Special Relativity Introduction for General Relativity 1
Friday 01.27.2017

Relawavity I.: A wavy introduction to relativity

Learning about sin (and cos…tan…and trig. road maps)
- Hyper-Trigonometric algebra and phasors in space-time
  - Single Continuous Wave (1CW) functions and phasors

Per-space-per-time vs Space-time
- Wave velocity formulas

Introducing Doppler shifting
- Why is $c=299{,}792{,}458\ m\cdot s^{-1}$ constant?!
  - Introducing Doppler Arithmetic and rapidity ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity
- Details of 2CW functions in rest frame
  - Pulse Waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation
- Analyzing wave velocity by per-space-per-time *and* space-time graphs
  - 16 coefficients of relativistic 2CW interference
    - Two "famous-name" coefficients and Lorentz transform
      - "Doppler Jeopardy" and Thales geometry of Lorentz transformation

See also related text and articles:

Classical Mechanics with a BANG! Unit 8

Relawavity: Intro SR&QM by Ruler&Compass

Mac-pages pdf *TeX-pdf*

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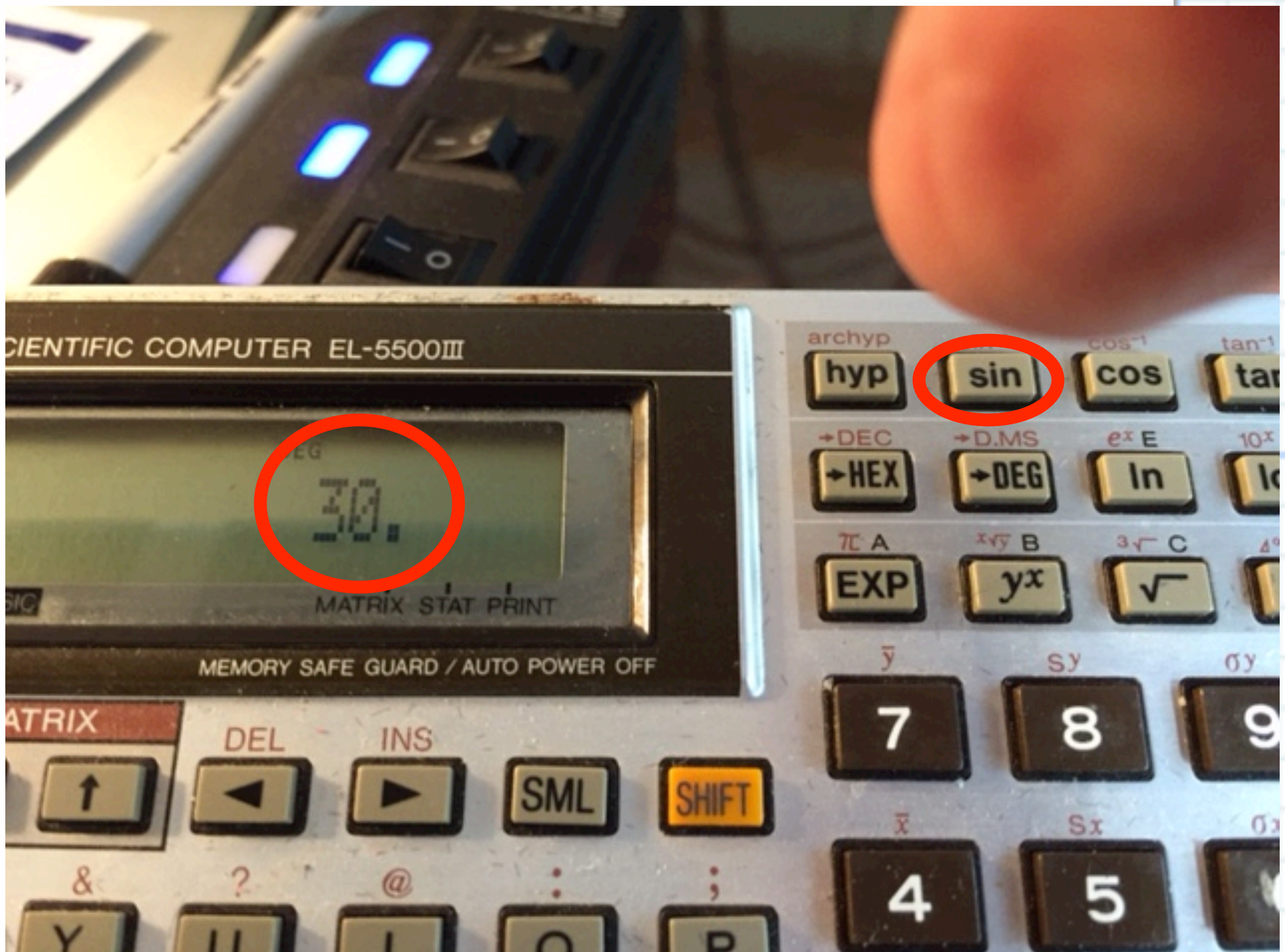
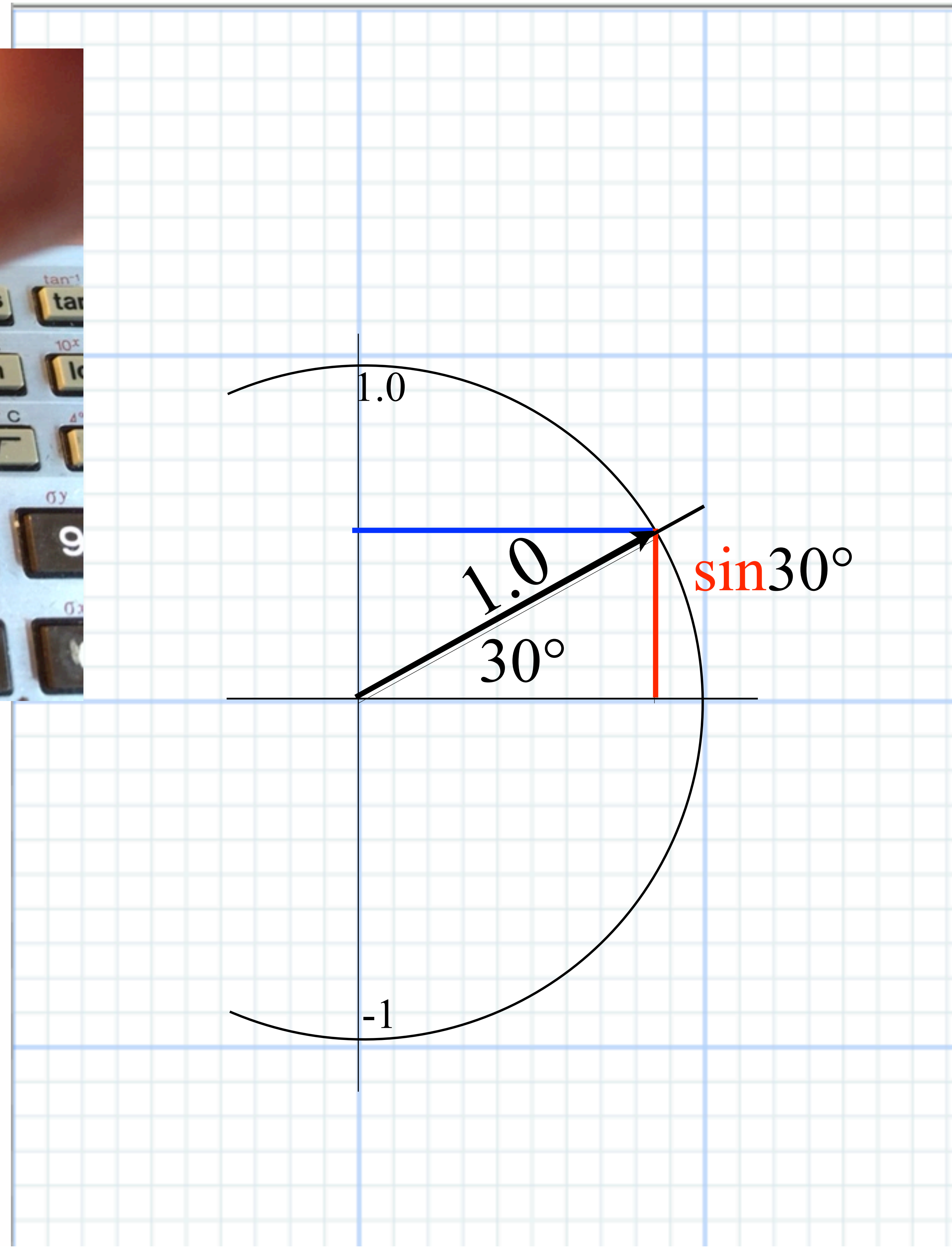
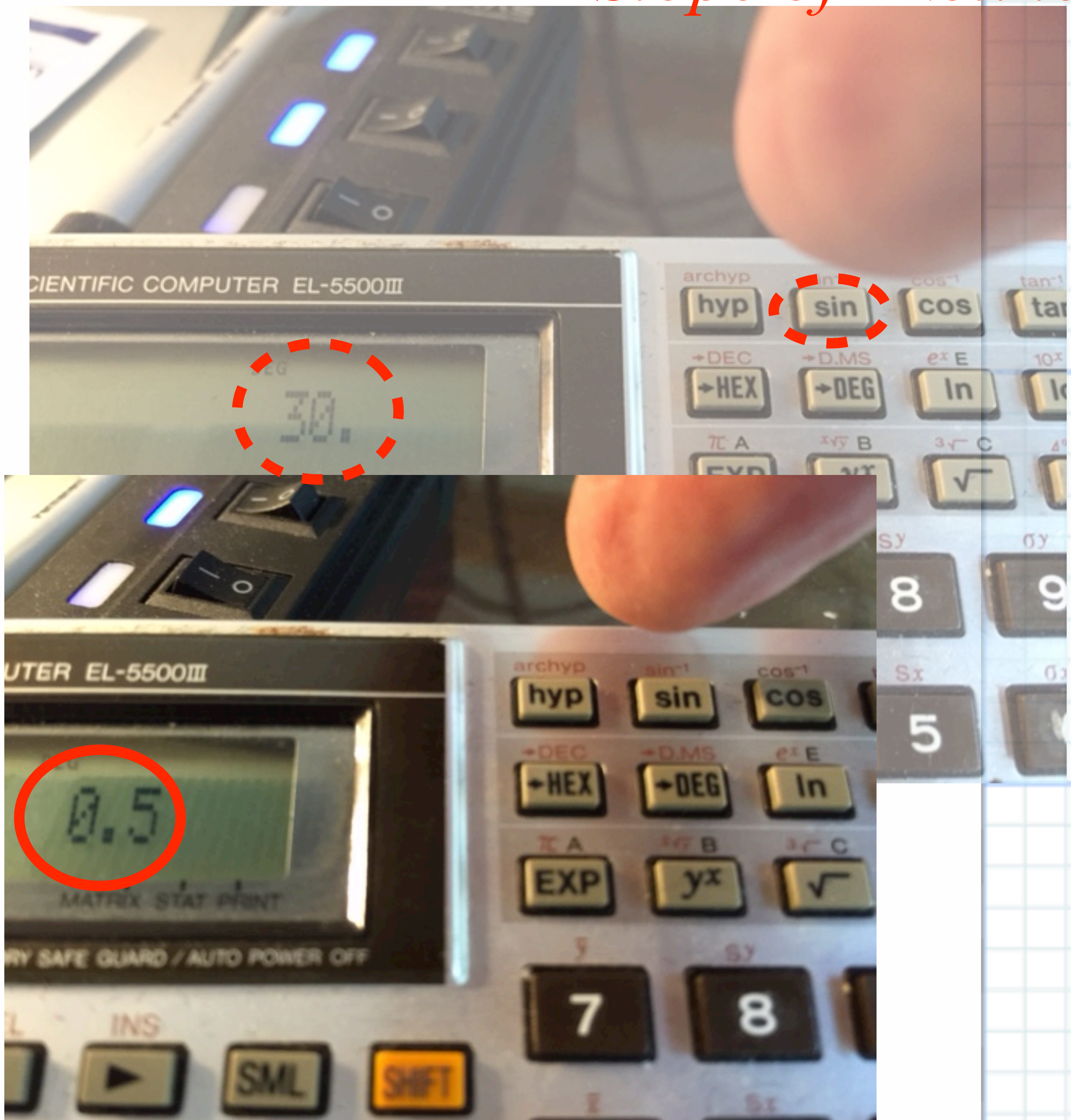
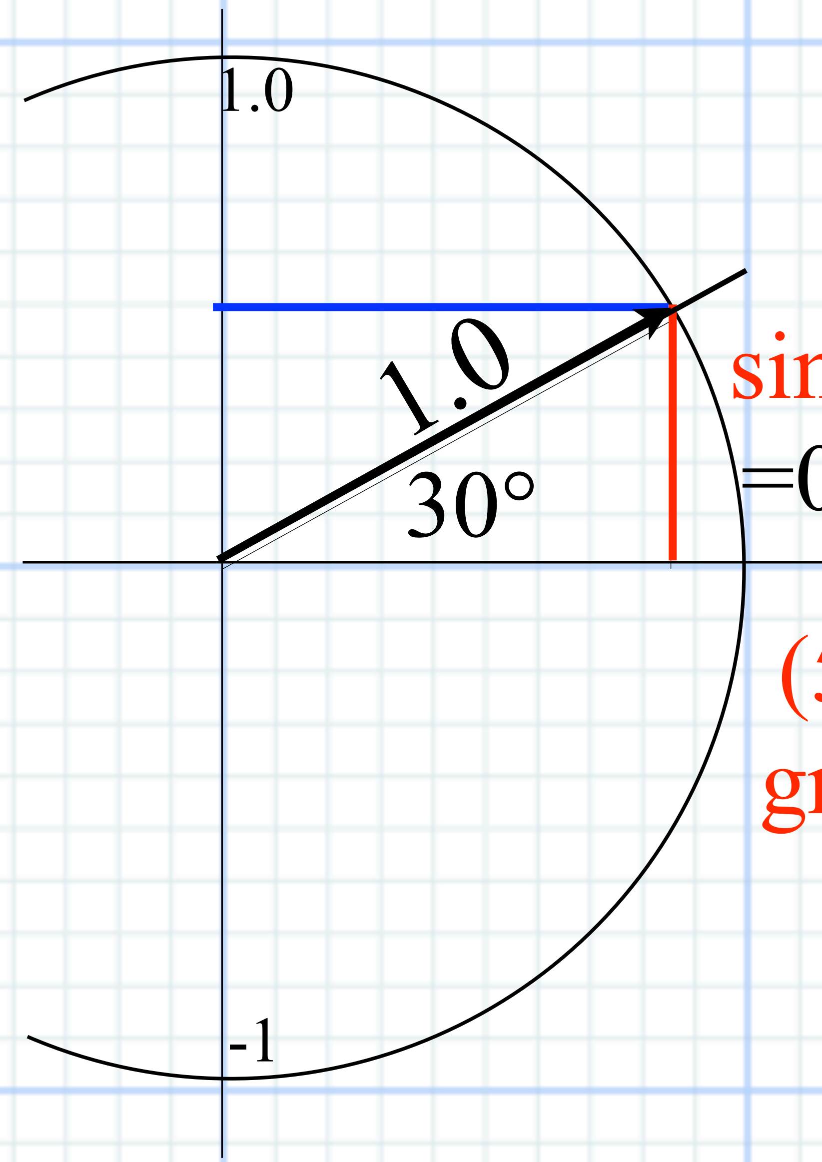
Learning about SIN





Learning about SIN
"Slope of INcline"



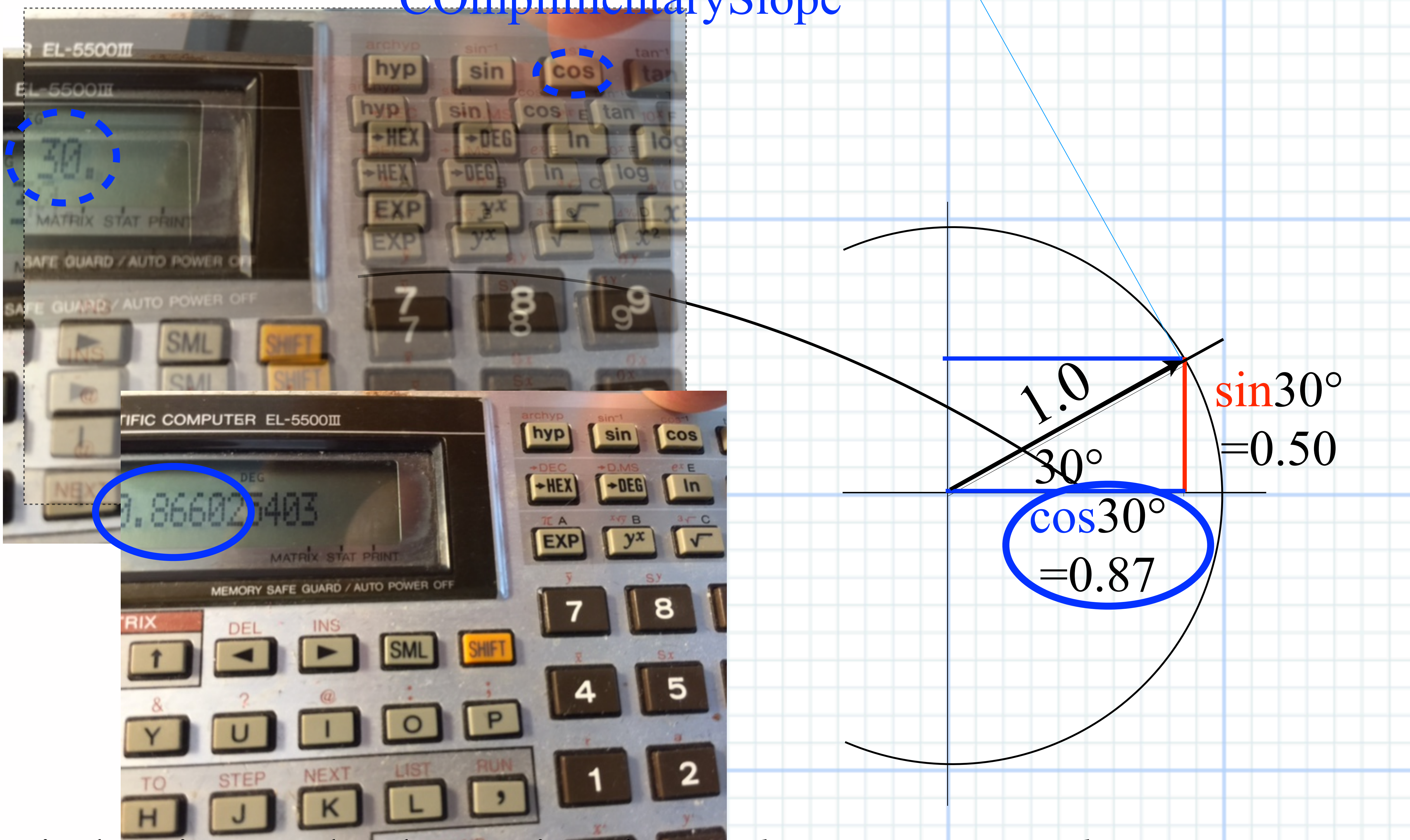


Circle-trig mostly about triangles *and sine-waves*

Learning about SIN and the COSin

"Slope of INcline"

"COmplimentarySlope"



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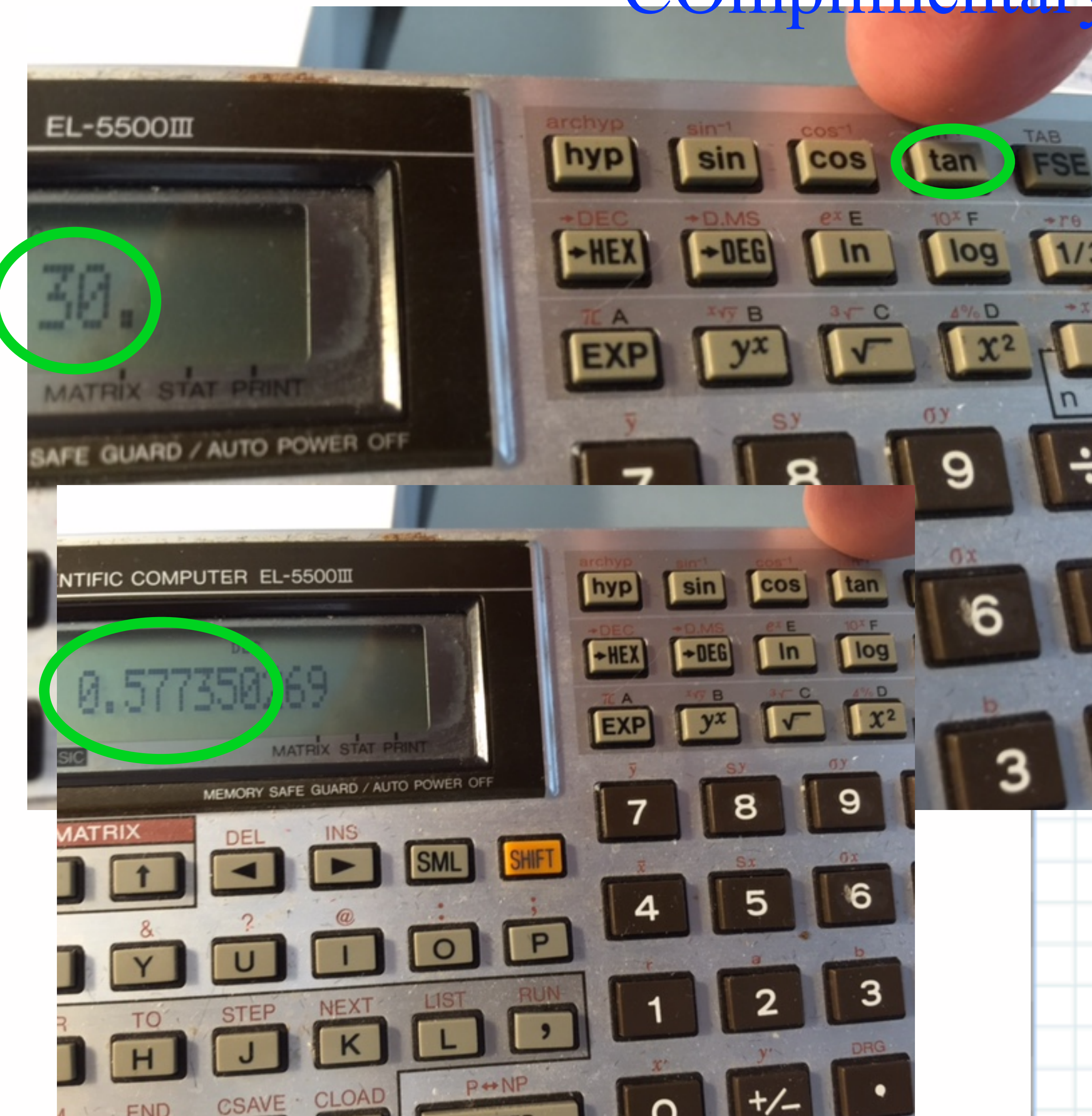

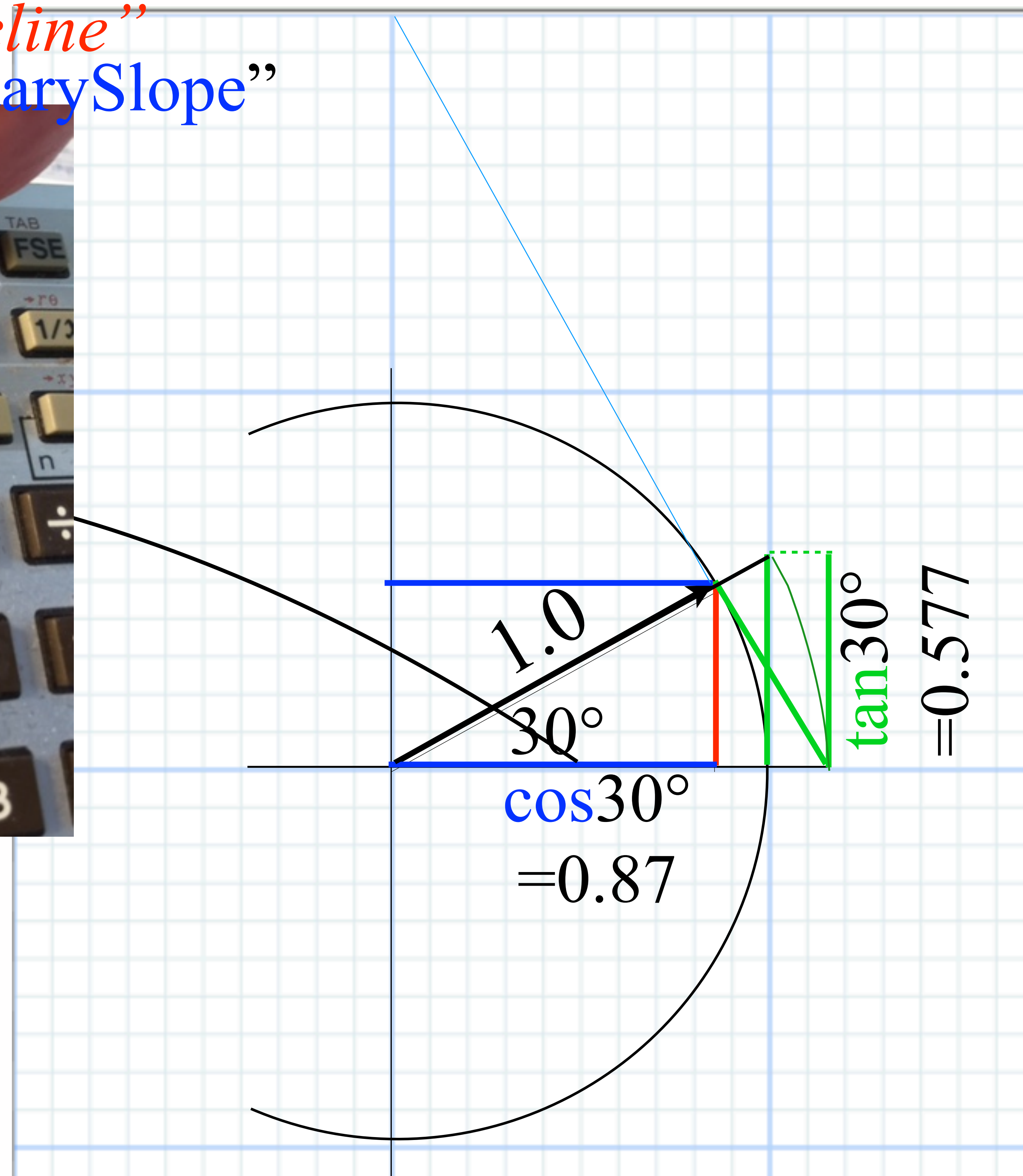

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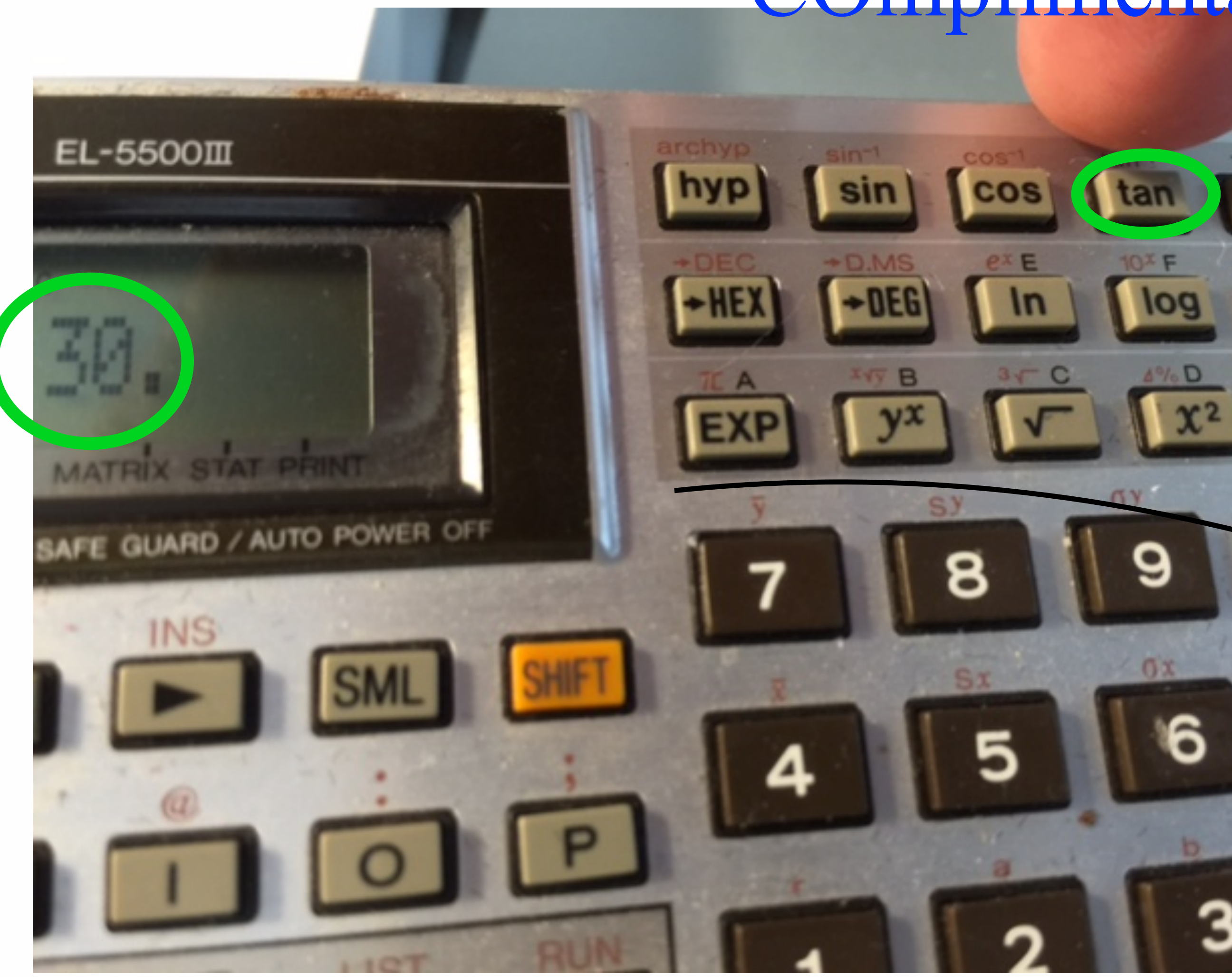

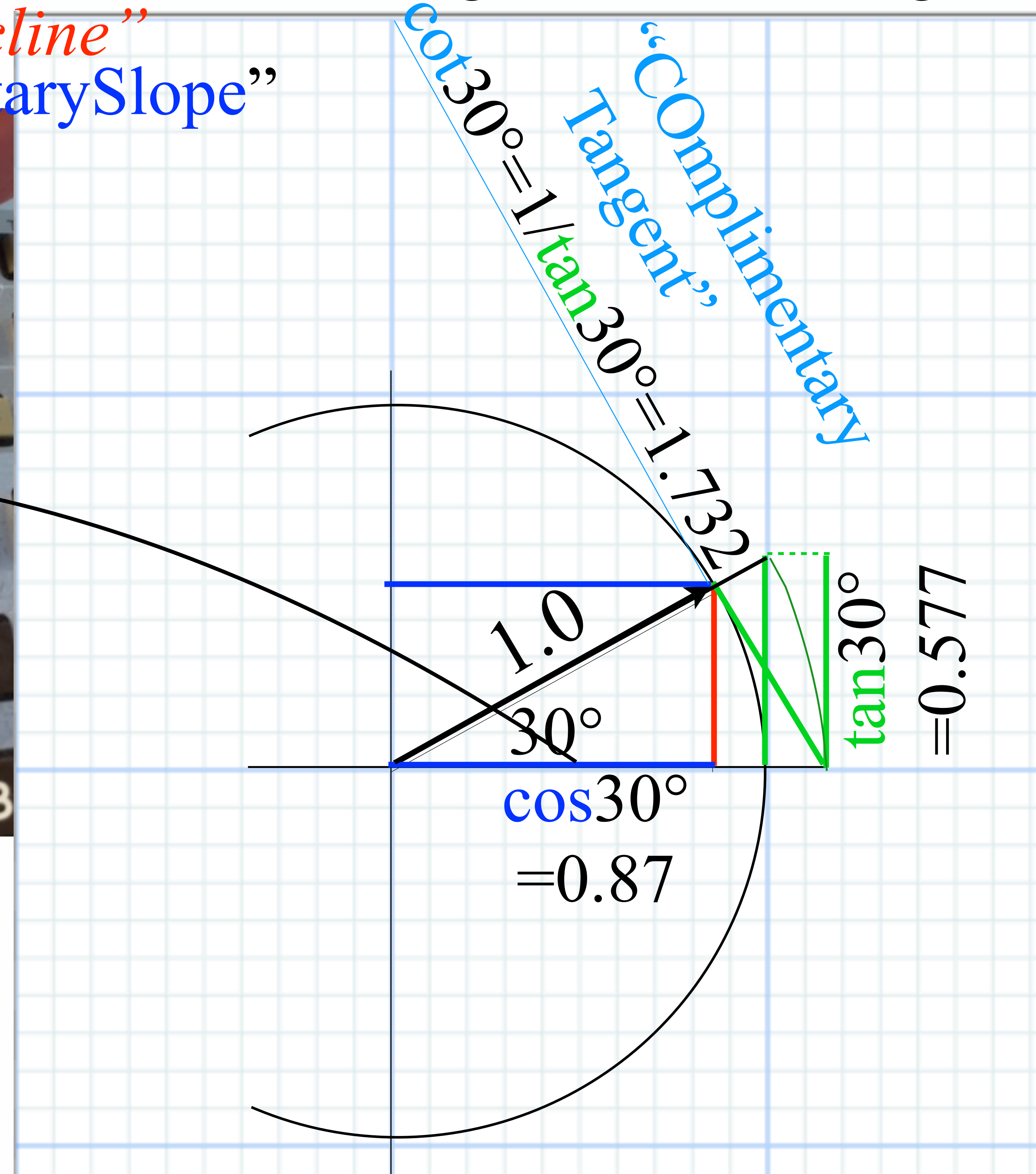

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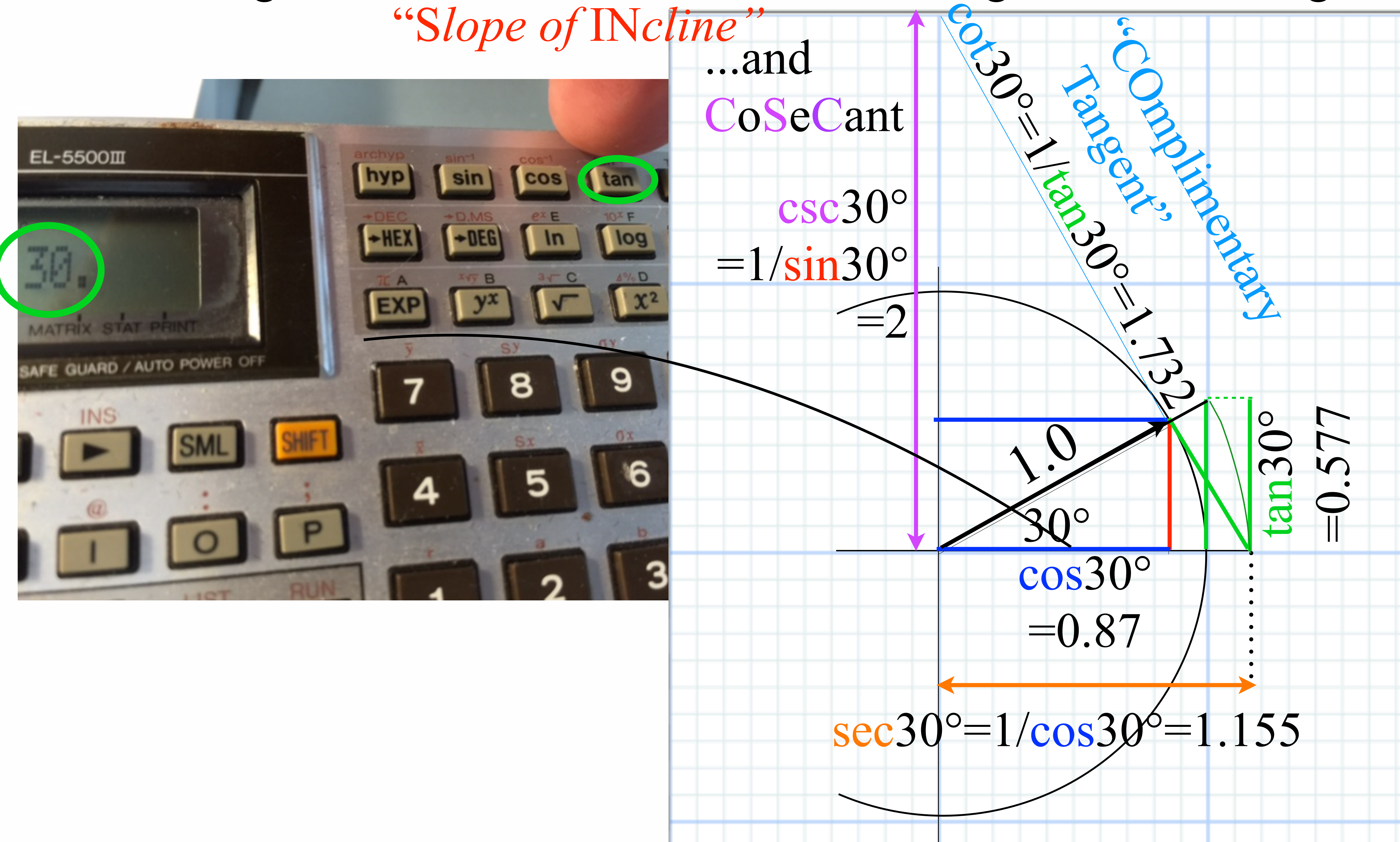

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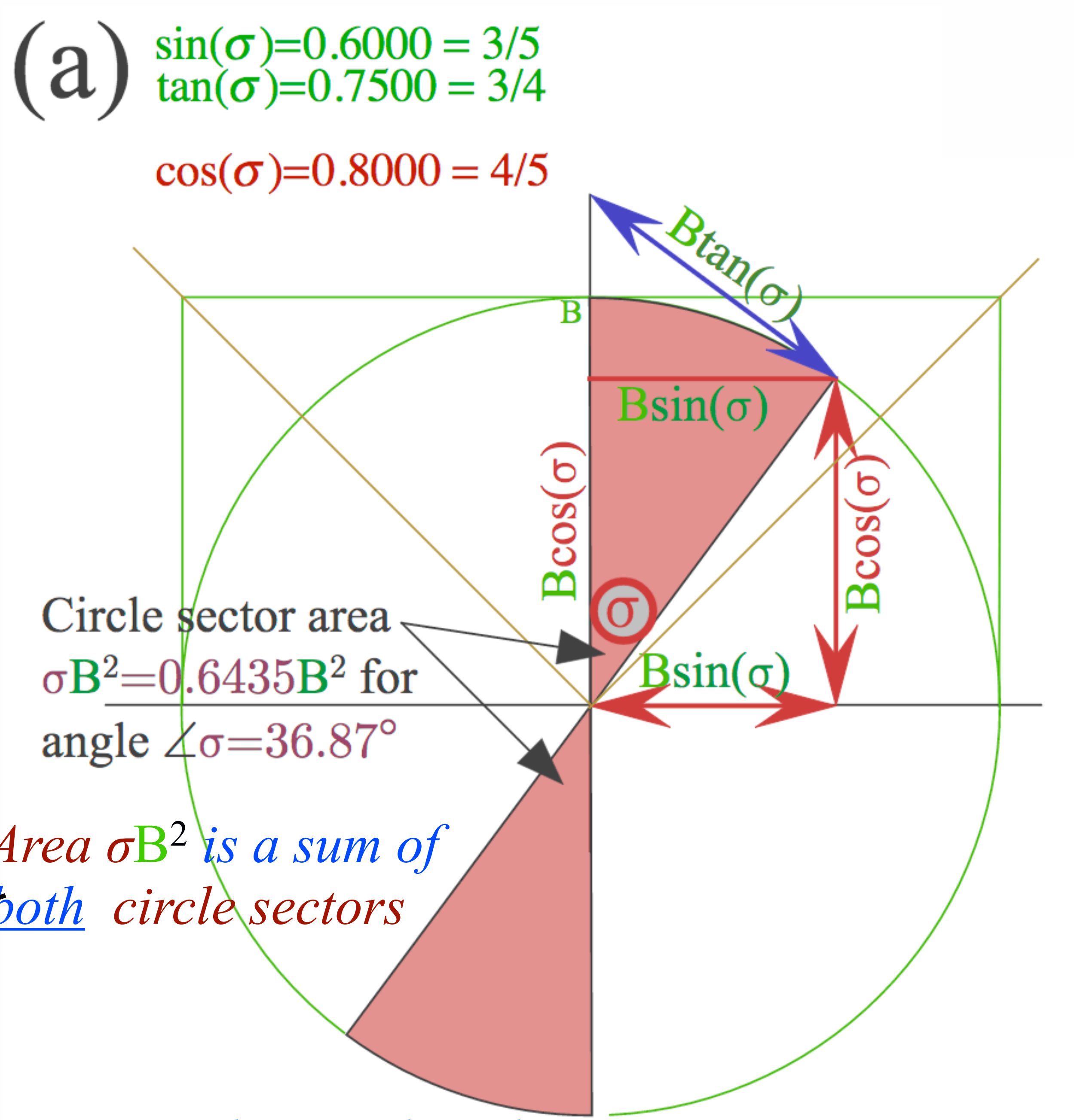
Trigonometric road maps

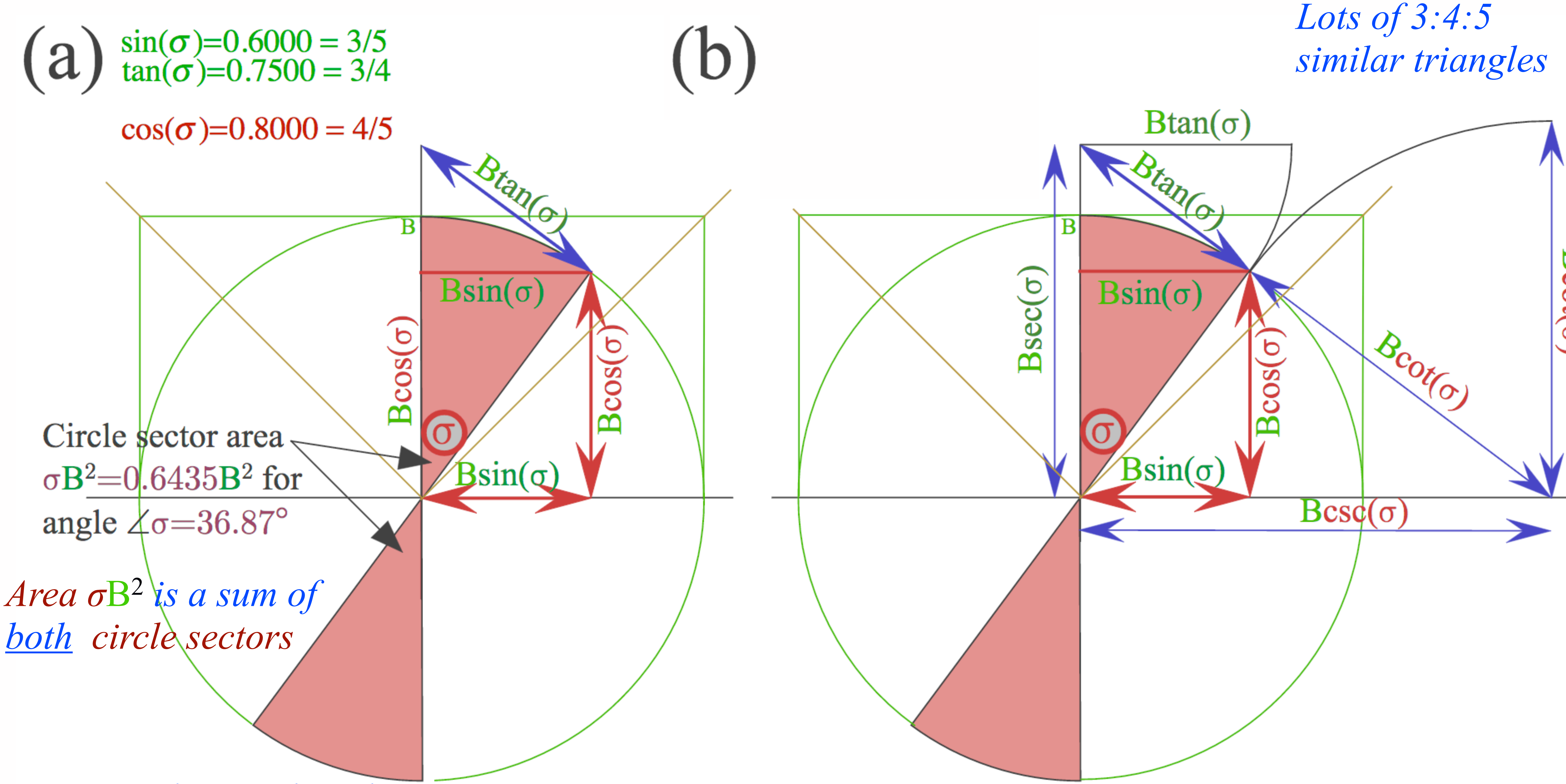


Area σB² is a sum of <u>both</u> circle sectors

σB is either arc length
σ is either angle in radians

Trigonometric road maps

Lots of 3:4:5 similar triangles



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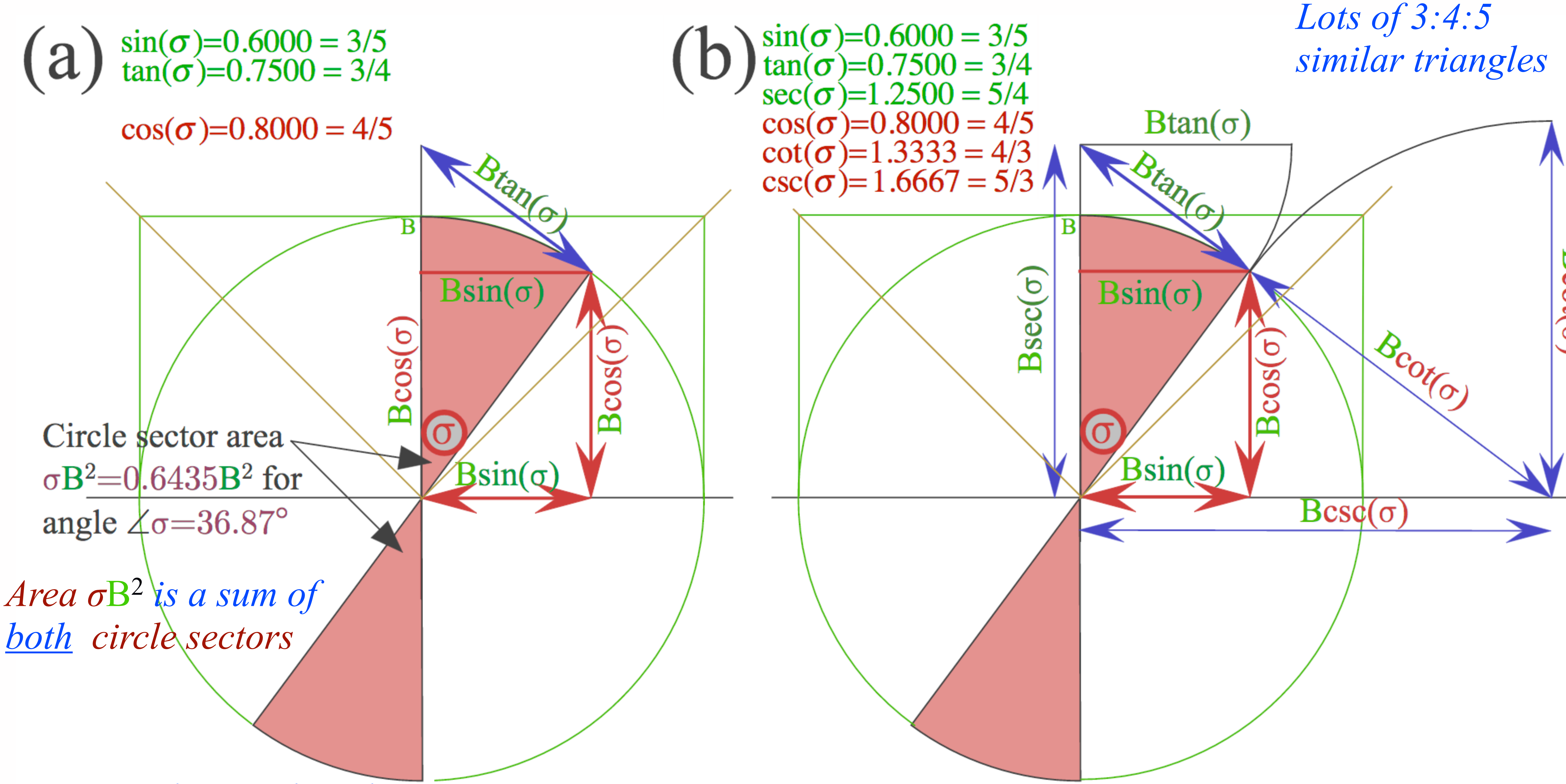

