Group Theory in Quantum Mechanics

Lecture 12.5 (2.24.15)

Symmetry and Dynamics of C_N cyclic systems(contd.)

(Geometry of U(2) characters - Ch. 6-9 of Unit 3)
(Principles of Symmetry, Dynamics, and Spectroscopy - Sec. 3-7 of Ch. 2)

Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Phase velocity for simple wave $e^{i(kx-\omega t)}$: Newton's "corpuscle" tracks vs.wave-zero paths

Slow *L*-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

Fast *R*-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$

Phase velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: Half-sum factor $S=e^{i(\mathbf{L}+\mathbf{R})/2}$

Group velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: Half-difference factor $D=e^{i(\mathbf{L}-\mathbf{R})/2}+e^{-i(\mathbf{L}-\mathbf{R})/2}$

Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams

L-laser 600THz and **R**-laser 600THZ (Laser lab frame)

Phase **P**-vector and group **G**-vector span Cartesian spacetime coordinates

L′-laser 300THz and **R′**-laser 1200THZ (Doppler shifted in moving frame)

Doppler shifted **L′**-vector and **R′**-vector in (**L**, **R**)-per-spacetime

Vectors of *phase* $\mathbf{P}'=(\mathbf{R}'+\mathbf{L}')/2$ and *group* $\mathbf{G}'=(\mathbf{R}'-\mathbf{L}')/2$

Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

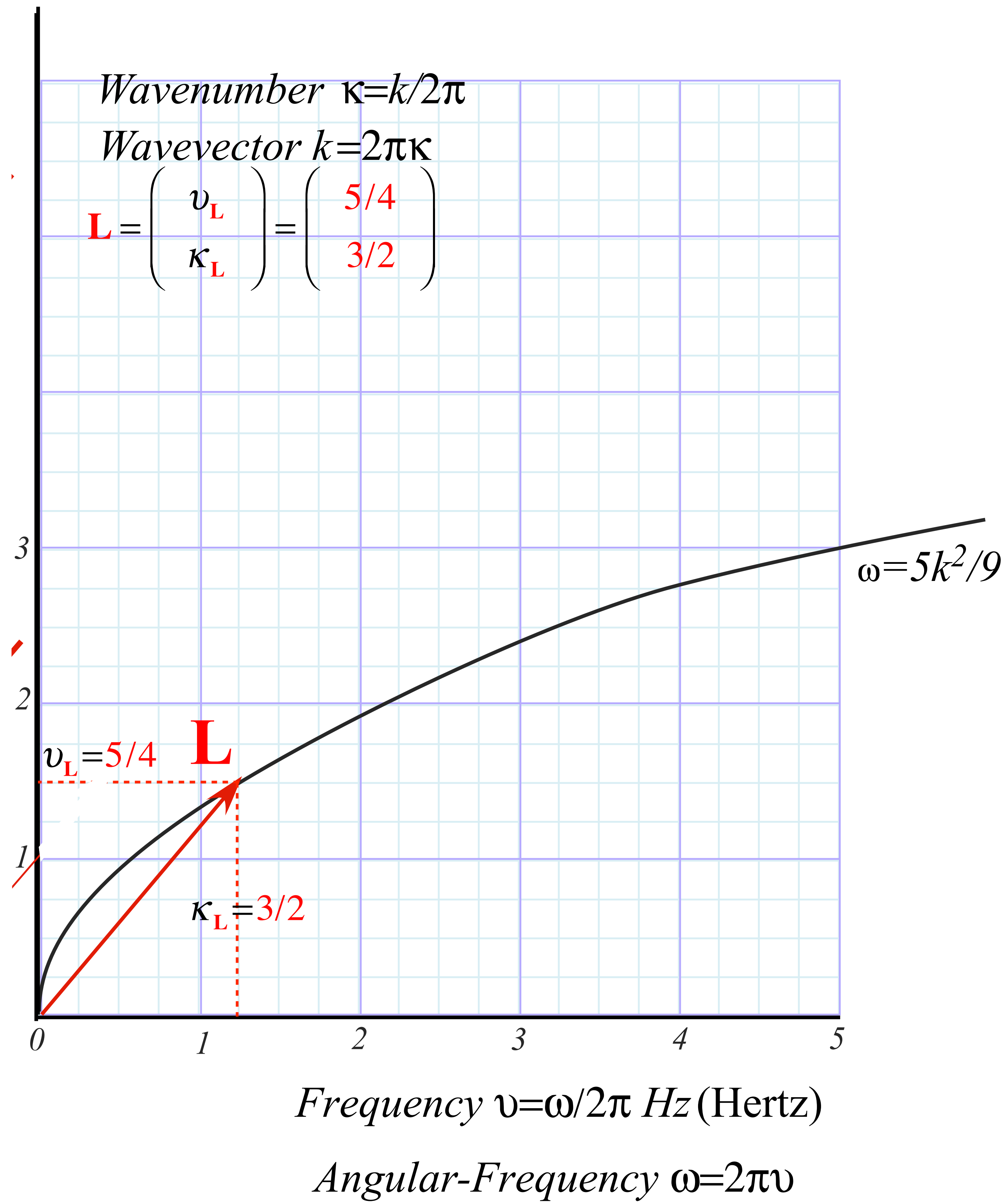
Brief tour of and relativistic mechanics by geometry

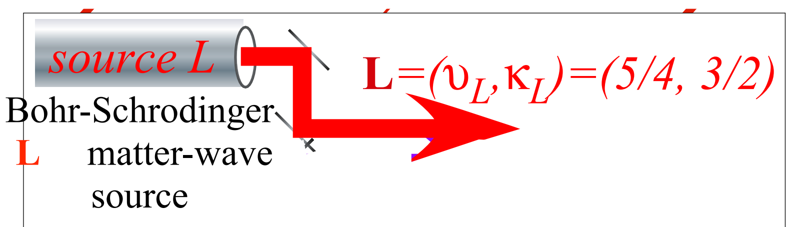
Summary of optical wave parameters for relativity and QM

Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Spacetime (x,t)

Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$





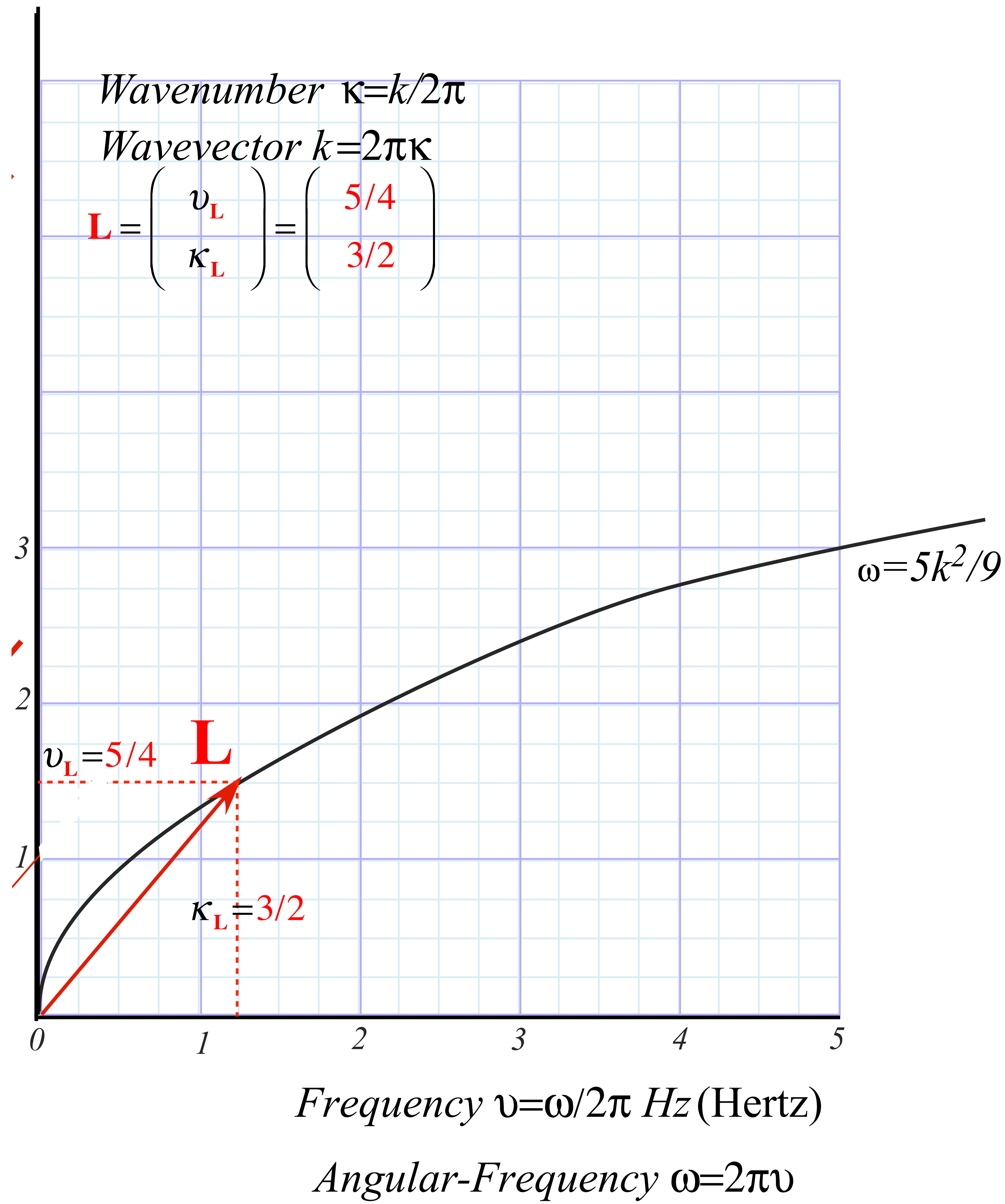
Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

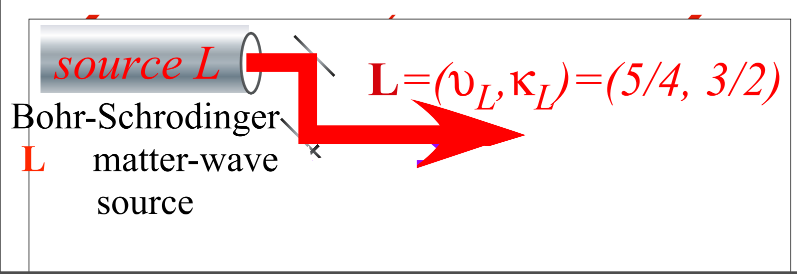
Spacetime (x,t) *Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$*

Phase velocity for simple wave $e^{i(kx-\omega t)}$
is $V=\omega/k=\upsilon/\kappa$ where:
υ =waves per second
and
κ =waves per meter

or:

$\omega=2\pi\upsilon$ =radians per second
and
$k=2\pi\kappa$ =radians per meter





Wave coordinates in spacetime and per-spacetime for Bohr-Schrodinger Dispersion

Phase velocity for simple wave $e^{i(kx-\omega t)}$*: Newton's "corpuscle" tracks vs.wave-zero paths*

→ *Slow L-wave* $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$

Phase velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$*: Half-sum factor* $S=e^{i(\mathbf{L}+\mathbf{R})/2}$

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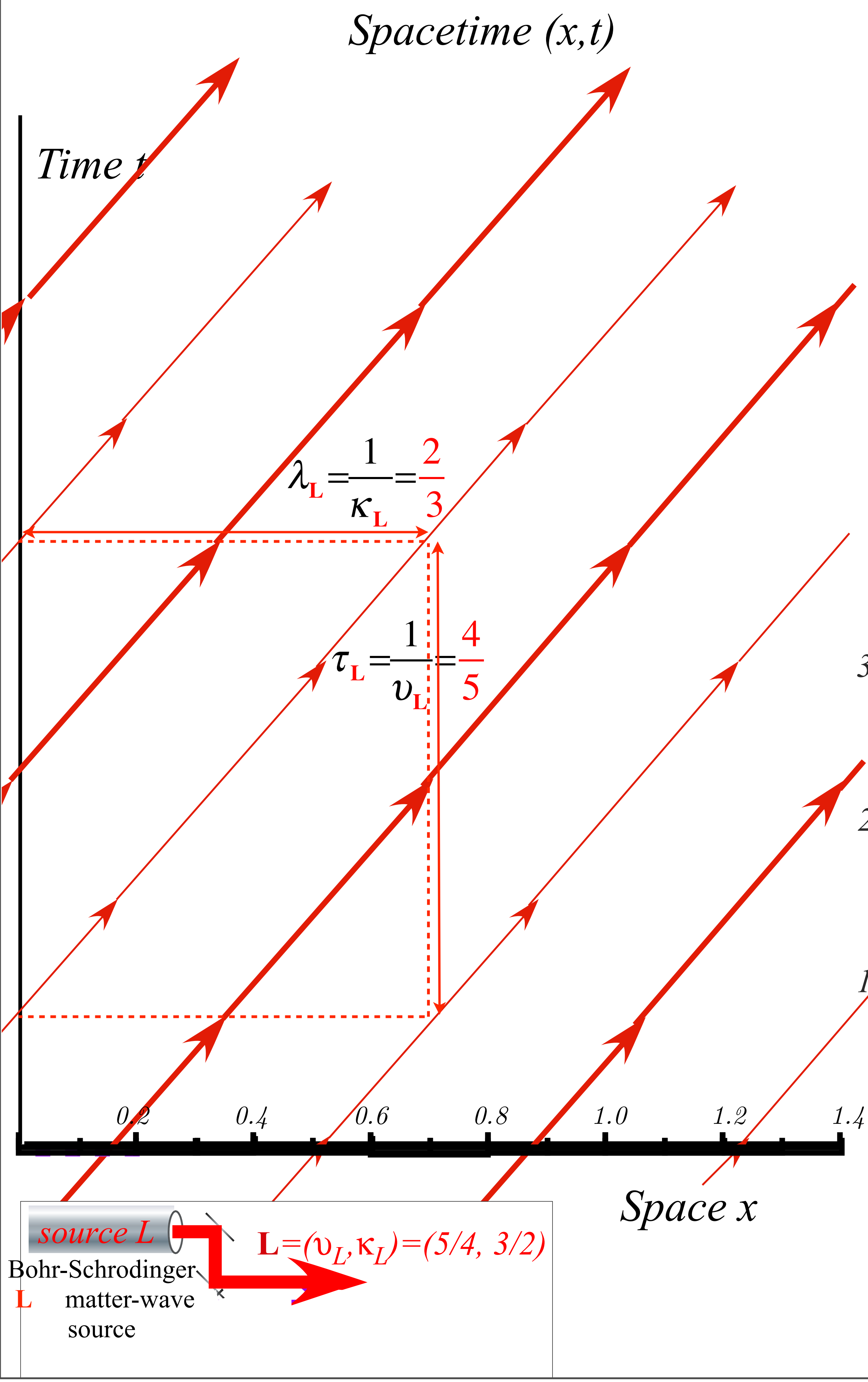
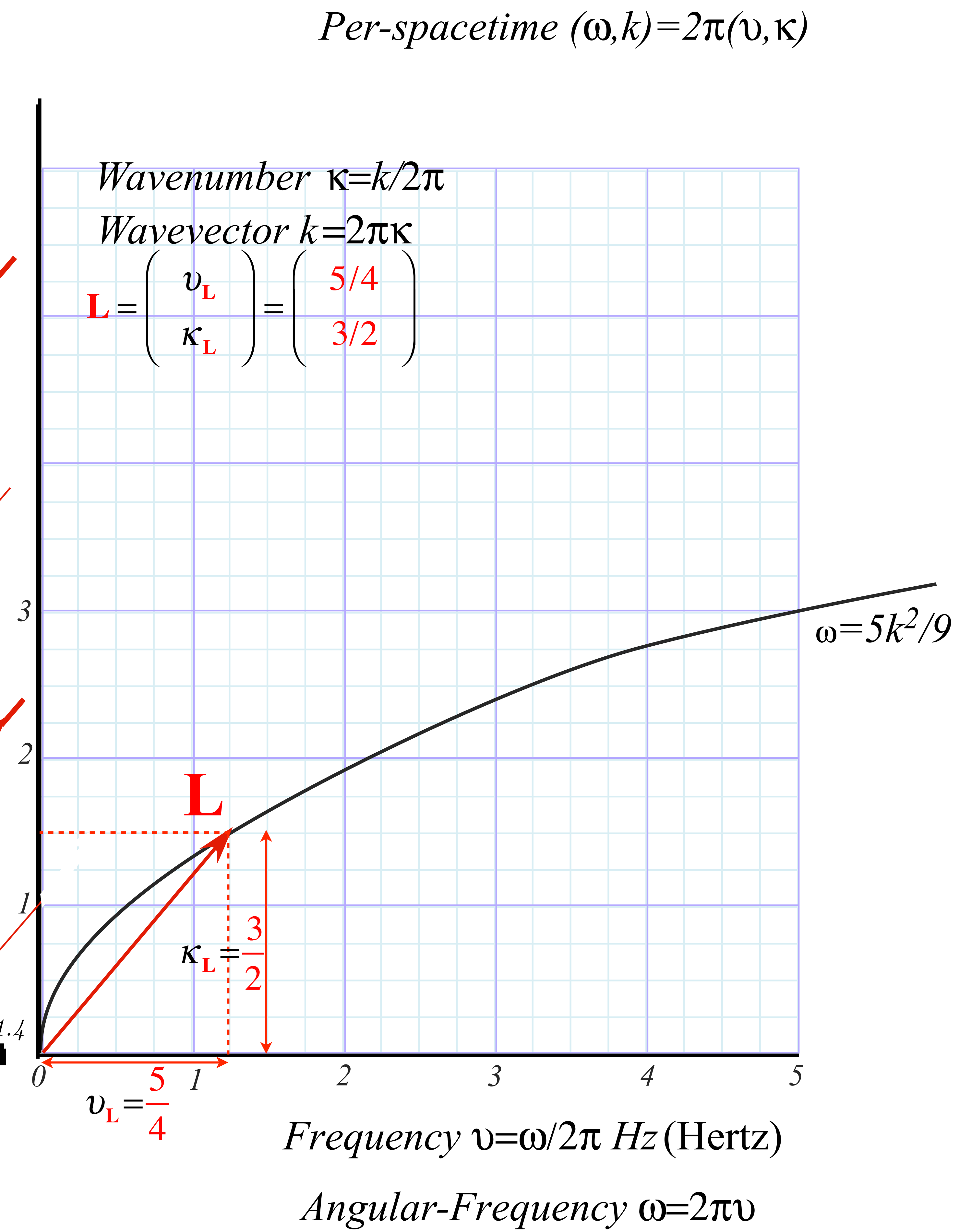
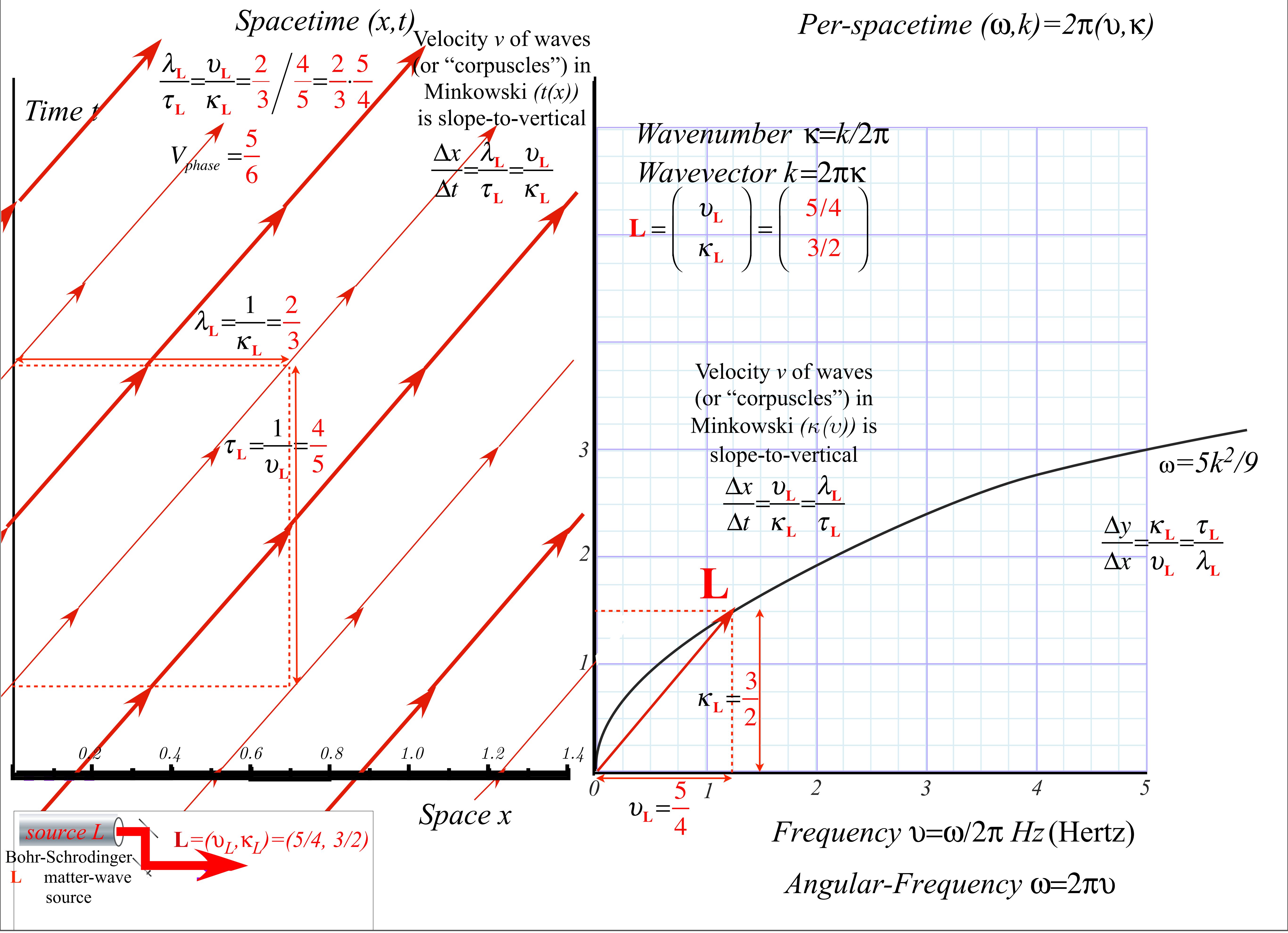
Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$



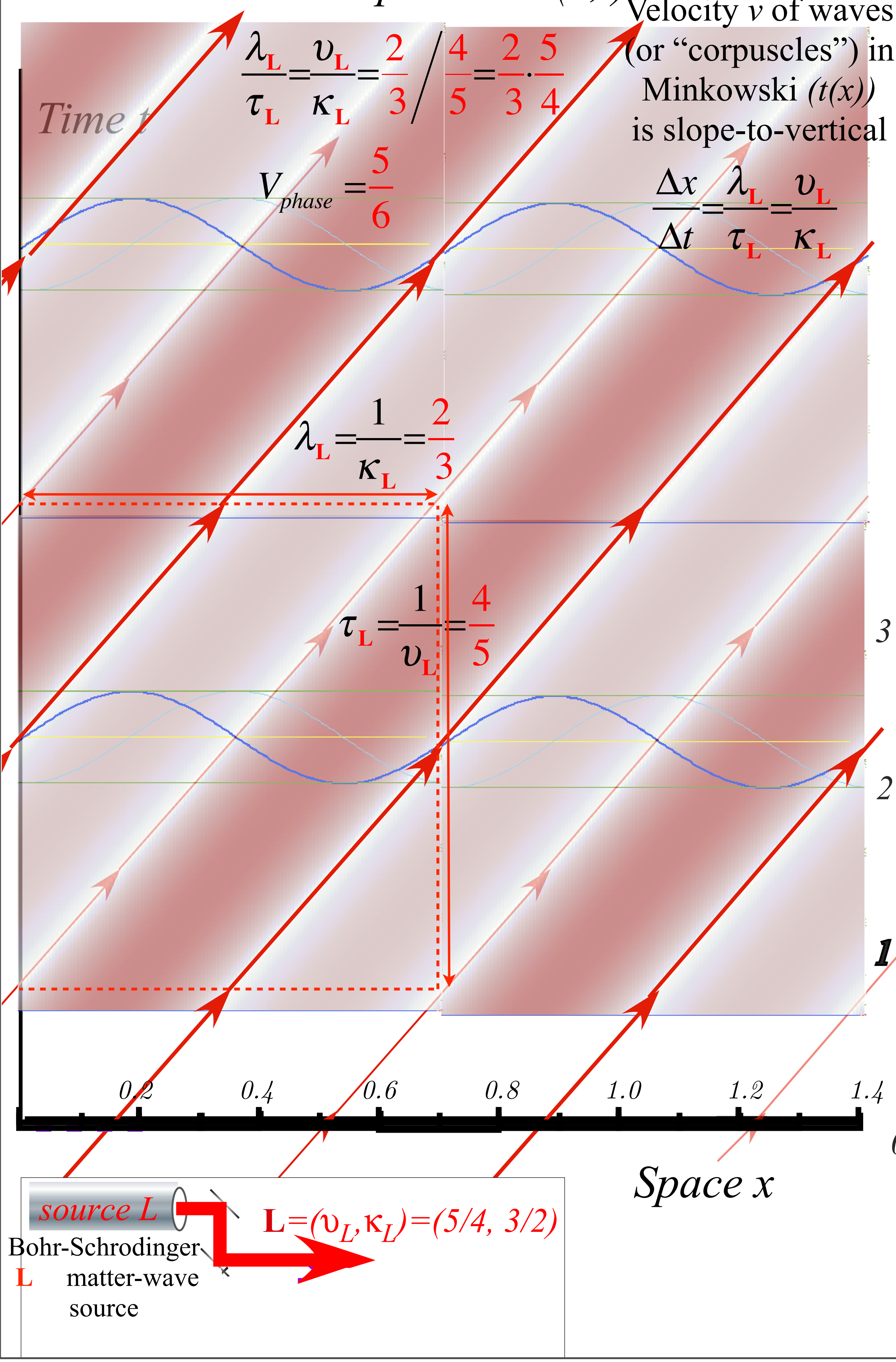


Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

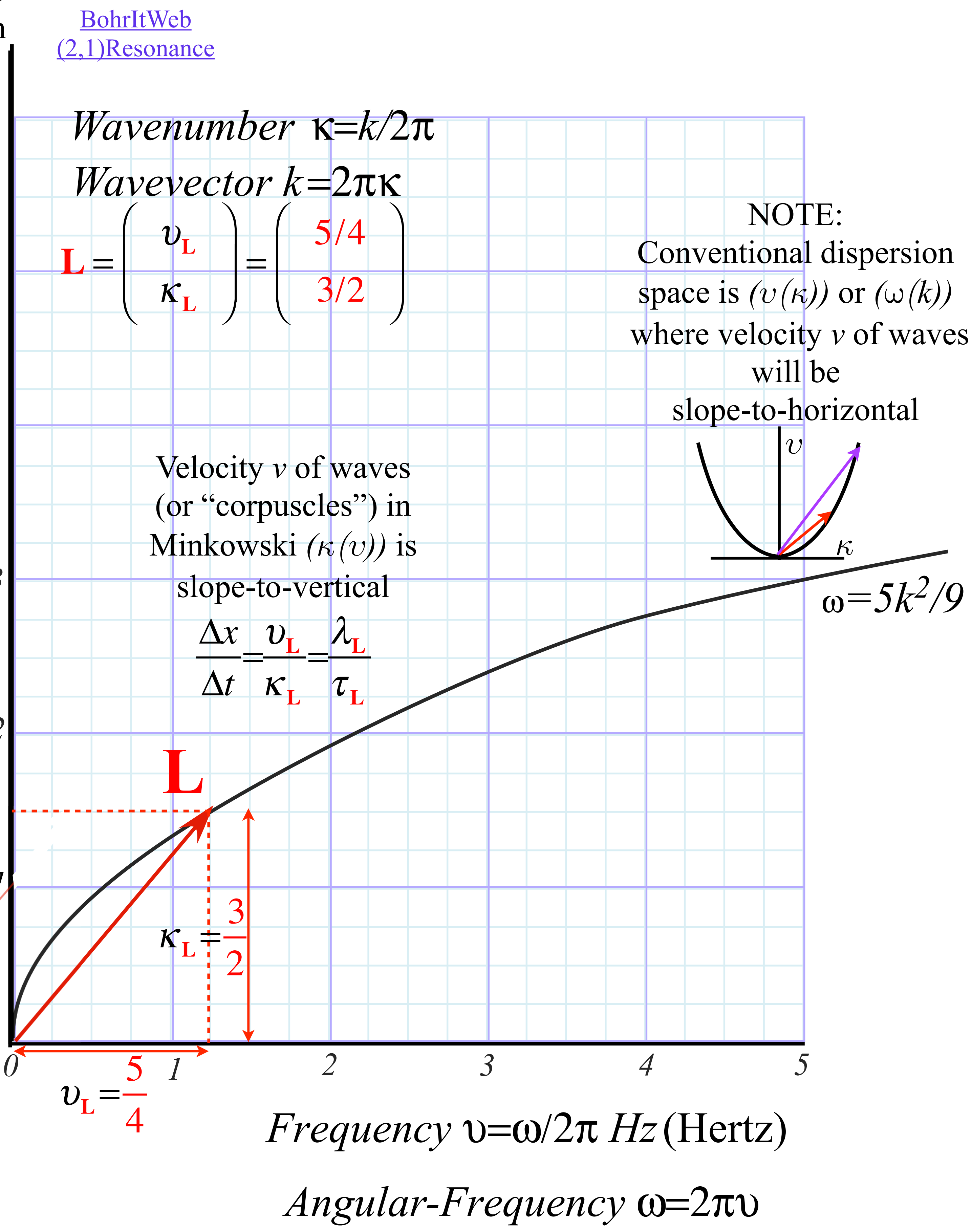


Slow L-wave $e^{iL}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$

Spacetime (x,t)



