Special Relativity Introduction for General Relativity 2
Monday 01.30.2017

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
    - Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes g-acceleration grid
  - Analysis of constant-g grid compared to zero-g Minkowsi grid
  - Animation of mechanics and metrology of constant-g grid

Lecture 31
Thur. 12.08.2016

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Learning about sin *and* cos *and...*

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Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
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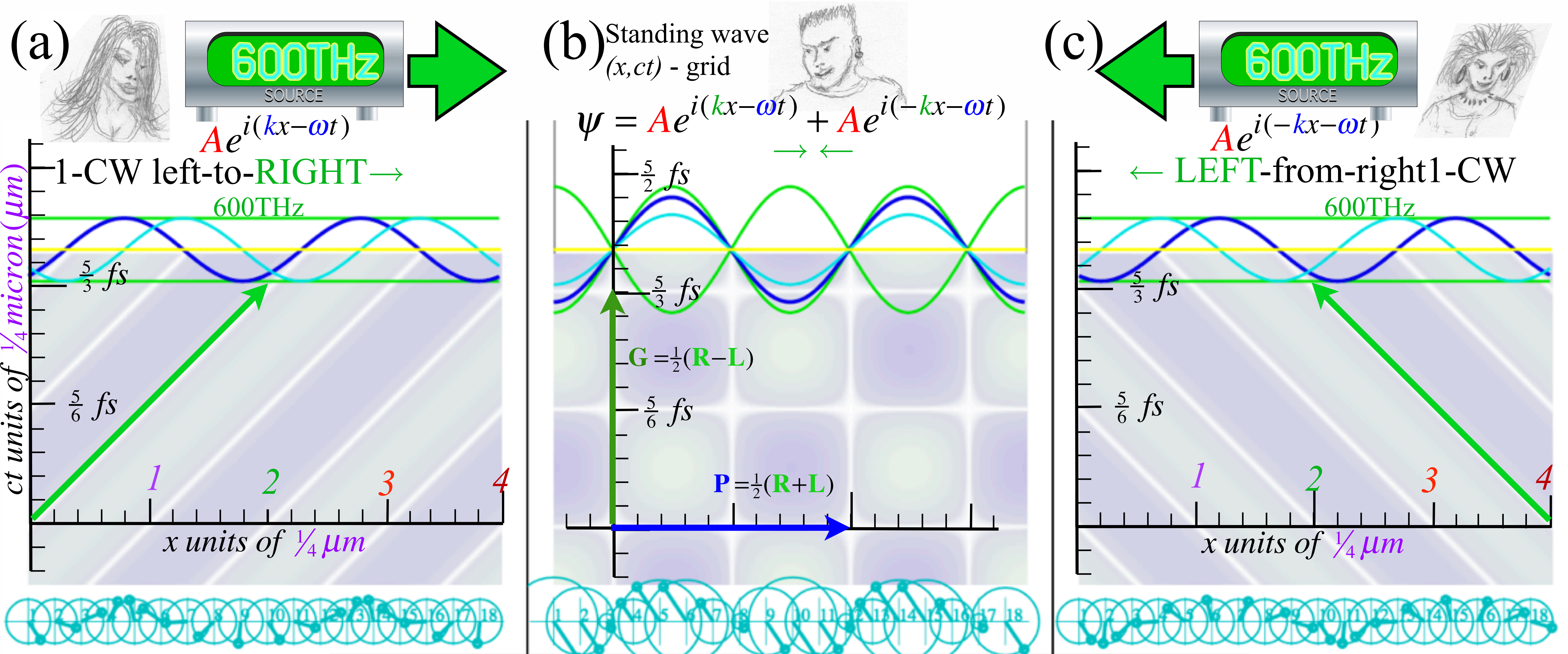
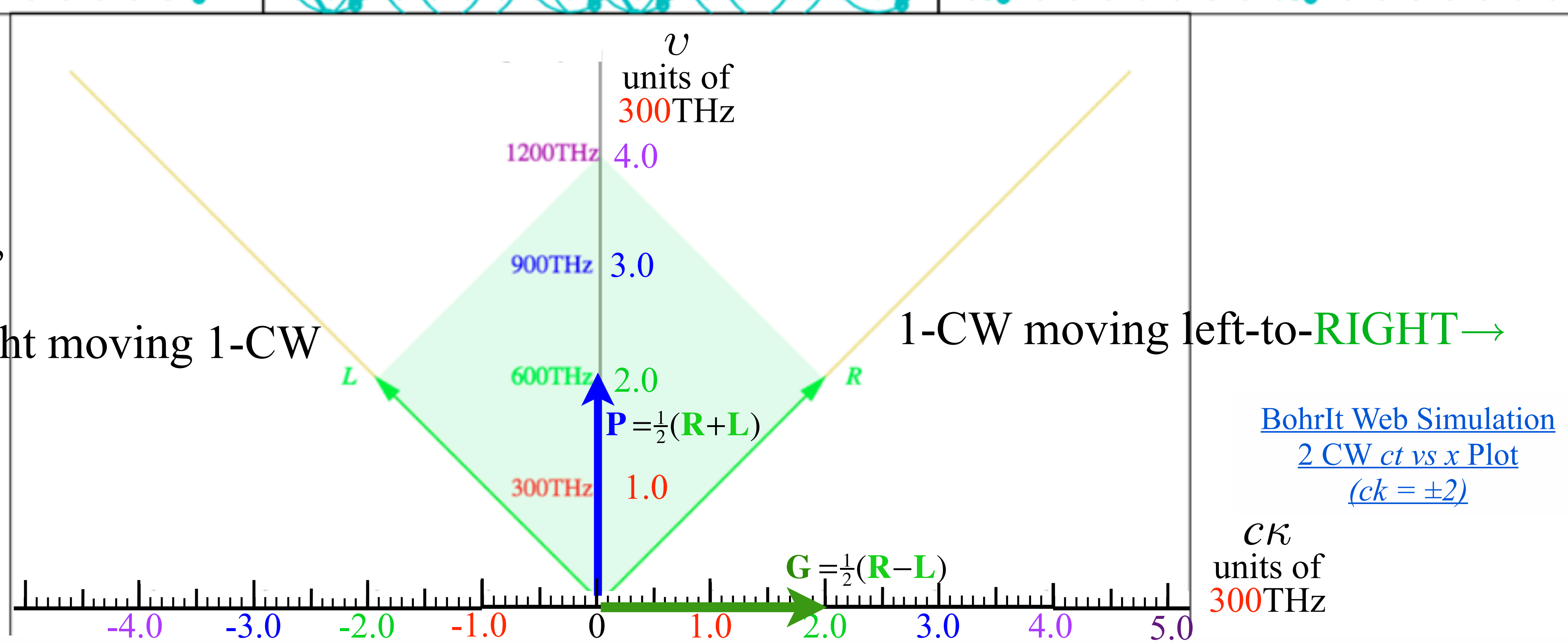
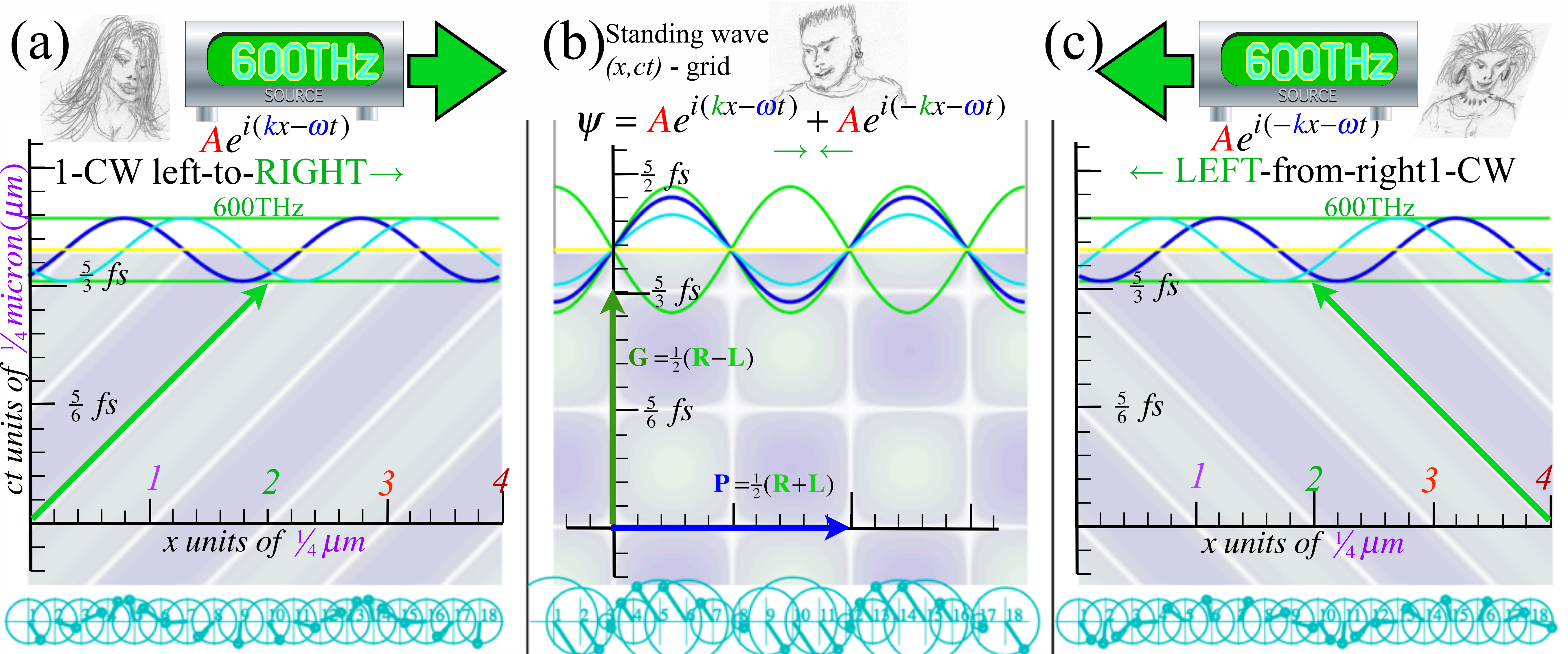


(d)

Introducing optical space-time grids and per-space-time "baseball-diamonds"

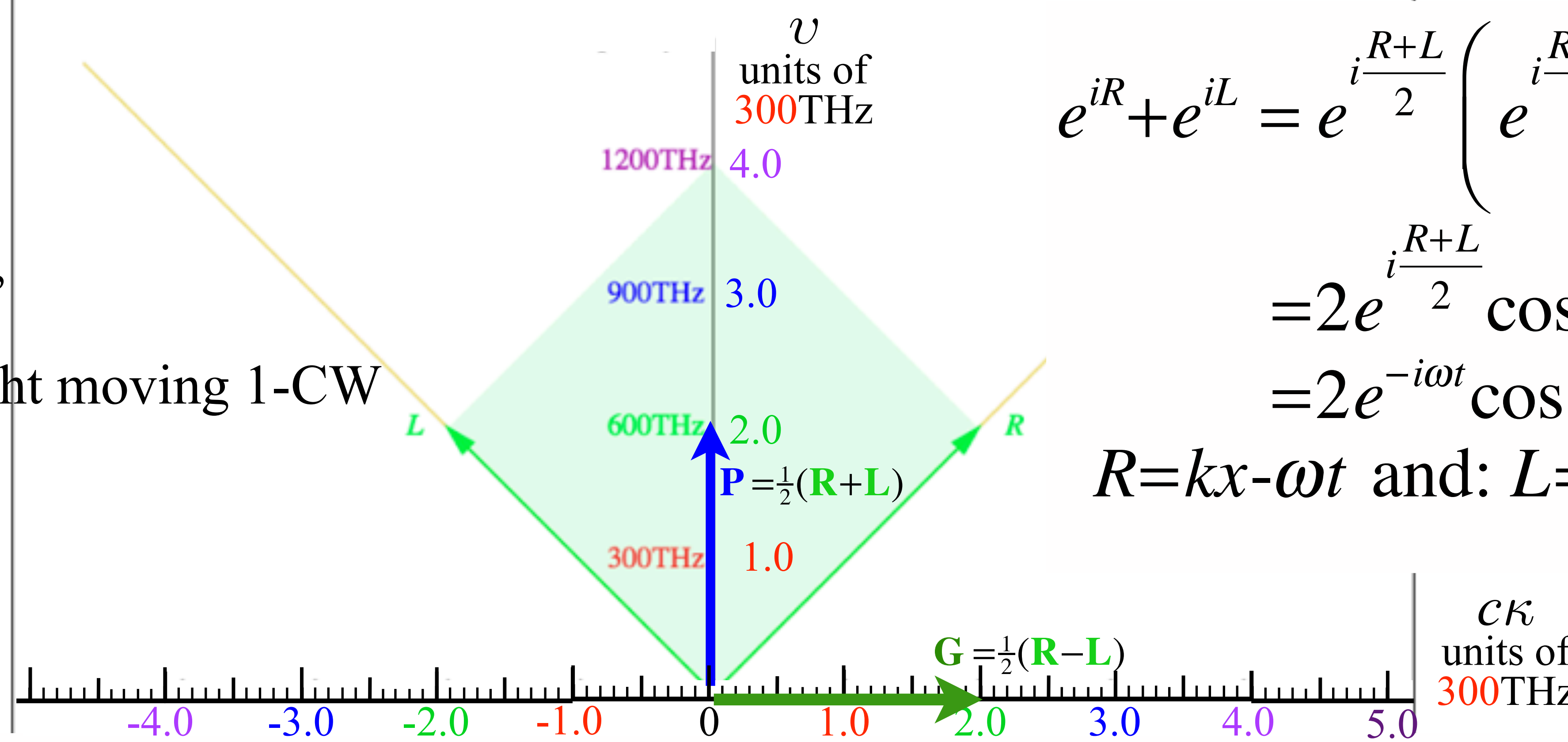




(d)

Introducing optical space-time grids and per-space-time "baseball-diamonds"

← LEFT-from-right moving 1-CW



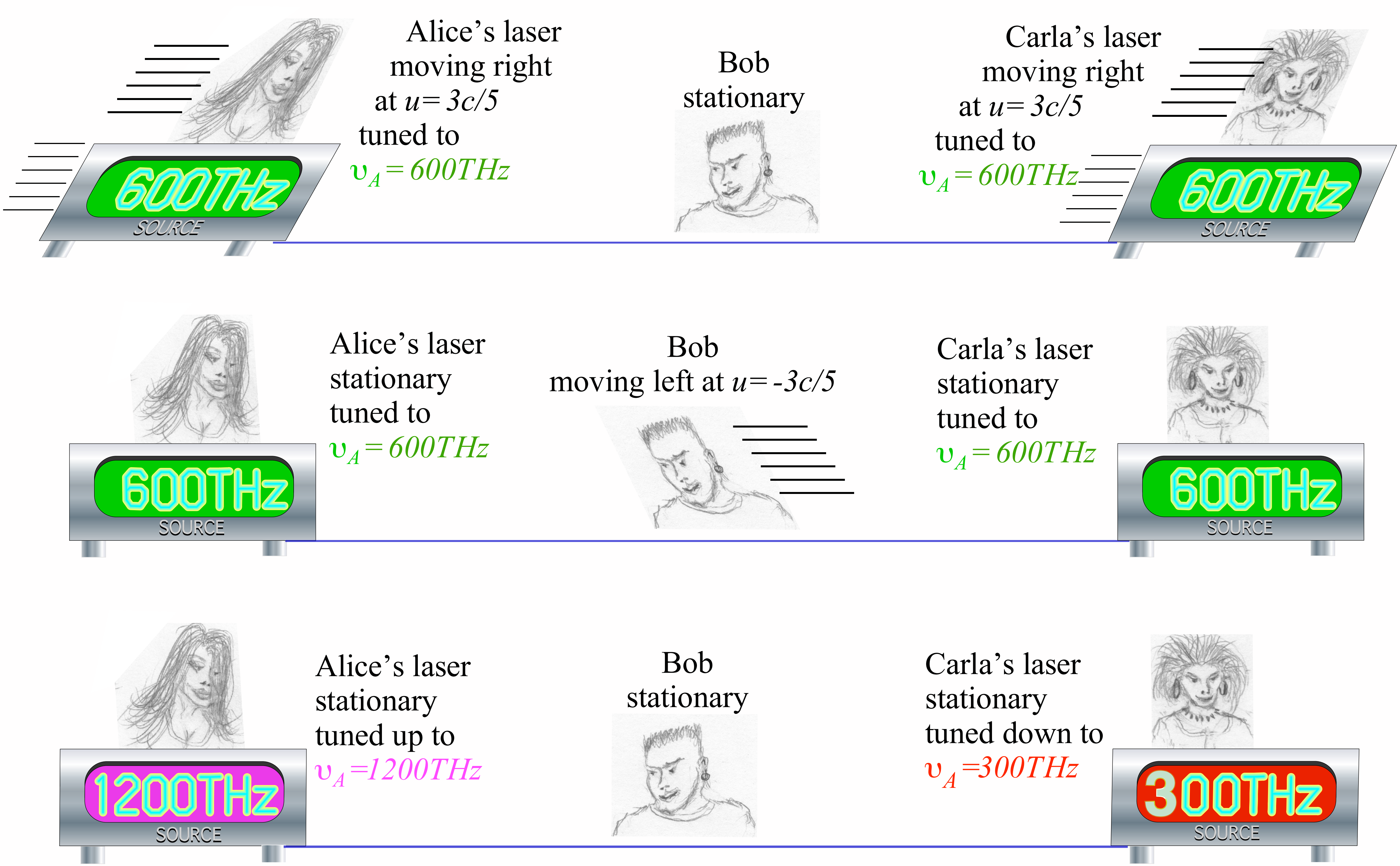
$$e^{iR}+e^{iL} = e^{i\frac{R+L}{2}}\left(e^{i\frac{R-L}{2}}+e^{-i\frac{R-L}{2}}\right)$$

$$=2e^{i\frac{R+L}{2}}\cos\frac{R-L}{2}$$

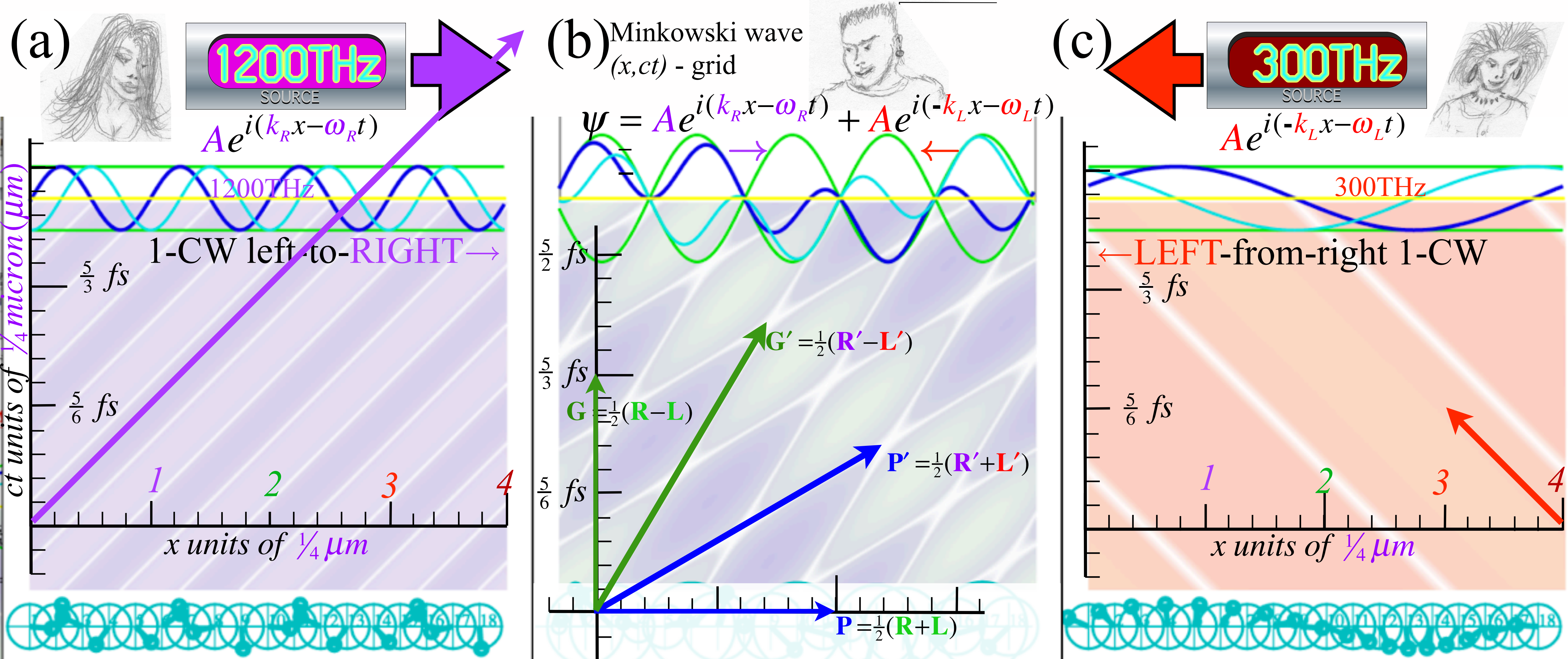
$$=2e^{-i\omega t}\cos kx$$

$R=kx-\omega t$ and: $L=-kx-\omega t$

Three scenarios that look the same to Bob



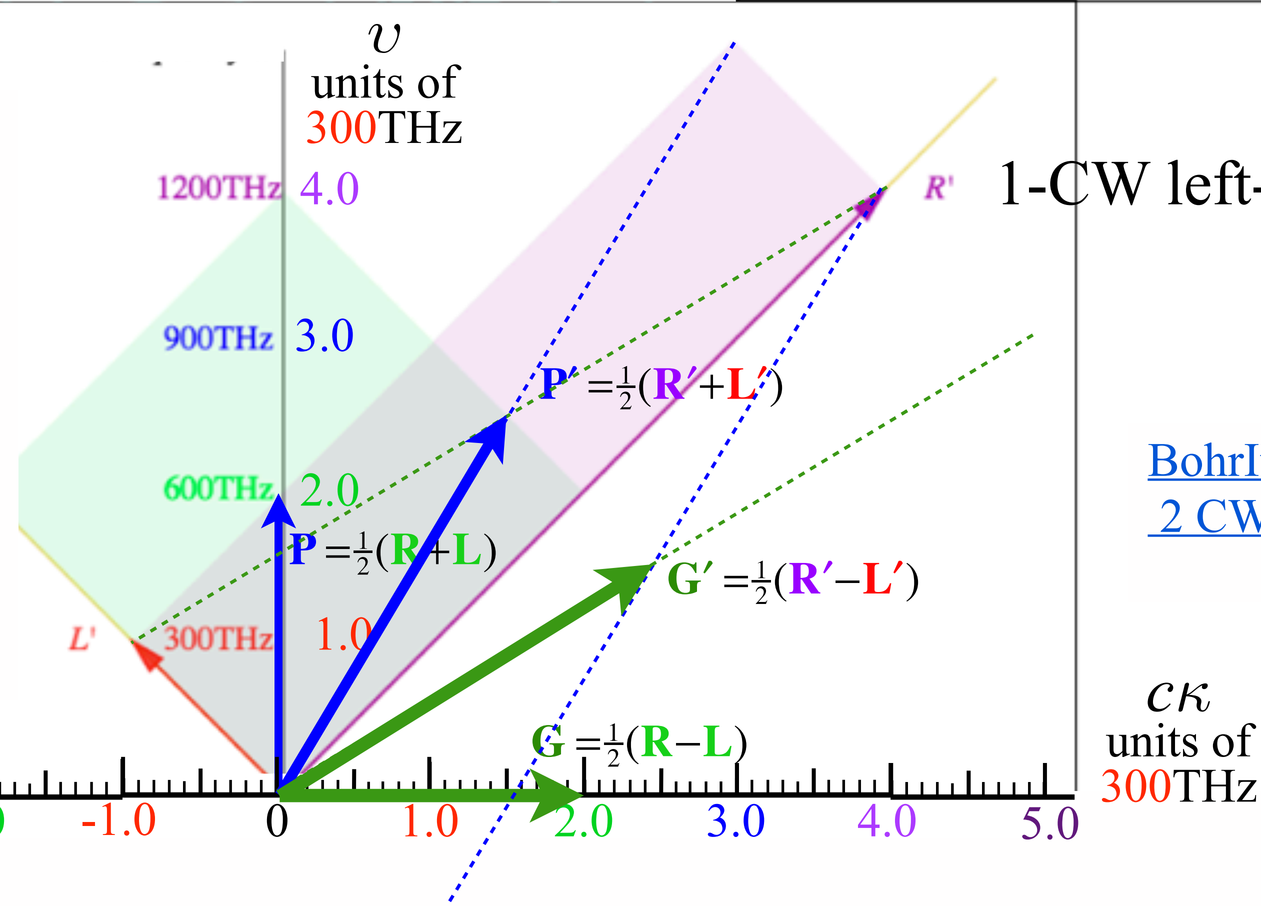
Much cheaper to do this one!$!



(d)

$$e^{iR'}+e^{iL'}=e^{i\frac{R'+L'}{2}}\left(e^{i\frac{R'-L'}{2}}+e^{-i\frac{R'-L'}{2}}\right)$$
$$=e^{i\frac{R'+L'}{2}}\,2\cos\frac{R'-L'}{2}$$
$$=\psi'_{phase}\psi'_{group}$$

$R'=k_R x\text{-}\omega_R t$ and: $L'=\text{-}k_L x\text{-}\omega_L t$



BohrIt Web Simulation
2 CW *Minkowski* Plot
(ck = -1, +4)

Fig. 10 in text
Relawavity...

The 16 dimensions of 2CW interference

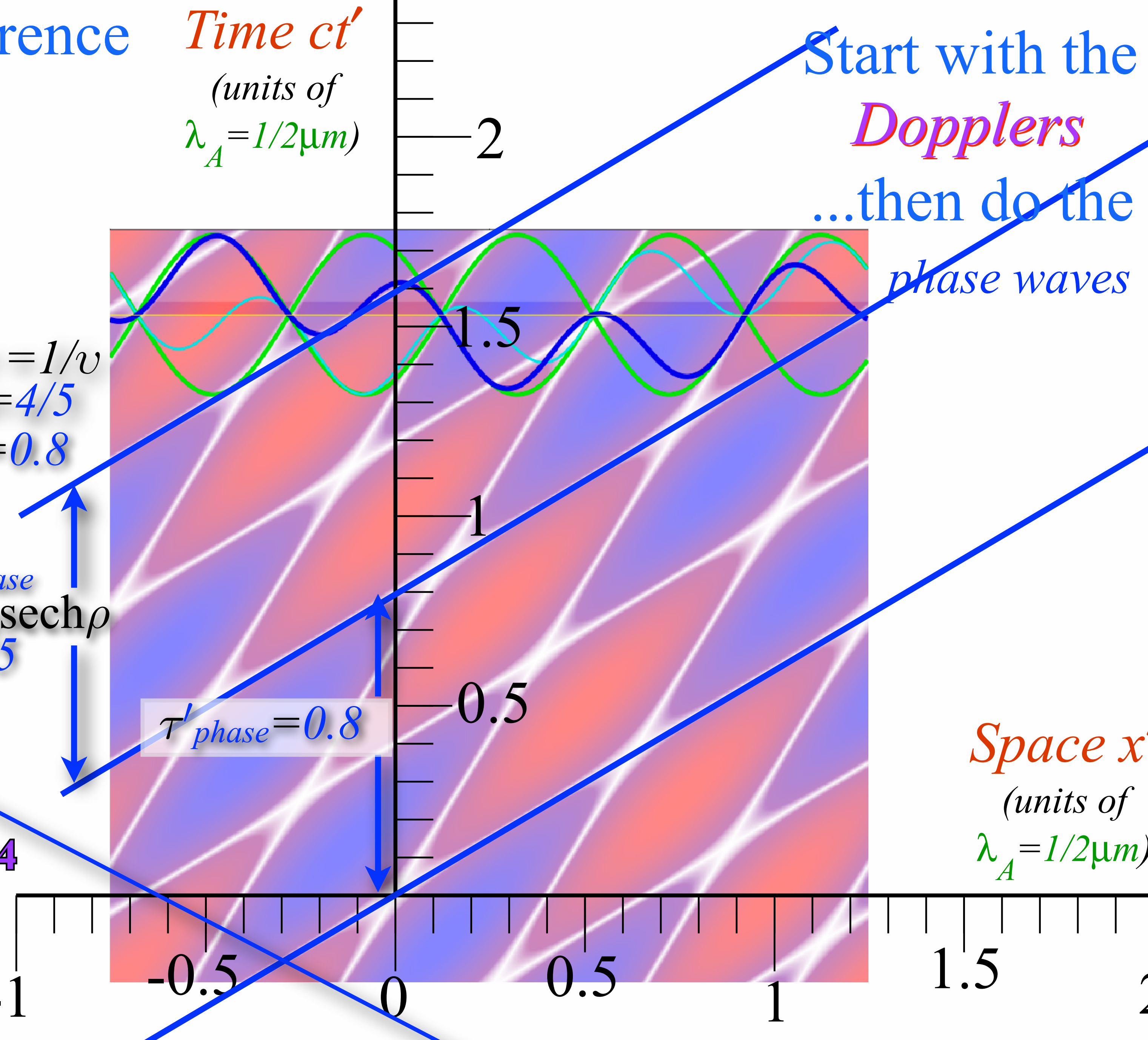
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

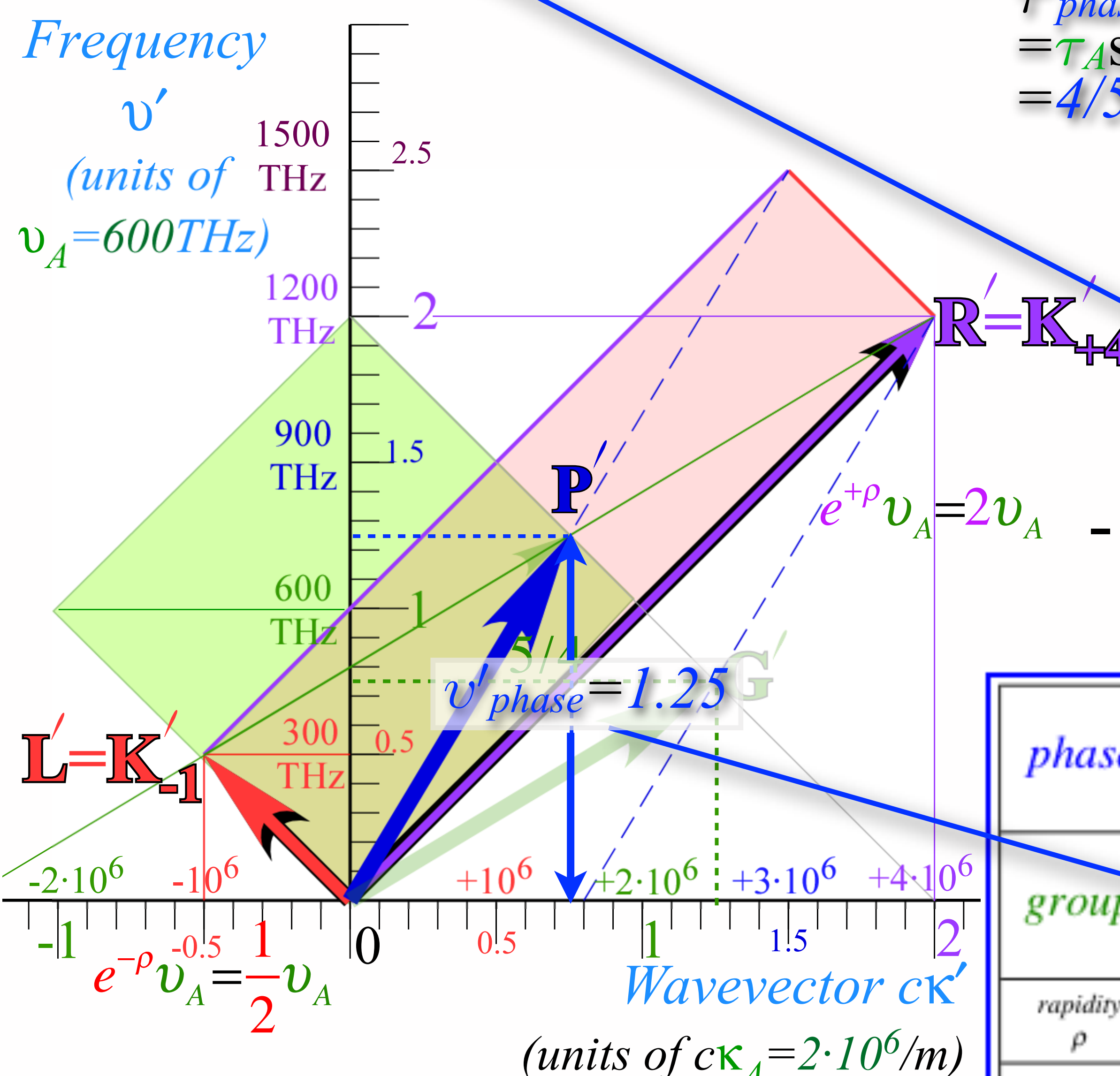
Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$

flips to

Phase period $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$

Start with the *Dopplers* ...then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

The 16 dimensions of 2CW interference

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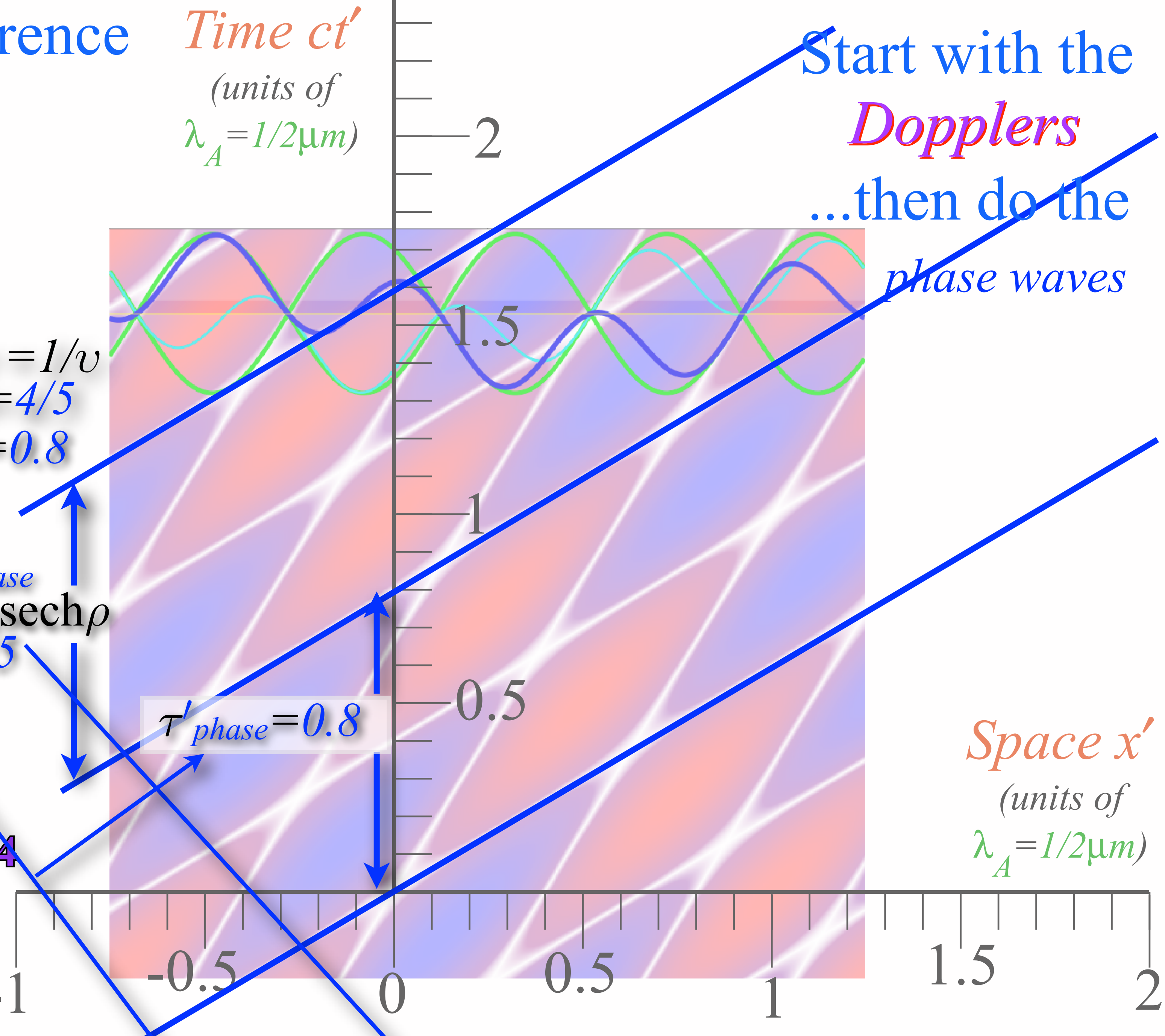
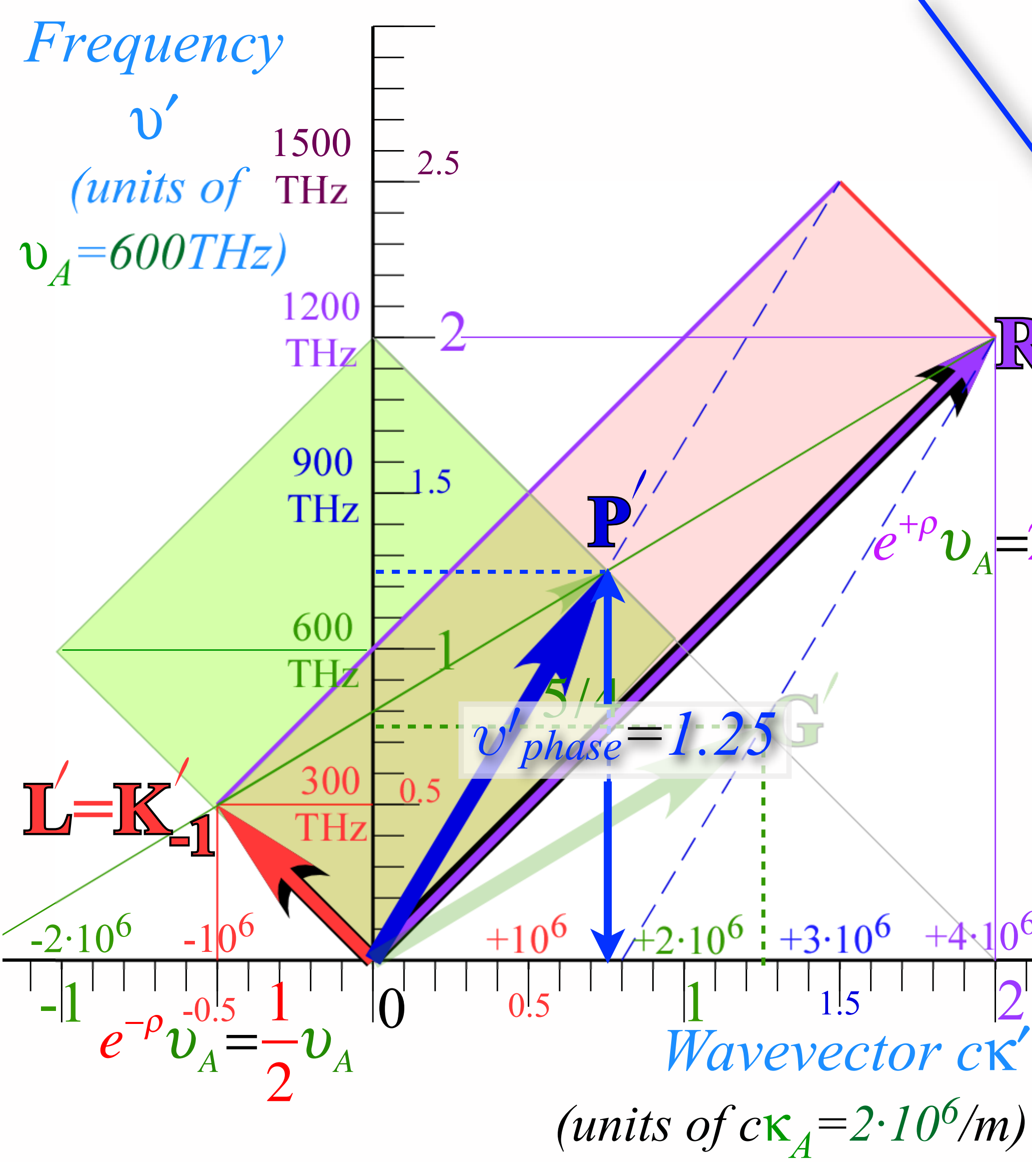
Start with the *Dopplers*
...then do the *phase waves*





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Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

