Lecture 10
Thur. 2.18.2016

Dynamics of Potentials and Force Fields

(Ch. 6 and part of Ch. 7 of Unit 1)

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to superball force law
Geometry and dynamics of single ball bounce
(a) Constant force F=-k (linear potential V=kx)
Some physics of dare-devil diving 80 ft. into kidee pool
(b) Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))
(c) Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)
A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier
Advantages of a geometric m_1, m_2, m_3,… series
A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions

Elastic examples: Western buckboard
Bouncing columns and Newton's cradle
Inelastic examples: "Zig-zag geometry" of freeway crashes
*Super-elastic examples: This really **is** "Rocket-Science"*

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General Non-linear force (like superball-floor or ball-bearing-anvil)

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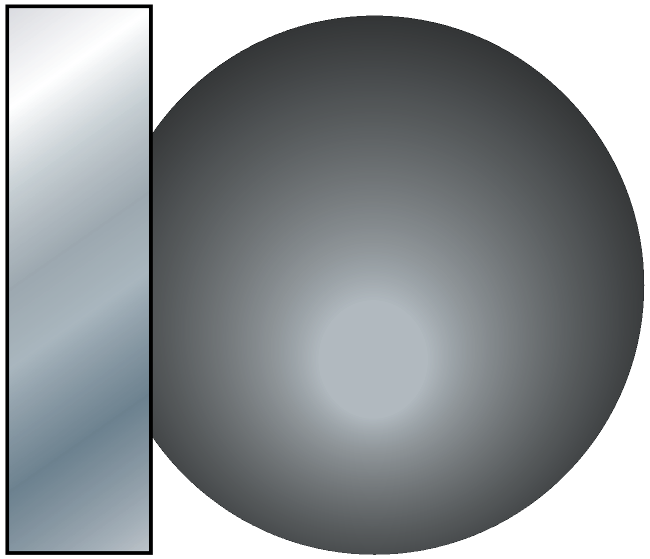
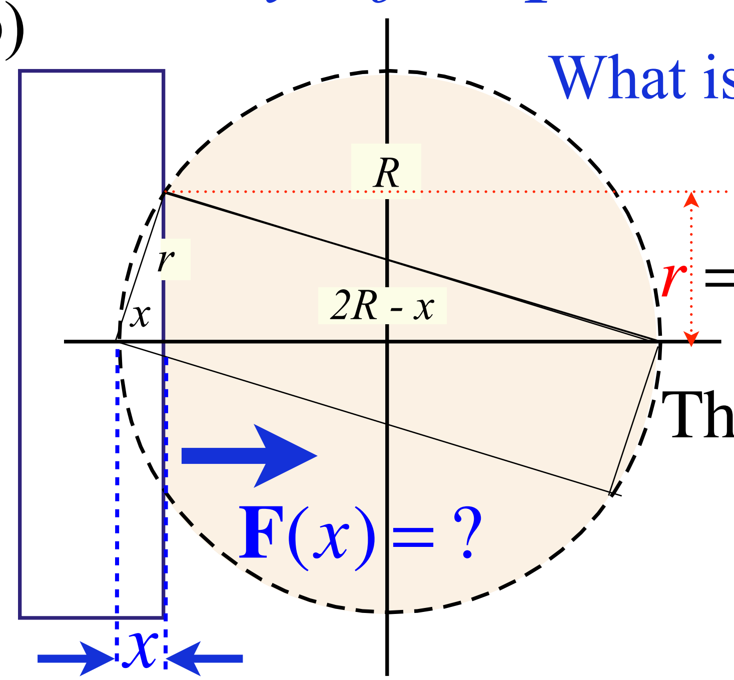
Potential Energy Geometry of Superballs and Related things

(a)



Unit 1
Fig. 6.1
(modified)

(b)



What is superball bounce force law $\mathbf{F}(x)$?

$$r = \sqrt{x(2R-x)} \quad \left(\approx \sqrt{2Rx} \quad for: \quad x << R\right)$$

Thales' geometry and "Sagittal†" approx.

$$\frac{x}{r} = \frac{r}{2R-x}$$

† "bow"