Per-spacetime $(\omega,k)=2\pi(\upsilon,\kappa)$



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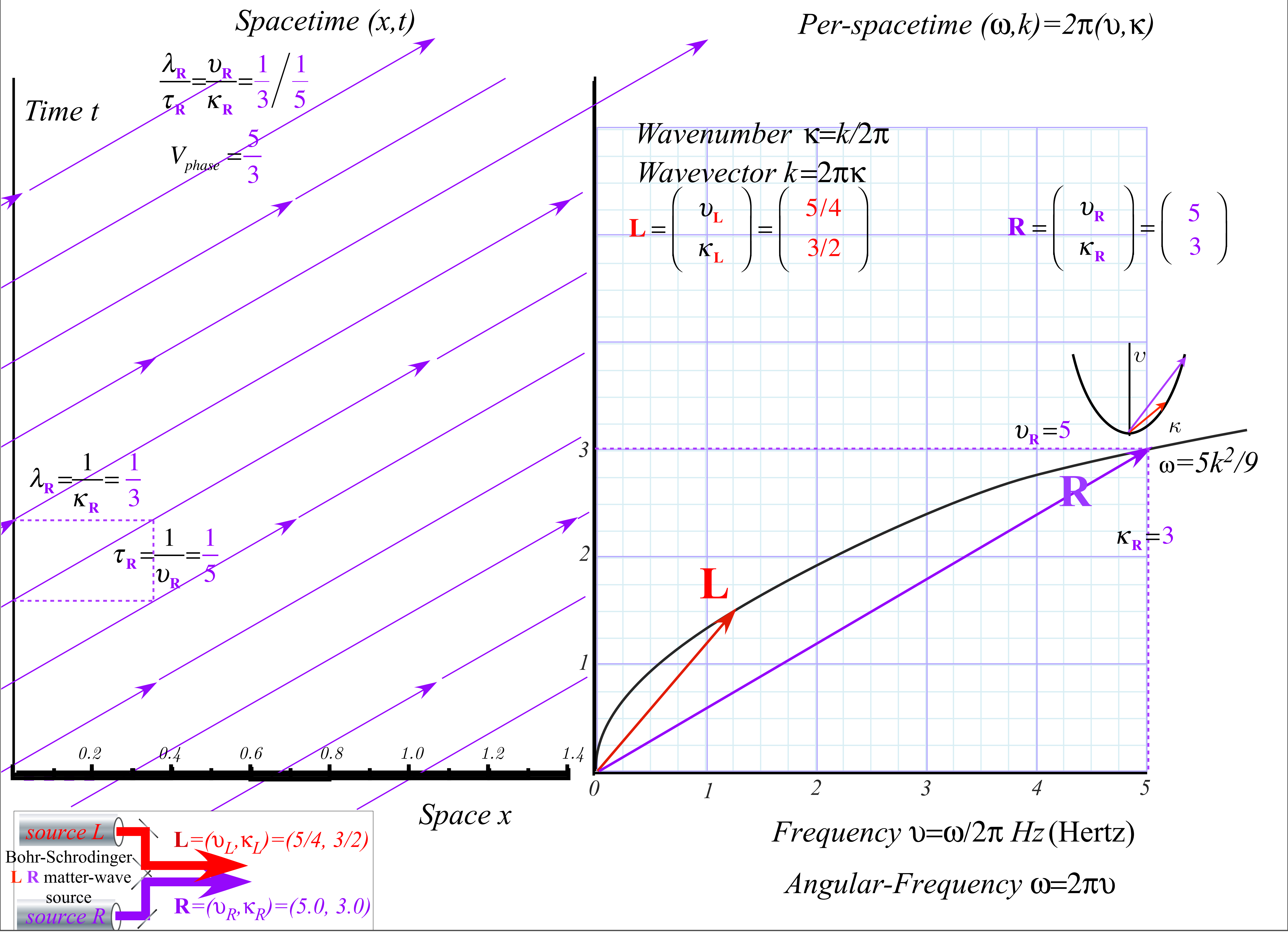
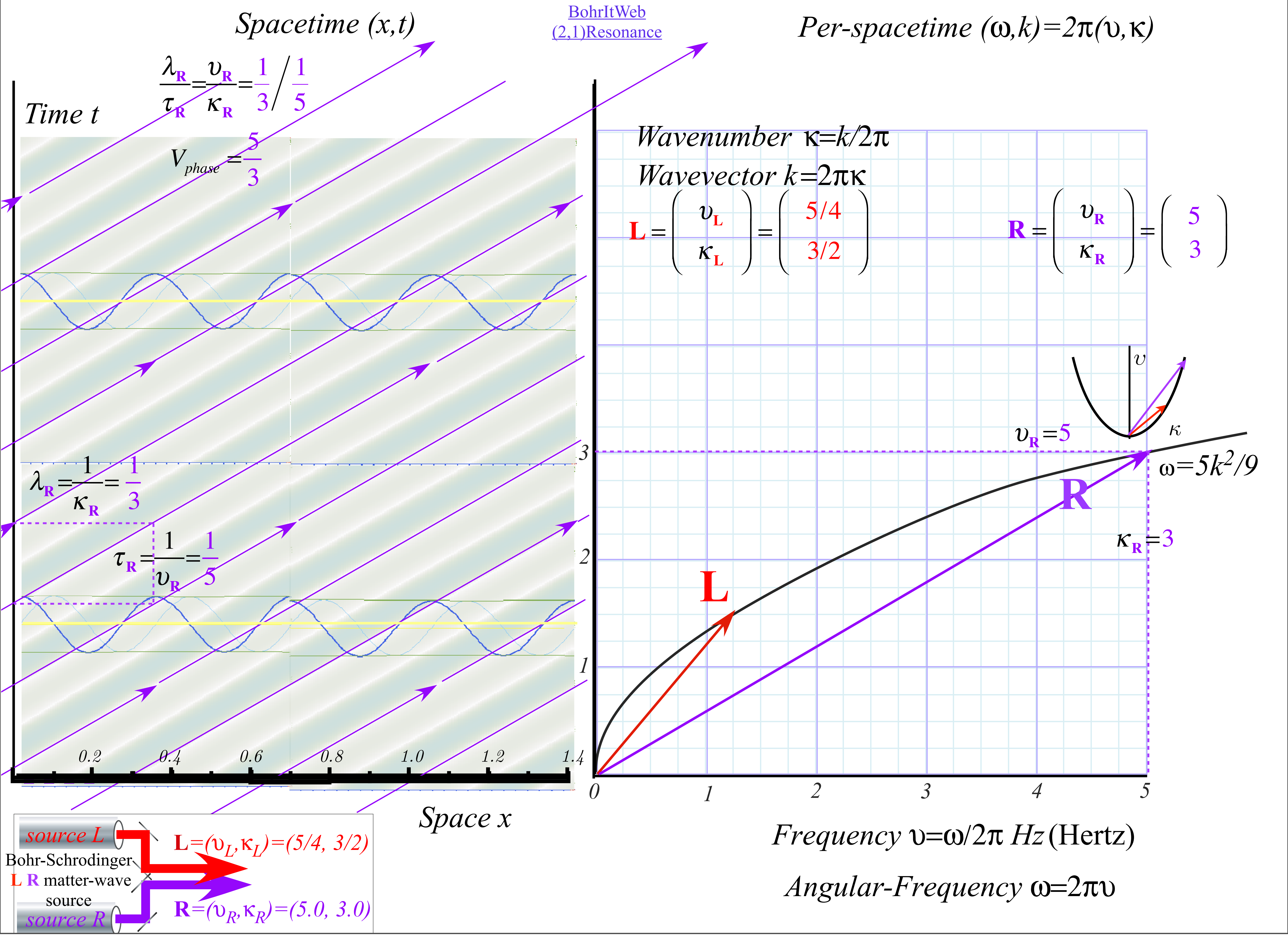
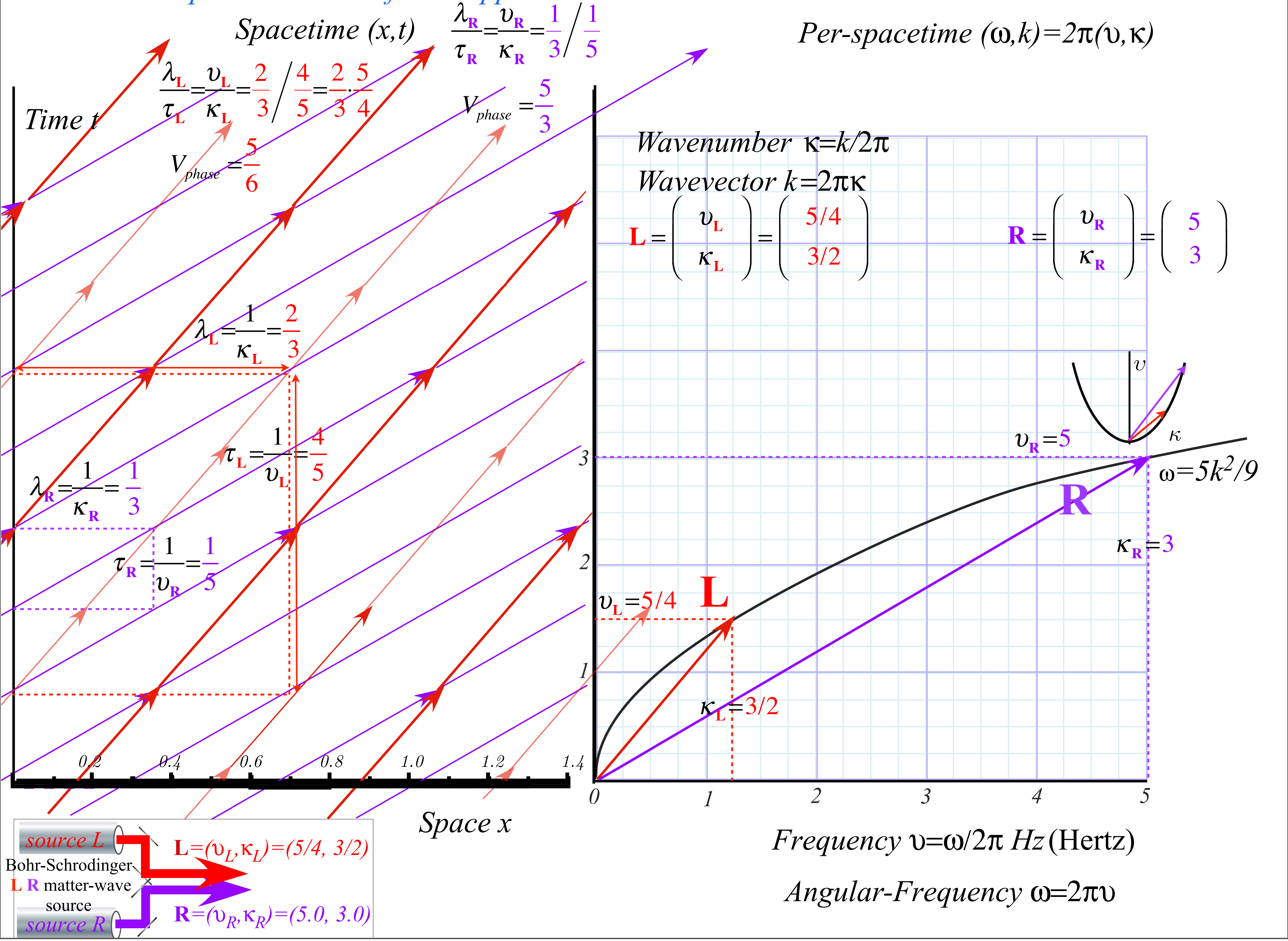
Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$



Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$



Newton's "corpuscle" tracks of overlapped Slow L-wave $e^{i\mathbf{L}}=e^{i(k(L)\cdot x-\omega(L)\cdot t)}$ & Fast R-wave $e^{i\mathbf{R}}=e^{i(k(R)\cdot x-\omega(R)\cdot t)}$



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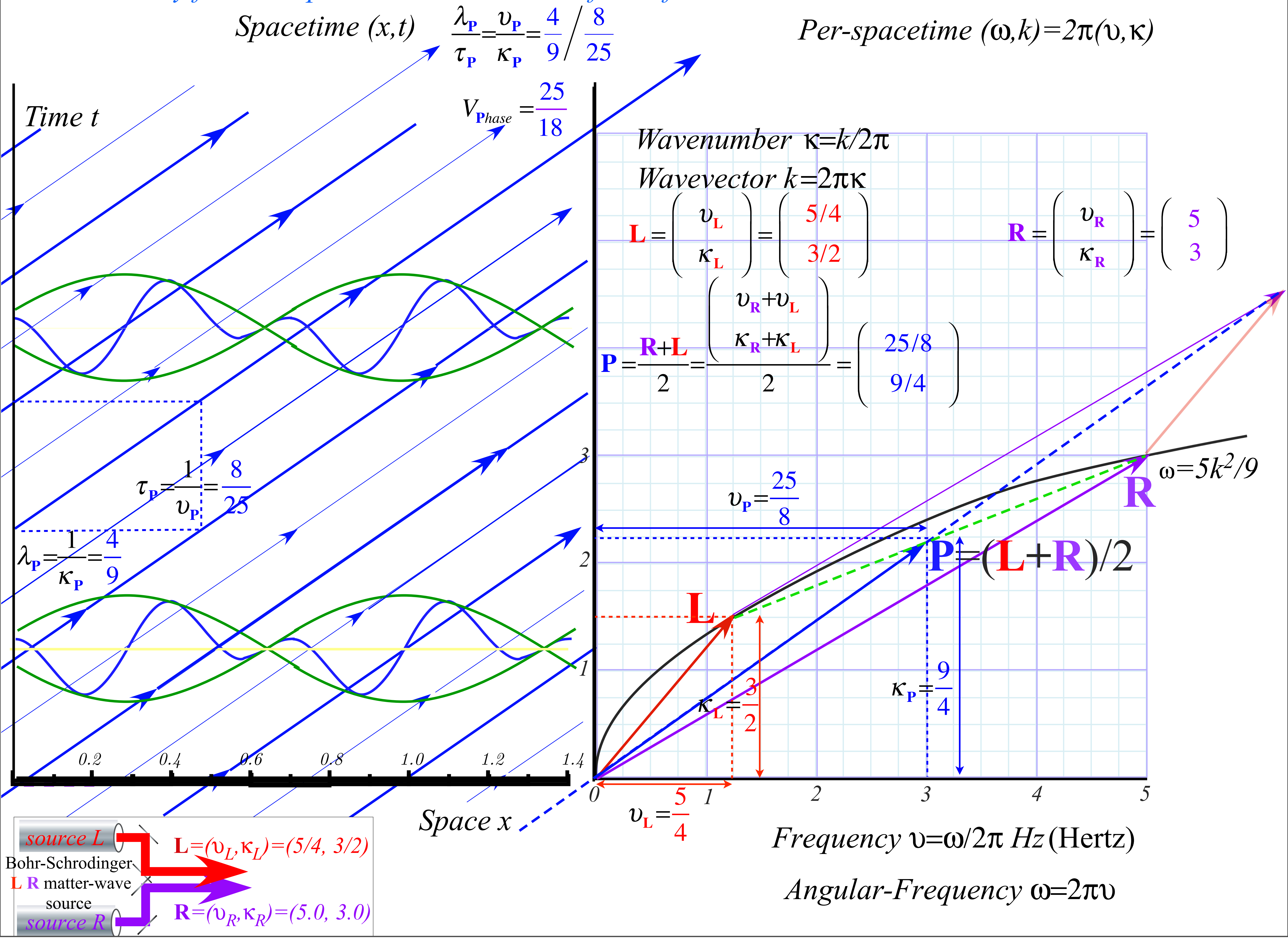
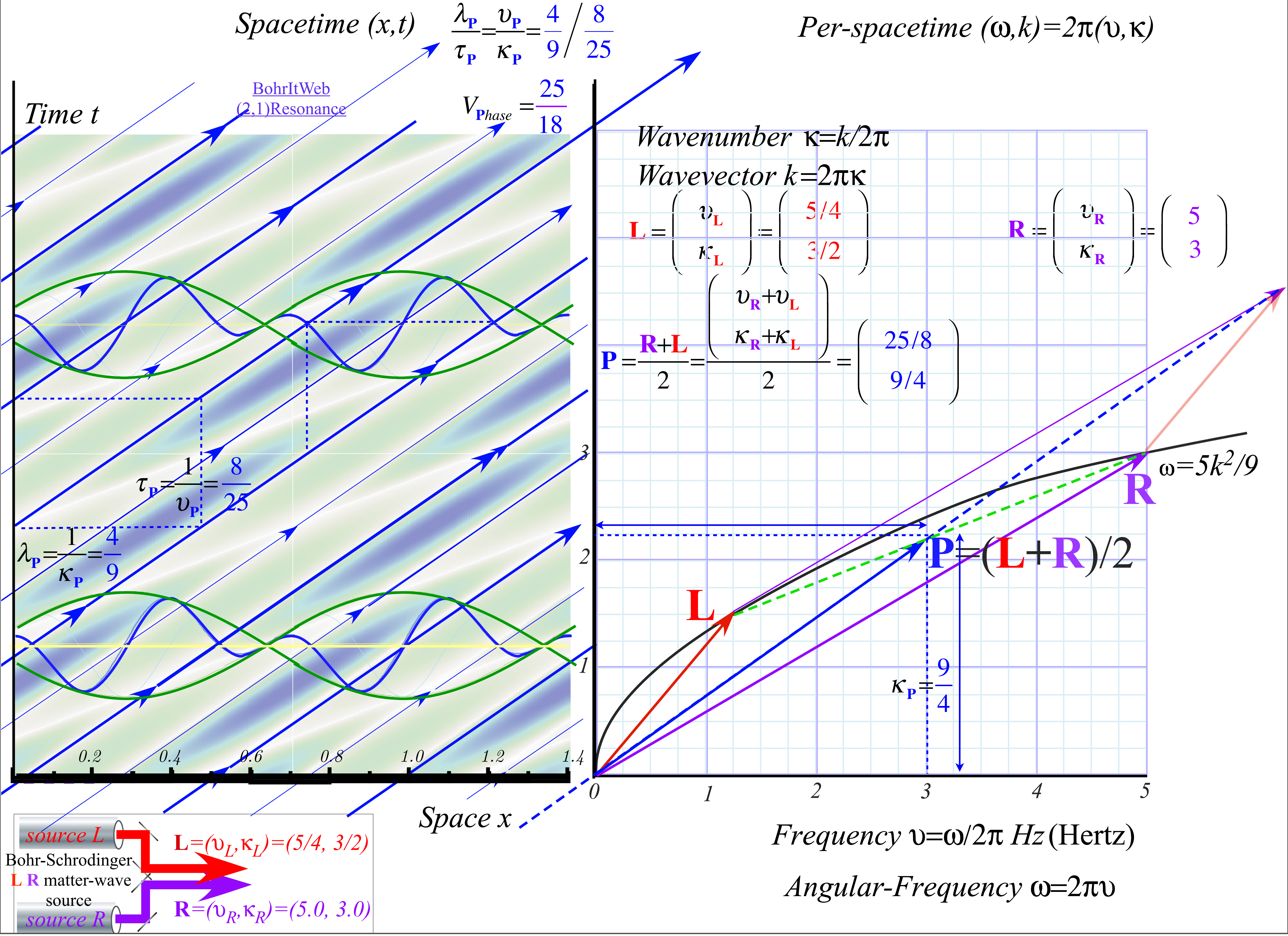
Phase velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: Half-sum factor $S=e^{i(\mathbf{L}+\mathbf{R})/2}$



Phase velocity for wave pair $e^{iL}+e^{iR}=S\cdot D$: Half-sum factor $S=e^{i(L+R)/2}$



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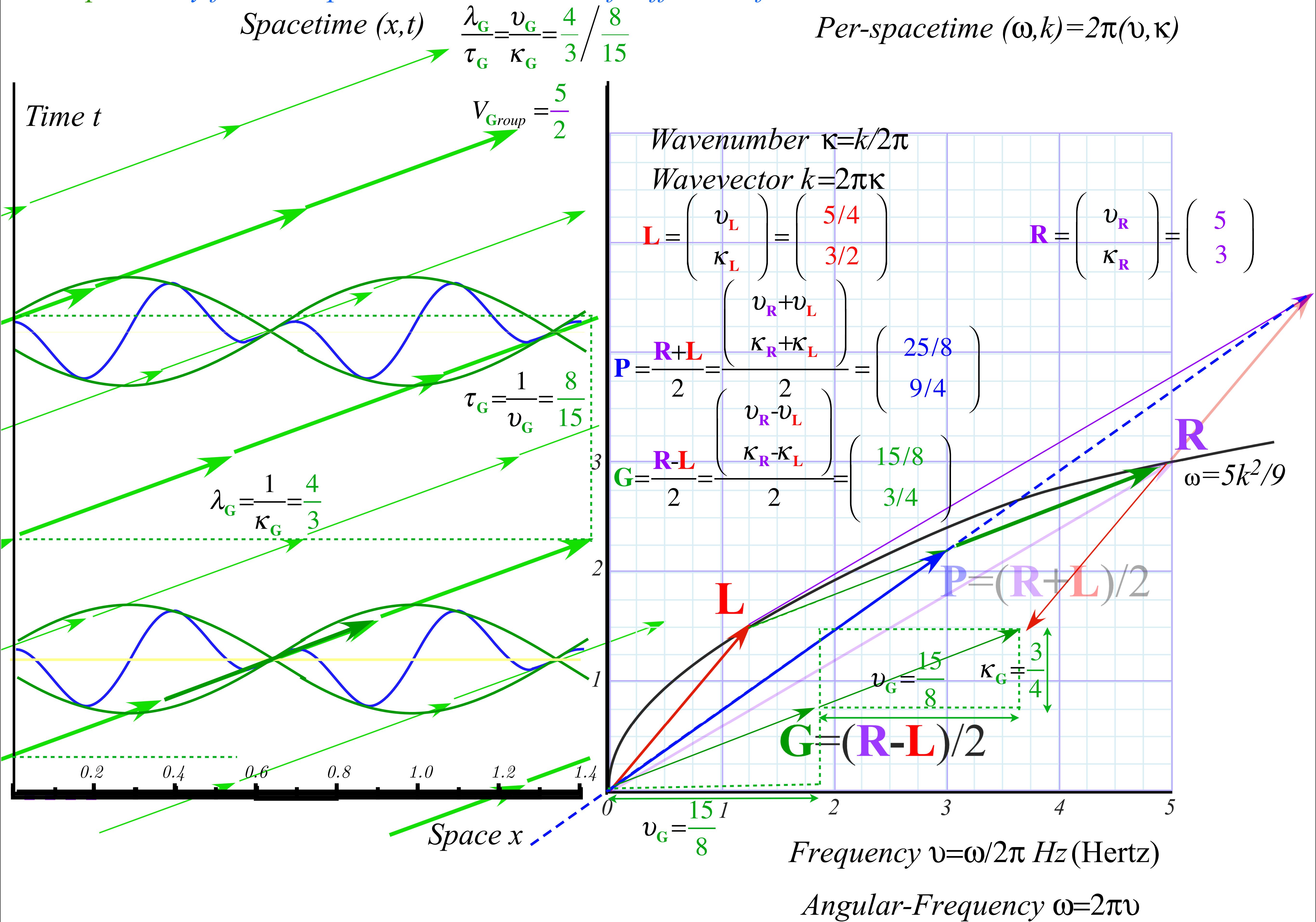
Group velocity for wave pair $e^{i\mathbf{L}}+e^{i\mathbf{R}}=S\cdot D$: Half-difference factor $D=e^{i(\mathbf{L}-\mathbf{R})/2}+e^{-i(\mathbf{L}-\mathbf{R})/2}$



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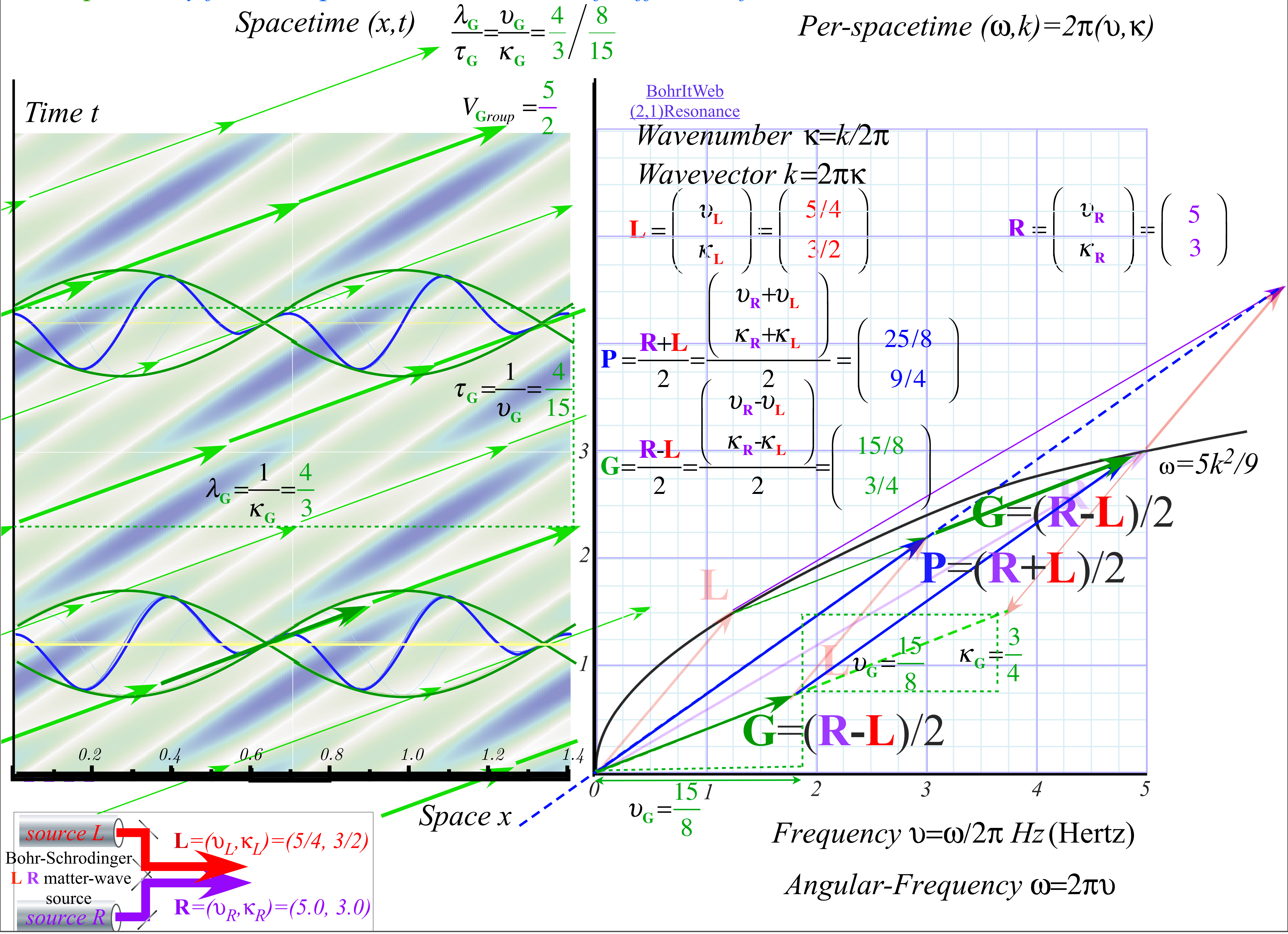

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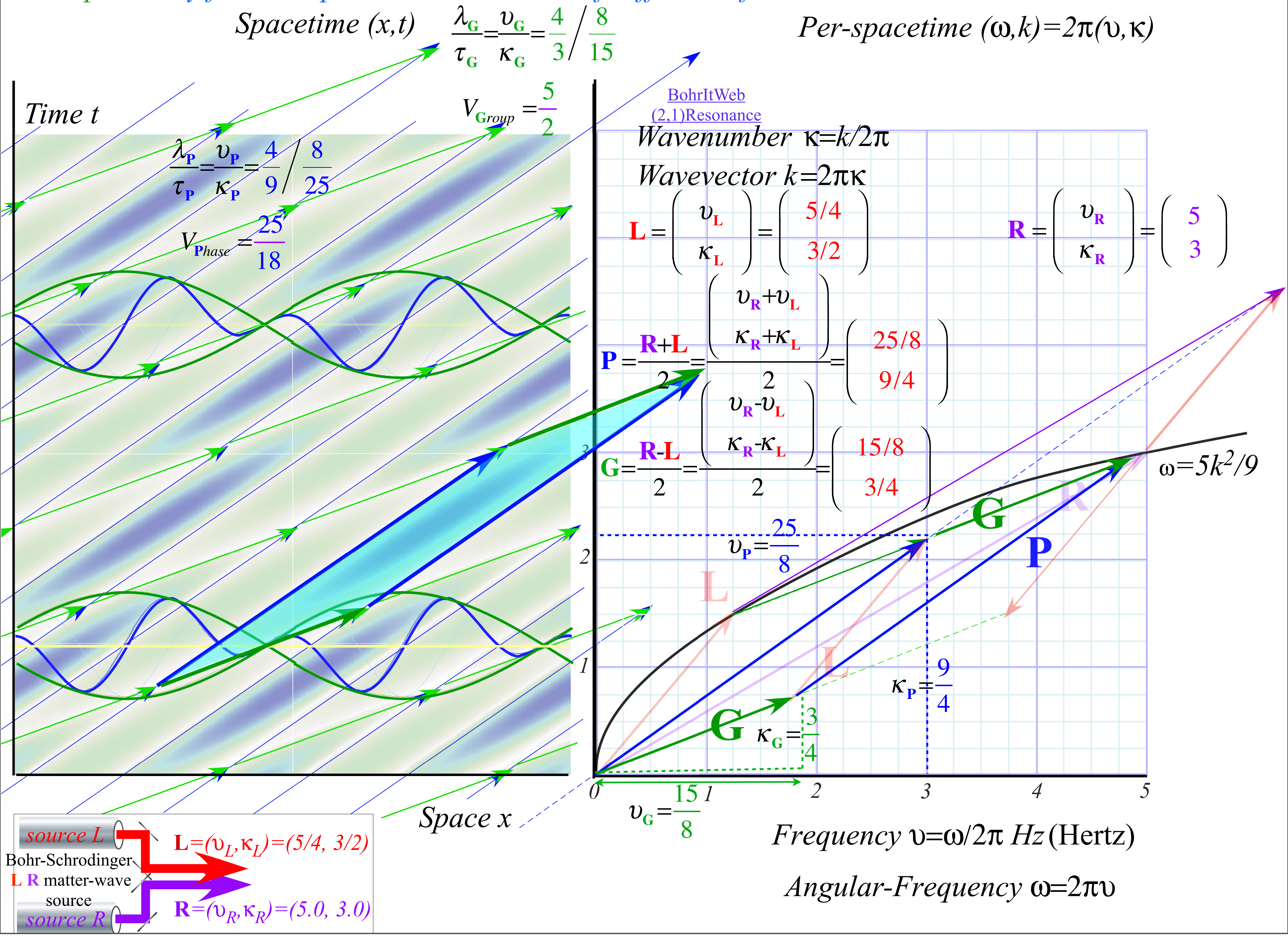

