Group Theory in Quantum Mechanics

Lecture 2 (1.17.13)

Quantum amplitudes, analyzers, and axioms

(Quantum Theory for Computer Age - Ch. 1 of Unit 1)
(Principles of Symmetry, Dynamics, and Spectroscopy - Sec. 1-2 of Ch. 1)

Review: "Abstraction" of bra and ket vectors from a Transformation Matrix
- Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)
- Did Max Planck Goof? What's 1-photon worth?
- Feynman amplitude axiom 1

What comes out of a beam sorter channel or branch-b?
- Sample calculations
- Feynman amplitude axioms 2-3

Beam analyzers: Sorter-unsorters
- The "Do-Nothing" analyzer
- Feynman amplitude axiom 4
- Some "Do-Something" analyzers
  - Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

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"Abstraction" of bra and ket vectors from a Transformation Matrix

Ket or column vectors

Bra or row vectors

Given Transformation Matrix $T_{m,n'}$:

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

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$\Downarrow$

$\Downarrow$

$$|x'\rangle = \begin{pmatrix} \langle x|x'\rangle \\ \langle y|x'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

Abstracting ket $|\,n'\rangle$ state vectors
from
Transformation Matrix
$T_{m,n'} = \langle m|\,n'\rangle$

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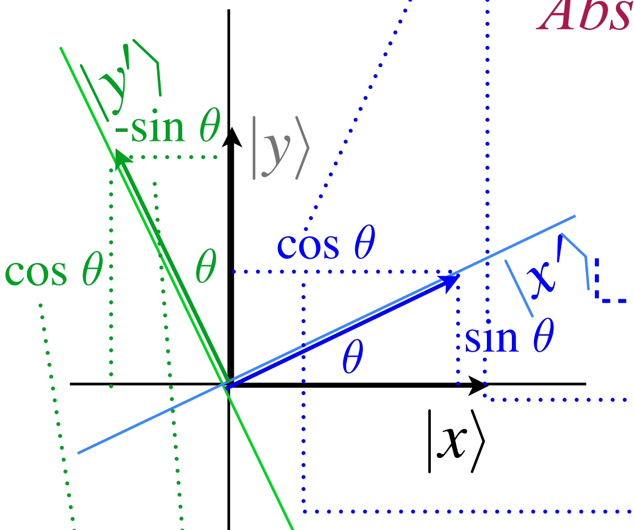
$$|x'\rangle=\begin{pmatrix}\langle x|x'\rangle\\ \langle y|x'\rangle\end{pmatrix}=\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix},\ |y'\rangle=\begin{pmatrix}\langle x|y'\rangle\\ \langle y|y'\rangle\end{pmatrix}=\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}$$

Abstracting ket $|\,n'\rangle$ state vectors
from
Transformation Matrix
$T_{m,n'}=\langle m|\,n'\rangle$



$$|x'\rangle=|x\rangle\langle x|x'\rangle\ +|y\rangle\langle y|x'\rangle$$
$$=|x\rangle(\cos\theta)+|y\rangle(\sin\theta)$$

$$|y'\rangle=|x\rangle\langle x|y'\rangle\ +|y\rangle\langle y|y'\rangle$$
$$=|x\rangle(-\sin\theta)+|y\rangle(\cos\theta)$$

(θ=+30°)-Rotated kets $\{|\mathbf{x}'\rangle,|\mathbf{y}'\rangle\}$ or $\{\mathbf{x}',\mathbf{y}'\}$ represented in page-aligned $\{|\mathbf{x}\rangle,|\mathbf{y}\rangle\}$ basis.

Ket vector algebra has the order of $T_{m,n'}$ transposed

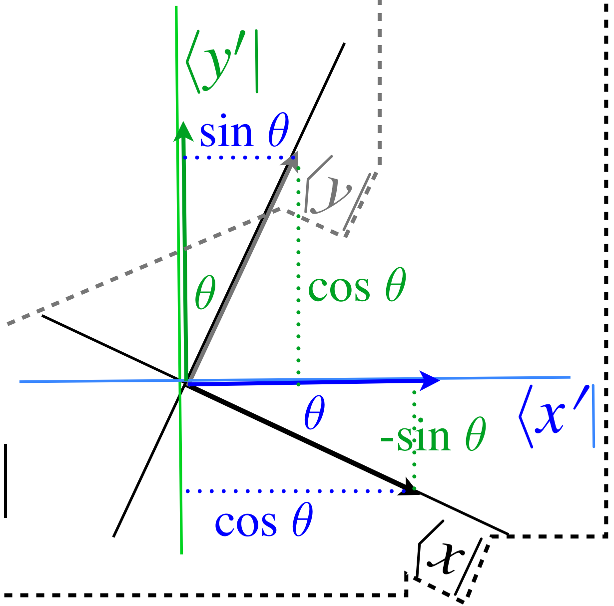
$$|x'\rangle=|x\rangle\langle x|x'\rangle+|y\rangle\langle y|x'\rangle=|x\rangle(\cos\theta)\ \ +|y\rangle(\sin\theta)$$
$$|y'\rangle=|x\rangle\langle x|y'\rangle+|y\rangle\langle y|y'\rangle=|x\rangle(-\sin\theta)+|y\rangle(\cos\theta)$$

Bra or row vectors

⇒ $\langle x|=(\langle x|x'\rangle\ \ \langle x|y'\rangle)=(\cos\theta\ \ -\sin\theta)$

⇒ $\langle y|=(\langle y|x'\rangle\ \ \langle y|y'\rangle)=(\sin\theta\ \ \cos\theta)$

Abstracting bra $\langle m|$ state vectors
from
Transformation Matrix
$T_{m,n'}=\langle m|\,n'\rangle$



$$\langle y|=\ \langle y|x'\rangle\langle x'|+\langle y|y'\rangle\langle y'|$$
$$=\ (\sin\theta)\langle x'|+(\cos\theta)\langle y'|$$

$$\langle x|=\ \langle x|x'\rangle\langle x'|+\ \langle x|y'\rangle\langle y'|$$
$$=(\cos\theta)\langle x'|+(-\sin\theta)\langle y'|$$

(θ=-30°)-Rotated bras $\{\langle\mathbf{x}|,\langle\mathbf{y}|\}$ or $\{\mathbf{x},\mathbf{y}\}$ represented in page-aligned $\{|\mathbf{x}'\rangle,|\mathbf{y}'\rangle\}$ basis.

Bra vector algebra has the same order as $T_{m,n'}$

$$\langle x|=\ \langle x|x'\rangle\langle x'|+\ \langle x|y'\rangle\langle y'|=(\cos\theta)\langle x'|+(-\sin\theta)\langle y'|$$
$$\langle y|=\ \langle y|x'\rangle\langle x'|+\langle y|y'\rangle\langle y'|=\ (\sin\theta)\langle x'|+(\cos\theta)\ \langle y'|$$

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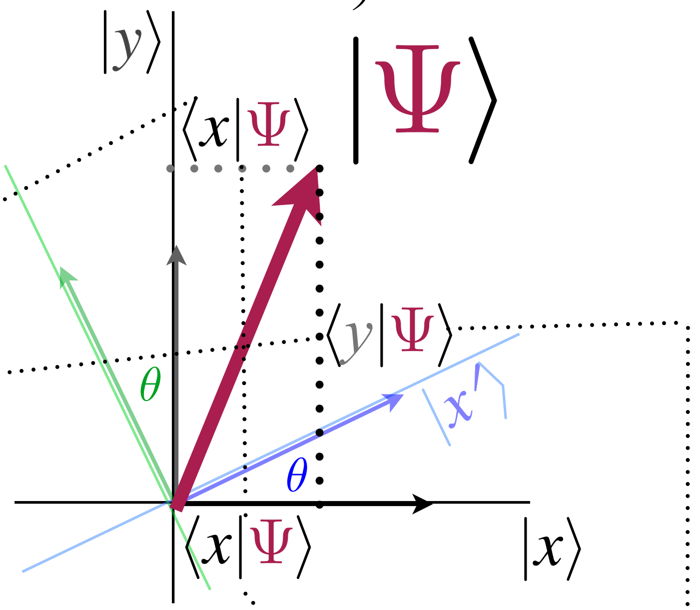
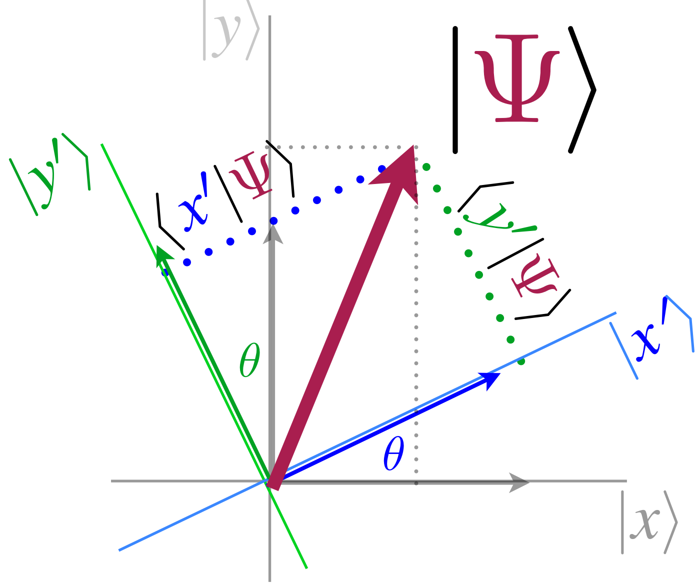
Transformation matrix $T_{m,n'}=\langle m|\,n'\rangle$ is array of dot or *scalar products* (dot products) of unit vectors or *direction cosines*.

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} (\mathbf{x}\bullet\mathbf{x'}) & (\mathbf{x}\bullet\mathbf{y'}) \\ (\mathbf{y}\bullet\mathbf{x'}) & (\mathbf{y}\bullet\mathbf{y'}) \end{pmatrix}$$

$\{\langle x|,\langle y|\}$
components
of $|\Psi\rangle$:
$\langle x|\Psi\rangle=\Psi_x$
$\langle y|\Psi\rangle=\Psi_y$





$\{\langle x'|,\langle y'|\}$
components
of $|\Psi\rangle$:
$\langle x'|\Psi\rangle=\Psi_{x'}$
$\langle y'|\Psi\rangle=\Psi_{y'}$

Any state $|\Psi\rangle$ can be expanded in any basis $\{\langle x|,\langle y|\}$, or $\{\langle x'|,\langle y'|\}$, ...*etc*.

$$|\Psi\rangle=|x\rangle\langle x|\Psi\rangle + |y\rangle\langle y|\Psi\rangle = |x'\rangle\langle x'|\Psi\rangle + |y'\rangle\langle y'|\Psi\rangle$$

Transformation matrix $T_{m,n'}$ relates $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$ amplitudes to $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$.

$$\begin{pmatrix} \langle x|\Psi\rangle \\ \langle y|\Psi\rangle \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle \end{pmatrix} \quad \text{or:} \quad \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix} = \begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} \begin{pmatrix} \Psi_{x'} \\ \Psi_{y'} \end{pmatrix}$$

Hybrid
Gibbs-Dirac
notation
(Ug-ly!)

Proof: $\langle x|=\langle x|x'\rangle\langle x'|+\langle x|y'\rangle\langle y'|$ implies: $\langle x|\Psi\rangle=\langle x|x'\rangle\langle x'|\Psi\rangle+\langle x|y'\rangle\langle y'|\Psi\rangle$

$\langle y|=\langle y|x'\rangle\langle x'|+\langle y|y'\rangle\langle y'|$ implies: $\langle y|\Psi\rangle=\langle y|x'\rangle\langle x'|\Psi\rangle+\langle y|y'\rangle\langle y'|\Psi\rangle$

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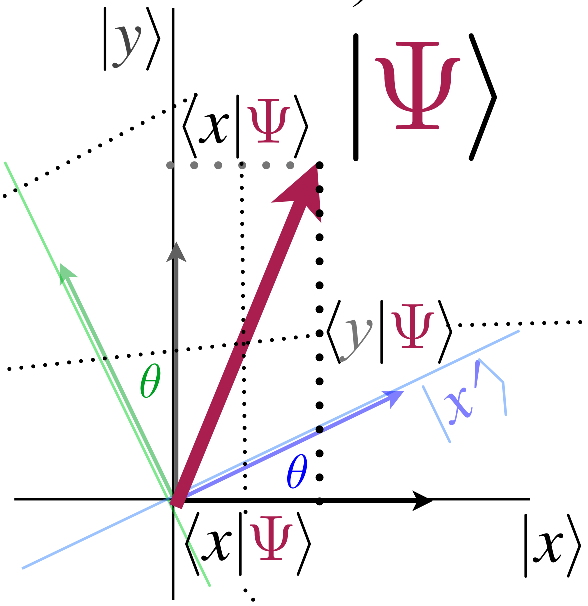

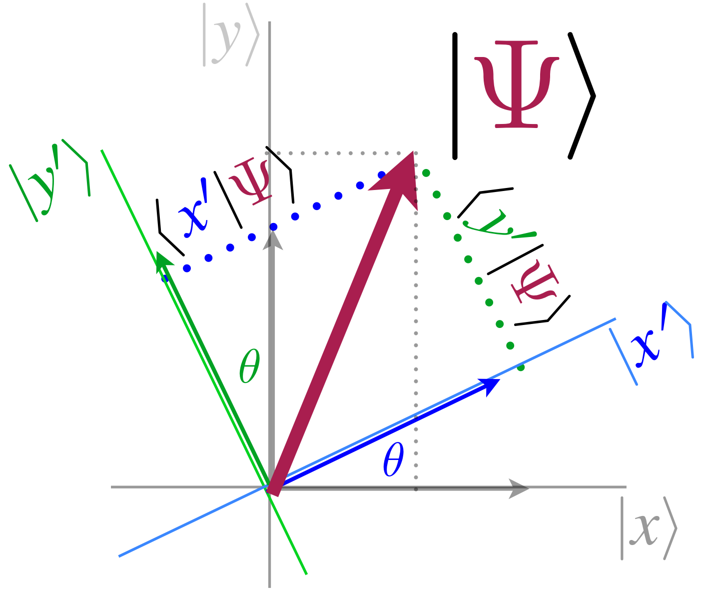

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Hybrid
Gibbs-Dirac
notation
(Ug-ly!)

Inverse $(\dagger=\top *=-1)$ matrix $T_{n',m}$ relates $\{\langle x'|\Psi\rangle,\langle y'|\Psi\rangle\}$ amplitudes to $\{\langle x|\Psi\rangle,\langle y|\Psi\rangle\}$.

$$\begin{pmatrix} \langle x'|\Psi\rangle \\ \langle y'|\Psi\rangle \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \langle x|\Psi\rangle \\ \langle y|\Psi\rangle \end{pmatrix} \quad \text{or:} \quad \begin{pmatrix} \Psi_{x'} \\ \Psi_{y'} \end{pmatrix} = \begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} \begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix}$$

Hybrid
Gibbs-Dirac
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Planck's energy and *N*-**quanta** *(Cavity/Beam of volume V with wave mode of frequency* $\omega=2\pi\nu$*)*

Planck axiom: ***E***-field energy density *U* in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

$h=2\pi\hbar=6.6310^{-34}Js$ *Planck constant*

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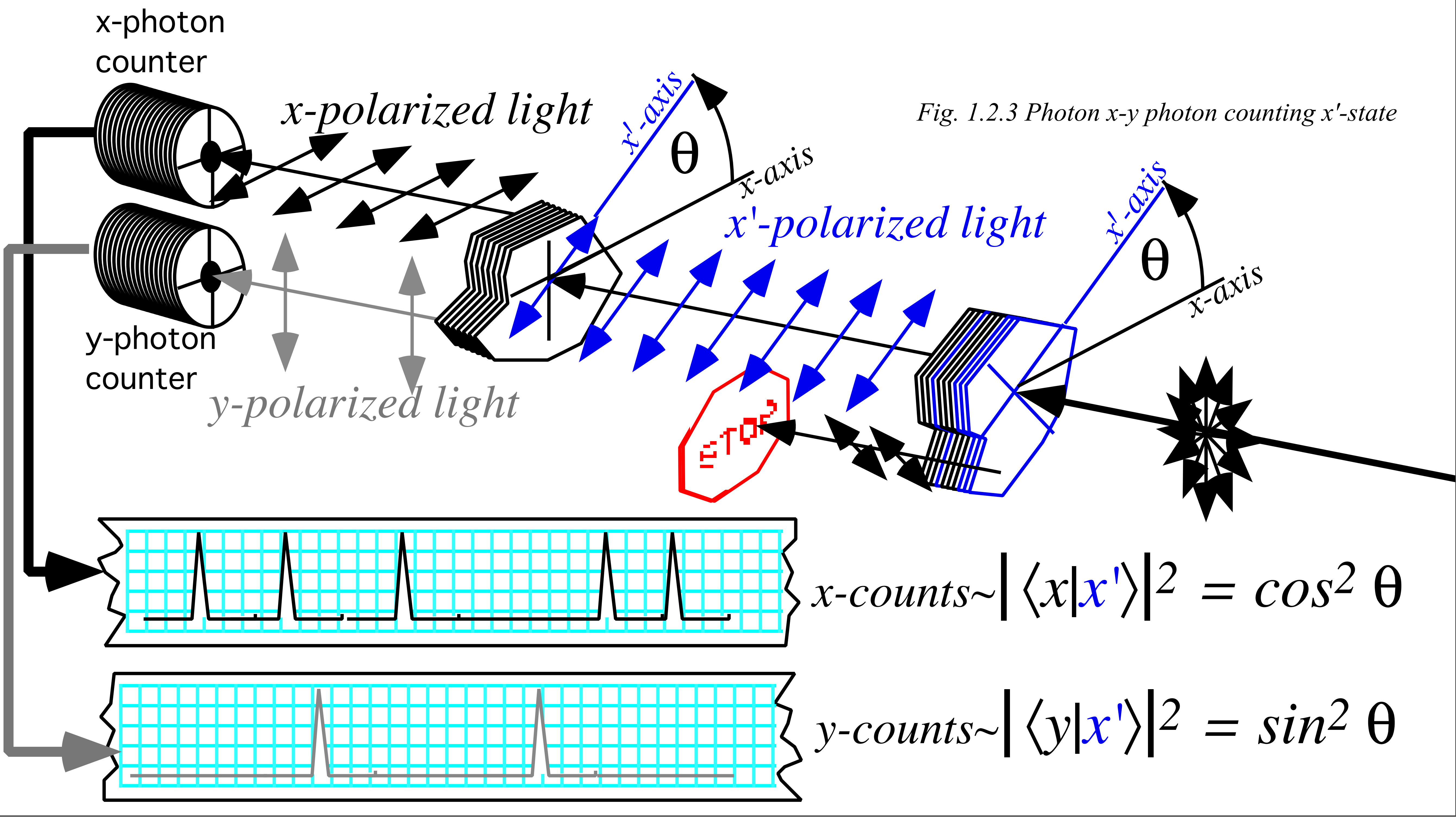

Fig. 1.2.3 Photon x-y photon counting x'-state

$$x\text{-counts}\sim\left|\langle x|x'\rangle\right|^2=\cos^2\theta$$

$$y\text{-counts}\sim\left|\langle y|x'\rangle\right|^2=\sin^2\theta$$

Planck's energy and *N*-quanta *(Cavity/Beam of volume V with wave mode of frequency $\omega=2\pi\nu$)*

Planck axiom: ***E***-field energy density U in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

E-field vector $(E_x, E_y) = f\cdot(\Psi_x, \Psi_y)$ where *quantum field proportionality constant is f*.

$h=2\pi\hbar=6.63\cdot10^{-34}Js$ *Planck constant*

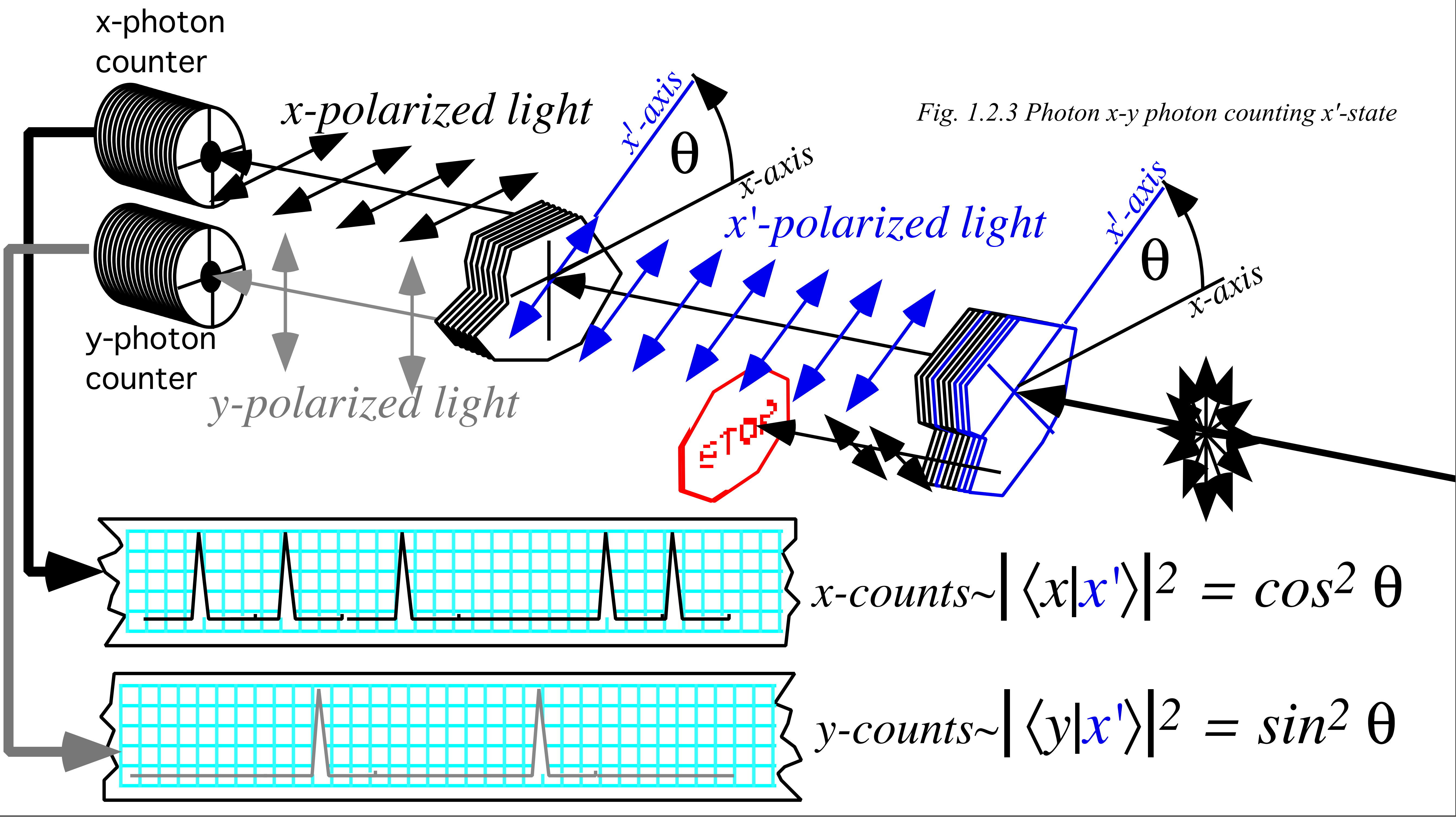

Fig. 1.2.3 Photon x-y photon counting x'-state

$$x\text{-counts}\sim\left|\langle x|x'\rangle\right|^2 = \cos^2\theta$$

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Planck's energy and *N*-quanta *(Cavity/Beam of volume V with wave mode of frequency $\omega=2\pi\nu$)*

