# Lecture 11
# Tue. 2.23.2016

*(Ch. 8 of Unit 1)*

## *Geometry of common power-law potentials*

*Geometric (Power) series*

*"Zig-Zag" exponential geometry*

*Projective or perspective geometry*

*Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

*Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields*

*Compare mks units of Coulomb Electrostatic vs. Gravity*

## *Geometry of idealized "Sophomore-physics Earth"*

*Coulomb field outside*　　*Isotropic Harmonic Oscillator (IHO) field inside*

*Contact-geometry of potential curve(s)*

*"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

*Earth matter vs nuclear matter:*

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

## *Introducing 2D IHO orbits and phasor geometry*

*Phasor "clock" geometry*

# *Geometry of common power-law potentials*

*Geometric (Power) series*

→ *"Zig-Zag" exponential geometry*

*Projective or perspective geometry*

*Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

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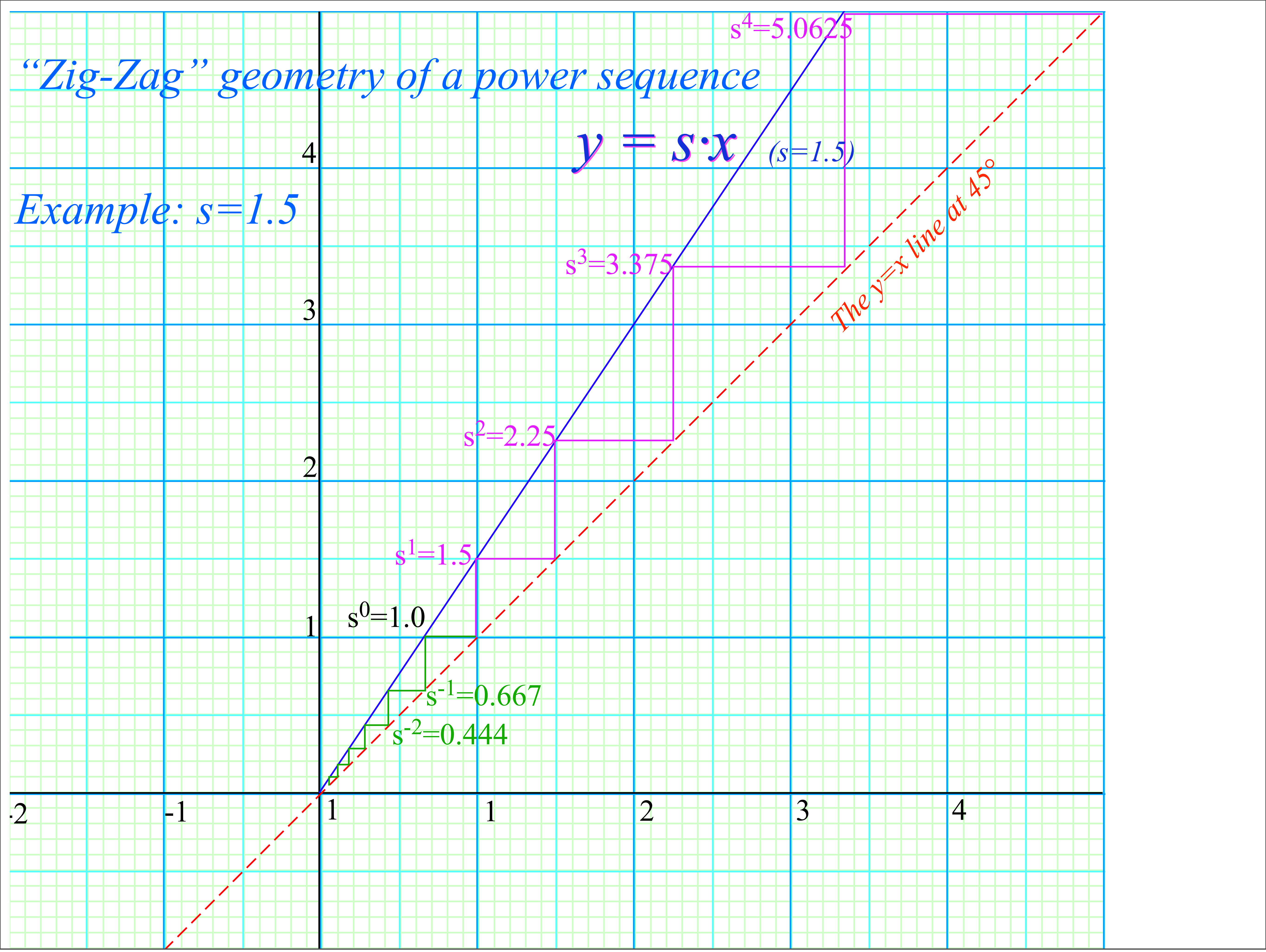
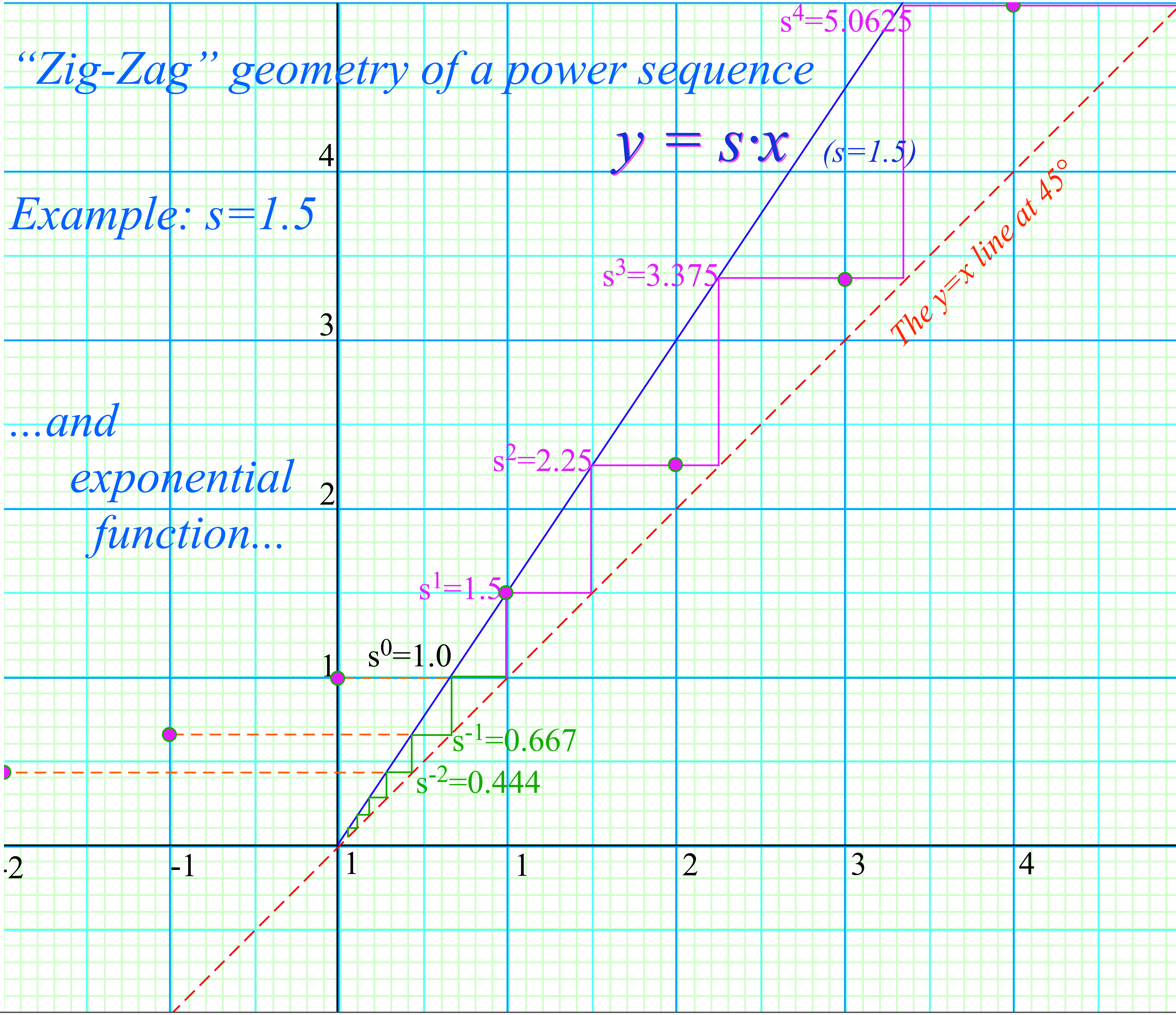
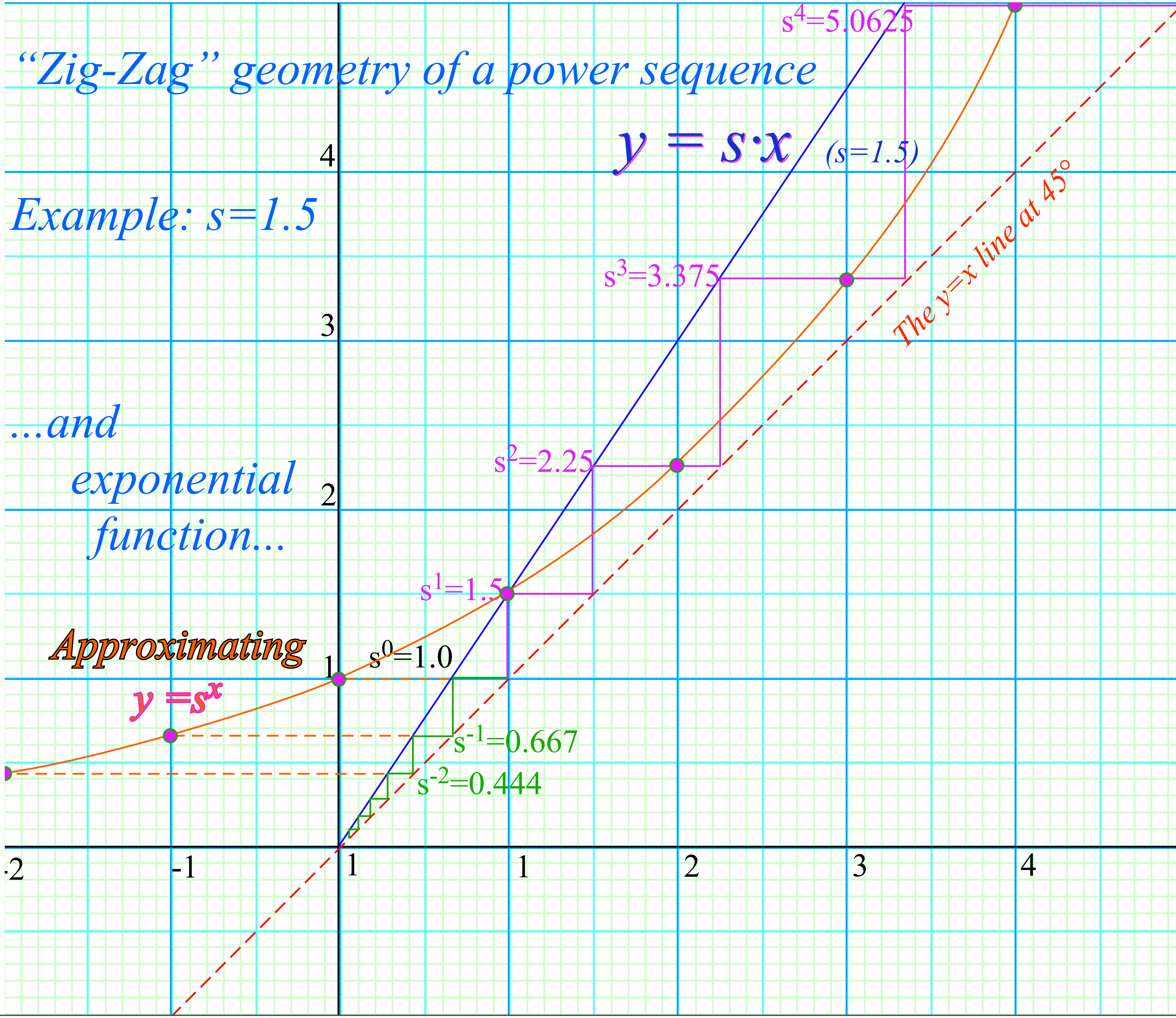
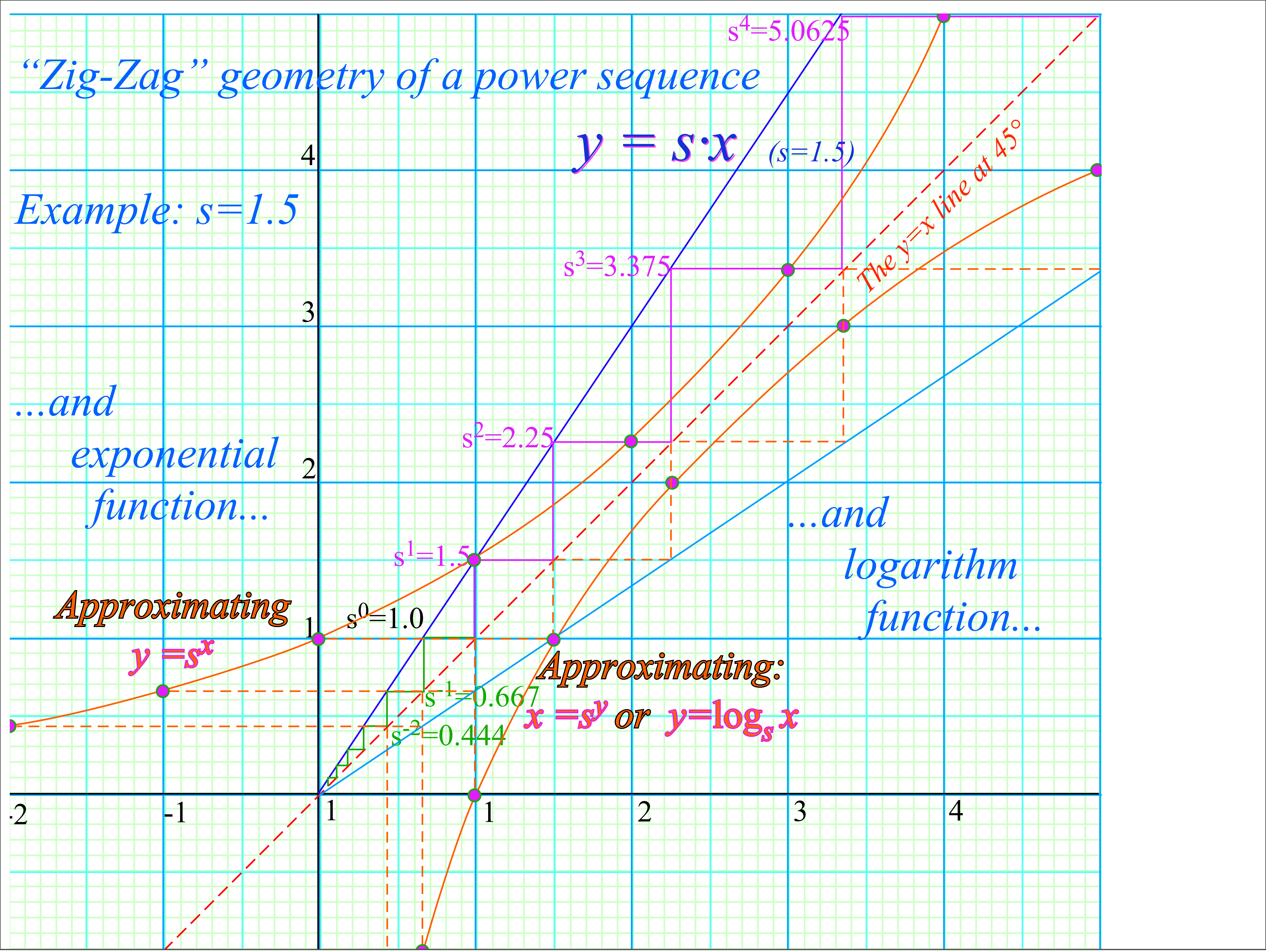

"Zig-Zag" geometry of a power sequence
Example: s=1.5
y = s·x
(s=1.5)
s4=5.0625
s3=3.375
s2=2.25
s1=1.5
s0=1.0
s-1=0.667
s-2=0.444
The y=x line at 45°
4
3
2
1
-2
-1
1
1
2
3
4

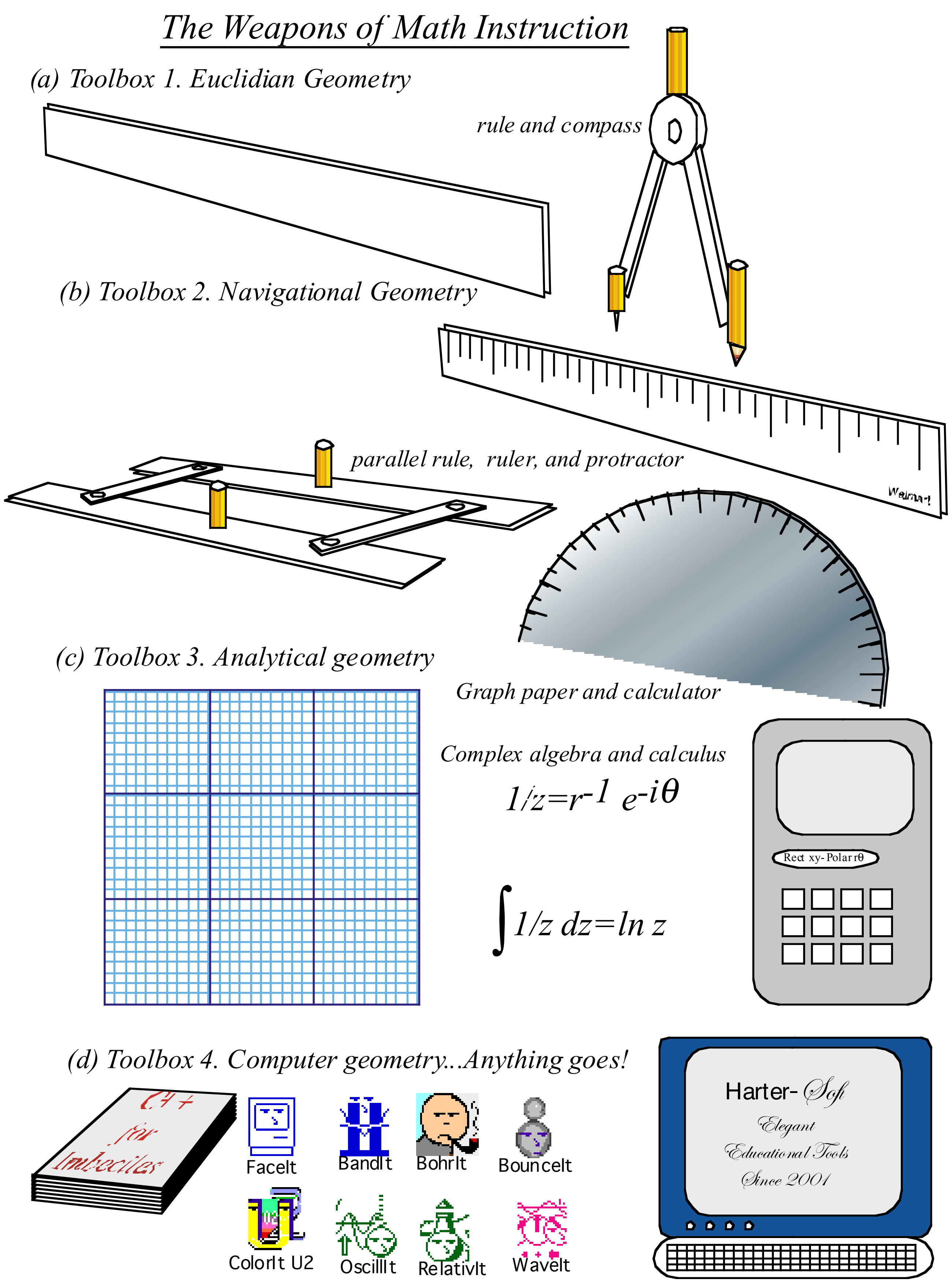

“Zig-Zag” geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
...and exponential function...
The y=x line at 45°
$s^4$=5.0625
$s^3$=3.375
$s^2$=2.25
$s^1$=1.5
$s^0$=1.0
$s^{-1}$=0.667
$s^{-2}$=0.444
4
3
2
1
-2
-1
1
1
2
3
4



“Zig-Zag” geometry of a power sequence
Example: s=1.5
...and
exponential
function...
Approximating
y = s^x
y = s·x
(s=1.5)
The y=x line at 45°
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
s^-1=0.667
s^-2=0.444
4
3
2
1
-2
-1
1
1
2
3
4



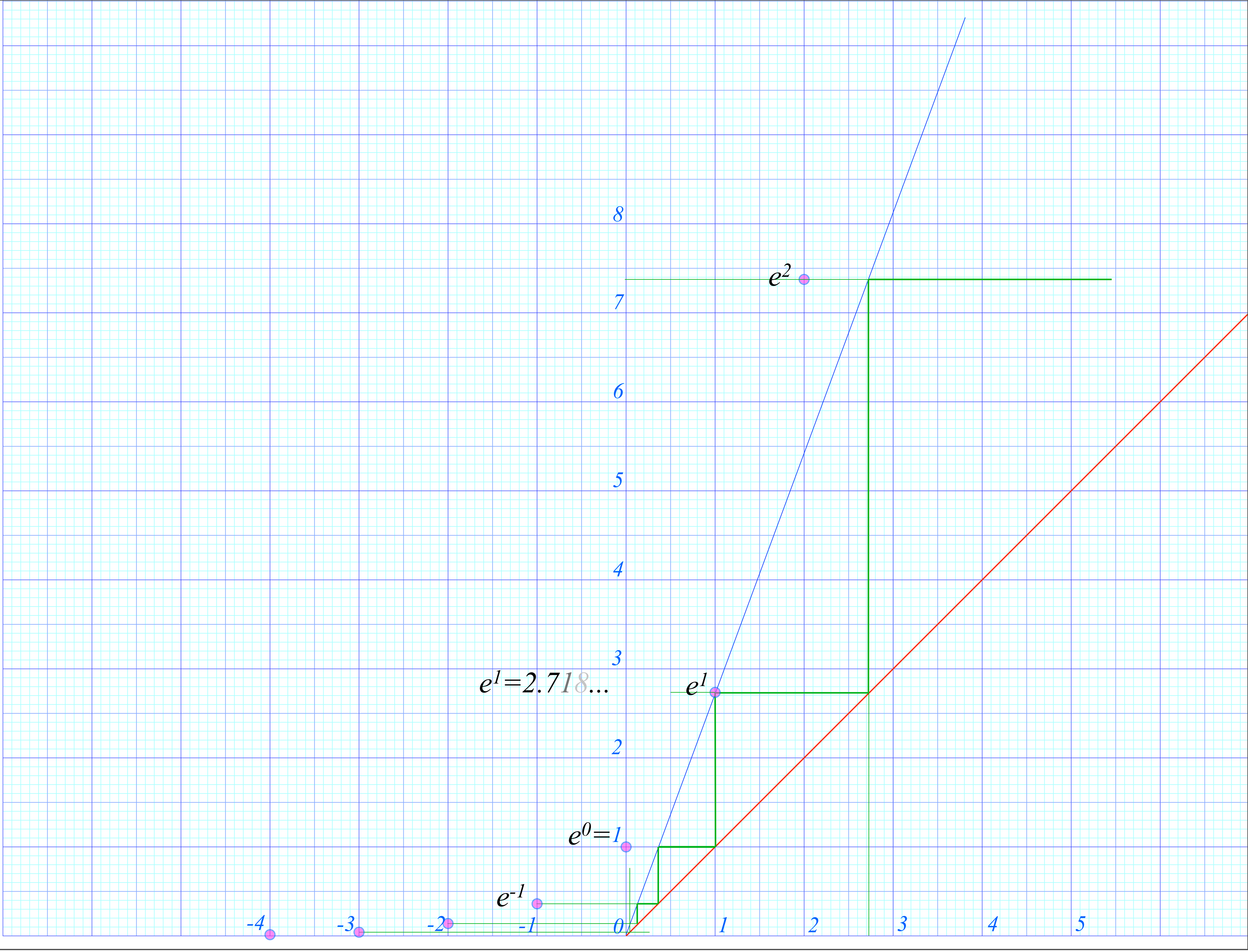
"Zig-Zag" geometry of a power sequence
Example: s=1.5
$y = s \cdot x$ (s=1.5)
$s^4$=5.0625
$s^3$=3.375
$s^2$=2.25
$s^1$=1.5
$s^0$=1.0
$s^{-1}$=0.667
$s^{-2}$=0.444
The y=x line at 45°
...and exponential function...
Approximating $y = s^x$
...and logarithm function...
Approximating: $x = s^y$ or $y = \log_s x$
4
3
2
1
-2
-1
1
1
2
3
4

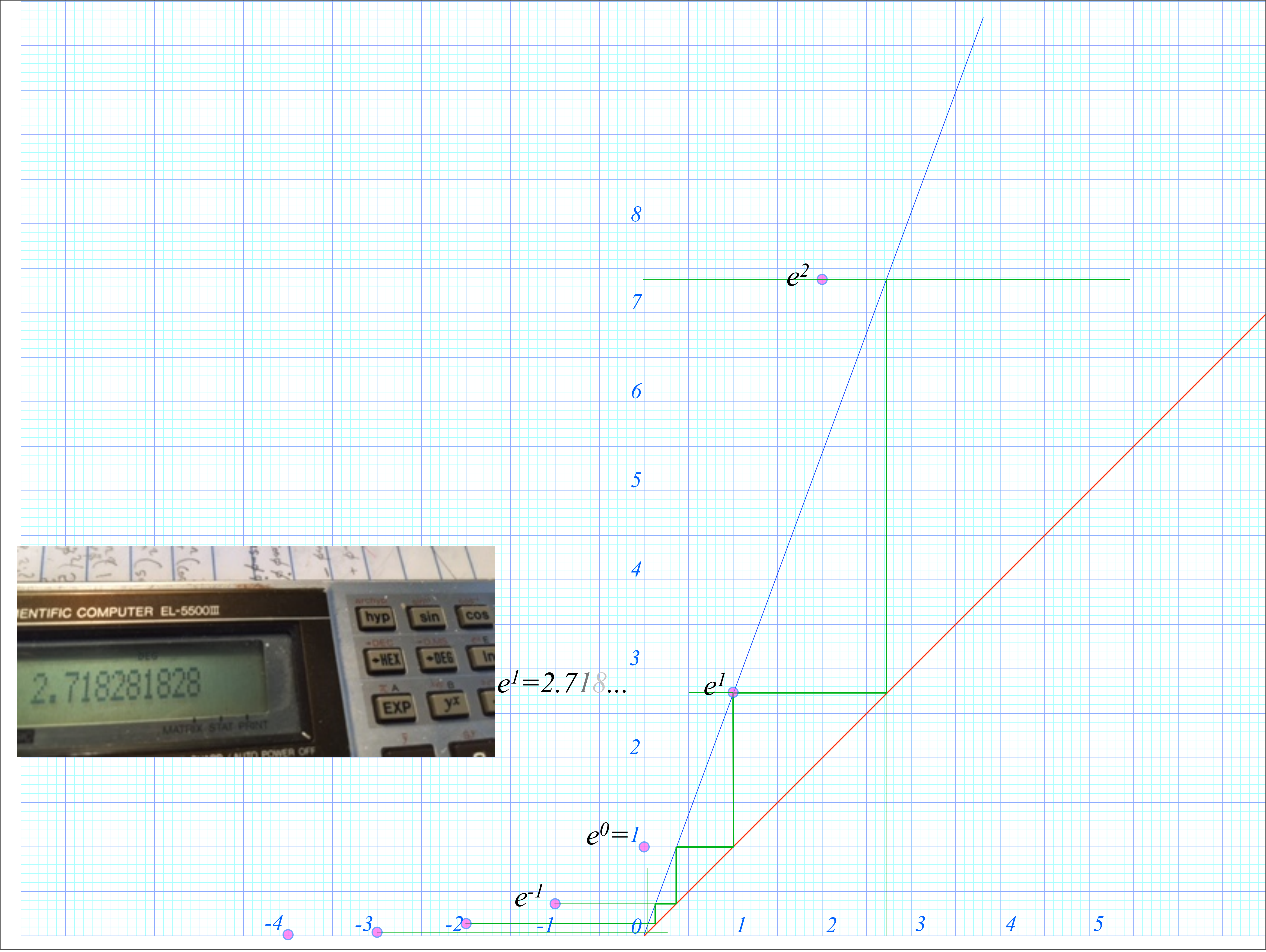



So far we mostly use Toolbox (a-b)

What follows uses Toolbox (c) ...