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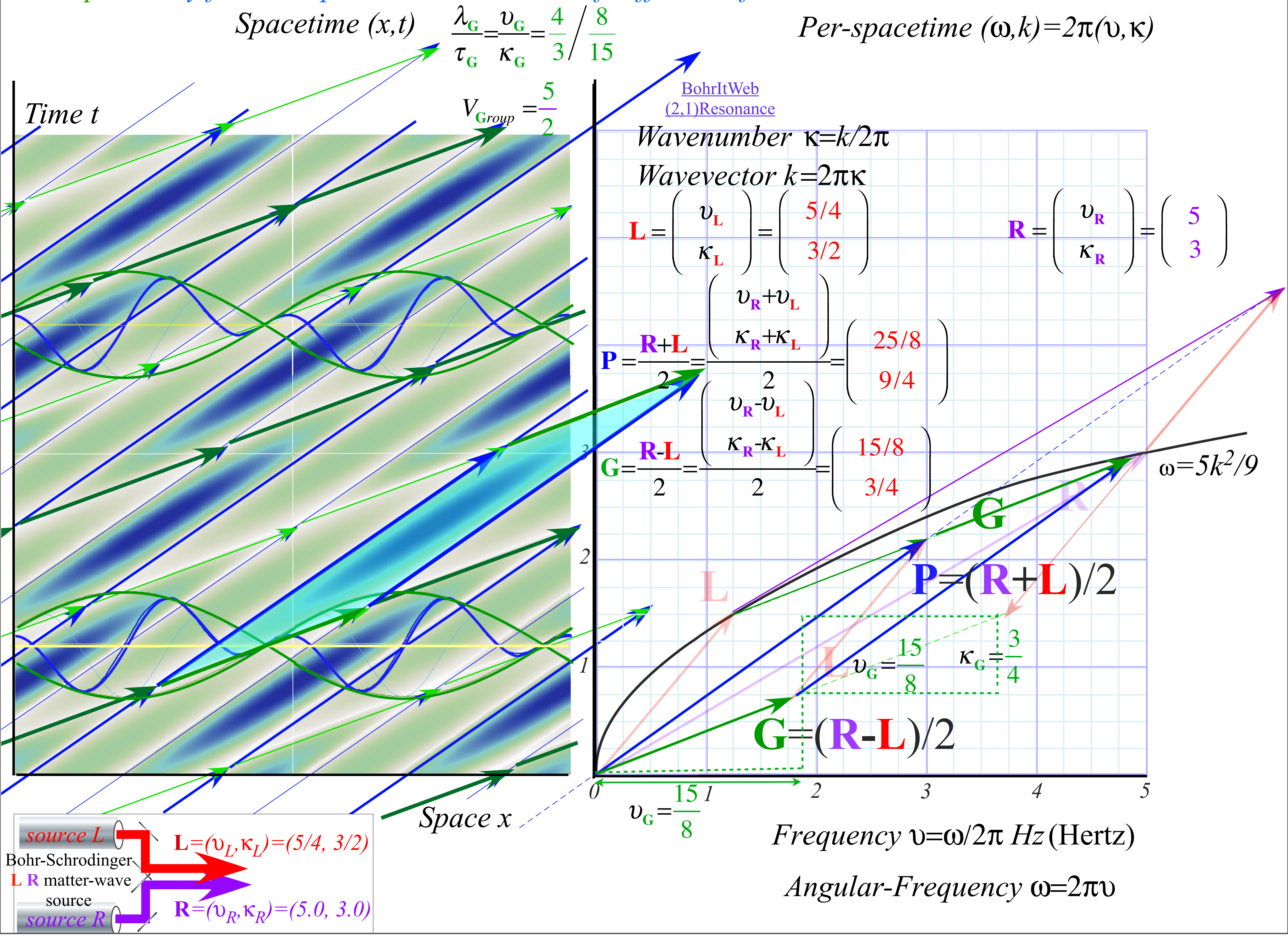
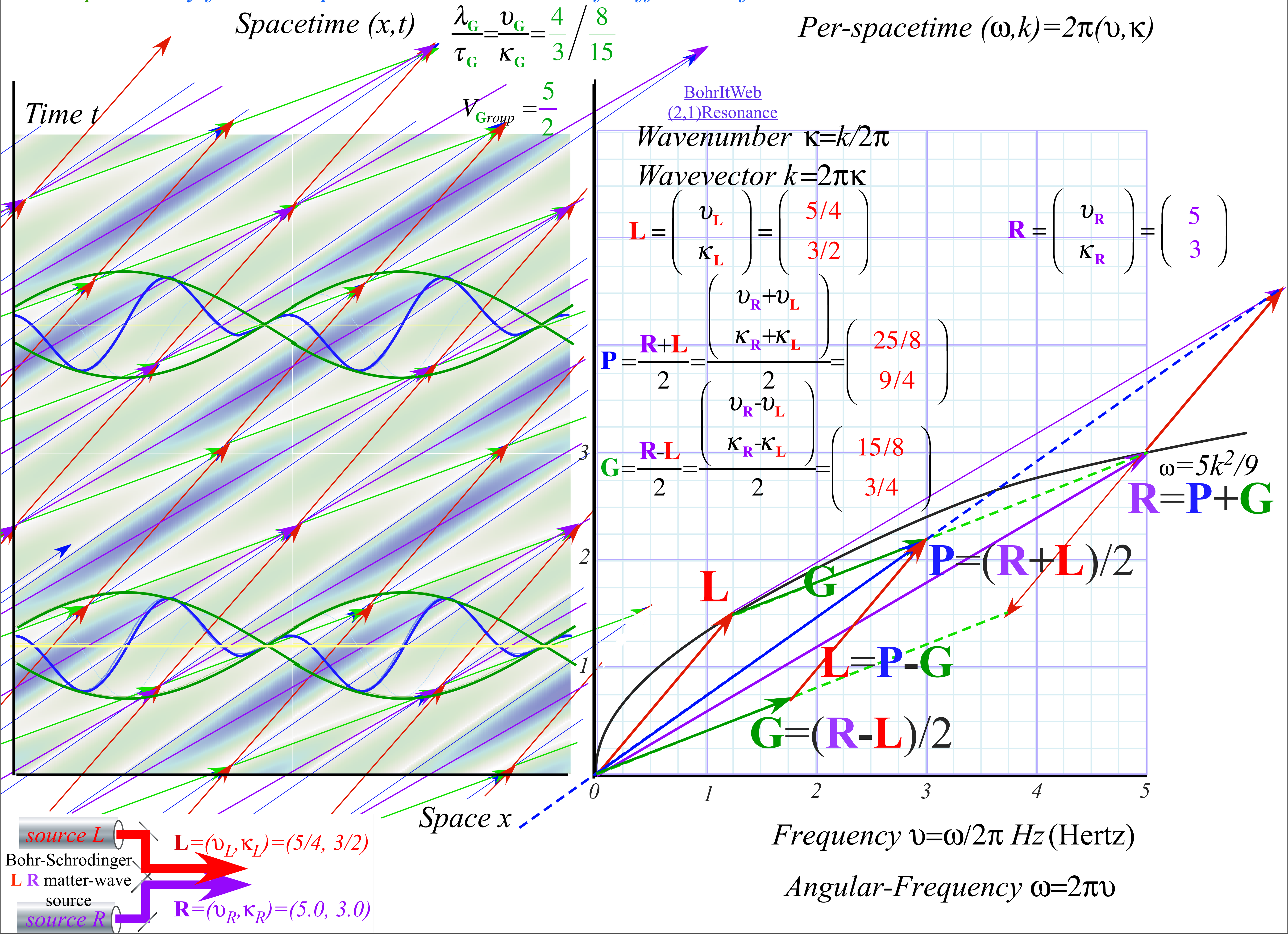


Group velocity for wave pair $e^{iL}+e^{iR}=S\cdot D$: Half-difference factor $D=e^{i(L-R)/2}+e^{-i(L-R)/2}$



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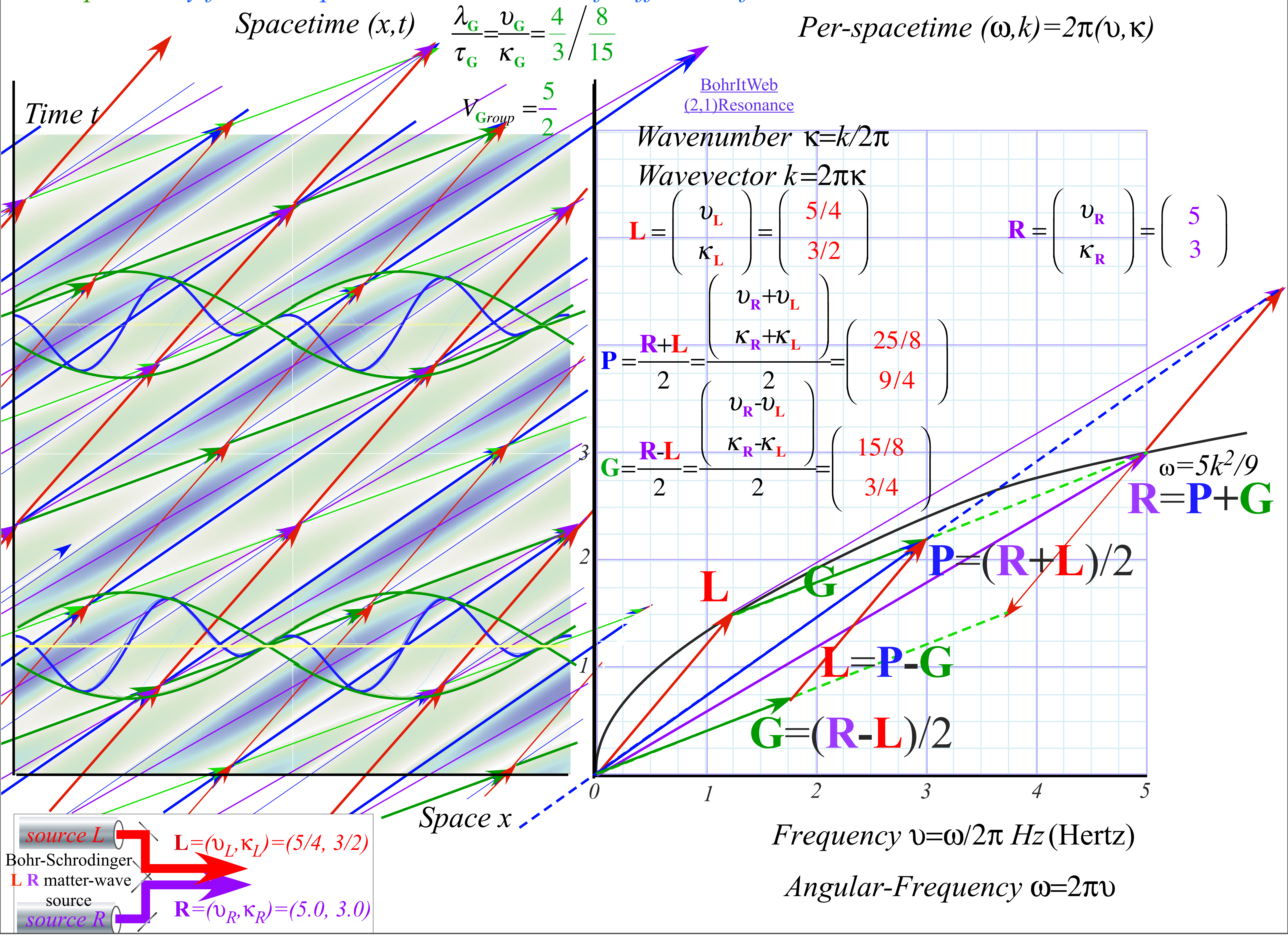


→ *Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R**′-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L**′-*vector and* **R**′-*vector in (***L**, **R***)-per-spacetime*

Vectors of phase **P**′*=(***R**′*+***L**′*)/2 and group* **G**′*=(***R**′*-***L**′*)/2*

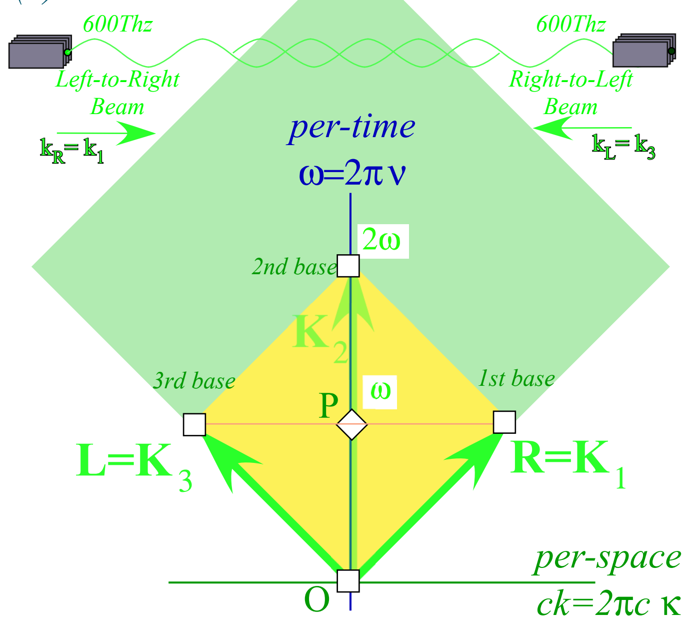
Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

Brief tour of and relativistic mechanics by geometry

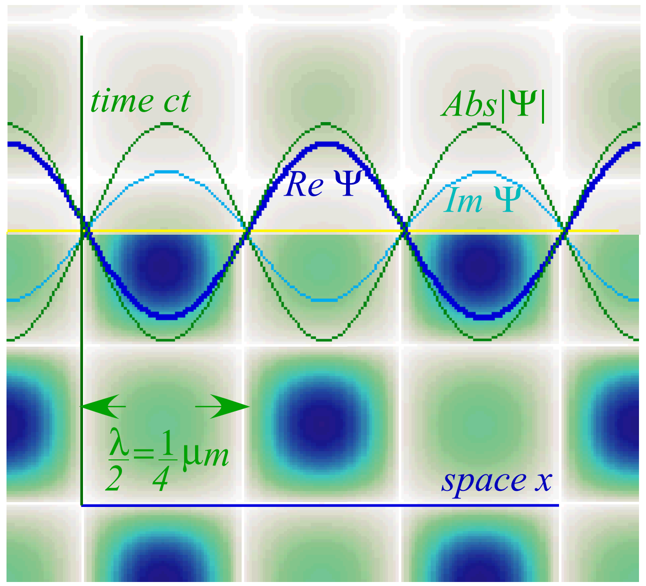
*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

u=0 space-time coordinates

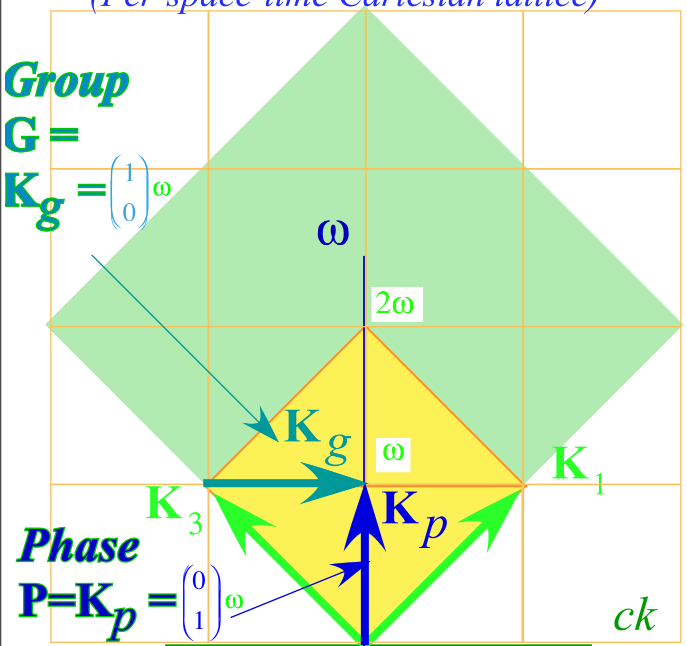
(a) Laser "Baseball Diamond"



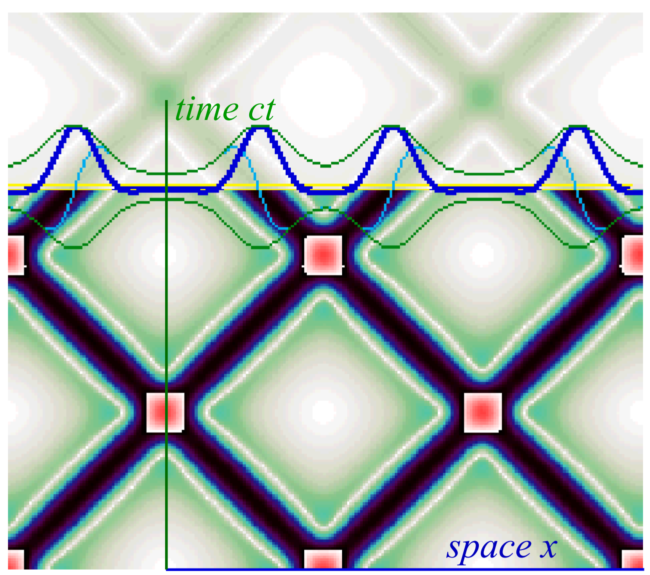
(c) Laser Coherent Wave (CW) paths
(Space-time Cartesian grid)



(b) Laser group and phase wavevectors
(Per-space-time Cartesian lattice)



(d) Laser Pulse Wave (PW) Paths
(Space-time Diamond grid)



Wave coordinates with Linear Dispersion

Continuous Wave (CW) coordinates discussed in following pages…

…starting with standing-wave case shown here

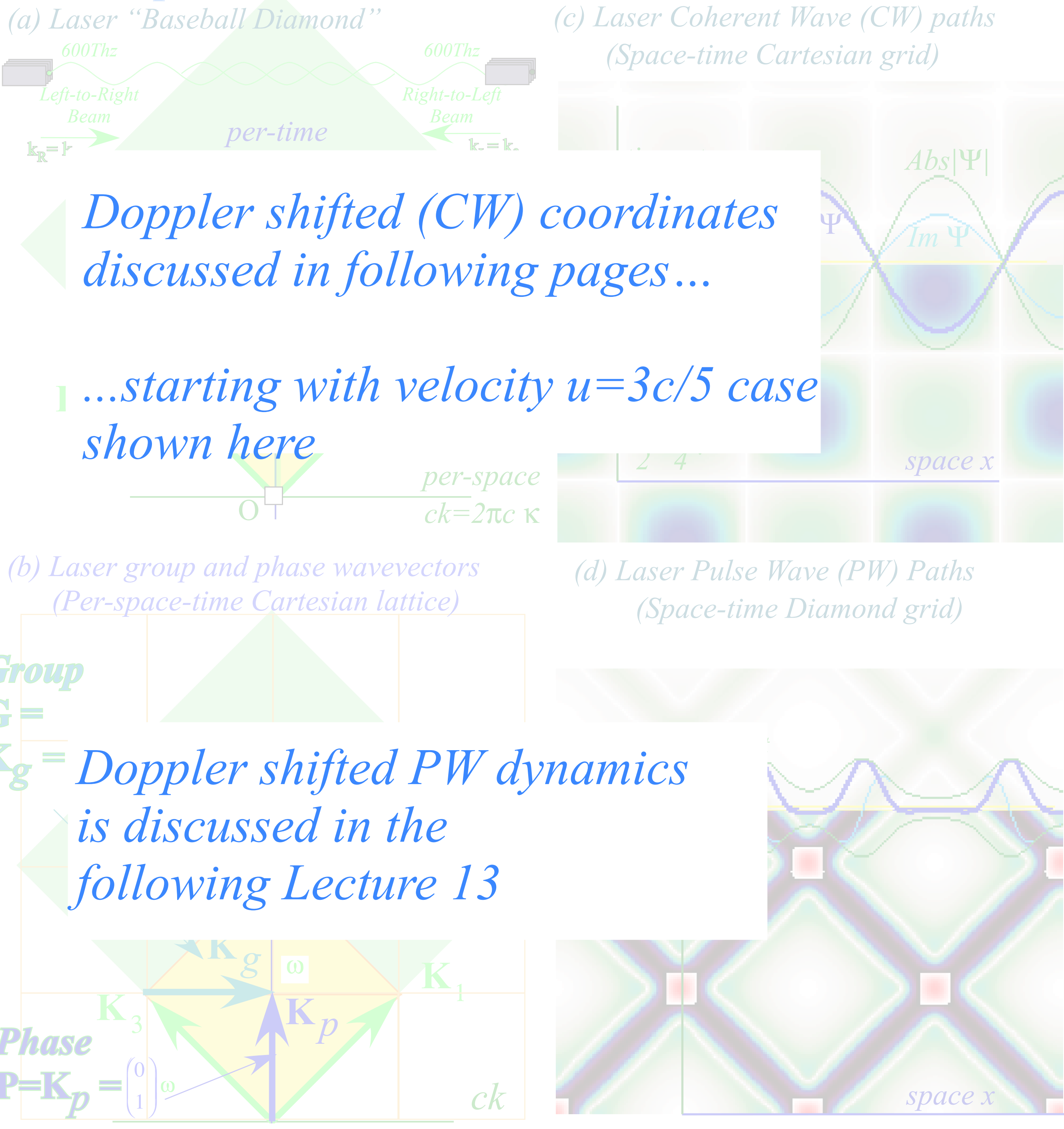
Pulse Wave (PW) coordinates ("Packet-Wave" or "particle-like") PW dynamics is discussed in the following Lecture 13

u=0 space-time pulse waves

CM with a BANG! Fig. 8.2.1 [*CM with a BANG! Unit 8*]

Introduction to wave coordinates by Left-moving and Right-moving laser beams

u=0 space-time coordinates



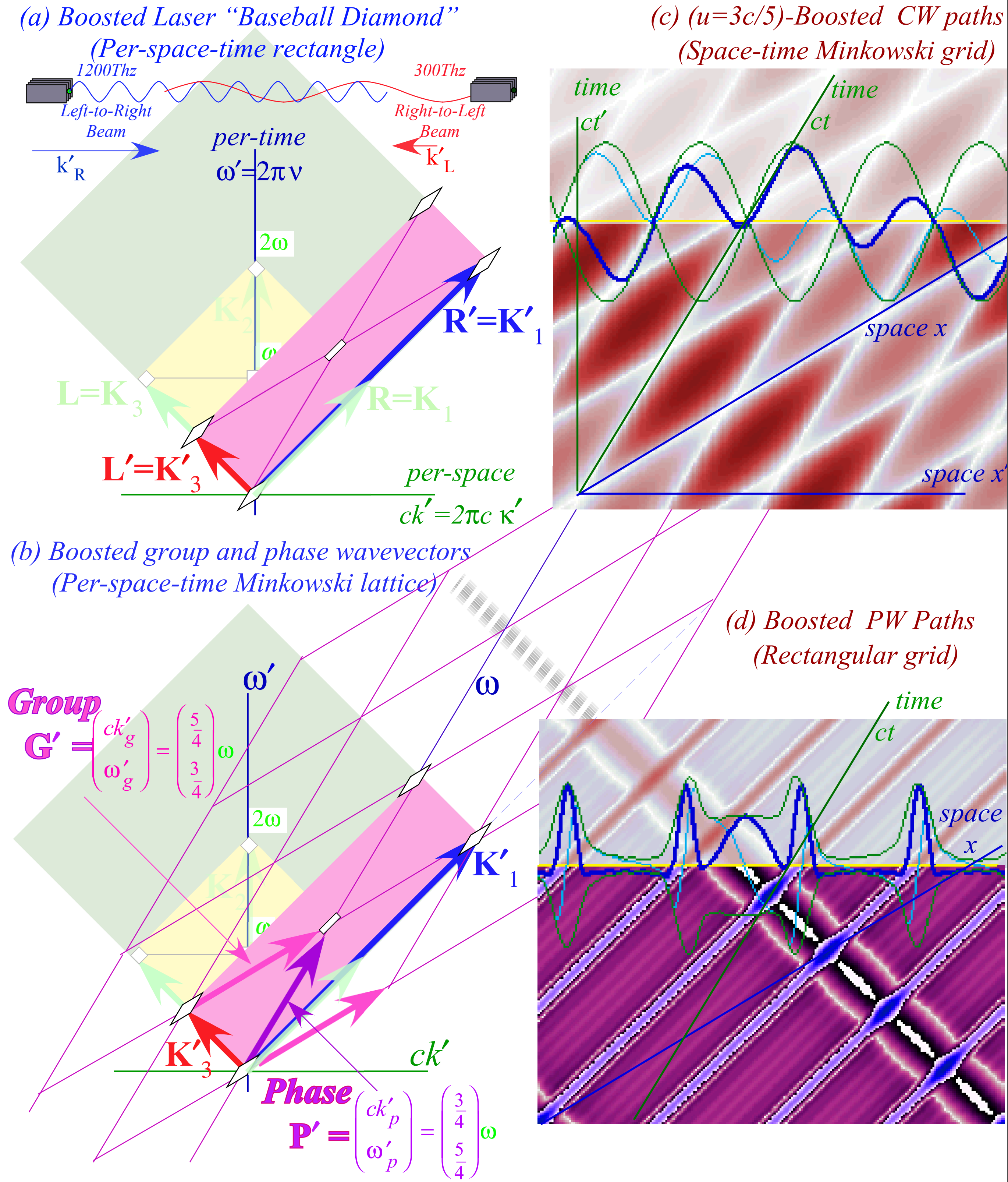
Doppler shifted (CW) coordinates discussed in following pages...

...starting with velocity u=3c/5 case shown here

Doppler shifted PW dynamics is discussed in the following Lecture 13

u=0 space-time pulse waves

Gives u=3c/5 Einstein-Lorentz-Minkowski coordinates



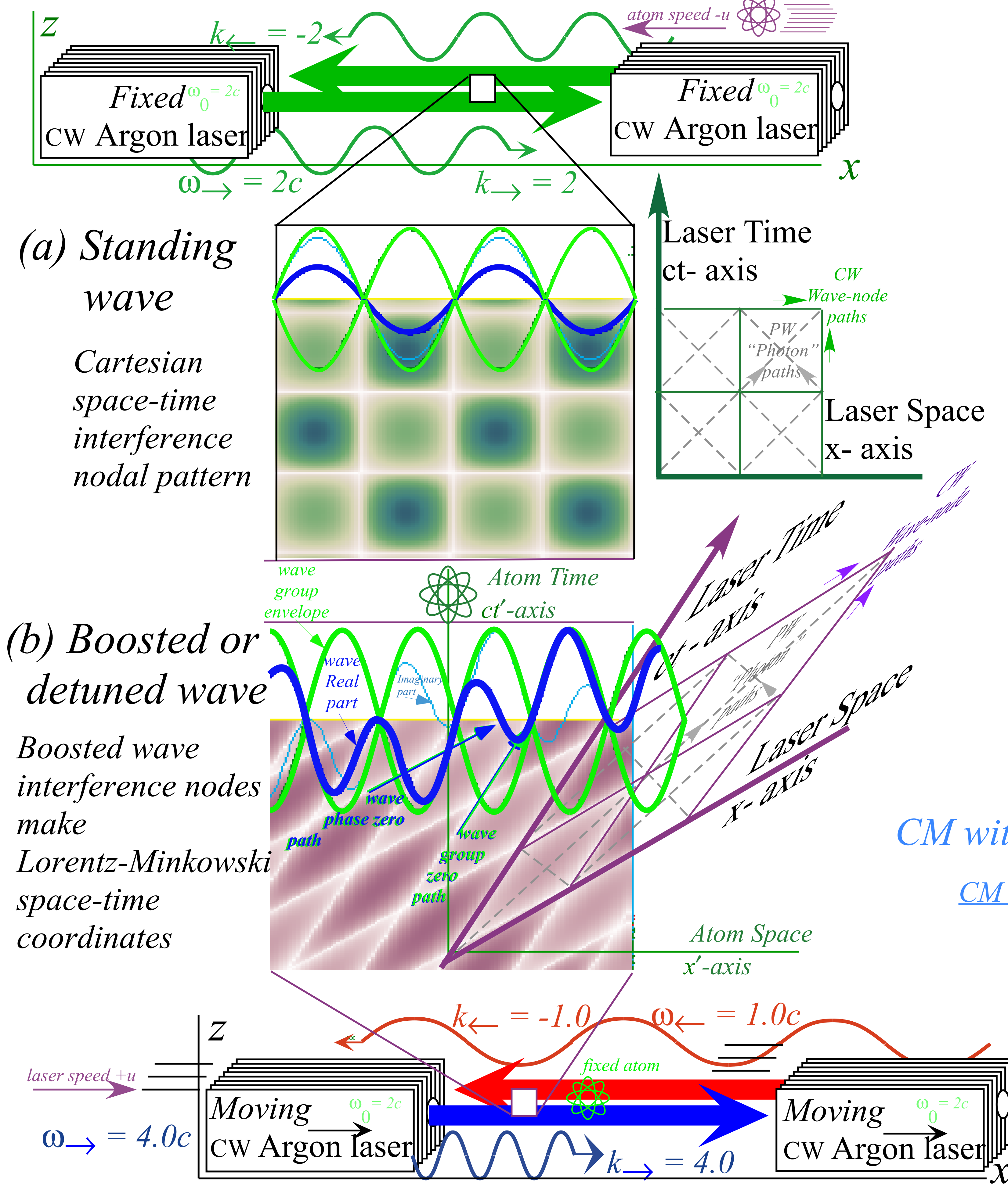
$$\text{Group } \mathbf{G}' = \begin{pmatrix} ck'_g \\ \omega'_g \end{pmatrix} = \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix} \omega$$

$$\text{Phase } \mathbf{P}' = \begin{pmatrix} ck'_p \\ \omega'_p \end{pmatrix} = \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix} \omega$$

u=3c/5 space-time pulse waves

CM with a BANG! Fig. 8.2.1 CM with a BANG! Unit 8 CM with a BANG! Fig. 8.2.2

*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*



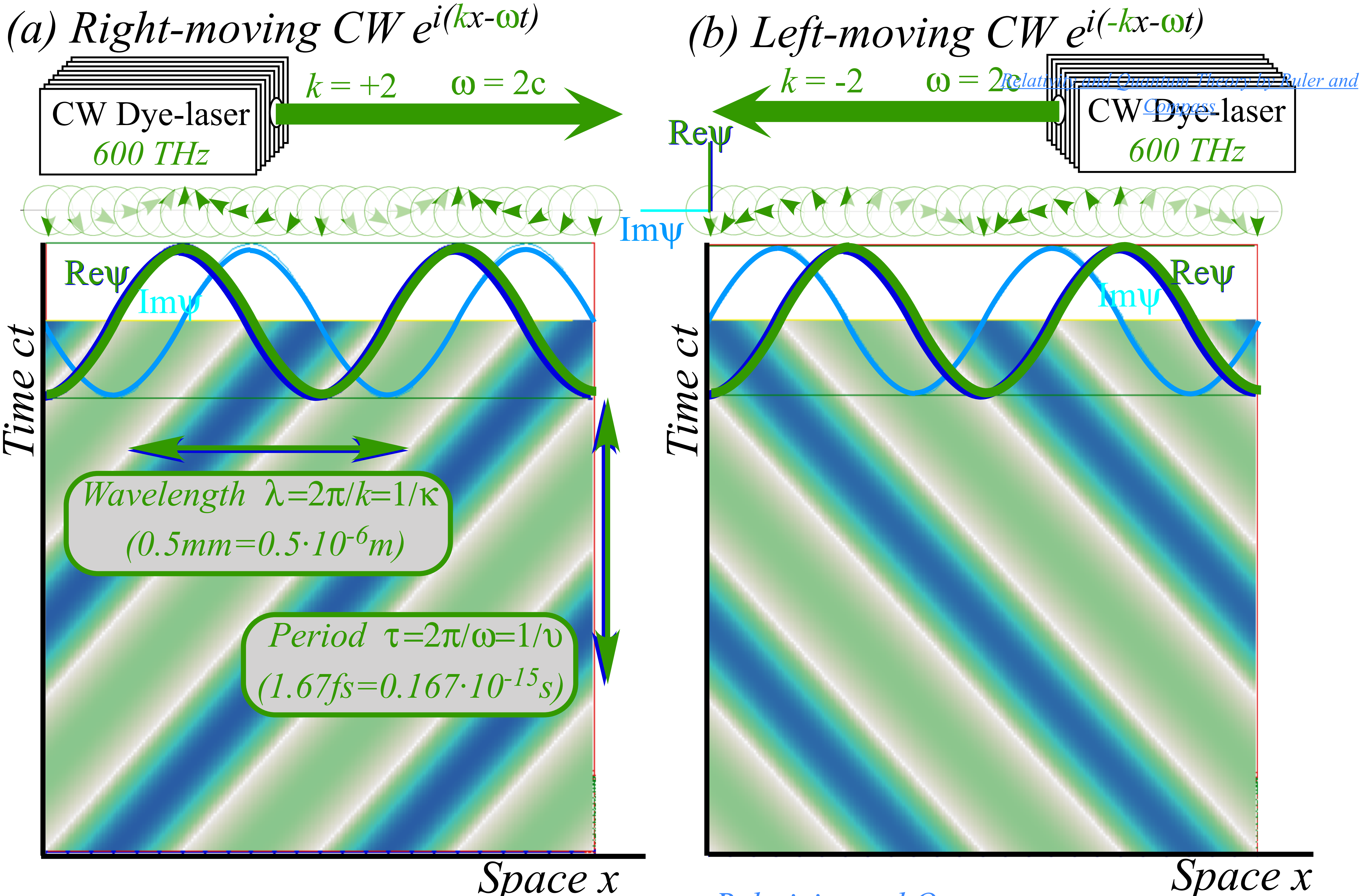
CM with a BANG! Fig. 8.2.3

CM with a BANG! Unit 8

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

- **L***-laser 600THz and* **R***-laser 600THZ (Laser lab frame)*
  - *Phase* **P***-vector and group* **G***-vector span Cartesian spacetime coordinates*
- **L′***-laser 300THz and* **R′***-laser 1200THZ (Doppler shifted in moving frame)*
  - *Doppler shifted* **L′***-vector and* **R′***-vector in (***L**, **R***)-per-spacetime*
  - *Vectors of phase* **P′***=(***R′***+***L′***)/2 and group* **G′***=(***R′***-***L′***)/2*
- *Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates*
  - *Brief tour of and relativistic mechanics by geometry*
- *Summary of optical wave parameters for relativity and QM*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*



