Lecture 9
Tue. 2.16.2016

Fractal behavior in matter-wave "Tiny-Big-Bang"

(Quantum wave analogies to classical "Monster-Mash" model in Ch. 5 of Unit 1)

Reviewing lightwave Fourier analysis - Pulse Waves (PW) versus Continuous Waves (CW)

- *Opposite-pair CW (colliding ±m=±2) Fourier components trace a Cartesian space-time grid*
- *Colliding PW lightwaves trace space-time "baseball diamonds"*
- *Introducing CW (colliding ±m=±2) Doppler shifted to (m=-1 and m=+4)*

NON-Lightwaves whose ω(k) dispersion functions are NOT straight lines

Animating PW made of CW that have quadratic (Bohr-Schrodinger) dispersion

- *Visualizing PW wave uncertainty relations for* space: $\Delta x \cdot \Delta \kappa = 1$ *and* time: $\Delta t \cdot \Delta \upsilon = 1$

Matter-wave fractal behavior in a "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)]; [Harter, Li IMSS (2013)]

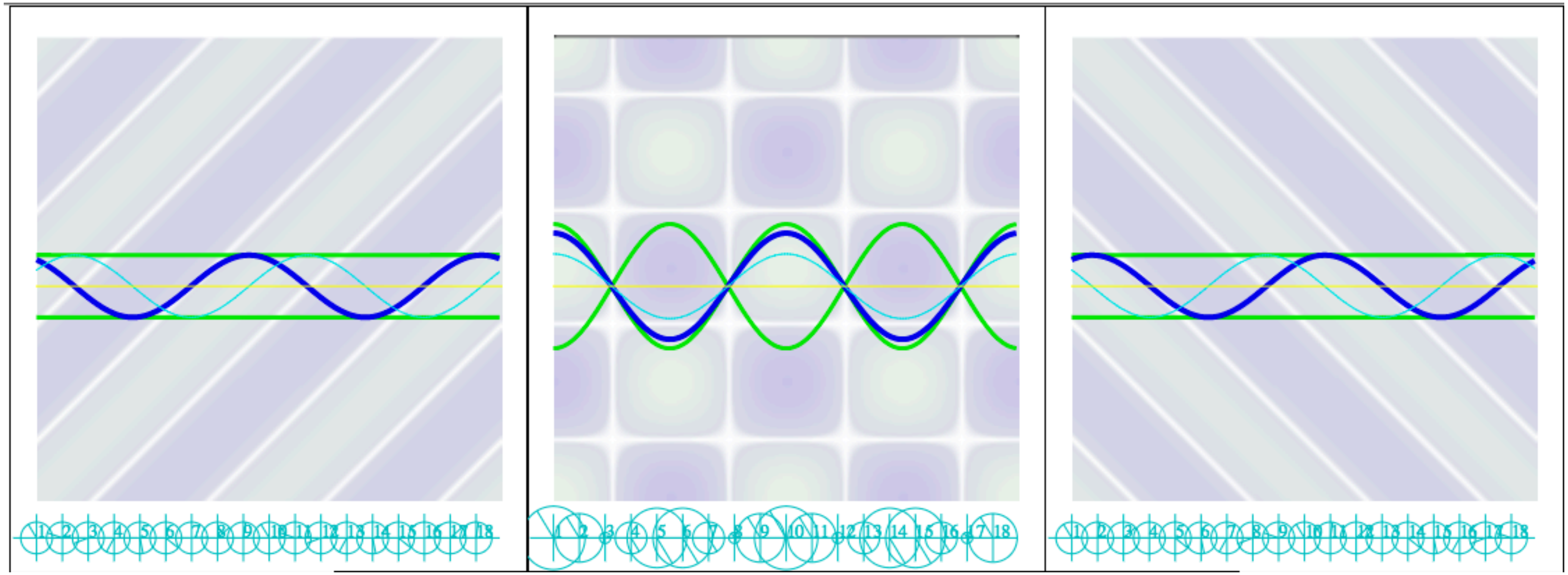
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]; [John Farey, Phil. Mag.(1816) Wolfram]; [Li, Harter, Chem.Phys.Letters (2015)]

Opposite-pair CW (colliding $\pm m=\pm 2$) Fourier components trace a Cartesian space-time grid

Spacetime animation of head-on collision of two υ=600THz CW modes of light

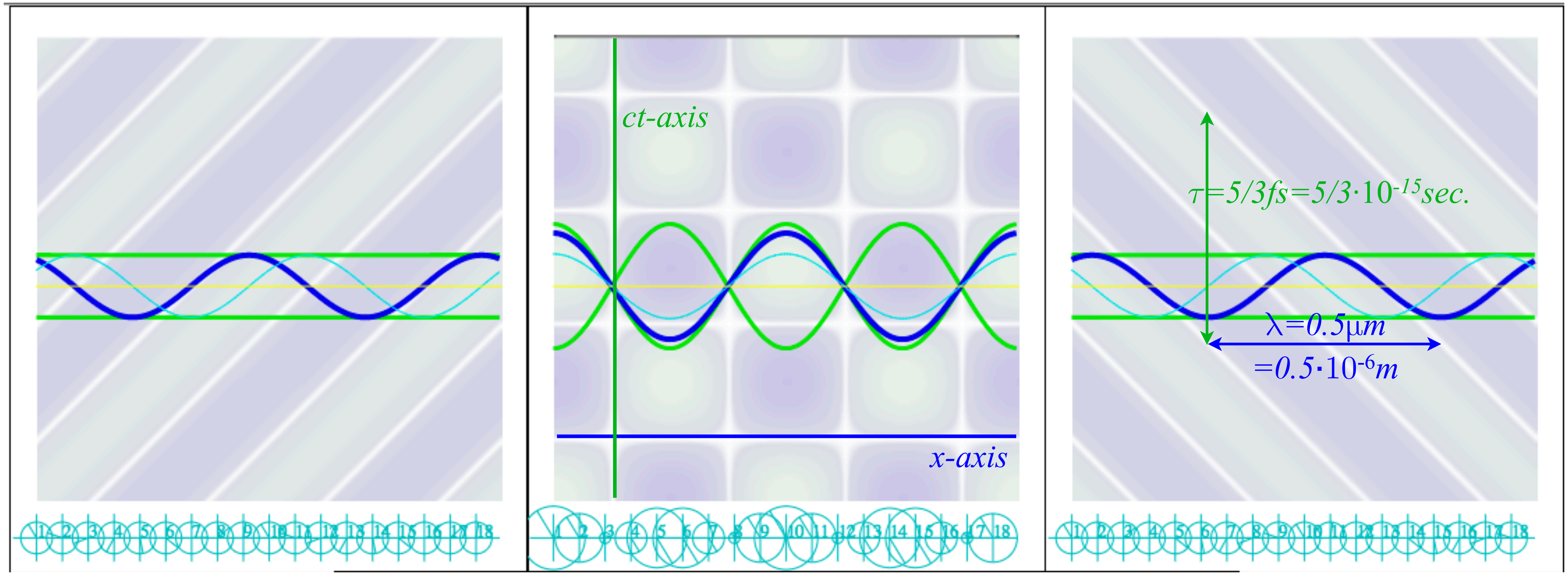
CW



[BohrIt Web Simulation](#)
[2 CW *ct vs x* Plot (ck = ±2)](#)

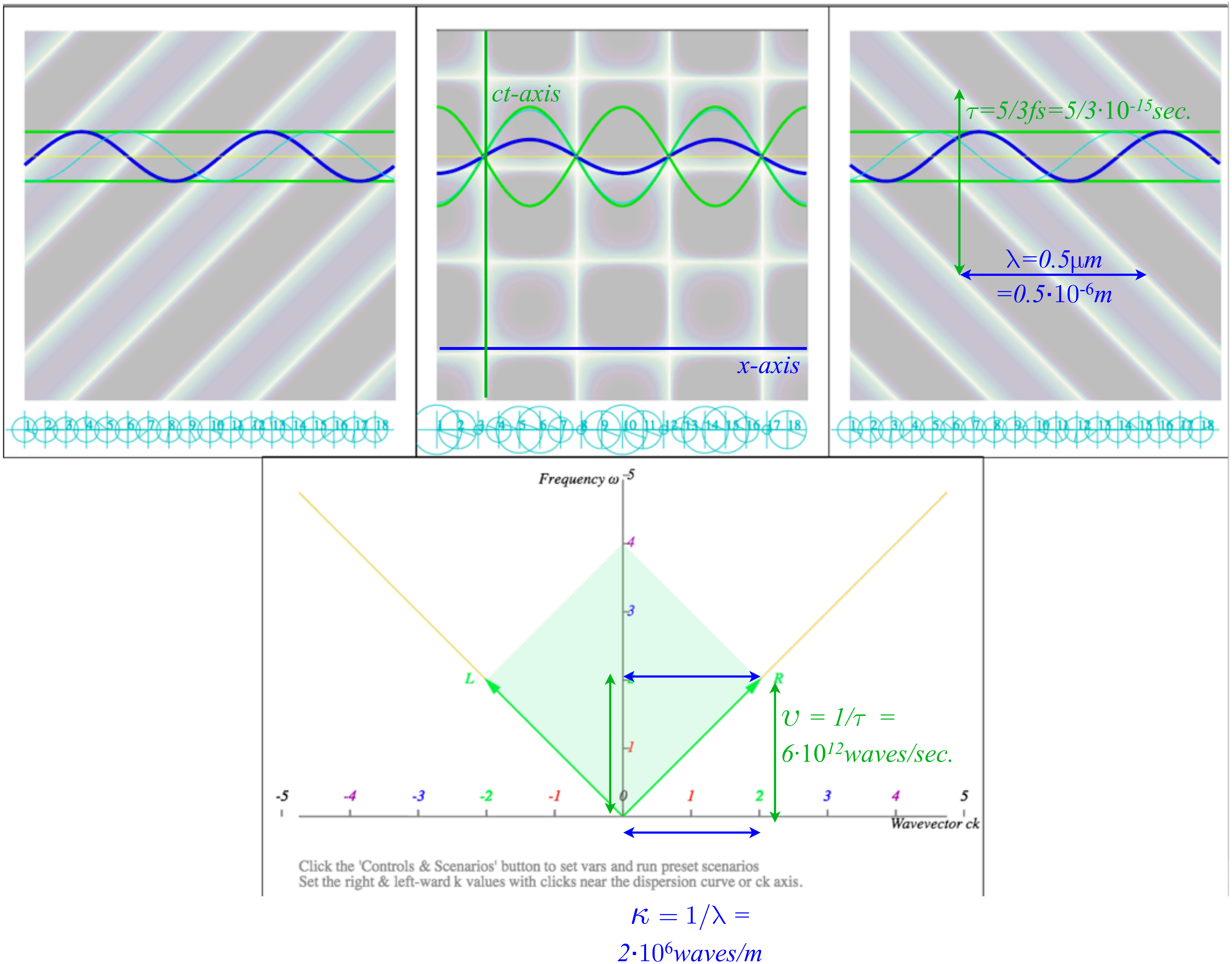
Spacetime animation of head-on collision of two υ=600THz CW modes of light

CW



BohrIt Web Simulation

2 CW *ct vs x* Plot (ck = ±2)



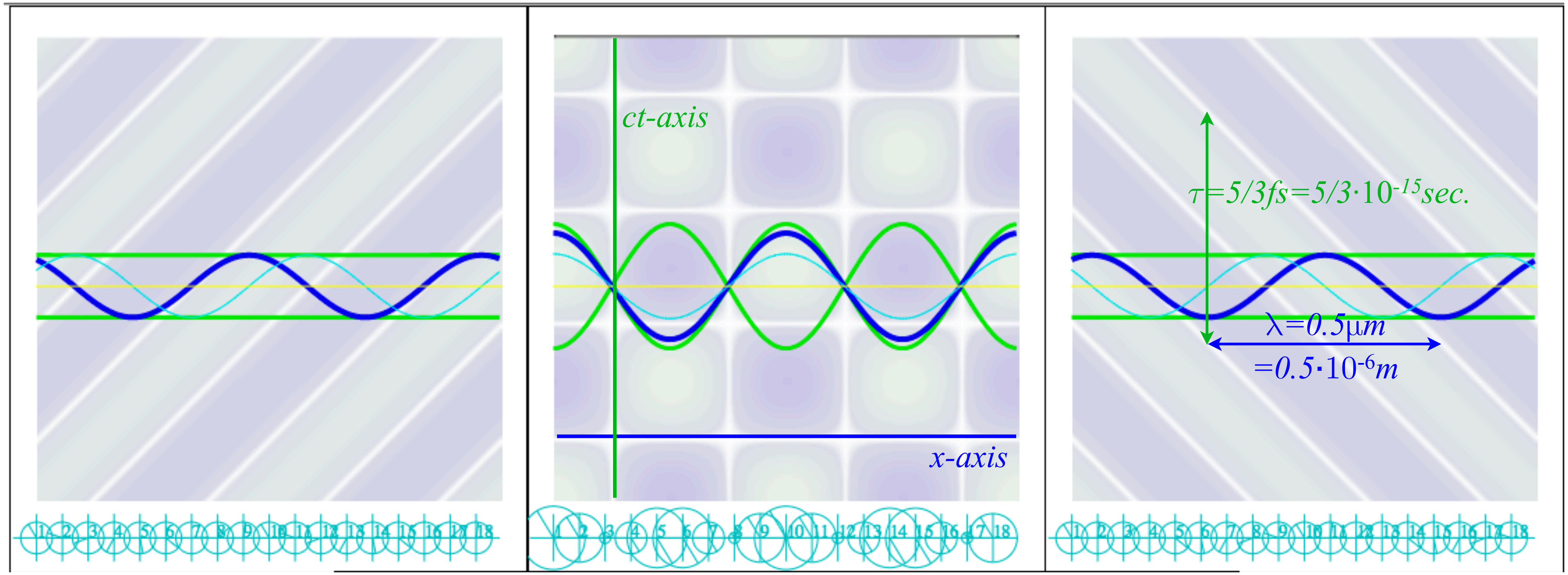
BohrIt Web Simulation
2 CW *ct vs x* Plot (ck = ±2)

Opposite-pair CW (colliding $\pm m=\pm 2$) Fourier components trace a Cartesian space-time grid

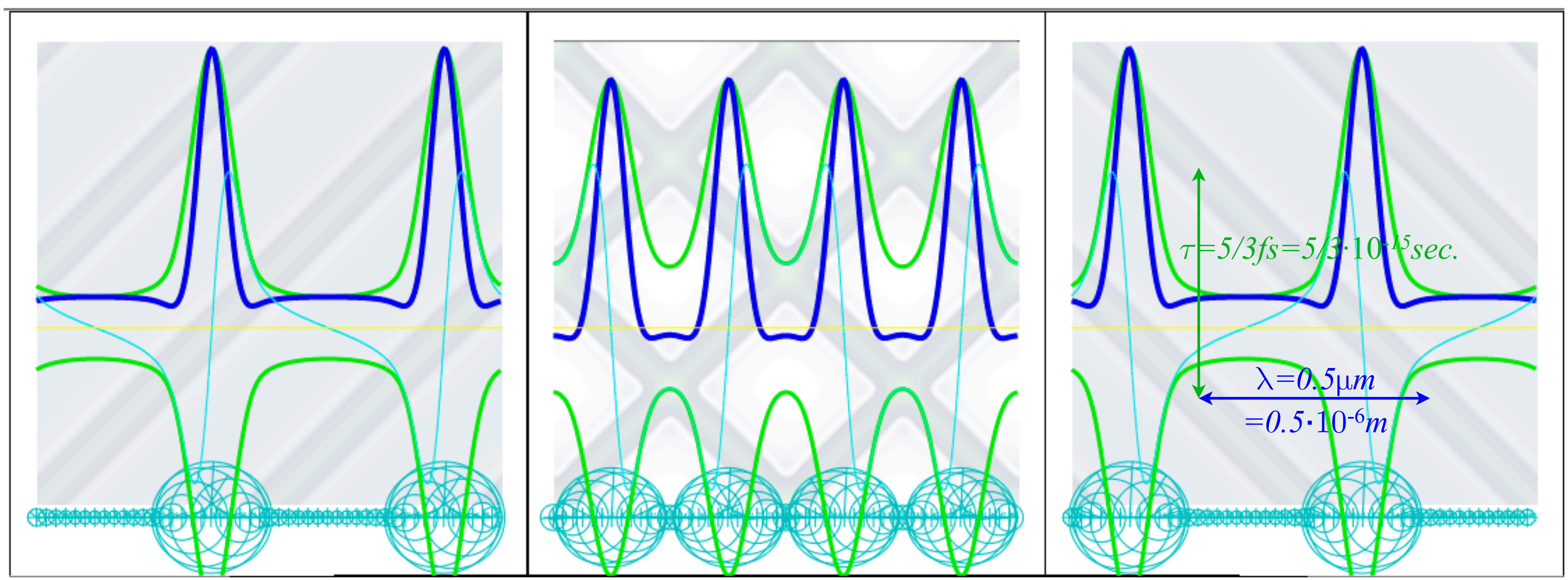
➔ *Colliding PW) Fourier components trace space-time "baseball diamonds"*

Spacetime animation of head-on collision of two υ=600THz CW modes of light

CW



Spacetime animation of head-on collision of two Nυ=600N THz PW modes of light



PW

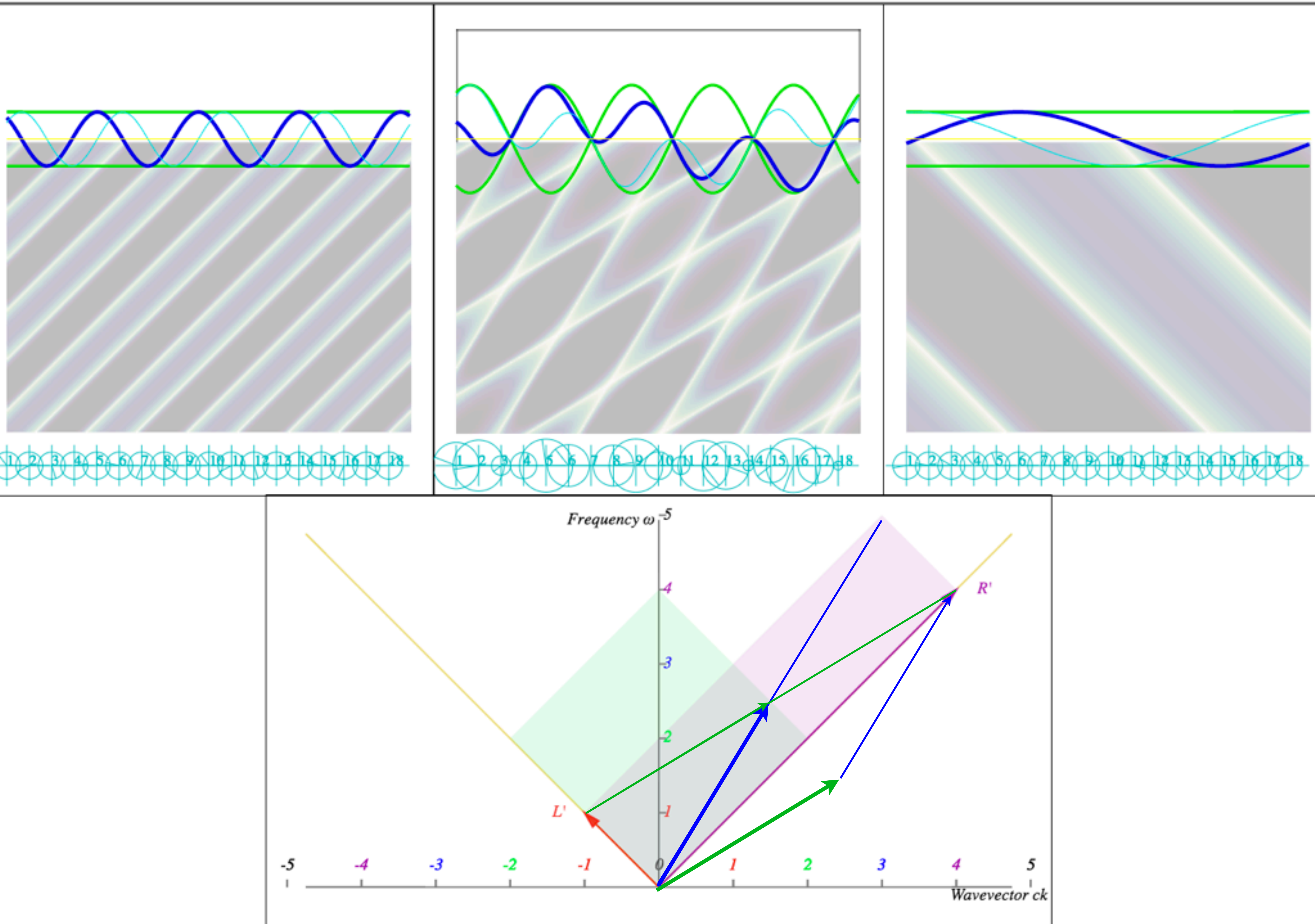
BohrIt Web Simulation
2 CW *ct vs x* Plot (ck = ±2)

BohrIt Web Simulation
2 PW *ct vs x* Plot (ck mod 2 = 0)

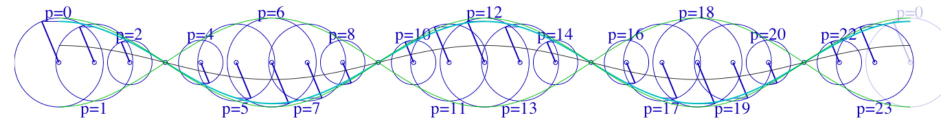
Opposite-pair CW (colliding ±m=±2) Fourier components trace a Cartesian space-time grid

Colliding PW lightwaves trace space-time "baseball diamonds"

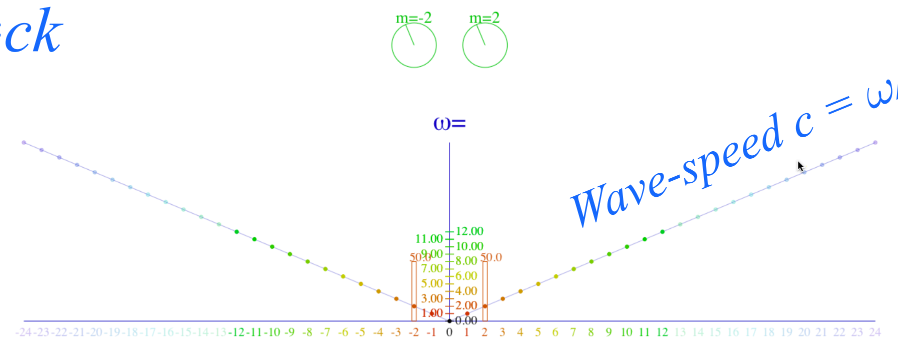
→ *Introducing CW (colliding ±m=±2) Doppler shifted to (m=-1 and m=+4)*



Click the 'Controls & Scenarios' button to set vars and run preset scenarios
Set the right & left-ward k values with clicks near the dispersion curve or ck axis.

