Lecture 12
Thur. 2.25.2016

Kepler Geometry of IHO *(Isotropic Harmonic Oscillator)* *Elliptical Orbits*

(Ch. 8 and Ch. 9 of Unit 1)

"Sophomore-Physics-Earth" models: 3 key energy "steps" and 4 key energy "levels"
"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"
Earth matter vs nuclear matter vs Schwarzchild singular matter:
Introducing the "neutron starlet" *("fingertip physics")*
*Fantasizing a completely crushed "***Black-Hole-Earth***"*
Introducing Isotropic Harmonic Oscillator (IHO) energy and frequency relations
Constructing 2D-IHO orbits using Kepler anomaly plots
Mean-anomaly and eccentric-anomaly geometry with web-app animation
Calculus and vector geometry of IHO orbits
Constructing 2D-IHO orbits using orbital phasor-clock plots
Phasor geometry of coordinate (x,y) and velocity (V_x,V_y) space with web-app animation

Kepler "laws" (Some that apply to all central (isotropic) F(r) force fields)
Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ *(Derived here)*
Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$ *(Derived later)*
Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$ *(Derived here)*
Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$ *(Derived later)*

Link ⇒ BoxIt simulation of IHO orbits
Link → IHO orbital time rates of change
Link → IHO Exegesis Plot

Geometry of idealized "Sophomore-physics Earth"

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

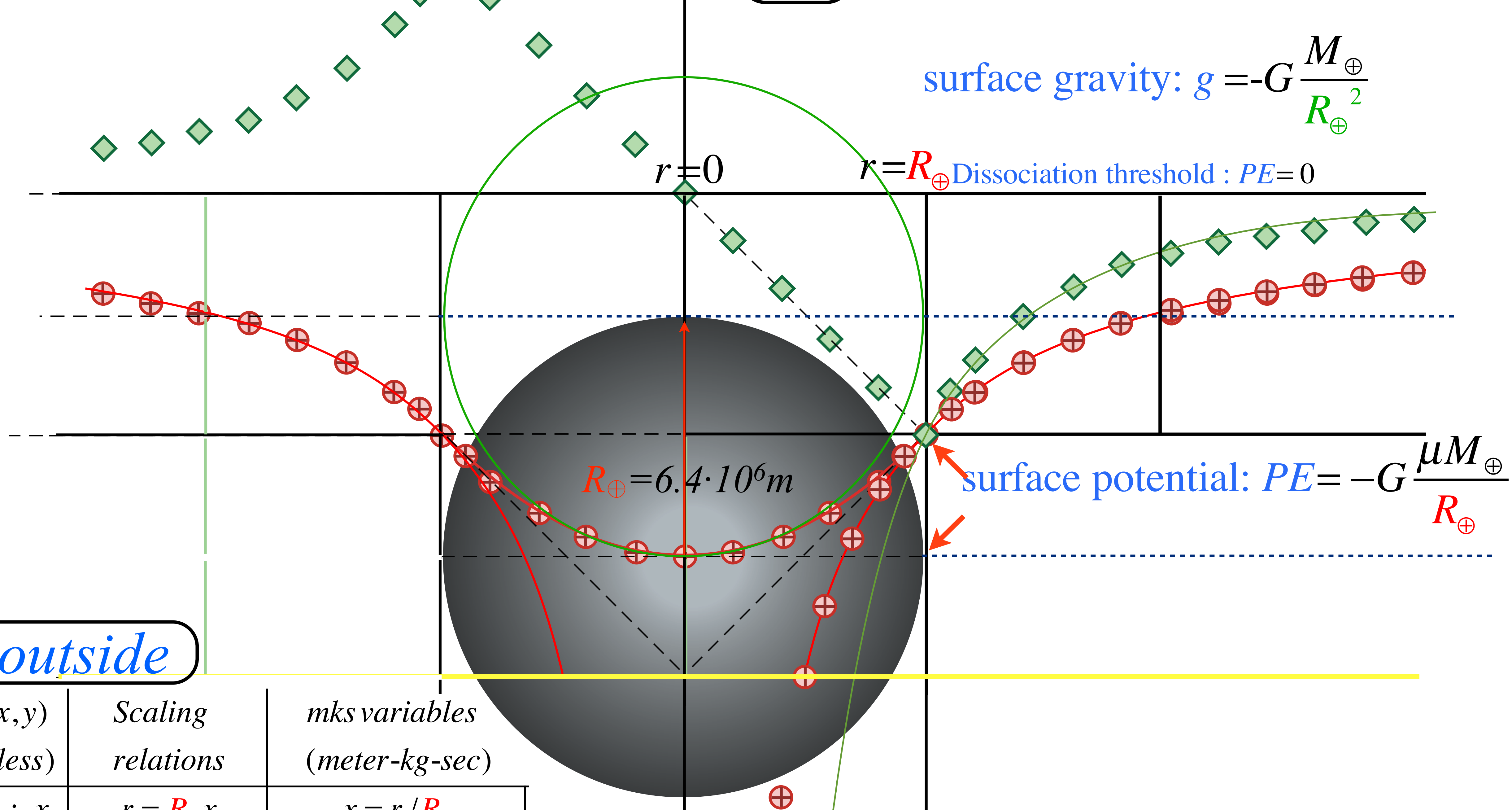
Contact-geometry of potential curve(s)

→ *"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

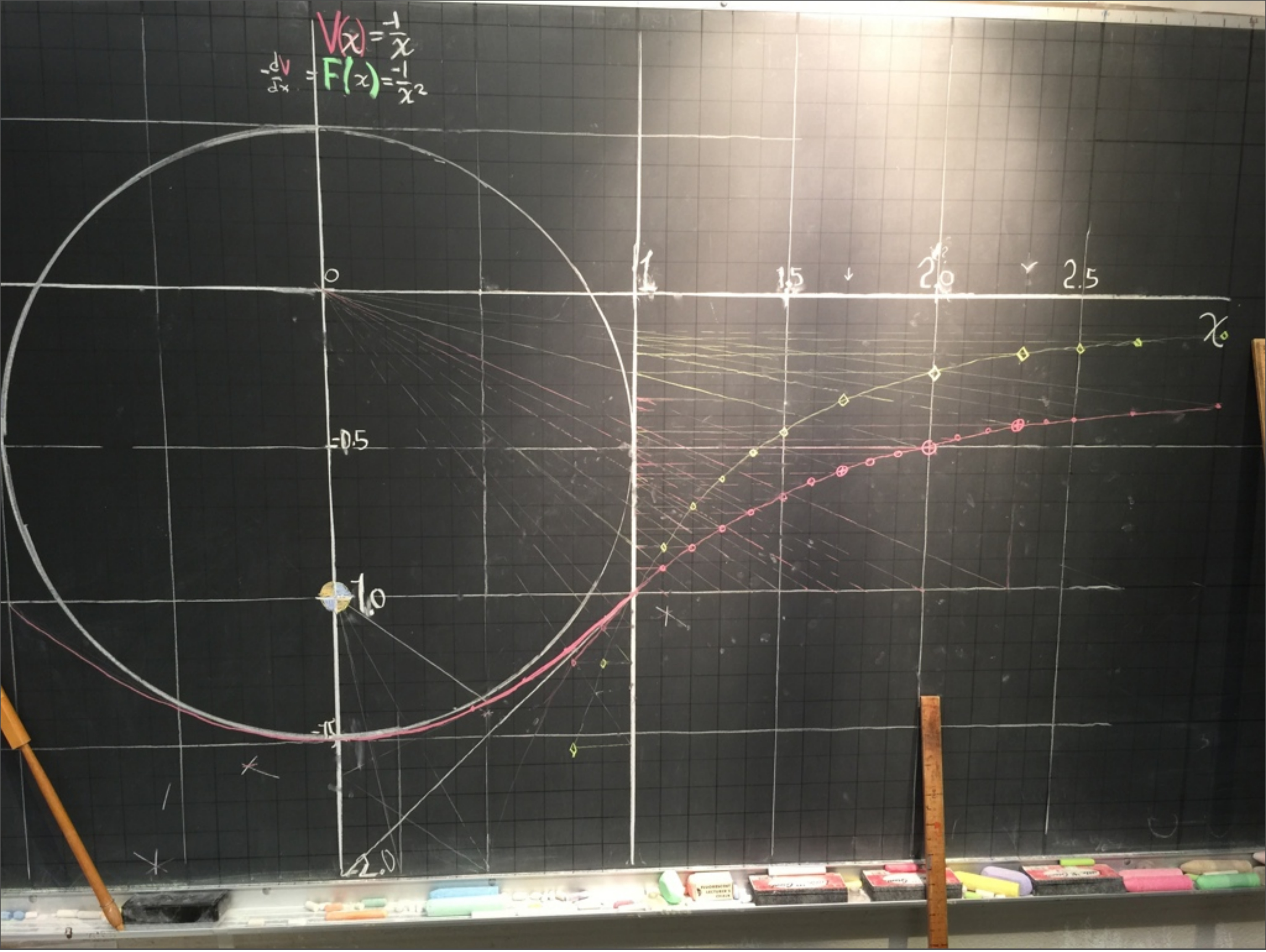
Earth matter vs nuclear matter:

*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

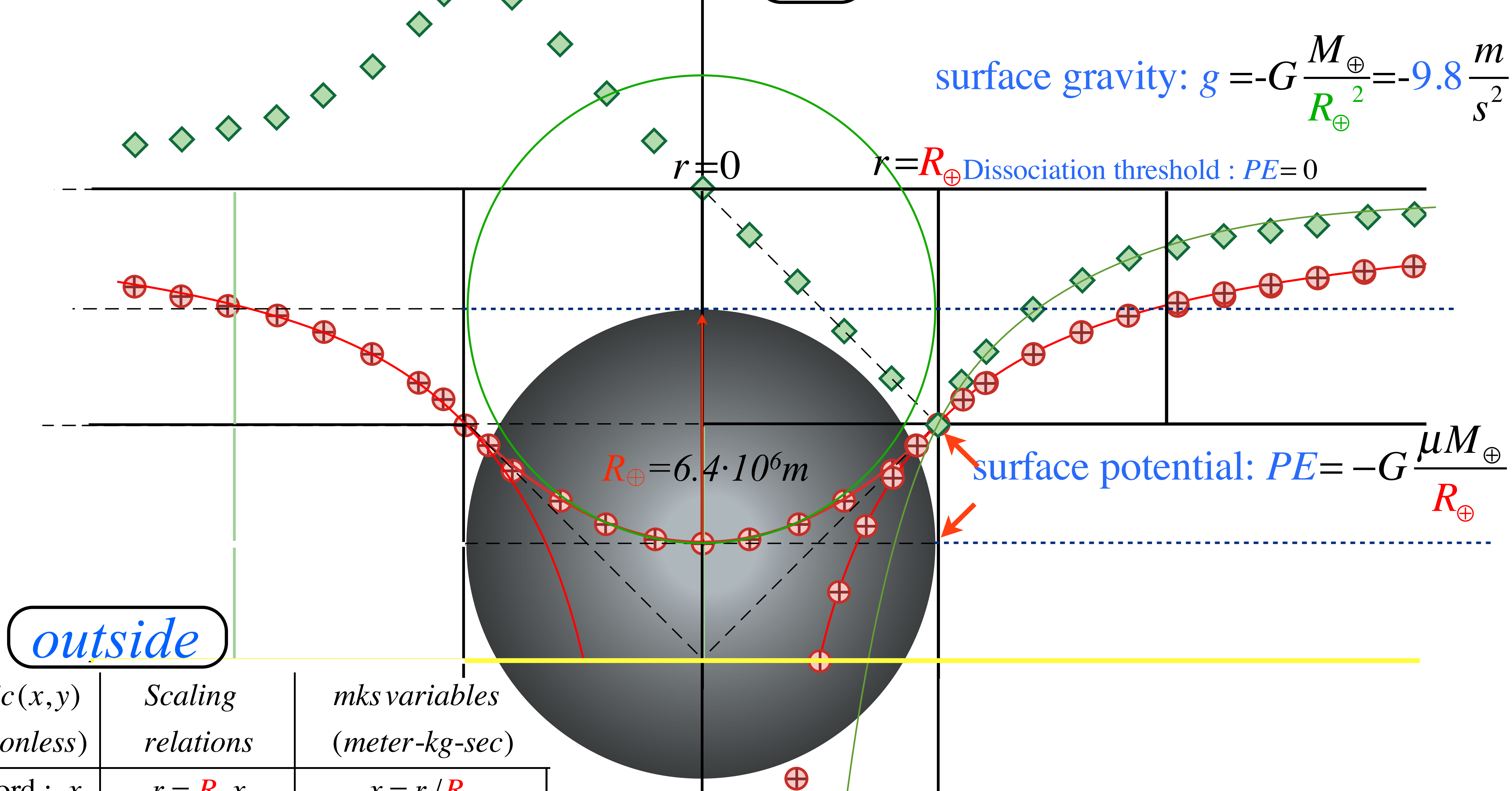
Sophomore-physics-Earth inside and out:



Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)
space coord.: x	$r = R_\oplus x$	$x = r / R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\dfrac{GM_\oplus\mu}{r} = -\dfrac{GM_\oplus\mu}{R_\oplus}\dfrac{1}{x}$

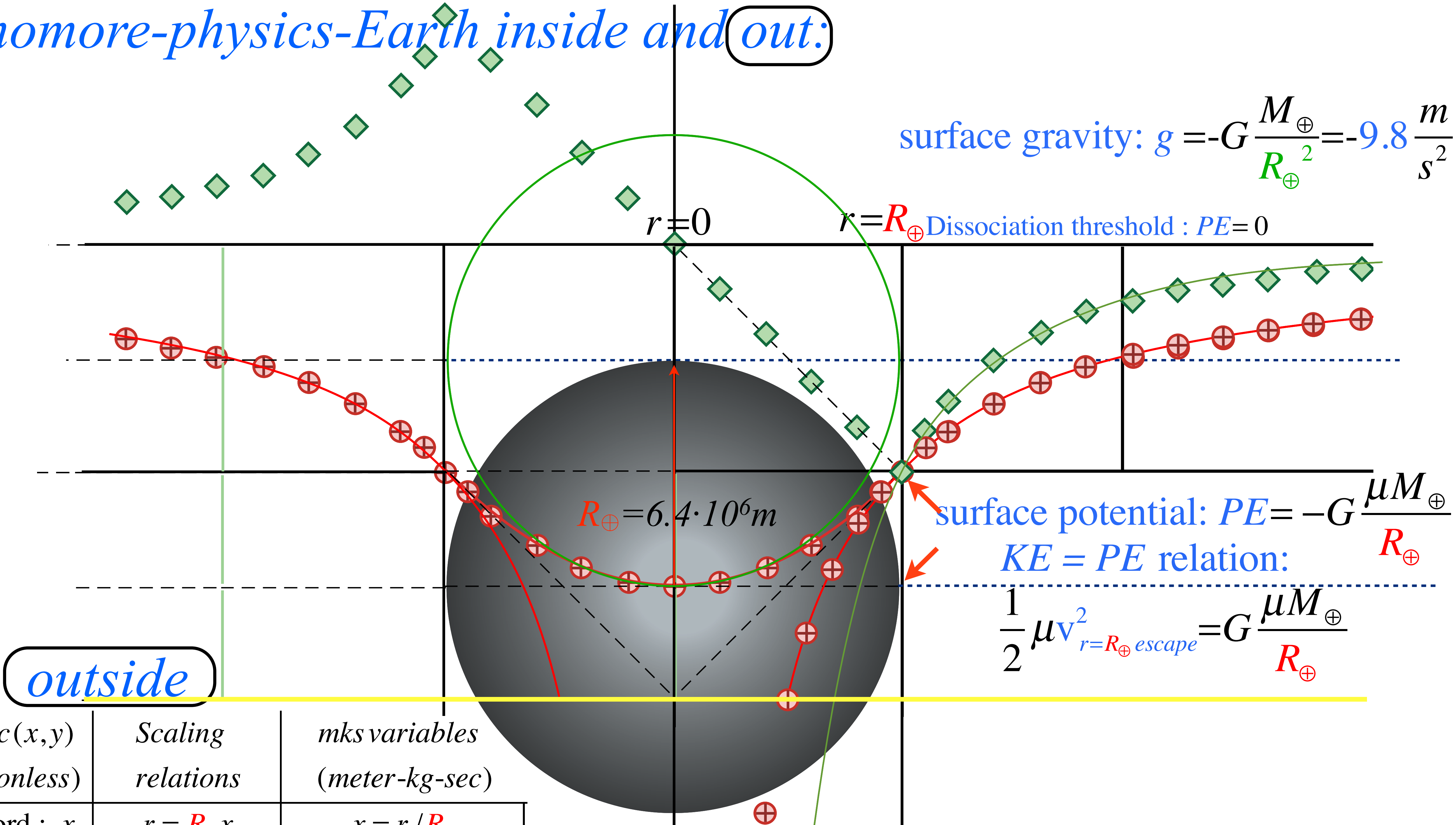

V(x) = -1/x
-dV/dx = F(x) = -1/x²
0
1
1.5
2.0
2.5
x
-0.5
-1.0
-1.5
-2.0

Sophomore-physics-Earth inside and out:



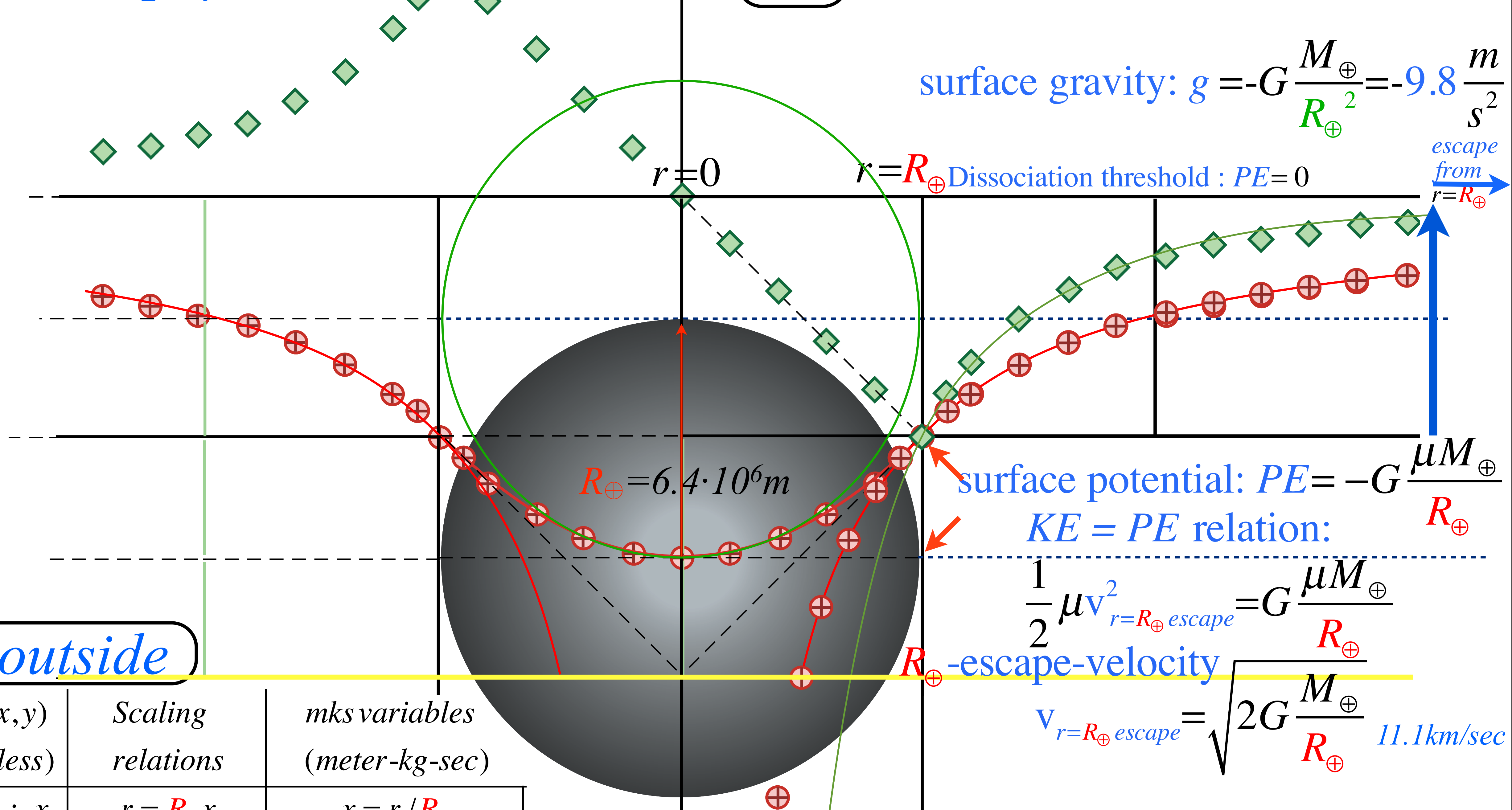
Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \ge 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\dfrac{GM_\oplus\mu}{r} = -\dfrac{GM_\oplus\mu}{R_\oplus}\dfrac{1}{x}$

Sophomore-physics-Earth inside and out:



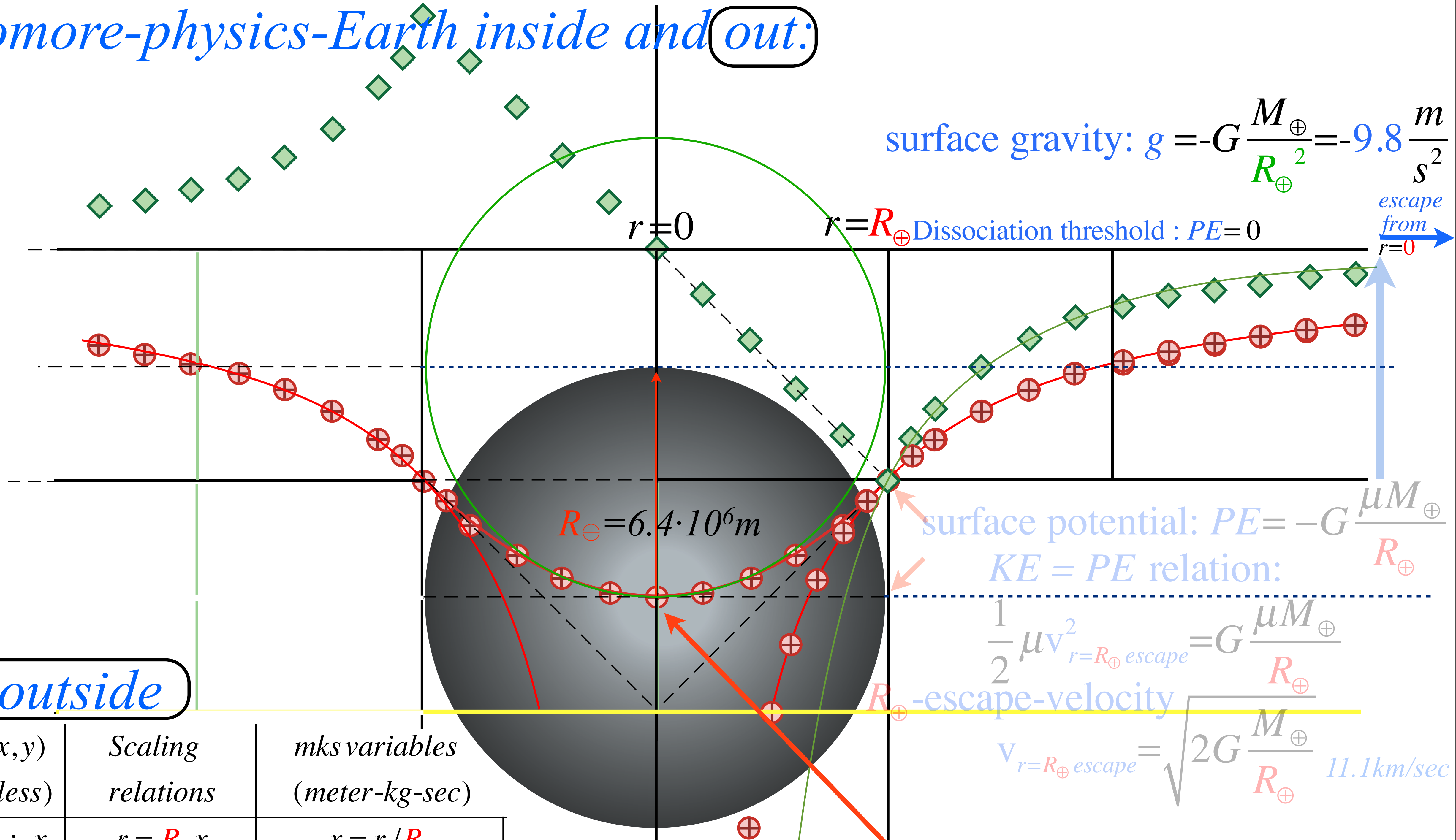
Geometric (x,y) (*Dimensionless*)	*Scaling relations*	*mks variables* (*meter-kg-sec*)
space coord.: x	$r = R_\oplus x$	$x = r / R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM_\oplus \mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM_\oplus \mu}{r} = -\frac{GM_\oplus \mu}{R_\oplus}\frac{1}{x}$

Sophomore-physics-Earth inside and out:



Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM_\oplus \mu}{R_\oplus} y^{PE}$	$PE^{mks}(r) = -\frac{GM_\oplus \mu}{r} = -\frac{GM_\oplus \mu}{R_\oplus}\frac{1}{x}$

Sophomore-physics-Earth inside and out:

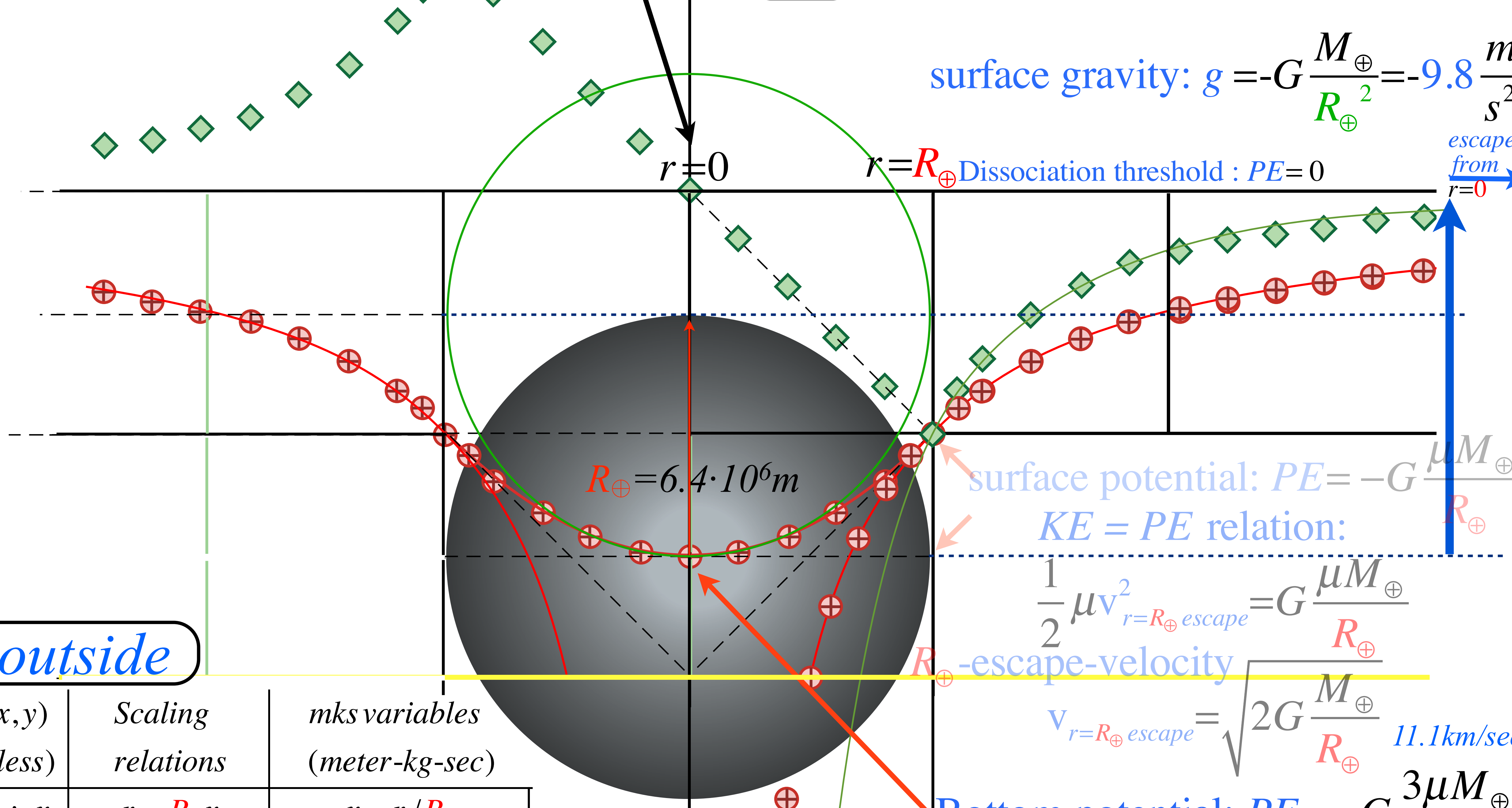


Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM_\oplus\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r) = -\frac{GM_\oplus\mu}{r} = -\frac{GM_\oplus\mu}{R_\oplus}\frac{1}{x}$

PE for $\lvert x\rvert < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$	$PE^{mks}(r) = \frac{GM_\oplus\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$

Sophomore-physics-Earth inside and out:

surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^{2}} = -9.8\,\dfrac{m}{s^2}$



surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\,escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

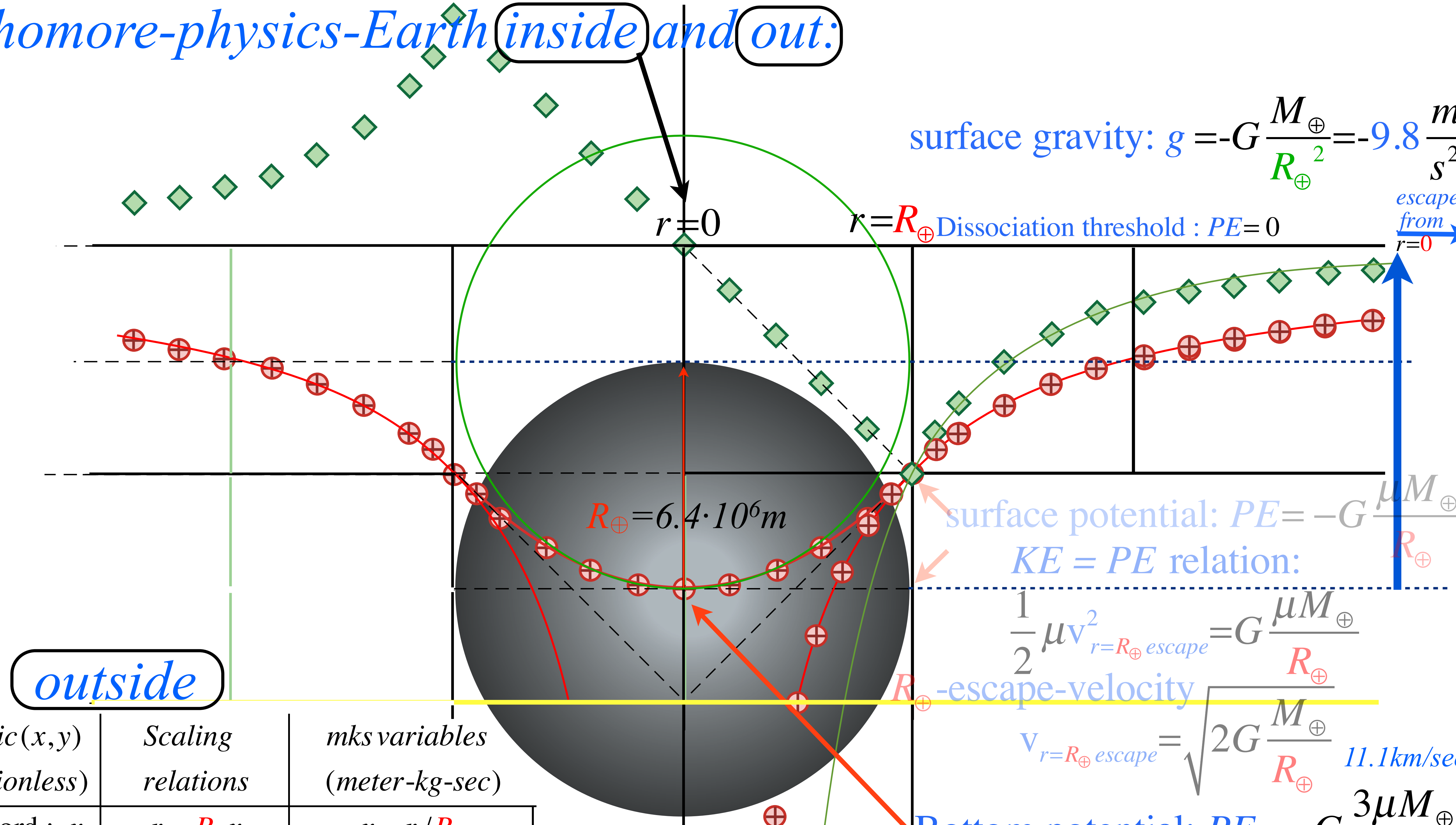
$$v_{r=R_\oplus\,escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}} \quad 11.1km/sec$$

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert\geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r) = -\dfrac{GM_\oplus\mu}{r} = -\dfrac{GM_\oplus\mu}{R_\oplus}\dfrac{1}{x}$

PE for $\lvert x\rvert<1$:	inside
$y^{PE} = \dfrac{x^2}{2} - \dfrac{3}{2}$	$PE^{mks}(r) = \boxed{\dfrac{GM_\oplus\mu}{R_\oplus}}\left(\dfrac{r^2}{2R_\oplus^2} - \boxed{\dfrac{3}{2}}\right)$

Sophomore-physics-Earth inside and out:



Bottom potential: $PE = -G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

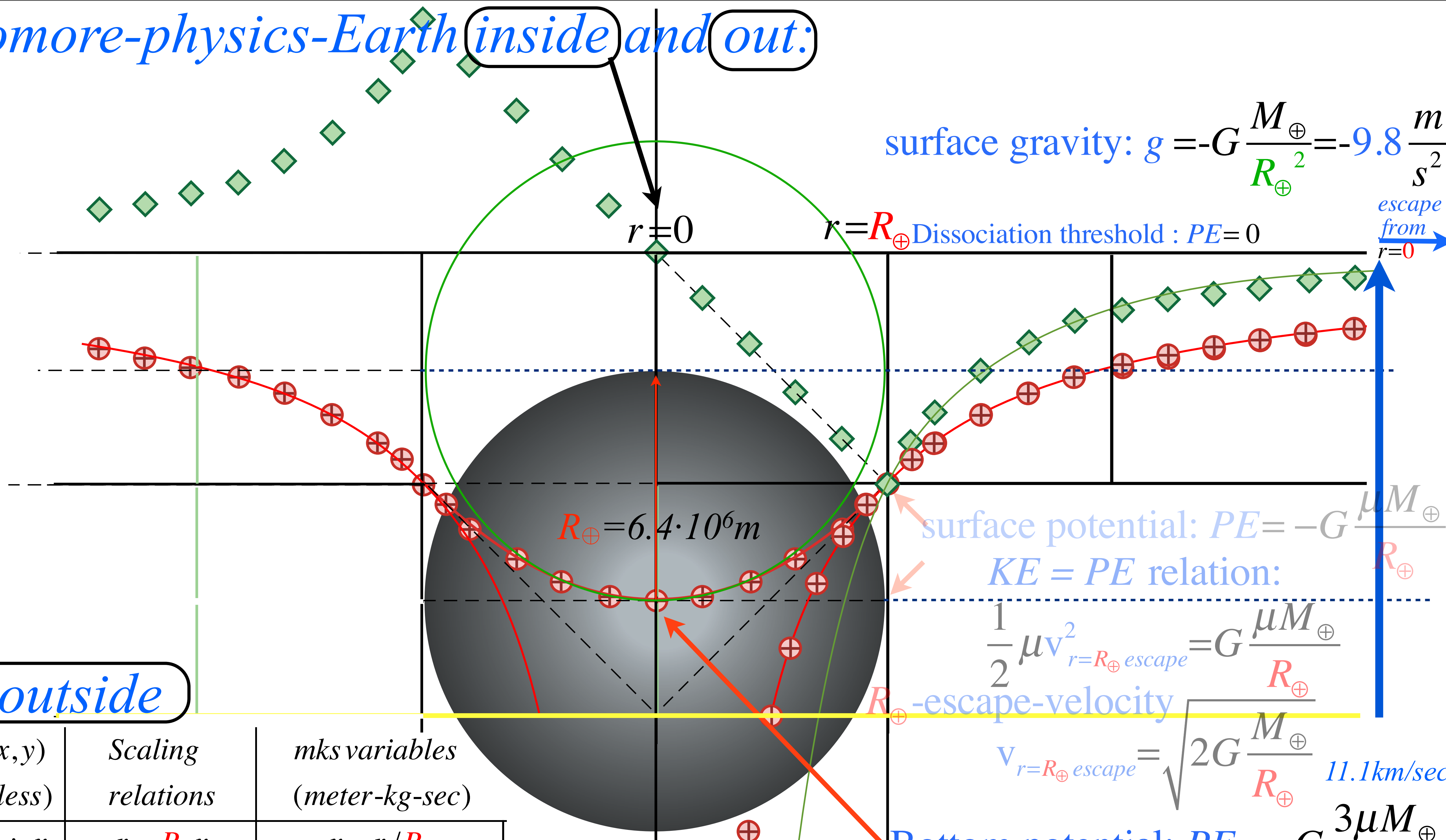
$$\frac{1}{2}\mu v^2_{bottom} = G\frac{3\mu M_\oplus}{2R_\oplus}$$

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \frac{-1}{x}$	$PE^{mks}(r) = \frac{GM_\oplus\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r) = -\frac{GM_\oplus\mu}{r} = -\frac{GM_\oplus\mu}{R_\oplus}\frac{1}{x}$

PE for $\lvert x\rvert < 1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$

$$PE^{mks}(r) = \frac{GM_\oplus\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$$

Sophomore-physics-Earth inside and out:



surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^{\,2}} = -9.8\,\dfrac{m}{s^2}$

surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\, escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\, escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v^2_{bottom} = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

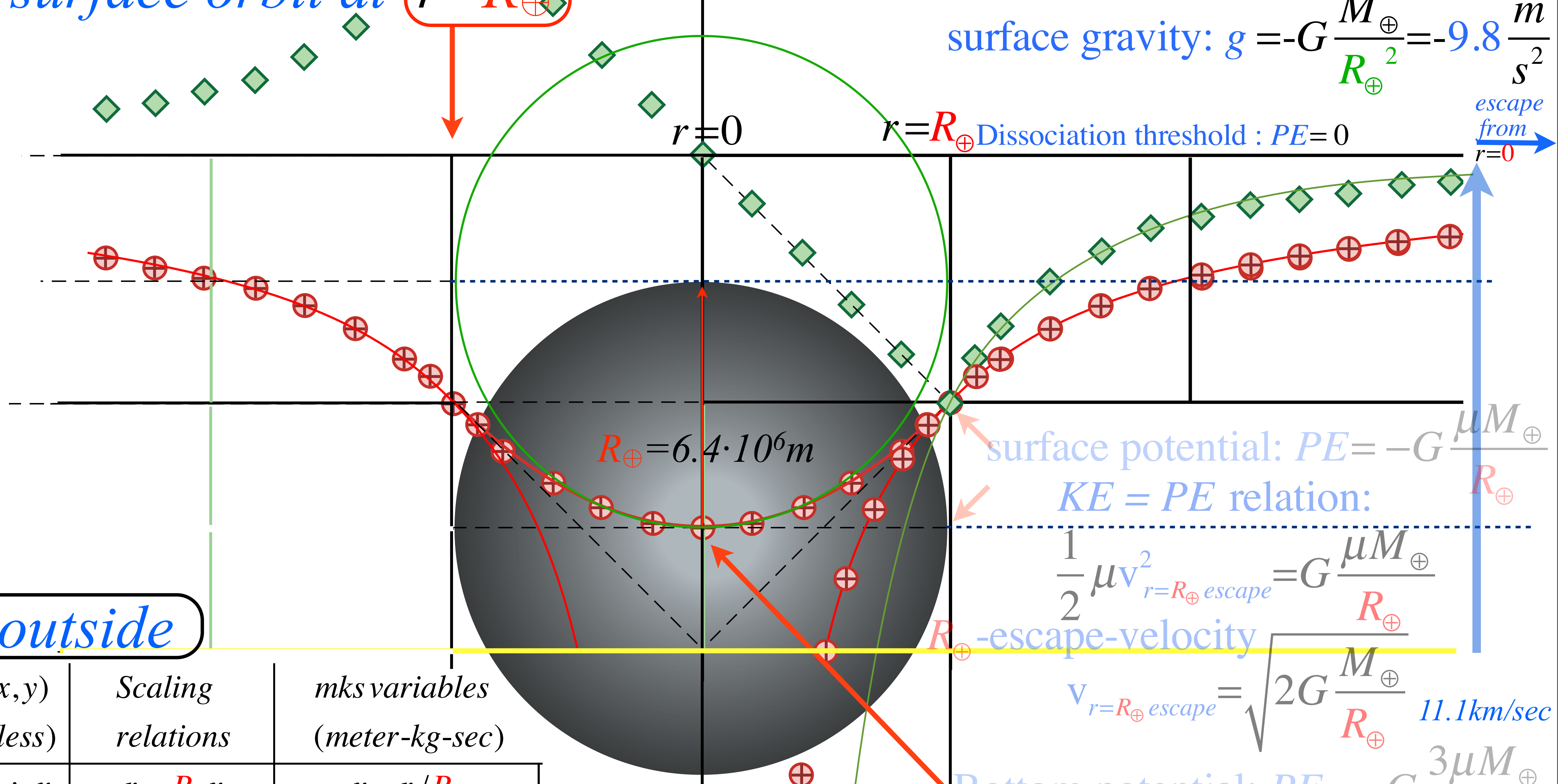
13.7km/sec

Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r) = -\dfrac{GM_\oplus\mu}{r} = -\dfrac{GM_\oplus\mu}{R_\oplus}\dfrac{1}{x}$

PE for $\lvert x\rvert < 1$: $y^{PE} = \dfrac{x^2}{2} - \dfrac{3}{2}$

$$PE^{mks}(r) = \frac{GM_\oplus\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$$

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$



surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^{\,2}} = -9.8\,\dfrac{m}{s^2}$

surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\,escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\,escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v^2_{bottom} = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Geometric (x,y) *(Dimensionless)*	*Scaling relations*	*mks variables (meter-kg-sec)*
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert\ge 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r) = -\dfrac{GM_\oplus\mu}{r} = -\dfrac{GM_\oplus\mu}{R_\oplus}\dfrac{1}{x}$
$Force$ for $\lvert x\rvert\ge 1$: $y^{Force} = \dfrac{-1}{x^2}$	$F^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus^2}y^{Force}$	$F^{mks}(r) = -\dfrac{GM_\oplus\mu}{r^2} = -\dfrac{GM_\oplus\mu}{R_\oplus^2}\dfrac{1}{x^2}$

PE for $\lvert x\rvert<1$: $y^{PE} = \dfrac{x^2}{2} - \dfrac{3}{2}$

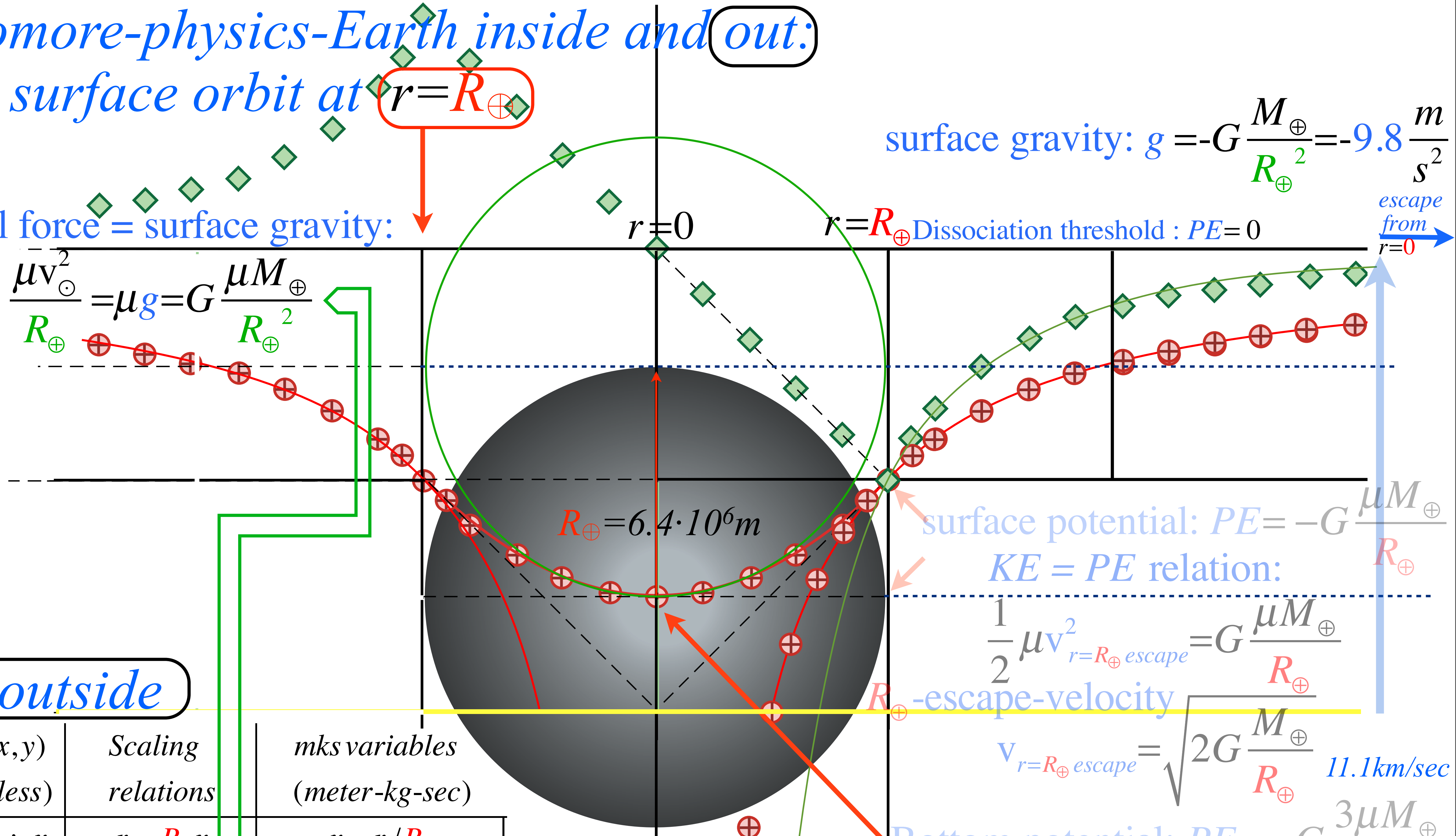
$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus}\left(\dfrac{r^2}{2R_\oplus^2} - \dfrac{3}{2}\right)$