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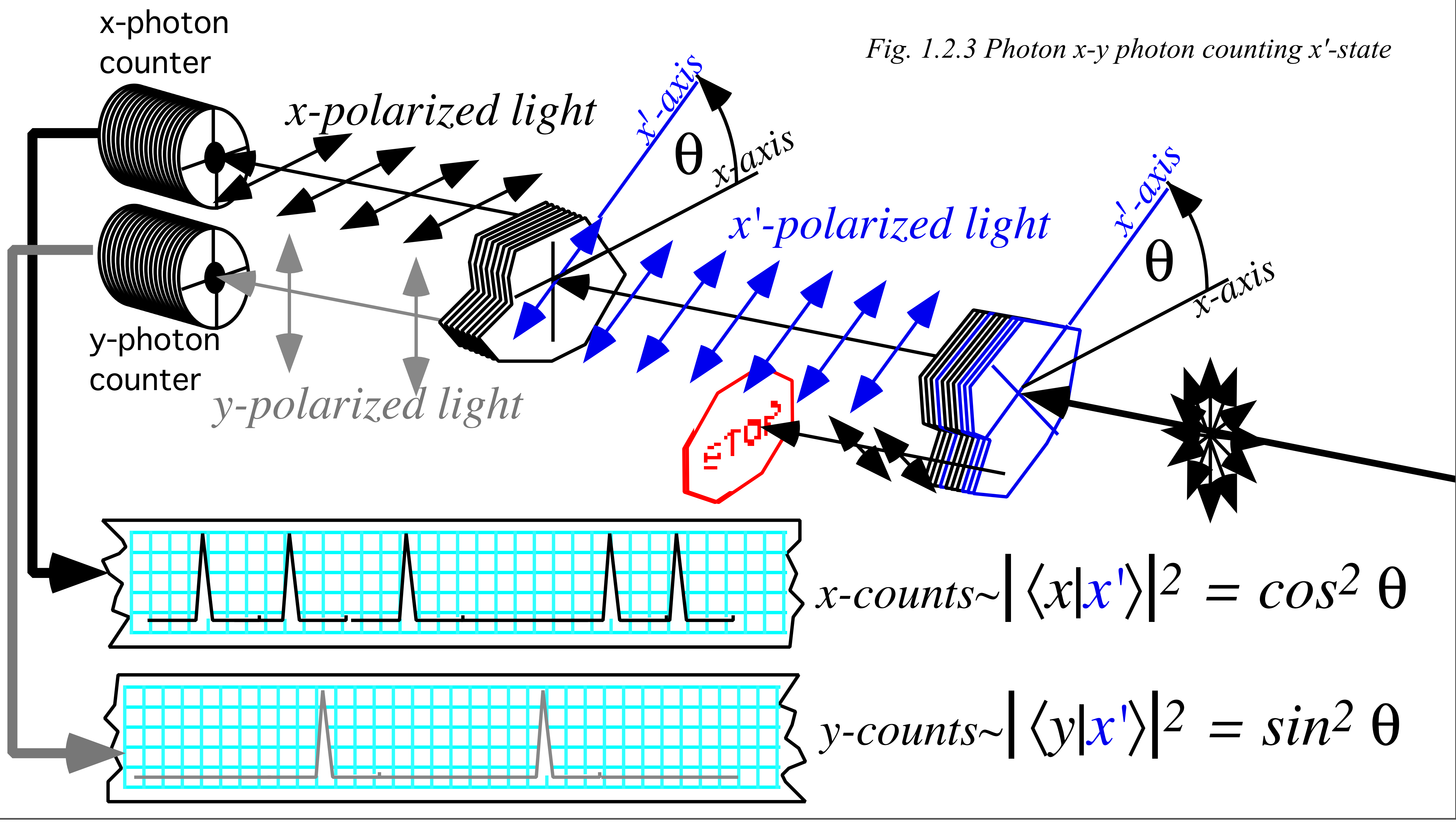
E-field vector $(E_x, E_y)=f\cdot(\Psi_x, \Psi_y)$ where *quantum field proportionality constant is f*.

$$\begin{pmatrix}E_x\\E_y\end{pmatrix}=\begin{pmatrix}E_x(0)e^{ikz-i\omega t}\\E_y(0)e^{ikz-i\omega t}\end{pmatrix}\cong\begin{pmatrix}E_x(0)e^{-i\omega t}\\E_y(0)e^{-i\omega t}\end{pmatrix}=f\begin{pmatrix}\Psi_x\\\Psi_y\end{pmatrix}$$

$h=2\pi\hbar=6.63\cdot10^{-34}Js$ Planck constant



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Planck's energy and *N*-quanta *(Cavity/Beam of volume V with wave mode of frequency ω=2πν)*

Planck axiom: **E**-field energy density U in cavity/beam mode-ω is: $U=N\hbar\omega/V=Nh\nu/V$ *(N "photons")*

E-field vector $(E_x, E_y)=f\cdot(\Psi_x, \Psi_y)$ where *quantum field proportionality constant is* $f=f(\hbar,\omega,V,\varepsilon_0)$.

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix}=\begin{pmatrix} E_x(0)e^{ikz-i\omega t} \\ E_y(0)e^{ikz-i\omega t} \end{pmatrix}\cong\begin{pmatrix} E_x(0)e^{-i\omega t} \\ E_y(0)e^{-i\omega t} \end{pmatrix}=f\begin{pmatrix} \Psi_x \\ \Psi_y \end{pmatrix}$$

$h=2\pi\hbar=6.626\ 075\cdot 10^{-34}Js$	*Planck constant*
$c=2.997\ 924\ 58\cdot 10^{8}\ ms^{-1}$	*Light speed*
$\varepsilon_0=8.854\cdot 10^{-12}C^2N^{-1}m^{-2}$	*Electrostatic constant*

Coulomb constant: $k=1/4\pi\varepsilon_0$

$=9\cdot 10^{9}\ J/C$

Ψ-amplitude-squares sum to *exciton-number N*. (...or *photon-number N*)

$$N=|\Psi_x|^2+|\Psi_y|^2=\langle x|\Psi\rangle^*\langle x|\Psi\rangle+\langle y|\Psi\rangle^*\langle y|\Psi\rangle$$

Quantum em wave theory

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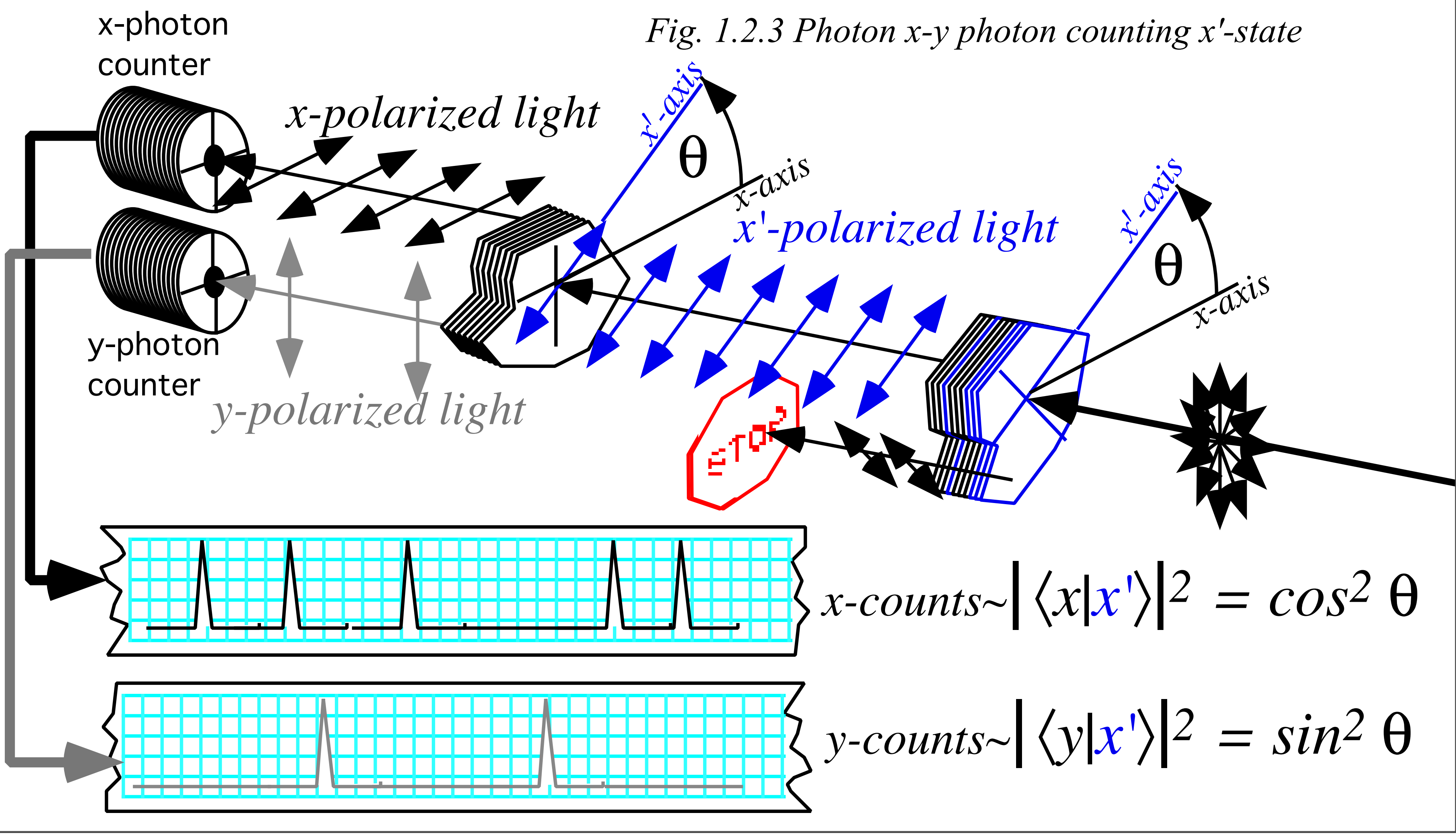

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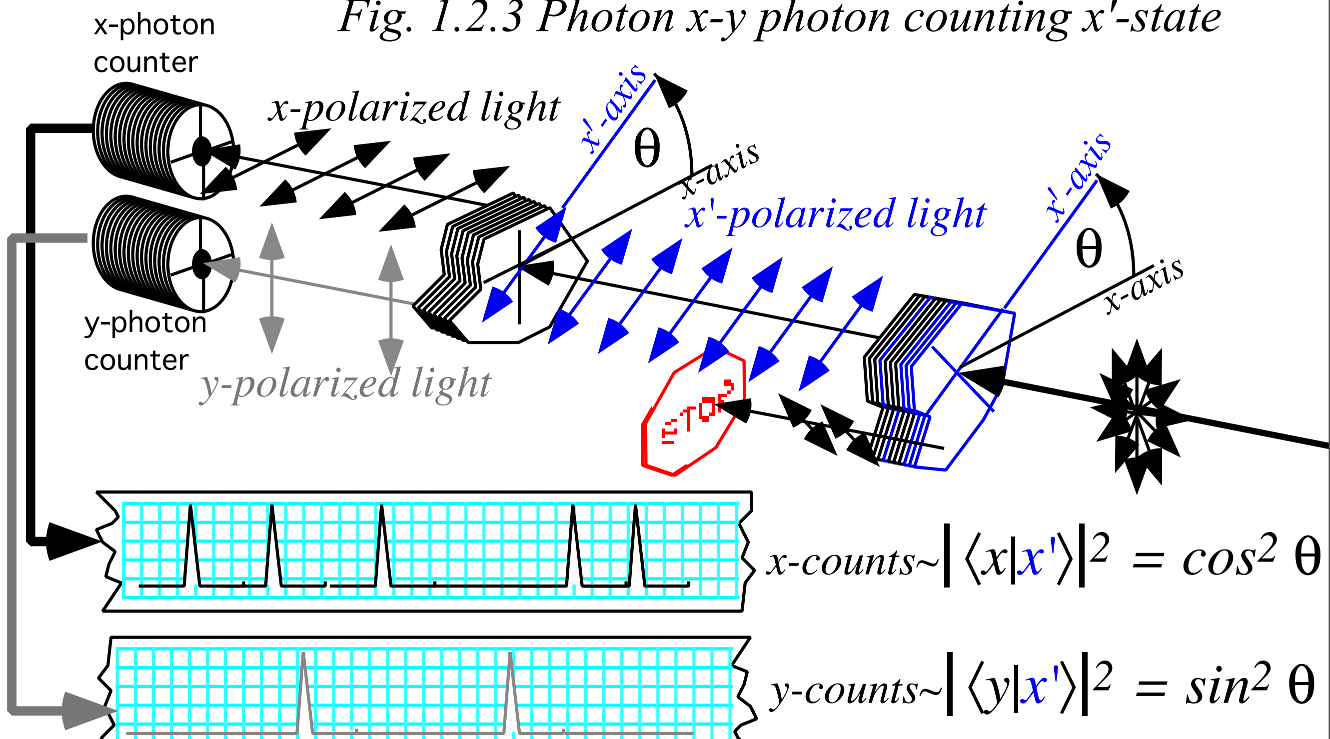
Quantum em wave theory

Poynting energy flux S(J/m²s) or *energy density U(J/m³)* of light beam.

$$S=cU,\ \text{ where: }\ U=\varepsilon_0\left(|E_x|^2+|E_y|^2\right)=\varepsilon_0\left(E_x^*E_x+E_y^*E_y\right)=\varepsilon_0\left(E_x(0)^2+E_y(0)^2\right)$$

Classical em wave theory

Fig. 1.2.3 Photon x-y photon counting x'-state



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Quantum em wave theory

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Classical em wave theory

Equate U to Planck's *N-"photon"* quantum energy density $N\hbar\omega/V$

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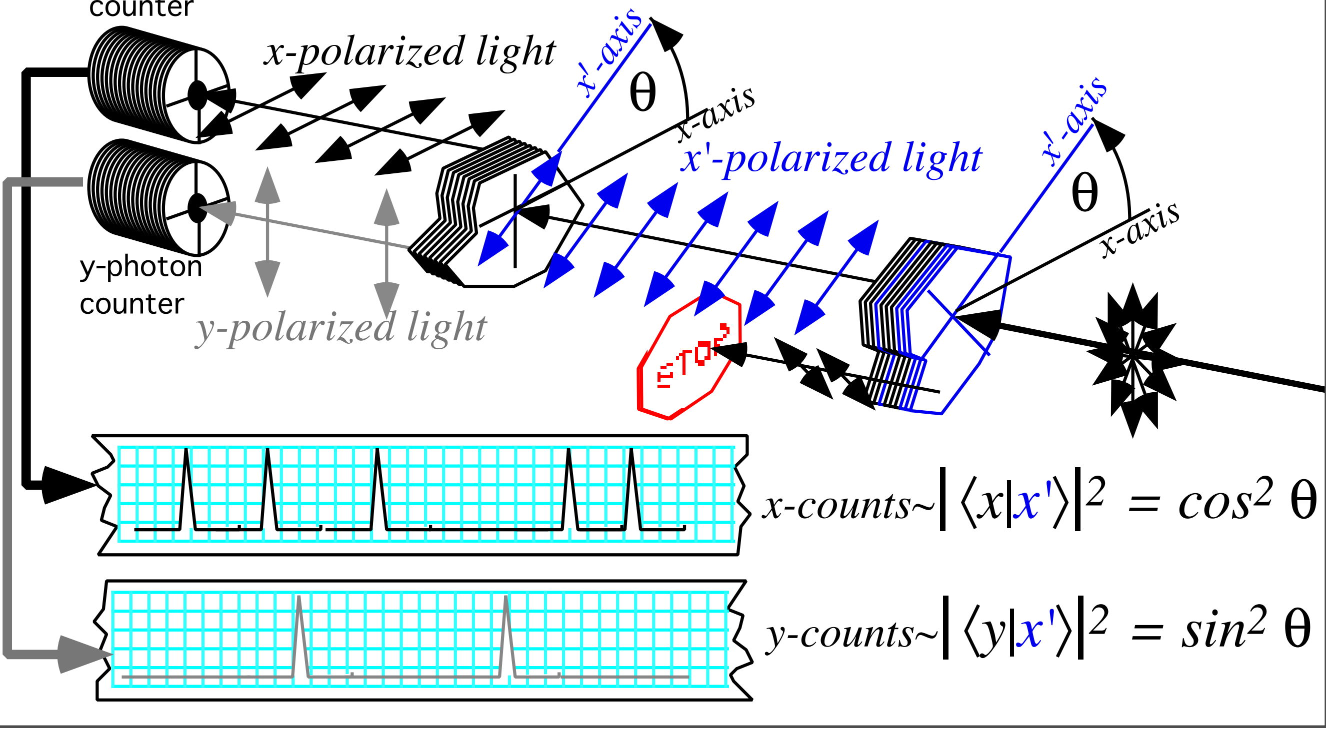

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$h=2\pi\hbar=6.626\,075\cdot10^{-34}Js$ *Planck constant*

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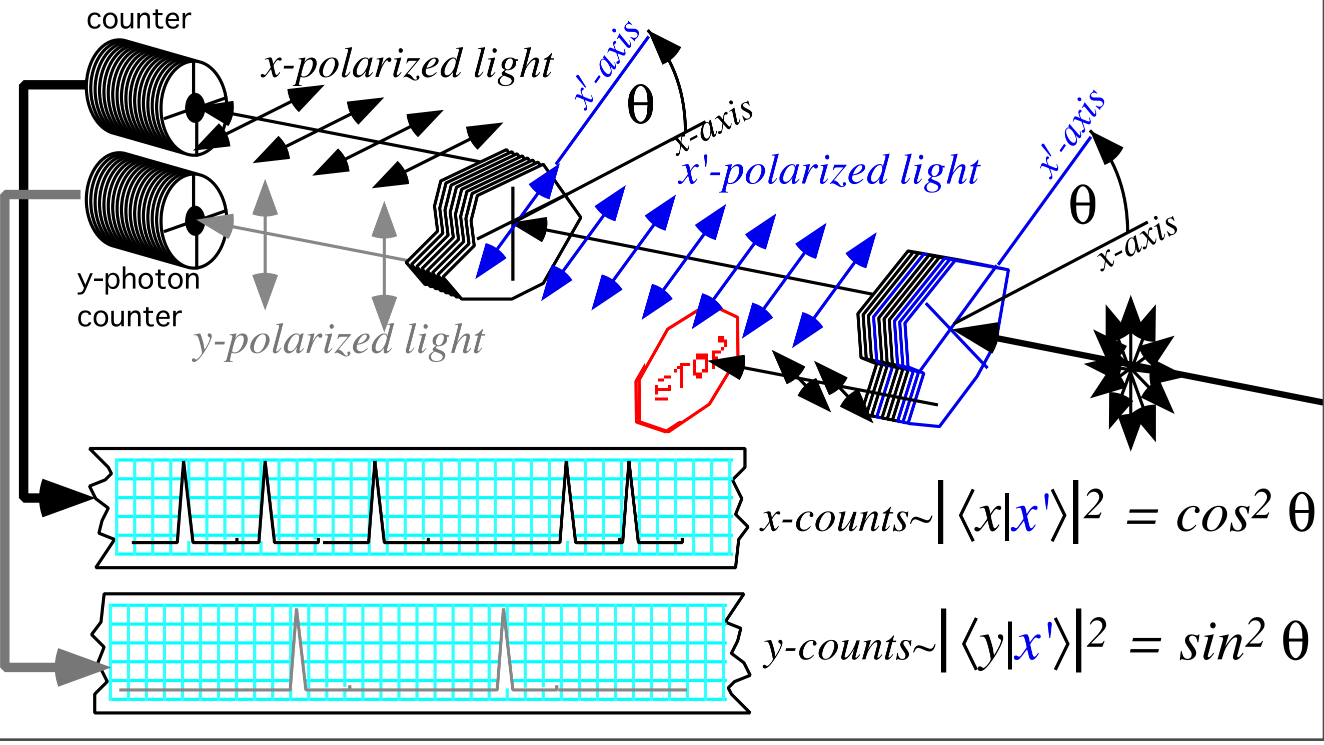

Fig. 1.2.3 Photon x-y photon counting x'-state

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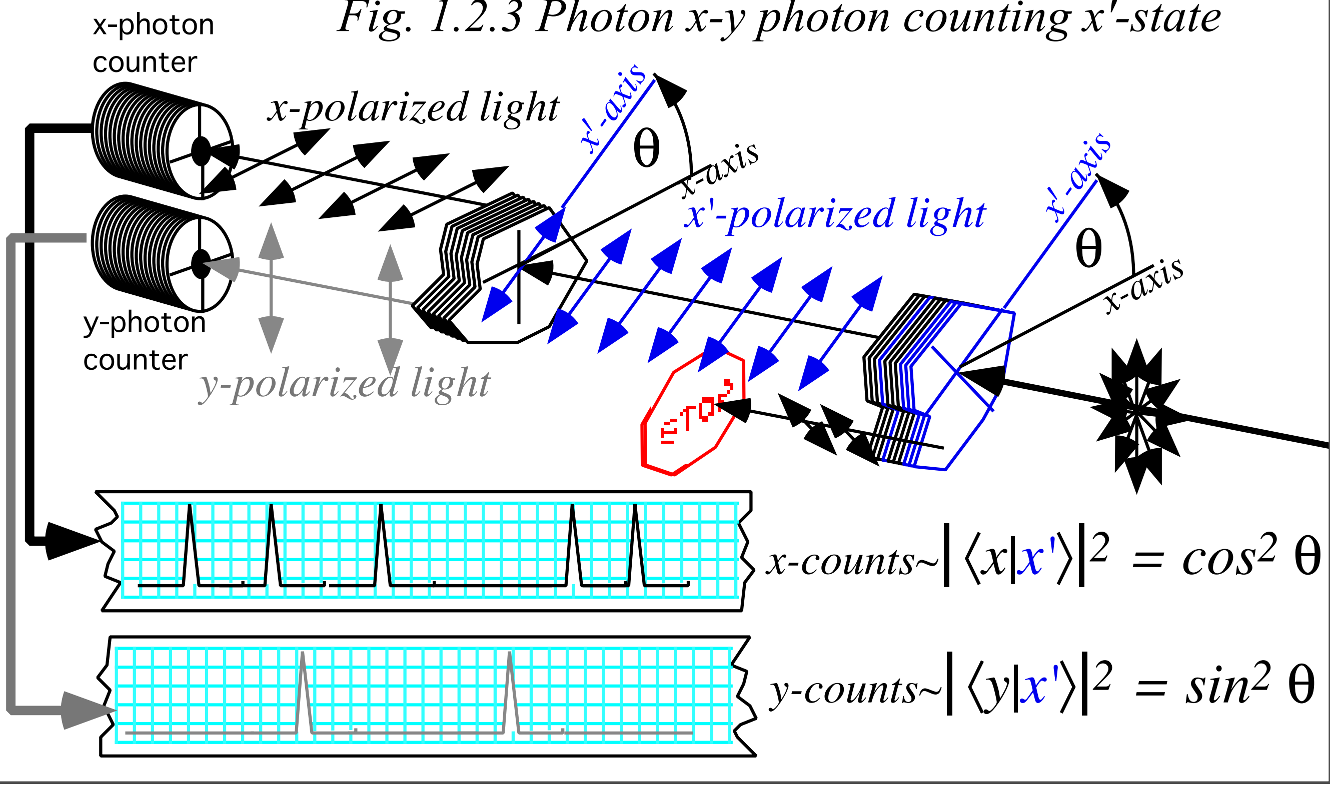

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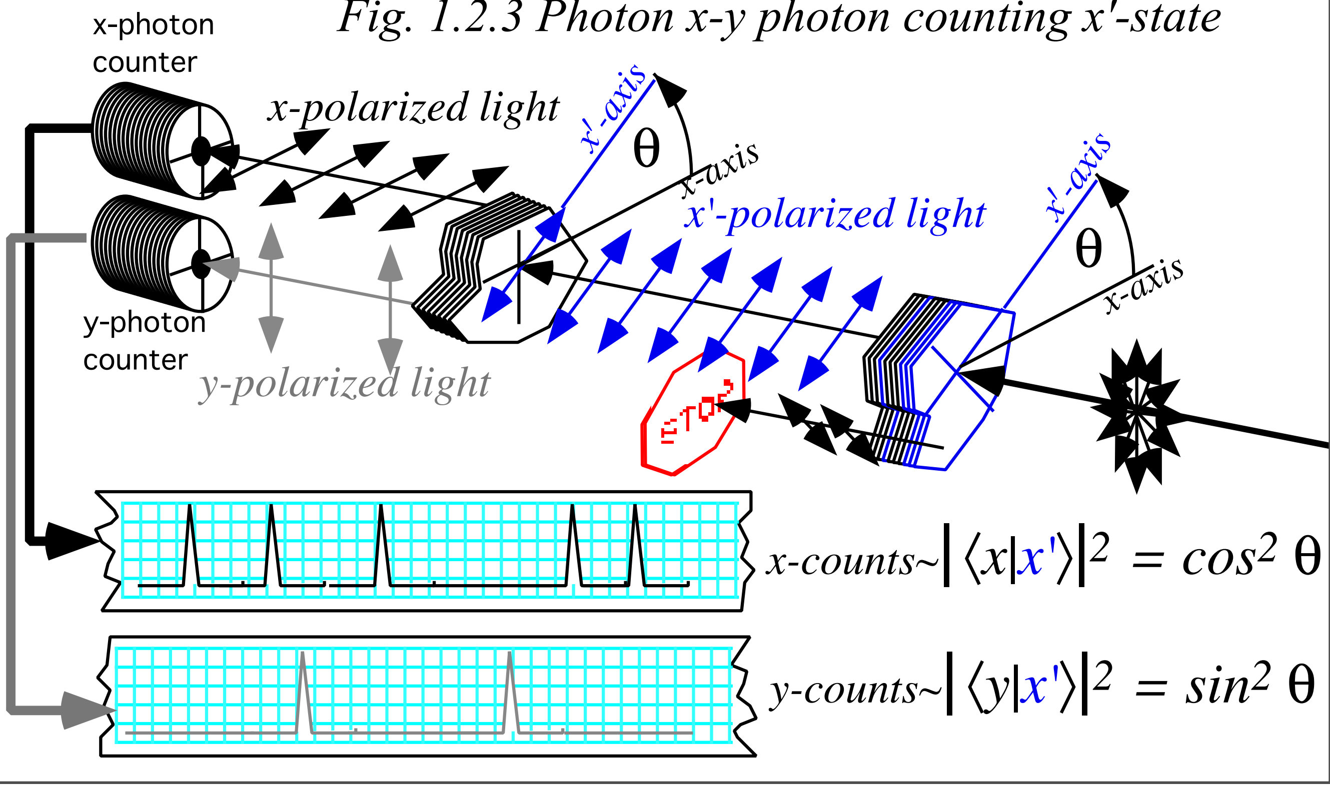

