Lecture 27 Relawavity-Applications 1
Tuesday 4.19.2016

Relawavity: Relativistic wave mechanics IV. Coordinate geometry

(Unit 3 p.19-32 - 4.19.16)

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

➔ 2005 and 2016 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

Lighthouse (x,y) frame: Dual concentric circular wavefronts serve as timing device

Ship frame: time dilation $\Delta = \cosh\rho = 1.15$ of Lighthouse blinks

Simultaneous events in Lighthouse (x,y) frame: Not so in Ship (x',y') frame

Lighthouse-square (x,ct) plots correlated with Ship-square (x',ct') plots

Overlapped Lighthouse (x,ct) and Ship (x',ct') frame Minkowski plots correlate inconsistencies

Ship (x',y') frame: Dual un-concentric circular wavefronts map space-time

Pythagorean derivation of time-dilation factor $\Delta = \cosh\rho$

Un-concentric derivation of stellar aberration **k**-angle σ

Per-spacetime 4-vector $(\omega_0, \omega_x, \omega_y, \omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

“Occam-sword” geometry: A pattern recognition aid

Relating velocity parameter $\beta = u/c$ to *rapidity* ρ to **k**-*angle* σ to *u/c-angle* ν

Circular arc-area σ vs. hyperbolic arc-area ρ

Each circular trig function has a hyperbolic “country-cousin” function

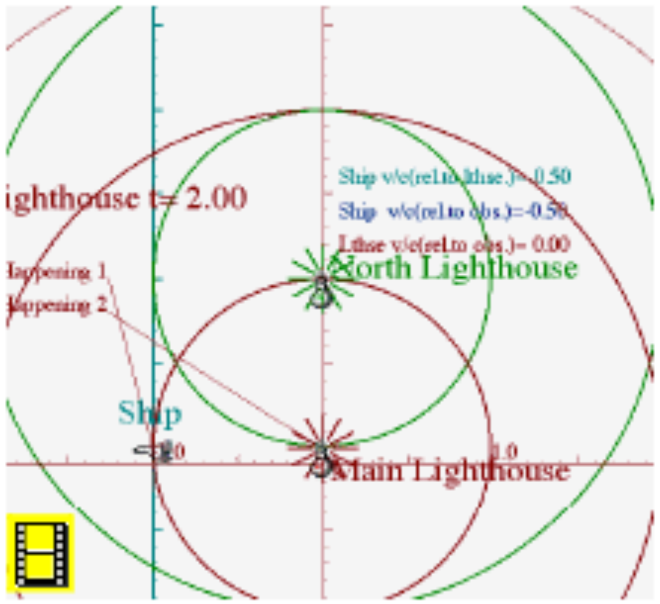
Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ

Simple 2005 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

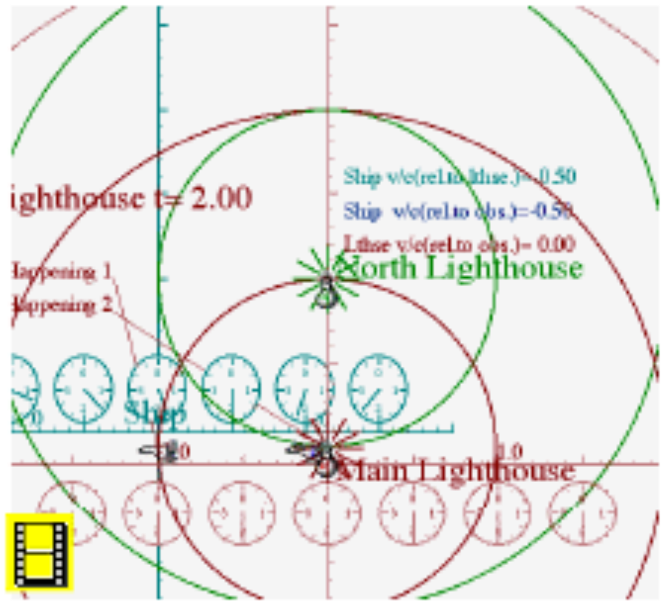
Relativistic Ship and Lighthouse Scenarios

Space-Space Movies

Ship passing two lighthouses in lighthouse rest frame



Two ships passing two lighthouses with clocks in both reference frames



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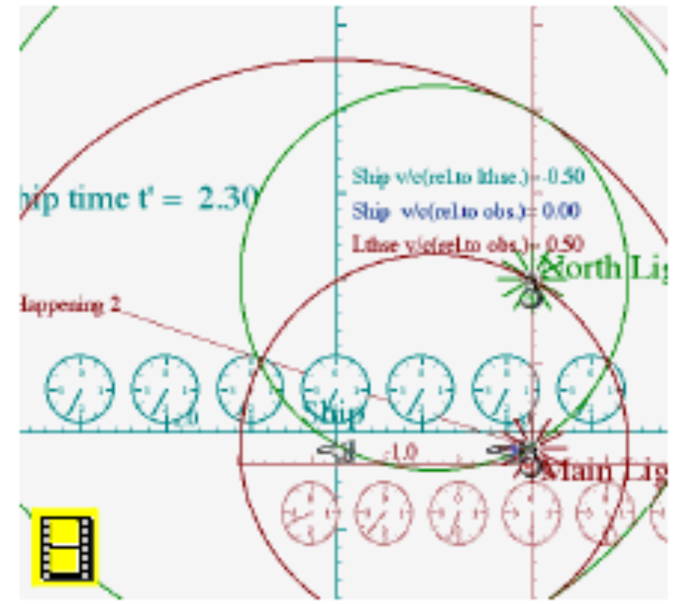
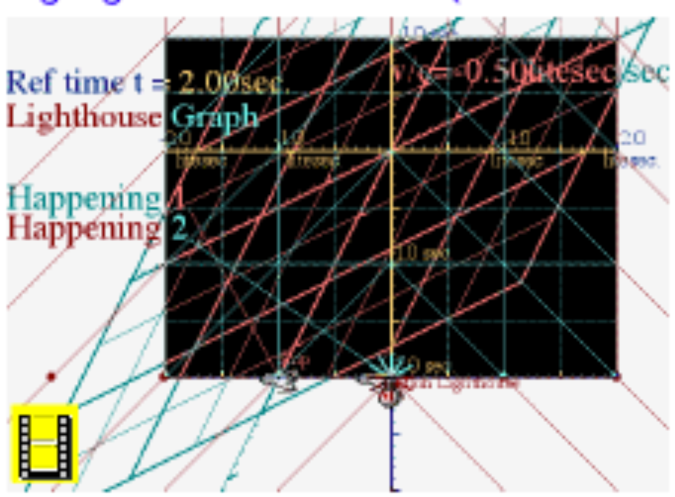
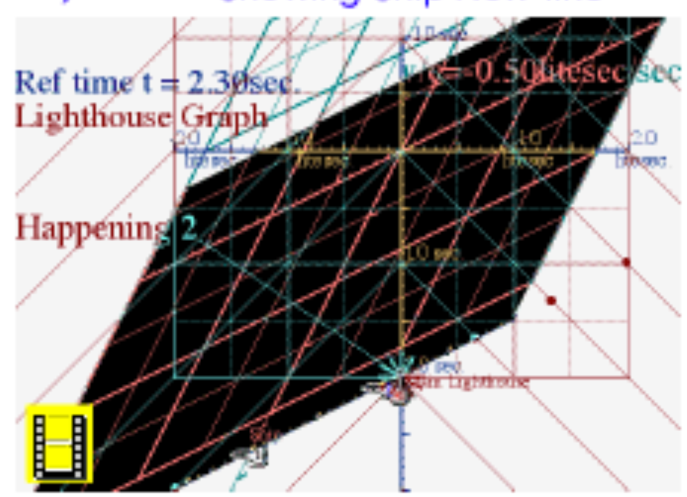


Space-Time Movies in Lighthouse Rest Frame

Showing Lighthouse Now-Line (Black terminator-line)



Showing Ship Now-line



<Home> <Back to Relativity Discussion>

www.uark.edu/ua/pirelli/php/lighthouse_scenarios.php

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

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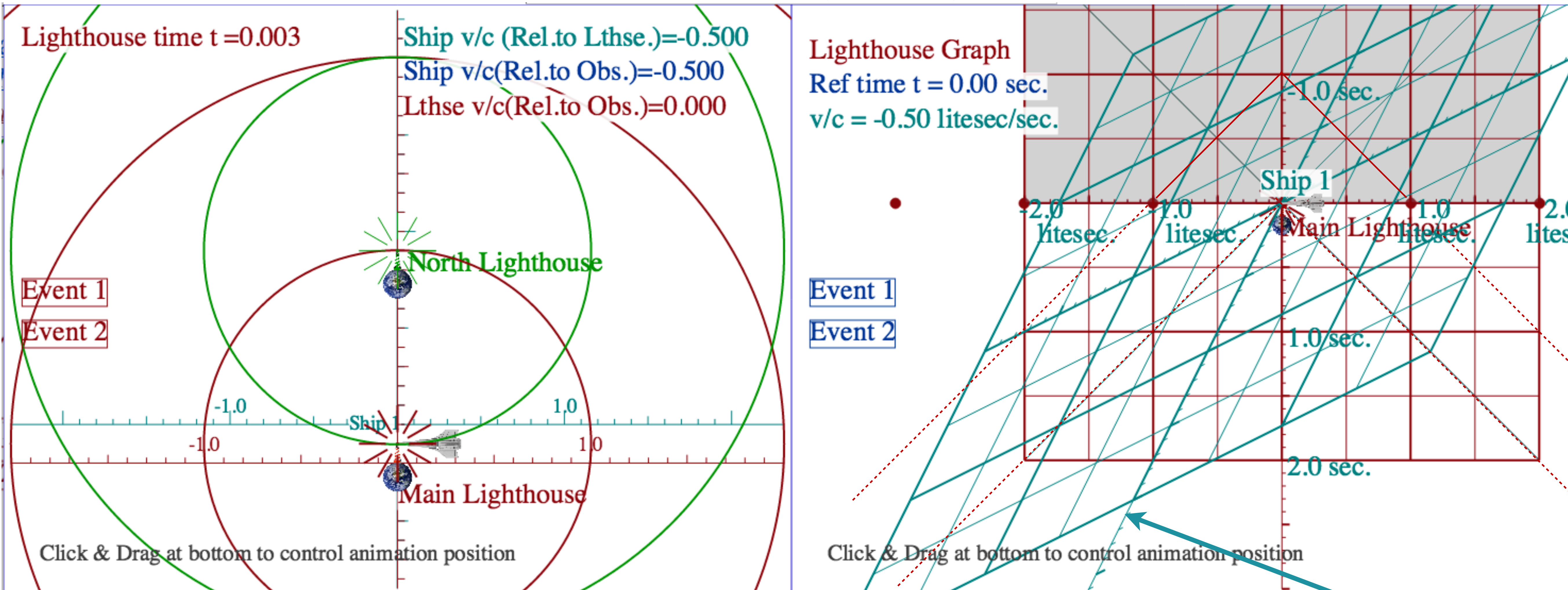
RelativIt Web Simulation
Relativistic Events in
Main Lighthouse's Frame

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$



$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$ To check whether to use matrix $\begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix}$ or else $\begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix}$ just check that $x' = 0 = 2x + 1ct$ or: $x = -ct/2$ gives correct path.

for: $\beta = \frac{1}{2}$ *or*: $e^{\rho} = \sqrt{3}$

$\cosh\rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 \quad = \Delta$

$\sinh\rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577$

Event 0: Ship passes Main Lighthouse.	Event 1: Ship gets hit by first blink from Main Lighthouse.	Event 2: Main Lighthouse blinks second time.
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http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=101

2015 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

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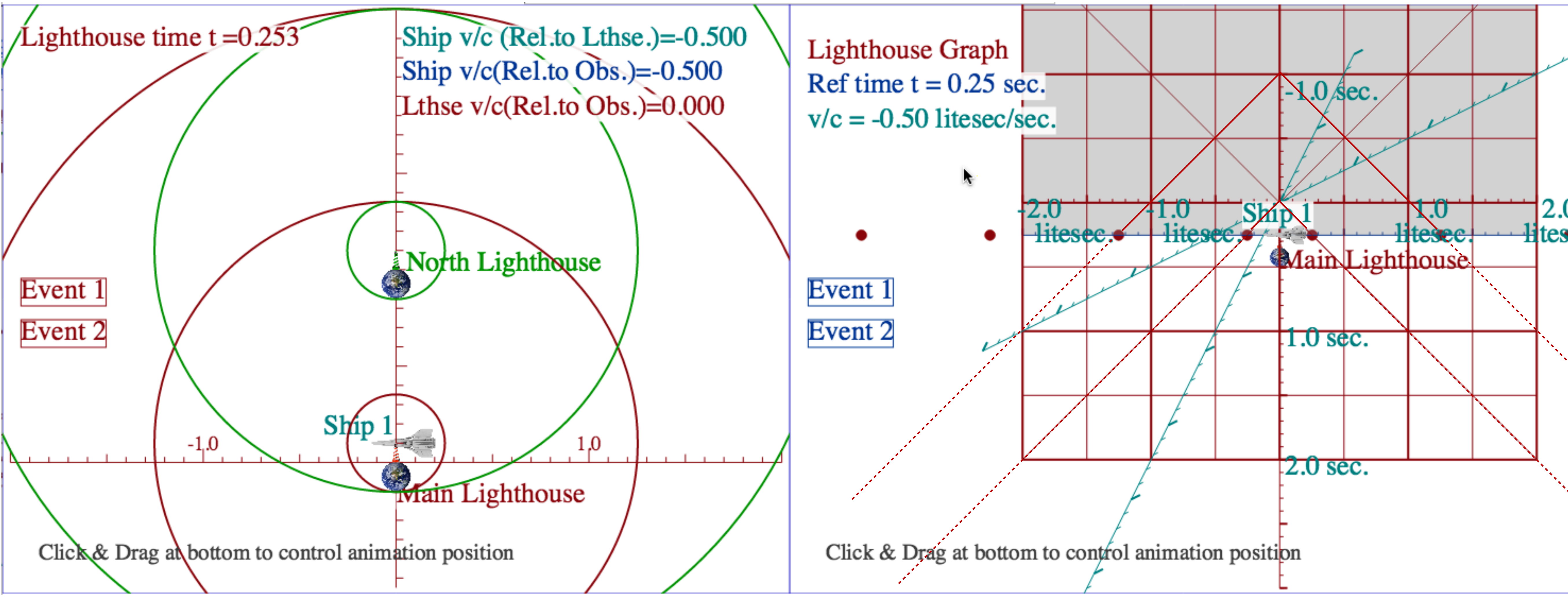

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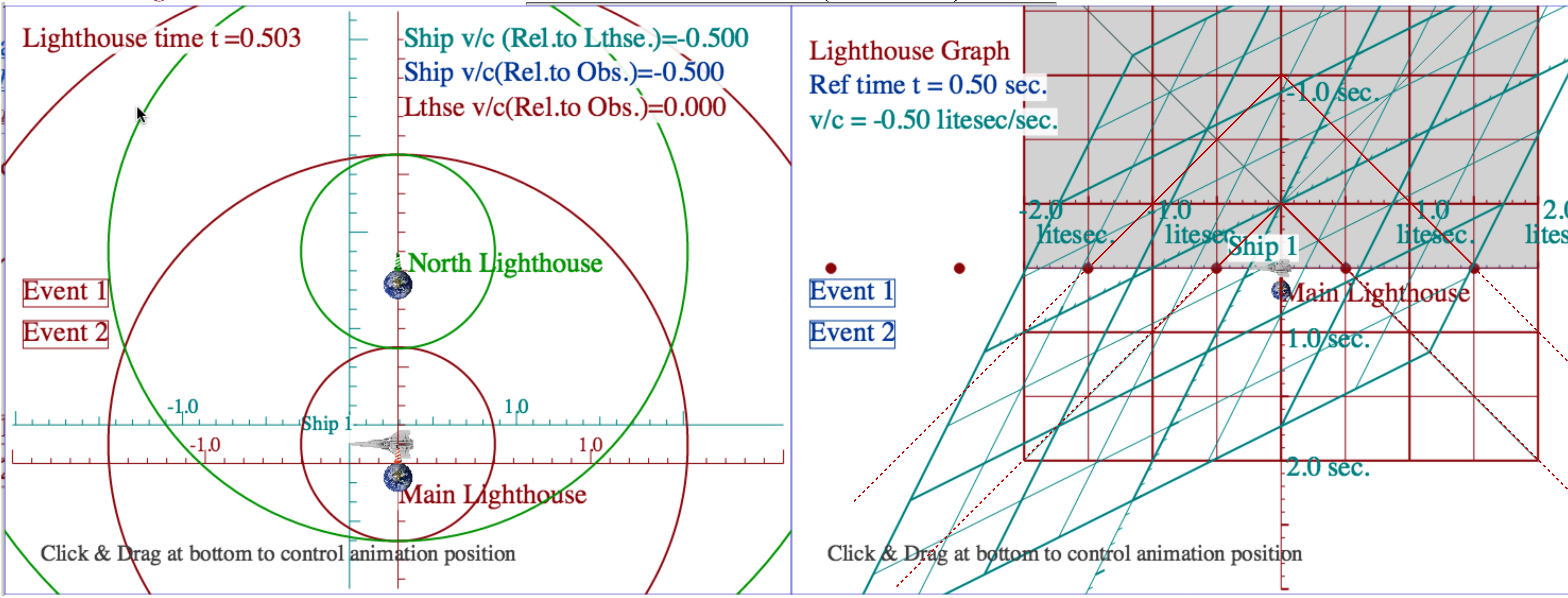

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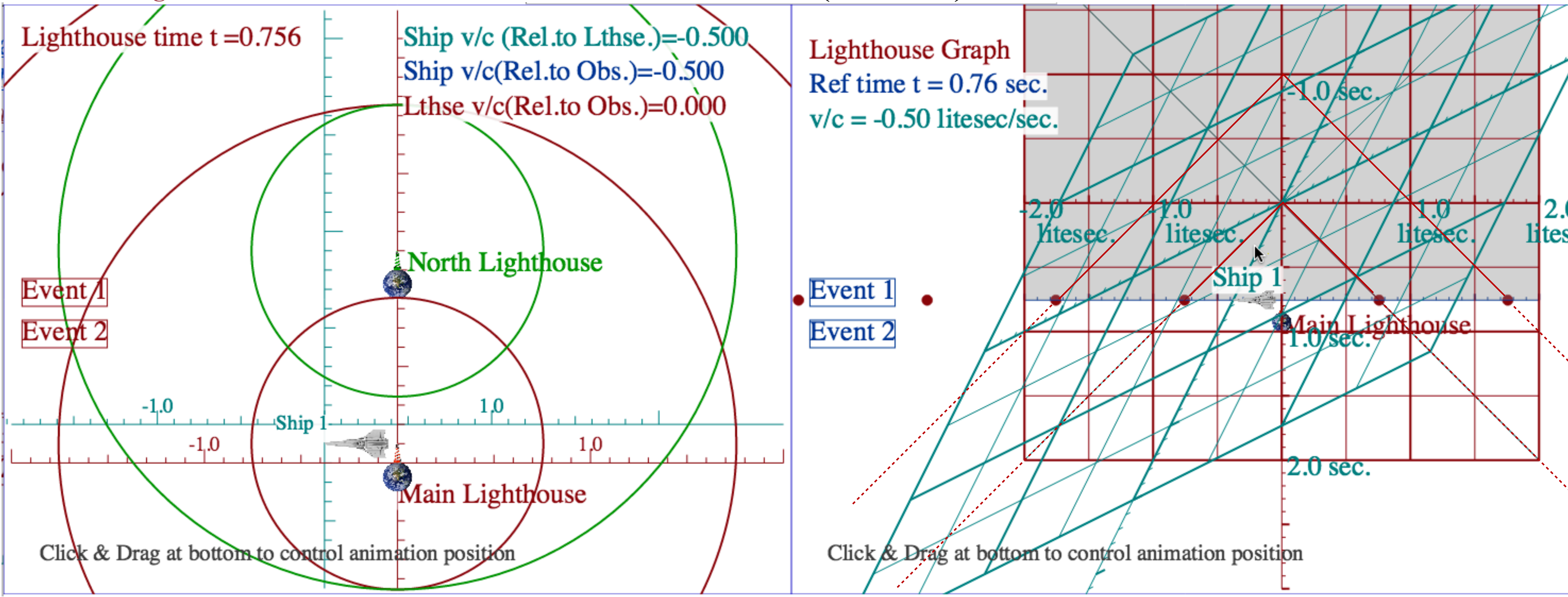

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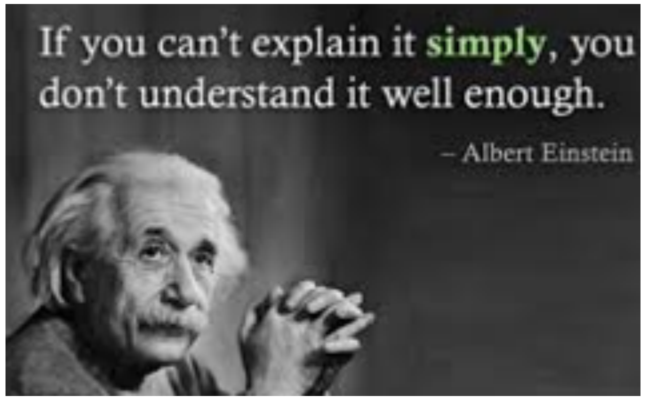
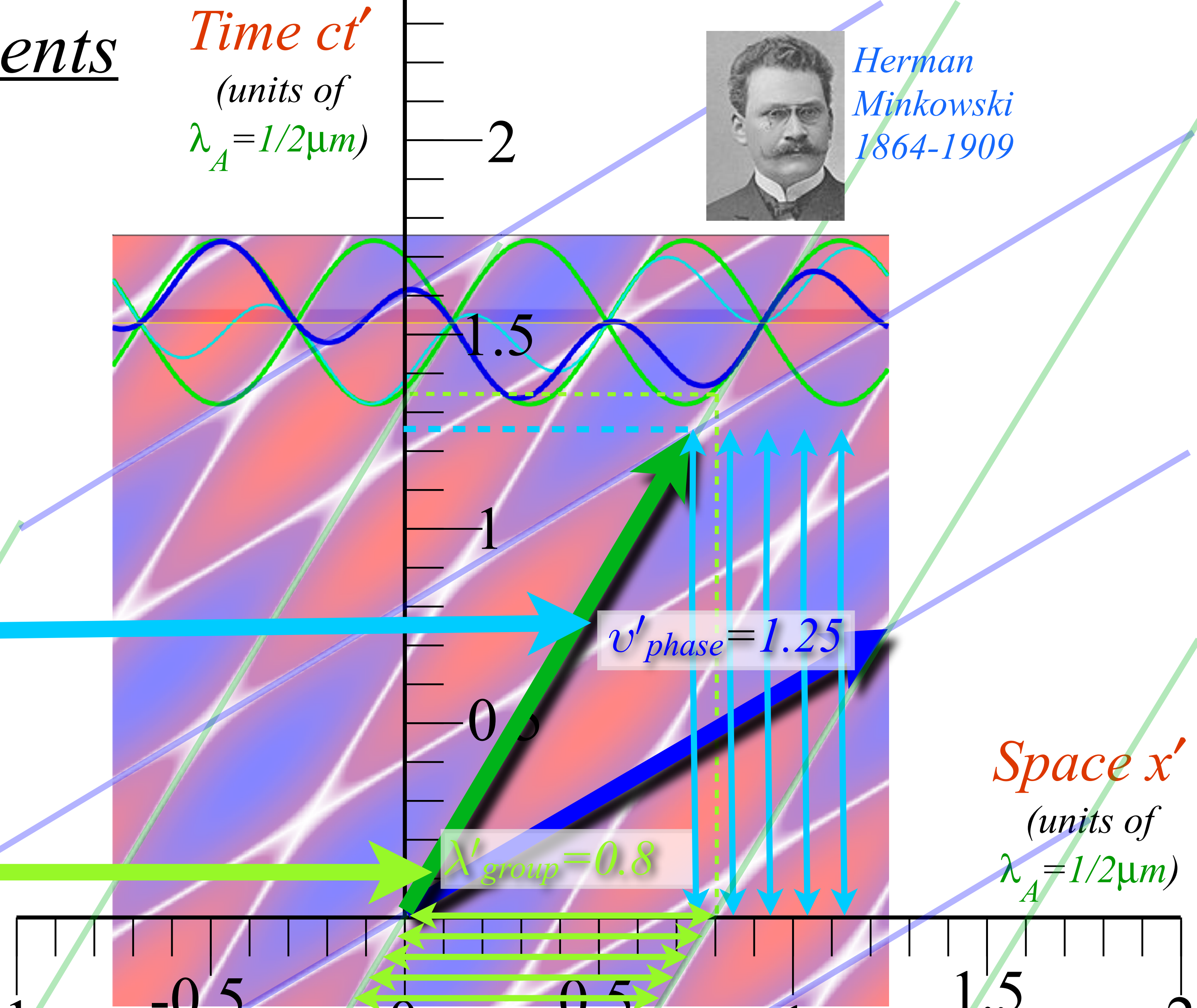
Two Famous-Name Coefficients

Albert Einstein 1859-1955

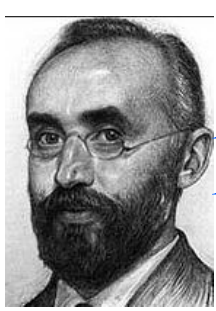


Herman Minkowski 1864-1909



This number is called an: ***Einstein time-dilation*** *(dilated by 25% here)*

This number is called a: ***Lorentz length-contraction*** *(contracted by 20% here)*



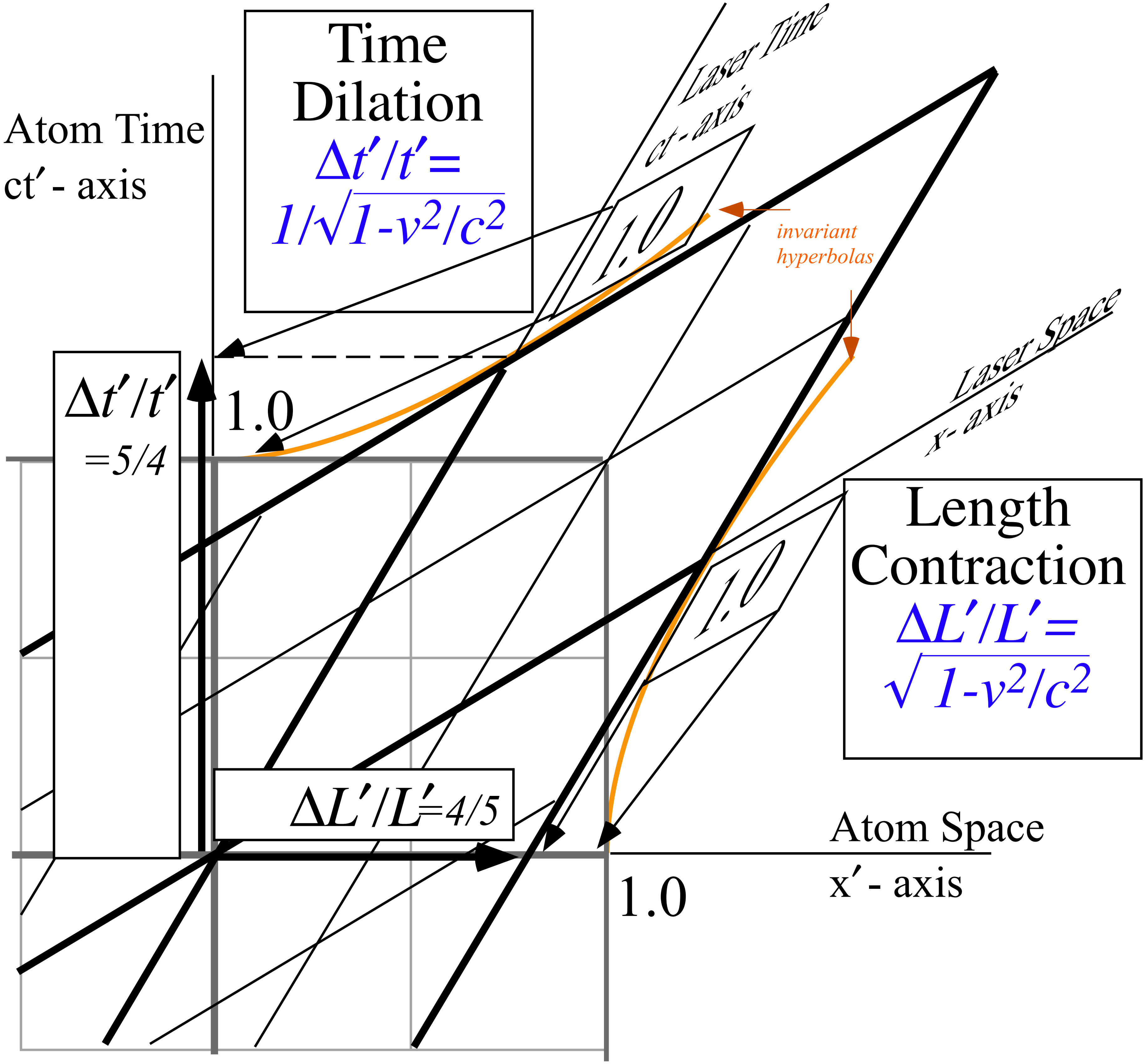
Hendrik A. Lorentz 1853-1928

Old-Fashioned Notation

phase	$b^{Doppler}_{RED}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$b^{Doppler}_{BLUE}$
group	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for β=3/5	$\frac{1}{2}$= 0.5	$\frac{3}{5}$ =0.6	$\frac{3}{4}$=0.75	$\frac{4}{5}$=0.80	$\frac{5}{4}$=1.25	$\frac{4}{3}$=1.33	$\frac{5}{3}$=1.67	$\frac{2}{1}$=2.0

RelaWavity Web Simulation

Relativistic *Terms (Dual plot w/expanded table)*



Space-time grid intersections mark Lorentz contraction and Einstein time dilation.

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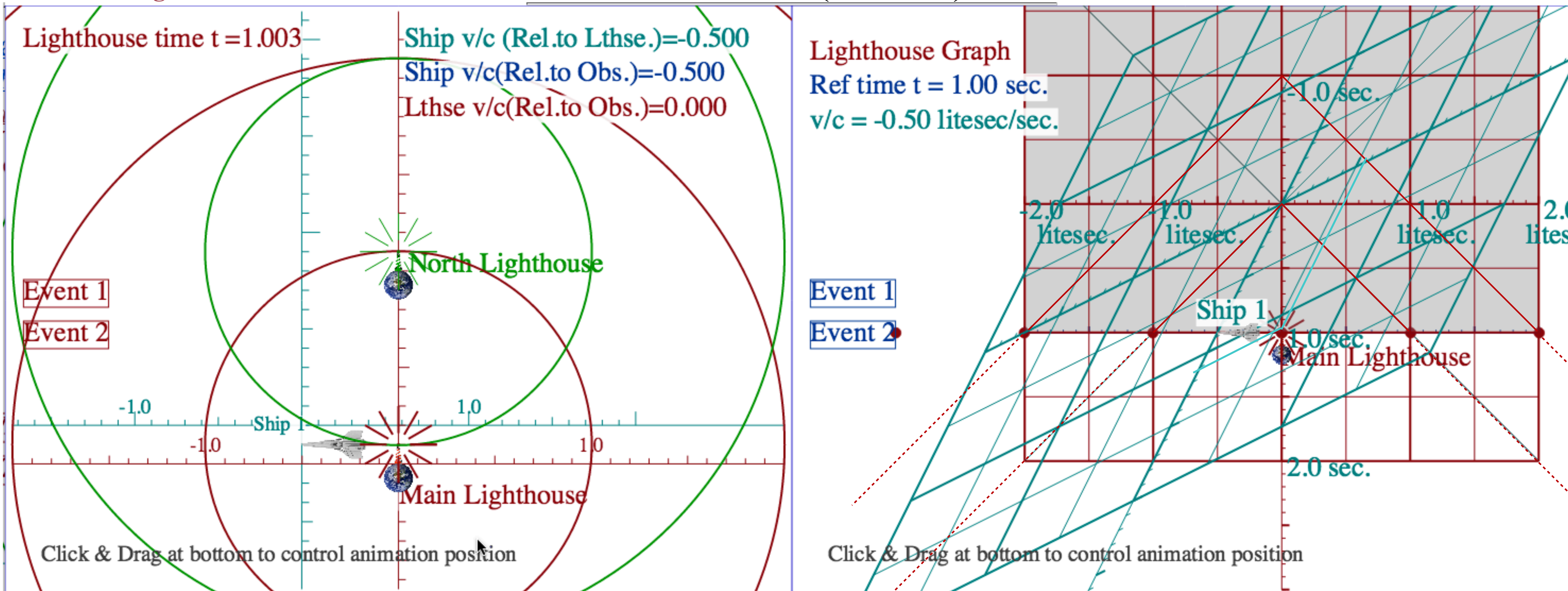

Ship registers 1st Lighthouse Blink

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$

at its position time $\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 0.577 \\ 1.155 \end{pmatrix} = \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}$

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http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=101

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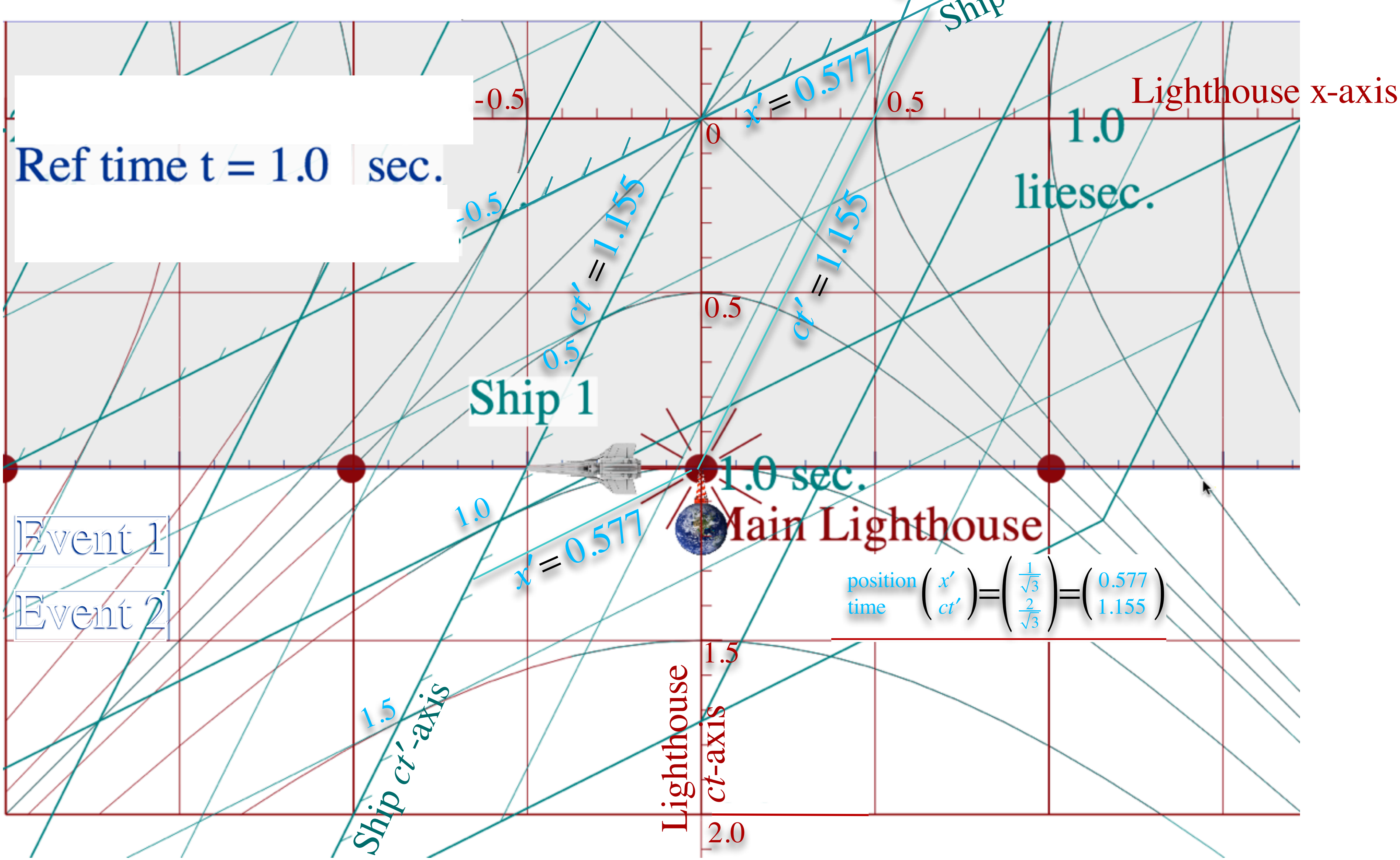
Ship frame: time dilation $\Delta = \cosh\rho = 1.155$ of Lighthouse blinks



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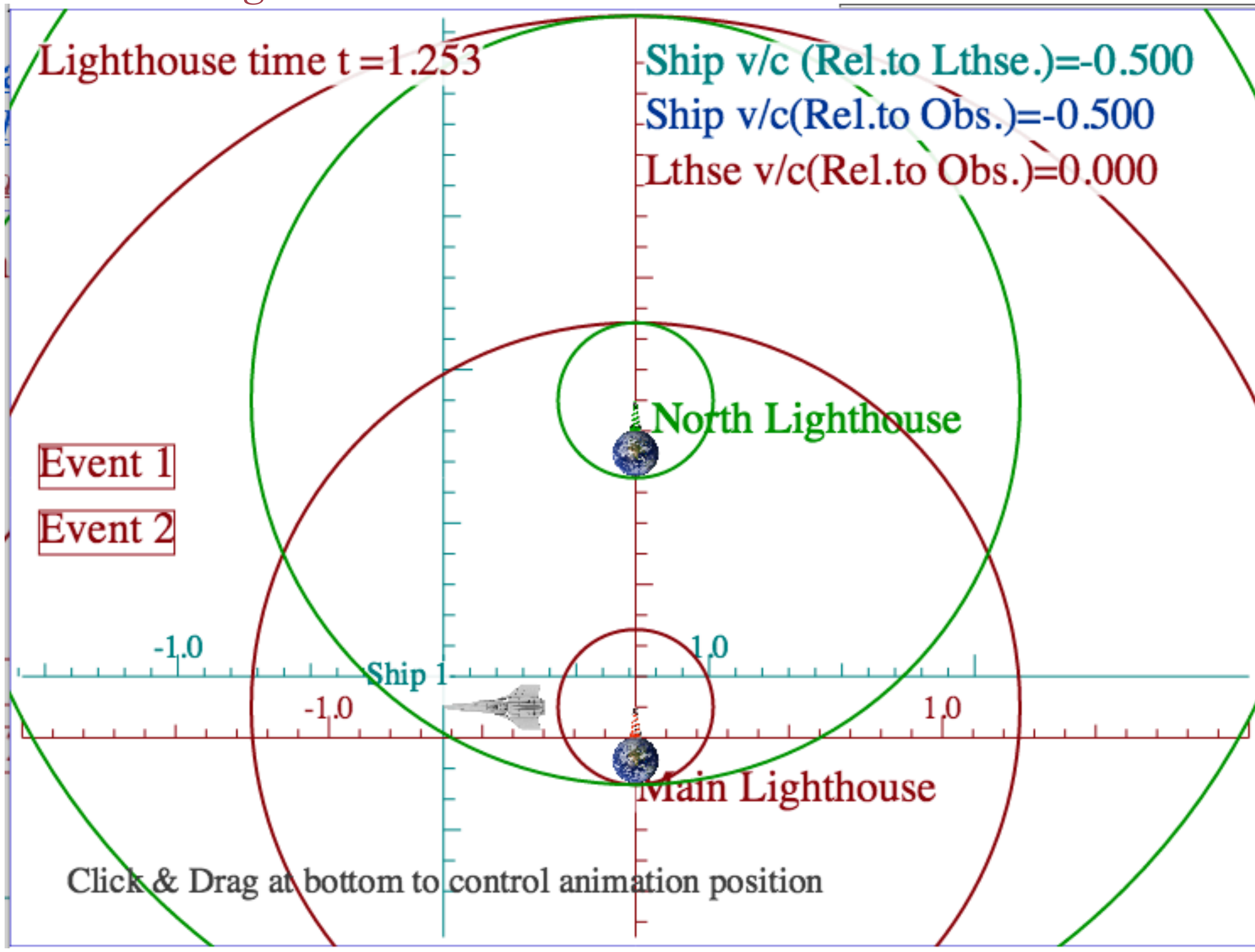