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$

surface gravity: $g=-G\frac{M_\oplus}{R_\oplus^2}=-9.8\frac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus^2}$$



Geometric (x,y) (Dimensionless)	Scaling relations	mks variables (meter-kg-sec)		
space coord.: x	$r=R_\oplus x$	$x=r/R_\oplus$		
PE for $\lvert x\rvert\ge1$: $y^{PE}=\frac{-1}{x}$	$PE^{mks}(r)=\frac{GM_\oplus\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r)=-\frac{GM_\oplus\mu}{r}=-\frac{GM_\oplus\mu}{R_\oplus}\frac{1}{x}$	PE for $\lvert x\rvert<1$: $y^{PE}=\frac{x^2}{2}-\frac{3}{2}$	$PE^{mks}(r)=\frac{GM_\oplus\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2}-\frac{3}{2}\right)$
$Force$ for $\lvert x\rvert\ge1$: $y^{Force}=\frac{-1}{x^2}$	$F^{mks}(r)=\frac{GM_\oplus\mu}{R_\oplus^2}y^{Force}$	$F^{mks}(r)=-\frac{GM_\oplus\mu}{r^2}=-\frac{GM_\oplus\mu}{R_\oplus^2}\frac{1}{x^2}$		

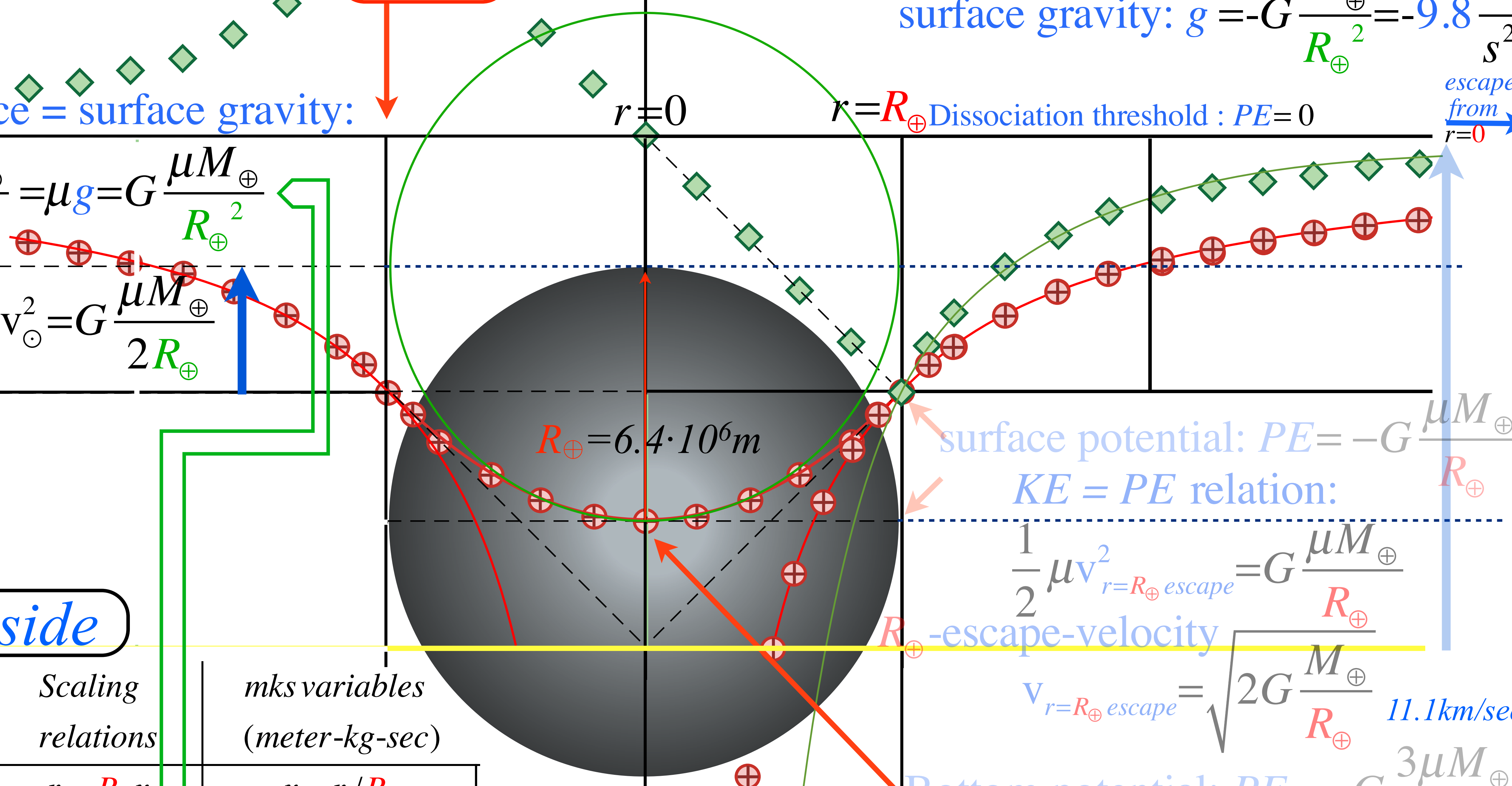
Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$

surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^{\,2}} = -9.8\,\dfrac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^{\,2}}$$

Orbit KE$=\dfrac{1}{2}\mu v_\odot^2 = G\dfrac{\mu M_\oplus}{2R_\oplus}$



surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\,escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\,escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{bottom} = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

outside

Geometric (x,y) (Dimensionless)	*Scaling relations*	*mks variables (meter-kg-sec)*
space coord.: x	$r = R_\oplus x$	$x = r/R_\oplus$
PE for $\lvert x\rvert \geq 1$: $y^{PE} = \dfrac{-1}{x}$	$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus}y^{PE}$	$PE^{mks}(r) = -\dfrac{GM_\oplus\mu}{r} = -\dfrac{GM_\oplus\mu}{R_\oplus}\dfrac{1}{x}$
Force for $\lvert x\rvert \geq 1$: $y^{Force} = \dfrac{-1}{x^2}$	$F^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus^2}y^{Force}$	$F^{mks}(r) = -\dfrac{GM_\oplus\mu}{r^2} = -\dfrac{GM_\oplus\mu}{R_\oplus^2}\dfrac{1}{x^2}$

inside

PE for $\lvert x\rvert < 1$: $y^{PE} = \dfrac{x^2}{2} - \dfrac{3}{2}$	$PE^{mks}(r) = \dfrac{GM\mu}{R_\oplus}\left(\dfrac{r^2}{2R_\oplus^2} - \dfrac{3}{2}\right)$

Sophomore-physics-Earth inside and out:

...and surface orbit at $r=R_\oplus$

outside

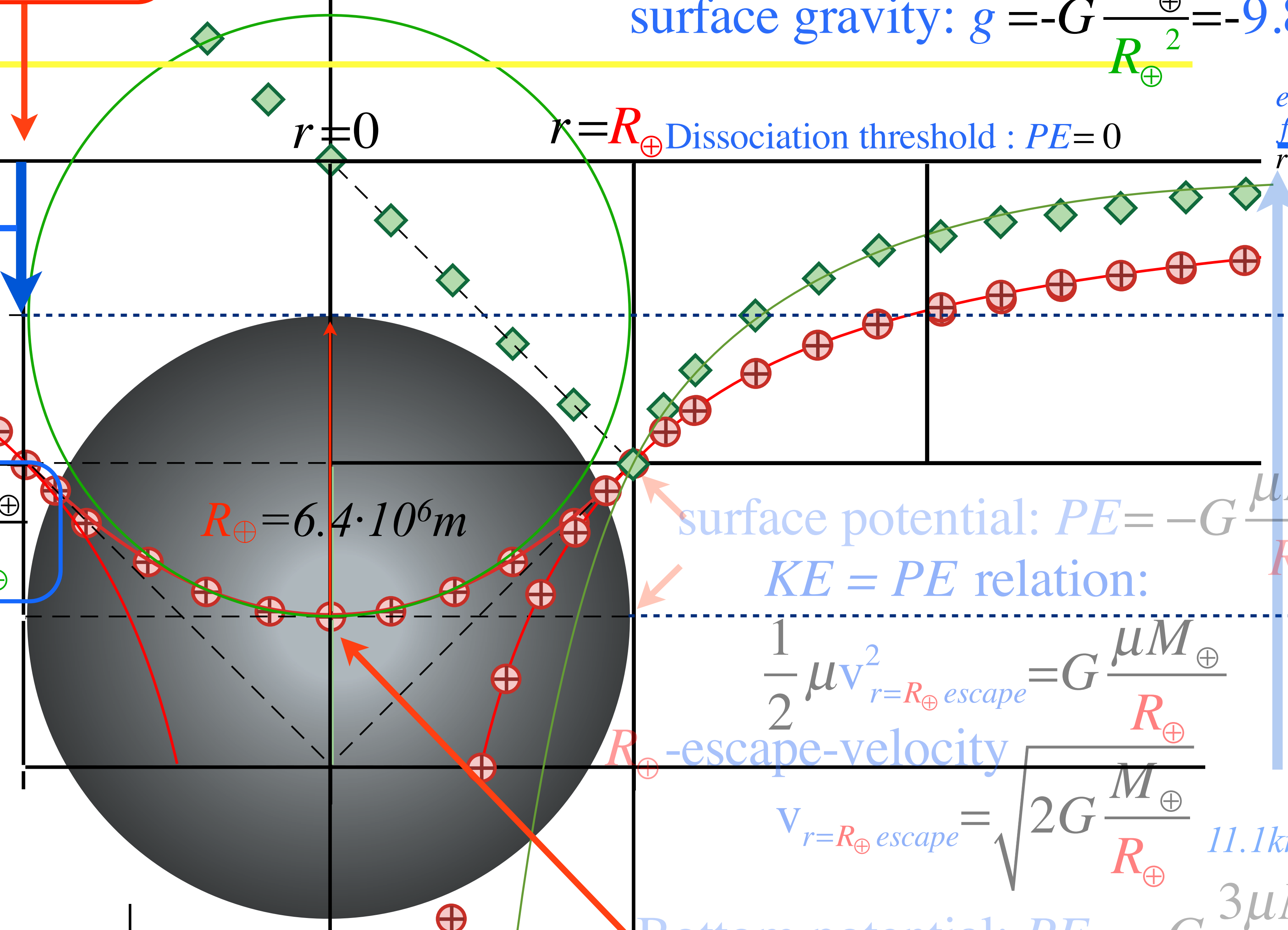
surface gravity: $g = -G\frac{M_\oplus}{R_\oplus^2} = -9.8\frac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^2}$$

$$\text{Orbit KE} = \frac{1}{2}\mu v_\odot^2 = G\frac{\mu M_\oplus}{2R_\oplus}$$

$$\text{Orbit } E_\odot^{Total} = \frac{1}{2}\mu v_\odot^2 - G\frac{\mu M_\oplus}{R_\oplus} = -G\frac{\mu M_\oplus}{2R_\oplus}$$



surface potential: $PE = -G\frac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\,escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\,escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE = -G\frac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{bottom} = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

inside

PE for $|x|<1$: $y^{PE} = \frac{x^2}{2} - \frac{3}{2}$

$$PE^{mks}(r) = \frac{GM\mu}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$$

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$ outside

surface gravity: $g=-G\dfrac{M_\oplus}{R_\oplus^{\,2}}=-9.8\dfrac{m}{s^2}$

Centifugal force = surface gravity:

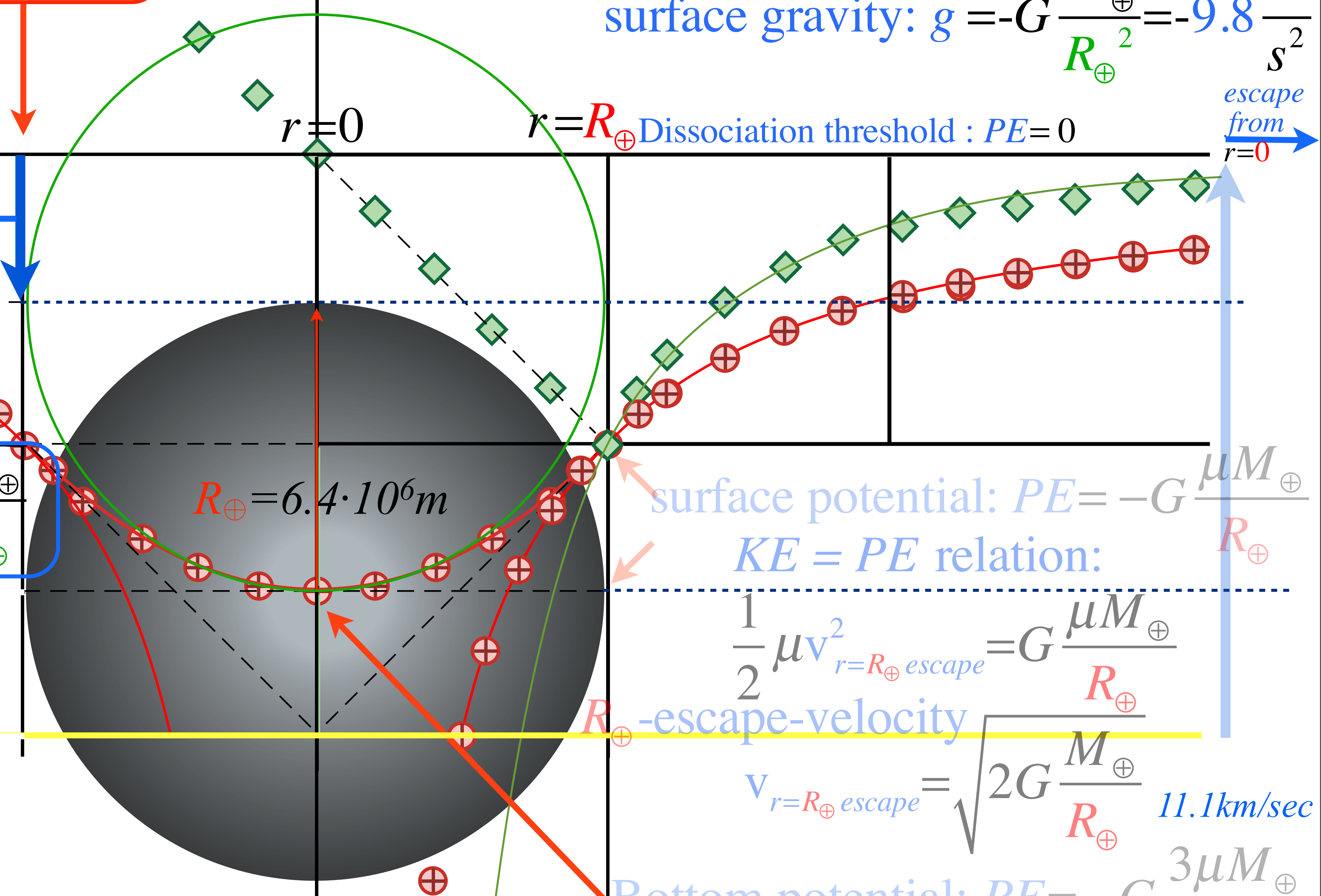
$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus^{\,2}}$$

Orbit KE$=\dfrac{1}{2}\mu v_\odot^2=G\dfrac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total}=\dfrac{1}{2}\mu v_\odot^2-G\dfrac{\mu M_\oplus}{R_\oplus}=-G\dfrac{\mu M_\oplus}{2R_\oplus}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus=G\frac{M_\oplus}{R_\oplus^2}\Rightarrow\omega_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$



surface potential: $PE=-G\dfrac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\,escape}=G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\,escape}=\sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE=-G\dfrac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{bottom}=G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Sophomore-physics-Earth inside and out: ...and surface orbit at $r=R_\oplus$ outside

surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^{\;2}} = -9.8\dfrac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^{\;2}}$$

Orbit KE $= \frac{1}{2}\mu v_\odot^2 = G\frac{\mu M_\oplus}{2R_\oplus}$

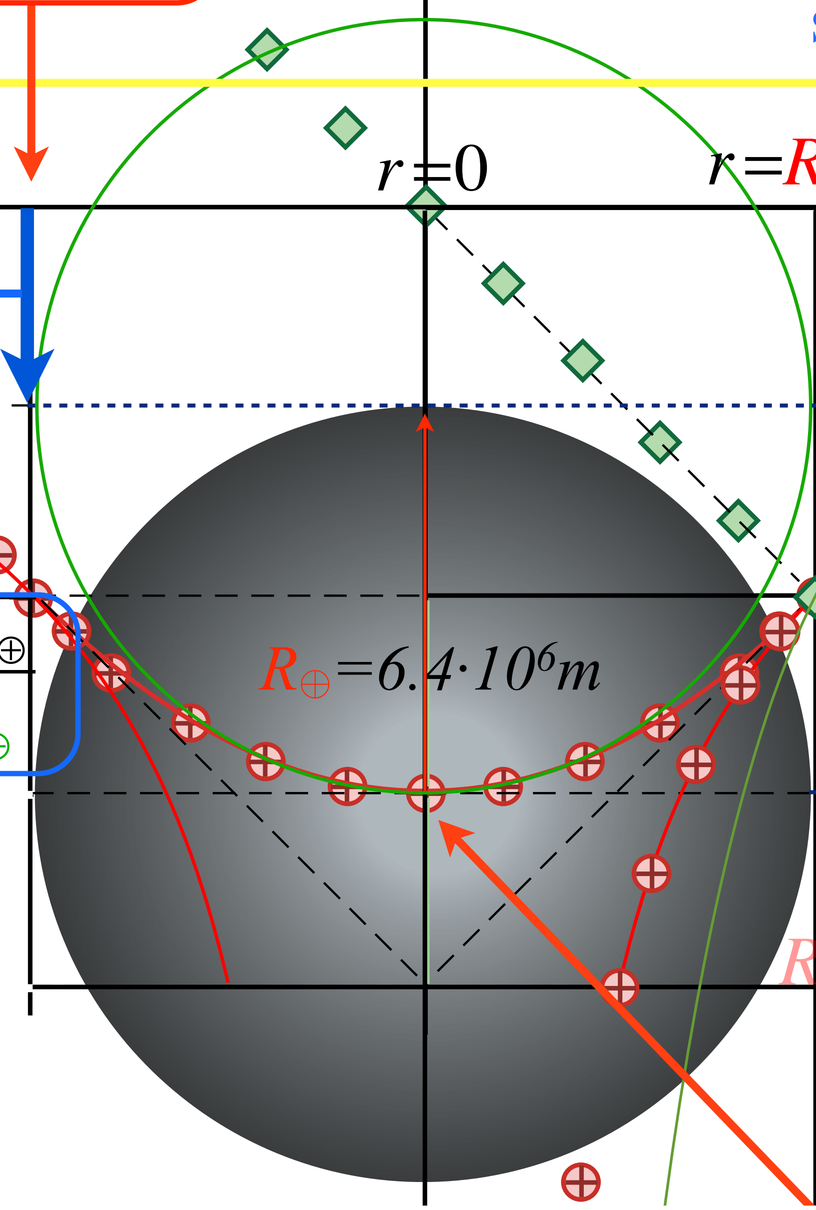
Orbit $E_\odot^{Total} = \frac{1}{2}\mu v_\odot^2 - G\frac{\mu M_\oplus}{R_\oplus} = \boxed{-G\frac{\mu M_\oplus}{2R_\oplus}}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

$(r=R_\oplus)$-orbit frequency:

$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$



Dissociation threshold : $PE = 0$

escape from $r=0$

surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\, escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\, escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{bottom} = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$ outside

surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus{}^2} = -9.8\dfrac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus{}^2}$$

Orbit KE $= \dfrac{1}{2}\mu v_\odot^2 = G\dfrac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total} = \dfrac{1}{2}\mu v_\odot^2 - G\dfrac{\mu M_\oplus}{R_\oplus} = \boxed{-G\dfrac{\mu M_\oplus}{2R_\oplus}}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

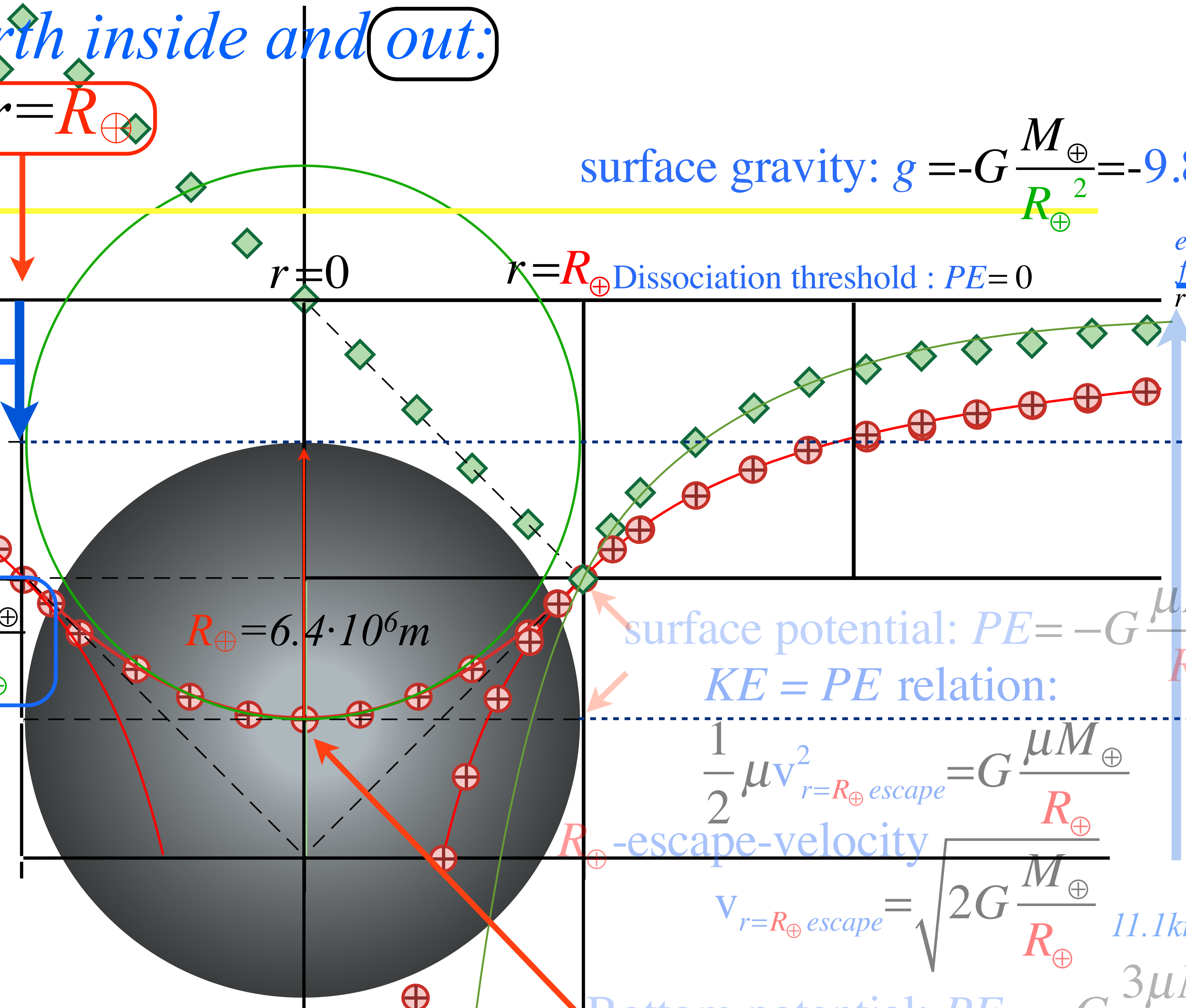
$(r=R_\oplus)$-orbit frequency:

$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

$(r=R_\oplus)$-orbit period:

$$\tau_\odot = 2\pi\sqrt{\frac{R_\oplus^3}{M_\oplus G}} = 2\pi\sqrt{\frac{R_\oplus}{g}}$$

$= 5062$ sec $= 84.4$ min



surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{r=R_\oplus\, escape} = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\, escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v^2_{bottom} = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

Sophomore-physics-Earth inside and out:
...and surface orbit at $r=R_\oplus$

outside

surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2} = -9.8\dfrac{m}{s^2}$

Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus} = \mu g = G\frac{\mu M_\oplus}{R_\oplus^2}$$

Orbit KE$=\dfrac{1}{2}\mu v_\odot^2 = G\dfrac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total} = \dfrac{1}{2}\mu v_\odot^2 - G\dfrac{\mu M_\oplus}{R_\oplus} = -G\dfrac{\mu M_\oplus}{2R_\oplus}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus = G\frac{M_\oplus}{R_\oplus^2} \Rightarrow \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

$(r=R_\oplus)$-orbit frequency:

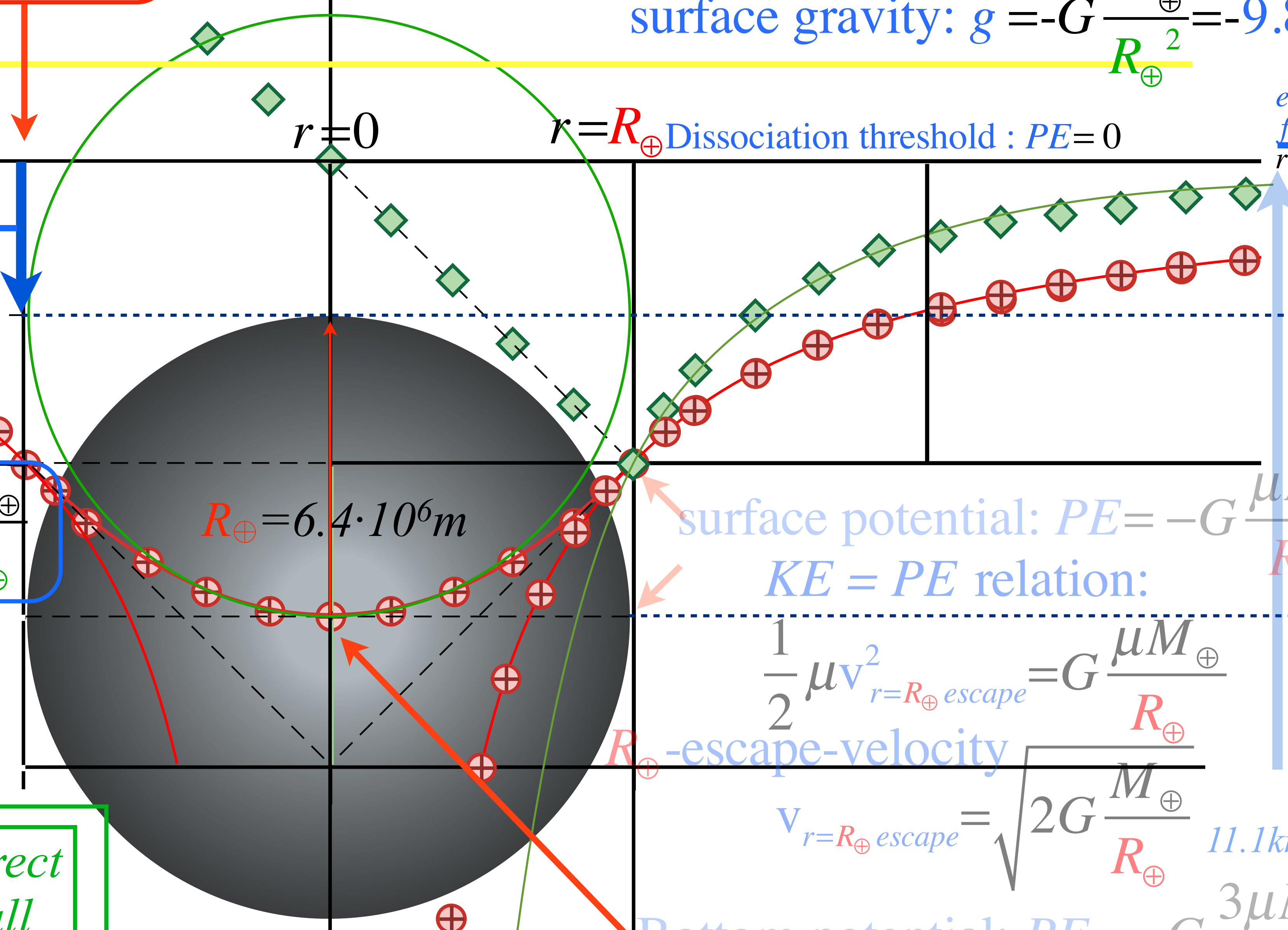
$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

$(r=R_\oplus)$-orbit period:

$$\tau_\odot = 2\pi\sqrt{\frac{R_\oplus^3}{M_\oplus G}} = 2\pi\sqrt{\frac{R_\oplus}{g}}$$

$= 5062$ sec $= 84.4$ min

Correct for all inside IHO orbits.



surface potential: $PE = -G\dfrac{\mu M_\oplus}{R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v_{r=R_\oplus\,escape}^2 = G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\,escape} = \sqrt{2G\frac{M_\oplus}{R_\oplus}}$$

11.1km/sec

Bottom potential: $PE = -G\dfrac{3\mu M_\oplus}{2R_\oplus}$

KE = PE relation:

$$\frac{1}{2}\mu v_{bottom}^2 = G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom} = \sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

inside

PE for $\lvert x\rvert<1$: $y^{PE} = \dfrac{x^2}{2} - \dfrac{3}{2}$	$PE^{mks}(r) = \dfrac{GM_\oplus\mu}{R_\oplus}\left(\dfrac{r^2}{2R_\oplus^2} - \dfrac{3}{2}\right)$
$Force$ for $\lvert x\rvert<1$: $y^{Force} = -x$	$F^{mks}(r) = -\boxed{\dfrac{GM_\oplus\mu}{R_\oplus^3}}\, r = k\mu r$

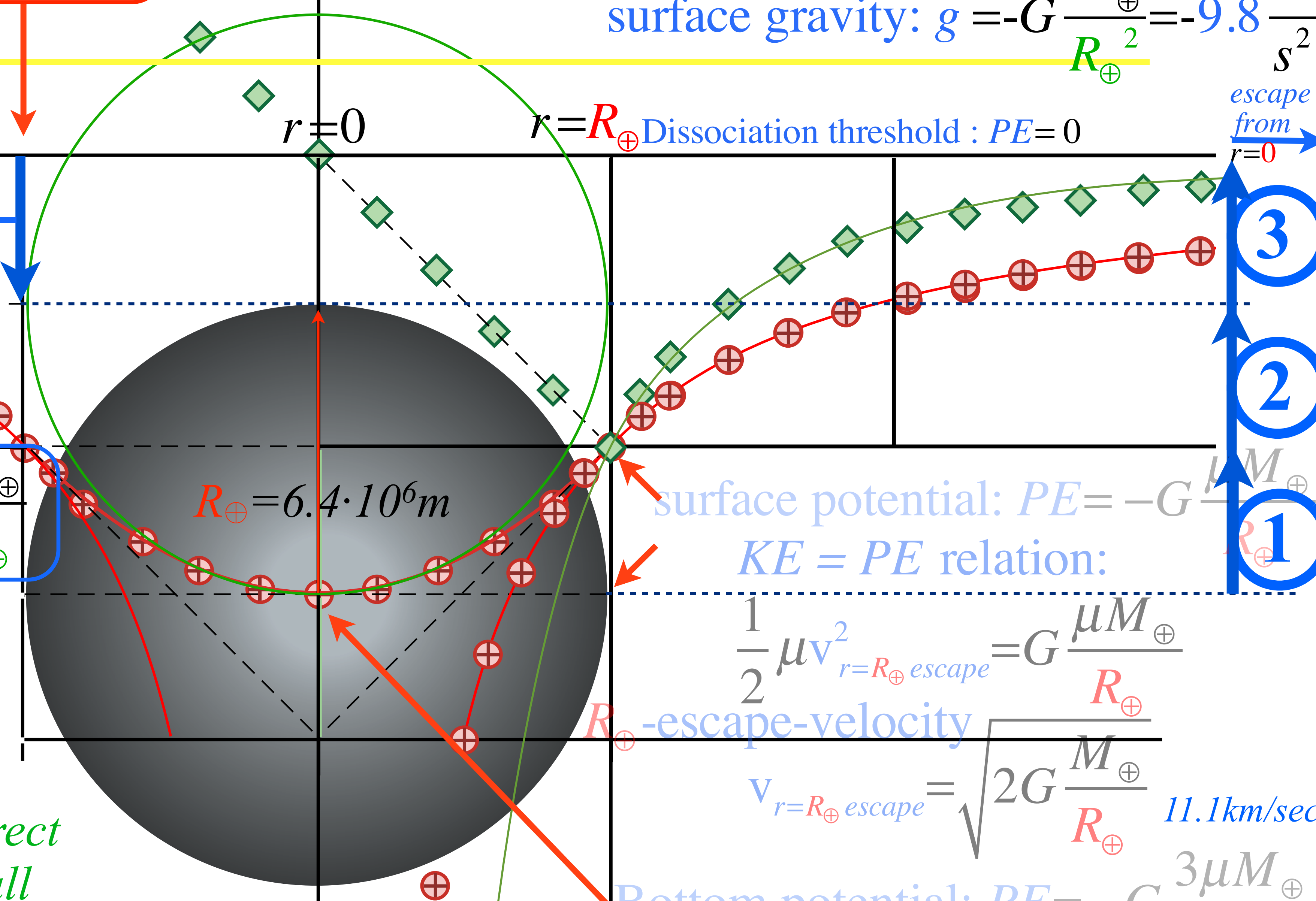
Hooke constant

$$k\mu = G\frac{4\pi}{3}\mu\rho_\oplus$$

Sophomore-physics-Earth inside and out: 3 (equal) steps out of (or into) Hell
...and surface orbit at $r=R_\oplus$

outside

surface gravity: $g=-G\frac{M_\oplus}{R_\oplus^2}=-9.8\frac{m}{s^2}$



Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus^2}$$

Orbit KE $=\frac{1}{2}\mu v_\odot^2=G\frac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total}=\frac{1}{2}\mu v_\odot^2-G\frac{\mu M_\oplus}{R_\oplus}=\boxed{-G\frac{\mu M_\oplus}{2R_\oplus}}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus=G\frac{M_\oplus}{R_\oplus^2}\Rightarrow\omega_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

$(r=R_\oplus)$-orbit frequency:

$$\upsilon_\odot=\frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}}=\frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

$(r=R_\oplus)$-orbit period:

$$\tau_\odot=2\pi\sqrt{\frac{R_\oplus^3}{M_\oplus G}}=2\pi\sqrt{\frac{R_\oplus}{g}}$$

$= 5062 \sec = 84.4 \min$

Correct for all inside IHO orbits.

surface potential: $PE=-G\frac{\mu M_\oplus}{R_\oplus}$

$KE=PE$ relation:

$$\frac{1}{2}\mu v_{r=R_\oplus\,escape}^2=G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\,escape}=\sqrt{2G\frac{M_\oplus}{R_\oplus}}$$ *11.1km/sec*

Bottom potential: $PE=-G\frac{3\mu M_\oplus}{2R_\oplus}$

$KE=PE$ relation:

$$\frac{1}{2}\mu v_{bottom}^2=G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

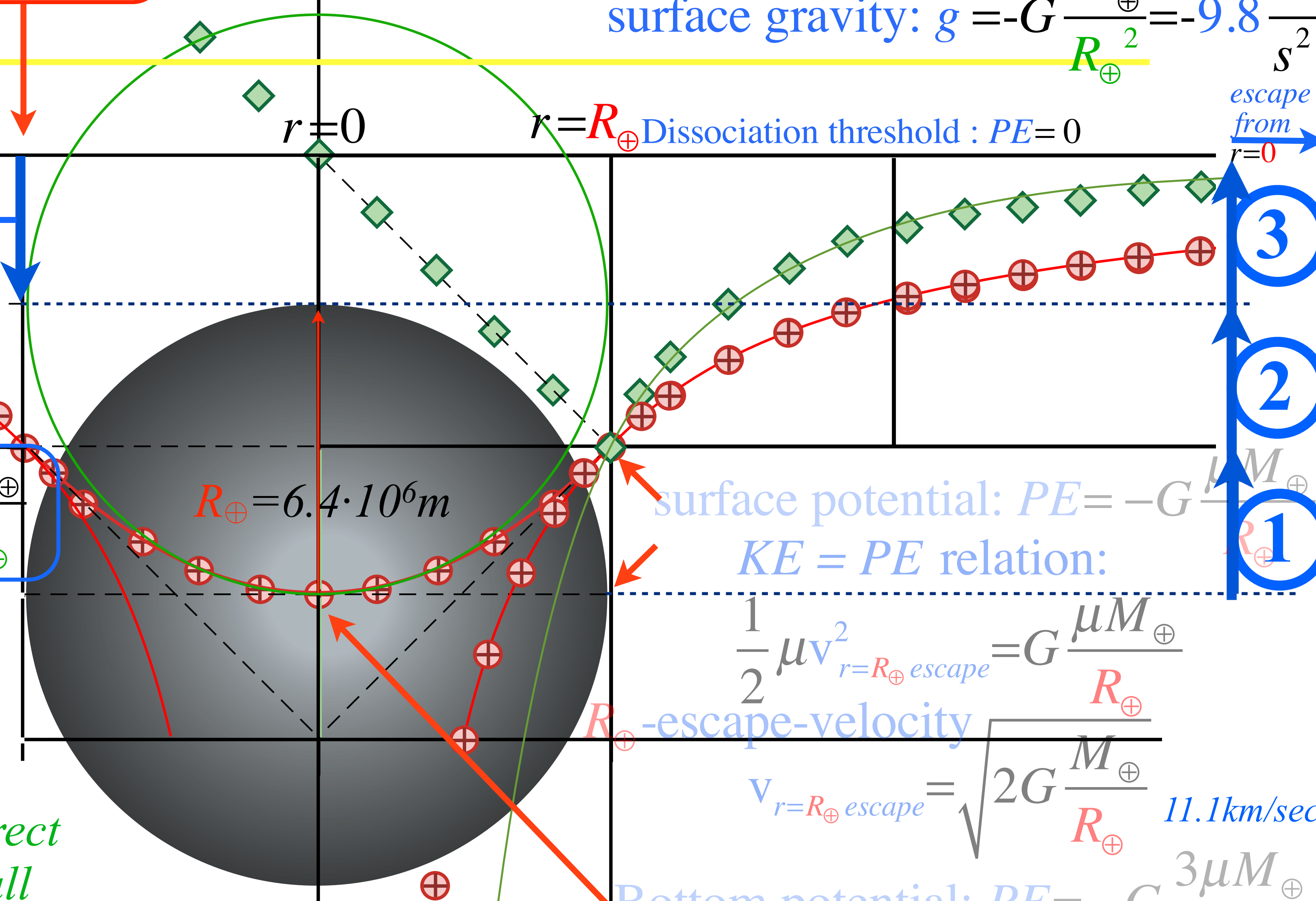
$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$ *13.7km/sec*

$(r=R_\oplus)$-circular orbit speed:

$$v_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus}}=\sqrt{gR_\oplus}$$ *=7.9km/sec*

Sophomore-physics-Earth inside and out: 3 (equal) steps out of (or into) Hell ...and surface orbit at $r=R_\oplus$ outside

surface gravity: $g=-G\dfrac{M_\oplus}{R_\oplus^2}=-9.8\dfrac{m}{s^2}$



Centifugal force = surface gravity:

$$\frac{\mu v_\odot^2}{R_\oplus}=\mu g=G\frac{\mu M_\oplus}{R_\oplus^2}$$

Orbit KE$=\dfrac{1}{2}\mu v_\odot^2=G\dfrac{\mu M_\oplus}{2R_\oplus}$

Orbit $E_\odot^{Total}=\dfrac{1}{2}\mu v_\odot^2-G\dfrac{\mu M_\oplus}{R_\oplus}=\boxed{-G\dfrac{\mu M_\oplus}{2R_\oplus}}$

$(r=R_\oplus)$-orbit angular frequency:

$$\omega_\odot^2 R_\oplus=G\frac{M_\oplus}{R_\oplus^2}\Rightarrow\omega_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

$(r=R_\oplus)$-orbit frequency:

$$\upsilon_\odot=\frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}}=\frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

$(r=R_\oplus)$-orbit period:

$$\tau_\odot=2\pi\sqrt{\frac{R_\oplus^3}{M_\oplus G}}=2\pi\sqrt{\frac{R_\oplus}{g}}$$

$= 5062\text{ sec} = 84.4\text{ min}$

Correct for all inside IHO orbits.