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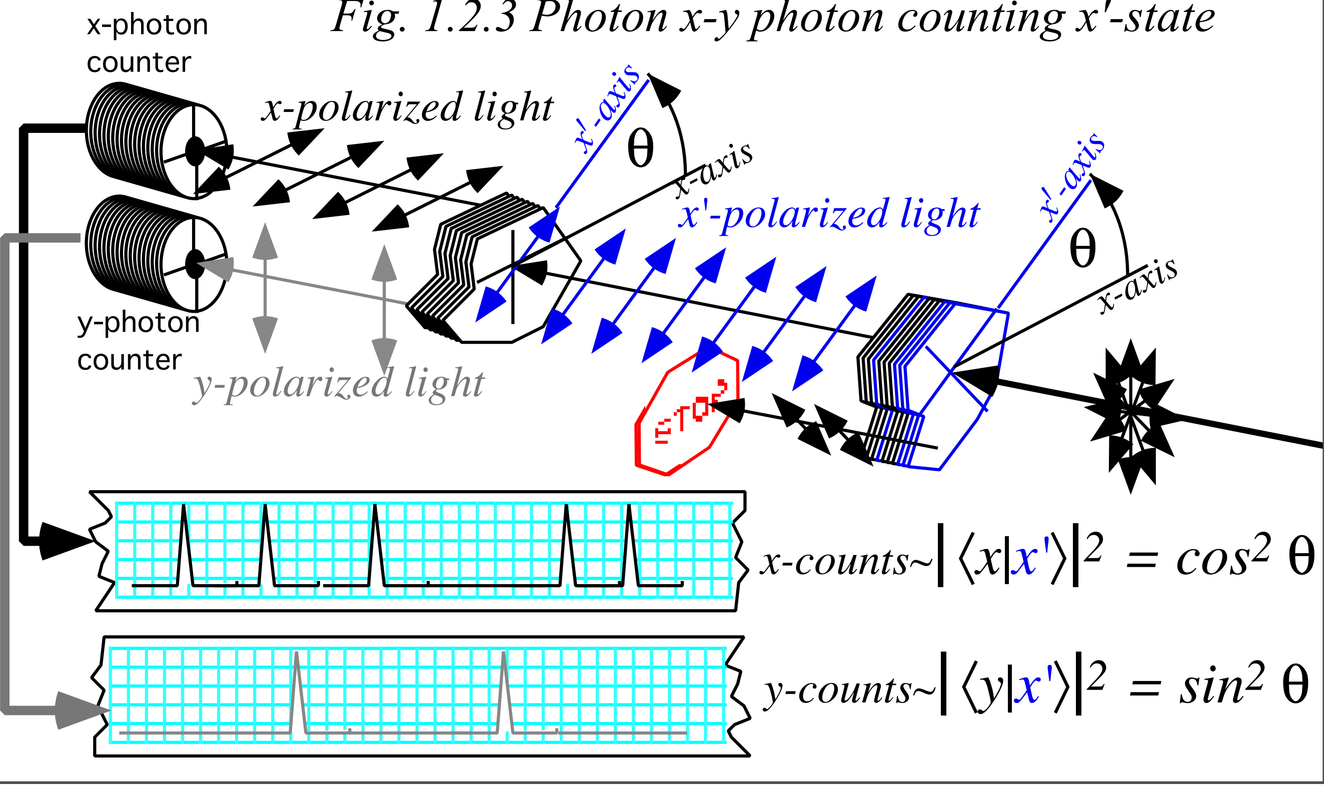

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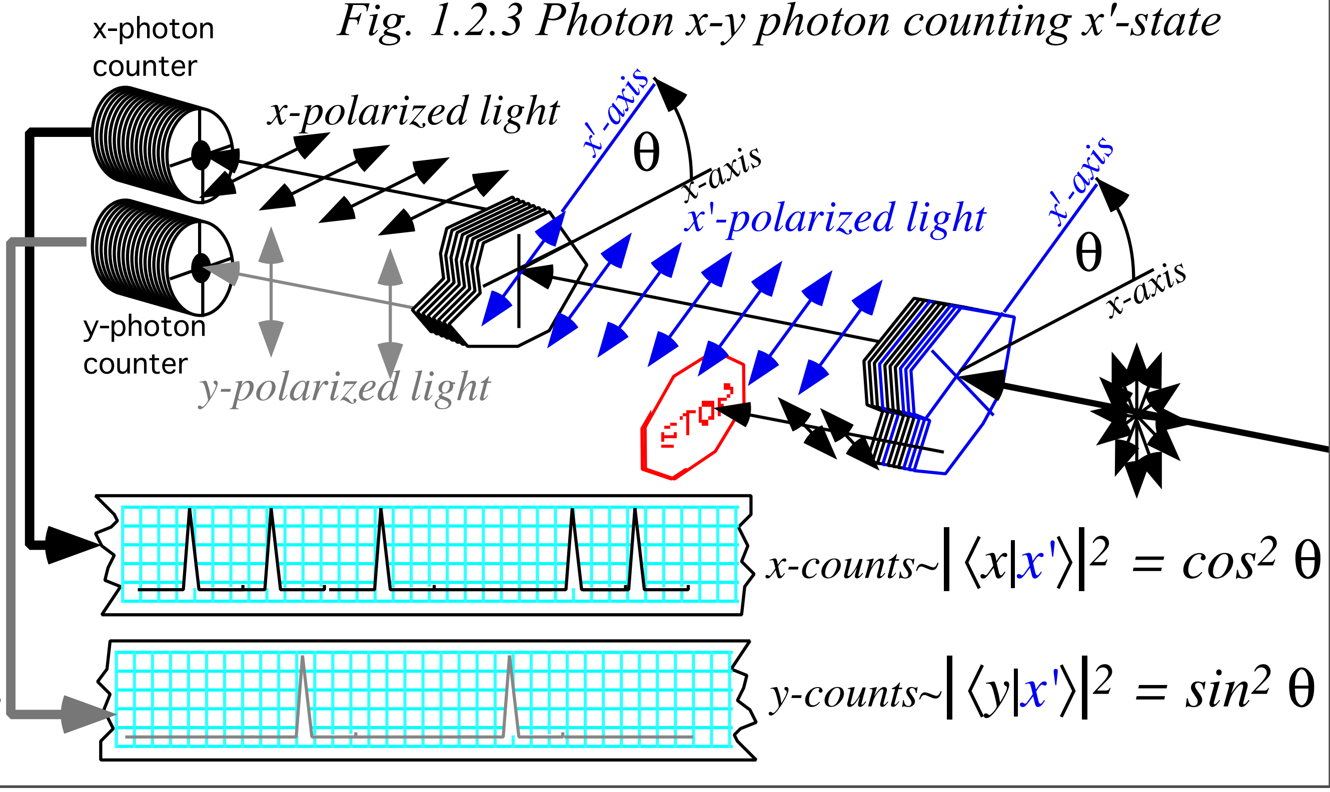

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Energy per $N=10^6$ photons $= 10^6\ h\upsilon = 10^6 6.63 \cdot 10^{-34} Js \cdot 600 \cdot 10^{12}/s = 4.0 \cdot 10^{-13} J = 2.5 MeV$

Was Planck's linear-in-frequency-υ energy axiom a Goof ?

Not at all. It's really not linear-in-frequency after all.

The $\varepsilon_0 \, |\mathbf{E}|^2 = h \, N \cdot \upsilon$ axiom IS in fact a product of TWO frequencies!

The 2nd "frequency" is COUNT RATE N.

As shown in Unit 2: Both frequencies transform by Relativistic Doppler factor $e^{\pm\rho}$:

Coherent frequency is light quality: $\upsilon' = e^{\pm\rho} \, \upsilon$. Incoherent frequency is light quantity: $N' = e^{\pm\rho} \, N$.

This would imply that the $|\mathbf{E}|$-field also transforms like a frequency: $|\mathbf{E}|' = = e^{\pm\rho} \, |\mathbf{E}|$.

Indeed, E-field amplitude is a frequency, too!

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Feynman amplitude axiom 1

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

=Amplitude of state-j after state-k′ is forced to choose from available m-type states

(1) The probability axiom

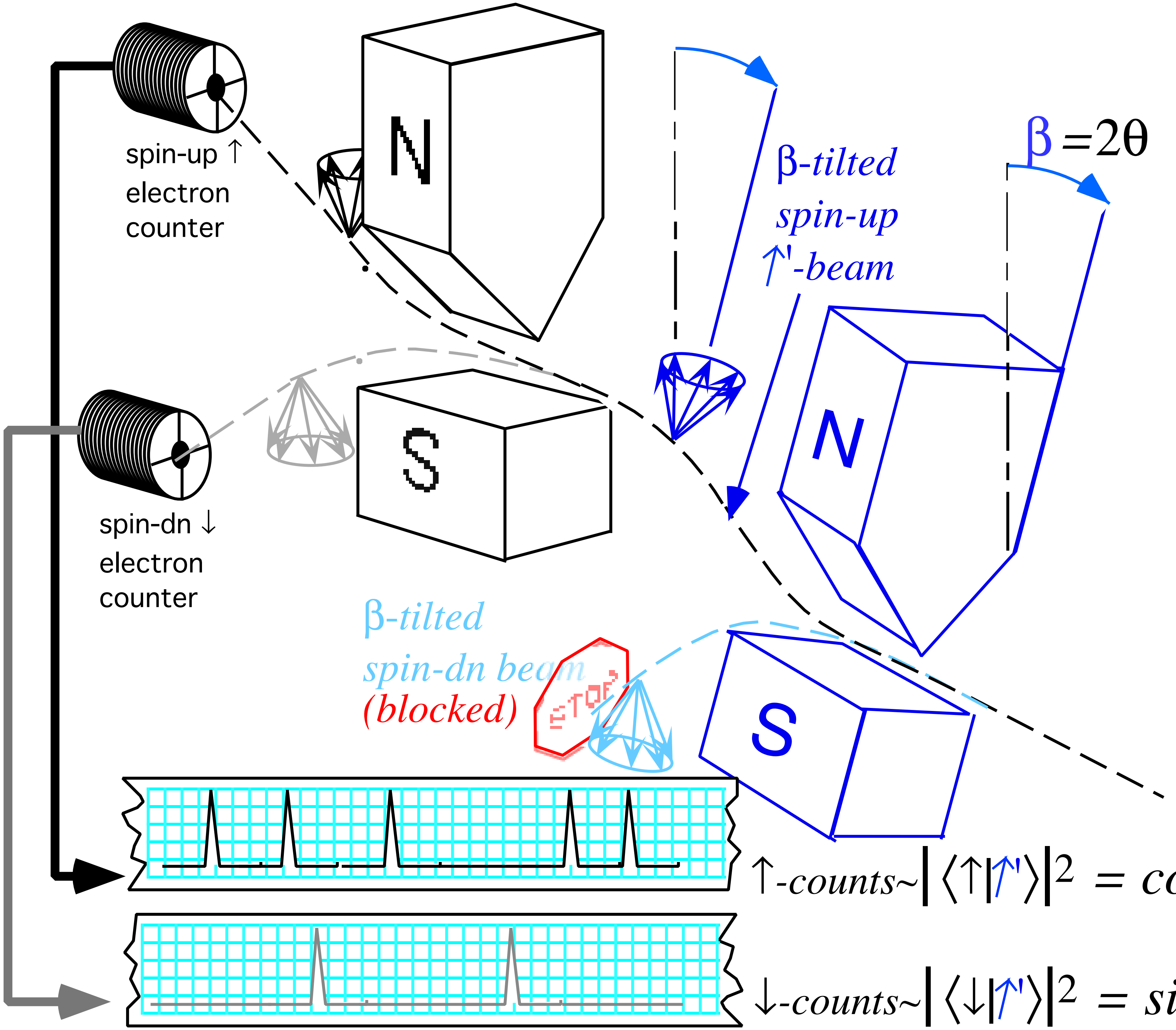
The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

Axiom 1: The absolute square $\left|\langle j | k' \rangle\right|^2 = \langle j | k' \rangle^* \langle j | k' \rangle$ *gives probability for occurrence in state-j of a system that started in state-k'=1',2',..,or n' from one sorter and then was forced to choose between states j=1,2,...,n by another sorter.*

probability

Amplitude axioms apply to all intensity-conserving systems

This includes, first of all, spin-1/2 electron, proton, ..., ^{13}C, ... particles (Fermions)



↑-counts~ $\left|\langle\uparrow|\uparrow'\rangle\right|^2 = \cos^2\beta/2 = \cos^2\theta$

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Fig. 1.2.4 Electron up-dn-spin counting of a tilted spin-up (↑′)-state

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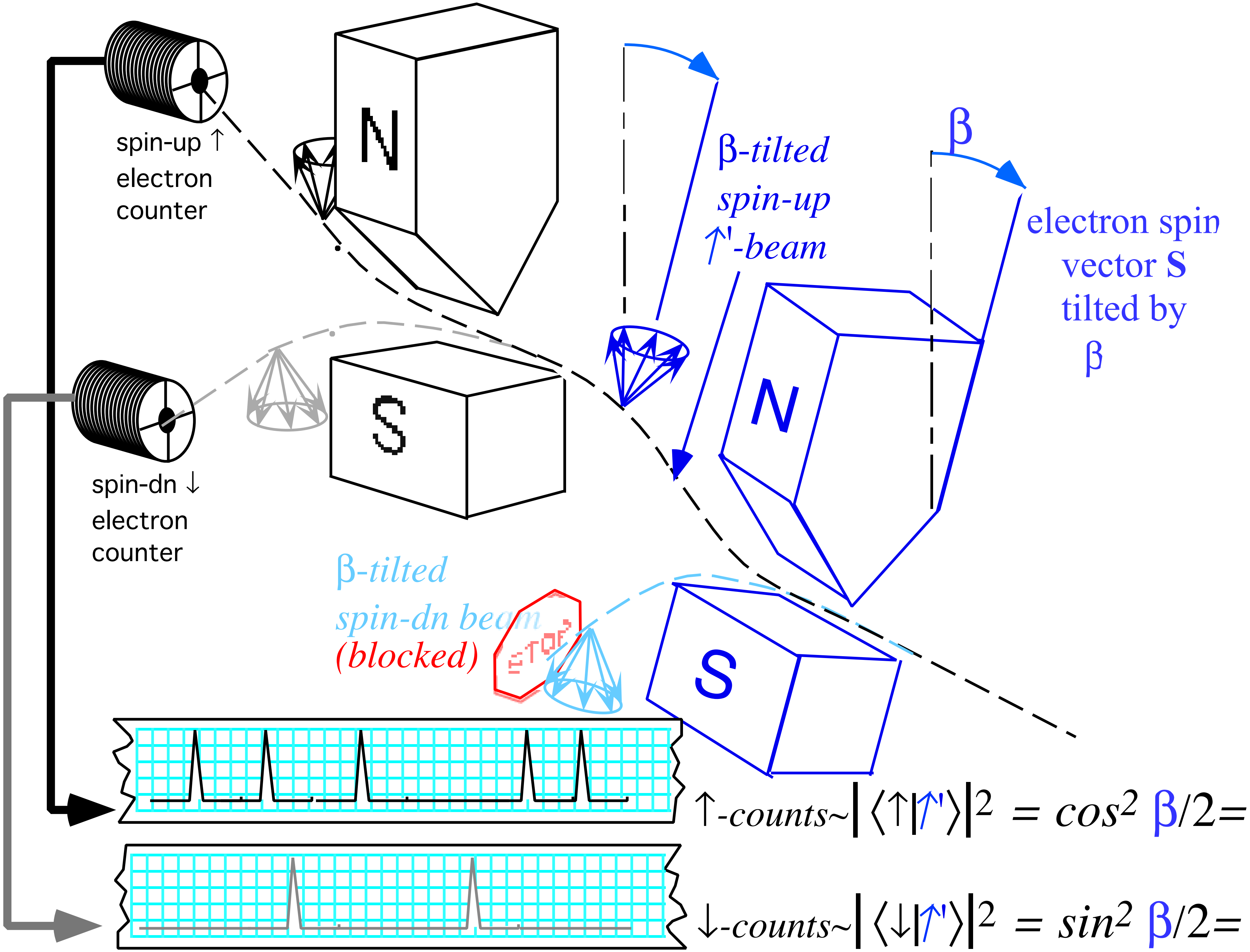

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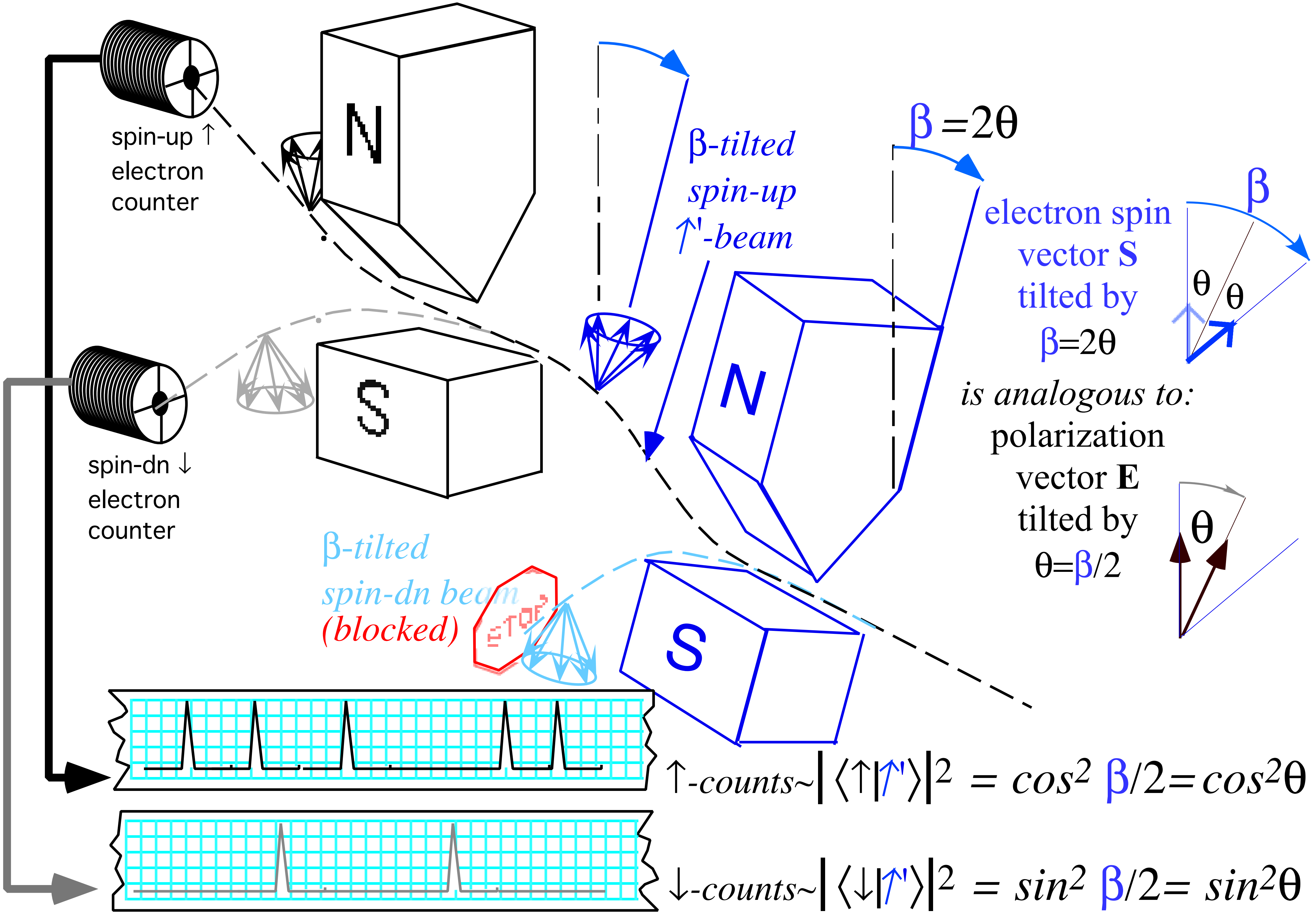

electron spin vector **S** tilted by β=2θ

is analogous to:

polarization vector **E** tilted by θ=β/2

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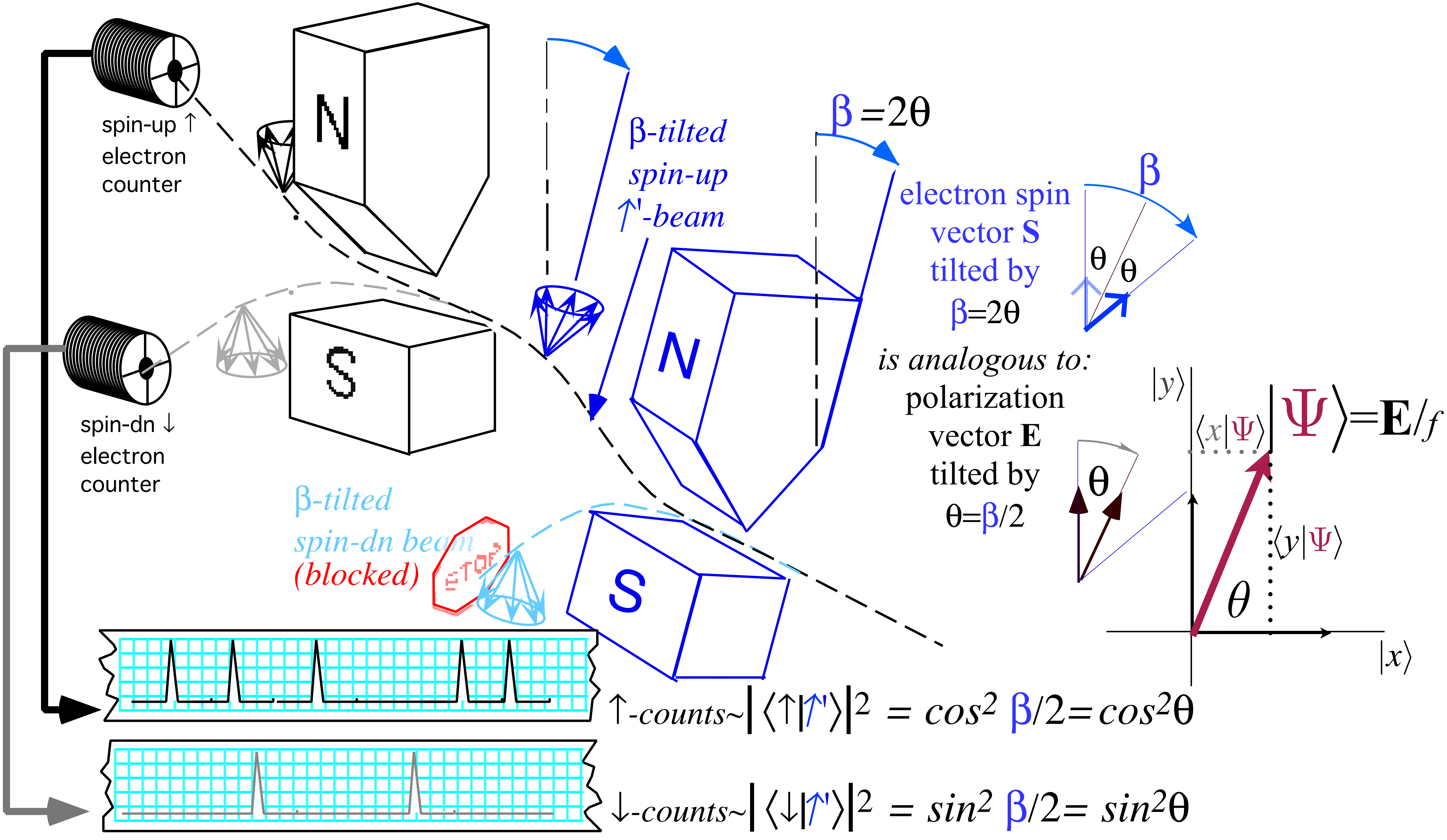