...and Toolbox (d)


8
7
6
5
4
3
2
e2
e1=2.718...
e1
e0=1
e-1
-4
-3
-2
-1
0
1
2
3
4
5



8
7
6
5
4
3
2
$e^2$
$e^1$
$e^1 = 2.718...$
$e^0 = 1$
$e^{-1}$
-4
-3
-2
-1
0
1
2
3
4
5
ENTIFIC COMPUTER EL-5500III
hyp
sin
cos
→HEX
→DEG
EXP
2.718281828
MATRIX STAT PRINT

# *Geometry of common power-law potentials*

*Geometric (Power) series*

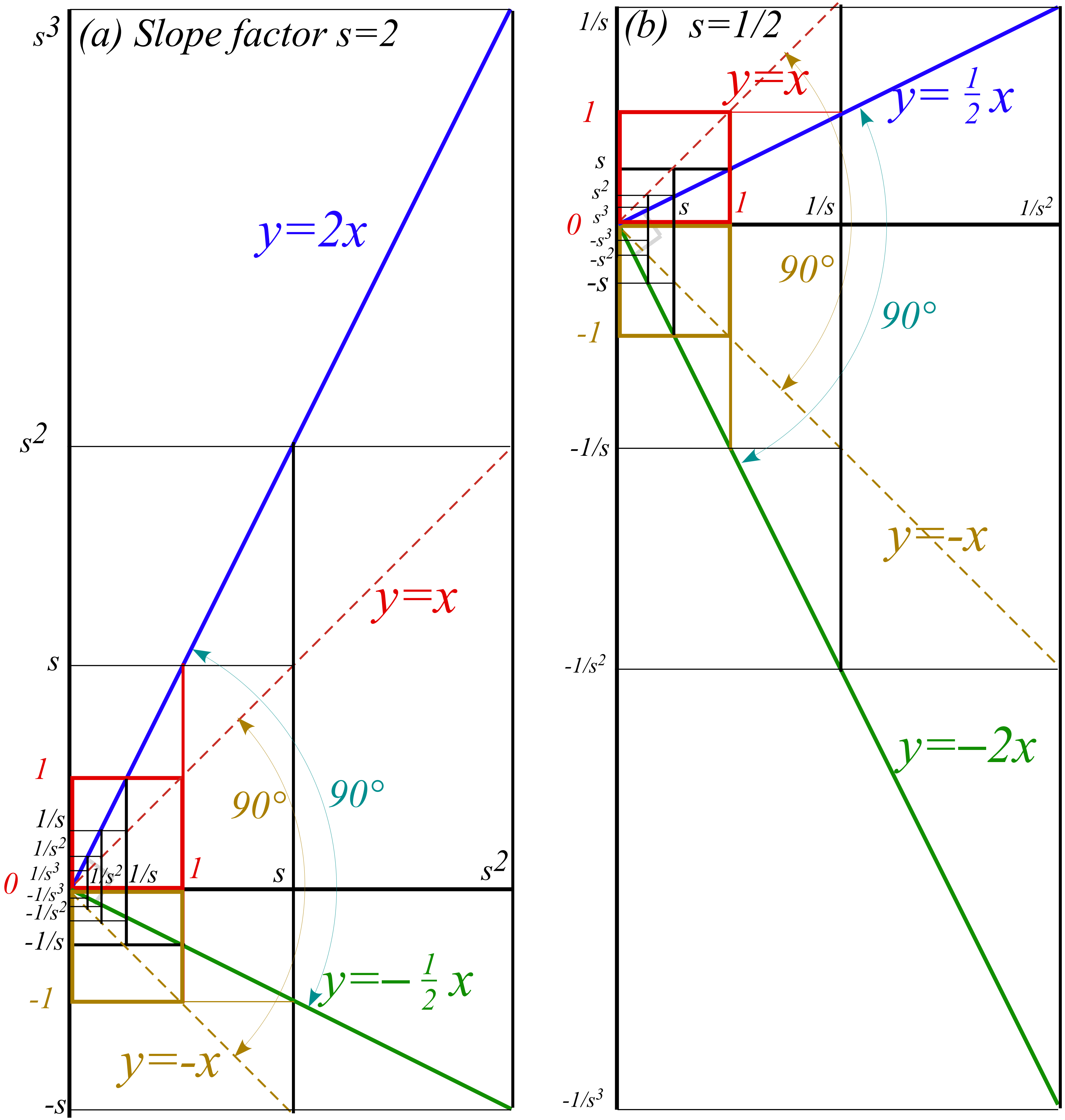
*"Zig-Zag" exponential geometry*

→ *Projective or perspective geometry*

*Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

*Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields*

*Compare mks units of Coulomb Electrostatic vs. Gravity*



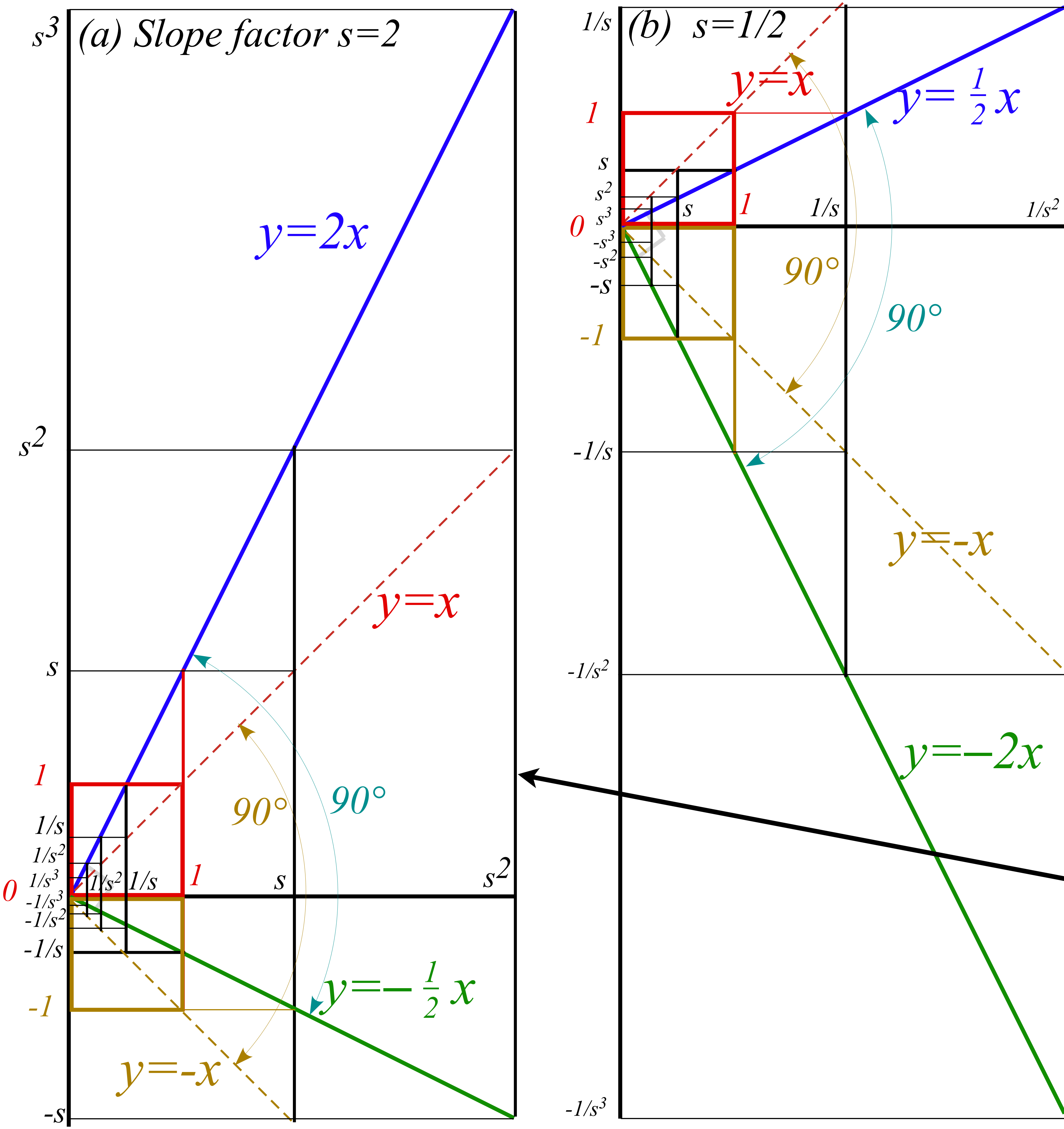

# "Zig-Zags" give perspective geometry

(1D-vanishing point )

Unit 1
Fig. 8.2

## "Zig-Zags" give perspective geometry

*(1D-vanishing point )*




Unit 1
Fig. 8.2

*1st-day-of-school perspective of 12th-grader*

*1st-day-of-school perspective of 1st-grader*

# *Geometry of common power-law potentials*

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*Projective or perspective geometry*

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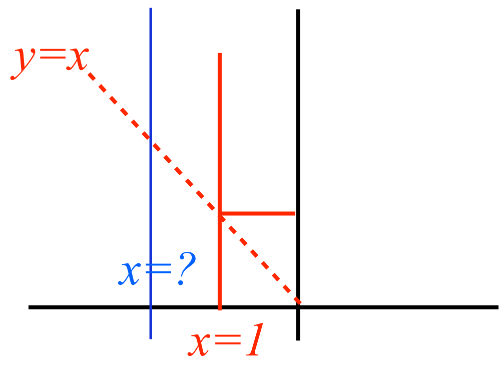
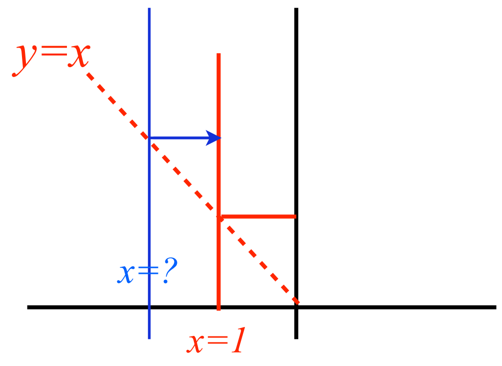
*Compare mks units of Coulomb Electrostatic vs. Gravity*

# *Each $y=x^2$ parabola point found by just one "Zig-Zag"*

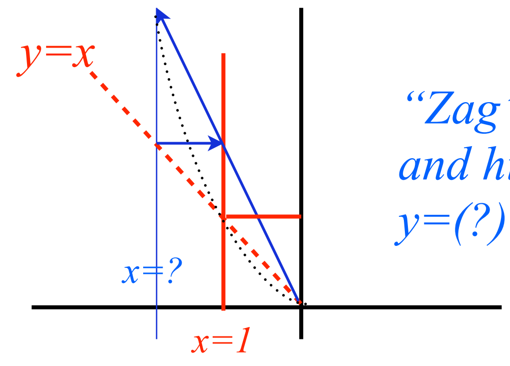
*1. Pick an (x=?)-line*




*2. "Zig" from its y=x intersection to x=1 line*




*3. "Zag" from origin back to (x=?)-line*



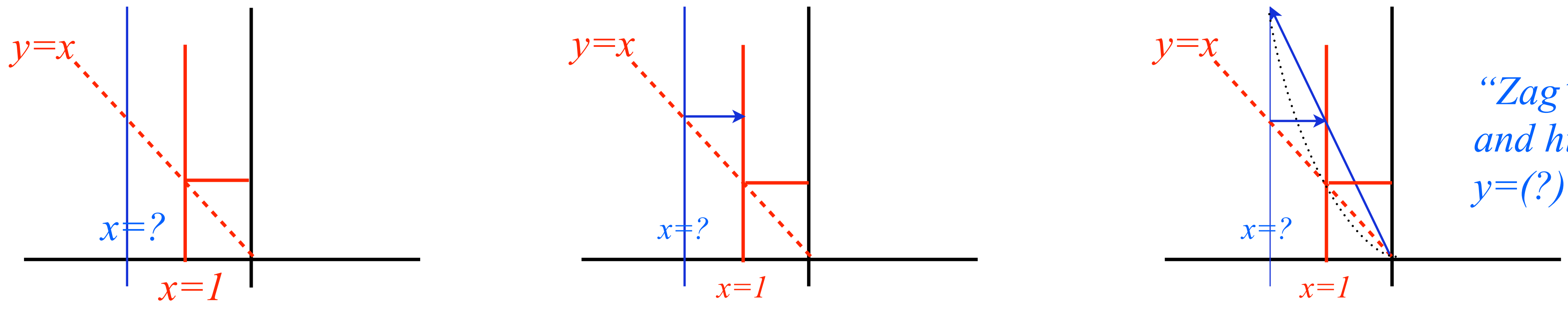

*"Zag" line is y=(?)·x and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$*

# Each $y=x^2$ parabola point found by just one "Zig-Zag"

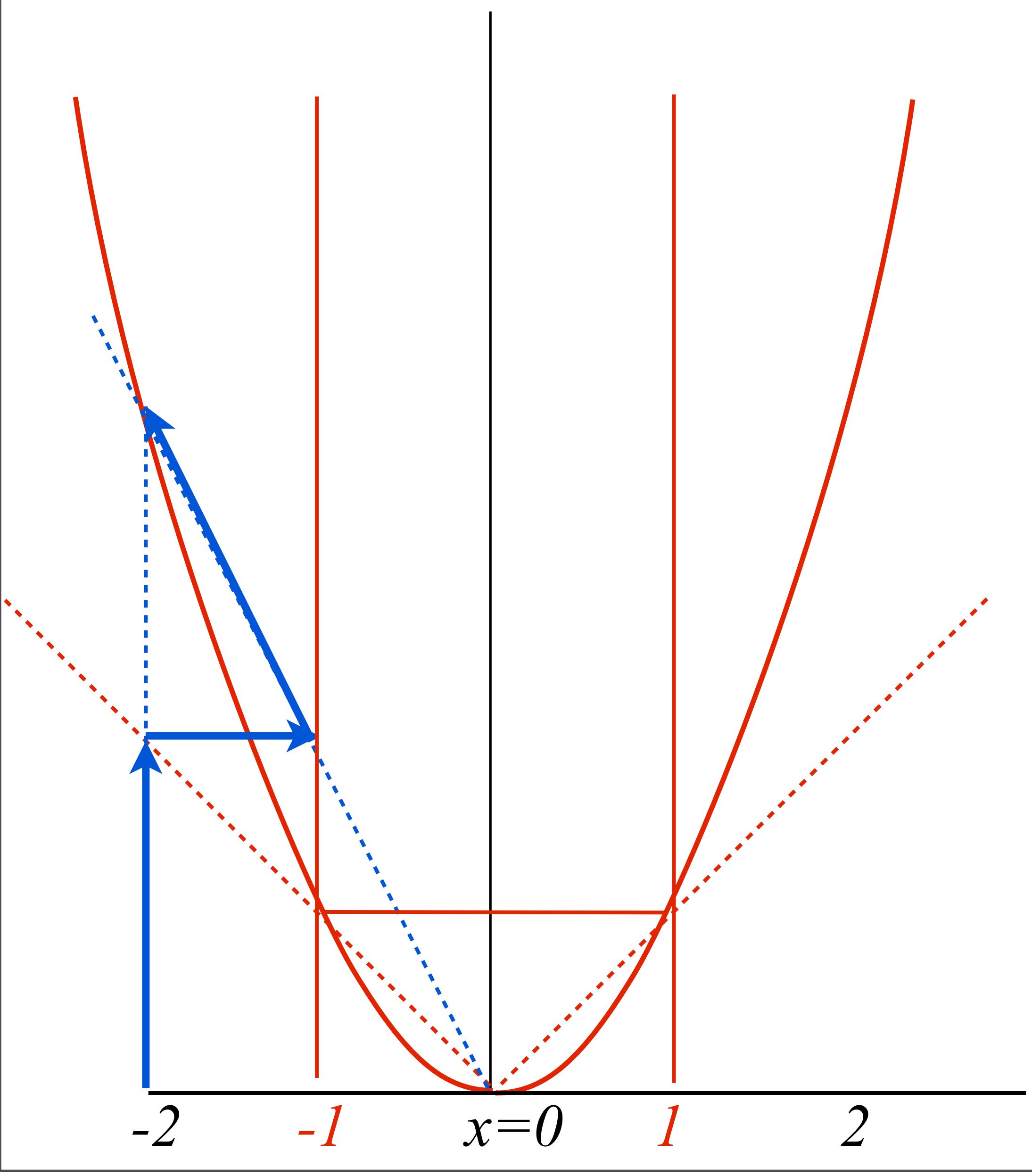
1. Pick an (x=?)-line

2. "Zig" from its y=x intersection to x=1 line

3. "Zag" from origin back to (x=?)-line




"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$




Unit 1
Fig. 9.1

# *Each $y=x^2$ parabola point found by just one "Zig-Zag"*

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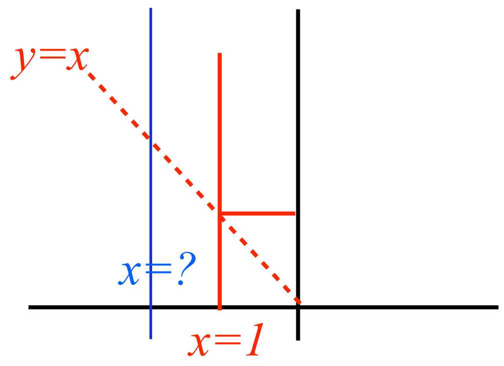
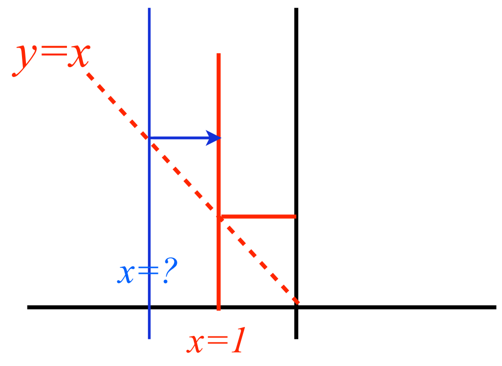
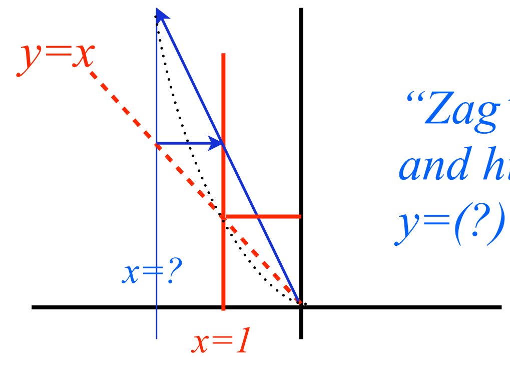
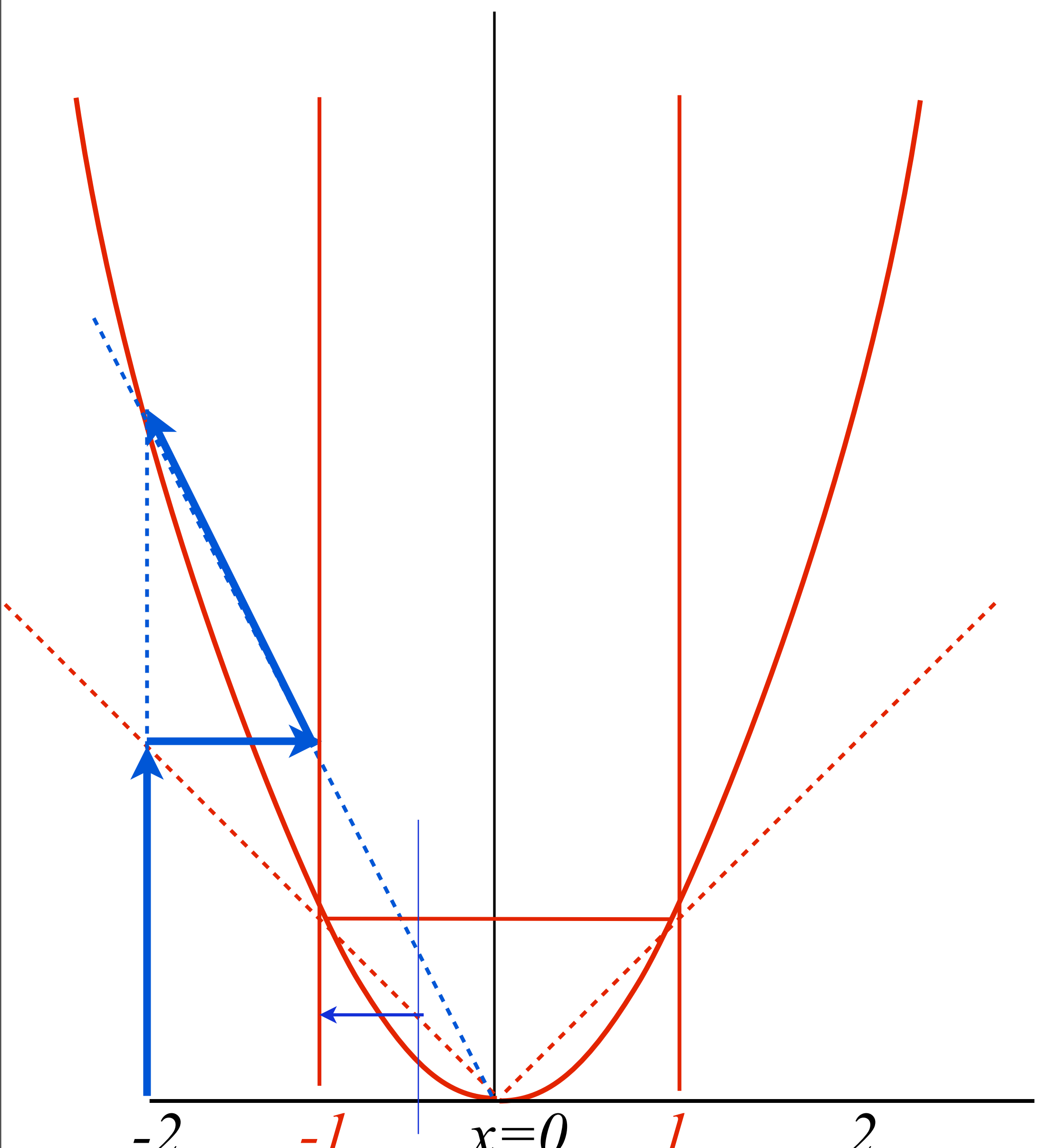



*"Zag" line is y=(?)·x and hits (x=?)-line at y=(?)·(?)=(?)²*




Unit 1
Fig. 9.1

# *Each $y=x^2$ parabola point found by just one "Zig-Zag"*

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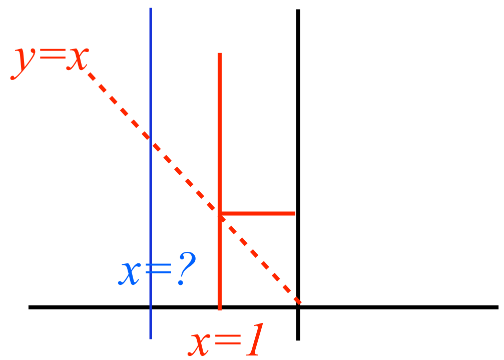
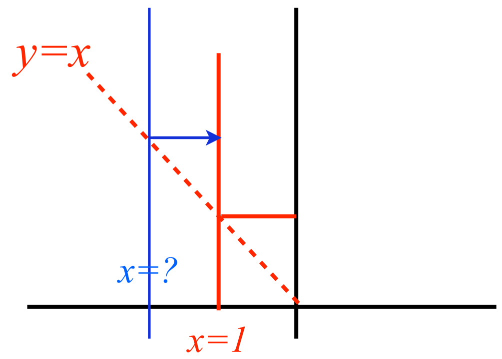
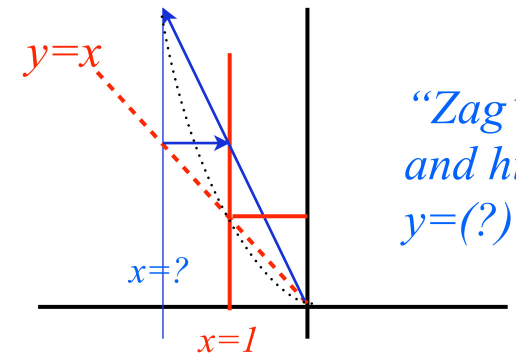
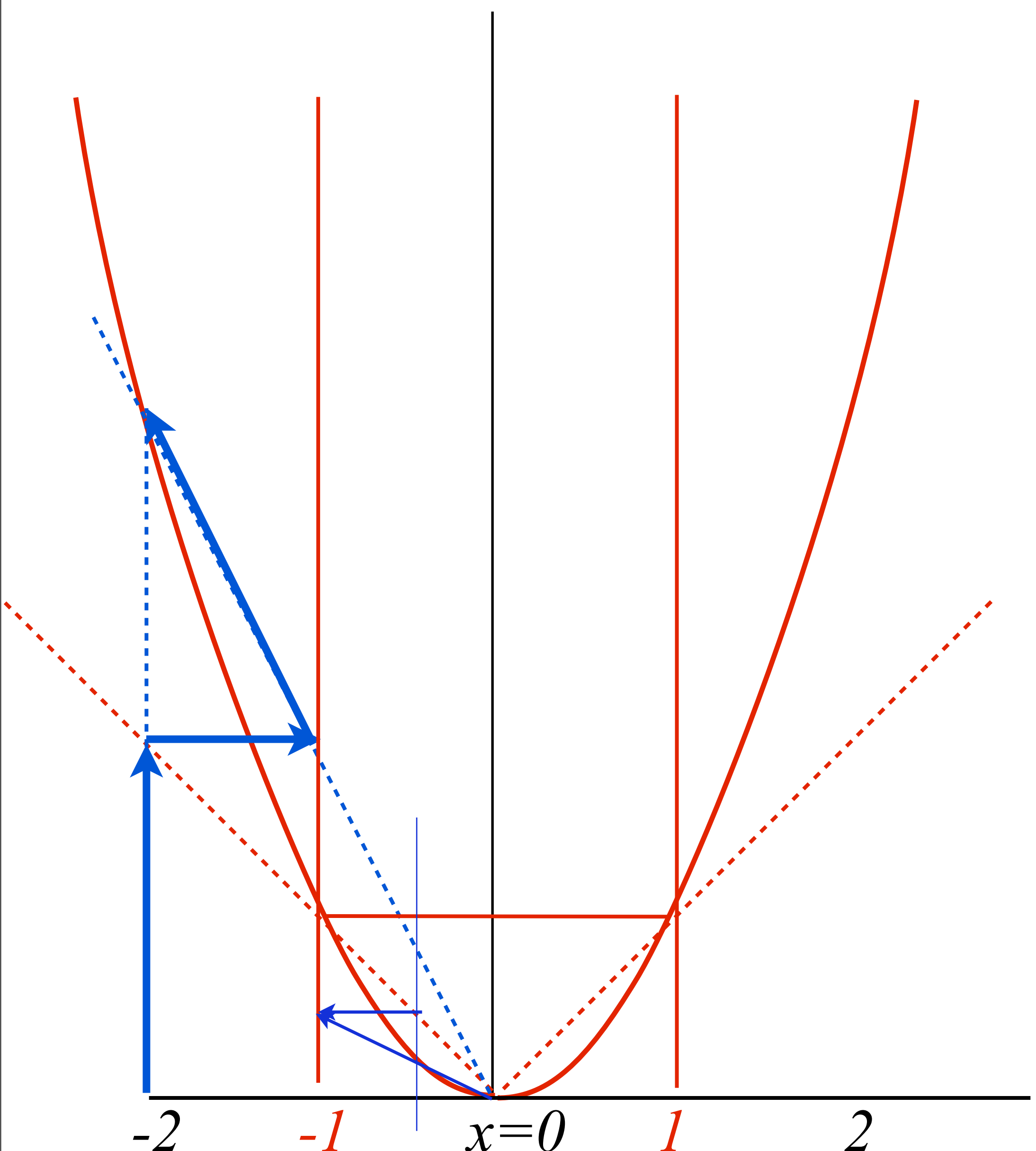



*"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$*




Unit 1
Fig. 9.1

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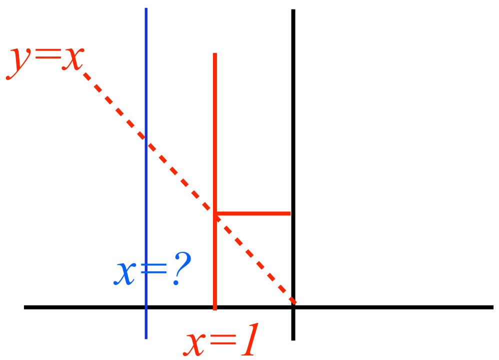


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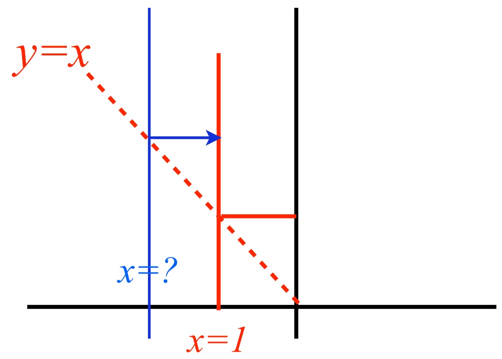


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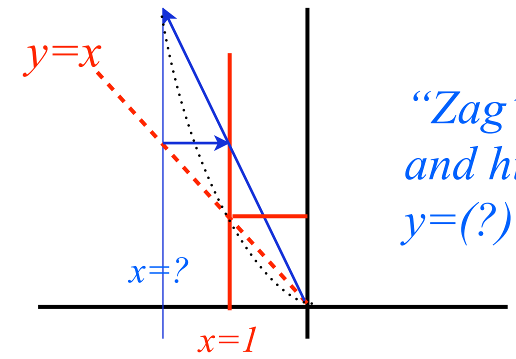
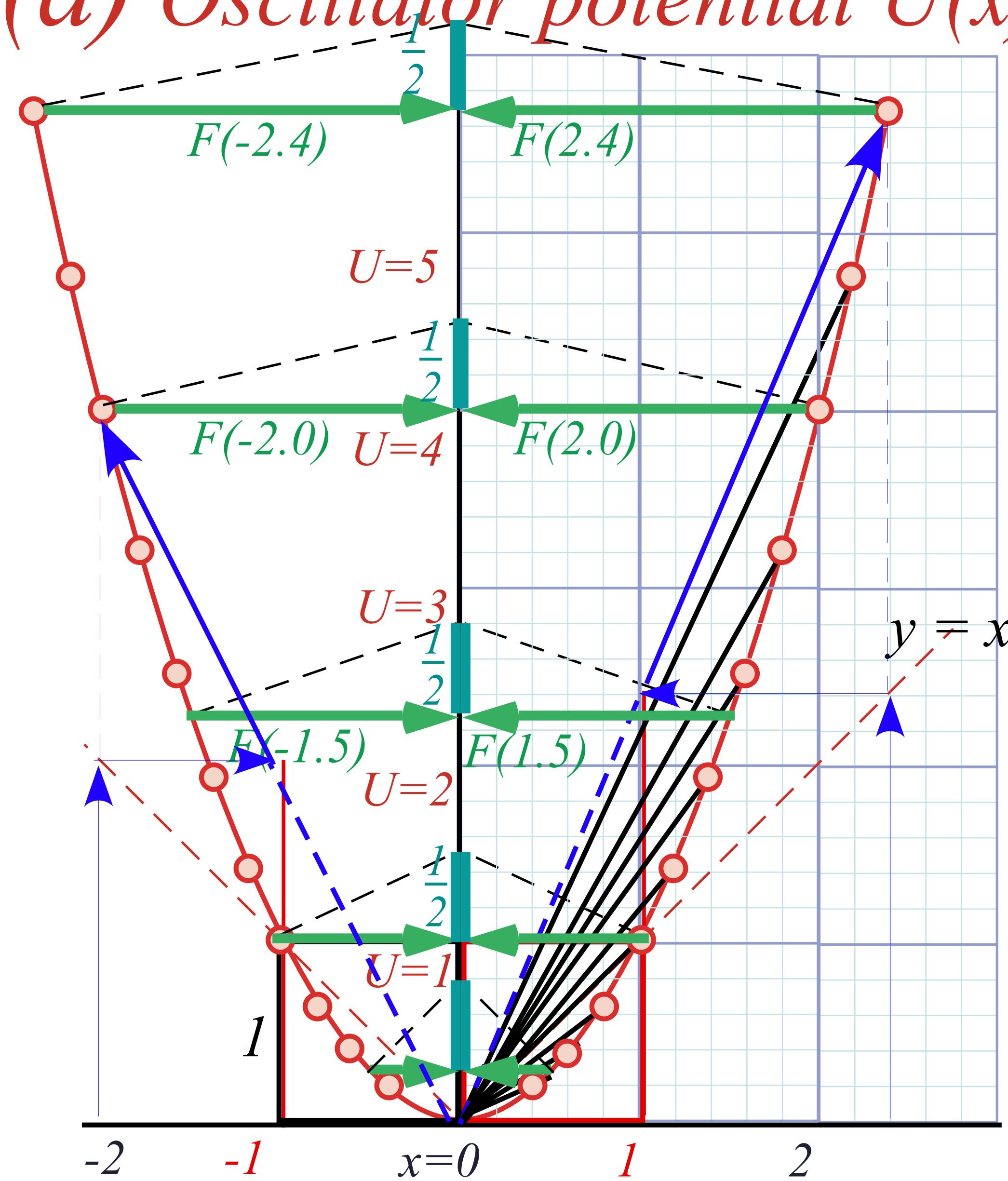
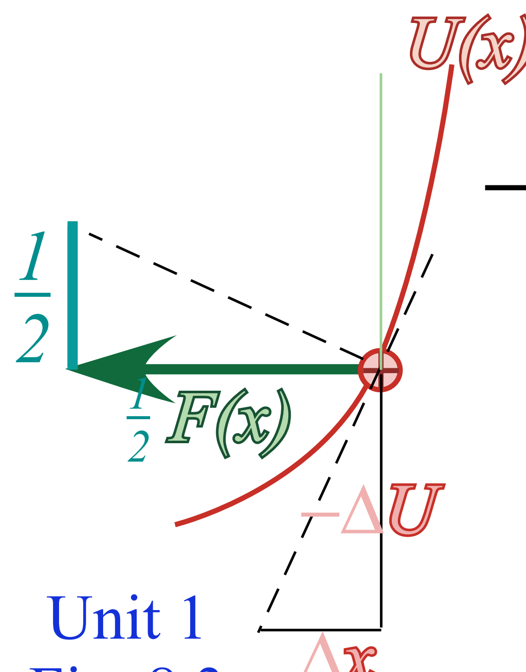



"Zag" line is $y=(?)\cdot x$ and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$