$(r=R_\oplus)$-circular orbit speed:

$$v_\odot=\sqrt{G\frac{M_\oplus}{R_\oplus}}=\sqrt{gR_\oplus}\quad =7.9\text{km/sec}$$

surface potential: $PE=-G\dfrac{\mu M_\oplus}{R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v_{r=R_\oplus\,escape}^2=G\frac{\mu M_\oplus}{R_\oplus}$$

$R_\oplus$-escape-velocity

$$v_{r=R_\oplus\,escape}=\sqrt{2G\frac{M_\oplus}{R_\oplus}}\quad 11.1\text{km/sec}$$

Bottom potential: $PE=-G\dfrac{3\mu M_\oplus}{2R_\oplus}$

$KE = PE$ relation:

$$\frac{1}{2}\mu v_{bottom}^2=G\frac{3\mu M_\oplus}{2R_\oplus}$$

$(r=0)$-escape-velocity

$$v_{bottom}=\sqrt{3G\frac{M_\oplus}{R_\oplus}}$$

13.7km/sec

IHO circular orbit velocity proportional to radius: $v_\odot=\omega_\odot r$

All IHO orbits (circular or elliptic) have the same frequency: $\omega_\odot r=\sqrt{\dfrac{g}{R_\oplus}}$

"Sophomore-Physics-Earth" models: 3 key energy "steps" and 4 key energy "levels"

→ *"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

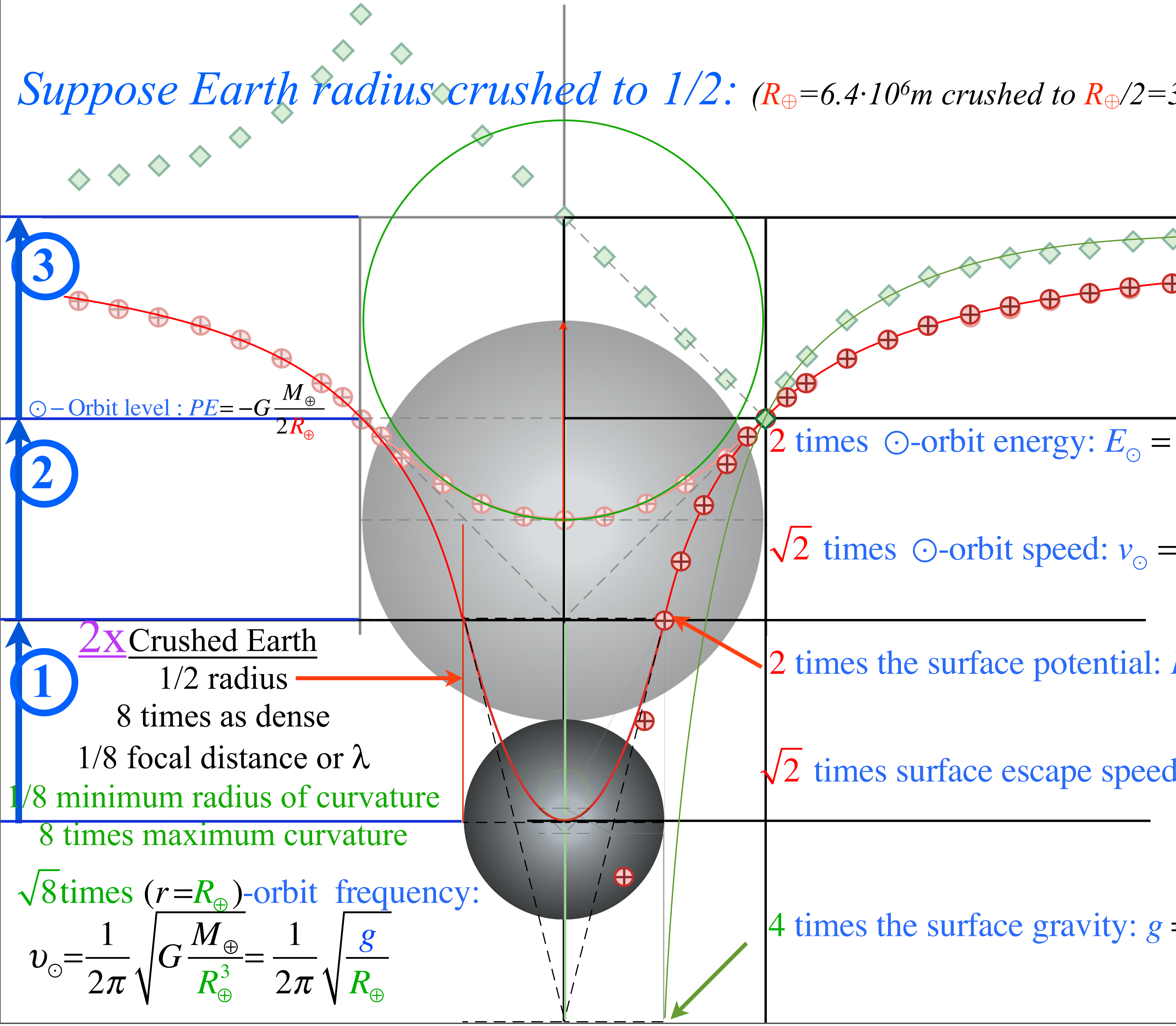
Earth matter vs nuclear matter vs Schwarzchild singular matter:

Introducing the "neutron starlet" *("fingertip physics")*

*Fantasizing a completely crushed "***Black-Hole-Earth***"*

Suppose Earth radius crushed to 1/2: *($R_\oplus$=6.4·10⁶m crushed to $R_\oplus/2$=3.2·10⁶m)*

All formulas identical to ones derived on p.11 to 25. Imagine reducing $R_\oplus$ to $R_\oplus/2$



2x Crushed Earth
1/2 radius
8 times as dense
1/8 focal distance or λ
1/8 minimum radius of curvature
8 times maximum curvature

$\sqrt{8}$ times ($r=R_\oplus$)-orbit frequency:

$$\upsilon_\odot = \frac{1}{2\pi}\sqrt{G\frac{M_\oplus}{R_\oplus^3}} = \frac{1}{2\pi}\sqrt{\frac{g}{R_\oplus}}$$

2 times $\odot$-orbit energy: $E_\odot = -G\dfrac{M_\oplus}{2R_\oplus}$

$\sqrt{2}$ times $\odot$-orbit speed: $v_\odot = \sqrt{G\dfrac{M_\oplus}{R_\oplus}}$

2 times the surface potential: $PE = -G\dfrac{M_\oplus}{R_\oplus}$

$\sqrt{2}$ times surface escape speed: $v_e = \sqrt{G\dfrac{2M_\oplus}{R_\oplus}}$

4 times the surface gravity: $g = -G\dfrac{M_\oplus}{R_\oplus^2}$

"Sophomore-Physics-Earth" models: 3 key energy "steps" and 4 key energy "levels"

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

→ *Earth matter vs nuclear matter vs Schwarzchild singular matter:*

Introducing the "neutron starlet" *("fingertip physics")*

*Fantasizing a completely crushed "***Black-Hole-Earth***"*

Examples of "crushed" matter

Earth matter

$Earth\,mass: M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ Density $\rho_{\oplus}$ $=??$

$Earth\,radius: R_{\oplus} = 6.371 \cdot 10^{6}\,m \simeq 6.4 \cdot 10^{6}\,m \quad Earth\,volume: (4\pi/3)R_{\oplus}^{3} \simeq 4 \cdot 262 \cdot 10^{18} \sim 10^{21}\,m^{3}$

$(6.4)^3 \sim 262$ and $(4\pi/3)262 = 1089 \sim 10^3$

Examples of “crushed” matter

$$\rho_{\oplus} = \frac{M_{\oplus}}{(4\pi/3)R_{\oplus}^{\ 3}}$$

Earth matter

$Earth\,mass: M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ $\boxed{\text{Density } \rho_{\oplus} \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^{3} kg/m^{3}}$

$Earth\,radius: R_{\oplus} = 6.371 \cdot 10^{6}\,m \simeq 6.4 \cdot 10^{6}\,m$ $\quad Earth\,volume: (4\pi/3)R_{\oplus}^{\ 3} \simeq 4 \cdot 262 \cdot 10^{18} \sim 10^{21}\,m^{3}$

$(6.4)^3 \sim 262$ *and* $(4\pi/3)262 = 1097 \sim 10^3$

Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus{}^3}$$

Earth matter

$Earth\, mass: M_\oplus = 5.9722 \times 10^{24} kg. \simeq 6.0 \cdot 10^{24} kg.$ $\boxed{\text{Density } \rho_\oplus \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 kg/m^3}$

$Earth\, radius: R_\oplus = 6.371 \cdot 10^6 m \simeq 6.4 \cdot 10^6 m$ $\quad Earth\, volume: (4\pi/3) R_\oplus{}^3 \simeq 4 \cdot 262 \cdot 10^{18} \sim 10^{21} m^3$

$(6.4)^3 \sim 262$ and $(4\pi/3)262 = 1097 \sim 10^3$

Density of solid Fe$=7.9 \cdot 10^3$kg/m^3
Density of liquid Fe$=6.9 \cdot 10^3$kg/m^3

Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus{}^3}$$

Earth matter

$Earth\,mass: M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ $\boxed{\text{Density } \rho_\oplus \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 kg/m^3}$

$Earth\,radius: R_\oplus = 6.371 \cdot 10^6\,m \simeq 6.4 \cdot 10^6\,m$ $\quad Earth\,volume: (4\pi/3)R_\oplus^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^3$

Nuclear matter

Nucleon mass $=1.67\cdot 10^{-27}kg. \sim 2\cdot 10^{-27}kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2\cdot 10^{-27}kg$.

Examples of "crushed" matter

$$\rho_{\oplus} = \frac{M_{\oplus}}{(4\pi/3)R_{\oplus}{}^3}$$

Earth matter

$Earth\,mass: M_{\oplus} = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ $\boxed{\text{Density } \rho_{\oplus} \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^{3} kg/m^{3}}$

$Earth\,radius: R_{\oplus} = 6.371 \cdot 10^{6}\,m \simeq 6.4 \cdot 10^{6}\,m$ $\quad Earth\,volume: (4\pi/3)R_{\oplus}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^{3}$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

$^{4\pi}/_{3}3^{3} = 36\pi = 113 \sim 10^{2}$

That's $100 \cdot 10^{-27} = 10^{-25}$ *kg* packed into a volume of $^{4\pi}/_{3} r^{3} = {}^{4\pi}/_{3}\, (3 \cdot 10^{-15})^{3}\ m^{3}$ or about $10^{-43}\ m^{3}$.

Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus{}^3}$$

Earth matter

$Earth\,mass: M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ $\boxed{\text{Density } \rho_\oplus \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 kg/m^3}$

$Earth\,radius: R_\oplus = 6.371 \cdot 10^6\,m \simeq 6.4 \cdot 10^6\,m \quad Earth\,volume: (4\pi/3)R_\oplus^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^3$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

$^{4\pi}/_3 3^3 = 36\pi = 113 \sim 10^2$

That's $100 \cdot 10^{-27} = 10^{-25}\ kg$ packed into a volume of $^{4\pi}/_3 r^3 = {}^{4\pi}/_3\ (3 \cdot 10^{-15})^3\ m^3$ or about $10^{-43}\ m^3$.

Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in a fingertip$(1\text{cm})^3$.

$(1cm)^3 = (10^{-2}m)^3 = 10^{-6}m^3$

Examples of "crushed" matter

$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus{}^3}$

Earth matter

$Earth\,mass: M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg.$ $\boxed{\text{Density } \rho_\oplus \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 kg/m^3}$

$Earth\,radius: R_\oplus = 6.371 \cdot 10^6\,m \simeq 6.4 \cdot 10^6\,m$ $\quad Earth\,volume: (4\pi/3)R_\oplus^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^3$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27}kg. \sim 2 \cdot 10^{-27}kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27}kg$.

$^{4\pi}/_3 3^3 = 36\pi = 113 \sim 10^2$

That's $100 \cdot 10^{-27} = 10^{-25}$ *kg* packed into a volume of $^{4\pi}/_3 r^3 = {}^{4\pi}/_3\,(3 \cdot 10^{-15})^3\ m^3$ or about $10^{-43}\ m^3$.

Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in a fingertip$(1\text{cm})^3$.

$(1cm)^3 = (10^{-2}m)^3 = 10^{-6}m^3$

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.

"Sophomore-Physics-Earth" models: 3 key energy "steps" and 4 key energy "levels"
"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"
Earth matter vs nuclear matter vs Schwarzchild singular matter:
→ *Introducing the "neutron starlet"* *("fingertip physics")*
*Fantasizing a completely crushed "***Black-Hole-Earth***"*

Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus{}^3}$$

Earth matter

Earth mass : $M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg$. Density $\rho_\oplus \sim 6.0 \cdot 10^{24\text{-}21} \sim 6 \cdot 10^3 kg/m^3$

Earth radius : $R_\oplus = 6.371 \cdot 10^6\,m \simeq 6.4 \cdot 10^6\,m$ *Earth volume* : $(4\pi/3)R_\oplus^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^3$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

${}^{4\pi}/_3 3^3 = 36\pi = 113 \sim 10^2$

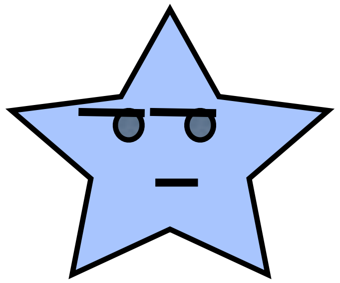
That's $100 \cdot 10^{-27} = 10^{-25}$ *kg* packed into a volume of ${}^{4\pi}/_3 r^3 = {}^{4\pi}/_3\, (3 \cdot 10^{-15})^3\ m^3$ or about $10^{-43}\ m^3$.

Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.

Introducing the "Neutron starlet"

1 cm³ of nuclear matter: mass= 10^{12} kg.



Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus^{\ 3}}$$

Earth matter

Earth mass : $M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg$. Density $\rho_\oplus \sim 6.0 \cdot 10^{24\text{-}21} \sim 6 \cdot 10^3 kg/m^3$

Earth radius : $R_\oplus = 6.371 \cdot 10^6\,m \simeq 6.4 \cdot 10^6\,m$ *Earth volume* : $(4\pi/3)R_\oplus^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^3$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27}kg. \sim 2 \cdot 10^{-27}kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27}kg$.

$^{4\pi}/_3 3^3 = 36\pi = 113 \sim 10^2$

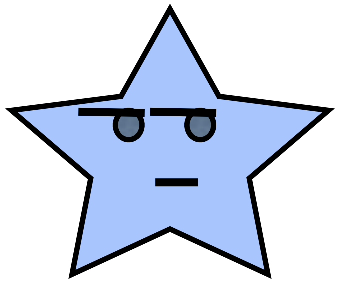
That's $100 \cdot 10^{-27} = 10^{-25}$ *kg* packed into a volume of $^{4\pi}/_3 r^3 = {}^{4\pi}/_3\,(3 \cdot 10^{-15})^3\ m^3$ or about $10^{-43}\ m^3$.

Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.

Introducing the "Neutron starlet"

1 cm^3 *of nuclear matter: mass*$= 10^{12}$ *kg.*



*Introducing the "***Black Hole Earth***"*

Suppose Earth is crushed so that its surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^8 m/s$.

$c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$$V_{escape} = \sqrt{(2GM/R_\otimes)}$$

(from p. 49)

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

"Sophomore-Physics-Earth" models: 3 key energy "steps" and 4 key energy "levels"

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter vs Schwarzchild singular matter:

→ *Introducing the "neutron starlet"* *("fingertip physics")*

*Fantasizing a completely crushed "***Black-Hole-Earth***"*

Examples of "crushed" matter

$$\rho_\oplus = \frac{M_\oplus}{(4\pi/3)R_\oplus^{\ 3}}$$

Earth matter

Earth mass : $M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0 \cdot 10^{24}\,kg$. Density $\rho_\oplus \sim 6.0 \cdot 10^{24-21} \sim 6 \cdot 10^3 kg/m^3$

Earth radius : $R_\oplus = 6.371 \cdot 10^6\,m \simeq 6.4 \cdot 10^6\,m$ *Earth volume* : $(4\pi/3)R_\oplus^3 \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\,m^3$

Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$ *("fingertip physics")*

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

$^{4\pi}/_3 3^3 = 36\pi = 113 \sim 10^2$

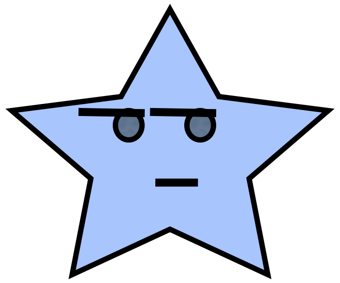
That's $100 \cdot 10^{-27} = 10^{-25}$ *kg* packed into a volume of $^{4\pi}/_3 r^3 = {}^{4\pi}/_3\,(3 \cdot 10^{-15})^3\ m^3$ or about $10^{-43}\ m^3$.

Nuclear density is $10^{-25+43} = 10^{18} kg/m^3$ or a *trillion* (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300m$ would approach neutron-star density.

Introducing the "Neutron starlet"

1 cm^3 *of nuclear matter: mass*$= 10^{12}$ *kg.*



*Introducing the "***Black Hole Earth***"*

Suppose Earth is crushed so that its surface escape velocity is the speed of light $c \cong 3.0 \cdot 10^8 m/s$.

$c \equiv 299{,}792{,}458 m/s$ (EXACTLY)

$$V_{escape} = \sqrt{(2GM/R_\otimes)}$$

(from p. 49)

$G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

$$c = \sqrt{(2GM/R_\otimes)}$$

$$R_\otimes = 2GM/c^2 = 8.9mm \sim 1cm$$

(fingertip size!)

Introducing Isotropic Harmonic Oscillator (IHO) energy and frequency relations

Constructing 2D-IHO orbits using Kepler anomaly plots

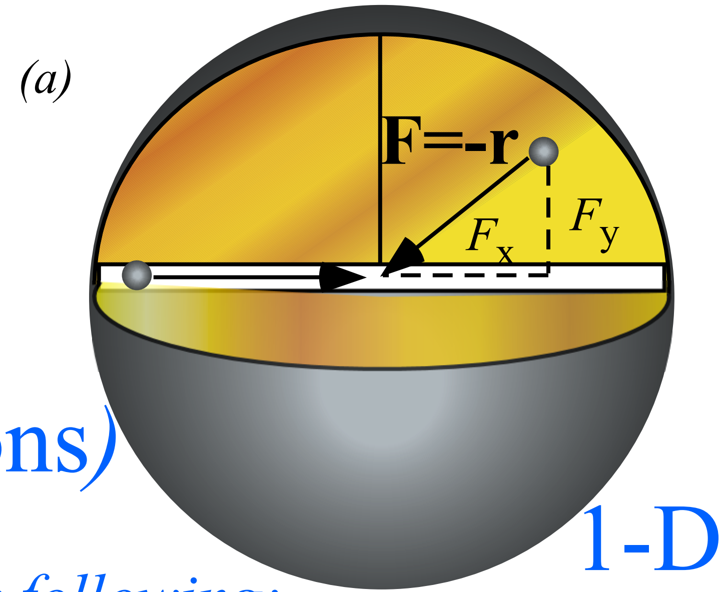
Mean-anomaly and eccentric-anomaly geometry with web-app animation

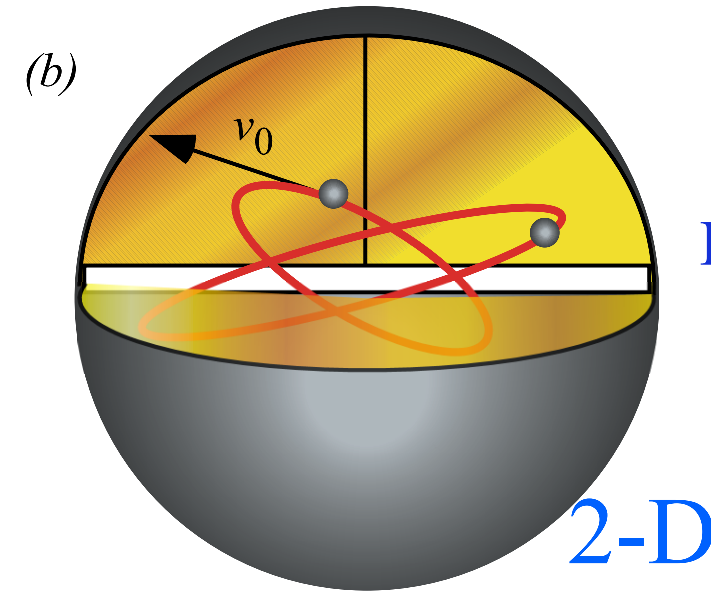
Calculus and vector geometry of IHO orbits

Constructing 2D-IHO orbits using orbital phasor-clock plots

Phasor geometry of coordinate (x,y) and velocity (V_x,V_y) space with web-app animation

Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 8.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

$F = -x$ (1-Dimension)

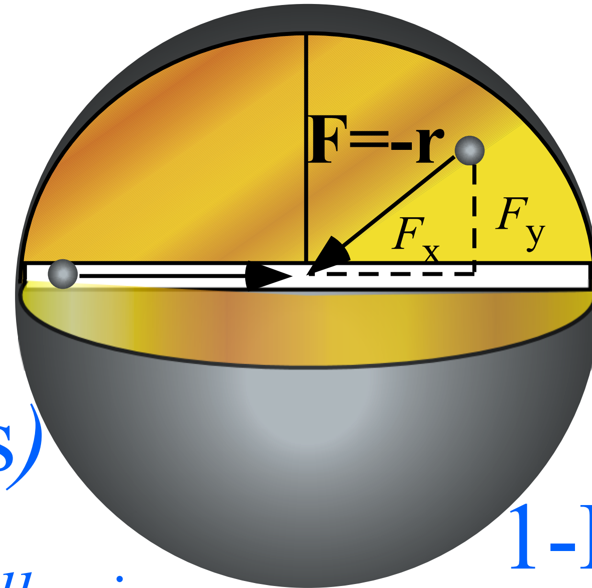
$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

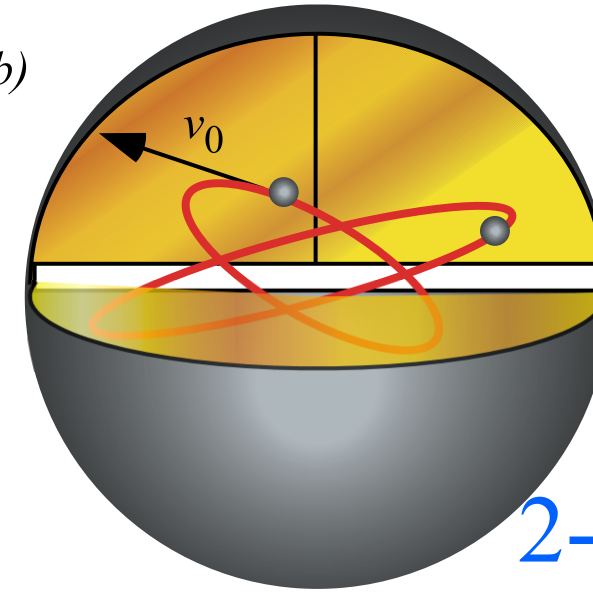
Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



1-D

(b)



Unit 1
Fig. 8.10

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

F =-x (1-Dimension)

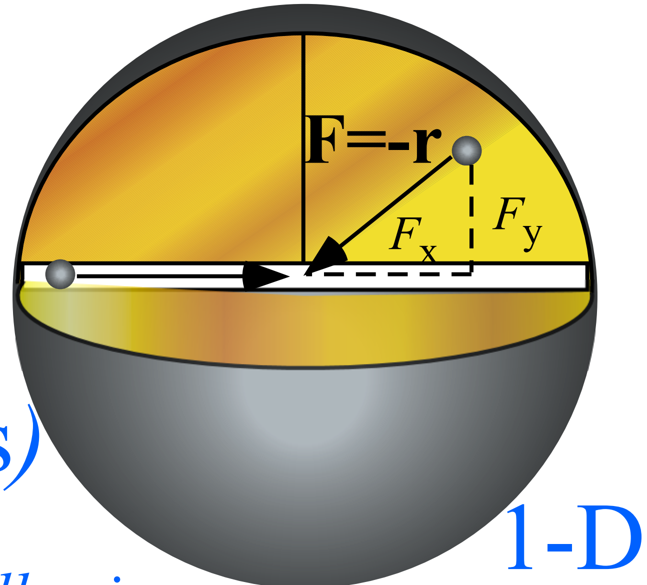
F =-r (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

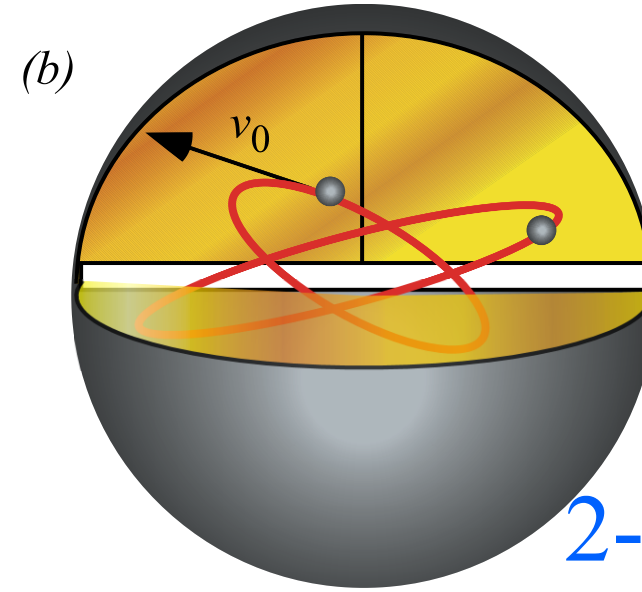
$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [$x(t)$ and $v_x=v(t)$] are given first. They apply as well to dimensions [$y(t)$ and $v_y=v(t)$] and [$z(t)$ and $v_z=v(t)$] in the ideal <u>*isotropic*</u> *case.*

Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D



2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1
Fig. 8.10

I.H.O. Force law

F =-*x* (1-Dimension)

F =-**r** (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

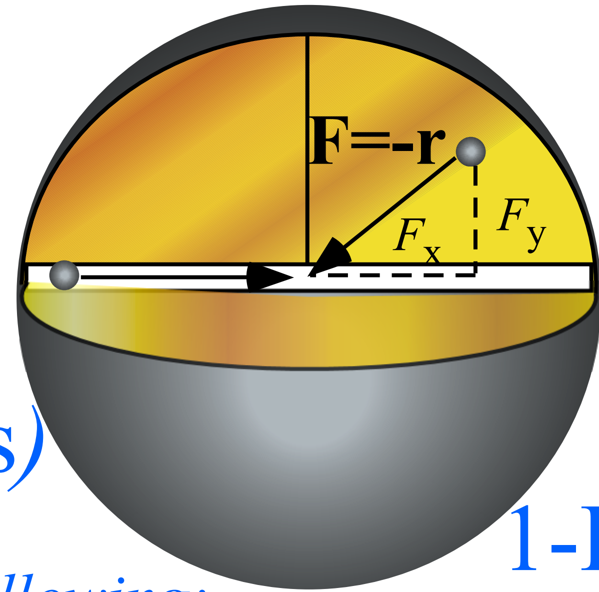
$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal <u>*isotropic*</u> *case.*

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

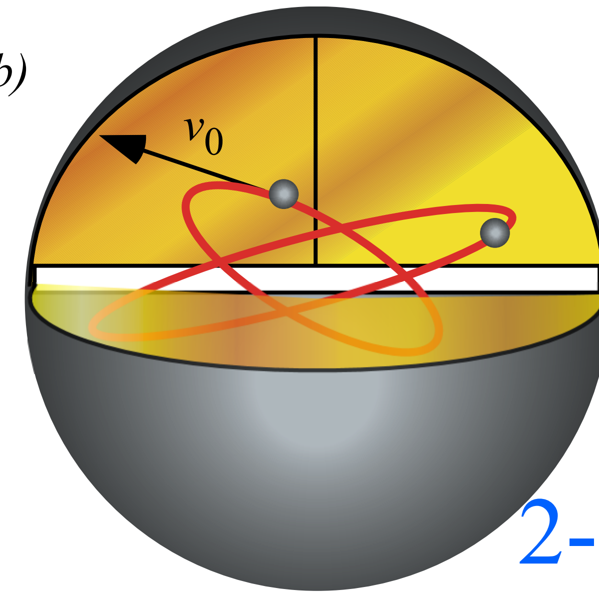
Isotropic Harmonic Oscillator phase dynamics in uniform-body

(a)



1-D

(b)



2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1
Fig. 8.10

I.H.O. Force law

F =-x (1-Dimension)

F =-r (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

velocity: *position:*

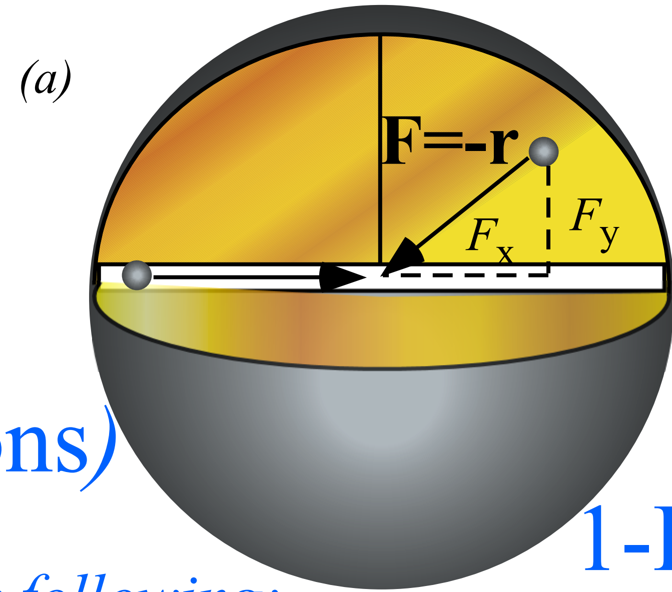
$$Let:\ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body

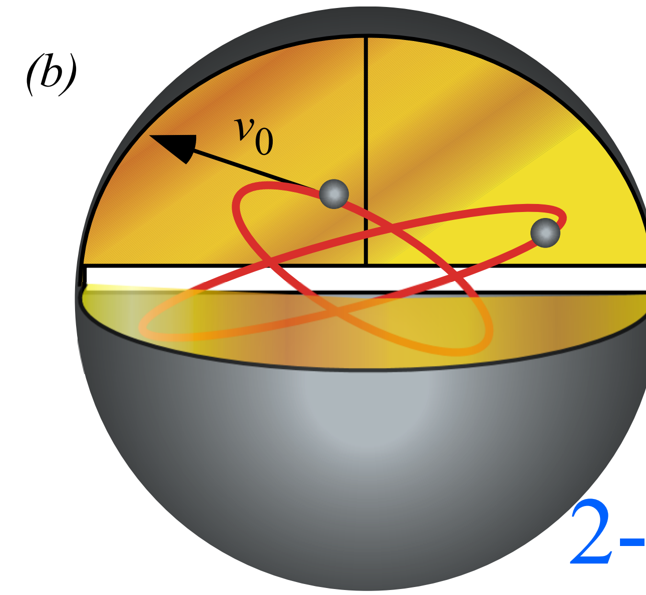
I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)



1-D



Unit 1
Fig. 8.10

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

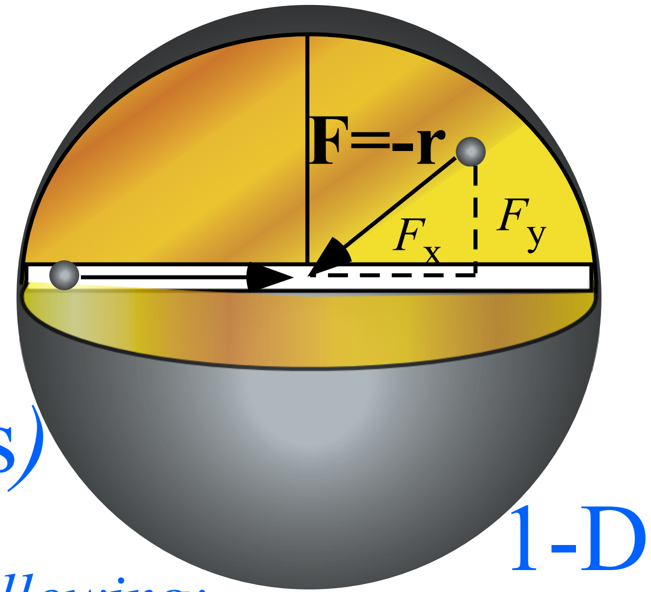
Another example of the old "scale-a-circle" trick...

velocity: *position:*

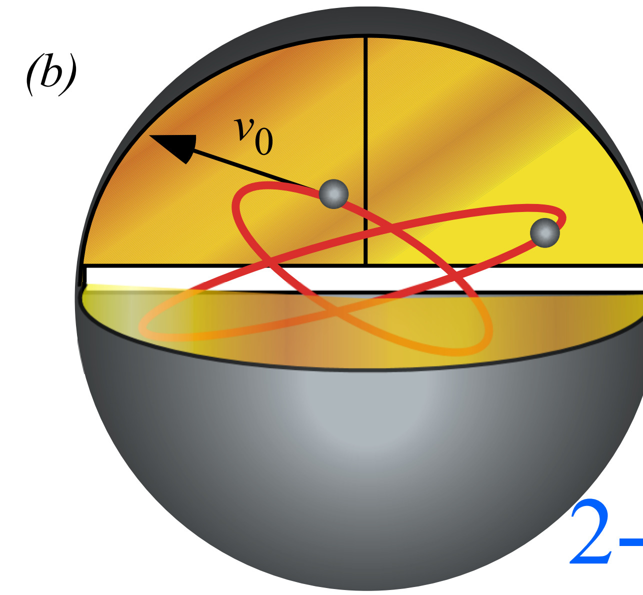
$$Let: \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and: \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta$$

$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta}_{by\ (1)} = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta}$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D



Unit 1
Fig. 8.10

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

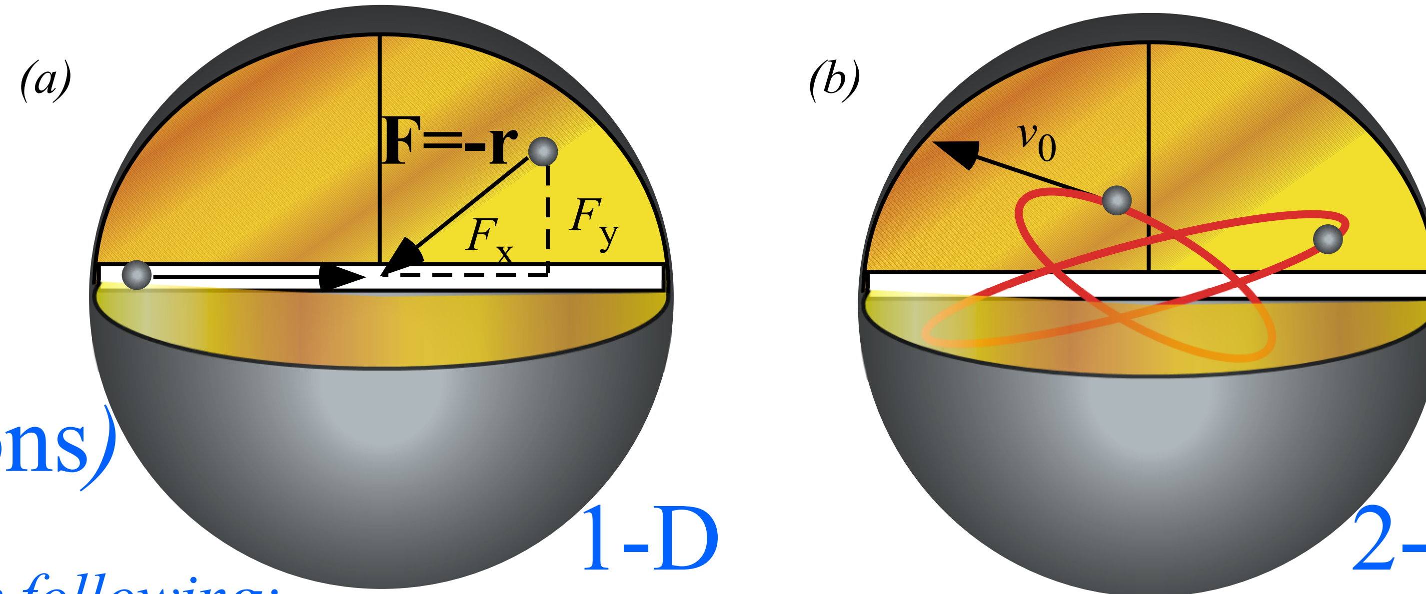
Another example of the old "scale-a-circle" trick...

velocity: *position:* *angular velocity:*

$$Let: \ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and: \ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \textbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta}$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1
Fig. 8.10

I.H.O. Force law

F =-x (1-Dimension)

F =-r (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

velocity: *position:* *angular velocity:*

$$Let:\ \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \mathbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}}_{by\ def.\ (3)}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta}$$

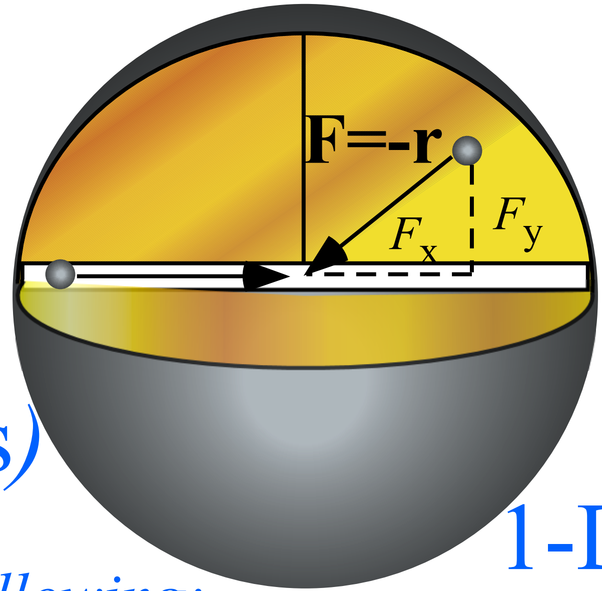
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

$F = -x$ (1-Dimension)

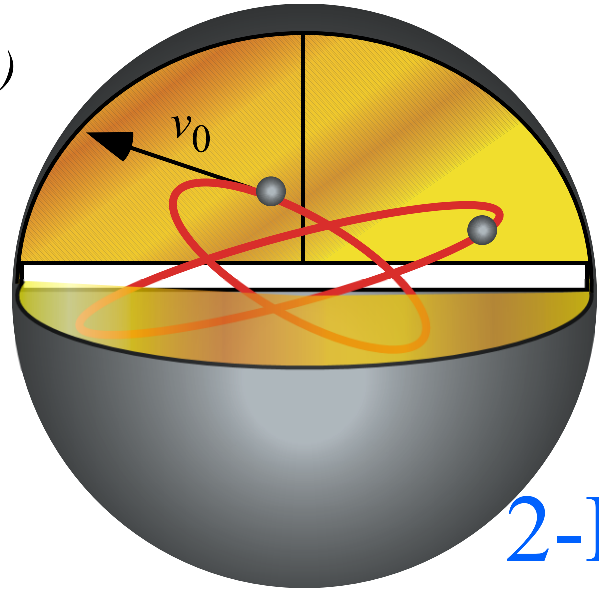
$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

(a)



1-D

(b)



2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Unit 1
Fig. 8.10

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion $[x(t)$ and $v_x=v(t)]$ are given first. They apply as well to dimensions $[y(t)$ and $v_y=v(t)]$ and $[z(t)$ and $v_z=v(t)]$ in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

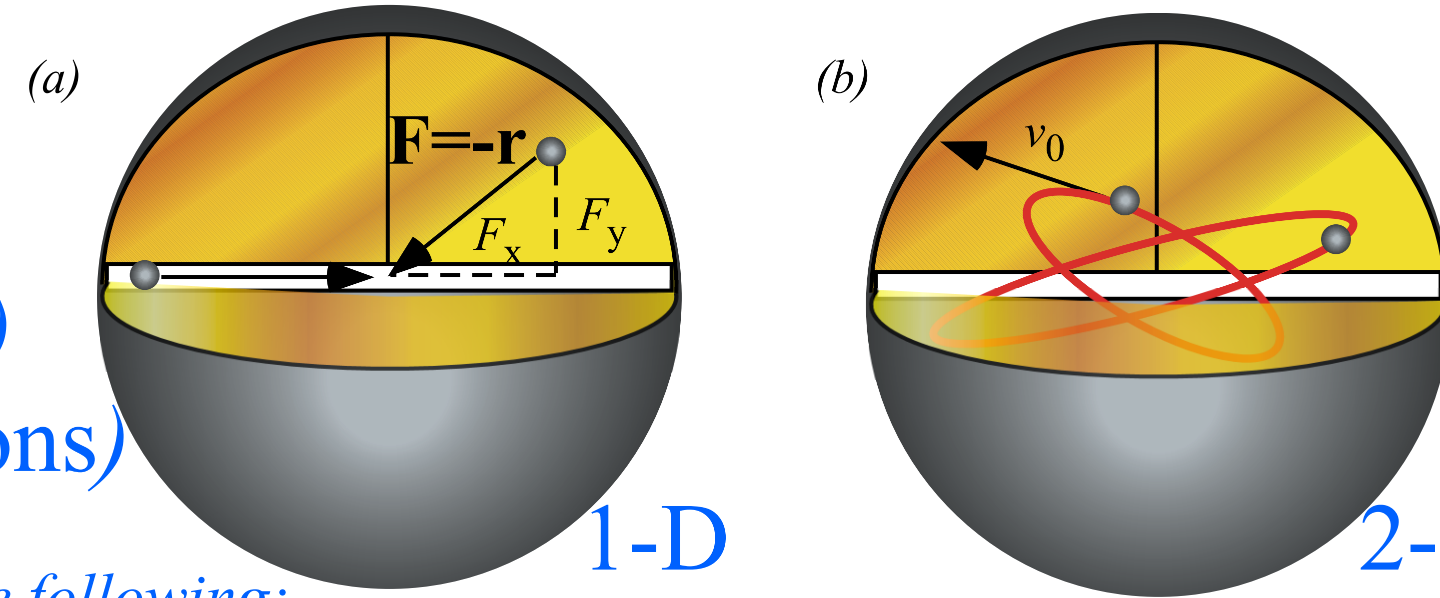
Another example of the old "scale-a-circle" trick...

velocity: *position:* *angular velocity:*

$$Let:\ \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \mathbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}\frac{dx}{d\theta} = \omega}_{by\ def.\ (3)}\frac{dx}{d\theta} = \underbrace{\omega\sqrt{\frac{2E}{k}}\cos\theta}_{by\ (2)}$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 8.10

I.H.O. Force law

F =-x (1-Dimension)

F =-r (2 or 3-Dimensions)

1-D

2-D or 3-D

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

velocity: *position:* *angular velocity:*

$$Let:\ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \textbf{(3)}\ \omega = \frac{d\theta}{dt}$$

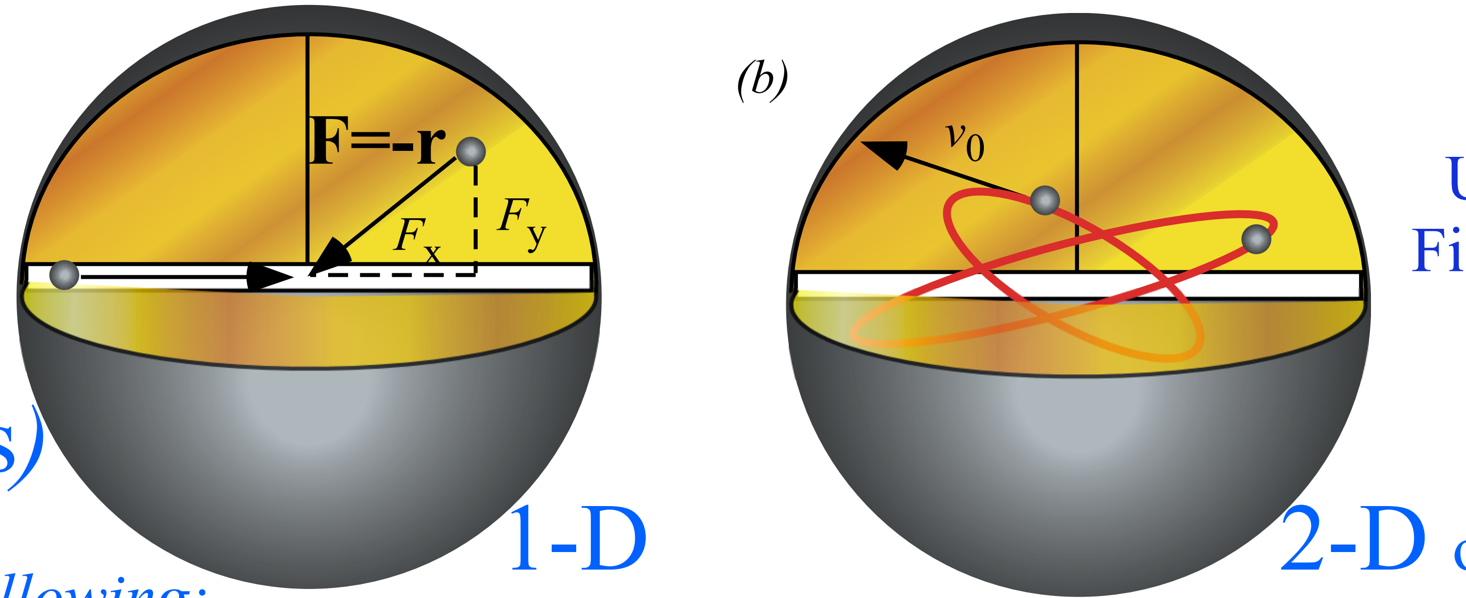
$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}}_{by\ def.\ (3)}\frac{dx}{d\theta} = \omega\underbrace{\frac{dx}{d\theta}}_{by\ (2)} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by def. **(3)**

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

divide this by **(1)**

Isotropic Harmonic Oscillator phase dynamics in uniform-body



Unit 1
Fig. 8.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

F =-*x* (1-Dimension)

F =-**r** (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

$$PE^{mks}(r) = \frac{GMm}{R_\oplus}\left(\frac{r^2}{2R_\oplus^2} - \frac{3}{2}\right)$$

HO"Spring-constant" (from p. 26)

$$\frac{1}{2}k = \frac{GM}{2R_\oplus^3} \quad \text{so:} \quad \omega_\odot = \sqrt{G\frac{M_\oplus}{R_\oplus^3}}$$

velocity: *position:* *angular velocity:*

$$Let: \ \textbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and: \ \textbf{(2)}\ x = \sqrt{2E/k}\sin\theta \qquad def.\ \textbf{(3)}\ \omega = \frac{d\theta}{dt}$$

$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}\frac{dx}{d\theta} = \omega}_{by\ def.\ (3)} \underbrace{\frac{dx}{d\theta}}_{by\ (2)} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

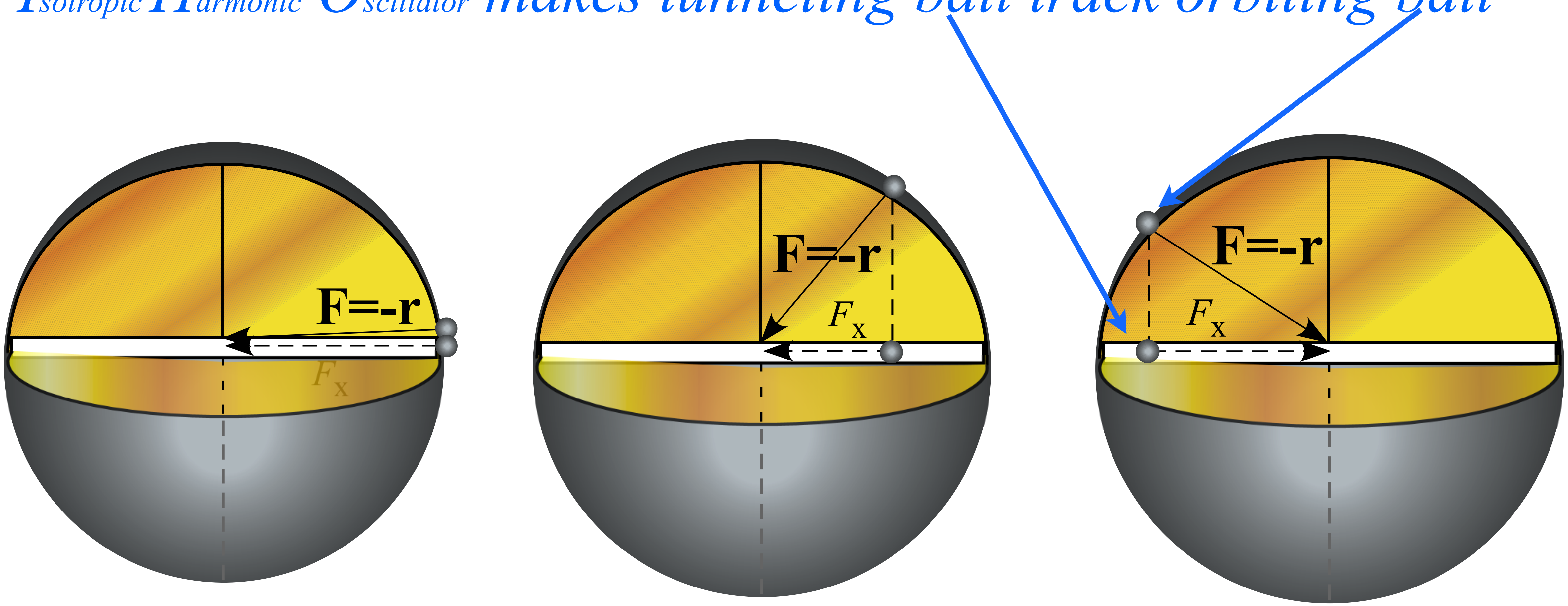
by def. **(3)**

$$\underbrace{\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}}_{divide\ this\ by\ (1)}$$