Relativity and Quantum Theory by Ruler and Compass Fig. 5

Introduction to wave coordinates by **L***eft-moving and* **R***ight-moving laser beams*

L*-laser 600THz and* **R***-laser 600THZ (Laser lab frame)*

➡ *Phase* **P***-vector and group* **G***-vector span Cartesian spacetime coordinates*

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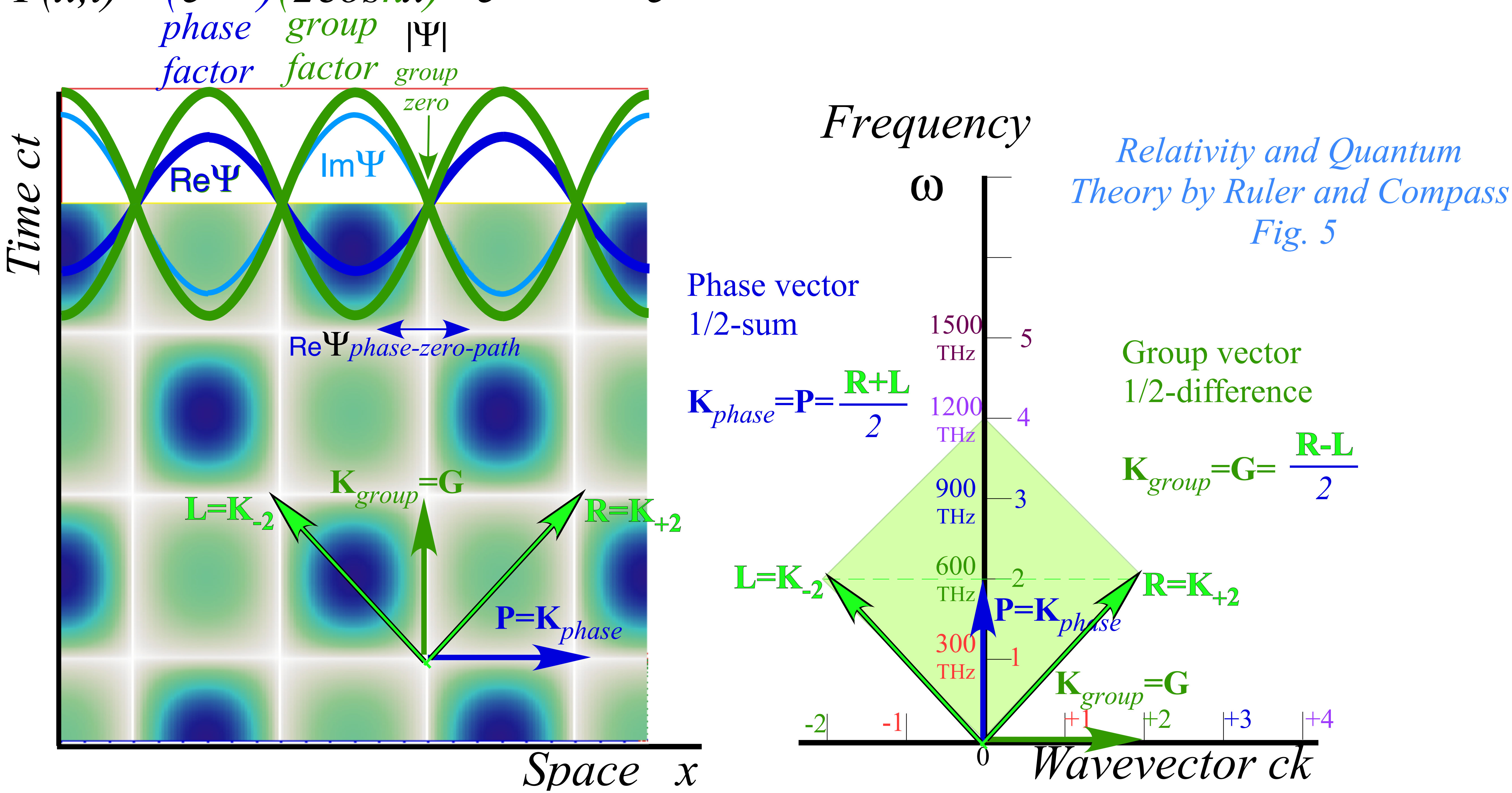
Relativity and Quantum Theory by Ruler and Compass

(c) Standing CW in space-time

$\Psi(x,t)=(e^{-i\omega t})(2\cos kx)=e^{i(kx-\omega t)}+e^{i(-kx-\omega t)}$

phase factor *group factor*

(d) Dispersion plot in per-space-time



*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

__L__-laser 600THz and __R__-laser 600THZ (Laser lab frame)

Phase __P__-vector and group __G__-vector span Cartesian spacetime coordinates

→ *__L′__-laser 300THz and __R′__-laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted __L′__-vector and __R′__-vector in (__L__, __R__)-per-spacetime

Vectors of phase __P′__=(__R′__+__L′__)/2 and group __G′__=(__R′__-__L′__)/2

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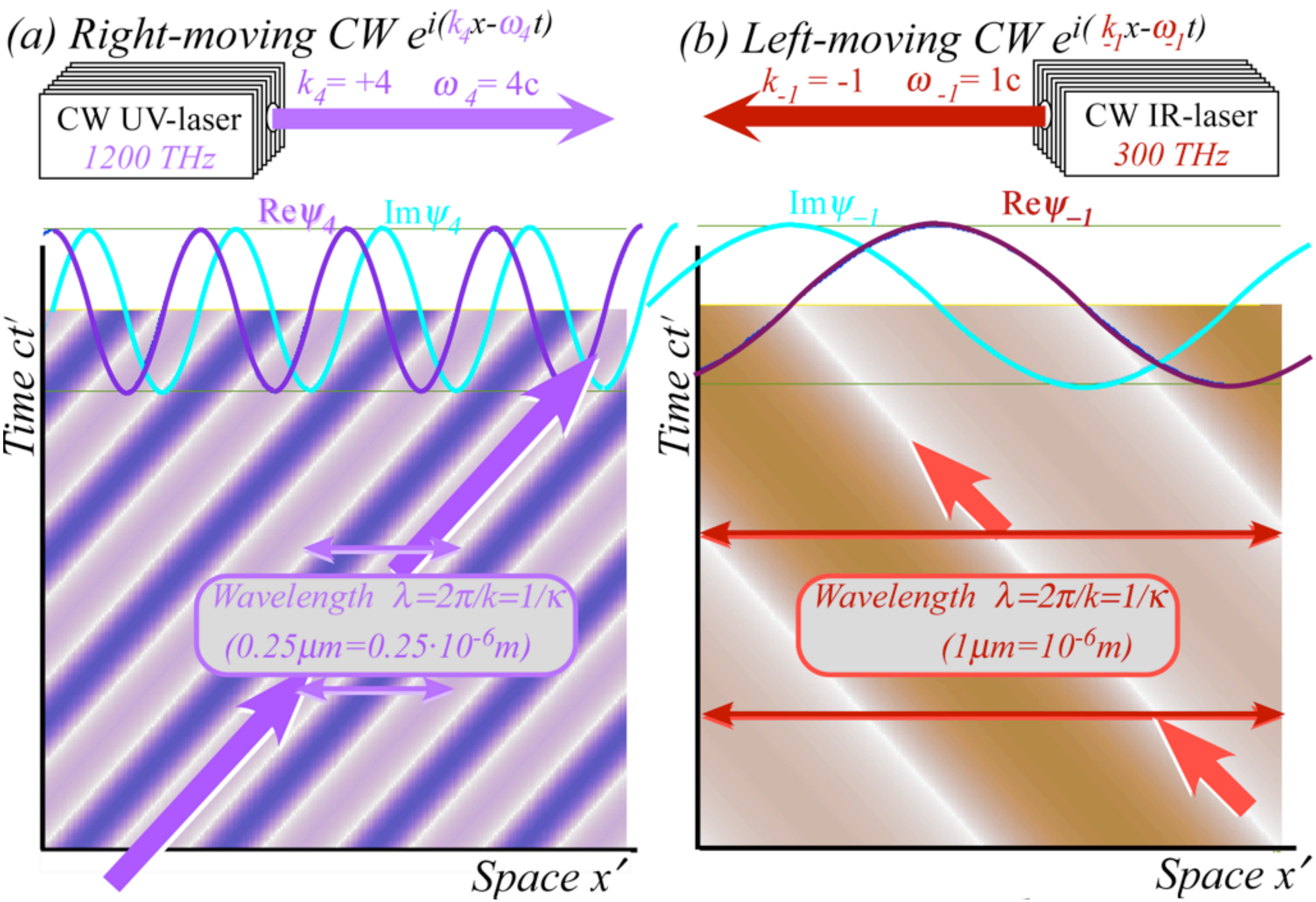
L′-laser 300THz and R′-laser 1200THZ (Doppler shifted in moving frame)



Relativity and Quantum Theory by Ruler and Compass Fig. 5

Relativity and Quantum Theory by Ruler and Compass

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Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

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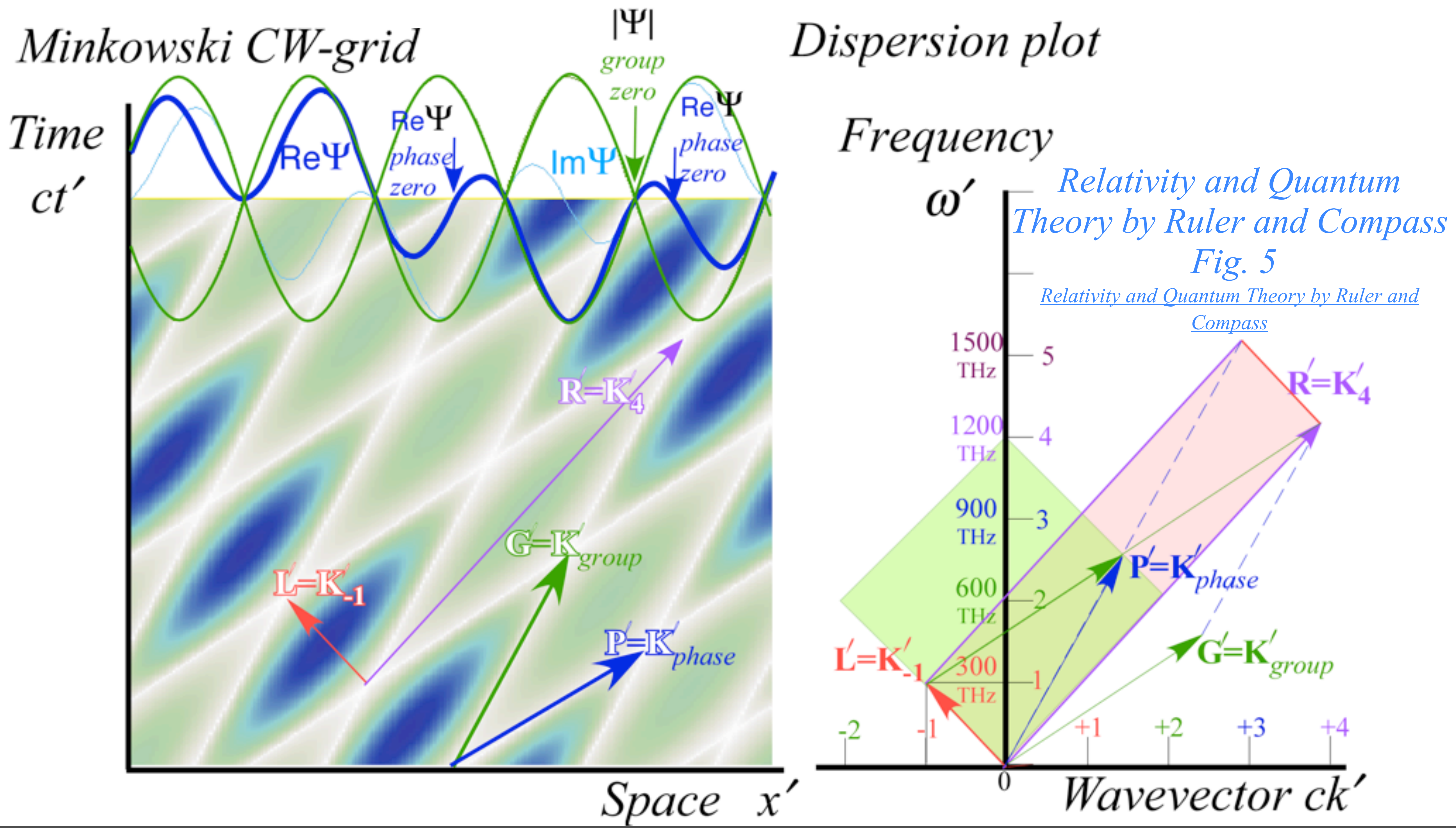
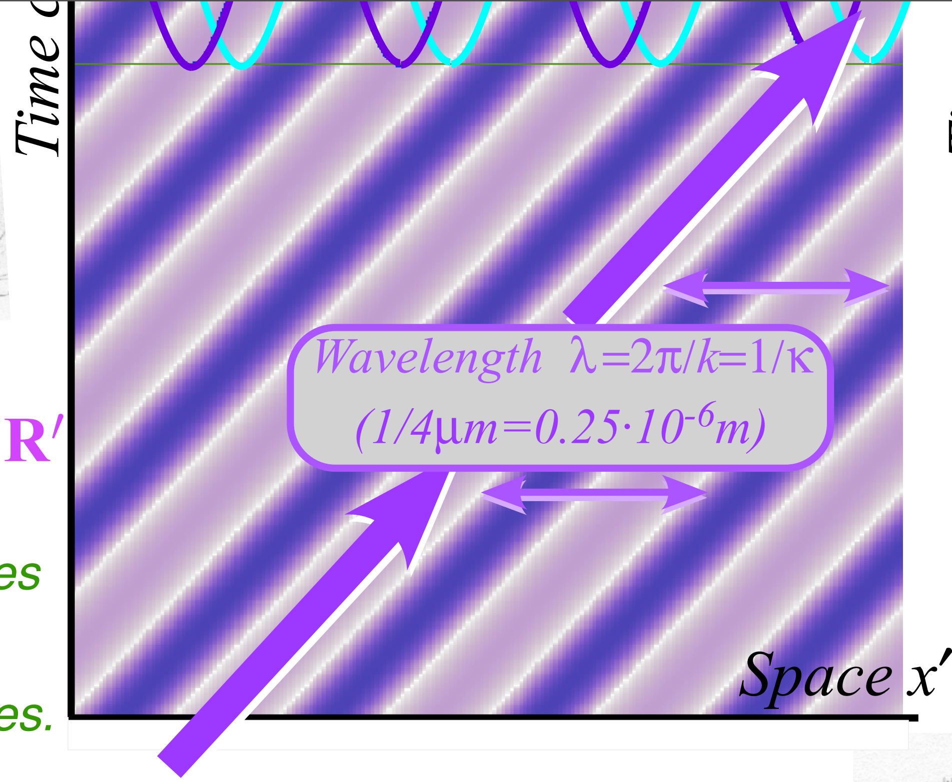
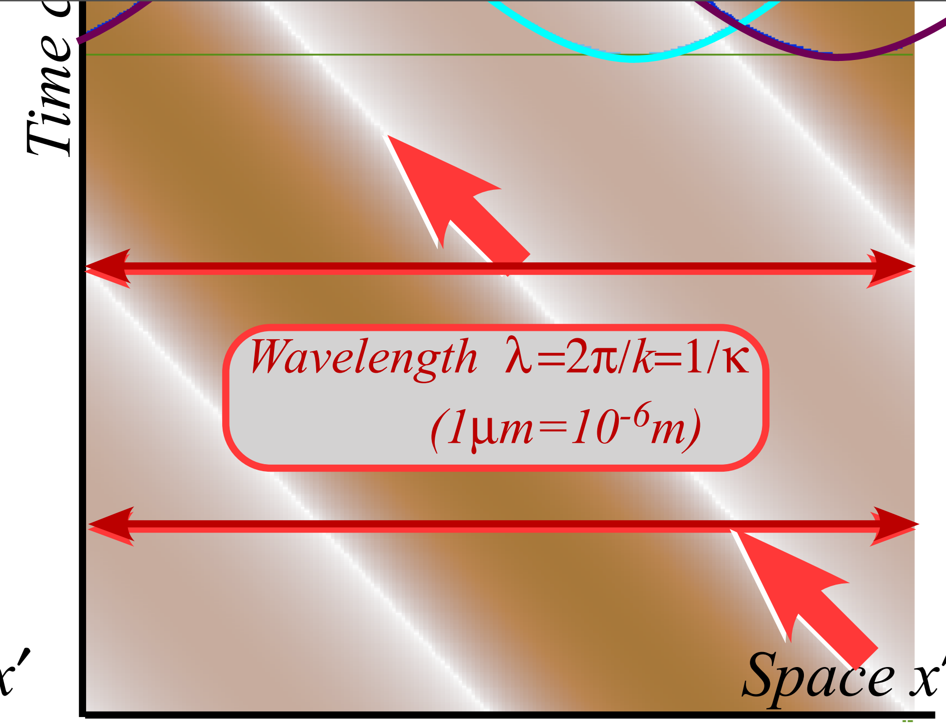
Summary of optical wave parameters for relativity and QM

Vectors of phase $\mathbf{P'}=(\mathbf{R'}+\mathbf{L'})/2$ *and group* $\mathbf{G'}=(\mathbf{R'}-\mathbf{L'})/2$

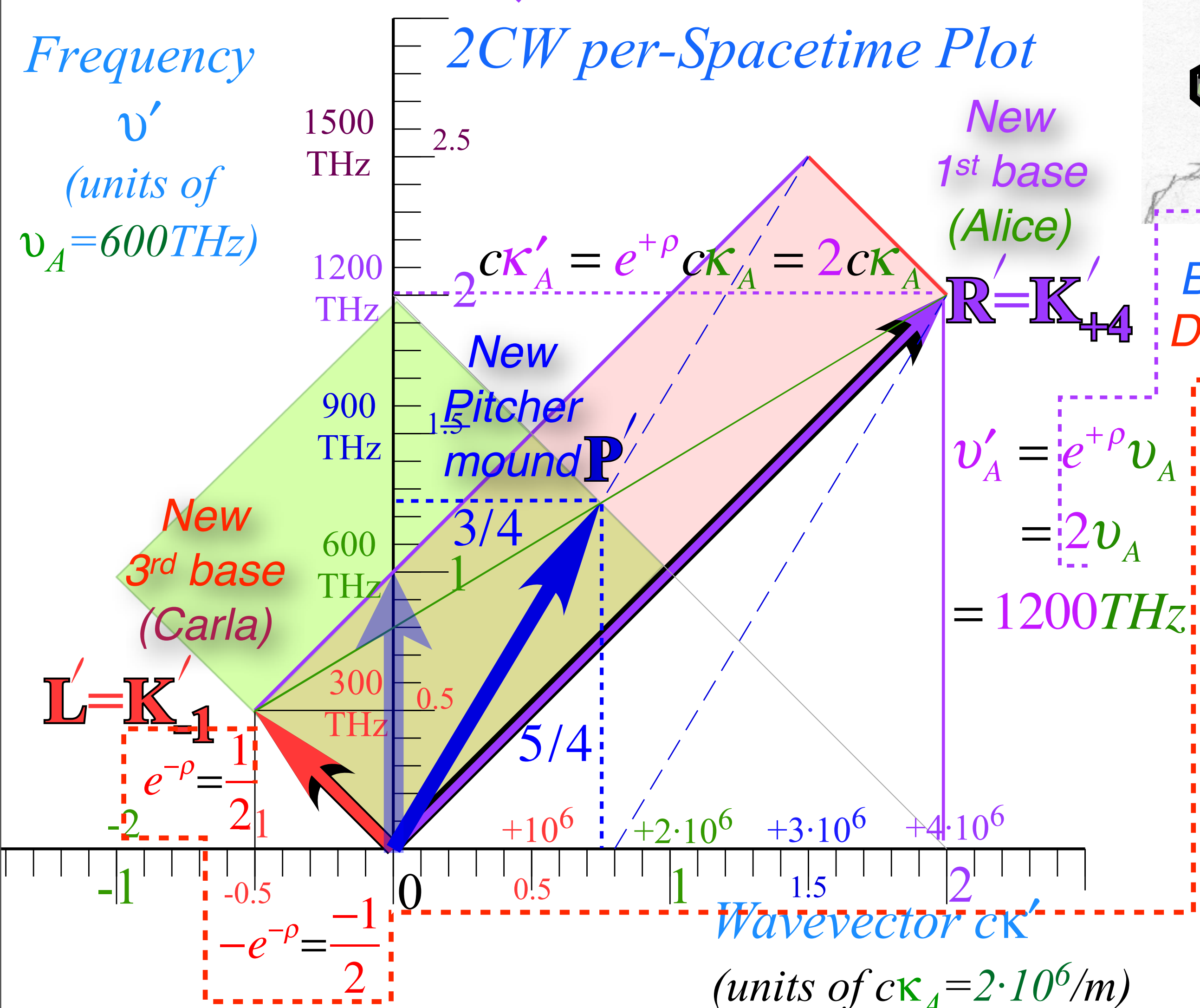


Alice: OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You'll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.



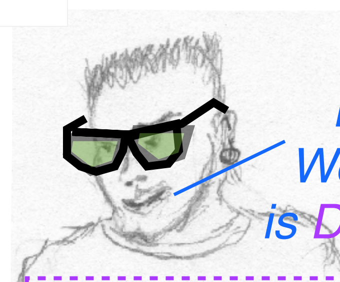


Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer!
(and half as long.)



$$\upsilon'_A = e^{+\rho}\upsilon_A = 2\upsilon_A = 1200THz$$

$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

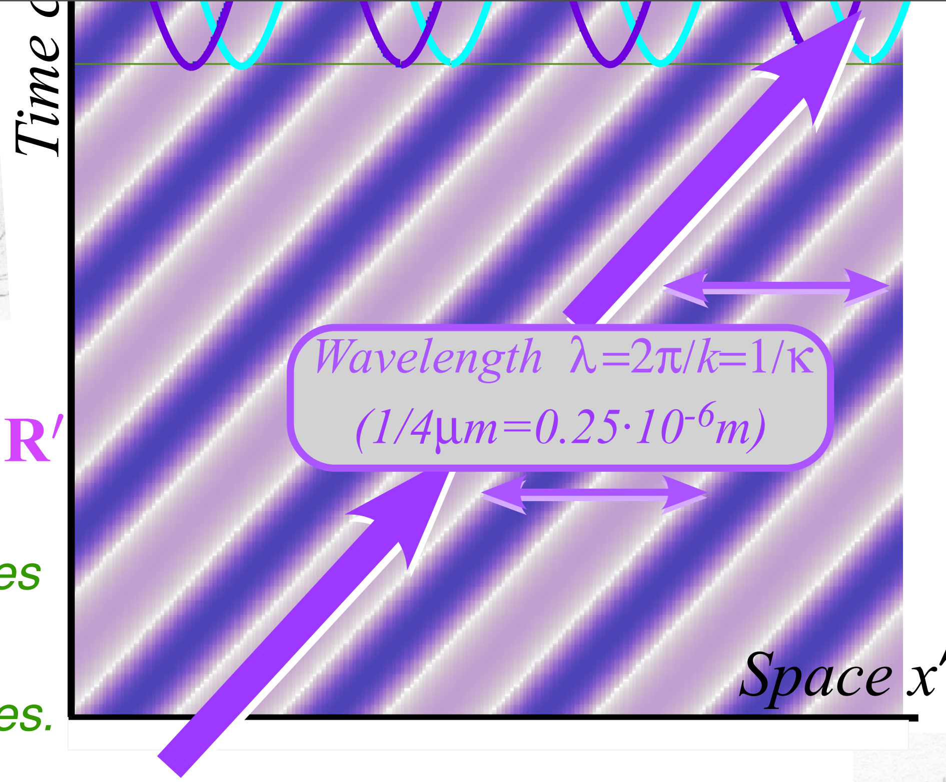
But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.

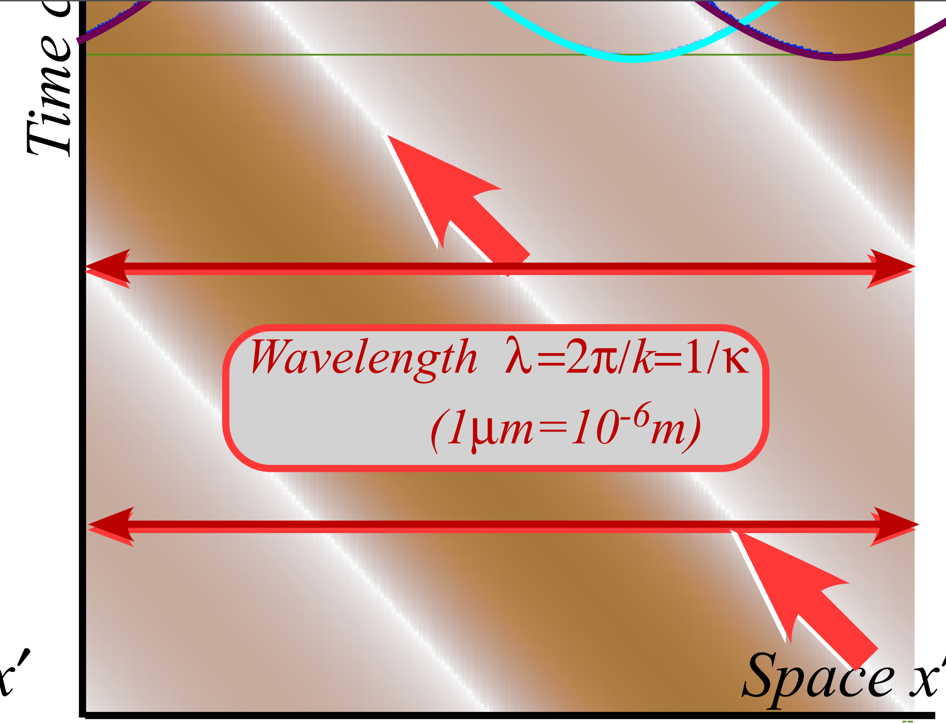
$$\mathbf{K}'_{phase} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2}$$

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

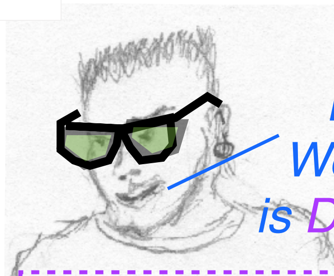
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} 2 \\ 2 \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -1/2 \\ +1/2 \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{2-1/2}{2} \\ \frac{2+1/2}{2} \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Alice: OK.
My UV 1200THz $\mathbf{R}'$
vector is fierce!
You"ll need glasses
to see $\mathbf{P}'$ and $\mathbf{G}'$
lines or coordinates.