## (a) Oscillator potential $U(x)=x^2$

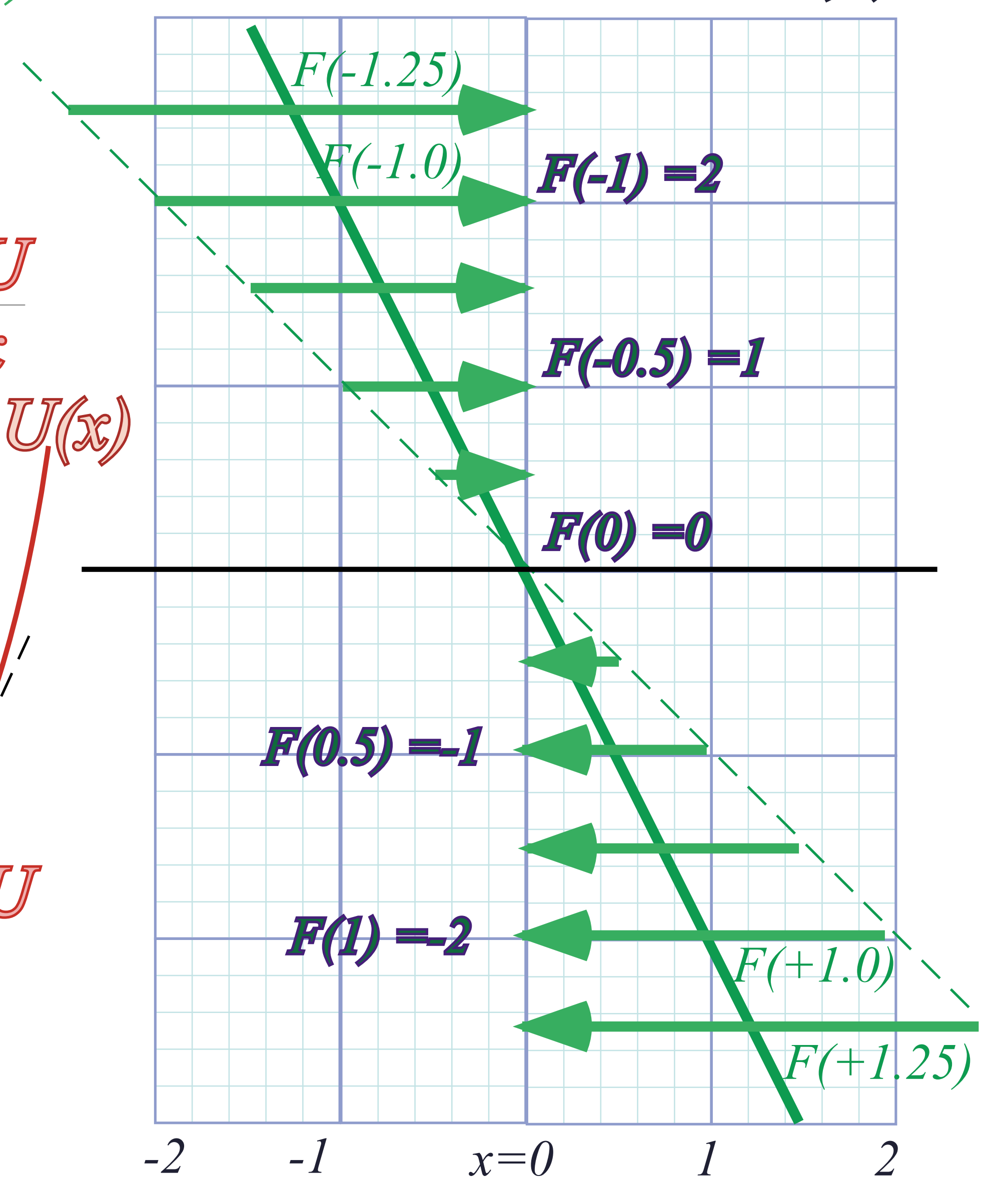



$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

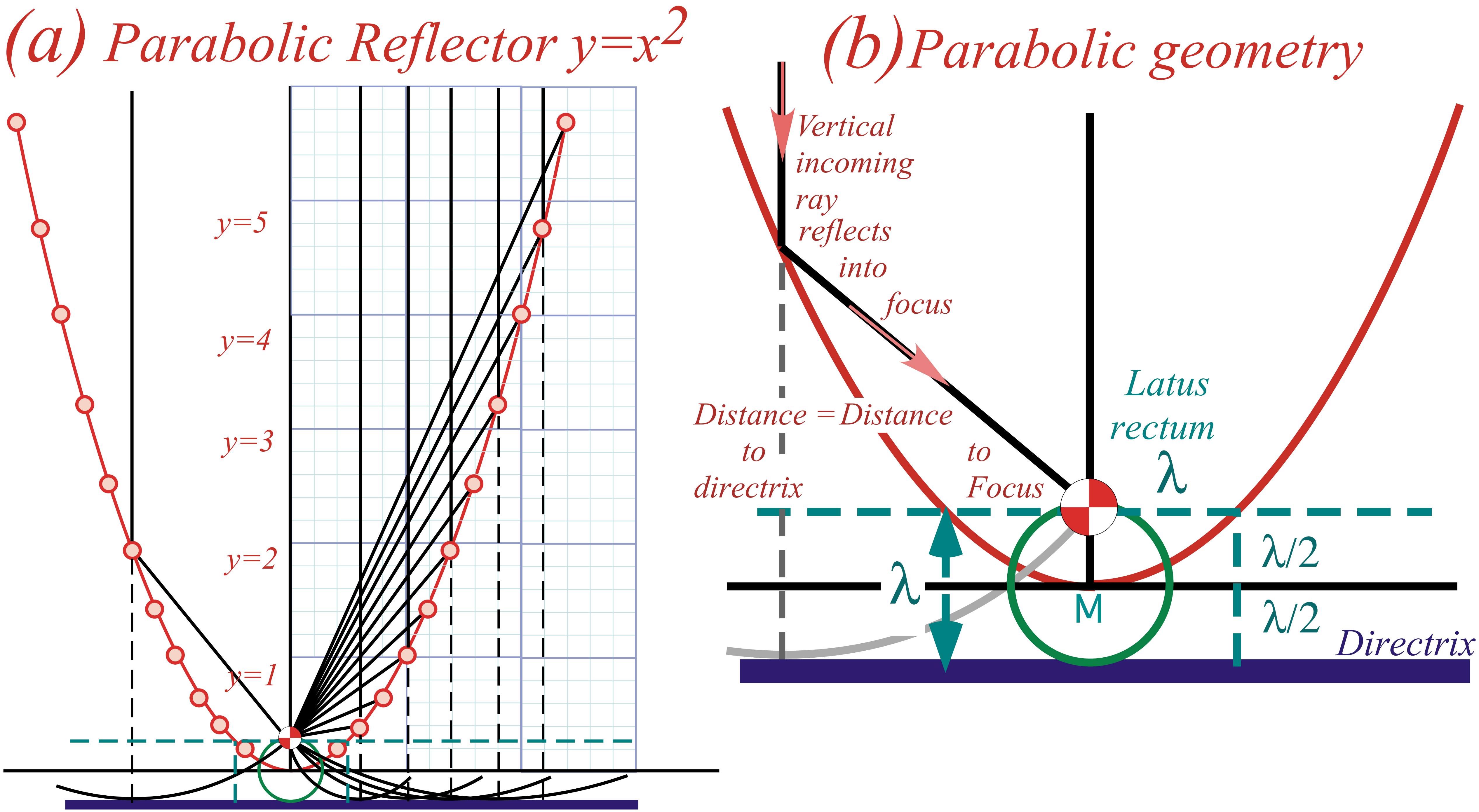



Unit 1
Fig. 8.3

## (b) Hooke-Law Force $F(x) = -2x$

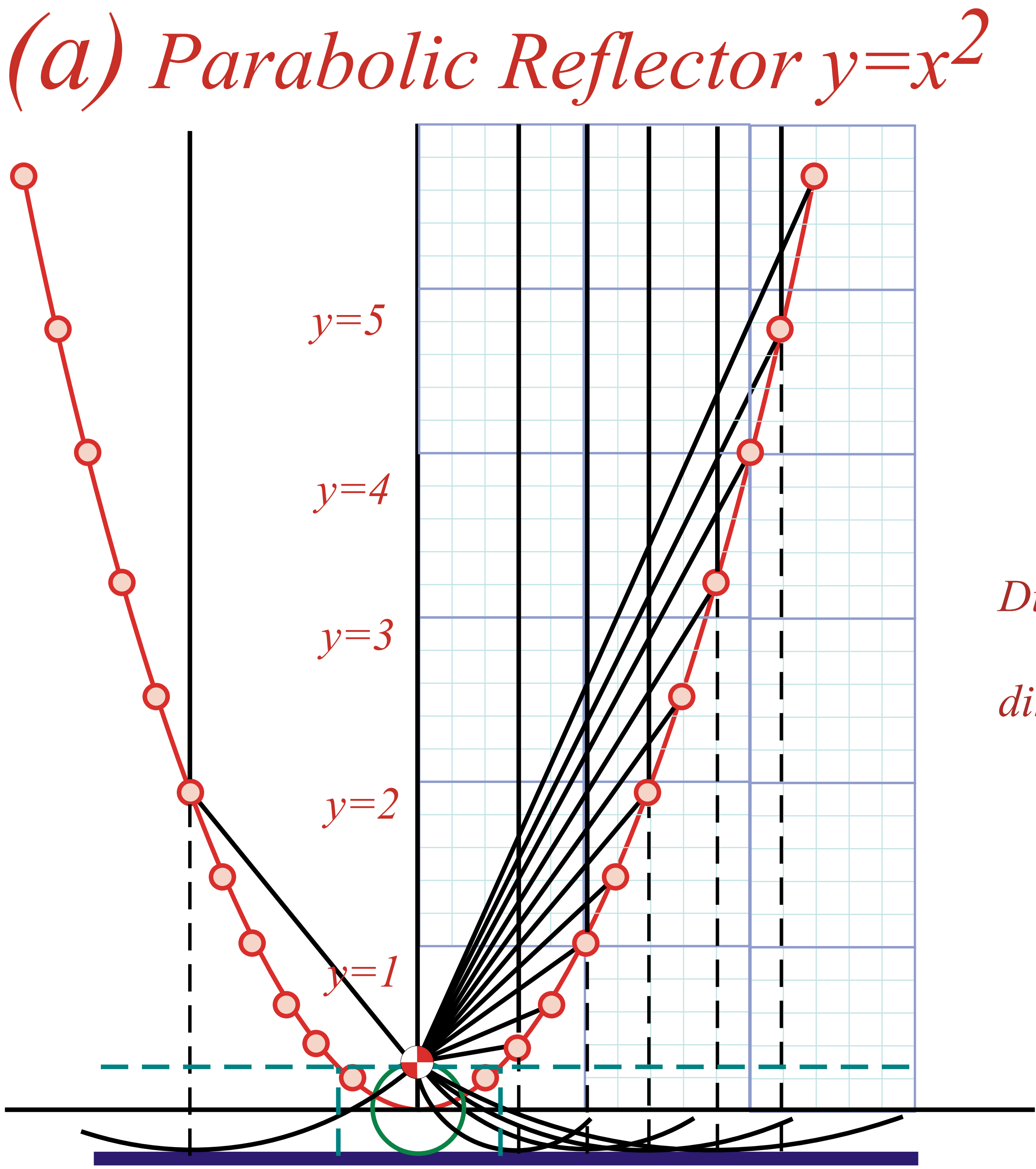


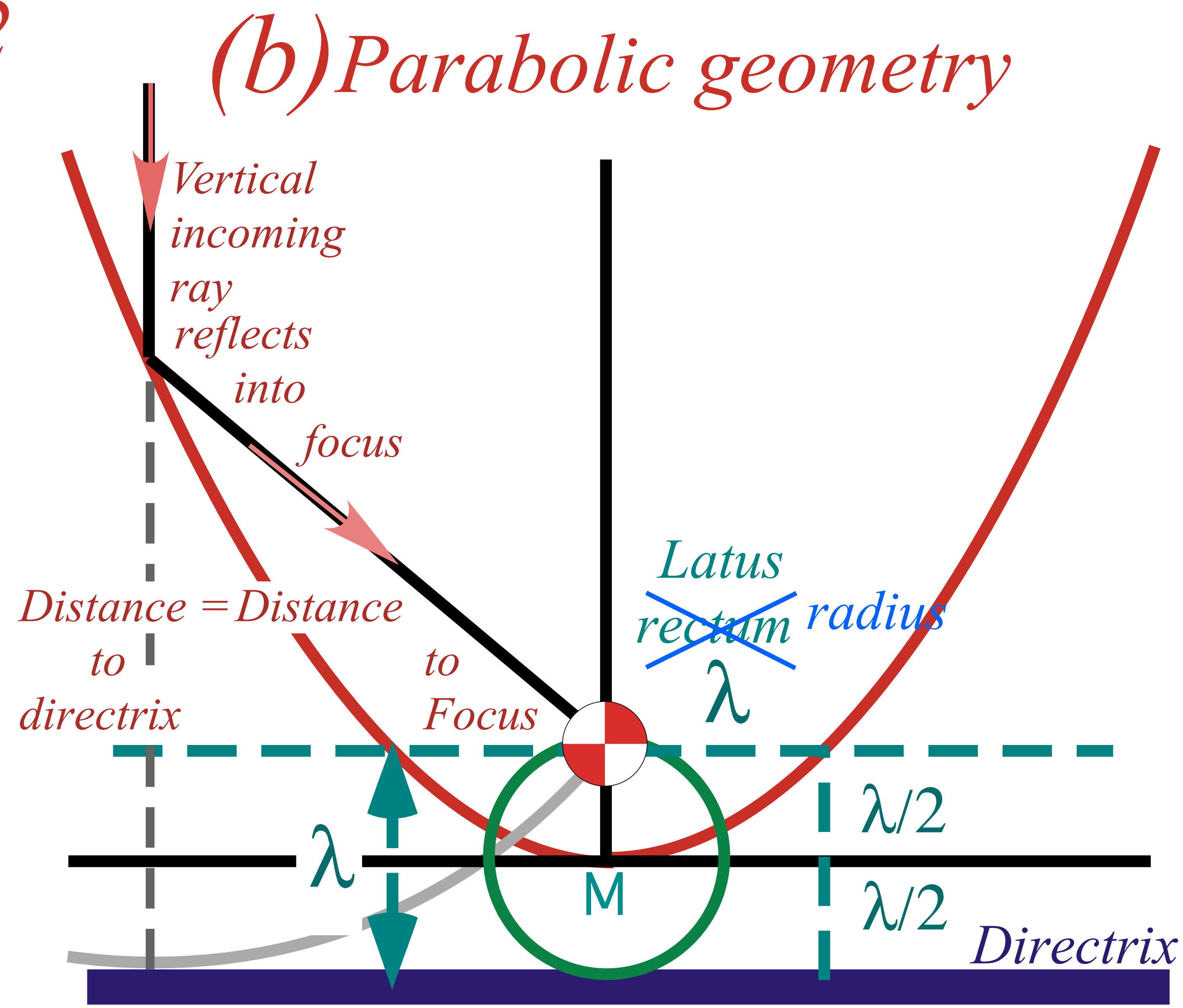
# *A more conventional parabolic geometry…(uses focal point)*




Unit 1
Fig. 8.3

# A more conventional parabolic geometry...



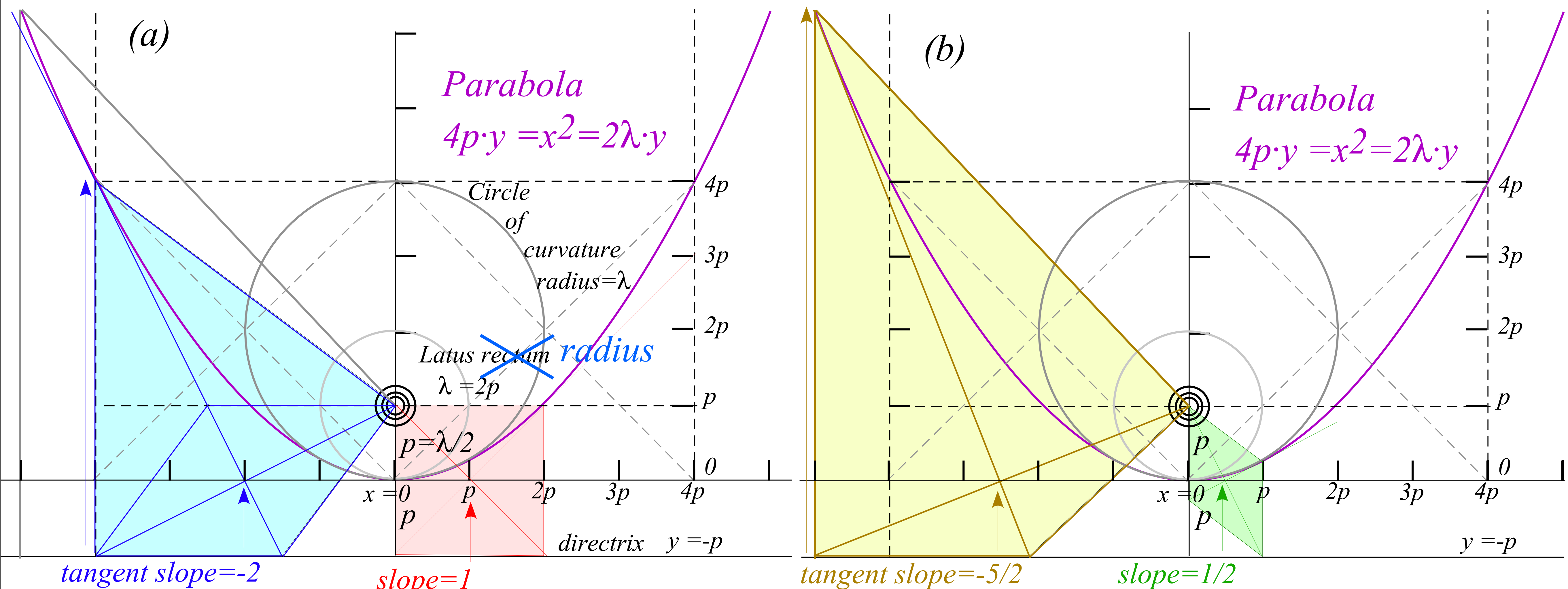




Better name† for λ : *latus radius*

†Old term *latus rectum* is exclusive copyright of
*X-Treme Roidrage Gyms*
Venice Beach, CA 90017

Unit 1
Fig. 8.3

# ...conventional parabolic geometry...carried to extremes...




Unit 1
Fig. 8.4

# *Geometry of common power-law potentials*

*Geometric (Power) series*

*"Zig-Zag" exponential geometry*

*Projective or perspective geometry*

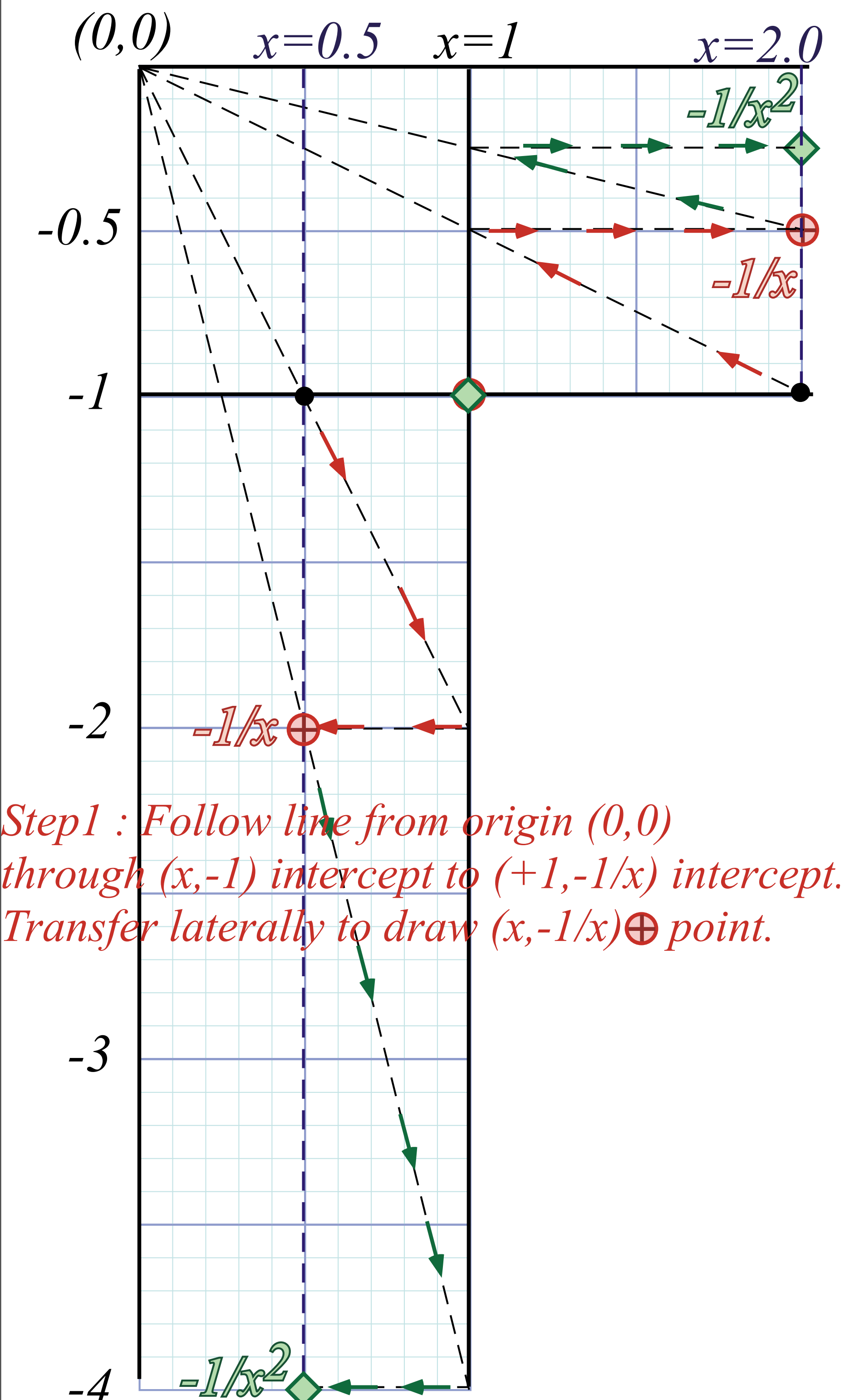
*Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

→ *Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields*

*Compare mks units of Coulomb Electrostatic vs. Gravity*

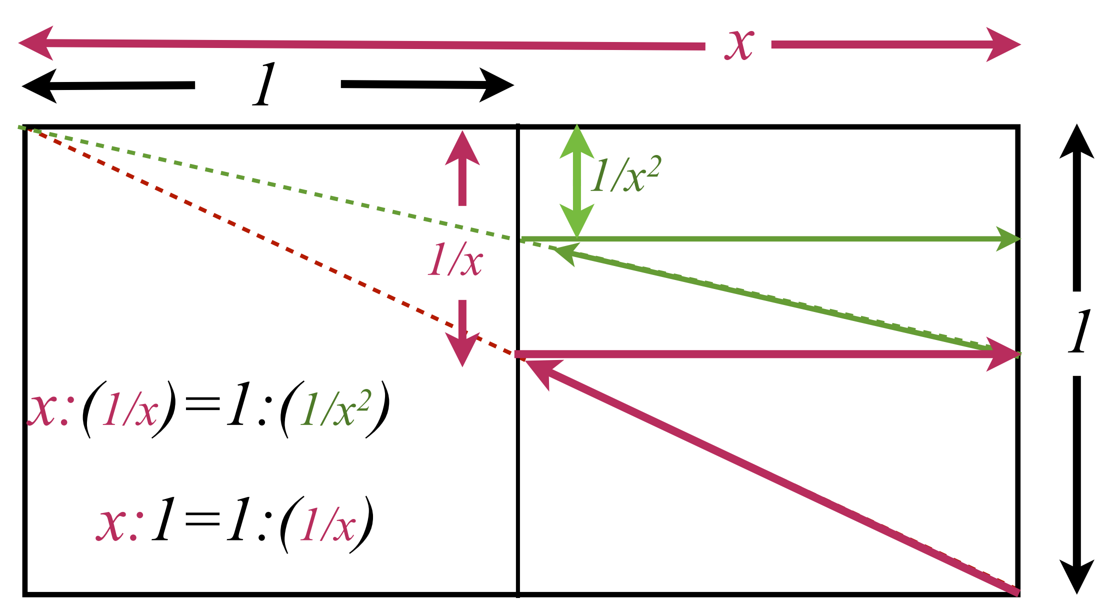
Unit 1
Fig. 8.4

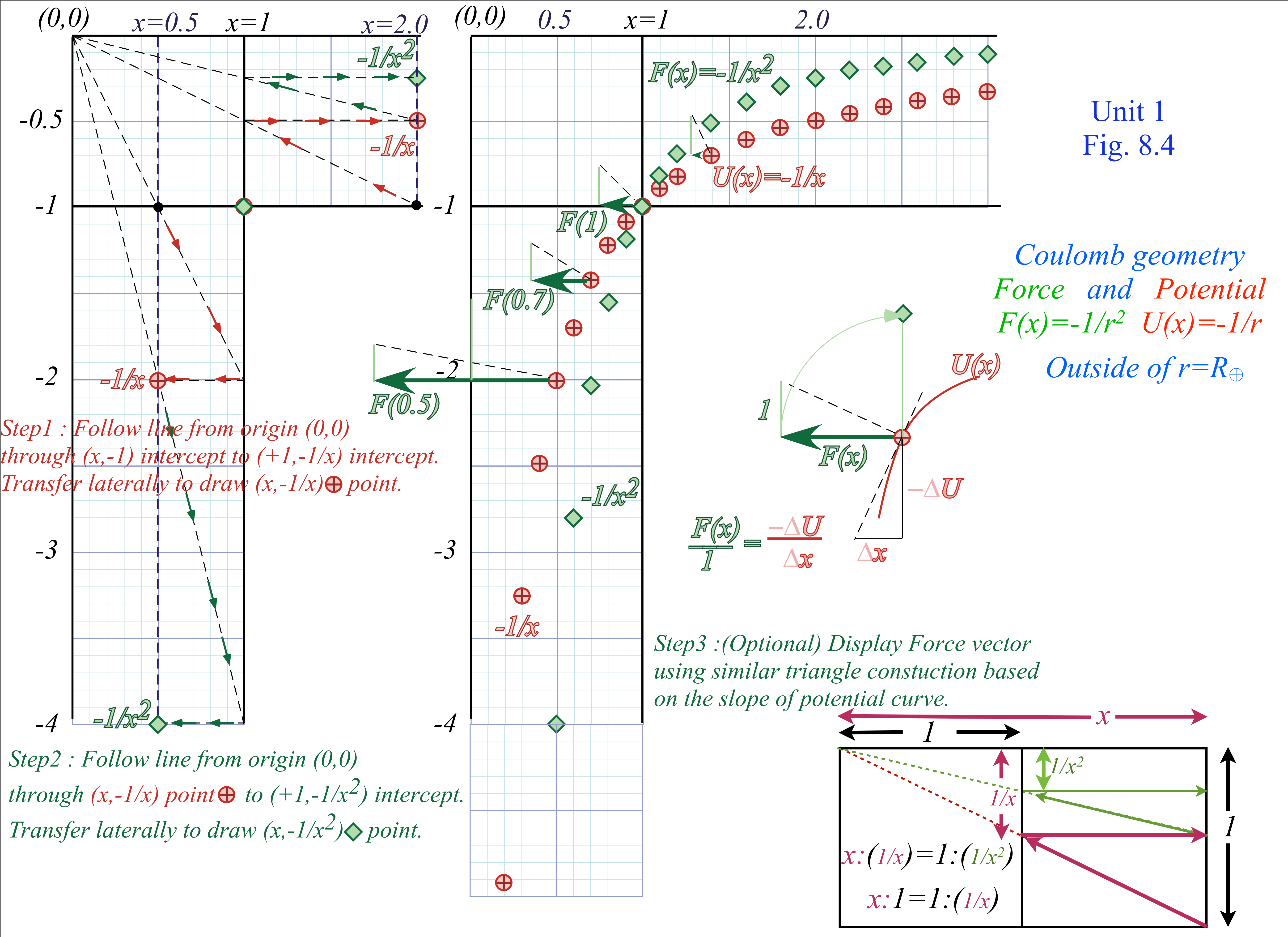
*Coulomb geometry*
*Force and Potential*
$F(x)=-1/r^2$ $U(x)=-1/r$

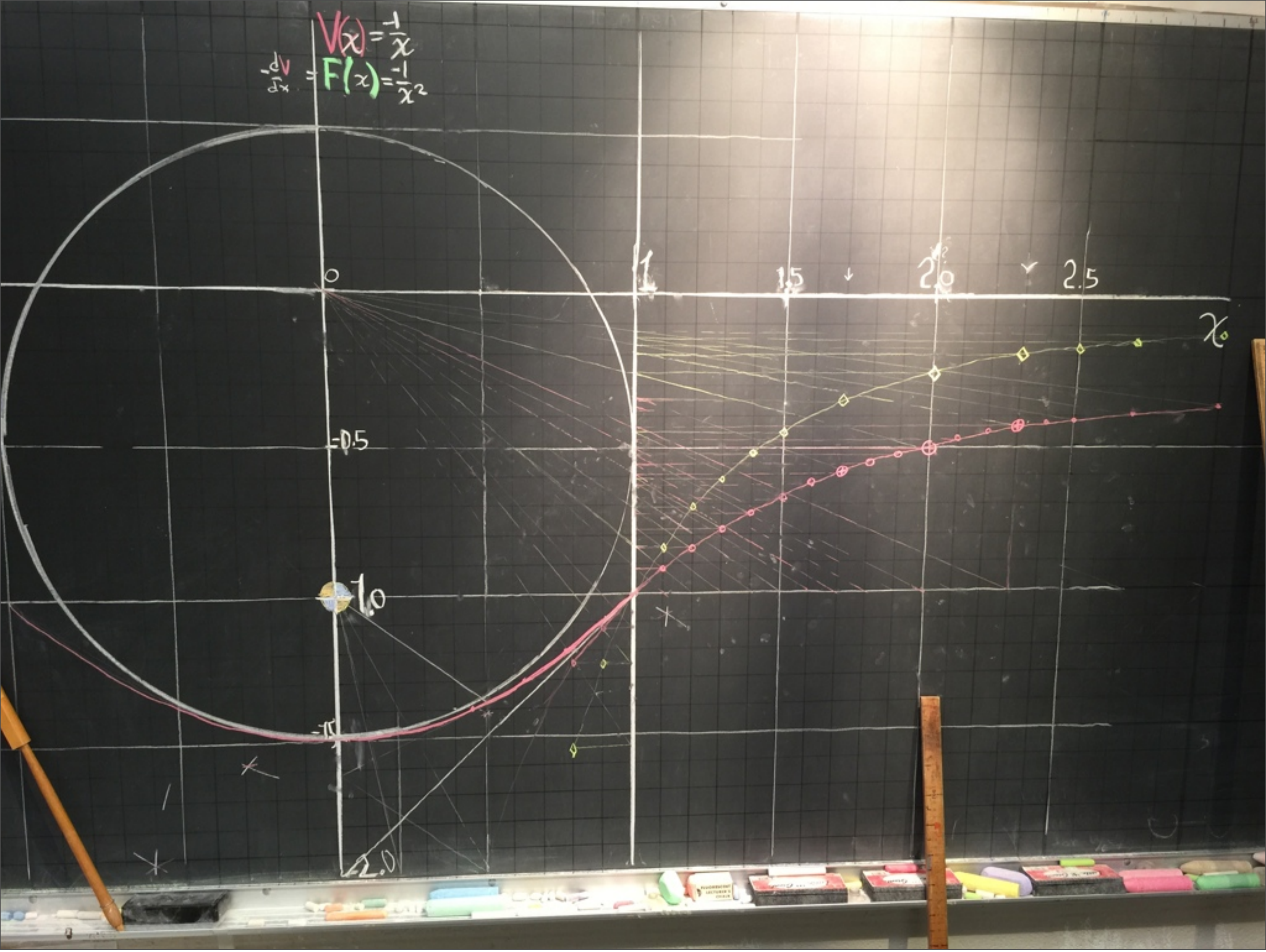



*Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.*

*Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x²) intercept. Transfer laterally to draw (x,-1/x²)◇ point.*




Unit 1
Fig. 8.4
Coulomb geometry
Force and Potential
F(x)=-1/r^2 U(x)=-1/r
Outside of r=R⊕
(0,0)
x=0.5
x=1
x=2.0
-1/x^2
-1/x
-0.5
-1
-2
-3
-4
0.5
2.0
F(x)=-1/x^2
U(x)=-1/x
F(1)
F(0.7)
F(0.5)
U(x)
1
F(x)
−ΔU
Δx
F(x)/1 = −ΔU/Δx
Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.
Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x^2) intercept. Transfer laterally to draw (x,-1/x^2)◇ point.
Step3 :(Optional) Display Force vector using similar triangle constuction based on the slope of potential curve.
x
1/x
1/x^2
x:(1/x)=1:(1/x^2)
x:1=1:(1/x)



$V(x) = \frac{-1}{x}$
$-\frac{dV}{dx} = F(x) = \frac{-1}{x^2}$
0
1
1.5
2.0
2.5
x
-0.5
1.0
-2.0

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*Projective or perspective geometry*

*Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

*Coulomb geometry of -1/r-potential and $-1/r^2$-force fields*

→ *Compare mks units of Coulomb Electrostatic vs. Gravity*

## Compare mks units for Coulomb fields

## 1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong \boxed{?.?\cdot 10^{?}} \frac{Newtons\cdot meter\cdot square}{per\ square\ Coulomb}$$

## *Compare mks units for Coulomb fields*

*1. Electrostatic force between q(Coulombs) and Q(C.)* !!!!