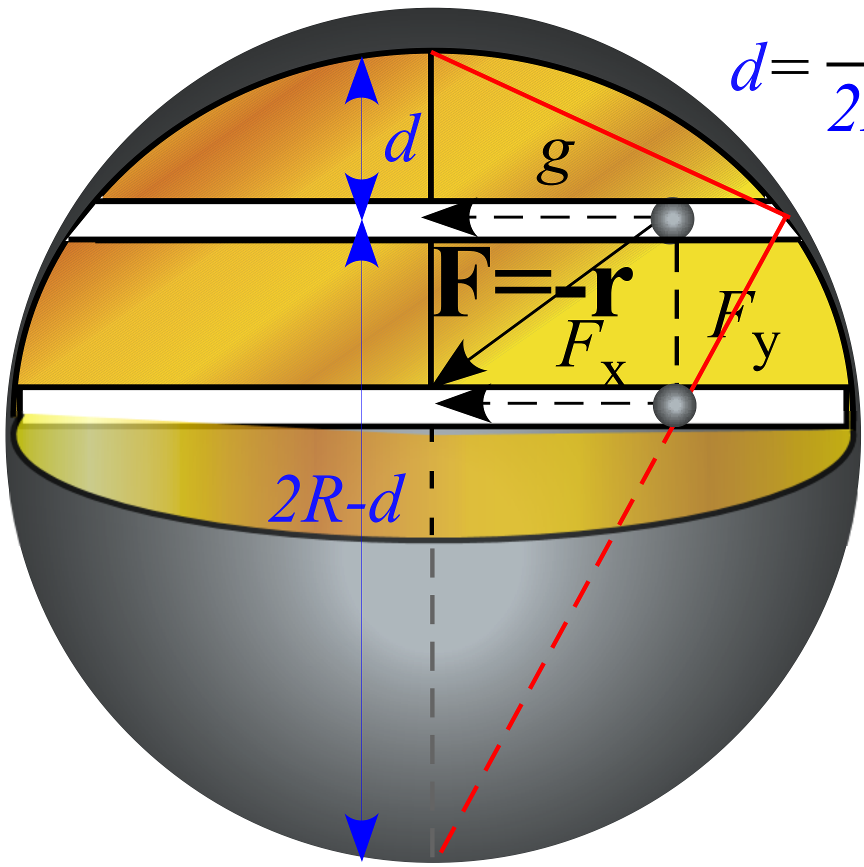
by integration for constant ω:

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

Isotropic Harmonic Oscillator makes tunneling ball track orbiting ball

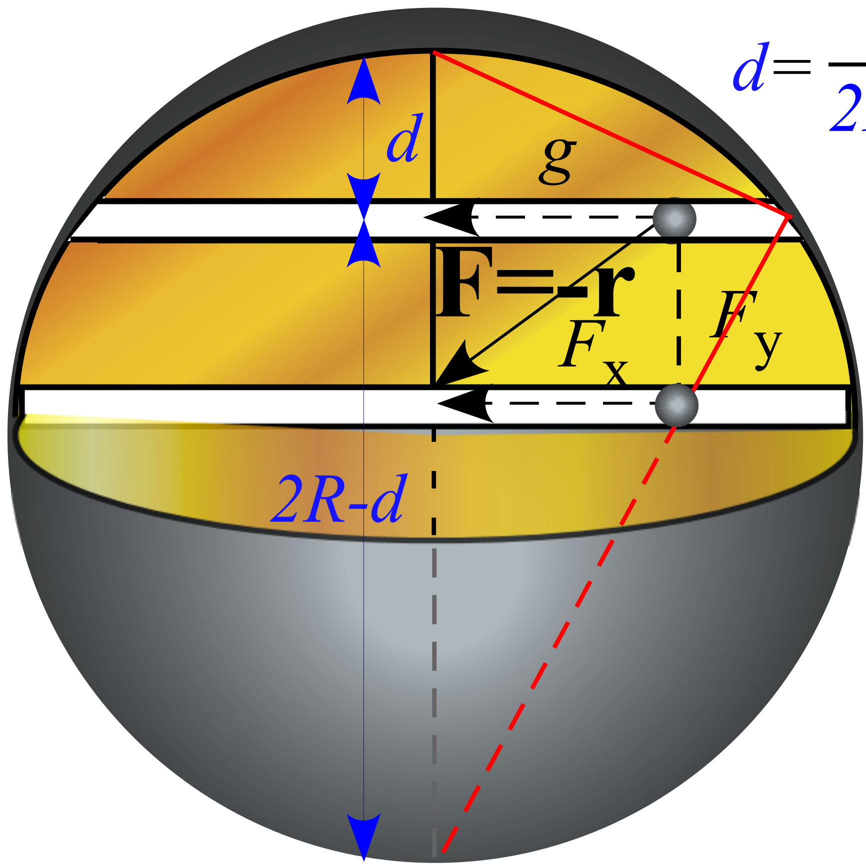


Isotropic Harmonic Oscillator makes balls in parallel tunnel track each other



$$d = \frac{g^2}{2R-d} \sim \frac{g^2}{2R}$$

*I*sotropic *H*armonic *O*scillator *makes balls in parallel tunnels track each other...*



$d = \frac{g^2}{2R-d} \sim \frac{g^2}{2R}$

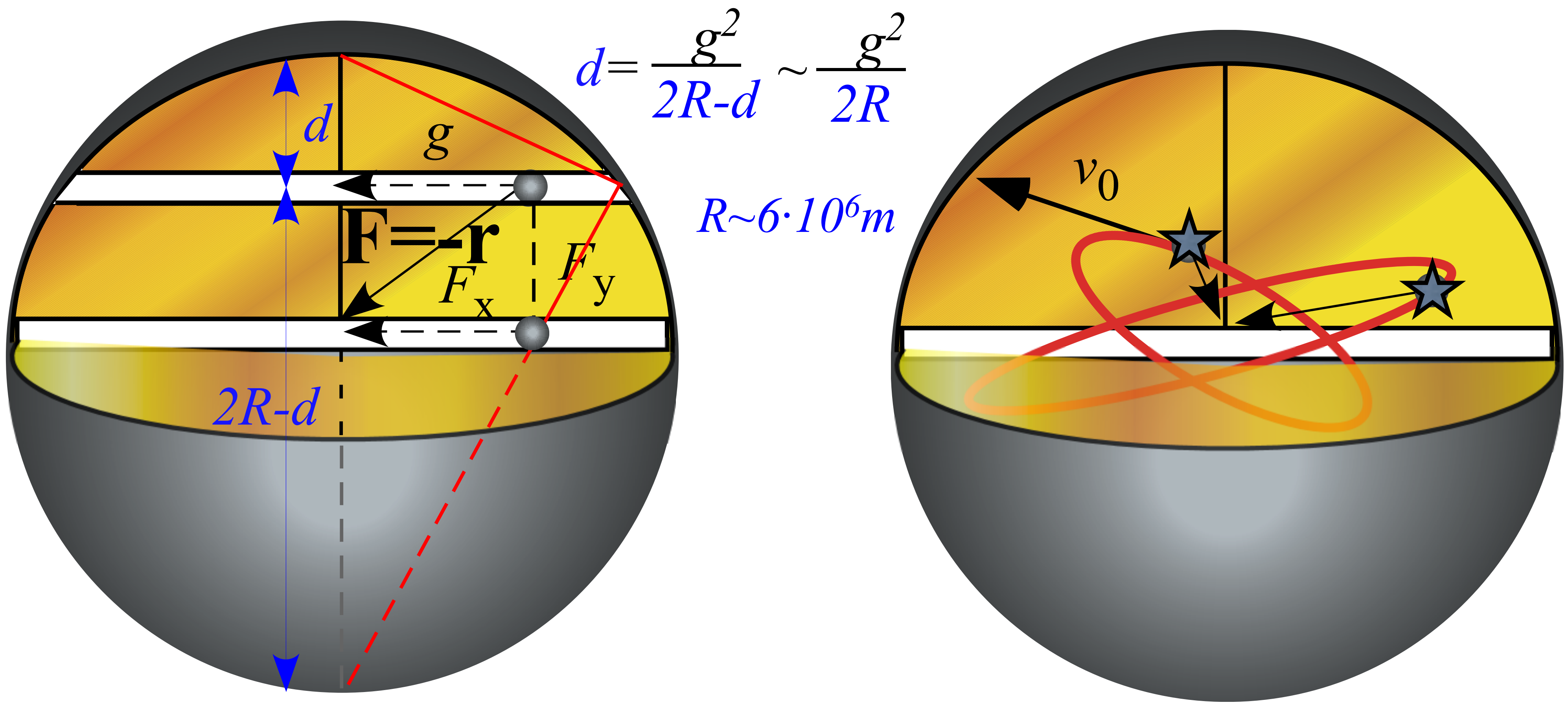
$R \sim 6 \cdot 10^6 m$

$d \sim \frac{1}{2R}$

...even if track length is just g *=1m so d~(1/12)micron*

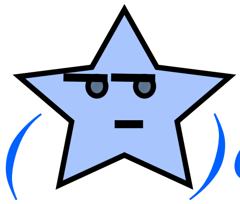
They all take about 84 minutes to go from right to left and back, again.

Isotropic Harmonic Oscillator makes balls in parallel tunnels track each other...



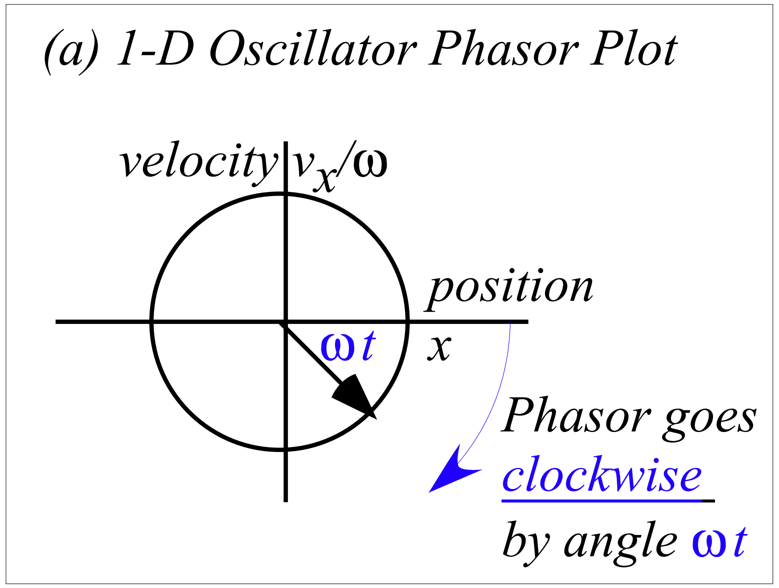
...even if track length is just g =1m *so* d=(1/12)micron

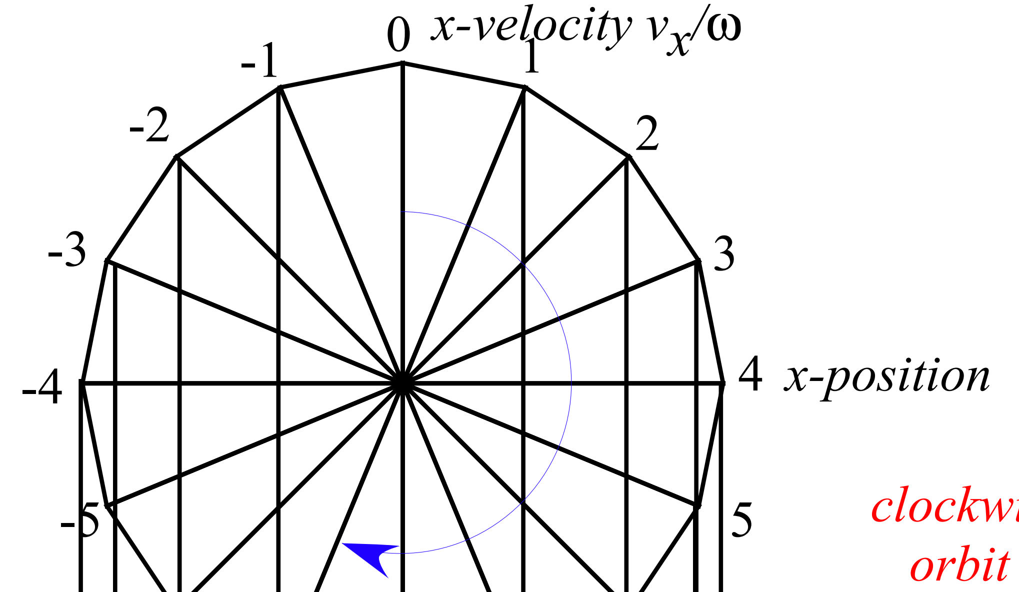
The all take about 84 minutes to go from right to left and back, again.

Most neutron starlet ( *)orbits are centered ellipses*

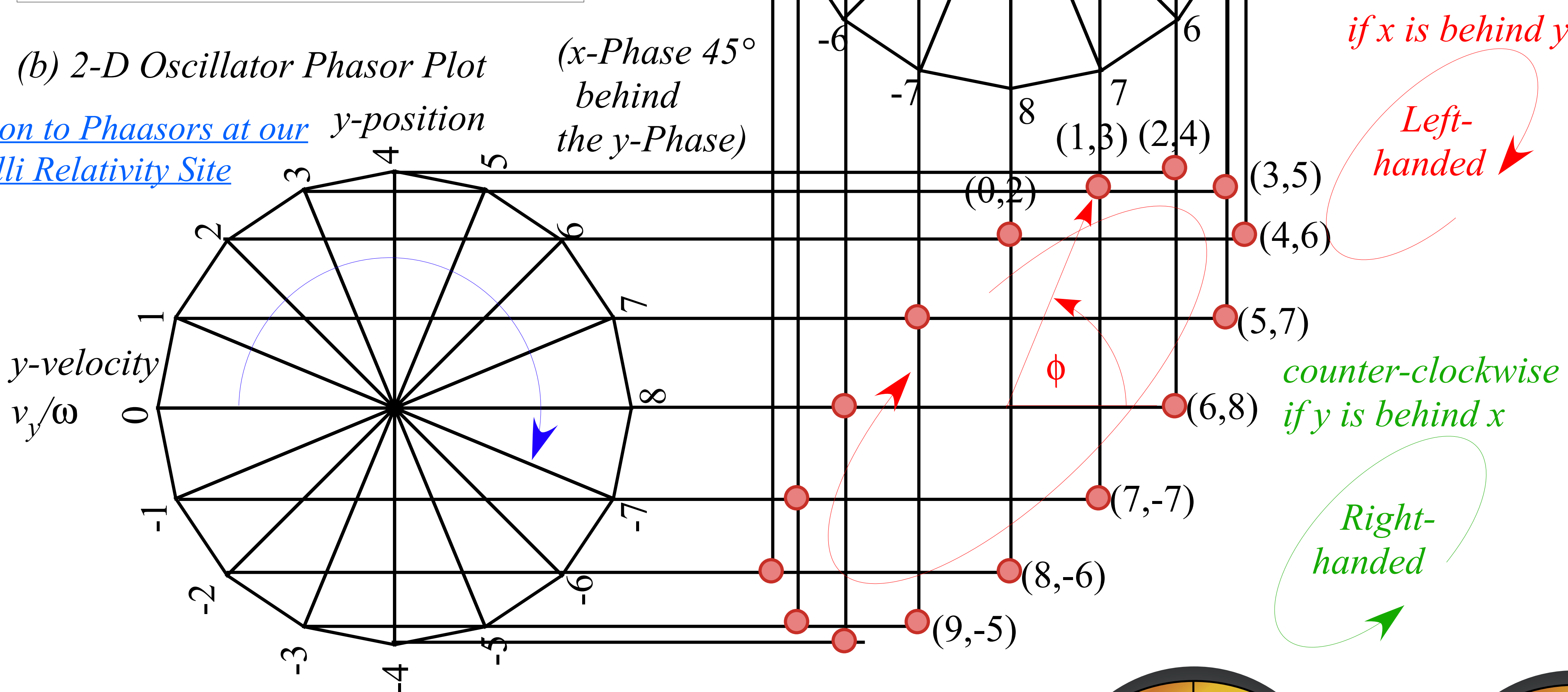
Isotropic Harmonic Oscillator phase dynamics in uniform-body

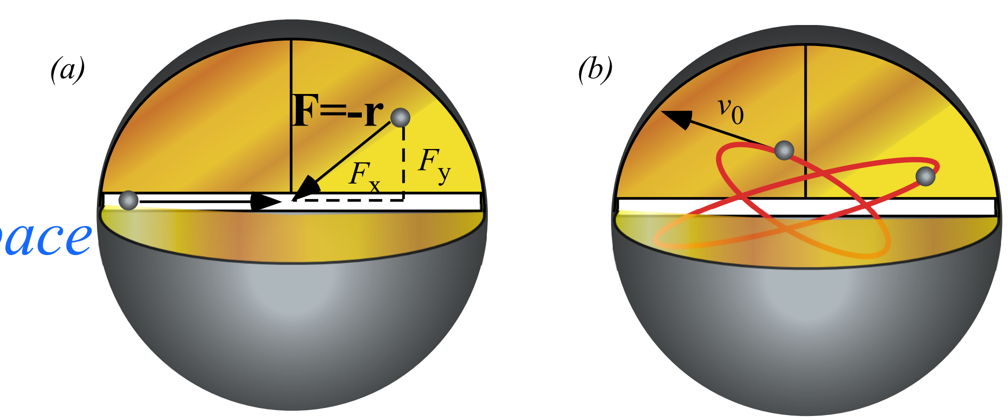
Unit 1
Fig. 8.10





[Introduction to Phaasors at our Pirelli Relativity Site](#)





Phasor geometry of coordinate (x,y) and velocity (V_x,V_y) space with animation apps described ahead on p.69

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html

Introducing Isotropic Harmonic Oscillator (IHO) energy and frequency relations

➔ *Constructing 2D-IHO orbits using Kepler anomaly plots*

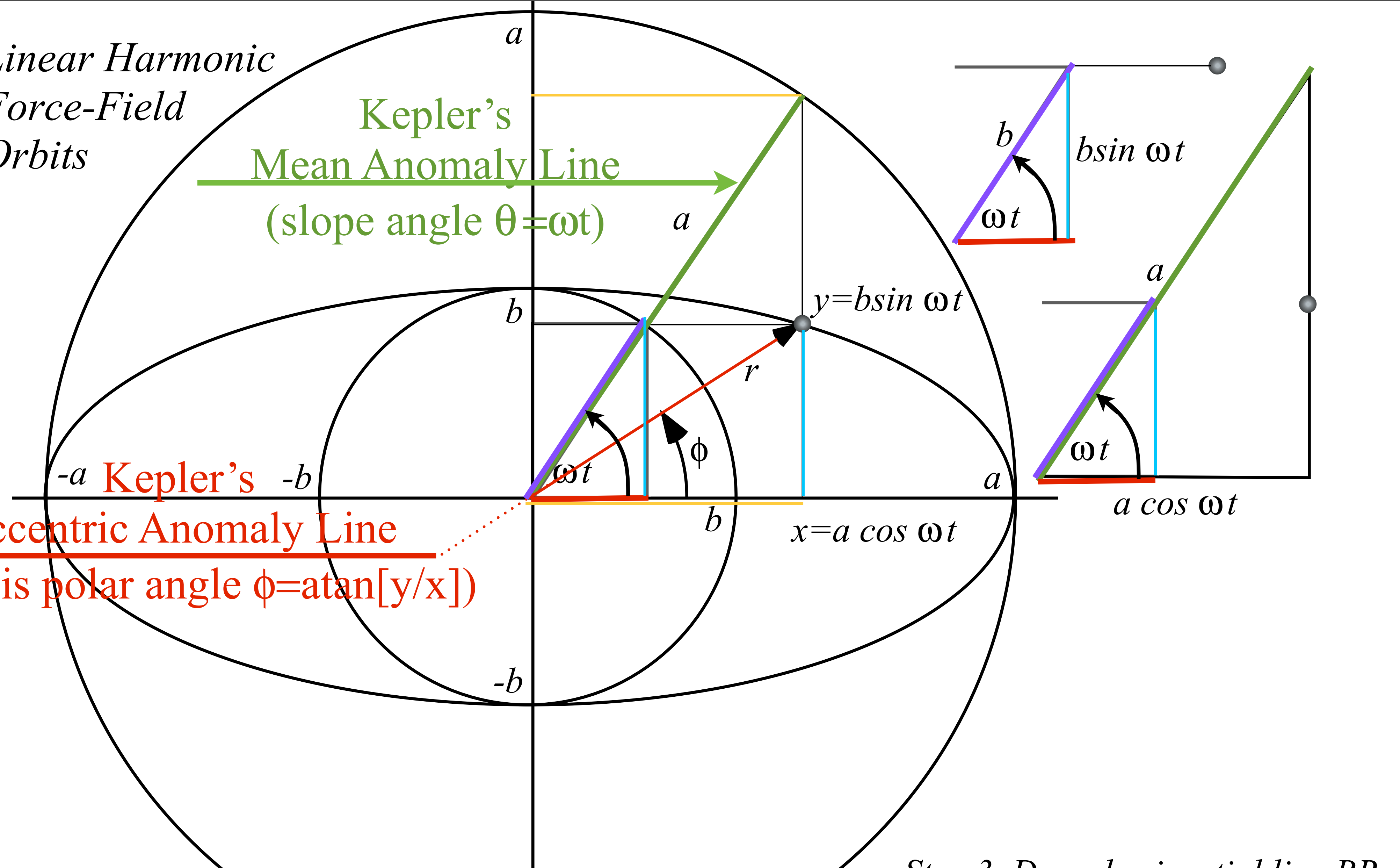
Mean-anomaly and eccentric-anomaly geometry with web-app animation

Calculus and vector geometry of IHO orbits

Constructing 2D-IHO orbits using orbital phasor-clock plots

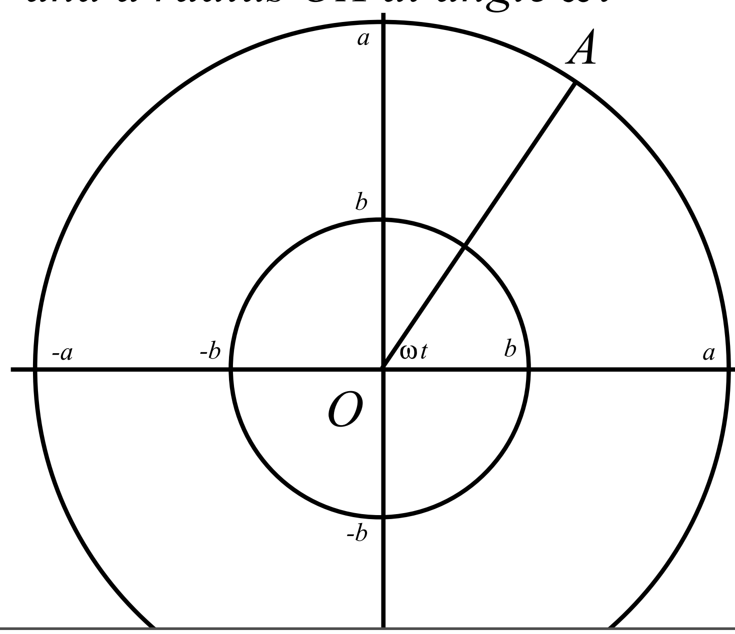
Phasor geometry of coordinate (x,y) and velocity (V_x,V_y) space with web-app animation

Linear Harmonic Force-Field Orbits

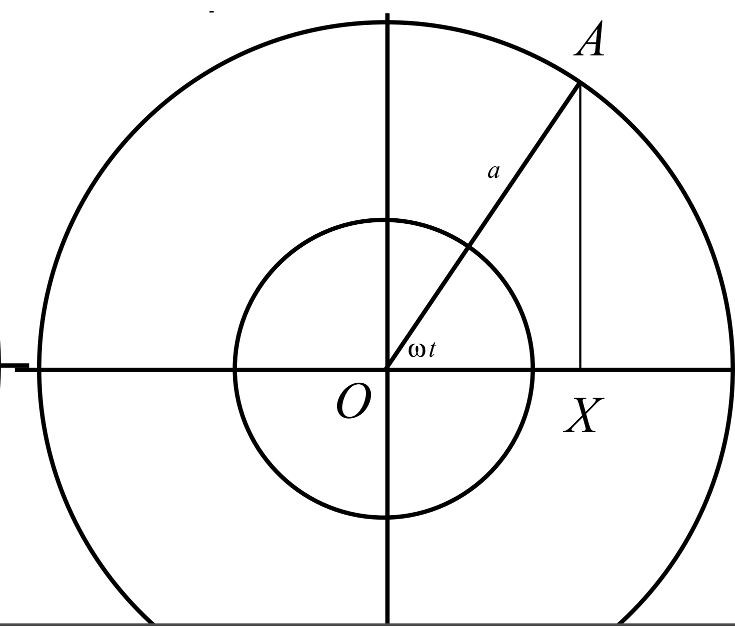


Unit 1
Fig. 9.1
(top 2/3's)

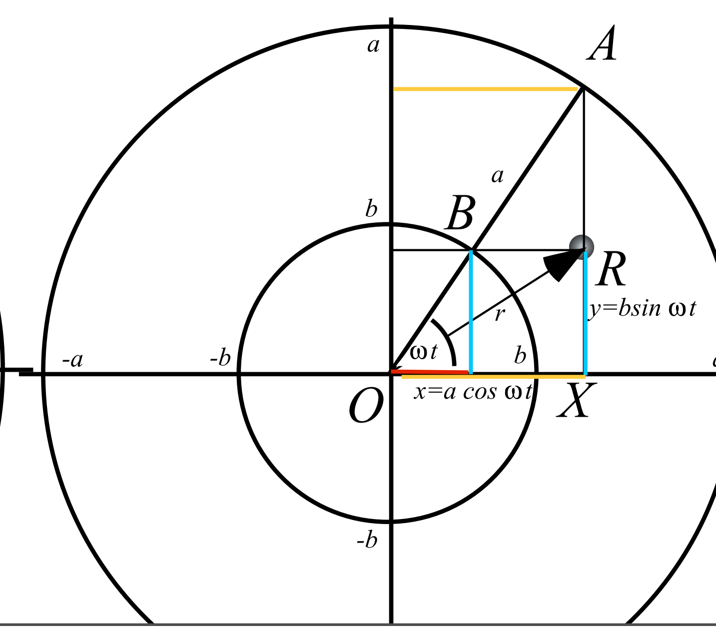
Step 1. Draw concentric circles of radius a and b and a radius OA at angle ωt

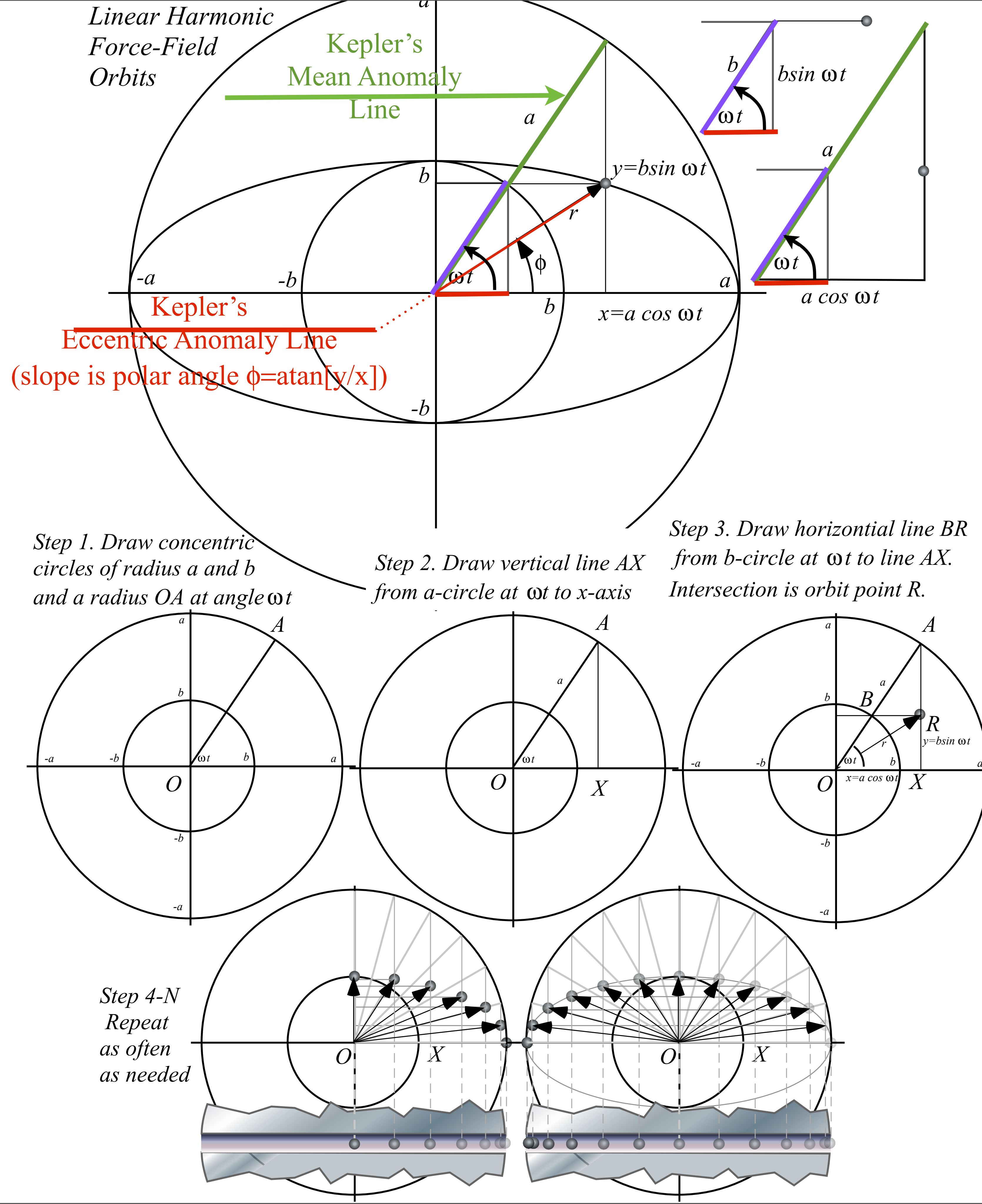


Step 2. Draw vertical line AX from a-circle at ωt to x-axis



Step 3. Draw horizontial line BR from b-circle at ωt to line AX. Intersection is orbit point R.





Linear Harmonic
Force-Field
Orbits
Kepler's
Mean Anomaly
Line
Kepler's
Eccentric Anomaly Line
(slope is polar angle ϕ=atan[y/x])
y=bsin ωt
x=a cos ωt
bsin ωt
a cos ωt
Step 1. Draw concentric
circles of radius a and b
and a radius OA at angle ωt
Step 2. Draw vertical line AX
from a-circle at ωt to x-axis
Step 3. Draw horizontial line BR
from b-circle at ωt to line AX.
Intersection is orbit point R.
Step 4-N
Repeat
as often
as needed

Unit 1
Fig. 9.1

Introducing Isotropic Harmonic Oscillator (IHO) energy and frequency relations

Constructing 2D-IHO orbits using Kepler anomaly plots

Mean-anomaly and eccentric-anomaly geometry with web-app animation

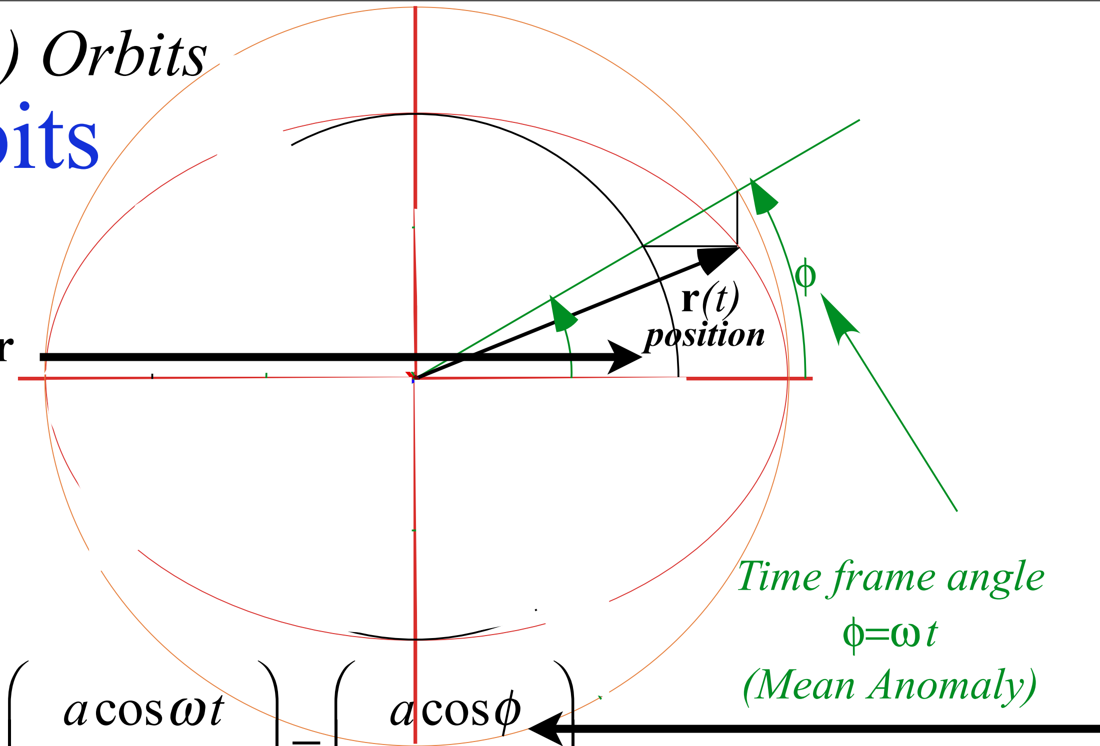
➔ *Calculus and vector geometry of IHO orbits*

Constructing 2D-IHO orbits using orbital phasor-clock plots

Phasor geometry of coordinate (x,y) and velocity (V_x,V_y) space with web-app animation

Calculus of IHO orbits

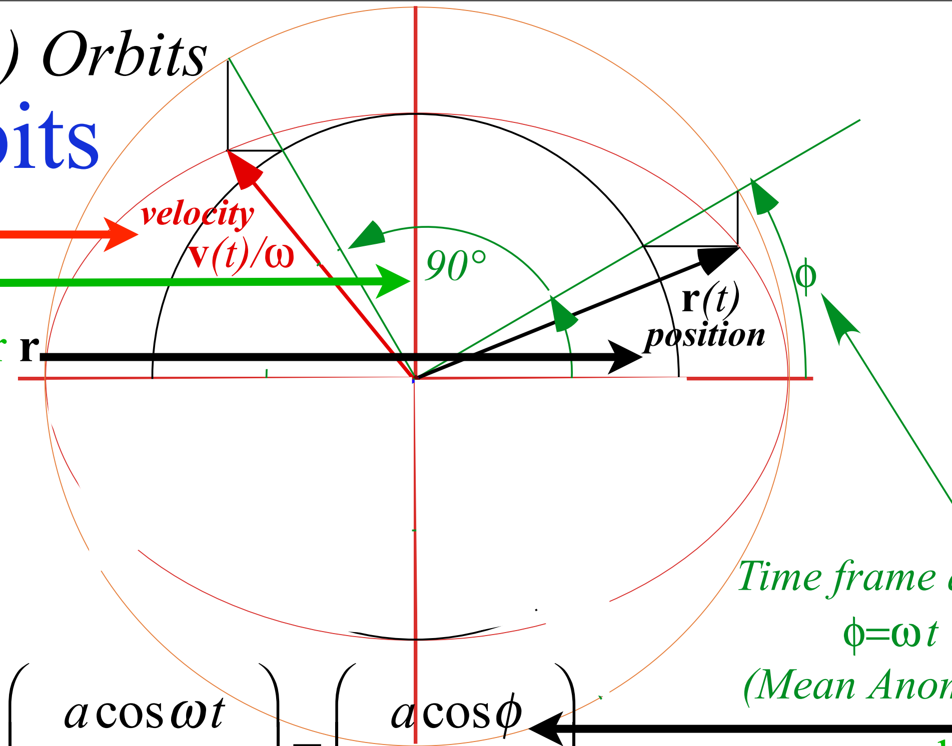
mean-anomaly ϕ of position vector **r**



Unit 1
Fig. 9.5

$$\textit{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

Calculus of IHO orbits



To make velocity vector **v**
just rotate by π/2 or 90°
the mean-anomaly ϕ of position vector **r**

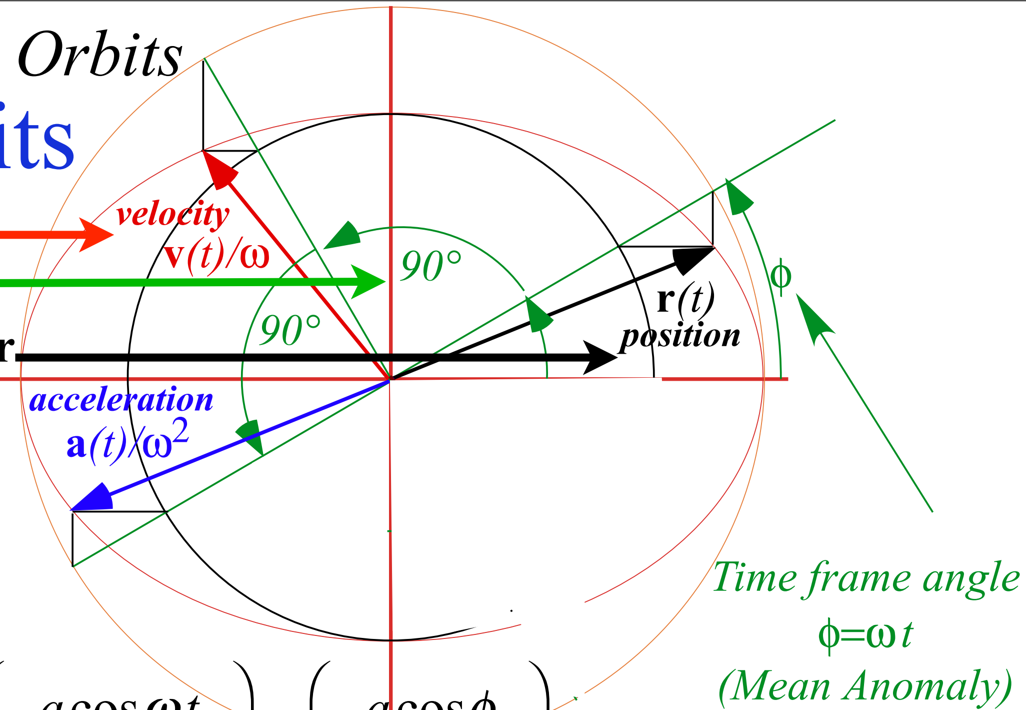
$$radius\ vector: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r**
rotated by π/2 or 90° is *m.a.* of vector **v**

$$velocity\ vector: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (for\ \omega = 1)$$

Unit 1
Fig. 9.5

Calculus of IHO orbits



To make velocity vector **v**
just rotate by π/2 or 90°
the mean-anomaly φ of position vector **r**

Unit 1
Fig. 9.5

$$\text{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly φ of position vector **r** rotated by π/2 or 90° is *m.a.* of vector **v**

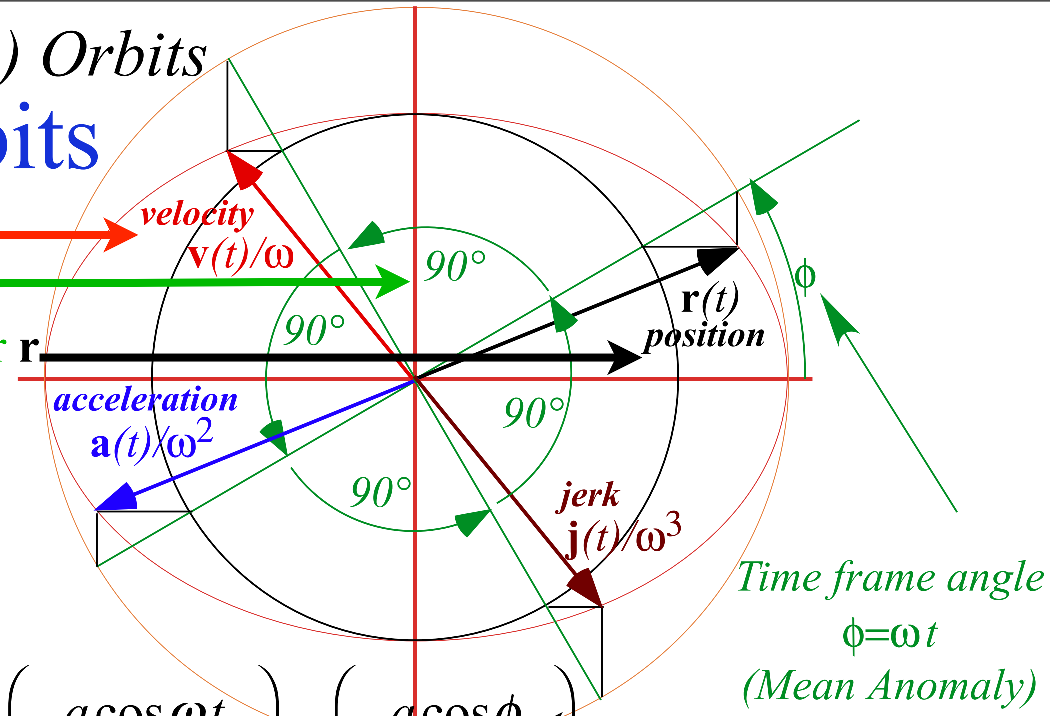
$$\text{velocity vector}: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (\text{for } \omega = 1)$$

m.a. φ+π/2 of vector **v** rotated by another π/2 is *m.a.* of vector **a**

$$\text{acceleration or force vector}: \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi + \frac{2\pi}{2}\right) \\ b\sin\left(\phi + \frac{2\pi}{2}\right) \end{pmatrix}$$

or change of velocity

Calculus of IHO orbits



To make velocity vector **v**
just rotate by π/2 or 90°
the mean-anomaly ϕ of position vector **r**

Unit 1
Fig. 9.5

$$\textit{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r**
rotated by π/2 or 90° is *m.a.* of vector **v**

$$\textit{velocity vector}: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (\textit{for } \omega = 1)$$

m.a. ϕ+π/2 of vector **v** rotated by
another π/2 is *m.a.* of vector **a**

$$\textit{acceleration or force vector}: \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi + \frac{2\pi}{2}\right) \\ b\sin\left(\phi + \frac{2\pi}{2}\right) \end{pmatrix}$$

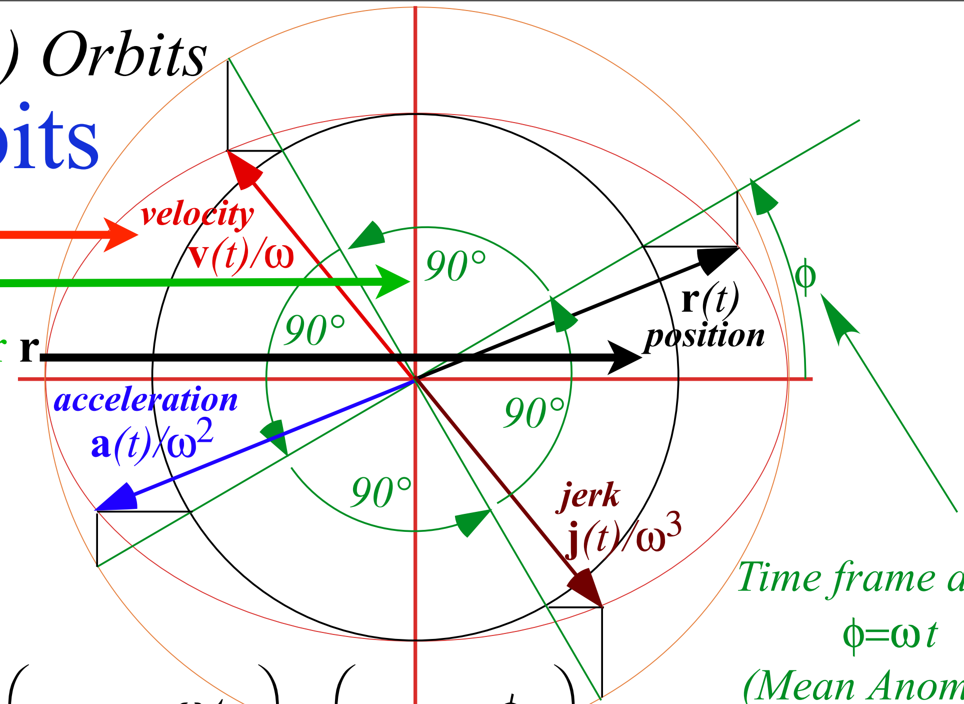
or change of velocity

$$\textit{jerk or change of acceleration}: \mathbf{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} +a\omega^3\sin\omega t \\ -b\omega^3\cos\omega t \end{pmatrix} = \frac{d\mathbf{a}}{dt} = \dot{\mathbf{a}} = \ddot{\mathbf{v}} = \dddot{\mathbf{r}} = \frac{d^3\mathbf{r}}{dt^3} = \begin{pmatrix} a\cos\left(\phi + \frac{3\pi}{2}\right) \\ b\sin\left(\phi + \frac{3\pi}{2}\right) \end{pmatrix}$$

...and so forth...