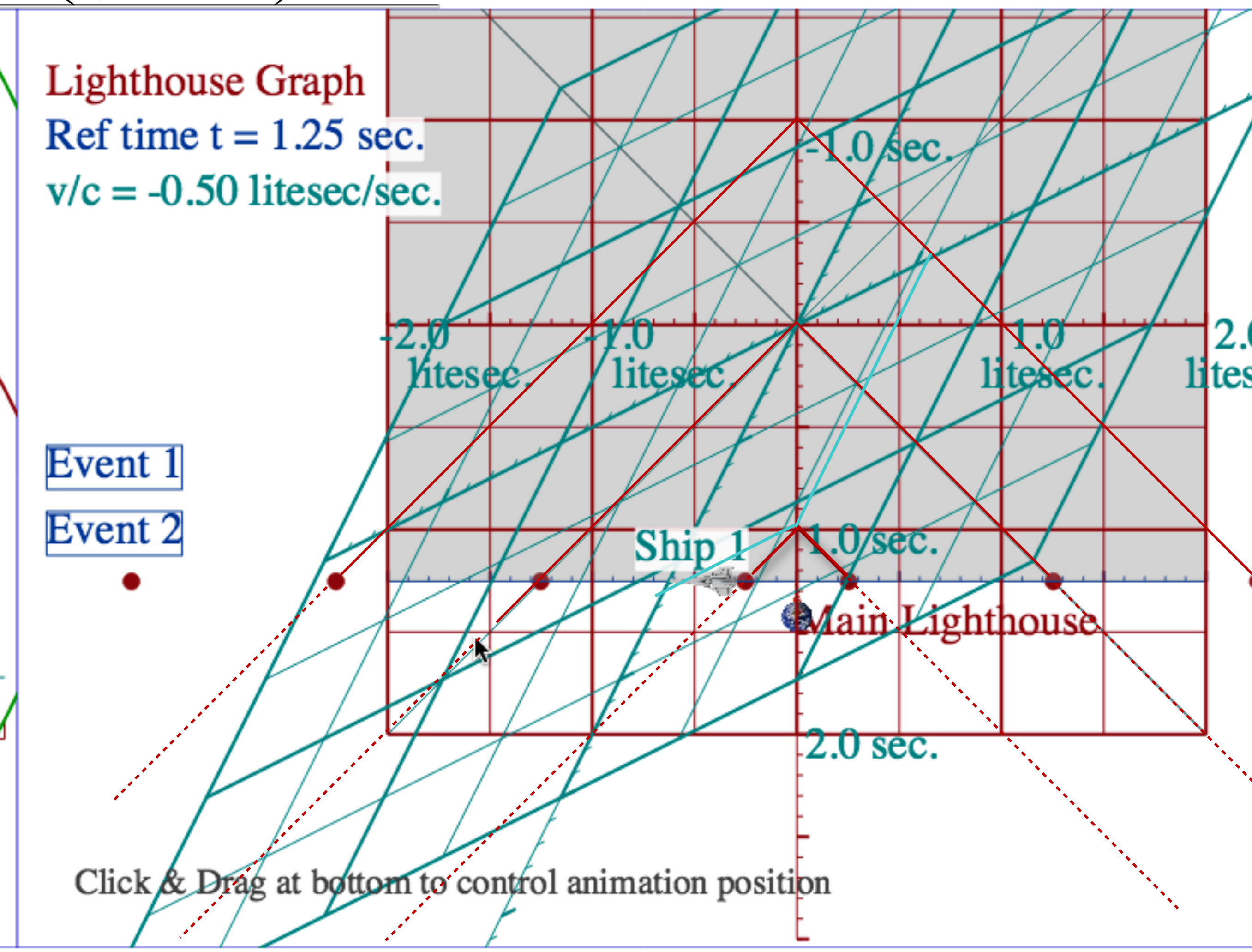

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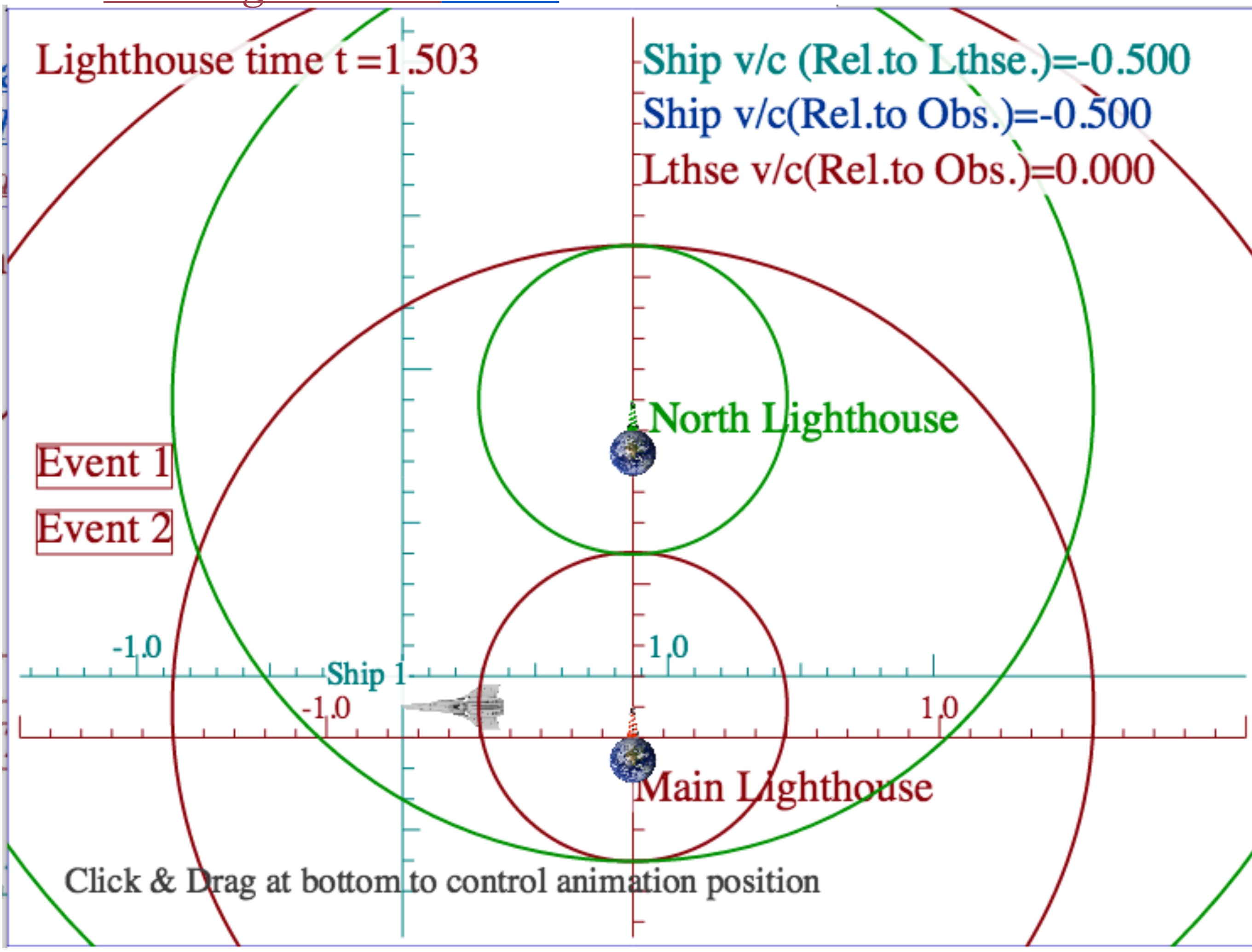
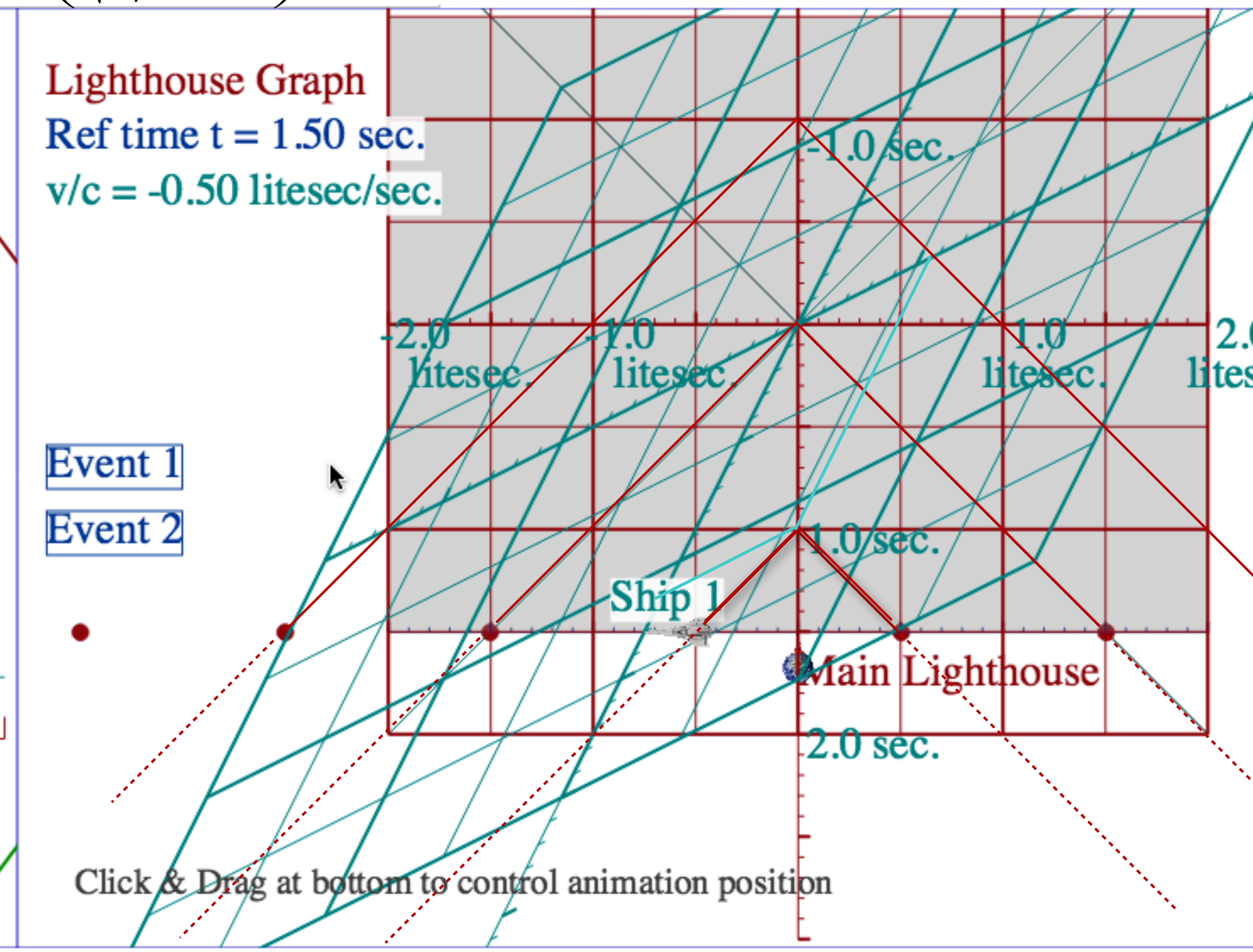
RelativIt Web Simulation

Relativistic Events in

Main Lighthouse's Frame

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$





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for: $\beta = \frac{1}{2}$ *or:* $e^{\rho} = \sqrt{3}$

$$\cosh\rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 \quad = \Delta$$

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Event 0: Ship passes Main Lighthouse.	Event 1: Ship gets hit by first blink from Main Lighthouse.	Event 2: Main Lighthouse blinks second time.
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2015 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

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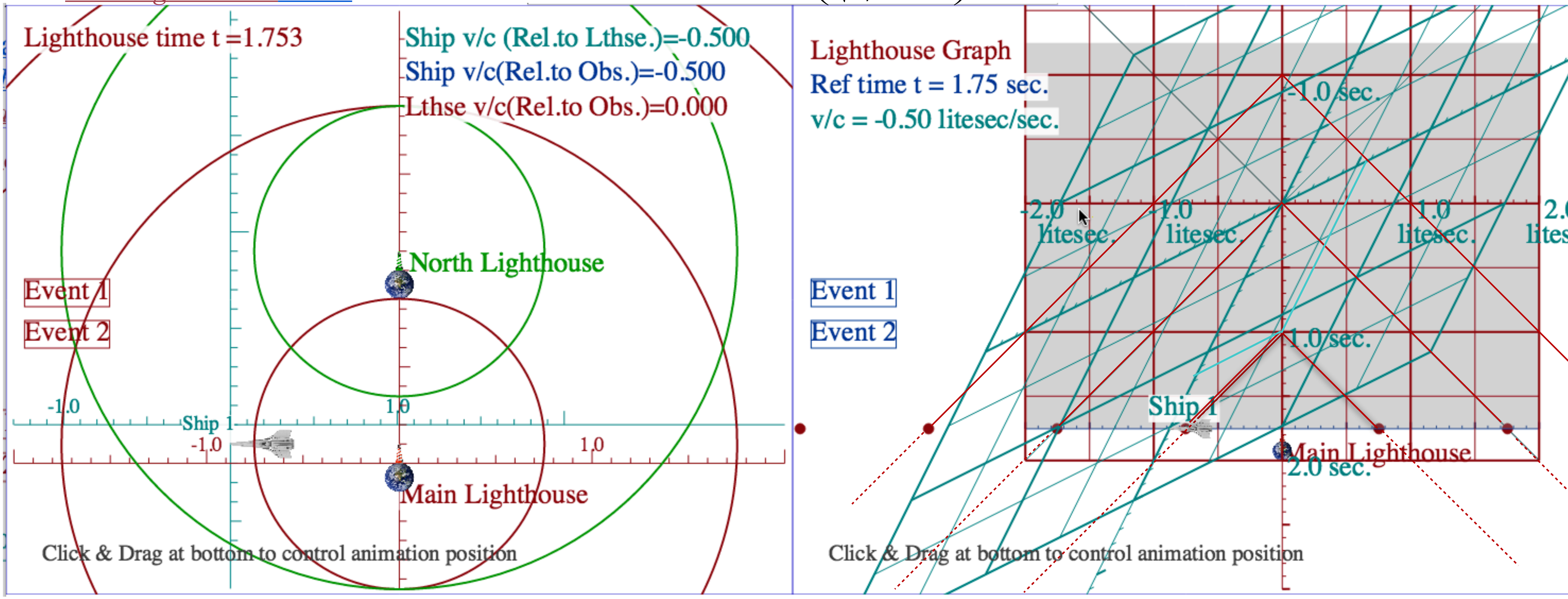

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Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

2005 and 2016 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

Lighthouse (x,y) frame: Dual concentric circular wavefronts serve as timing device

Ship frame: time dilation $\Delta=\cosh\rho=1.15$ of Lighthouse blinks

➔ Simultaneous events in Lighthouse (x,y) frame: Not so in Ship (x',y') frame

Lighthouse-square (x,ct) plots correlated with Ship-square (x',ct') plots

Overlapped Lighthouse (x,ct) and Ship (x',ct') frame Minkowski plots correlate inconsistencies

Ship (x',y') frame: Dual un-concentric circular wavefronts map space-time

Pythagorean derivation of time-dilation factor $\Delta=\cosh\rho$

Un-concentric derivation of stellar aberration **k**-angle σ

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega,ck_x,ck_y,ck_z)$ transformation

"Occam-sword" geometry: A pattern recognition aid

Relating velocity parameter $\beta=u/c$ to *rapidity* ρ to **k**-*angle* σ to *u/c-angle* ν

Circular arc-area σ vs. hyperbolic arc-area ρ

Each circular trig function has a hyperbolic "country-cousin" function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ

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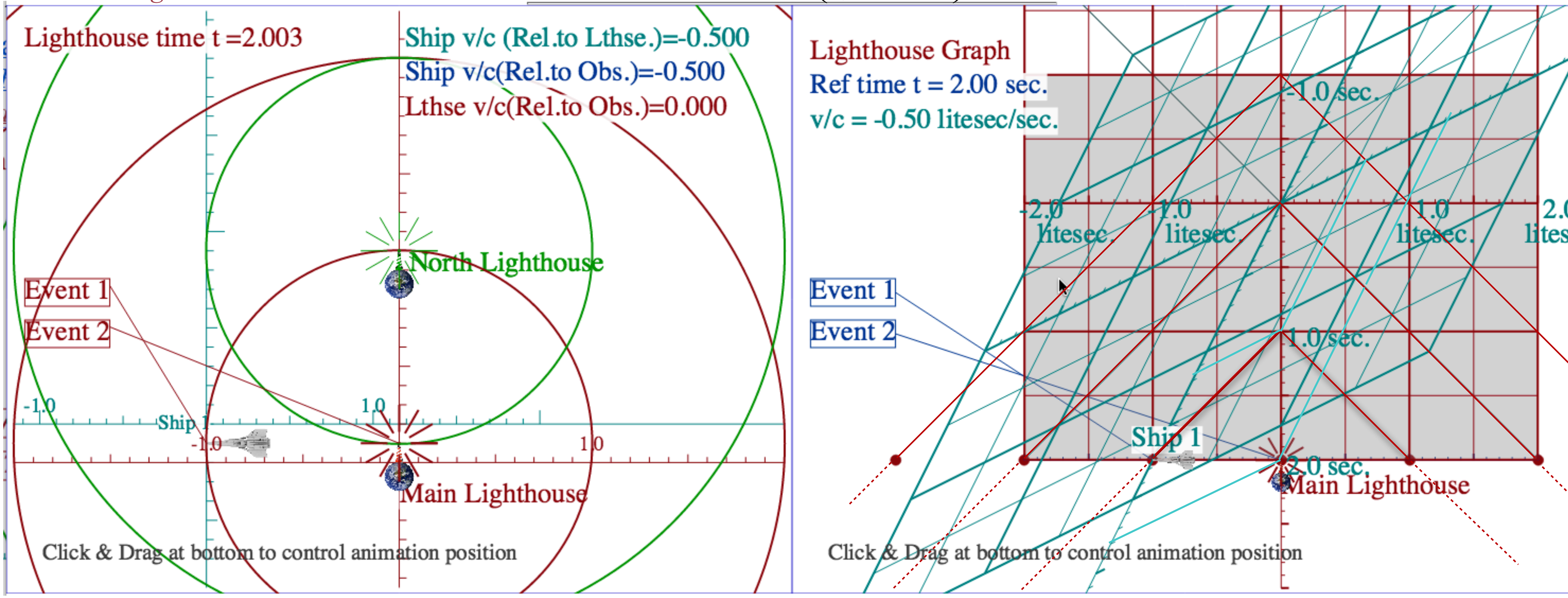

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http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=101

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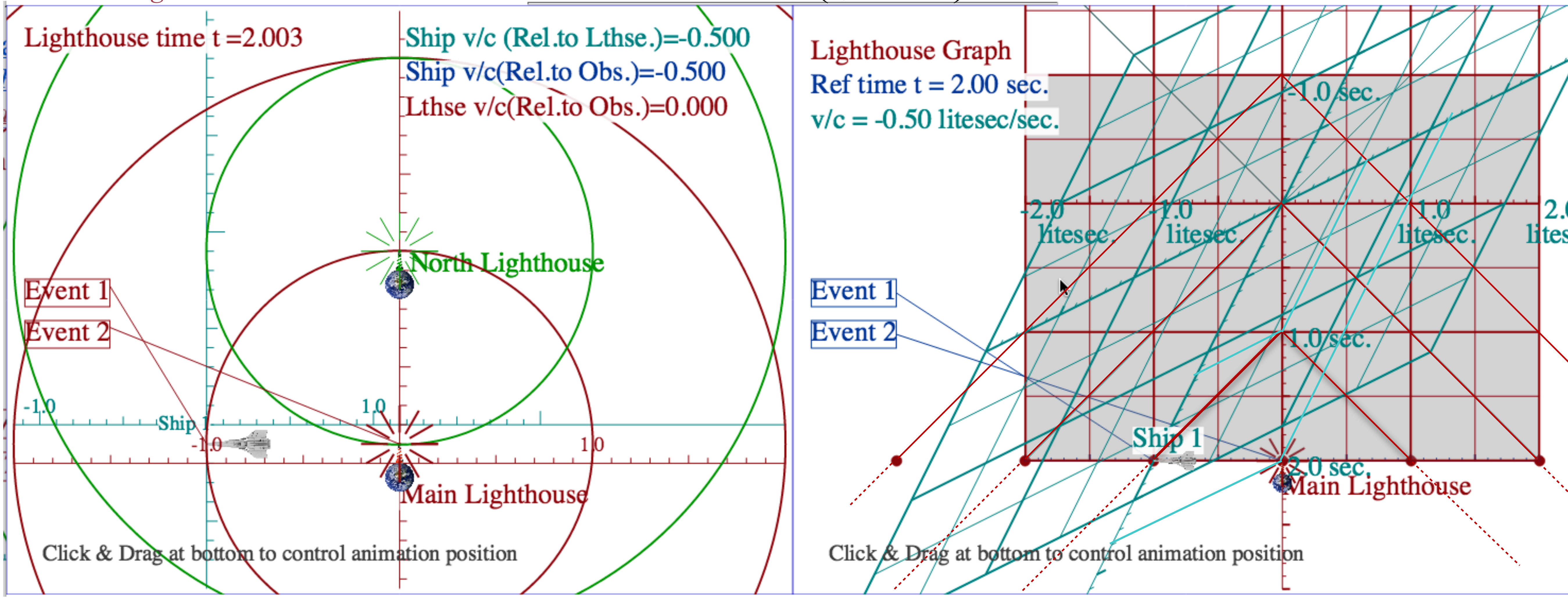

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Key point: Any event happening to you has your x-value set to zero!

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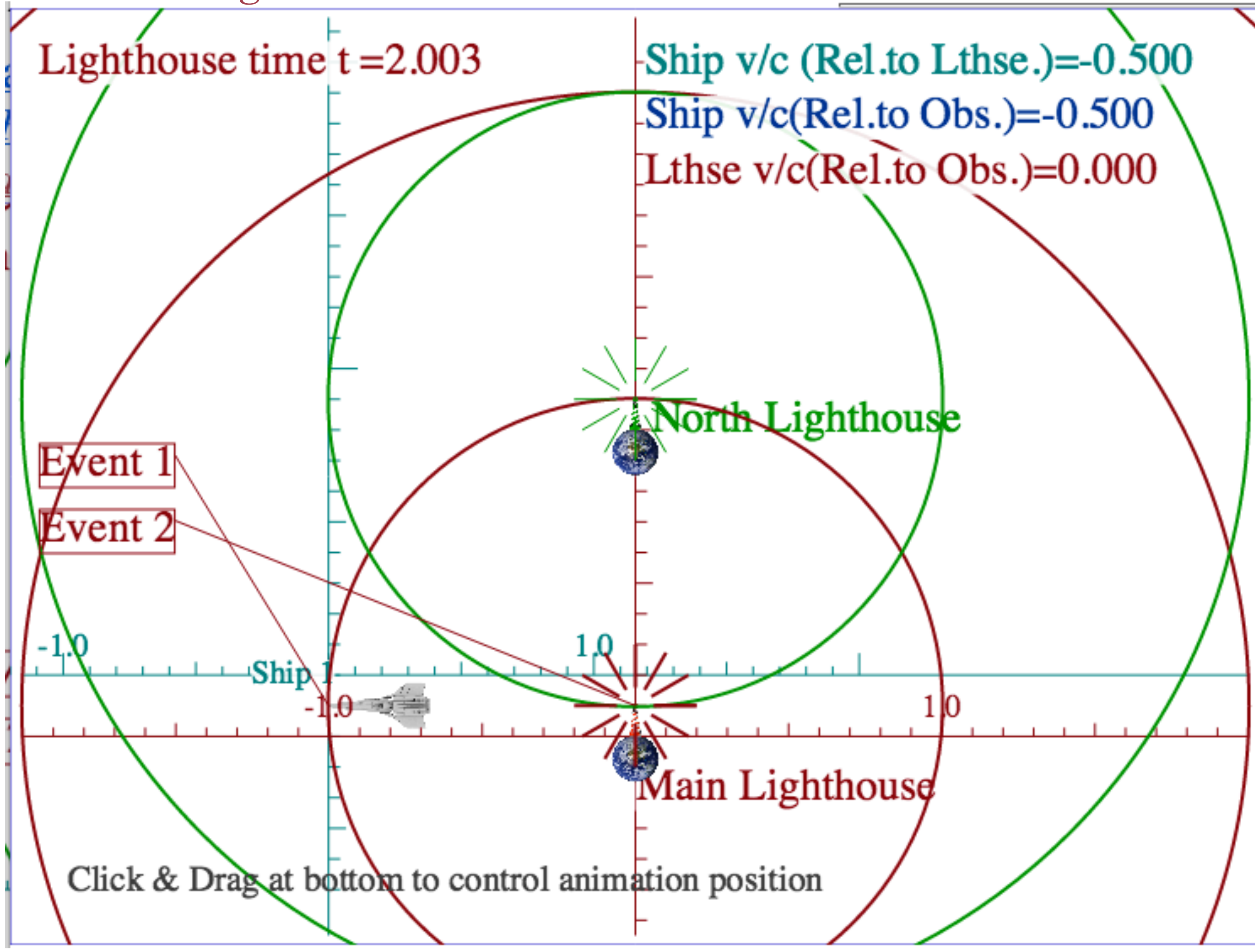

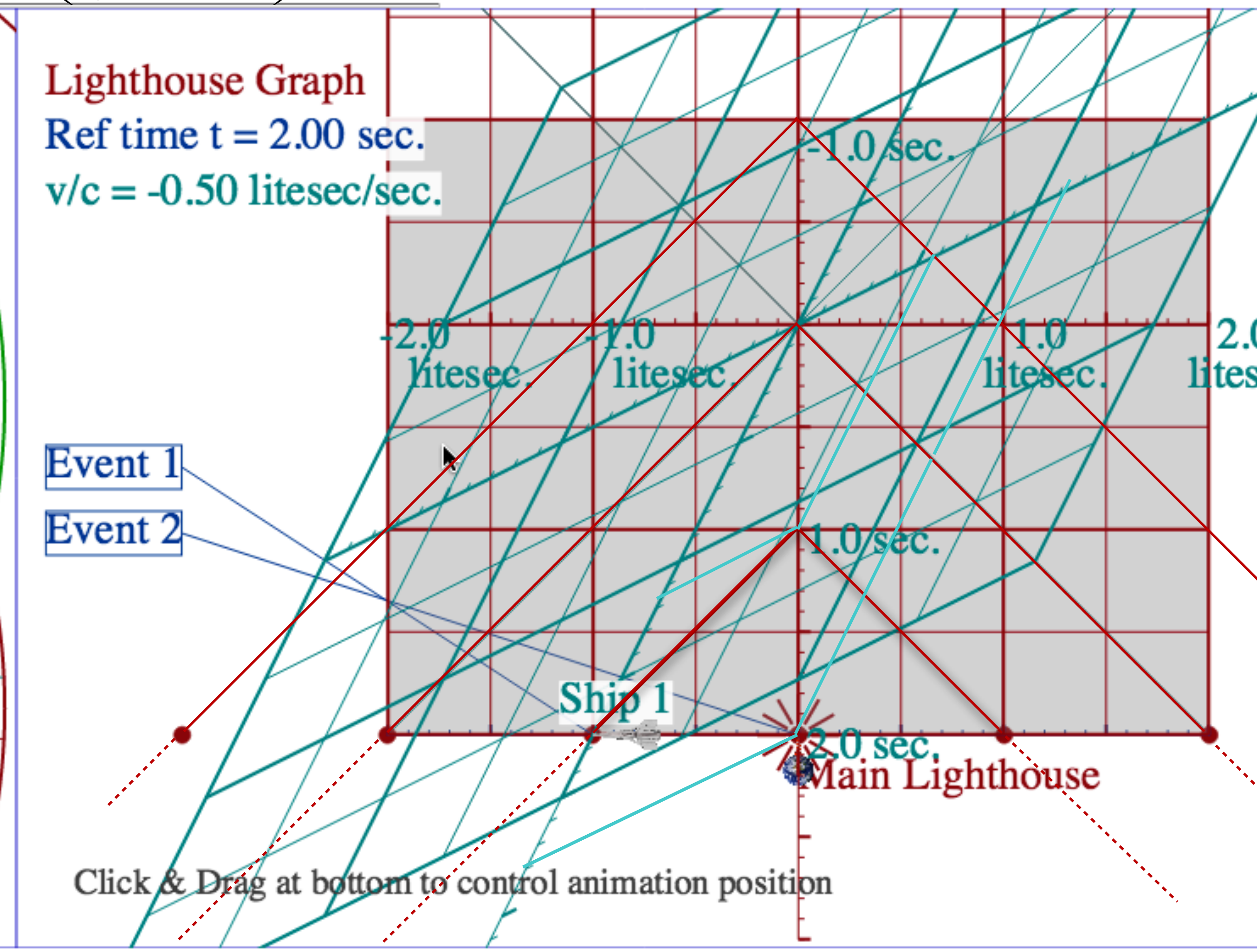

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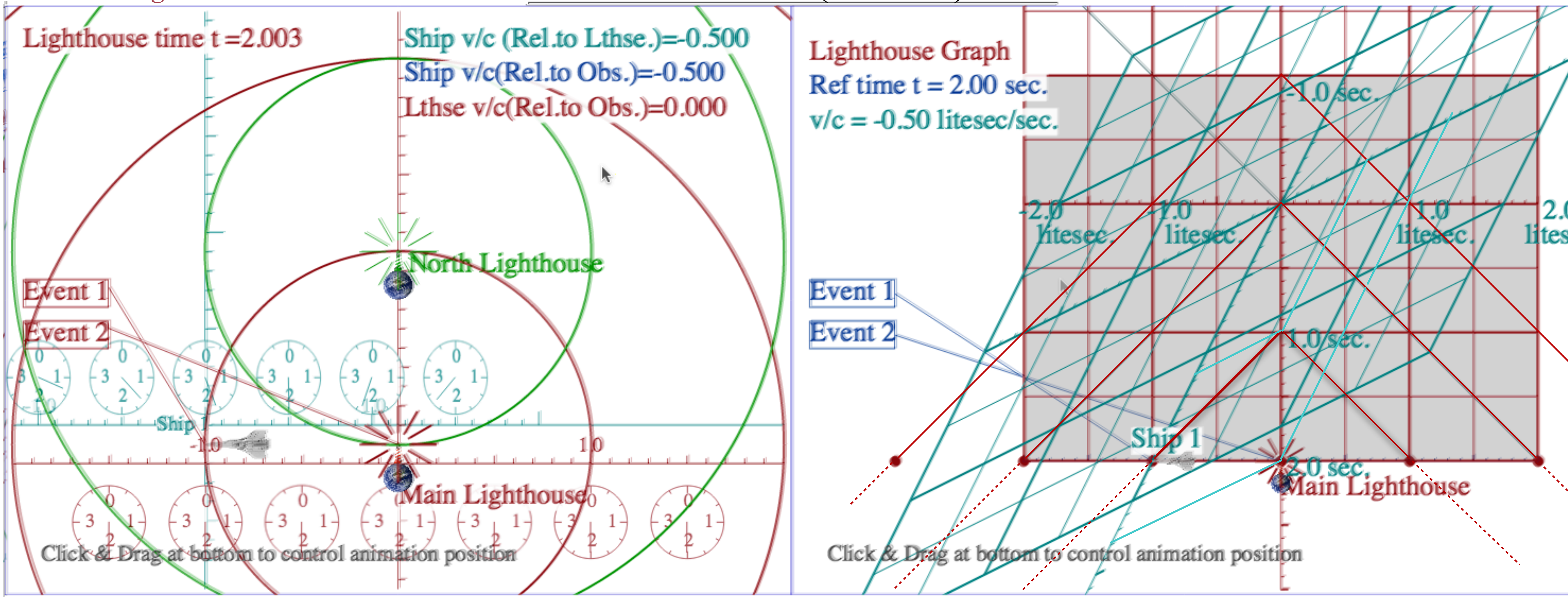

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2015 animations of lighthouses and ships in (*x*,*y*) scenarios and Minkowski (*x*,*ct*) plots

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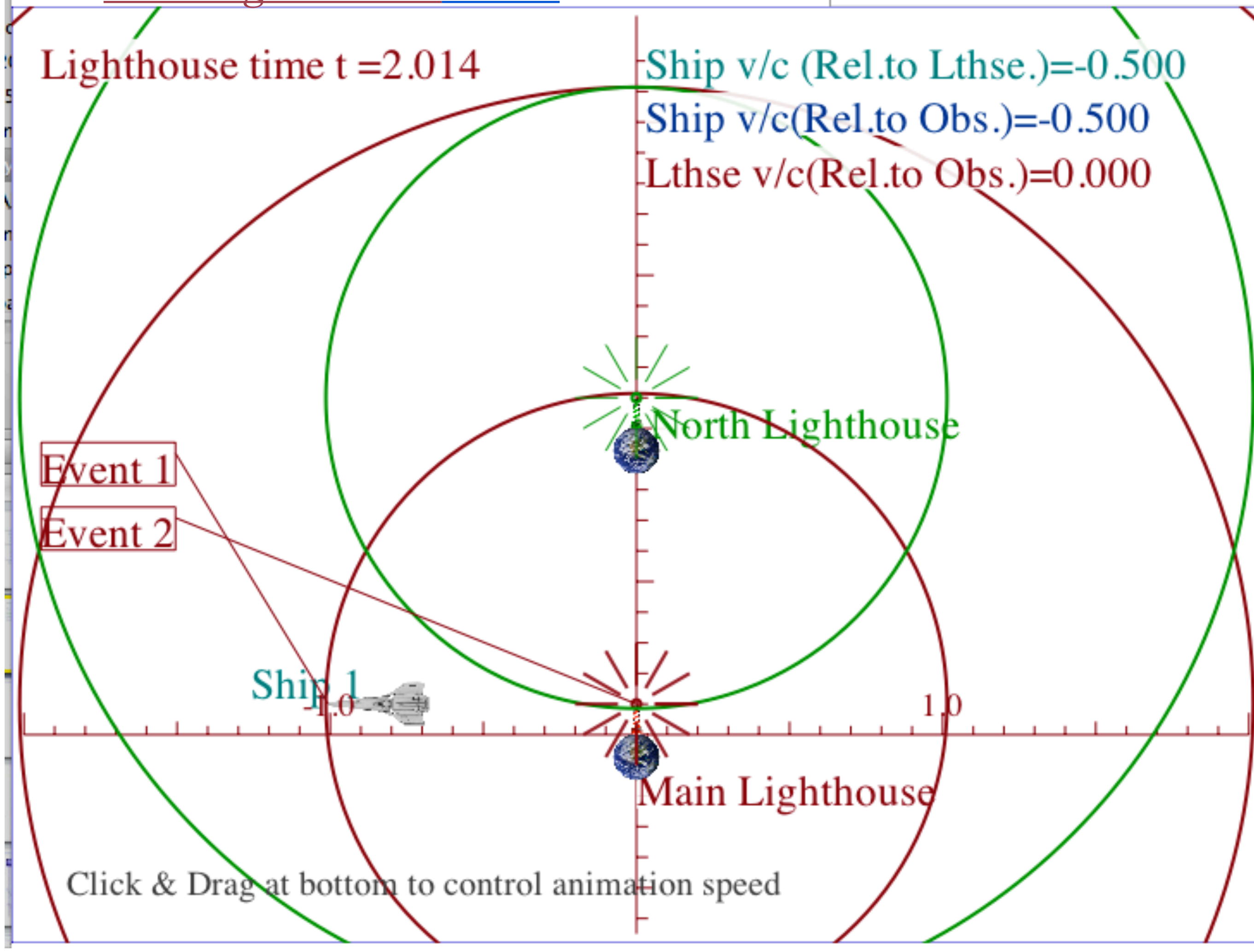

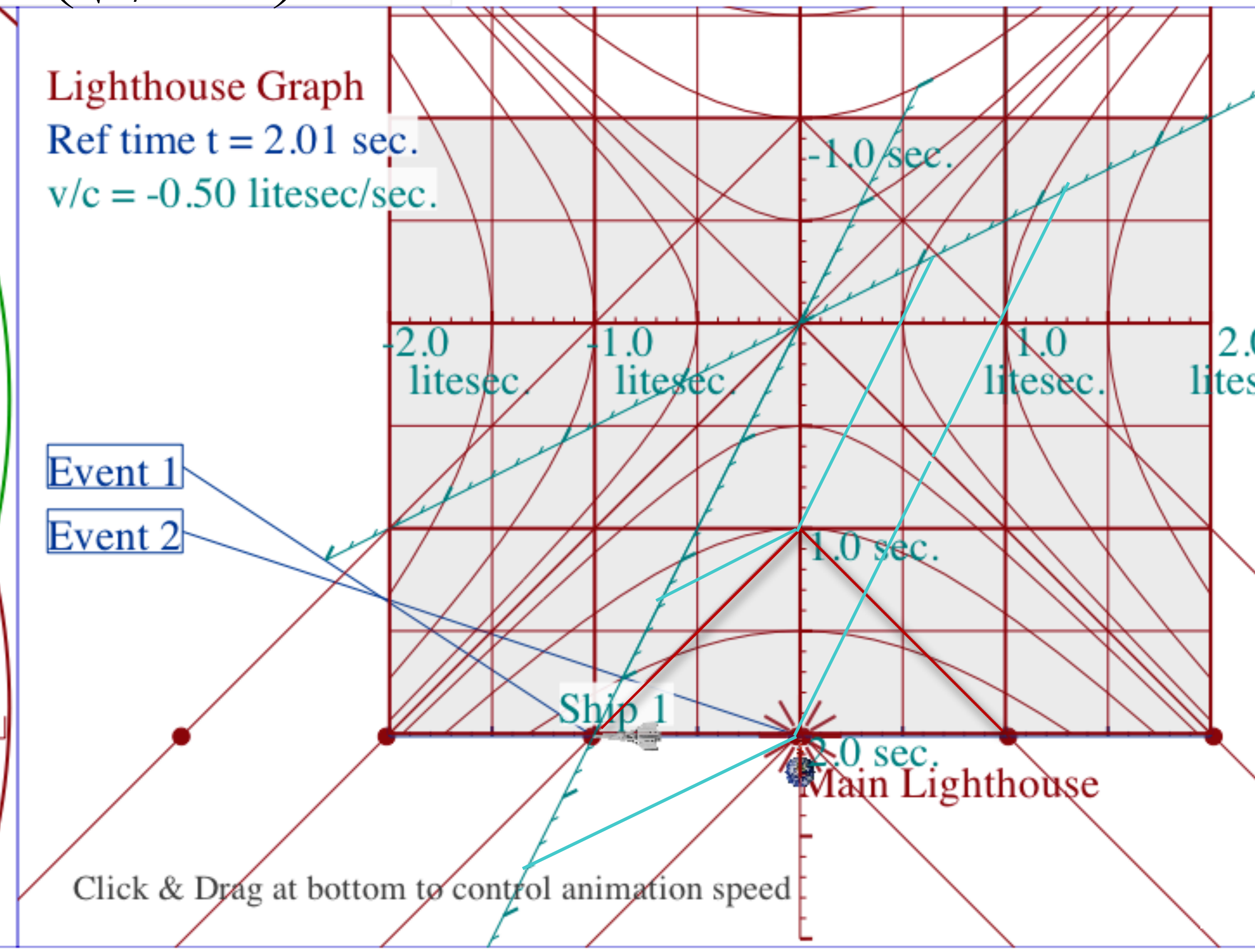

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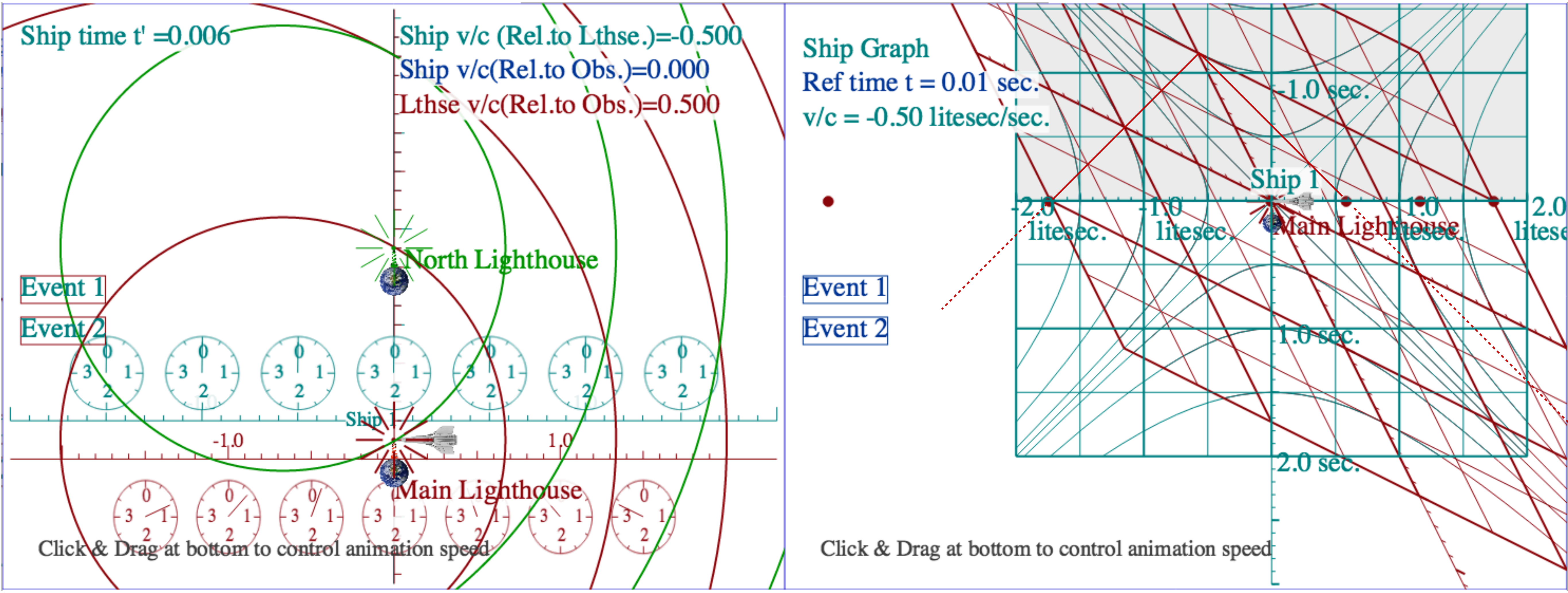

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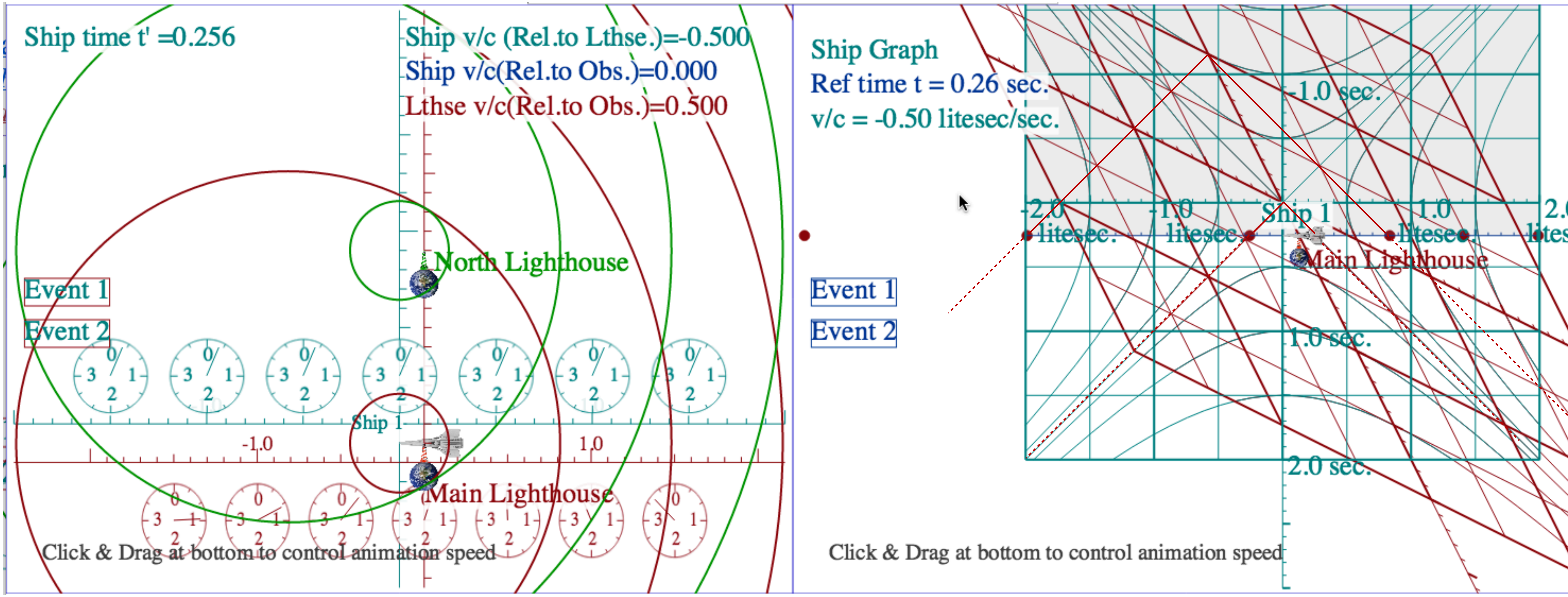
http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104

Simple 2015 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

RelativIt Web Simulation
Relativistic Events in
Main Lighthouse's Frame

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix}\begin{pmatrix} x \\ ct \end{pmatrix}$$



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$$\cosh\rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 = \Delta$$

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Simple 2015 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

[RelativIt Web Simulation](http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104)
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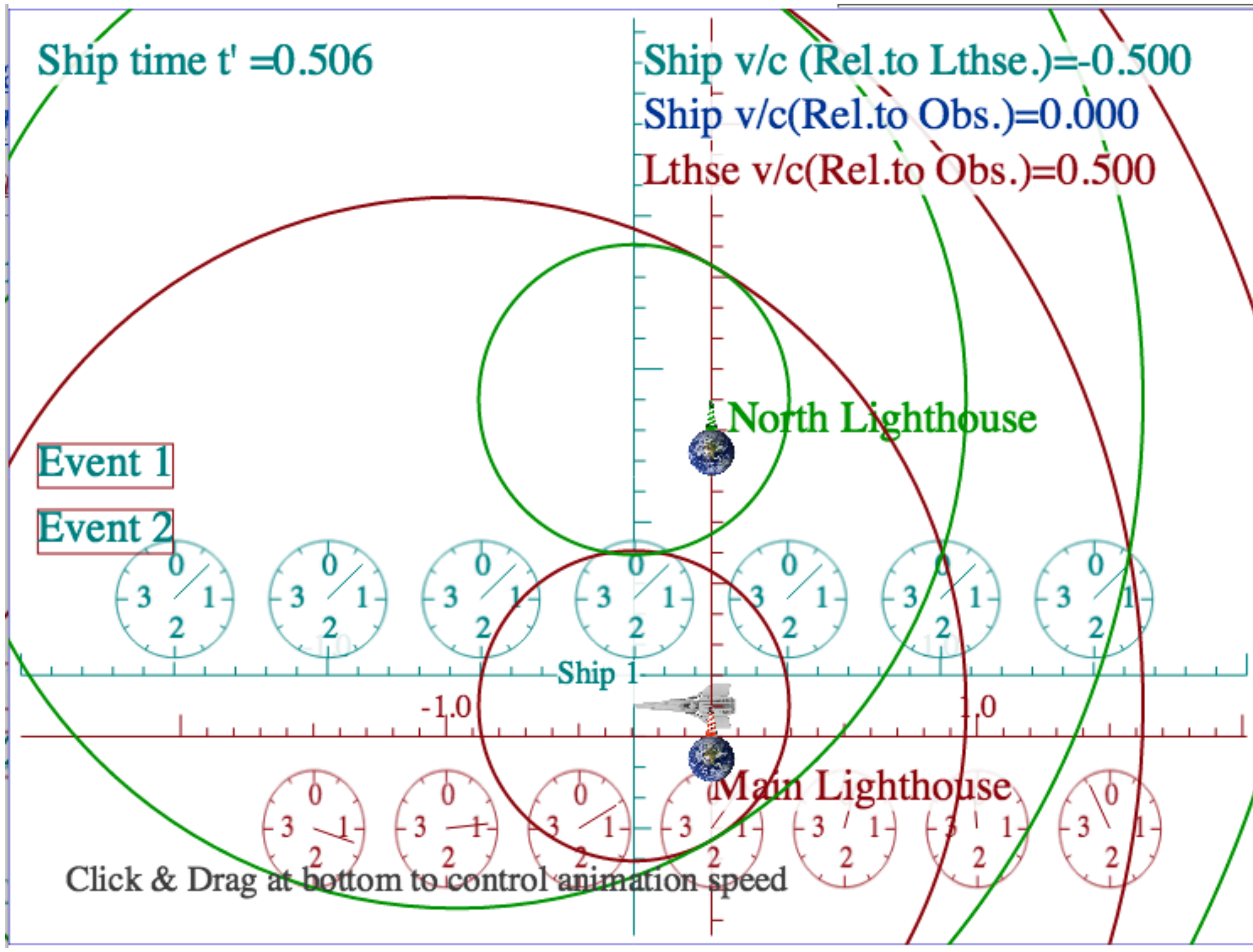

