Current understanding of relativity and QM at UAF

NWAT photo by David Gottschalk

Current understanding of relativity and QM at UAF

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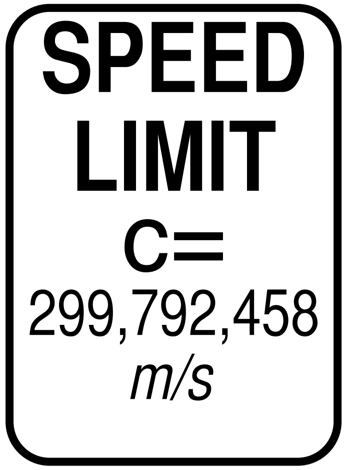
Is a clearer understanding possible…?

Level 1 Secrets *(which really shouldn't be secrets at all!)*

Special relativity and quantum mechanics

are very much a story of

the geometry of light-wave motion

- How badly does Galilean relativity fail for light waves?
- How do you make sense of light-wave 

axiom(s)?

The *Einstein Pulse Wave* (PW) axiom

versus

The *Evenson Continuous Wave* (CW) axiom

Good approximation:
c=300 million m/s

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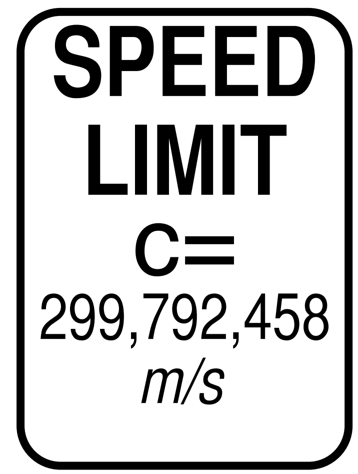
- How do you make sense of light-wave  axiom(s)?

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  (*wavelength* λ – *period* τ) and/or (*wavenumber* κ – *frequency* υ)

  ($\lambda = 1/\kappa$ and $\tau = 1/\upsilon$) ($\kappa = 1/\lambda$ and $\upsilon = 1/\tau$)

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(λ = *meters per wave* and τ = *seconds per wave*) (κ = *waves per meter* and υ = *waves per second*)

Heinreich Hertz 1857-1894

1Hz=1sec^{-1}

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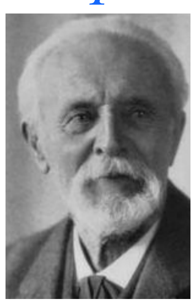
1Hz=1sec^{-1}

Greek "L" for Length

Greek "t" for time

Greek "k" for Kayser (or "kinks")



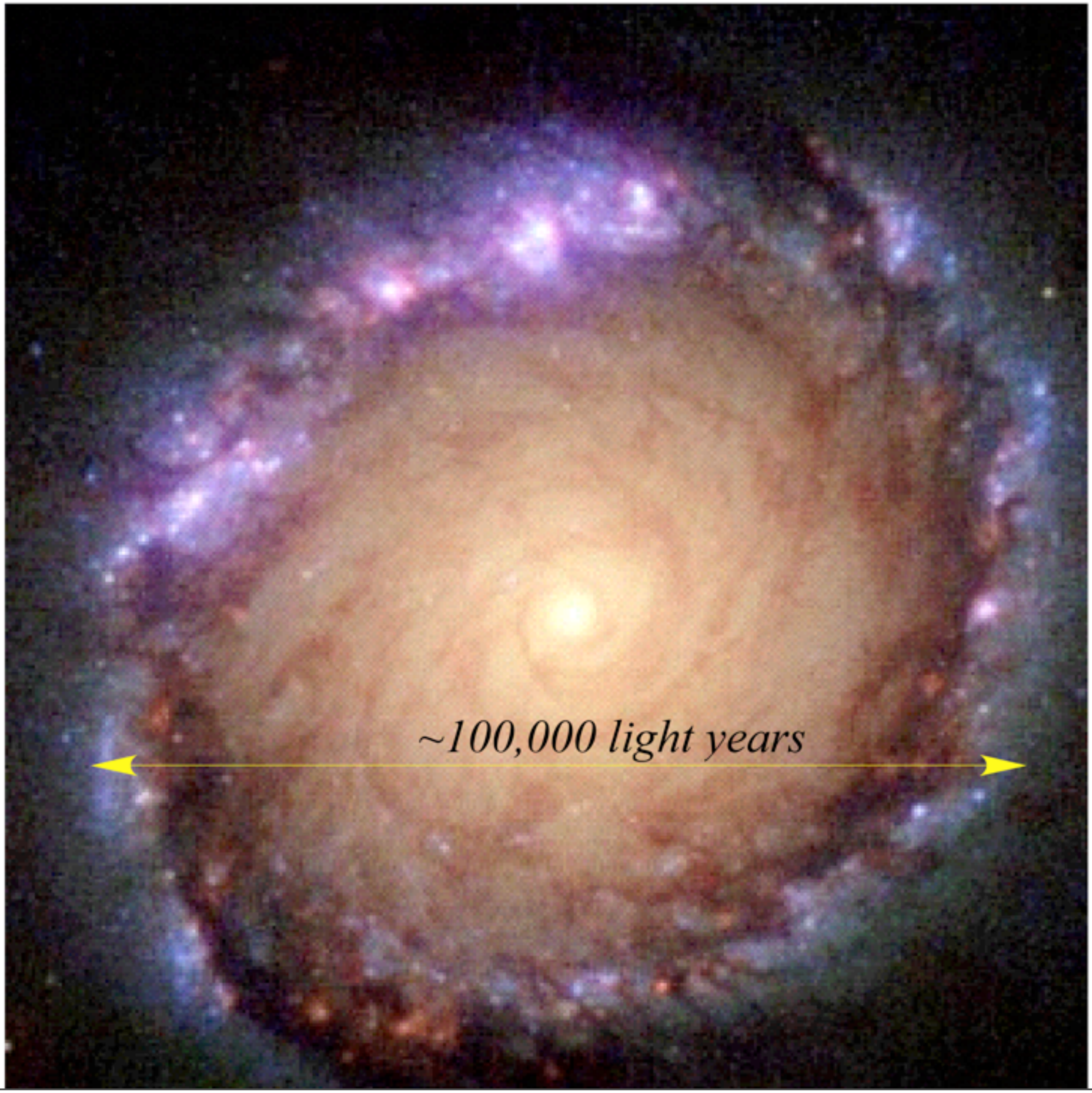
Heinreich Kayser 1853-1940

1Kayser=1cm^{-1}

Greek "n" for number of waves per second or Hertz (Hz)

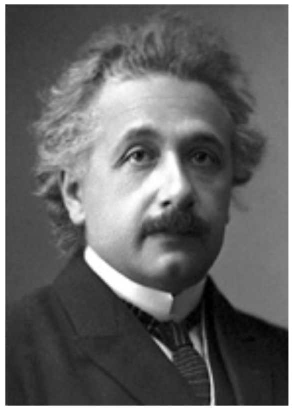
How fast is light? Light goes one foot in a nano-second .
This may seem quite fast to us.
But, on a cosmic scale lightspeed is positively sub-glacial.

In your lifetime light cannot move across one pixel (.) of a Hubble deep-sky photo.



- How badly does Galilean relativity fail for light waves?

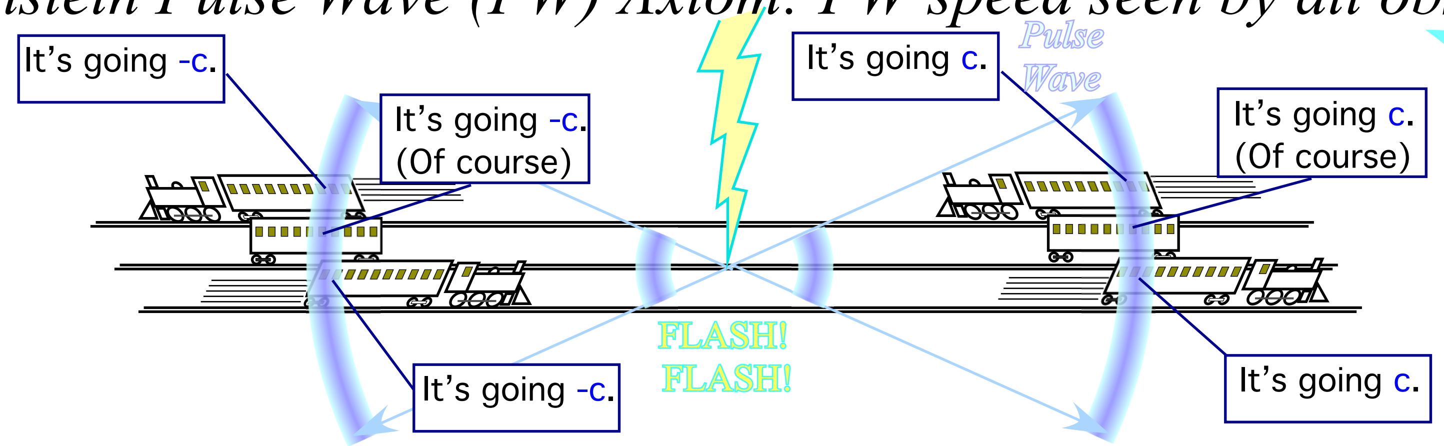
Albert Einstein



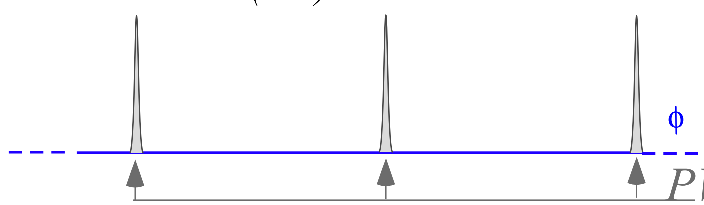
1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c

A “road-runner” axiom is a “show-stopper”



Pulse wave (PW) train



$A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots$

Complicated

PW peaks precisely locate places where wave is.

• How badly does Galilean relativity fail for light waves?

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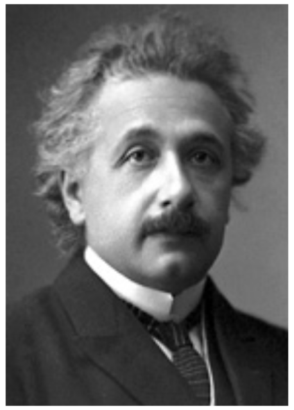

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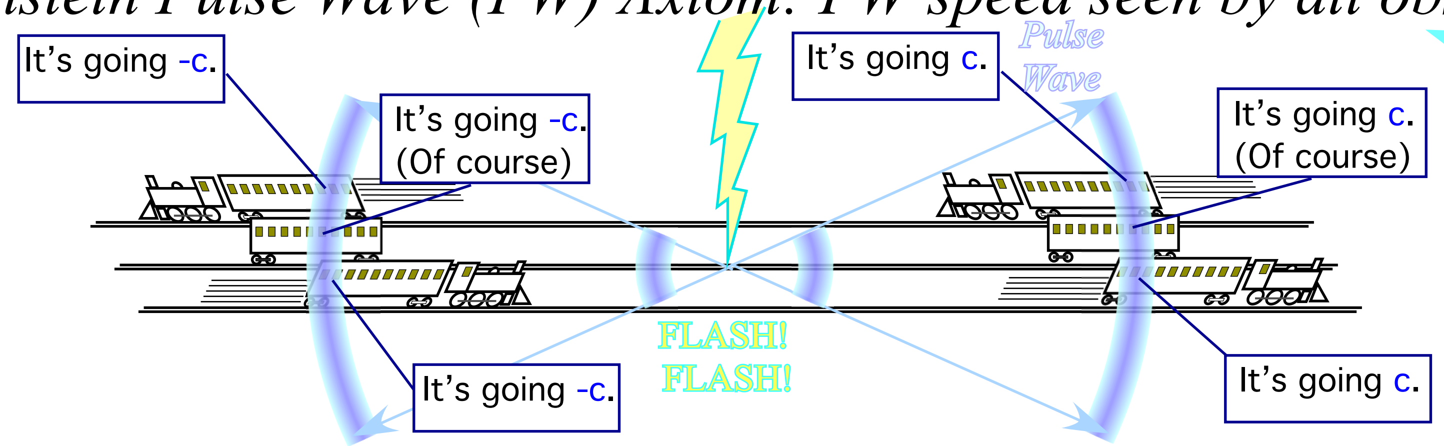

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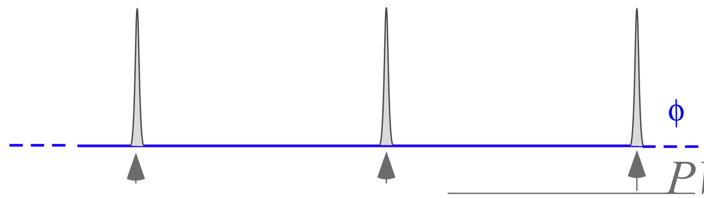
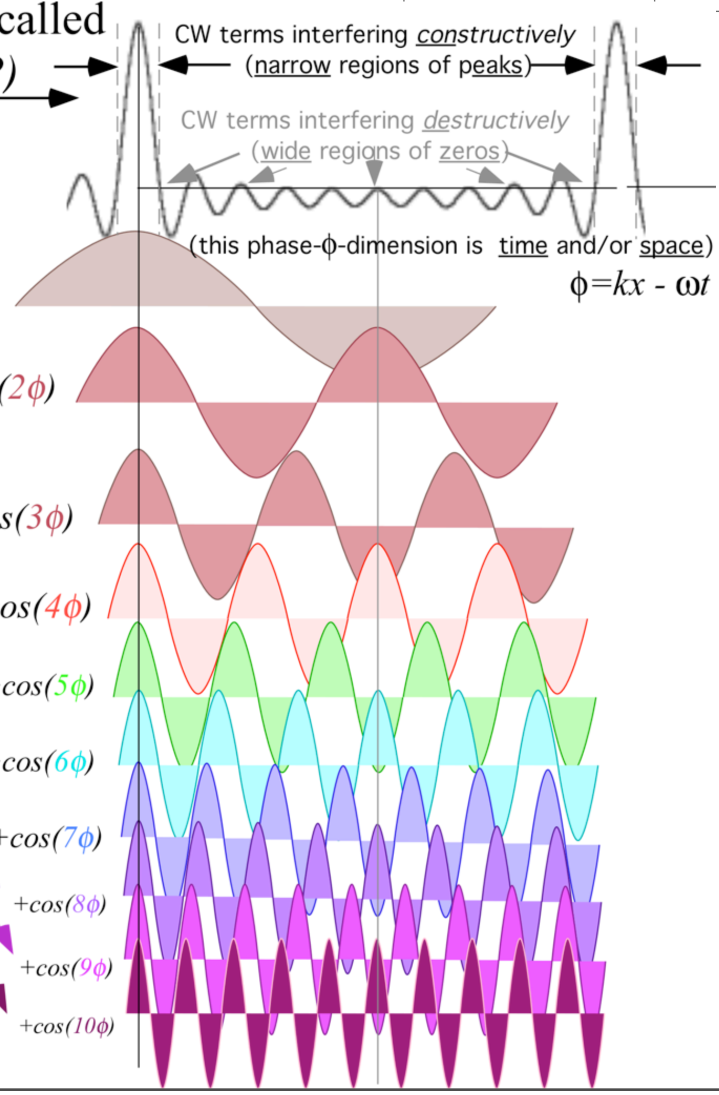


$A_1\cos\omega t + A_2\cos 2\omega t + A_3\cos 3\omega t + A_4\cos 4\omega t + \ldots$ ← Complicated

PW peaks precisely locate places where wave is.

PW forms are also called *Wave Packets (WP)* since they are interfering sums of many *CW* terms (10-Cosine Waves make up this pulse)



CW terms are also called *Color Waves* or *Fourier Spectral Components*

• How badly does Galilean relativity fail for light waves?

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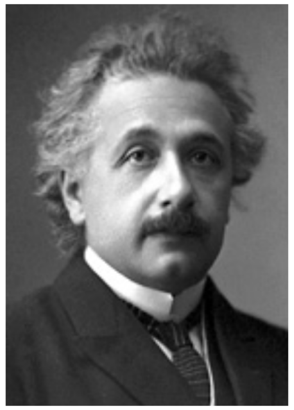

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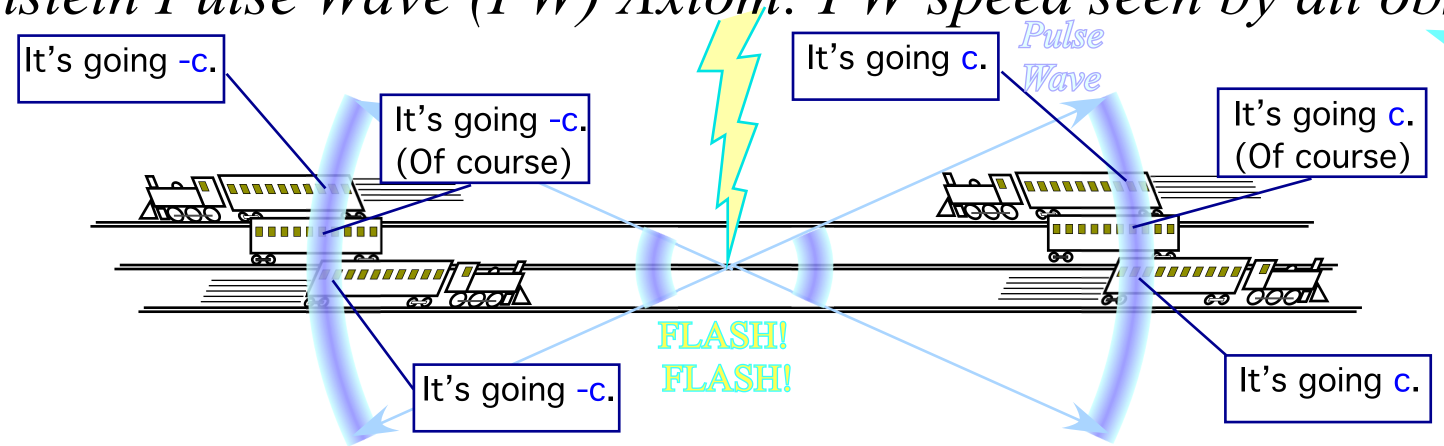

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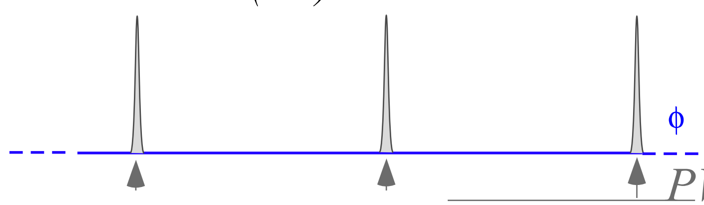

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PW peaks precisely locate places where wave is.

PW widths reduce proportionally with more *CW* terms (greater *Spectral* width)

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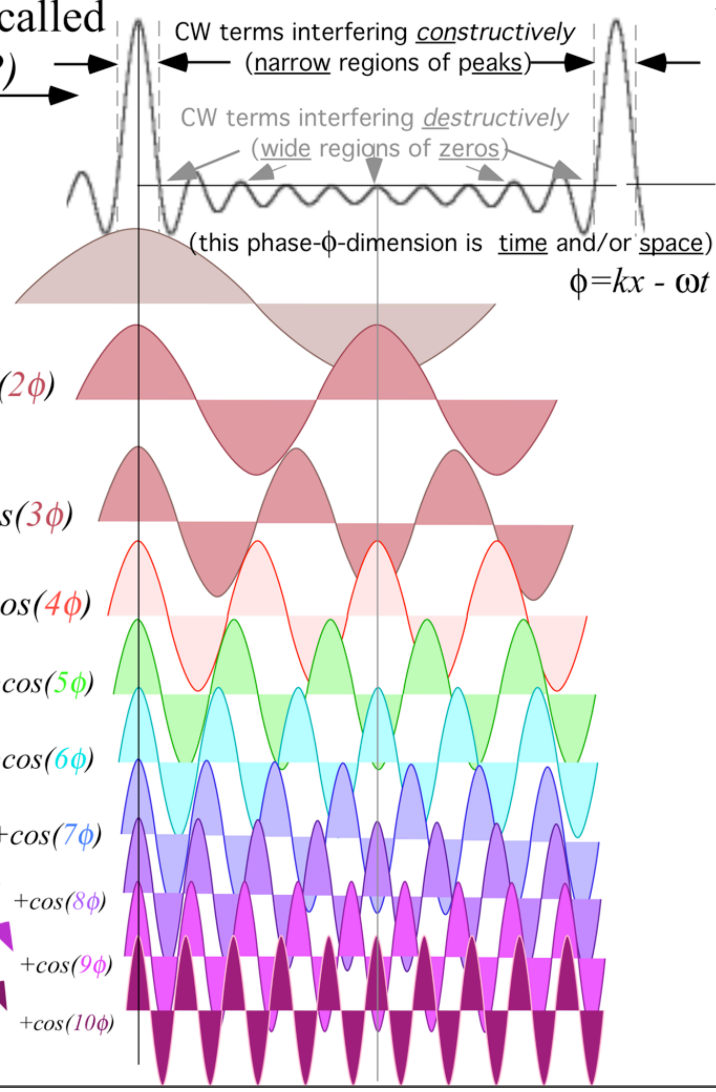
(10-Cosine Waves make up this pulse)



CW terms are also called *Color Waves* or *Fourier Spectral Components*

Space-time width (pulse width)

Spectral width (harmonic frequency range)

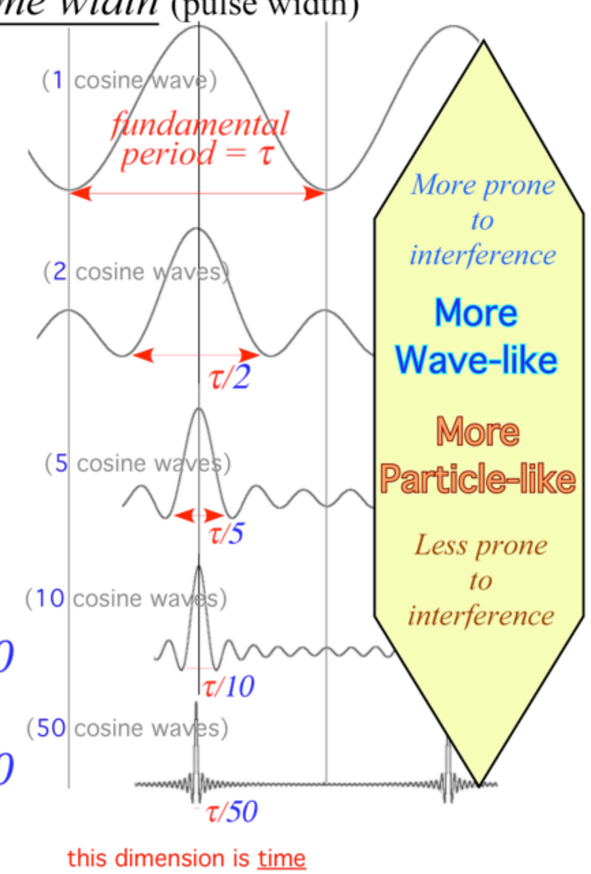
$\Delta t = \tau$

$\Delta t = \tau/2$

$\Delta t = \tau/5$

$\Delta t = \tau/10$

$\Delta t = \tau/50$



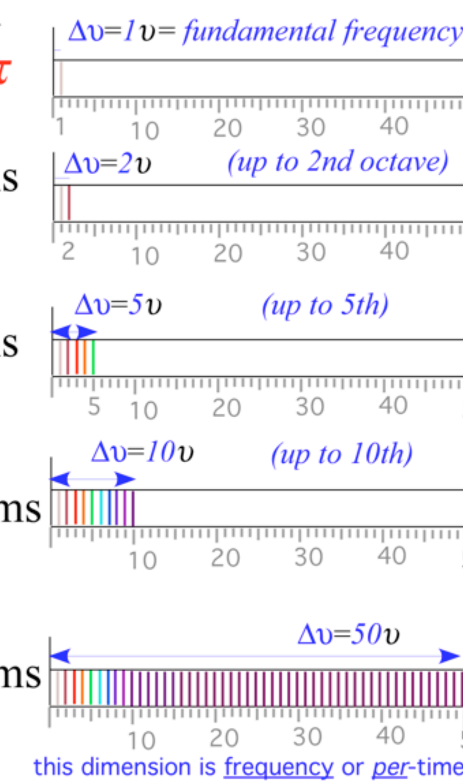
1 *CW* term $\Delta\upsilon = \upsilon = 1/\tau$

2 *CW* terms $\Delta\upsilon = 2\upsilon$

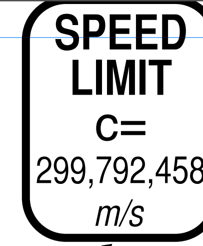
5 *CW* terms $\Delta\upsilon = 5\upsilon$

10 *CW* terms $\Delta\upsilon = 10\upsilon$

50 *CW* terms $\Delta\upsilon = 50\upsilon$



Fourier-Heisenberg product: $\Delta t \cdot \Delta\upsilon = 1$ (time-frequency uncertainty relation)

• How do you make sense of light-wave  axiom(s)?

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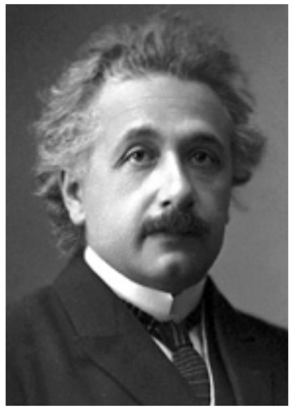

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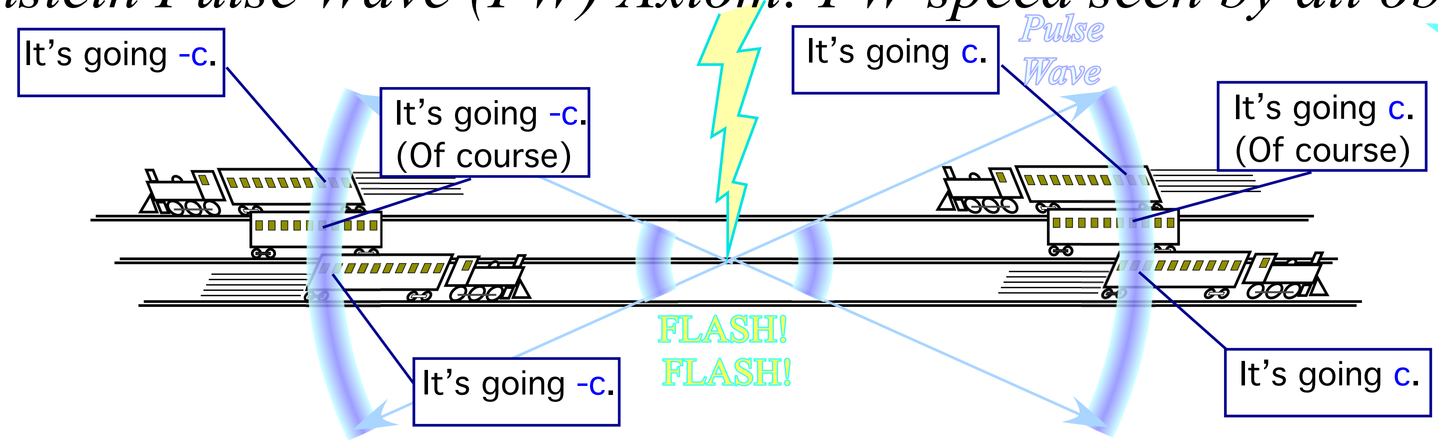


A "road-runner" axiom is a "show-stopper"

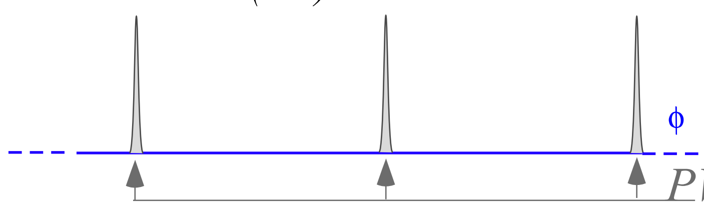
William of Ockham

1285-1349

Using Occam's Razor

(and Evenson's lasers)

Pulse wave (PW) train



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Complicated

...many waves and Amplitude parameters

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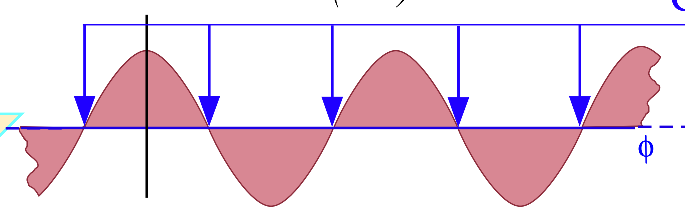
CW zeros precisely locate places where wave is not.

Continuous wave (CW) train

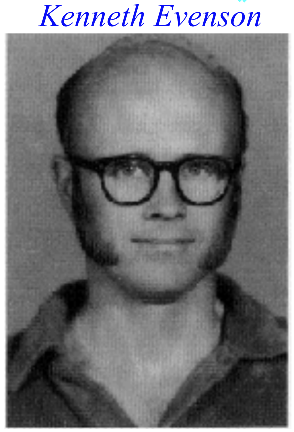


$A\cos\omega t$

Simpler

...just one wave (a 1CW)

Kenneth Evenson



1929-2002
c=299,792,458 m/s

Cut a *PW* to just *one* Continuous Wave (*1CW*)

• How do you make sense of light-wave SPEED LIMIT c= 299,792,458 m/s axiom(s)?

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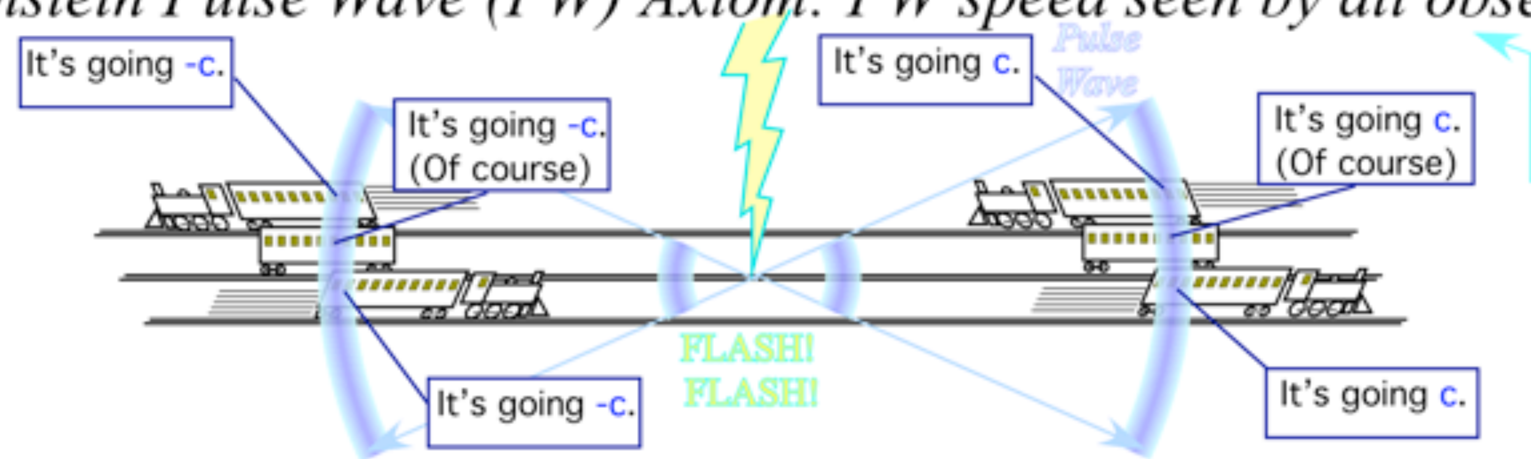

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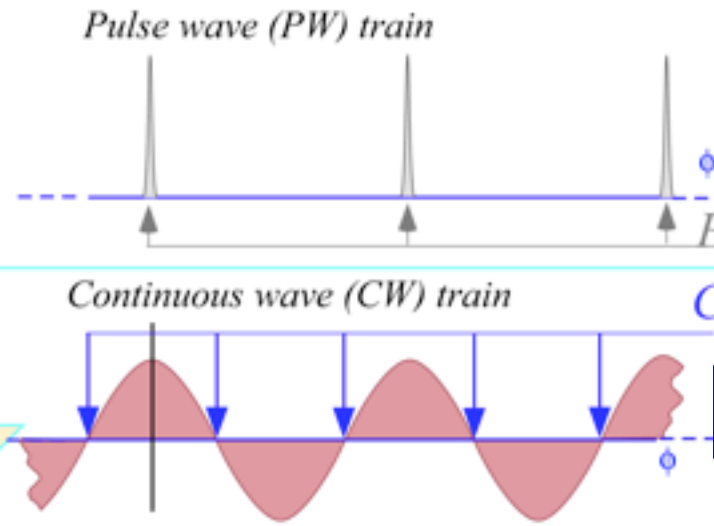

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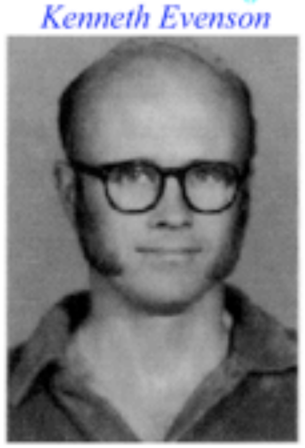
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Kenneth Evenson



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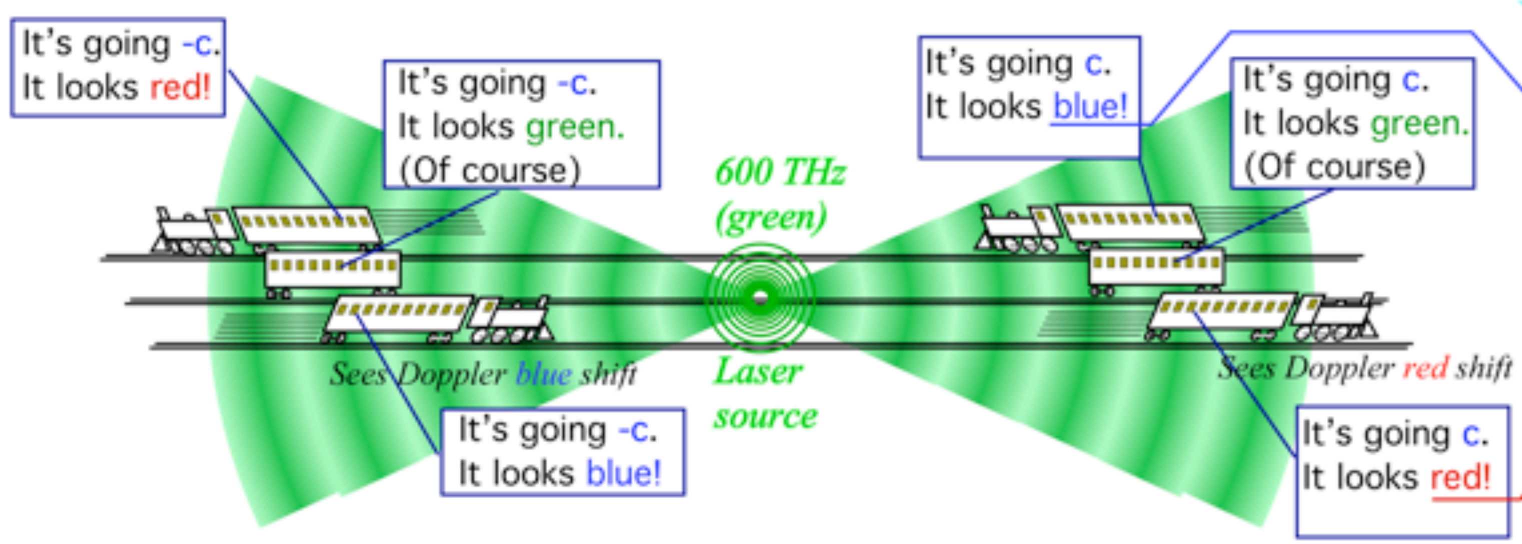

CW affected by 1st order Doppler
Blue shifts $b=e^{+\rho}$
and
Red shifts $r=e^{-\rho}$
of frequency υ
and wavenumber κ

Cut a *PW* to *one* Continuous Wave (*1CW*) that changes Color if you accelerate!

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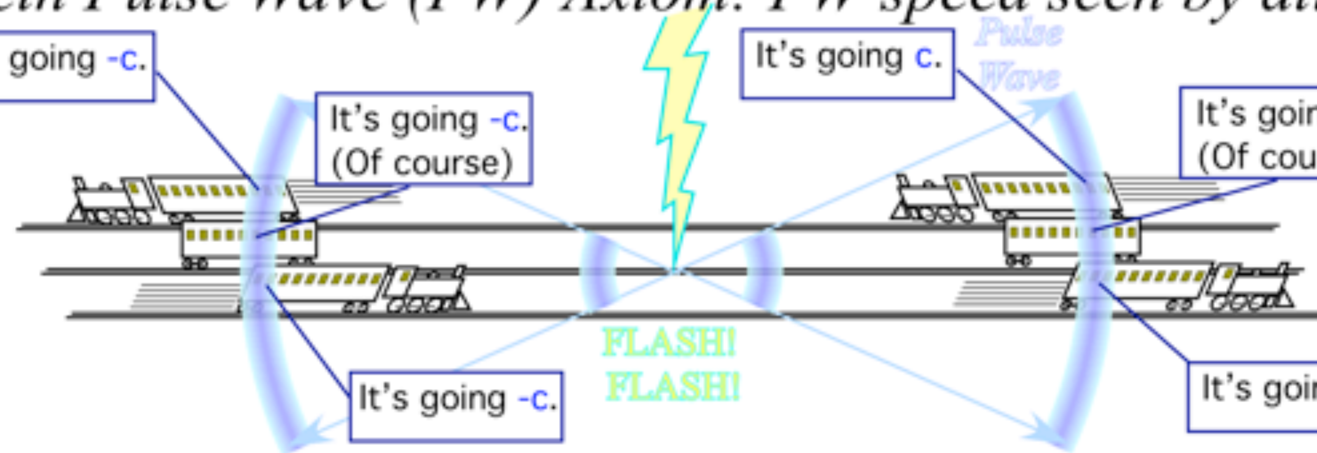

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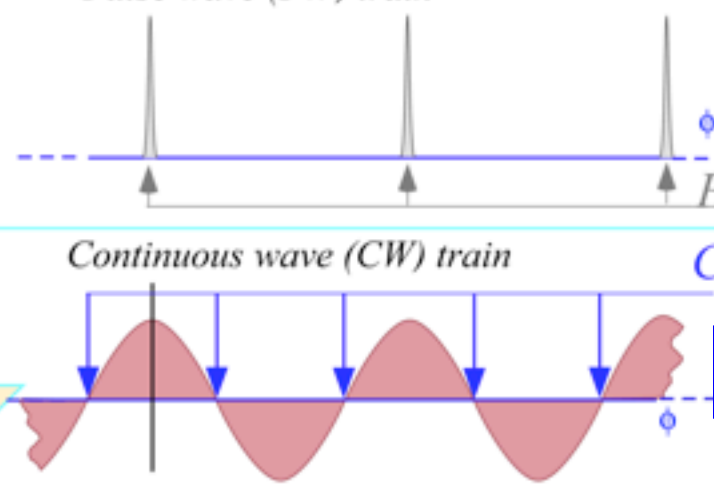

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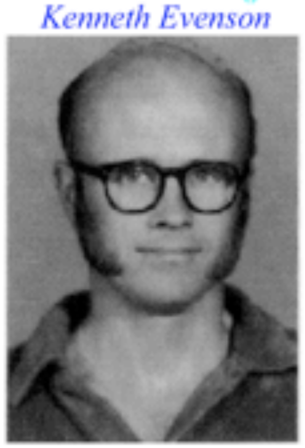

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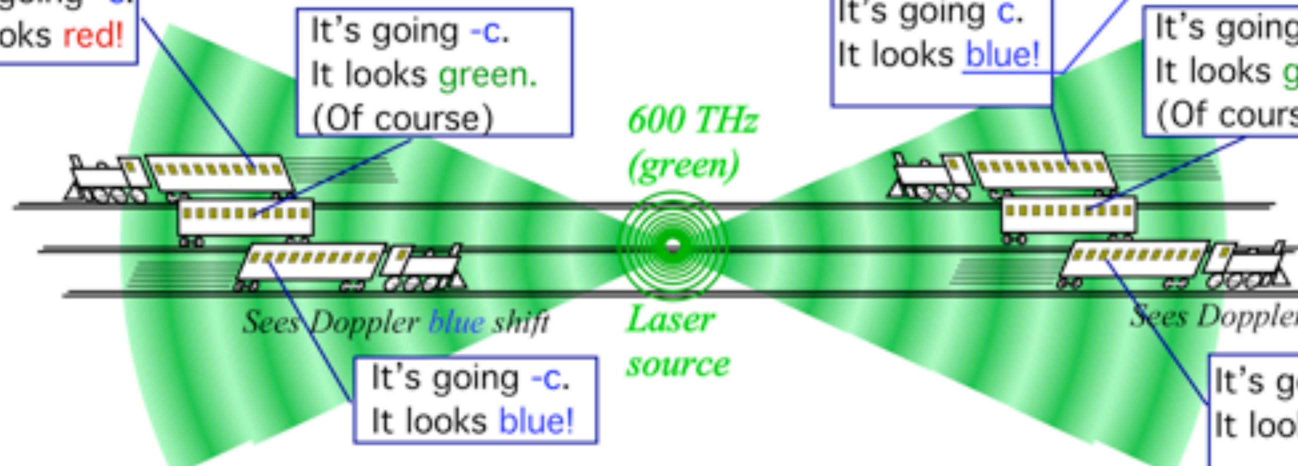

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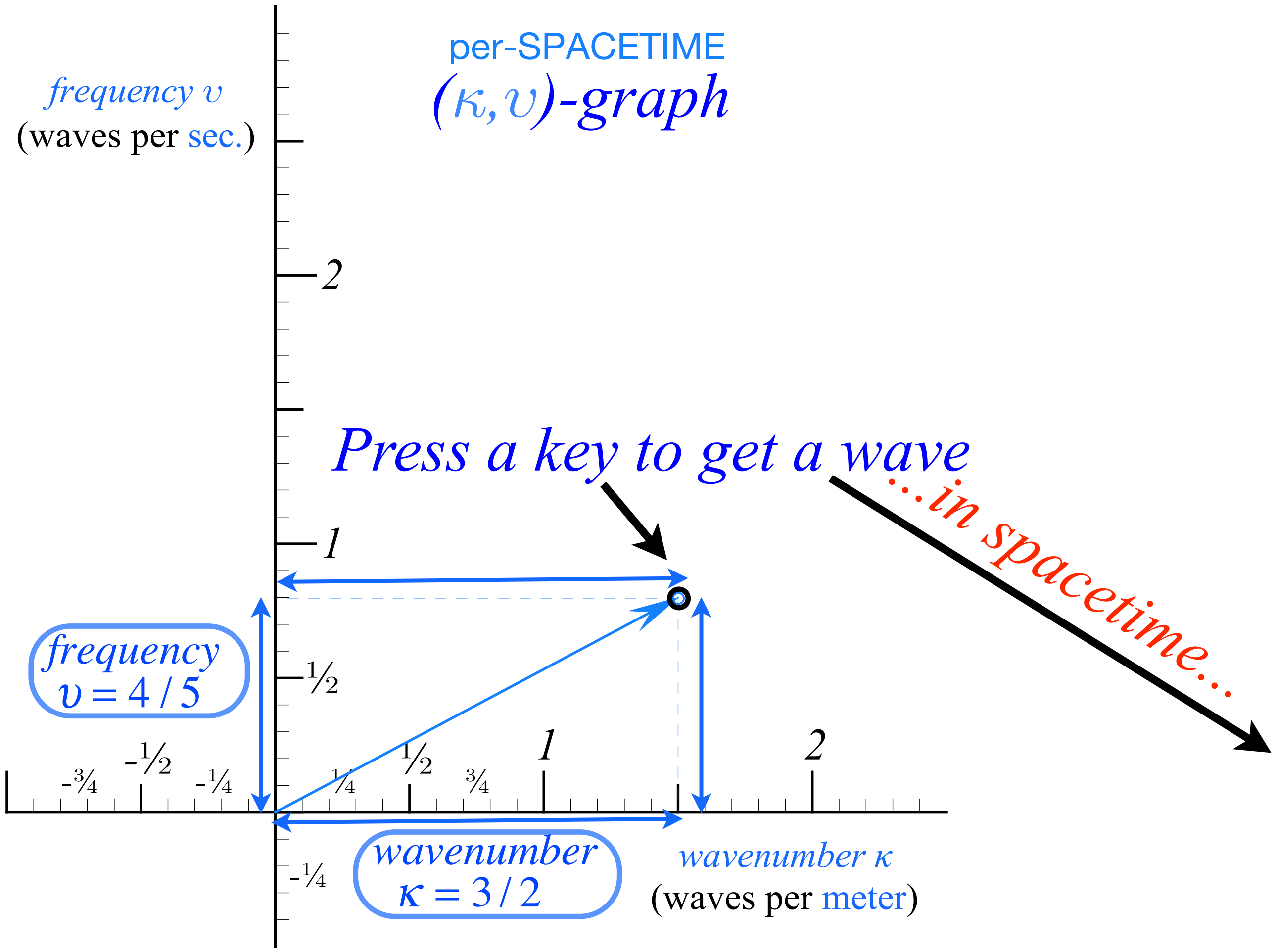
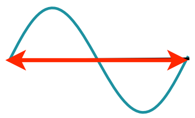
Cut a *PW* to *one* Continuous Wave (*1CW*) that changes Color if you accelerate!

CW also stands for "Cosine Wave" or "Coherent Wave" or "Colored Wave" (all helpful things!)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs





"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

Ways to quantify general waves

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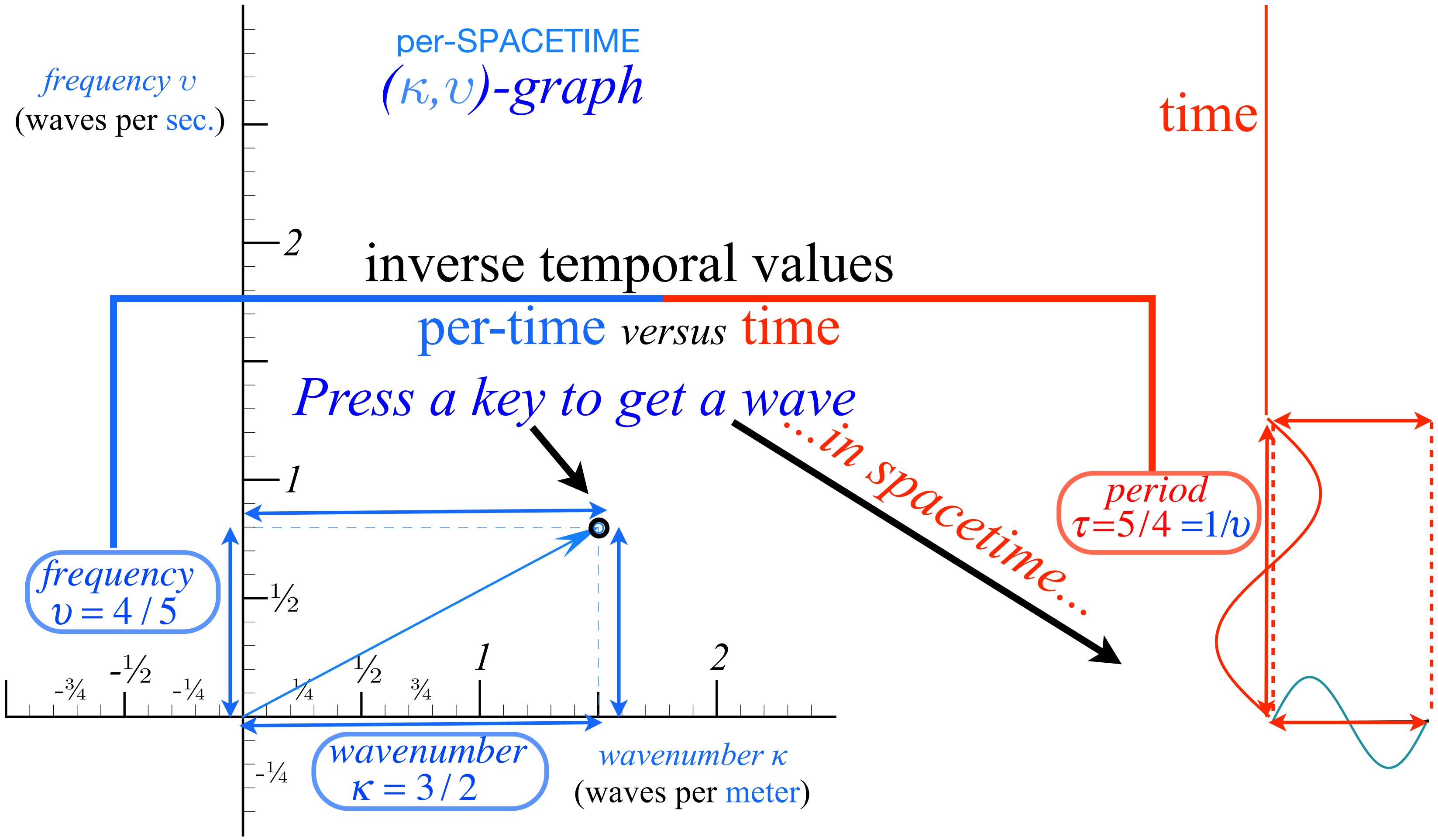

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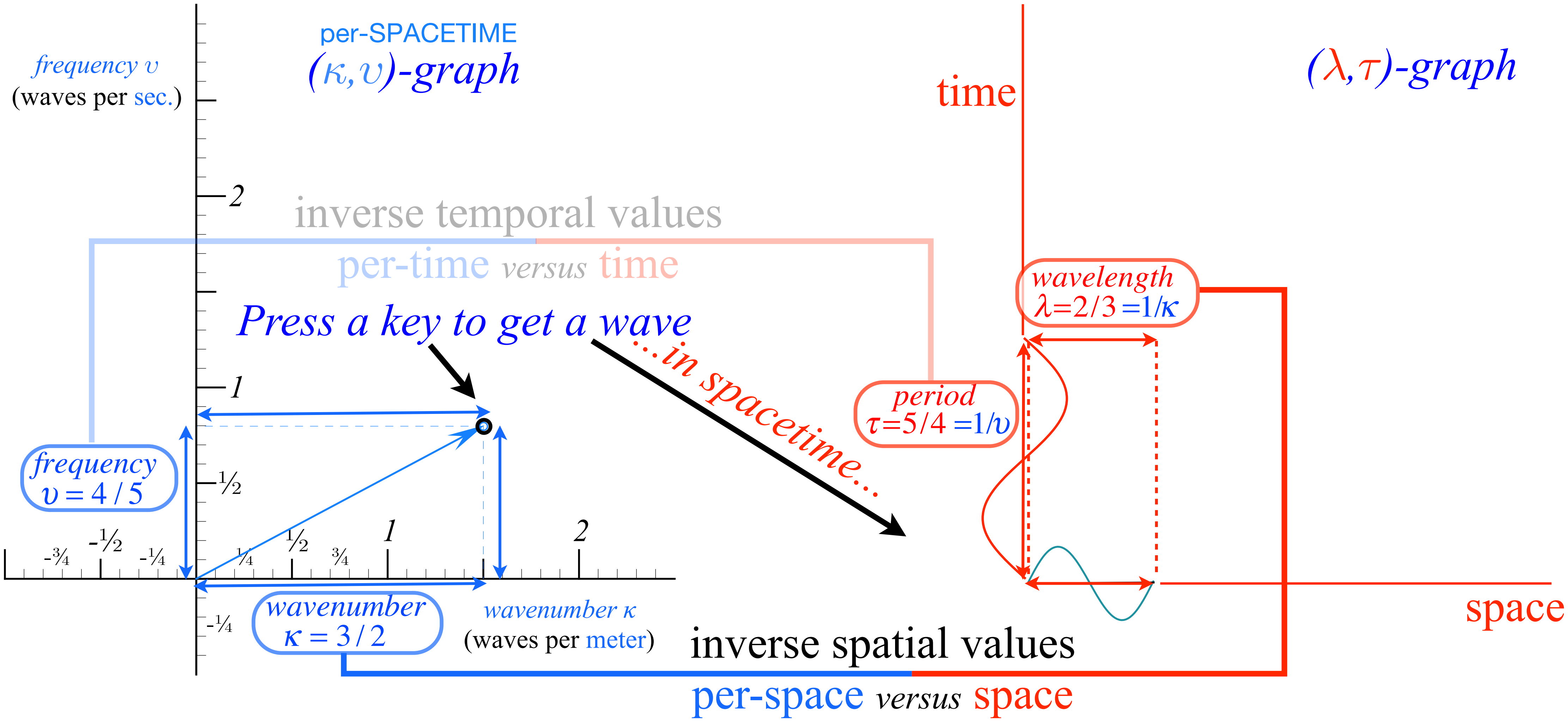

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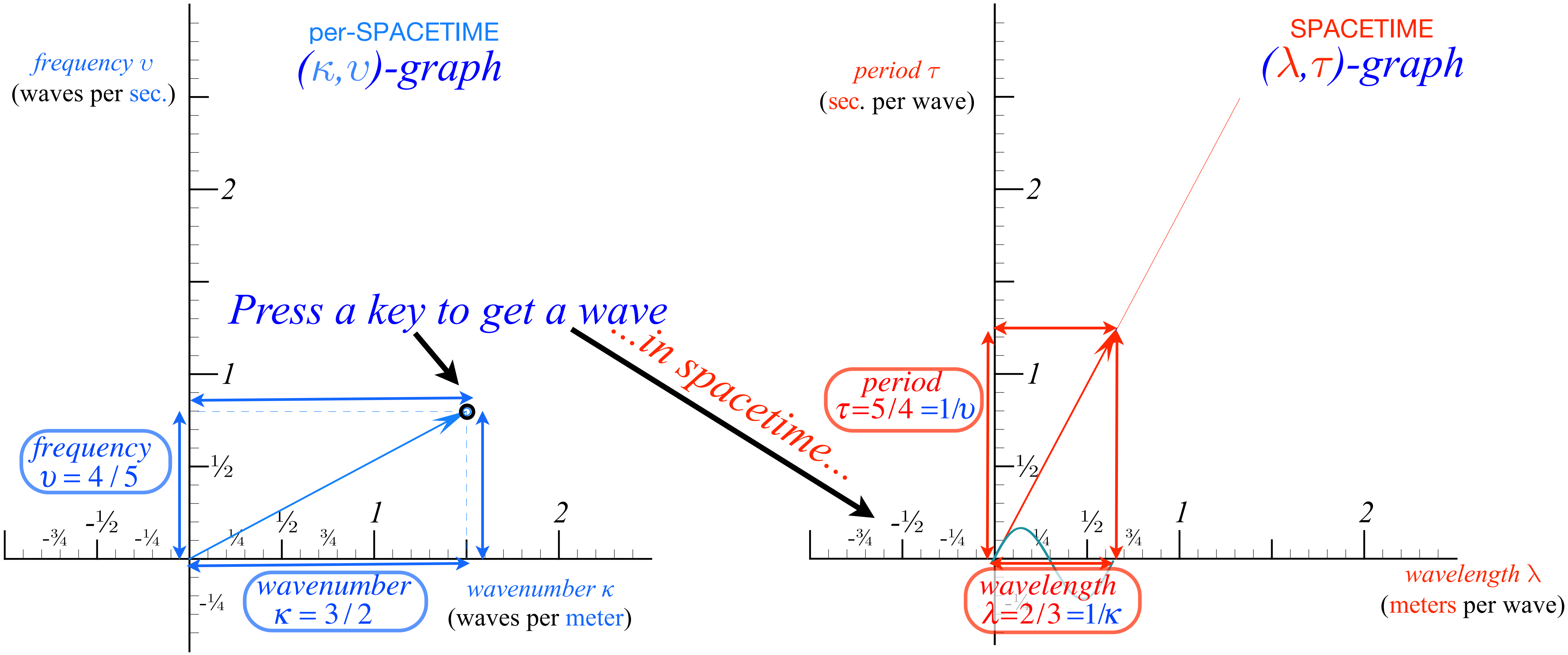

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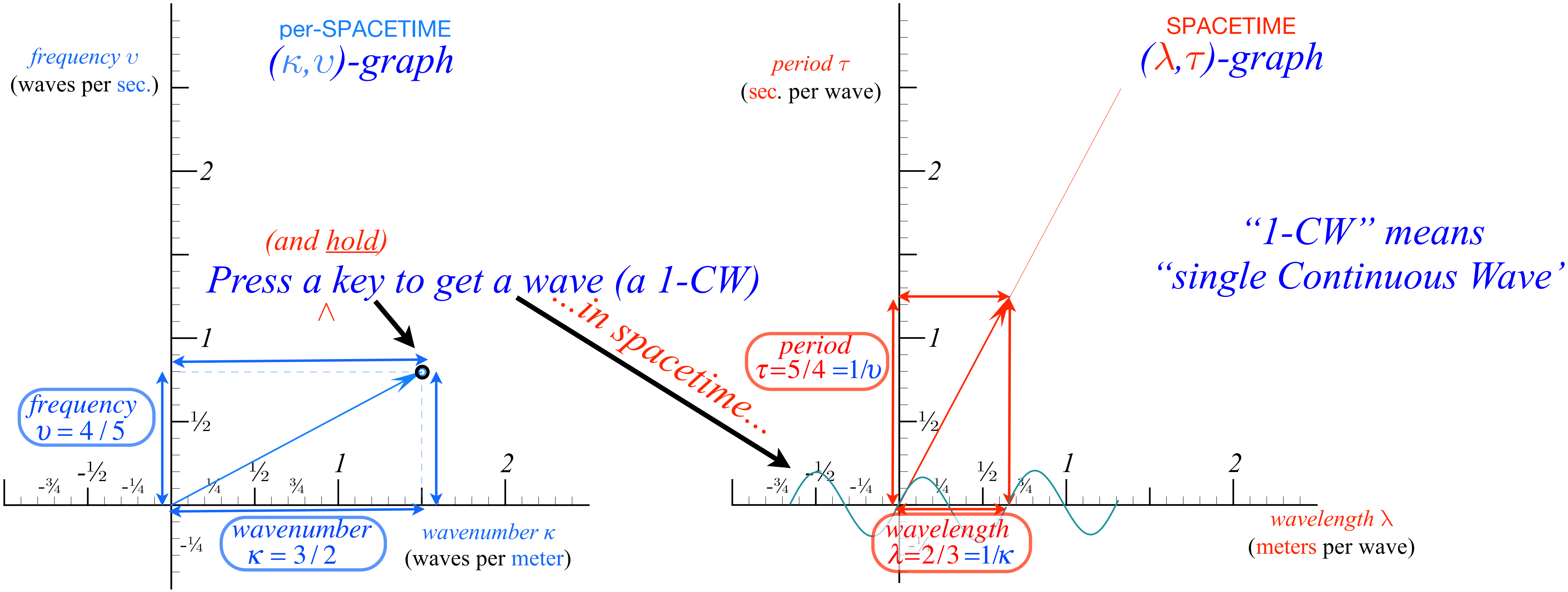

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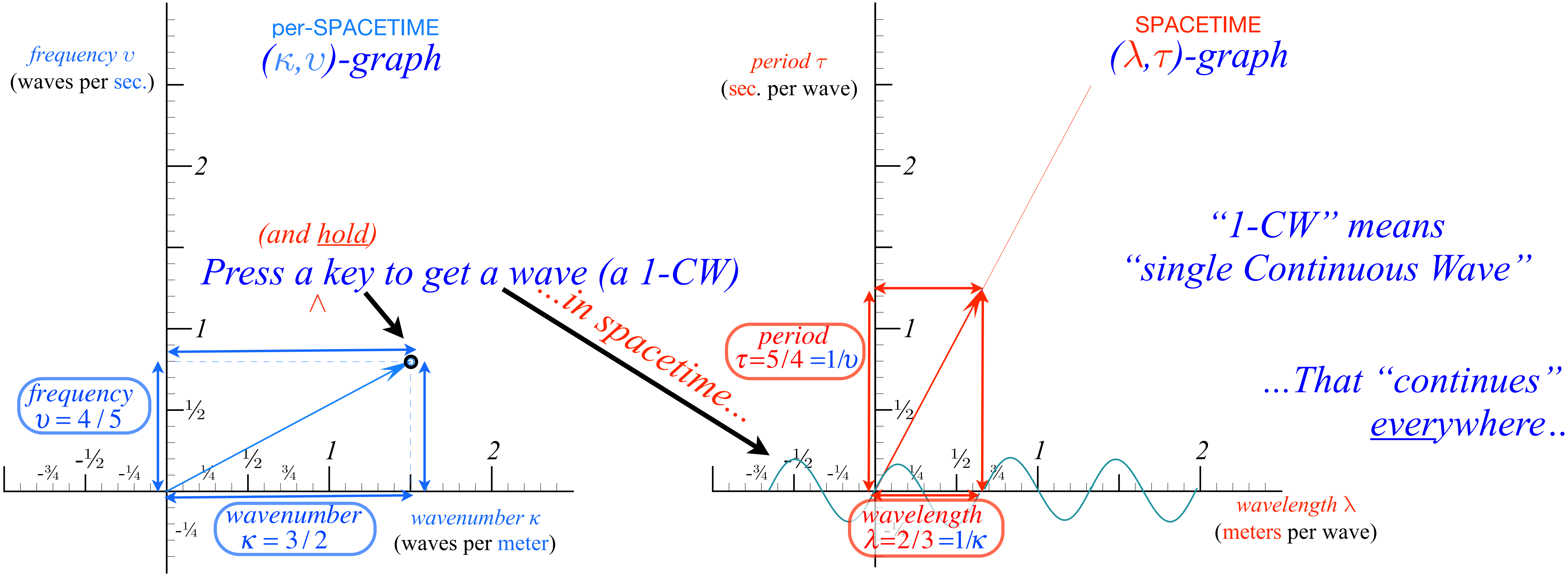

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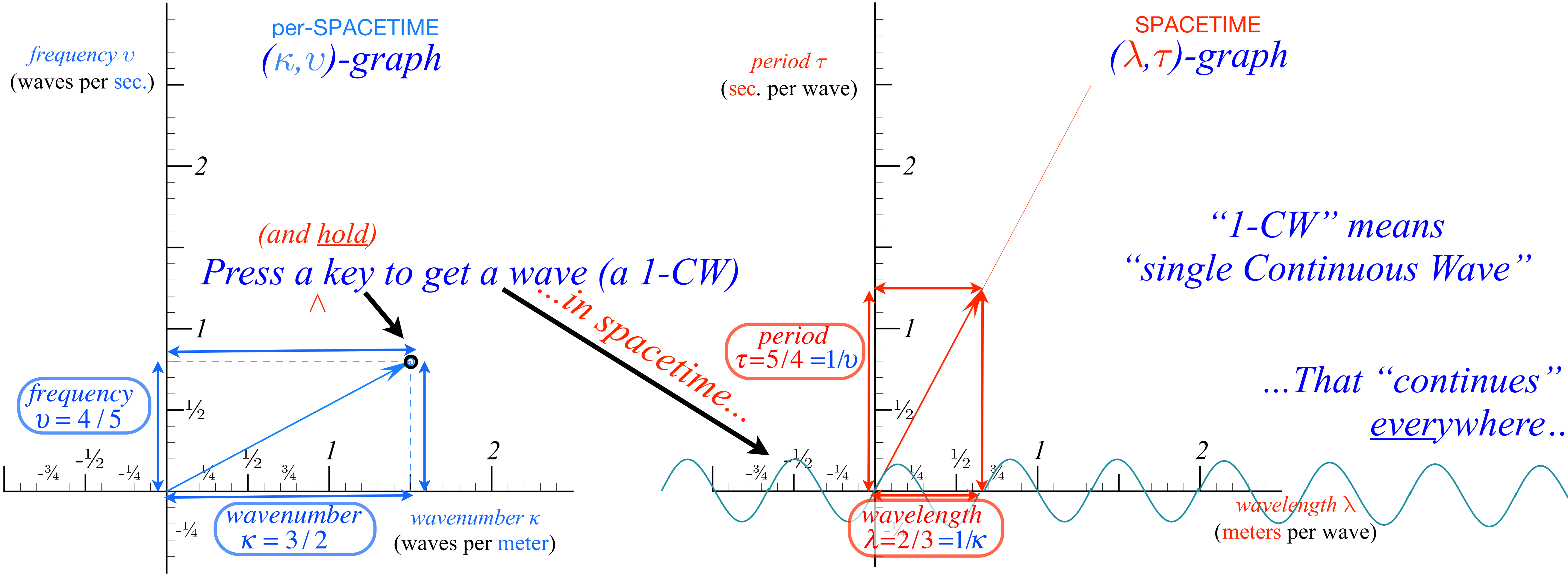

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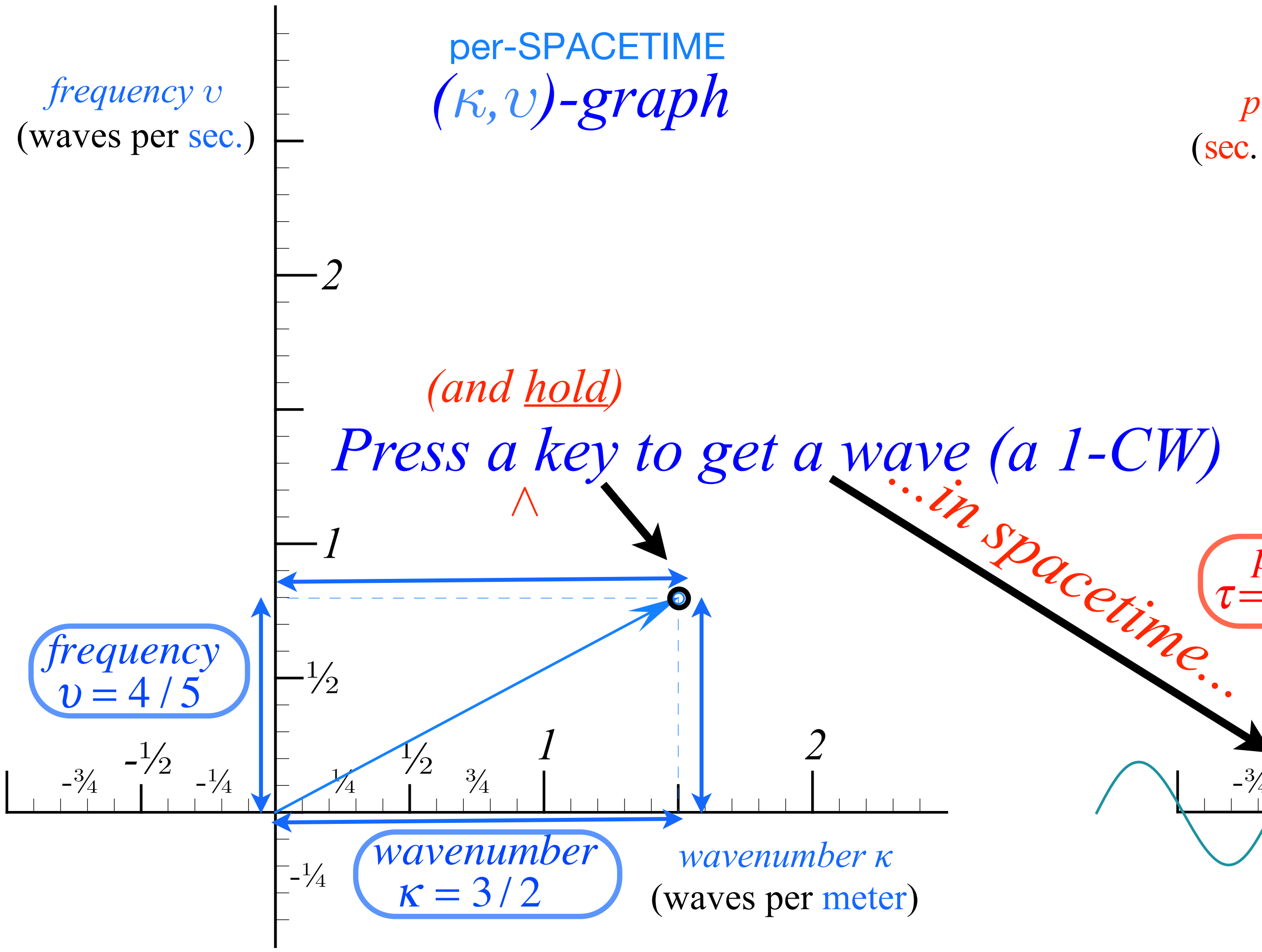
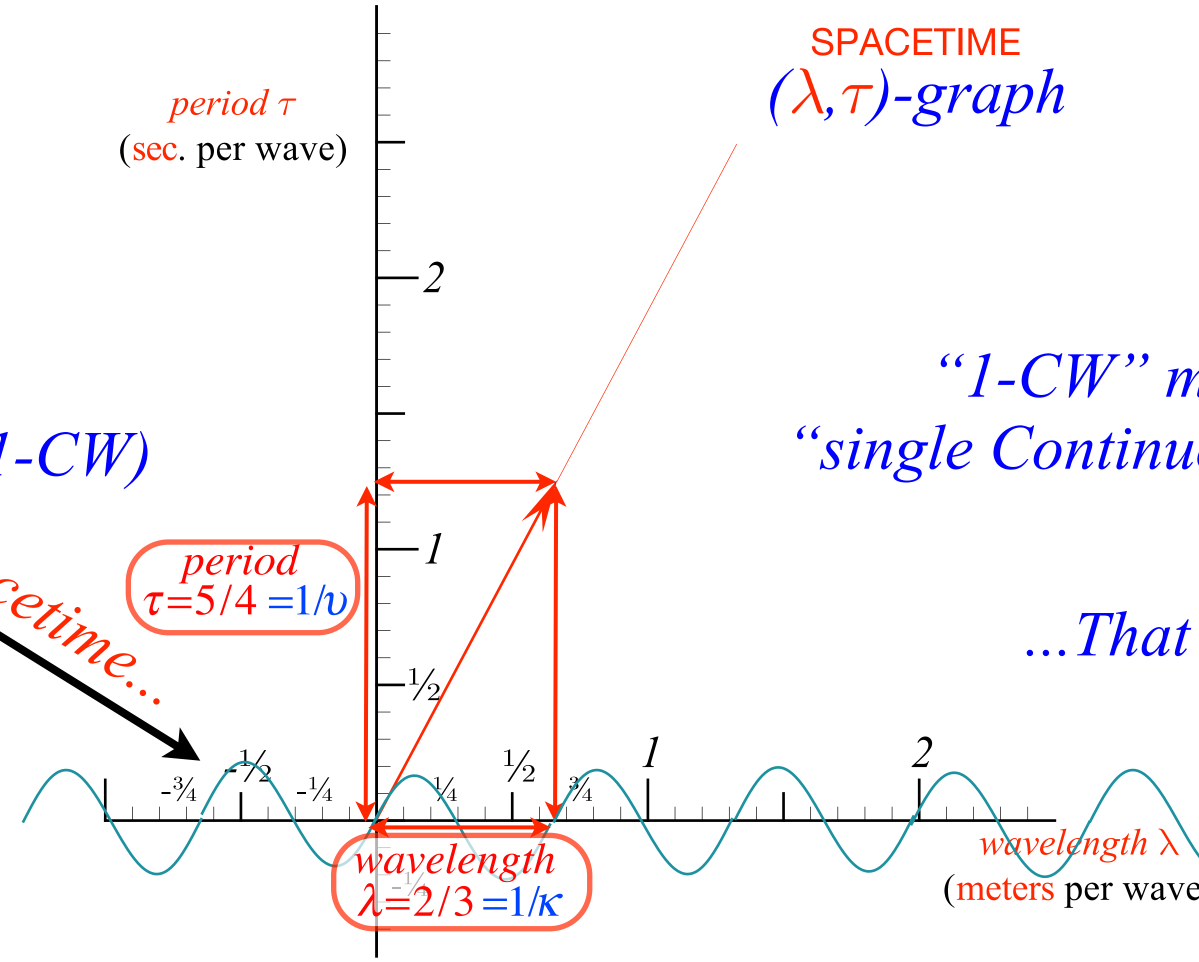
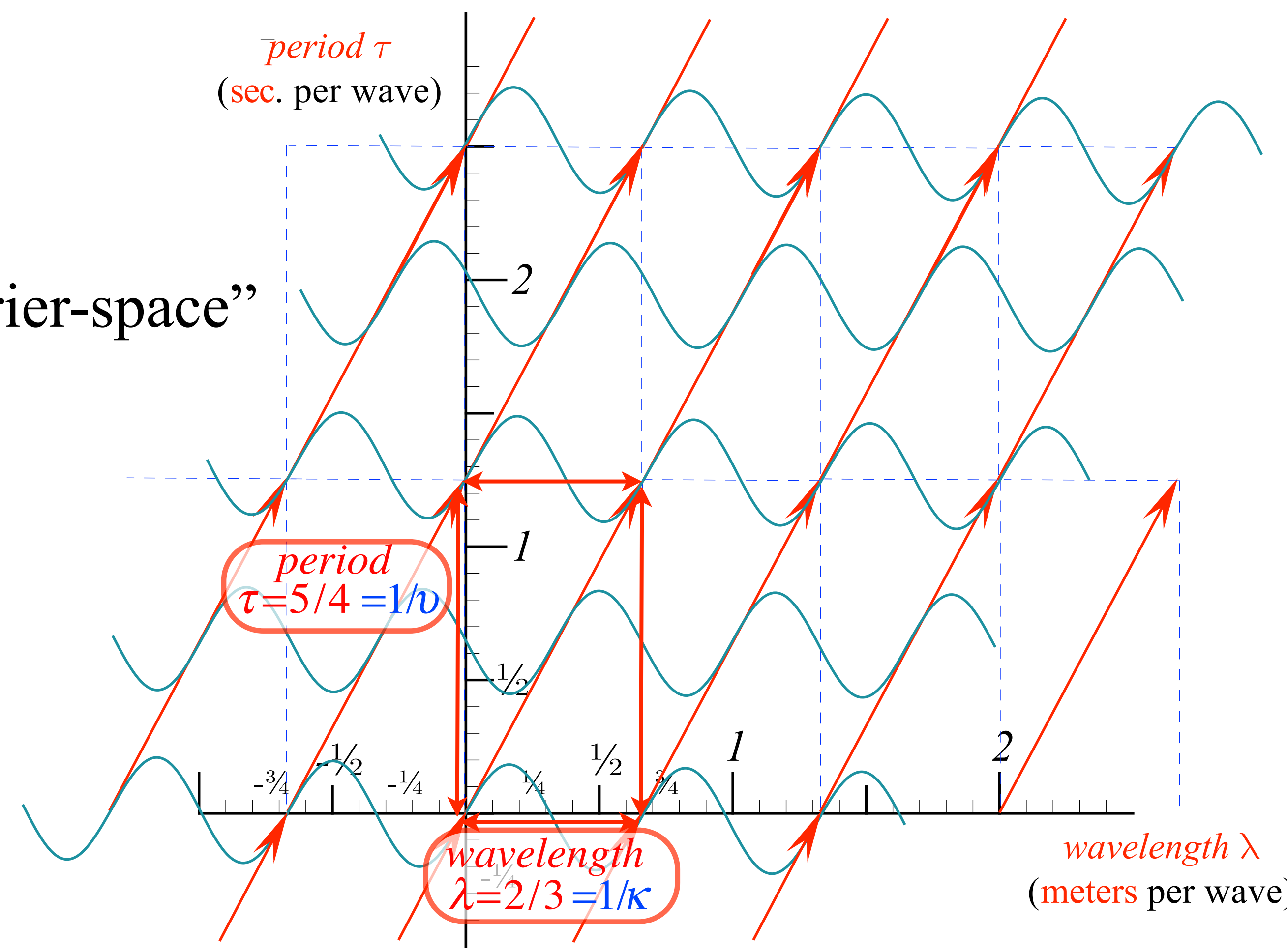




"1-CW" means
"single Continuous Wave"

...That "continues" everywhere..



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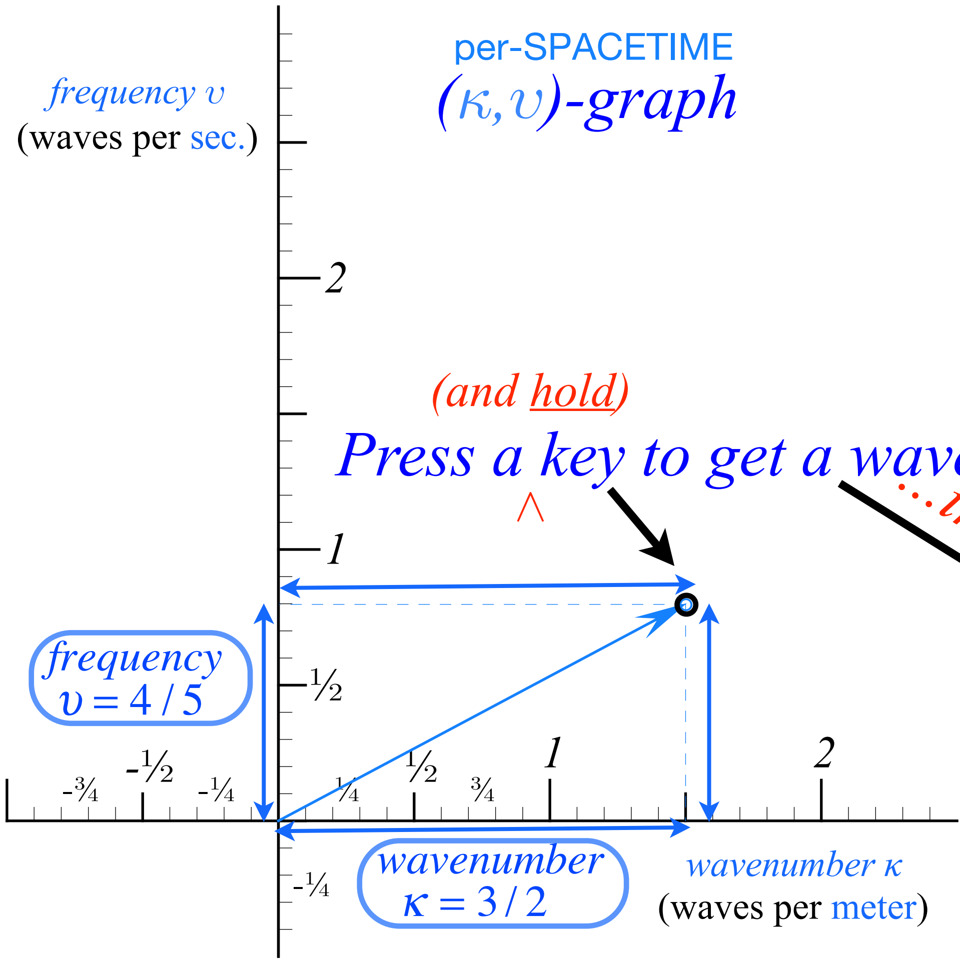
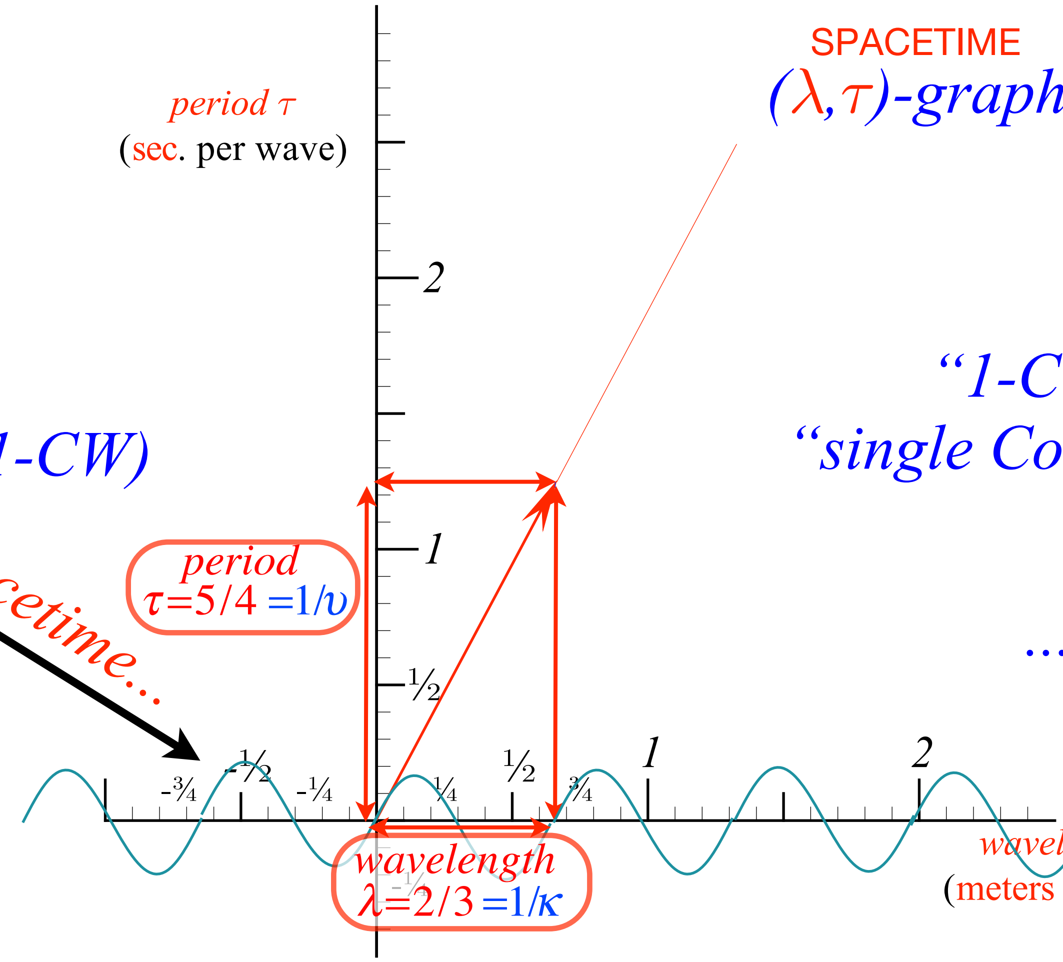


(and hold)

Press a key to get a wave (a 1-CW)

...in spacetime...



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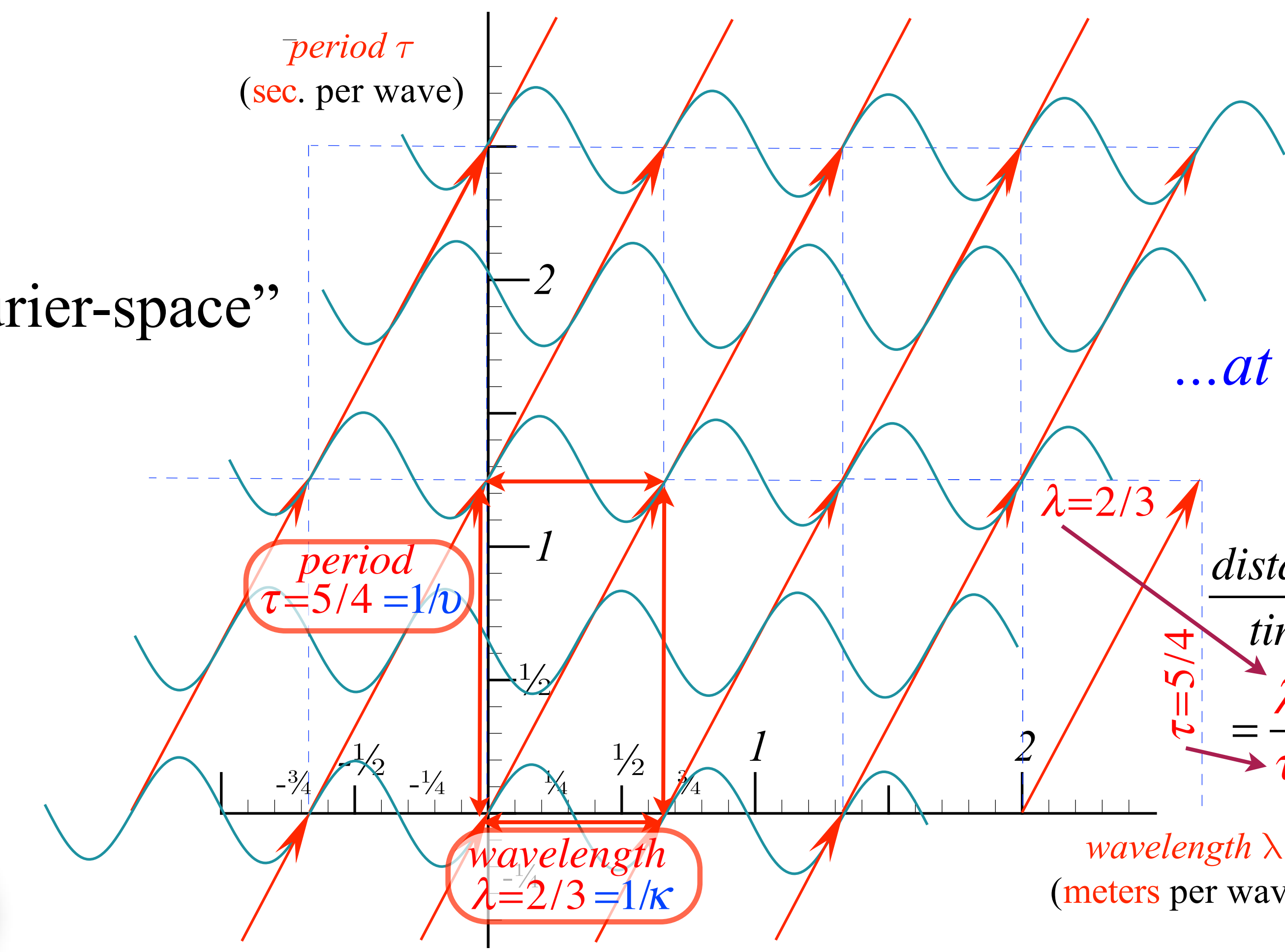
...That "continues" everywhere...

...for all time...



...at a speed of:

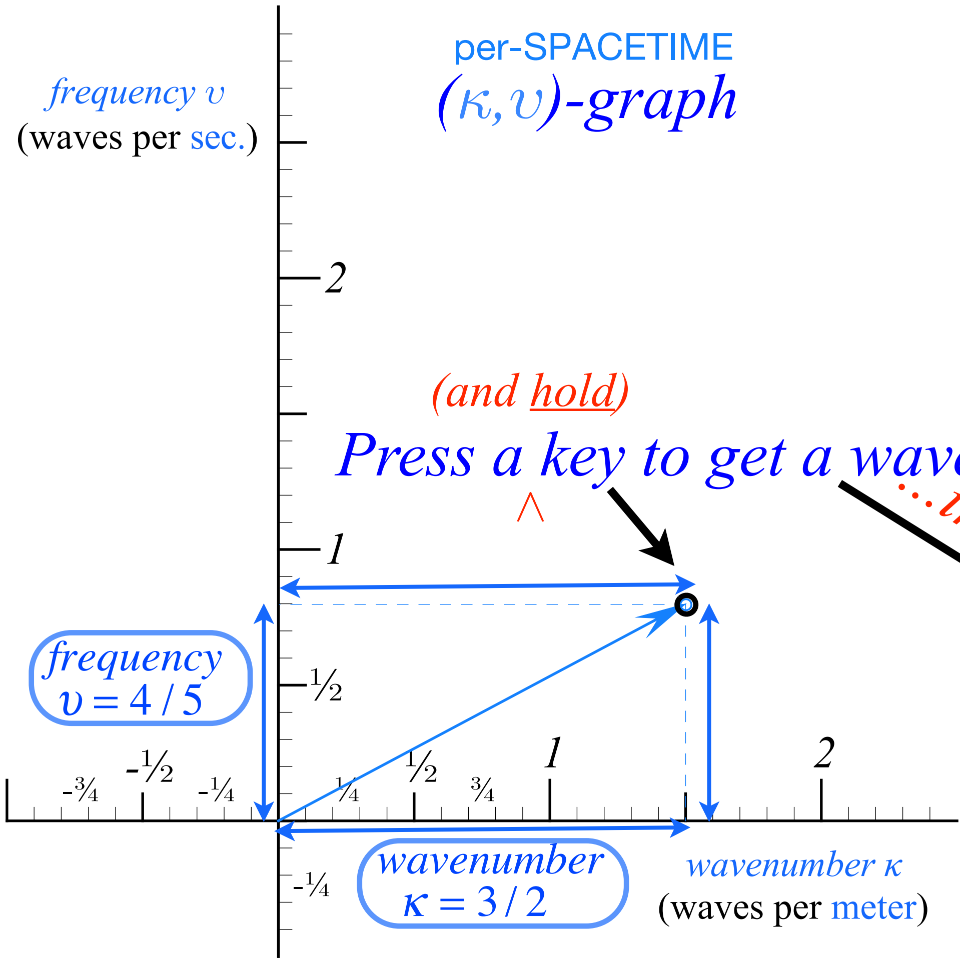
$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau}$$

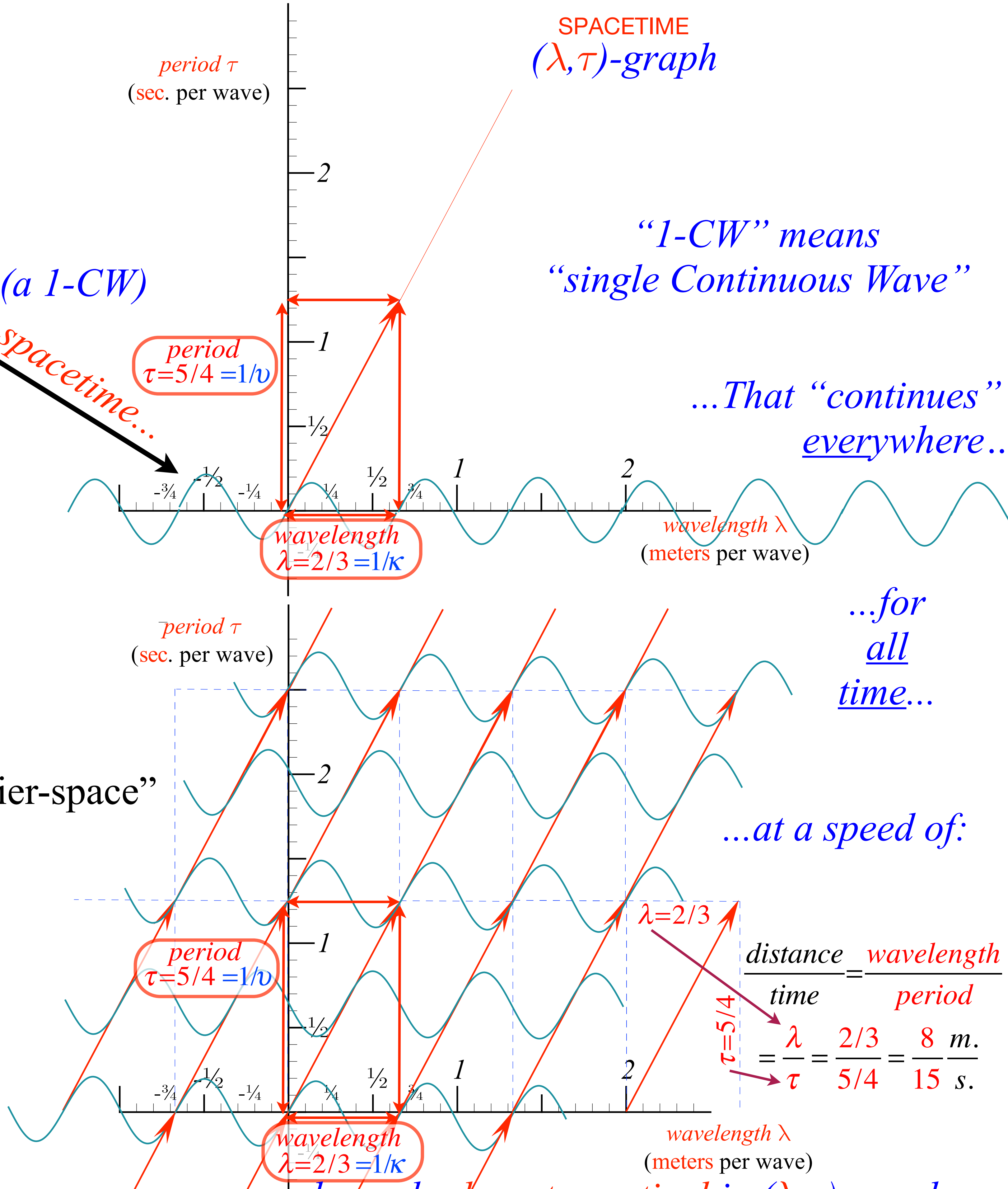
"Keyboard of the gods" is known as "Fourier-space"

Jean-Baptiste Joseph Fourier 1768-1830

Ways to quantify general waves

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs





"1-CW" means "single Continuous Wave"

...That "continues" everywhere..

