

Read Unit 3 (*SRQM by Ruler&Compass*) thru page 28. Study Lecture 25-26

Space-time and per-space-time

Lorentz-Minkowski space-time (or per-space-time) coordinate system graphs (similar to the ones made in class for $u = \frac{3}{5}c$) are attached and available on-line. Let 1 inch squares correspond to (*light-sec., sec.*) in space-time (x, ct) or to the per-space-time units (*light-Hz, Hz*) on a (κ, ν) graph. Space-time graphs for the problems below require ± 5 values for space and time. Graphs can be flipped so either Lighthouse or else Ship can have square (rest frame) axes. You should do one of each.

NOTE: For this assignment it is recommended you follow the Newtonian graph convention: +x-to-the-right and +ct-down. This conforms to the animations on two of our main relativity web apps: *Pirelli Relativity Challenge* and *RelativIt*.

Follow these links to go to them directly:

- http://www.uark.edu/ua/pirelli/html/lighthouse_scenarios.html
- <http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=22>
- <http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=24>

Space-time Terrorism

1 (a) Complete the following happening tables using the Lorentz transformation between ship space-time coordinates (x', ct') and lighthouse coordinates (x, ct) given that the ship is traveling from right to left at a speed of $u = \frac{3}{5}c$ and passes the lighthouse at $t=0=t'$. Calculate answers needed below by algebra and then make a $u = \frac{3}{5}c$ plot to check the results.

Ship emits light	Explosion #1	Explosion #2	Explosion #3
x = 3 litesec. t = -5 sec.	x = t =	x = -1 litesec. t = -1 sec.	x = t = 1 sec.
x' = t' =	x' = -1 litesec. t' = -3 sec.	x' = t' =	x' = -3 litesec. t' =

Draw the space-time paths of light waves emitted right and left from explosions #1 and #2 on the space-time graph and answer the following questions.

If lighthouse broadcasts 100 Mhz what ν does ship tune to receive it at $t=-1$? MHz at $t=+1$? Mhz

- (a) What is rapidity of ship relative to lab $\rho_{SvsL} = \underline{\hspace{2cm}}$? ... lab relative to ship $\rho_{LvsS} = \underline{\hspace{2cm}}$?
- (b) When does light from explosion #1 hit the lighthouse? (Lighthouse time)
- (c) When does light from explosion #1 hit the lighthouse? (Ship time)
- (d) When does light from explosion #2 hit the lighthouse? (Lighthouse time)
- (e) When does light from explosion #2 hit the lighthouse? (Ship time)
- (f) Draw paths of fragments from explosions #1 and #2 for fragment speed $c/2$ or $-c/2$ relative to the ship.

B.I.G.A.N.N. Investigates

2 Explosions in problem 1 lead to an investigation by B.I.G.A.N.N. (Bureau of Intergalactic Aids to Navigation at Night) .

- (a) When does the first fragment from explosion #1 hit the lighthouse? (Lighthouse time)
- (b) When does a second fragment from explosion #1 hit the lighthouse? (Lighthouse time)
- (c) When does a fragment from explosion #1 hit the ship? (Ship time)
- (d) When does a fragment from explosion #2 hit the ship? (Ship time)
- (e) When does a fragment from explosion #2 hit the Lighthouse? (Lighthouse time)
- (f) ...lighthouse says 1st fragment goes c using **addition formula of rapidity** $\rho_{FvsL} = \rho_{FvsS} + \rho_{SvsL}$ and of **velocity**:
- (g) ...lighthouse says 2nd fragment goes c

The authorities of BIGANN have spotted a causal (as opposed to acausal) connection between all the explosions. To whom does it point?

SOLUTIONS

Spacetime and per-space-time

1 (a) Using ruler and compass construct on graph paper a $\sqrt{3}/5c$ Lorentz-Minkowski space-time and per-space-time coordinate system graph similar to the ones made in class. Let 1 inch squares correspond to (light-sec., sec.) in space-time (x,ct) or to the per-space-time units (light-Hz, Hz) on a (ck,v) graph.

