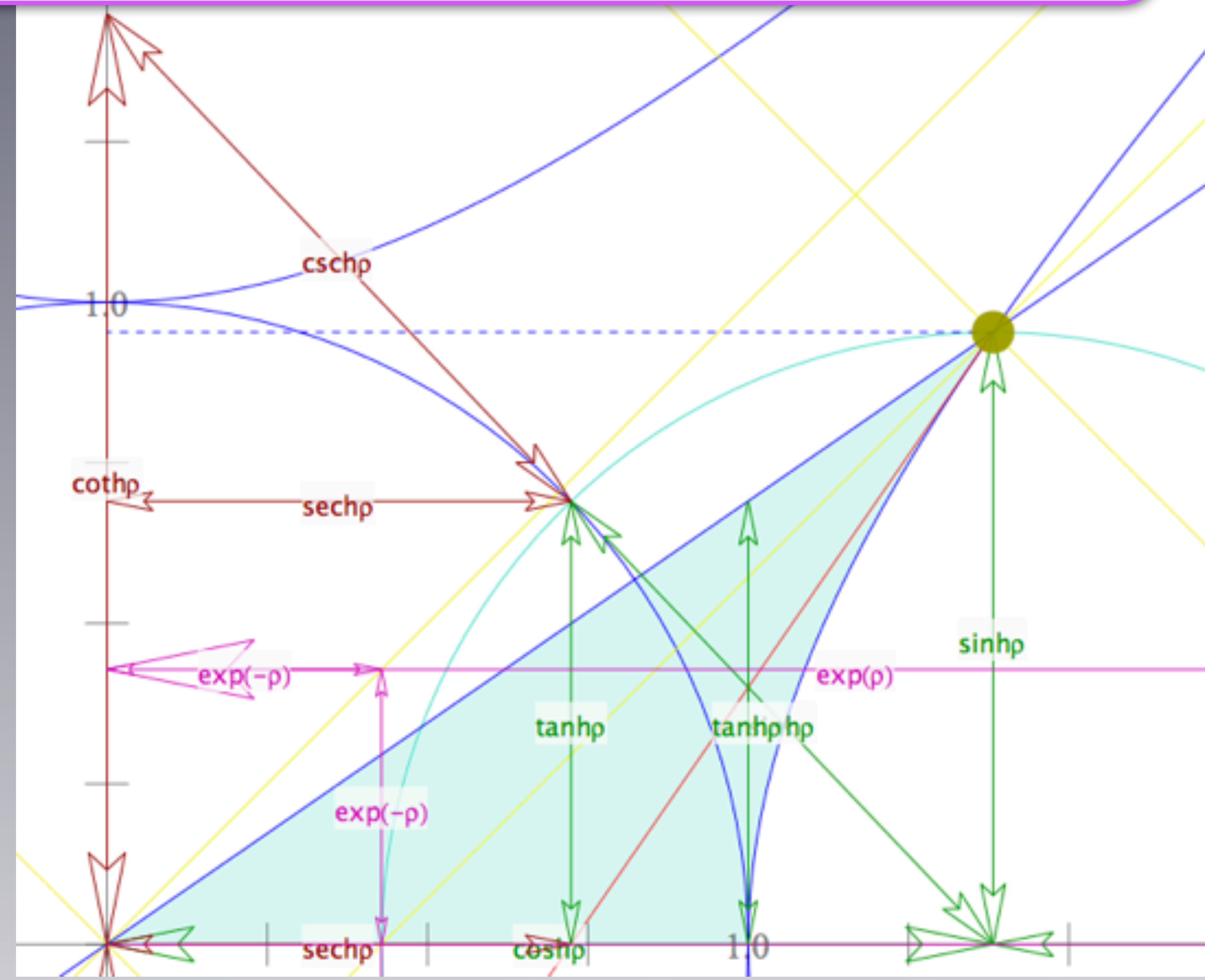
$\tan\sigma = \sinh\rho$

$\sin\sigma = \tanh\rho$

$\cos\sigma = \mathrm{sech}\,\rho$

Circular Geometry of Lagrangian Functions *versus* *Hyperbolic Geometry of Hamiltonian Functions*





The Circular Functions "Urban elite" *The Hyperbolic Functions "Country-cousins"*

They're related by Legendre contact transformation $L = \mathbf{p}\bullet\mathbf{v} - H$