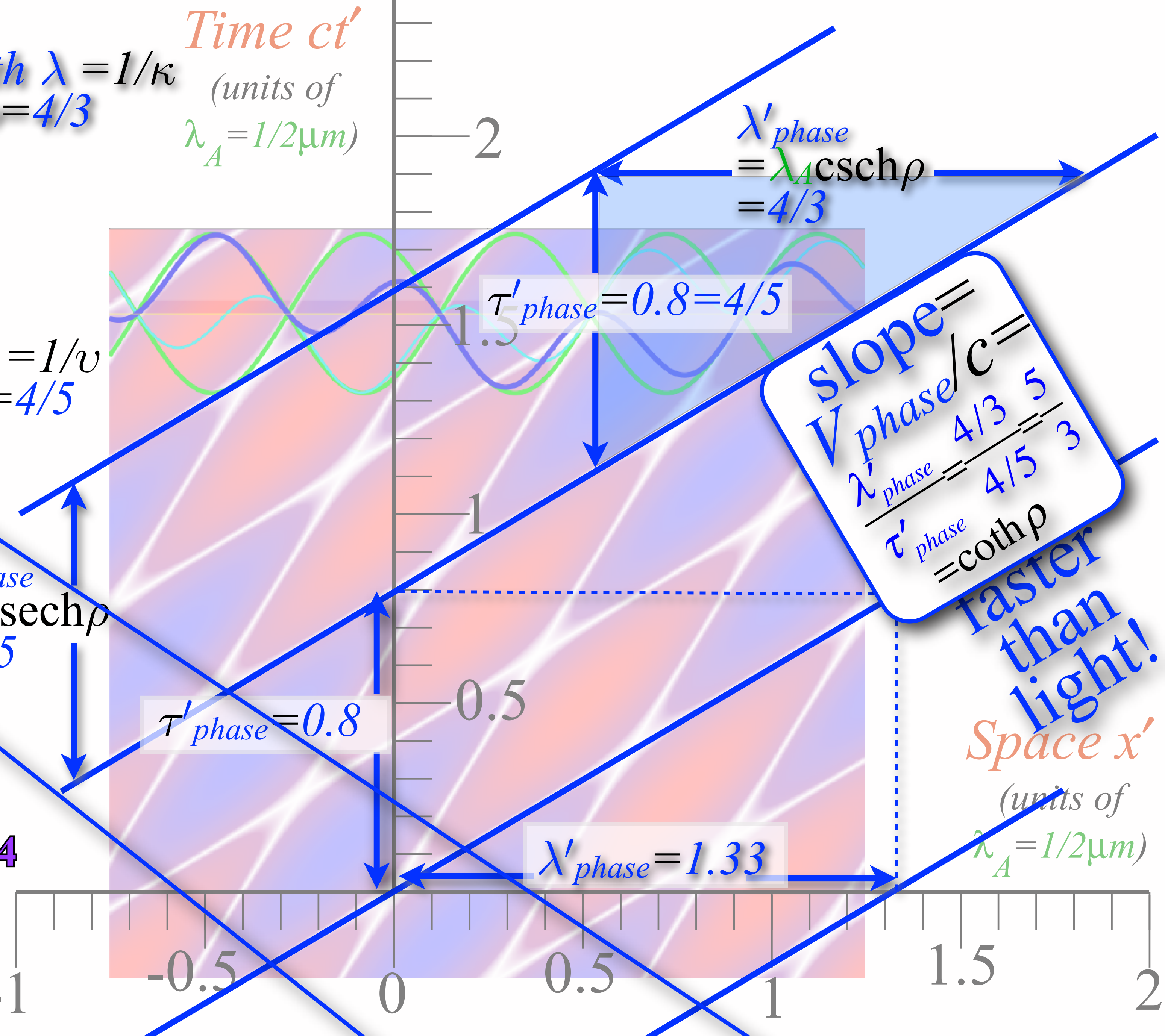
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

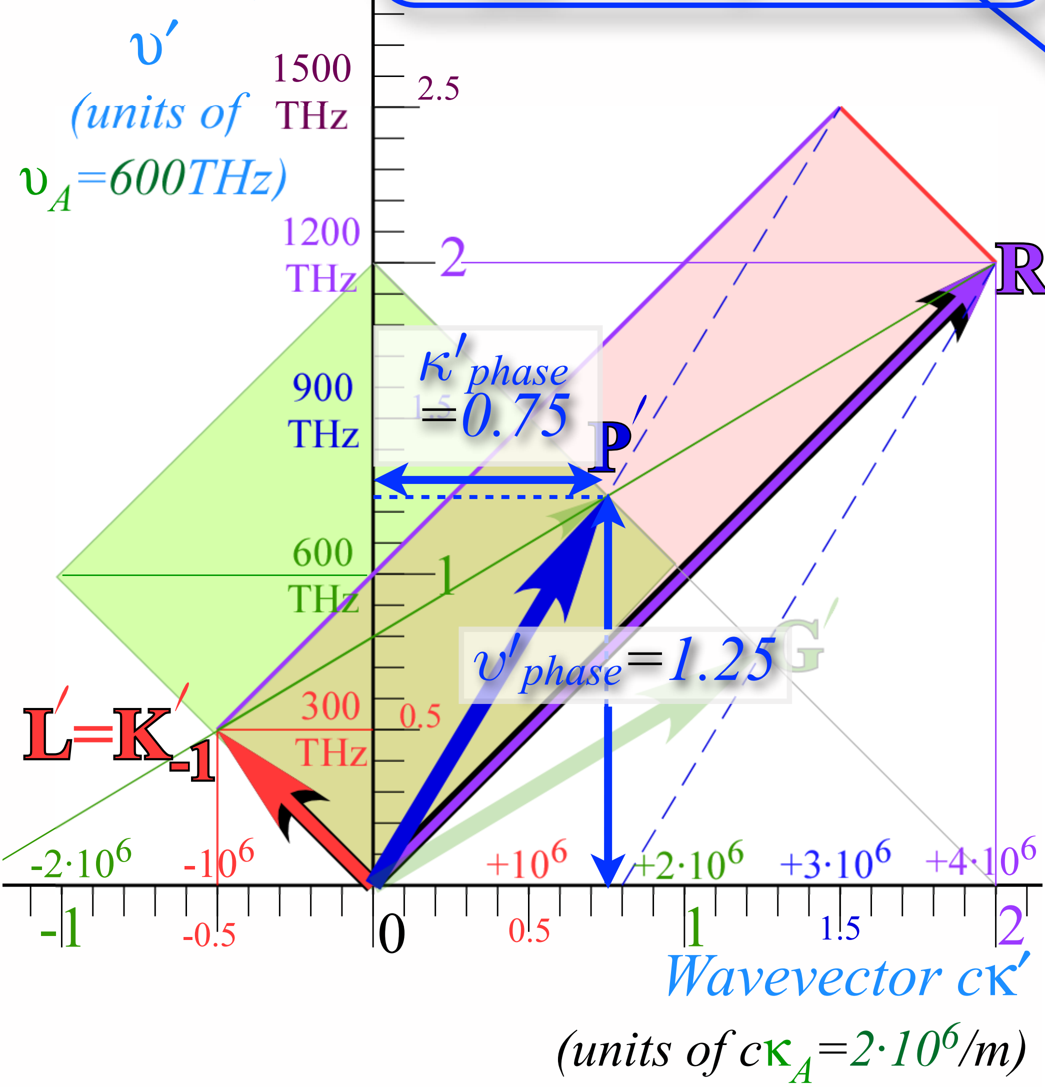
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

$$\mathbf{P}\text{-slope} = V_{phase}/c = \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
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Group wavenumber
$\kappa'_{group}=\kappa_A\cosh\rho=5/4$
$=1.25$

Group wavelength $\lambda=1/\kappa$
$\lambda'_{group}=\lambda_A\text{sech}\rho=4/5$
$=0.8$

$$\mathbf{G}'=\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group}\end{pmatrix}=\upsilon_A\begin{pmatrix}\cosh\rho \\ \sinh\rho\end{pmatrix}=\upsilon_A\begin{pmatrix}5/4 \\ 3/4\end{pmatrix}$$

Group frequency
$\upsilon'_{group}=\upsilon_A\sinh\rho=3/4$
$=0.75$

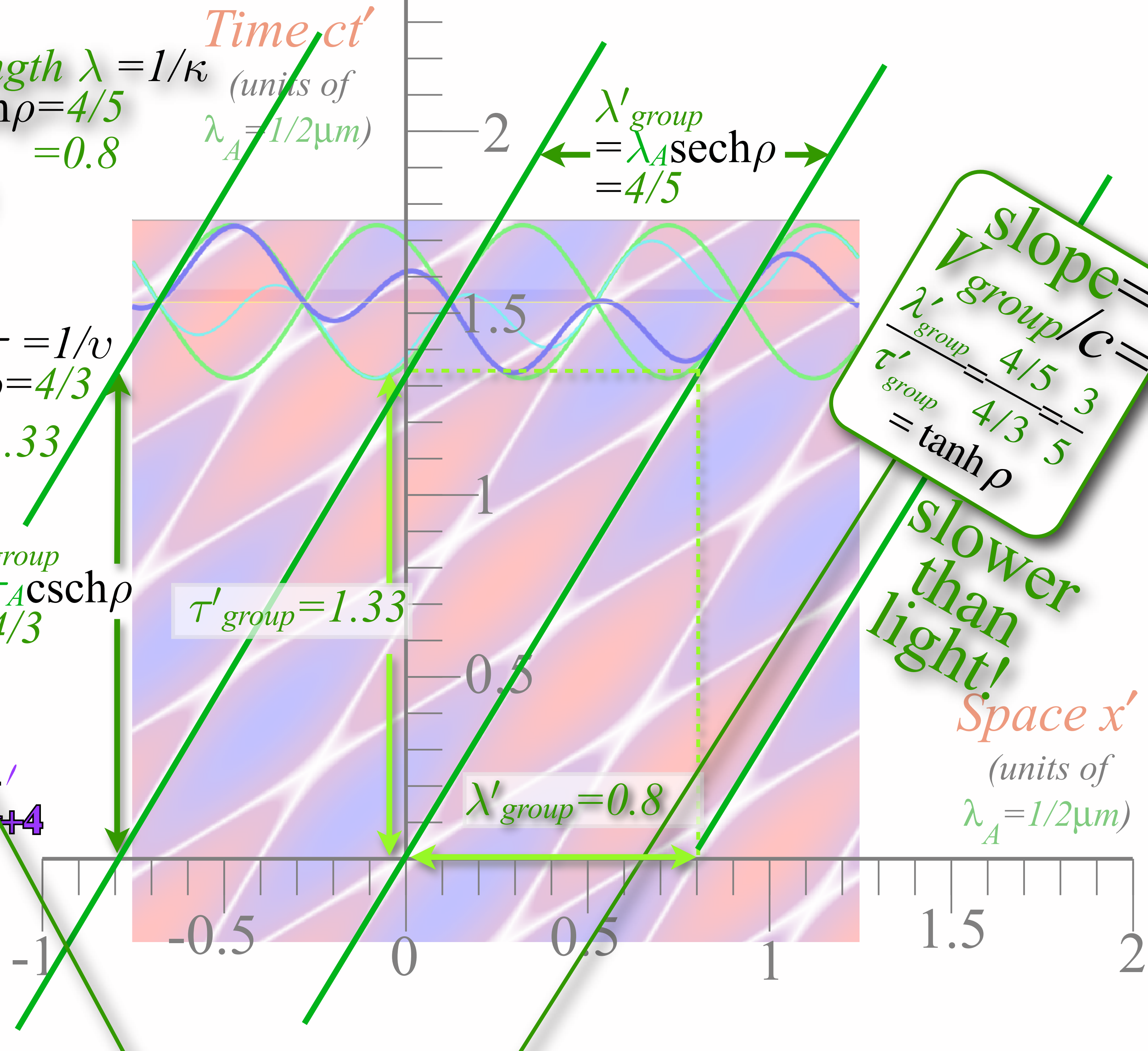
flips to

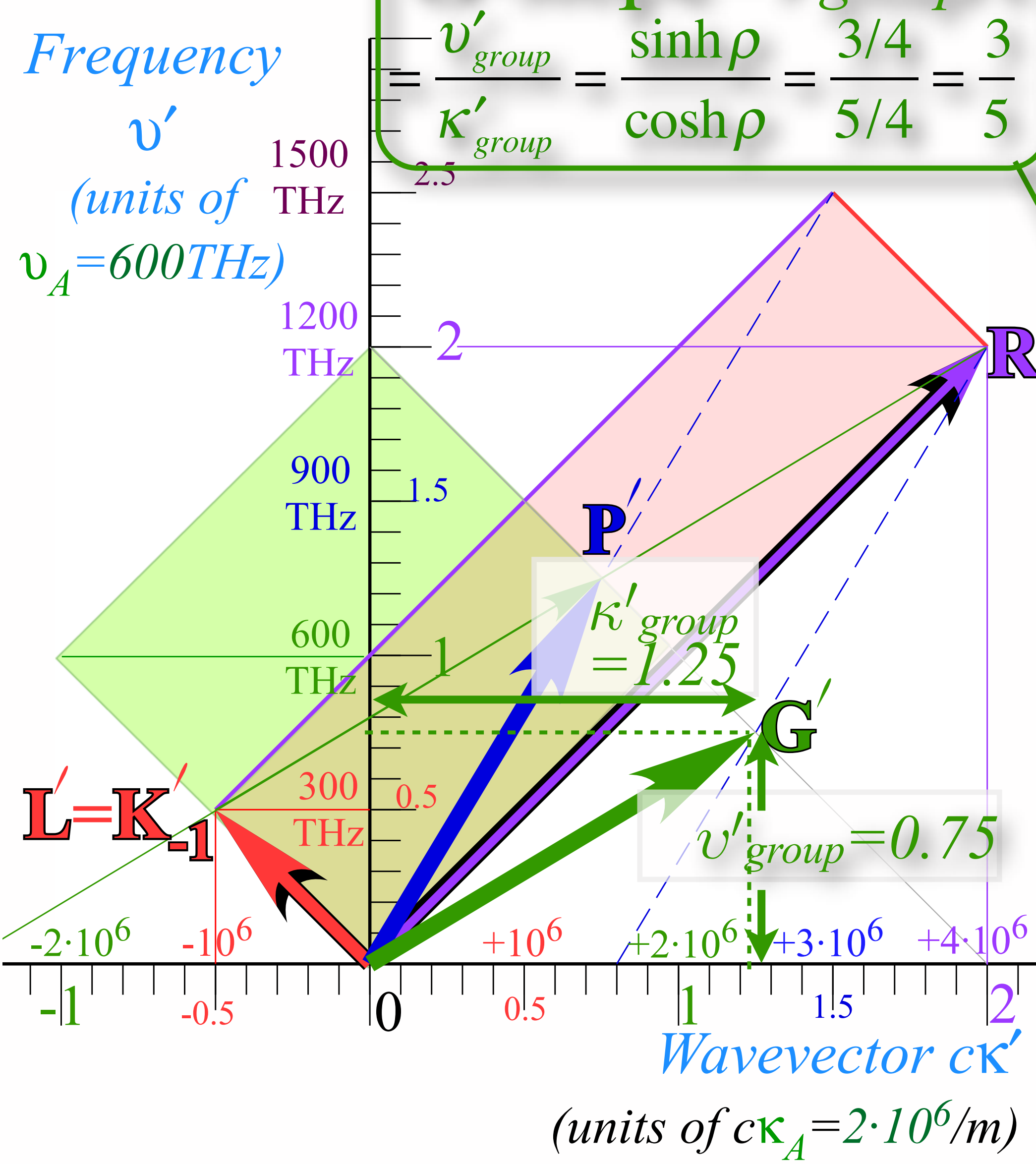
Group period $\tau=1/\upsilon$
$\tau'_{group}=\tau_A\text{csch}\rho=4/3$
$=1.33$

G-slope= V_{group}/c

$$=\frac{\upsilon'_{group}}{\kappa'_{group}}=\frac{\sinh\rho}{\cosh\rho}=\frac{3/4}{5/4}=\frac{3}{5}$$

τ'_{group}
$=\tau_A\text{csch}\rho$
$=4/3$





phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
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Lorentz transformations...

write $\mathbf{G}'$ *and* $\mathbf{P}'$ *in terms of* $\mathbf{G}$ *and* $\mathbf{P}$ *using* $\cosh\rho$ *and* $\sinh\rho$

$$\mathbf{G}'=\begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cosh\rho + \upsilon_A \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sinh\rho$$

$$\mathbf{G}' = \mathbf{G} \cosh\rho + \mathbf{P} \sinh\rho$$

$$\mathbf{P}'=\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

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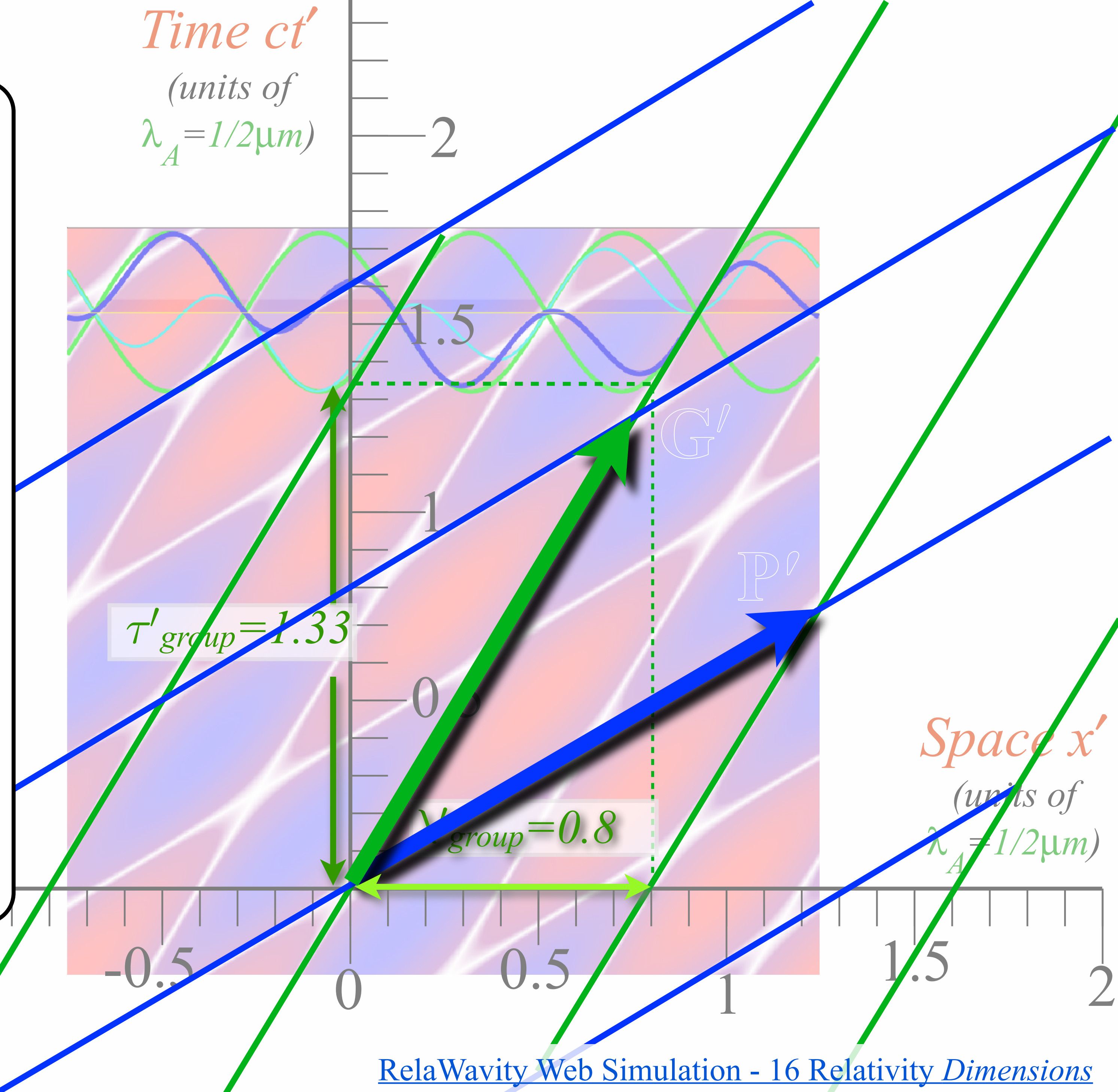

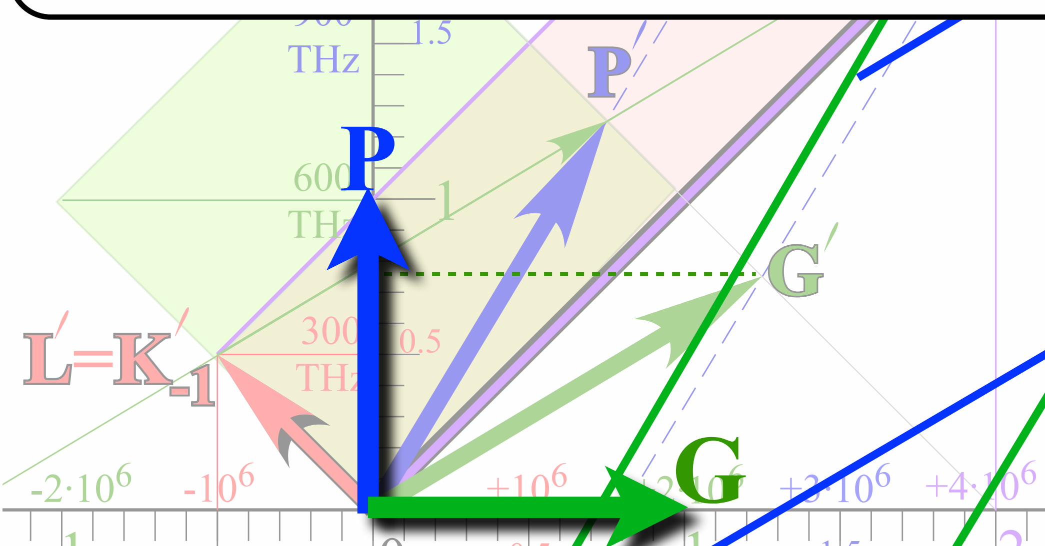

$$\begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}$$ Lorentz transform matrix

RelaWavity Web Simulation - 16 Relativity *Dimensions*

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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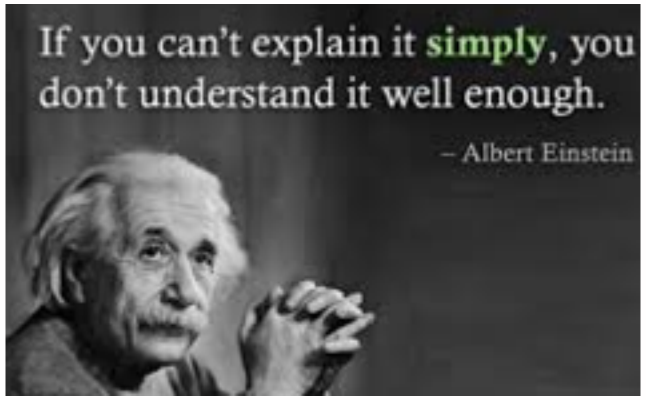
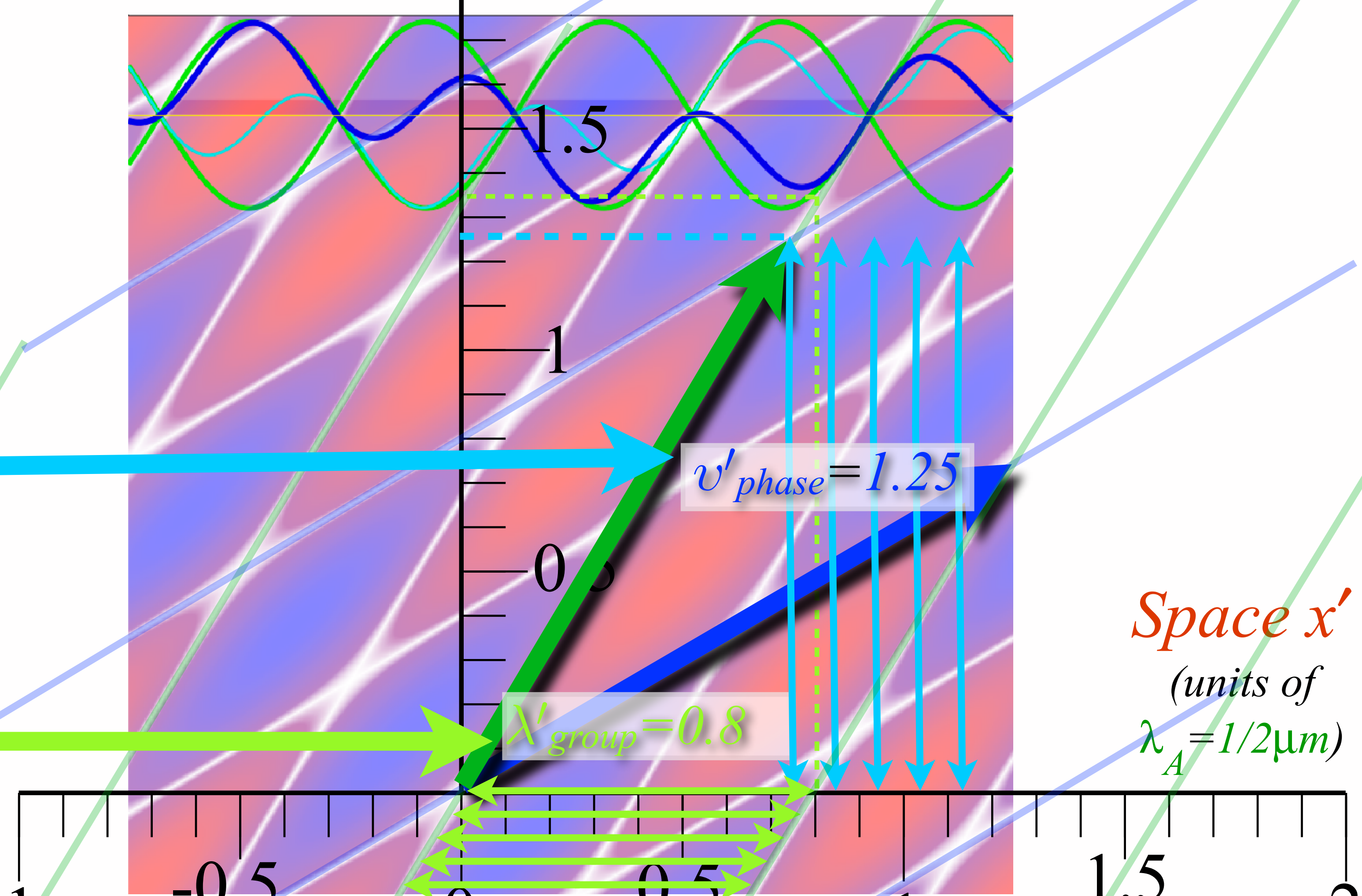
Two Famous-Name Coefficients

Review of Lect. 30 p.106



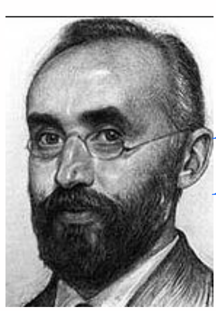
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: ***Einstein time-dilation*** *(dilated by 25% here)*

This number is called a: ***Lorentz length-contraction*** *(contracted by 20% here)*



Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
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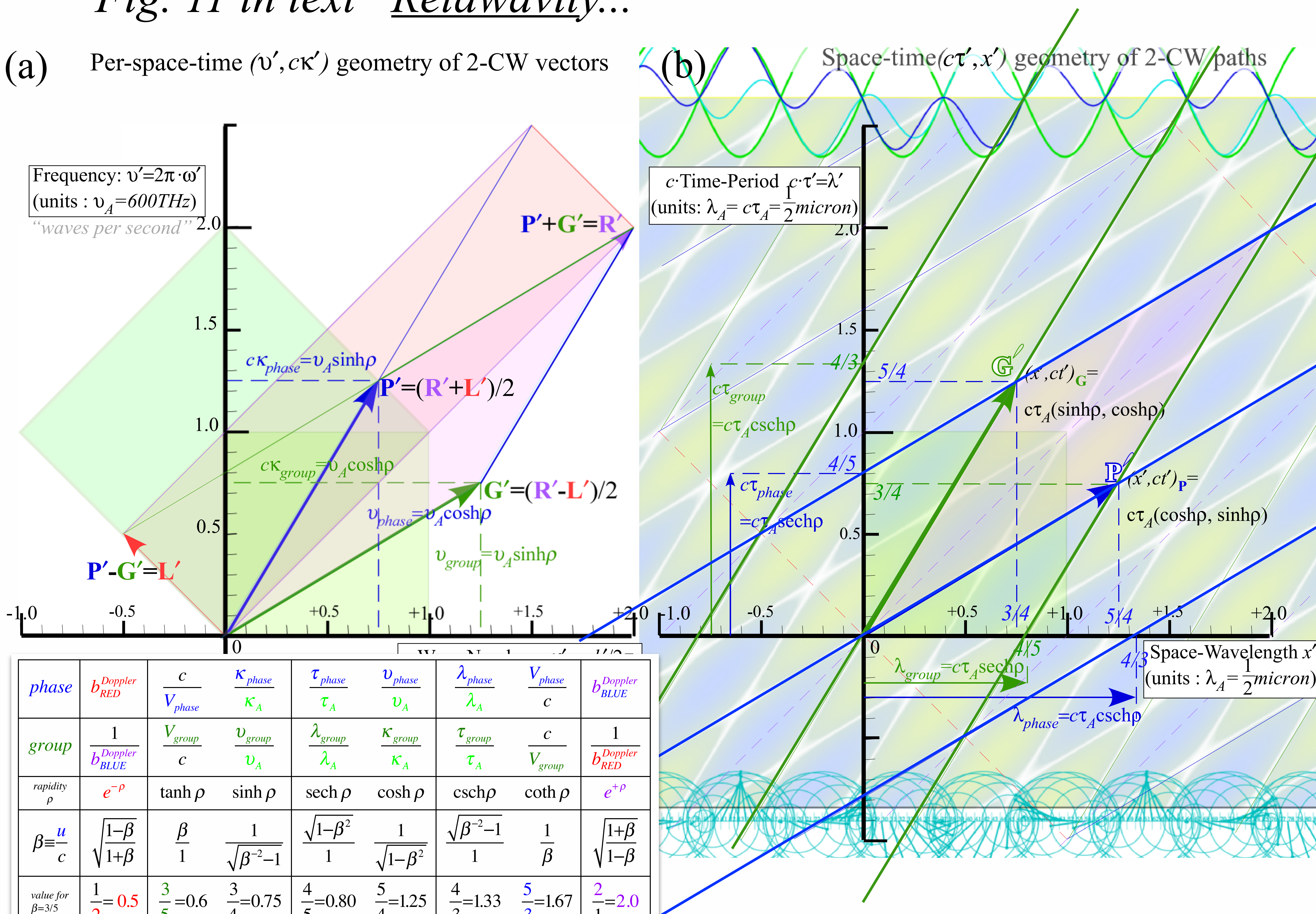
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation - Relativistic *Terms*
(Expanded Table)

Fig. 11 in text *Relawavity...*



phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
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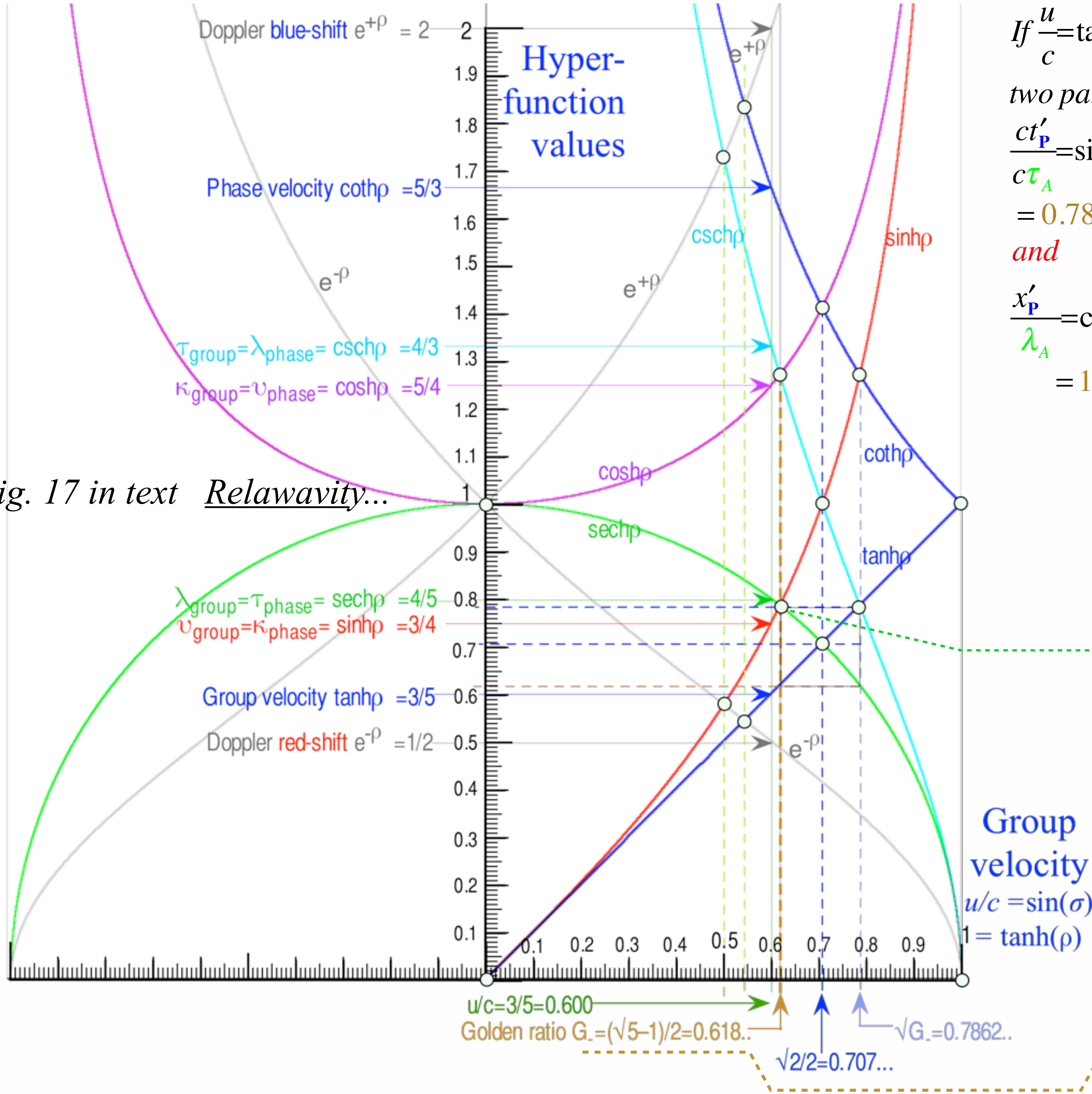

Fig. 17 in text *Relawavity...*

$$If\ \frac{u}{c}=\tanh\rho=0.618...(Golden\text{-}Mean\ G\_)$$

two parameters become exactly equal :

$$\frac{ct'_P}{c\tau_A}=\sinh\rho=\frac{\lambda_{group}}{\lambda_A}=\frac{\tau_{phase}}{\tau_A}=\text{sech}\rho$$

$$=0.786..=\sqrt{G\_}\qquad =0.786..$$

and

$$\frac{x'_P}{\lambda_A}=\cosh\rho=\frac{\lambda_{phase}}{\lambda_A}=\frac{\tau_{group}}{\tau_A}=\text{csch}\rho$$

$$=1.272..=1/\sqrt{G\_}\qquad =1.272..$$

Solve :

$$\text{sech}\rho=\sinh\rho$$

or:

$$\sinh\rho\cosh\rho=1$$

or:

$$\sinh 2\rho=2$$

$$\rho=\frac{1}{2}\sinh^{-1}2=0.7218...$$

$$\tanh\rho=0.618...=\frac{\sqrt{5}-1}{2}$$

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Thur. 12.08.2016

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Doppler Jeopardy

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency υ_E?

(2.) What is that frequency υ_E?

Doppler Jeopardy

$\omega_R = 2\pi\upsilon_R$ υ_R=600THz

υ_L=300THz $\omega_L = 2\pi\upsilon_L$

(1.) To what velocity u_E must Bob accelerate so he sees beams with equal frequency υ_E?

(2.) What is that frequency υ_E?

Query (1.) has a Jeopardy-style answer-by-question: What is beam group velocity?

$$u_E = V_{group} = \frac{\upsilon_{group}}{\kappa_{group}} = \frac{\upsilon_R - \upsilon_L}{\kappa_R - \kappa_L} = c\frac{\upsilon_R - \upsilon_L}{\upsilon_R + \upsilon_L}$$

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Query (2.) similarly: What υ_E is blue-shift $b\upsilon_L$ of υ_L *and* red-shift υ_R/b of υ_R?

$$\upsilon_E = b\,\upsilon_L = \upsilon_R/b \quad \Rightarrow \quad b = \sqrt{\upsilon_R/\upsilon_L} \quad \Rightarrow \quad \upsilon_E = \sqrt{\upsilon_R \cdot \upsilon_L}$$

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↑

Geometric mean

Doppler Jeopardy

$\omega_R = 2\pi\upsilon_R$ υ_R=600THz

υ_L=300THz $\omega_L = 2\pi\upsilon_L$

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$$\upsilon_E = b\upsilon_L = \upsilon_R/b \quad\Rightarrow\quad b = \sqrt{\upsilon_R/\upsilon_L} \quad\Rightarrow\quad \upsilon_E = \sqrt{\upsilon_R \cdot \upsilon_L}$$

↑ *Geometric mean*

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$$\upsilon_E = \sqrt{\upsilon_R \cdot \upsilon_L} = \sqrt{180000} = 424$$

Geometric mean

Doppler Jeopardy

$\omega_R = 2\pi\upsilon_R$ υ_R=600THz

υ_L=300THz $\omega_L = 2\pi\upsilon_L$

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$$\upsilon_E = \sqrt{\upsilon_R\cdot\upsilon_L} = \sqrt{180000} = 424$$

V_{group}/c is ratio of difference mean $\upsilon_{group} = \frac{\upsilon_R - \upsilon_L}{2}$ to arithmetic mean $\upsilon_{phase} = \frac{\upsilon_R + \upsilon_L}{2}$. Frequency $\upsilon_E = B$ is the geometric mean $\sqrt{\upsilon_R\cdot\upsilon_L}$ of left and right-moving frequencies defining the geometry

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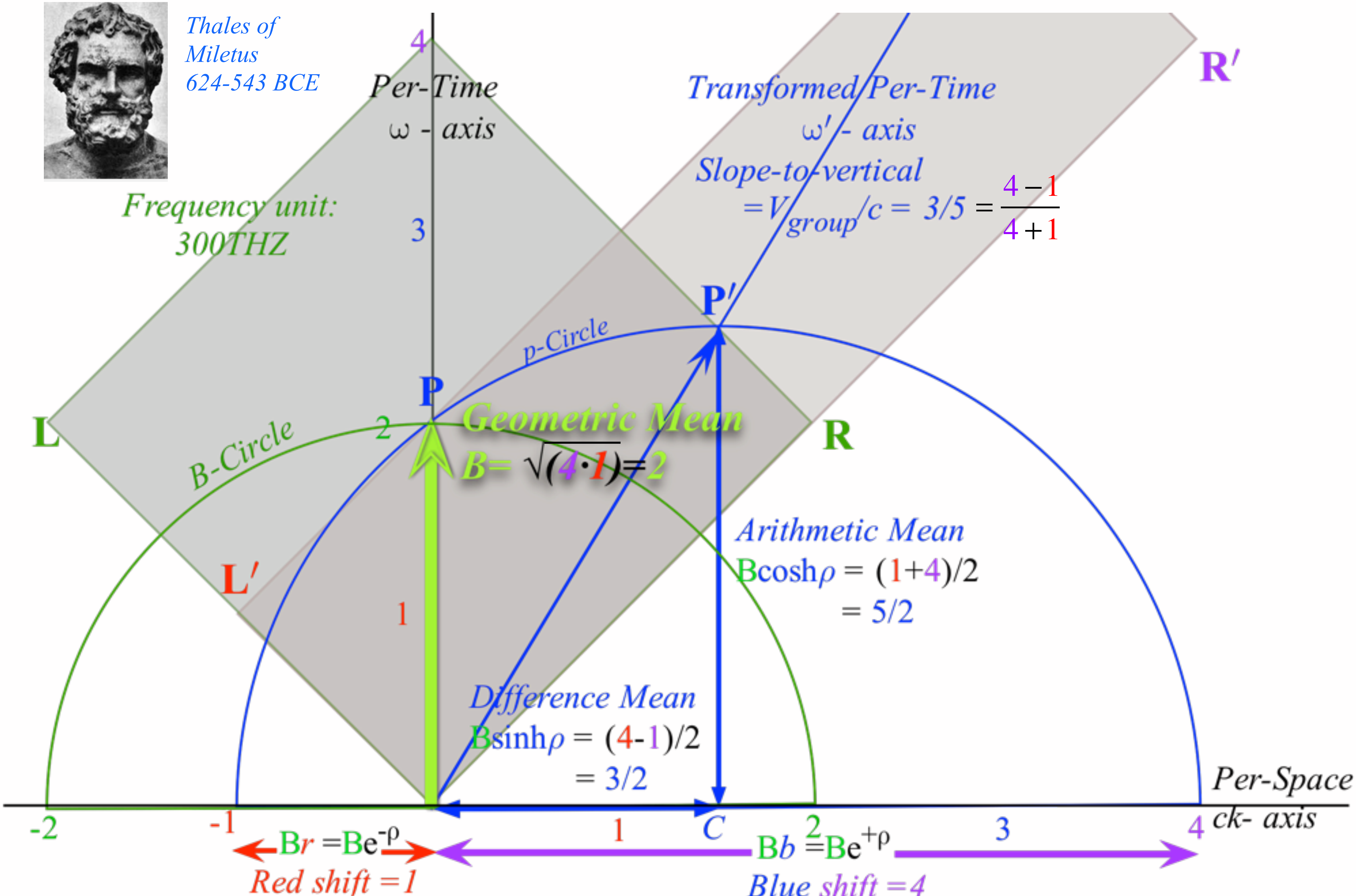
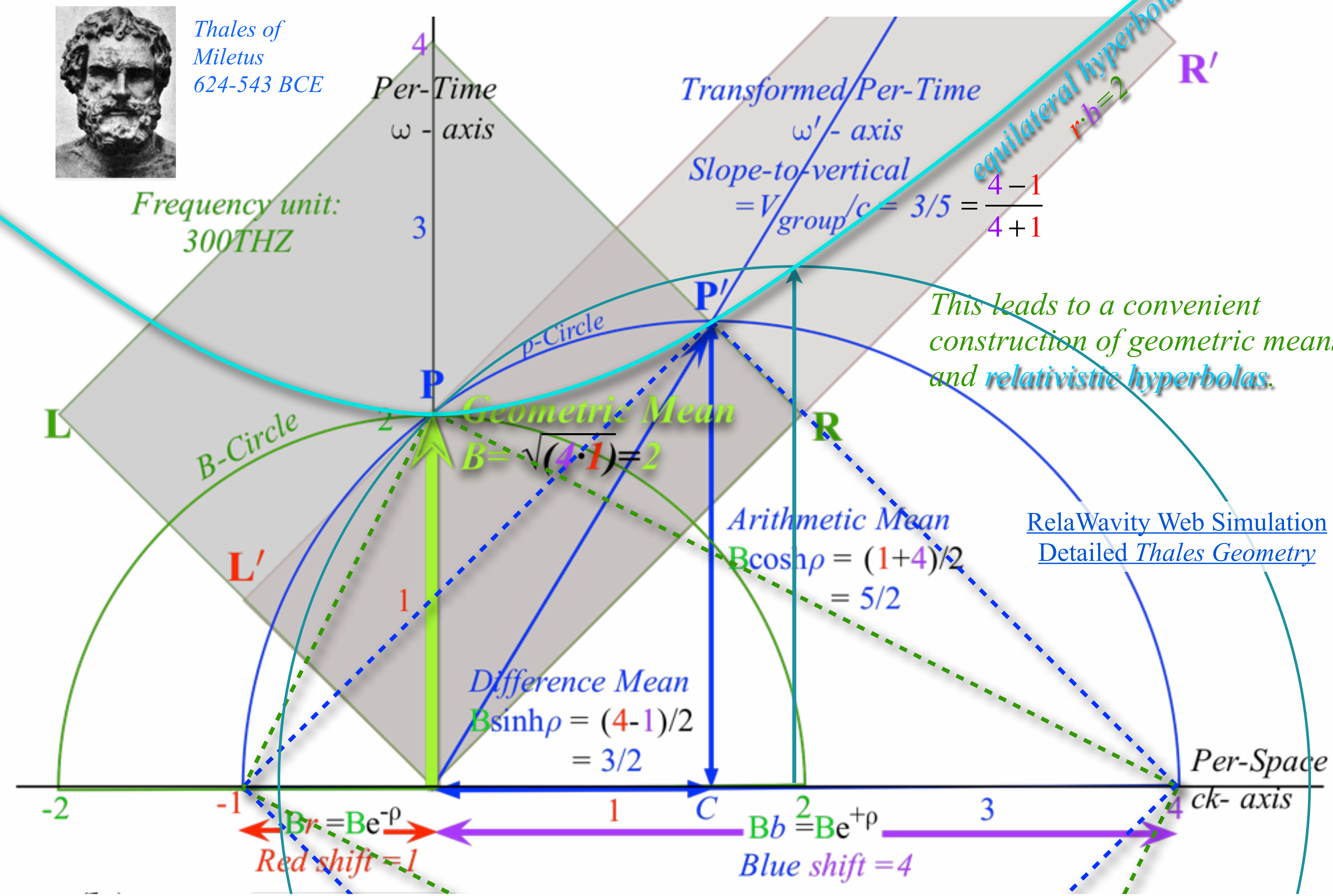
Thales Mean Geometry (600BCE)

helps "Relawavity"