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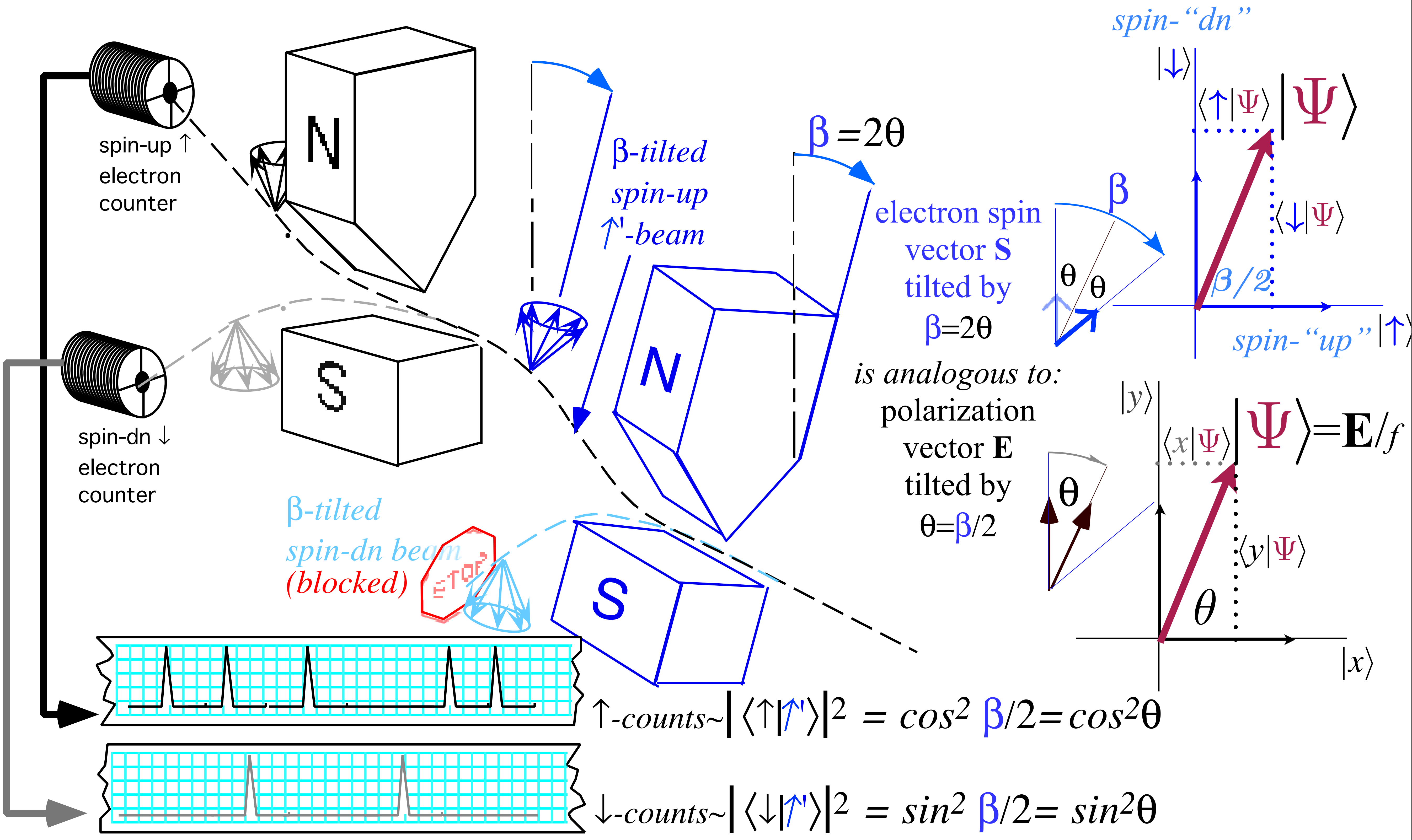

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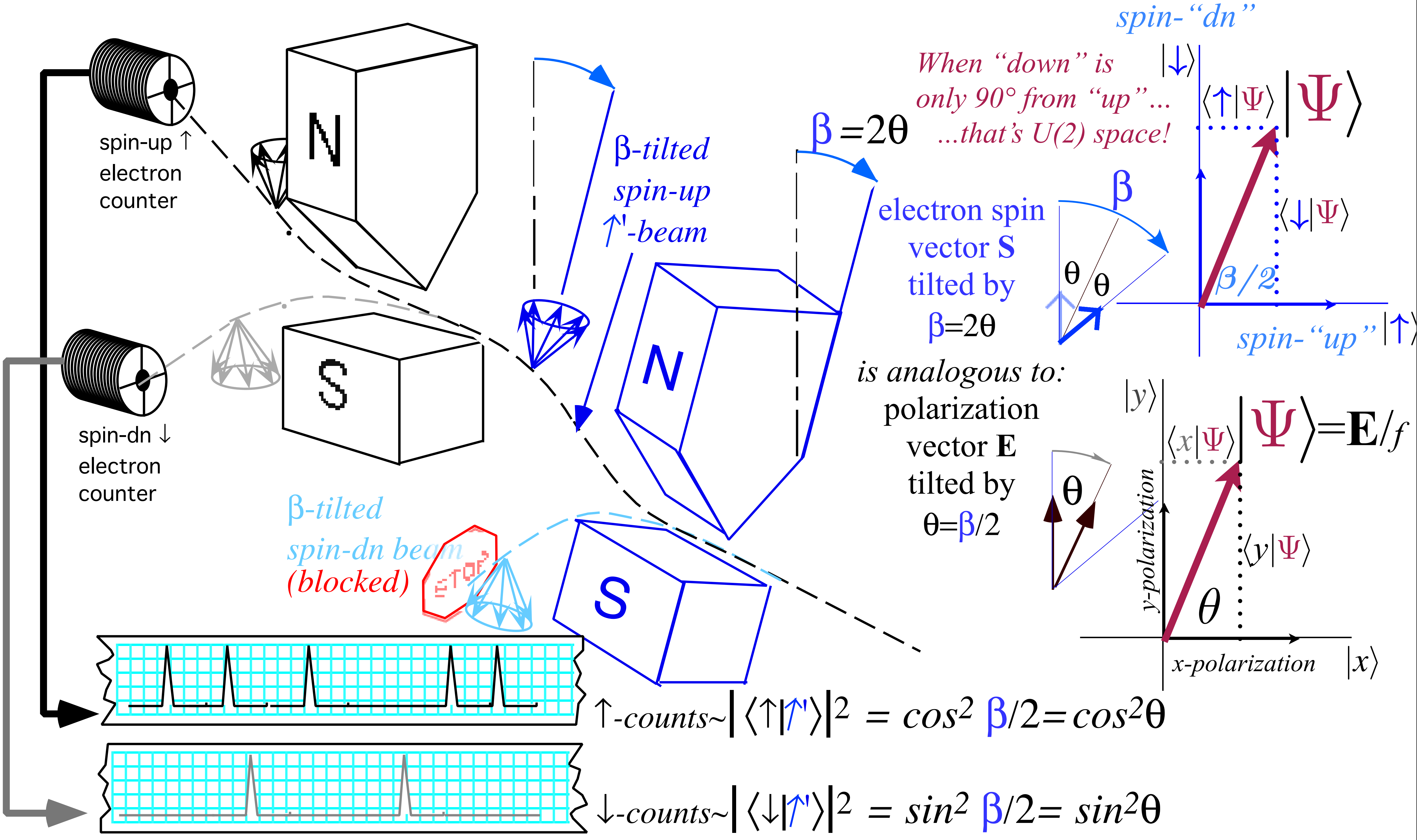

When "down" is only 90° from "up"...
...that's U(2) space!

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A: Want to determine or calculate:

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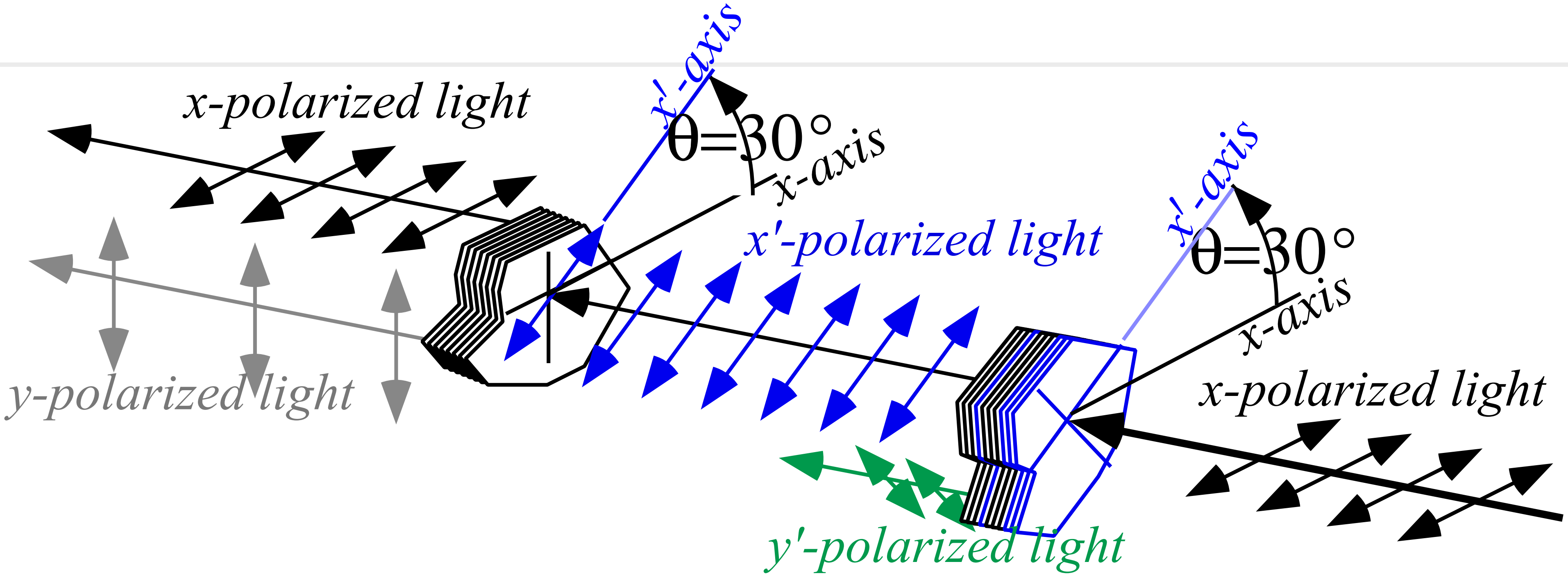

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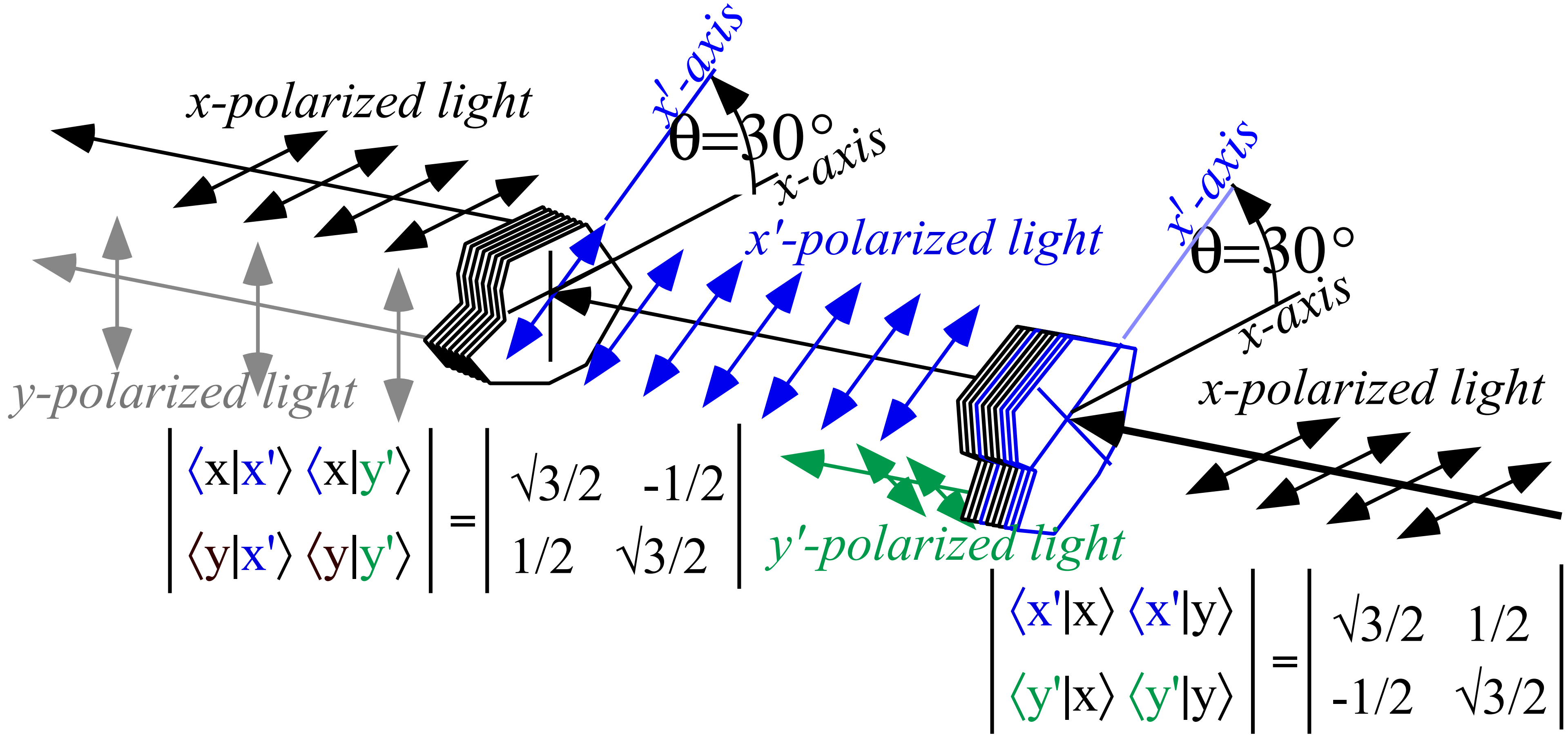

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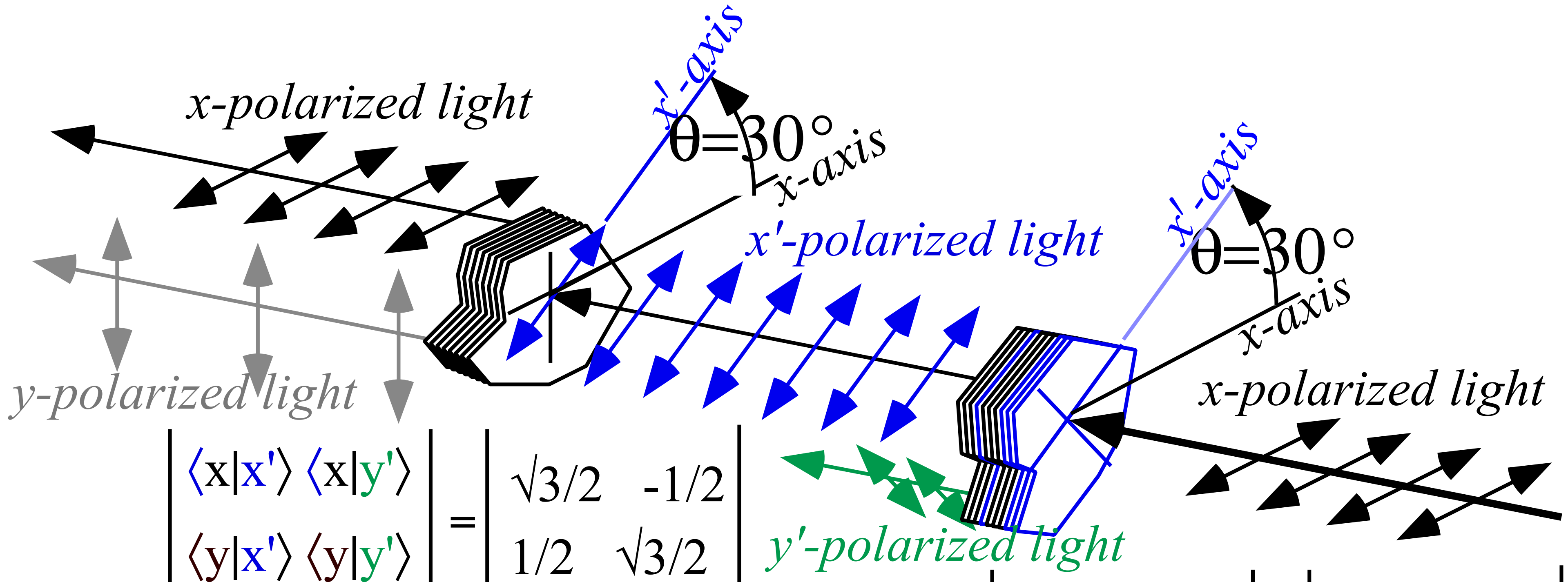

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Base
State
$|x\rangle$

Amplitude=
1.0
Probability=
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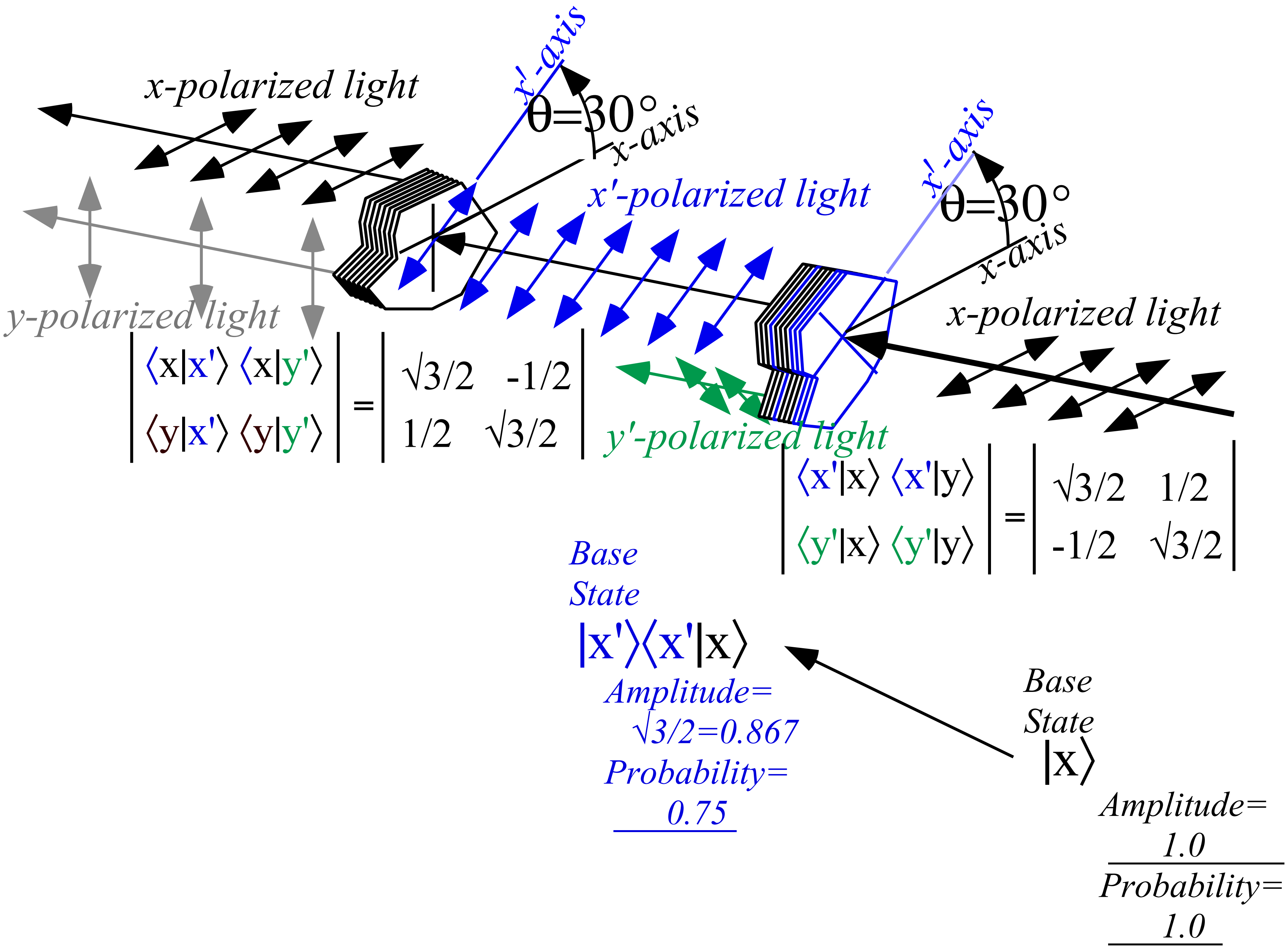

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Base State

$|x'\rangle\langle x'|x\rangle$

Amplitude=
√3/2=0.867

Probability=
0.75

Base State

$|x\rangle$

Amplitude=
1.0

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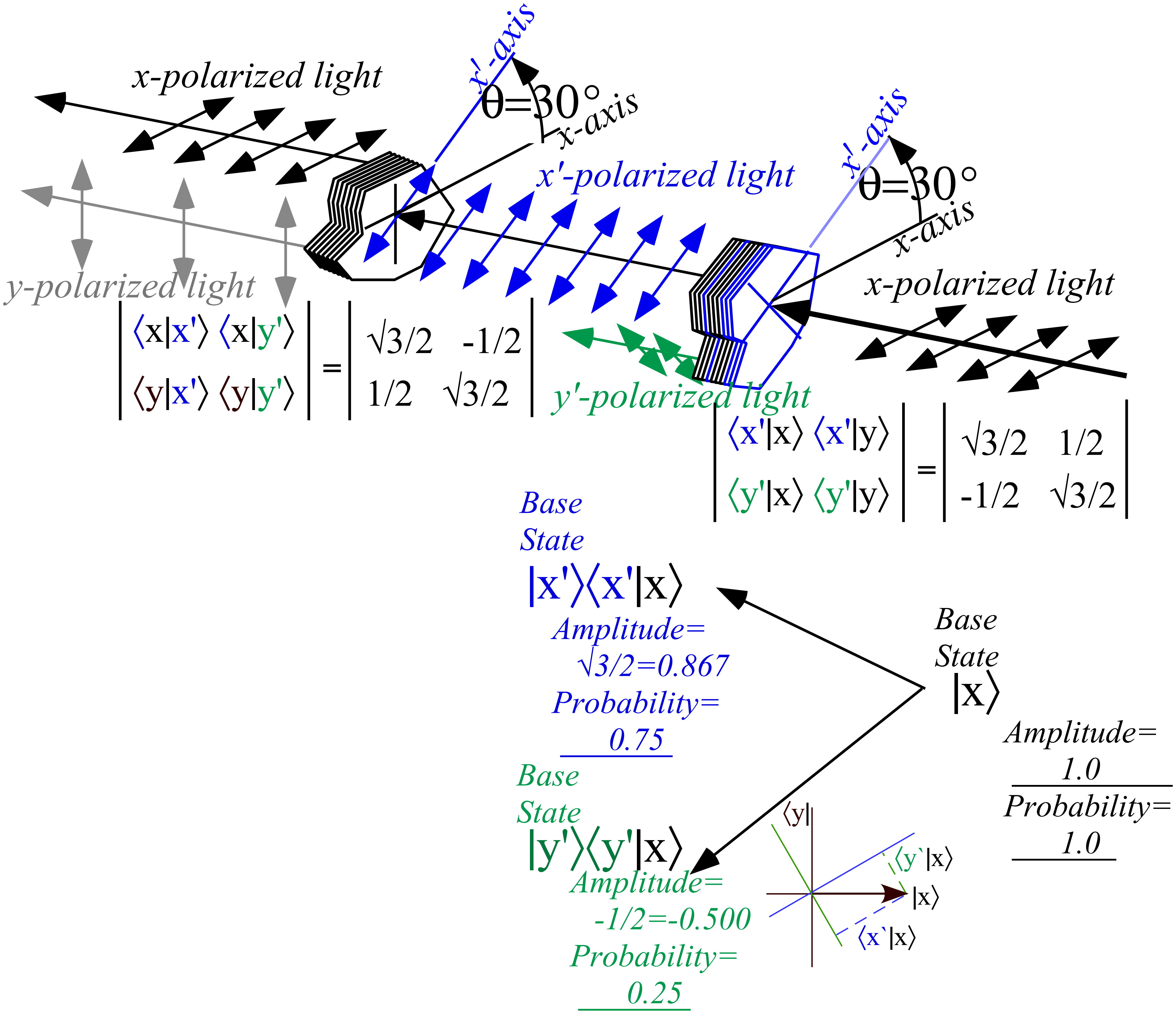