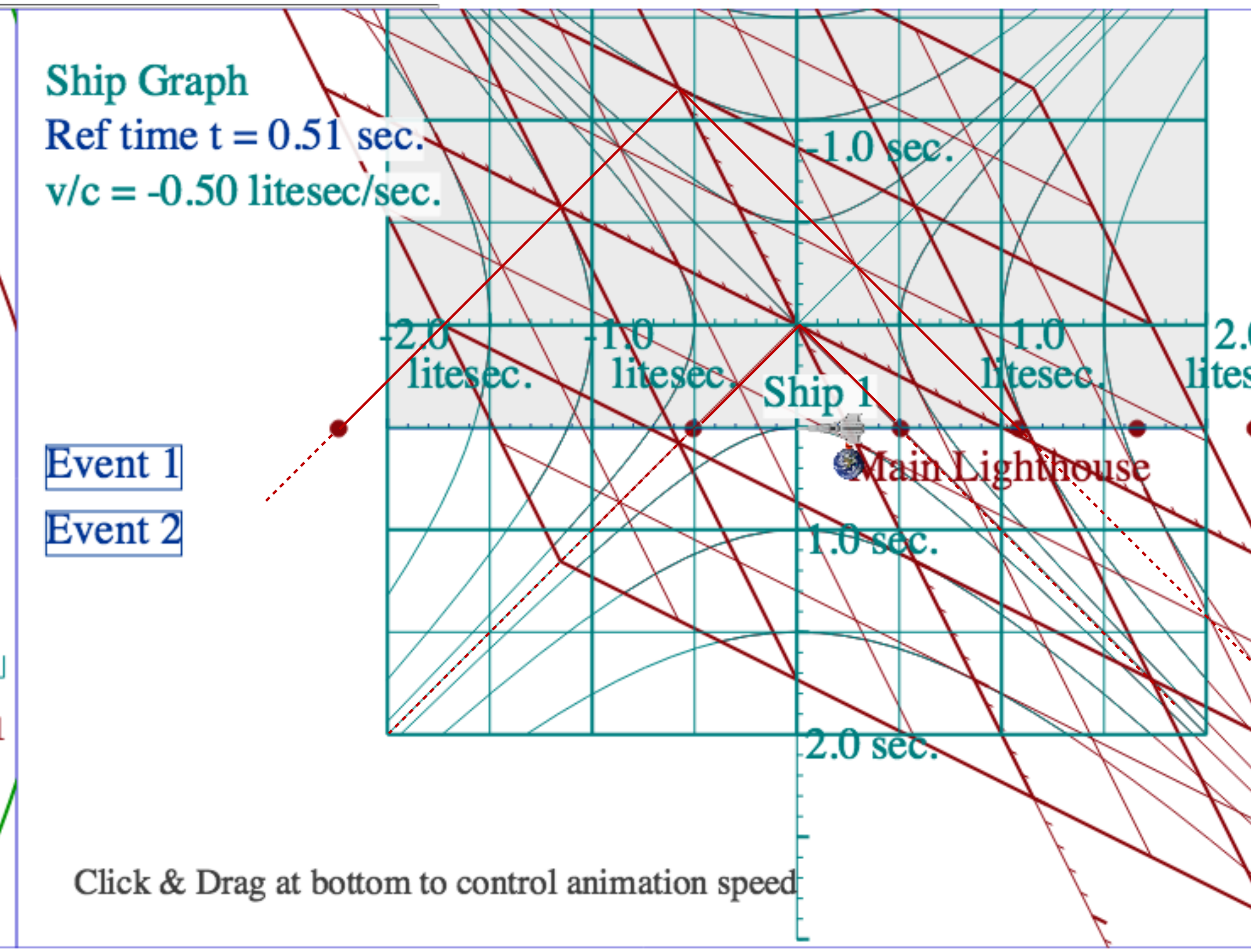

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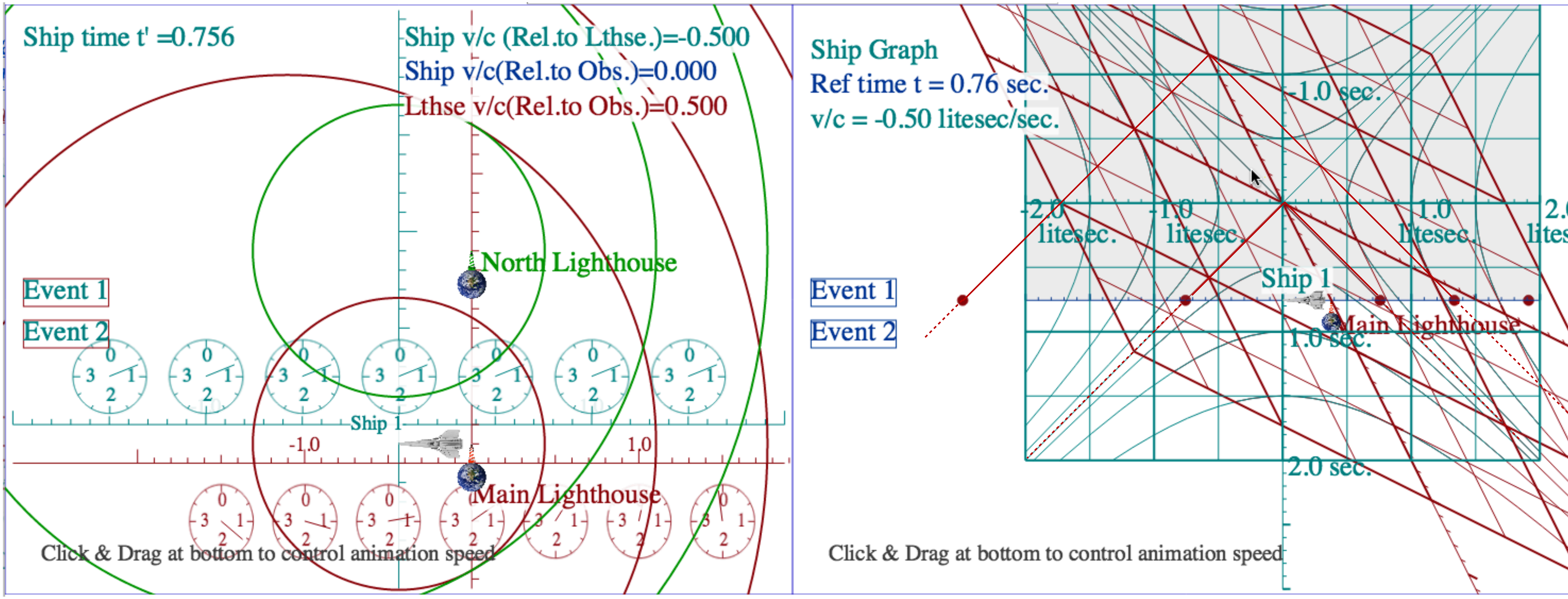

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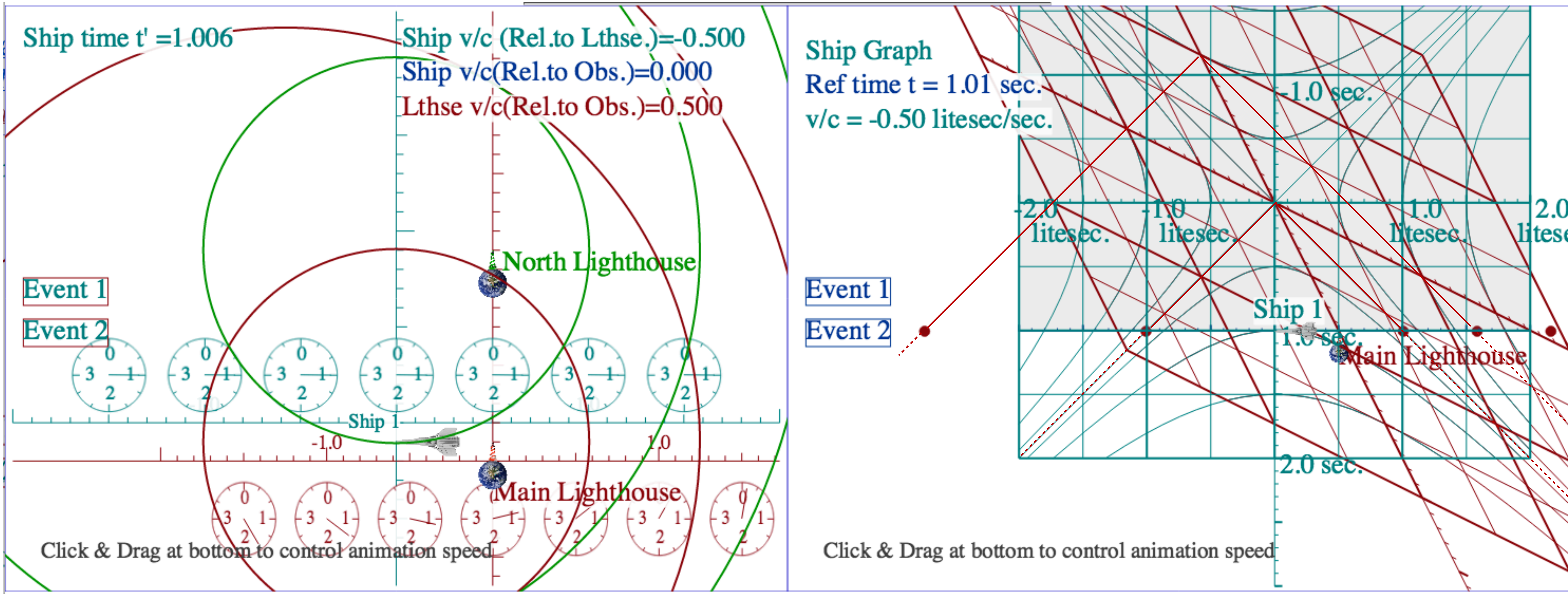

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Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

2005 and 2016 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

Lighthouse (x,y) frame: Dual concentric circular wavefronts serve as timing device

Ship frame: time dilation $\Delta=\cosh\rho=1.15$ of Lighthouse blinks

Simultaneous events in Lighthouse (x,y) frame: Not so in Ship (x',y') frame

Lighthouse-square (x,ct) plots correlated with Ship-square (x',ct') plots

Overlapped Lighthouse (x,ct) and Ship (x',ct') frame Minkowski plots correlate inconsistencies

➔ Ship (x',y') frame: Dual un-concentric circular wavefronts map space-time

Pythagorean derivation of time-dilation factor $\Delta=\cosh\rho$

Un-concentric derivation of stellar aberration **k**-angle σ

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) =(\omega,ck_x,ck_y,ck_z)$ transformation

"Occam-sword" geometry: A pattern recognition aid

Relating velocity parameter $\beta=u/c$ to *rapidity* ρ to **k**-*angle* σ to *u/c-angle* ν

Circular arc-area σ vs. hyperbolic arc-area ρ

Each circular trig function has a hyperbolic "country-cousin" function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ

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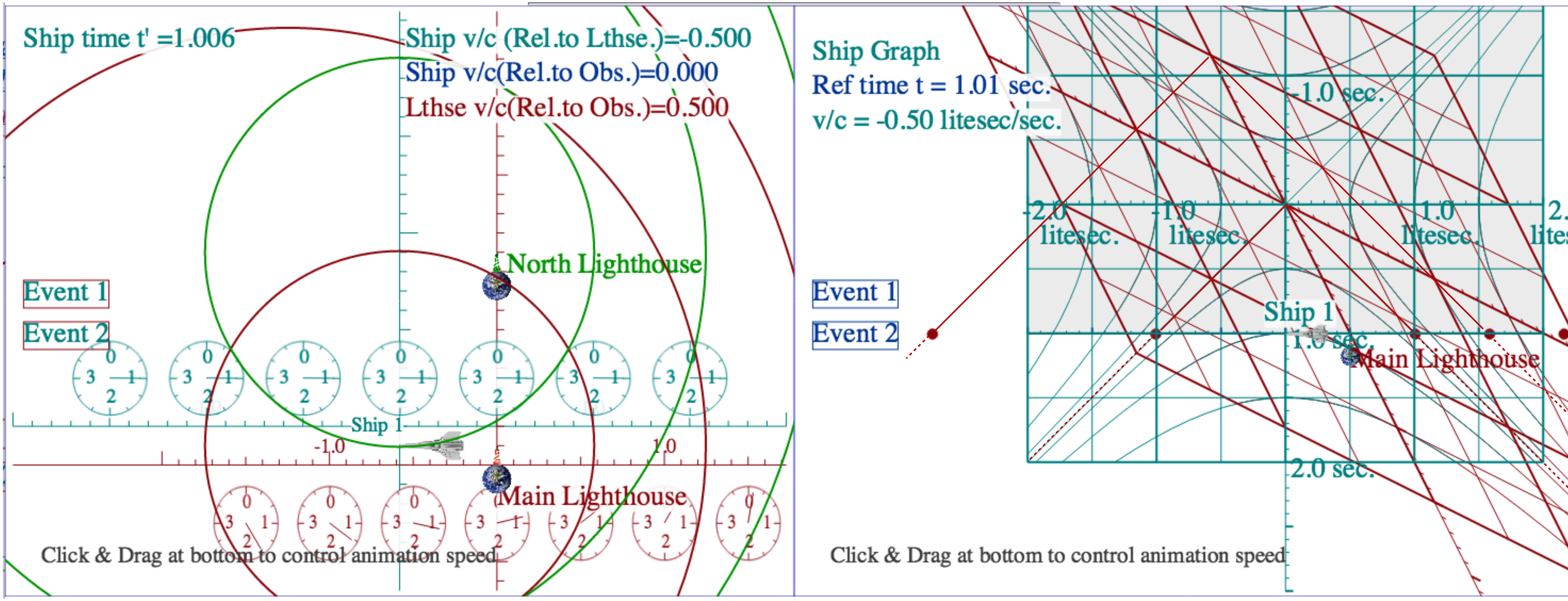

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Caution:May be confusing
http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=103

Simple 2015 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

[RelativIt Web Simulation](#)
[Relativistic Events in Main Lighthouse's Frame](#)

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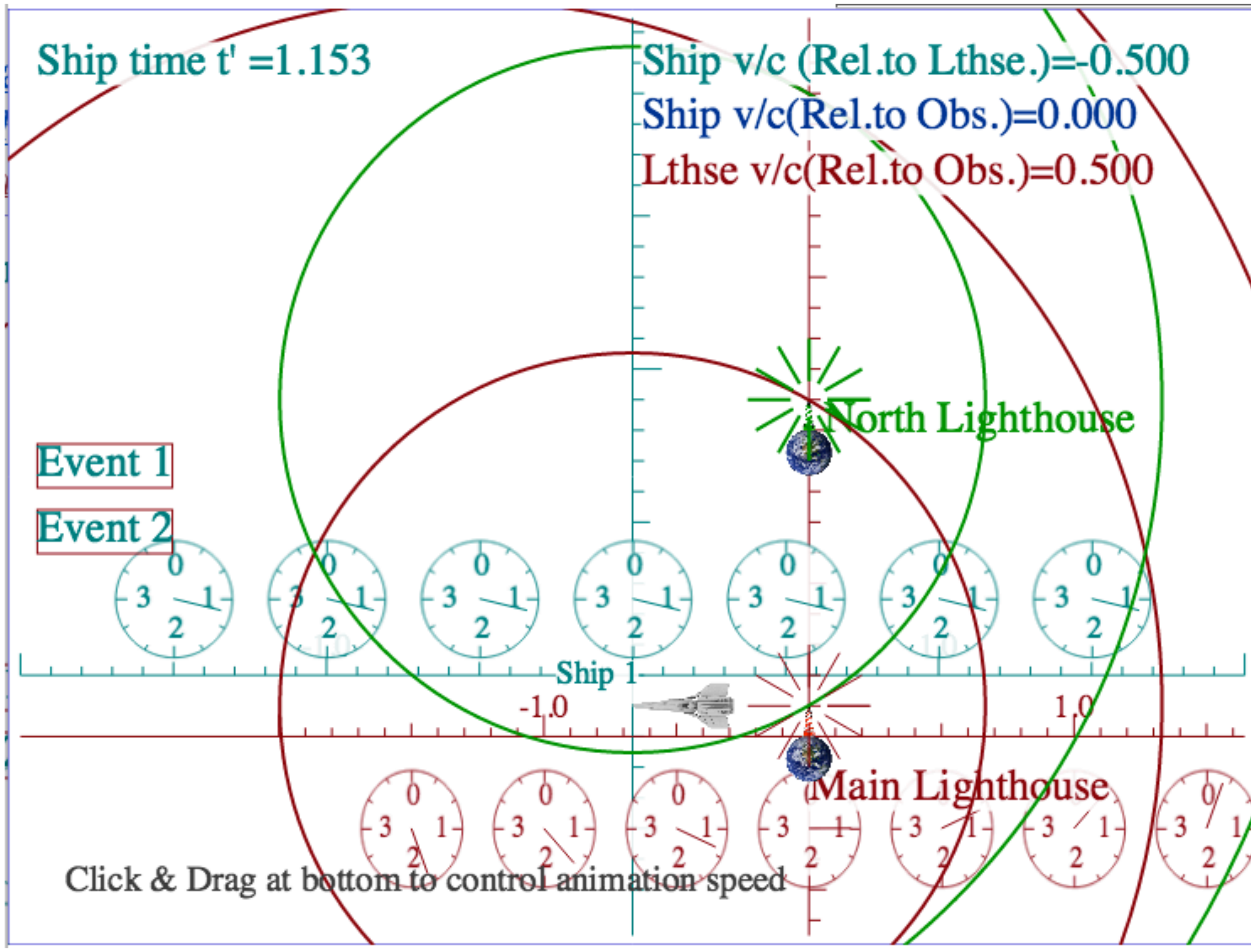

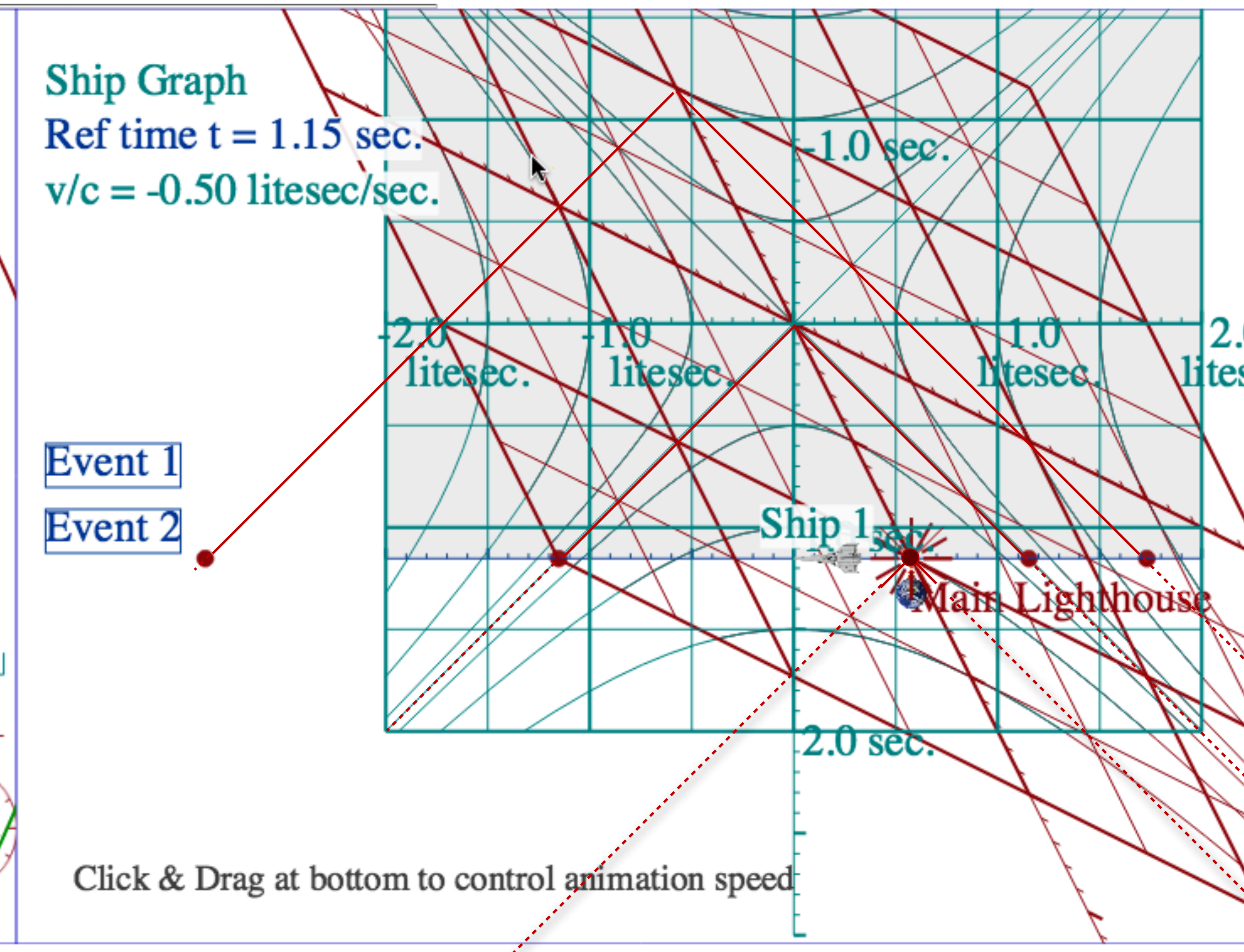

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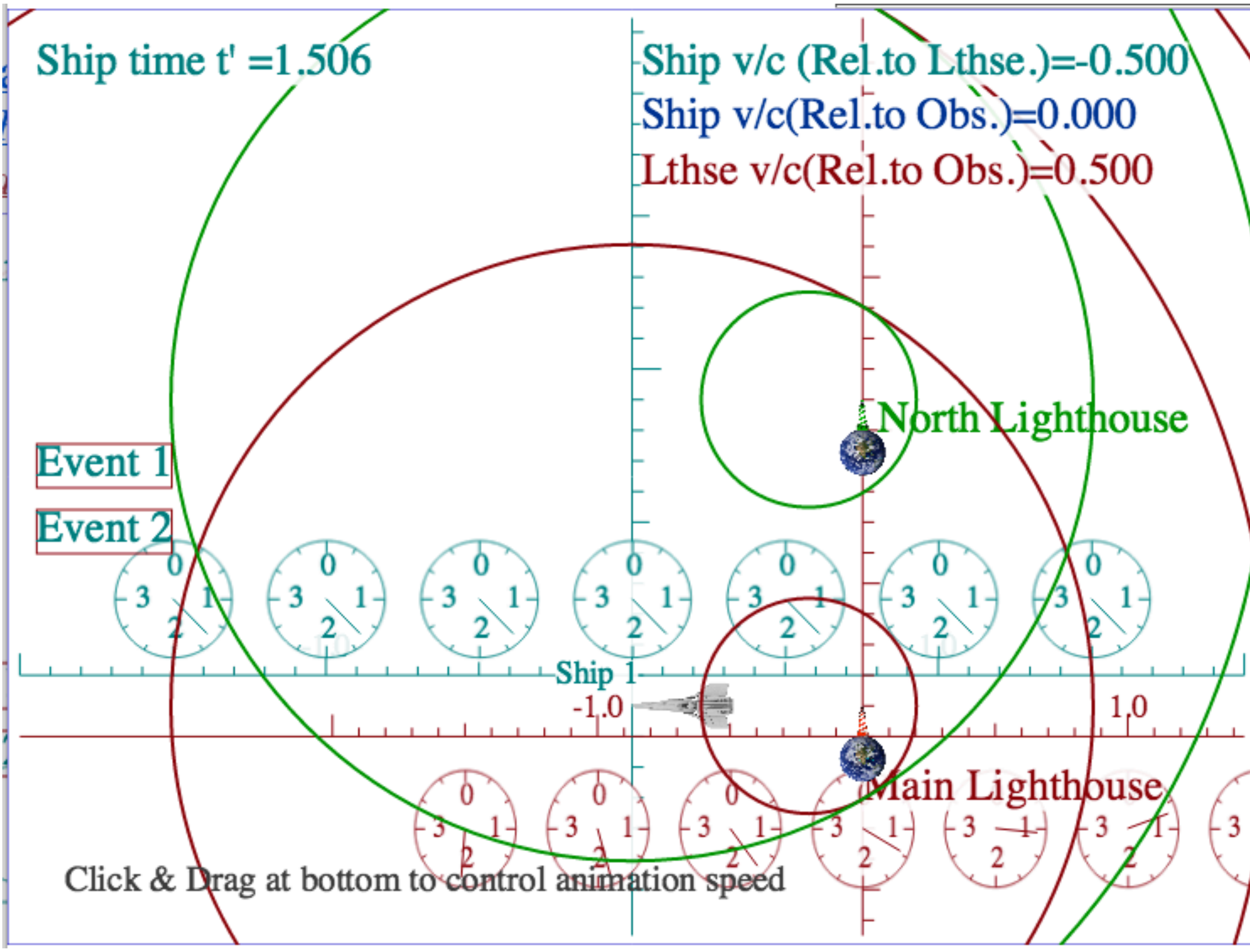

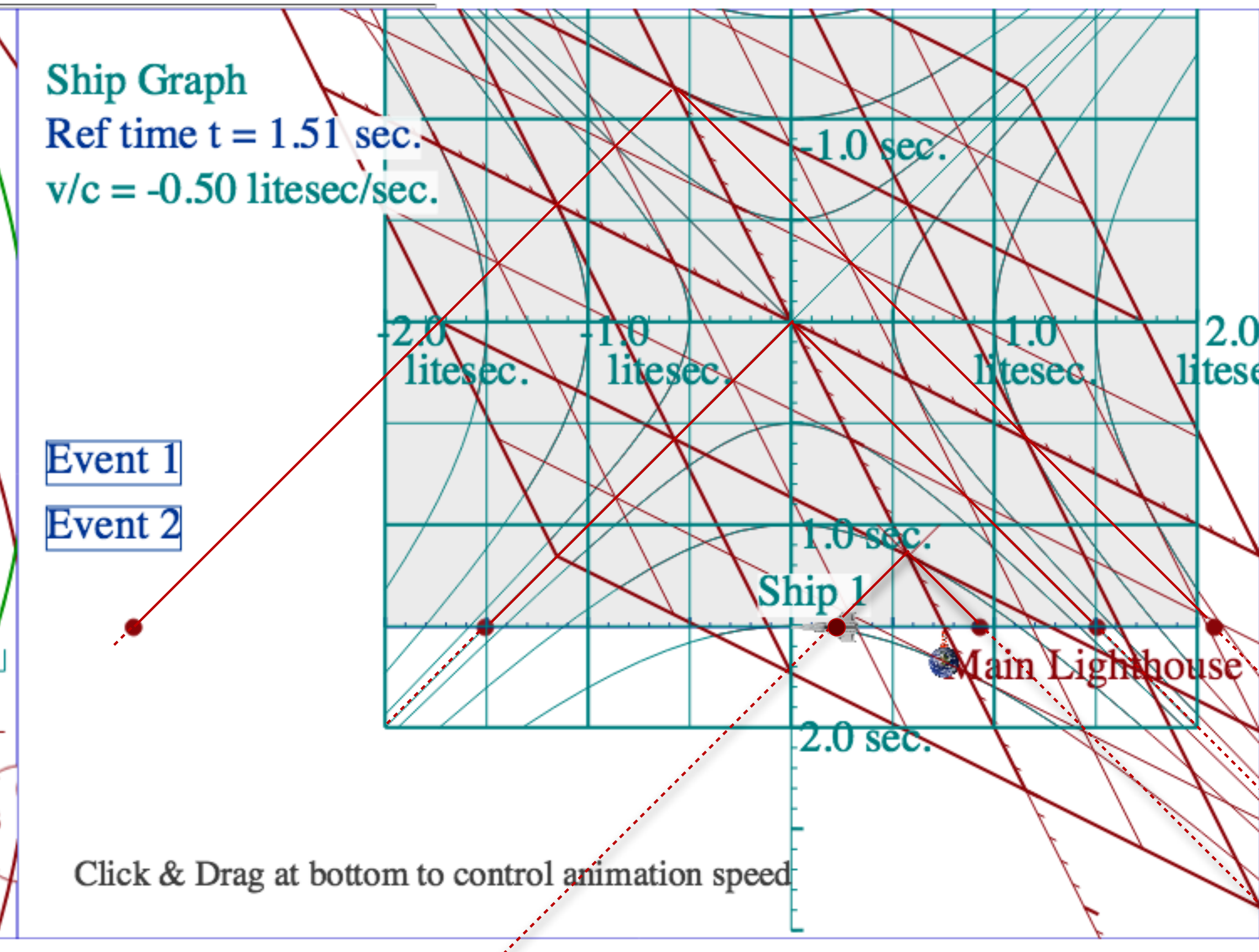

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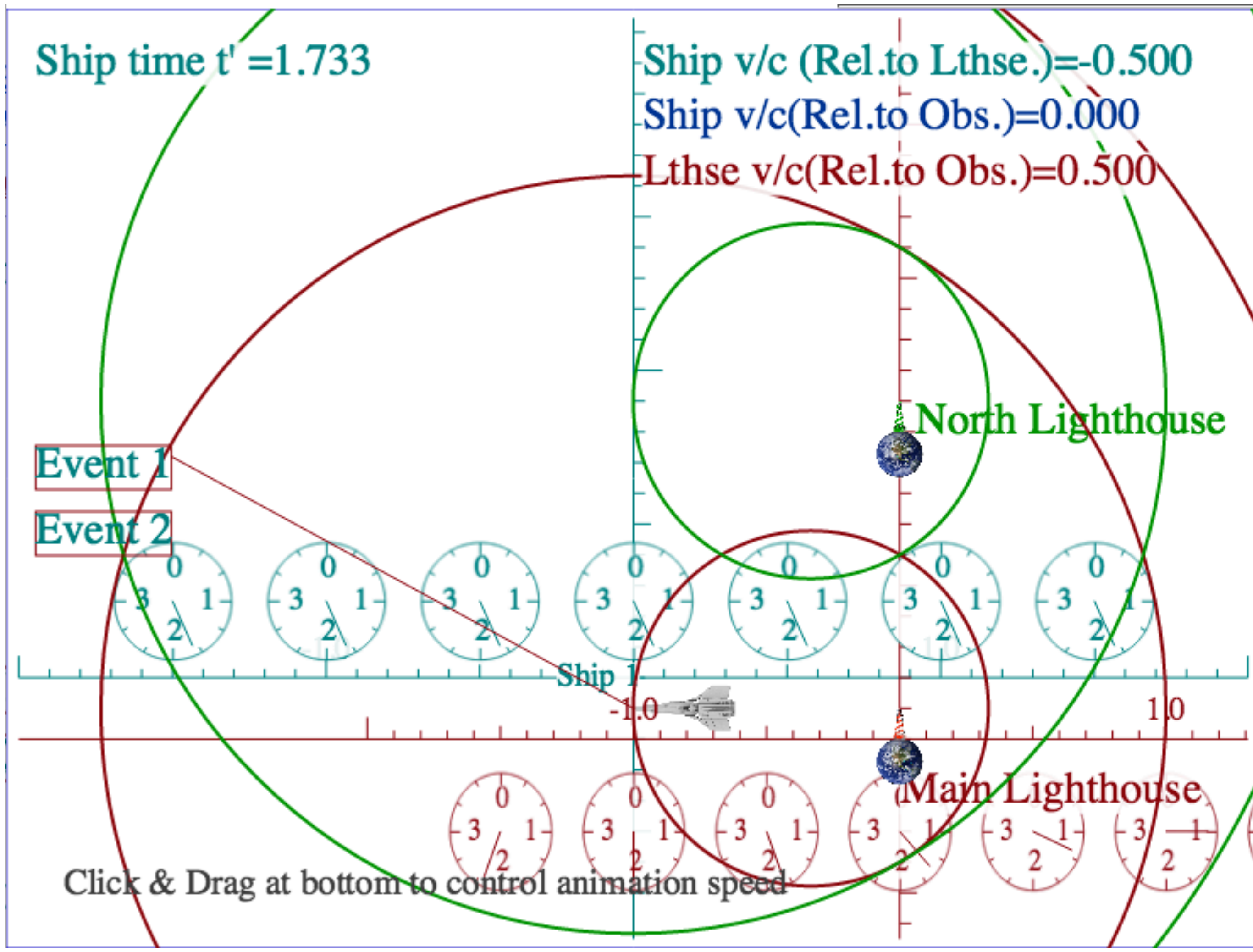

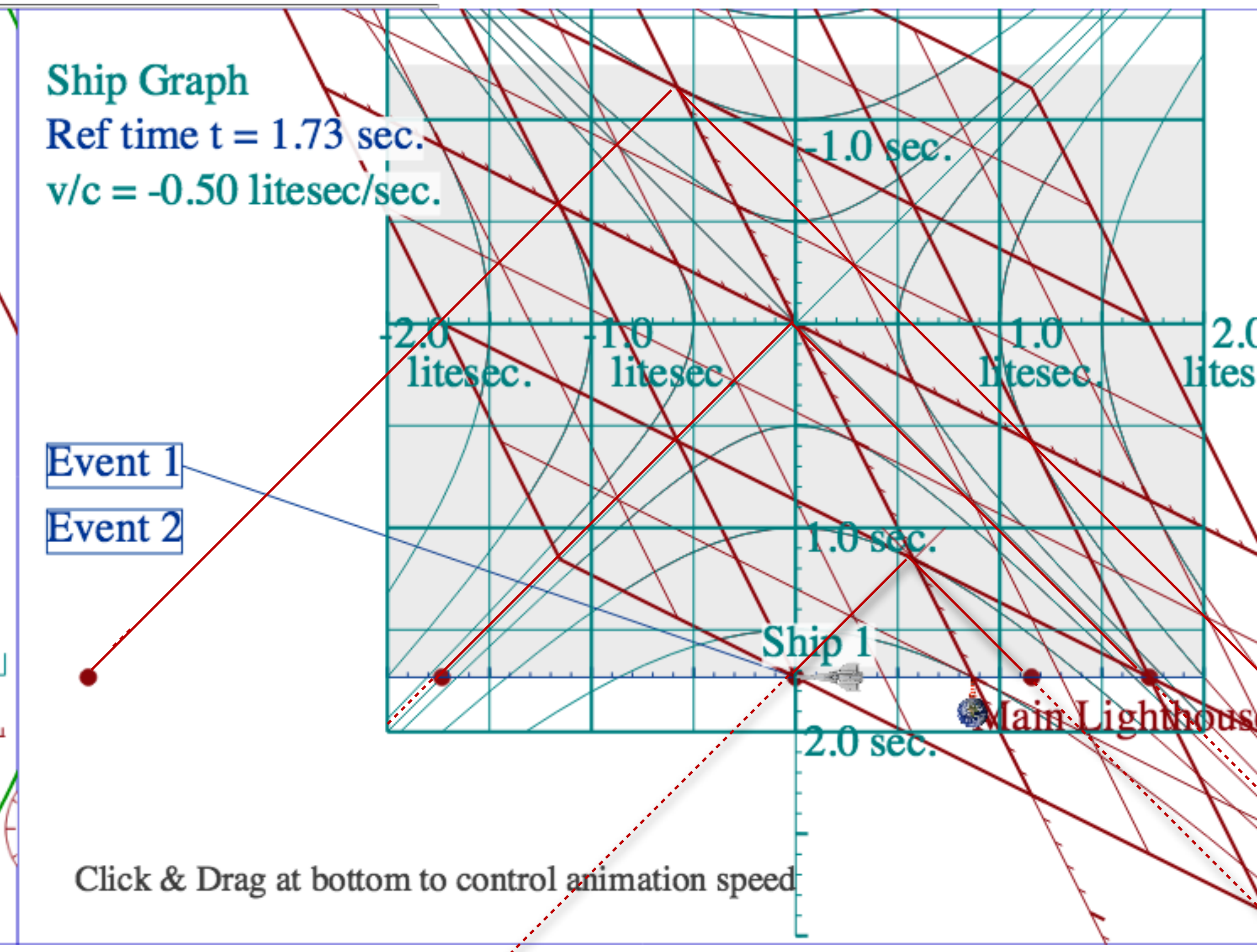

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Relativistic Events in
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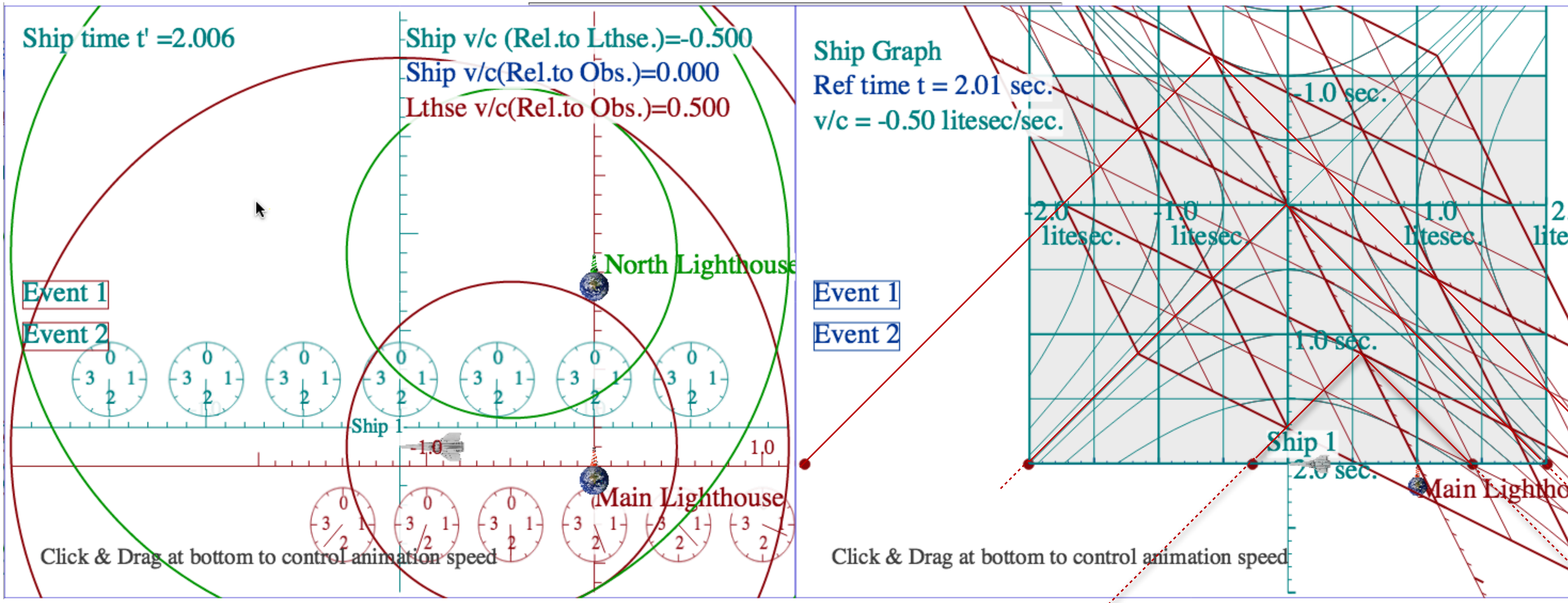

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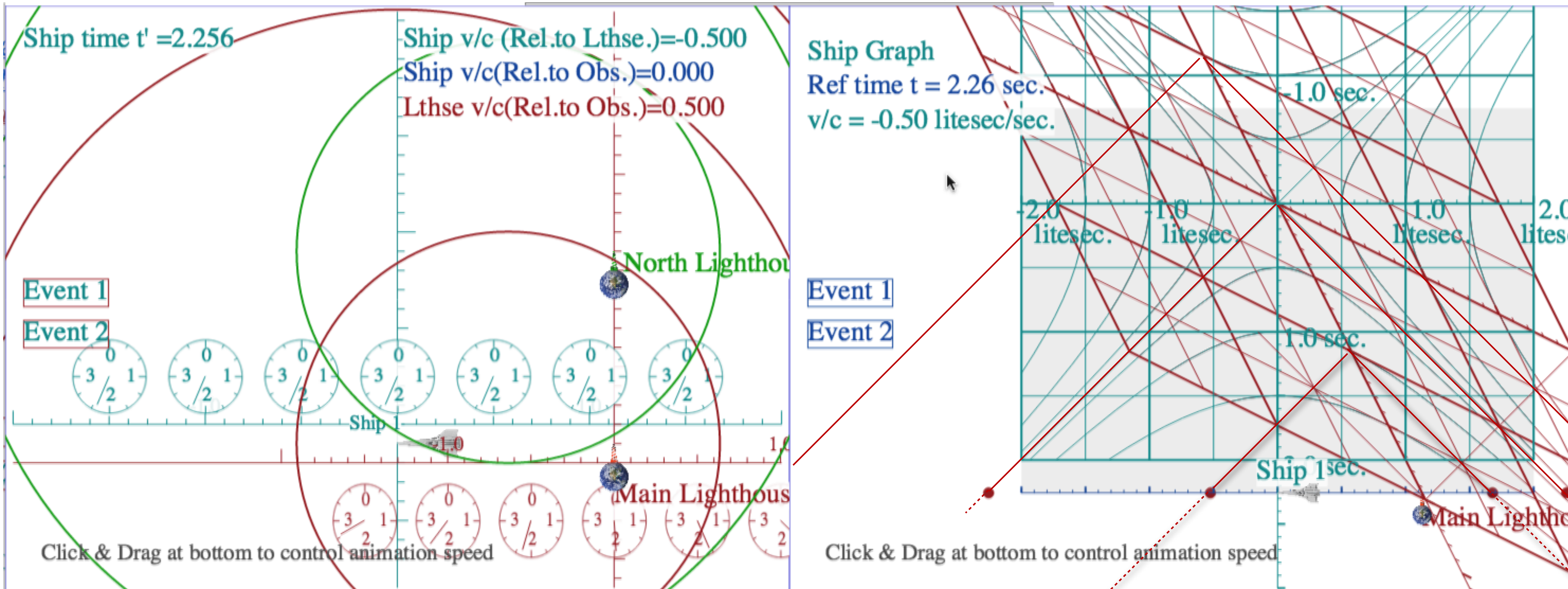

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http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=104

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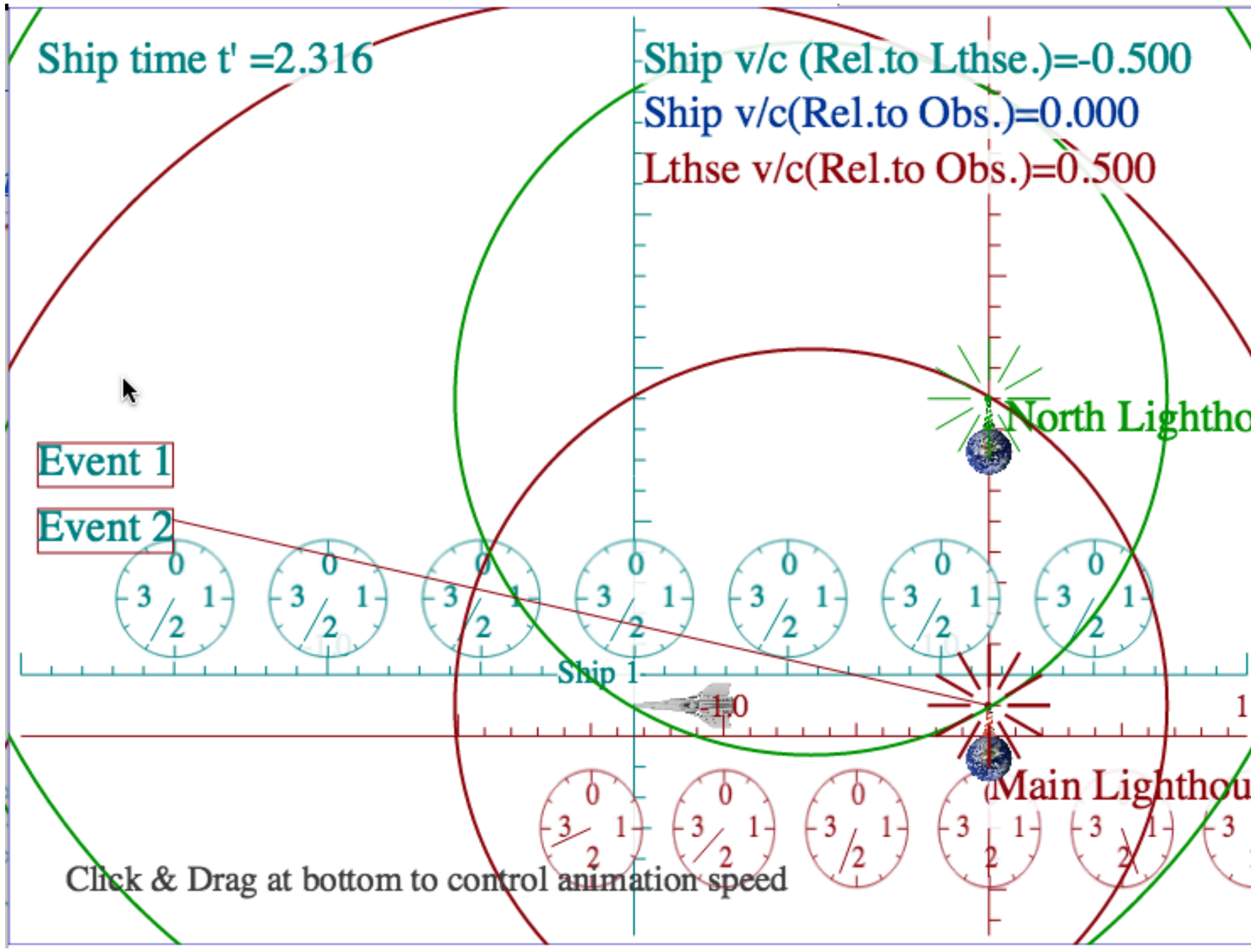