Thales Mean Geometry (600BCE)

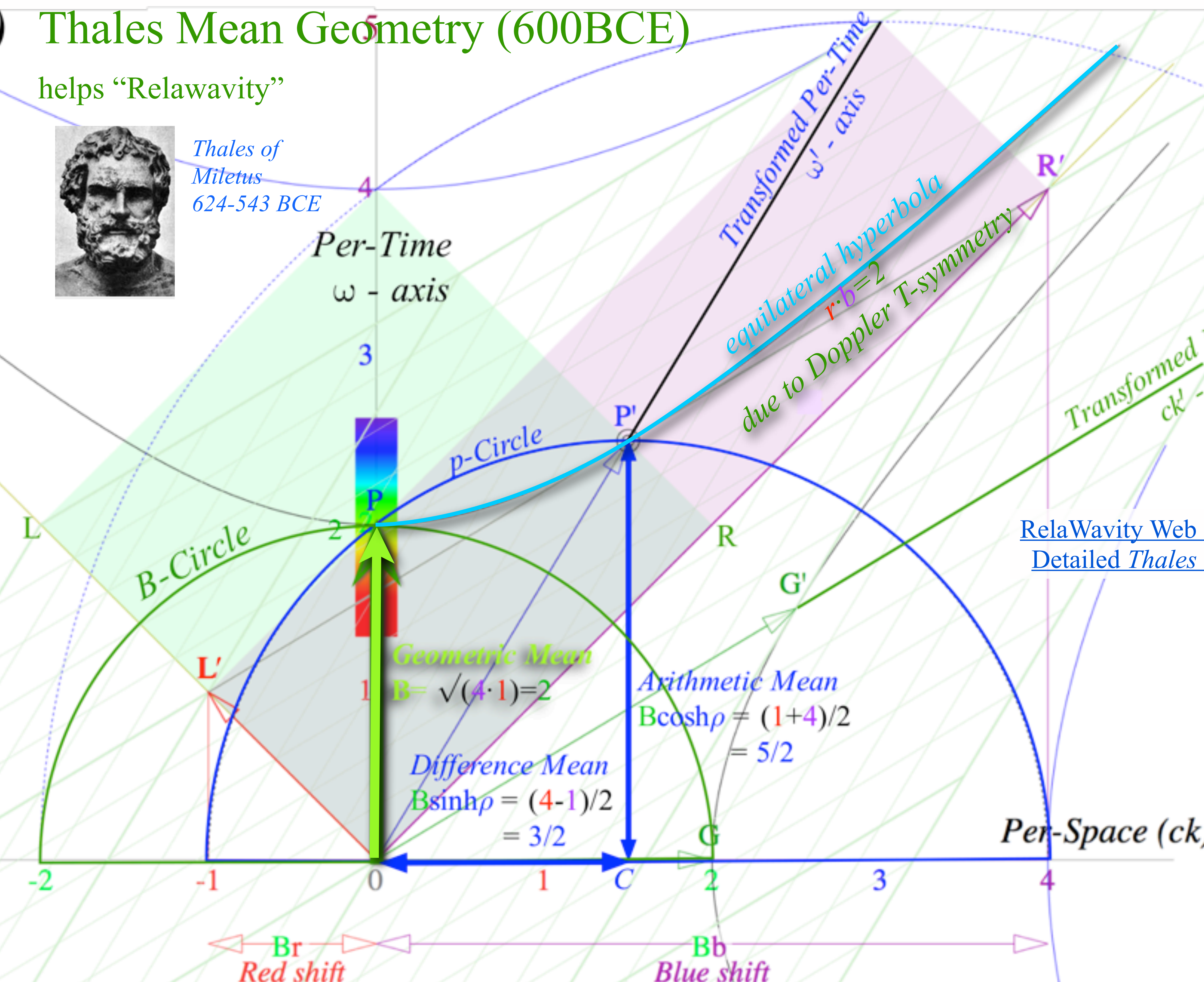
helps "Relawavity" *Thales showed a circle diameter subtends a right angle with any circle point P*



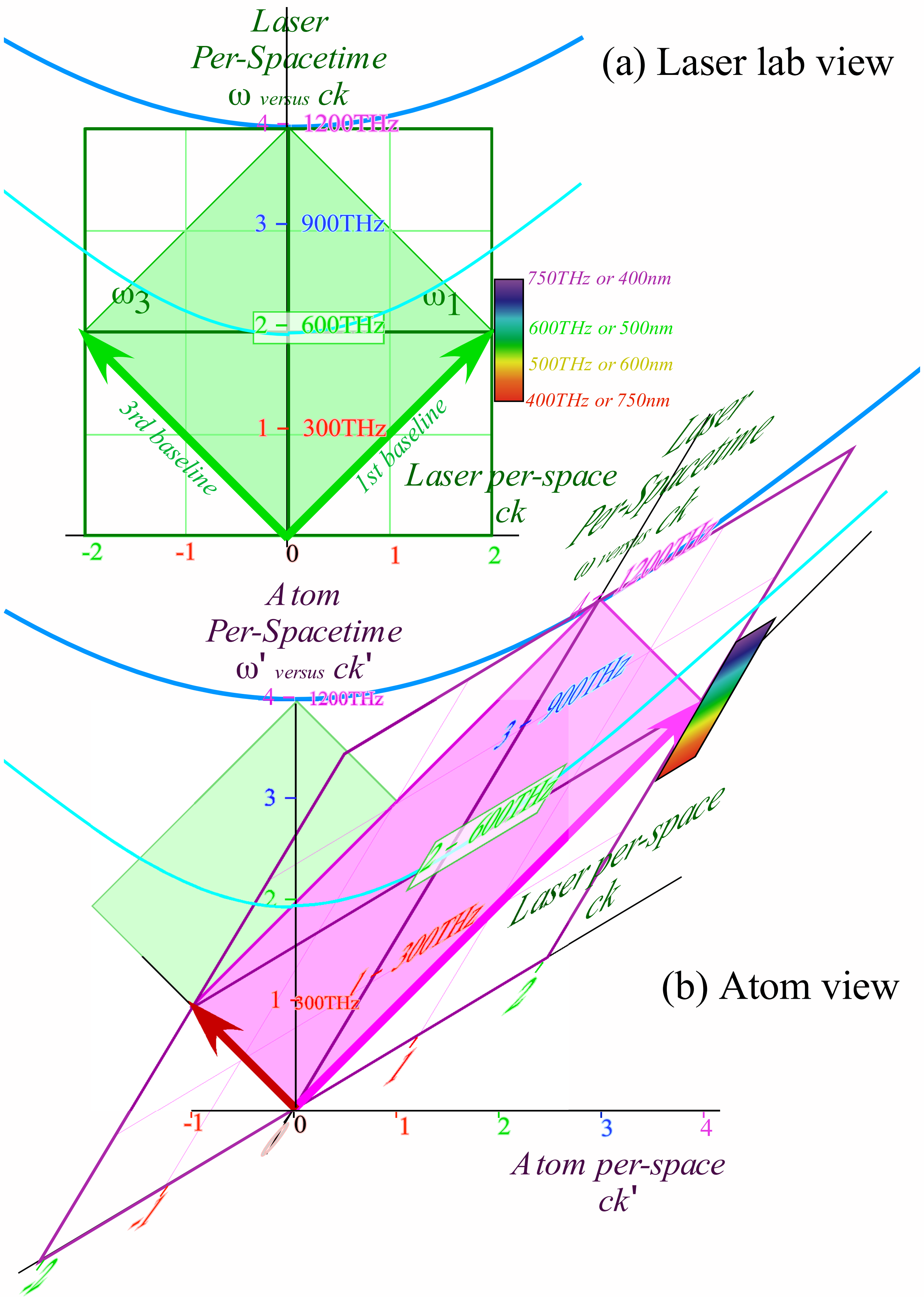
Thales Mean Geometry (600BCE)

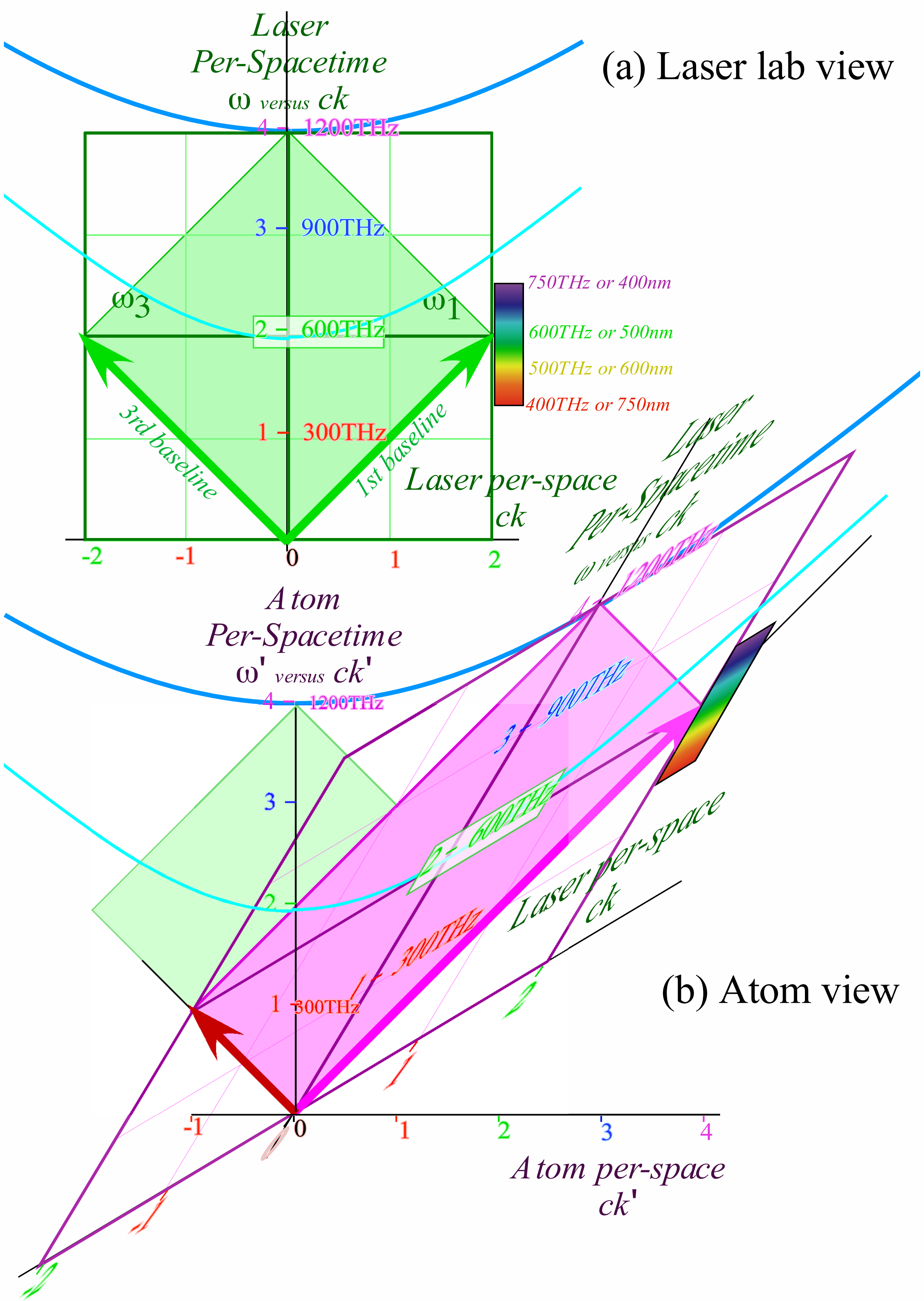
helps "Relawavity"

Thales of Miletus 624-543 BCE



[RelaWavity Web Simulation](#)
[Detailed *Thales Geometry*](#)


(a) Laser lab view
Laser
Per-Spacetime
ω versus ck
4 – 1200THz
3 – 900THz
2 – 600THz
1 – 300THz
ω3
ω1
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
3rd baseline
1st baseline
Laser per-space
ck
-2
-1
0
1
2
Atom
Per-Spacetime
ω' versus ck'
4 – 1200THz
3
2
1 300THz
3 – 900THz
2 – 600THz
1 – 300THz
Laser
Per-Spacetime
ω versus ck
Laser per-space
ck
(b) Atom view
-1
0
1
2
3
4
Atom per-space
ck'



Laser
Per-Spacetime
ω versus ck
(a) Laser lab view
4 – 1200THz
3 – 900THz
2 – 600THz
1 – 300THz
ω3
ω1
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
3rd baseline
1st baseline
Laser per-space
ck
-2
-1
0
1
2
Atom
Per-Spacetime
ω' versus ck'
4 – 1200THz
3
2
1
300THz
Laser
Per-Spacetime
ω versus ck
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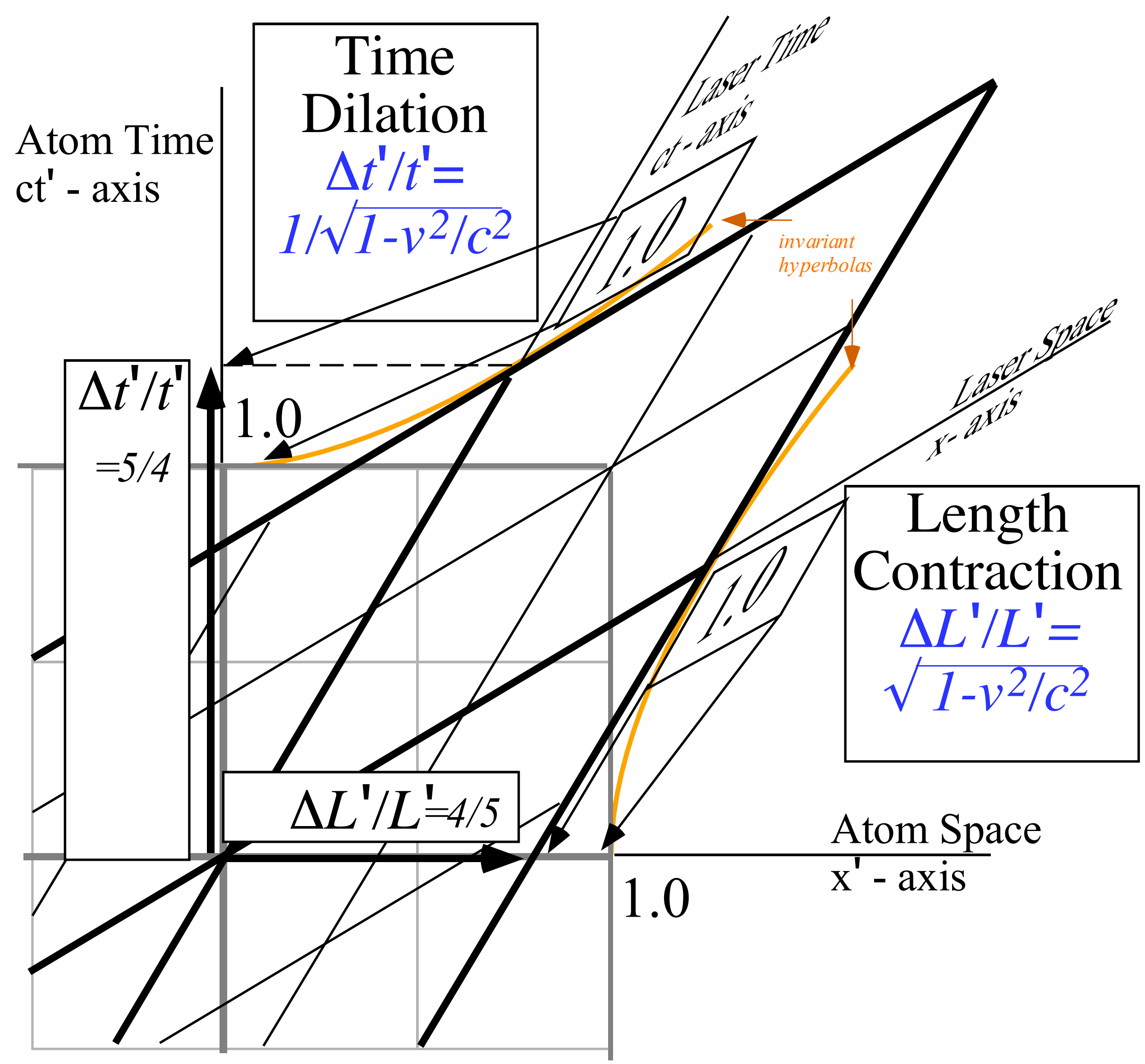
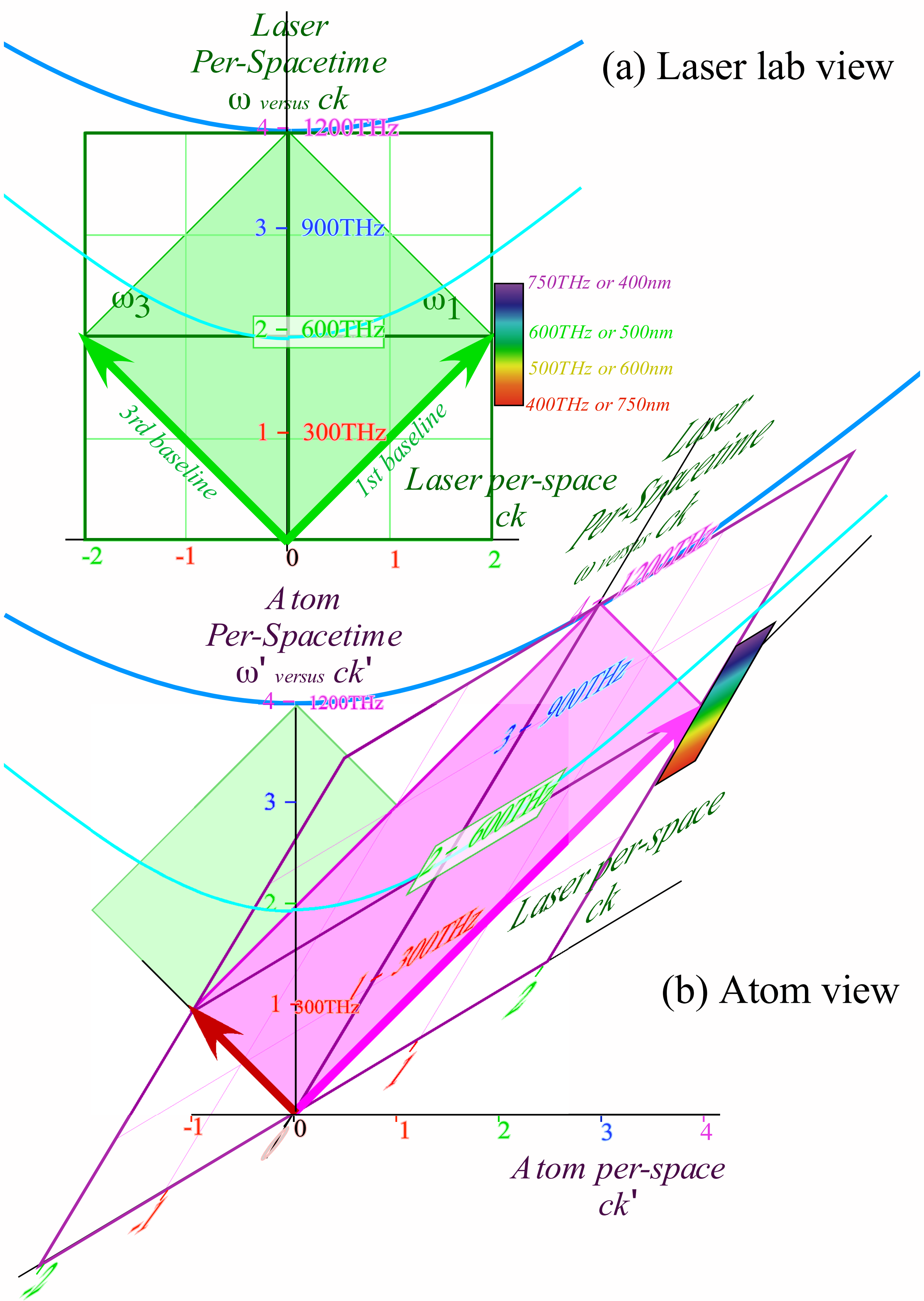
900THz
600THz
300THz
Laser per-space
ck
(b) Atom view
-1
0
1
2
3
4
Atom per-space
ck'



Atom Time
ct' - axis
Time
Dilation
Δt'/t'=
1/√1-v²/c²
Laser Time
ct - axis
1.0
invariant
hyperbolas
Δt'/t'
=5/4
1.0
Laser Space
x- axis
Length
Contraction
ΔL'/L'=
√1-v²/c²
1.0
ΔL'/L'=4/5
Atom Space
x' - axis
1.0

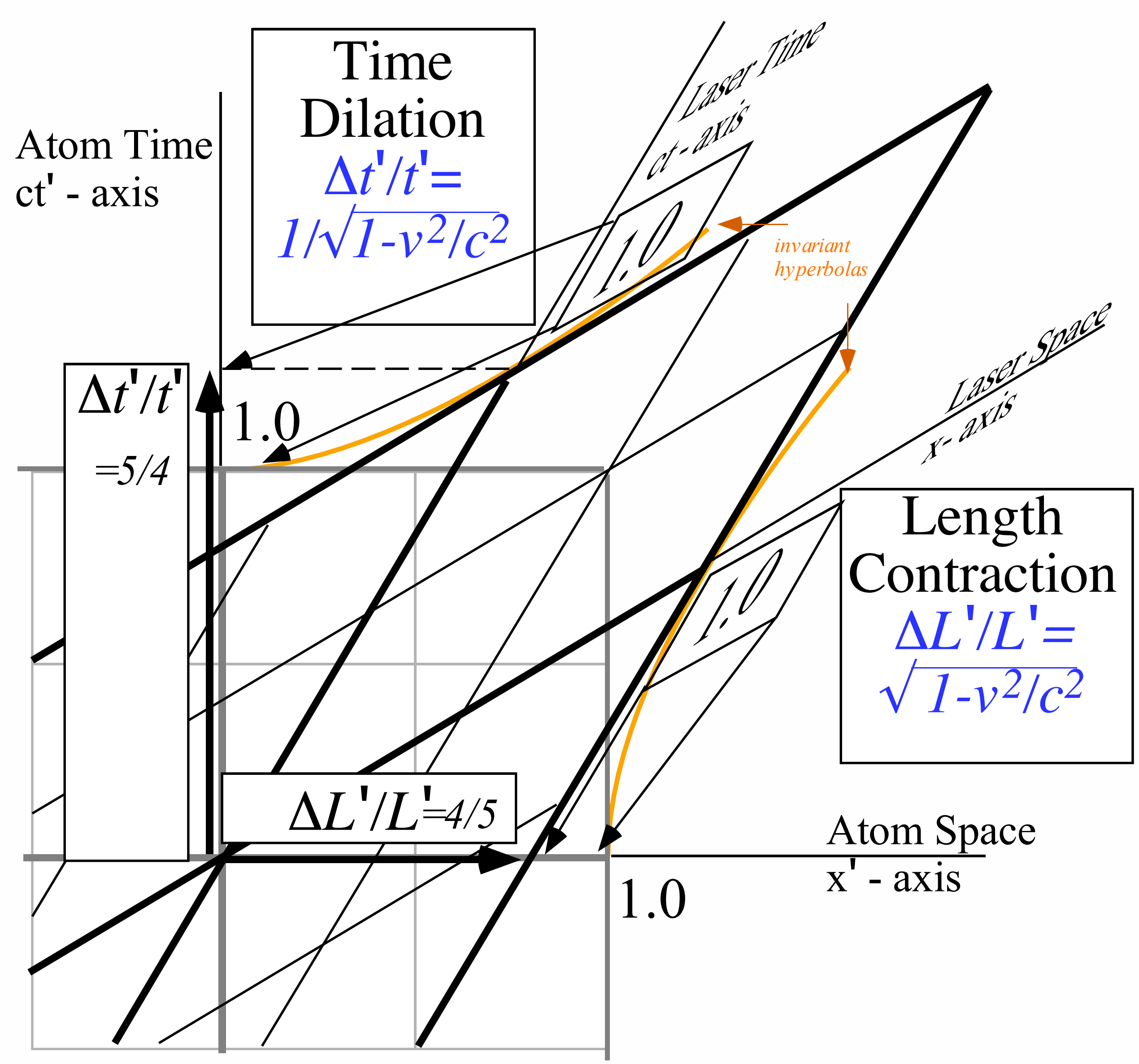


OK! But...

What about "Time Contraction"?

or

"Length dilation"?



Lecture 31
Thur. 12.08.2016

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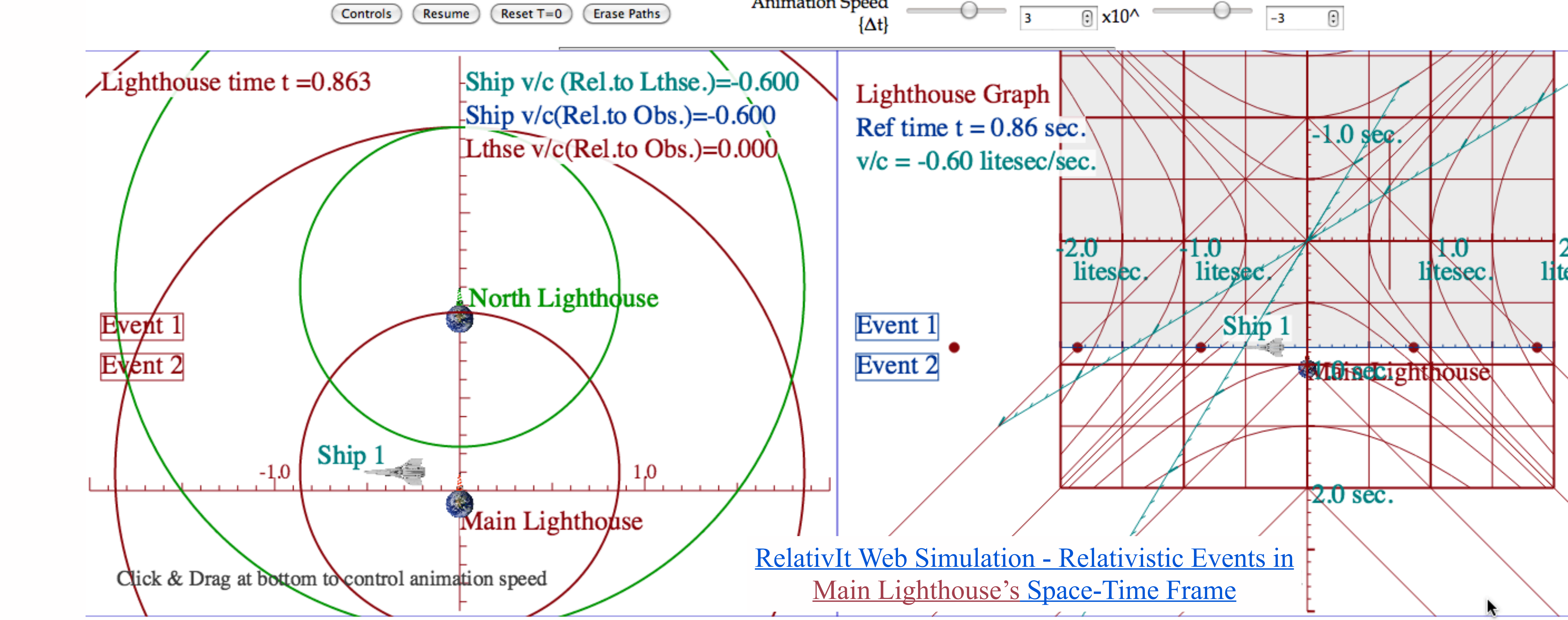
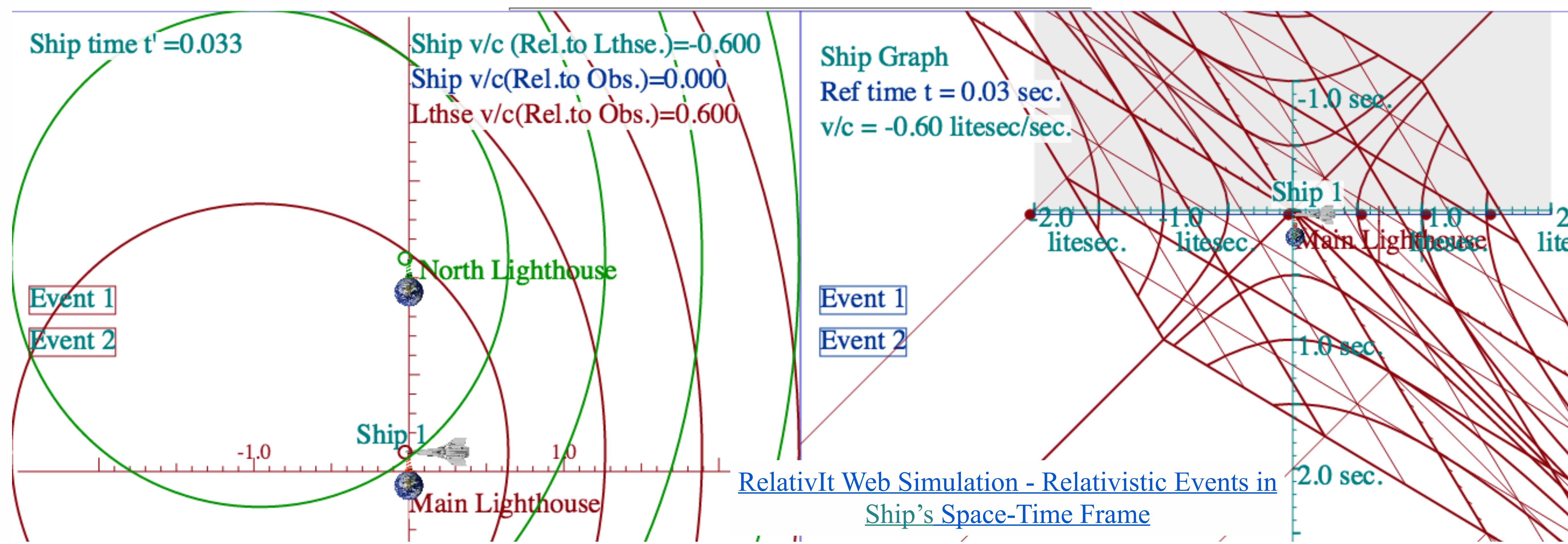

Controls
Resume
Reset T=0
Erase Paths
Animation Speed
{Δt}
x10^
Lighthouse time t =0.863
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=-0.600
Lthse v/c(Rel.to Obs.)=0.000
North Lighthouse
Event 1
Event 2
Ship 1
-1.0
1.0
Main Lighthouse
Click & Drag at bottom to control animation speed
Lighthouse Graph
Ref time t = 0.86 sec.
v/c = -0.60 litesec/sec.
-1.0 sec.
-2.0 litesec.
-1.0 litesec.
1.0 litesec.
2.0 sec.
RelativIt Web Simulation - Relativistic Events in
Main Lighthouse's Space-Time Frame


Ship time t' =0.033
Ship v/c (Rel.to Lthse.)=-0.600
Ship v/c(Rel.to Obs.)=0.000
Lthse v/c(Rel.to Obs.)=0.600
North Lighthouse
Event 1
Event 2
Ship 1
-1.0
1.0
Main Lighthouse
Ship Graph
Ref time t = 0.03 sec.
v/c = -0.60 litesec/sec.
-1.0 sec.
-2.0 litesec.
-1.0 litesec.
1.0 sec.
2.0 sec.
RelativIt Web Simulation - Relativistic Events in
Ship's Space-Time Frame

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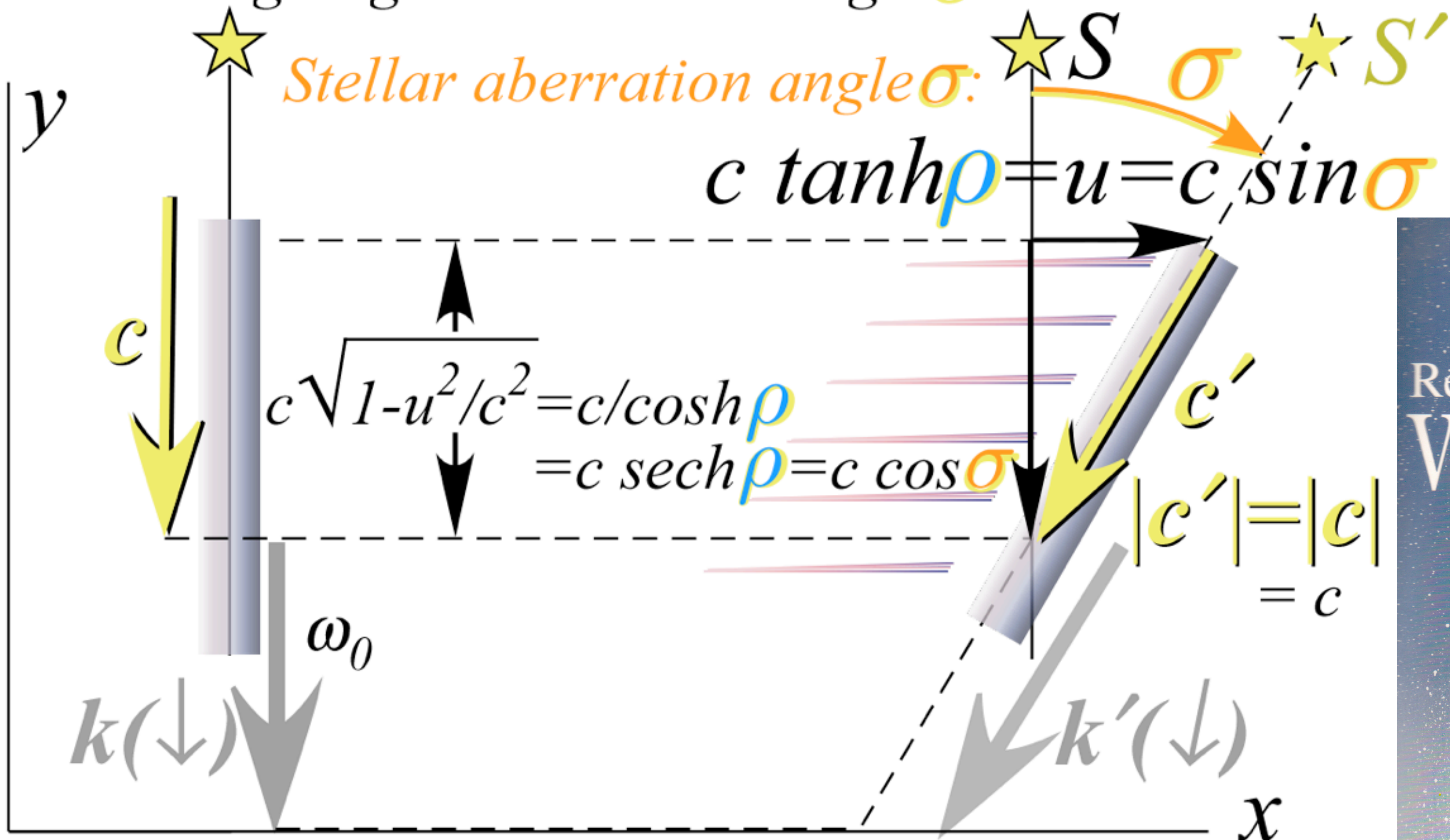
to a Transverse*relativity parameter: Stellar aberration angle σ

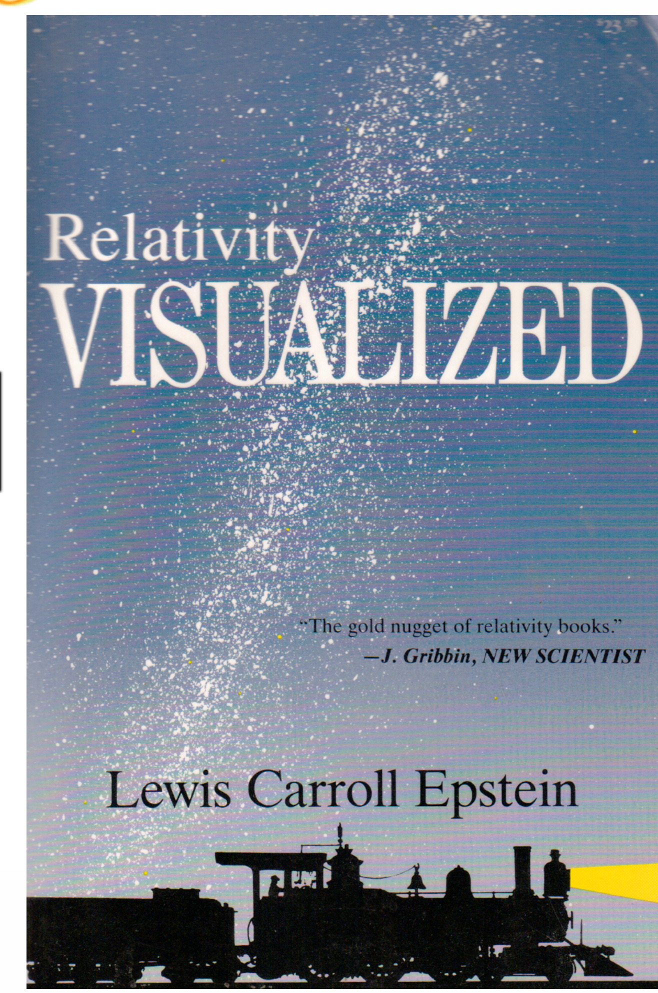
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*



We used notion σ for stellar-ab-angle, (a "flipped-out" ρ). Epstein not interested in ρ analysis or in relation of σ and ρ.