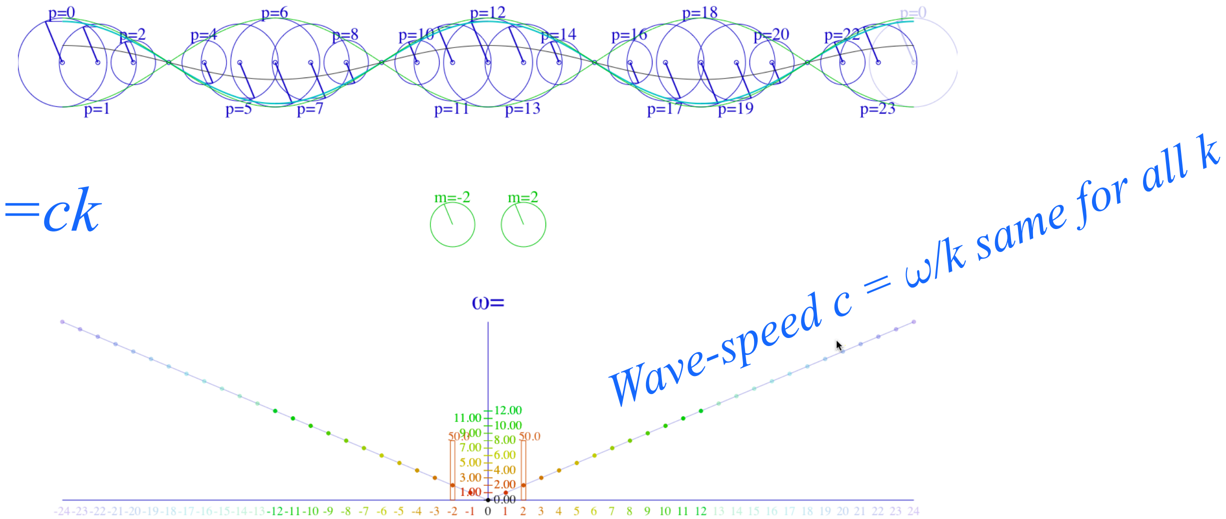


Example 0: Light-like ω(k)=ck
(Linear dispersion-free)



Wave-speed c = ω/k same for all k

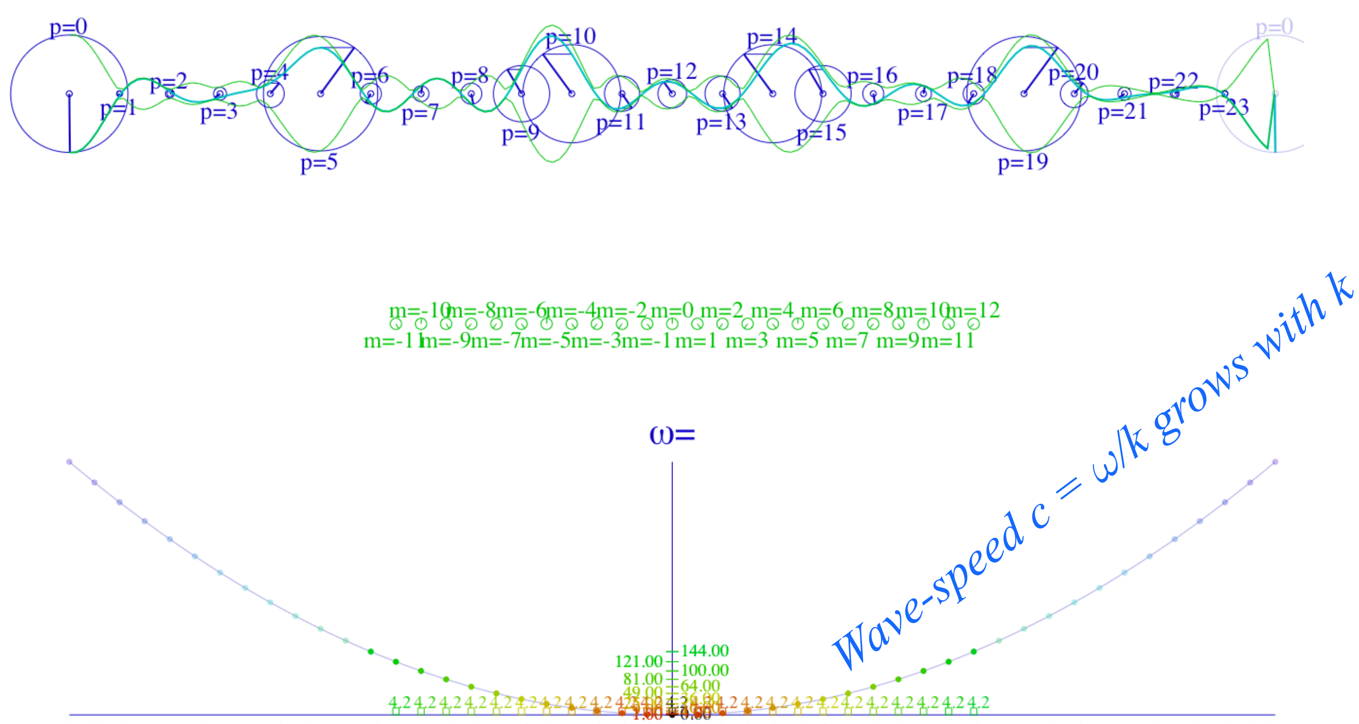
Now consider NON-Lightwaves
(whose ω(k) dispersion functions are NOT straight lines)



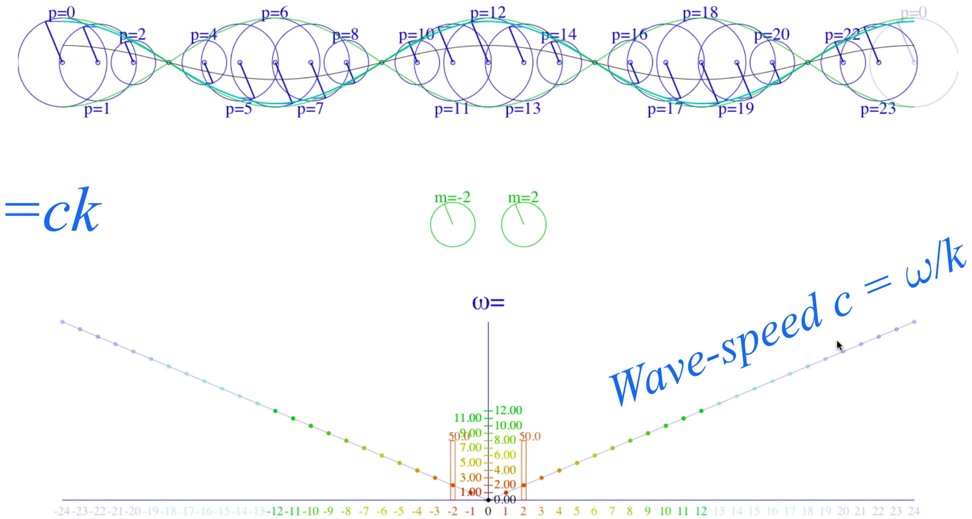
Example 0: Light-like ω(k)=ck
(Linear dispersion-free)

Now consider NON-Lightwaves
(whose ω(k) dispersion functions are NOT straight lines)

Example 1: Bohr-Schrodinger ω(k)=Bk²



Counter prop.
http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=2PW_QuadDisp_2016HP

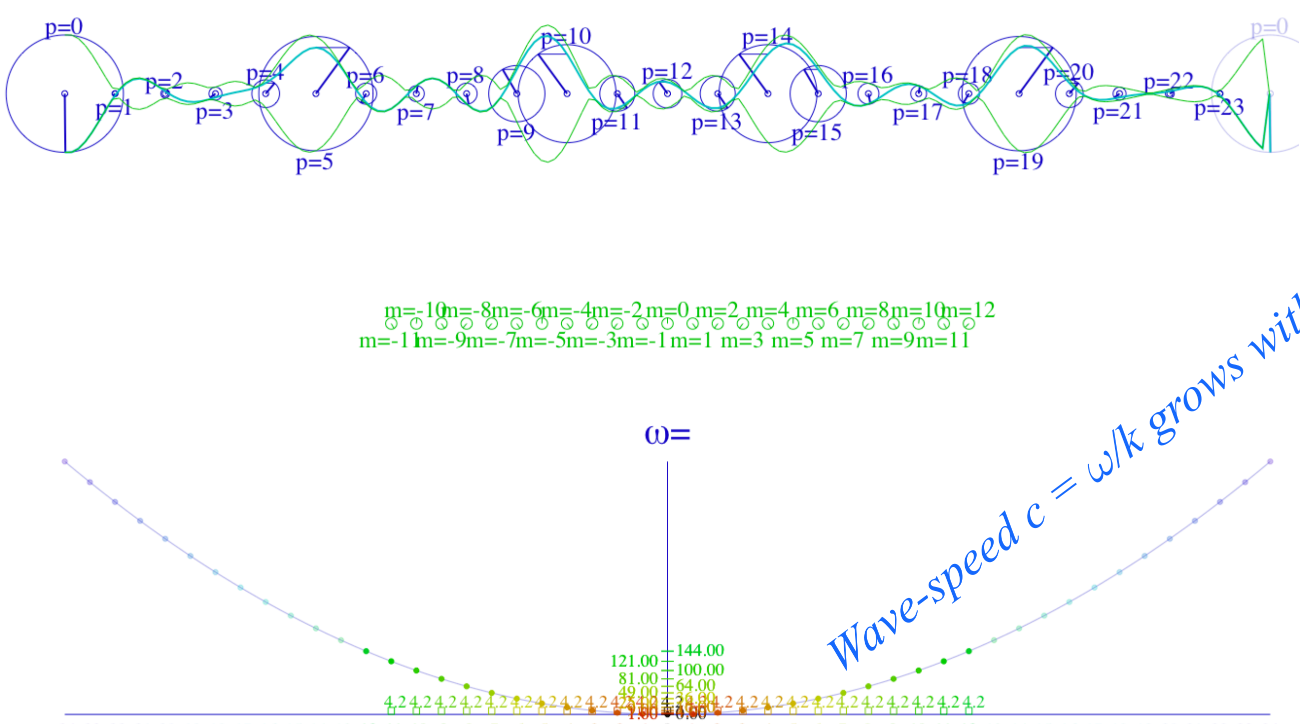


Example 0: Light-like $\omega(k)=ck$
(Linear dispersion-free)

Wave-speed $c = \omega/k$ same for all k

Now consider NON-Lightwaves
(whose $\omega(k)$ dispersion functions are NOT straight lines)

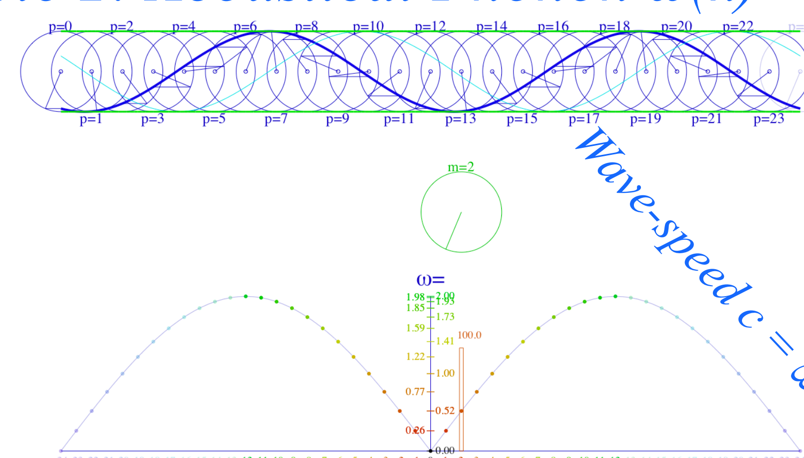
Example 1: Bohr-Schrodinger $\omega(k)=Bk^2$



Wave-speed $c = \omega/k$ grows with k

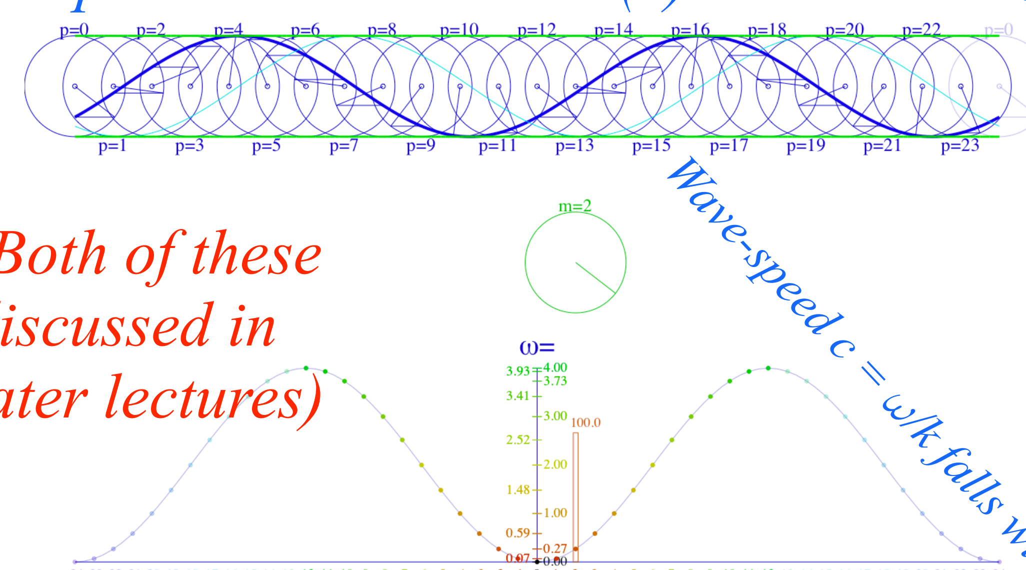
Counter prop.
http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=2PW_QuadDisp_2016HP

Example 2: Acoustical Phonon $\omega(k)=\sqrt{|\cos ak|}$



Wave-speed $c = \omega/k$ falls with k

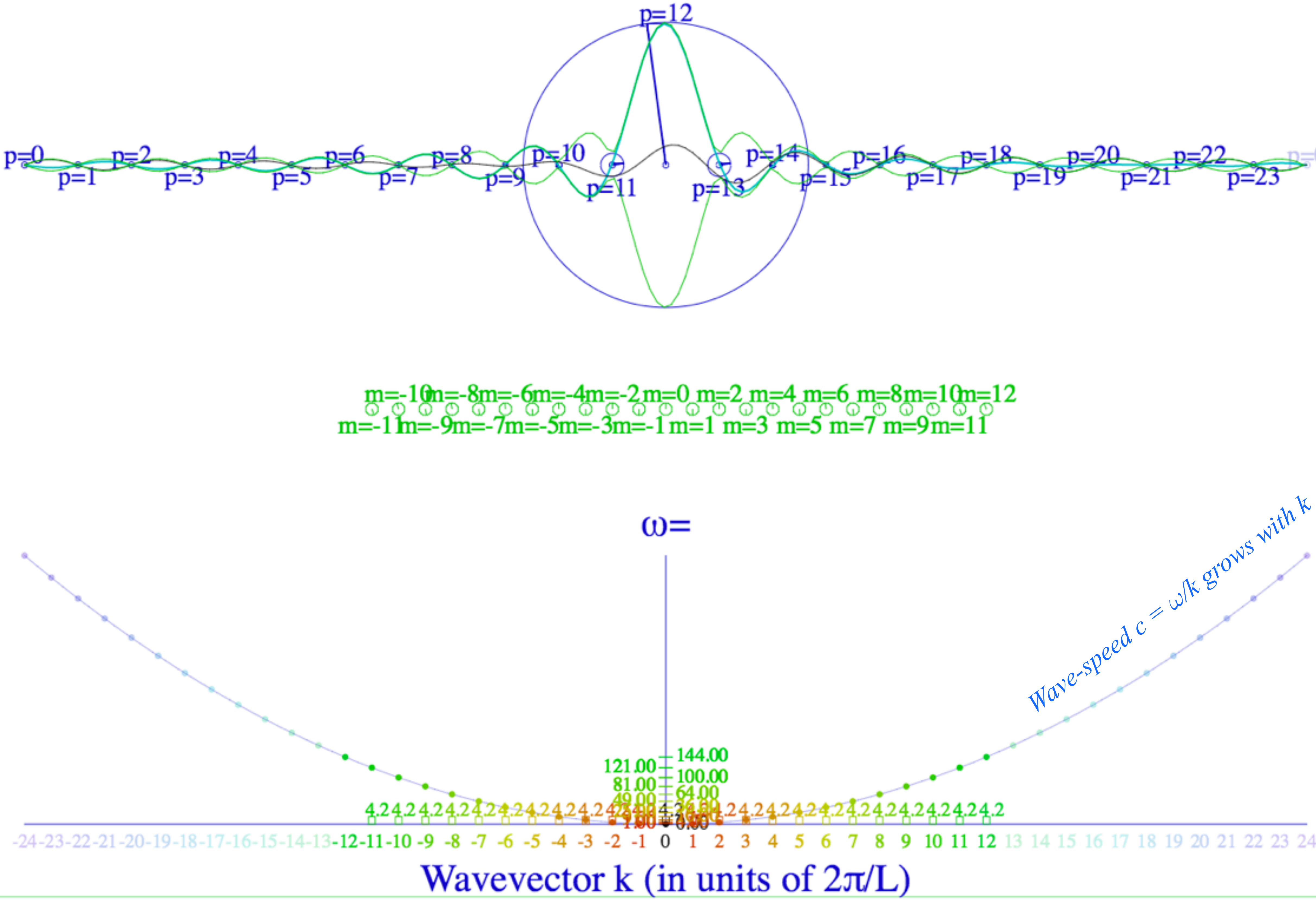
Example 3: Bloch waves $\omega(k)= A\cos ak$



Wave-speed $c = \omega/k$ falls with k

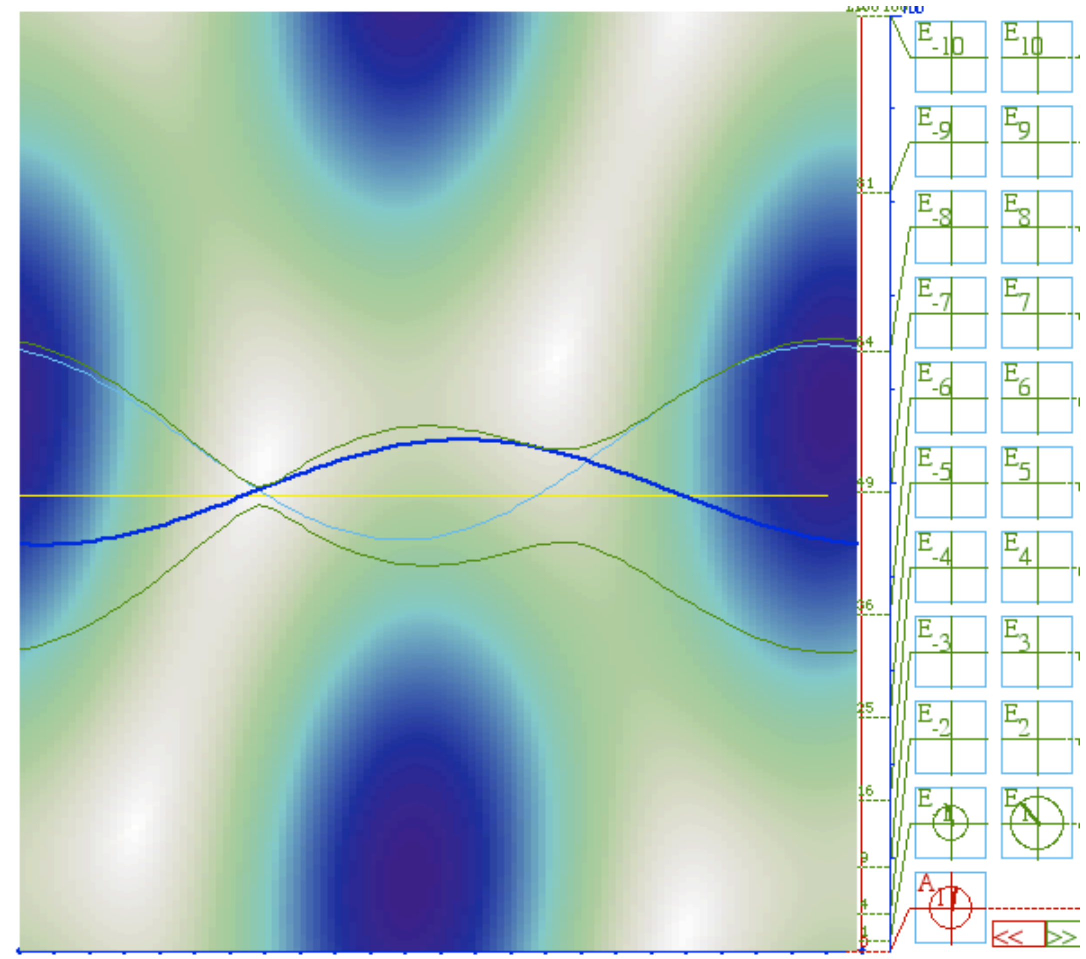
(Both of these discussed in later lectures)

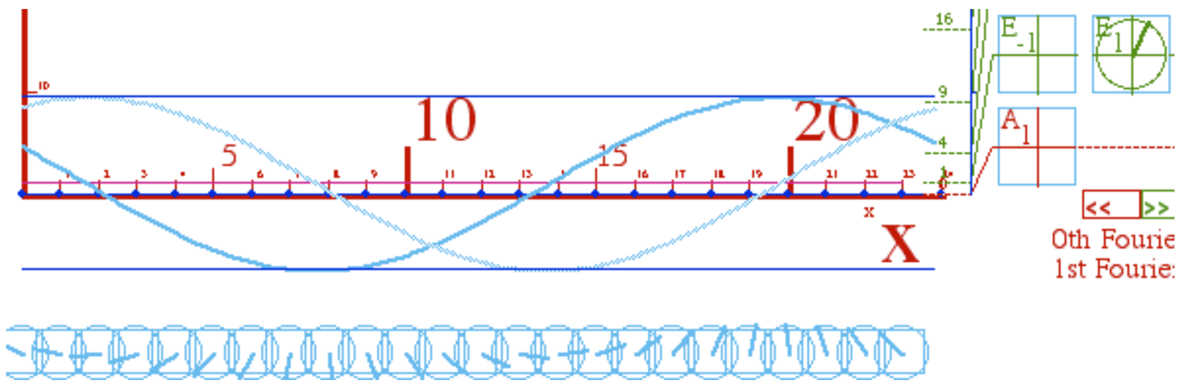
Example 1: Bohr-Schrodinger $\omega(k)=B\,k^2$