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$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.}$$

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wave-speed equals slope-to-vertical in (λ,τ)-graph

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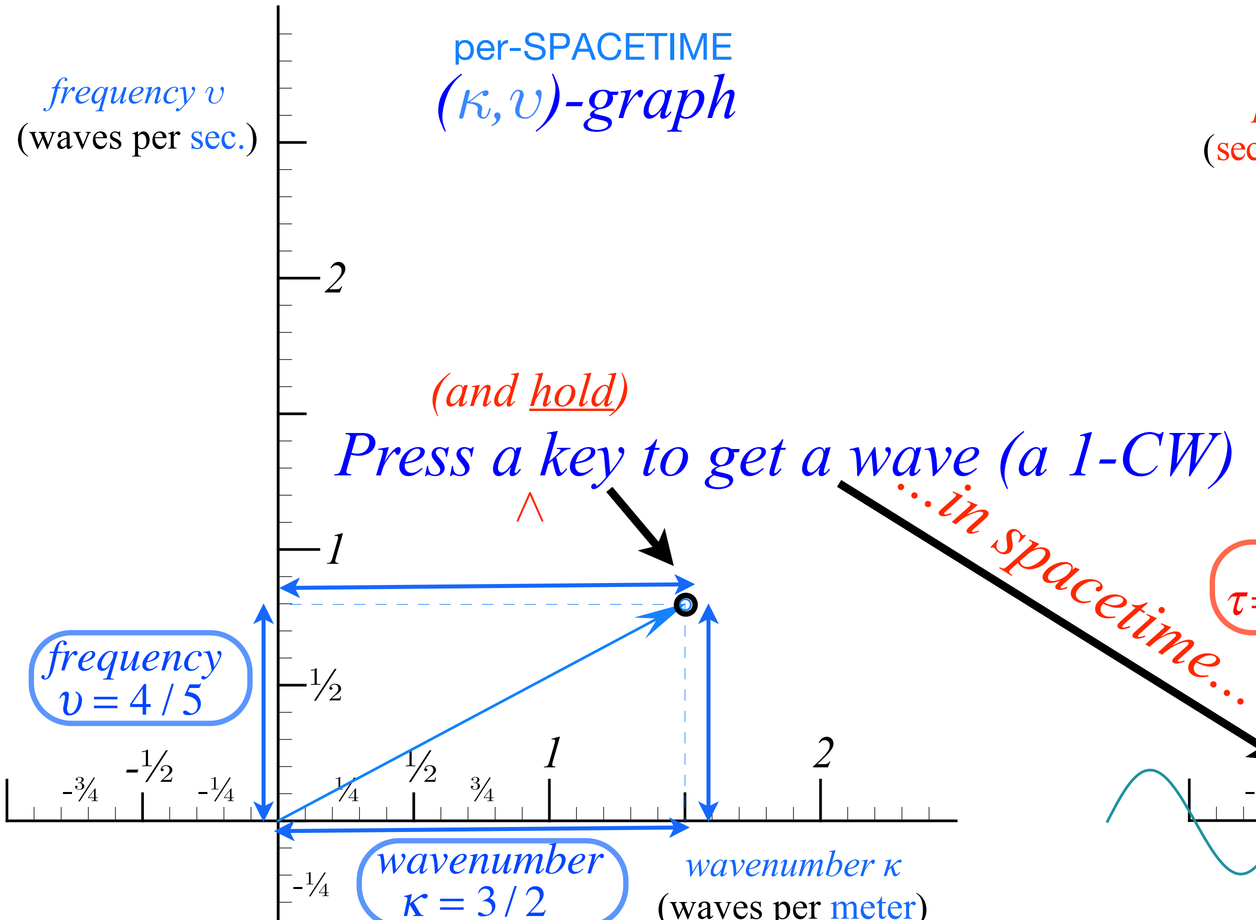
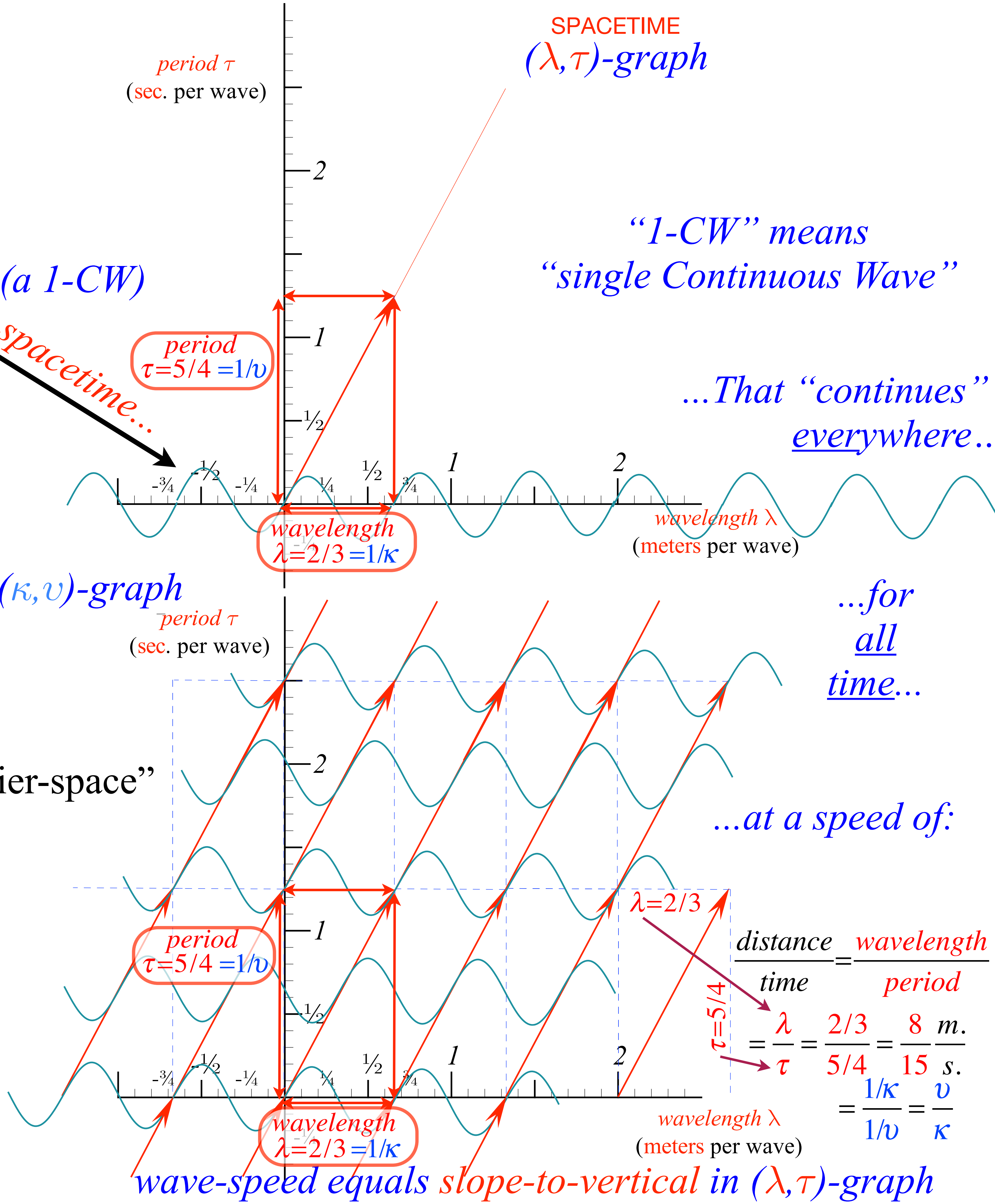


wave-speed equals slope-to-horizontal in (κ,υ)-graph



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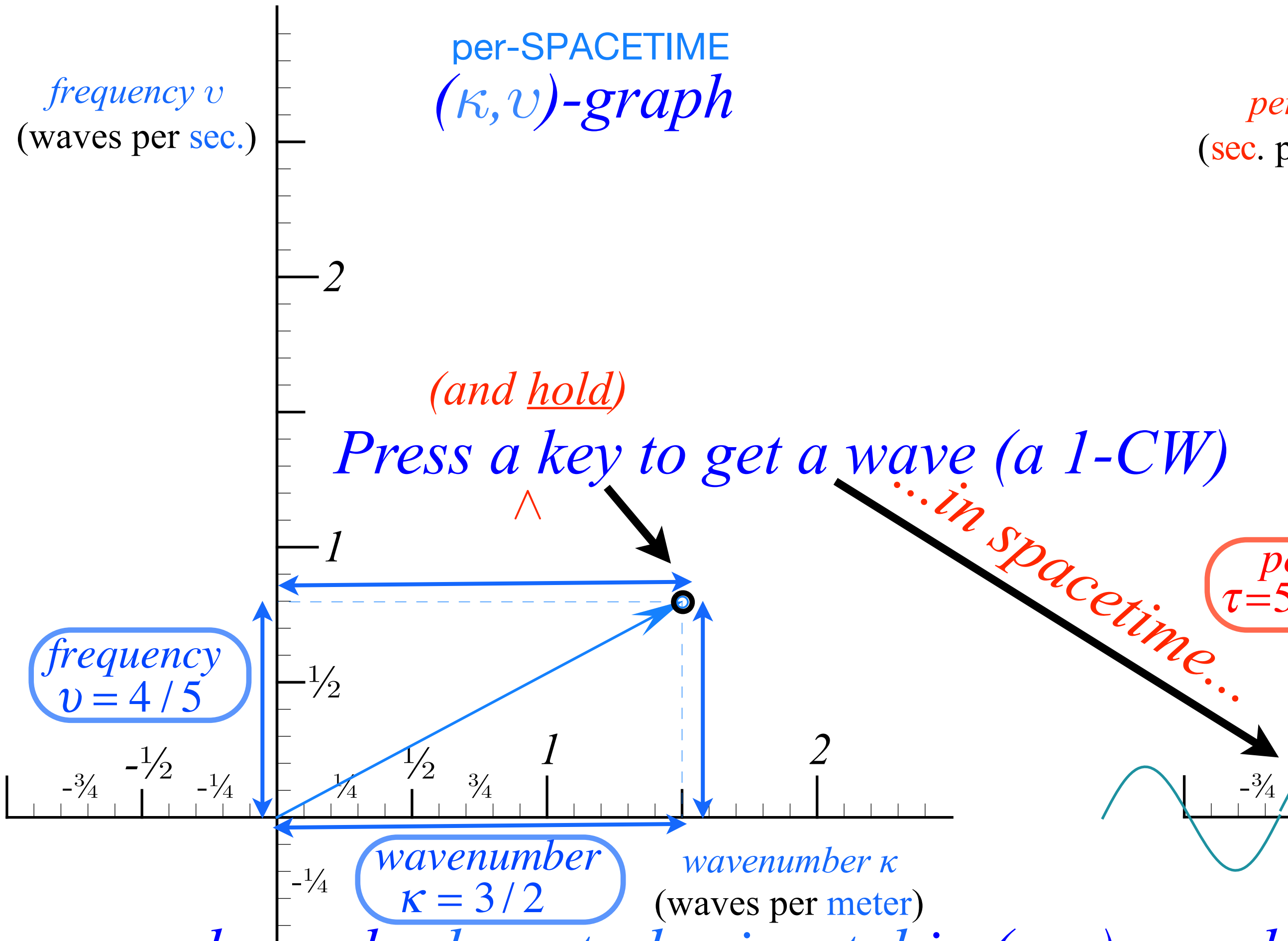

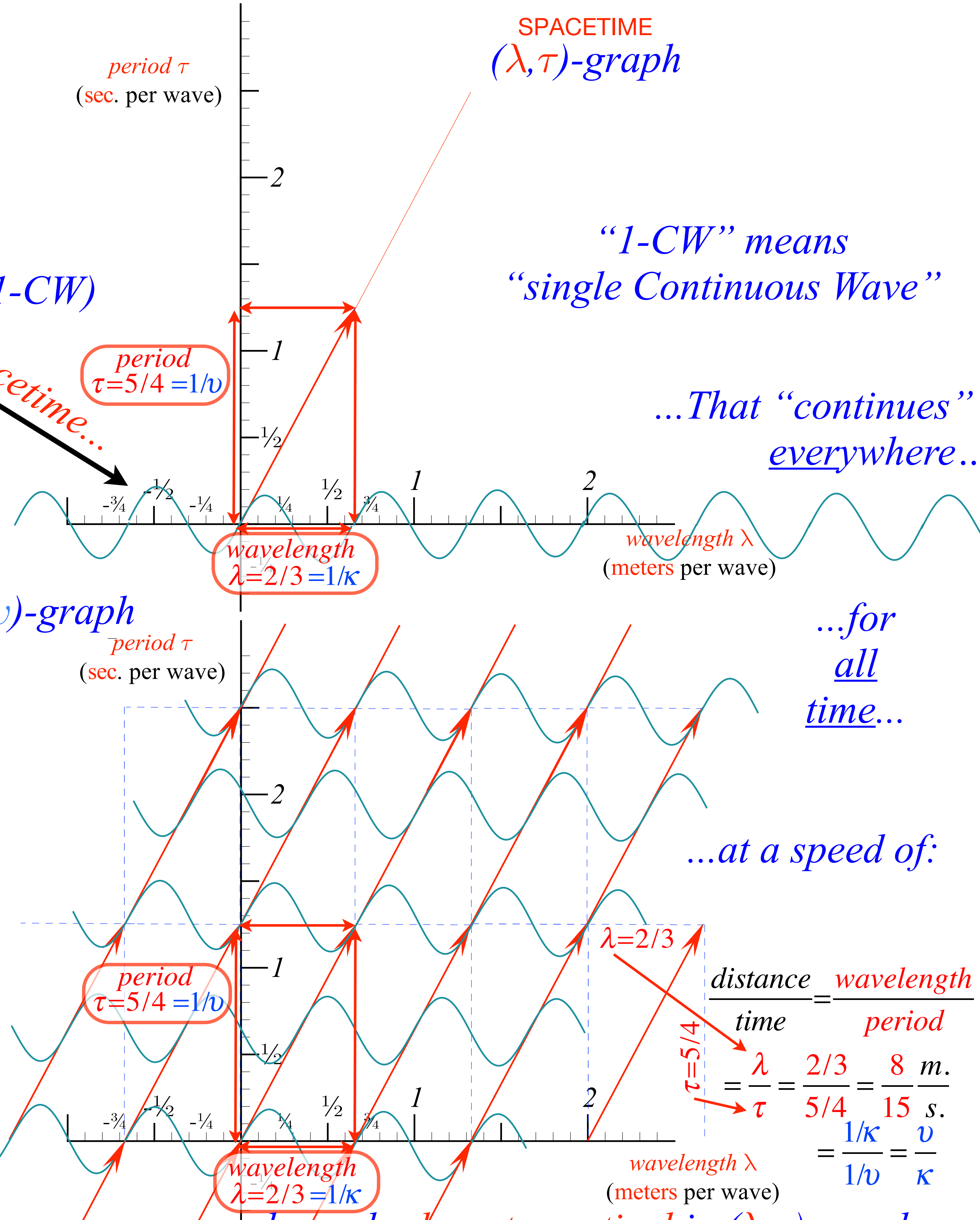

wave-speed equals slope-to-horizontal in (κ,υ)-graph

wave-velocity formula

$$\frac{distance}{time}=\frac{wavelength}{period}=\frac{frequency}{wavenumber}$$

$$V_{wave}=\frac{\lambda}{\tau}=\frac{1/\kappa}{1/\upsilon}=\frac{\upsilon}{\kappa}=\frac{1/\tau}{1/\lambda}$$

$$=\frac{2/3}{5/4}=\frac{4/5}{3/2}=\frac{8}{15}\frac{m.}{s.}$$

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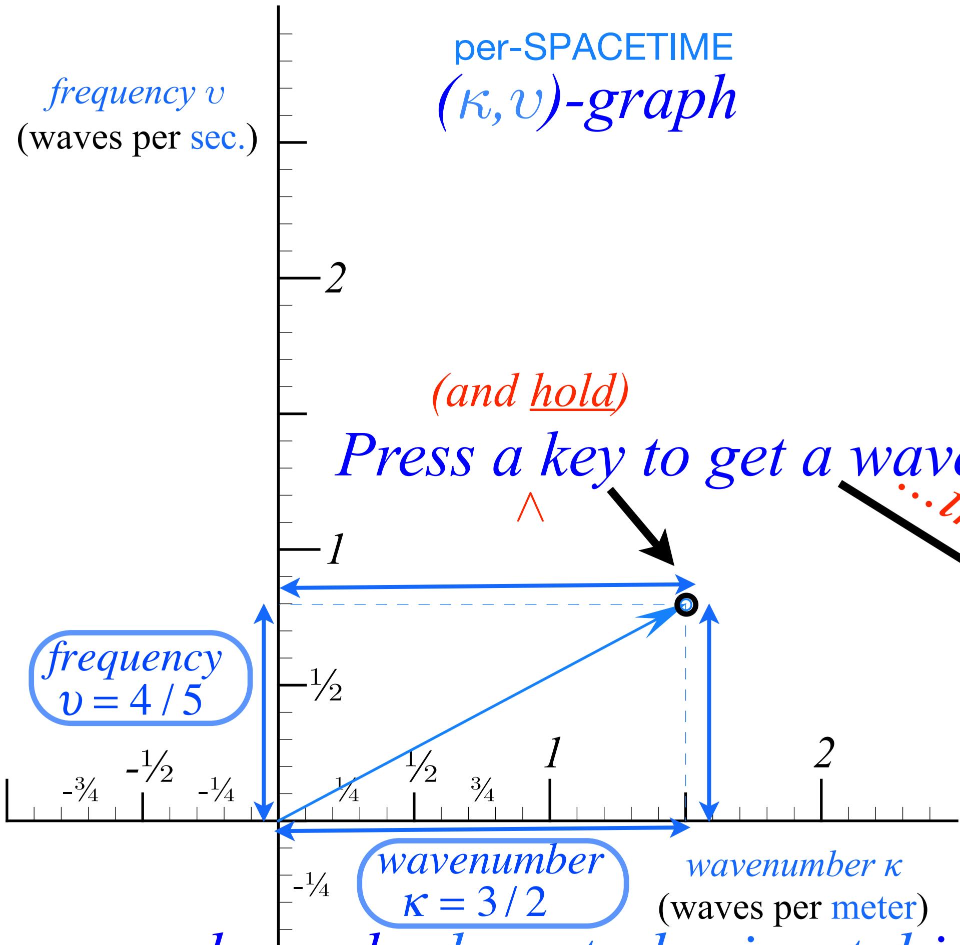
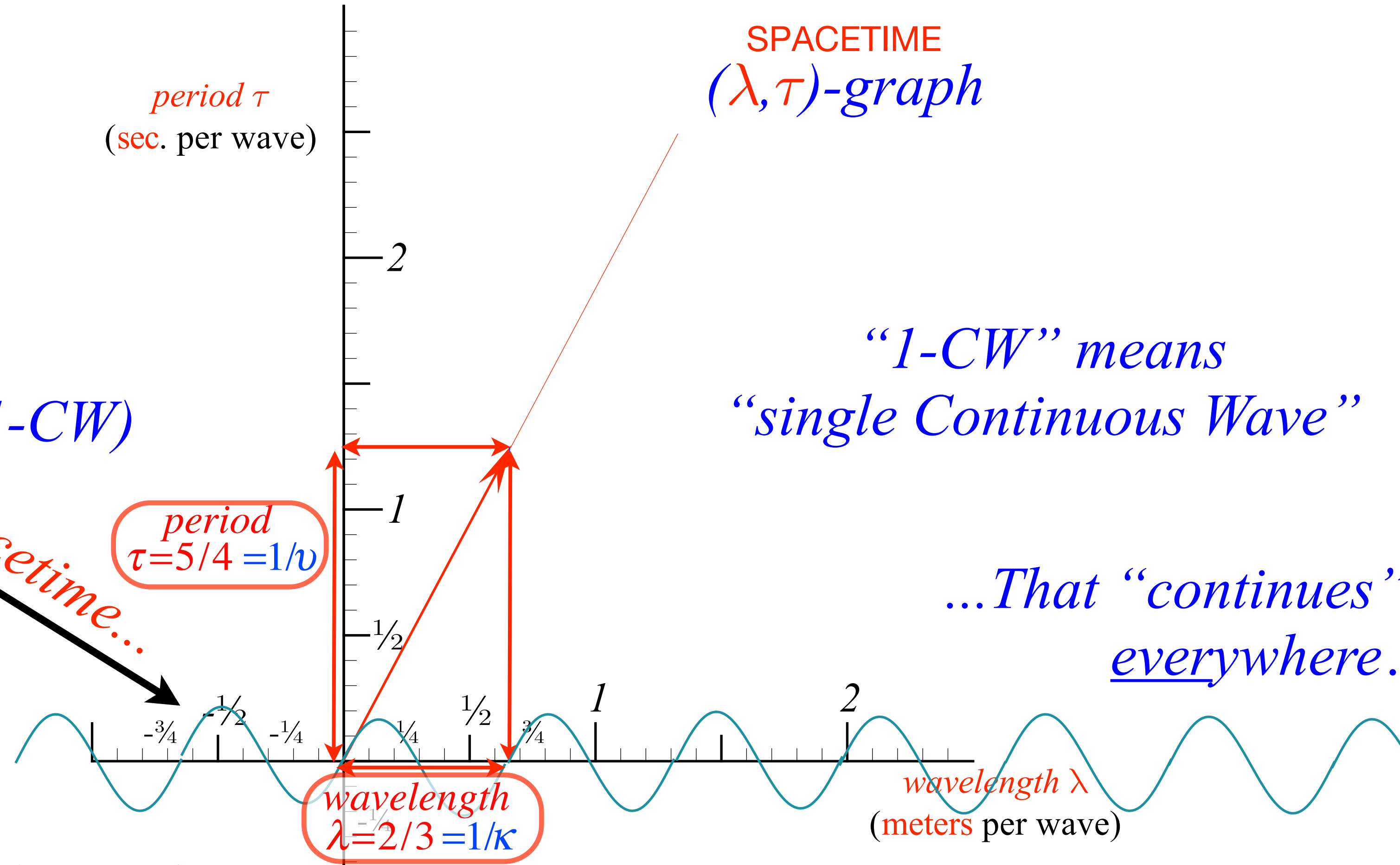


...in spacetime...



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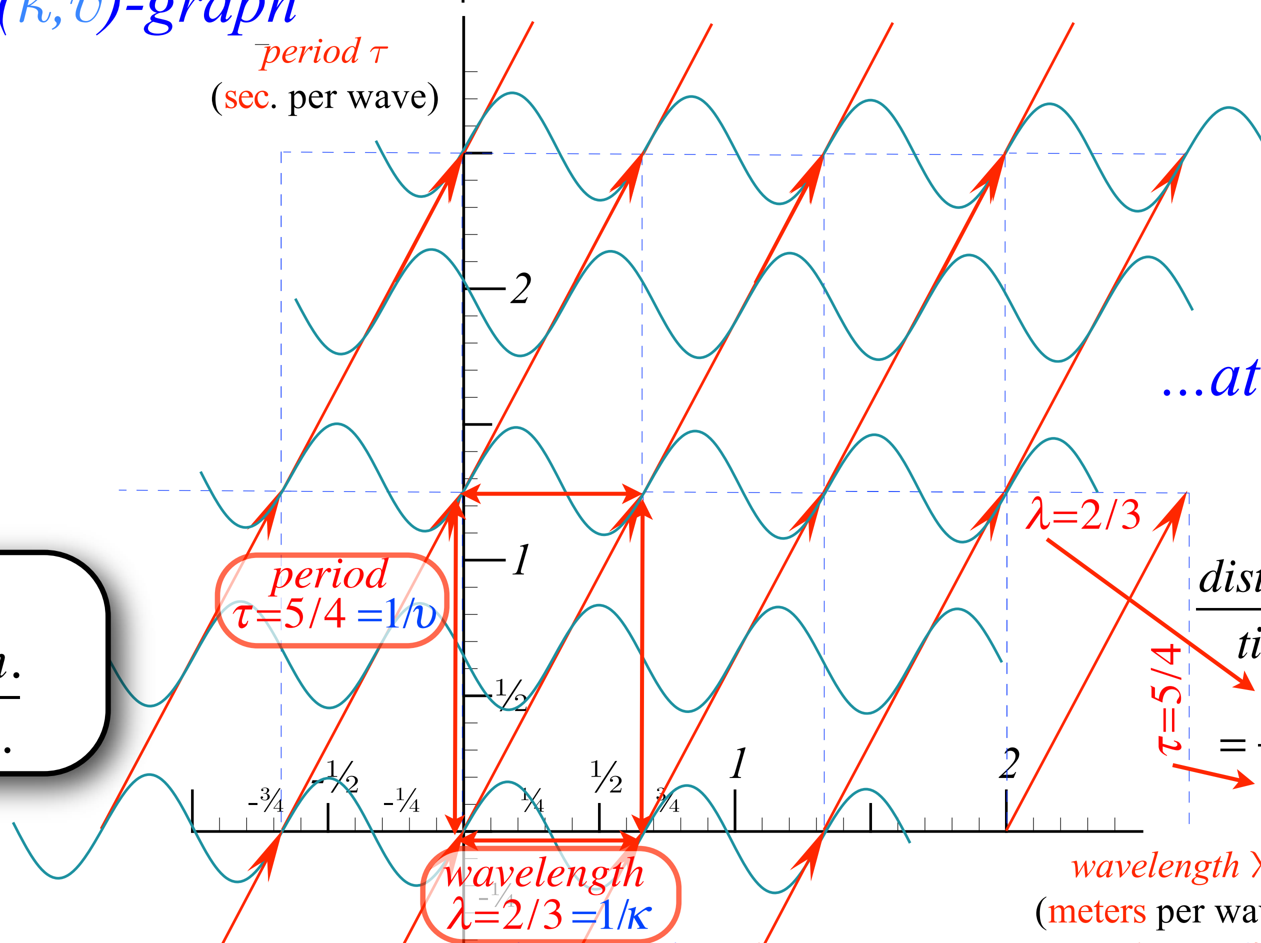
Light wave-velocity c (our main topic)

$$V_{light} = c = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda} = 299{,}792{,}458\frac{m.}{s.}$$

(Next up:) Ways to quantify light waves

...for all time...



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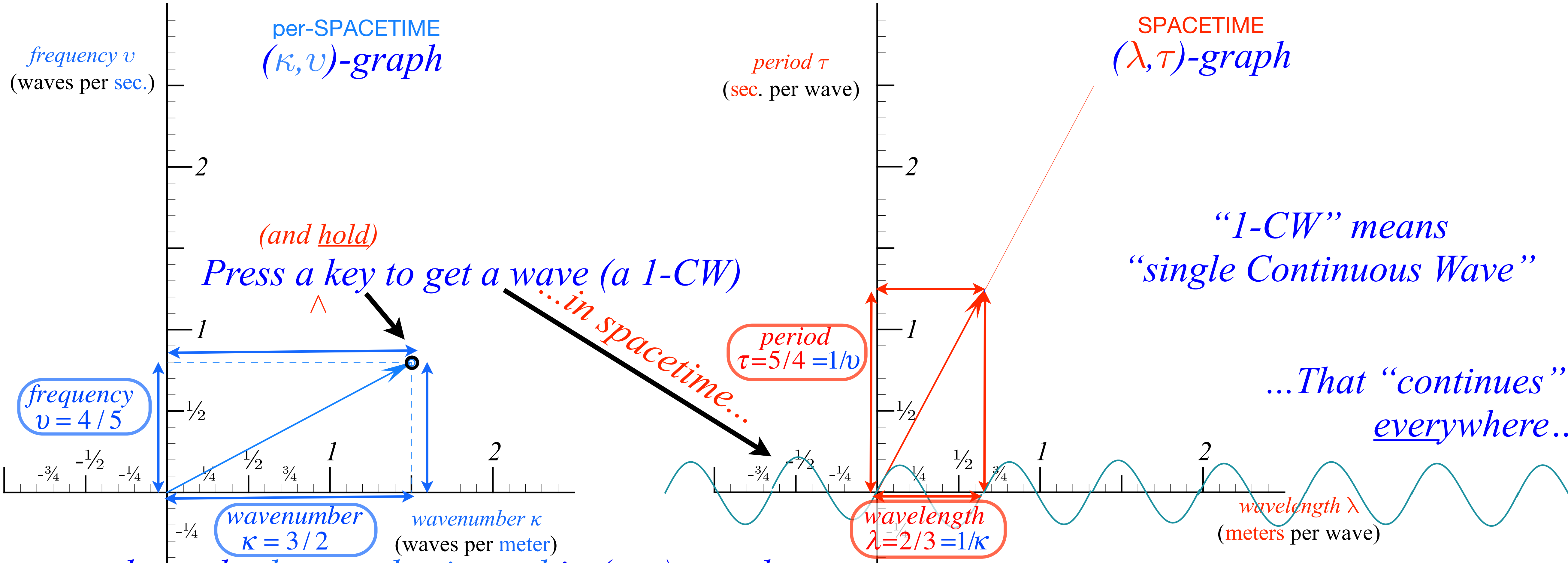
The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



wave-speed equals slope-to-horizontal in (κ,υ)*-graph*

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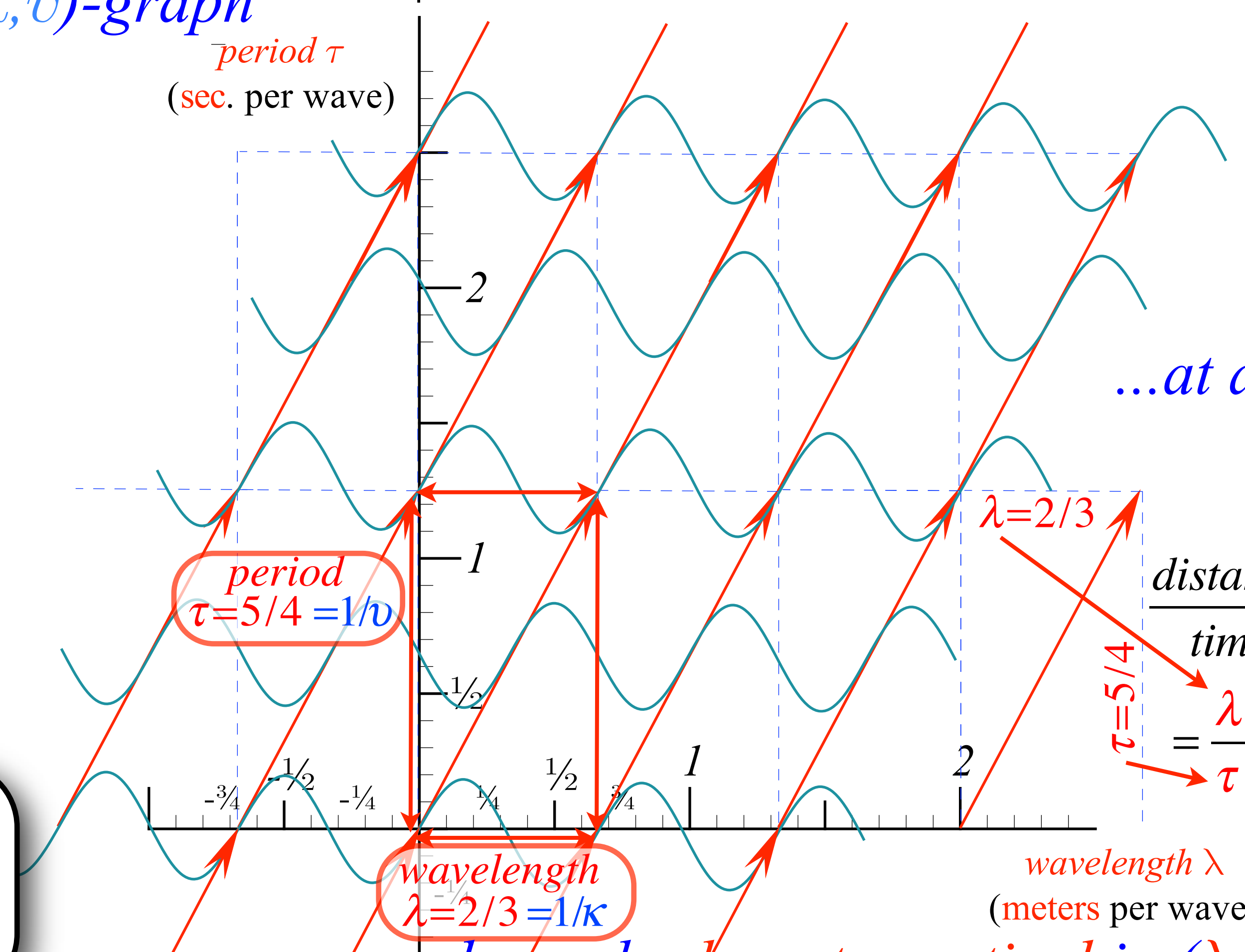
Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c}=\frac{\lambda}{c\tau}=\frac{1/\kappa}{c/\upsilon}=\frac{\upsilon}{c\kappa}=\frac{1/\tau}{c/\lambda}=1$$



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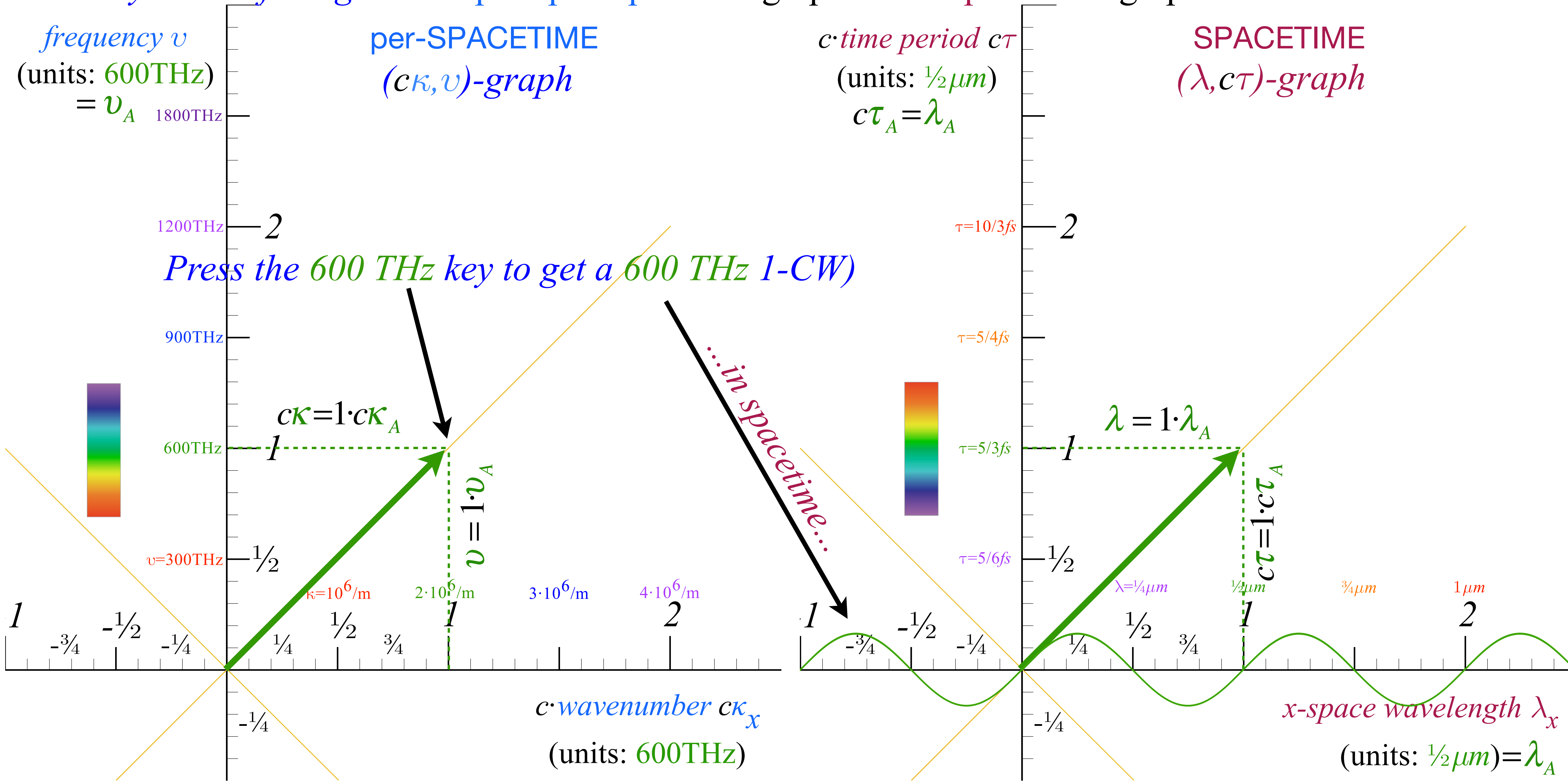
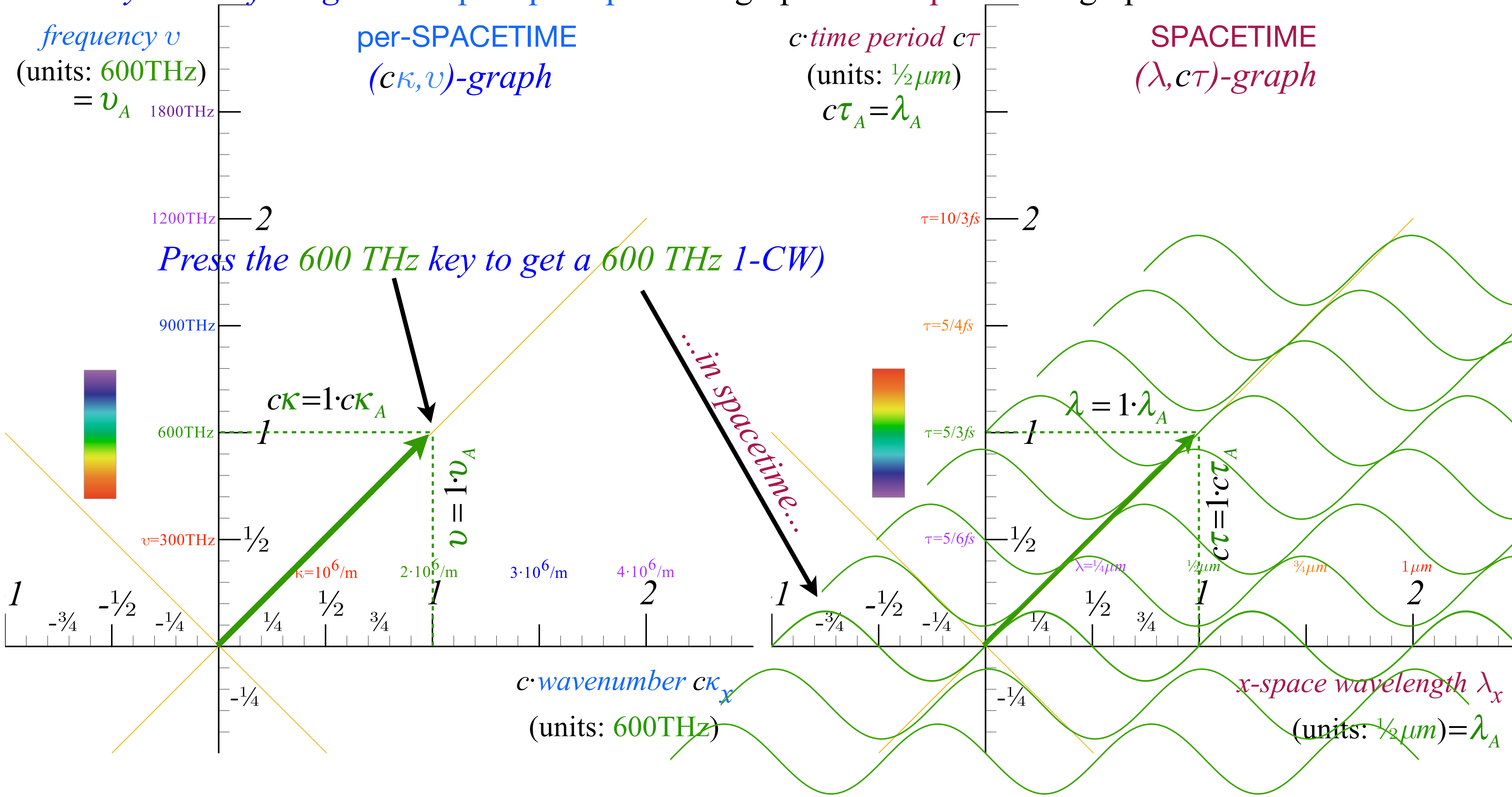


*Ways to quantify **light** waves (600 THz example)*

The *"Keyboard of the gods"* or per-space-per-time graphs *versus* space-time graphs



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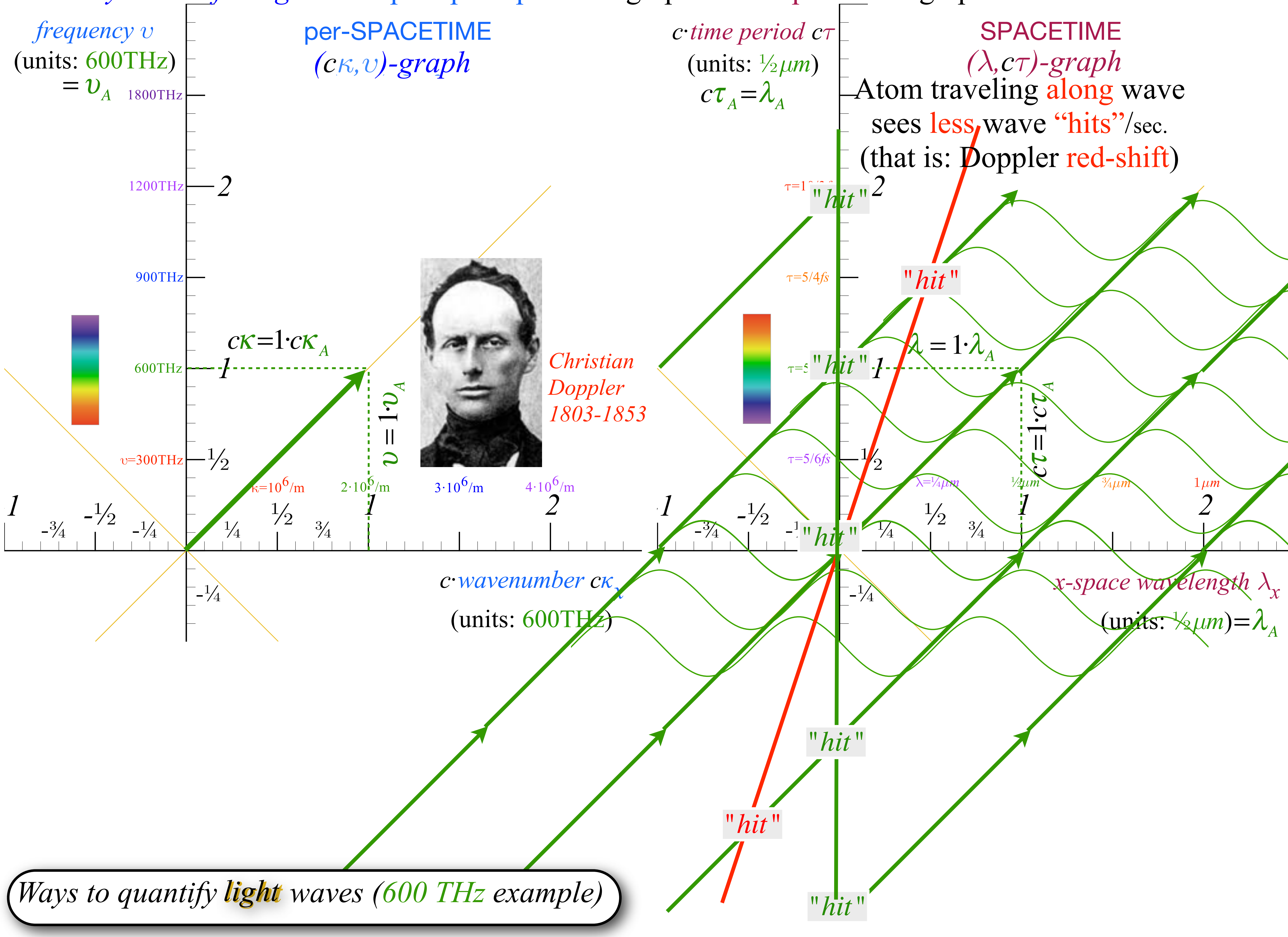

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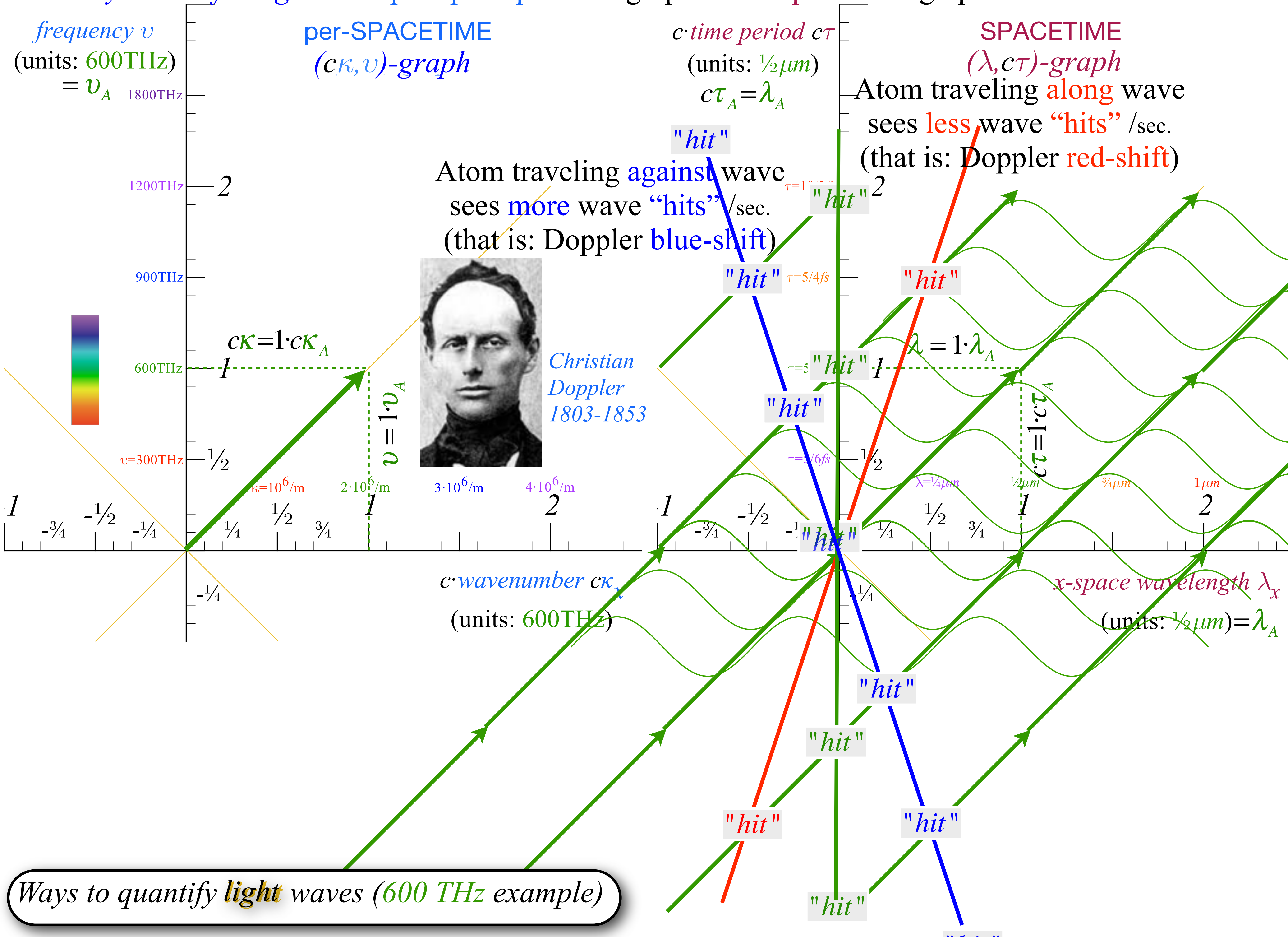

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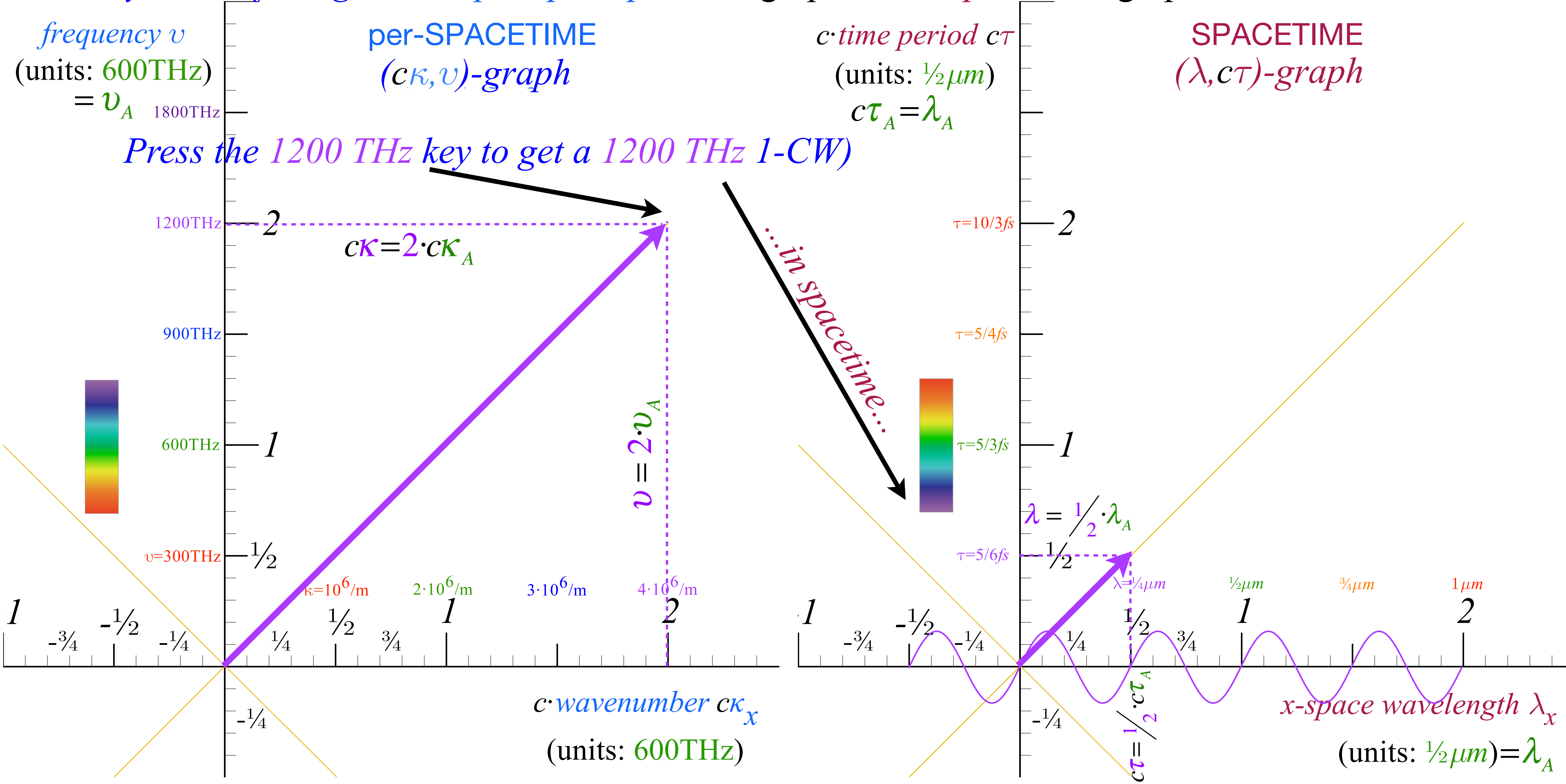
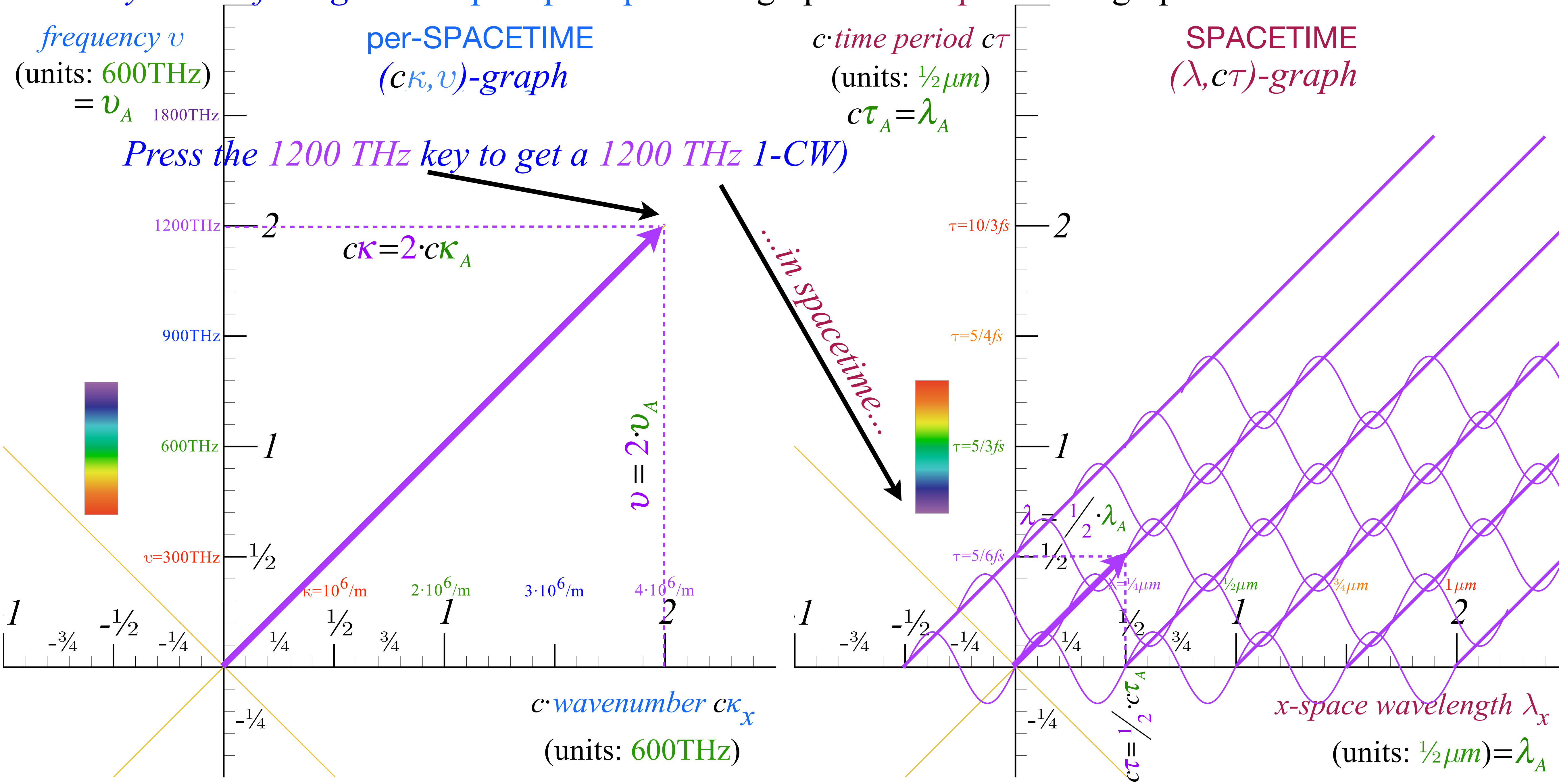


Ways to quantify light waves (1200 THz example)

The "Keyboard of the gods" or per-space-per-time graphs *versus* space-time graphs



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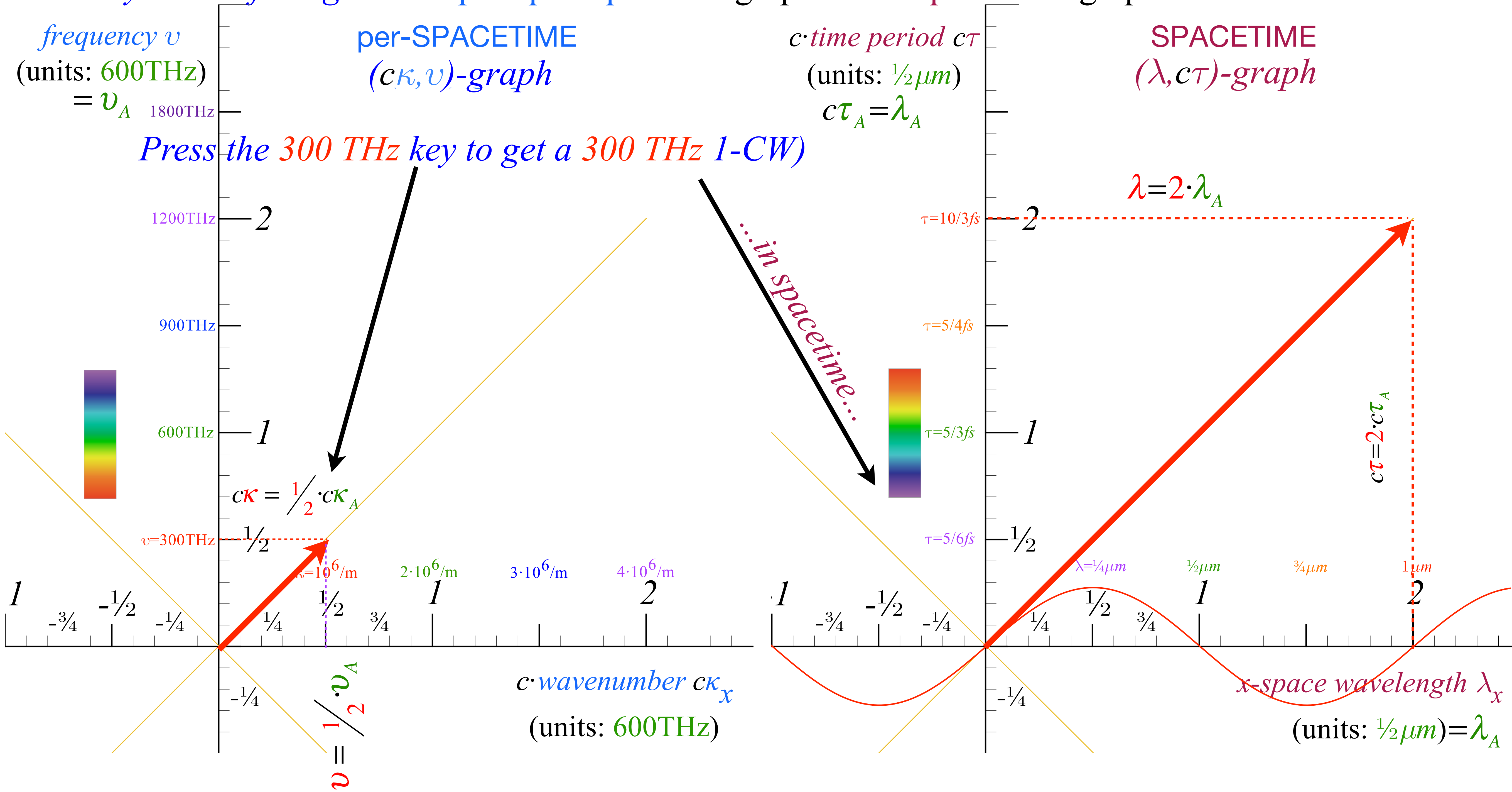

Ways to quantify light waves (300 THz example)

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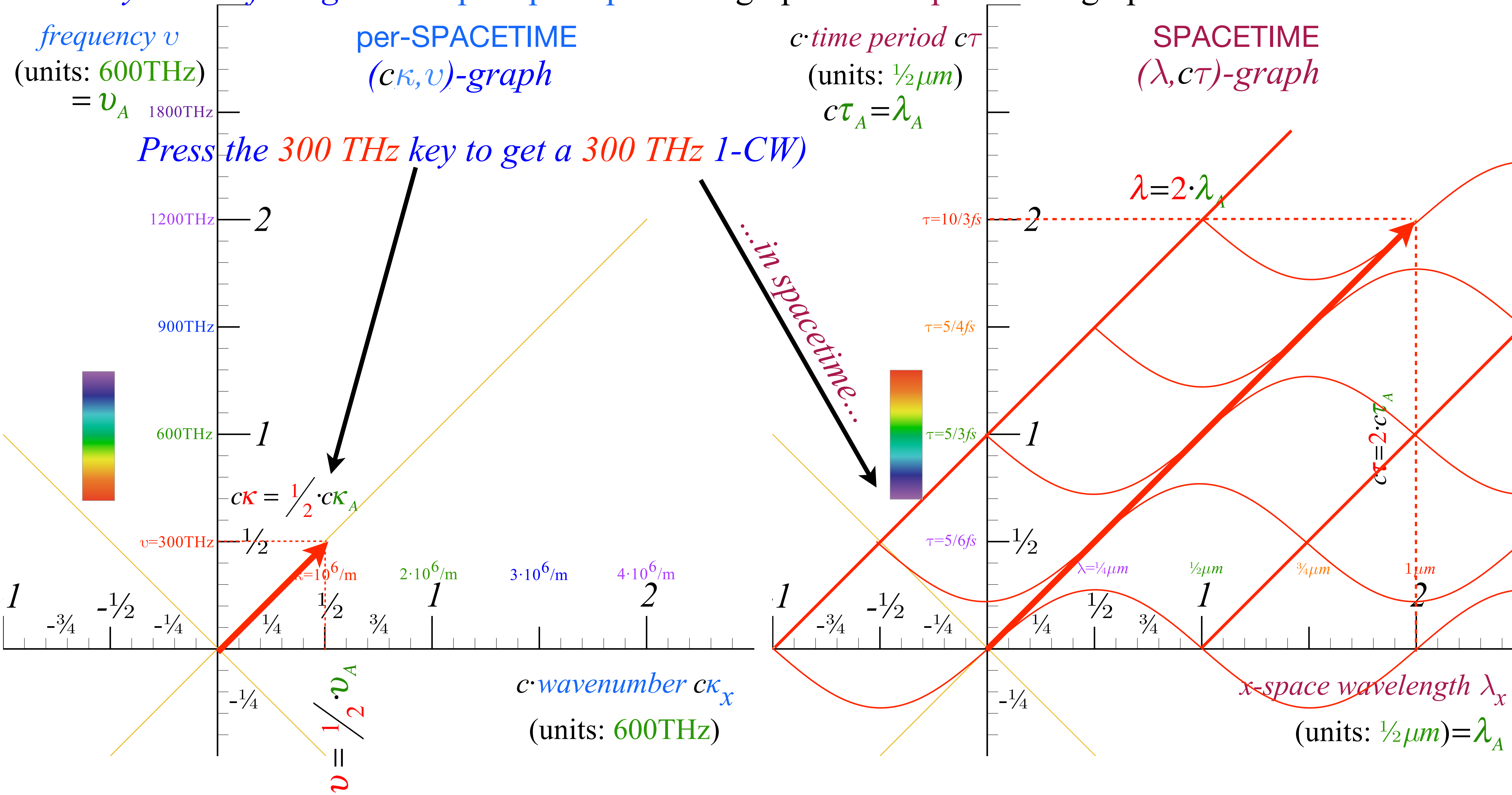

Ways to quantify light waves (300 THz example)

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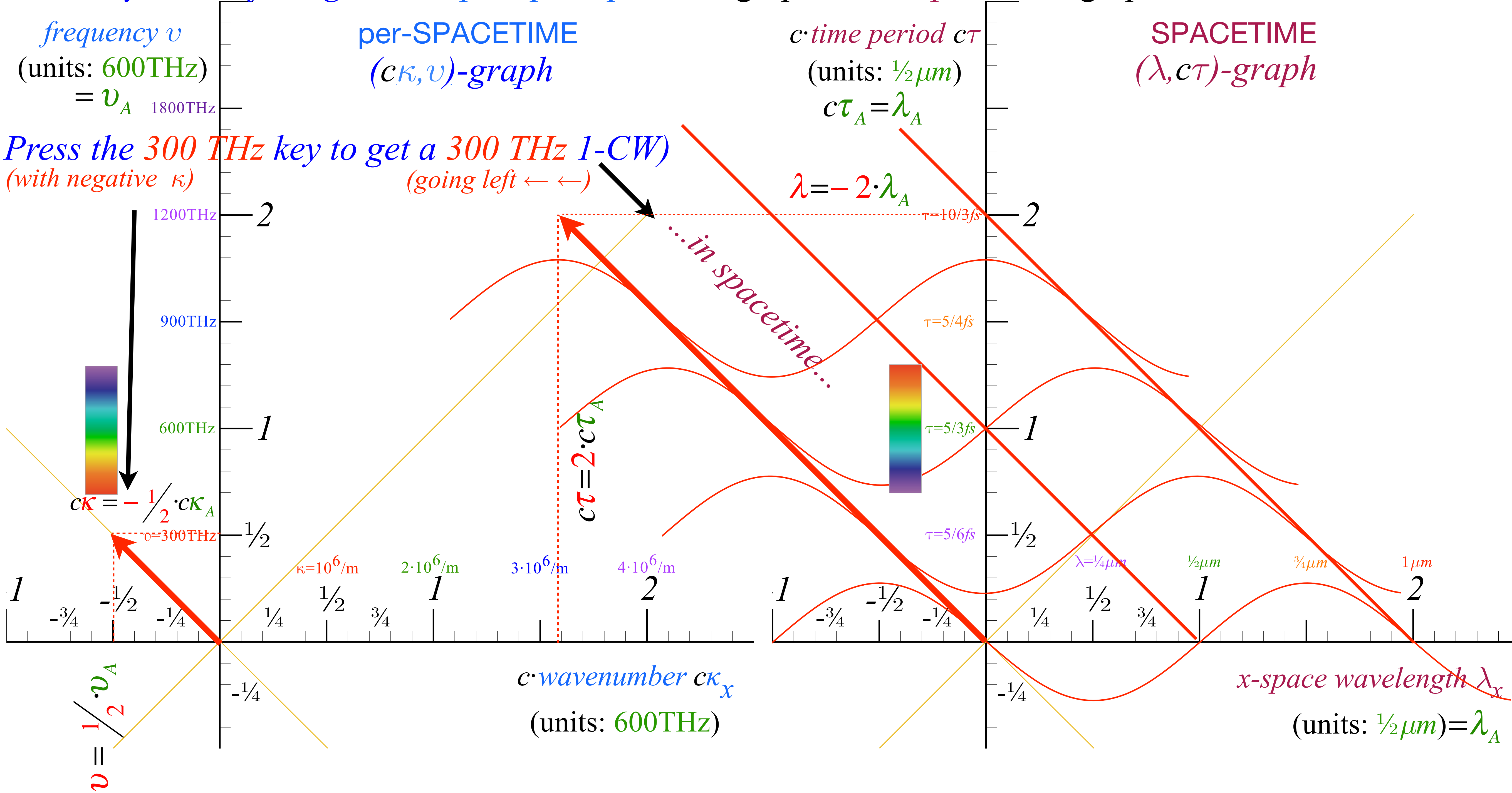
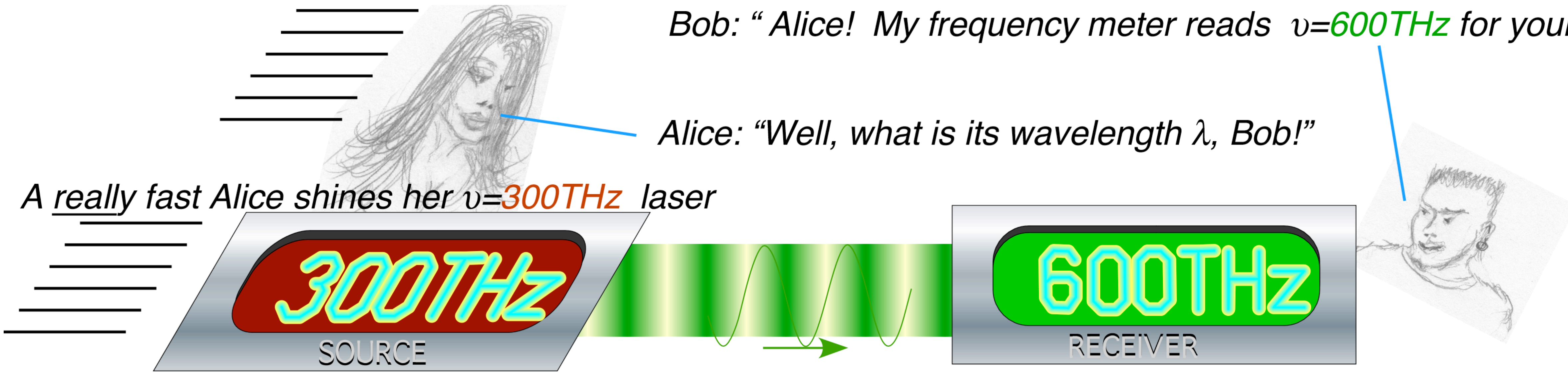


Ways to quantify light waves (300 THz example)

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)



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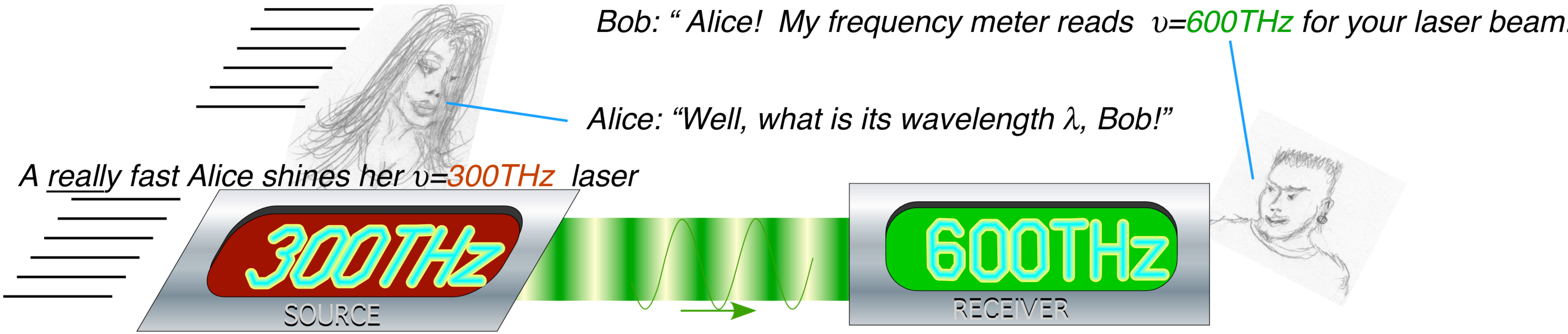

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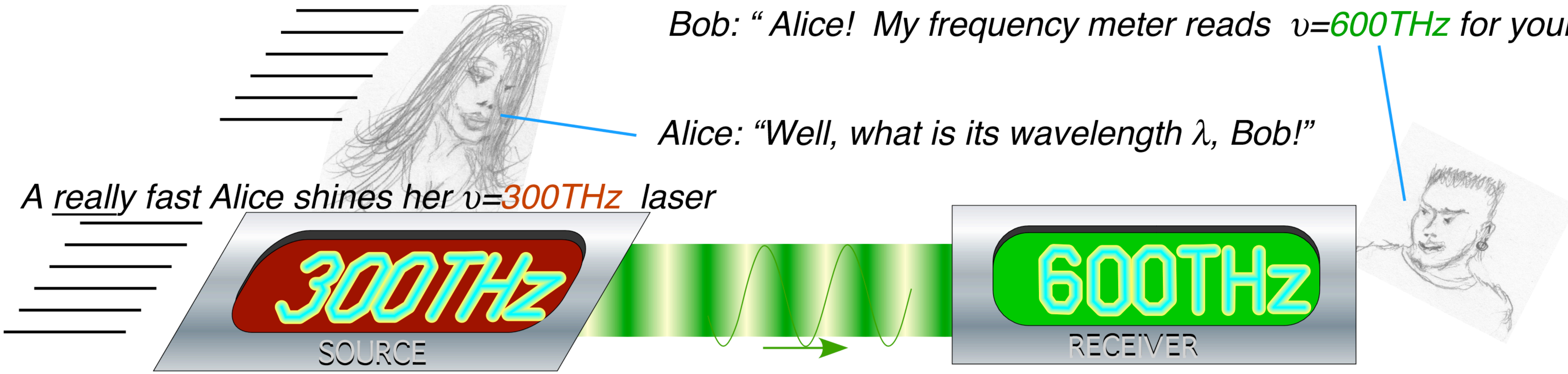

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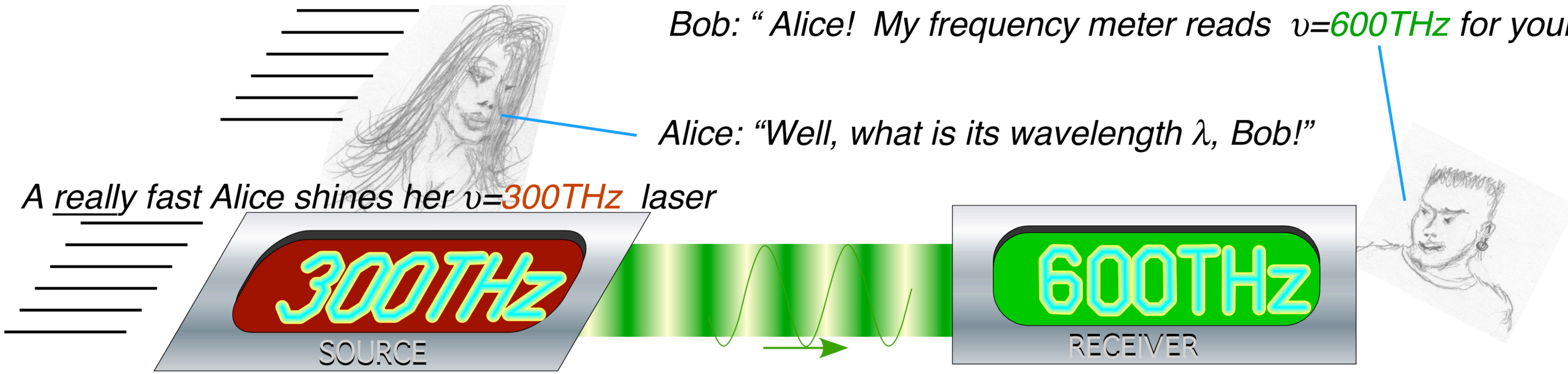

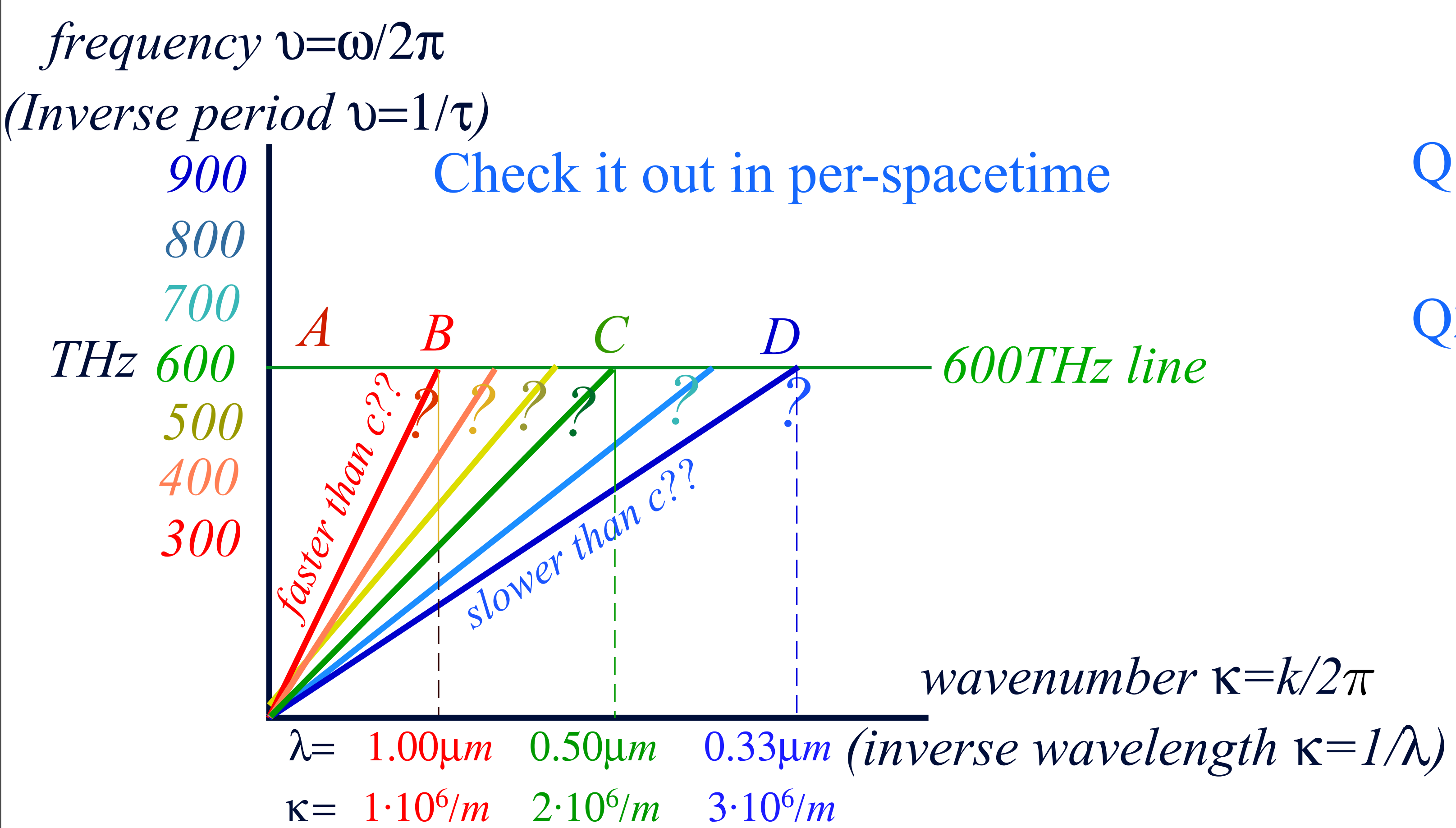

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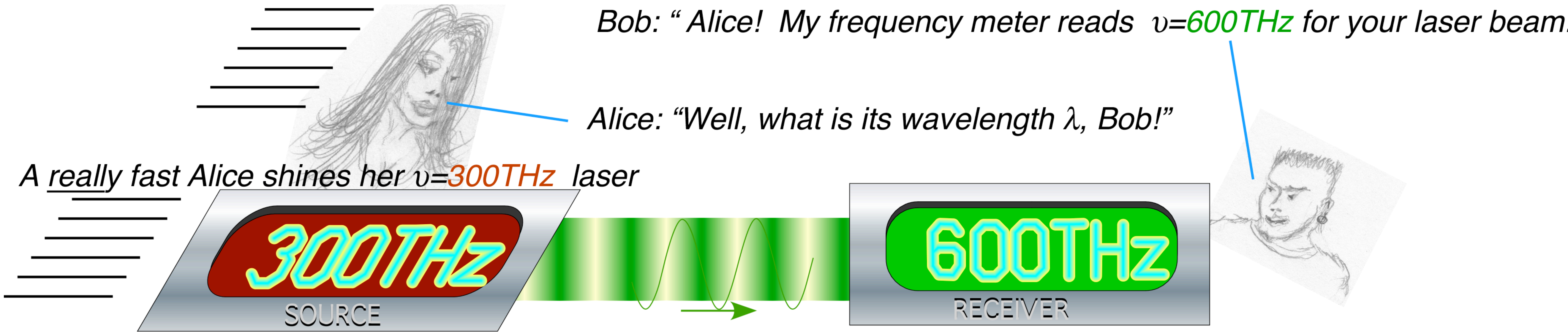

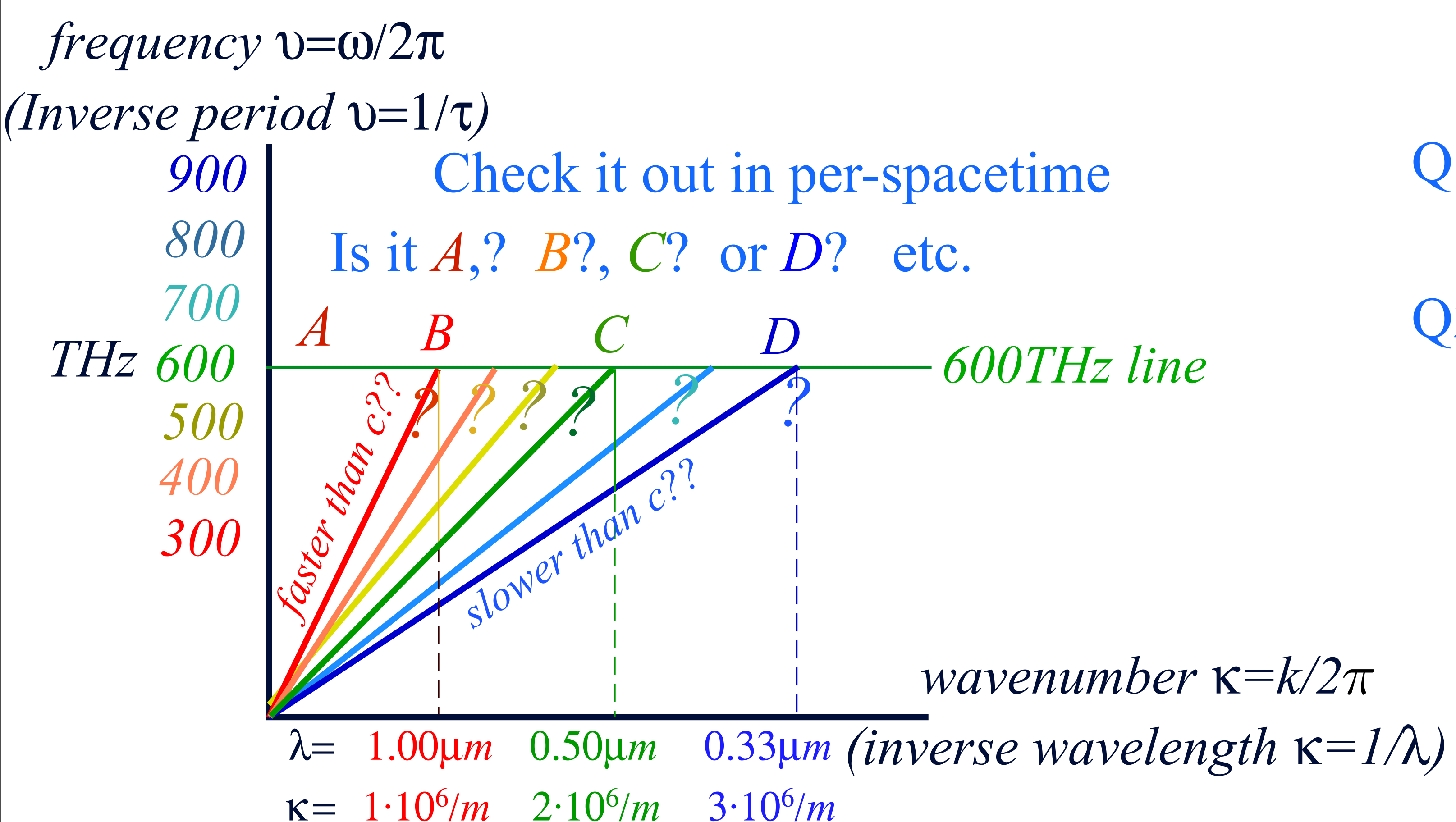

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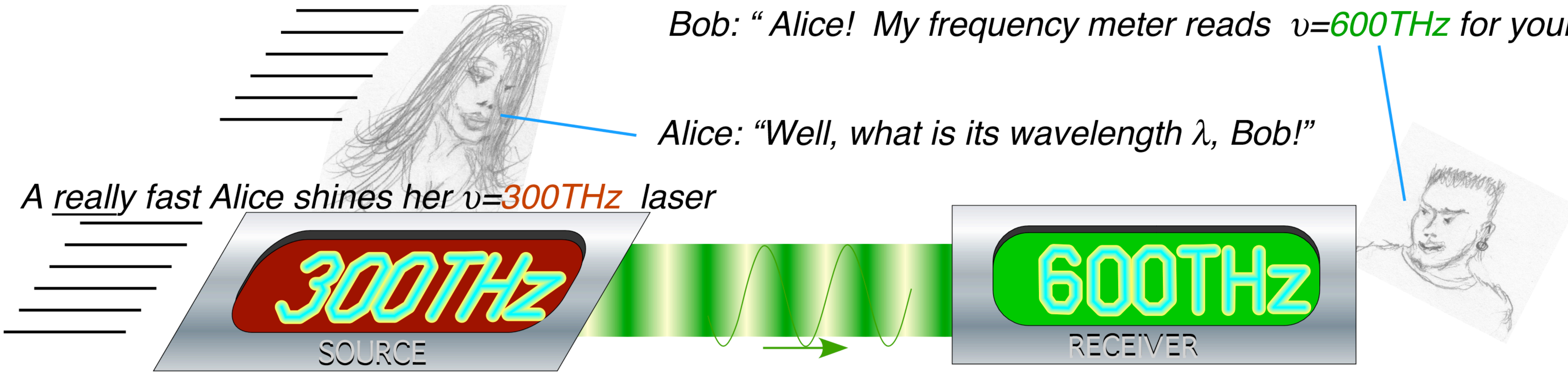

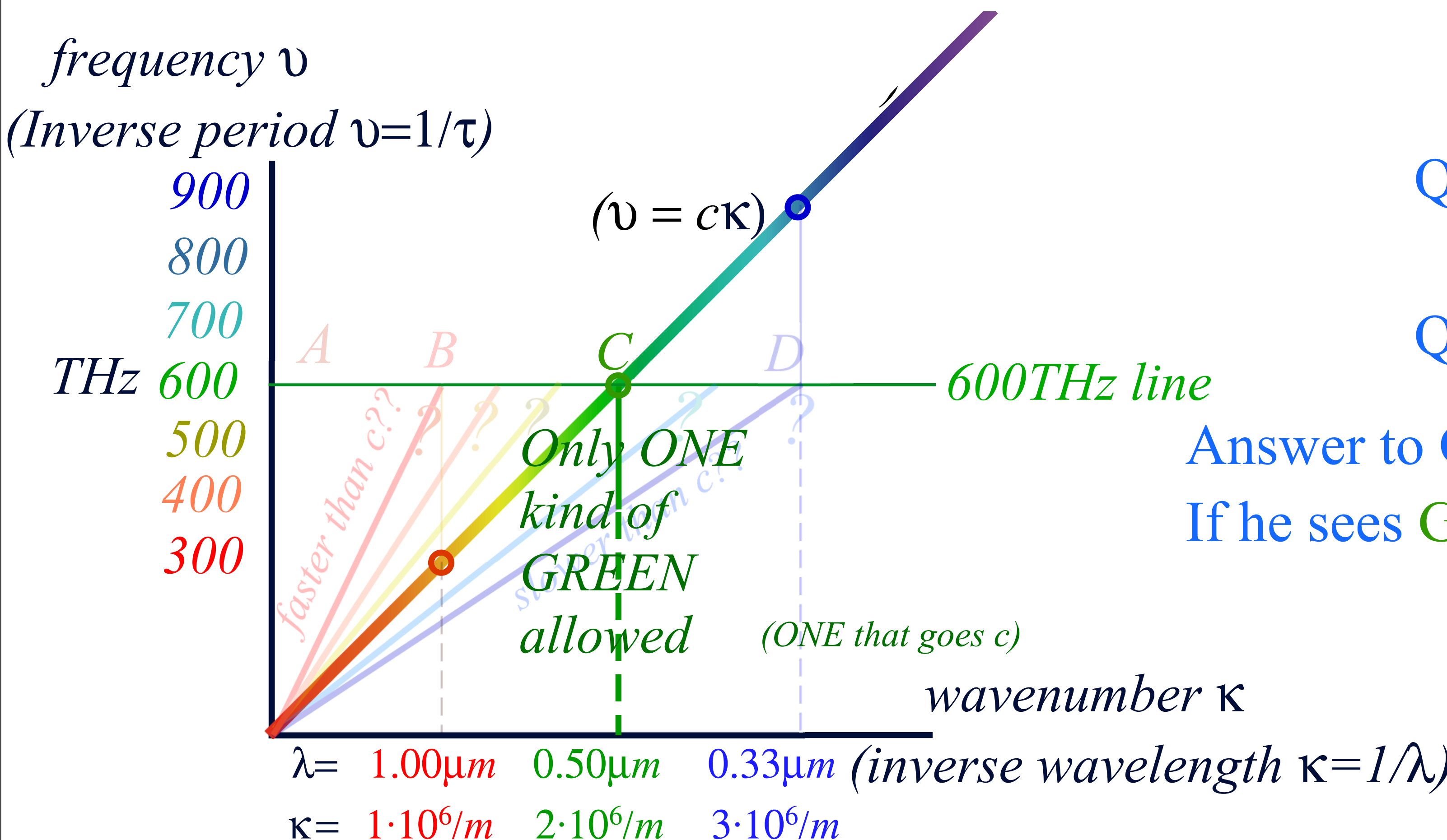

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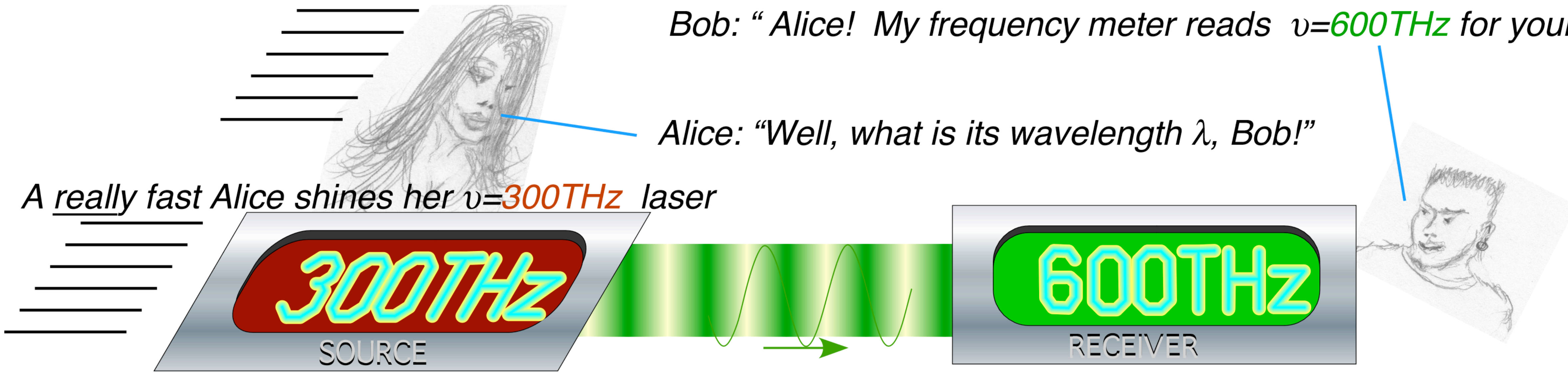

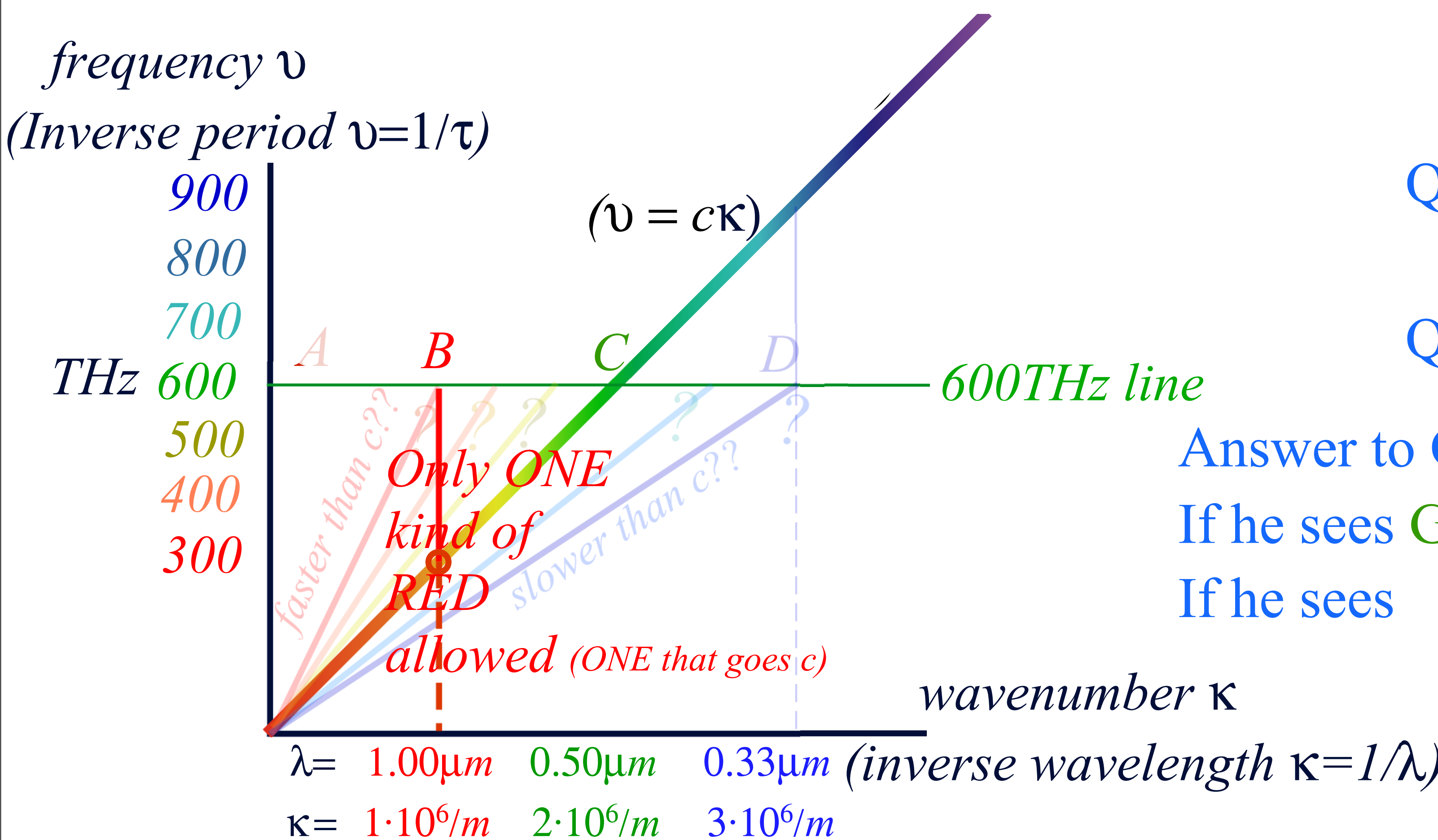

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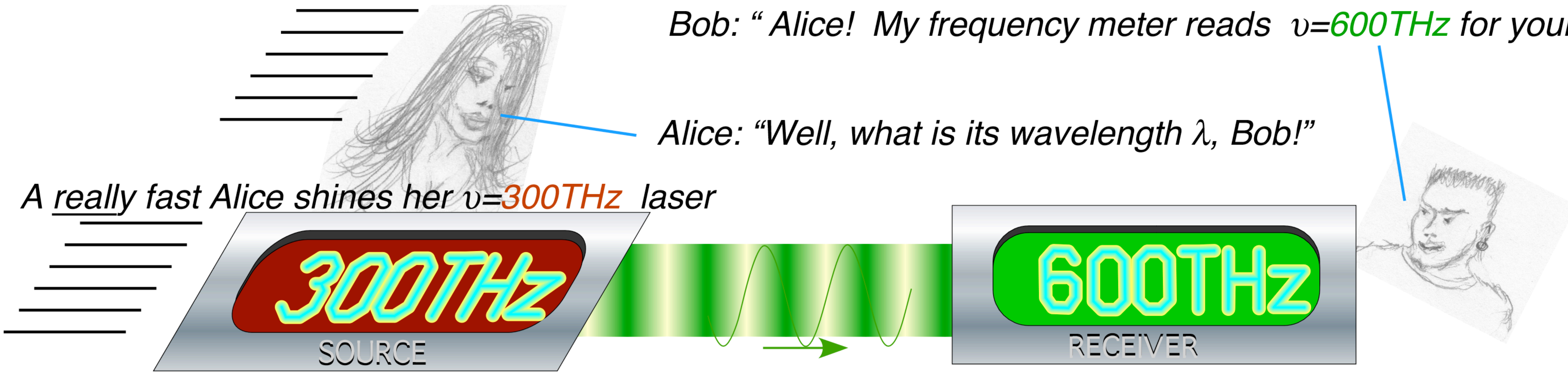
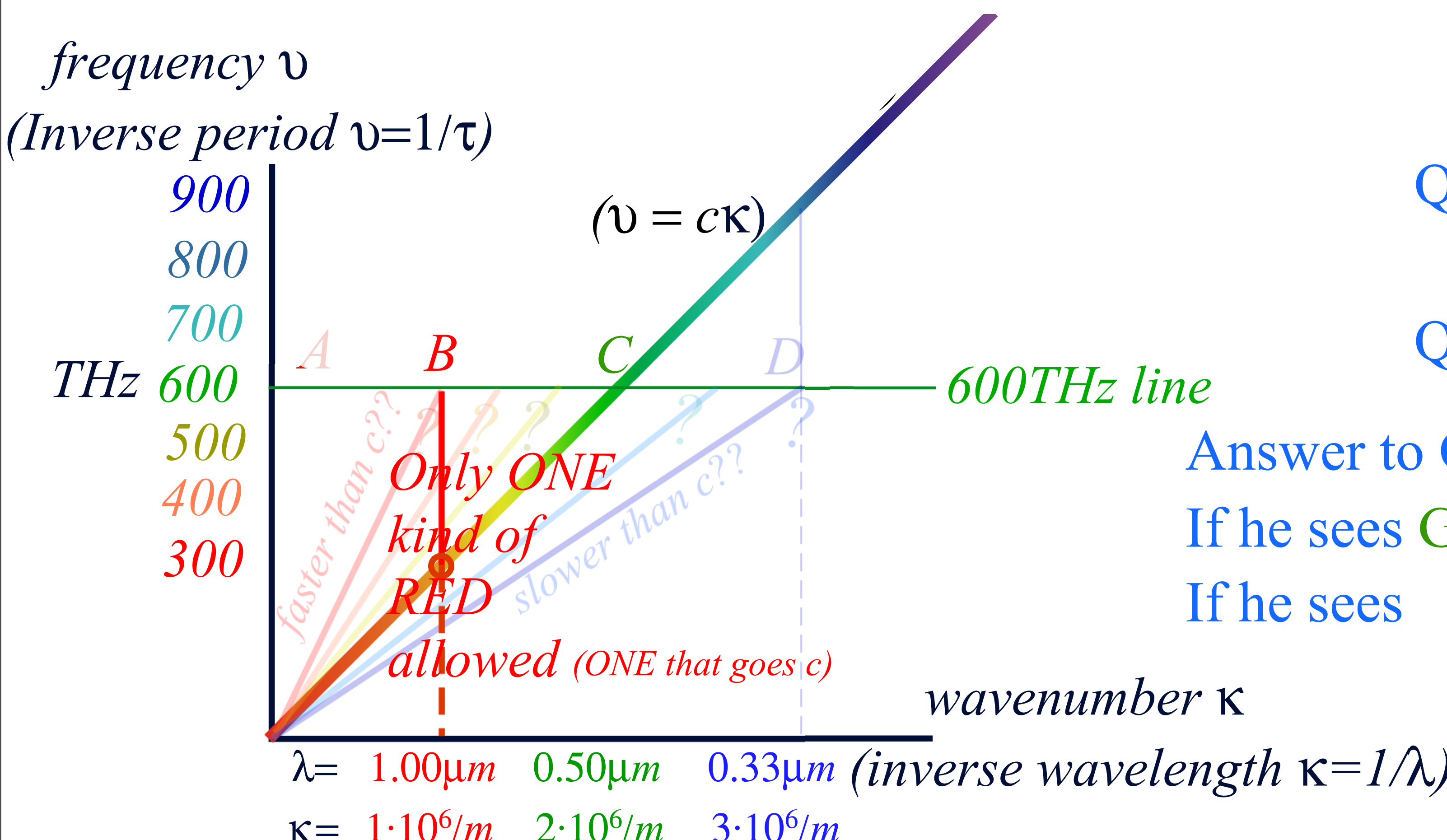
Alice tries to fool Bob that she's shining a 600THz laser. (Bob's unaware she's moving *really* fast...)