Calculus of IHO orbits

To make velocity vector $\mathbf{v}$
just rotate by $\pi/2$ or 90°
the mean-anomaly ϕ of position vector $\mathbf{r}$



Time frame angle
$\phi=\omega t$
(Mean Anomaly)

Unit 1
Fig. 9.5

$$radius\ vector: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector $\mathbf{r}$
rotated by $\pi/2$ or 90° is *m.a.* of vector $\mathbf{v}$

$$velocity\ vector: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi + \frac{\pi}{2}\right) \\ b\sin\left(\phi + \frac{\pi}{2}\right) \end{pmatrix} (for\ \omega = 1)$$

m.a. $\phi+\pi/2$ of vector $\mathbf{v}$ rotated by
another $\pi/2$ is *m.a.* of vector $\mathbf{a}$

$$acceleration\ or\ force\ vector: \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi + \frac{2\pi}{2}\right) \\ b\sin\left(\phi + \frac{2\pi}{2}\right) \end{pmatrix}$$
$$or\ change\ of\ velocity$$

$$jerk\ or\ change\ of\ acceleration: \mathbf{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} +a\omega^3\sin\omega t \\ -b\omega^3\cos\omega t \end{pmatrix} = \frac{d\mathbf{a}}{dt} = \dot{\mathbf{a}} = \ddot{\mathbf{v}} = \dddot{\mathbf{r}} = \frac{d^3\mathbf{r}}{dt^3} = \begin{pmatrix} a\cos\left(\phi + \frac{3\pi}{2}\right) \\ b\sin\left(\phi + \frac{3\pi}{2}\right) \end{pmatrix}$$

...and so forth...

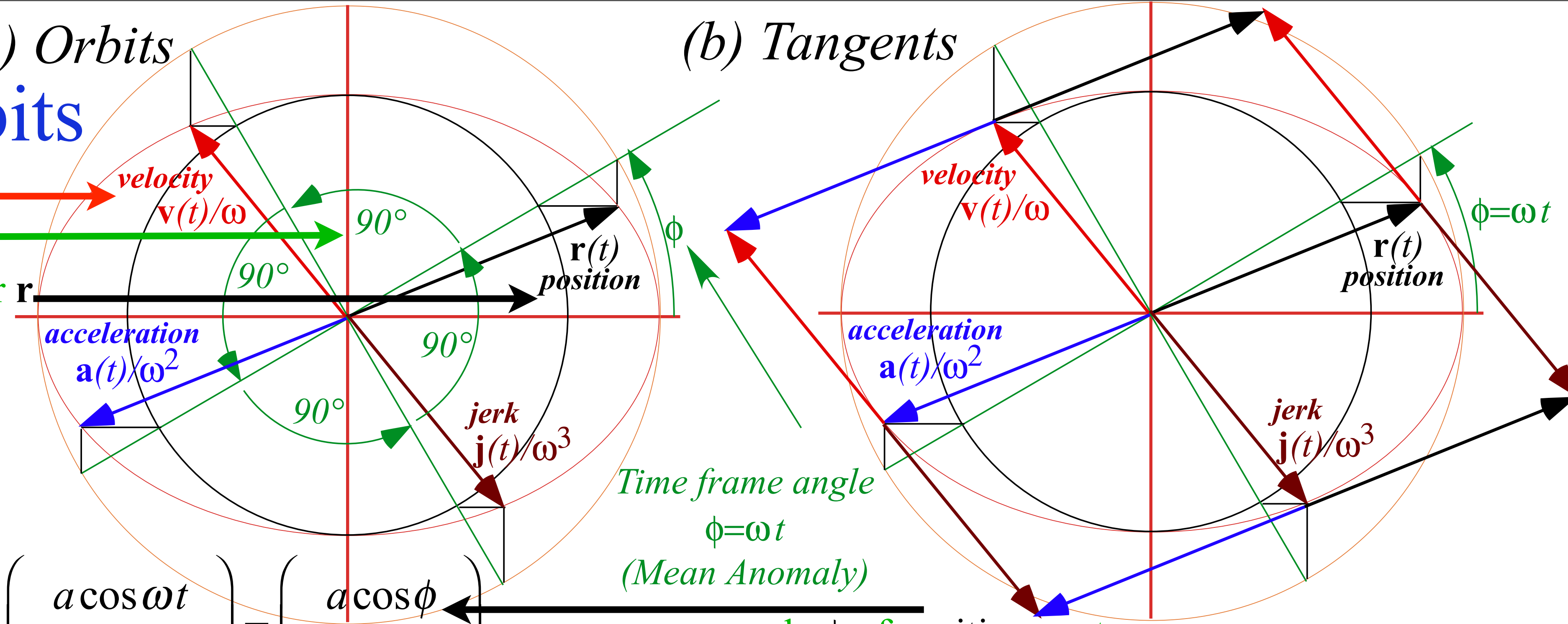
$$inauguration\ or\ change\ of\ jerk: \mathbf{i} = \begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} +a\omega^4\cos\omega t \\ +b\omega^4\sin\omega t \end{pmatrix} = \frac{d\mathbf{j}}{dt} = \dot{\mathbf{j}} = \ddot{\mathbf{a}} = \dddot{\mathbf{v}} = \ddddot{\mathbf{r}} = \frac{d^4\mathbf{r}}{dt^4} = \begin{pmatrix} a\cos\left(\phi + \frac{4\pi}{2}\right) \\ b\sin\left(\phi + \frac{4\pi}{2}\right) \end{pmatrix}$$

...and so on...
...But, now it
repeats after 4
t-derivatives

Calculus of IHO orbits

To make velocity vector **v** just rotate by $\pi/2$ or 90° the mean-anomaly ϕ of position vector **r**

Link ⇒ [BoxIt simulation of IHO orbits]



$$\text{radius vector}: \mathbf{r} = \begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} = \begin{pmatrix} a\cos\phi \\ b\sin\phi \end{pmatrix}$$

mean-anomaly ϕ of position vector **r** rotated by $\pi/2$ or 90° is *m.a.* of vector **v**

Unit 1
Fig. 9.5

Link → [IHO Exegesis Plot]

Link → [IHO orbital time rates of change]

$$\text{velocity vector}: \mathbf{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix} = \frac{d\mathbf{r}}{dt} = \dot{\mathbf{r}} = \begin{pmatrix} a\cos\left(\phi+\frac{\pi}{2}\right) \\ b\sin\left(\phi+\frac{\pi}{2}\right) \end{pmatrix} (\text{for } \omega = 1)$$

m.a. $\phi+\pi/2$ of vector **v** rotated by another $\pi/2$ is *m.a.* of vector **a**

$$\text{acceleration or force vector}: \frac{\mathbf{F}}{m} = \mathbf{a} = \begin{pmatrix} a_x \\ a_y \end{pmatrix} = \begin{pmatrix} -a\omega^2\cos\omega t \\ -b\omega^2\sin\omega t \end{pmatrix} = \frac{d\mathbf{v}}{dt} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \frac{d^2\mathbf{r}}{dt^2} = \begin{pmatrix} a\cos\left(\phi+\frac{2\pi}{2}\right) \\ b\sin\left(\phi+\frac{2\pi}{2}\right) \end{pmatrix}$$

or change of velocity

$$\text{jerk or change of acceleration}: \mathbf{j} = \begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} +a\omega^3\sin\omega t \\ -b\omega^3\cos\omega t \end{pmatrix} = \frac{d\mathbf{a}}{dt} = \dot{\mathbf{a}} = \ddot{\mathbf{v}} = \dddot{\mathbf{r}} = \frac{d^3\mathbf{r}}{dt^3} = \begin{pmatrix} a\cos\left(\phi+\frac{3\pi}{2}\right) \\ b\sin\left(\phi+\frac{3\pi}{2}\right) \end{pmatrix}$$

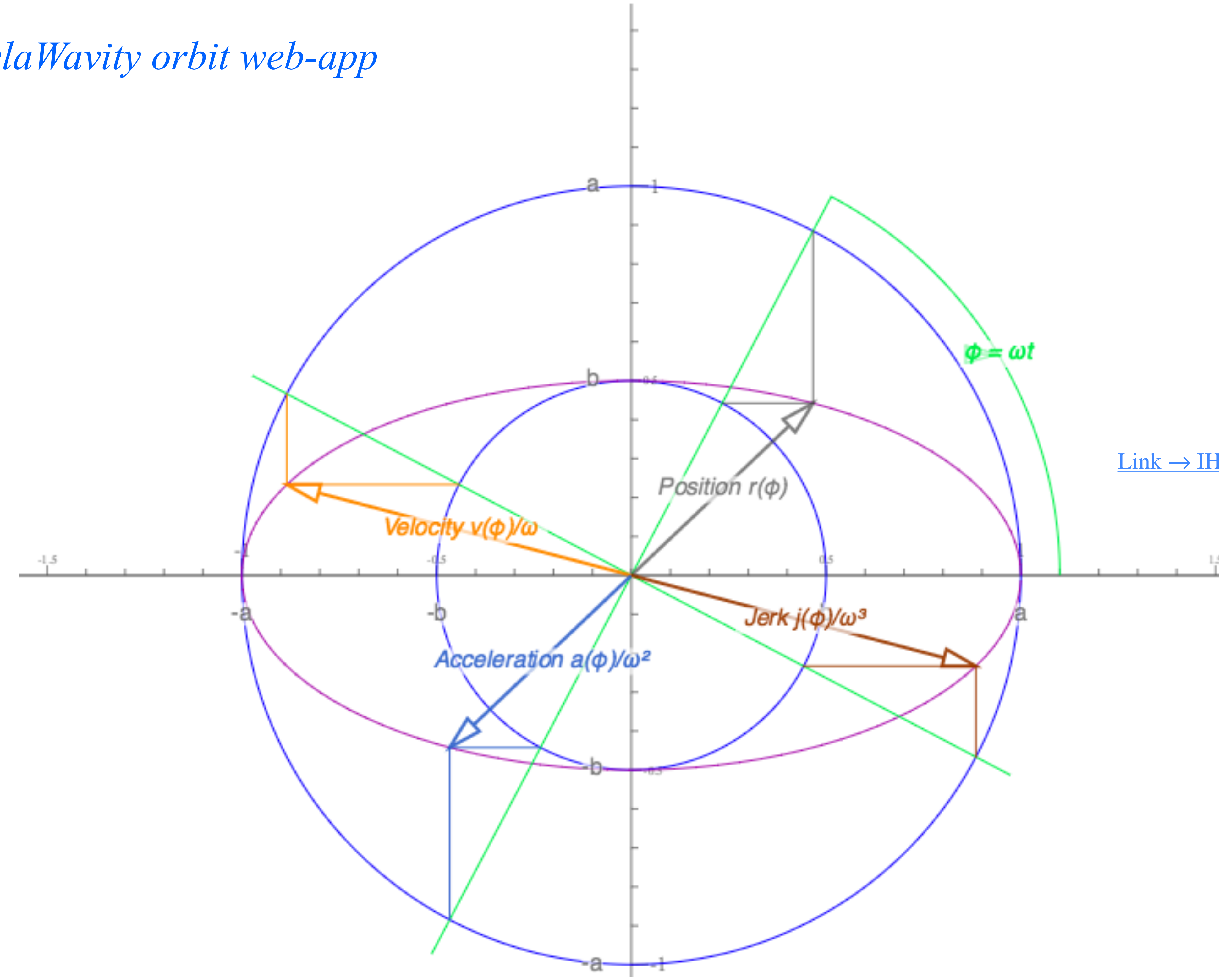
...and so forth...

$$\text{inauguration or change of jerk}: \mathbf{i} = \begin{pmatrix} i_x \\ i_y \end{pmatrix} = \begin{pmatrix} +a\omega^4\cos\omega t \\ +b\omega^4\sin\omega t \end{pmatrix} = \frac{d\mathbf{j}}{dt} = \dot{\mathbf{j}} = \ddot{\mathbf{a}} = \dddot{\mathbf{v}} = \ddddot{\mathbf{r}} = \frac{d^4\mathbf{r}}{dt^4} = \begin{pmatrix} a\cos\left(\phi+\frac{4\pi}{2}\right) \\ b\sin\left(\phi+\frac{4\pi}{2}\right) \end{pmatrix}$$

...and so on...
...But, now it repeats after 4 *t*-derivatives

Elliptical | Orbits | Controls | Contextual | Set ISM | User's Guide

RelaWavity orbit web-app

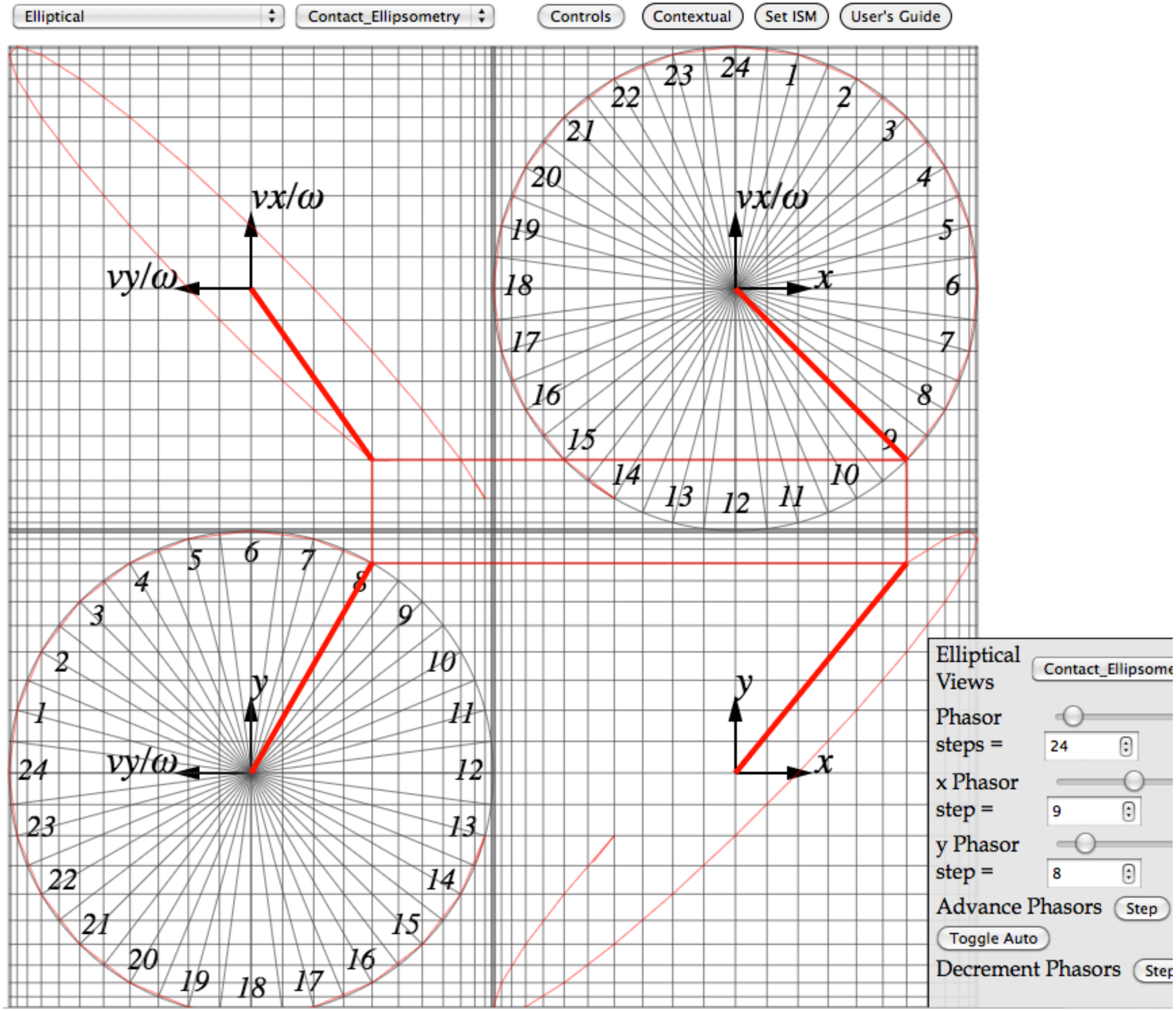


[Link → IHO orbital time rates of change]

Geometry of Kepler anomalies for vectors [$\mathbf{r}(\phi)$, $\mathbf{v}(\phi)$, $\mathbf{a}(\phi)$, $\mathbf{j}(\phi)$,] *in coordinate (x,y) space rendered by animation web-apps BoxIt and RelaWavity.*

RelaWavity
ellipsometry
web-app

[RelaWavity Web Simulation](http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=1I2)
[*Ellipsometry*](http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=1I2)

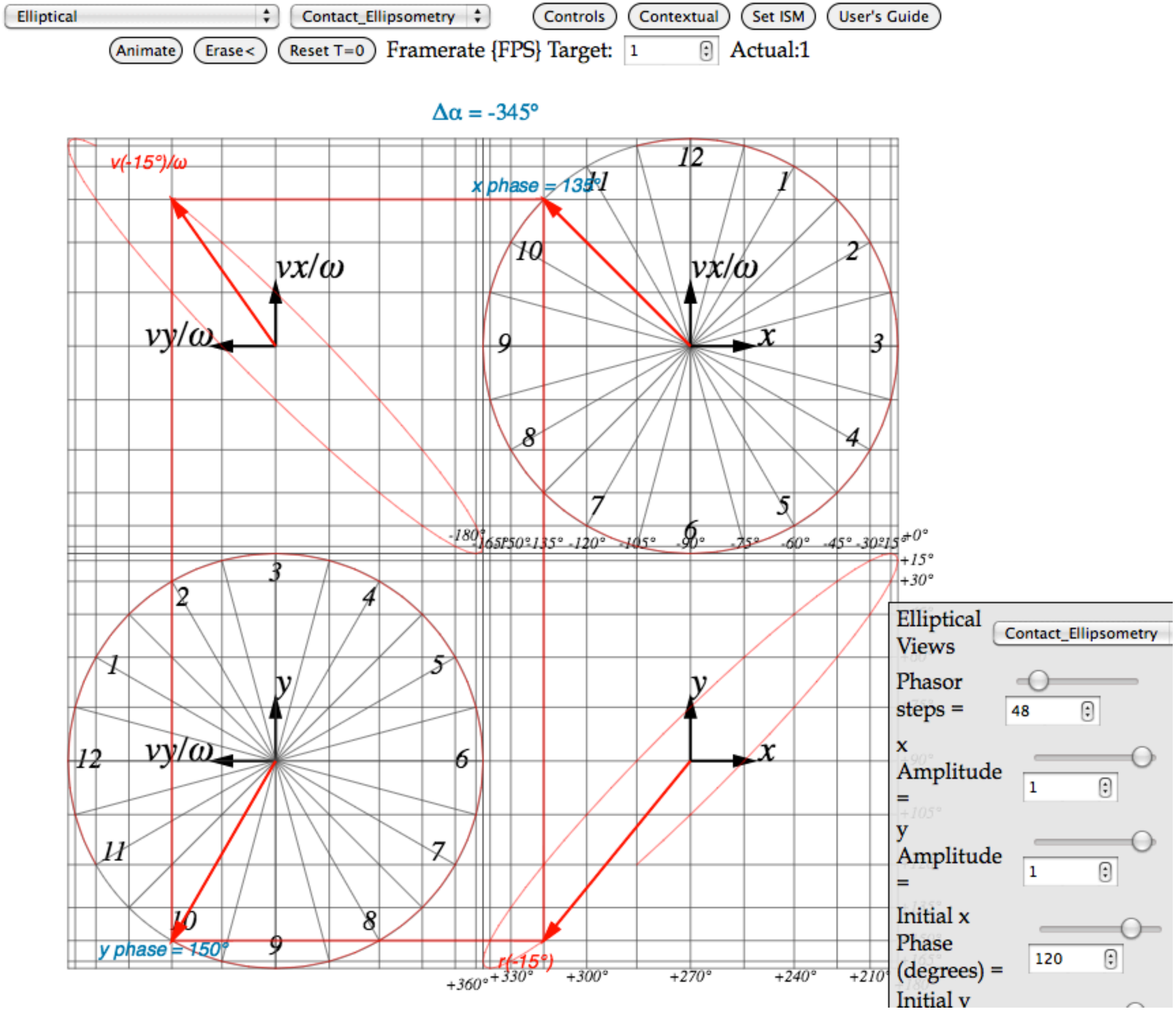


Geometry of Kepler anomalies for vectors [**r**(ϕ), **v**(ϕ)] *in coordinate (x,y) space rendered by animation web-apps BoxIt and RelaWavity described below after p.70.*

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html?plotType=1I2

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=1I2

RelaWavity ellipsometry web-app

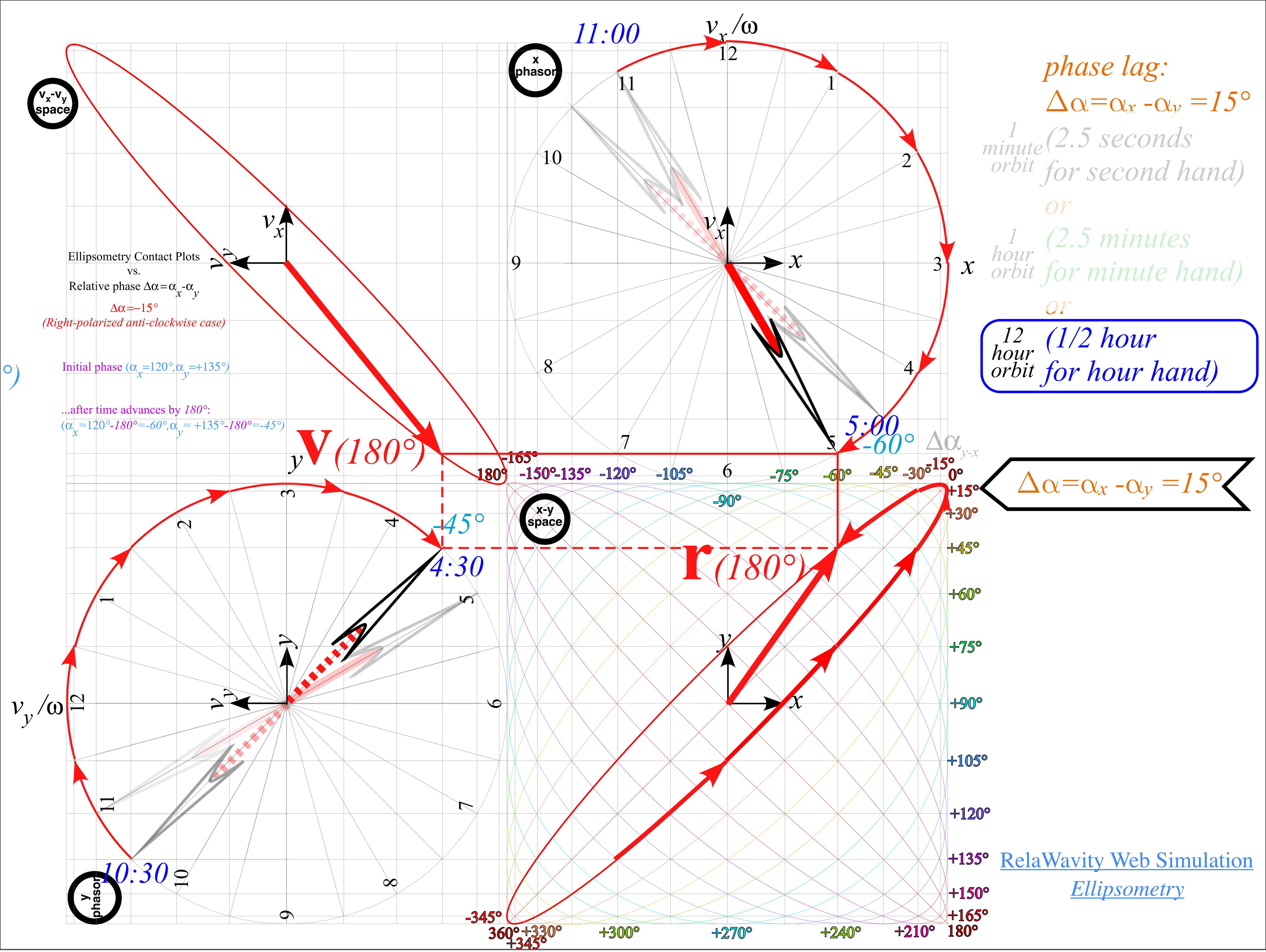


[RelaWavity Web Simulation *Ellipsometry*](http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=1l2)

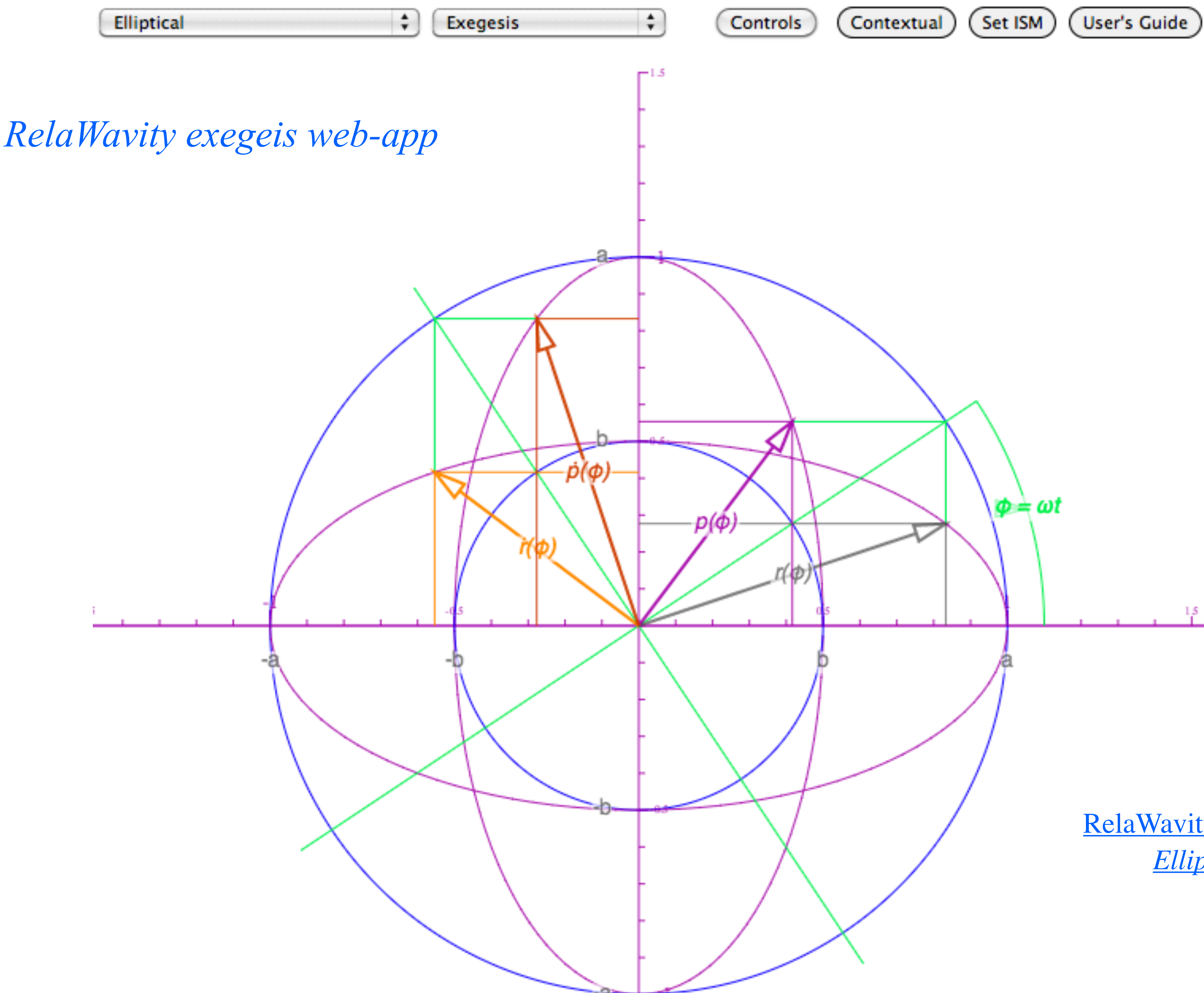
Geometry of Kepler anomalies for vectors [$\mathbf{r}(\phi)$, $\mathbf{v}(\phi)$] *in coordinate (x,y) space rendered by animation web-apps BoxIt and RelaWavity described below after p.70.*

http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html?plotType=1l2

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=1l2

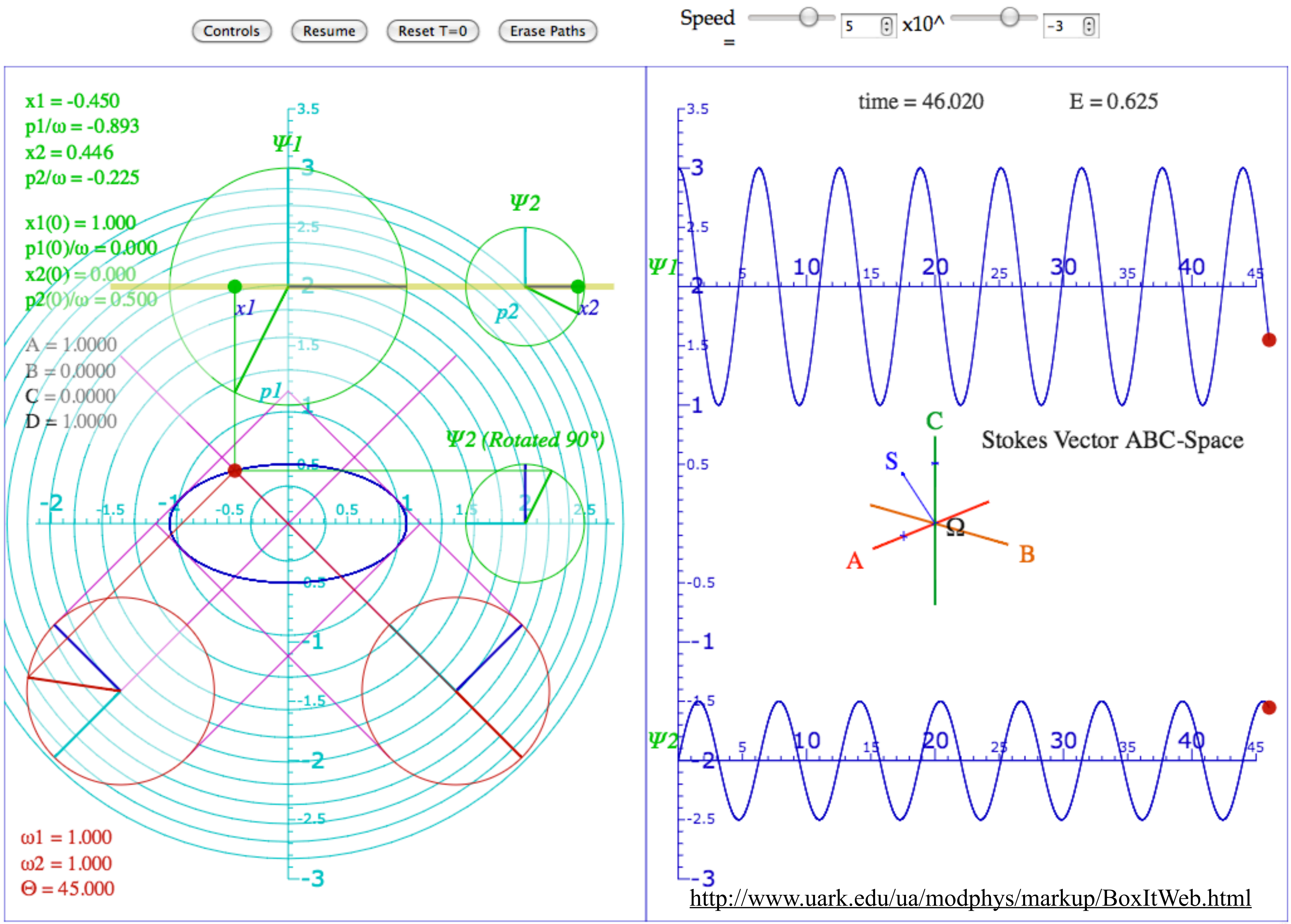


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 180°:
(α_x=120°-180°=-60°,α_y=+135°-180°=-45°)
v_x-v_y space
x phasor
y phasor
x-y space
11:00
5:00
4:30
10:30
v_x/ω
v_y/ω
V(180°)
r(180°)
-60°
-45°
Δα_{y-x}
phase lag:
Δα=α_x -α_y =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Δα=α_x -α_y =15°
RelaWavity Web Simulation
Ellipsometry



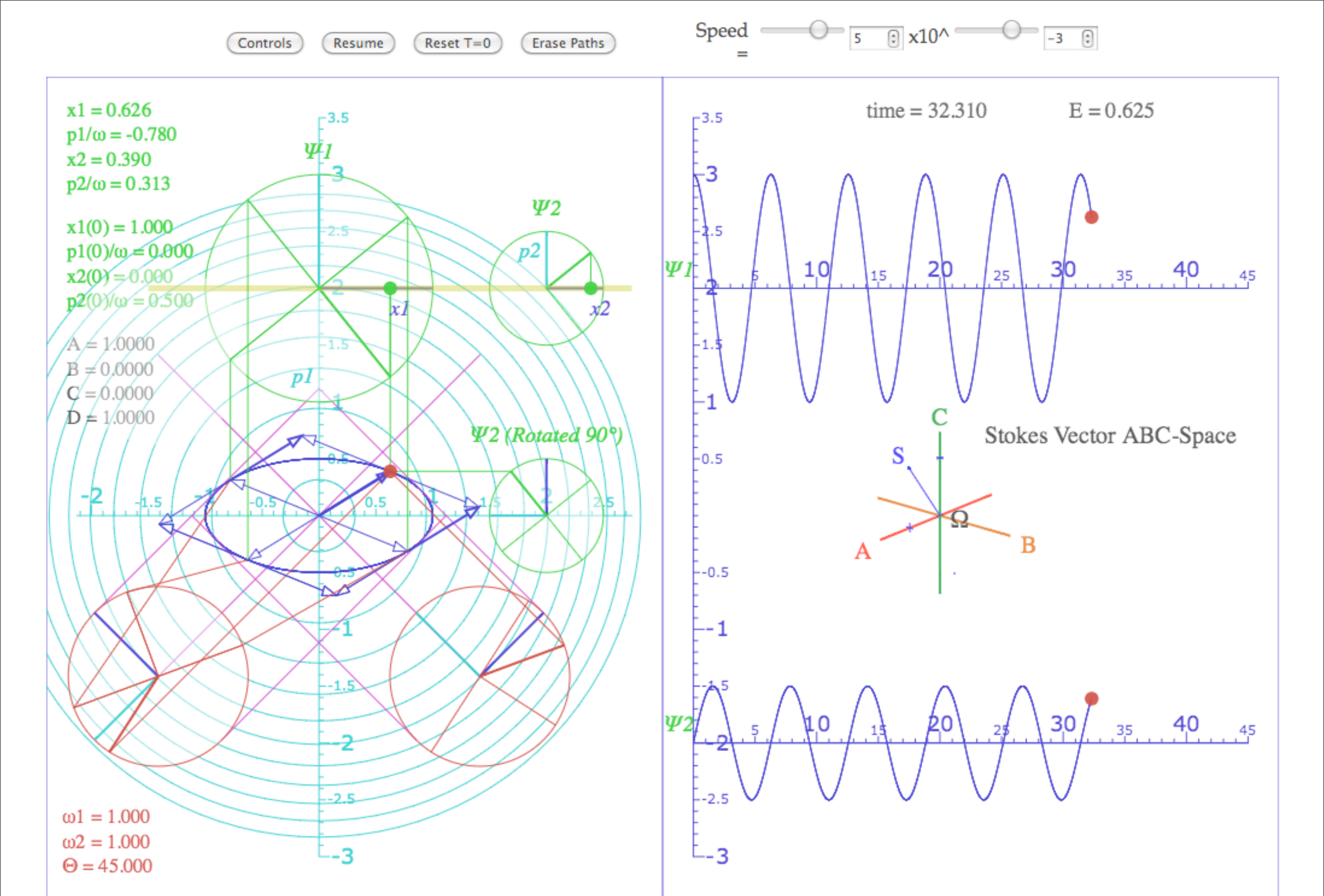
[RelaWavity Web Simulation](#)
[Ellipse/Exegesis](#)

Geometry of dual ellipse Kepler anomalies for vectors [$\mathbf{r}(\phi)$, $\mathbf{p}(\phi)$] and $^{d}/_{dt}$[$\mathbf{r}(\phi)$, $\mathbf{p}(\phi)$,] *in coordinate (x,y) space rendered by animation web-app in RelaWavity and described in Lect. 12-advanced.*

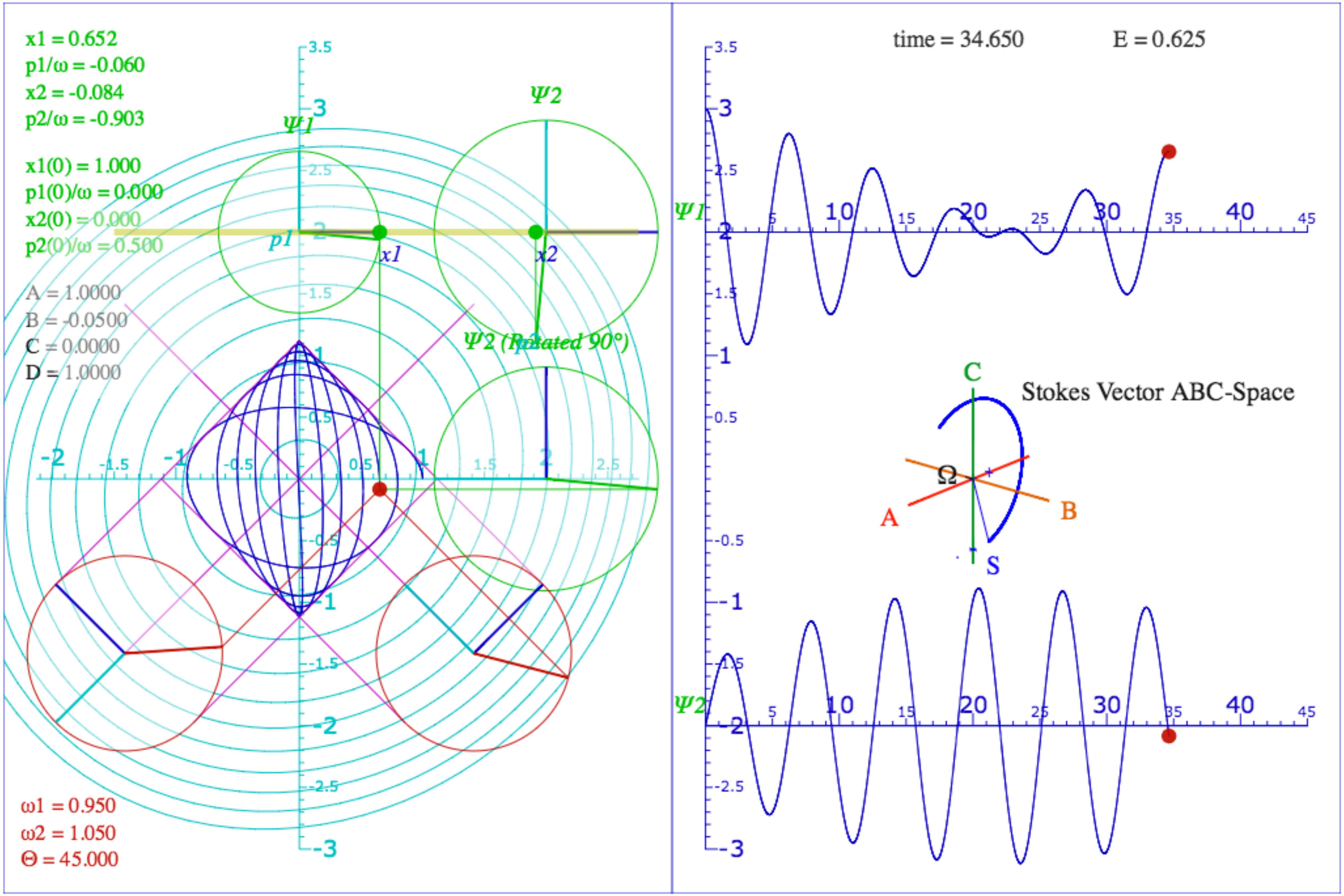


Geometry of Kepler anomalies for vectors [**r**(*ϕ*)] *in coordinate (x,y) space and 2-particle* (x_1,x_2) *space rendered by animation web-apps BoxIt.*

[*BoxIt Web Simulation*](http://www.uark.edu/ua/modphys/markup/BoxItWeb.html)



Geometry of Kepler anomalies for vectors [**r**(ϕ), **v**(ϕ), **a**(ϕ), **j**(ϕ),] *in coordinate (x,y) space and 2-particle* (x_1,x_2) *space rendered by animation web-apps BoxIt.* [BoxIt Web Simulation - *w/Derivatives*]



Geometry of vectors [**r**(*φ*), **p**(*φ*)] *and quantum spin S-space and 2-particle* (x_1,x_2) *space rendered by animation web-apps BoxIt.*

BoxIt Web Simulation - *B-Type Motion*

Introducing Isotropic Harmonic Oscillator (IHO) energy and frequency relations

Constructing 2D-IHO orbits using Kepler anomaly plots

Mean-anomaly and eccentric-anomaly geometry with web-app animation

Calculus and vector geometry of IHO orbits

→ *Constructing 2D-IHO orbits using orbital phasor-clock plots*

Phasor geometry of coordinate (x,y) and velocity (V_x,V_y) space with web-app animation

Introducing Isotropic Harmonic Oscillator (IHO) energy and frequency relations

Constructing 2D-IHO orbits using Kepler anomaly plots

Mean-anomaly and eccentric-anomaly geometry with web-app animation

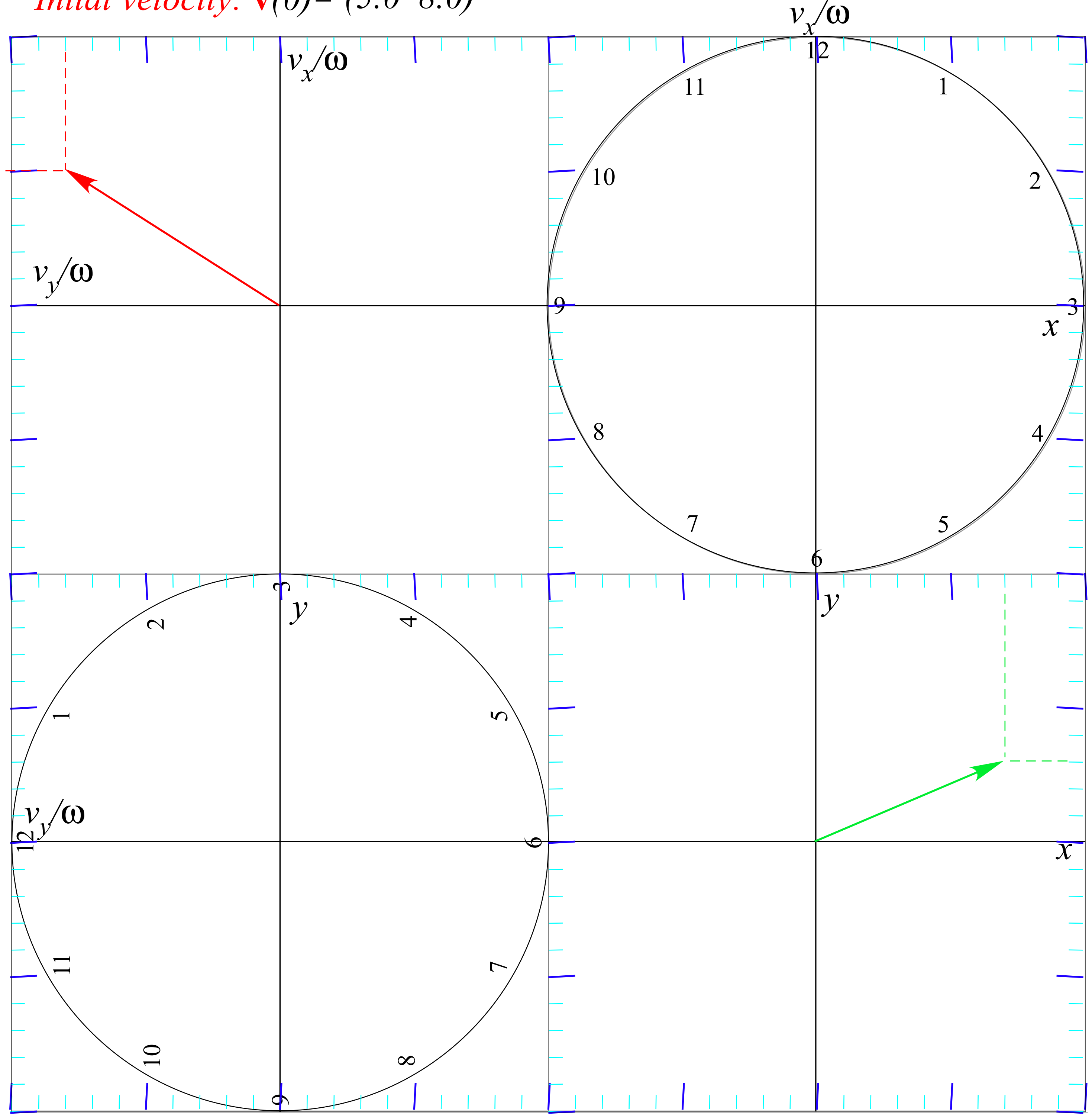
Calculus and vector geometry of IHO orbits

Constructing 2D-IHO orbits using orbital phasor-clock plots

➔ *Phasor geometry of coordinate (x,y) and velocity (V_x,V_y) space with web-app animation*