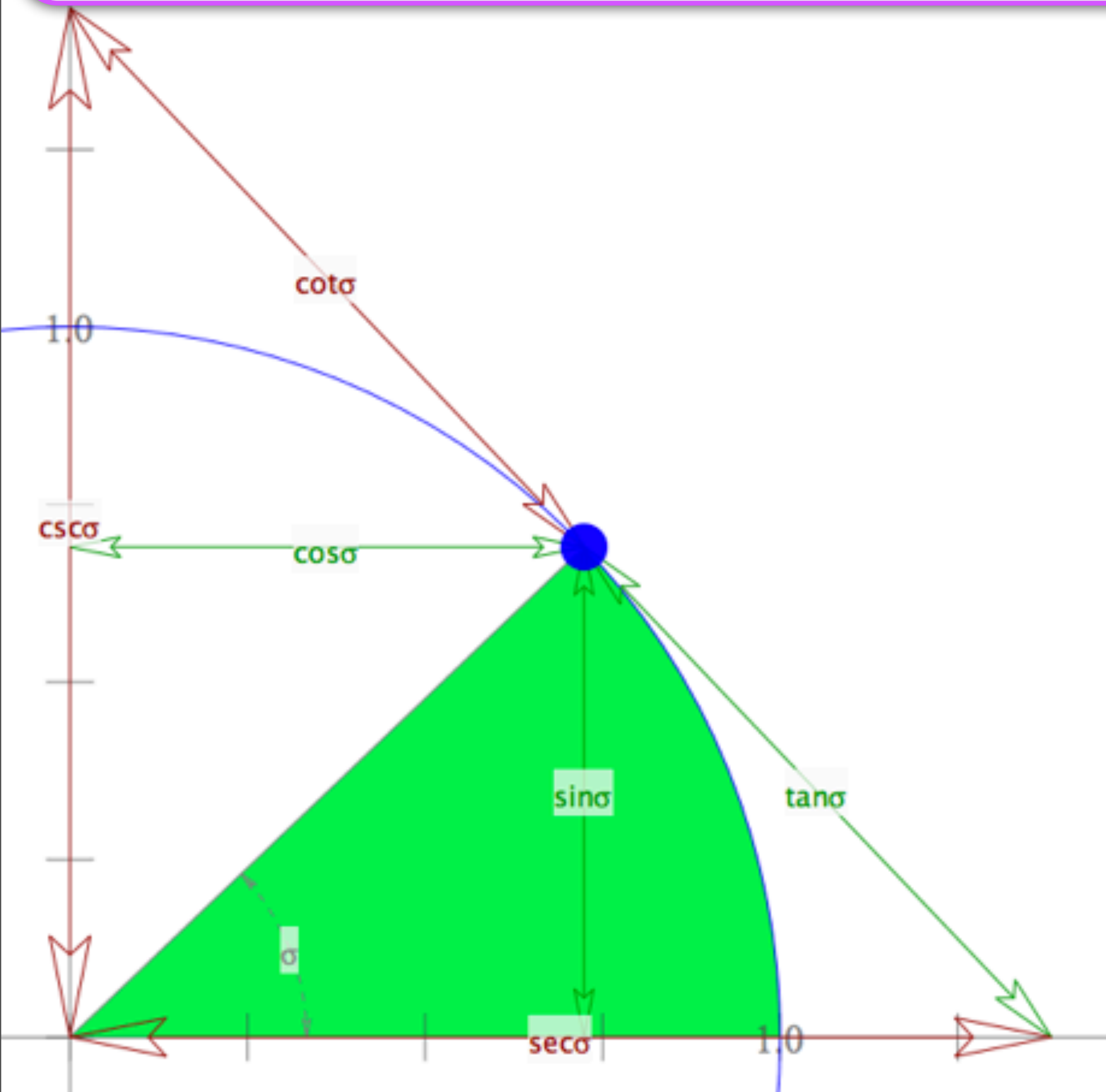
$\tan\sigma = \sinh\rho$

$\sin\sigma = \tanh\rho$

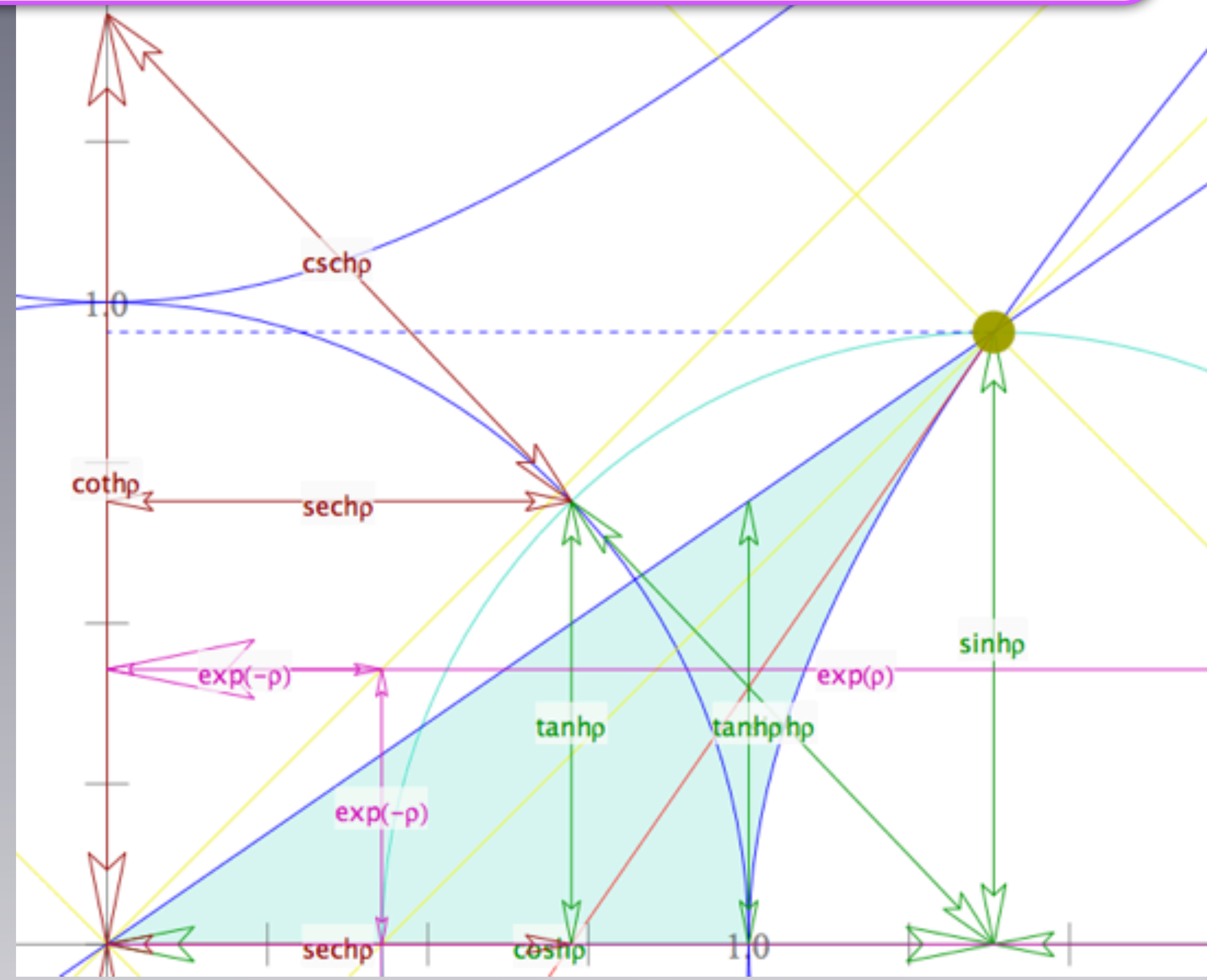
$\cos\sigma = \operatorname{sech}\rho$

$\sec\sigma = \cosh\rho$

Circular Geometry of Lagrangian Functions *versus* *Hyperbolic Geometry of Hamiltonian Functions*



The Circular Functions "Urban elite"