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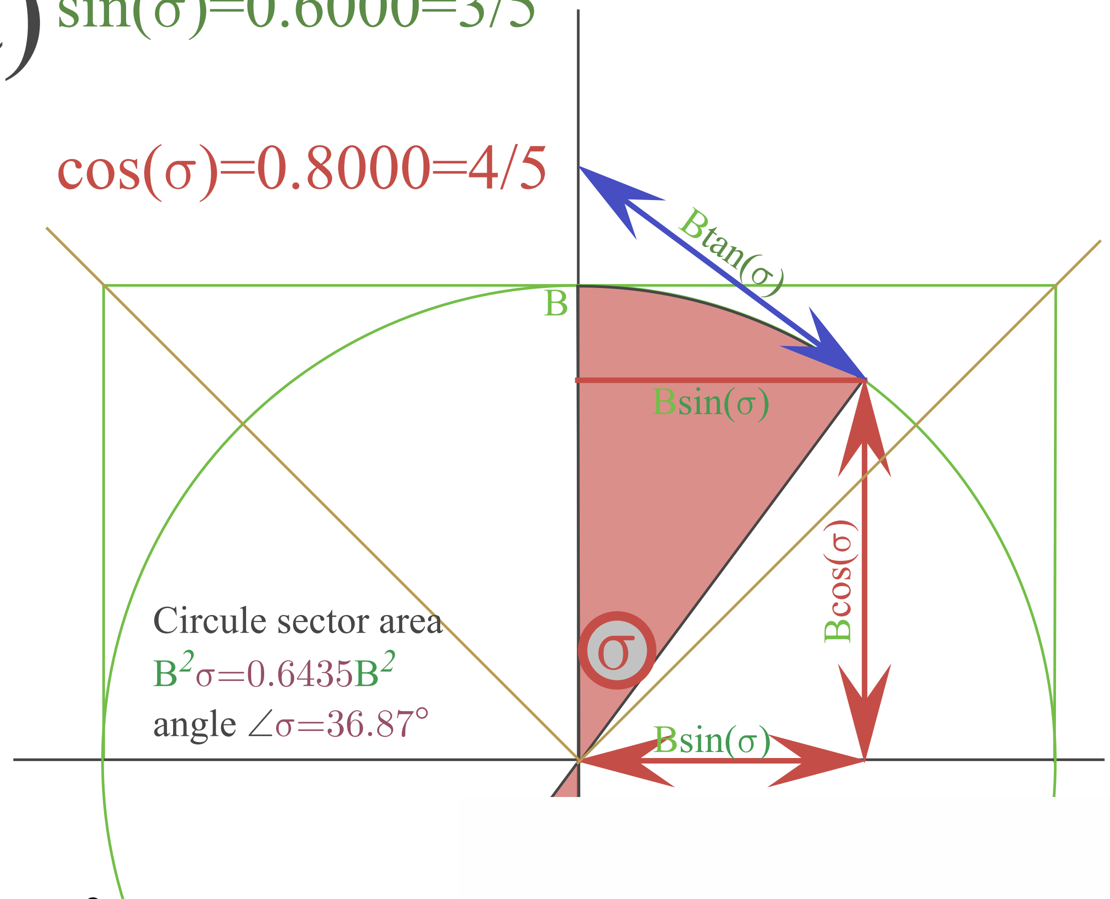
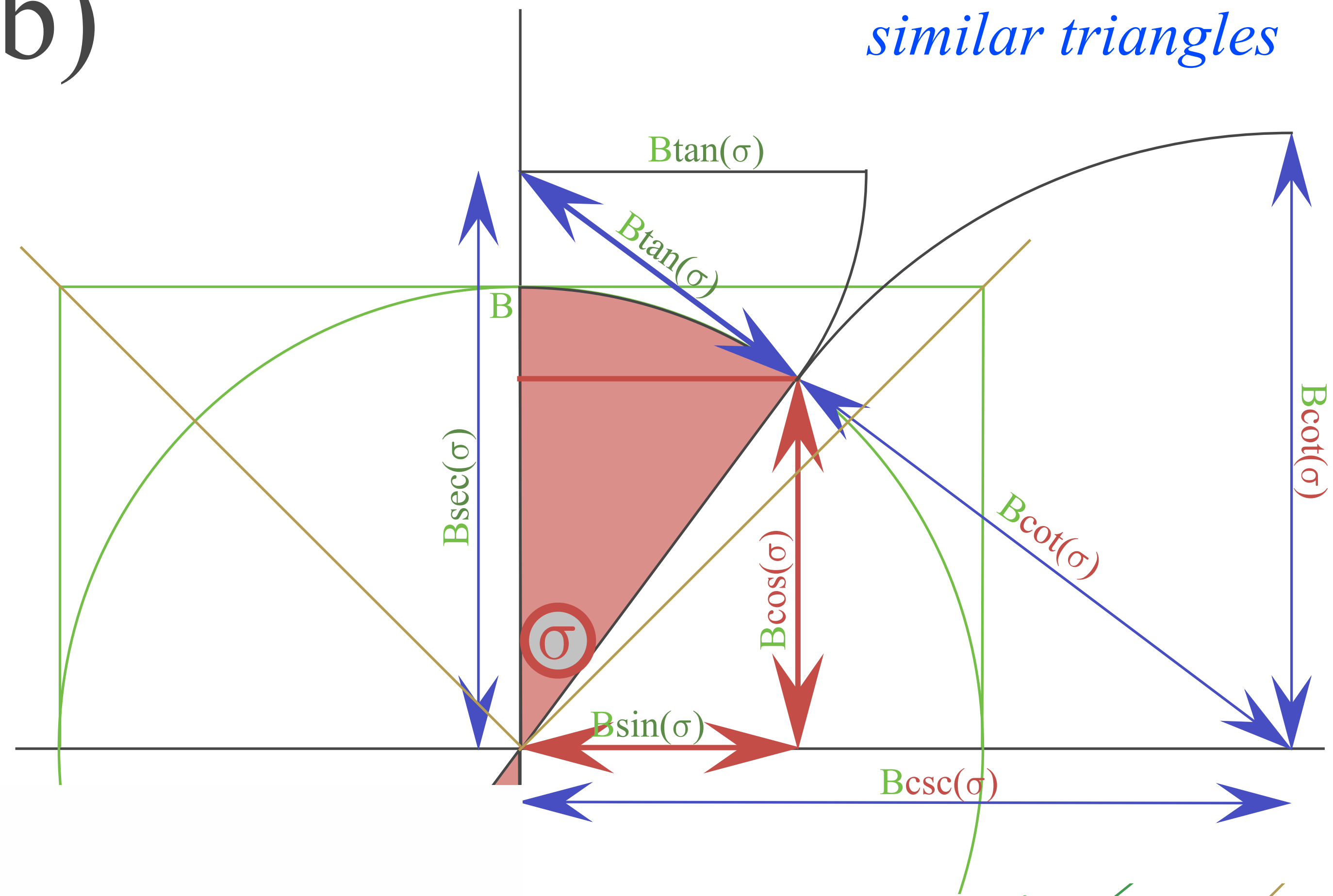


(b)

Lots of 3:4:5 similar triangles



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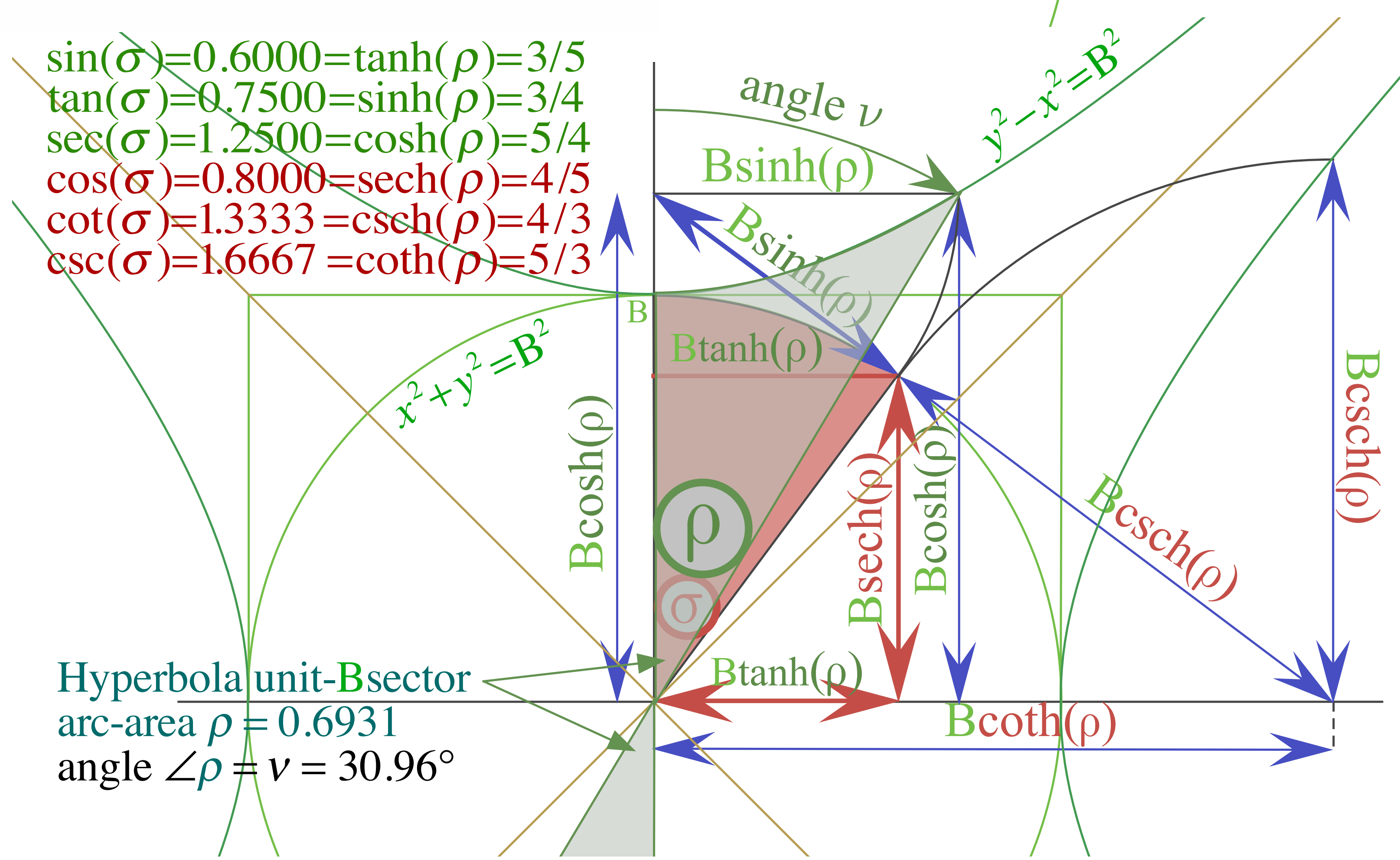
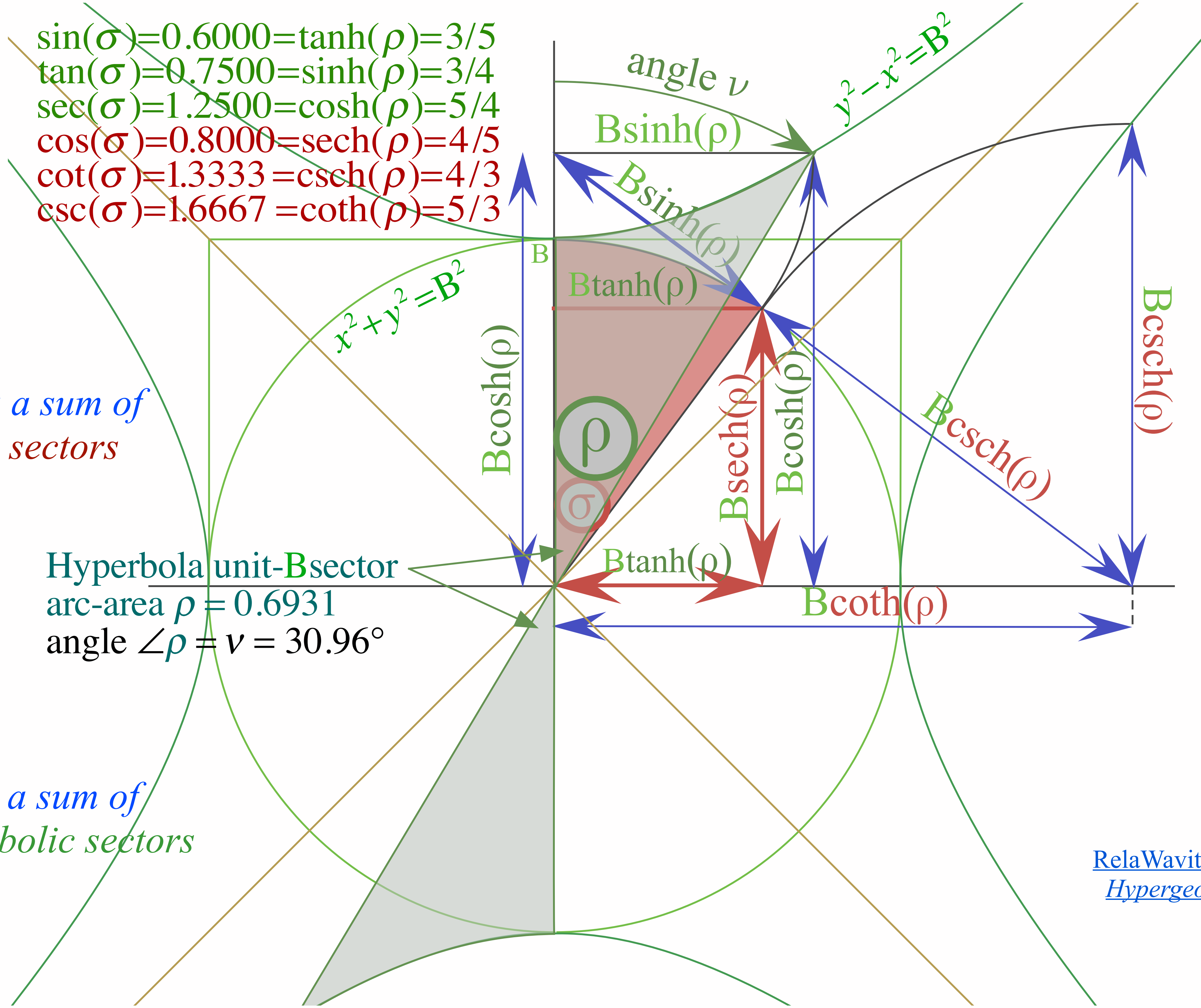


σB is either arc length
σ is either angle in radians

Area ρB² is a sum of
both hyperbolic sectors

Trigonometric road maps



Area σB^2 *is a sum of* *both* *circle sectors*

Area ρB^2 *is a sum of* *both* *hyperbolic sectors*

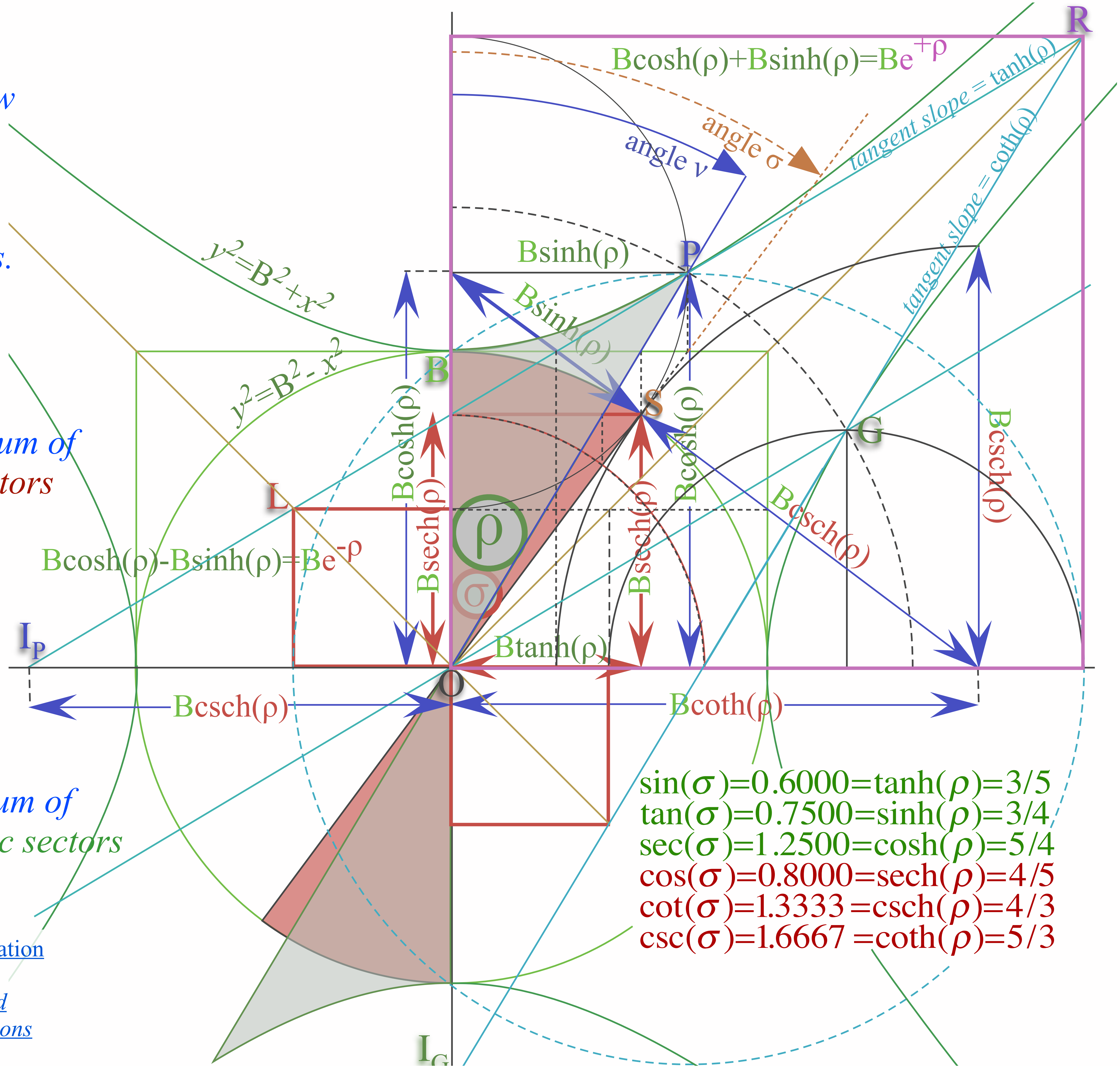
RelaWavity Web Simulation
Hypergeometric functions

Trigonometric road maps

Need to see how trig road maps match the physical maps on next 2 pages.

Area σB^2 is a sum of *both* *circle sectors*

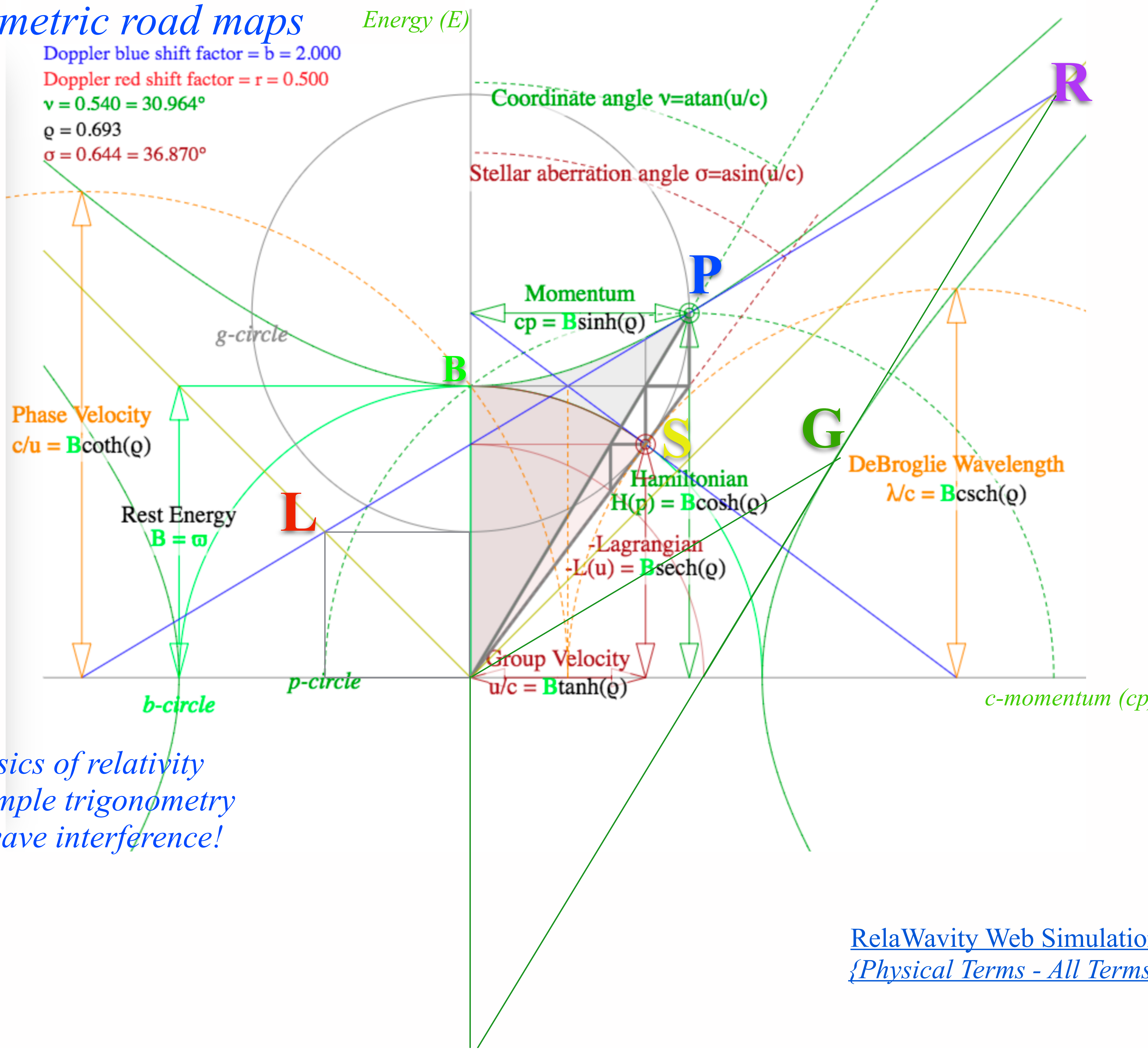
Area ρB^2 is a sum of *both* *hyperbolic sectors*



$\sin(\sigma)=0.6000=\tanh(\rho)=3/5$
$\tan(\sigma)=0.7500=\sinh(\rho)=3/4$
$\sec(\sigma)=1.2500=\cosh(\rho)=5/4$
$\cos(\sigma)=0.8000=\text{sech}(\rho)=4/5$
$\cot(\sigma)=1.3333=\text{csch}(\rho)=4/3$
$\csc(\sigma)=1.6667=\coth(\rho)=5/3$

RelaWavity Web Simulation
Relations between Hypergeometric and Hypergeometric functions

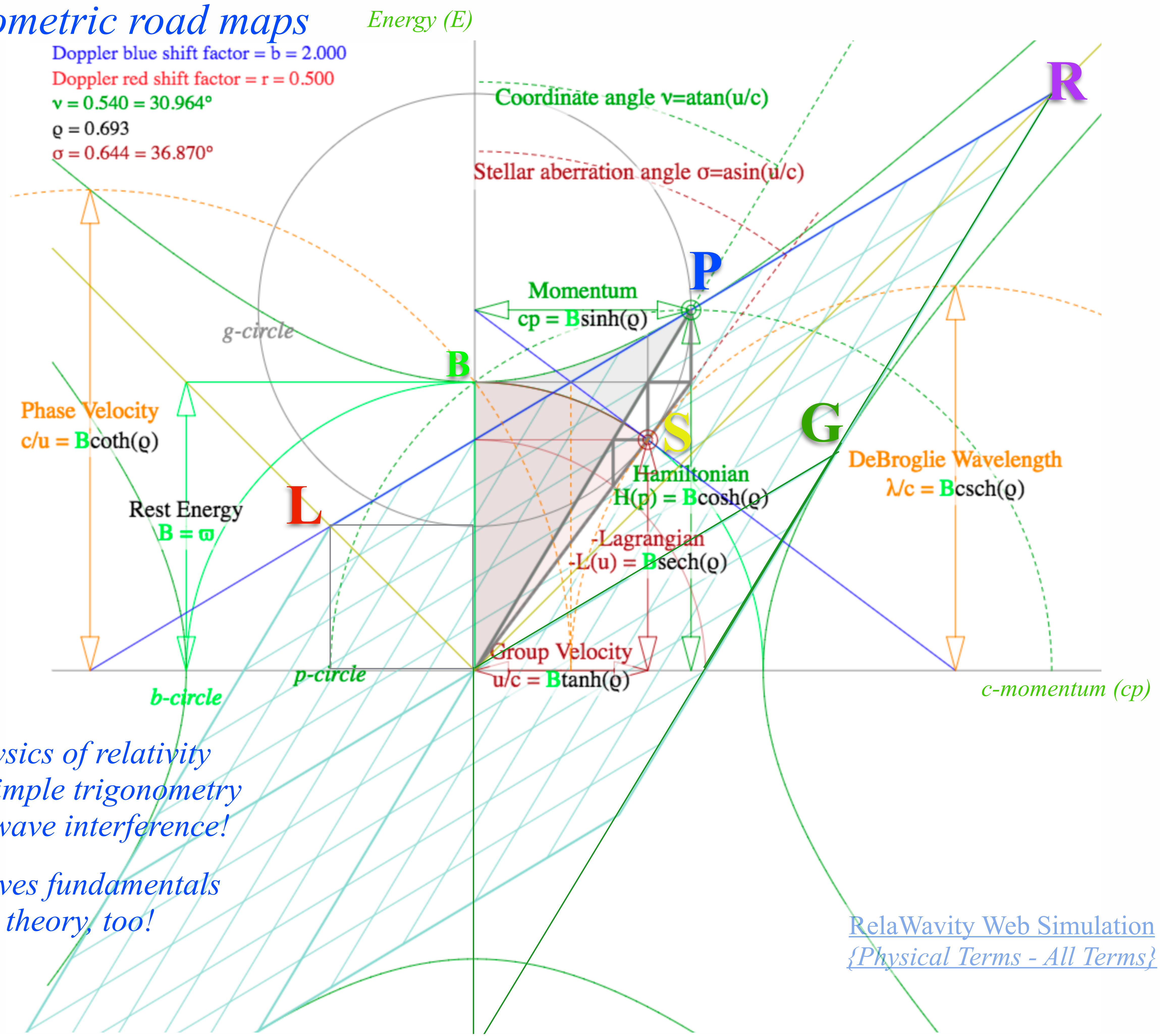
Trigonometric road maps



All this physics of relativity is mostly simple trigonometry of optical wave interference!

RelaWavity Web Simulation
{Physical Terms - All Terms}

Trigonometric road maps



All this physics of relativity is mostly simple trigonometry of optical wave interference!

And, it derives fundamentals of quantum theory, too!

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Calculus of trig-map geometry uses an infinite-n-compounding limit of the interest rate-r formula.

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$$e^{rt} = 1 + rt + \frac{(rt)^2}{2} + \frac{(rt)^3}{2\cdot 3} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^5}{2\cdot 3\cdot 4\cdot 5} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} + \ldots$$

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Half-sum and half difference of $e^{\pm rt}$ series define the hyperbolic cosine (cosh(rt)) and sine (sinh(rt)).

$$\frac{e^{+rt}+e^{-rt}}{2} = 1 + \frac{(rt)^2}{2} + \frac{(rt)^4}{2\cdot 3\cdot 4} + \frac{(rt)^6}{2\cdot 3\cdot 4\cdot 5\cdot 6} - \ldots = \cosh(rt)$$

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Replace rate r with imaginary rate ir ($i=\sqrt{-1}$) powers $i^0=1$, $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$, $i^5=i$, $i^6=-1$, $i^7=-i$,…

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Sum and difference of these pairs gives Euler-DeMoivre formulae.

$$e^{+i\sigma} = \cos(\sigma) + i\sin(\sigma), \qquad e^{-i\sigma} = \cos(\sigma) - i\sin(\sigma).$$

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$$e^{+i\sigma} = \cos(\sigma) + i\sin(\sigma), \qquad e^{-i\sigma} = \cos(\sigma) - i\sin(\sigma).$$

Don't forget these! $e^{+\rho} = \cosh(\rho) + \sinh(\rho)$, $e^{-\rho} = \cosh(\rho) - \sinh(\rho)$.

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1CW Laser-phasor wave function

$$\psi = A \cdot e^{i(kx-\omega t)} = A \cdot \cos(kx-\omega t) + iA \cdot \sin(kx-\omega t)$$

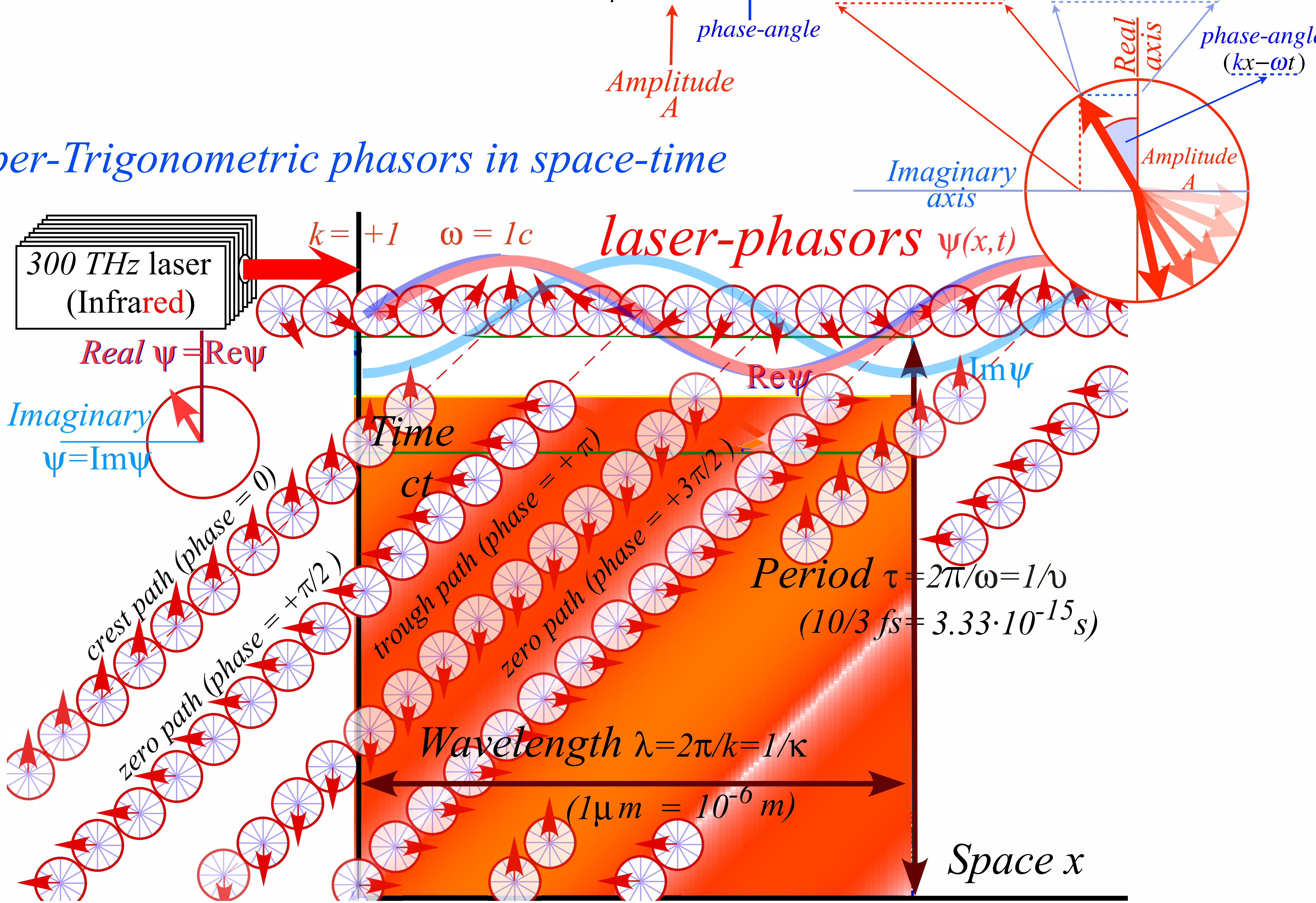
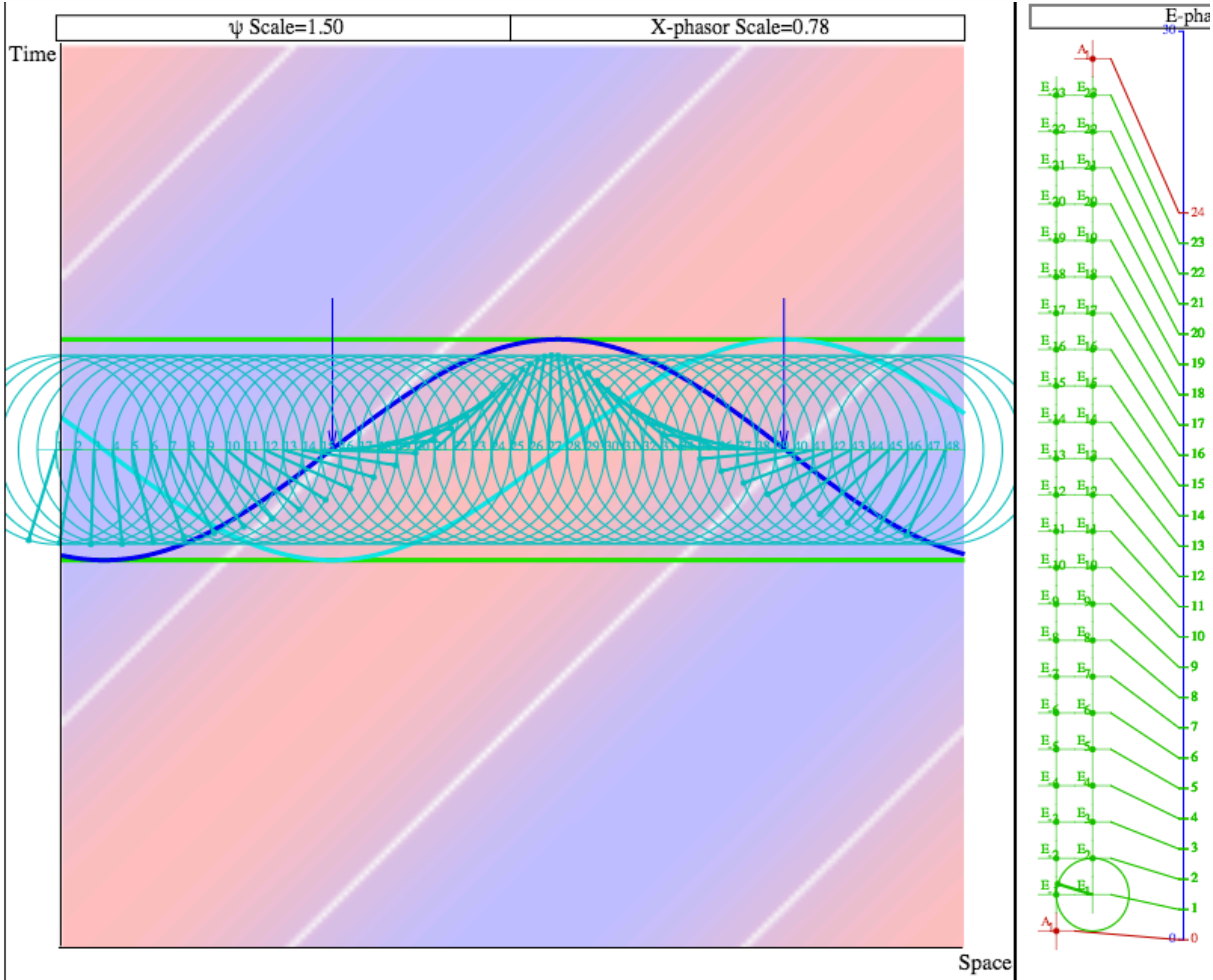


Fig. 4(a) Single-phasor plot of wave-function at (x,ct). (b) Array of phasors at many (x,ct)-points.

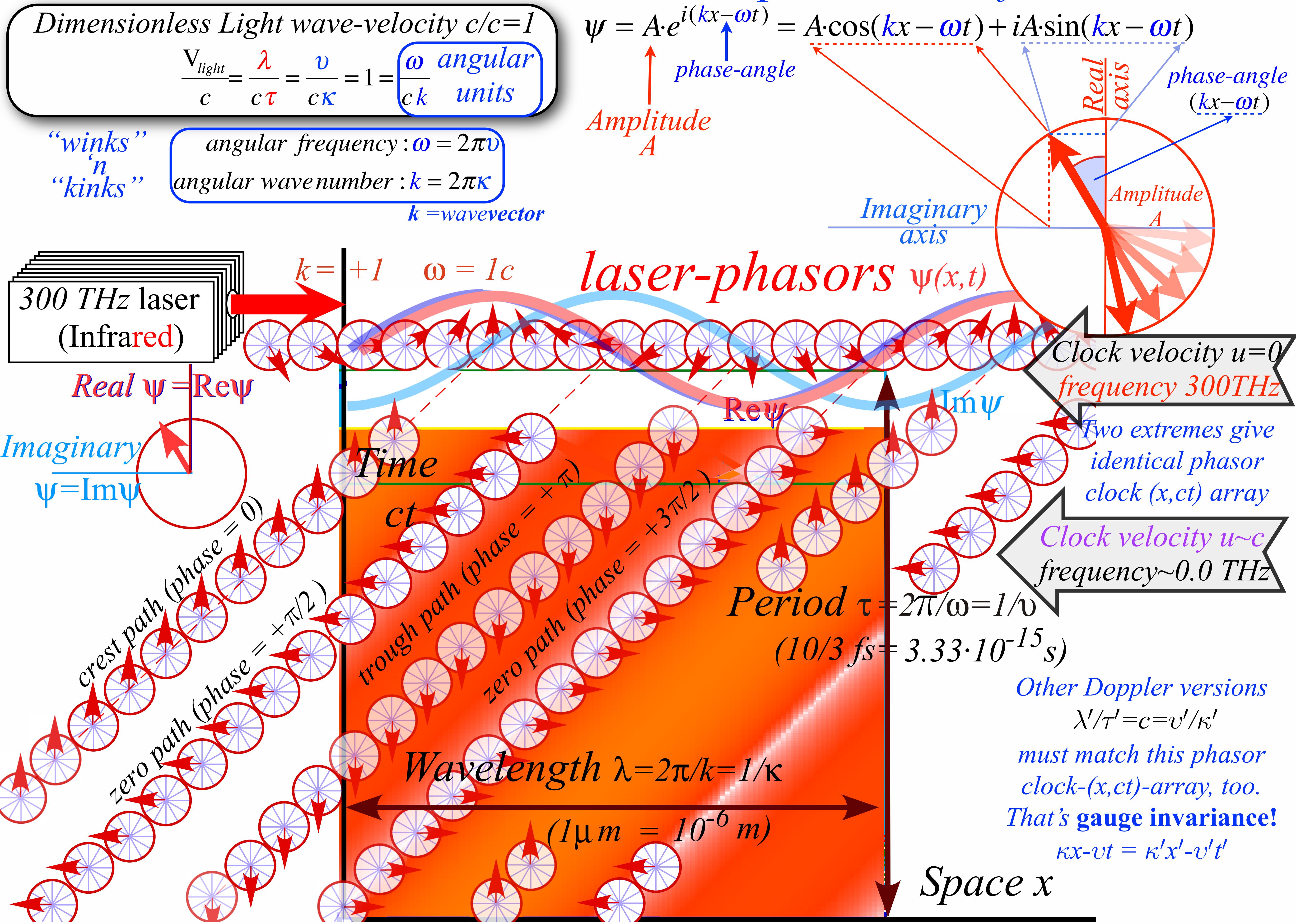


[BohrIt Web Simulation](#)

[1 CW ct vs x Plot (ck = +1)](#)

[Single panel with Zero Tracers](#)

1CW Laser-phasor wave function



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The "Keyboard of the gods" : per-space-per-time plot *versus* space-time Minkowski plot

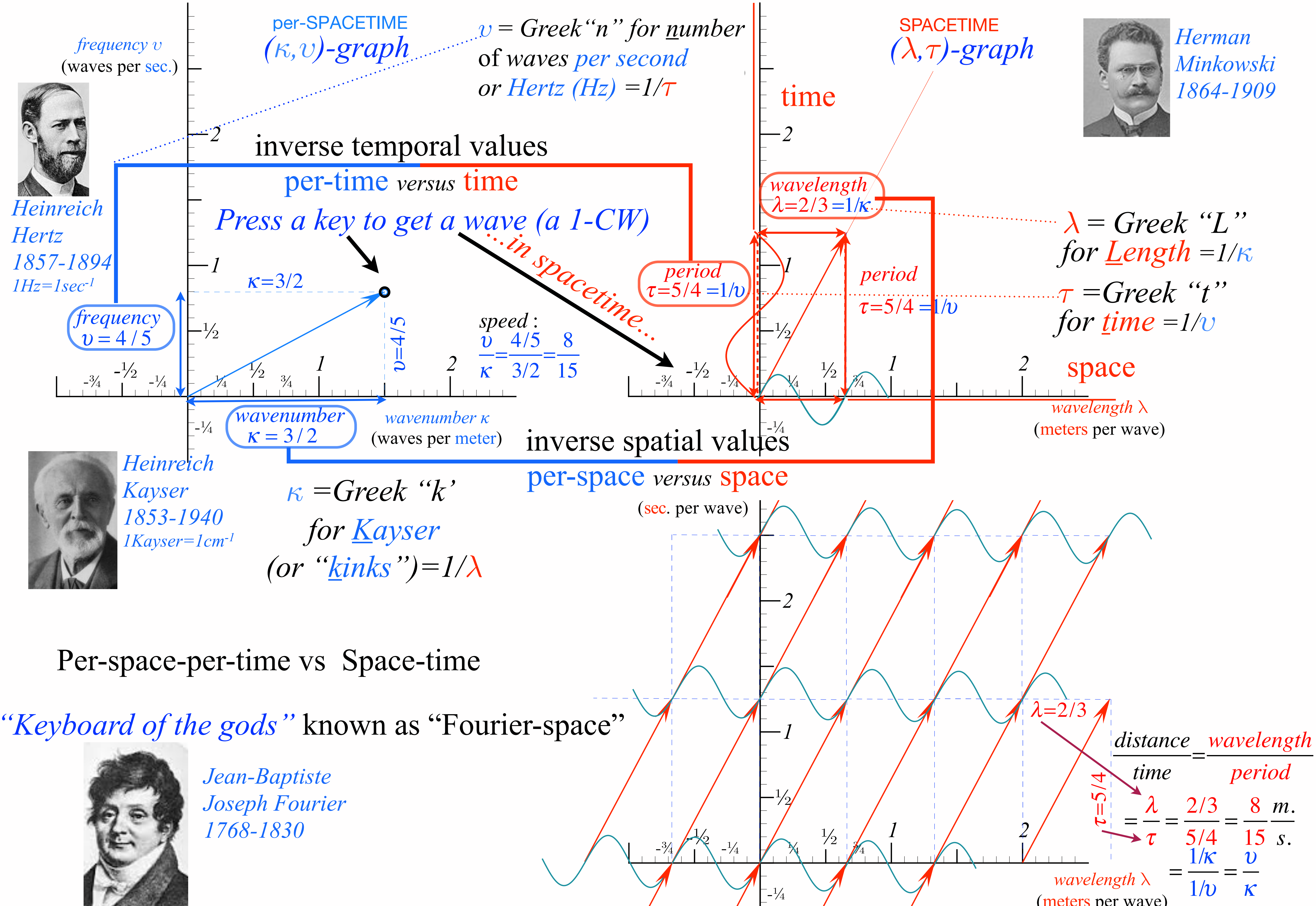
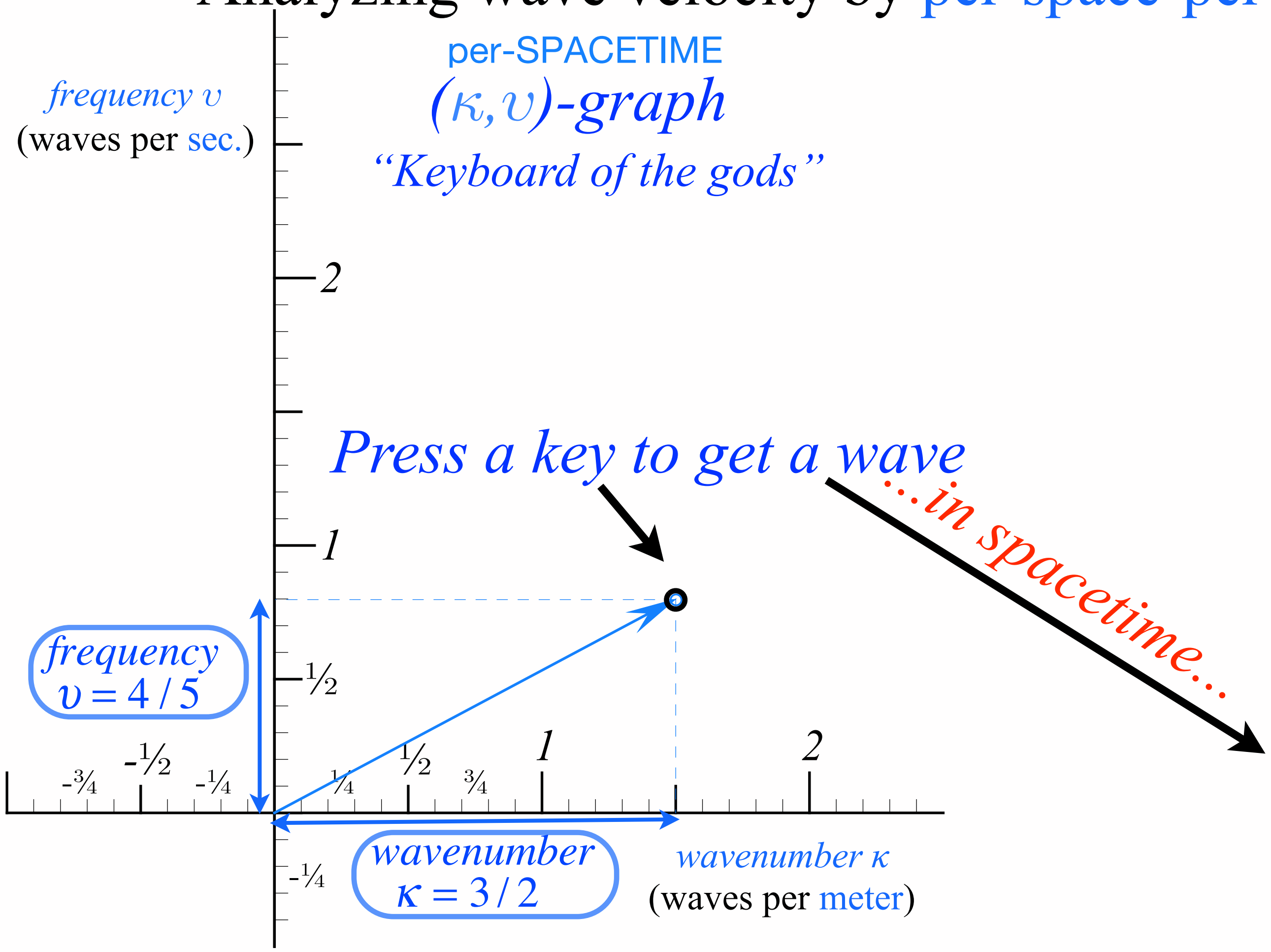
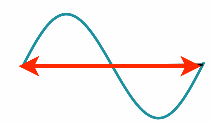


Fig. 5 Comparing a wave point in Kaiser-Hertz per-space-time to its Minkowski space-time view.