Bob: " Alice! My frequency meter reads υ=600THz for your laser beam.

Alice: "Well, what is its wavelength λ, Bob!"

A really fast Alice shines her υ=300THz laser





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Light carries no birth-certificate!

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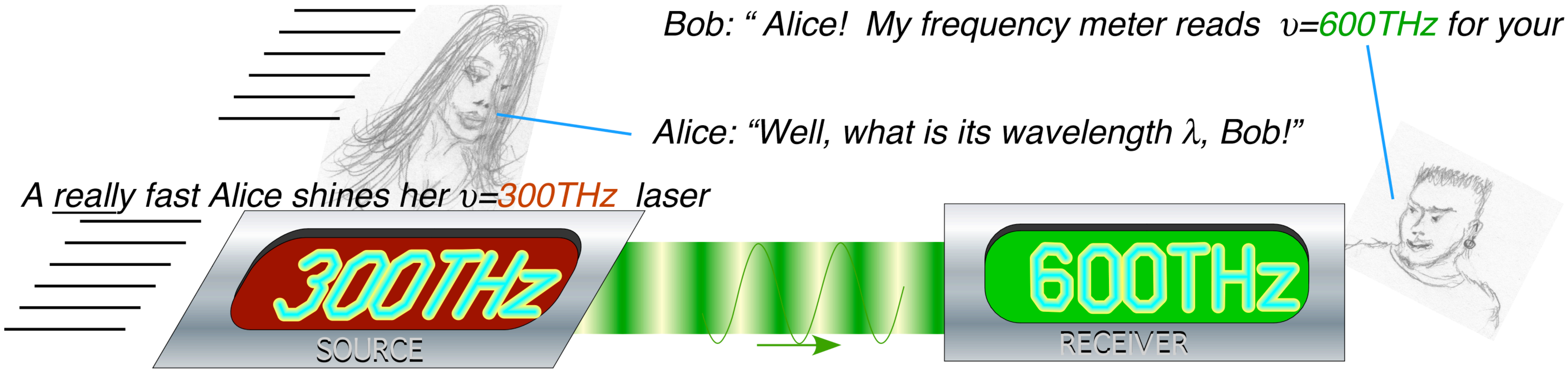

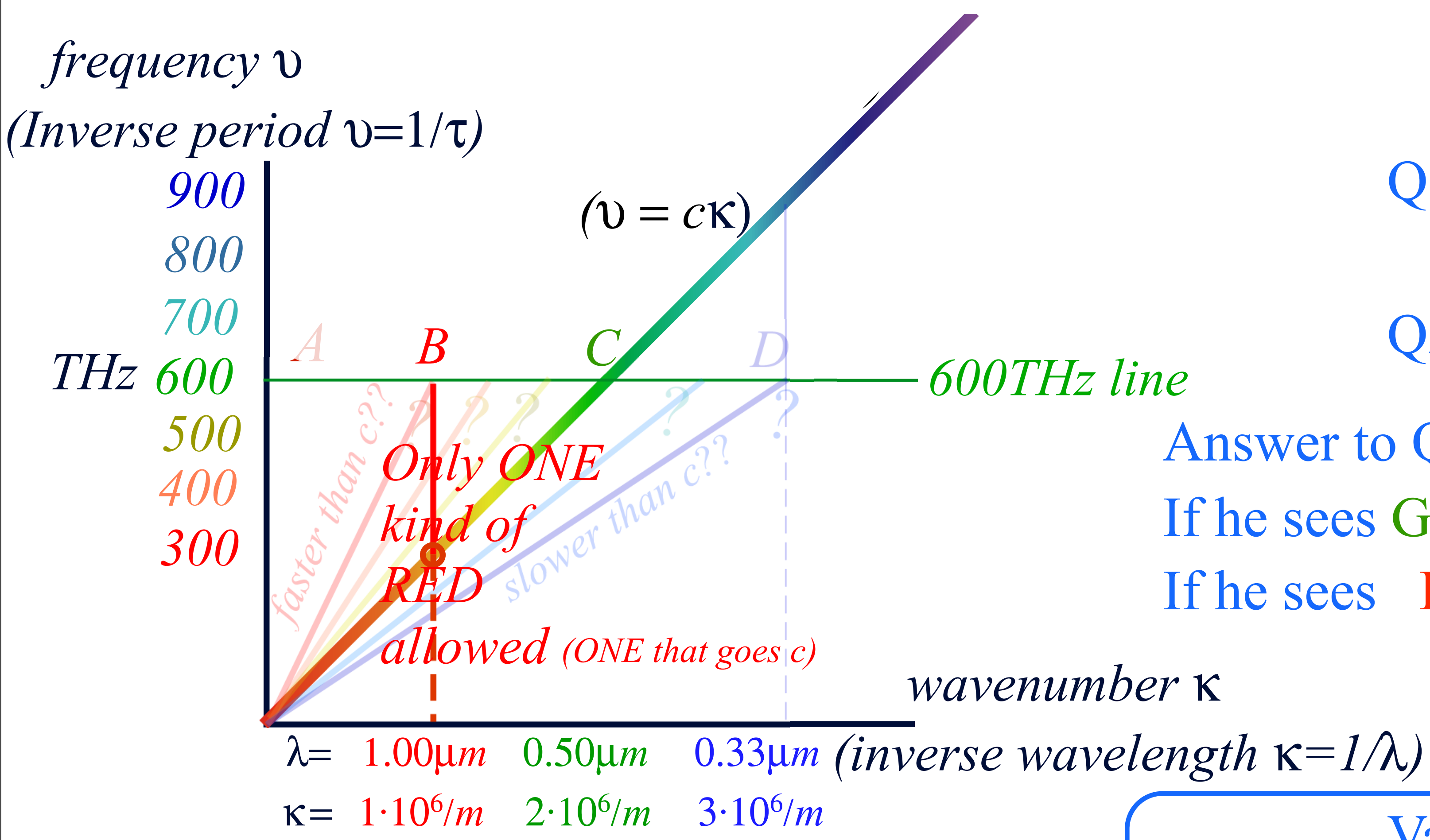

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Answer to Q1is **NO!**

Light carries **no** birth-certificate!

Vacuum only makes one λ for each υ.*

"All colors go $c = \lambda\upsilon = \upsilon/\kappa$ "

Then *Evenson's axiom* holds:

*for each beam and polarization orientation

Clarify *Evenson's CW Axiom (All colors go c) by Doppler effects*

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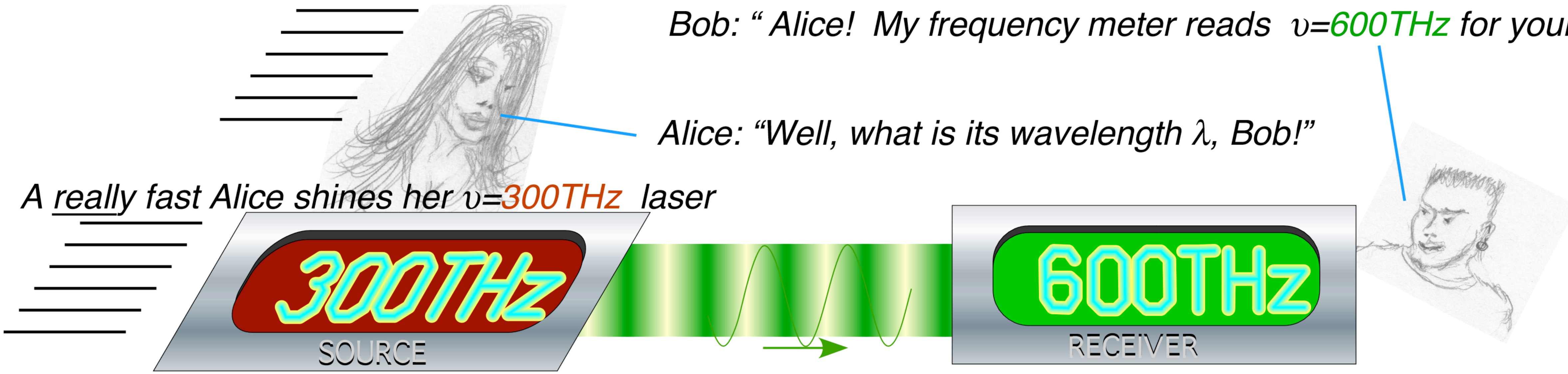

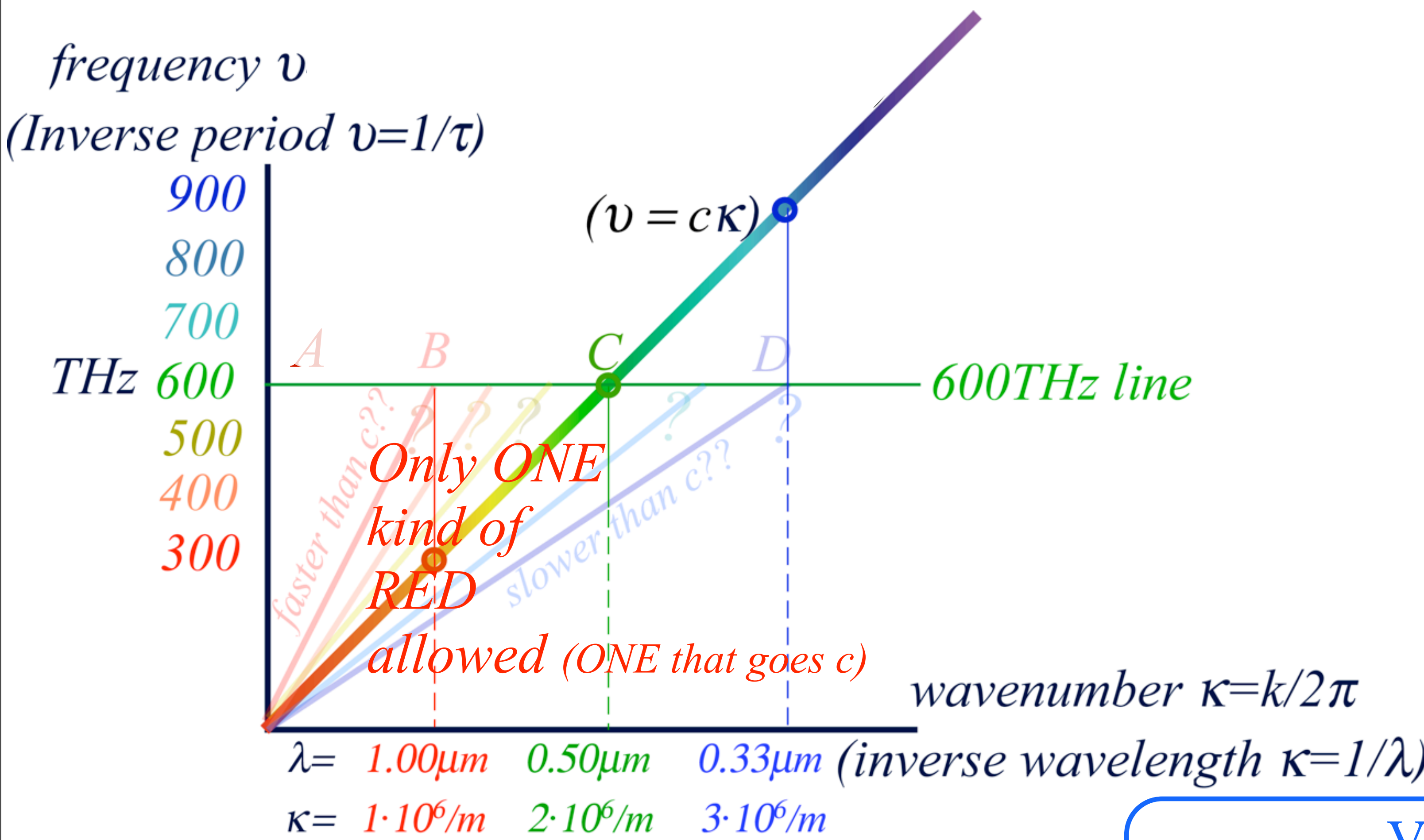

Also could be labeled :

Linear-(non)-dispersion axiom: $\upsilon = c\kappa$

Vacuum only makes one λ for each υ.

"All colors go $c = \lambda\upsilon = \upsilon/\kappa$"

Then *Evenson's axiom* holds:

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_anim.php

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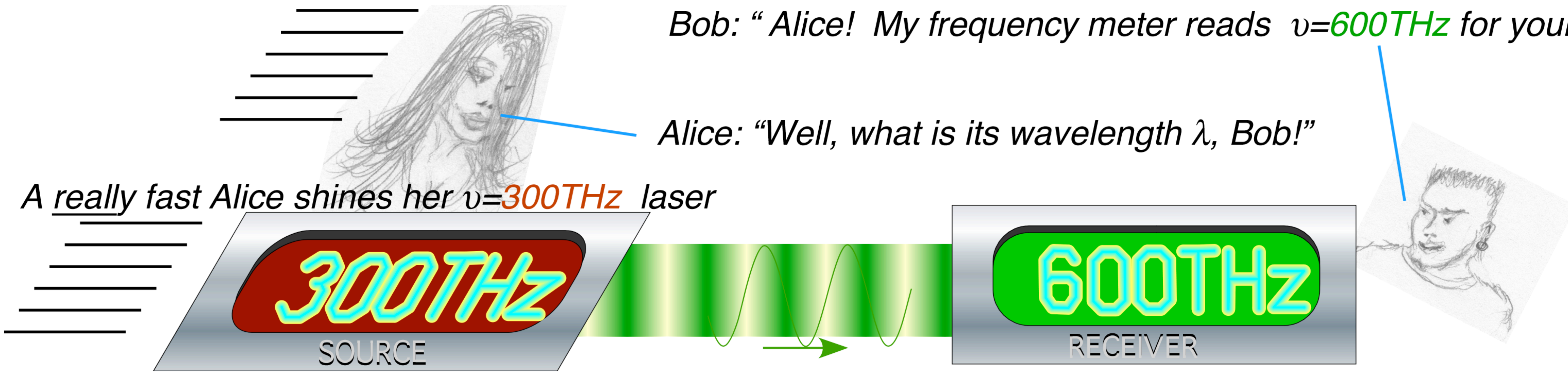

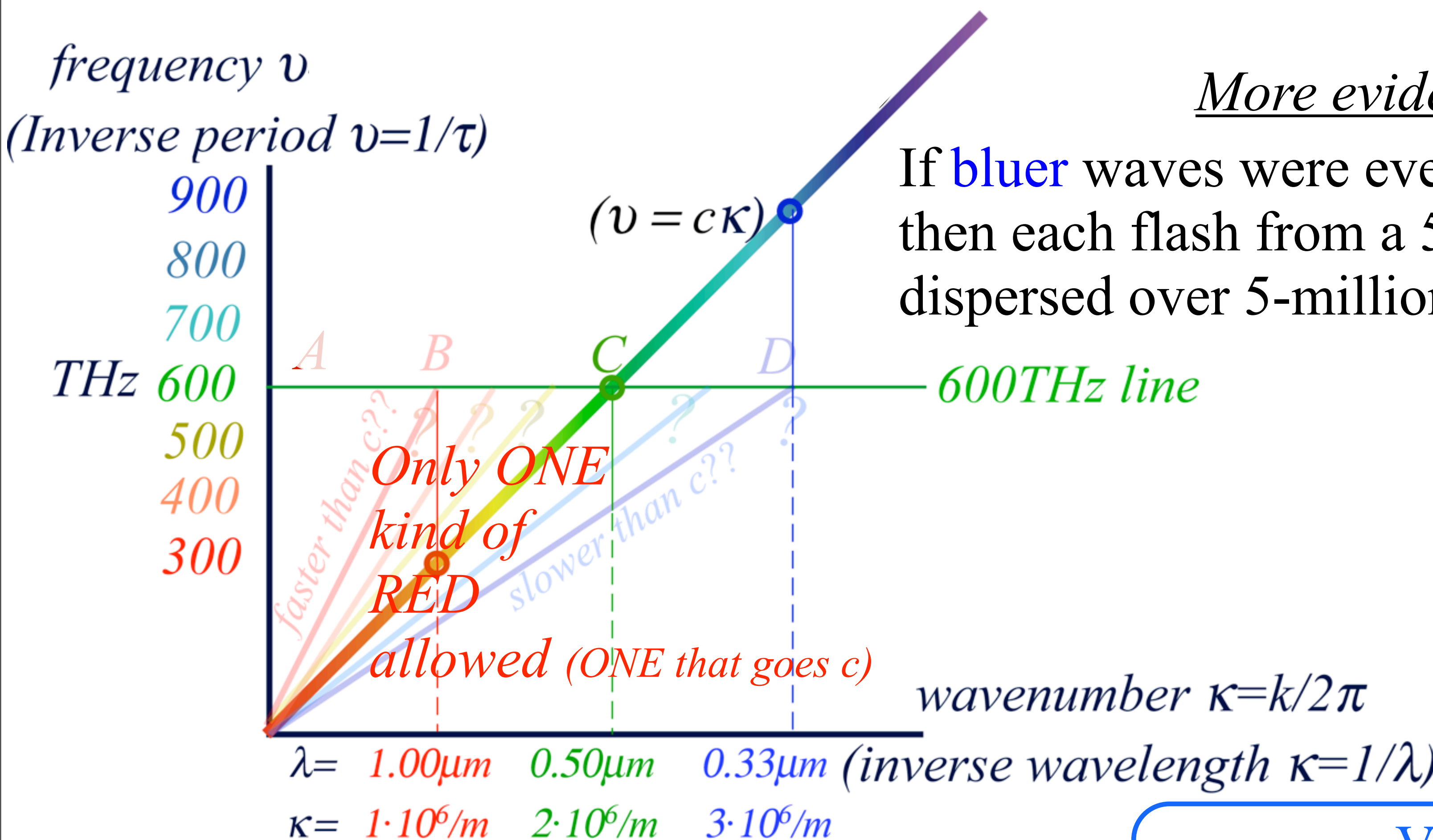

More evidence supporting Evenson's axiom

If bluer waves were even 0.1% faster (or slower) than redder ones then each flash from a 5-billion light-year distant galaxy shows up dispersed over 5-million years. (*Goodbye galactic astronomy!*)

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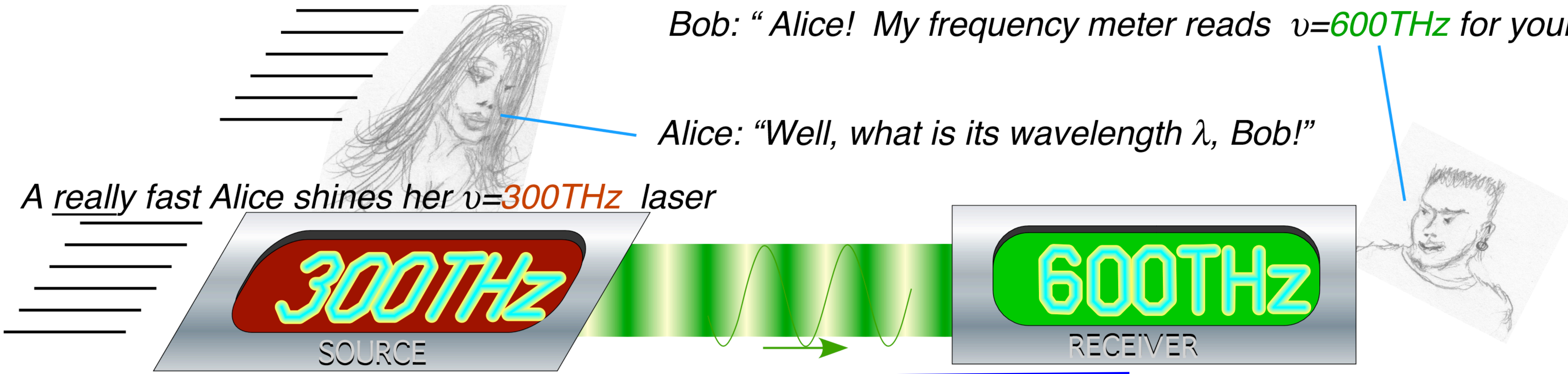
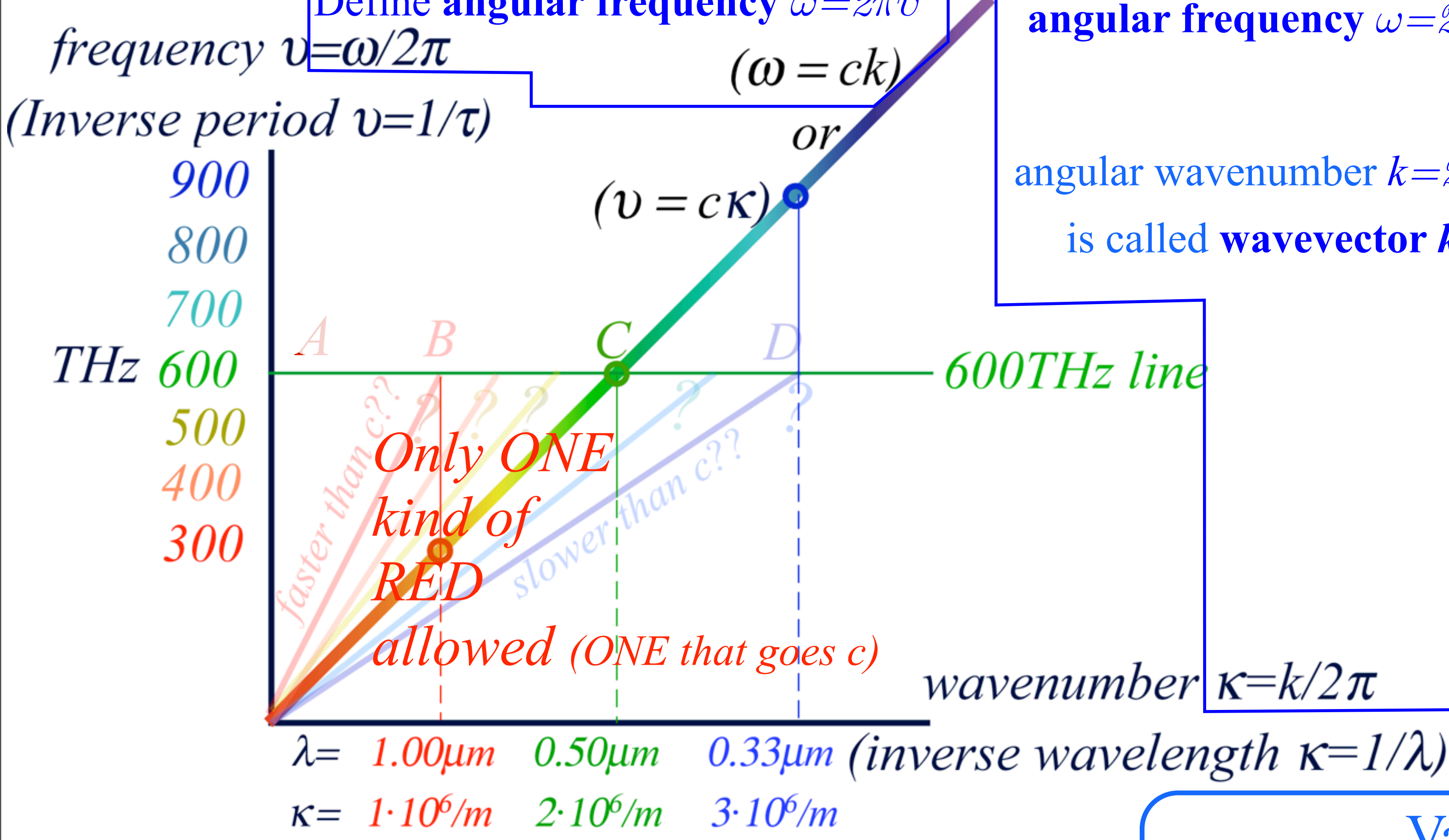


Define **angular frequency** $\omega=2\pi\upsilon$

angular frequency $\omega=2\pi\upsilon$

angular wavenumber $k=2\pi\kappa$ is called **wavevector *k***



Coming Soon:
Introduction of Laser-Phasor clock Parameters ω and k

Also could be labeled :

Linear-(non)-dispersion axiom: $\upsilon = c\kappa$ *or:* $\omega=ck$

Vacuum only makes one λ for each υ.

"All colors go $c = \lambda\upsilon = \upsilon/\kappa = \omega/k$"

Then *Evenson's axiom* holds:

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_anim.php

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Doppler shift-ratios and rapidity

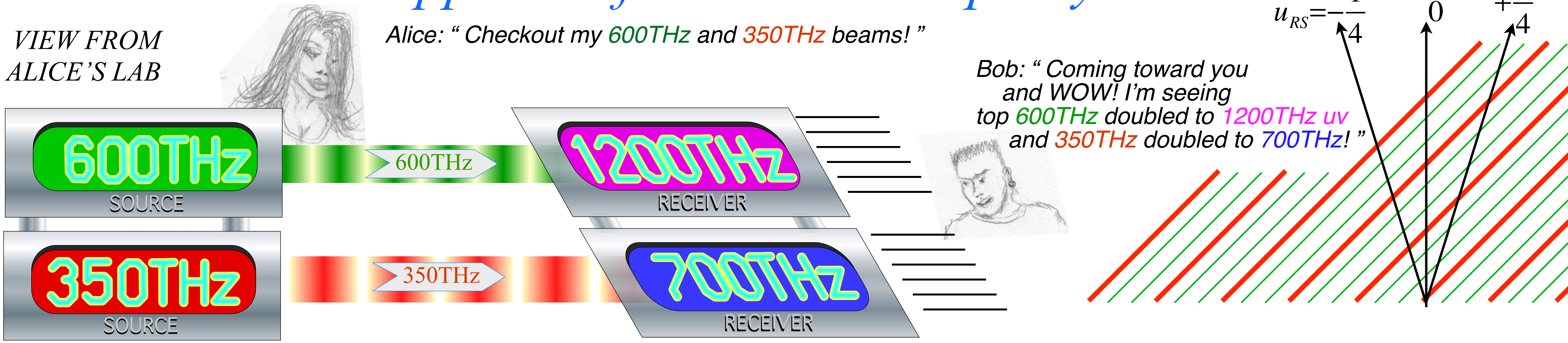
Evenson's axiom is: "All frequencies march in lock-step." Hence, *Doppler shift ratio* $\langle R|S\rangle = \frac{\upsilon_R}{\upsilon_S}$ depends on relative velocity u_{RS} of RECEIVER R vs. SOURCE S but not on source frequency υ_S:

$$\boxed{\upsilon_{RECEIVER} = \langle R|S\rangle \upsilon_{SOURCE}}$$

Light is GEOMETRIC

(If light were ARITHMETRIC then $\upsilon_{RECEIVER} = \upsilon_{SOURCE} \pm \Delta_{RS}$ *might be convenient.)*

Doppler shift-ratios and rapidity



Evenson's axiom is: "All frequencies march in lock-step." Hence, *Doppler shift ratio* $\langle R|S\rangle = \frac{\upsilon_R}{\upsilon_S}$ depends on relative velocity u_{RS} of RECEIVER R vs. SOURCE S but not on source frequency υ_S:

$$\upsilon_{RECEIVER} = \langle R|S\rangle \upsilon_{SOURCE}$$

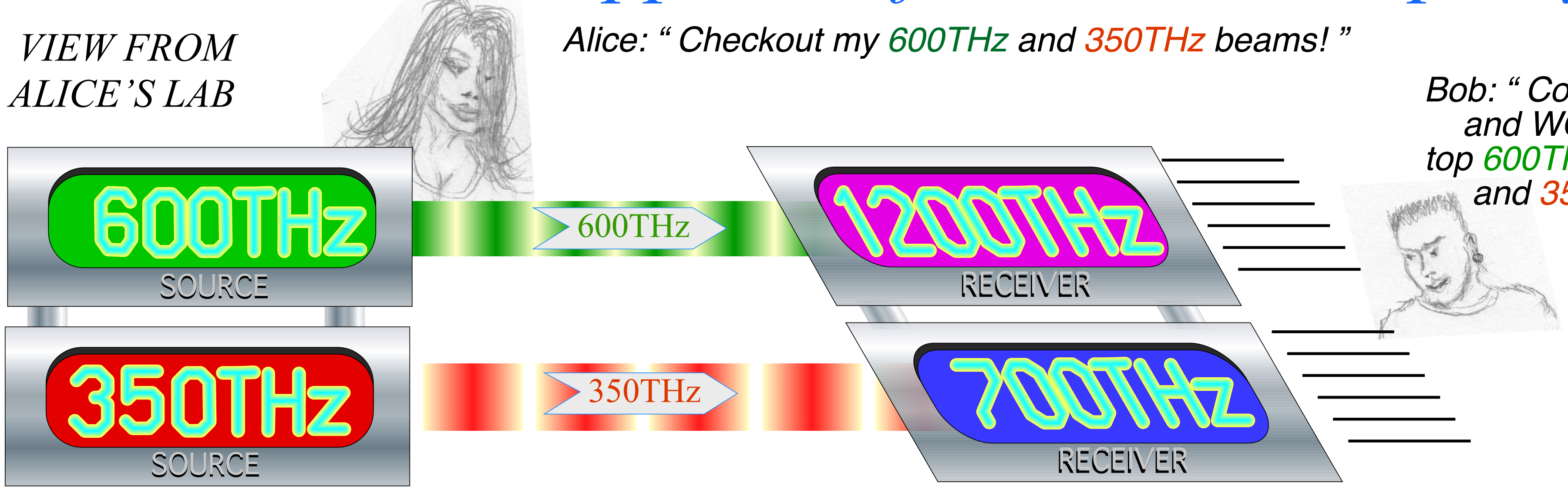
If Source-Receiver distance is *contracting*:

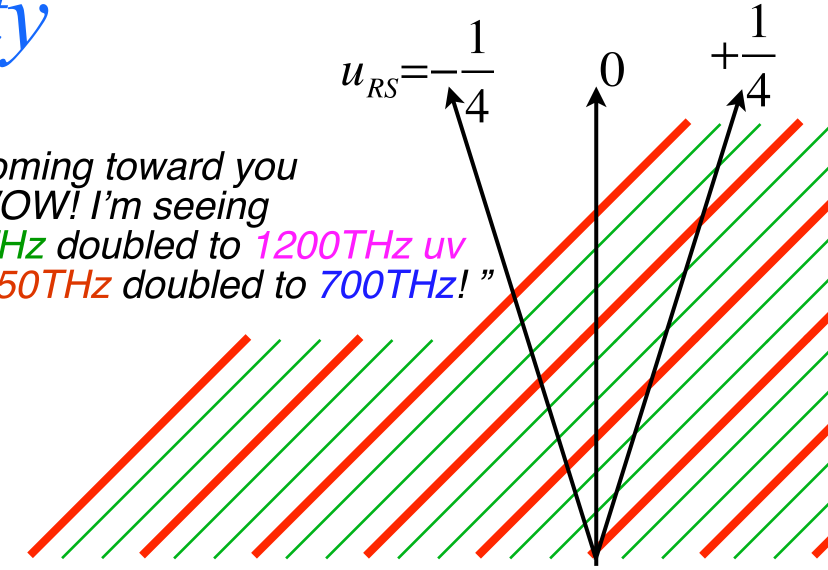
$$\frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}} = \textit{Blue shift} = \langle R|S\rangle > 1$$

If Source-Receiver distance is *expanding*:

$$\frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}} = \textit{Red shift} = \langle R|S\rangle < 1$$

Doppler shift-ratios and rapidity





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$$\frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}=\textit{Red shift}=\langle R|S\rangle<1$$

Logarithm of $\langle R|S\rangle$ known as *Rapidity*: $\rho_{RS}=\log_e\langle R|S\rangle$ or: $\langle R|S\rangle=e^{\rho_{RS}}$

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Logarithm of $\langle R|S\rangle$ known as $\boxed{\textit{Rapidity}:\ \rho_{RS}=\log_e\langle R|S\rangle\quad\text{or:}\quad\langle R|S\rangle=e^{\rho_{RS}}}$

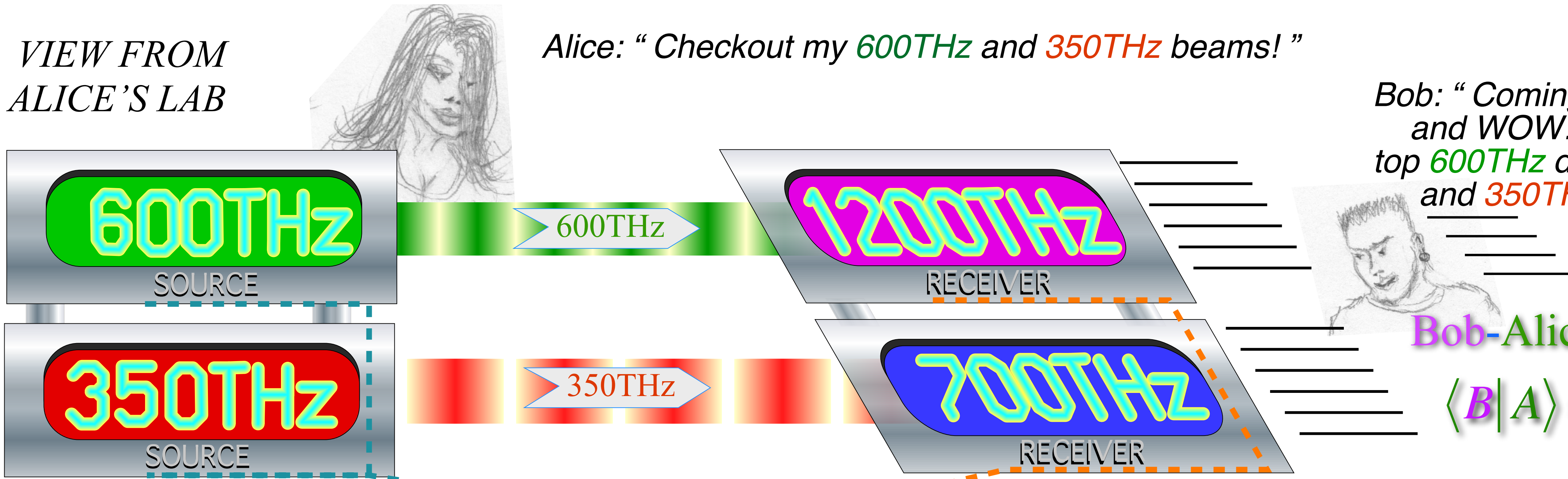
$\langle R|S\rangle=e^{\rho_{RS}}\textit{with}:\ \rho_{RS}>0$ for *contraction*, $\langle R|S\rangle=e^{\rho_{RS}}\textit{with}:\ \rho_{RS}<0$ for *expansion*.

Doppler time-reversal symmetry

VIEW FROM ALICE'S LAB

Alice: " Checkout my 600THz and 350THz beams! "

Bob: " Coming toward you and WOW! I'm seeing top 600THz doubled to 1200THz uv and 350THz doubled to 700THz!



Bob-Alice Doppler ratio:

$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{1200}{600} = \frac{2}{1}$$

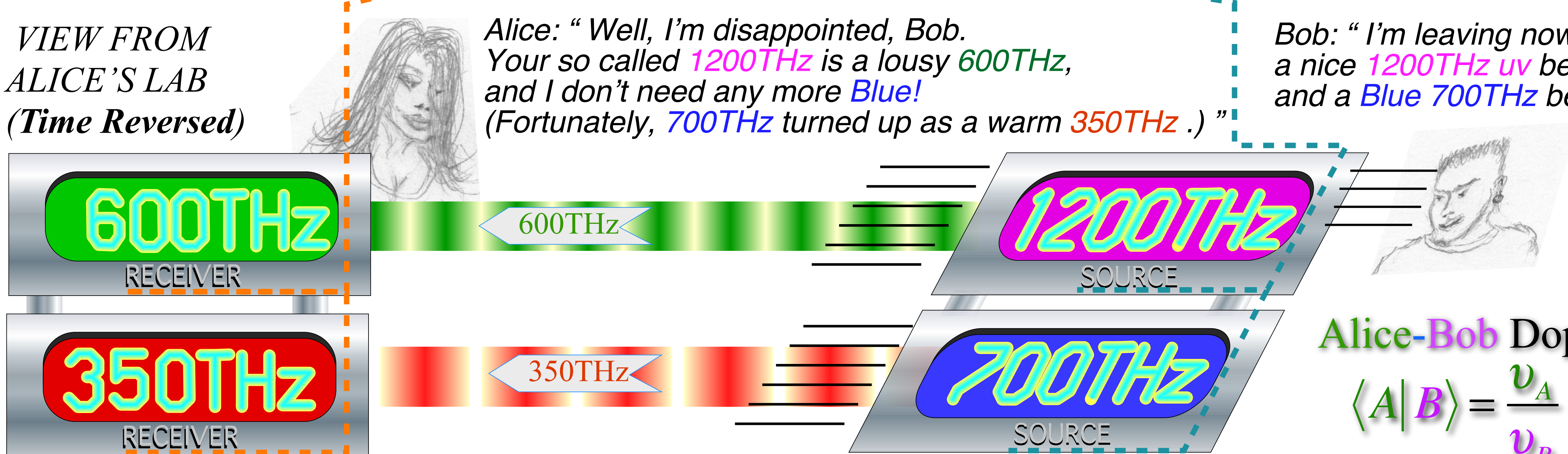
Note: time-reversal switches SOURCE *with* RECEIVER, *reverses motion of Bob's lasers and laser beams.*

(But, digital frequency readouts remain ***unchanged****.)*

VIEW FROM ALICE'S LAB ***(Time Reversed)***

Alice: " Well, I'm disappointed, Bob. Your so called 1200THz is a lousy 600THz, and I don't need any more Blue! (Fortunately, 700THz turned up as a warm 350THz .) "

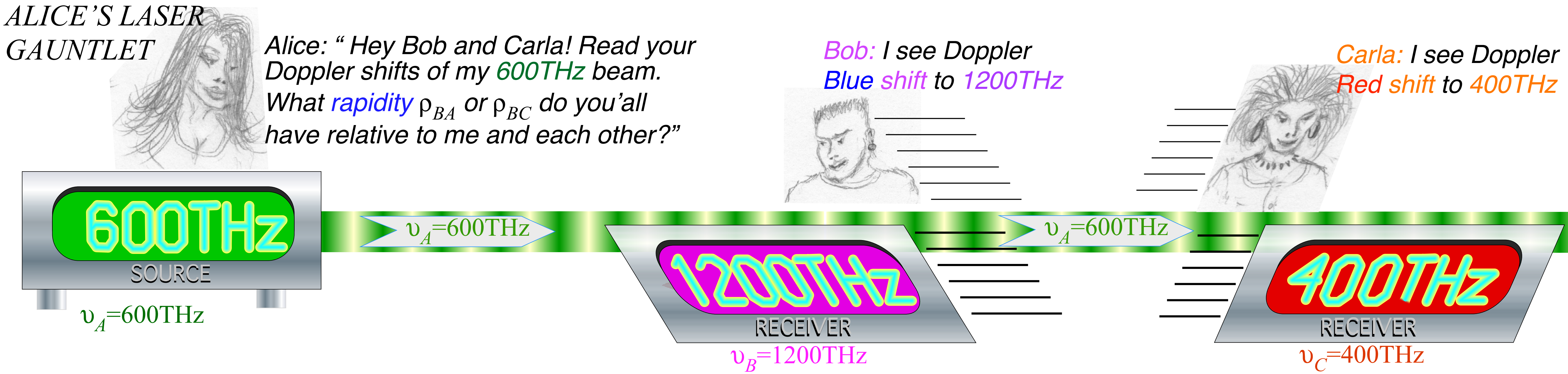
Bob: " I'm leaving now! But, I'll send you a nice 1200THz uv beam and a Blue 700THz beam. "



Alice-Bob Doppler ratio:

$$\langle A|B\rangle = \frac{\upsilon_A}{\upsilon_B} = \frac{600}{1200} = \frac{1}{2}$$

Easy Doppler-shift and Rapidity calculation



Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

Bob-Alice Doppler ratio:

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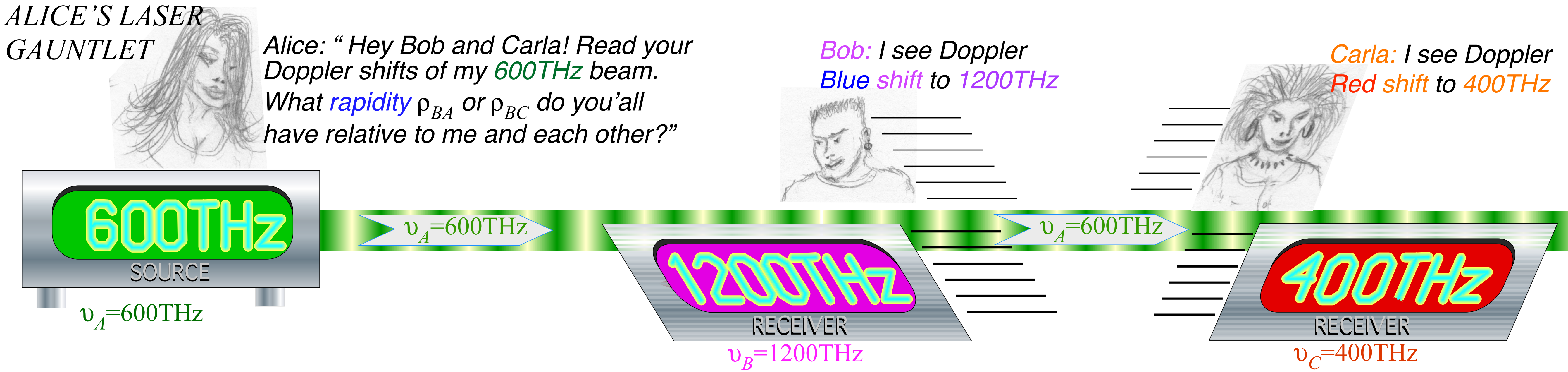
Carla-Alice Doppler ratio:

$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{600} = \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation



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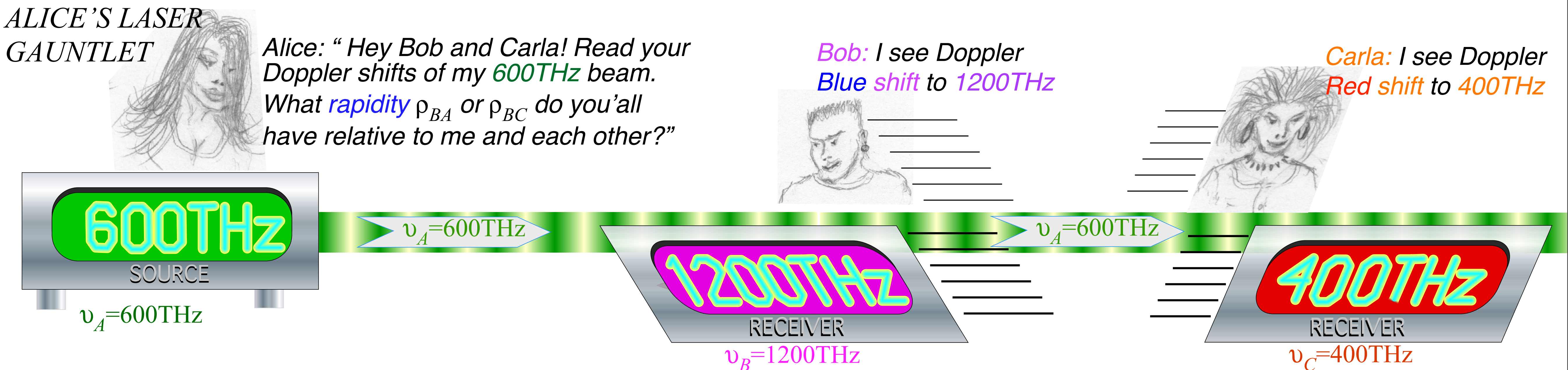
rapidity:

$$\rho_{RS} = \log_e \langle R|S\rangle$$

Definition of Rapidity

Easy Doppler-shift and Rapidity calculation



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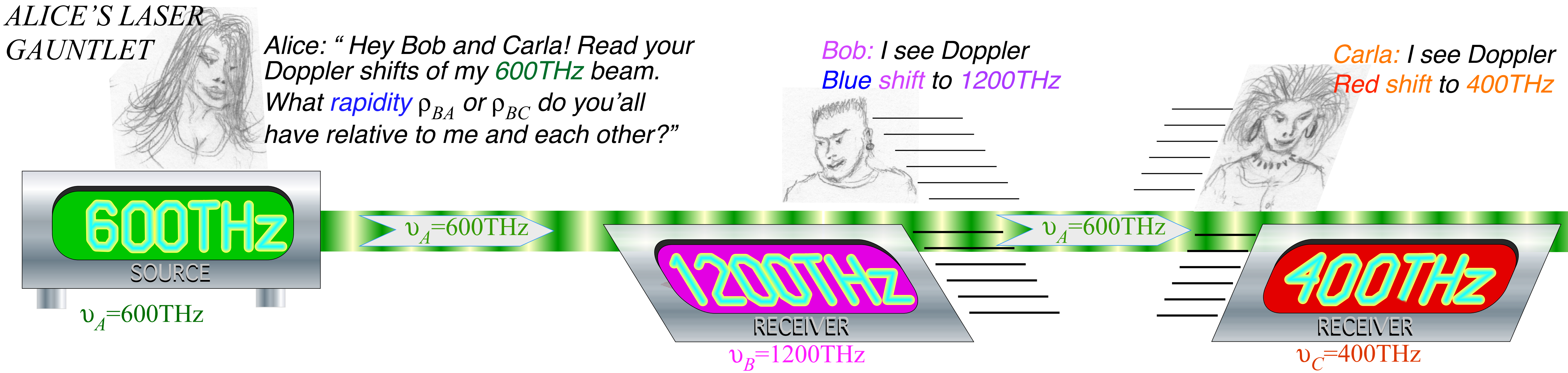

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Carla-Alice Doppler ratio:

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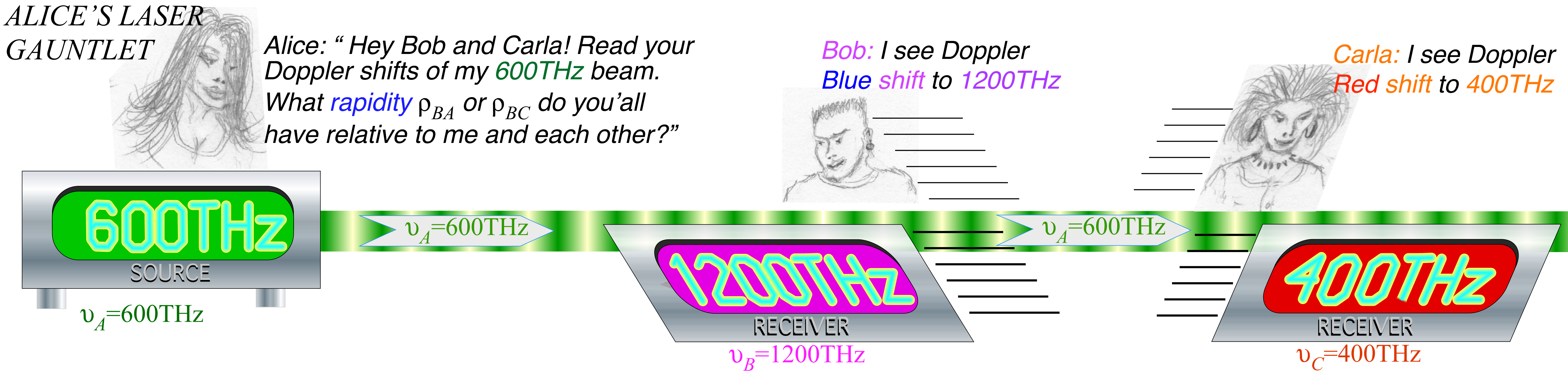
Carla-Alice rapidity:

$$\rho_{CA} = \log_e \langle C|A\rangle = \log_e \frac{2}{3}$$

Easy Doppler-shift and Rapidity calculation



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$$\rho_{BA} = 0.69 \qquad (so{:}\rho_{AB} = -0.69)$$

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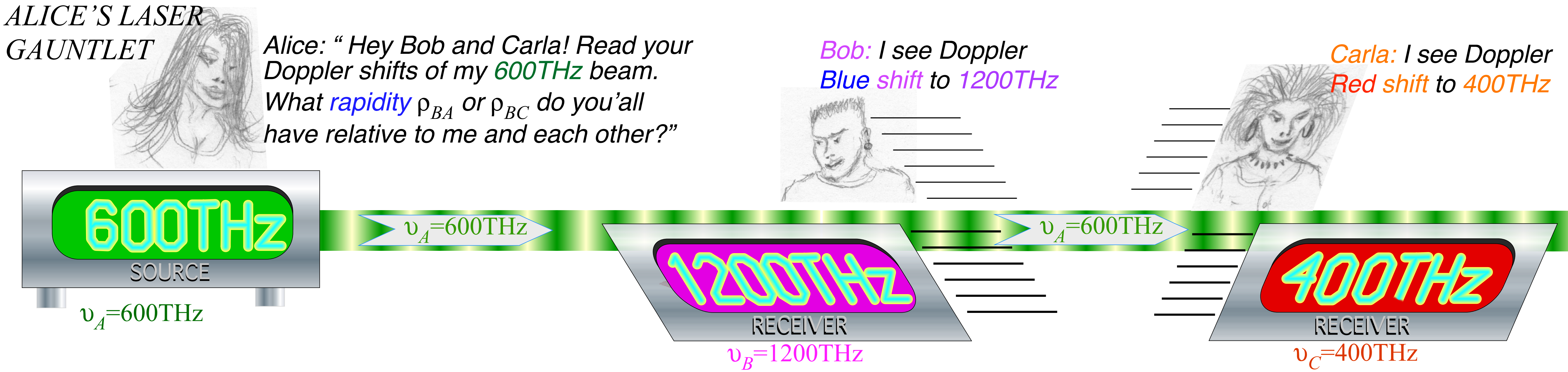

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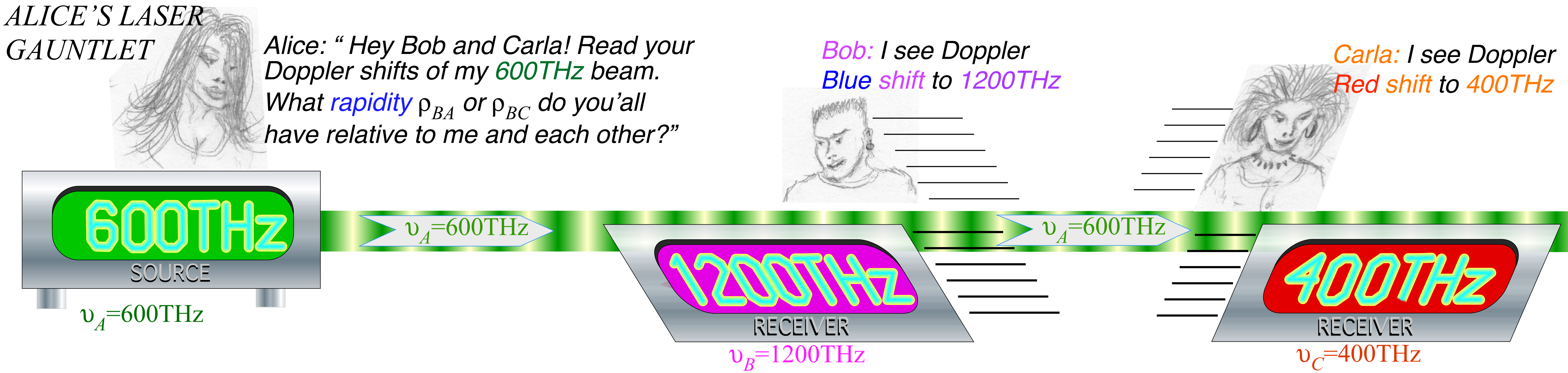

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$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}}$$

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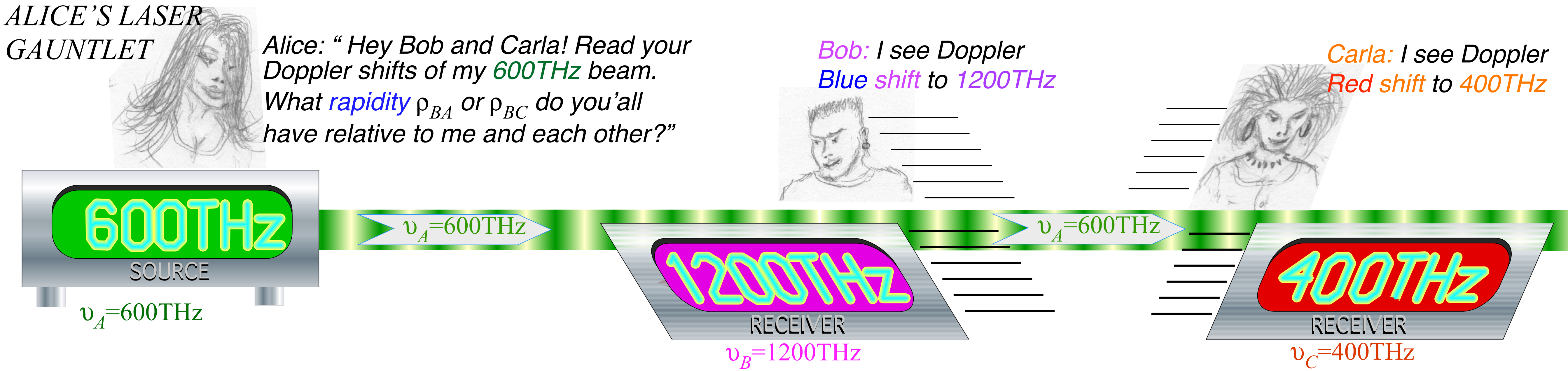

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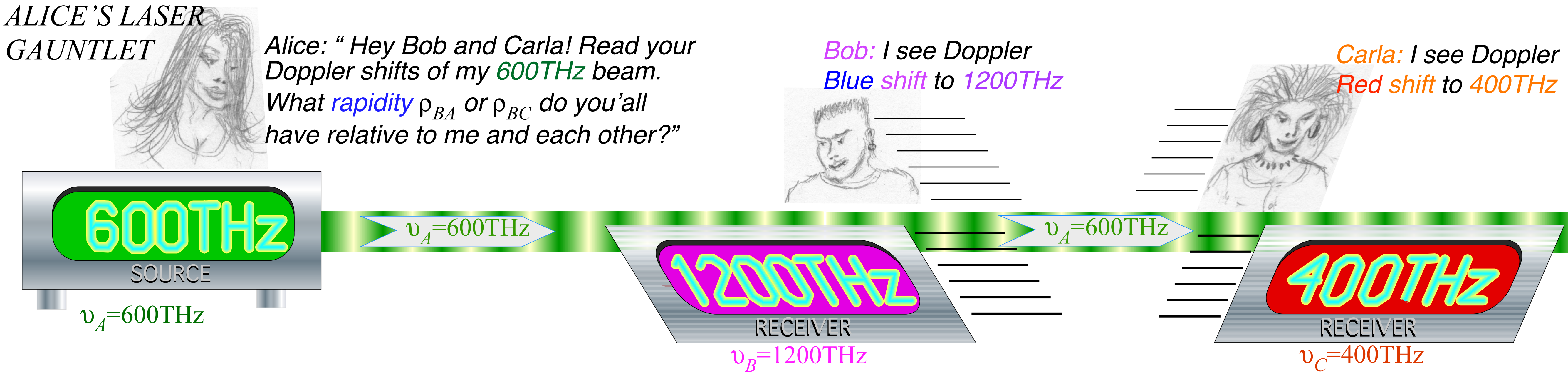
Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}} \quad \text{implies:} \quad \rho_{CB} = \rho_{CA} + \rho_{AB}$$

$$= -0.41 - 0.69 = -1.10$$

Easy Doppler-shift and Rapidity calculation



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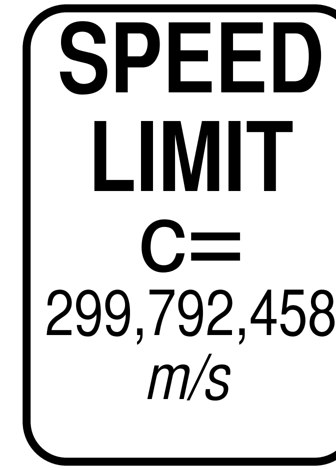
Carla-Alice rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}} e^{\rho_{AB}}$$ implies:

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CB} = \rho_{CA} + \rho_{AB}$$
$$= -0.41 - 0.69 = -1.10$$



<u>Level 2 Secrets</u>*(which also shouldn't be secrets!)*

Special relativity and quantum mechanics

are very much a story of

the geometry of light-wave motion

- How do we measure space and time with light waves?

  Use *1CW laser-phasors* for a *phase-based* theory
- How do we make spacetime coordinate graph with light waves?

  Use *2CW laser-phasors* and wave interference geometry

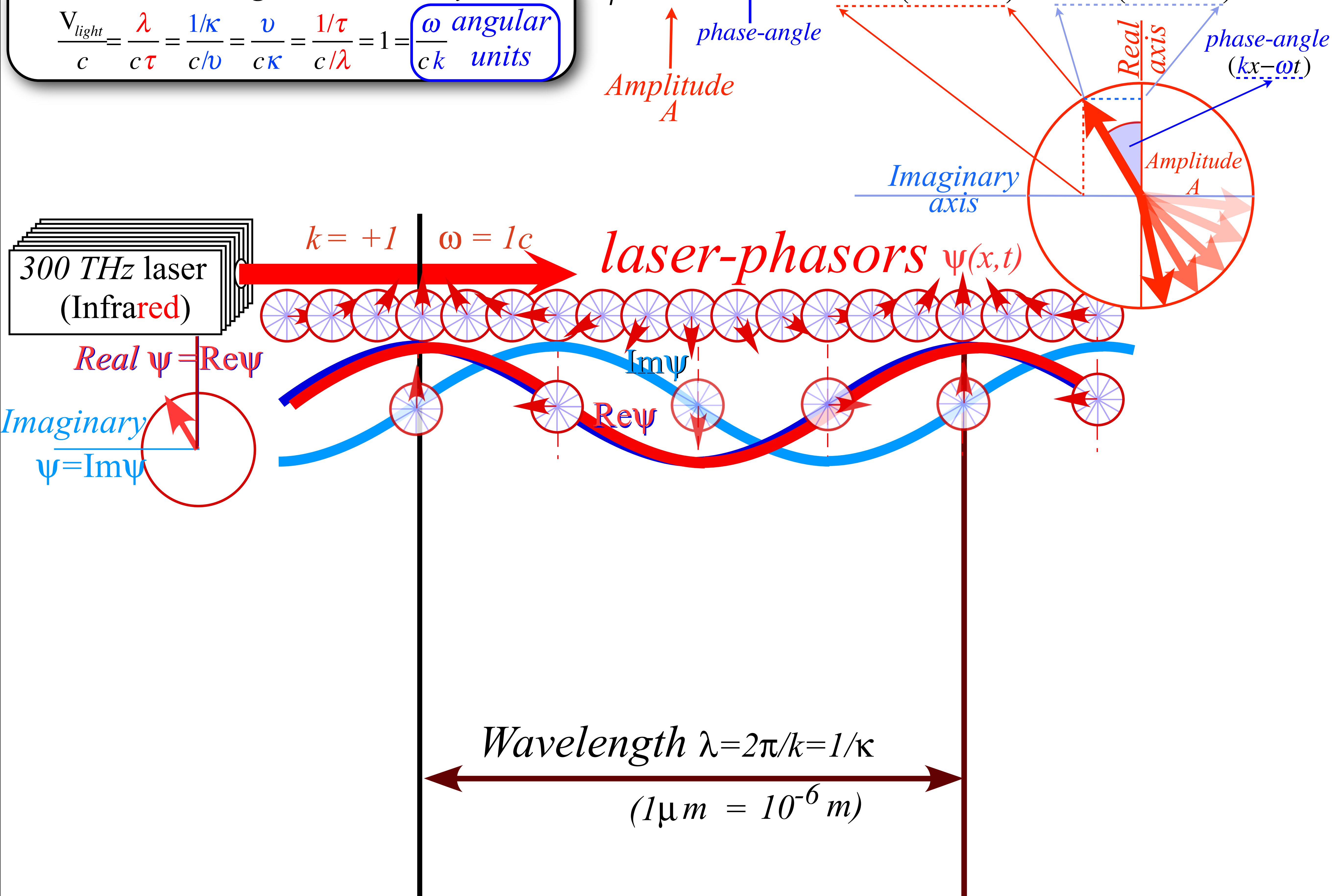
  Get Einstein-Lorentz-Minkowski graphs for free!

1CW Laser-phasor wave function

Dimensionless Light wave-velocity c/c=1

$$\frac{V_{light}}{c} = \frac{\lambda}{c\tau} = \frac{1/\kappa}{c/\upsilon} = \frac{\upsilon}{c\kappa} = \frac{1/\tau}{c/\lambda} = 1 = \frac{\omega}{ck} \text{ angular units}$$

$$\psi = A \cdot e^{i(kx-\omega t)} = A\cdot\cos(kx-\omega t) + iA\cdot\sin(kx-\omega t)$$



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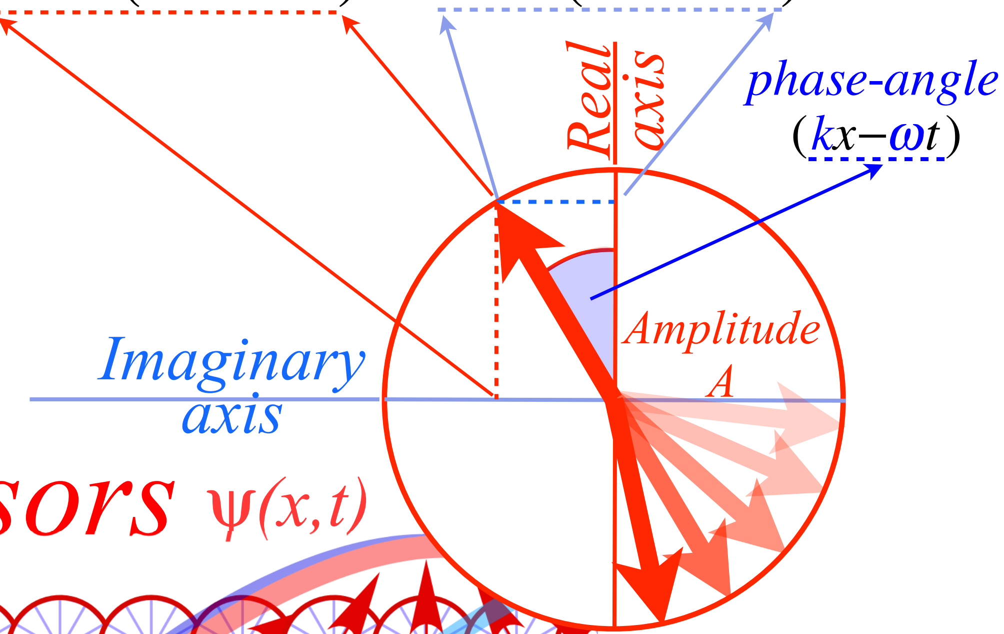
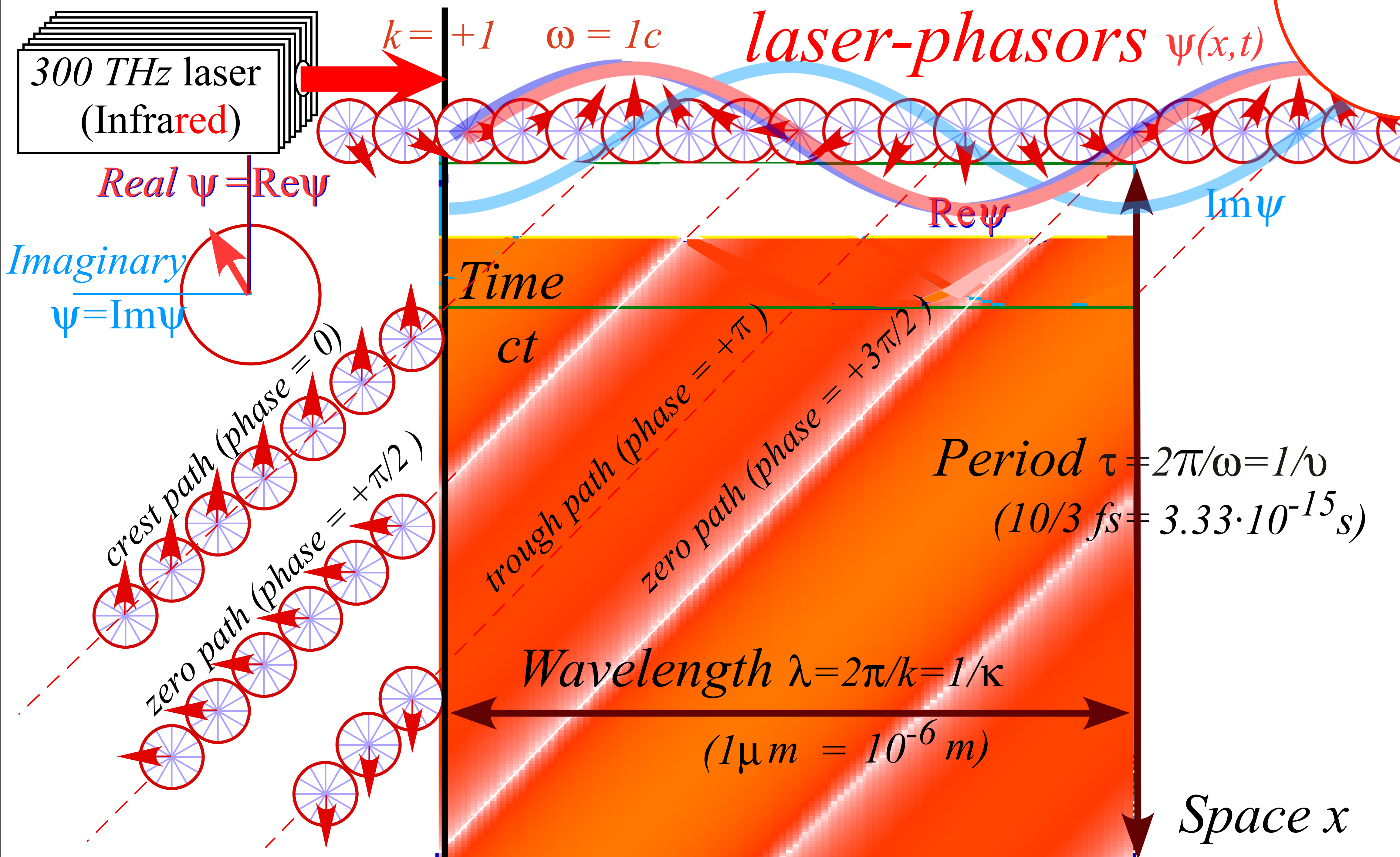
Amplitude A — phase-angle

Q: Where is phase=$(kx-\omega t)$=0?

A: It is wherever this is: $\frac{x}{t} = \frac{\omega}{k}$





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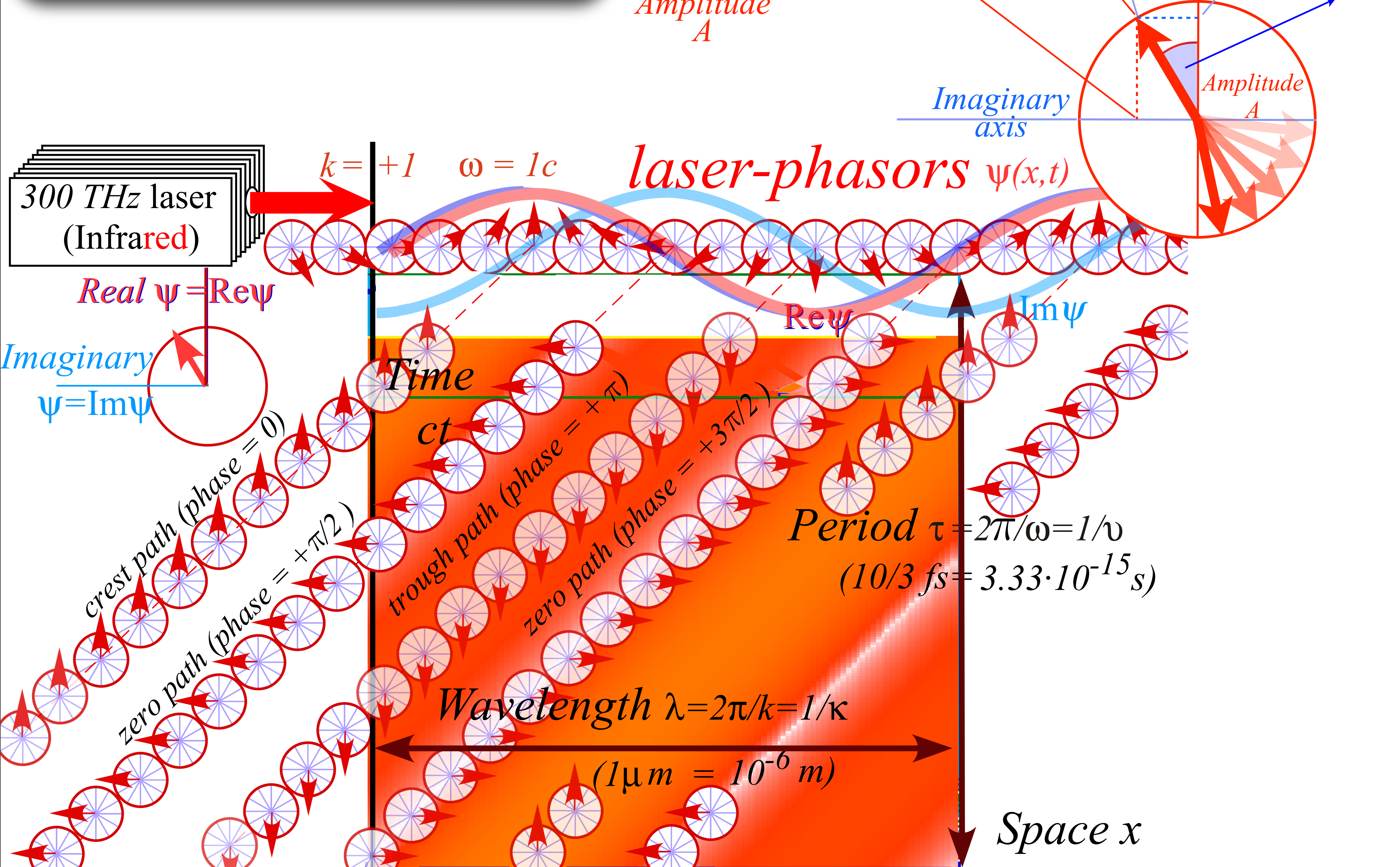
Amplitude A — phase-angle — phase-angle $(kx-\omega t)$ — Real axis



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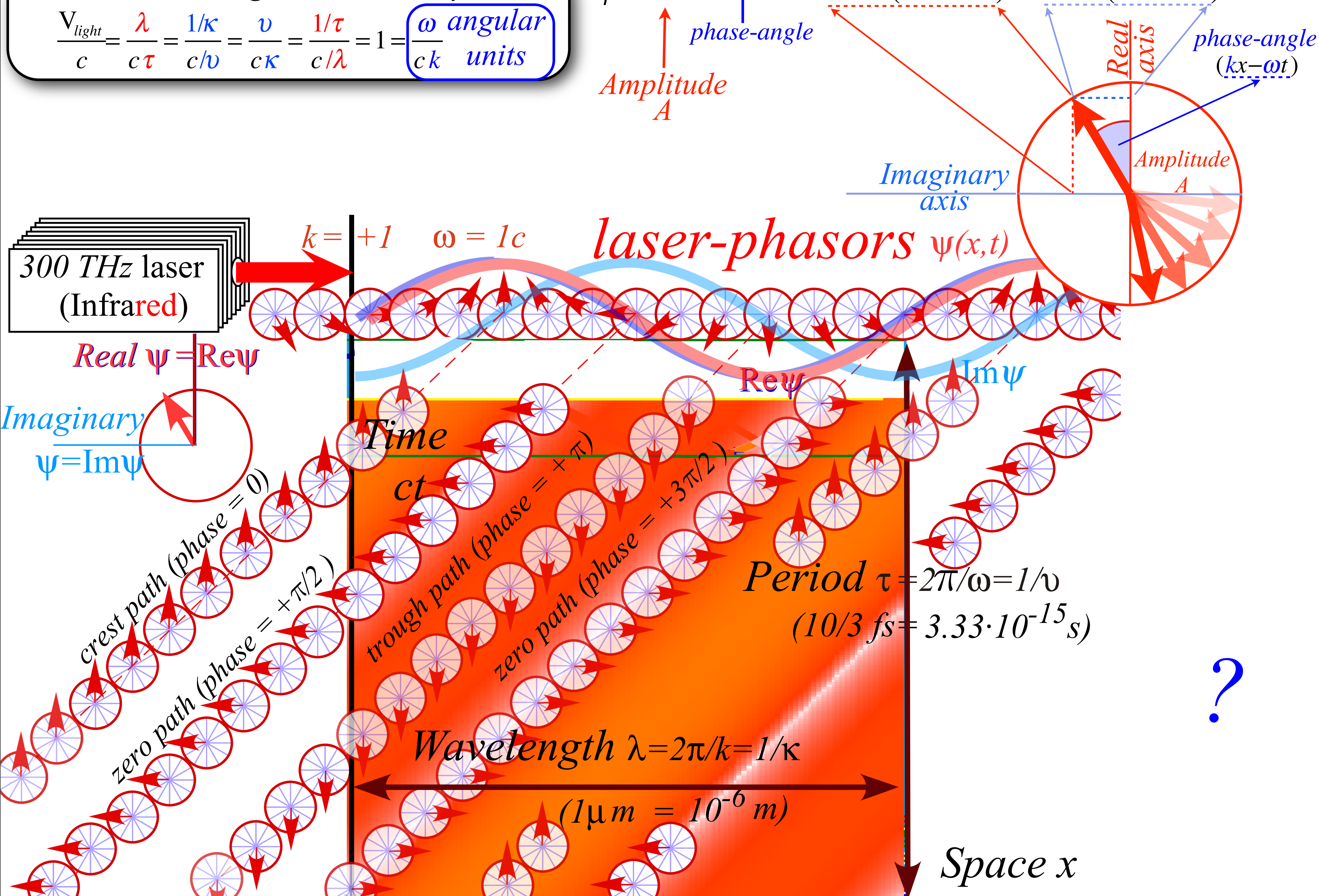
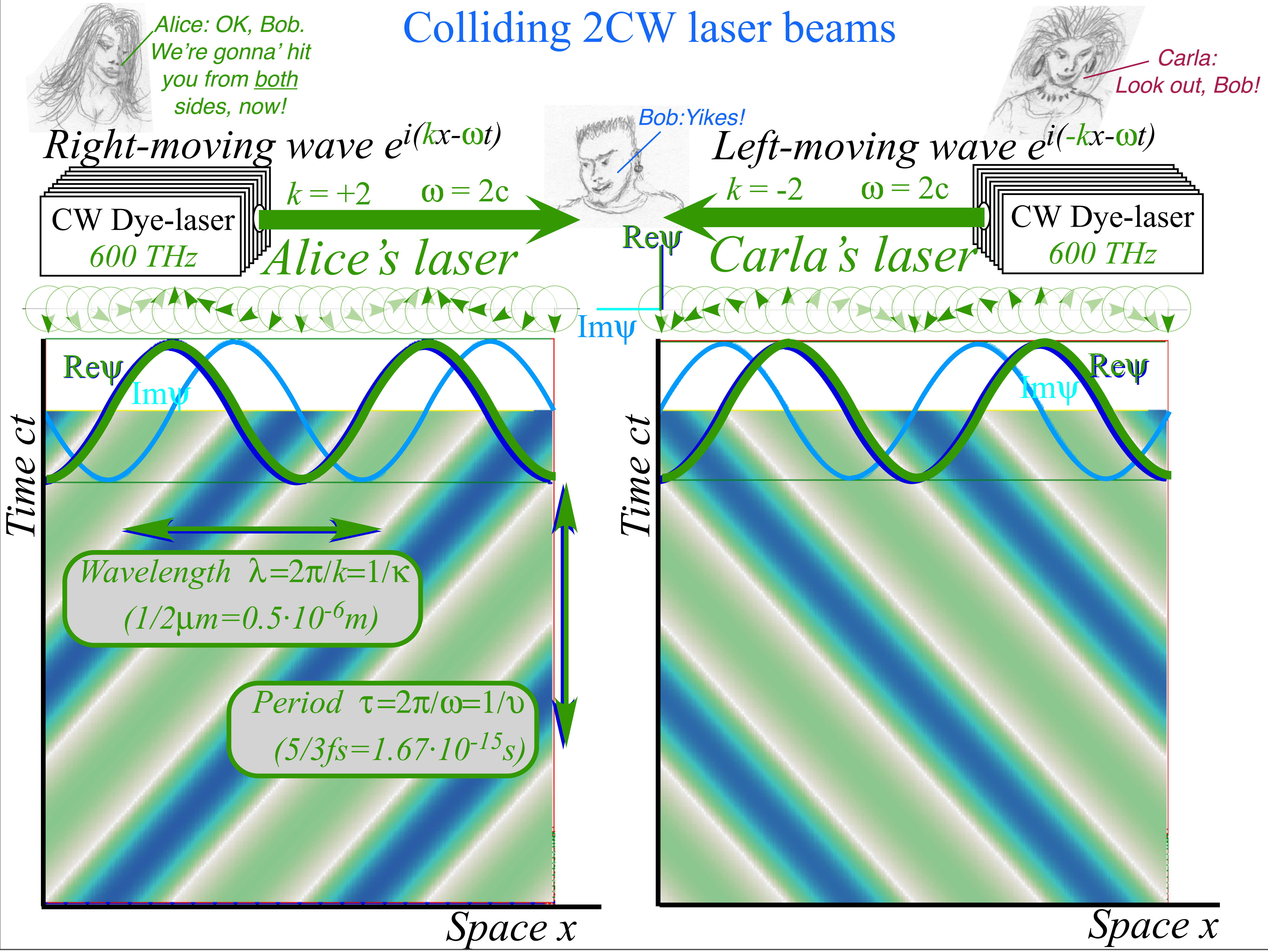
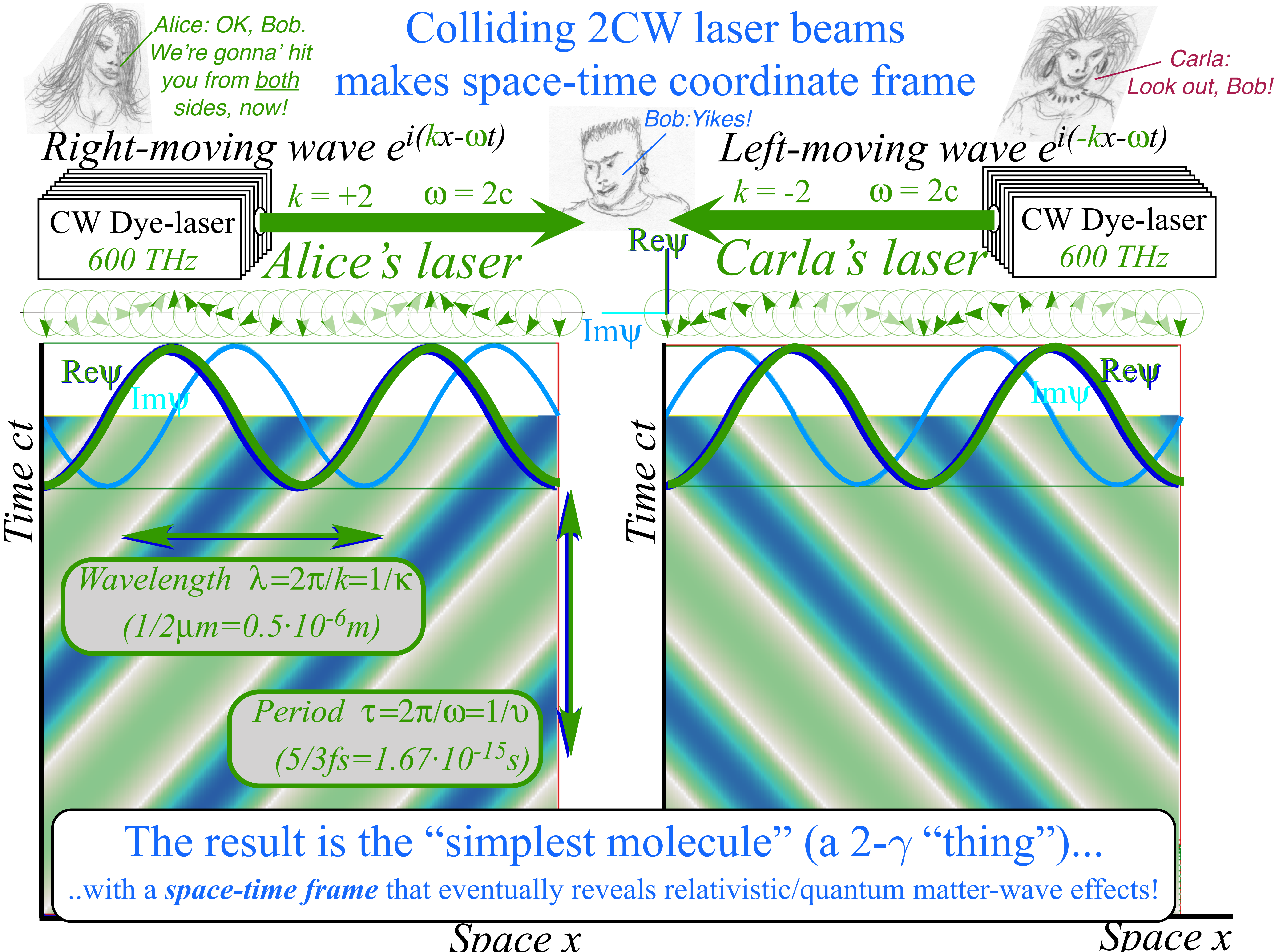



Colliding 2CW laser beams
Alice: OK, Bob. We're gonna' hit you from both sides, now!
Bob:Yikes!
Carla: Look out, Bob!
Right-moving wave $e^{i(kx-\omega t)}$
Left-moving wave $e^{i(-kx-\omega t)}$
$k = +2$
$\omega = 2c$
$k = -2$
$\omega = 2c$
CW Dye-laser
600 THz
Alice's laser
CW Dye-laser
600 THz
Carla's laser
Reψ
Imψ
Reψ
Imψ
Reψ
Imψ
Time ct
Time ct
Wavelength $\lambda=2\pi/k=1/\kappa$
$(1/2\mu m=0.5\cdot 10^{-6}m)$
Period $\tau=2\pi/\omega=1/\upsilon$
$(5/3fs=1.67\cdot 10^{-15}s)$
Space x
Space x



Colliding 2CW laser beams
makes space-time coordinate frame
Alice: OK, Bob. We're gonna' hit you from both sides, now!
Bob:Yikes!
Carla: Look out, Bob!
Right-moving wave $e^{i(kx-\omega t)}$
Left-moving wave $e^{i(-kx-\omega t)}$
$k = +2$ $\omega = 2c$
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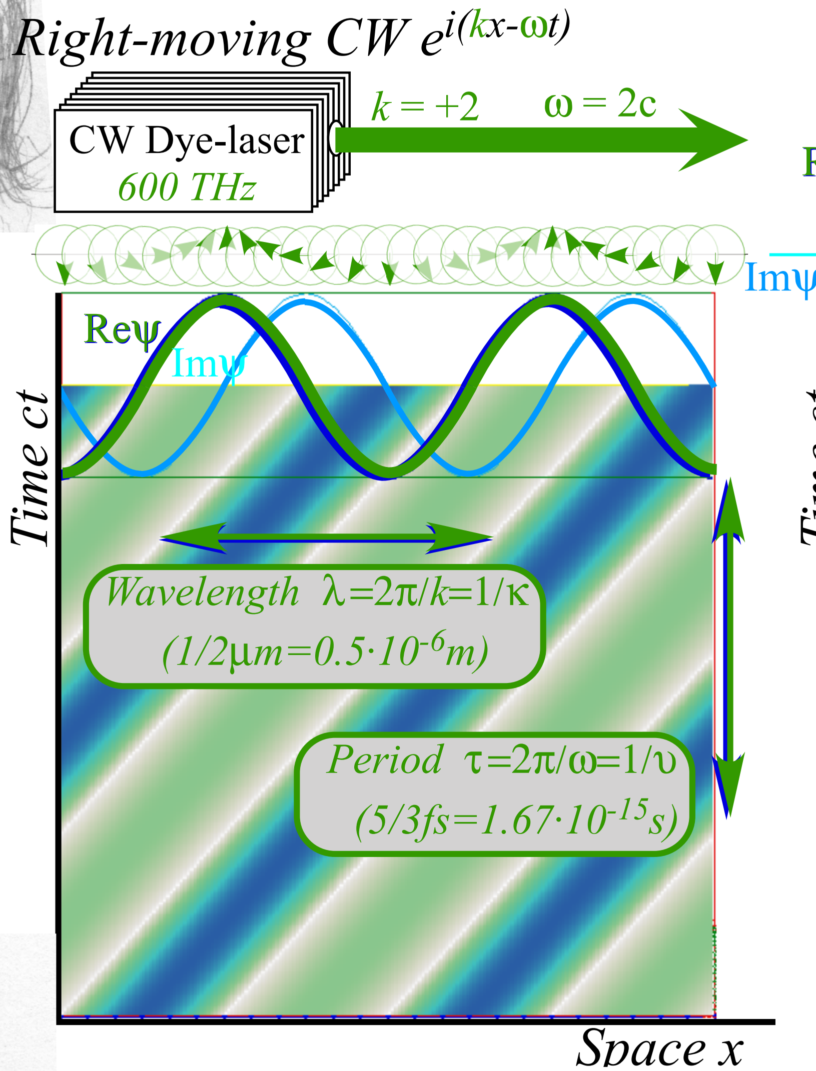
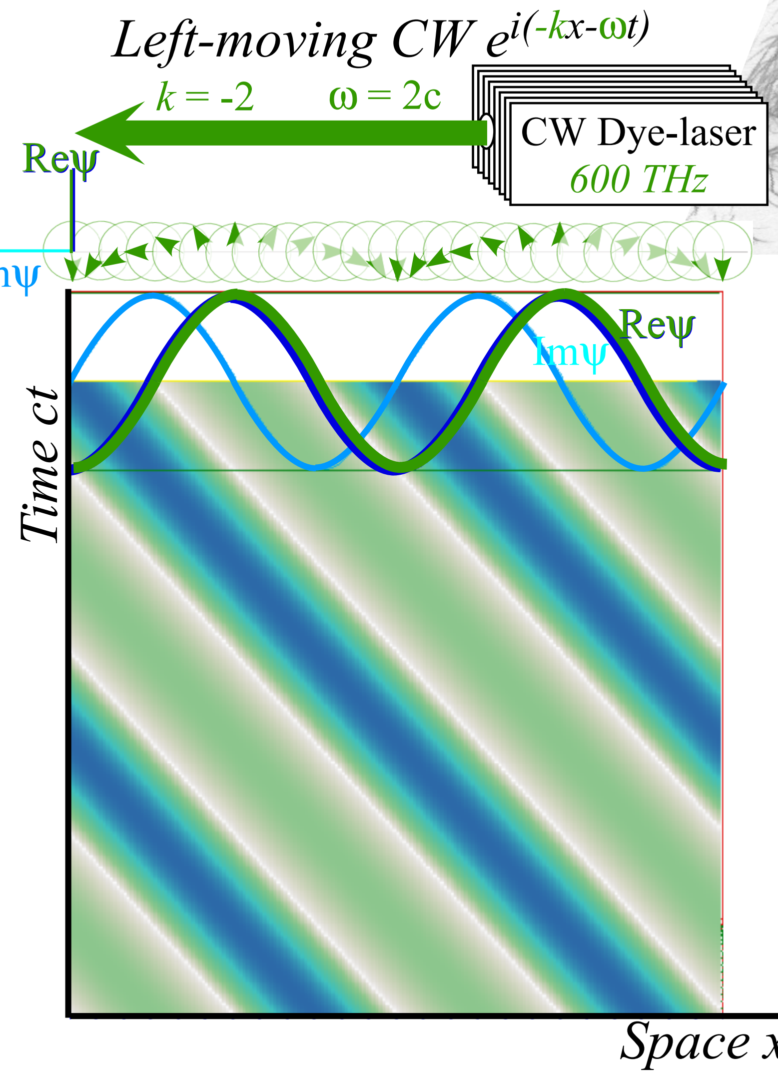
CW Dye-laser
600 THz
Alice's laser
CW Dye-laser
600 THz
Carla's laser
Reψ
Imψ
Reψ
Imψ
Reψ
Imψ
Time ct
Time ct
Wavelength $\lambda=2\pi/k=1/\kappa$
$(1/2\mu m=0.5\cdot 10^{-6}m)$
Period $\tau=2\pi/\omega=1/\upsilon$
$(5/3fs=1.67\cdot 10^{-15}s)$
The result is the "simplest molecule" (a 2-γ "thing")...
..with a space-time frame that eventually reveals relativistic/quantum matter-wave effects!
Space x
Space x

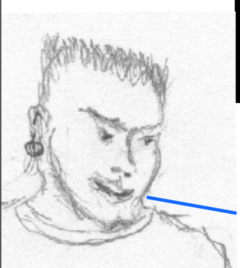




Carla:

Easy!