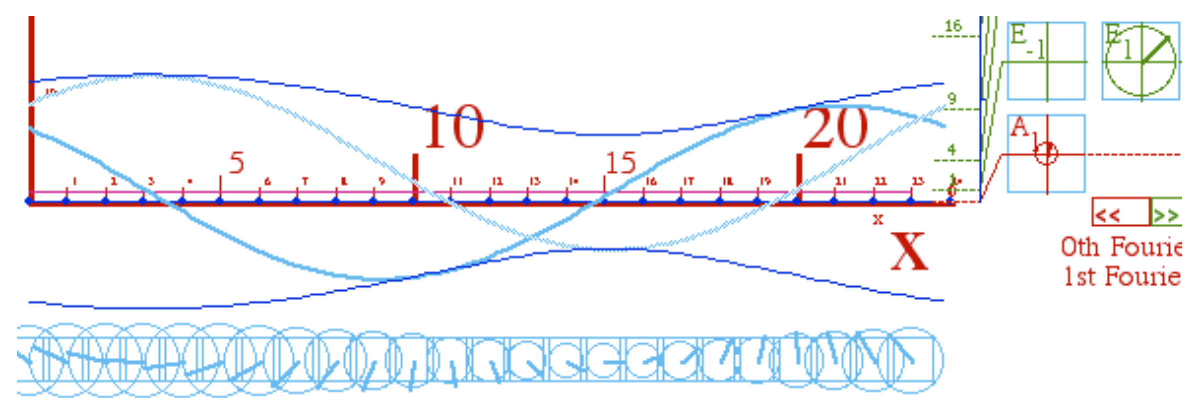


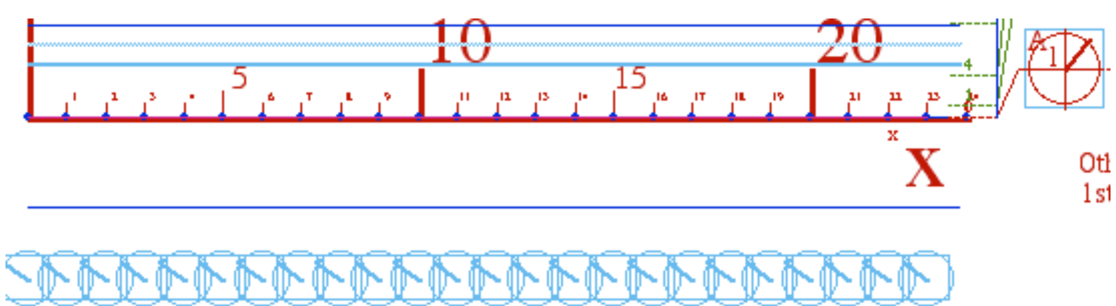
Counter prop.
http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=2PW_QuadDisp_2016HP

Examples of interference beats of quantum waves obeying Bohr-Schrodinger $\omega(k)=B\,k^2$









By Harter-Soft and University of Arkansas Physics *Elegant Educational Tools Since 2001*

Matter-wave fractal behavior in a "Tiny-Big-Bang" [*Harter, J. Mol. Spec. 210, 166-182 (2001)*]; [*Harter, Li IMSS (2013)*]
A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums
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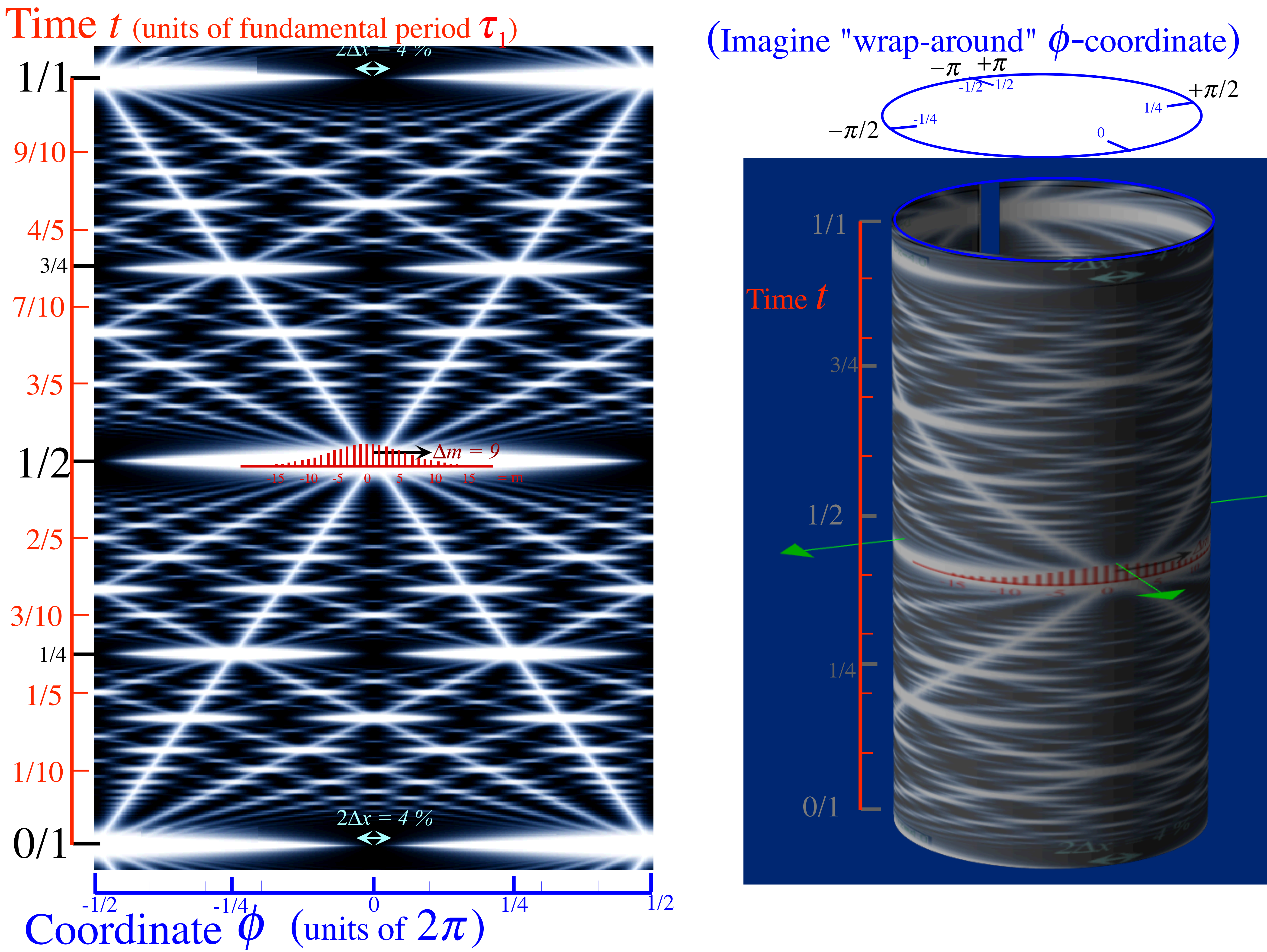
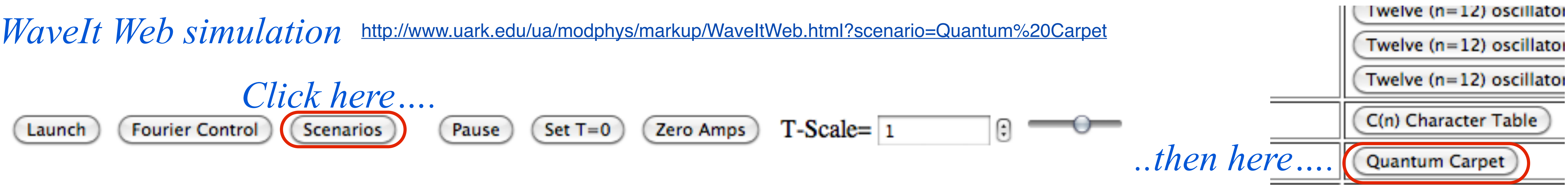
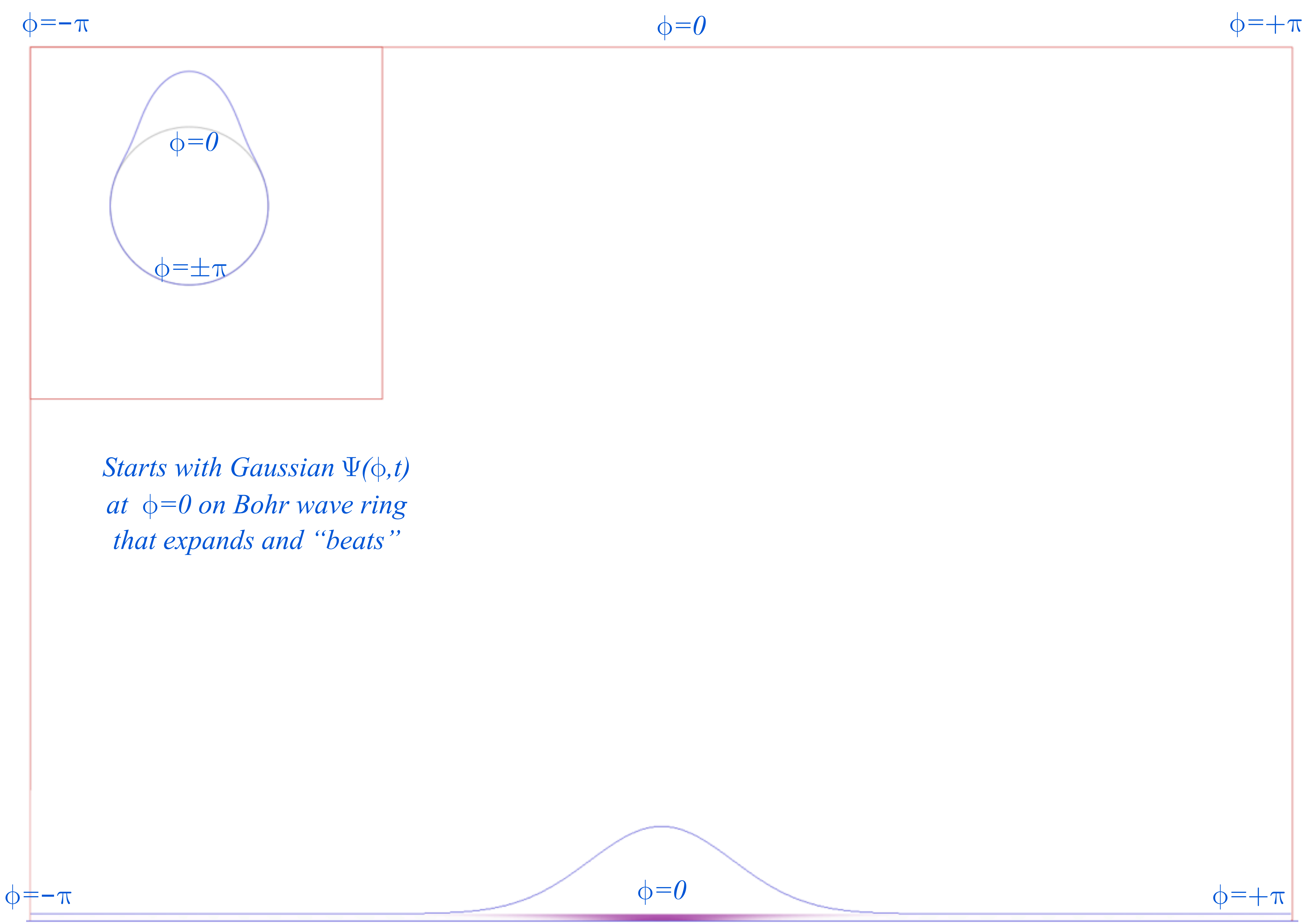

Time t (units of fundamental period τ_1)
1/1
9/10
4/5
3/4
7/10
3/5
1/2
2/5
3/10
1/4
1/5
1/10
0/1
$2\Delta x = 4$ %
$\Delta m = 9$
-15 -10 -5 0 5 10 15 = m
-1/2 -1/4 0 1/4 1/2
Coordinate ϕ (units of 2π)
(Imagine "wrap-around" ϕ-coordinate)
$-\pi$ $+\pi$
-1/2 1/2
$+\pi/2$
1/4
$-\pi/2$
-1/4
0
Time t
1/1
3/4
1/2
1/4
0/1

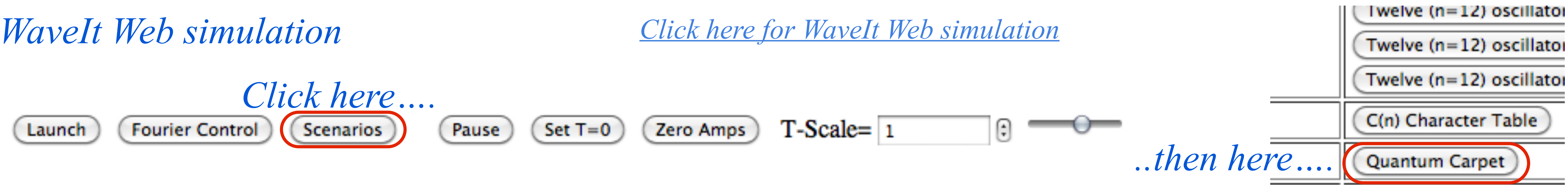
WaveIt Web simulation http://www.uark.edu/ua/modphys/markup/WaveItWeb.html?scenario=Quantum%20Carpet