Analyzing wave velocity by per-space-per-time *and* space-time graphs



SPACETIME
(λ,τ)-graph



"Keyboard of the gods" is known as "Fourier-space"

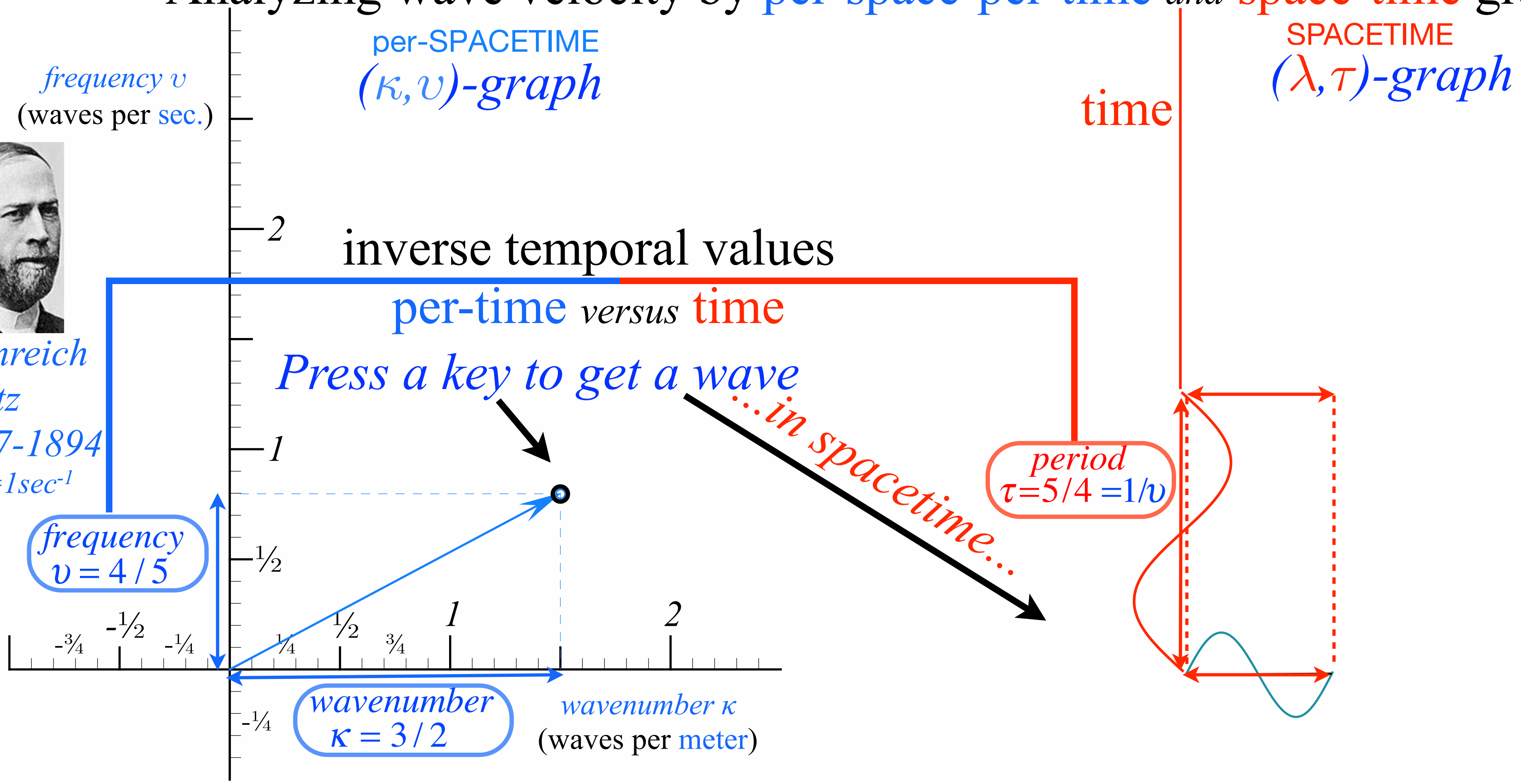
Jean-Baptiste
Joseph Fourier
1768-1830

•How to understand waves
and
wave velocity V_{wave}

[RelaWavity Web Simulation](#)
[*Keyboard of the Gods*](#)
[*(*Dual Plot*)*](#)

Analyzing wave velocity by per-space-per-time *and* space-time graphs


Heinreich Hertz 1857-1894
$1Hz=1sec^{-1}$



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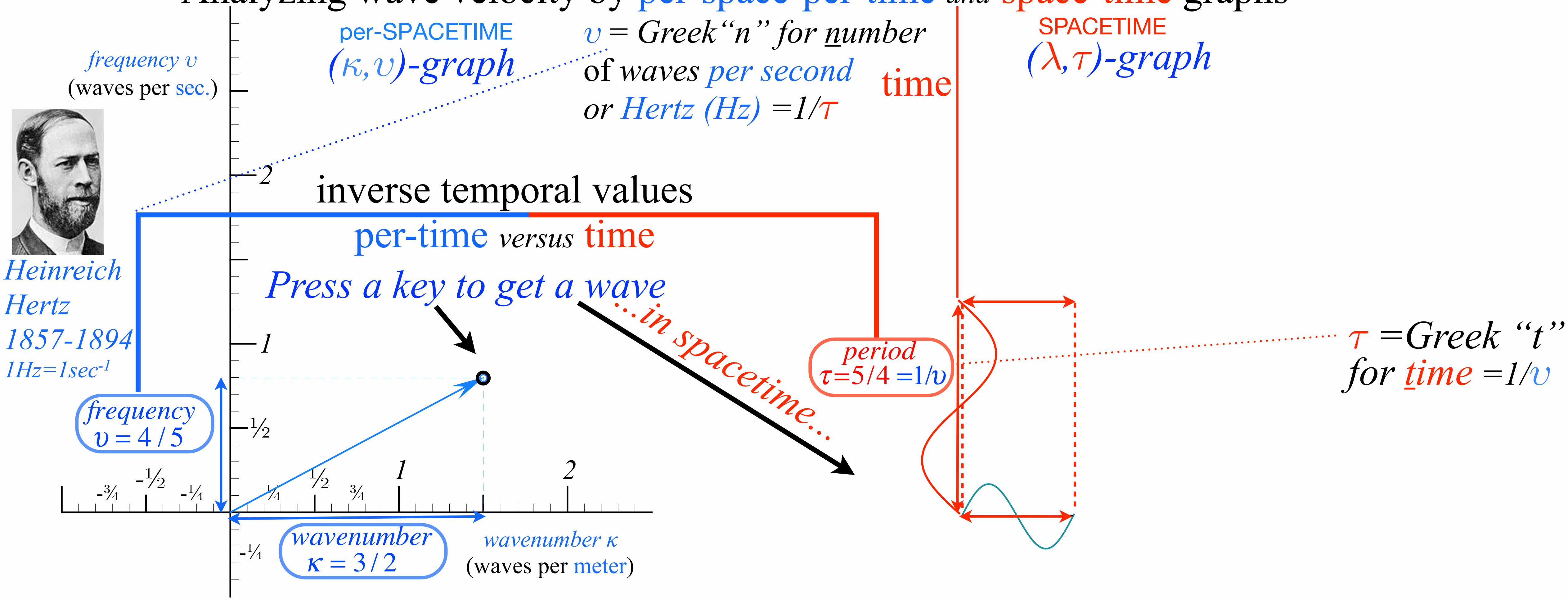

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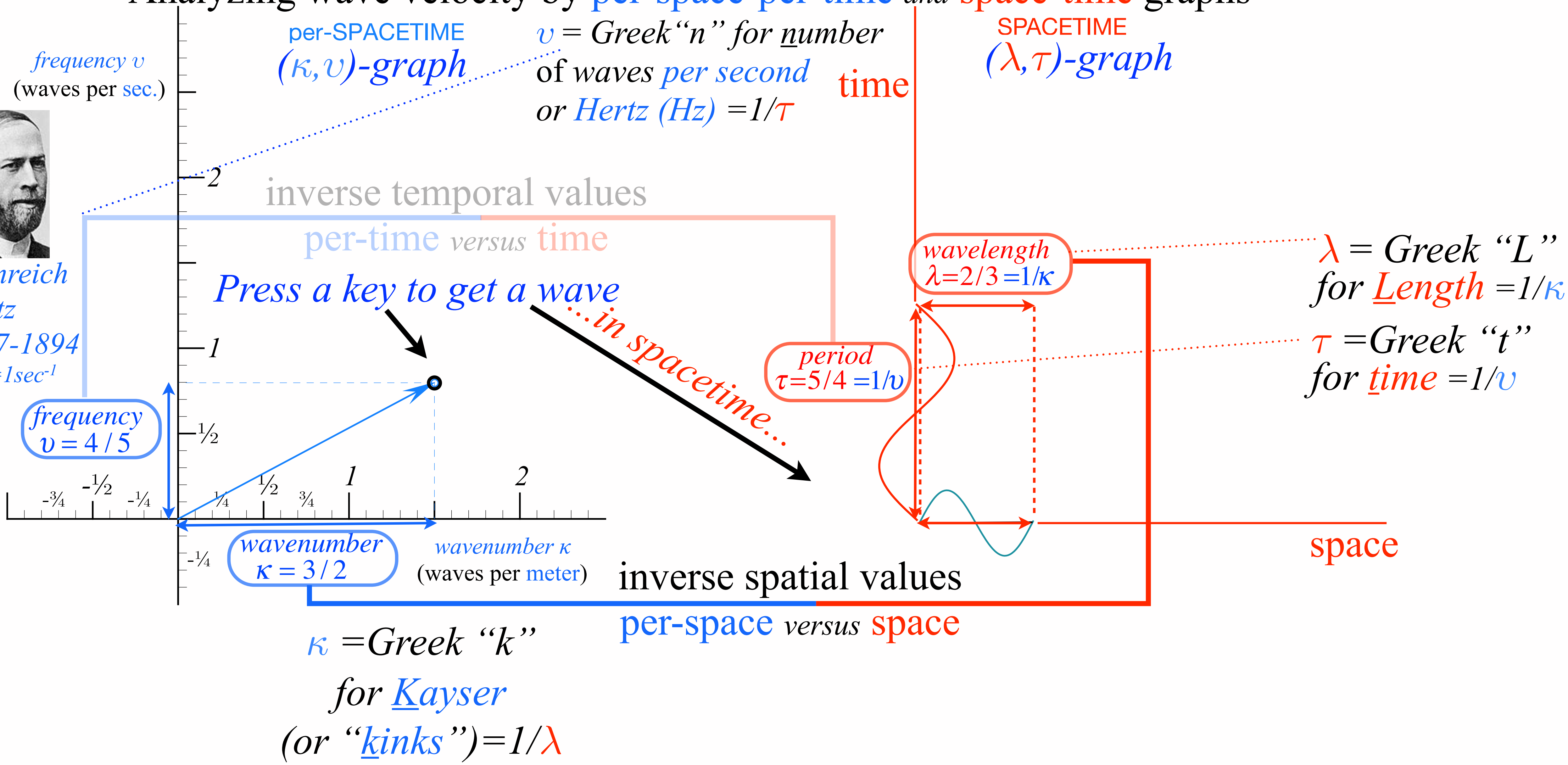

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1Hz=1sec⁻¹

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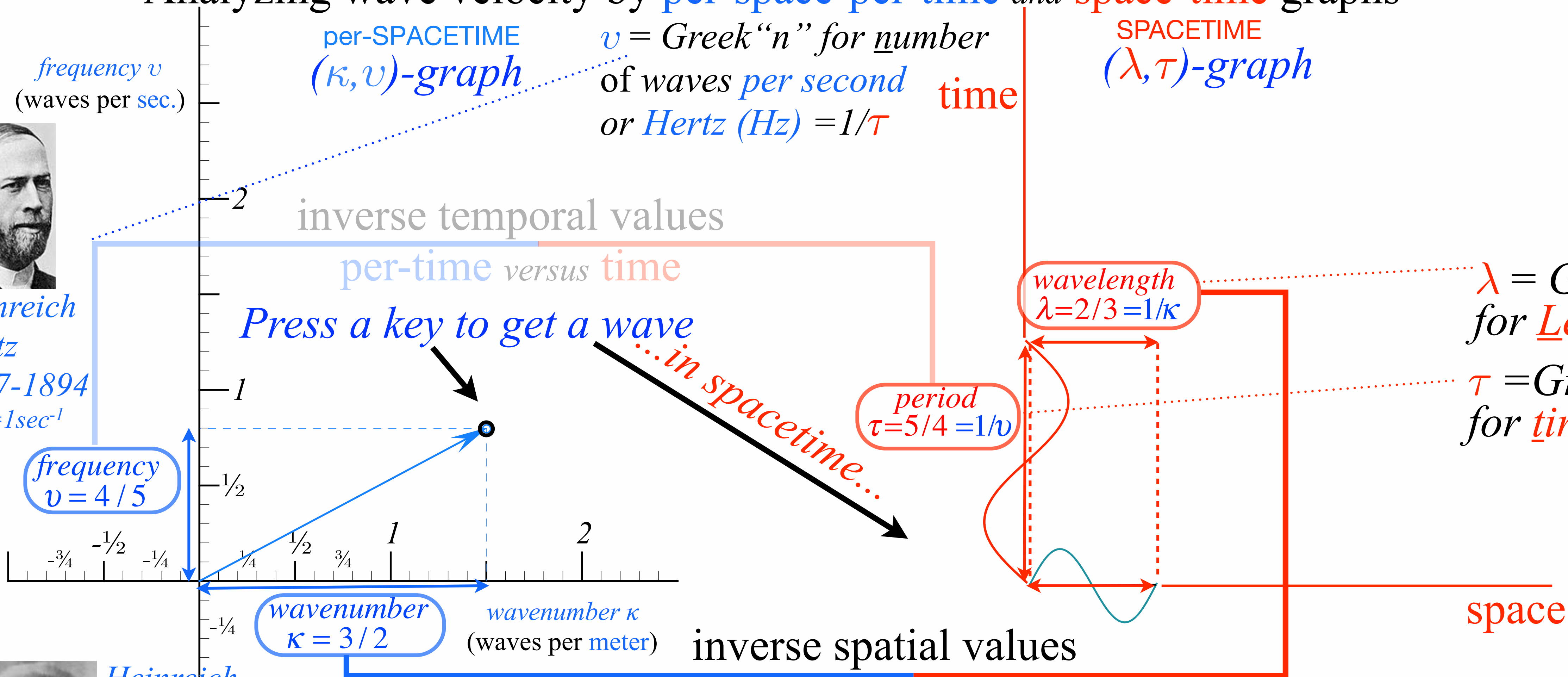

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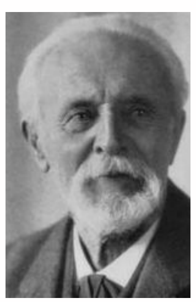

Heinreich Kayser 1853-1940
1Kayser=1cm⁻¹

κ =Greek "k"
for Kayser
(or "kinks")=1/λ

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•*How to understand waves and wave velocity* V_{wave}

RelaWavity Web Simulation
Keyboard of the Gods
(Dual Plot)

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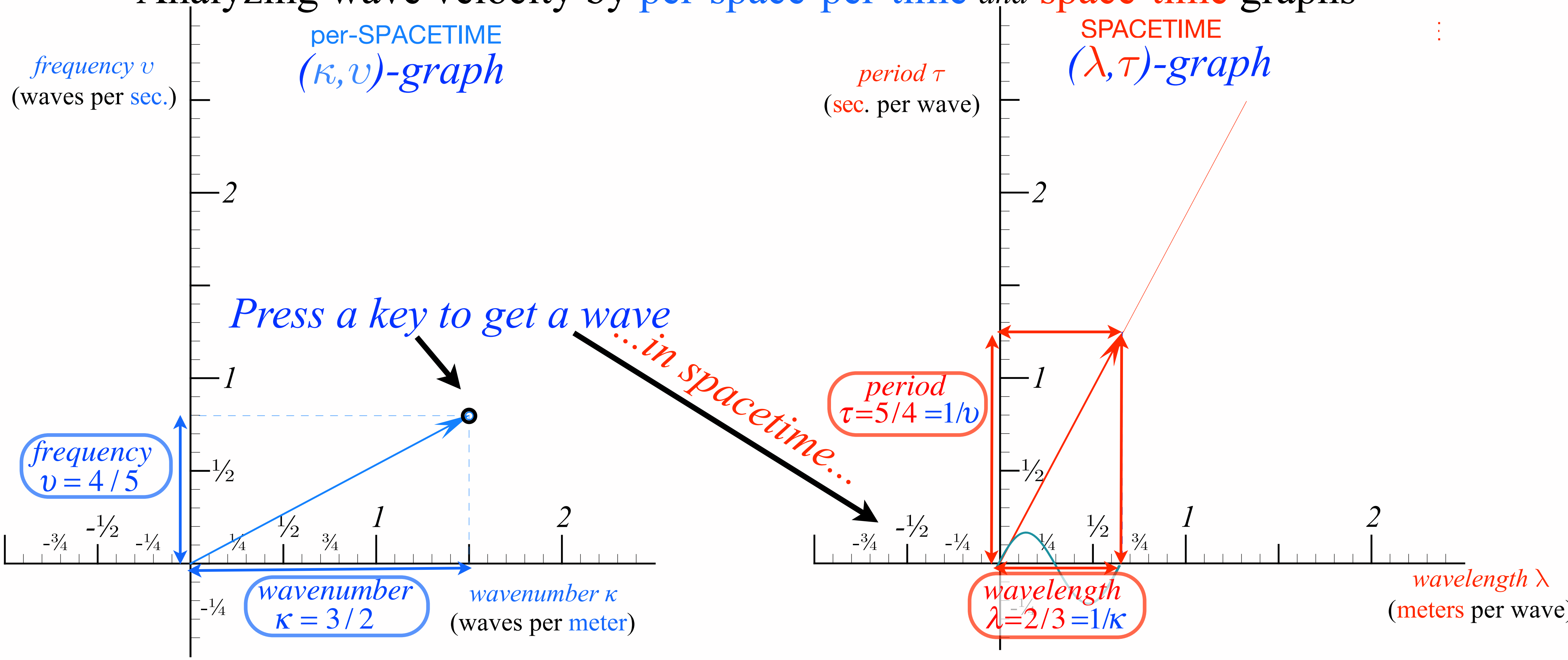

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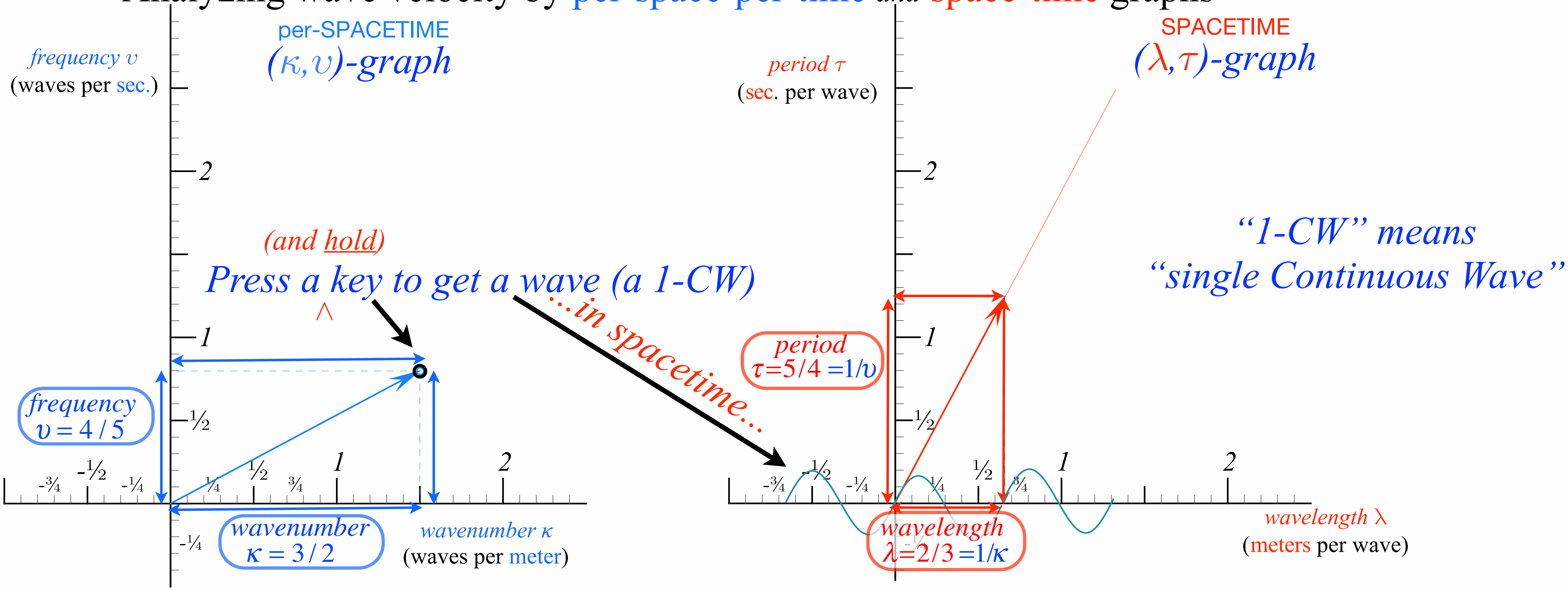

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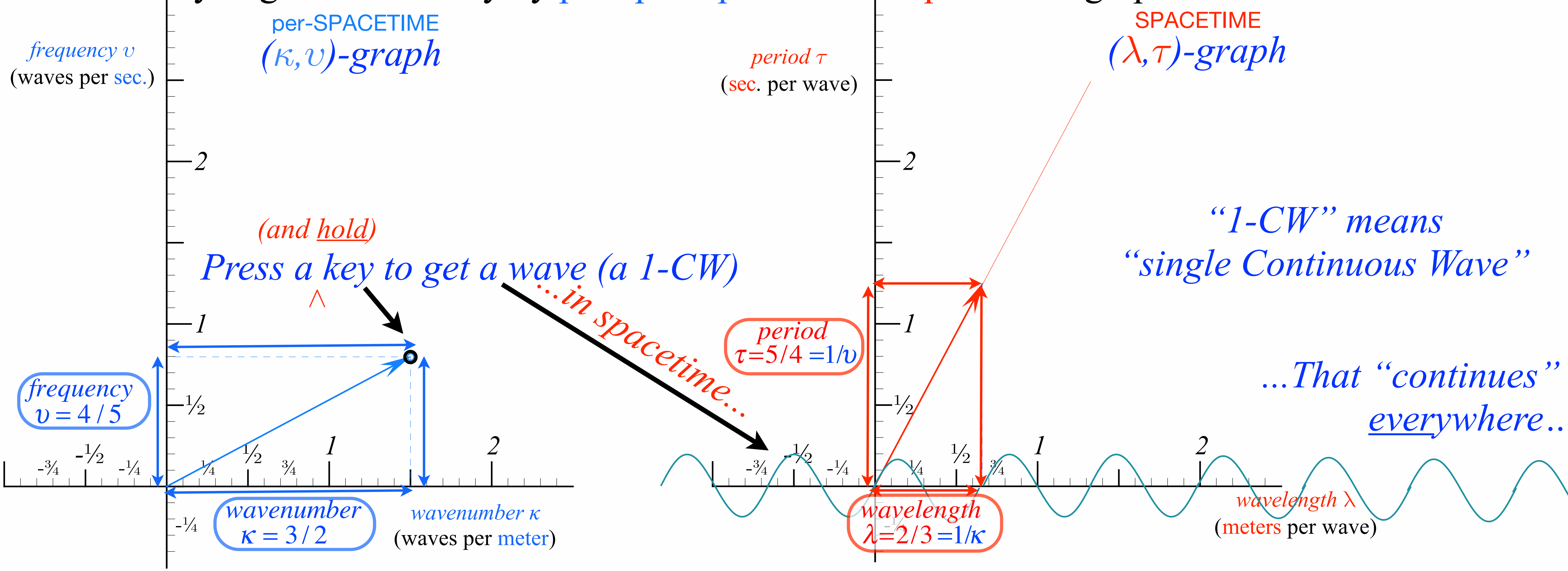

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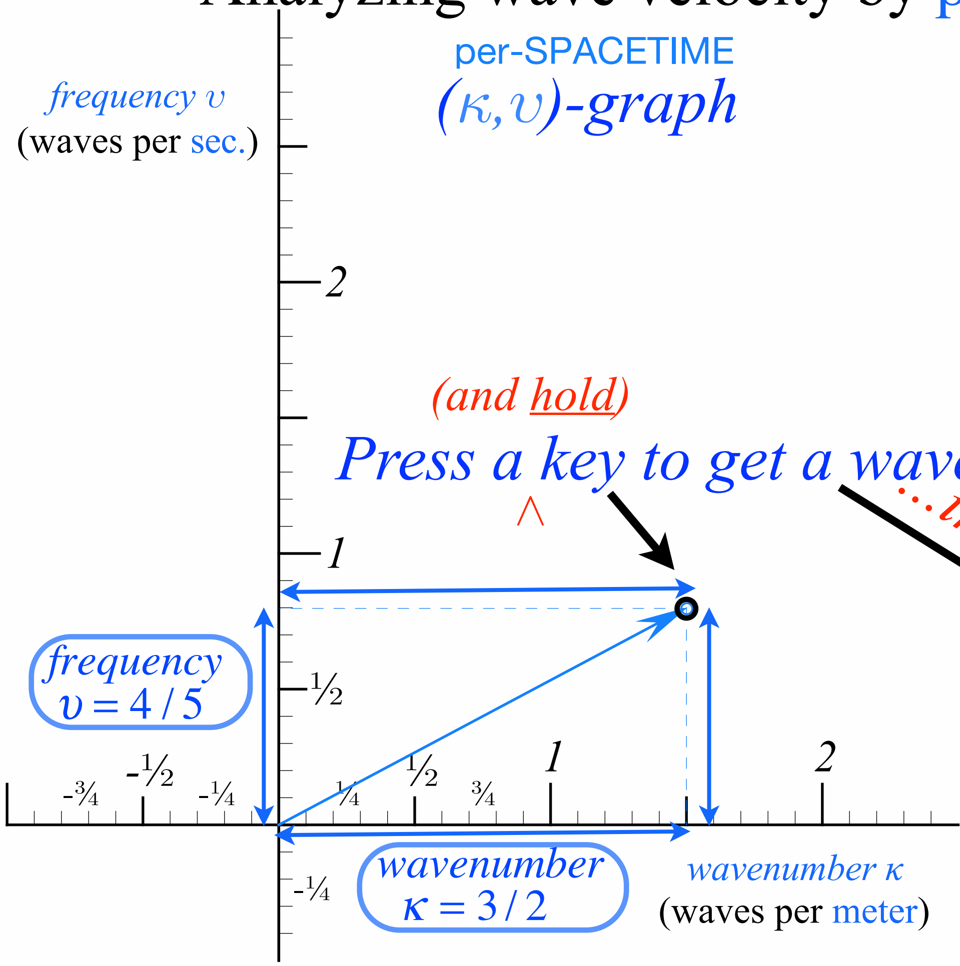

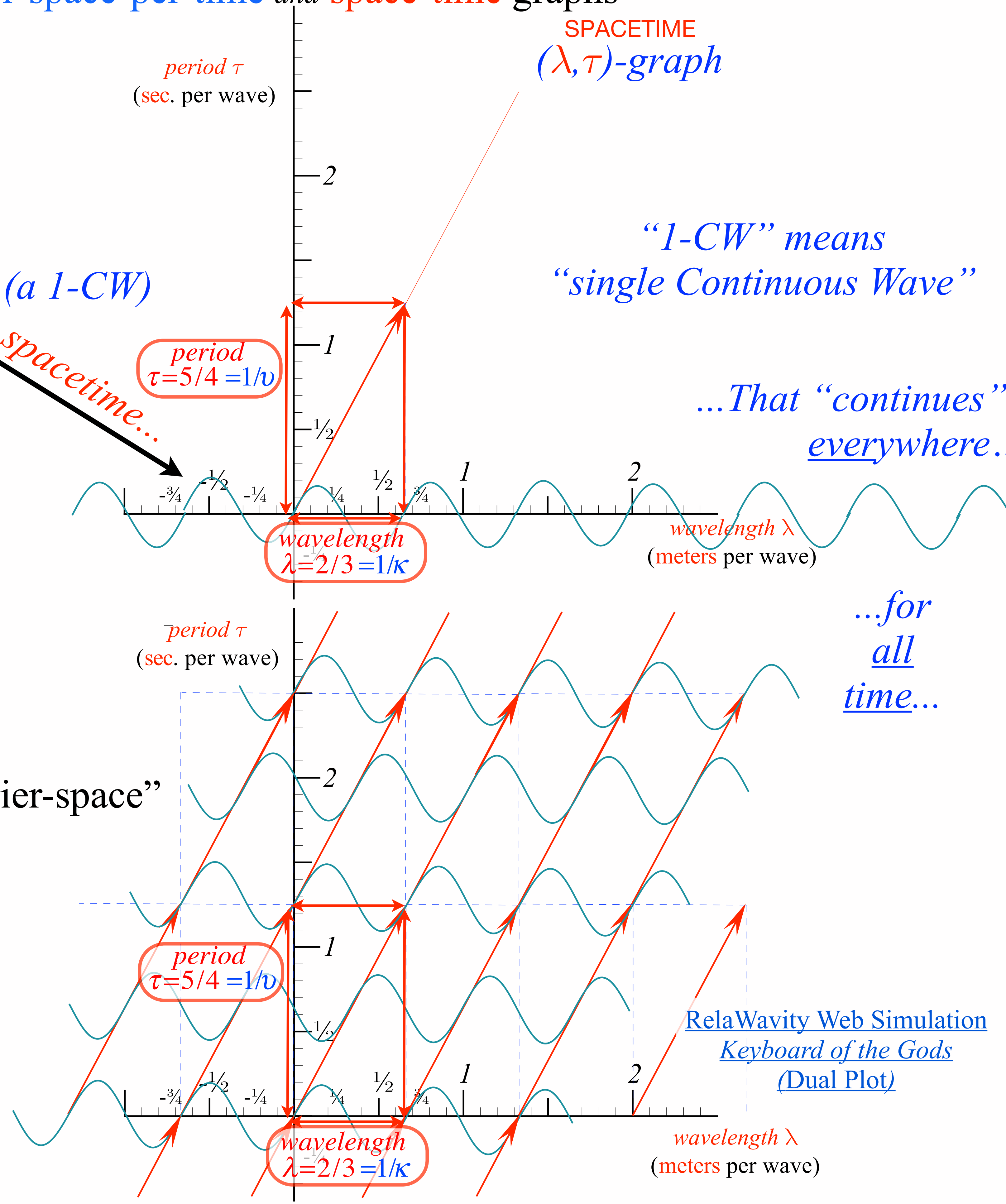

"1-CW" means "single Continuous Wave"

...That "continues" everywhere..

...for all time...

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RelaWavity Web Simulation
Keyboard of the Gods (Dual Plot)

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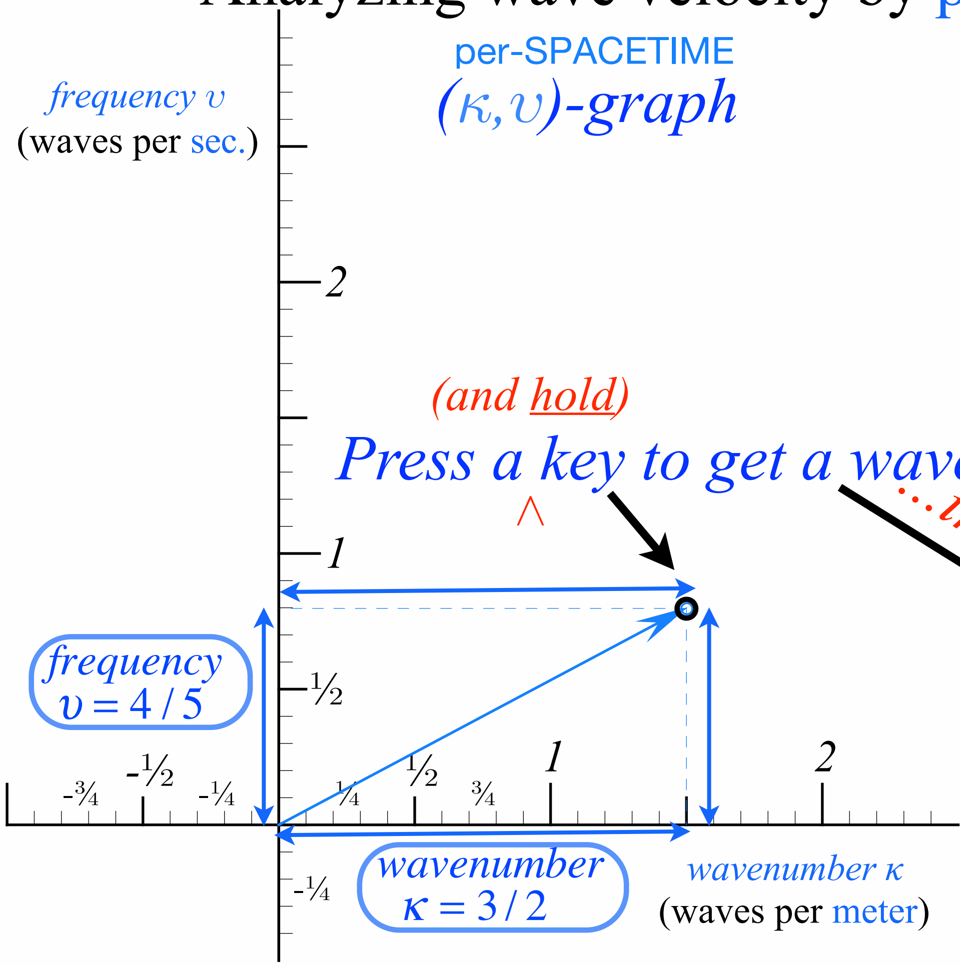
Analyzing wave velocity by per-space-per-time *and* space-time graphs

per-SPACETIME

(κ,υ)-graph



(and hold)

Press a key to get a wave (a 1-CW)

...in spacetime...

SPACETIME

(λ,τ)-graph

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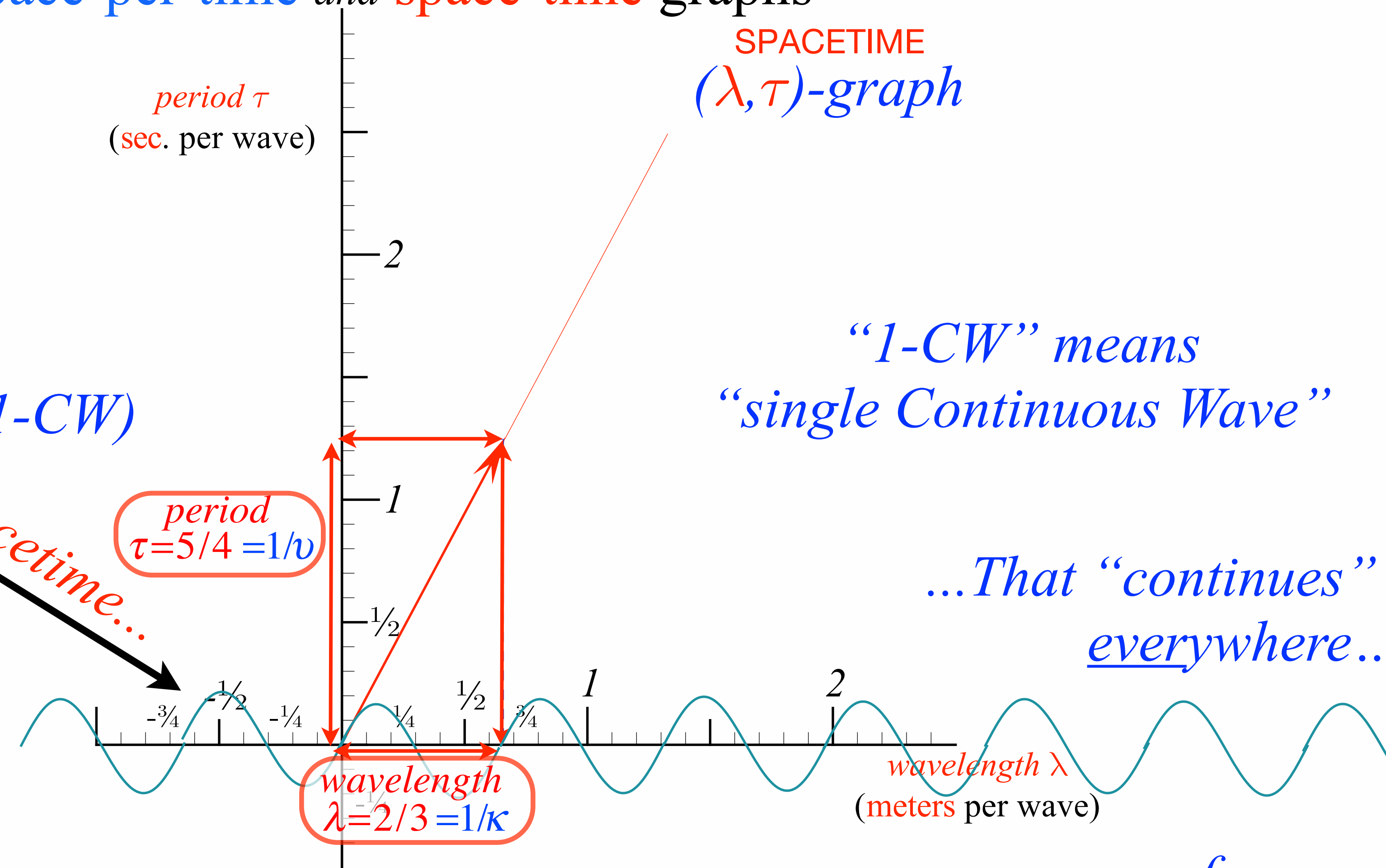

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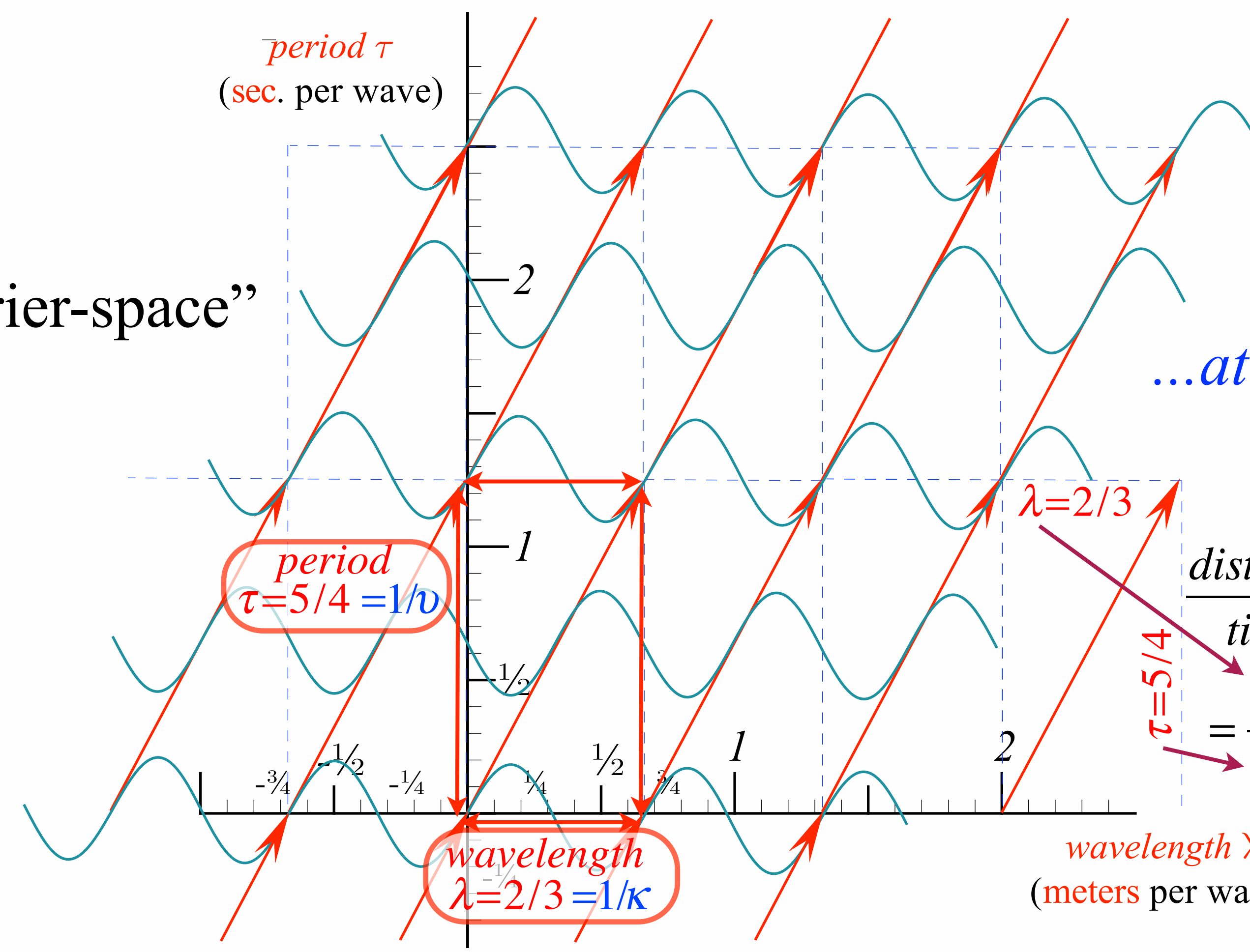

...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau}$$

•How to understand waves and wave velocity V_{wave}

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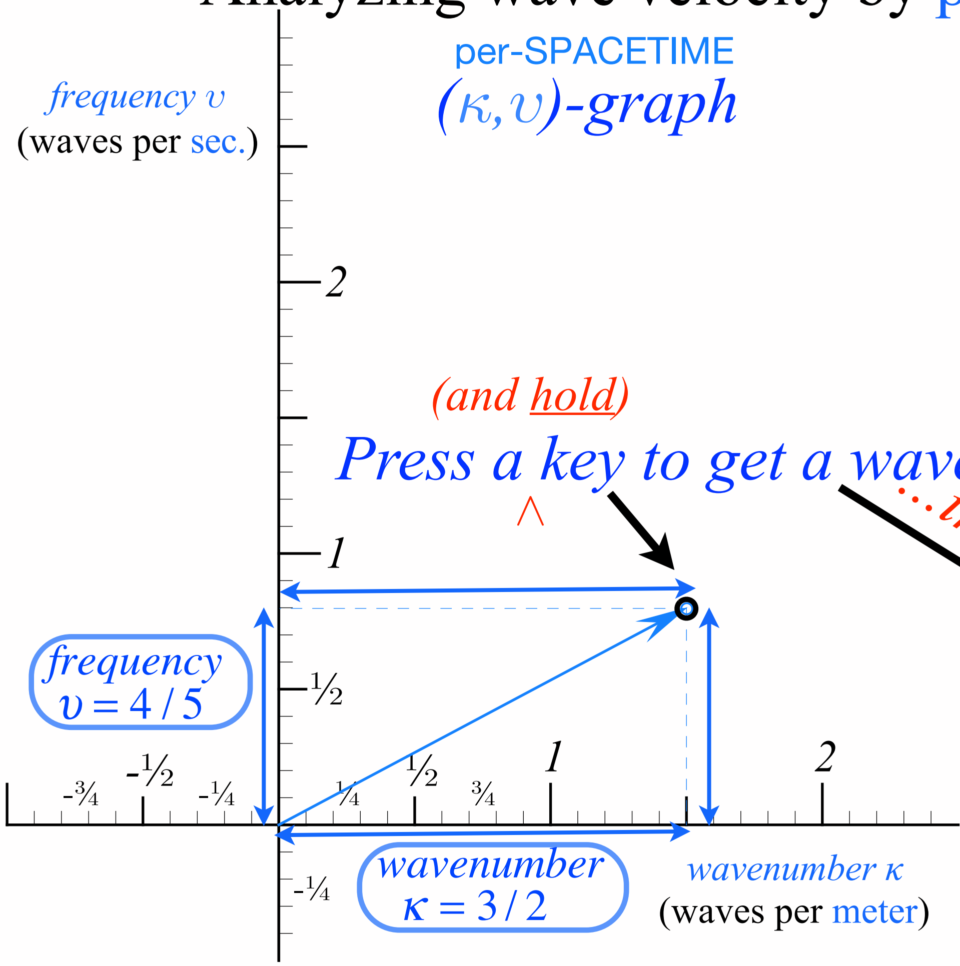

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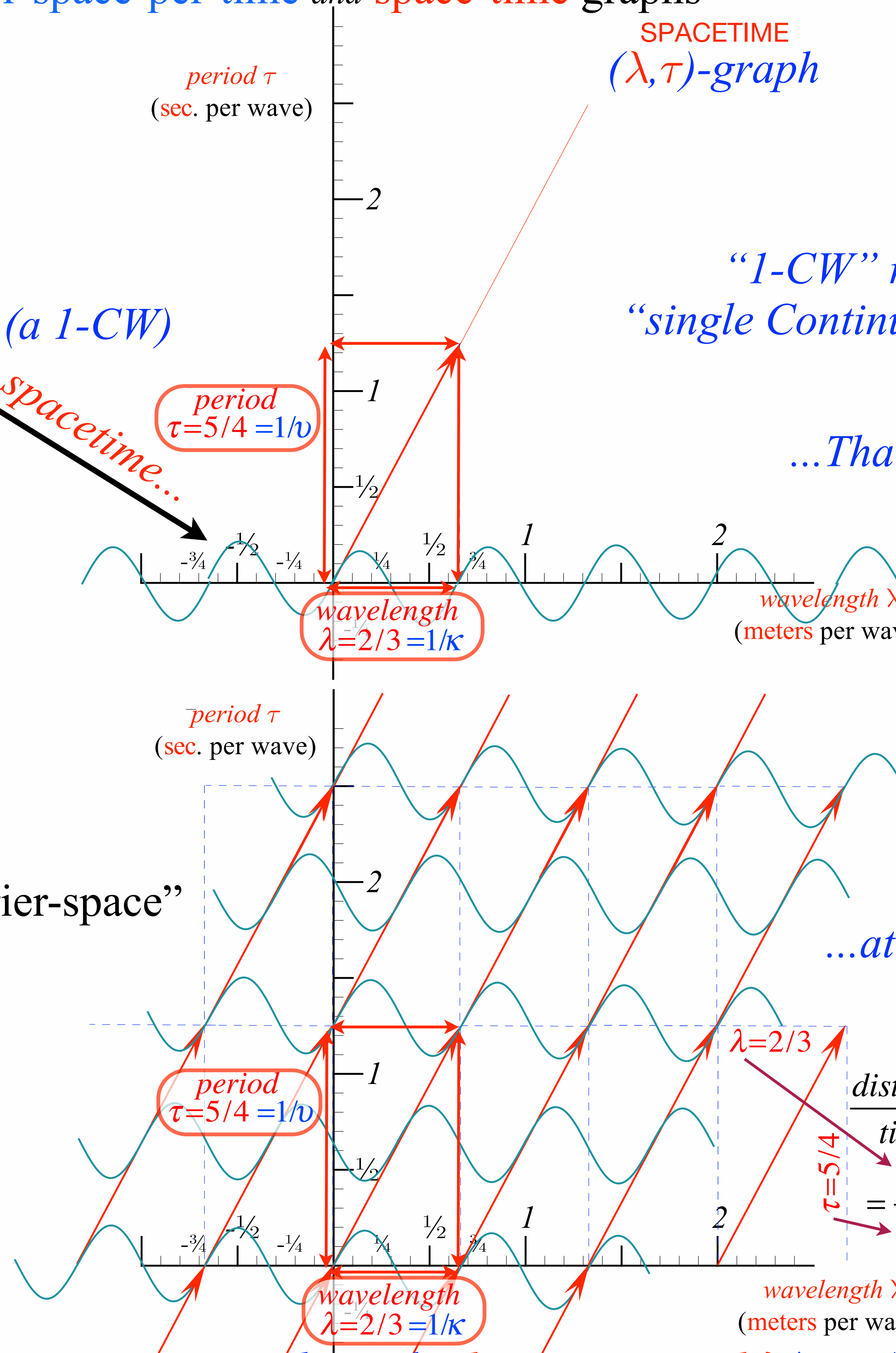

"1-CW" means "single Continuous Wave"

...That "continues" everywhere..