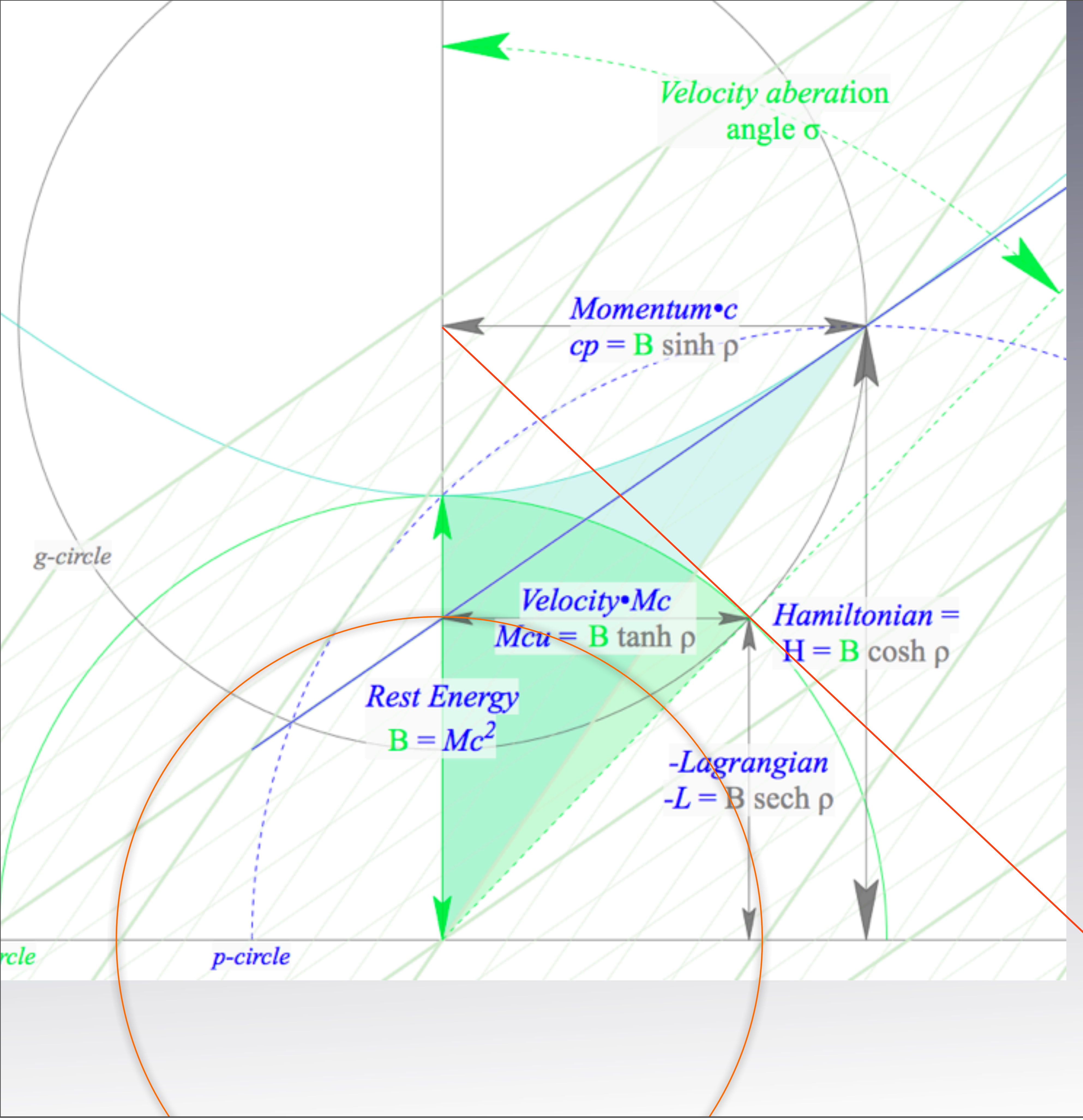


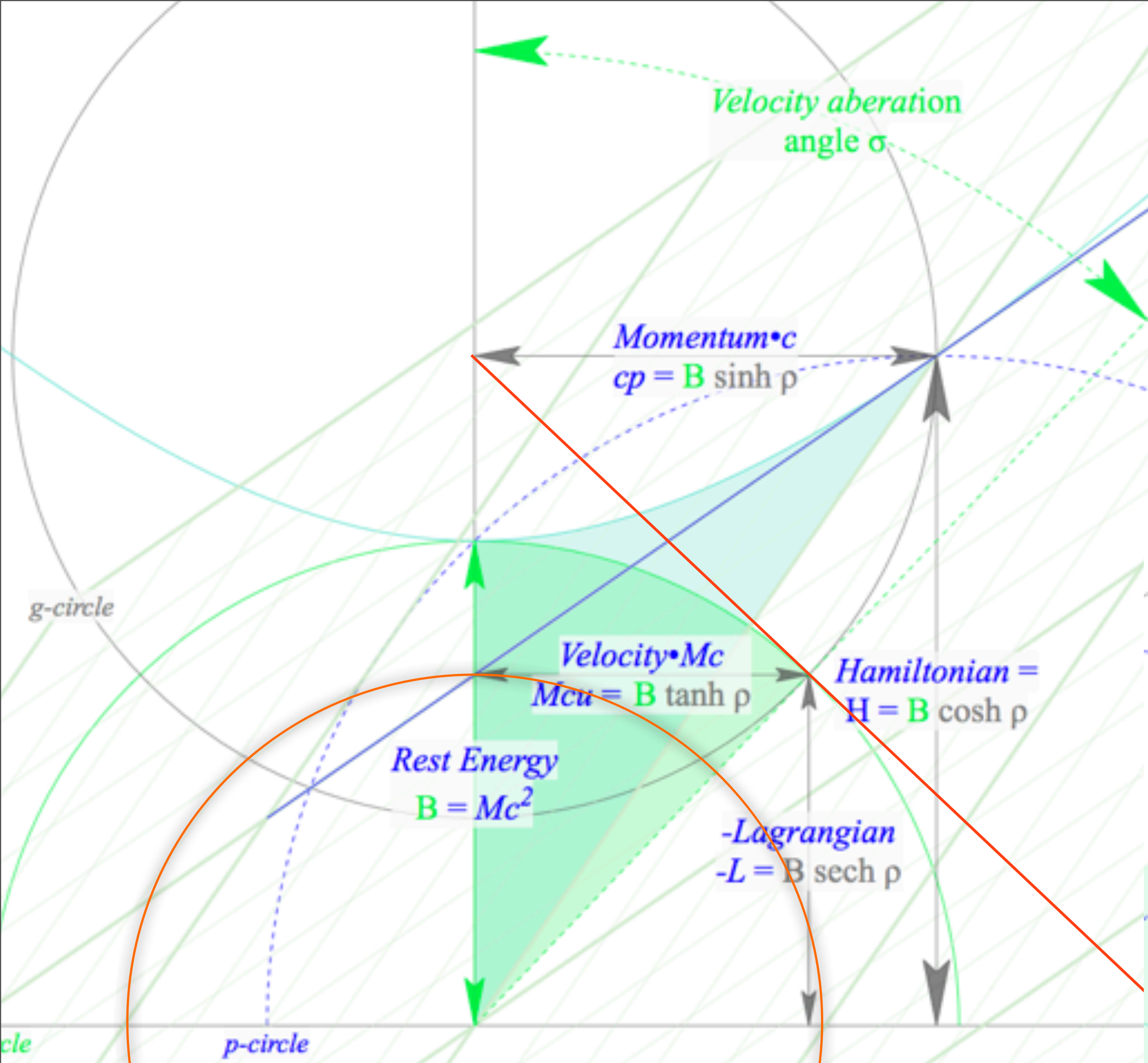
The Hyperbolic Functions "Country-cousins"