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$$\begin{vmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{vmatrix}$$

Base State

$|x'\rangle\langle x'|x\rangle$

Amplitude=

$\sqrt{3}/2=0.867$

Probability=

0.75

Base State

$|x\rangle$

Amplitude=

1.0

Probability=

1.0

Base State

$|y'\rangle\langle y'|x\rangle$

Amplitude=

-1/2=-0.500

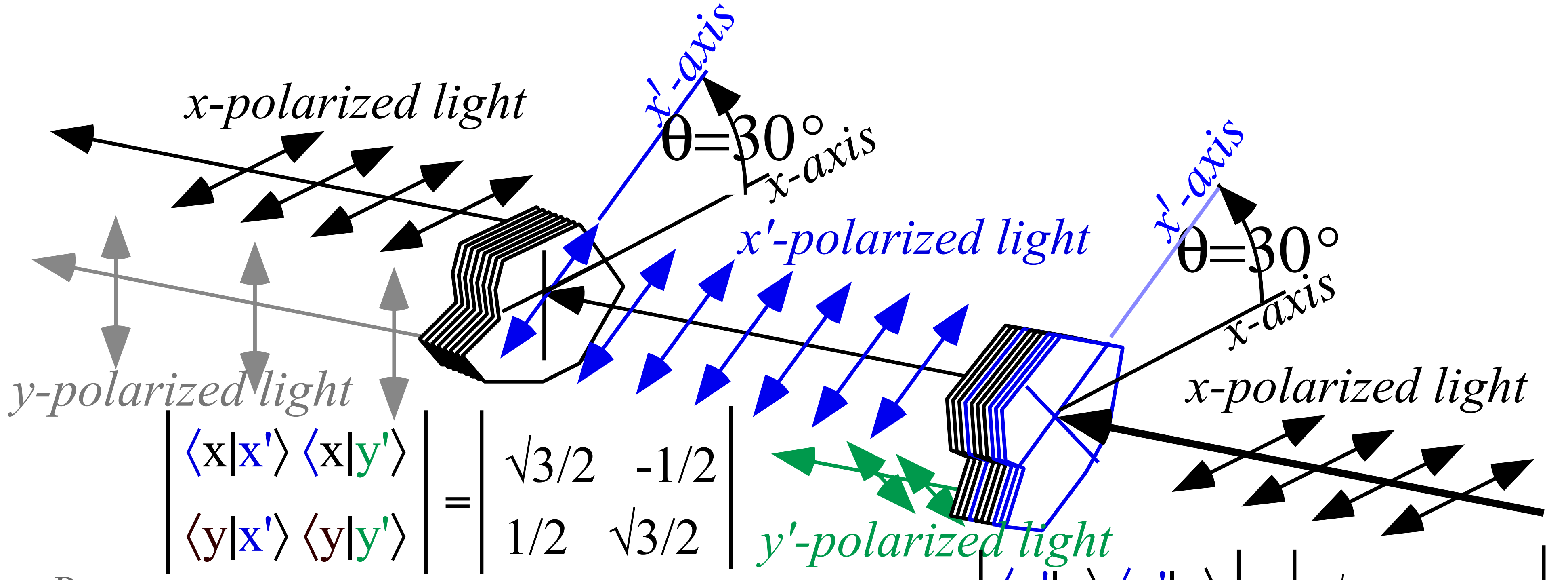
Probability=

0.25

Q:What comes out of an analyzer channel or branch-b?

A: Want to determine or calculate:

its *base state* $|b\rangle$, its *amplitude* $\langle b|\Psi\rangle$, and its *probability* $|\langle b|\Psi\rangle|^2$ using *T*-matrices



$$\begin{vmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{vmatrix}$$

$$\begin{vmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{vmatrix}$$

Base State

$|x\rangle$

Amplitude=

1.0

Probability=

1.0

Base State

$|x'\rangle\langle x'|x\rangle$

Amplitude=

√3/2=0.867

Probability=

0.75

Base State

$|x\rangle\langle x|x'\rangle\langle x'|x\rangle$

Amplitude=

(√3/2)(√3/2)=0.750

Probability=

0.5625

Base State

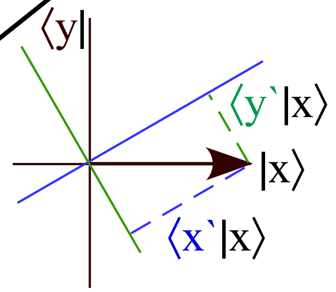
$|y'\rangle\langle y'|x\rangle$

Amplitude=

-1/2=-0.500

Probability=

0.25



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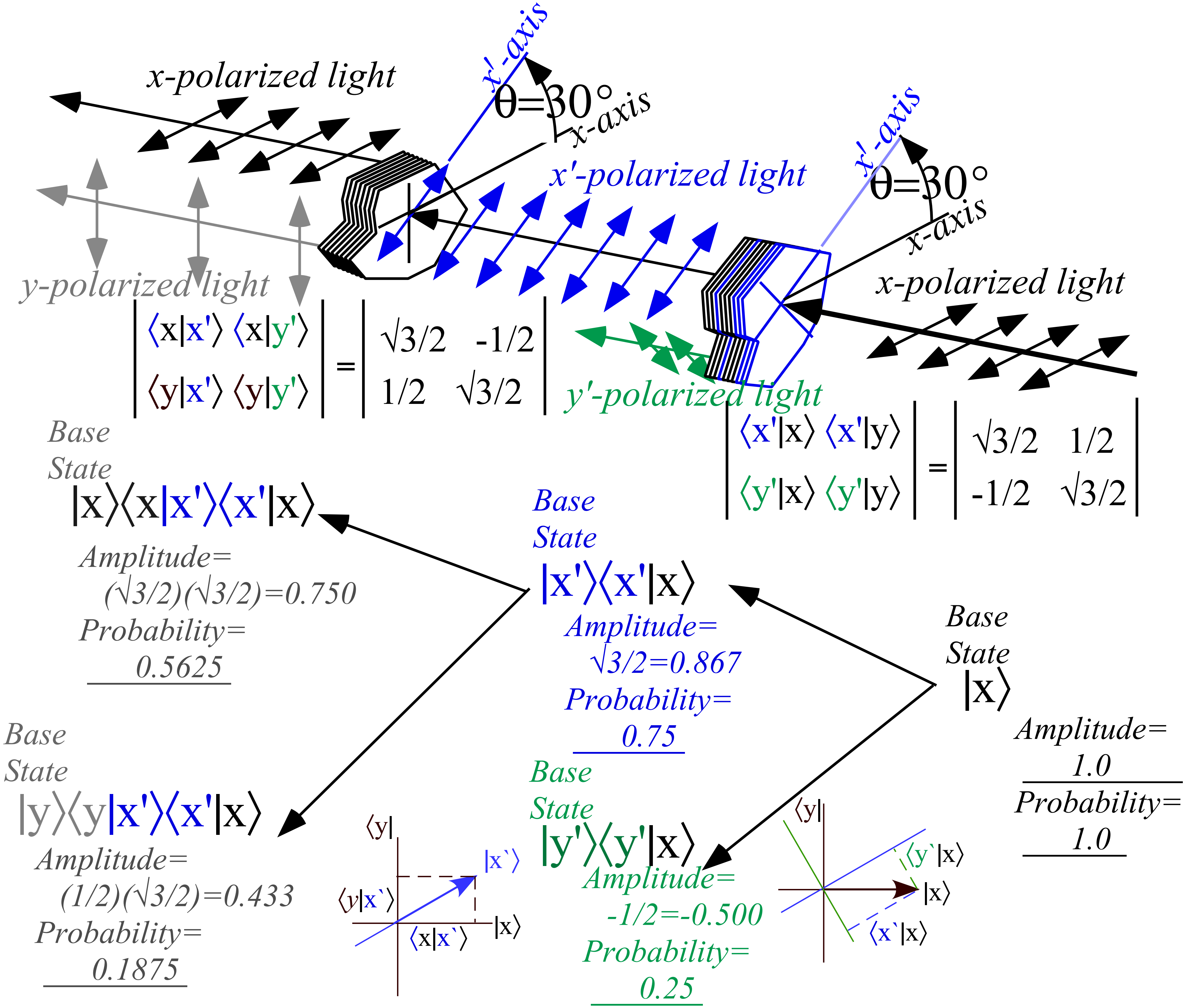


$$\begin{vmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{vmatrix}$$

$$\begin{vmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{vmatrix} = \begin{vmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{vmatrix}$$

Base State
$|x\rangle$
Amplitude=
1.0
Probability=
1.0

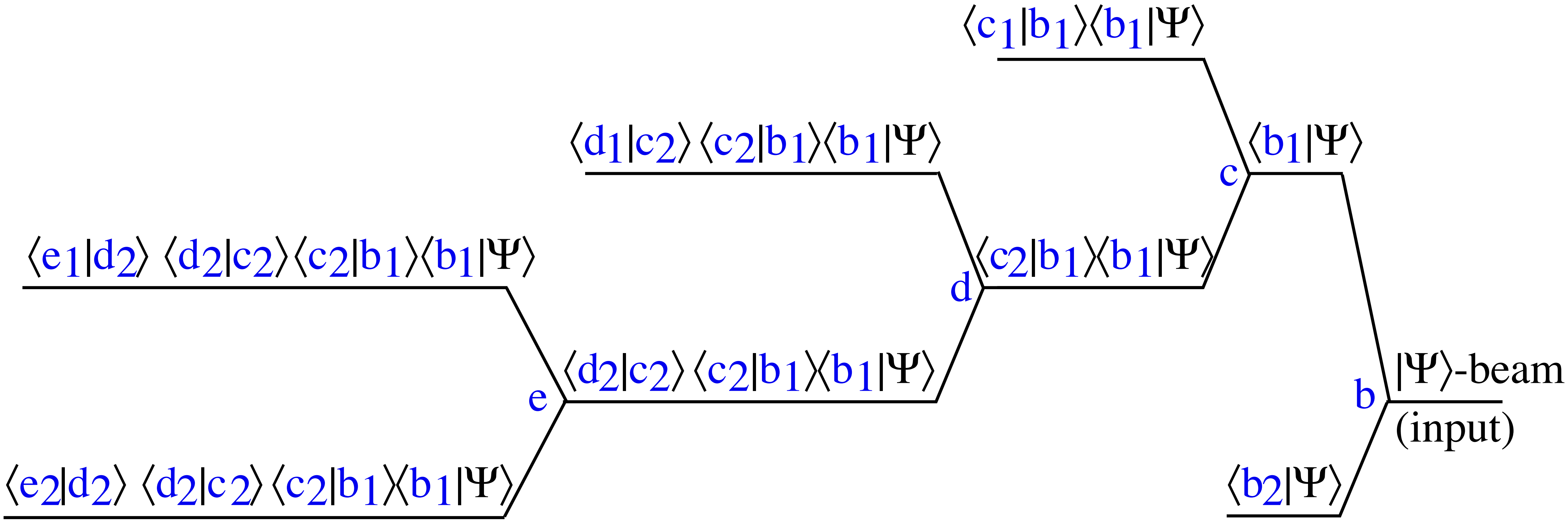
Base State
$|x'\rangle\langle x'|x\rangle$
Amplitude=
$\sqrt{3}/2$=0.867
Probability=
0.75

Base State
$|y'\rangle\langle y'|x\rangle$
Amplitude=
-1/2=-0.500
Probability=
0.25

Base State
$|x\rangle\langle x|x'\rangle\langle x'|x\rangle$
Amplitude=
($\sqrt{3}/2$)($\sqrt{3}/2$)=0.750
Probability=
0.5625

Base State
$|y\rangle\langle y|x'\rangle\langle x'|x\rangle$
Amplitude=
(1/2)($\sqrt{3}/2$)=0.433
Probability=
0.1875

$$\left\langle e_p \middle| D_o C_n B_m \middle| \Psi \right\rangle = \left\langle e_p \middle| d_o \right\rangle \left\langle d_o \middle| c_n \right\rangle \left\langle c_n \middle| b_m \right\rangle \left\langle b_m \middle| \Psi \right\rangle$$



$\langle c_1|b_1\rangle\langle b_1|\Psi\rangle$

$\langle d_1|c_2\rangle\langle c_2|b_1\rangle\langle b_1|\Psi\rangle$

$\langle b_1|\Psi\rangle$

c

$\langle e_1|d_2\rangle\langle d_2|c_2\rangle\langle c_2|b_1\rangle\langle b_1|\Psi\rangle$

$\langle c_2|b_1\rangle\langle b_1|\Psi\rangle$

d

$\langle d_2|c_2\rangle\langle c_2|b_1\rangle\langle b_1|\Psi\rangle$

e

b

$|\Psi\rangle$-beam

(input)

$\langle e_2|d_2\rangle\langle d_2|c_2\rangle\langle c_2|b_1\rangle\langle b_1|\Psi\rangle$

$\langle b_2|\Psi\rangle$

Fig. 1.3.10 Beams-amplitude products for successive beam sorting

- *Review: "Abstraction" of bra and ket vectors from a Transformation Matrix*
  - *Introducing scalar and matrix products*
- *Planck's energy and N-quanta (Cavity/Beam wave mode)*
  - *Did Max Planck Goof? What's 1-photon worth?*
  - *Feynman amplitude axiom 1*
- *What comes out of a beam sorter channel or branch-b?*
  - *Sample calculations*
  - ➡ *Feynman amplitude axioms 2-3*
- *Beam analyzers: Sorter-unsorters*
  - *The "Do-Nothing" analyzer*
  - *Feynman amplitude axiom 4*
  - *Some "Do-Something" analyzers*
    - *Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate*

Feynman amplitude axiom 1 *(Given above p.35)*

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

=Amplitude of ***state-j*** *after state-k′ is forced to choose from available* ***m-type*** *states*

(1) The probability axiom

The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

Axiom 1: The absolute square $\left|\langle j | k' \rangle\right|^2 = \langle j | k' \rangle^* \langle j | k' \rangle$ *gives probability for occurrence in* ***state-j*** *of a system that started in state-k′=1′,2′,..,or n′ from one sorter and then was forced to choose between states* ***j=1,2,...,n*** *by another sorter.*

Feynman amplitude axioms 1-2

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

*=Amplitude of **state-j** after state-k′ is forced to choose from available **m-type** states*

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(2) The conjugation or inversion axiom *(time reversal symmetry)*

The second axiom concerns going backwards through a sorter or the reversal of amplitudes.

Axiom 2: The complex conjugate $\langle j | k' \rangle^*$ *of an amplitude* $\langle j | k' \rangle$ *equals its reverse:* $\langle j | k' \rangle^* = \langle k' | j \rangle$

Feynman amplitude axioms 1-3

Feynman-Dirac Interpretation of

$$\langle j | k' \rangle$$

*=Amplitude of state-j after state-k′ is forced to choose from available **m-type** states*

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The first axiom deals with physical interpretation of amplitudes $\langle j | k' \rangle$.

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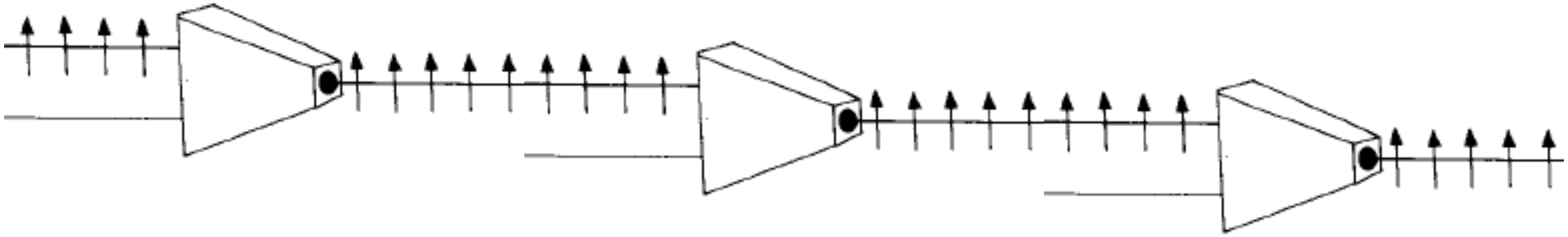
(3) The orthonormality or identity axiom

The third axiom concerns the amplitude for "re measurement" by the same analyzer.

Axiom 3: If identical analyzers are used twice or more the amplitude for a passed state-k is one, and for all others it is zero:

$$\langle j | k \rangle = \delta_{jk} = \left\{ \begin{array}{c} 1 \text{ if: } j{=}k \\ 0 \text{ if: } j \neq k \end{array} \right\} = \langle j' | k' \rangle$$



Review: "Abstraction" of bra and ket vectors from a Transformation Matrix

Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)

Did Max Planck Goof? What's 1-photon worth?

Feynman amplitude axiom 1

What comes out of a beam sorter channel or branch-b?

Sample calculations

Feynman amplitude axioms 2-3

➡ *Beam analyzers: Sorter-unsorters*

The "Do-Nothing" analyzer

Feynman amplitude axiom 4

Some "Do-Something" analyzers

Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

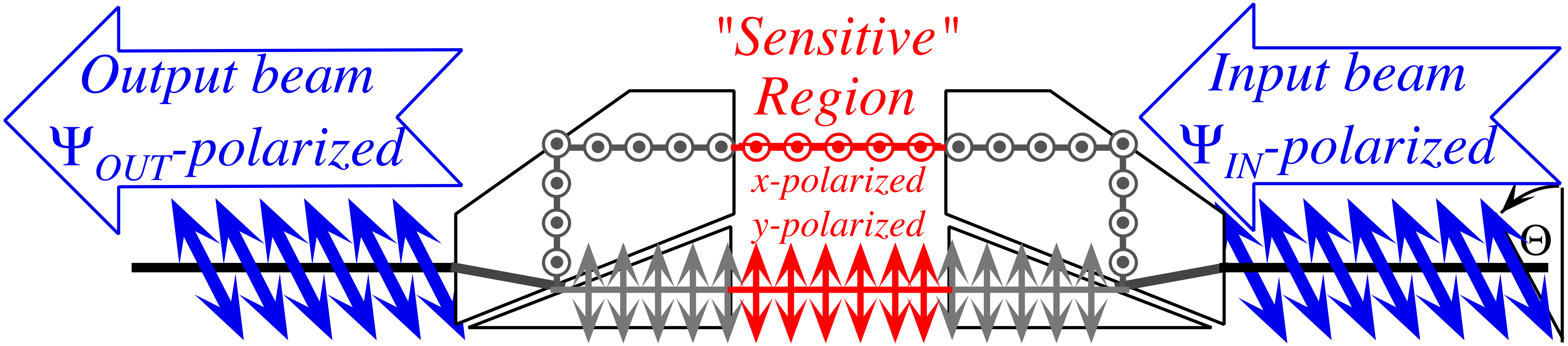
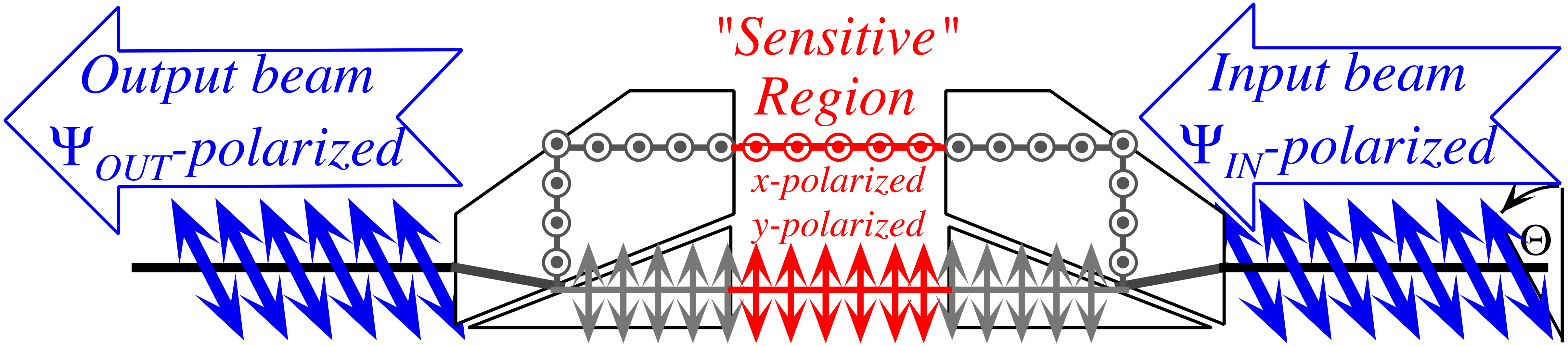


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)



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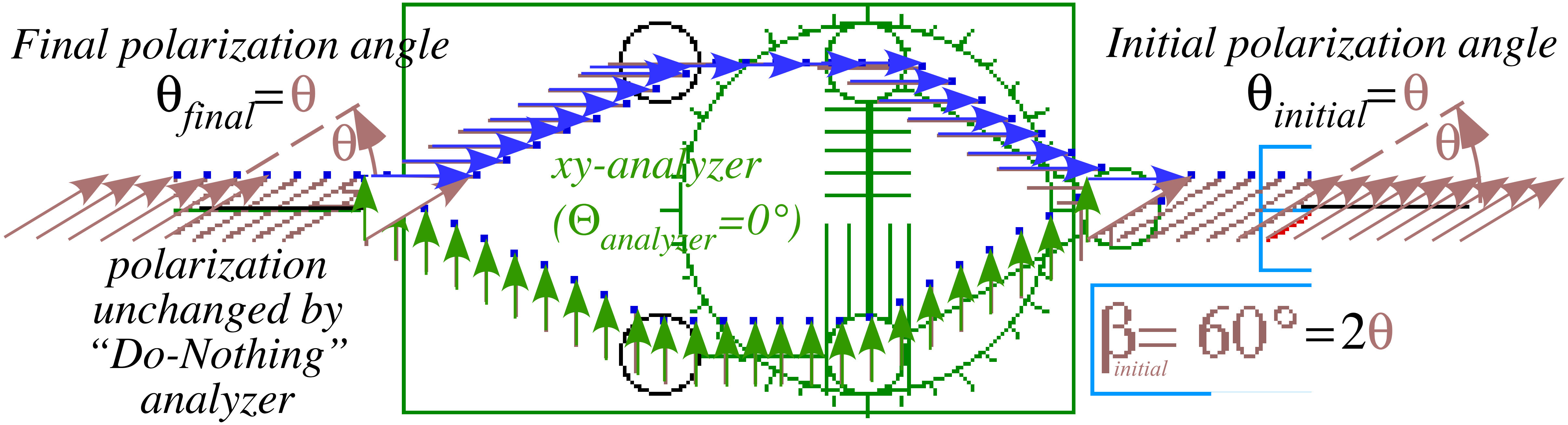


Fig. 1.3.2 Computer sketch of simulated polarization analyzer in "do-nothing" mode

Review: *"Abstraction" of bra and ket vectors from a Transformation Matrix*

Introducing scalar and matrix products

Planck's energy and N-quanta (Cavity/Beam wave mode)

Did Max Planck Goof? What's 1-photon worth?

Feynman amplitude axiom 1

What comes out of a beam sorter channel or branch-b?