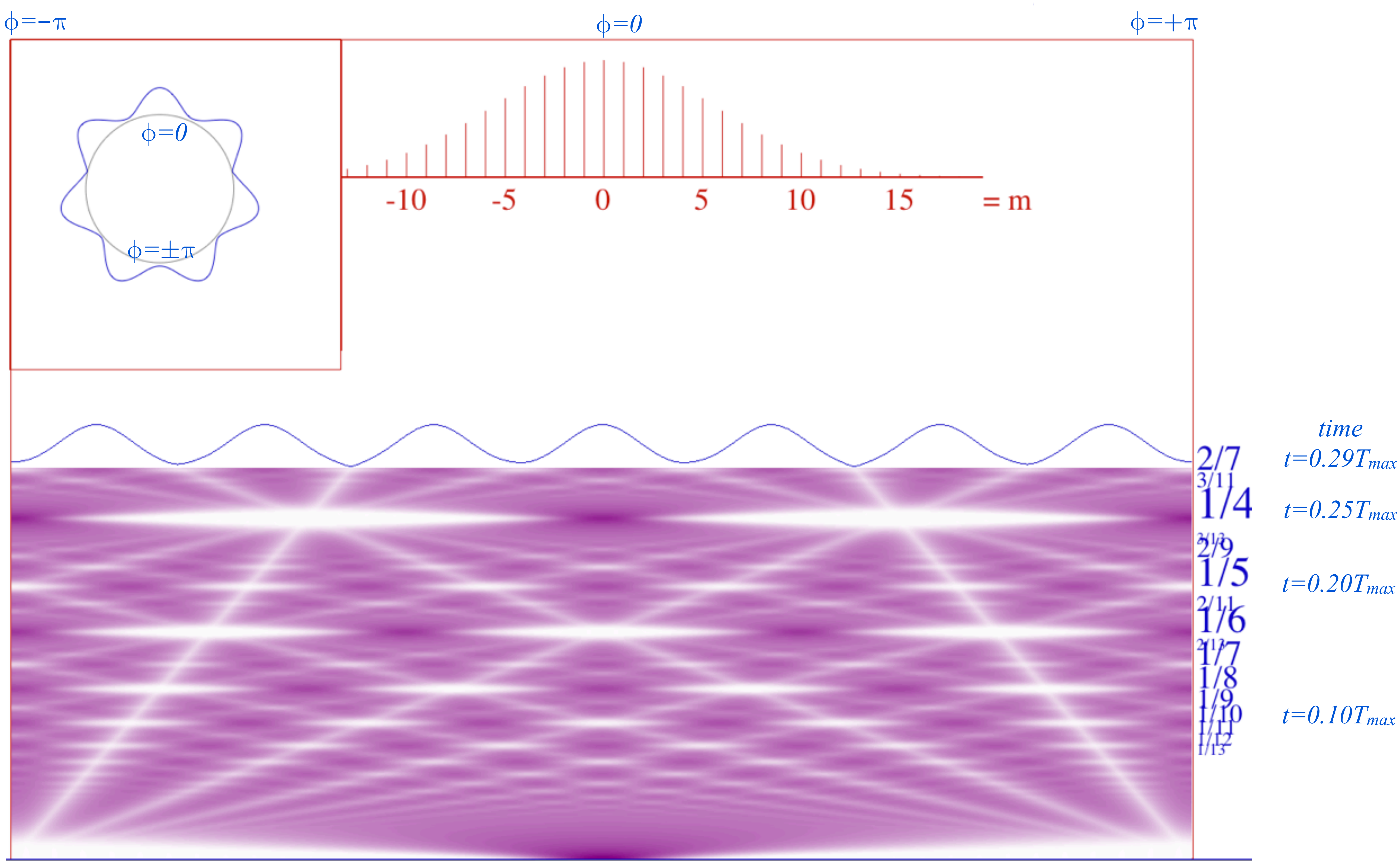


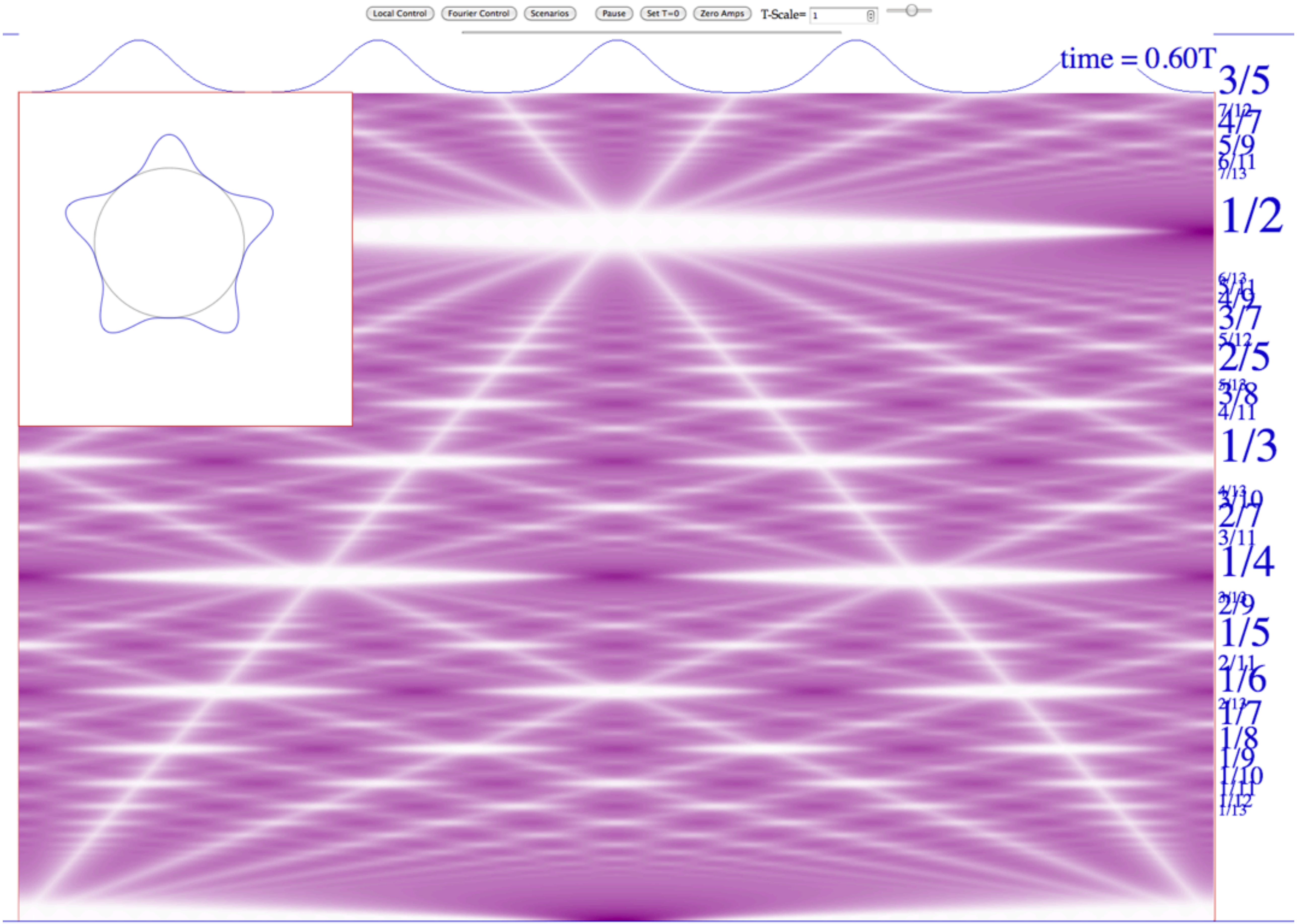


WaveIt Web simulation

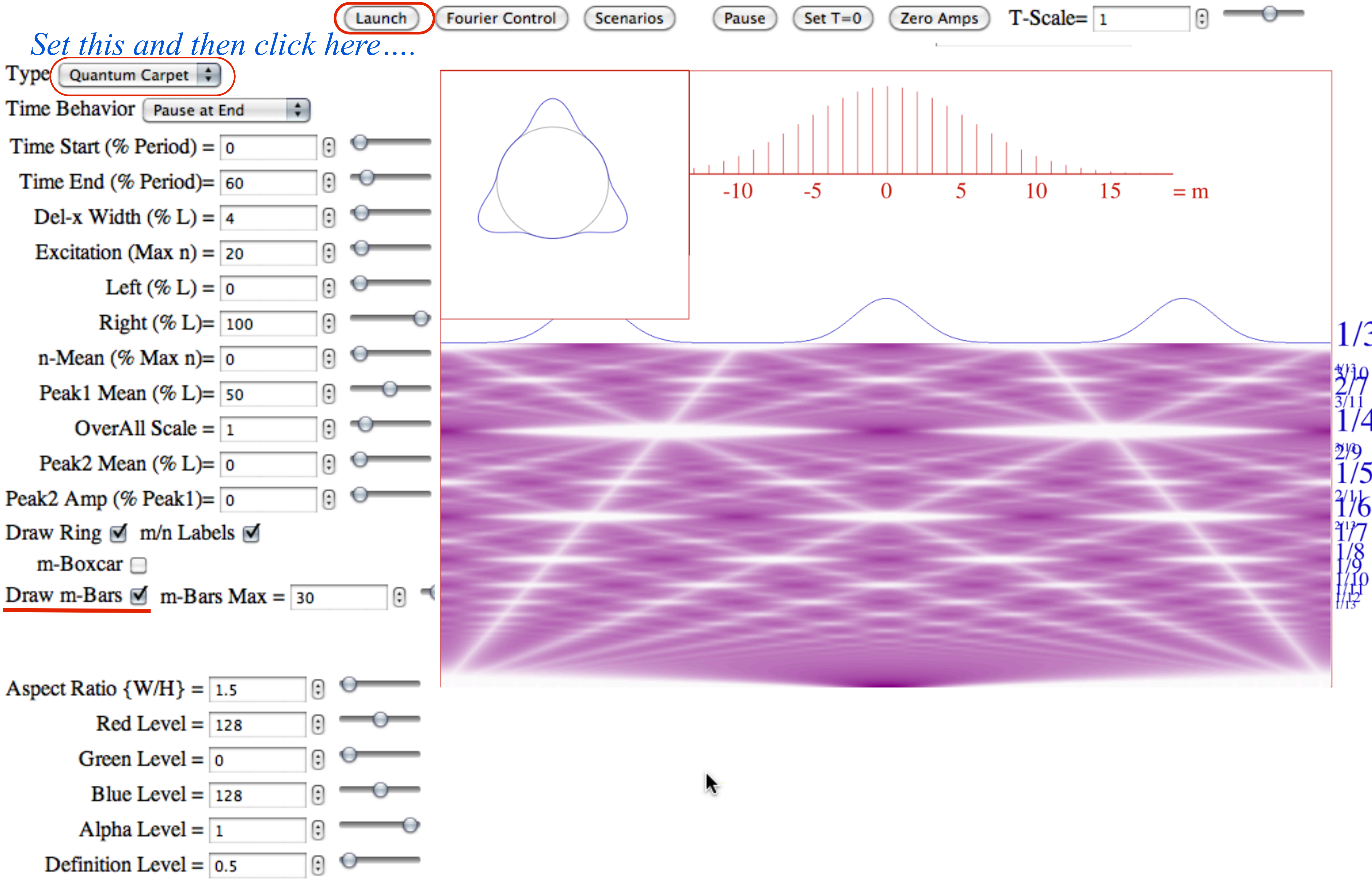
Click here for WaveIt Web simulation

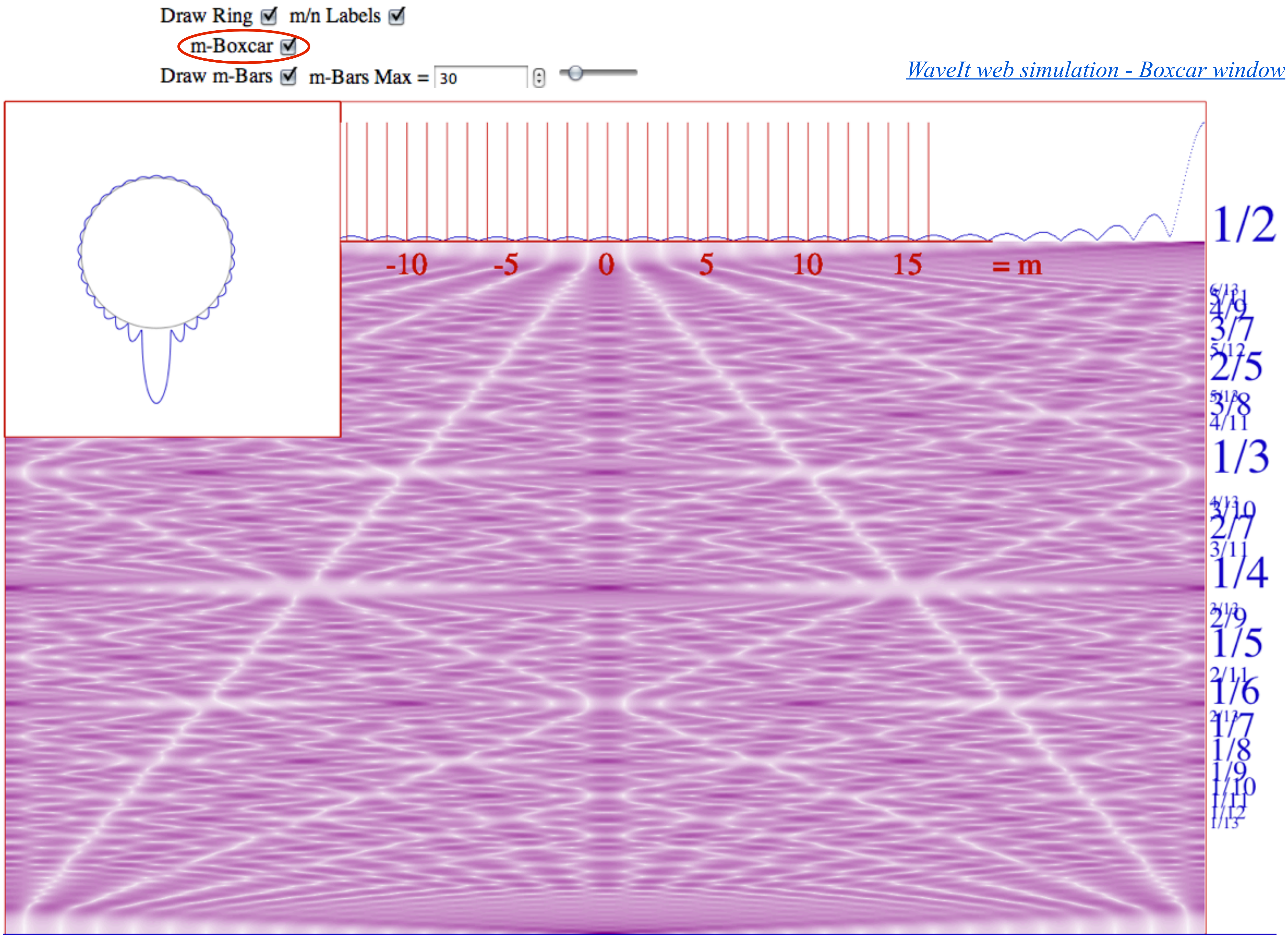





Local Control
Fourier Control
Scenarios
Pause
Set T=0
Zero Amps
T-Scale=
1
time = 0.60T
3/5
7/12
4/7
5/9
6/11
7/13
1/2
6/13
4/9
3/7
5/12
2/5
3/8
4/11
1/3
3/10
2/7
3/11
1/4
2/9
1/5
2/11
1/6
1/7
1/8
1/9
1/10
1/11
1/12
1/13

Set this and then click here....



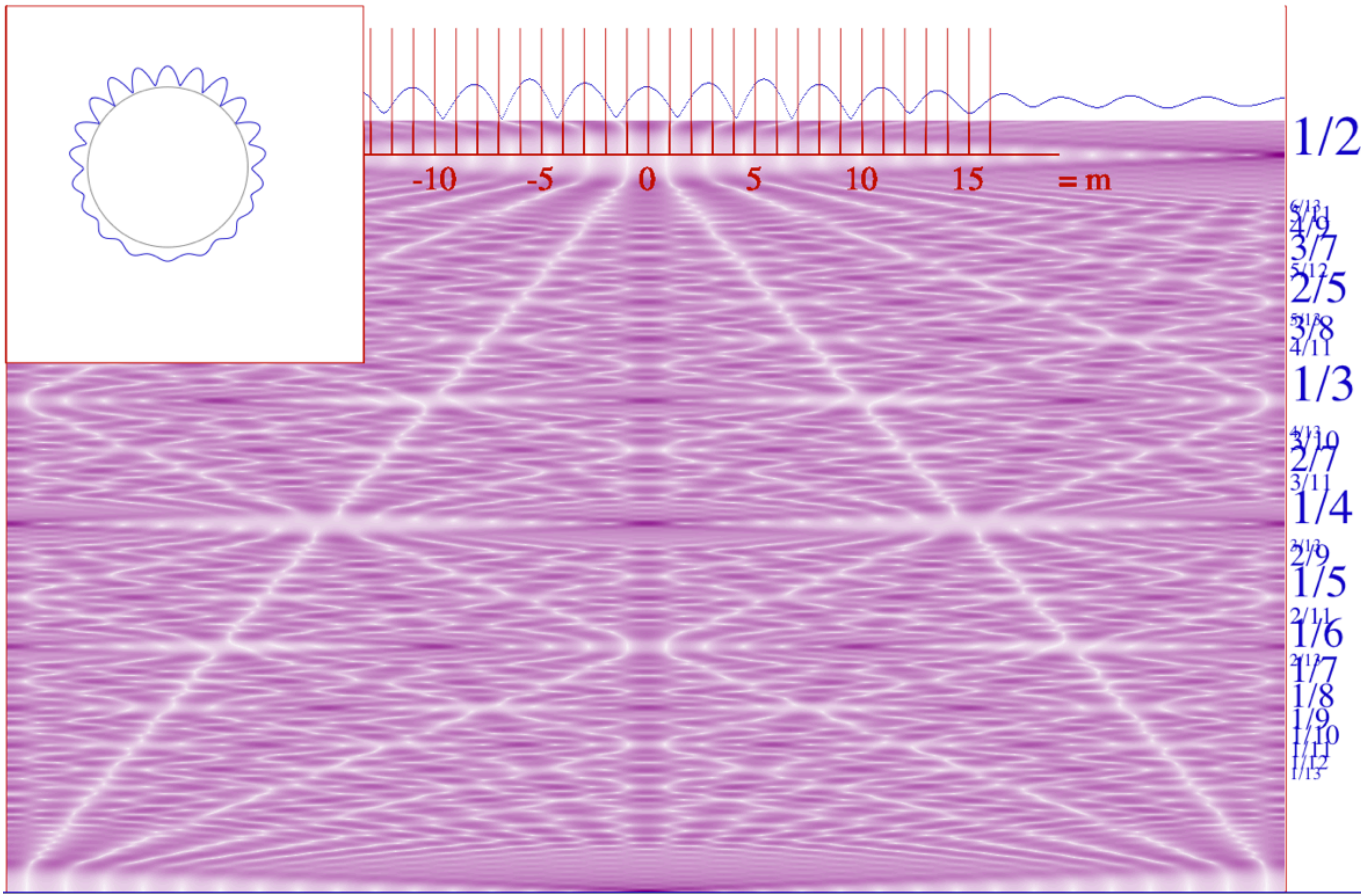

Draw Ring m/n Labels
m-Boxcar
Draw m-Bars m-Bars Max = 30
WaveIt web simulation - Boxcar window
-10 -5 0 5 10 15 = m
1/2
3/7
2/5
3/8
1/3
2/7
1/4
1/5
1/6
1/7
1/8

Draw Ring ☑ m/n Labels ☑

m-Boxcar ☑

Draw m-Bars ☑ m-Bars Max = 30

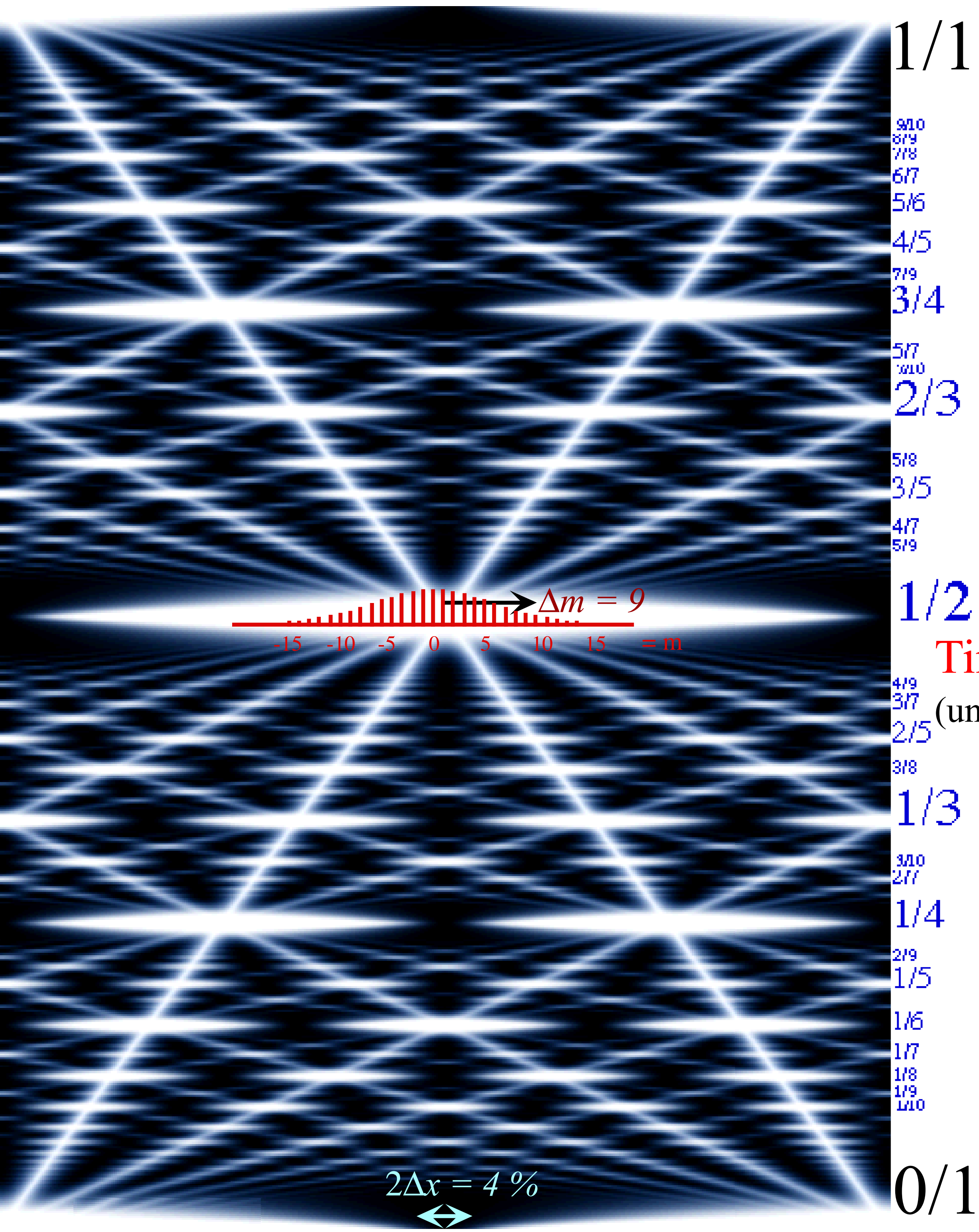
[WaveIt web simulation - Boxcar window]



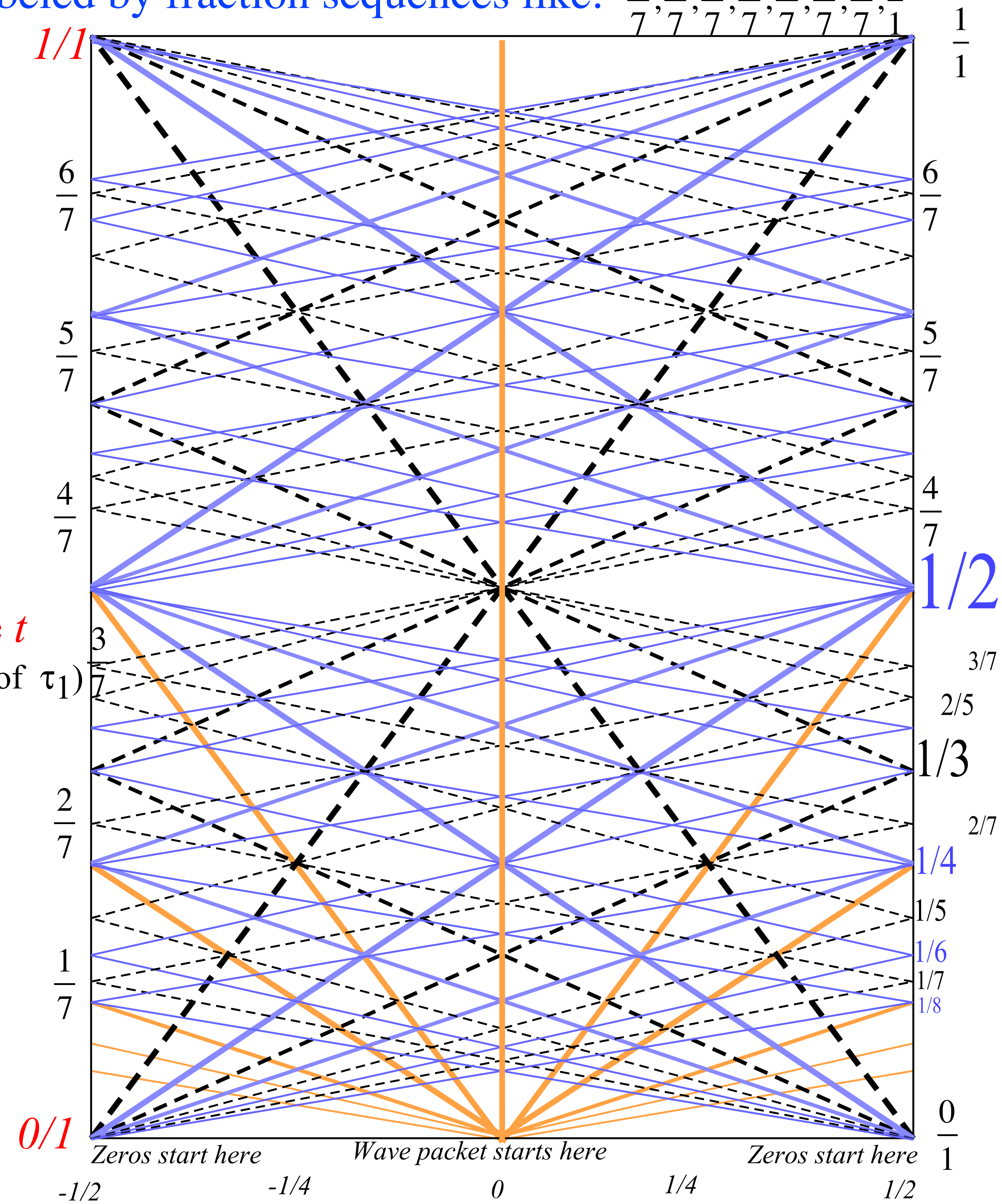
N-level-system and revival-beat wave dynamics

(9 or10-levels (0, ±1, ±2, ±3, ±4,..., ±9, ±10, ±11...) excited)

Zeros (clearly) and "particle-packets" (faintly) have paths labeled by fraction sequences like: $\frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{1}{1}$



Time *t* (units of τ_1)



Coordinate ϕ (units of 2π)