Potential Energy Geometry of Superballs and Related things

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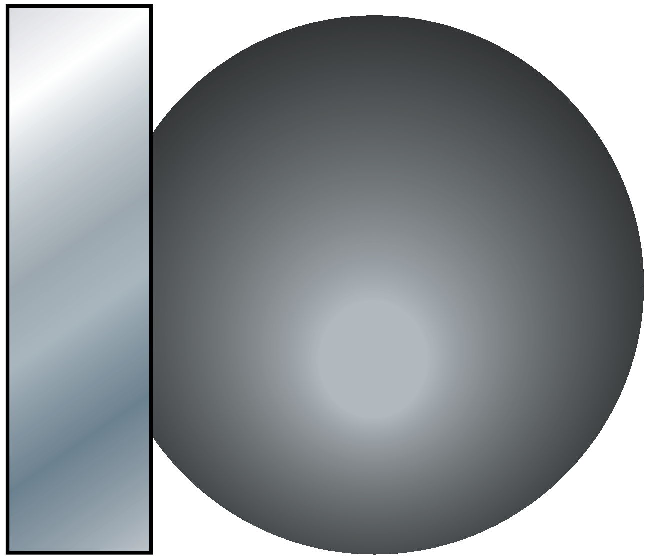

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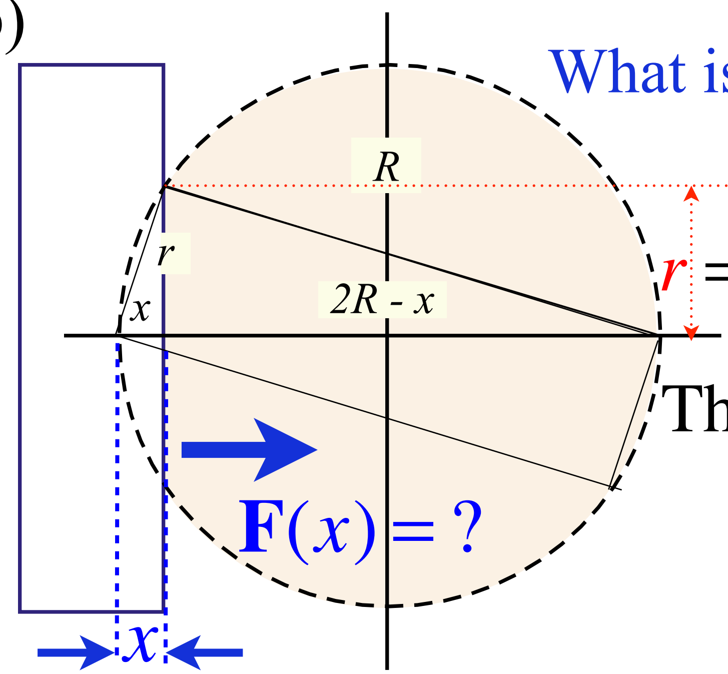