They're related by Legendre contact transformation $L = \mathbf{p} \bullet \mathbf{v} - H$

In spacetime:	*asimultaneity factor*	$\tan\sigma$	=	$\sinh\rho$	*In per-spacetime:* *momentum*
	velocity u/c	$\sin\sigma$	=	$\tanh\rho$	*group velocity*
	Lorentz contraction	$\cos\sigma$	=	$\text{sech}\,\rho$	*-Lagrangian*
	Einstein time dilation	$\sec\sigma$	=	$\cosh\rho$	*Hamiltonian*
		$\cot\sigma$	=	$\text{csch}\,\rho$	
		$\csc\sigma$	=	$\coth\rho$	

Old-fashioned notation:
$\sin\sigma = \tanh\rho = u/c$
$\sec\sigma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$


Velocity aberration
angle σ
Momentum•c
cp = B sinh ρ
g-circle
Velocity•Mc
Mcu = B tanh ρ
Hamiltonian =
H = B cosh ρ
Rest Energy
B = Mc2
-Lagrangian
-L = B sech ρ
rcle
p-circle


Velocity aberration
angle σ
Momentum•c
cp = B sinh ρ
g-circle
Velocity•Mc
Mcu = B tanh ρ
Hamiltonian =
H = B cosh ρ
Rest Energy
B = Mc²
-Lagrangian
-L = B sech ρ
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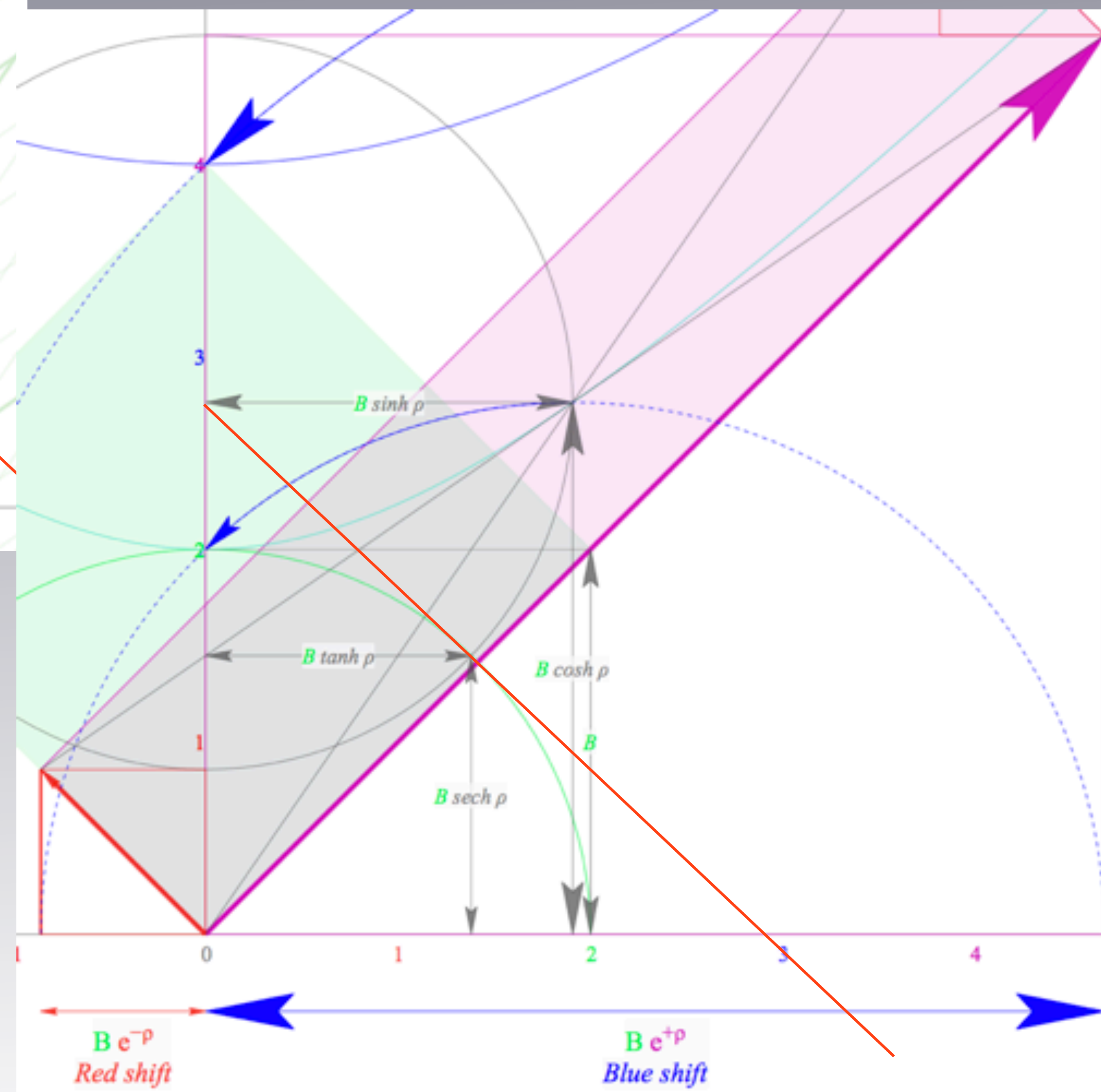
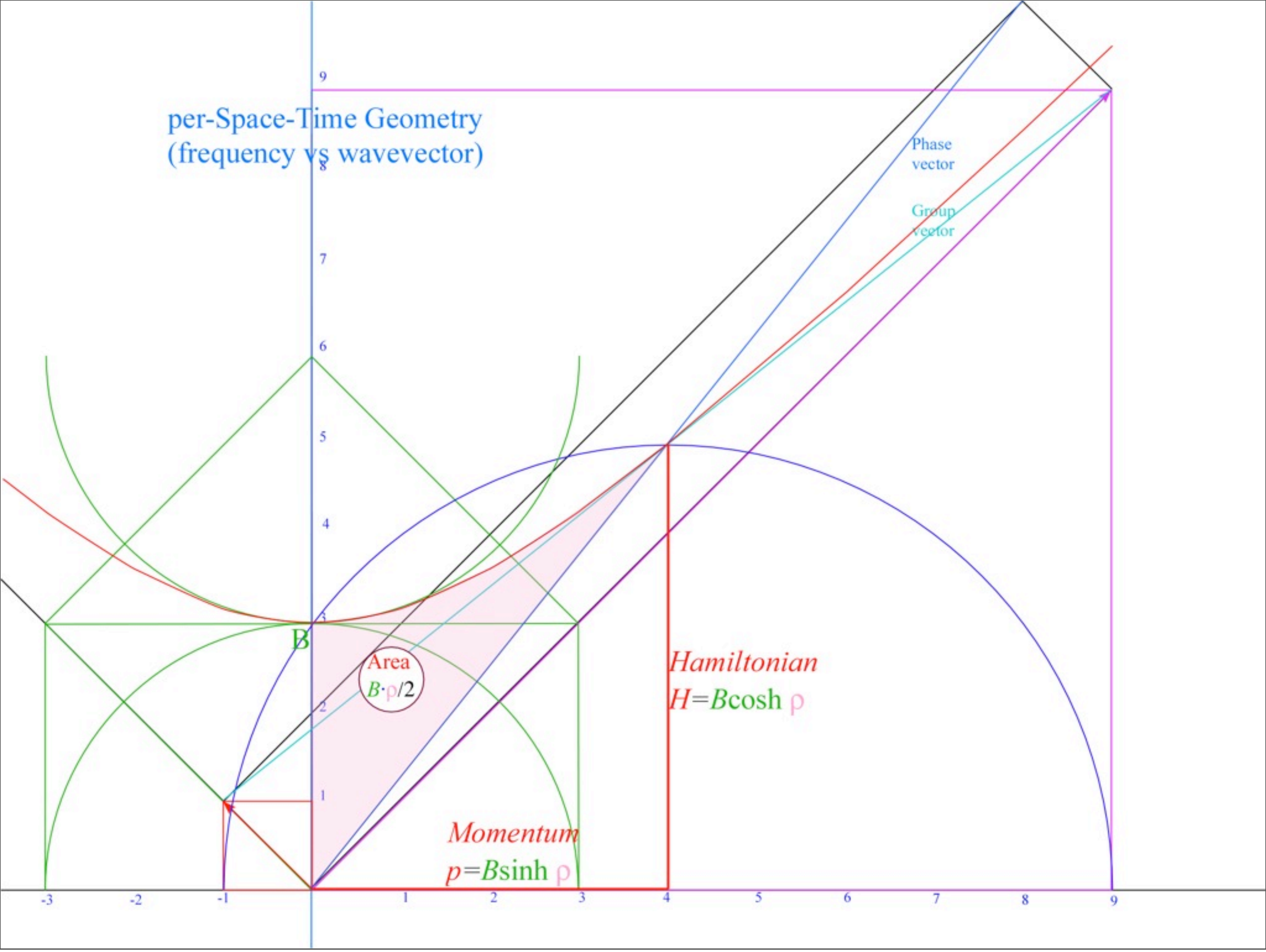
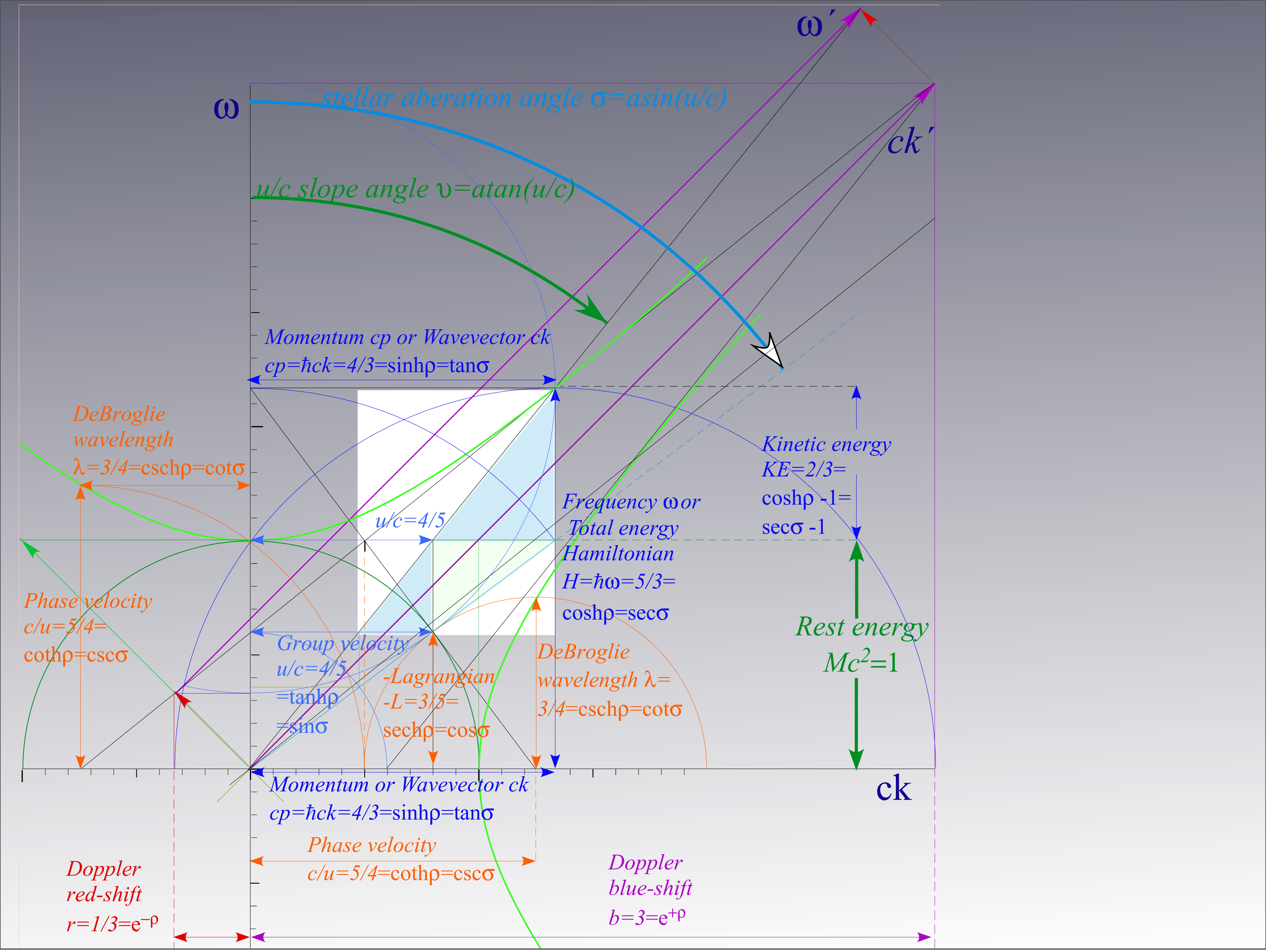
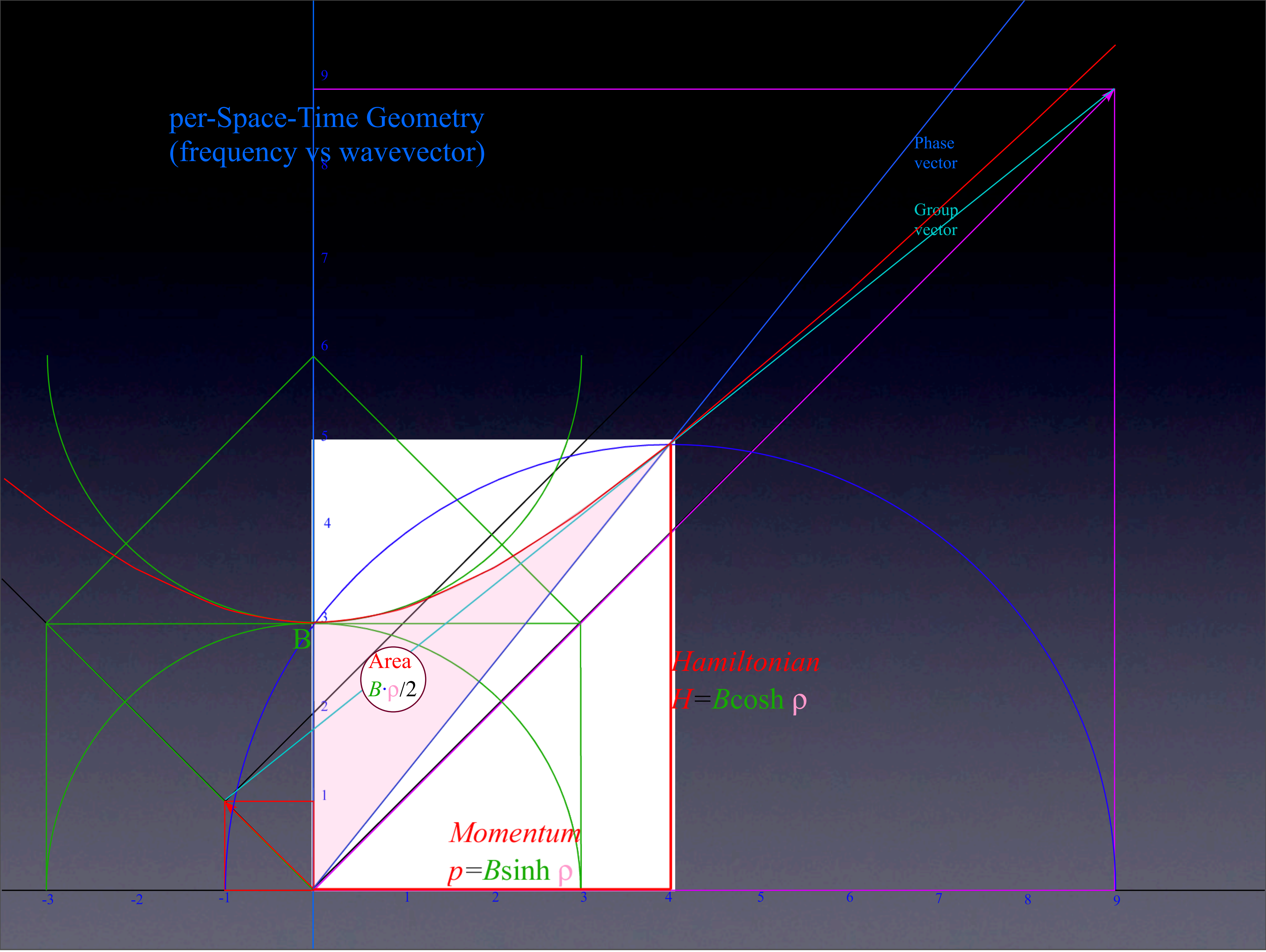