Sample calculations

Feynman amplitude axioms 2-3

Beam analyzers: Sorter-unsorters

The "Do-Nothing" analyzer

→ *Feynman amplitude axiom 4*

Some "Do-Something" analyzers

Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate

Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

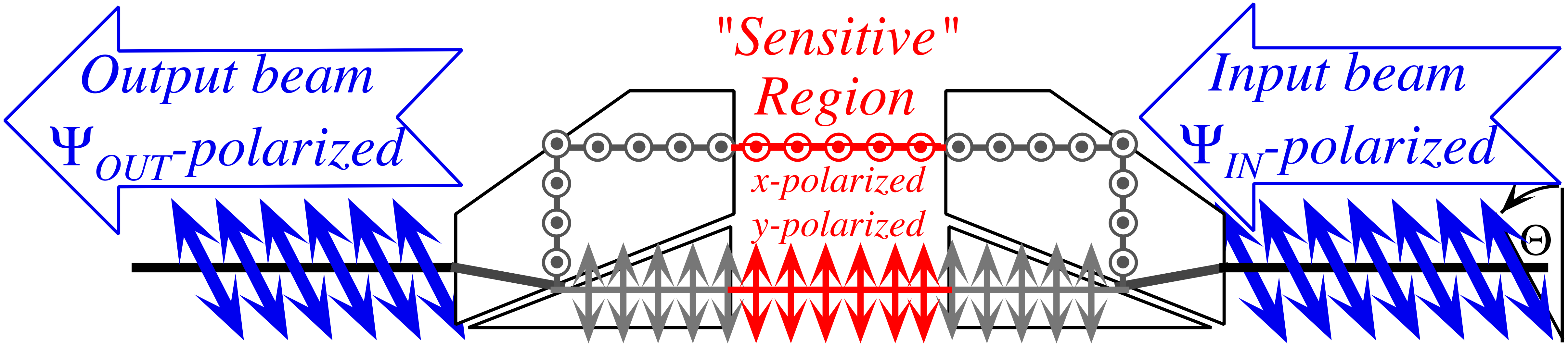


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

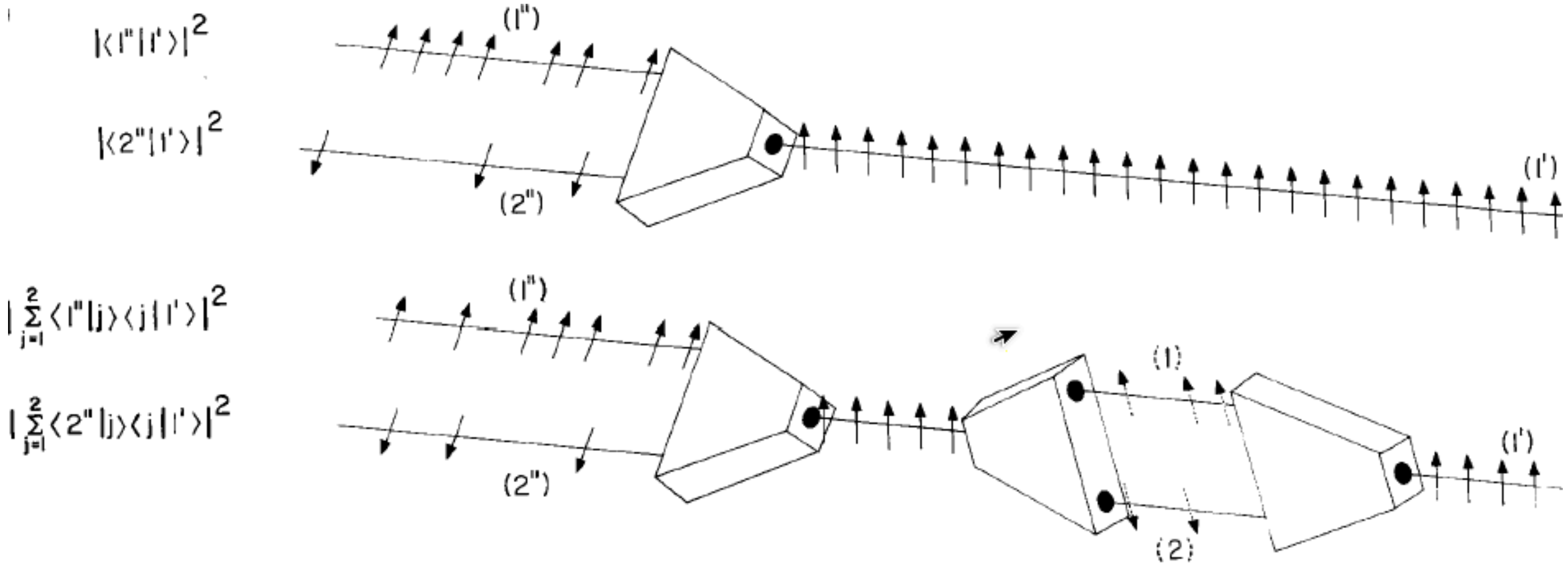
Feynman amplitude axiom 4

(4) The completeness or closure axiom

The fourth axiom concerns the "*Do-nothing*" property of an ideal analyzer, that is, a sorter followed by an "unsorter" or "put-back-togetherer" as sketched above.

Axiom 4. Ideal sorting followed by ideal recombination of amplitudes has no effect:

$$\langle j''|m'\rangle = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|m'\rangle$$



Beam Analyzers: Fundamental Quantum Processes (Sorter-Unsorter pairs)

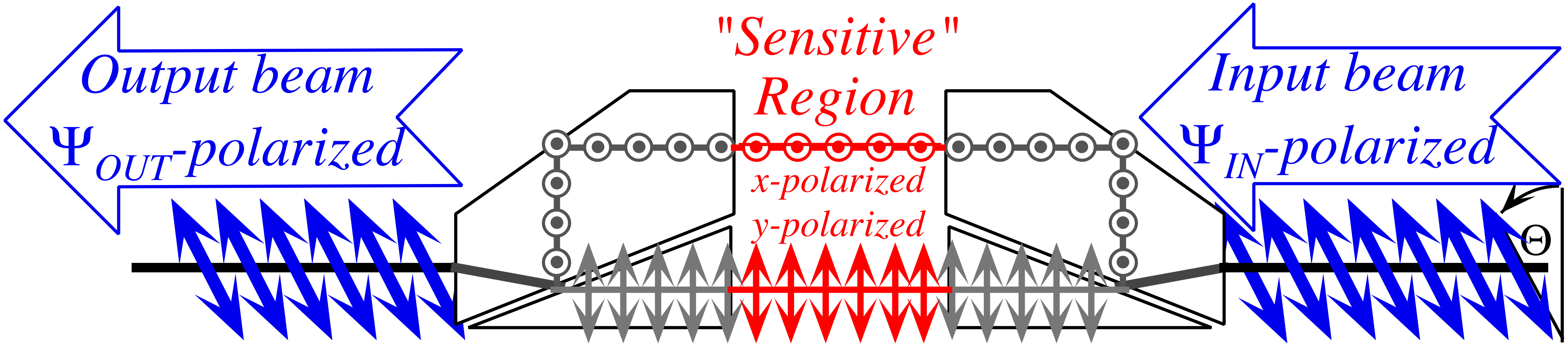


Fig. 1.3.1 Anatomy of ideal optical polarization analyzer in its "Do nothing" mode

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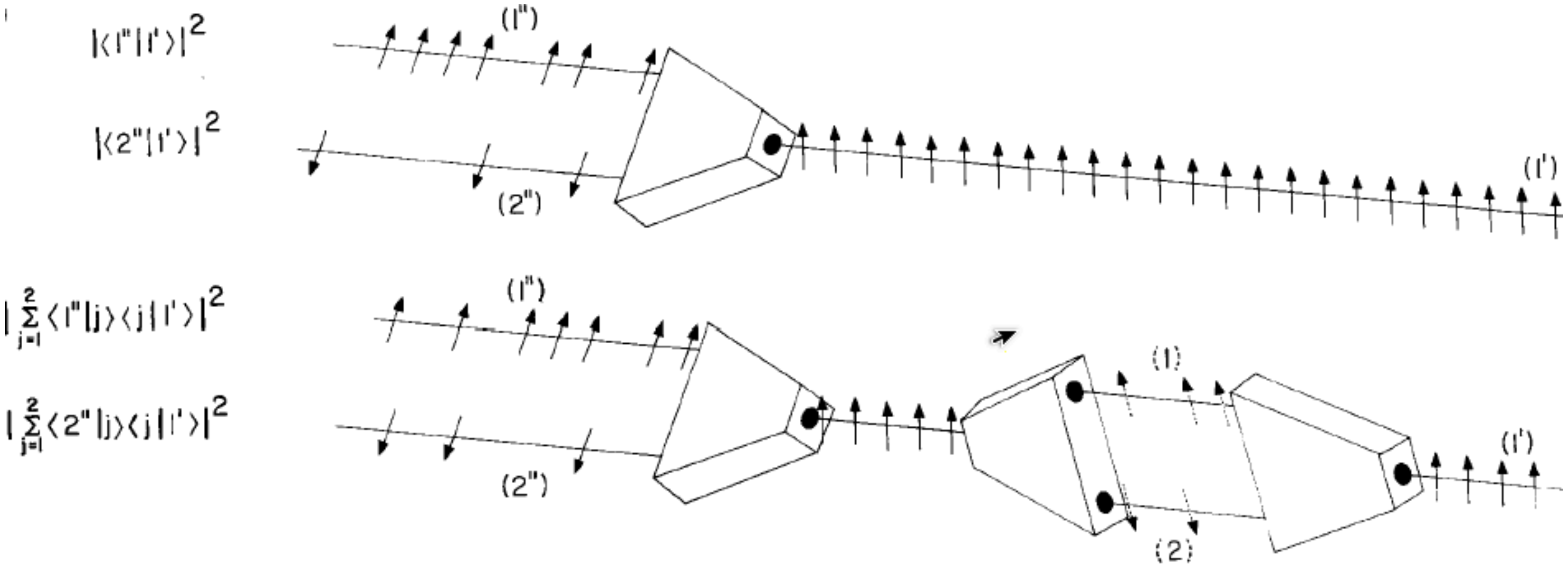

May use axioms 1-3 to prove special case:

$$1 = \langle m'|m'\rangle \text{ by 3}$$

$$\Updownarrow_{\text{by 1}}$$

$$1 = \sum_{k=1}^{n} |\langle k|m'\rangle|^2$$

$$\Updownarrow_{\text{by 1}}$$

$$1 = \sum_{k=1}^{n} \langle k|m'\rangle^* \langle k|m'\rangle$$

$$\Updownarrow_{\text{by 2}}$$

$$\sum_{k=1}^{n} \langle m'|k\rangle\langle k|m'\rangle$$

$$= \langle m'|m'\rangle$$

Feynman amplitude axioms 1-4

Feynman-Dirac Interpretation of

$$\langle j|k'\rangle$$

*=Amplitude of **state-j** after state-k′ is forced to choose from available **m-type** states*

(1) The probability axiom

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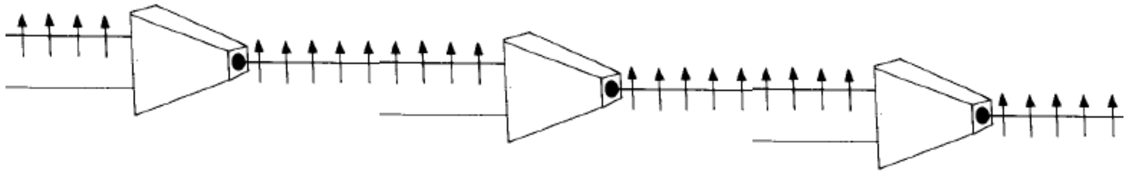

(4) The completeness or closure axiom

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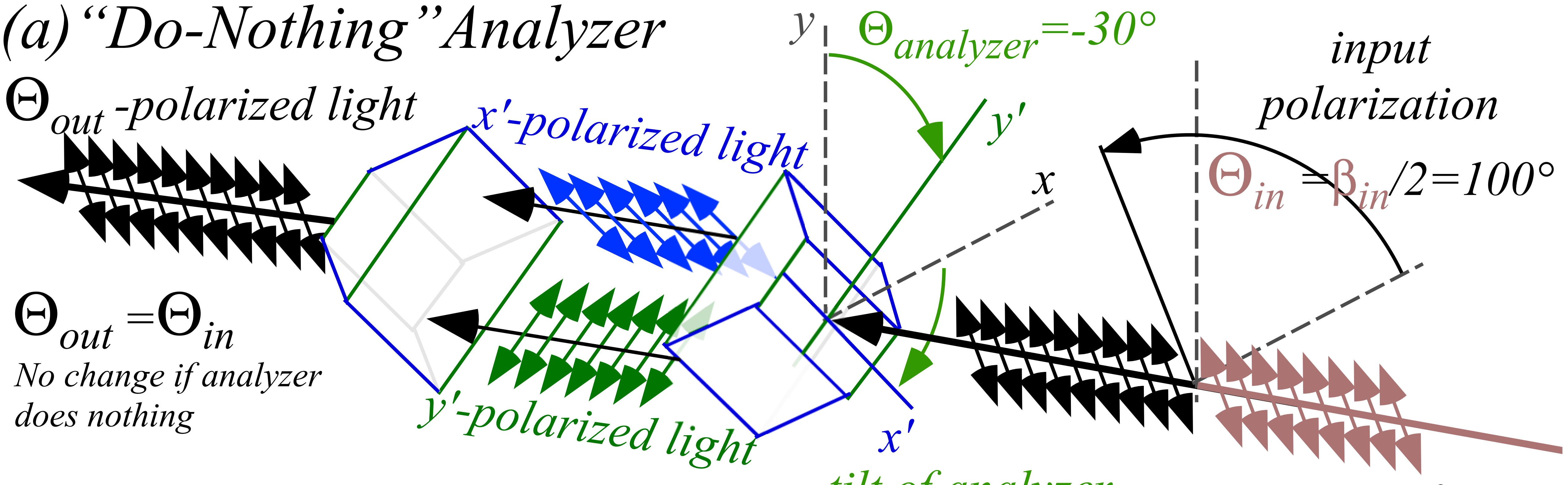
$$\langle j''|m'\rangle = \sum_{k=1}^{n} \langle j''|k\rangle\langle k|m'\rangle$$

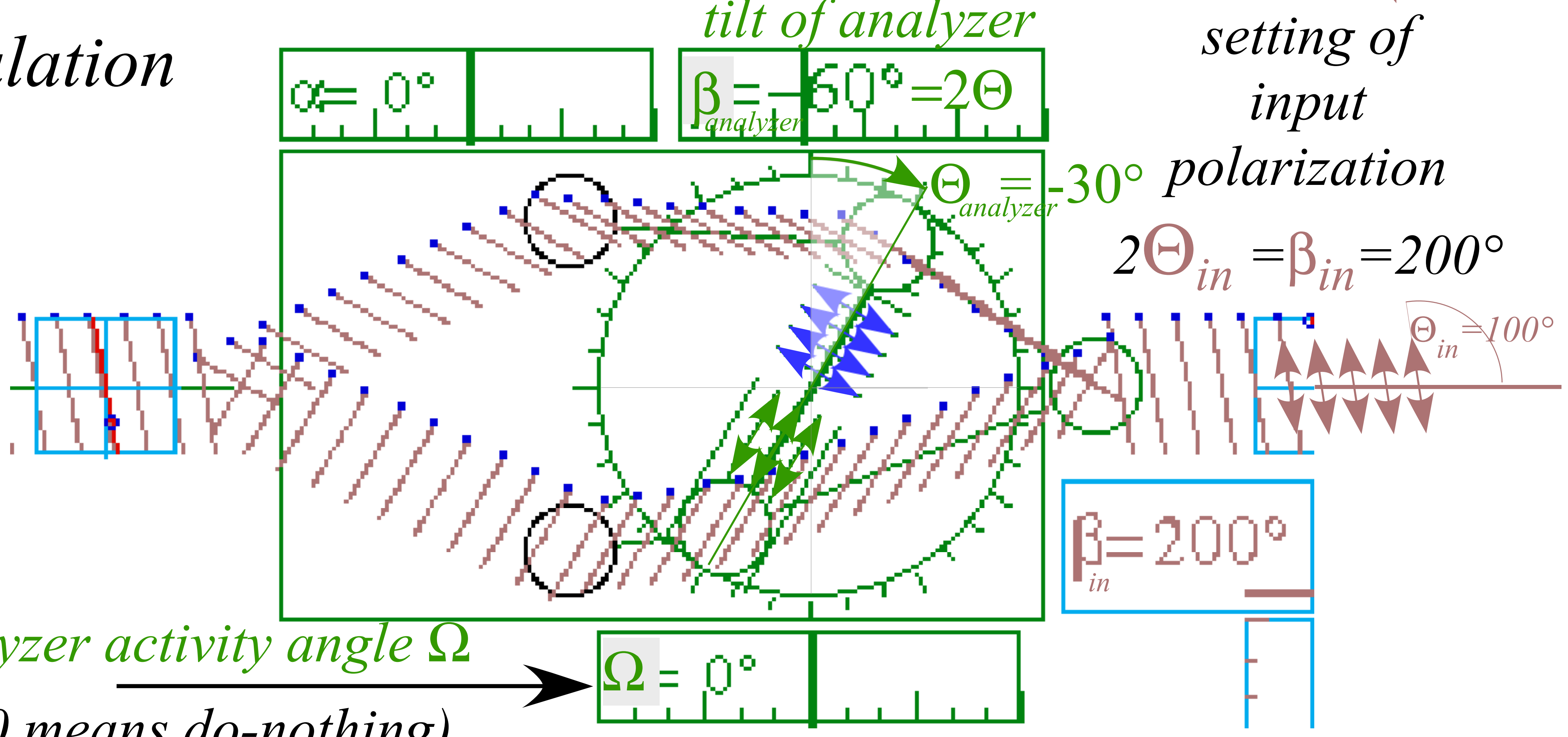
(a) "Do-Nothing" Analyzer



$\Theta_{out}=\Theta_{in}$

No change if analyzer does nothing

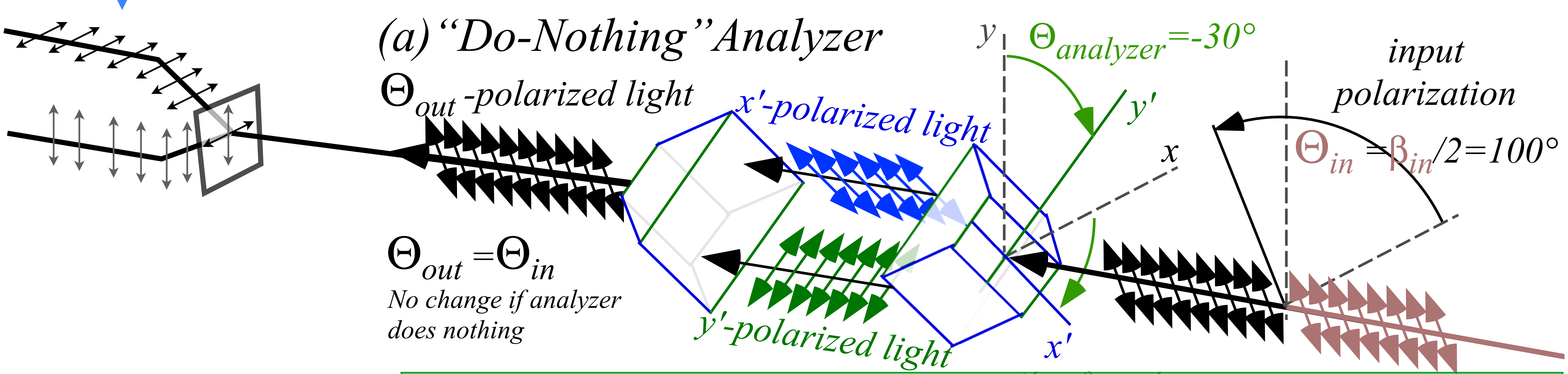
(b) Simulation

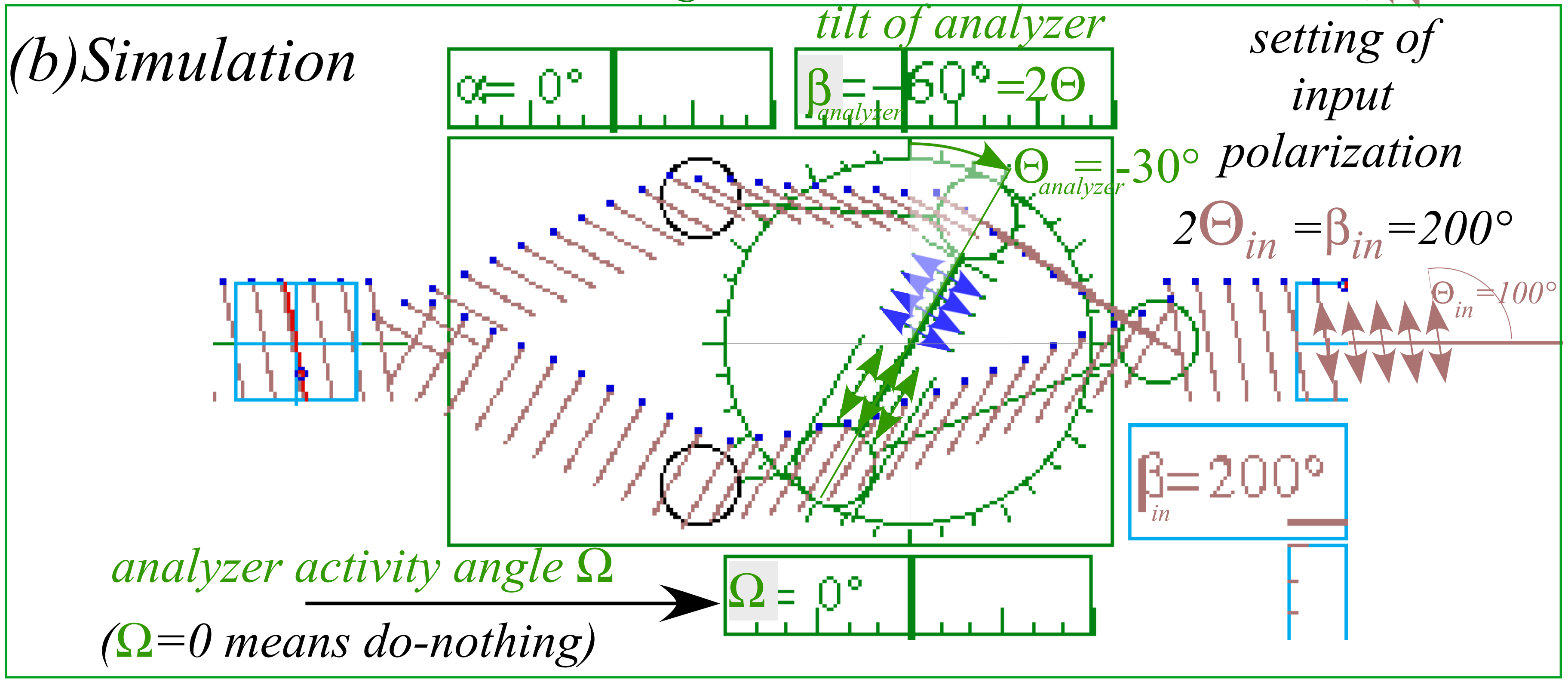


analyzer activity angle Ω

(Ω=0 means do-nothing)

Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.





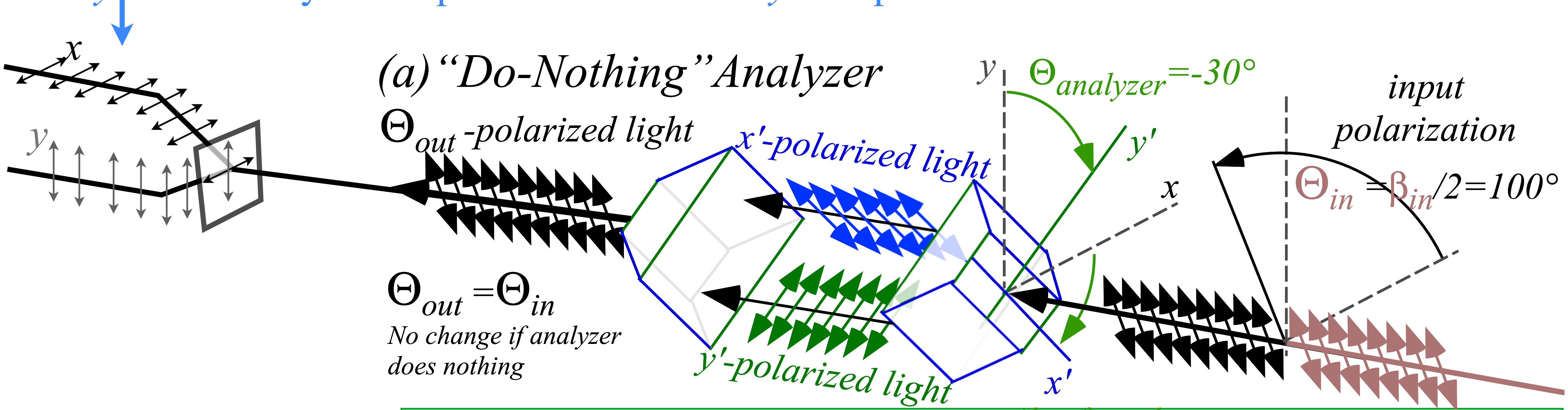
Amplitude in *x* or *y*-channel is sum over x' and y'-amplitudes

$\langle x'|\Theta_{in}\rangle=\cos(\Theta_{in}-\Theta)$

$\langle y'|\Theta_{in}\rangle=\sin(\Theta_{in}-\Theta)$

with relative angle $\Theta_{in}-\Theta$ of Θ_{in} to Θ-analyzer axes-(x',y')

Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.



