The following is to acquaint you with of some lesser known properties of exponentials and logarithms

1.8.1 *Backsides of exponentials*

(a) Follow zig-zag scheme shown at the beginning of Lect. 11 to make plots of exponential $y=e^x$ at as many integer points x= -2, -1, 0, 1, 2,.. as is practical on full page graph paper provided online or in lab. Then add to the plot precise half-way points x= -2.5, -1.5, -0.5, etc... as is practical. Show how a plot of $y=\log_e x$ function is obtained from the graph

(b) By algebra or geometry find tangent lines and their slope at integer points x=-2, -1, 0, 1, 2,.. (This is equivalent to solving the part (c) of this exercise.)

(c) As a roller-coaster car moves down a track $y=e^x$ it shines one laser headlight beam along the track and another droplight beam vertically downward so both make spots on baseline $y=0$. Find the distance between spots as function of x.

1.8.2 *Sophomore-Physics-Earth*

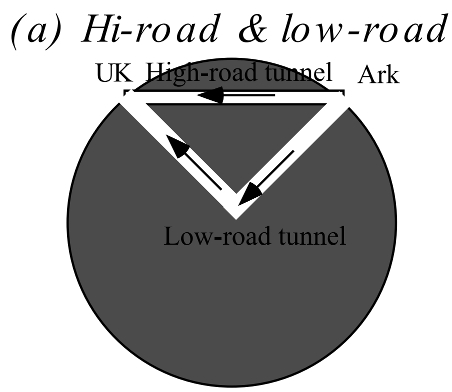
(a) Follow the zig-zag scheme in Lect. 11 (or in Fig. 8.5 and 8.7 of text) to construct the potential and force curves of the Ideal Uniform Density Earth inside ($PE(x)=kx^2/2+PE(0)$) and outside ($PE(x)=-x^{-1}$).

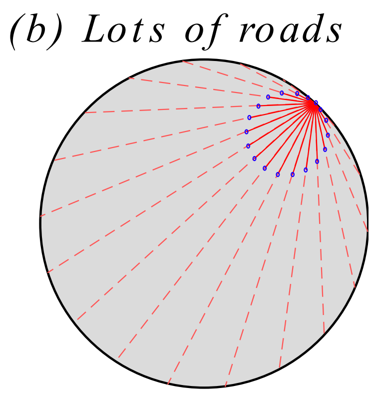
(b) On graph show focal point, latus-radius , and directrix of the inside PE parabola. Draw as accurately as possible the parabola's circle of curvature contacting it at $x=0$.

(c) Draw a "kite" (see Fig. 8.4 in text) tangent to parabola at $x=1$ and another tangent at $x=½$.

1.8.3 Tunnels to UK (5600 miles as an earthworm crawls) are shown below. One high-road is a direct route. A low-road turns at the Earth center. (Travel and turn-around are assumed frictionless and survivable.)

(a) What is the time for each trip? Discuss using geometry or algebra arguments.

