Lectures 5 to 6

Tue. 2.2.16 to Thur 2.4.2016

Kinetic Derivation of 1D Potentials and Force Fields

(Ch. 5 of Unit 1)

Review of $(V_1,V_2)\rightarrow(y_1,y_2)$ relations *High mass ratio $M_1/m_2 = 49$*

Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)$=const./y and the 1D-Adiabatic force field $F(y)$=const./y^3

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)$=const./y^2

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)$=const. $\ln(y)$

"Monster Mash" classical analog of Heisenberg action relations

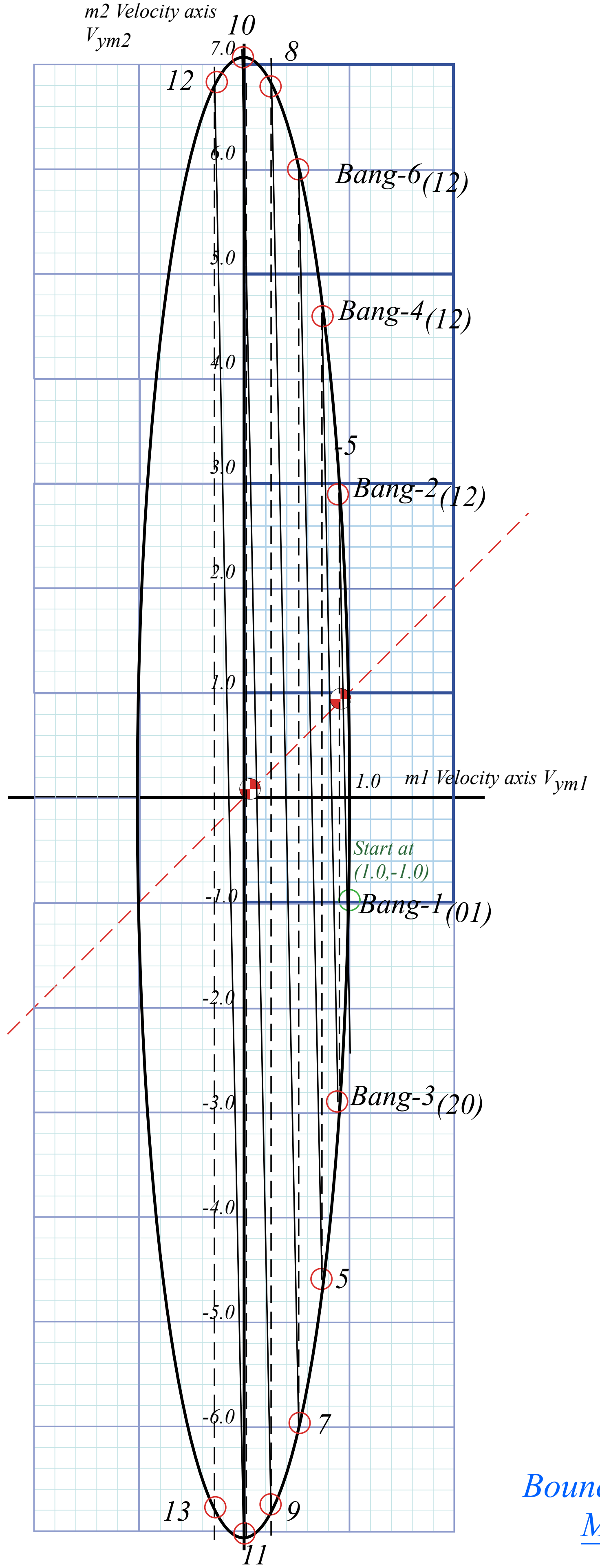
Example of very very large M_1 ball-wall(s) crushing a poor little m_2

How does m_2 conserve action ($\Delta x \Delta p$ or $\int p\cdot dx$) as its KE changes?

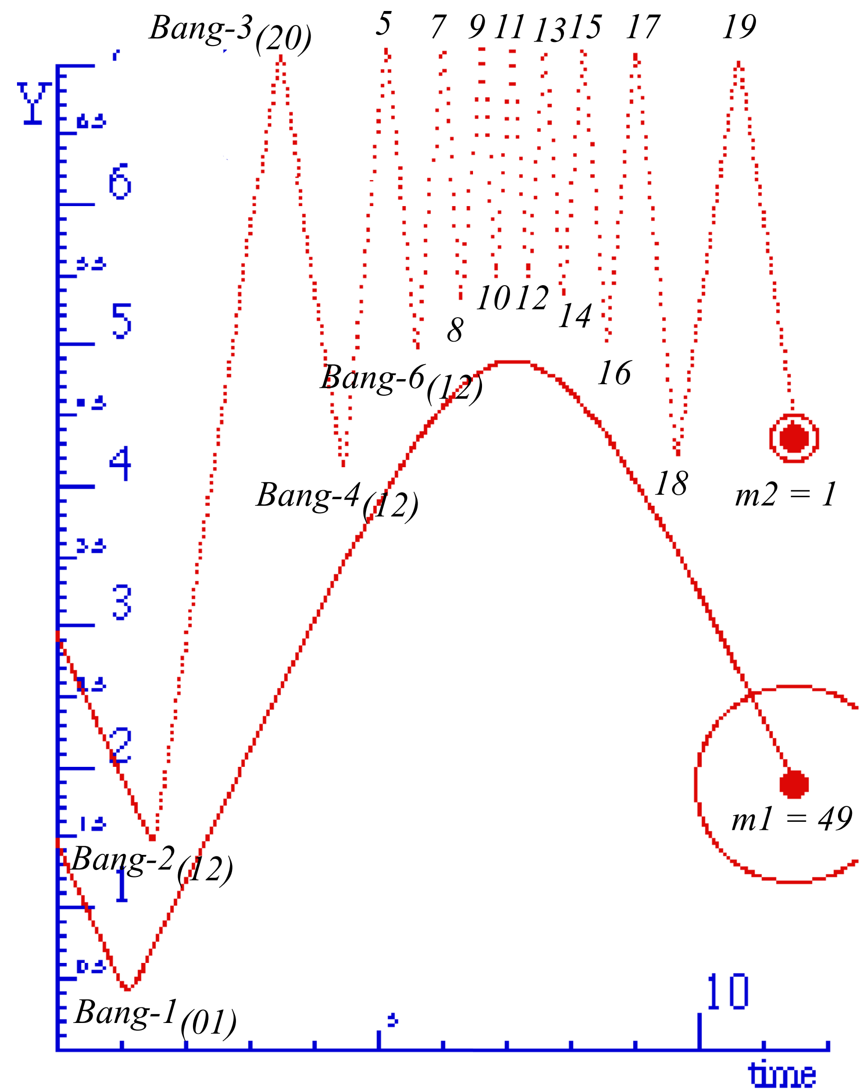
Review of $(V_1,V_2) \rightarrow (y_1,y_2)$ relations

→ *High mass ratio $M_1/m_2 = 49$*

Geometric "Integration" (Converting Velocity data to Space-time trajectory)



Example with masses: m_1=49 and m_2=1



Kinetic Energy Ellipse

$$KE = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{49}{2} + \frac{1}{2} = 25$$

$$1 = \frac{v_1^2}{2KE/m_1} + \frac{v_2^2}{2KE/m_2} = \frac{x_1^2}{a_1^2} + \frac{x_2^2}{a_2^2}$$

Ellipse radius 1

$$a_1 = \sqrt{2KE/m_1} = \sqrt{2KE/49} = \sqrt{50/49} = 1.01$$

Ellipse radius 2

$$a_2 = \sqrt{2KE/m_2} = \sqrt{2KE/1} = \sqrt{50/1} = 7.07$$

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with Newtonian time plot]

Fig. 4.1
Unit 1

BounceIt Superball Collision Web Simulator:
[M_1=49, M_2=1 with V_2 vs V_1 plot]

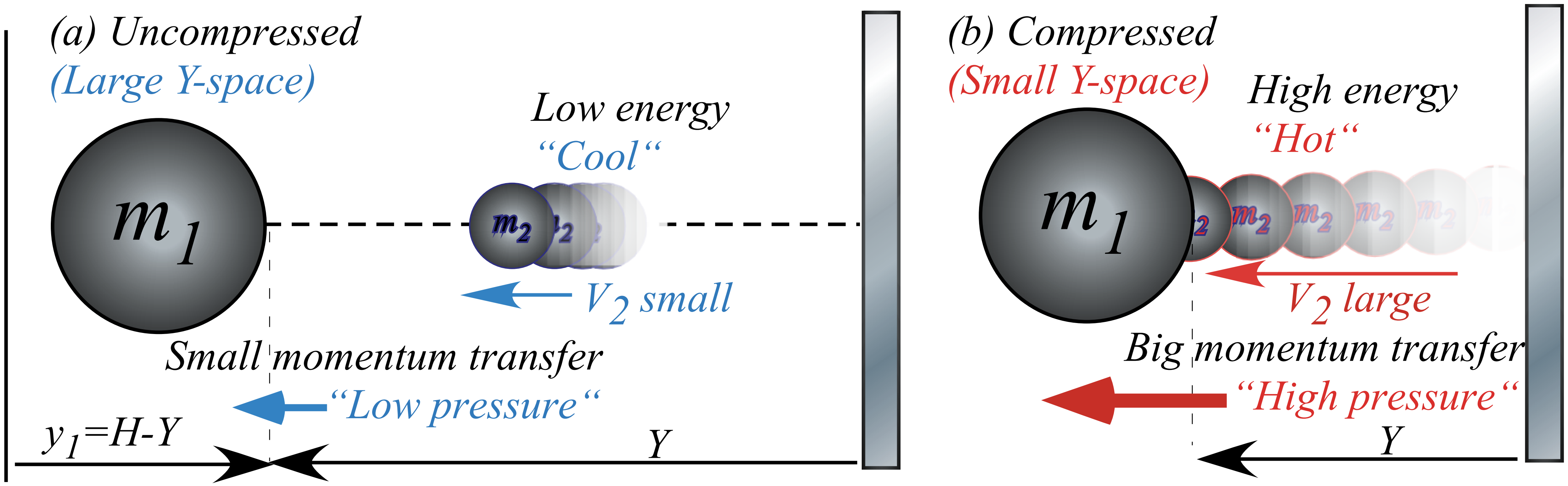
Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

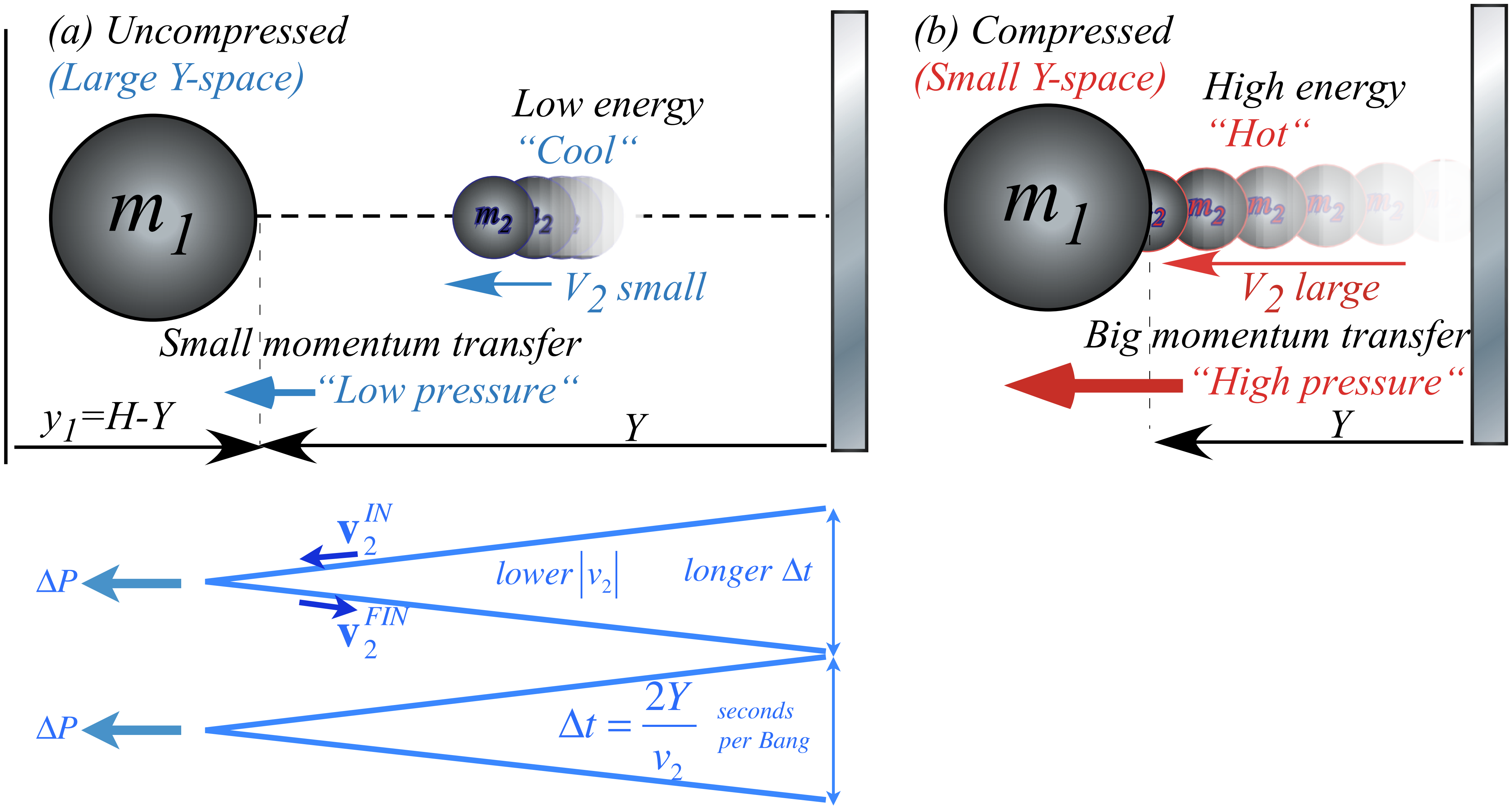
Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Fig. 5.1
Unit 1



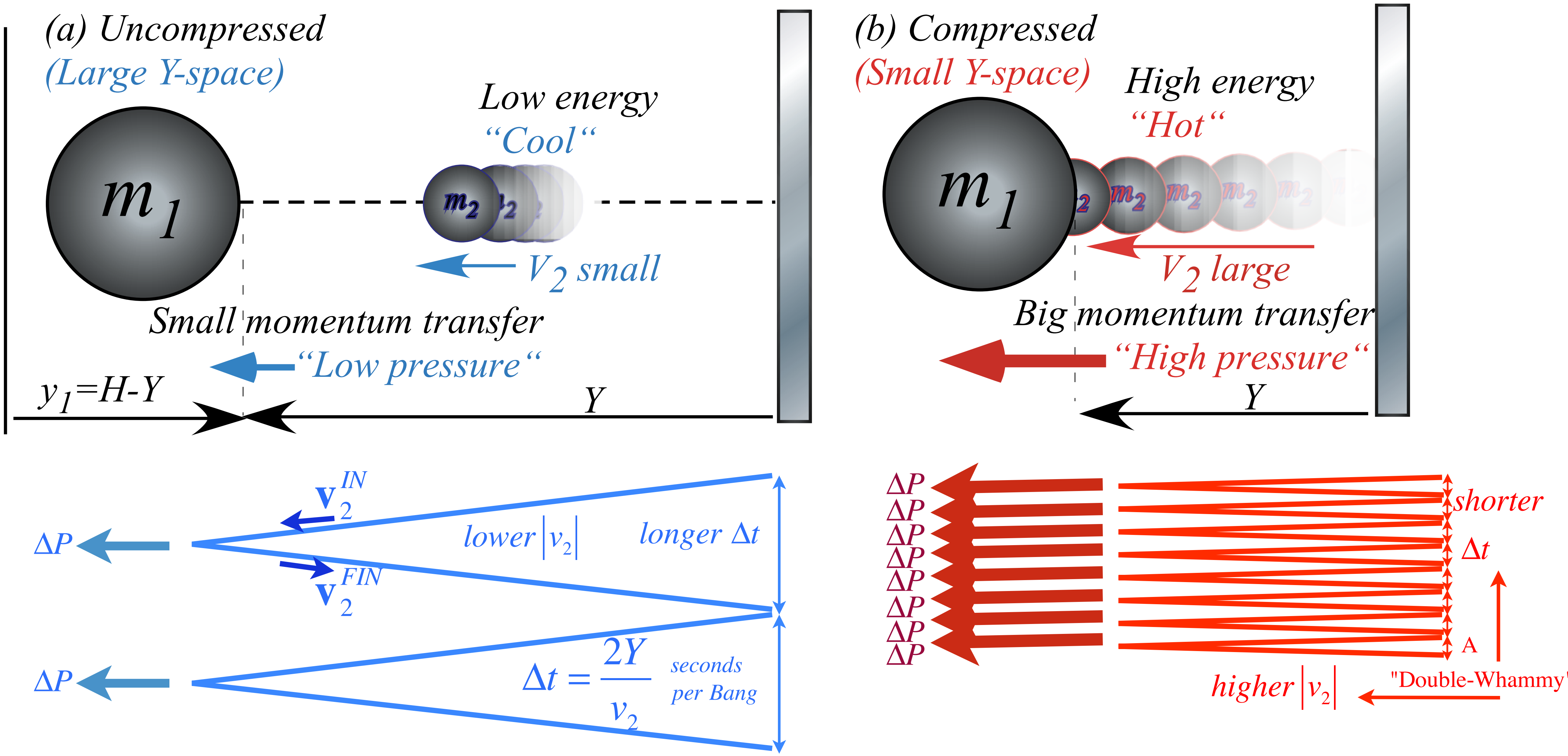
Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Fig. 5.1
Unit 1



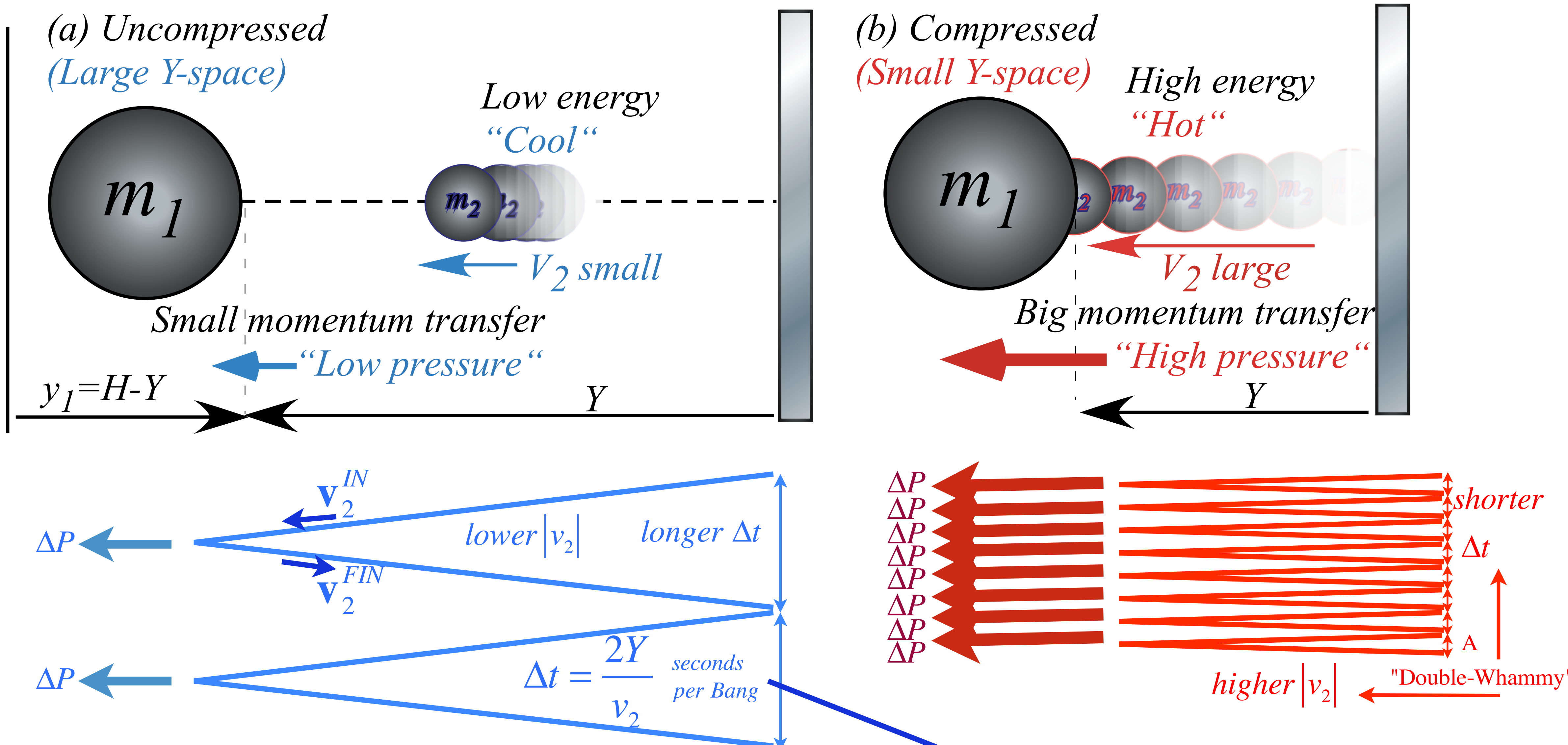
Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Fig. 5.1
Unit 1



Big mass-m_1 ball feeling "force-field" or "pressure" of small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

Fig. 5.1
Unit 1



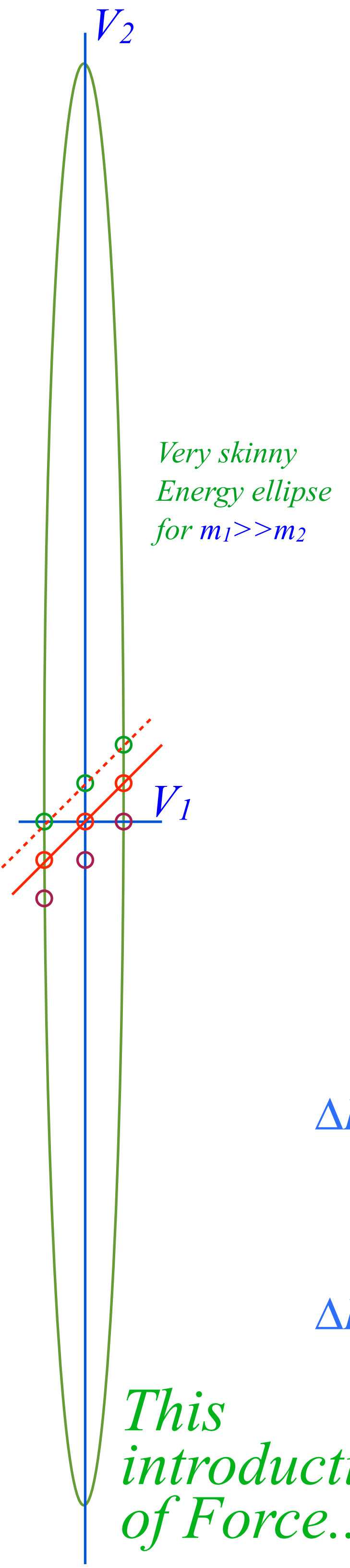
This introduction of Force...