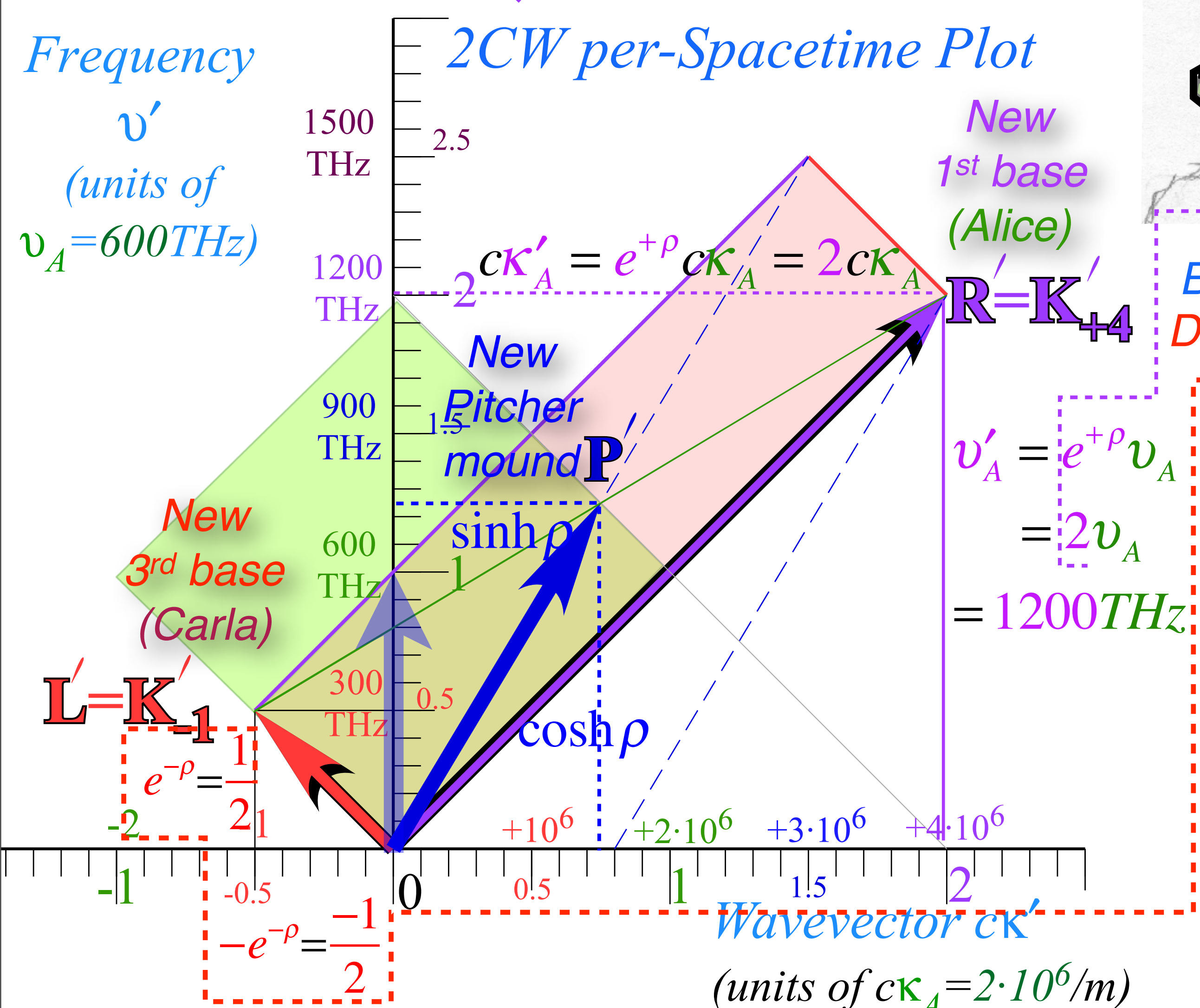


Carla: My
UV 300THz $\mathbf{L}'$
3rd baseline
is a lot nicer!
(and half as long.)



Bob: Sunglasses help.
Wow! Your 1st baseline $\mathbf{R}'$
is Doppler blued up by $e^{+\rho}=2$.

$\mathbf{K}'_{phase}=\mathbf{P}'=\dfrac{\mathbf{R}'+\mathbf{L}'}{2}$



$e^{-\rho}=\dfrac{1}{2}$

$-e^{-\rho}=\dfrac{-1}{2}$

$\upsilon'_A = e^{+\rho}\upsilon_A$
$= 2\upsilon_A$
$= 1200THz$

$\upsilon'_C = e^{-\rho}\upsilon_A = \dfrac{1}{2}\upsilon_A = 300THz$

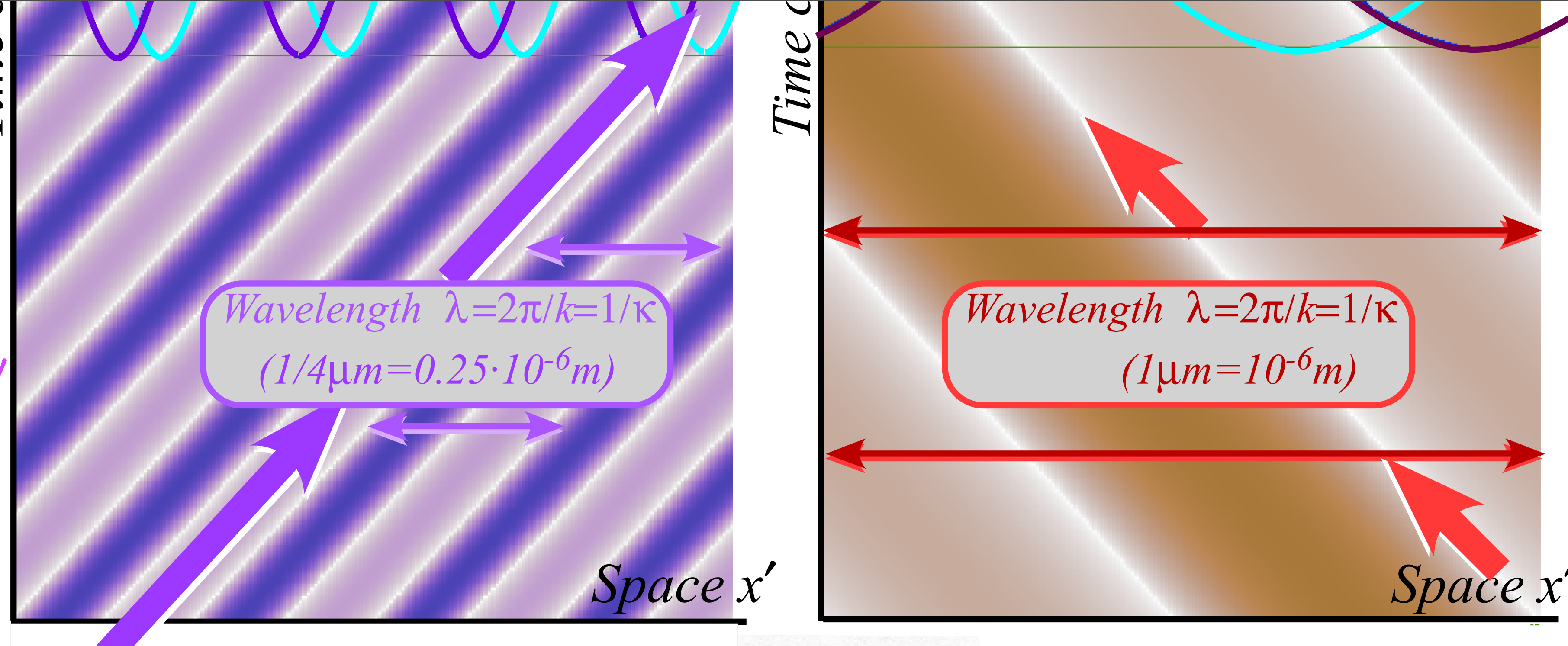
But, Carla's 3rd baseline $\mathbf{L}'$ is
Doppler red shifted by $e^{-\rho}=1/2$.

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.)
is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

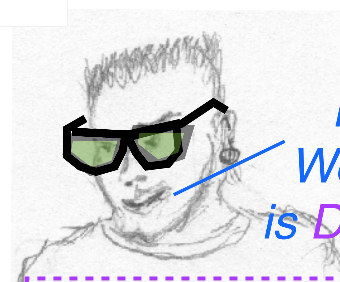
$$\begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho} \\ e^{+\rho} \end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho} \\ +e^{-\rho} \end{pmatrix} = \upsilon_A \begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2} \\ \frac{e^{+\rho}+e^{-\rho}}{2} \end{pmatrix}$$

$$= \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Alice:OK.
My UV 1200THz $\mathbf{R}'$ vector is fierce!
You"ll need glasses to see $\mathbf{P}'$ and $\mathbf{G}'$ lines or coordinates.

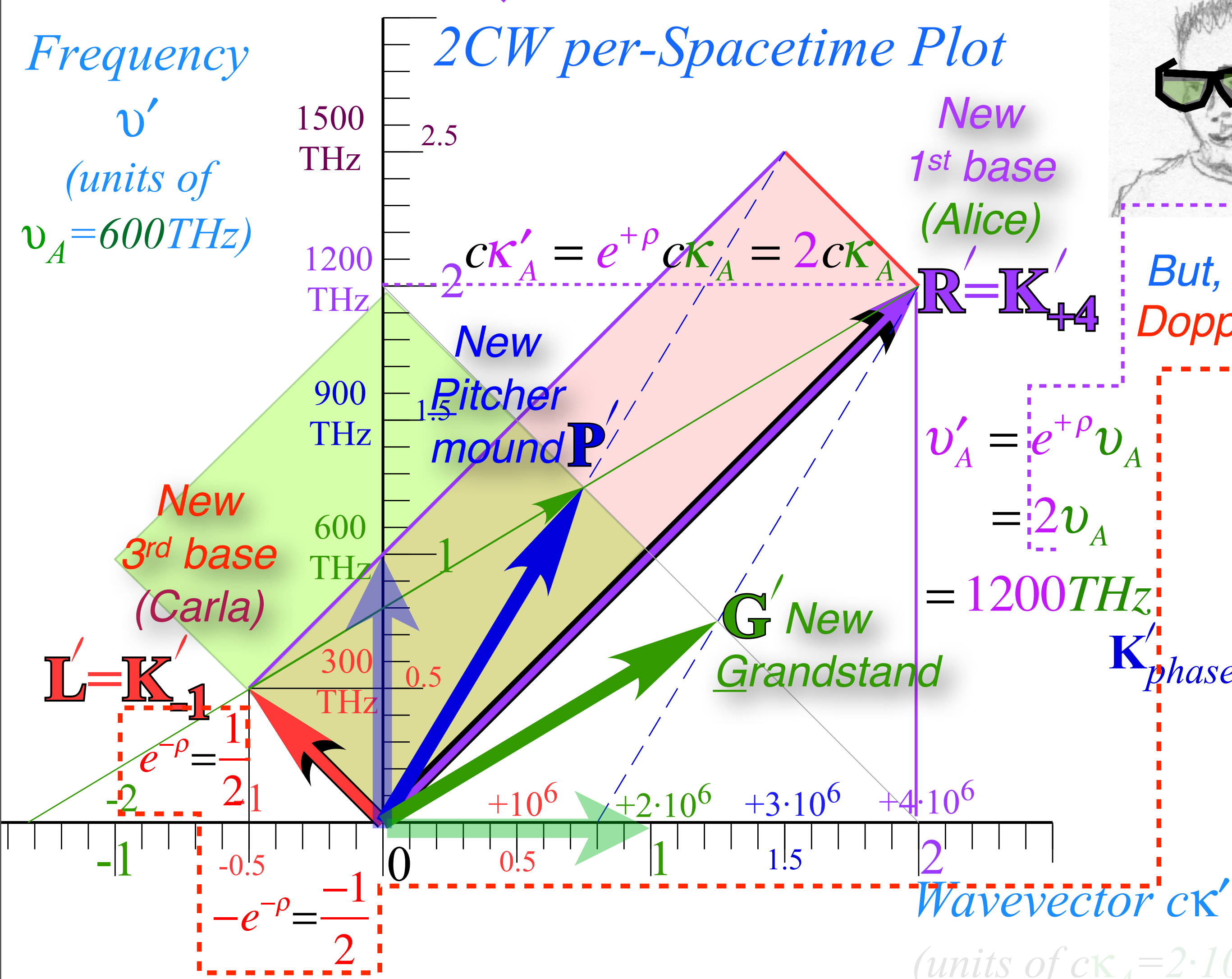


Carla: My UV 300THz $\mathbf{L}'$ 3rd baseline is a lot nicer! (and half as long.)



Bob: Sunglasses help. Wow! Your 1st baseline $\mathbf{R}'$ is Doppler blued up by $e^{+\rho}=2$.

But, Carla's 3rd baseline $\mathbf{L}'$ is Doppler red shifted by $e^{-\rho}=1/2$.



$$\upsilon'_C = e^{-\rho}\upsilon_A = \frac{1}{2}\upsilon_A = 300THz$$

New "Pitcher-mound" $\mathbf{P}'$ (Phase pt.) is 1/2-sum $(\mathbf{R}'+\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{phase}\\ \upsilon'_{phase}\end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho}\\ e^{+\rho}\end{pmatrix} + \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho}\\ +e^{-\rho}\end{pmatrix} = \upsilon_A\begin{pmatrix} \frac{e^{+\rho}-e^{-\rho}}{2}\\ \frac{e^{+\rho}+e^{-\rho}}{2}\end{pmatrix}$$

$$\mathbf{K}'_{phase} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \upsilon_A\begin{pmatrix} \sinh\rho\\ \cosh\rho\end{pmatrix}$$

New "Grandstand" $\mathbf{G}'$ (Group pt.) is 1/2-difference $(\mathbf{R}'-\mathbf{L}')/2$:

$$\begin{pmatrix} c\kappa'_{group}\\ \upsilon'_{group}\end{pmatrix} = \frac{\upsilon_A}{2}\begin{pmatrix} e^{+\rho}\\ e^{+\rho}\end{pmatrix} - \frac{\upsilon_A}{2}\begin{pmatrix} -e^{-\rho}\\ +e^{-\rho}\end{pmatrix} = \upsilon_A\begin{pmatrix} \frac{e^{+\rho}+e^{-\rho}}{2}\\ \frac{e^{+\rho}-e^{-\rho}}{2}\end{pmatrix}$$

Group vector $\mathbf{G}$
1/2-diff vector

$$\mathbf{K}'_{group} = \mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = \upsilon_A\begin{pmatrix} \cosh\rho\\ \sinh\rho\end{pmatrix} = \upsilon_A\begin{pmatrix} 5/4\\ 3/4\end{pmatrix}$$

[RelaWavity WebApp]

PerSpace-PerTime | Group & Phase Vectors | Controls | Contextual | Set ISM | User's Guide

Laser frequency = B = 2 = 600THz
Doppler blue shift factor = b = 1.488
Doppler red shift factor = r = 0.672
ρ = 0.397

CW Light Axioms

All colors go c: ω/k = c or L&R on diagonals

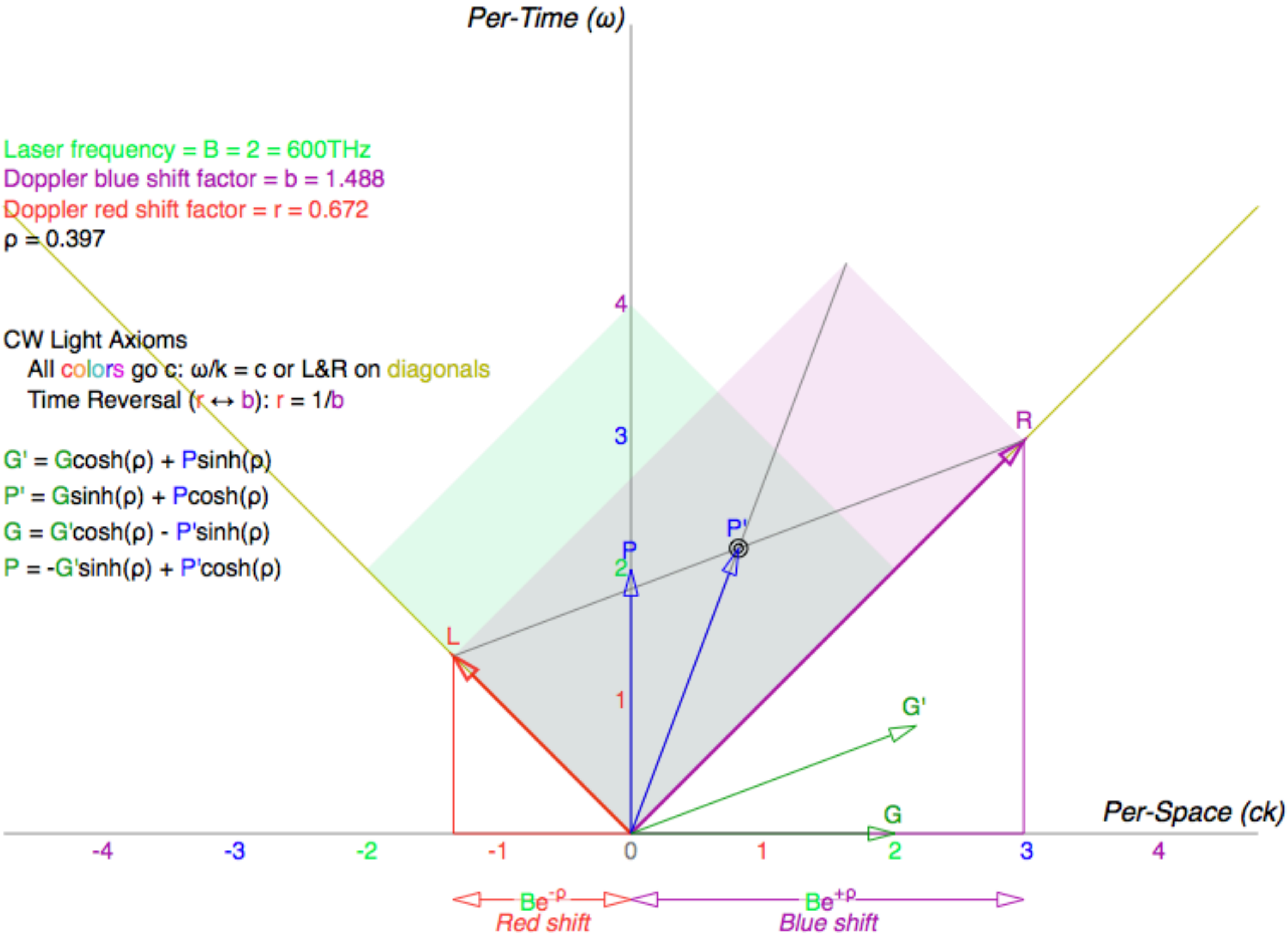
Time Reversal (r ↔ b): r = 1/b

G' = Gcosh(ρ) + Psinh(ρ)

P' = Gsinh(ρ) + Pcosh(ρ)

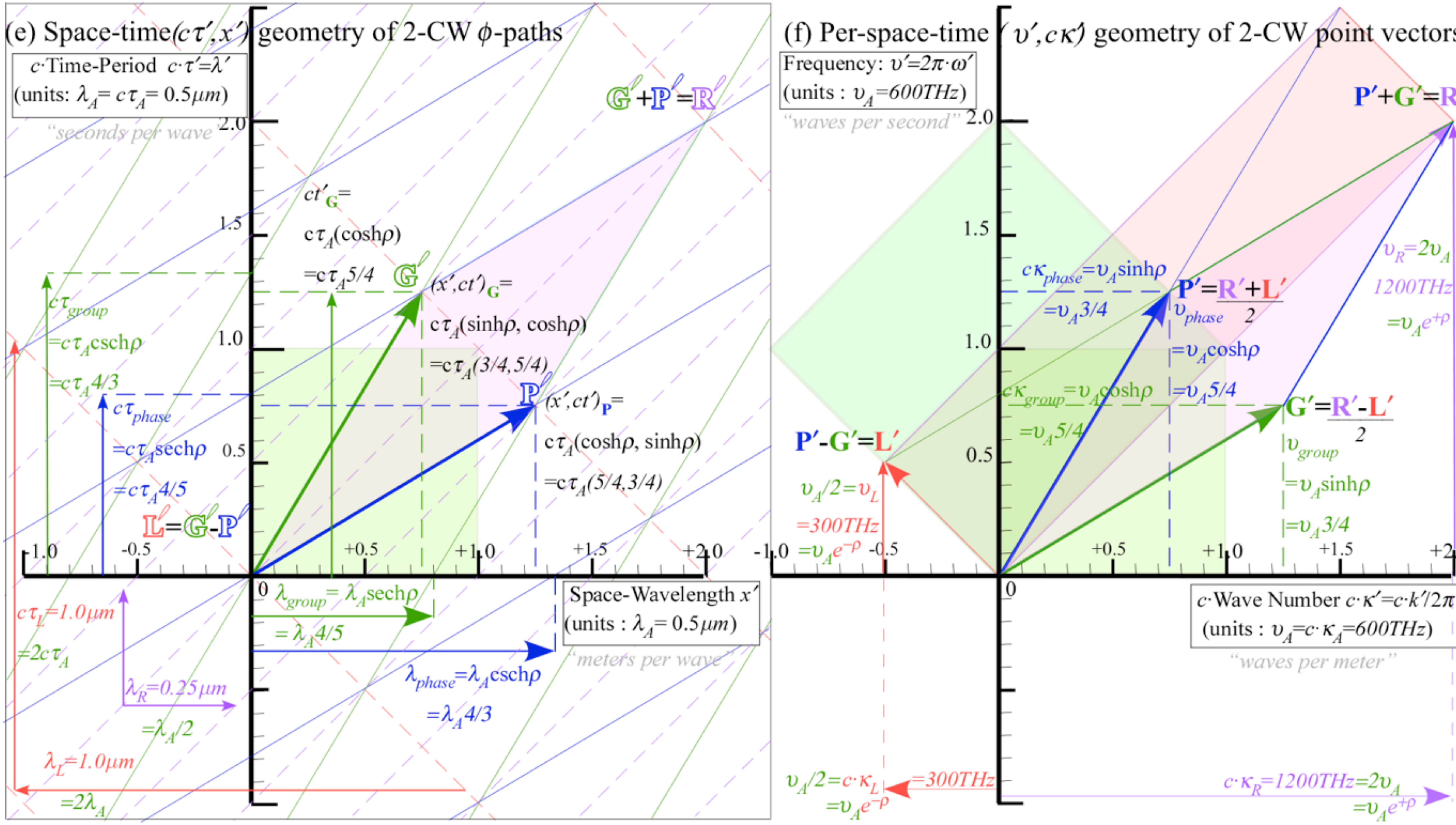
G = G'cosh(ρ) - P'sinh(ρ)

P = -G'sinh(ρ) + P'cosh(ρ)



Relativity and Quantum Theory by Ruler and Compass
Fig. 7-8

[SR&QM by Ruler and Compass]



[Relativity and Quantum Theory by Ruler and Compass]

*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

L-*laser 600THz and* **R**-*laser 600THZ (Laser lab frame)*

Phase **P**-*vector and group* **G**-*vector span Cartesian spacetime coordinates*

L′-*laser 300THz and* **R′**-*laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L′**-*vector and* **R′**-*vector in (***L**, **R***)-per-spacetime*

Vectors of phase **P′**=(**R′**+**L′**)/2 *and group* **G′**=(**R′**-**L′**)/2

➡ *Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates*

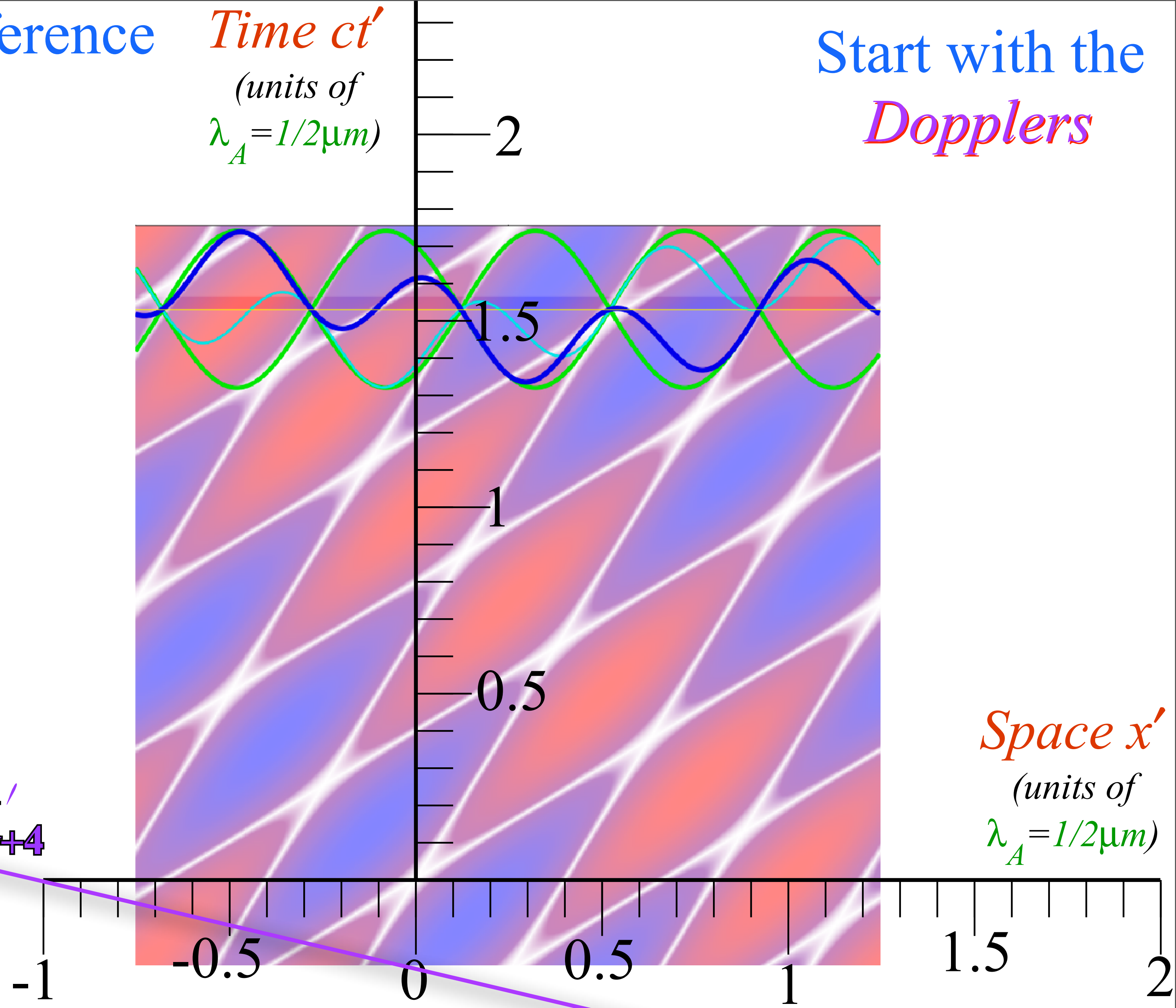
Brief tour of and relativistic mechanics by geometry

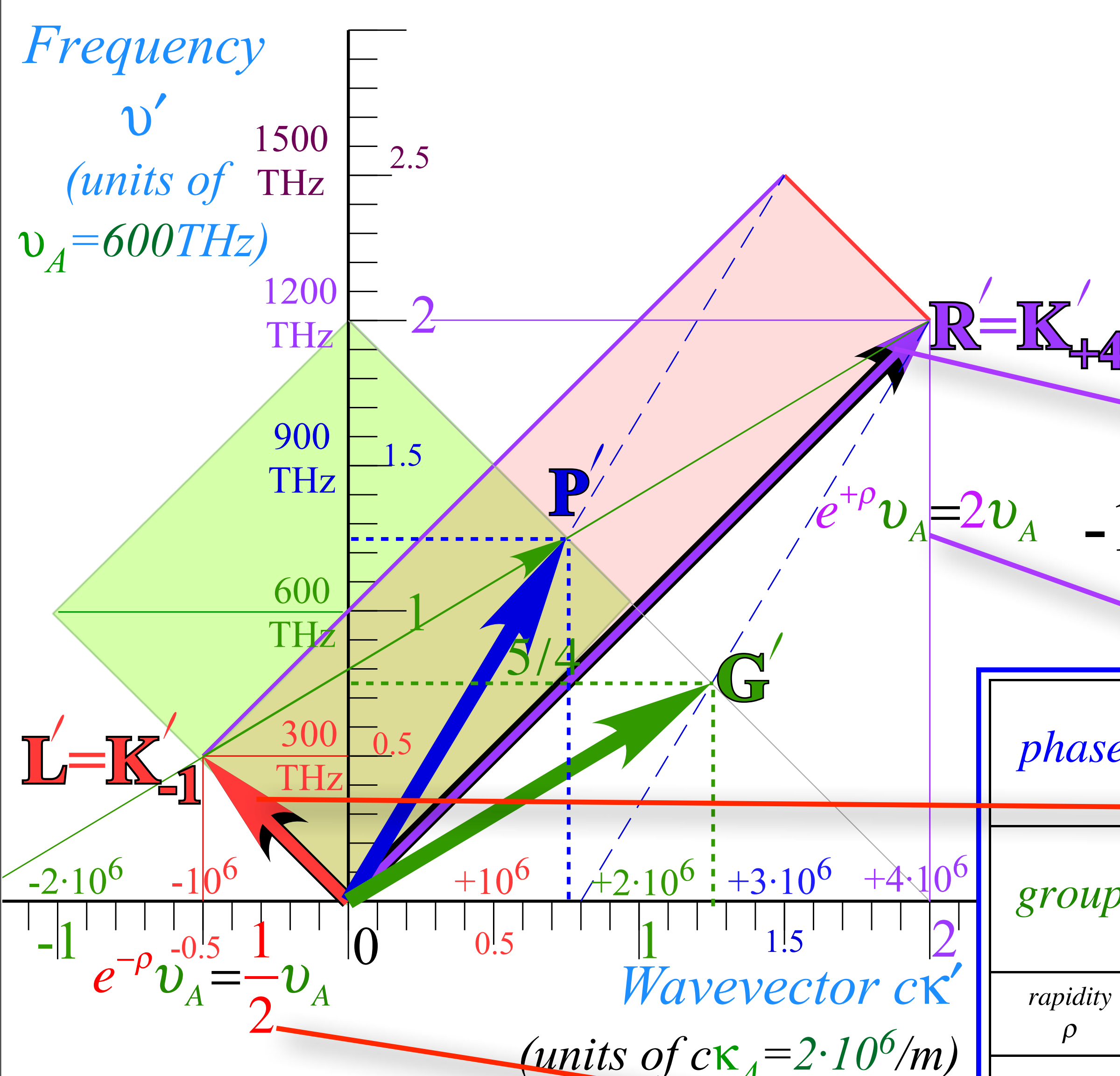
Summary of optical wave parameters for relativity and QM