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...at a speed of:

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\,\frac{m.}{s.}$$

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•How to understand waves and wave velocity V_{wave}

wave-speed equals slope-to-vertical λ/τ in (λ,τ)-graph

Learning about sin (and cos…tan…and trig. road maps)
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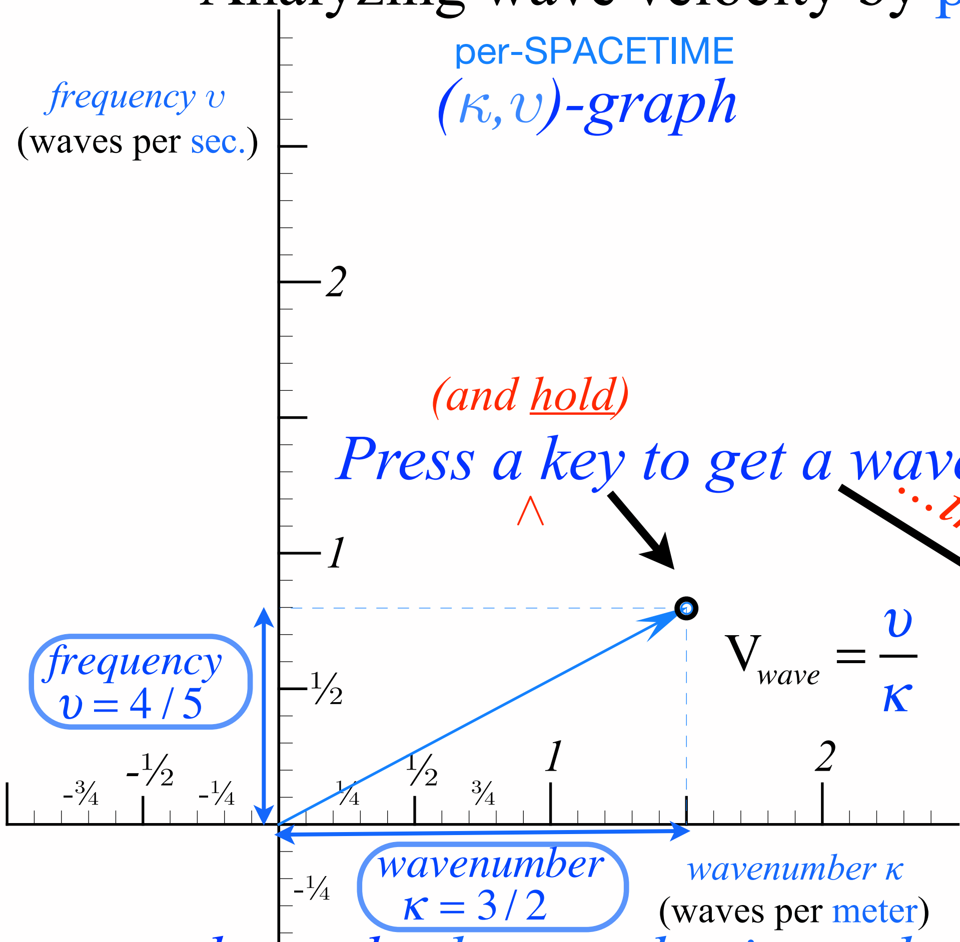
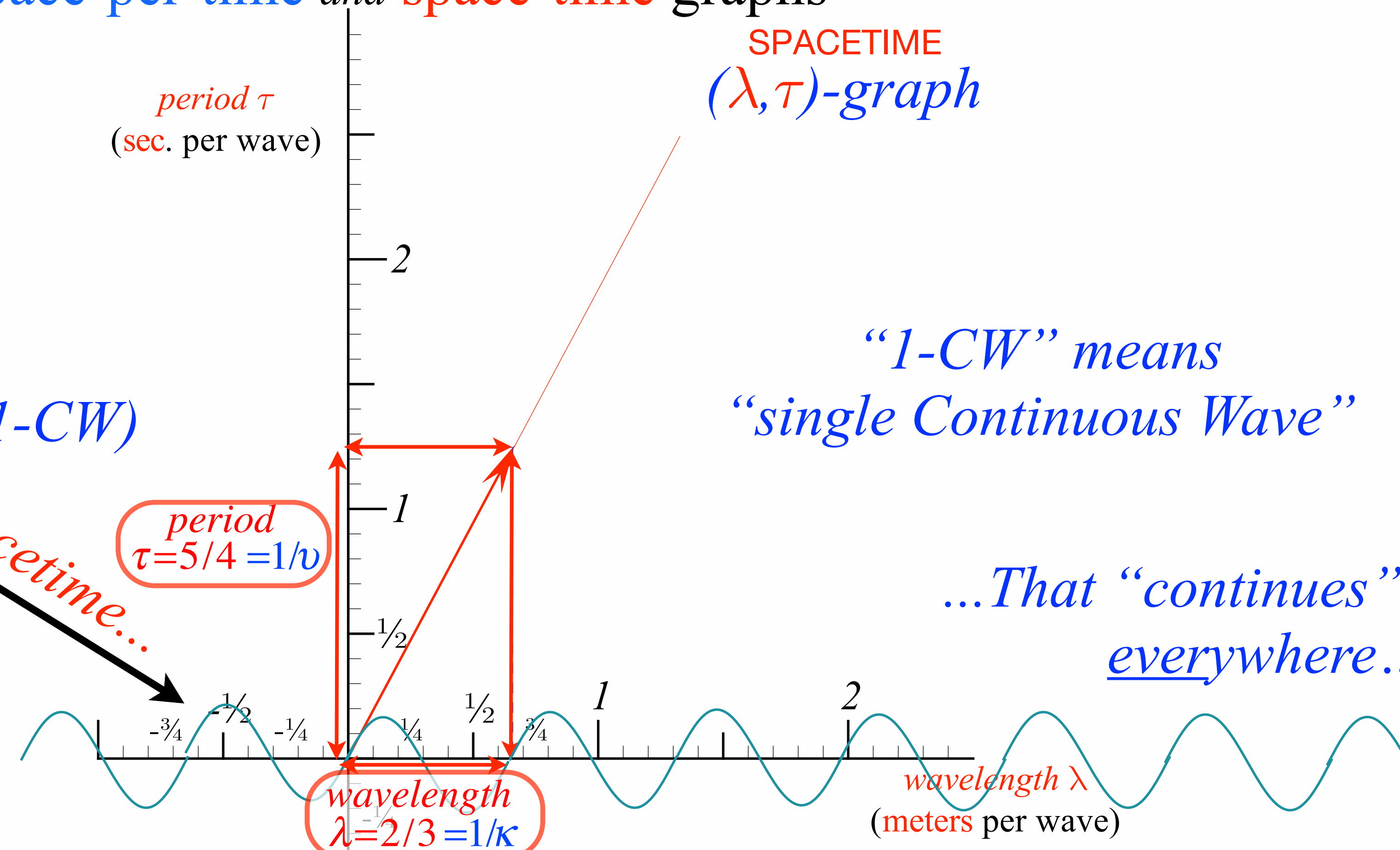


wave-speed equals slope-to-horizontal υ/κ in (κ,υ)-graph



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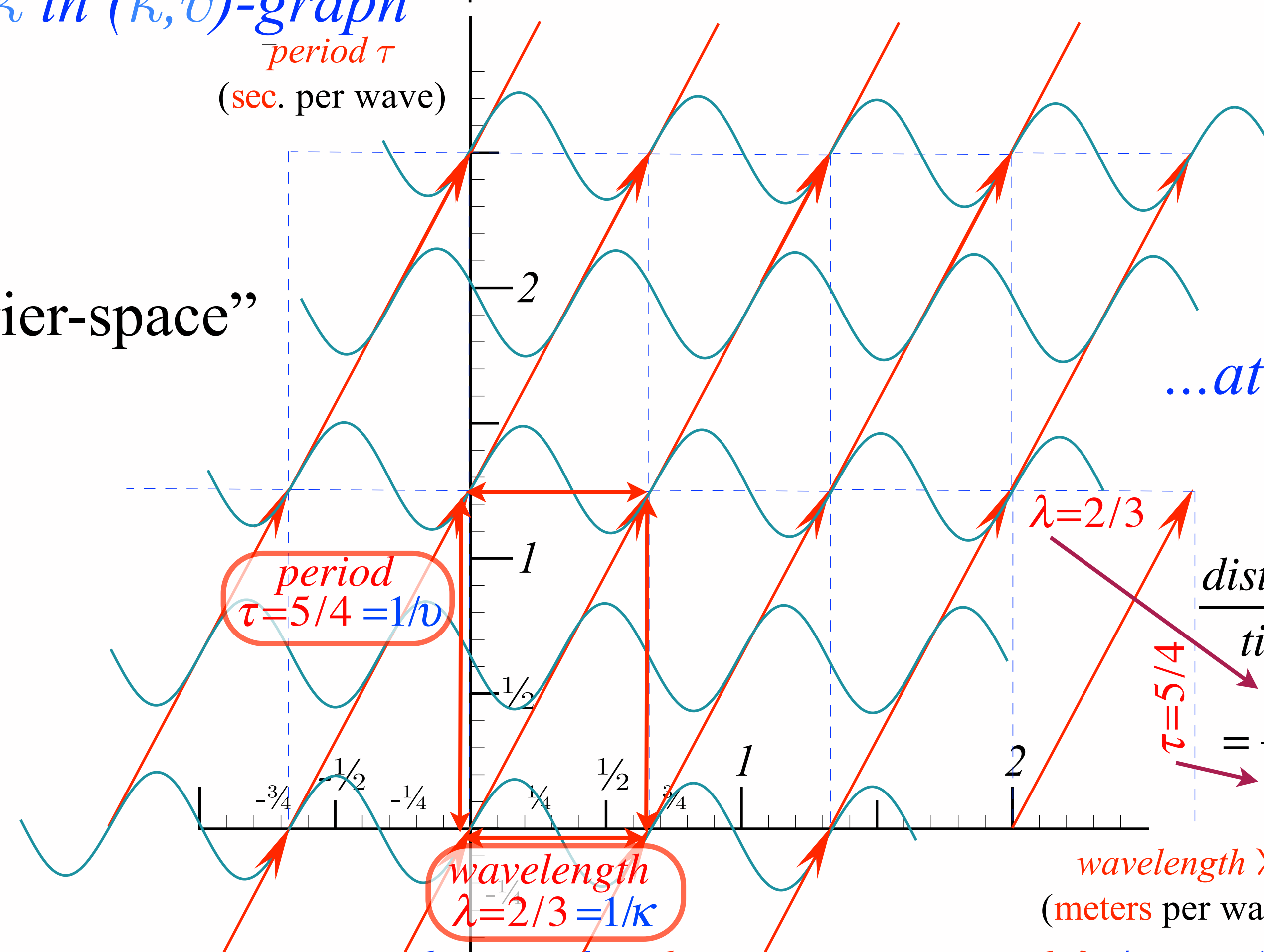

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$$\frac{\text{distance}}{\text{time}} = \frac{\text{wavelength}}{\text{period}} = \frac{\lambda}{\tau} = \frac{2/3}{5/4} = \frac{8}{15}\frac{m.}{s.} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa}$$

•*How to understand waves*
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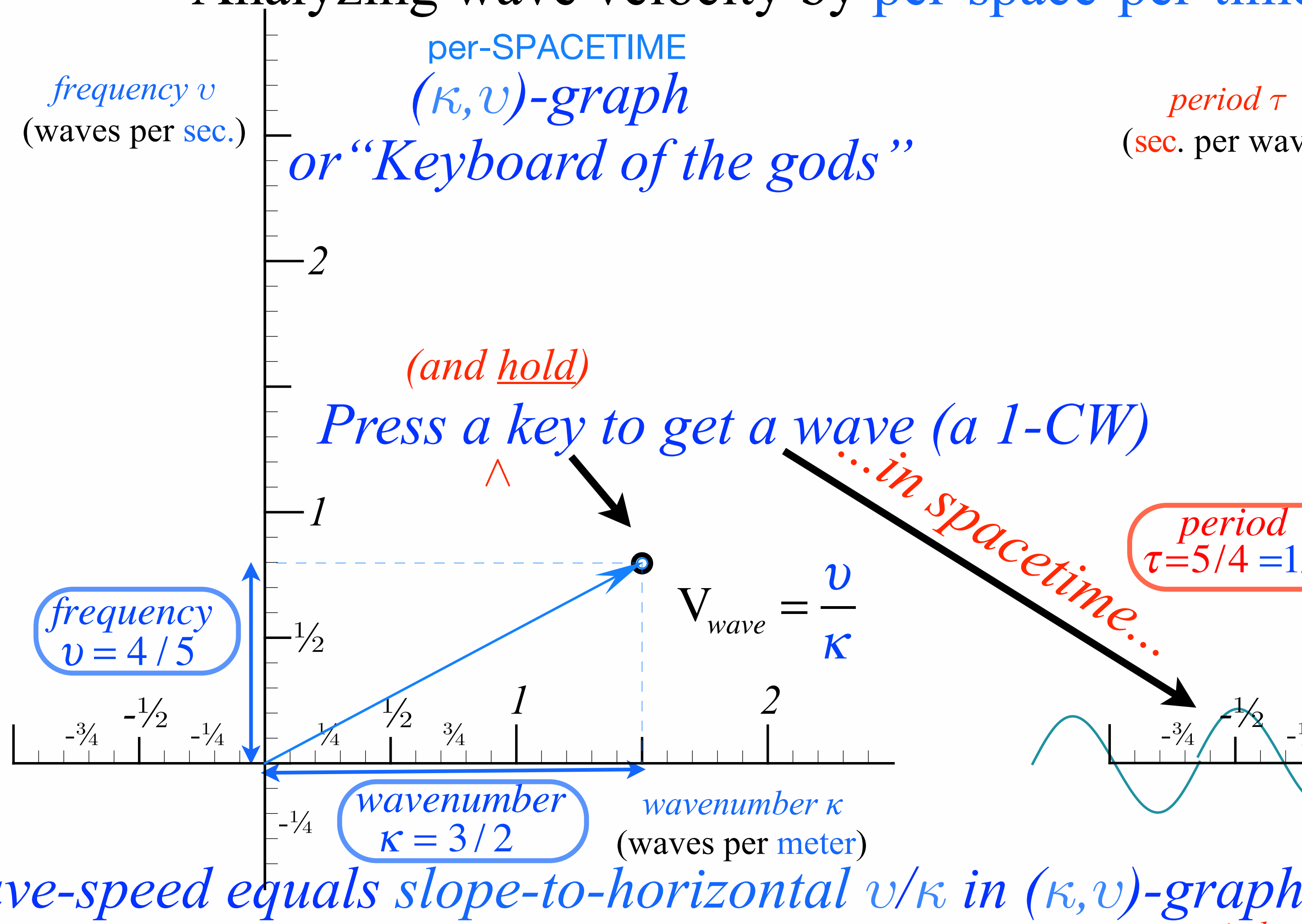

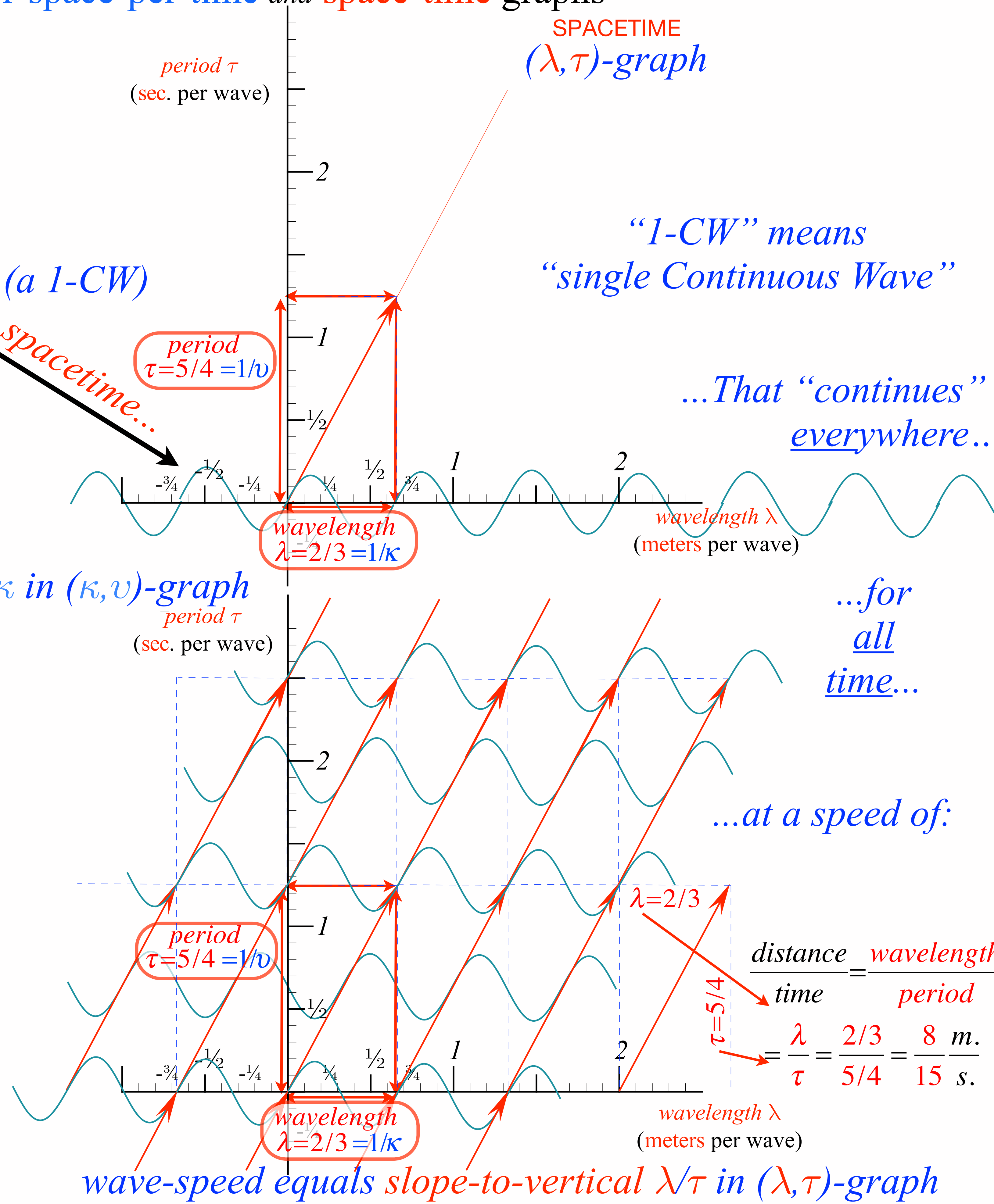

wave-velocity formulas

$$\frac{distance}{time} = \frac{wavelength}{period} = \frac{frequency}{wavenumber}$$

$$V_{wave} = \frac{\lambda}{\tau} = \frac{1/\kappa}{1/\upsilon} = \frac{\upsilon}{\kappa} = \frac{1/\tau}{1/\lambda}$$

$$= \frac{2/3}{5/4} = \frac{4/5}{3/2} = \frac{8}{15} \frac{m.}{s.}$$

wave arithmetic is simpler to explain using fractions

- *How to understand waves and "1st quantization"*

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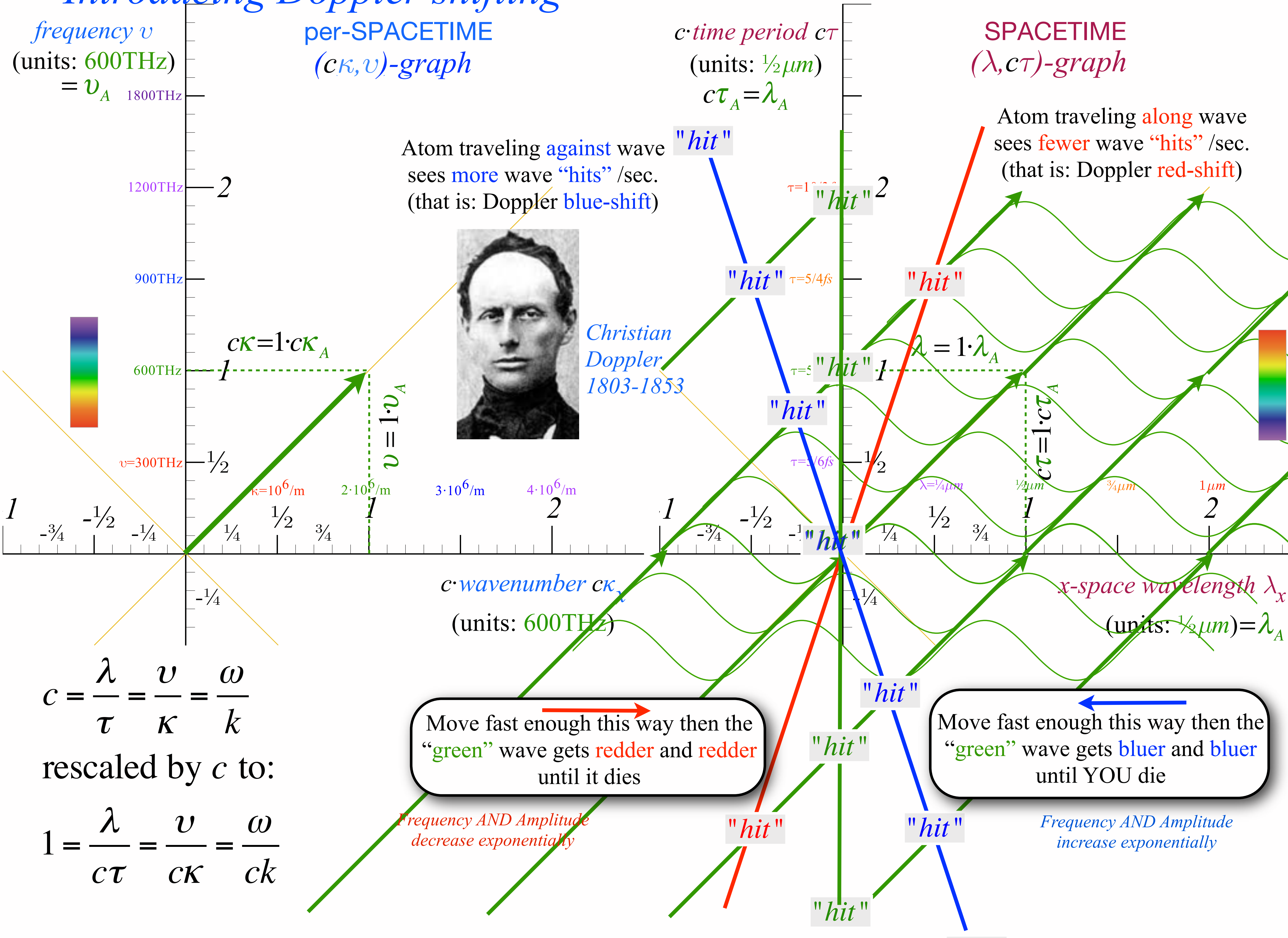

$$c = \frac{\lambda}{\tau} = \frac{\upsilon}{\kappa} = \frac{\omega}{k}$$

rescaled by c to:

$$1 = \frac{\lambda}{c\tau} = \frac{\upsilon}{c\kappa} = \frac{\omega}{ck}$$

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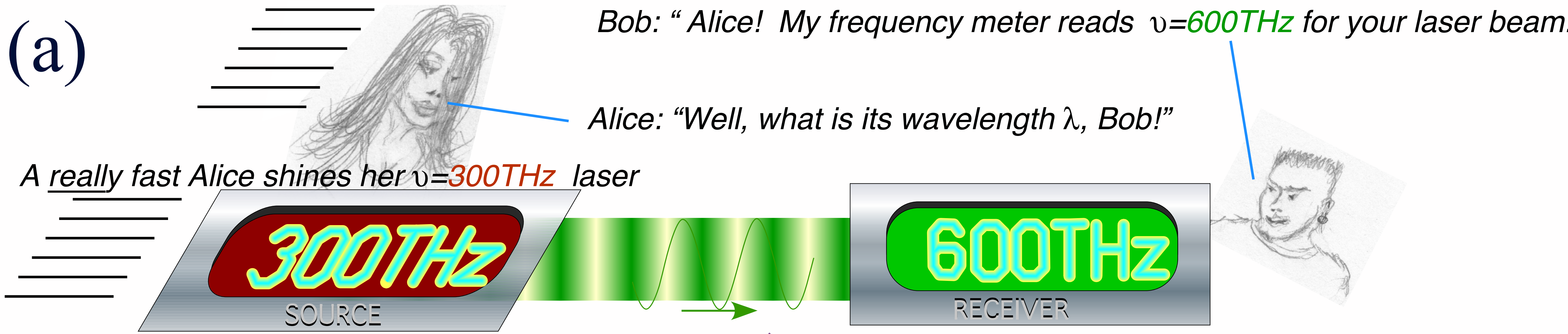
Introducing Doppler shifting and why c is *really* constant



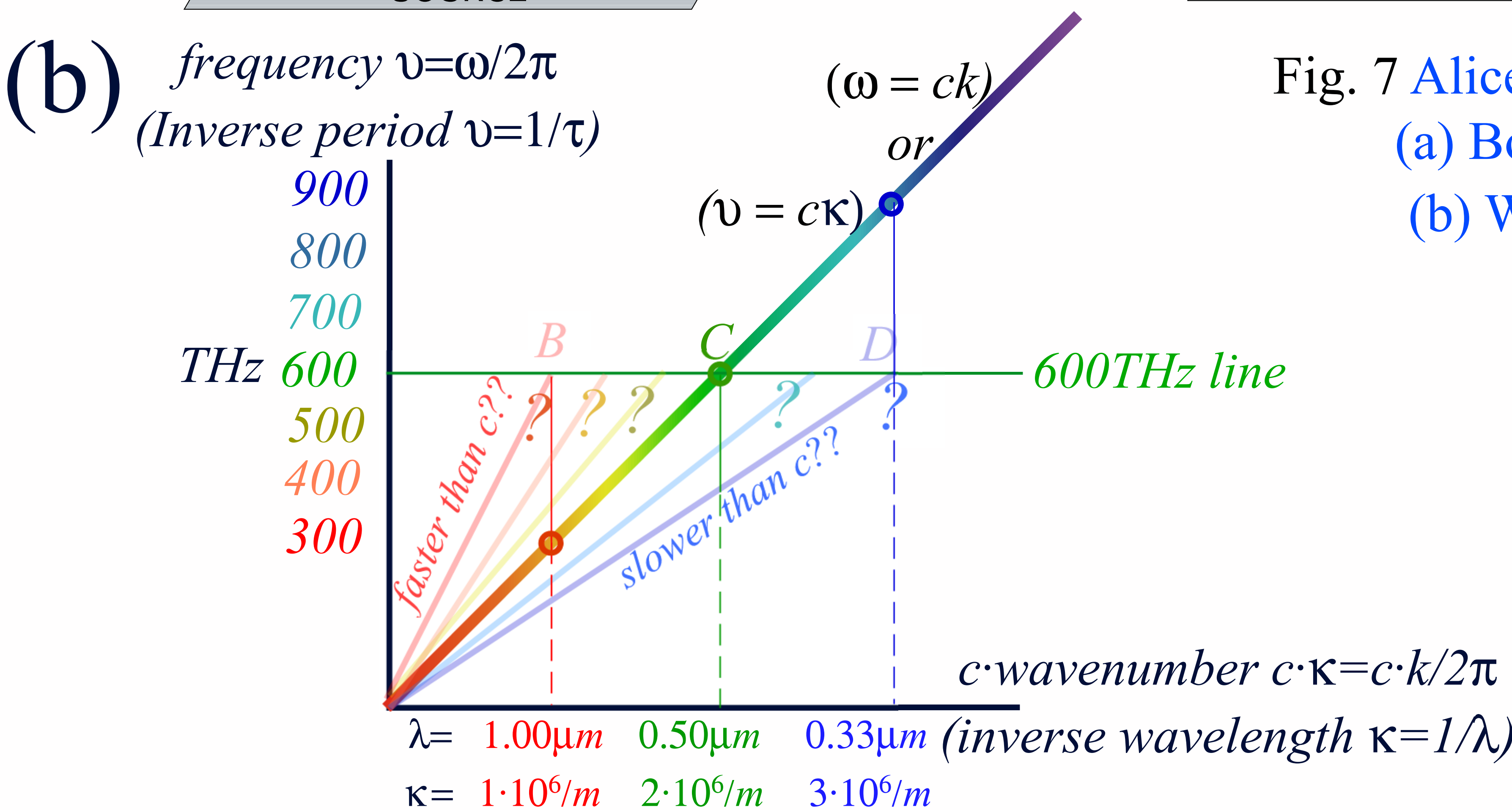
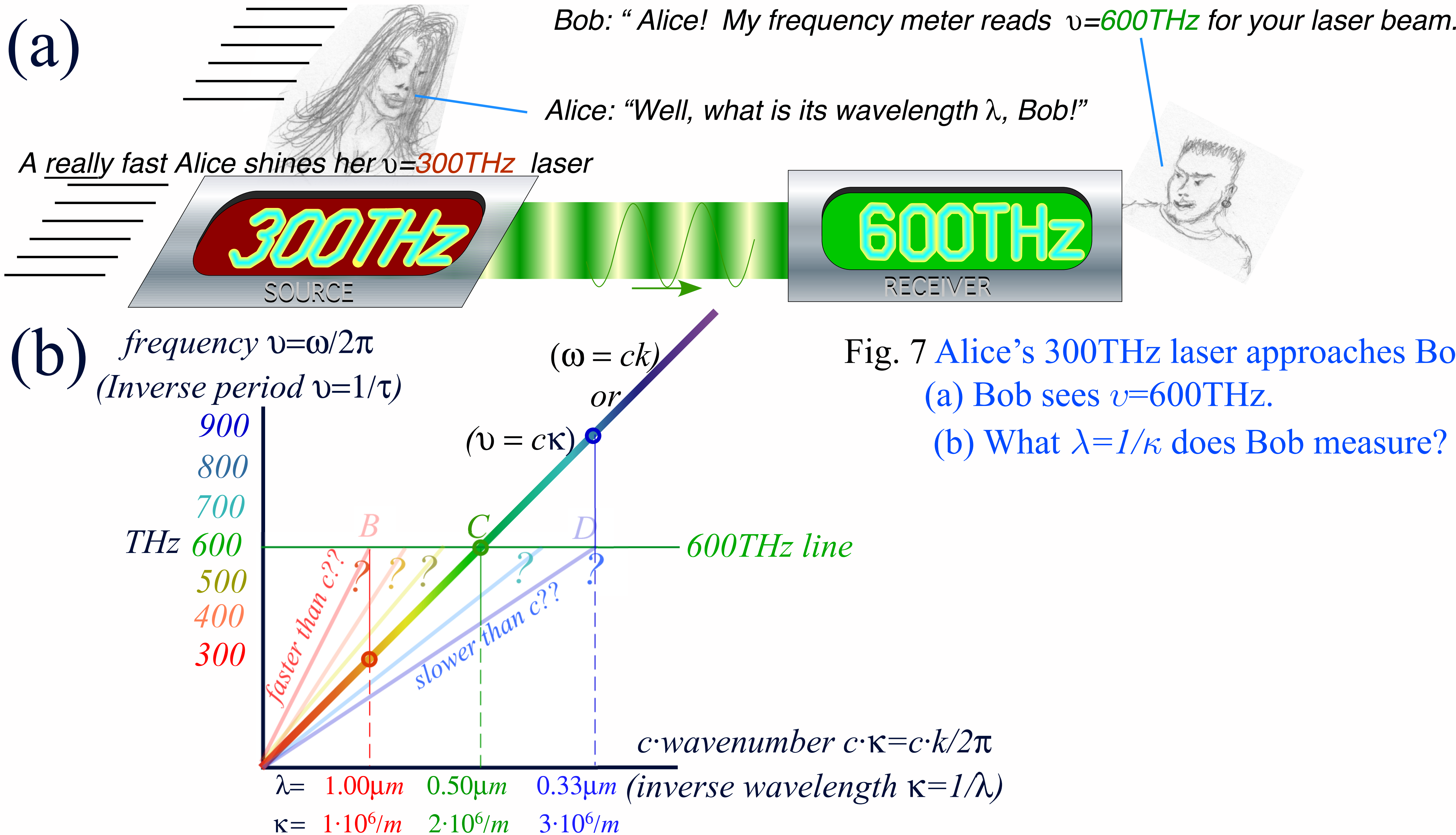


Fig. 7 Alice's 300THz laser approaches Bob.
(a) Bob sees υ=600THz.
(b) What $\lambda=1/\kappa$ does Bob measure?

Introducing Doppler shifting and why c is *really* constant



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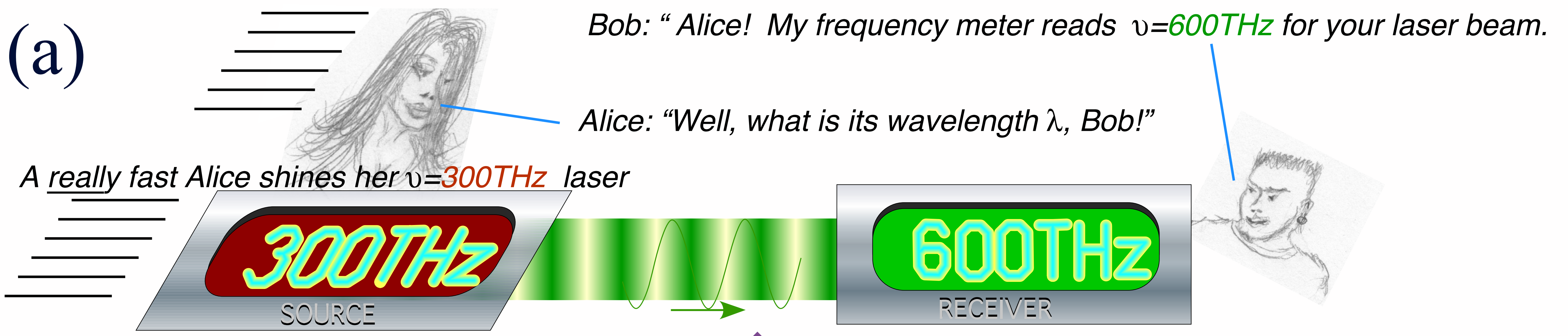
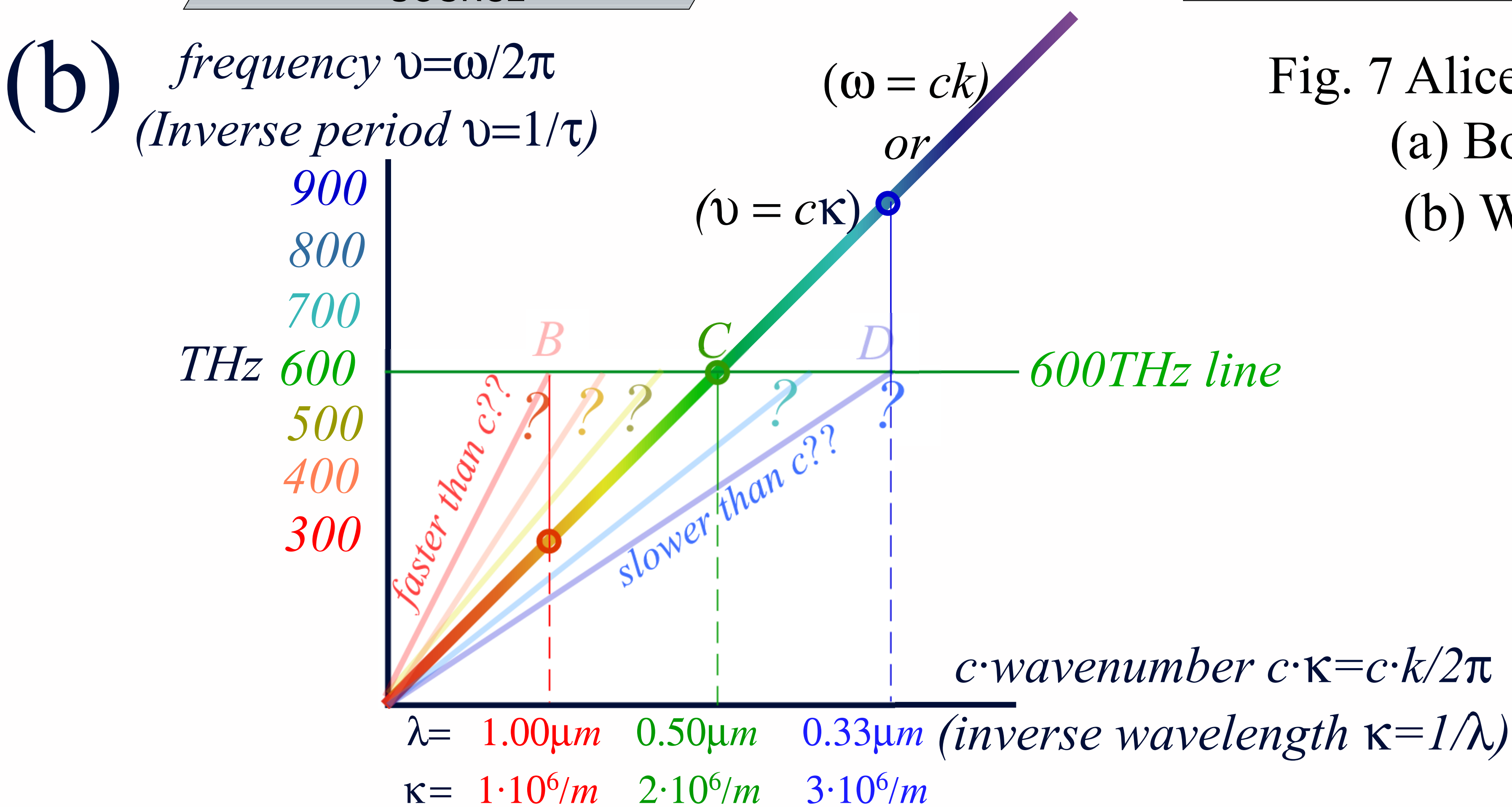
Years of spectroscopy rule out 'phony' 600THz blue-green that do not have wavelength λ=0.5*micron*.
The only choice is *C*.

Introducing Doppler shifting and why c is *really* constant





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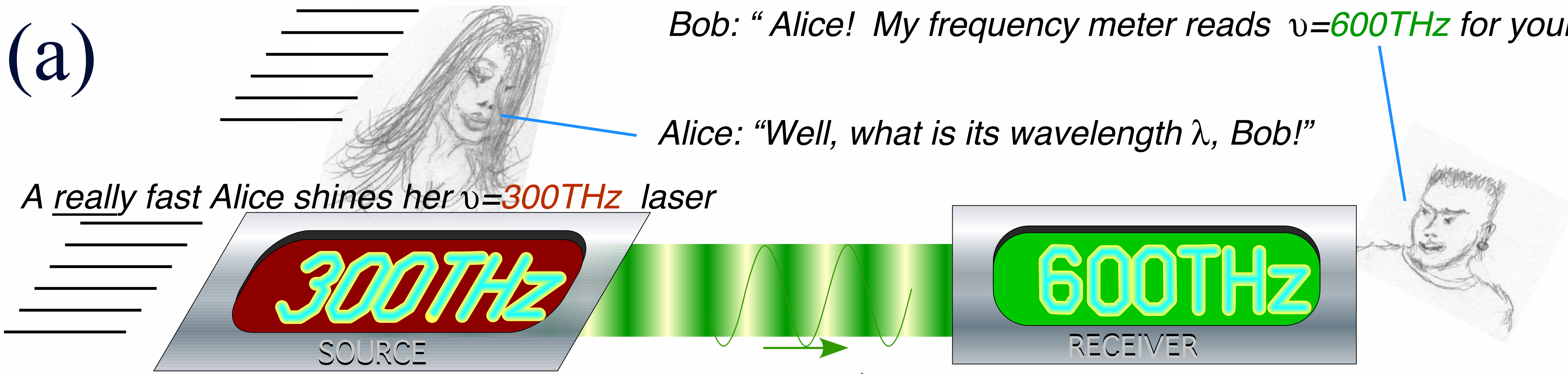
Introducing Doppler shifting and why c is *really* constant

(a)



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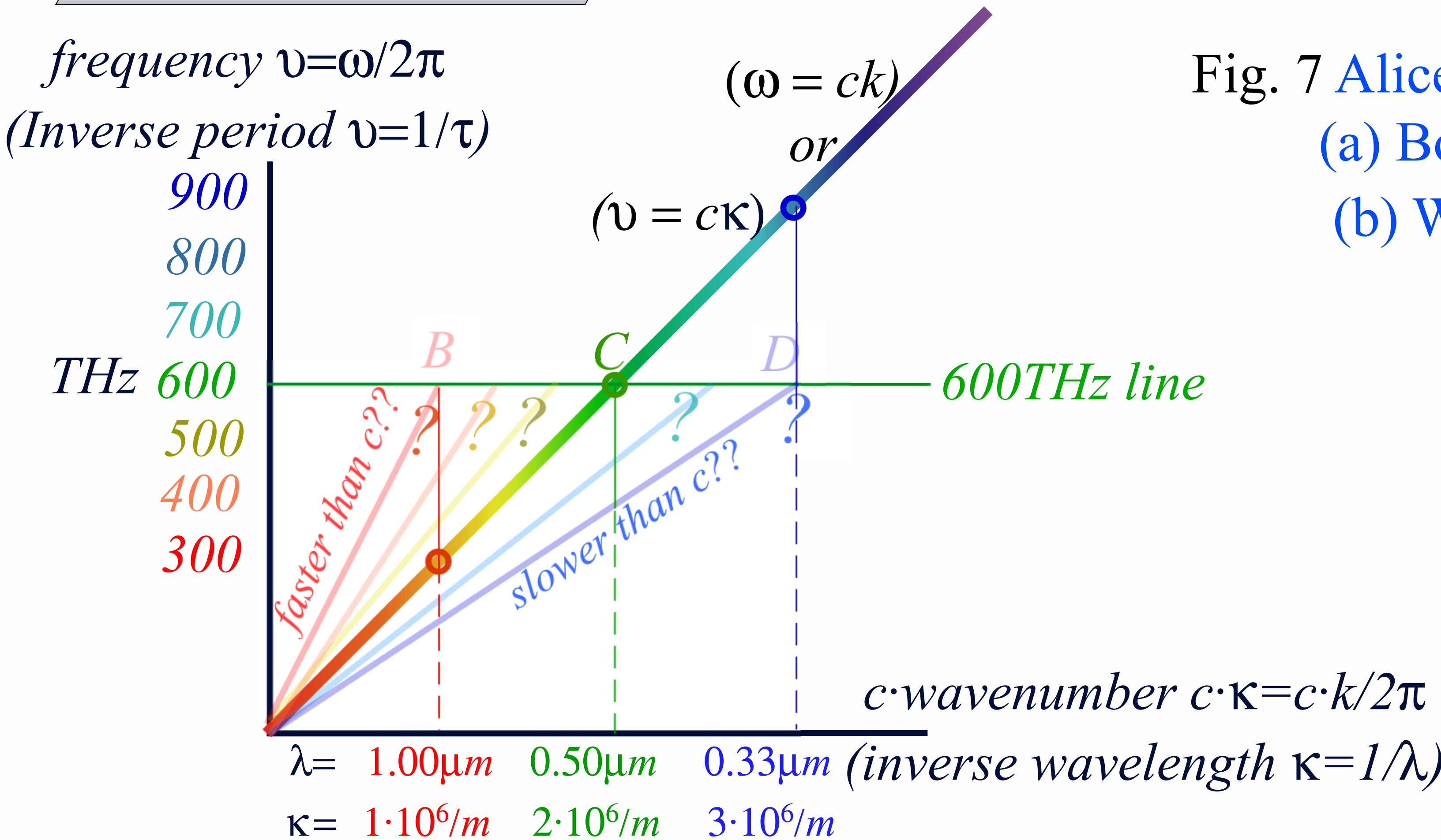

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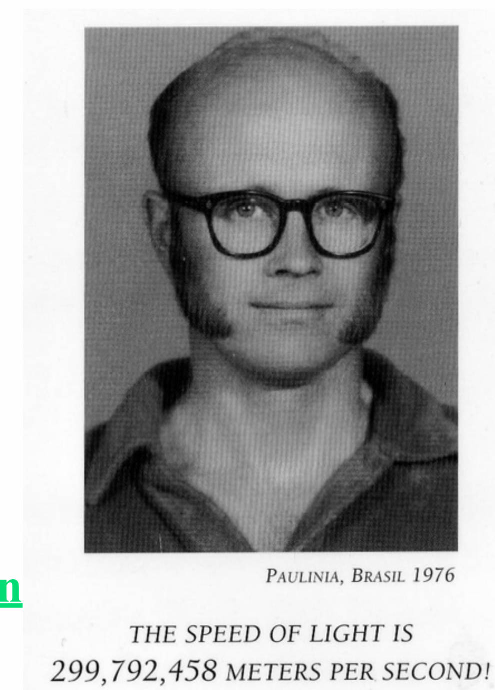

Ken Evenson
1932-2002

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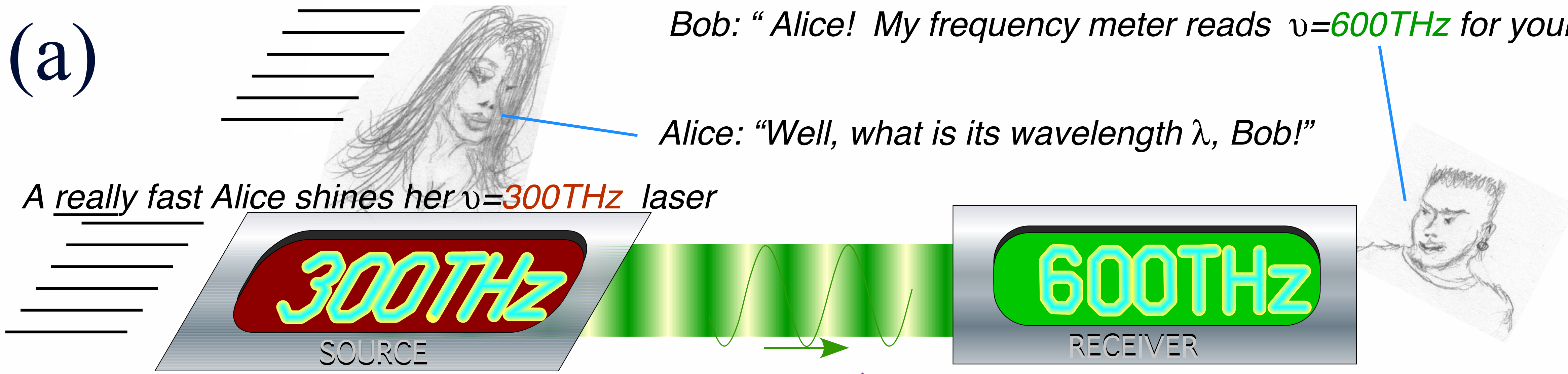
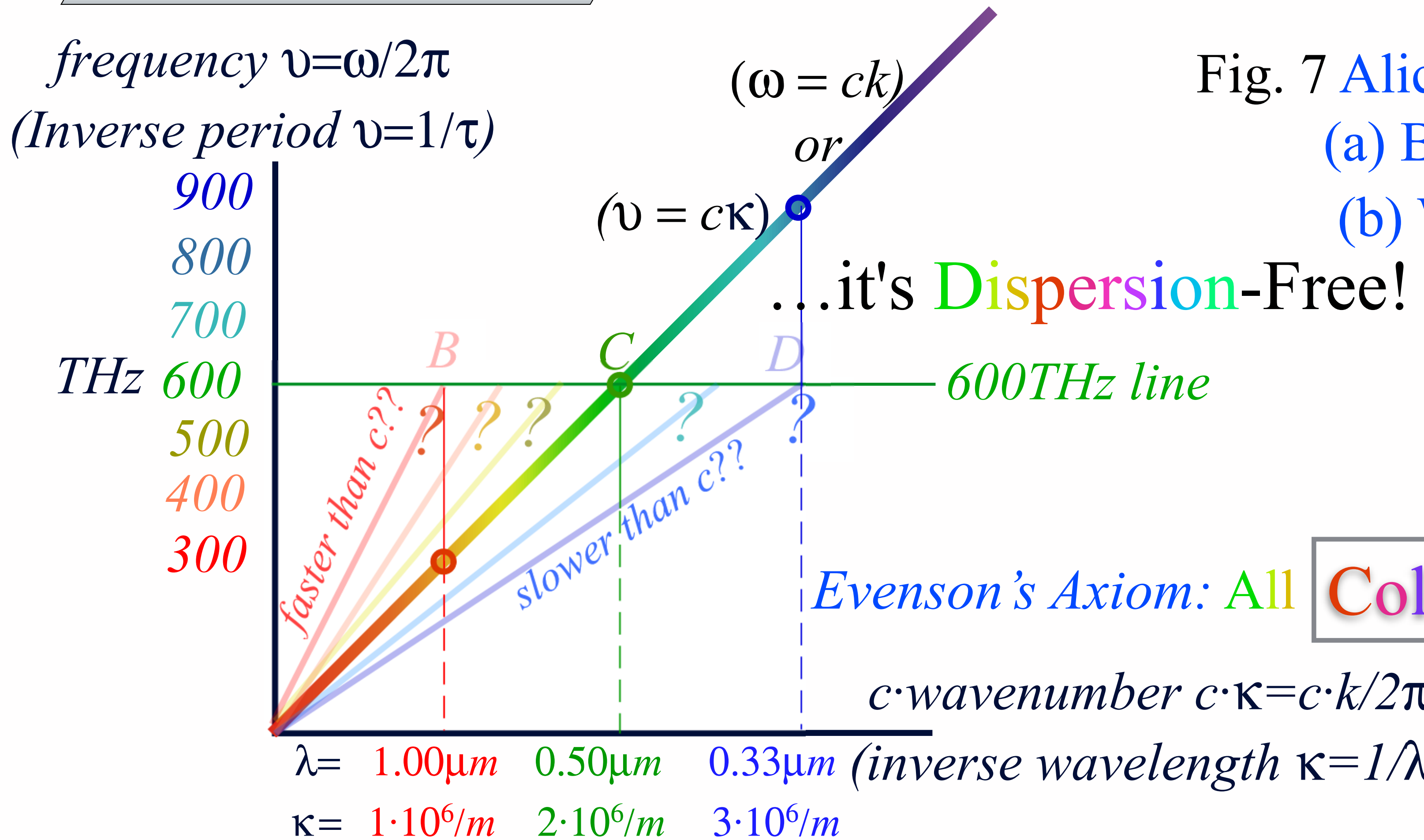
Actually: $2.99792458\cdot 10^{8}\mathrm{m\cdot s^{-1}}$

Introducing Doppler shifting and why *c* is *really* constant





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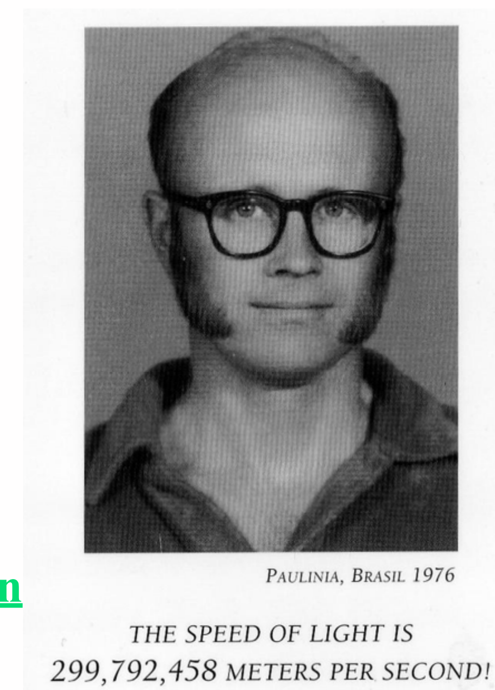

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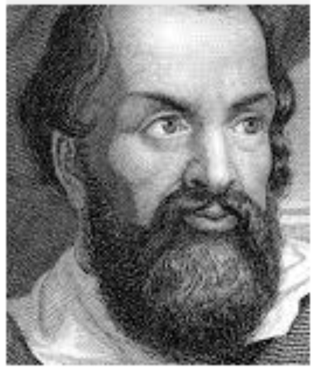
Galileo Galilei



1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like Galilean velocity

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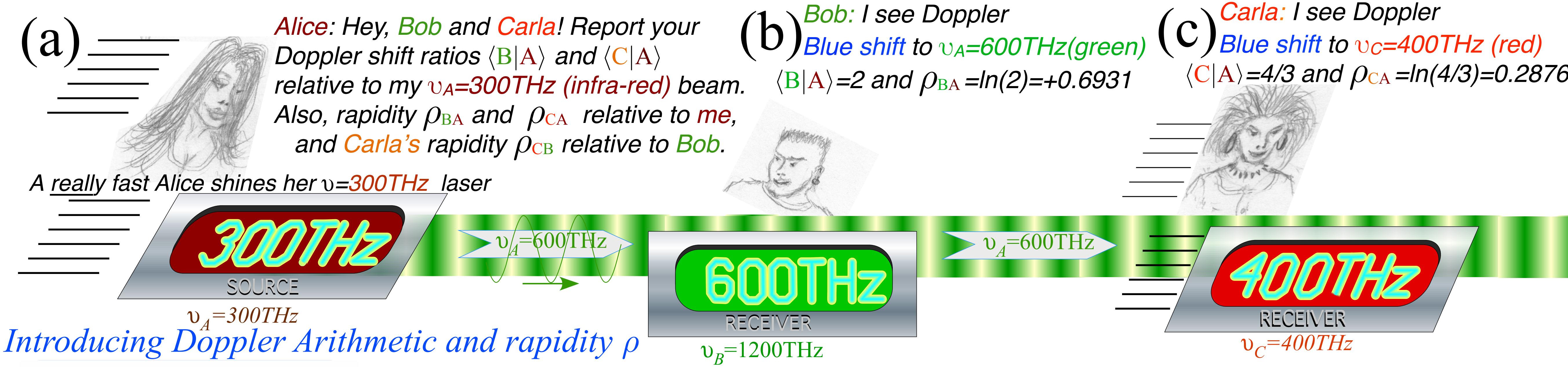


Introducing Doppler Arithmetic and rapidity ρ

(a) *Alice: Hey, Bob and Carla! Report your Doppler shift ratios ⟨B|A⟩ and ⟨C|A⟩ relative to my υ_A=300THz (infra-red) beam. Also, rapidity ρ_{BA} and ρ_{CA} relative to me, and Carla's rapidity ρ_{CB} relative to Bob.*

(b) *Bob: I see Doppler Blue shift to υ_A=600THz(green)*
⟨B|A⟩=2 and ρ_{BA} =ln(2)=+0.6931

(c) *Carla: I see Doppler Blue shift to υ_C=400THz (red)*
⟨C|A⟩=4/3 and ρ_{CA} =ln(4/3)=0.2876

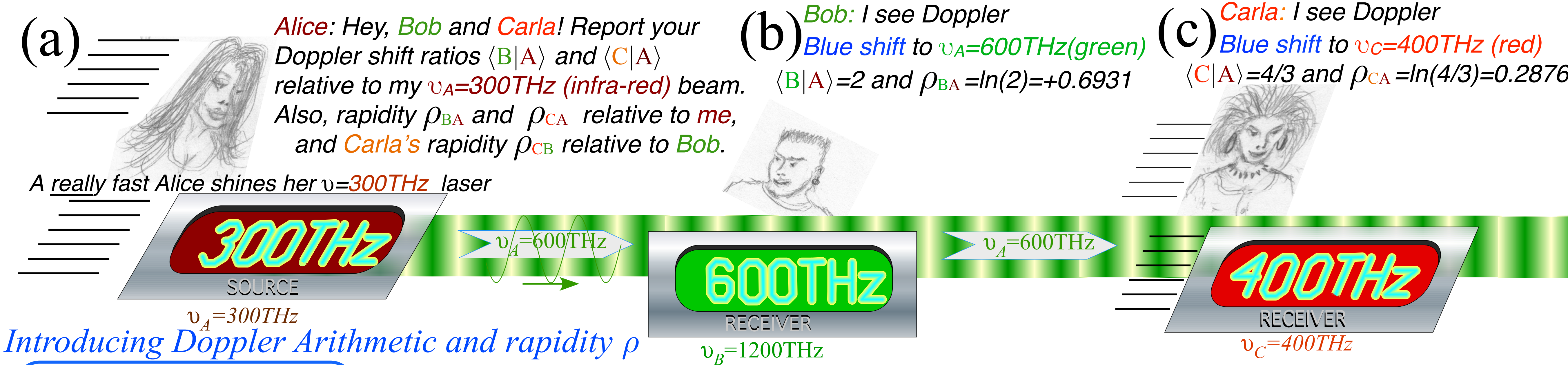
A really fast Alice shines her υ=300THz laser

υ_A=300THz υ_B=1200THz υ_C=400THz

Introducing Doppler Arithmetic and rapidity ρ

Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$



Introducing Doppler Arithmetic and rapidity ρ

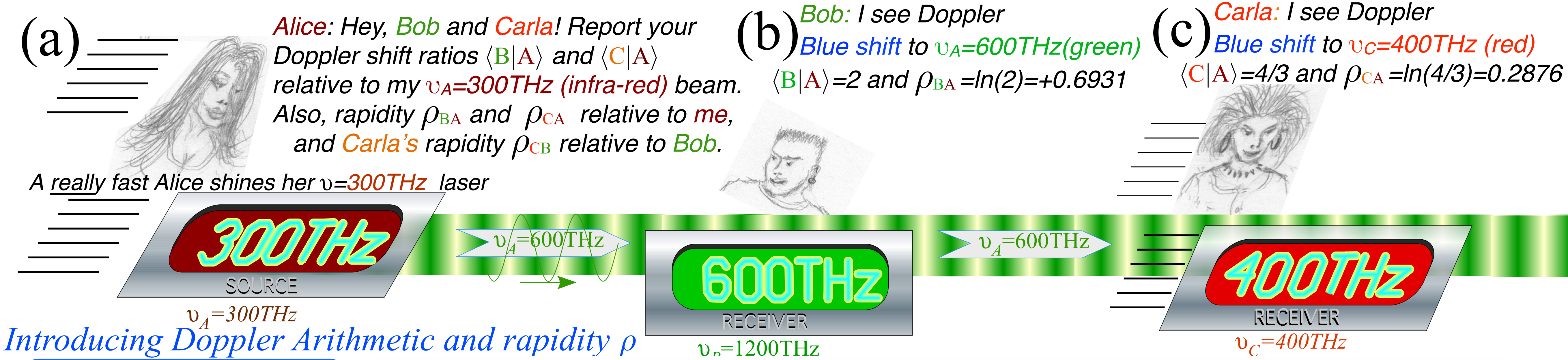
Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$



Introducing Doppler Arithmetic and rapidity ρ

Doppler ratio:

$$\langle R|S\rangle = \frac{\upsilon_{RECEIVER}}{\upsilon_{SOURCE}}$$

$$\rho_{RS} = \ln\langle R|S\rangle$$

or:

$$\langle R|S\rangle = e^{\rho_{RS}} = e^{-\rho_{SR}}$$

Definition of Rapidity ρ_{RS}

Rapidity is most convenient!