Type: Graphs

Time Behavior: Pause at End

Time Start (% Period) = 0

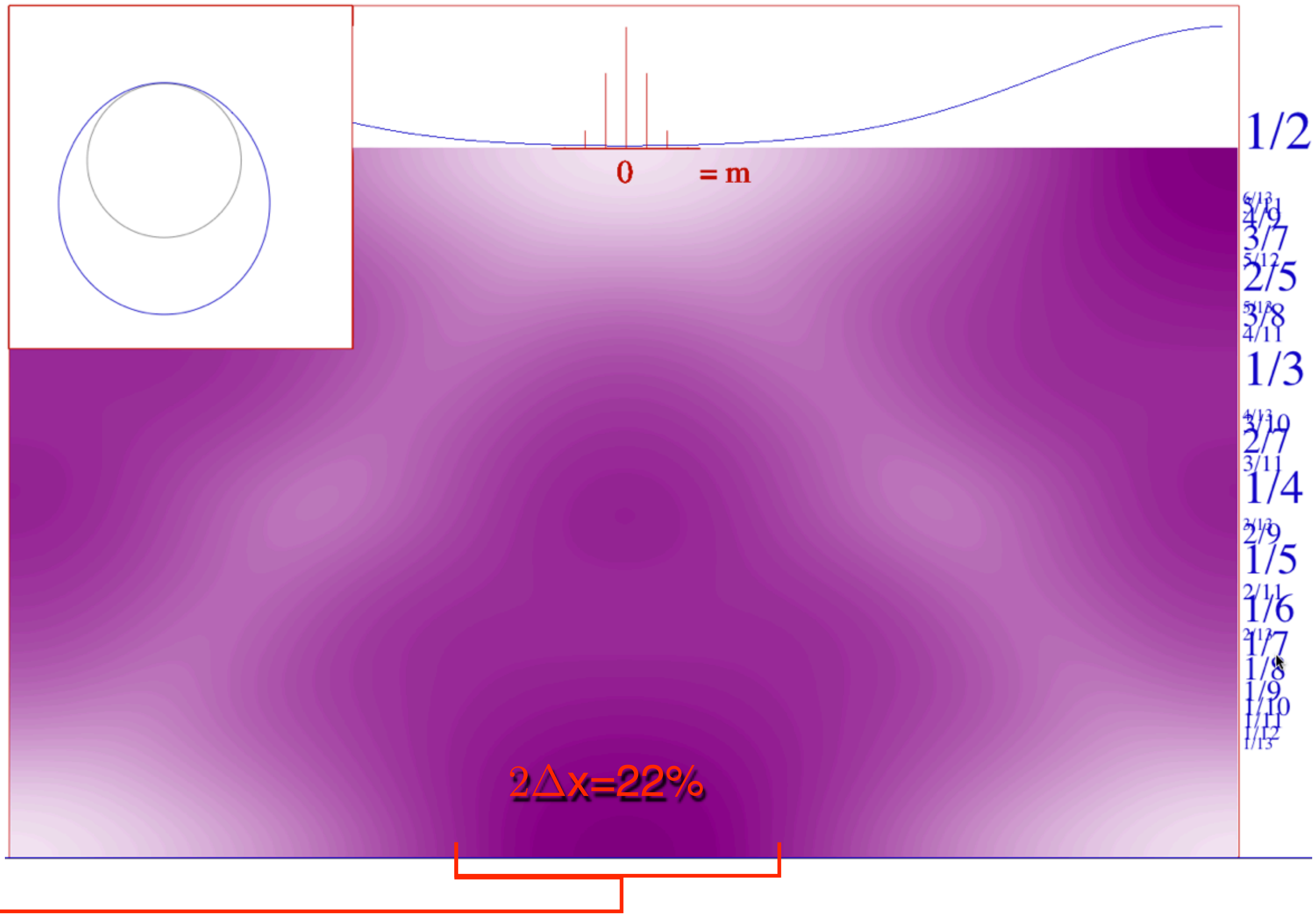
Time End (% Period)= 60

Del-x Width (% L) = 22

Excitation (Max n) = 20

Left (% L) = 0

Right (% L)= 100



Type: Graphs

Time Behavior: Pause at End

Time Start (% Period) = 0

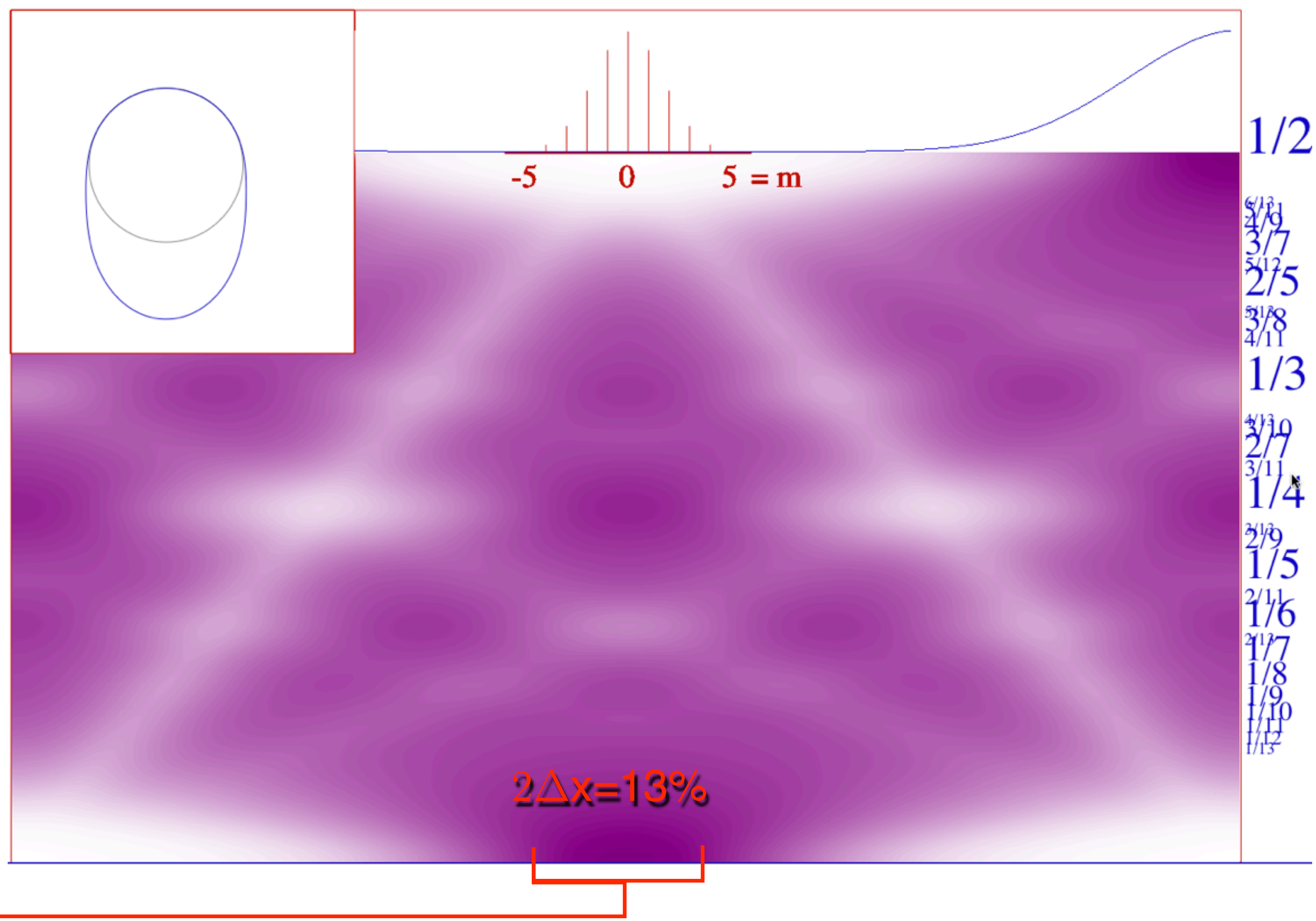
Time End (% Period)= 60

Del-x Width (% L) = 13

Excitation (Max n) = 20

Left (% L) = 0

Right (% L)= 100



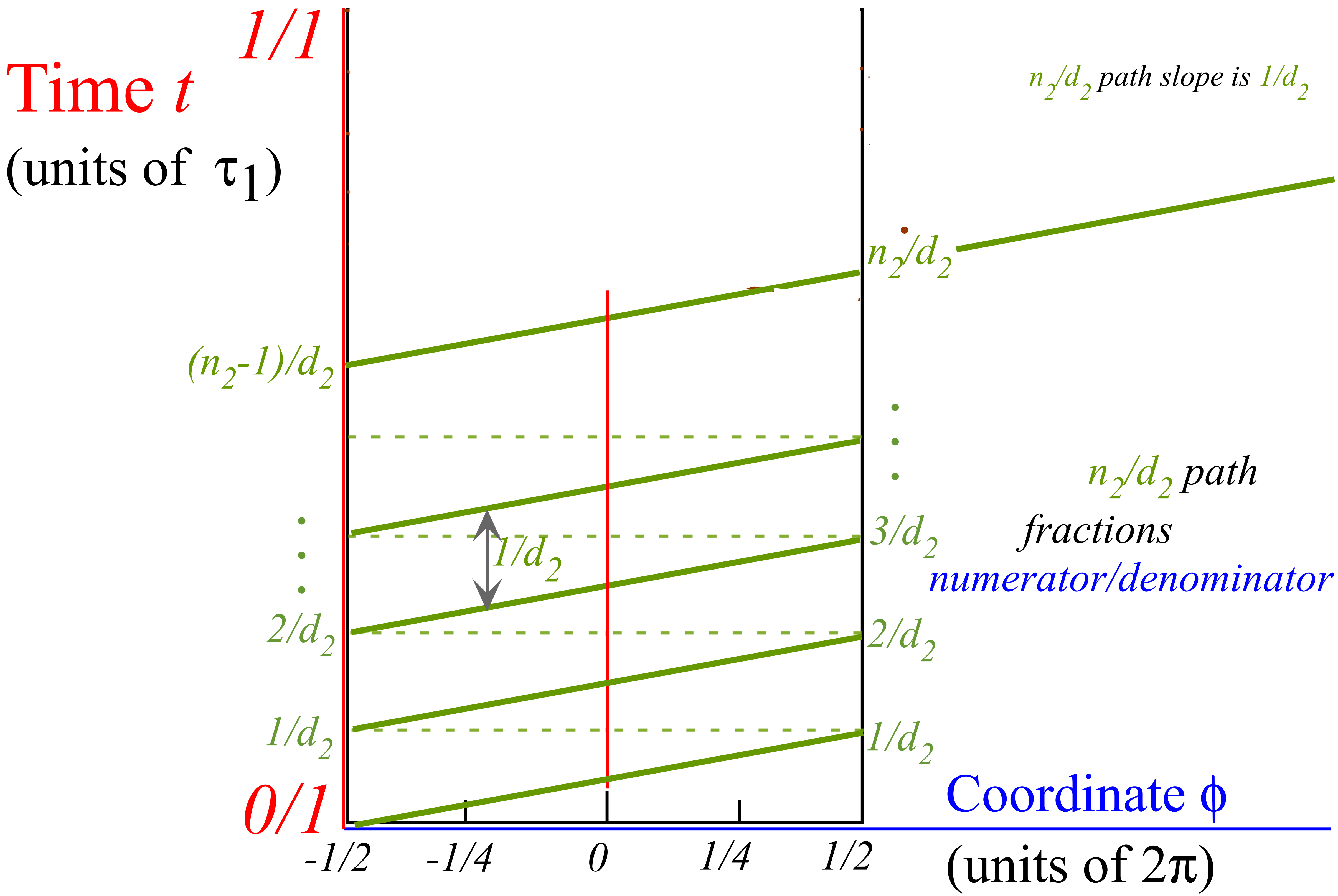
Matter-wave fractal behavior in a "Tiny-Big-Bang" [Harter, J. Mol. Spec. 210, 166-182 (2001)]; [Harter, Li IMSS (2013)]

A lesson in geometry of fractions and fractals: Ford Circles and Farey Sums

[Lester. R. Ford, Am. Math. Monthly 45,586(1938)]; [John Farey, Phil. Mag.(1816) Wolfram]; [Li, Harter, Chem.Phys.Letters (2015)]

Farey Sum algebra of revival-beat wave dynamics

Label by *numerators N* and *denominators D* of rational fractions *N/D*



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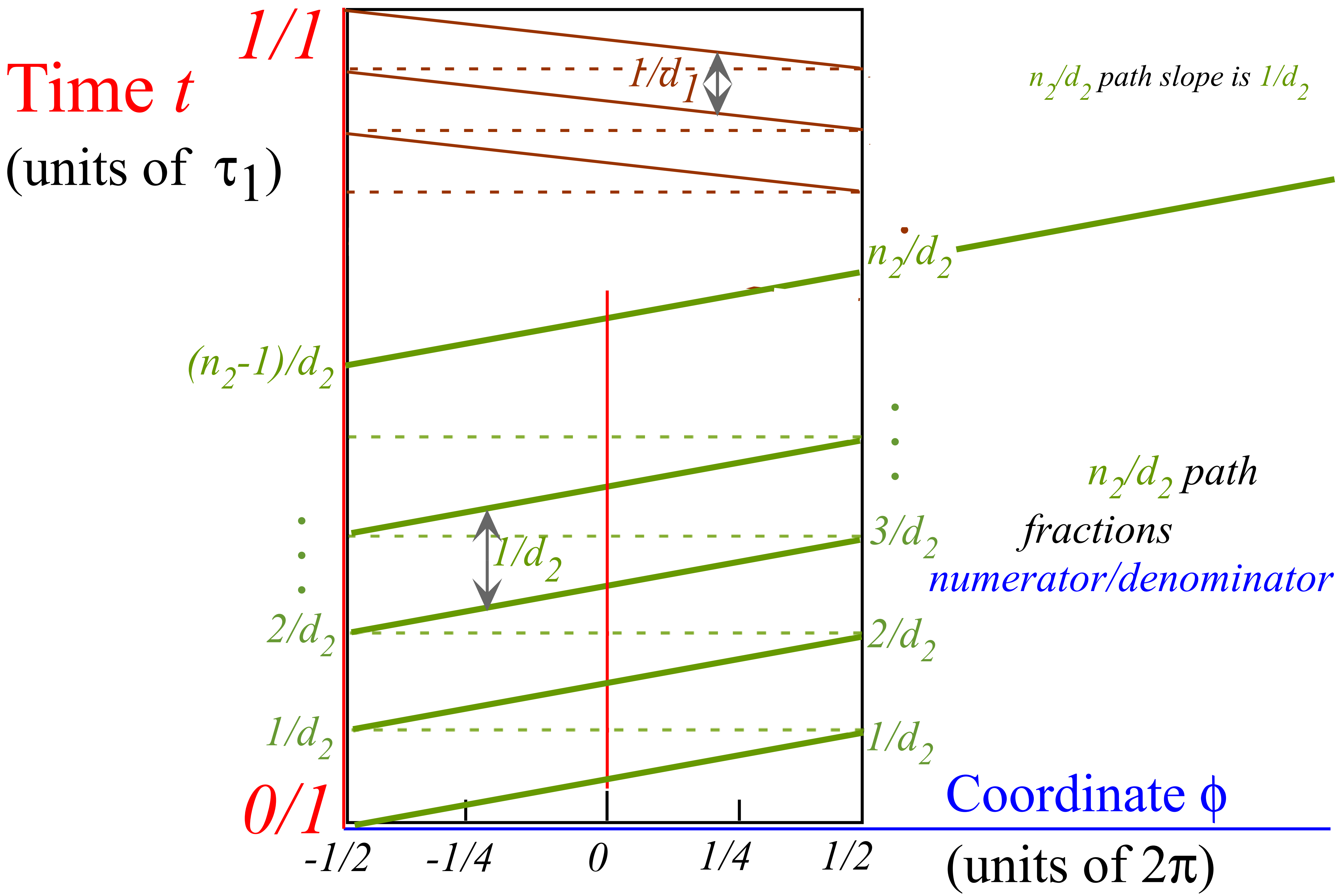

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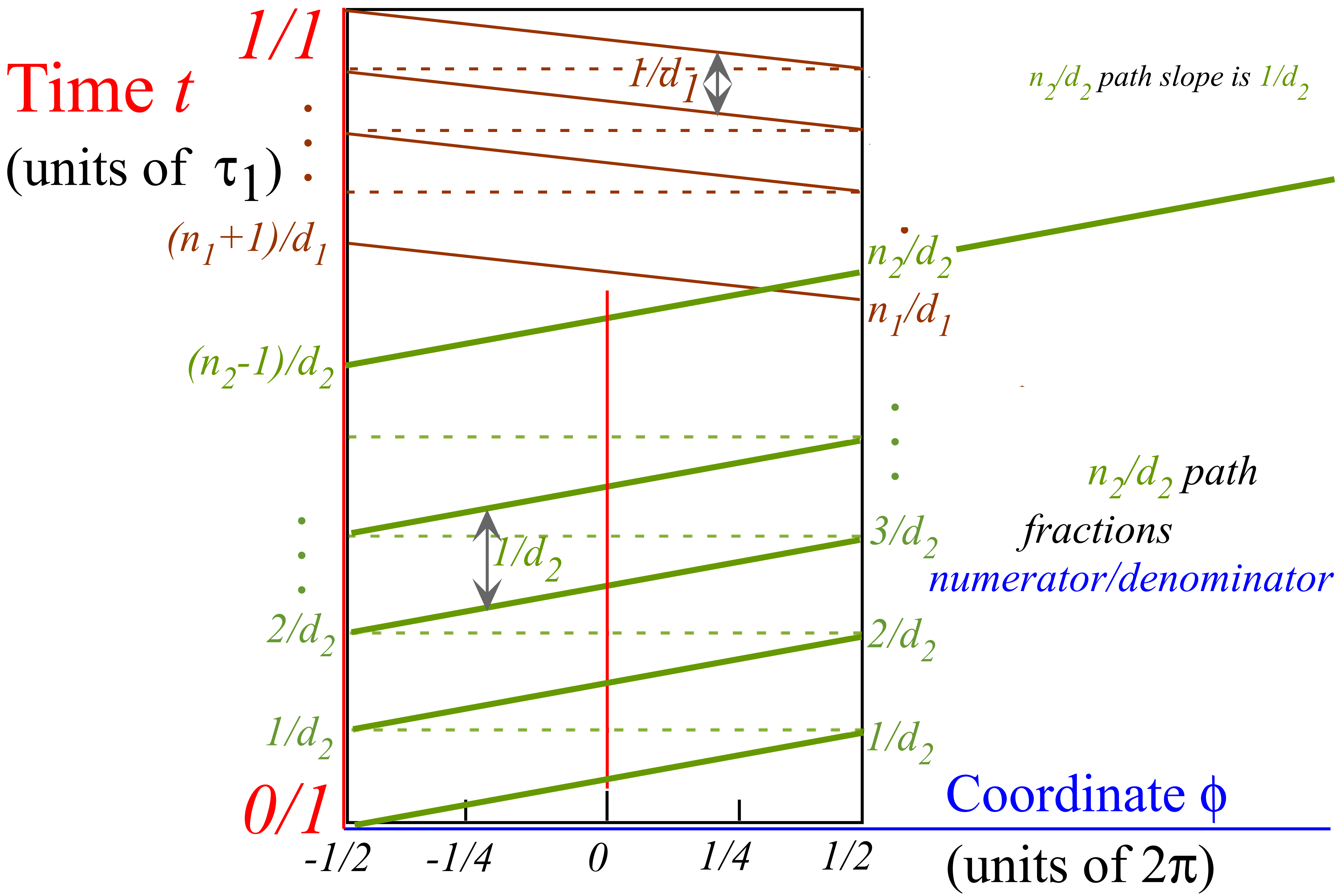

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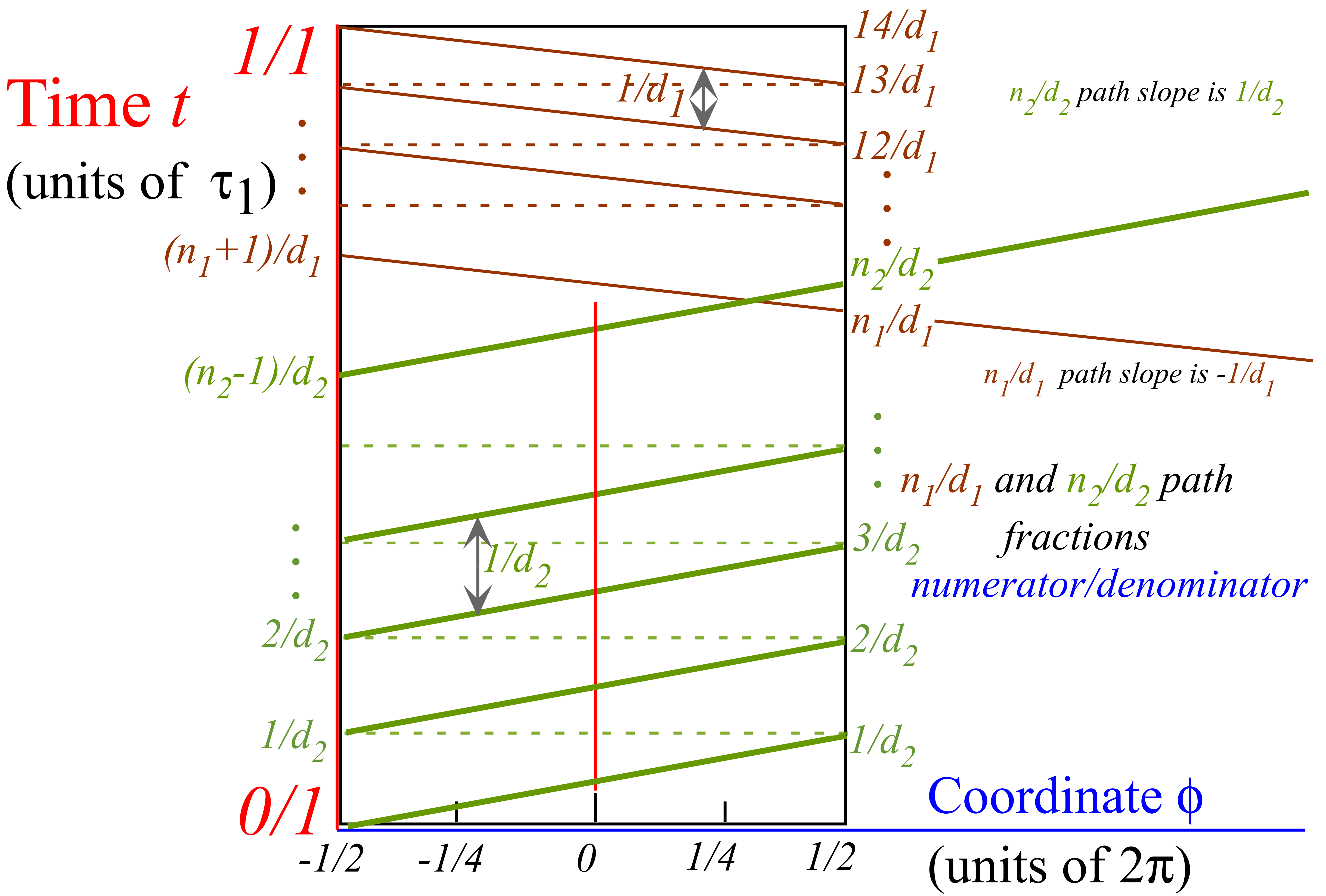

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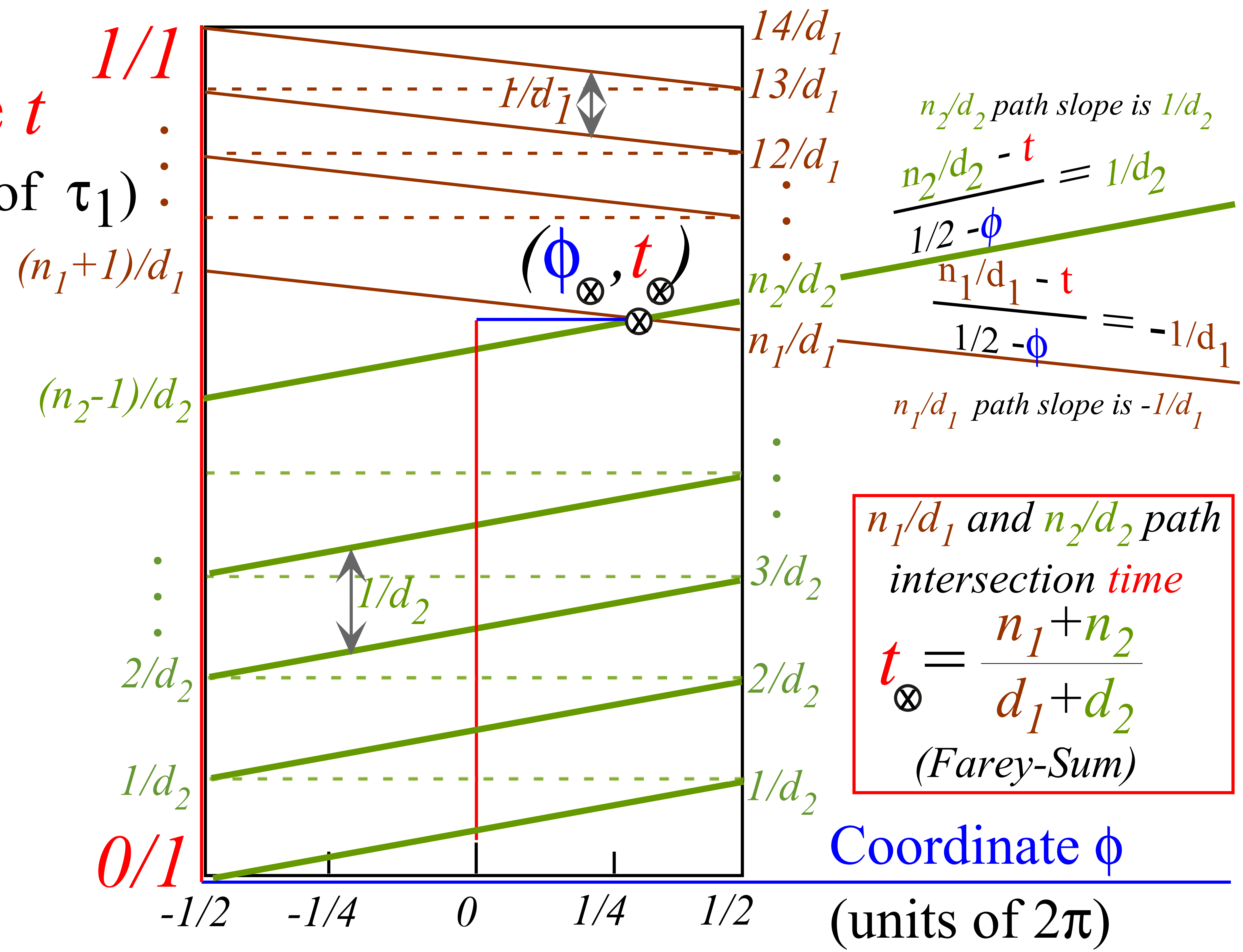