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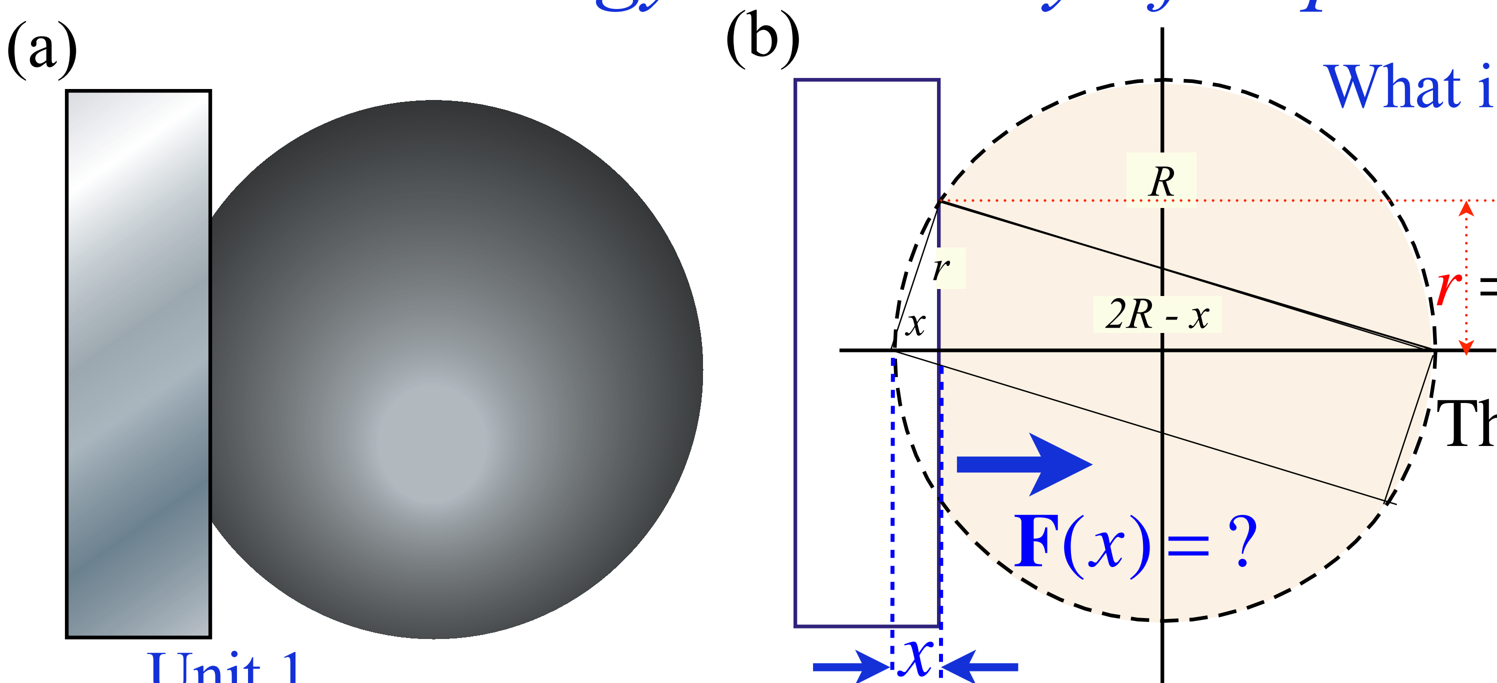
If superball was a balloon its bounce force law would be linear $F=-k\cdot x$ *(Hooke Law)*

(Pressure)·(Area)

$$F_{balloon}(x) = P\cdot A = P\cdot\pi r^2$$

$$\approx P\cdot\pi 2Rx$$

Potential Energy Geometry of Superballs and Related things



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$$\approx P \cdot \pi 2Rx = P \cdot 2\pi R x \quad \text{(Hooke spring constant } k\text{)}$$

$$= kx$$

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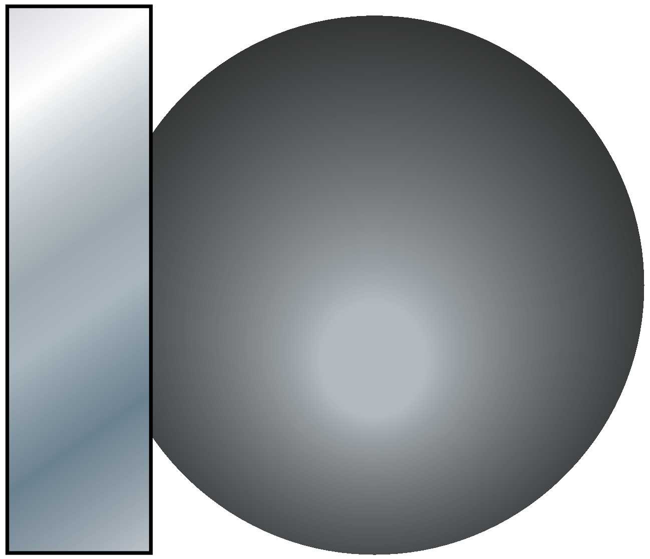

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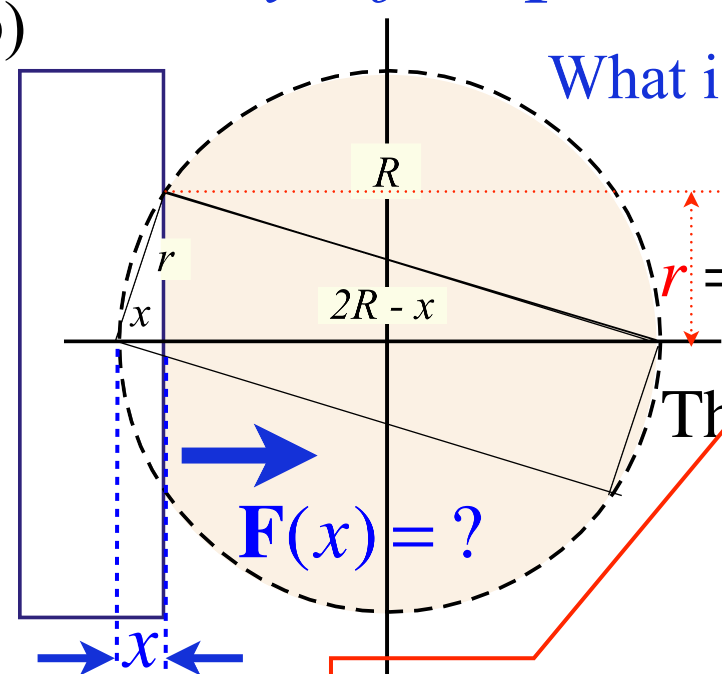