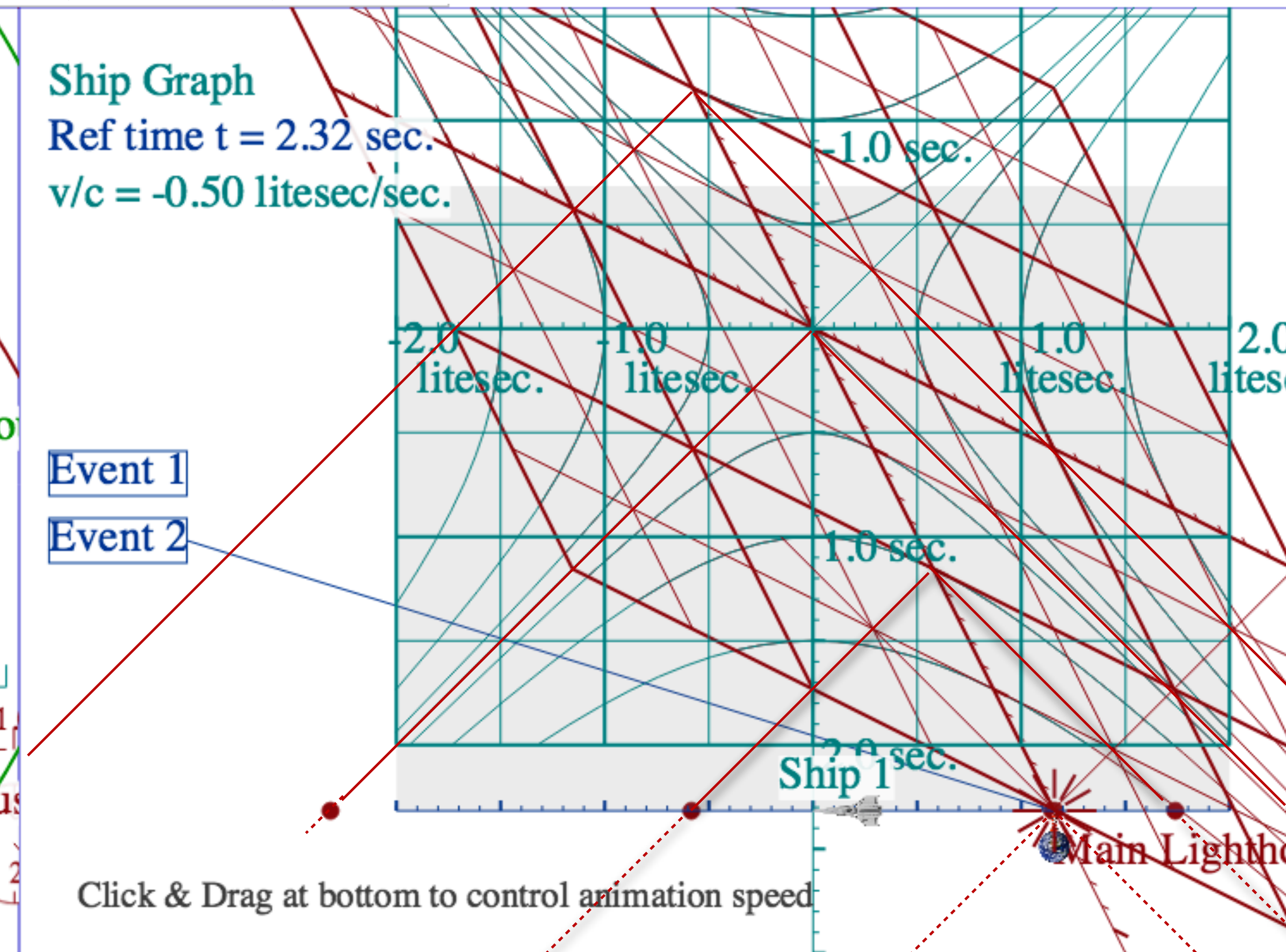

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Relativistic Events in
Main Lighthouse's Frame

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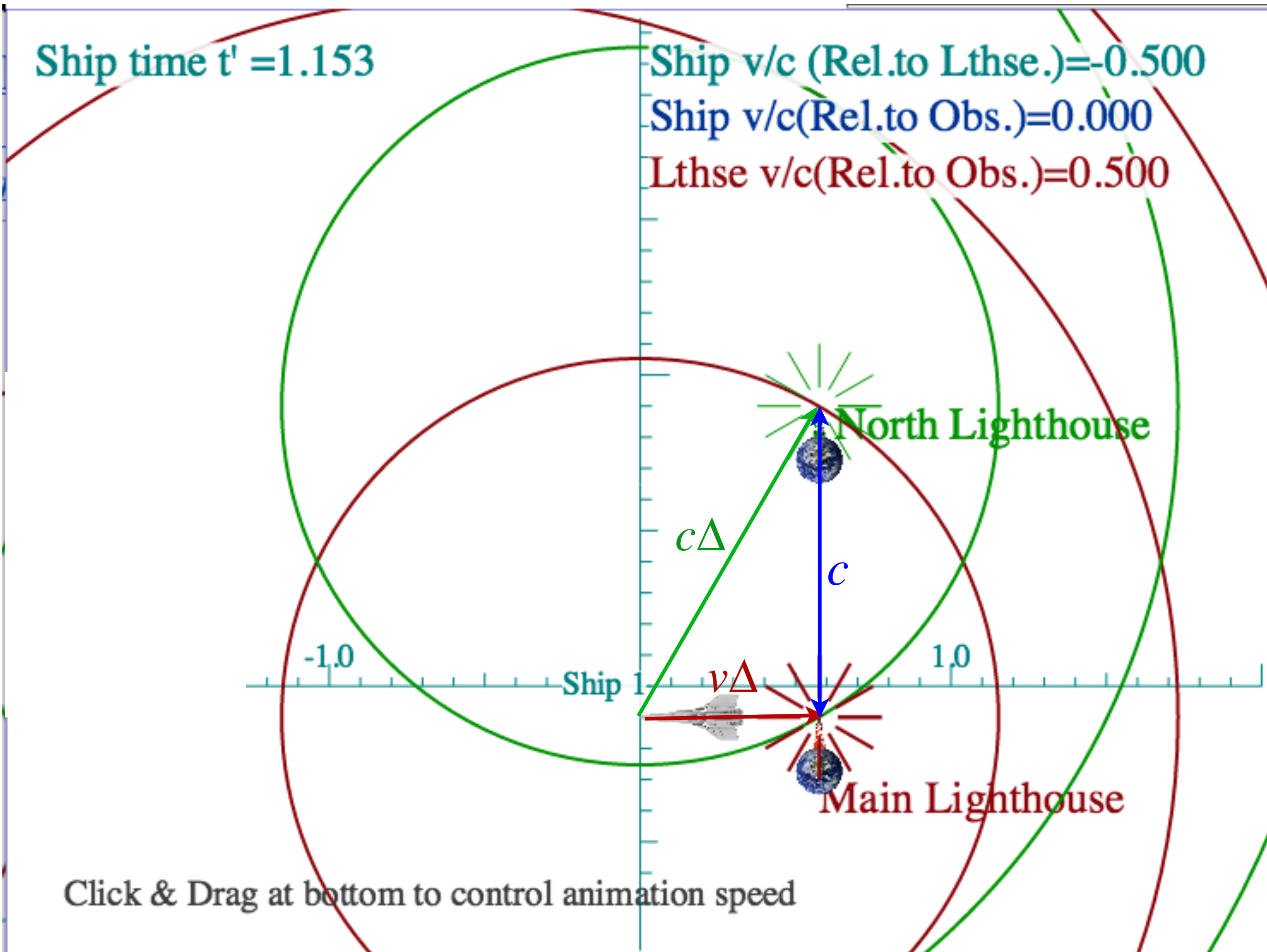

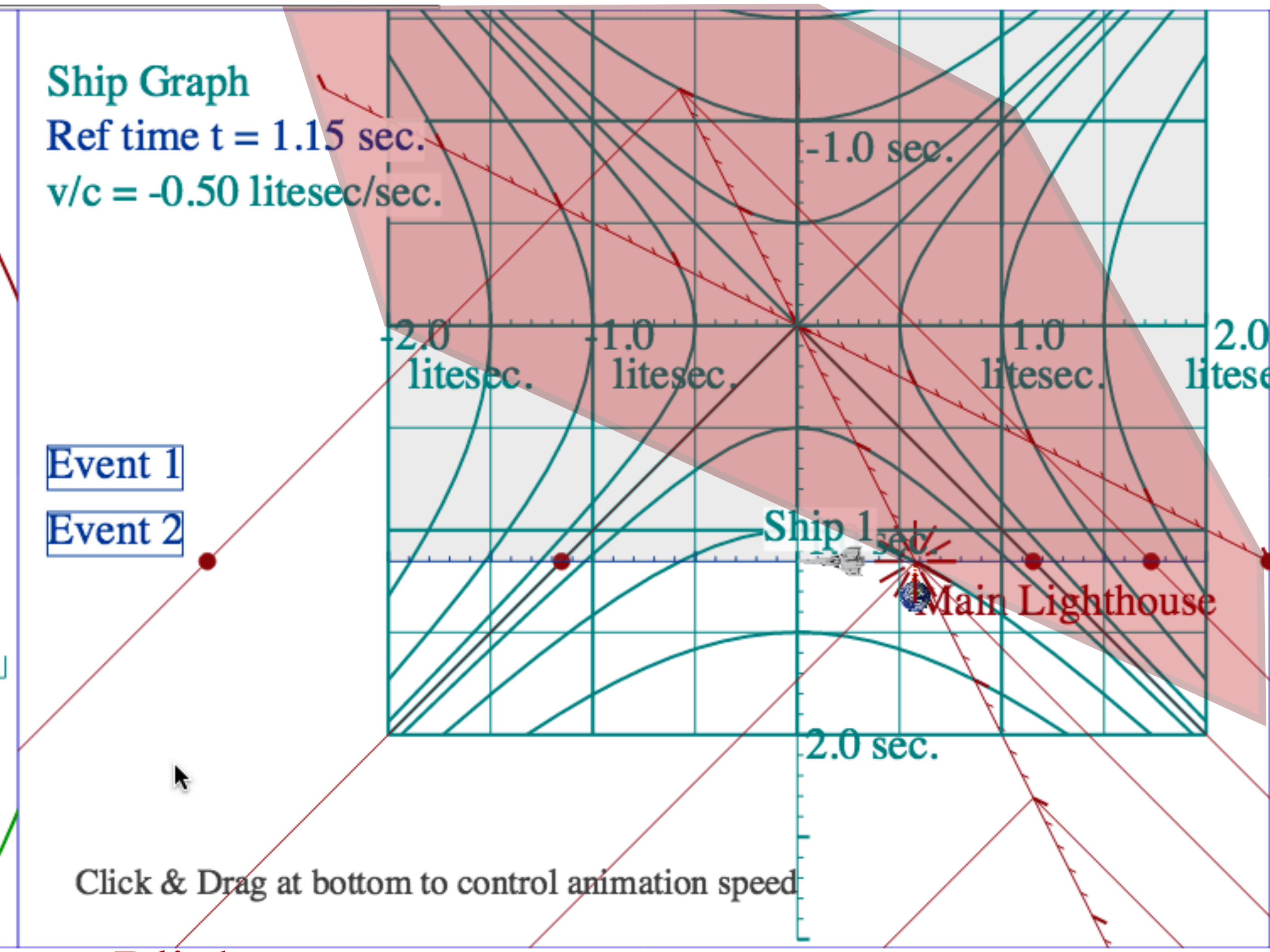

Ship registers 1st Lighthouse Blink

at its position, time $\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 0.577 \\ 1.155 \end{pmatrix} = \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}$

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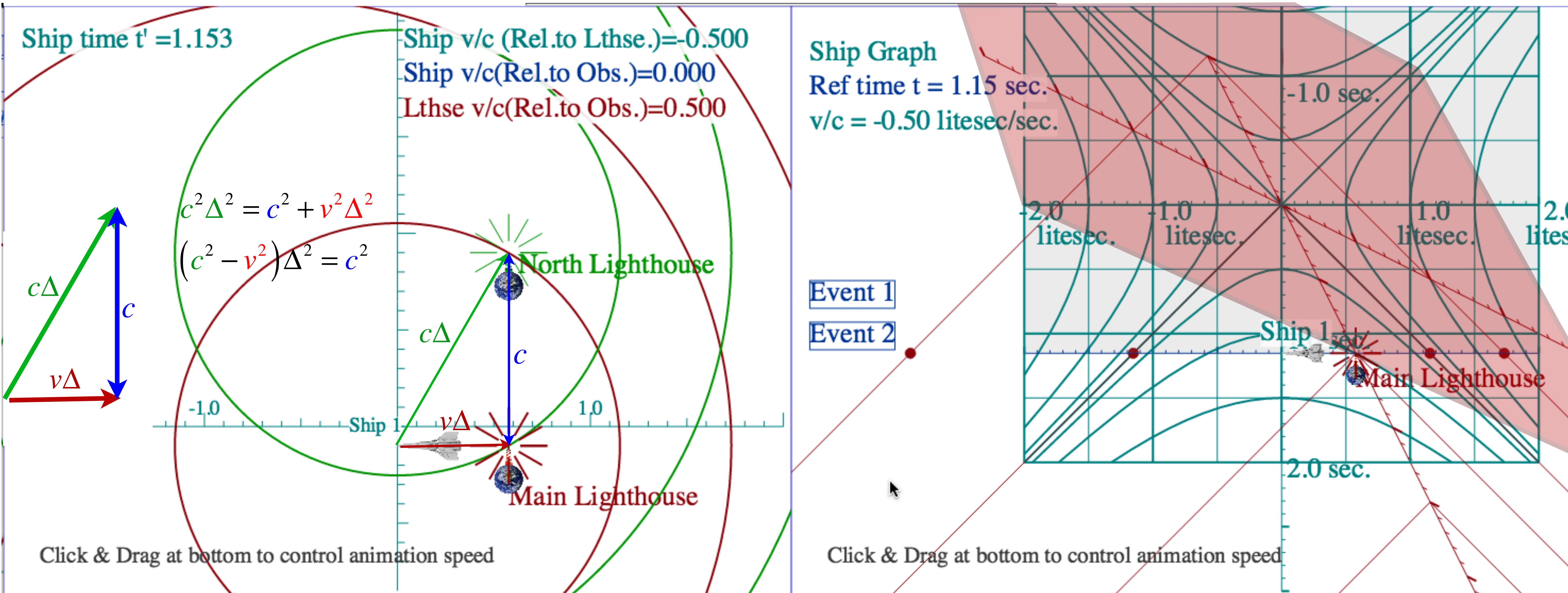

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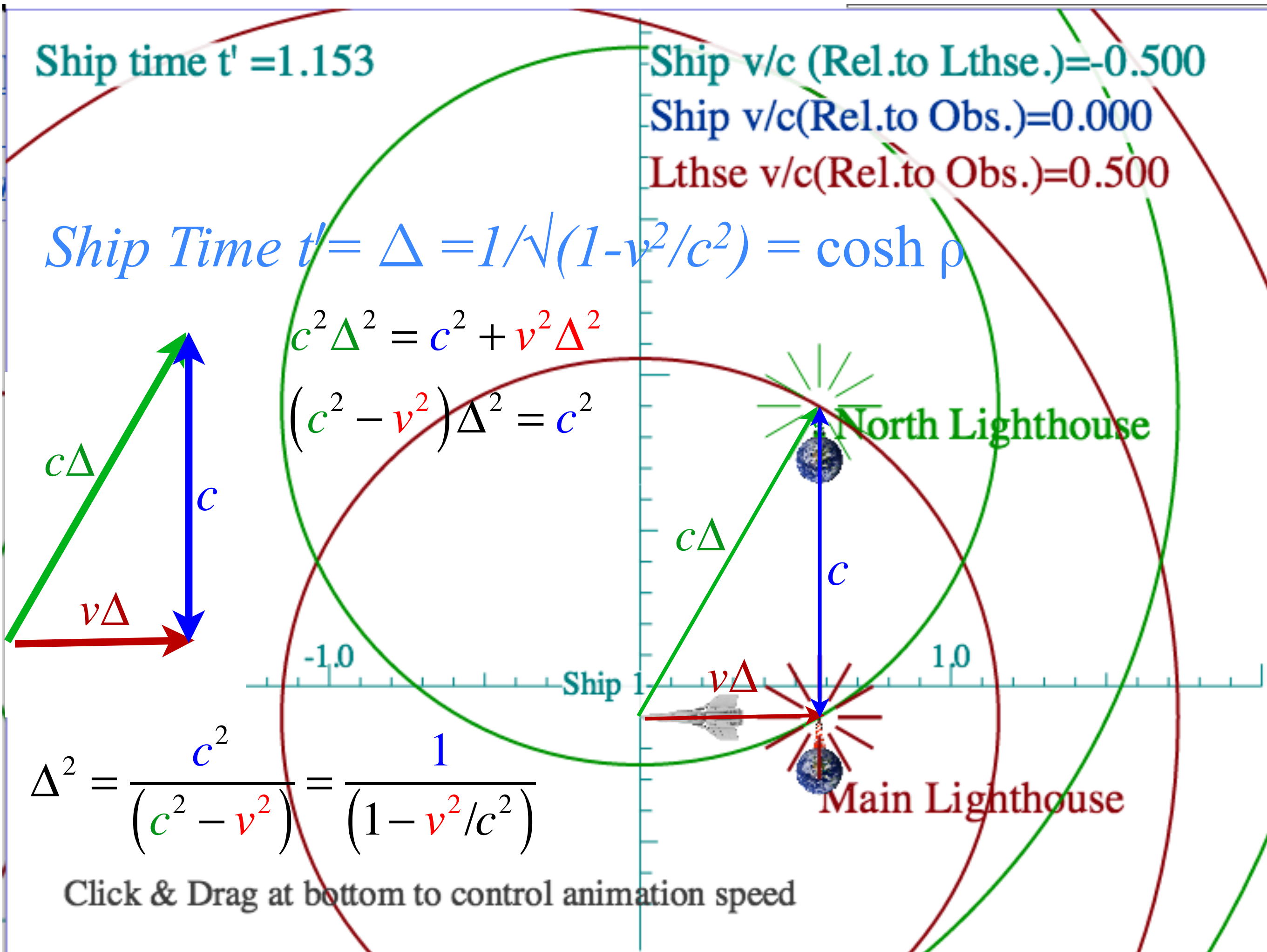

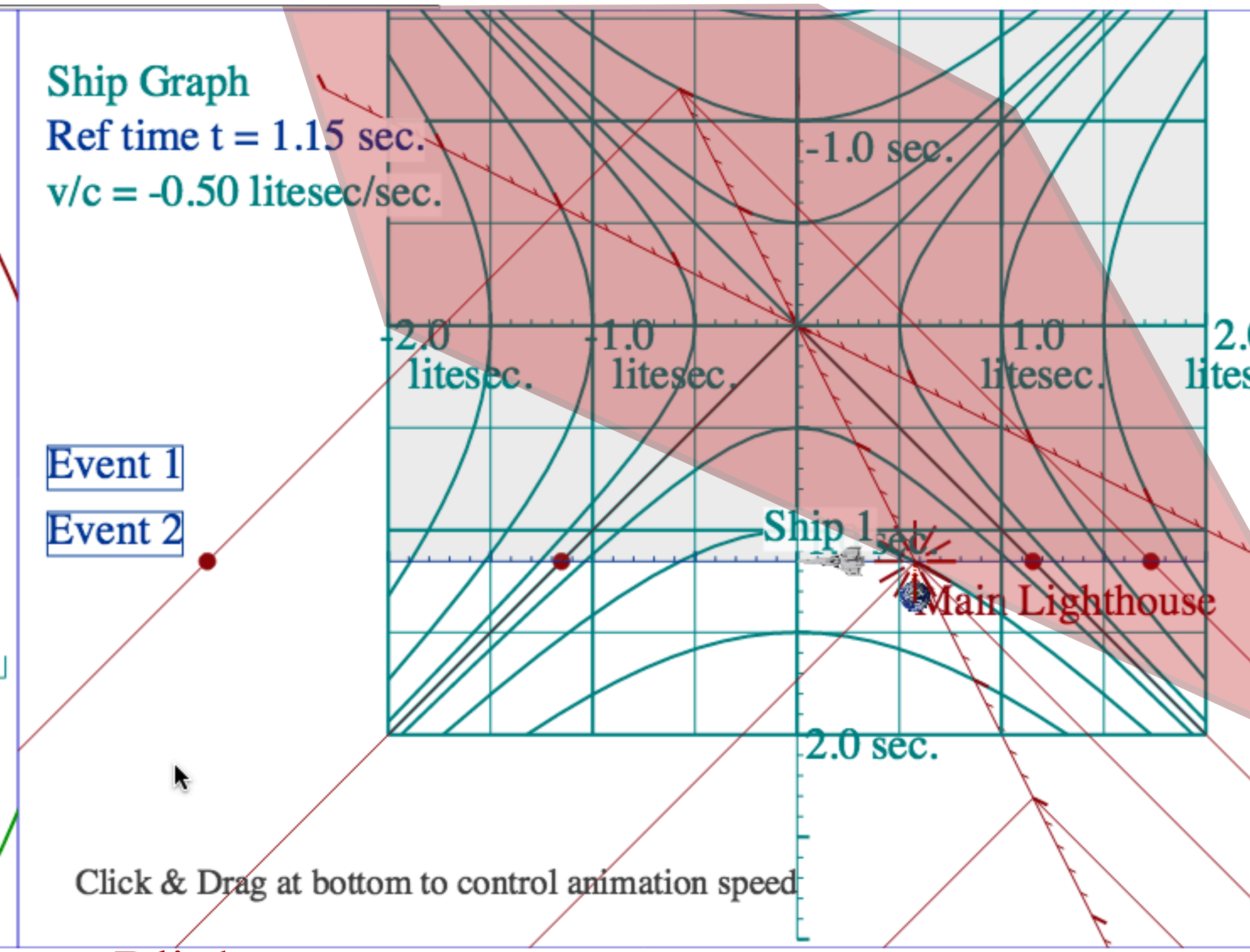

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[RelativIt Web Simulation](#)

[*Relativistic Events in Ship's Space-Time Frame*](#)

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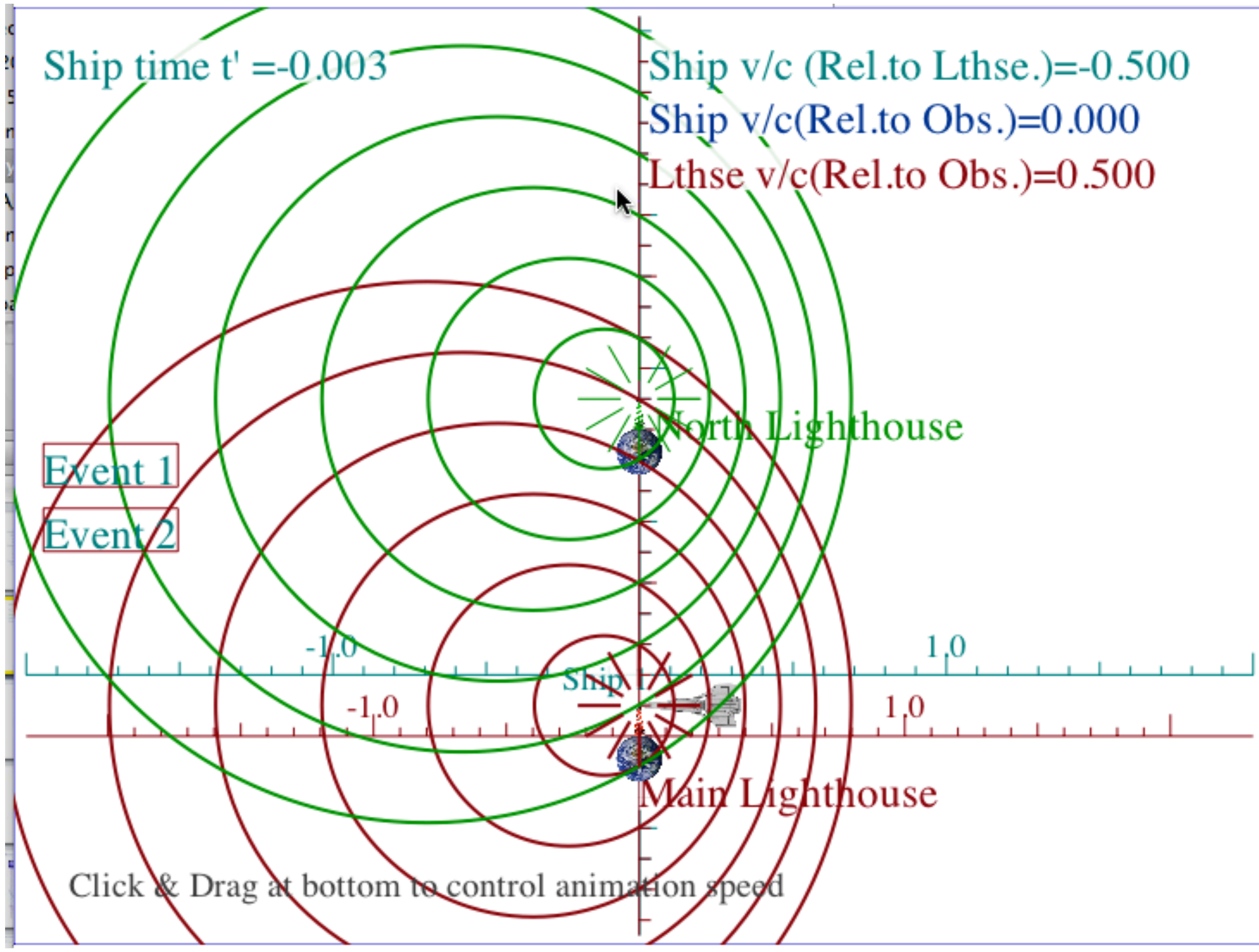

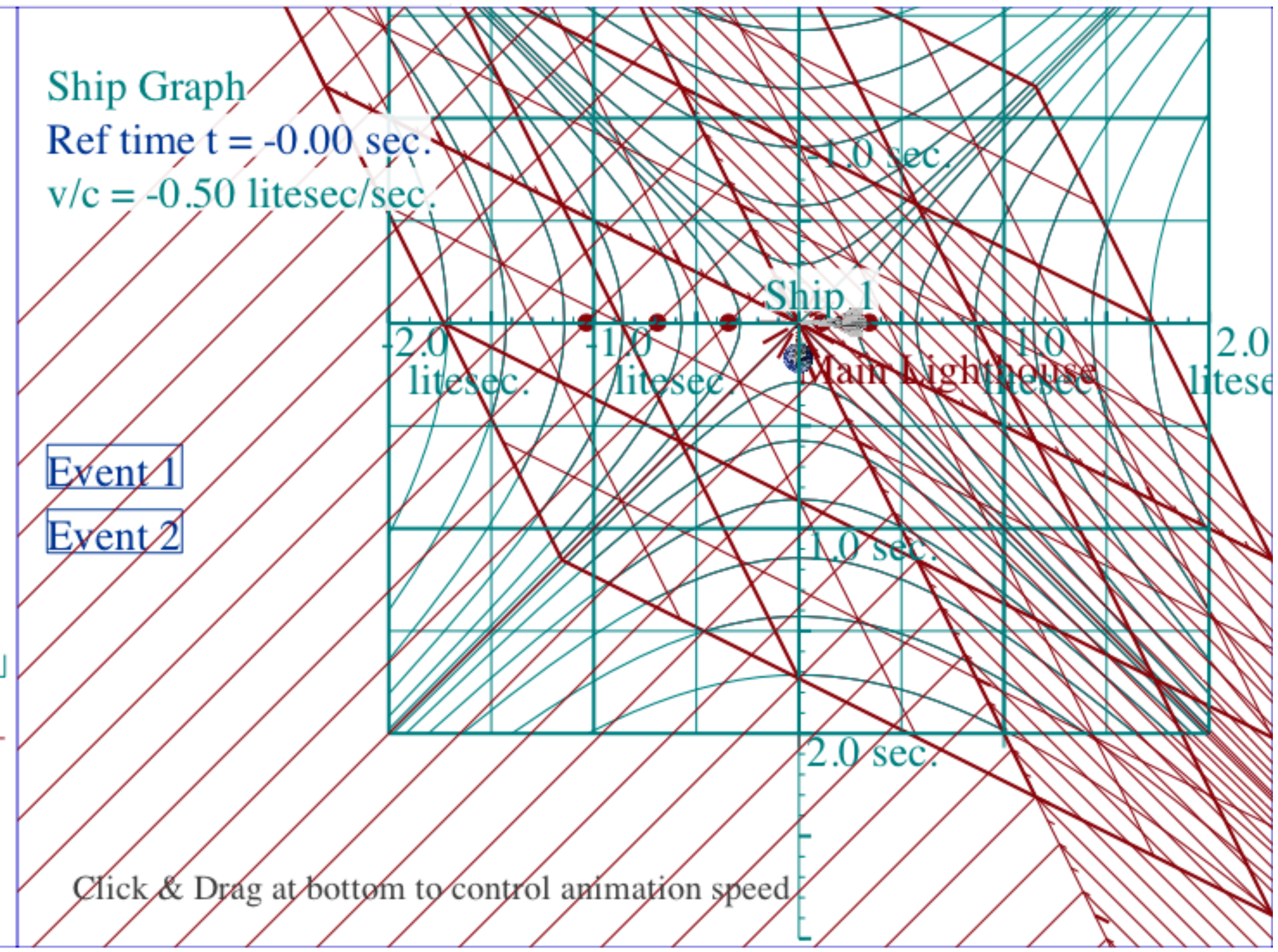

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for: $\beta = \frac{1}{2}$ *or*: $e^{\rho} = \sqrt{3}$

$\cosh\rho = \frac{1}{\sqrt{1-\beta^2}} = \frac{2}{\sqrt{3}} = 1.155 \quad = \Delta$

$\sinh\rho = \frac{\beta}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{3}} = 0.577$

Event 0: Ship passes Main Lighthouse.	Event 1: Ship gets hit by first blink from Main Lighthouse.	Event 2: Main Lighthouse blinks second time.
(Lighthouse space) $x = 0$ (Lighthouse time) $ct = 0$	$x = -1.00$ $ct = 2.00$	$x = 0$ $ct = 2.00$
(Ship space) $x' = 0$ (Ship time) $ct' = 0$	$x' = 0$ $ct' = 1.73$	$x' = c\,\Delta$ $ct' = 2\Delta = 2.30$

2015 animations of lighthouses and ships in (x,y) scenarios and Minkowski (x,ct) plots

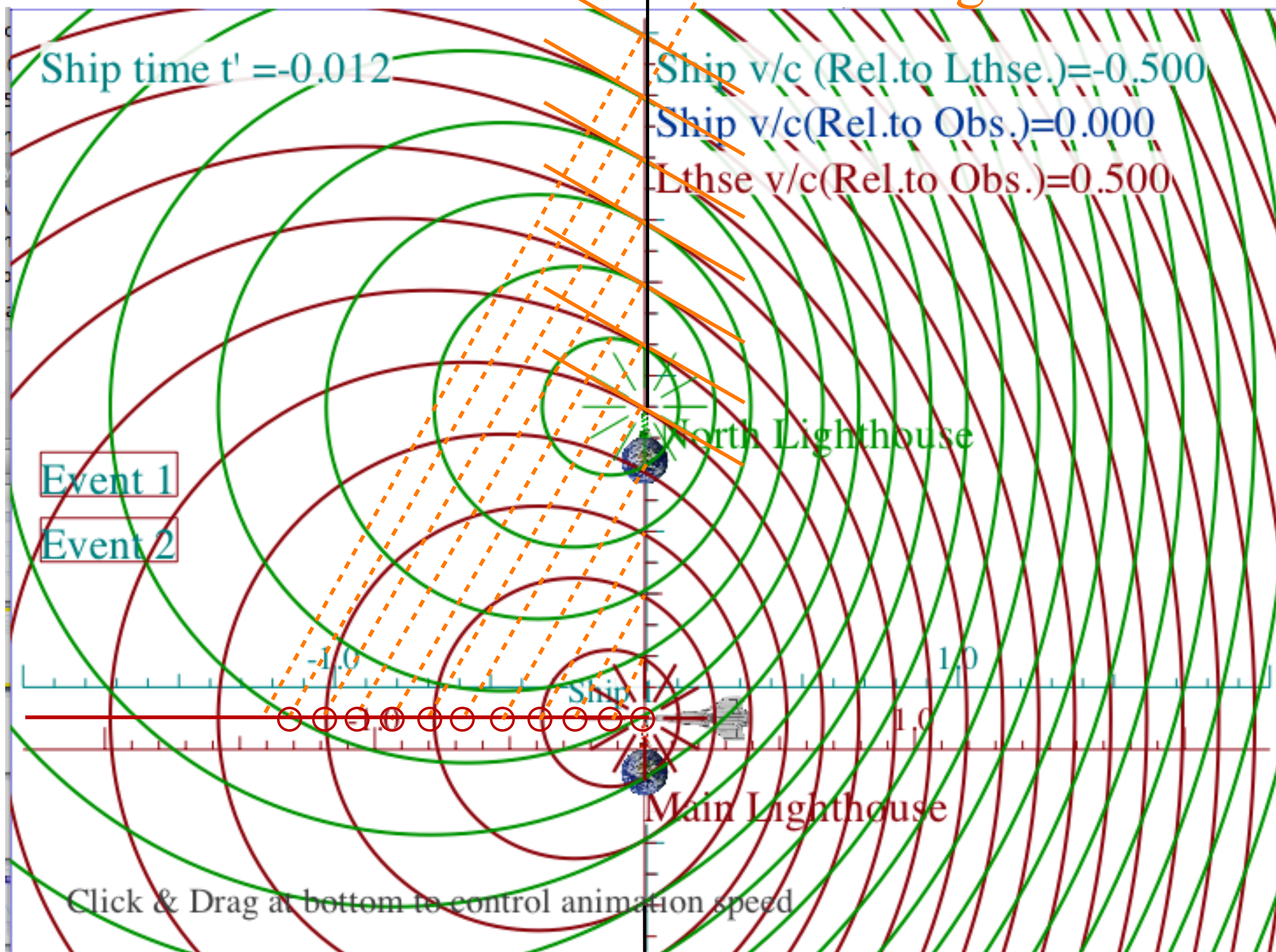
[RelativIt Web Simulation](#)

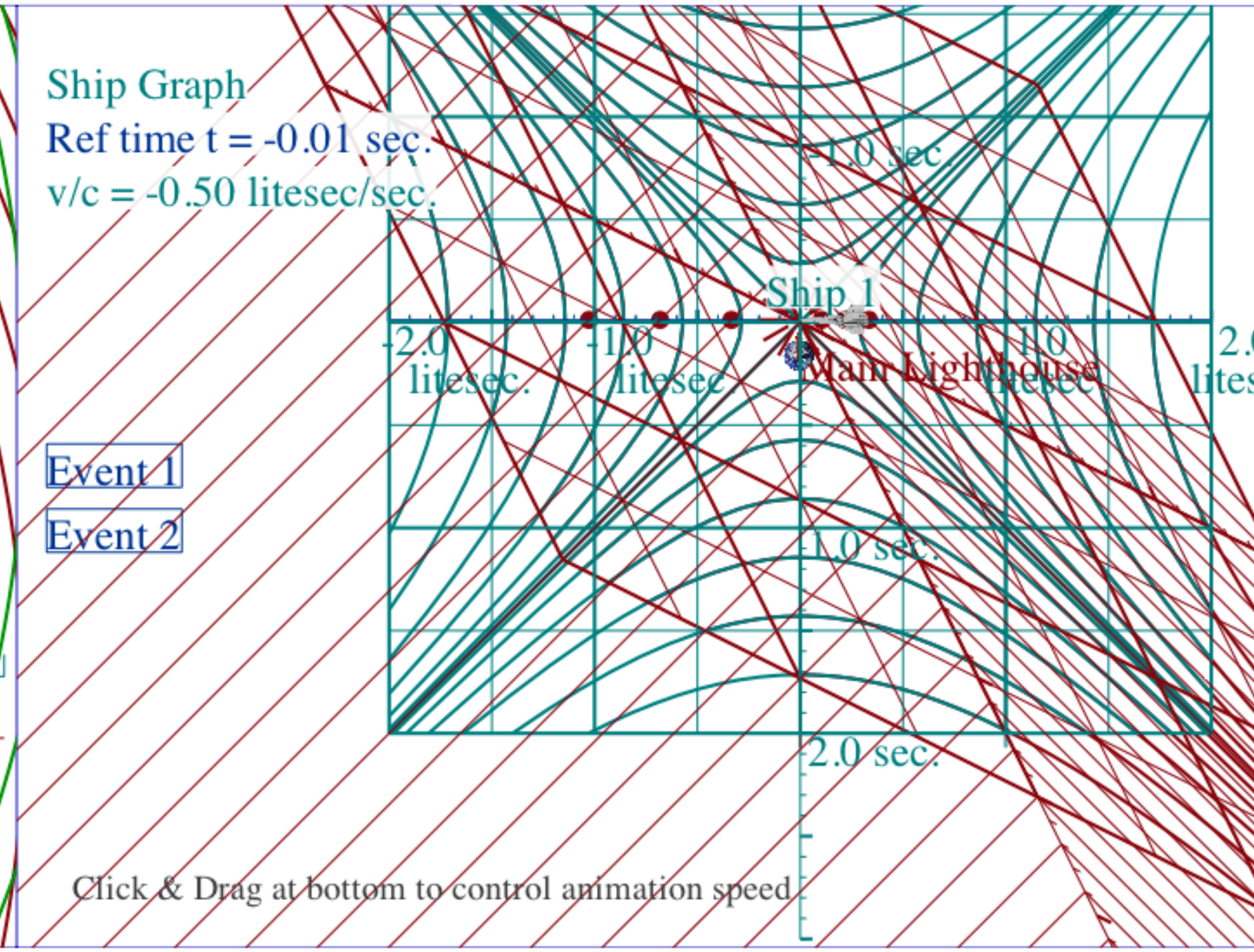
Relativistic Events in Ship's Space-Time Frame

$\sigma=30°$

Stellar angle σ

$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{1-\beta^2}} & \frac{\beta}{\sqrt{1-\beta^2}} \\ \frac{\beta}{\sqrt{1-\beta^2}} & \frac{1}{\sqrt{1-\beta^2}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} 1.155 & 0.577 \\ 0.577 & 1.155 \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix}$$





$$\begin{pmatrix} x' \\ ct' \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} -\frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} -1.0 \\ 2.0 \end{pmatrix} = \begin{pmatrix} -\frac{2}{\sqrt{3}}+2\frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{3}}+2\frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 0 \\ \sqrt{3} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{2}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2\frac{1}{\sqrt{3}} \\ 2\frac{2}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix} \frac{1}{\sqrt{3}} = \begin{pmatrix} 1.15 \\ 2.30 \end{pmatrix}$$

for: $\beta=\frac{1}{2}$ *or:* $e^{\rho}=\sqrt{3}$

$$\cosh\rho=\frac{1}{\sqrt{1-\beta^2}}=\frac{2}{\sqrt{3}}=1.155$$

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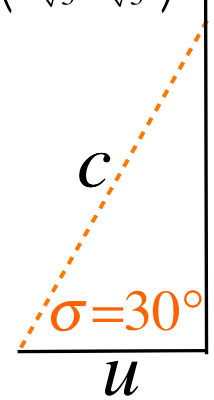

$$\frac{u}{c}=\frac{1}{2}=\sin\sigma$$

Event 0: Ship passes Main Lighthouse.	Event 1: Ship gets hit by first blink from Main Lighthouse.	Event 2: Main Lighthouse blinks second time.
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http://www.uark.edu/ua/modphys/markup/RelativItWeb.html?scenario=105

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Un-concentric derivation of stellar aberration **k**-angle σ

➔ Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega, ck_x, ck_y, ck_z)$ transformation

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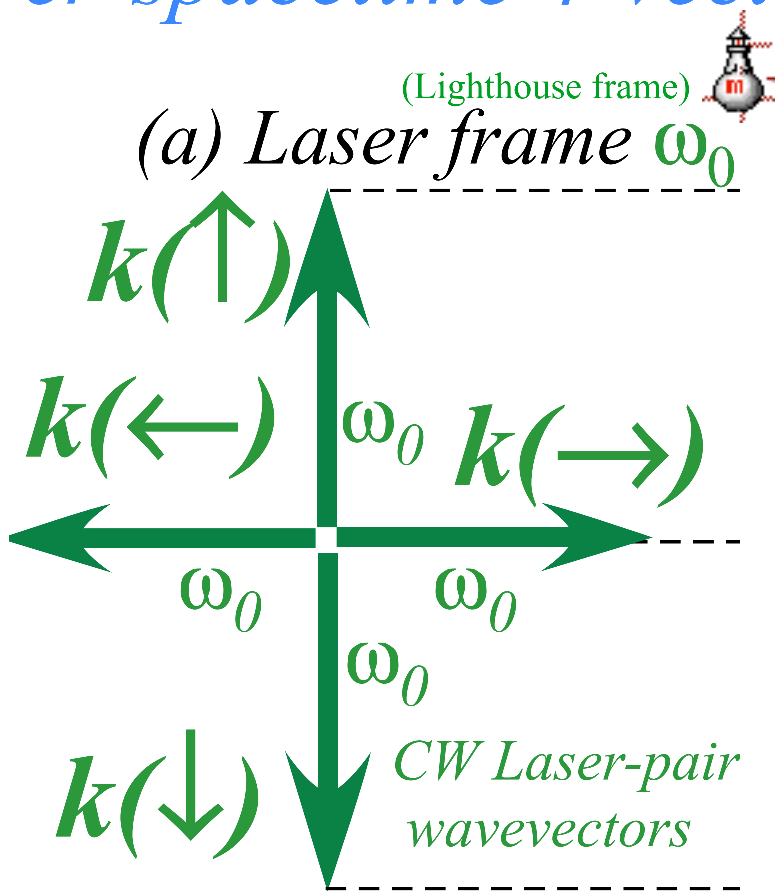

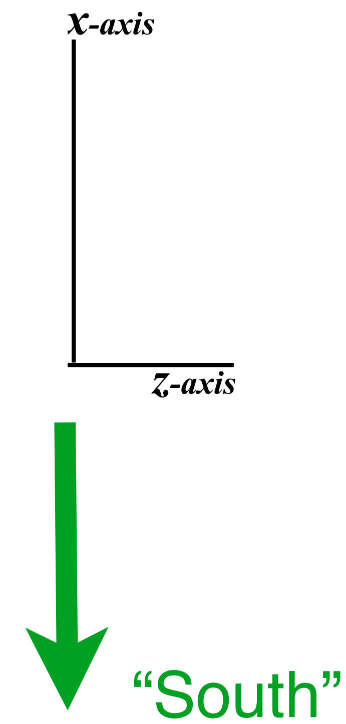

Suppose starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow}, ck_{x\downarrow}, ck_{y\downarrow}, ck_{y\downarrow}\right) = \left(\omega_0, -\omega_0, 0, 0\right)$

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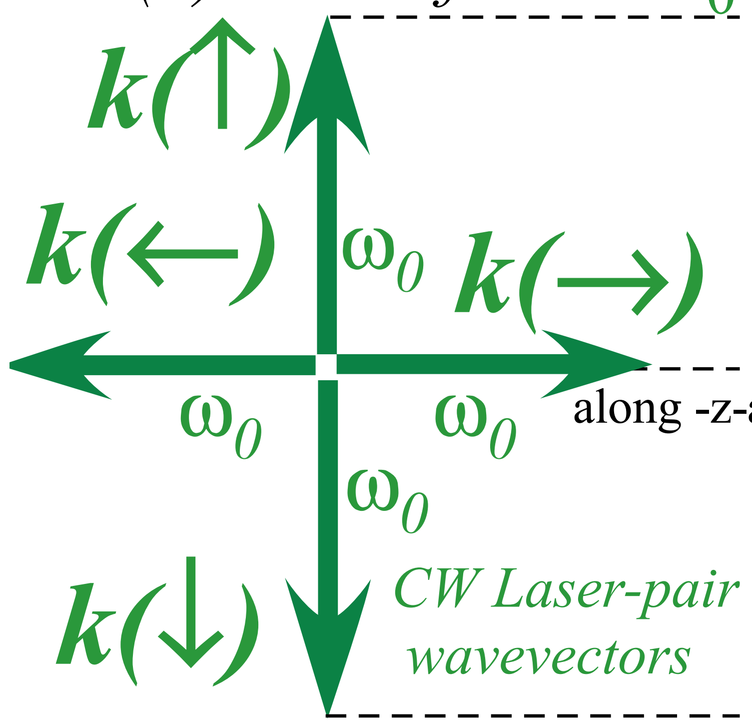
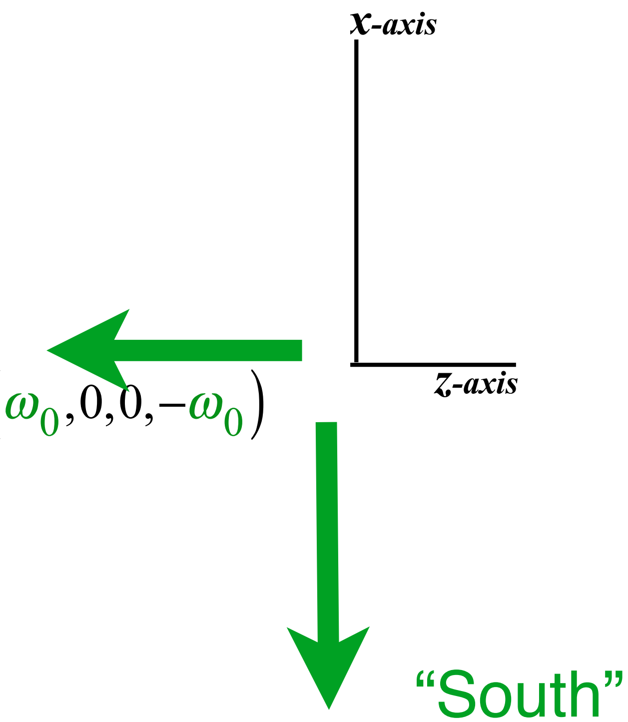
(Lighthouse frame)