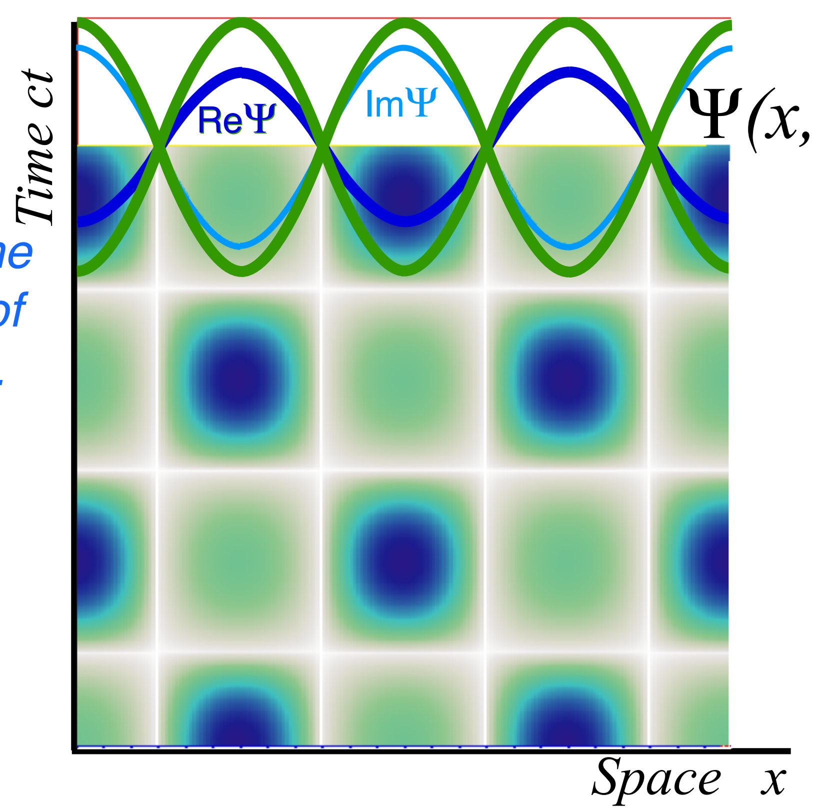
You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.



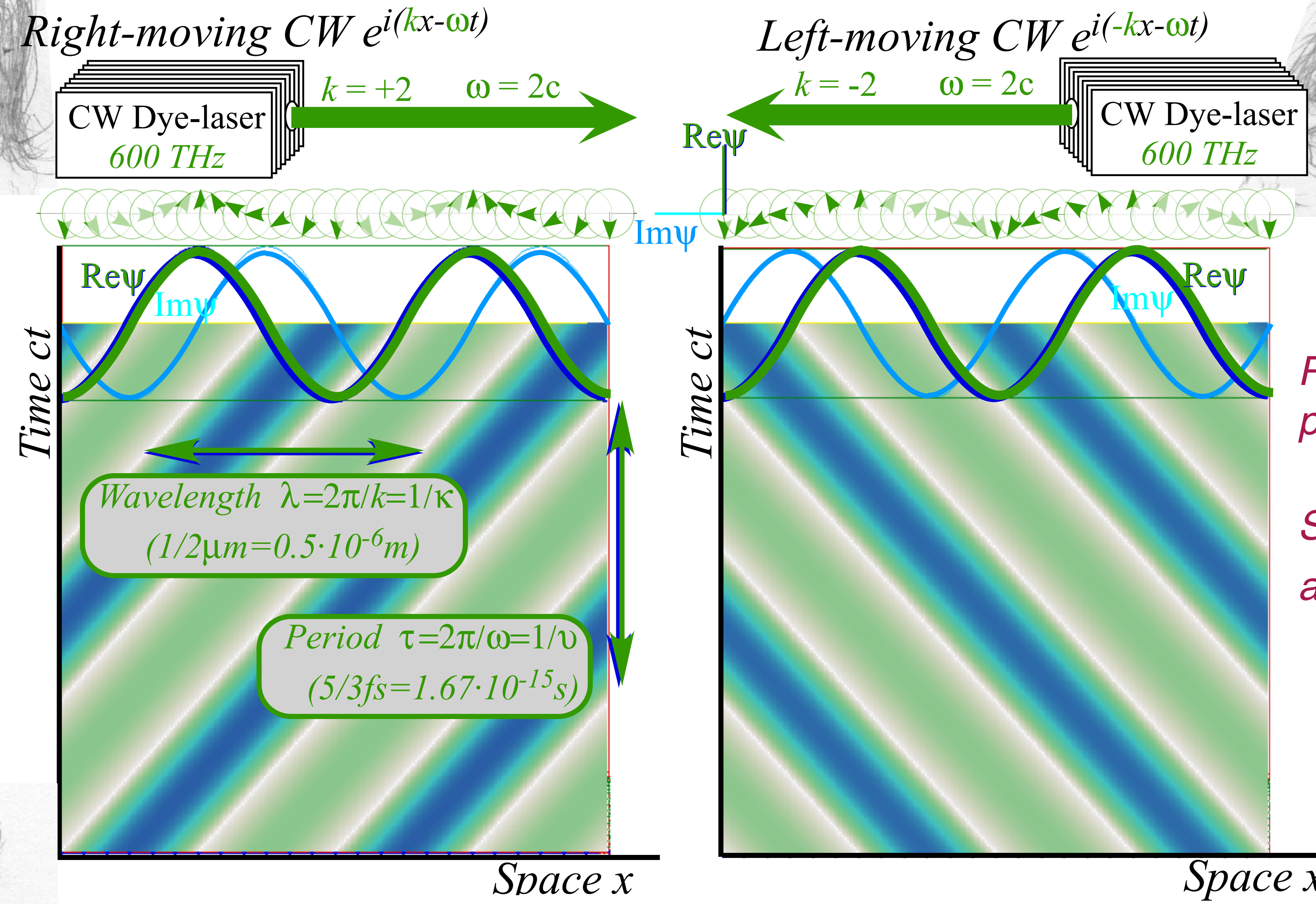
Bob:

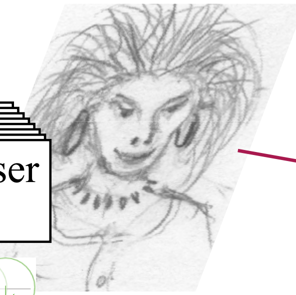
Cool!
You guys made me a space-time graph out of real zeros.

How'd it **do** that?



$$\Psi(x,t)=e^{ia}+e^{ib}, \quad a = kx-\omega t, \quad b = -kx-\omega t$$





Carla:

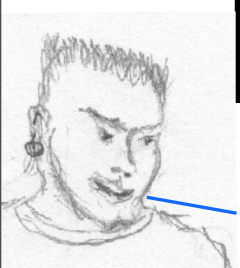
Easy!

You get zeros of any wave-sum $e^{ia}+e^{ib}$ by factoring it into *phase* and *group* parts.

Remember your algebra? Exponents of products add.

So, half-sum $\frac{a+b}{2}$ plus half-diff $\frac{a-b}{2}$ gives a,

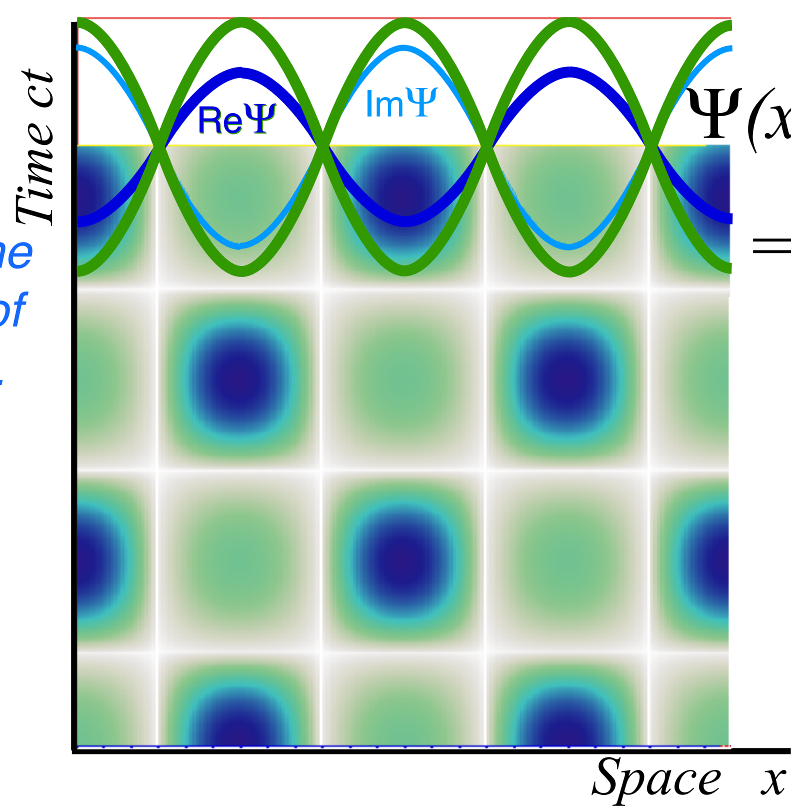
and half-sum $\frac{a+b}{2}$ minus half-diff $\frac{a-b}{2}$ gives b.



Bob:

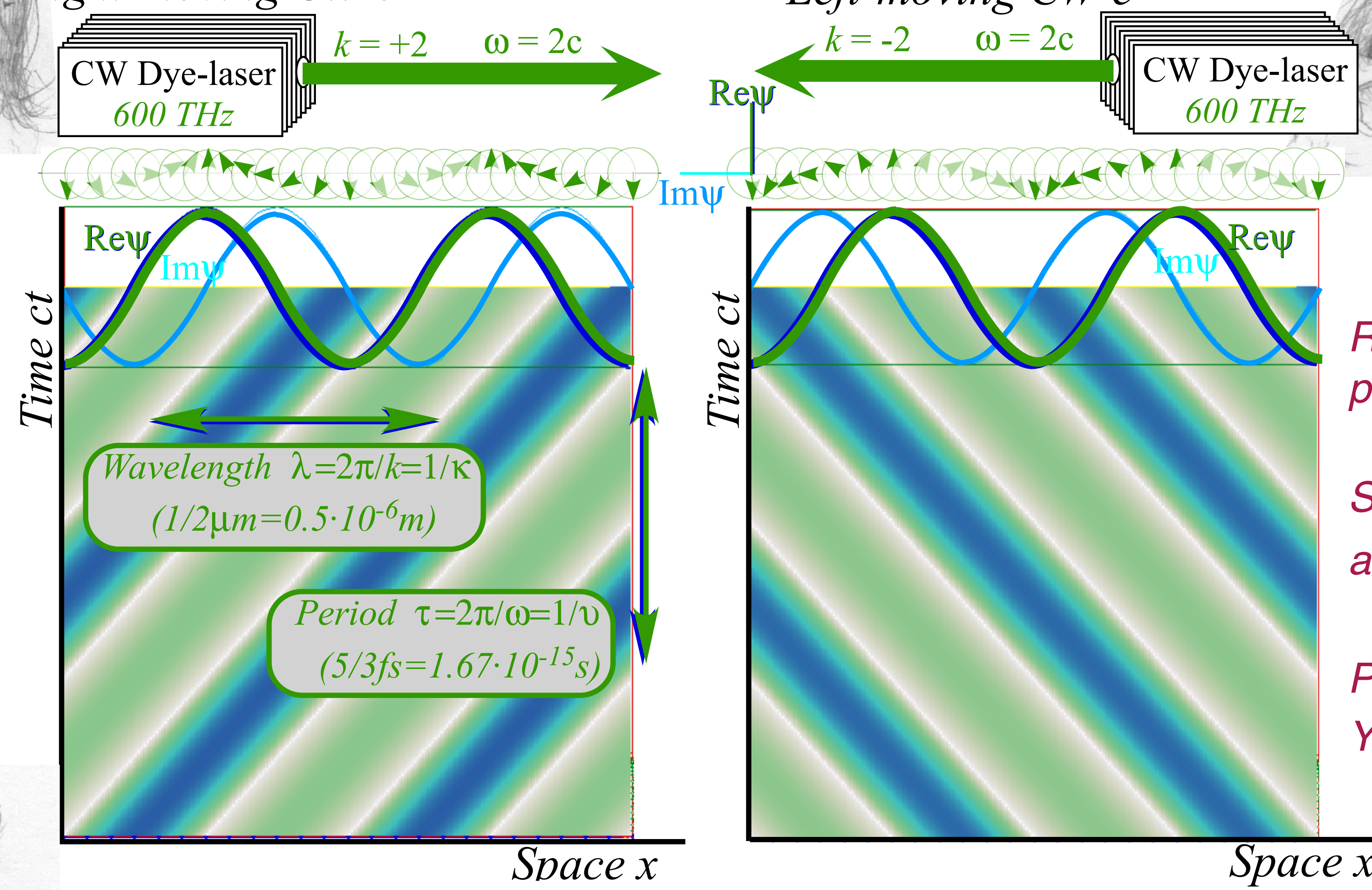
Cool!
You guys made me a space-time graph out of real zeros.

How'd it **do** that?



$$\Psi(x,t)=e^{ia}+e^{ib}, \quad a = kx-\omega t, \quad b = -kx-\omega t$$

$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$



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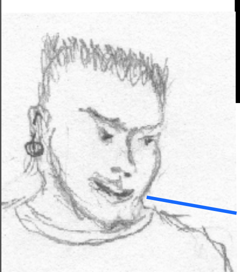
Presto!
You factor $e^{ia}+e^{ib}$ into $e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$



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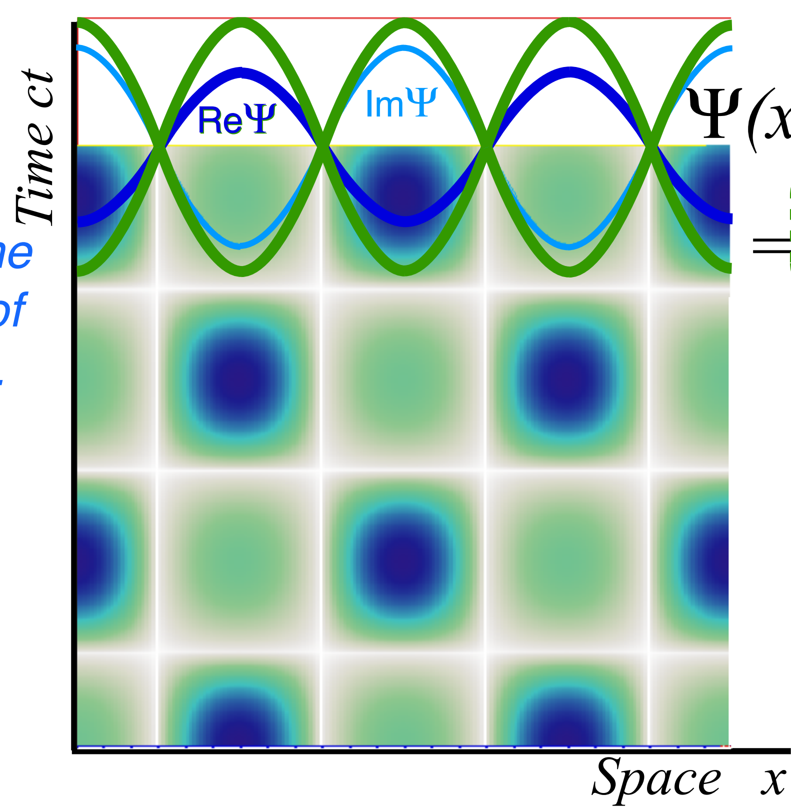
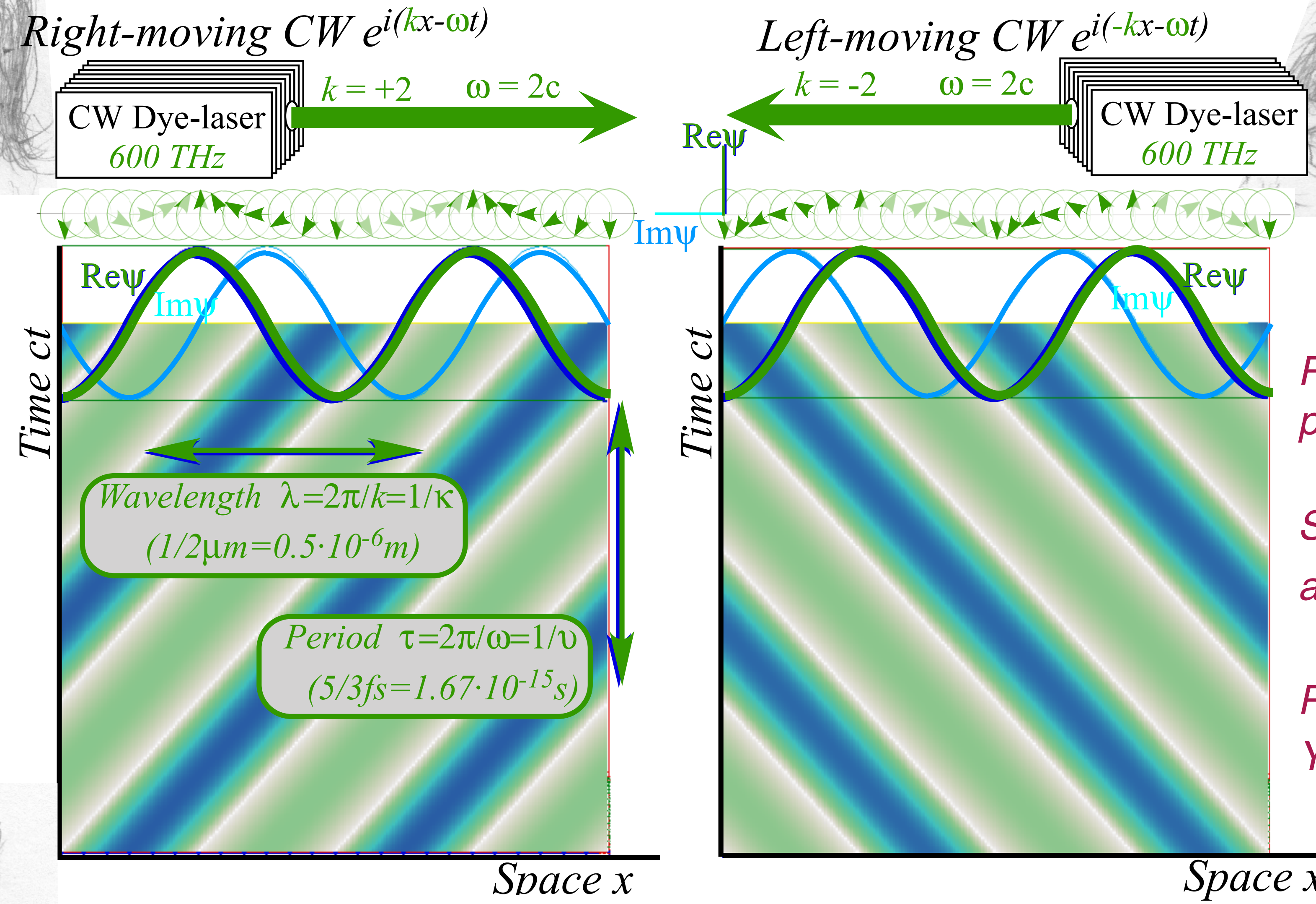
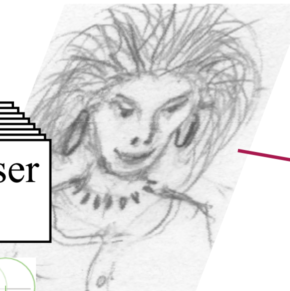
*How'd it **do** that?*



$$\Psi(x,t)=e^{ia}+e^{ib} \quad (a = kx-\omega t,\ b = -kx-\omega t)$$

$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$





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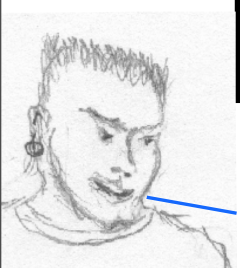
Alice 1CW phase: $a = kx-\omega t$
Carla 1CW phase: $b = -kx-\omega t$



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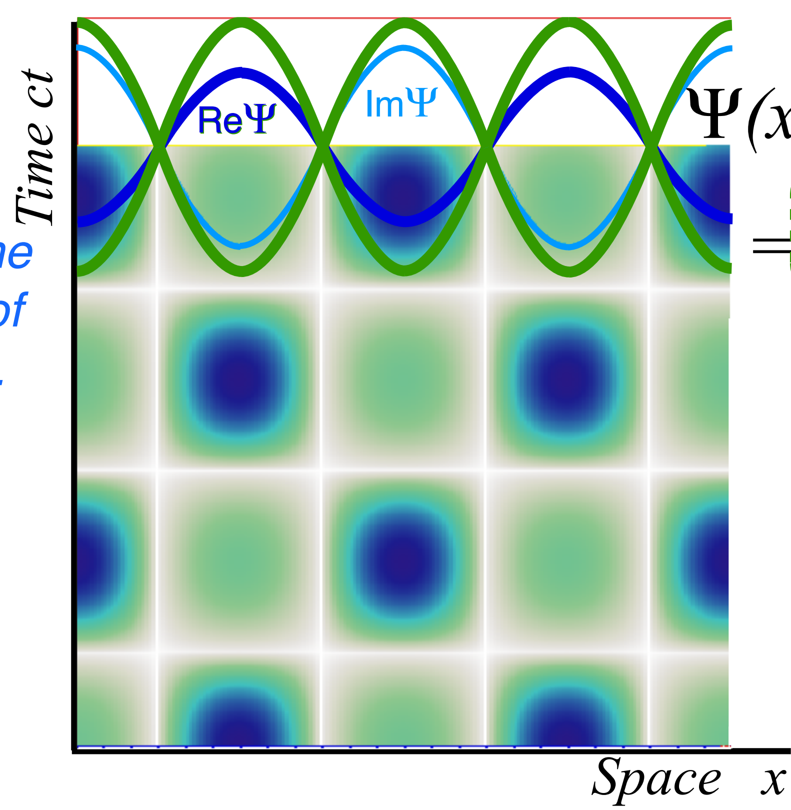
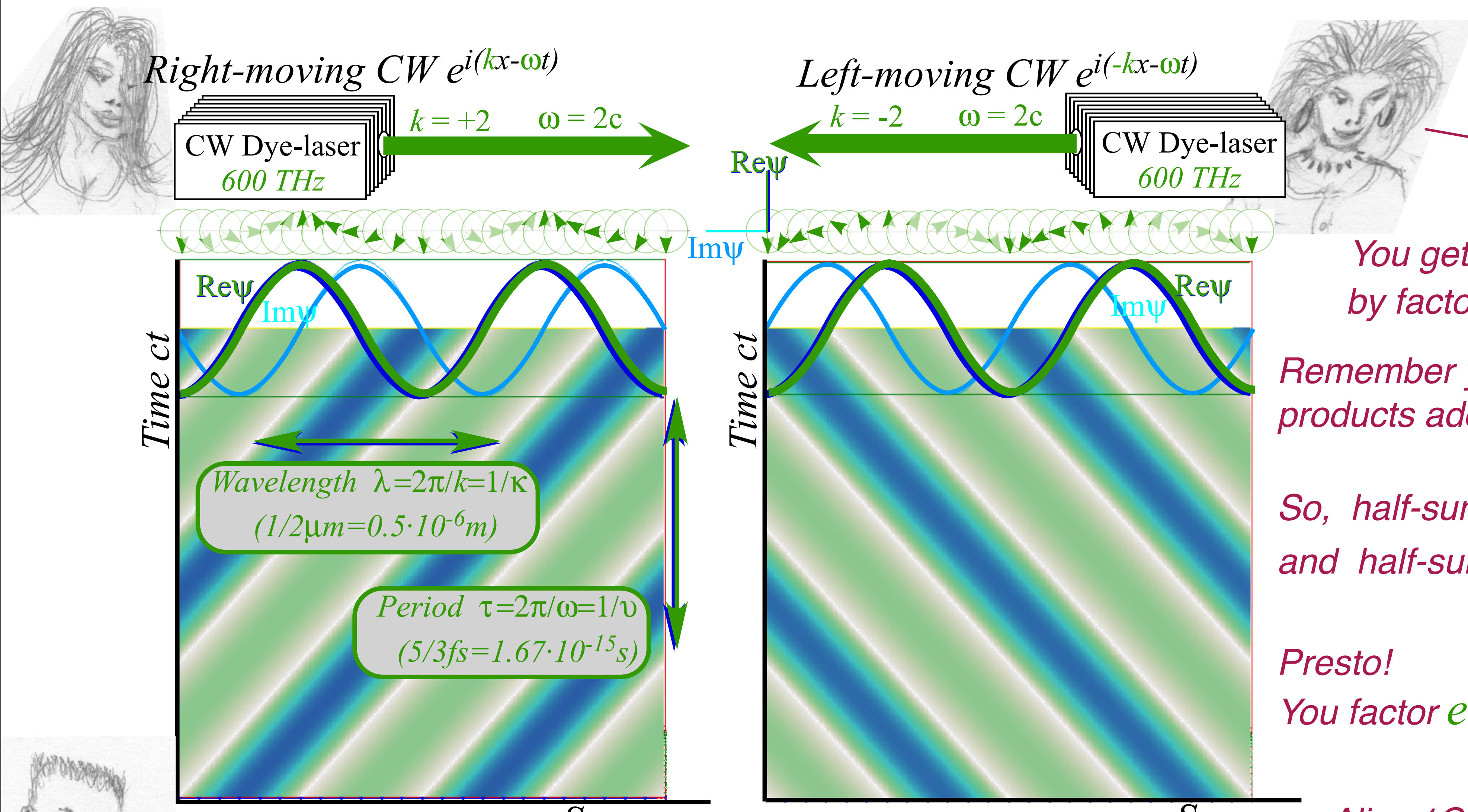
How'd it **do** that?



$$\Psi(x,t)=\underbrace{e^{ia}}_{kx-\omega t}+\underbrace{e^{ib}}_{-kx-\omega t}$$
$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$



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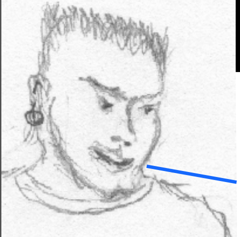

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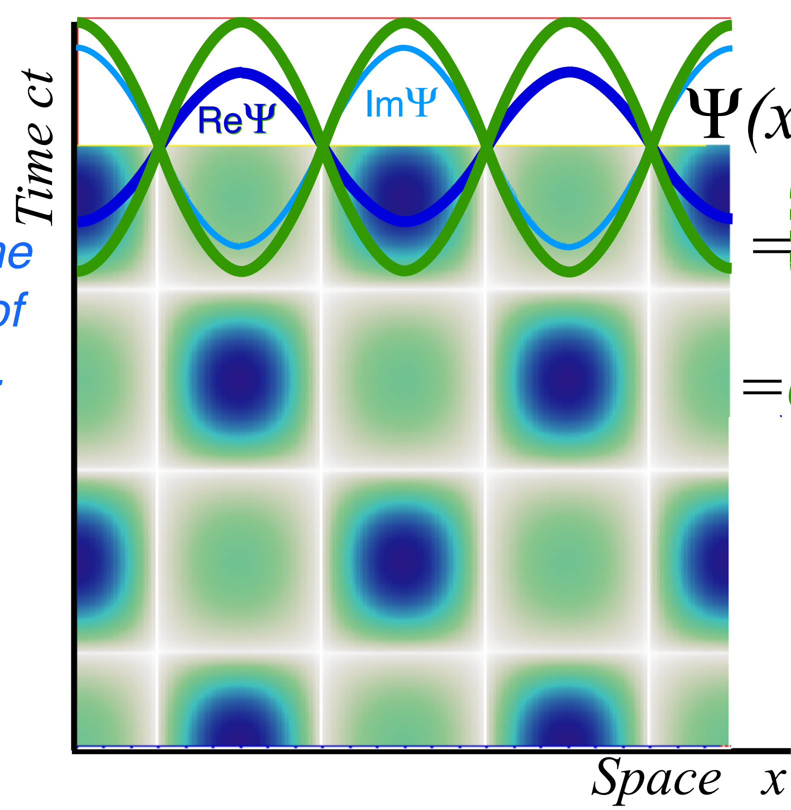
How'd it **do** that?



$$\Psi(x,t)=\underbrace{e^{ia}}_{kx-\omega t}+\underbrace{e^{ib}}_{-kx-\omega t}$$

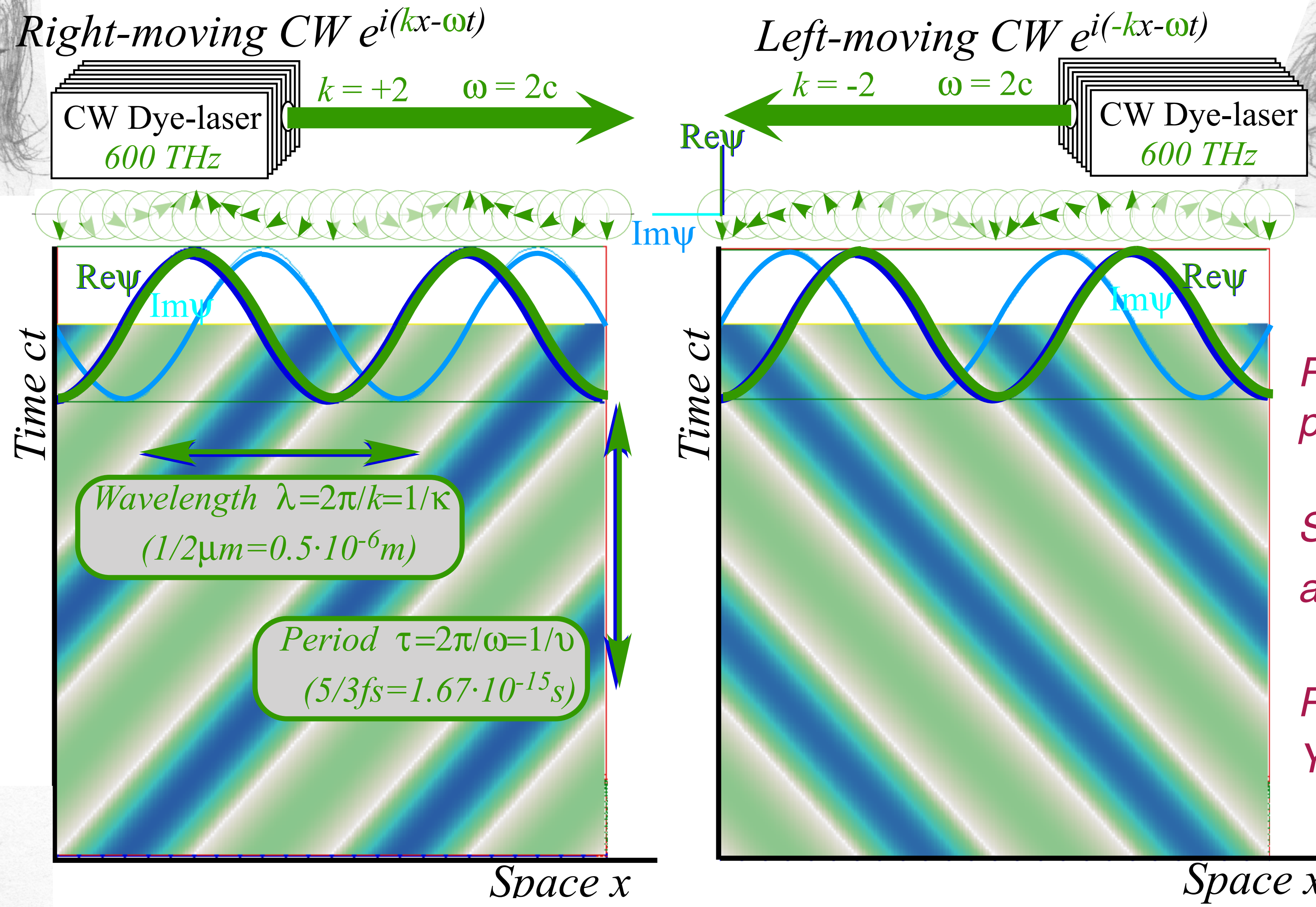
$$=e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

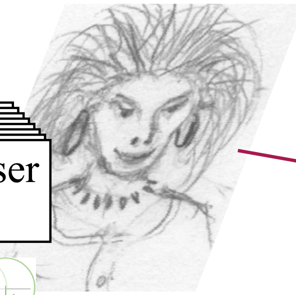
$$=e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)$$

Bob's 2CW **Group**-phase: $+k=\frac{a-b}{2}$
Wave

Group wave: $e^{-ikx}+e^{-ikx}=2\cos kx$

is *standing* wave (does not vary with time t)





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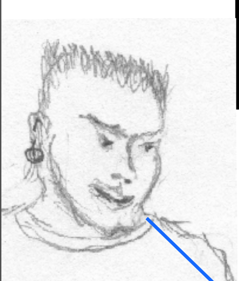
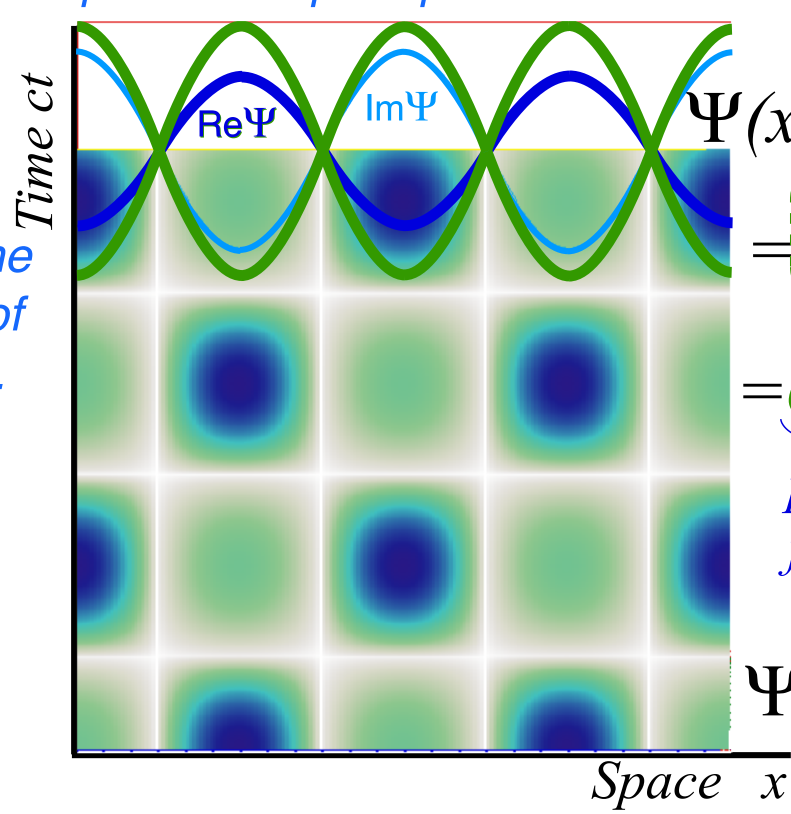


Bob: Let's plot this in per-spacetime?!

Cool!
You guys made me a space-time graph out of real zeros.

How'd it **do** that?



$$\Psi(x,t) = e^{ia} + e^{ib} \quad (a = kx-\omega t,\ b = -kx-\omega t)$$

$$= e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right)$$

$$= e^{-i\omega t}\left(e^{ikx}+e^{-ikx}\right)$$

phase factor $e^{-i\omega t}$, *group factor* $\left(e^{ikx}+e^{-ikx}\right)$

$$\Psi(x,t) = e^{-i\omega t}\, 2\cos kx$$

Bob's 2CW **Group Wave**-phase: $+k = \frac{a-b}{2}$

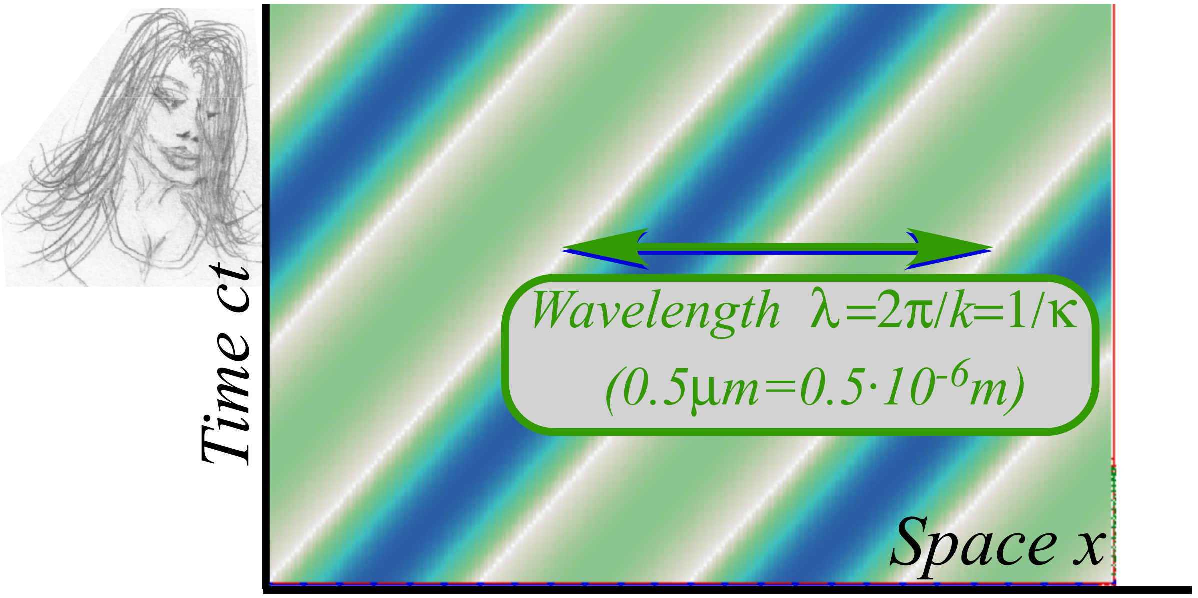
Group wave: $e^{-ikx} + e^{-ikx} = 2\cos kx$

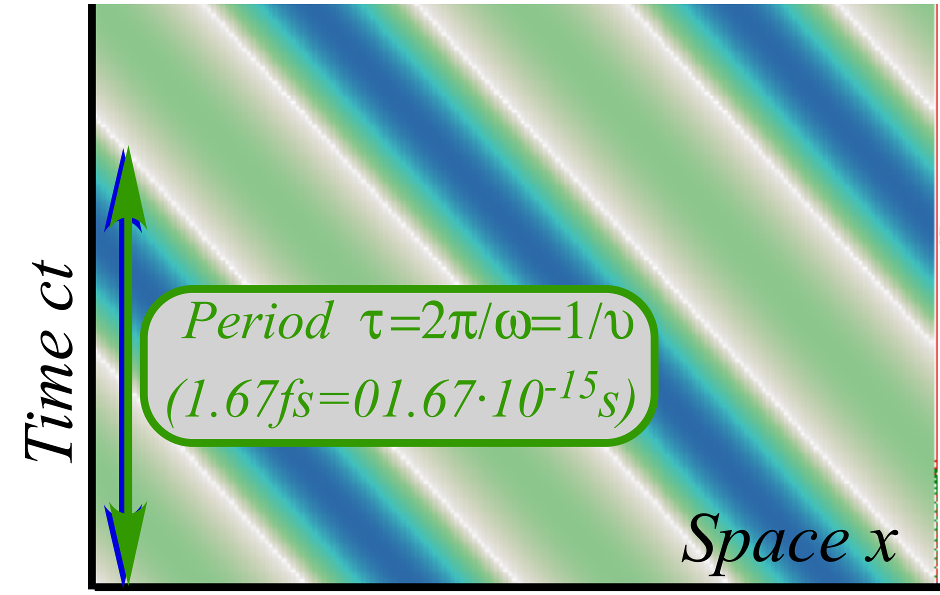
is *standing* wave (does not vary with time t)

Bob's 2CW **Phase Wave**-phase: $-\omega = \frac{a+b}{2}$

Phase wave real part: $\mathrm{Re}(e^{-i\omega t}) = \cos(\omega t)$

is *"instanton"* wave (does not vary in space x)



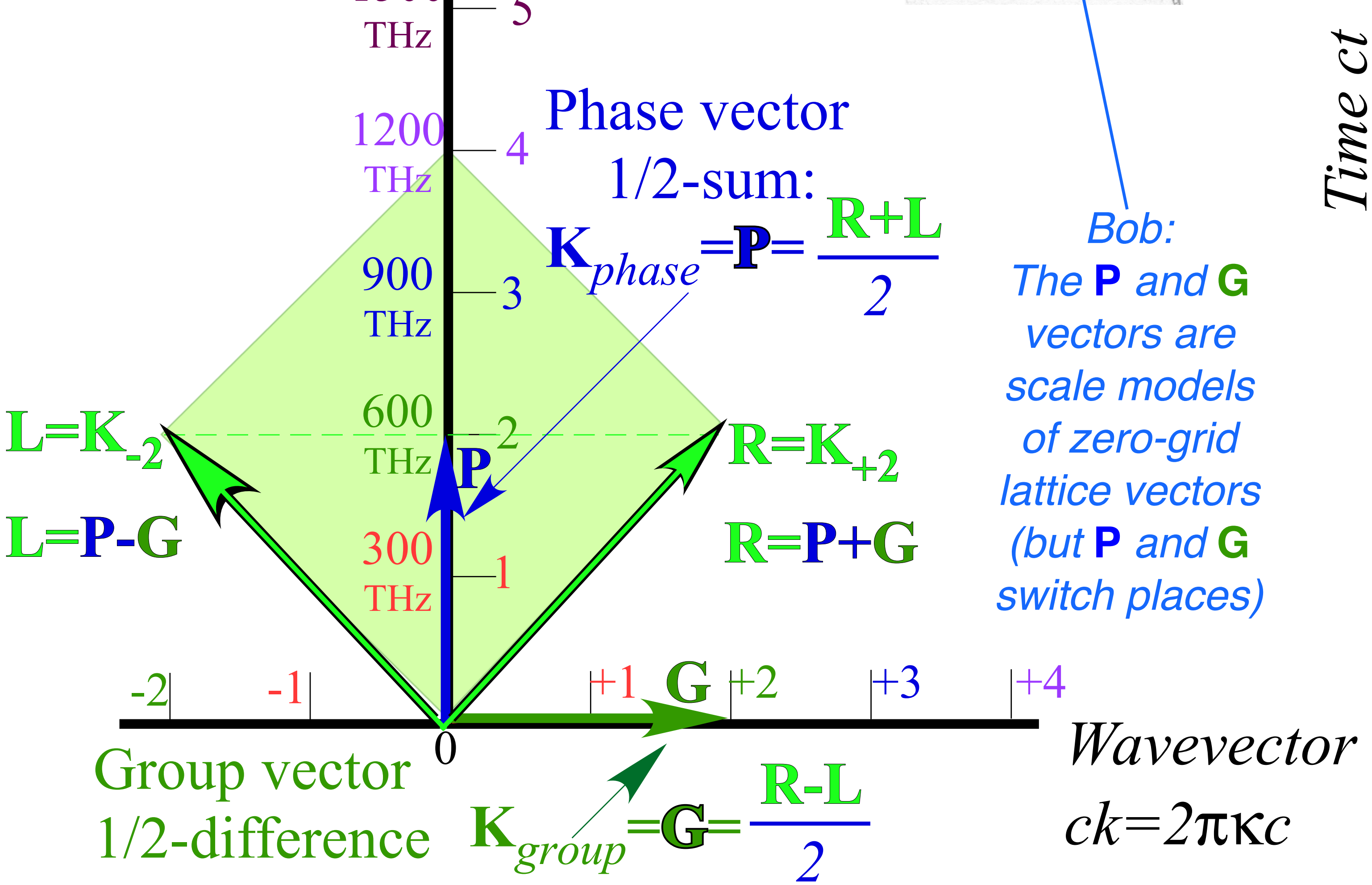


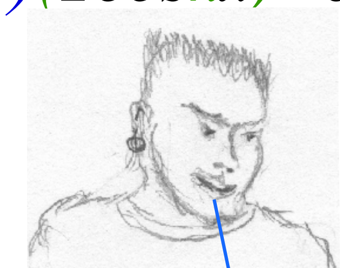
*Carla: OK, Bob! It looks like a baseball diamond with **P** at Pitcher's mound and **G** at the Grandstand*. I'm on 1st base! (**R**)*

**Thanks, Woody!*

$$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$$

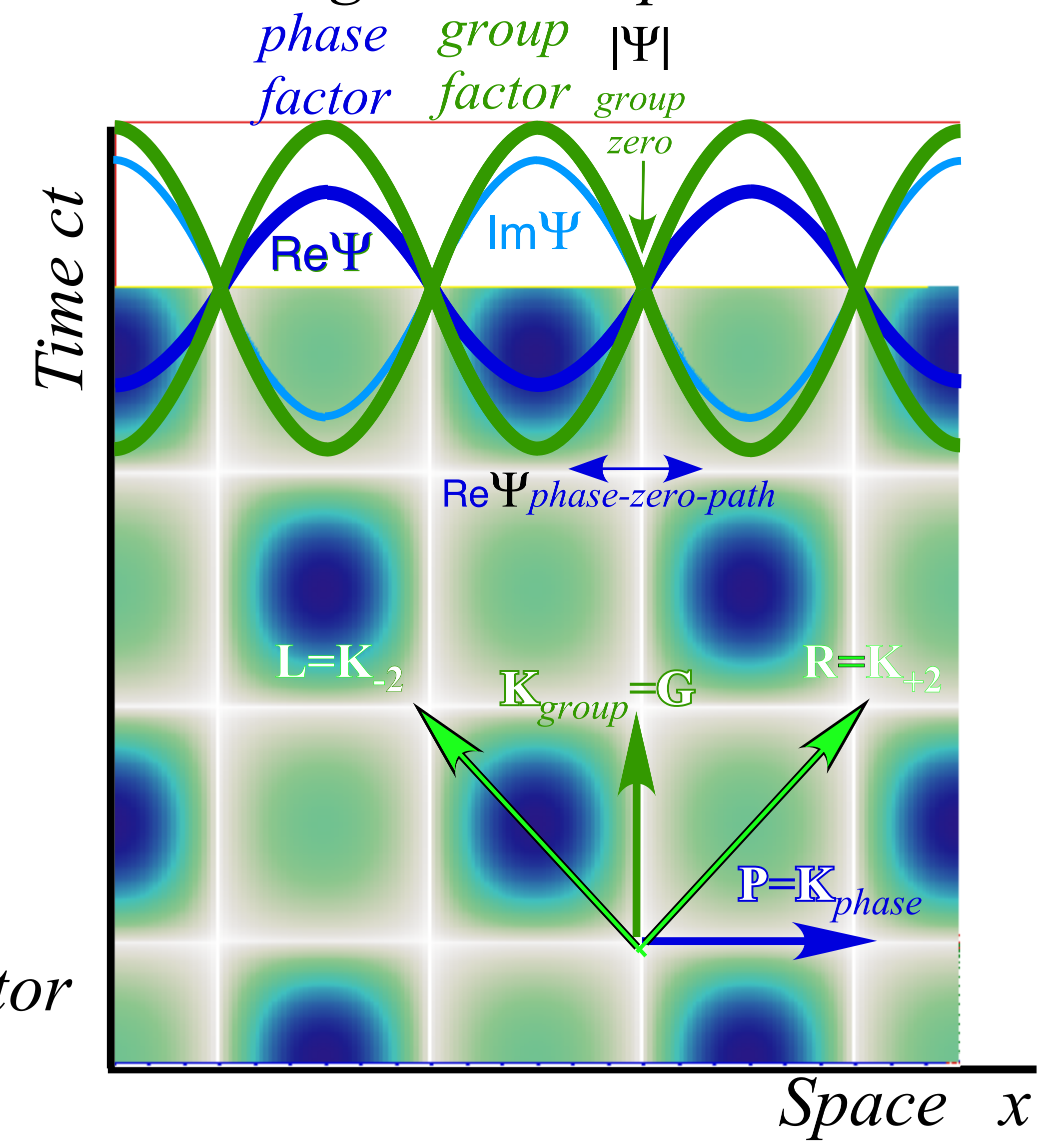
Standing 2CW in per-space-time

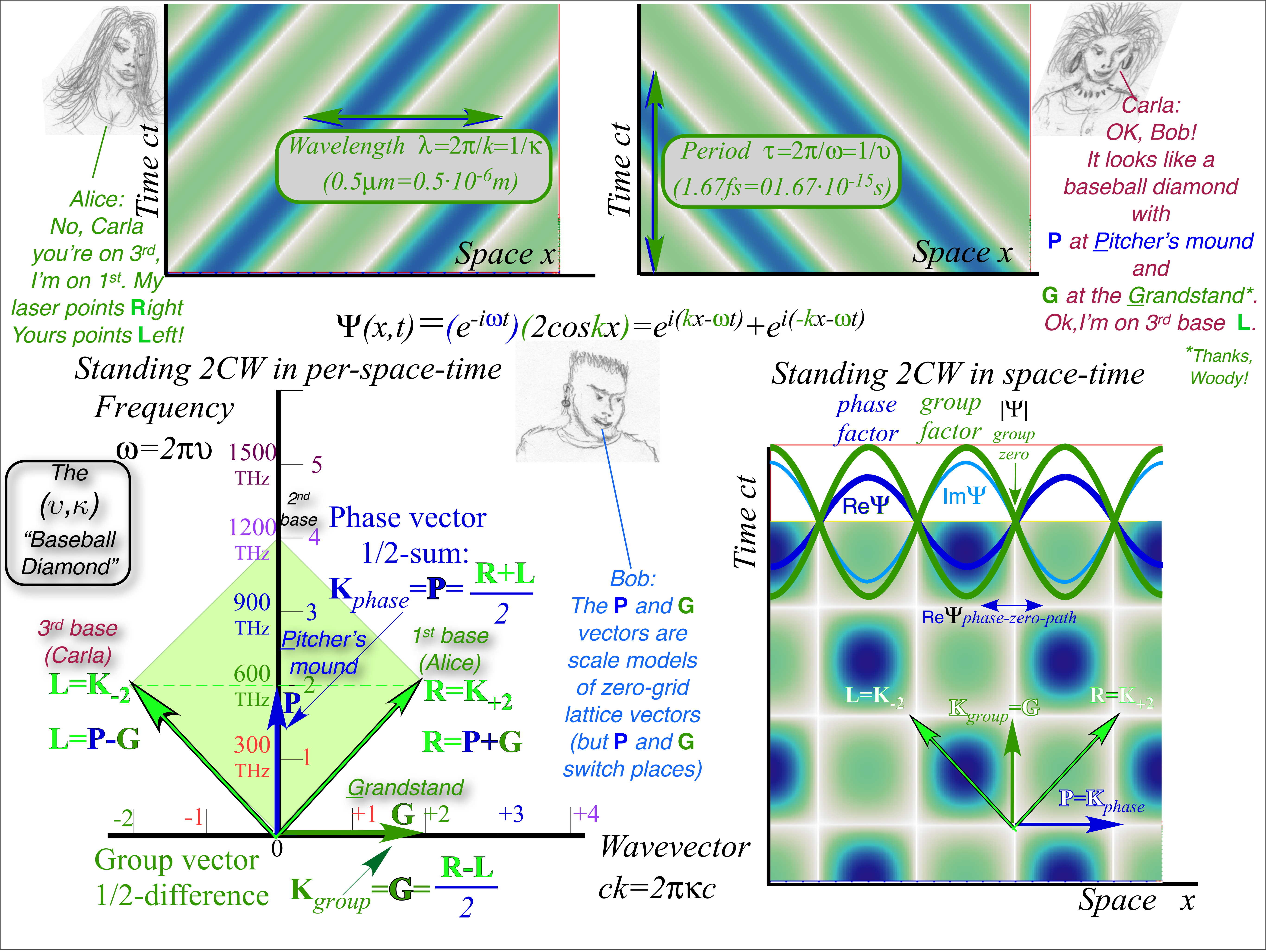




*Bob: The **P** and **G** vectors are scale models of zero-grid lattice vectors (but **P** and **G** switch places)*

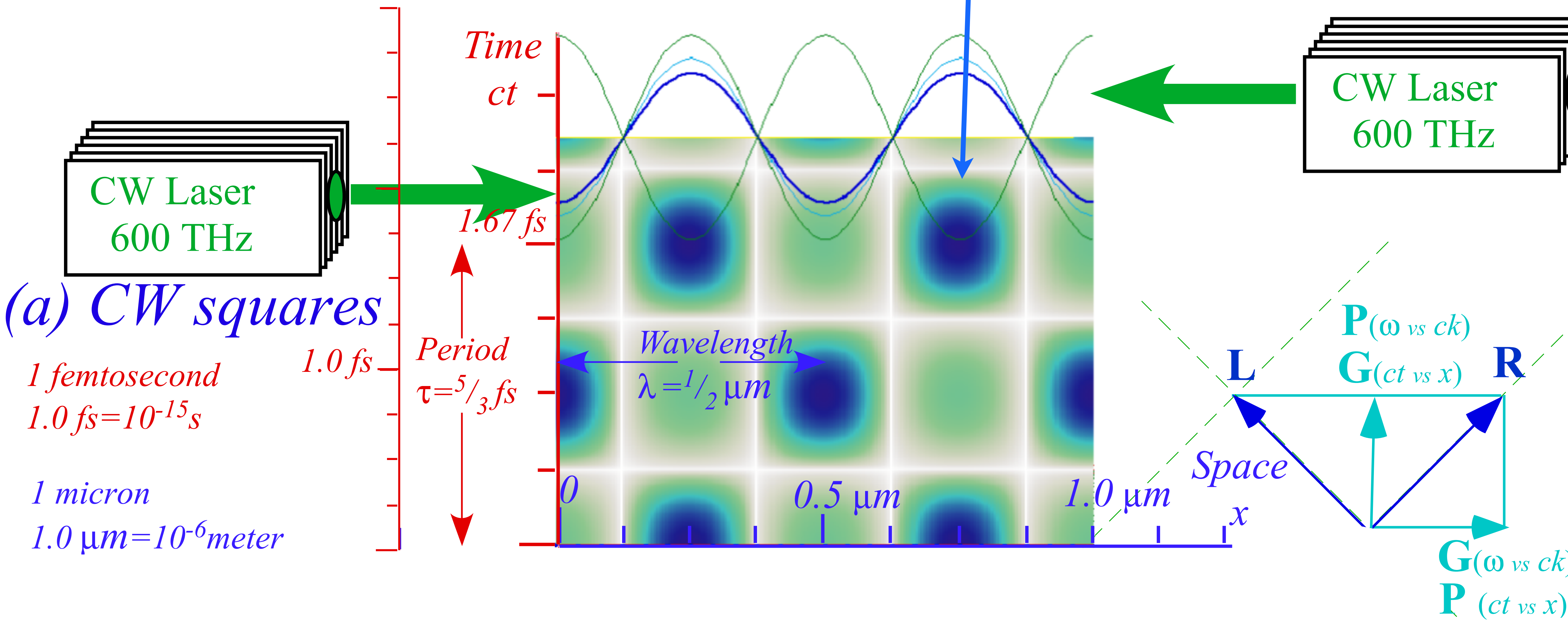
Standing 2CW in space-time



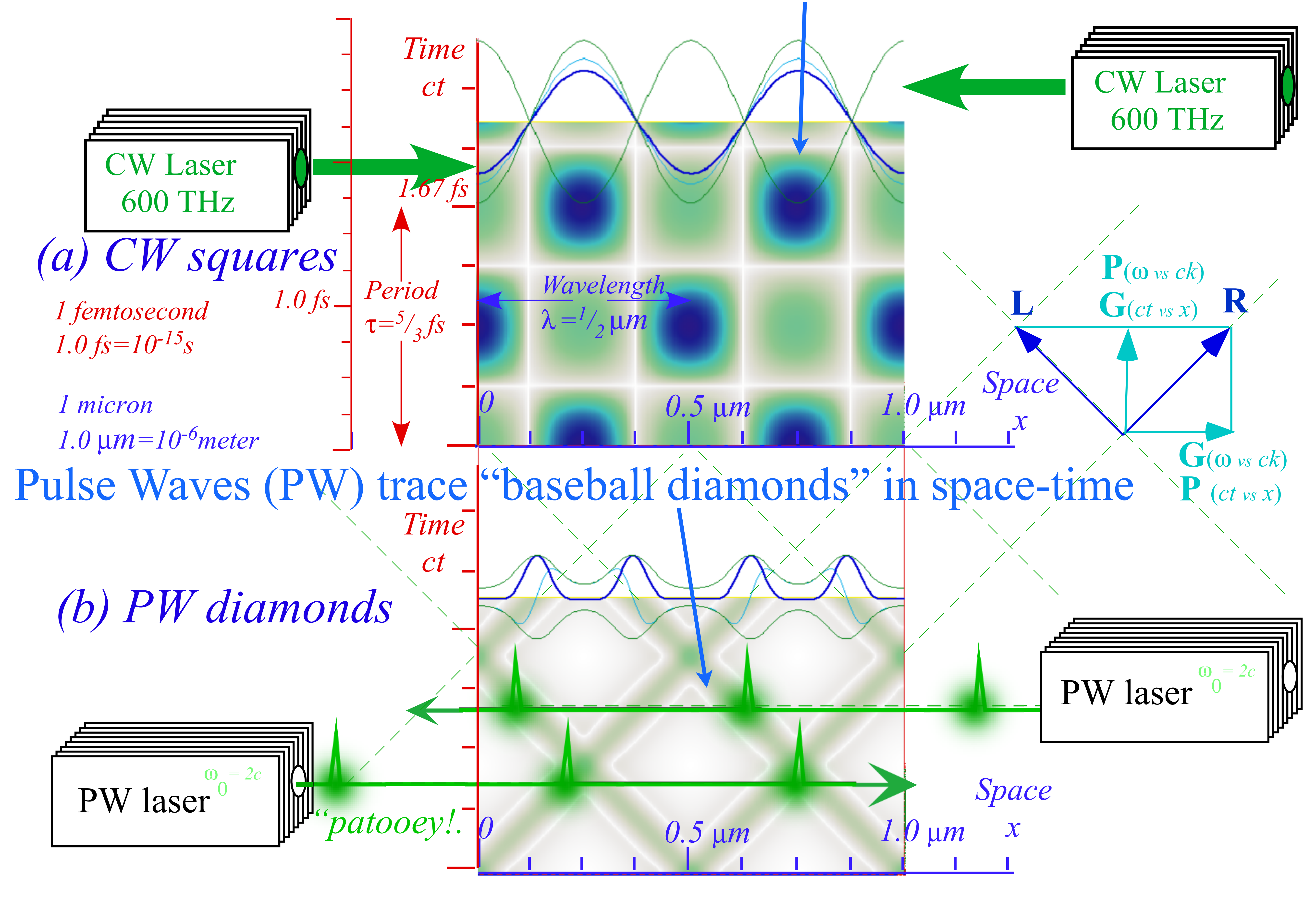


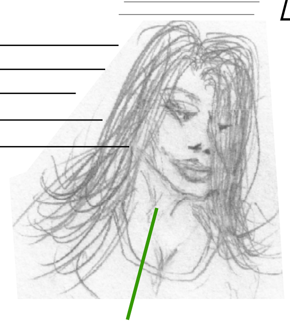
Time ct
Wavelength λ=2π/k=1/κ
(0.5μm=0.5·10^-6m)
Space x
Time ct
Period τ=2π/ω=1/υ
(1.67fs=01.67·10^-15s)
Space x
Alice:
No, Carla
you're on 3rd,
I'm on 1st. My
laser points Right
Yours points Left!
Carla:
OK, Bob!
It looks like a
baseball diamond
with
P at Pitcher's mound
and
G at the Grandstand*.
Ok,I'm on 3rd base L.
*Thanks,
Woody!
$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$
Standing 2CW in per-space-time
Frequency
ω=2πυ
1500 THz
5
1200 THz
4
2nd base
900 THz
3
600 THz
2
300 THz
1
The (υ,κ) "Baseball Diamond"
Phase vector
1/2-sum:
$K_{phase}=P=\frac{R+L}{2}$
3rd base
(Carla)
L=K_-2
L=P-G
Pitcher's
mound
P
1st base
(Alice)
R=K_+2
R=P+G
Grandstand
-2
-1
0
+1
G
+2
+3
+4
Group vector
1/2-difference
$K_{group}=G=\frac{R-L}{2}$
Wavevector
ck=2πκc
Bob:
The P and G
vectors are
scale models
of zero-grid
lattice vectors
(but P and G
switch places)
Standing 2CW in space-time
phase
factor
group
factor
|Ψ|
group
zero
ReΨ
ImΨ
ReΨ phase-zero-path
Time ct
L=K_-2
K_group=G
R=K_+2
P=K_phase
Space x

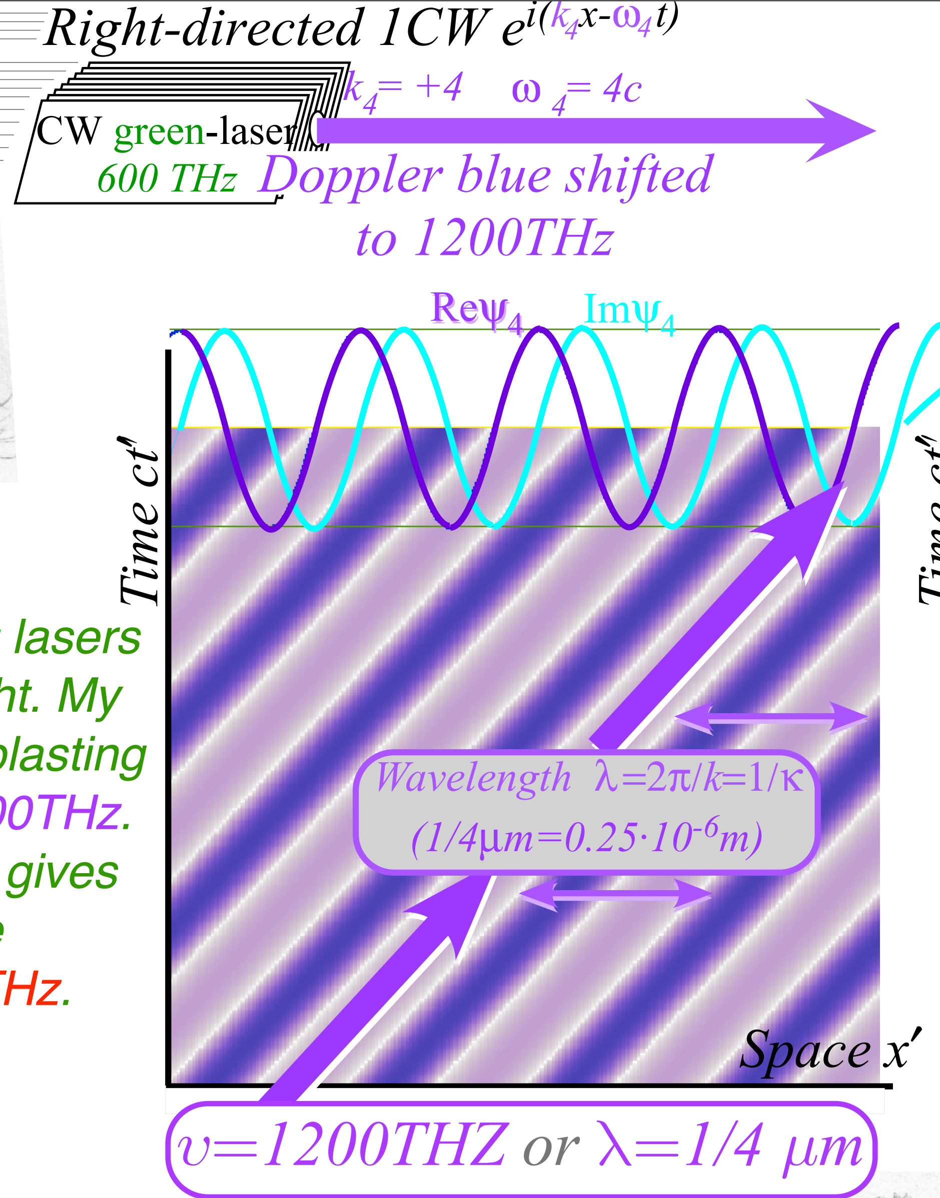
Continuous Waves (CW) trace "Cartesian squares" in space-time

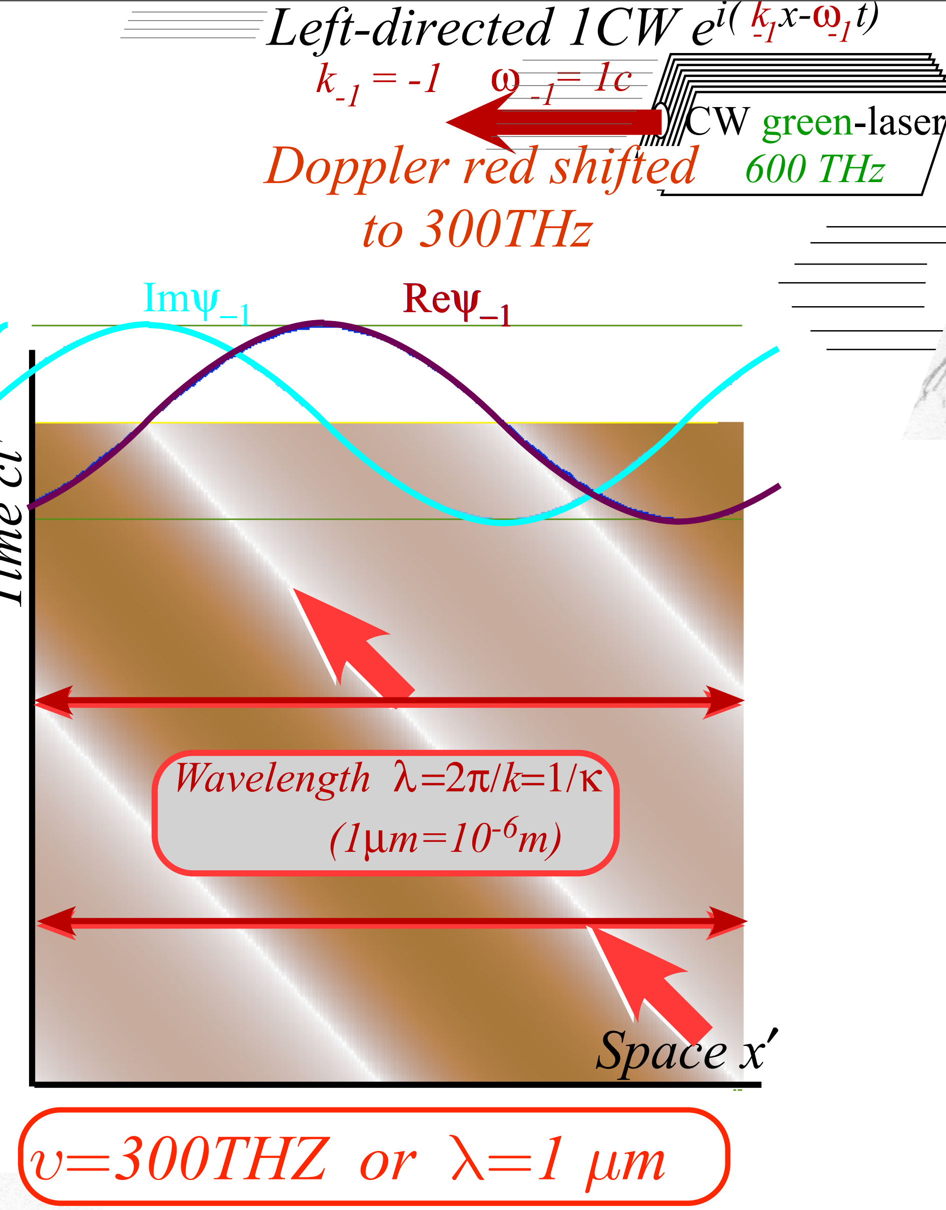


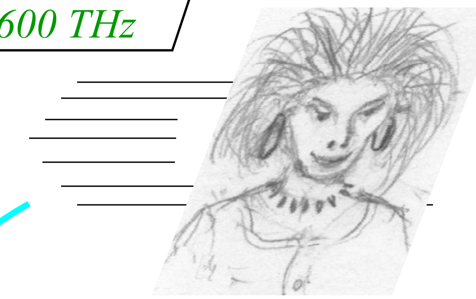
Continuous Waves (CW) trace "Cartesian squares" in space-time



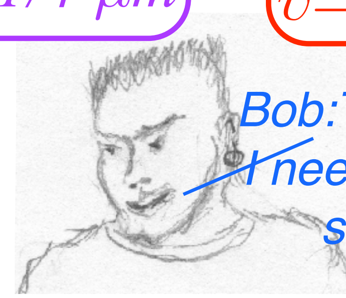






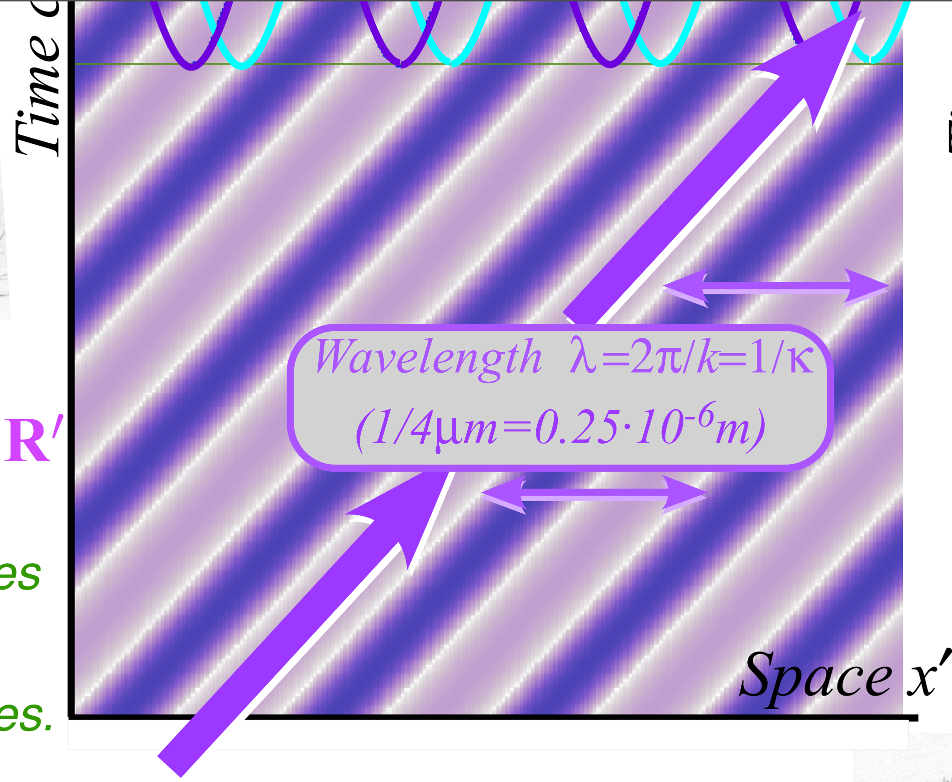


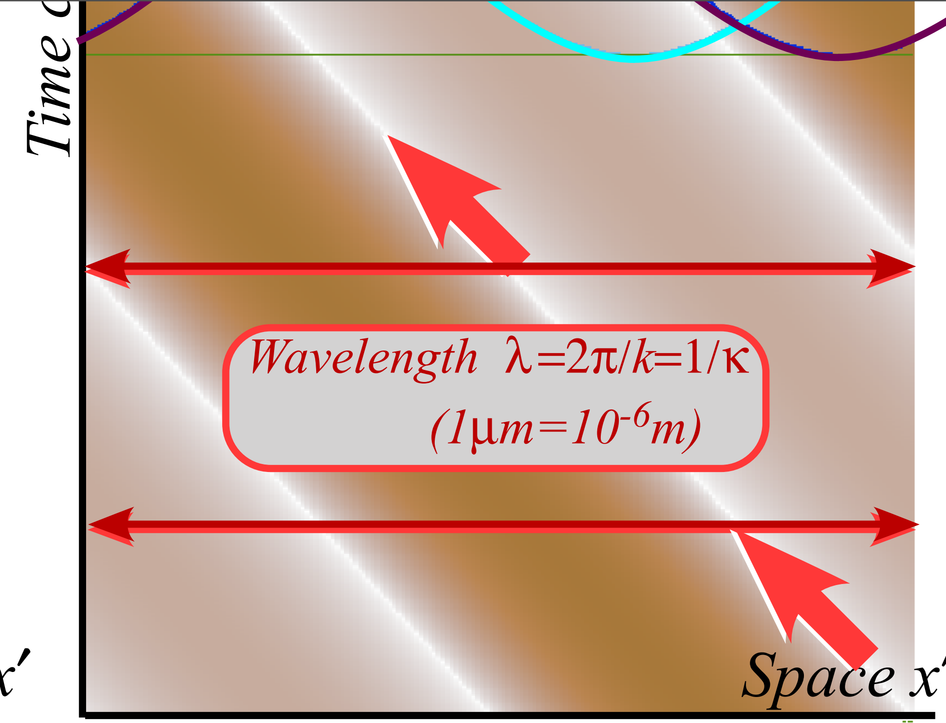
Alice:
Now our 600THz lasers move left-to-right. My 600THz laser is blasting you with UV 1200THz. Carla's 600THz gives you a nice infrared 300THz.

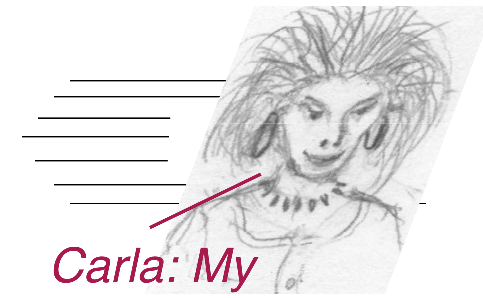


Bob:That UV burns! I need to put on my sunglasses.

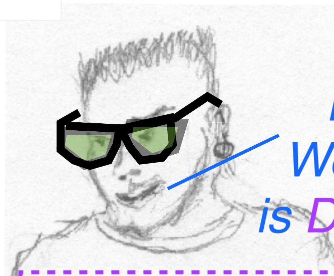
Alice: OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You'll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



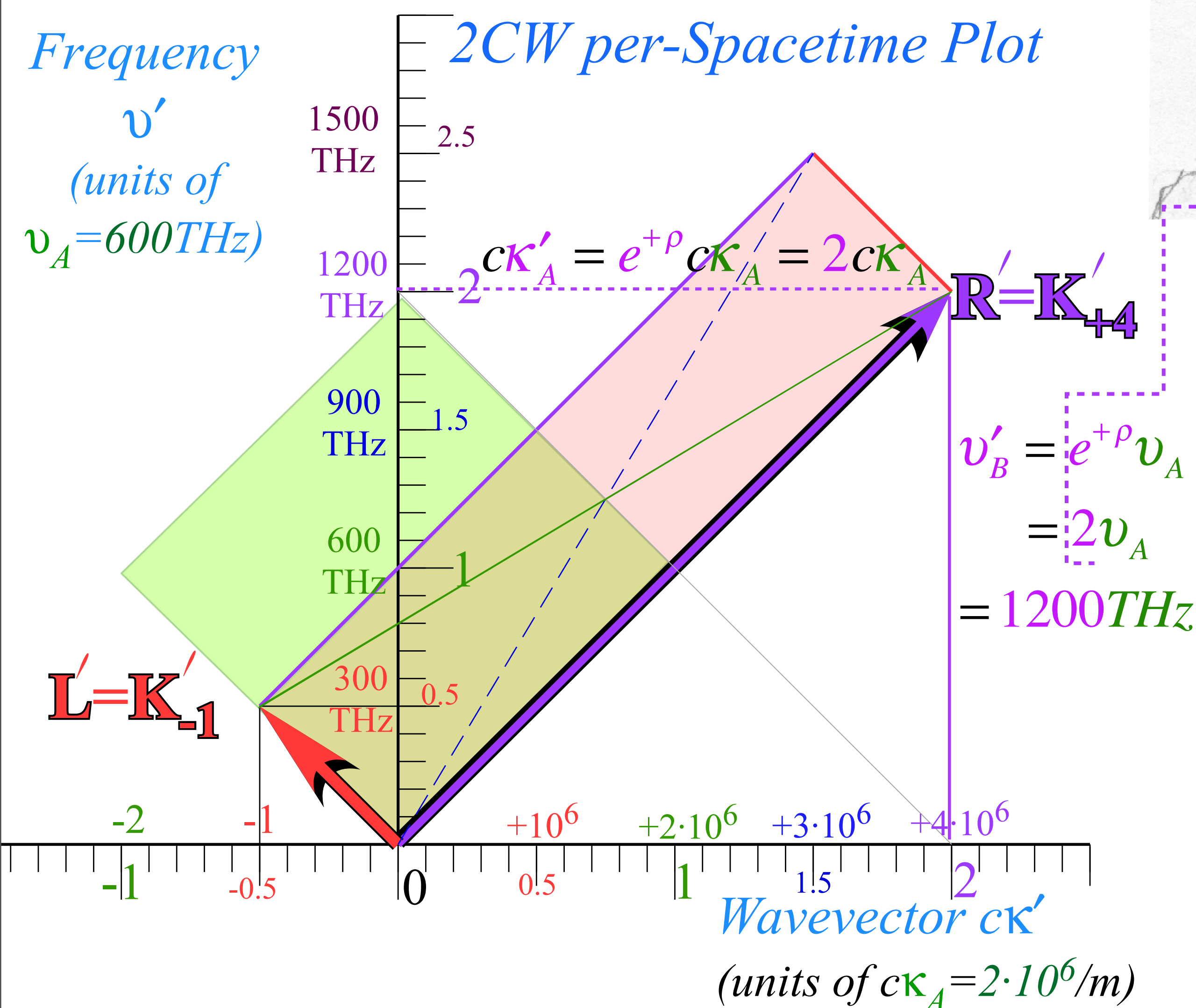


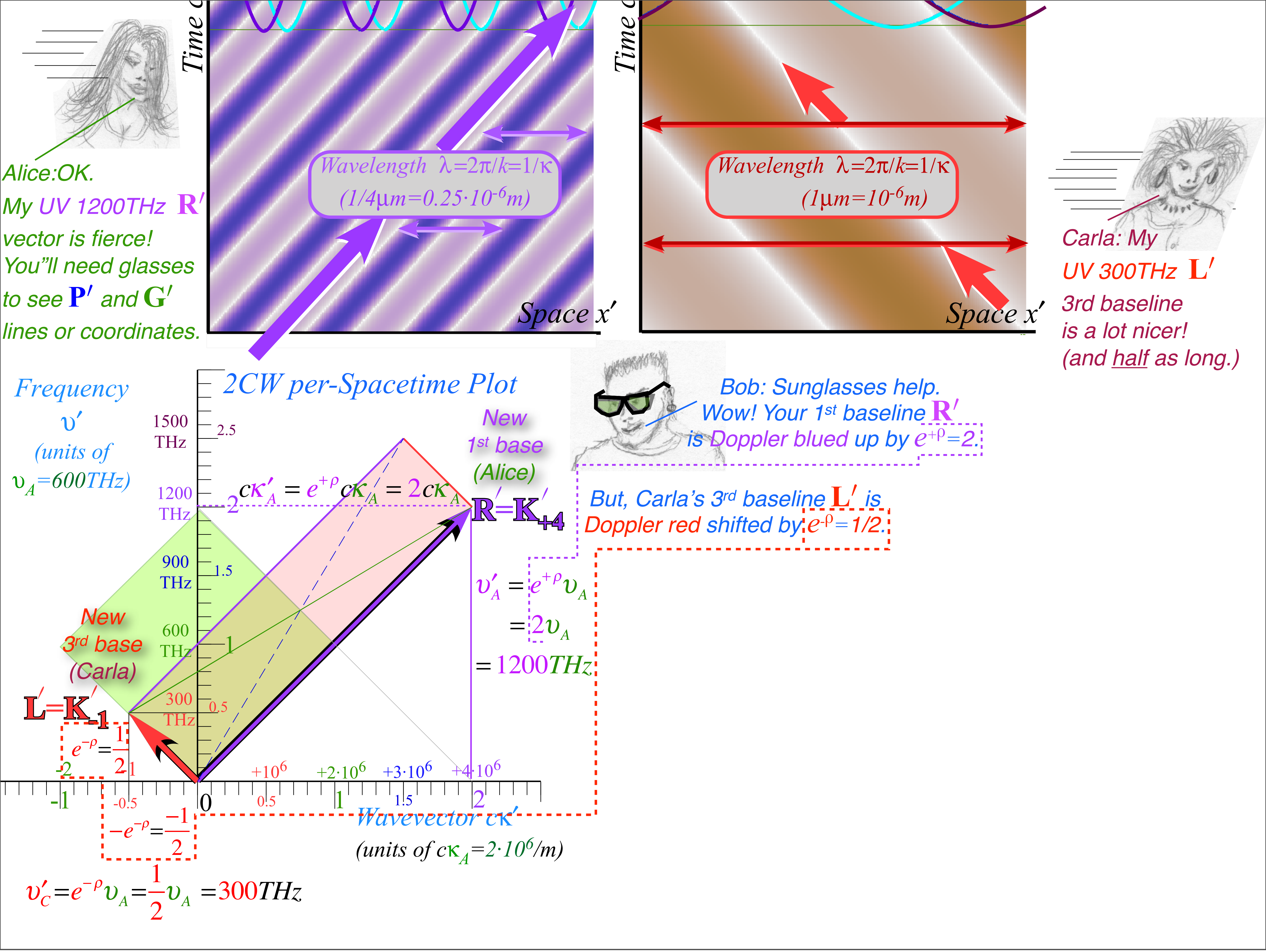


Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer!