~9E9~$10^{10}$

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

# Compare mks units for Coulomb fields

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*More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2$ $\sim 9 \cdot 10^9 \sim 10^{10}$*

*quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb*

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

*...but 1 Ampere = 1 Coulomb/sec.*

# Compare mks units for Coulomb fields

## 1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong \overset{\sim 9E9 \sim 10^{10}}{9{,}000{,}000{,}000} \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

*More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2 \sim 9 \cdot 10^9 \sim 10^{10}$*

*quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb*

↑

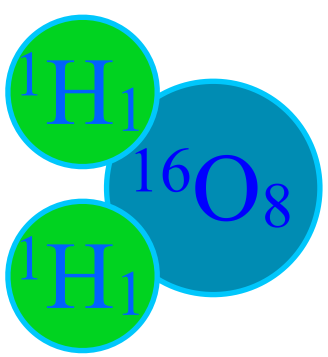
Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

*...but 1 Ampere = 1 Coulomb/sec.*

*"Fingertip Physics" of Ch. 8 notes that 1 $(cm)^3$ =1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules* *Avogodro's Number*

*$\sim 0.3 \cdot 10^{23}$*

*$H_2O$ Molecular weight~18*



# Compare mks units for Coulomb fields

## 1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

~9E9~$10^{10}$

*More precise value for electrostatic constant :* $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2$ ~$9 \cdot 10^9$~$10^{10}$

*quantum of charge:* $|e| = 1.6022 \cdot 10^{-19}$ *Coulomb*

↑

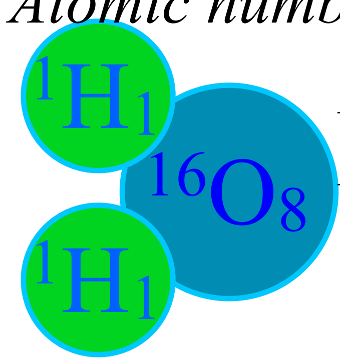
Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

*...but 1 Ampere = 1 Coulomb/sec.*

*"Fingertip Physics" of Ch. 9 notes that 1 (cm)$^3$ =1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules or ~ $3 \cdot 10^{23}$ electrons*

*~$0.3 \cdot 10^{23}$ and ~ $3 \cdot 10^{23}$ protrons.*

*$H_2O$ Molecular weight~18*
*Atomic number = 10*




*10 electrons*
*10 protons*

# Compare mks units for Coulomb fields

## 1. Electrostatic force between q(Coulombs) and Q(C.)     !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong \overset{\sim 9E9\sim 10^{10}}{9,000,000,000}\, \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

*More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987,551 \cdot 10^9 Nm^2/C^2$ $\sim 9 \cdot 10^9 \sim 10^{10}$*

*quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb*

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

*...but 1 Ampere = 1 Coulomb/sec.*

*"Fingertip Physics" of Ch. 9 notes that 1 $(cm)^3$ =1gm of water (1/18 Mole) has (1/18) $6 \cdot 10^{23}$ molecules or $\sim 3 \cdot 10^{23}$ electrons*
*$\sim 0.3 \cdot 10^{23}$     and $\sim 3 \cdot 10^{23}$ protrons.*

*$H_2O$   Molecular weight~18*
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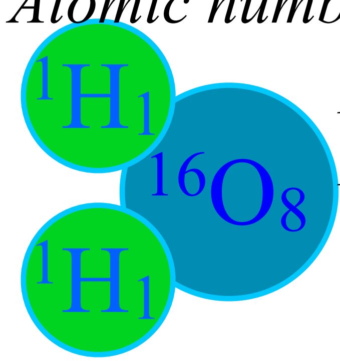


*10 electrons*   *That is $\sim -3 \cdot 10^{23} 1.6022 \cdot 10^{-19}$ Coulomb or about $-0.5 \cdot 10^{+5}$ C or − 50,000 Coulomb*

*10 protons*   *plus $\sim +3 \cdot 10^{23} 1.6022 \cdot 10^{-19}$ Coulomb or about $+0.5 \cdot 10^{+5}$ C or +50,000 Coulomb*

*Equals zero total charge*

# *Compare mks units for Coulomb fields*

## *1. Electrostatic force between q(Coulombs) and Q(C.)* !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

~9E9~$10^{10}$

*More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2$* ~$9 \cdot 10^9$~$10^{10}$

*quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb*

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)
vs
*Always Attractive (so far)*

↓

↕ COMPARE! ↕

BIG
vs
*small*

## *2. Gravitational force between m(kilograms) and M(kg.)*

$$F^{grav.}(r) = -G\frac{mM}{r^2} \quad where: G = \boxed{?.? \cdot 10^{?}} \frac{Newtons \cdot meter \cdot square}{per\ square\ kilogram}$$

# Compare mks units for Coulomb fields

## 1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$\sim 9E9 \sim 10^{10}$

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

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Repulsive (+)(+) or (-)(-)
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↕ COMPARE! ↕

BIG
vs
*small*

## 2. Gravitational force between m(kilograms) and M(kg.) !!!!

$\sim {}^2/_3 10^{-10} \sim 10^{-10}$

$$F^{grav.}(r) = -G\frac{mM}{r^2} \quad where: G = 0.000{,}000{,}000{,}067 \frac{Newtons \cdot meter \cdot square}{per\ square\ kilogram}$$

# Compare mks units for Coulomb fields

## 1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

~9E9~$10^{10}$

More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2$ ~$9 \cdot 10^9$~$10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ Coulomb

↑
Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)
vs
*Always Attractive (so far)*
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BIG
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More precise value for gravitational constant : $G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2$ ~$(2/3)10^{-10}$

## Compare mks units for Coulomb fields

## 1. Electrostatic force between q(Coulombs) and Q(C.) !!!!

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong \overset{\sim 9E9 \sim 10^{10}}{9{,}000{,}000{,}000} \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

*quantum of charge: $|e|=1.6022\cdot 10^{-19}$ Coulomb*

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Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

*Discussion of repulsive force and PE in Ch. 8...*

## 1(a). Electrostatic potential energy between q(Coulombs) and Q(C.)

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} \quad where: \frac{1}{4\pi\varepsilon_0} \cong \overset{\sim 9E9 \sim 10^{10}}{9{,}000{,}000{,}000} \frac{Joule}{per\ square\ Coulomb}$$ !!!!

# Compare mks units for Coulomb fields

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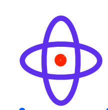
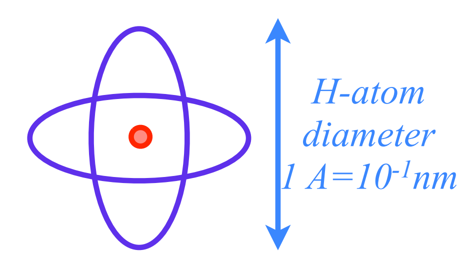


Nuclear size ~ $10^{-15}$ m = 1 femtometer =1fm

Atomic size ~ 1 Angstrom = $10^{-10}$ m



# Compare mks units for Coulomb fields

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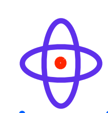

Nuclear size ~ $10^{-15}$ m = 1 femtometer =1fm

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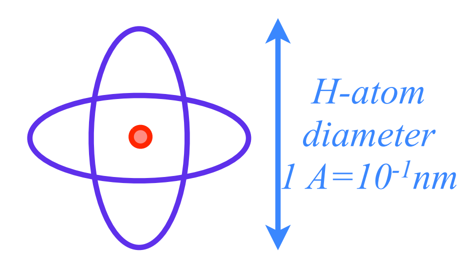
Big molecule ~ 10 Angstrom = $10^{-9}$ m = 1nanometer=1nm



# Compare mks units for Coulomb fields

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Repulsive (+)(+) or (-)(-)
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*quantum of charge: $|e|=1.6022\cdot 10^{-19}$ Coulomb*

*Discussion of repulsive force and PE in Ch. 8...*

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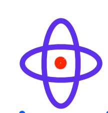

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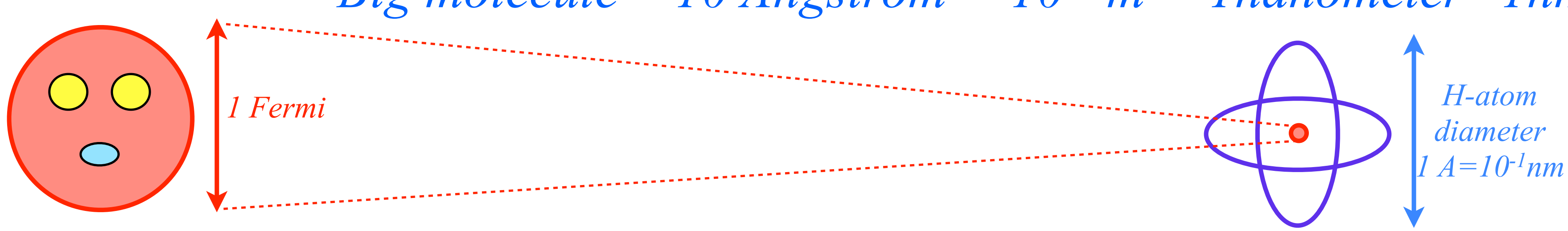
*also:1fm = $10^{-13}$ cm =1Fermi =1Fm*



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~9E9~$10^{10}$ !!!!

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

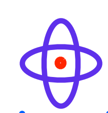
*quantum of charge:* $|e| = 1.6022 \cdot 10^{-19}$ *Coulomb*

*Discussion of repulsive force and PE in Ch. 8...*

## 1(a). Electrostatic potential energy between q(Coulombs) and Q(C.)

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Joule}{per\ square\ Coulomb}$$

~9E9~$10^{10}$ !!!!



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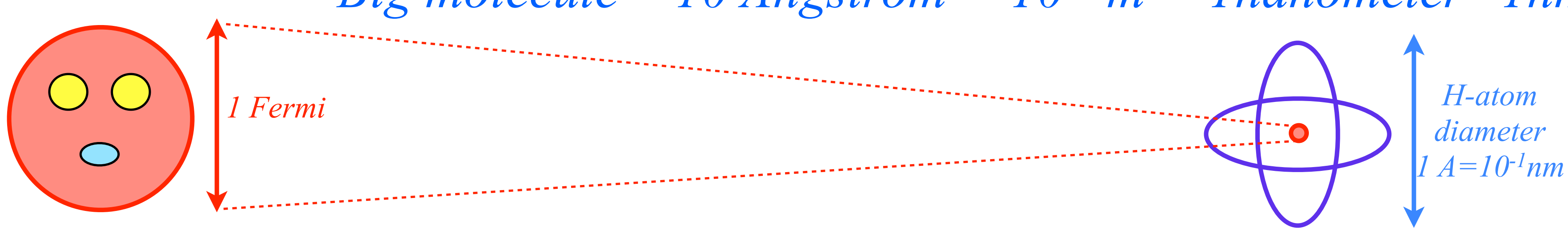



*nuclear radii are 100,000 to 1,000,000 times smaller than atomic/chemical radii*

*...so nuclear qQ/r energy 100,000 to 1,000,000 times* **bigger** *that of atomic/chemical...*

# *Geometry of idealized "Sophomore-physics Earth"*

*Coulomb field outside* *Isotropic Harmonic Oscillator (IHO) field inside*

*Contact-geometry of potential curve(s)*

*"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

*Earth matter vs nuclear matter:*

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

# *Coulomb force vanishes inside-spherical shell* (Gauss-law)

Unit 1
Fig. 8.6

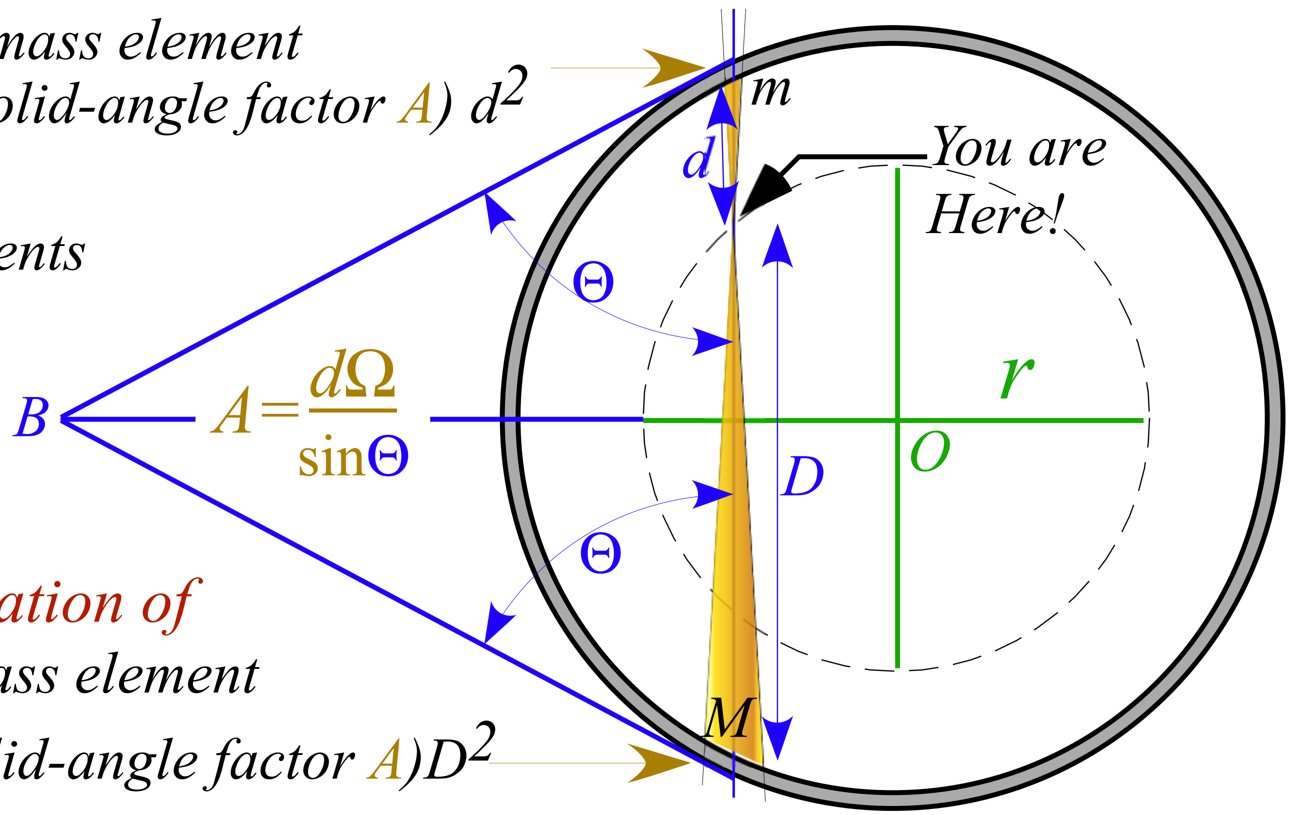
*Shell mass element*
*m* =(solid-angle factor *A*) $d^2$

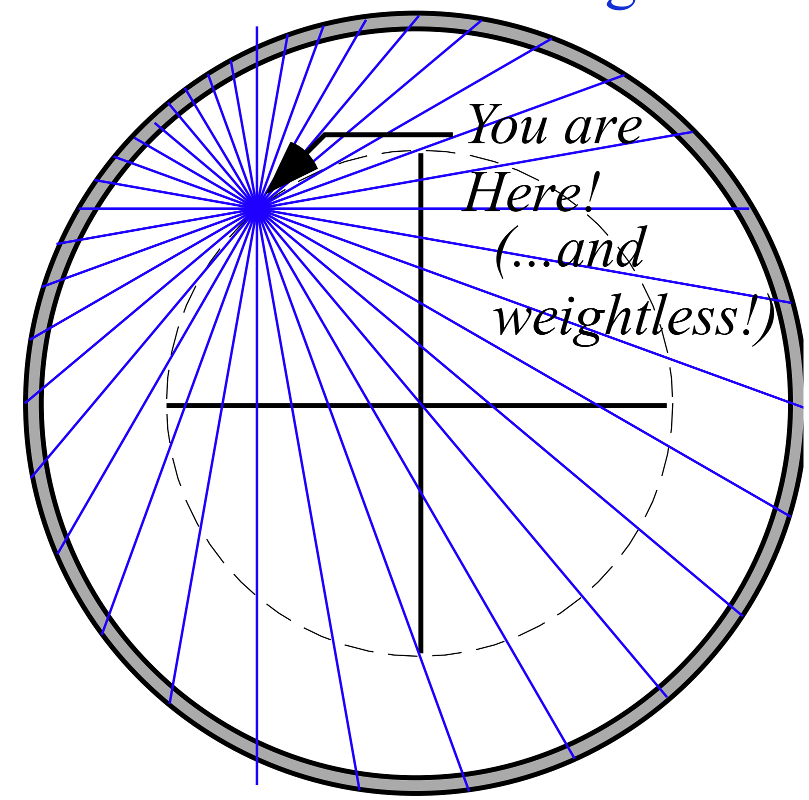
*Gravity at* $r$
*due to shell mass elements*

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

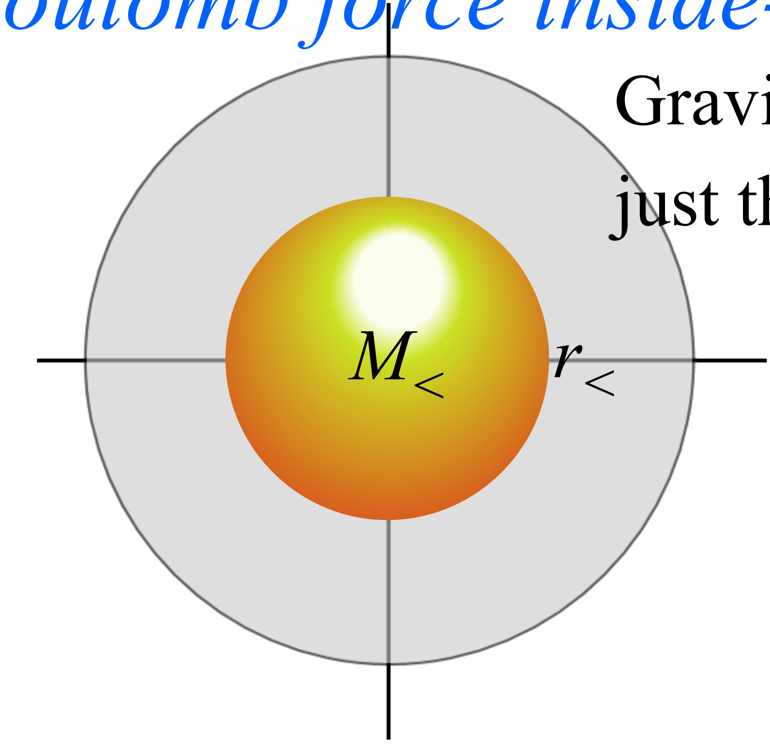
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

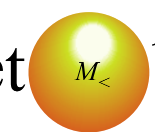
*Cancellation of*
*Shell mass element*
*M* =(solid-angle factor *A*)$D^2$

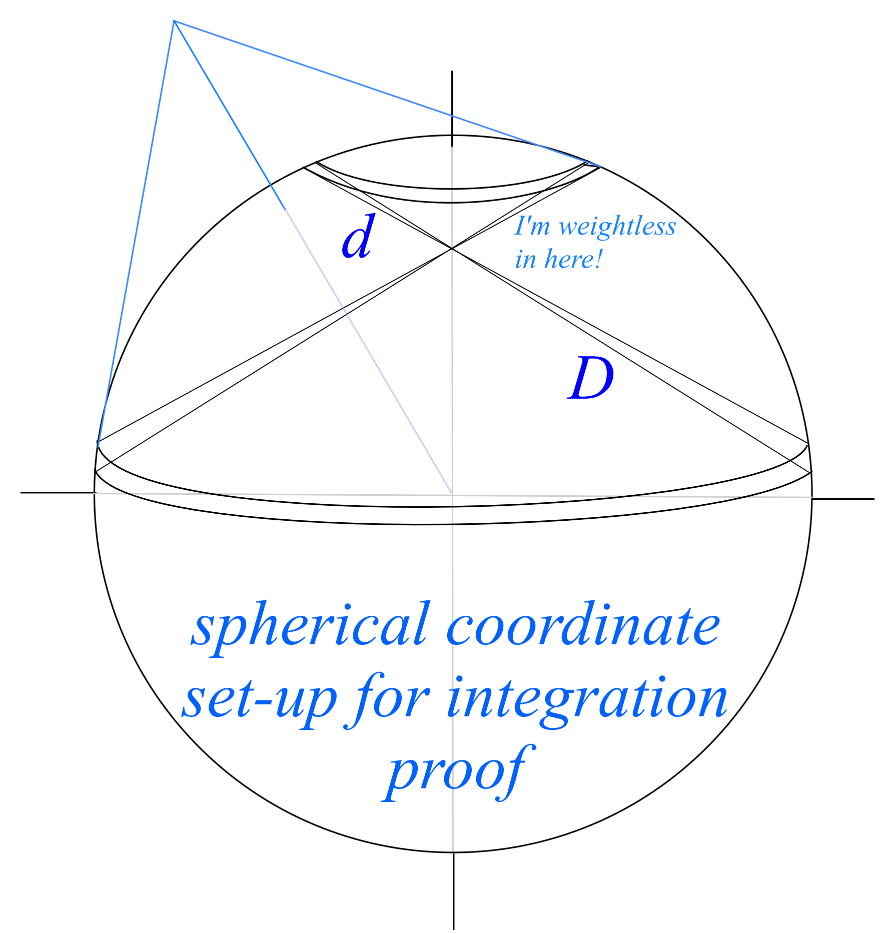



# *Coulomb force inside-spherical body due to stuff* <u>*below*</u> *you,* <u>*only*</u>.




Gravitational force at $r_<$ is just that of planet  below $r_<$

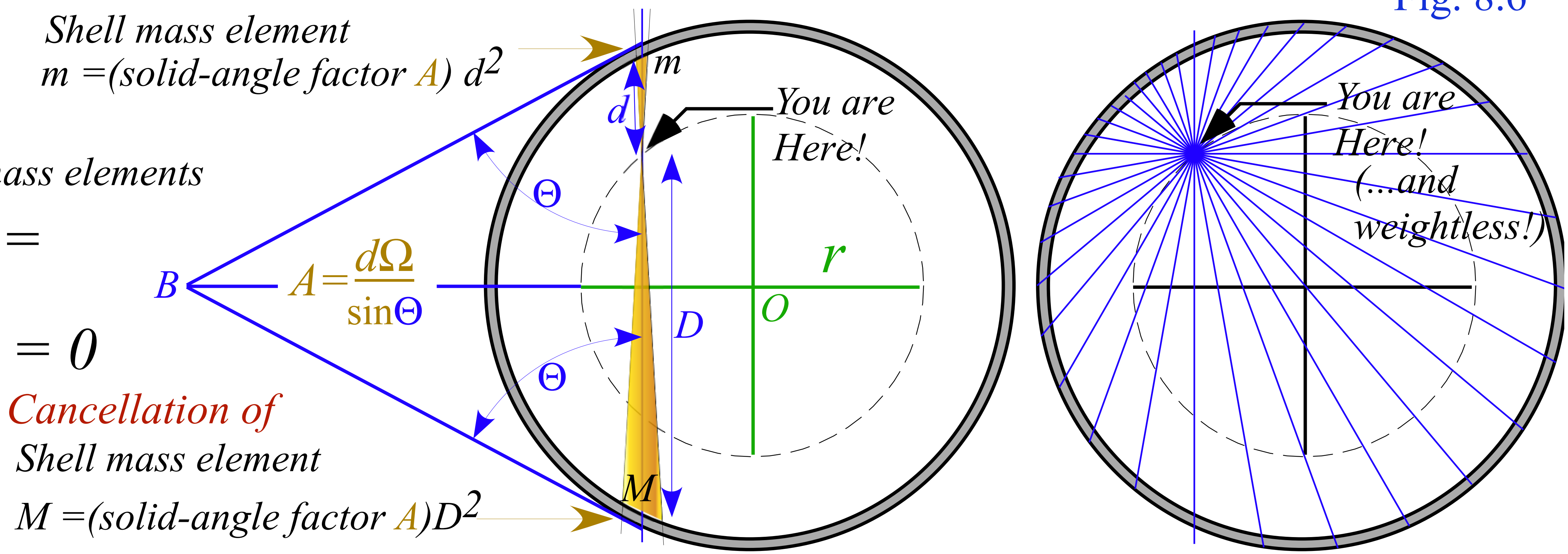


# Coulomb force vanishes inside-spherical shell *(Gauss-law)*

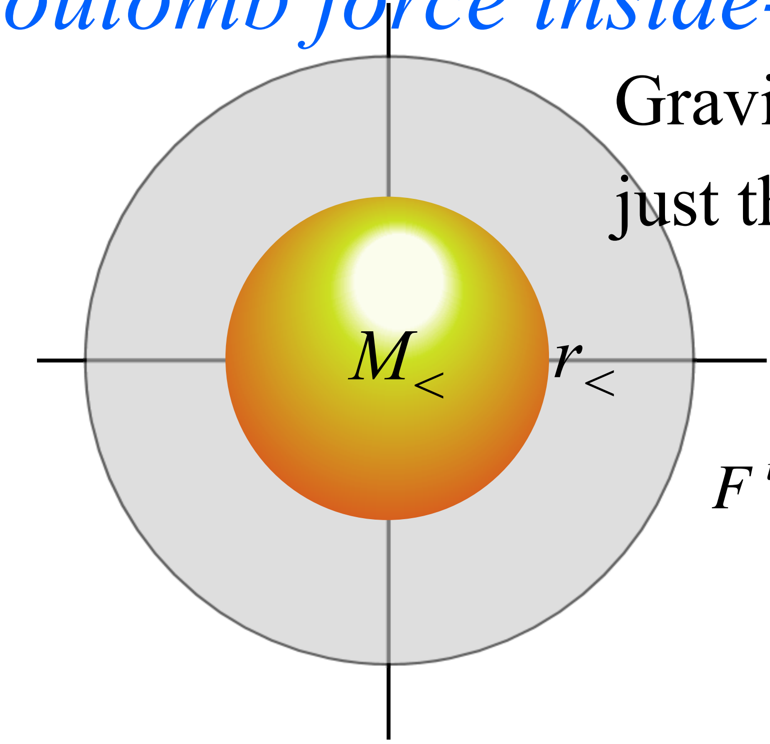
Unit 1
Fig. 8.6

*Gravity at $r$*
*due to shell mass elements*

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} = \left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$




# Coulomb force inside-spherical body due to stuff <u>below</u> you, <u>only</u>.




Gravitational force at $r_<$ is just that of planet $M_<$ below $r_<$

*Note:*
*Hooke's (linear) force law for $r_<$ inside uniform body*

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

# *Coulomb force vanishes inside-spherical shell* (Gauss-law)

Unit 1
Fig. 8.6

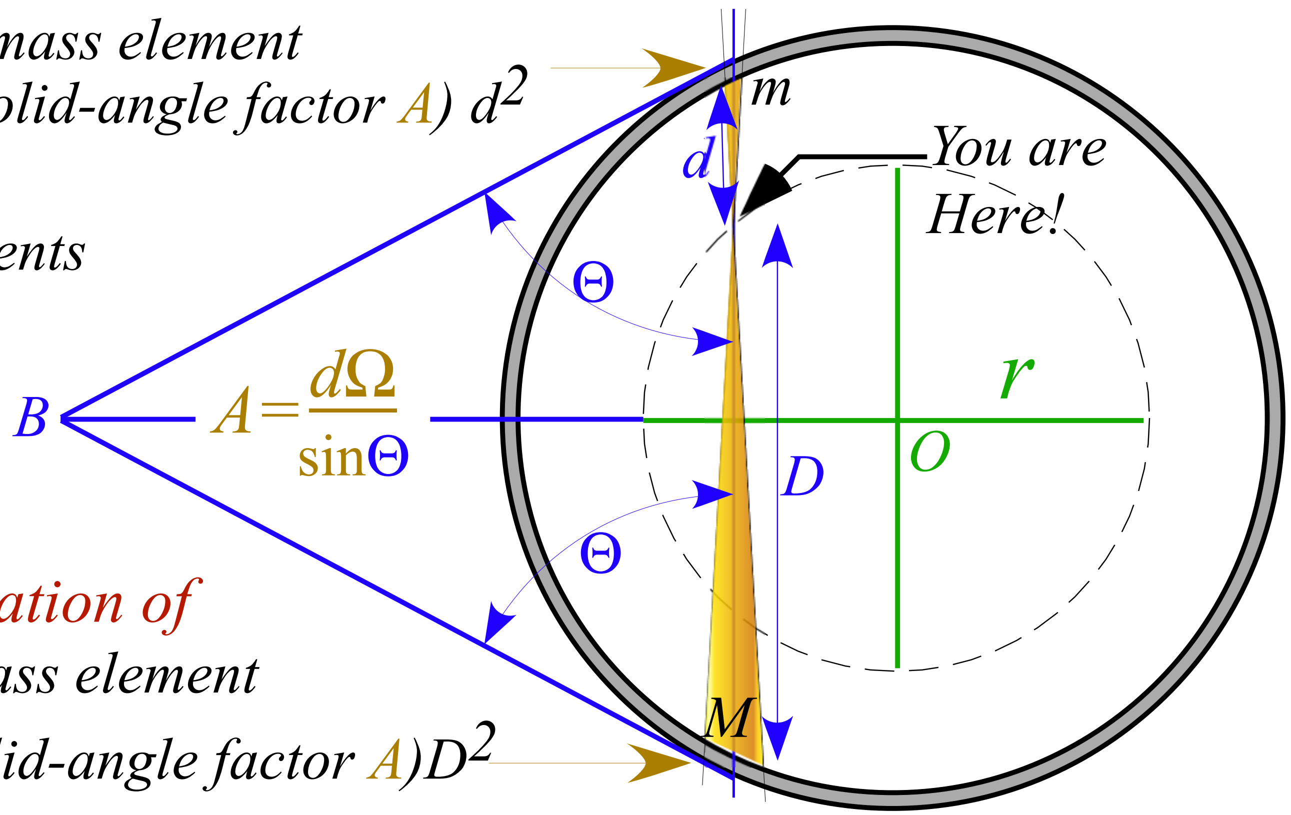
*Shell mass element*
*m =(solid-angle factor A) $d^2$*