The 16 dimensions of 2CW interference

UAF Colloquium Nov. 14 2014

Start with the *Dopplers*

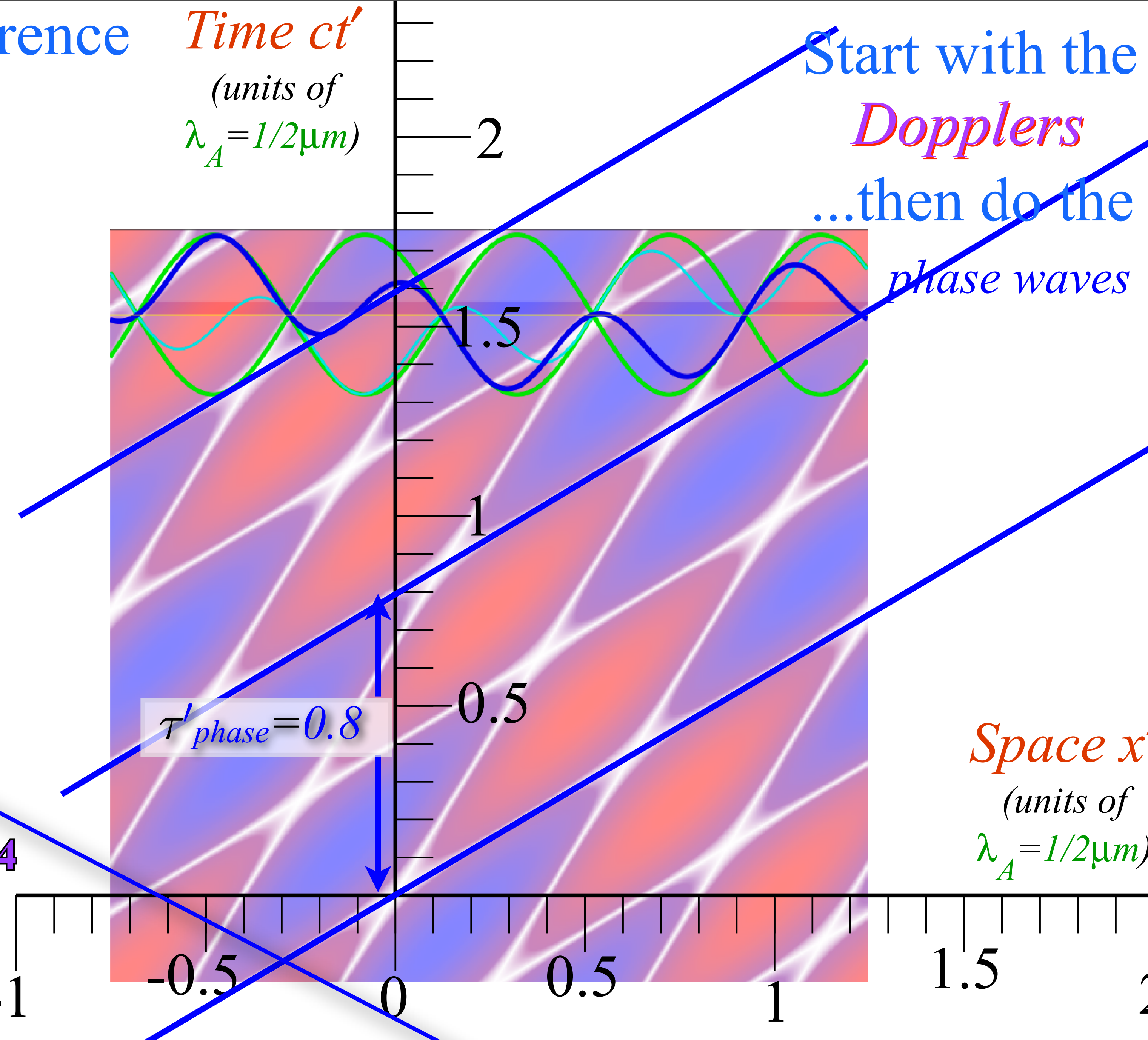




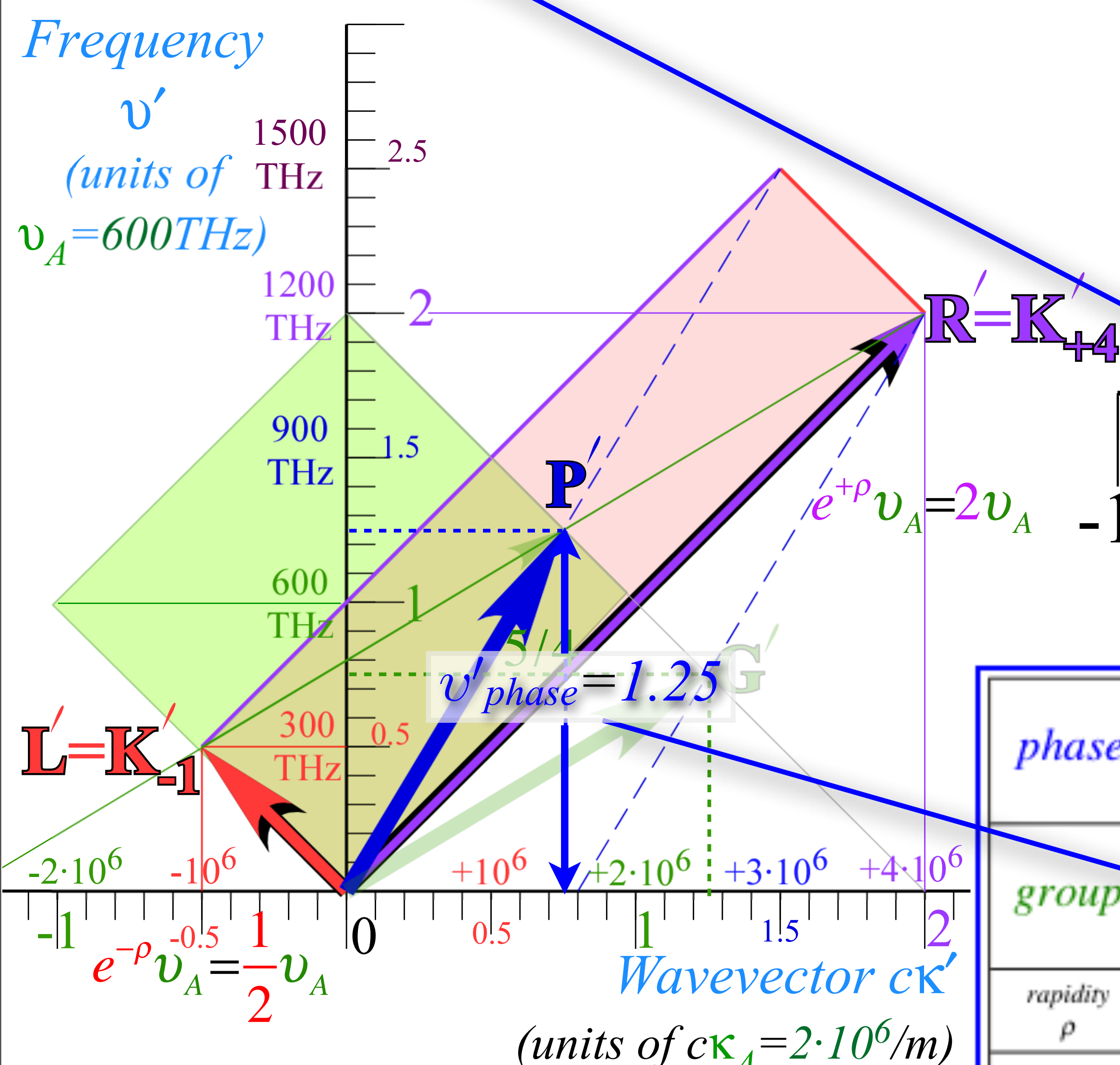
phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

The 16 dimensions of 2CW interference

$$\cosh\rho = \frac{e^{\rho}+e^{-\rho}}{2} = \frac{2+1/2}{2} = \frac{5}{4}$$



Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$
$= 1.25$



phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$	$\mathrm{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

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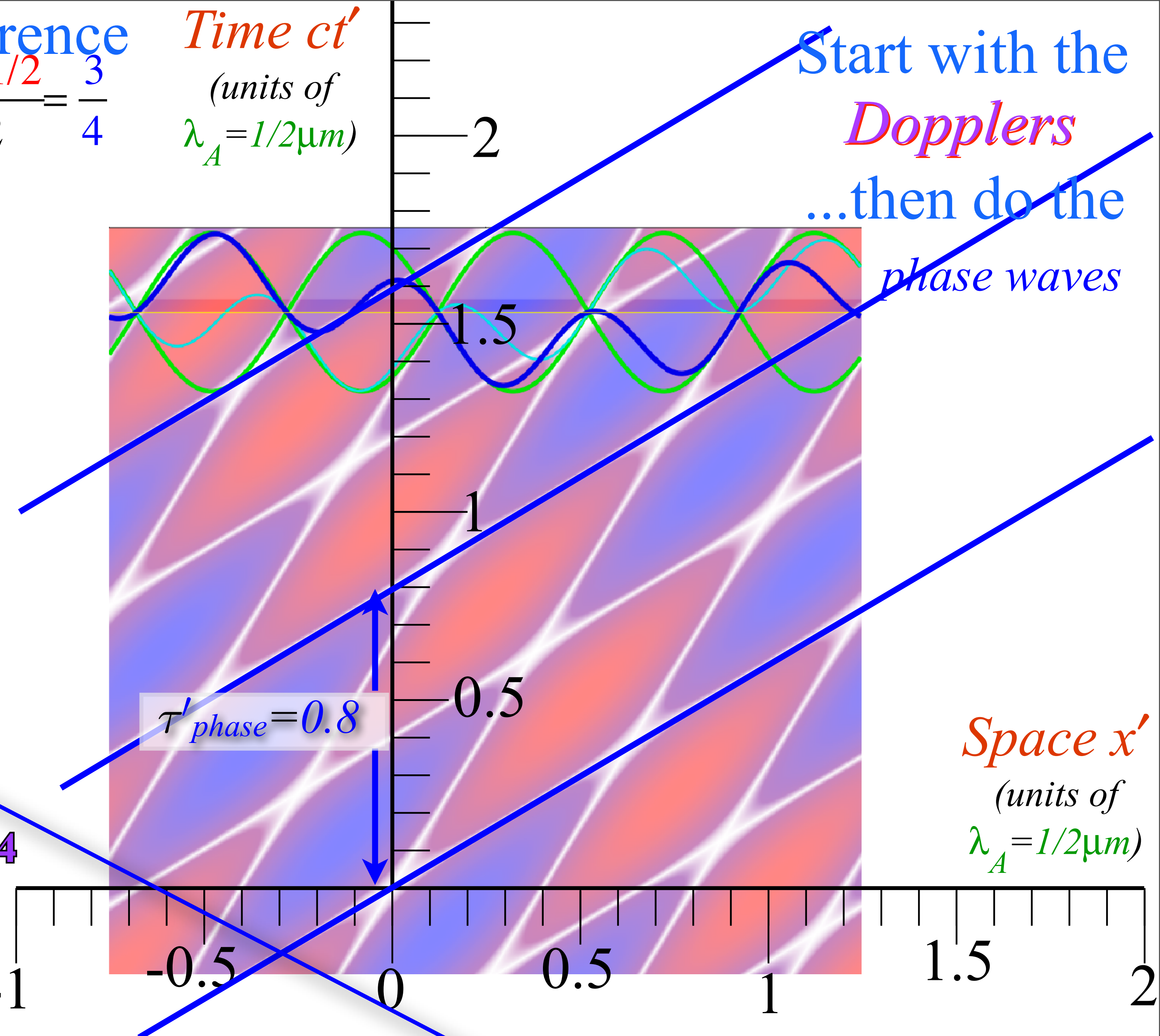
The 16 dimensions of 2CW interference

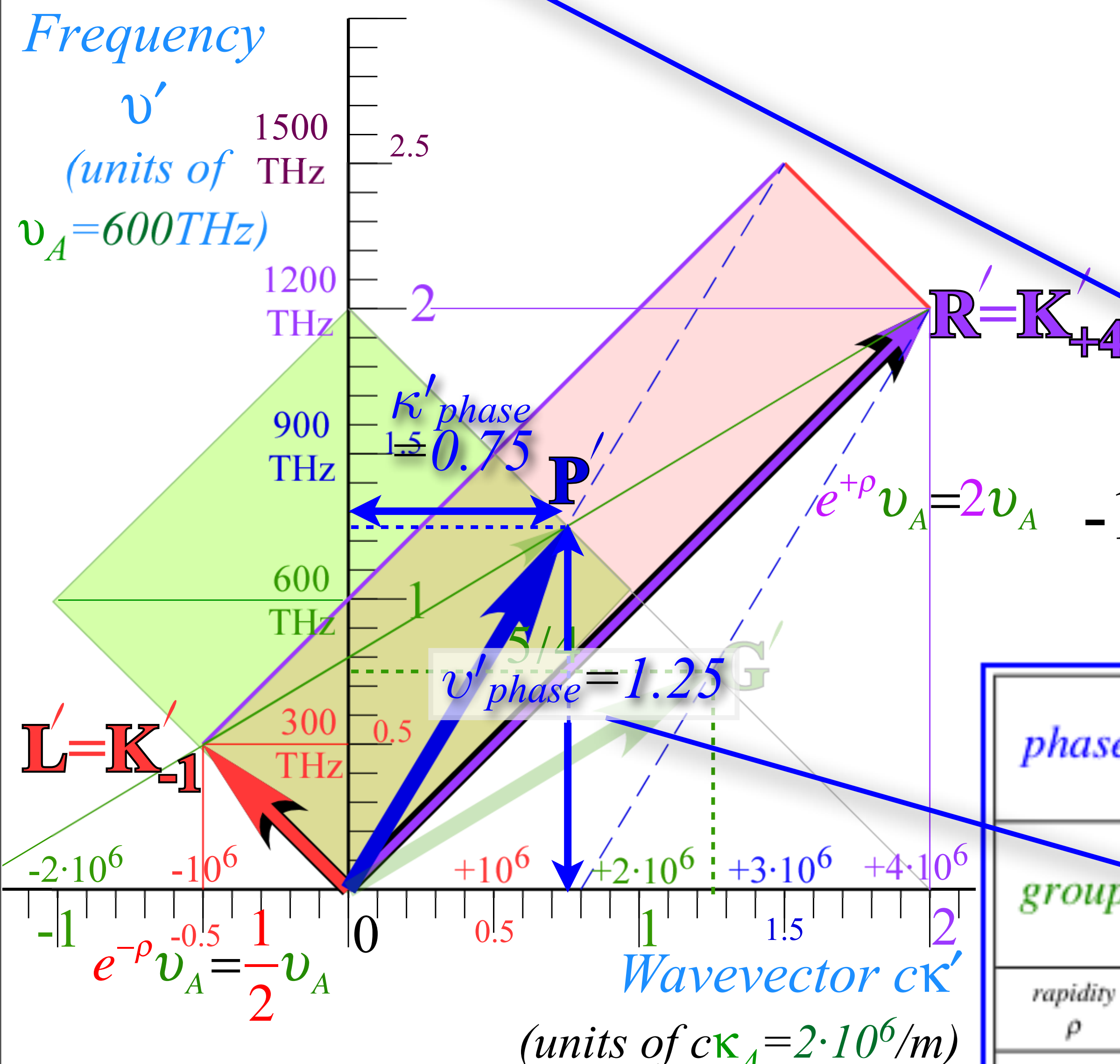
$\cosh\rho = \frac{e^{\rho}+e^{-\rho}}{2} = \frac{2+1/2}{2} = \frac{5}{4}$ $\sinh\rho = \frac{e^{\rho}-e^{-\rho}}{2} = \frac{2-1/2}{2} = \frac{3}{4}$

Phase wavenumber
$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4 = 0.75$

Phase frequency
$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$
$= 1.25$

Start with the *Dopplers* ...then do the *phase waves*





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

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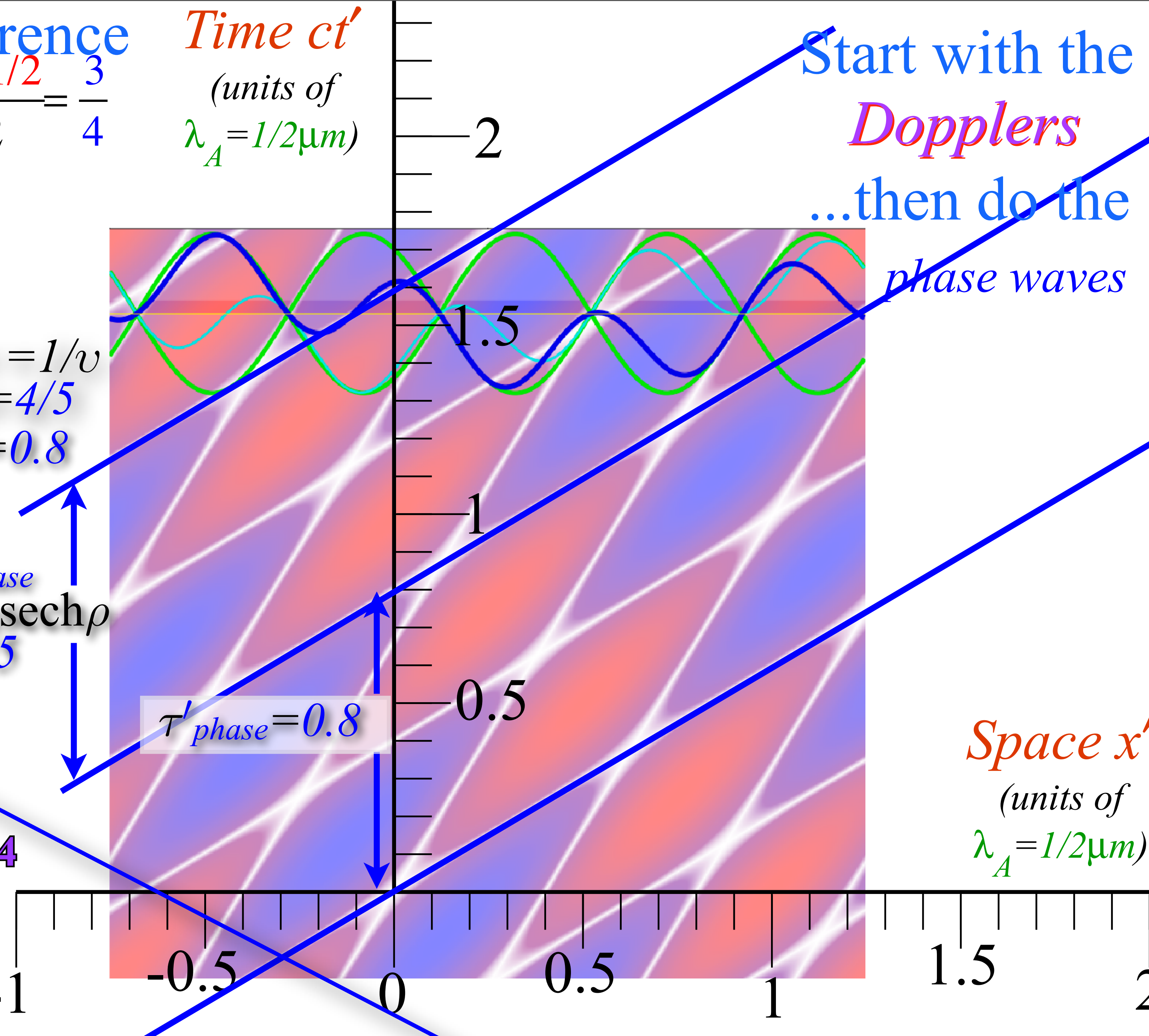
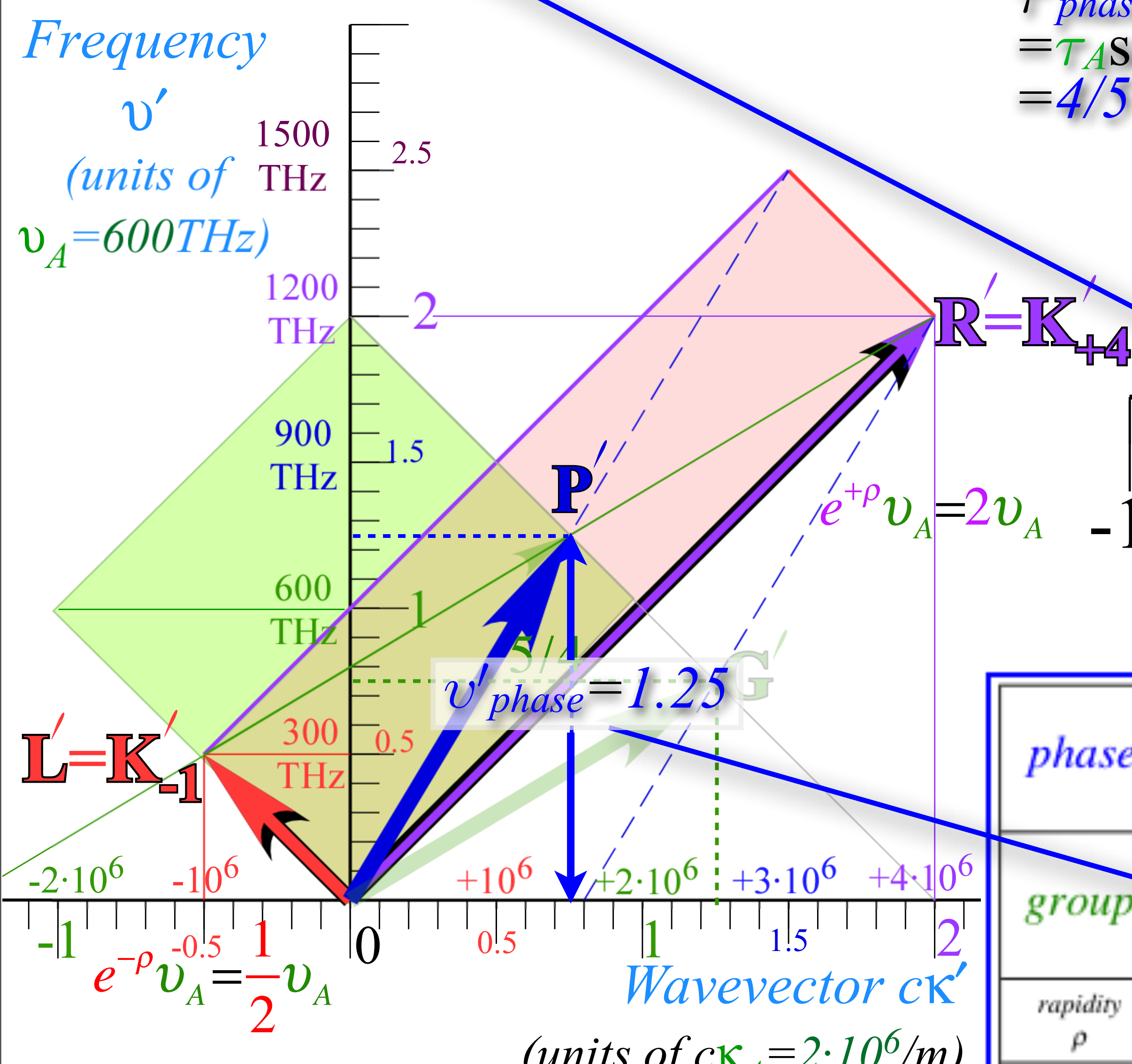
The 16 dimensions of 2CW interference

$\cosh\rho = \frac{e^{\rho}+e^{-\rho}}{2} = \frac{2+1/2}{2} = \frac{5}{4}$ $\sinh\rho = \frac{e^{\rho}-e^{-\rho}}{2} = \frac{2-1/2}{2} = \frac{3}{4}$

$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

Phase frequency $\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4 = 1.25$ flips to *Phase period* $\tau = 1/\upsilon$ $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5 = 0.8$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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Phase wavenumber flips to *Phase wavelength* $\lambda = 1/\kappa$

$\kappa'_{phase} = \kappa_A \sinh\rho = 3/4$ flips to $\lambda'_{phase} = \lambda_A \operatorname{csch}\rho = 4/3$

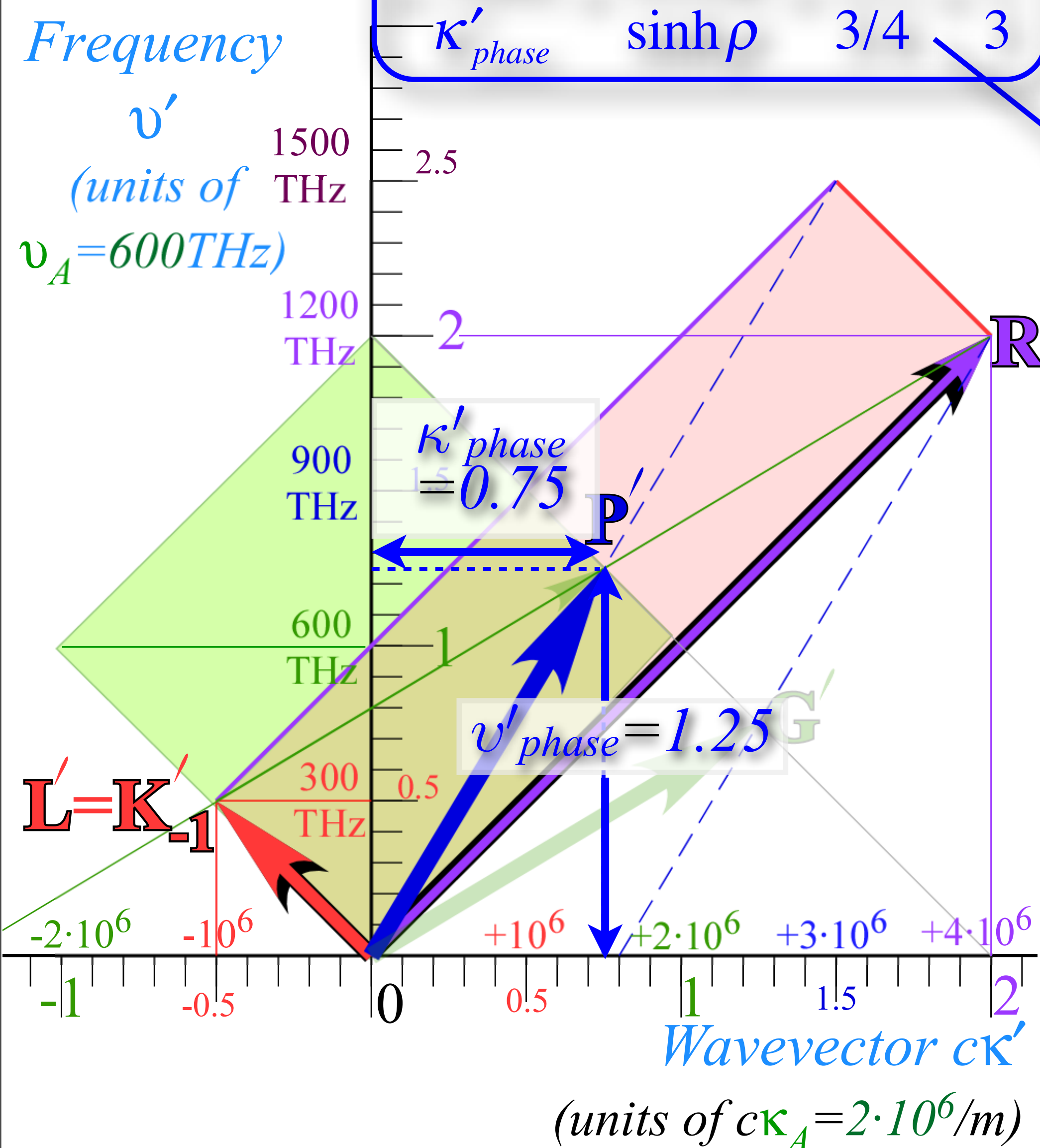
$$\mathbf{P}' = \begin{pmatrix} c\kappa'_{phase} \\ \upsilon'_{phase} \end{pmatrix} = \upsilon_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 3/4 \\ 5/4 \end{pmatrix}$$

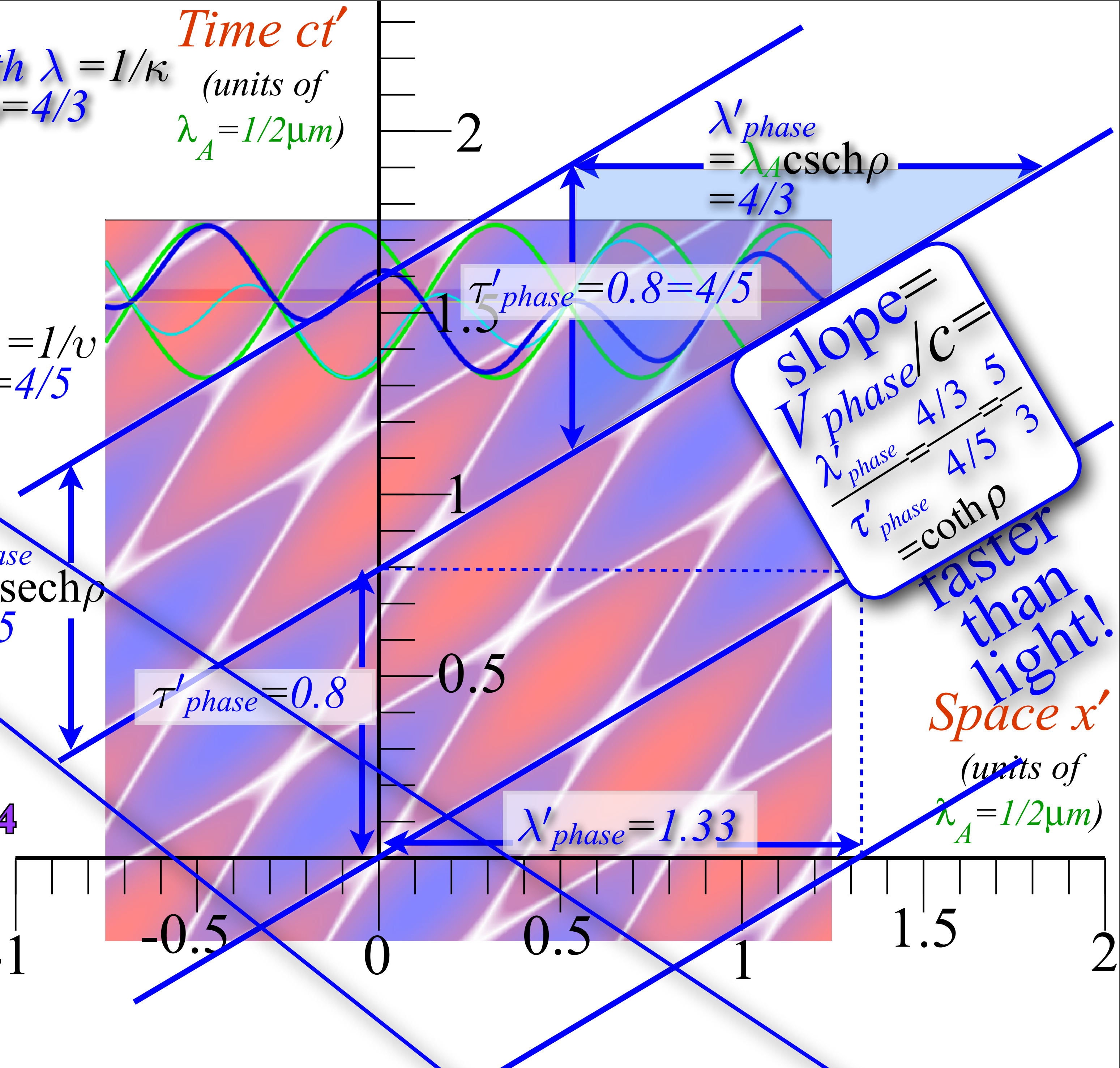
Phase frequency flips to *Phase period* $\tau = 1/\upsilon$