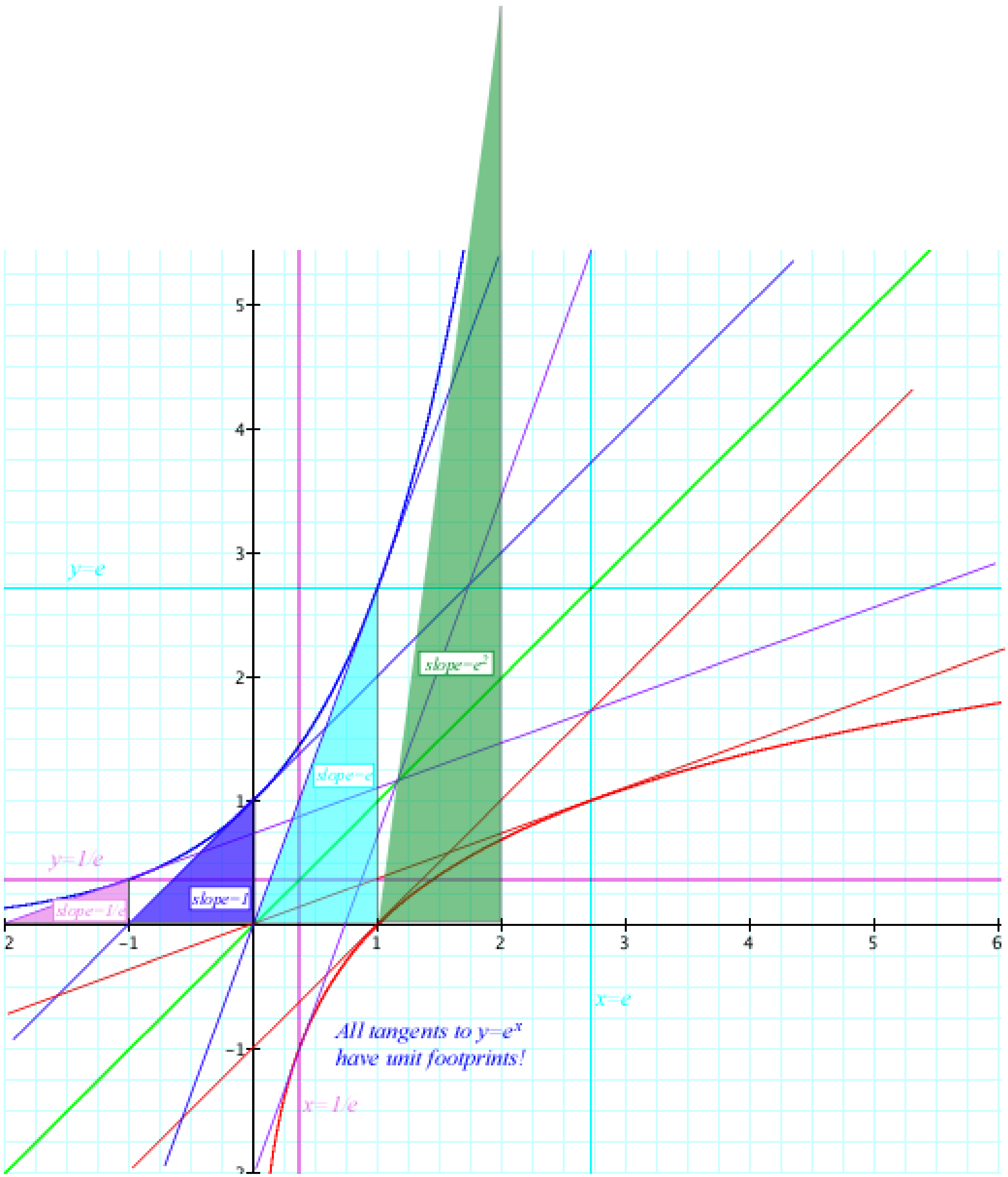


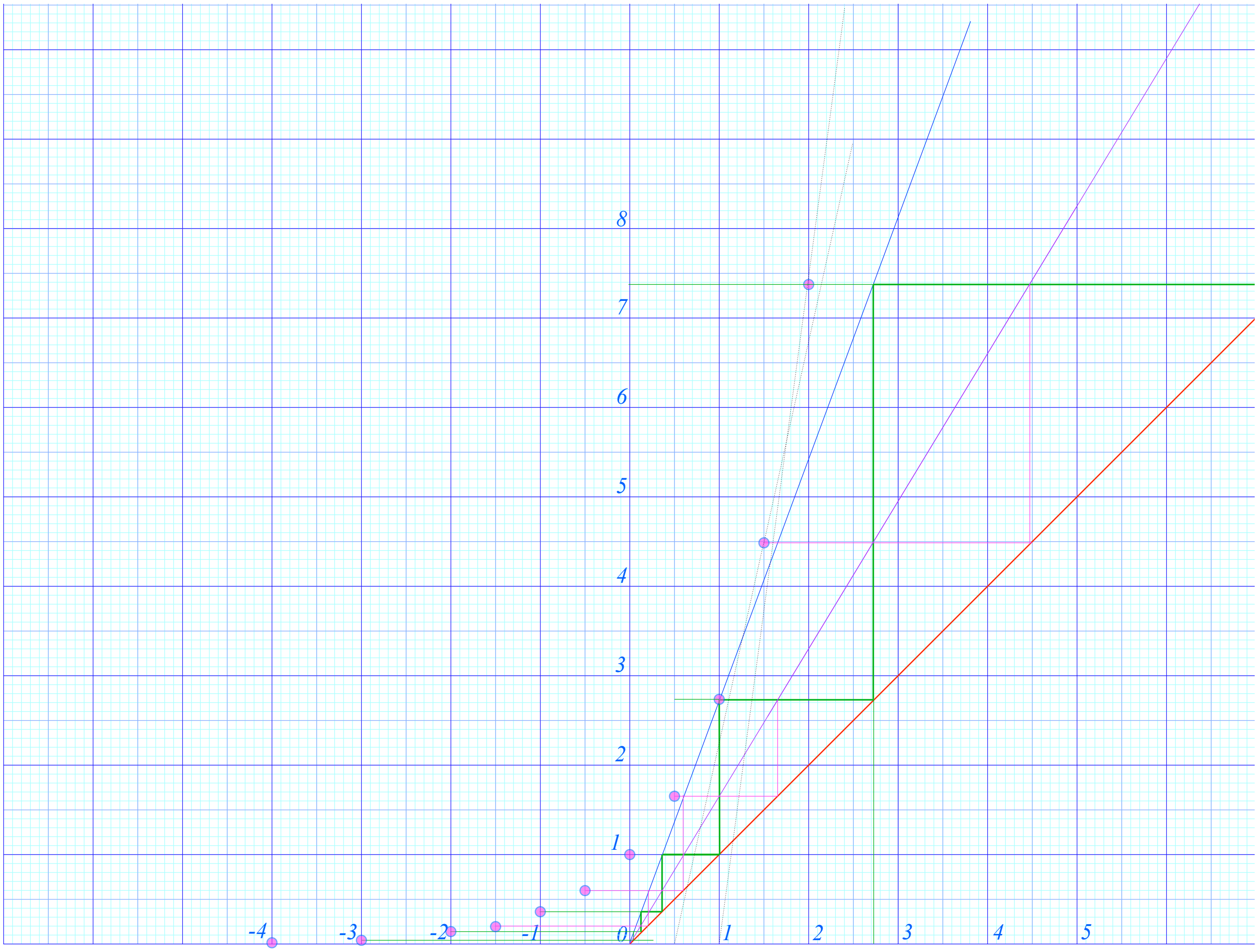


(b) Assume cars in subway tunnels depart Ark. at time $t=0$ as indicated above. Describe curve (thru dots shown) locating car positions at a later mid-trip time t before arrival and at arrival. (Thales geometry of circular chords may help. Recall superball figure 6.1 in text.)

(c) What if the half-way turn-around point is above the Earth-center. Is trip quicker or slower?

(a) Follow zig-zag scheme shown at the beginning of Lect. 11 to make plots of exponential $y=e^x$ at as many integer points x= -2, -1, 0, 1, 2,.. as is practical on full page graph paper provided online or in lab. Then add to the plot precise half-way points x= -2.5, -1.5, -0.5, etc... as is practical. Show how a plot of $y=\log_e x$ function is obtained from the graph