B sinh ρ
B tanh ρ
B cosh ρ
B
B sech ρ
B e^-ρ
Red shift
B e^+ρ
Blue shift


per-Space-Time Geometry
(frequency vs wavevector)
Phase
vector
Group
vector
B
Area
B·ρ/2
Hamiltonian
H=Bcosh ρ
Momentum
p=Bsinh ρ
9
8
7
6
5
4
3
2
1
-3
-2
-1
1
2
3
4
5
6
7
8
9


ω´
ω
ck´
stellar aberation angle σ=asin(u/c)
u/c slope angle υ=atan(u/c)
Momentum cp or Wavevector ck
cp=ħck=4/3=sinhρ=tanσ
DeBroglie
wavelength
λ=3/4=cschρ=cotσ
Kinetic energy
KE=2/3=
coshρ -1=
secσ -1
u/c=4/5
Frequency ω or
Total energy
Hamiltonian
H=ħω=5/3=
coshρ=secσ
Phase velocity
c/u=5/4=
cothρ=cscσ
Rest energy
Mc²=1
Group velocity
u/c=4/5
=tanhρ
=sinσ
-Lagrangian
-L=3/5=
sechρ=cosσ
DeBroglie
wavelength λ=
3/4=cschρ=cotσ
ck
Momentum or Wavevector ck
cp=ħck=4/3=sinhρ=tanσ
Phase velocity
c/u=5/4=cothρ=cscσ
Doppler
red-shift
r=1/3=e^−ρ
Doppler
blue-shift
b=3=e^+ρ


per-Space-Time Geometry
(frequency vs wavevector)
Phase vector
Group vector
B
Area
B·ρ/2
Hamiltonian
H=Bcosh ρ
Momentum
p=Bsinh ρ
9
8
7
6
5
4
3
2
1
-3
-2
-1
1
2
3
4
5
6
7
8
9