Amplitude in *x* or *y*-channel is sum over *x′* and *y′*-amplitudes

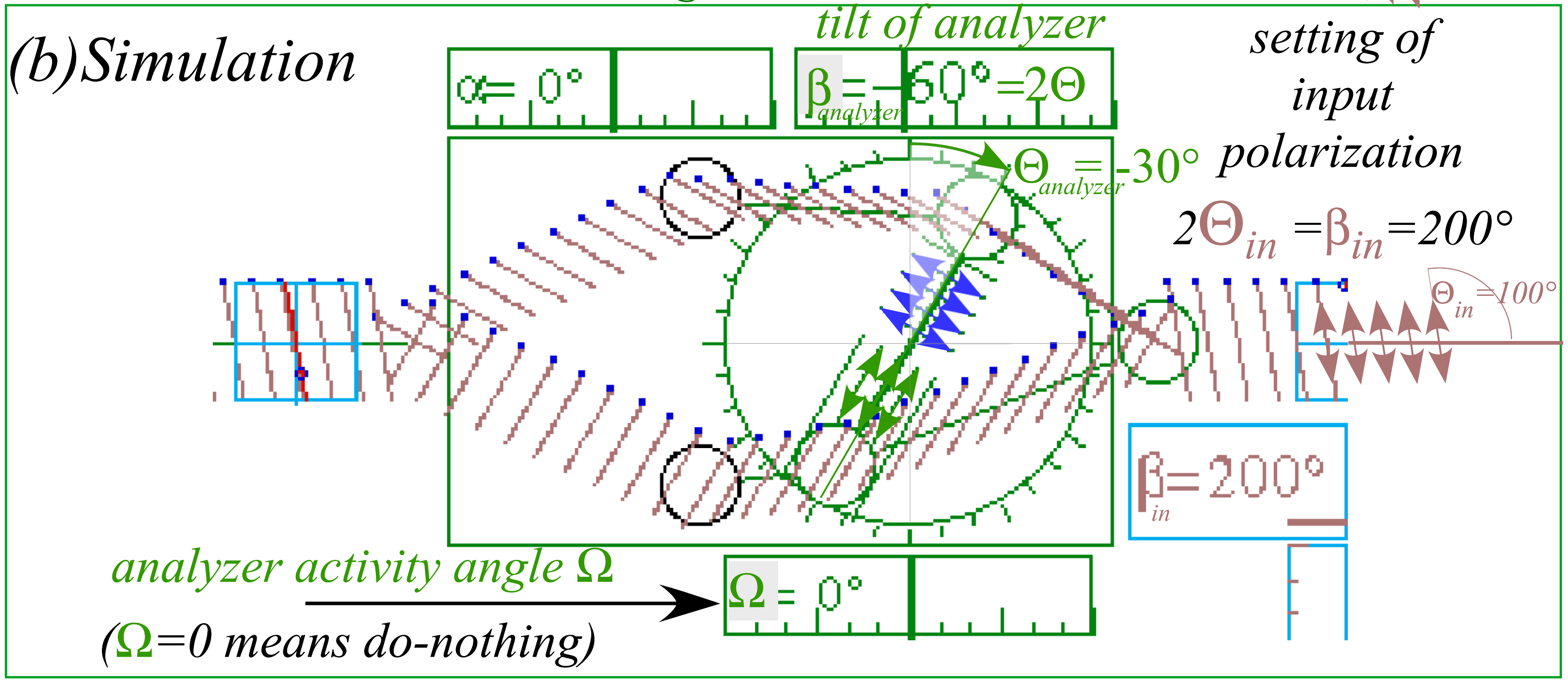
$\langle x'|\Theta_{in}\rangle=\cos(\Theta_{in}-\Theta)$

$\langle y'|\Theta_{in}\rangle=\sin(\Theta_{in}-\Theta)$

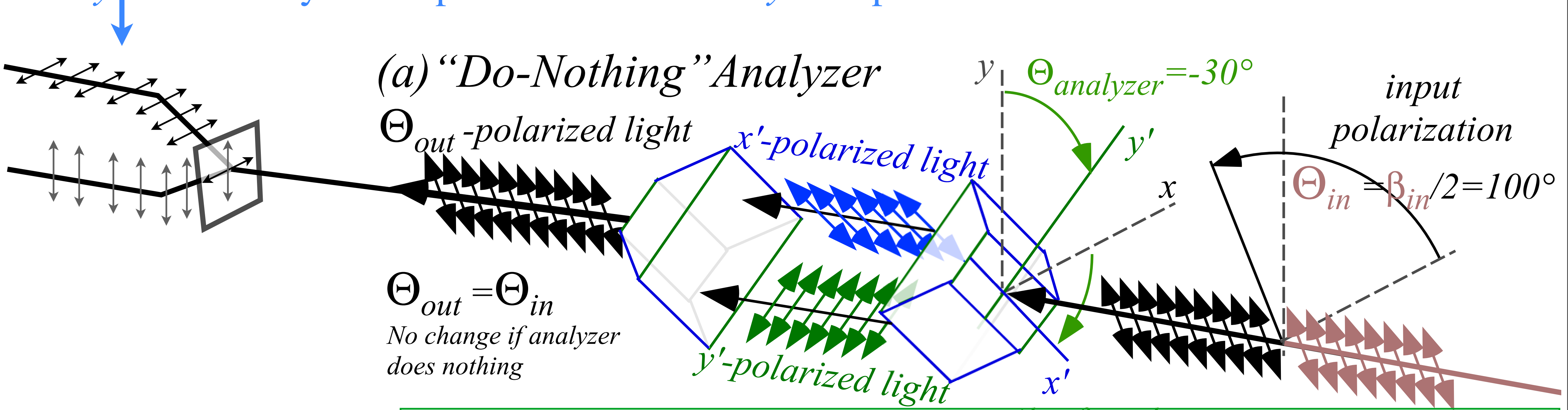
with relative angle $\Theta_{in}-\Theta$ of Θ_{in} to Θ-analyzer axes-(x',y') in products with final *xy*-sorter:

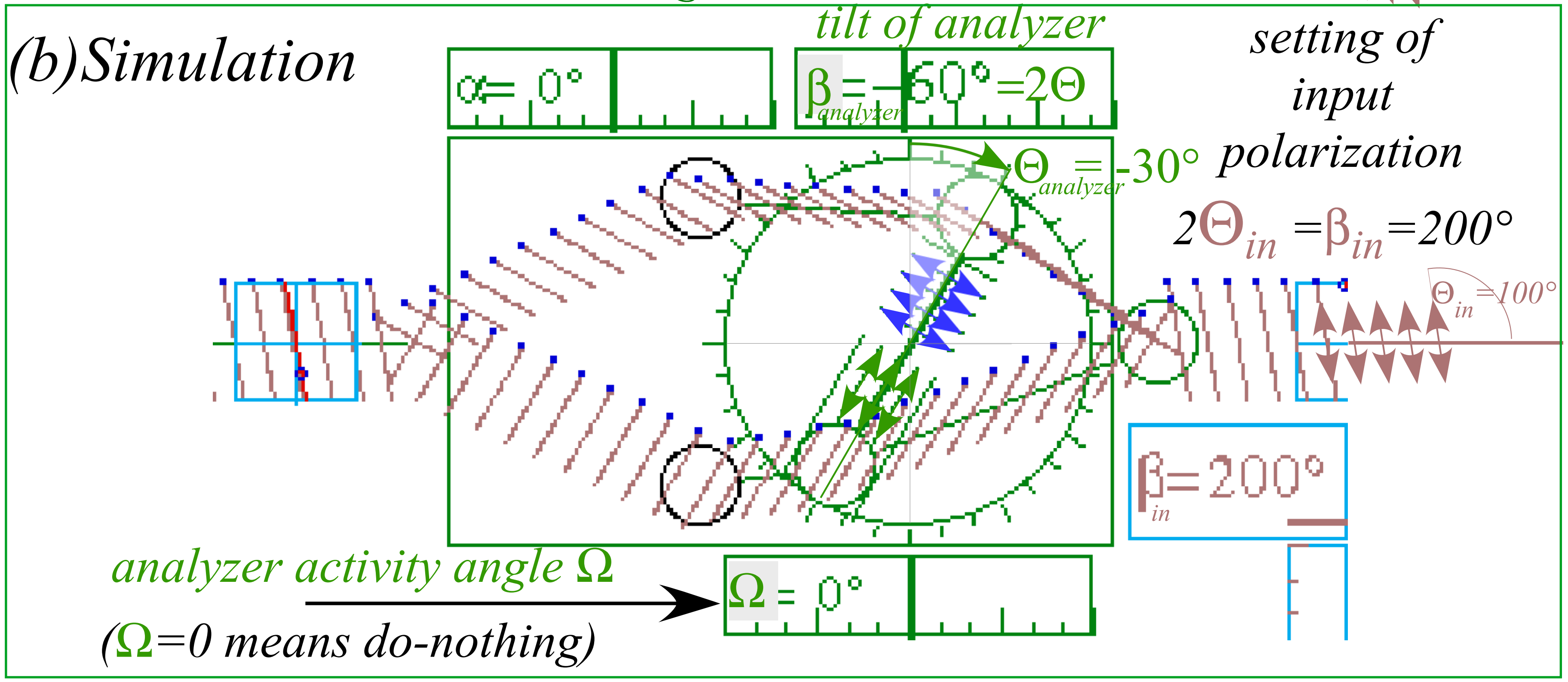
lab x-axis: $\langle x|x'\rangle=\cos\Theta=\langle y|y'\rangle$

y-axis: $\langle y|x'\rangle=\sin\Theta=-\langle x|y'\rangle$.



Imagine final *xy*-sorter analyzes output beam into *x* and *y*-components.





Amplitude in *x* or *y*-channel is sum over *x'* and *y'*-amplitudes

$\langle x'|\Theta_{in}\rangle = \cos(\Theta_{in}-\Theta)$

$\langle y'|\Theta_{in}\rangle = \sin(\Theta_{in}-\Theta)$

with relative angle $\Theta_{in}-\Theta$

of Θ_{in} to Θ-analyzer axes-(x',y')

in products with final *xy*-sorter:

lab x-axis: $\langle x|x'\rangle = \cos\Theta = \langle y|y'\rangle$

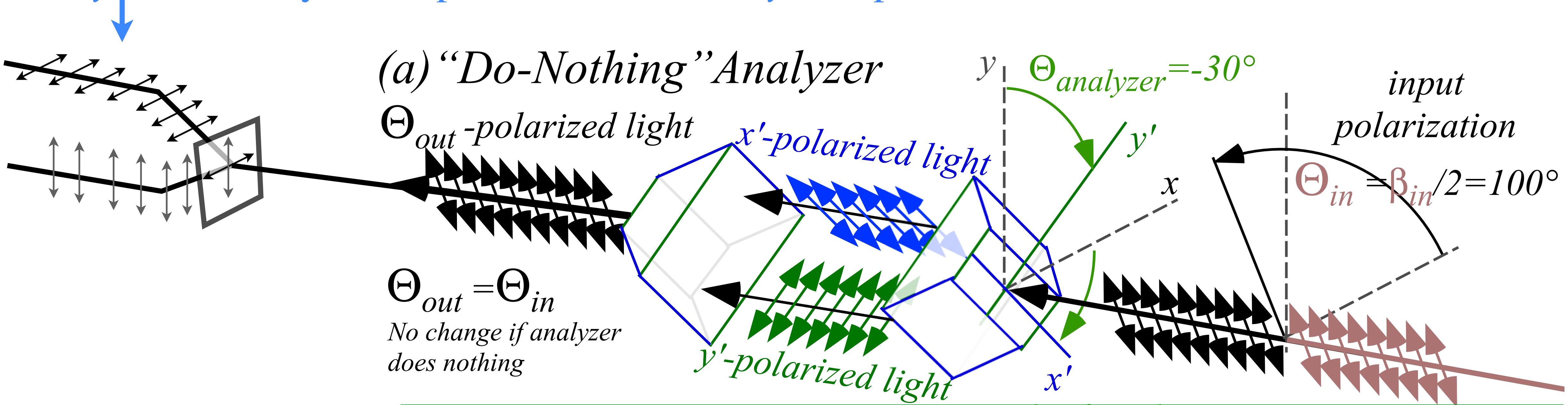
y-axis: $\langle y|x'\rangle = \sin\Theta = -\langle x|y'\rangle$.

x-Output is: $\langle x|\Theta_{out}\rangle = \langle x|x'\rangle\langle x'|\Theta_{in}\rangle + \langle x|y'\rangle\langle y'|\Theta_{in}\rangle = \cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta) = \cos\Theta_{in}$

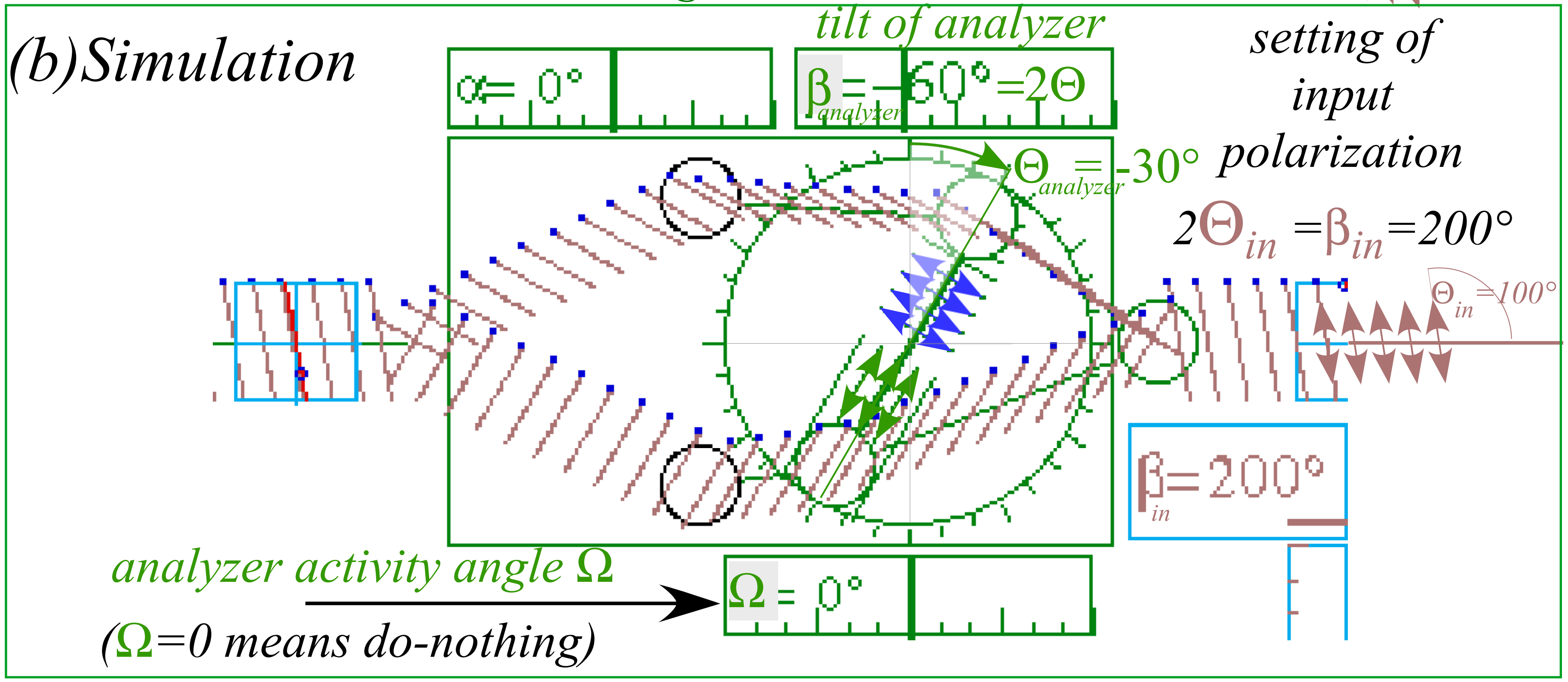
y-Output is: $\langle y|\Theta_{out}\rangle = \langle y|x'\rangle\langle x'|\Theta_{in}\rangle + \langle y|y'\rangle\langle y'|\Theta_{in}\rangle = \sin\Theta\cos(\Theta_{in}-\Theta) - \cos\Theta\sin(\Theta_{in}-\Theta) = \sin\Theta_{in}$.

(Recall $\cos(a+b)=\cos a \cos b - \sin a \sin b$ and $\sin(a+b)=\sin a \cos b + \cos a \sin b$)

Imagine final xy-sorter analyzes output beam into x and y-components.



Amplitude in x or y-channel is
sum over x' and y'-amplitudes
$\langle x'|\Theta_{in}\rangle=\cos(\Theta_{in}-\Theta)$
$\langle y'|\Theta_{in}\rangle=\sin(\Theta_{in}-\Theta)$
with relative angle $\Theta_{in}-\Theta$
of Θ_{in} to Θ-analyzer axes-(x',y')
in products with final xy-sorter:
lab x-axis: $\langle x|x'\rangle=\cos\Theta=\langle y|y'\rangle$
y-axis: $\langle y|x'\rangle=\sin\Theta=-\langle x|y'\rangle$.



x-Output is: $\langle x|\Theta_{out}\rangle=\langle x|x'\rangle\langle x'|\Theta_{in}\rangle+\langle x|y'\rangle\langle y'|\Theta_{in}\rangle=\cos\Theta\cos(\Theta_{in}-\Theta)-\sin\Theta\sin(\Theta_{in}-\Theta)=\cos\Theta_{in}$

y-Output is: $\langle y|\Theta_{out}\rangle=\langle y|x'\rangle\langle x'|\Theta_{in}\rangle+\langle y|y'\rangle\langle y'|\Theta_{in}\rangle=\sin\Theta\cos(\Theta_{in}-\Theta)-\cos\Theta\sin(\Theta_{in}-\Theta)=\sin\Theta_{in}$.

(Recall $\cos(a+b)=\cos a\cos b-\sin a\sin b$ and $\sin(a+b)=\sin a\cos b+\cos a\sin b$)

Conclusion:

$\langle x|\Theta_{out}\rangle=\cos\Theta_{out}=\cos\Theta_{in}$ or: $\Theta_{out}=\Theta_{in}$ so “Do-Nothing” Analyzer in fact does nothing.

Review: *"Abstraction" of bra and ket vectors from a Transformation Matrix*
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Planck's energy and N-quanta (Cavity/Beam wave mode)
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Beam analyzers: Sorter-unsorters
- *The "Do-Nothing" analyzer*
- *Feynman amplitude axiom 4*
- *Some "Do-Something" analyzers*
  - ➡ *Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate*

(1) Optical analyzer in sorter-counter configuration

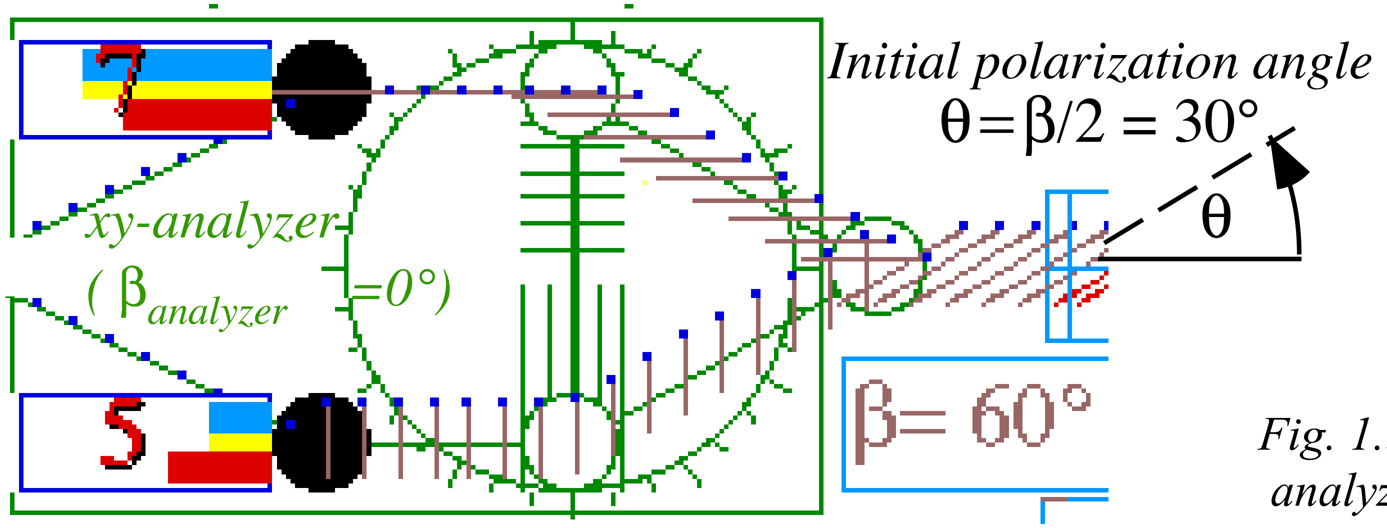
Analyzer reduced to a simple sorter-counter by blocking output of *x*-high-road and *y*-low-road with counters

$$x\text{-counts}\sim\left|\langle x|x'\rangle\right|^2$$

$$=\cos^2\theta=0.75$$

$$y\text{-counts}\sim\left|\langle y|x'\rangle\right|^2$$

$$=\sin^2\theta=0.25$$



Initial polarization angle
$\theta=\beta/2=30°$

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(1) Optical analyzer in sorter-counter configuration

Analyzer reduced to a simple sorter-counter by blocking output of *x*-high-road and *y*-low-road with counters

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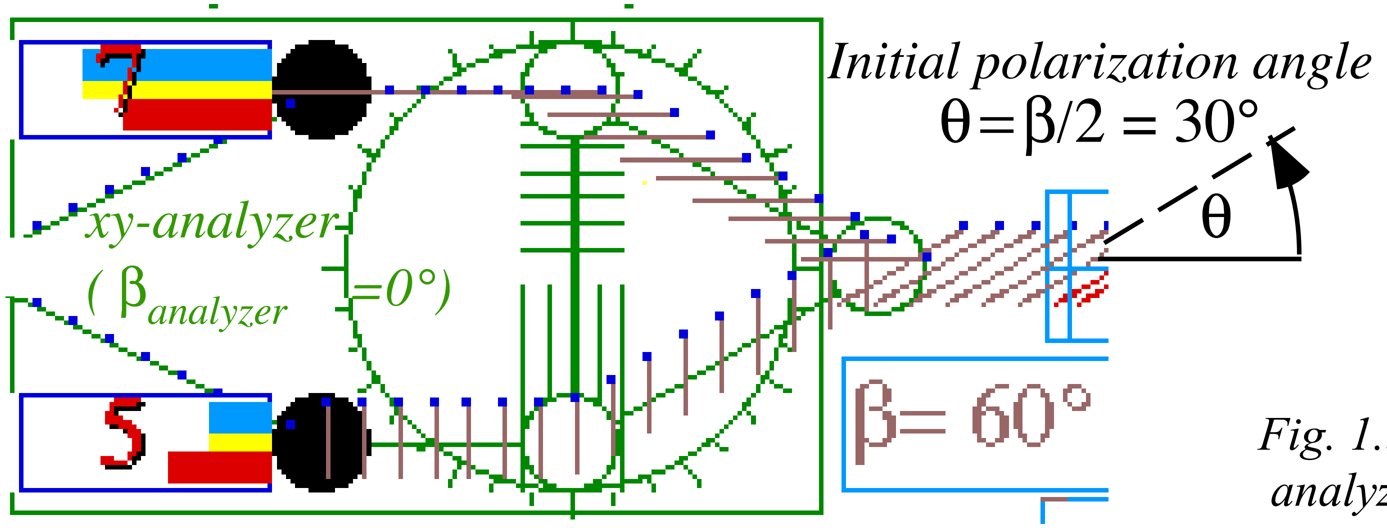


Initial polarization angle
$\theta=\beta/2 = 30°$

Fig. 1.3.3 Simulated polarization analyzer set up as a sorter-counter

(2) Optical analyzer in a filter configuration (Polaroid© sunglasses)

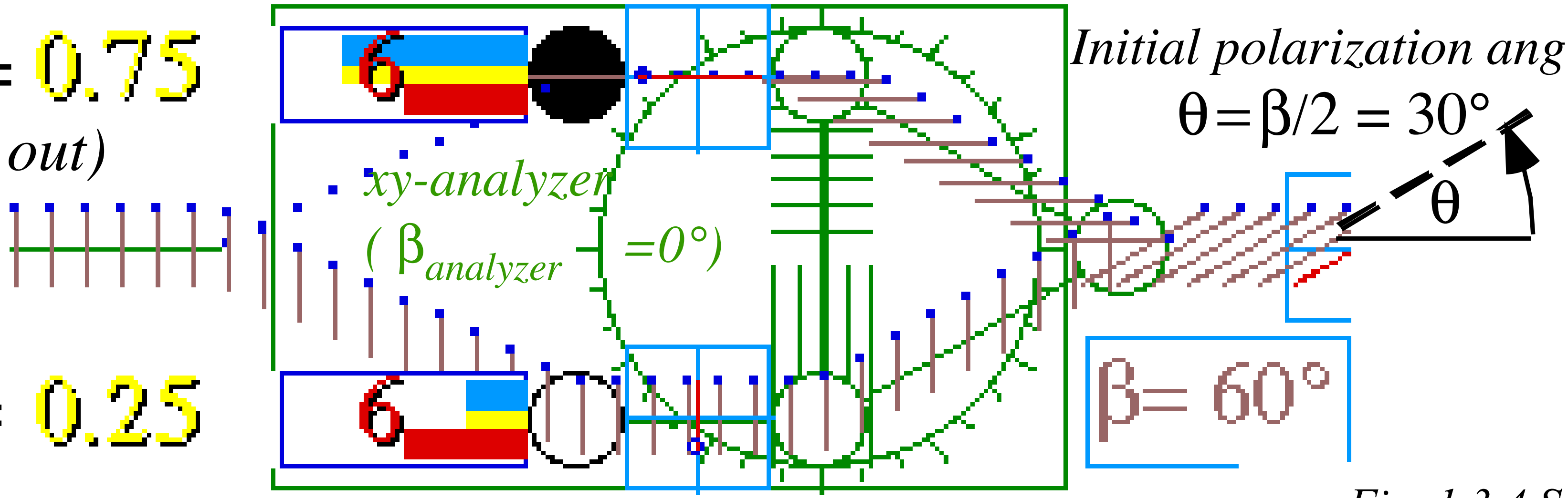
Analyzer blocks one path which may have photon counter without affecting function.