Purchase at: AllBookstores.com amazon

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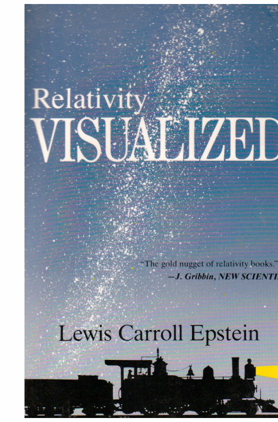
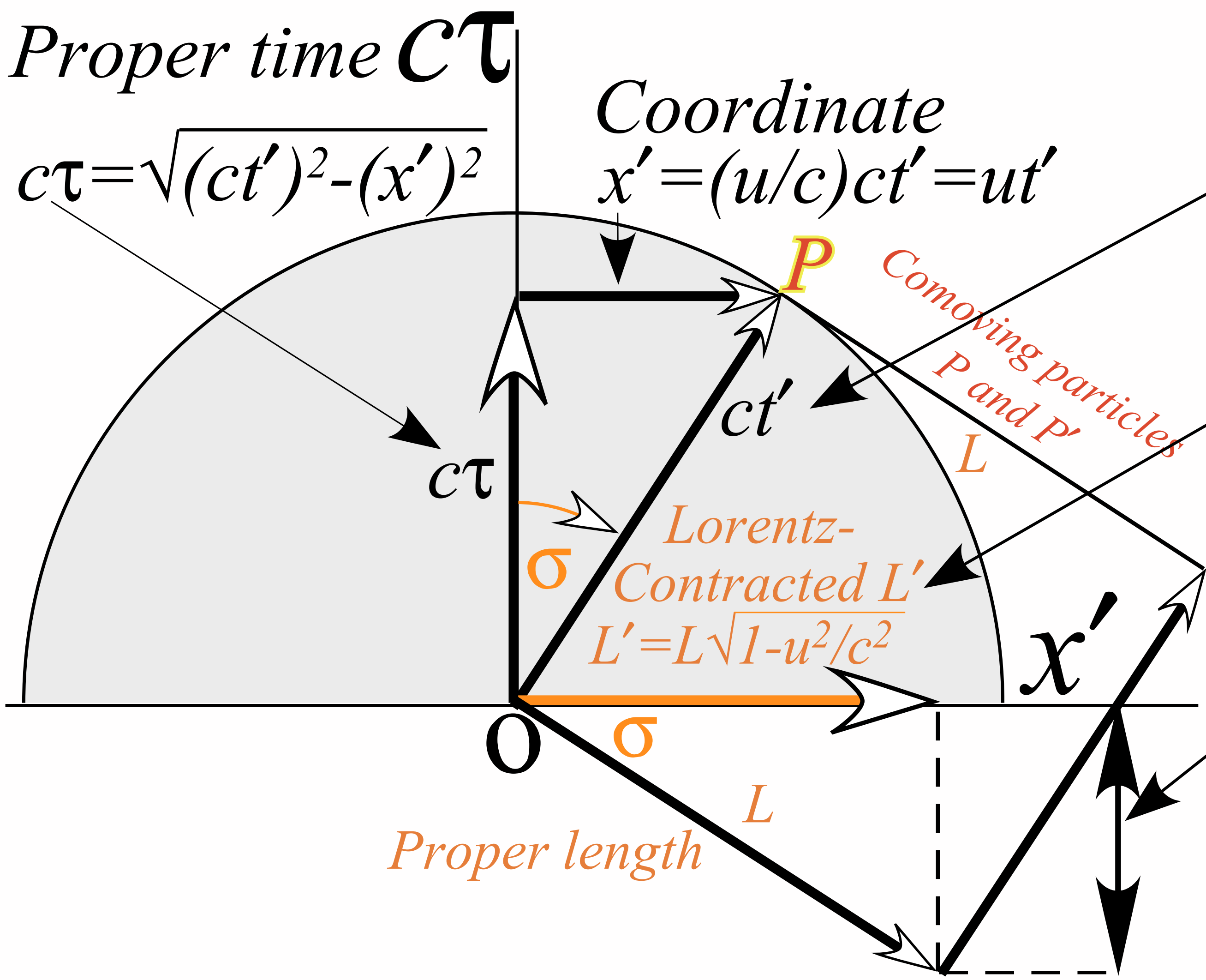


Proper time $c\tau$ vs. coordinate space x - *(L. C. Epstein's "Cosmic Speedometer")*

Particles P and P' have speed u in (x',ct') and speed c in $(x, c\tau)$



Einstein time dilation:

$$ct'=c\tau\sec\sigma=c\tau\cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L'= L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to turn a hyperbolic form $c\tau=\sqrt{(ct')^2-(x')^2}$ into a circular form: $\sqrt{(c\tau)^2+(x')^2}=(ct')$

Comparing Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

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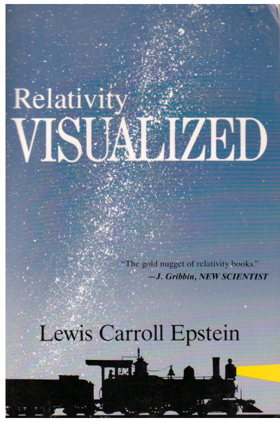
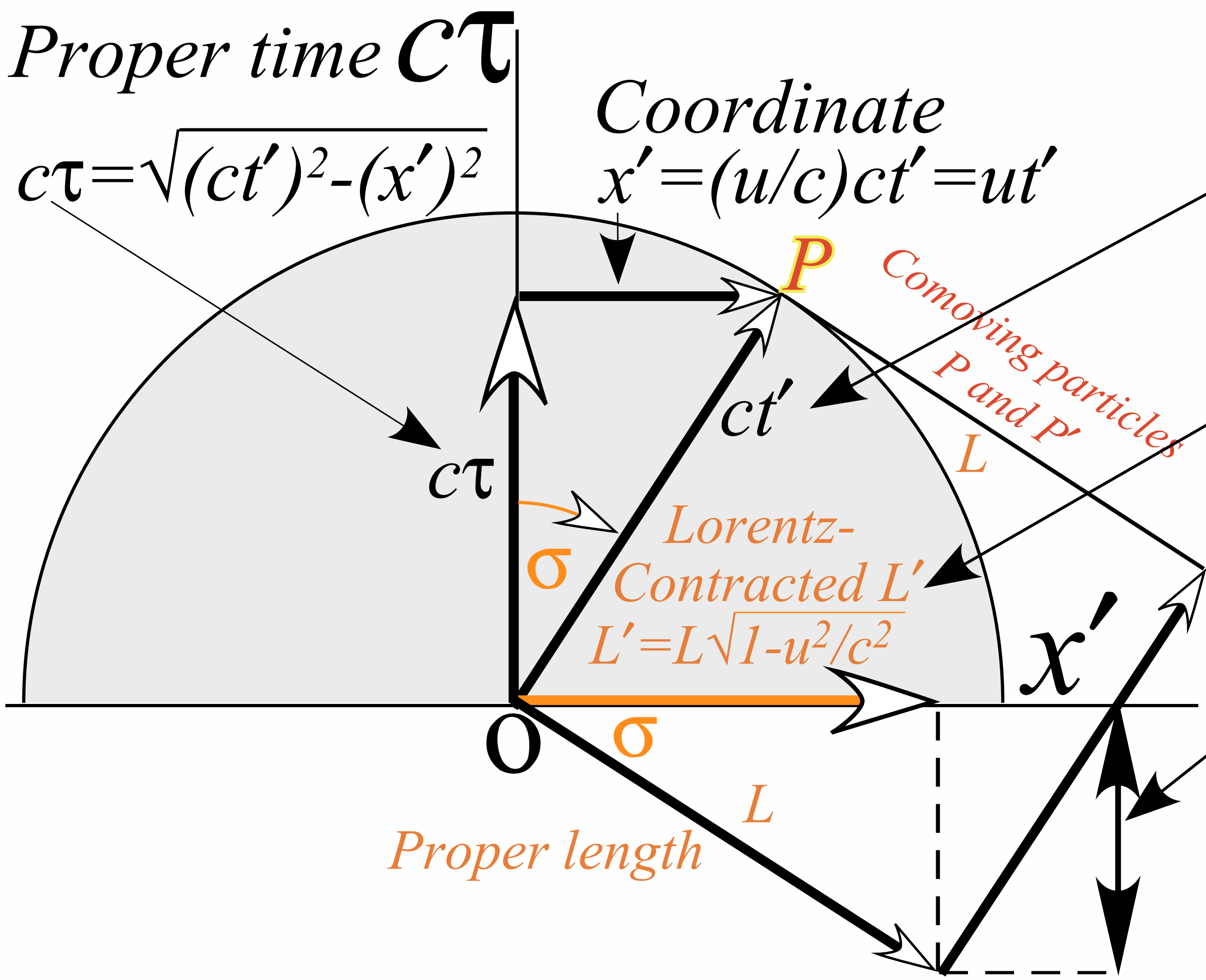


Proper time cτ vs. coordinate space x - *(L. C. Epstein's "Cosmic Speedometer")*

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



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$$= L\cos\sigma\tan\sigma$$

$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Epstein's trick is to
turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$ into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$
Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

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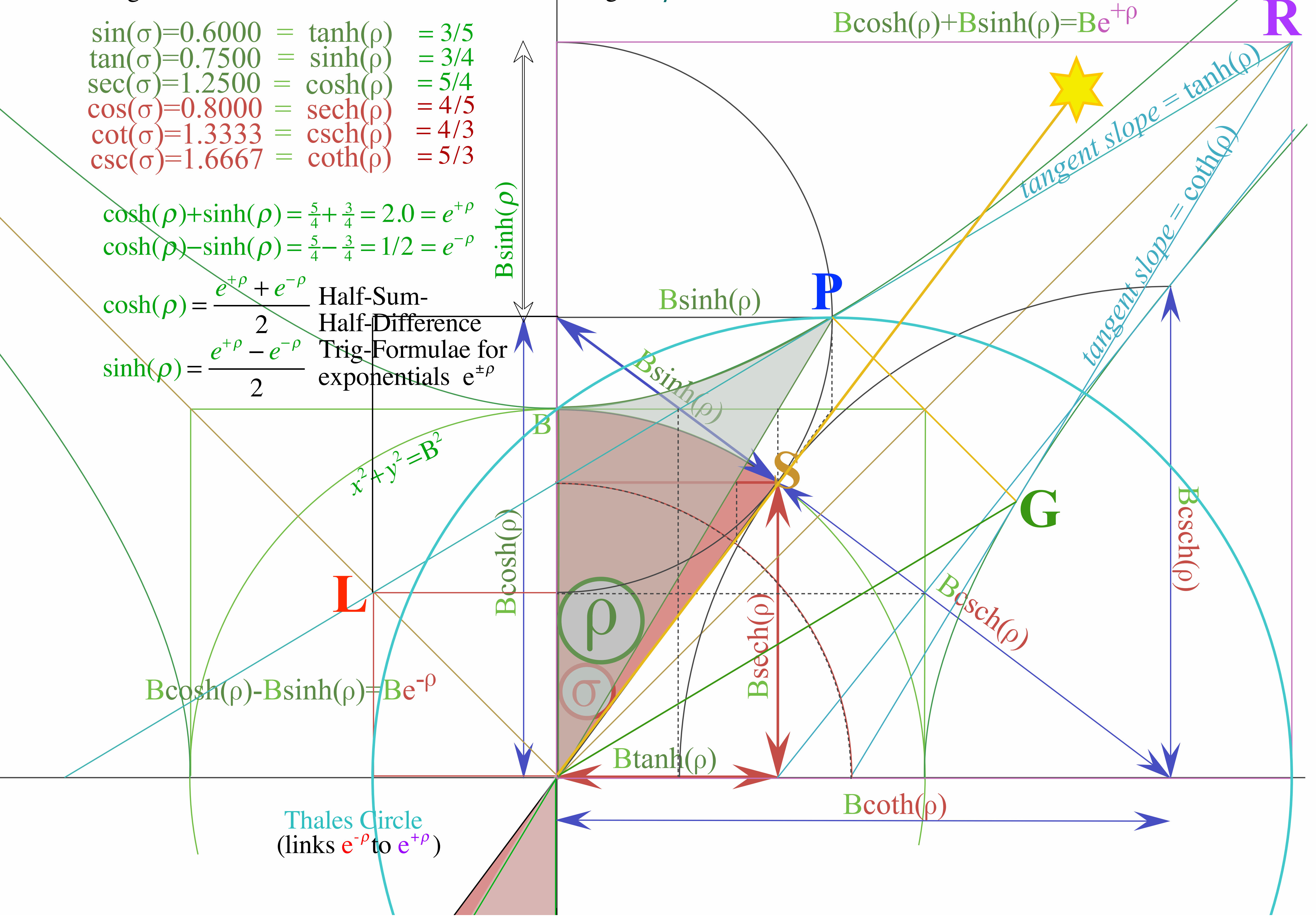
This map has circle sector arc-area $\sigma = 0.6435$ set to angle $\angle\sigma = 36.87° = 0.6435\,radian$

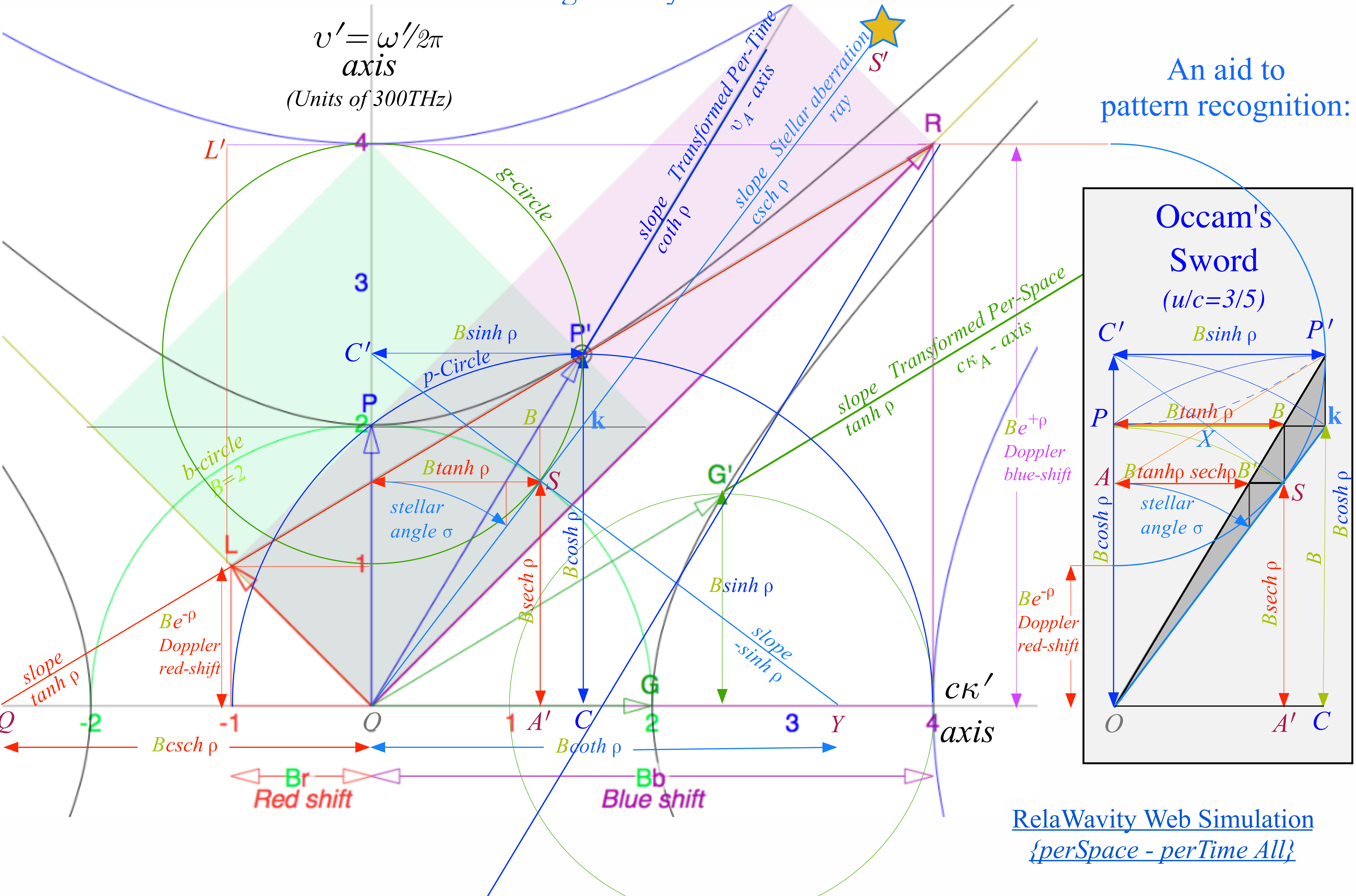
Also it is set to hyperbola sector arc-area $\rho = 0.6931$ angle $\angle\rho = \nu = 30.96°$



Summary of optical wave parameters for relativity and QM

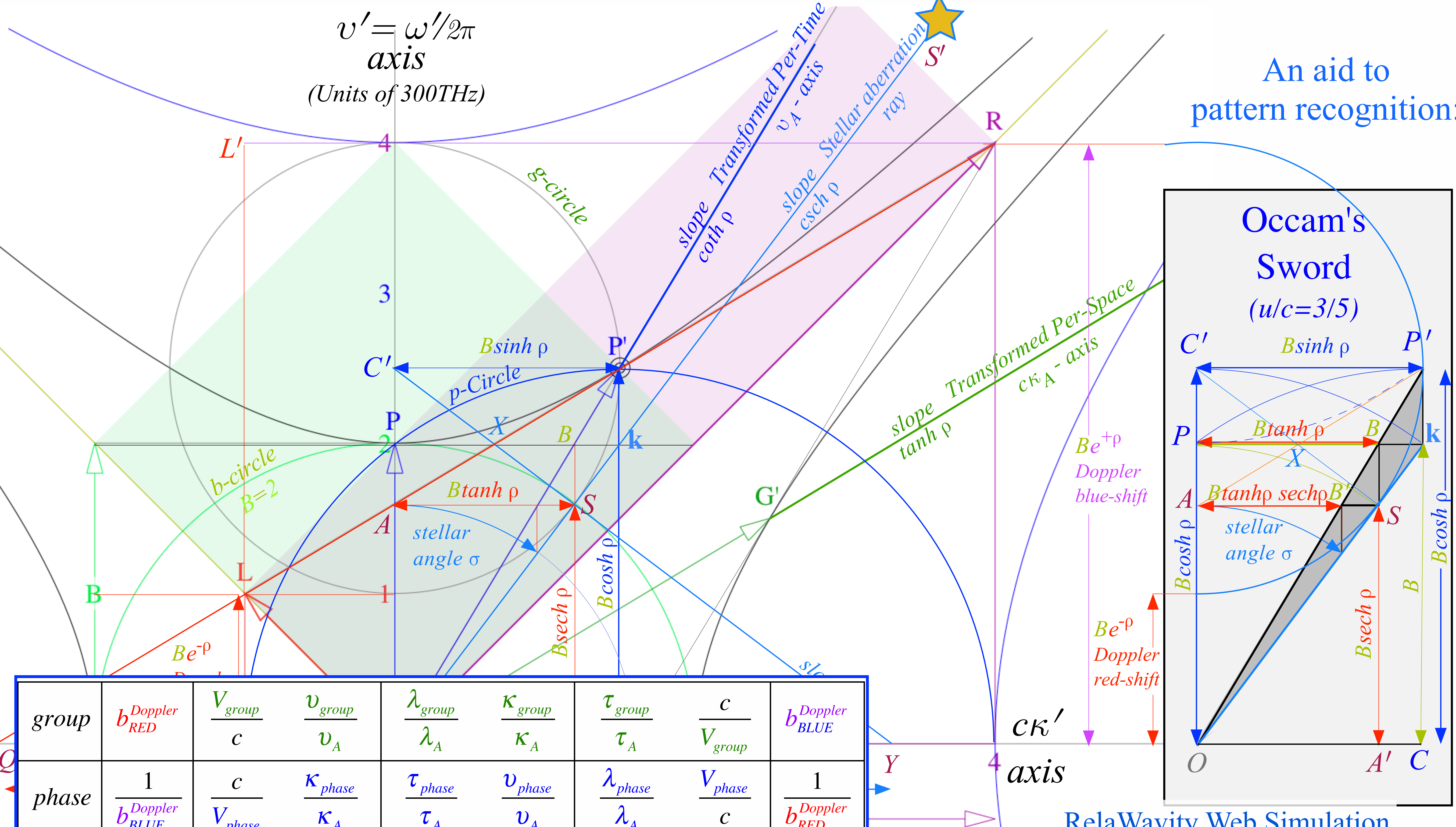
...and their geometry

An aid to pattern recognition:



RelaWavity Web Simulation
{perSpace - perTime All}

An aid to pattern recognition:



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

RelaWavity Web Simulation
{perSpace - perTime All}

Table of 12 wave parameters (includes inverses) for relativity

...and values for $u/c=3/5$

RelaWavity Web Simulation
Expanded *Table of Relativistic* Relations

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

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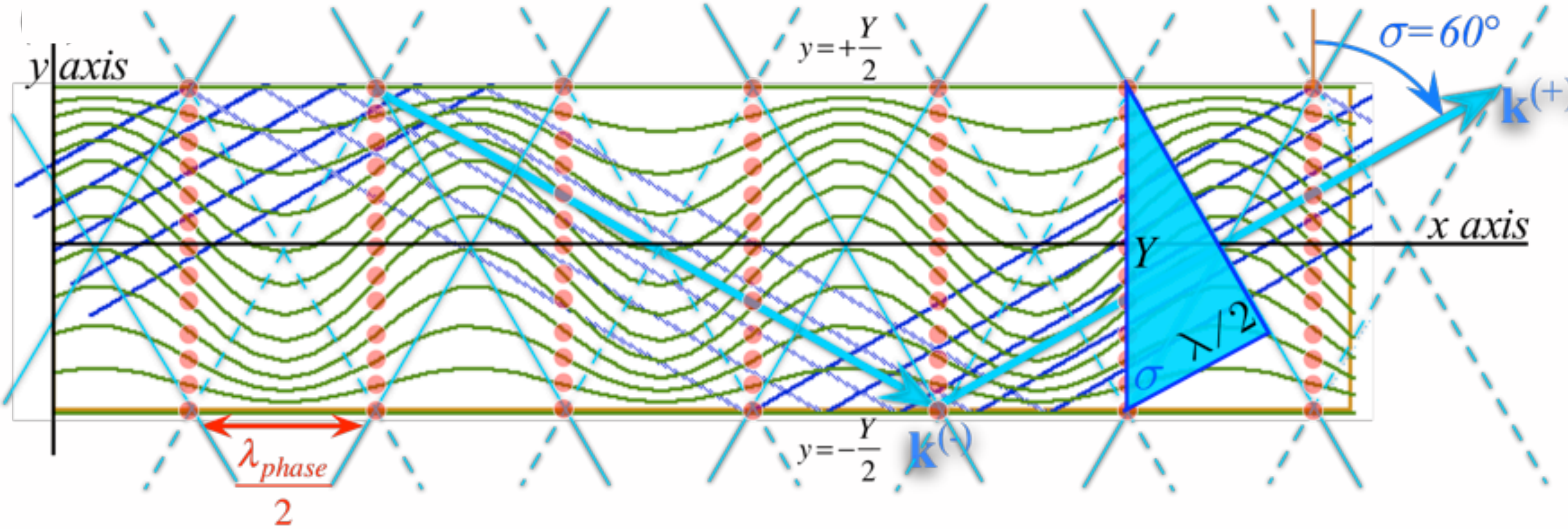
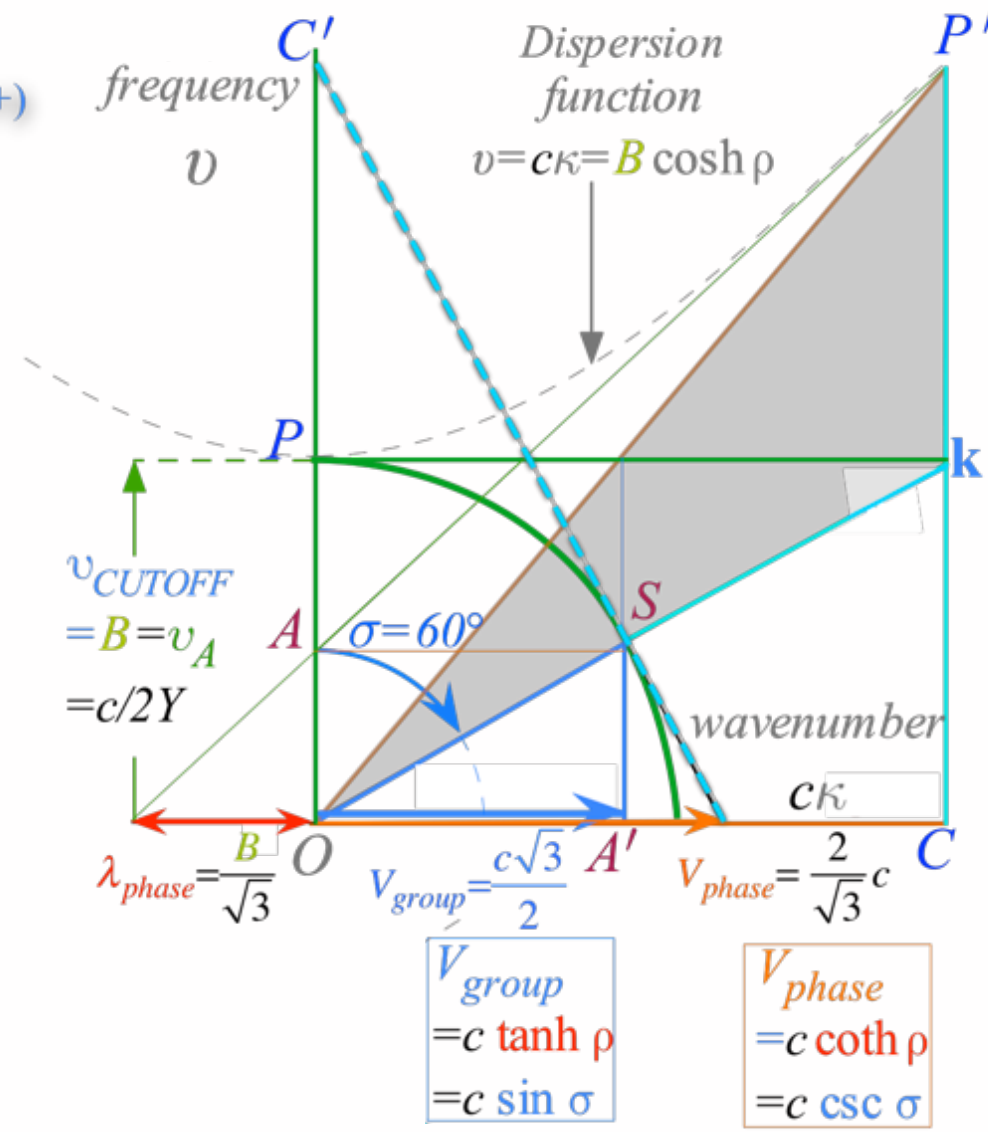
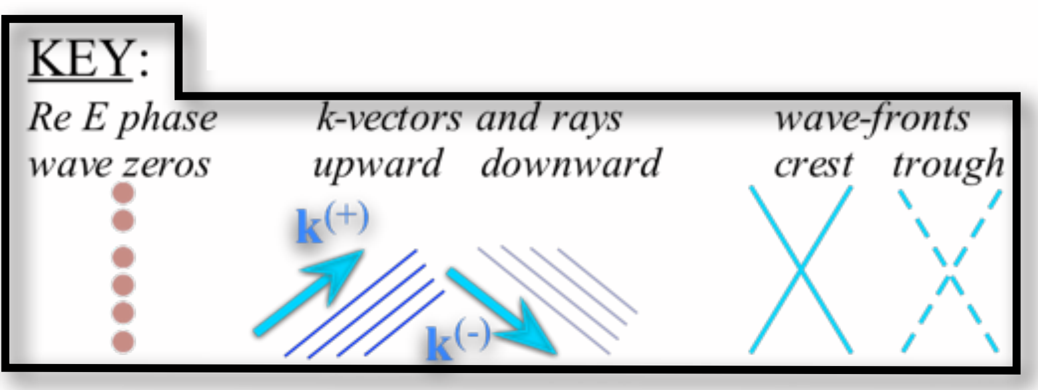
Optical wave guide relativistic geometry aided by Occam's Sword

geometry applies to (x,y) space-space
to (k_x,k_y)per-space-per-space
to (x,ct) space-time

Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)







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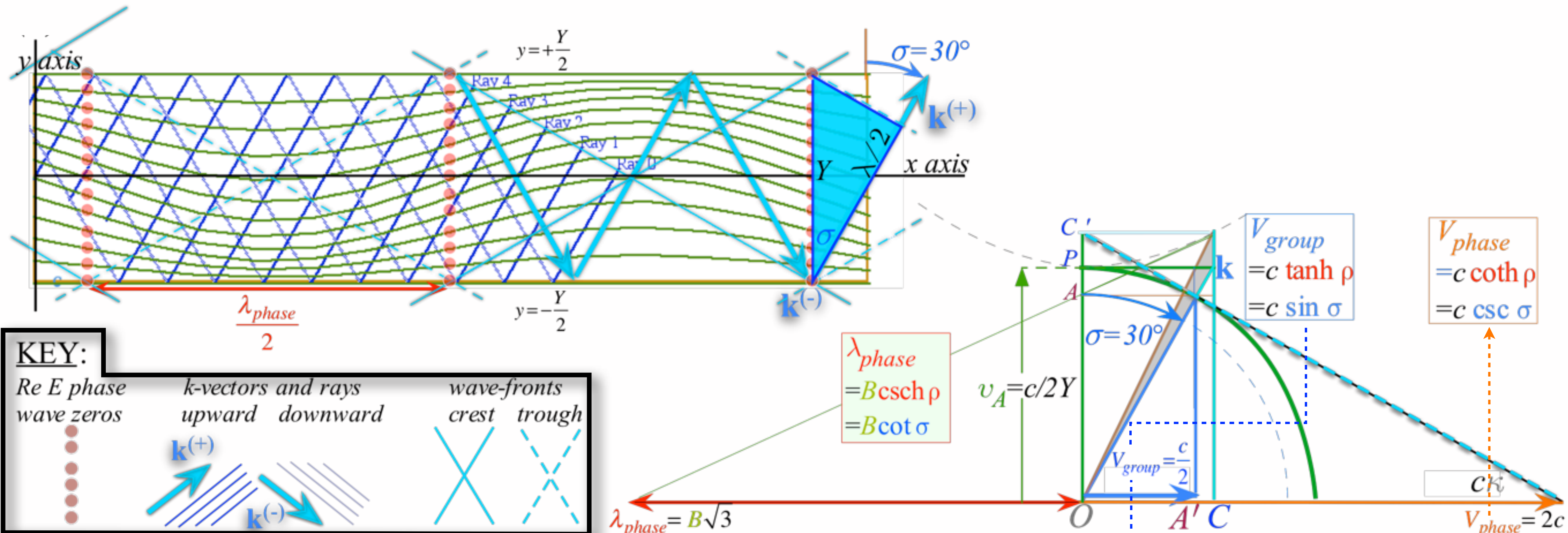

Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)

Optical wave guide relativistic geometry aided by Occam's Sword

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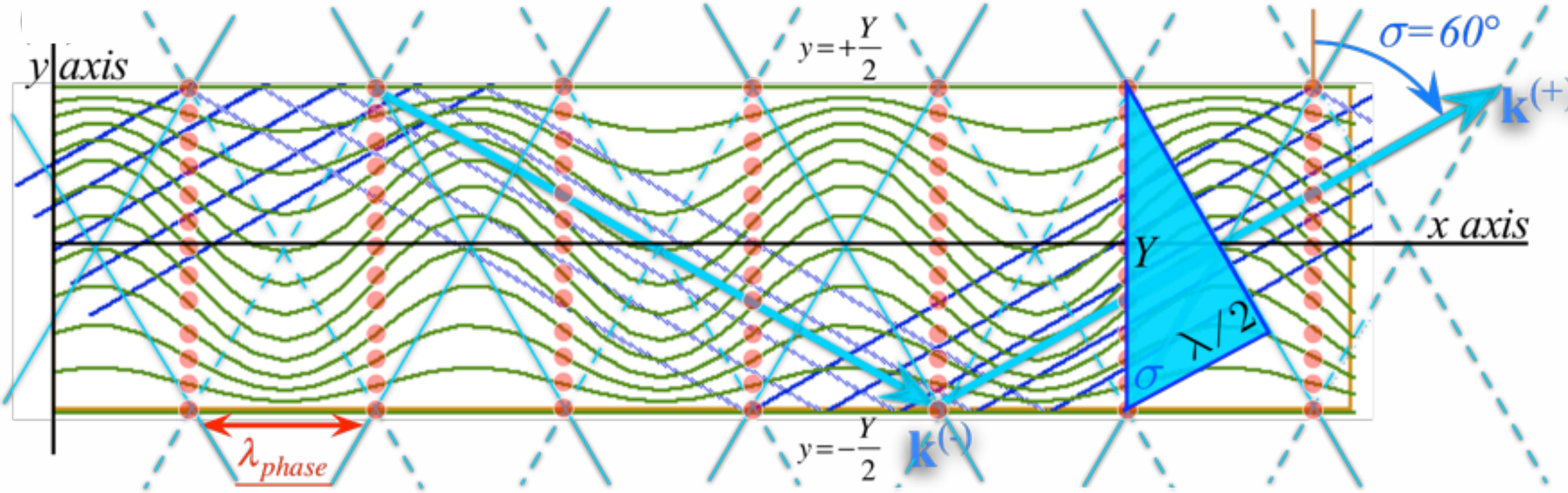
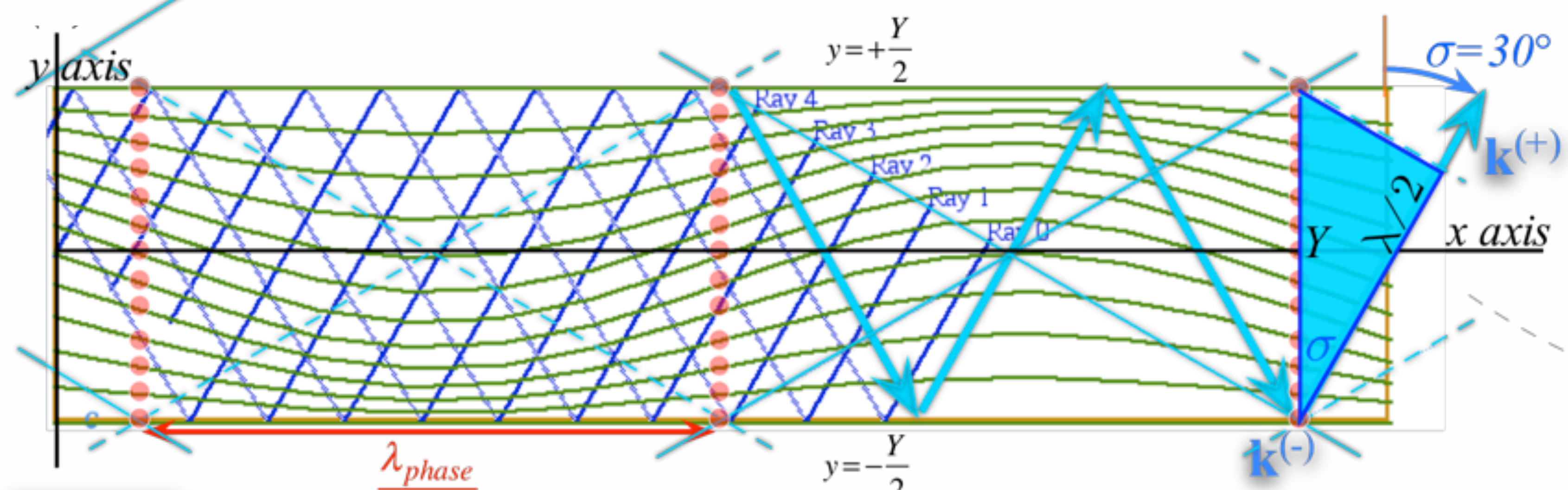
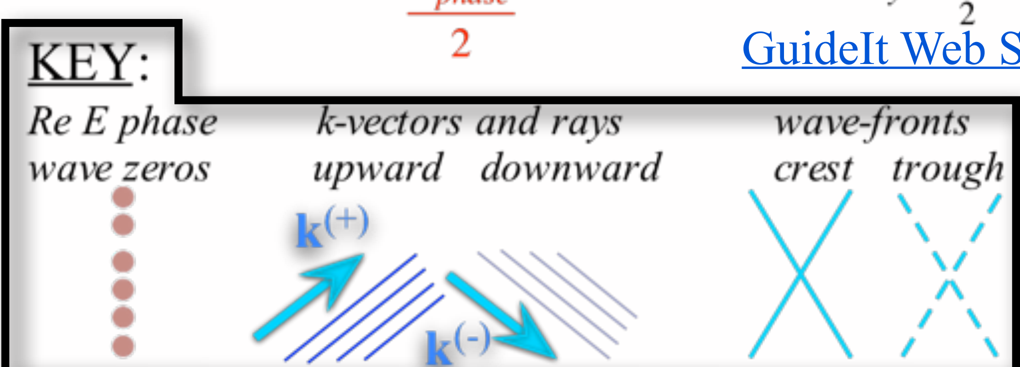
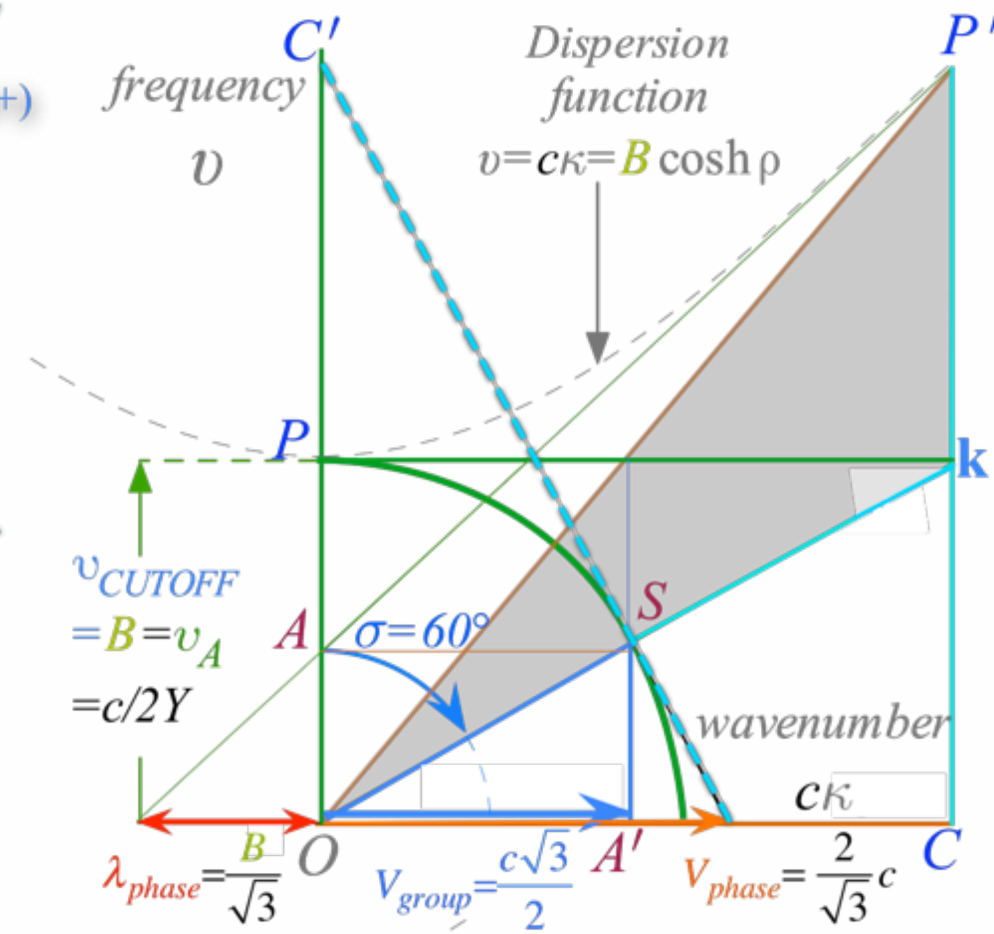