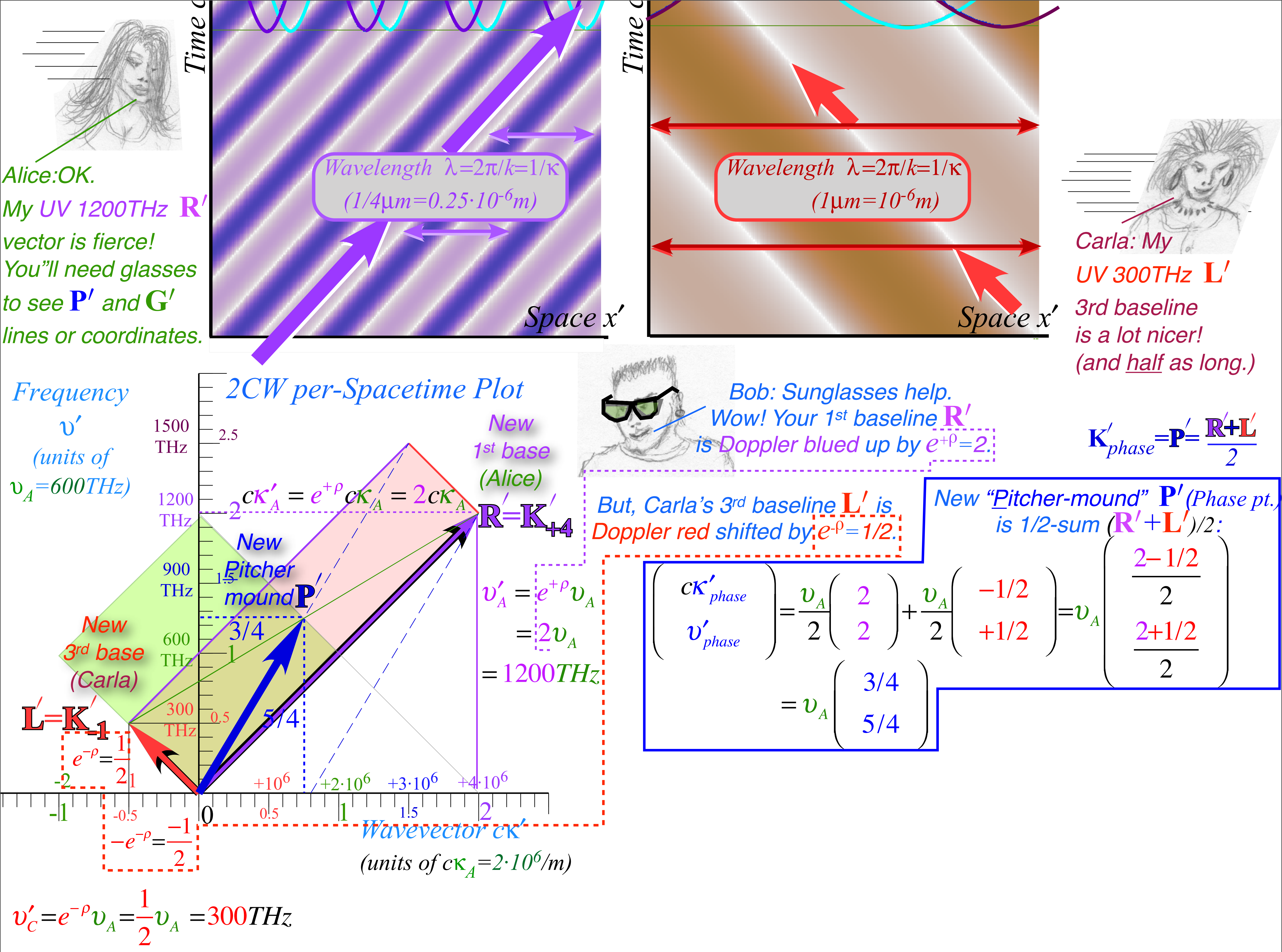


Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.





Time
Space x′
Wavelength λ=2π/k=1/κ
(1/4μm=0.25·10^-6m)
Wavelength λ=2π/k=1/κ
(1μm=10^-6m)
Alice:OK.
My UV 1200THz R′
vector is fierce!
You"ll need glasses
to see P′ and G′
lines or coordinates.
Carla: My
UV 300THz L′
3rd baseline
is a lot nicer!
(and half as long.)
2CW per-Spacetime Plot
Frequency
υ′
(units of
υ_A=600THz)
1500 THz
1200 THz
900 THz
600 THz
300 THz
cκ′_A = e^+ρ cκ_A = 2cκ_A
New
1st base
(Alice)
R′=K′_+4
Bob: Sunglasses help.
Wow! Your 1st baseline R′
is Doppler blued up by e^+ρ=2.
But, Carla's 3rd baseline L′ is
Doppler red shifted by e^-ρ=1/2.
υ′_A = e^+ρ υ_A
= 2υ_A
= 1200THz
New
3rd base
(Carla)
L′=K′_-1
e^-ρ = 1/2
-e^-ρ = -1/2
+10^6
+2·10^6
+3·10^6
+4·10^6
Wavevector cκ′
(units of cκ_A=2·10^6/m)
υ′_C = e^-ρ υ_A = 1/2 υ_A = 300THz



Time
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(1/4μm=0.25·10^-6m)
Space x′
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Wavelength λ=2π/k=1/κ
(1μm=10^-6m)
Space x′
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υ′
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1500 THz
1200 THz
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600 THz
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cκ′_A = e^{+ρ} cκ_A = 2cκ_A
New
1st base
(Alice)
R′=K′_{+4}
New
Pitcher
mound P′
3/4
5/4
New
3rd base
(Carla)
L′=K′_{-1}
e^{-ρ} = 1/2
−e^{-ρ} = −1/2
+10^6
+2·10^6
+3·10^6
+4·10^6
Wavevector cκ′
(units of cκ_A=2·10^6/m)
υ′_A = e^{+ρ} υ_A
= 2υ_A
= 1200THz
υ′_C = e^{-ρ} υ_A = 1/2 υ_A = 300THz
Bob: Sunglasses help.
Wow! Your 1st baseline R′
is Doppler blued up by e^{+ρ}=2.
But, Carla's 3rd baseline L′ is
Doppler red shifted by e^{-ρ}=1/2.
K′_phase = P′ = (R′+L′)/2
New "Pitcher-mound" P′ (Phase pt.)
is 1/2-sum (R′+L′)/2:
(cκ′_phase, υ′_phase) = υ_A/2 (2, 2) + υ_A/2 (−1/2, +1/2) = υ_A ((2−1/2)/2, (2+1/2)/2)
= υ_A (3/4, 5/4)

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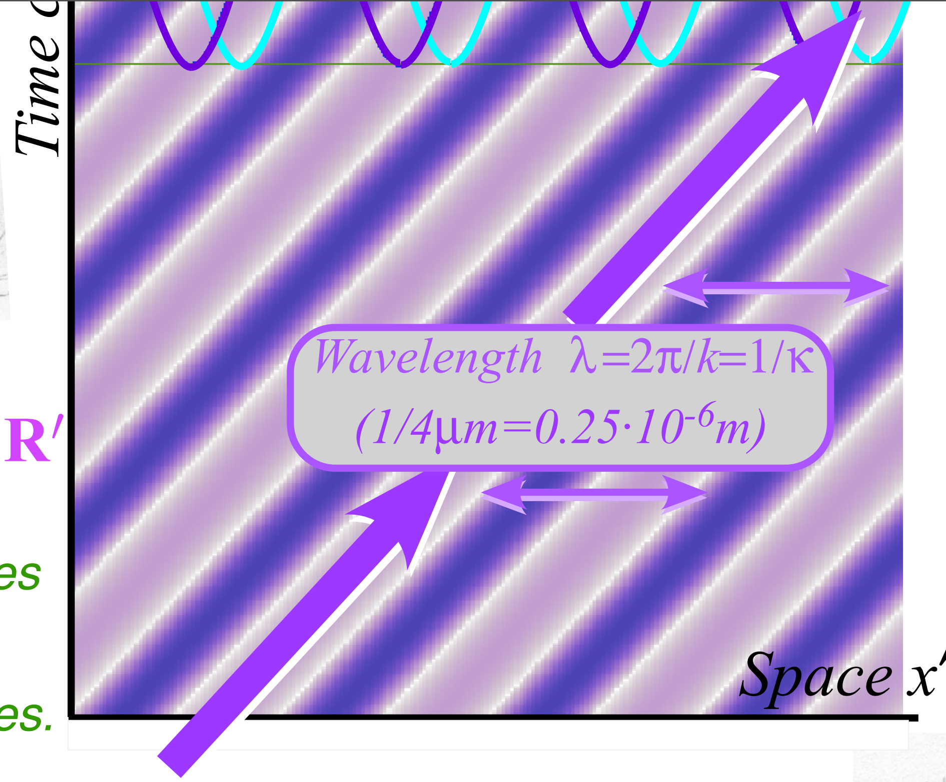
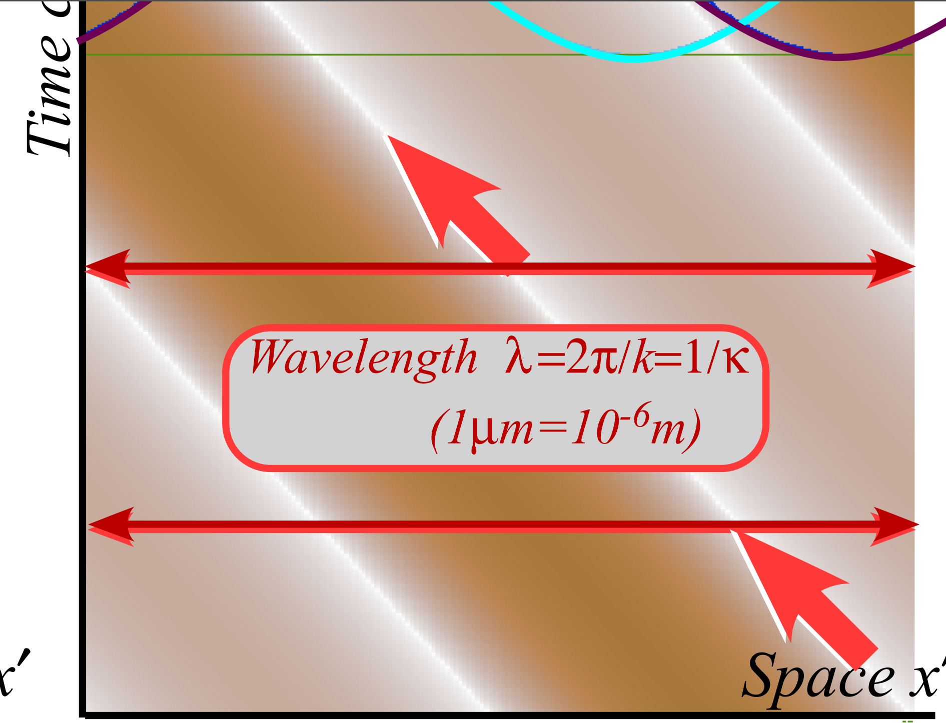
My UV 1200THz $\mathbf{R}'$
vector is fierce!
You"ll need glasses
to see $\mathbf{P}'$ and $\mathbf{G}'$
lines or coordinates.





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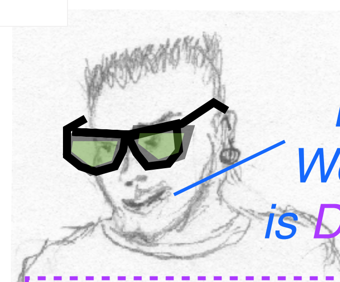
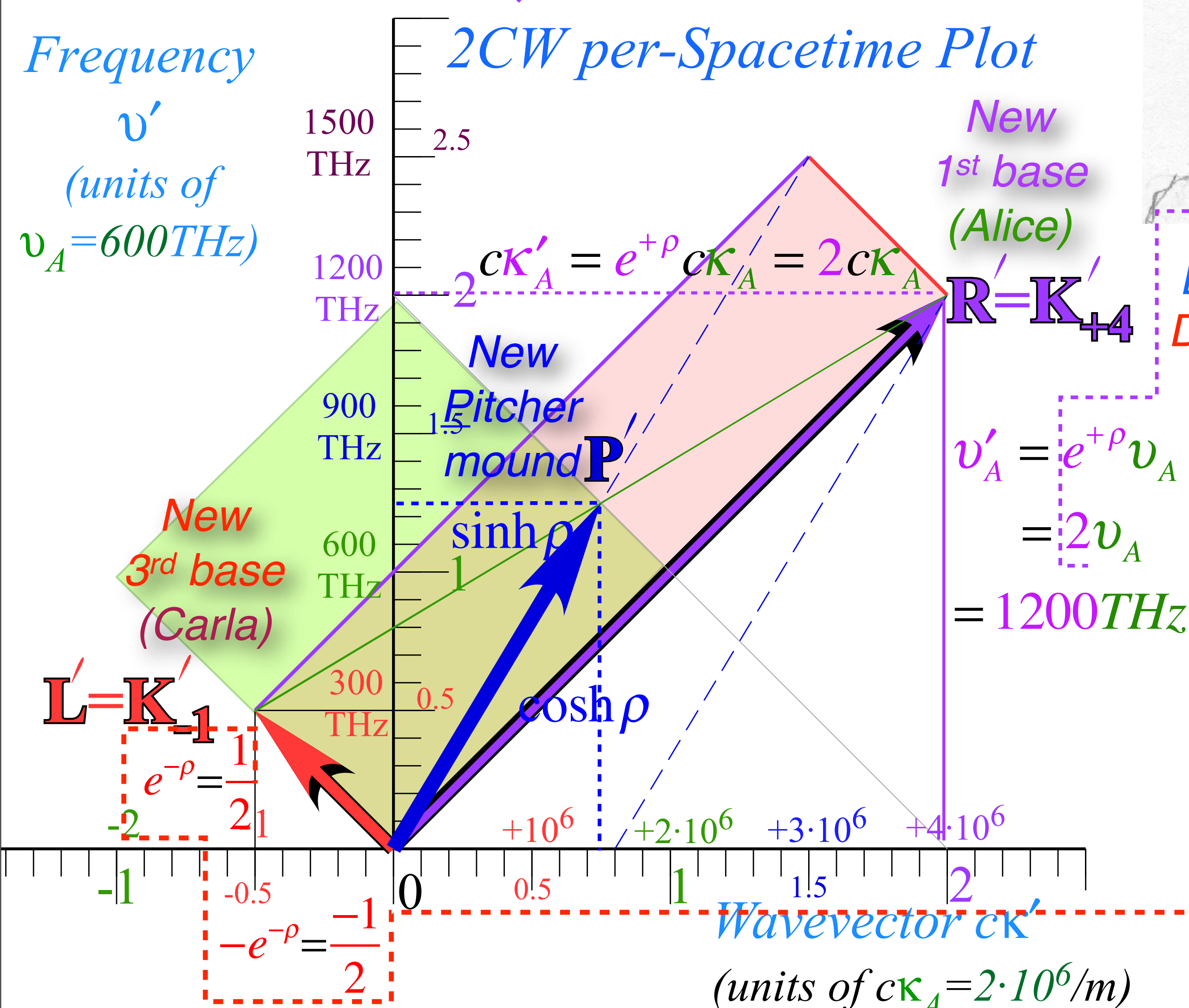


Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$
is Doppler blued up by $e^{+\rho}=2$.

$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$



$\upsilon'_A = e^{+\rho}\upsilon_A$
$= 2\upsilon_A$
$= 1200THz$

But, Carla's 3rd baseline $\mathbf{L}'$ is
Doppler red shifted by $e^{-\rho}=1/2$.

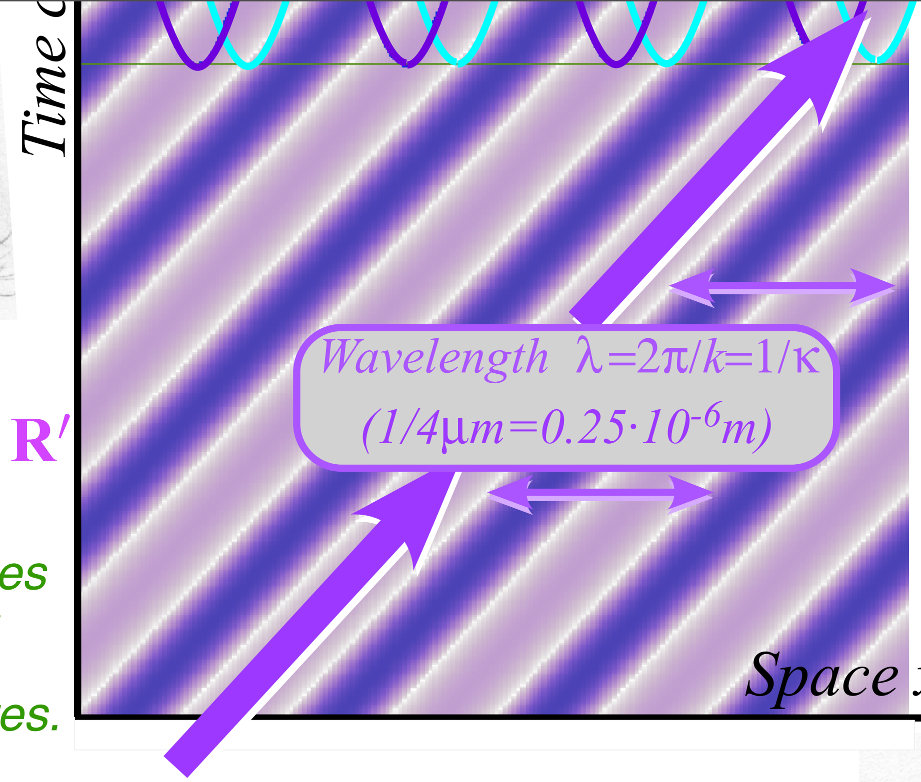
New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.)
is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

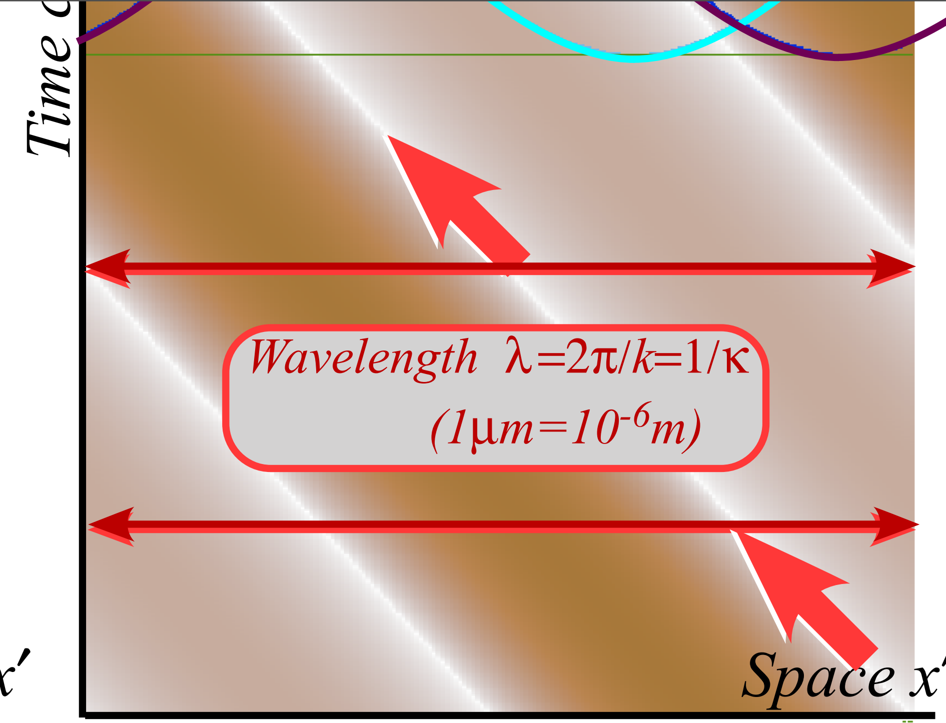
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

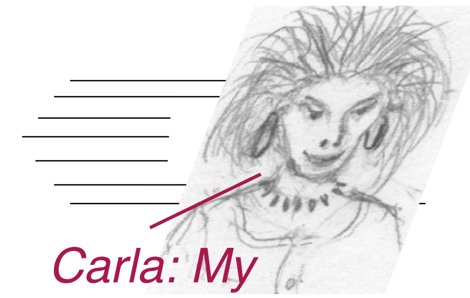
$$= \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$

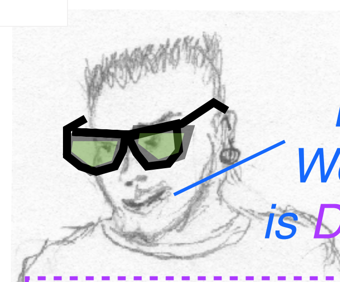
Alice: OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.







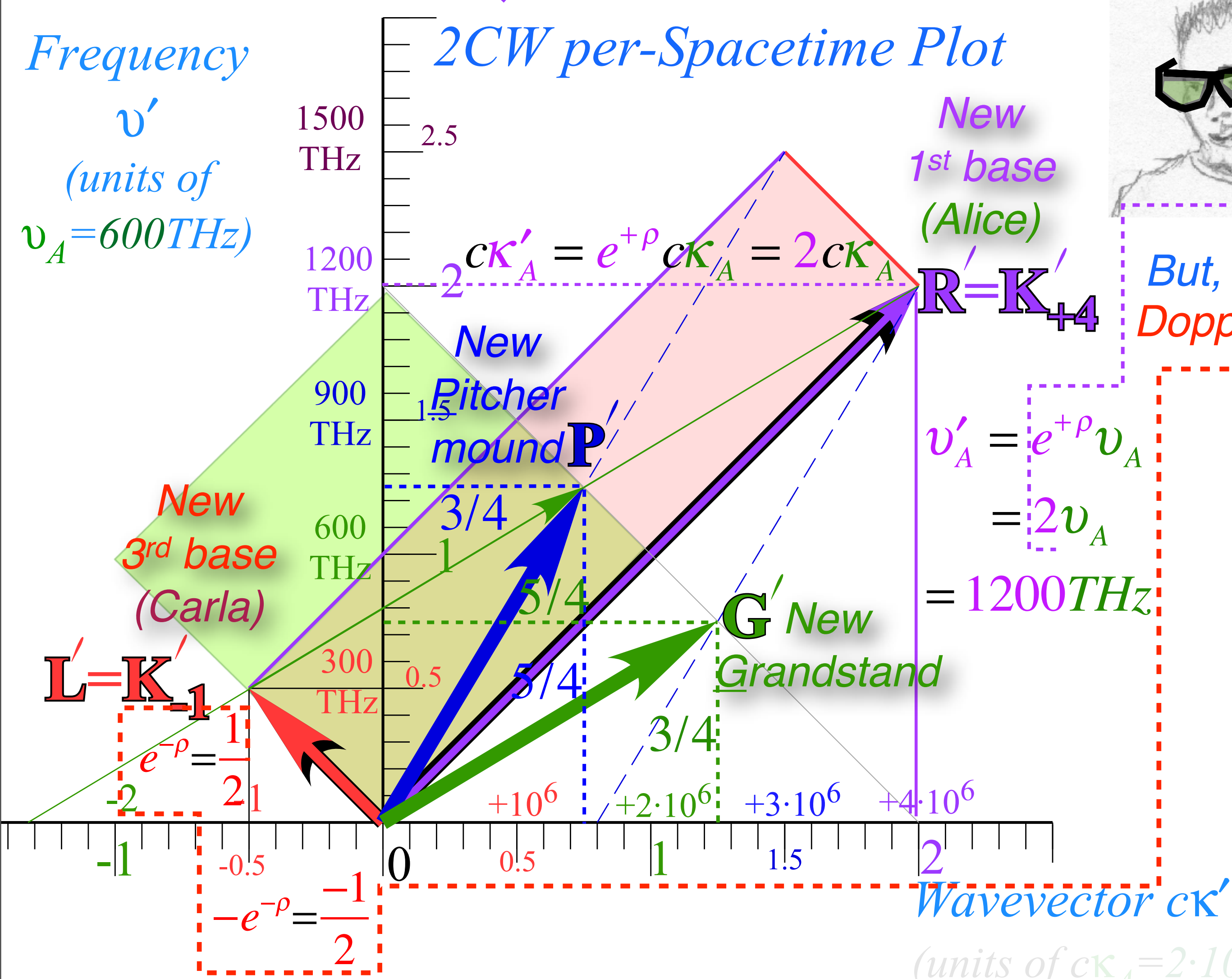
Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)



Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

$$K'_{phase} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2}$$

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.



$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

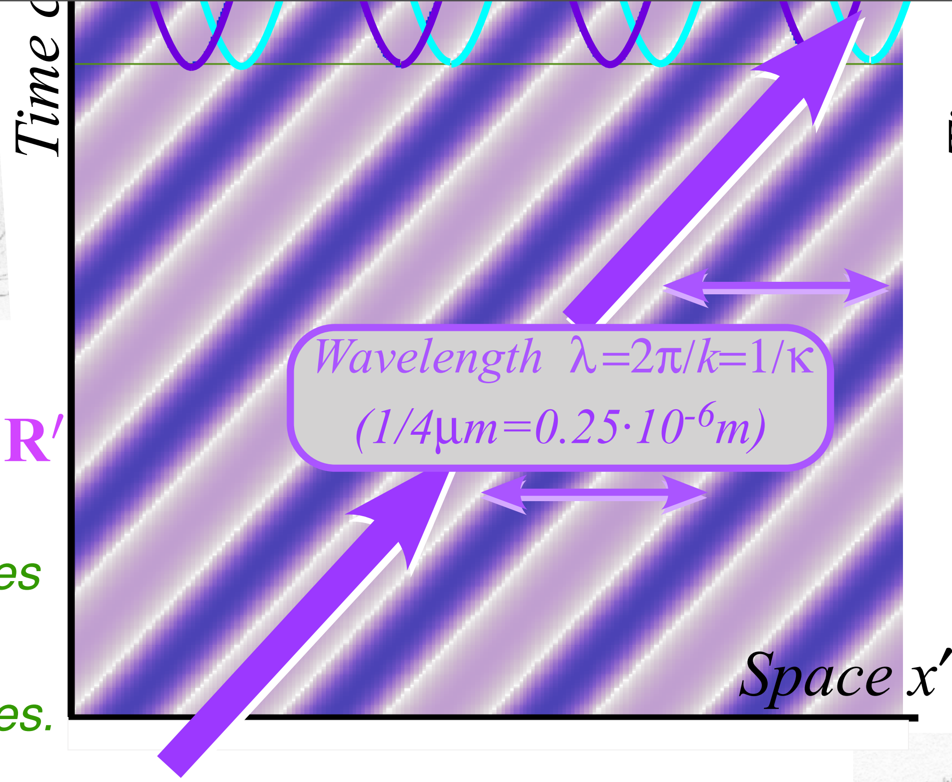
New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

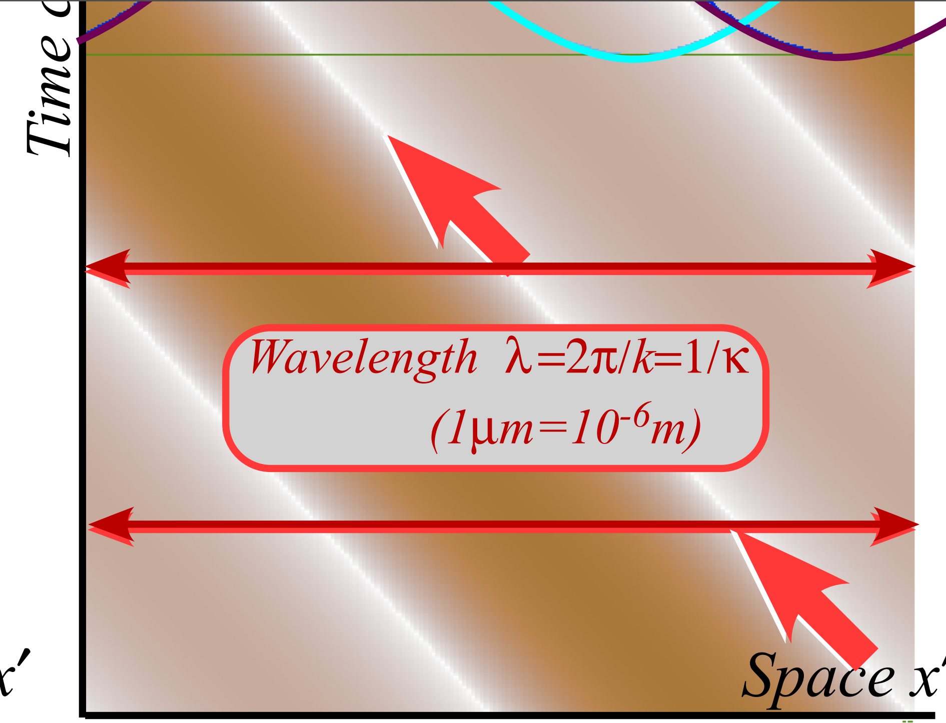
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2-1/2}{2} \\ \frac{2+1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

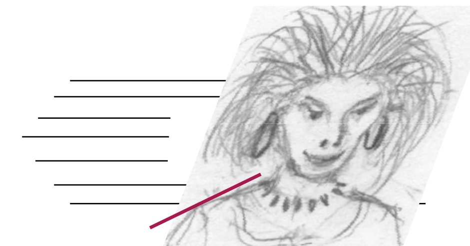
New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2+1/2}{2} \\ \frac{2-1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

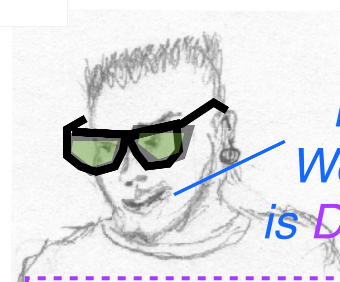
Alice: OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.







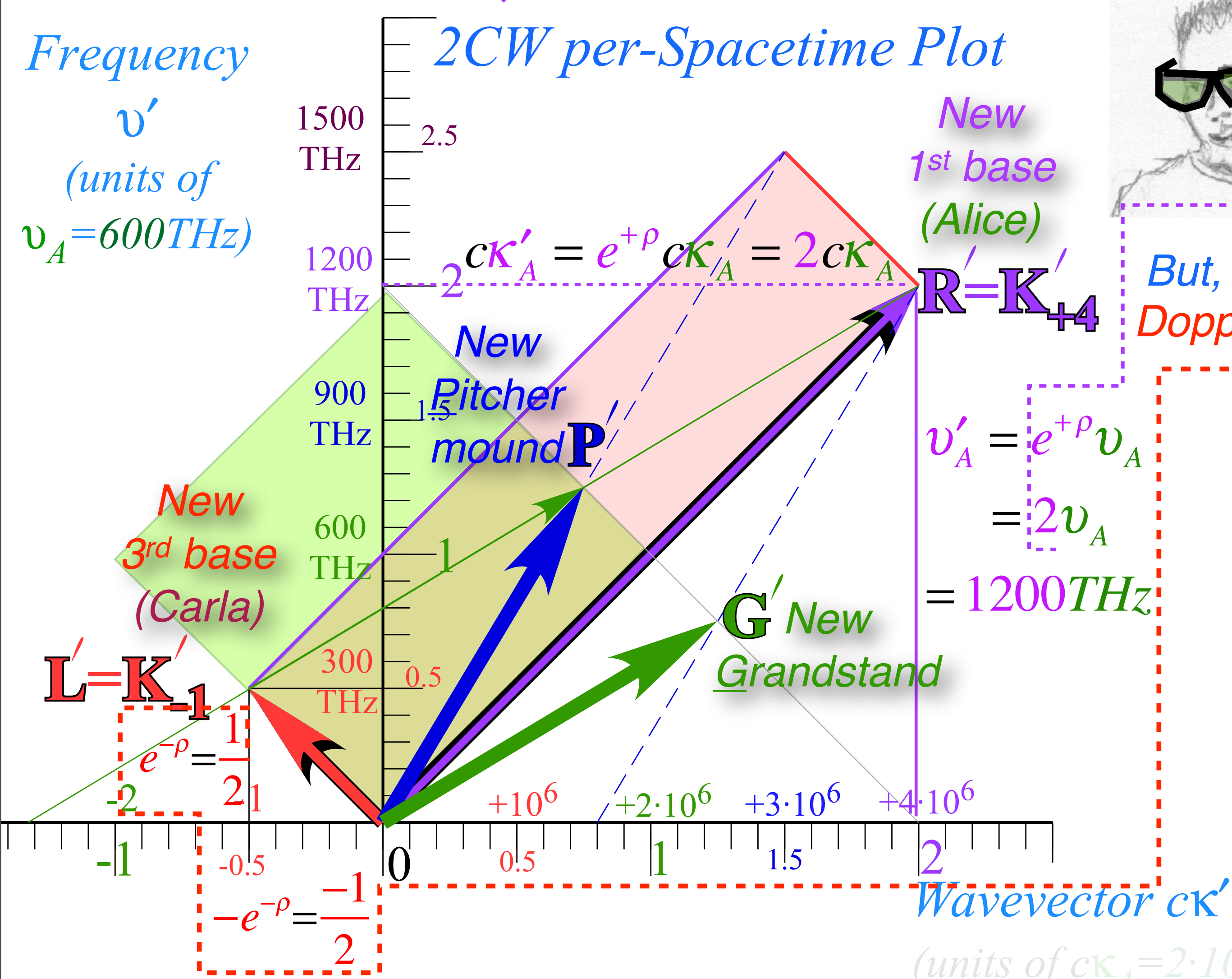
Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)



Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

$$\mathbf{K}'_{phase}=\mathbf{P}'=\frac{\mathbf{R}'+\mathbf{L}'}{2}$$

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.



$$\upsilon'_A=e^{+\rho}\upsilon_A=2\upsilon_A=1200THz$$

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase}\end{pmatrix}=\frac{\upsilon_A}{2}\begin{pmatrix}2\\2\end{pmatrix}+\frac{\upsilon_A}{2}\begin{pmatrix}-1/2\\+1/2\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{2-1/2}{2}\\ \frac{2+1/2}{2}\end{pmatrix}=\upsilon_A\begin{pmatrix}3/4\\5/4\end{pmatrix}$$

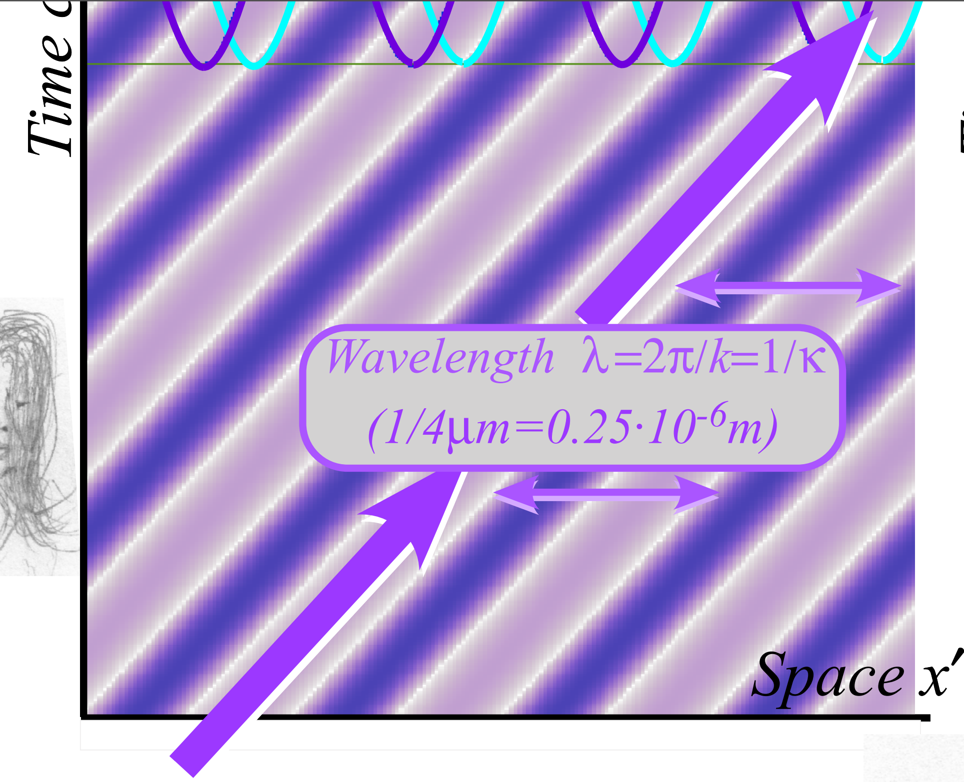
New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

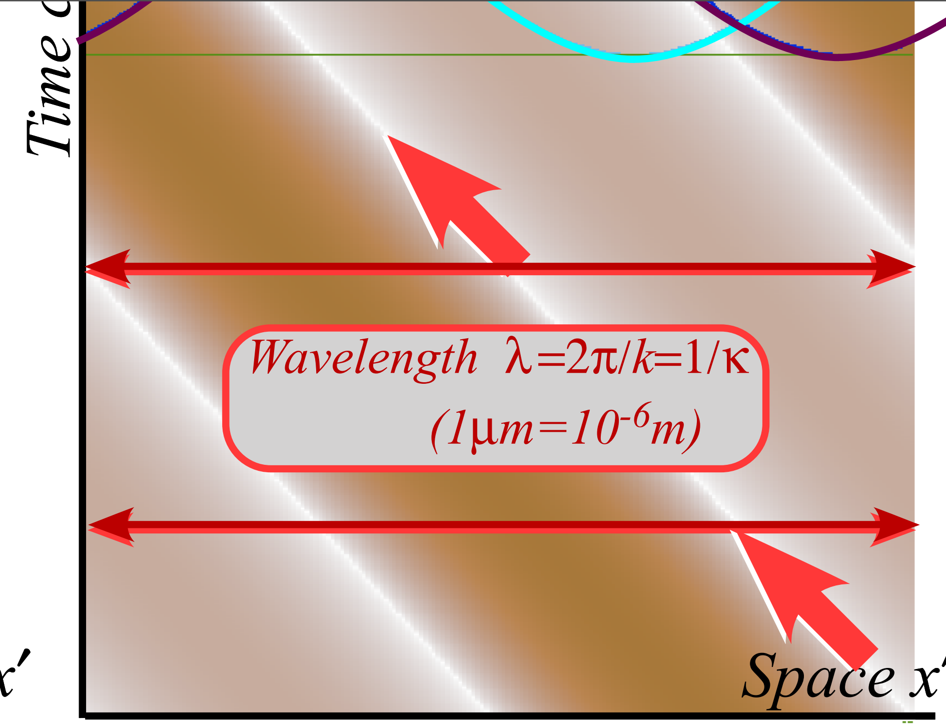
$$\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group}\end{pmatrix}=\frac{\upsilon_A}{2}\begin{pmatrix}e^{+\rho}\\e^{+\rho}\end{pmatrix}-\frac{\upsilon_A}{2}\begin{pmatrix}-e^{-\rho}\\+e^{-\rho}\end{pmatrix}=\upsilon_A\begin{pmatrix}\frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix}$$

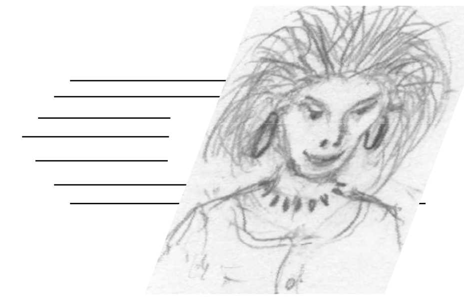
$$\mathbf{K}'_{group}=\mathbf{G}'=\frac{\mathbf{R}'-\mathbf{L}'}{2}\quad=\upsilon_A\begin{pmatrix}\cosh\rho\\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}5/4\\3/4\end{pmatrix}$$

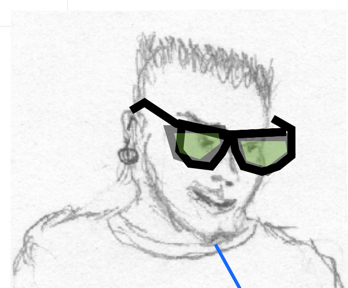
$$\upsilon'_C=e^{-\rho}\upsilon_A=\frac{1}{2}\upsilon_A=300THz$$

Group vector **G**
1/2-diff vector

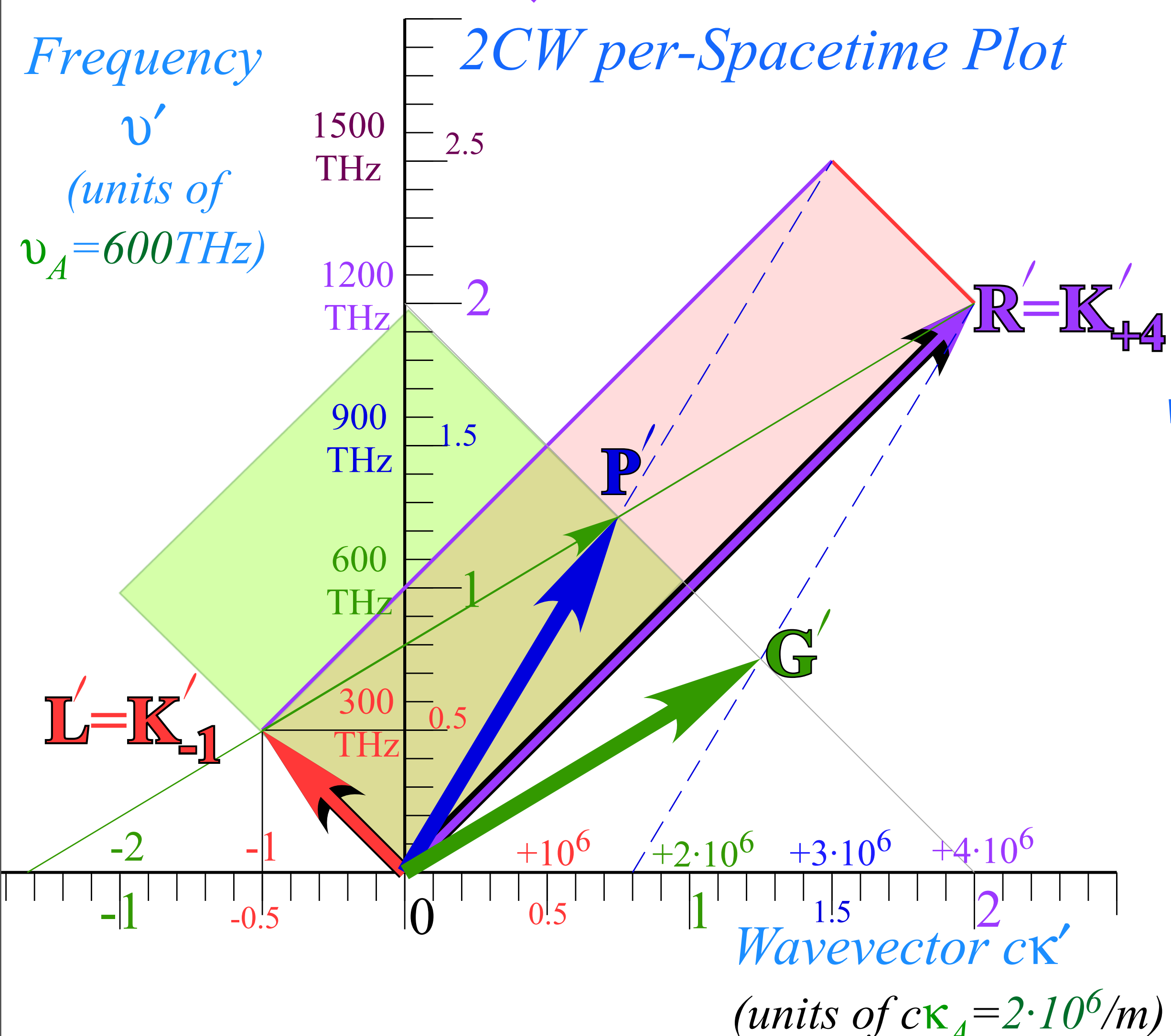


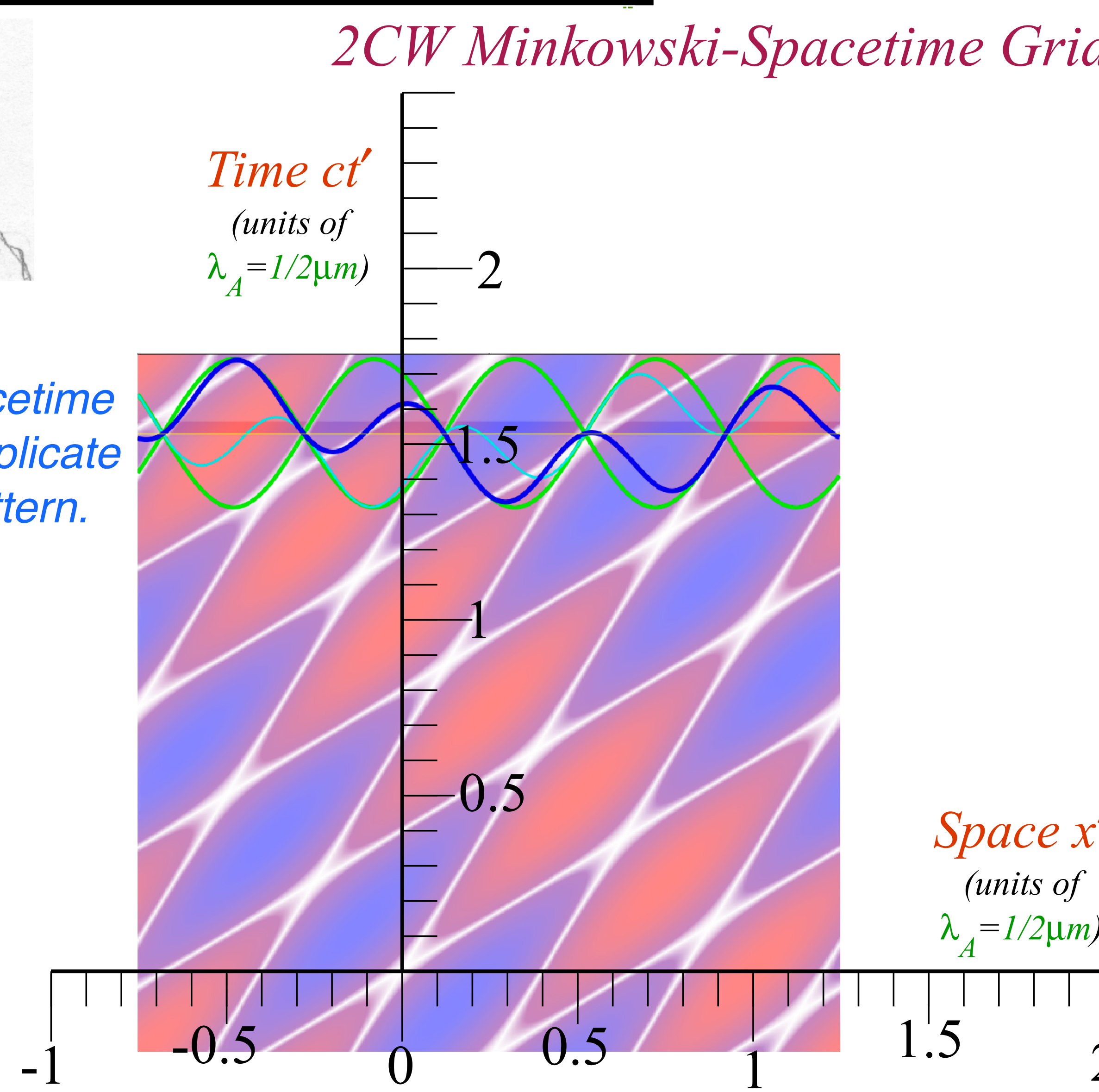






Bob: The spacetime wave-zeros replicate the same pattern.



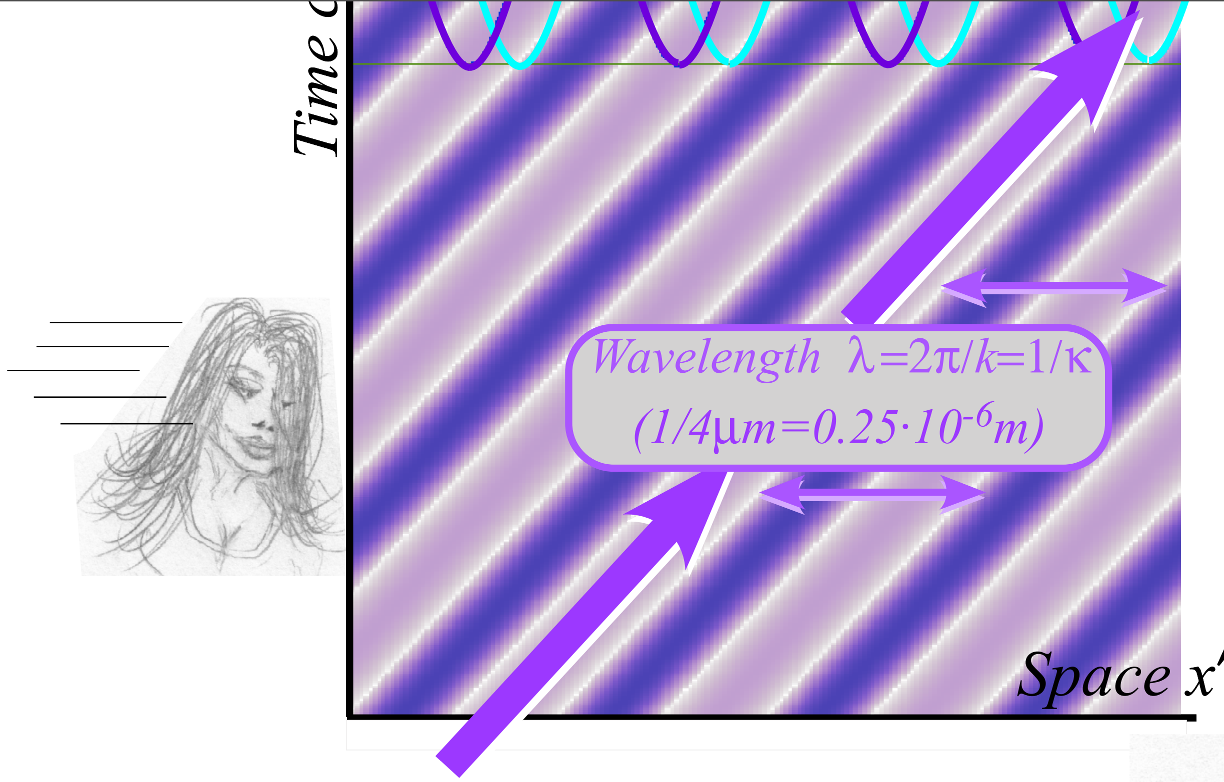


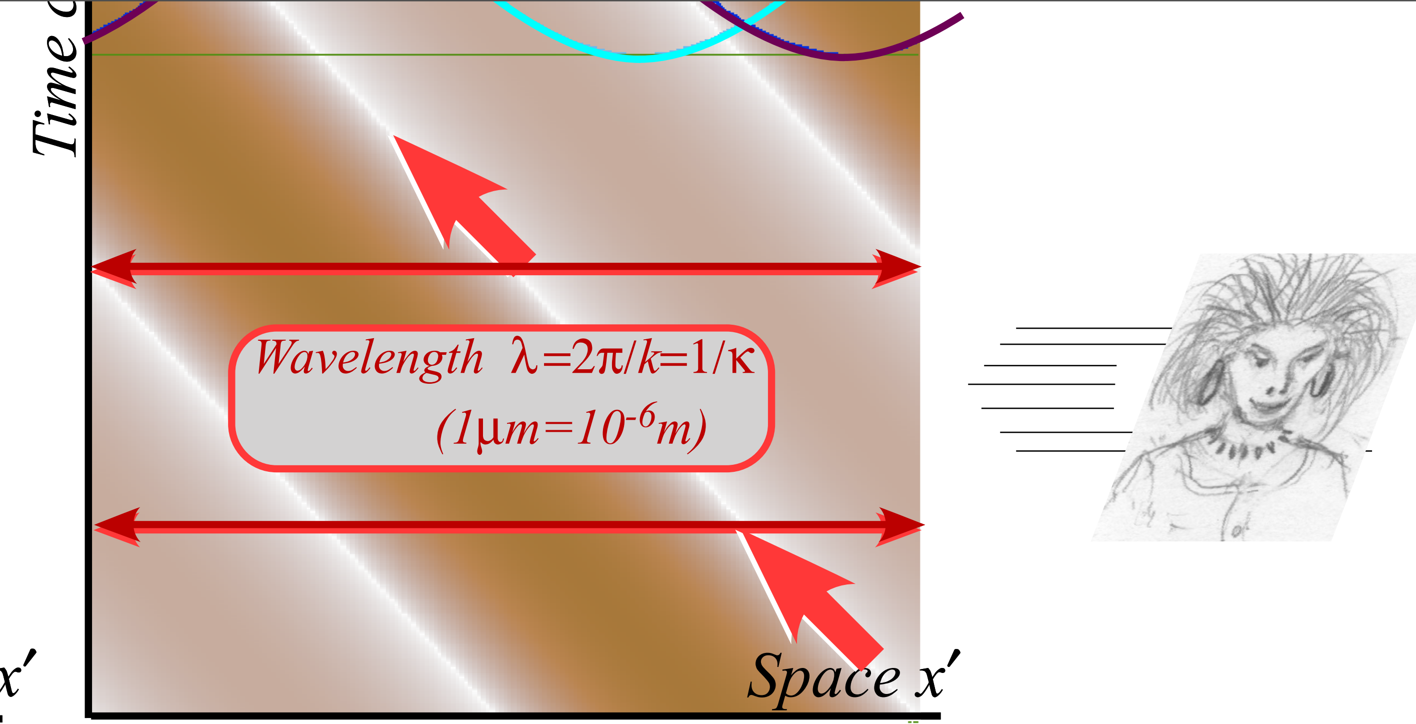
Phase vector **P**
1/2-sum vector

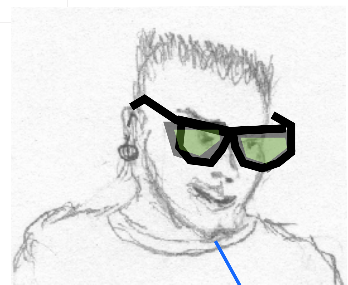
$K'_{phase}=P'=\frac{R'+L'}{2}$

Group vector **G**
1/2-diff vector

$K'_{group}=G'=\frac{R'-L'}{2}$

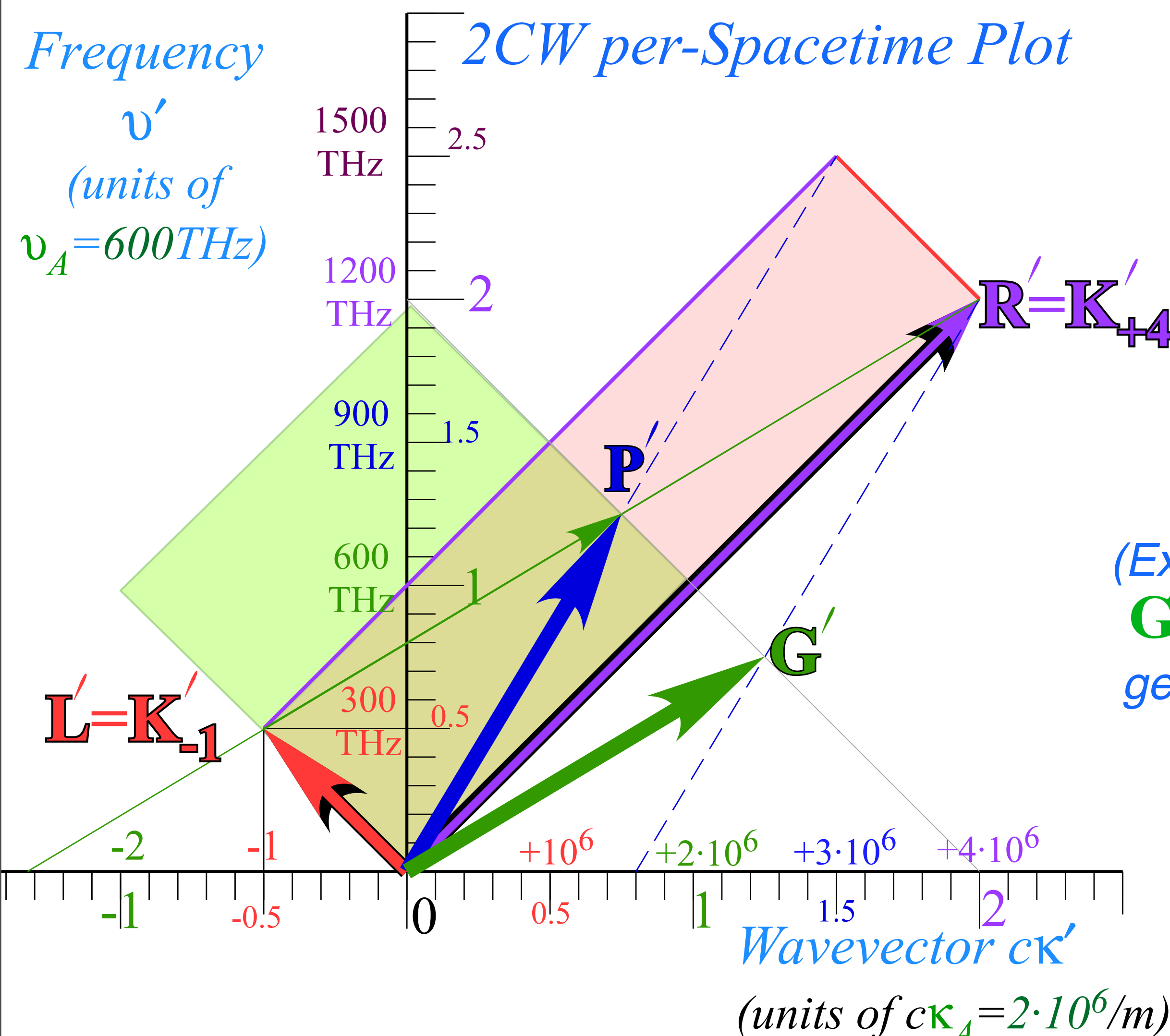


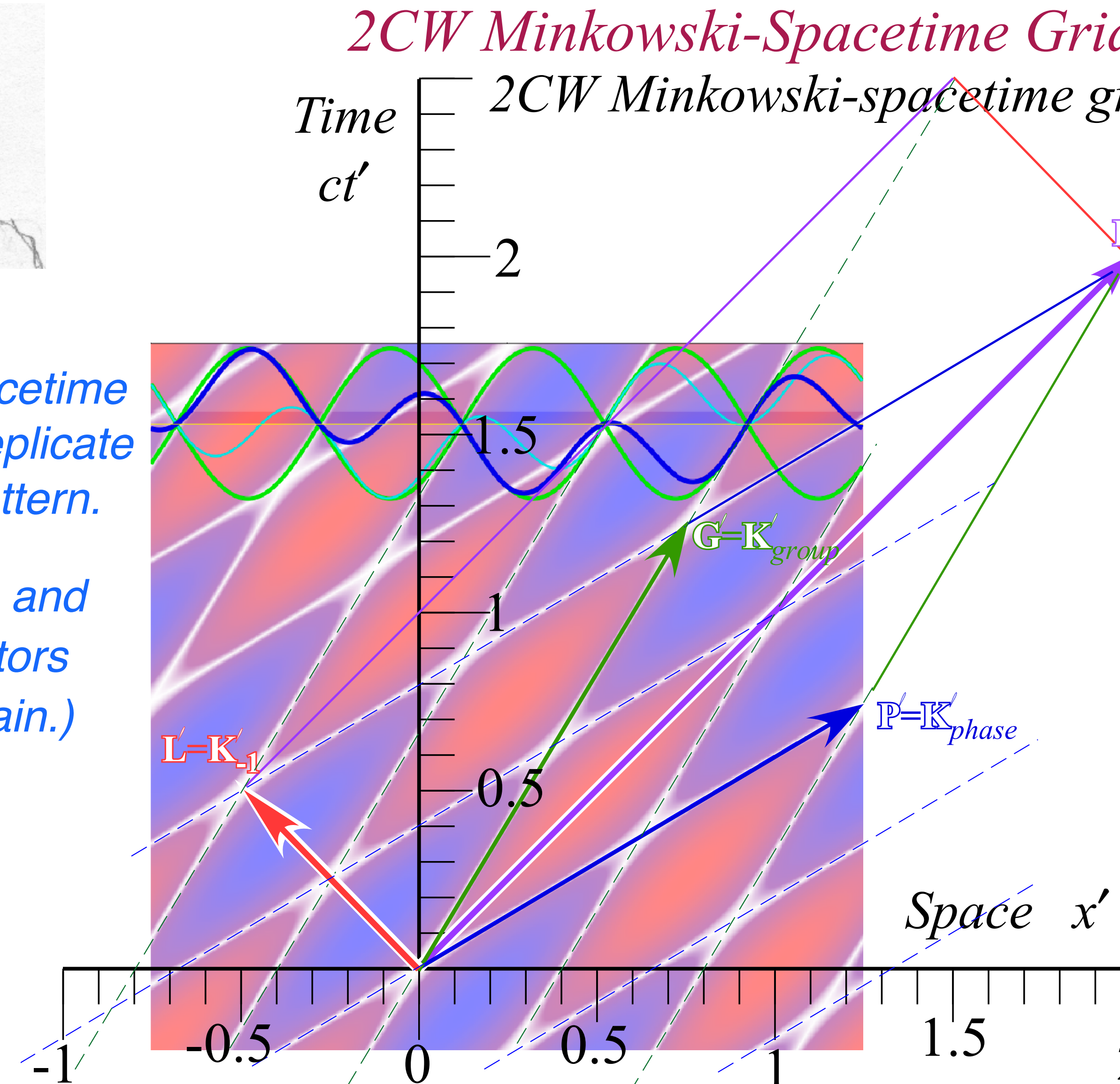




Bob: The spacetime wave-zeros replicate the same pattern.

(Except **P′**-*phase* and **G′**-*group* indicators get switched again.)



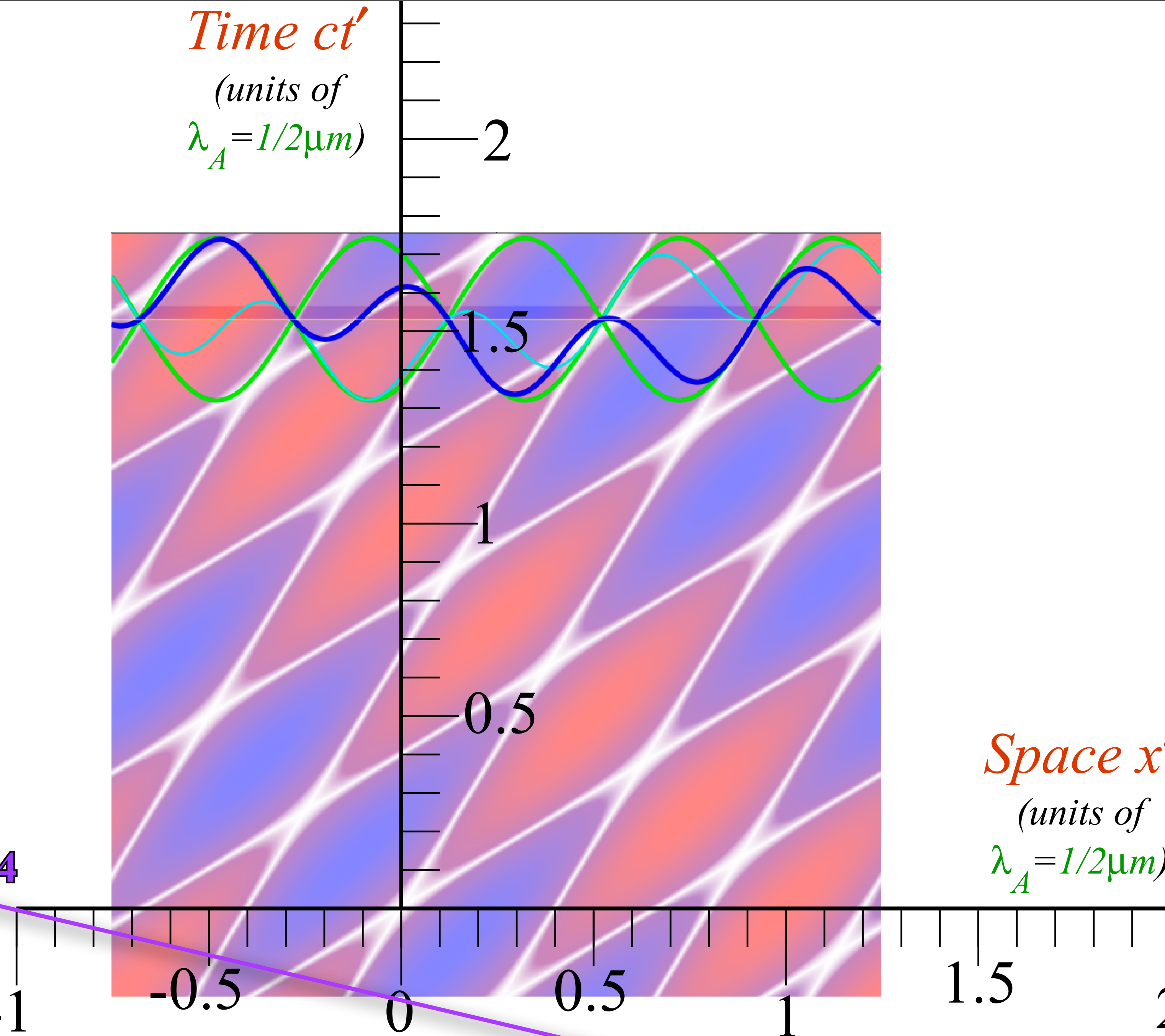


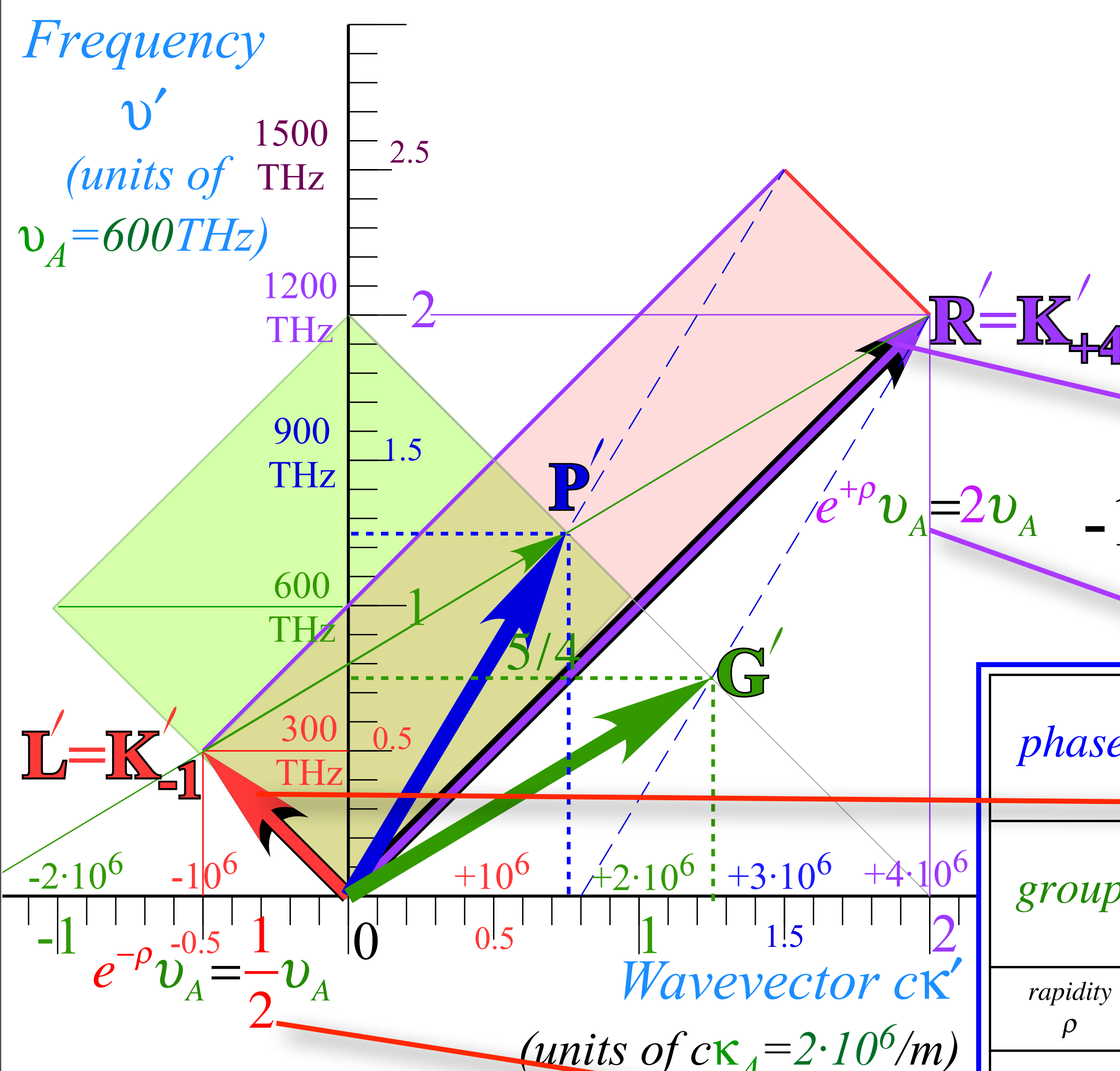
Phase vector **P**
1/2-sum vector

$$K'_{phase} = P' = \frac{R' + L'}{2}$$

Group vector **G**
1/2-diff vector

$$K'_{group} = G' = \frac{R' - L'}{2}$$





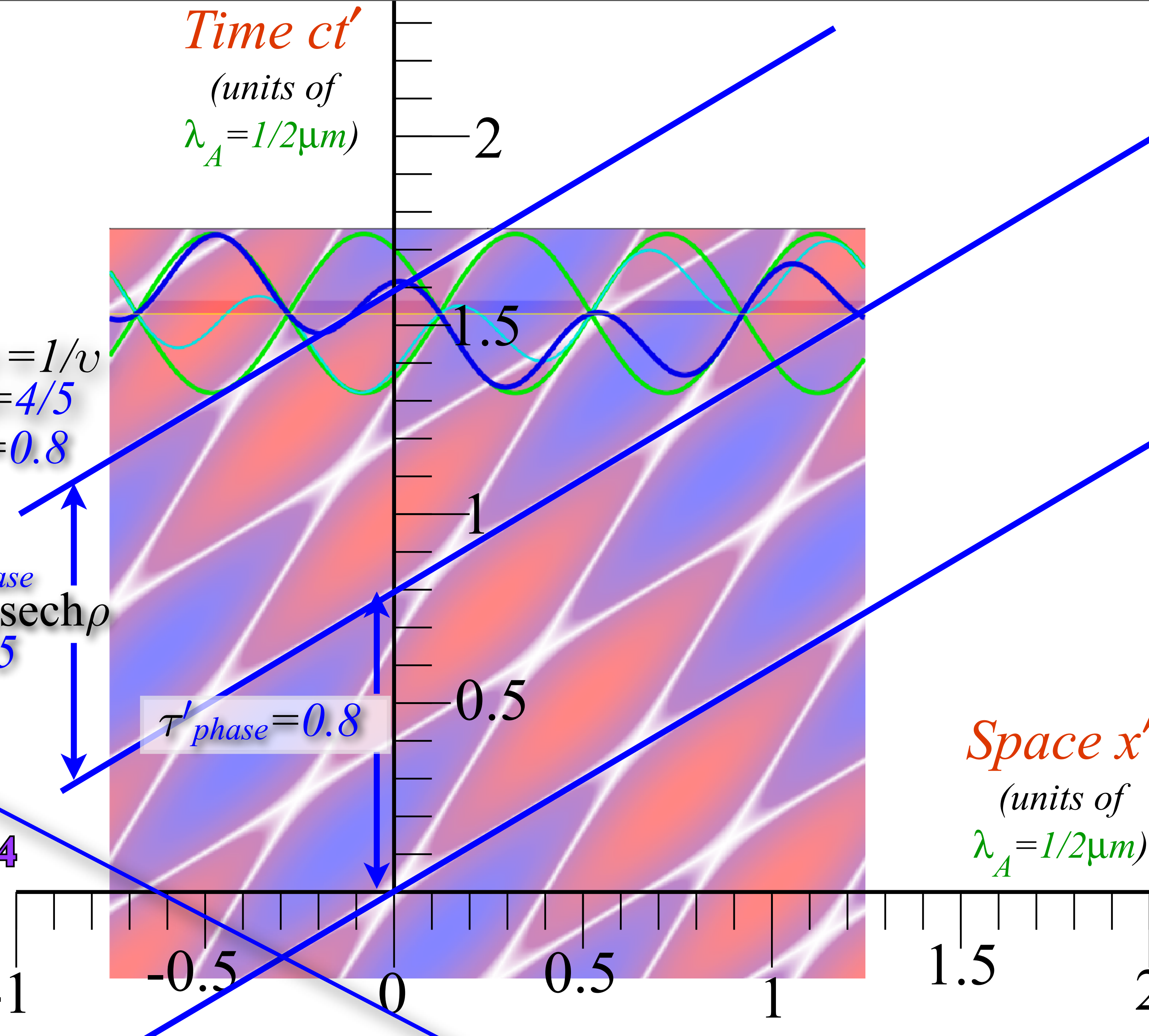
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

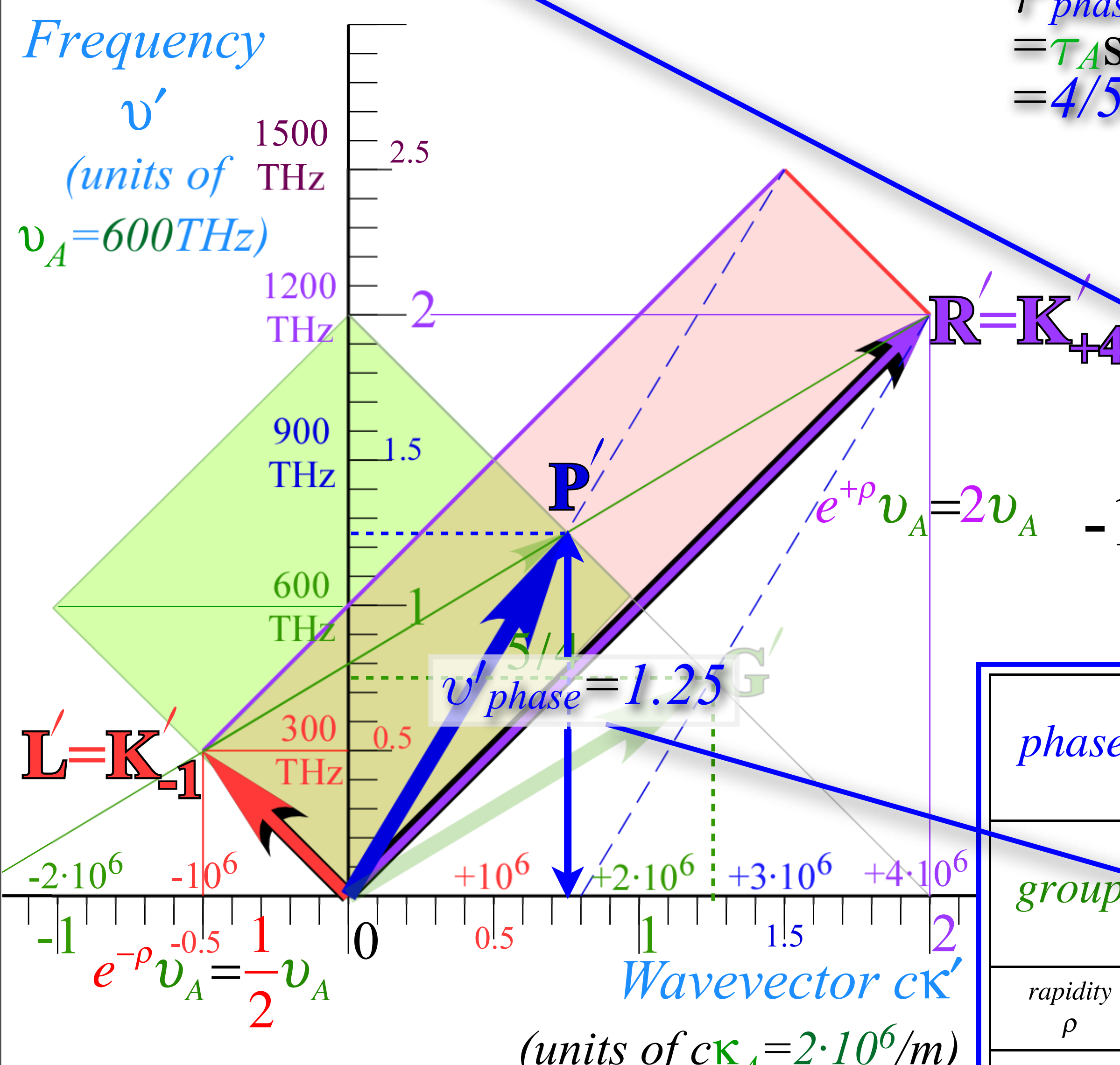
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





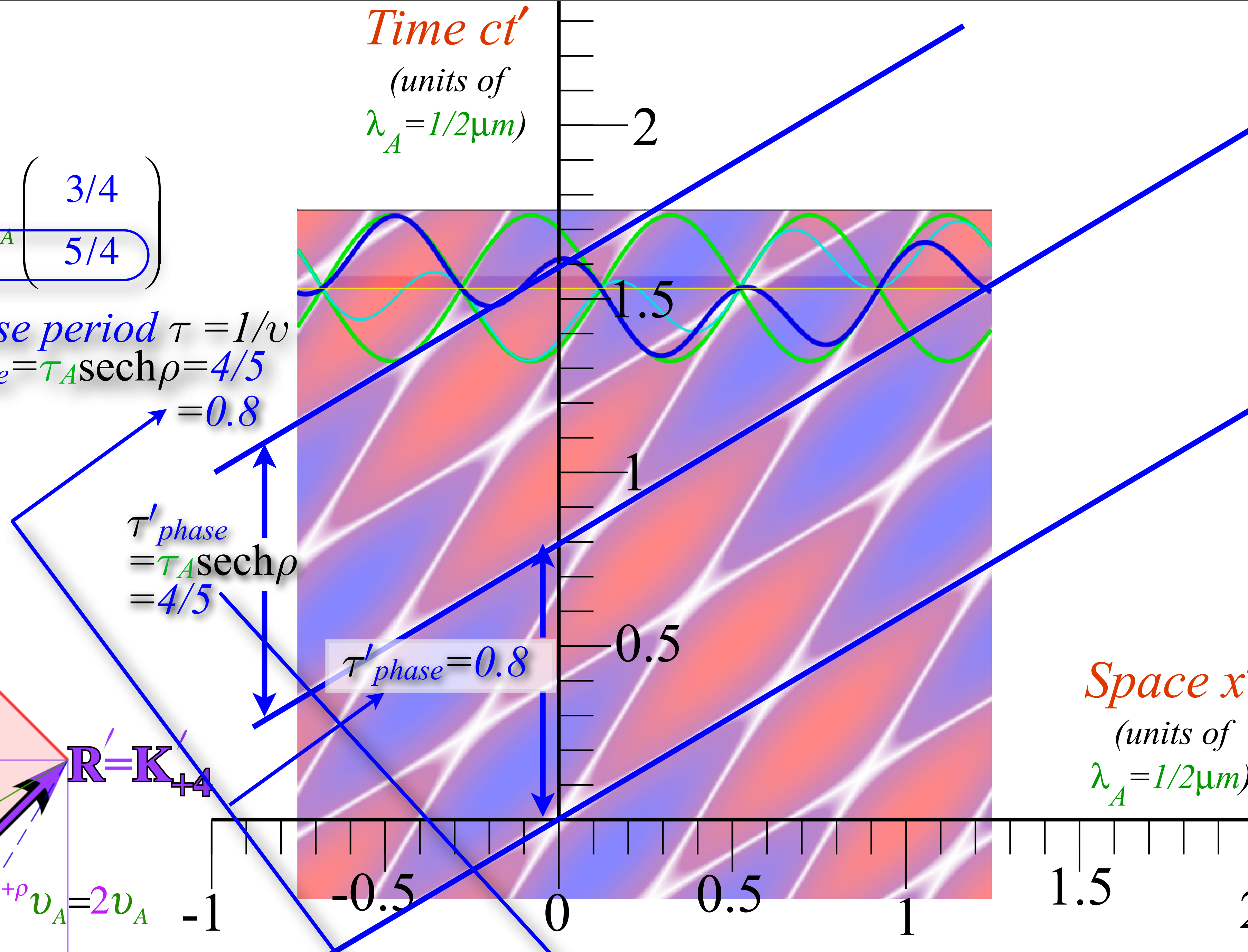
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

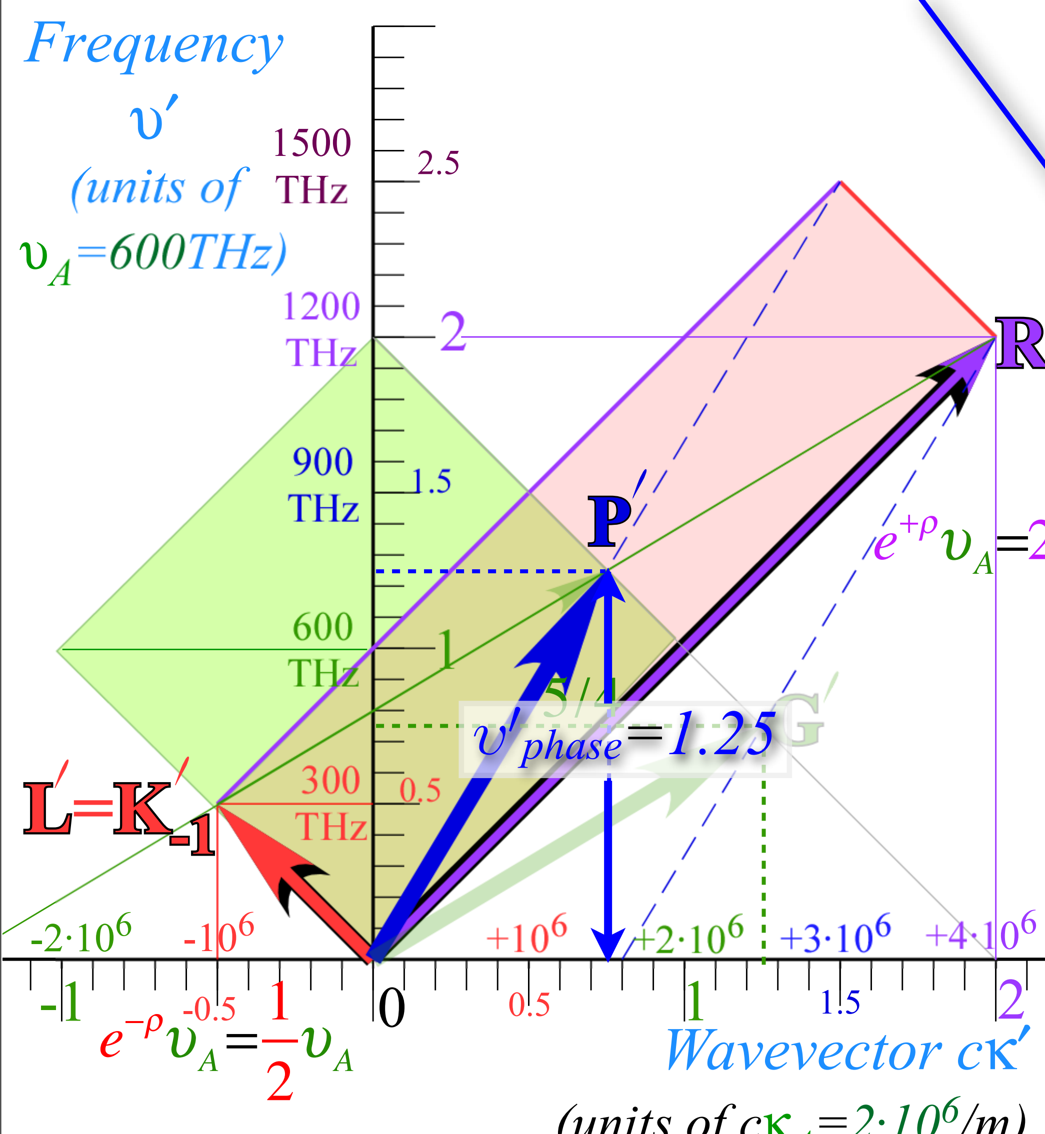
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\lambda_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

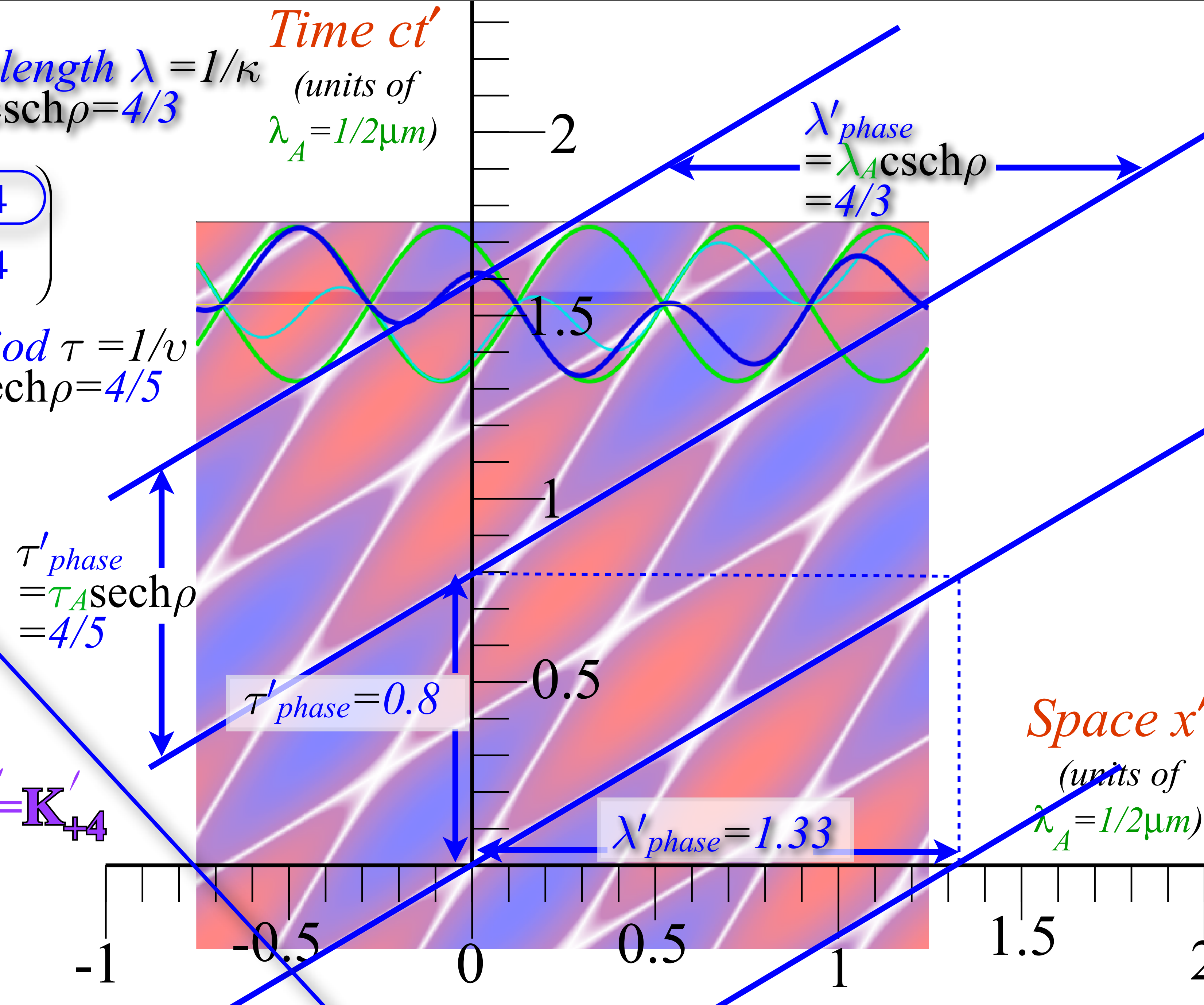
Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