Spacetime Terrorism

2 (a) Complete the following happening tables using the Lorentz transformation between ship space-time coordinates (x',ct') and lighthouse coordinates (x,ct) given that the ship is traveling from right to left at a speed of $\sqrt{3}/5c$ and passes the lighthouse at $t=0=t'$. Calculate and use a $u/c = 3/5$ graph (exercise 1) to plot and check the results.

Ship emits light	Explosion #1	Explosion #2	Explosion #3
x = 3 litesec. t = -5 sec.	x = 1 t = -3	x = -1 litesec. t = -1 sec.	x = -3 t = 1 sec.
x' = 0 t' = -4	x' = -1 litesec. t' = -3 sec.	x' = -2 t' = -2	x' = -3 litesec. t' = -1

(a) Draw the space-time paths of light waves emitted right and left from explosions #1 and #2 on the space-time graph and answer the following questions.

(a) If lighthouse broadcasts 100 Mhz what v does ship tune to receive it at $t=-1$? **200MHz** at $t=+1$? **50Mhz** **Before**

passing: blue shift ratio: $b = \sqrt{\frac{1+u/c}{1-u/c}} = \sqrt{\frac{1+3/5}{1-3/5}} = \frac{2}{1}$ After passing: red shift ratio: $r = \sqrt{\frac{1-u/c}{1+u/c}} = \sqrt{\frac{1-3/5}{1+3/5}} = \frac{1}{2}$

(b) What is rapidity of ship relative to lab $\rho_{SvsL} = \ln(1/2) = -0.6931$? ... lab relative to ship $\rho_{LvsS} = \ln(2) = +0.6931$?

(c) When does light from explosion #1 hit the lighthouse? -2.0 (Lighthouse time)

(d) When does light from explosion #1 hit the lighthouse? -2.5 (Ship time)

(e) When does light from explosion #2 hit the lighthouse? 0.0 (Lighthouse time)

(f) When does light from explosion #2 hit the lighthouse? 0.0 (Ship time)

(g) Draw paths of fragments from explosions #1 and #2 for fragment speed $c/2$ or $-c/2$ relative to the ship.

B.I.G.A.N.N. Investigates

3 The explosions in problem 2 lead to an investigation by B.I.G.A.N.N. (Bureau of Intergalactic Aids to Navigation at Night) headed by Rollah H. Ann Hoover (secret granddaughter of J. Edgar Hoover).

(a) When does the first fragment from explosion #1 hit the lighthouse? -20/11 = -1.81 (Lighthouse time)

(b) When does a second fragment from explosion #1 hit the lighthouse? 4 (Lighthouse time)

(c) When does a fragment from explosion #1 hit the ship? -1 (Ship time)

(d) When does a fragment from explosion #2 hit the ship? 2 (Ship time)

(e) When does a fragment from explosion #2 hit the Lighthouse? ∞ 12th hour of never (Lighthouse time)

(f) ...lighthouse says 1st fragment goes -11/13c using **addition formula of rapidity** $\rho_{FvsL} = \rho_{FvsS} + \rho_{SvsL}$ or of **velocity**:

$$\frac{u_{FvsL}}{c} = \tanh(\rho_{FvsL}) = \tanh(\rho_{FvsS} + \rho_{SvsL}) = \frac{\tanh(\rho_{FvsS}) + \tanh(\rho_{SvsL})}{1 + \tanh(\rho_{FvsS})\tanh(\rho_{SvsL})}$$