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Instead superball force law depends on bulk *volume* modulus and is non-linear $F\sim x^{p?}+?$ *(Power Law?)*

$$Volume(X) = \int_0^X \pi r^2\, dx = \int_0^X \pi x(2R-x)\, dx$$

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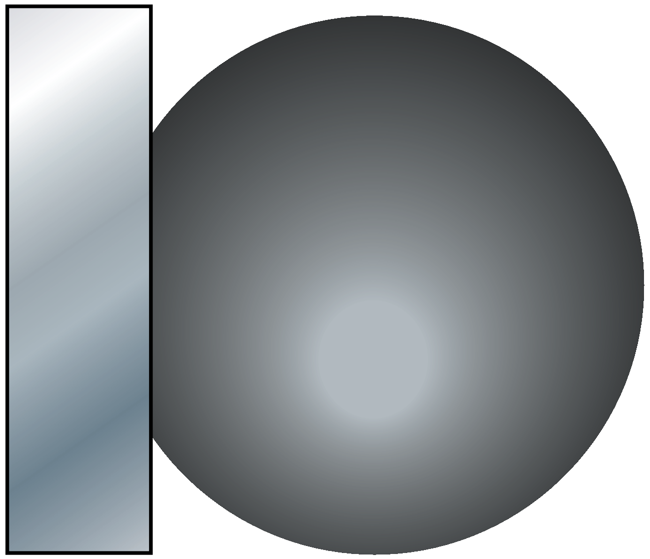

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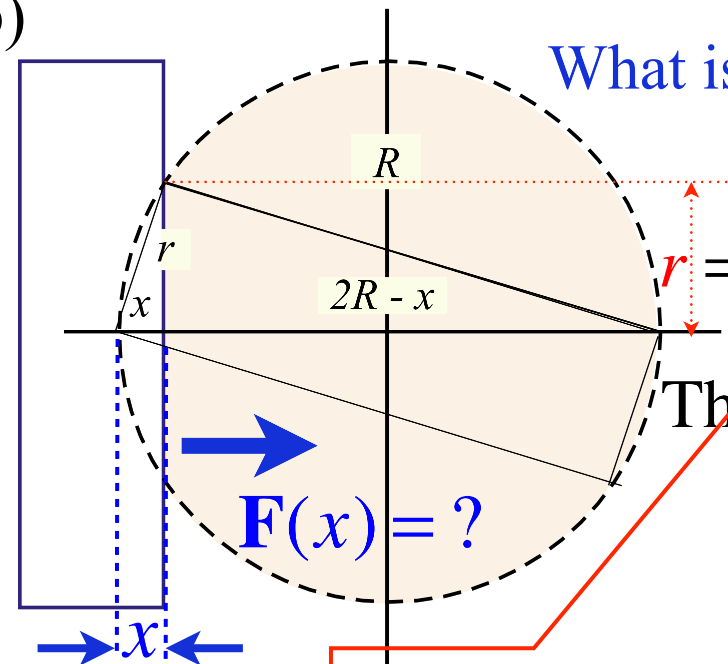

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(Hooke spring constant k)

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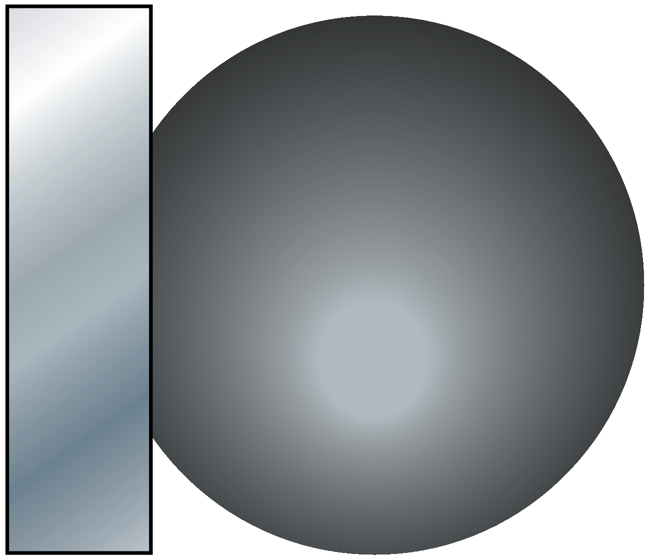