[Harter, J. Mol. Spec. 210, 166-182 (2001)]

[John Farey, Phil. Mag.(1816)]

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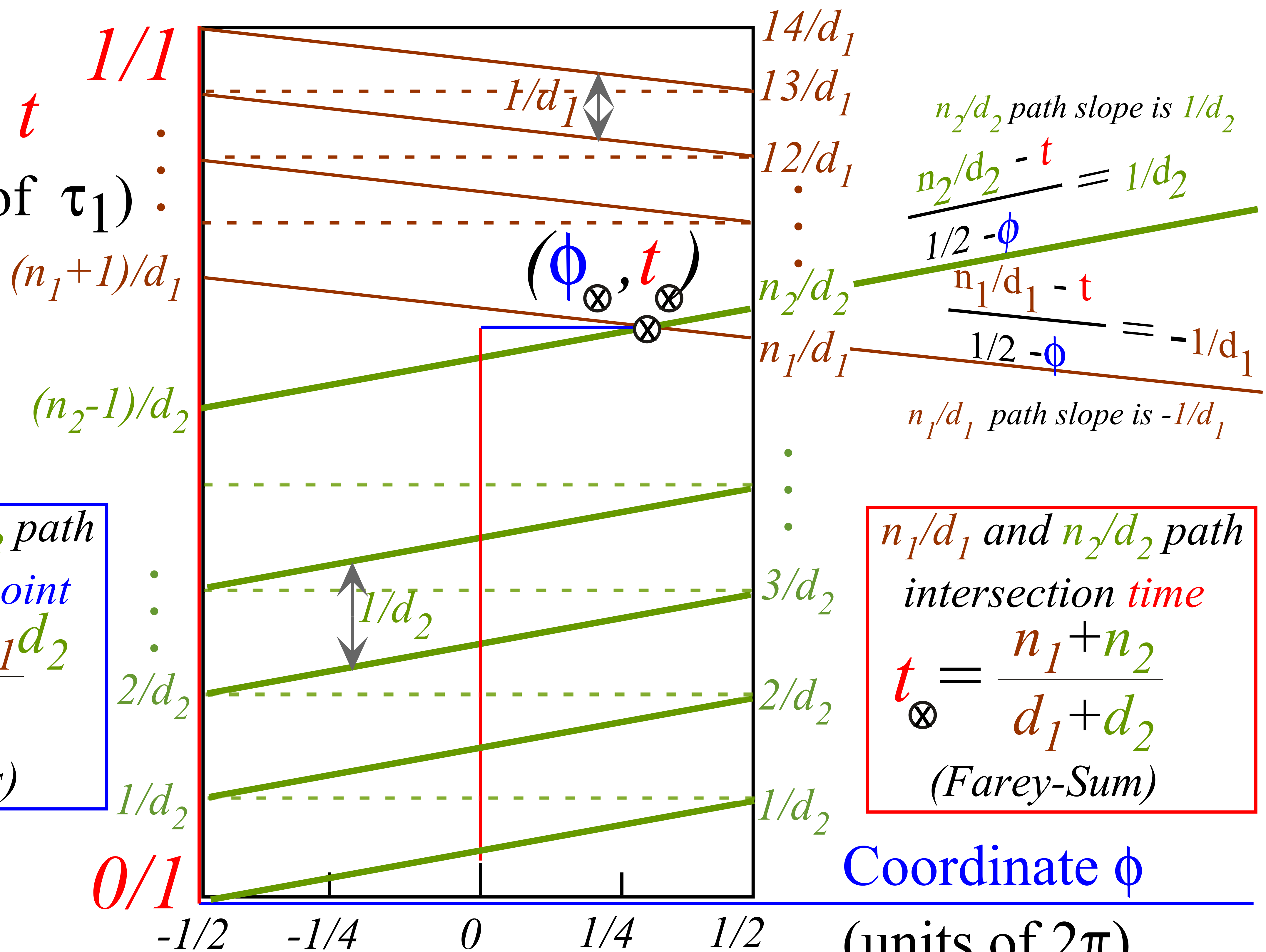

n_1/d_1 and n_2/d_2 path intersection point

$$\phi_\otimes = \frac{d_1 n_2 - n_1 d_2}{d_1 + d_2}$$

(Ford-Cross)

n_1/d_1 and n_2/d_2 path intersection time

$$t_\otimes = \frac{n_1 + n_2}{d_1 + d_2}$$

(Farey-Sum)

[*Lester. R. Ford, Am. Math. Monthly 45,586(1938)*]

[*John Farey, Phil. Mag.(1816)*]

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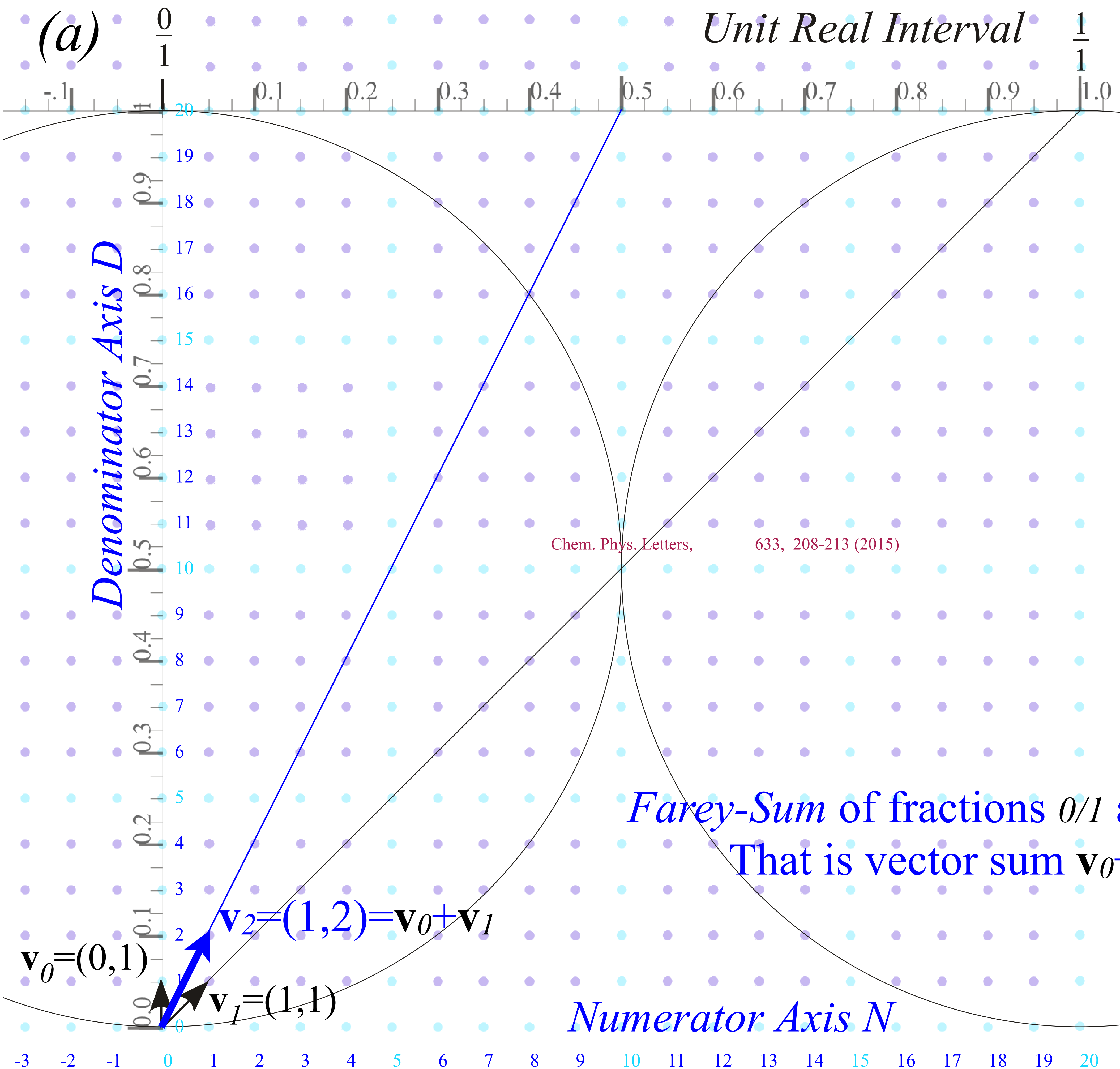

Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*

Farey-Sum of fractions *0/1* and *1/1* is *1/2*

That is vector sum $\mathbf{v}_0+\mathbf{v}_1 = (1,2) = \mathbf{v}_2$

[Li, Harter, Chem.Phys.Letters (2015)]

[Li, Harter, Chem.Phys.Letters **633**, 208-213(2015)]

Farey Sum related to vector sum and *Ford Circles*

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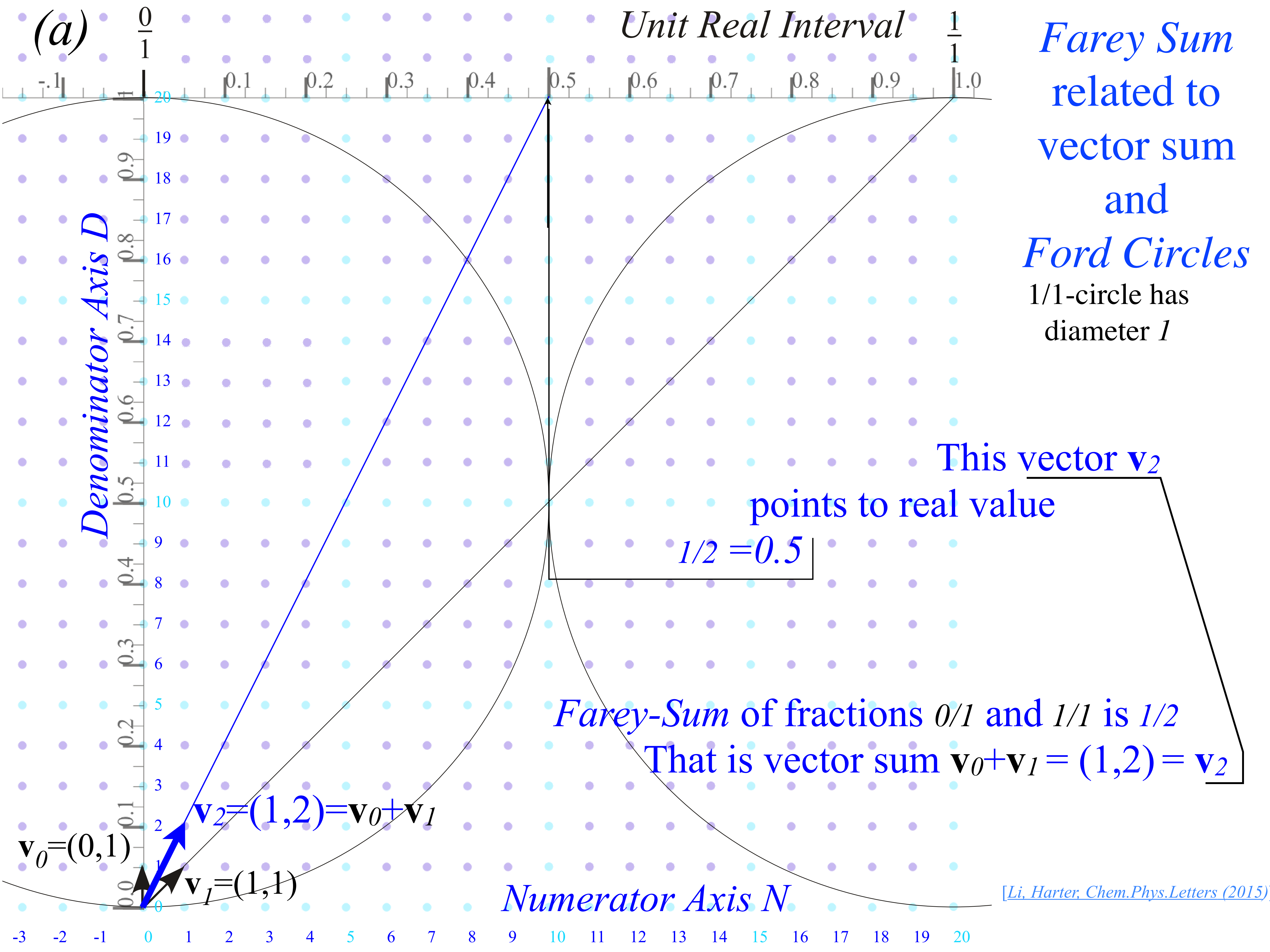
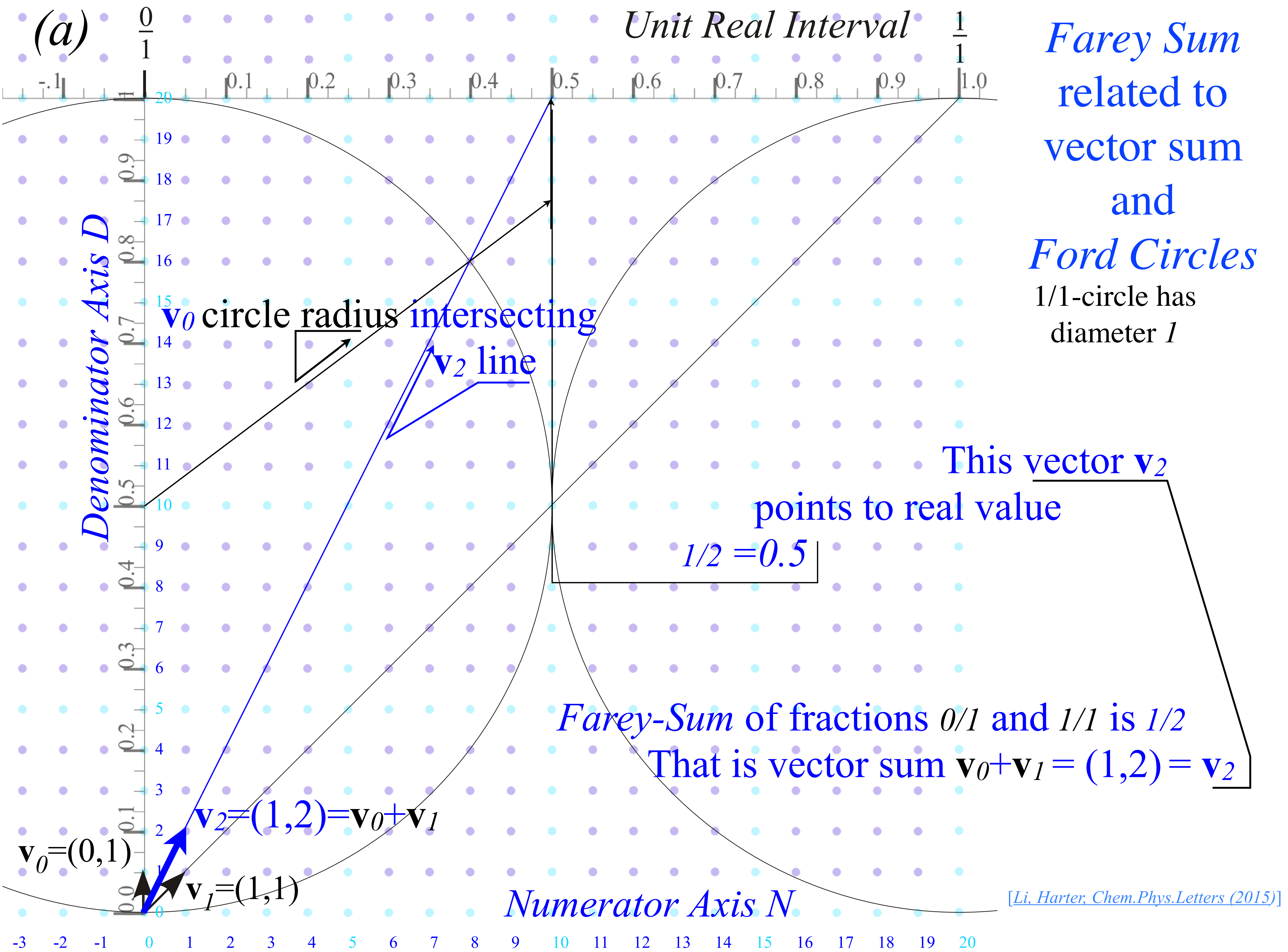
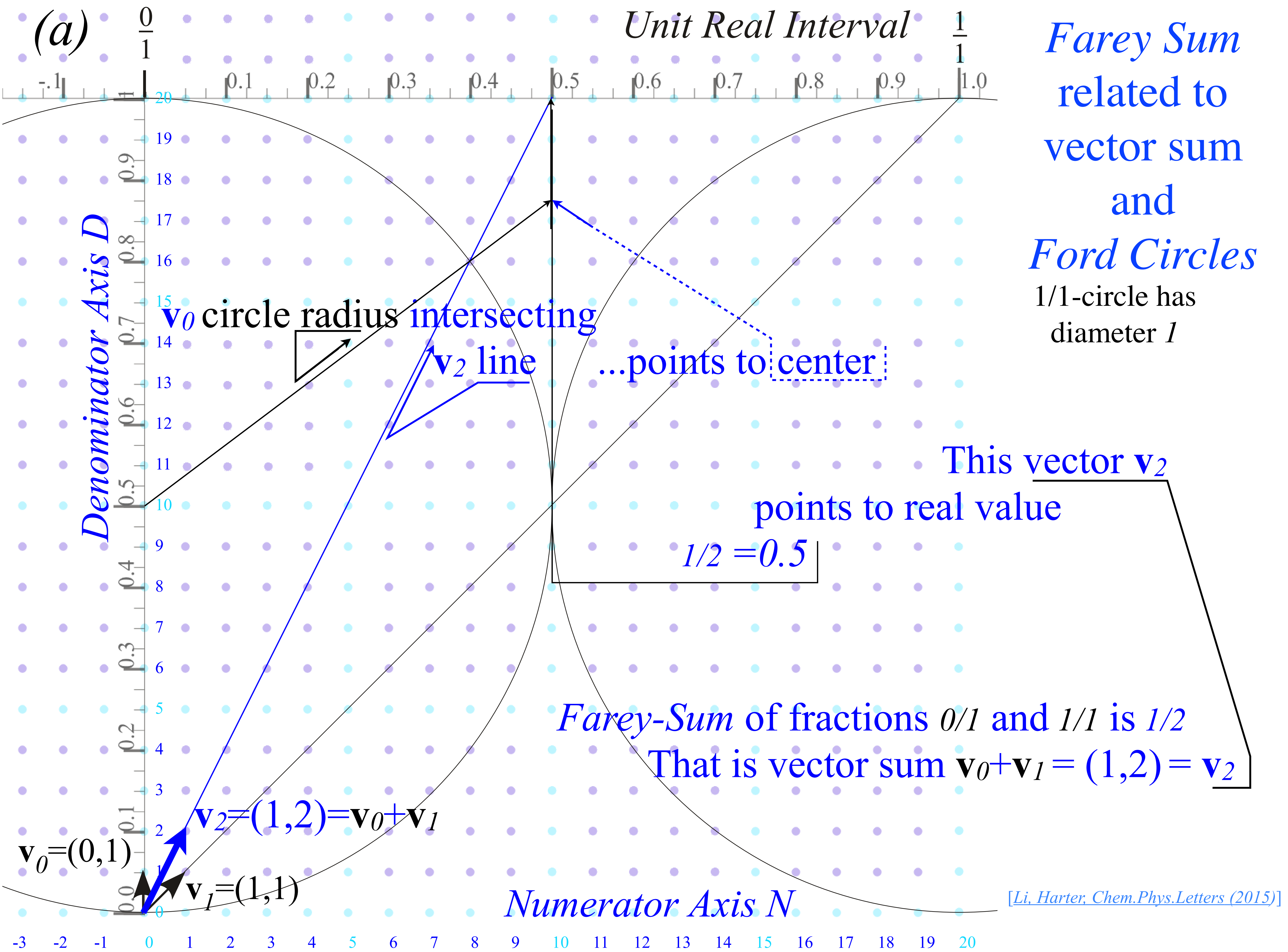
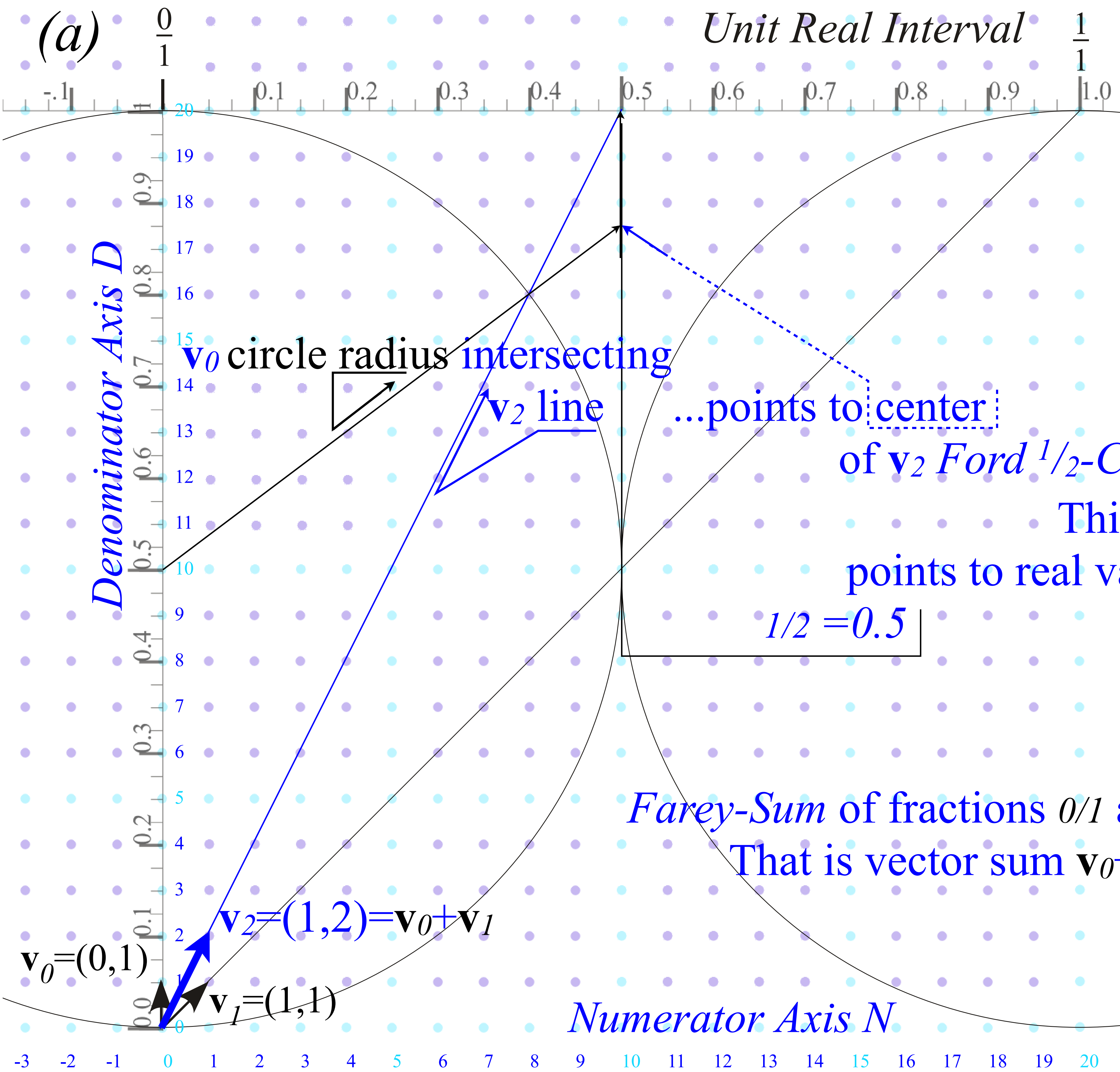