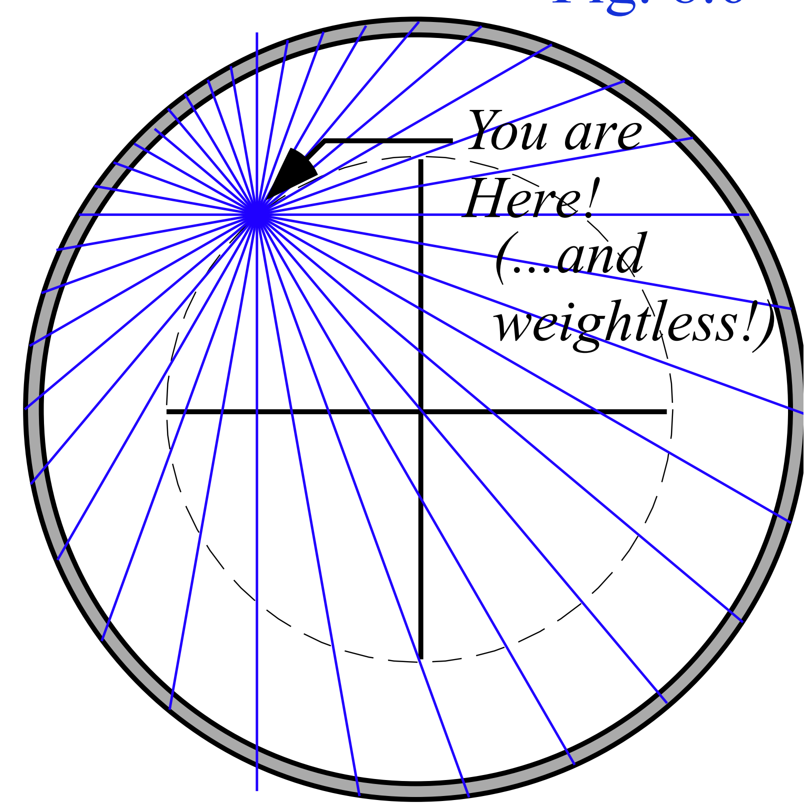
*Gravity at $r$*
*due to shell mass elements*

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

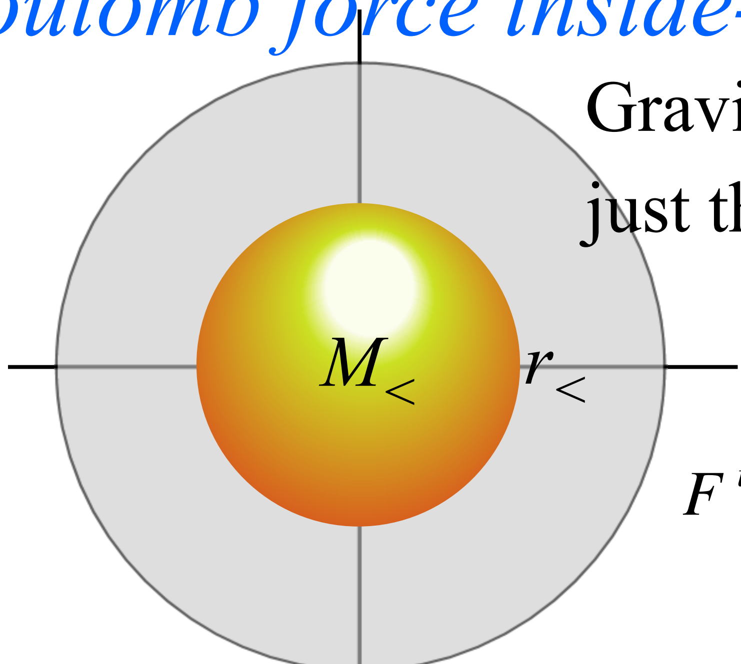
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

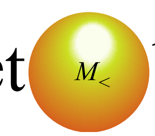
*Cancellation of*
*Shell mass element*
*M =(solid-angle factor A)$D^2$*




# *Coulomb force inside-spherical body due to stuff below you, only.*




Gravitational force at $r_<$ is
just that of planet  below $r_<$

*Note:*
*Hooke's (linear) force law*
*for $r_<$ inside uniform body*

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg \cdot x$$

Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s^2$

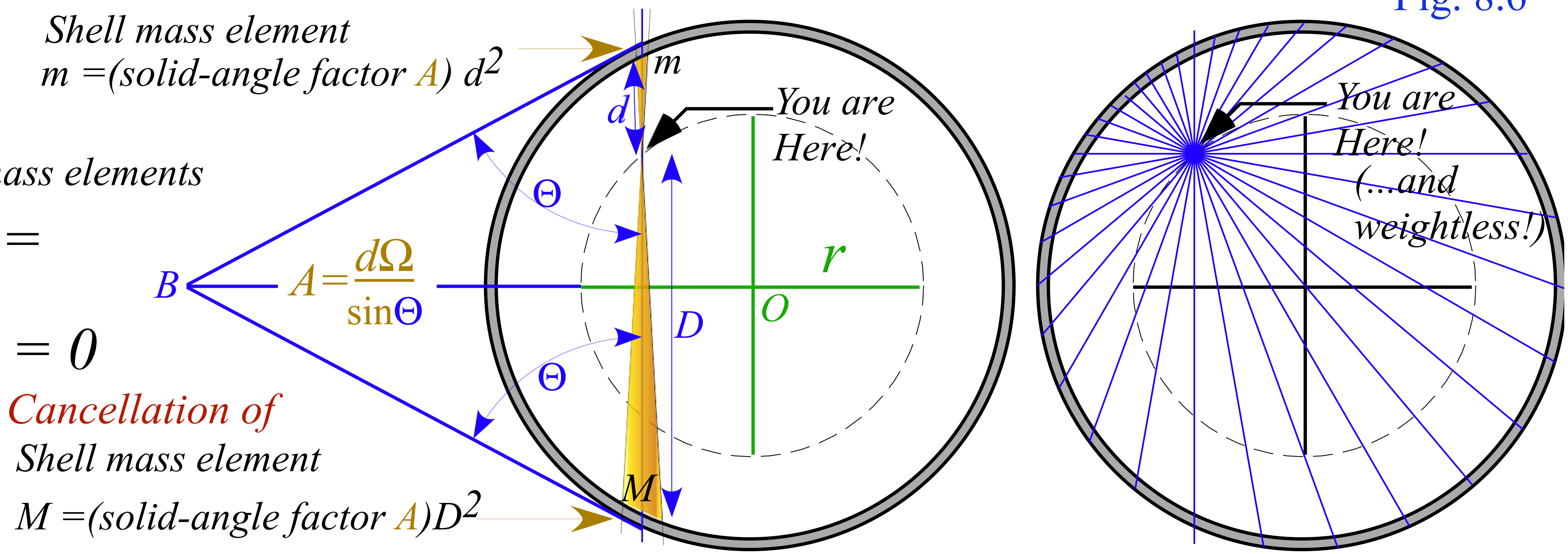
*$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$*

# *Coulomb force vanishes inside-spherical shell (Gauss-law)*

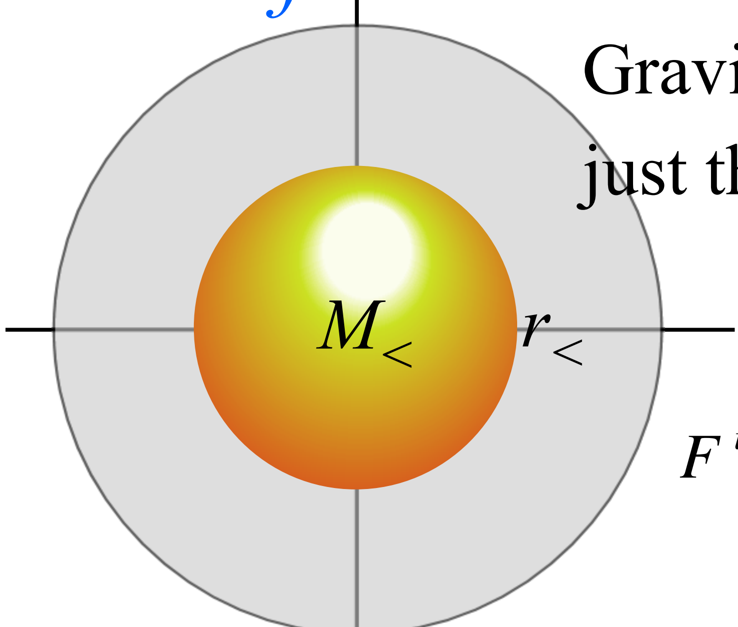
Unit 1
Fig. 8.6

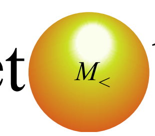
*Gravity at $r$*
*due to shell mass elements*

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} = \left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$




# *Coulomb force inside-spherical body due to stuff below you, only.*




Gravitational force at $r_<$ is just that of planet  below $r_<$

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Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s^2$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

$Earth\,radius: R_\oplus = 6.371\cdot 10^6\,m \simeq 6.4\cdot 10^6\,m$

$Earth\,mass: M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0\cdot 10^{24}\,kg.$

$Solar\,radius: R_\odot = 6.955 \times 10^8\,m. \simeq 7.0\cdot 10^8\,m.$

$Solar\,mass: M_\odot = 1.9889 \times 10^{30}\,kg. \simeq 2.0\cdot 10^{30}\,kg.$

# *Geometry of idealized "Sophomore-physics Earth"*

*Coulomb field* ***outside*** *Isotropic Harmonic Oscillator (IHO) field* ***inside***

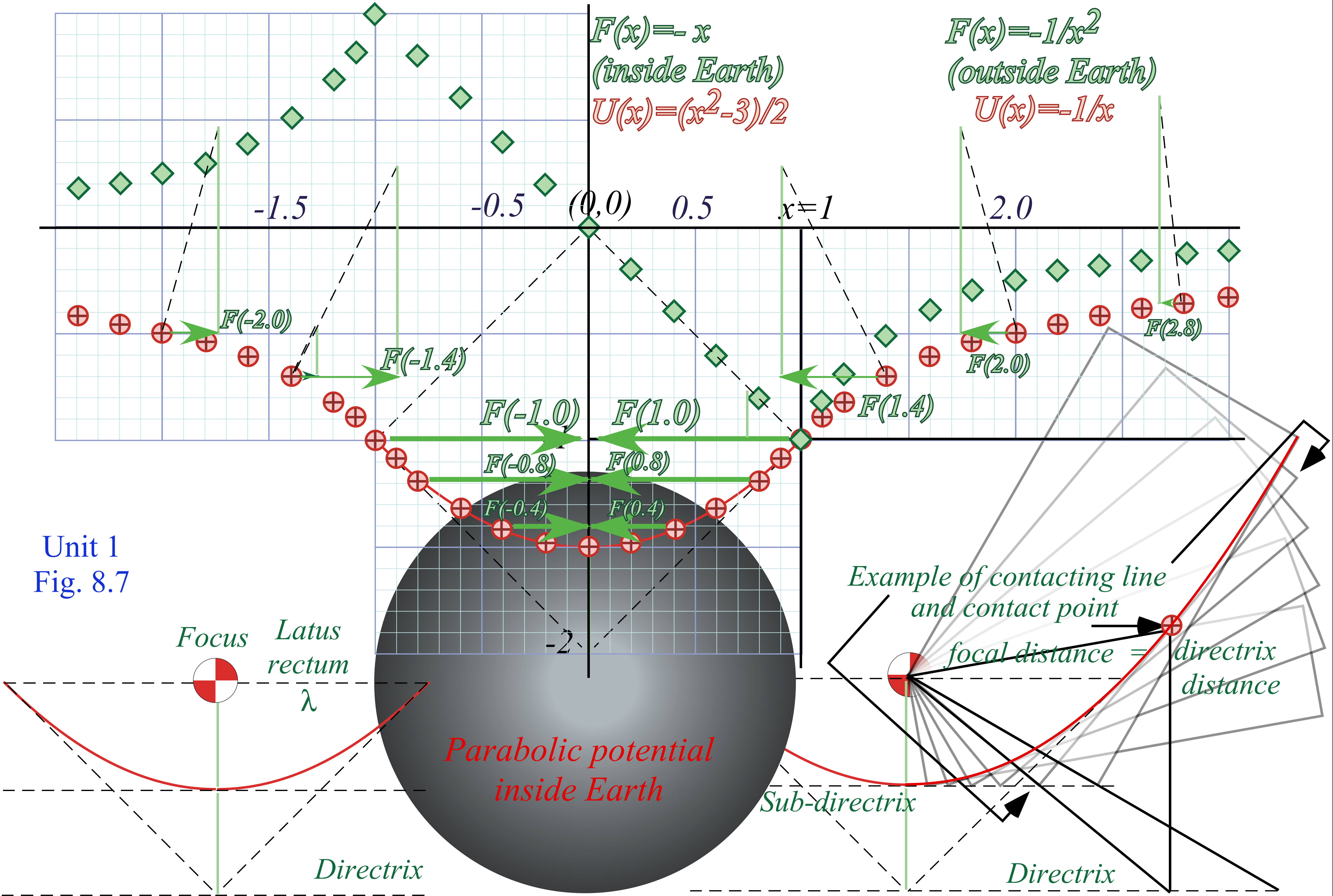
→ *Contact-geometry of potential curve(s)*

*"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

*Earth matter vs nuclear matter:*

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

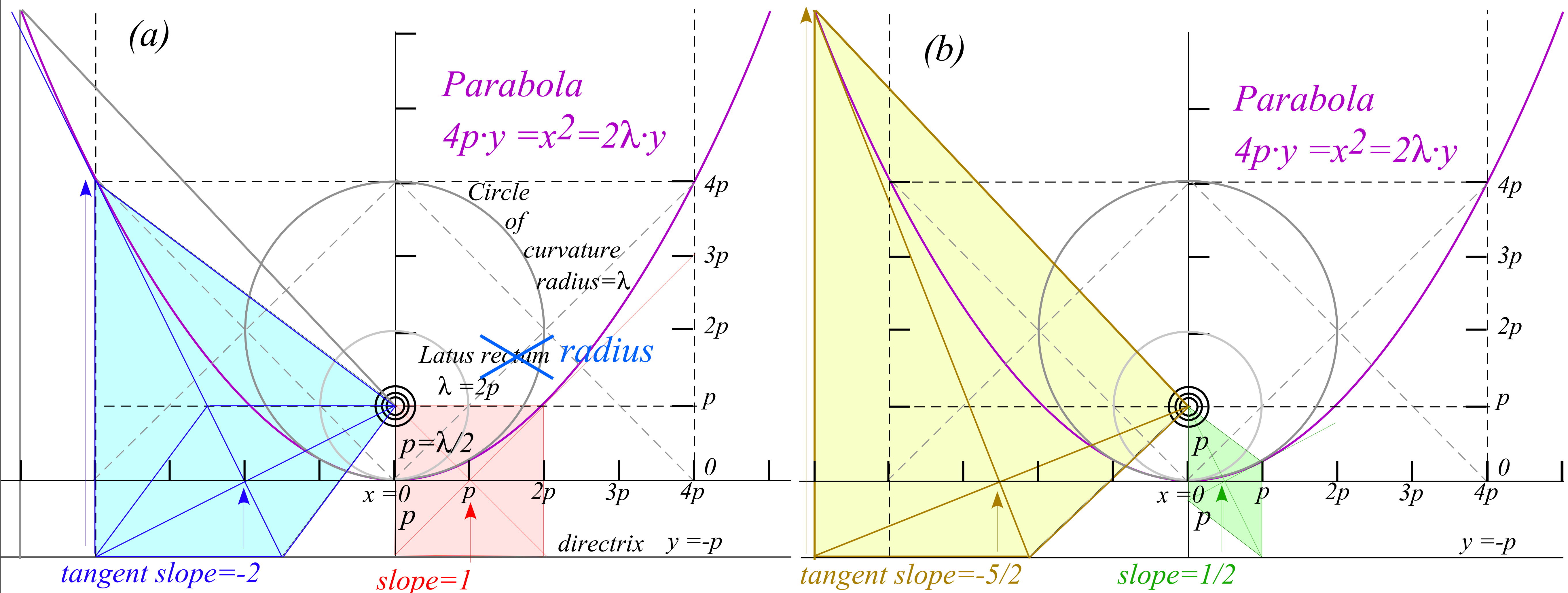
# The ideal "Sophomore-Physics-Earth" model of geo-gravity




Unit 1
Fig. 8.7

# ...conventional parabolic geometry...carried to extremes...

# (From p.21)




Unit 1
Fig. 8.4

# *Geometry of idealized "Sophomore-physics Earth"*

*Coulomb field* <u>*outside*</u> *Isotropic Harmonic Oscillator (IHO) field* <u>*inside*</u>

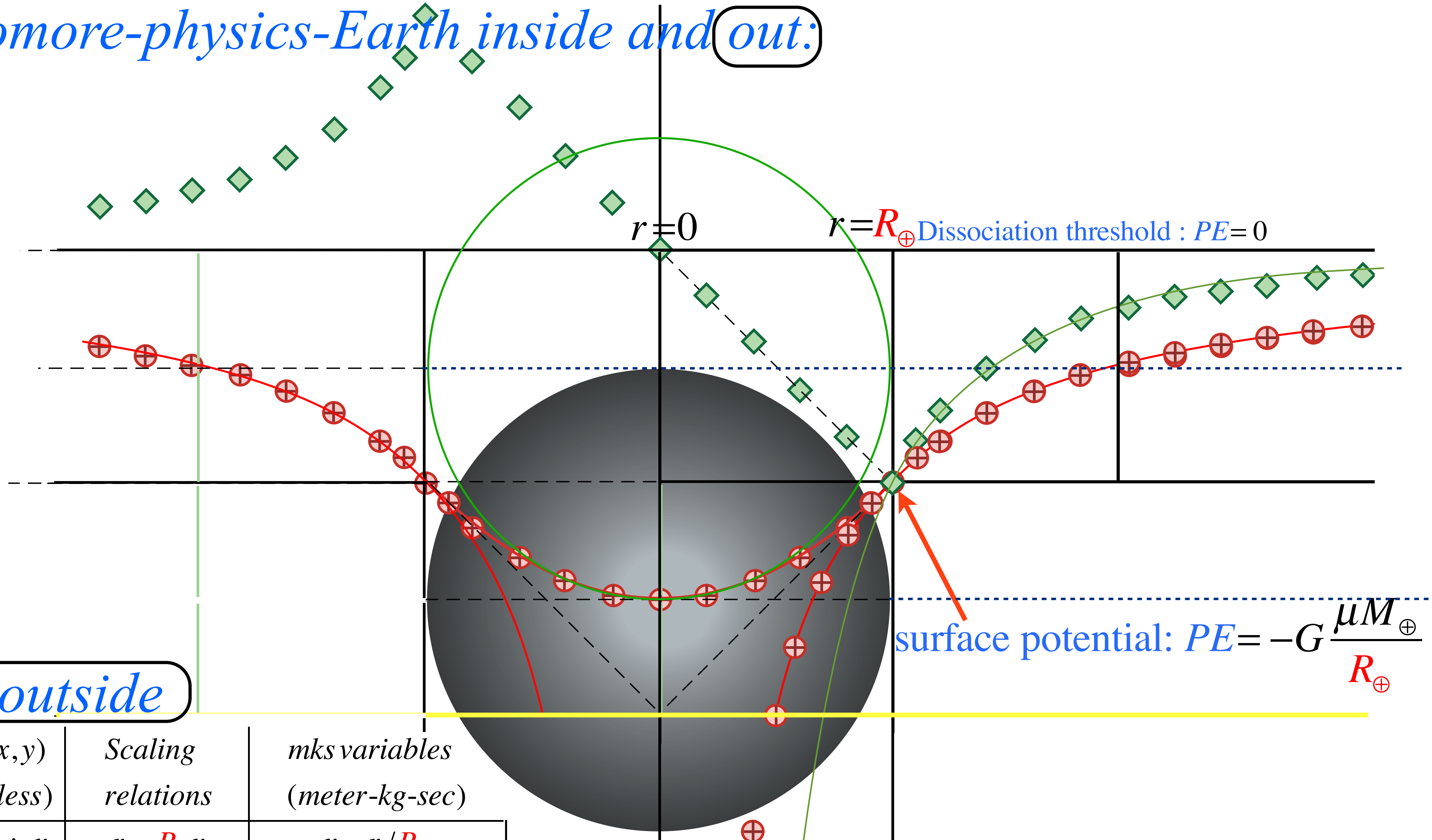
*Contact-geometry of potential curve(s)*

→ *"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

*Earth matter vs nuclear matter:*

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

# Sophomore-physics-Earth inside and out:




## outside

| *Geometric* $(x,y)$ *(Dimensionless)* | *Scaling relations* | *mks variables (meter-kg-sec)* |
|---|---|---|
| space coord.: $x$ | $r = R_\oplus x$ | $x = r / R_\oplus$ |
| $PE$ for $\lvert x \rvert \geq 1$: $y^{PE} = \frac{-1}{x}$ | $PE^{mks}(r) = \frac{GM\mu}{R_\oplus} y^{PE}$ | $PE^{mks}(r) = -\frac{GM\mu}{r} = -\frac{GM\mu}{R_\oplus}\frac{1}{x}$ |

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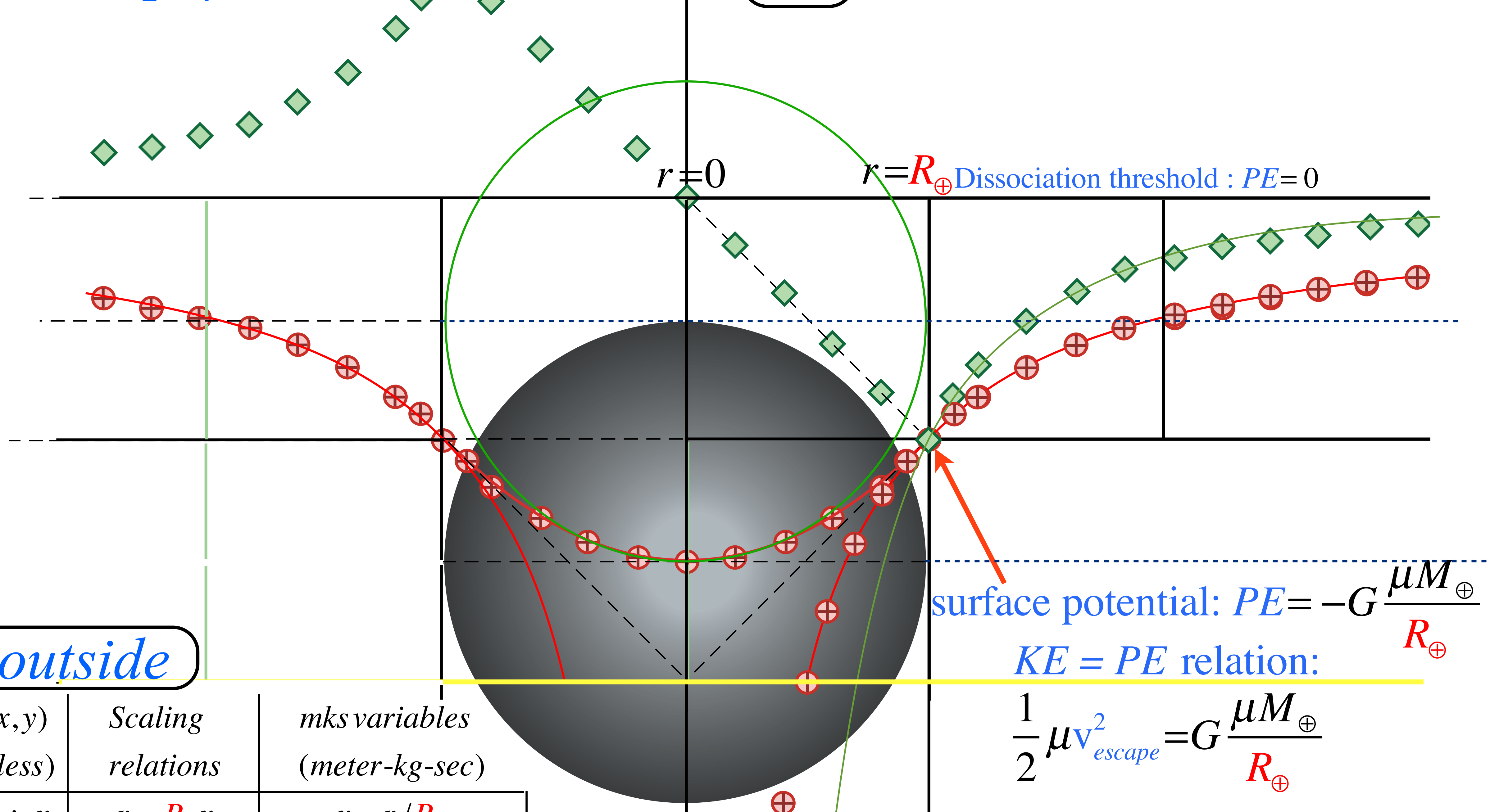


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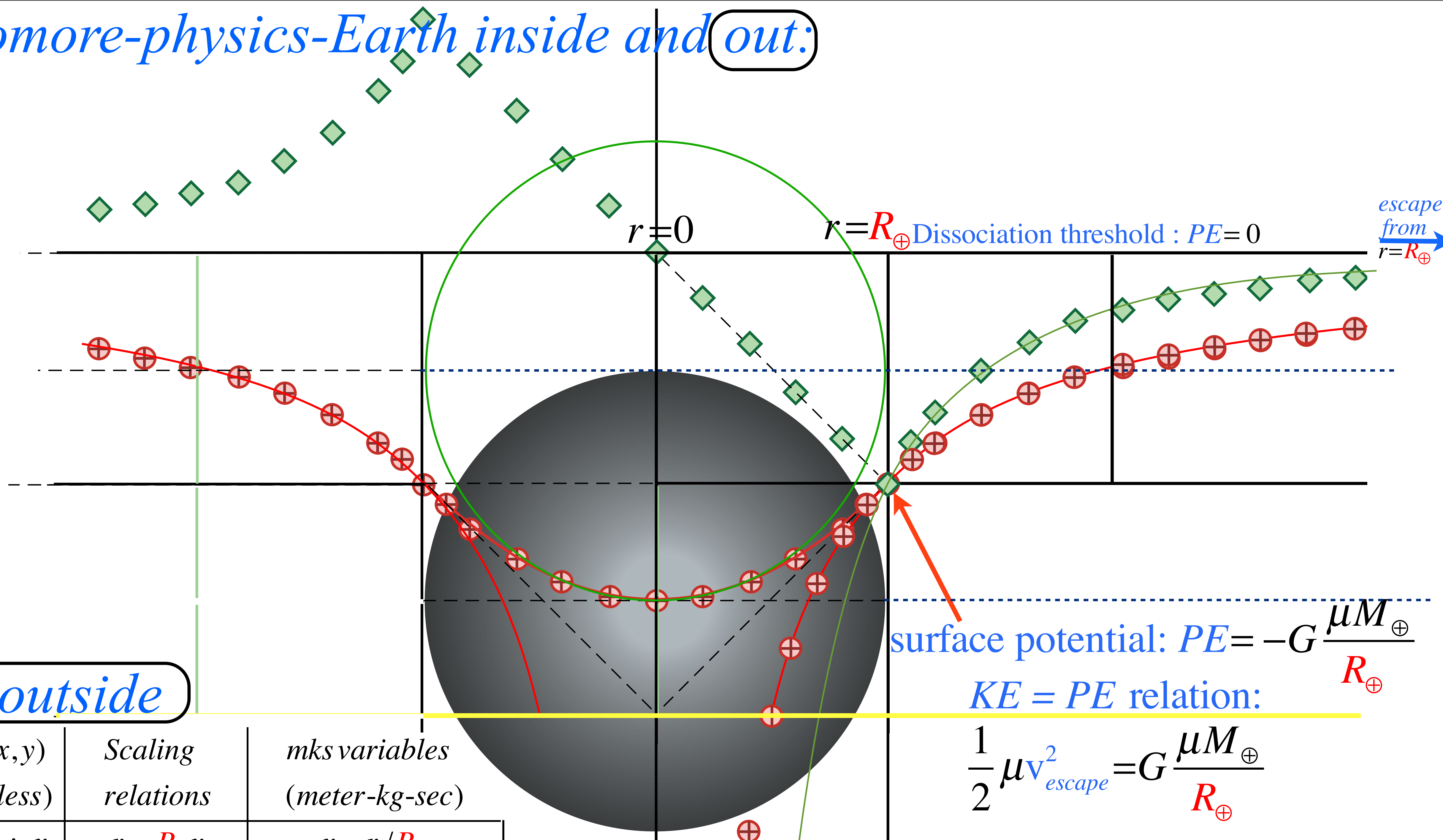


surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v^2_{escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

| Geometric $(x,y)$ (Dimensionless) | Scaling relations | mks variables (meter-kg-sec) |
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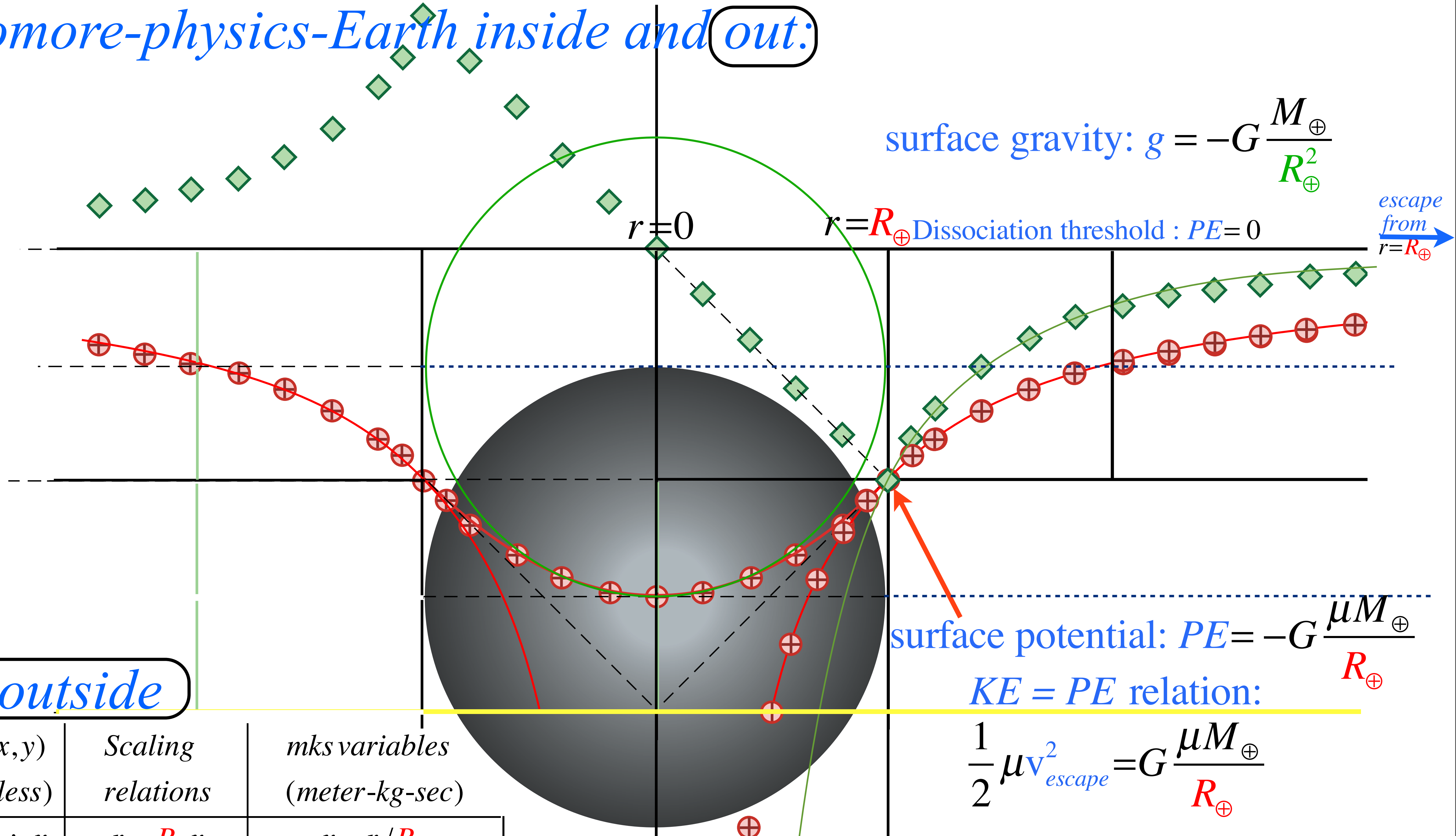
$R_\oplus$-escape-velocity