(a) Laser frame ω_0





along -z-axis : $\left(\omega_{\leftarrow}, ck_{x\leftarrow}, ck_{y\leftarrow}, ck_{z\leftarrow}\right) = \left(\omega_0, 0, 0, -\omega_0\right)$

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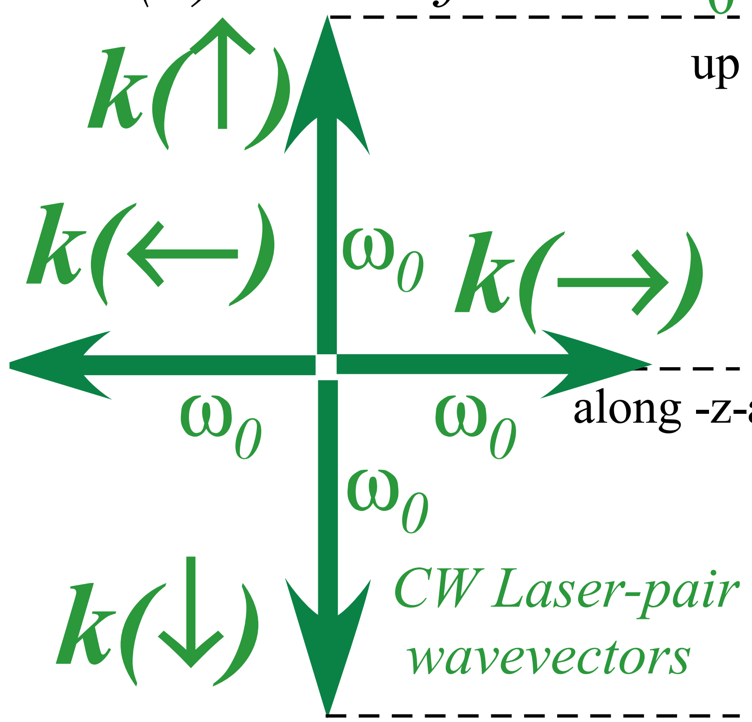

up +x-axis : $\left(\omega_{\uparrow}, ck_{x\uparrow}, ck_{y\uparrow}, ck_{z\uparrow}\right) = \left(\omega_0, +\omega_0, 0, 0\right)$

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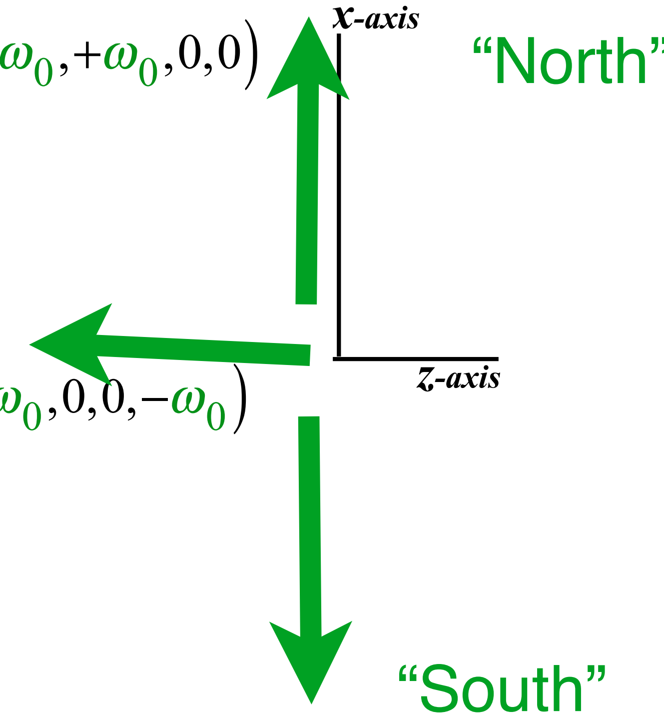

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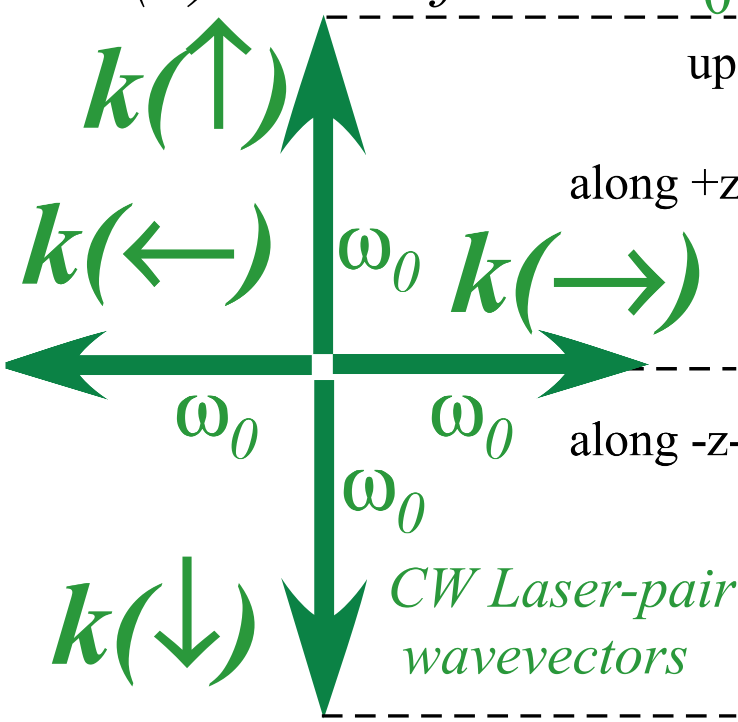
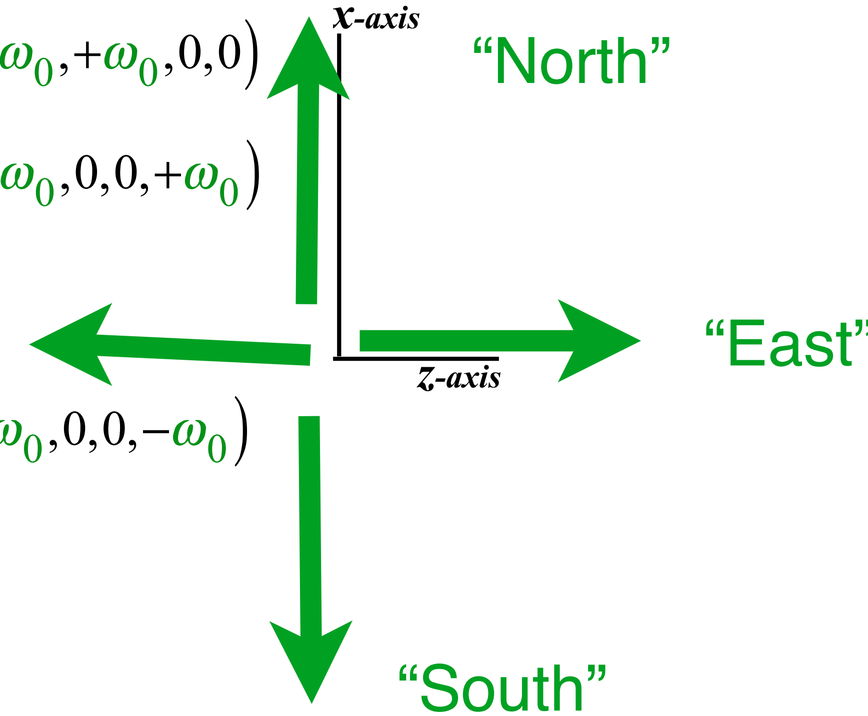


up +x-axis : $\left(\omega_{\uparrow}, ck_{x\uparrow}, ck_{y\uparrow}, ck_{z\uparrow}\right) = \left(\omega_0, +\omega_0, 0, 0\right)$

along +z-axis : $\left(\omega_{\rightarrow}, ck_{x\rightarrow}, ck_{y\rightarrow}, ck_{z\rightarrow}\right) = \left(\omega_0, 0, 0, +\omega_0\right)$

along -z-axis : $\left(\omega_{\leftarrow}, ck_{x\leftarrow}, ck_{y\leftarrow}, ck_{z\leftarrow}\right) = \left(\omega_0, 0, 0, -\omega_0\right)$



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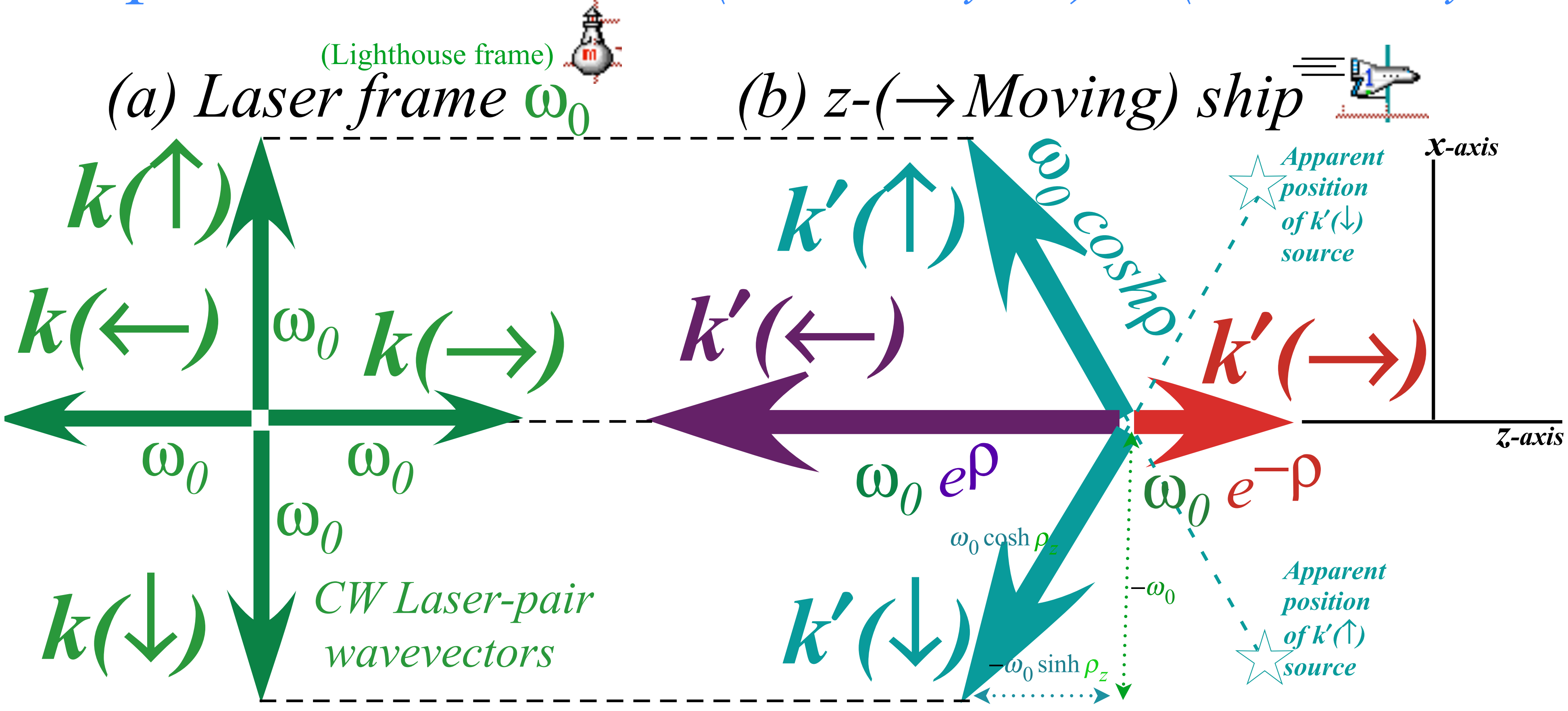

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$$\begin{pmatrix}\omega'_{\downarrow}\\ ck'_{x\downarrow}\\ ck'_{y\downarrow}\\ ck'_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_{\downarrow}\\ ck_{x\downarrow}\\ ck_{y\downarrow}\\ ck_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ -\omega_0\\ 0\\ 0\end{pmatrix}=\begin{pmatrix}\omega_0\cosh\rho_z\\ -\omega_0\\ 0\\ -\omega_0\sinh\rho_z\end{pmatrix}$$

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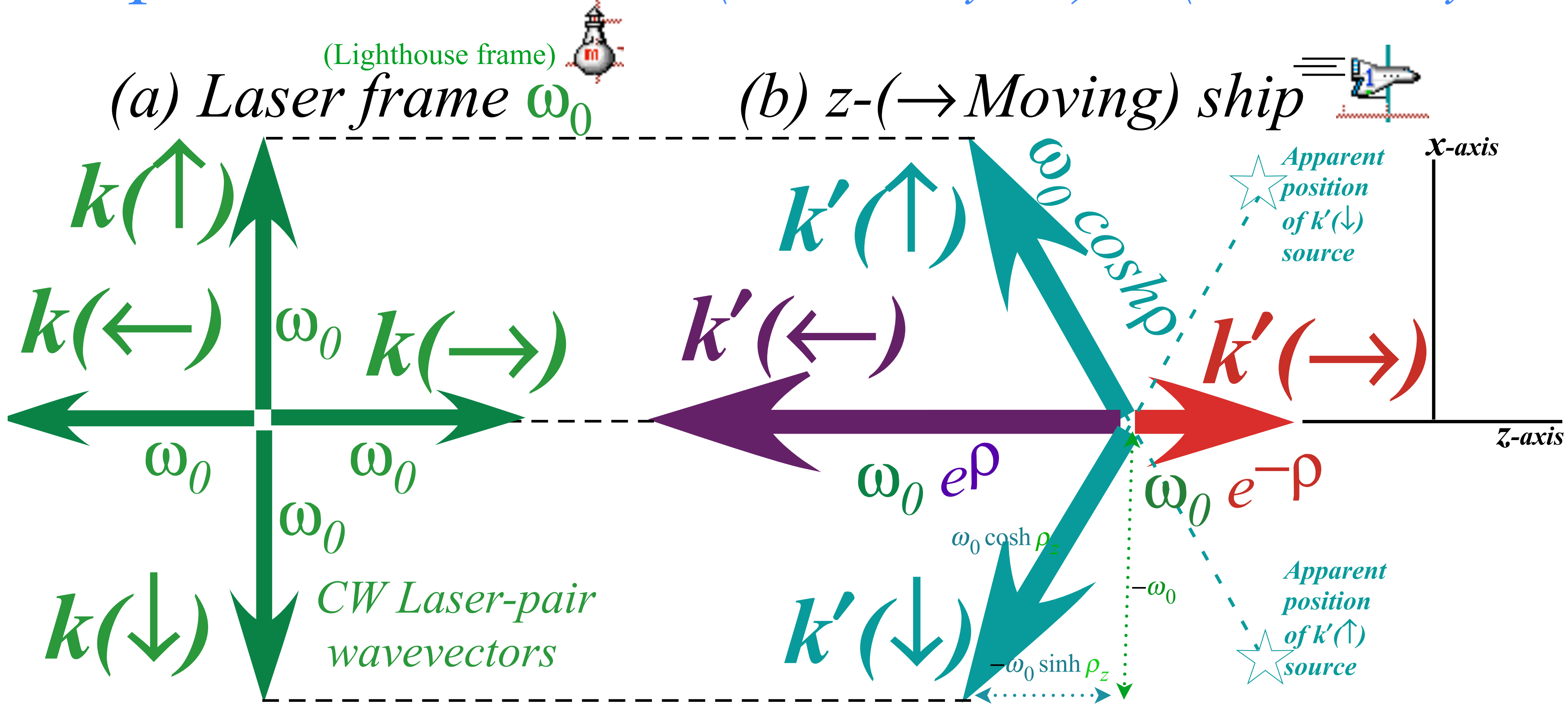

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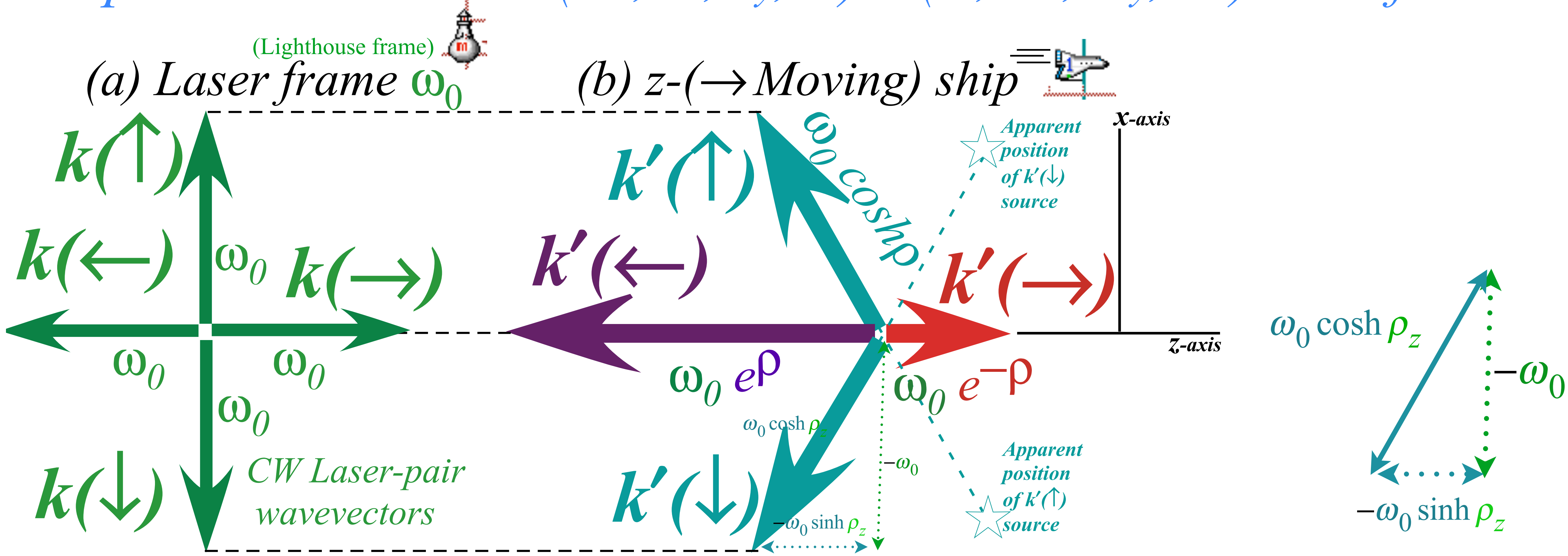
Alternative ordering to $\left(\omega, ck_x, ck_y, ck_z\right)$: $\left(\omega, ck_A, ck_B, ck_C\right) = \left(\omega, ck_z, ck_x, ck_y\right)$

You can simplify notation by using $\{\omega,z,x,y\}$ *or* $\{\omega,A,B,C\}$ ordering of 4-vector.

$$\begin{pmatrix} \omega'_\downarrow \\ ck'_{z\downarrow} \\ ck'_{x\downarrow} \\ ck'_{y\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & -\sinh\rho_z & \cdot & \cdot \\ -\sinh\rho_z & \cosh\rho_z & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{pmatrix} \omega_\downarrow \\ ck_{z\downarrow} \\ ck_{x\downarrow} \\ ck_{y\downarrow} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & -\sinh\rho_z & \cdot & \cdot \\ -\sinh\rho_z & \cosh\rho_z & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & 1 \end{pmatrix} \begin{pmatrix} \omega_0 \\ 0 \\ -\omega_0 \\ 0 \end{pmatrix} = \begin{pmatrix} \omega_0\cosh\rho_z \\ -\omega_0\sinh\rho_z \\ -\omega_0 \\ 0 \end{pmatrix}$$

(But, we won't do that,now)

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z) = (\omega,ck_x,ck_y,ck_z)$ transformation



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After the 4-vector transformation, $\omega_0=\omega_{\downarrow}$ is *transverse Doppler shifted* to $\omega_0 \cosh\rho_z$, while $ck_z=0$ becomes $ck_z' = -\omega_0 \sinh\rho_z$.
(The x-component is unchanged: $ck_x' = -\omega_0 = ck_x$ and so is y-component: $ck_y' = 0 = ck_y$.)

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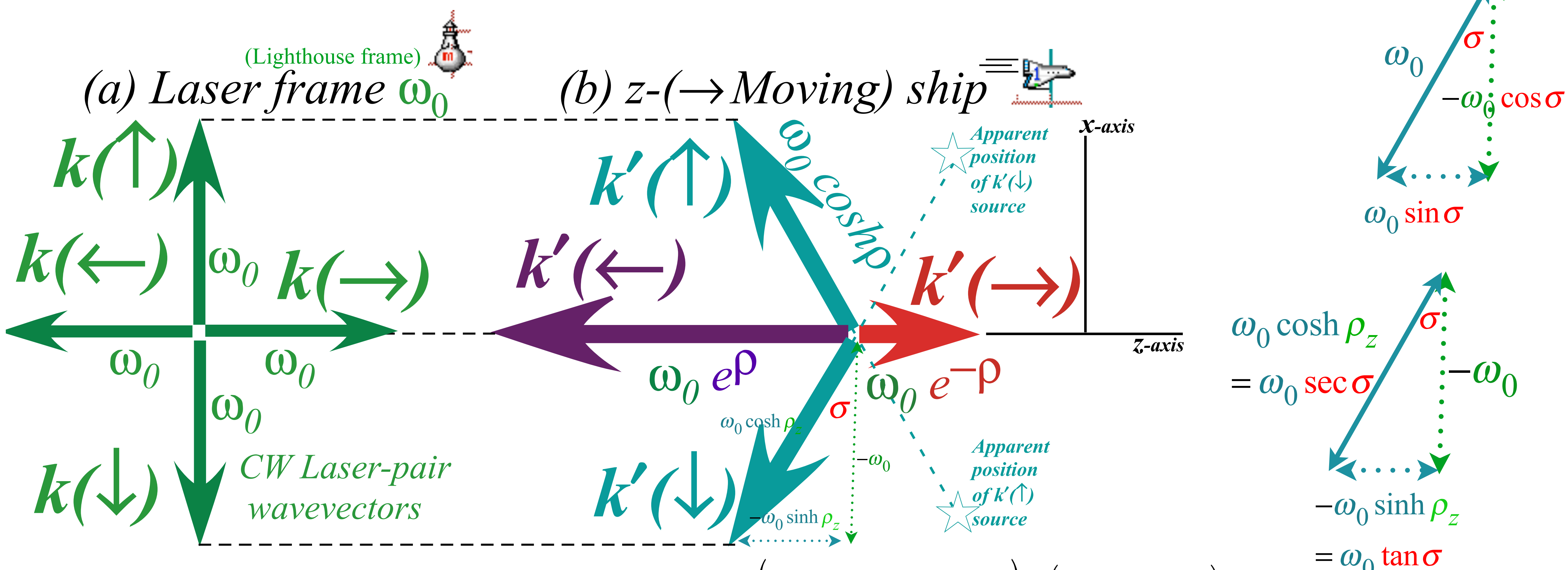

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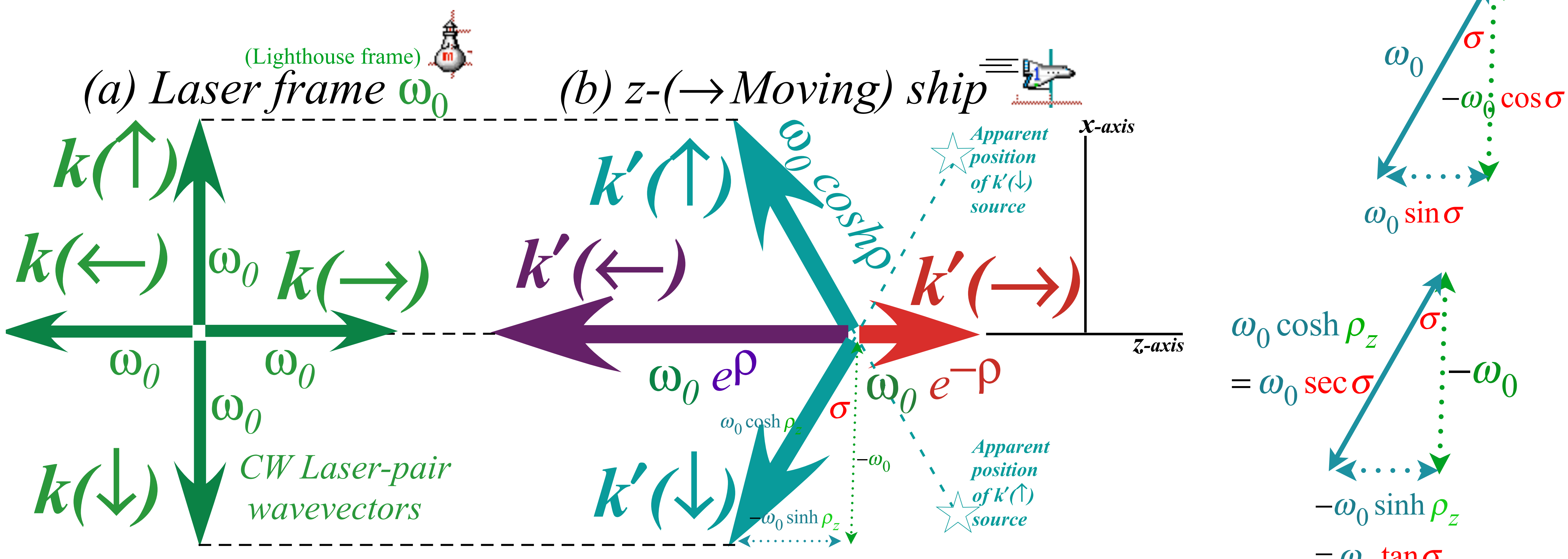

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(The x-component is unchanged: $ck_x{}' = -\omega_0 = ck_x$ and so is y-component: $ck_y{}' = 0 = ck_y$.)

Recall hyperbolic invariant to Lorentz transform: $\omega^2 - c^2k^2 = \omega'^2 - c^2k'^2$ (=0 for 1-CW light)
The 4-vector form of this is: $\omega^2 - c^2\mathbf{k}\bullet\mathbf{k} = \omega'^2 - c^2\mathbf{k}'\bullet\mathbf{k}'$ (=0 " " ")

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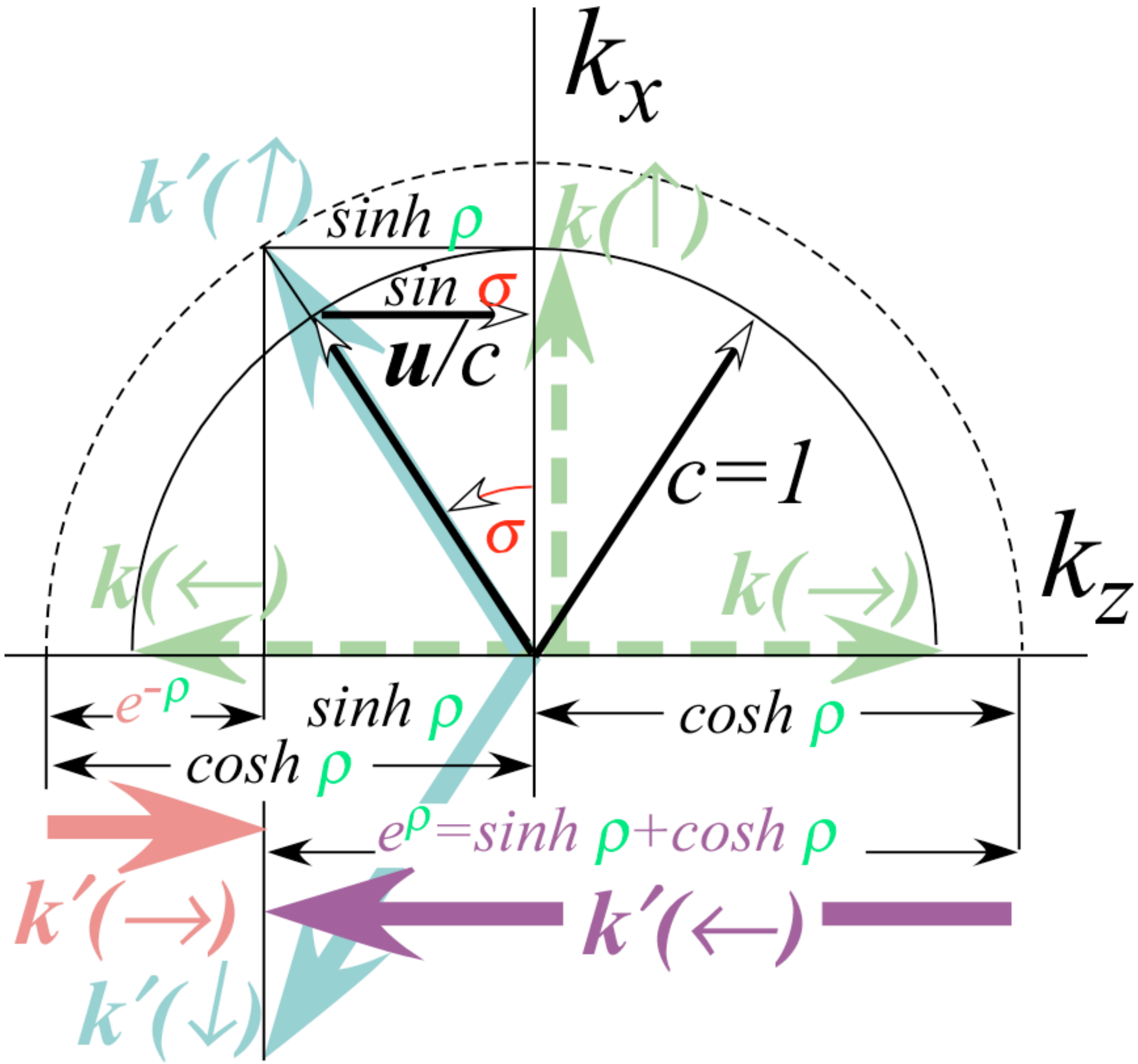
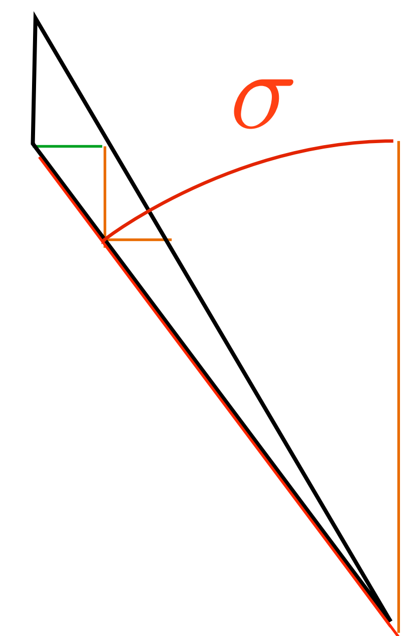
Fig. 5.10 CW cosmic speedometer.

Geometry of Lorentz boost of counter-propagating waves.





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Un-concentric derivation of stellar aberration **k**-angle σ

Per-spacetime 4-vector $(\omega_0,\omega_x,\omega_y,\omega_z)=(\omega,ck_x,ck_y,ck_z)$ transformation

"Occam-sword" geometry: A pattern recognition aid

➔ Relating velocity parameter $\beta=u/c$ to *rapidity* ρ to **k**-*angle* σ to *u/c-angle* ν

Circular arc-area σ vs. hyperbolic arc-area ρ

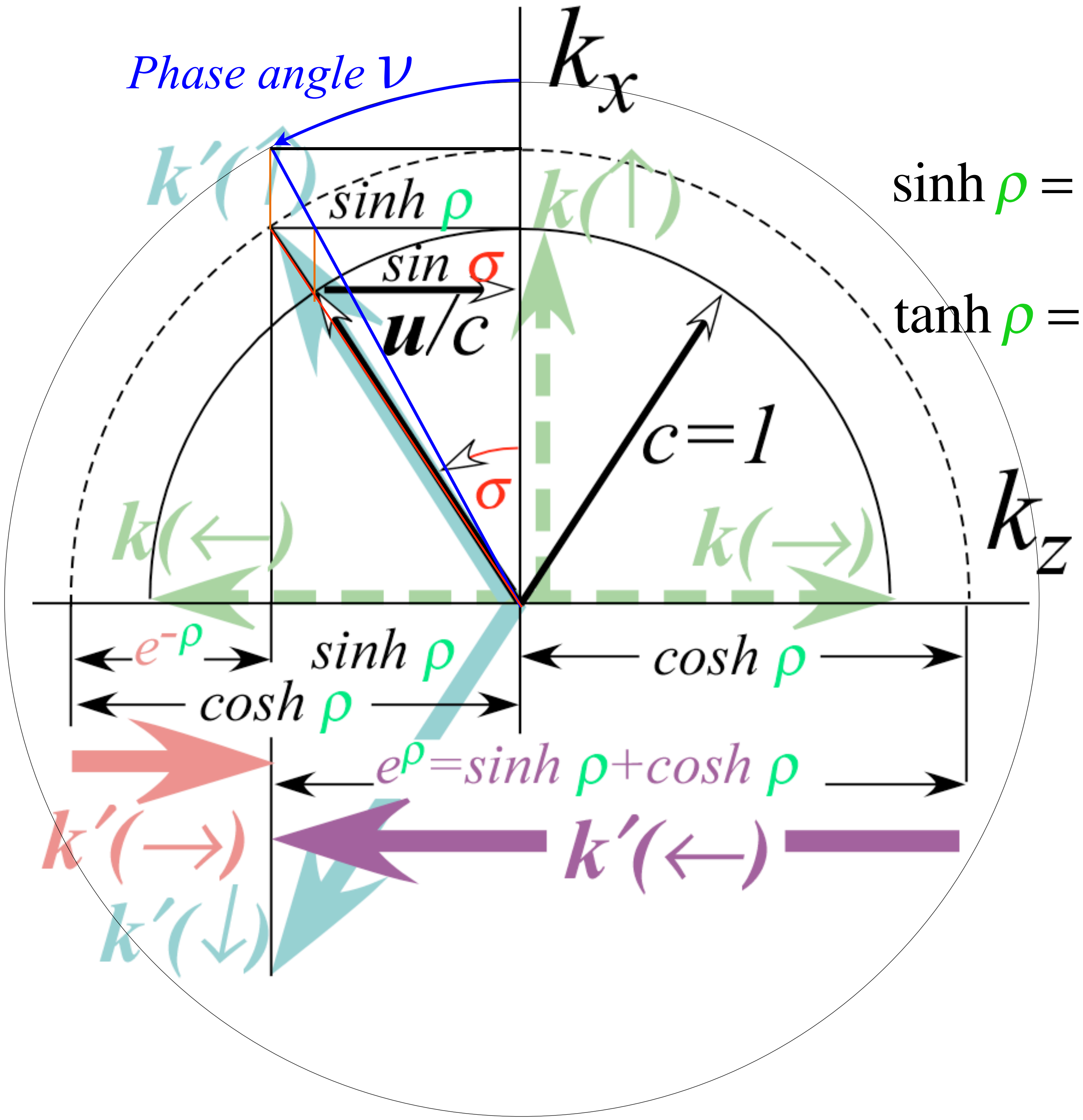
Each circular trig function has a hyperbolic "country-cousin" function

Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ

Fig. 5.10 CW cosmic speedometer.

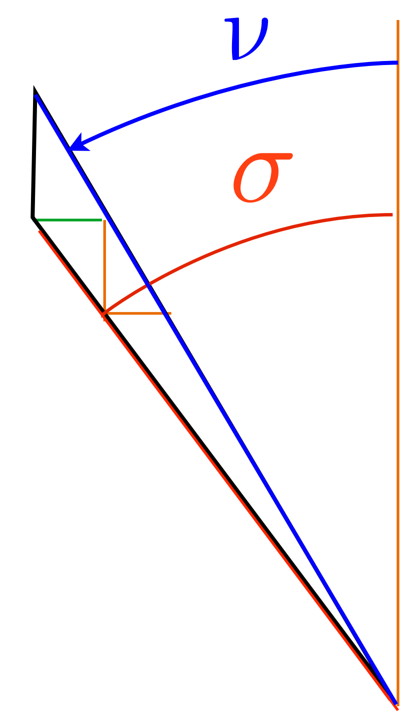
Geometry of Lorentz boost of counter-propagating waves.

Pattern recognition aid: "Occam's Sword"



$$\sinh\rho = \tan\sigma$$

$$\tanh\rho = \sin\sigma = \frac{u}{c} = \tan\nu$$



Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

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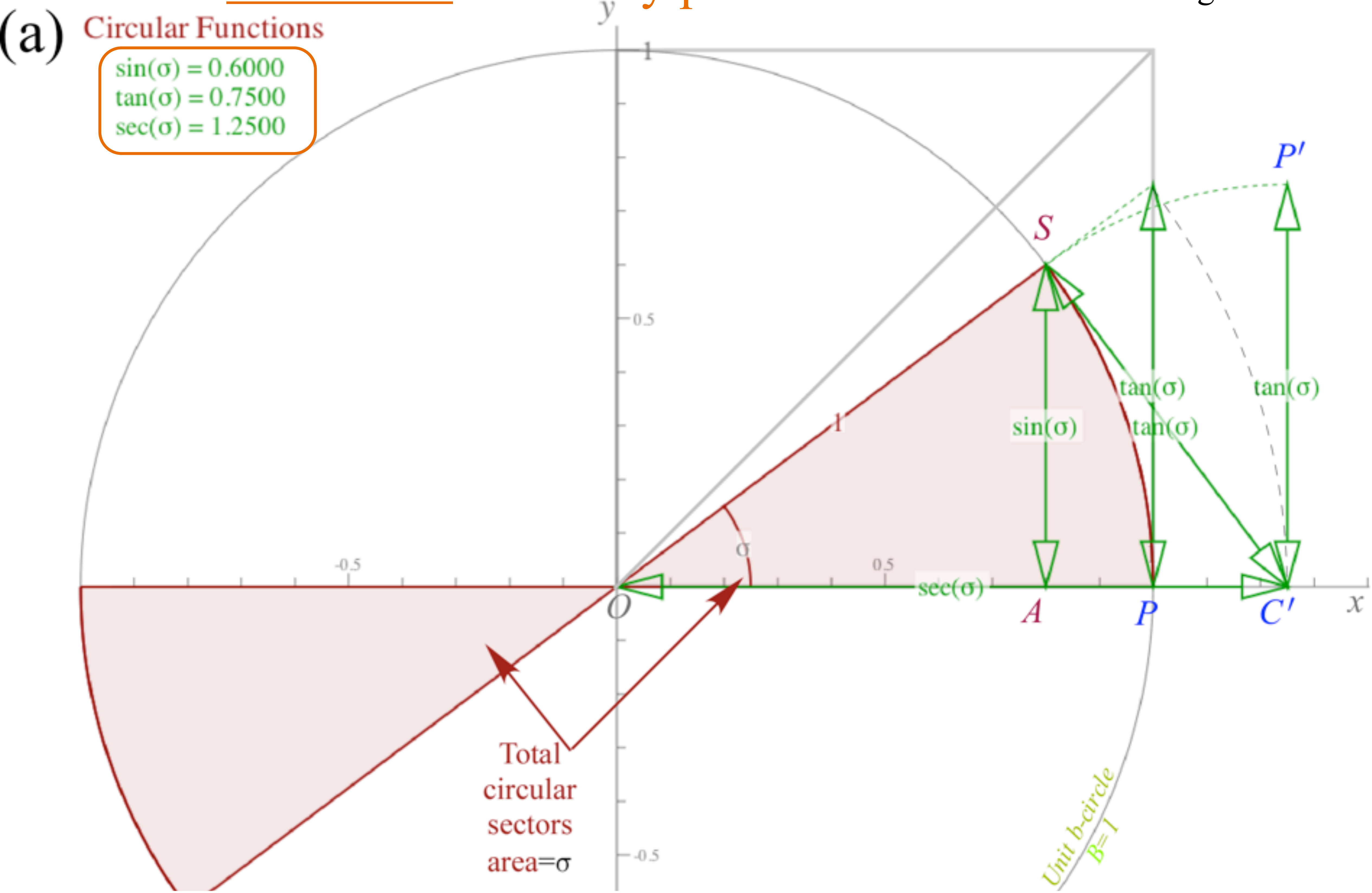
Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to Transverse relativity parameter: Stellar aberration angle σ



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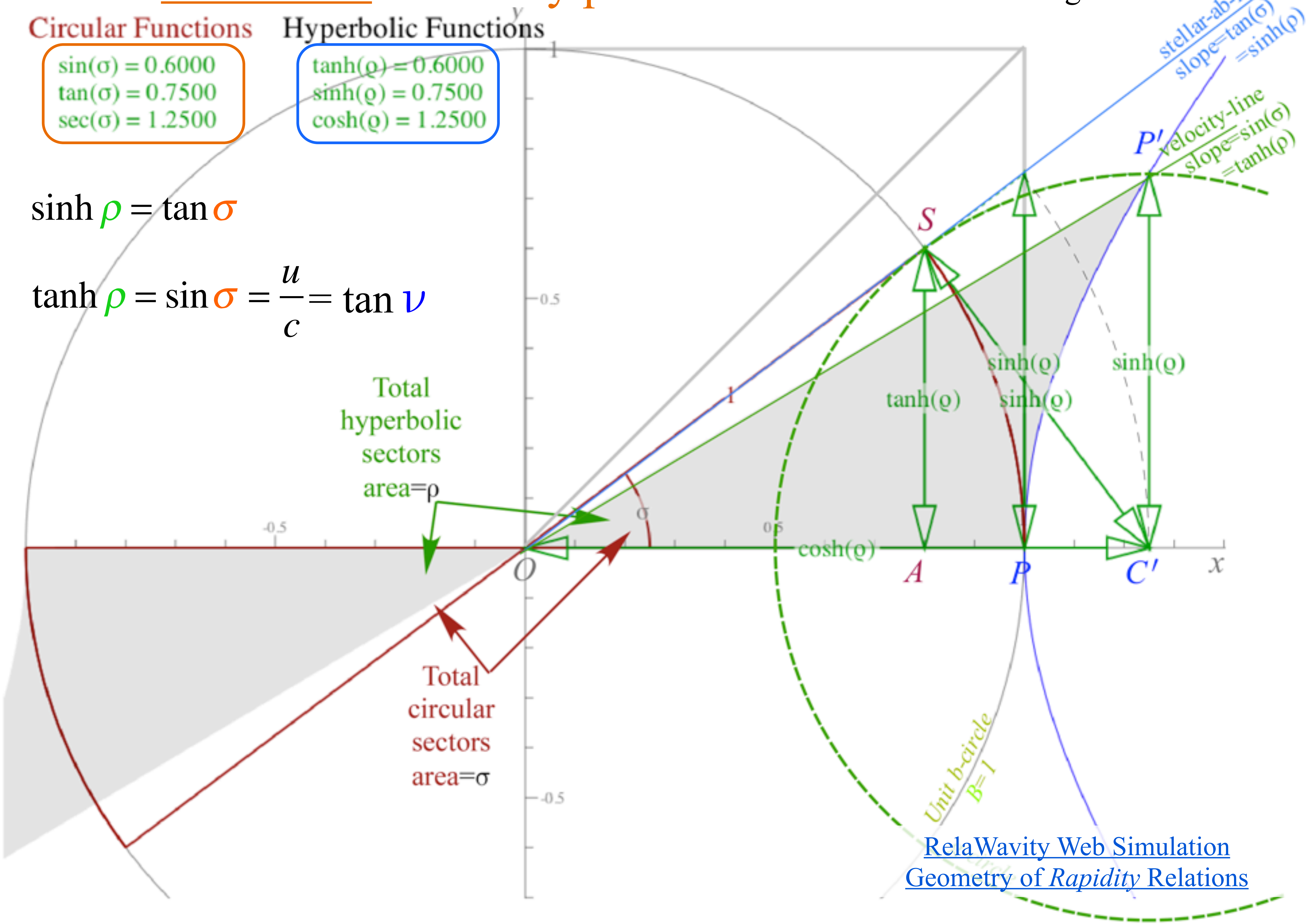
Geometry of Stellar Aberation Angle

Relating Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

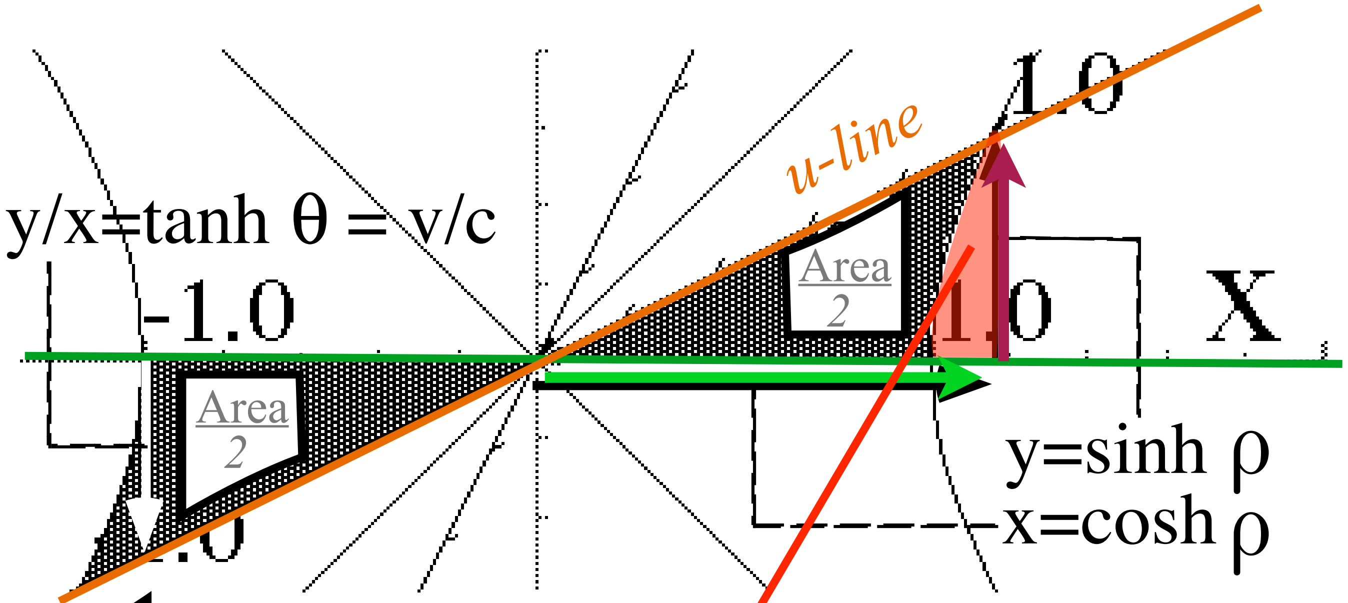
to Transverse relativity parameter: Stellar aberration angle σ

$$\sinh \rho = \tan \sigma$$

$$\tanh \rho = \sin \sigma = \frac{u}{c} = \tan \nu$$



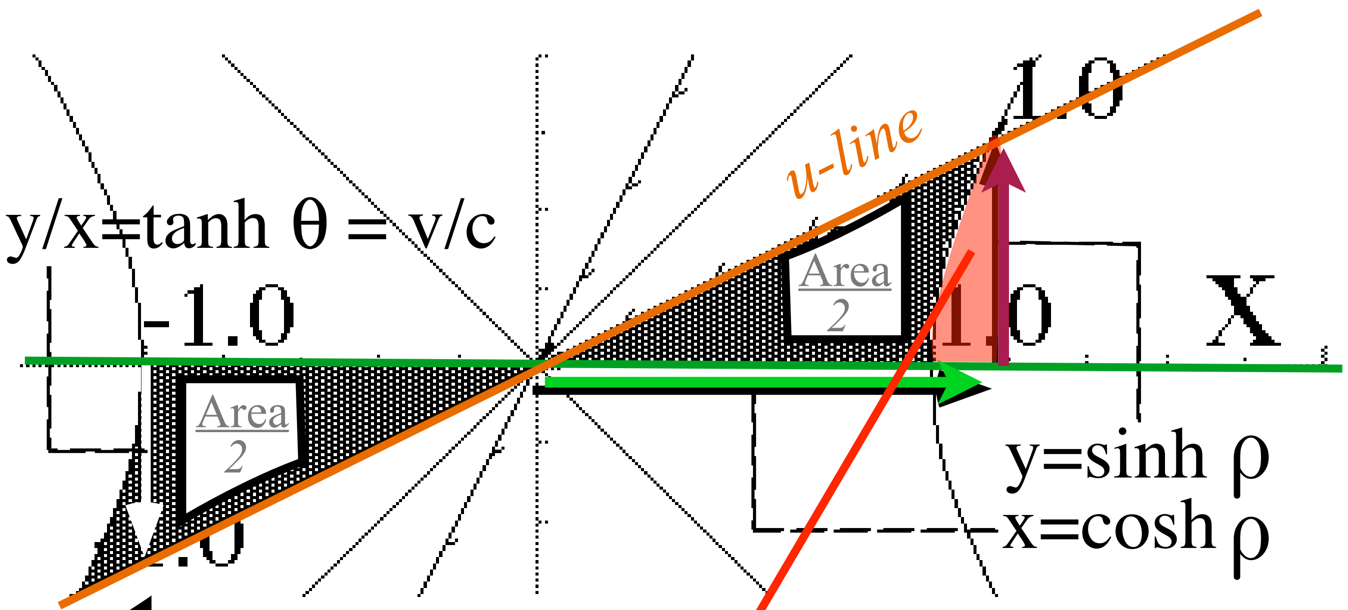
The straight scoop on "angle" and "rapidity" (They're area!)



$$\frac{\text{Area}}{2} = \frac{1}{2}\,\textit{base}\cdot\textit{altitude} - \text{area under curve} = \frac{1}{2}xy - \int y\,dx$$

The "Area" being calculated is the *total* **Gray Area** *between hyperbola pairs, X axis, and sloping u-line*

The straight scoop on "angle" and "rapidity" (They're area!)