$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

...is more of a <u>definition</u> than another <u>axiom</u>

Double-Bang Sequences for $m_1 >> m_2$



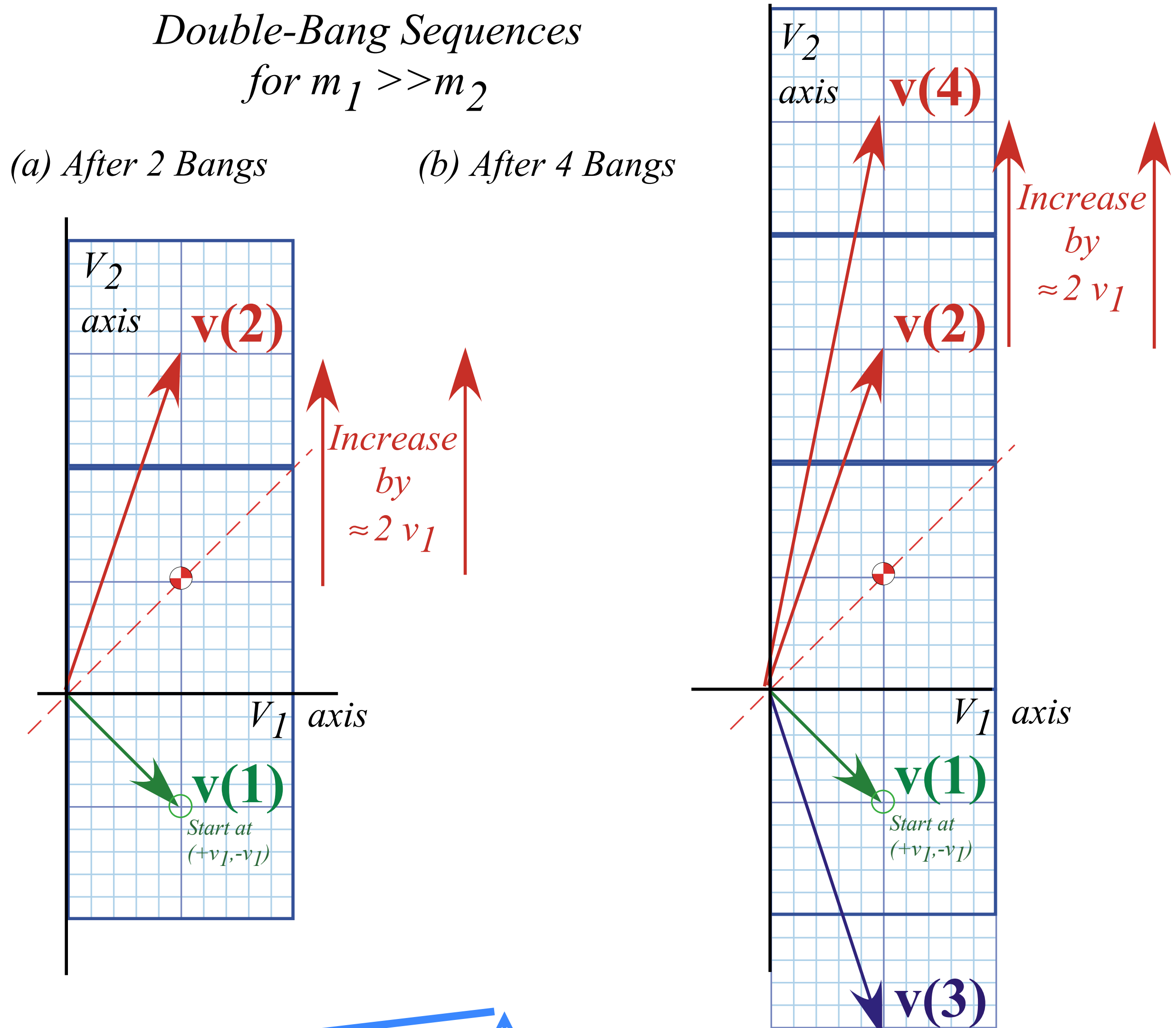
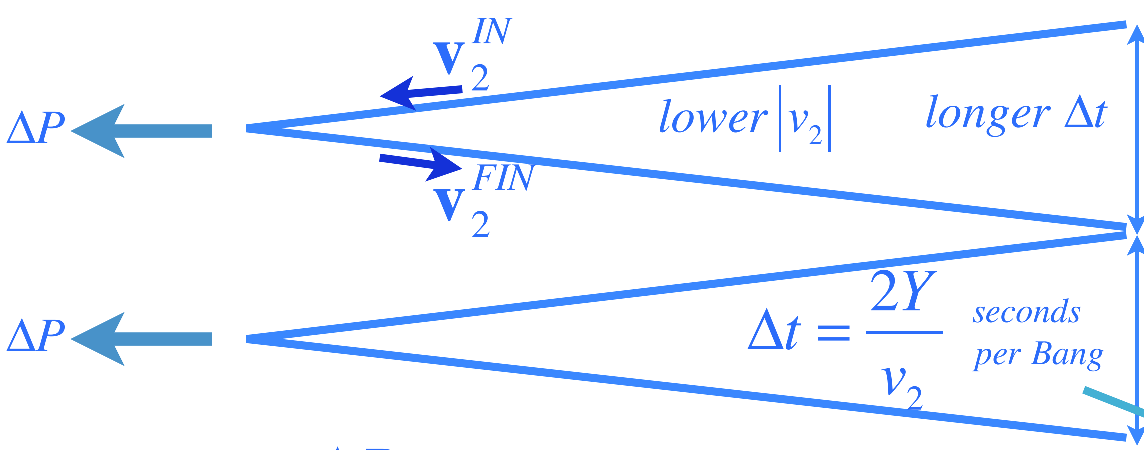


Fig. 5.2 Unit 1



$$\Delta t = \frac{2Y}{v_2} \text{ seconds per Bang}$$

$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

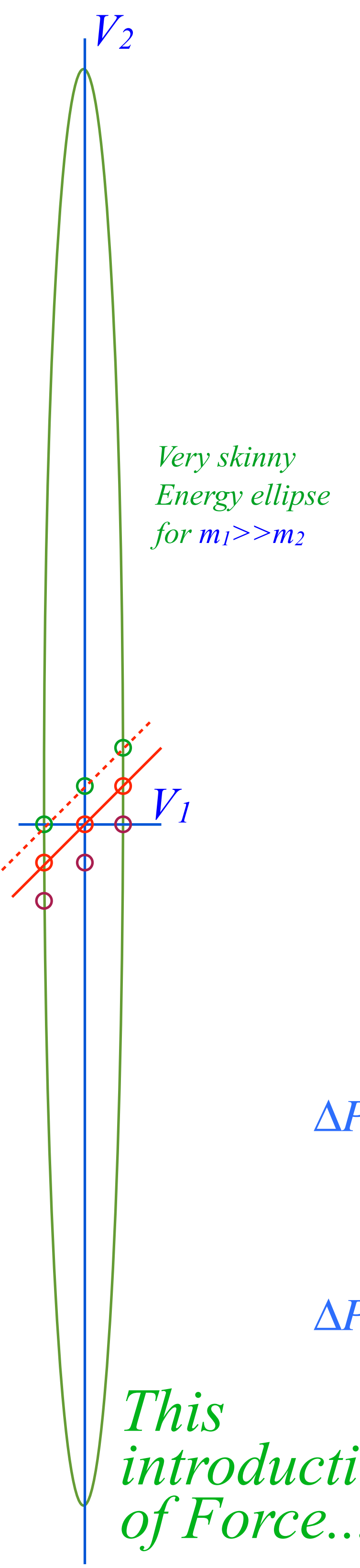
This introduction of Force...

$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

...is more of a <u>definition</u> than another <u>axiom</u>

Double-Bang Sequences for $m_1 >> m_2$



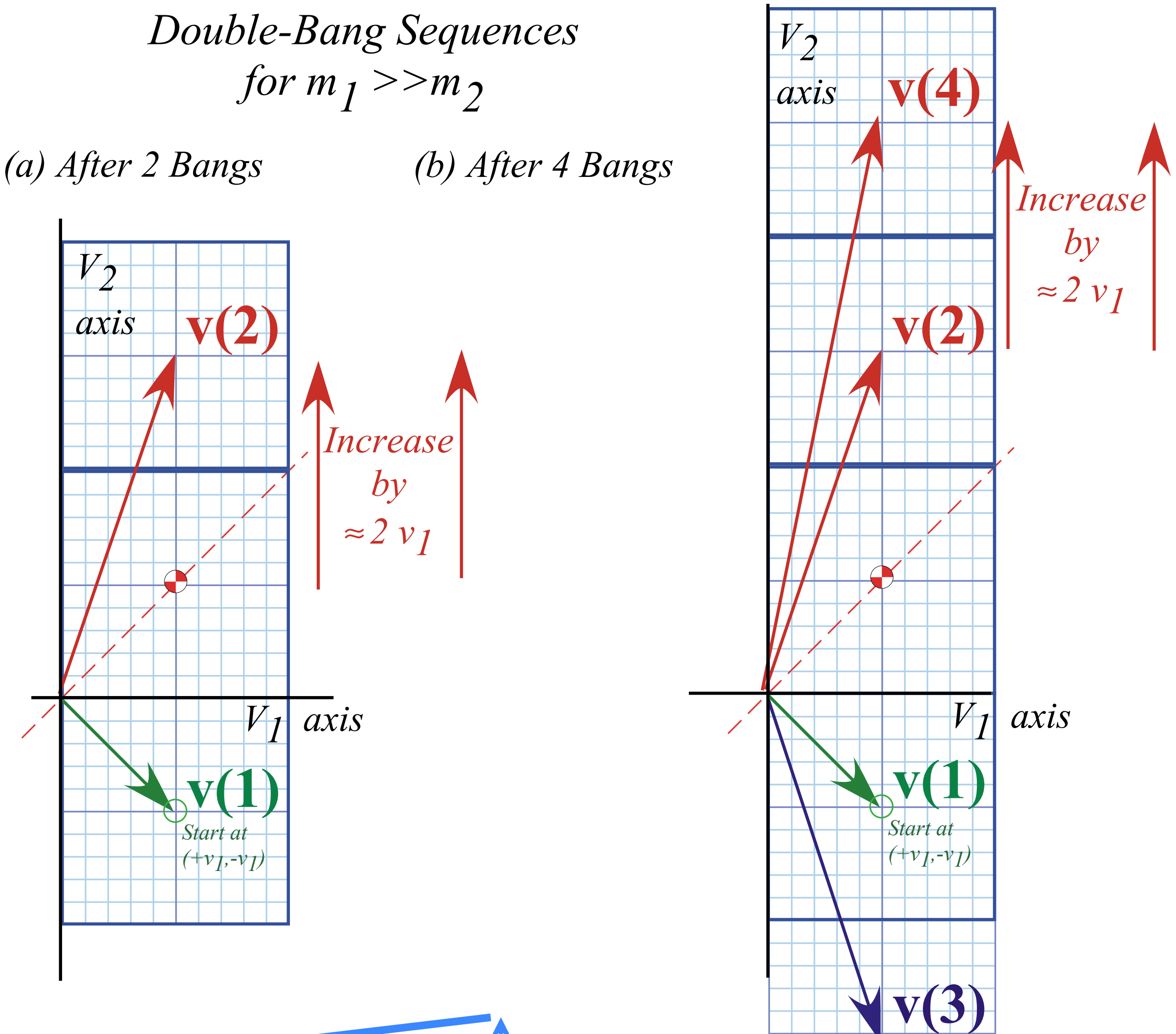
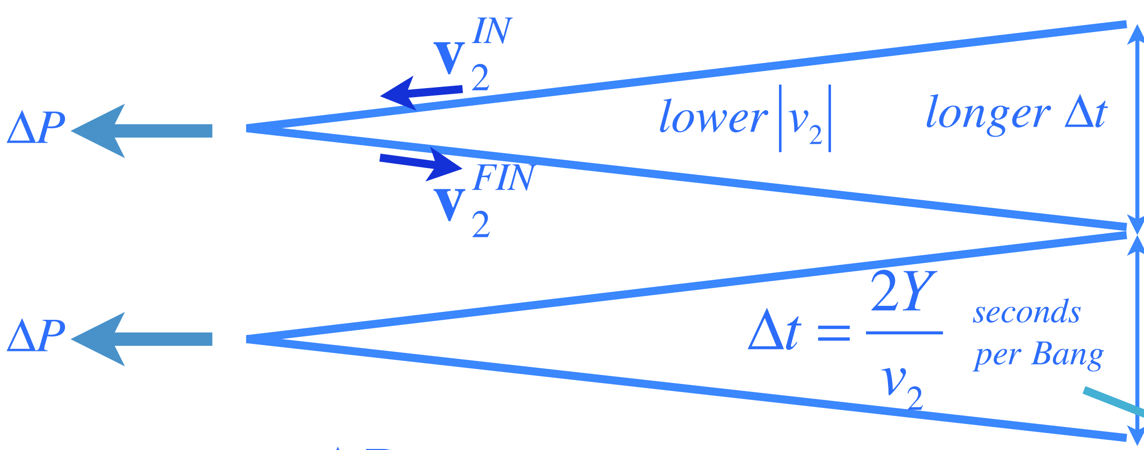


Fig. 5.2 Unit 1



$$|v_2^{FIN}| = |v_2^{IN}| + |2v_1| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

This introduction of Force...

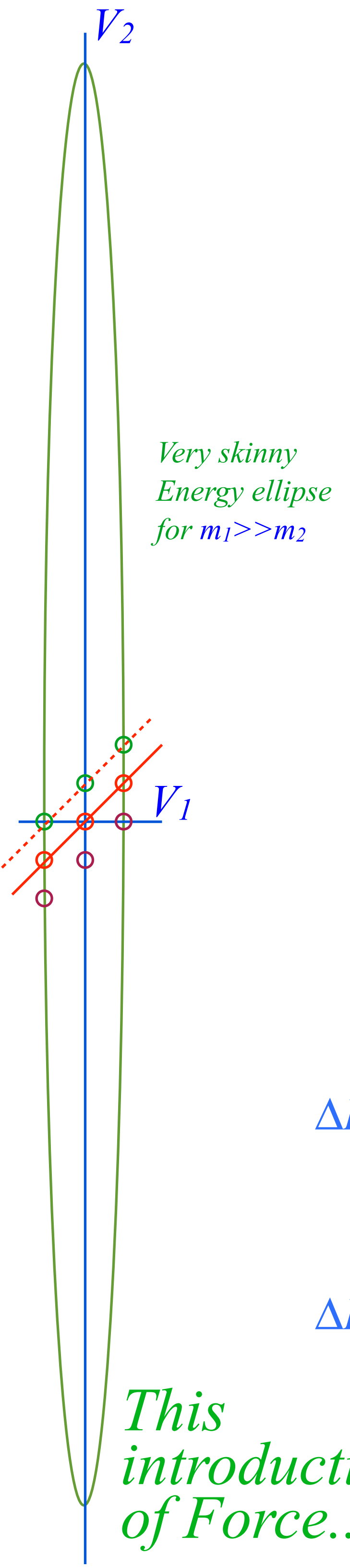
$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1$$

...is more of a <u>definition</u> than another <u>axiom</u>

Double-Bang Sequences for $m_1 >> m_2$



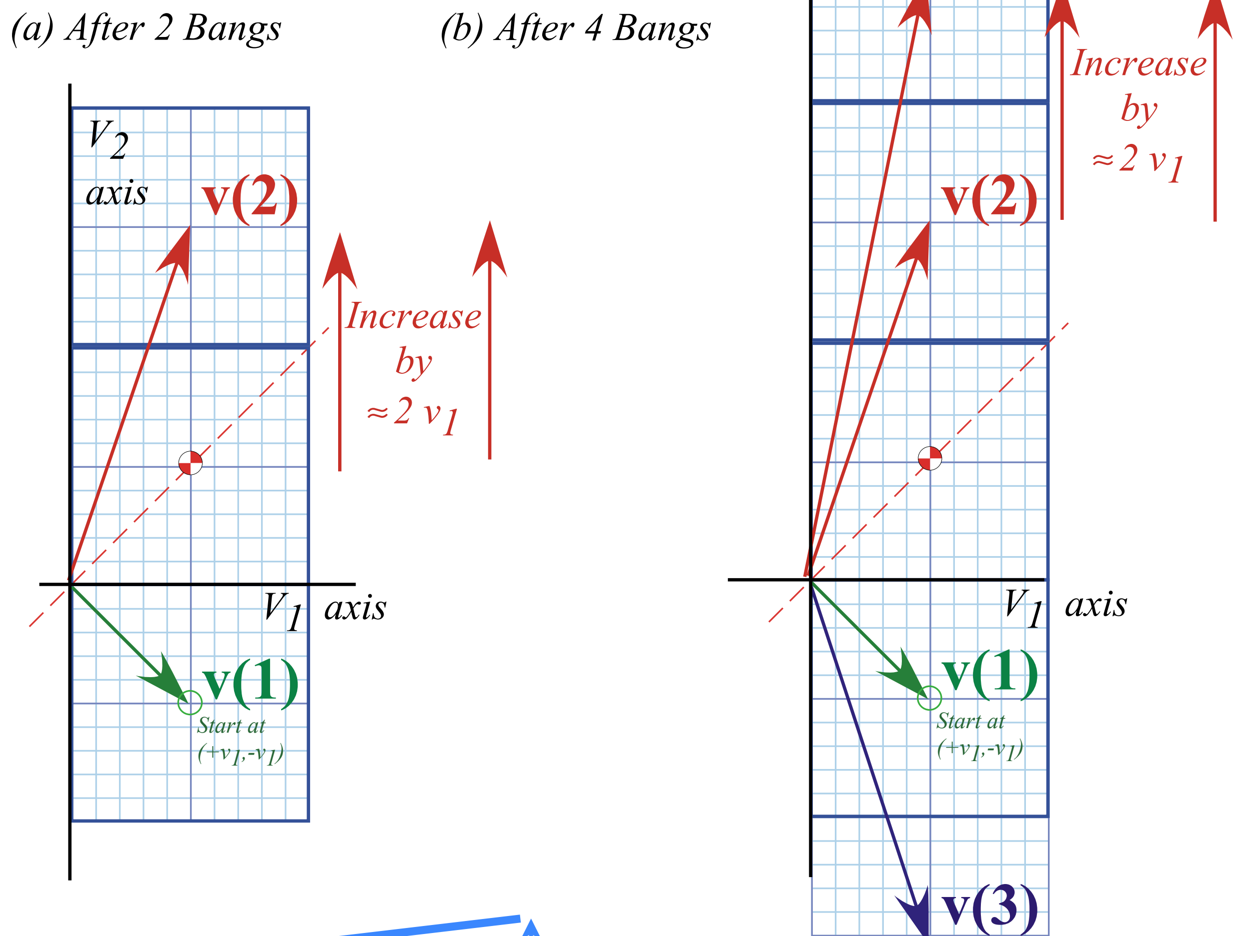
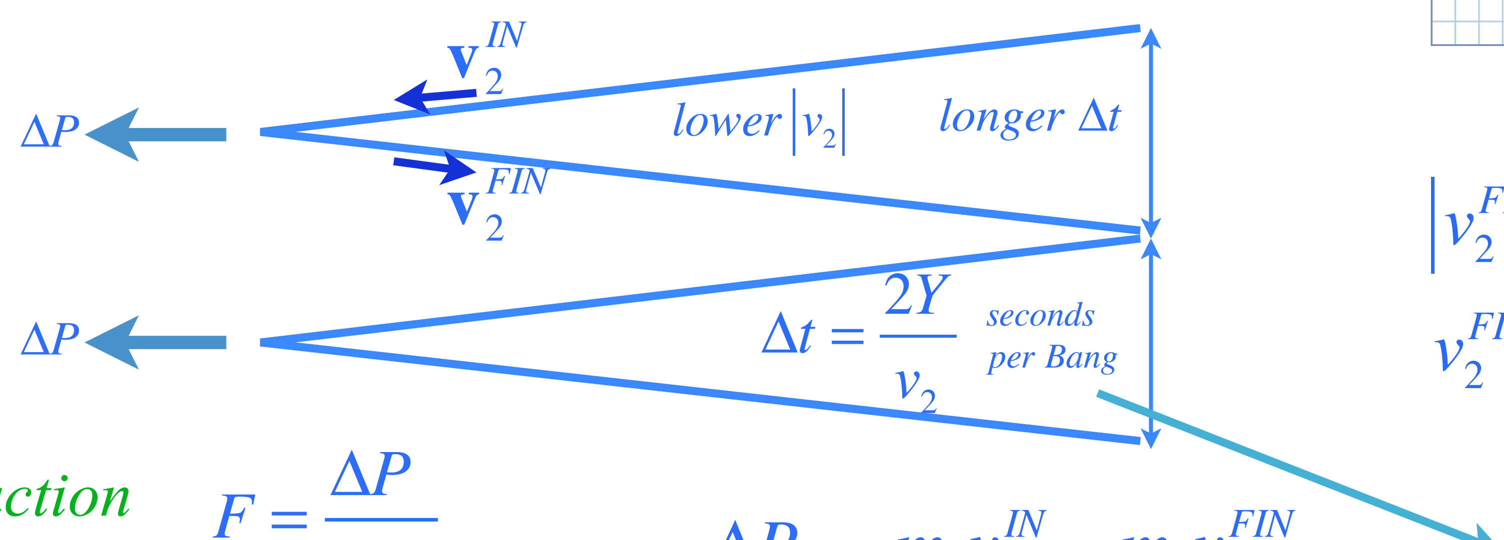


Fig. 5.2
Unit 1



$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

This introduction of Force...

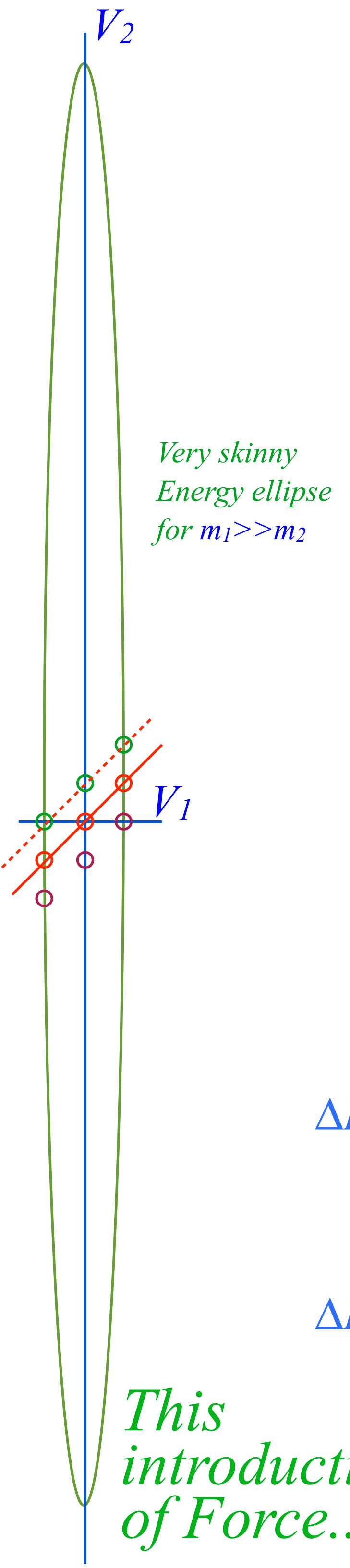
$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1 \approx 2m_2 v_2^{IN}$$

...is more of a <u>definition</u> than another <u>axiom</u>

Assuming slow m_1 : $v_1 \ll v_2$



Double-Bang Sequences for $m_1 >> m_2$

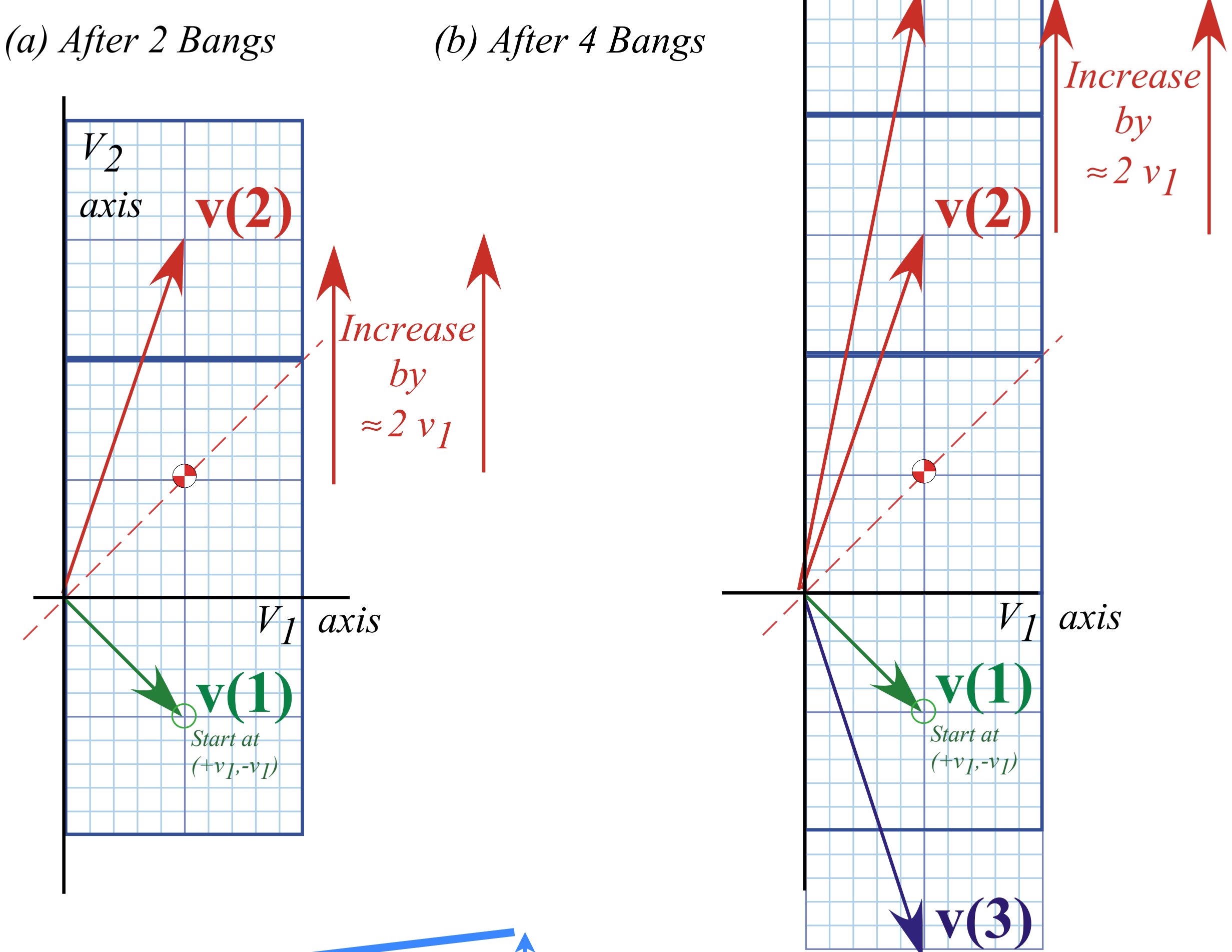
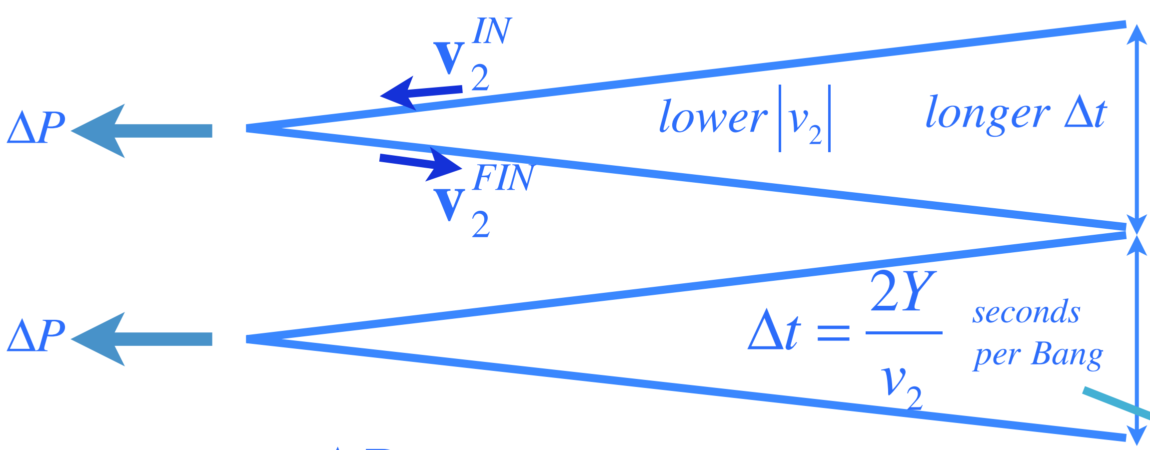