$\upsilon'_{phase} = \upsilon_A \cosh\rho = 5/4$ flips to $\tau'_{phase} = \tau_A \operatorname{sech}\rho = 4/5$

P-slope $= V_{phase}/c$

$$= \frac{\upsilon'_{phase}}{\kappa'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5/4}{3/4} = \frac{5}{3}$$





phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

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Group wavenumber
$\kappa'_{group} = \kappa_A \cosh\rho = 5/4 = 1.25$

Group wavelength $\lambda = 1/\kappa$
$\lambda'_{group} = \lambda_A \operatorname{sech}\rho = 4/5 = 0.8$

$$\mathbf{G}' = \begin{pmatrix} c\kappa'_{group} \\ \upsilon'_{group} \end{pmatrix} = \upsilon_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \upsilon_A \begin{pmatrix} 5/4 \\ 3/4 \end{pmatrix}$$

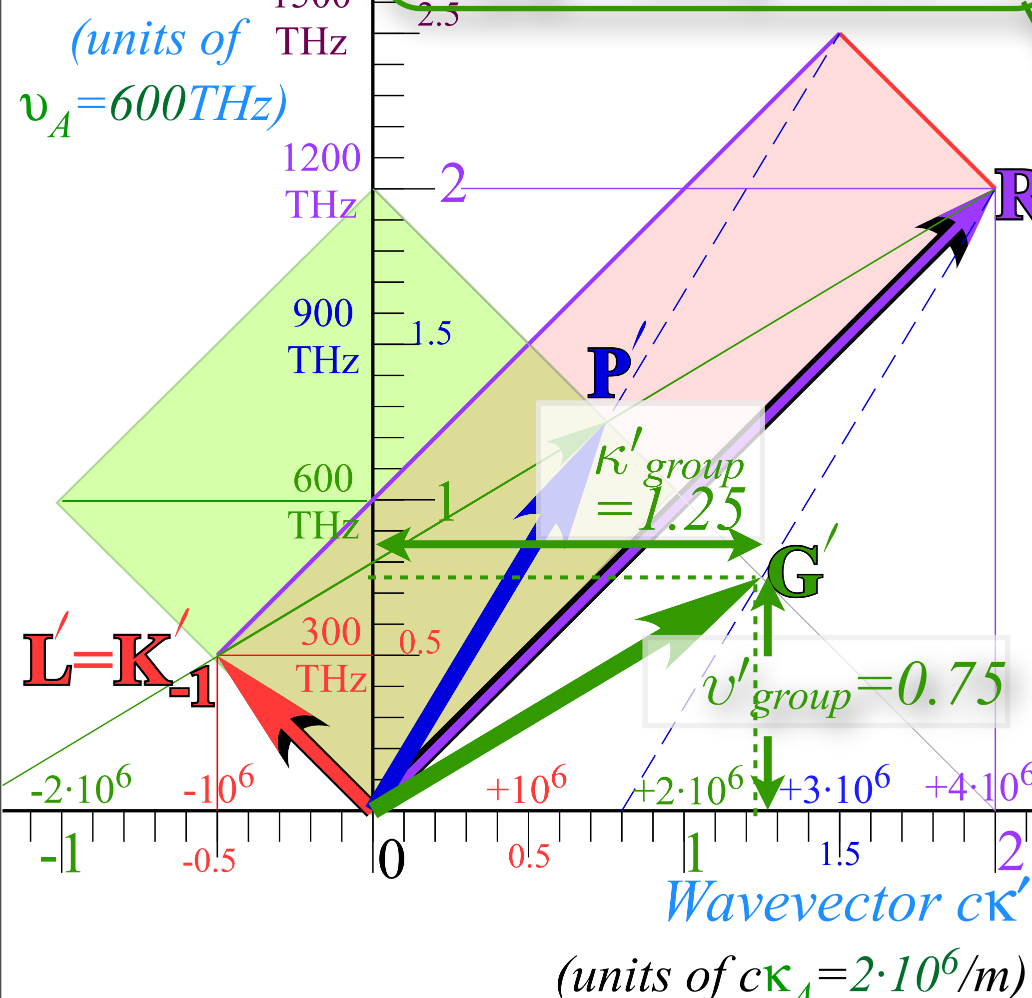
Group frequency
$\upsilon'_{group} = \upsilon_A \sinh\rho = 3/4 = 0.75$

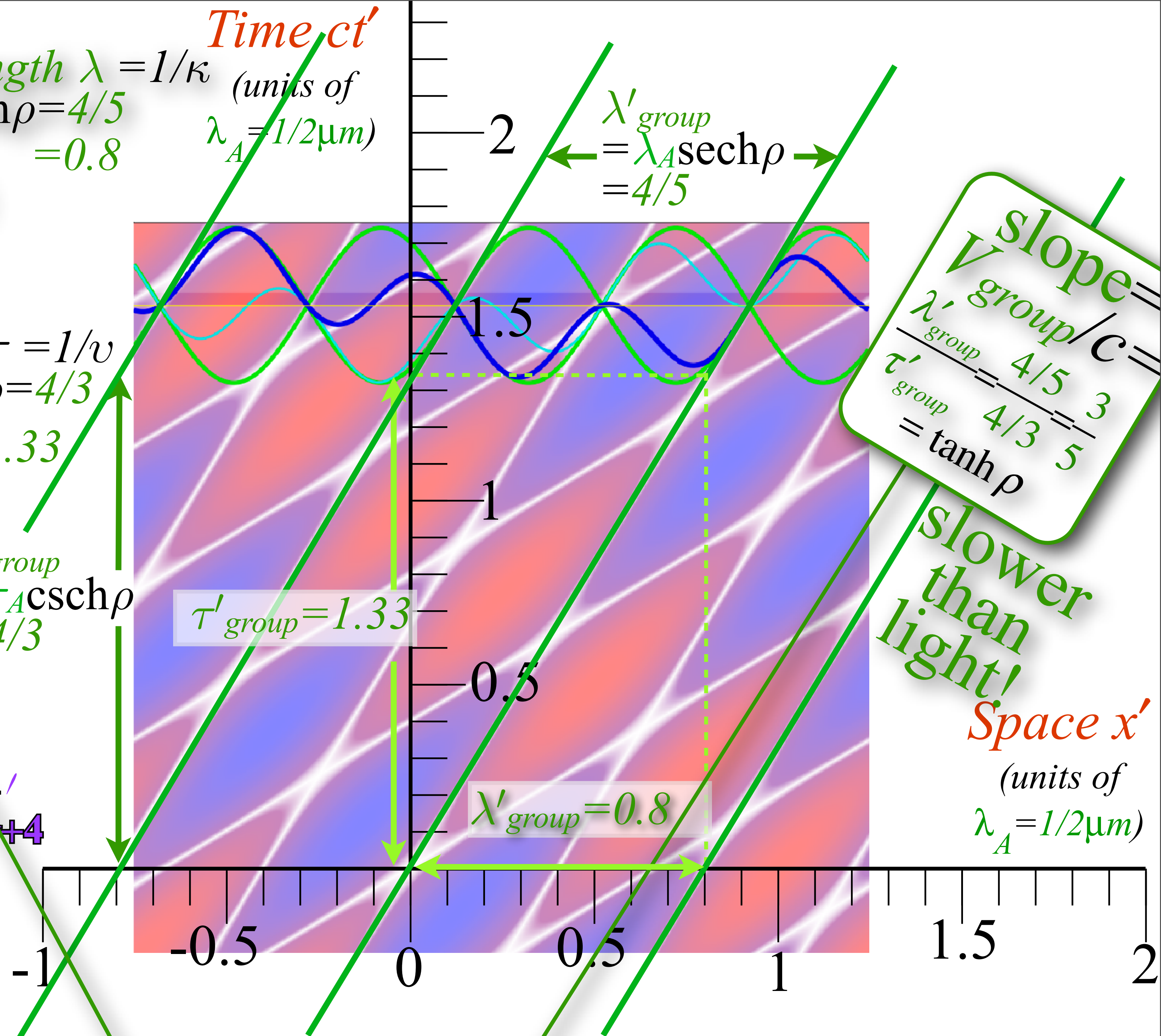
flips to

Group period $\tau = 1/\upsilon$
$\tau'_{group} = \tau_A \operatorname{csch}\rho = 4/3 = 1.33$

G-slope $= V_{group}/c$

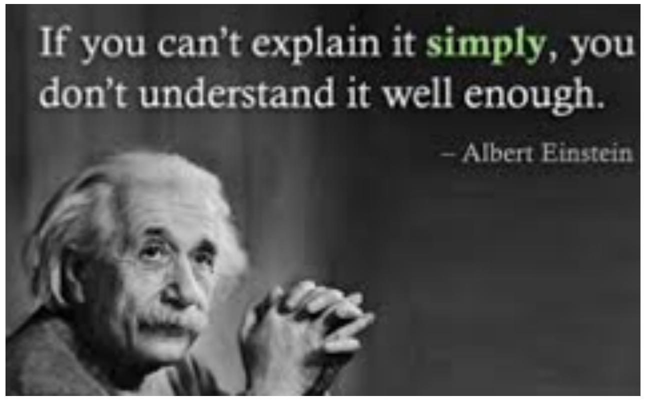
$$= \frac{\upsilon'_{group}}{\kappa'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3/4}{5/4} = \frac{3}{5}$$





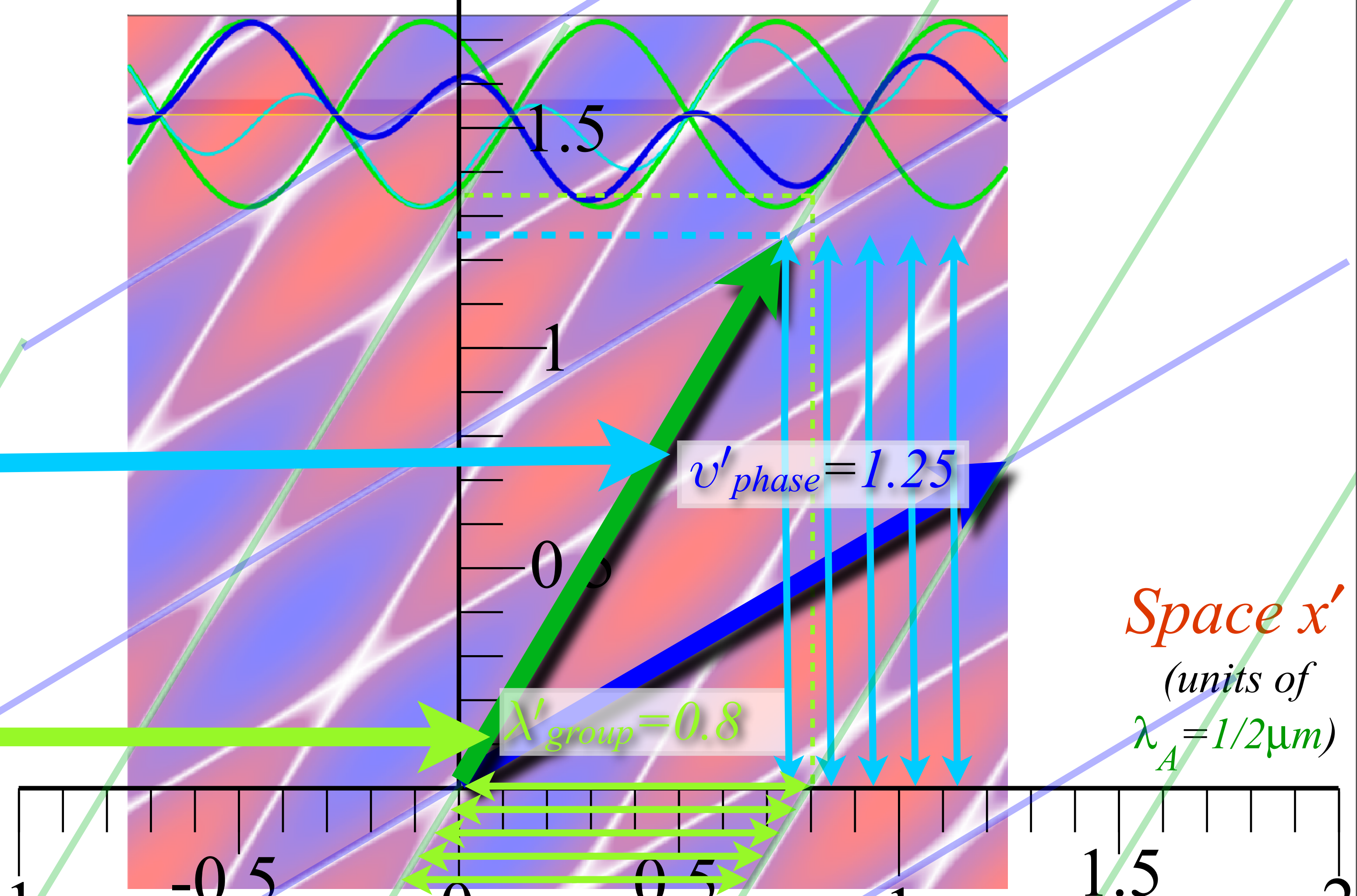
phase	$b_{RED}^{Doppler}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
group	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
value for $\beta=3/5$	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

Two Famous-Name Coefficients



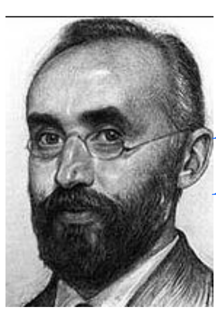
Albert Einstein 1859-1955

Herman Minkowski 1864-1909



This number is called an: **Einstein time-dilation** (dilated by 25% here)

This number is called a: **Lorentz length-contraction** (contracted by 20% here)



Hendrik A. Lorentz 1853-1928

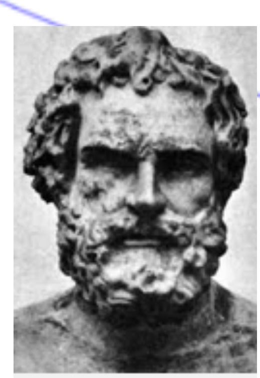
Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$=0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

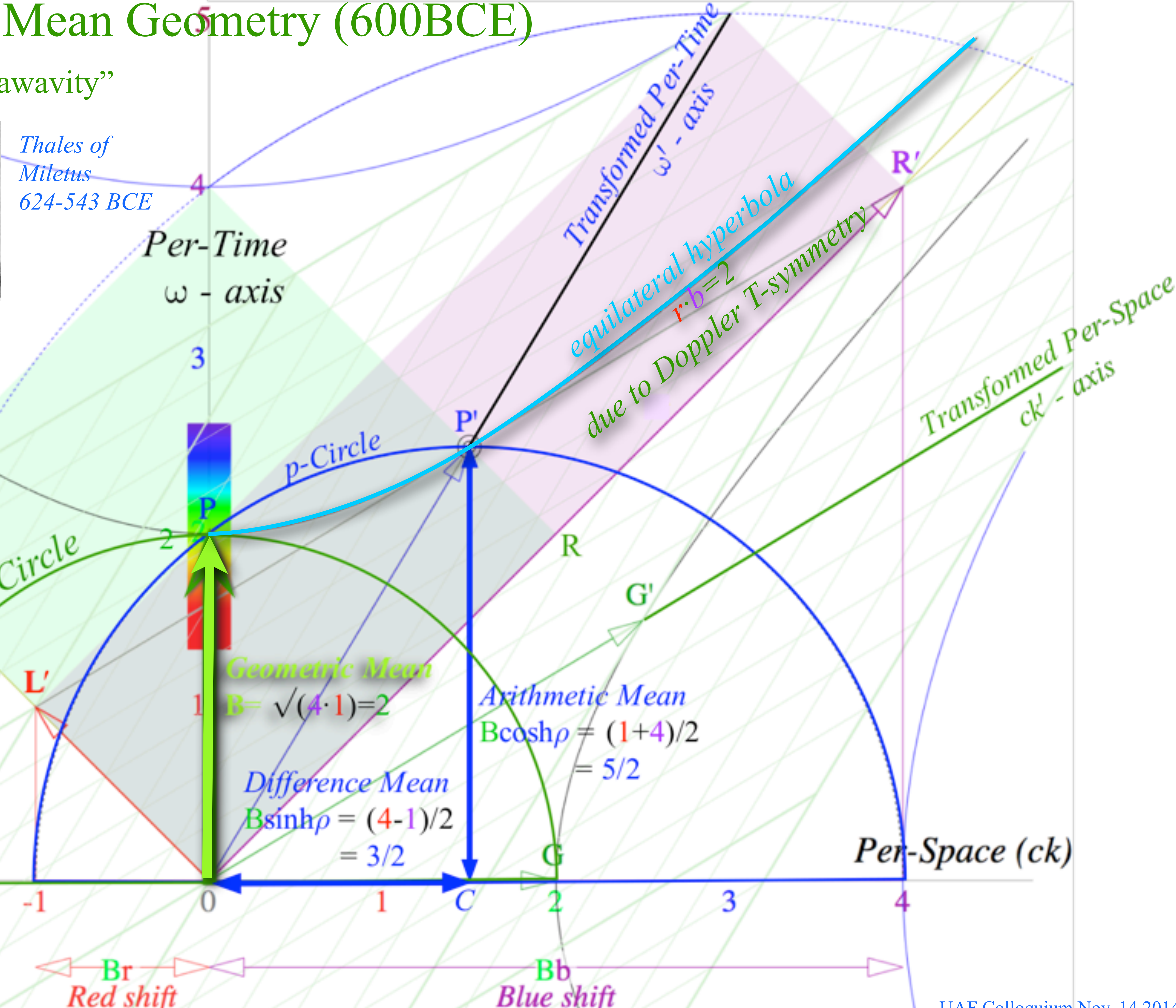
UAF Colloquium Nov. 14 2014

Thales Mean Geometry (600BCE)

helps "Relawavity"



Thales of Miletus 624-543 BCE



UAF Colloquium Nov. 14 2014

*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

L*-laser 600THz and* **R***-laser 600THZ (Laser lab frame)*

Phase **P***-vector and group* **G***-vector span Cartesian spacetime coordinates*

L′*-laser 300THz and* **R′***-laser 1200THZ (Doppler shifted in moving frame)*

Doppler shifted **L′***-vector and* **R′***-vector in (***L***,* **R***)-per-spacetime*

Vectors of phase **P′***=(***R′***+***L′***)/2 and group* **G′***=(***R′***-***L′***)/2*

Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

→ *Brief tour of and relativistic mechanics by geometry*

Summary of optical wave parameters for relativity and QM

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

At low speeds:

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2$

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$B = \upsilon_A$

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group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$

Resembles: $const. + \frac{1}{2}Mu^2$

Resembles: Mu

time	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	$b_{BLUE}^{Doppler}$
space	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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Resembles: Mu

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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b_{RED}^{Doppler}}$
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(old-fashioned notation)

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Planck (1900)

$$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$$

Einstein (1905)

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Max Planck 1858-1947

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$= \text{Total Energy: } E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

This motivates the "particle" normalization $\int \Psi^*\Psi\, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

group	$b_{RED}^{Doppler}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b_{BLUE}^{Doppler}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$			
stellar angle $\forall\ \sigma$	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2} = 0.5$	$\frac{3}{5} = 0.6$	$\frac{3}{4} = 0.75$	$\frac{4}{5} = 0.80$	$\frac{5}{4} = 1.25$	$\frac{4}{3} = 1.33$	$\frac{5}{3} = 1.67$	$\frac{2}{1} = 2.0$

Big worry: Is not oscillator energy quadratic in frequency υ?

HO energy $= \frac{1}{2}A^2\upsilon^2$

Resolution and dirty secret: E, N, and υ_{phase} are all frequencies!

Using (some) wave parameters to develop relativistic quantum theory

$\upsilon_{phase} = B\cosh\rho \approx B + \frac{1}{2}B\rho^2$ (for $u \ll c$)

$c\kappa_{phase} = B\sinh\rho \approx B\rho$ (for $u \ll c$)

$\frac{u}{c} = \tanh\rho \approx \rho$ (for $u \ll c$)

$\cosh\rho \approx 1 + \frac{1}{2}\rho^2 \approx 1 + \frac{1}{2}\frac{u^2}{c^2}$

$\sinh\rho \approx \rho \approx \frac{u}{c}$

Max Planck 1858-1947

Louis DeBroglie 1892-1987

$B = \upsilon_A$

$B = \upsilon_A = c\kappa_A$

At low speeds:

$$\upsilon_{phase} \approx B + \frac{1}{2}\frac{B}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad \kappa_{phase} \approx \frac{B}{c^2}u$$

υ_{phase} and κ_{phase} resemble formulae for Newton's kinetic energy $\frac{1}{2}Mu^2$ and momentum Mu.

Rescale υ_{phase} by h so: $M = \frac{hB}{c^2}$ or: $hB = Mc^2$

(The famous Mc^2 shows up here!)

So attach scale factor h *(or hN)* to match units.

$$h\upsilon_{phase} \approx hB + \frac{1}{2}\frac{hB}{c^2}u^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx \frac{hB}{c^2}u$$

$$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \quad \Leftarrow \text{for } (u \ll c) \Rightarrow \quad h\kappa_{phase} \approx Mu$$

~~Lucky coincidences??~~ Natural wave conspiracy ~~Cheap~~ Expensive trick??

...Try exact υ_{phase} and κ_{phase}...

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Planck (1900)

Need to replace h with hN *to match e.m. energy density* $\varepsilon_0 \mathbf{E}^* \cdot \mathbf{E} = hN\upsilon_{phase}$

= Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ angle σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$=0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

$\frac{1}{\sqrt{\beta^{-2}-1}} = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$ (old-fashioned notation)

$cp = \frac{Muc}{\sqrt{1-u^2/c^2}}$

Momentum: $h\kappa_{phase} = p = \frac{Mu}{\sqrt{1-u^2/c^2}}$

DeBroglie (1921)

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So attach scale factor h (or hN) to match units.

$h\upsilon_{phase} \approx Mc^2 + \frac{1}{2}Mu^2 \Leftarrow$ for $(u \ll c) \Rightarrow h\kappa_{phase} \approx Mu$

~~Lucky coincidences~~ Natural wave conspiracy?? ~~Cheap~~ Expensive trick??

...Try *exact* υ_{phase} and κ_{phase}...

UAF Colloquium Nov. 14 2014

$h\upsilon_{phase} = hB\cosh\rho = Mc^2\cosh\rho$

Need to replace h with hN to match e.m. energy density $\varepsilon_0 \mathbf{E}^* \bullet \mathbf{E} = hN\upsilon_{phase}$

Planck (1900)

= Total Energy: $E = \frac{Mc^2}{\sqrt{1-u^2/c^2}}$

Einstein (1905)

This motivates the "particle" normalization $\int \Psi^*\Psi \, dV = N$ $\quad \Psi = \sqrt{\frac{\varepsilon_0}{h\upsilon}}E$

$hc\kappa_{phase} = hB\sinh\rho = Mc^2\sinh\rho$

group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{V_{phase}}{c}$	
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$			
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\mathrm{sech}\,\rho$	$\cosh\rho$			
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$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
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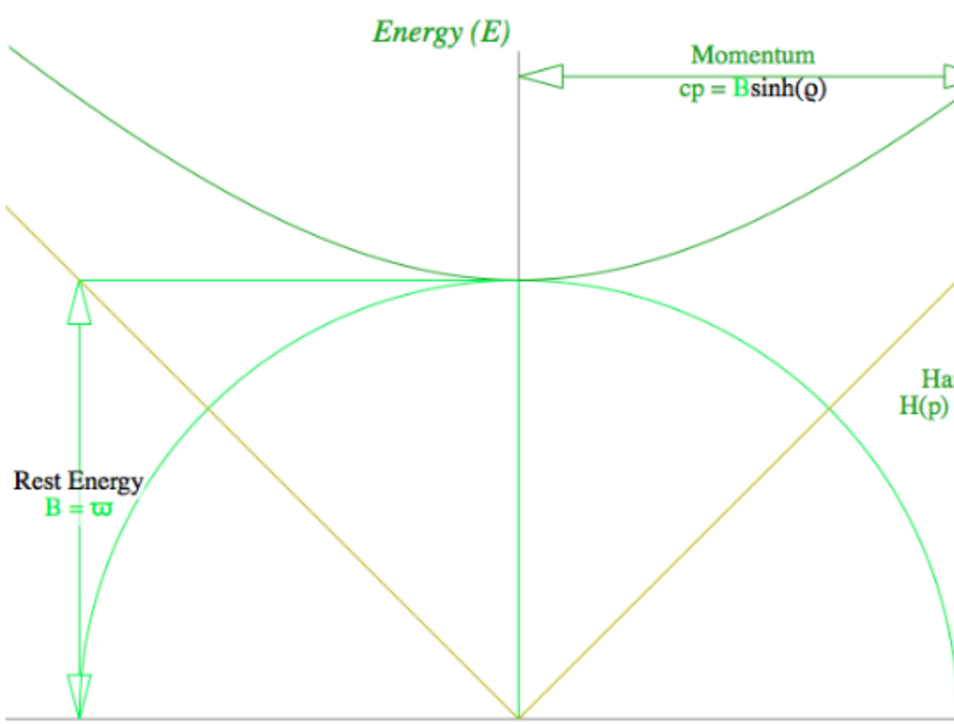
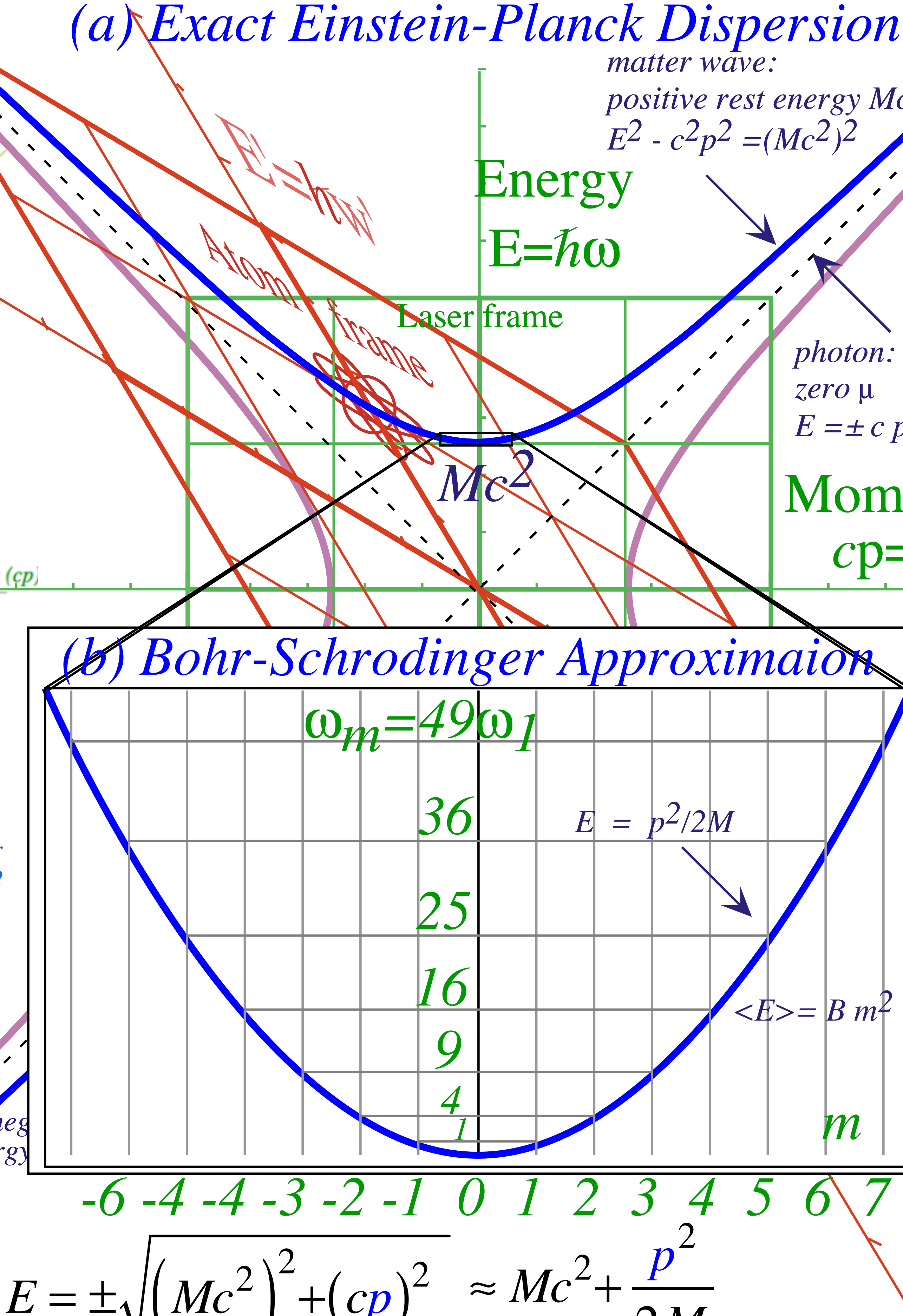
DeBroglie (1921)