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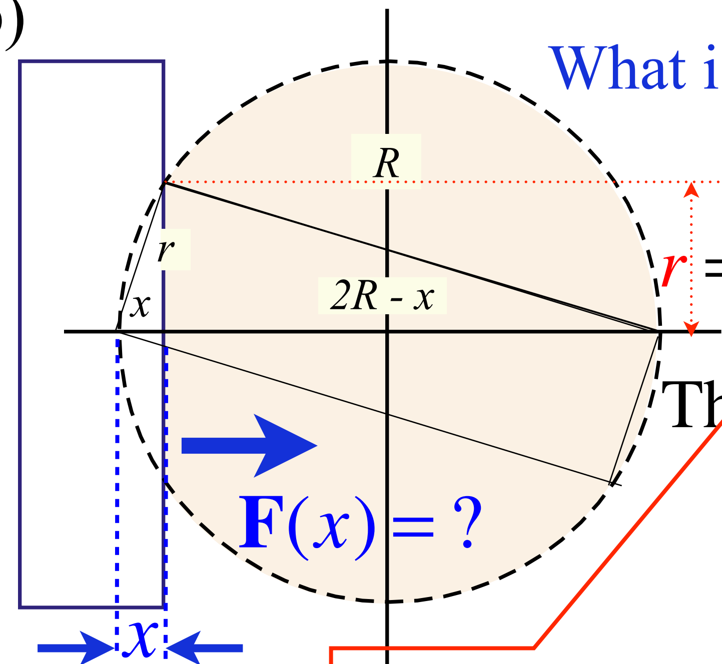

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It also depends on velocity $\dot{x} = \frac{dx}{dt}$. *Adiabatic* differs from *Isothermal* as shown by *"Project-Ball*"*

* *Am. J. Phys.* **39**, 656 (1971)

(Discussed after p. 33)

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→ *Geometry and dynamics of single ball bounce*