1TeV proton has

$u=0.999995598\cdot c$ (Pain in the A)

or: $\langle R|S\rangle=2131.6$ (Better)

or: $\rho_{RS}=7.6646$ (Best)

For low velocity $u<<c$ rapidity ρ_{RS} approaches u/c

(a)

Alice: Hey, *Bob* and *Carla*! Report your Doppler shift ratios ⟨B|A⟩ and ⟨C|A⟩ relative to my υ_A=300THz (infra-red) beam. Also, rapidity ρ_{BA} and ρ_{CA} relative to *me*, and *Carla's* rapidity ρ_{CB} relative to *Bob*.

(b) *Bob: I see Doppler Blue shift to* υ_A*=600THz(green)*
⟨B|A⟩=2 and ρ_{BA} =ln(2)=+0.6931

(c) *Carla: I see Doppler Blue shift to* υ_C*=400THz (red)*
⟨C|A⟩=4/3 and ρ_{CA} =ln(4/3)=0.2876

A really fast Alice shines her υ*=300THz laser*

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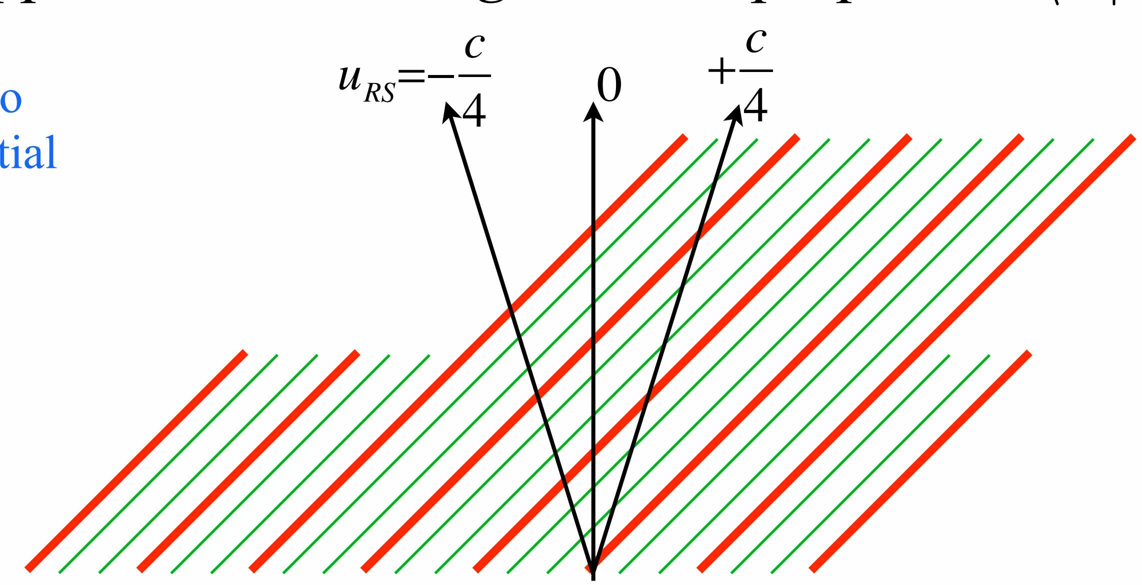
IMPORTANT POINTS:

Evenson axiom says Blue, Green, Red, etc. all march in lockstep and so *all* frequencies Doppler shift in same *geometric* proportion ⟨R|S⟩.

Geometric phenomena tend to involve logarithmic/exponential functionality!



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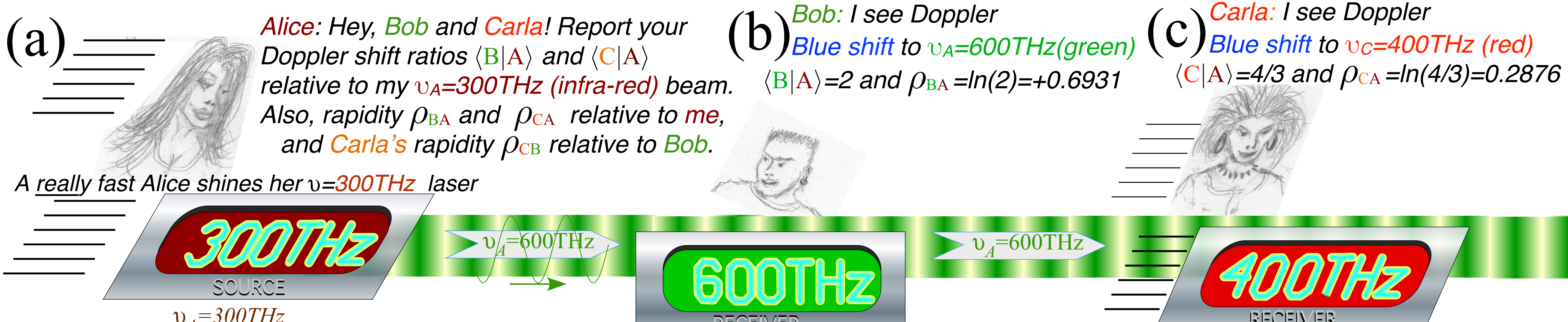

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$$\langle B|A\rangle = \frac{\upsilon_B}{\upsilon_A} = \frac{600}{300} = \frac{2}{1}$$

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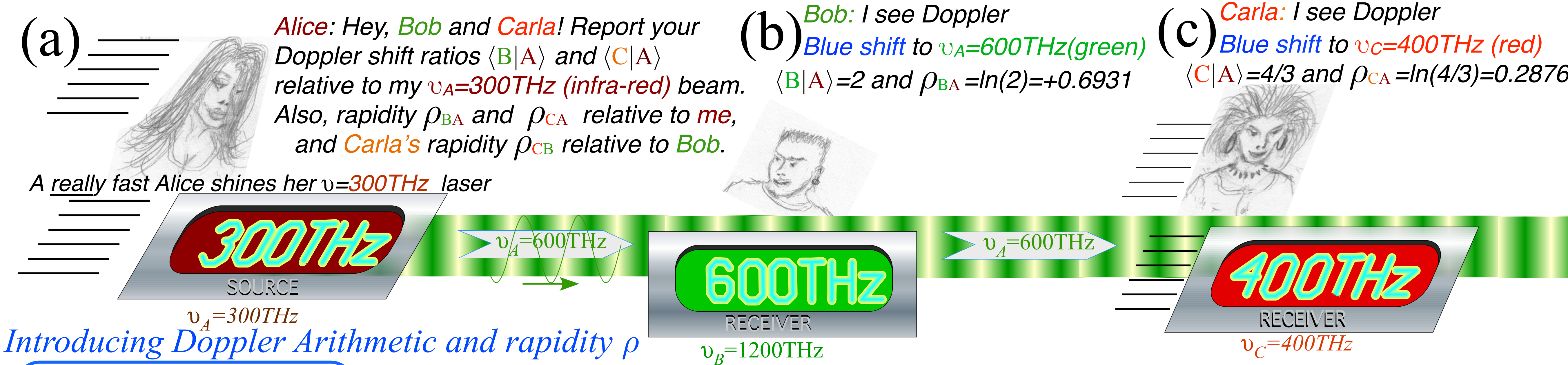

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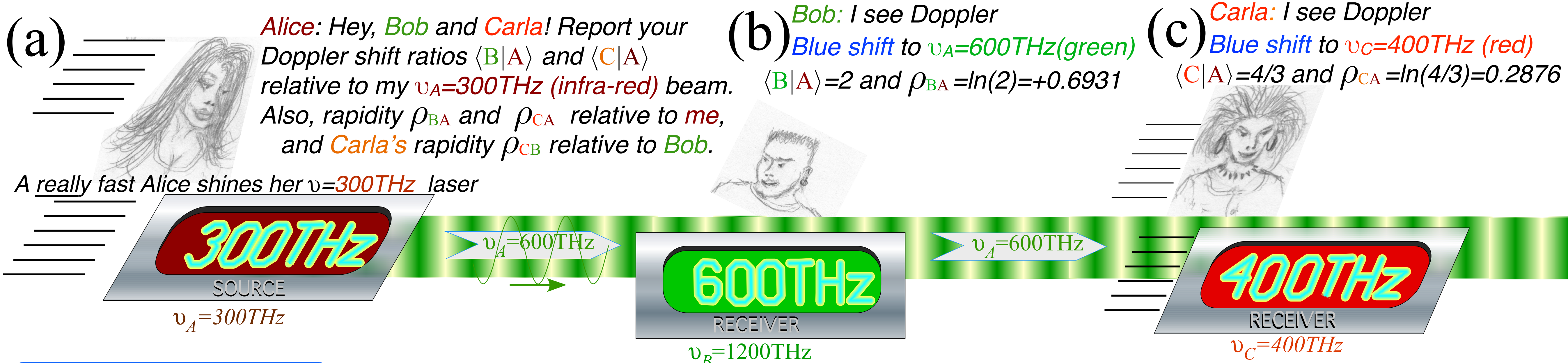
$$\langle C|A\rangle = \frac{\upsilon_C}{\upsilon_A} = \frac{400}{300} = \frac{4}{3}$$

Carla-Alice rapidity:

$$\rho_{CA} = \ln\langle C|A\rangle = \ln\frac{4}{3} = 0.2876$$



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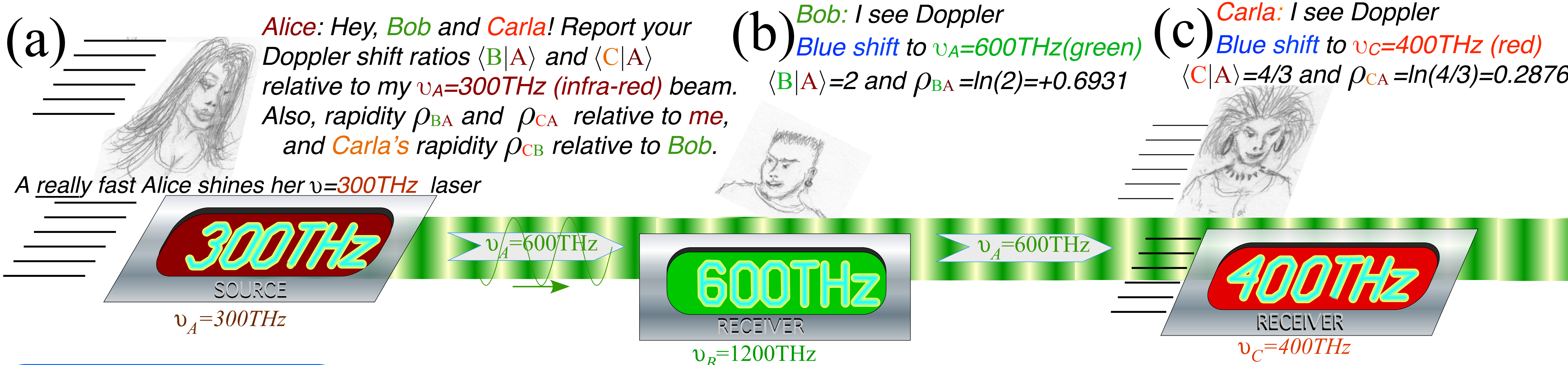

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Carla-Bob rapidity:

$$e^{\rho_{CB}} = e^{\rho_{CA}}e^{\rho_{AB}} = e^{\rho_{CA}+\rho_{AB}}$$

$$\rho_{CB} = \rho_{CA} + \rho_{AB} = 0.2876 - 0.6931 = -0.4055$$

$$= \ln\frac{4}{3} + \ln\frac{1}{2} = \ln\frac{2}{3}$$

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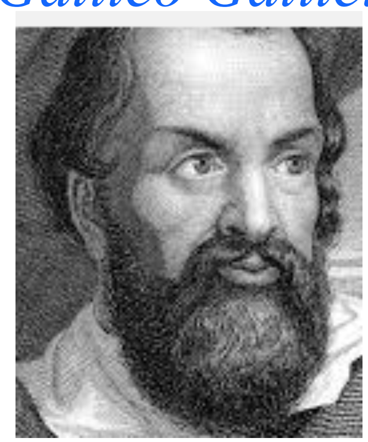
Galileo Galilei



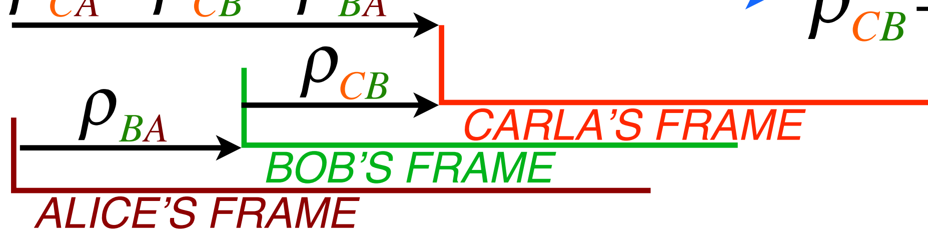
1564-1642

Happy now?

Galileo's Revenge (part 1)

Rapidity adds just like Galilean velocity

$$\rho_{CA} = \rho_{CB} + \rho_{BA}$$



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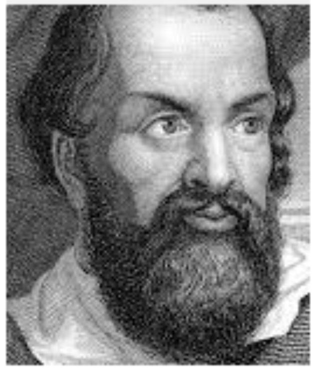

1564-1642

Galileo's Revenge (part 1)
Rapidity adds just like Galilean velocity

Galileo's Revenge (part 2)
Phasor angular velocity adds just like Galilean velocity

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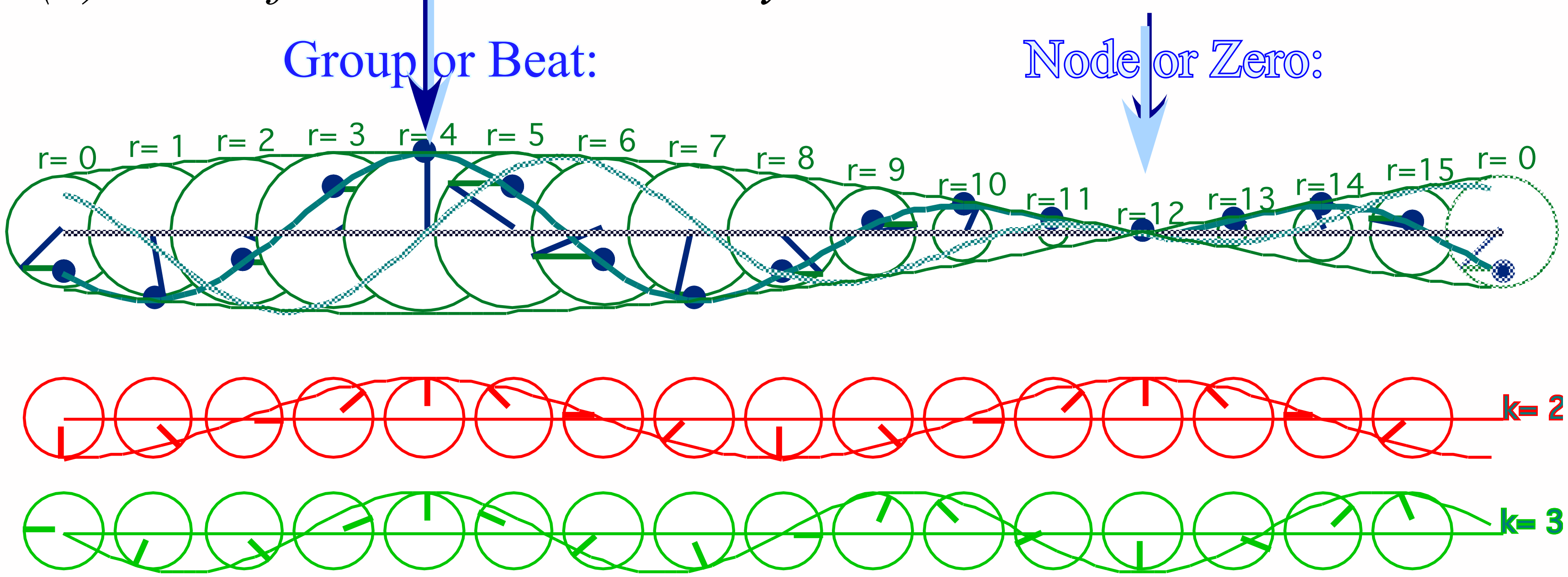
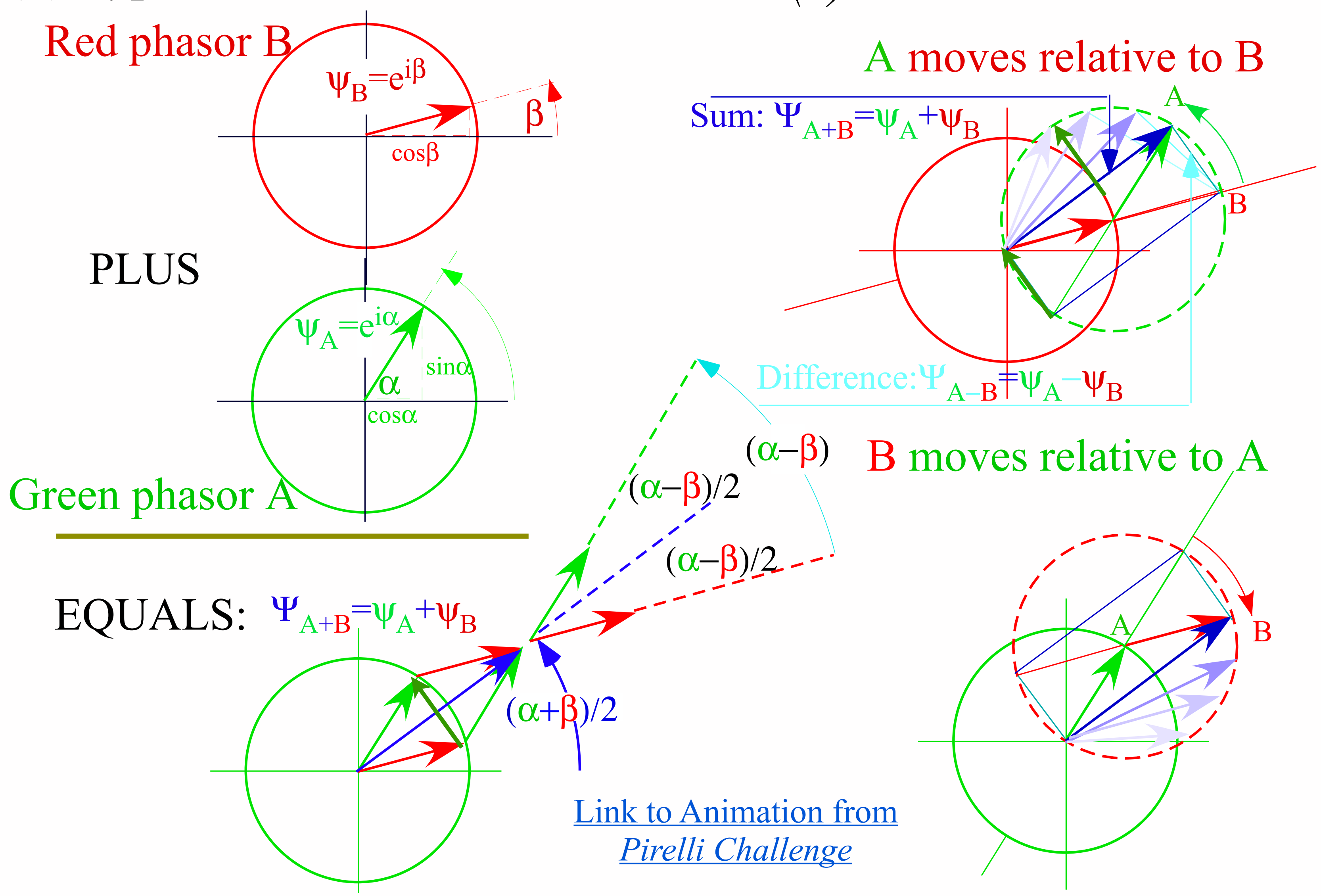
(a) Sum of Wave Phasor Array



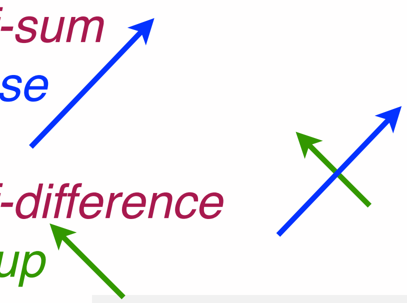
(b) Typical Phasor Sum:

(c) Phasor-relative views

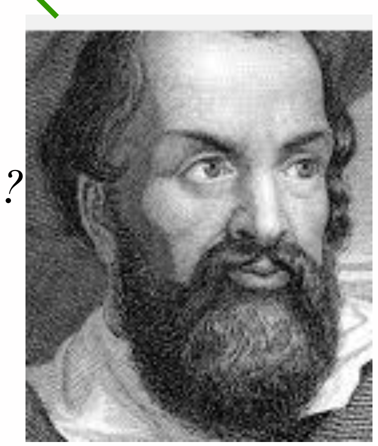


[Link to Animation from *Pirelli Challenge*]

Geometry of the Half-sum Phase and Half-difference Group



Happy now?



Galileo's Revenge (part 2)
Phasor angular velocity
adds just like
Galilean velocity

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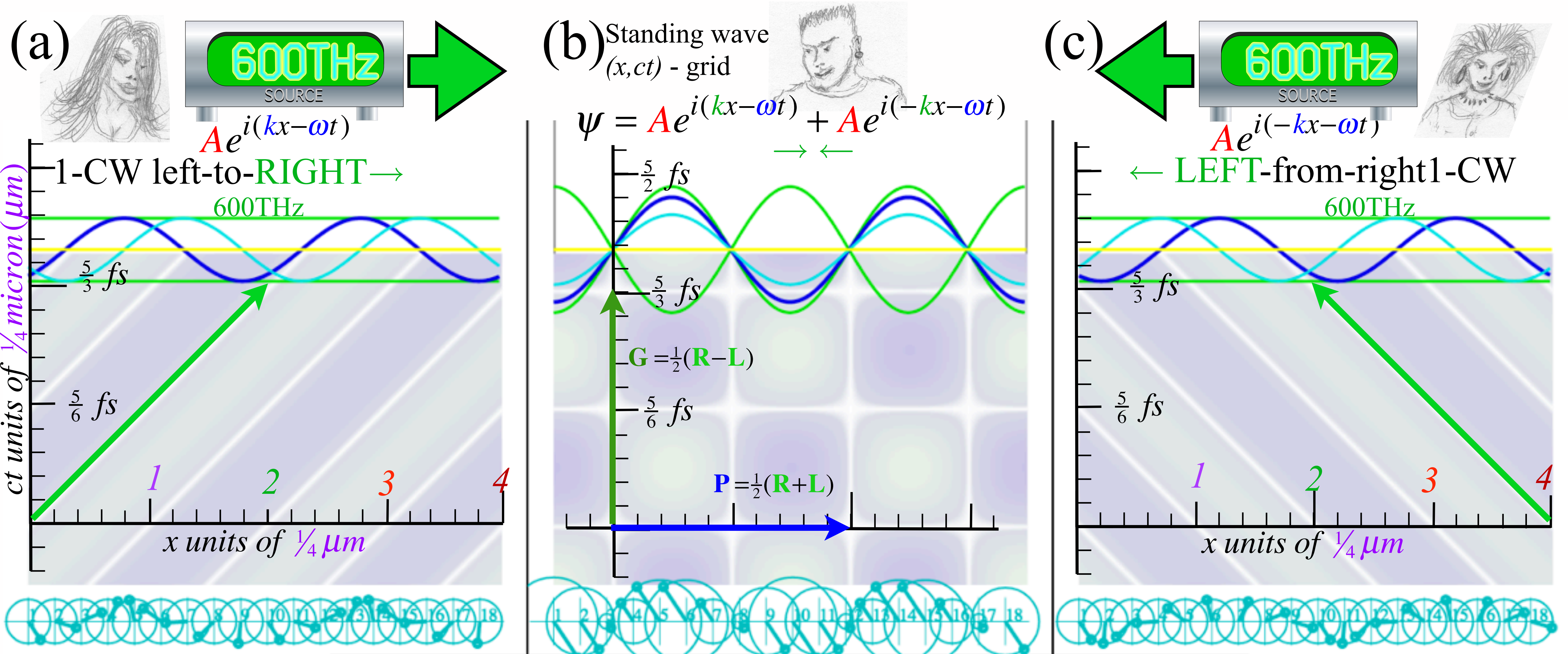
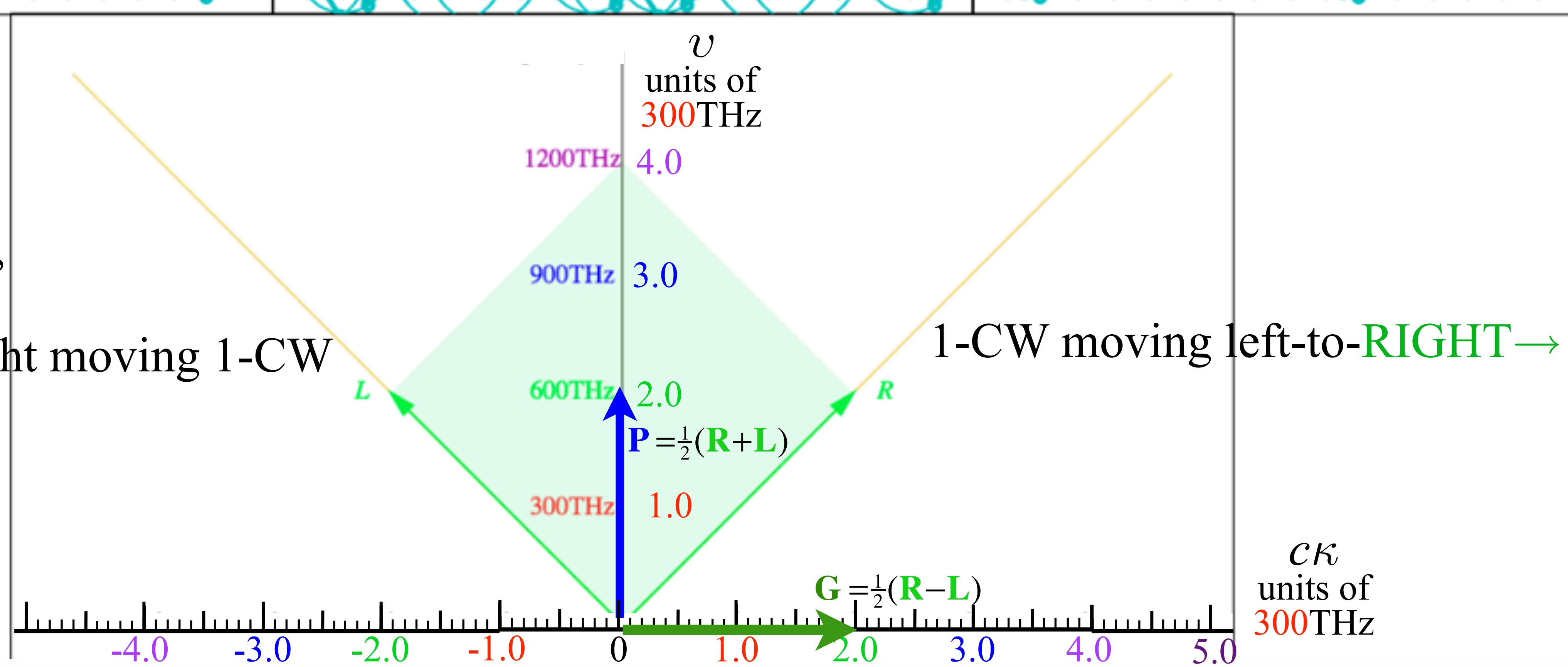
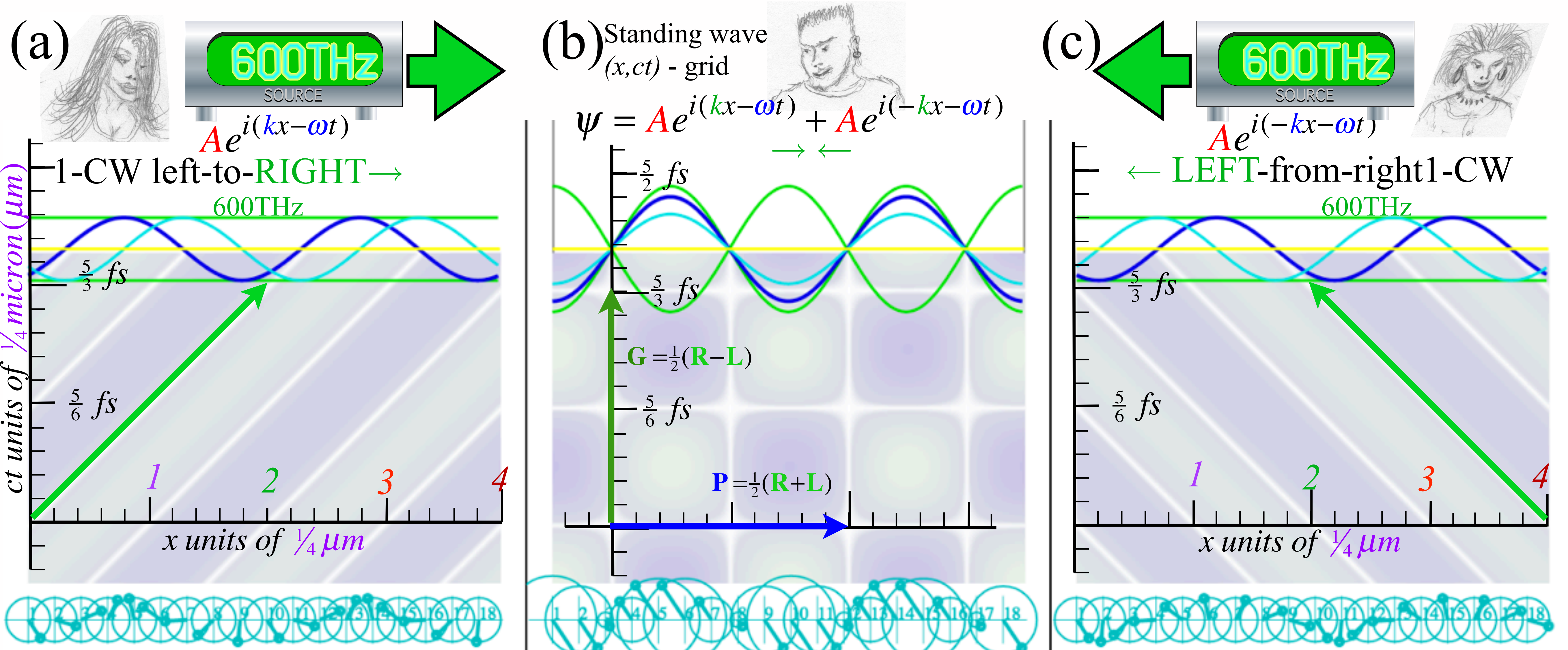


(d)

Introducing optical space-time grids and per-space-time “baseball-diamonds”

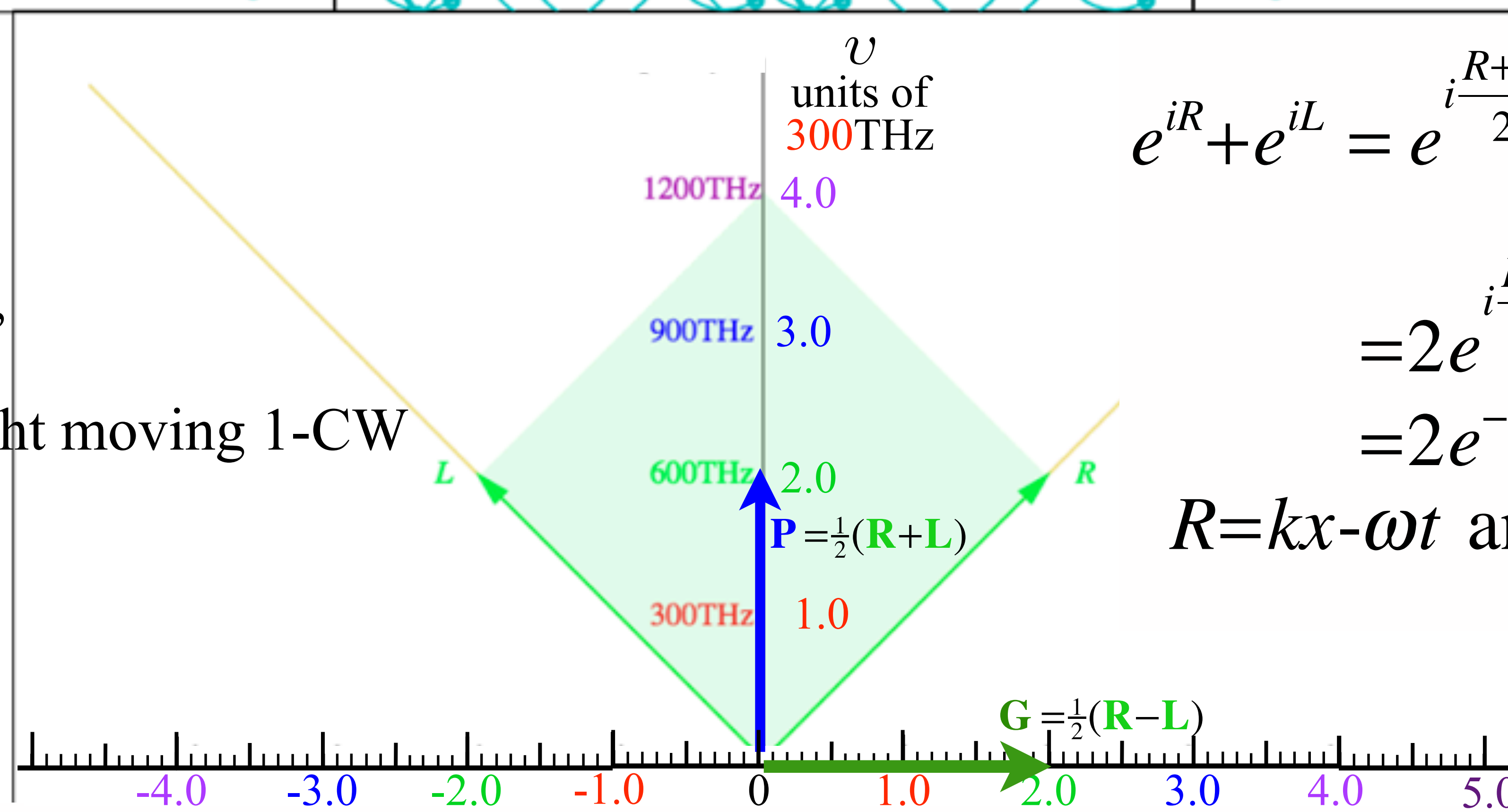




(d)

Introducing optical space-time grids and per-space-time "baseball-diamonds"

← LEFT-from-right moving 1-CW



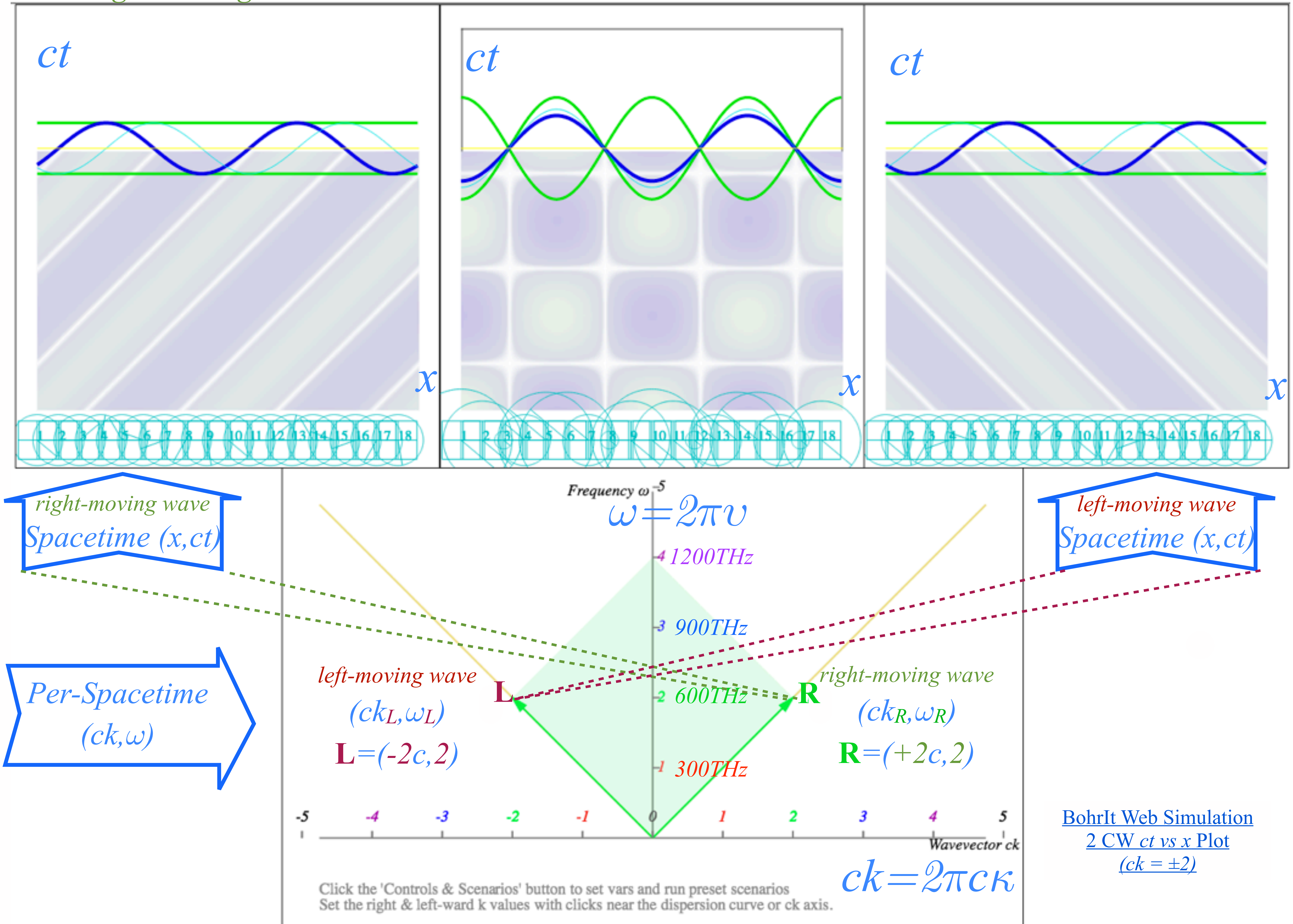
$$e^{iR}+e^{iL} = e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$

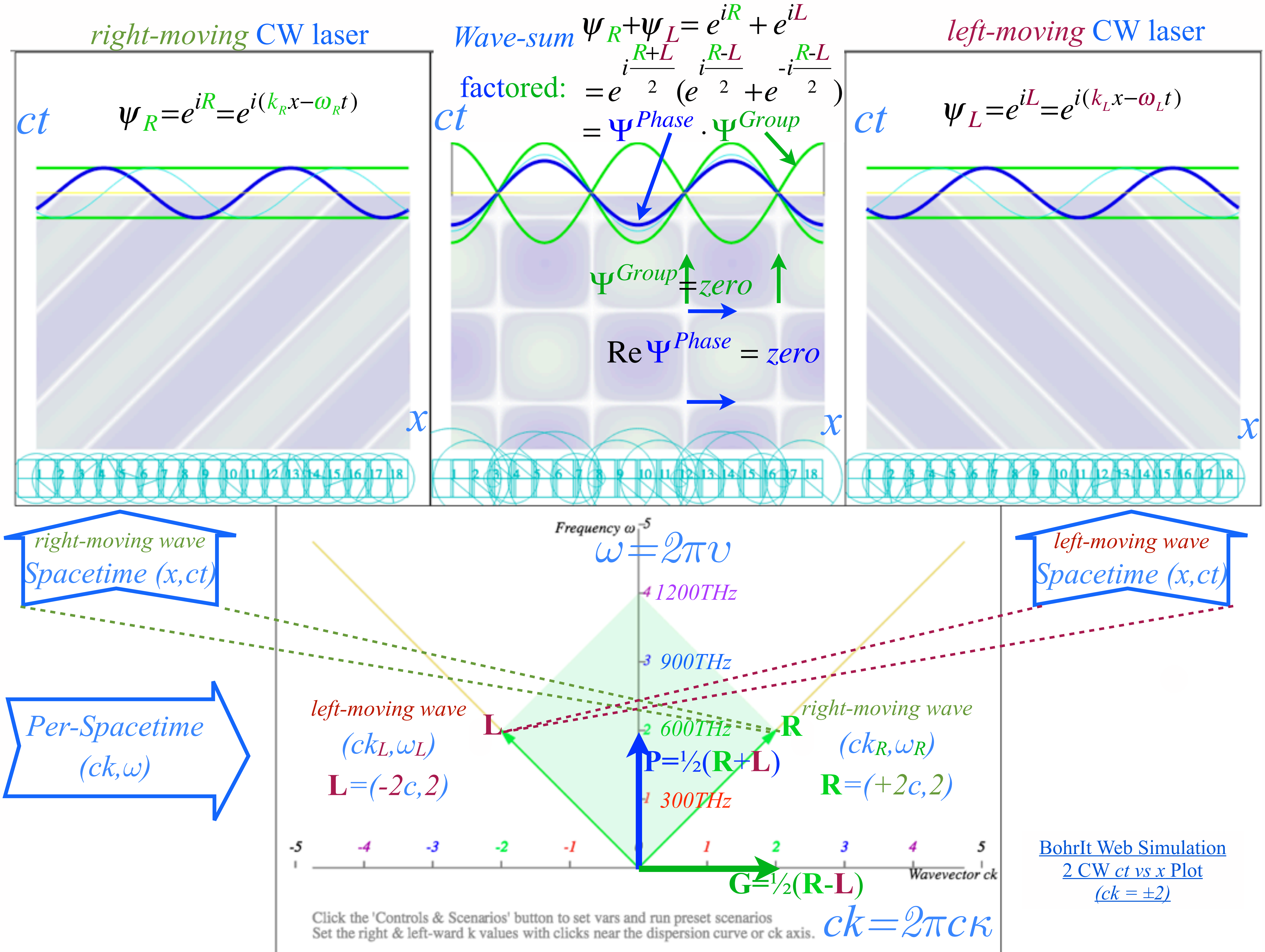
$$=2e^{i\frac{R+L}{2}}\cos\frac{R-L}{2}$$

$$=2e^{-i\omega t}\cos kx$$

$R=kx\text{-}\omega t$ and: $L=\text{-}kx\text{-}\omega t$

right-moving CW laser | Colliding 2CW laser beams | *left-moving* CW laser




right-moving CW laser
Wave-sum $\psi_R + \psi_L = e^{iR} + e^{iL}$
factored: $= e^{i\frac{R+L}{2}} (e^{i\frac{R-L}{2}} + e^{-i\frac{R-L}{2}})$
$= \Psi^{Phase} \cdot \Psi^{Group}$
left-moving CW laser
ct
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
x
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
$\Psi^{Group} = zero$
$\mathrm{Re}\,\Psi^{Phase} = zero$
right-moving wave
Spacetime (x,ct)
left-moving wave
Spacetime (x,ct)
Per-Spacetime
(ck,ω)
Frequency ω
$\omega = 2\pi\upsilon$
1200THz
900THz
600THz
300THz
left-moving wave
(ck_L, ω_L)
L=(-2c,2)
right-moving wave
(ck_R, ω_R)
R=(+2c,2)
P=½(R+L)
G=½(R-L)
Wavevector ck
$ck = 2\pi c\kappa$
Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.
BohrIt Web Simulation
2 CW ct vs x Plot
(ck = ±2)

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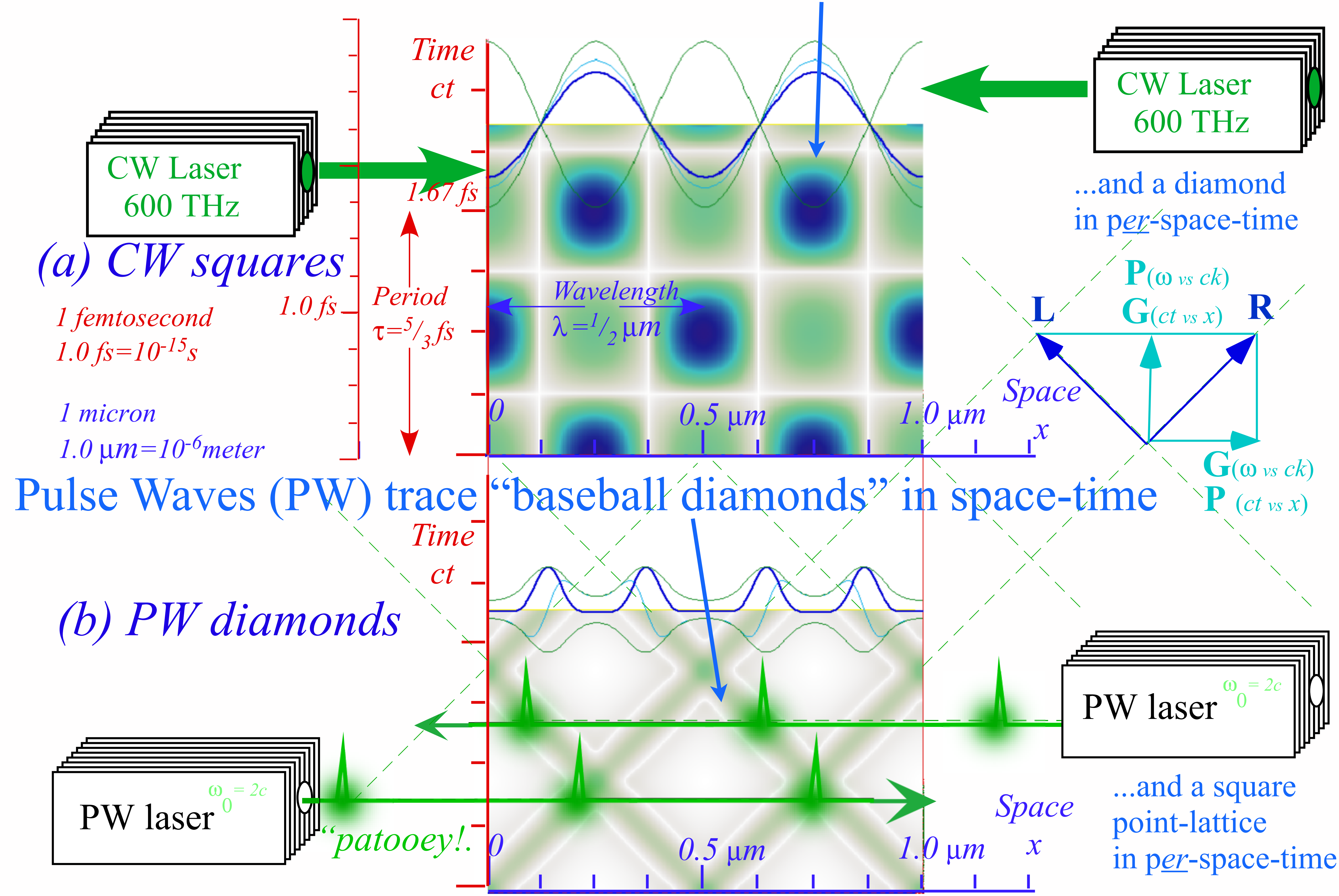
Continuous Waves (CW) trace "Cartesian squares" in space-time