$$v_{escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

# Sophomore-physics-Earth inside and out:




surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2}$

surface potential: $PE = -G\frac{\mu M_\oplus}{R_\oplus}$

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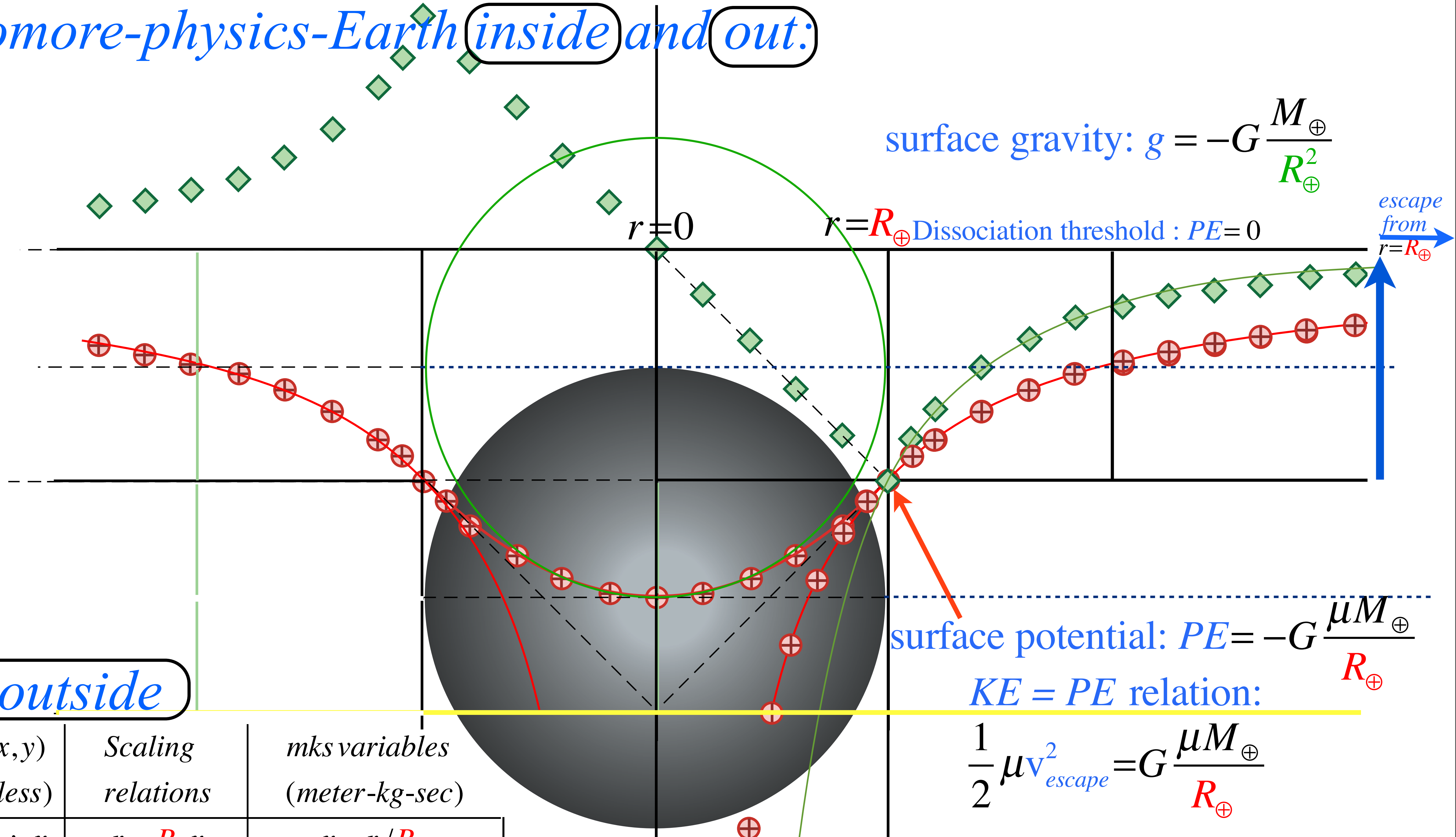


| *Geometric (x,y) (Dimensionless)* | *Scaling relations* | *mks variables (meter-kg-sec)* | | *inside* |
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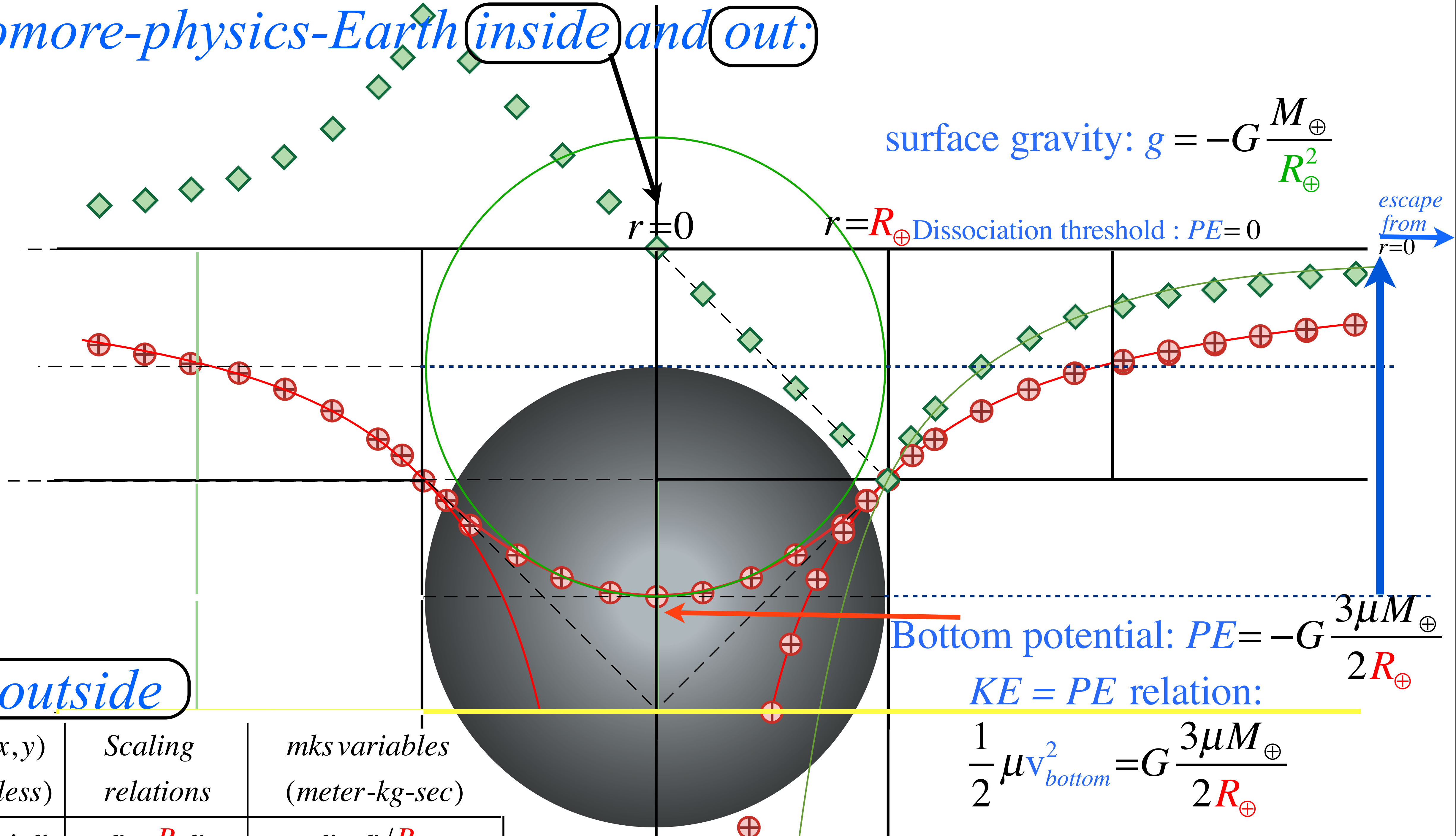


surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2}$

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

*KE = PE* relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

| *Geometric* $(x,y)$ (*Dimensionless*) | *Scaling relations* | *mks variables* (*meter-kg-sec*) | | |
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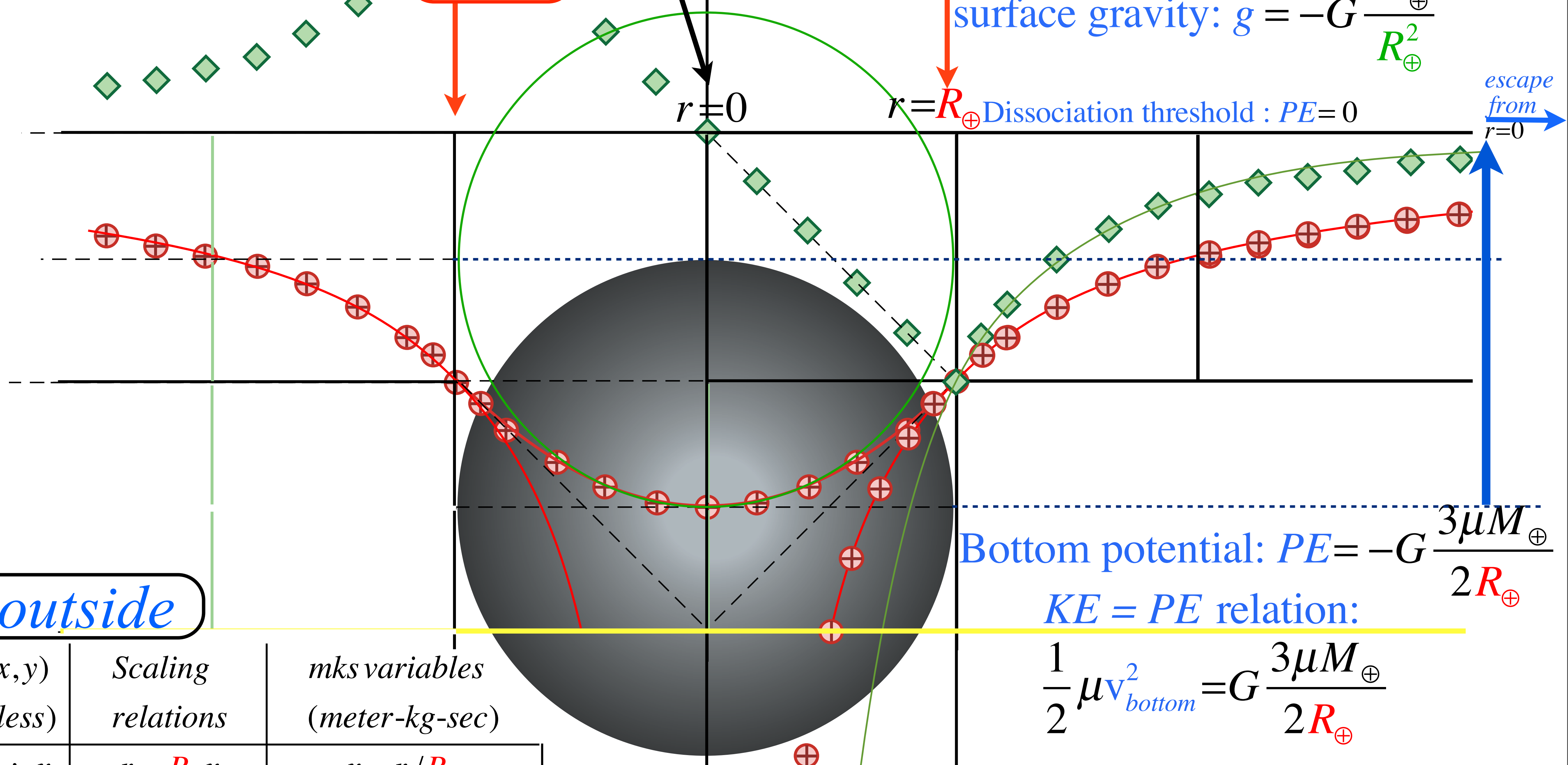
# Sophomore-physics-Earth inside and out:
## ...and surface orbit at $r=R_\oplus$




surface gravity: $g=-G\frac{M_\oplus}{R_\oplus^2}$

Bottom potential: $PE=-G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

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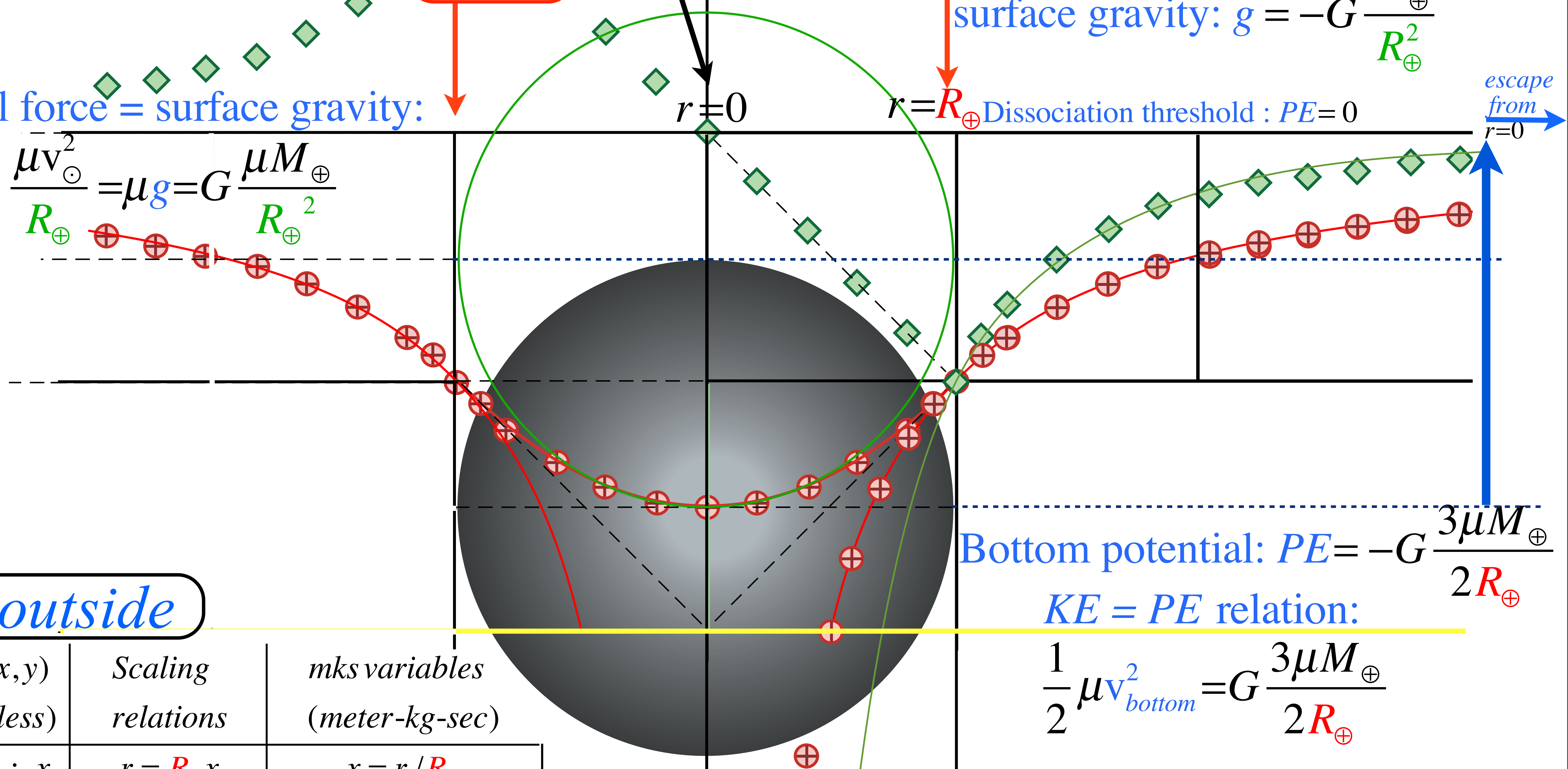
$(r=0)$-escape-velocity

$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

# Sophomore-physics-Earth inside and out:
# ...and surface orbit at $r=R_\oplus$




$(r=0)$-escape-velocity

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| *Geometric* $(x,y)$ (*Dimensionless*) | *Scaling relations* | *mks variables (meter-kg-sec)* | | |
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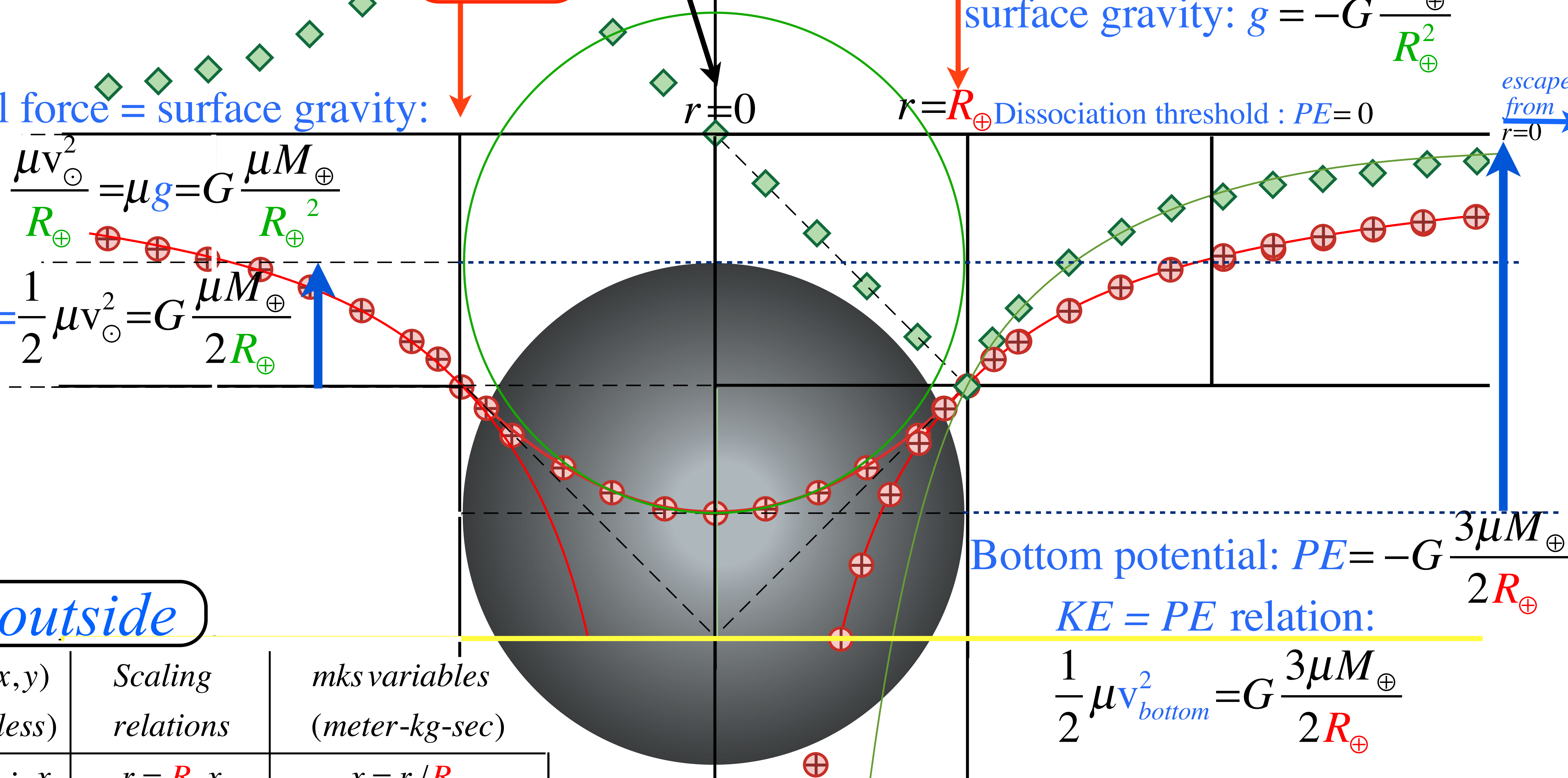
# *Sophomore-physics-Earth inside and out:*
# *...and surface orbit at $r=R_\oplus$*

surface gravity: $g=-G\dfrac{M_\oplus}{R_\oplus^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus{}^2}$$

Orbit KE $=\dfrac{1}{2}\mu v_\odot^2=G\dfrac{\mu M_\oplus}{2R_\oplus}$




Bottom potential: $PE=-G\dfrac{3\mu M_\oplus}{2R_\oplus}$

*KE = PE* relation:

$$\frac{1}{2}\mu v_{bottom}^2=G\frac{3\mu M_\oplus}{2R_\oplus}$$

*outside*

*inside*

| *Geometric* $(x,y)$ *(Dimensionless)* | *Scaling relations* | *mks variables (meter-kg-sec)* | | |
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$(r=0)$-escape-velocity

$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

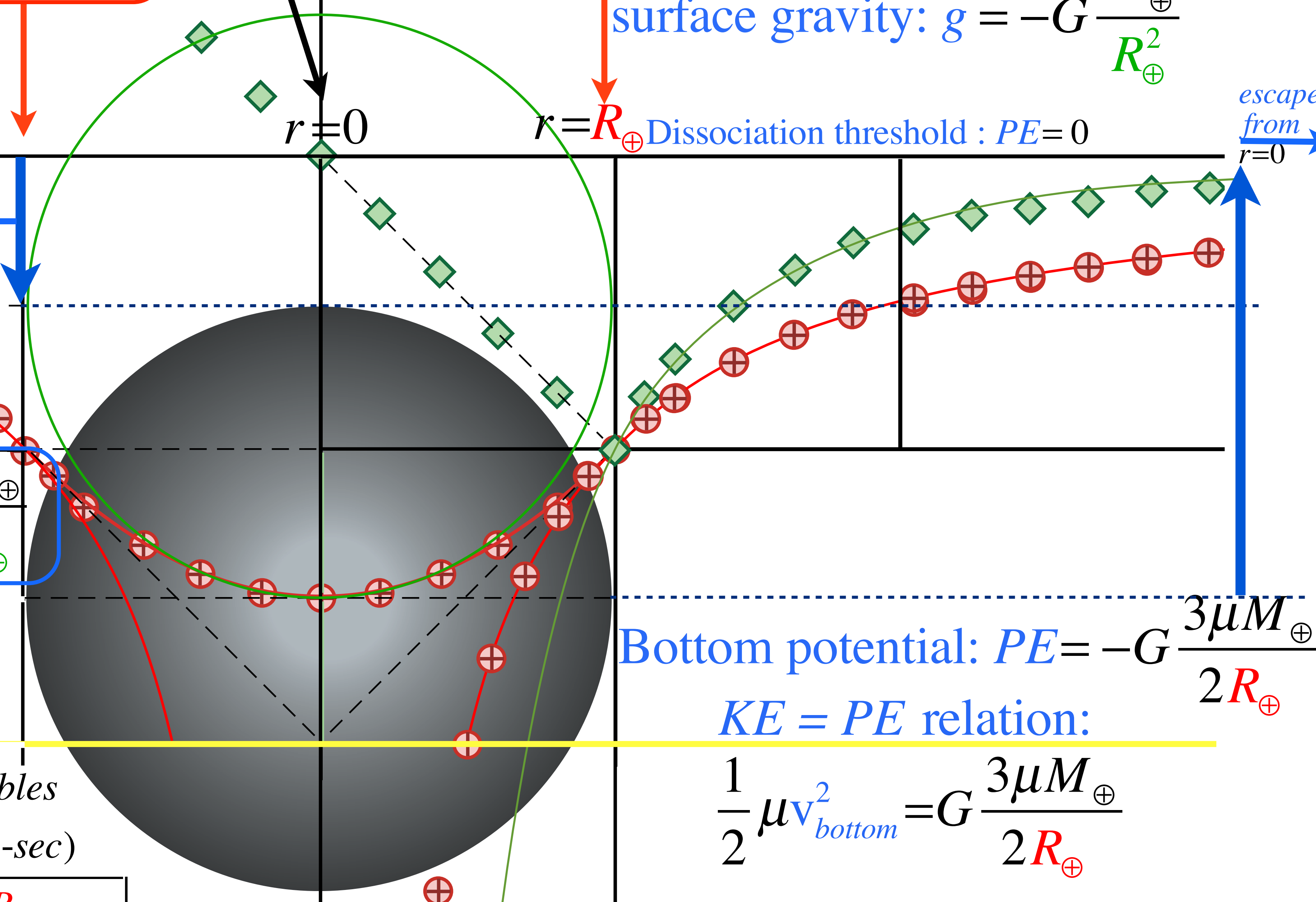
# Sophomore-physics-Earth inside and out:
# ...and surface orbit at $r=R_\oplus$

Centifugal force = surface gravity: $\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^{\ 2}}$

Orbit KE $=\frac{1}{2}\mu v_\odot^2 = G\frac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total} = \frac{1}{2}\mu v_\odot^2 - G\frac{\mu M_\oplus}{R_\oplus} = -G\frac{\mu M_\oplus}{2R_\oplus}$

surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2}$




Bottom potential: $PE = -G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation: $\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$

outside

inside

$(r=0)$-escape-velocity $v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$

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# *Sophomore-physics-Earth inside and out: "3-steps out of (or into) Hell"*
## *...and surface orbit at $r=R_\oplus$*

surface gravity: $g=-G\dfrac{M_\oplus}{R_\oplus^2}$

Centifugal force = surface gravity:

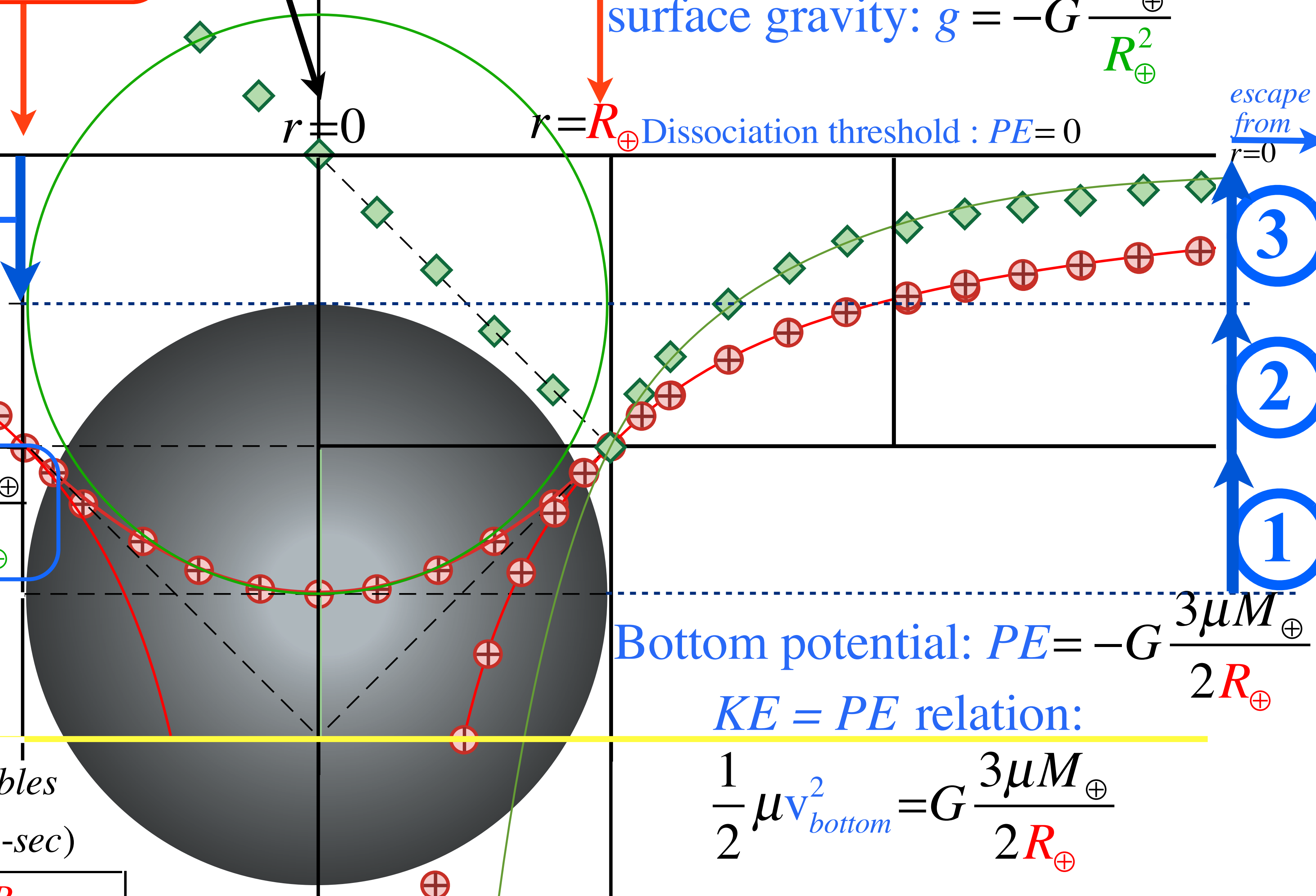
$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus{}^2}$$

Orbit KE $=\dfrac{1}{2}\mu v_\odot^2=G\dfrac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total}=\dfrac{1}{2}\mu v_\odot^2-G\dfrac{\mu M_\oplus}{R_\oplus}=-G\dfrac{\mu M_\oplus}{2R_\oplus}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus=G\frac{M_\oplus}{R_\oplus^2}\Rightarrow\omega_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$




Bottom potential: $PE=-G\dfrac{3\mu M_\oplus}{2R_\oplus}$

*KE = PE* relation:

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$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