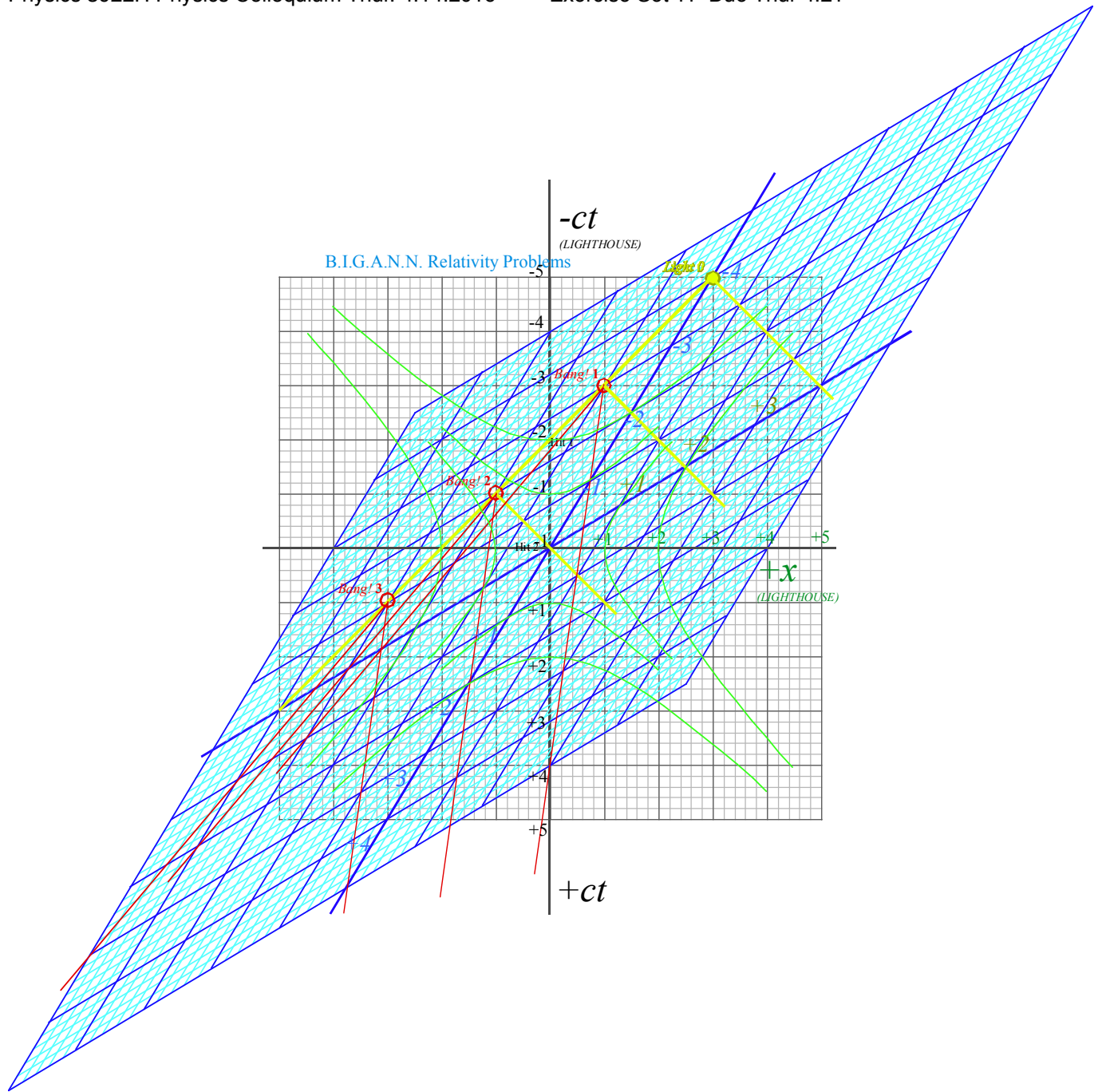
$$u_{F_1vsL} = \frac{u_{F_1vsS} + v_{SvsL}}{1 + \frac{u}{c} \cdot \frac{v}{c}} = \frac{(-\frac{1}{2}) + (-\frac{3}{5})}{1 + (-\frac{1}{2})(-\frac{3}{5})} c = \frac{-5-6}{10+3} c = \frac{-11}{13} c$$