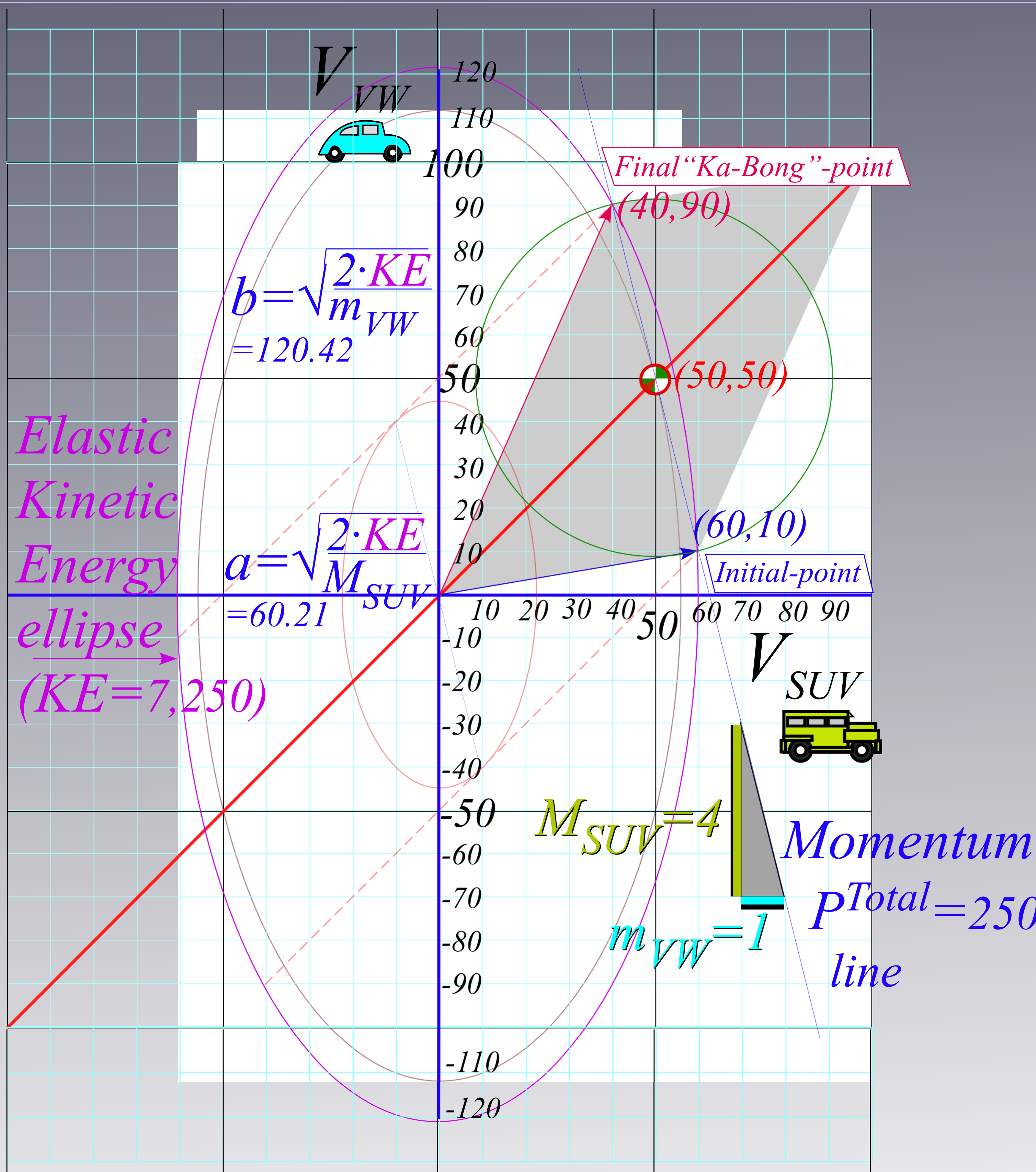


Fig. 3.1 a
in Unit 1

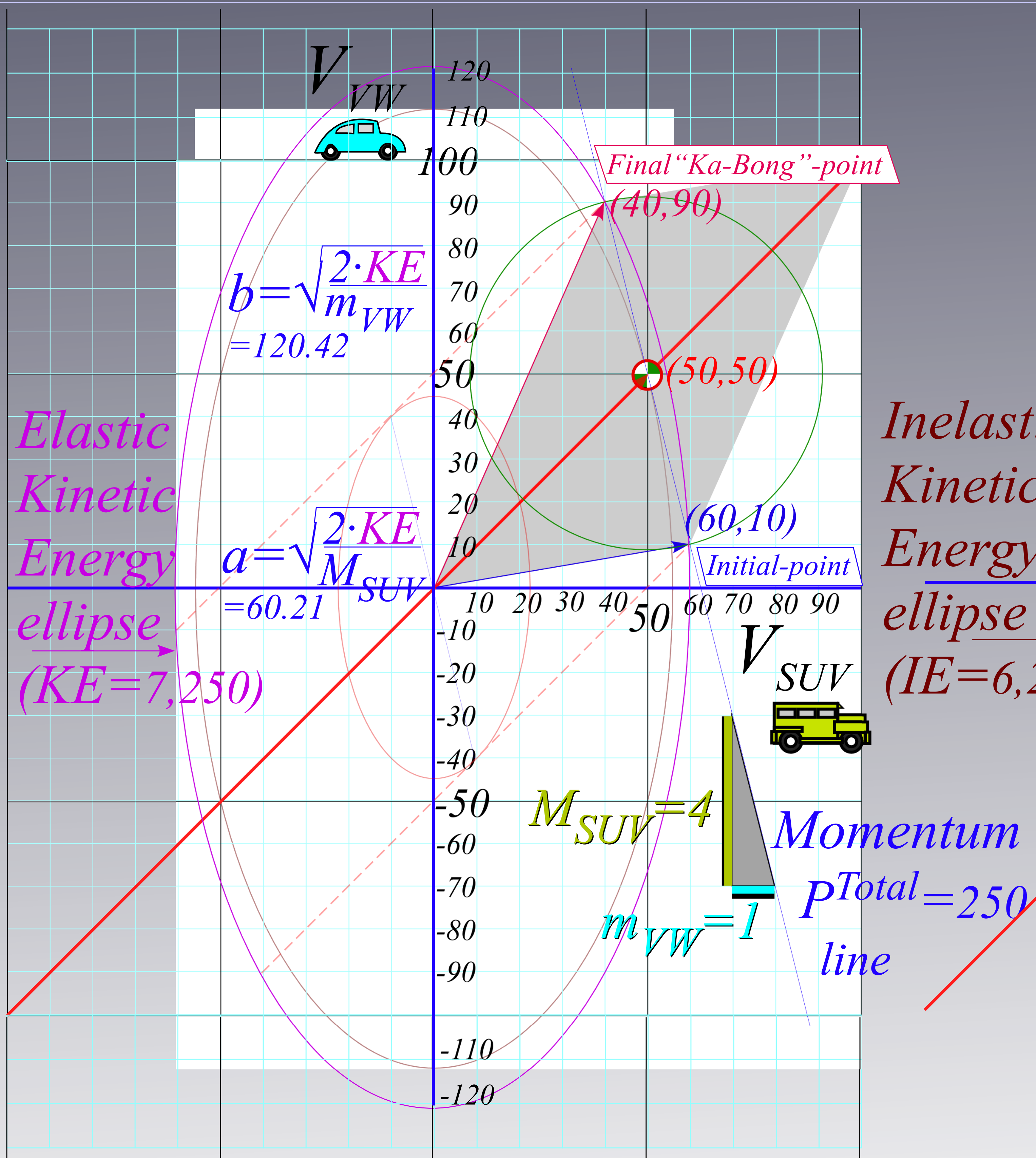