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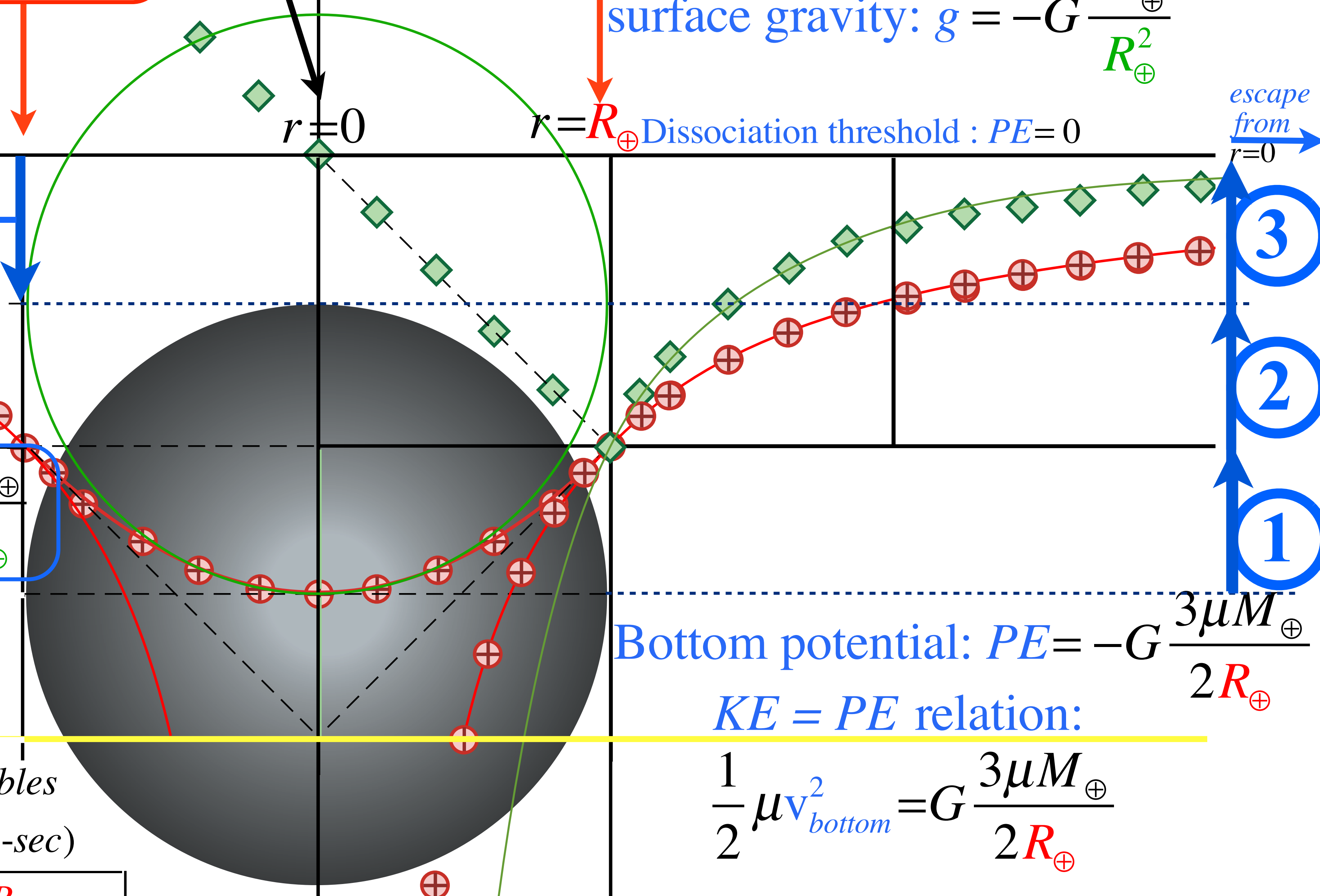


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Orbit KE $=\dfrac{1}{2}\mu v_\odot^2=G\dfrac{\mu M_\oplus}{2R_\oplus}$

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$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus=G\frac{M_\oplus}{R_\oplus^2}\Rightarrow\omega_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

Bottom potential: $PE=-G\dfrac{3\mu M_\oplus}{2R_\oplus}$

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$(r=0)$-escape-velocity

$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

$(r=R_\oplus)$-orbit speed:

$$v_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus}}=\sqrt{gR_\oplus}$$

*7.9km/sec*

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# *...and surface orbit at $r=R_\oplus$*

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$(r=R_\oplus)$-orbit angular frequency:

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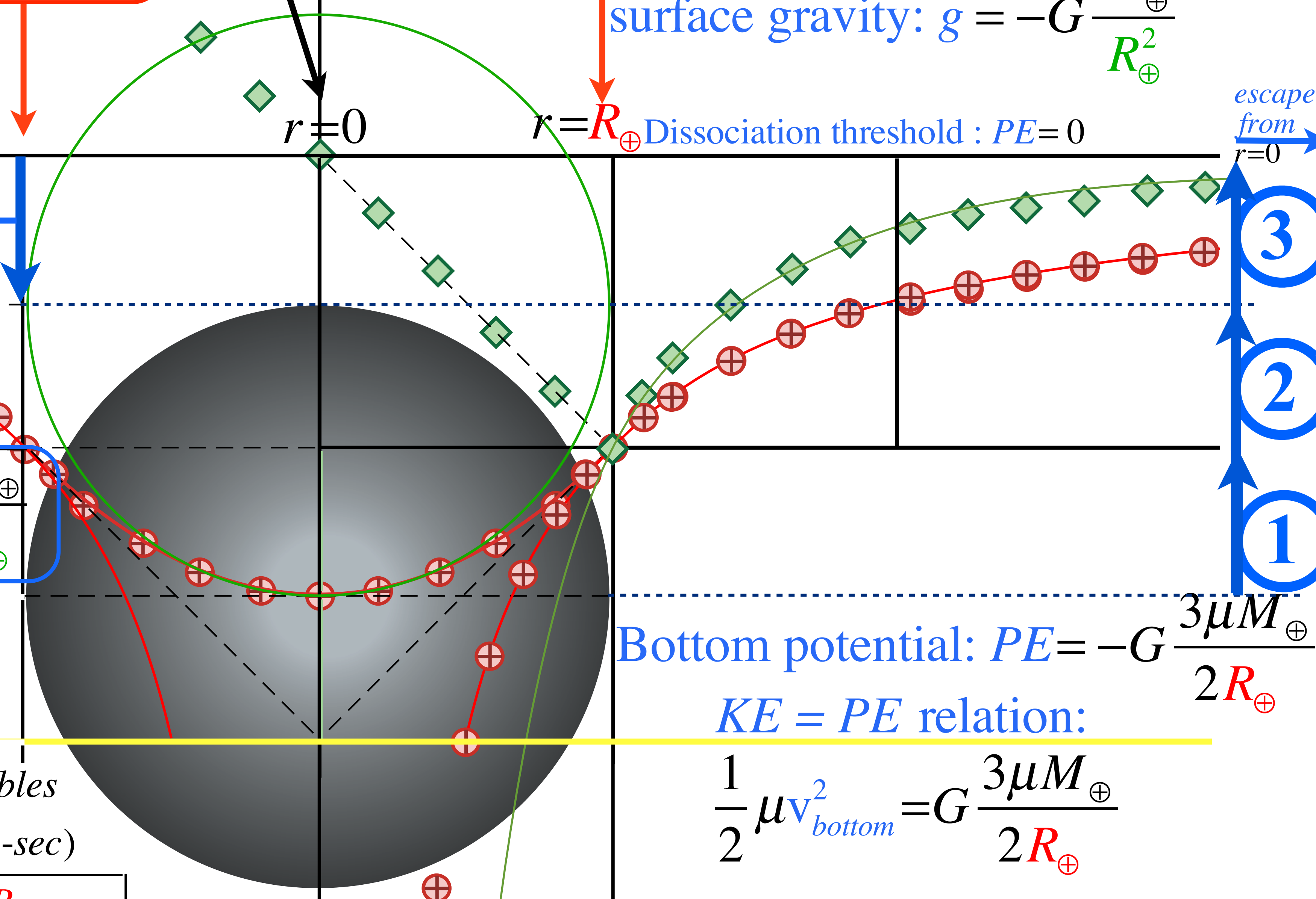


Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

*KE = PE* relation:

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$(r=0)$-escape-velocity

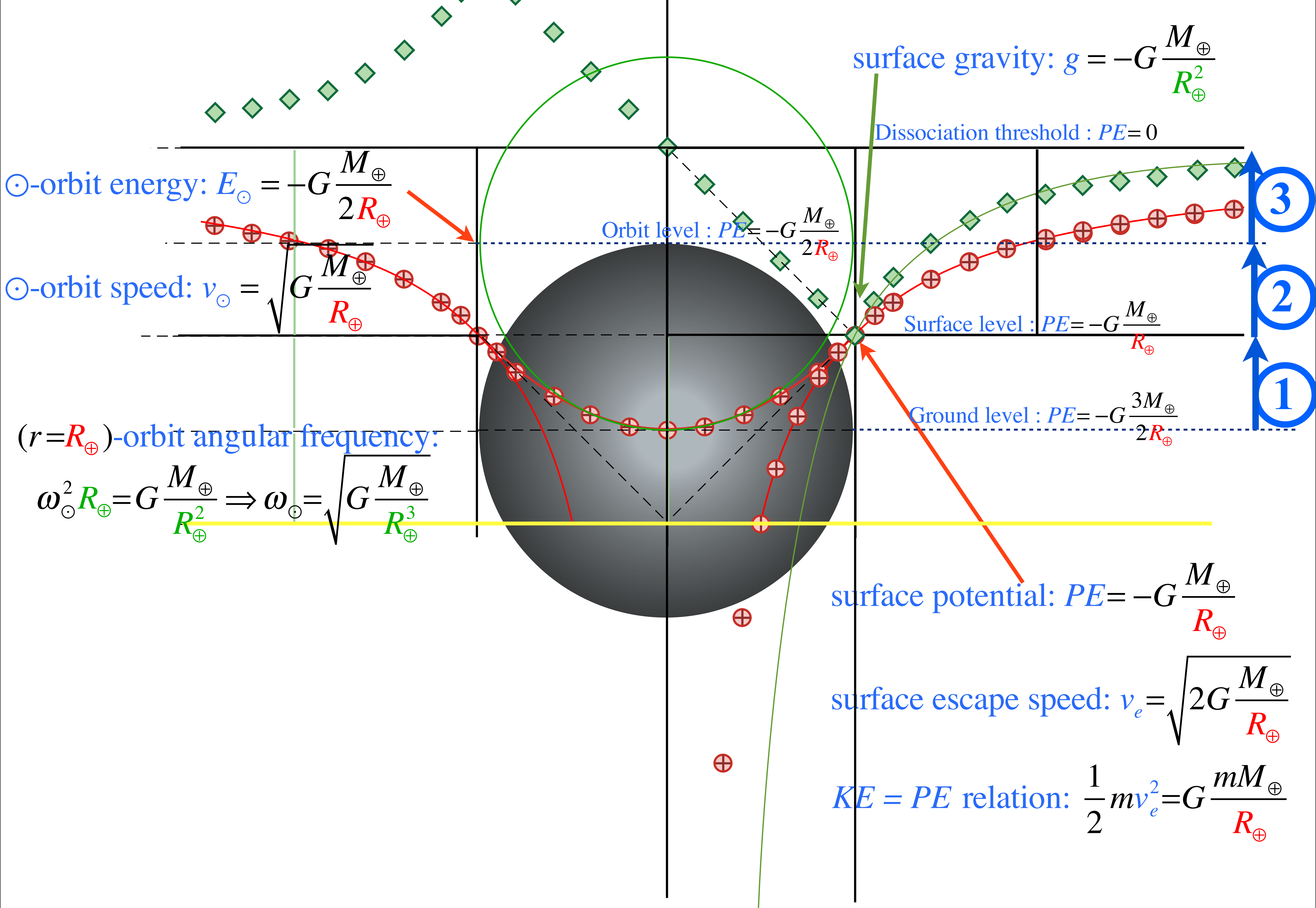
$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

$(r=R_\oplus)$-escape velocity:

$$v_{escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

*11.1km/sec*

# *The "Three (equal) steps from Hell"*

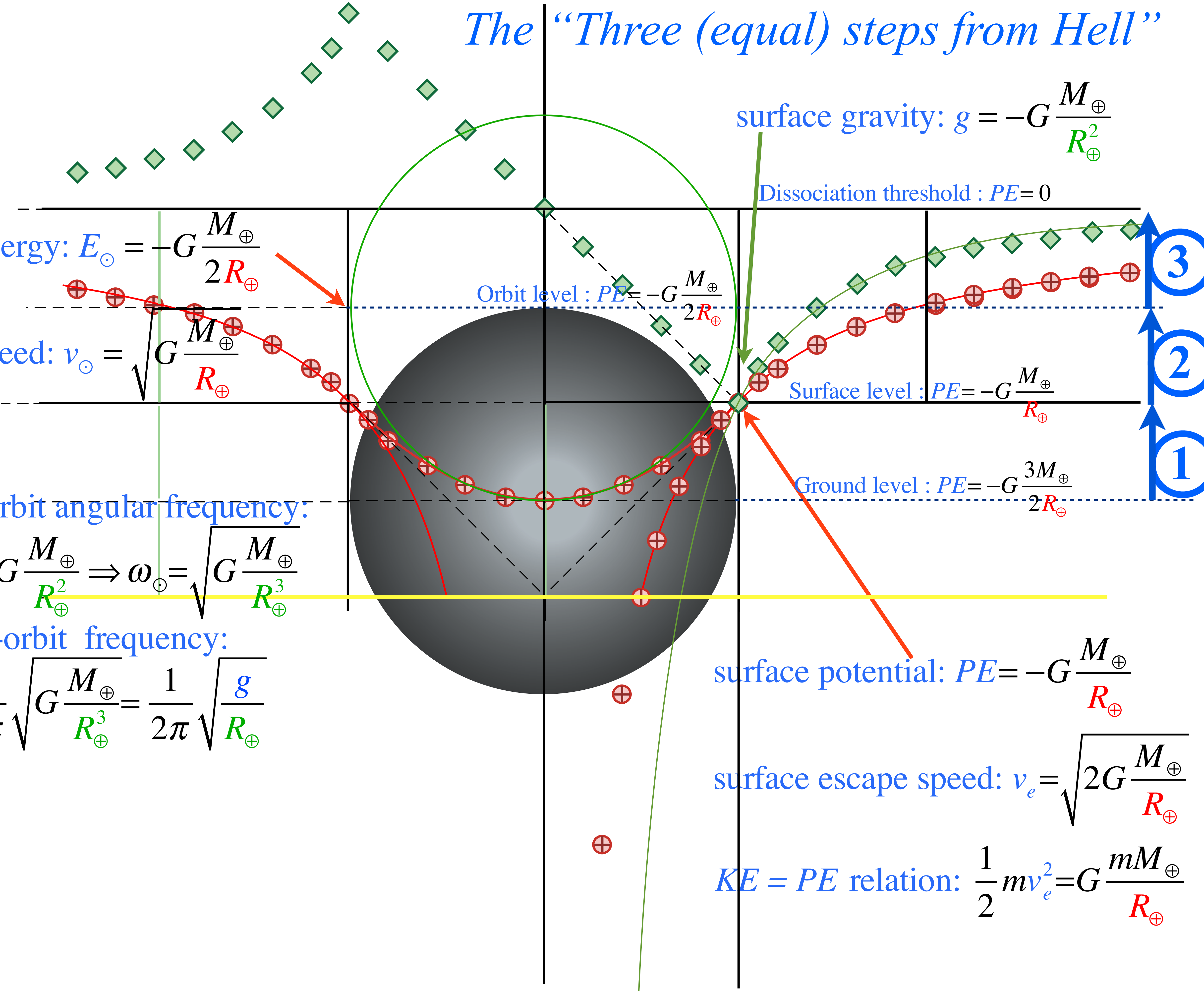


# The "Three (equal) steps from Hell"

surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2}$

$\odot$-orbit energy: $E_\odot = -G\dfrac{M_\oplus}{2R_\oplus}$

$\odot$-orbit speed: $v_\odot = \sqrt{G\dfrac{M_\oplus}{R_\oplus}}$




$(r = R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

$(r = R_\oplus)$-orbit frequency:

$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

surface potential: $PE = -G\dfrac{M_\oplus}{R_\oplus}$

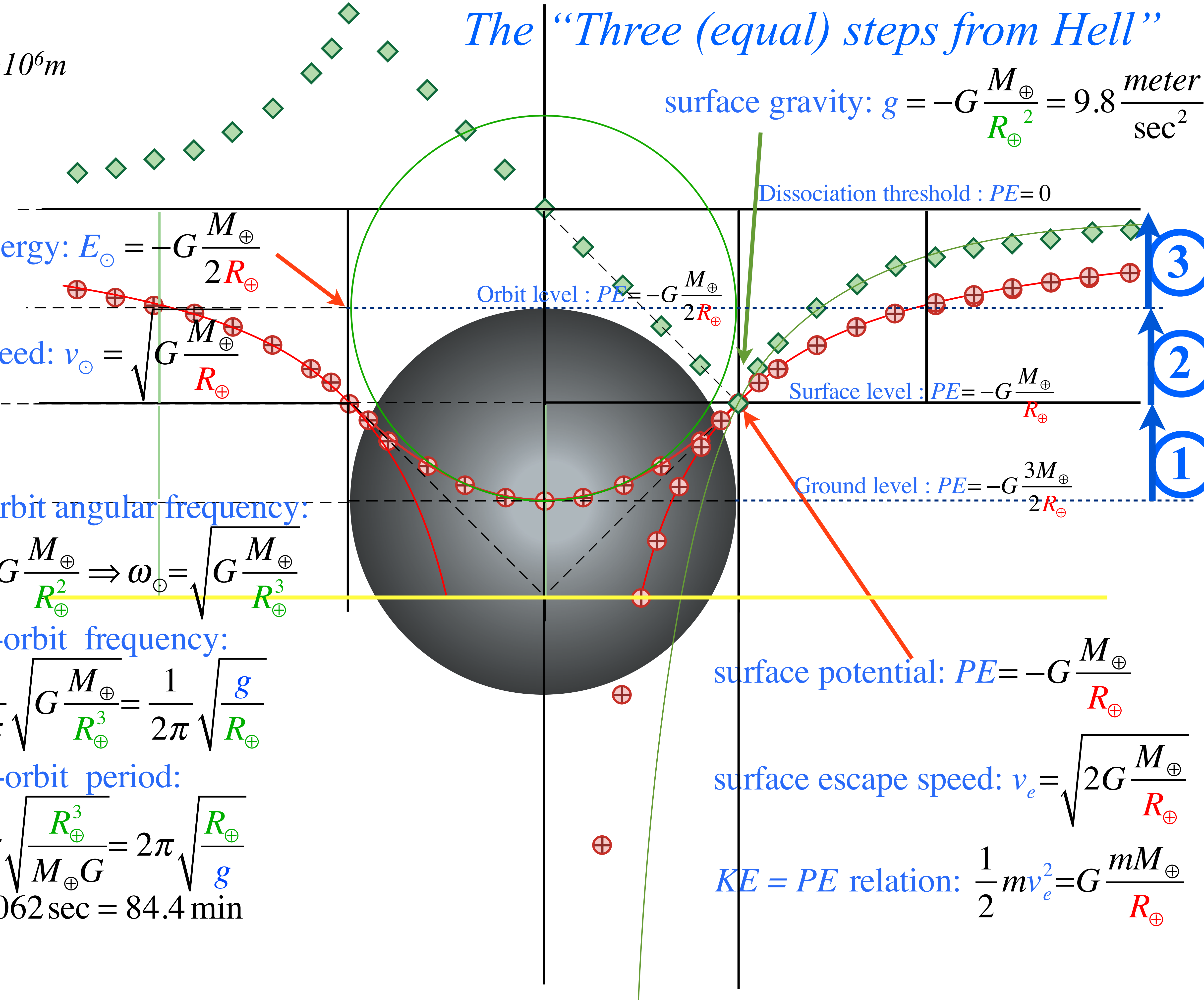
surface escape speed: $v_e = \sqrt{2G\dfrac{M_\oplus}{R_\oplus}}$

$KE = PE$ relation: $\dfrac{1}{2}mv_e^2 = G\dfrac{mM_\oplus}{R_\oplus}$

# The "Three (equal) steps from Hell"

$R_\oplus = 6.4\cdot 10^6 m$

surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^{\ 2}} = 9.8\frac{meter}{\sec^2}$




⊙-orbit energy: $E_\odot = -G\frac{M_\oplus}{2R_\oplus}$

⊙-orbit speed: $v_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus}}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

$(r=R_\oplus)$-orbit frequency:

$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

$(r=R_\oplus)$-orbit period:

$$\upsilon_\odot = 2\pi\sqrt{\frac{R_\oplus^3}{M_\oplus G}} = 2\pi\sqrt{\frac{R_\oplus}{g}}$$

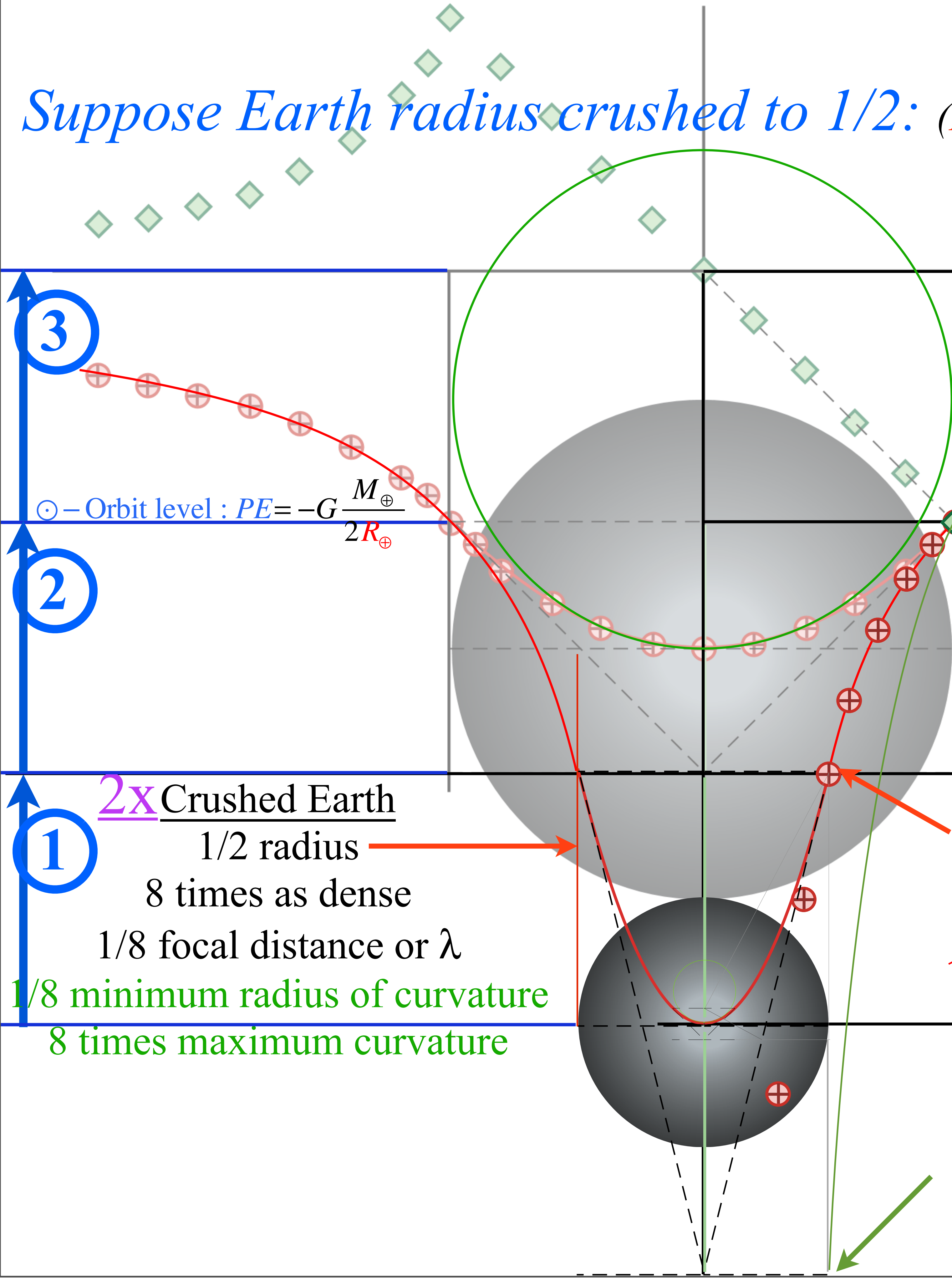
$$= 5062\sec = 84.4\min$$

surface potential: $PE = -G\frac{M_\oplus}{R_\oplus}$

surface escape speed: $v_e = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$

KE = PE relation: $\frac{1}{2}mv_e^2 = G\frac{mM_\oplus}{R_\oplus}$

# *Suppose Earth radius crushed to 1/2:* *($R_\oplus$=6.4·10$^6$m crushed to $R_\oplus$/2=3.2·10$^6$m )*




*All formulas identical to ones derived on p.50 to 63. Imagine reducing $R_\oplus$ to $R_\oplus/2$*

2 times $\odot$-orbit energy: $E_\odot = -G\dfrac{M_\oplus}{2R_\oplus}$

$\sqrt{2}$ times $\odot$-orbit speed: $v_\odot = \sqrt{G\dfrac{M_\oplus}{R_\oplus}}$

2 times the surface potential: $PE = -G\dfrac{M_\oplus}{R_\oplus}$

$\sqrt{2}$ times surface escape speed: $v_e = \sqrt{G\dfrac{2M_\oplus}{R_\oplus}}$

4 times the surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2}$

# *Geometry of idealized "Sophomore-physics Earth"*

*Coulomb field <u>outside</u>* *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

*Contact-geometry of potential curve(s)*

*"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

*Earth matter vs nuclear matter:*

→ *Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

# Examples of "crushed" matter

## Earth matter

$Earth\ mass: M_{\oplus} = 5.9722 \times 10^{24}\ kg. \simeq 6.0 \cdot 10^{24}\ kg.$ Density $\rho_{\oplus} = ??$

$Earth\ radius: R_{\oplus} = 6.371 \cdot 10^{6}\ m \simeq 6.4 \cdot 10^{6}\ m \quad Earth\ volume: (4\pi / 3) R_{\oplus}^{3} \simeq 4 \cdot 262 \cdot 10^{18} \sim 10^{21}\ m^{3}$

$(6.4)^3 \sim 262$ and $(4\pi/3)262 = 1089 \sim 10^3$

# Examples of "crushed" matter

## Earth matter

Earth mass : $M_{\oplus} = 5.9722 \times 10^{24} kg. \simeq 6.0 \cdot 10^{24} kg.$ Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 kg/m^3$

Earth radius : $R_{\oplus} = 6.371 \cdot 10^6 m \simeq 6.4 \cdot 10^6 m$ Earth volume : $(4\pi / 3) R_{\oplus}^3 \simeq 4 \cdot 262 \cdot 10^{18} \sim 10^{21} m^3$

$(6.4)^3 \sim 262$ and $(4\pi/3)260 = 1089 \sim 10^3$

# *Examples of "crushed" matter*

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Density of solid Fe=$7.9 \cdot 10^3$kg/m$^3$
Density of liquid Fe=$6.9 \cdot 10^3$kg/m$^3$

# *Examples of "crushed" matter*

## *Earth matter*

*Earth mass* : $M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^{3} kg/m^{3}$

*Earth radius* : $R_{\oplus} = 6.371 \cdot 10^{6}\,m \simeq 6.4 \cdot 10^{6}\,m$ *Earth volume* : $(4\pi / 3) R_{\oplus}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^{3}$

## *Nuclear matter*

*Nucleon mass* $= 1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

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$36\pi = 113 \sim 10^{2}$

# *Examples of "crushed" matter*

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Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* $(10^{12})$ kilograms in the size of a fingertip (1cc).

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300 m$ would approach neutron-star density.