Fig. 5.2
Unit 1



$$\Delta t = \frac{2Y}{v_2}$$ seconds per Bang

$$\left|v_2^{FIN}\right| = \left|v_2^{IN}\right| + \left|2v_1\right| \quad \text{for: } m_1 >> m_2$$

$$v_2^{FIN} = -v_2^{IN} - 2v_1$$

This introduction of Force...

$$F = \frac{\Delta P}{\Delta t} \qquad \Delta P = m_2 v_2^{IN} - m_2 v_2^{FIN} \qquad \frac{1}{\Delta t} = \frac{v_2}{2Y}$$

Force F on m_1 = (Momentum per sec.) = (Momentum per Bang)·(Bangs per second)

$$\Delta P = m_2 v_2^{IN} - m_2(-v_2^{IN} - 2v_1) = 2m_2 v_2^{IN} + 2m_2 v_1 \approx 2m_2 v_2^{IN}$$

...is more of a <u>definition</u> than another <u>axiom</u>

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

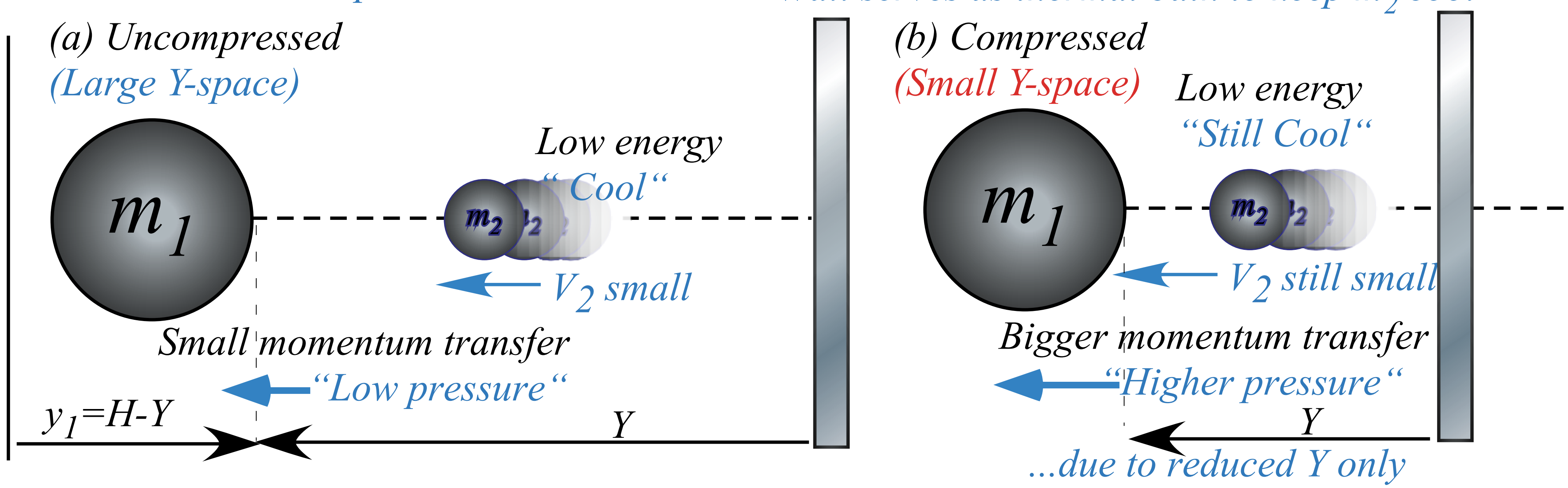
Assuming slow $m_1 : v_1 \ll v_2$

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2 m_2 v_2\right) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Not a "Double-Whammy"...
...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

Isothermal expansion or contraction: *Wall serves as thermal bath to keep m_2 cool*



Force "field" or "pressure" due to many small bounces

Force defined as momentum transfer rate

The 1D-Isothermal force field $F(y)=const./y$ and the 1D-Adiabatic force field $F(y)=const./y^3$

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

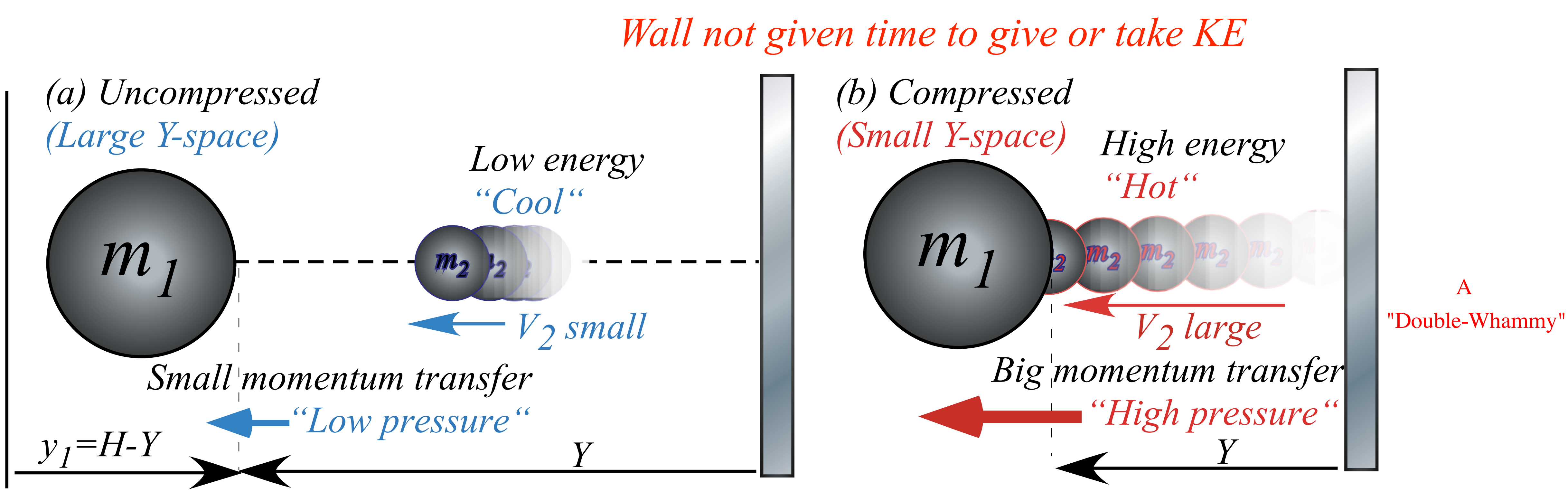
Not a "Double-Whammy"... ...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B = v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y}$$



$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Not a "Double-Whammy"... ...only a "Single-Whammy"

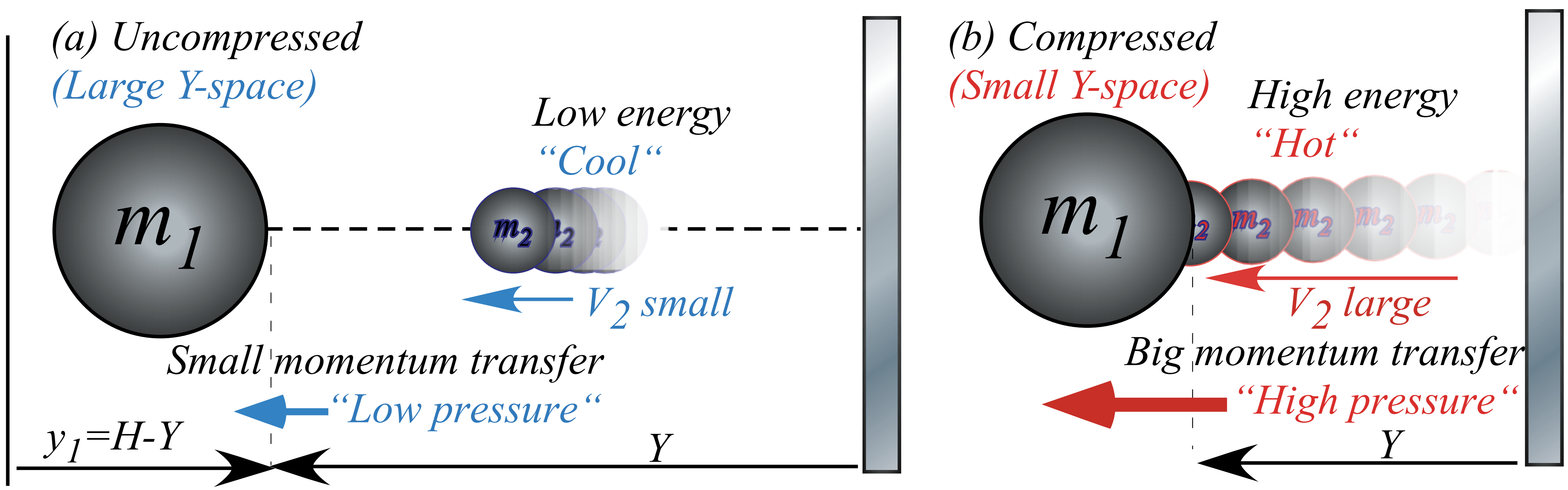
1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B = v_2/2Y$.

$$\frac{dv_2}{\cancel{dt}} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -\cancel{2}\frac{dY}{\cancel{dt}}\frac{v_2}{\cancel{2}Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2} = -\frac{dY}{Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right)\cdot\left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Not a "Double-Whammy"... ...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

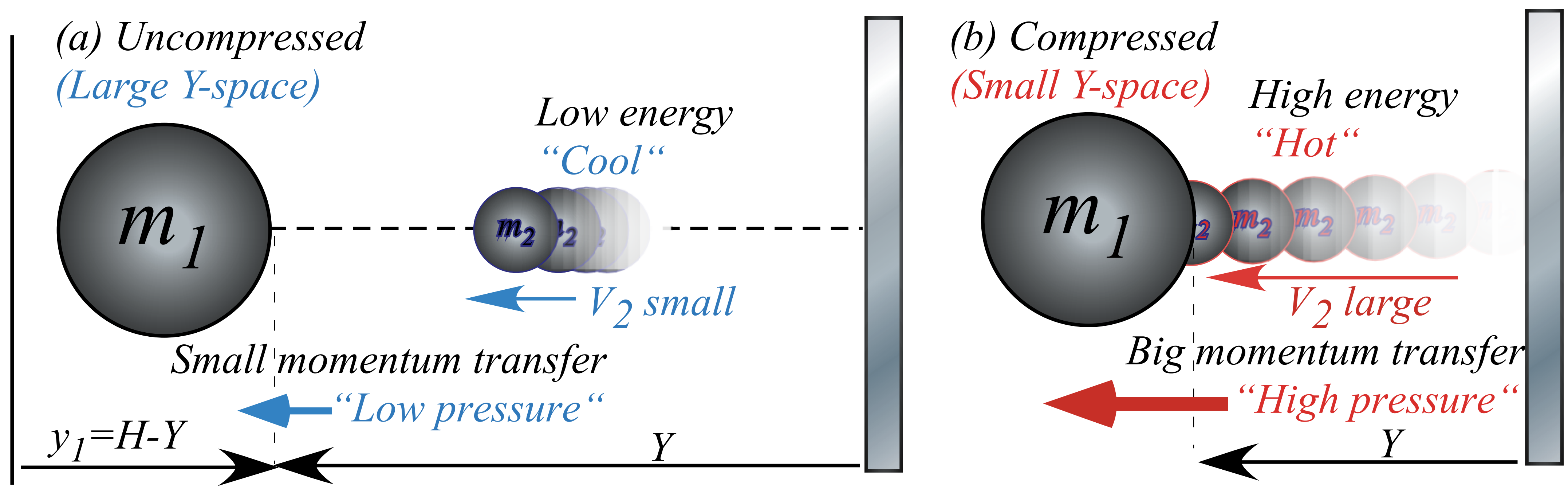
When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2} = -\frac{dY}{Y}$$

Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e(e^C x)$

$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \quad \text{integrates to: } \ln v_2 = -\ln Y + C \quad \text{or: } \ln v_2 = \ln\frac{const.}{Y} \quad \text{or: } v_2 = \frac{const.}{Y}$$

Wall not given time to give or take KE



A "Double-Whammy"

$$F=\frac{\Delta P}{\Delta t}=\left(\Delta P\approx 2m_2v_2\right)\cdot\left(\frac{1}{\Delta t}\approx\frac{v_2}{2Y}\right)\approx\frac{m_2v_2^2}{Y}$$

Not a "Double-Whammy"... ...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt}\equiv v_1=-\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B=v_2/2Y$.

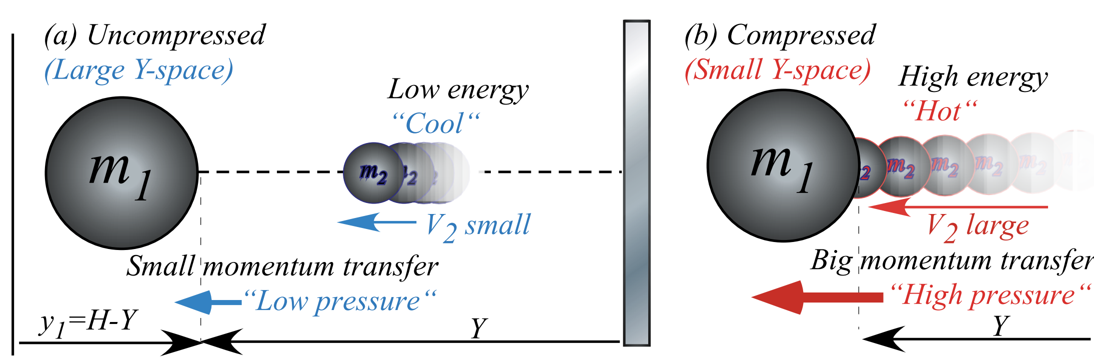
$$\frac{dv_2}{dt}=2v_1B=2v_1\frac{v_2}{2Y}=-2\frac{dY}{dt}\frac{v_2}{2Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2}=-\frac{dY}{Y}$$

Differential equation results and has logarithmic integral. $\int\frac{dx}{x}=\ln x+C=\log_e x+\log_e e^C=\log_e(e^C x)$

$$\frac{dv_2}{v_2}=-\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2=-\ln Y+C \quad \text{or:} \quad \ln v_2=\ln\frac{const.}{Y} \quad \text{or:} \quad v_2=\frac{const.}{Y}$$

Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2=\frac{const.}{Y}=\frac{v_2^{IN}Y(t=0)}{Y}$*):* $\boxed{F=\frac{m_2\left(v_2^{IN}Y(t=0)\right)^2}{Y^3}=\frac{const.}{Y^3}}$



A "Double-Whammy"

$$F = \frac{\Delta P}{\Delta t} = \left(\Delta P \approx 2m_2 v_2\right) \cdot \left(\frac{1}{\Delta t} \approx \frac{v_2}{2Y}\right) \approx \frac{m_2 v_2^2}{Y}$$

Not a "Double-Whammy"... ...only a "Single-Whammy"

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

However, if ceiling is elastic, v_2 isn't constant if m_1 changes bounce range Y: $\frac{dy_1}{dt} \equiv v_1 = -\frac{dY}{dt}$

When m_1 collides with m_2 it adds twice its velocity ($2v_1$) to v_2. This occurs at "bang-rate" $B = v_2/2Y$.

$$\frac{dv_2}{dt} = 2v_1 B = 2v_1 \frac{v_2}{2Y} = -2\frac{dY}{dt}\frac{v_2}{2Y} \qquad \text{simplifies to:} \quad \frac{dv_2}{v_2} = -\frac{dY}{Y}$$

Differential equation results and has logarithmic integral. $\int \frac{dx}{x} = \ln x + C = \log_e x + \log_e e^C = \log_e(e^C x)$

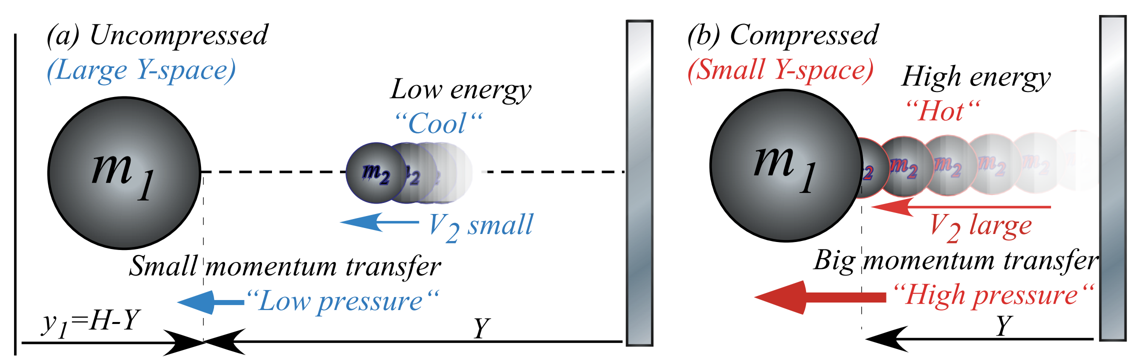
$$\frac{dv_2}{v_2} = -\frac{dY}{Y} \quad \text{integrates to:} \quad \ln v_2 = -\ln Y + C \quad \text{or:} \quad \ln v_2 = \ln\frac{const.}{Y} \quad \text{or:} \quad v_2 = \frac{const.}{Y}$$

Force law with this variable v_2 is called *adiabatic* or not-*diabatic* or not-gradual.

1D-Adiabatic Force Law (assume v_2 varies: $v_2 = \frac{const.}{Y} = \frac{v_2^{IN} Y(t=0)}{Y}$*):* $\boxed{F = \frac{m_2\left(v_2^{IN} Y(t=0)\right)^2}{Y^3} = \frac{const.}{Y^3}}$

$$\boxed{F = \frac{m_2\left(\bar{v}_2^{IN} Y_0\right)^2}{Y^3} = \frac{const.}{Y^3}}$$

1D-Adiabatic Force Law



A "Double-Whammy"

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

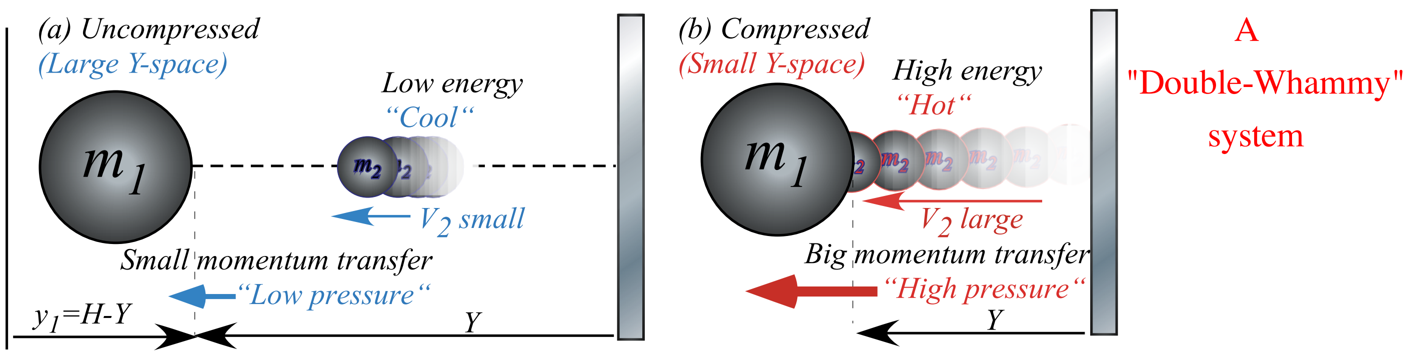
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $\boxed{KE(v_1)}$ *vs* $\boxed{\textit{Potential energy } PE(Y)=U(Y)}$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$



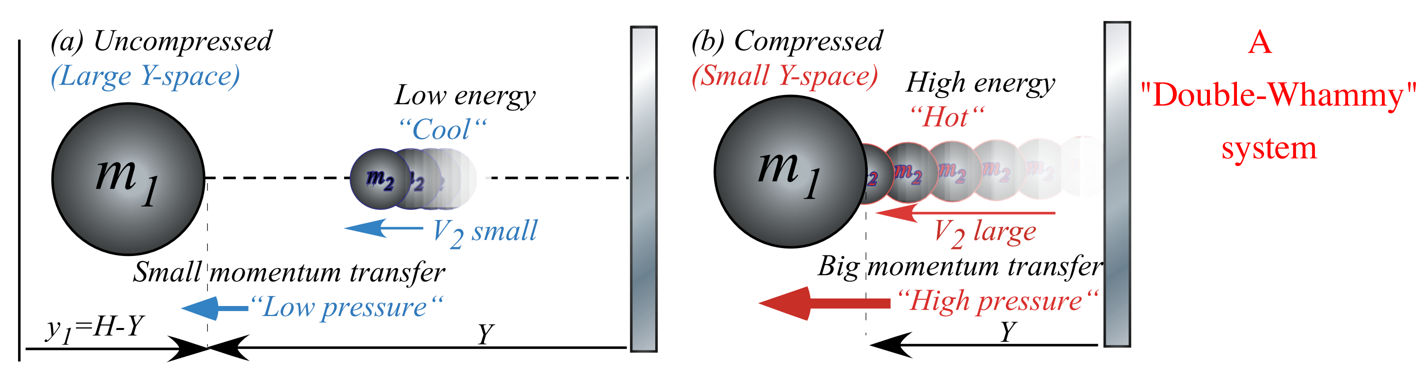
Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F \cdot dY = \pm dU$



Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

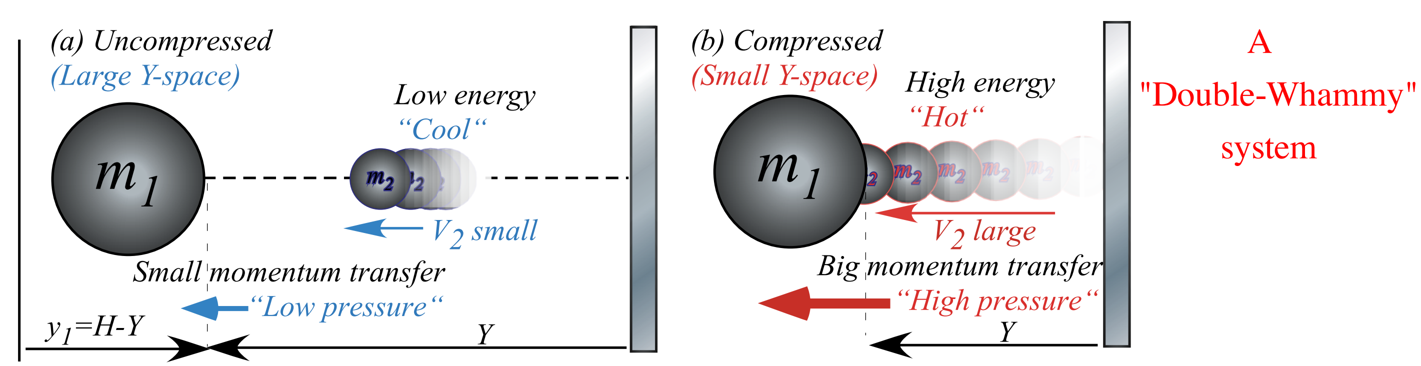
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $\boxed{KE(v_1)}$ *vs* $\boxed{\textit{Potential energy } PE(Y)=U(Y)}$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?*



Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

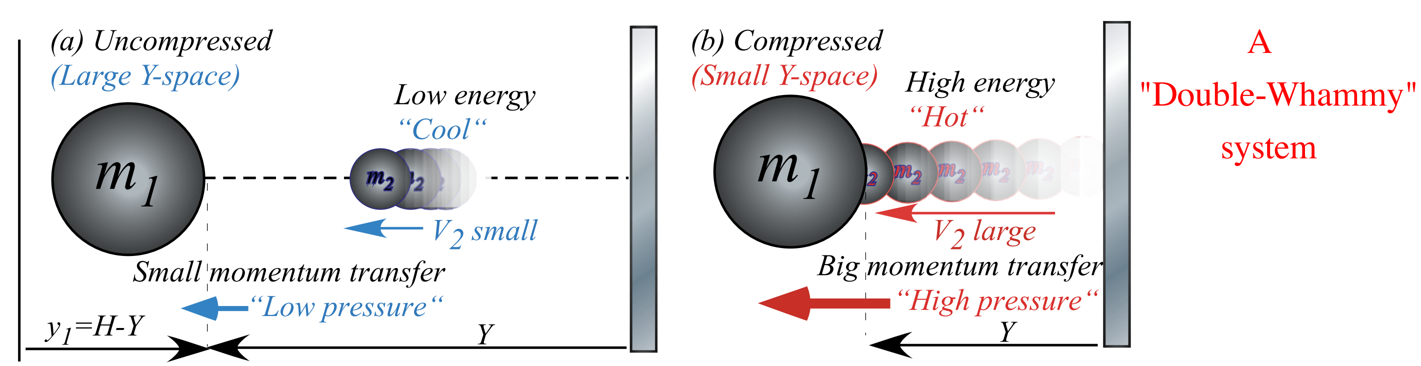
In adiabatic case where $v_2 = \dfrac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?* A: No.



Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

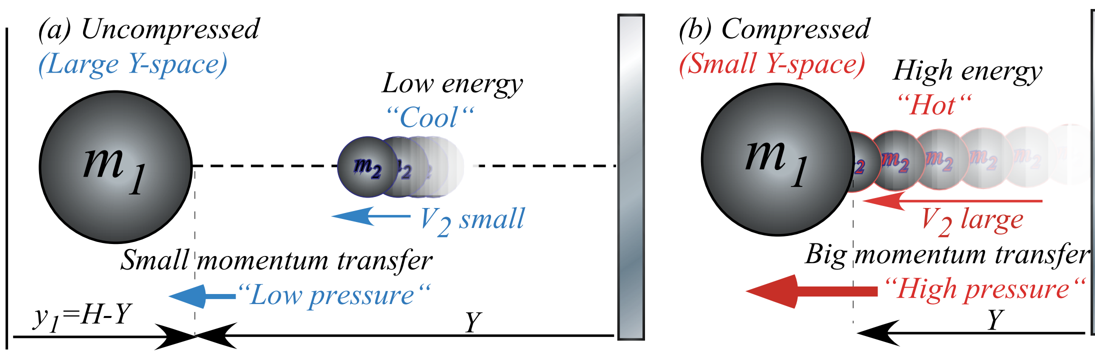
Define for big mass m_1: *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

Q*?Another axiom?* A: No.

$$\int F\cdot dY = \int \frac{dp}{dt}\cdot dY = \int \frac{dY}{dt}\cdot dp = \int V\cdot dp = \int V\cdot d(mV) = m\frac{V^2}{2} + const = U$$

(Here: $V = v_2$)



A "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy KE(v_1)* vs *Potential energy PE(Y)=U(Y)*

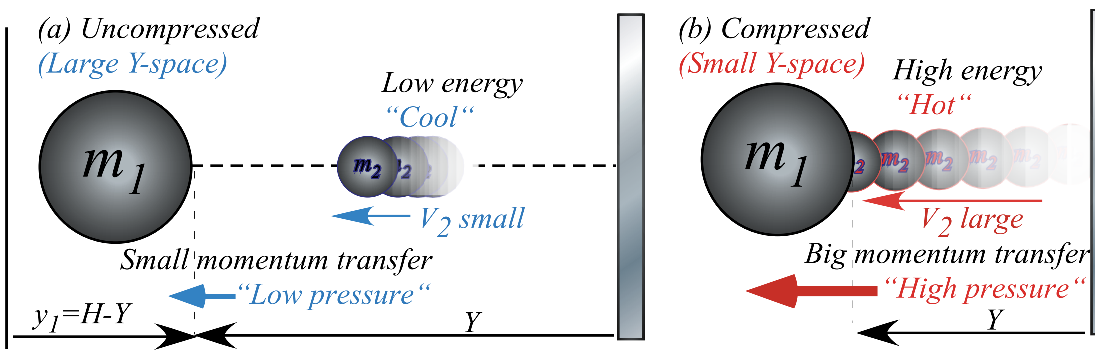
Potential energy PE(Y)=U(Y)$=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force F(Y)* thru *Work relations* $F \cdot dY = \pm dU$

Q*?Another axiom?* A: No.

$$\int F \cdot dY = \int \frac{dp}{dt} \cdot dY = \int \frac{dY}{dt} \cdot dp = \int V \cdot dp = \int V \cdot d(mV) = m\frac{V^2}{2} + const = U$$

or else :

$$F \cdot \frac{dY}{dt} = \frac{dp}{dt} \cdot V = \frac{d(mV)}{dt} \cdot V = \frac{d(mV^2)/2}{dt} = \frac{dU}{dt} \qquad \text{(Here: } V = v_2\text{)}$$



A "Double-Whammy" system

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

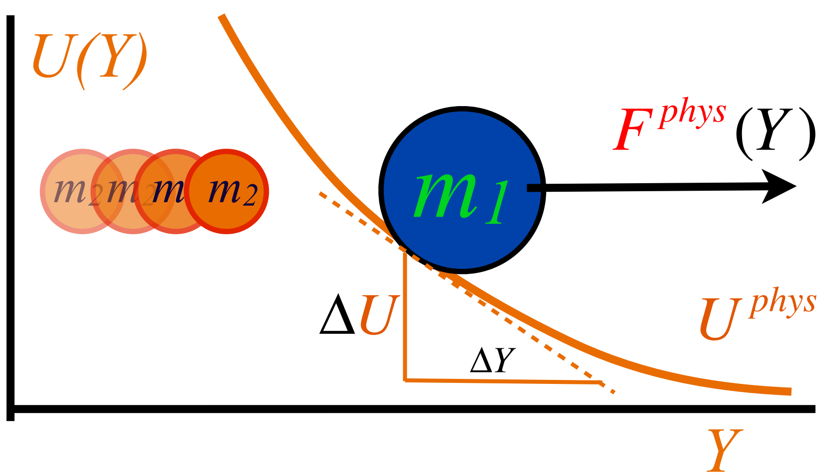
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1: *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

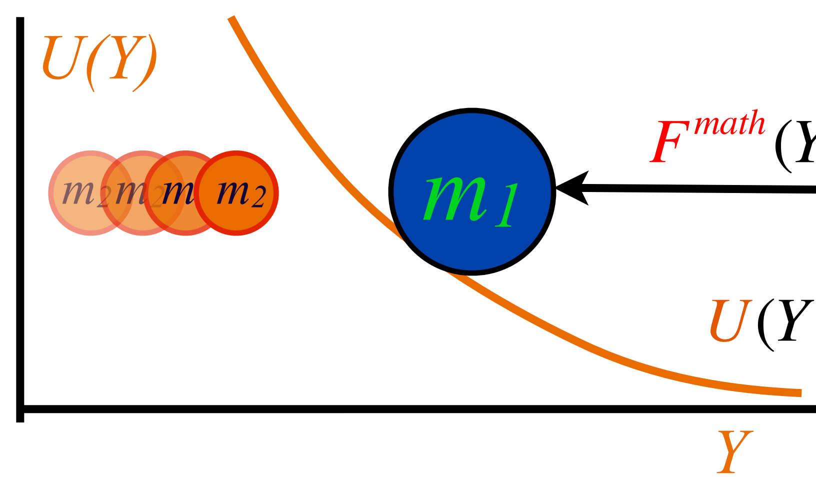
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

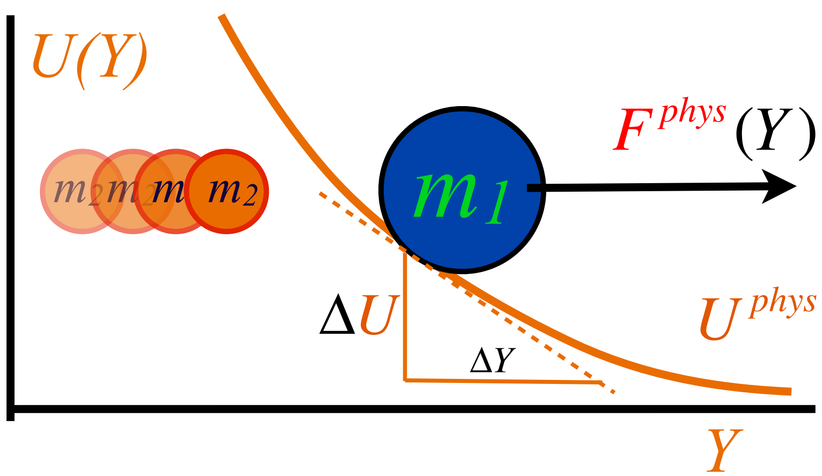
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

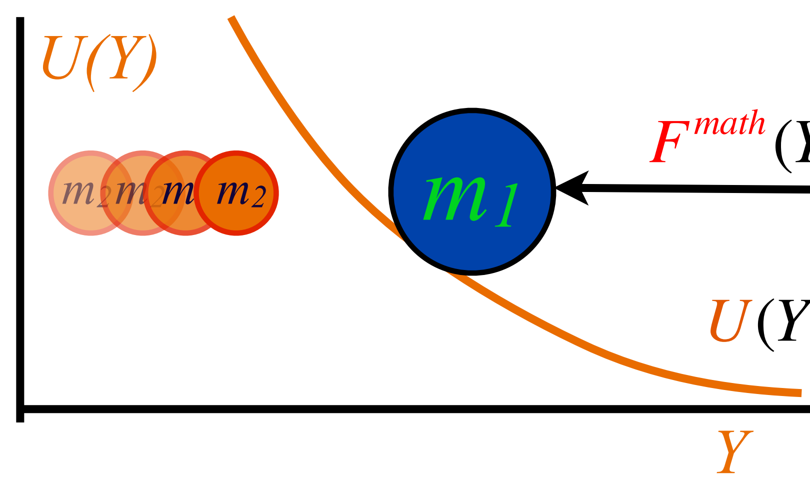
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\, dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\, dY$$

(OK, But, does it work?)

For the "Double-Whammy" system

Big mass-m_1 ball feeling "potential-field" or "gradient" due to small ($m_2 << m_1$) rapidly ($v_2 >> v_1$) bouncing ball

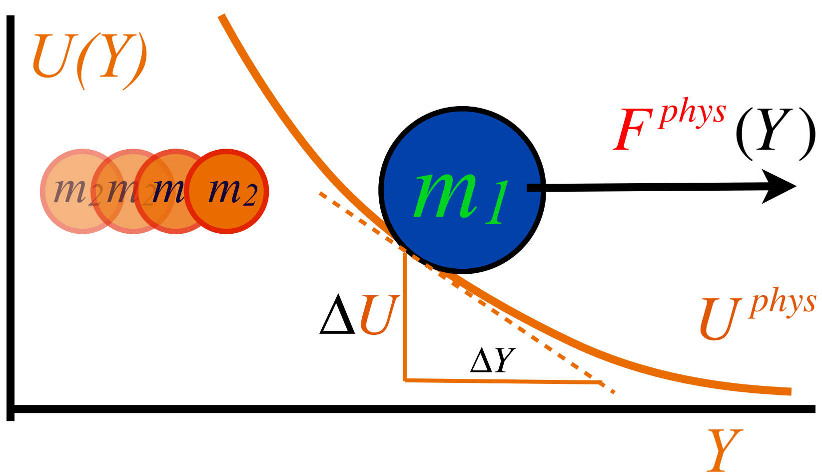
In adiabatic case where $v_2 = \frac{const.}{Y}$ the total energy E is strictly conserved.

$$const. = E = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \boxed{\frac{1}{2}m_1 v_1^2} + \boxed{\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2}$$

Define for big mass m_1 : *Kinetic energy* $KE(v_1)$ *vs Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

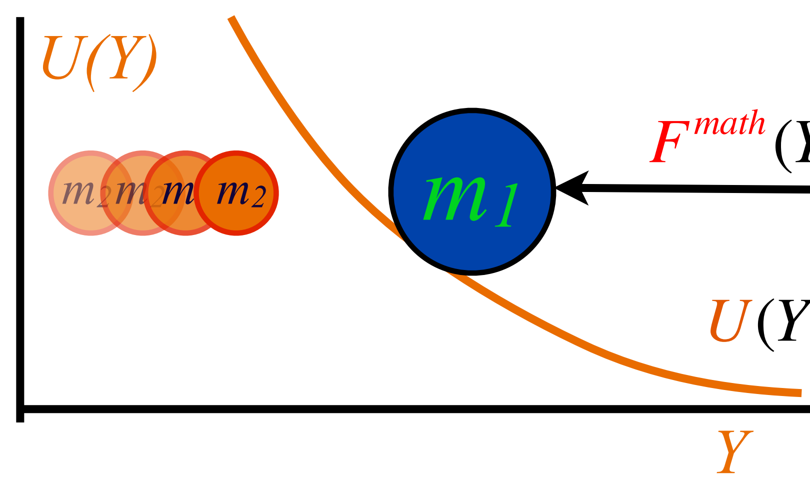
The "Physicist" View of Force



$$F^{phys}(Y) = -\frac{dU}{dY}$$

$$U^{phys}(Y) = -\int F^{phys}\,dY$$

The "Mathematician" View of Force



$$F^{math}(Y) = +\frac{dU}{dY}$$

$$U(Y) = +\int F^{phys}\,dY$$

(OK, But, does it work?)

For the "Double-Whammy" system

$$F^{phys} = m_2\frac{(const.)^2}{Y^3}$$

consistent with :

$$F^{phys} = -\frac{\Delta U}{\Delta Y} = -\frac{d}{dY}\frac{1}{2}m_2\left(\frac{const.}{Y}\right)^2 = m_2\frac{(const.)^2}{Y^3}$$

(Hurrah!)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

$1D$-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$

Not a "Double-Whammy"... ...only a "Single-Whammy"

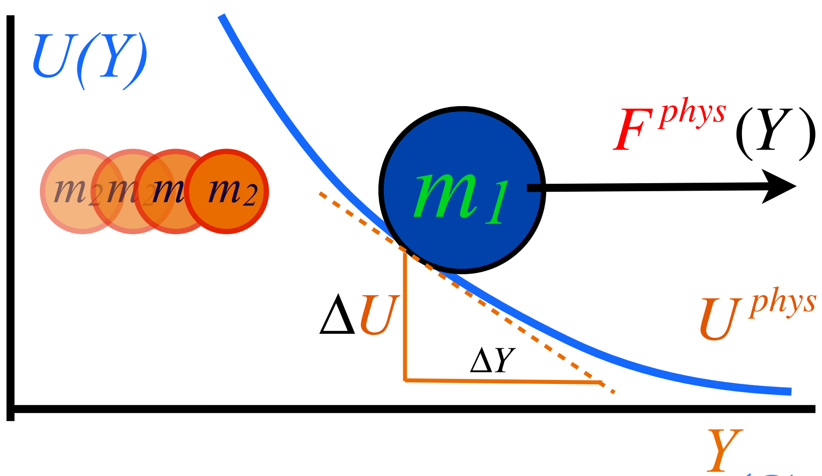
$$F^{phys}=\frac{m_2v_2{}^2}{Y}=-\frac{\Delta U}{\Delta Y}\quad implies: \quad U(Y)=\int -F^{phys}dY=\int -\frac{m_2v_2{}^2}{Y}dY=-m_2v_2{}^2\ln(Y)$$

$$const.=E=\boxed{\frac{1}{2}m_1v_1{}^2}+\boxed{U(Y)\quad where: U(Y)=-m_2v_2{}^2\ln(Y)}$$

Define for big mass m_1: *Kinetic energy* $KE(v_1)$ *vs* *Potential energy* $PE(Y)=U(Y)$

Potential energy $PE(Y)=U(Y)=-m_2v_2{}^2\ln(Y)$ relates to *Force* $F(Y)$ thru *Work relations* $F\cdot dY=\pm dU$

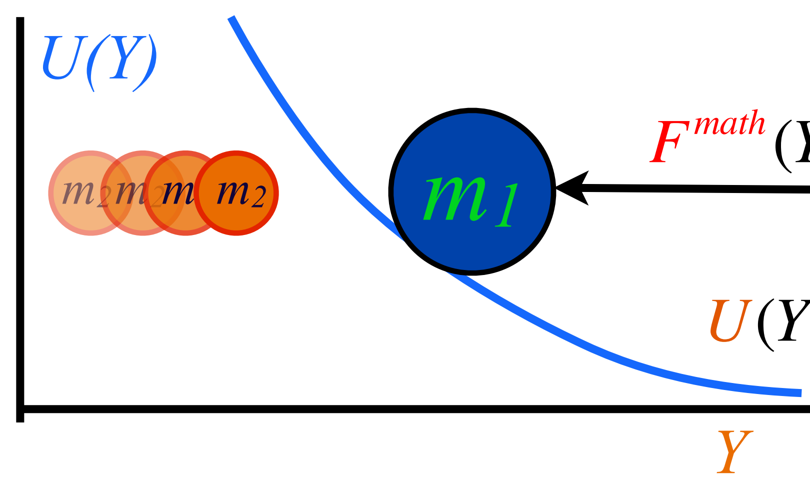
The "Physicist" View of Force



$$F^{phys}(Y)=-\frac{dU}{dY}$$

$$U^{phys}(Y)=-\int F^{phys}\,dY$$

The "Mathematician" View of Force



$$F^{math}(Y)=+\frac{dU}{dY}$$

$$U(Y)=+\int F^{phys}\,dY$$

(Same integral/differential relations)

$$F^{phys}=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}\qquad \textit{consistent with:}\qquad F^{phys}=-\frac{\Delta U}{\Delta Y}=-\frac{d}{dY}(-const.\ln(Y))=\frac{const.}{Y}$$

(Hurrah! again)

Potential field due to many small bounces

Example of 1D-Adiabatic potential $U(y)=const./y^2$

Physicist's Definition $F=-\Delta U/\Delta y$ vs. Mathematician's Definition $F=+\Delta U/\Delta y$

Example of 1D-Isothermal potential $U(y)=const.\ ln(y)$

Example of oscillator with opposing Isothermal potentials

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \qquad \text{implies :} \qquad U = -m_2 v_2^2 \ln(Y)$$

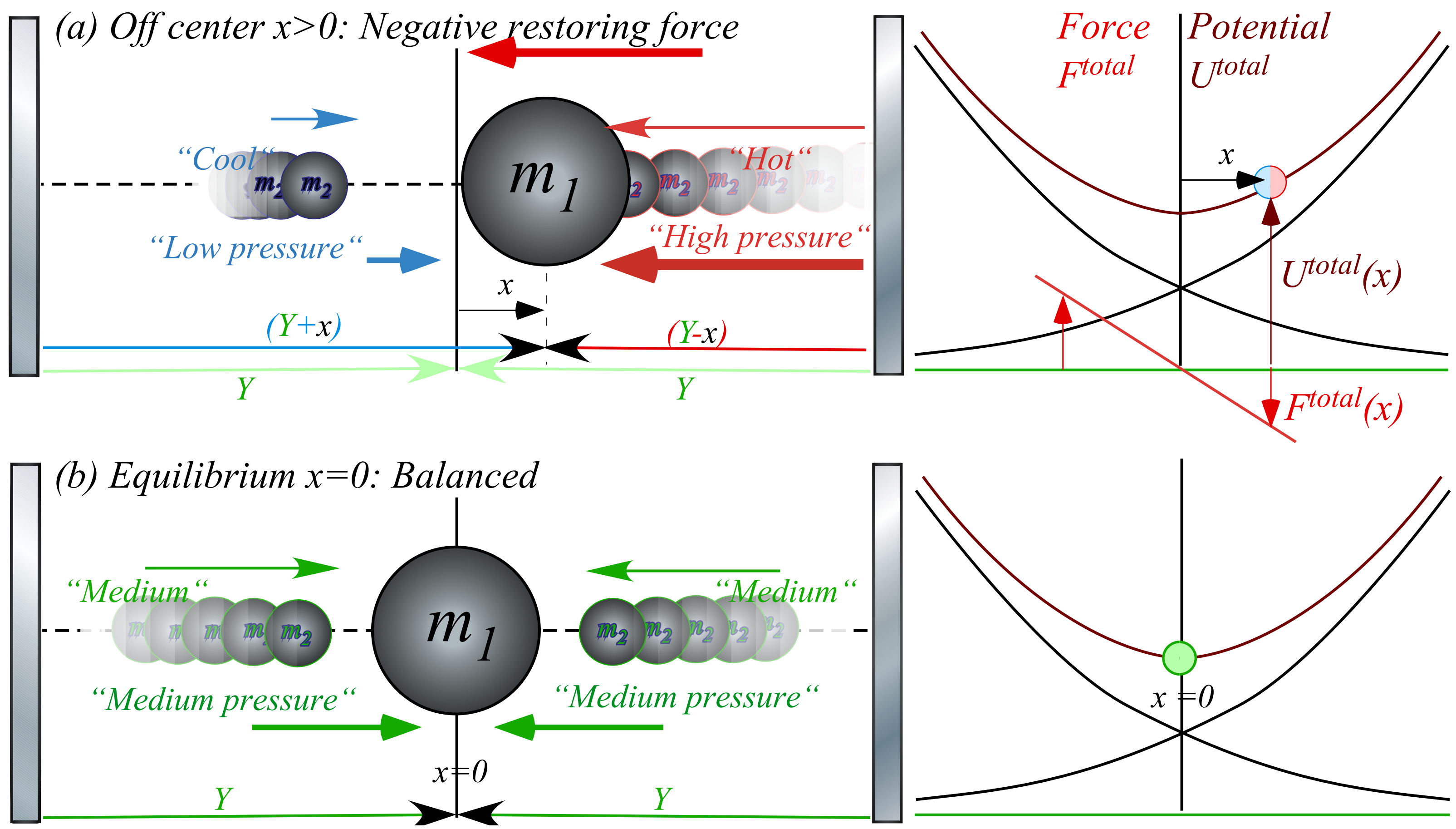


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x}$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \quad \text{implies :} \quad U = -m_2 v_2^2 \ln(Y)$$

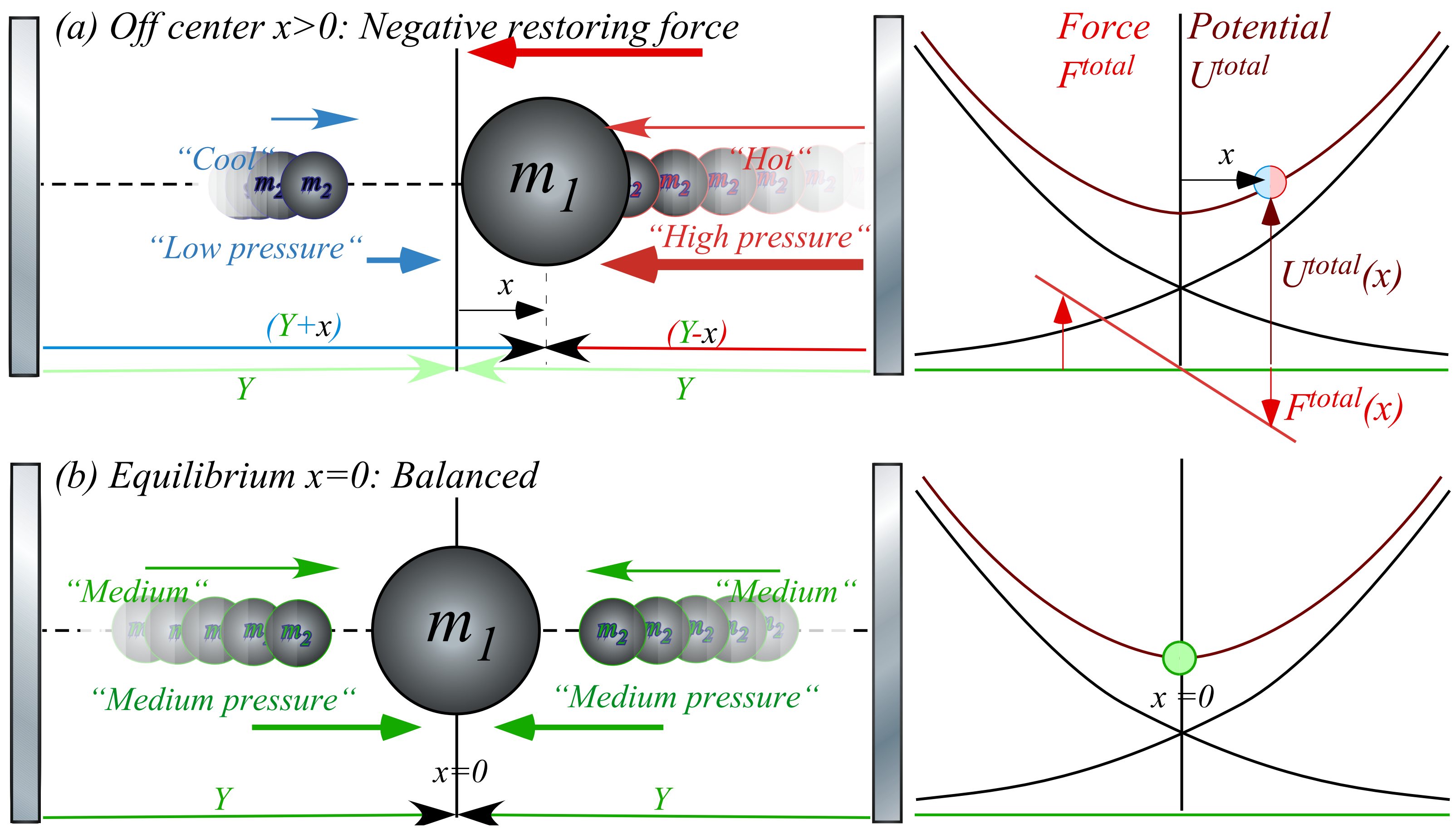


Fig. 5.3 Unit 1

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x}$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots$$

$$\boxed{(Y_0 + x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n-1)}{2}Y_0^{n-2}x^2 + \frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3 + \frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4 \ldots}$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \qquad \textit{implies :} \qquad U=-m_2v_2^2\ln(Y)$$