Review of IHO orbital phasor "clock" dynamics in uniform-body

Inital velocity: **v**(0)= (5.0 8.0)



Initial position: **r**(0)=(7.0 3.0)

RelaWavity Web Simulation

Ellipsometry

BoxIt simulation of U(2) orbits

http://www.uark.edu/ua/modphys/markup/BoxItWeb.html

Review of IHO orbital phasor "clock" dynamics in uniform-body

phase lag:

$\Delta\alpha=\alpha_x - \alpha_y = 30°$

1 minute orbit (5 seconds for second hand)

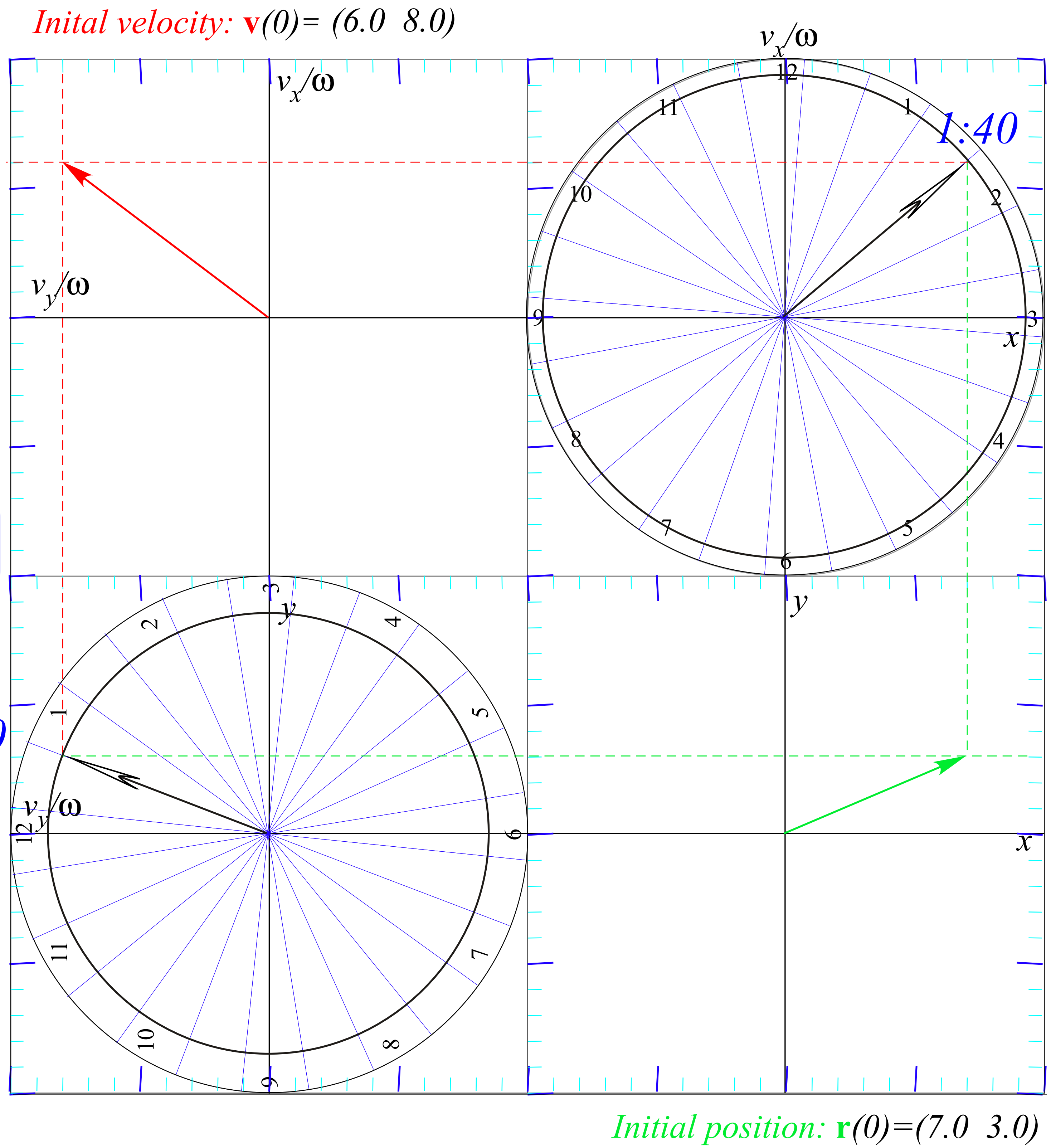
or

1 hour orbit (5 minutes for minute hand)

or

12 hour orbit (1 hour for hour hand)

12:40



Arbitrary initial position
r(0)=(x(0),y(0))

and initial velocity
v(0)=(v_x(0),v_y(0))

Usually have x and y
phasor circles of unequal size

Review of IHO orbital phasor "clock" dynamics in uniform-body

Inital velocity: $\mathbf{v}(0)=(6.0\ \ 8.0)$

phase lag:

$\Delta\alpha=\alpha_x-\alpha_y=30°$

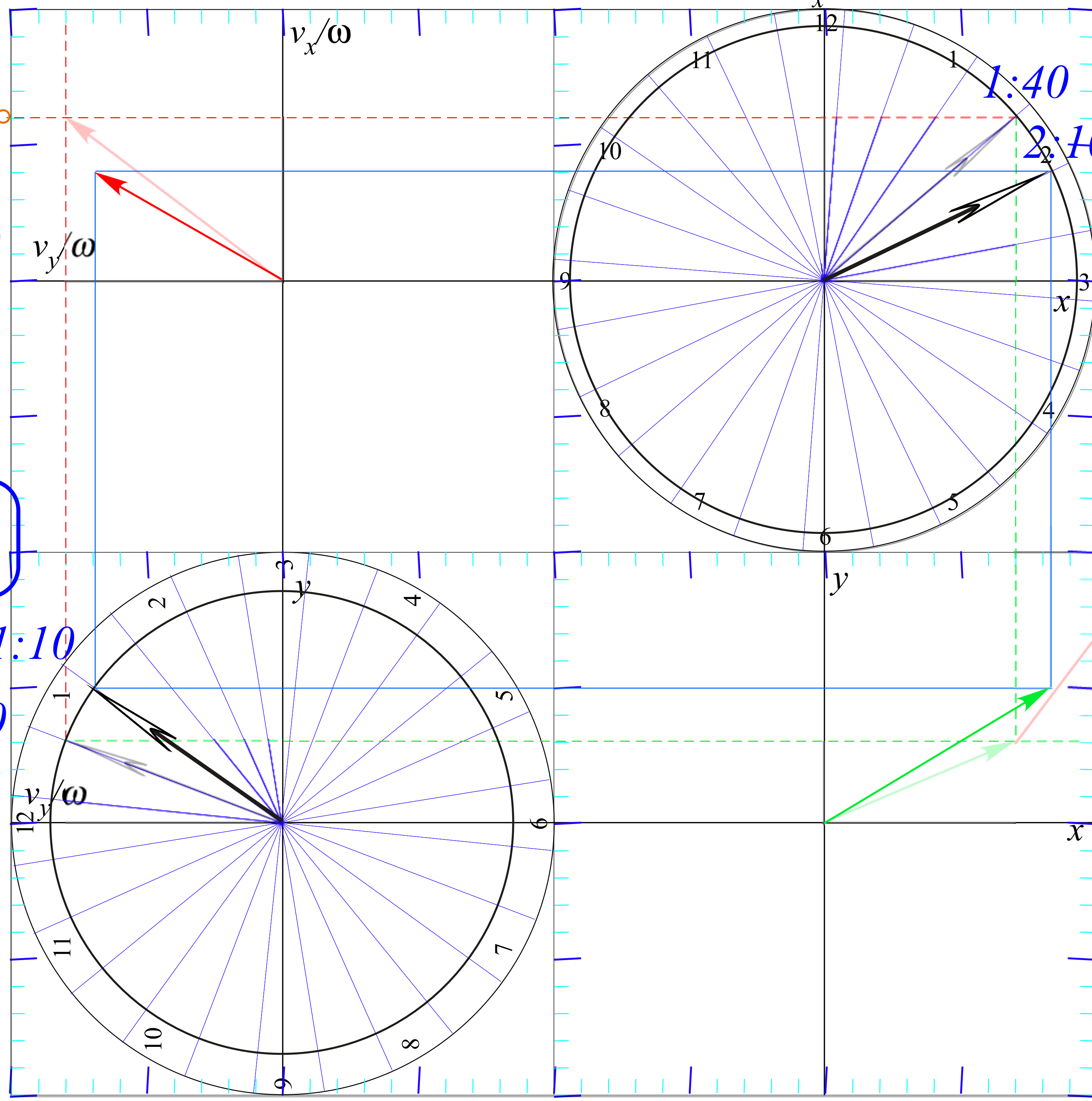
1 minute orbit (5 seconds for second hand)

or

1 hour orbit (5 minutes for minute hand)

or

12 hour orbit (1 hour for hour hand)



Arbitrary initial position $\mathbf{r}(0)=(x(0),y(0))$

and initial velocity $\mathbf{v}(0)=(v_x(0),v_y(0))$

Usually have x and y phasor circles of unequal size

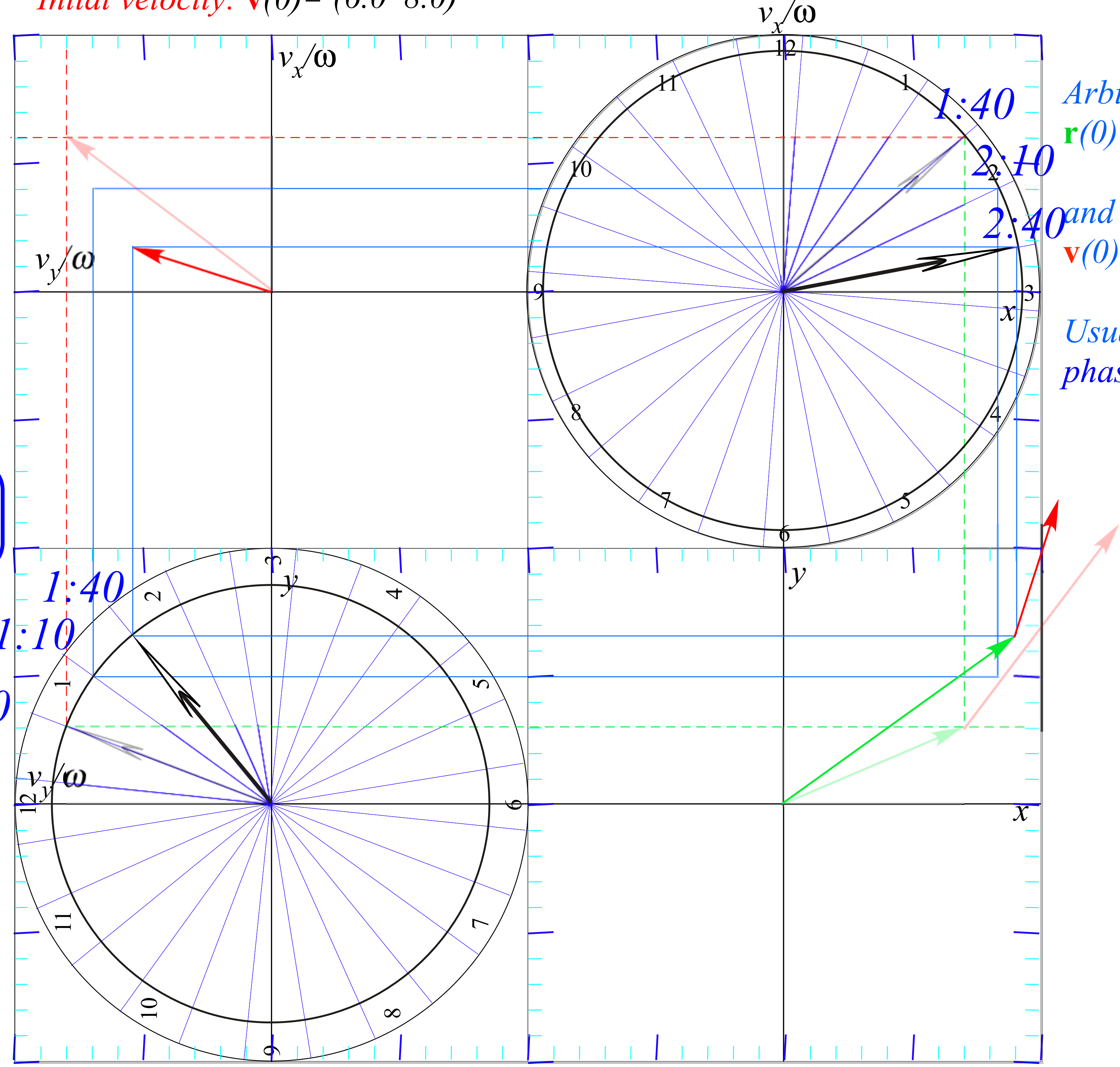
Initial position: $\mathbf{r}(0)=(7.0\ \ 3.0)$

Review of IHO orbital phasor "clock" dynamics in uniform-body

Inital velocity: $\mathbf{v}(0)=(6.0\ \ 8.0)$

phase lag:
$\Delta\alpha=\alpha_x-\alpha_y=30°$

1 minute orbit (5 seconds for second hand)
or
1 hour orbit (5 minutes for minute hand)
or
12 hour orbit (1 hour for hour hand)



Arbitrary initial position
$\mathbf{r}(0)=(x(0),y(0))$

and initial velocity
$\mathbf{v}(0)=(v_x(0),v_y(0))$

Usually have x and y
phasor circles of unequal size

Initial position: $\mathbf{r}(0)=(7.0\ \ 3.0)$

Review of IHO orbital phasor "clock" dynamics in uniform-body

phase lag:

$\Delta\alpha = \alpha_x - \alpha_y = 30°$

1 minute orbit (5 seconds for second hand)

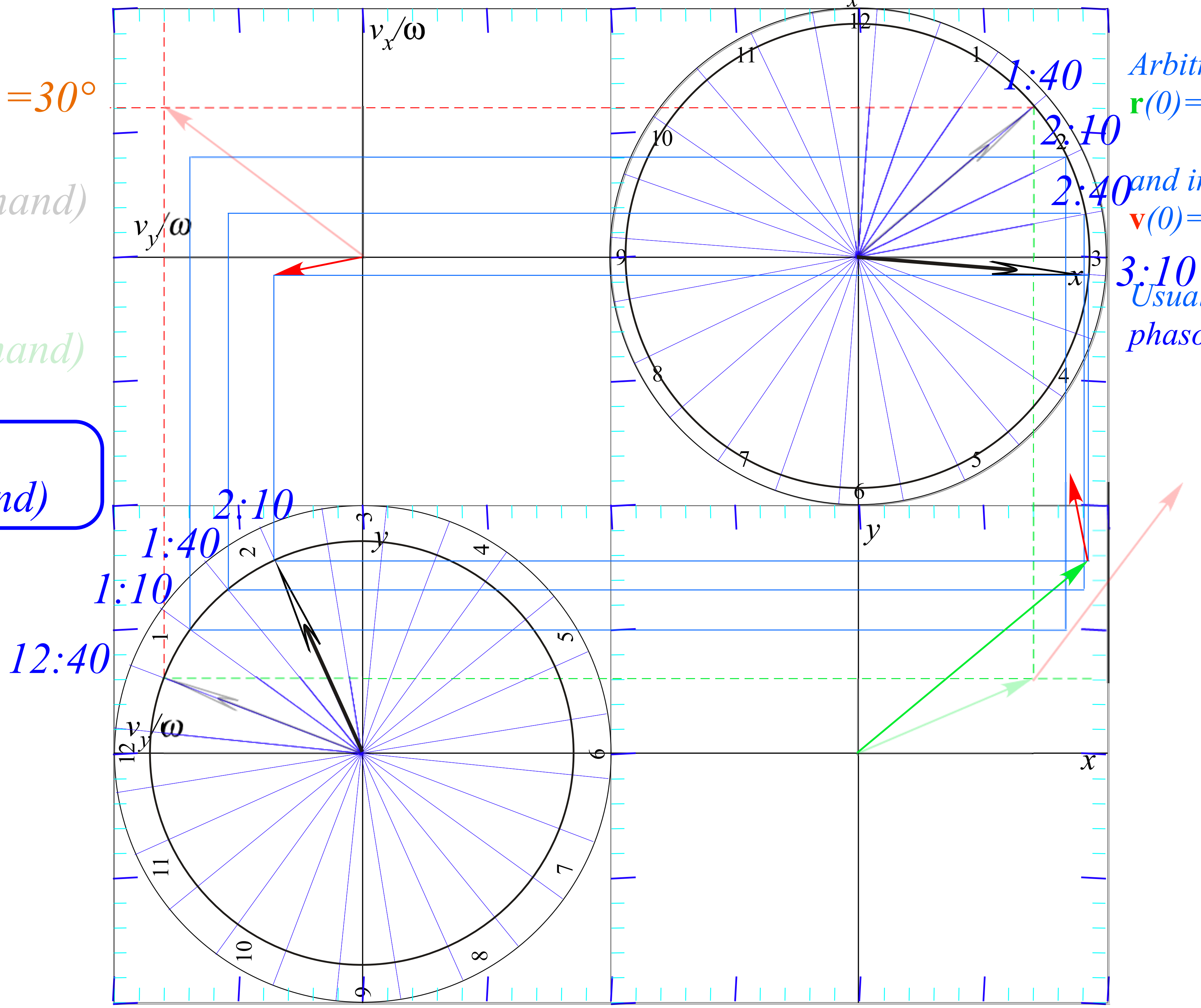
or

1 hour orbit (5 minutes for minute hand)

or

12 hour orbit (1 hour for hour hand)

Inital velocity: $\mathbf{v}(0) = (6.0\ \ 8.0)$



Arbitrary initial position $\mathbf{r}(0) = (x(0), y(0))$

and initial velocity $\mathbf{v}(0) = (v_x(0), v_y(0))$

Usually have x and y phasor circles of unequal size

Initial position: $\mathbf{r}(0) = (7.0\ \ 3.0)$

Review of IHO orbital phasor "clock" dynamics in uniform-body

Inital velocity: $\mathbf{v}(0)=(6.0\ \ 8.0)$

phase lag:

$\Delta\alpha=\alpha_x-\alpha_y=30°$

1 minute orbit (5 seconds for second hand)

or

1 hour orbit (5 minutes for minute hand)

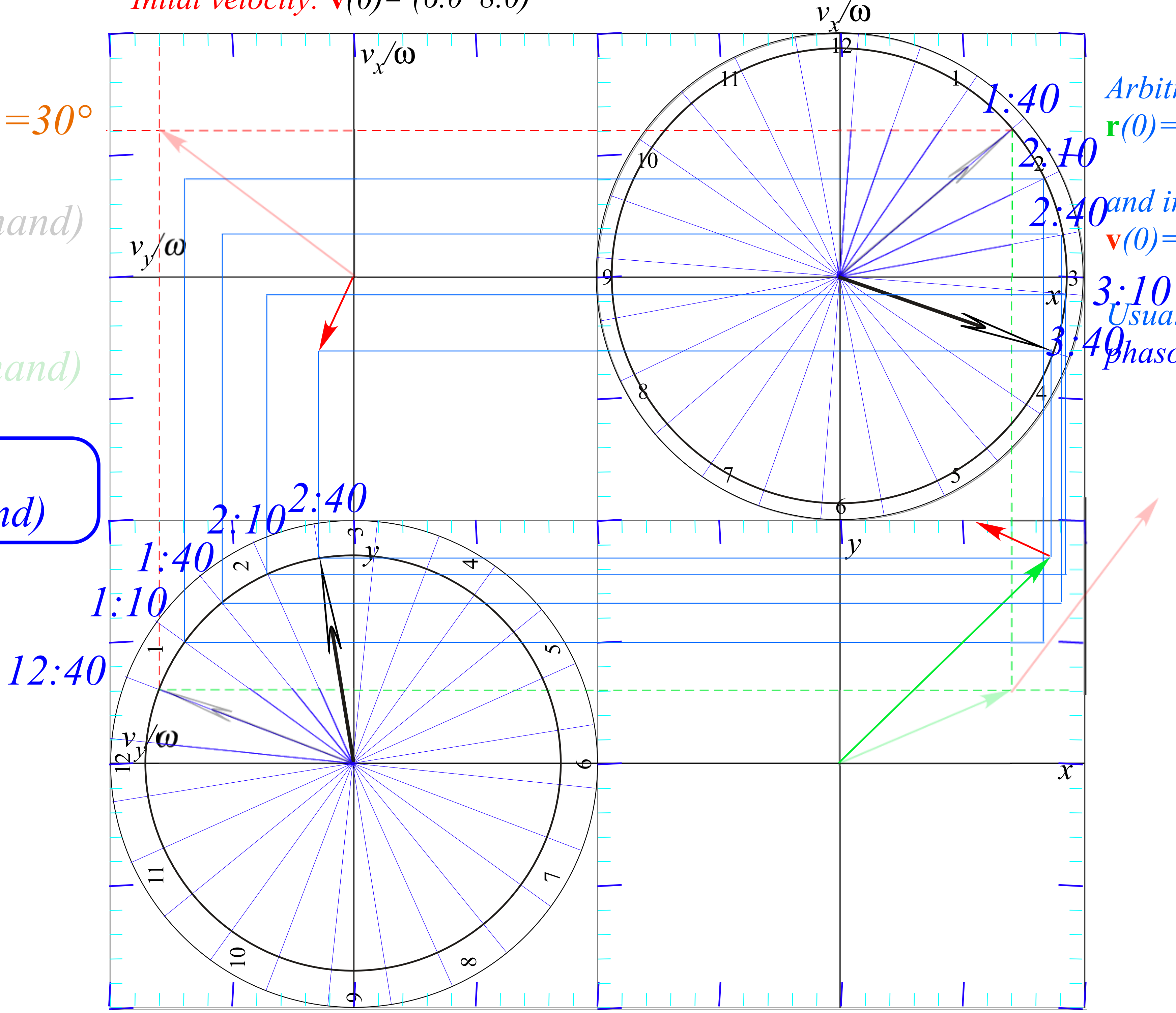
or

12 hour orbit (1 hour for hour hand)

Arbitrary initial position $\mathbf{r}(0)=(x(0),y(0))$

and initial velocity $\mathbf{v}(0)=(v_x(0),v_y(0))$

Usually have x and y phasor circles of unequal size



Initial position: $\mathbf{r}(0)=(7.0\ \ 3.0)$

Review of IHO orbital phasor "clock" dynamics in uniform-body

phase lag:

$\Delta\alpha = \alpha_x - \alpha_y = 30°$

1 minute orbit (5 seconds for second hand)

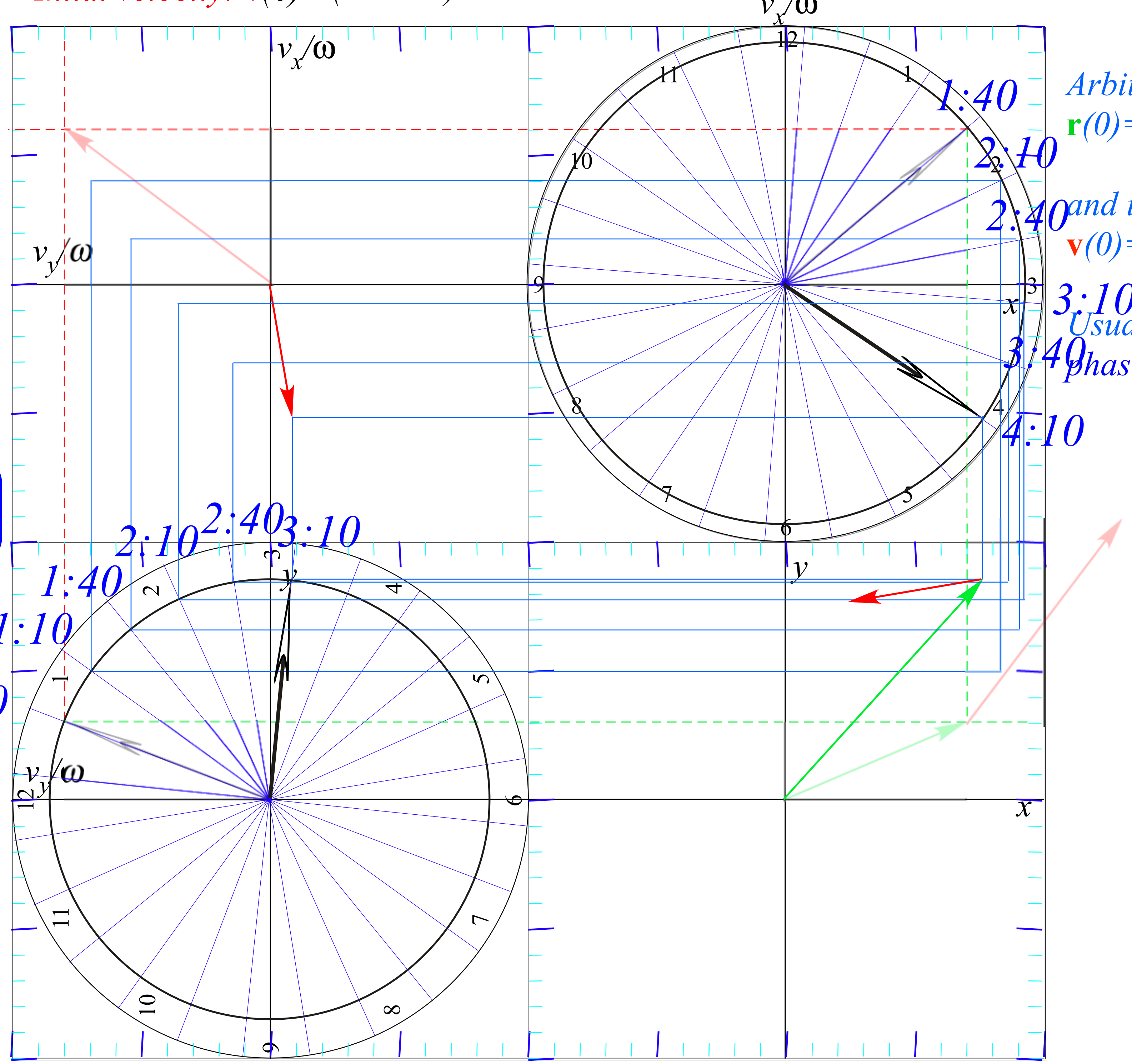
or

1 hour orbit (5 minutes for minute hand)

or

12 hour orbit (1 hour for hour hand)

Inital velocity: $\mathbf{v}(0) = (6.0\ \ 8.0)$



Arbitrary initial position $\mathbf{r}(0)=(x(0),y(0))$

and initial velocity $\mathbf{v}(0)=(v_x(0),v_y(0))$

Usually have x and y phasor circles of unequal size

Initial position: $\mathbf{r}(0)=(7.0\ \ 3.0)$

Review of IHO orbital phasor "clock" dynamics in uniform-body

Inital velocity: **v**(0)= (6.0 8.0)

phase lag:

$\Delta\alpha=\alpha_x - \alpha_y = 30°$

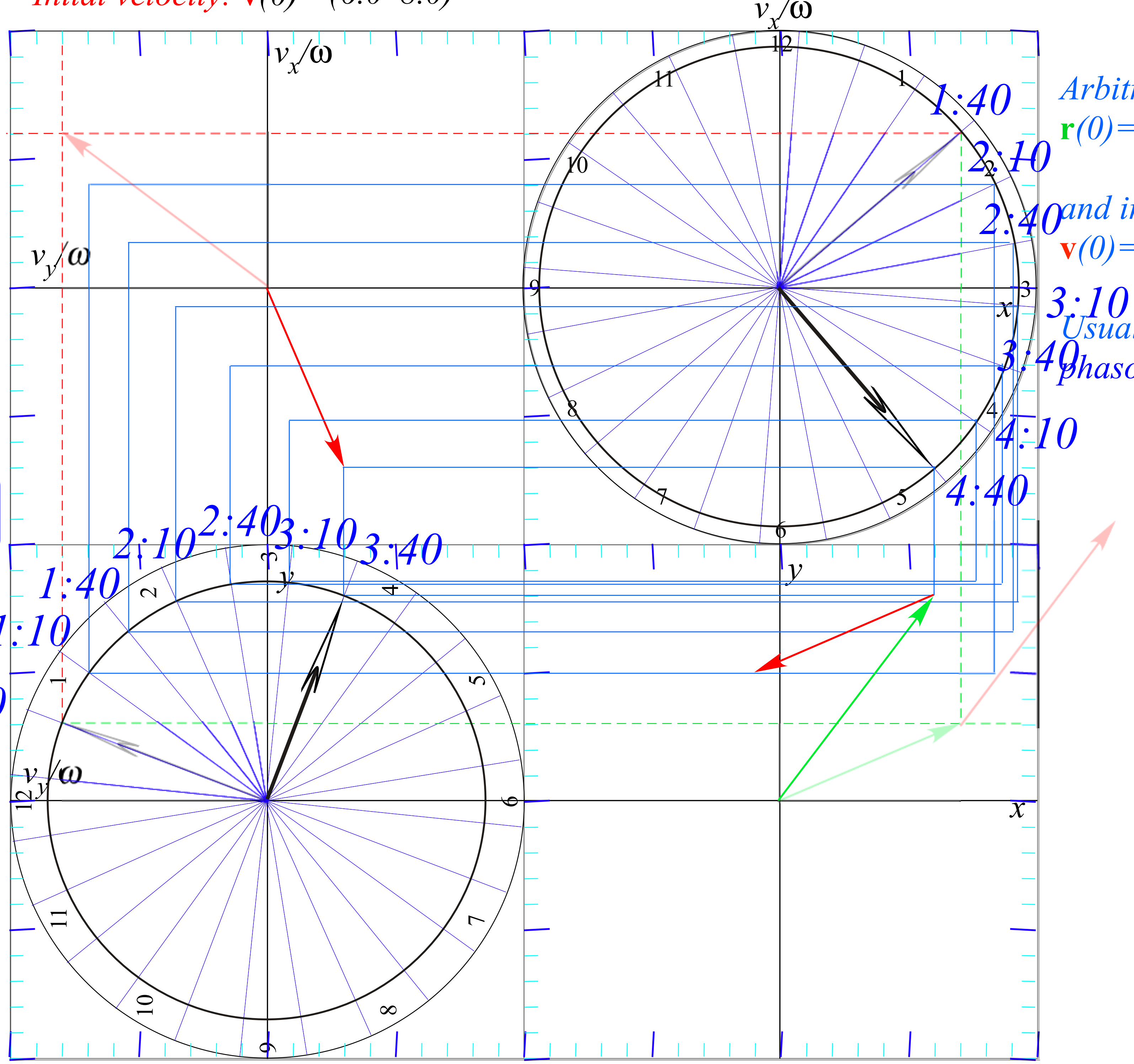
1 minute orbit (5 seconds for second hand)

or

1 hour orbit (5 minutes for minute hand)

or

12 hour orbit (1 hour for hour hand)



Arbitrary initial position
r(0)=(x(0),y(0))

and initial velocity
v(0)=(v_x(0),v_y(0))

Usually have x and y phasor circles of unequal size

Initial position: **r**(0)=(7.0 3.0)

Review of IHO orbital phasor "clock" dynamics in uniform-body

Inital velocity: $\mathbf{v}(0)=(6.0\ \ 8.0)$

phase lag:

$\Delta\alpha=\alpha_x-\alpha_y=30°$

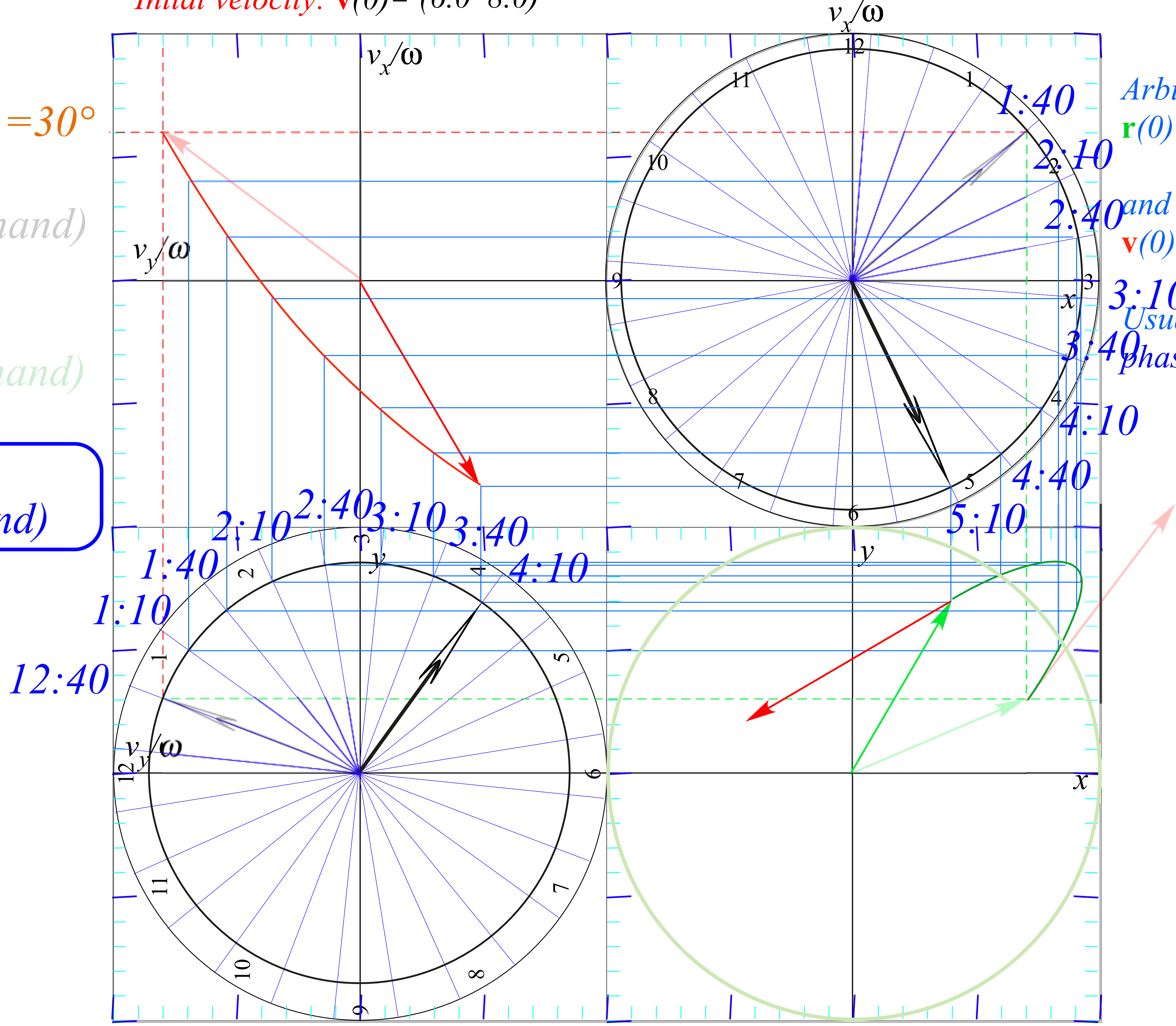
1 minute orbit *(5 seconds for second hand)*

or

1 hour orbit *(5 minutes for minute hand)*

or

12 hour orbit *(1 hour for hour hand)*



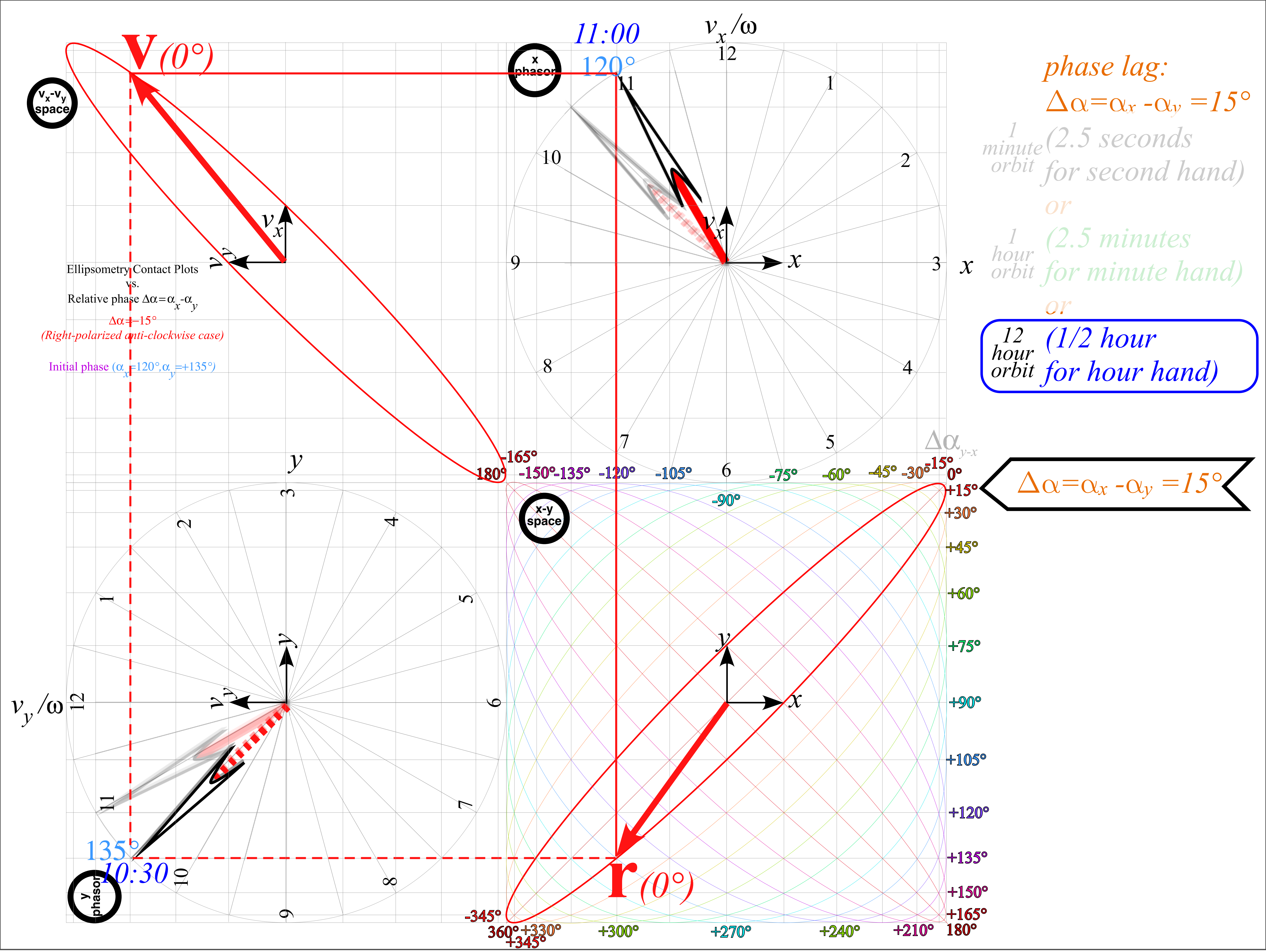
Initial position: $\mathbf{r}(0)=(7.0\ \ 3.0)$

Arbitrary initial position
$\mathbf{r}(0)=(x(0),y(0))$

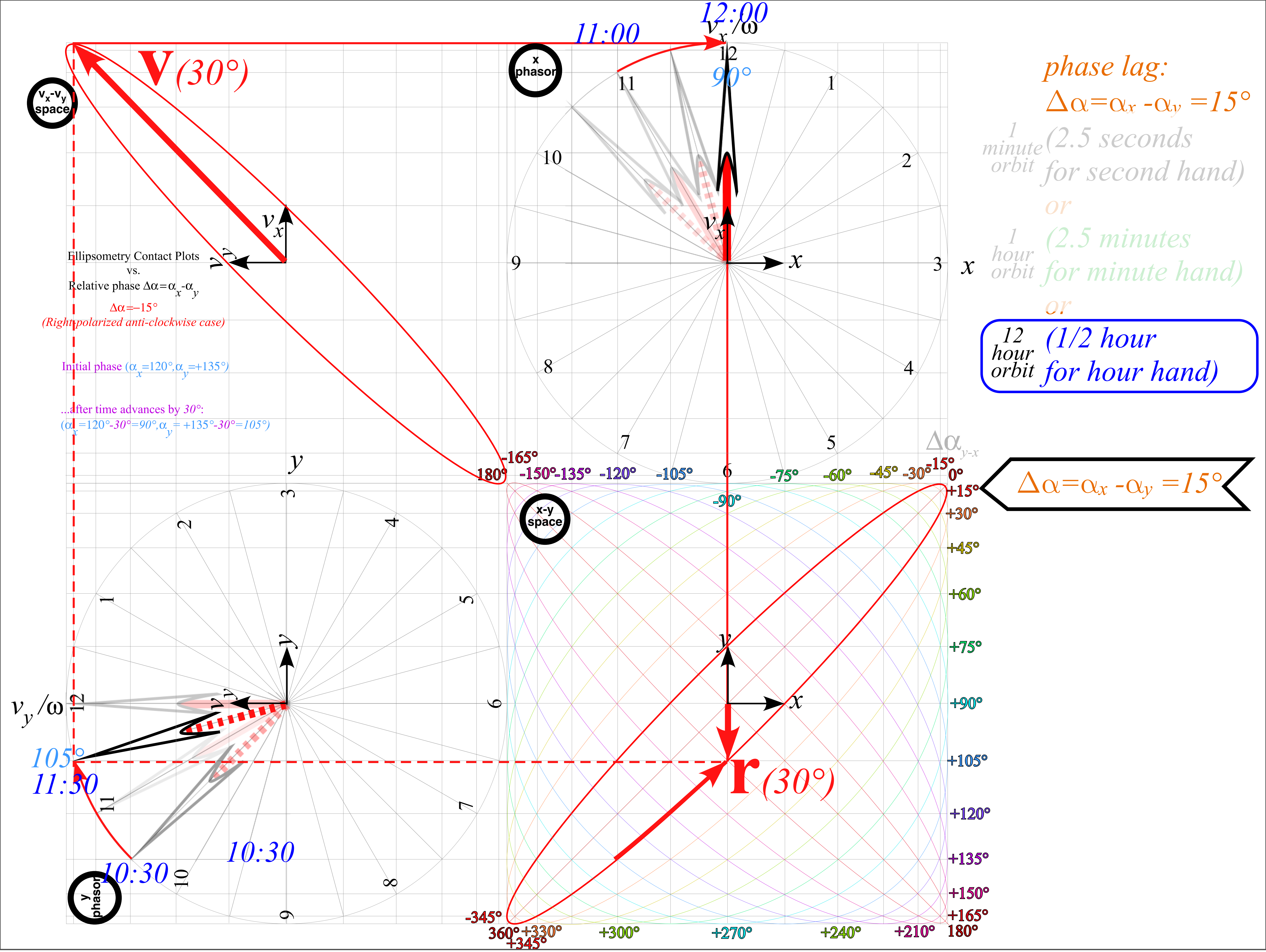
and initial velocity
$\mathbf{v}(0)=(v_x(0),v_y(0))$

Usually have x and y phasor circles of unequal size

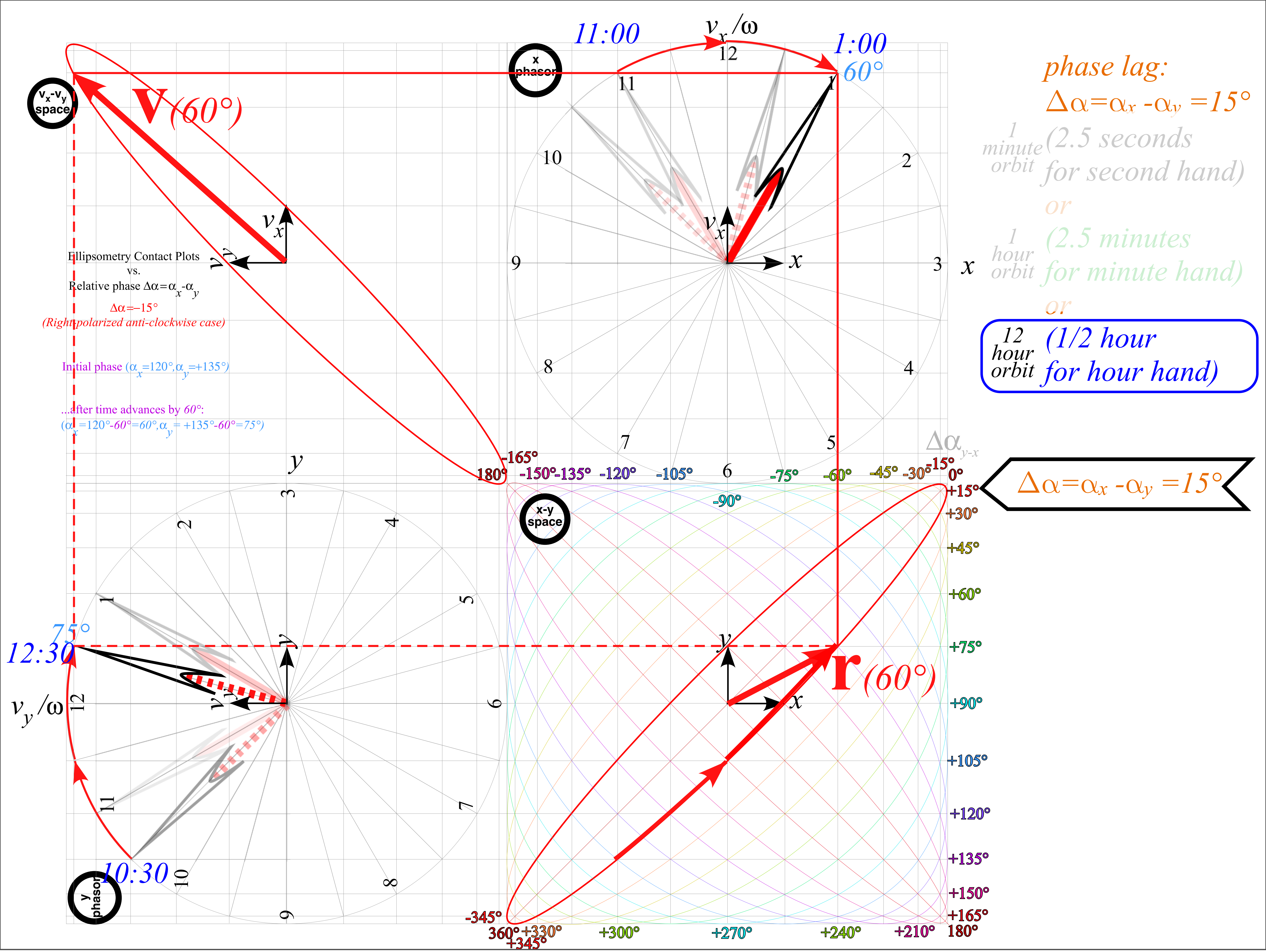
Review of IHO orbital phasor "clock" dynamics in uniform-body with "home-made movie" examples