Using (some) wave coordinates for relativistic quantum theory

UAF Colloquium Nov. 14 2014





Mass (resting)

$$hB = h\upsilon_A = Mc^2 = hc\kappa_A$$

Energy

$$h\upsilon_{phase} = E = h\upsilon_A \cosh\rho$$

Neils Bohr
1885-1962

Erwin Schrodinger
1887-1961

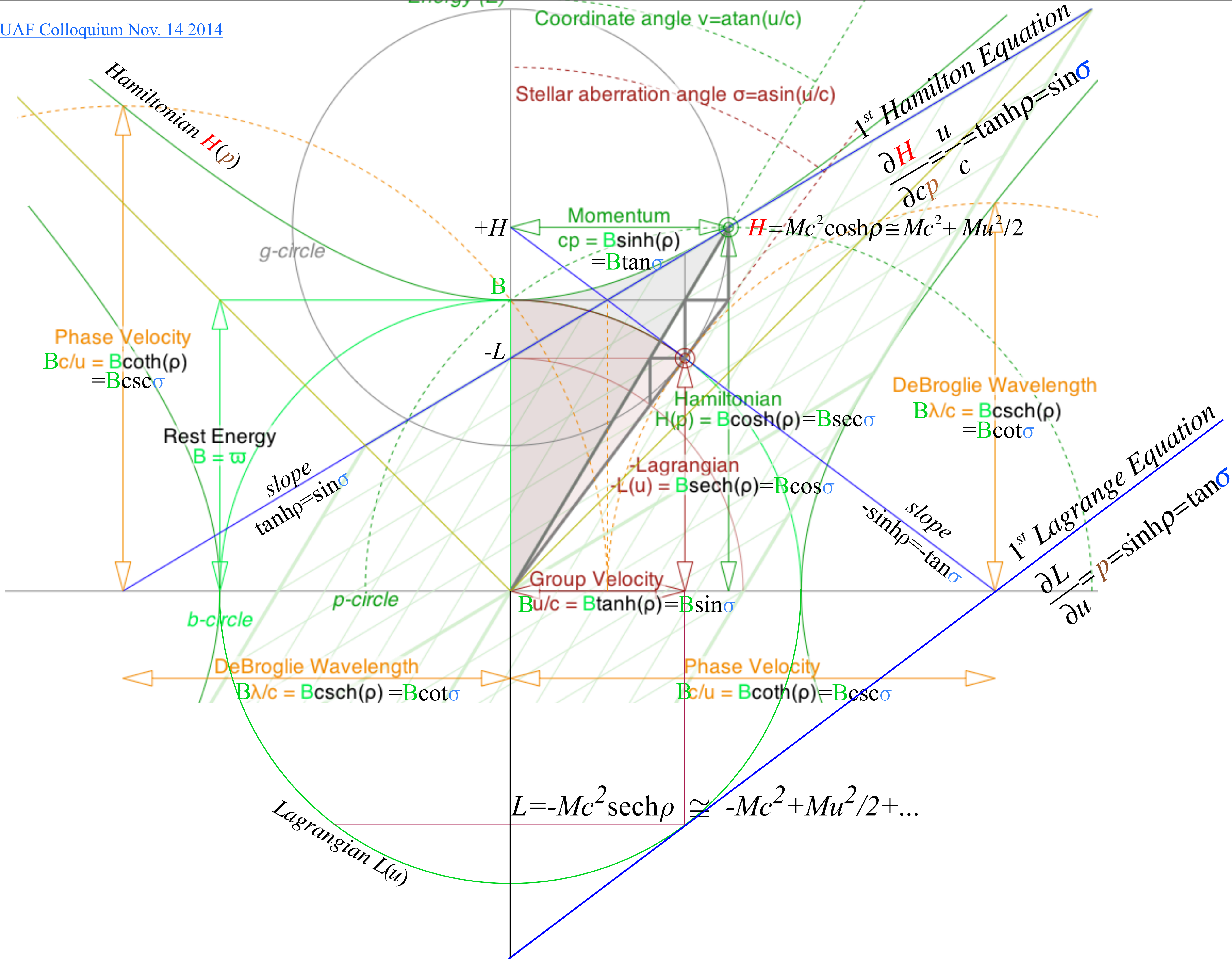
Momentum

$$hc\kappa_{phase} = cp = hc\kappa_A \sinh\rho = h\upsilon_A \sinh\rho$$

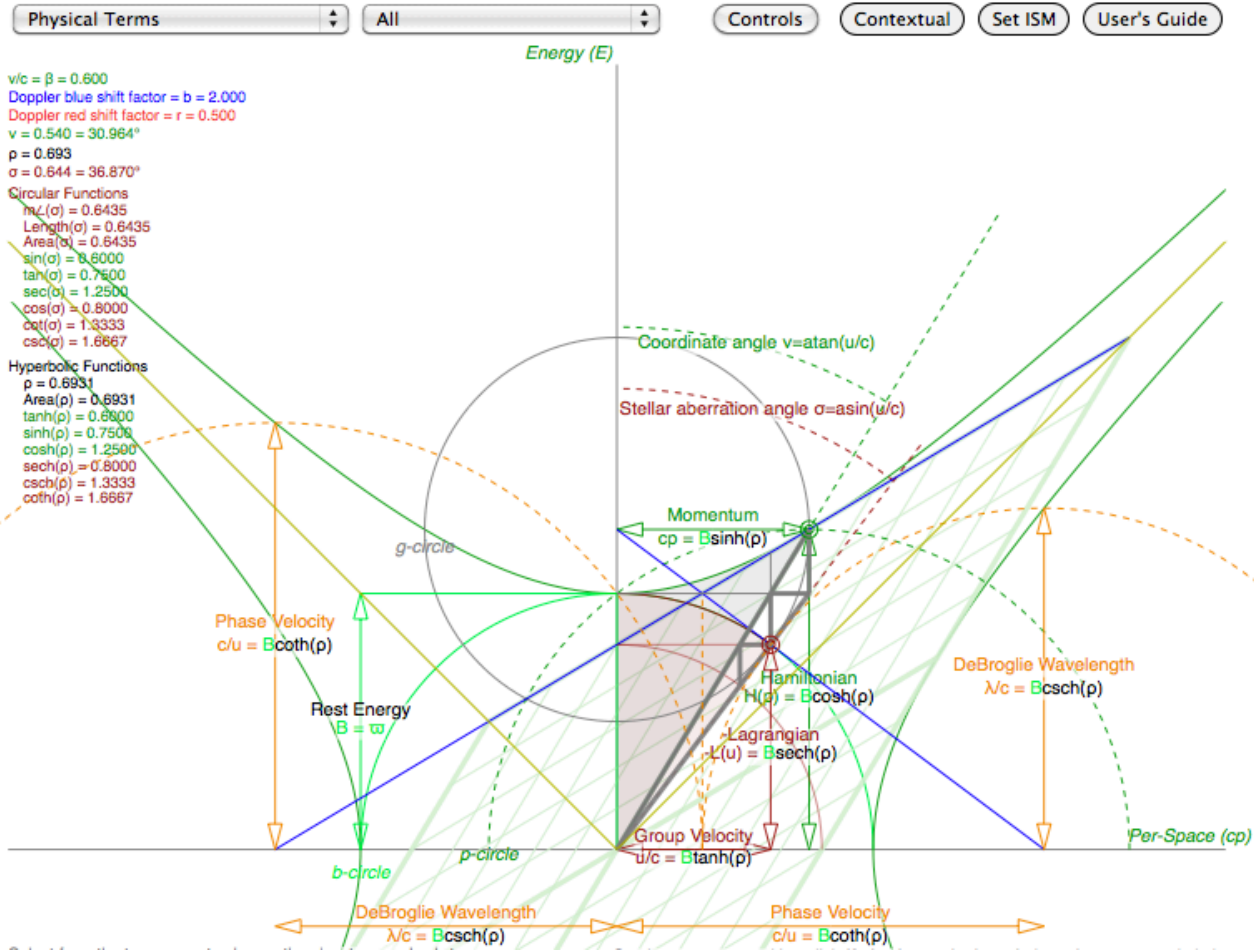
Energy versus Momentum

$$E^2 = \left(Mc^2\right)^2 \cosh^2\rho$$

$$= \left(Mc^2\right)^2\left(1+\sinh^2\rho\right) = \left(Mc^2\right)^2 + (cp)^2 \Rightarrow E = \pm\sqrt{\left(Mc^2\right)^2 + (cp)^2} \approx Mc^2 + \frac{p^2}{2M}$$


Energy (E)
Coordinate angle v=atan(u/c)
Stellar aberration angle σ=asin(u/c)
Hamiltonian H(p)
1st Hamilton Equation
∂H/∂cp = u/c = tanhρ=sinσ
+H
Momentum
cp = Bsinh(ρ)
=Btanσ
H=Mc²coshρ≅Mc²+ Mu²/2
g-circle
B
-L
Phase Velocity
Bc/u = Bcoth(ρ)
=Bcscσ
Hamiltonian
H(ρ) = Bcosh(ρ)=Bsecσ
DeBroglie Wavelength
Bλ/c = Bcsch(ρ)
=Bcotσ
Rest Energy
B = ϖ
slope
tanhρ=sinσ
-Lagrangian
-L(u) = Bsech(ρ)=Bcosσ
slope
-sinhρ=-tanσ
1st Lagrange Equation
∂L/∂u = p=sinhρ=tanσ
Group Velocity
Bu/c = Btanh(ρ)=Bsinσ
p-circle
b-circle
DeBroglie Wavelength
Bλ/c = Bcsch(ρ) =Bcotσ
Phase Velocity
Bc/u = Bcoth(ρ)=Bcscσ
L=-Mc²sechρ ≅ -Mc²+Mu²/2+...
Lagrangian L(u)

[Relawavity WebApp]



*Introduction to wave coordinates by **L**eft-moving and **R**ight-moving laser beams*

***L**-laser 600THz and **R**-laser 600THZ (Laser lab frame)*

*Phase **P**-vector and group **G**-vector span Cartesian spacetime coordinates*

***L**′-laser 300THz and **R**′-laser 1200THZ (Doppler shifted in moving frame)*

*Doppler shifted **L**′-vector and **R**′-vector in (**L**, **R**)-per-spacetime*

*Vectors of phase **P**′=(**R**′+**L**′)/2 and group **G**′=(**R**′-**L**′)/2*

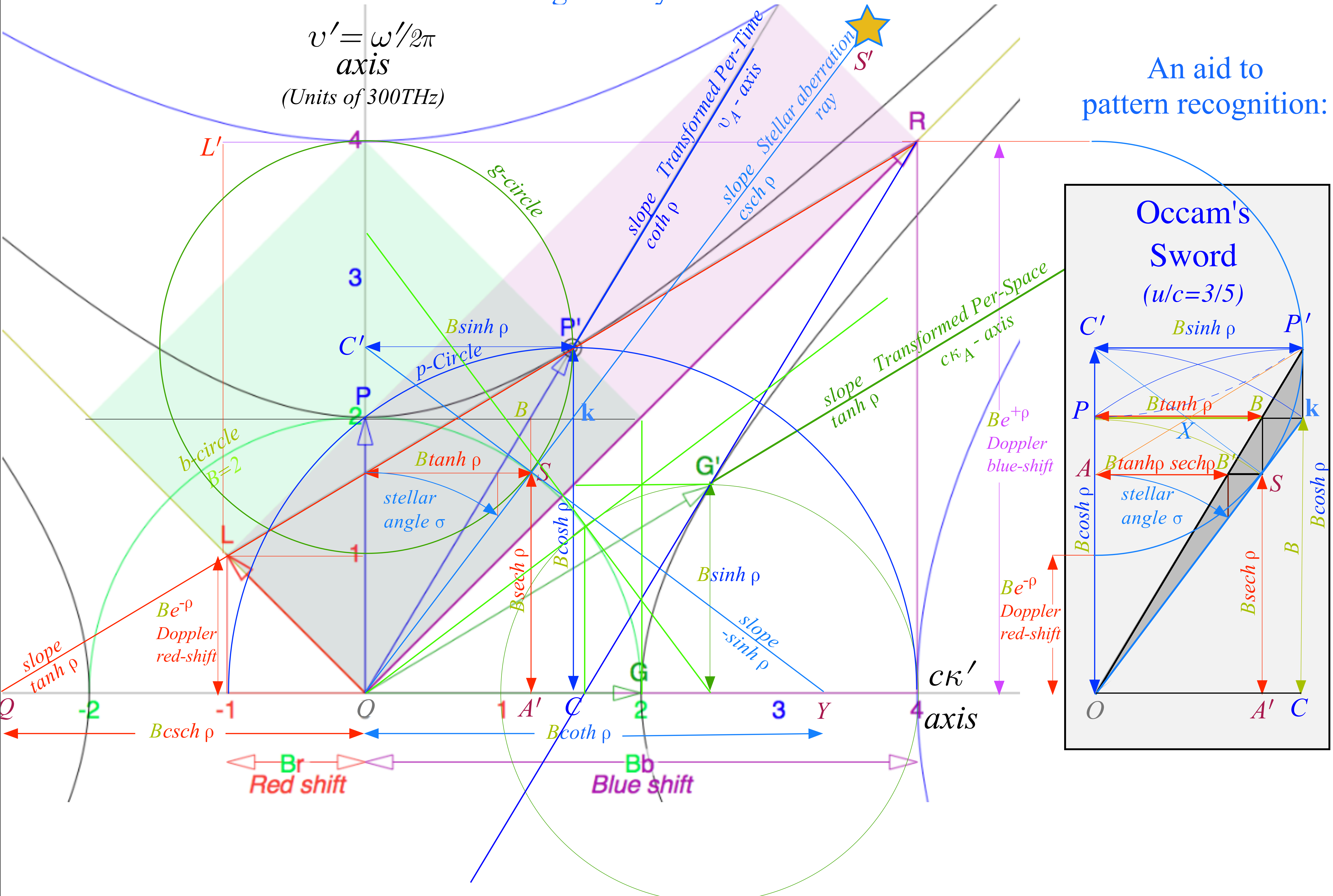
Einstein-Lorentz-Minkowski "Relawavity" spacetime coordinates

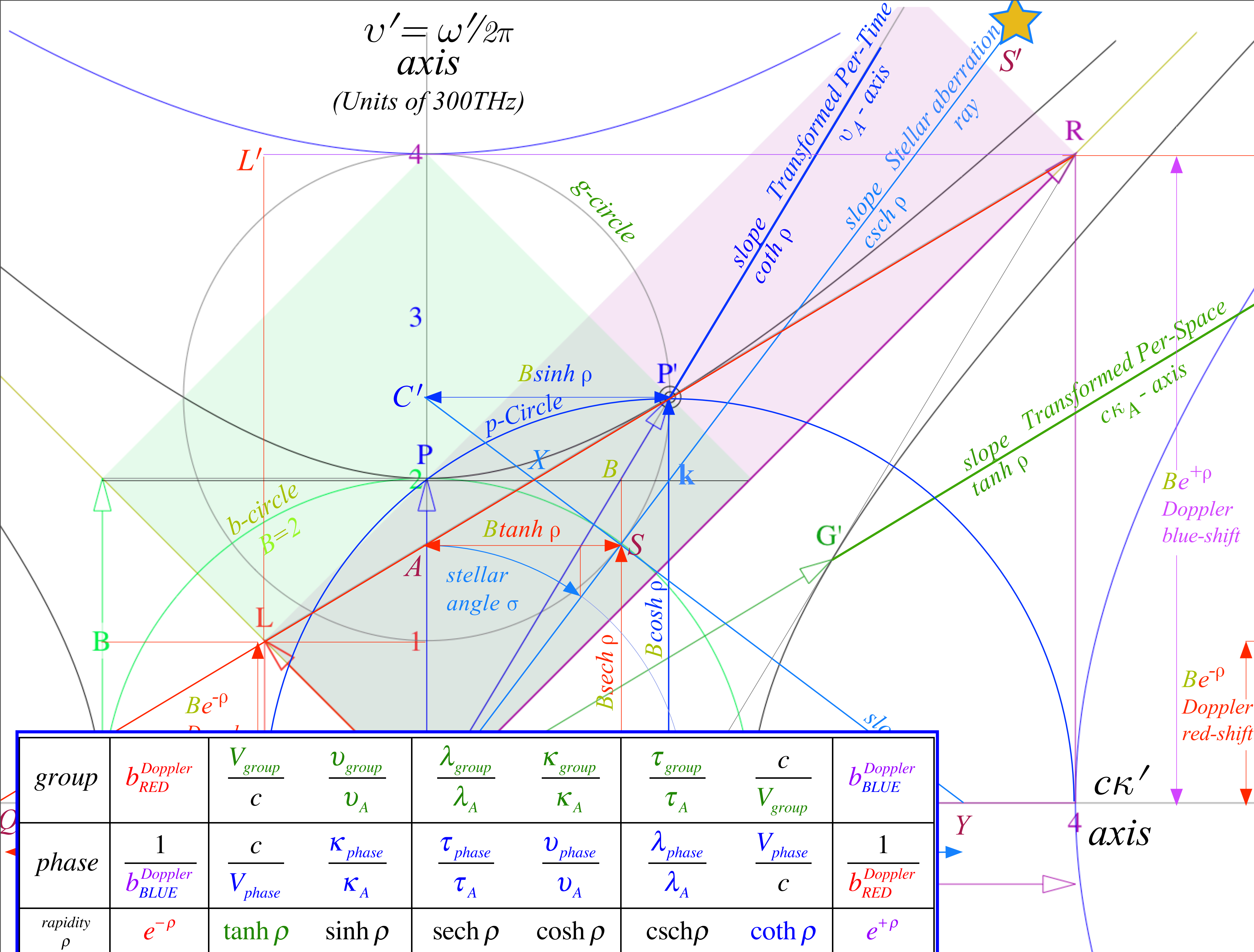
Brief tour of and relativistic mechanics by geometry

→ *Summary of optical wave parameters for relativity and QM*

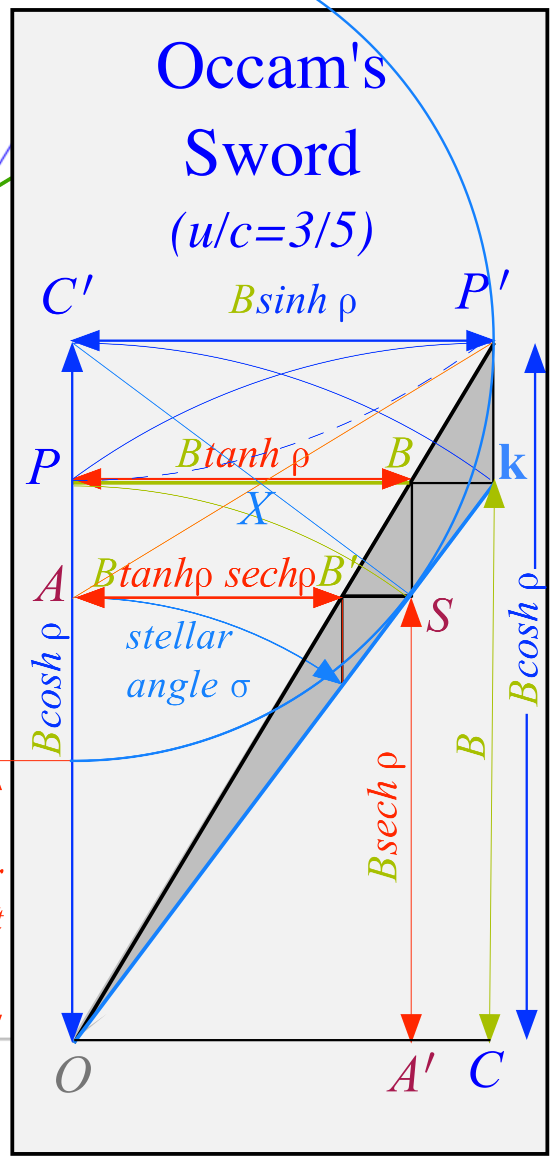
Summary of optical wave parameters for relativity and QM

...and their geometry





An aid to pattern recognition:



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\text{sech}\,\rho$	$\cosh\rho$	$\text{csch}\,\rho$	$\coth\rho$	$e^{+\rho}$
stellar ∀ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta \equiv \frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

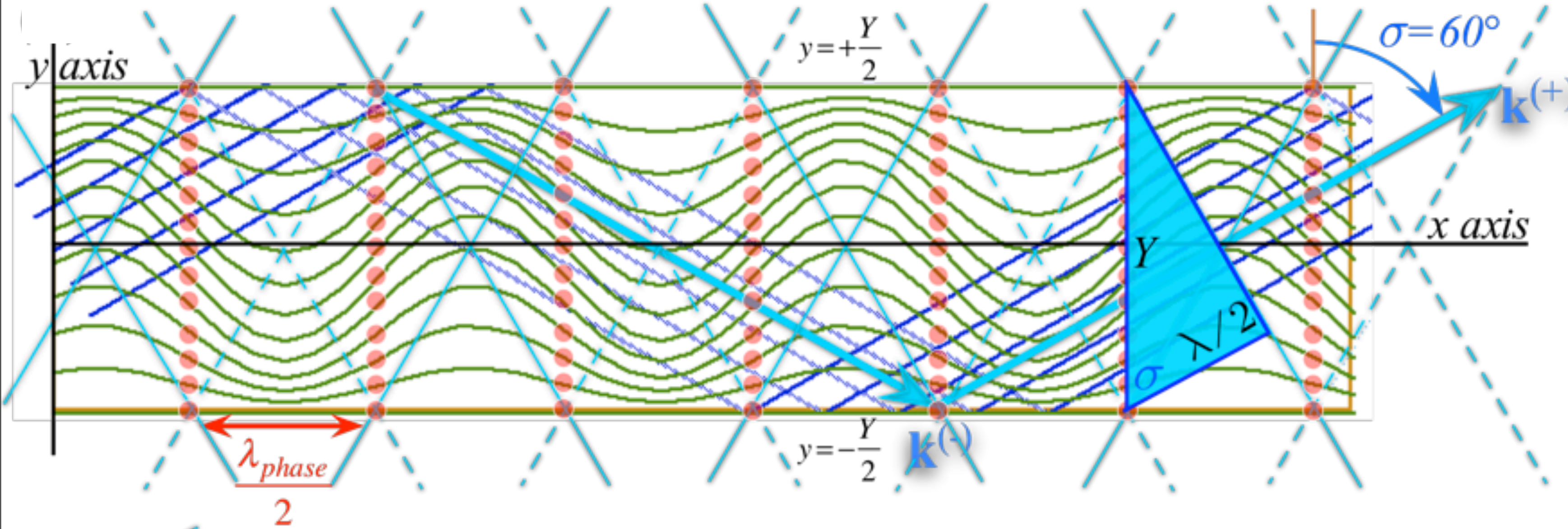
Table of 12 wave parameters (includes inverses) for relativity

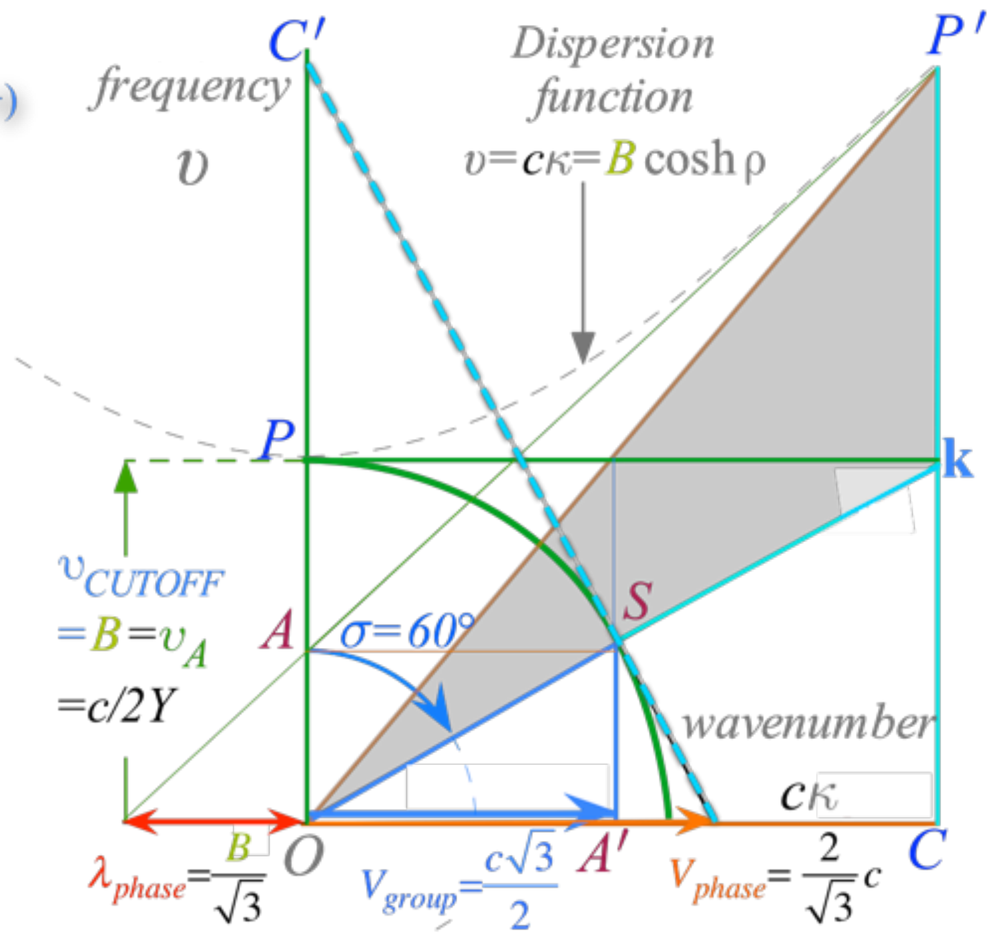
...and values for *u/c=3/5*

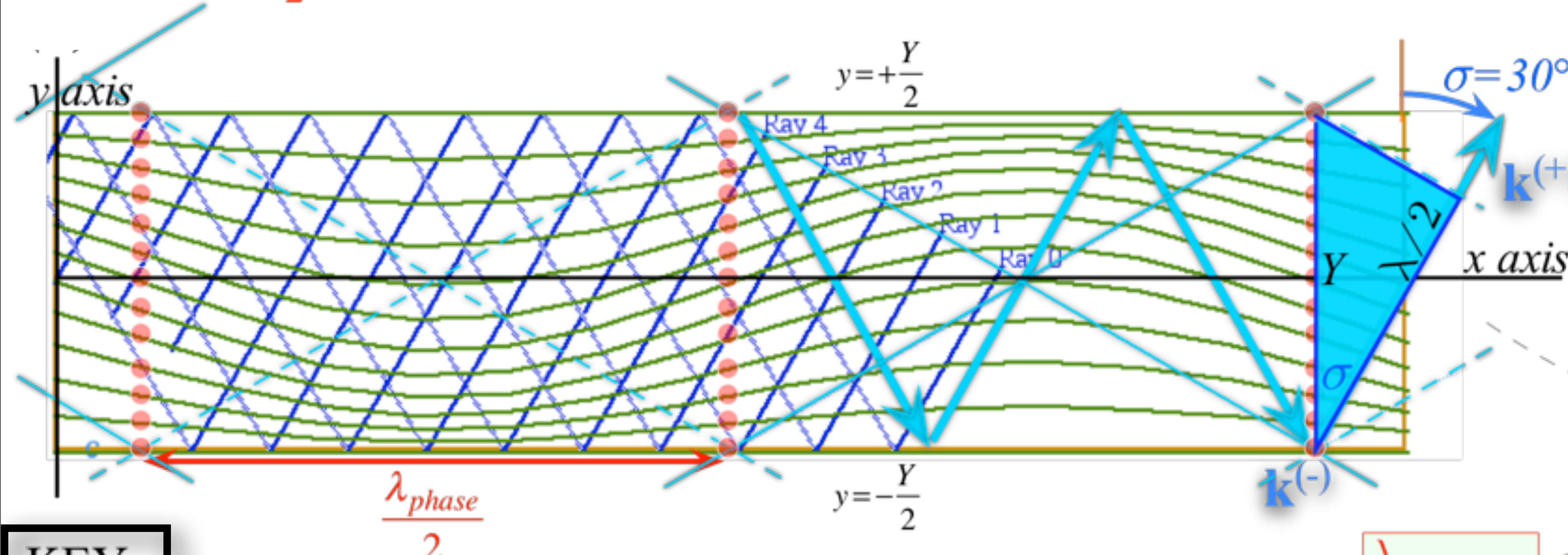
Optical wave guide relativistic geometry aided by Occam's Sword

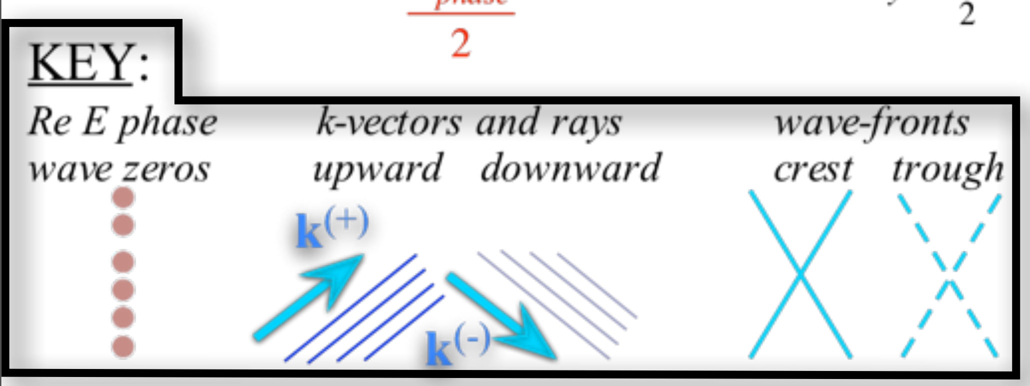
geometry applies to (x,y) space-space
to (k_x,k_y) per-space-per-space
to (x,ct) space-time

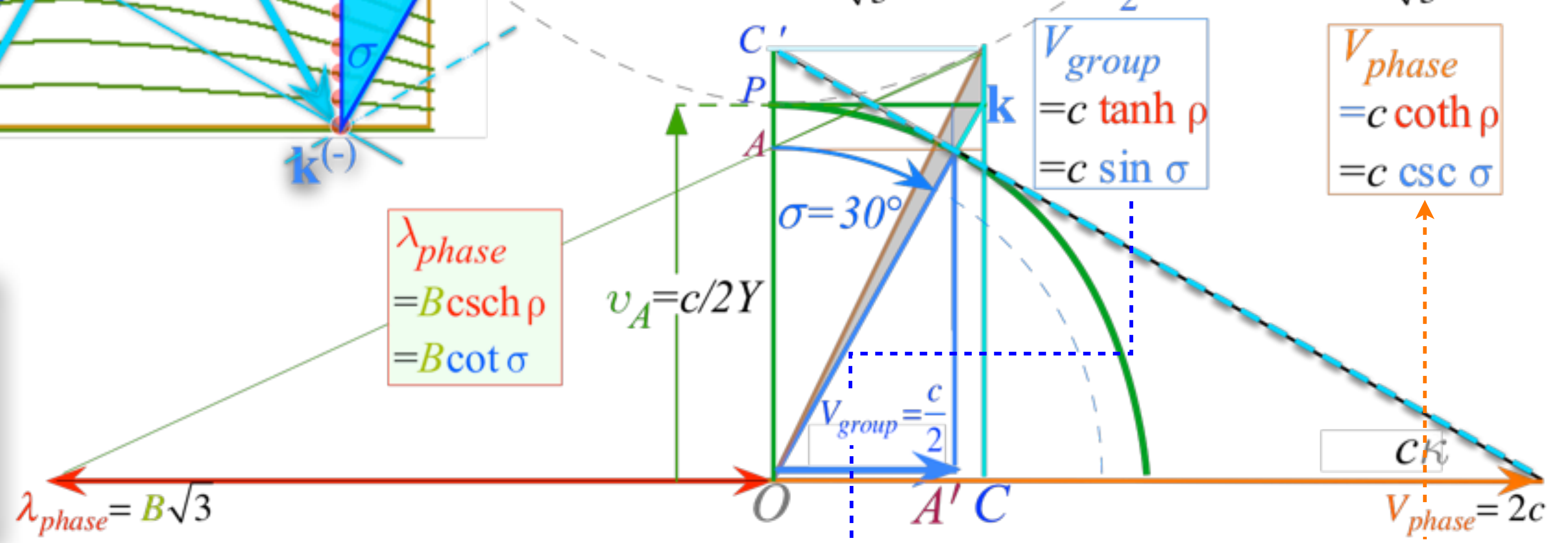
Relativistic mode with near-c $V_{group}=c/2$ and $V_{phase}=2c$. (Low dispersion.)











Example of near-cut-off mode with low $V_{group}=c/2$ and high $V_{phase}=2c$. (High dispersion.)