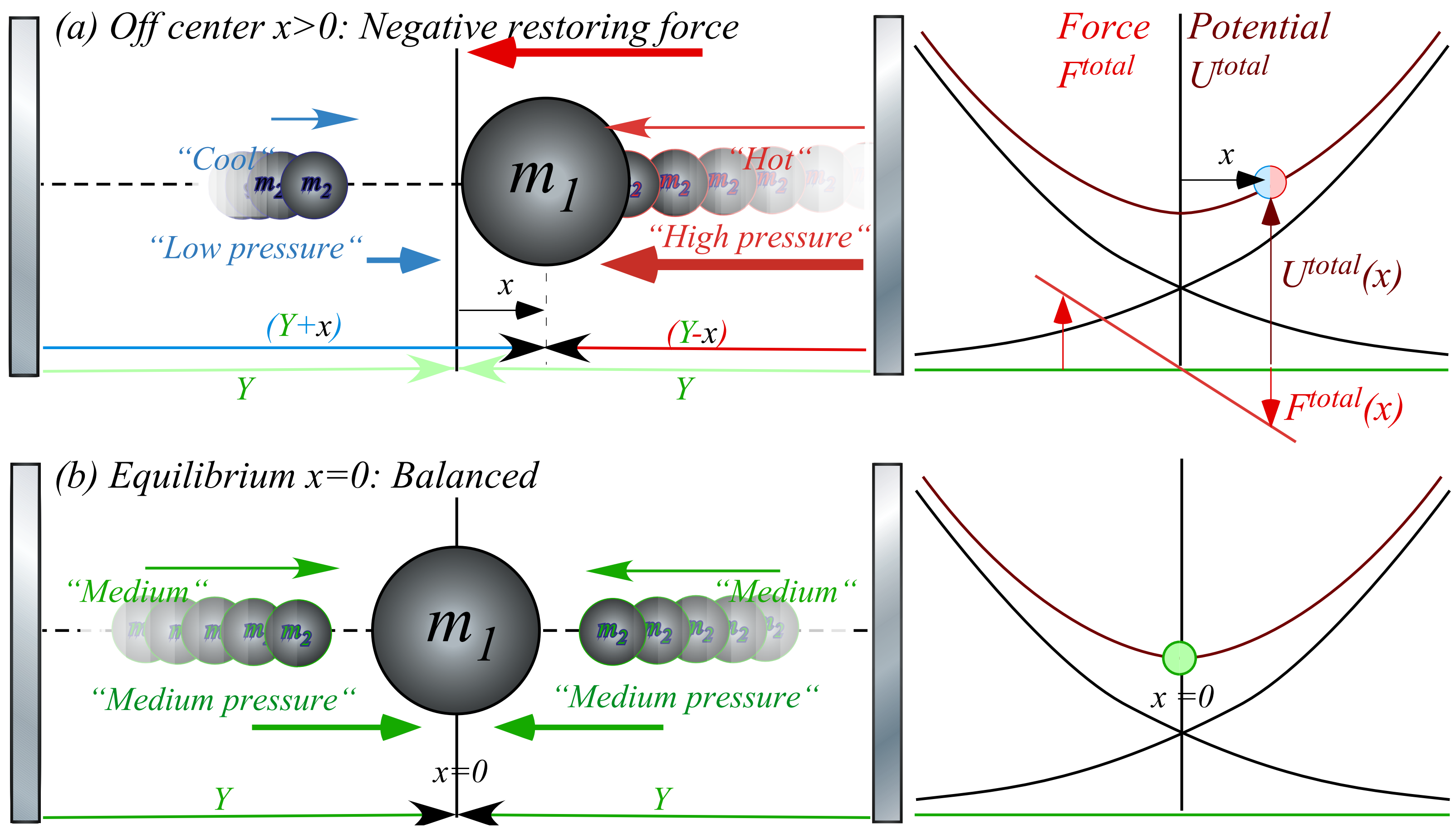


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\frac{1}{Y_0}-\frac{x}{Y_0^2}+\frac{x^2}{Y_0^3}-\frac{x^3}{Y_0^4}+\cdots\right]-\ldots$$

$$(Y_0+x)^{-1}=Y_0^{-1}-xY_0^{-2}+x^2Y_0^{-3}-x^3Y_0^{-4}\ldots$$

$$\boxed{(Y_0+x)^n=Y_0^n+nY_0^{n-1}x+\frac{n(n-1)}{2}Y_0^{n-2}x^2+\frac{n(n-1)(n-2)}{2\cdot 3}Y_0^{n-3}x^3+\frac{n(n-1)(n-2)(n-3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4\ldots}$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y): $\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \qquad \textit{implies :} \qquad U = -m_2 v_2^2 \ln(Y)$$

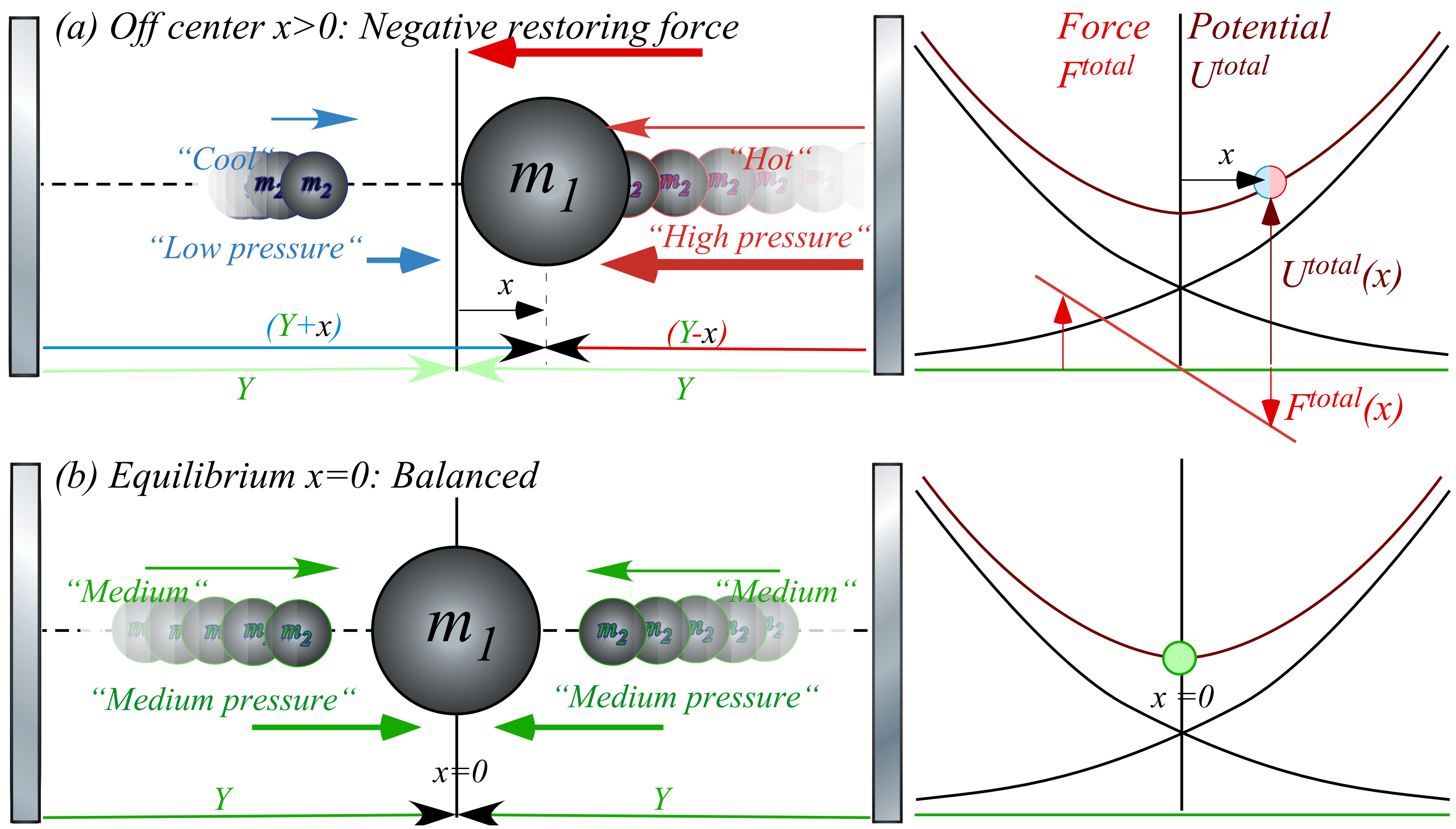


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\frac{1}{Y_0} - \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\frac{1}{Y_0} + \frac{x}{Y_0^2} + \frac{x^2}{Y_0^3} + \frac{x^3}{Y_0^4} + \cdots\right]$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots \qquad (Y_0 - x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2 Y_0^{-3} + x^3 Y_0^{-4} \ldots$$

$$\boxed{(Y_0 + x)^n = Y_0^n + nY_0^{n-1} x + \frac{n(n\text{-}1)}{2} Y_0^{n-2} x^2 + \frac{n(n\text{-}1)(n\text{-}2)}{2\cdot 3} Y_0^{n-3} x^3 + \frac{n(n\text{-}1)(n\text{-}2)(n\text{-}3)}{2\cdot 3\cdot 4} Y_0^{n-4} x^4 \ldots}$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \qquad \textit{implies :} \qquad U = -m_2 v_2^2 \ln(Y)$$

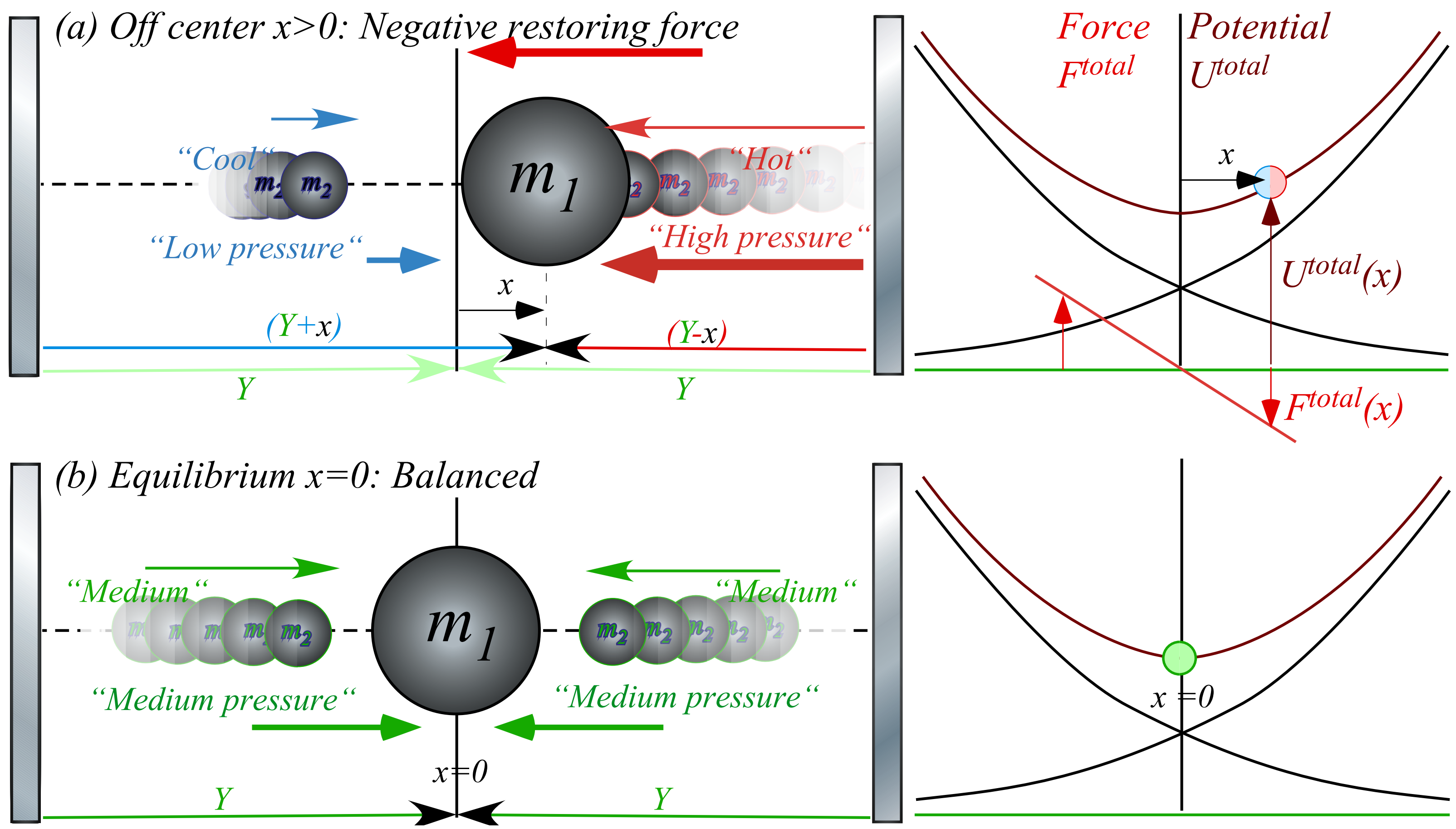


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\cancel{\frac{1}{Y_0}} - \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\cancel{\frac{1}{Y_0}} + \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} + \frac{x^3}{Y_0^4} + \cdots\right]$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots \qquad (Y_0 - x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2 Y_0^{-3} + x^3 Y_0^{-4} \ldots$$

$$(Y_0 + x)^n = Y_0^n + nY_0^{n-1} x + \frac{n(n-1)}{2} Y_0^{n-2} x^2 + \frac{n(n-1)(n-2)}{2 \cdot 3} Y_0^{n-3} x^3 + \frac{n(n-1)(n-2)(n-3)}{2 \cdot 3 \cdot 4} Y_0^{n-4} x^4 \ldots$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F = \frac{m_2 v_2^2}{Y} = \frac{const.}{Y}}$$

$$F^{phys} = \frac{m_2 v_2^2}{Y} = -\frac{\Delta U}{\Delta Y} \qquad implies: \qquad U = -m_2 v_2^2 \ln(Y)$$

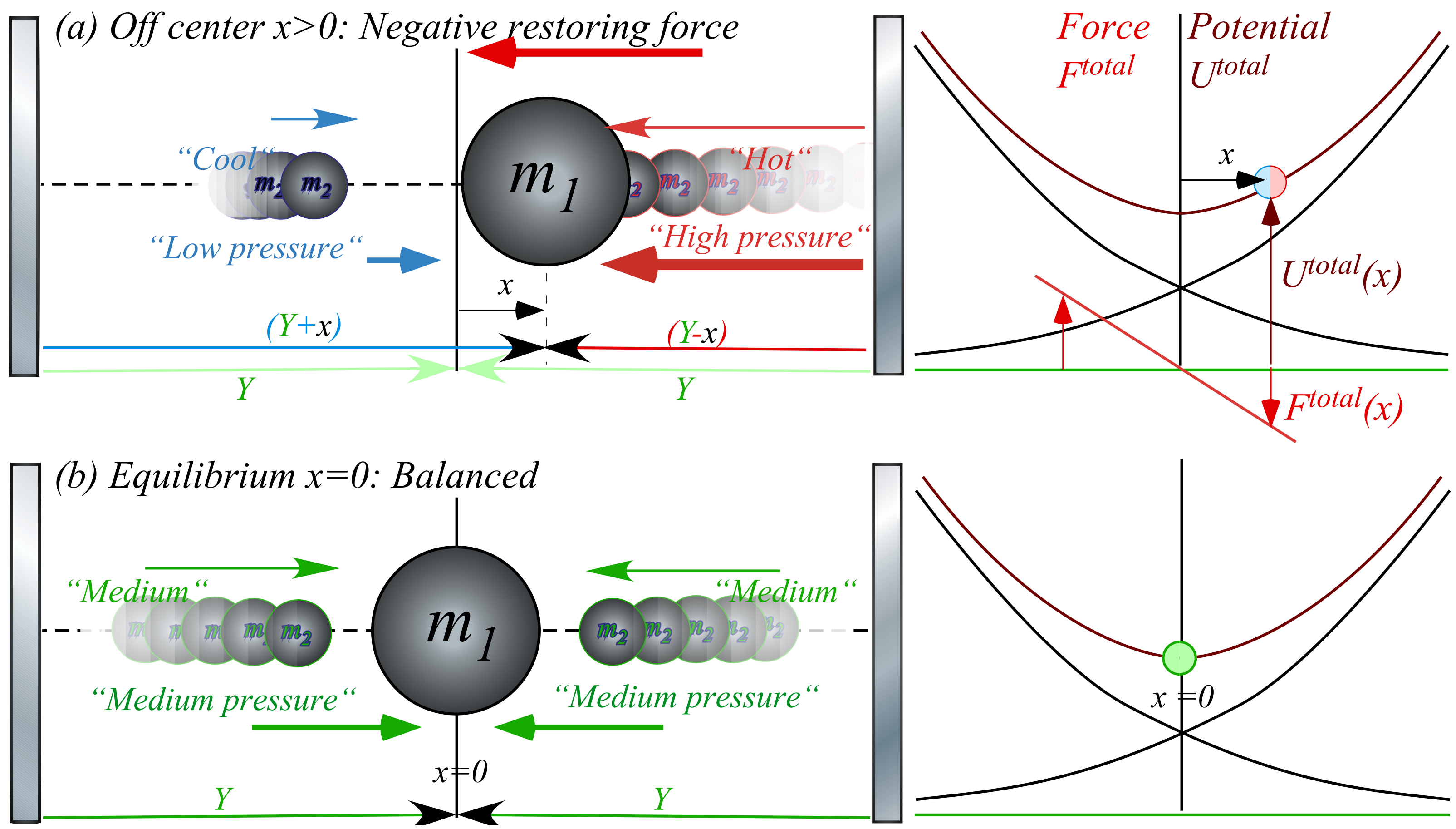


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total} = \frac{f}{Y_0 + x} - \frac{f}{Y_0 - x} = f\left[\cancel{\frac{1}{Y_0}} - \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} - \frac{x^3}{Y_0^4} + \cdots\right] - f\left[\cancel{\frac{1}{Y_0}} + \frac{x}{Y_0^2} + \cancel{\frac{x^2}{Y_0^3}} + \frac{x^3}{Y_0^4} + \cdots\right] = -2f\frac{x}{Y_0^2} - 2f\frac{x^3}{Y_0^4} - \ldots$$

$$(Y_0 + x)^{-1} = Y_0^{-1} - xY_0^{-2} + x^2 Y_0^{-3} - x^3 Y_0^{-4} \ldots \qquad (Y_0 - x)^{-1} = Y_0^{-1} + xY_0^{-2} + x^2 Y_0^{-3} + x^3 Y_0^{-4} \ldots$$

$$\boxed{(Y_0 + x)^n = Y_0^n + nY_0^{n-1}x + \frac{n(n\text{-}1)}{2}Y_0^{n-2}x^2 + \frac{n(n\text{-}1)(n\text{-}2)}{2\cdot 3}Y_0^{n-3}x^3 + \frac{n(n\text{-}1)(n\text{-}2)(n\text{-}3)}{2\cdot 3\cdot 4}Y_0^{n-4}x^4 \ldots}$$

Binomial Theorem

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \qquad \textit{implies :} \qquad U=-m_2v_2^2\ln(Y)$$

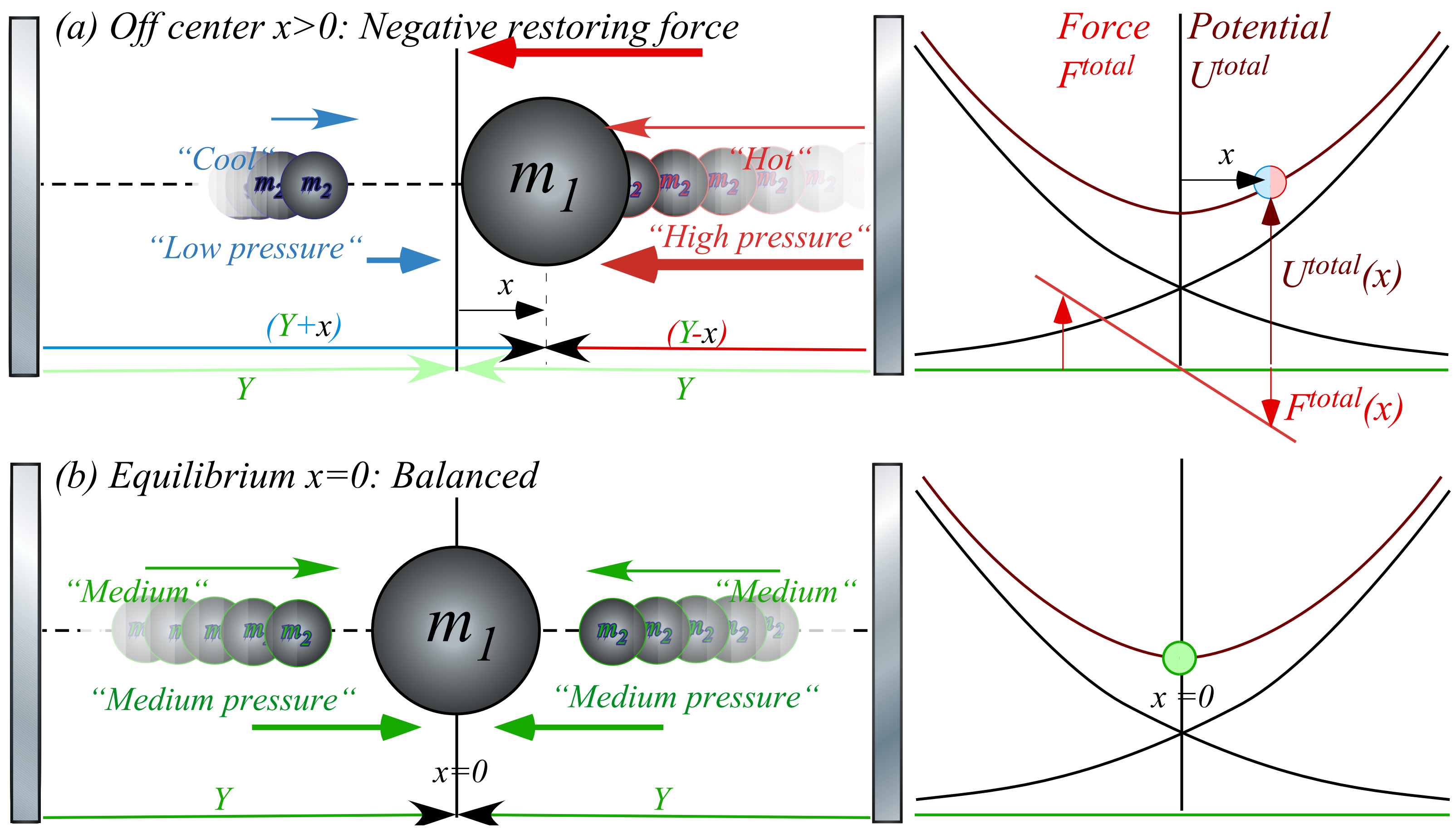


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Anharmonic oscillator terms...