BohrIt Web Simulation: 2 PW *ct vs x* Plot

Friday, January 27, 2017

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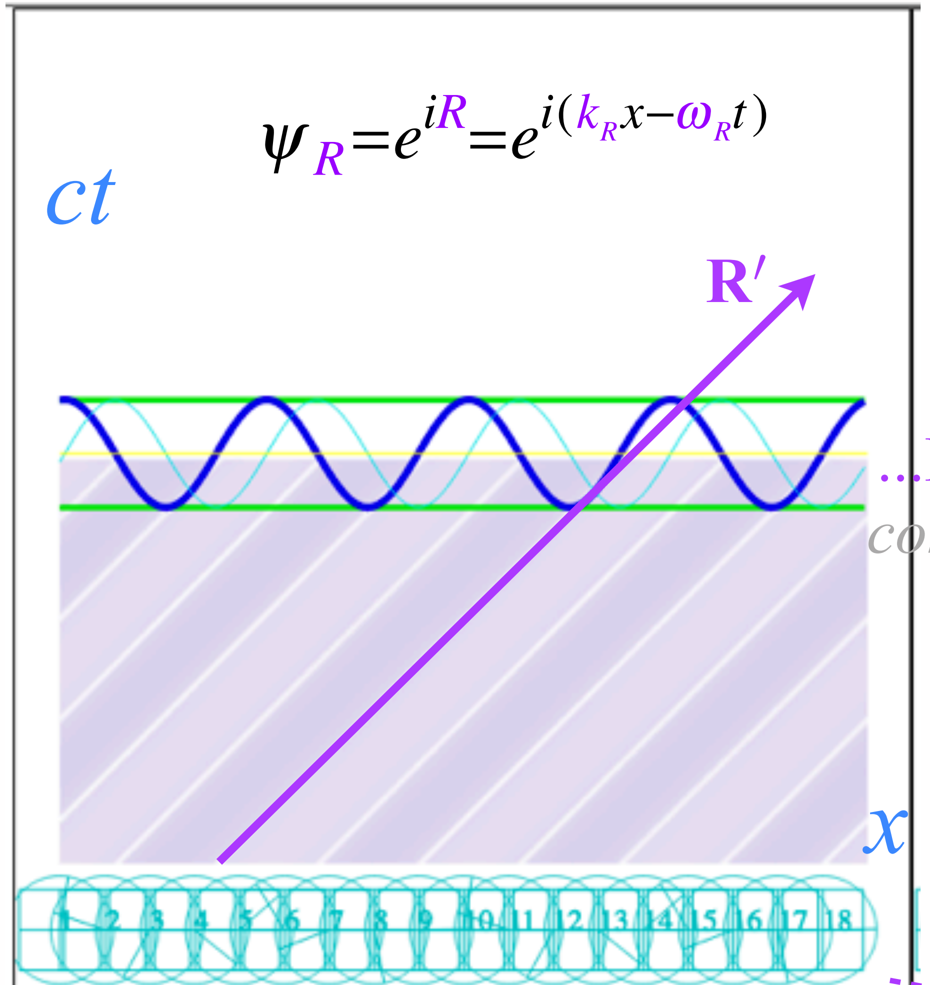
right-moving Doppler blue shifted wave

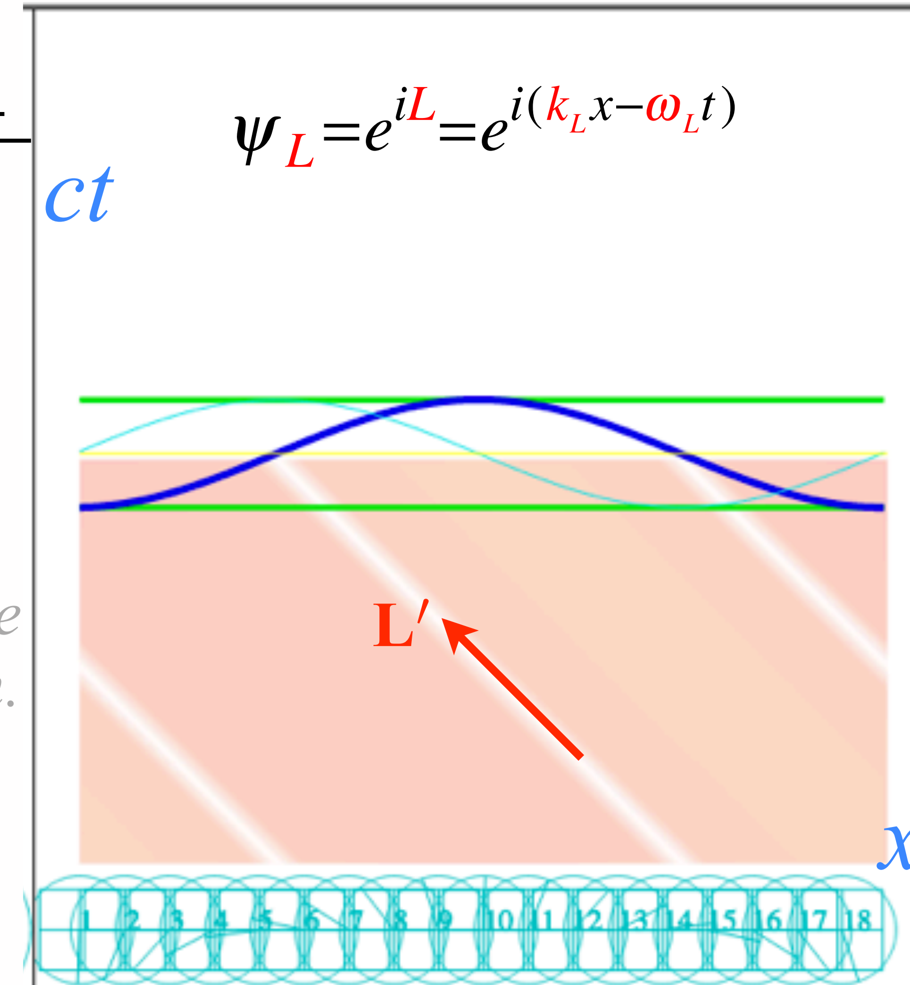
left-moving Doppler red shifted wave



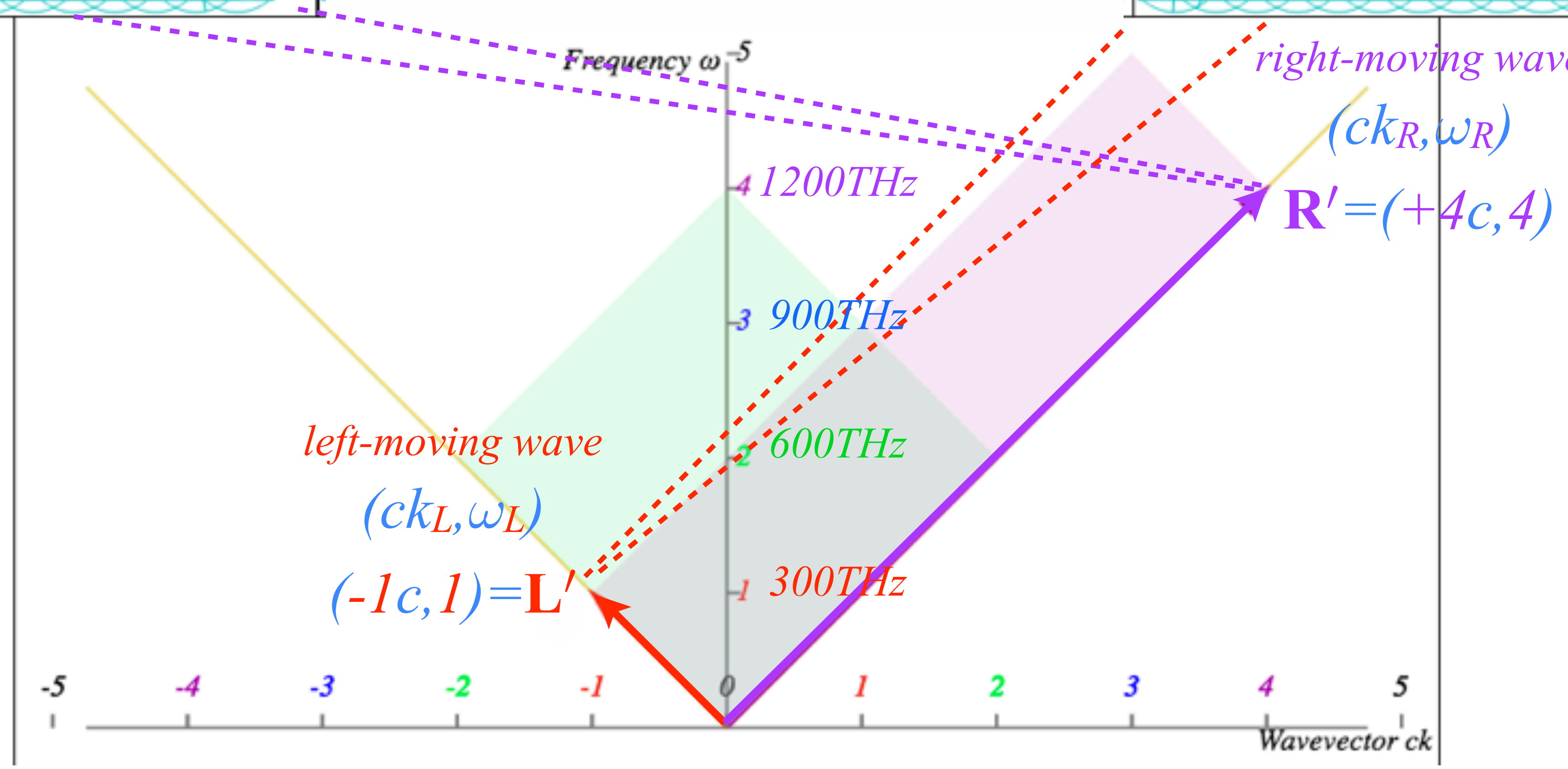
Rapidly moving Bob sees...

...Blue shifted *wave coming at him and...*

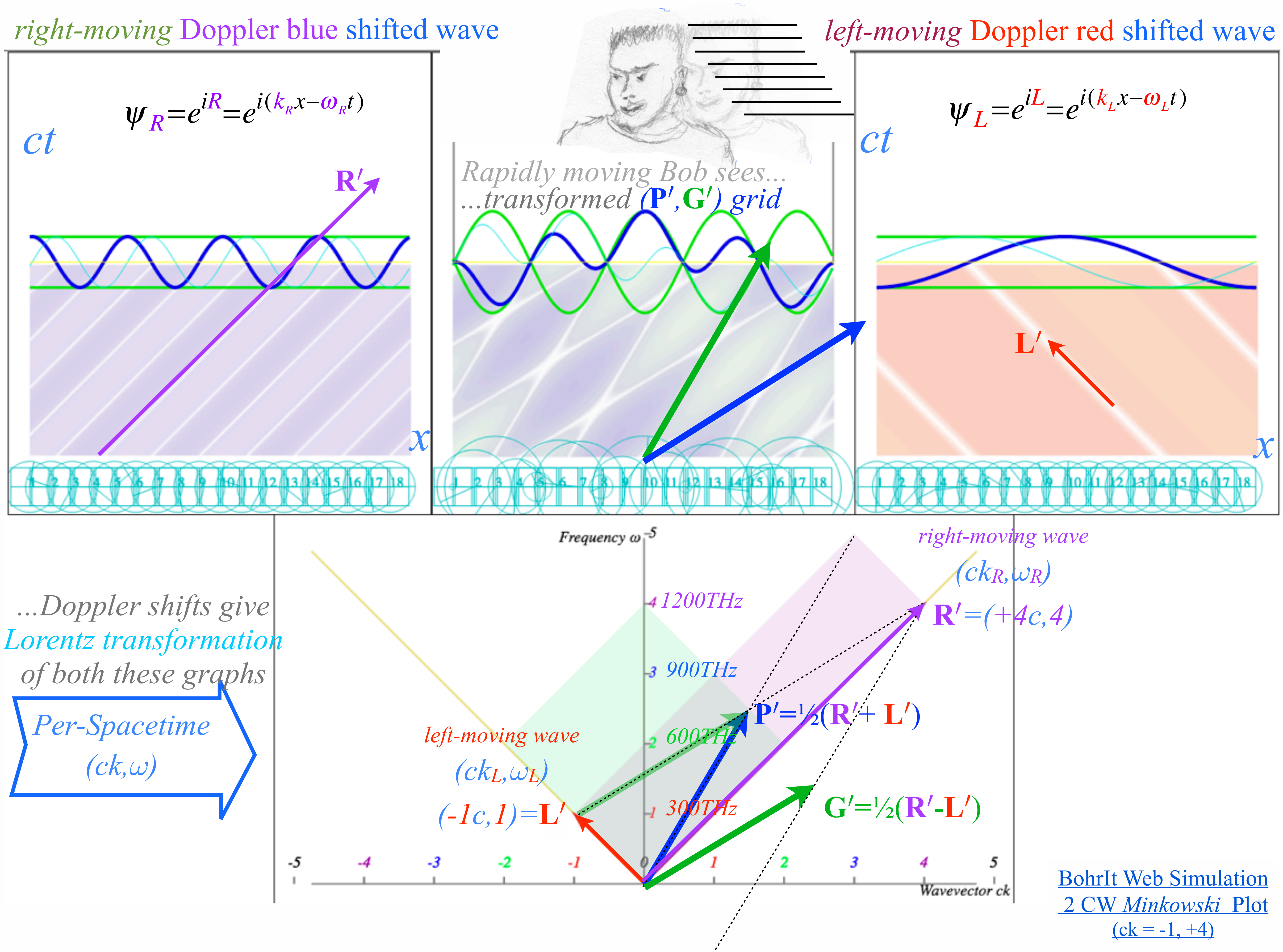
...Red shifted *wave behind him.*



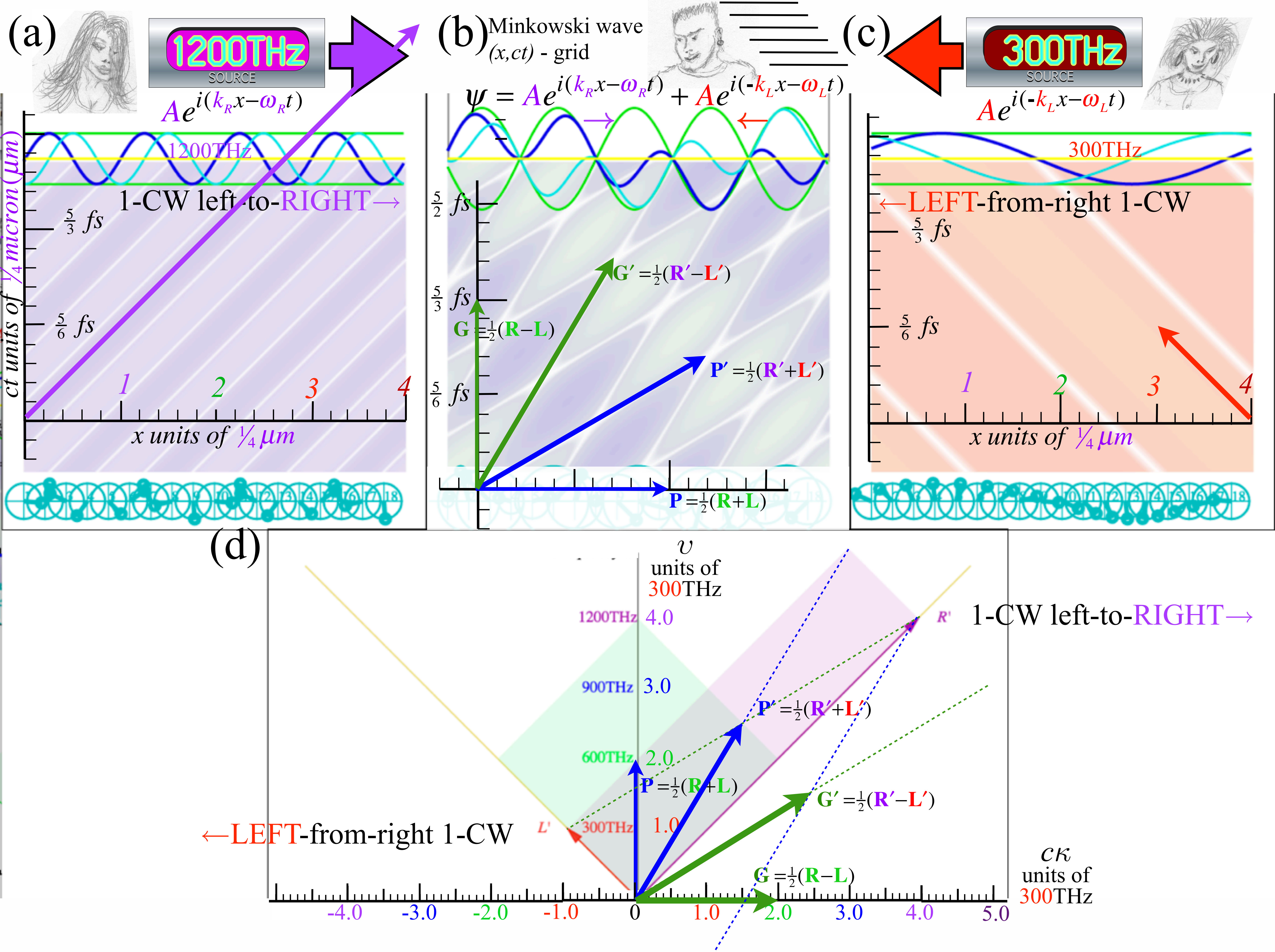
Web Simulation
1 CW *ct vs x* Plot
(ck = +4)



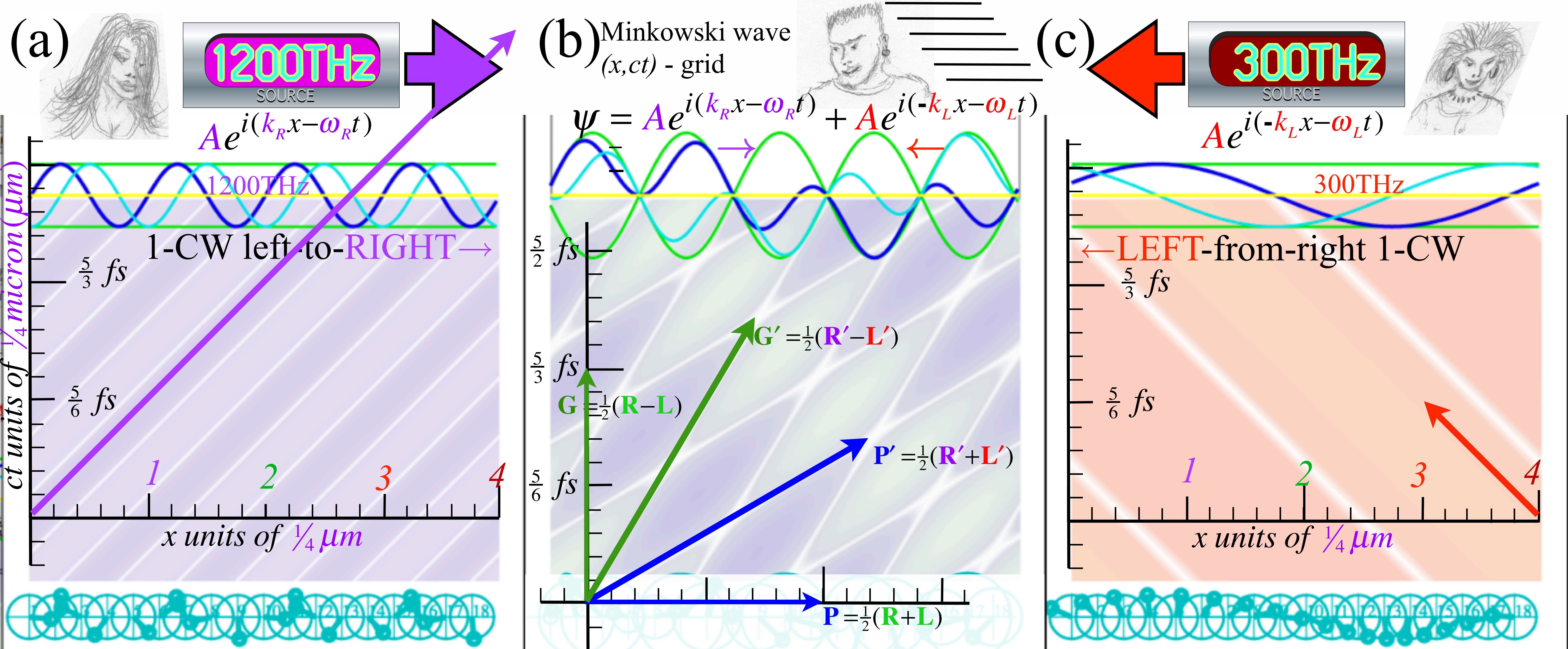
Web Simulation
1 CW *ct vs x* Plot
(ck = -1)



right-moving Doppler blue shifted wave
left-moving Doppler red shifted wave
$\psi_R = e^{iR} = e^{i(k_R x - \omega_R t)}$
$\psi_L = e^{iL} = e^{i(k_L x - \omega_L t)}$
ct
x
R′
L′
Rapidly moving Bob sees...
...transformed (P′,G′) grid
...Doppler shifts give
Lorentz transformation
of both these graphs
Per-Spacetime
(ck,ω)
Frequency ω
1200THz
900THz
600THz
300THz
Wavevector ck
right-moving wave
(ck_R, ω_R)
R′=(+4c,4)
P′=½(R′+ L′)
G′=½(R′-L′)
left-moving wave
(ck_L, ω_L)
(-1c,1)=L′
BohrIt Web Simulation
2 CW Minkowski Plot
(ck = -1, +4)



(a)
1200THz
SOURCE
$Ae^{i(k_R x - \omega_R t)}$
1200THz
1-CW left-to-RIGHT→
ct units of ¼ micron (μm)
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(b) Minkowski wave (x,ct) - grid
$\psi = Ae^{i(k_R x - \omega_R t)} + Ae^{i(-k_L x - \omega_L t)}$
→
←
$\frac{5}{2}$ fs
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
G′ =½(R′−L′)
G =½(R−L)
P′ =½(R′+L′)
P =½(R+L)
(c)
300THz
SOURCE
$Ae^{i(-k_L x - \omega_L t)}$
300THz
←LEFT-from-right 1-CW
$\frac{5}{3}$ fs
$\frac{5}{6}$ fs
1
2
3
4
x units of ¼ μm
(d)
υ units of 300THz
1200THz 4.0
900THz 3.0
600THz 2.0
300THz 1.0
R′
1-CW left-to-RIGHT→
P′ =½(R′+L′)
P =½(R+L)
G′ =½(R′−L′)
G =½(R−L)
L′
←LEFT-from-right 1-CW
cκ units of 300THz
-4.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
4.0
5.0



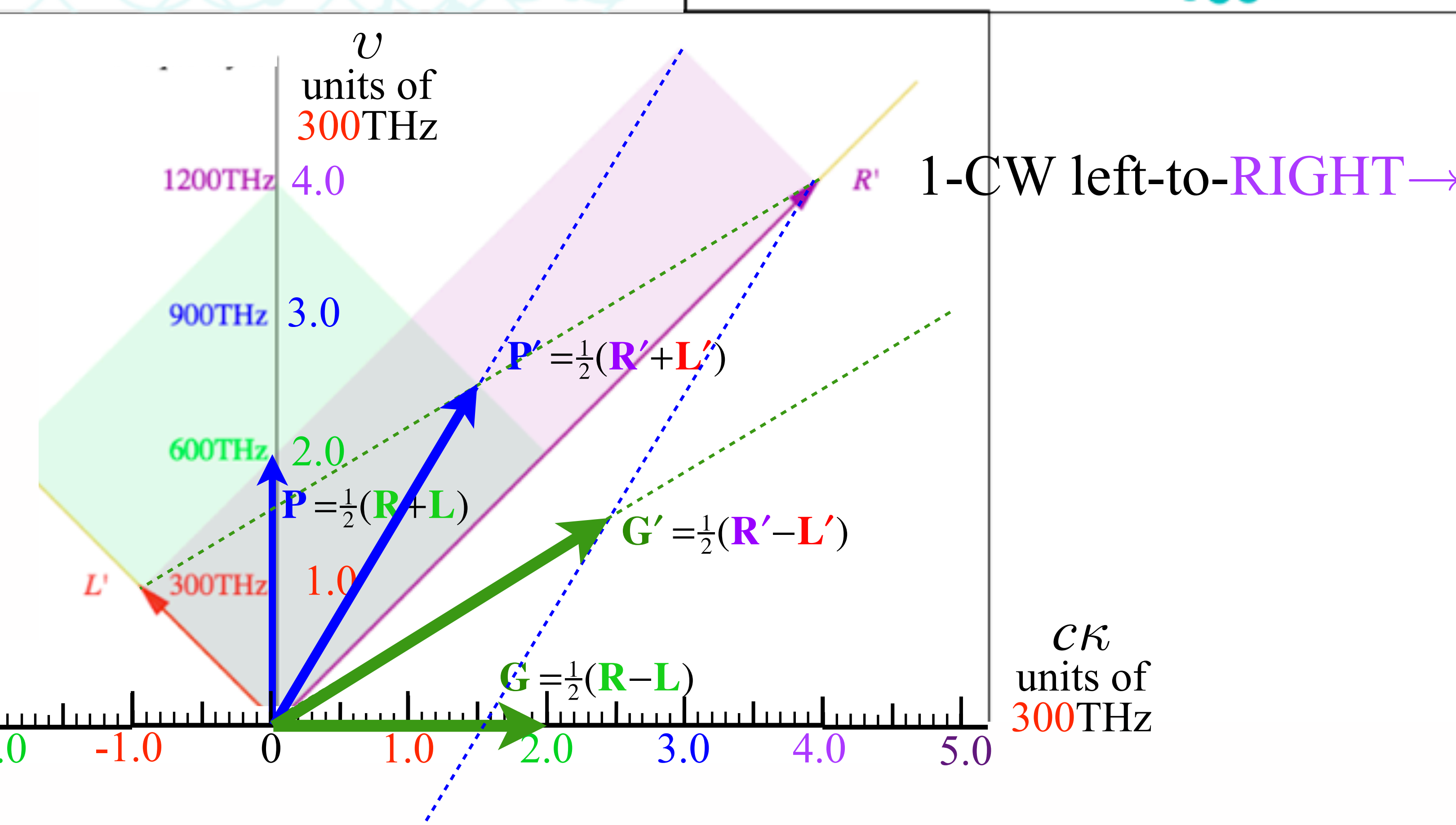
(d)

$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$

$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$

$$=\psi'_{phase}\psi'_{group}$$

$R'=k_R x\text{-}\omega_R t$ and: $L'=\text{-}k_L x\text{-}\omega_L t$



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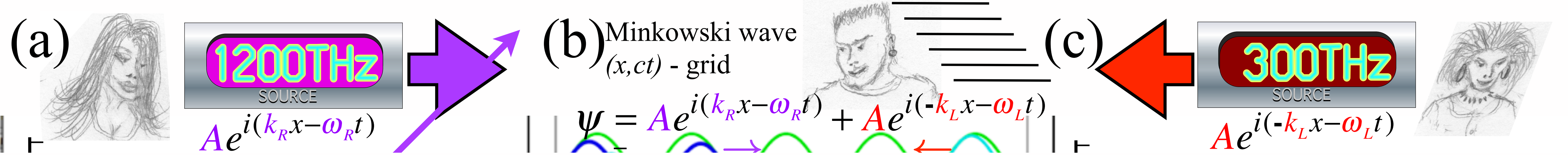

$$\mathbf{P}' = \begin{pmatrix} \upsilon'_{phase} \\ c\kappa'_{phase} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'+\mathbf{L}') = \upsilon_A \begin{pmatrix} \frac{1}{2}(e^{\rho}+e^{-\rho}) \\ \frac{1}{2}(e^{\rho}-e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}_{\substack{Bob's \\ View}} \text{ or: } \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix}_{\substack{Alice's \\ View}}$$

$$\mathbf{G}' = \begin{pmatrix} \upsilon'_{group} \\ c\kappa'_{group} \end{pmatrix} = \tfrac{1}{2}(\mathbf{R}'-\mathbf{L}') = \upsilon_A \begin{pmatrix} \frac{1}{2}(e^{\rho}-e^{-\rho}) \\ \frac{1}{2}(e^{\rho}+e^{-\rho}) \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}_{\substack{Bob's \\ View}} \text{ or: } \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix}_{\substack{Alice's \\ View}}$$

(d)

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$$= e^{i\frac{R'+L'}{2}}\, 2\cos\frac{R'-L'}{2}$$
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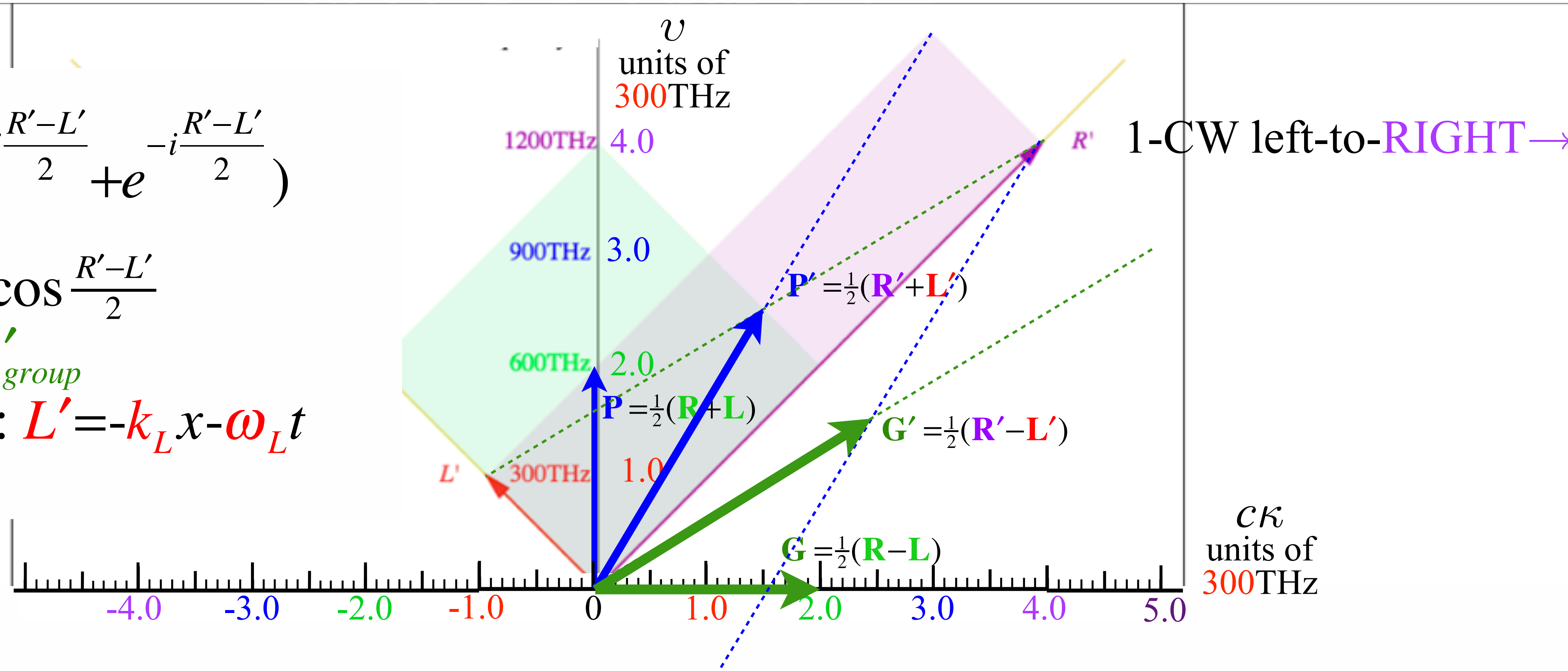
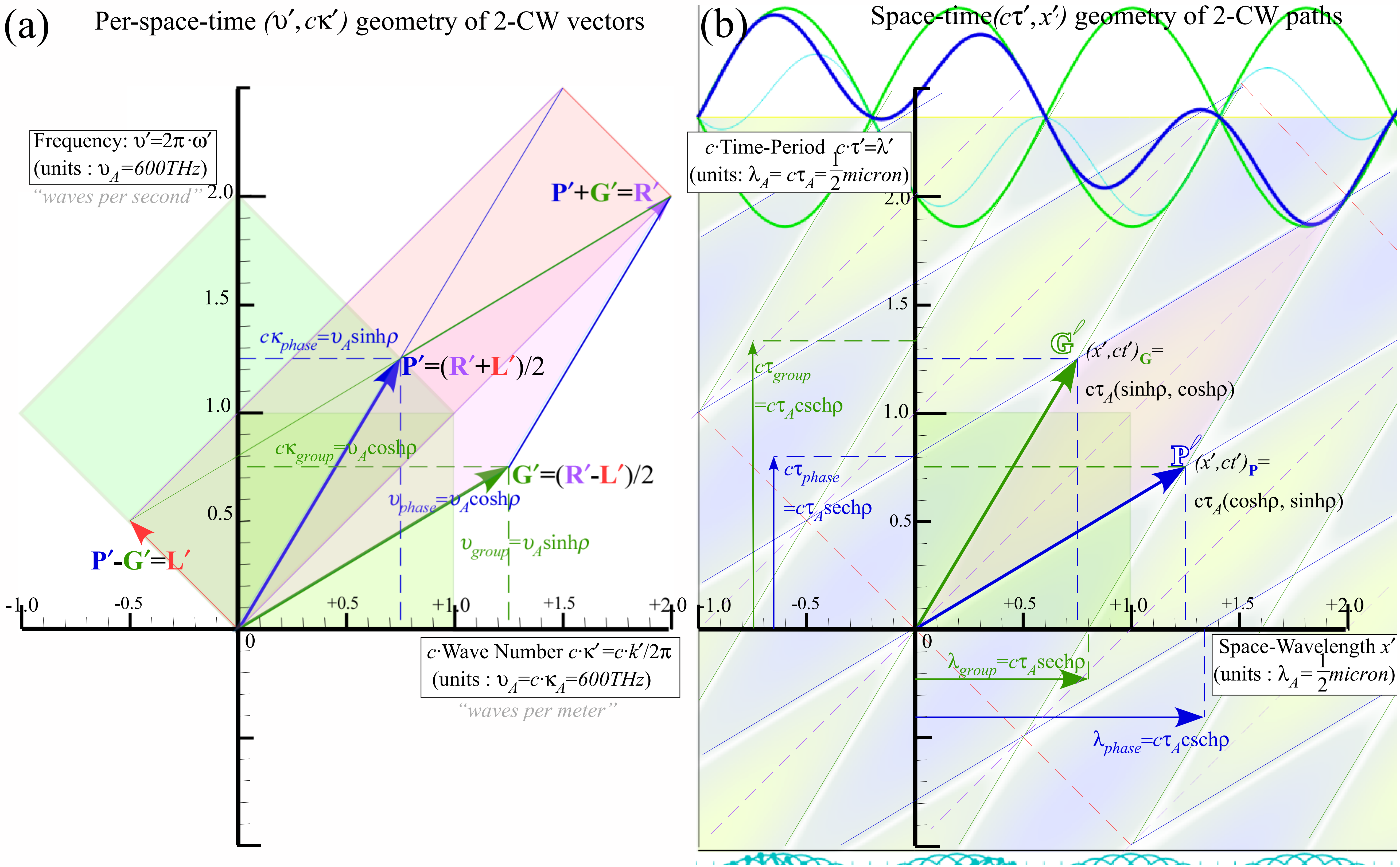
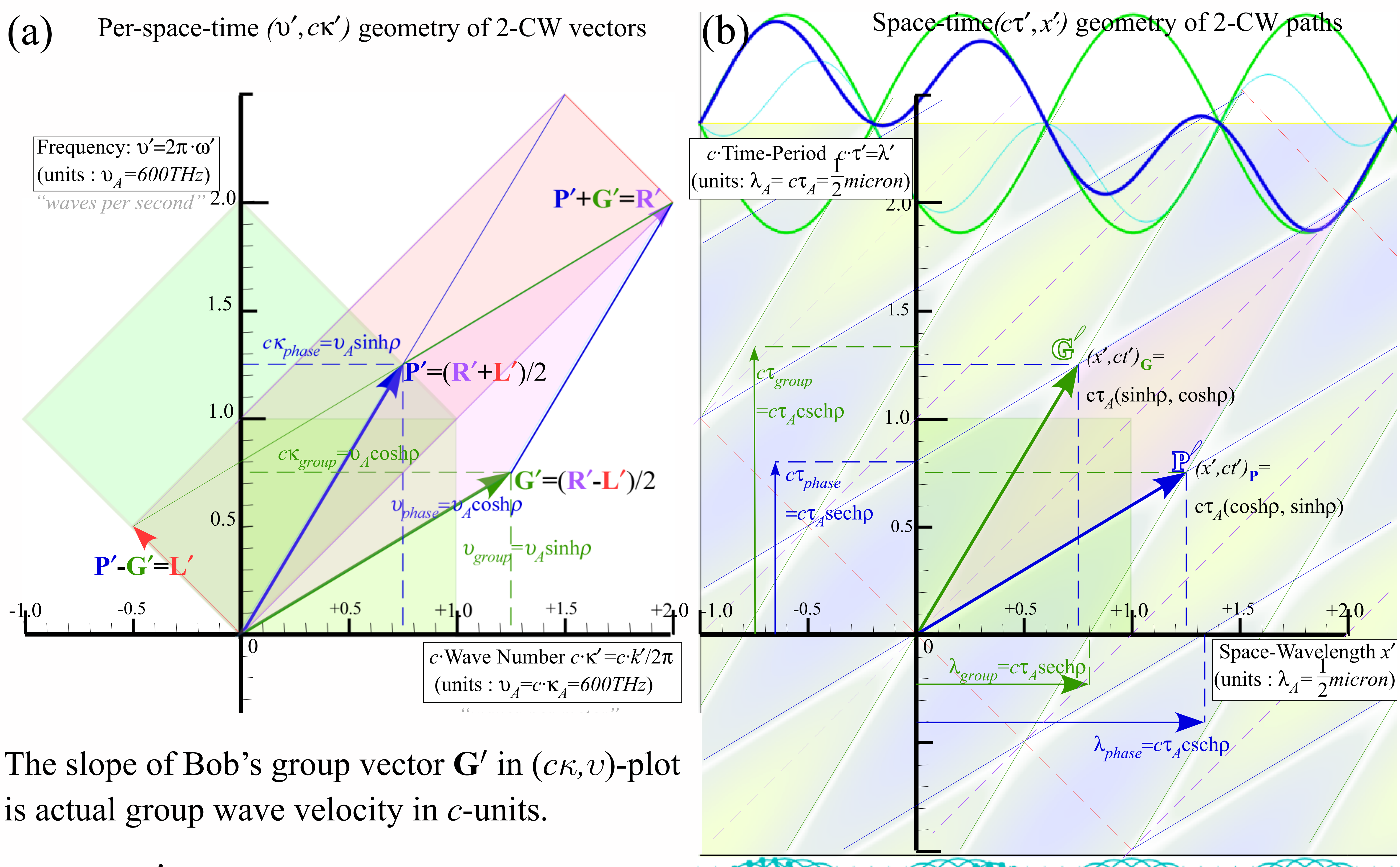
$R' = k_R x - \omega_R t$ and: $L' = -k_L x - \omega_L t$




(a)
Per-space-time (υ′, cκ′) geometry of 2-CW vectors
Frequency: υ′=2π·ω′
(units : υ_A=600THz)
"waves per second"
P′+G′=R′
cκ_phase=υ_A sinhρ
P′=(R′+L′)/2
cκ_group=υ_A coshρ
G′=(R′-L′)/2
υ_phase=υ_A coshρ
υ_group=υ_A sinhρ
P′-G′=L′
c·Wave Number c·κ′=c·k′/2π
(units : υ_A=c·κ_A=600THz)
"waves per meter"
(b)
Space-time (cτ′, x′) geometry of 2-CW paths
c·Time-Period c·τ′=λ′
(units: λ_A= cτ_A=1/2 micron)
cτ_group
=cτ_A cschρ
cτ_phase
=cτ_A sechρ
G′ (x′,ct′)_G=
cτ_A(sinhρ, coshρ)
P′ (x′,ct′)_P=
cτ_A(coshρ, sinhρ)
λ_group=cτ_A sechρ
λ_phase=cτ_A cschρ
Space-Wavelength x′
(units : λ_A=1/2 micron)



The slope of Bob's group vector **G**′ in ($c\kappa,\upsilon$)-plot is actual group wave velocity in c-units.

$$\frac{V^{group}}{c} = \frac{\upsilon'_{group}}{c\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \tanh\rho = \frac{\frac{3}{2}}{\frac{5}{2}} = \frac{3}{5} \equiv \frac{u}{c} \equiv \beta$$

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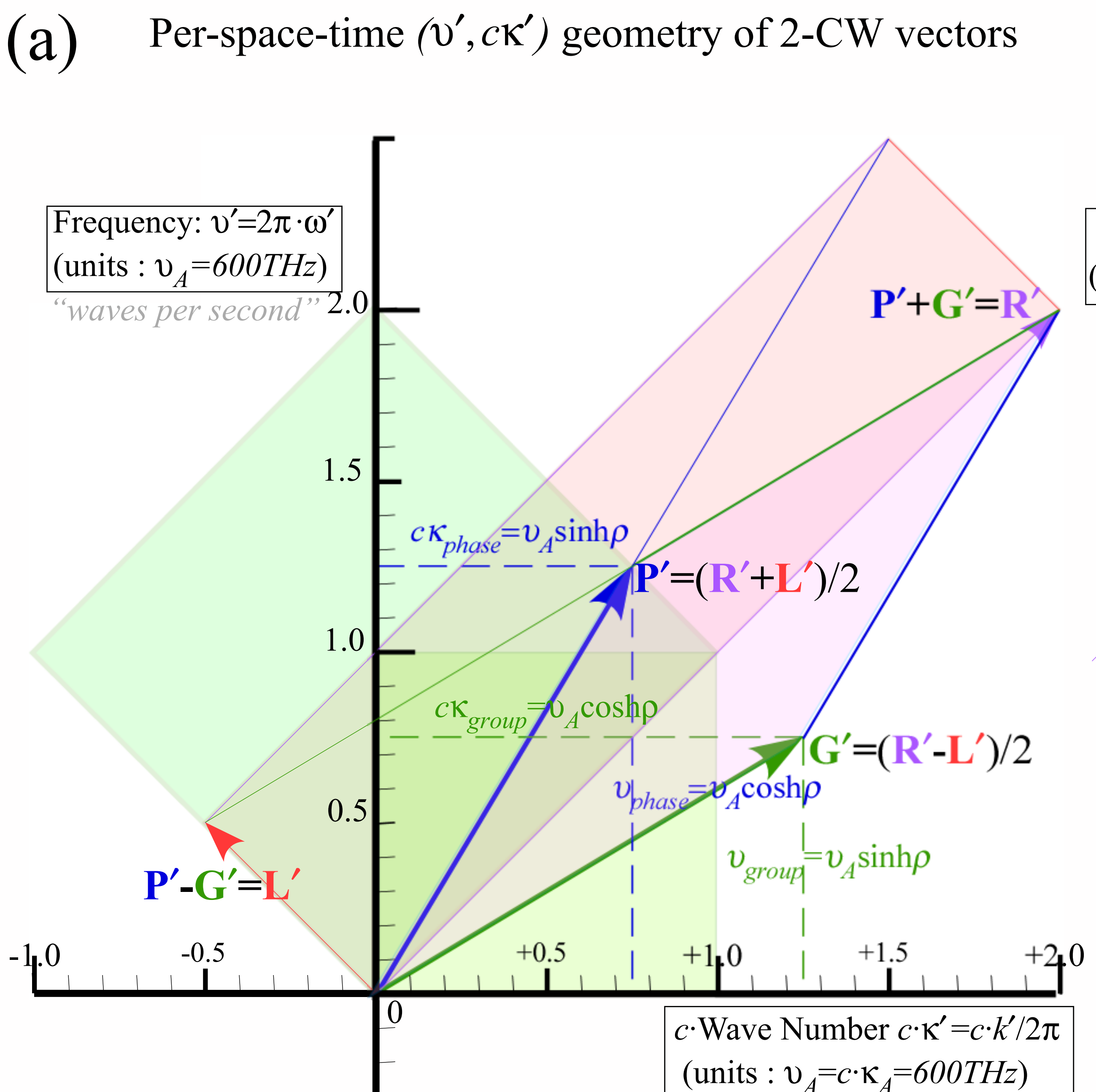
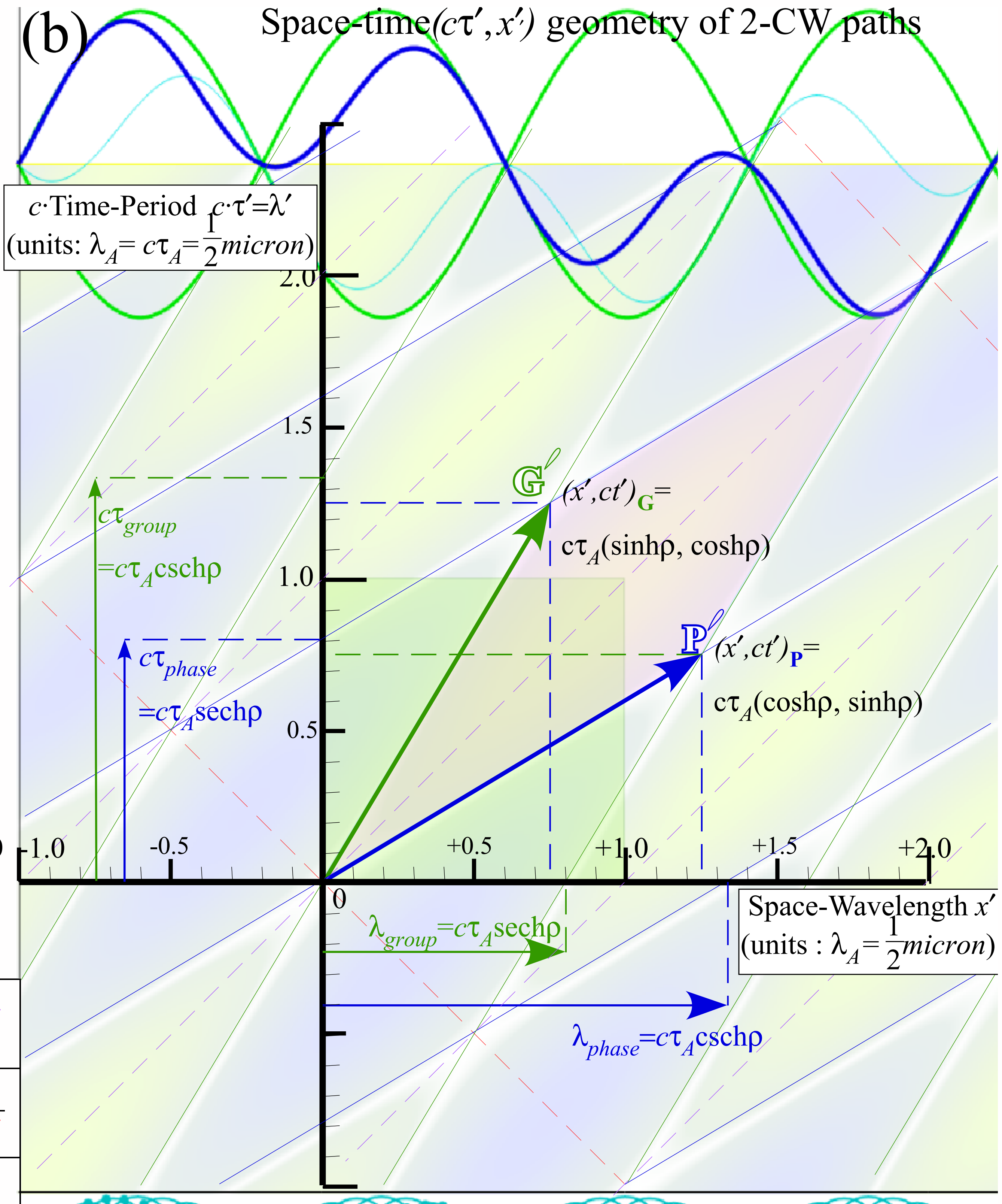




phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

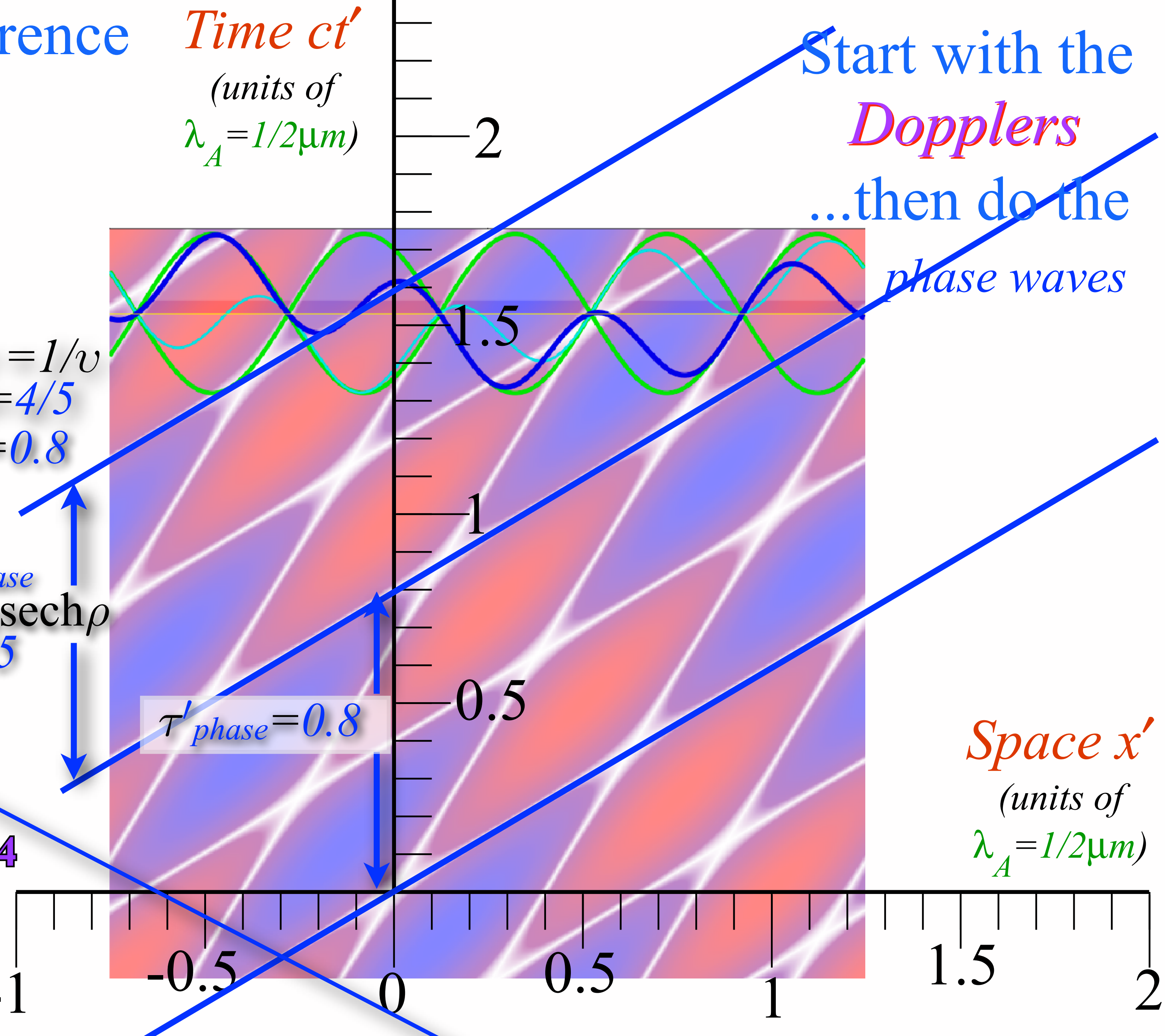
$$\mathbf{P'} = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