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$$\frac{Area}{2} = \frac{1}{2} base \cdot altitude - \text{area under curve} = \frac{1}{2}xy - \int y\,dx$$

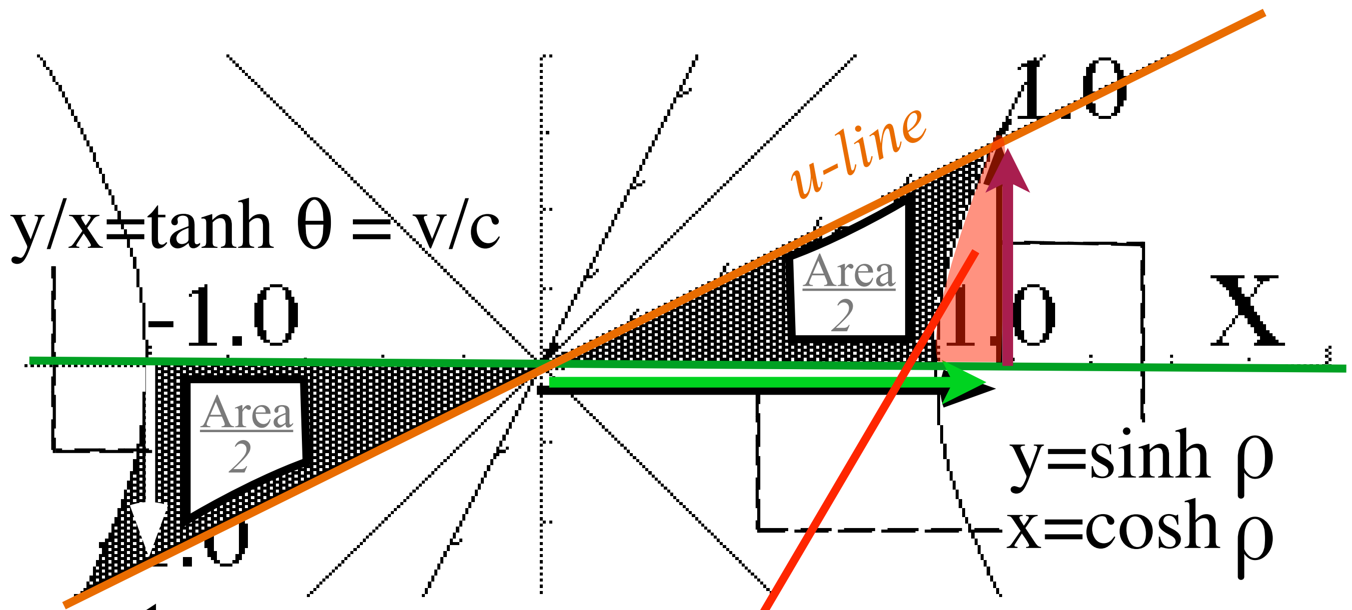
$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh\rho\, d(\cosh\rho)$$

Useful hyperbolic identities

$$\sinh^2\rho = \left(\frac{e^{\rho}-e^{-\rho}}{2}\right)^2 = \frac{1}{4}\left(e^{2\rho}+e^{-2\rho}-2\right) = \frac{\cosh 2\rho - 1}{2}$$

$$\sinh\rho\cosh\rho = \left(\frac{e^{\rho}-e^{-\rho}}{2}\right)\left(\frac{e^{\rho}+e^{-\rho}}{2}\right) = \frac{1}{4}\left(e^{2\rho}-e^{-2\rho}\right) = \frac{1}{2}\sinh 2\rho$$

The straight scoop on "angle" and "rapidity" (They're area!)



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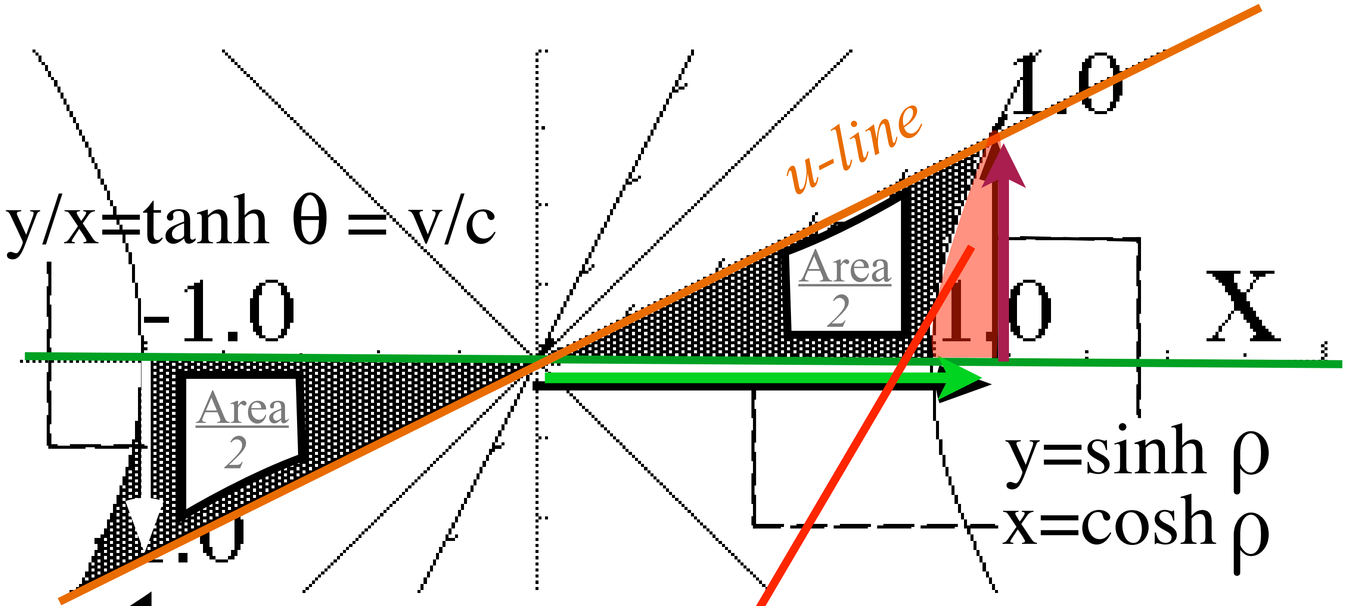
$$\sinh\theta\cosh\theta = \left(\frac{e^{\theta}-e^{-\theta}}{2}\right)\left(\frac{e^{\theta}+e^{-\theta}}{2}\right) = \frac{1}{4}\left(e^{2\theta}-e^{-2\theta}\right) = \frac{1}{2}\sinh 2\theta$$

$$\int \cosh a\rho\, d\rho = \frac{1}{a}\sinh a\rho$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh^2\rho\, d\rho = \frac{1}{4}\sinh 2\rho - \int \frac{\cosh 2\rho - 1}{2}\, d\rho$$

The straight scoop on "angle" and "rapidity" (They're area!)



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$$\int \cosh a\theta\, d\theta = \frac{1}{a}\sinh a\theta$$

$$\frac{Area}{2} = \frac{1}{2}\sinh\rho\cosh\rho - \int \sinh^2\rho\, d\rho = \frac{1}{4}\sinh 2\rho - \int \frac{\cosh 2\rho - 1}{2}\, d\rho$$

$$= \frac{1}{4}\sinh 2\rho - \frac{1}{4}\sinh 2\rho + \int \frac{1}{2}\, d\rho$$

$$= \frac{\rho}{2}$$

Amazing result: ***Area*** $= \rho$ *is rapidity*

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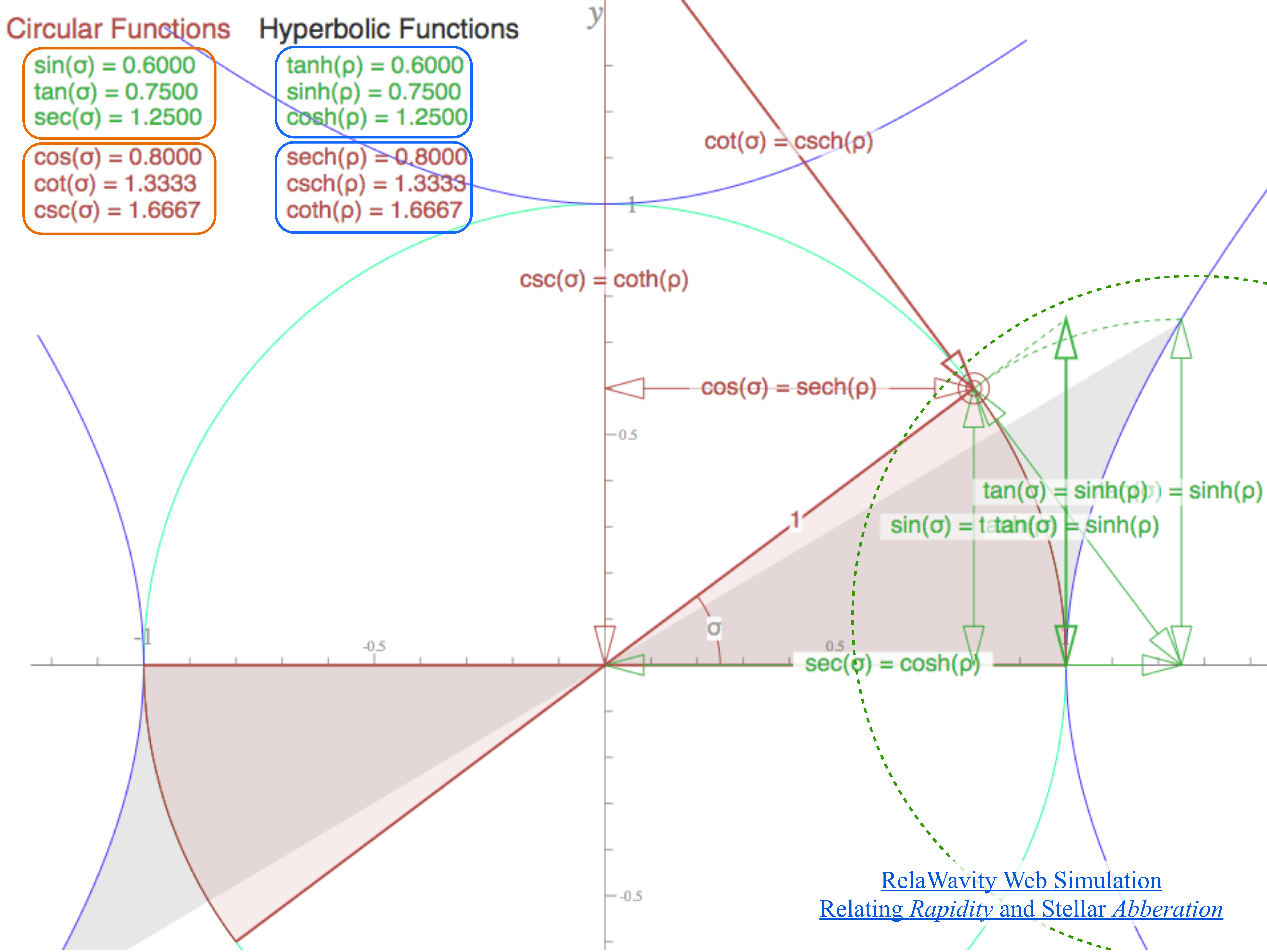
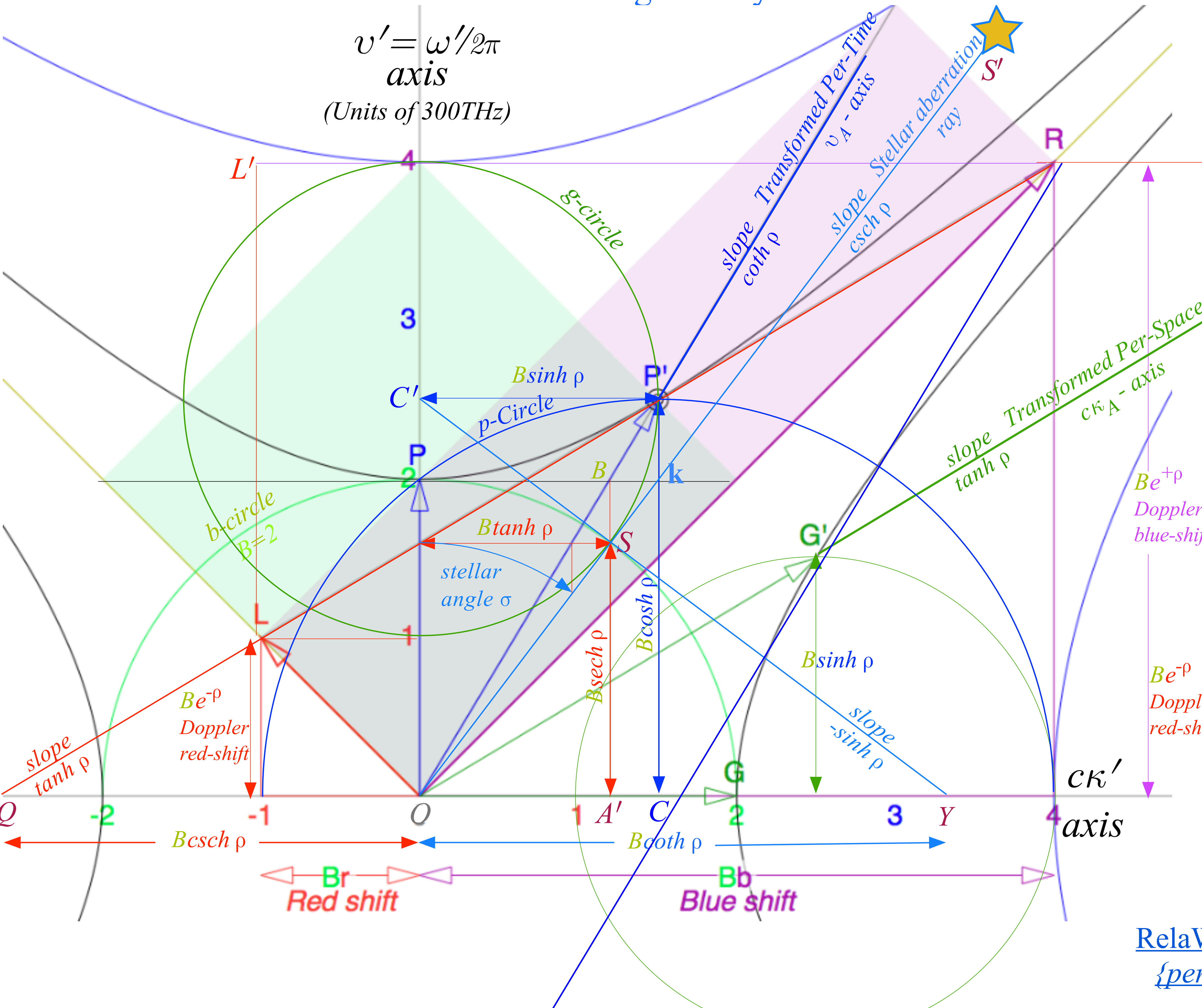
Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration **k**-angle σ



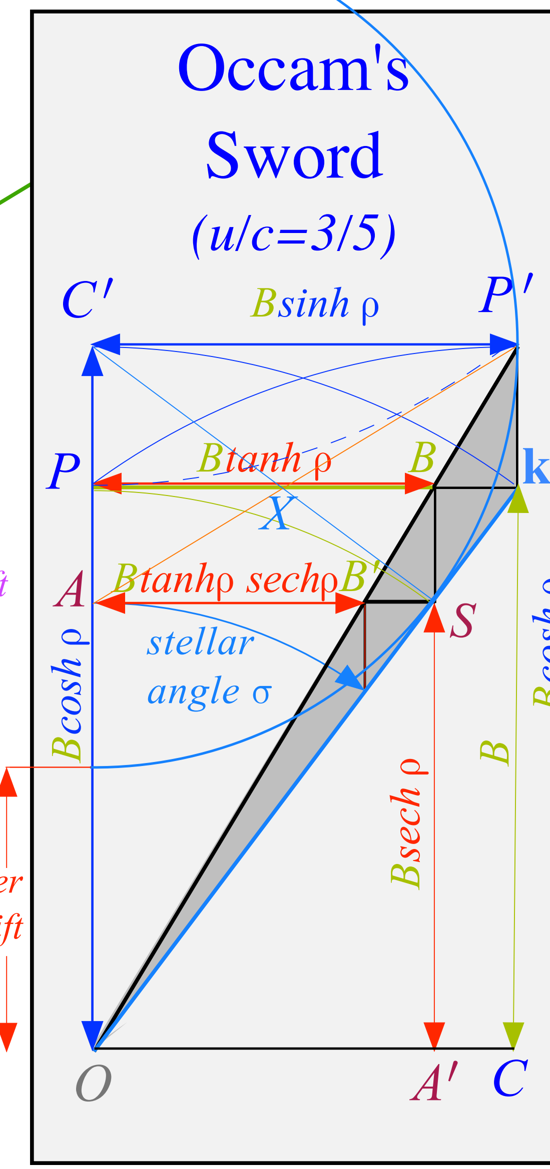
[RelaWavity Web Simulation](#)
[Relating *Rapidity* and Stellar *Abberation*](#)

Summary of optical wave parameters for relativity and QM

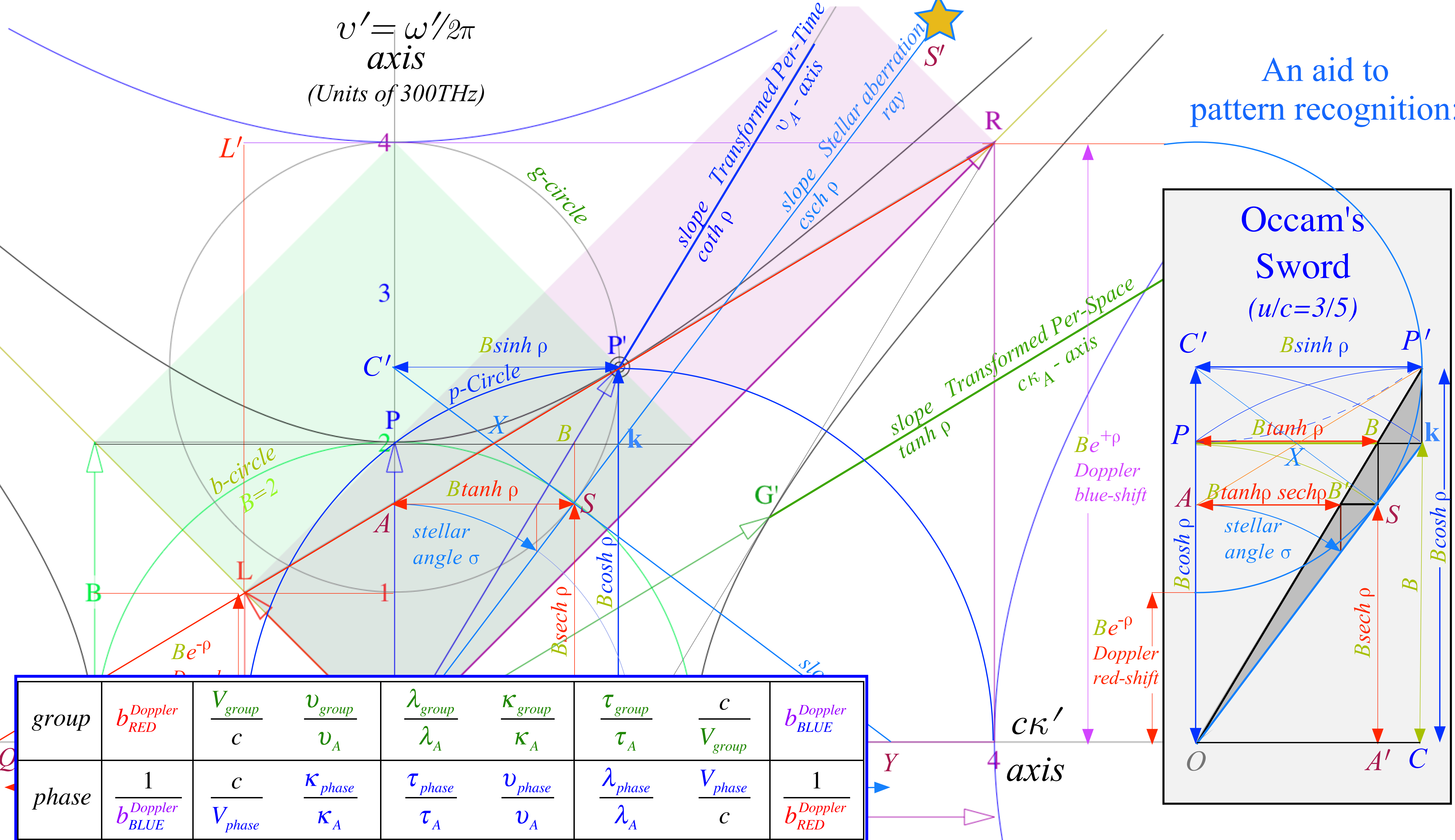
...and their geometry



An aid to pattern recognition:



[RelaWavity Web Simulation {perSpace - perTime All}]



group	$b^{Doppler}_{RED}$	$\frac{V_{group}}{c}$	$\frac{\upsilon_{group}}{\upsilon_A}$	$\frac{\lambda_{group}}{\lambda_A}$	$\frac{\kappa_{group}}{\kappa_A}$	$\frac{\tau_{group}}{\tau_A}$	$\frac{c}{V_{group}}$	$b^{Doppler}_{BLUE}$
phase	$\frac{1}{b^{Doppler}_{BLUE}}$	$\frac{c}{V_{phase}}$	$\frac{\kappa_{phase}}{\kappa_A}$	$\frac{\tau_{phase}}{\tau_A}$	$\frac{\upsilon_{phase}}{\upsilon_A}$	$\frac{\lambda_{phase}}{\lambda_A}$	$\frac{V_{phase}}{c}$	$\frac{1}{b^{Doppler}_{RED}}$
rapidity ρ	$e^{-\rho}$	$\tanh\rho$	$\sinh\rho$	$\operatorname{sech}\rho$	$\cosh\rho$	$\operatorname{csch}\rho$	$\coth\rho$	$e^{+\rho}$
stellar $\forall$ *angle* σ	$1/e^{+\rho}$	$\sin\sigma$	$\tan\sigma$	$\cos\sigma$	$\sec\sigma$	$\cot\sigma$	$\csc\sigma$	$1/e^{-\rho}$
$\beta\equiv\frac{u}{c}$	$\sqrt{\frac{1-\beta}{1+\beta}}$	$\frac{\beta}{1}$	$\frac{1}{\sqrt{\beta^{-2}-1}}$	$\frac{\sqrt{1-\beta^2}}{1}$	$\frac{1}{\sqrt{1-\beta^2}}$	$\frac{\sqrt{\beta^{-2}-1}}{1}$	$\frac{1}{\beta}$	$\sqrt{\frac{1+\beta}{1-\beta}}$
value for $\beta=3/5$	$\frac{1}{2}=0.5$	$\frac{3}{5}=0.6$	$\frac{3}{4}=0.75$	$\frac{4}{5}=0.80$	$\frac{5}{4}=1.25$	$\frac{4}{3}=1.33$	$\frac{5}{3}=1.67$	$\frac{2}{1}=2.0$

Table of 12 wave parameters (includes inverses) for relativity

...and values for *u/c=3/5*

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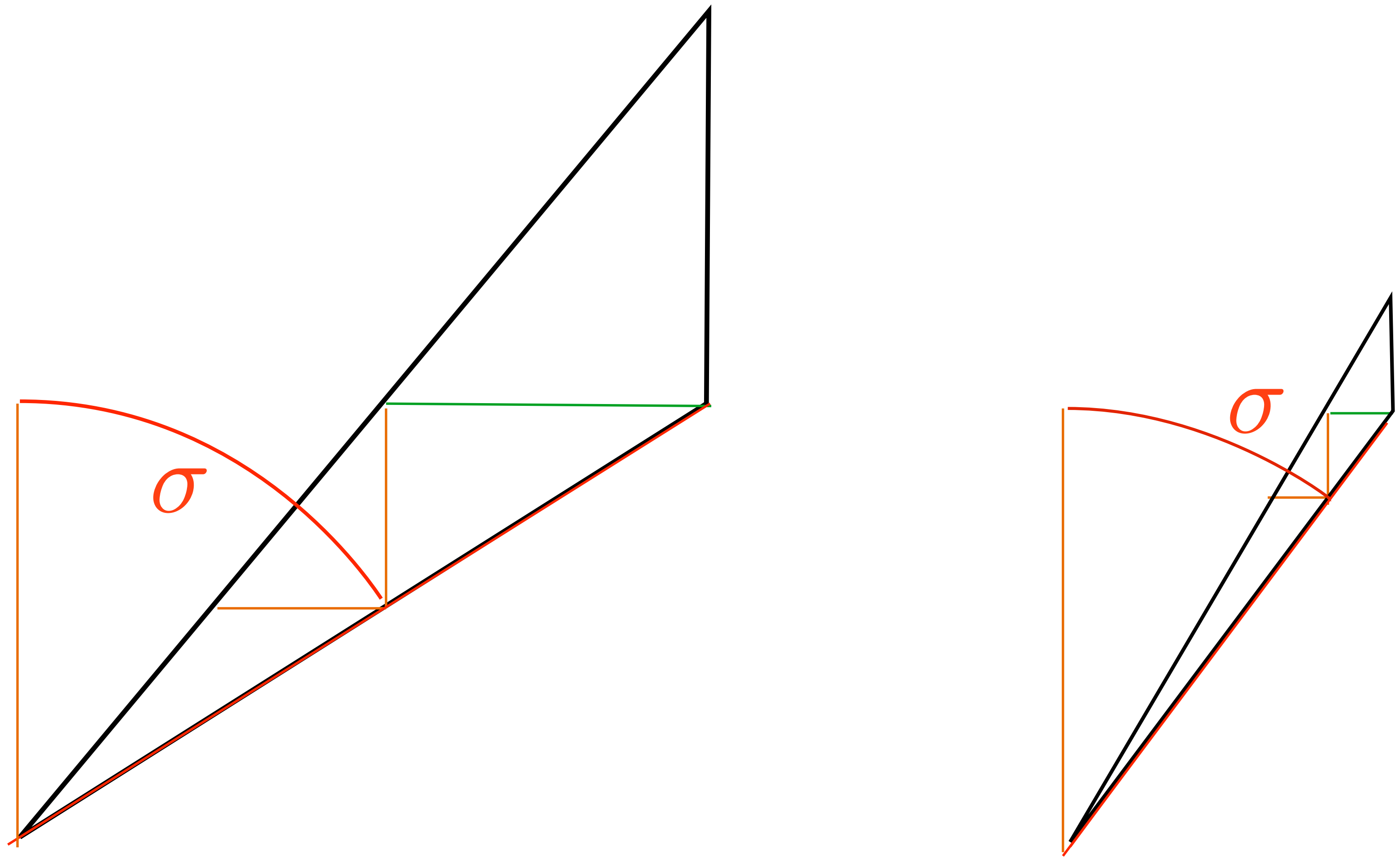
Relativistic *Terms (Dual plot w/expanded table)*

Pattern recognition aid: "Occam's Sword"



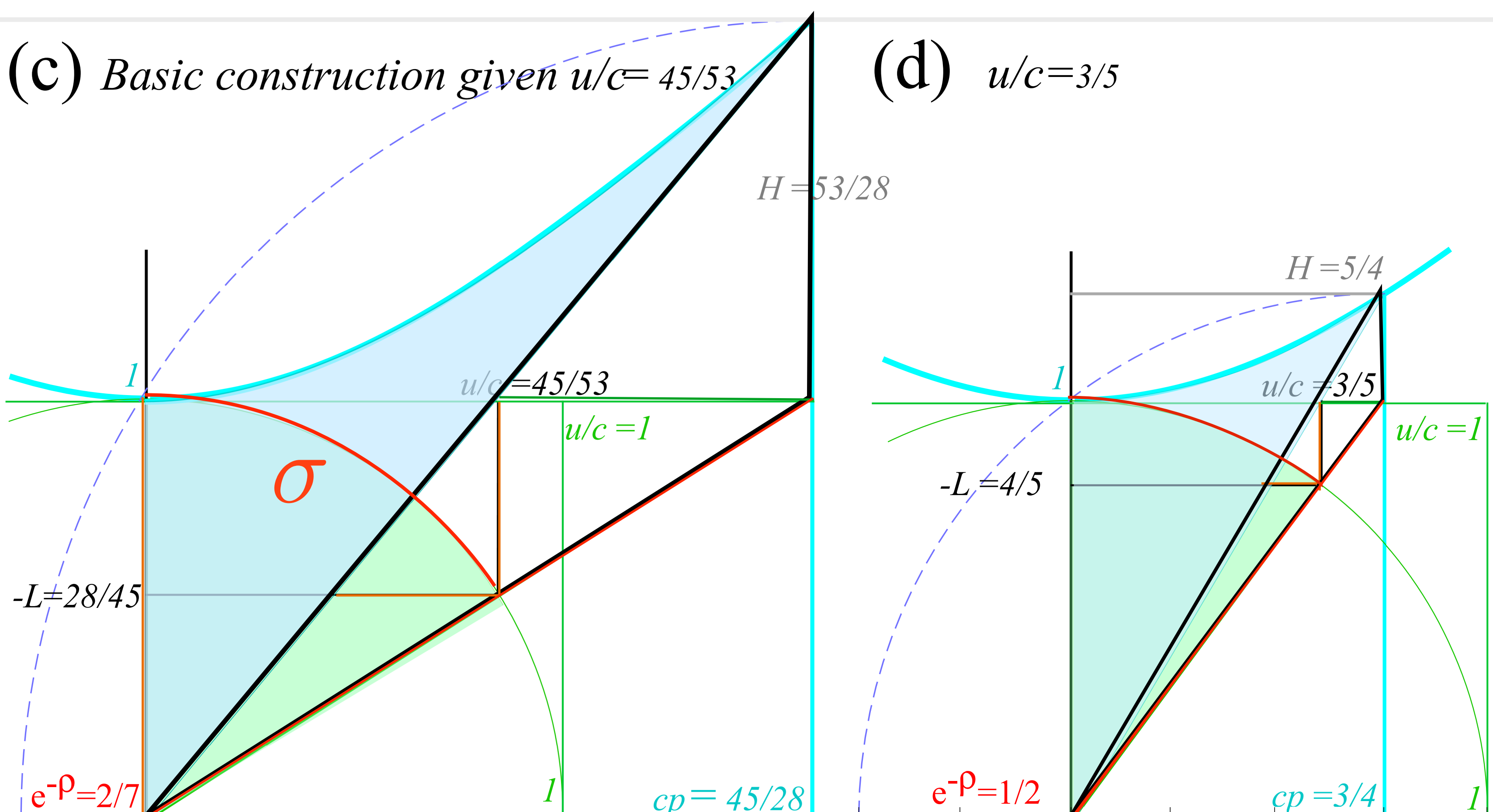
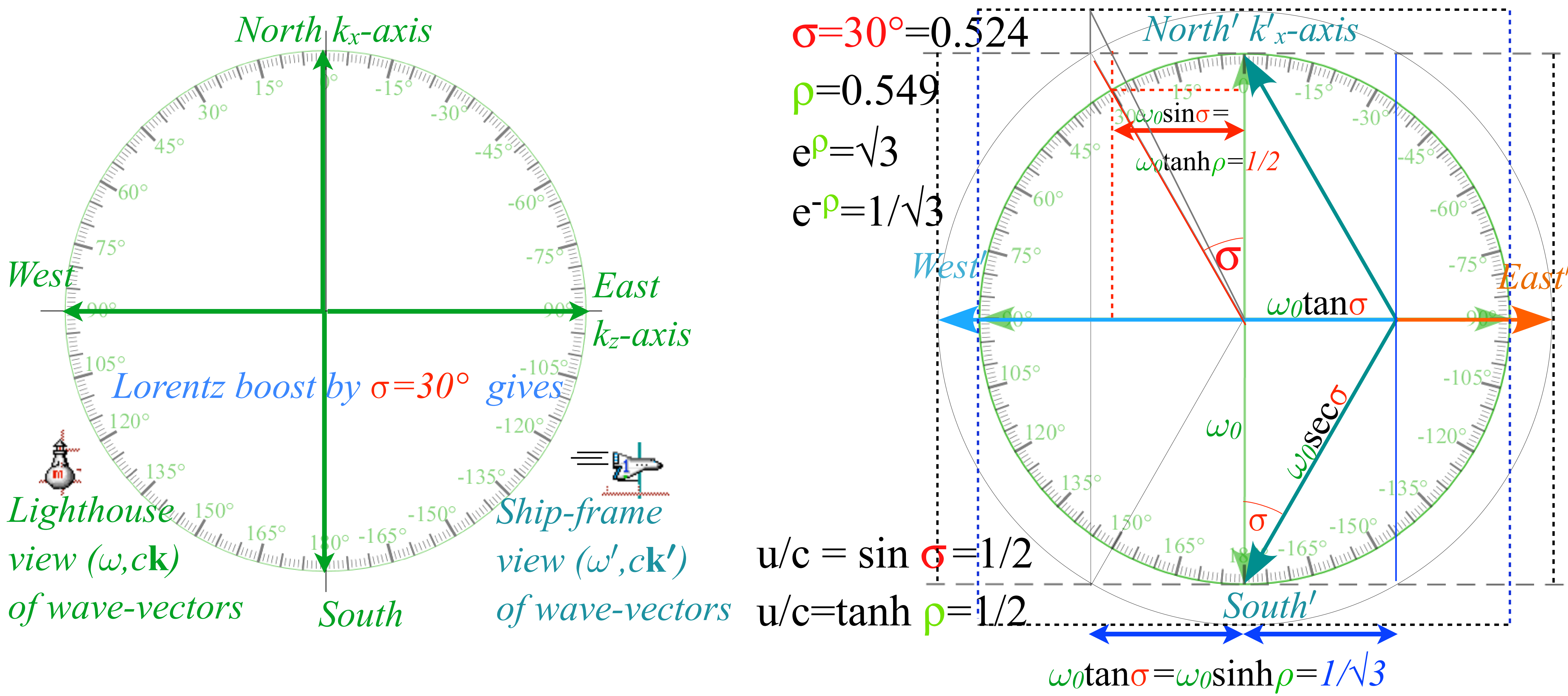


Fig. 5.5
Relativistic wave mechanics geometry.
(a) Overview.

(b-d) Details of contacting tangents.