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\overbrace{\frac{x}{Y_0^2}}^{\text{Harmonic oscillator term}}-2f\frac{x^3}{Y_0^4}-\dots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \quad \textit{implies :} \quad U=-m_2 v_2^2\ln(Y)$$

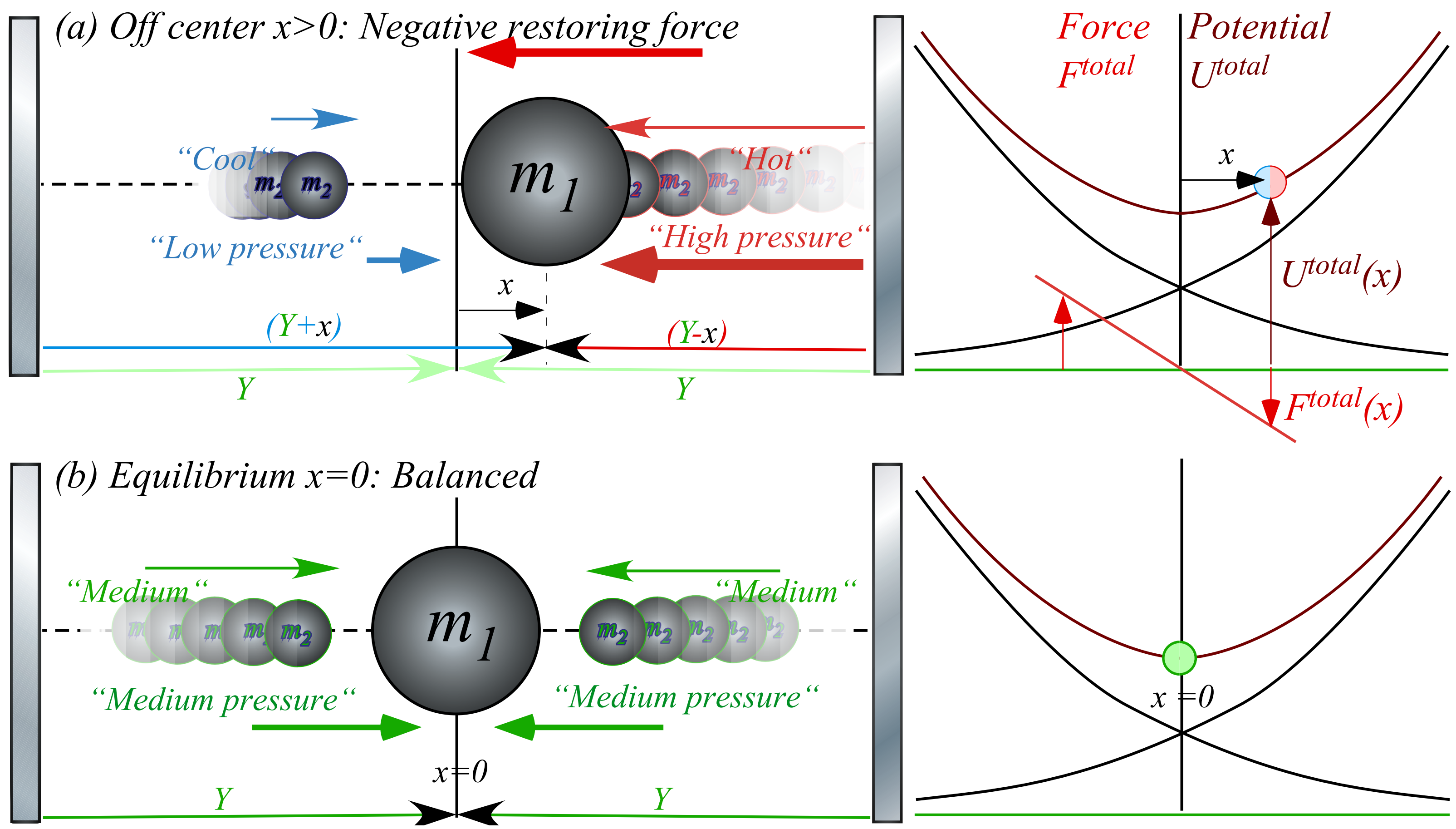


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\frac{x}{Y_0^2}-2f\frac{x^3}{Y_0^4}-\ldots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

Example of oscillator with opposing *Isothermal potentials*

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \qquad \textit{implies :} \qquad U=-m_2 v_2^2 \ln(Y)$$

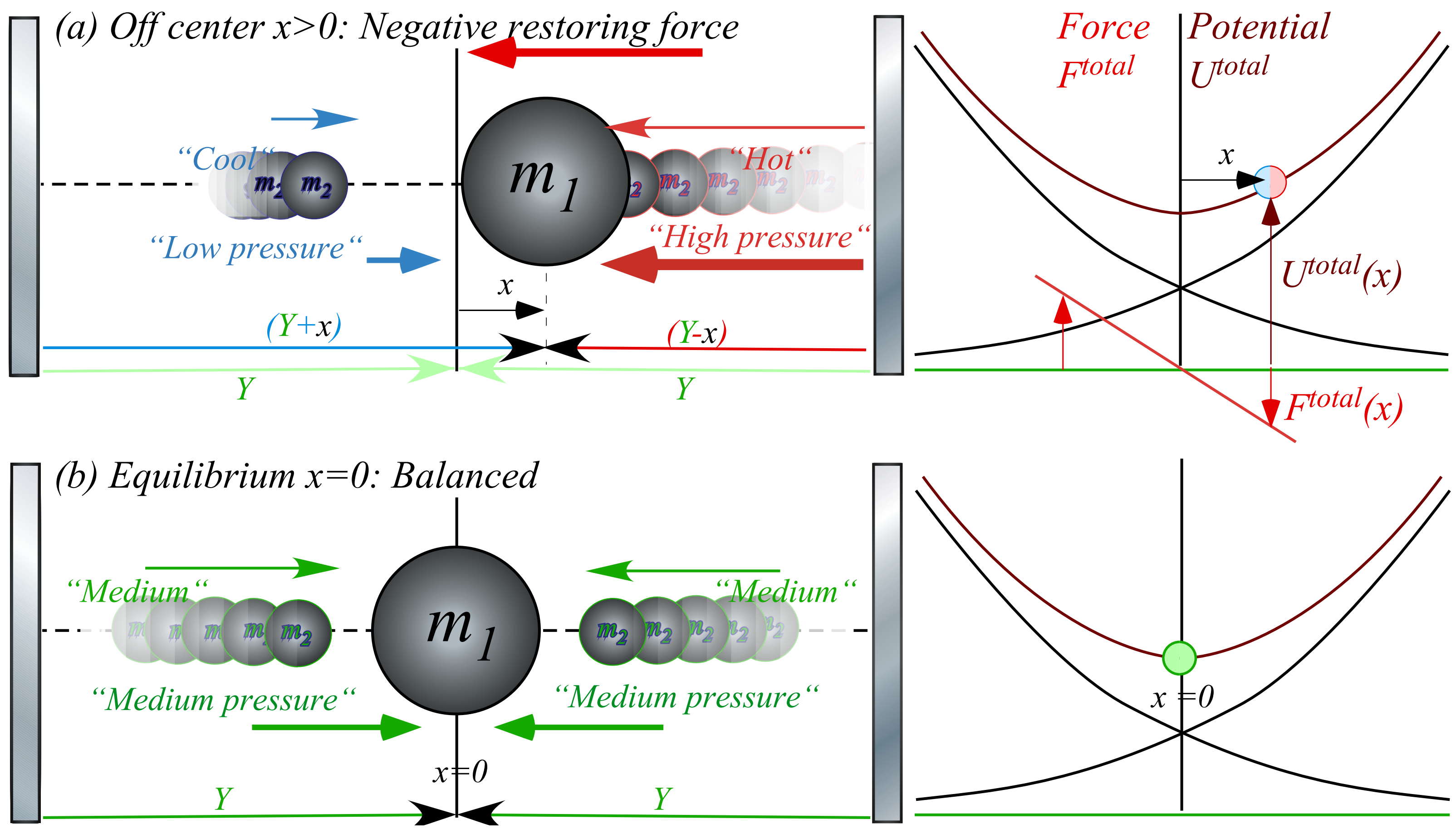


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\underbrace{\frac{x}{Y_0^2}}-2f\frac{x^3}{Y_0^{-4}}-\ldots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2v_2^2/Y_0^2$

Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

Potential

$$U^{HO}=\frac{1}{2}k\cdot x^2=-\int F^{HO}\,dx$$

Example of oscillator with opposing Isothermal potentials

1D-Isothermal Force Law (assume v_2 is constant for all Y):

$$\boxed{F=\frac{m_2 v_2^2}{Y}=\frac{const.}{Y}}$$

$$F^{phys}=\frac{m_2 v_2^2}{Y}=-\frac{\Delta U}{\Delta Y} \qquad \textit{implies :} \qquad U=-m_2 v_2^2 \ln(Y)$$

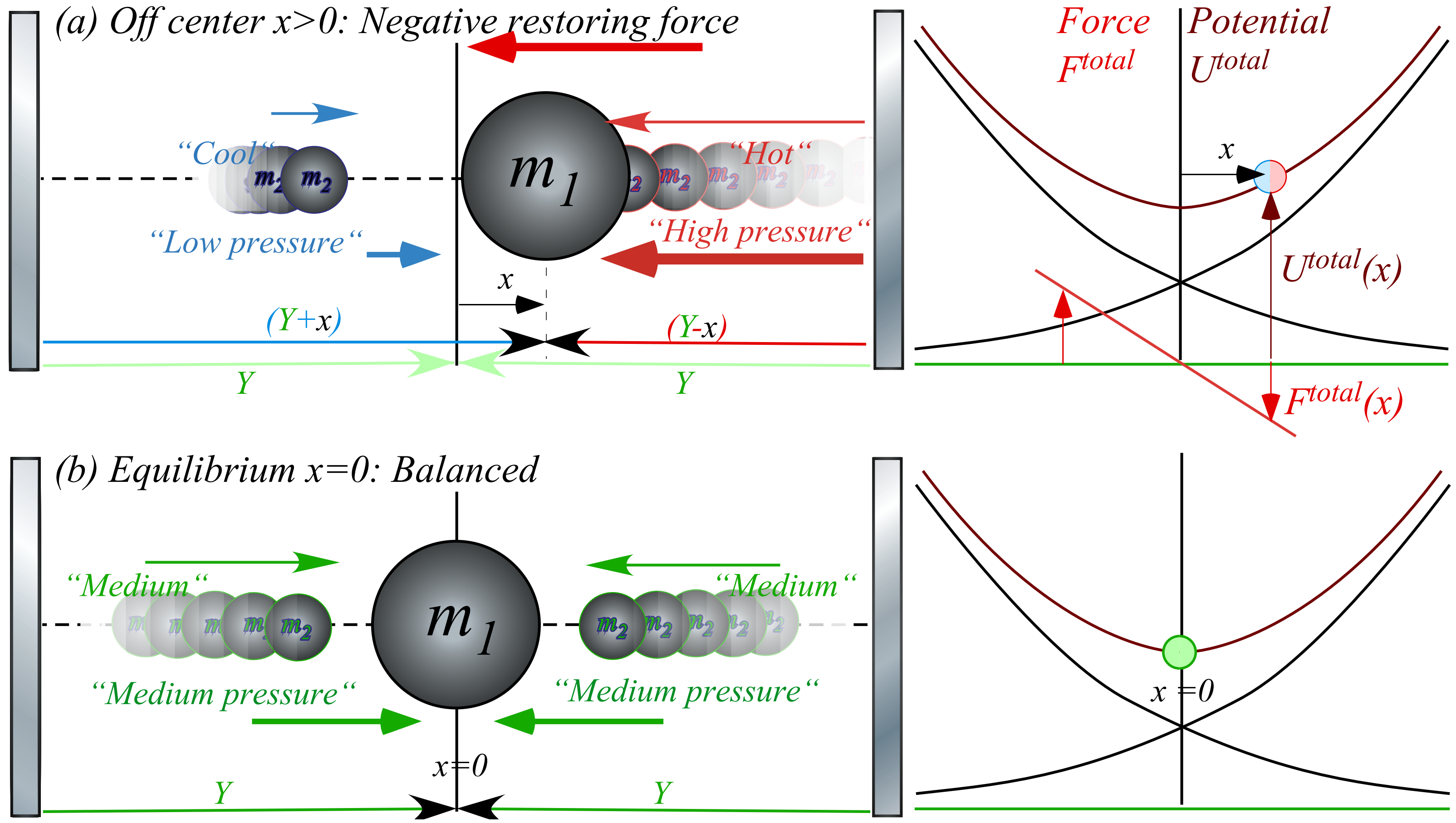


Fig. 5.3
Unit 1

Two opposing 1D-Isothermal Force fields approximate harmonic oscillator

Harmonic oscillator term

Anharmonic oscillator terms...

$$F^{total}=\frac{f}{Y_0+x}-\frac{f}{Y_0-x}=f\left[\cancel{\frac{1}{Y_0}}-\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}-\frac{x^3}{Y_0^4}+\cdots\right]-f\left[\cancel{\frac{1}{Y_0}}+\frac{x}{Y_0^2}+\cancel{\frac{x^2}{Y_0^3}}+\frac{x^3}{Y_0^4}+\cdots\right]=-2f\frac{x}{Y_0^2}-2f\frac{x^3}{Y_0^4}-\ldots$$

Harmonic oscillator force constant : $k=2f/Y_0^2=2m_2 v_2^2/Y_0^2$

Harmonic Oscillator Force

$$F^{HO}=-k\cdot x=-\frac{\partial U^{HO}}{\partial x}$$

Potential

$$U^{HO}=\frac{1}{2}k\cdot x^2=-\int F^{HO}\,dx$$

Frequency

HO ∡frequency: $\omega=\sqrt{\frac{k}{m_1}}=\sqrt{\frac{2m_2}{m_1}}\frac{v_2}{Y_0}=2\pi\upsilon$

What does *Harmonic* mean?

Given total energy $E = KE+PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* constant for *any* amplitude A of sine-oscillation where:

$Y = A\sin\omega t$ with velocity $V = A\omega\cos\omega t$

What does *Harmonic* mean?

Given total energy $E = KE{+}PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* constant for *any* amplitude A of sine-oscillation where:

$$Y = A\sin\omega t \quad \text{with velocity} \quad V = A\omega\cos\omega t$$

Because then: $E = \frac{1}{2}m\left(A\omega\cos\omega t\right)^2 + \frac{1}{2}k\left(A\sin\omega t\right)^2$

What does *Harmonic* mean?

Given total energy $E = KE{+}PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* constant for *any* amplitude A of sine-oscillation where:

$Y = A\sin\omega t$ with velocity $V = A\omega\cos\omega t$

Because then: $E = \frac{1}{2}m\left(A\omega\cos\omega t\right)^2 + \frac{1}{2}k(A\sin\omega t)^2$

$$= \frac{1}{2}m\omega^2 A^2\left(\cos\omega t\right)^2 + \frac{1}{2}kA^2(\sin\omega t)^2$$

What does *Harmonic* mean?

Given total energy $E = KE{+}PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* constant for *any* amplitude A of sine-oscillation where:

$Y = A\sin\omega t$ with velocity $V = A\omega\cos\omega t$

Because then: $E = \frac{1}{2}m\left(A\omega\cos\omega t\right)^2 + \frac{1}{2}k(A\sin\omega t)^2$

$$= \frac{1}{2}m\omega^2 A^2\left(\cos\omega t\right)^2 + \frac{1}{2}kA^2(\sin\omega t)^2$$

$$= \frac{1}{2}m\omega^2 A^2\left(\cos^2\omega t + \sin^2\omega t\right)^2 \quad \text{if: } m\omega^2 = k$$

What does *Harmonic* mean?

Given total energy $E = KE+PE = \frac{1}{2}mV^2 + \frac{1}{2}kY^2$

E is *same* constant for *any* amplitude A of sine-oscillation where:

$Y = A\sin\omega t$ with velocity $V = A\omega\cos\omega t$

Because then: $E = \frac{1}{2}m\left(A\omega\cos\omega t\right)^2 + \frac{1}{2}k(A\sin\omega t)^2$

$$= \frac{1}{2}m\omega^2A^2\left(\cos\omega t\right)^2 + \frac{1}{2}kA^2(\sin\omega t)^2$$

$$= \frac{1}{2}m\omega^2A^2\left(\cos^2\omega t + \sin^2\omega t\right)^2 \quad \text{if: } m\omega^2 = k$$

$$= \frac{1}{2}m\omega^2A^2 \qquad \text{if: } \omega = \sqrt{\frac{k}{m}}$$

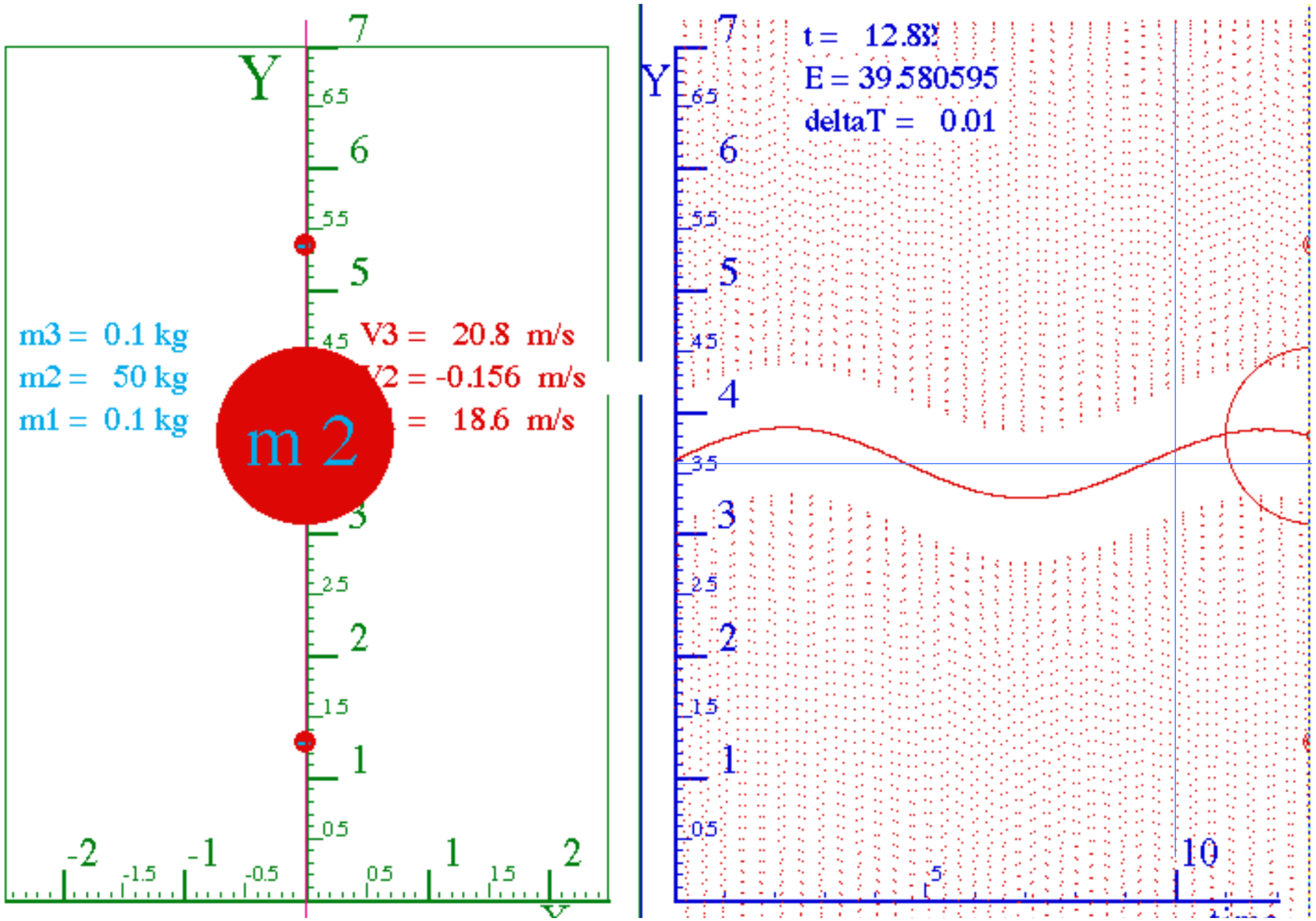


Fig. 5.3
Unit 1

Simulation of
the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute frequency and/or period*

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$

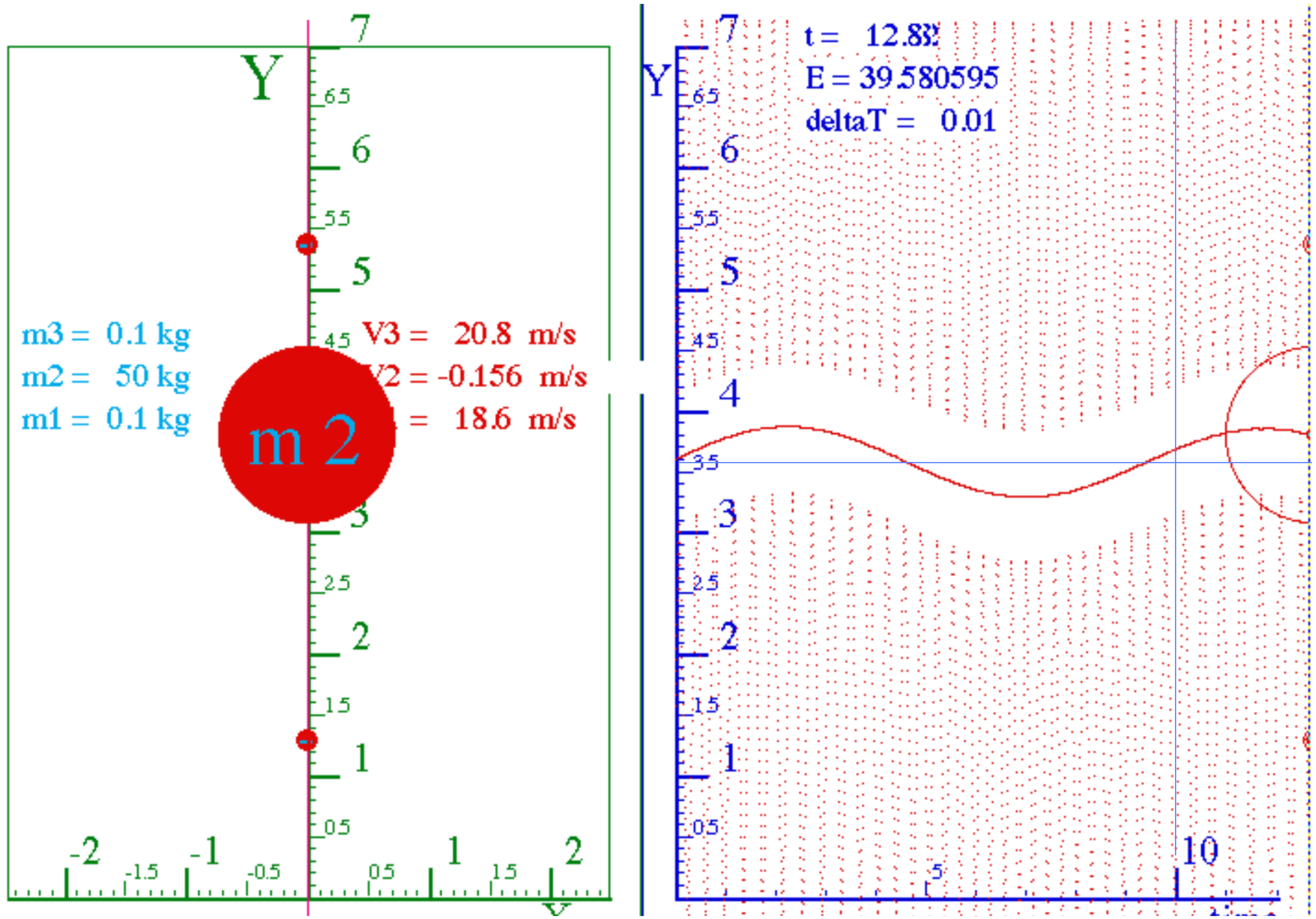


Fig. 5.3 Unit 1

Simulation of the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute frequency and/or period*

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

Frequency

HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$

Switch
$m_1=m_3$
with
m_2
to match
formula

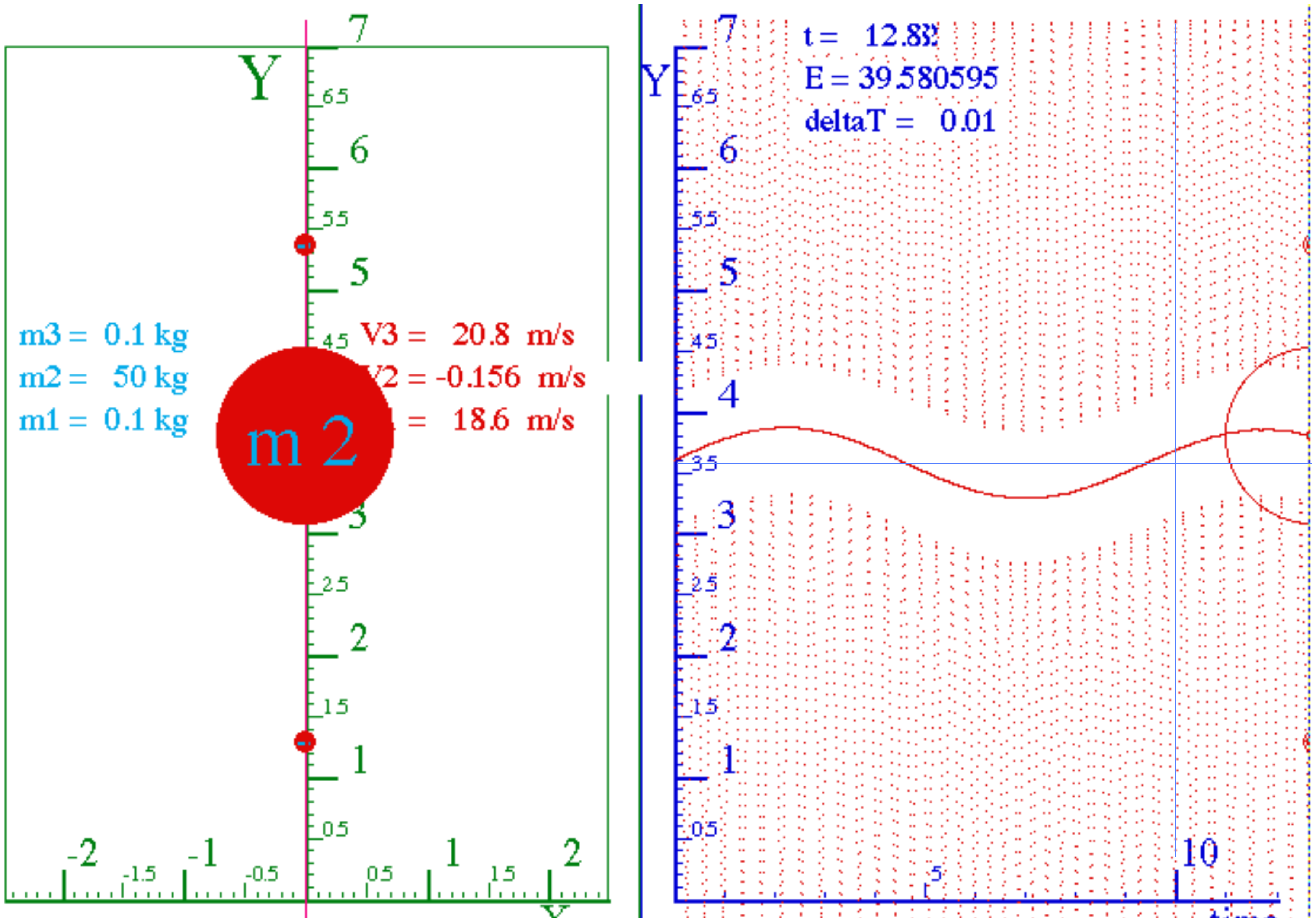


Fig. 5.3
Unit 1

Simulation of
the adiabatic case

[BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)]()