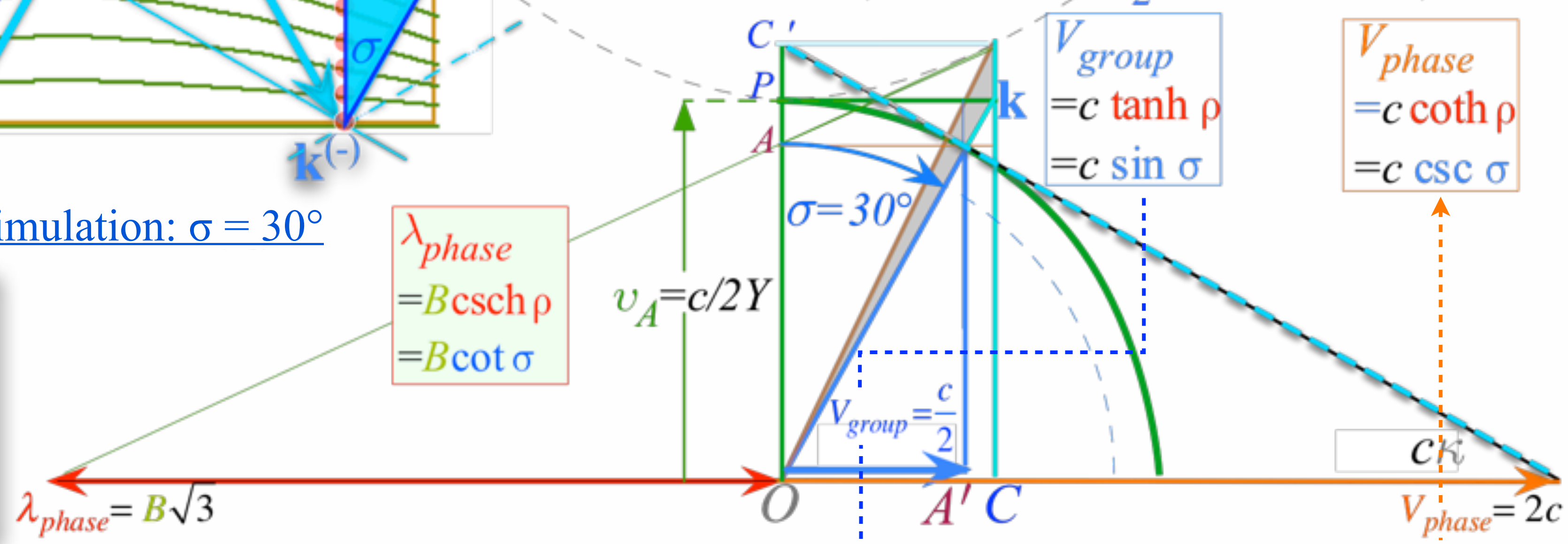
GuideIt Web Simulation: σ = 60°



GuideIt Web Simulation: σ = 30°







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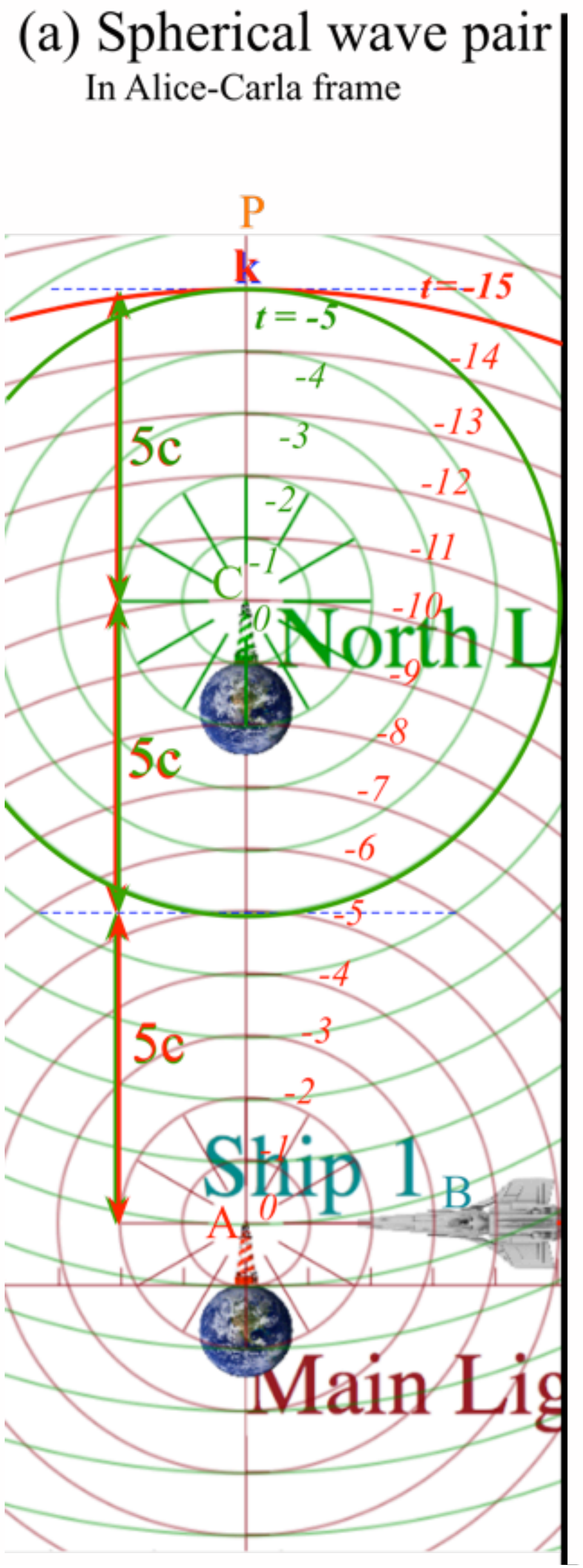
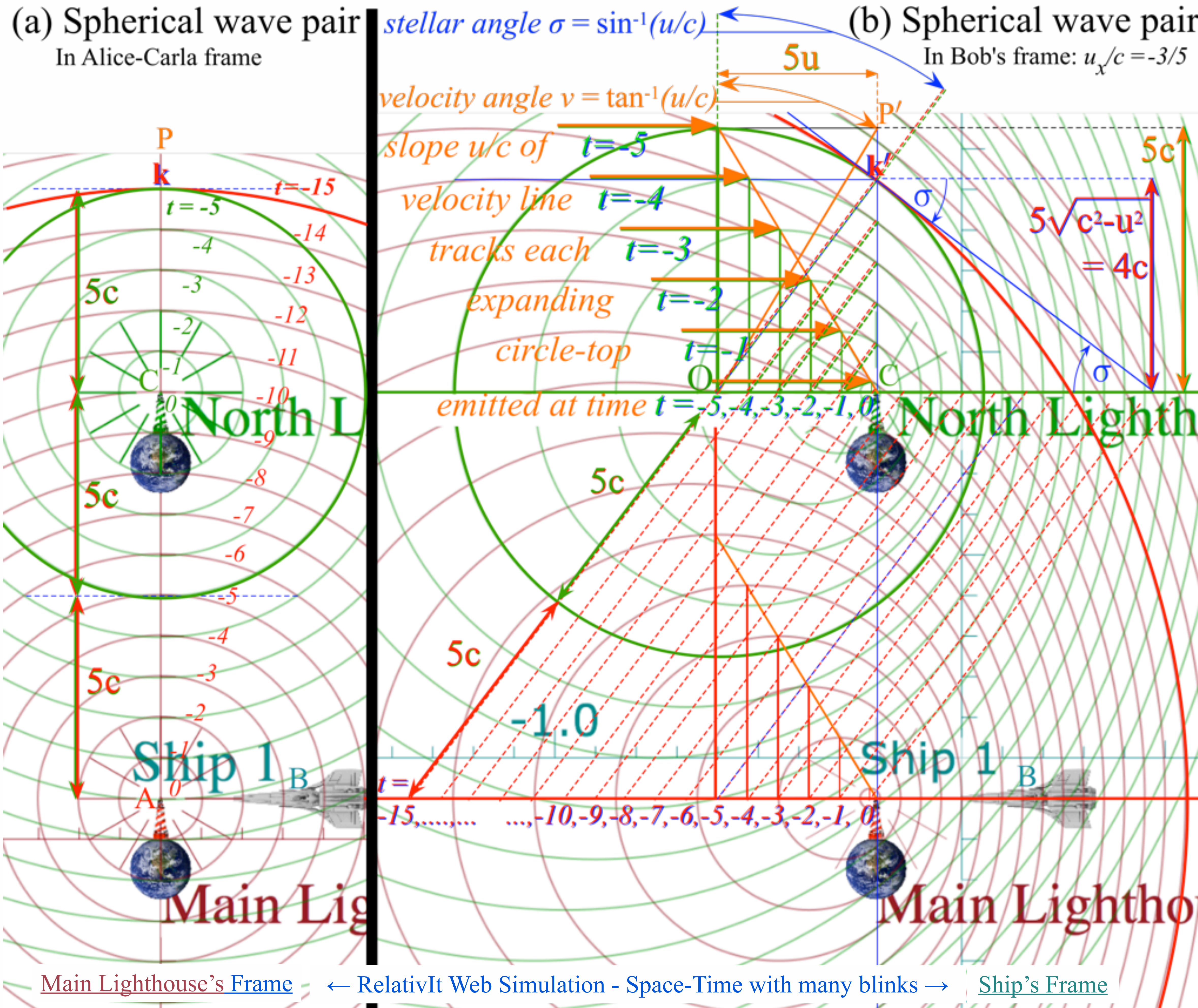
Spherical wave relativistic geometry

Also, aided by Occam's Sword

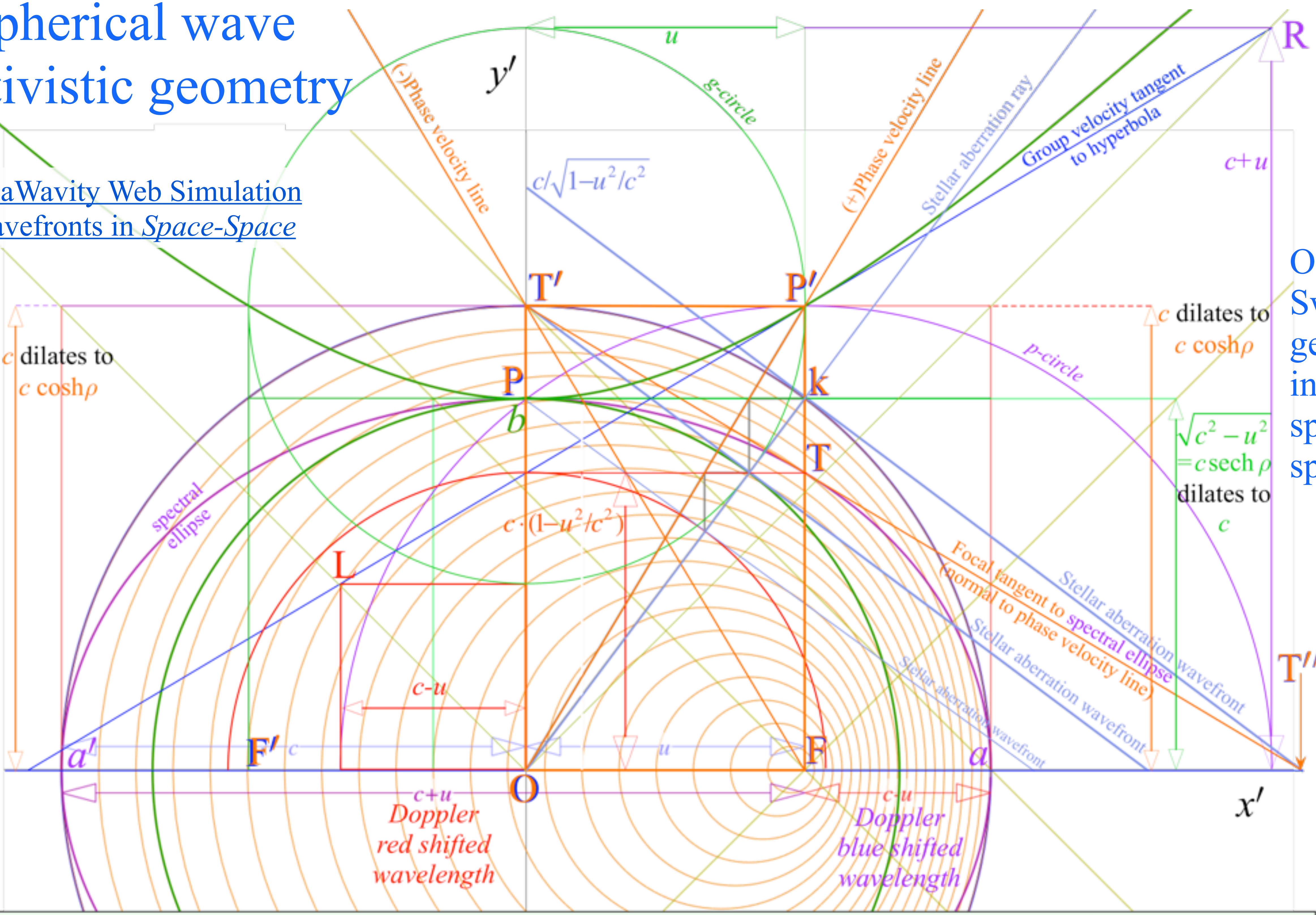




Occam Sword geometry in (x,y) space-space

Spherical wave relativistic geometry

RelaWavity Web Simulation
Wavefronts in *Space-Space*



Occam Sword geometry in (x,y) space-space

Doppler Red λ=c+u
dilates to: $(c+u)\cosh\rho = c\sqrt{\frac{c+u}{c-u}} = ce^{+\rho}$

ellipse focal length **FO**= $u = c\tanh\rho$
dilates to: $u\cosh\rho = c\sinh\rho$

Doppler Blue λ=$c-u$
dilates to: $(c-u)\cosh\rho = c\sqrt{\frac{c-u}{c+u}} = ce^{-\rho}$

ellipse major radius a=**OF**a=c
dilates to: $c\cosh\rho = c/\sqrt{1-u^2/c^2}$

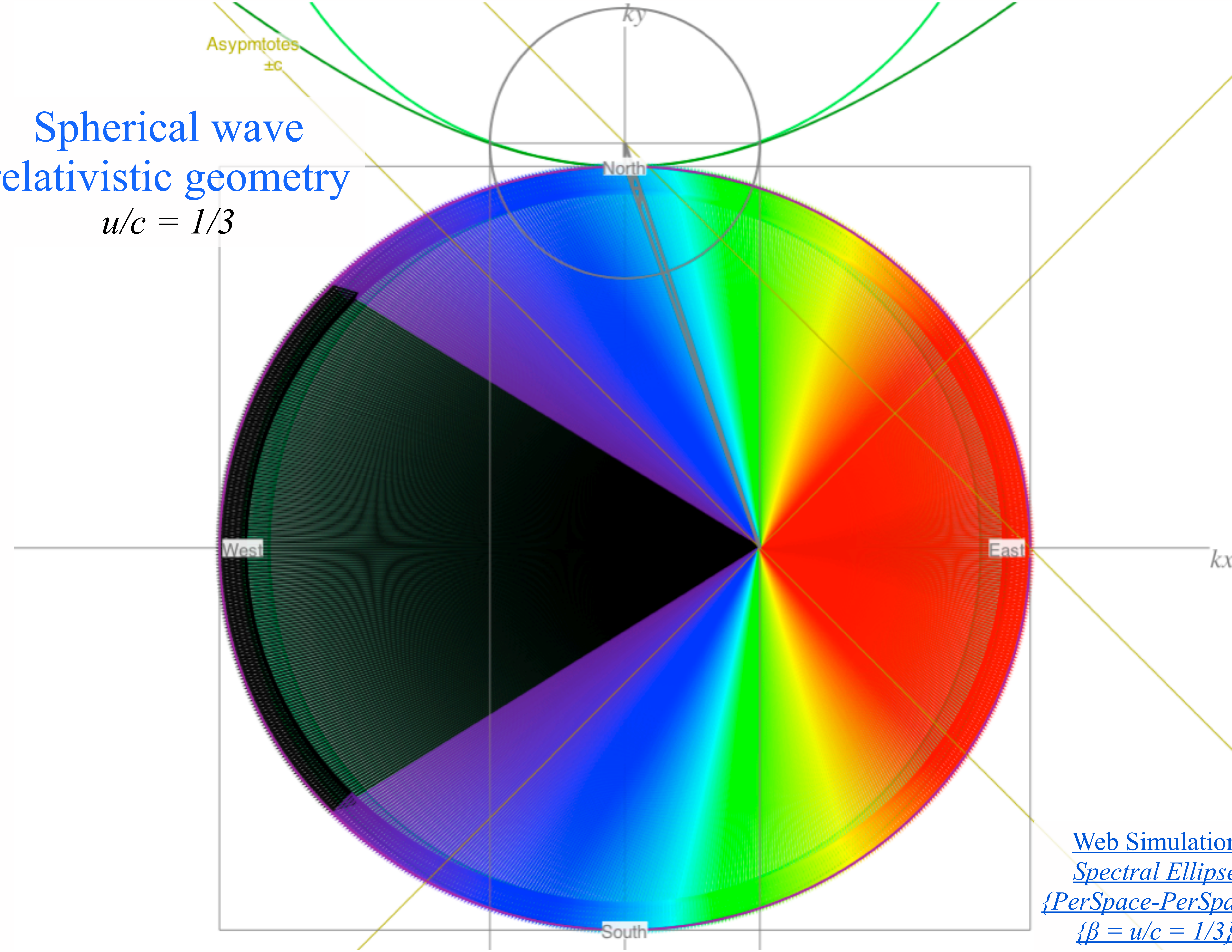
Applications of Einstein dilation factor:
$\gamma = \cosh\rho = 1/\sqrt{1-u^2/c^2}$

ellipse latus radius **FT**=$c(1-u^2/c^2)$
dilates to: $c(1-u^2/c^2)\cosh\rho$
$= c\sqrt{1-u^2/c^2} = c\ \mathrm{sech}\rho$

Base height **FTk**=$\sqrt{c^2-u^2}$
dilates to: $\sqrt{c^2-u^2}\ \cosh\rho = c$
(equal to ellipse minor radius b)

Spherical wave relativistic geometry

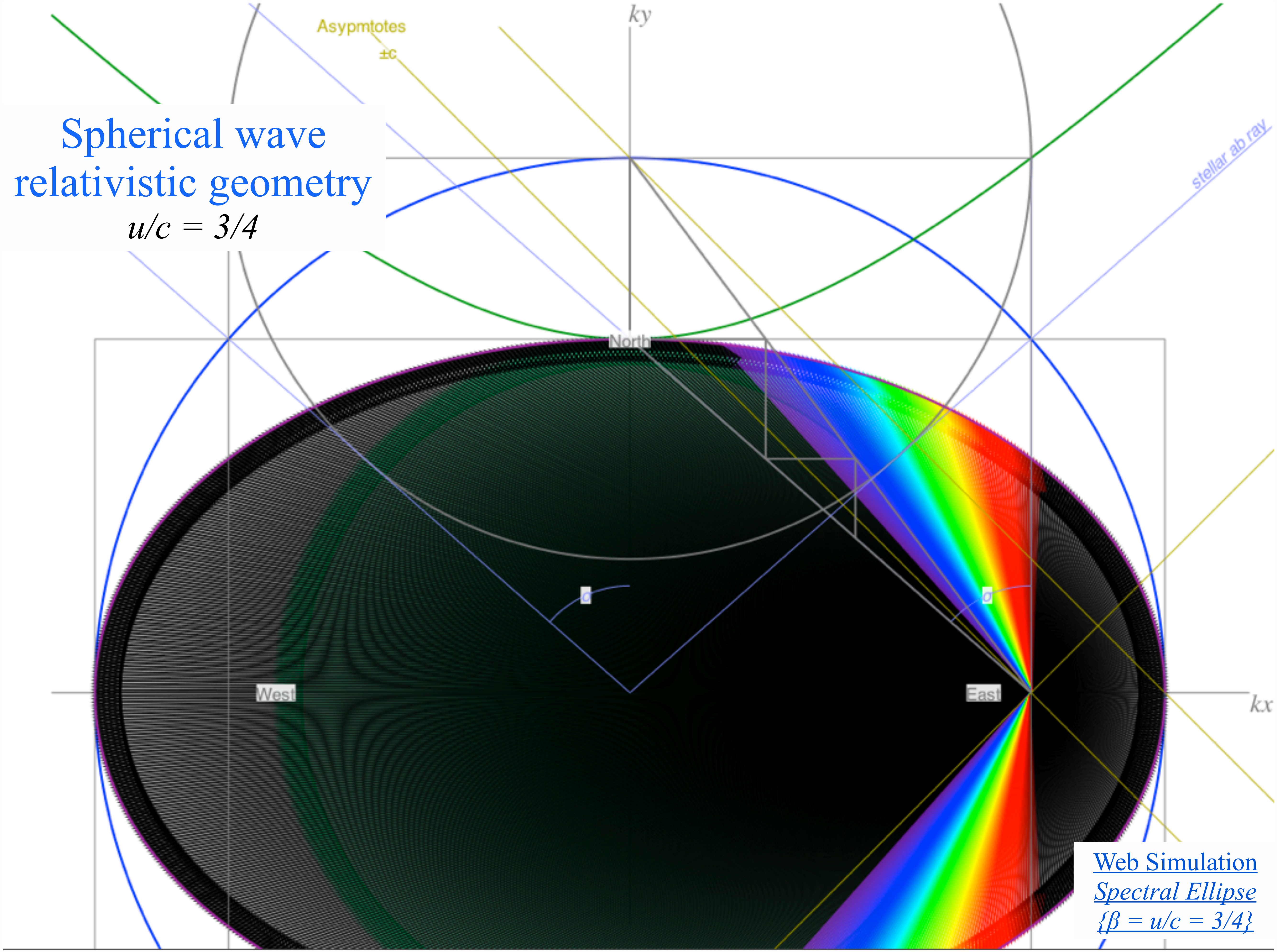
u/c = 1/3



[Web Simulation](#)
[*Spectral Ellipse*](#)
[*{PerSpace-PerSpace}*](#)
[*{β = u/c = 1/3}*](#)

Spherical wave relativistic geometry

$u/c = 3/4$



[Web Simulation](#)
[*Spectral Ellipse*](#)
[*{β = u/c = 3/4}*](#)

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Learning about sin! *and* cos *and...*

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Using (some) wave parameters to develop relativistic quantum theory

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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RelaWavity Web Simulation - Relativistic *Terms (Expanded Table)*

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
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time	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
space	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy and momentum

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
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So attach scale factor h to match units.

Resembles: $const. + \frac{1}{2}Mu^2$

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group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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RelaWavity Web Simulation - Relativistic *Terms* *(Expanded Table)*

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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(old-fashioned notation)

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

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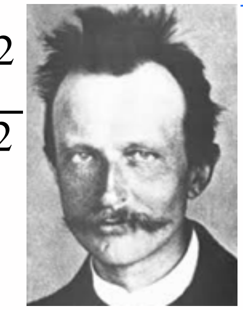

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phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
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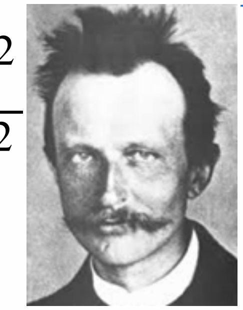

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For more visit the Pirelli Challenge Site
Quantized amplitude

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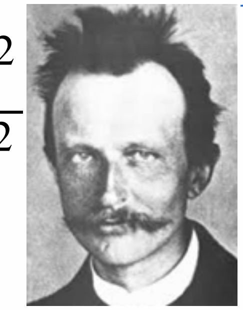

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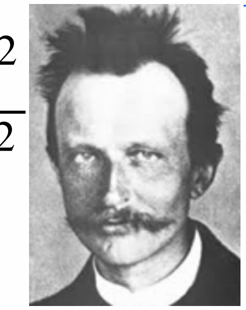
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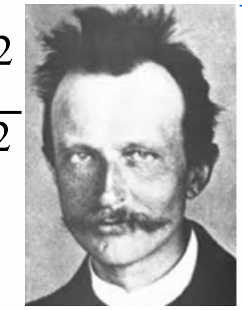

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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Using (some) wave parameters to develop relativistic quantum theory

$$\upsilon_{phase} = B\cosh\rho \approx B + \tfrac{1}{2}B\rho^2 \quad (\text{for } u \ll c)$$

$$c\kappa_{phase} = B\sinh\rho \approx B\rho \quad (\text{for } u \ll c)$$

$$\frac{u}{c} = \tanh\rho \approx \rho \quad (\text{for } u \ll c)$$

$$\cosh\rho \approx 1 + \tfrac{1}{2}\rho^2 \approx 1 + \tfrac{1}{2}\frac{u^2}{c^2}$$

$$\sinh\rho \approx \rho \approx \frac{u}{c}$$

Max Planck
1858-1947

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \dfrac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

So attach scale factor h (or hN) to match units.

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

Lucky coincidences?? *Cheap trick??*

...Try exact υ_{phase} *and* κ_{phase}*...*

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^*\!\cdot\mathbf{E} = hN\upsilon_{phase}$

$$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$$

Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$cp = \frac{Mcu}{\sqrt{1-u^2/c^2}}$$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$$ (old-fashioned notation)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	[illegible]
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Louis DeBroglie 1892-1987

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So attach scale factor h (or hN) to match units.

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~~Natural wave conspiracy~~ ~~Lucky coincidences??~~ Expensive ~~Cheap~~ trick??

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Need to replace h with hN to match e.m. energy density $\varepsilon_0 \mathbf{E}^*\cdot\mathbf{E} = hN\upsilon_{phase}$

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$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

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$$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$$

$$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$$

$$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}} \quad \text{(old-fashioned notation)}$$

$$\text{Momentum: } h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$$

DeBroglie (1921)

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Planck (1900)

$=$ Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

This motivates the "particle" normalization $\int \Psi^*\Psi \, dV = N \qquad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$

$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$

$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$ (old-fashioned notation)

Momentum: $h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$

DeBroglie (1921)

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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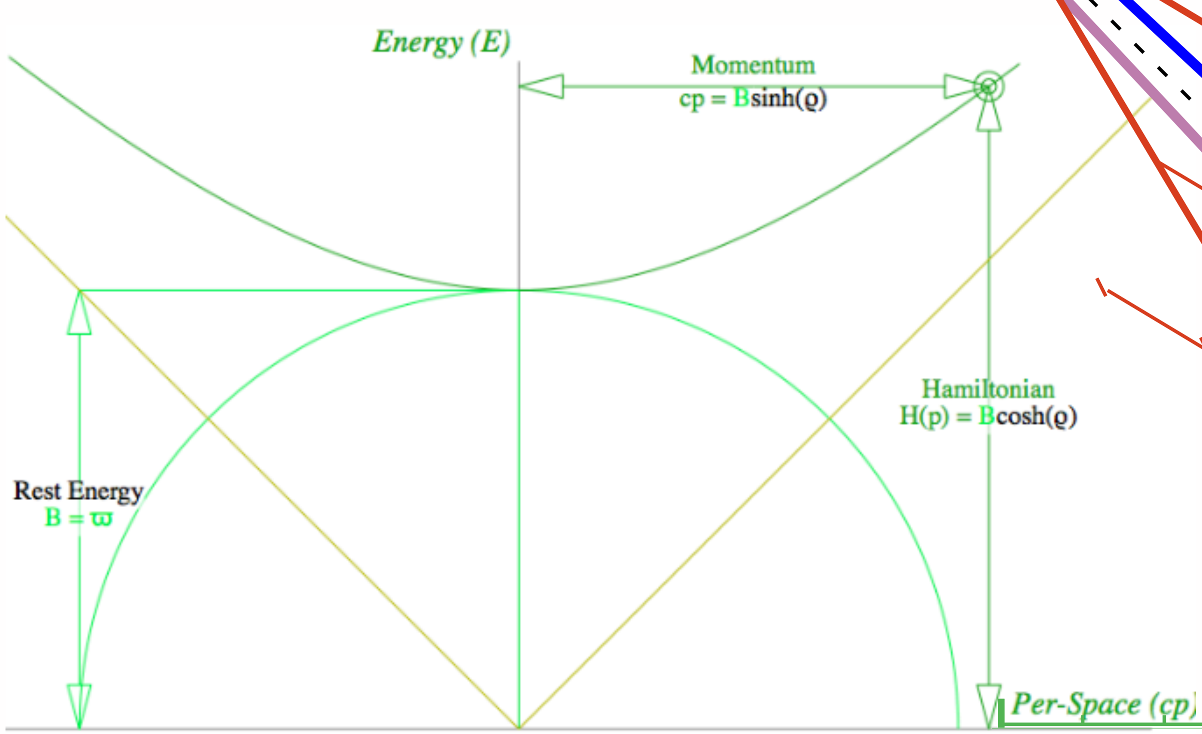
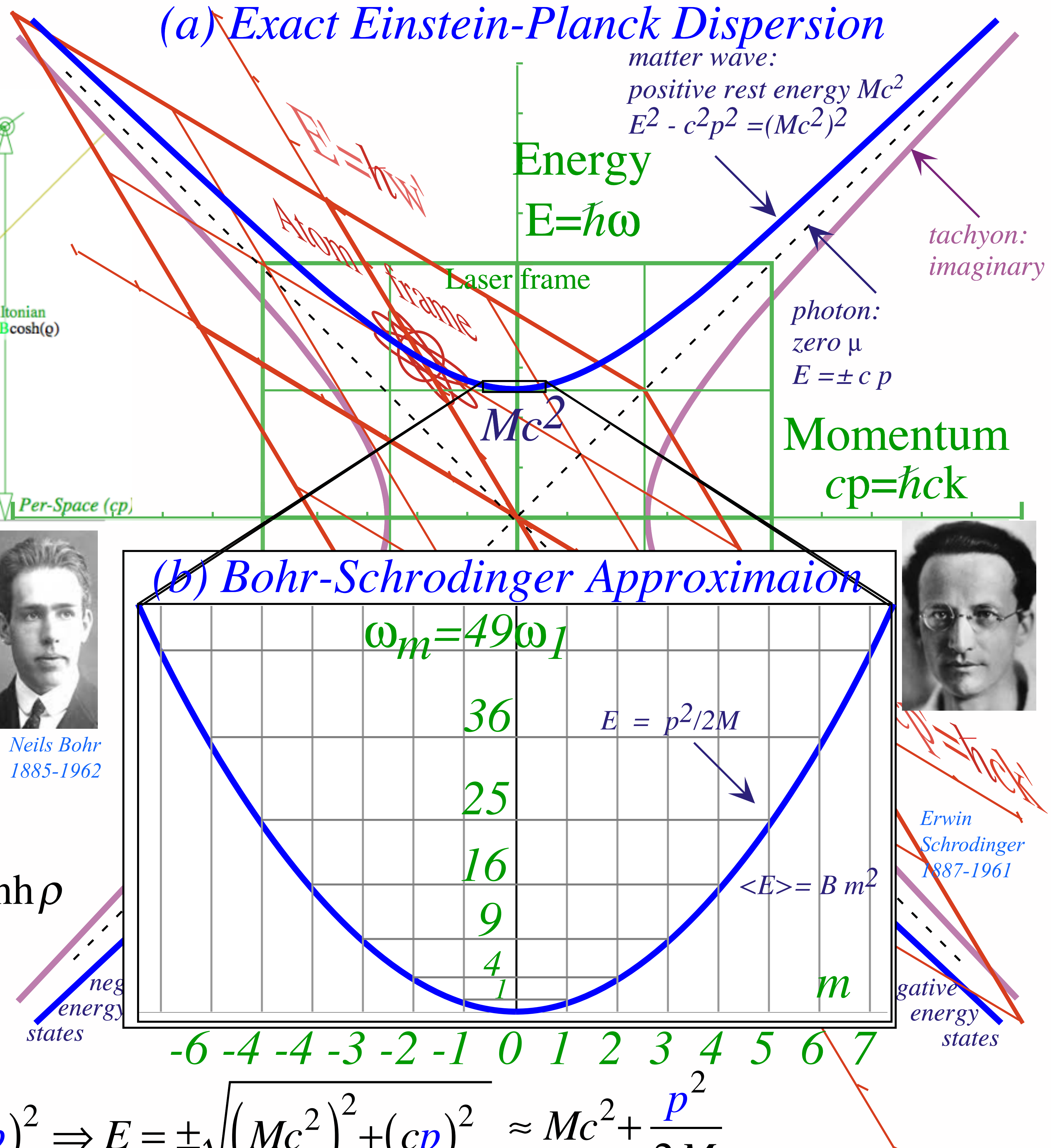
Using (some) wave coordinates for relativistic quantum theory





Neils Bohr
1885-1962

Erwin
Schrodinger
1887-1961

Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Momentum

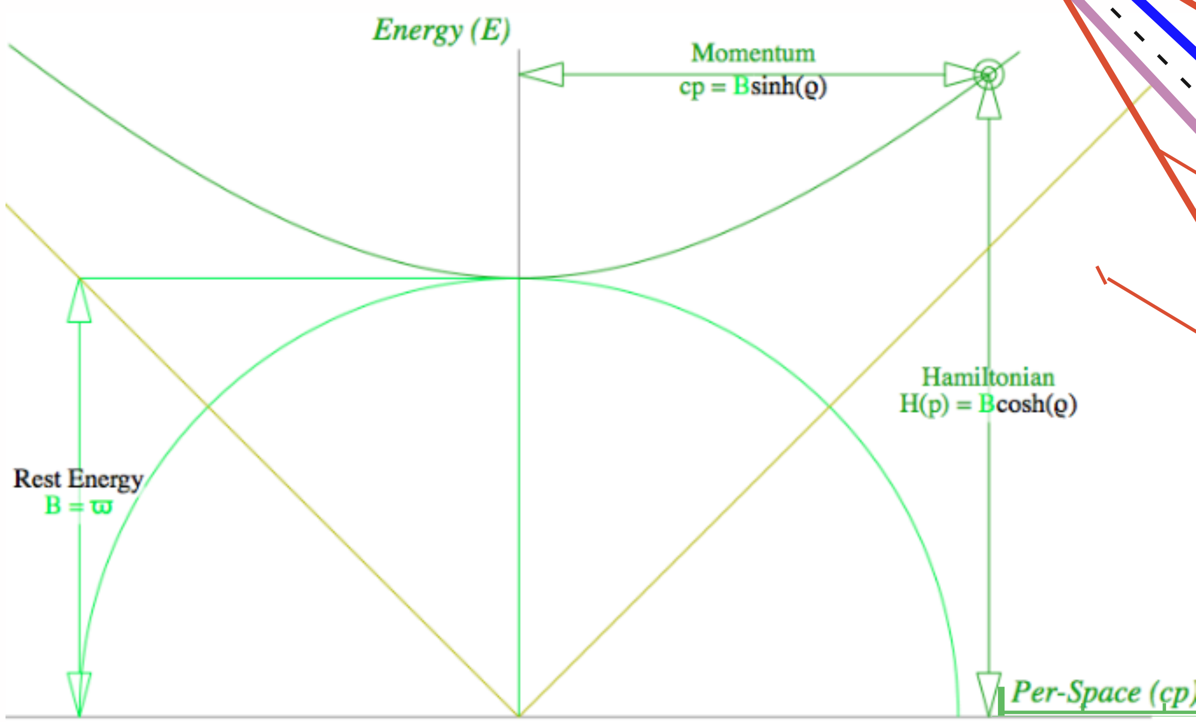
$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

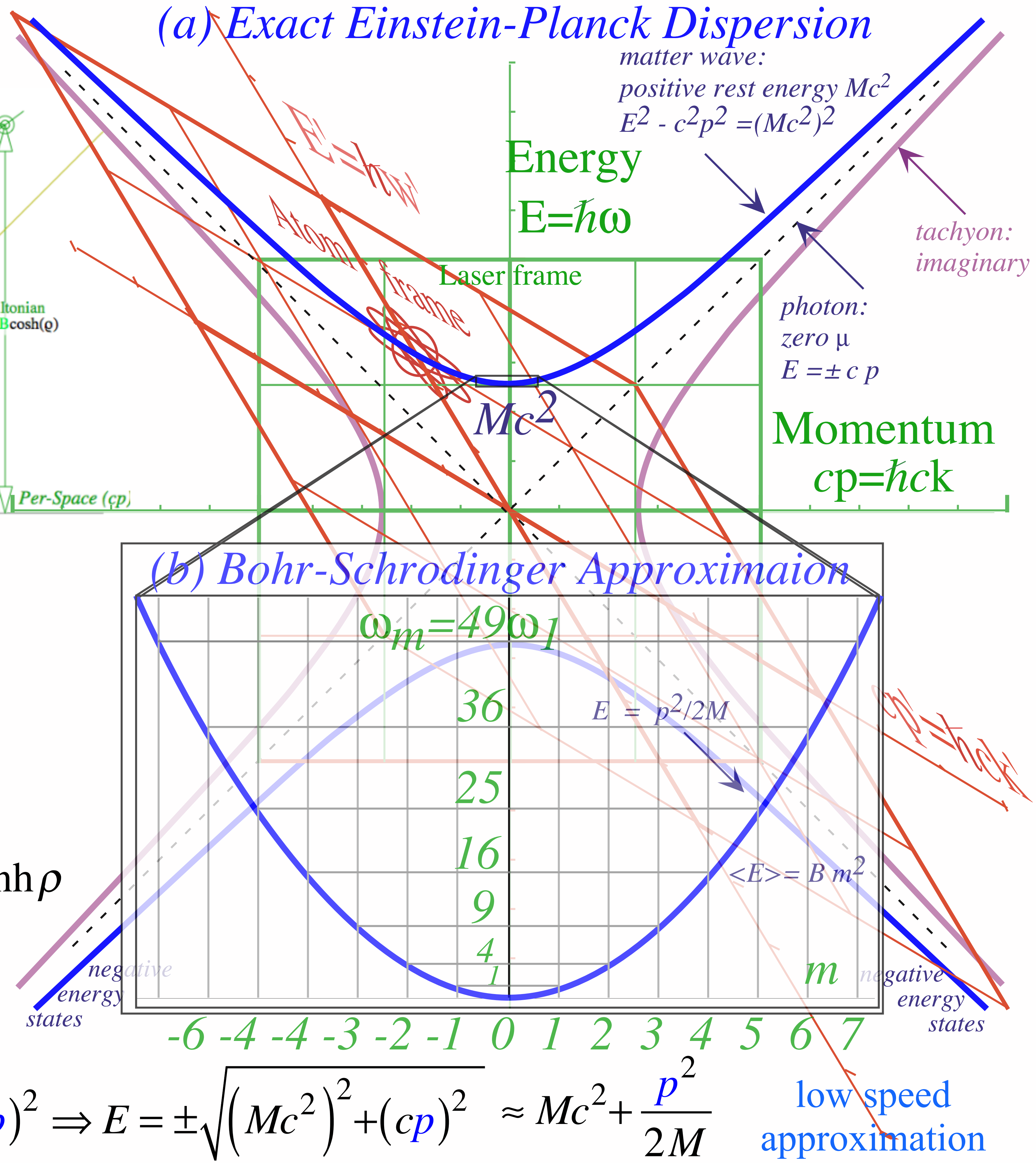
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$

Using (some) wave coordinates for relativistic quantum theory





Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

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Momentum

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Relawavity variable tables

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$
effects	$b^{Doppler}_{RED}$	V_{group}	*past-future asymmetry* (*off-diagonal Lorentz-transform*)	*x-contraction*(Lorentz) τ_{phase}*-contraction*	*t-dilation*(Einstein) υ_{phase}*-dilation* (*on-diagonal Lorentz-transform*)	*inverse asymmetry*	V_{phase}	$b^{Doppler}_{BLUE}$

Relativistic quantum mechanics variable tables

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{\beta}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{1-\beta^2}}{\beta}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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functions		$V_{group}=$ $c\tanh\rho$	*momentum* $cp=Mc^2\sinh\rho$	*-Lagrangian* $L=-Mc^2\operatorname{sech}\rho$	*Hamiltonian* $H=Mc^2\cosh\rho$	*DeBroglie* $\lambda=\alpha\operatorname{csch}\rho$	$V_{phase}=$ $c\coth\rho$	

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
➔ What's the matter with mass? Shining some light on the Elephant in the room
Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$

$= hc\kappa_{phase}$

velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

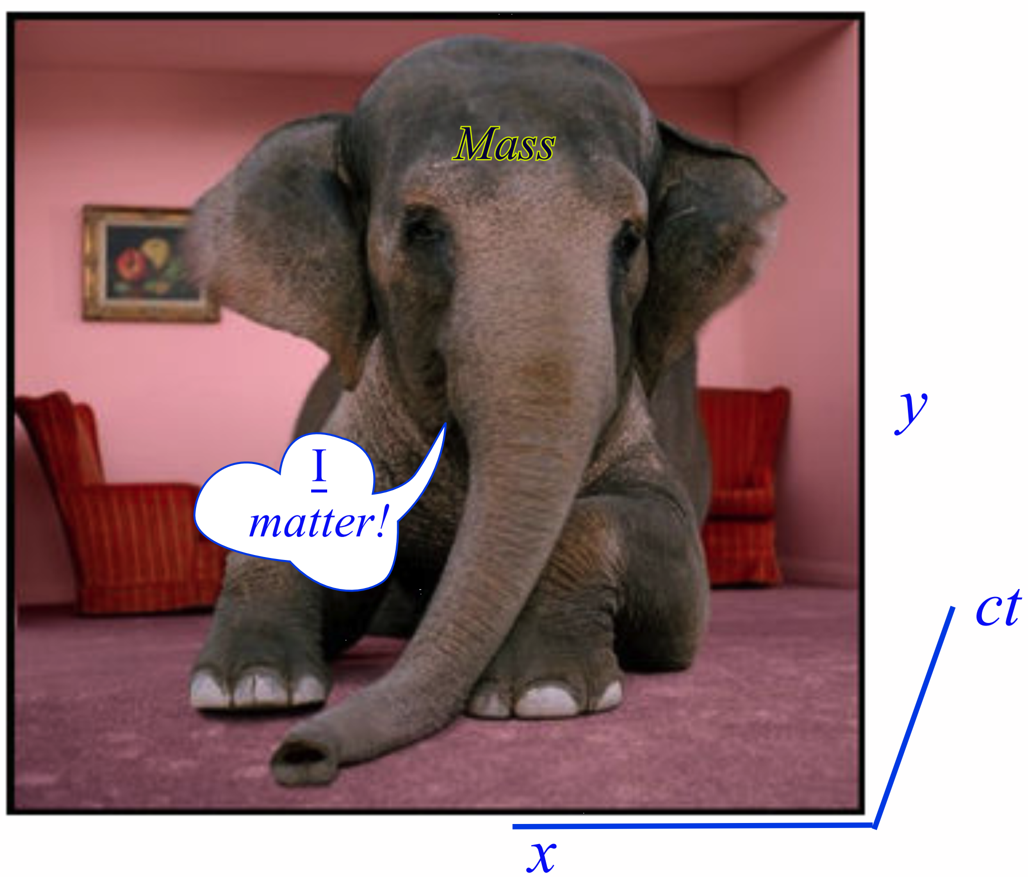
Rest Mass M_{rest} *(Einstein's mass)*

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

- ***What's the matter with Mass?***



Shining some light on the elephant in the spacetime room

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$$\frac{h\upsilon_{phase}}{c^2} = M_{rest}$$

Rest Mass

Defines invariant hyperbola(s)

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$$\frac{h\upsilon_{phase}}{c^2} = M_{rest}$$ *<u>Rest Mass</u>*

<u>Momentum Mass</u> M_{mom} (Galileo's mass) Defined by ratio *p/u* of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}c\sinh\rho}{c\tanh\rho}$$

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$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}$$ *Momentum Mass*

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest} e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Definition(s) of mass for relativity/quantum

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$= h\upsilon_{phase}$

momentum: $cp = Mc^2 \sinh\rho$
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Rest Mass M_{rest} *(Einstein's mass)*

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Defines invariant hyperbola(s)

$$E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2}$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest}$$ *Rest Mass*

Momentum Mass M_{mom} *(Galileo's mass)* Defined by ratio p/u of relativistic momentum to group velocity.

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c \sinh\rho}{c \tanh\rho}$$

$$= M_{rest}\cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}}$$ *Momentum Mass*

Limiting cases:

$$M_{mom} \xrightarrow[u\to c]{} M_{rest}\, e^{\rho}/2$$

$$M_{mom} \xrightarrow[u\ll c]{} M_{rest}$$

Effective Mass M_{eff} *(Newton's mass)* Defined by ratio $F/a = dp/du$ of relativistic force to acceleration.