Spectral details of Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

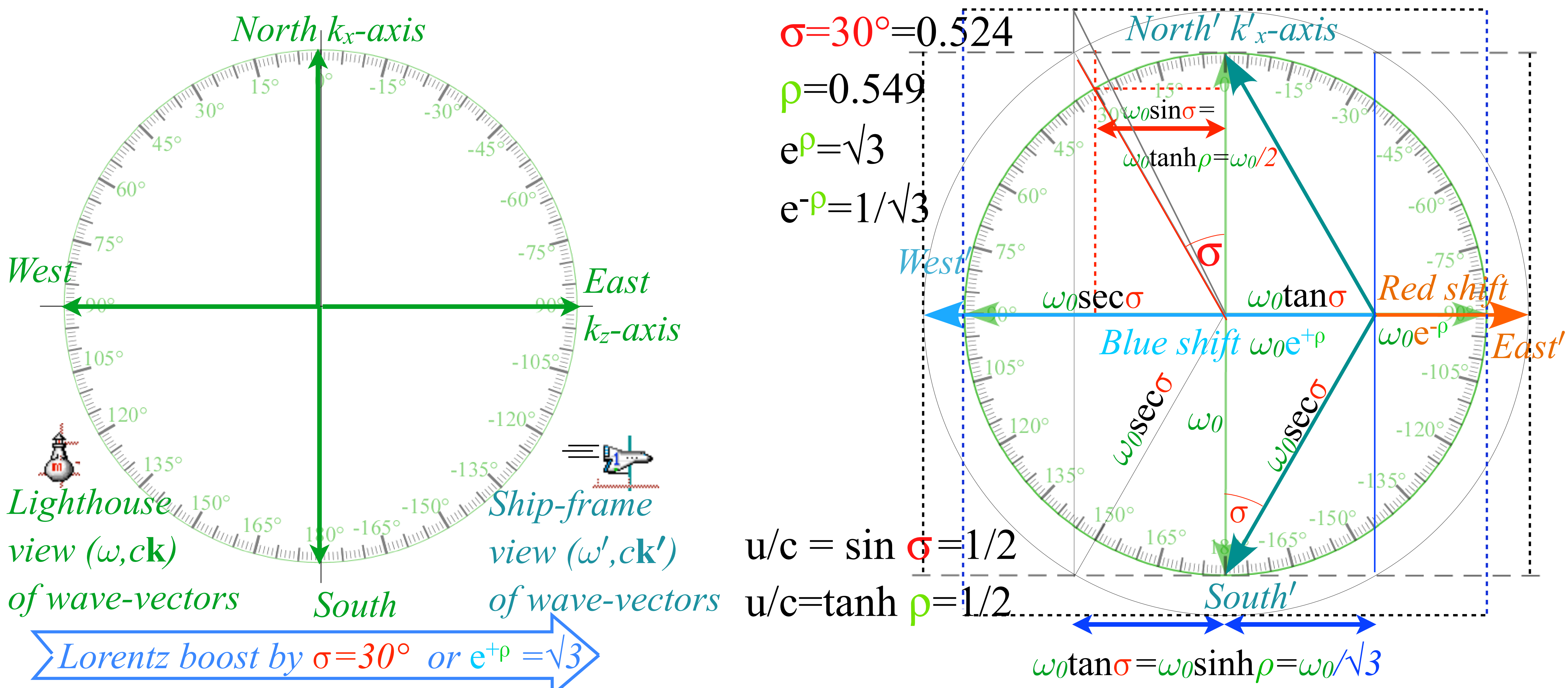


South starlight in lighthouse frame is straight down x-axis : $\left(\omega_{\downarrow},ck_{x\downarrow},ck_{y\downarrow},ck_{z\downarrow}\right)=\left(\omega_0,-\omega_0,0,0\right)$

$+\rho_z$-rapidity ship frame sees starlight Lorentz transformed to :$\left(\omega'_{\downarrow},ck'_{x\downarrow},ck'_{y\downarrow},ck'_{z\downarrow}\right)=\left(\omega_0\cosh\rho_z,-\omega_0,0,-\omega_0\sinh\rho_z\right)$

$$\begin{pmatrix}\omega'_{\downarrow}\\ ck'_{x\downarrow}\\ ck'_{y\downarrow}\\ ck'_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_{\downarrow}\\ ck_{x\downarrow}\\ ck_{y\downarrow}\\ ck_{z\downarrow}\end{pmatrix}=\begin{pmatrix}\cosh\rho_z & \cdot & \cdot & -\sinh\rho_z\\ \cdot & 1 & \cdot & \cdot\\ \cdot & \cdot & 1 & \cdot\\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z\end{pmatrix}\begin{pmatrix}\omega_0\\ -\omega_0\\ 0\\ 0\end{pmatrix}=\begin{pmatrix}\omega_0\cosh\rho_z\\ -\omega_0\\ 0\\ -\omega_0\sinh\rho_z\end{pmatrix}=\begin{pmatrix}\omega_0\sec\sigma\\ -\omega_0\\ 0\\ -\omega_0\tan\sigma\end{pmatrix}$$

Lecture 27 discusses Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0, \omega_x, \omega_y, \omega_z)$



For ship going $u=c\tanh\rho$ along z-axis

West starlight $(\omega_0, 0, 0, -\omega_0)$ is *blue shifted by* $e^{+\rho}=\cosh\rho+\sinh\rho$

$$\begin{pmatrix} \omega'_{\leftarrow} \\ ck'_{x\leftarrow} \\ ck'_{y\leftarrow} \\ ck'_{z\leftarrow} \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z \\ 0 \\ 0 \\ -\sinh\rho_z - \cosh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 e^{+\rho_z} \\ 0 \\ 0 \\ -\omega_0 e^{+\rho_z} \end{pmatrix}$$

and *East* starlight $(\omega_0, 0, 0, +\omega_0)$ is *red shifted by* $e^{-\rho}=\cosh\rho-\sinh\rho$

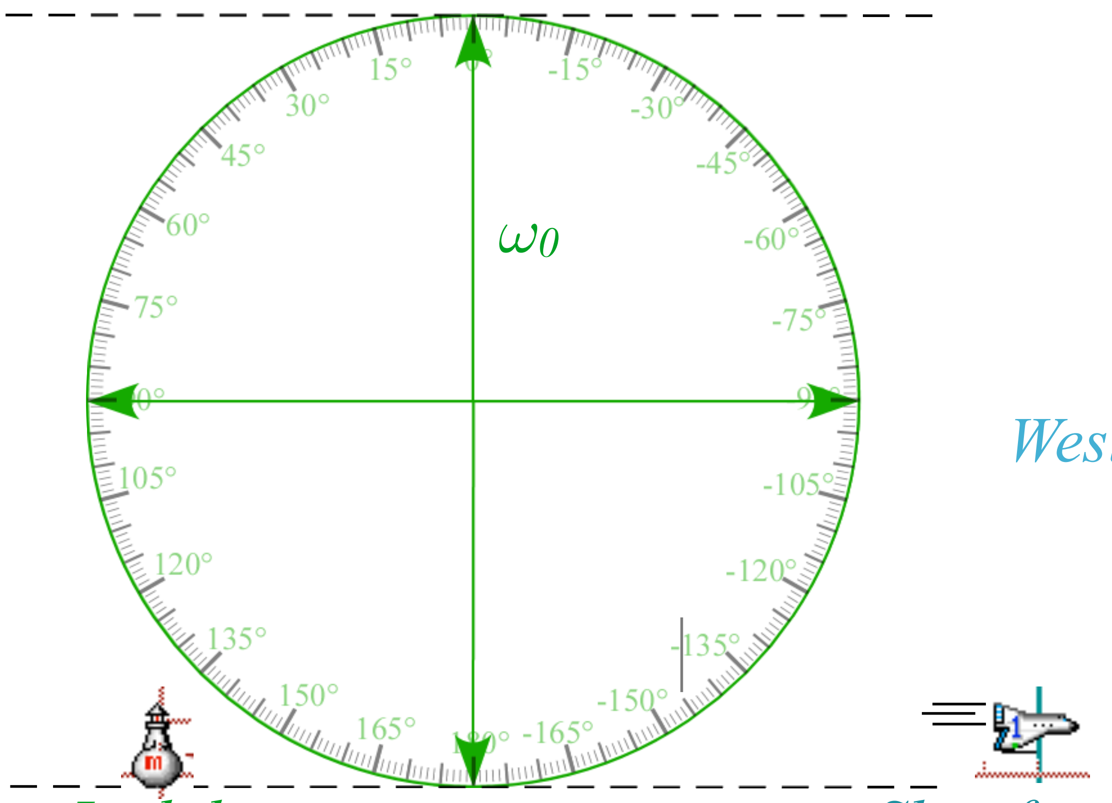
$$\begin{pmatrix} \omega'_{\rightarrow} \\ ck'_{x\rightarrow} \\ ck'_{y\rightarrow} \\ ck'_{z\rightarrow} \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z - \sinh\rho_z \\ 0 \\ 0 \\ -\sinh\rho_z + \cosh\rho_z \end{pmatrix} = \begin{pmatrix} \omega_0 e^{-\rho_z} \\ 0 \\ 0 \\ -\omega_0 e^{-\rho_z} \end{pmatrix}$$

Blue shift factor is $e^{+\rho}=\cosh\rho+\sinh\rho=\sec\sigma+\tan\sigma$

Red shift factor is $e^{-\rho}=\cosh\rho-\sinh\rho=\sec\sigma-\tan\sigma$

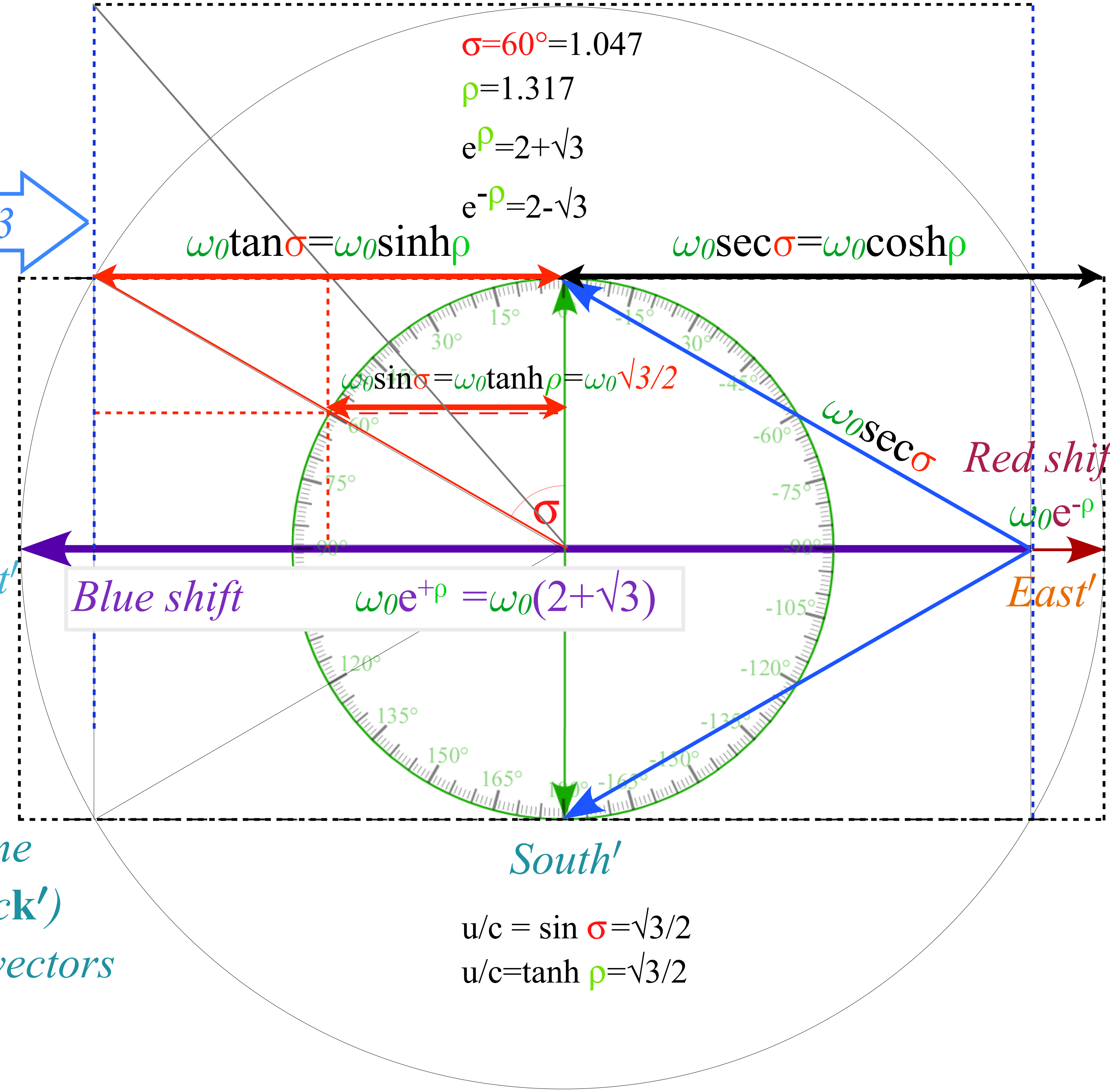
Faster Lorentz boost of
North-South-East-West
plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$

Lorentz boost by $\sigma=60°$ *or* $e^{+\rho}=2+\sqrt{3}$



Lighthouse view $(\omega, c\mathbf{k})$ *of wave-vectors*

Ship-frame view $(\omega', c\mathbf{k}')$ *of wave-vectors*



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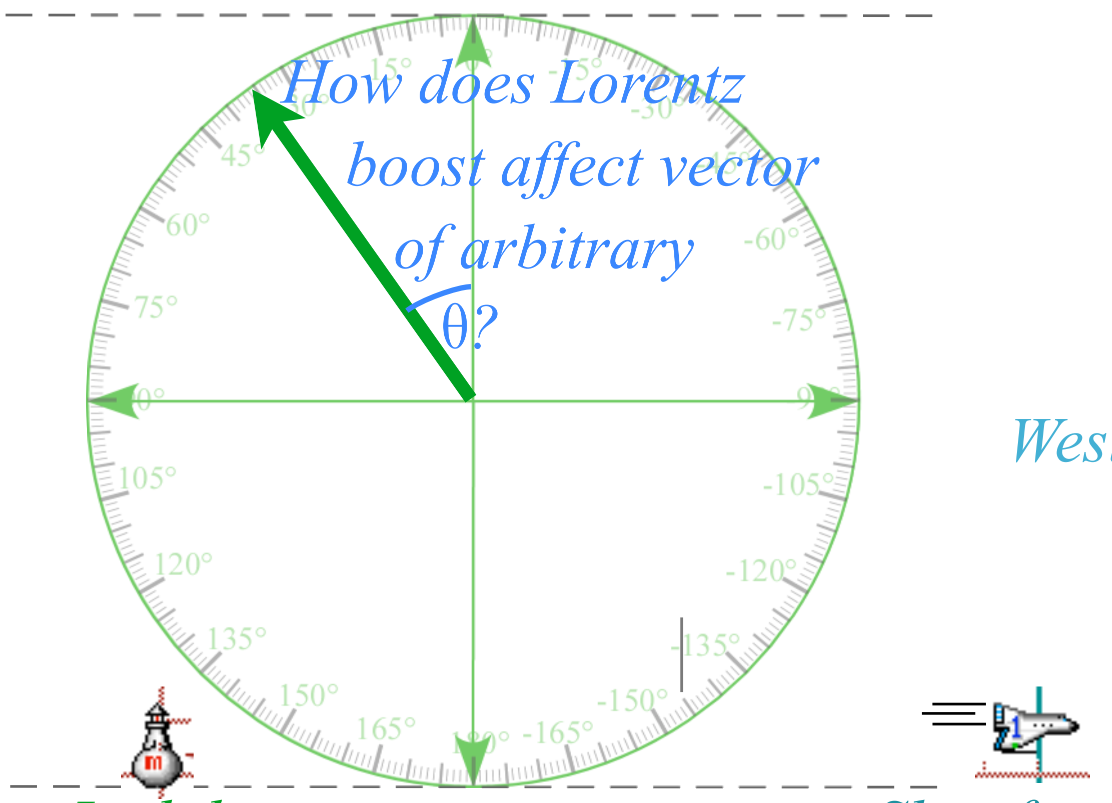

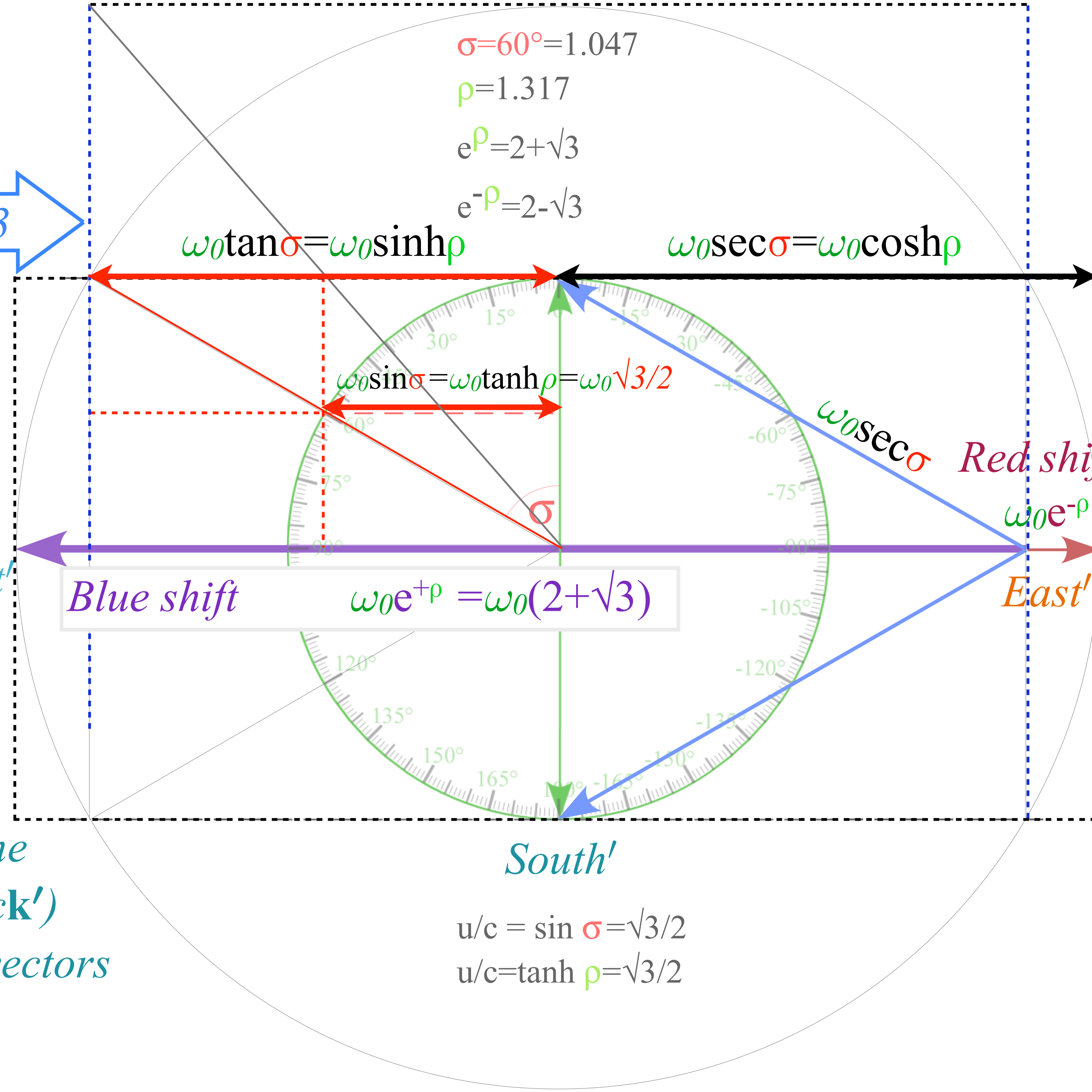

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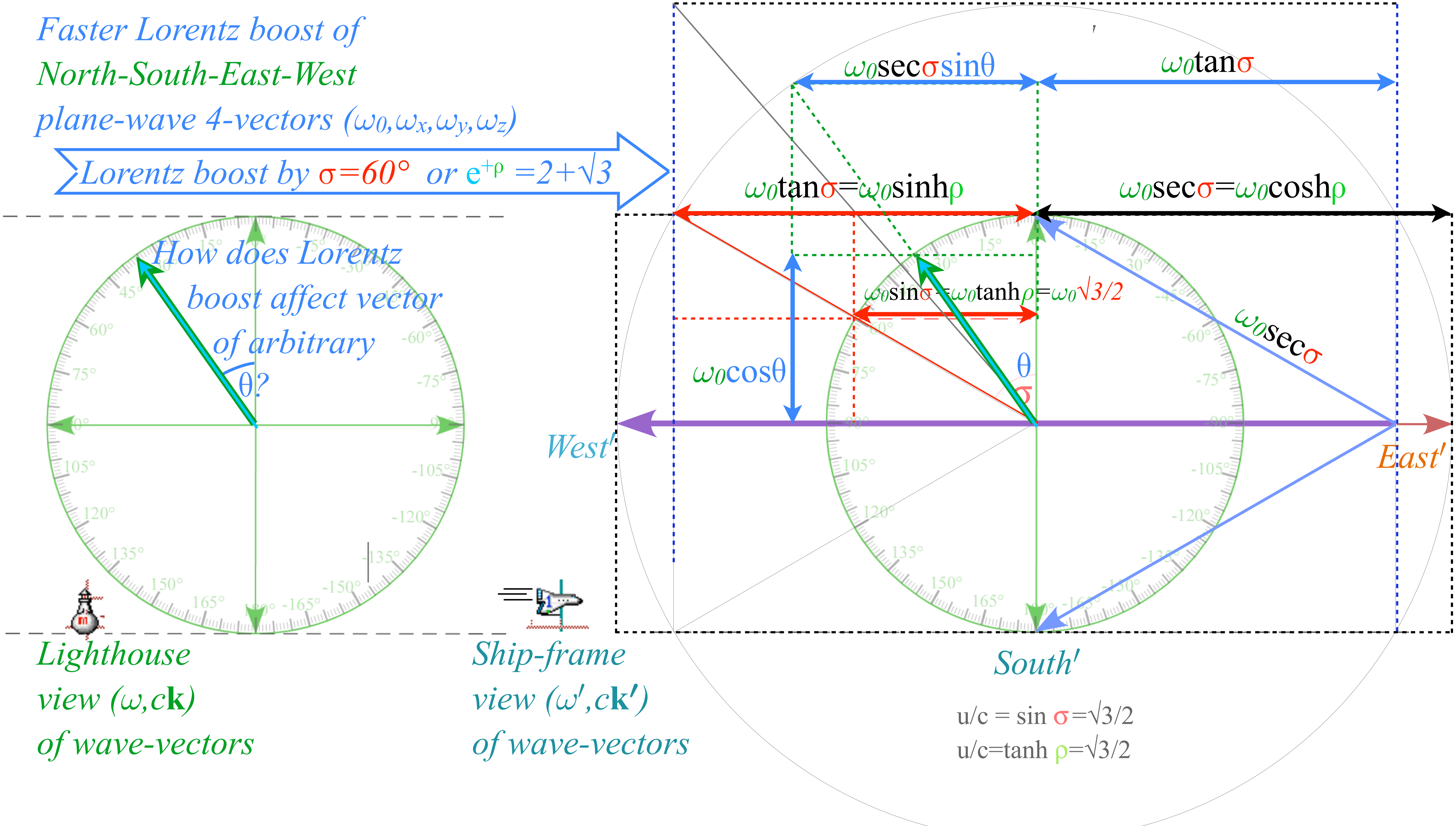
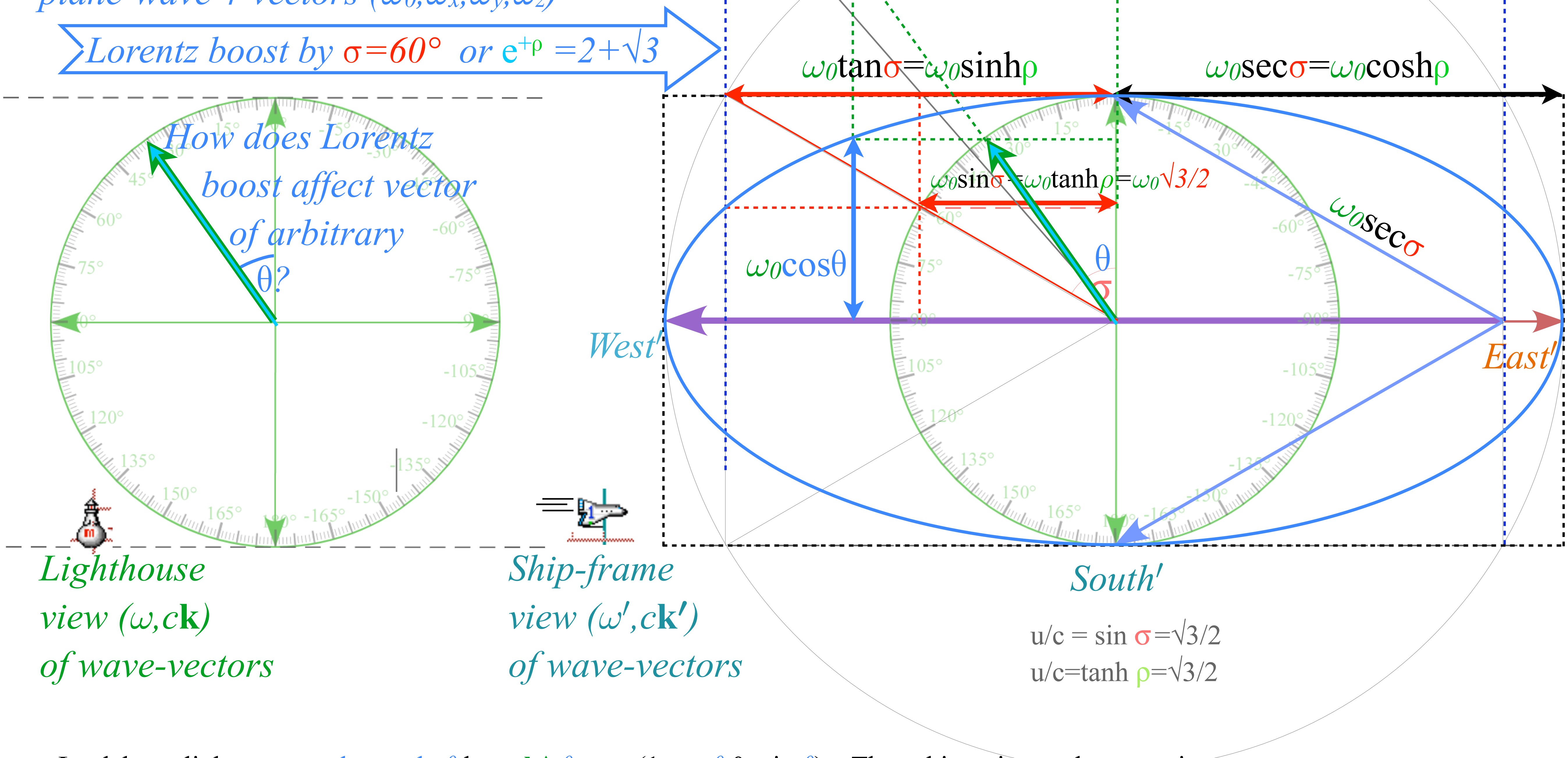


Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0\,(1, \cos\theta, 0, -\sin\theta)$. Then ship going $\mathbf{u}$ along z-axis sees :

$$\begin{pmatrix} \omega'_{\uparrow\theta} \\ ck'_{x\uparrow\theta} \\ ck'_{y\uparrow\theta} \\ ck'_{z\uparrow\theta} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_0\cos\theta \\ 0 \\ -\omega_0\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z\sin\theta \\ \cos\theta \\ 0 \\ -\sinh\rho_z - \cosh\rho_z\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \sec\sigma + \tan\sigma\sin\theta \\ \cos\theta \\ 0 \\ -\tan\sigma - \sec\sigma\sin\theta \end{pmatrix}$$

Faster Lorentz boost of North-South-East-West plane-wave 4-vectors $(\omega_0,\omega_x,\omega_y,\omega_z)$



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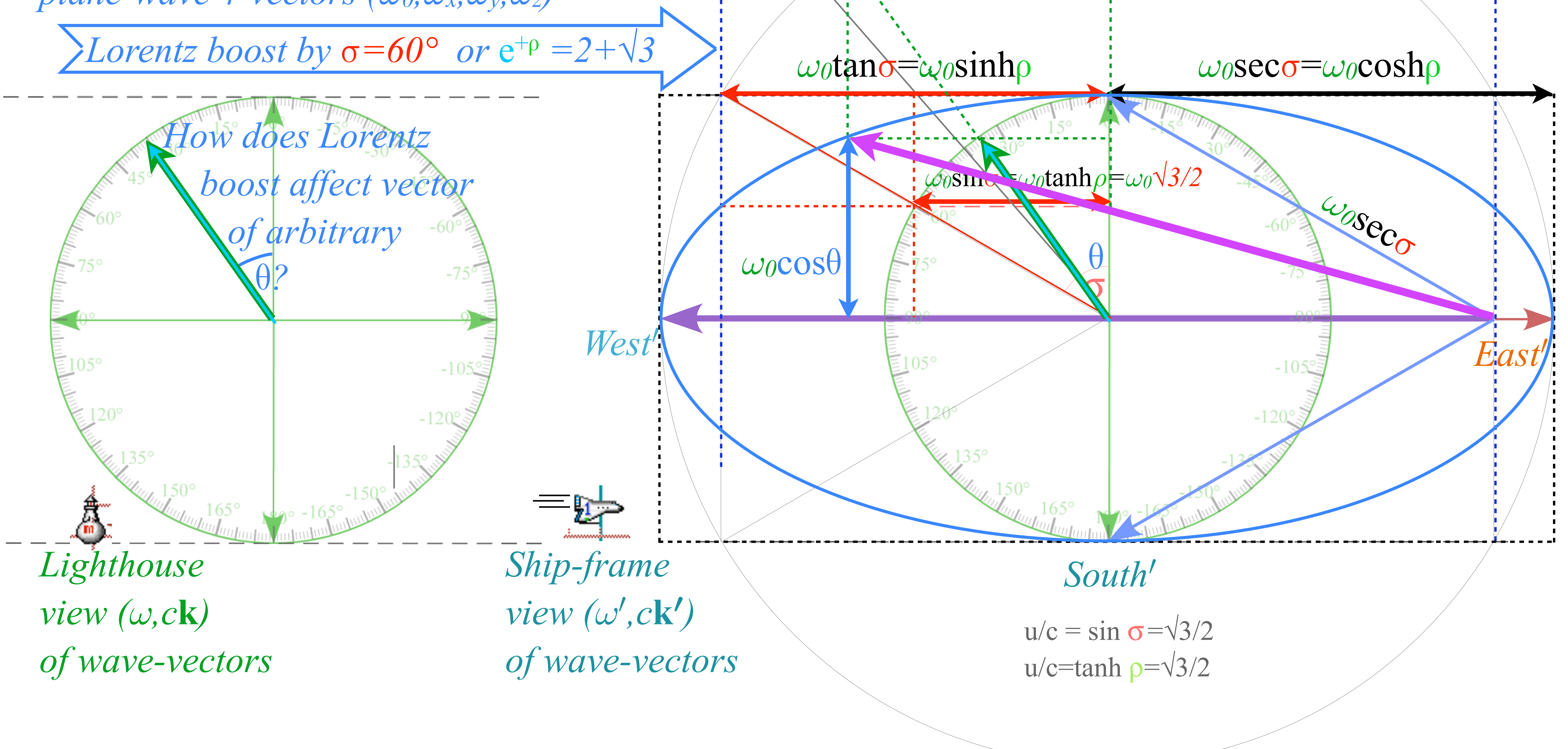
North-South-East-West
plane-wave 4-vectors $(\omega_0, \omega_x, \omega_y, \omega_z)$



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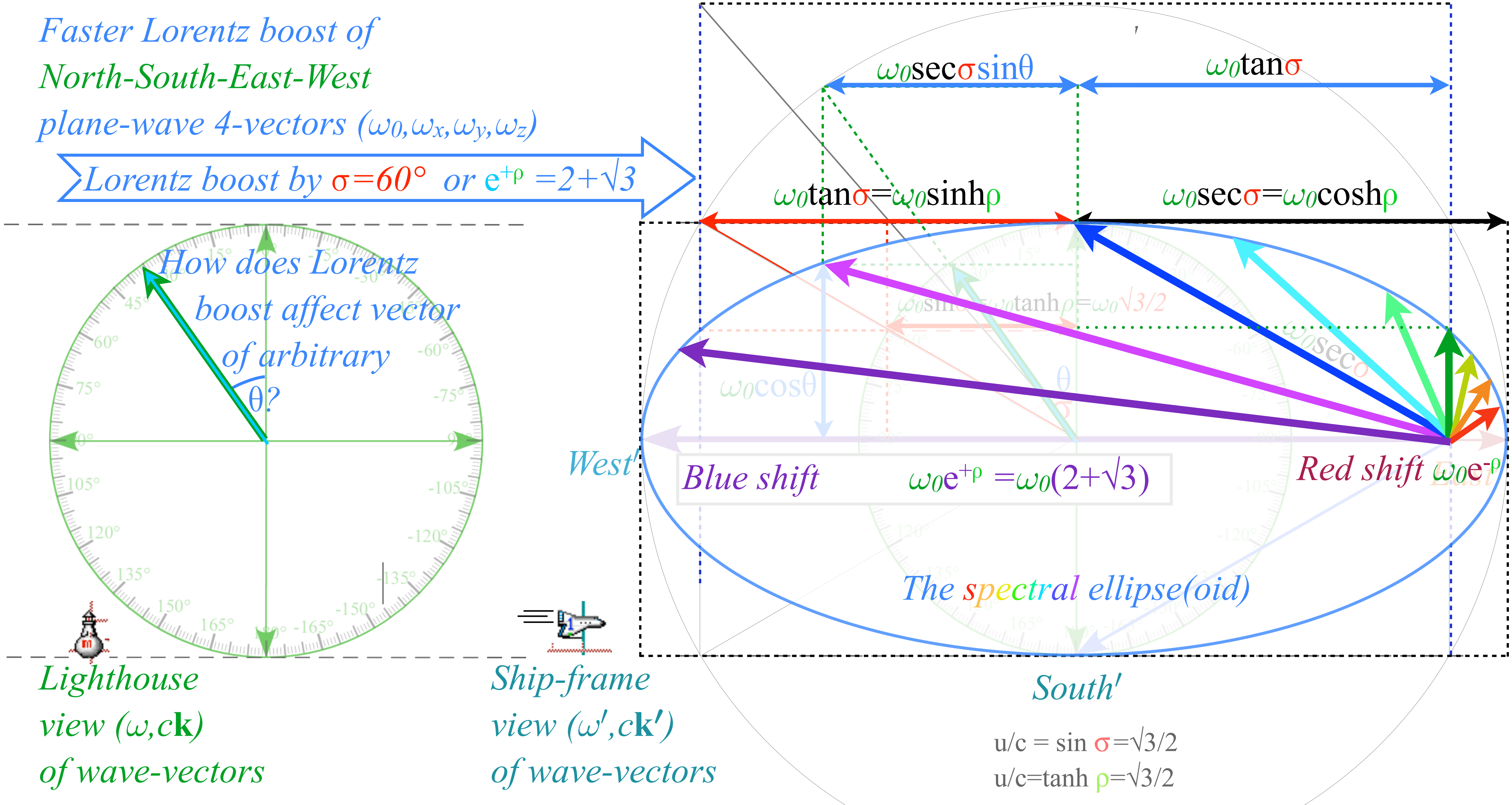
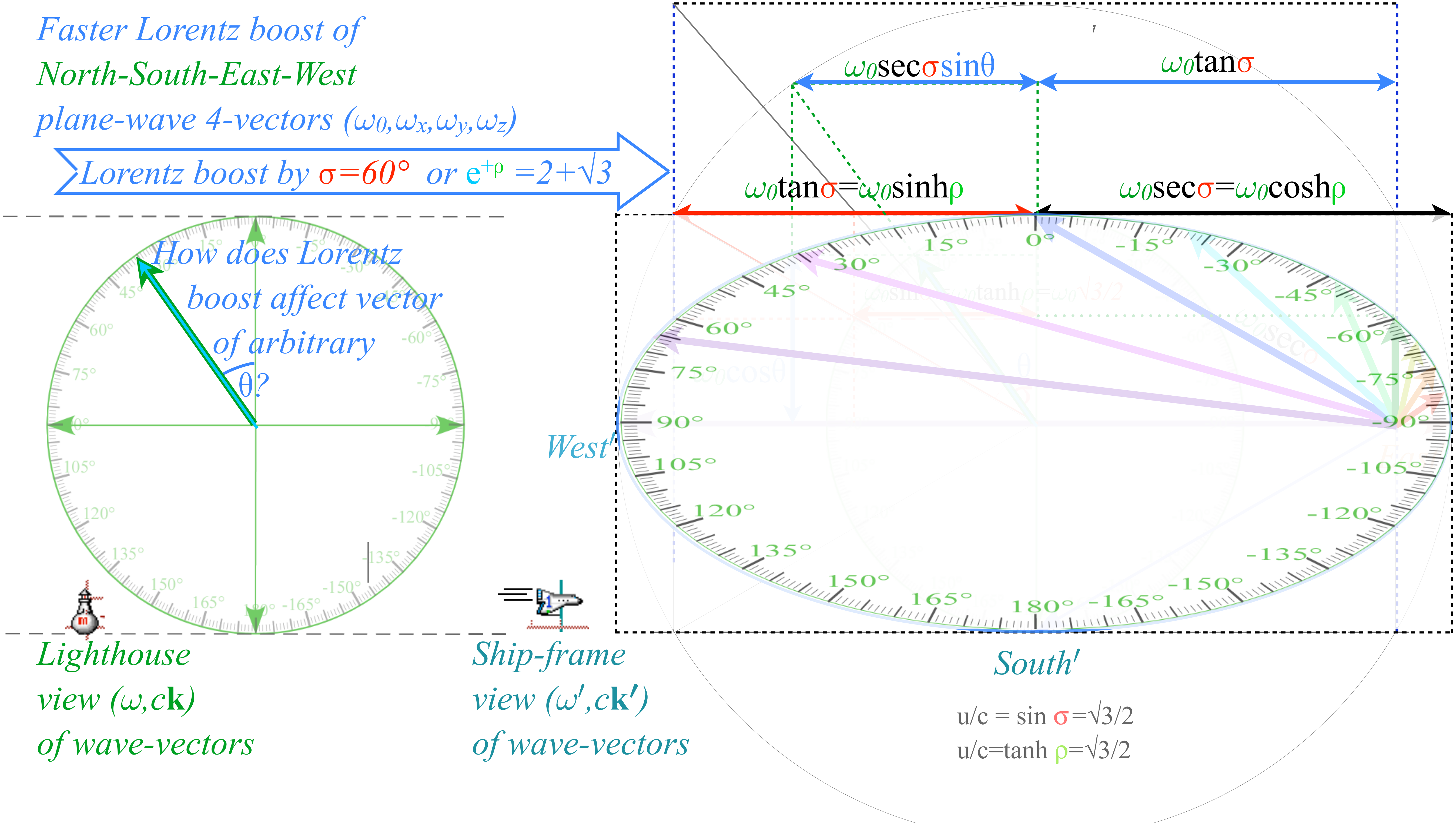


Let lab starlight ray at polar angle θ have $\mathbf{k}\uparrow\theta = \omega_0(1,\cos\theta,0,-\sin\theta)$. Then ship going **u** along z-axis sees :

$$\begin{pmatrix} \omega'_{\uparrow\theta} \\ ck'_{x\uparrow\theta} \\ ck'_{y\uparrow\theta} \\ ck'_{z\uparrow\theta} \end{pmatrix} = \begin{pmatrix} \cosh\rho_z & \cdot & \cdot & -\sinh\rho_z \\ \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot \\ -\sinh\rho_z & \cdot & \cdot & \cosh\rho_z \end{pmatrix} \begin{pmatrix} \omega_0 \\ \omega_0\cos\theta \\ 0 \\ -\omega_0\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \cosh\rho_z + \sinh\rho_z\sin\theta \\ \cos\theta \\ 0 \\ -\sinh\rho_z - \cosh\rho_z\sin\theta \end{pmatrix} = \omega_0 \begin{pmatrix} \sec\sigma + \tan\sigma\sin\theta \\ \cos\theta \\ 0 \\ -\tan\sigma - \sec\sigma\sin\theta \end{pmatrix}$$



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Review of geometric construction , per-space-time (ω,ck) dispersion hyperbola ω = Bcoshρ...

A quick flip to space-time *(ct,x)* construction: Minkowski coordinate grid

Lorentz transformations of Phase vector $\mathbf{P'}$ and Group vector $\mathbf{G'}$ in per-space-time

Lorentz matrix transformation of *(x,ct)* space-time coordinates

Two Famous-Name Coefficients: Lorentz space contraction and Einsein time dilation

Heighway Paradoxes: A relativistic "*He said-She-said...*" argument

Phase invariance…derives Lorentz transformations...and vice-versa

Another view of phas*or*-invariance

Geometry of invariant hyperbolas

Algebra of invariant hyperbolas

Proper time τ_0 and proper frequency ω_0

A politically incorrect analogy of rotation to Lorentz transformation

➔ Yet another view: The Epstein space-proper-time approach to SR uses stellar aberration angle σ

Relating rapidity ρ to stellar aberration angle σ and circular or hyperbolic arc-area

Each circular trig function has a hyperbolic "country-cousin" function

Ship vs Lighthouse sagas and the Bureau of Inter-Galactic Aids to Navigation at Night (Our 1st *RelativIt* animations).

Relating Longitudinal relativity parameter: Rapidity $\rho = \log_e$(Doppler Shift)

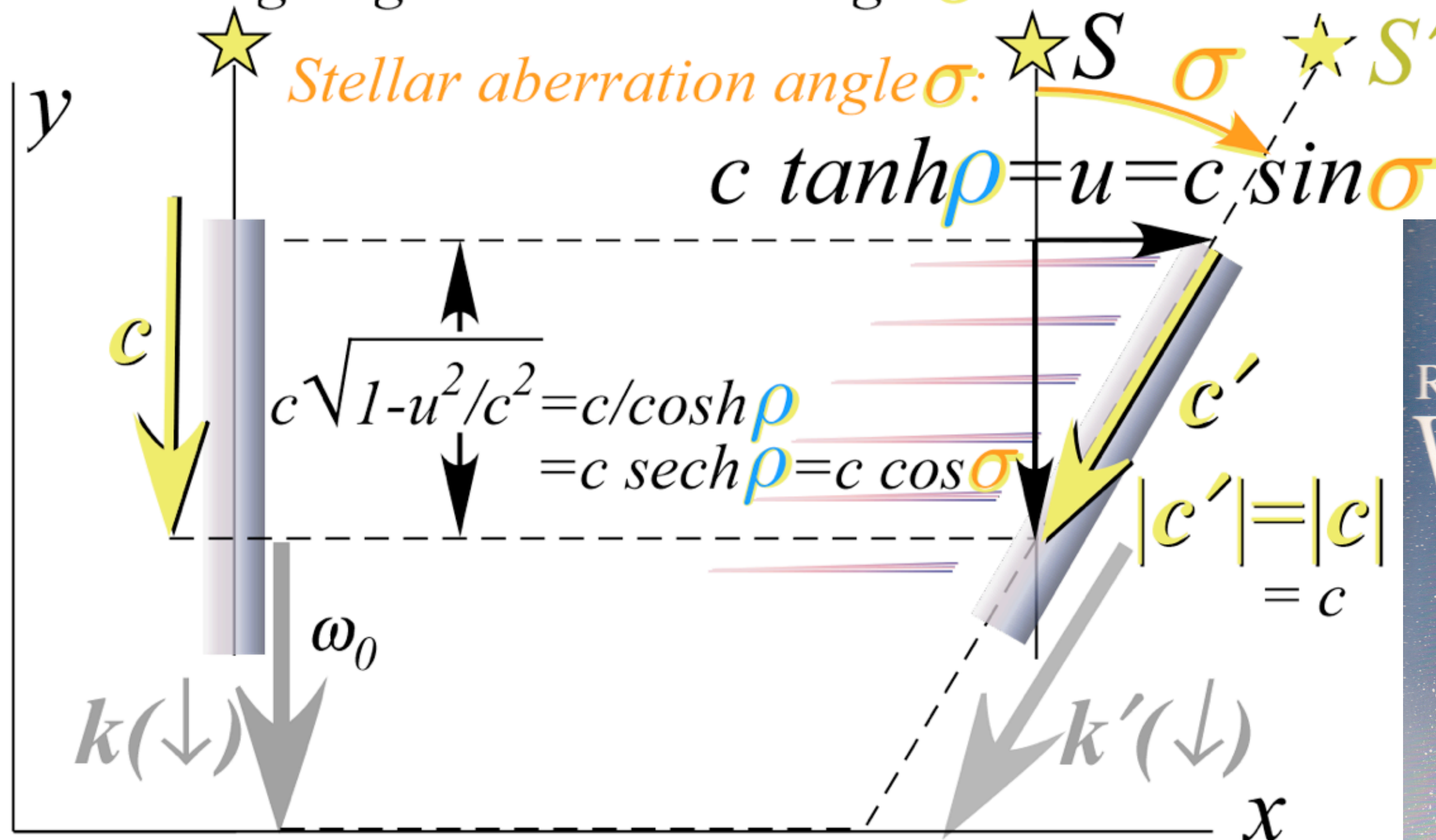
to a Transverse relativity parameter*: Stellar aberration angle σ

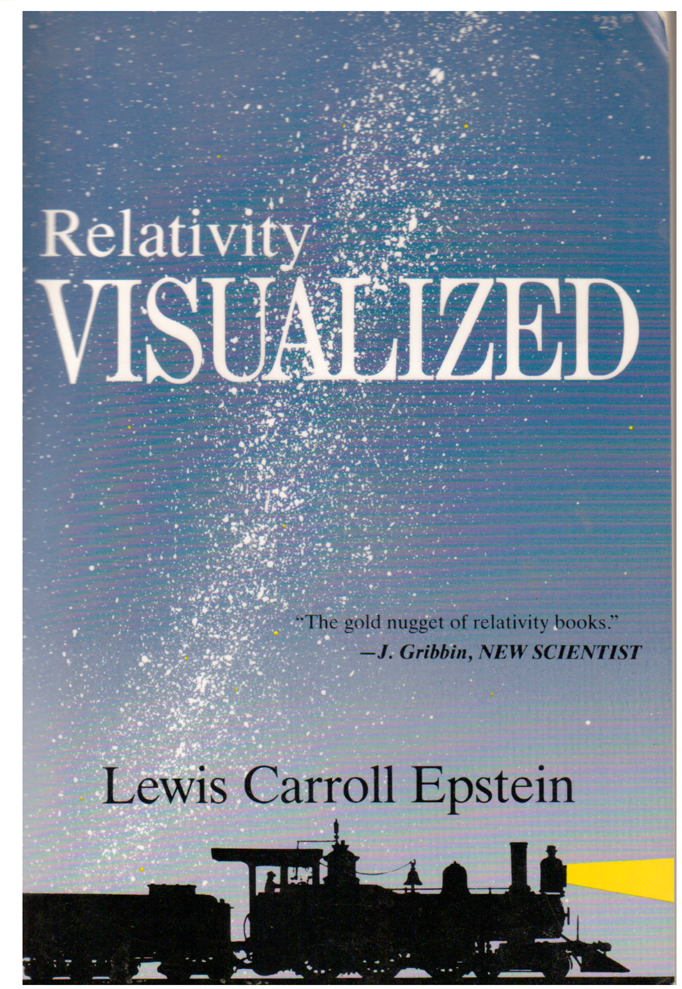
*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-

Observer fixed below star sees it directly overhead.

*Observer going **u** sees star at angle σ in **u** direction.*

We used notion σ for stellar-ab-angle, (a "flipped-out" ρ).





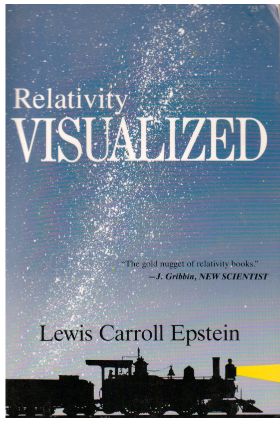
Epstein seemed uninterested in ρ analysis or in relation of σ and ρ.