Sample problem: *Compute period given $m_1=50$, $m_2=0.1=m_3$, $v_2=20$, $Y_0=3.5$*

Period :

$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

$$=17.38$$

Period : $\tau = \dfrac{1}{\upsilon} = 2\pi\sqrt{\dfrac{m_1}{k}} = 2\pi\sqrt{\dfrac{m_1}{2m_2}\dfrac{Y_0}{v_2}}$

Frequency

HO ∡frequency: $\omega = \sqrt{\dfrac{k}{m_1}} = \sqrt{\dfrac{2m_2}{m_1}\dfrac{v_2}{Y_0}} = 2\pi\upsilon$

Switch $m_1=m_3$ with m_2 to match formula

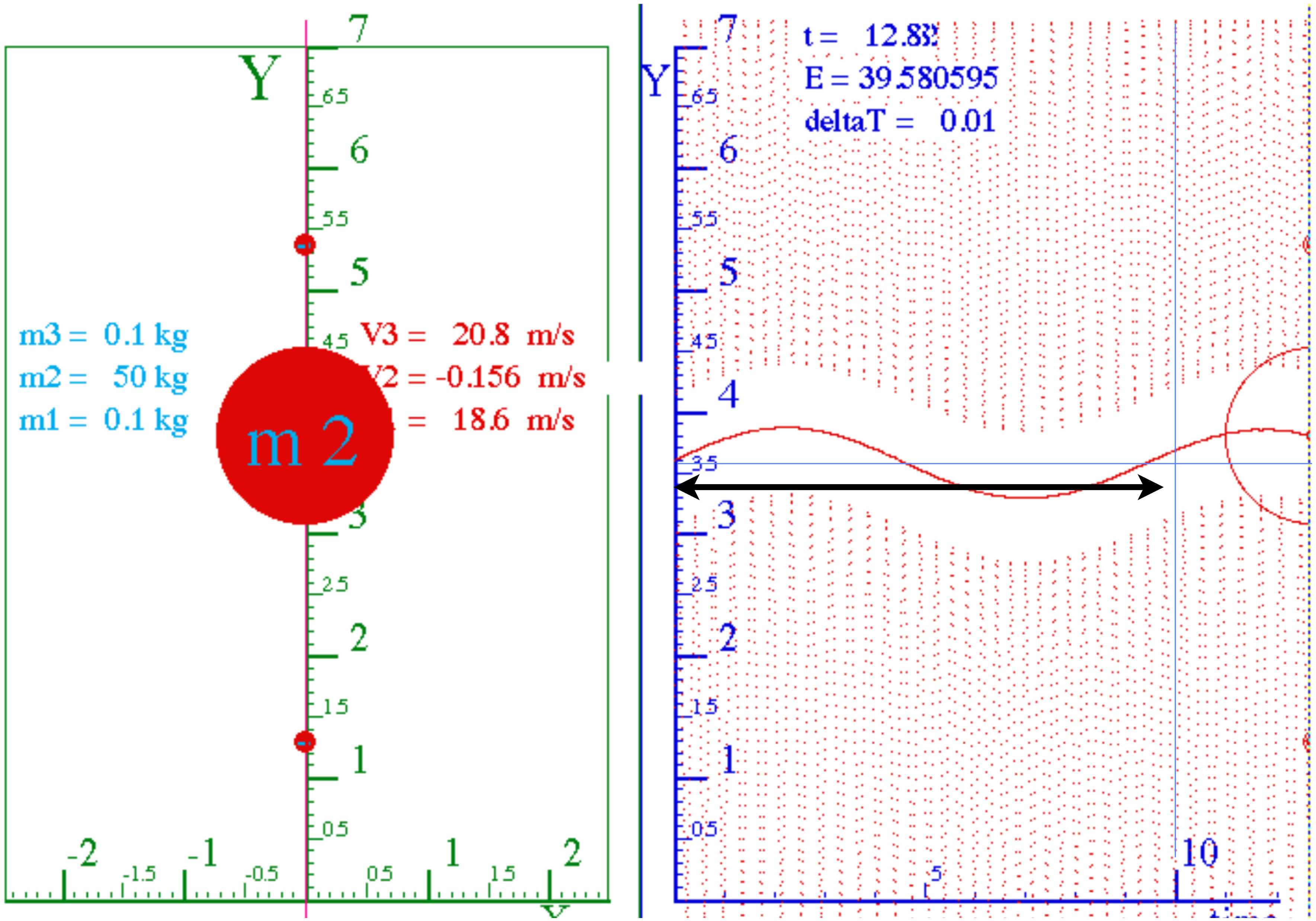


Fig. 5.3 Unit 1

Simulation of the adiabatic case

[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

Sample problem: *Compute period given $m_1=50$, $m_2=0.1=m_3$, $v_2=20$, $Y_0=3.5$*

Period : $\tau = \frac{1}{\upsilon} = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}}$

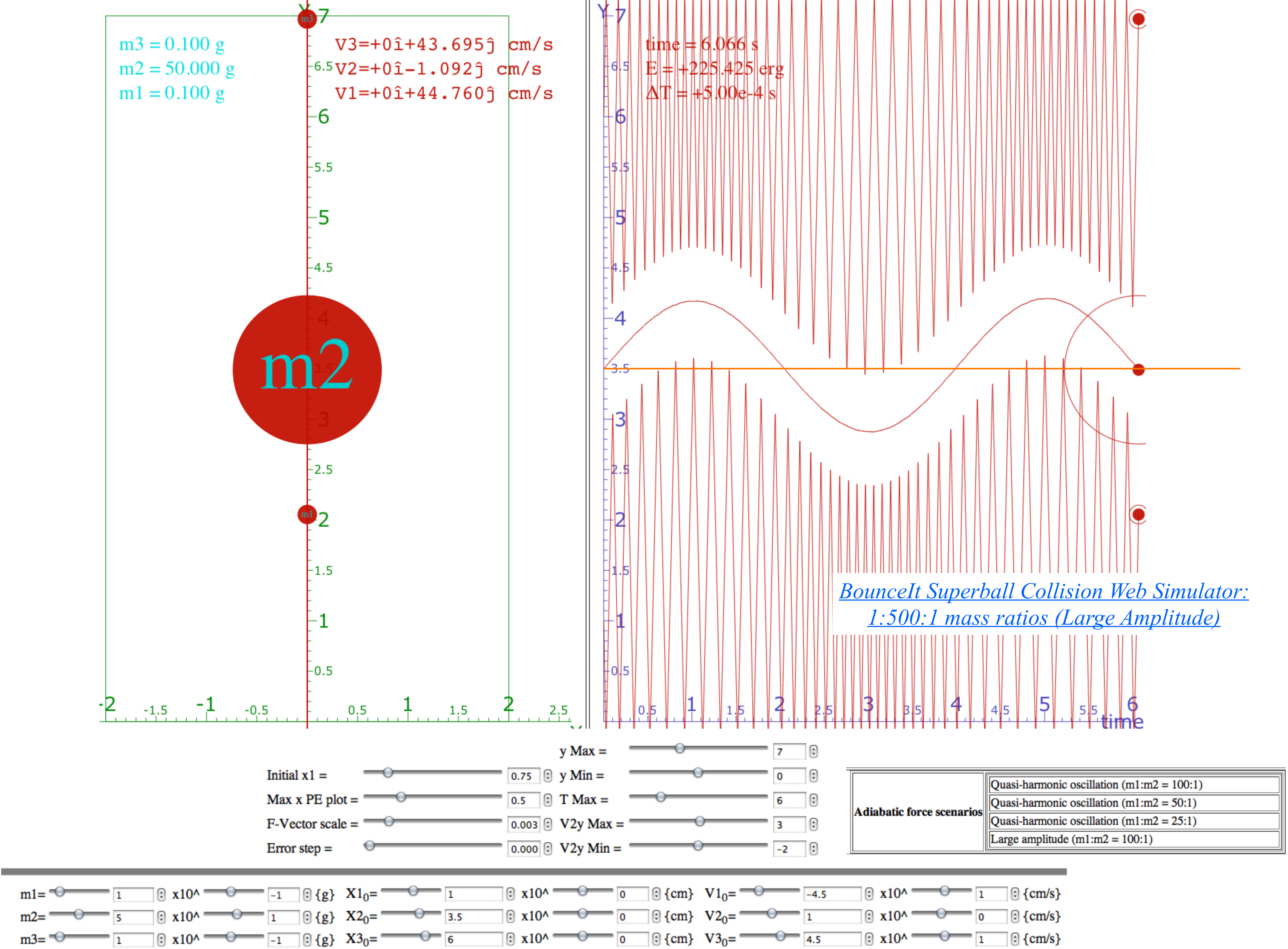
Period :

$$\tau = 2\pi\sqrt{\frac{m_1}{2m_2}\frac{Y_0}{v_2}} = 6.28\sqrt{\frac{50}{2\cdot(0.1)}\frac{3.5}{20}}$$

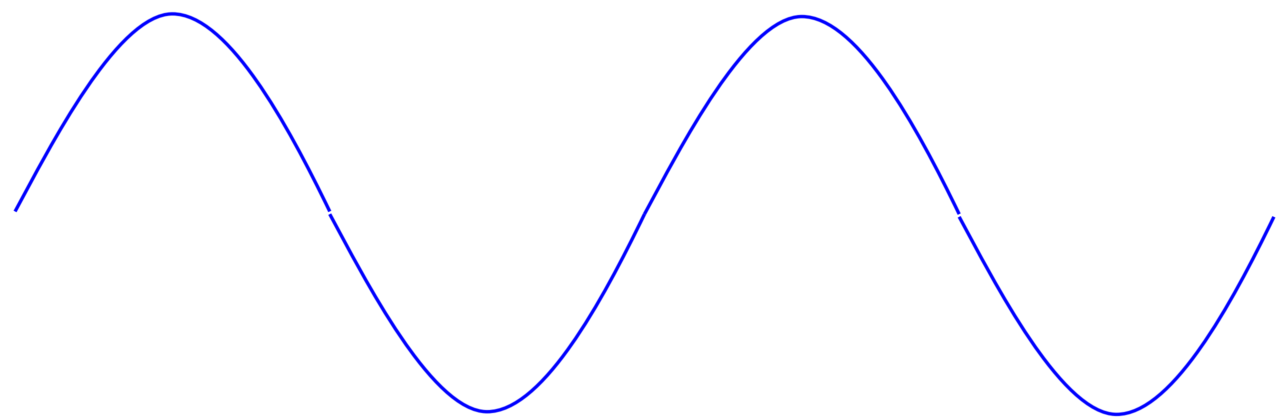
$=17.38$ *That's about $\sqrt{3}$ times too big!*

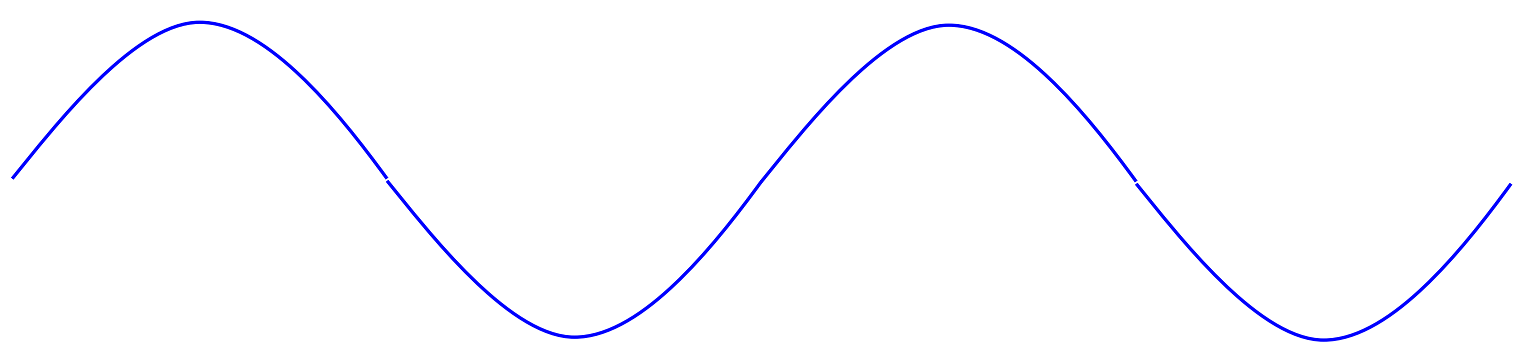
Frequency

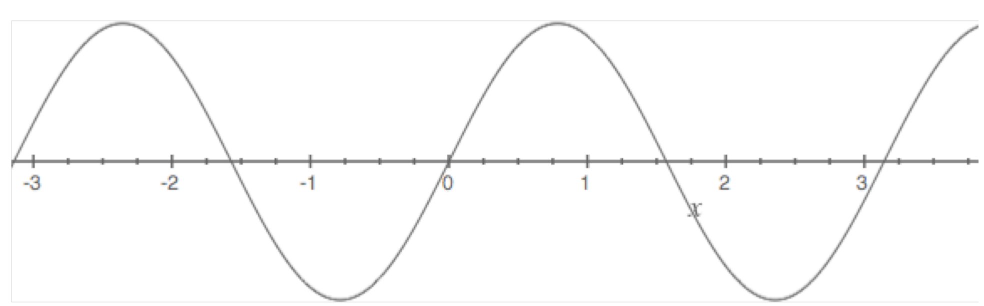
HO ∡frequency: $\omega = \sqrt{\frac{k}{m_1}} = \sqrt{\frac{2m_2}{m_1}\frac{v_2}{Y_0}} = 2\pi\upsilon$



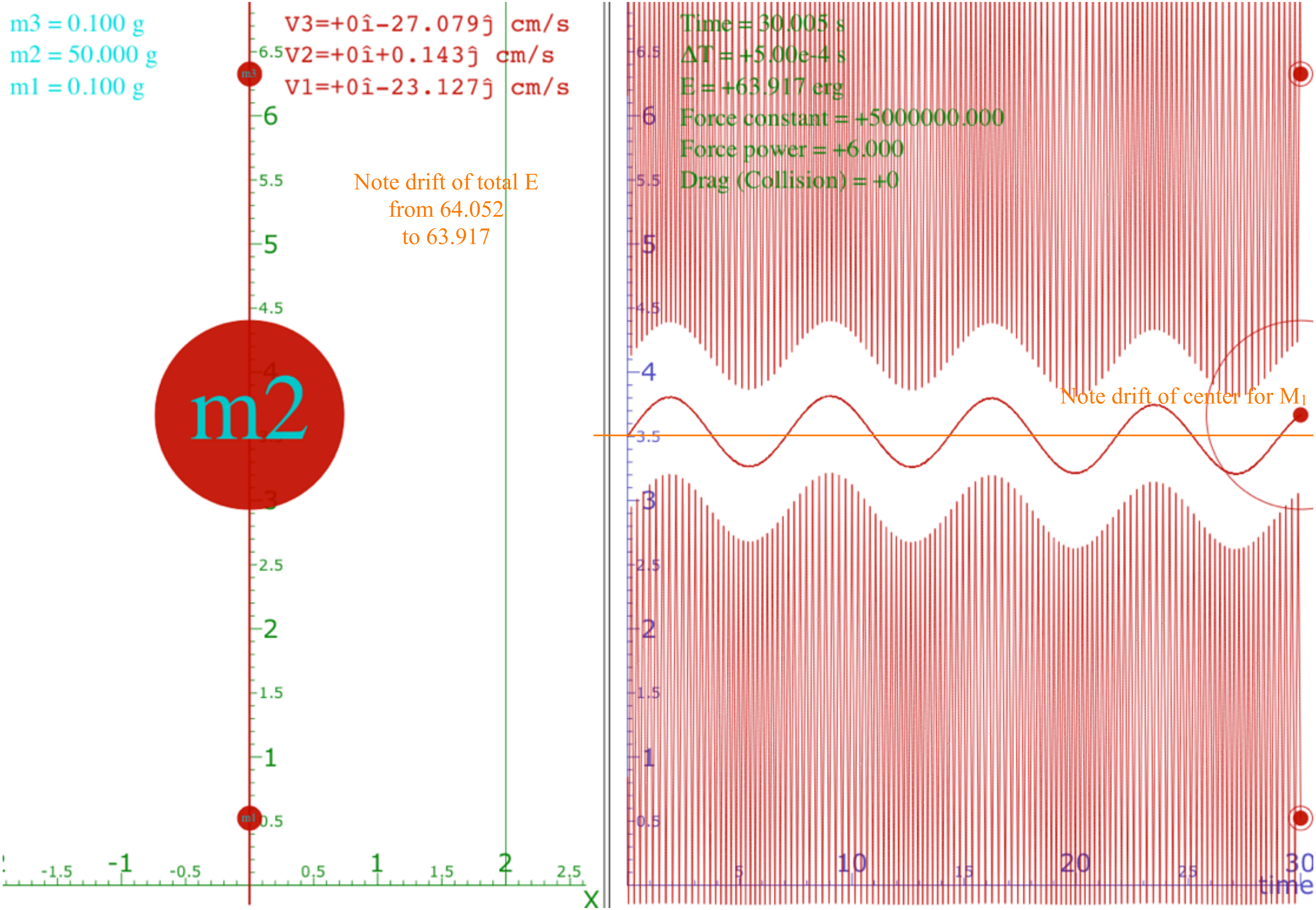
BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Large Amplitude)







[*BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)*]

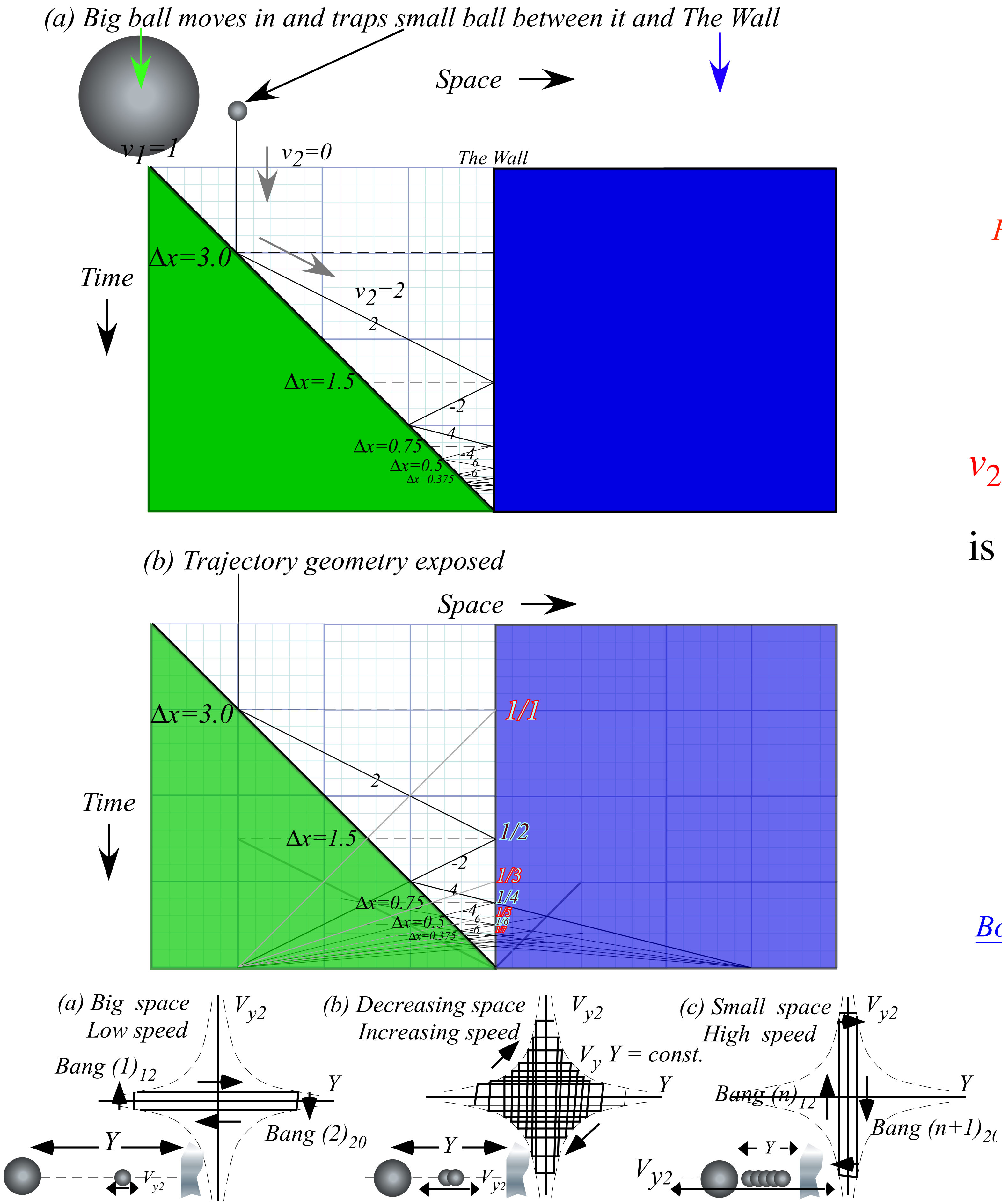


BounceIt Superball Collision Web Simulator: 1:500:1 mass ratios (Small Amplitude)

"Monster Mash" classical analog of Heisenberg action relations

Example of very very large M_1 ball-wall(s) crushing a poor little m_2

How does m_2 conserve action ($\Delta x \Delta p$ or $\int p \cdot dx$) as its KE changes?