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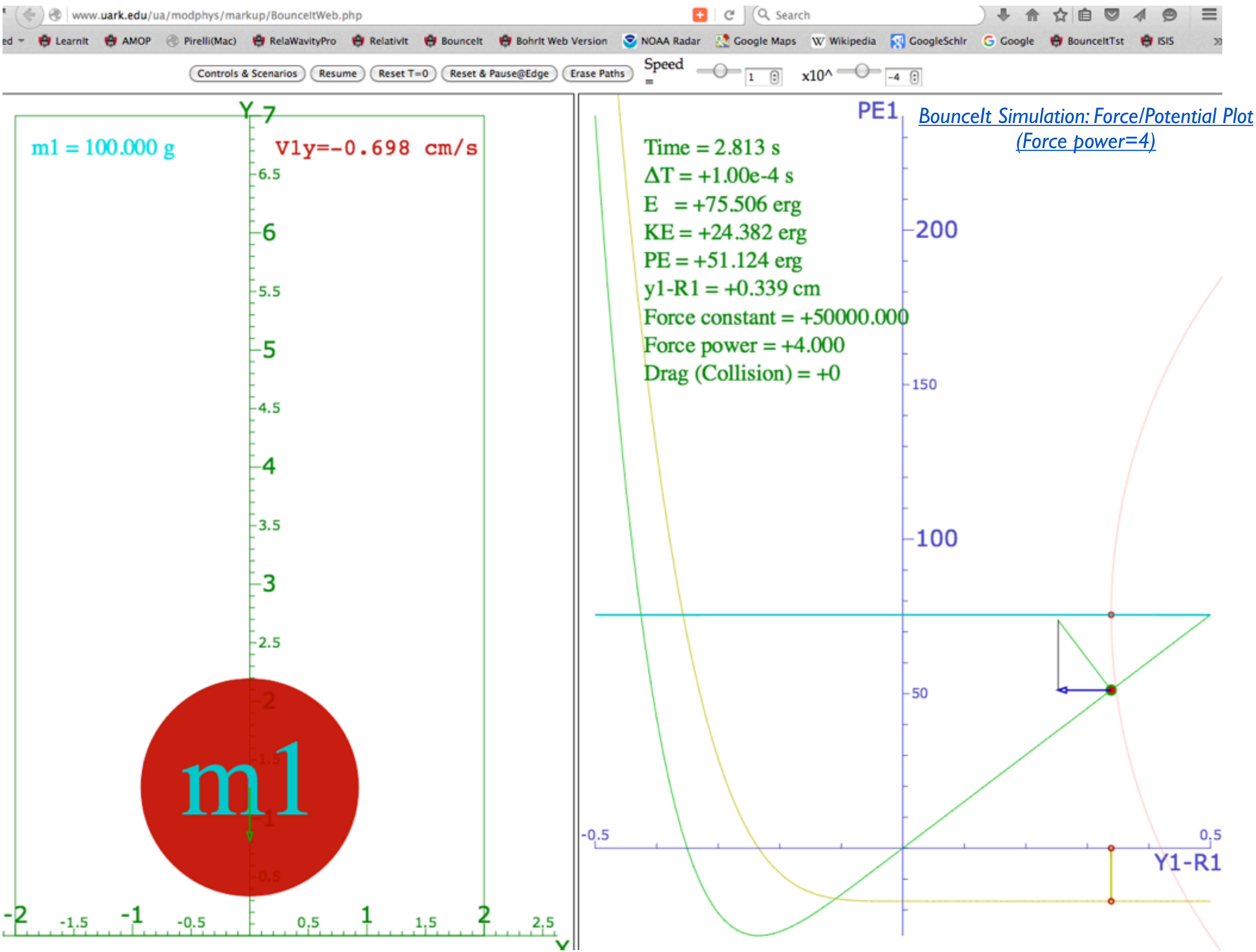

www.uark.edu/ua/modphys/markup/BounceItWeb.php
Search
LearnIt
AMOP
Pirelli(Mac)
RelaWavityPro
RelativIt
BounceIt
BohrIt Web Version
NOAA Radar
Google Maps
Wikipedia
GoogleSchlr
Google
BounceItTst
ISIS
Controls & Scenarios
Resume
Reset T=0
Reset & Pause@Edge
Erase Paths
Speed =
x10^
m1 = 100.000 g
V1y=-0.698 cm/s
m1
Time = 2.813 s
ΔT = +1.00e-4 s
E = +75.506 erg
KE = +24.382 erg
PE = +51.124 erg
y1-R1 = +0.339 cm
Force constant = +50000.000
Force power = +4.000
Drag (Collision) = +0
PE1
Y1-R1
BounceIt Simulation: Force/Potential Plot
(Force power=4)

Main Control Panel

Start | Resume

This is the generic BounceIt URL (or address): http://www.uark.edu/ua/modphys/markup/BounceItWeb.html

BounceIt Simulation: Force/Potential Plot (Force power=4)

Number of masses: 1 Balls

- ⦿ Let mouse set: (x,y,Vx,Vy)
- ○ Let mouse set force: F(t)
- ○ Plot solid paths
- ⦿ Plot dotted paths
- ○ Plot no paths
- ○ Plot V1 vs. V2
- ○ Plot Y1(t), Y2(t), ...
- ⦿ Plot PE of m1 vs. Y1
- ○ Plot Y2 vs. Y1
- ○ Plot user defined i.e - Y1 vs. Y2
- ○ Balls initially falling
- ○ Balls initially fixed
- ⦿ No preset initial values

Acceleration of gravity: 1.51 100x{cm/s^2} — *Sets gravity*

- ☑ Draw force vectors
- ☐ Pause (once) at top
- ☑ Constrain motion to Y-axis
- ☑ Plot v2 vs v1
- ☐ Plot p2 vs p1
- ☐ Plot V2 vs V1
- ☐ Plot Ellipses
- ☐ Plot Bisector Lines
- ☐ Old Color Scheme
- ☑ Show right panel information
- ☑ Show left panel information
- ☐ Set Initial positions