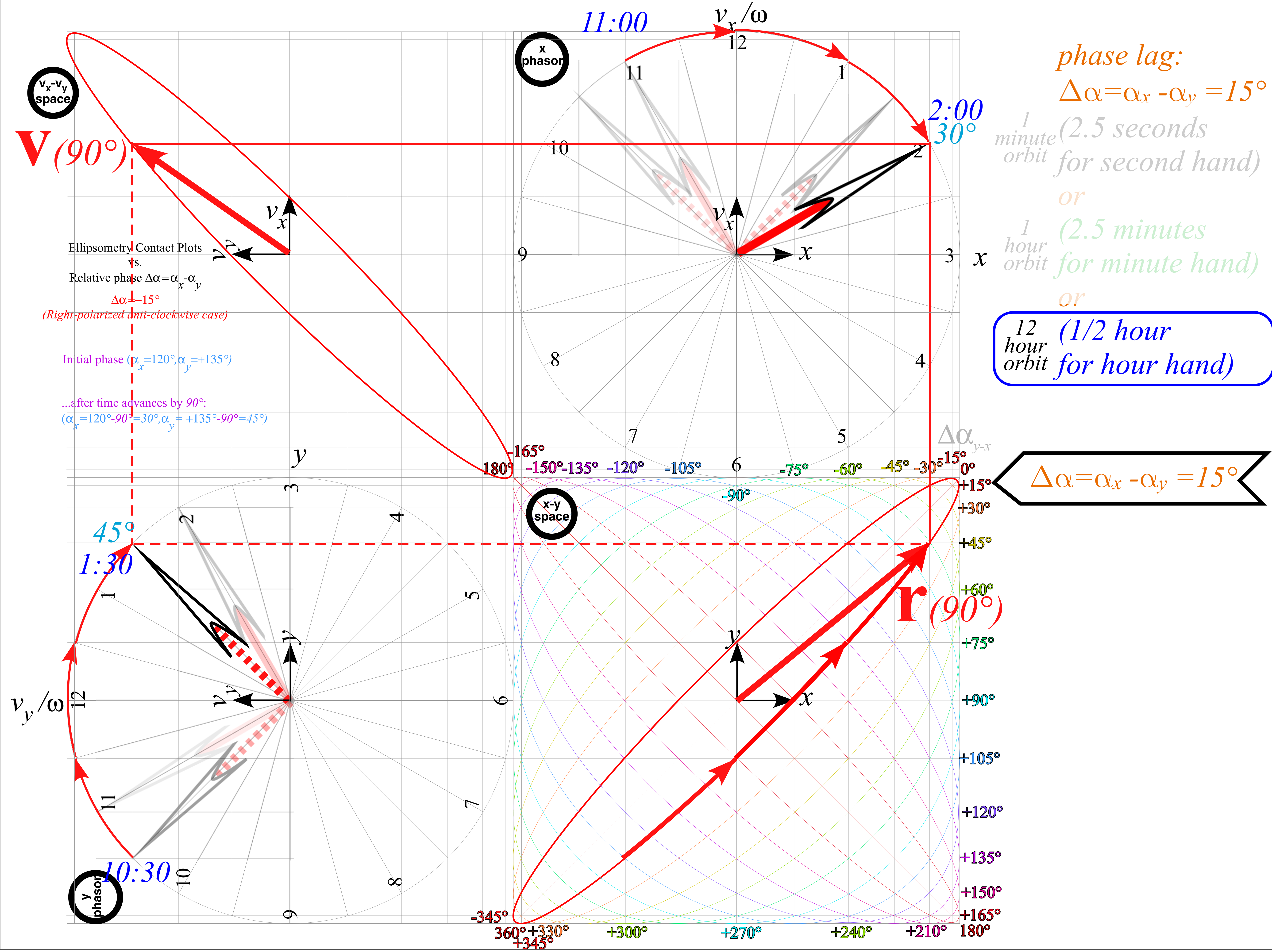
V (0°)
v_x-v_y space
x phasor
11:00
120°
v_x/ω
phase lag:
$\Delta\alpha=\alpha_x-\alpha_y=15°$
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Ellipsometry Contact Plots vs. Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°, α_y=+135°)
$\Delta\alpha_{y-x}$
$\Delta\alpha=\alpha_x-\alpha_y=15°$
x-y space
y phasor
135°
10:30
v_y/ω
r (0°)



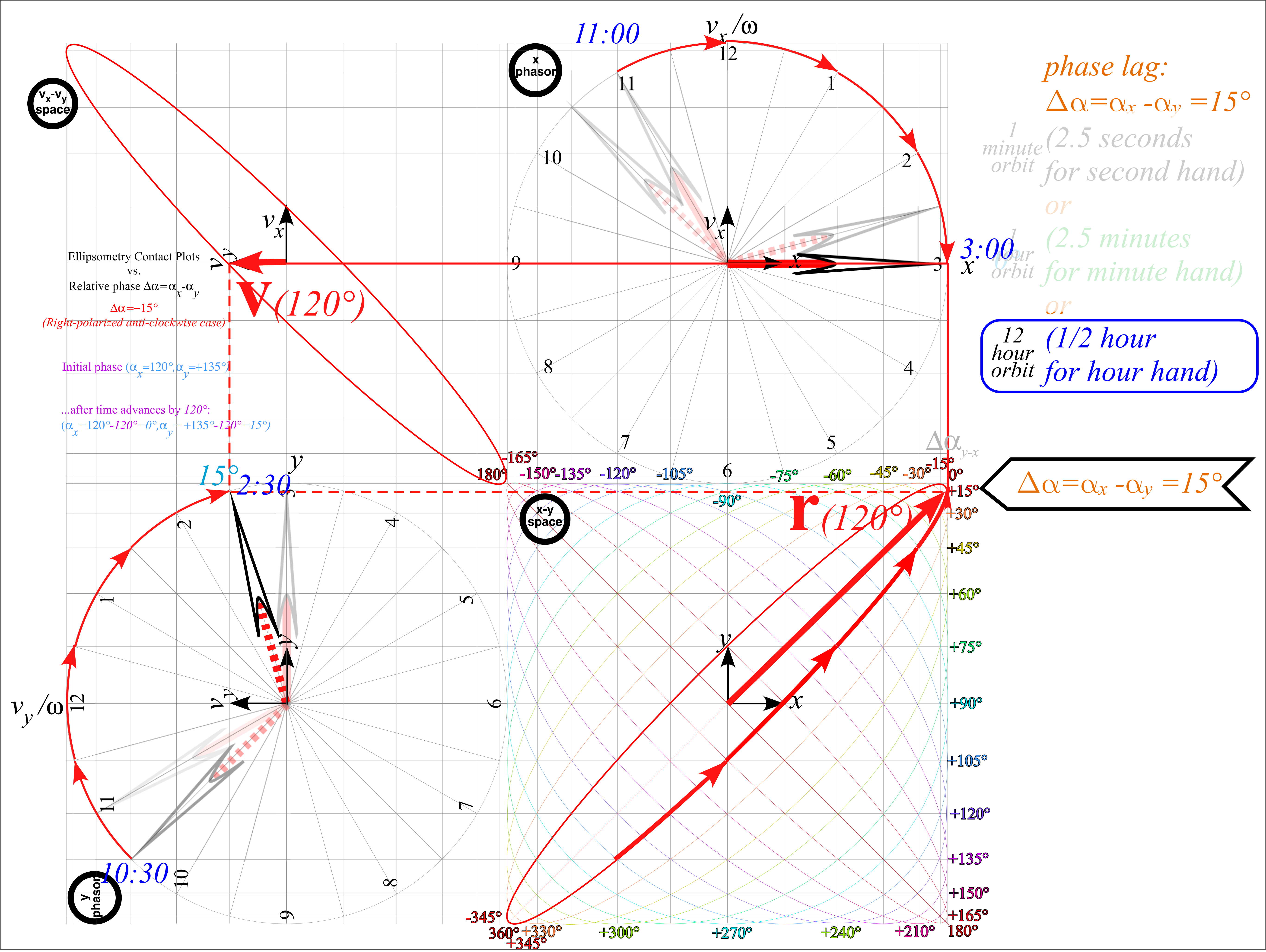
V (30°)
11:00
12:00
x phasor
v_x/ω
v_x-v_y space
phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Ellipsometry Contact Plots vs. Relative phase Δα=αx-αy
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 30°:
(αx=120°-30°=90°,αy = +135°-30°=105°)
90°
x-y space
Δα=αx -αy =15°
Δαy-x
r (30°)
y phasor
v_y/ω
105°
11:30
10:30
10:30



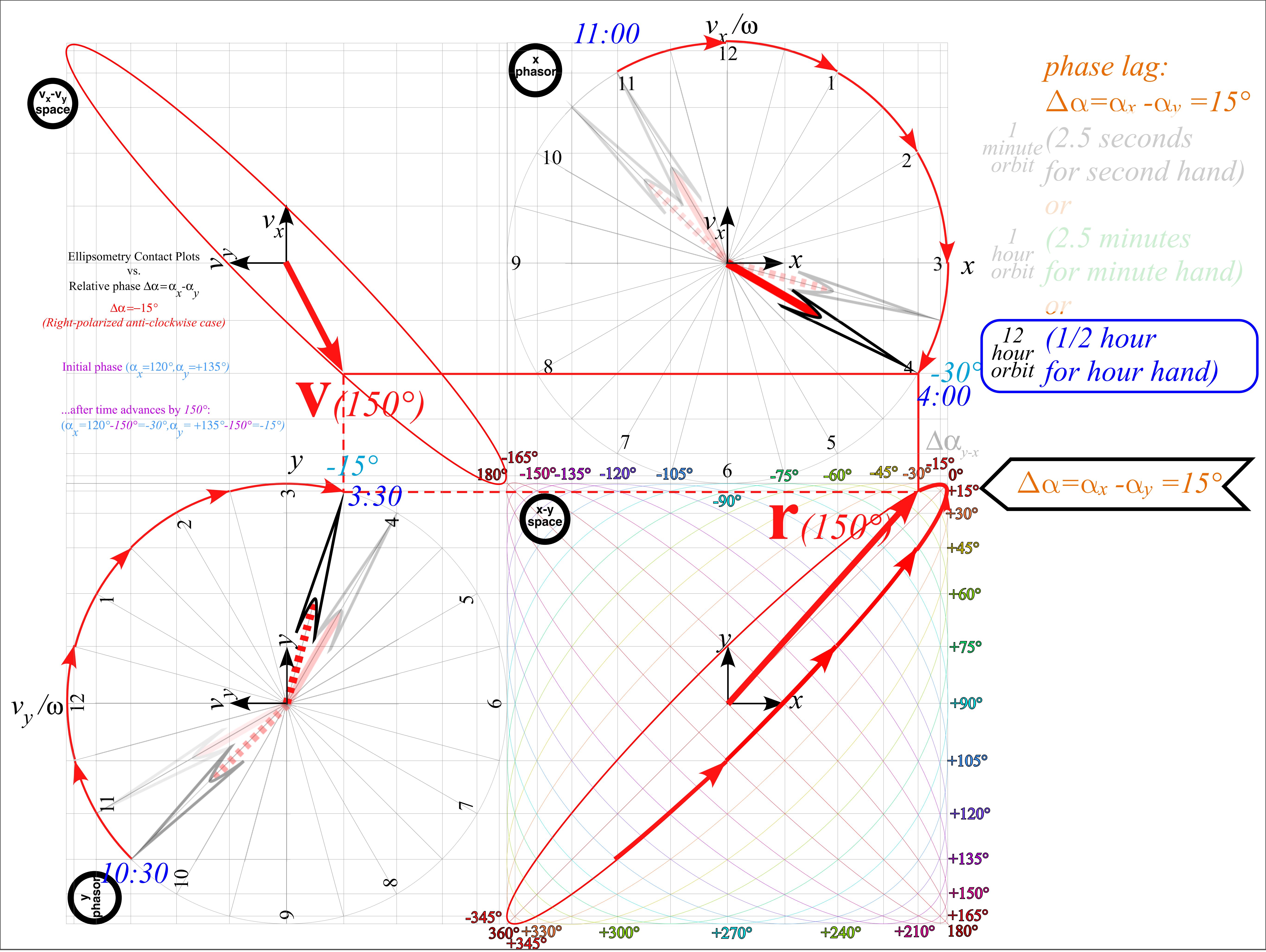
phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Ellipsometry Contact Plots vs. Relative phase Δα=αx-αy
Δα=−15°
(Right polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 60°:
(αx=120°-60°=60°,αy = +135°-60°=75°)
V(60°)
r(60°)
x phasor
y phasor
vx-vy space
x-y space
vx/ω
vy/ω
11:00
1:00
60°
12:30
75°
10:30
Δαy-x
Δα=αx -αy =15°



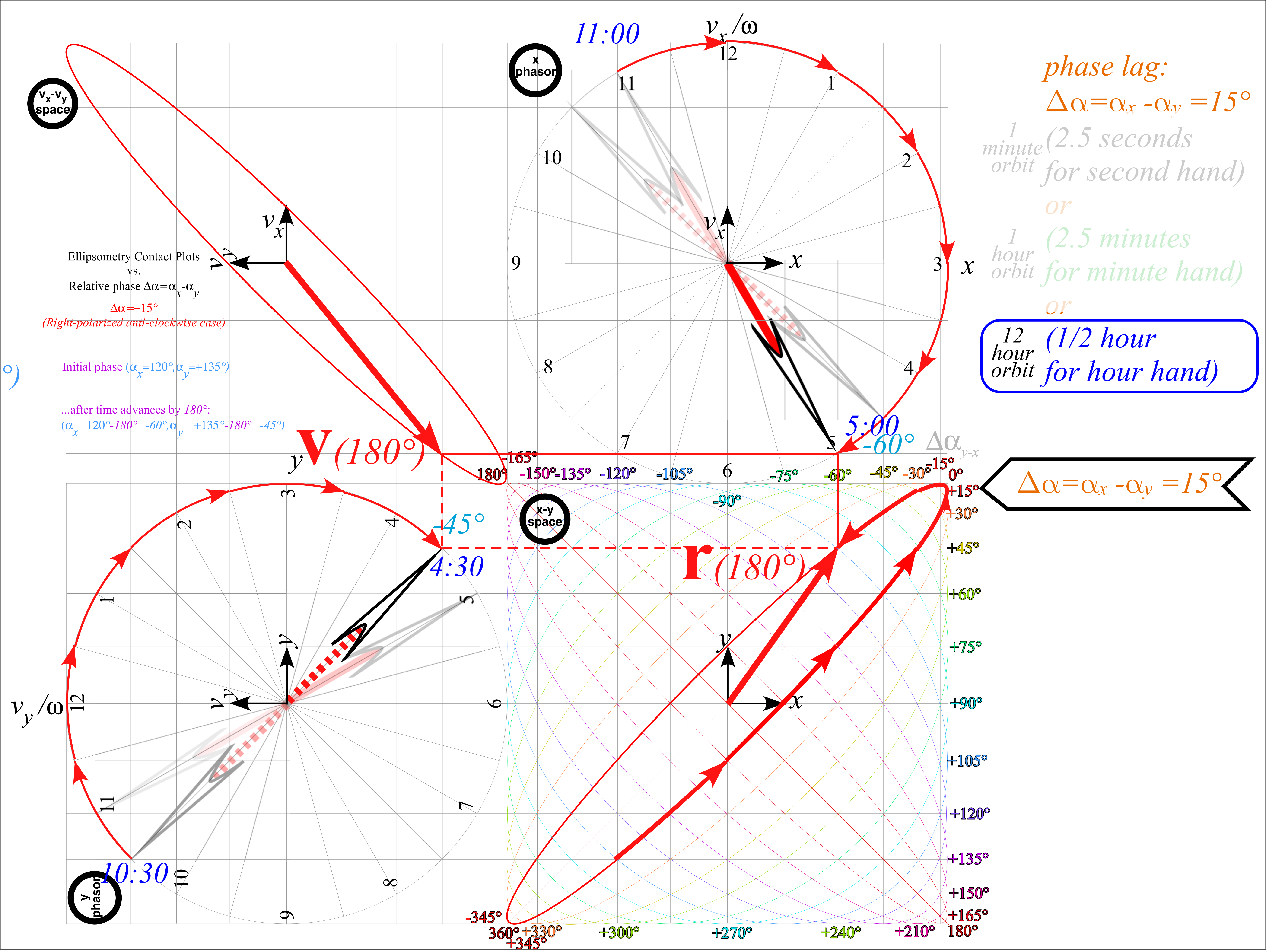
phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Ellipsometry Contact Plots vs. Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 90°:
(αx=120°-90°=30°,αy=+135°-90°=45°)
Δα=αx -αy =15°
vx-vy space
x phasor
y phasor
x-y space
V(90°)
r(90°)
11:00
2:00
30°
45°
1:30
10:30
vx/ω
vy/ω

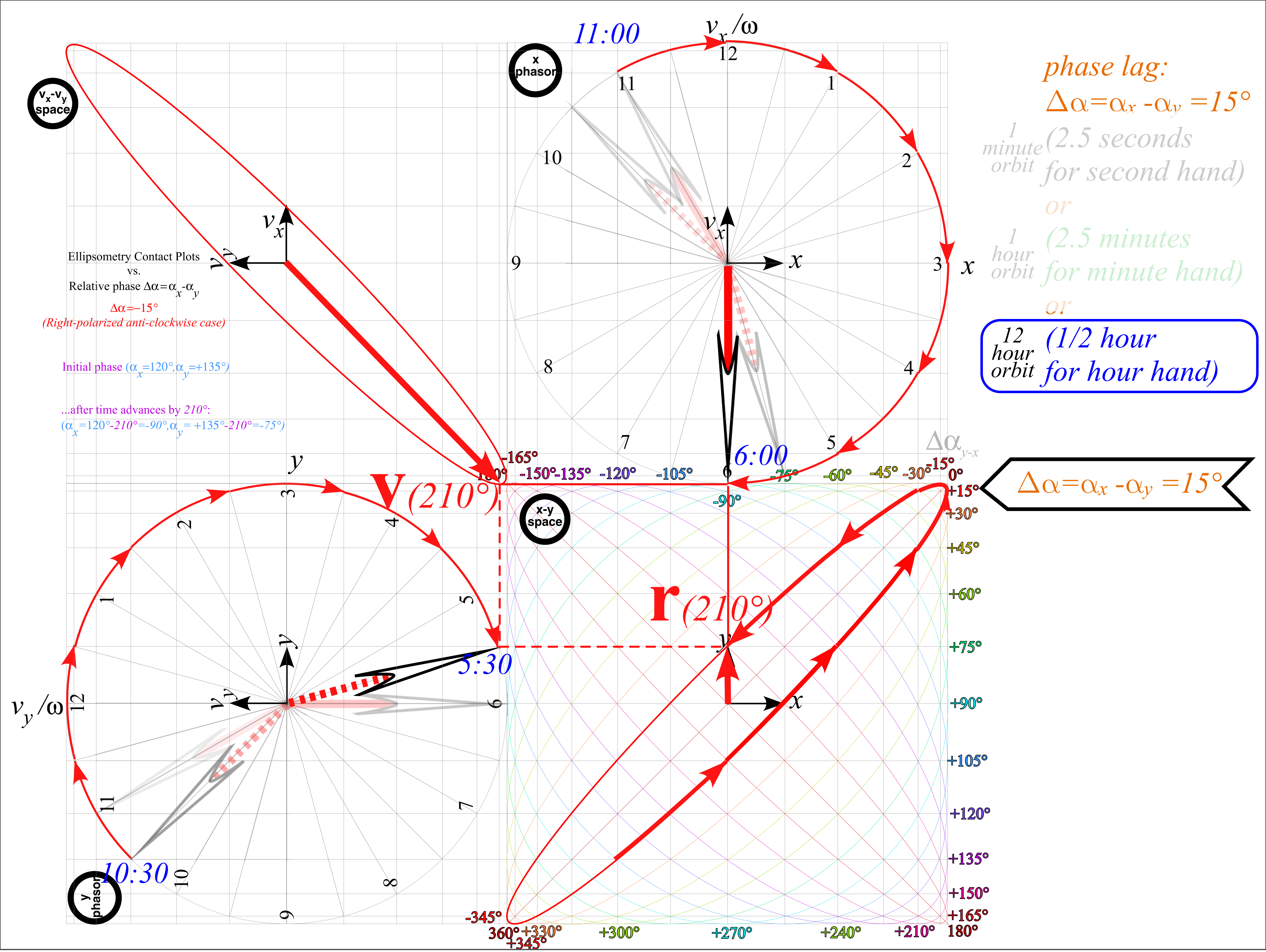


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 120°:
(α_x=120°-120°=0°,α_y=+135°-120°=15°)
v_x-v_y space
x phasor
y phasor
x-y space
11:00
3:00
2:30
10:30
15°
v_x/ω
v_y/ω
V(120°)
r(120°)
phase lag:
Δα=α_x -α_y =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Δα=α_x -α_y =15°
Δα_y-x

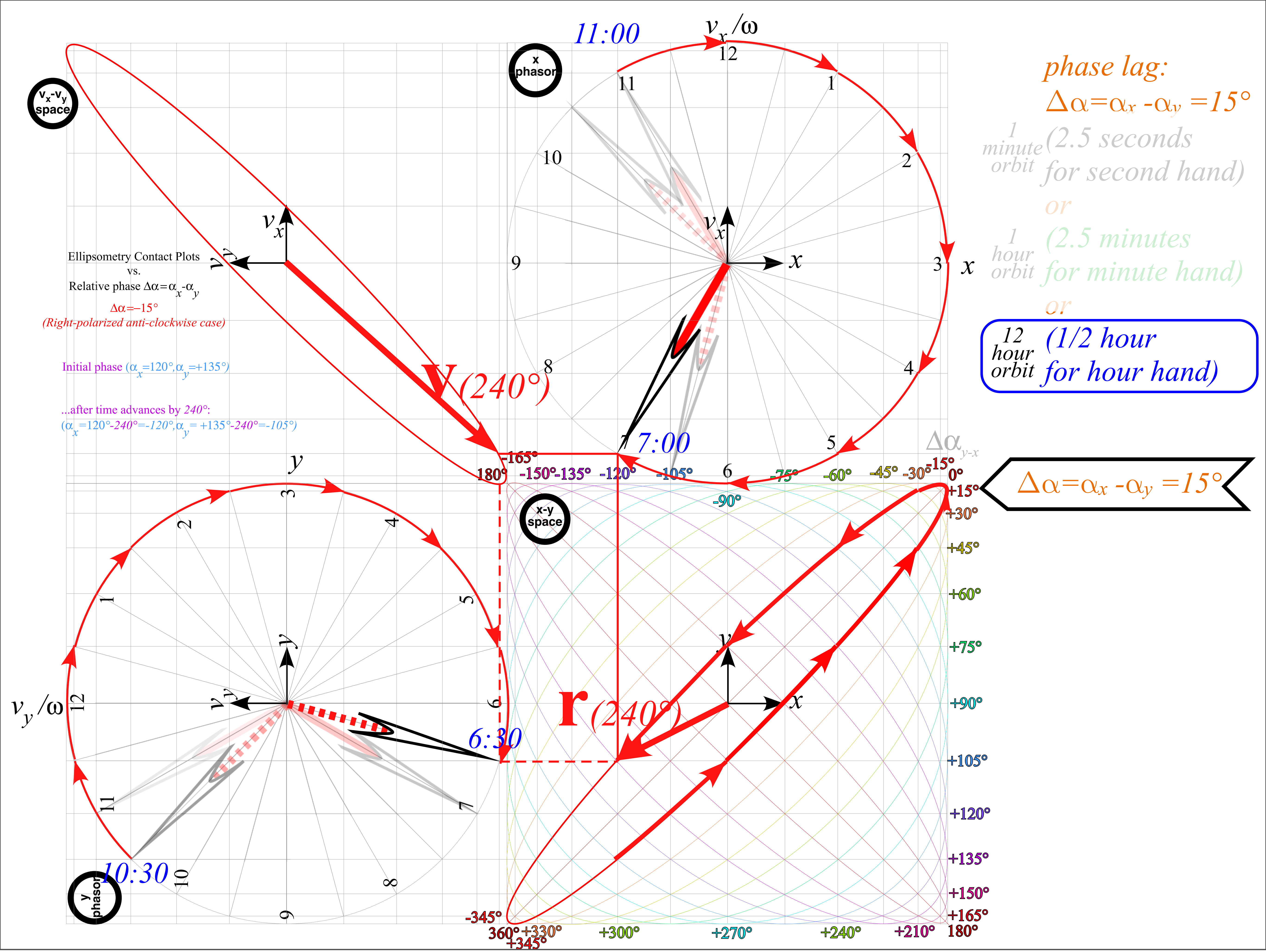


Ellipsometry Contact Plots vs. Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase ($\alpha_x=120°, \alpha_y=+135°$)
...after time advances by 150°:
($\alpha_x=120°-150°=-30°, \alpha_y=+135°-150°=-15°$)
v_x-v_y space
x phasor
y phasor
x-y space
11:00
4:00
3:30
10:30
V(150°)
r(150°)
phase lag:
$\Delta\alpha=\alpha_x-\alpha_y=15°$
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
$\Delta\alpha_{y-x}$
$\Delta\alpha=\alpha_x-\alpha_y=15°$

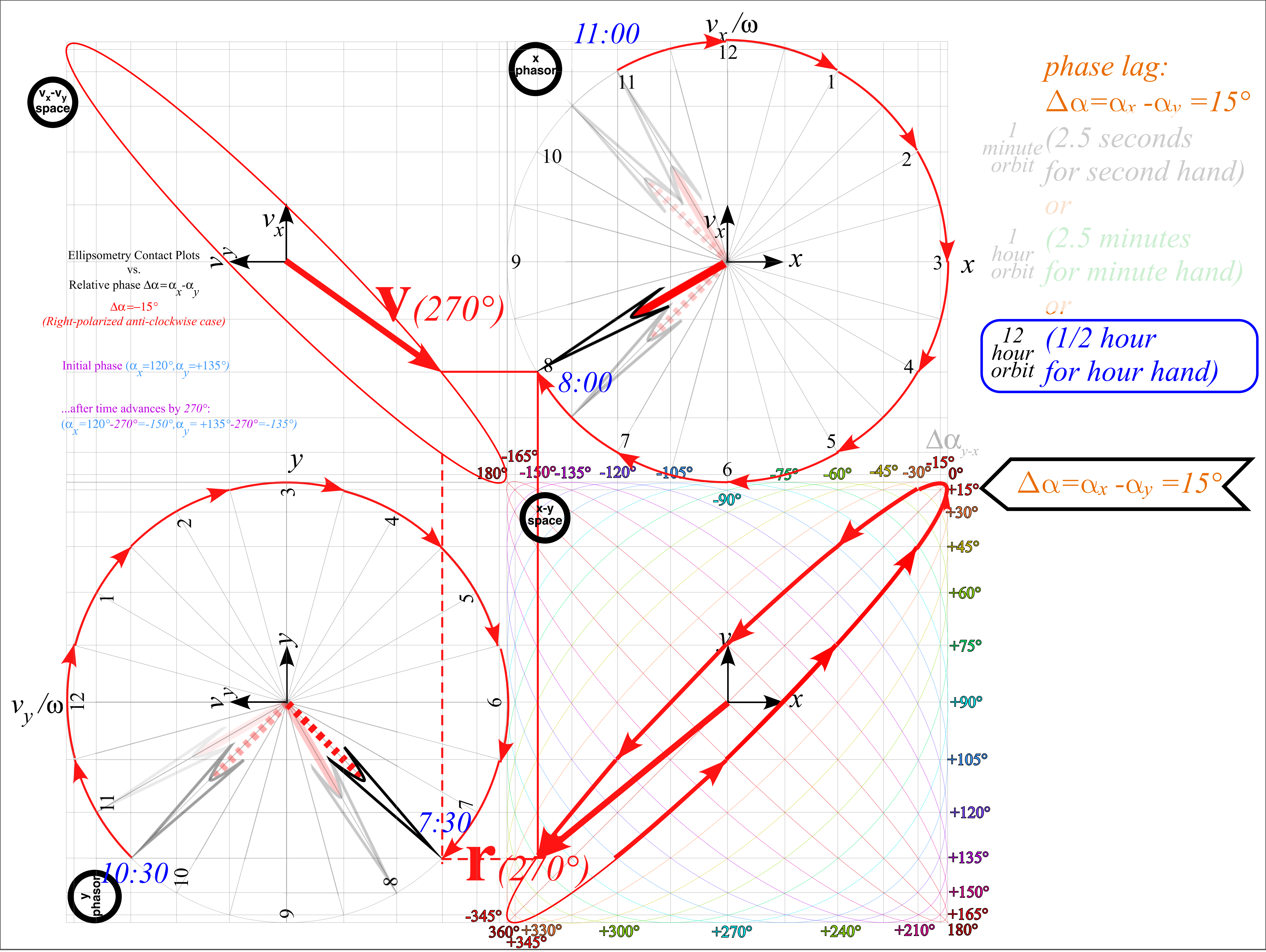

Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 180°:
(αx=120°-180°=-60°,αy=+135°-180°=-45°)
phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Δα=αx -αy =15°
11:00
5:00
4:30
10:30
V(180°)
r(180°)
-60°
-45°
x phasor
y phasor
vx-vy space
x-y space
vx/ω
vy/ω



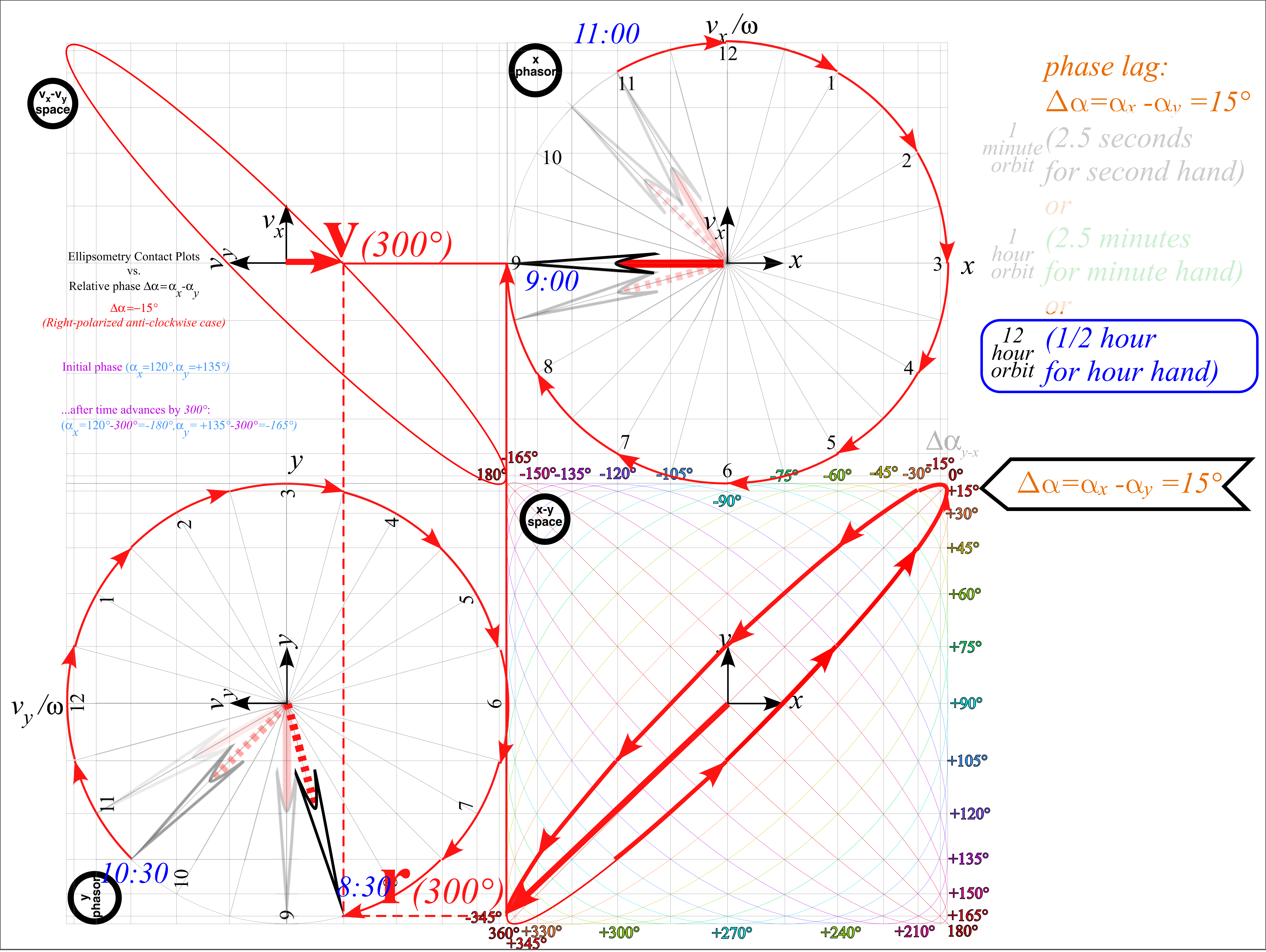
phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Δα=αx -αy =15°
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 210°:
(αx=120°-210°=-90°,αy= +135°-210°=-75°)
vx-vy space
x phasor
y phasor
x-y space
11:00
6:00
5:30
10:30
V(210°)
r(210°)
vx/ω
vy/ω
Δαy-x



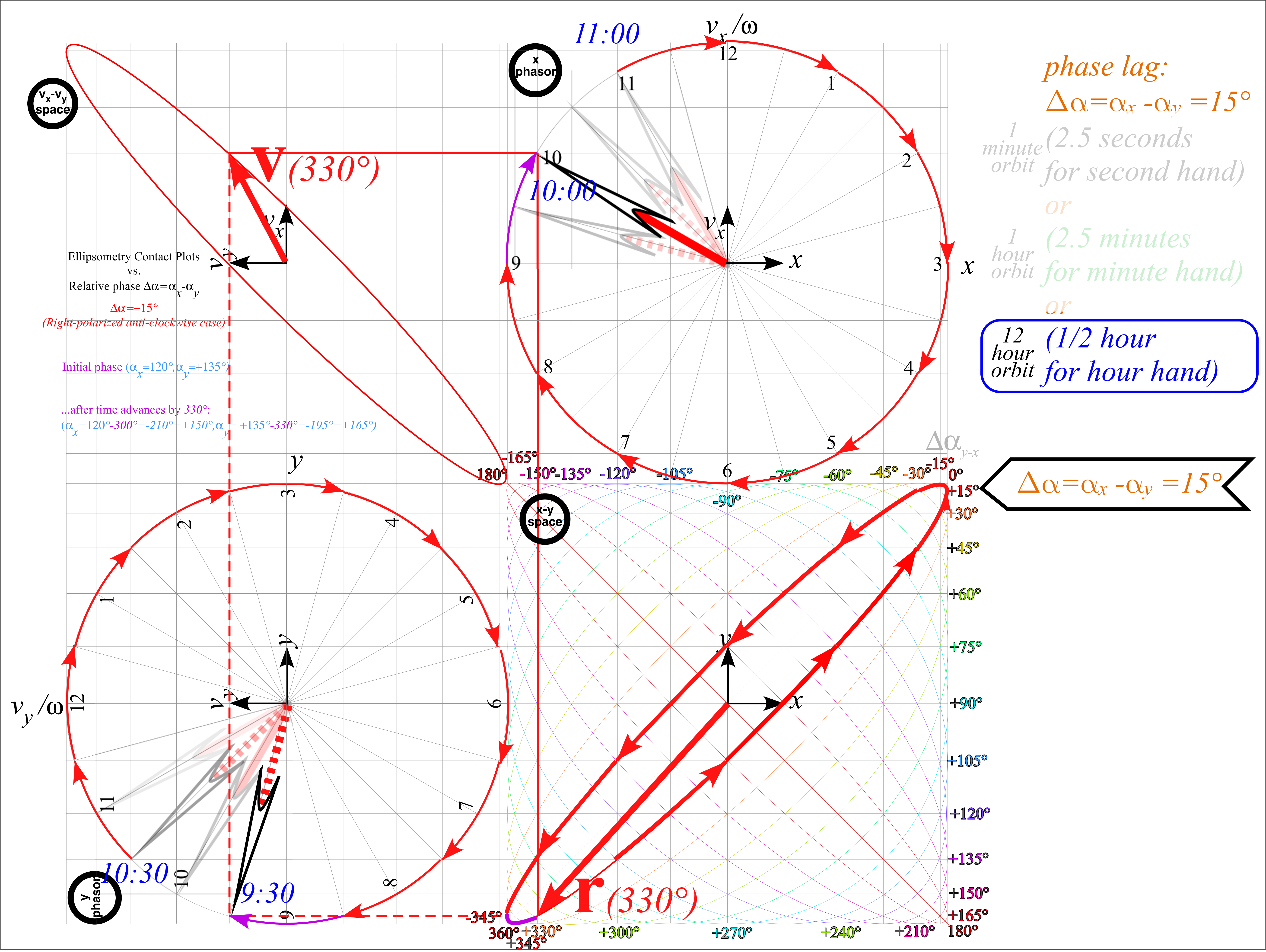
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 240°:
(αx=120°-240°=-120°,αy = +135°-240°=-105°)
vx-vy space
x phasor
y phasor
x-y space
V(240°)
r(240°)
11:00
7:00
6:30
10:30
vx/ω
vy/ω
phase lag:
Δα=αx -αy =15°
1 minute orbit
(2.5 seconds for second hand)
or
1 hour orbit
(2.5 minutes for minute hand)
or
12 hour orbit
(1/2 hour for hour hand)
Δαy-x
Δα=αx -αy =15°



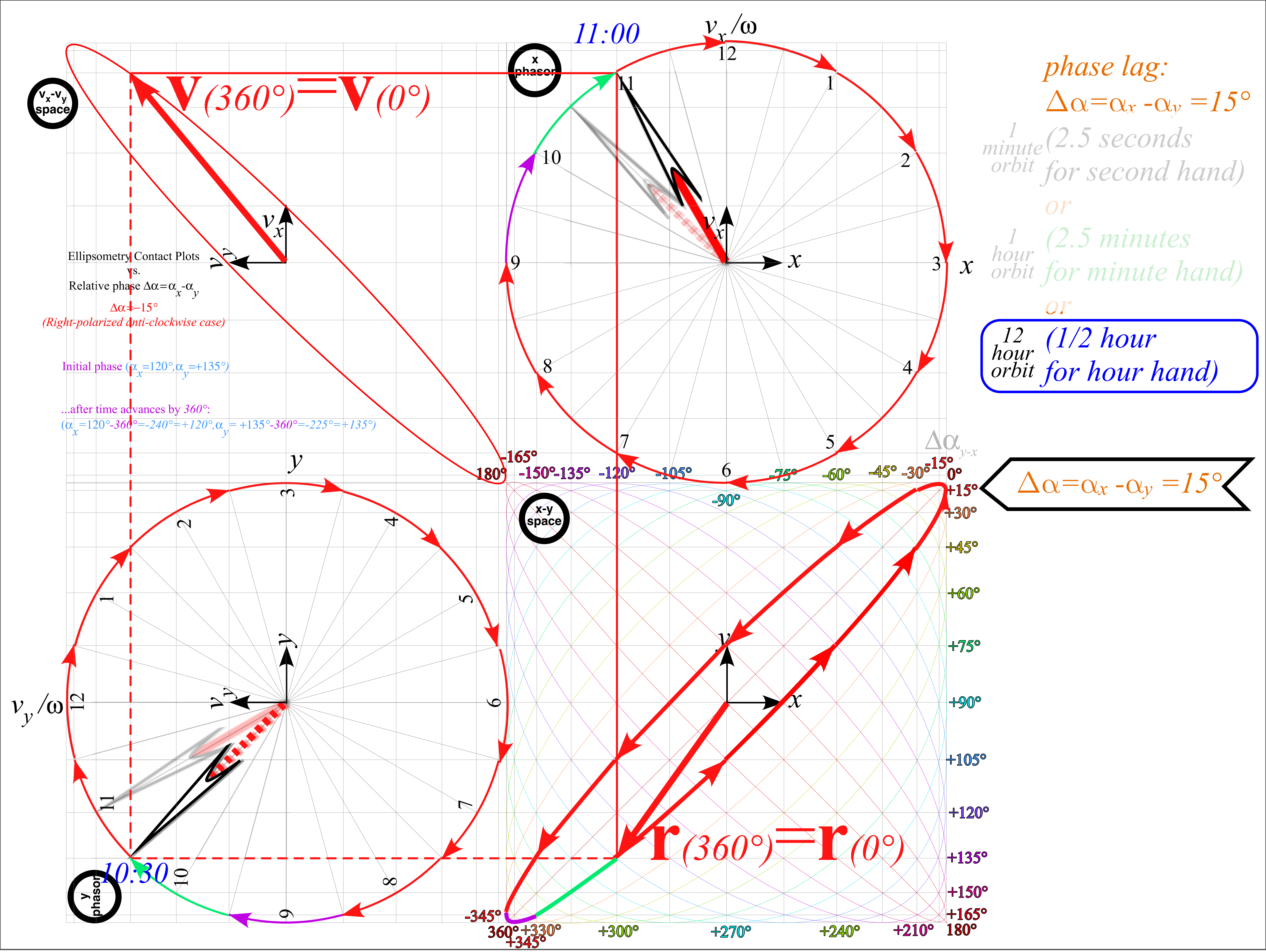
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 270°:
(αx=120°-270°=-150°,αy = +135°-270°=-135°)
vx-vy space
x phasor
y phasor
x-y space
V(270°)
r(270°)
11:00
8:00
7:30
10:30
vx/ω
vy/ω
phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Δαy-x
Δα=αx -αy =15°



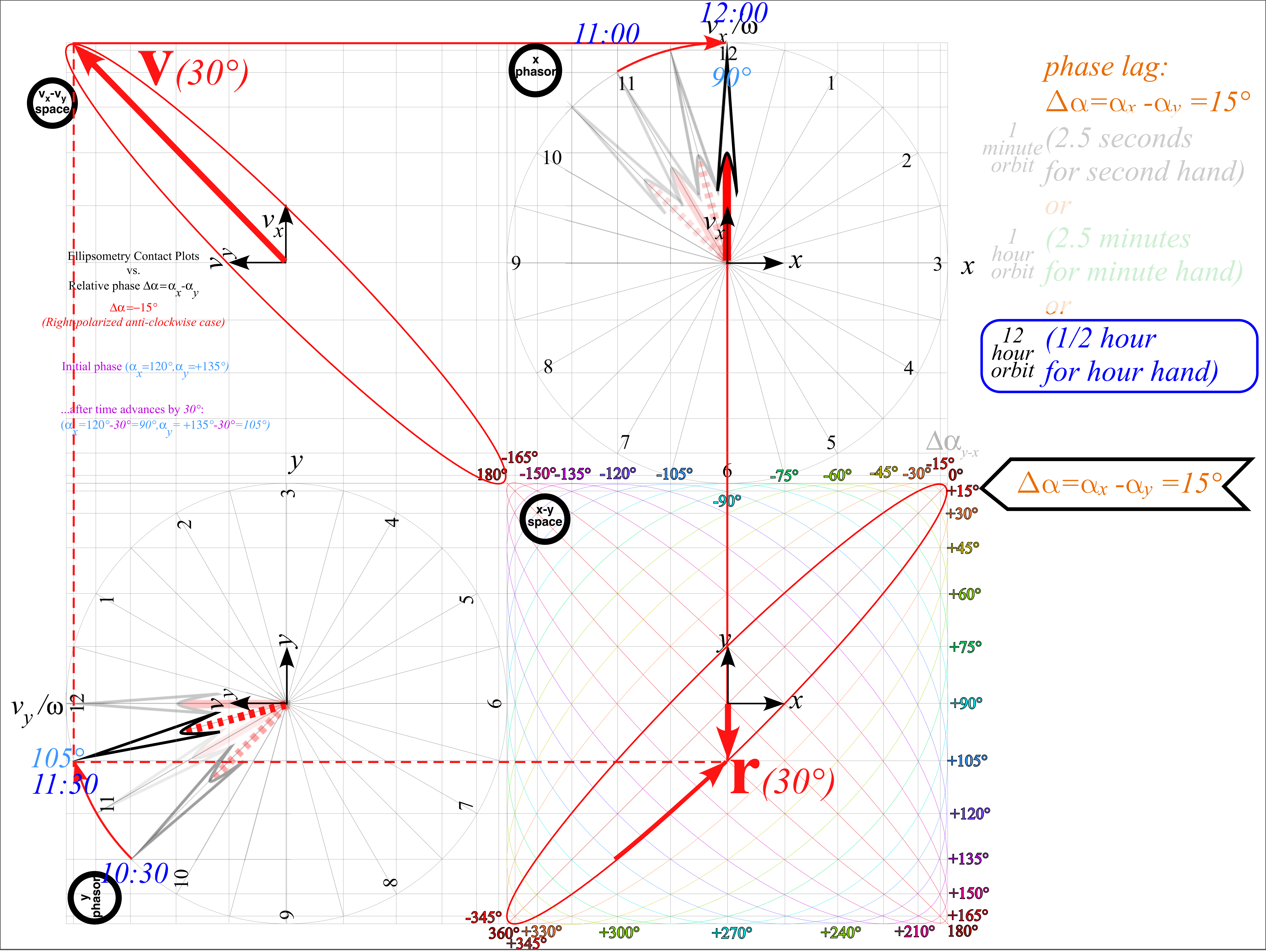
phase lag:
$\Delta\alpha=\alpha_x-\alpha_y=15°$
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Ellipsometry Contact Plots vs. Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase ($\alpha_x=120°, \alpha_y=+135°$)
...after time advances by 300°:
($\alpha_x=120°-300°=-180°, \alpha_y=+135°-300°=-165°$)
v_x-v_y space
x phasor
y phasor
x-y space
V (300°)
r (300°)
11:00
9:00
10:30
8:30
v_x/ω
v_y/ω
$\Delta\alpha_{y-x}$
$\Delta\alpha=\alpha_x-\alpha_y=15°$