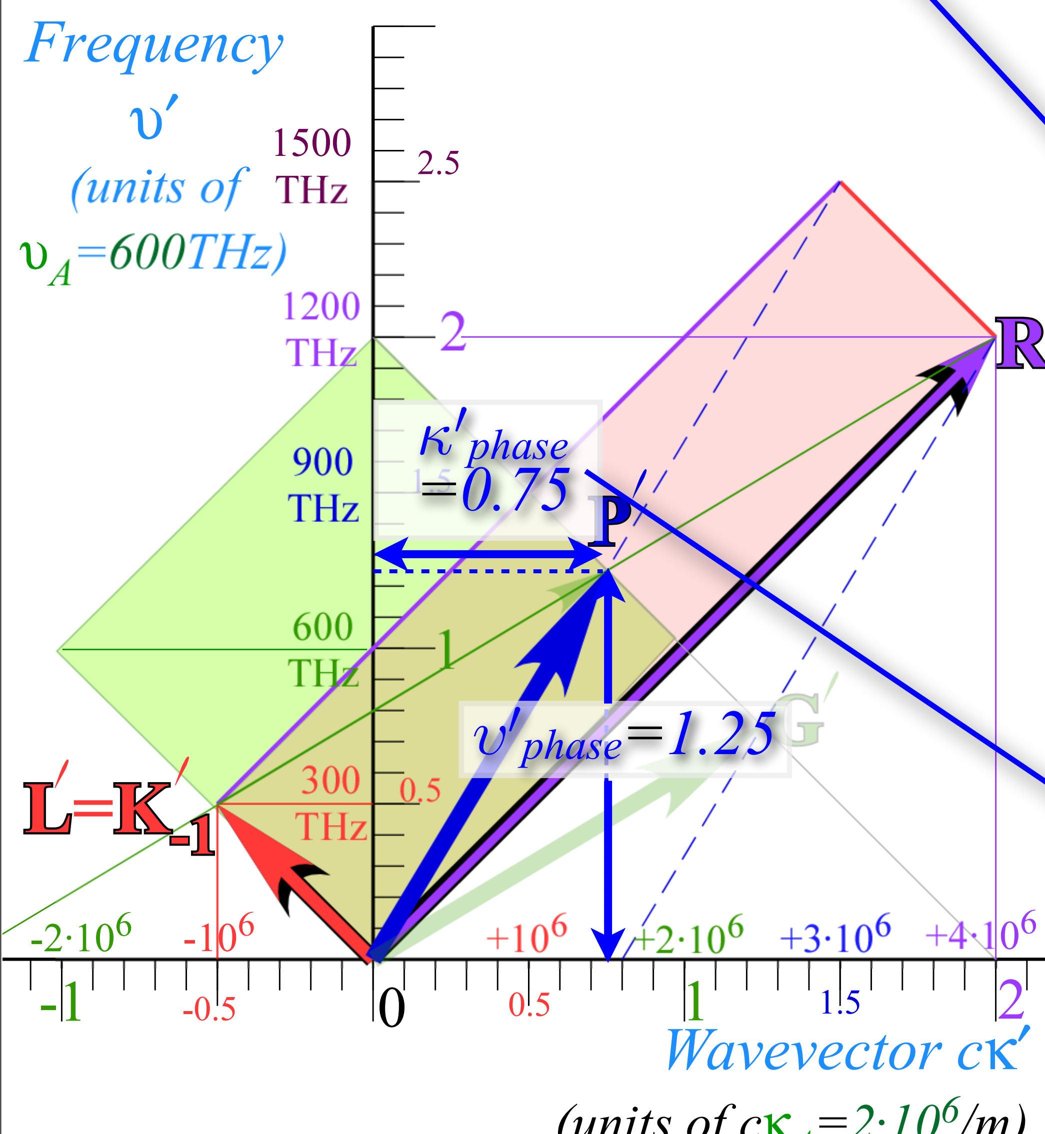
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

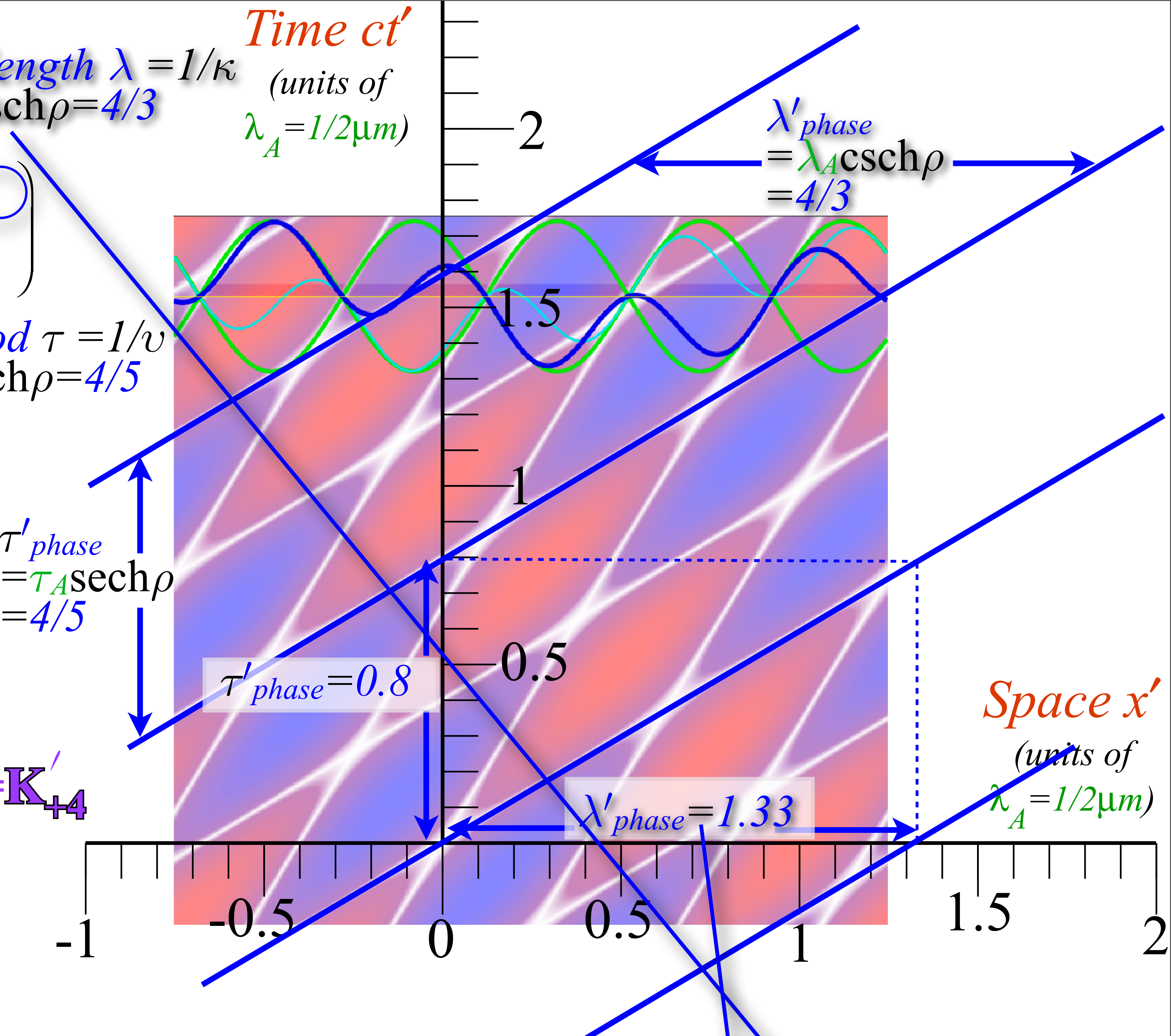
Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

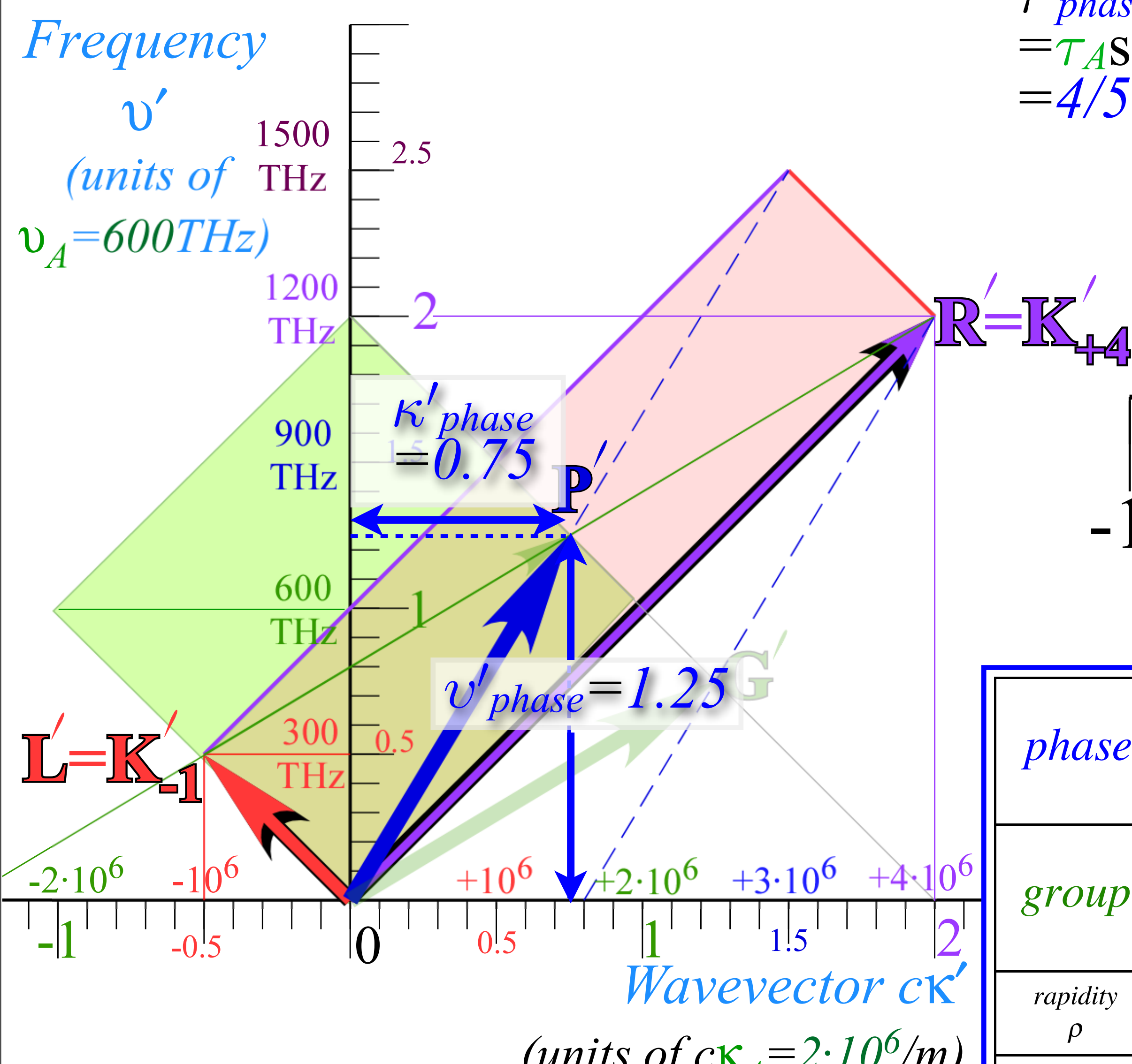
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

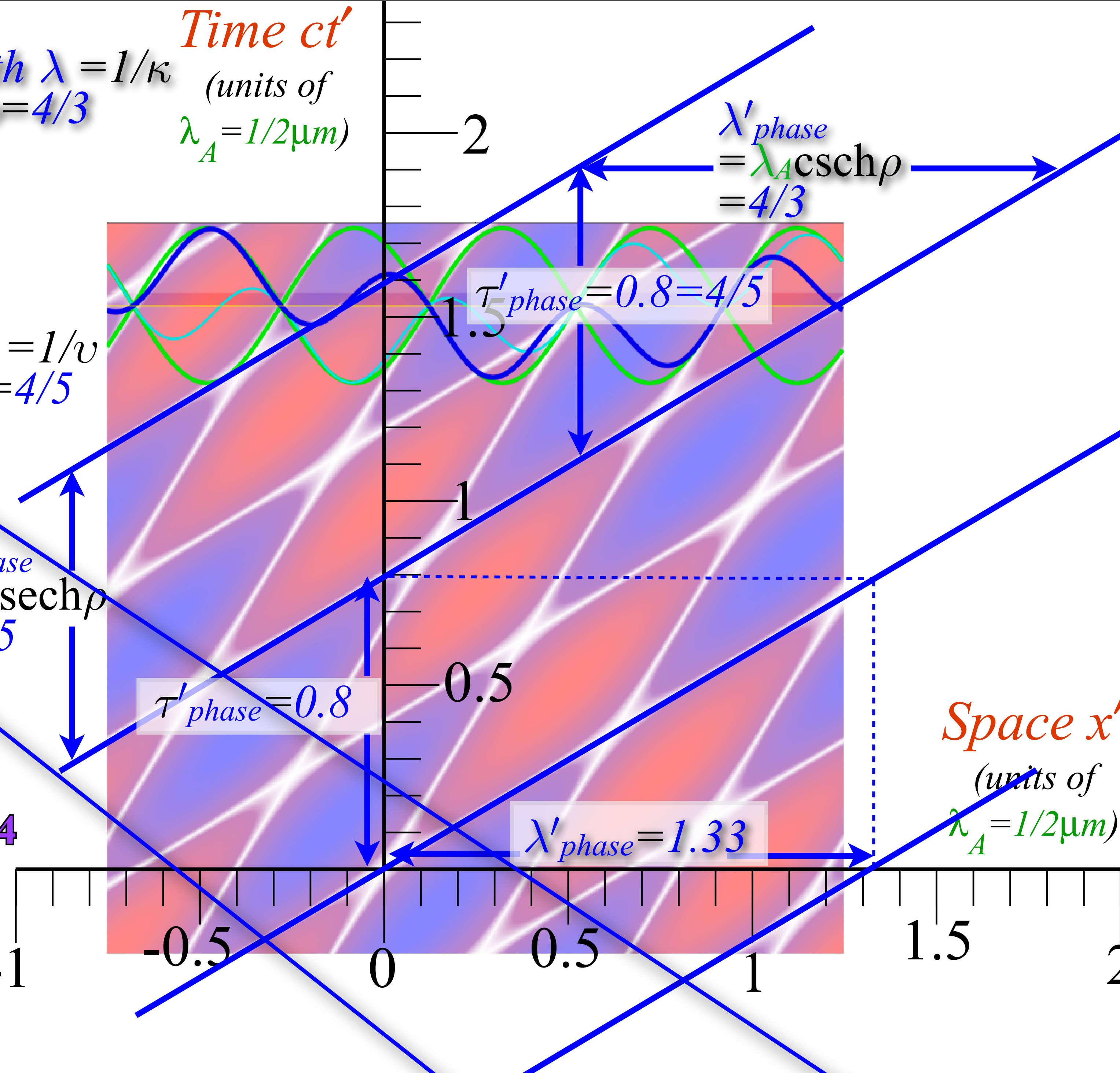
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

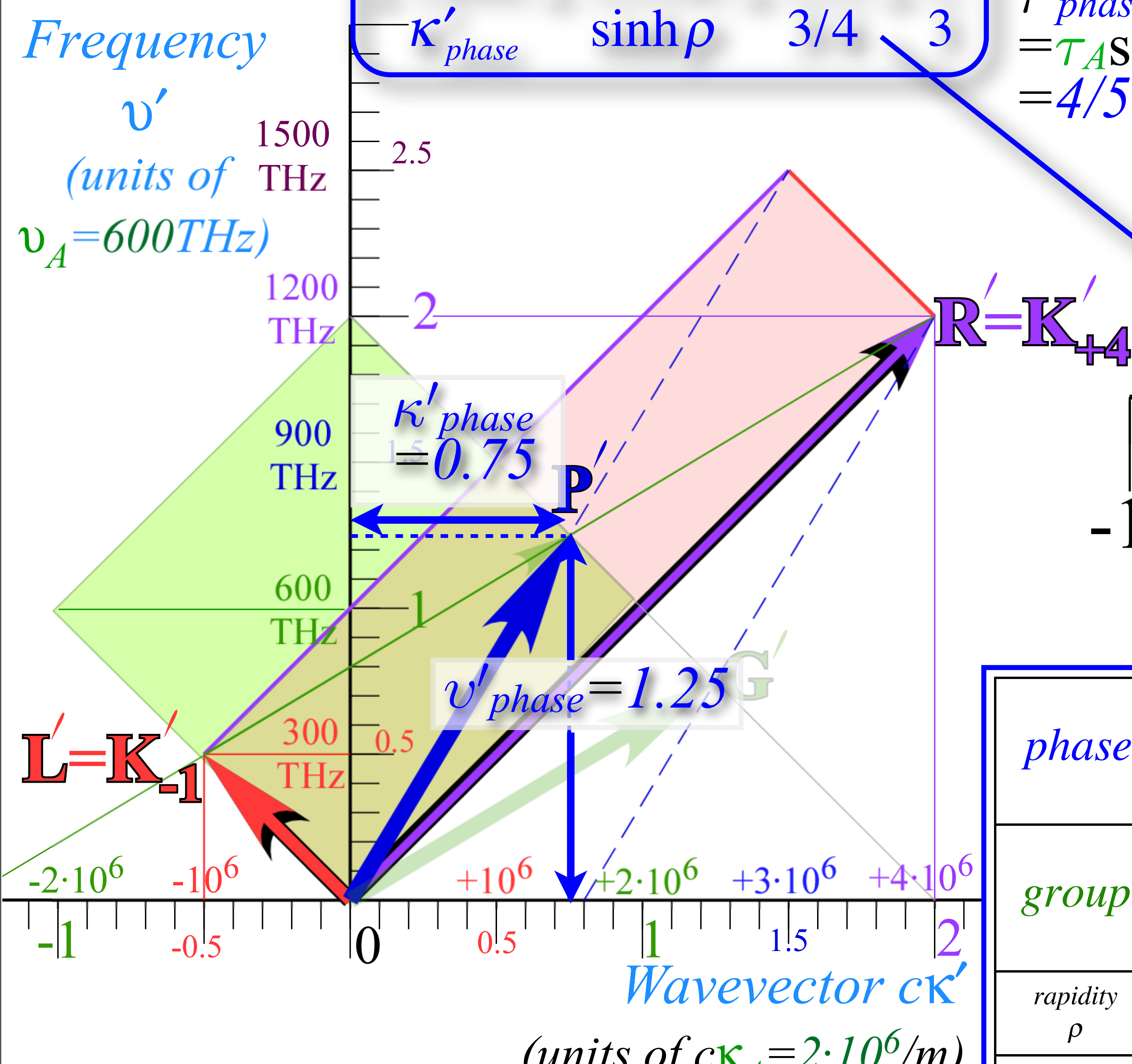
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

P-slope $= V_{phase}/c$

$$= \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

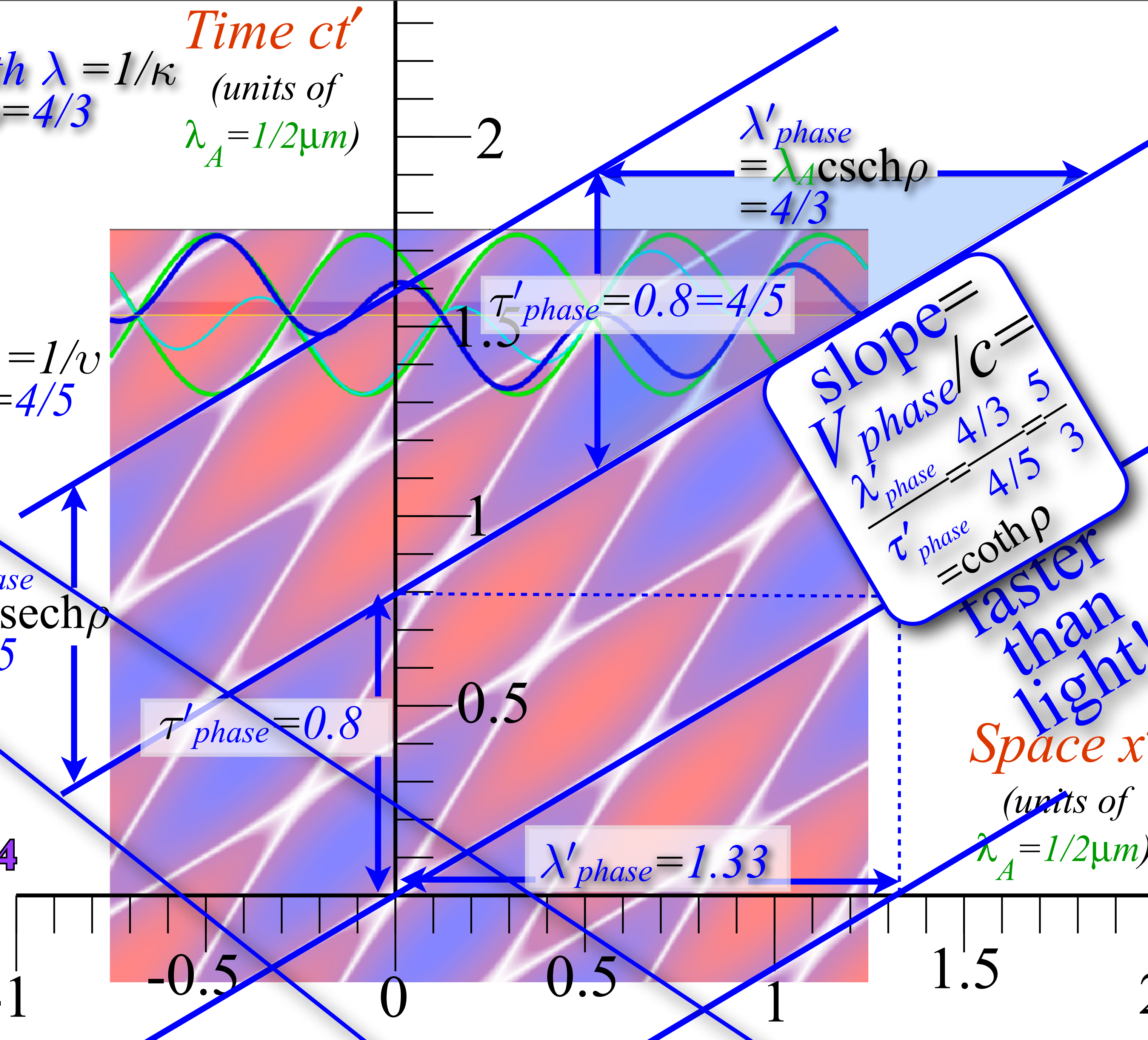
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

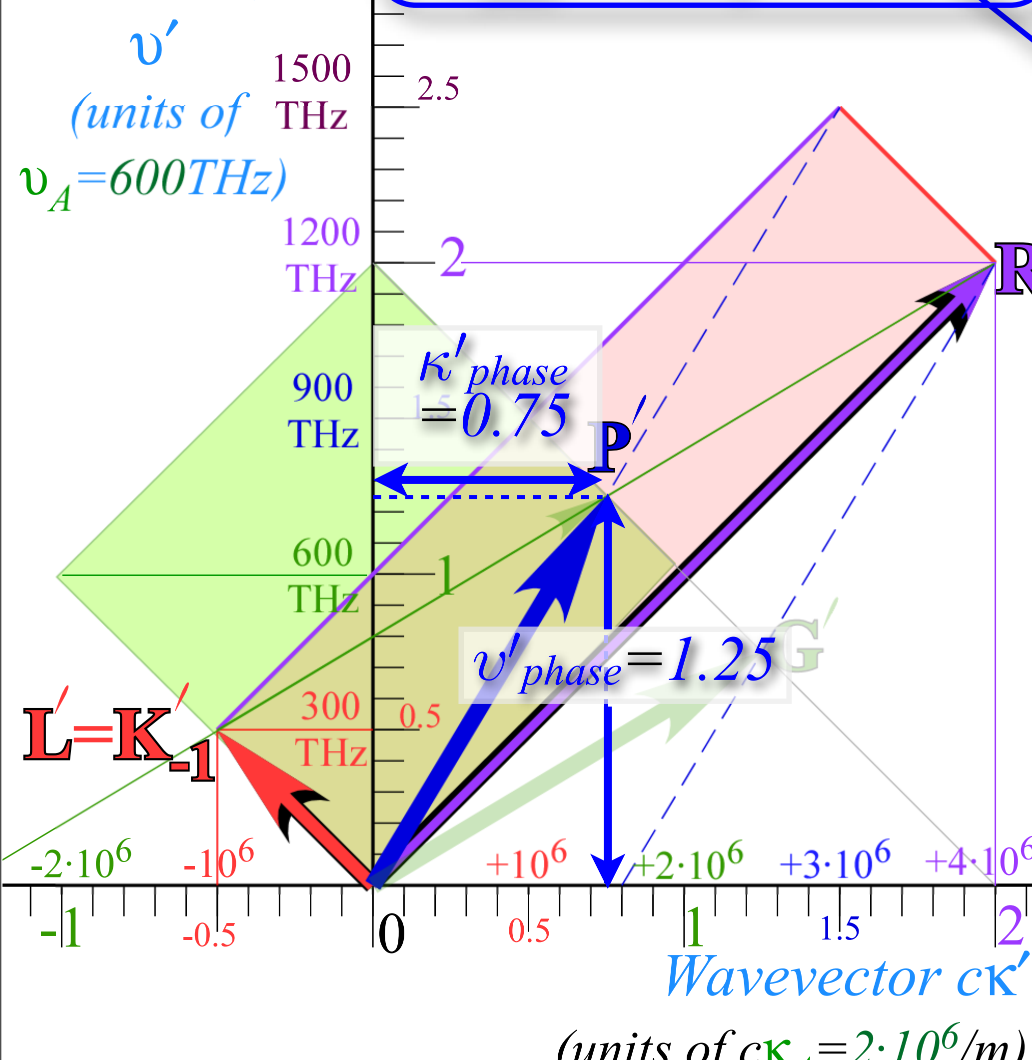
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





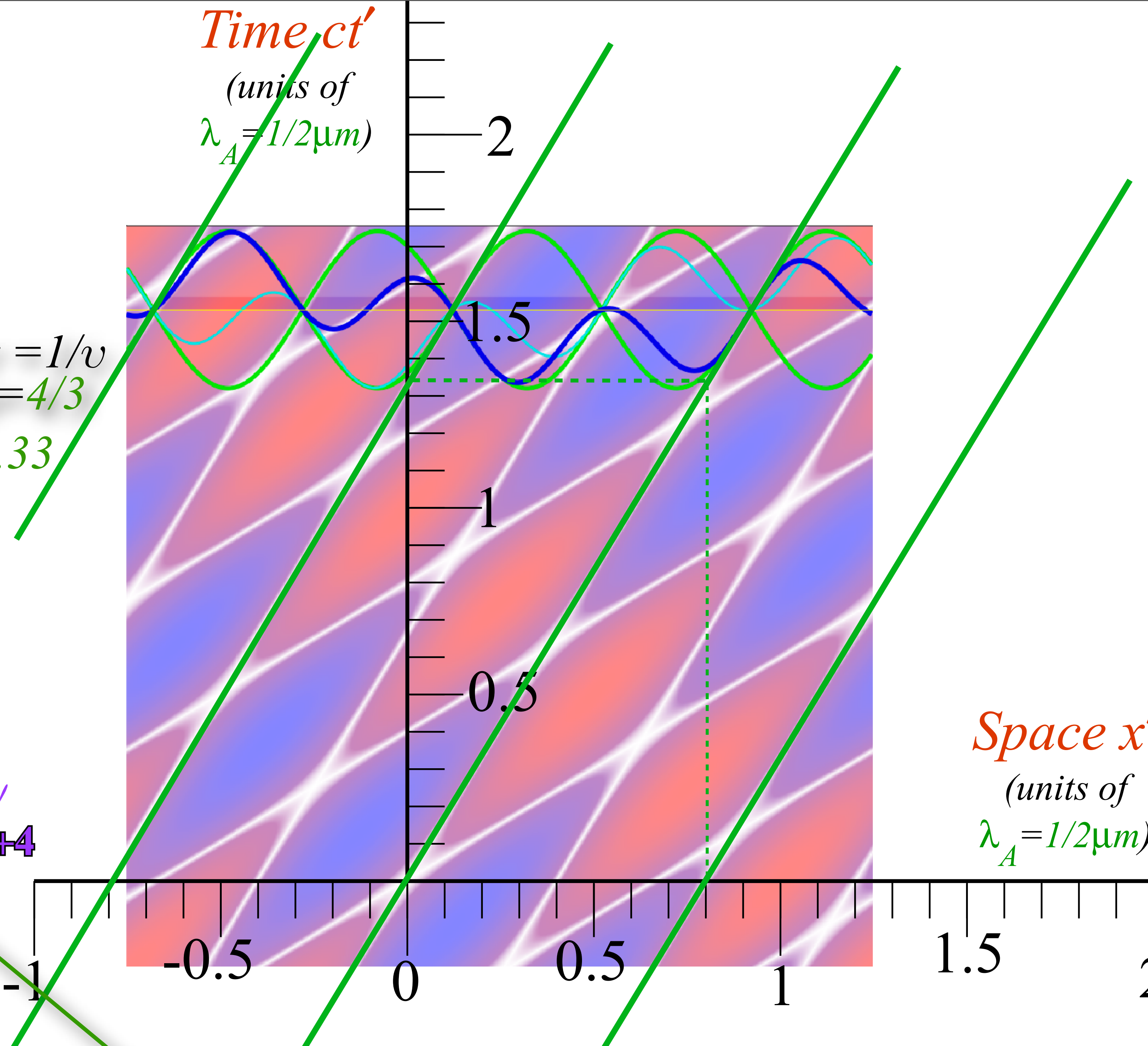
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}= 0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

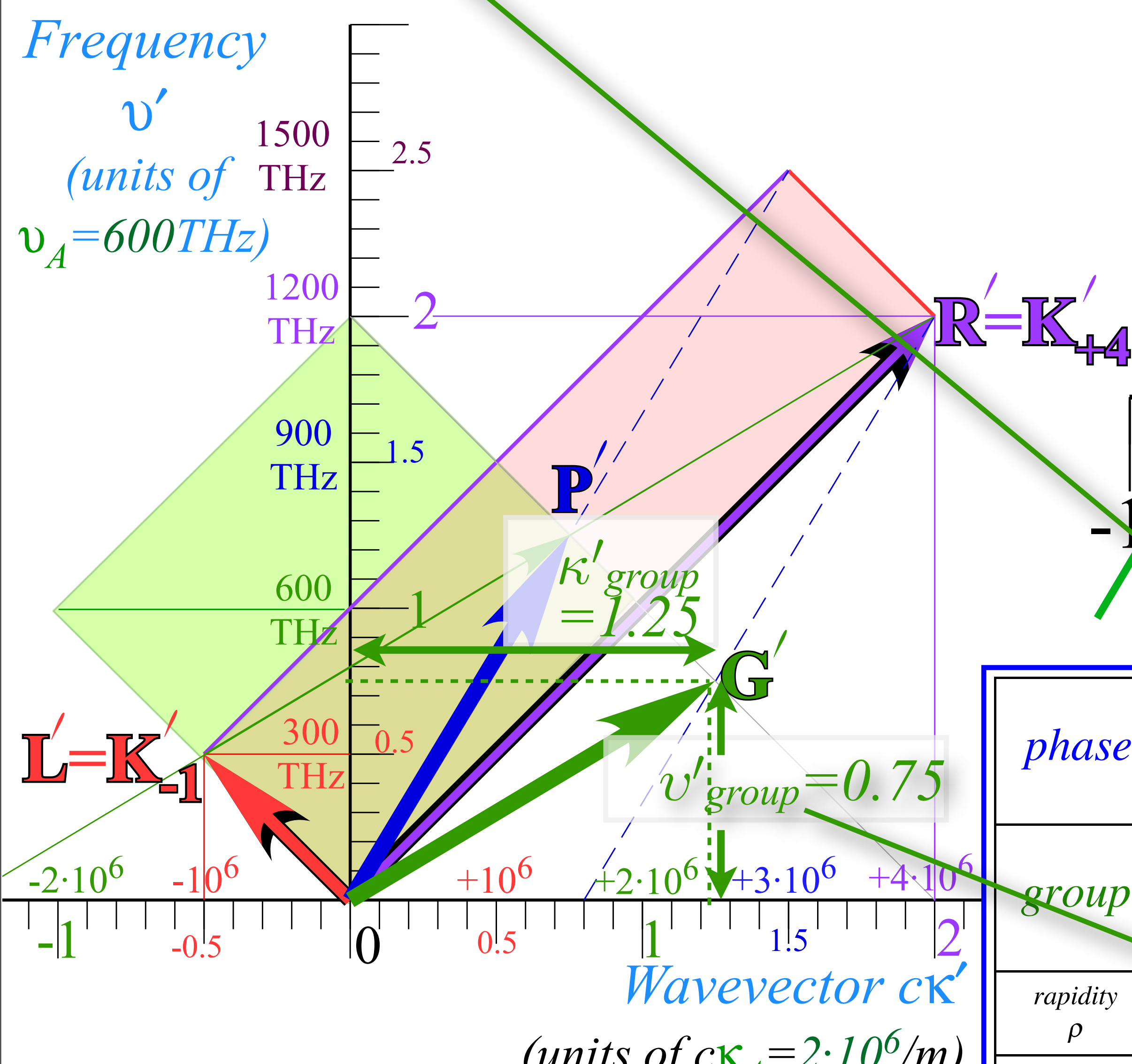
$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

flips to

Group period $\tau = 1/\upsilon$ $\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$





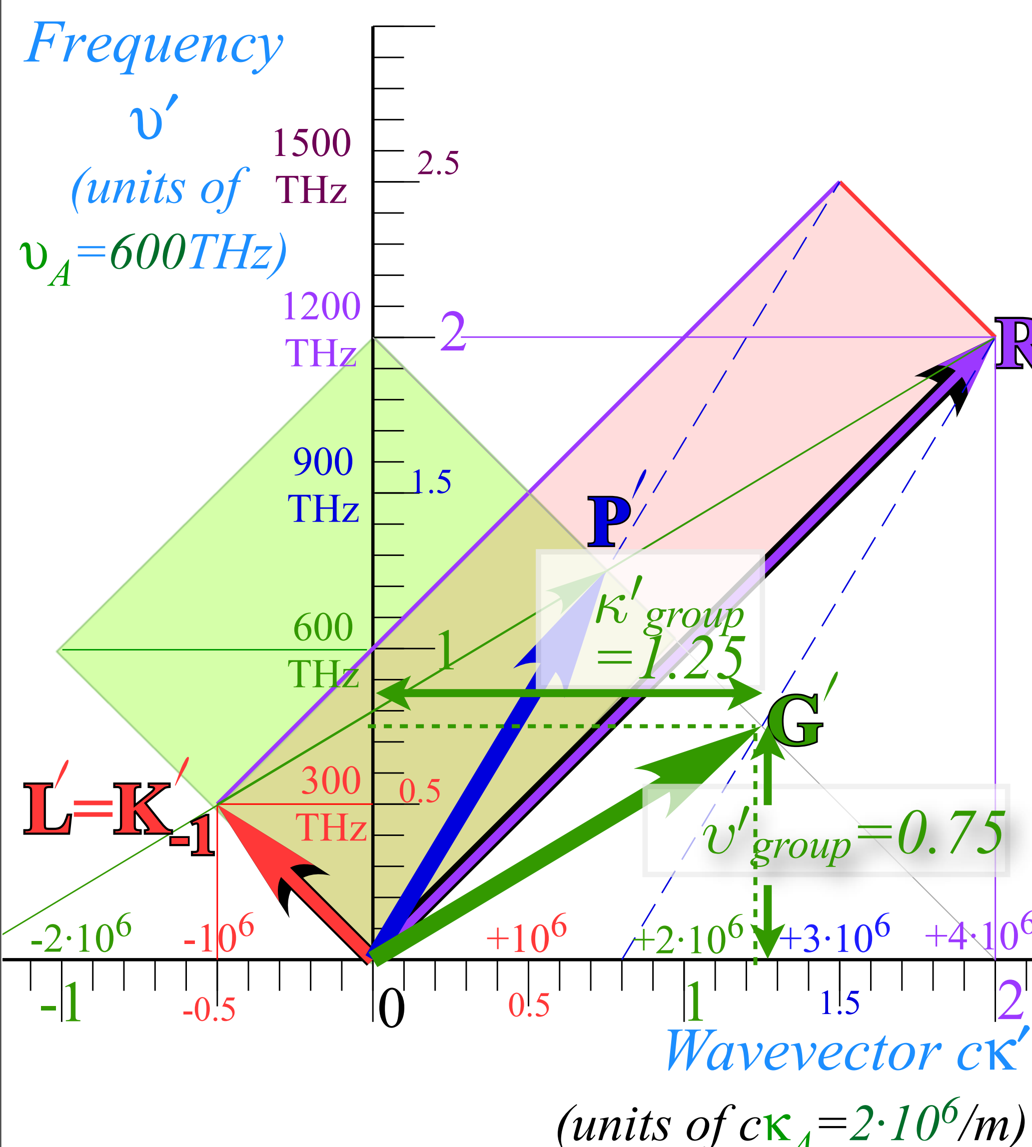
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

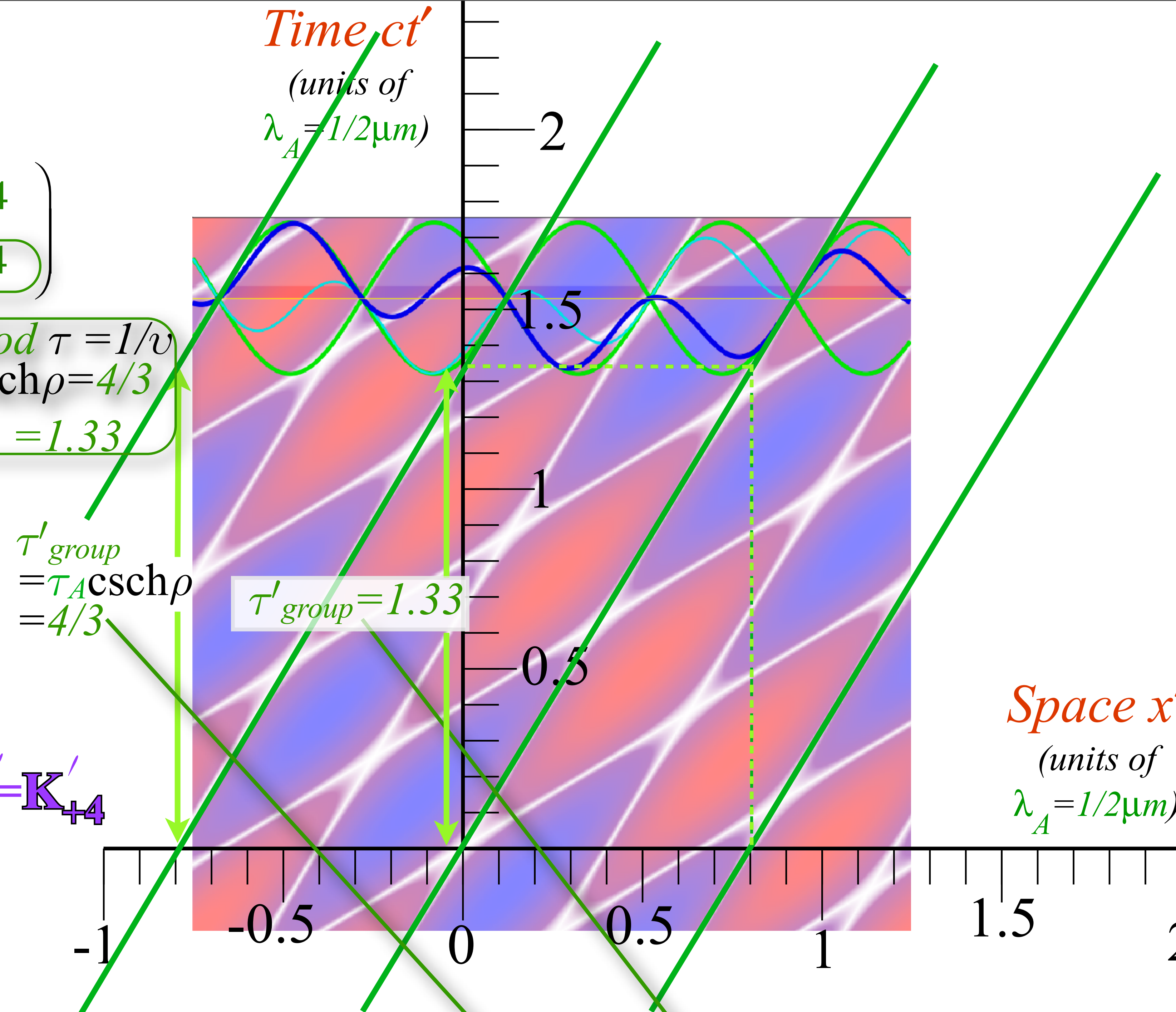
$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

Group frequency $\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

flips to

Group period $\tau = 1/\upsilon$: $\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

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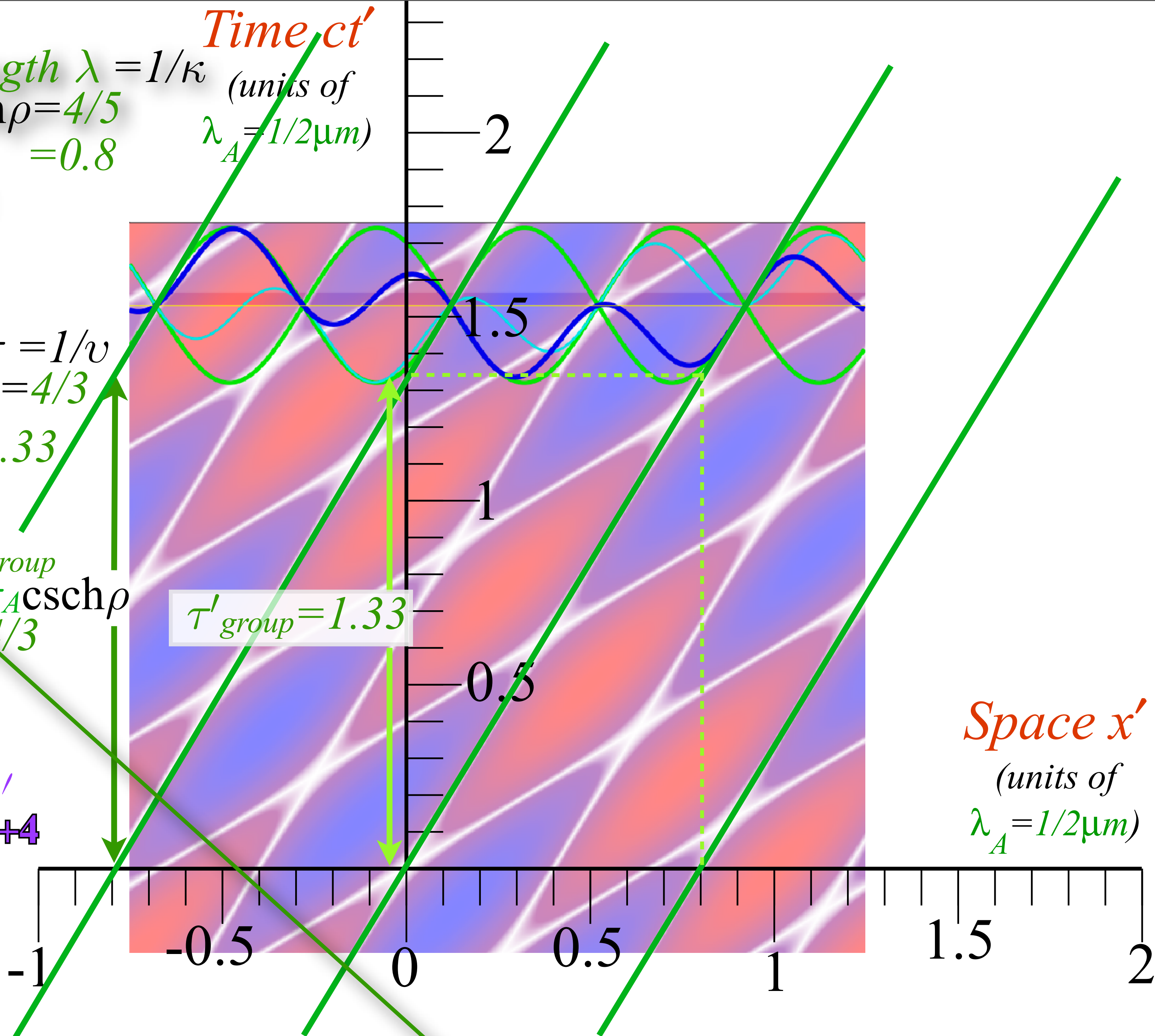

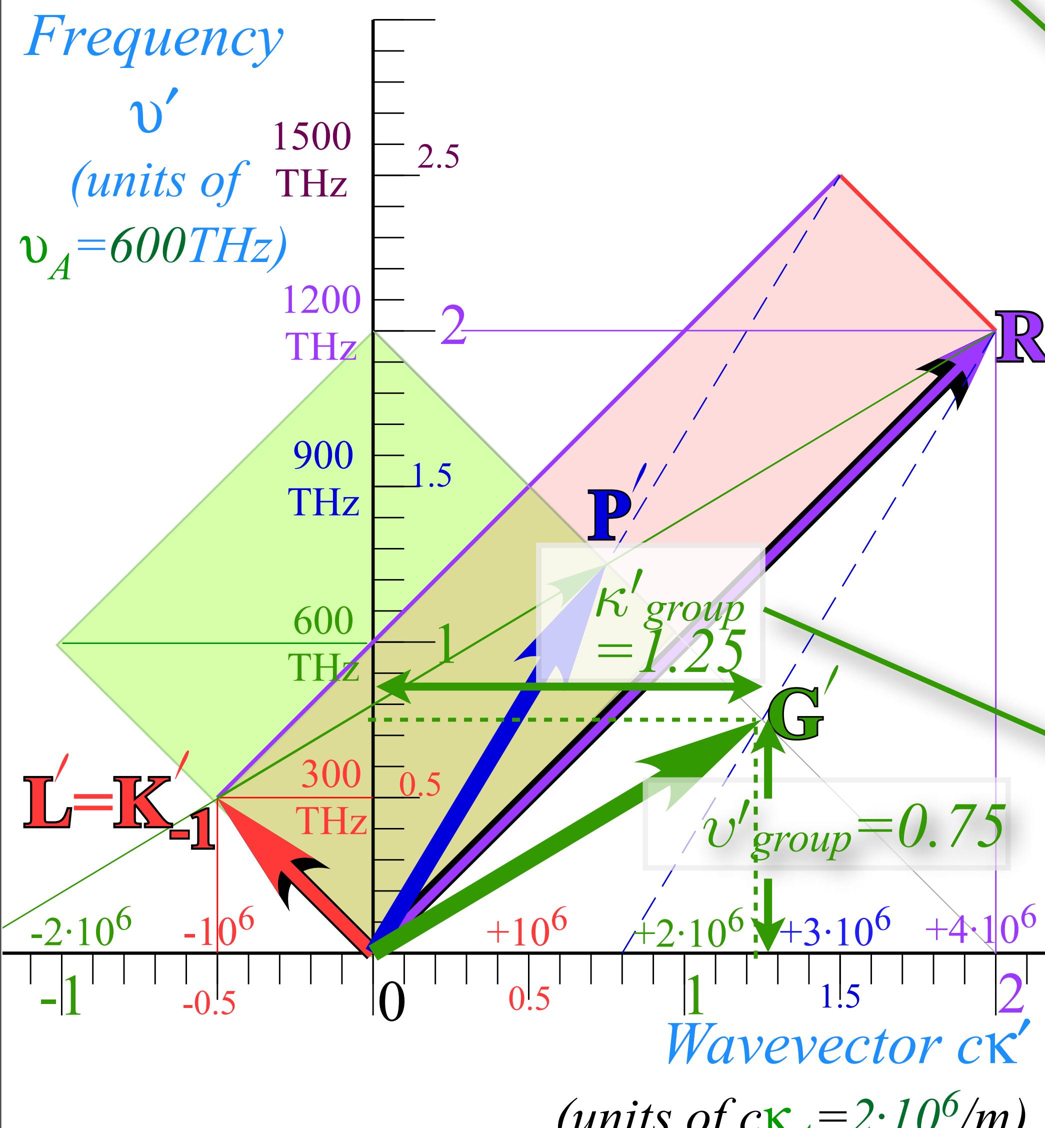

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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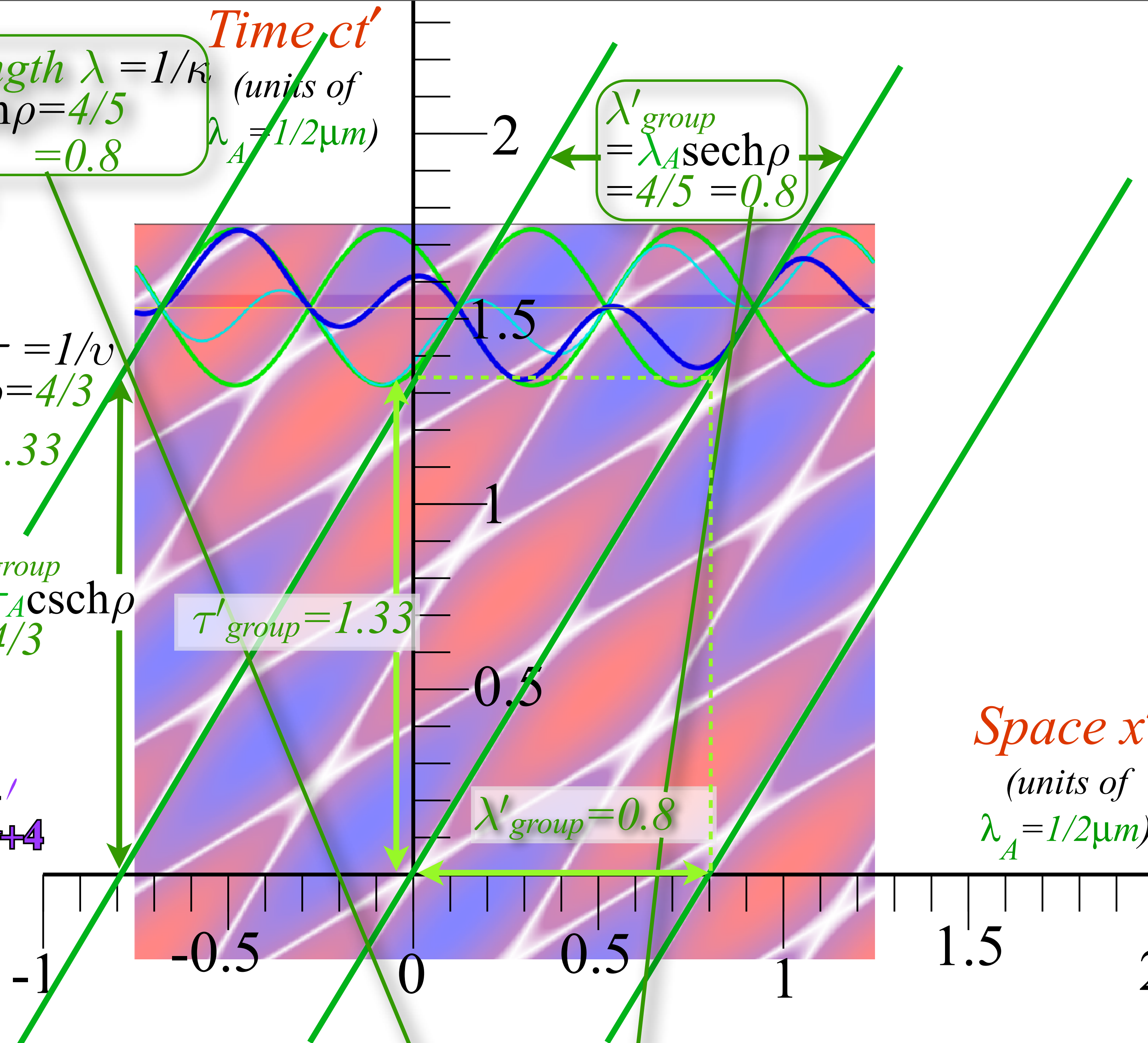

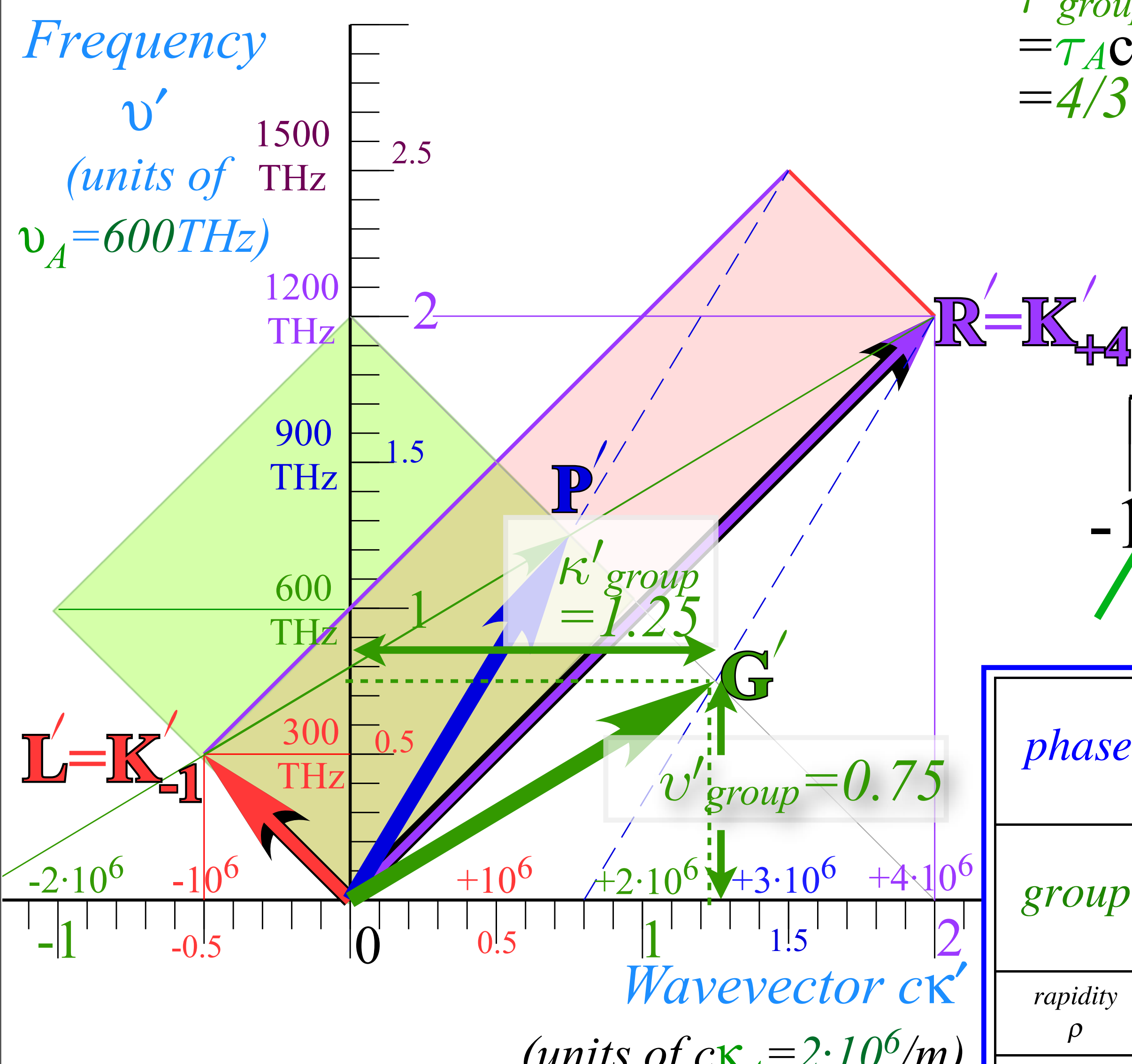

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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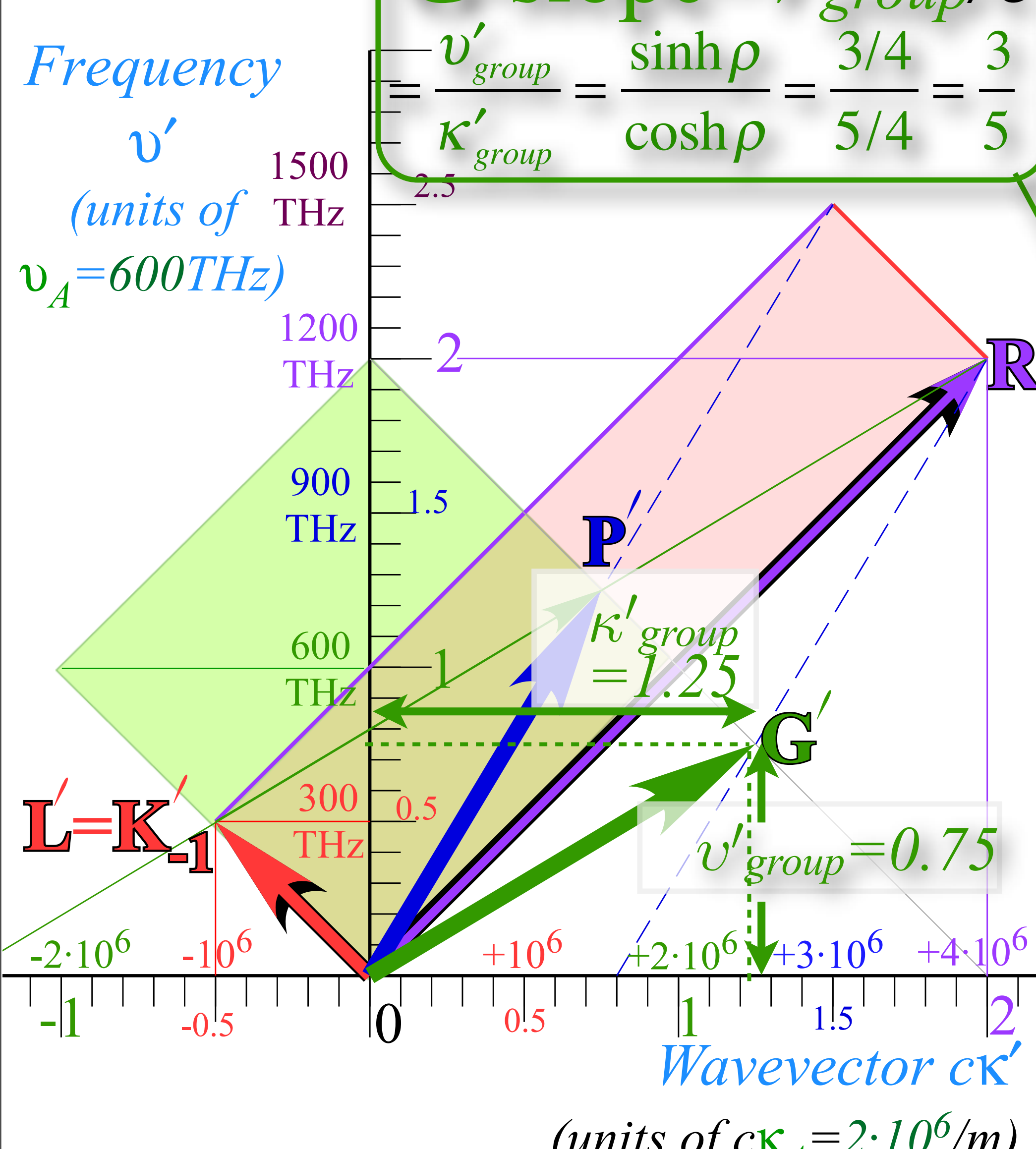
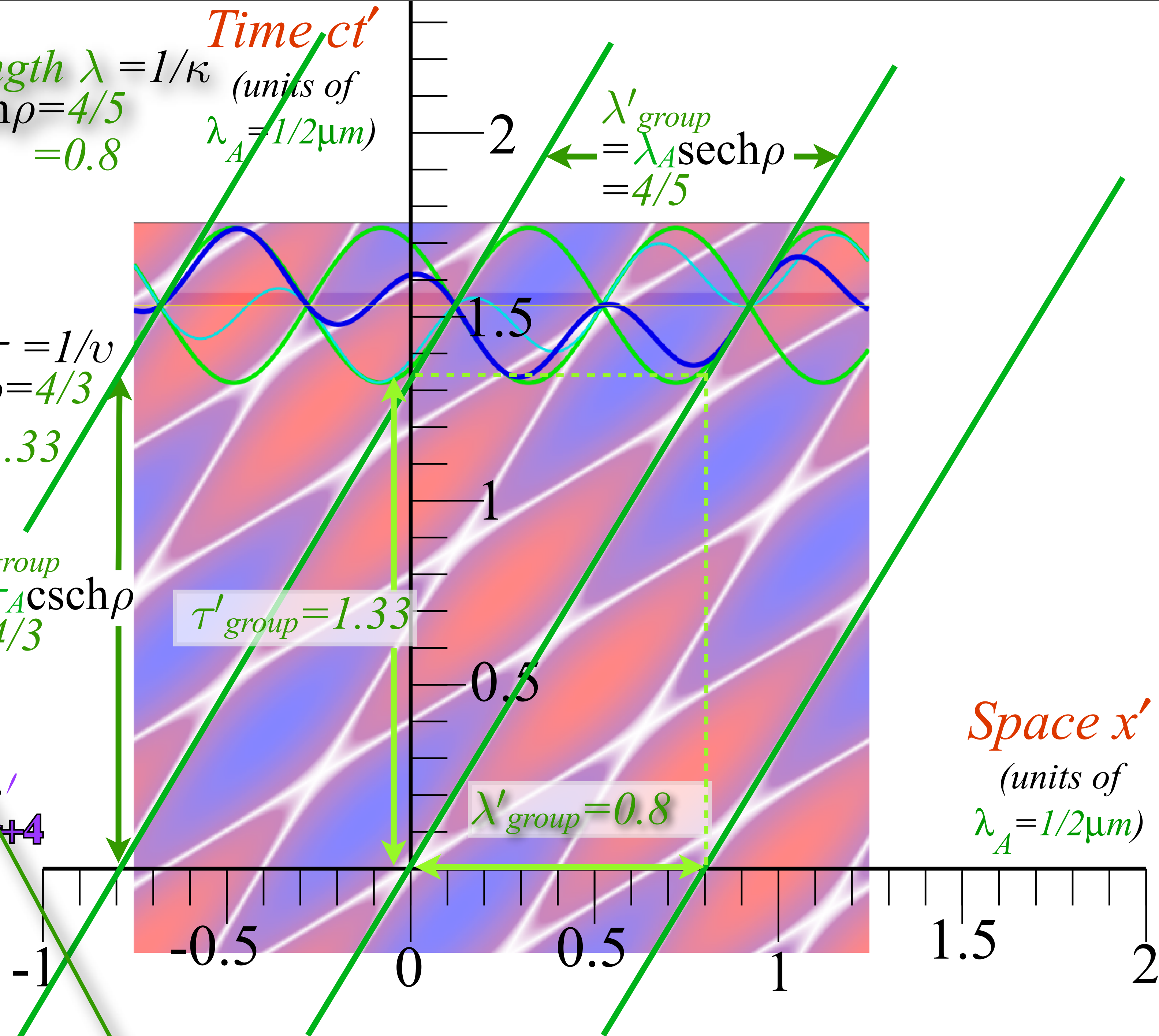
$$\text{G-slope} = V_{group}/c = \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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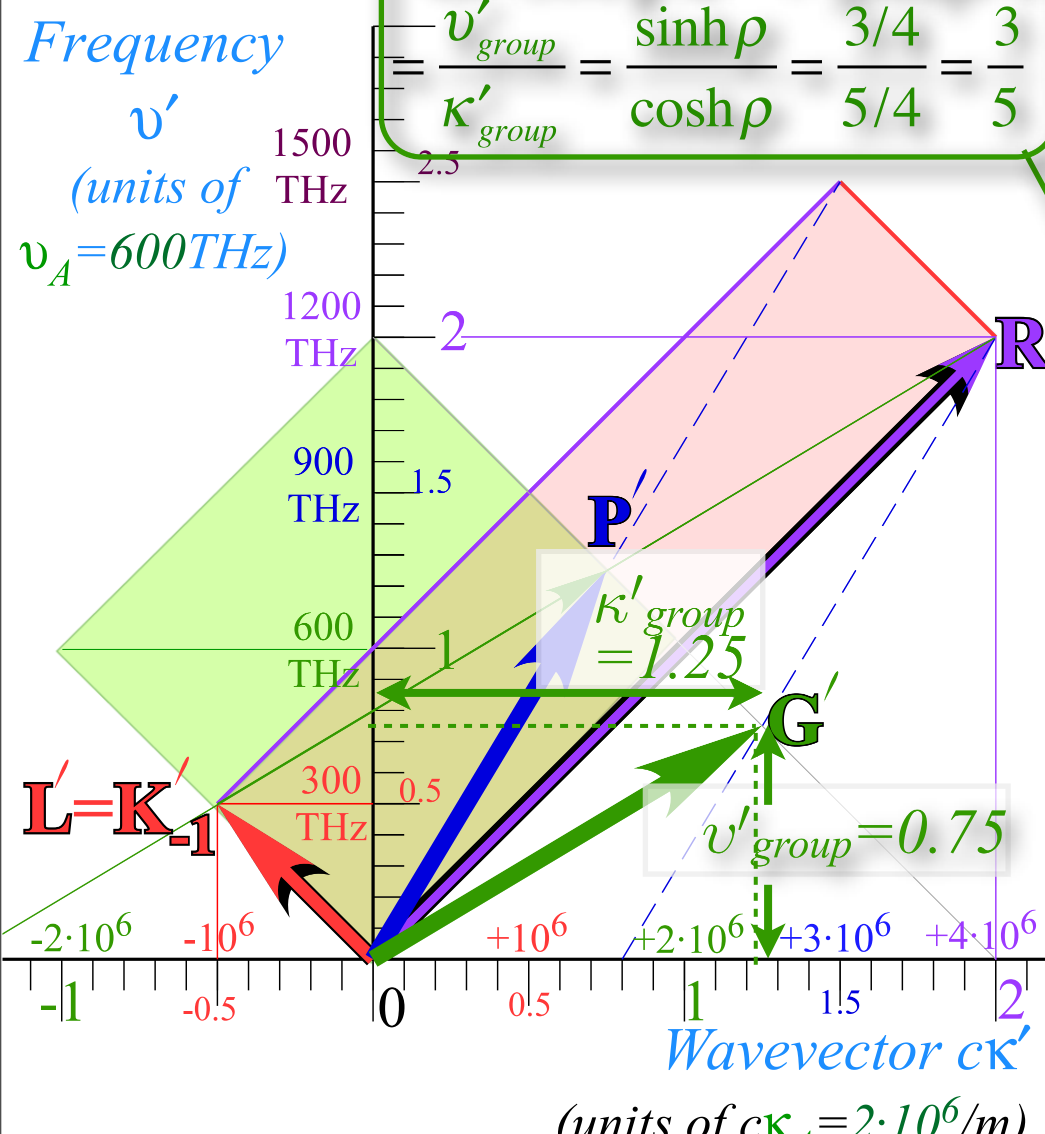
Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

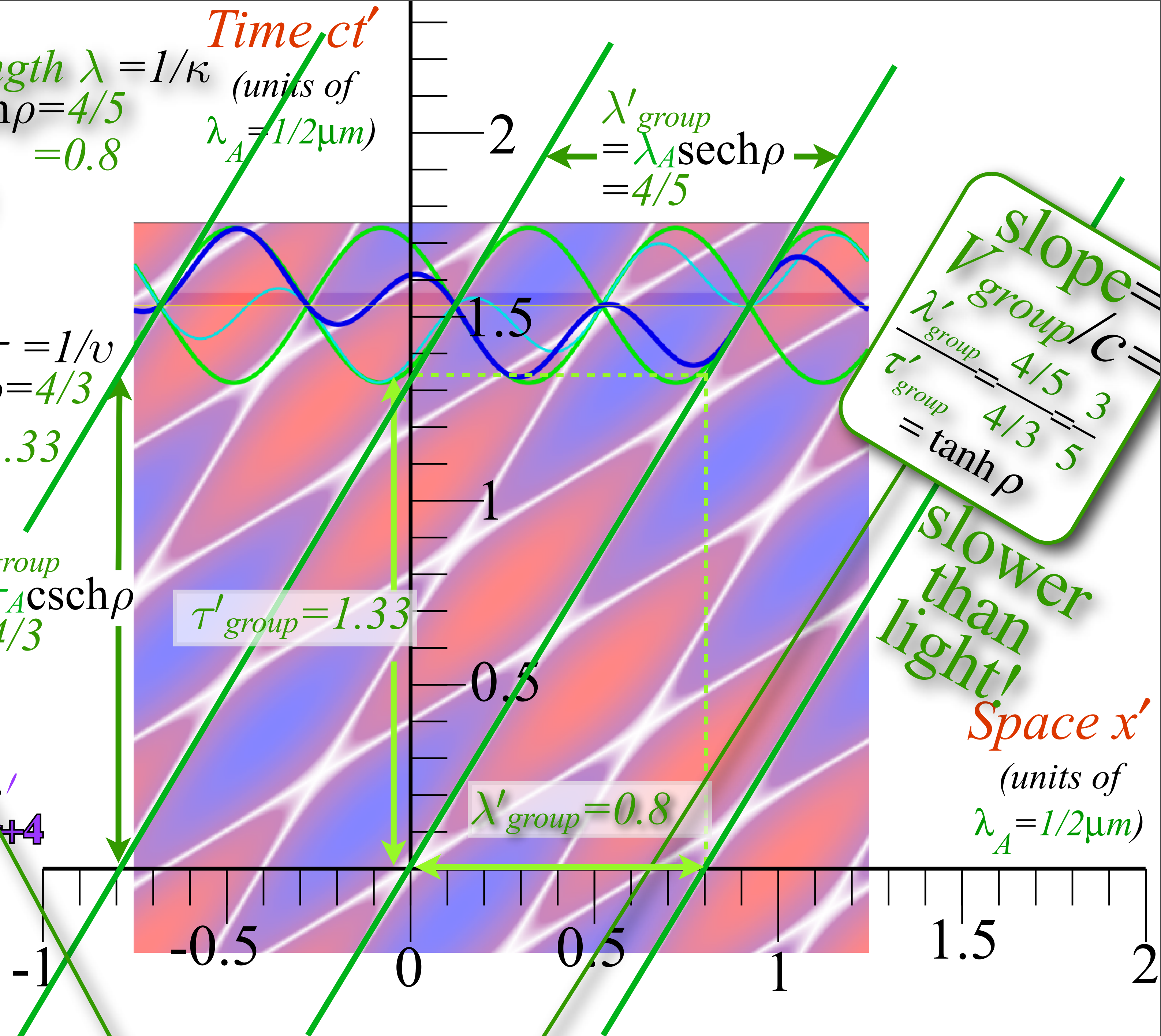
flips to

Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

G-slope $= V_{group}/c$
$$= \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$

$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Lorentz transformations...

write **G′** *and* **P′** *in terms of* **G** *and* **P** *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

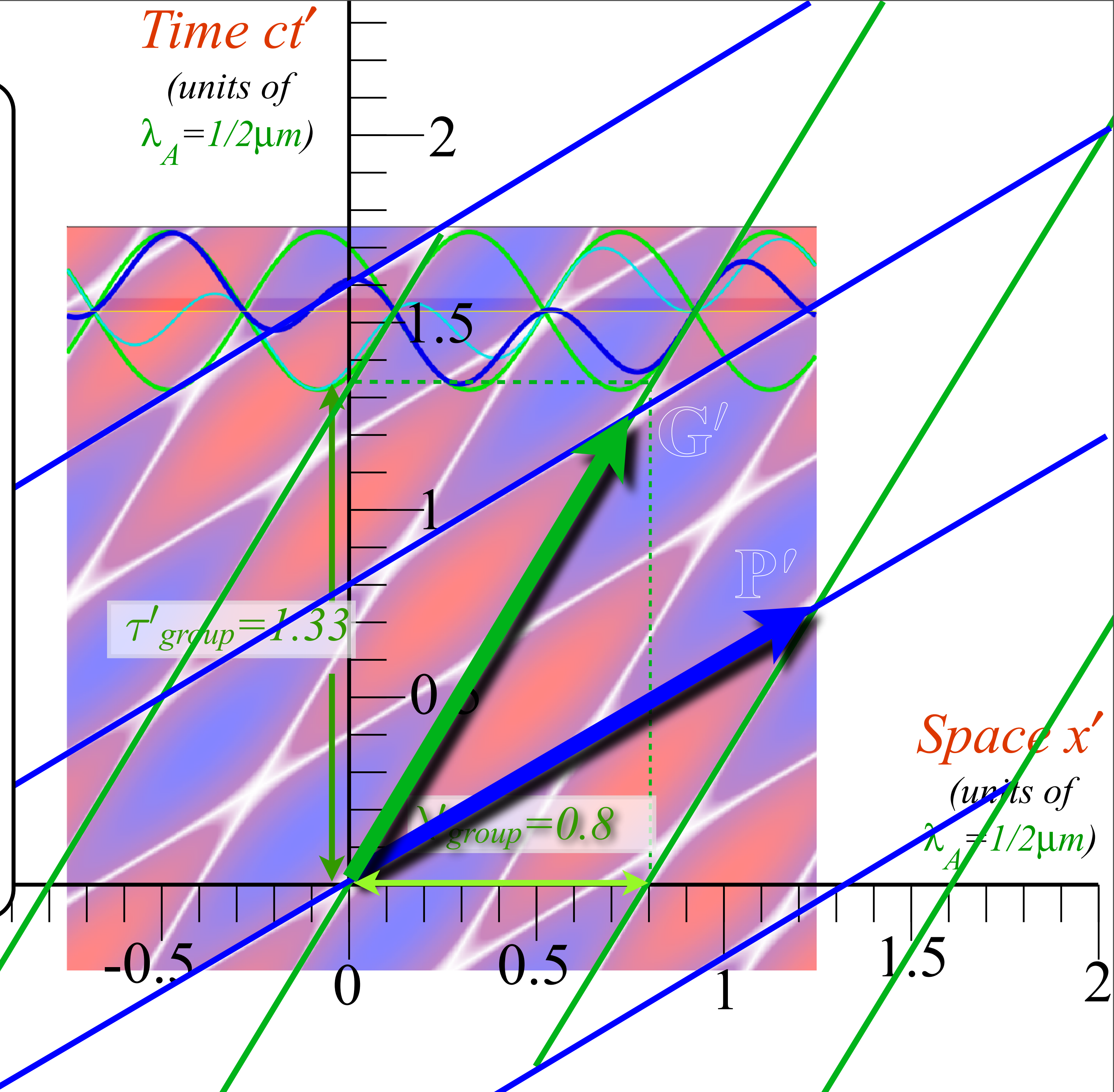
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

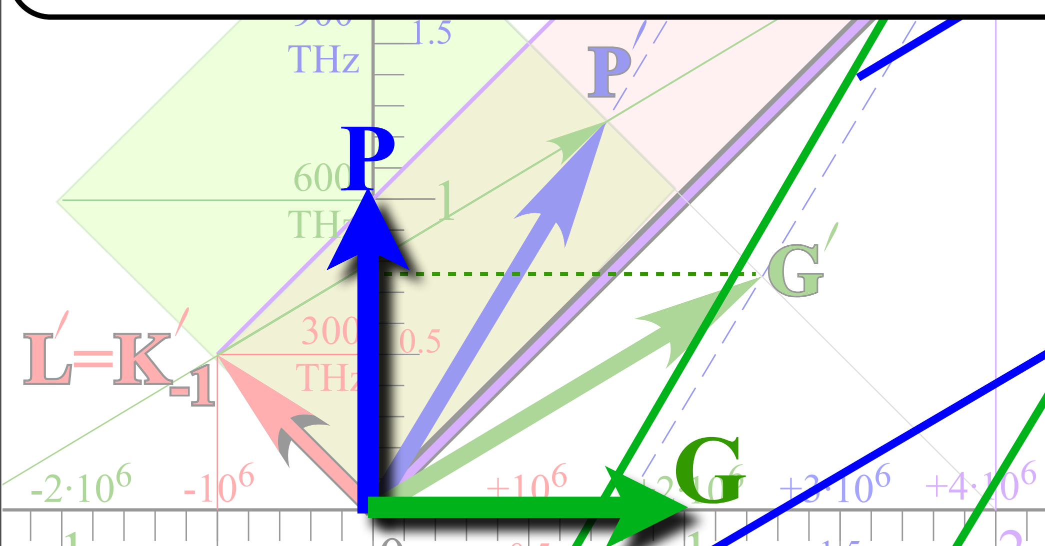
$$\mathbf{G}' = \mathbf{G}\ \cosh\rho + \mathbf{P}\ \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G}\ \sinh\rho + \mathbf{P}\ \cosh\rho$$

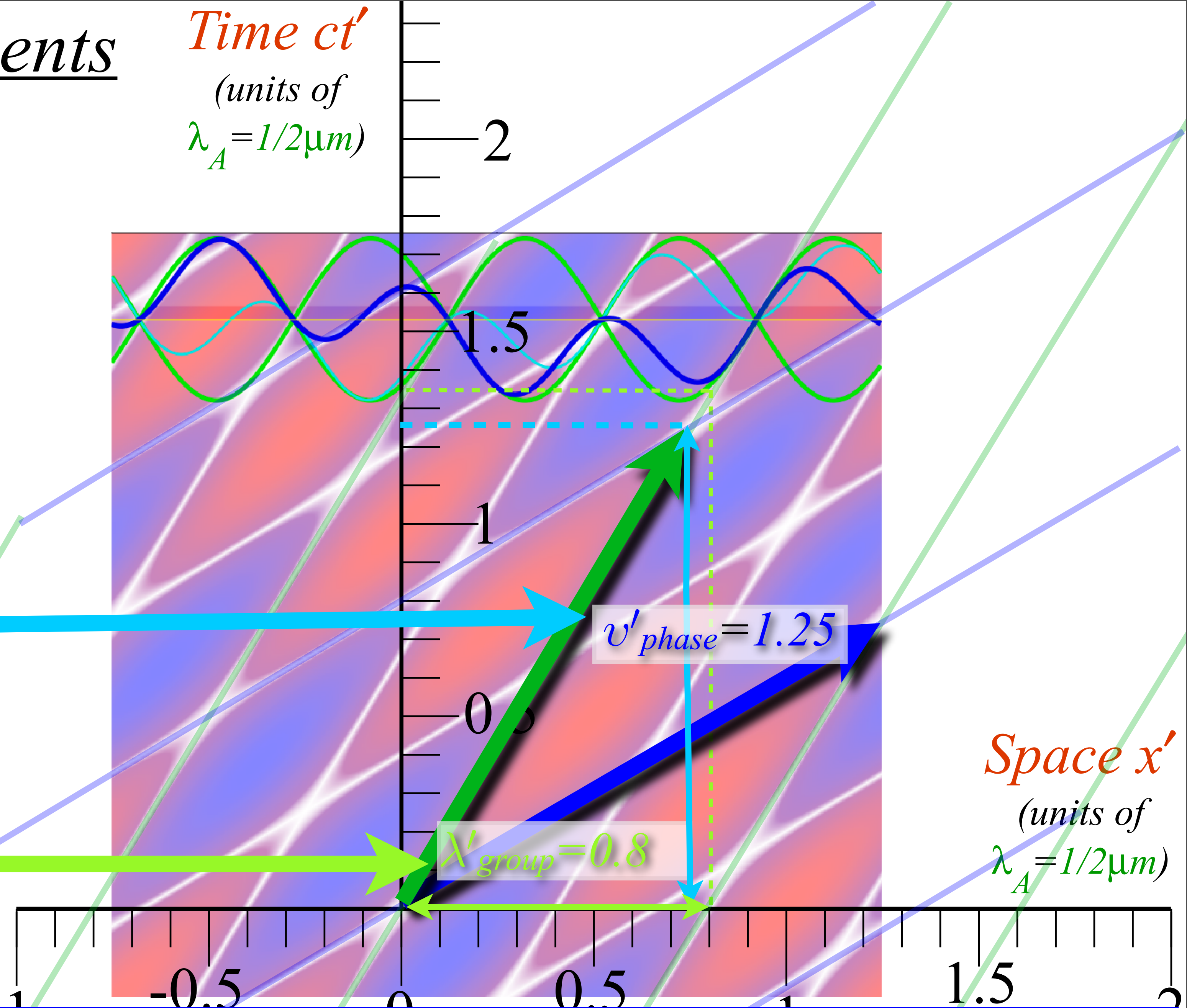




$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$ Lorentz transform matrix

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Two Famous-Name Coefficients



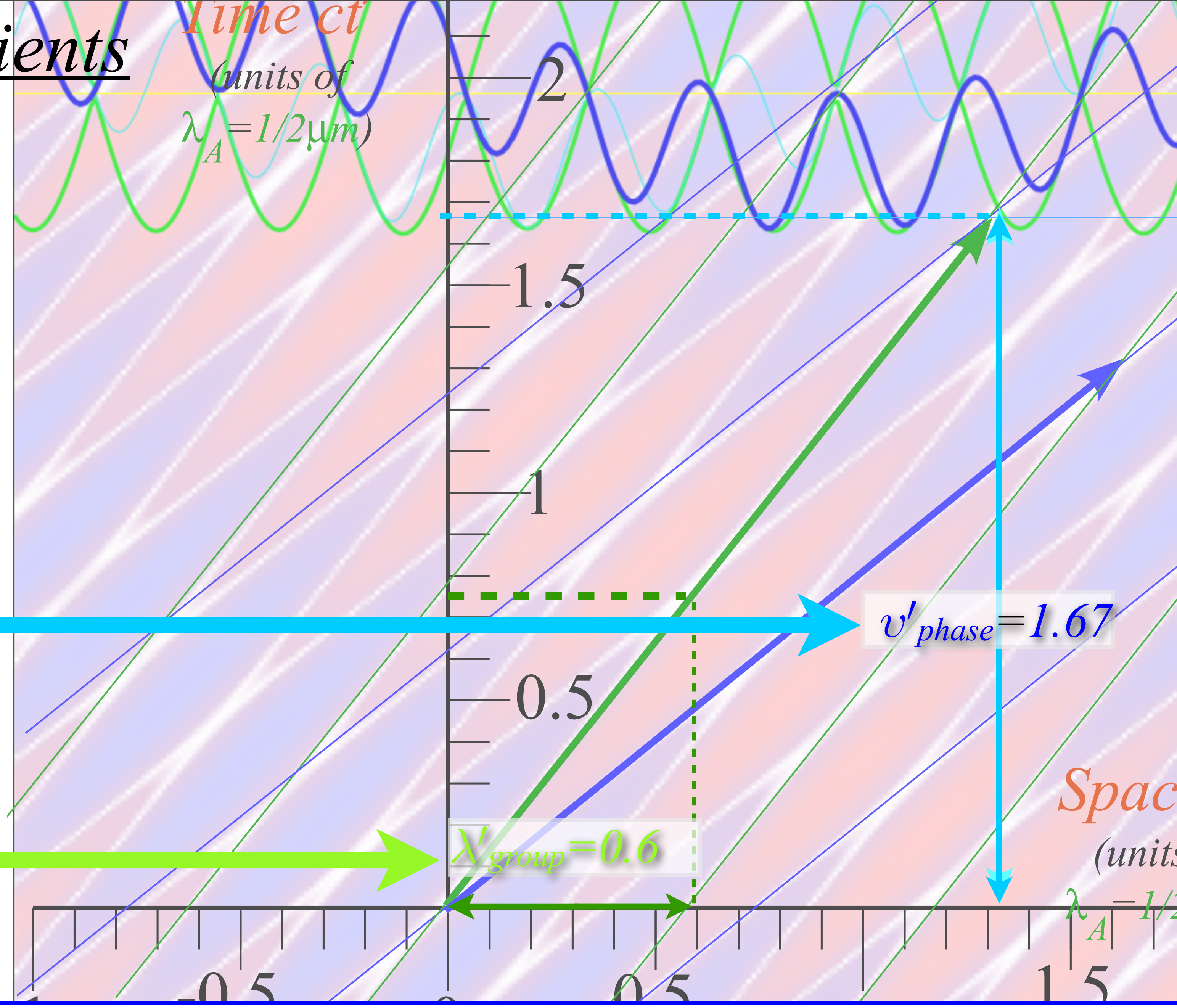
This number is called an: ***Einstein time-dilation*** *(dilated by 25% here)*

This number is called a: ***Lorentz length-contraction*** *(contracted by 20% here)*

Old-Fashioned Notation

phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Two Famous-Name Coefficients

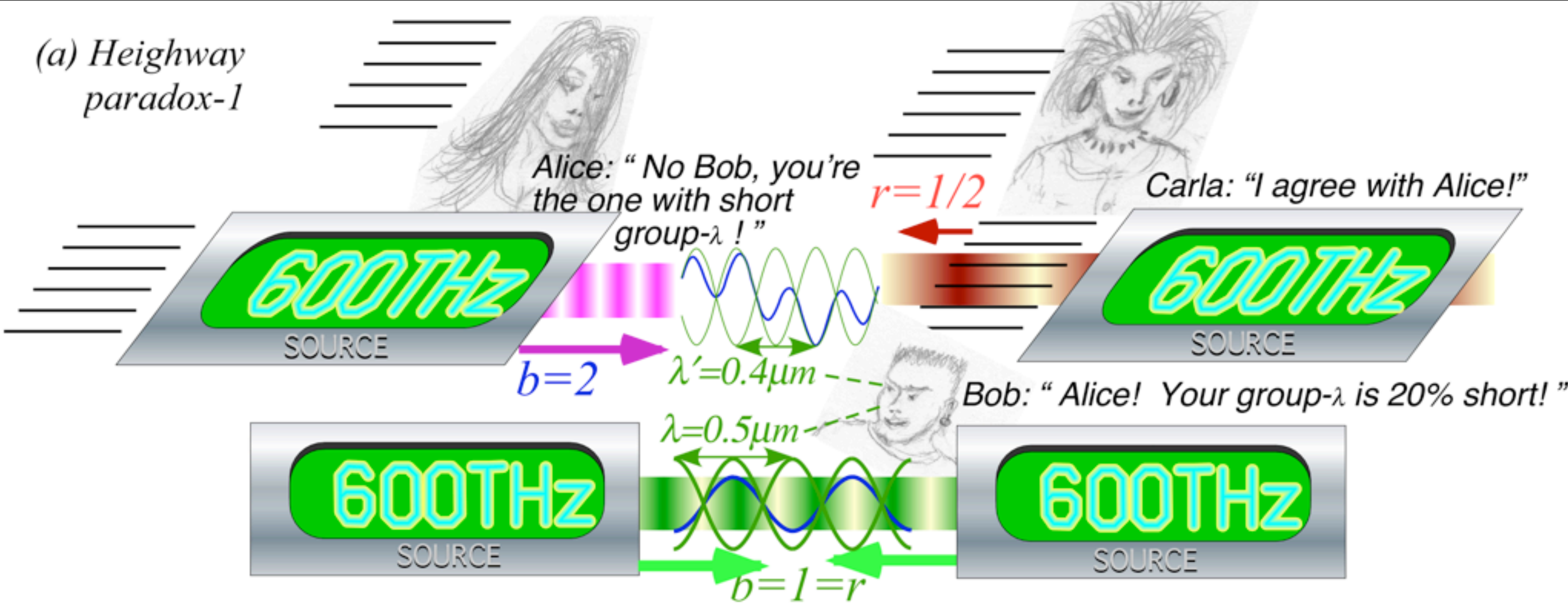


This number
is called an: ***Einstein time-dilation***
(dilated by 67% here)

This number
is called a: ***Lorentz length-contraction***
(contracted by 40% here)

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=4/5	$\frac{1}{3}$= 0.33	$\frac{4}{5}$ =0.8	$\frac{4}{3}$=1.33	$\frac{3}{5}$=0.60	$\frac{5}{3}$=1.67	$\frac{3}{4}$=0.75	$\frac{5}{4}$=1.25	$\frac{3}{1}$=3.0

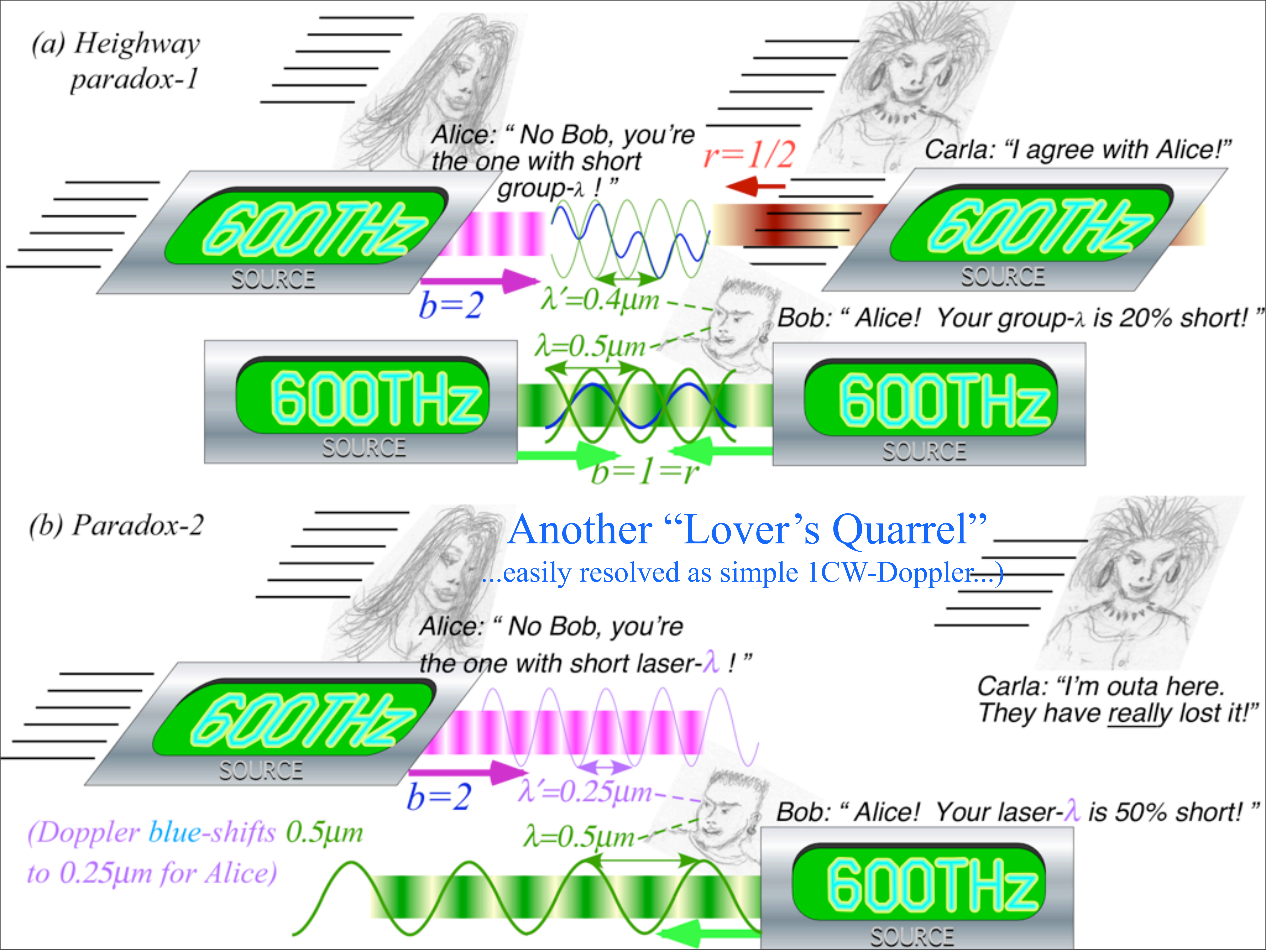


A "Lover's Quarrel" about a 20% Lorentz contraction $\lambda'_{group}=0.8\ \lambda_A$

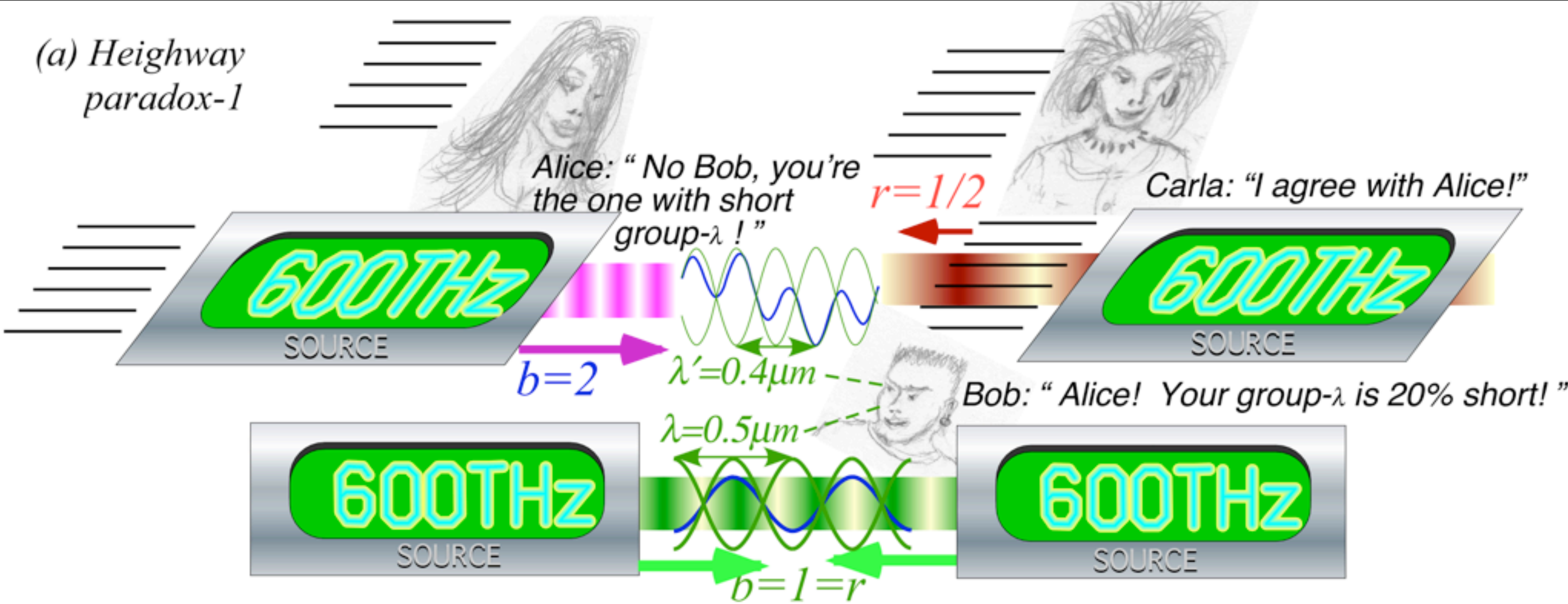
(You're short! No, *YOU'RE* short!!, *etc.*)

…(The worst kind of quarrel is when *both* are right *and* wrong)

$\frac{3}{5}c$



(a) Heighway paradox-1
Alice: " No Bob, you're the one with short group-λ ! "
r=1/2
Carla: "I agree with Alice!"
600THz
SOURCE
b=2
λ′=0.4μm
Bob: " Alice! Your group-λ is 20% short! "
λ=0.5μm
b=1=r
(b) Paradox-2
Another "Lover's Quarrel"
...easily resolved as simple 1CW-Doppler...)
Alice: " No Bob, you're the one with short laser-λ ! "
Carla: "I'm outa here. They have really lost it!"
λ′=0.25μm
Bob: " Alice! Your laser-λ is 50% short! "
(Doppler blue-shifts 0.5μm to 0.25μm for Alice)



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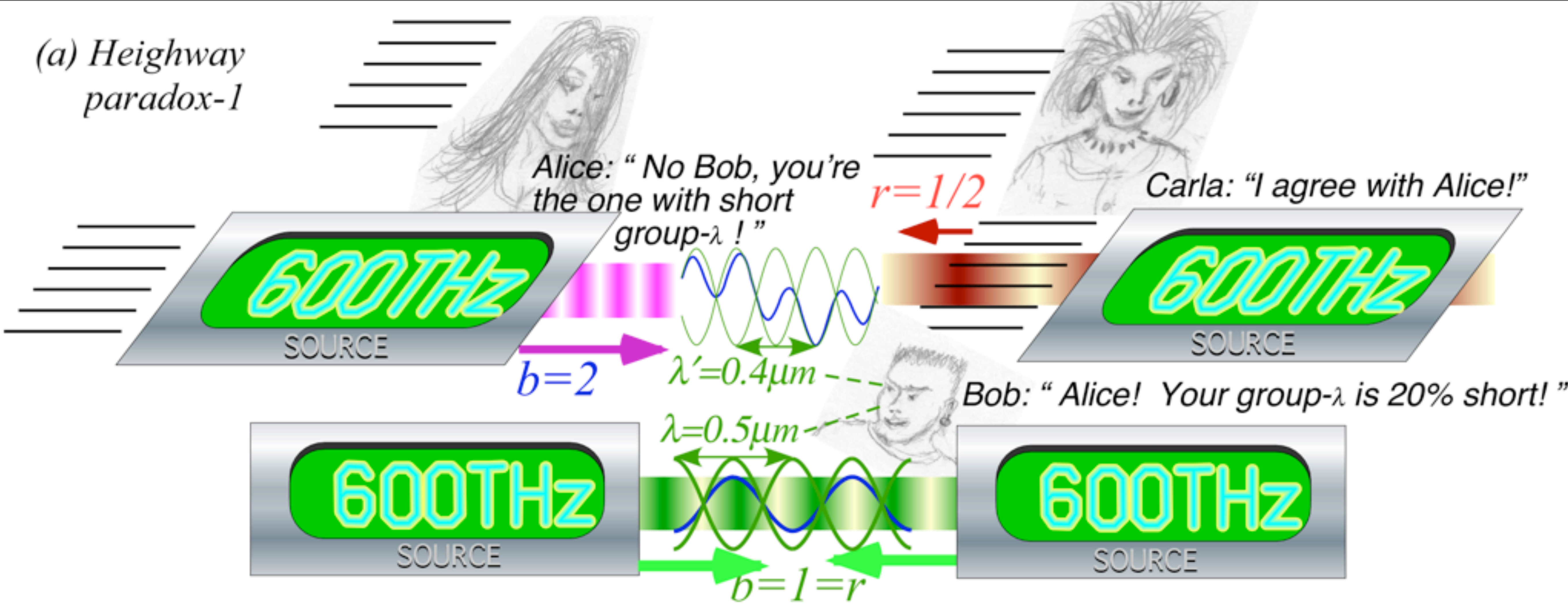
So we learn to accept that a group-wave shortens by 20% at this enormous speed of $\frac{3}{5}c$.