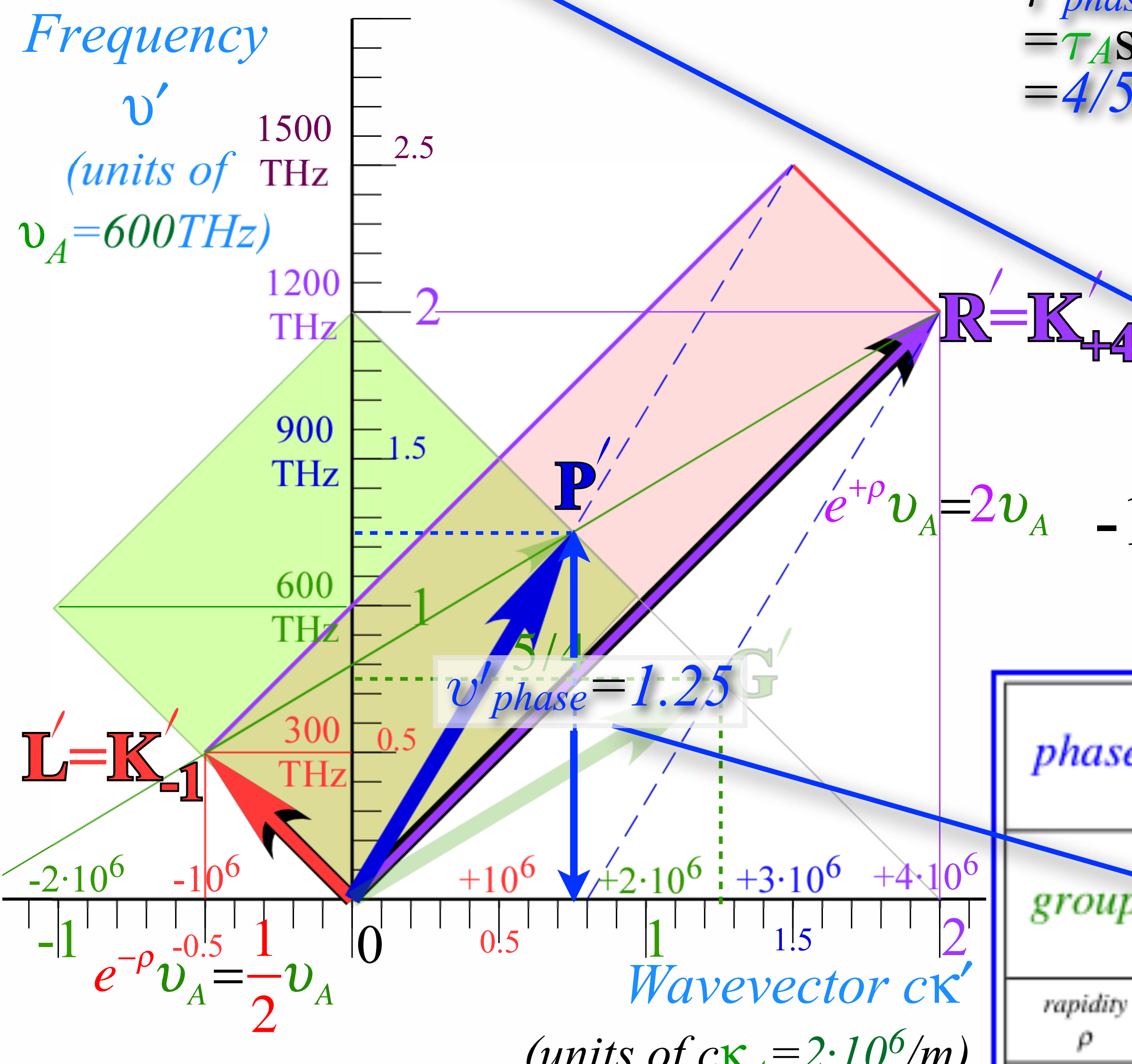
Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* ...then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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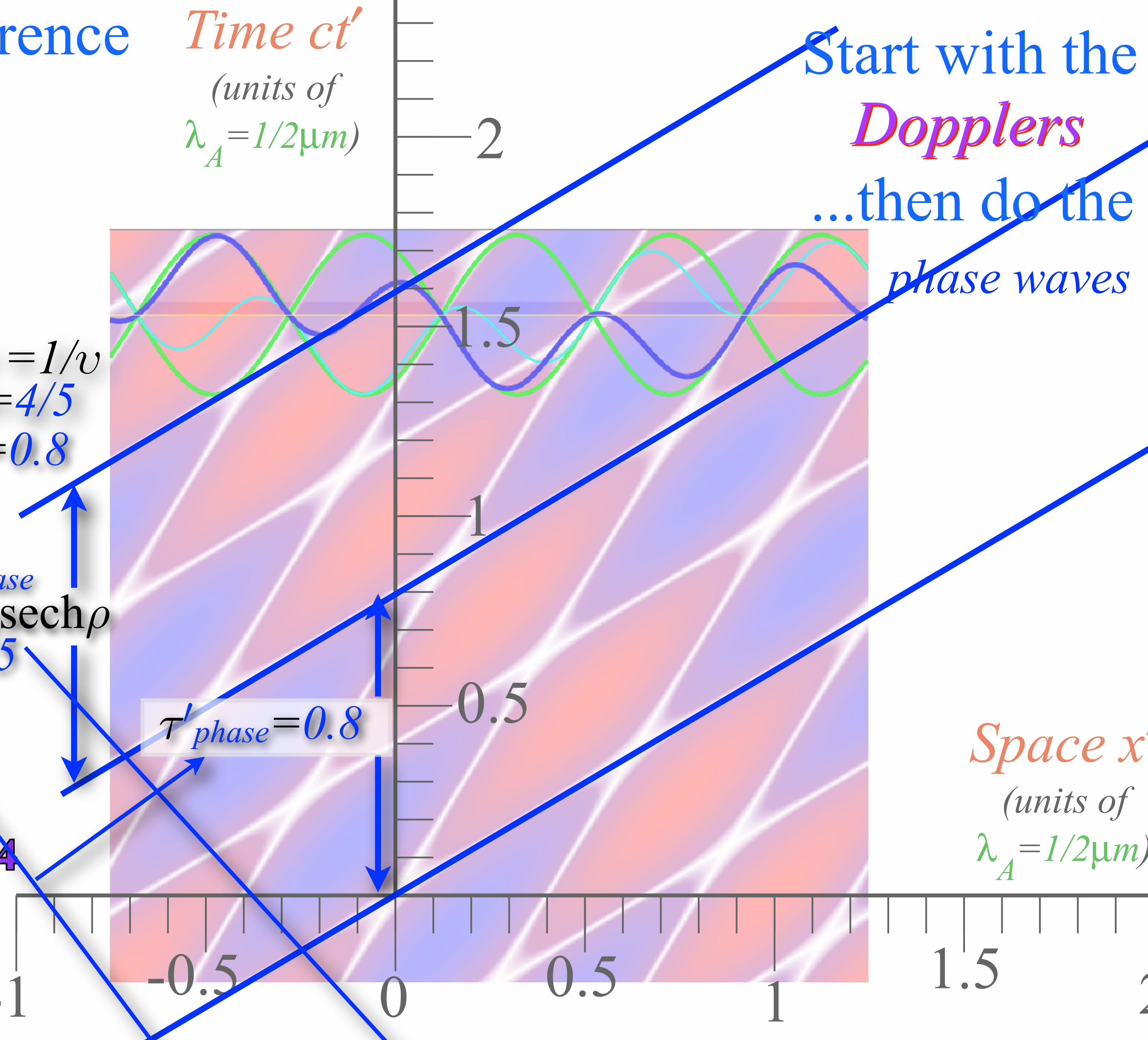
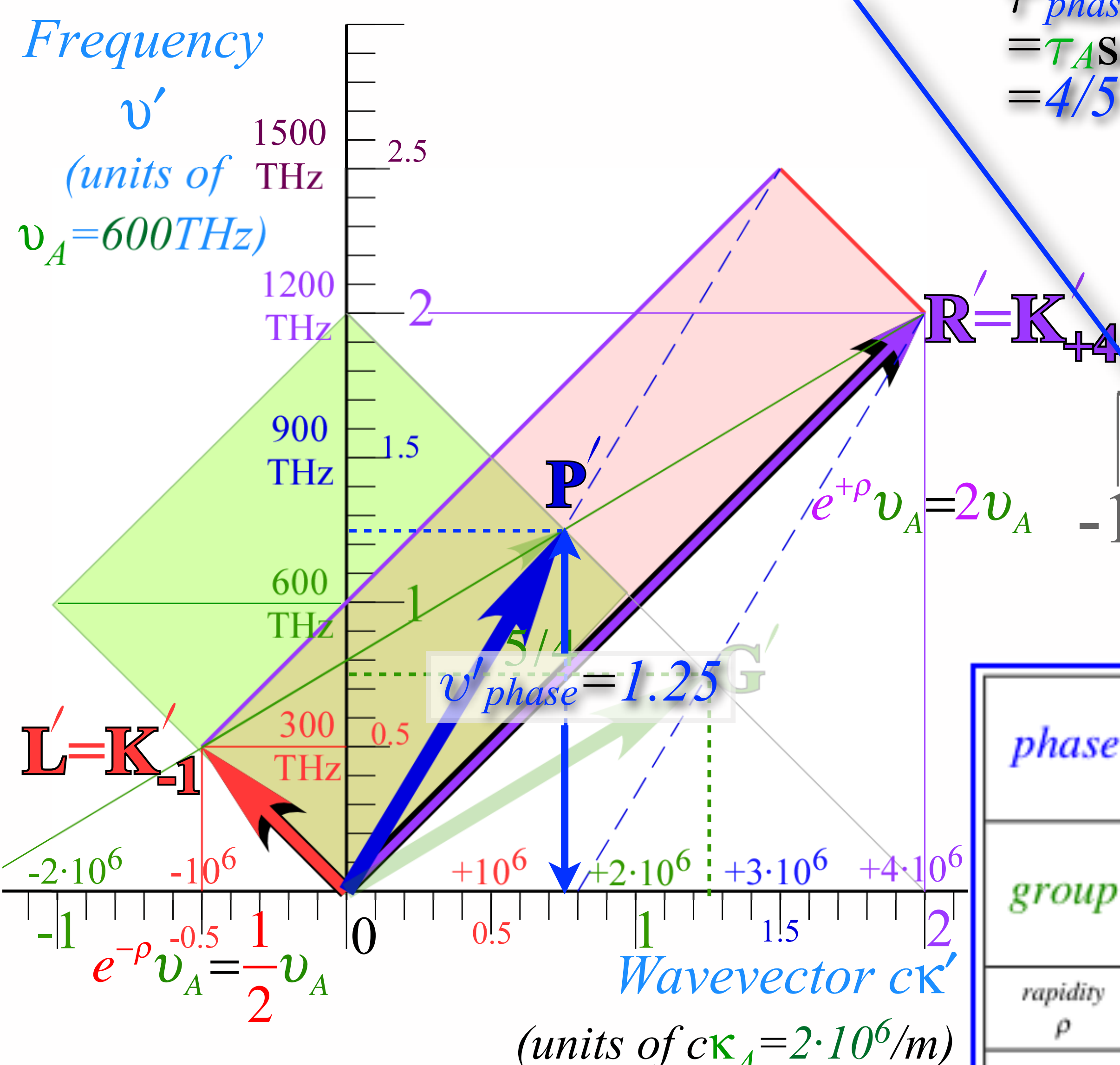
Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$ flips to *Phase period* $\tau = 1/\upsilon$: $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* ...then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

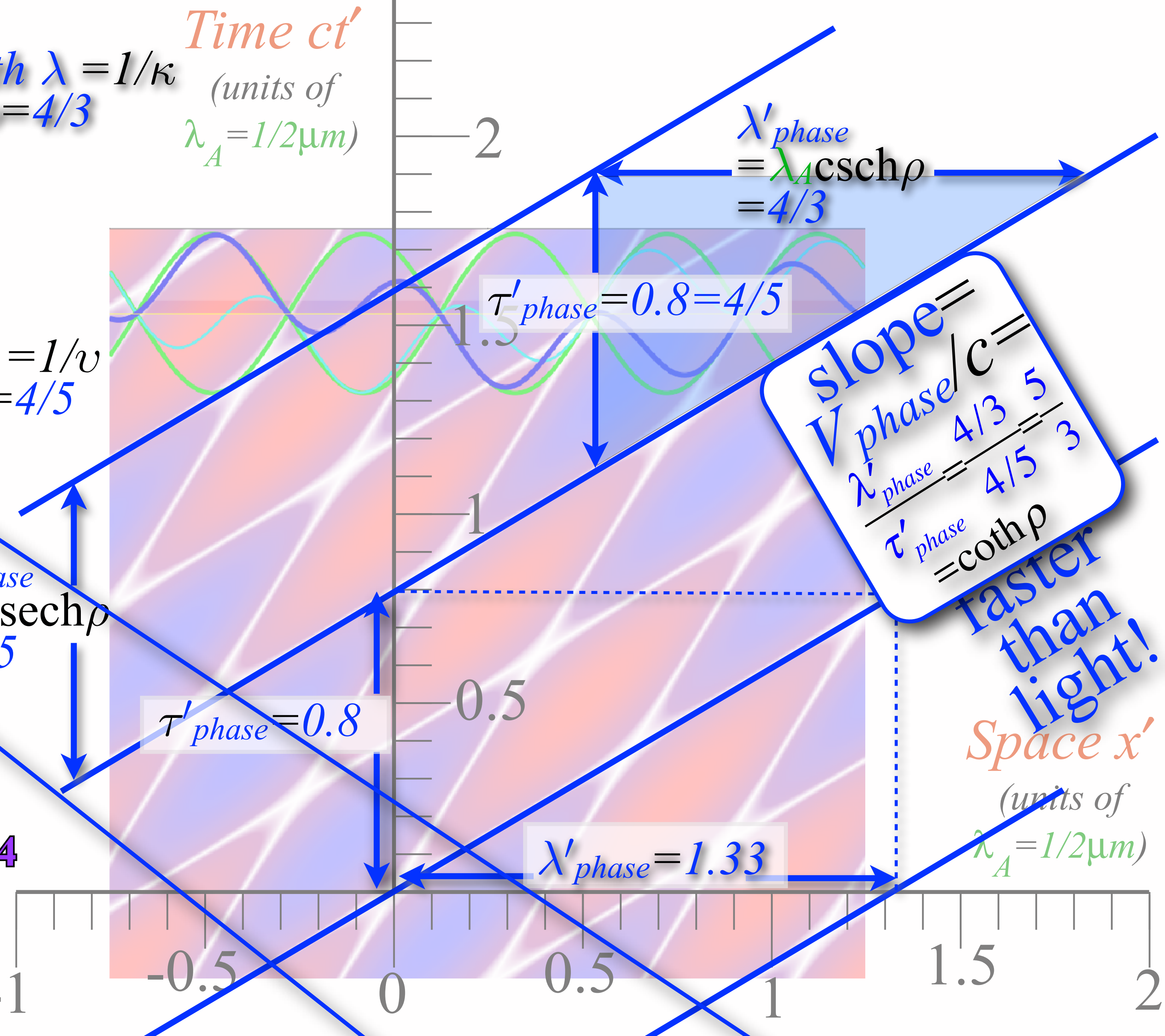
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

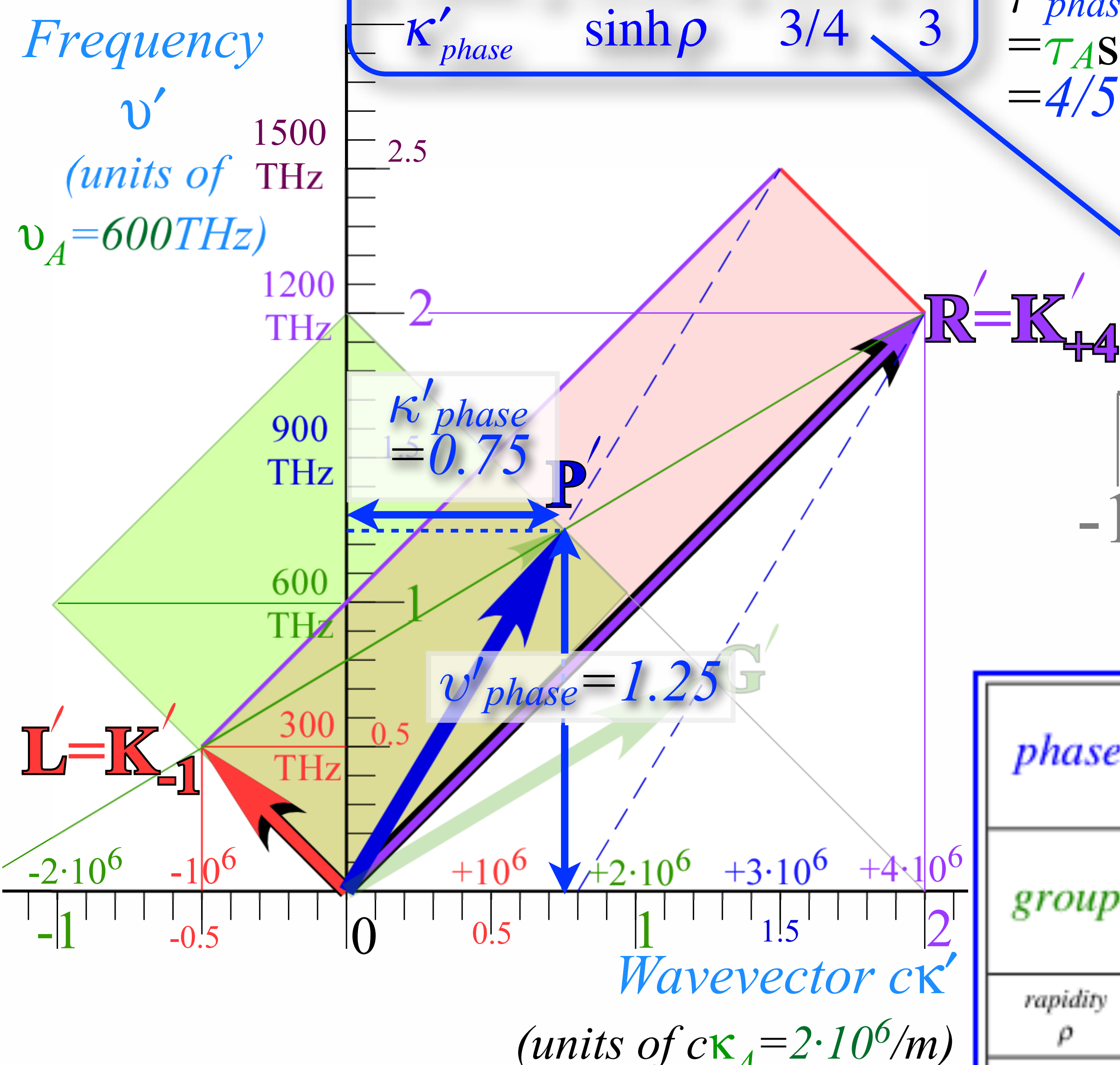
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

P-slope $= V_{phase}/c$

$$= \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

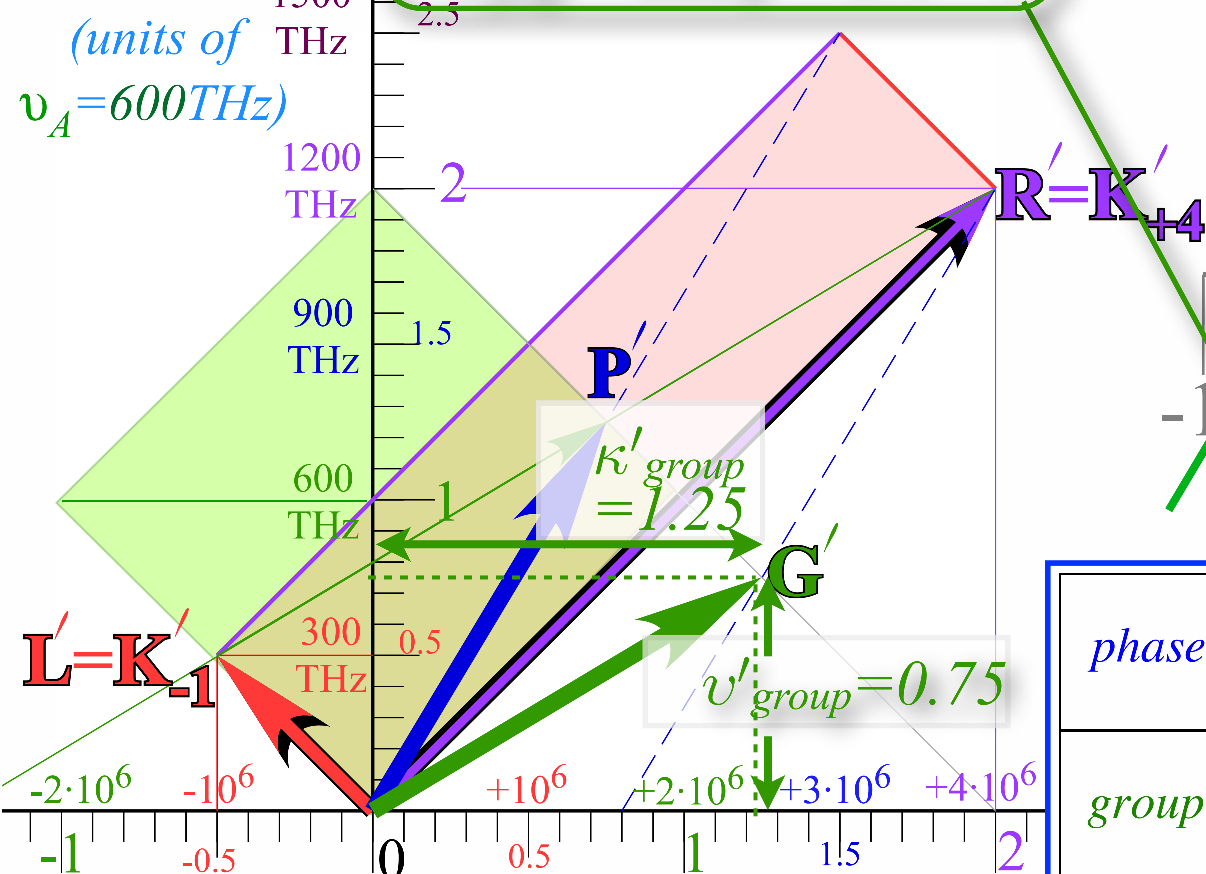
Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

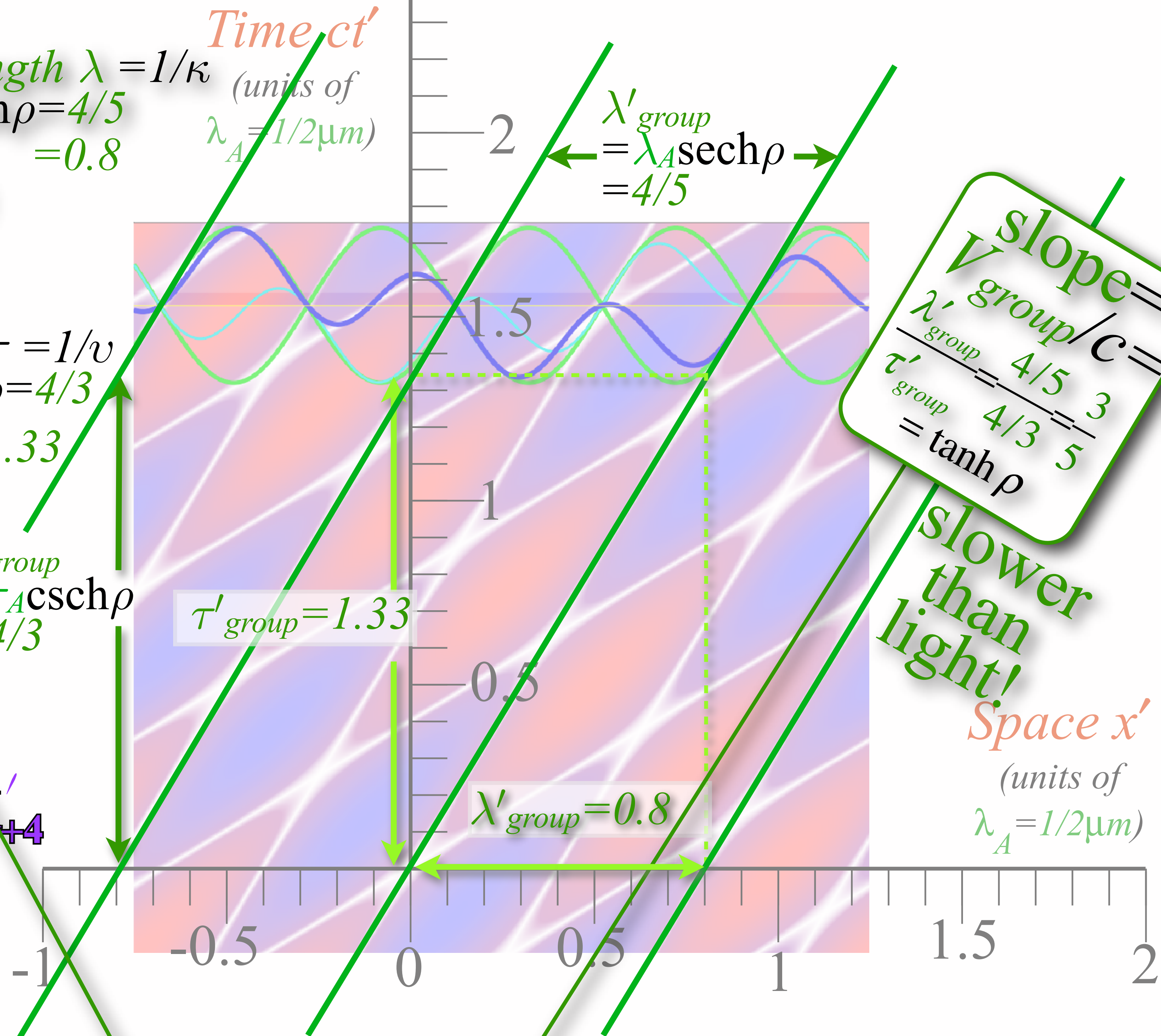
flips to

Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

G-slope $= V_{group}/c = \dfrac{\upsilon'_{group}}{\kappa'_{group}} = \dfrac{\sinh\rho}{\cosh\rho} = \dfrac{3/4}{5/4} = \dfrac{3}{5}$

$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3$





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group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

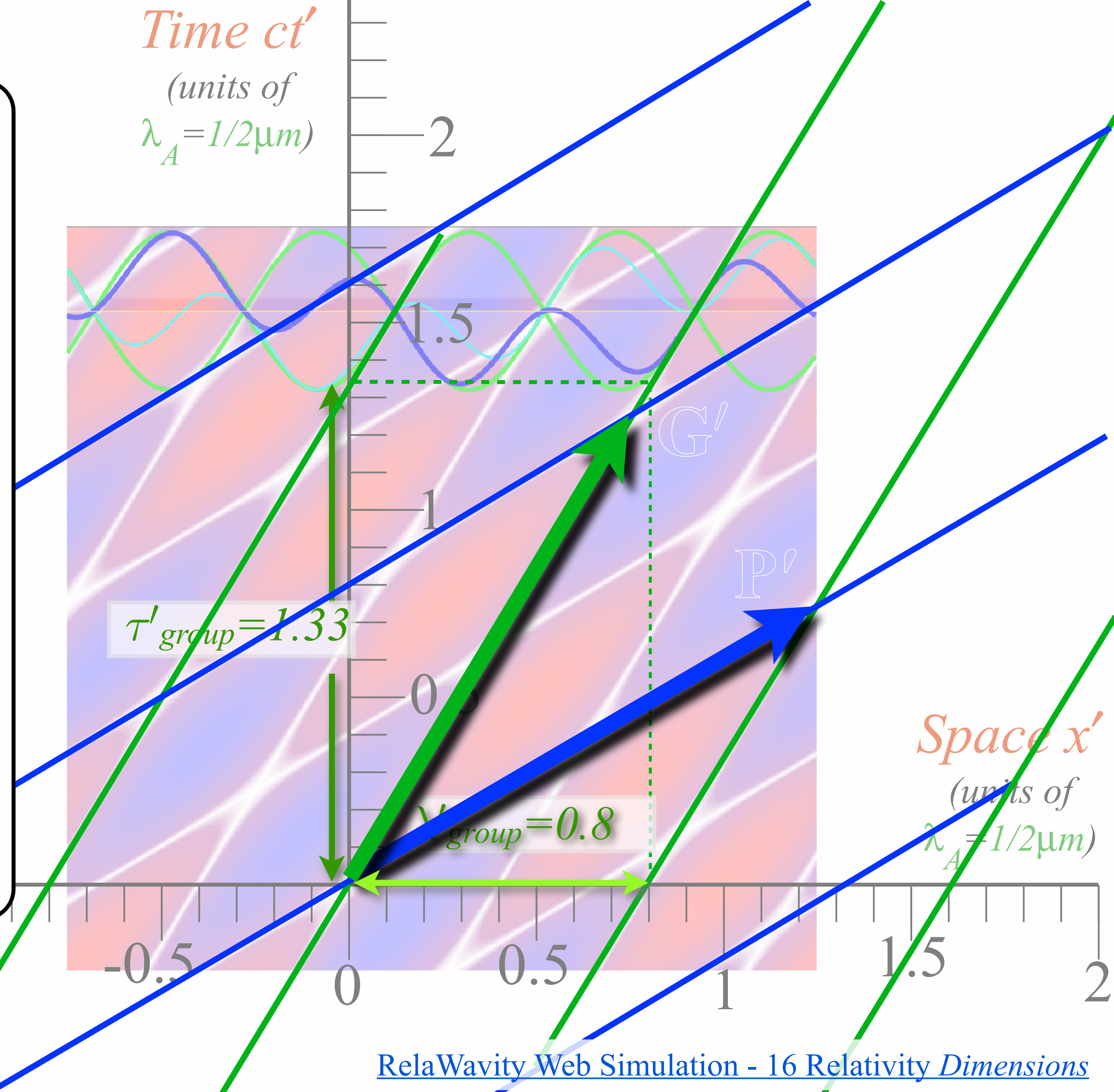
$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

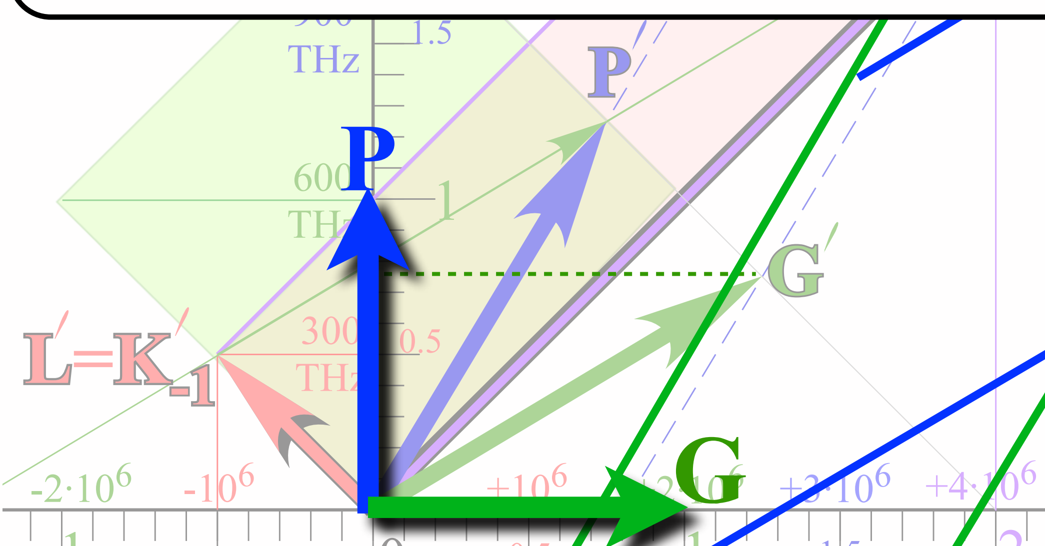
$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sinh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cosh\rho$$

$$\mathbf{P}' = \mathbf{G} \sinh\rho + \mathbf{P} \cosh\rho$$



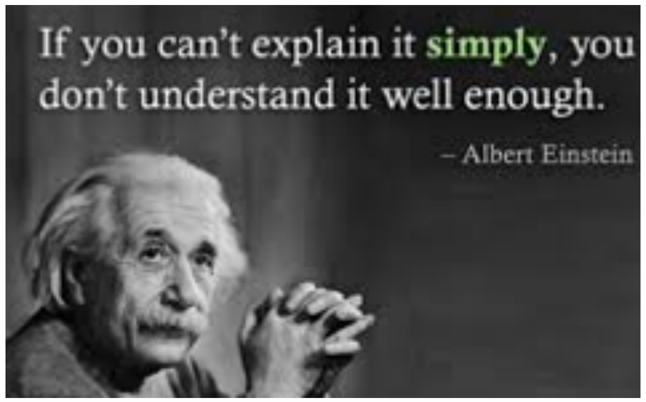


$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$ Lorentz transform matrix

RelaWavity Web Simulation - 16 Relativity *Dimensions*

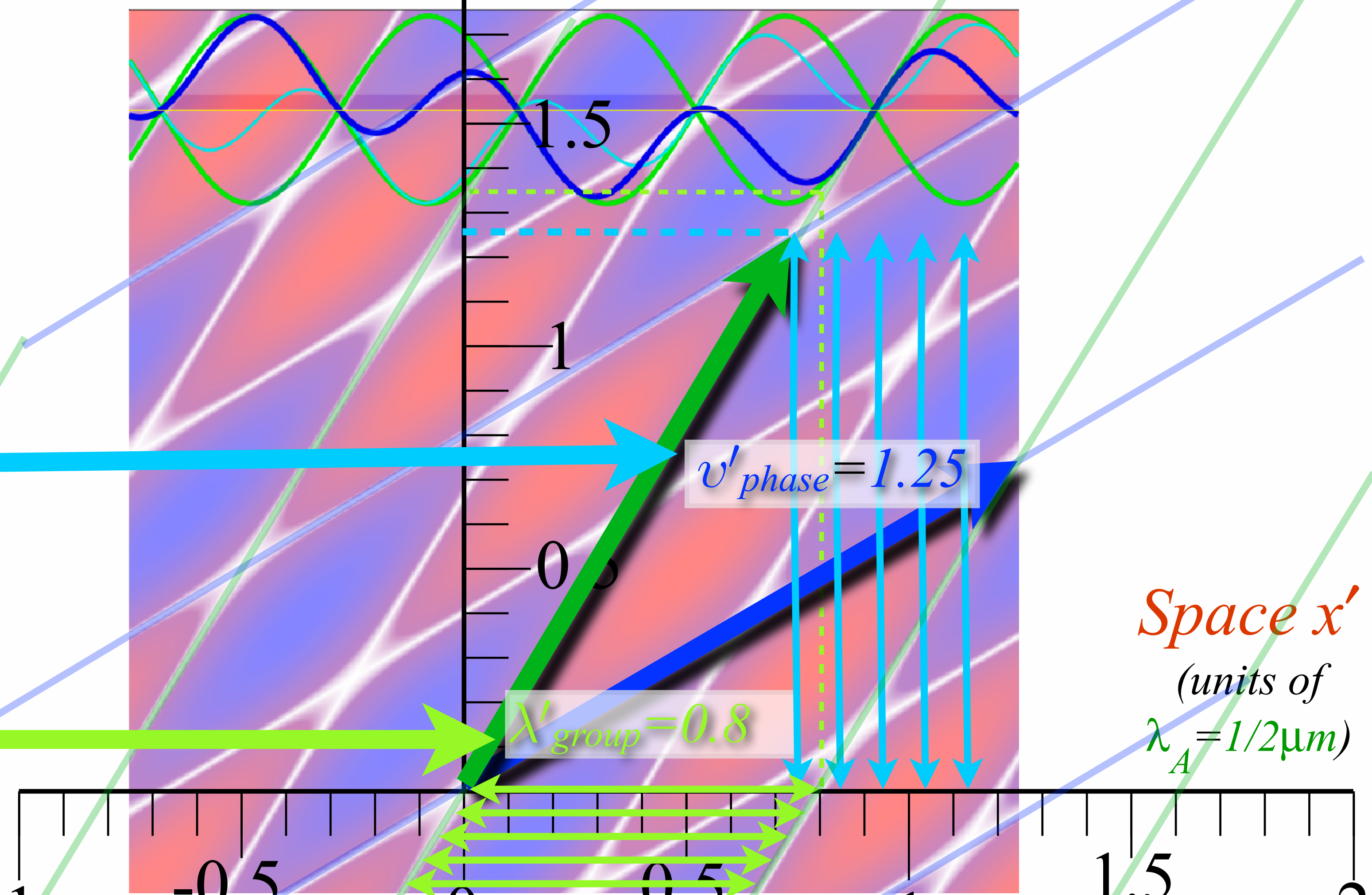
phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Two Famous-Name Coefficients



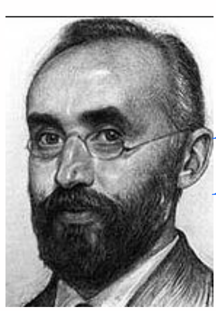
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation** (dilated by 25% here)

This number is called a: **Lorentz length-contraction** (contracted by 20% here)

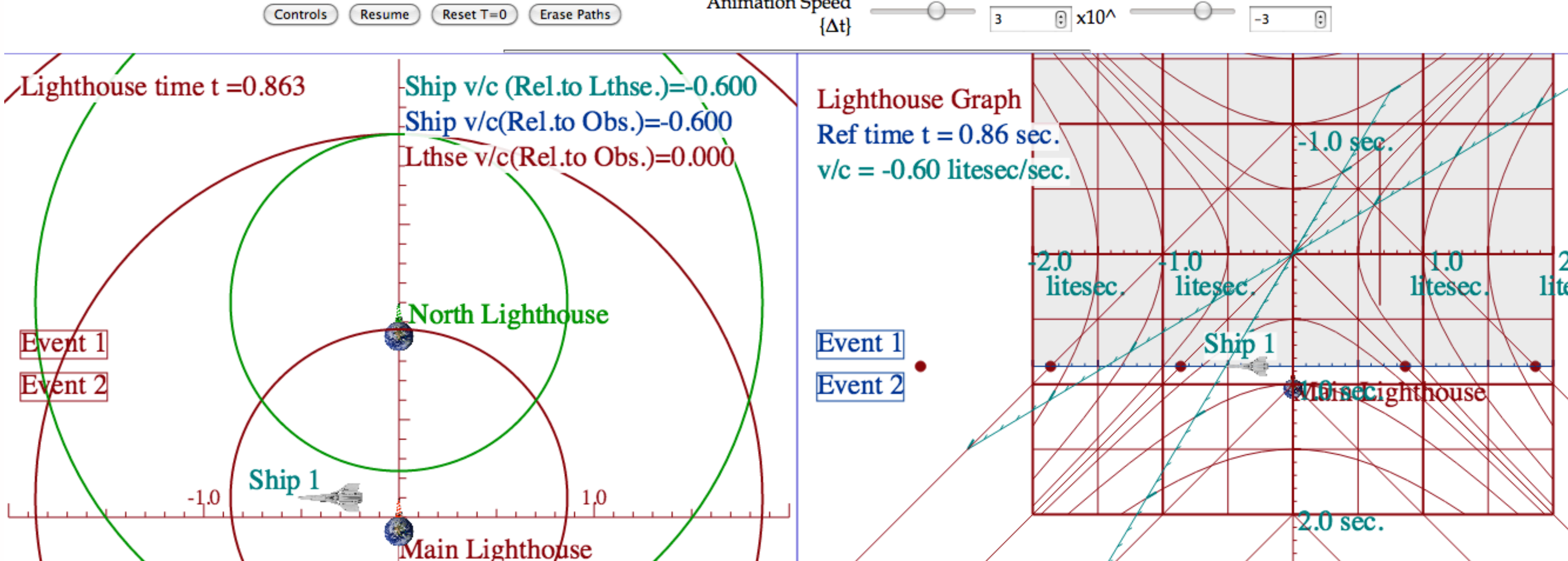


Hendrik A. Lorentz 1853-1928

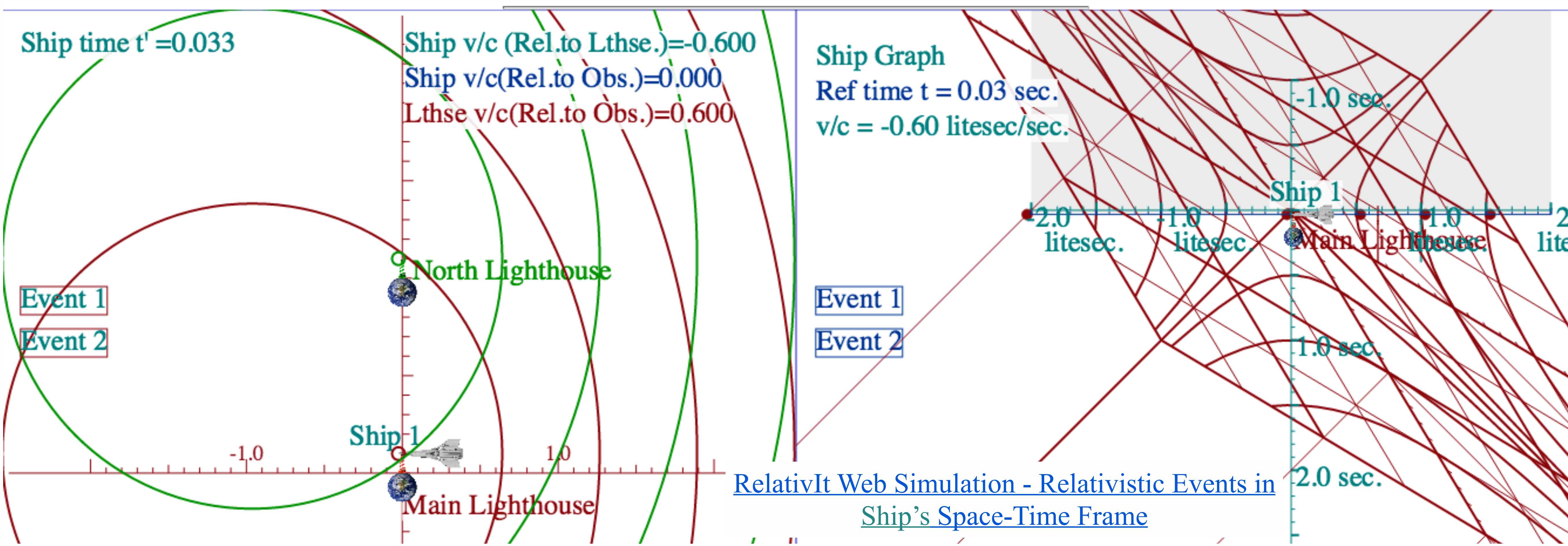
Old-Fashioned Notation

RelaWavity Web Simulation - Relativistic *Terms* (Expanded Table)

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0



RelativIt Web Simulation - Relativistic Events in Main Lighthouse's Space-Time Frame



RelativIt Web Simulation - Relativistic Events in Ship's Space-Time Frame

Learning about sin! *and* cos *and…Trigonometric road maps*

Hyper-Trigonometric algebra and phasors in space-time

1CW wavefunctions and phasors

Per-space-per-time vs Space-time

Wave velocity formulas

Introducing Doppler shifting

Why c is constant?!

Introducing Doppler Arithmetic and rapidity ρ

Optical interference "baseball-diamond" displays *phase* and *group* velocity

Details of 2CW wavefunctions in rest frame

Pulse waves (PW) versus Continuous Waves (CW)

Doppler shifted "baseball-diamond" displays Lorentz frame transformation

Analyzing wave velocity by per-space-per-time *and* space-time graphs

16 coefficients of relativistic 2CW interference

Two "famous-name" coefficients and the Lorentz transformation

➔ Thales geometry of Lorentz transformation

Rapidity ρ related to *stellar aberration angle σ* and L. C. Epstein's approach to relativity

Longitudinal hyperbolic *ρ*-geometry connects to transverse circular *σ*-geometry

"Occams Sword" and geometry of 16 parameter functions of *ρ* and *σ*

Application to TE-Waveguide modes

Learning about sin (and cos…tan…and trig. road maps)
Hyper-Trigonometric algebra and phasors in space-time
Single Continuous Wave (1CW) functions and phasors

Per-space-per-time vs Space-time
Wave velocity formulas

Introducing Doppler shifting
Why is $c=299,792,458\ m\cdot s^{-1}$ constant?!
Introducing Doppler Arithmetic and rapidity ρ

For an introductory, web based development of this and other concepts in special relativity see our entrant in the 2005 Pirelli Challenge*:*

A Colorful Road to Relativity Using Occam's Razors and Evenson's Lasers

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Two "famous-name" coefficients and Lorentz transform
Doppler Jeopardy" and Thales geometry of Lorentz transformation

Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency ω_E?

(2.) What is that frequency ω_E ?

Doppler Jeopardy

$\omega_R = 2\pi\upsilon_R$ υ_R=600THz

υ_L=300THz $\omega_L = 2\pi\upsilon_L$

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency υ_E?