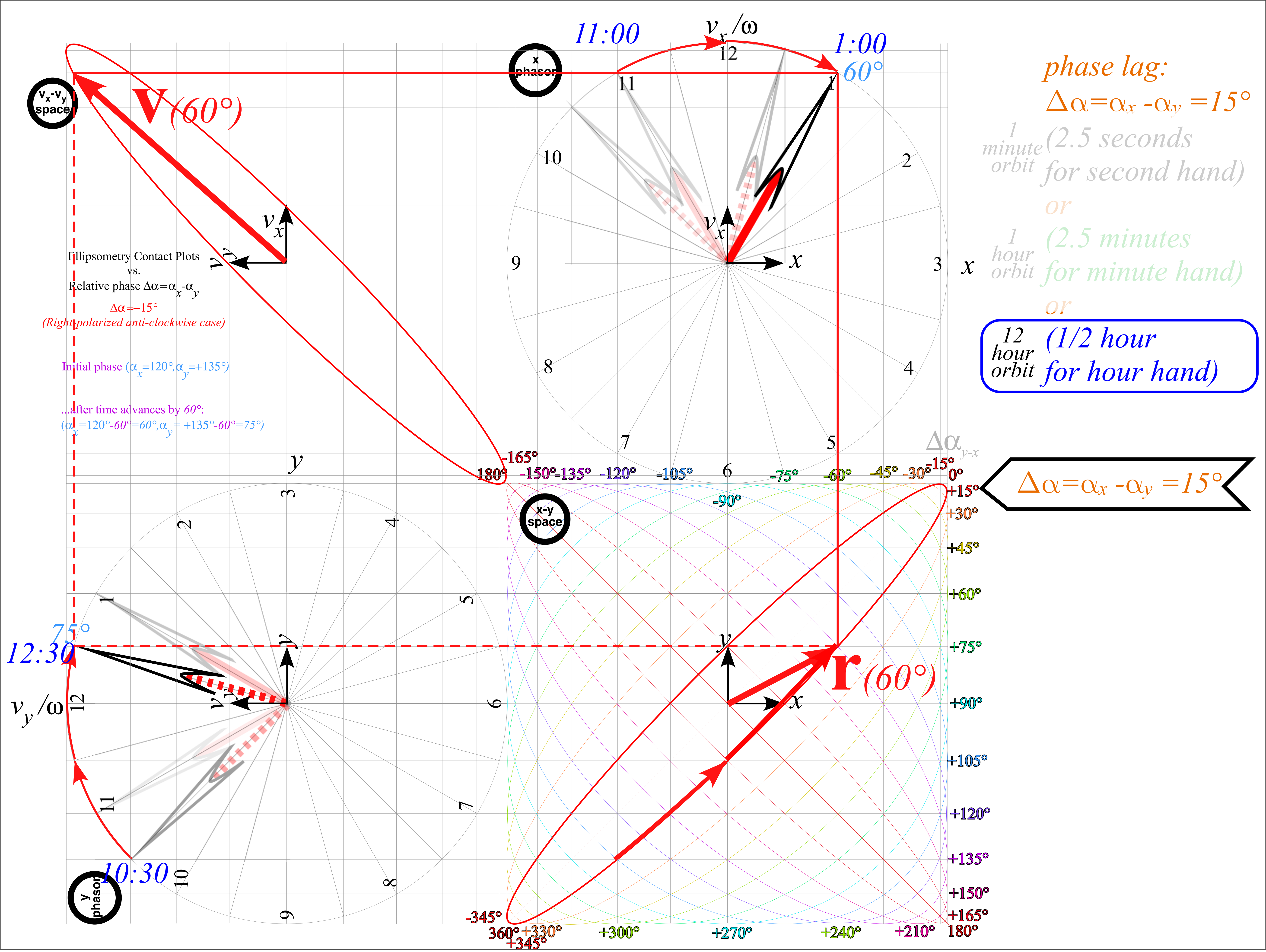
11:00
v_x/ω
x phasor
v_x-v_y space
V (330°)
10:00
phase lag:
$\Delta\alpha=\alpha_x-\alpha_y=15°$
1 minute orbit
(2.5 seconds for second hand)
or
1 hour orbit
(2.5 minutes for minute hand)
or
12 hour orbit
(1/2 hour for hour hand)
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 330°:
(α_x=120°-300°=-210°=+150°,α_y= +135°-330°=-195°=+165°)
$\Delta\alpha_{y-x}$
$\Delta\alpha=\alpha_x-\alpha_y=15°$
x-y space
v_y/ω
y phasor
10:30
9:30
r (330°)



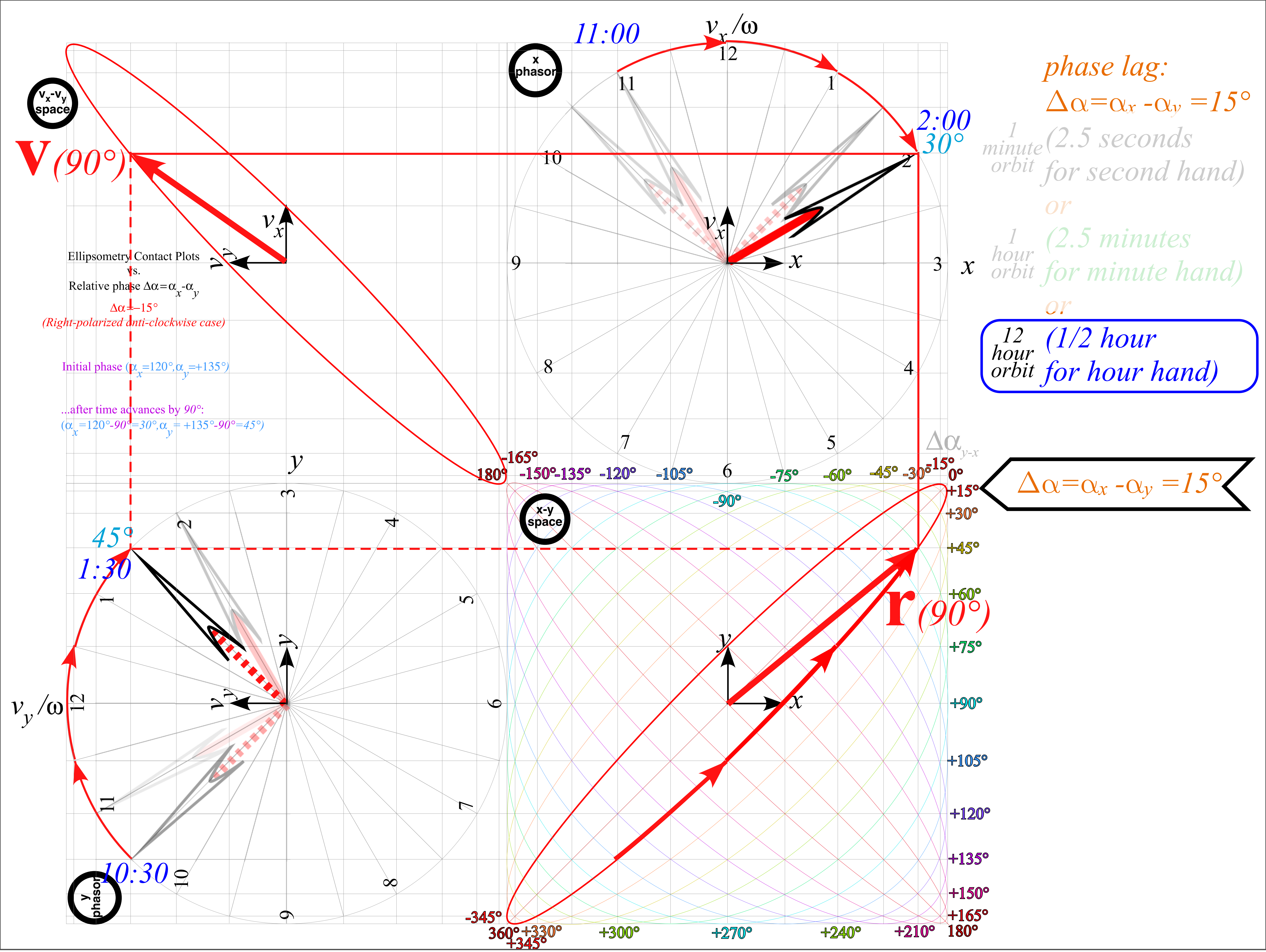
11:00
v_x/ω
x phasor
v_x-v_y space
V(360°)=V(0°)
phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Ellipsometry Contact Plots vs. Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 360°:
(αx=120°-360°=-240°=+120°,αy= +135°-360°=-225°=+135°)
Δα=αx -αy =15°
x-y space
r(360°)=r(0°)
v_y/ω
10:30
y phasor
Δαy-x



V (30°)
v_x-v_y space
x phasor
11:00
12:00
v_x/ω
90°
phase lag:
$\Delta\alpha=\alpha_x-\alpha_y=15°$
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Ellipsometry Contact Plots vs. Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 30°:
(α_x=120°-30°=90°,α_y= +135°-30°=105°)
$\Delta\alpha_{y-x}$
$\Delta\alpha=\alpha_x-\alpha_y=15°$
x-y space
r (30°)
v_y/ω
105°
11:30
10:30
y phasor



Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right polarized anti-clockwise case)
Initial phase (α_x=120°, α_y=+135°)
...after time advances by 60°:
(α_x=120°-60°=60°, α_y = +135°-60°=75°)
phase lag:
$\Delta\alpha=\alpha_x-\alpha_y=15°$
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
$\Delta\alpha=\alpha_x-\alpha_y=15°$
V(60°)
r(60°)
11:00
1:00
60°
12:30
75°
10:30
v_x/ω
v_y/ω
x phasor
y phasor
v_x-v_y space
x-y space
$\Delta\alpha_{y-x}$



phase lag:
Δα=αx -αy =15°
1 minute orbit (2.5 seconds for second hand)
or
1 hour orbit (2.5 minutes for minute hand)
or
12 hour orbit (1/2 hour for hour hand)
Δα=αx -αy =15°
Ellipsometry Contact Plots vs. Relative phase Δα=αx-αy
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 90°:
(αx=120°-90°=30°,αy= +135°-90°=45°)
vx-vy space
x phasor
x-y space
y phasor
V(90°)
r(90°)
11:00
2:00
30°
45°
1:30
10:30

Kepler "laws" (Some that apply to <u>all</u> central (isotropic) F(r) force fields)

→ *Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$* *(Derived here)*

Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$ *(Derived in Unit 5)*

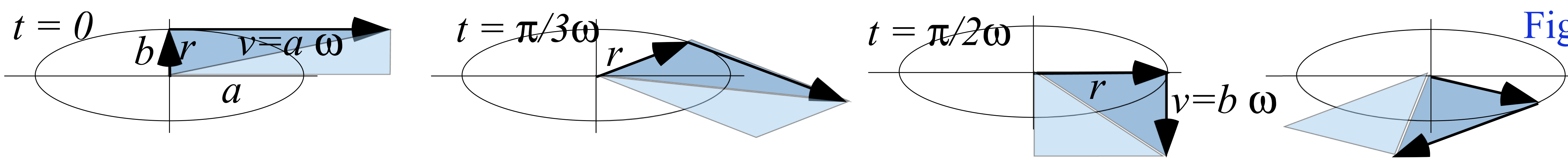
Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$ *(Derived here)*

Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$ *(Derived in Unit 5)*

Some Kepler's "laws" for central (isotropic) force F(r)

...and certainly apply to the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ (Recall p.19: $k=G\frac{4\pi}{3}m\rho_{\oplus}$)

Unit 1
Fig. 9.8



1. Area of triangle $\measuredangle^{\mathbf{v}}_{\mathbf{r}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - b\sin\omega t\cdot(-a\omega\sin\omega t) = ab\cdot\omega\left(\cos^2\omega t+\sin^2\omega t\right)$$

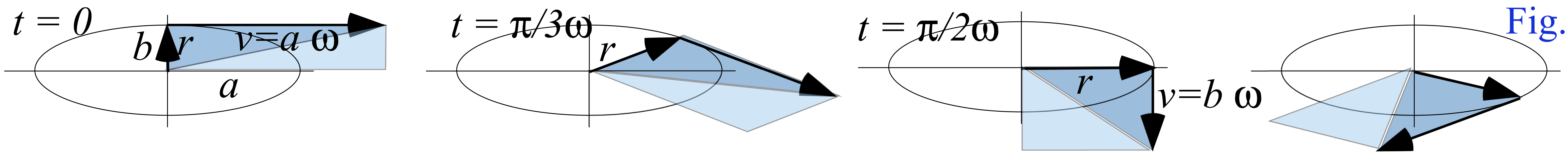
✓ *for IHO*

$$\begin{pmatrix} r_x \\ r_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix} \qquad \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix}$$

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ (Recall p.19: $k=G\frac{4\pi}{3}m\rho_{\oplus}$)

Unit 1
Fig. 9.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - a\sin\omega t\cdot(-b\omega\sin\omega t) = ab\cdot\omega$$

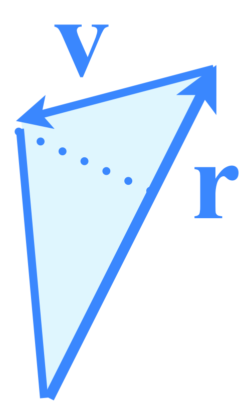
✓ *for IHO*

2. *Angular momentum* $\mathbf{L} = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,|\mathbf{r}\times\mathbf{v}| = m\left(r_x v_y - r_y v_x\right) = m\cdot ab\cdot\omega$$

✓ *for IHO*

$$|\mathbf{r}\times\mathbf{v}| = r\cdot v\cdot\sin\measuredangle_r^v$$



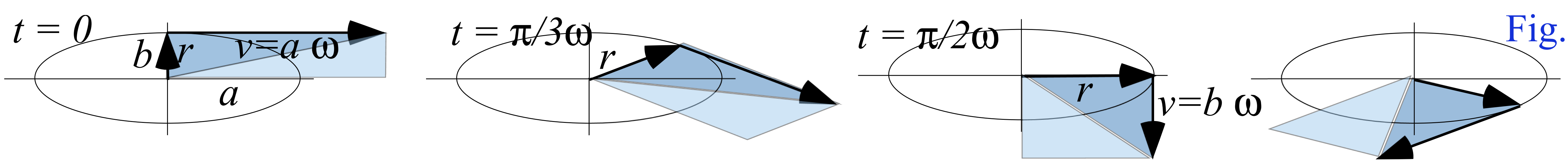
$$|\mathbf{r}\bullet\mathbf{v}| = r\cdot v\cdot\cos\measuredangle_r^v$$

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

(Recall p.19: $k=G\frac{4\pi}{3}m\rho_{\oplus}$)

Unit 1
Fig. 9.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - a\sin\omega t\cdot(-b\omega\sin\omega t) = ab\cdot\omega$$

for IHO

2. *Angular momentum* $\mathbf{L} = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,|\mathbf{r}\times\mathbf{v}| = m\left(r_x v_y - r_y v_x\right) = m\cdot ab\cdot\omega$$

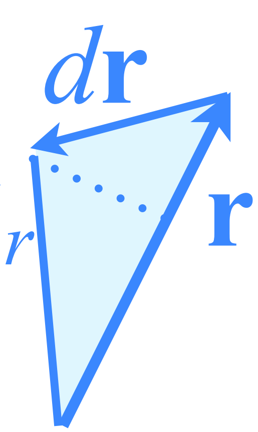
for IHO

3. Equal area is swept by radius vector in each equal time interval T

$$A_T = \int_0^T \frac{\mathbf{r}\times d\mathbf{r}}{2} = \int_0^T \frac{\mathbf{r}\times\frac{d\mathbf{r}}{dt}}{2}dt = \int_0^T \frac{\mathbf{r}\times\mathbf{v}}{2}dt = \frac{L}{\underbrace{2m}_{\boxed{\text{by 2.}}}}\int_0^T dt = \frac{L}{2m}T$$

for IHO

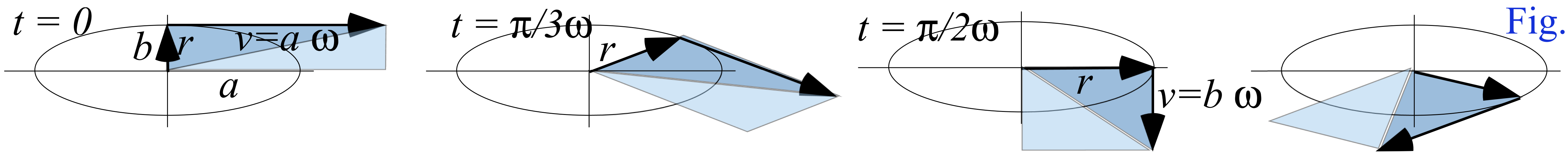
$$|\mathbf{r}\times d\mathbf{r}| = r\cdot dr\cdot\sin\measuredangle_{r}^{dr}$$



Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ (Recall p.19: $k=G\frac{4\pi}{3}m\rho_{\oplus}$)

Unit 1 Fig. 9.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - a\sin\omega t\cdot(-b\omega\sin\omega t) = ab\cdot\omega$$ ✓ *for IHO*

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m(r_x v_y - r_y v_x) = m\cdot ab\cdot\omega = m\cdot ab\cdot\frac{2\pi}{\tau}$$ ✓ *for IHO*

3. Equal area is swept by radius vector in each equal time interval T

$$A_T = \int_0^T \frac{\mathbf{r}\times d\mathbf{r}}{2} = \int_0^T \frac{\mathbf{r}\times\frac{d\mathbf{r}}{dt}}{2}dt = \int_0^T \frac{\mathbf{r}\times\mathbf{v}}{2}dt = \frac{L}{2m}\int_0^T dt = \frac{L}{2m}T$$ ✓ *for IHO*

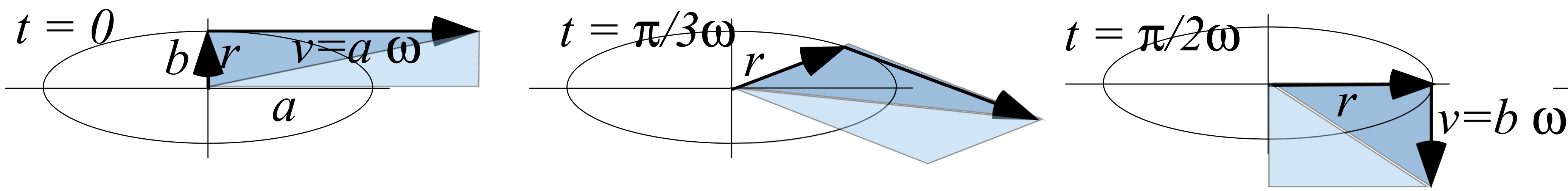
In one period: $\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L}$ the area is: $A_\tau = \frac{L\tau}{2m}$ $(= ab\cdot\pi$ for ellipse orbit)

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

...and certainly apply to the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

(Recall p.19: $k=G\frac{4\pi}{3}m\rho_{\oplus}$)

Unit 1
Fig. 9.8



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = a\cos\omega t\cdot(b\omega\cos\omega t) - a\sin\omega t\cdot(-b\omega\sin\omega t) = ab\cdot\omega$$

✓ *for IHO*

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m(r_x v_y - r_y v_x) = m\cdot ab\cdot\omega = m\cdot ab\cdot\frac{2\pi}{\tau}$$

✓ *for IHO*

3. Equal area is swept by radius vector in each equal time interval T

$$A_T = \int_0^T \frac{\mathbf{r}\times d\mathbf{r}}{2} = \int_0^T \frac{\mathbf{r}\times\frac{d\mathbf{r}}{dt}}{2}dt = \int_0^T \frac{\mathbf{r}\times\mathbf{v}}{2}dt = \frac{L}{2m}\int_0^T dt = \frac{L}{2m}T$$

✓ *for IHO*

In one period: $\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L}$ the area is: $A_\tau = \frac{L\tau}{2m}$ $(= ab\cdot\pi$ for ellipse orbit)

(Recall from Lecture 7: $\omega = \sqrt{k/m} = \sqrt{G\rho_{\oplus}4\pi/3}$)

Kepler "laws" (Some that apply to <u>all</u> central (isotropic) F(r) force fields)

→ *Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$* *(Derived here)*

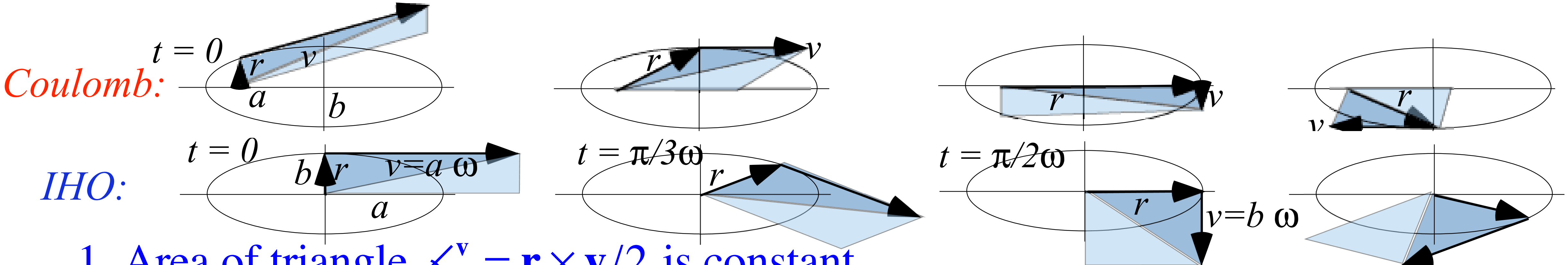
Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$ *(Derived in Unit 5)*

Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$ *(Derived here)*

Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$ *(Derived in Unit 5)*

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ and Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

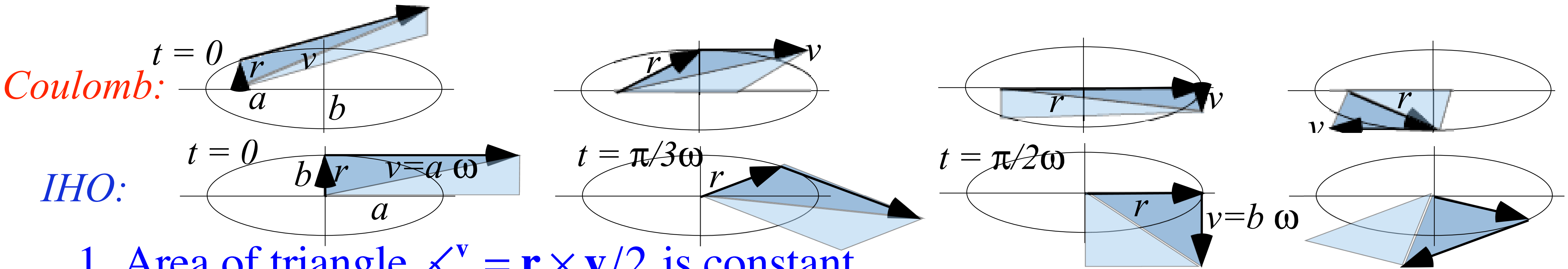
$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_{\oplus}4\pi/3} & \text{for IHO} \\ a^{-1/2}b\sqrt{GM_{\oplus}} & \text{for Coul.} \end{cases}$$

✓ *for IHO*

(Derived in Unit 5) ✓ *for Coul.*

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ and Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

✓ for IHO

(Derived in Unit 5) ✓ for Coul.

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m\left(r_x v_y - r_y v_x\right) = \begin{cases} m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ m\cdot a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

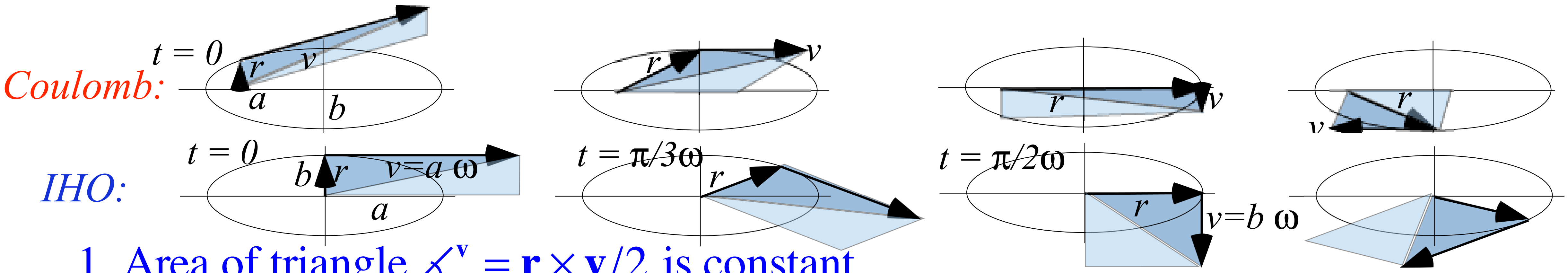
(... in Unit 5)

✓ for IHO

✓ for Coul.

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ and Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

(Derived in Unit 5) *(Derived in Unit 5)* ✓ *for IHO* ✓ *for Coul.*

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m\left(r_x v_y - r_y v_x\right) = \begin{cases} m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ m\cdot a^{-1/2}b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

(... in Unit 5) ✓ *for IHO* ✓ *for Coul.*

3. Equal area is swept by radius vector in each equal time interval T

In one period:

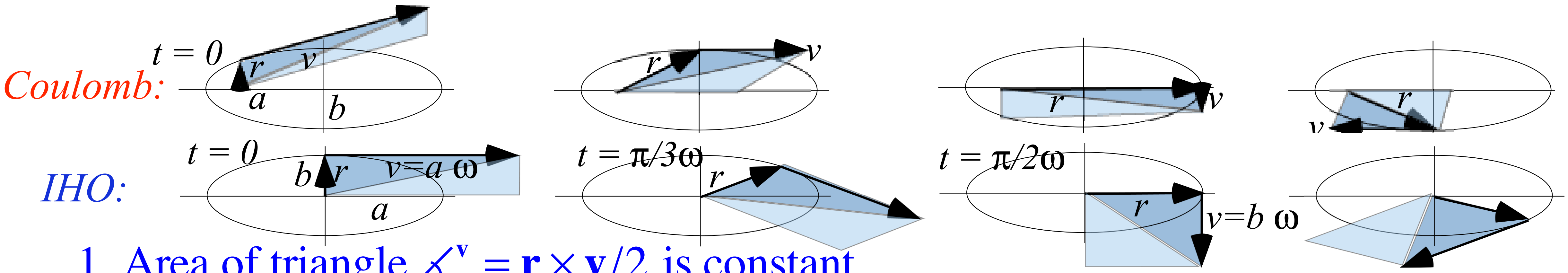
$$\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L} = \frac{2m\cdot ab\cdot\pi}{L} = \begin{cases} \dfrac{2m\cdot ab\cdot\pi}{m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3}} \\ \dfrac{2m\cdot ab\cdot\pi}{m\cdot a^{-1/2}b\sqrt{GM_\oplus}} \end{cases}$$

Applies to any central F(r)

Applies to IHO and Coulomb

Some Kepler's "laws" that apply to any central (isotropic) force F(r)

Apply to IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ and Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$



1. Area of triangle $\measuredangle_{\mathbf{r}}^{\mathbf{v}} = \mathbf{r}\times\mathbf{v}/2$ is constant

$$\mathbf{r}\times\mathbf{v} = r_x v_y - r_y v_x = \begin{cases} ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ a^{-1/2} b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

(Derived in Unit 5) (Derived in Unit 5)

✓ for IHO

✓ for Coul.

2. *Angular momentum* $L = m\,\mathbf{r}\times\mathbf{v}$ is conserved

$$L = m\,\mathbf{r}\times\mathbf{v} = m\left(r_x v_y - r_y v_x\right) = \begin{cases} m\cdot ab\cdot\sqrt{G\rho_\oplus 4\pi/3} & \text{for IHO} \\ m\cdot a^{-1/2} b\sqrt{GM_\oplus} & \text{for Coul.} \end{cases}$$

(... in Unit 5)

✓ for IHO

✓ for Coul.

3. Equal area is swept by radius vector in each equal time interval T

In one period:

$$\tau = \frac{1}{\upsilon} = \frac{2\pi}{\omega} = \frac{2mA_\tau}{L} = \frac{2m\cdot ab\cdot\pi}{L} = \begin{cases} \dfrac{2m\cdot\cancel{ab}\cdot\pi}{m\cdot\cancel{ab}\cdot\sqrt{G\rho_\oplus 4\pi/3}} = \dfrac{2\pi}{\sqrt{G\rho_\oplus 4\pi/3}} & \text{for IHO} \\ \dfrac{2m\cdot a\cancel{b}\cdot\pi}{m\cdot a^{-1/2}\cancel{b}\sqrt{GM_\oplus}} = \dfrac{2\pi}{a^{-3/2}\sqrt{GM_\oplus}} & \text{for Coul.} \end{cases}$$

$\frac{2mA_\tau}{L}$: *Applies to any central F(r)*

$\frac{2m\cdot ab\cdot\pi}{L}$: *Applies to IHO and Coulomb*

(not a function of *a* or *b*)

that is ω_{IHO}

that is ω_{Coul}

(not a function of *b*)

Kepler "laws" (Some that apply to all central (isotropic) $F(r)$ force fields)

Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ *(Derived here)*

Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$ *(Derived in Unit 5)*

Total energy $E=KE+PE$ invariance of IHO: $F(r)=-k\cdot r$ *(Derived here)*

Total energy $E=KE+PE$ invariance of Coulomb: $F(r)=-GMm/r^2$ *(Derived in Unit 5)*

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total energy=$KE+PE$ is constant

$$KE+PE=\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}+\frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$=\frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix}+\begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}kr_x^2+\frac{1}{2}kr_y^2$$

$$=\frac{1}{2}m(-a\omega\sin\omega t)^2+\frac{1}{2}m(b\omega\cos\omega t)^2+\frac{1}{2}k(a\cos\omega t)^2+\frac{1}{2}k(b\sin\omega t)^2$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix}=\begin{pmatrix} -a\omega\sin\omega t \\ b\omega\cos\omega t \end{pmatrix}$$

$$\begin{pmatrix} r_x \\ r_y \end{pmatrix}=\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} a\cos\omega t \\ b\sin\omega t \end{pmatrix}$$

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total *IHO* energy=$KE+PE$ is constant

$$KE+PE=\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}+\frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$=\frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix}+\begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}kr_x^2+\frac{1}{2}kr_y^2$$

$$=\frac{1}{2}m(-a\omega\sin\omega t)^2+\frac{1}{2}m(b\omega\cos\omega t)^2+\frac{1}{2}k(a\cos\omega t)^2+\frac{1}{2}k(b\sin\omega t)^2$$

$$=\frac{1}{2}ma^2\omega^2(\sin^2\omega t)+\frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2+\frac{1}{2}ka^2(\cos^2\omega t)+\frac{1}{2}kb^2(\sin^2\omega t)$$

$$=\frac{1}{2}m\omega^2(a^2+b^2)$$

$$Given: k=m\omega^2$$

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: F(r)=-k·r with U(r)=k·r²/2

Total *IHO* energy=$KE+PE$ is constant

$$KE+PE = \frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v} + \frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$= \frac{1}{2}\begin{pmatrix} v_x & v_y \end{pmatrix}\bullet\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\bullet\begin{pmatrix} v_x \\ v_y \end{pmatrix} + \begin{pmatrix} r_x & r_y \end{pmatrix}\bullet\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}\bullet\begin{pmatrix} r_x \\ r_y \end{pmatrix}$$

$$= \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}kr_x^2 + \frac{1}{2}kr_y^2$$

$$= \frac{1}{2}m(-a\omega\sin\omega t)^2 + \frac{1}{2}m(b\omega\cos\omega t)^2 + \frac{1}{2}k(a\cos\omega t)^2 + \frac{1}{2}k(b\sin\omega t)^2$$

$$= \frac{1}{2}ma^2\omega^2(\sin^2\omega t) + \frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2 + \frac{1}{2}ka^2(\cos^2\omega t) + \frac{1}{2}kb^2(\sin^2\omega t)$$

$$= \frac{1}{2}m\omega^2(a^2+b^2) \qquad Given: k = m\omega^2$$

$$E = KE+PE = \frac{1}{2}m\omega^2(a^2+b^2) = \frac{1}{2}k(a^2+b^2) \quad \text{since: } \omega = \sqrt{\frac{k}{m}} = \sqrt{G\rho_{\oplus}4\pi/3} \quad \text{or: } m\omega^2 = k$$

Kepler "laws" (Some that apply to <u>all</u> central (isotropic) F(r) force fields)

Angular momentum invariance of IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$ *(Derived here)*

Angular momentum invariance of Coulomb: $F(r)=-GMm/r^2$ with $U(r)=-GMm\cdot/r$ *(Derived in Unit 5)*

Total energy E=KE+PE invariance of IHO: $F(r)=-k\cdot r$ *(Derived here)*

→ *Total energy E=KE+PE invariance of Coulomb: $F(r)=-GMm/r^2$* *(Derived in Unit 5)*

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total *IHO* energy$=KE+PE$ is constant

$$KE+PE=\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}+\frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$=\frac{1}{2}\begin{pmatrix} v_x & v_y\end{pmatrix}\bullet\begin{pmatrix} m & 0\\ 0 & m\end{pmatrix}\bullet\begin{pmatrix} v_x\\ v_y\end{pmatrix}+\begin{pmatrix} r_x & r_y\end{pmatrix}\bullet\begin{pmatrix} k & 0\\ 0 & k\end{pmatrix}\bullet\begin{pmatrix} r_x\\ r_y\end{pmatrix}$$

$$=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}kr_x^2+\frac{1}{2}kr_y^2$$

$$=\frac{1}{2}m(-a\omega\sin\omega t)^2+\frac{1}{2}m(b\omega\cos\omega t)^2+\frac{1}{2}k(a\cos\omega t)^2+\frac{1}{2}k(b\sin\omega t)^2$$

$$=\frac{1}{2}ma^2\omega^2(\sin^2\omega t)+\frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2+\frac{1}{2}ka^2(\cos^2\omega t)+\frac{1}{2}kb^2(\sin^2\omega t)$$

$$=\frac{1}{2}m\omega^2(a^2+b^2)$$

$$Given: k=m\omega^2$$

$E=KE+PE=\frac{1}{2}m\omega^2(a^2+b^2)=\frac{1}{2}k(a^2+b^2)$ since: $\omega=\sqrt{\frac{k}{m}}=\sqrt{G\rho_{\oplus}4\pi/3}$ or: $m\omega^2=k$

We'll see that the Coul. orbits are simpler: (*like the period*...not a function of *b*)

Kepler laws involve ∡-momentum conservation in isotropic force F(r)

Now consider orbital energy conservation of the IHO: $F(r)=-k\cdot r$ with $U(r)=k\cdot r^2/2$

Total *IHO* energy=$KE+PE$ is constant

$$KE+PE=\frac{1}{2}\mathbf{v}\bullet\mathbf{M}\bullet\mathbf{v}+\frac{1}{2}\mathbf{r}\bullet\mathbf{K}\bullet\mathbf{r}$$

$$=\frac{1}{2}\begin{pmatrix} v_x & v_y\end{pmatrix}\bullet\begin{pmatrix} m & 0\\ 0 & m\end{pmatrix}\bullet\begin{pmatrix} v_x\\ v_y\end{pmatrix}+\begin{pmatrix} r_x & r_y\end{pmatrix}\bullet\begin{pmatrix} k & 0\\ 0 & k\end{pmatrix}\bullet\begin{pmatrix} r_x\\ r_y\end{pmatrix}$$

$$=\frac{1}{2}mv_x^2+\frac{1}{2}mv_y^2+\frac{1}{2}kr_x^2+\frac{1}{2}kr_y^2$$

$$=\frac{1}{2}m(-a\omega\sin\omega t)^2+\frac{1}{2}m(b\omega\cos\omega t)^2+\frac{1}{2}k(a\cos\omega t)^2+\frac{1}{2}k(b\sin\omega t)^2$$

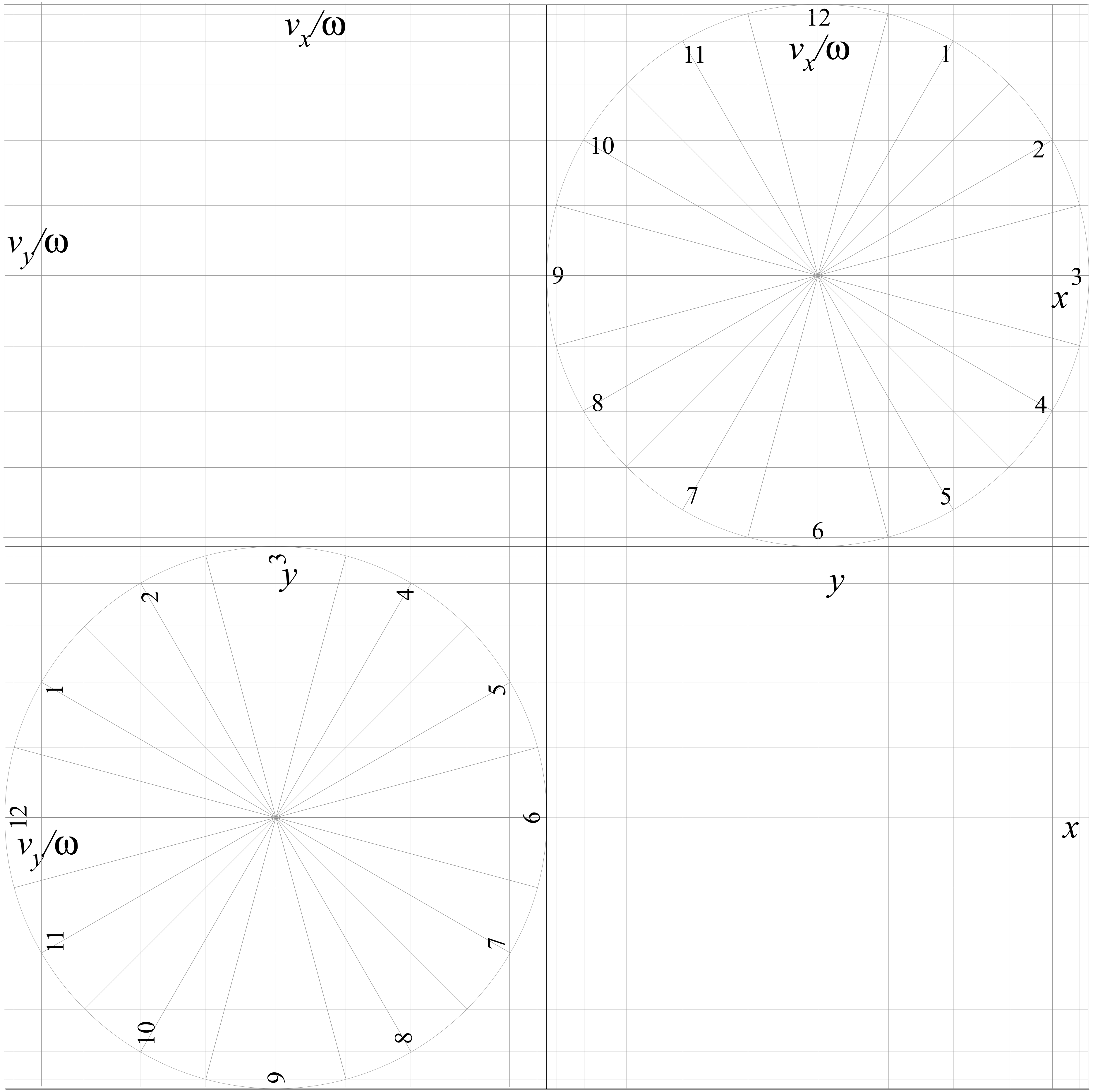
$$=\frac{1}{2}ma^2\omega^2(\sin^2\omega t)+\frac{1}{2}mb^2\omega^2(\cos^2\omega t)^2+\frac{1}{2}ka^2(\cos^2\omega t)+\frac{1}{2}kb^2(\sin^2\omega t)$$

$$=\frac{1}{2}m\omega^2(a^2+b^2)\qquad Given: k=m\omega^2$$

$E=KE+PE=\frac{1}{2}m\omega^2(a^2+b^2)=\frac{1}{2}k(a^2+b^2)$ since: $\omega=\sqrt{\frac{k}{m}}=\sqrt{G\rho_\oplus 4\pi/3}$ or: $m\omega^2=k$

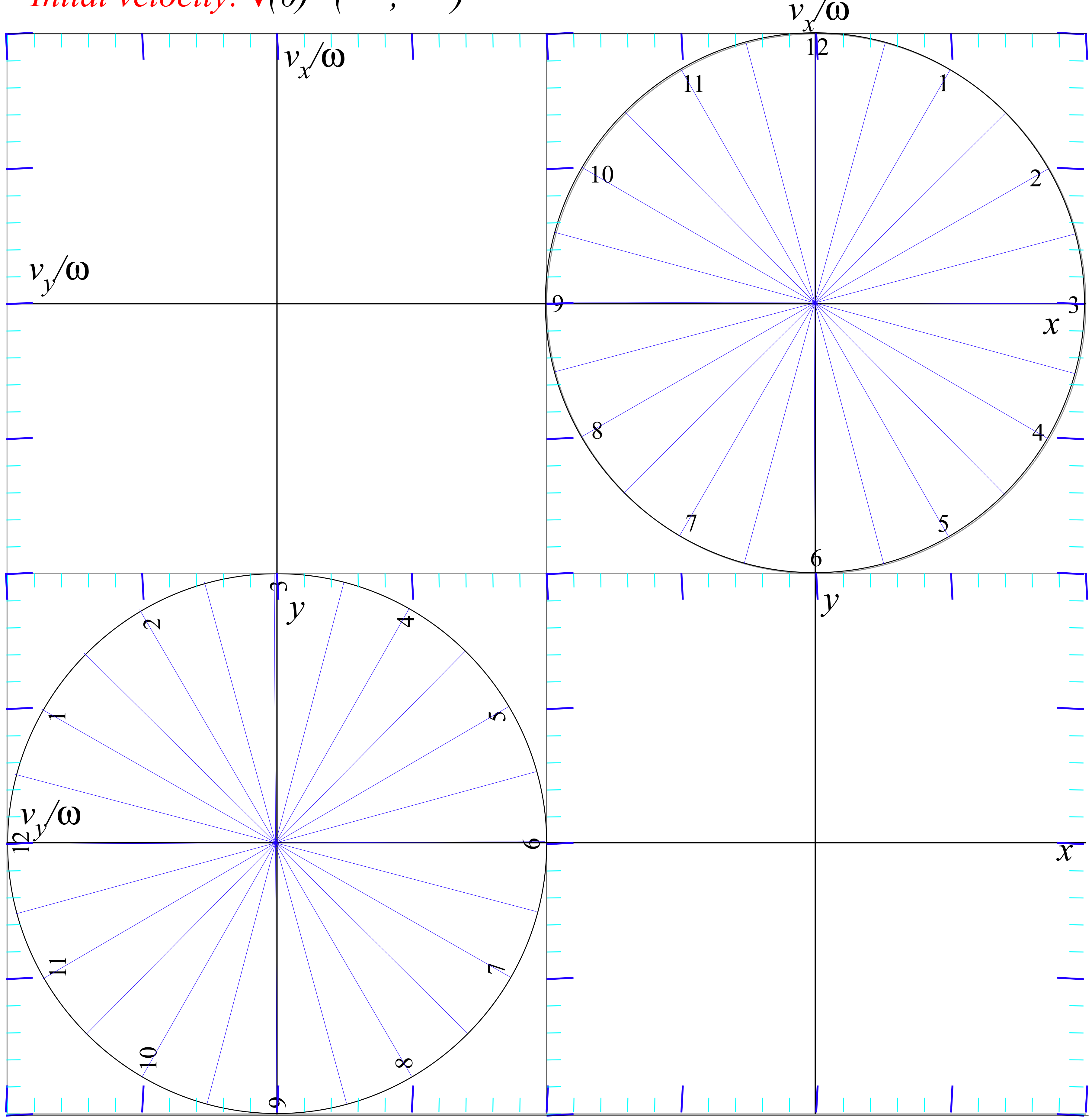
We'll see that the Coul. orbits are simpler: (*like the period*...not a function of *b*)

$$E=KE+PE=\frac{1}{2}mv^2{}_x+\frac{1}{2}mv^2{}_y-\frac{k}{r}=\frac{1}{2}mv^2{}_x+\frac{1}{2}mv^2{}_y-\frac{GM_\oplus m}{r}=-\frac{GM_\oplus m}{a}$$



v_x/ω
v_y/ω
12
11
1
10
2
9
3
8
4
7
5
6
x
y

Inital velocity: **v***(0)=(,   )*



Initial position: **r***(0)=(,   )*