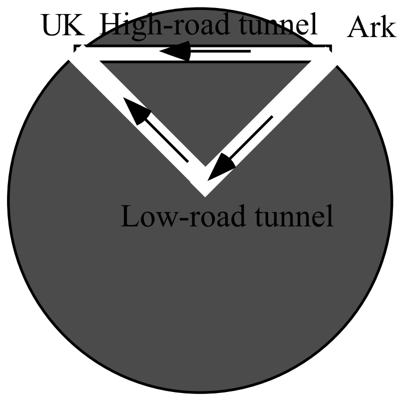


Each slope line intersects y=0 exactly -1 unit distance from their x-coordinate point.

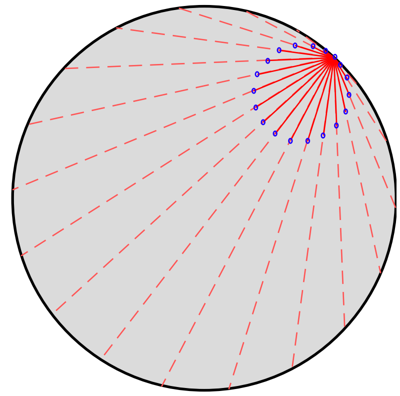
If the graph is expanded it clearly shows that there is unit distance ($\Delta x=1$) between x-axis intersections of any tangent to point (x, $y=e^x$) and the vertical line $x=x_1$ going thru that point.
Quite remarkable! All tangents to $y=e^x$ have unit footprints.

Exercise 1.8.3. Tunnels to UK (5600 miles away as an earthworm crawls) are shown below. One high-road is a direct route. The other low-road turns around at the Earth center. Travel and turn-around are assumed frictionless and survivable. (a) How long is each trip? Discuss. Both the same.