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Doppler Jeopardy

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Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\upsilon_{group}}{\kappa_{group}} = \frac{\upsilon_R - \upsilon_L}{\kappa_R - \kappa_L} = c\,\frac{\upsilon_R - \upsilon_L}{\upsilon_R + \upsilon_L}$$

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Query (2.) similarly: What υ_E is blue-shift $b\upsilon_L$ of υ_L *and* red-shift υ_R/b of υ_R?

$$\upsilon_E = b\,\upsilon_L = \upsilon_R/b \quad \Rightarrow \quad b = \sqrt{\upsilon_R/\upsilon_L} \quad \Rightarrow \quad \upsilon_E = \sqrt{\upsilon_R \cdot \upsilon_L}$$

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Geometric mean

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↑

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$$\upsilon_E = \sqrt{\upsilon_R \cdot \upsilon_L} = \sqrt{180000} = 424$$

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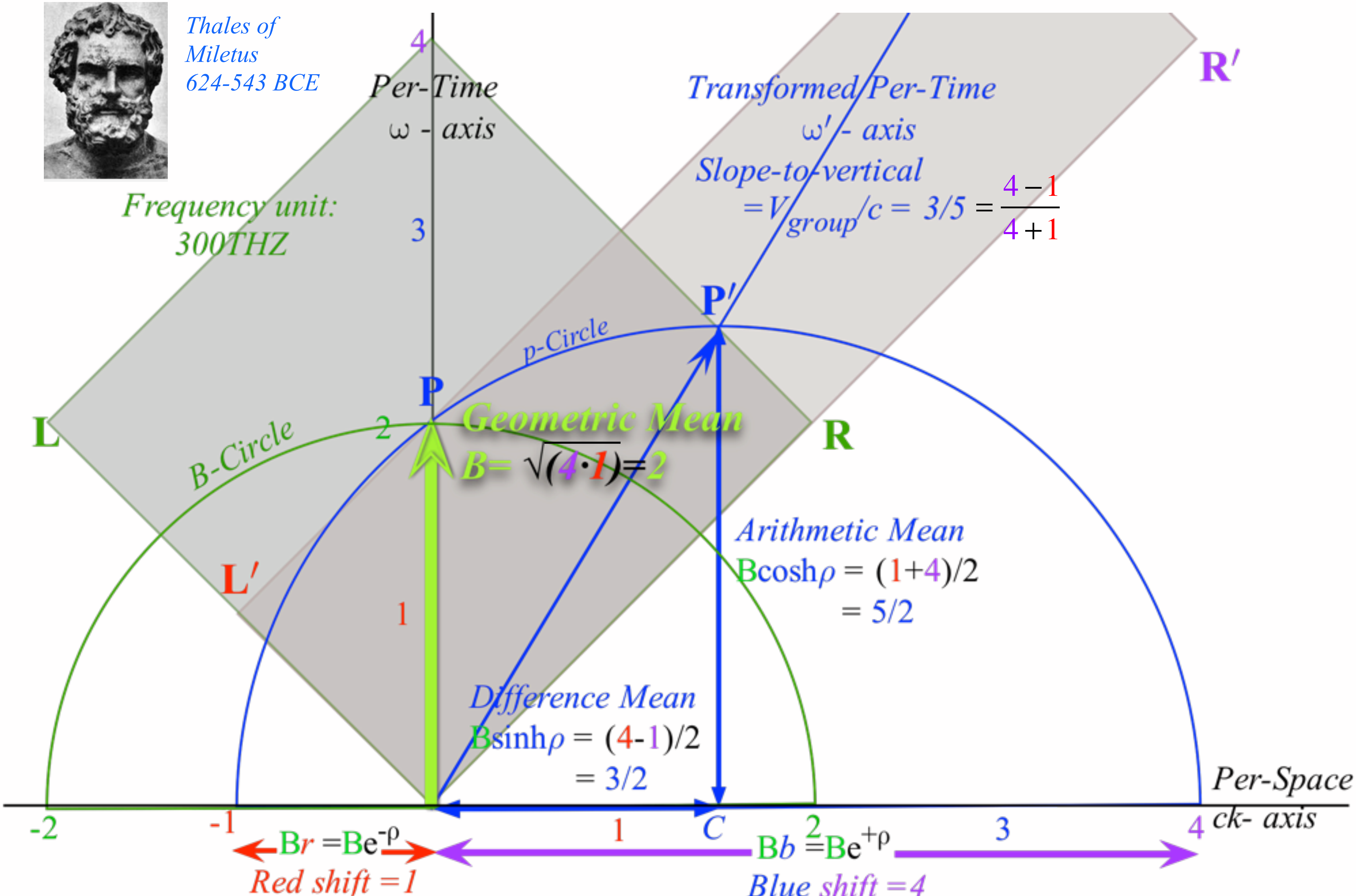
$$\upsilon_E = \sqrt{\upsilon_R \cdot \upsilon_L} = \sqrt{180000} = 424$$

V_{group}/c is ratio of difference mean $\upsilon_{group} = \frac{\upsilon_R - \upsilon_L}{2}$ to arithmetic mean $\upsilon_{phase} = \frac{\upsilon_R + \upsilon_L}{2}$. Frequency $\upsilon_E = B$ is the geometric mean $\sqrt{\upsilon_R \cdot \upsilon_L}$ of left and right-moving frequencies defining the geometry

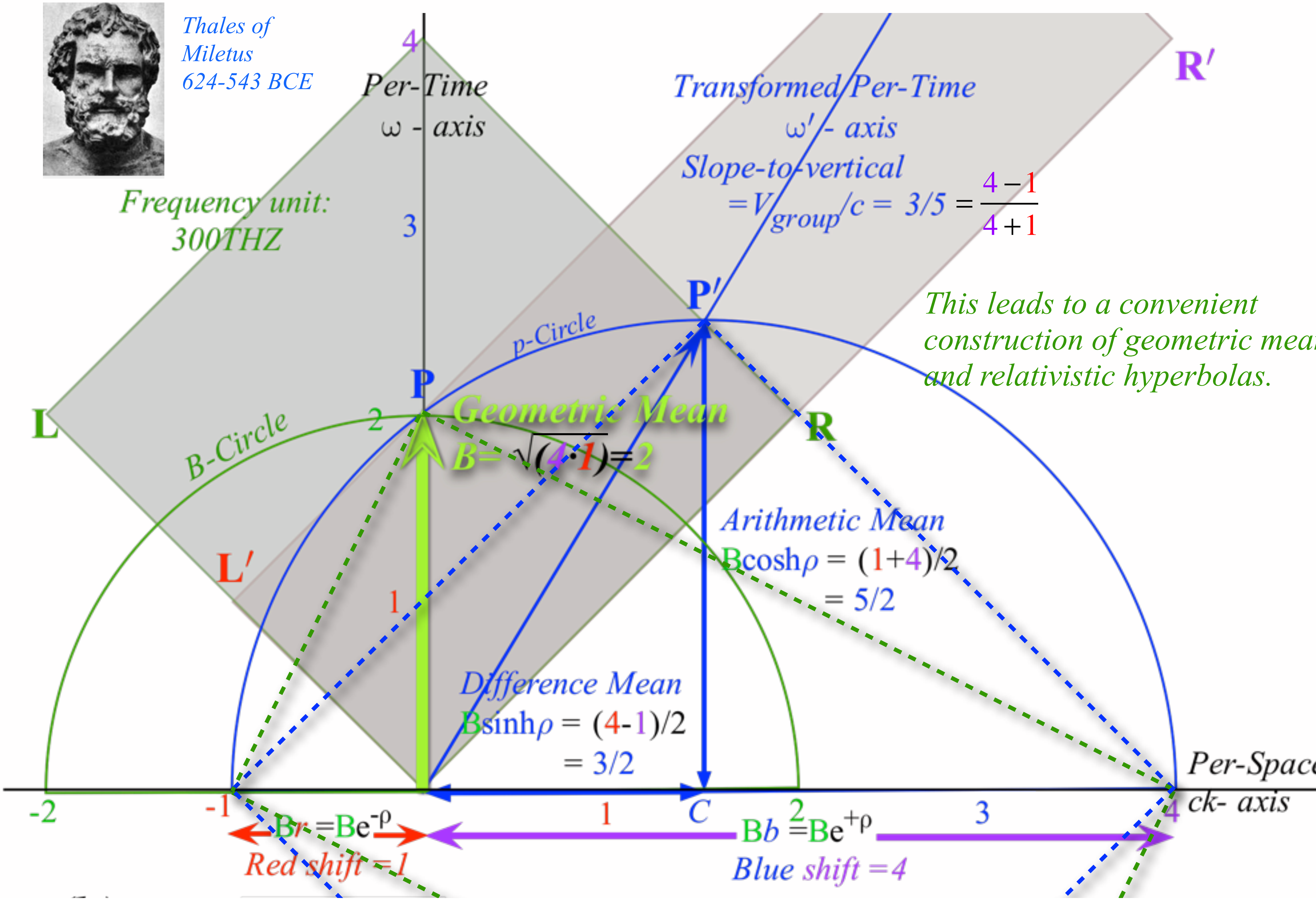
Thales Mean Geometry (600BCE)

helps "Relawavity"



Thales Mean Geometry (600BCE)

helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*



This leads to a convenient construction of geometric means and relativistic hyperbolas.

Learning about sin! *and* cos *and…Trigonometric road maps*

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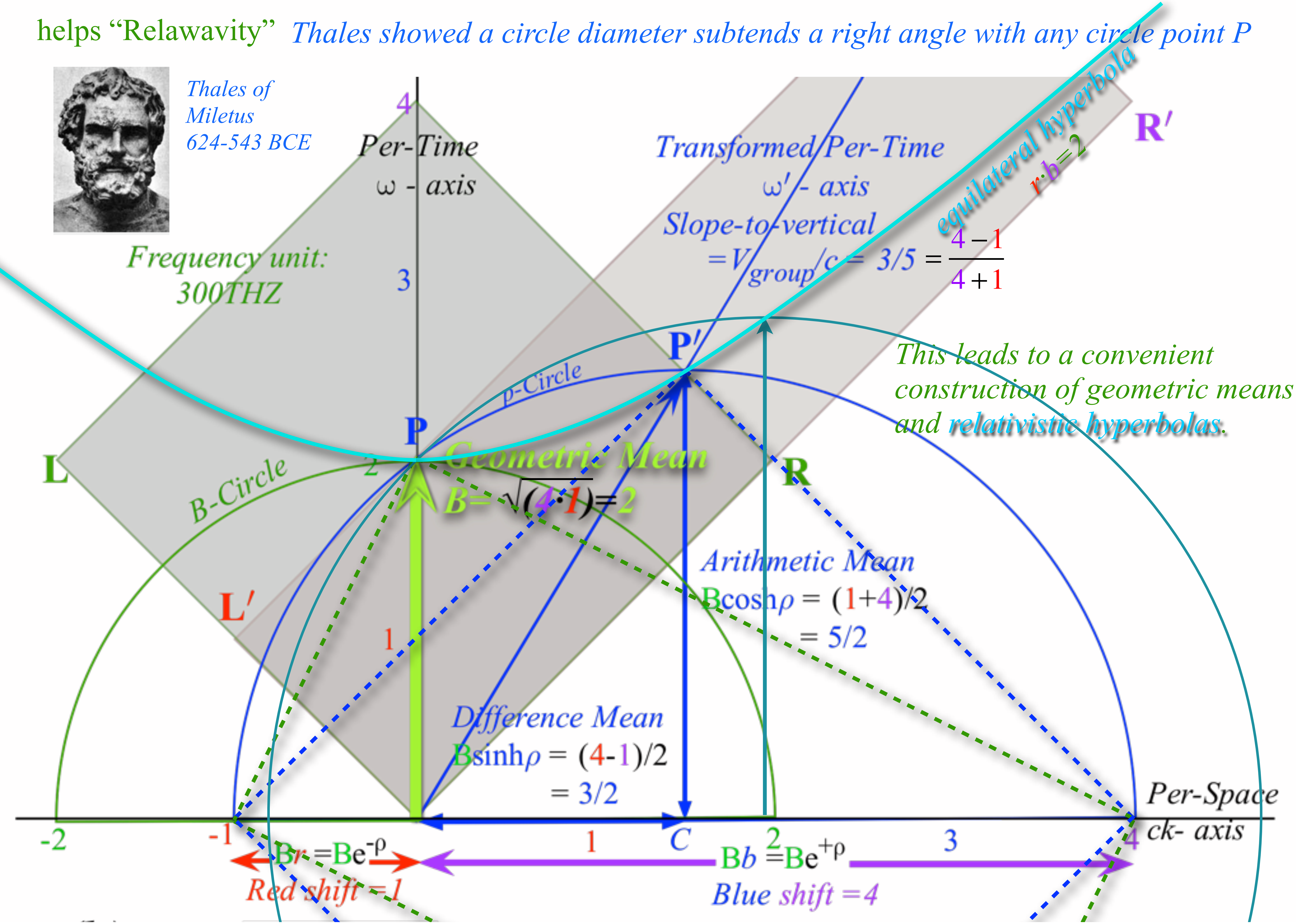

Thales Mean Geometry (600BCE)

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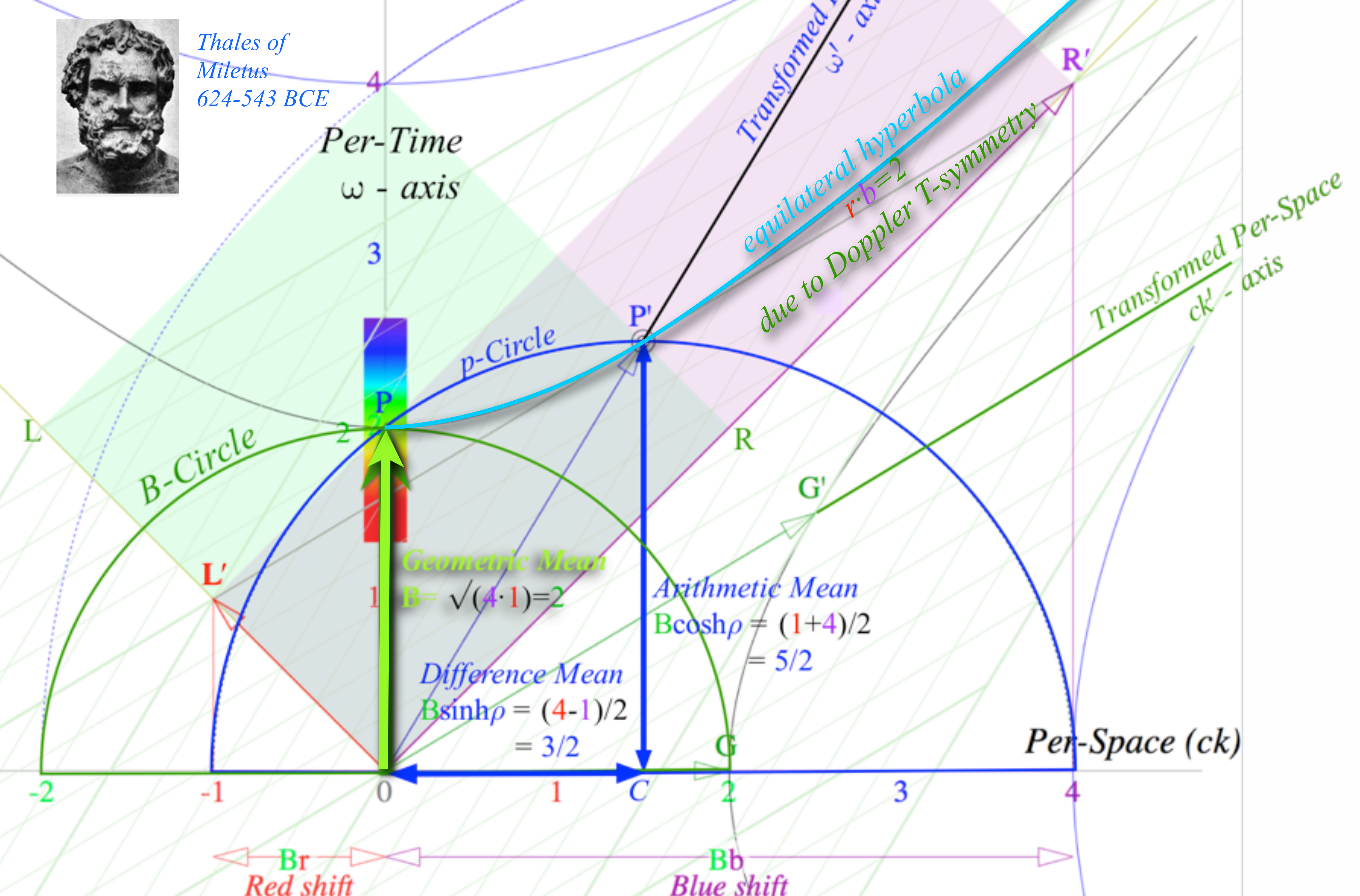

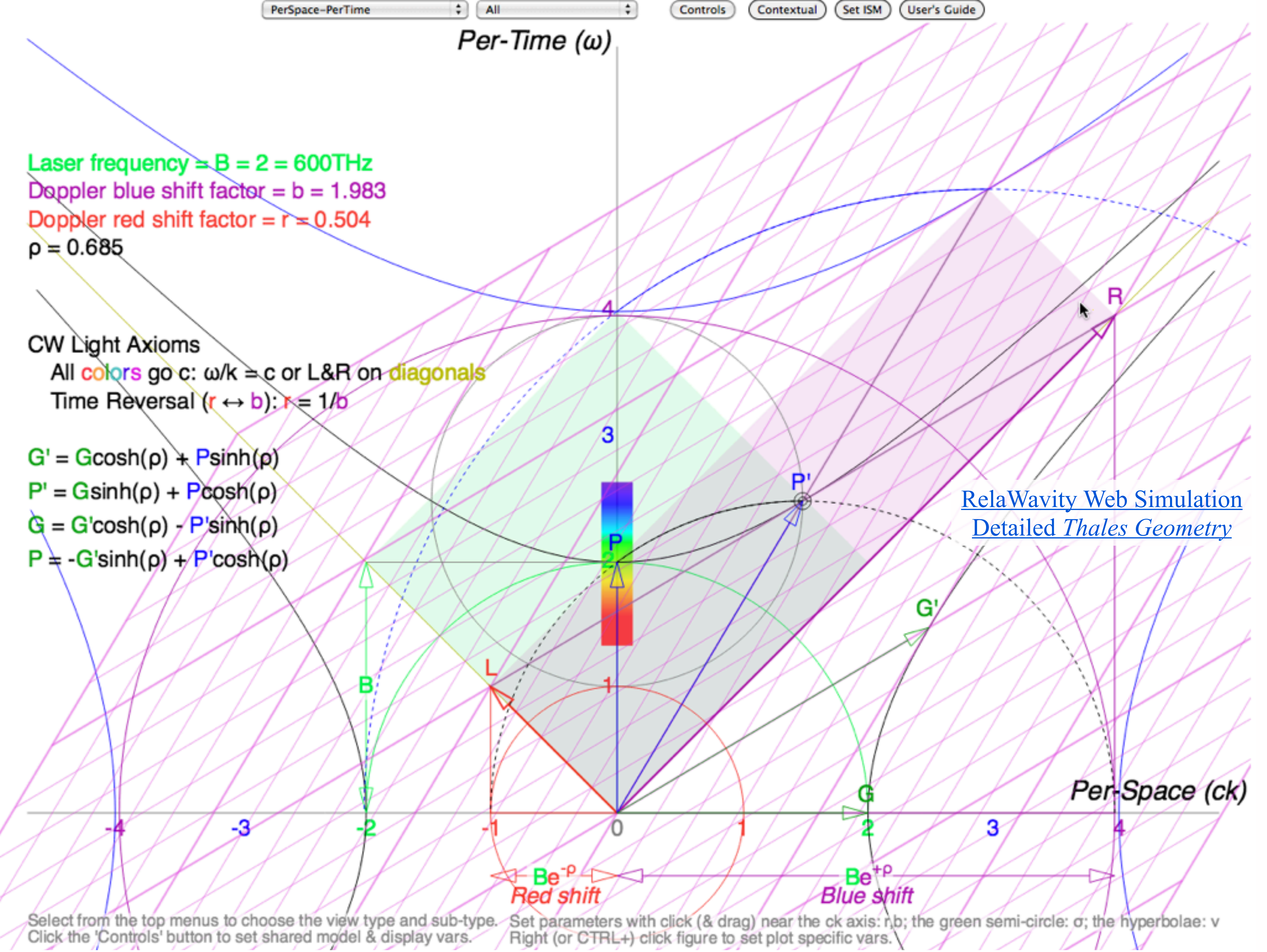
PerSpace-PerTime
All
Controls
Contextual
Set ISM
User's Guide
Per-Time (ω)
Laser frequency = B = 2 = 600THz
Doppler blue shift factor = b = 1.983
Doppler red shift factor = r = 0.504
ρ = 0.685
CW Light Axioms
All colors go c: ω/k = c or L&R on diagonals
Time Reversal (r ↔ b): r = 1/b
G' = Gcosh(ρ) + Psinh(ρ)
P' = Gsinh(ρ) + Pcosh(ρ)
G = G'cosh(ρ) - P'sinh(ρ)
P = -G'sinh(ρ) + P'cosh(ρ)
R
P'
P
G'
L
B
G
RelaWavity Web Simulation
Detailed Thales Geometry
Per-Space (ck)
-4
-3
-2
-1
0
1
2
3
4
$Be^{-\rho}$
Red shift
$Be^{+\rho}$
Blue shift
Select from the top menus to choose the view type and sub-type.
Click the 'Controls' button to set shared model & display vars.
Set parameters with click (& drag) near the ck axis: r,b; the green semi-circle: σ; the hyperbolae: v
Right (or CTRL+) click figure to set plot specific vars.

Learning about sin! *and* cos *and…Trigonometric road maps*

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➔ Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry

"Occams Sword" and geometry of 16 parameter functions of ρ and σ

Application to TE-Waveguide modes

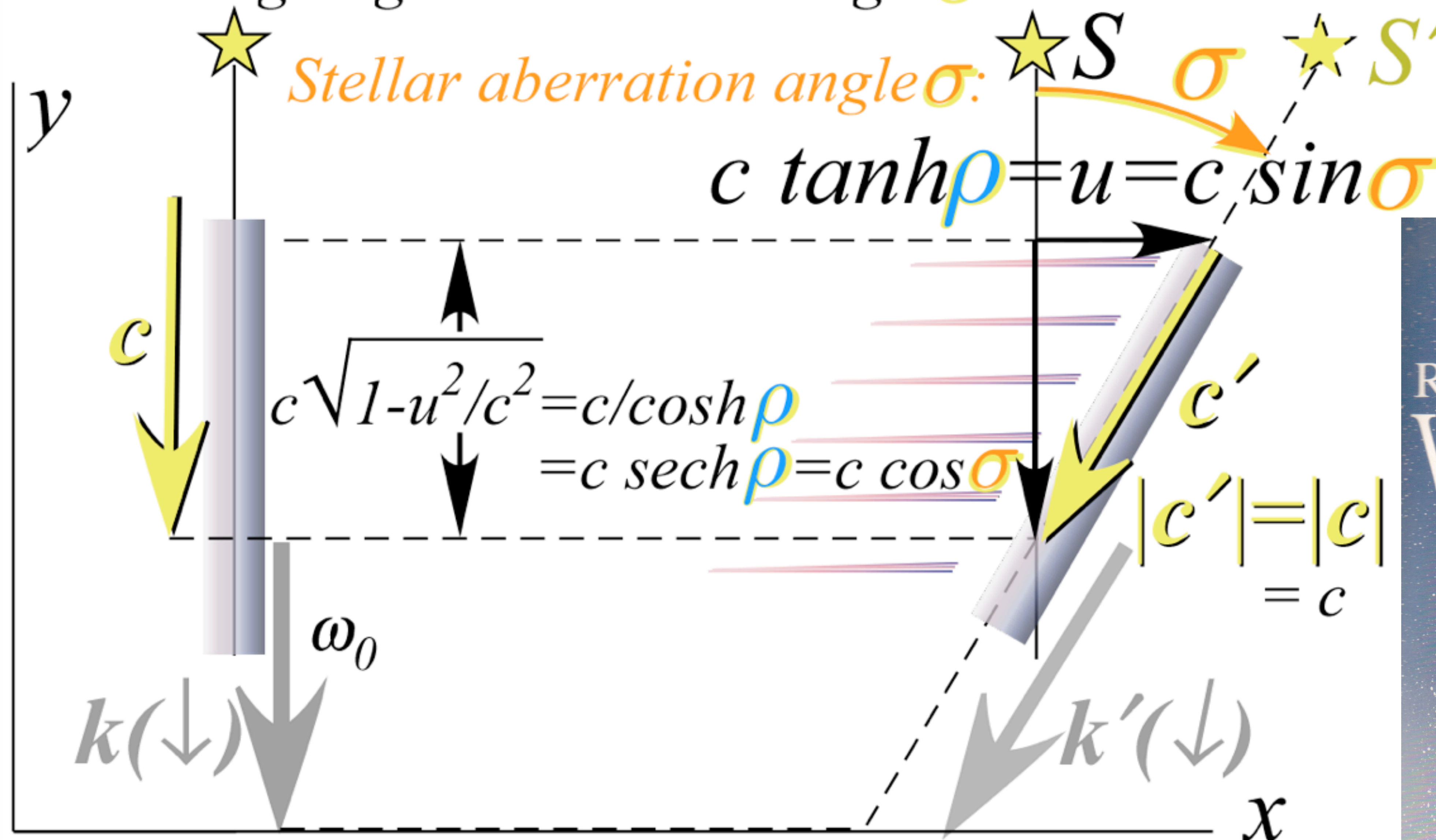
Comparing Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

to a Transverse*relativity parameter: Stellar aberration angle σ

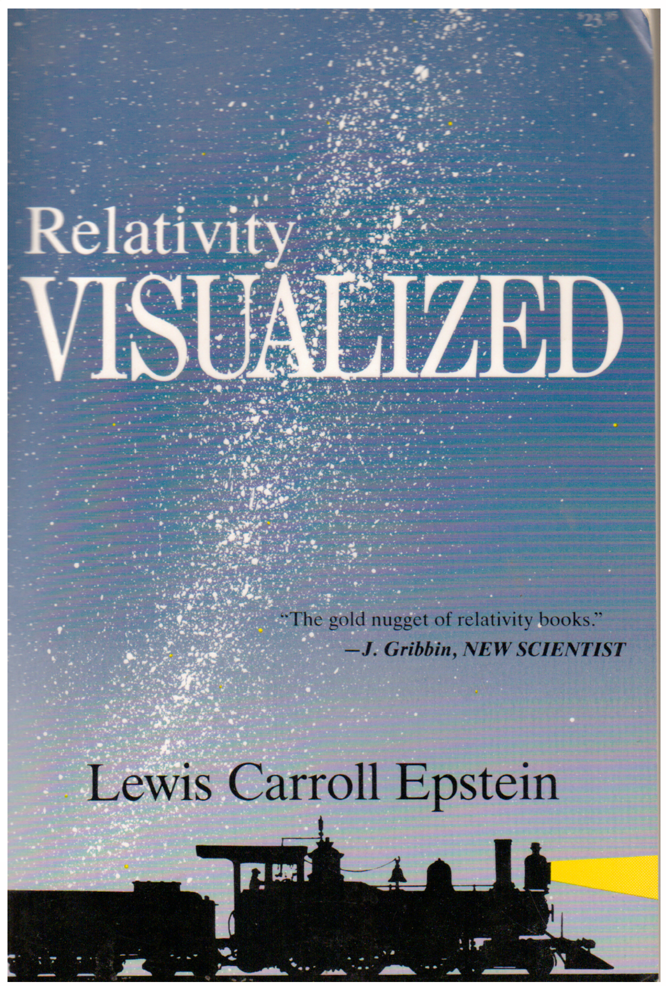
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



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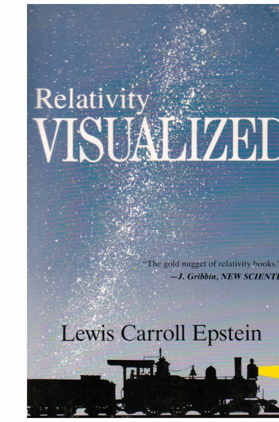
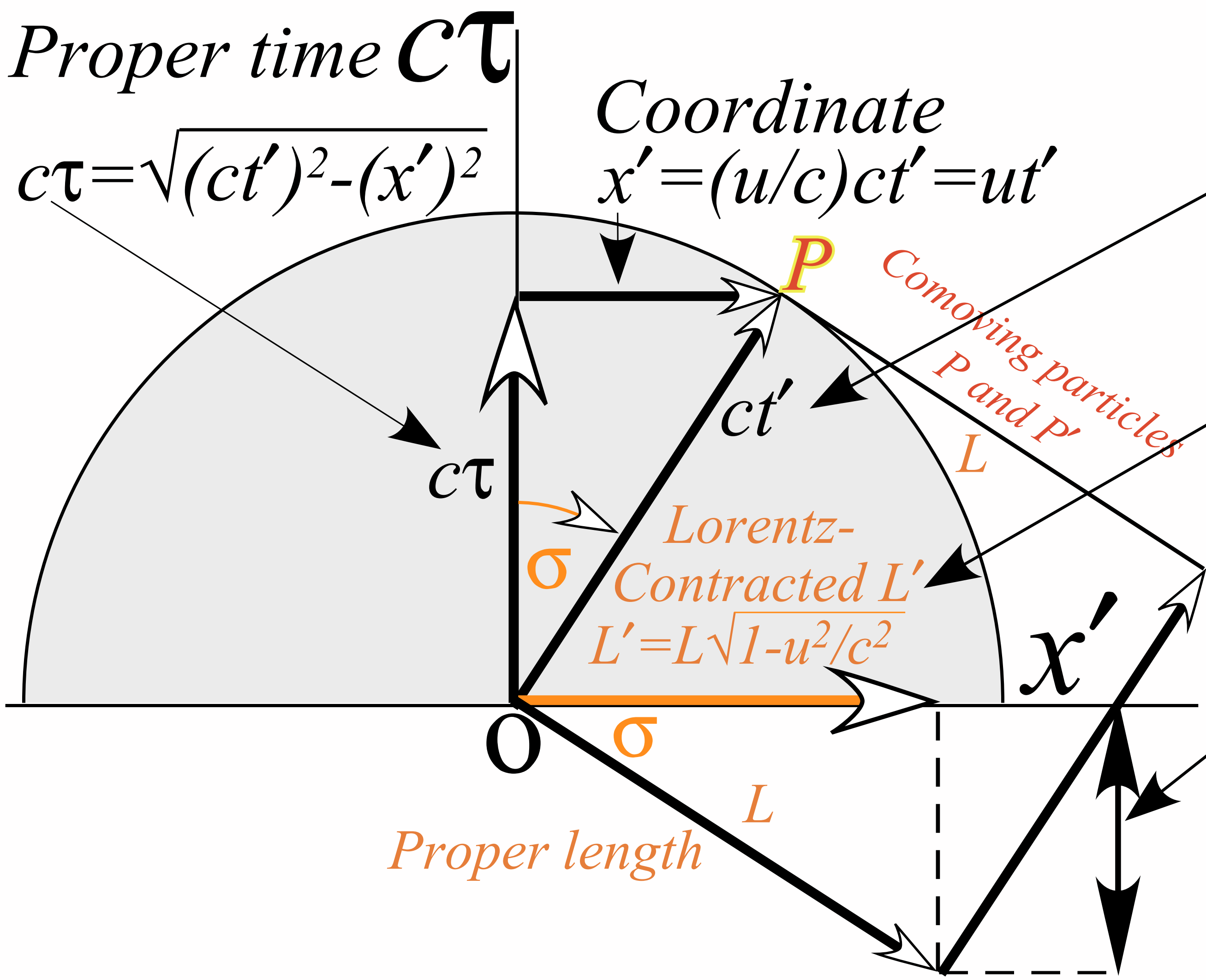


Proper time $c\tau$ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x', ct') and speed c in $(x, c\tau)$



Einstein time dilation:

$$ct' = c\tau \sec\sigma = c\tau \cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L' = L\,\text{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

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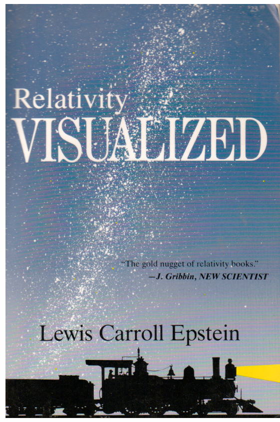
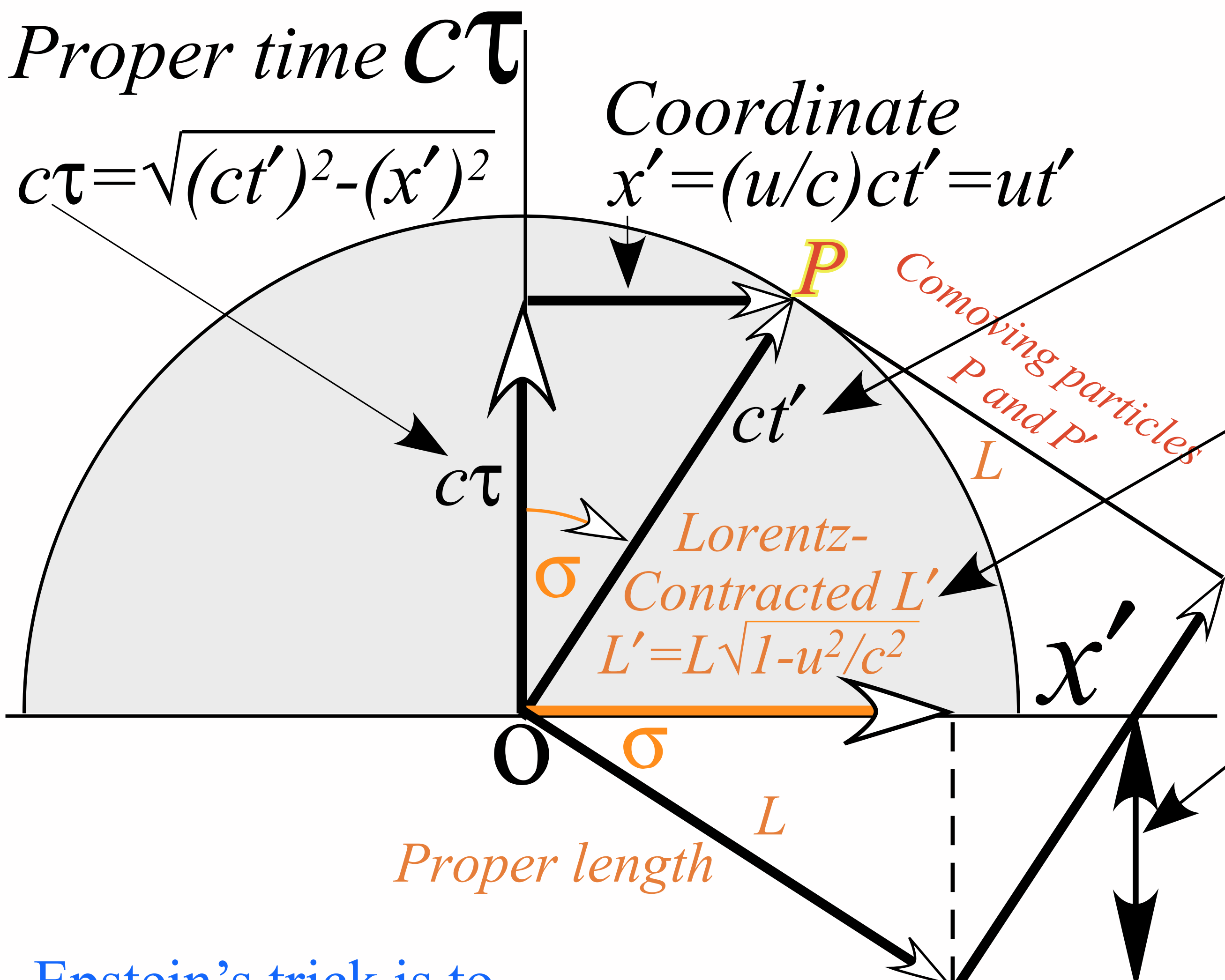


Proper time cτ vs. coordinate space x - *(L. C. Epstein's "Cosmic Speedometer")*

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



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$$= L \cos\sigma \tan\sigma$$
$$= L \sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed *c* through $(x', c\tau)$ space!

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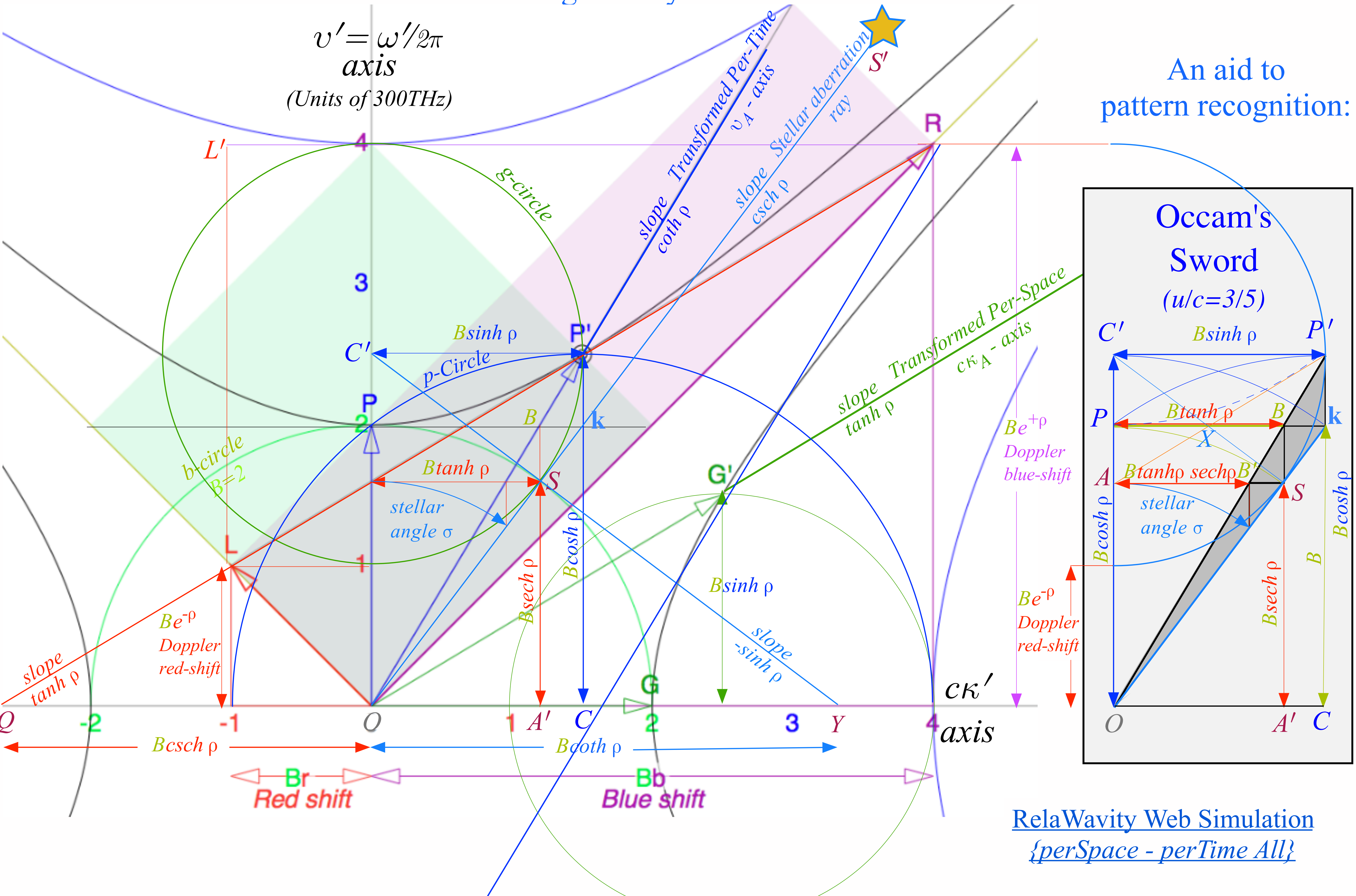
➔ Application to TE-Waveguide modes

Summary of optical wave parameters for relativity and QM

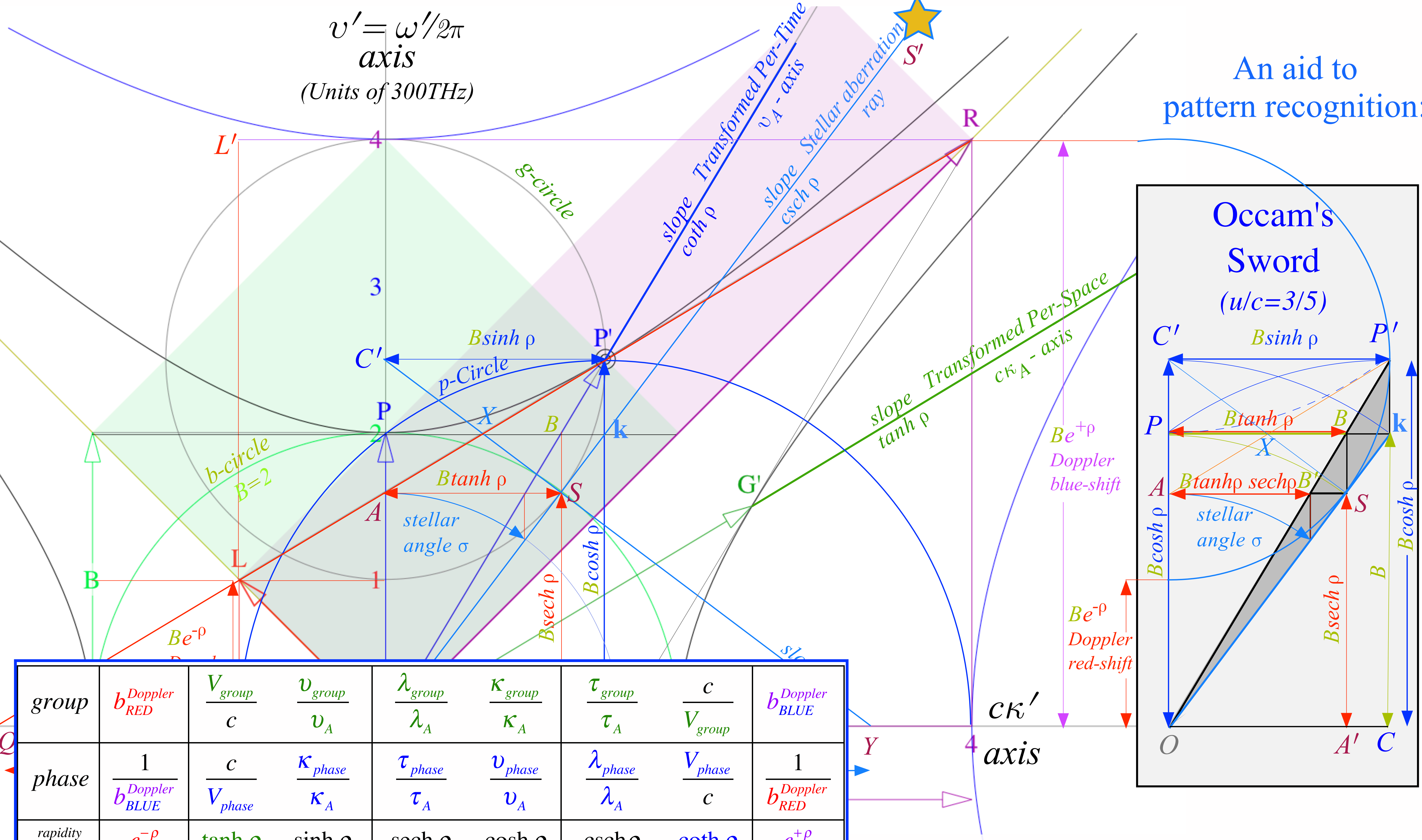
...and their geometry

An aid to pattern recognition:



RelaWavity Web Simulation
{perSpace - perTime All}

An aid to pattern recognition:



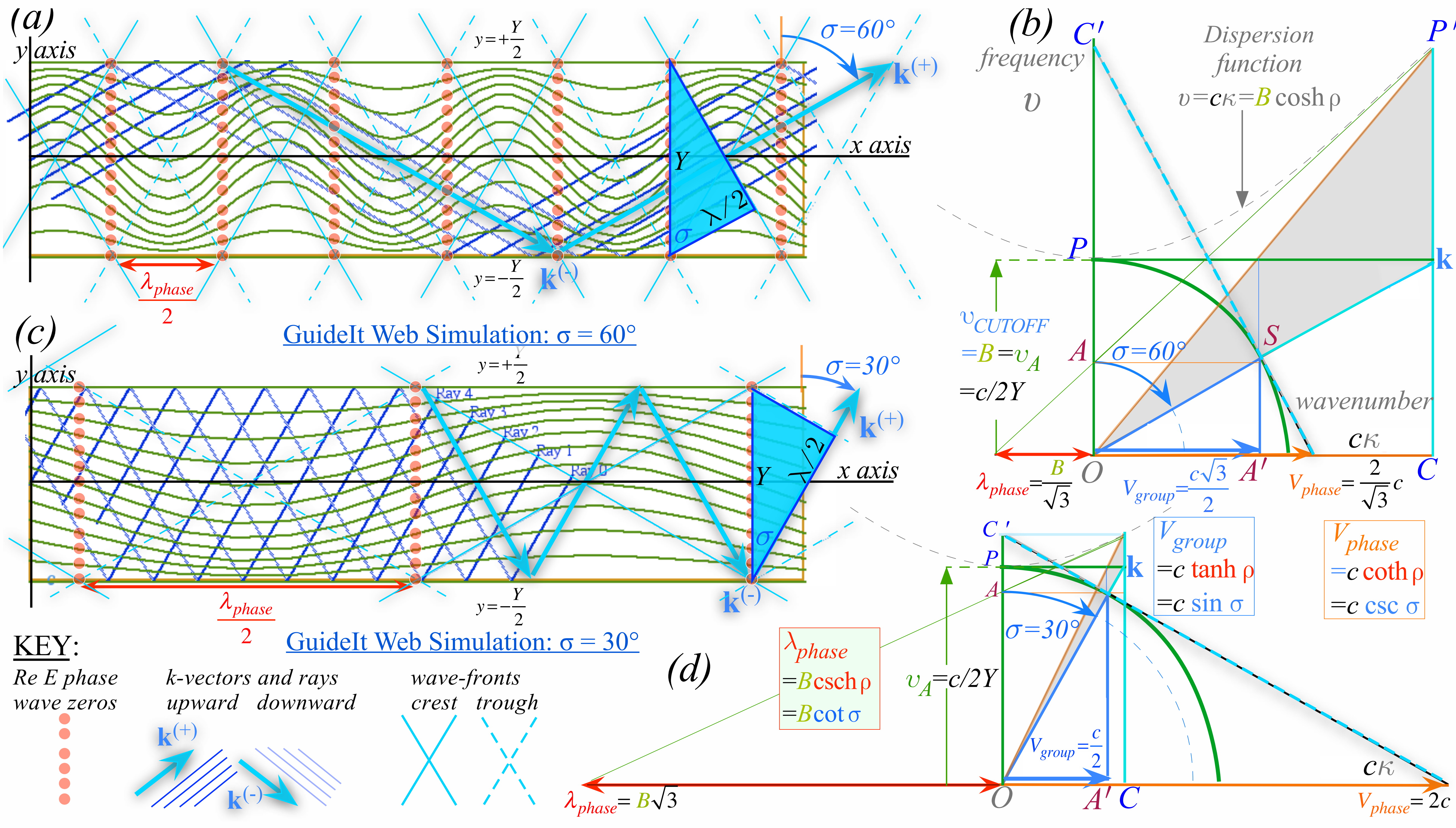
group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Table of 12 wave parameters (includes inverses) for relativity

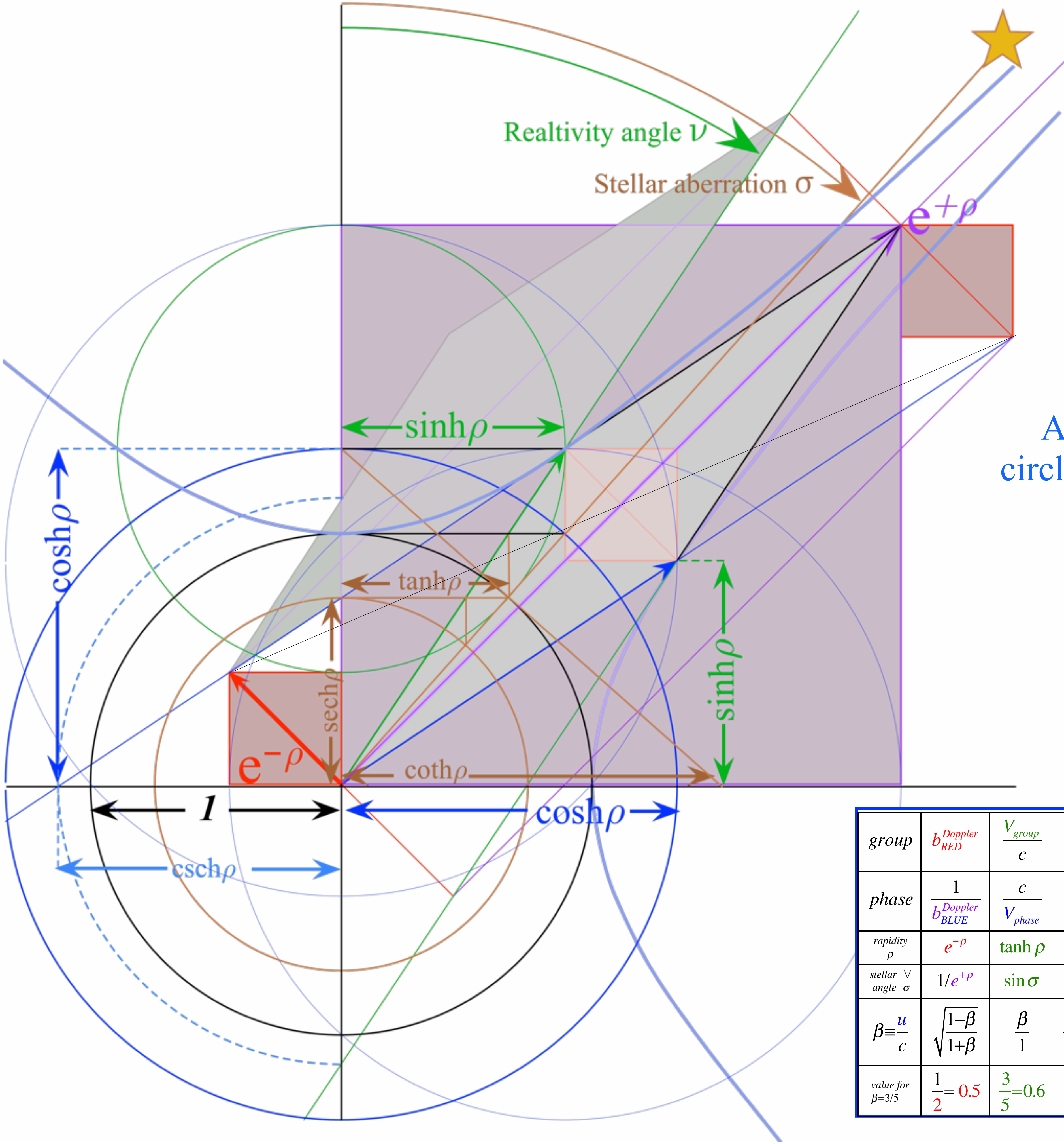
...and values for $u/c=3/5$

RelaWavity Web Simulation

Expanded *Relativistic* Relations



(a)
y axis
x axis
$y=+\frac{Y}{2}$
$y=-\frac{Y}{2}$
σ=60°
$\mathbf{k}^{(+)}$
$\mathbf{k}^{(-)}$
Y
λ/2
σ
$\frac{\lambda_{phase}}{2}$
(b)
frequency
υ
Dispersion function
$\upsilon=c\kappa=B\cosh\rho$
C′
P′
P
k
A
S
σ=60°
υ_{CUTOFF}
$=B=\upsilon_A$
$=c/2Y$
wavenumber
$c\kappa$
$\lambda_{phase}=\frac{B}{\sqrt{3}}$
O
$V_{group}=\frac{c\sqrt{3}}{2}$
A′
$V_{phase}=\frac{2}{\sqrt{3}}c$
C
(c)
GuideIt Web Simulation: σ = 60°
y axis
x axis
$y=+\frac{Y}{2}$
$y=-\frac{Y}{2}$
Ray 4
Ray 3
Ray 2
Ray 1
Ray 0
σ=30°
$\mathbf{k}^{(+)}$
$\mathbf{k}^{(-)}$
Y
λ/2
σ
$\frac{\lambda_{phase}}{2}$
GuideIt Web Simulation: σ = 30°
KEY:
Re E phase wave zeros
k-vectors and rays upward downward
wave-fronts crest trough
$\mathbf{k}^{(+)}$
$\mathbf{k}^{(-)}$
(d)
C′
P
A
k
σ=30°
V_{group} $=c\tanh\rho$ $=c\sin\sigma$
V_{phase} $=c\coth\rho$ $=c\csc\sigma$
λ_{phase} $=B\operatorname{csch}\rho$ $=B\cot\sigma$
$\upsilon_A=c/2Y$
$V_{group}=\frac{c}{2}$
$c\kappa$
$\lambda_{phase}=B\sqrt{3}$
O
A′
C
$V_{phase}=2c$



A more compact circle-based geometry

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$