Definition(s) of mass for relativity/quantum

Given: Energy: $E = Mc^2 \cosh\rho$

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momentum: $cp = Mc^2 \sinh\rho$

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velocity: $u = c\tanh\rho = \frac{d\upsilon}{d\kappa}$

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Limiting cases: $M_{eff} \xrightarrow[u\to c]{} M_{rest}e^{3\rho}/2$

$M_{eff} \xrightarrow[u\ll c]{} M_{rest}$

More common derivation using group velocity: $u \equiv V_{group} = \frac{d\omega}{dk} = \frac{d\upsilon}{d\kappa}$

$$M_{eff} \equiv \frac{dp}{du} = \frac{\hbar dk}{dV_{group}} = \frac{\hbar}{\frac{d}{dk}\frac{d\omega}{dk}} = \frac{\hbar}{\frac{d^2\omega}{dk^2}} = \frac{M_{rest}}{\left(1-u^2/c^2\right)^{3/2}}$$

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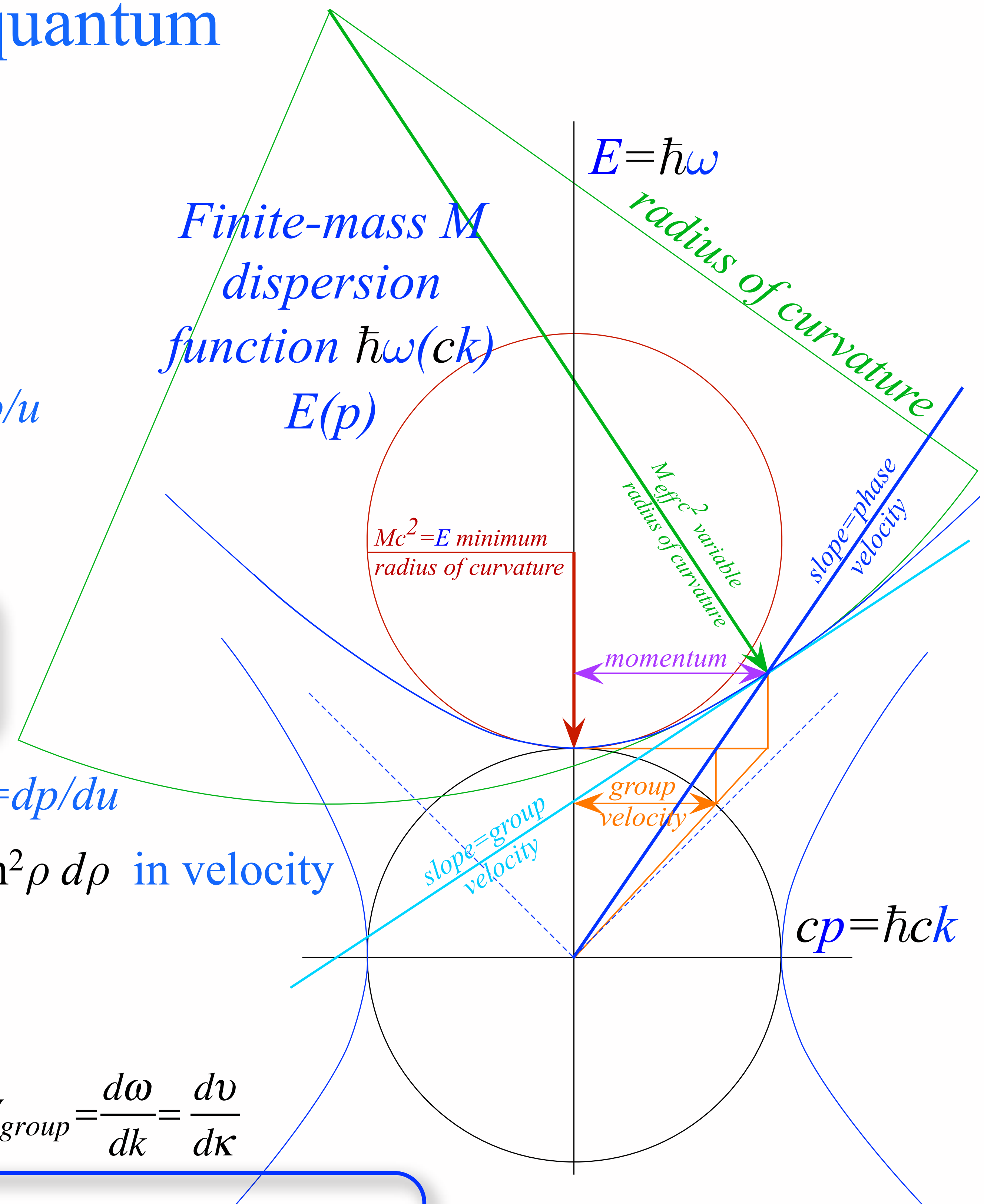
general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Definition(s) of mass for relativity/quantum



Rest Mass M_{rest} (Einstein's mass)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

$$\frac{h\upsilon_{phase}}{c^2} = M_{rest} = \frac{hc\kappa_{phase}}{c^2} \quad \textit{Rest Mass}$$

Momentum Mass M_{mom} (Galileo's mass) Defined by p/u

$$M_{mom} \equiv \frac{p}{u} = \frac{M_{rest}\, c \sinh\rho}{c \tanh\rho}$$

$$= M_{rest} \cosh\rho = \frac{M_{rest}}{\sqrt{1-u^2/c^2}} \quad \textit{Momentum Mass}$$

Effective Mass M_{eff} (Newton's mass) Defined by $F/a=dp/du$

That is ratio of $dp=Mc \cosh\rho \, d\rho$ to change $du=c\, \mathrm{sech}^2\rho \, d\rho$ in velocity

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general wave formula to accompany $V_{group} = \frac{d\omega}{dk}$

Effective mass is proportional to the *radius of curvature* of $\omega(k)$ *dispersion.*

Definition(s) of mass for relativity/quantum

How much mass does a γ-photon have?

Rest Mass (a) γ-rest mass: $M^{\gamma}_{rest} = 0$,

Momentum Mass (b) γ-momentum mass: $M^{\gamma}_{mom} = \frac{p}{c} = \frac{h\kappa}{c} = \frac{h\upsilon}{c^2}$,

Effective Mass (c) γ-effective mass: $M^{\gamma}_{eff} = \infty$.

Newton complained about *his* "corpuscles" of light having *"fits"* (going *crazy*).

(All *this* would be evidence of *triple Schizophrenia*.)

$$M^{\gamma}_{mom} = \frac{h\upsilon}{c^2} = \upsilon(1.2 \cdot 10^{-51})kg \cdot s = 4.5 \cdot 10^{-36}\,kg \quad \text{(for: } \upsilon\text{=600THz)}$$

Lecture 31
Thur. 12.10.2015

Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
"Occams Sword" and summary of 16 parameter functions of ρ and σ
Applications to optical waveguide, spherical waves, and accelerator radiation

Derivation of relativistic quantum mechanics
What's the matter with mass? Shining some light on the Elephant in the room
➔ Relativistic action and Lagrangian-Hamiltonian relations
Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
Feynman diagram geometry
Compton recoil related to rocket velocity formula
Comparing 2^{nd}-quantization "photon" number N and 1^{st}-quantization wavenumber κ

Relawavity in accelerated frames
Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
Animation of mechanics and metrology of constant-g grid

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar \omega$$

$$\hbar \equiv \frac{h}{2\pi}$$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

←linear Hz format

angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar ck_A$

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$$L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

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Legendre transformation

Use *Group velocity* : $u = \frac{dx}{dt} = c \tanh\rho$

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Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$

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$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

$$L \text{ is}: Mc^2 \frac{-1}{\cosh\rho} = -Mc^2\operatorname{sech}\rho$$

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$$L = pu - H = (Mc \sinh\rho)(c \tanh\rho) - Mc^2 \cosh\rho$$

Note: $Mcu = Mc^2 \tanh\rho$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

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Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh\rho$$

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Use *Group velocity* : $u = \frac{dx}{dt} = c \tanh\rho$

$$p = \hbar k = Mc \sinh\rho \qquad E = \hbar\omega = Mc^2 \cosh\rho = H$$

$$L = pu - H = (Mc\sinh\rho)(c\tanh\rho) - Mc^2\cosh\rho$$

$$= Mc^2 \frac{\sinh^2\rho - \cosh^2\rho}{\cosh\rho} = -Mc^2 \operatorname{sech}\rho$$

Note: $Mcu = Mc^2 \tanh\rho$

Also: $cp = Mc^2 \sinh\rho$

Compare *Lagrangian L*

$$L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh\rho$$

$$= Mc^2\sqrt{1 + \sinh^2\rho} = Mc^2\sqrt{1 + (cp)^2}$$

$\hbar = h/2\pi$

Prior wave relations

$h\upsilon_A = Mc^2 = hc\kappa_A$

$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$

$hc\kappa_{phase} = cp = h\upsilon_A \sinh\rho$

←linear Hz format

angular phasor→ format

$\hbar\omega_A = Mc^2 = \hbar c k_A$

$\hbar\omega_{phase} = E = \hbar\omega_A \cosh\rho$

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$$\hbar \equiv \frac{h}{2\pi}$$

Relativistic action S and Lagrangian-Hamiltonian relations

Define *Lagrangian L* using invariant wave phase $\Phi = kx - \omega t = k'x' - \omega' t'$ for wave of $k = k_{phase}$ and $\omega = \omega_{phase}$.

Use DeBroglie-momentum $p = \hbar k$ relation and Planck-energy $E = \hbar\omega$ relation to define *Hamiltonian* $H = E$

$$\frac{dS}{dt} = L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$

$= c\sin\sigma$

$$p = \hbar k = Mc\sinh\rho \qquad E = \hbar\omega = Mc^2\cosh\rho = H$$

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Note: $Mcu = Mc^2\tanh\rho$

$= Mc^2\sin\sigma$

Also: $cp = Mc^2\sinh\rho$

$= \hbar ck = Mc^2\tan\sigma$

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1-\frac{u^2}{c^2}} = Mc^2\cosh\rho = Mc^2\sec\sigma$$

$$= Mc^2\sqrt{1+\sinh^2\rho} = Mc^2\sqrt{1+(cp)^2}$$

Define *Action* $S = \hbar\Phi$

Including stellar angle σ

Prior wave relations

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←linear Hz format

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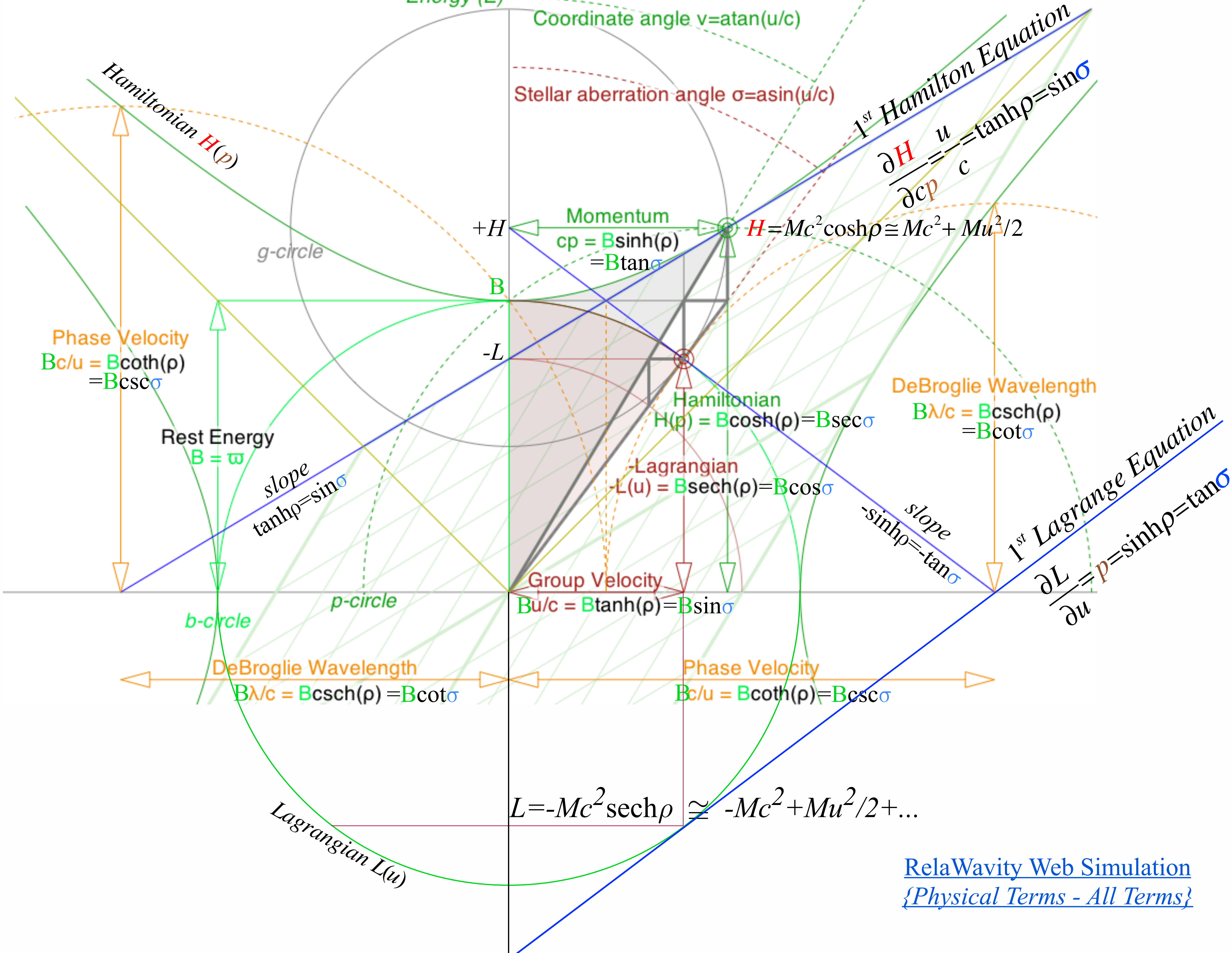

Energy (E)
Coordinate angle ν=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Hamiltonian H(p)
1st Hamilton Equation
$\frac{\partial H}{\partial cp}=\frac{u}{c}=\tanh\rho=\sin\sigma$
Momentum
cp = Bsinh(ρ)
=Btanσ
+H
$H=Mc^2\cosh\rho\cong Mc^2+Mu^2/2$
g-circle
B
-L
Phase Velocity
Bc/u = Bcoth(ρ)
=Bcscσ
Rest Energy
B = ϖ
slope
tanhρ=sinσ
Hamiltonian
H(p) = Bcosh(ρ)=Bsecσ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
slope
-sinhρ=-tanσ
1st Lagrange Equation
$\frac{\partial L}{\partial u}=p=\sinh\rho=\tan\sigma$
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
$L=-Mc^2\text{sech}\rho\cong -Mc^2+Mu^2/2+\ldots$
Lagrangian L(u)
RelaWavity Web Simulation
{Physical Terms - All Terms}

Lecture 31
Thur. 12.10.2015

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$$\frac{dS}{dt} = L \equiv \hbar \frac{d\Phi}{dt} = \hbar k \frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2 \operatorname{sech}\rho = -Mc^2 \cos\sigma$$

with *Hamiltonian* $H = E$

$$H = \hbar\omega = Mc^2 \Big/ \sqrt{1 - \frac{u^2}{c^2}} = Mc^2 \cosh\rho = Mc^2 \sec\sigma$$

$$= Mc^2\sqrt{1 + \sinh^2\rho} = Mc^2\sqrt{1 + (cp)^2}$$

Define *Action* $S = \hbar\Phi$

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←linear Hz format

angular phasor→ format

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$$\frac{dS}{dt} = L \equiv \hbar\frac{d\Phi}{dt} = \hbar k\frac{dx}{dt} - \hbar\omega = p\frac{dx}{dt} - E \equiv p\dot{x} - E \equiv pu - H = L$$

Legendre transformation

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\, dx - \hbar\omega\, dt = p\,dx - H\,dt$$

Poincare Invariant action differential

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1-\frac{u^2}{c^2}} = -Mc^2\operatorname{sech}\rho = -Mc^2\cos\sigma$$

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Use *Group velocity*: $u = \frac{dx}{dt} = c\tanh\rho$

$$dS \equiv L\,dt \equiv \hbar\, d\Phi = \hbar k\,dx - \hbar\omega\,dt = p\,dx - H\,dt$$

Poincare Invariant action differential

$$\frac{\partial S}{\partial x} = p \qquad \frac{\partial S}{\partial t} = -H$$

Hamilton-Jacobi equations

Compare *Lagrangian L*

$$\dot{S} = L = \hbar\dot{\Phi} = -Mc^2\sqrt{1 - \frac{u^2}{c^2}} = -Mc^2\,\mathrm{sech}\rho = -Mc^2\cos\sigma$$

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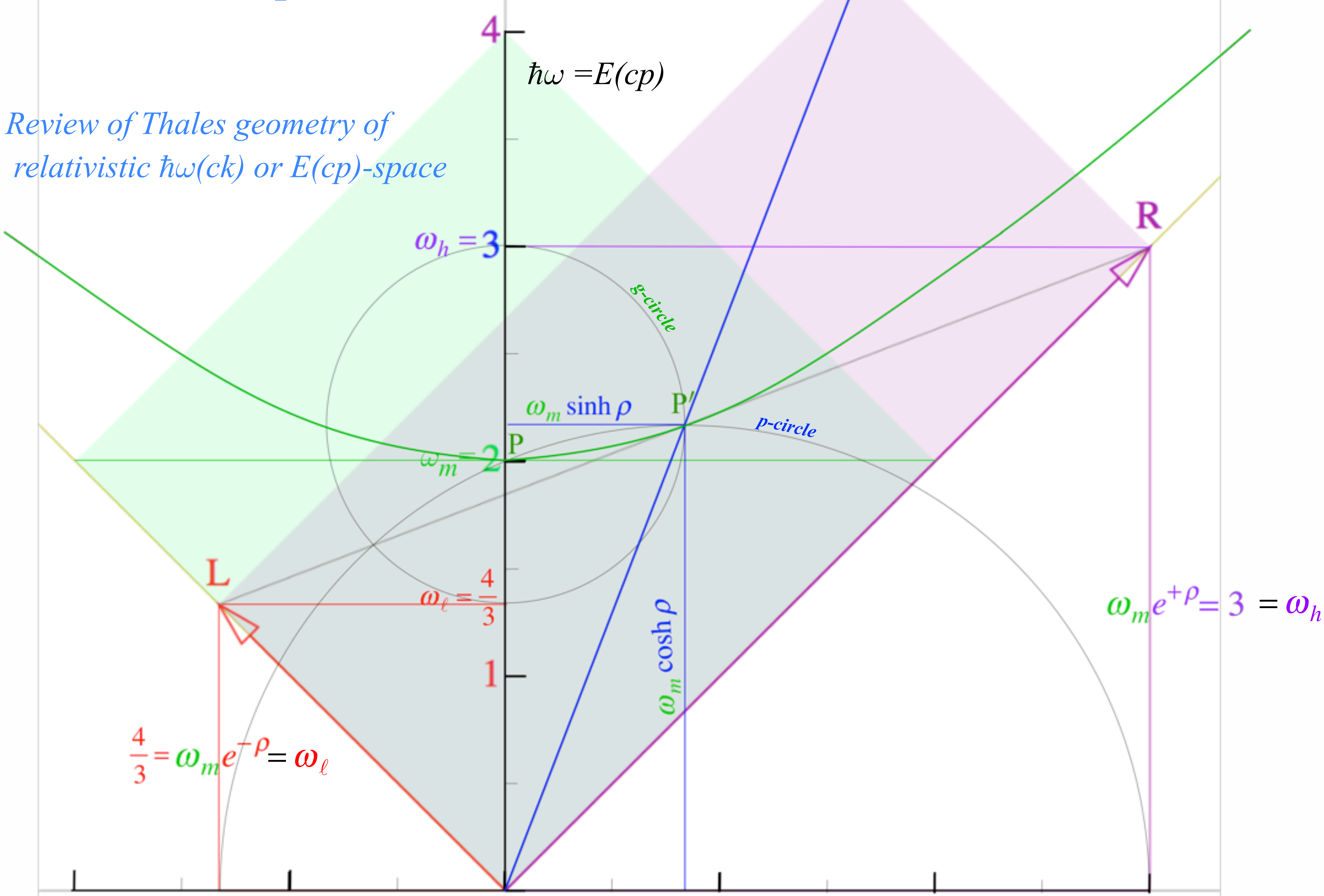
Review of Thales geometry of relativistic ħω(ck) or E(cp)-space



Doppler RED factor: $\frac{2}{3} = e^{-\rho}$ Doppler BLUE factor: $\frac{3}{2} = e^{+\rho}$

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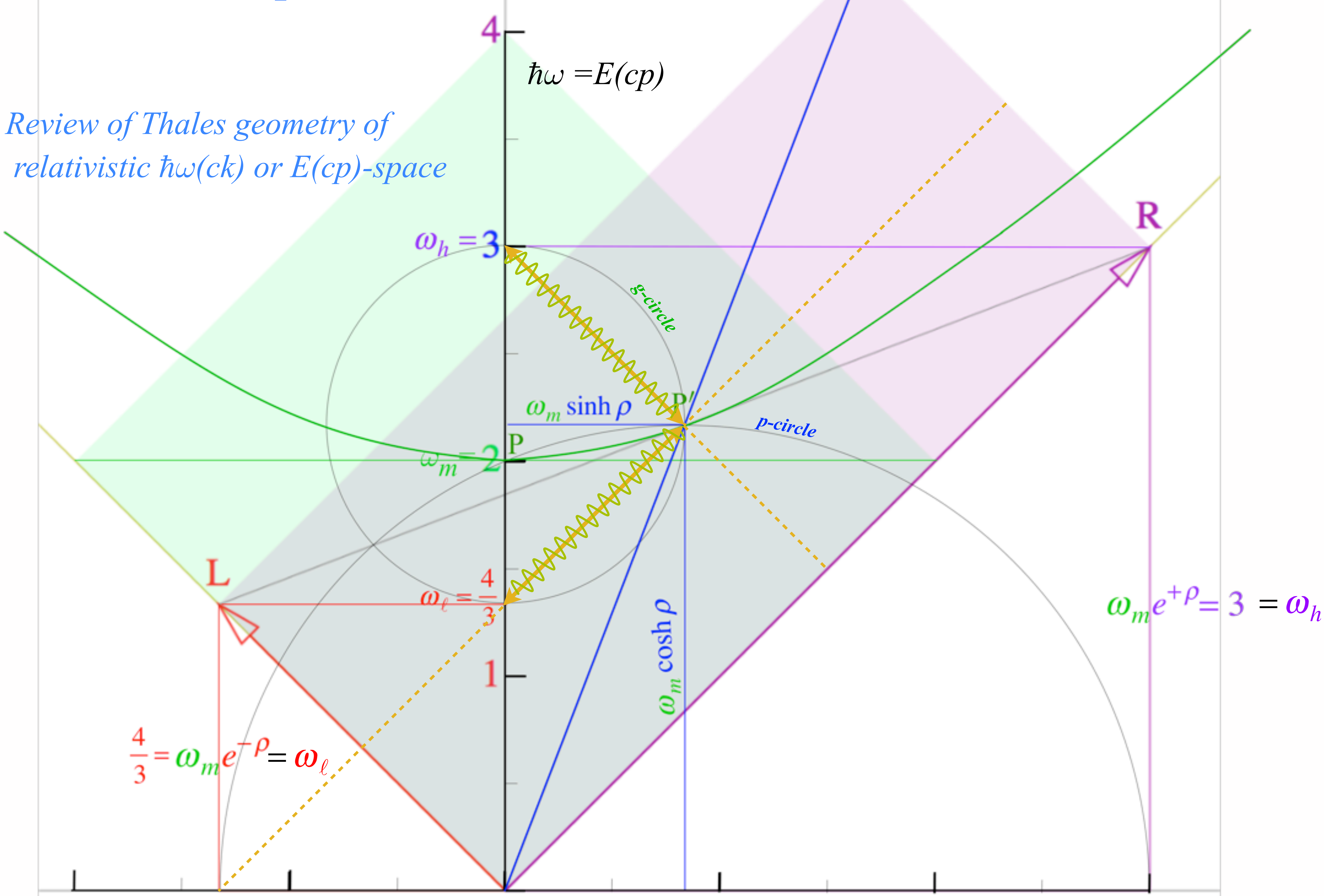

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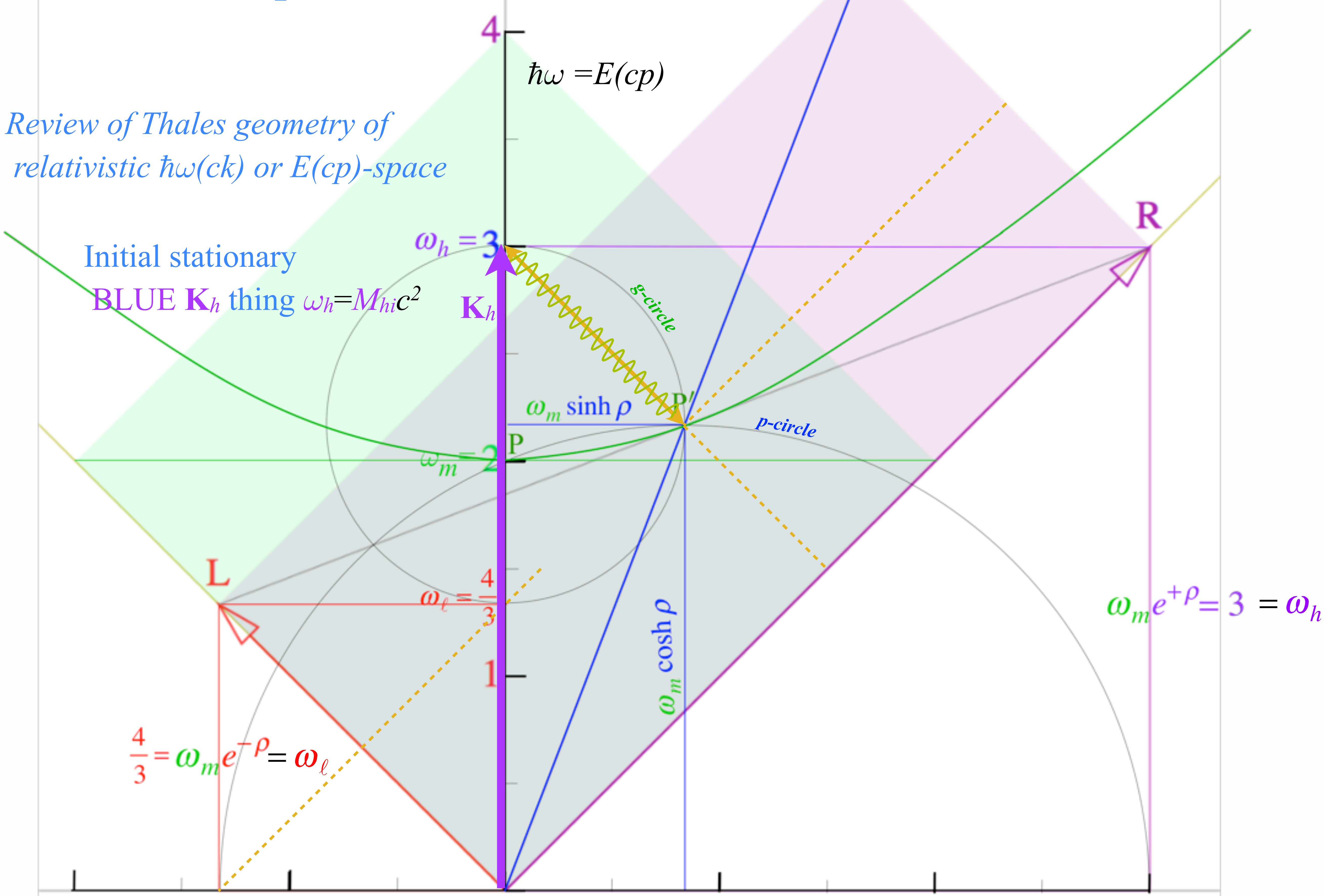

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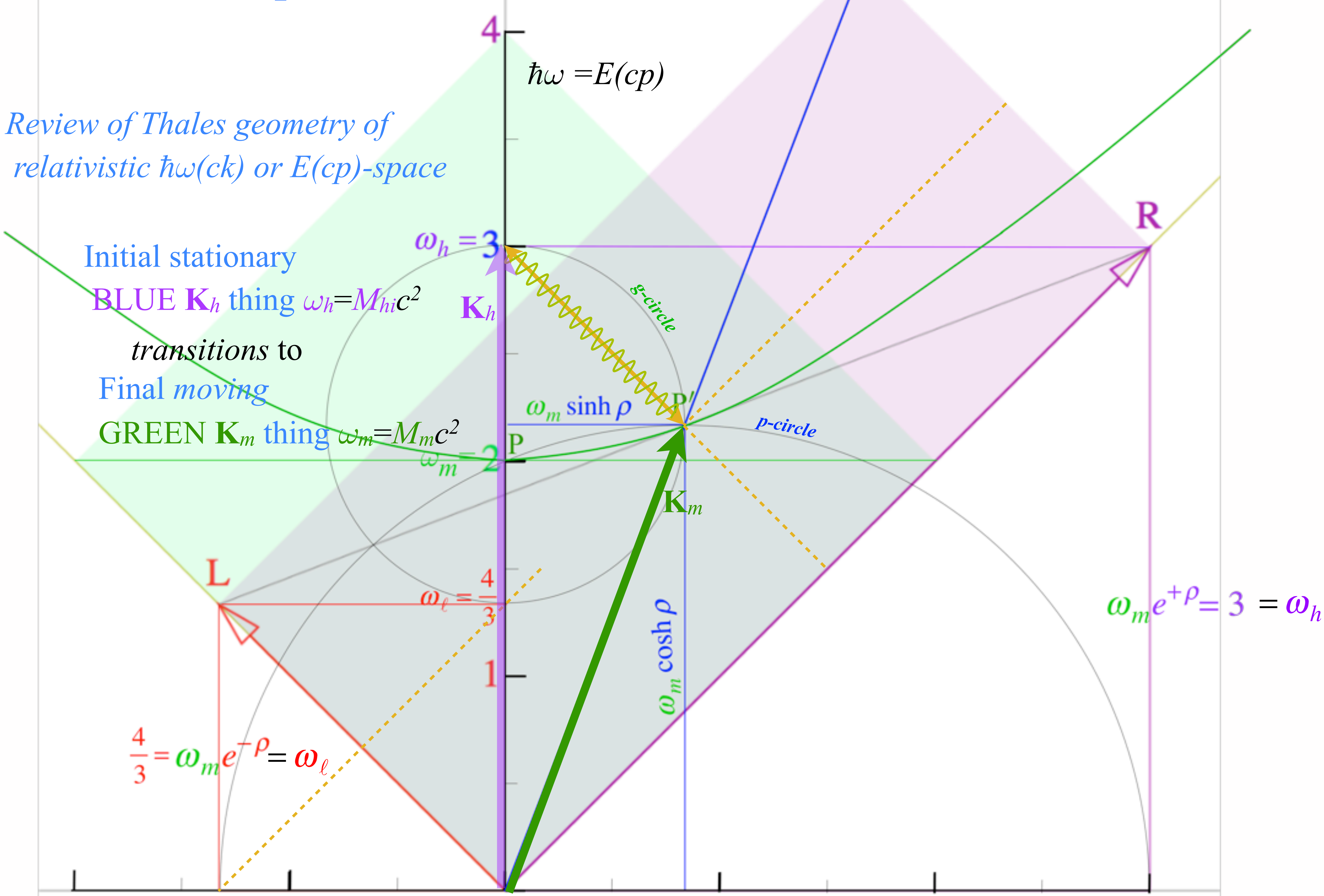

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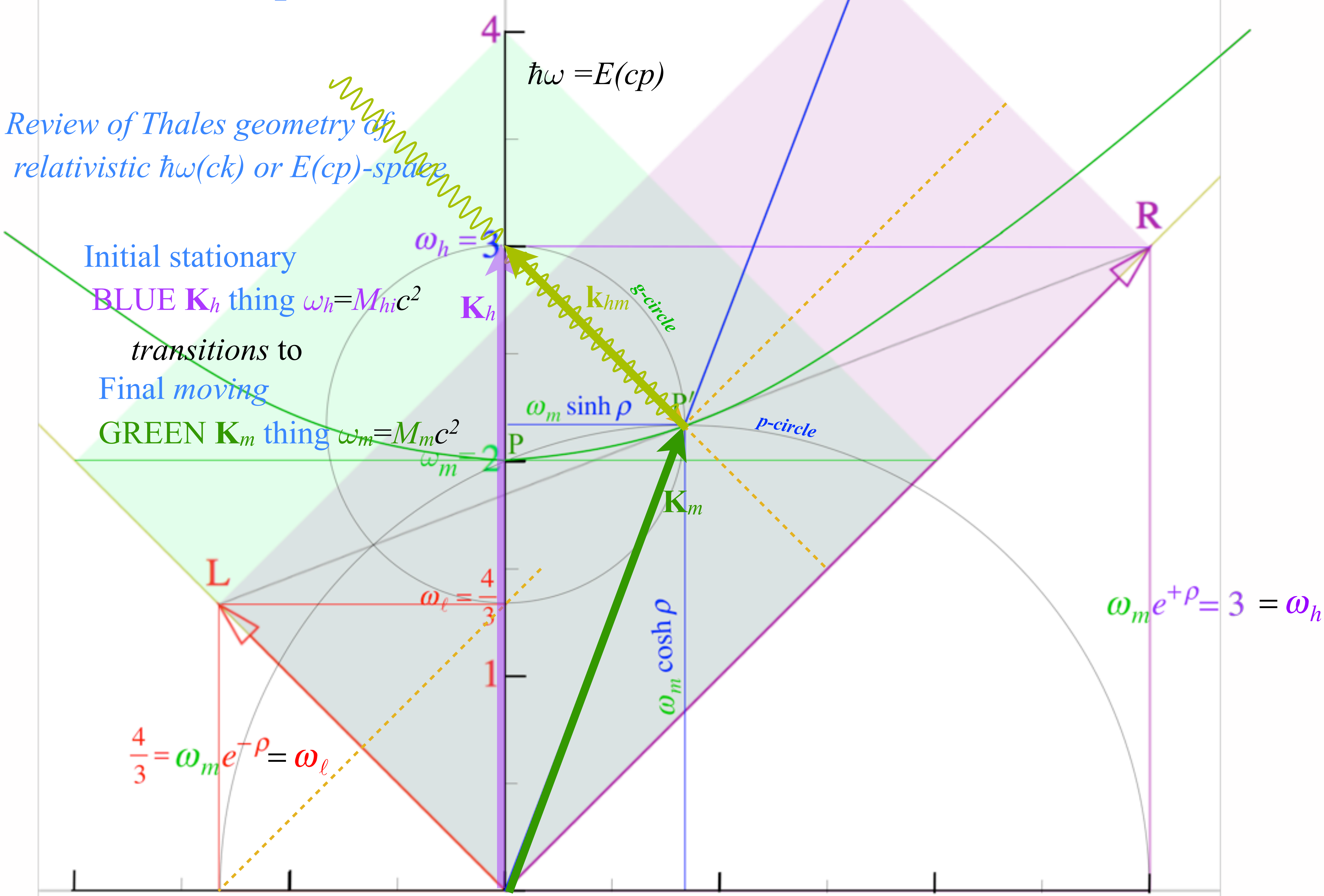

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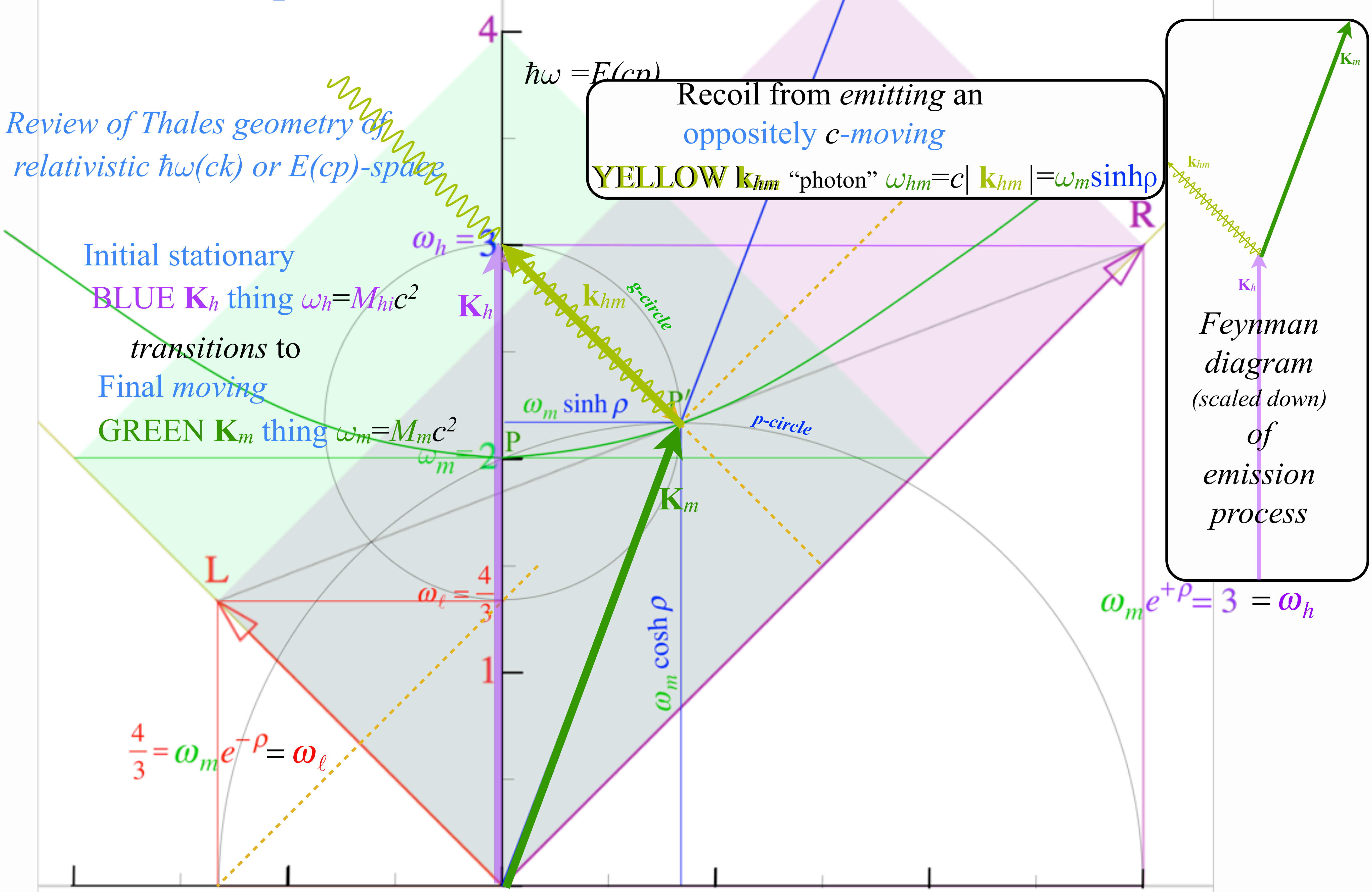

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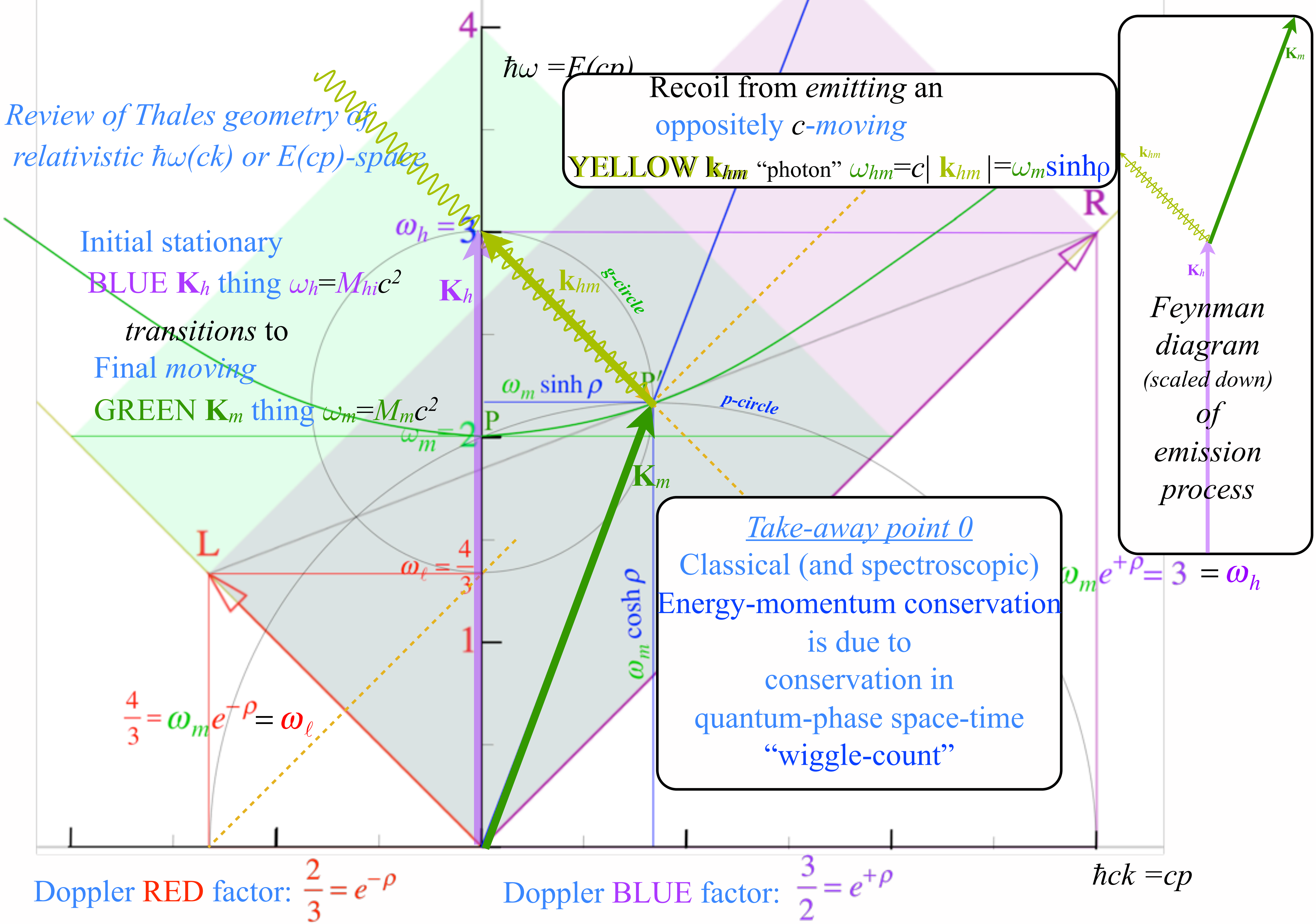