(a)
Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting v2 line
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]



(a)
0/1
Unit Real Interval
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
v0 circle radius intersecting
v2 line
...points to center
This vector v2 points to real value
1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
Numerator Axis N
[Li, Harter, Chem.Phys.Letters (2015)]



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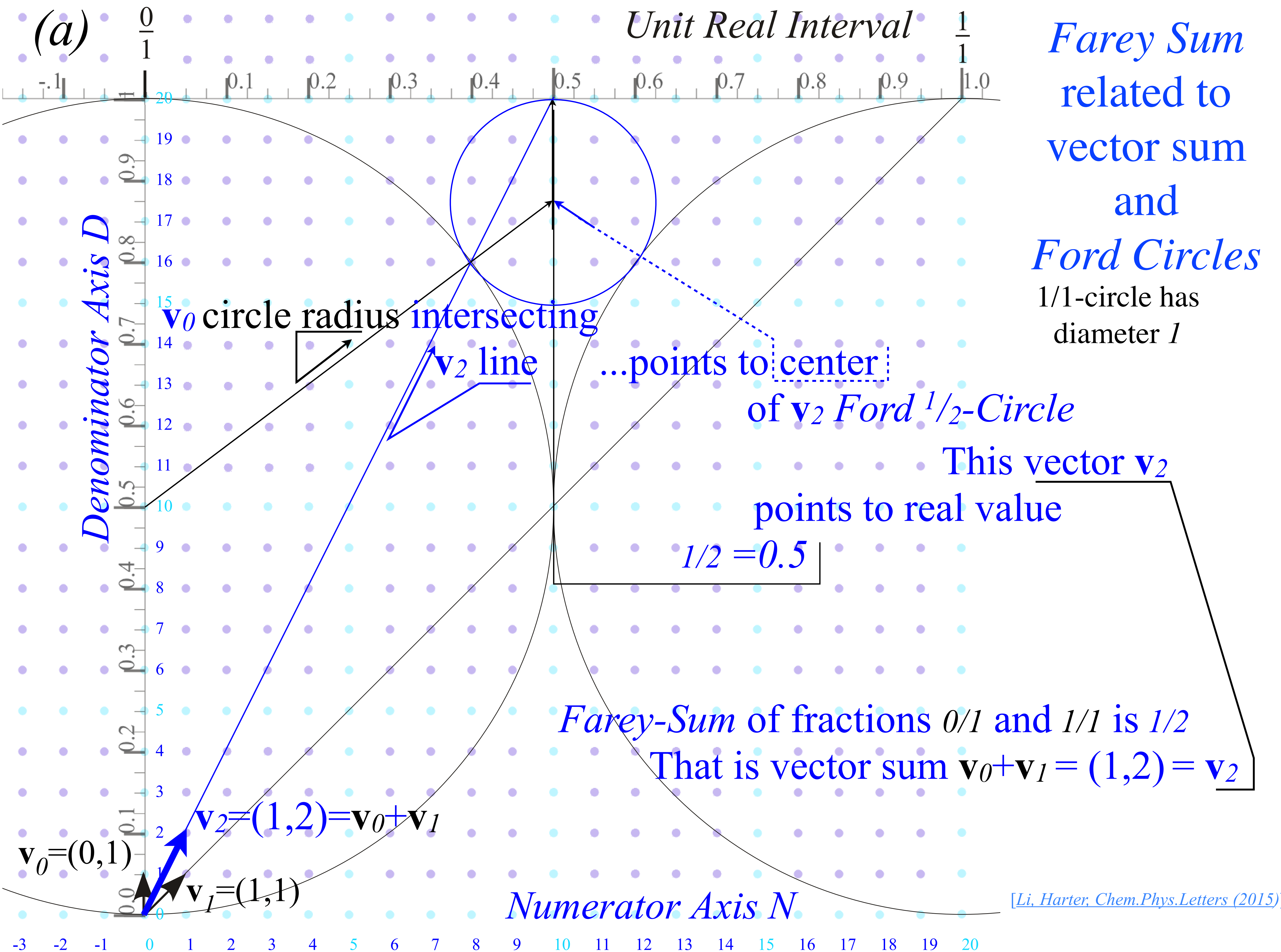
1/1-circle has diameter *1*

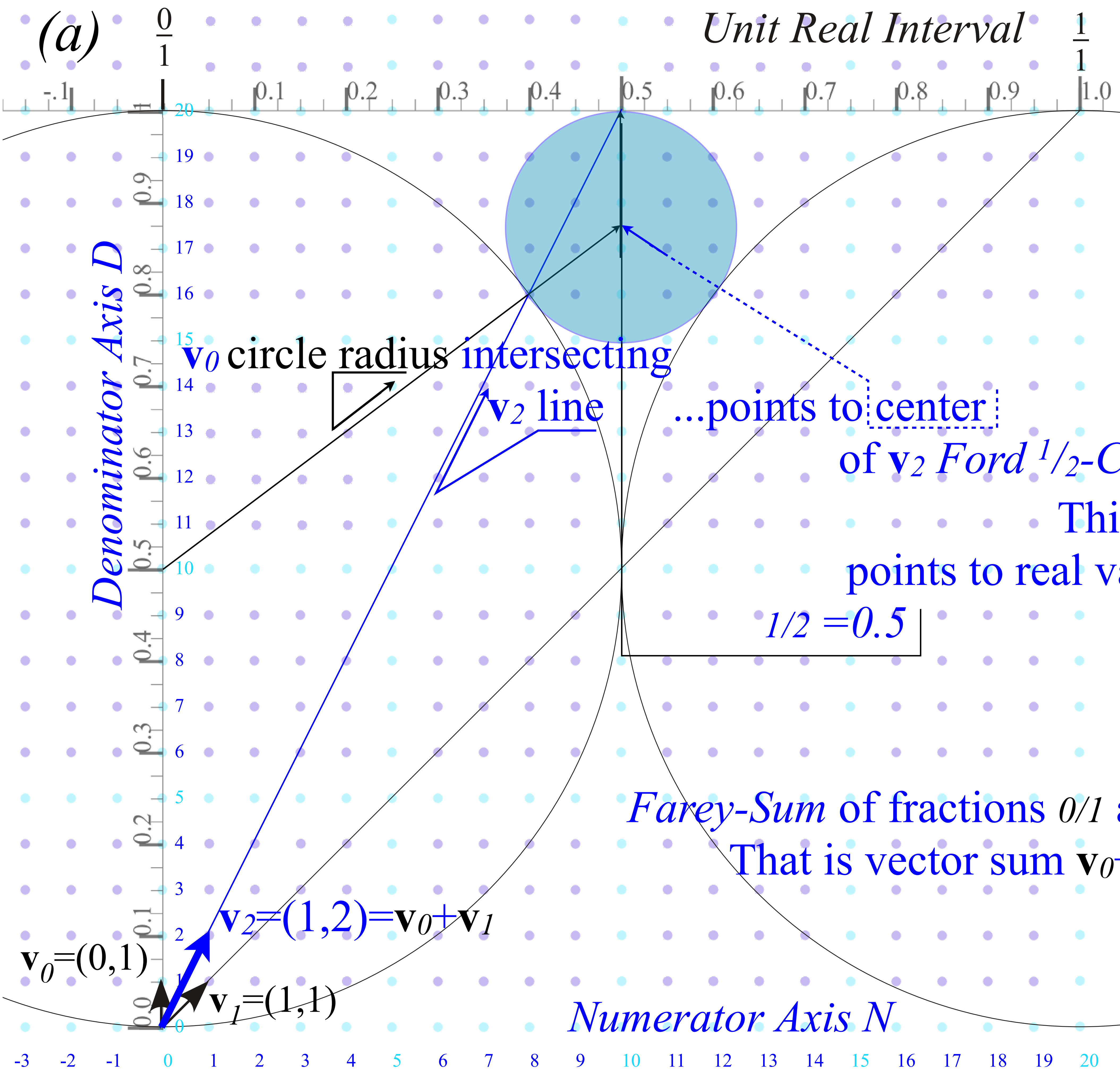
[Li, Harter, Chem.Phys.Letters (2015)]

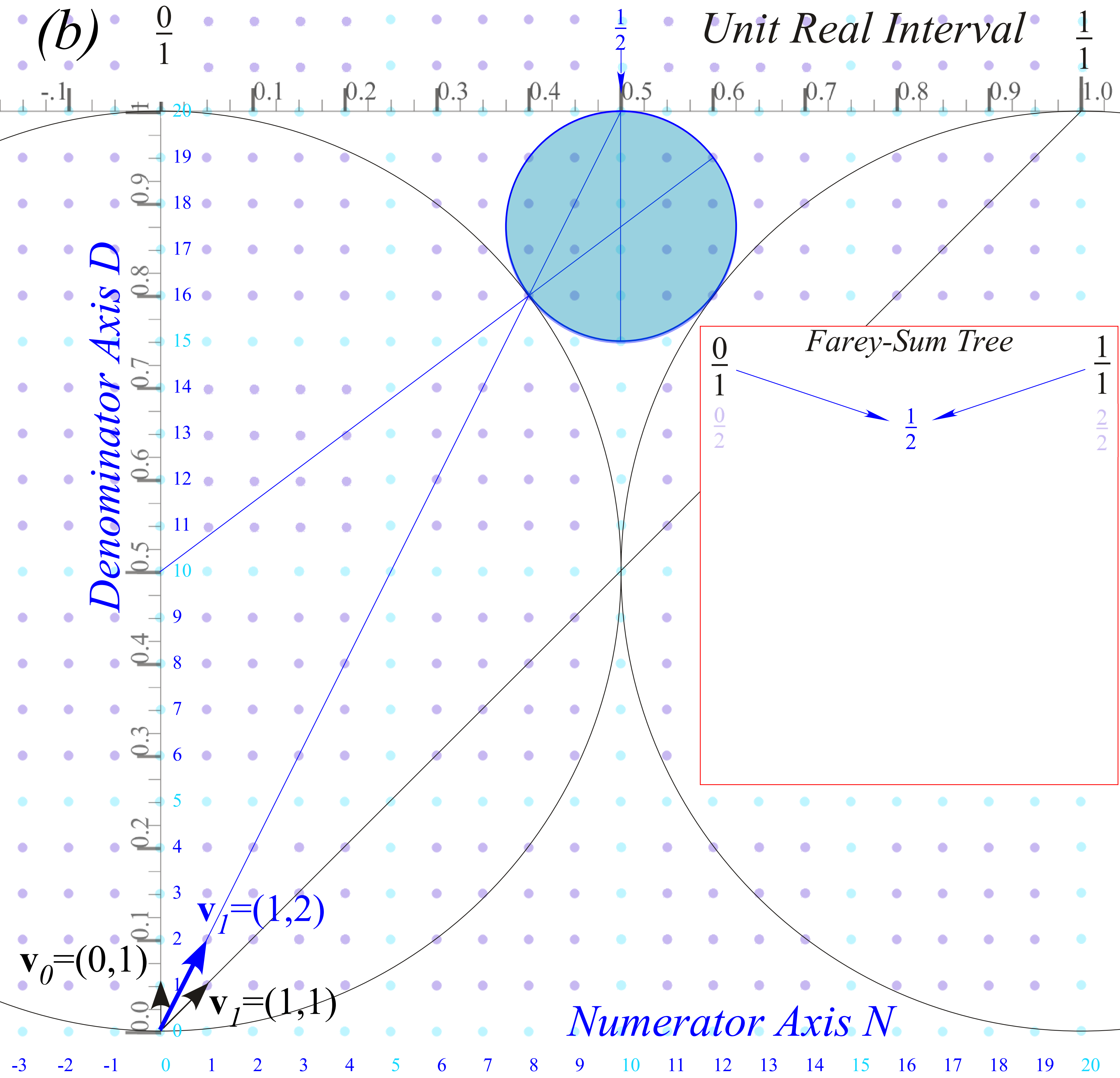


(a)
Unit Real Interval
0/1
1/1
Farey Sum related to vector sum and Ford Circles
1/1-circle has diameter 1
Denominator Axis D
Numerator Axis N
v0 circle radius intersecting
v2 line
...points to center of v2 Ford 1/2-Circle
This vector v2 points to real value 1/2 =0.5
Farey-Sum of fractions 0/1 and 1/1 is 1/2
That is vector sum v0+v1 = (1,2) = v2
v2=(1,2)=v0+v1
v0=(0,1)
v1=(1,1)
[Li, Harter, Chem.Phys.Letters (2015)]

Farey Sum related to vector sum and *Ford Circles*

1/1-circle has diameter *1*



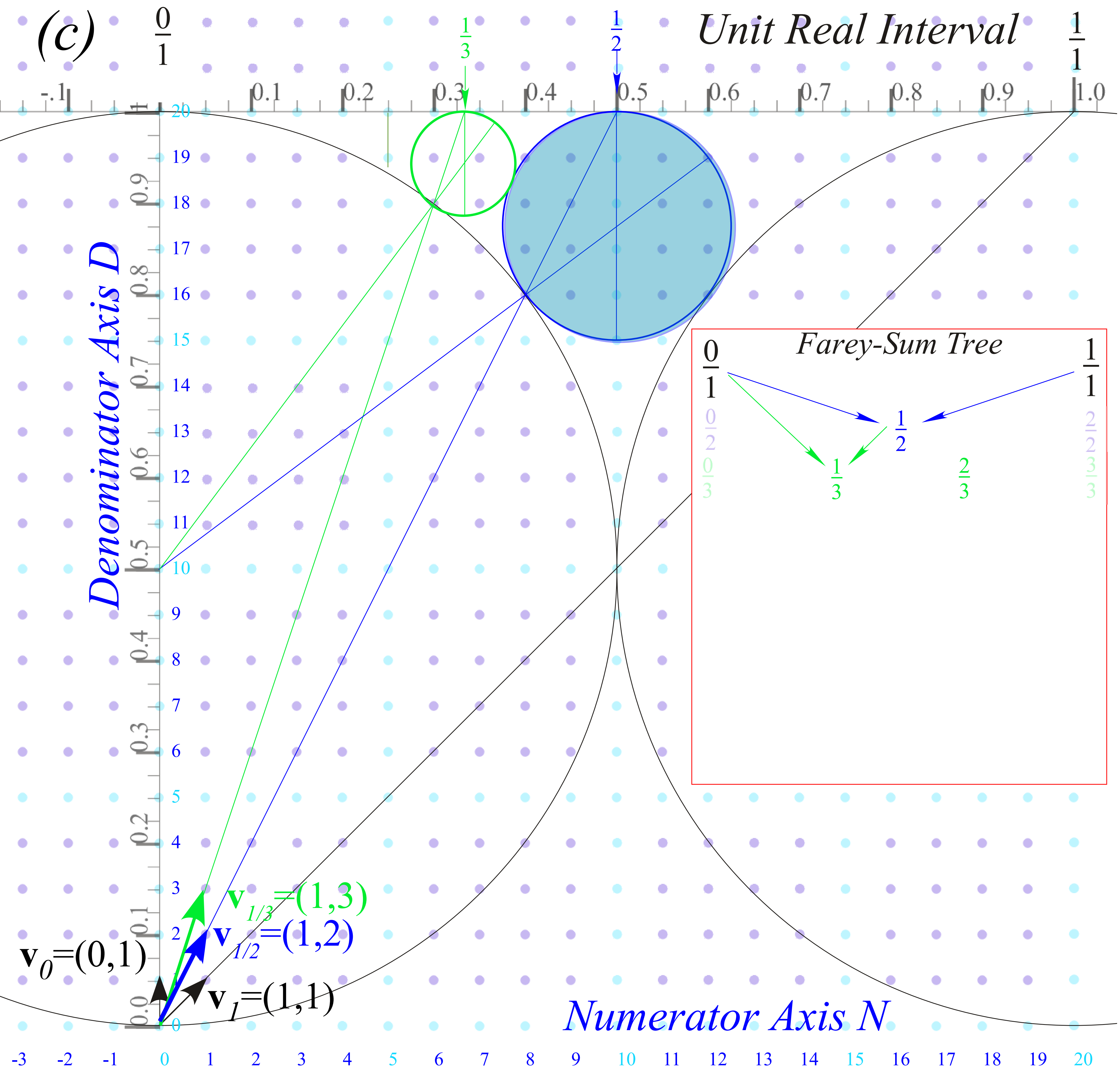


Farey Sum related to vector sum and Ford Circles

1/1-circle has diameter *1*

1/2-circle has diameter $1/2^2=1/4$

[Li, Harter, Chem.Phys.Letters (2015)]

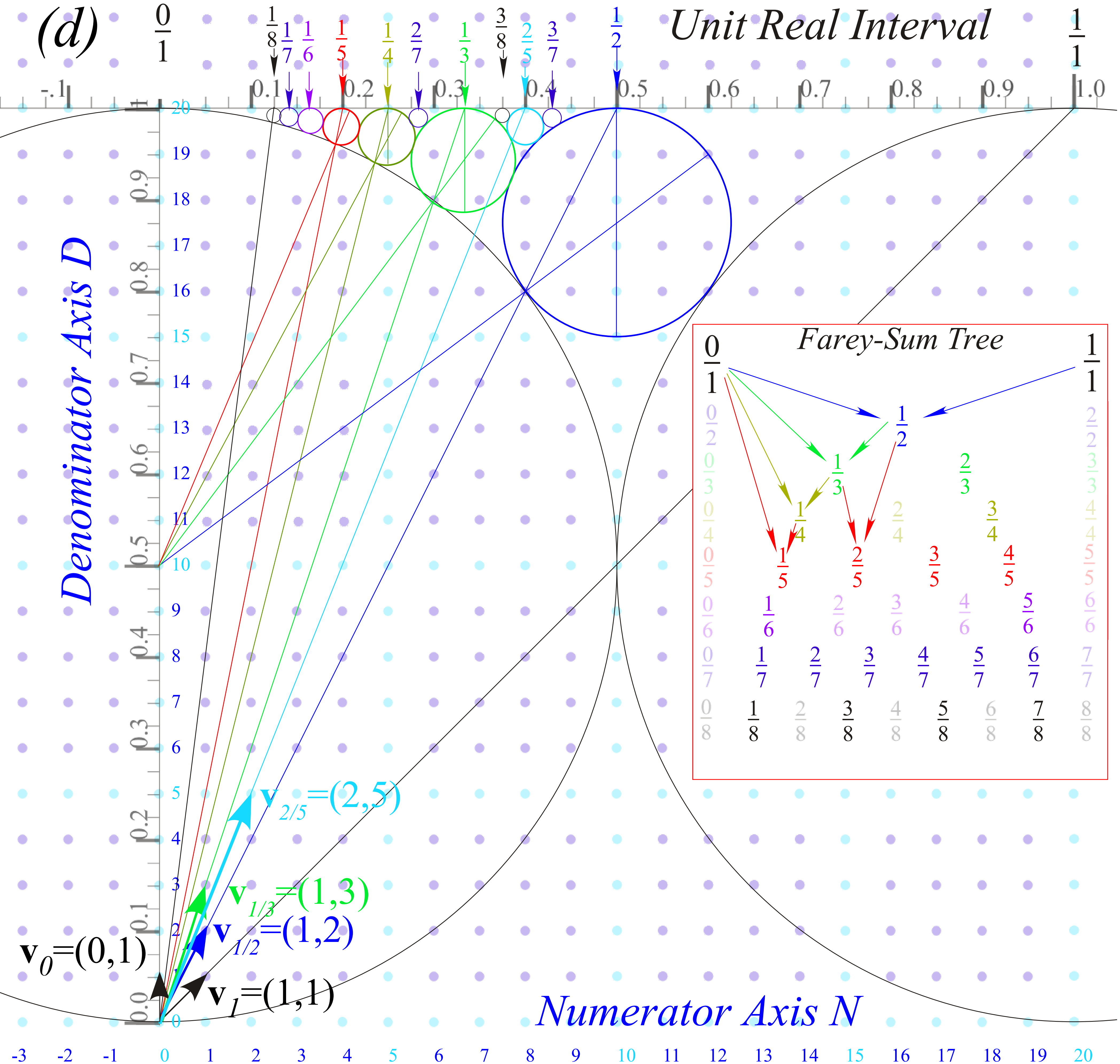


Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

[Li, Harter, Chem.Phys.Letters (2015)]