Purchase at: AllBookstores.com amazon

Relating Longitudinal relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

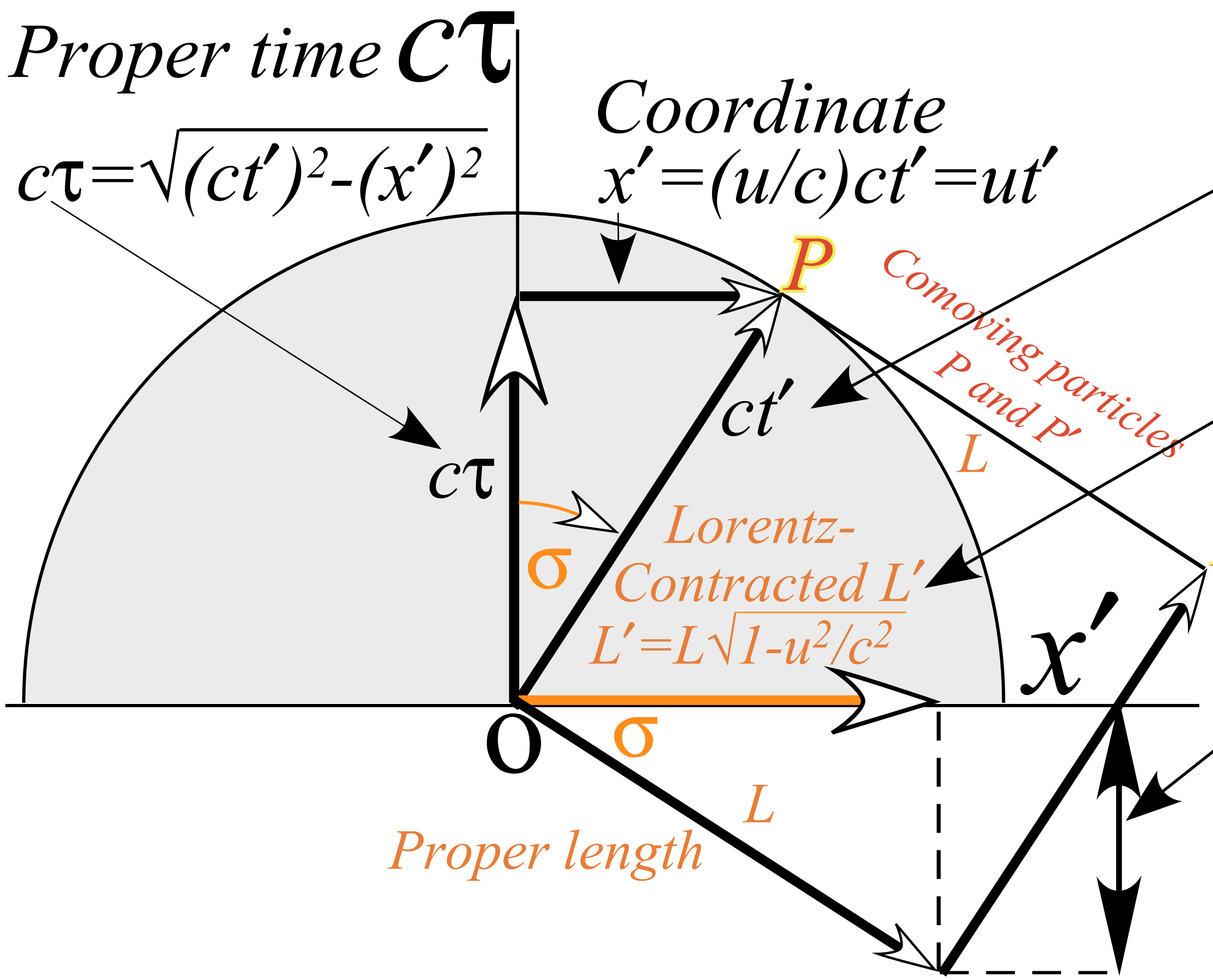
to Transverse relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



Proper time cτ vs. coordinate space x - (L. C. Epstein's "Cosmic Speedometer")

Particles P and P′ have speed u in (x′,ct′) and speed c in (x, cτ)



Einstein time dilation:

$$ct'=c\tau\sec\sigma=c\tau\cosh\rho = c\tau/\sqrt{1-u^2/c^2}$$

Lorentz length contraction:

$$L'= L\,\mathrm{sech}\rho = L\cos\sigma = L\cdot\sqrt{1-u^2/c^2}$$

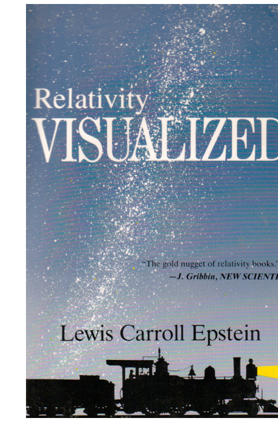
Proper Time asimultaneity:

$$c\,\Delta\tau = L'\sinh\rho = L\cos\sigma\sinh\rho$$
$$= L\cos\sigma\tan\sigma$$
$$= L\sin\sigma \quad = L/\sqrt{c^2/u^2-1} \sim L\,u/c$$

Relating <u>Longitudinal</u> relativity parameter: Rapidity ρ = $\log_e$(Doppler Shift)

to <u>Transverse</u> relativity parameter: Stellar aberration angle σ

*Lewis Carroll Epstein, ***Relativitätstheorie***, Birkhäuser, (2004) Earlier English version (1985)-



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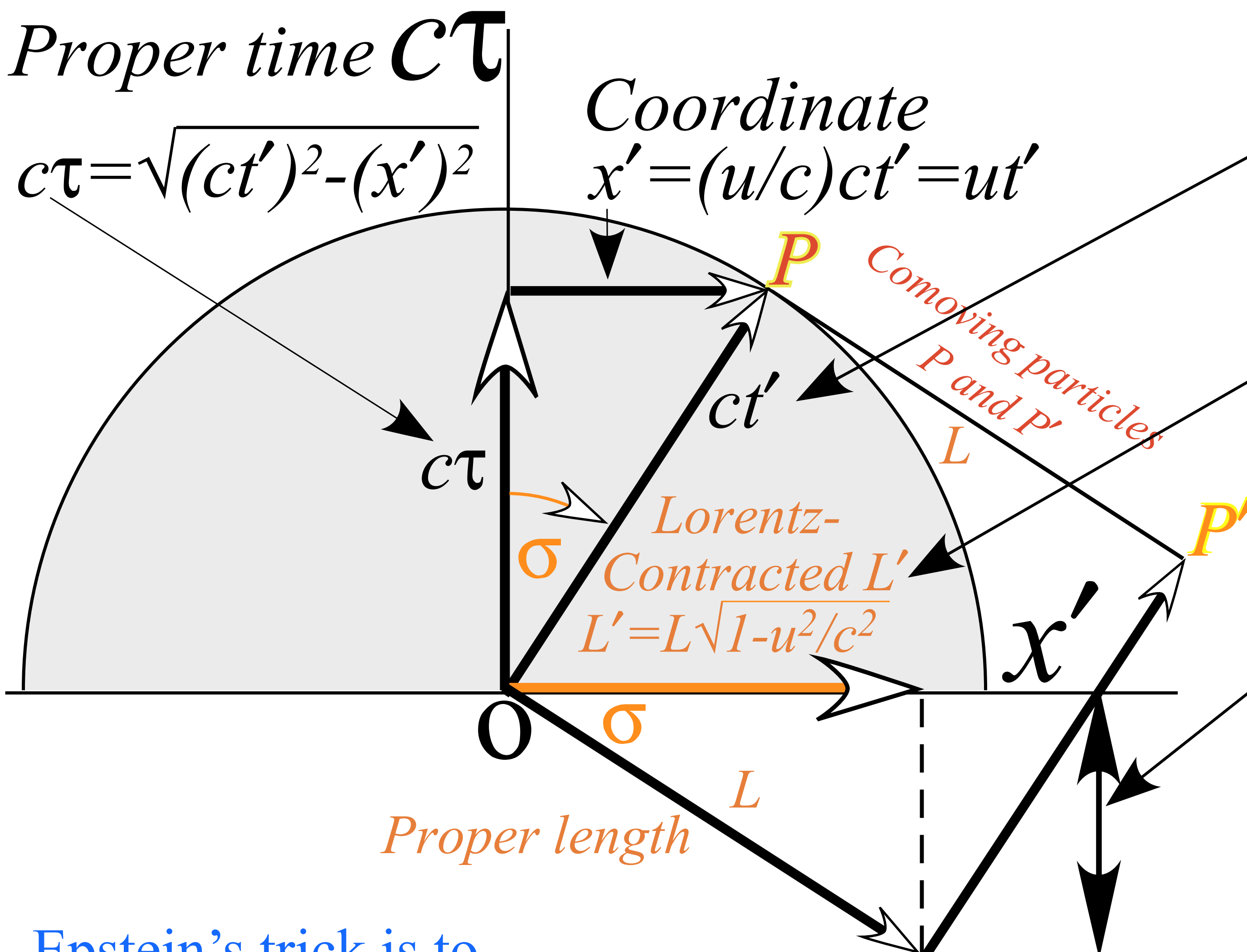

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Epstein's trick is to turn a hyperbolic form $c\tau = \sqrt{(ct')^2 - (x')^2}$

into a circular form: $\sqrt{(c\tau)^2 + (x')^2} = (ct')$

Then everything (and everybody) always goes speed c through $(x', c\tau)$ space!

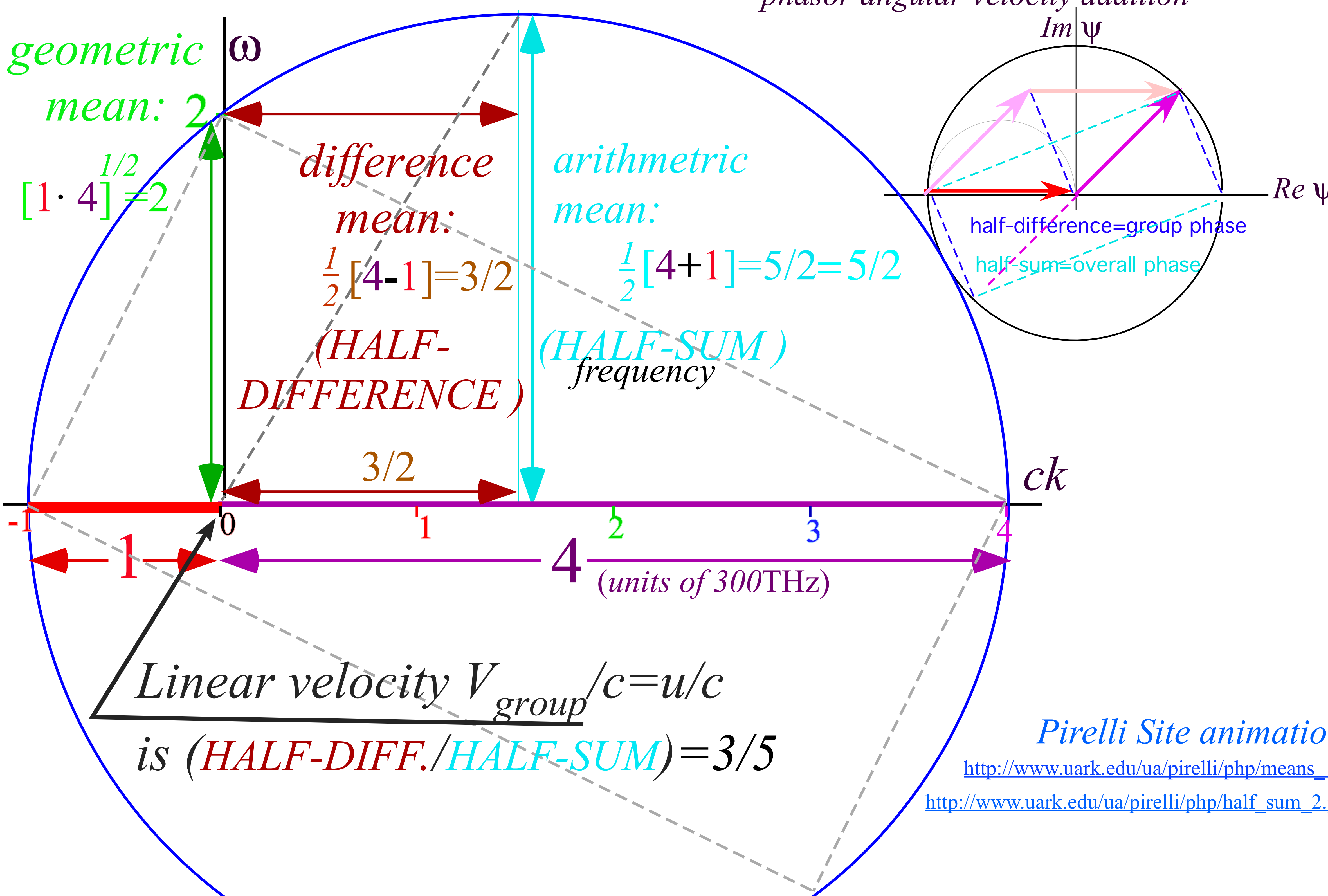
Geometry of invariant hyperbolas

Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition



Pirelli Site animations:

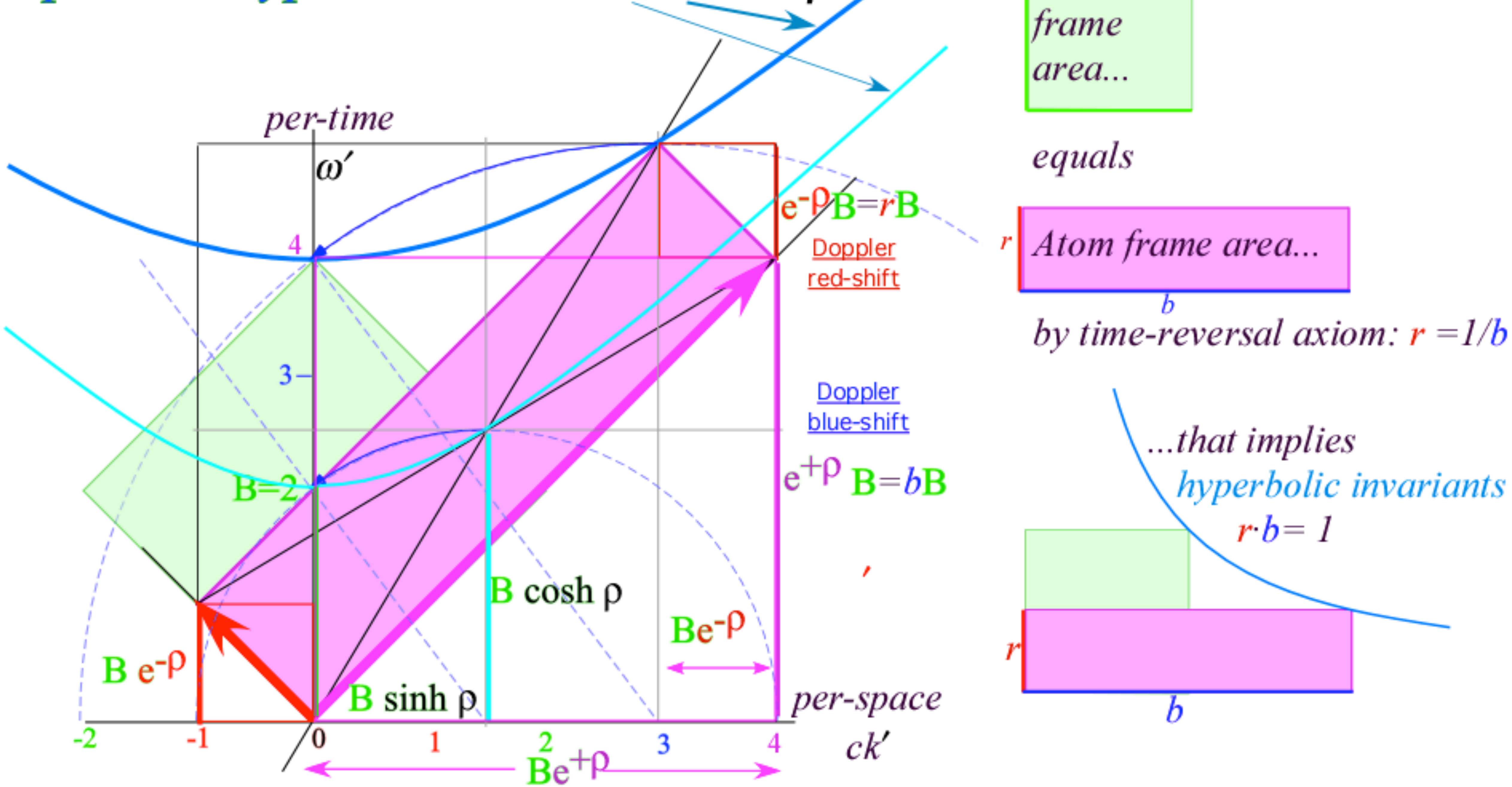
http://www.uark.edu/ua/pirelli/php/means_1.php

http://www.uark.edu/ua/pirelli/php/half_sum_2.php

Fig. 10a Euclidian mean geometry for counter-moving waves of frequency 1 and 4. (300THz units).

Geometry of invariant hyperbolas

Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas: $\omega = n B \cosh \rho$



$$B \sinh \rho = (B e^{+\rho} - B e^{-\rho})/2 \qquad B \cosh \rho = (B e^{+\rho} + B e^{-\rho})/2$$

Time $r=1/b$ symmetry shows geometry of 2-CW grid transformation that leaves hyperbolas invariant.

http://www.uark.edu/ua/modphys/markup/RelaWavityWeb.html?plotType=3l5&minkGridPosCells=2

Algebra of invariant hyperbolas: Proper time τ_0 and proper frequency ω_0

$$\begin{pmatrix} ck \\ \omega \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} ck' \\ \omega' \end{pmatrix} \qquad \begin{pmatrix} x \\ ct \end{pmatrix} = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x' \\ ct' \end{pmatrix}$$

Hyperbolic invariants to Lorentz transformation

Per-space-time invariant:

$$\omega_0^2 = \omega^2 - (ck)^2 = \omega'^2 - (ck')^2$$

ω_0 is called "proper frequency" or rate of "aging"

$$\omega_0 = \omega\sqrt{1 - \frac{k^2}{(c\omega)^2}} = \omega'\sqrt{1 - \frac{k'^2}{(c\omega')^2}}$$

$$= \omega\sqrt{1 - \frac{u^2}{c^2}} = \omega'\sqrt{1 - \frac{u'^2}{c^2}}$$

Space-time invariant:

$$(c\tau_0)^2 = (ct)^2 - x^2 = (ct')^2 - (x')^2$$

τ_0 is called "proper time" or "age":

$$\tau_0 = t\sqrt{1 - \frac{x^2}{(ct)^2}} = t'\sqrt{1 - \frac{x'^2}{(ct')^2}}$$

$$= t\sqrt{1 - \frac{u^2}{c^2}} = t'\sqrt{1 - \frac{u'^2}{c^2}}$$

The "grand-daddy-of 'em all" invariant

Phase invariance:

$$\Phi_0 = k\cdot x - \omega\cdot t = k'\cdot x' - \omega'\cdot t'$$

$$ck\cdot x\cdot\cosh^2 - ck\cdot x\cdot\sinh^2 = ck\cdot x$$

$$\omega\cdot ct\cdot\sinh^2 - \omega\cdot ct\cdot\cosh^2 = -\omega\cdot ct$$

Proof: $ck' \cdot x' \quad - \quad \omega' \cdot ct'$

	$x\cdot\cosh$	$ct\cdot\sinh$		$x\cdot\sinh$	$ct\cdot\cosh$
$ck\cdot\cosh$	$ck\cdot x\cdot\cosh^2$	~~$ck\cdot ct\cdot\cosh\cdot\sinh$~~	$ck\cdot\sinh$	$ck\cdot x\cdot\sinh^2$	~~$ck\cdot ct\cdot\sinh\cdot\cosh$~~
$\omega\cdot\sinh$	~~$\omega\cdot x\cdot\sinh\cdot\cosh$~~	$\omega\cdot ct\cdot\sinh^2$	$\omega\cdot\cosh$	~~$\omega\cdot x\cdot\cosh\cdot\sinh$~~	$\omega\cdot ct\cdot\cosh^2$

A politically incorrect analogy of rotation to Lorentz transformation

Fig. 2.B.1 Town map according to a "tipsy" surveyor.

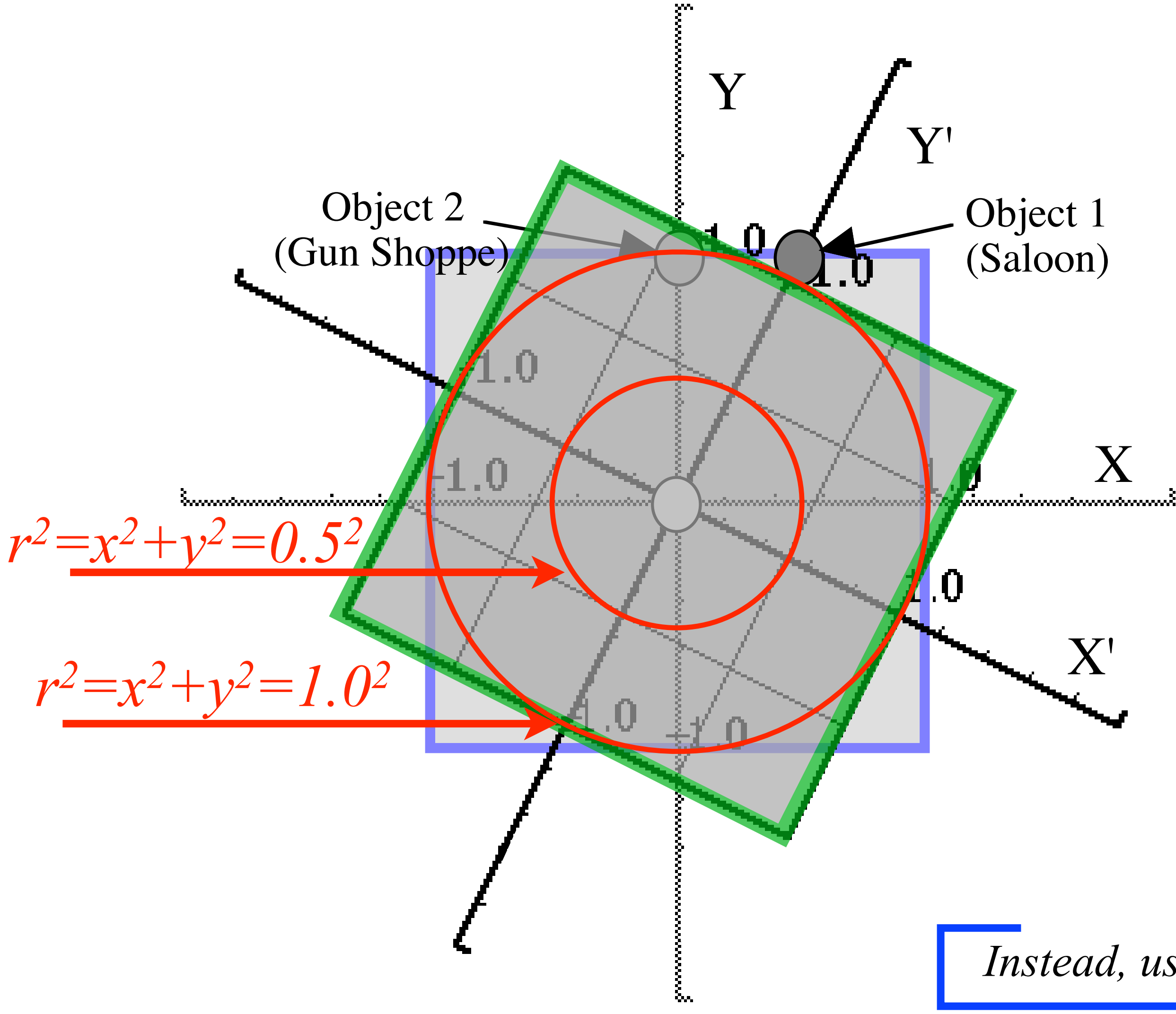
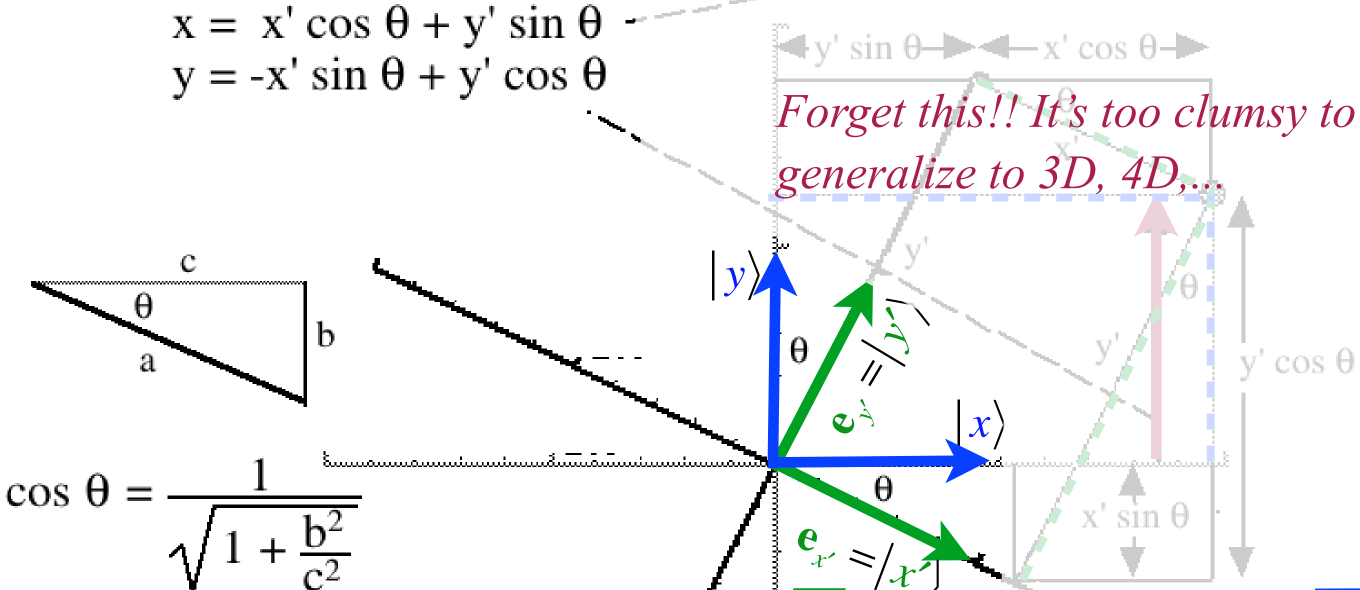


Fig. 2.B.2 Diagram and formulas for reconciliation of the two surveyor's data.

Reminder: Component-based derivation is clumsy!

$$x = x' \cos\theta + y' \sin\theta$$
$$y = -x' \sin\theta + y' \cos\theta$$

Forget this!! It's too clumsy to generalize to 3D, 4D,...



$$\cos\theta = \frac{1}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$\sin\theta = \frac{b/c}{\sqrt{1+\frac{b^2}{c^2}}}$$

Instead, use Dirac unit vectors $|x\rangle, |y\rangle$ *and* $|x'\rangle, |y'\rangle$

$$\mathbf{e}_{x'} = |x'\rangle = \cos\theta|x\rangle - \sin\theta|y\rangle$$
$$\mathbf{e}_{y'} = |y'\rangle = \sin\theta|x\rangle + \cos\theta|y\rangle$$

or the inverse relation:

$$\mathbf{e}_{x} = |x\rangle = \cos\theta|x'\rangle + \sin\theta|y'\rangle$$
$$\mathbf{e}_{y} = |y\rangle = -\sin\theta|x'\rangle + \cos\theta|y'\rangle$$

Object 0: Town Square.	Object 1: Saloon.	Object 2: Gun Shoppe.
(US surveyor) $x = 0$, $y = 0$	$x = 0.5$, $y = 1.0$	$x = 0$, $y = 1.0$
(2nd surveyor) $x' = 0$, $y' = 0$	$x' = 0$, $y' = 1.1$	$x' = -0.45$, $y' = 0.89$

You may apply (Jacobian) transform matrix:

$$\begin{pmatrix} \langle x|x'\rangle & \langle x|y'\rangle \\ \langle y|x'\rangle & \langle y|y'\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

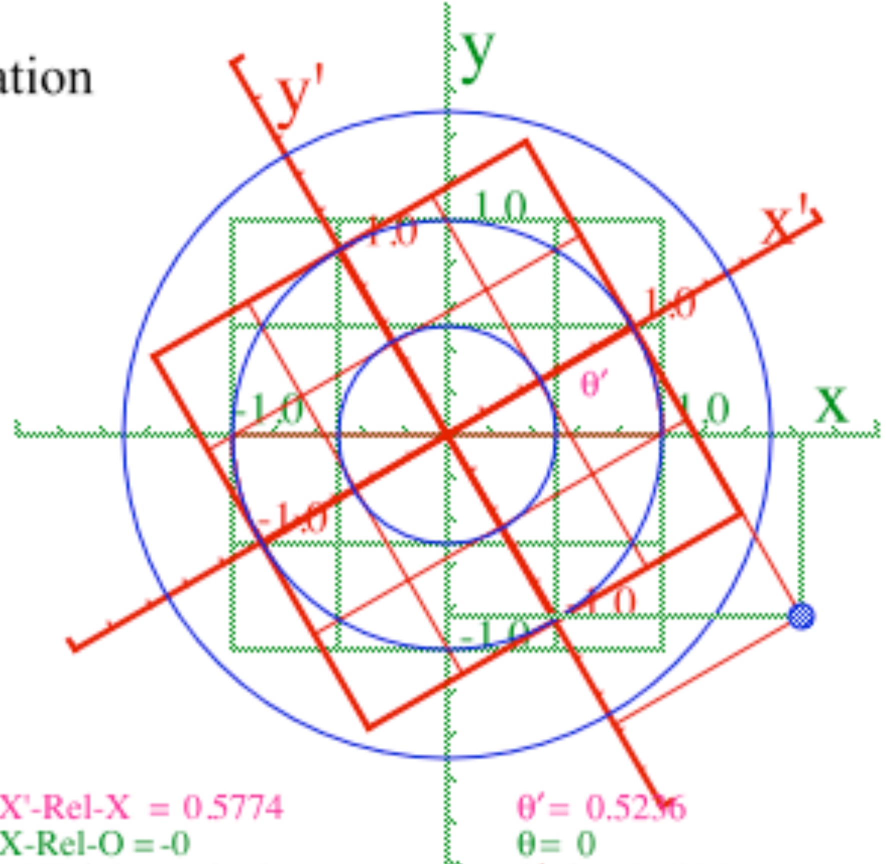
or the inverse (Kajobian) transformation:

$$\begin{pmatrix} \langle x'|x\rangle & \langle x'|y\rangle \\ \langle y'|x\rangle & \langle y'|y\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

to *any* vector $\mathbf{V} = |V\rangle = |x\rangle\langle x|V\rangle + |y\rangle\langle y|V\rangle$
$= |x'\rangle\langle x'|V\rangle + |y'\rangle\langle y'|V\rangle$

Circular invariants $r^2 = x^2 + y^2$

(a) Rotation Transformation and Invariants



$x = 1.65$
$y = -0.85$
$x^2+y^2 = 3.43$
$x' = 1.00$
$y' = -1.56$
$x'^2+y'^2 = 3.43$

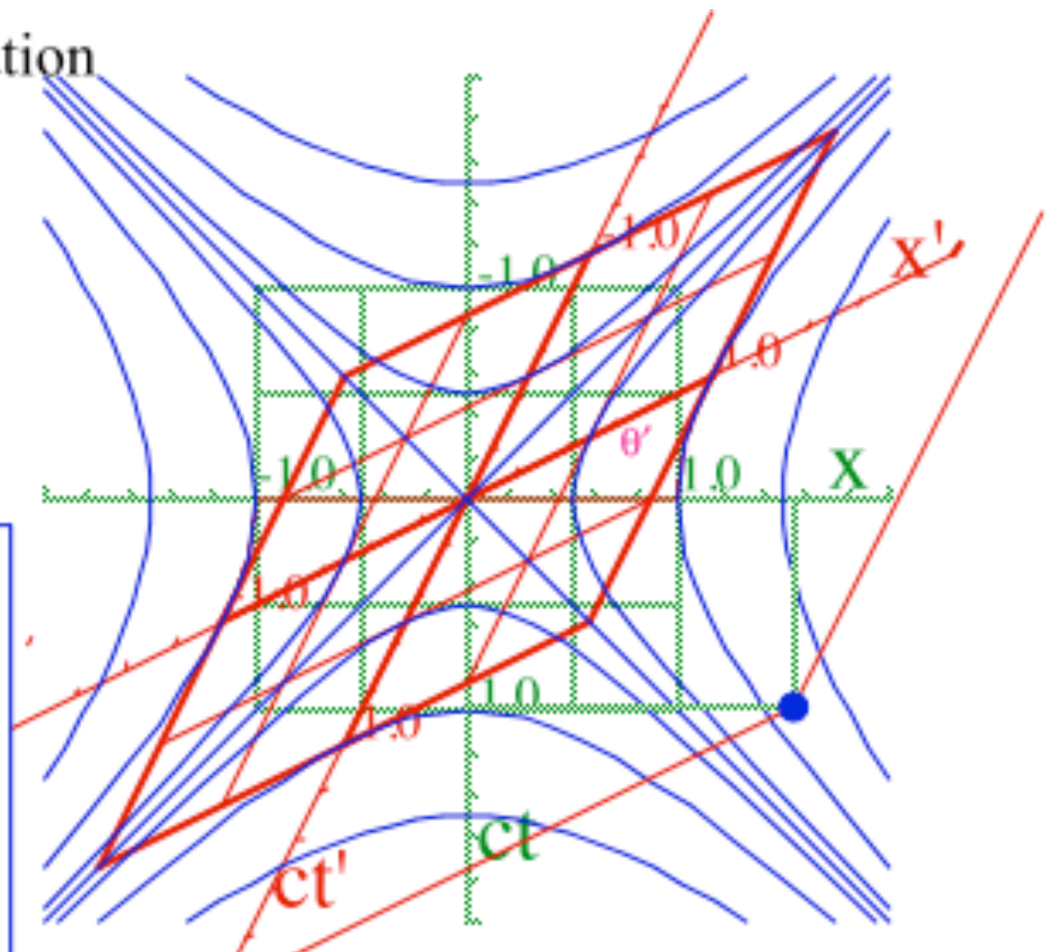
SlopeX'-Rel-X = 0.5774
SlopeX-Rel-O = -0
SlopeX'-Rel-O = 0.5774

θ′ = 0.5236
θ = 0
θ′+θ = 0.5236

$$x' = x\cos\theta - y\sin\theta = \frac{x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{-(b/c)y}{\sqrt{1+\frac{b^2}{c^2}}}$$

$$y' = x\sin\theta + y\cos\theta = \frac{(b/c)x}{\sqrt{1+\frac{b^2}{c^2}}} + \frac{y}{\sqrt{1+\frac{b^2}{c^2}}}$$

(b) Lorentz Transformation and Invariants



$x = 1.5453$
$ct = 0.9819$
$x^2-(ct)^2 = 1.42$
$x' = 2.3512$
$ct' = 2.0260$
$x'^2-(ct')^2 = 1.42$

v'/c X' Relative to X = -0.5
v/c X Relative to O = 0
v'/c X' Relative to O = -0.5

θ′ = −0.5493
θ = 0
θ+θ′ = −0.5493

$$x' = \frac{x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{\frac{v}{c}ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\cosh\rho + y\sinh\rho$$

$$ct' = \frac{\frac{v}{c}x}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{ct}{\sqrt{1-\frac{v^2}{c^2}}} = x\sinh\rho + y\cosh\rho$$

Light-cone-sections are hyperbolas

Main Lighthouse on (x=0,y=0) time line

North Lighthouse-1 on (x=0,y=1) time line

Main Lighthouse blinks trace x=±ct "V"-lines thru each time tic

North Lighthouse-1 blinks trace $x^2-(ct)^2=1$ hyperbolas thru each tic

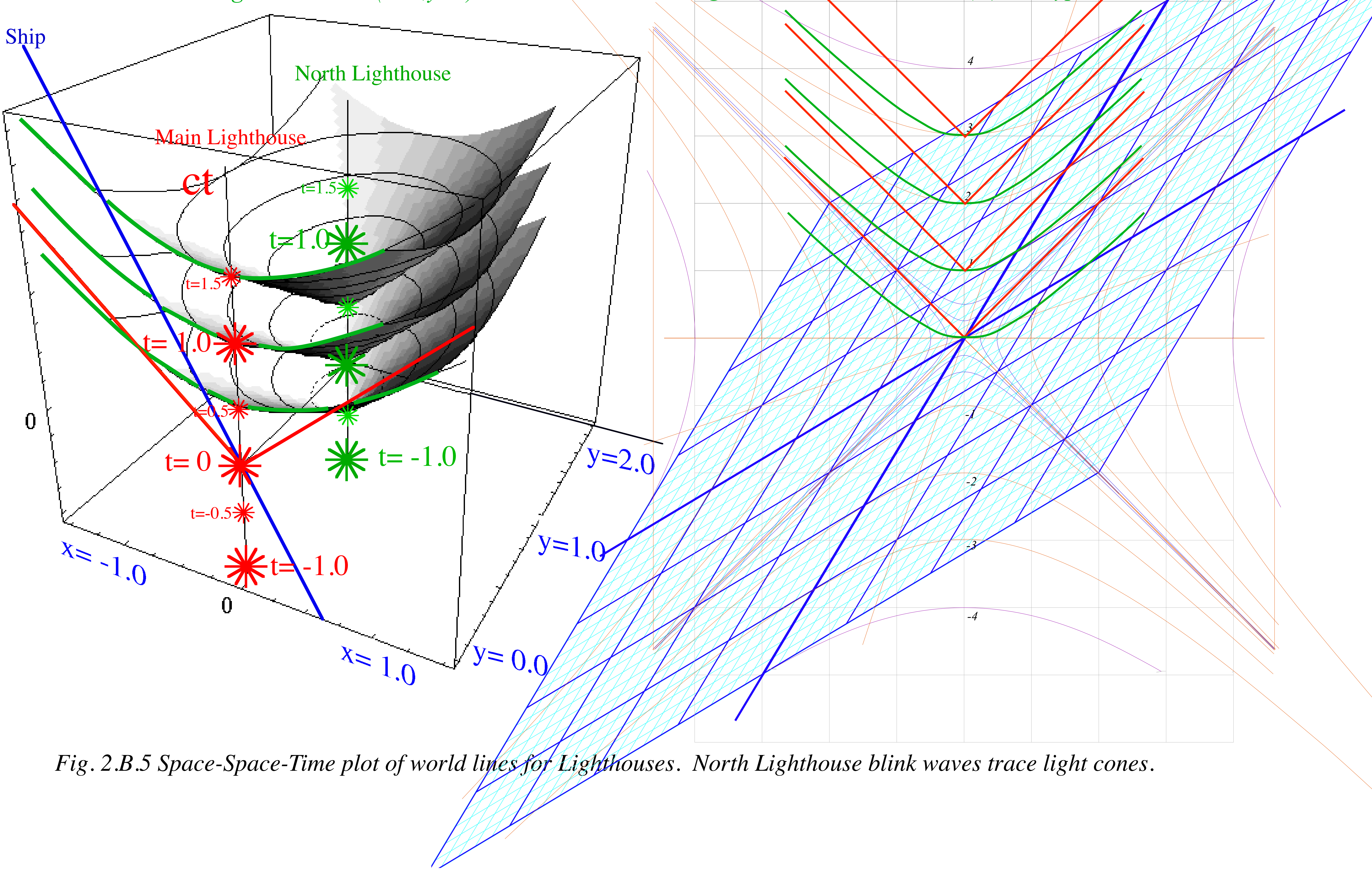
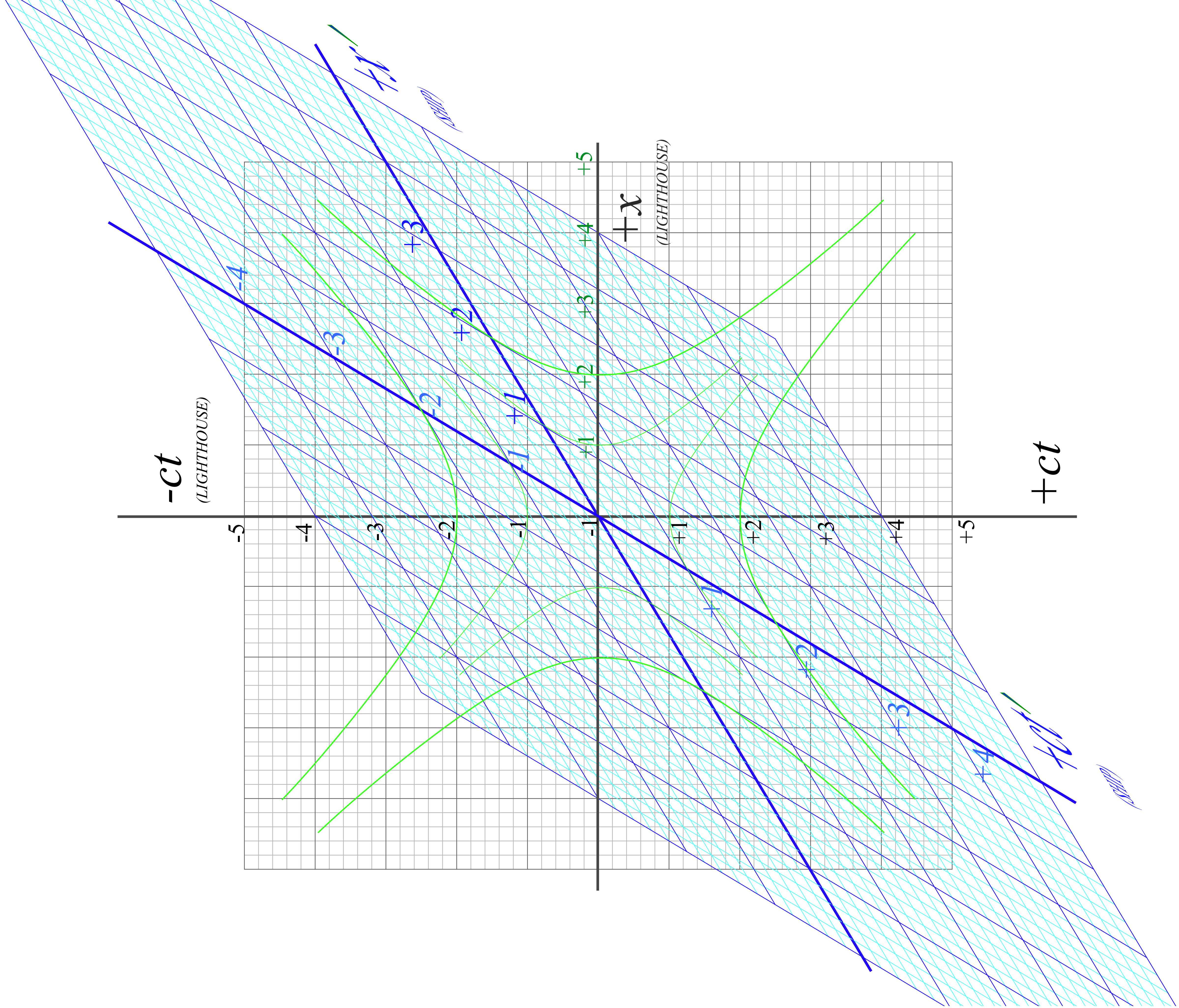
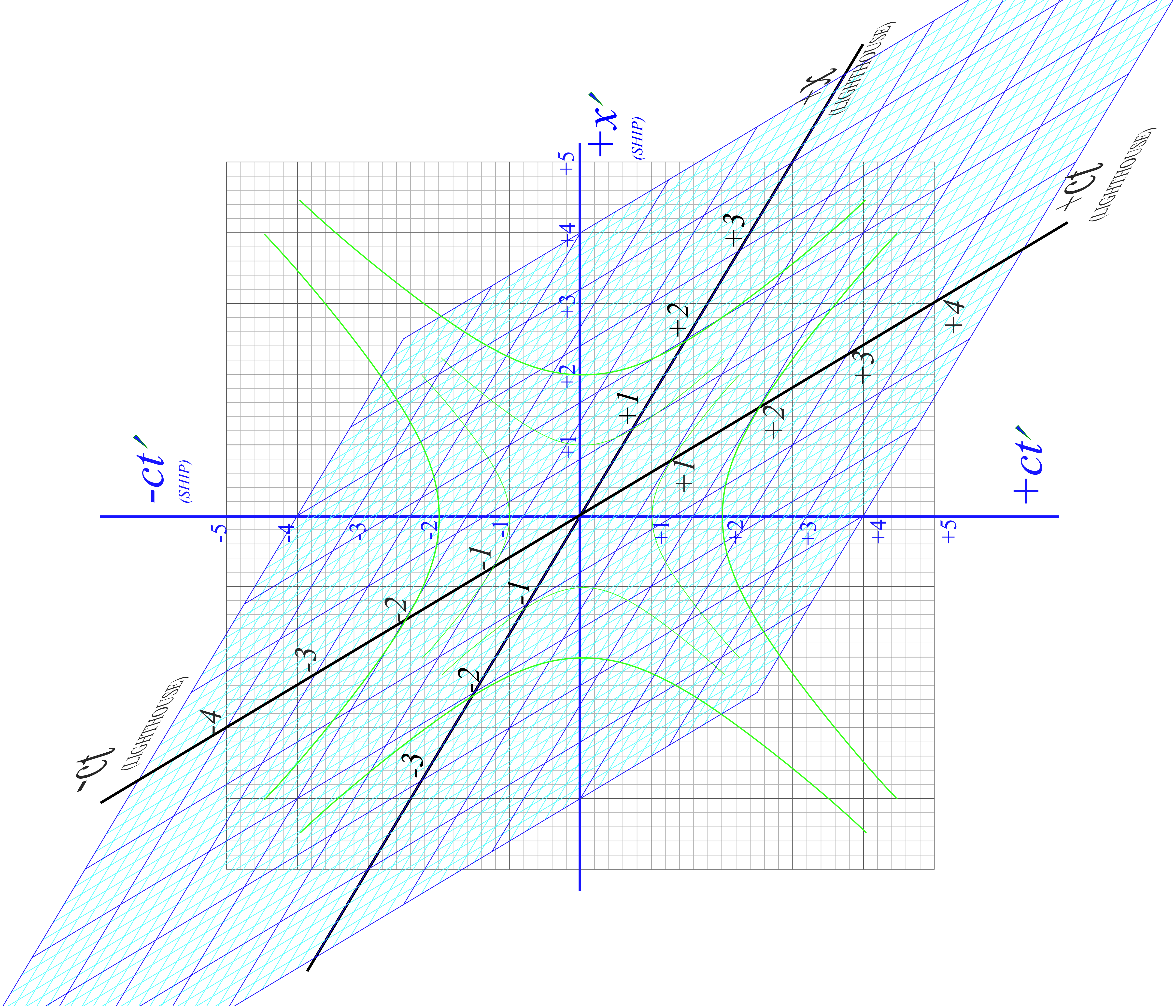


Fig. 2.B.5 Space-Space-Time plot of world lines for Lighthouses. North Lighthouse blink waves trace light cones.


+ct
-ct
(LIGHTHOUSE)
+x
(LIGHTHOUSE)
+x'
(SHIP)
+ct'
(SHIP)


+x′
(SHIP)
-ct′
(SHIP)
+ct′
+x
(LIGHTHOUSE)
+ct
(LIGHTHOUSE)
-ct
(LIGHTHOUSE)


+10
+9
+8
+7
+6
+5
+4
+3
+2
+1
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O
+1
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+3
+4
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+6
+7
+8
+9
+10