$x\text{-counts} \sim \left|\langle y|x'\rangle\right|^2 = 0.75$

(Blocked and filtered out)

$y\text{-output} \sim \left|\langle y|x'\rangle\right|^2$

$= \sin^2\theta = 0.25$



Initial polarization angle
$\theta=\beta/2 = 30°$

Fig. 1.3.4 Simulated polarization analyzer set up to filter out the x-polarized photons

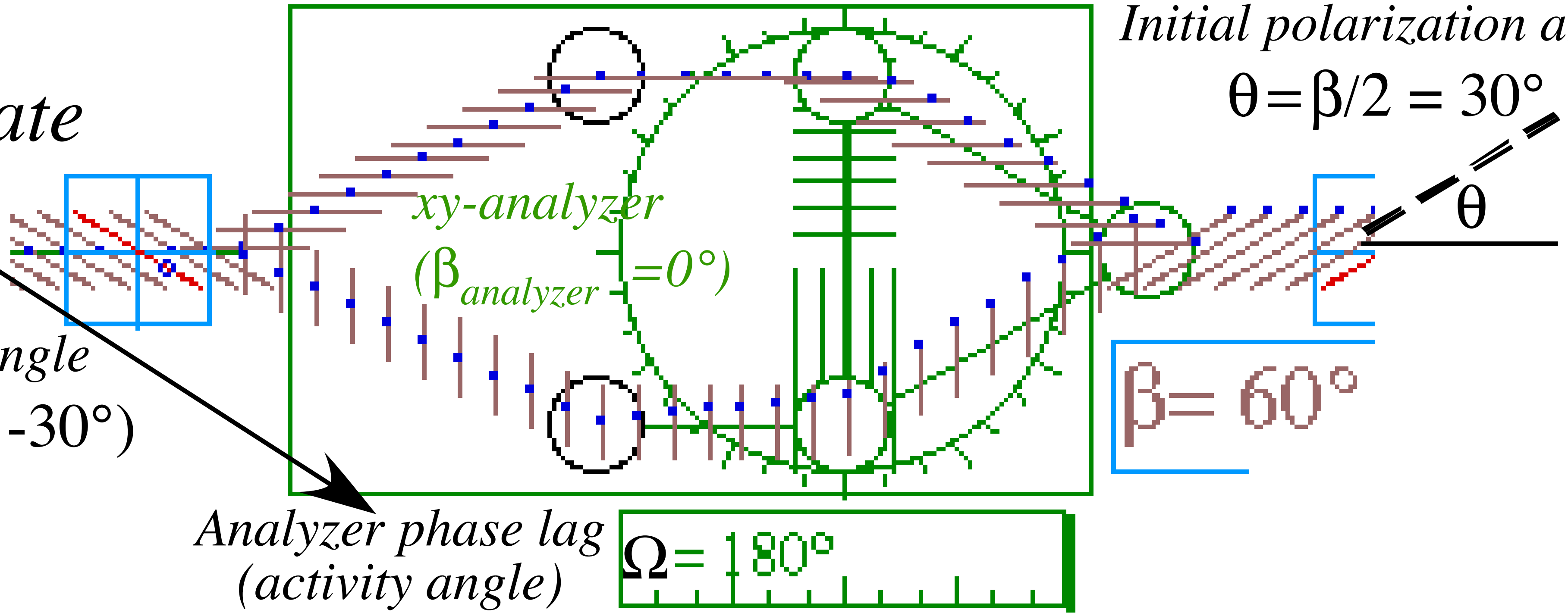
- *Review: "Abstraction" of bra and ket vectors from a Transformation Matrix*
    - *Introducing scalar and matrix products*
- *Planck's energy and N-quanta (Cavity/Beam wave mode)*
    - *Did Max Planck Goof? What's 1-photon worth?*
    - *Feynman amplitude axiom 1*
- *What comes out of a beam sorter channel or branch-b?*
    - *Sample calculations*
    - *Feynman amplitude axioms 2-3*
- *Beam analyzers: Sorter-unsorters*
    - *The "Do-Nothing" analyzer*
    - *Feynman amplitude axiom 4*
    - *Some "Do-Something" analyzers*
        - *Sorter-counter, Filter, 1/2-wave plate, 1/4-wave plate*

(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates

(a)

Half-wave plate

$(\Omega=\pi)$



Initial polarization angle

$\theta=\beta/2 = 30°$

Final polarization angle

$\theta=\beta/2 = 150°$(or -30°)

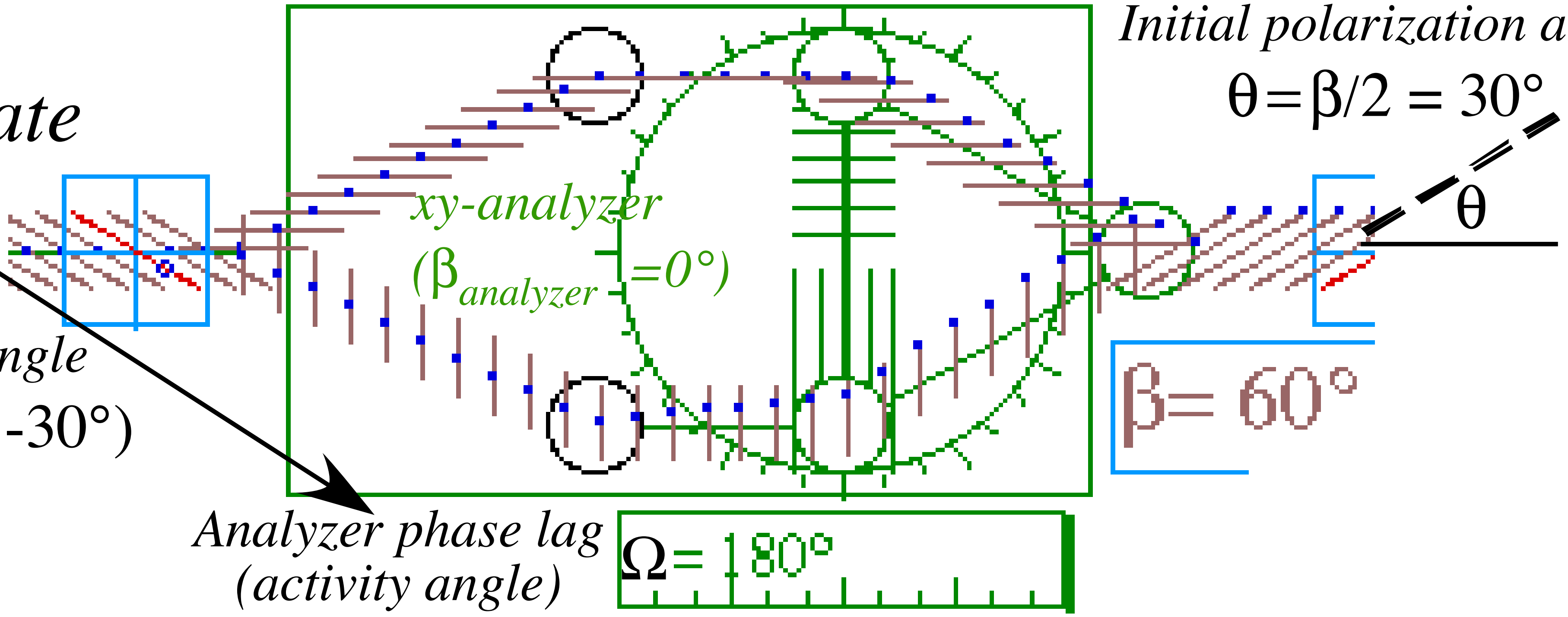
Analyzer phase lag
(activity angle)

(3) Optical analyzers in the "control" configuration: Half or Quarter wave plates

(a)

Half-wave plate

$(\Omega=\pi)$



Initial polarization angle

$\theta=\beta/2 = 30°$

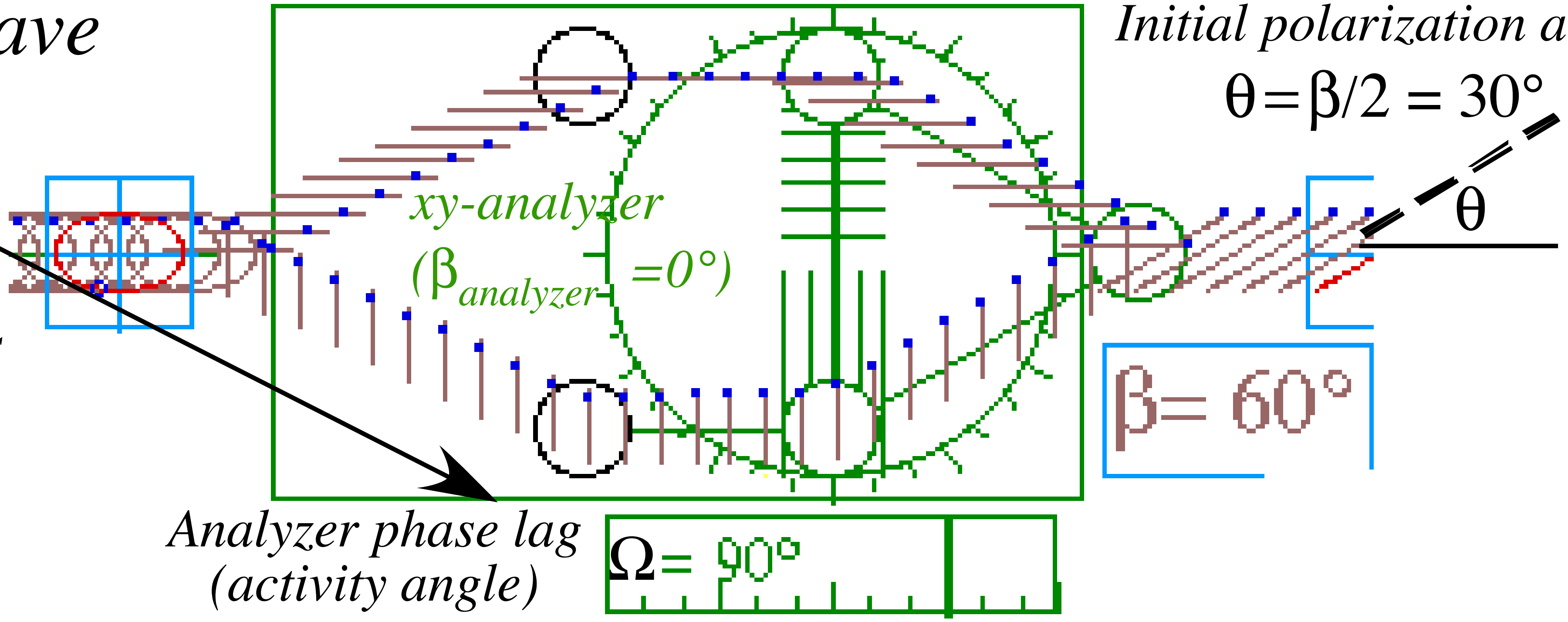
Final polarization angle

$\theta=\beta/2 = 150°$(or -30°)

Analyzer phase lag
(activity angle)

(b) Quarter-wave plate

$(\Omega=\pi/2)$



Initial polarization angle

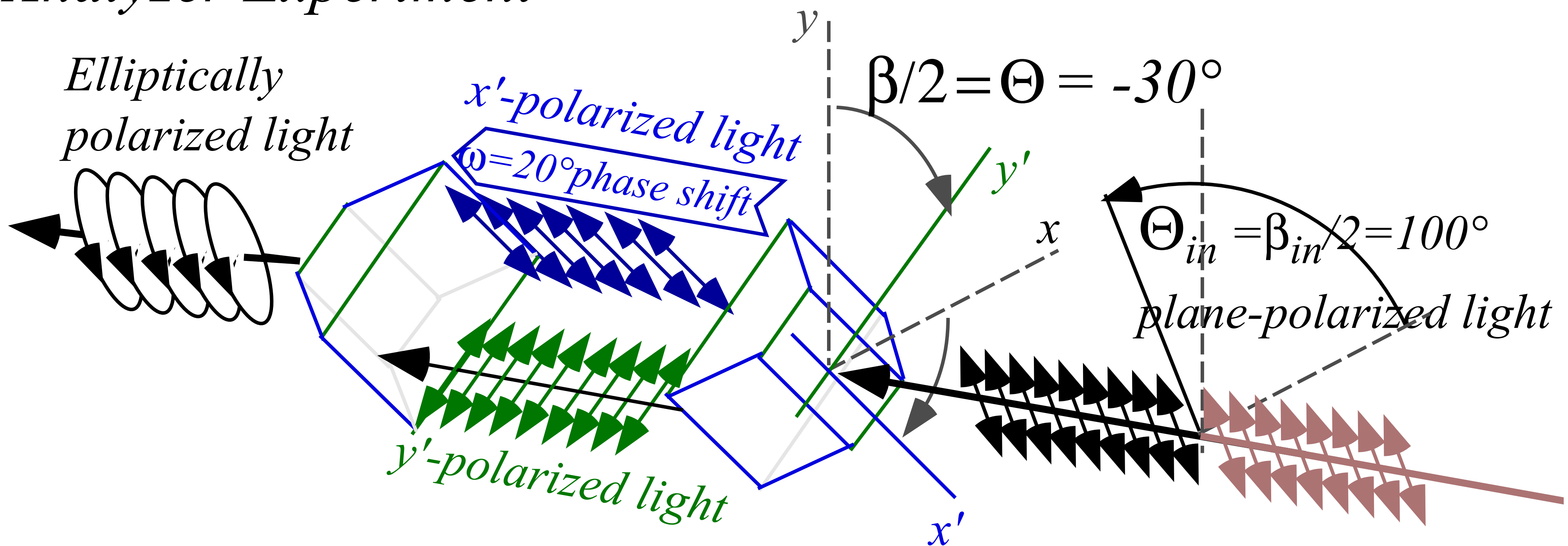
$\theta=\beta/2 = 30°$

Final polarization is
untilted elliptical

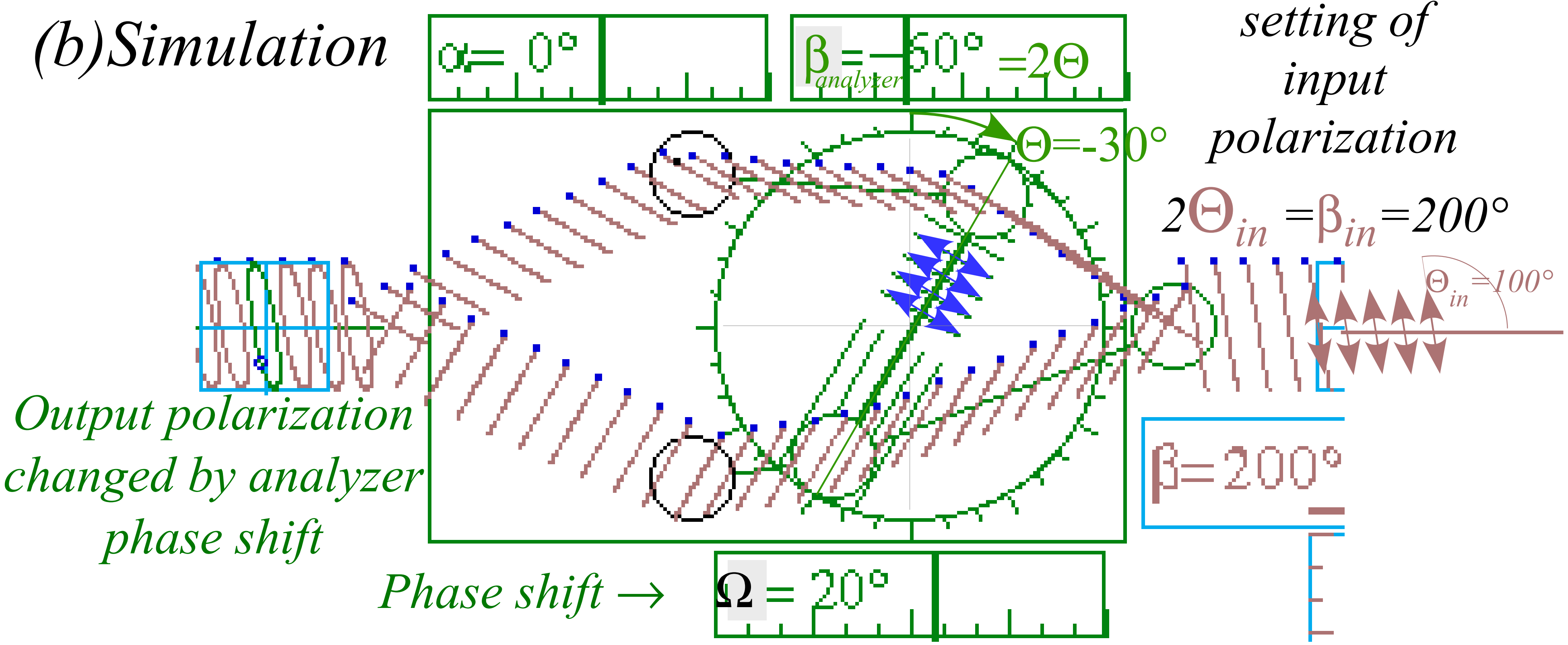
Analyzer phase lag
(activity angle)

Fig. 1.3.5 Polarization control set to shift phase by (a) Half-wave ($\Omega=\pi$) , (b) Quarter wave ($\Omega=\pi/2$)

(a)Analyzer Experiment



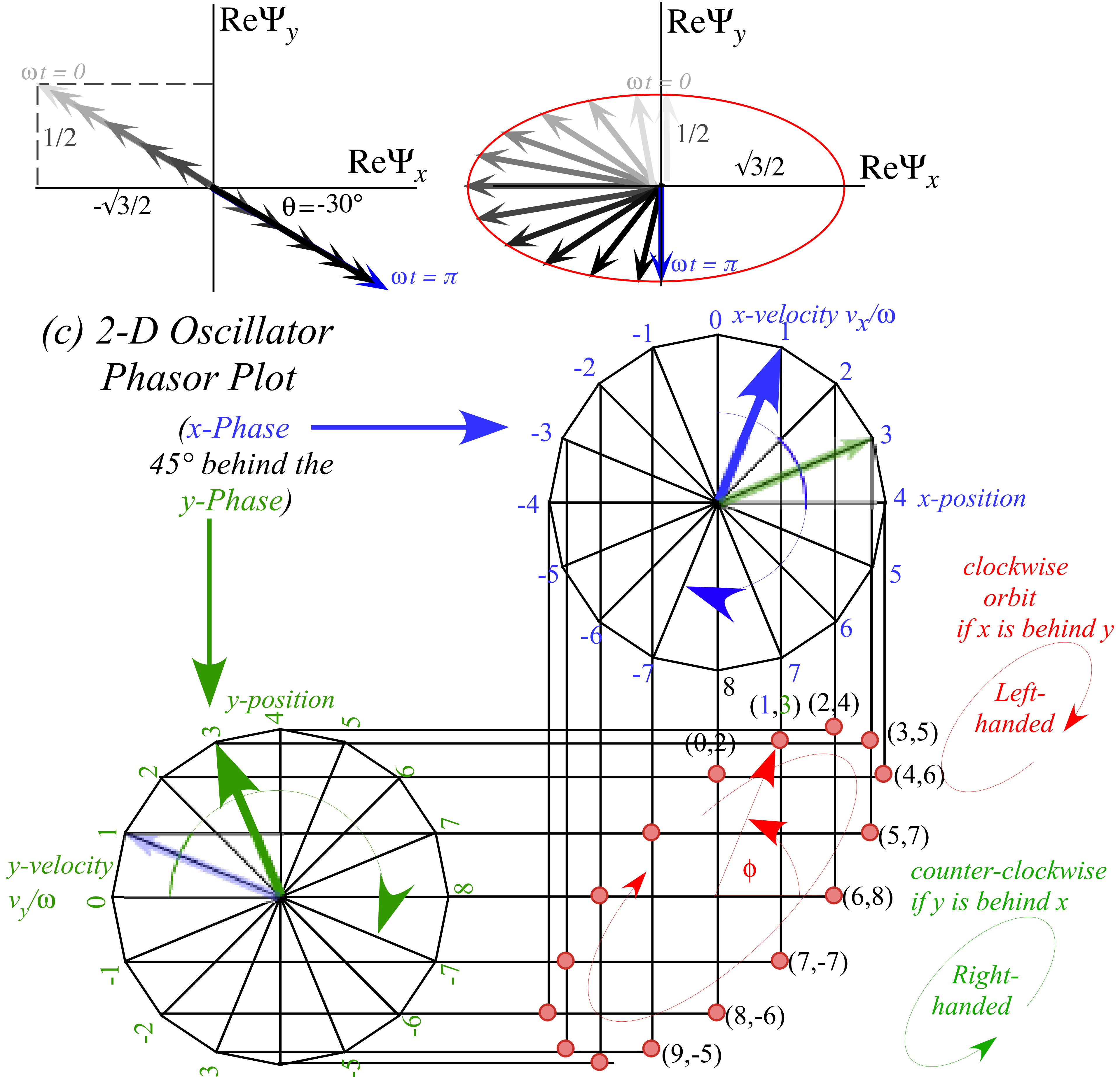
(b)Simulation



Similar to "do-nothing" analyzer but has extra phase factor $e^{-i\Omega} = 0.94-i\ 0.34$ on the x'-path .

x-output: $$\langle x|\Psi_{out}\rangle = \langle x|x'\rangle e^{-i\Omega}\langle x'|\Psi_{in}\rangle + \langle x|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega}\cos\Theta\cos(\Theta_{in}-\Theta) - \sin\Theta\sin(\Theta_{in}-\Theta)$$

y-output: $$\langle y|\Psi_{out}\rangle = \langle y|x'\rangle e^{-i\Omega}\langle x'|\Psi_{in}\rangle + \langle y|y'\rangle\langle y'|\Psi_{in}\rangle = e^{-i\Omega}\sin\Theta\cos(\Theta_{in}-\Theta) + \cos\Theta\sin(\Theta_{in}-\Theta)$$



(c) 2-D Oscillator Phasor Plot

(x-Phase 45° behind the y-Phase)

clockwise orbit if x is behind y

Left-handed

counter-clockwise if y is behind x

Right-handed

Fig. 1.3.6 Polarization states for (a) Half-wave ($\Omega=\pi$) , (b) Quarter wave ($\Omega=\pi/2$) (c) ($\Omega=-\pi/4$)