Physics 3922H Physics Colloquium Thur. 4.26.2016 Exercise Set 12 Due Thur 4.28

Read Unit 3 (*SRQM by Ruler&Compass*) thru page 36. Study Lecture 25-29

H*ow long does it take to get to* α*-Centauri in 6 months?*

1. Suppose we define a velocity we will call $v_{ignorant}$ as that speed that someone ignorant of relativity would say a spaceship had to go to get to a distant star in a given time. For example, if we ask how fast a ship would have to go to get to α-Centauri (~4 light years away) in 6 months then the "ignorant" person would say it had to go $v_{ignorant} = 8c$, that is, eight times the speed of light. So if super-luminal travel is prohibited, then 6 months seems too short.

But the relativity expert says that there is a speed v_{expert} which <u>will</u> get the ship to α-Centauri in 6 months according to the ship's passengers, who, after all, are the ones really counting their time.

(a) Compute v_{expert} for α-Centauri trip and derive general algebraic relations giving v_{expert} in terms of $v_{ignorant}$ and vice-versa.

(b) How long does it really take to get to α-Centauri in 6 months? (Lighthouse time.)

2. Consider a more realistic project discussed recently in the New York Times.
This involves sending a package that would get to α-Centauri in 16 years (its time) by blasting it up to necessary speed with powerful lasers. This one can be plotted on a Minkowski graph such as can be made using the Relawavity website.
Do a plot and make an event table involving departure and arrival space-time events. Find $v_{ignorant}$ and v_{expert}.
If upon arrival the package sends a message back, when should we expect to hear from it if we had sent it out today?

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=8l8&velocity=-0.25

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2. Suppose we define a velocity we will call $v_{ignorant}$ as that speed that someone ignorant of relativity would say a spaceship had to go to get to a distant star in a given time. For example, if we ask how fast a ship would have to go to get to α-Centauri (~4 light years away) in 6 months then the "ignorant" person would say it had to go $v_{ignorant} = 8c$, that is, eight times the speed of light. So if super-luminal travel is prohibited, then 6 months seems too short.

But the relativity expert says that there is a speed v_{expert} which <u>will</u> get the ship to α-Centauri in 6 months according to the ship's passengers, who, after all, are the ones really counting their time.

(a) Compute v_{expert} for this 6-month α-Centauri trip and derive general algebraic relations giving v_{expert} in terms of $v_{ignorant}$ and vice-versa.

(b) How long does it really take to get to α-Centauri in 6 months? (Lighthouse time.)

Problems like these are helped by a happening or *event table* that tabulates *(x,ct)* and *(x,′ct′)* values for key events.

α−Centauri Trip Event Table	Departure from L.H.	Arrival at α−C.
Lighthouse coordinates	$x_D=0$ $ct_D=0$	$x_A=4.0c-yrs$ $ct_A=?$
Ship coordinates	$x'_D=0$ $ct'_D=0$	$x'_A=0$ $ct'_A=0.5c-yrs$

We put down the numbers we know and leave the unknowns with a ?-mark. The first column just sets the coincidence of the space and time origins of ship and LH at the departure event where they pass.
In the second column we know LH measured distance (4.0 lite-yrs) where arrival will happen for the ship. Since it happens to the ship it must be ship-space-zero ($x'_A=0$) since the ship (like us all) always carries its origin with it.