Q: But, does the *steel laser cavity* holding the wave *also shorten* by 20%??

A: ...



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So we learn to accept that a group-wave shortens by 20% at this enormous speed of $\frac{3}{5}c$.

Q: But, does the *steel laser cavity* holding the wave *also shorten* by 20%??

A: Yes, or else laser does not resonate! *Steel is made of waves, too.*

Contraction is what waves do.

Let's do the A$_{lice}$B$_{ob}$C$_{arla}$ problem backwards...

Suppose Bob sees beam of frequency υ_L coming from the *LEFT*
and
opposing beam of frequency υ_R coming from the *RIGHT*.

Question 1: To what velocity u_E must Bob accelerate to see beams of *EQUAL* frequency υ_E ?

Question 2: What is frequency υ_E ?

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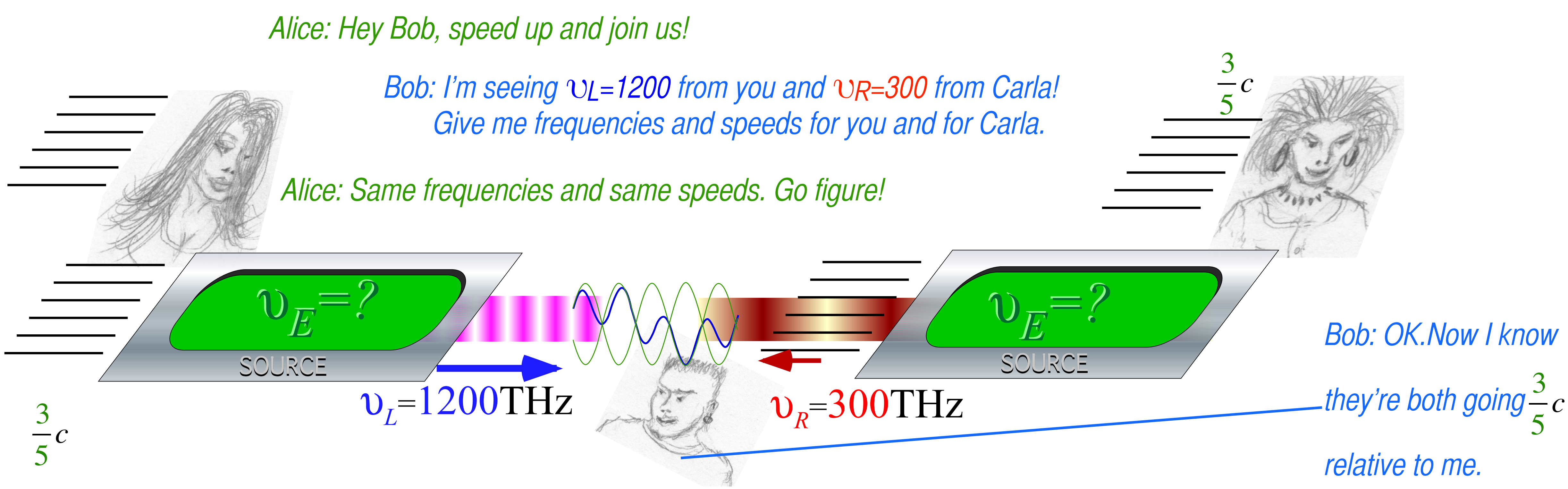
$$\frac{\text{Difference Mean}}{\text{Arithmetic Mean}} = \frac{1200 - 300}{1200 + 300}c = \frac{900}{1500}c = \frac{3}{5}c$$



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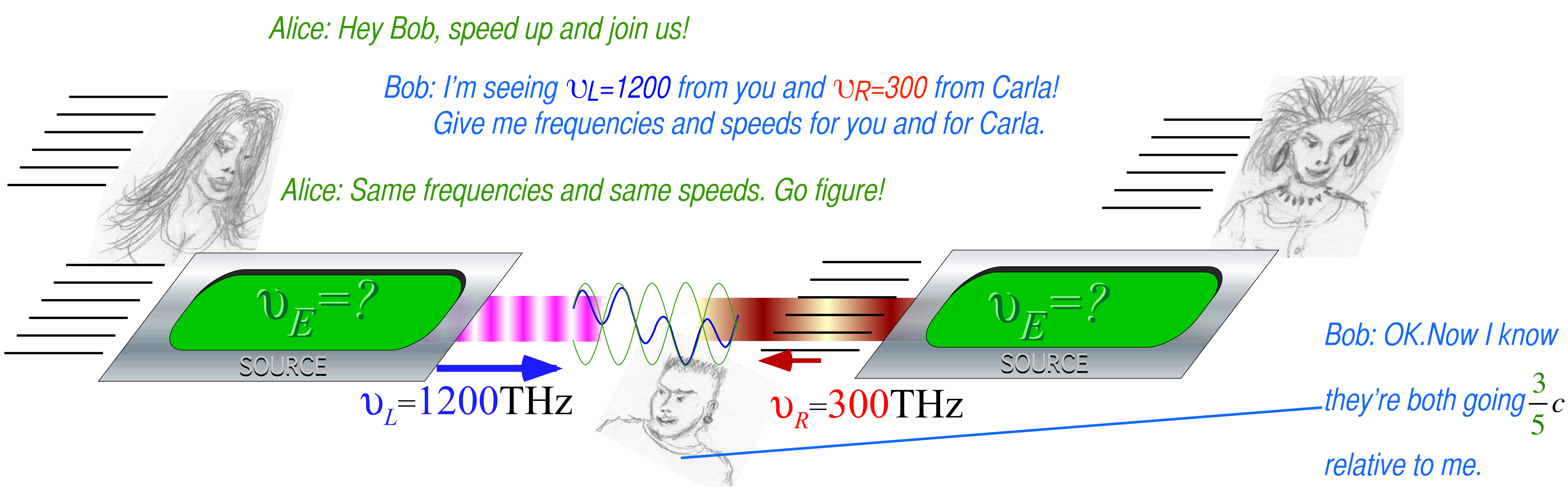
Question 2. ...similarly: *What* υ_E *is blue-shift* $b\upsilon_R$ *of* υ_R *AND red-shift* $r\upsilon_L = \upsilon_L/b$ *of* υ_L*?*



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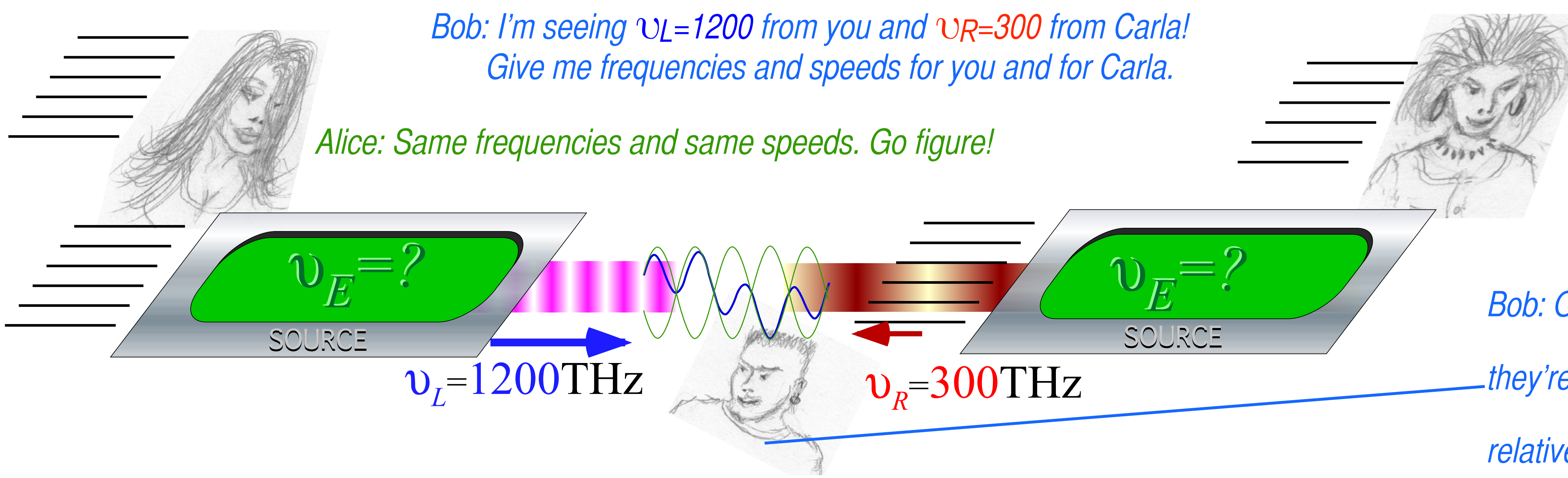
$$\upsilon_E = b\upsilon_R = r\upsilon_L = \frac{\upsilon_L}{b} \text{ implies: } b^2 = \frac{\upsilon_L}{\upsilon_R} \text{ or: } b = \sqrt{\frac{\upsilon_L}{\upsilon_R}}$$

Alice: Hey Bob, speed up and join us!

Bob: I'm seeing υ_L=1200 from you and υ_R=300 from Carla!
Give me frequencies and speeds for you and for Carla.

Alice: Same frequencies and same speeds. Go figure!



Bob: OK.Now I know
they're both going $\frac{3}{5}c$
relative to me.

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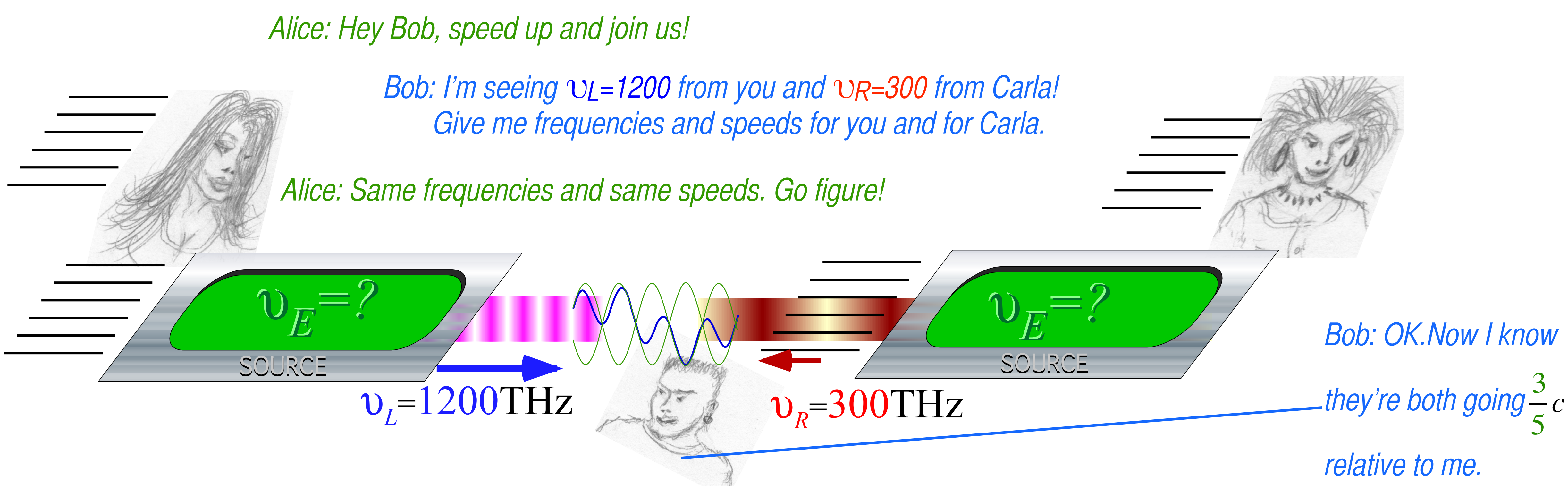

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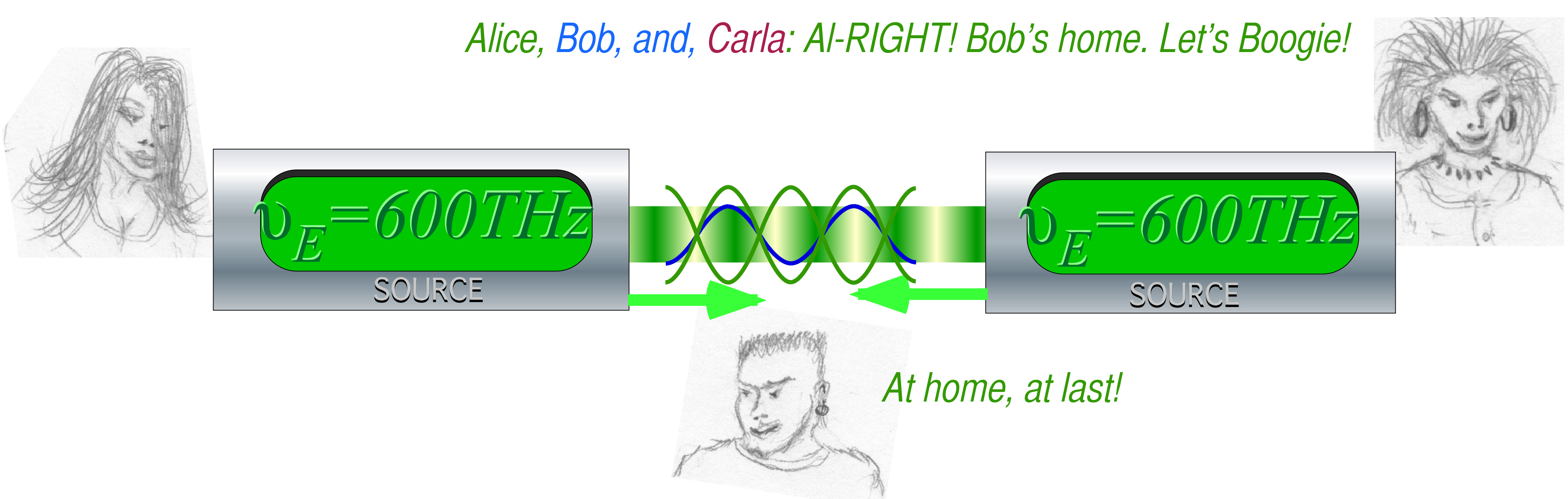
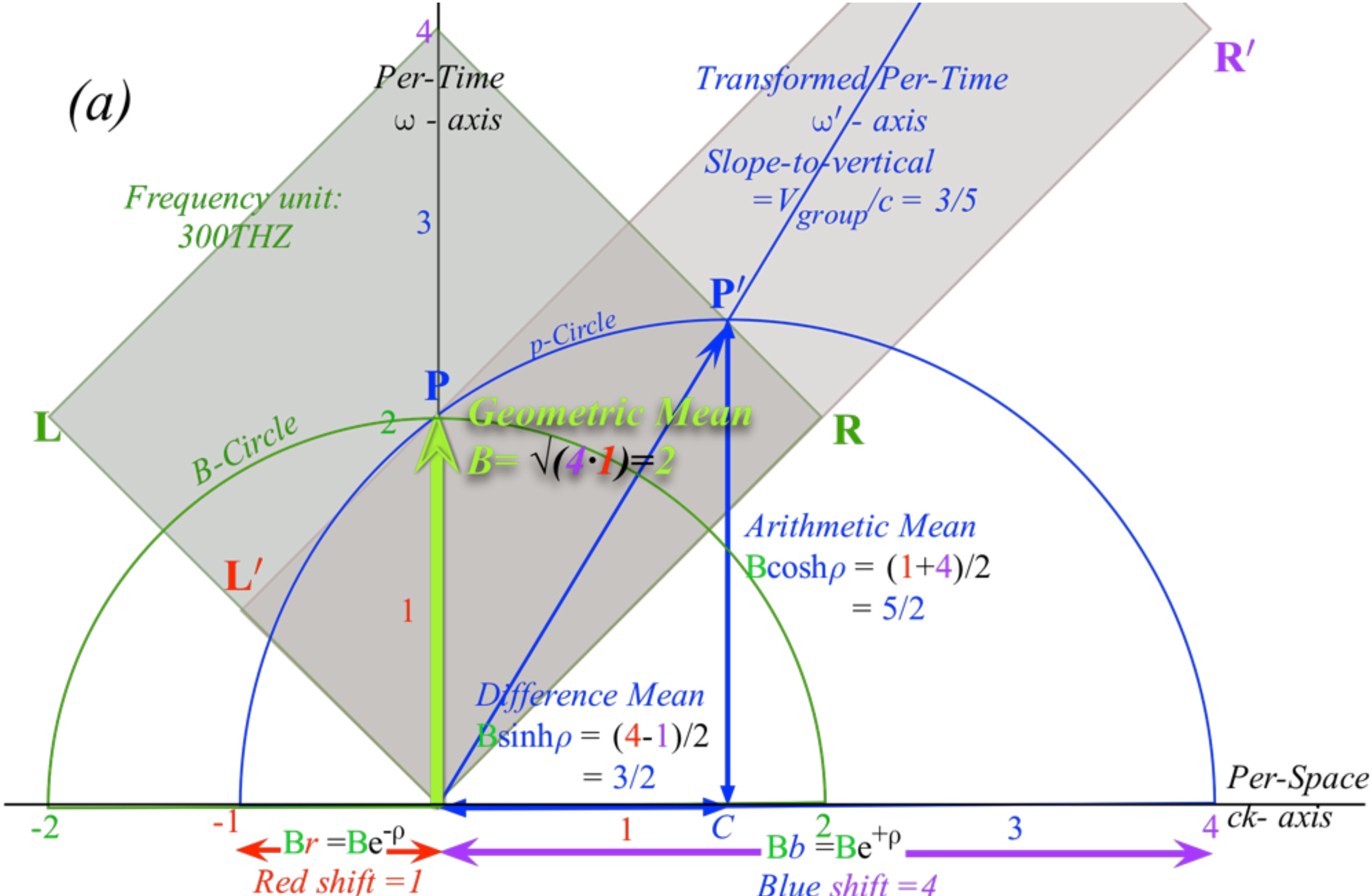
Geometric Mean

$$= \sqrt{1200 \cdot 300}$$
$$= \sqrt{360000}$$
$$= 600Thz$$



Thales Mean Geometry (600BCE)

helps “Relawavity”



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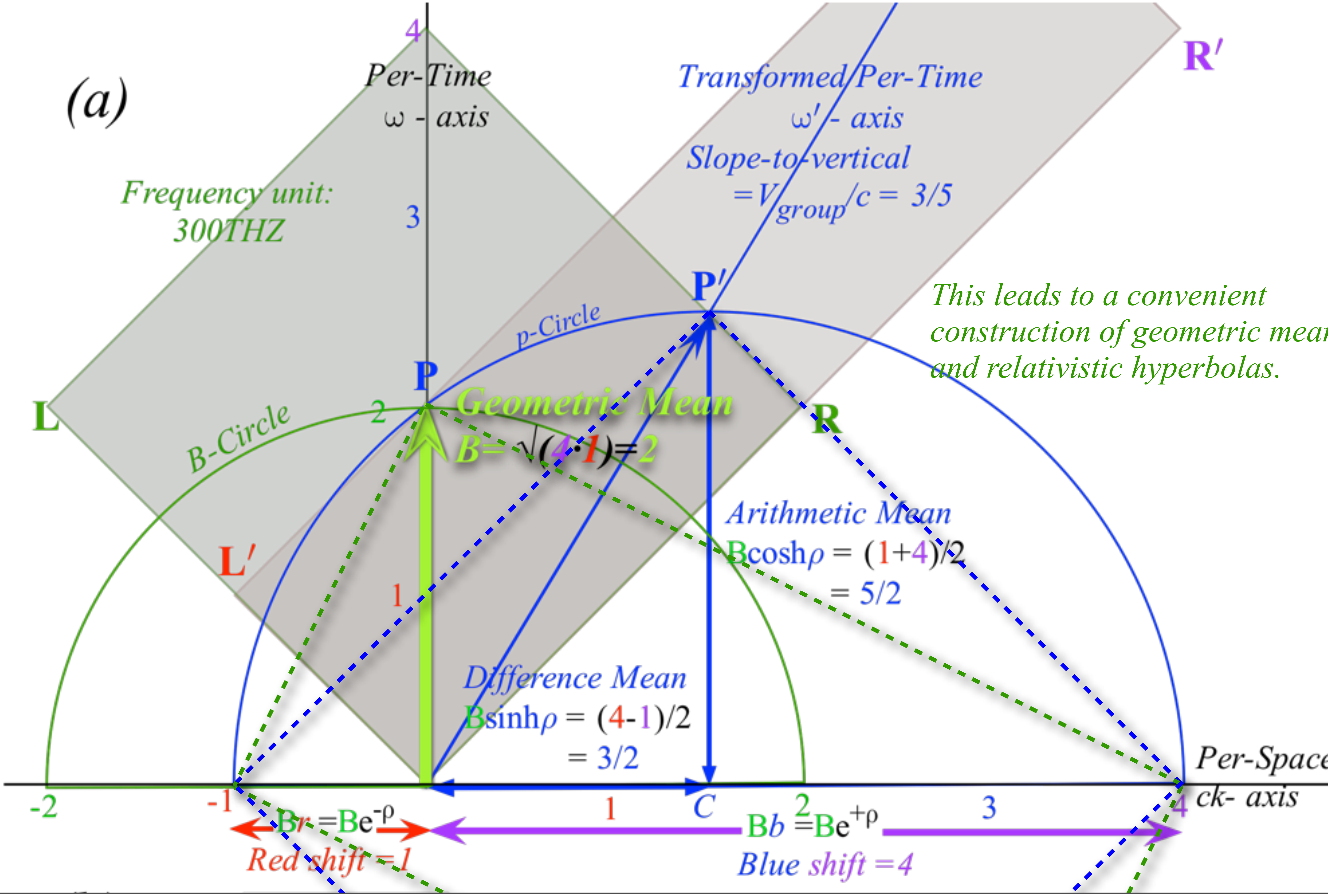

This leads to a convenient construction of geometric means and relativistic hyperbolas.

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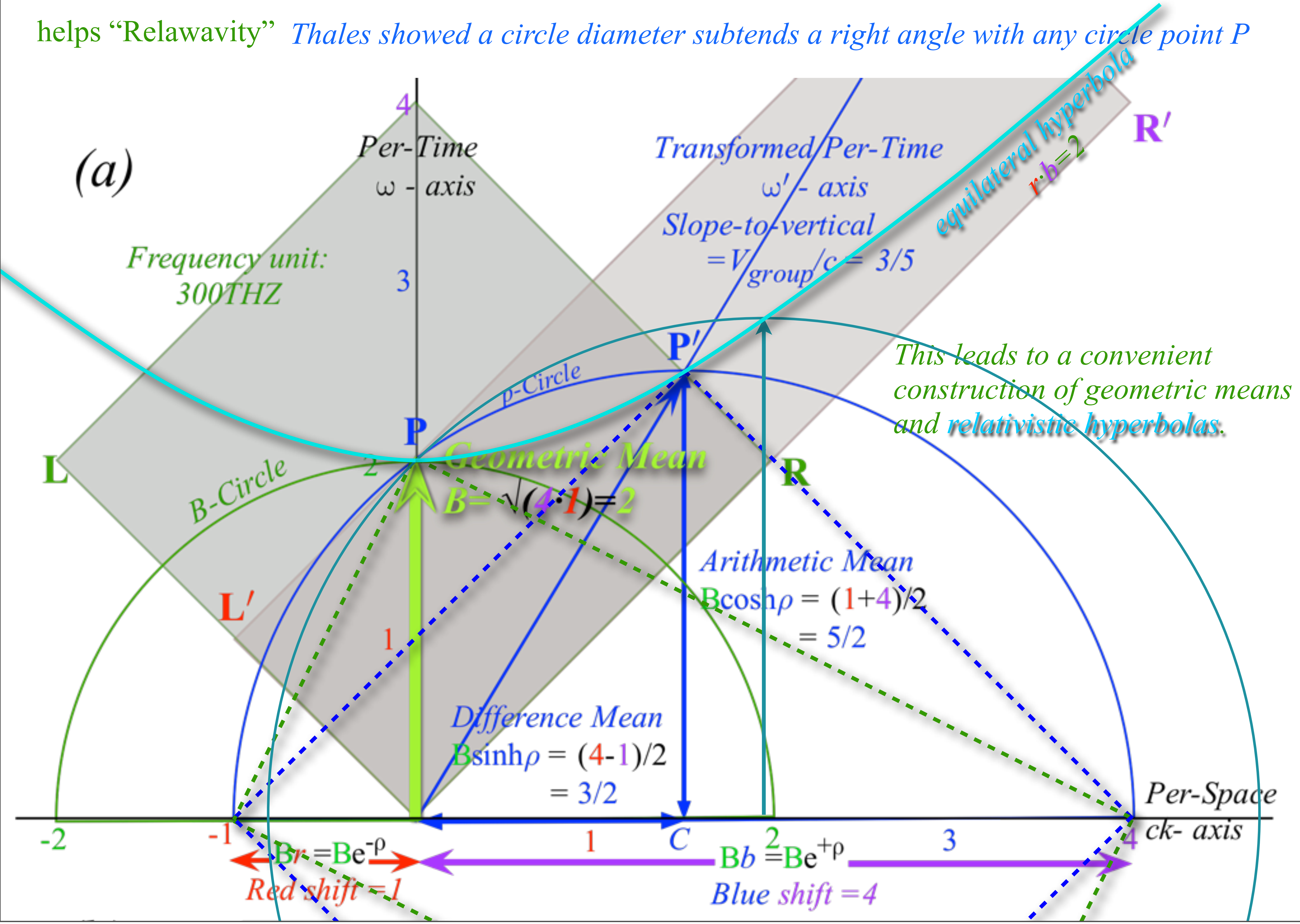

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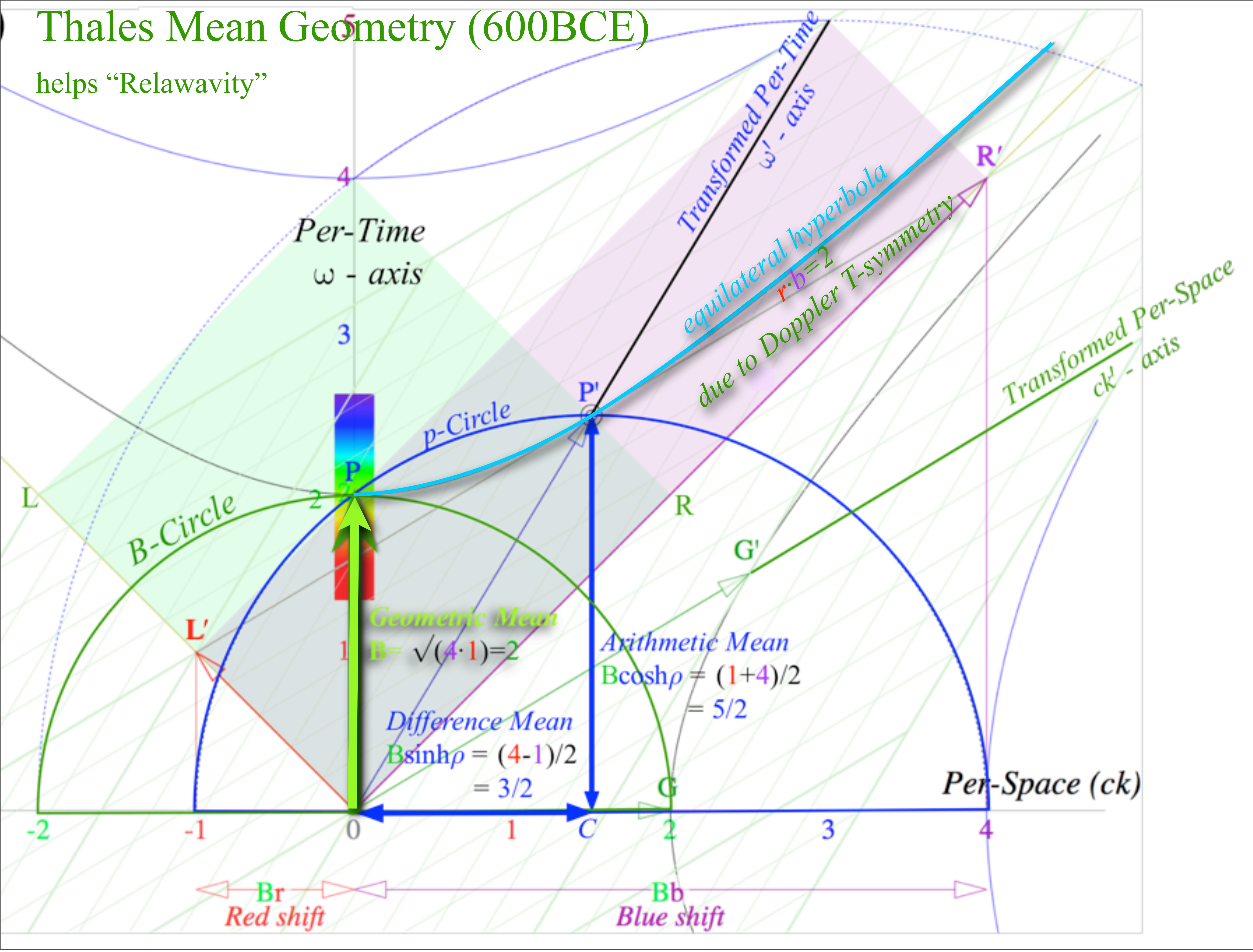


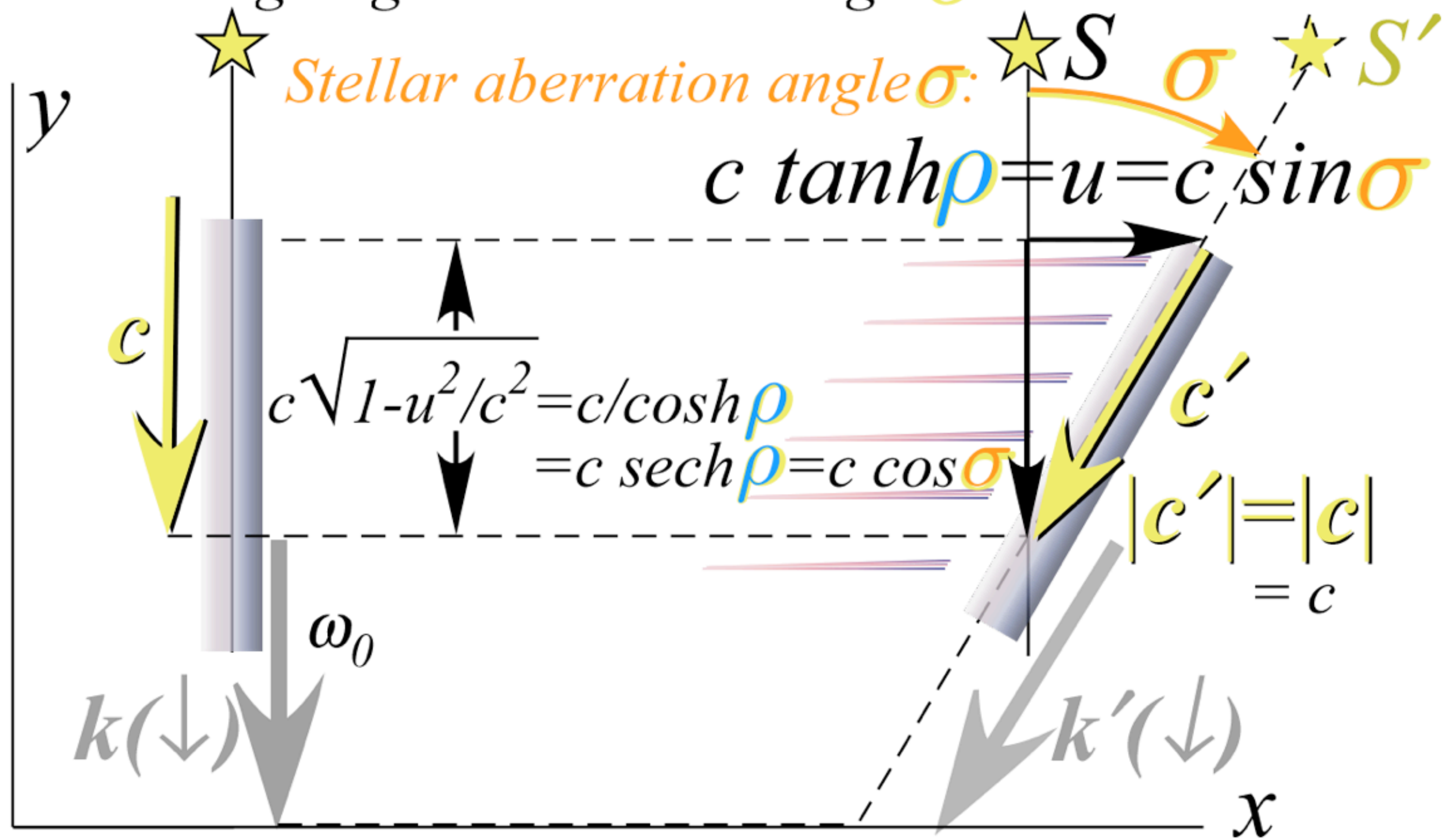
Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



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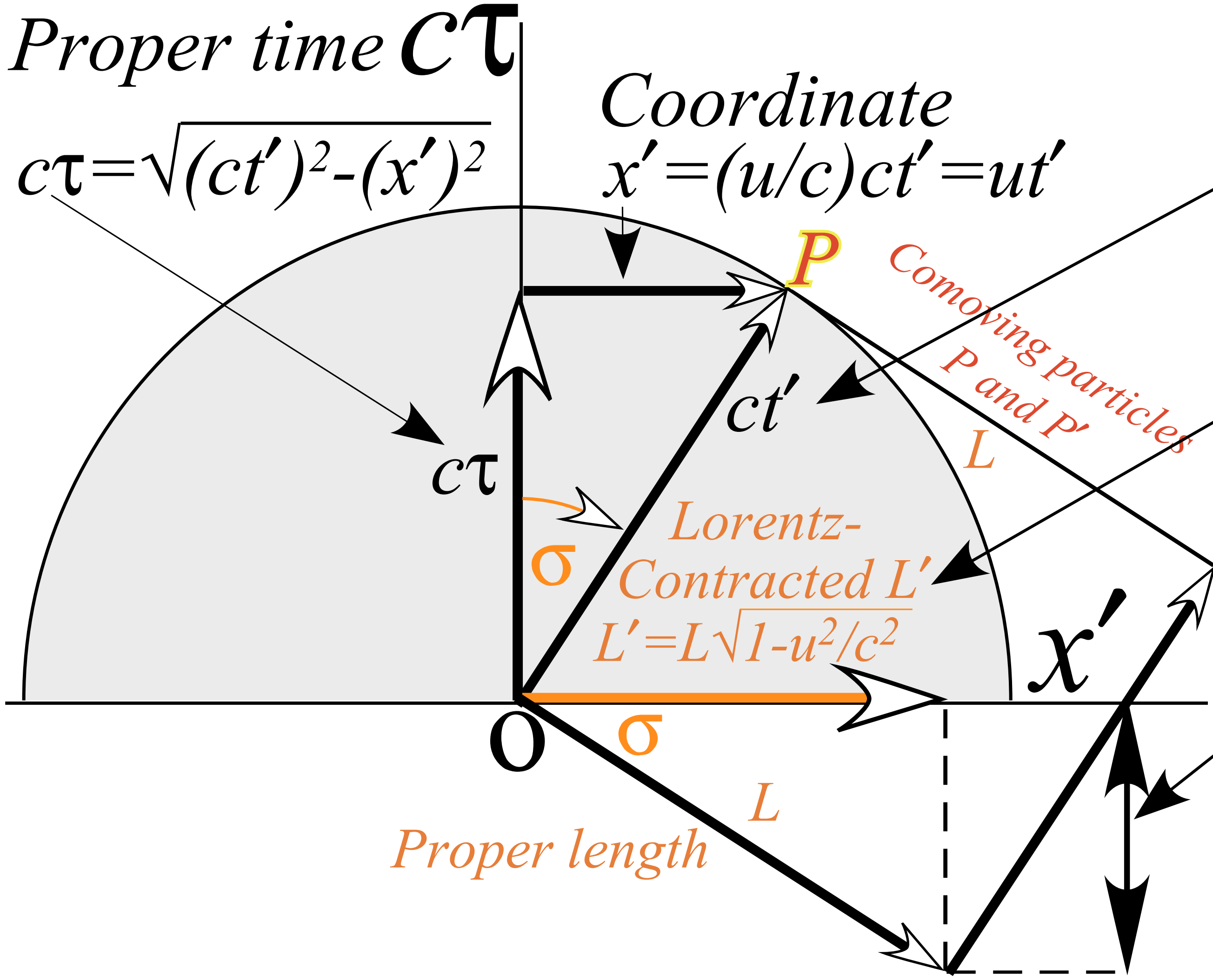
Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x',ct') and speed c in $(x, c\tau)$



Einstein time dilation:

$$ct' = c\tau\,\sec\sigma = c\tau\,\cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$

$$= L\cos\sigma\tan\sigma$$

$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

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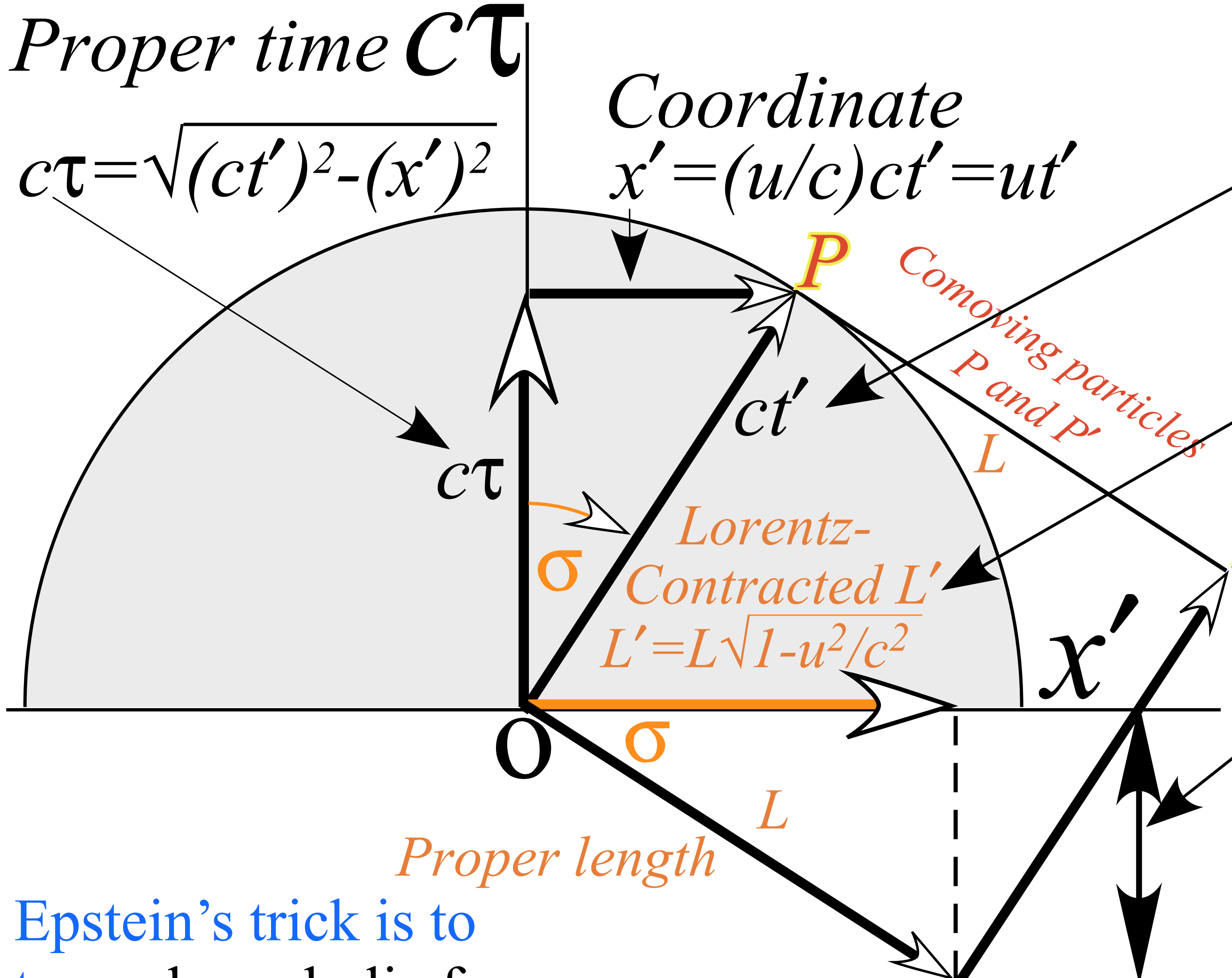

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Epstein's trick is to turn a hyperbolic form into a circular form:

$$\sqrt{(c\tau)^2+(x')^2} = (ct')$$

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

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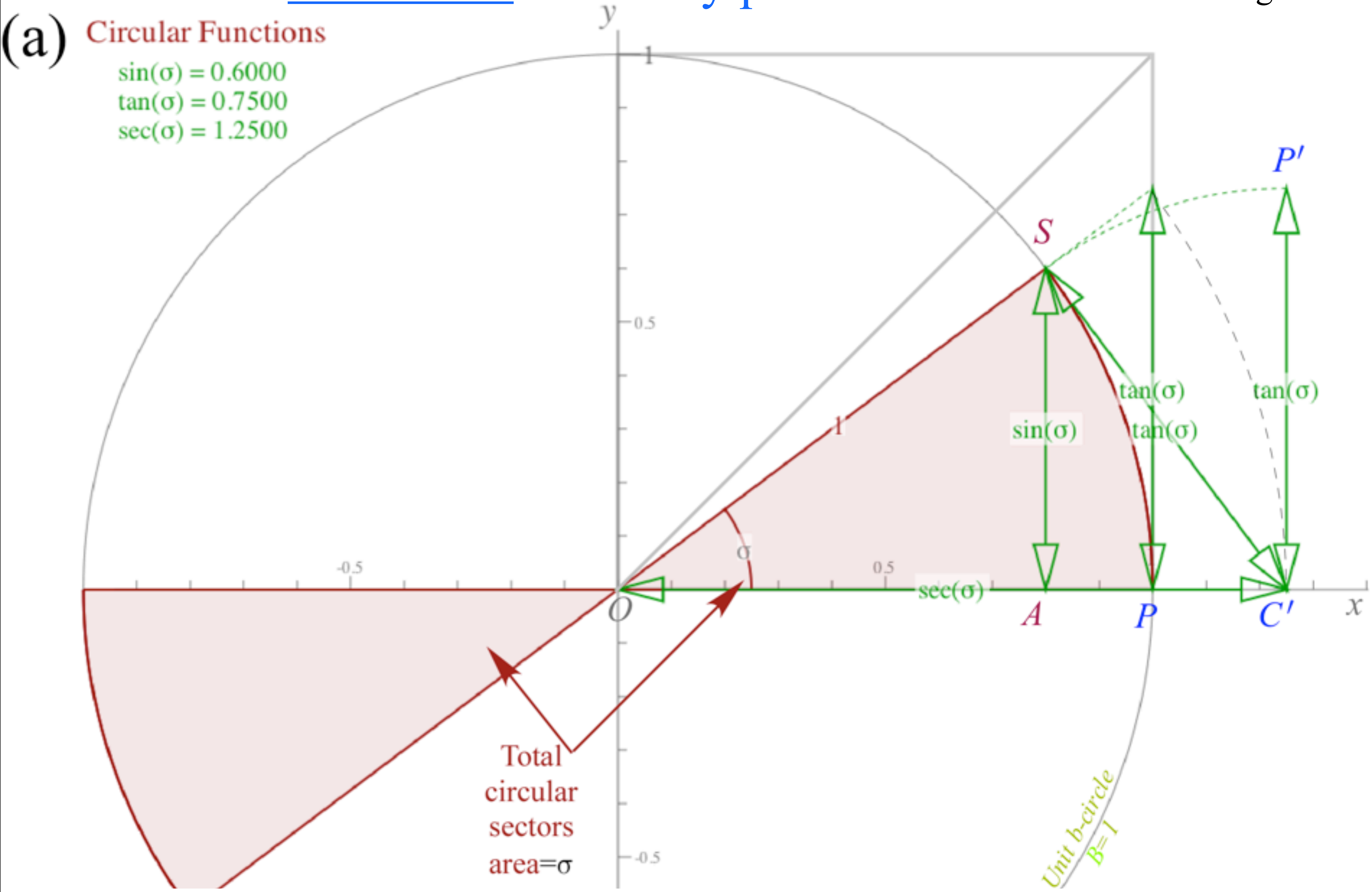

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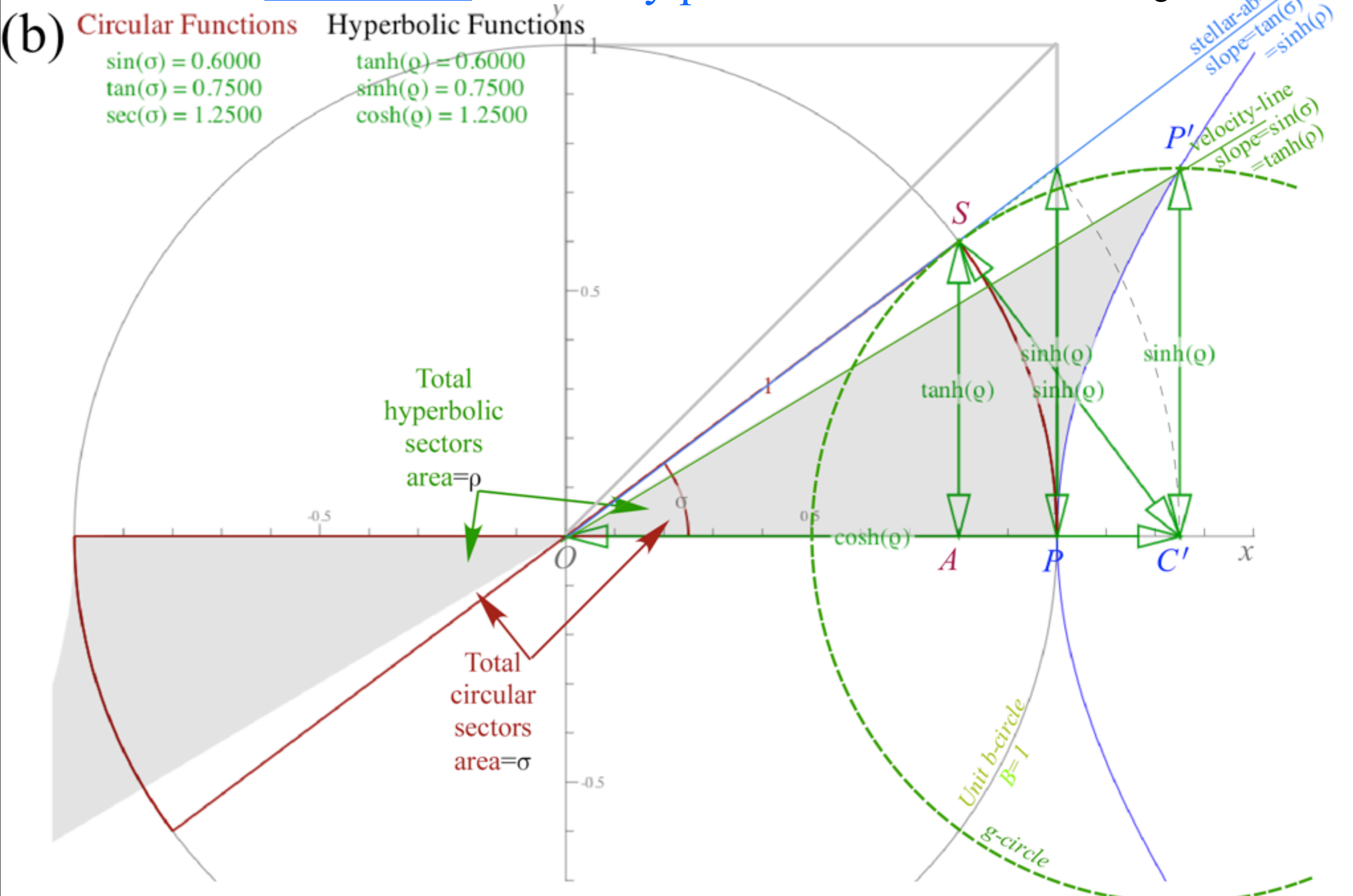

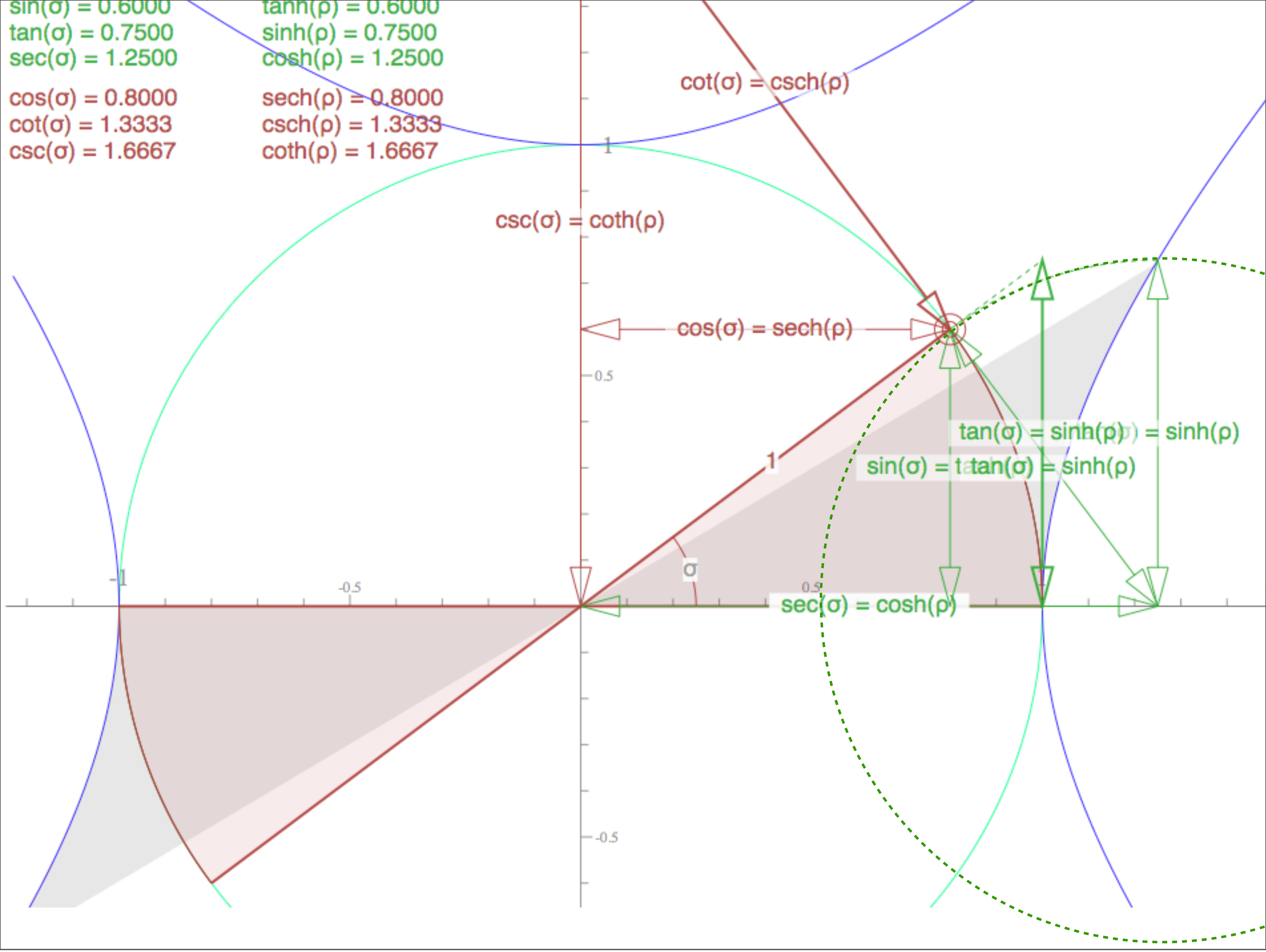

sin(σ) = 0.6000
tan(σ) = 0.7500
sec(σ) = 1.2500
cos(σ) = 0.8000
cot(σ) = 1.3333
csc(σ) = 1.6667
tanh(ρ) = 0.6000
sinh(ρ) = 0.7500
cosh(ρ) = 1.2500
sech(ρ) = 0.8000
csch(ρ) = 1.3333
coth(ρ) = 1.6667
cot(σ) = csch(ρ)
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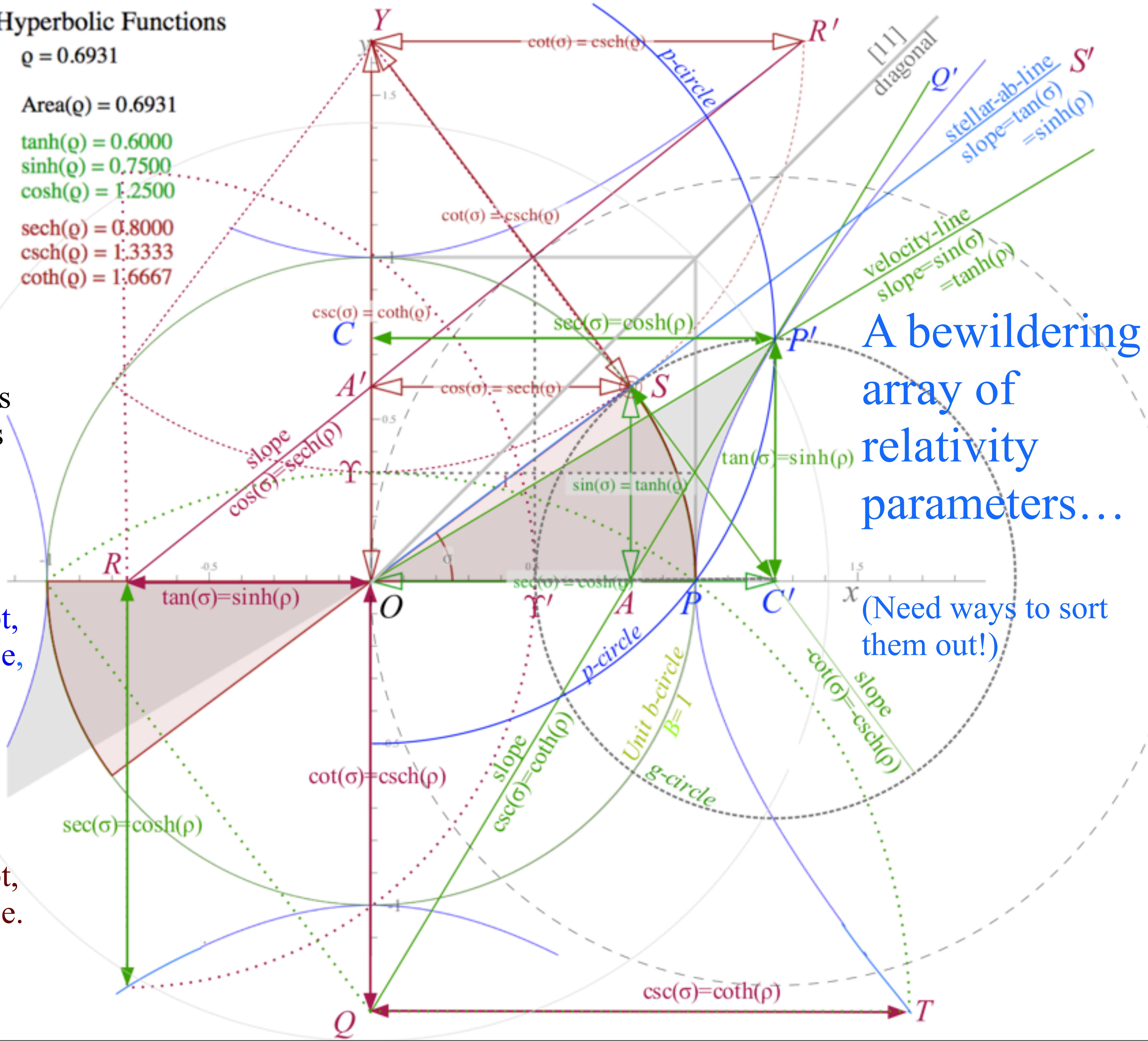
cos(σ) = sech(ρ)
tan(σ) = sinh(ρ)
tan(σ) = sinh(ρ)
sec(σ) = cosh(ρ)
1
σ

Circular Functions	Hyperbolic Functions
m∠(σ) = 0.6435	ϱ = 0.6931
Length(σ) = 0.6435	
Area(σ) = 0.6435	Area(ϱ) = 0.6931
sin(σ) = 0.6000	tanh(ϱ) = 0.6000
tan(σ) = 0.7500	sinh(ϱ) = 0.7500
sec(σ) = 1.2500	cosh(ϱ) = 1.2500
cos(σ) = 0.8000	sech(ϱ) = 0.8000
cot(σ) = 1.3333	csch(ϱ) = 1.3333
csc(σ) = 1.6667	coth(ϱ) = 1.6667

Each of 6 trig (or trig*h*) functions serves at least once as a hyperbolic *x*,*y*, and *z* coordinate, *x*,*y*, and *z* tangent intercept, and tangent slope, and a circular *x*,*y*, and *z* coordinate, *x*,*y*, and *z* tangent intercept, and tangent slope.

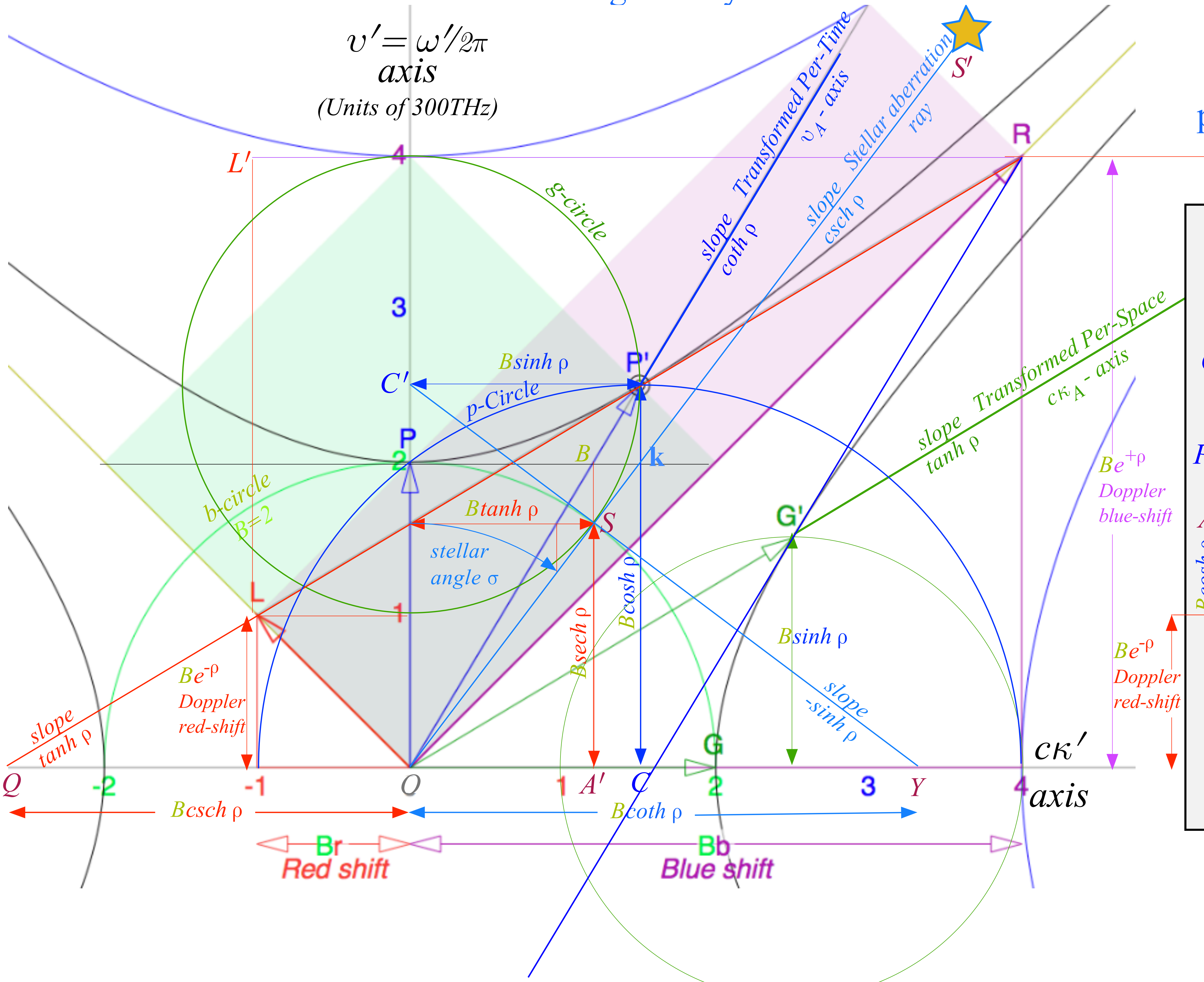


A bewildering array of relativity parameters…

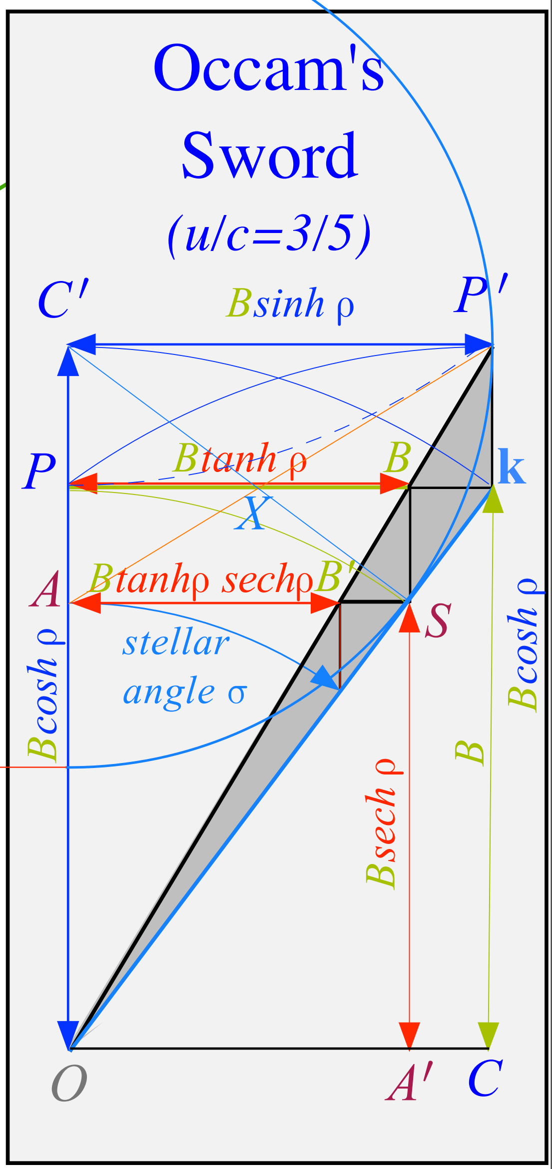
(Need ways to sort them out!)

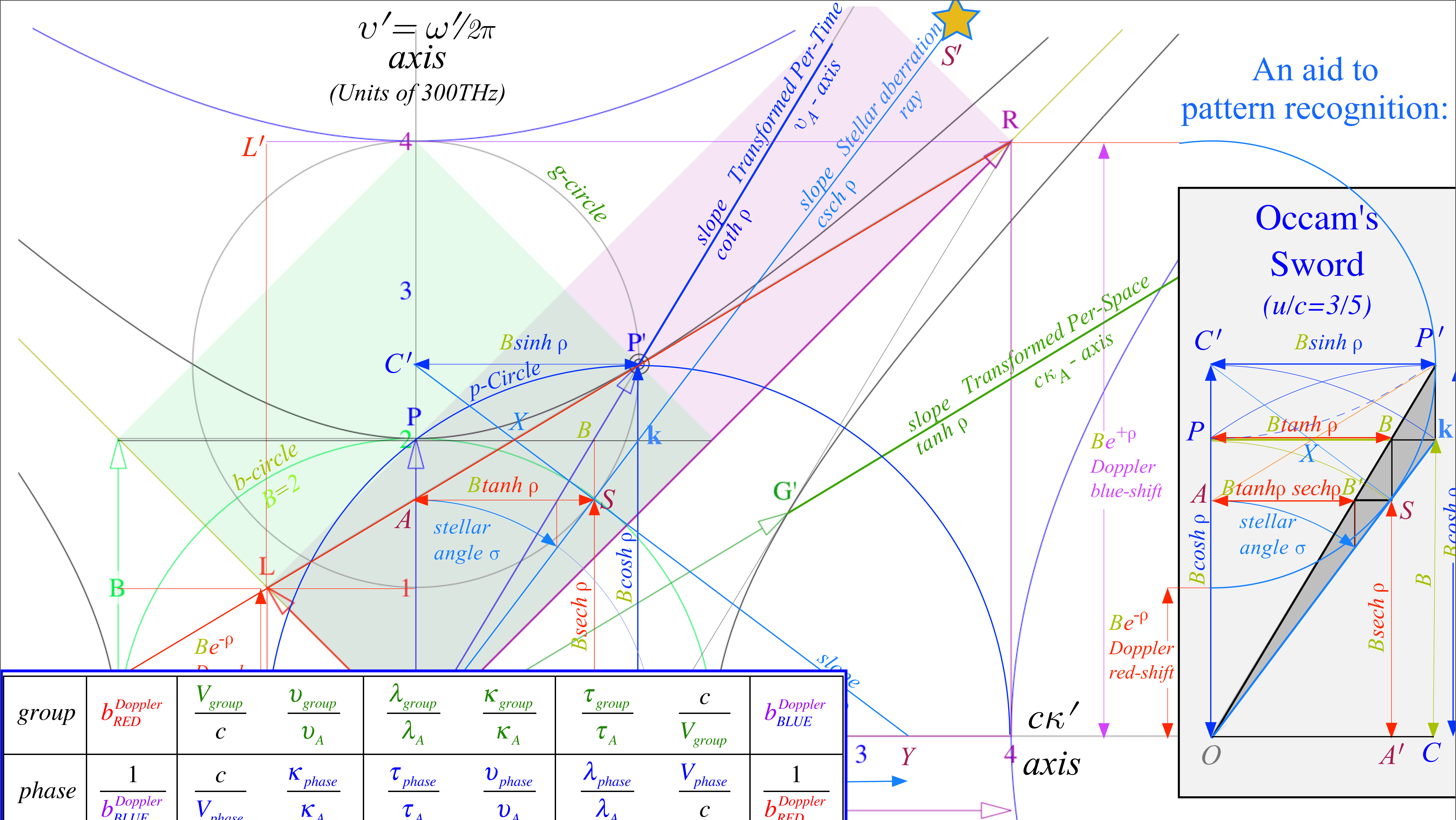
Summary of optical wave parameters for relativity and QM

...and their geometry



An aid to
pattern recognition:





group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 12 wave parameters (includes inverses) for relativity

...and values for $u/c=3/5$

Using (some) wave parameters for relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \text{ (for } u \ll c\text{)}$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad \text{(for } u \ll c\text{)}$$

At low speeds:

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2$$

$$\sinh\rho \approx \rho$$

$$B = \upsilon_A$$

$$B = \upsilon_A = c\kappa_A$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave parameters for relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

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group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Resembles: $const. + \frac{1}{2}Mu^2$

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Einstein (1905)

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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$$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$$

$$\text{Momentum: } h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$$

DeBroglie (1921)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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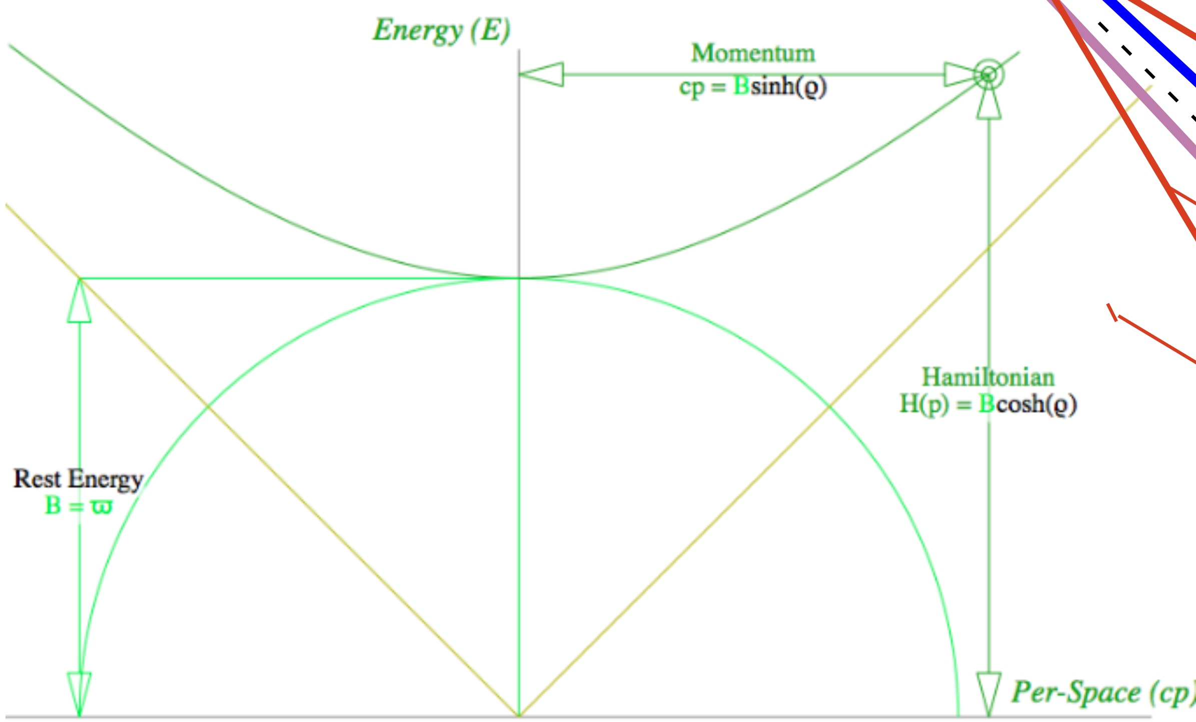
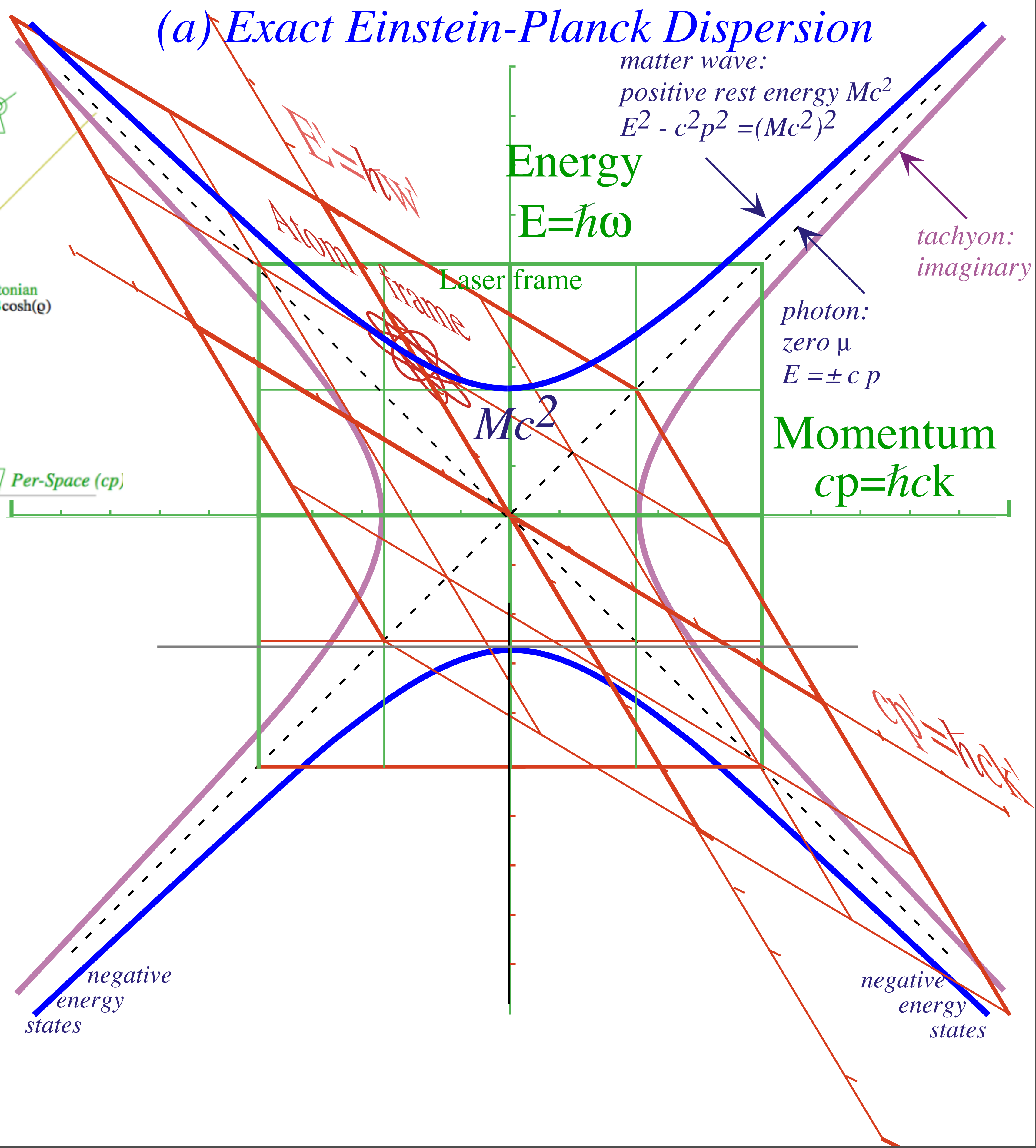
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Using (some) wave coordinates for relativistic quantum theory





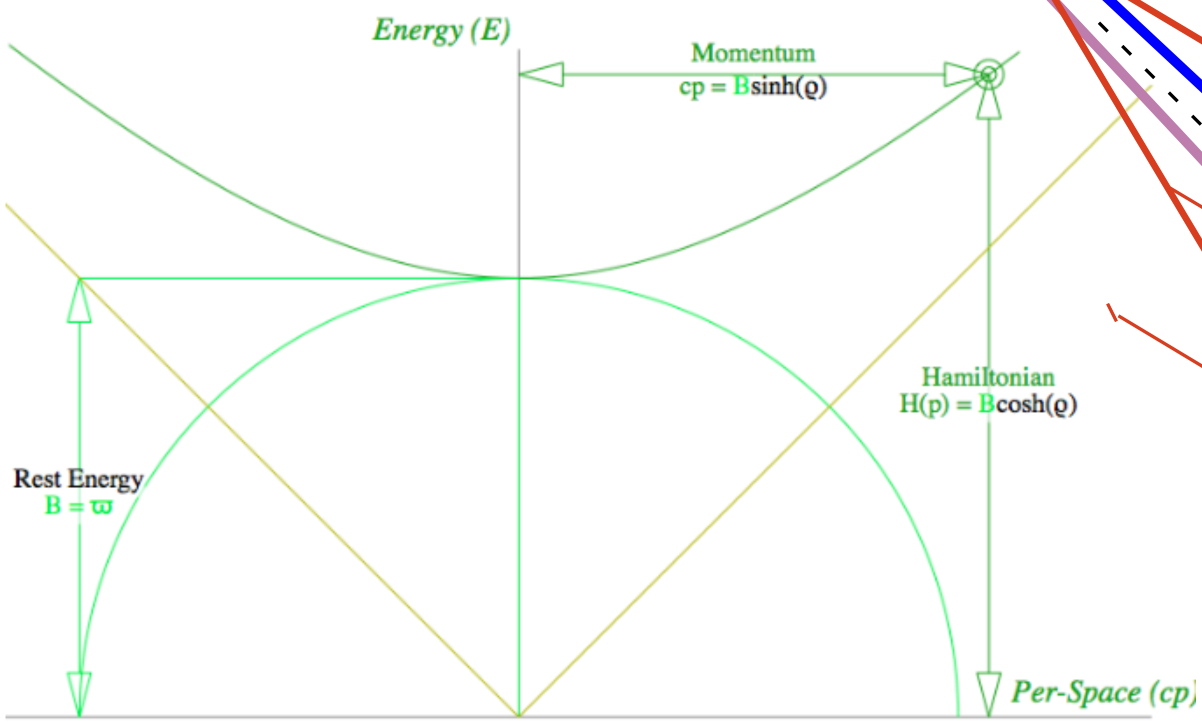
Mass (resting)

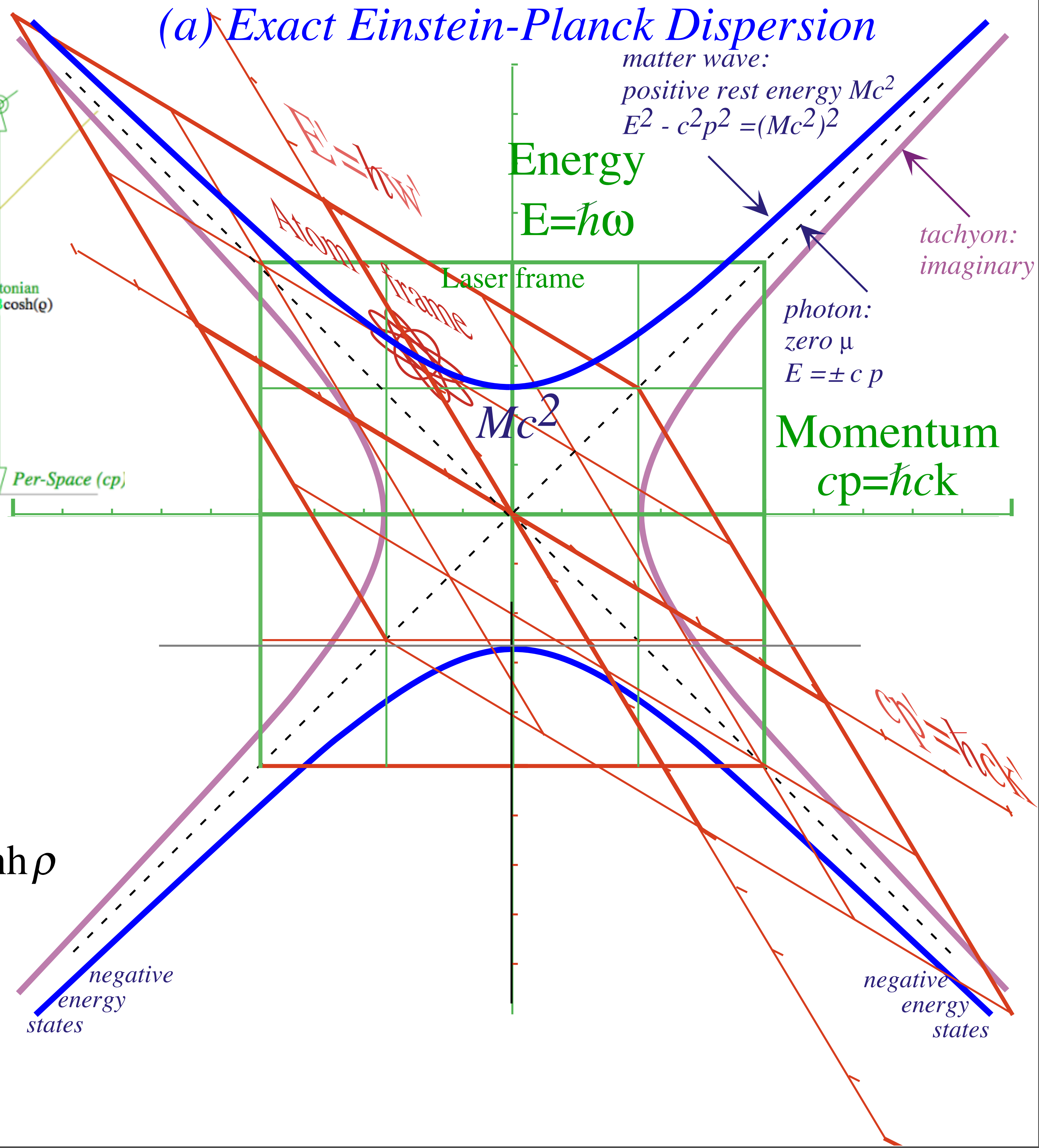
$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Using (some) wave coordinates for relativistic quantum theory





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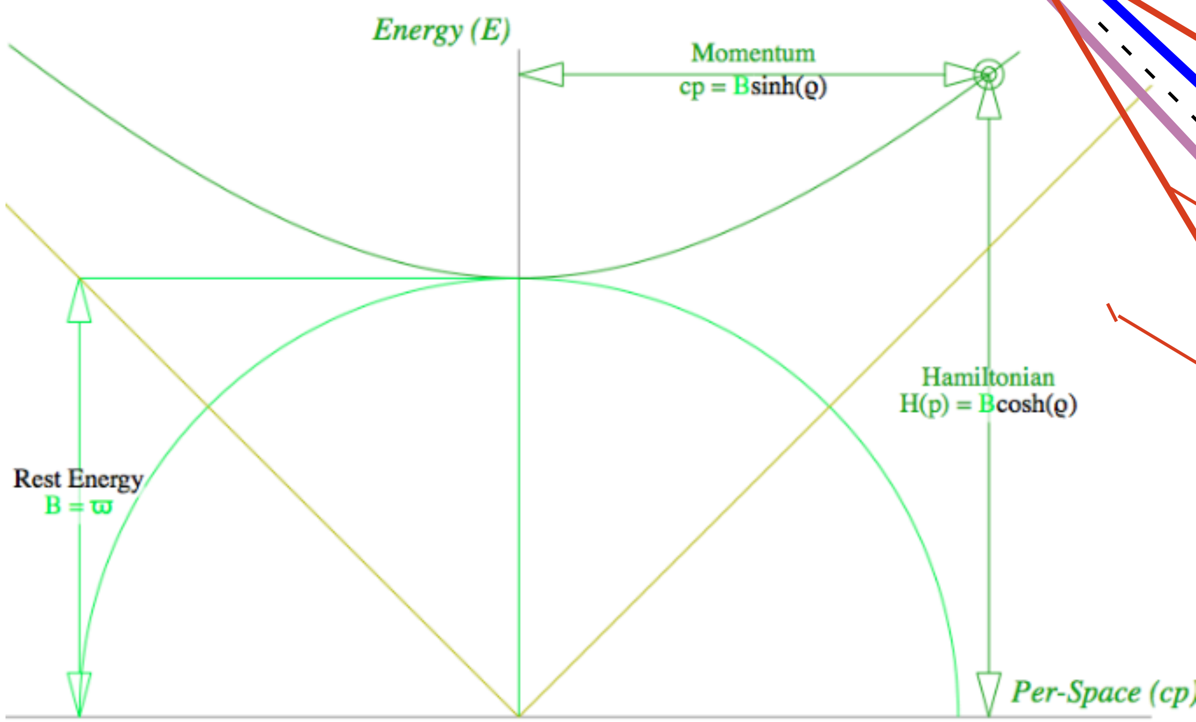