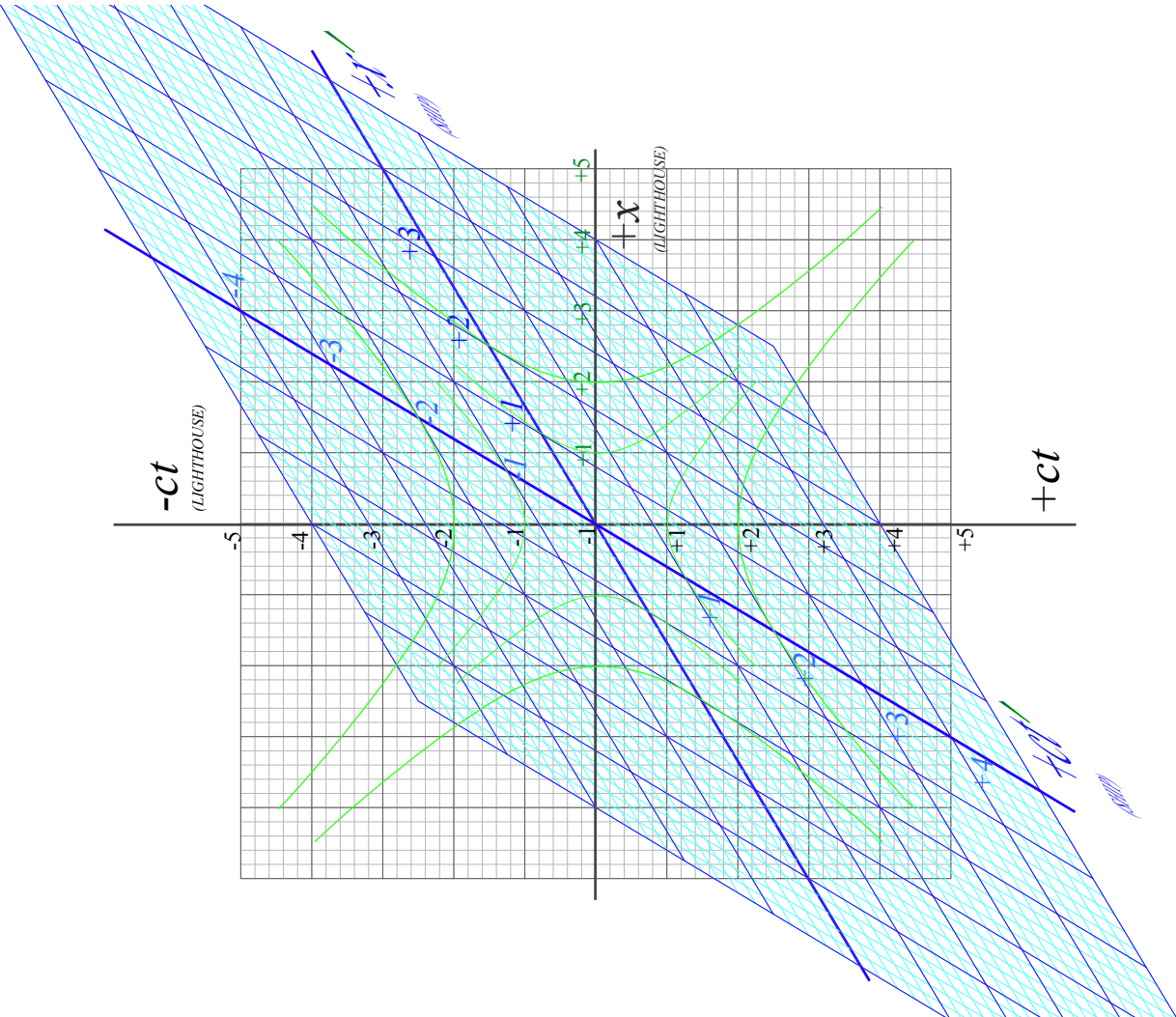
(g) ...lighthouse says 2nd fragment goes -1/7c

$$u_{F_2vsL} = \frac{u_{F_2vsS} + v_{SvsL}}{1 + \frac{u}{c} \cdot \frac{v}{c}} = \frac{(+\frac{1}{2}) + (-\frac{3}{5})}{1 + (+\frac{1}{2})(-\frac{3}{5})} c = \frac{+5-6}{10-3} c = \frac{-1}{7} c$$