Fig. 3.1 a
in Unit 1

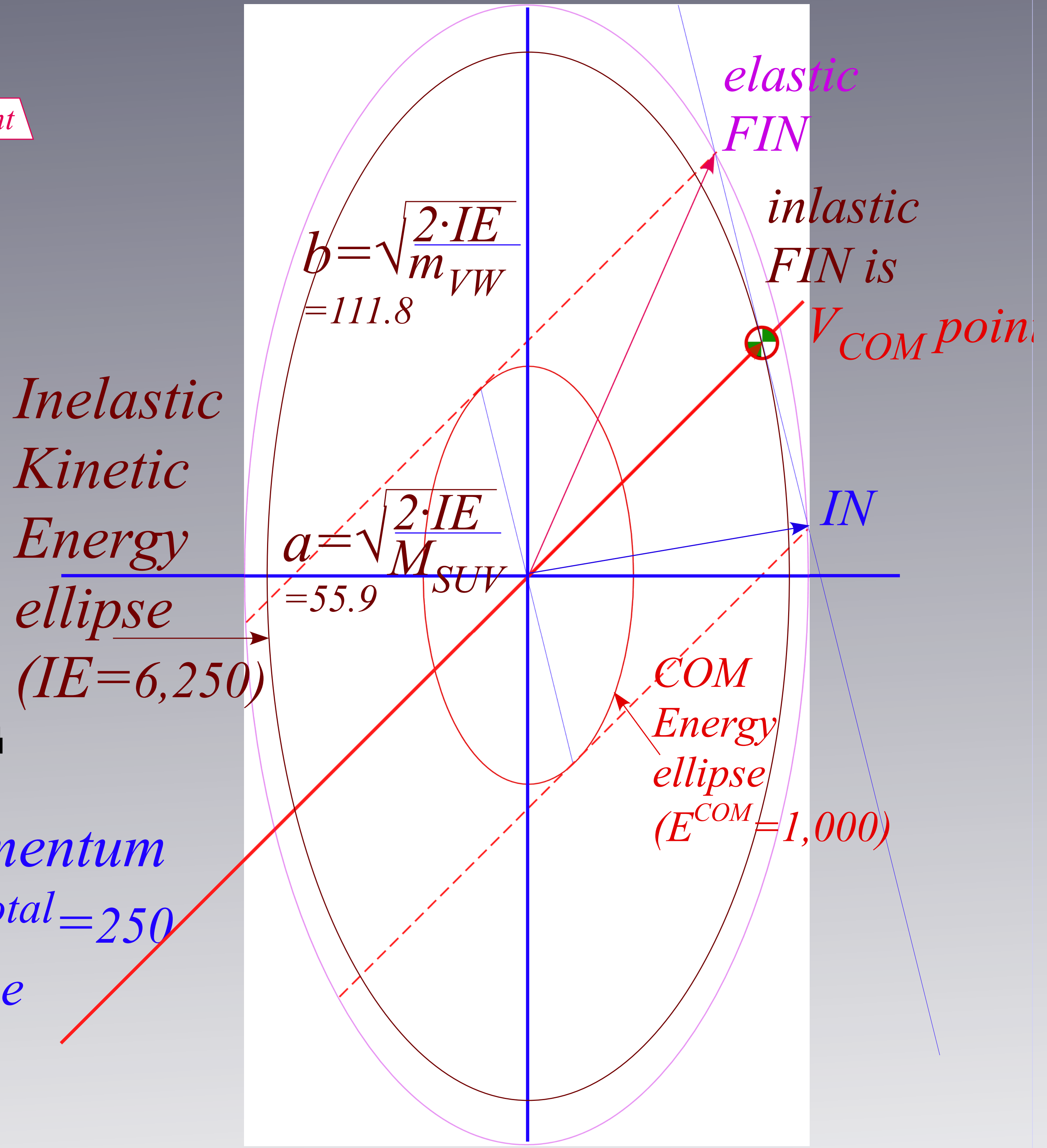


Fig. 3.1 b
in Unit 1