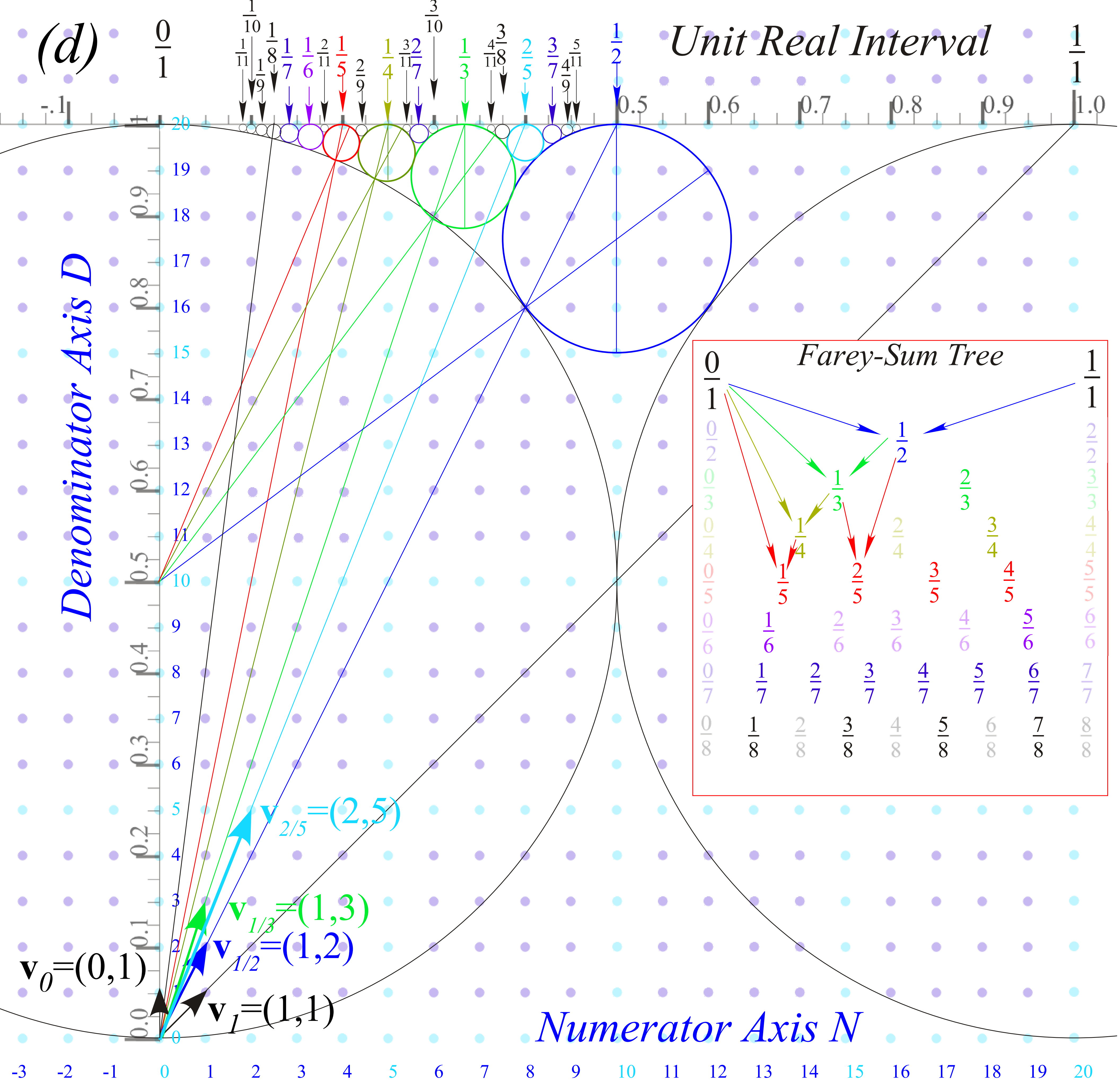
Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

[Li, Harter, Chem.Phys.Letters (2015)]



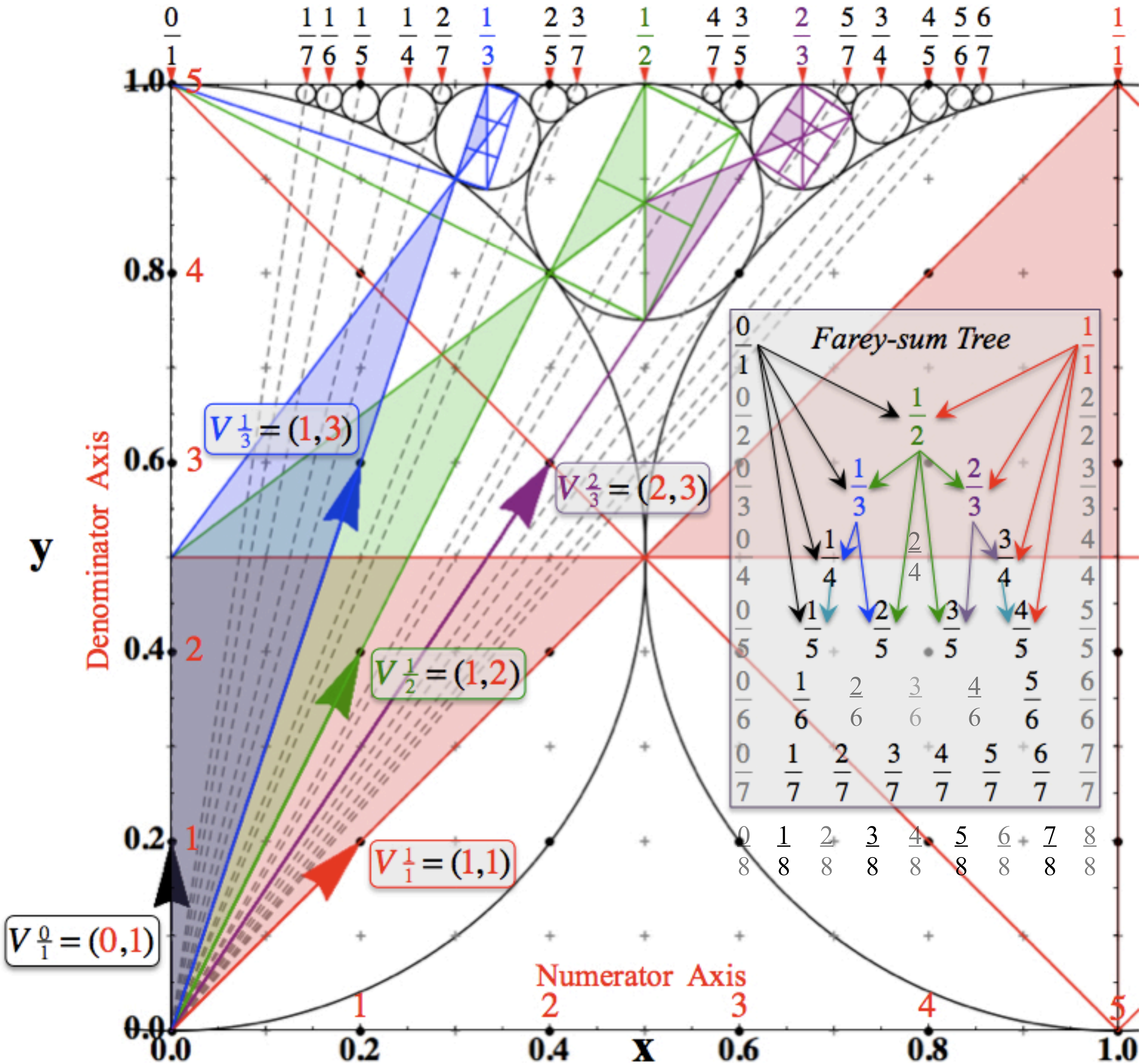
Farey Sum related to vector sum and *Ford Circles*

1/2-circle has diameter $1/2^2=1/4$

1/3-circles have diameter $1/3^2=1/9$

n/d-circles have diameter $1/d^2$

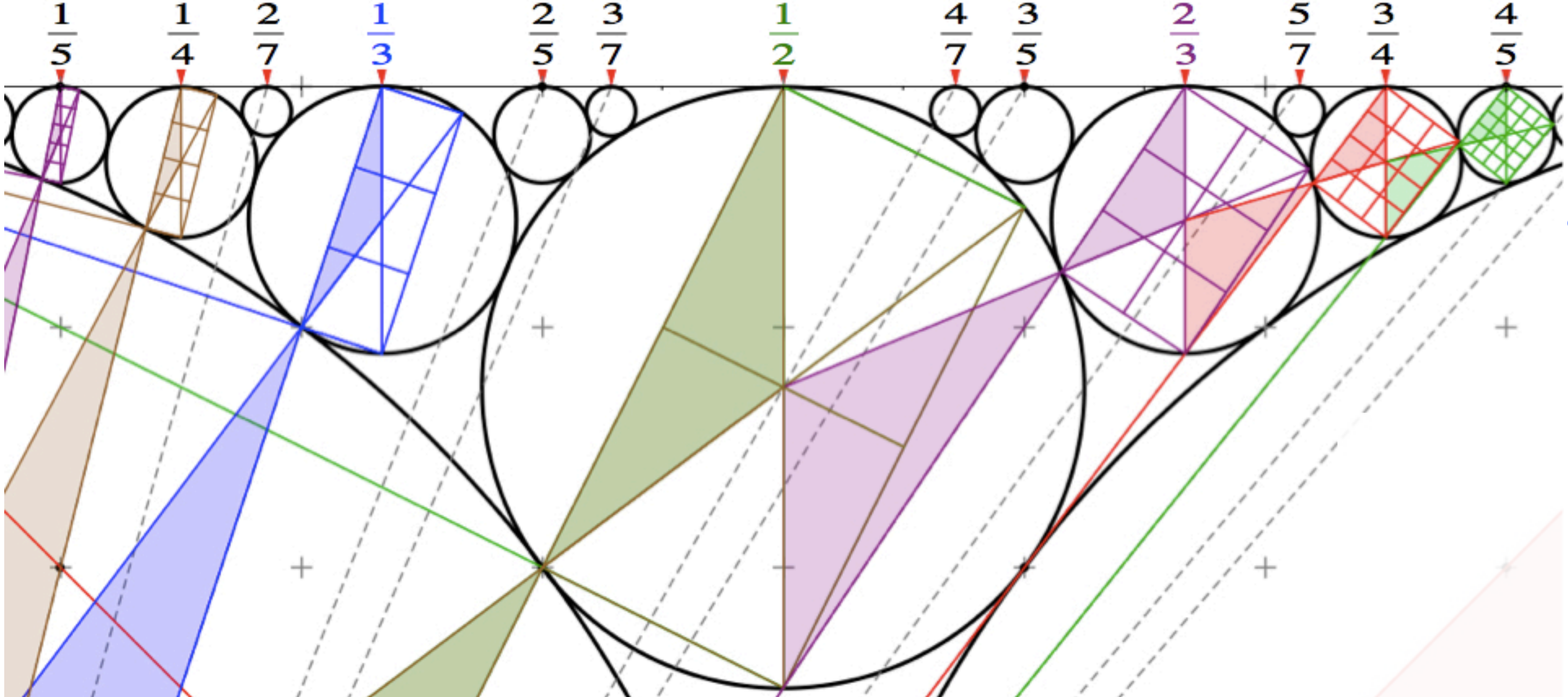
[Li, Harter, Chem.Phys.Letters (2015)]



Thales Rectangles

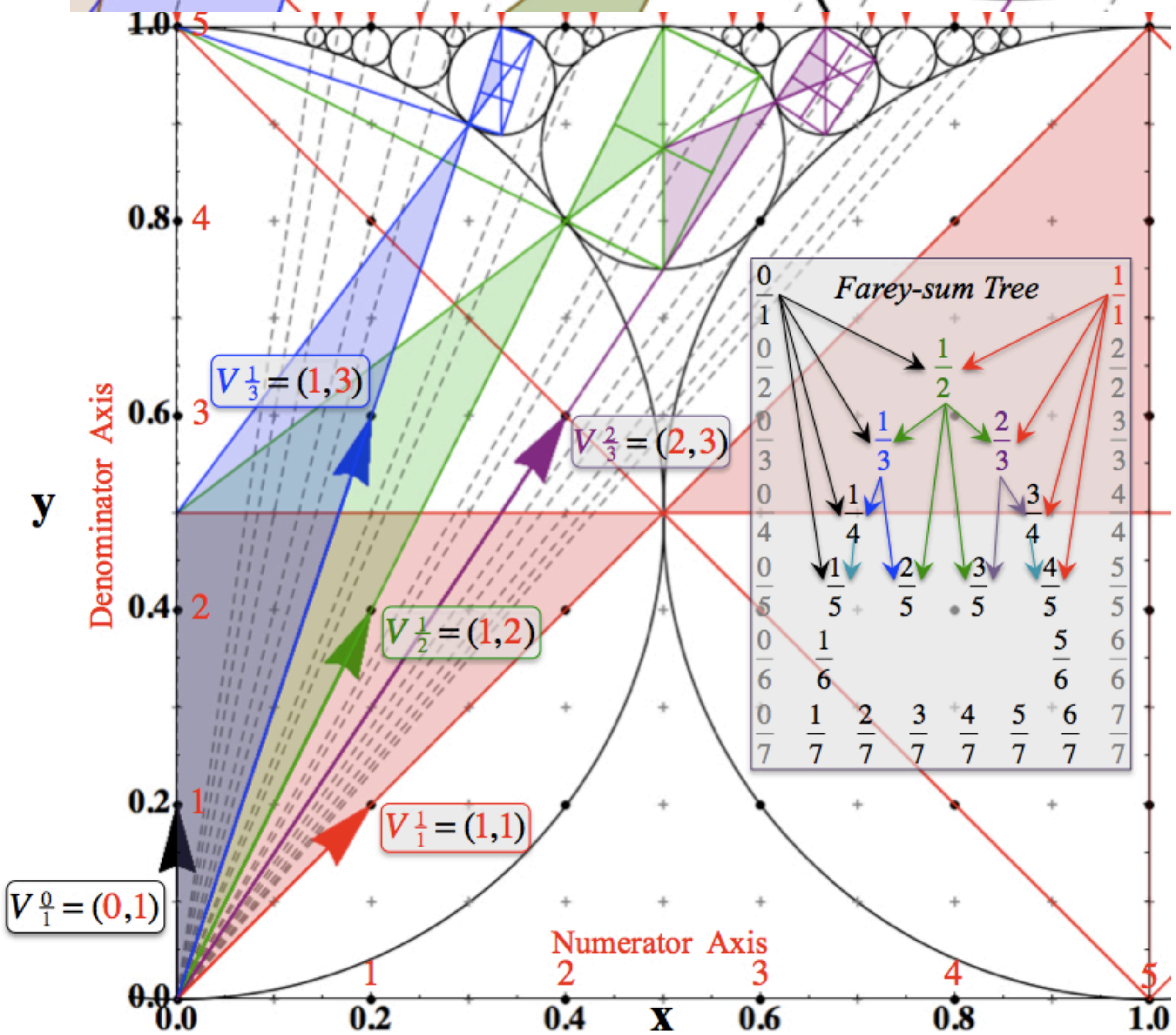
provide
analytic geometry
of
fractal structure

[Li, Harter, Chem.Phys.Letters (2015)]



"Quantized" Thales Rectangles

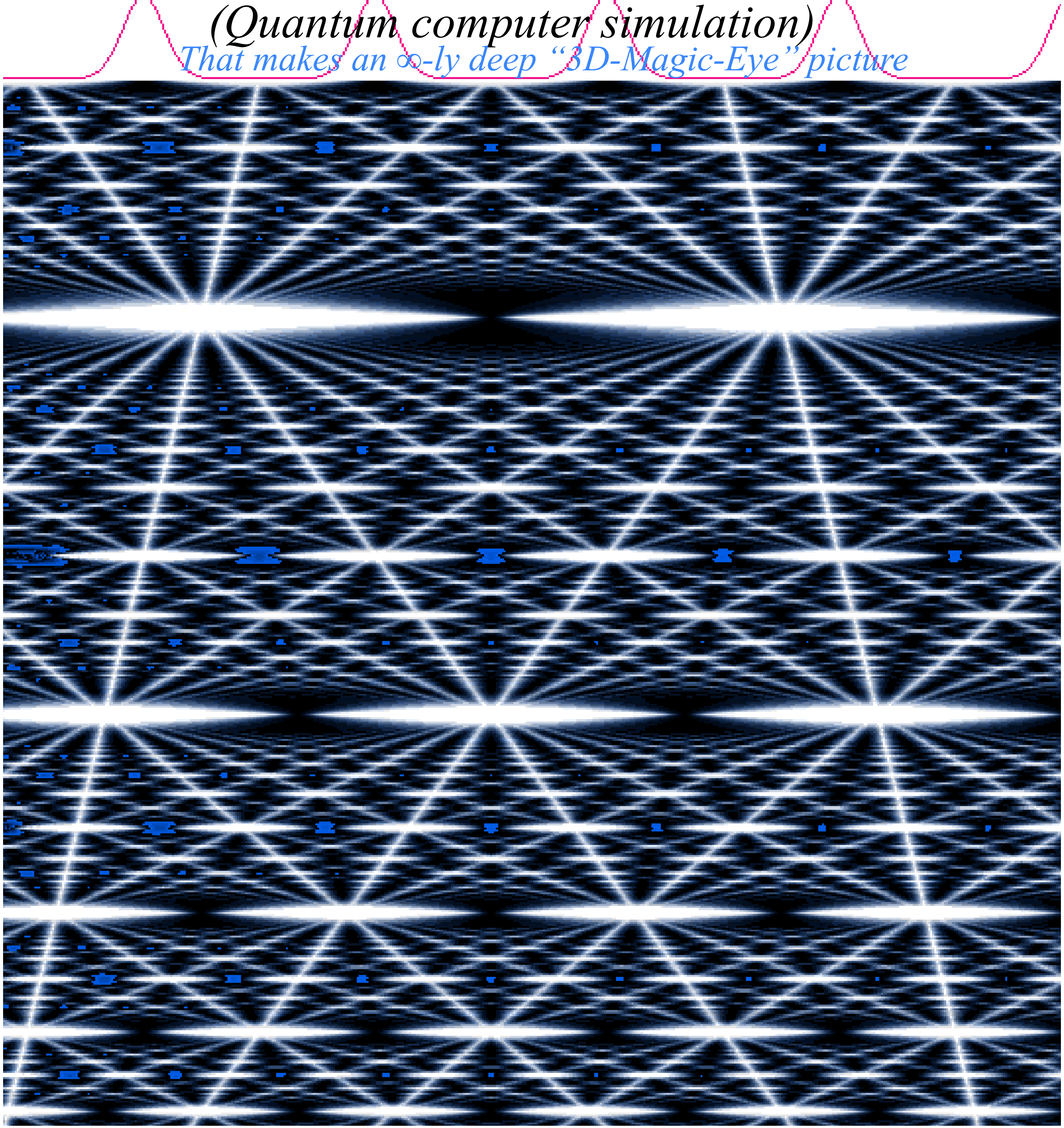
provide analytic geometry of fractal structure

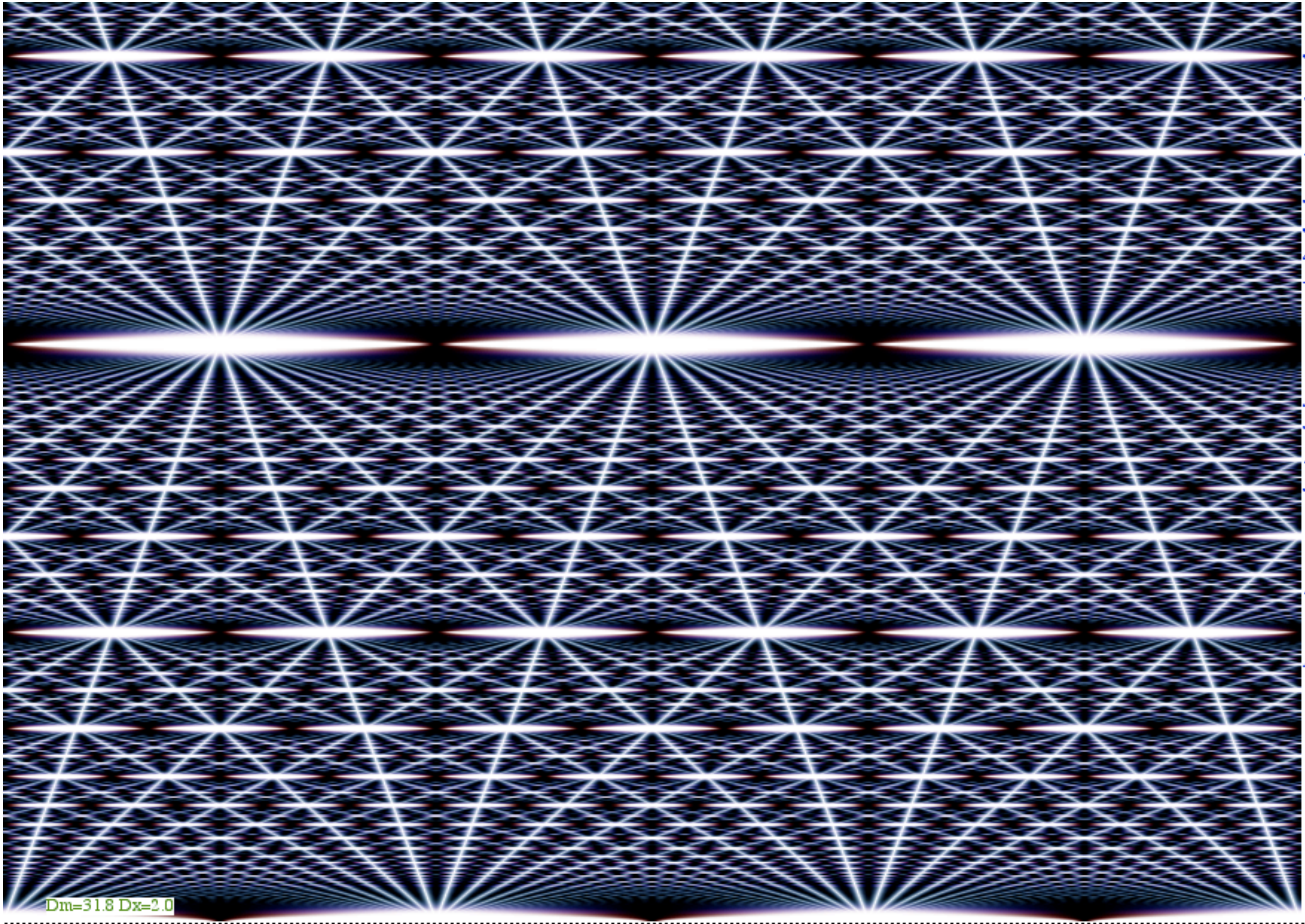


[Li, Harter, Chem.Phys.Letters (2015)]

(Quantum computer simulation)

That makes an ∞-ly deep "3D-Magic-Eye" picture




Dm=31.8 Dx=2.0

An ∞-ly deep "3D-Magic-Eye" using boxcar spectrum