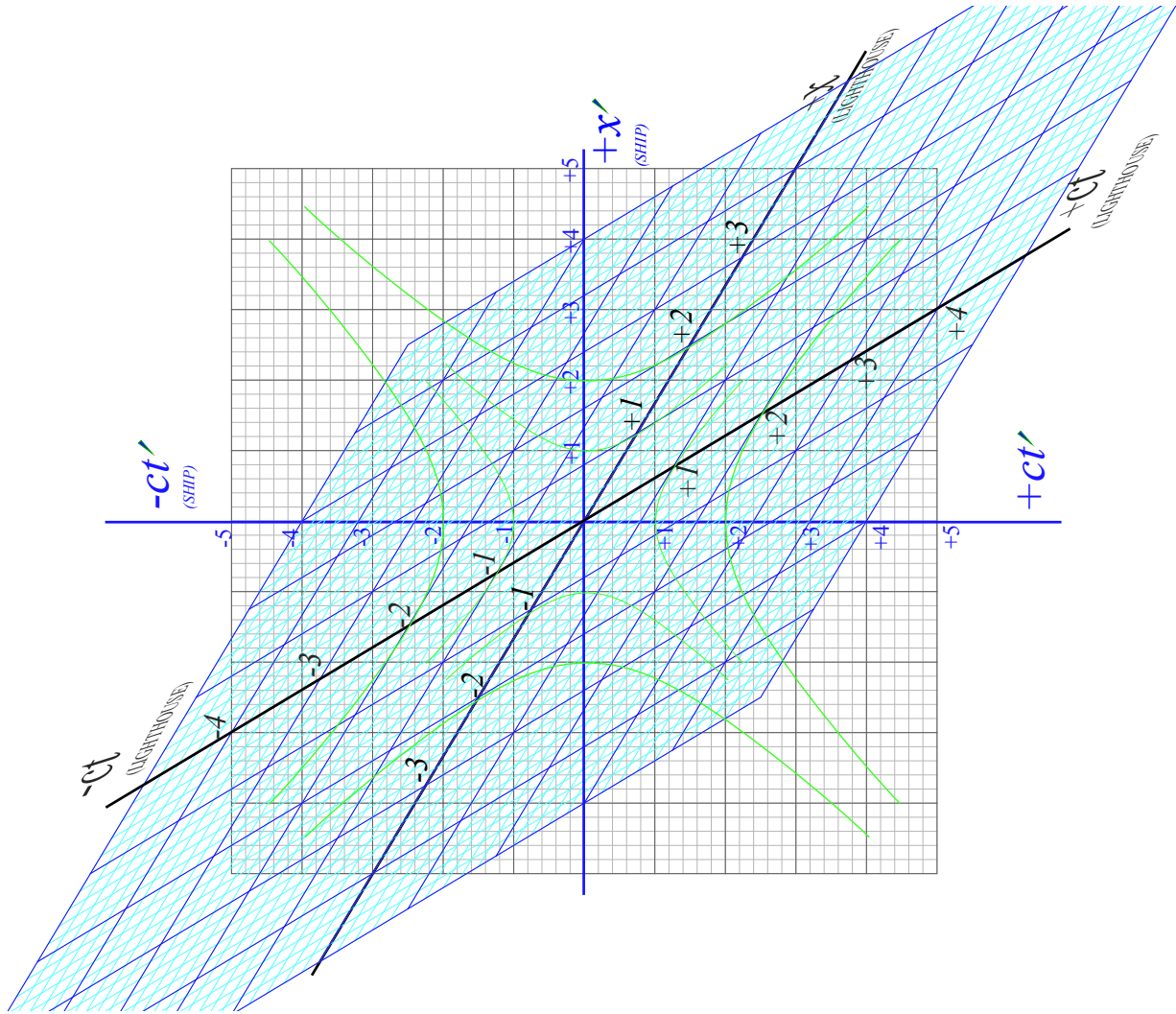
(a) The authorities of BIGANN have spotted a causal (as opposed to acausal) connection between all the explosions. To whom does it point? **Ship looks very guilty starting with that light burst at $t=-5$ sec. Was it a set-up? Was he framed?**

Ship is traveling from right to left at a speed of $\sqrt{3}/5c$ and passes the lighthouse at $t=0=t'$.





Better version of Lighthouse-square graph available in class or online.



Better version of Ship-square graph available in class or online.