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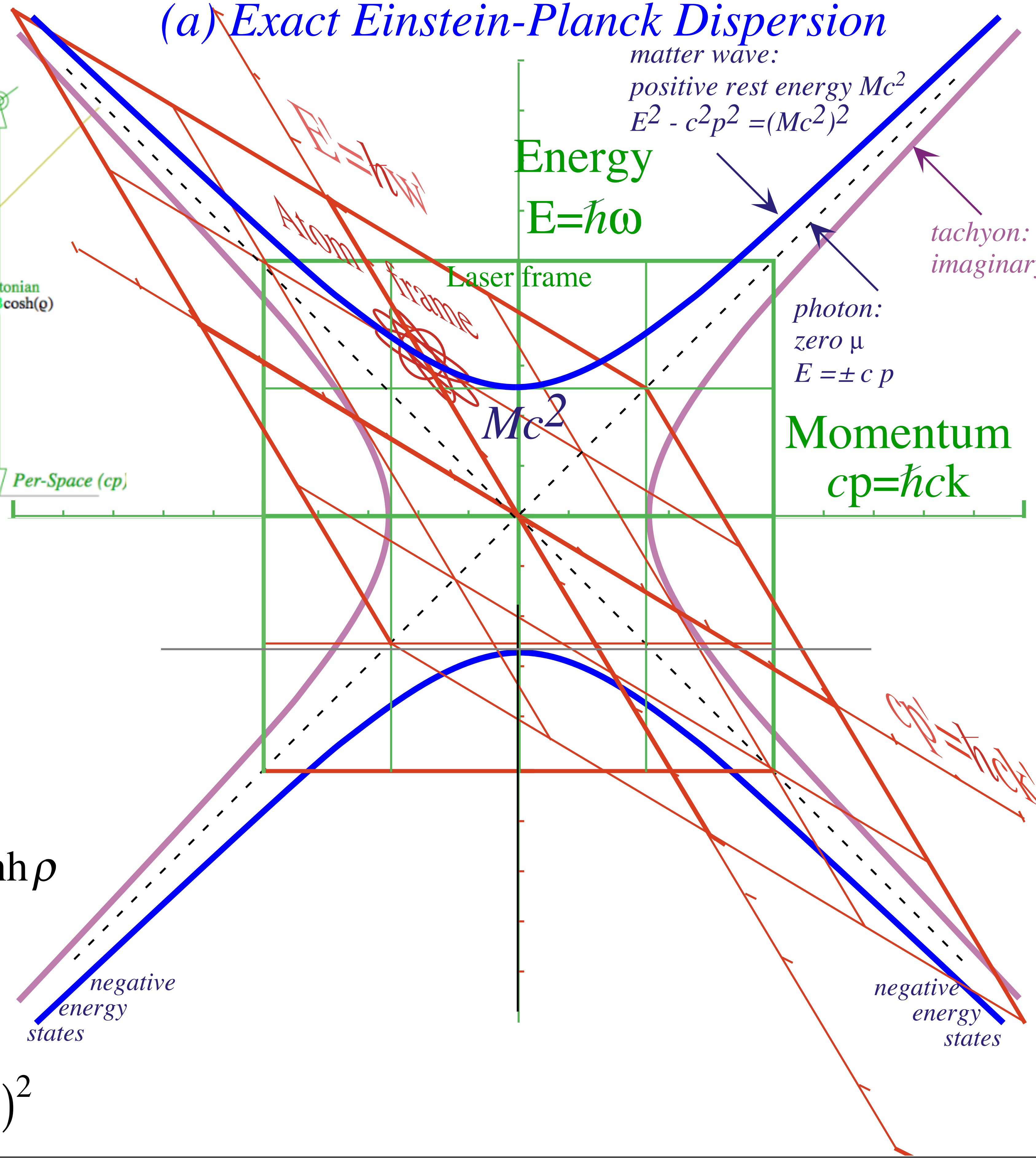
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2$$



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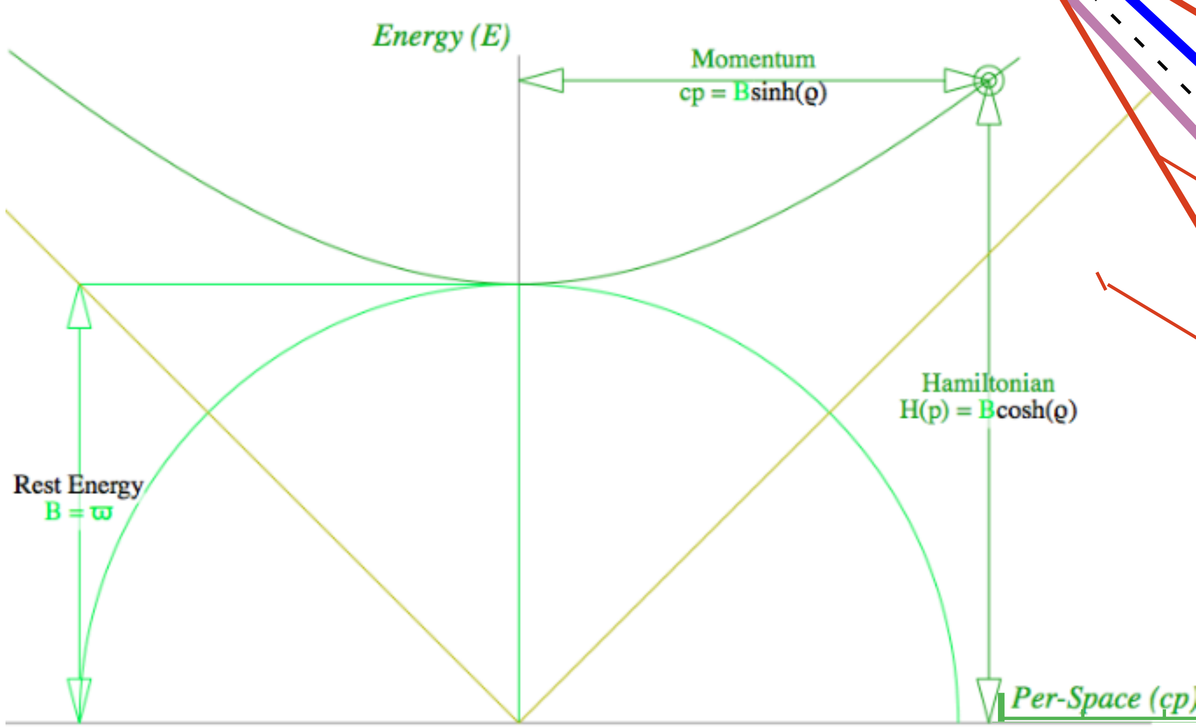

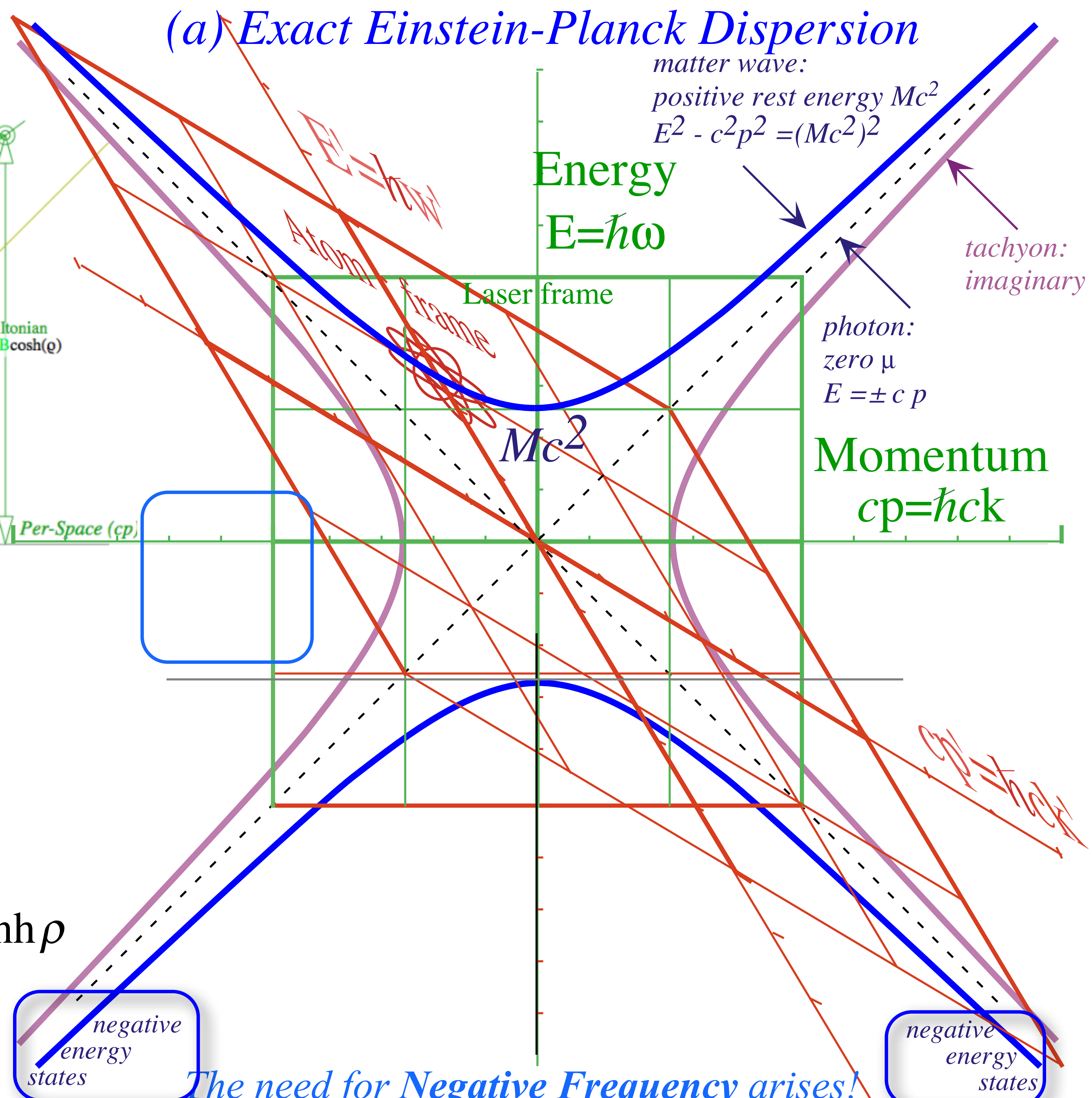

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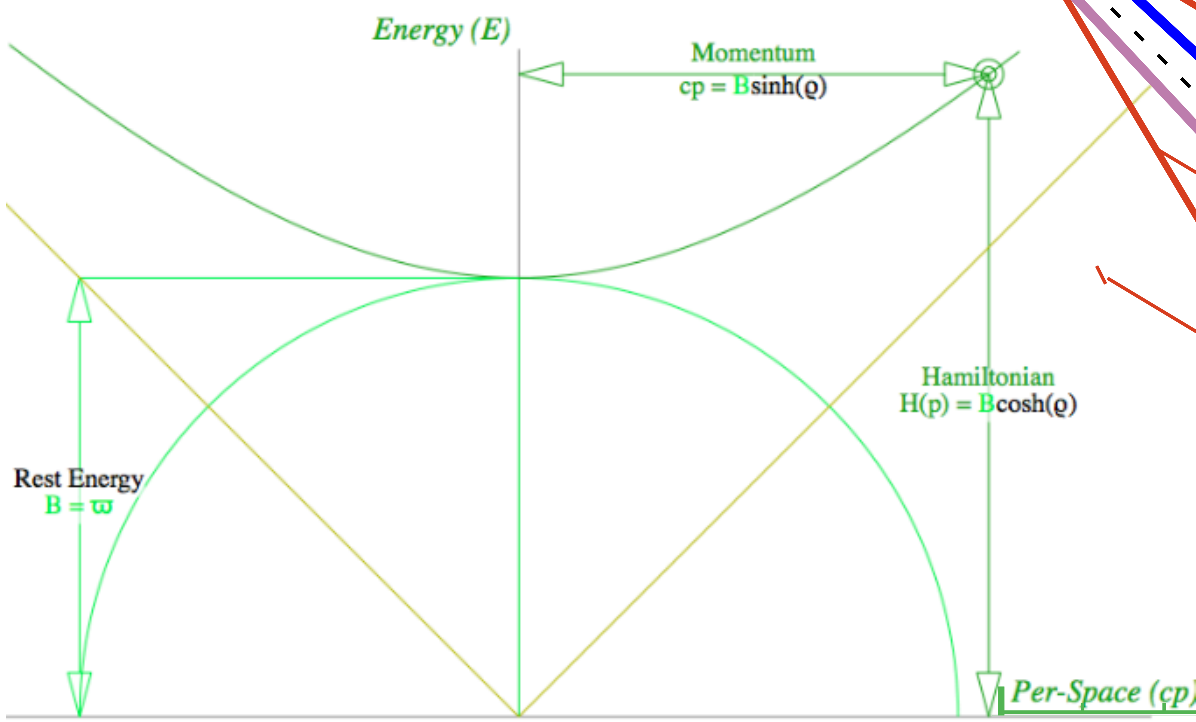
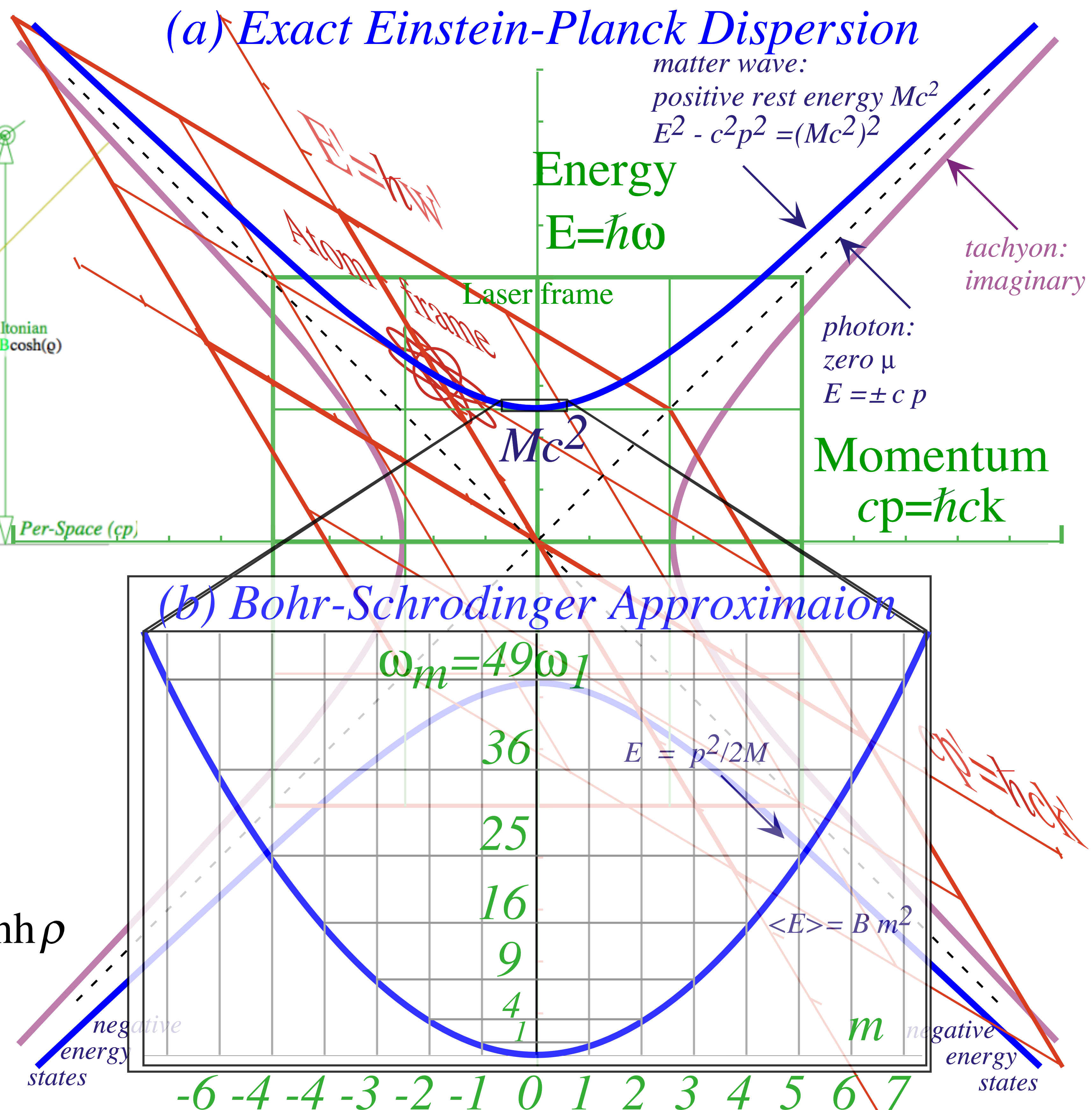
*The need for **Negative Frequency** arises!*

Phase conjugate light!

Counter-clockwise phasors!

Using (some) wave coordinates for relativistic quantum theory





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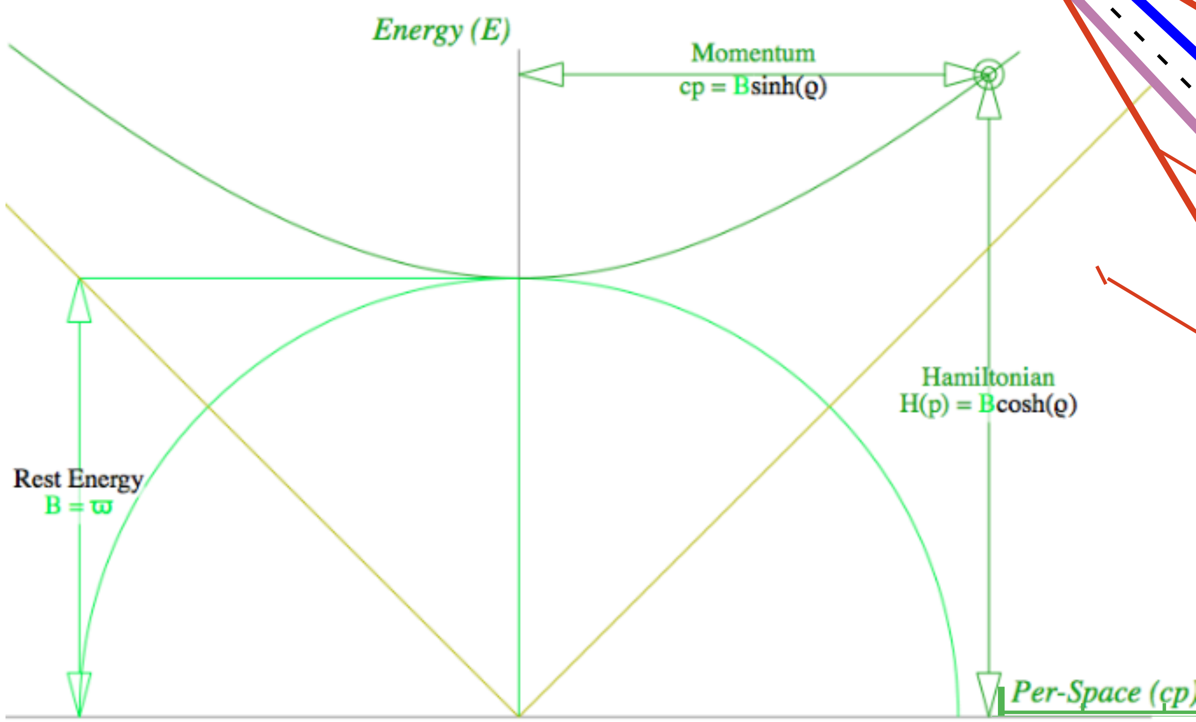
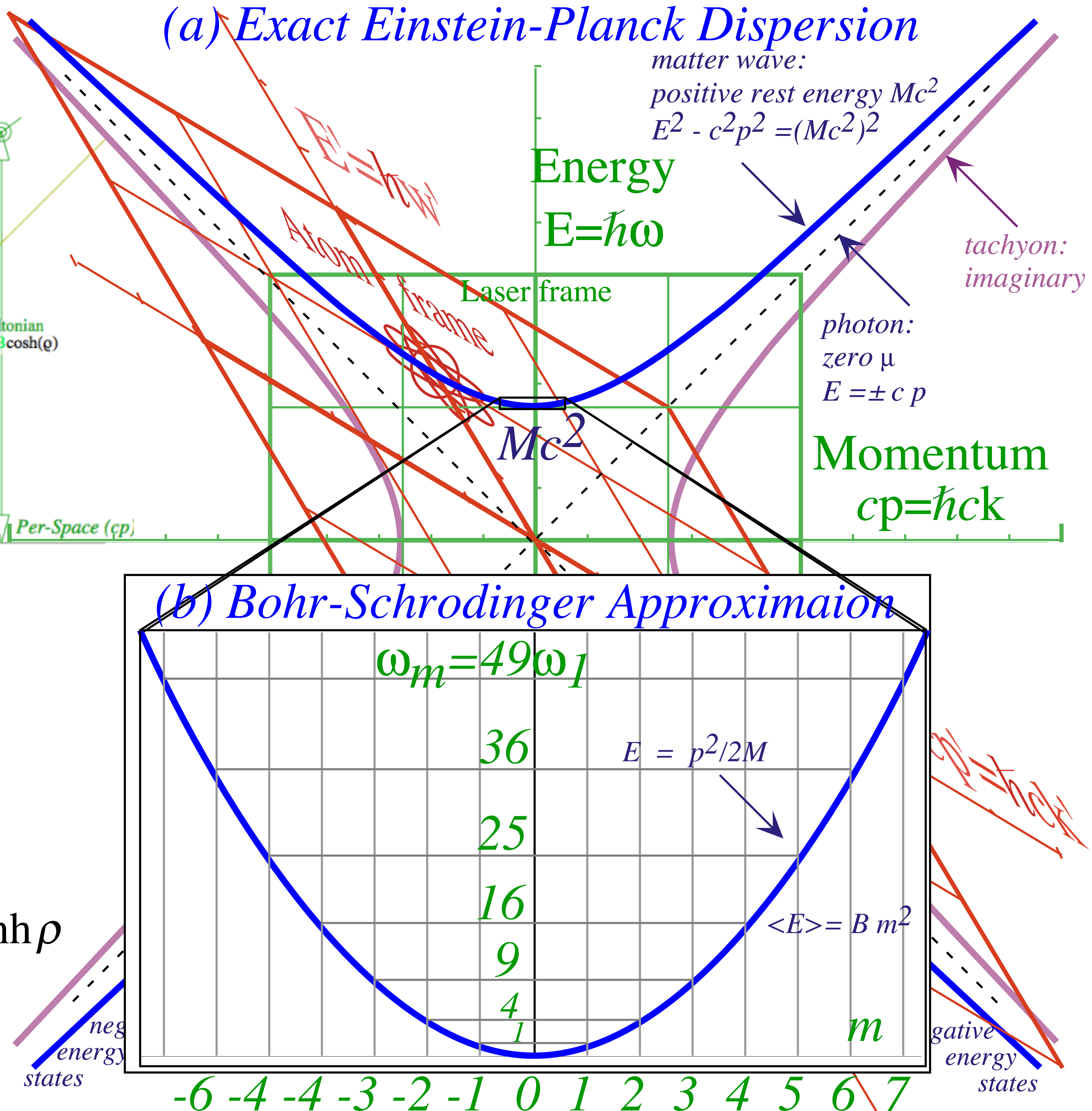
Using (some) wave coordinates for relativistic quantum theory





<u>Mass</u> *(resting)*

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low speed approximation

Definition(s) of mass for relativity/quantum

Given: *Energy:* $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + \left(cp\right)^2}$$

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$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

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Limiting cases:

$$M_{mom} \xrightarrow[u \to c]{} M_{rest}e^{\rho}/2$$

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Rest Mass M_{rest} *(Einstein's mass)*

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Definition(s) of mass for relativity/quantum

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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

$(a)\gamma$-rest mass: $M^{\gamma}_{rest} = 0,$

$(b)\gamma$-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2},$

$(c)\gamma$-effective mass: $M^{\gamma}_{eff} = \infty.$

Newton complained about *his* "corpuscles" of light having "fits" (going crazy).

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2\cdot 10^{-51})kg\cdot s = 4.5\cdot 10^{-36}\,kg \quad \text{(for: } \upsilon\text{=600THz)}$$

Definition(s) of mass for relativity/quantum

How much does a γ-photon weigh?

$(a)\gamma$-rest mass: $M^{\gamma}_{rest} = 0,$

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Newton complained about *his* "corpuscles" of light having "fits" (going crazy).
For him this would be evidence of optical-triple-schizophrenia!

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2 \cdot 10^{-51})kg \cdot s = 4.5 \cdot 10^{-36}\,kg \quad \text{(for: } \upsilon\text{=600THz)}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

←linear Hz format

angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar c k_A$

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Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega$$

$$p = \hbar k = Mc \sinh\rho$$

$$E = \hbar\omega = Mc^2 \cosh\rho$$

Prior wave relations

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Relativistic action S and Lagrangian-Hamiltonian relations

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Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E$$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho$$

Prior wave relations

$$h\upsilon_A = Mc^2 = hc\kappa_A$$
$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$
$$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$$

←linear Hz format

angular phasor format→

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Legendre transformation

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$$= Mc^2\frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

$$L \text{ is} : Mc^2\frac{-1}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

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Also: $cp = Mc^2\sinh\rho$

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$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho$$

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Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$ $= c\sin\sigma$

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Note: $Mcu = Mc^2\tanh\rho$

$= Mc^2\sin\sigma$

Also: $cp = Mc^2\sinh\rho$

$= \hbar ck = Mc^2\tan\sigma$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H = \hbar\omega = -Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

$$= Mc^2\sqrt{1+\sinh^2\rho} = Mc^2\sqrt{1+(cp)^2}$$

Including stellar angle σ

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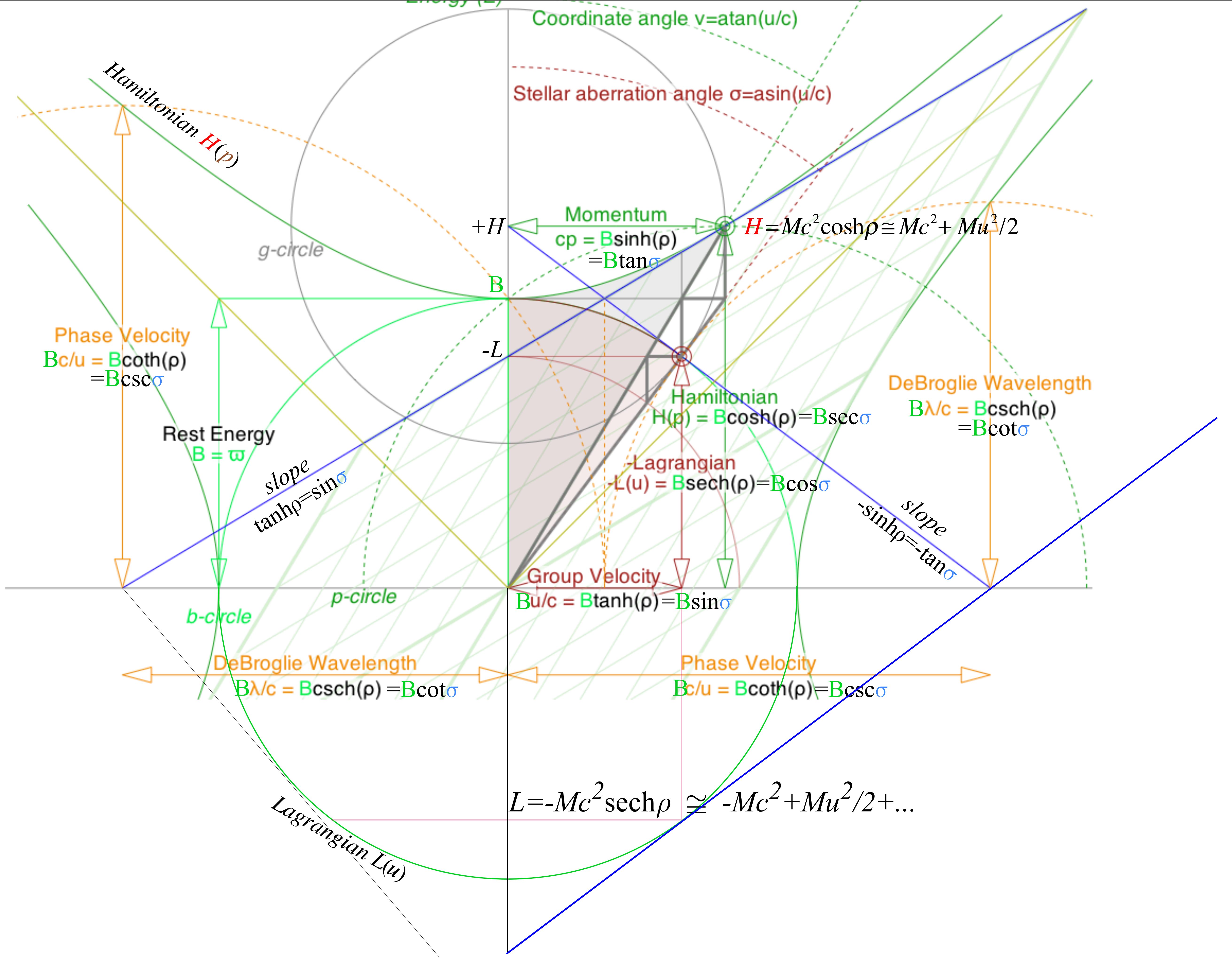
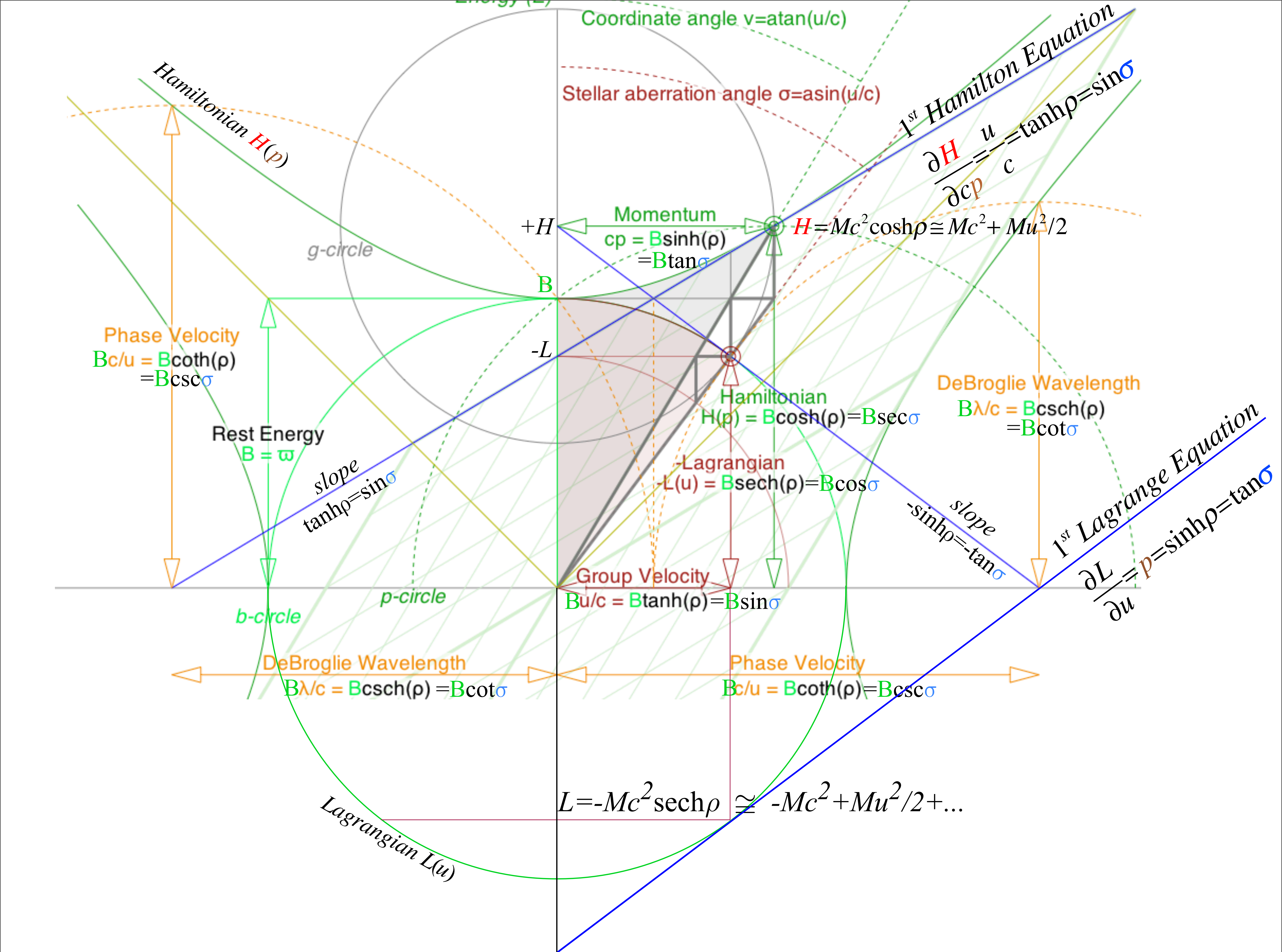

Energy (E)
Coordinate angle v=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Hamiltonian H(p)
Momentum
cp = Bsinh(ρ)
=Btanσ
+H
H=Mc²coshρ≅Mc²+ Mu²/2
g-circle
B
-L
Phase Velocity
Bc/u = Bcoth(ρ)
=Bcscσ
Hamiltonian
H(ρ) = Bcosh(ρ)=Bsecσ
DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
Rest Energy
B = ϖ
slope
tanhρ=sinσ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
slope
-sinhρ=-tanσ
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
L=-Mc²sechρ ≅ -Mc²+Mu²/2+...
Lagrangian L(u)



Coordinate angle v=atan(u/c)
Stellar aberration angle σ=asin(u/c)
1st Hamilton Equation
∂H/∂cp = u/c = tanhρ=sinσ
Hamiltonian H(p)
Momentum
cp = Bsinh(ρ)
=Btanσ
+H
H=Mc²coshρ≅Mc²+ Mu²/2
g-circle
B
-L
Phase Velocity
Bc/u = Bcoth(ρ)
=Bcscσ
DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
Hamiltonian
H(ρ) = Bcosh(ρ)=Bsecσ
Rest Energy
B = ϖ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
slope
tanhρ=sinσ
slope
-sinhρ=-tanσ
1st Lagrange Equation
∂L/∂u = p=sinhρ=tanσ
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
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Lagrangian L(u)

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Legendre transformation

Compare *Lagrangian L*

$$\dot{S}=L=\hbar\dot{\Phi}=-Mc^2\sqrt{1-\frac{u^2}{c^2}}=-Mc^2\operatorname{sech}\rho=-Mc^2\cos\sigma$$

with *Hamiltonian H=E*

$$H=\hbar\omega=-Mc^2\Big/\sqrt{1-\frac{u^2}{c^2}}=Mc^2\cosh\rho=Mc^2\sec\sigma$$

$$=Mc^2\sqrt{1+\sinh^2\rho}=Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S=\hbar\Phi$

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←linear Hz format

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Legendre transformation

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\, dx - H\, dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\,\mathrm{sech}\rho = -Mc^2\cos\sigma$$

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$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2\,\mathrm{sech}\rho = -Mc^2\cos\sigma$$

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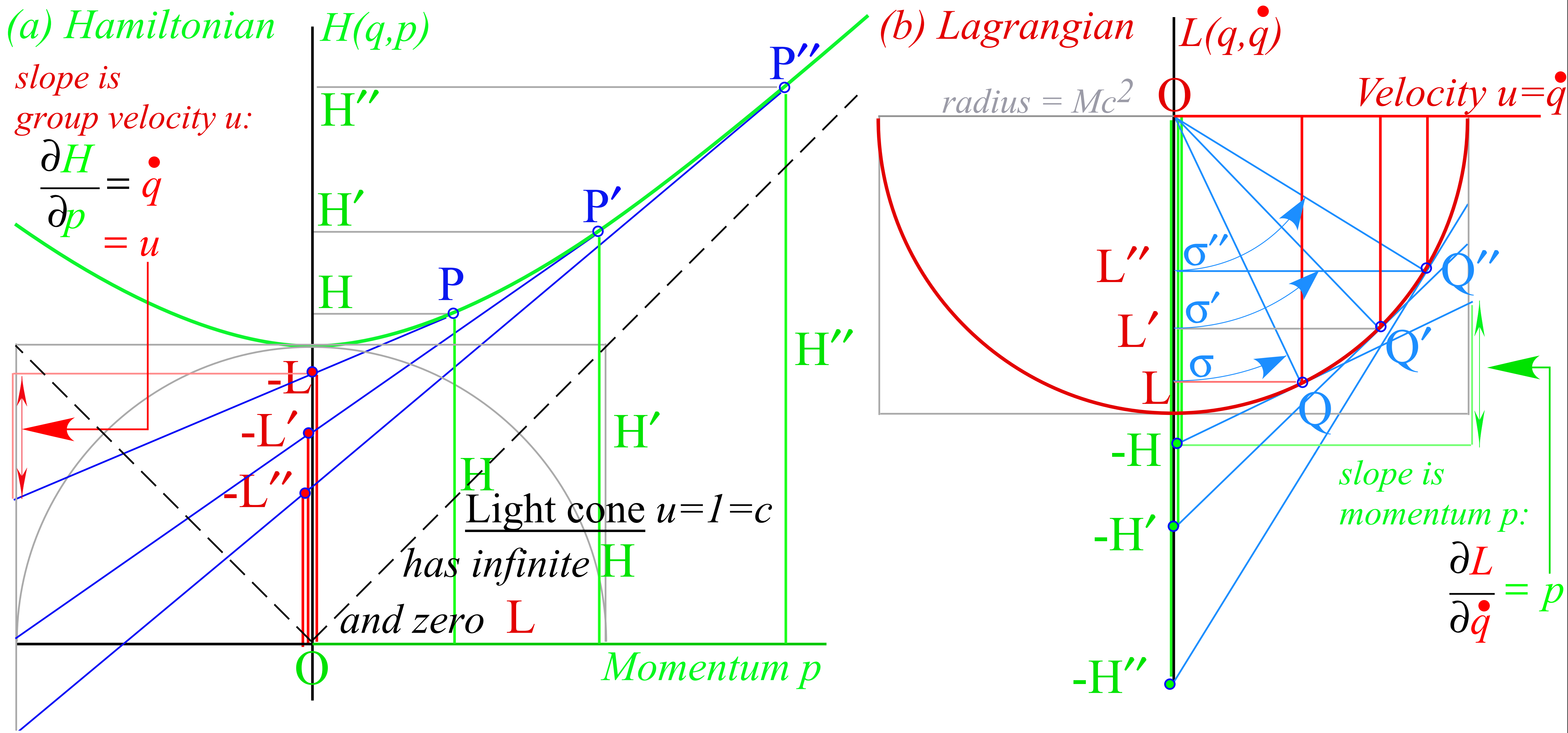
Poincare Invariant Action $dS = L\,dt = p\,dq - H\,dt = \hbar d\Phi$ *(phase)*

Hamiltonian $H(p,q) = p\dot{q} - L$ vs. *Lagrangian* $L(\dot{q}q) = p\dot{q} - H$

Contact transformation: *(slope,-intercept)* of *H* (or *L*) *tangent* determines the *(X,Y coordinates)* of *L* (or *H*).

(Also, called a *Legendre contact transformation* which is a special case of a *Huygens transformation* that uses contacting tangent *curves* instead of *lines*.)



Here *slope* is group velocity $u=\dot{q}$

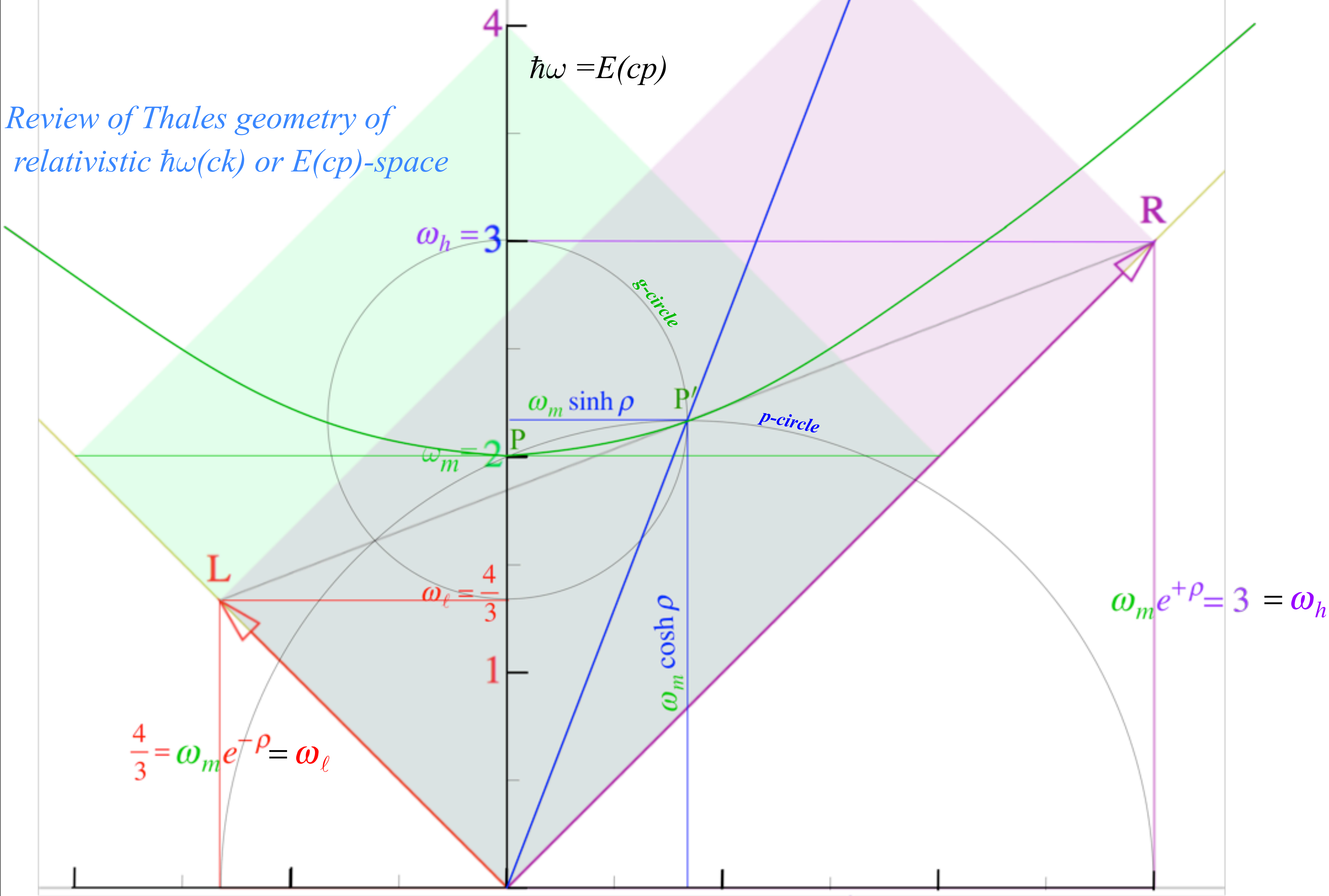
Y-coordinate is *energy* $H=\hbar\omega$

Here *slope* is momentum p

Y-coordinate is *phase rate* $L=\hbar\Phi$

Relativistic optical transitions $|high\rangle=|\omega_h\rangle \rightleftarrows |mid\rangle=|\omega_m\rangle \rightleftarrows |low\rangle=|\omega_\ell\rangle$

Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

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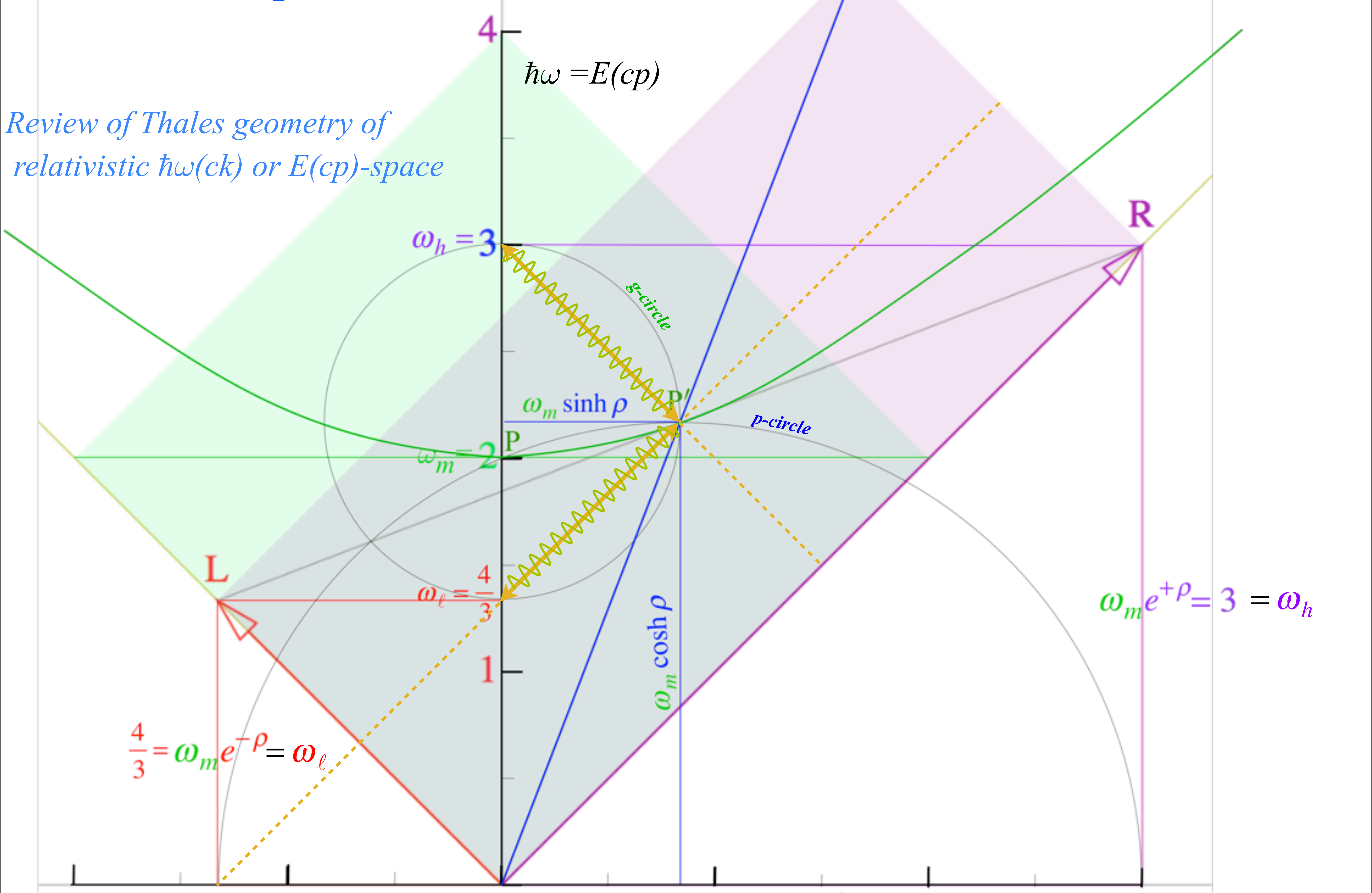

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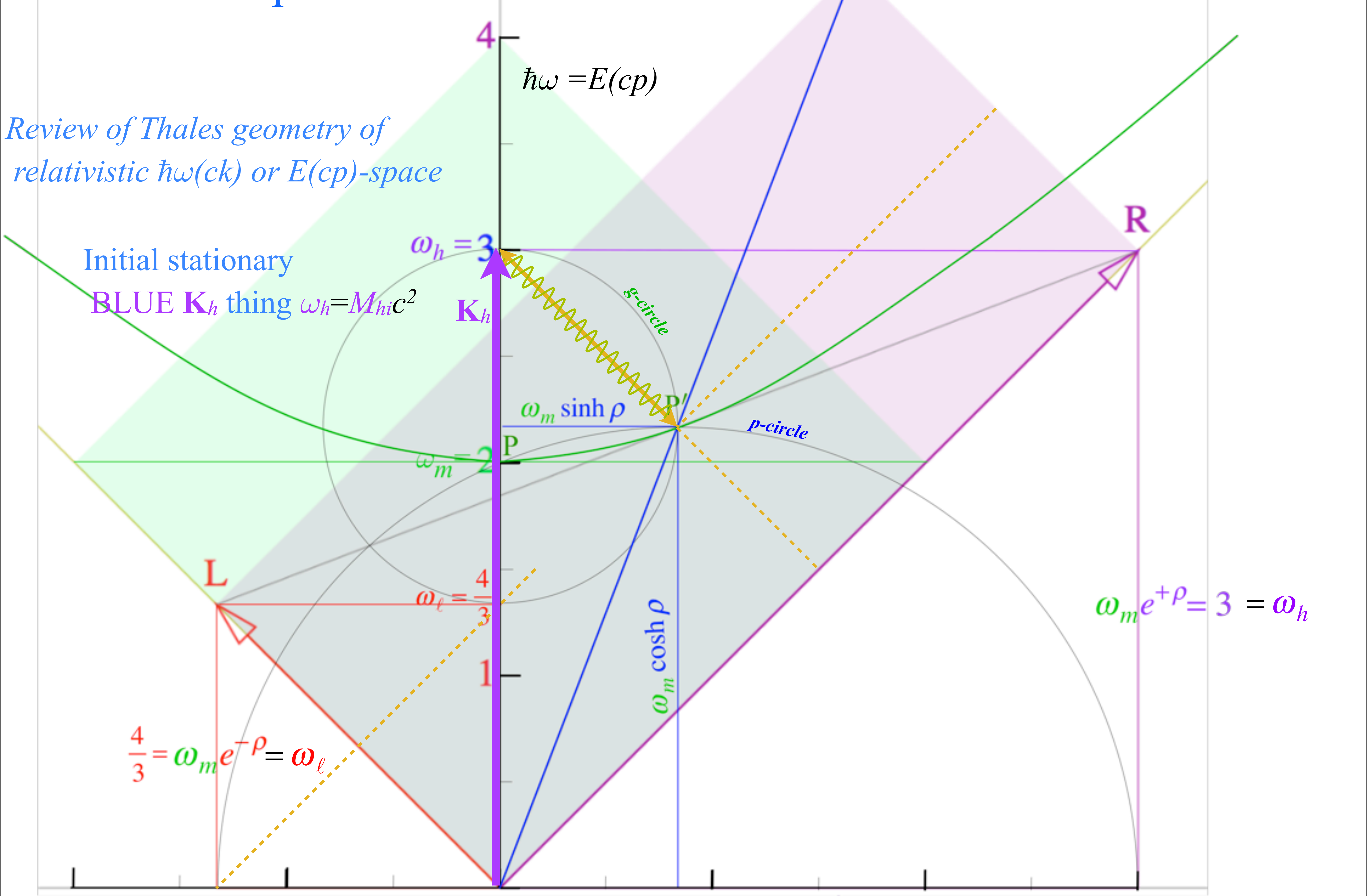
Initial stationary BLUE $\mathbf{K}_h$ thing $\omega_h = M_{hi}c^2$



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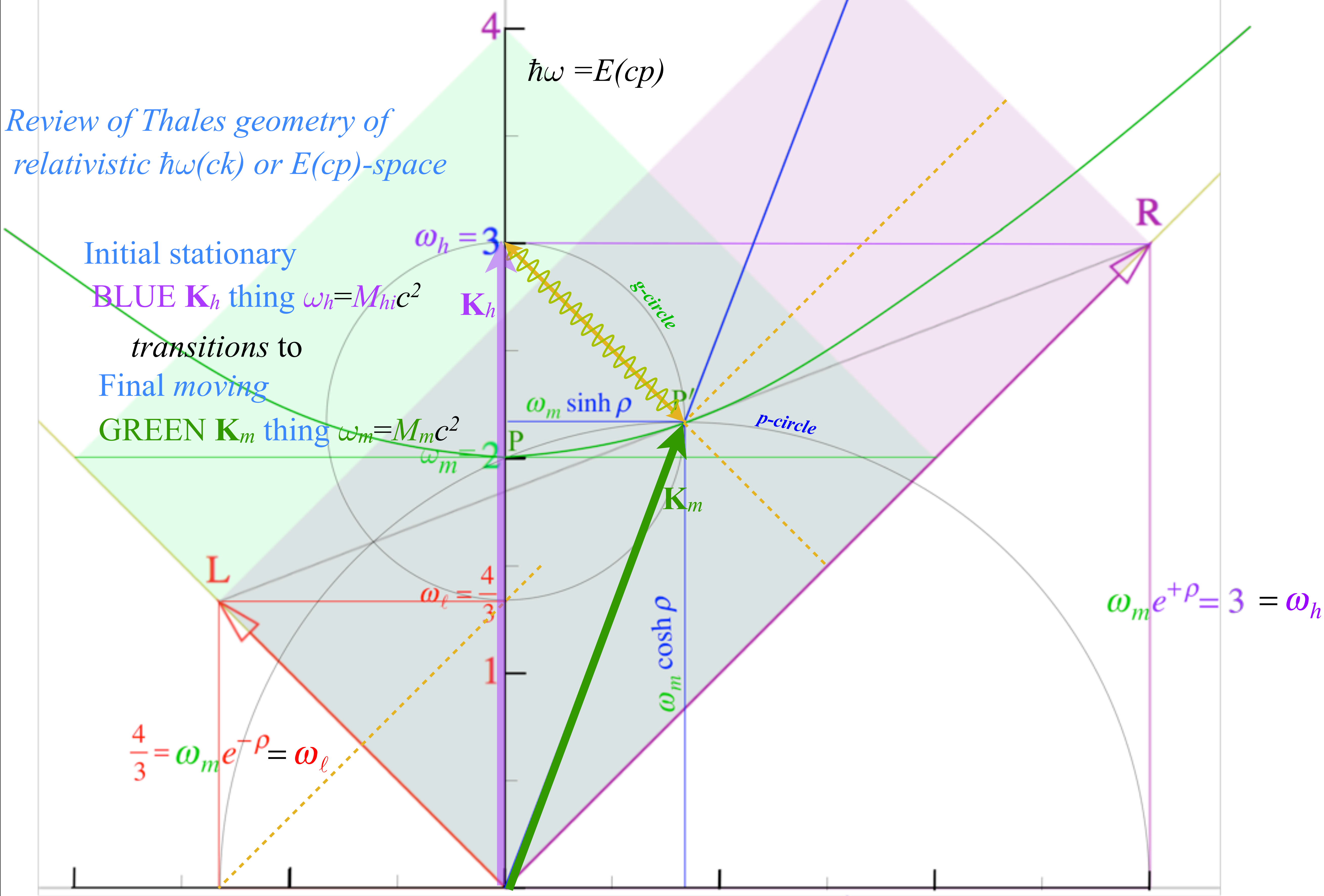
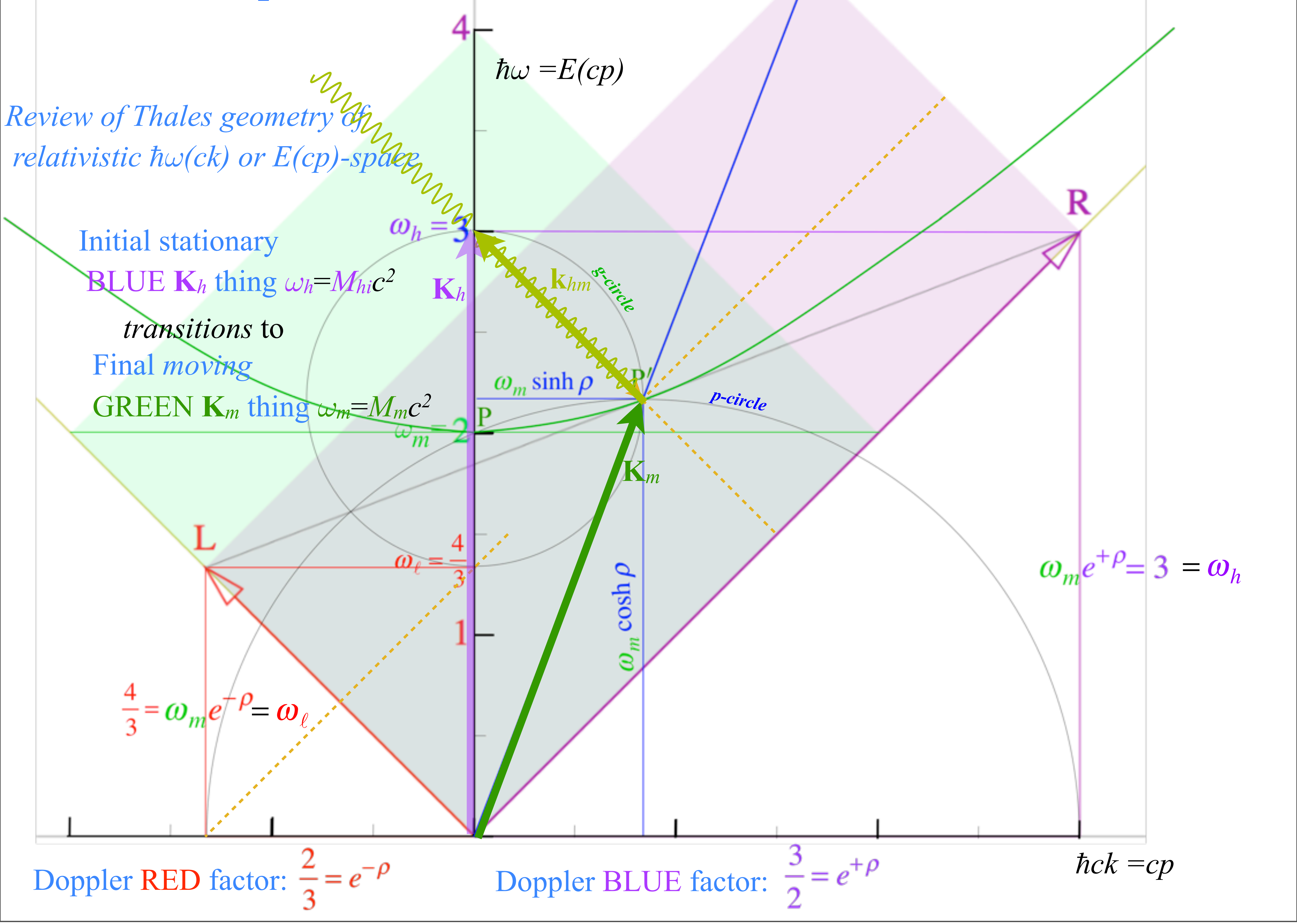
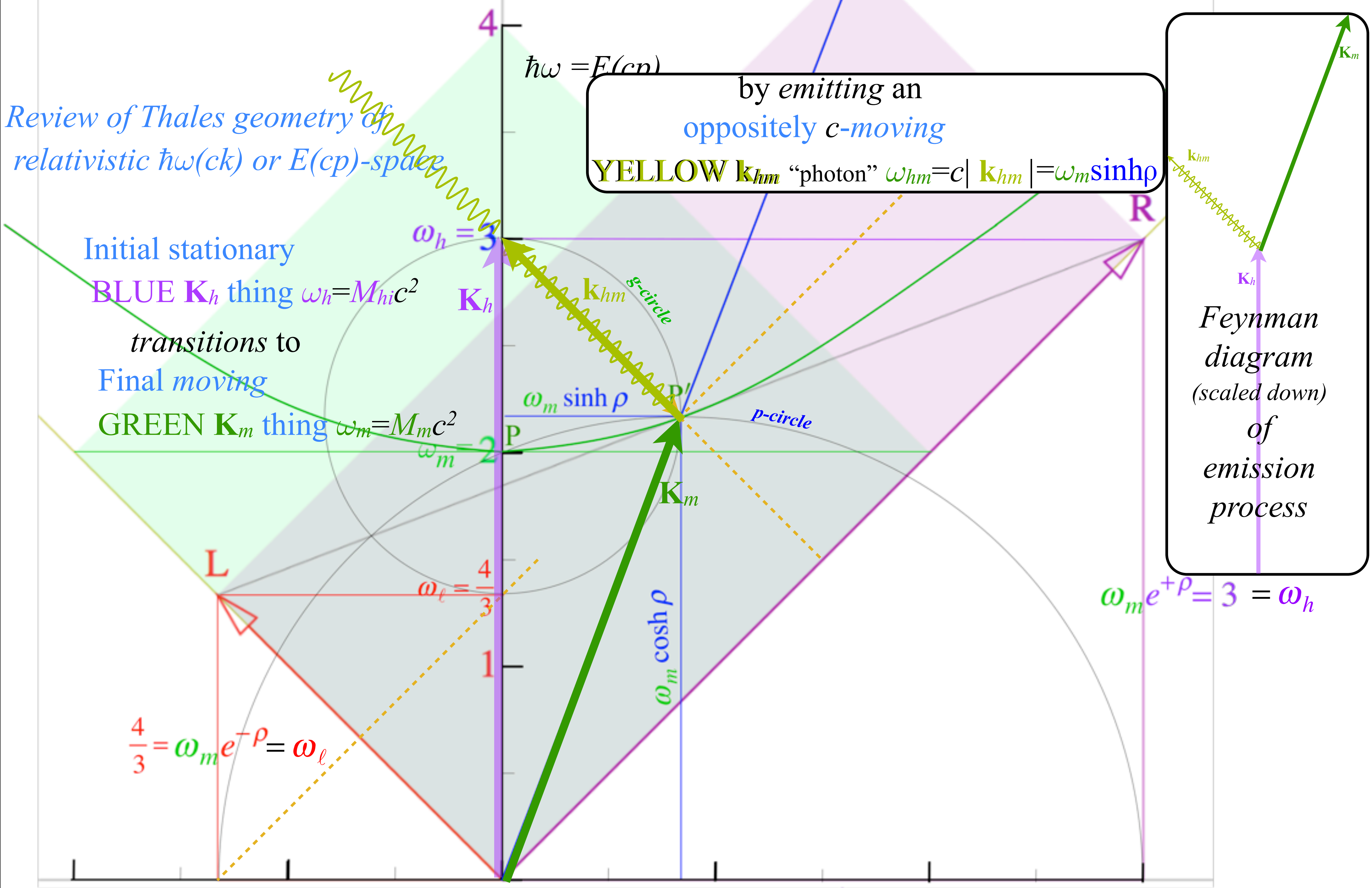


Doppler RED factor: $\frac{2}{3}=e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2}=e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3}=e^{-\rho}$

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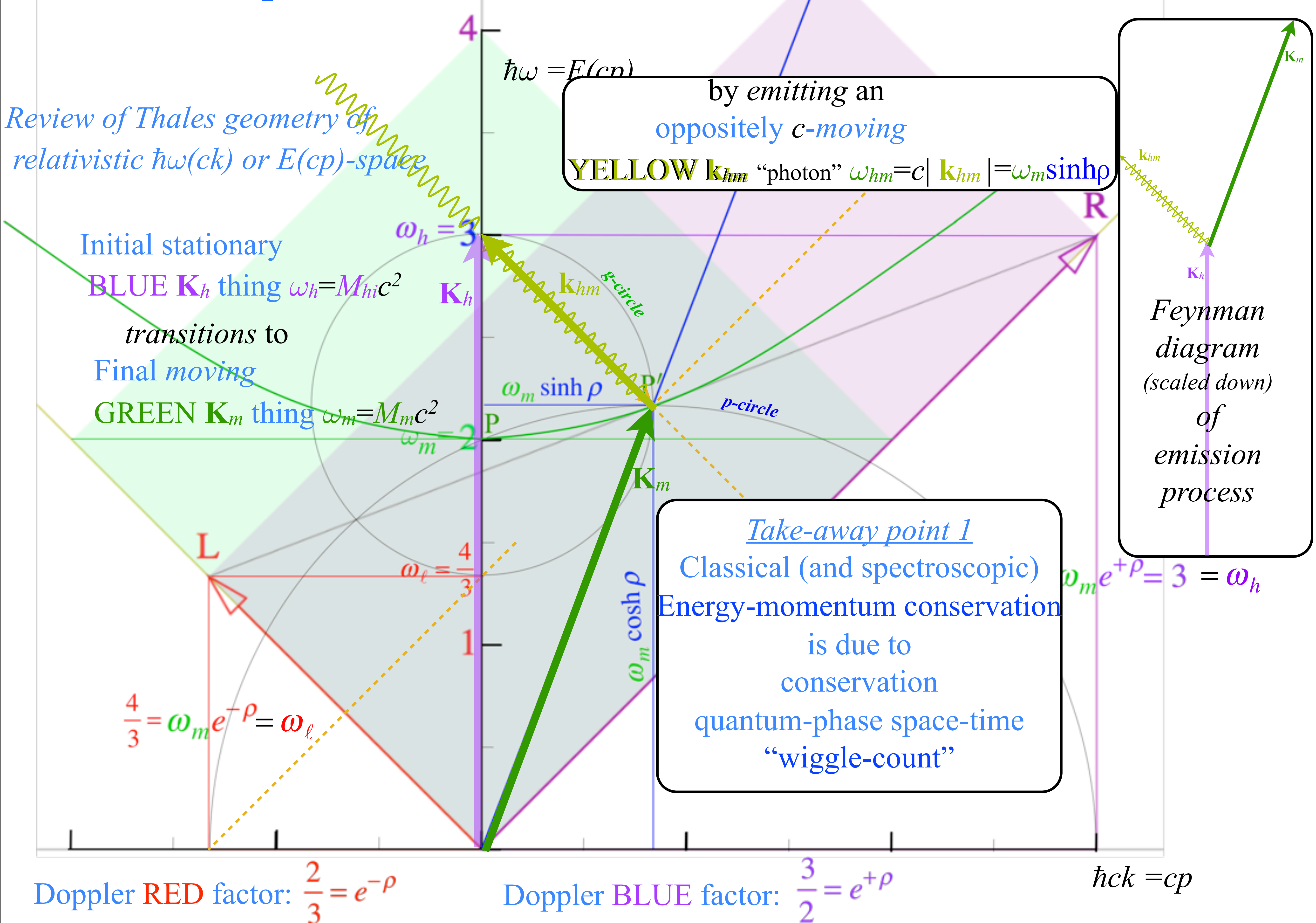
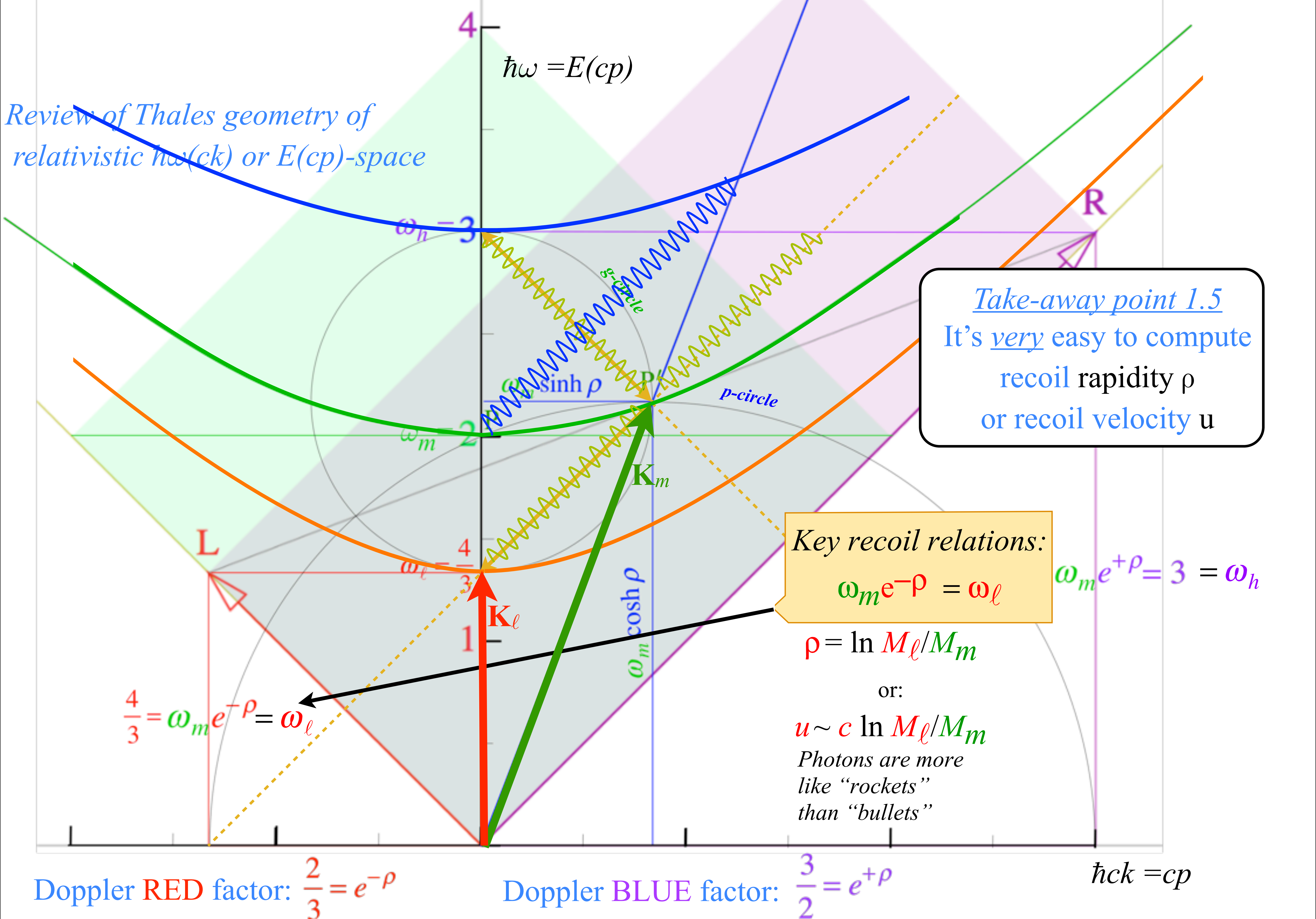
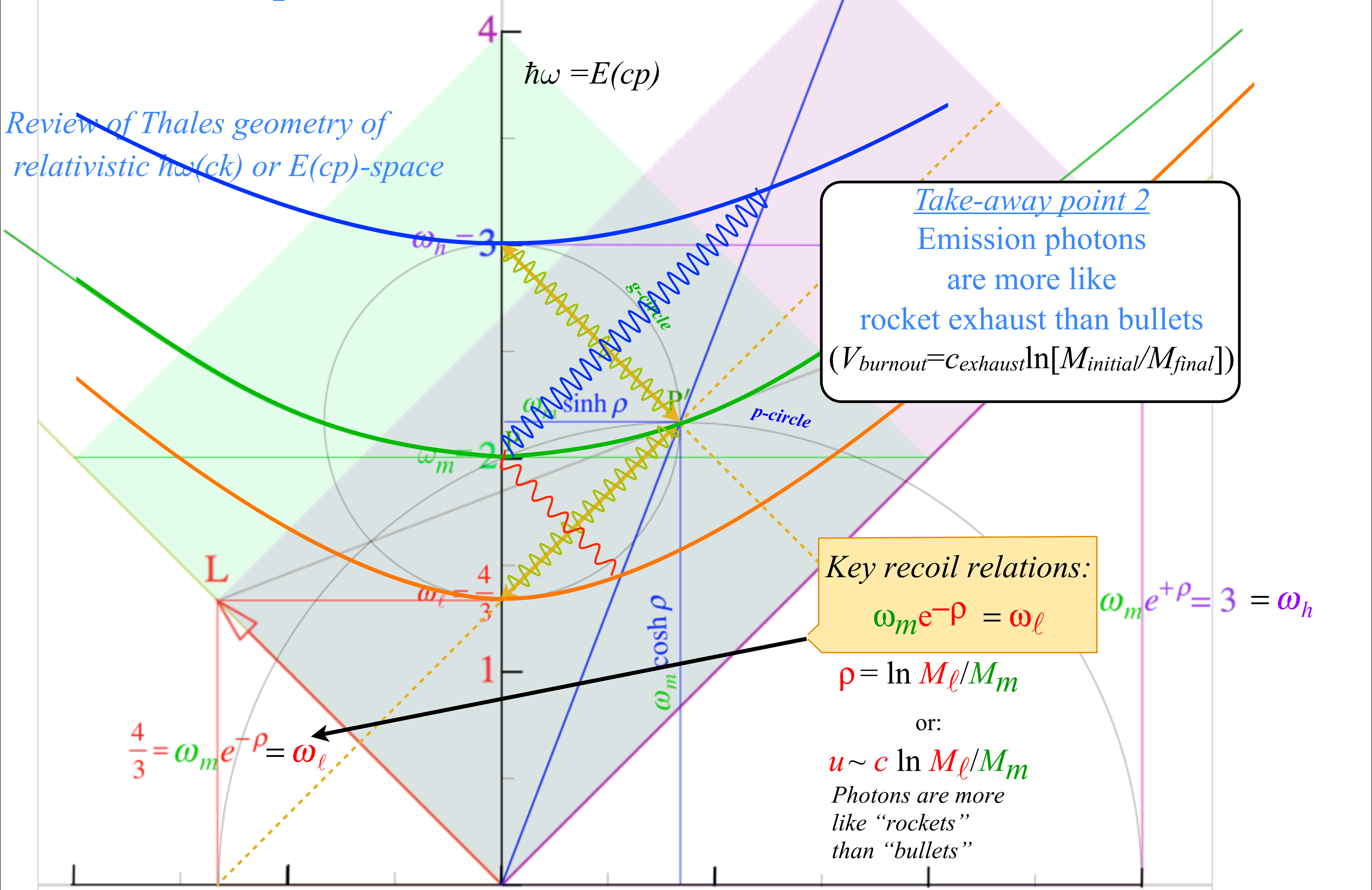
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



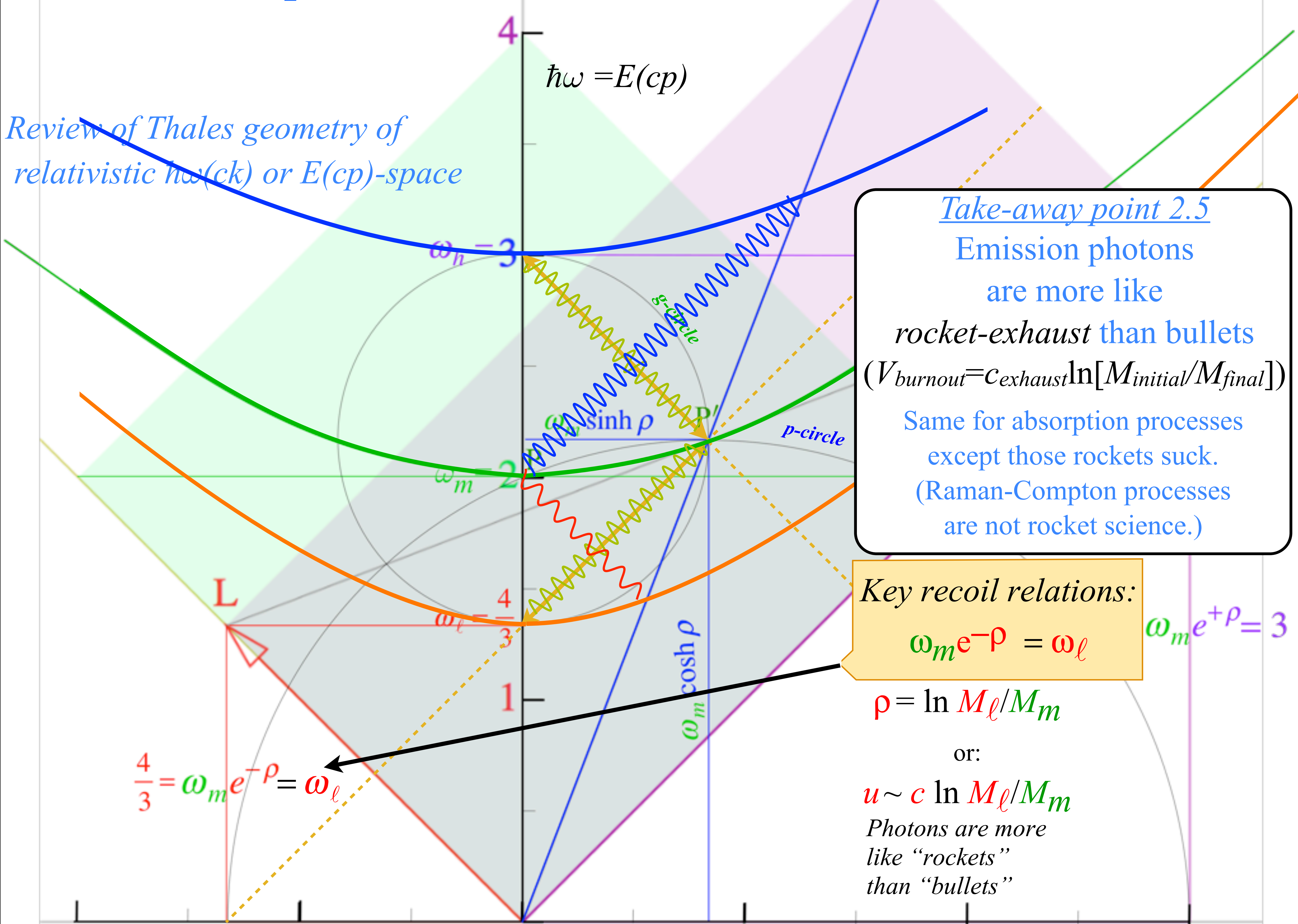
Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

Relativistic optical transitions $|high\rangle = |\omega_h\rangle \rightleftarrows |mid\rangle = |\omega_m\rangle \rightleftarrows |low\rangle = |\omega_\ell\rangle$

Review of Thales geometry of relativistic $\hbar\omega(ck)$ or $E(cp)$-space



Take-away point 2.5

Emission photons are more like *rocket-exhaust* than bullets

$(V_{burnout} = c_{exhaust} \ln[M_{initial}/M_{final}])$

Same for absorption processes except those rockets suck.

(Raman-Compton processes are not rocket science.)

Key recoil relations:

$\omega_m e^{-\rho} = \omega_\ell$

$\rho = \ln M_\ell / M_m$

or:

$u \sim c \ln M_\ell / M_m$

Photons are more like "rockets" than "bullets"

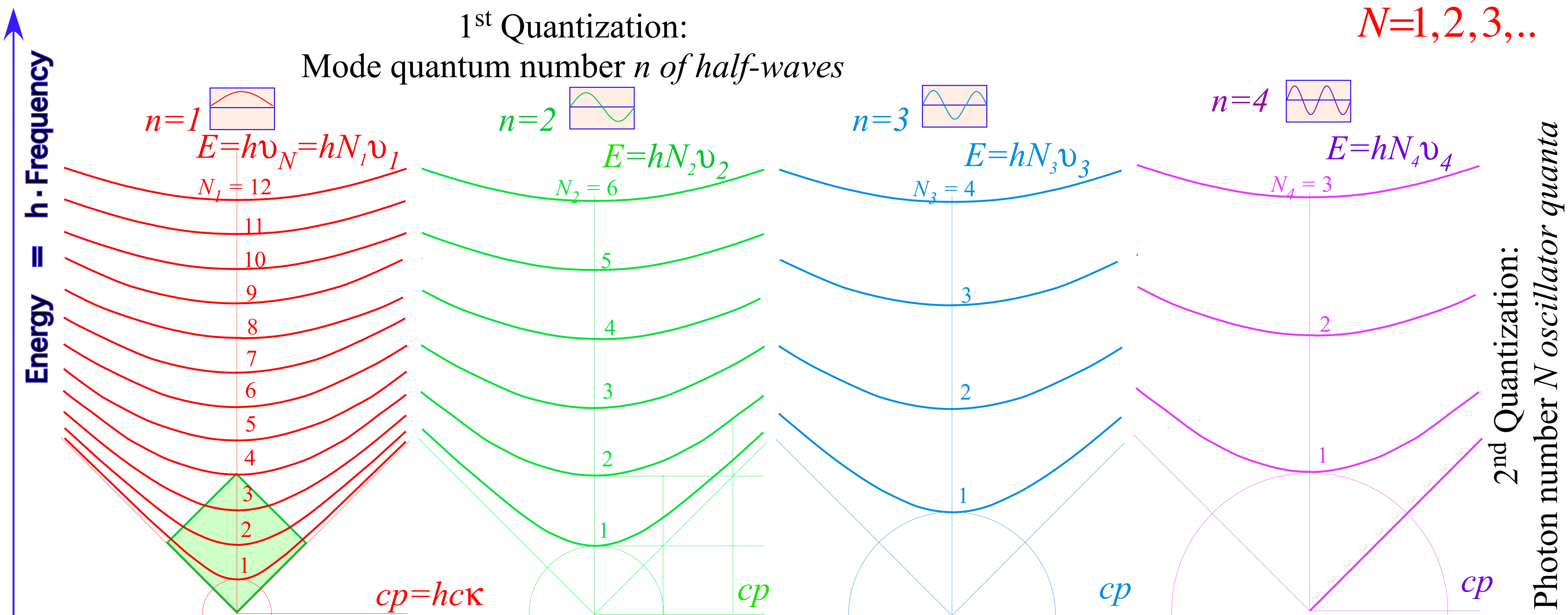
Doppler RED factor: $\frac{2}{3} = e^{-\rho}$

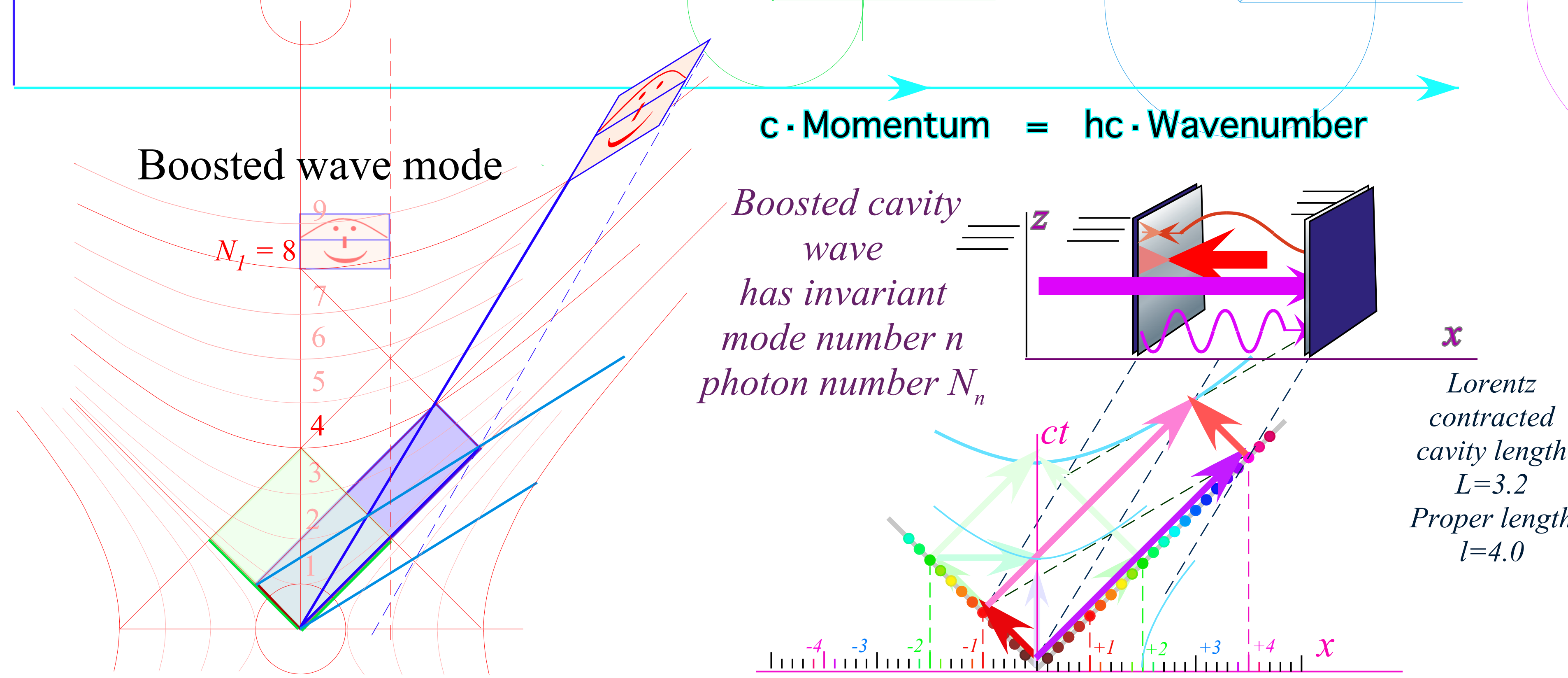
Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

2^{nd} Quantization: NEWS FLASH!!! $h\upsilon$ is actually $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho$ with *quantum* numbers$)$

$N=1,2,3,..$



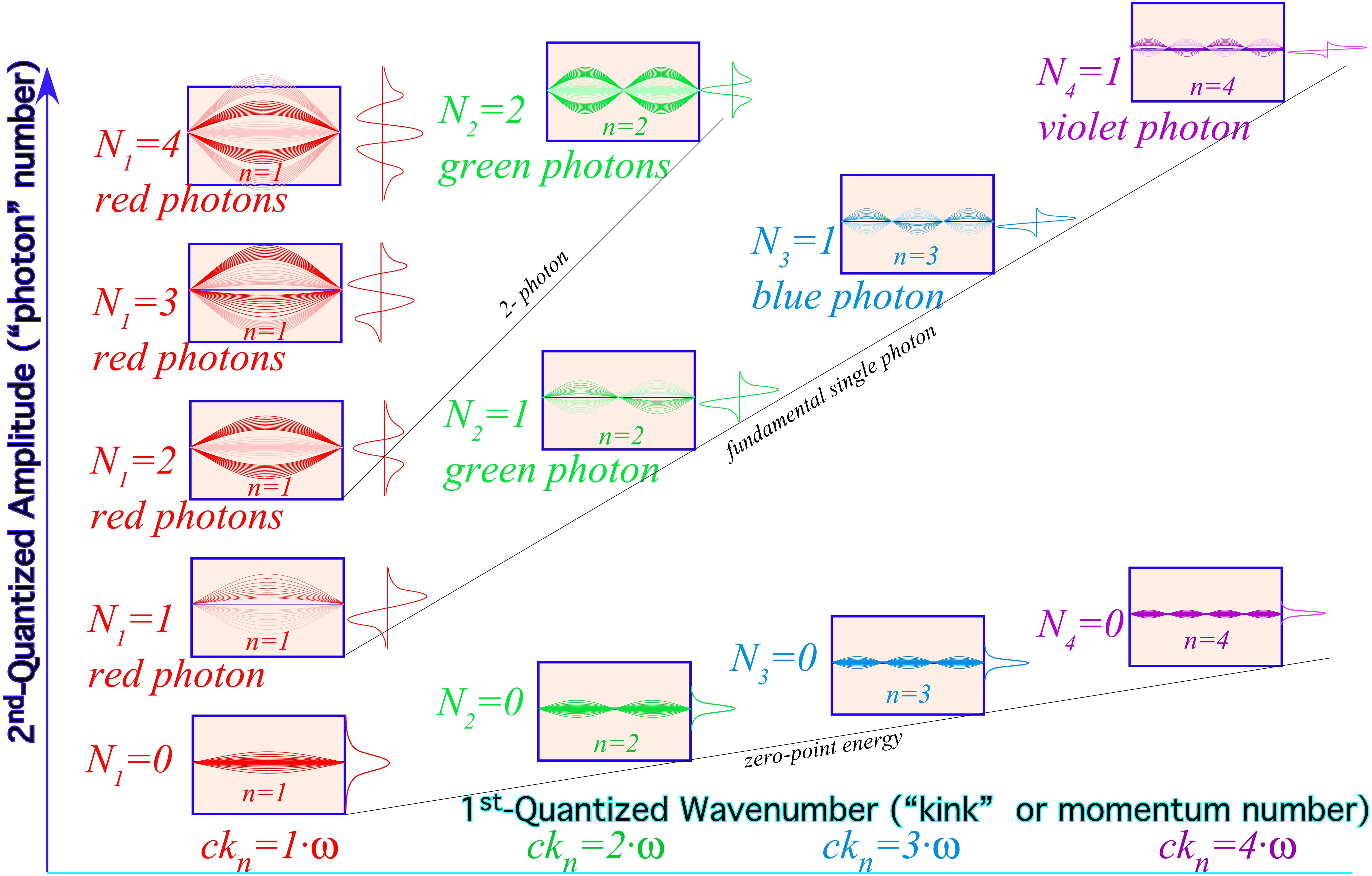


Take-away point 3

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2^{nd} Quantization: NEWS FLASH!!! $h\upsilon$ *is actually* $hN\upsilon$

$(h\upsilon_{phase}=E=h\upsilon_A\cosh\rho)$ *is actually* $(\ hN\upsilon_{phase}=E_N=hN\upsilon_A\cosh\rho \quad (N=1,2,..)\ )$



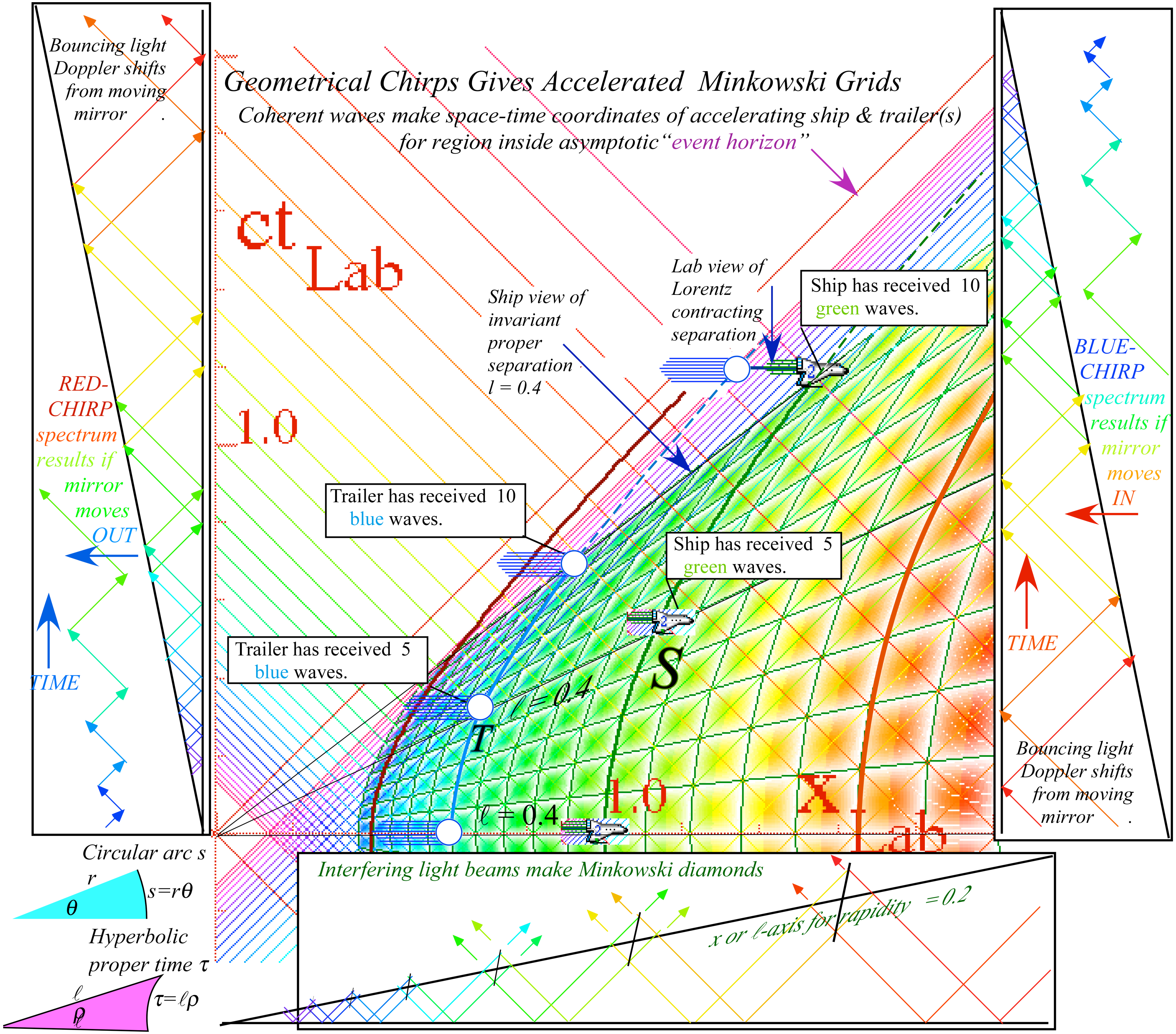


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light