The Classical "Monster Mash"

Classical introduction to

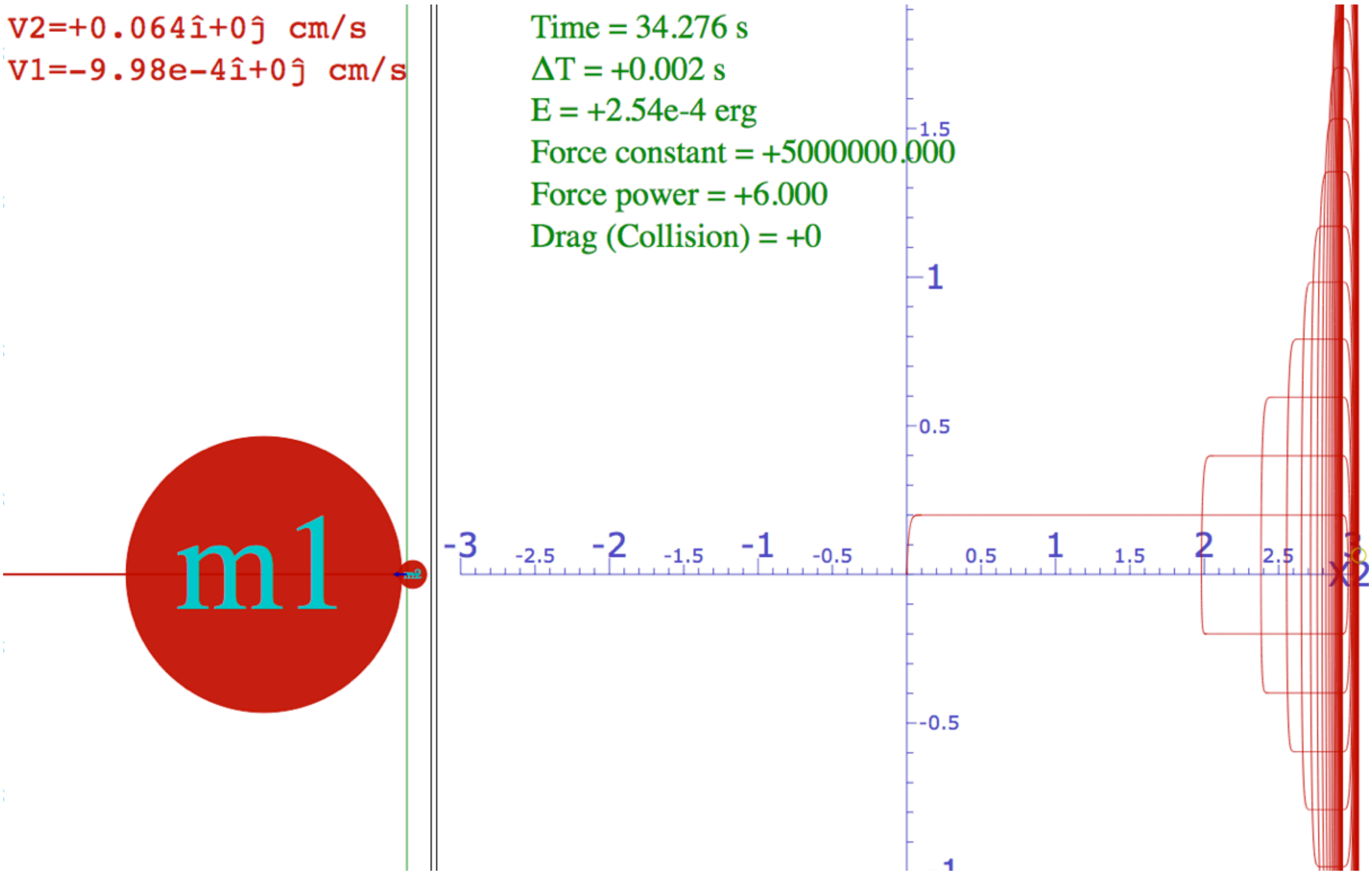
Heisenberg "Uncertainty" Relations

$$v_2 = \frac{const.}{Y} \quad \text{or:} \quad Y \cdot v_2 = const.$$

is analogous to: $\Delta x \cdot \Delta p = N \cdot \hbar$

Fig. 5.4
Unit 1

BounceIt "Monster Mash" $x_2(t)$ animation
(Note: Time sense is inverted)



BounceIt "Monster Mash" Vx_2 vs x_2 animation

Double "Monster Mash"

Realizes reflection symmetry of perfect wall bounces

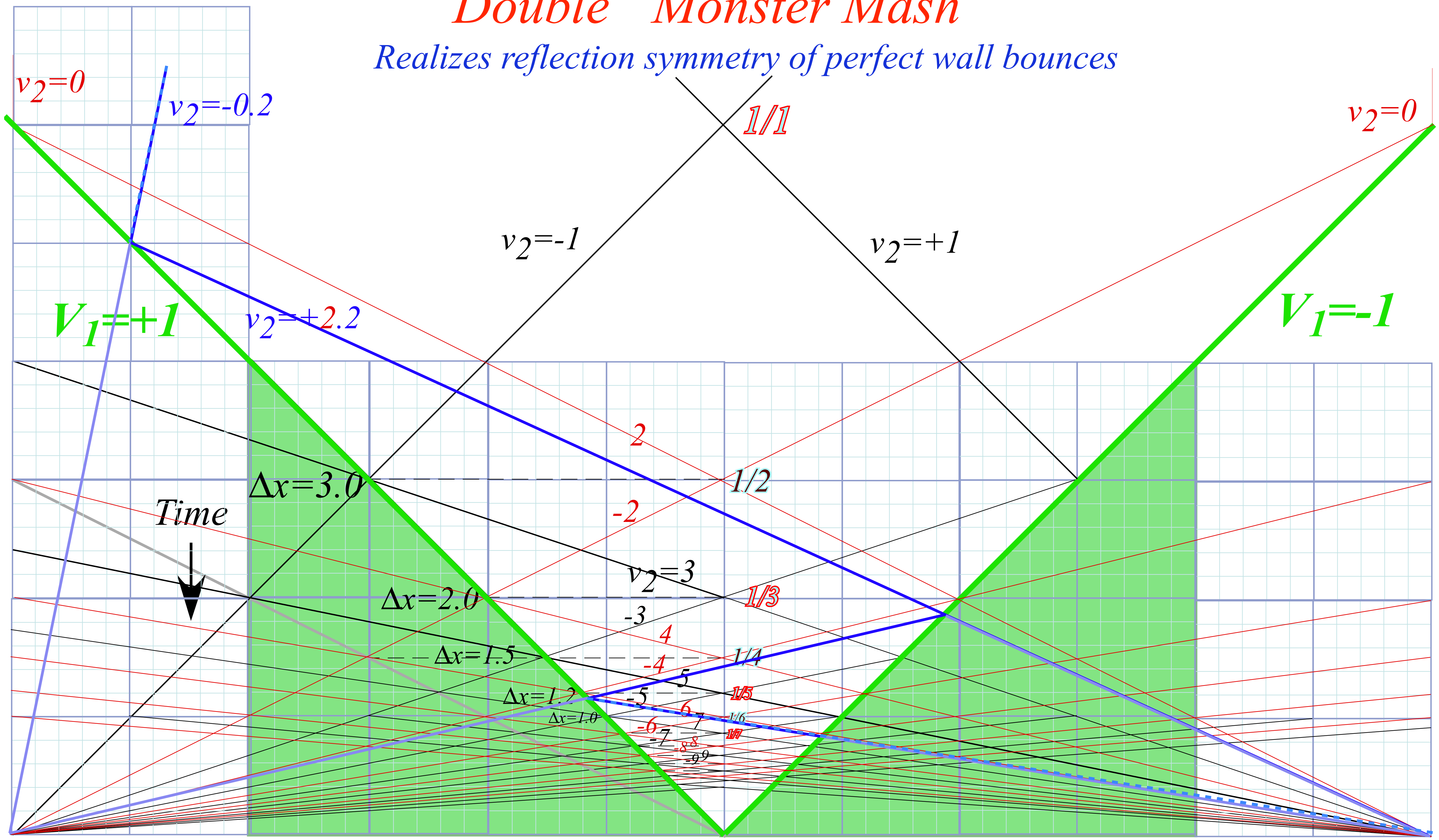


Fig. 5.5
Unit 1

See Homework problem 1.6.2: *Construct related spacetime case*

Double "Monster Mash"

Realizes reflection symmetry of perfect wall bounces

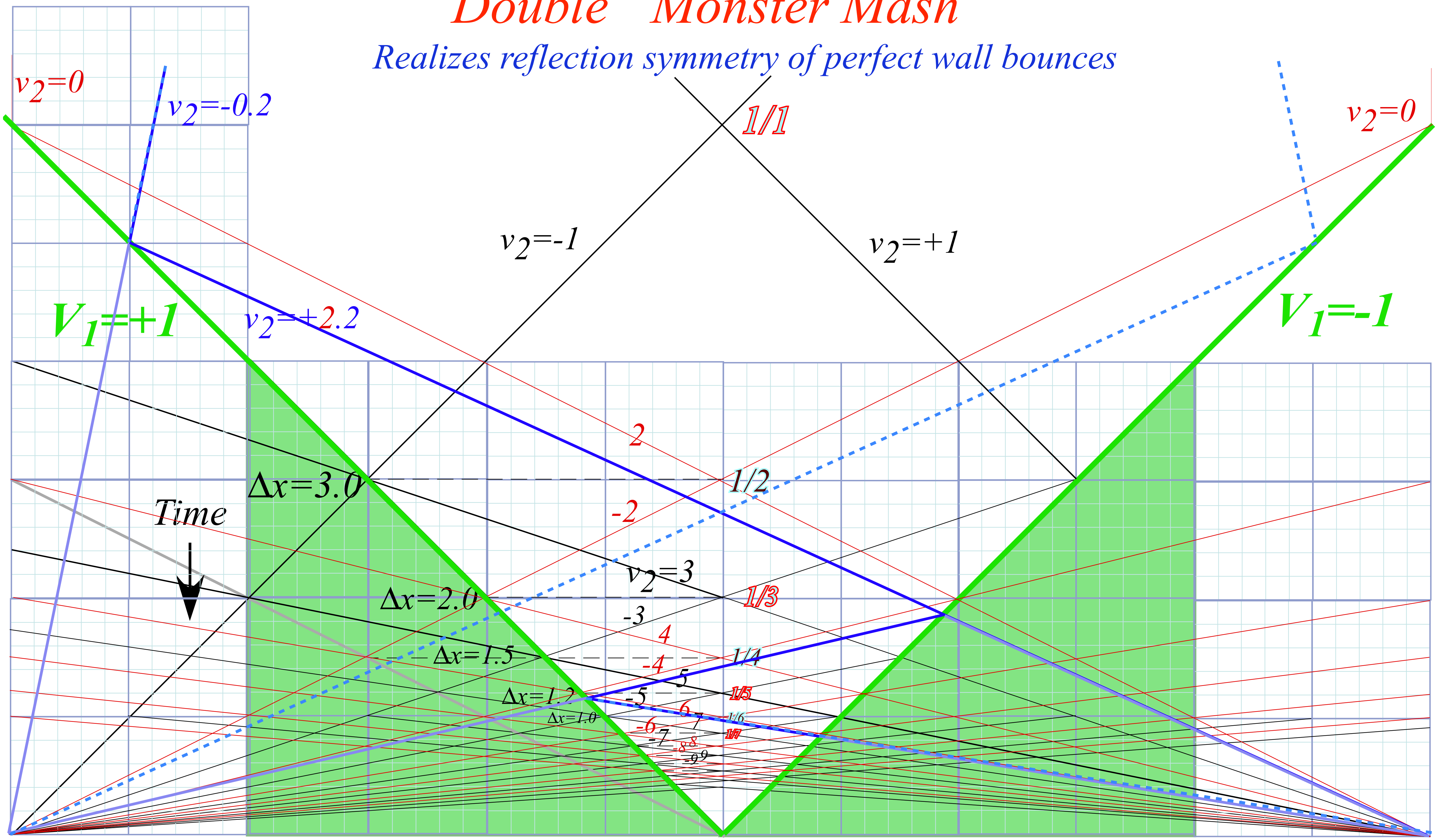


Fig. 5.5
Unit 1

See Homework problem 1.6.2: *Construct related spacetime case*

Double "Monster Mash"

Realizes reflection symmetry of perfect wall bounces

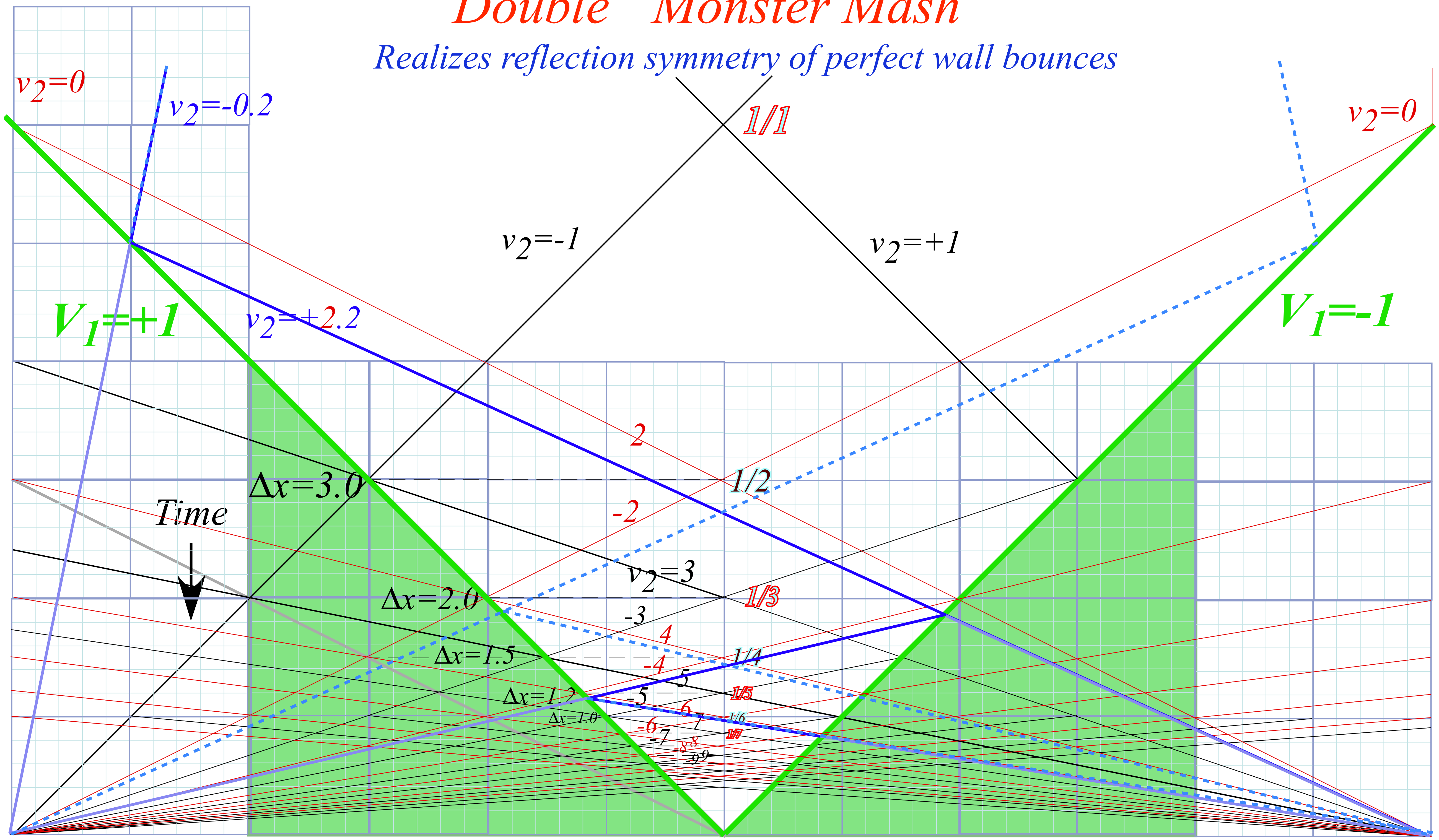


Fig. 5.5
Unit 1

See Homework problem 1.6.2: *Construct related spacetime case*

Double "Monster Mash"

Realizes reflection symmetry of perfect wall bounces

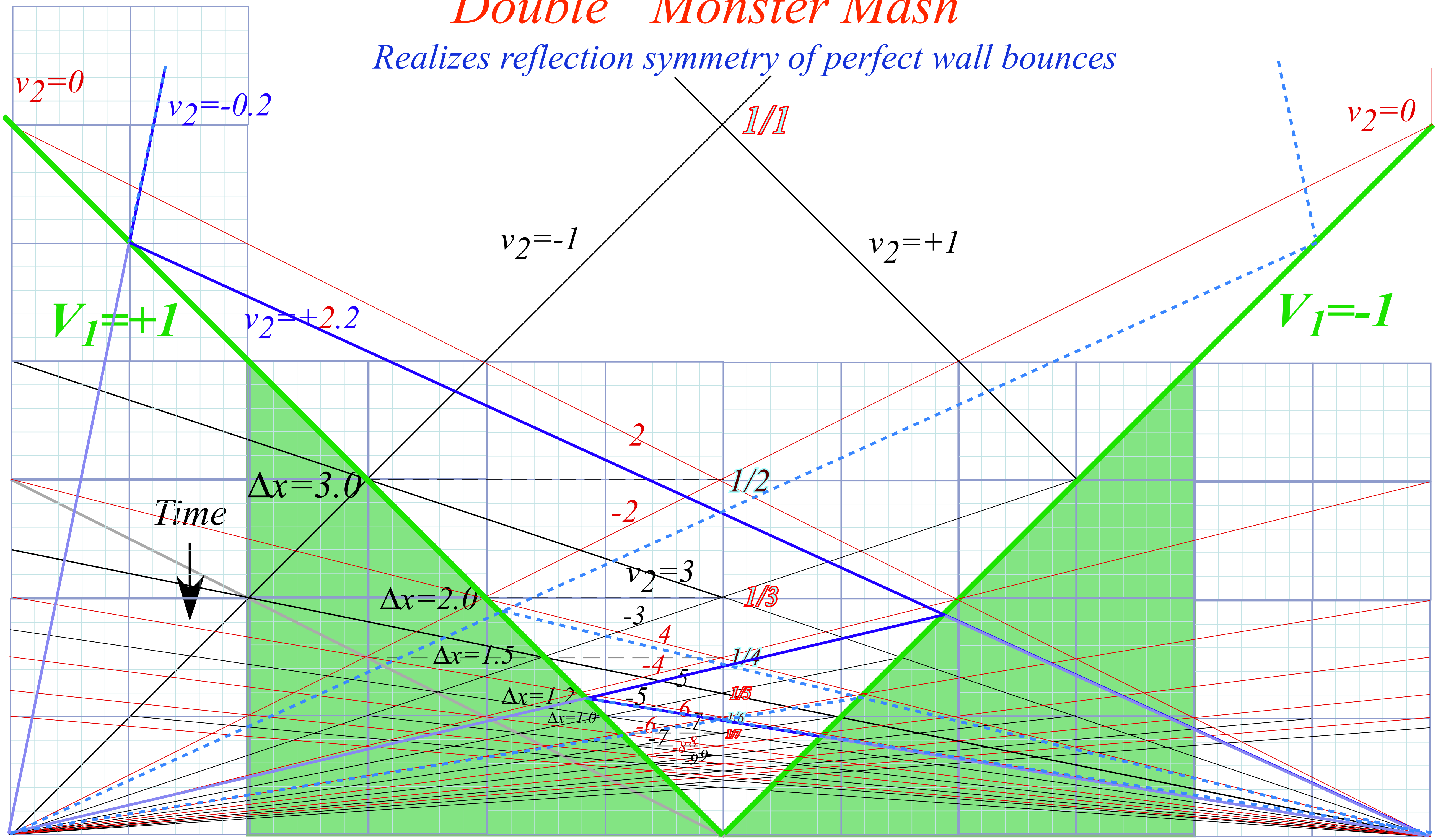
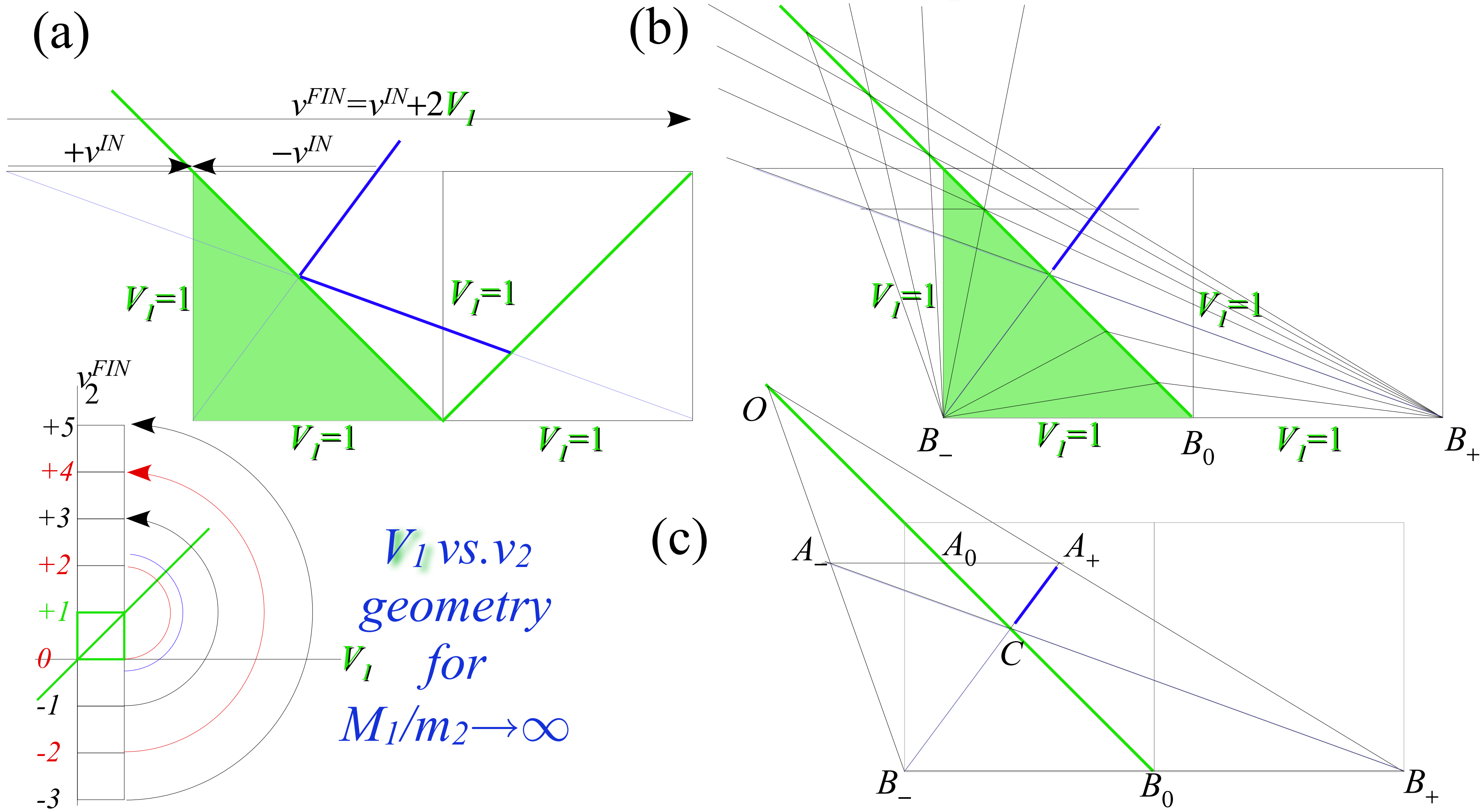


Fig. 5.5
Unit 1

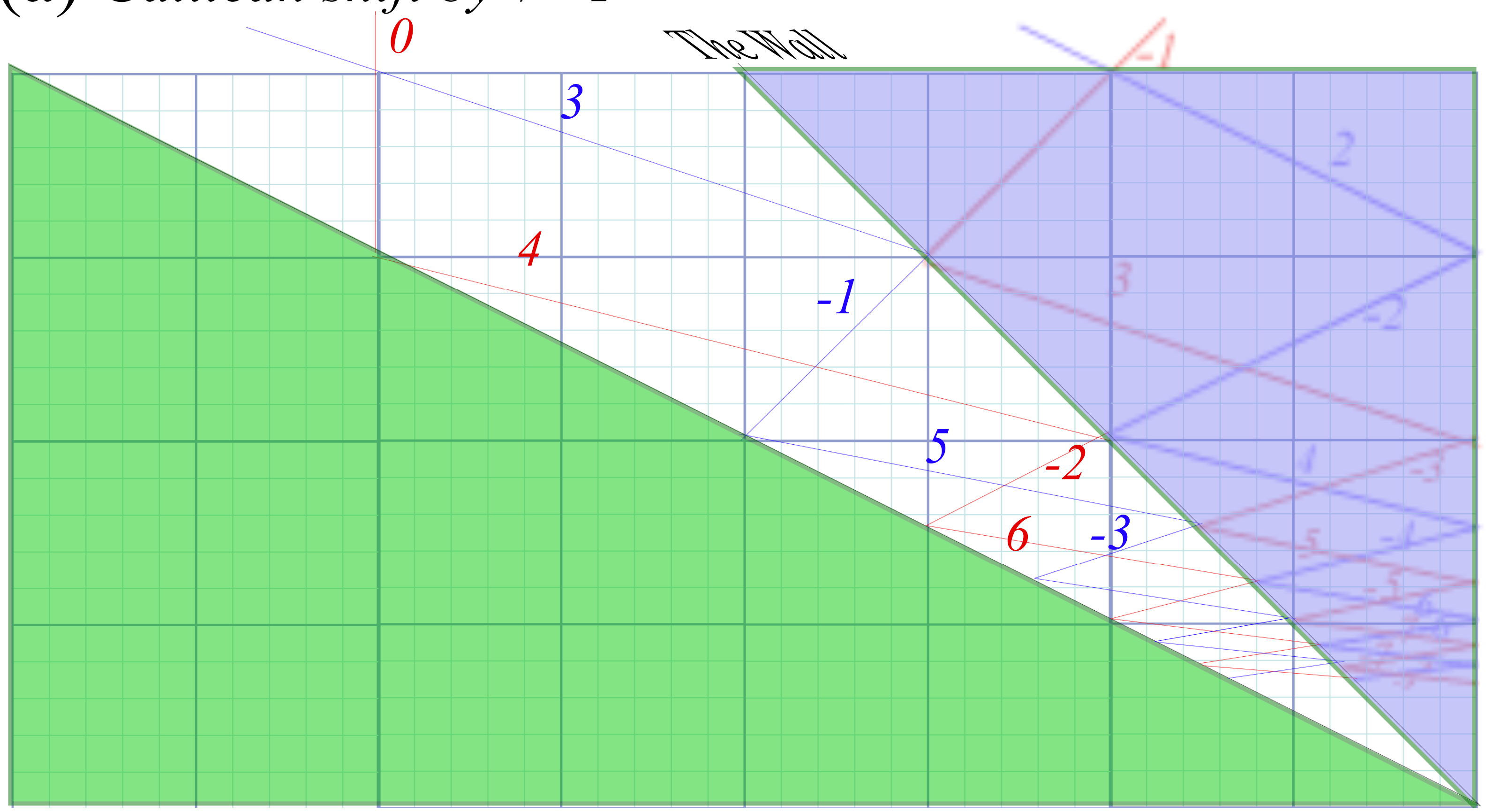
Exercise 1.6.2: *Construct similar spacetime case*

Geometry of reflection $-v_2^{IN} \longrightarrow {}_+v_2^{IN}$ followed by adding $2V_1$ to ${}_+v_2^{IN}$

Fig. 5.6 and Fig. 5.7 Unit 1



(a) *Galilean shift by V=1*



(b)