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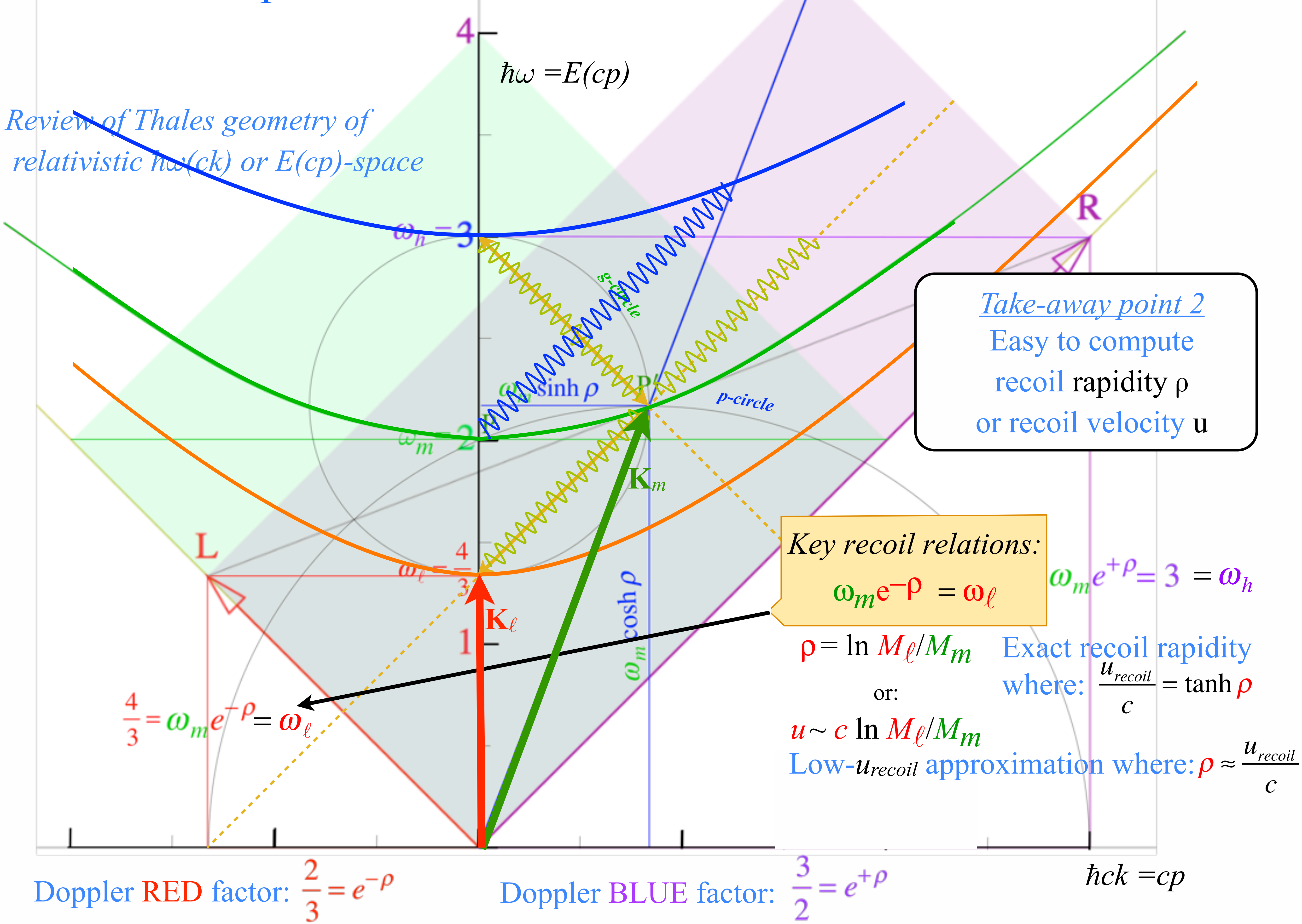

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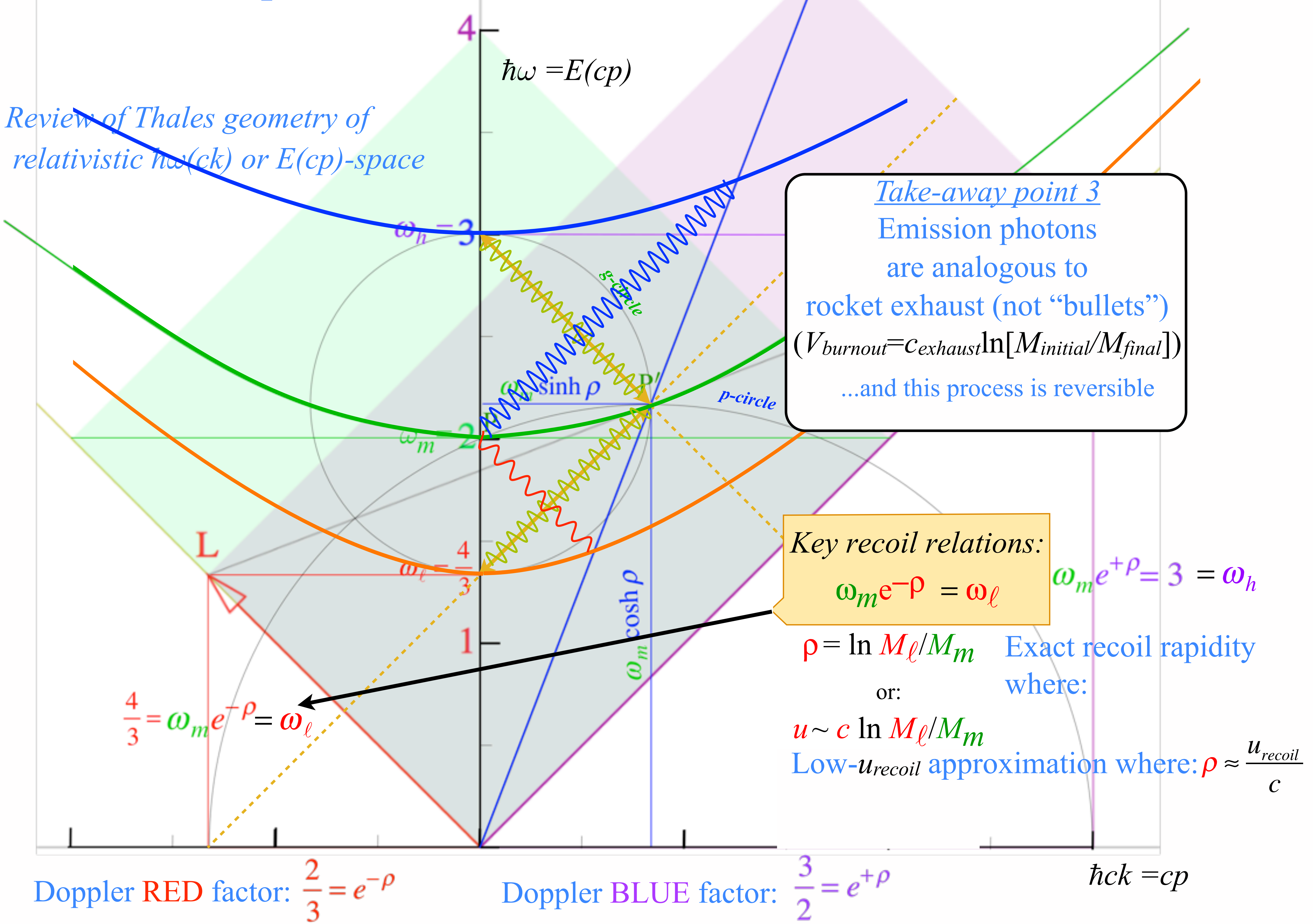

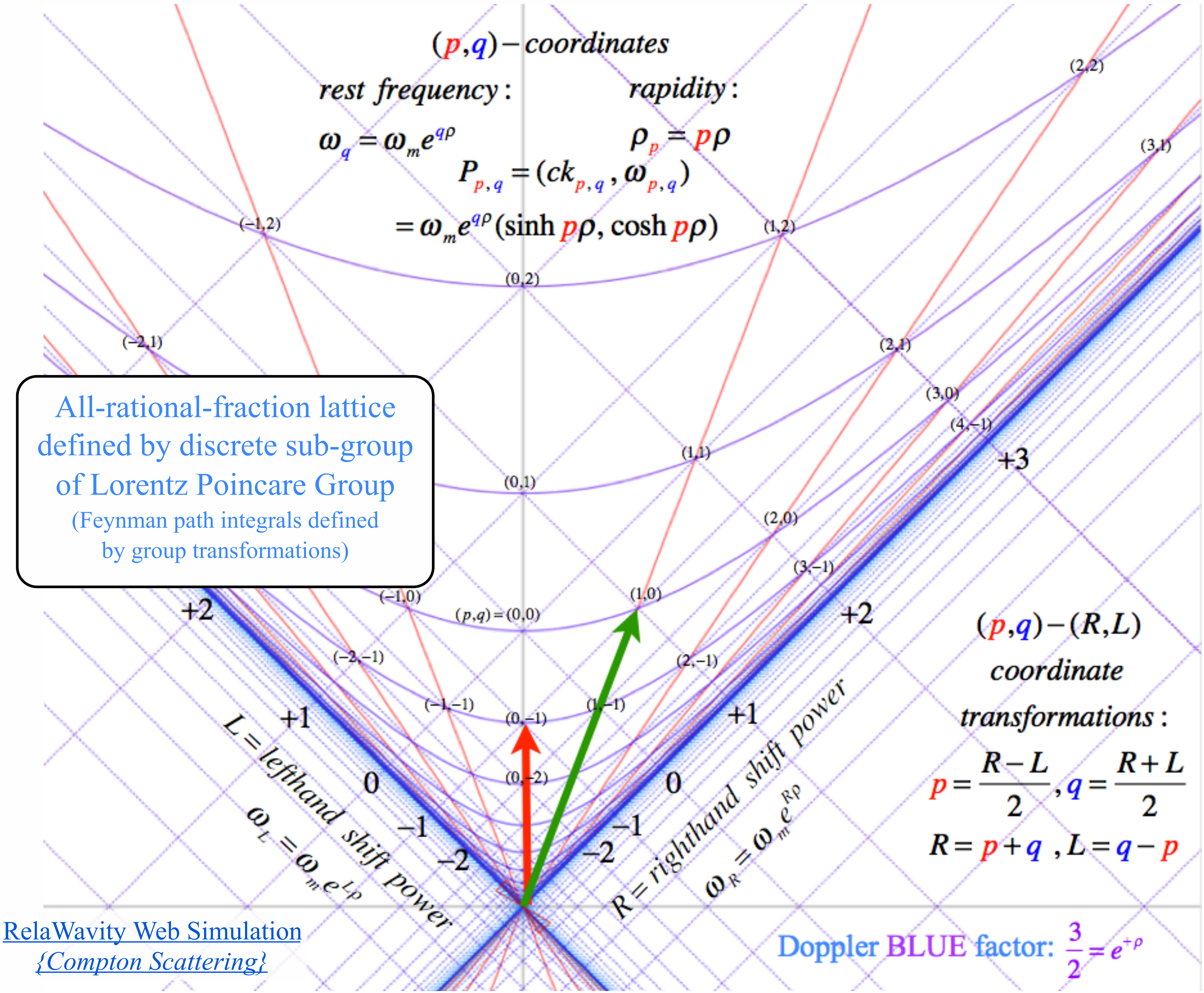

RelaWavity Web Simulation
{Compton Scattering}

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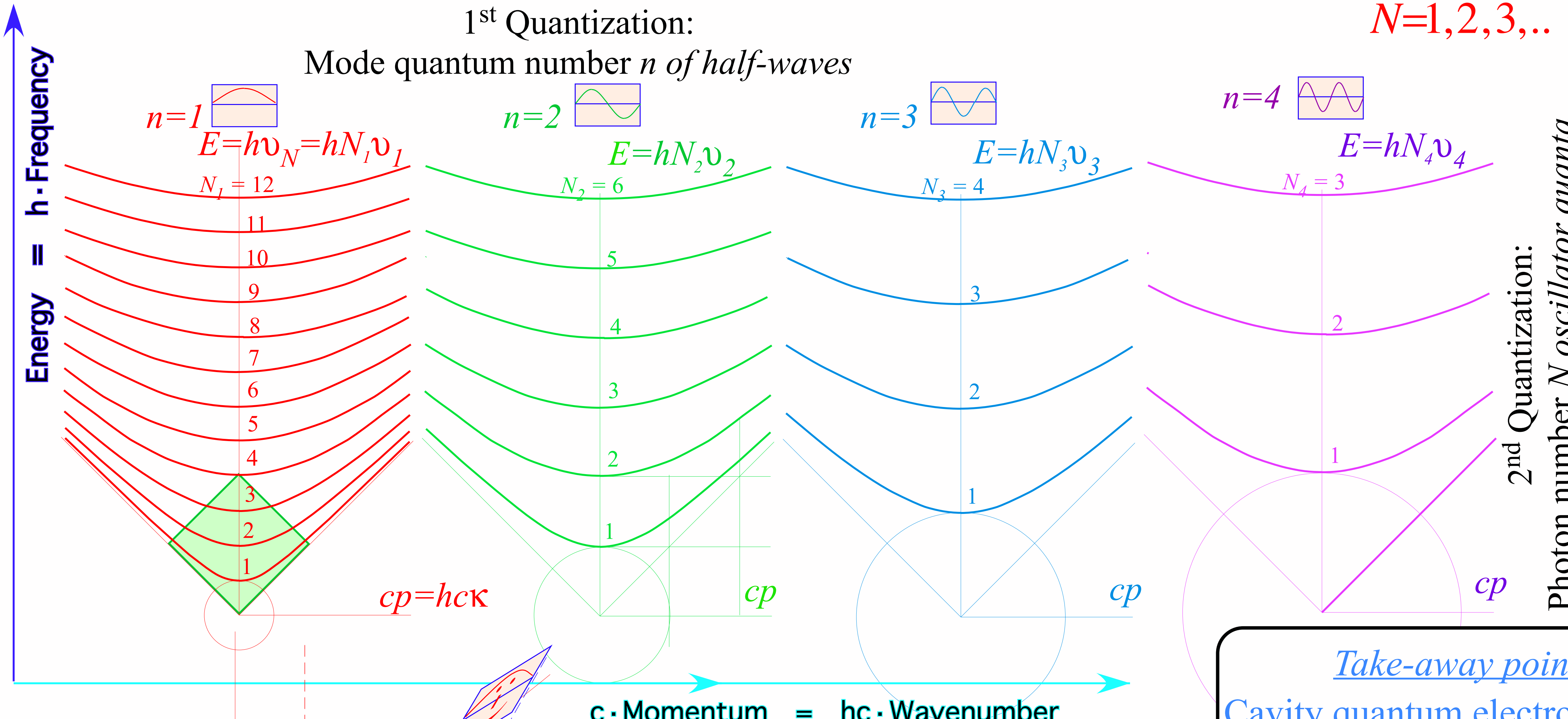
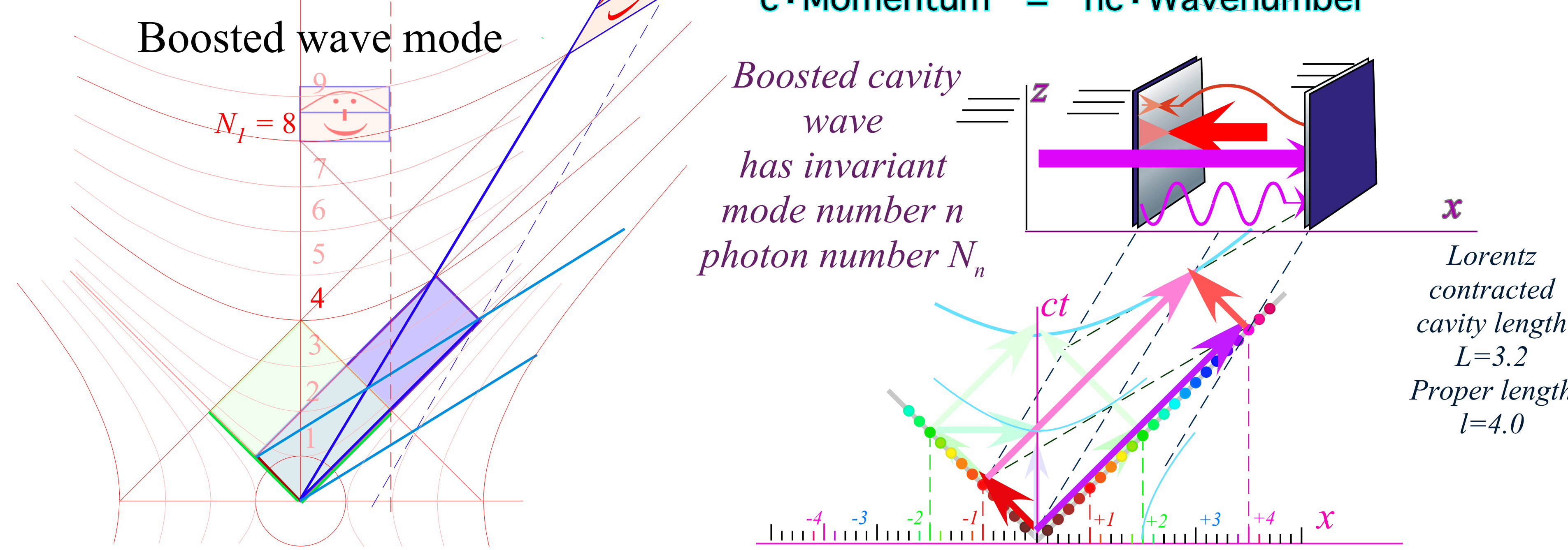
2^{nd} Quantization: $h\upsilon$ *is actually* $hN\upsilon$

($h\upsilon_{phase}=E=h\upsilon_A \cosh\rho$) *is actually* ($hN\upsilon_{phase}=E_N=hN\upsilon_A \cosh\rho$ with *quantum* numbers)

$N=1,2,3,..$





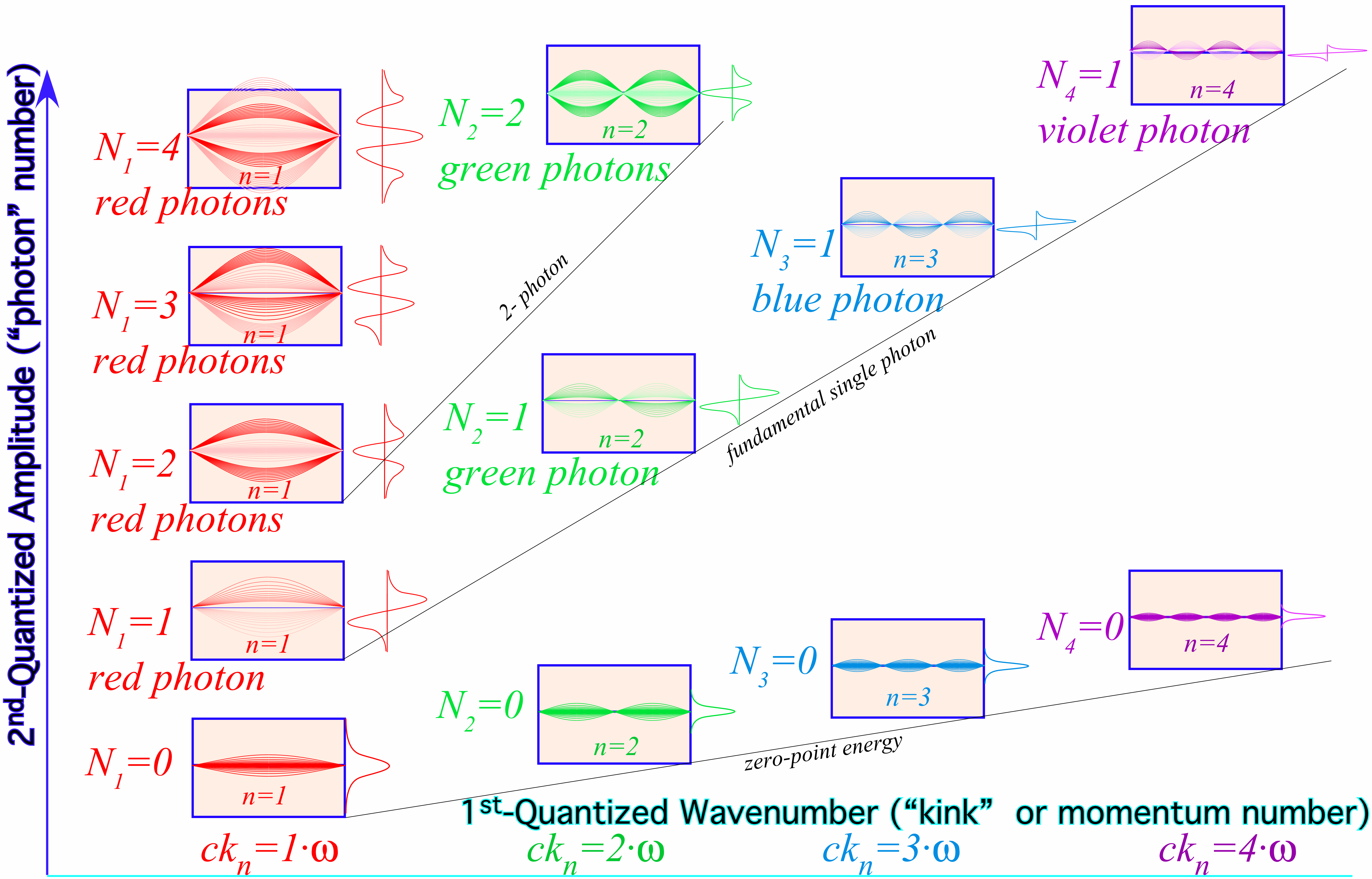
Take-away point 4

Cavity quantum electrodynamics (CQED) and spectra are analogous to molecular rovibronic dynamics with rotation-vibration algebra replaced by Lorentz-Poincare-Dirac algebra (and geometry!)

2^{nd} Quantization:

$h\upsilon$ *is actually* $hN\upsilon$

$\left(h\upsilon_{phase}=E=h\upsilon_A \cosh\rho\right)$ *is actually* $\left(hN\upsilon_{phase}=E_N=hN\upsilon_A \cosh\rho \quad (N=1,2,..) \right)$



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Learning about sin! *and* cos *and...*

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Acceleration by chirping laser pairs

Varying acceleration (General case)

From Lect. 35
ModPhys (2012)

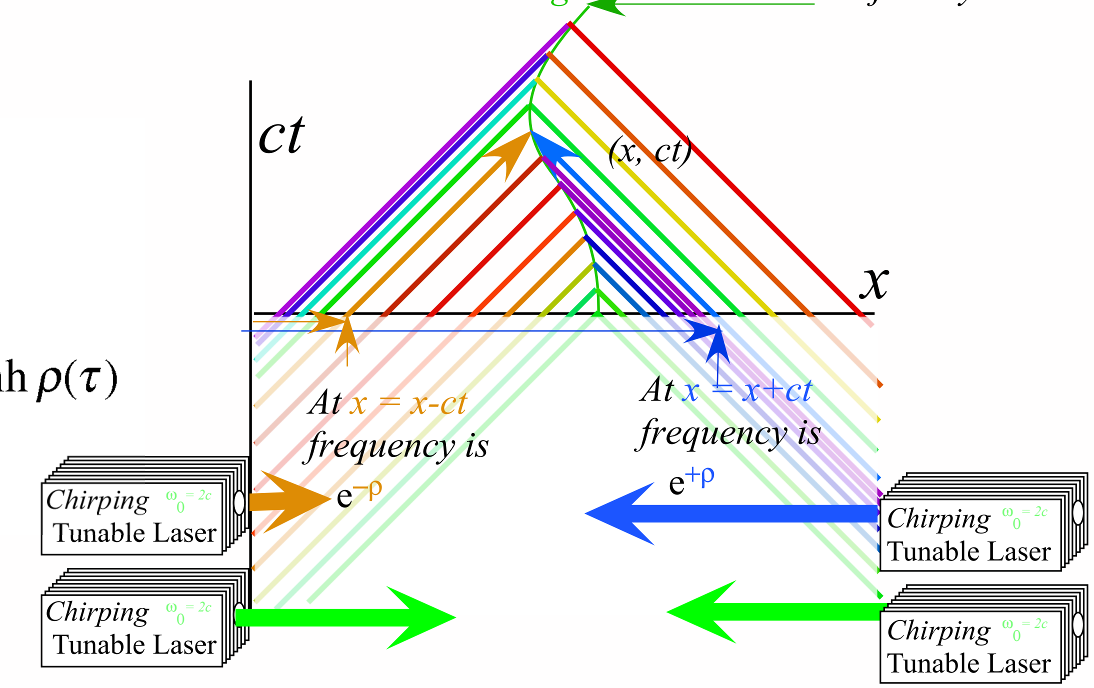
Varying local acceleration $\rho = \rho(\tau)$

$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

$$ct = c\int \cosh\rho(\tau)\,d\tau \qquad x = c\int \sinh\rho(\tau)\,d\tau$$

Only green-light is seen by observers on the green accelerated trajectory



Constant local acceleration $\rho = \frac{g\tau}{c}$ *"Einstein Elevator"*

$$ct = c\int \cosh\frac{g\tau}{c}\,d\tau \qquad x = c\int \sinh\frac{g\tau}{c}\,d\tau$$

$$= \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$

Only green-light is seen by observers on the green constant-g hyberbola

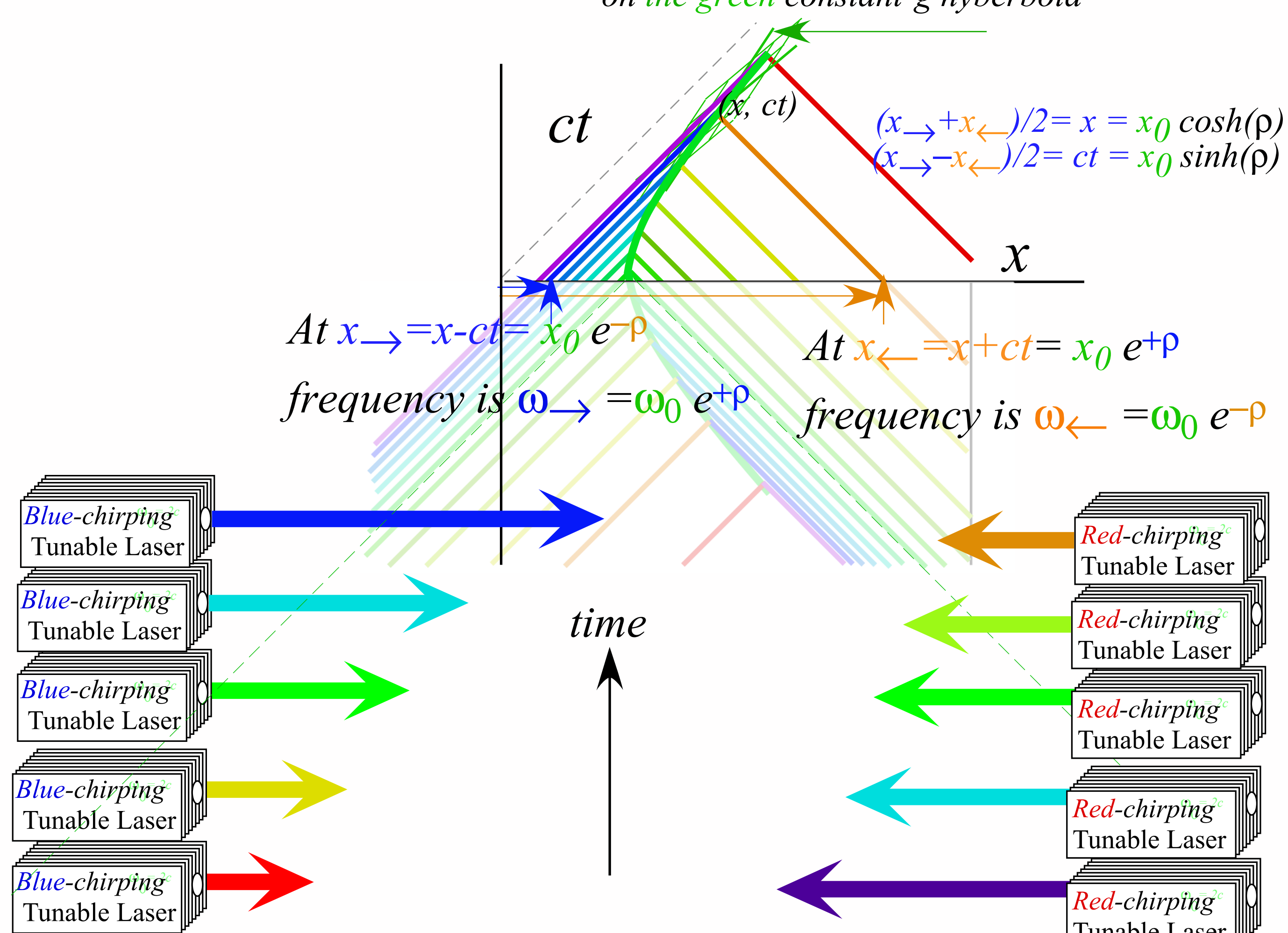


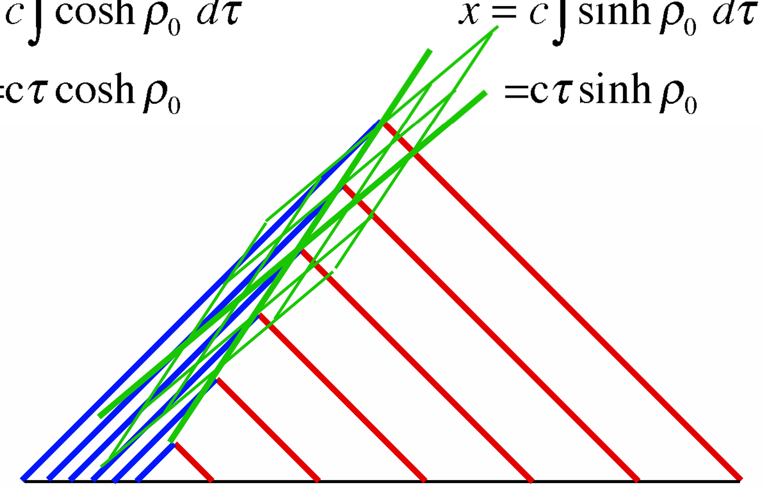
Fig. 8.1 Optical wave frames by red-and-blue-chirped lasers (a)Varying acceleration (b)Constant g

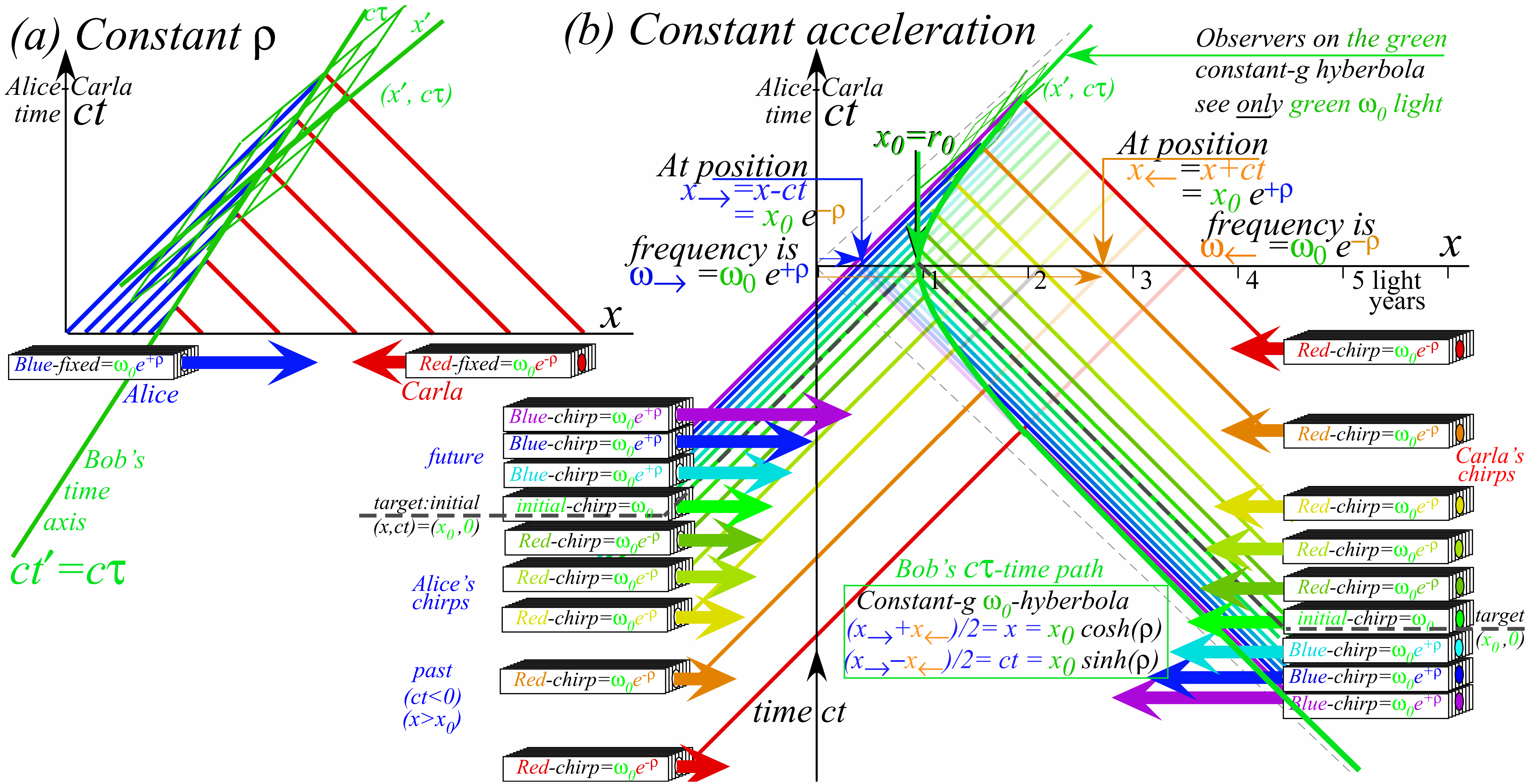
Previous examples involved constant velocity

Constant velocity $\rho = \rho_0 = const.$ *"Lorentztransformation"*

$$ct = c\int \cosh\rho_0\,d\tau \qquad x = c\int \sinh\rho_0\,d\tau$$

$$= c\tau\cosh\rho_0 \qquad = c\tau\sinh\rho_0$$

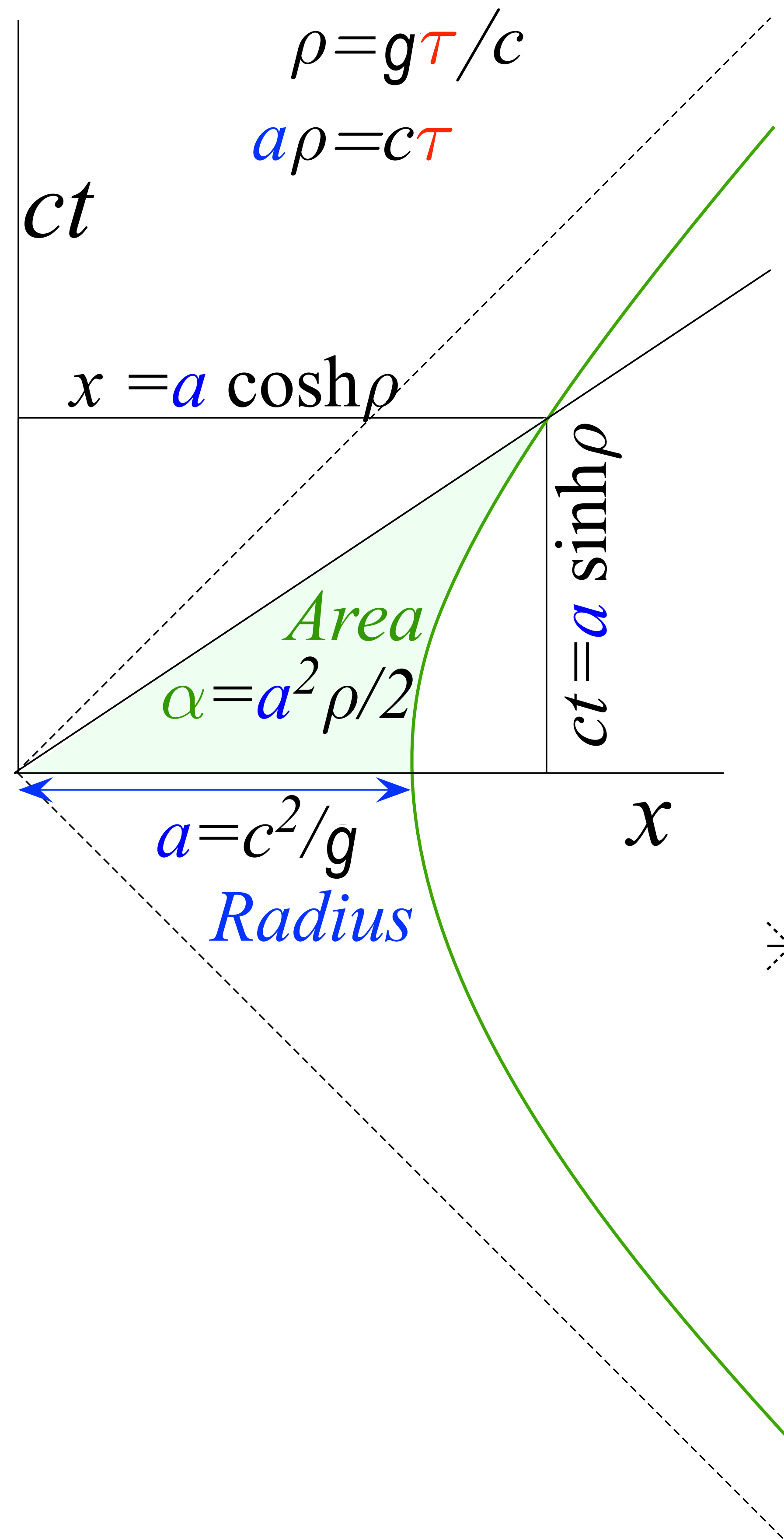




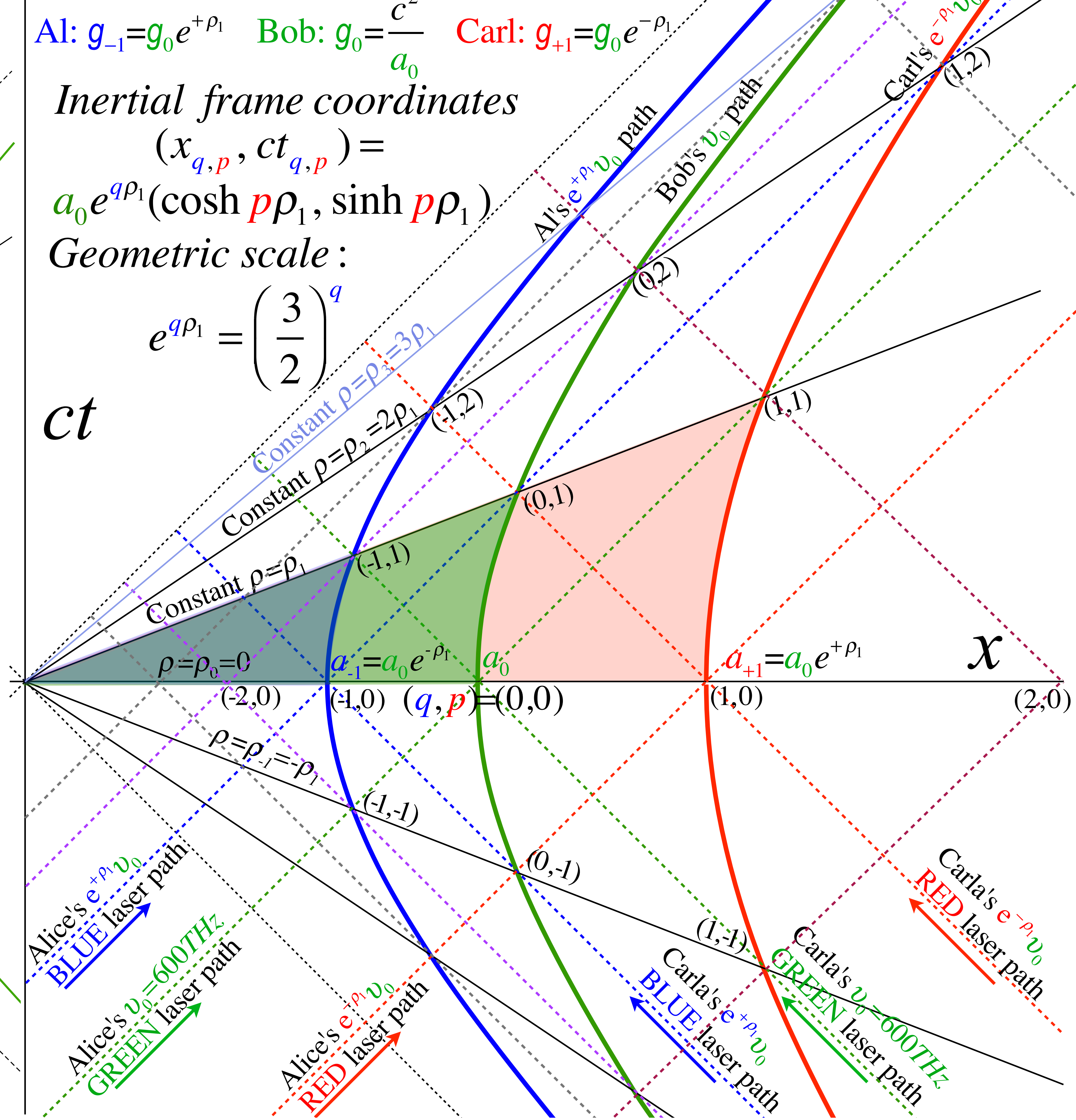
(a) Constant ρ
Alice-Carla time ct
x
(x′, cτ)
Blue-fixed=ω0e+ρ
Alice
Red-fixed=ω0e-ρ
Carla
Bob's time axis
ct′=cτ
(b) Constant acceleration
Alice-Carla time ct
Observers on the green constant-g hyberbola see only green ω0 light
x0=r0
At position x→=x-ct = x0 e−ρ
frequency is ω→ =ω0 e+ρ
At position x←=x+ct = x0 e+ρ
frequency is ω← =ω0 e−ρ
1 2 3 4 5 light years
Blue-chirp=ω0e+ρ
future
target:initial (x,ct)=(x0,0)
initial-chirp=ω0
Red-chirp=ω0e-ρ
Alice's chirps
past (ct<0) (x>x0)
Carla's chirps
target (x0,0)
Bob's cτ-time path
Constant-g ω0-hyberbola
(x→+x←)/2= x = x0 cosh(ρ)
(x→−x←)/2= ct = x0 sinh(ρ)
time ct