Lorentz transformation is a pair of equations for two unknowns: (1) rapidity ρ or speed $u=c\tanh\rho$, and (2) LH time t_A.

$$x'_A = +x_A\cosh\rho - ct_A\sinh\rho \qquad \text{inverse pair of equations:} \quad x_A = +x'_A\cosh\rho + ct'_A\sinh\rho$$
$$ct'_A = -x_A\sinh\rho + ct_A\cosh\rho \qquad\qquad ct_A = +x'_A\sinh\rho + ct'_A\cosh\rho$$

Choose sign of ρ so ship origin ($x'_A=0$) travels at speed u according to LH. ($x_A=ct_A\sinh\rho/\cosh\rho=ct_A\tanh\rho=u\cdot t_A$)

Known values from the event table gives two independent equations (or inverses) to solve for ρ and time t_A.

$$0 \;= +4.0c\cosh\rho - ct_A\sinh\rho \qquad \text{inverse pair of equations:} \quad 4.0c = +0\cosh\rho + c0.5\sinh\rho$$
$$c0.5 = -4.0c\sinh\rho + ct_A\cosh\rho \qquad\qquad ct_A = +0\sinh\rho + c0.5\cosh\rho$$

First we get $4.0c/c0.5=\sinh\rho=8.0$ which means that $\cosh\rho=\sqrt{1+\sinh^2\rho}=\sqrt{1+8^2}=\sqrt{65}$. That gives the speed u/c.

$\frac{u}{c}=\tanh\rho=\frac{\sinh\rho}{\cosh\rho}=\frac{8}{\sqrt{65}}=0.992278$ (It's fast indeed! Rapidity $\rho=\tanh^{-1}\frac{8}{\sqrt{65}}=2.77647$ is big hyperbolic area.)

Ship sees α-Centauri *4.0literyr* distance shrink by a Lorentz factor sechρ=1/coshρ $=1/\sqrt{65}=0.1240355$ to 0.496139 *liteyrs*.
The LH time is $t_A=0.5\cosh\rho=0.5\sqrt{65}=4.03112\,yrs.$, less than 1% longer than *light by itself* takes to go to α- Centauri! Let's call $V_{ignorant}$ the LH-measured distance x_A divided by the ship-measured time t'_A for the trip.

$$\frac{V_{ignorant}}{c}=\frac{x_A}{ct'_A}=\frac{x'_A\cosh\rho+ct'_A\sinh\rho}{ct'_A}=\frac{0+ct'_A\sinh\rho}{ct'_A}=\sinh\rho=\frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$$

Solving this gives a formula for $u=V_{expert}$ in terms of $V_{ignorant}$ which is $4.0c/0.5=8c$ in this case.

$$\left(\frac{V_{ignorant}}{c}\right)^2=\frac{\frac{u^2}{c^2}}{1-\frac{u^2}{c^2}} \quad \text{so:} \left(1-\frac{u^2}{c^2}\right)\frac{V^2_{ignorant}}{c^2}=\frac{u^2}{c^2} \quad \text{or:} \;\frac{u^2}{c^2}\left(1+\frac{V^2_{ignorant}}{c^2}\right)=\frac{V^2_{ignorant}}{c^2} \quad \text{or:}\; u=\frac{V_{ignorant}}{\sqrt{1+\frac{V^2_{ignorant}}{c^2}}}=V_{expert}=\frac{8c}{\sqrt{65}}$$

That agrees with the previous result. $u=0.992278c$.

Ex. 2.

Assumed we want to get to α- Centauri in 16 years Lighthouse time.
Formula for $u = V_{expert}$ in terms of $V_{ignorant}$ which is $4.0c/16 = 0.25c$ in this case.

$$u = \frac{V_{ignorant}}{\sqrt{1 + \frac{V_{ignorant}^2}{c^2}}} = V_{expert} = \frac{c/4}{\sqrt{1 + \frac{1}{16}}} = \frac{c/4}{\sqrt{\frac{17}{16}}} = \frac{c}{\sqrt{17}} = 0.2425c$$ That is just below. $u = c/4$.

Given $u/c = \tanh\rho = 0.2425$ means that $\rho = \text{atanh}(0.2425) = 0.2475$ and also very close and just above c/4.

The distance seen by the ship (or particle/payload) is 4(LiteYr)sech ρ=4·0.9701(LiteYr) =3.88 (LiteYr). So it will experience a 12% reduction in the travel time of 16year or 0.9701·16=15.52yrs.