# *Examples of "crushed" matter*

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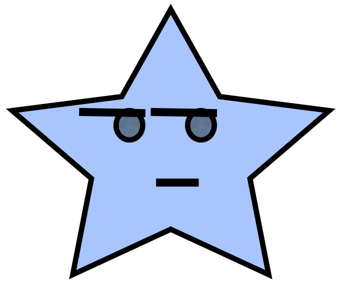
## *Introducing the "Neutron starlet"*

*1* $cm^{3}$ *of nuclear matter: mass*$= 10^{12}\, kg.$



# Examples of "crushed" matter

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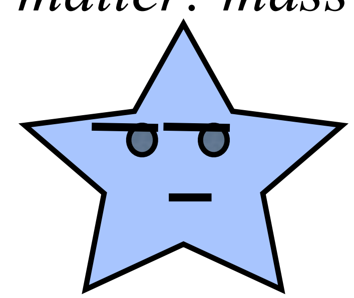
## Introducing the "Neutron starlet"
*1 cm³ of nuclear matter: mass=* $10^{12}$ *kg.*



## Introducing the "**Black Hole Earth**"
*Suppose Earth is crushed so that its surface escape velocity is the speed of light* $c \cong 3.0 \cdot 10^8 m/s$.   $c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$$V_{escape} = \sqrt{(2GM/R_\otimes)}$$

*( from p. 49 )*

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

# Examples of "crushed" matter

## Earth matter

*Earth mass* : $M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0\cdot 10^{24}\,kg.$ Density $\rho_{\oplus} \sim 6.0\cdot 10^{24\text{-}21} \sim 6\cdot 10^{3} kg/m^{3}$

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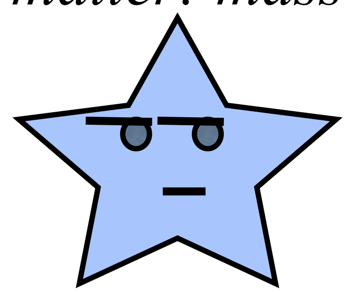

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$$V_{escape} = \sqrt{(2GM/R_{\otimes})}$$

*( from p. 49 )*

$$c = \sqrt{(2GM/R_{\otimes})}$$

$G = 6.67384(80)\cdot 10^{-11} Nm^{2}/C^{2} \sim (2/3)10^{-10}$

$R_{\otimes} = 2GM/c^{2} = 8.9mm \sim 1cm$ *(fingertip size!)*

→ *Introducing 2D IHO orbits and phasor geometry*

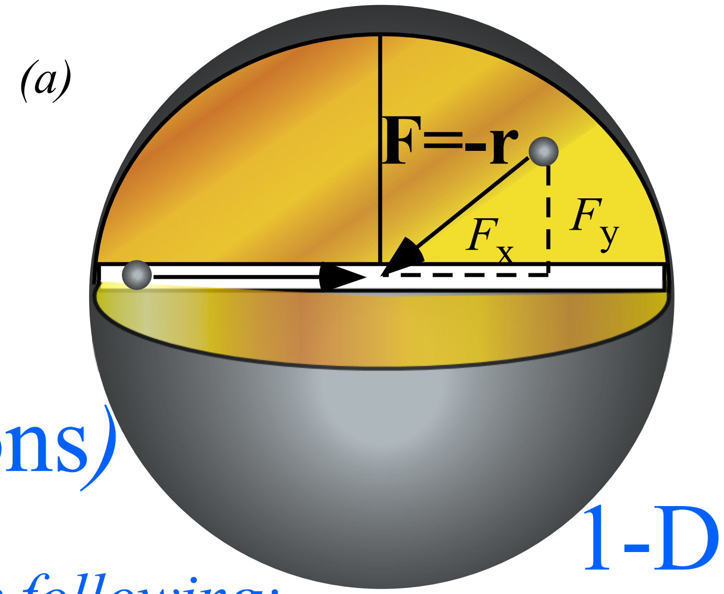
*Phasor "clock" geometry*

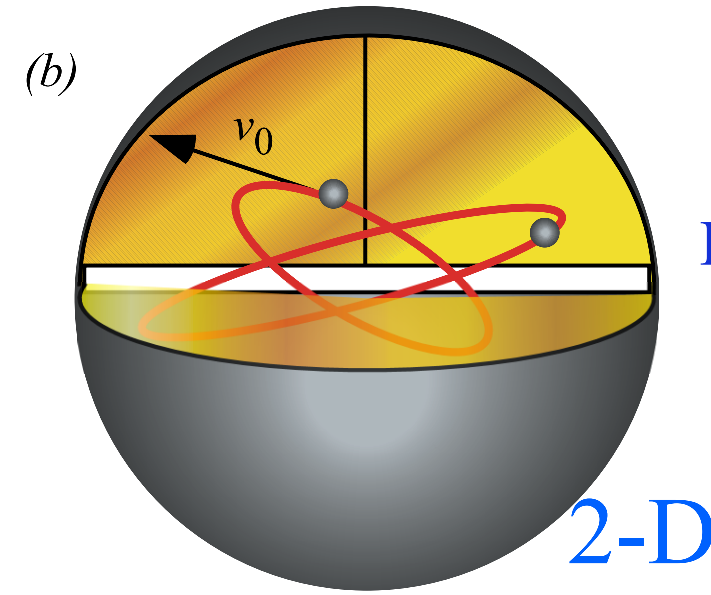
# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*

<u>*I.H.O. Force law*</u>

*F =-x* (1-Dimension)

**F** =-**r** (2 or 3-Dimensions)



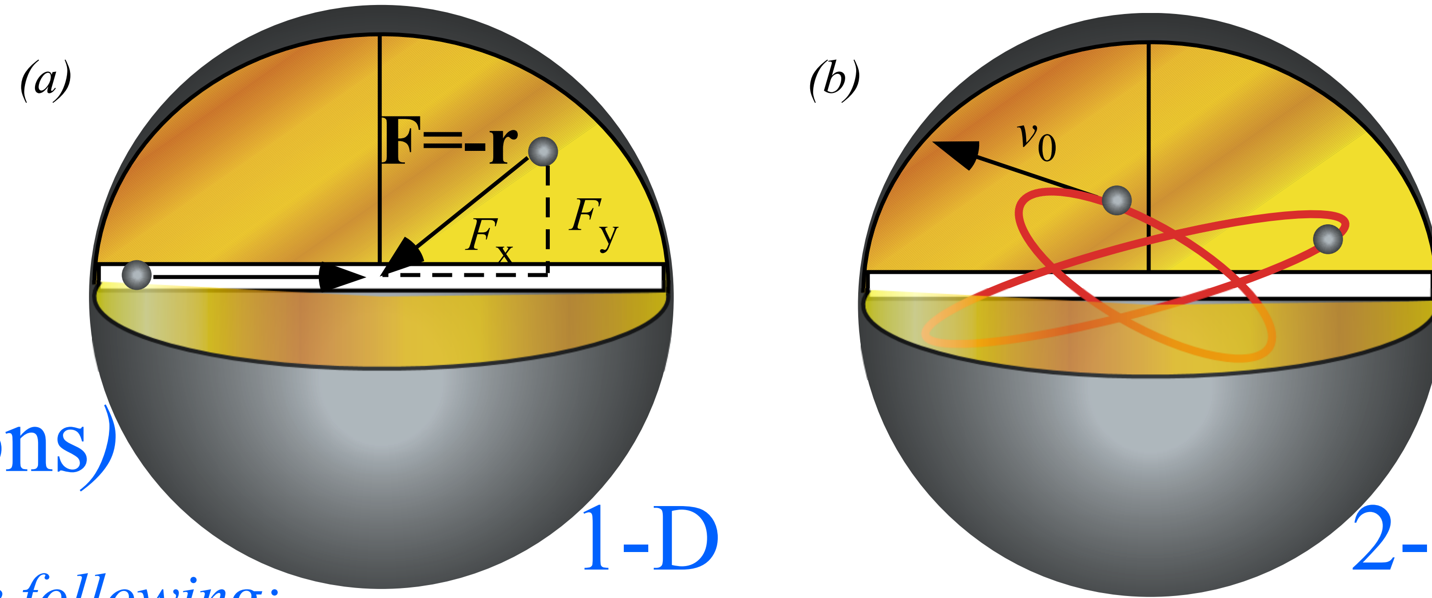

Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

*Each dimension x, y, or z obeys the following:*

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*

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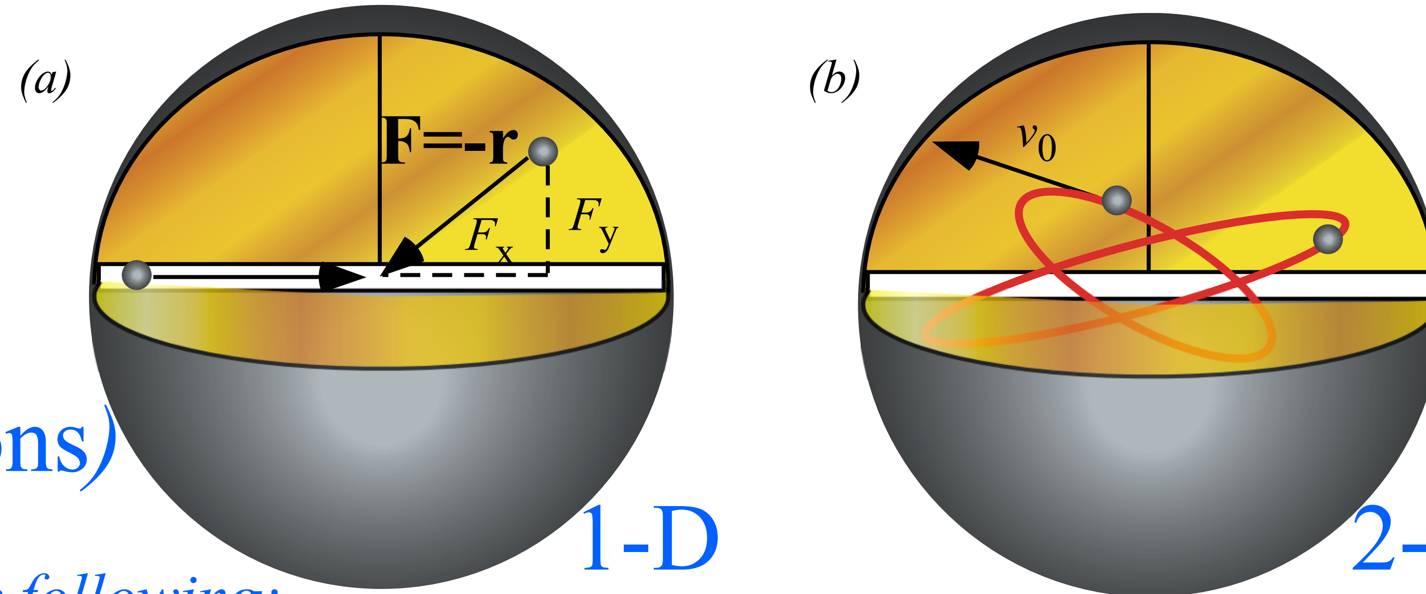
*Equations for x-motion [x(t) and $v_x$=v(t)] are given first. They apply as well to dimensions [y(t) and $v_y$=v(t)] and [z(t) and $v_z$=v(t)] in the ideal* <u>*isotropic*</u> *case.*

# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*




Unit 1
Fig. 9.10

1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

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$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

*Another example of the old "scale-a-circle" trick...*

$$Let: (1)\ v = \sqrt{2E/m}\cos\theta, \quad and: (2)\ x = \sqrt{2E/k}\sin\theta$$

# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*

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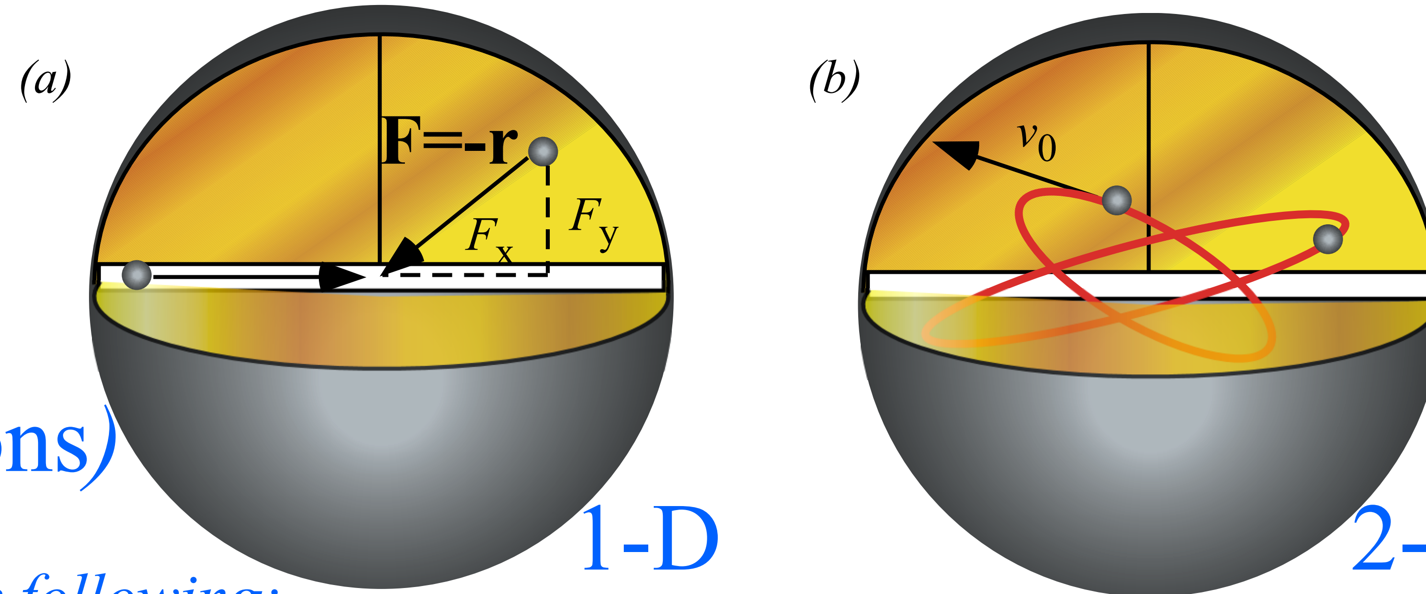


Unit 1
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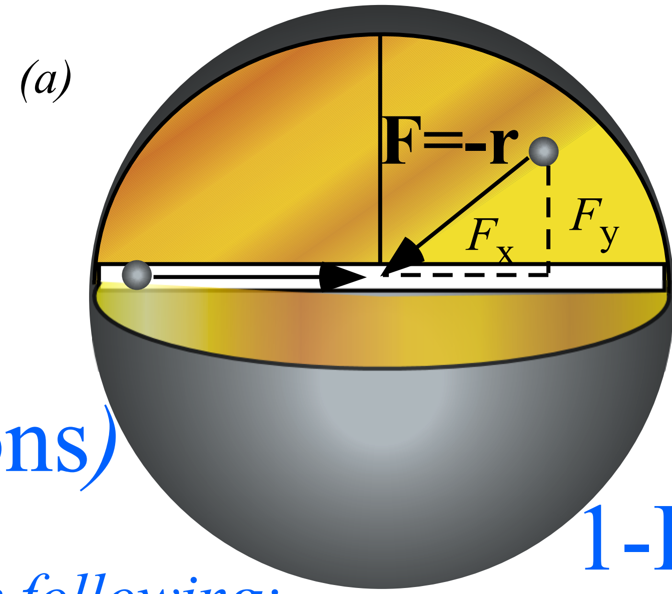
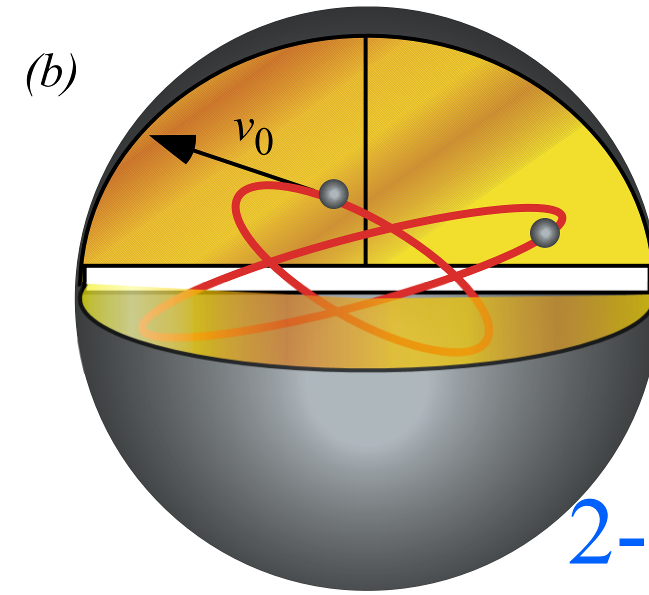
$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}\frac{dx}{d\theta} = \omega}_{by\ def.\ (3)}\frac{dx}{d\theta}$$

# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*




1-D




Unit 1
Fig. 9.10

2-D or 3-D

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## *I.H.O. Force law*

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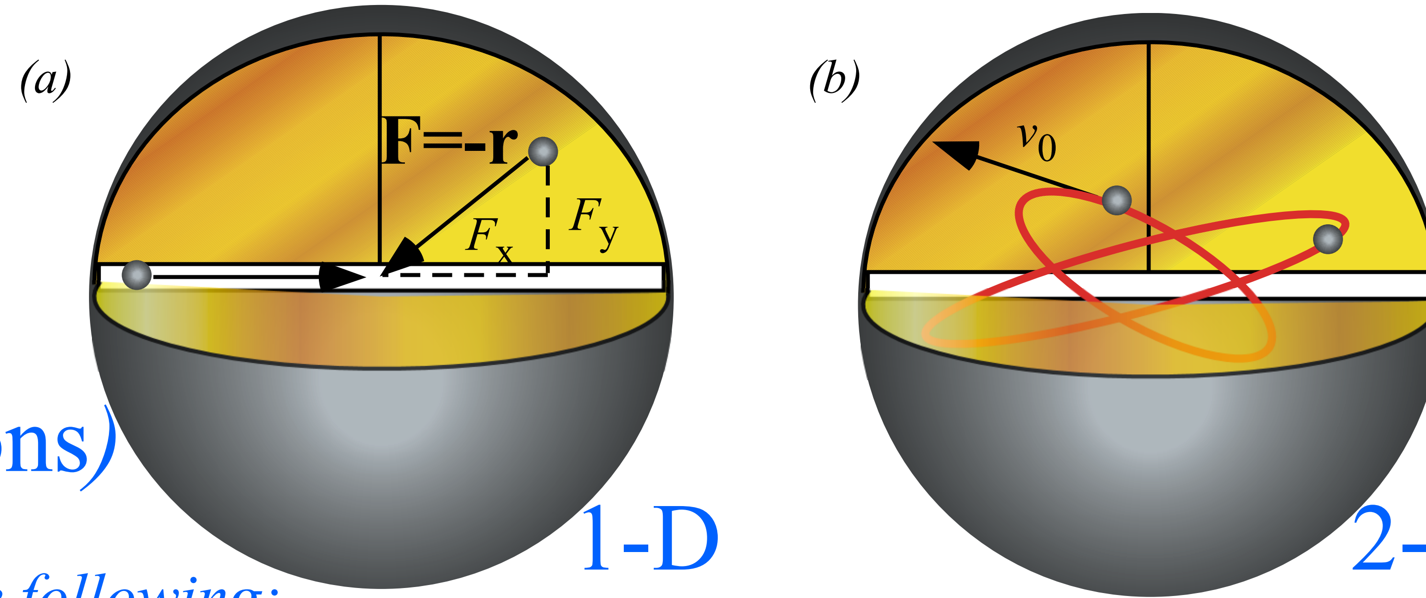
$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by (1) — by def. (3) — by (2)

# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*




Unit 1
Fig. 9.10

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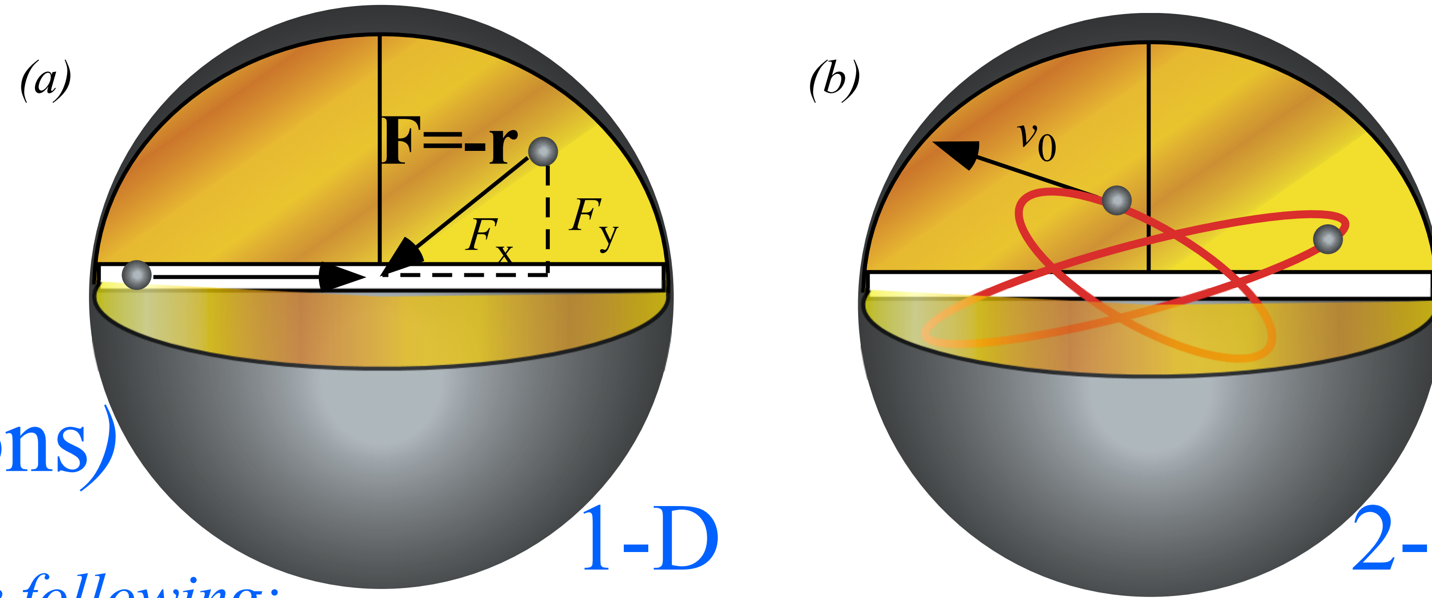
(by (1); by def. (3); by (2))

by def. (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

divide this by (1)

# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*




Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

## *I.H.O. Force law*

*F =-x* (1-Dimension)

**F** =-**r** (2 or 3-Dimensions)

*Each dimension x, y, or z obeys the following:*

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

*Equations for x-motion [x(t) and $v_x$=v(t)] are given first. They apply as well to dimensions [y(t) and $v_y$=v(t)] and [z(t) and $v_z$=v(t)] in the ideal isotropic case.*

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

*Another example of the old "scale-a-circle" trick...*

$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta \qquad def.\ (3)\ \omega = \frac{d\theta}{dt}$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

*by* (1) — *by def.* (3) — *by* (2)

*by def.* (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

*divide this by* (1)

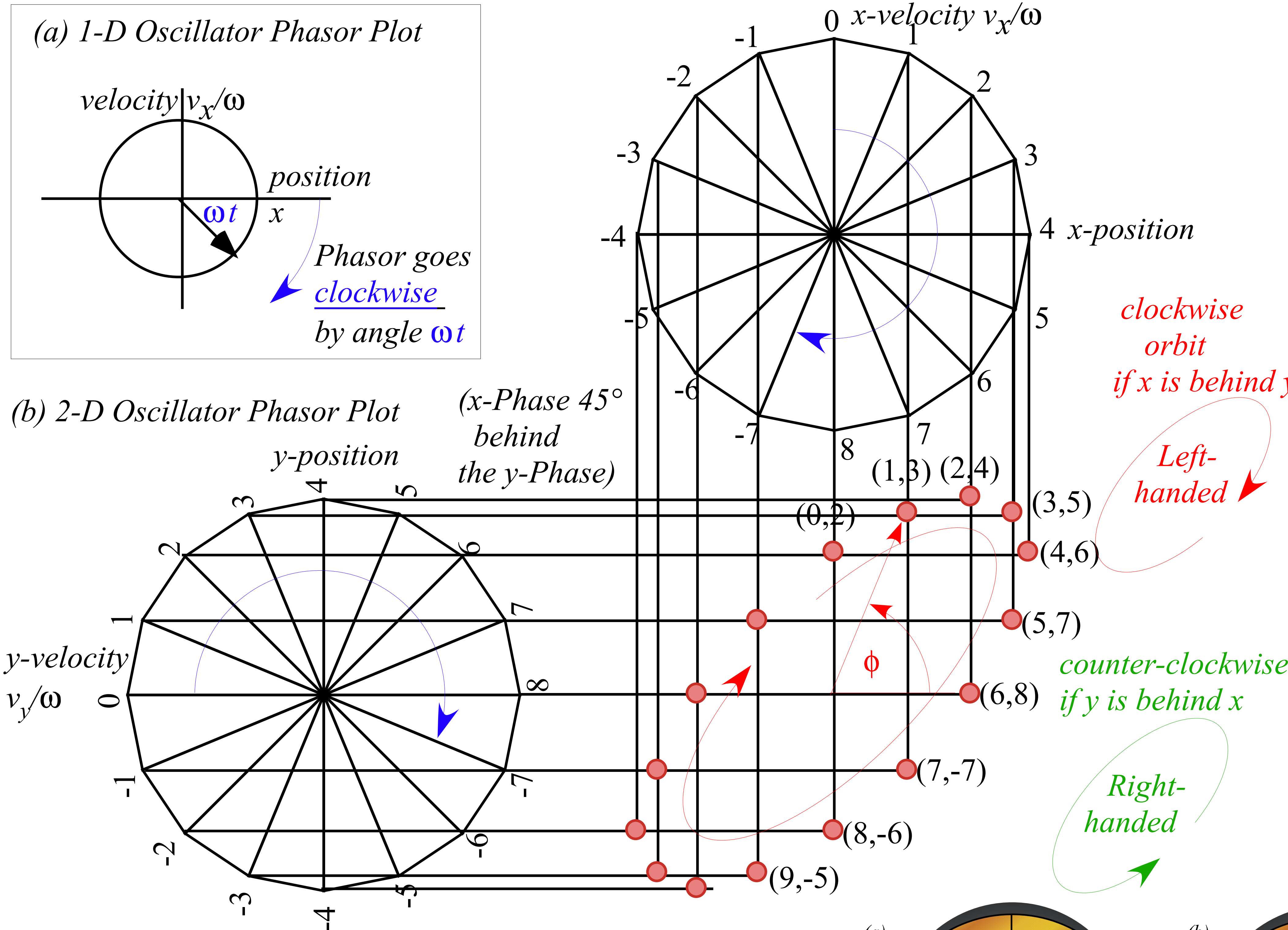
*by integration given constant ω:*

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

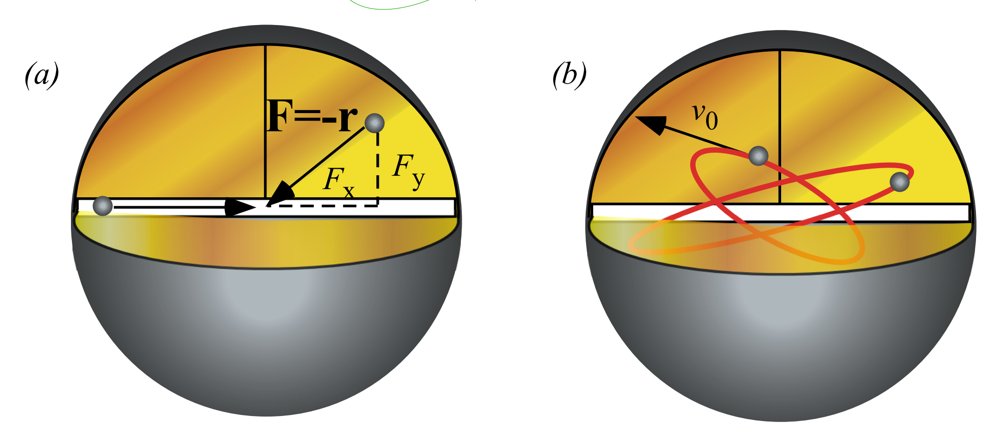
*Introducing 2D IHO orbits and phasor geometry*

→ *Phasor "clock" geometry*

# *Isotropic Harmonic Oscillator phase dynamics in uniform-body*



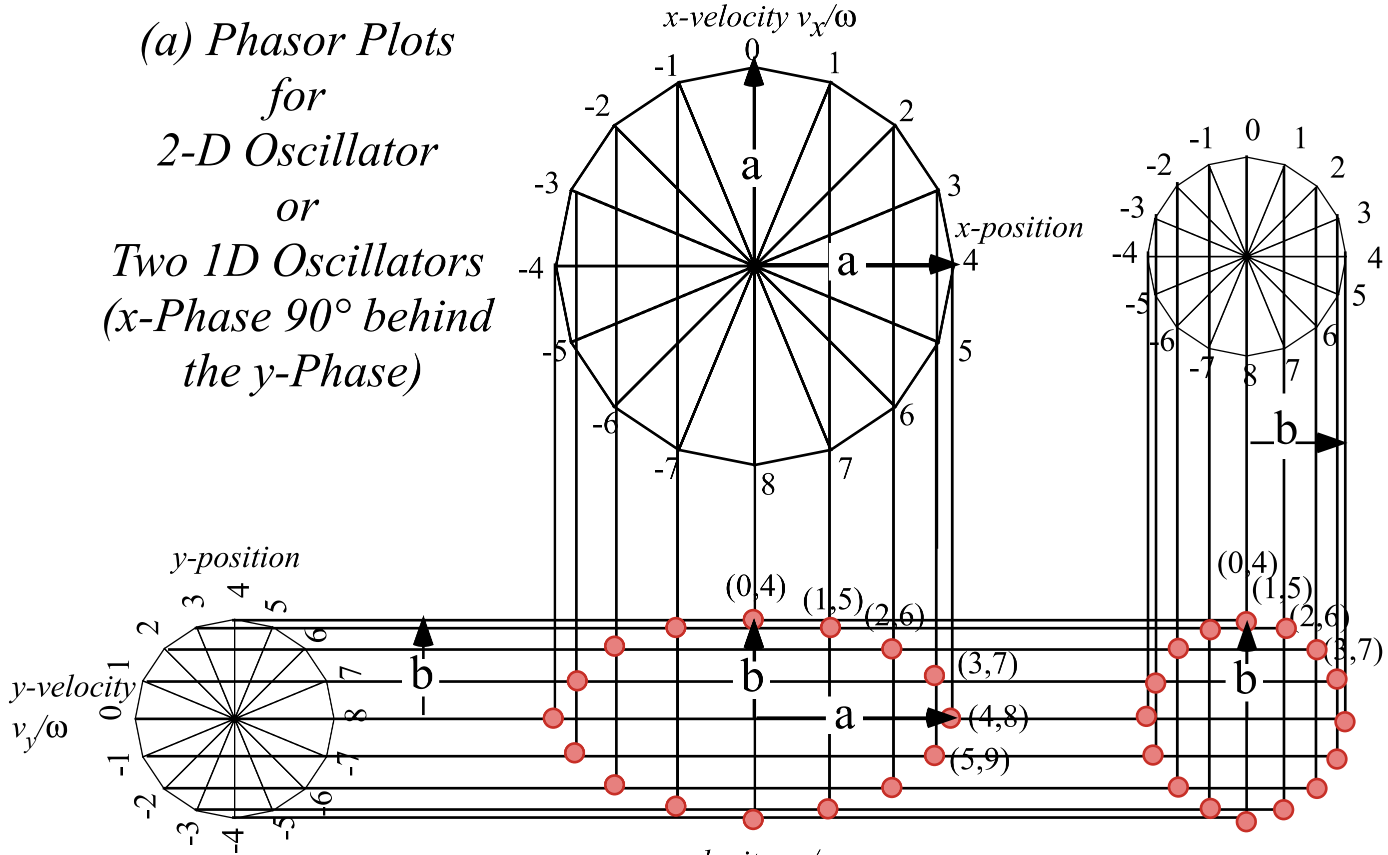

Unit 1
Fig. 8.10




RelaWavity web simulation
*Contact ellipsometry*

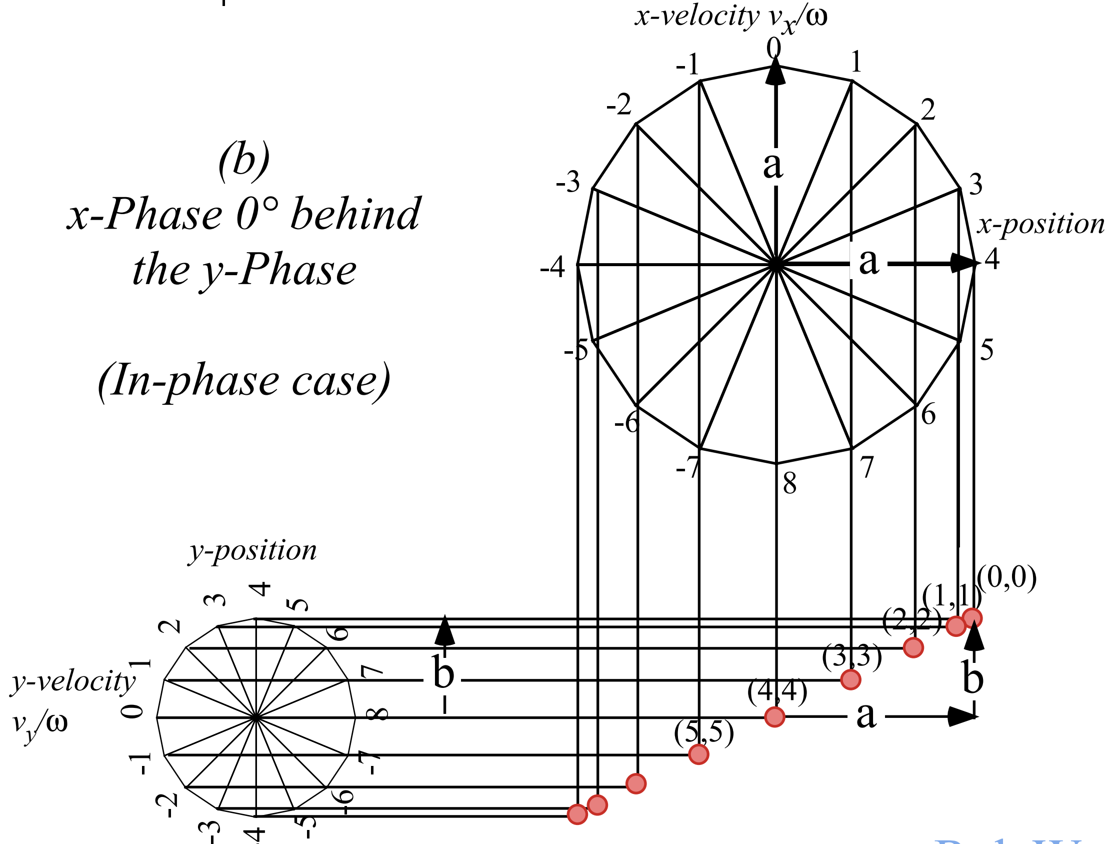
# Unit 1
# Fig. 9.12

*(a) Phasor Plots for 2-D Oscillator or Two 1D Oscillators (x-Phase 90° behind the y-Phase)*




*(b) x-Phase 0° behind the y-Phase*

*(In-phase case)*




*These are more generic examples with radius of x-phasor differing from that of the y-phasor.*

[RelaWavity web simulation](#)
[*Contact ellipsometry*](#)