(a) Constant acceleration g

Rapidity ρ vs proper time τ



(b) Traveler paths of acceleration g_q



Lecture 31
Thur. 12.10.2015

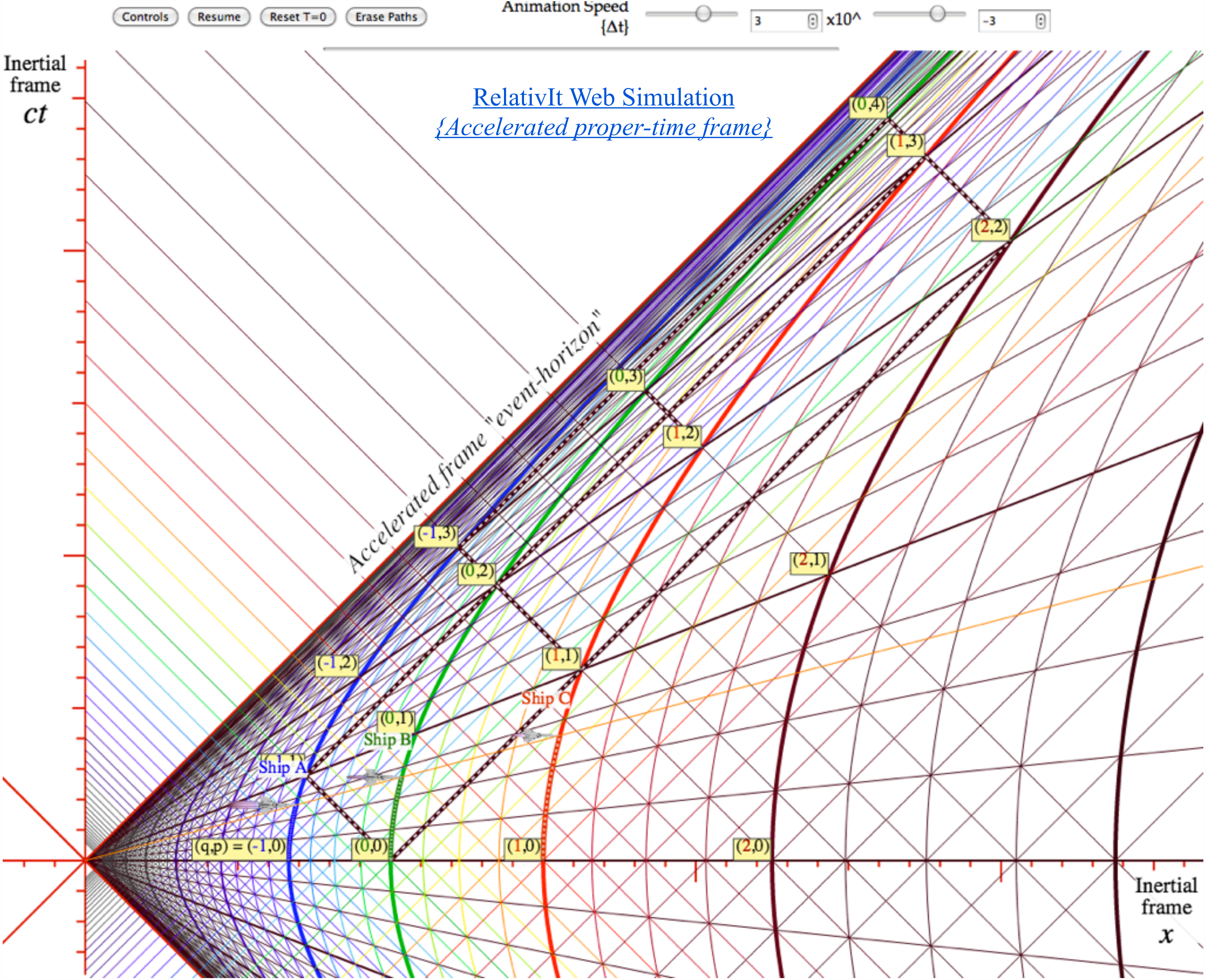
Review: Relawavity ρ functions Two famous ones Extremes and plot vs. ρ
Doppler jeopardy Geometric mean and Relativistic hyperbolas
Animation of $e^{\rho}=2$ spacetime and per-spacetime plots

Rapidity ρ related to *stellar aberration angle* σ and L. C. Epstein's approach to relativity
- Longitudinal hyperbolic ρ-geometry connects to transverse circular σ-geometry
  - "Occams Sword" and summary of 16 parameter functions of ρ and σ
  - Applications to optical waveguide, spherical waves, and accelerator radiation
    - *Learning about* sin! *and* cos *and...*

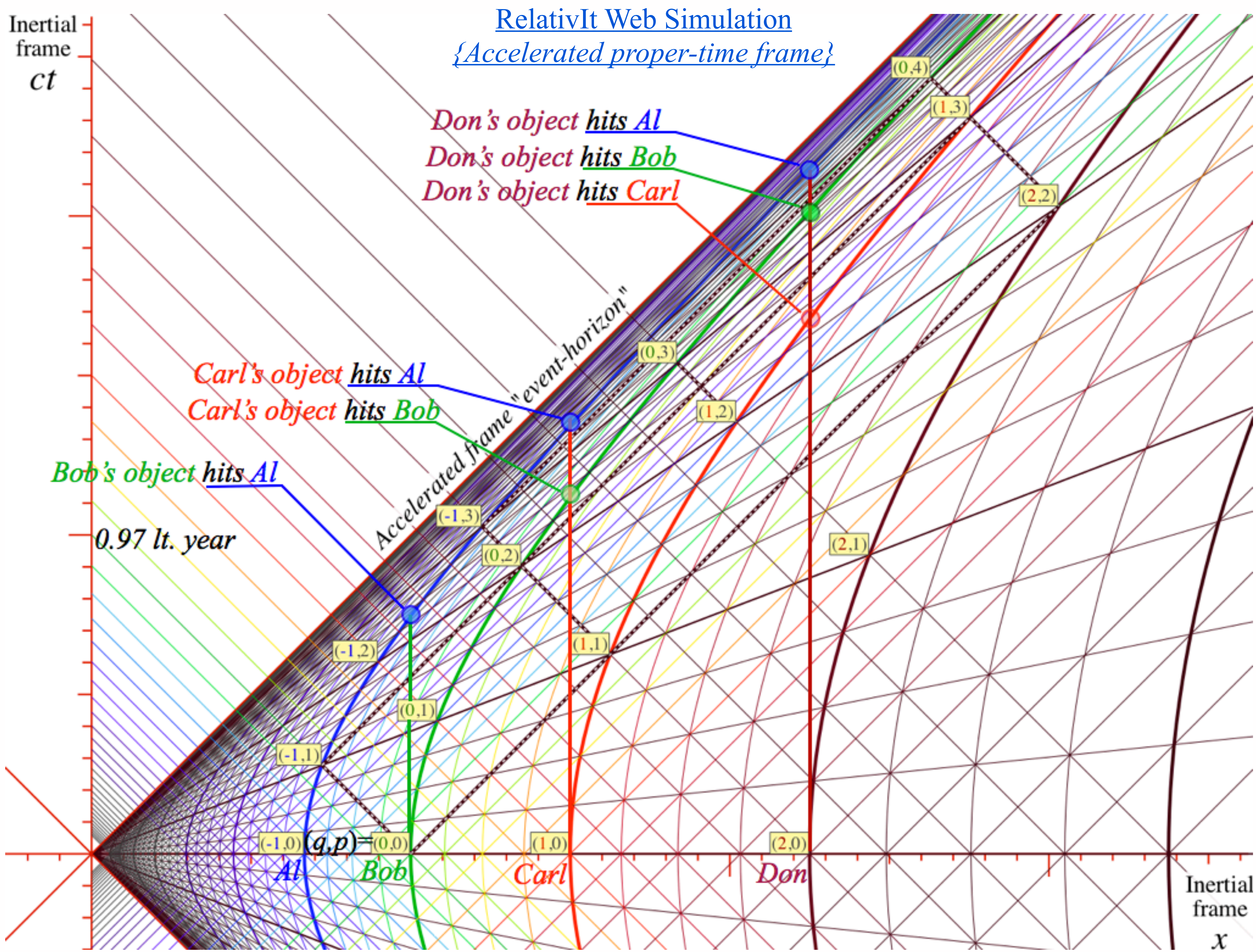
Derivation of relativistic quantum mechanics
- What's the matter with mass? Shining some light on the Elephant in the room
- Relativistic action and Lagrangian-Hamiltonian relations
  - Poincare' and Hamilton-Jacobi equations

Relativistic optical transitions and Compton recoil formulae
- Feynman diagram geometry
  - Compton recoil related to rocket velocity formula
  - Comparing 2nd-quantization "photon" number N and 1st-quantization wavenumber κ

Relawavity in accelerated frames
- Laser up-tuning by Alice and down-tuning by Carla makes *g*-acceleration grid
  - Analysis of constant-*g* grid compared to zero-*g* Minkowsi grid
  - ➔ Animation of mechanics and metrology of constant-g grid



Controls
Resume
Reset T=0
Erase Paths
Animation Speed
{Δt}
3
x10^
-3
Inertial
frame
ct
RelativIt Web Simulation
{Accelerated proper-time frame}
Accelerated frame "event-horizon"
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(1,1)
(-1,2)
Ship C
(0,1)
Ship B
Ship A
(q,p) = (-1,0)
(0,0)
(1,0)
(2,0)
Inertial
frame
x


RelativIt Web Simulation
{Accelerated proper-time frame}
Inertial frame
ct
Don's object hits Al
Don's object hits Bob
Don's object hits Carl
Carl's object hits Al
Carl's object hits Bob
Bob's object hits Al
Accelerated frame "event-horizon"
0.97 lt. year
(0,4)
(1,3)
(2,2)
(0,3)
(1,2)
(-1,3)
(0,2)
(2,1)
(-1,2)
(1,1)
(0,1)
(-1,1)
(-1,0)
(q,p)=(0,0)
(1,0)
(2,0)
Al
Bob
Carl
Don
Inertial frame
x

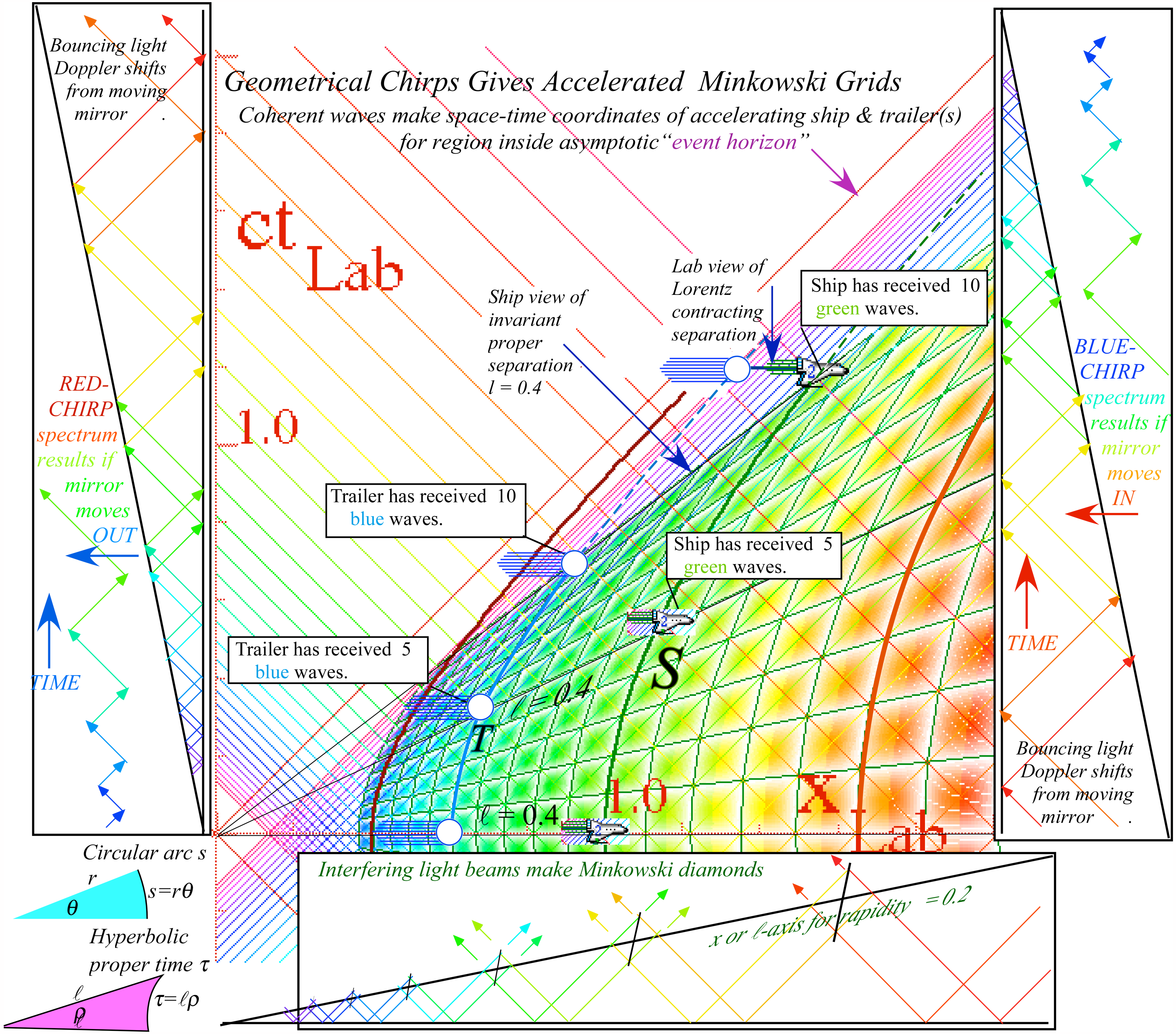
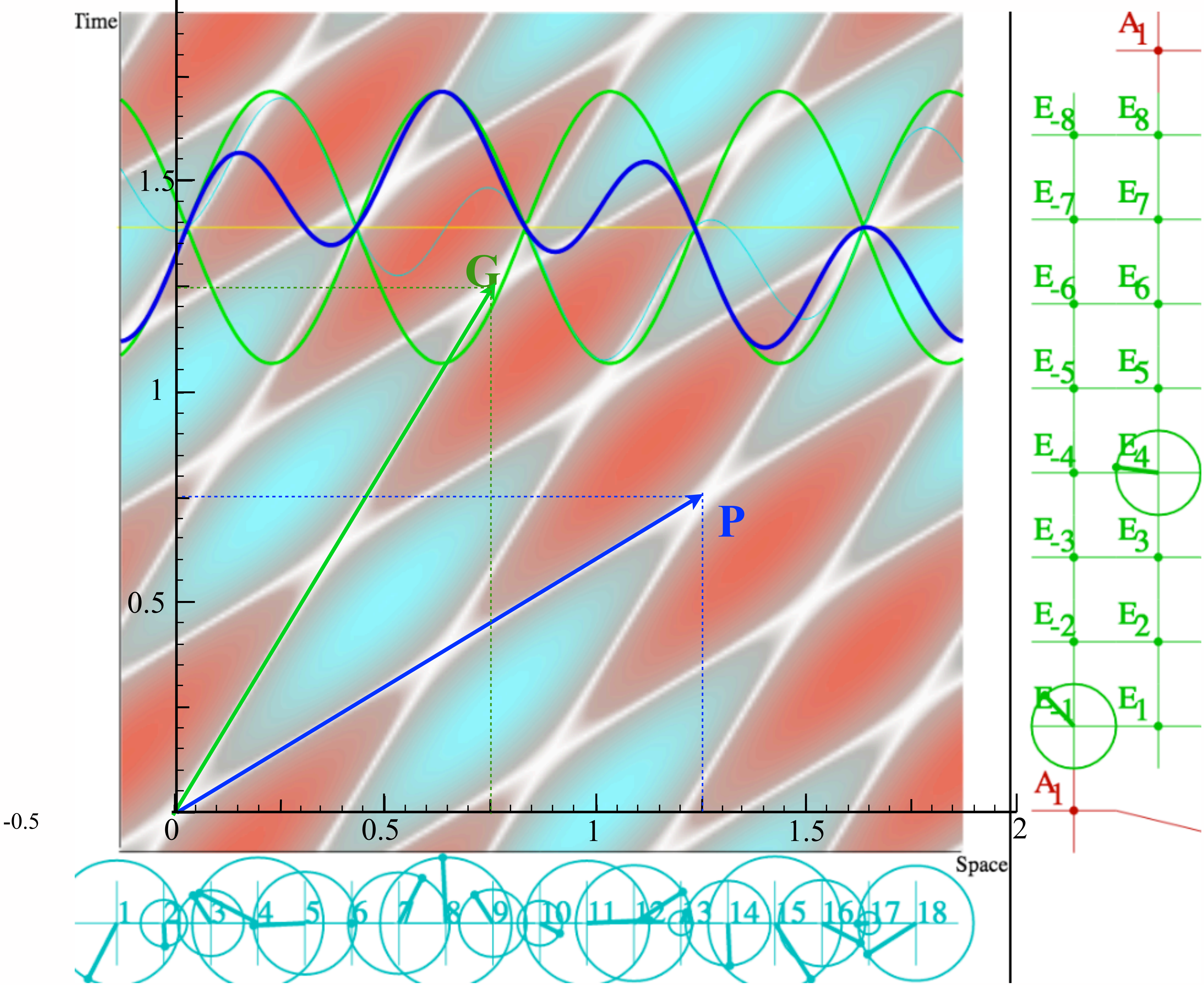
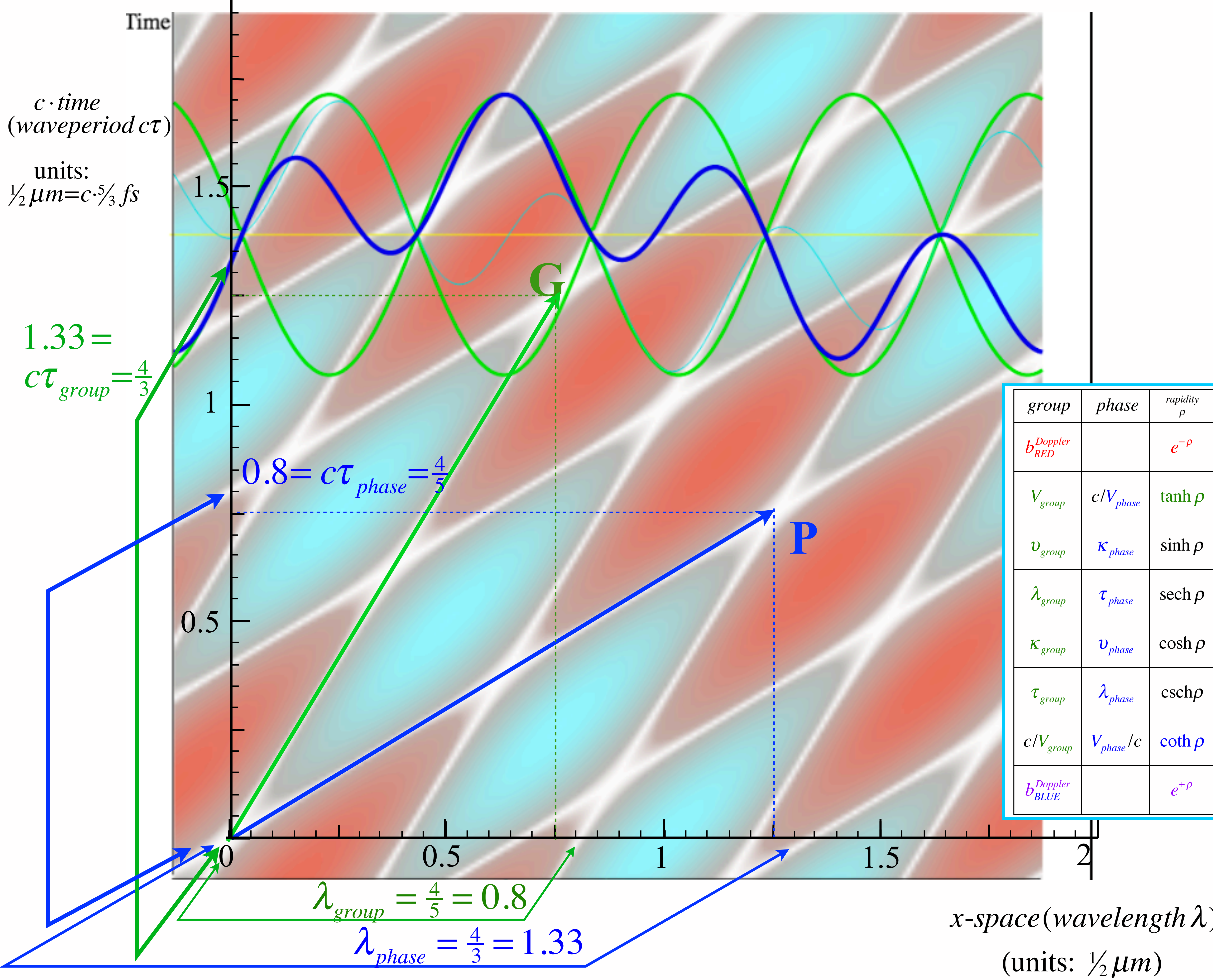
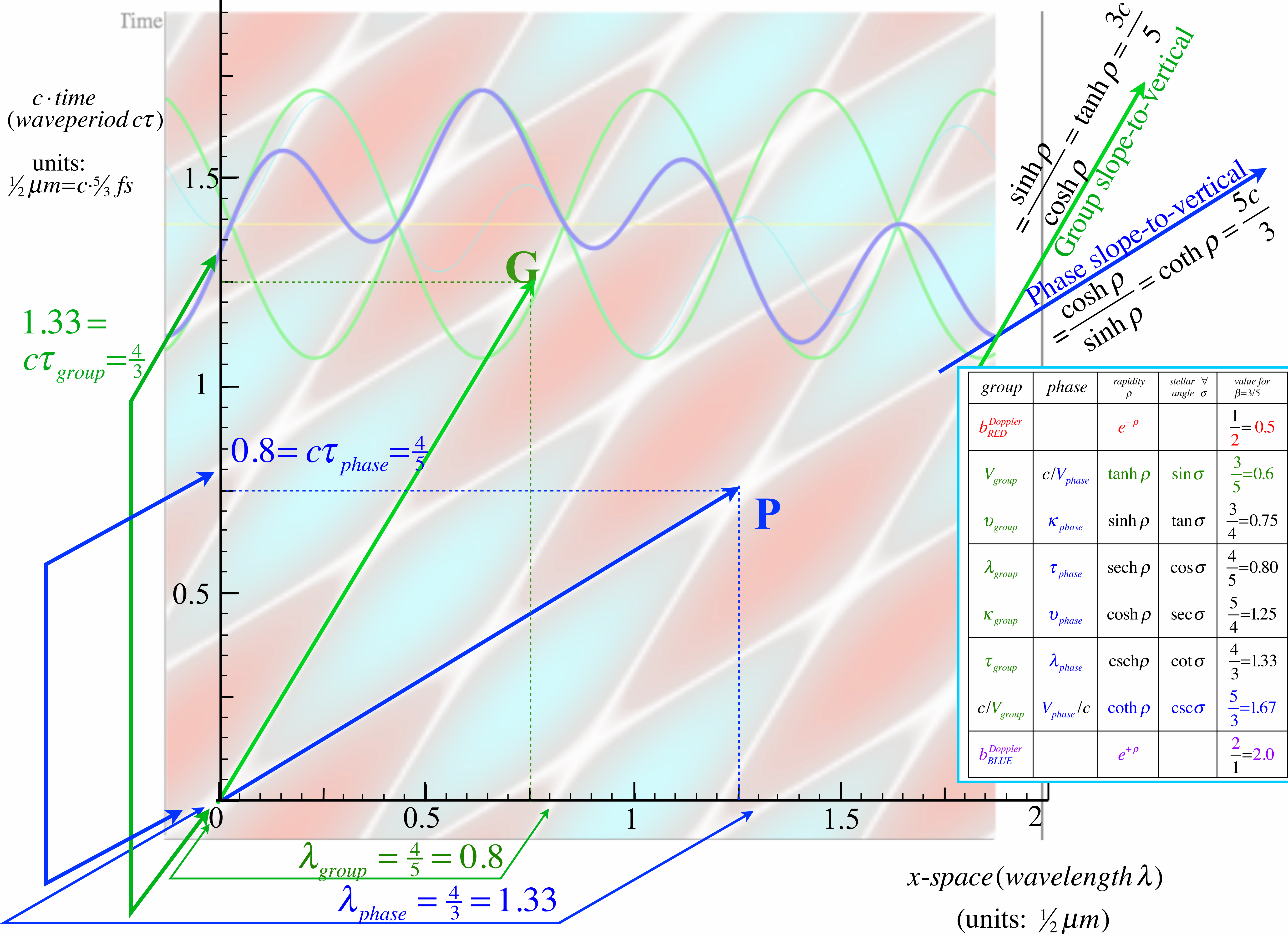


Fig. 8.2 Accelerated reference frames and their trajectories painted by chirped coherent light

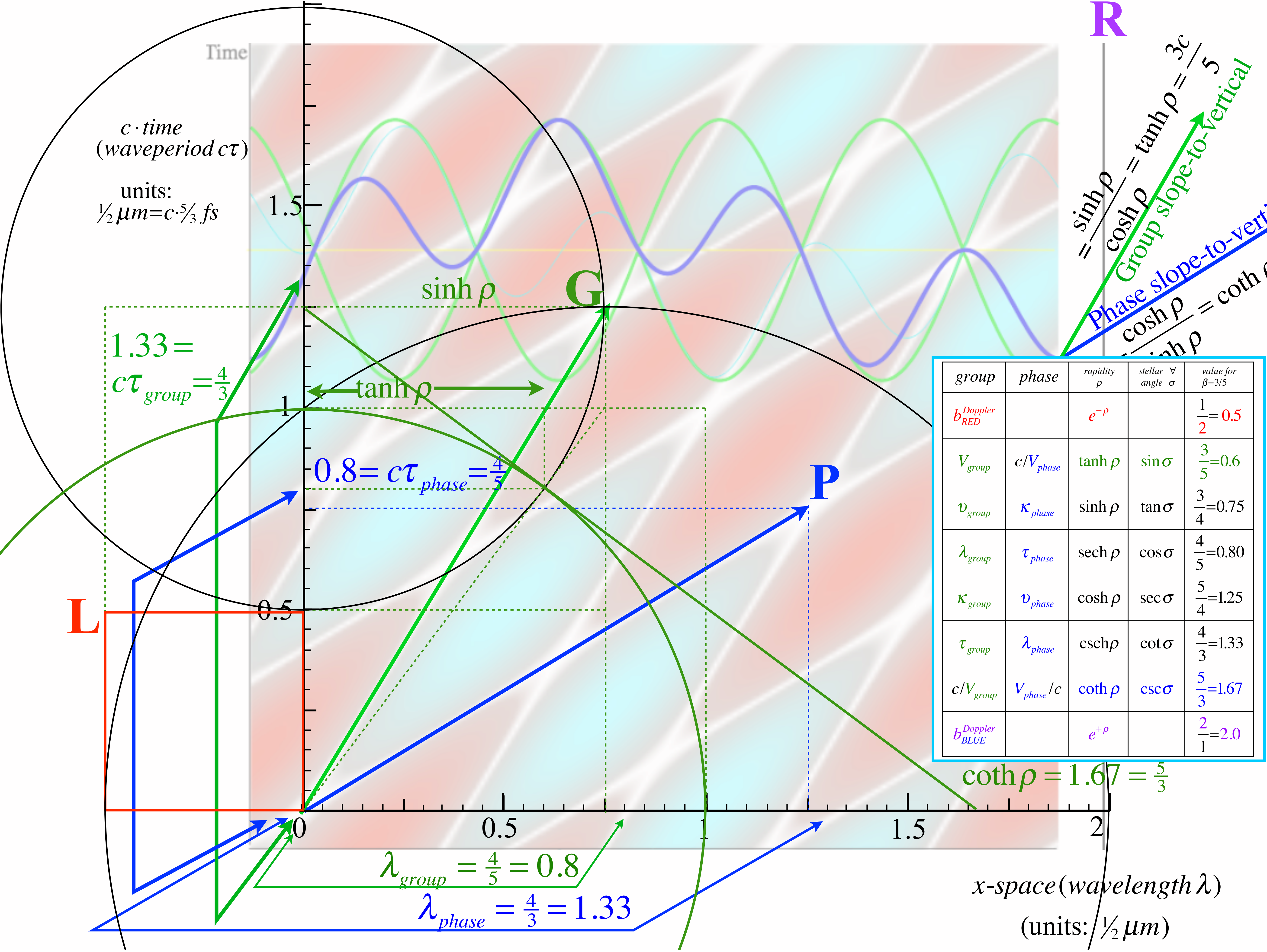

Time
1.5
1
0.5
-0.5
0
0.5
1
1.5
2
Space
G
P
A1
E-8 E8
E-7 E7
E-6 E6
E-5 E5
E-4 E4
E-3 E3
E-2 E2
E-1 E1
A1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18



group	*phase*	*rapidity* ρ	*stellar* ∀ *angle* σ	*value for* β=3/5
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}$= 0.5
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}$=0.6
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}$=0.75
λ_{group}	τ_{phase}	$\mathrm{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}$=0.80
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}$=1.25
τ_{group}	λ_{phase}	$\mathrm{csch}\rho$	$\cot\sigma$	$\frac{4}{3}$=1.33
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}$=1.67
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}$=2.0

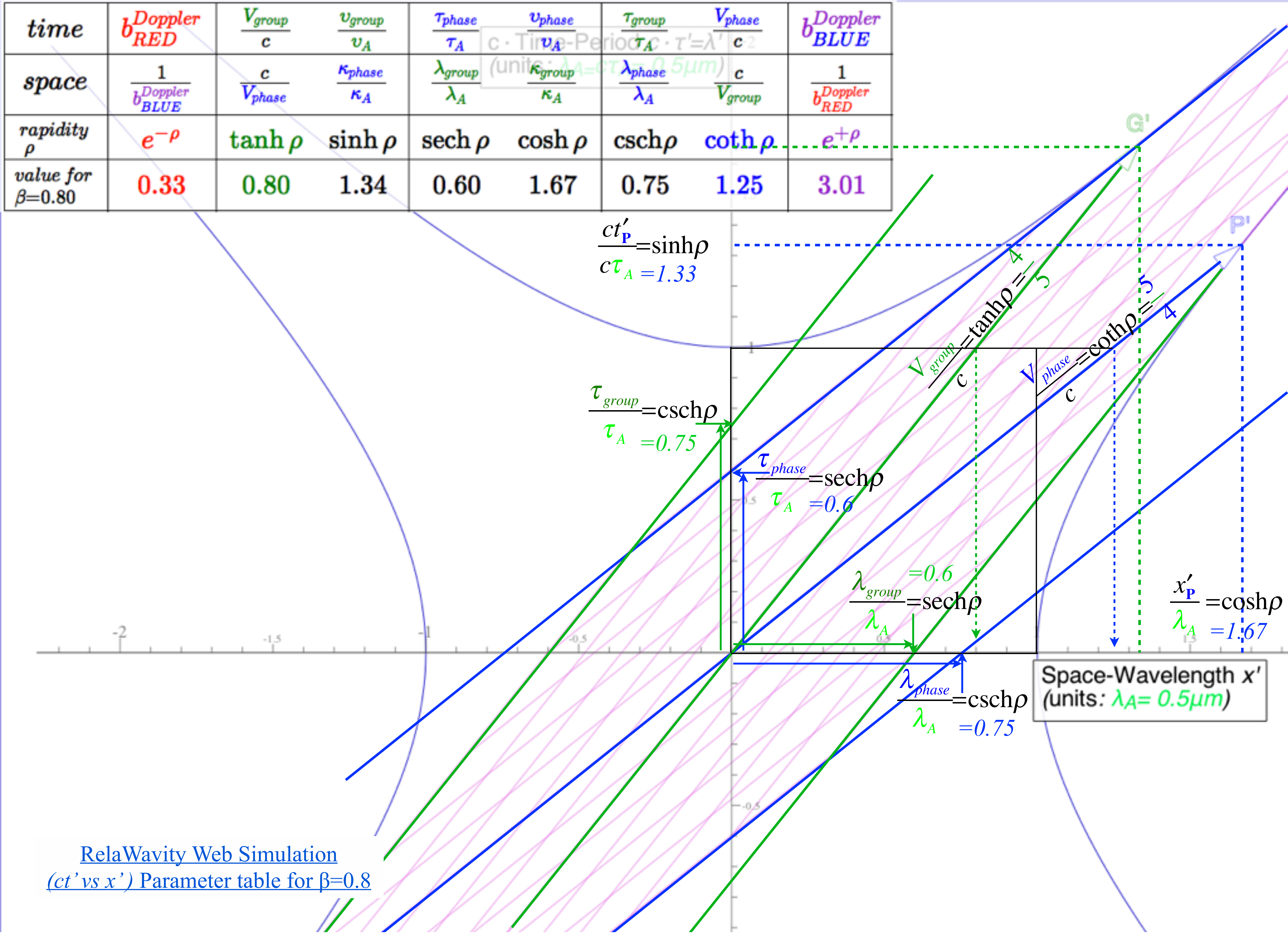


group	phase	rapidity ρ	stellar angle $\forall\ \sigma$	value for $\beta=3/5$
$b_{RED}^{Doppler}$		$e^{-\rho}$		$\frac{1}{2}=0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\mathrm{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\mathrm{csch}\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b_{BLUE}^{Doppler}$		$e^{+\rho}$		$\frac{2}{1}=2.0$

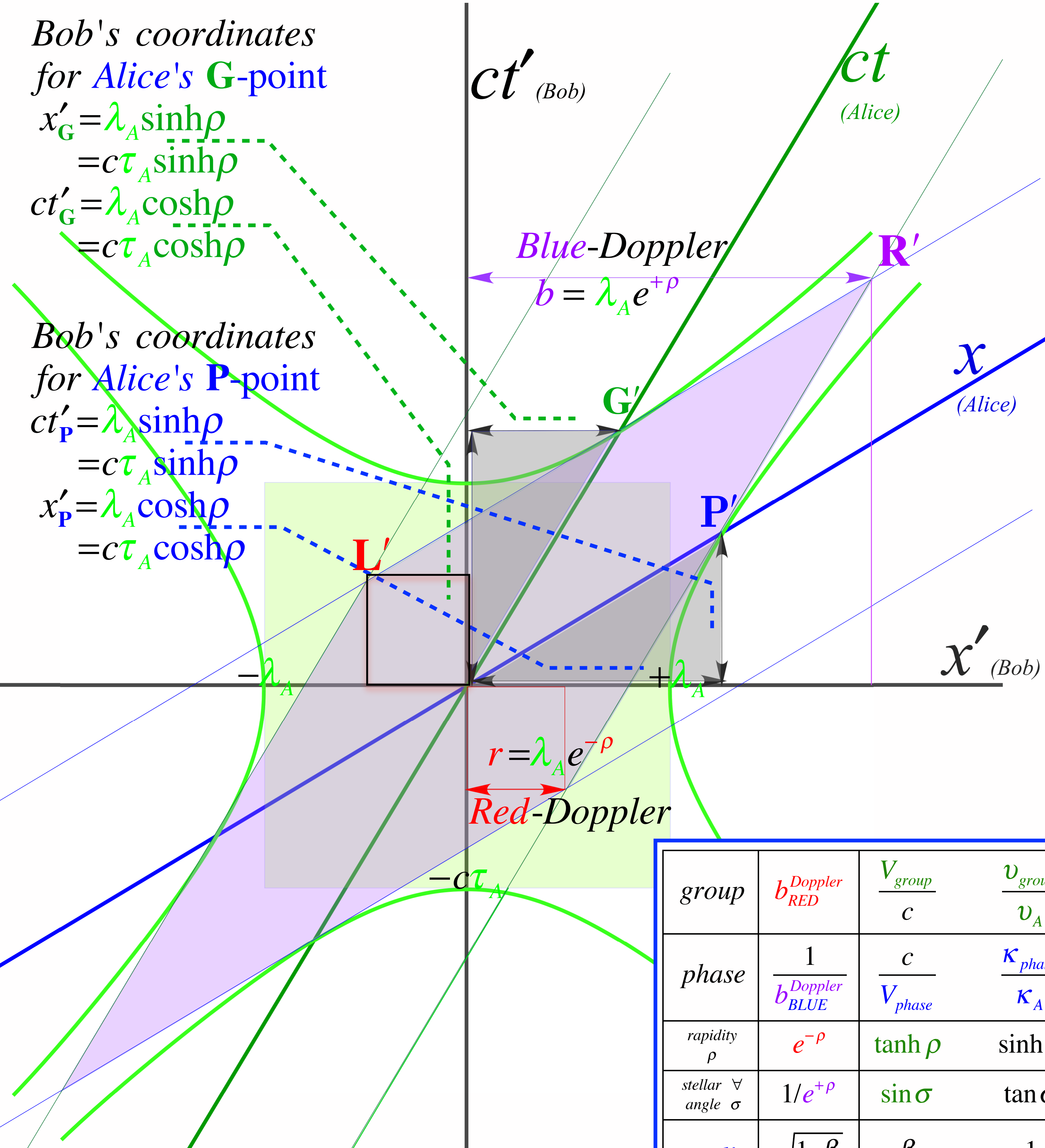


group	phase	rapidity ρ	stellar $\forall$ angle σ	value for $\beta=3/5$
$b^{Doppler}_{RED}$		$e^{-\rho}$		$\frac{1}{2}= 0.5$
V_{group}	c/V_{phase}	$\tanh\rho$	$\sin\sigma$	$\frac{3}{5}=0.6$
υ_{group}	κ_{phase}	$\sinh\rho$	$\tan\sigma$	$\frac{3}{4}=0.75$
λ_{group}	τ_{phase}	$\text{sech}\,\rho$	$\cos\sigma$	$\frac{4}{5}=0.80$
κ_{group}	υ_{phase}	$\cosh\rho$	$\sec\sigma$	$\frac{5}{4}=1.25$
τ_{group}	λ_{phase}	$\text{csch}\,\rho$	$\cot\sigma$	$\frac{4}{3}=1.33$
c/V_{group}	V_{phase}/c	$\coth\rho$	$\csc\sigma$	$\frac{5}{3}=1.67$
$b^{Doppler}_{BLUE}$		$e^{+\rho}$		$\frac{2}{1}=2.0$

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=0.80	0.33	0.80	1.34	0.60	1.67	0.75	1.25	3.01



[RelaWavity Web Simulation](#)
[*(ct' vs x')* Parameter table for β=0.8](#)



Space-time parameters

$\lambda_{phase} = \lambda_A \operatorname{csch}\rho$

$\lambda_{group} = \lambda_A \operatorname{sech}\rho$

$c\tau_{phase} = c\tau_A \operatorname{sech}\rho$

$c\tau_{group} = c\tau_A \operatorname{csch}\rho$

Per-space-time parameters

$c\kappa_{phase} = c\kappa_A \sinh\rho$

$c\kappa_{group} = c\kappa_A \cosh\rho$

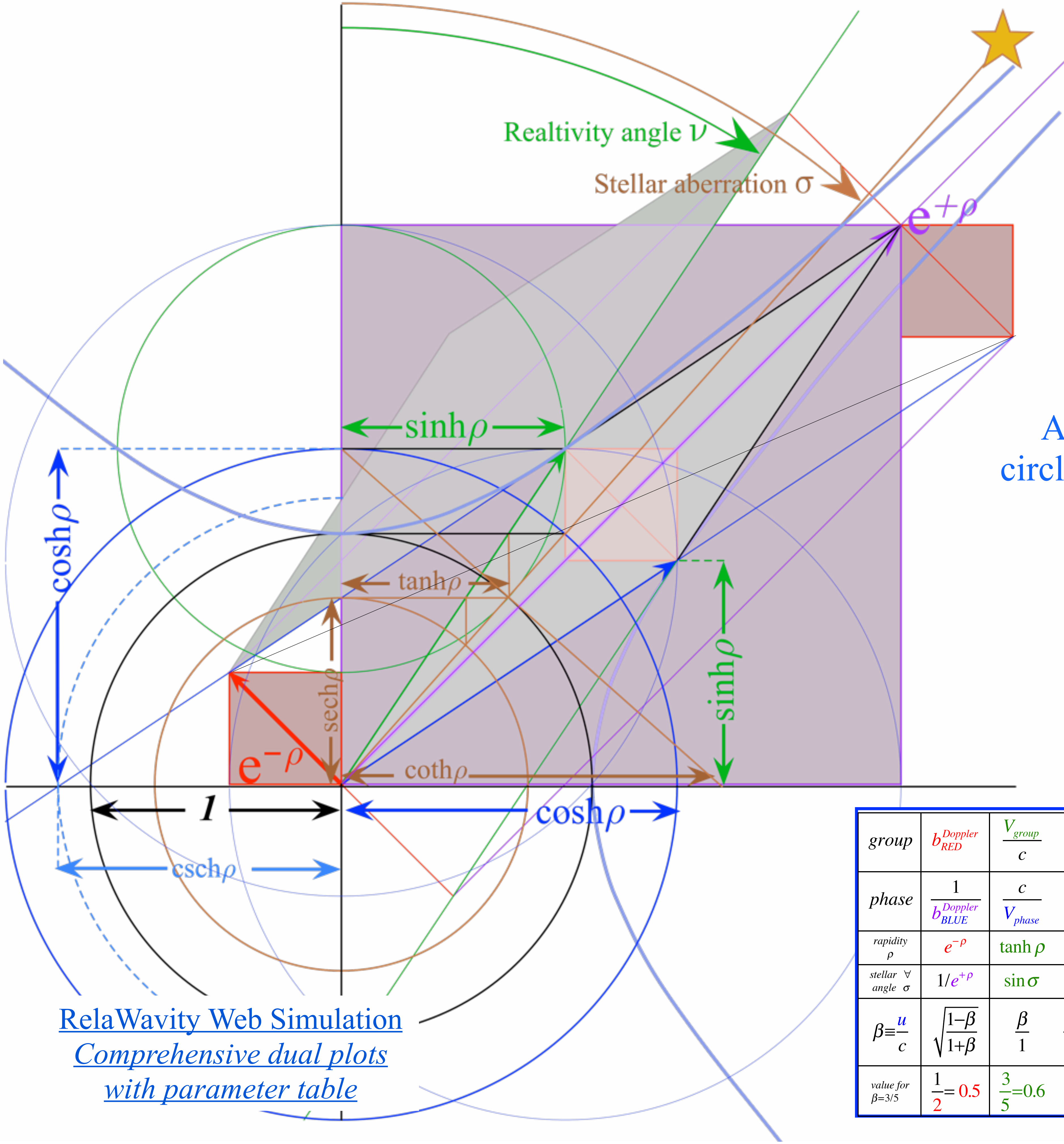
$\upsilon_{phase} = \upsilon_A \cosh\rho$

$\upsilon_{group} = \upsilon_A \sinh\rho$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0
effects	$b^{Doppler}_{RED}$	V_{group}	*past-future asymmetry* (off-diagonal Lorentz-transform)	*x-contraction*(Lorentz) τ_{phase}*-contraction*	*t-dilation*(Einstein) υ_{phase}*-dilation* (on-diagonal Lorentz-transform)	*inverse asymmetry*	V_{phase}	$b^{Doppler}_{BLUE}$

RelaWavity Web Simulation
Comprehensive dual plots with parameter table

RelaWavity Web Simulation
(ct' vs x') with parameter table



A more compact circle-based geometry

RelaWavity Web Simulation

Comprehensive dual plots with parameter table

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$