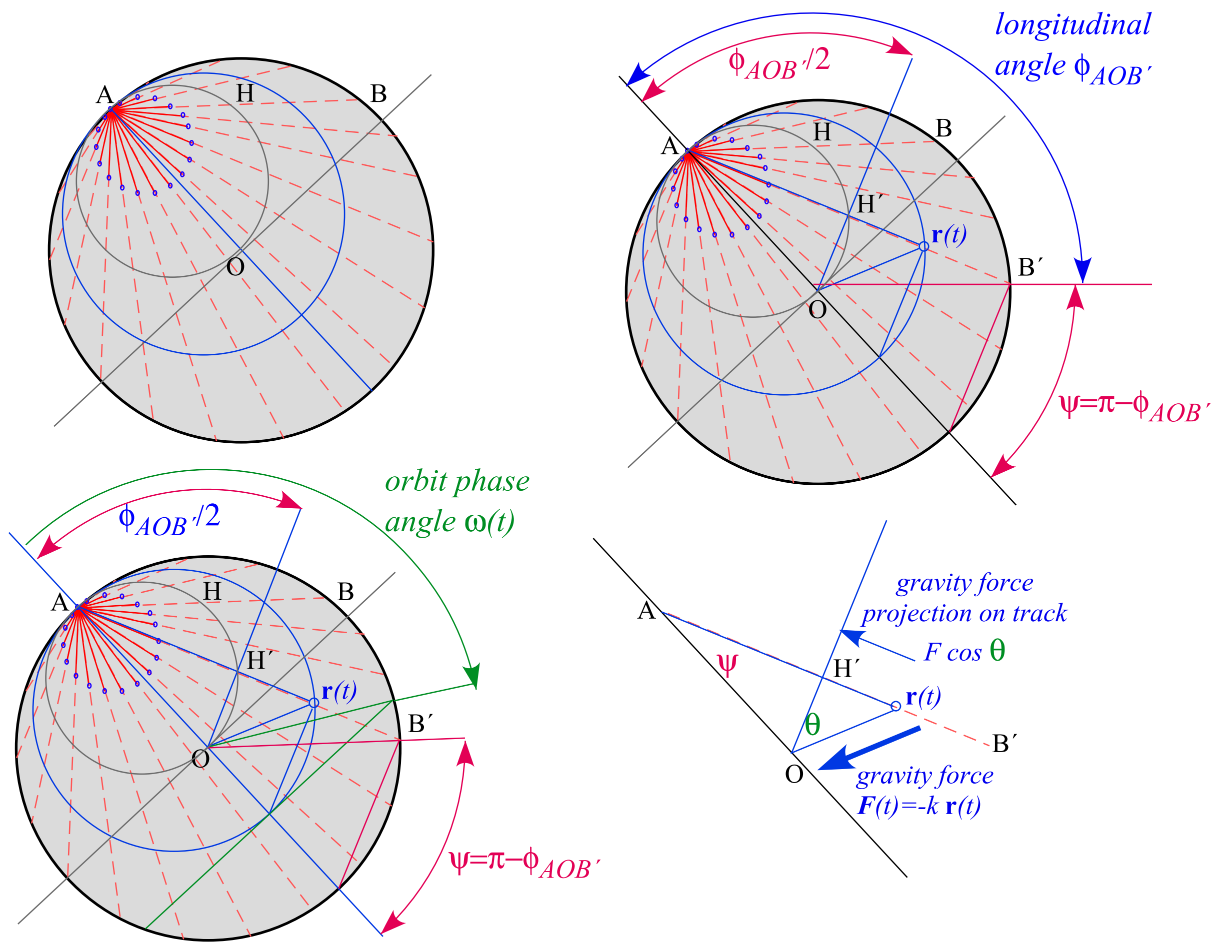
(a) Hi-road & low-road



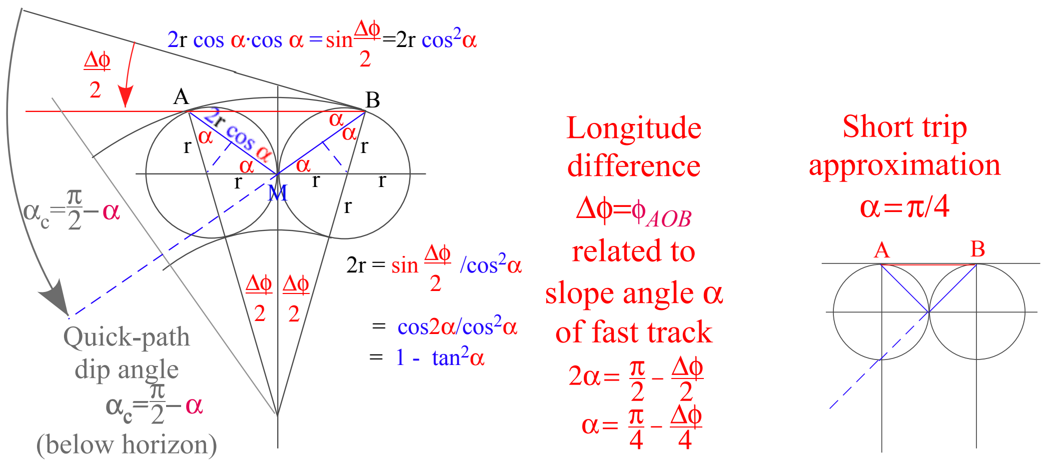
(b) Lots of roads



(b) A network of subways leaving Ark. at time t=0. What curve (like the dots) describe each moment?
Each is on a **circle** at distance $r_A = D\cos\theta$ from A with $D = R_{earth}(1-\cos\omega_{earth}t)$. θ is subway polar angle and $\pi/\omega_{earth} = 42$ minutes is the one-way surface-to-surface trip on each θ path having length $L = R_{earth}\cos\theta$.

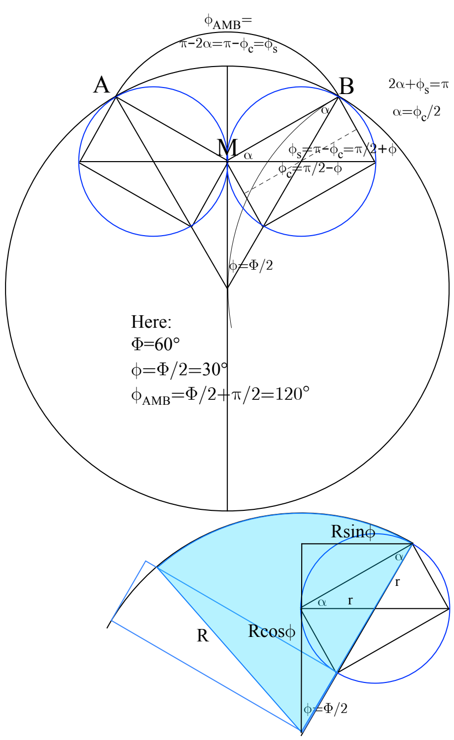


(c) What if the half-way turn-around point is above the Earth-center. Is trip quicker or slower?
There is a point nearly midway between the bend at Earth-center and the center of the straight Ark. to U.K. track where the bend should be to achieve a minimum travel time and shorter than the others'.



The more difficult problem of deep-V-tunnel global travel is solved similarly, but a geometric solution sketched below is quick (once you see the trick!). The trick is to imagine a pencil of competing tunnels going out from both point A and point B so that the trial runs form two expanding circles that finally touch on a tangent that bisects the A-to-B longitude angle $\phi_{AOB}=\Delta\phi$. We find the angle $\alpha=\pi/4-\Delta\phi/4$ between shortest path and *quickest* path. It approaches α=45° in the local limit $\Delta\phi\rightarrow 0$. The *AMB* vertex angle is $\phi_{AMB}=\pi/2+\phi_{AOB}/2$ and approaches a local *90°* limit. Half the *AMB* vertex angle is $\phi_{AMB}/2=\pi/2-\alpha=\alpha_C$ (compliment of α) that is also horizon dip angle between the horizon and the quickest path. For short trips: $\alpha=\alpha_C=45°$. For longer trips: $\alpha<45°$ and $\alpha_C>45°$.

Each circle diameter $D=2r$ (in units of Earth radius $R_\oplus$) expands as $D=1-\cos\theta$ where $\theta=\omega t$ is the circular orbit angle subtended by projecting the diameter point to the Earth circle. Travel time T is proportional to angle θ with θ=π corresponding to 42 minutes of a half-circle orbit and θ=π/2 to 21 min. (Going half-way between A and B by the straight tunnel takes 21 minutes.)



The following is a general solution with the $\phi=\Phi/2=45°$ case given numerically.