Collision friction (Viscosity): 0 x10^ 0 {g}

Initial gap between balls: 5.5 x10^ -1 {g}

Force Constant: 5 x10^ 4 {g} — *Usually need to decrease k for p =1*

Force power law exponent: 4 — *This is non-linear $F=-kx^4$ (Set p =1 for linear $F=-kx^1$)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0: 0.75

y Max = 7

Initial x1 = 0.5 | y Min = 0

Max x PE plot = 0.5 | T Max = 6

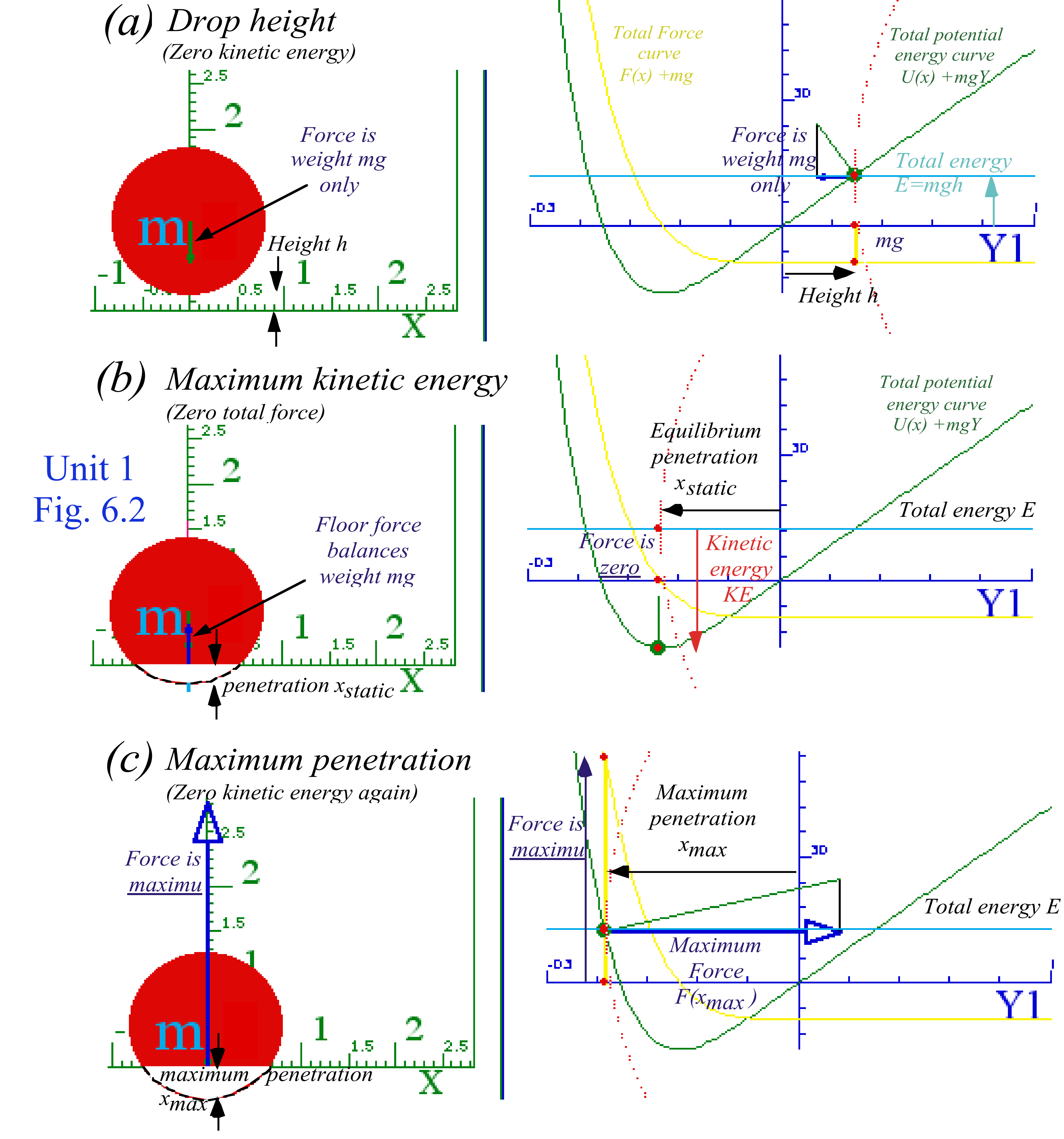
F-Vector scale = 0.003 | V2y Max = 3.1112

Error step = 0.0000 | V2y Min = -3.111

Mass factor = 1

M_1= 1 x10^ 2 {g}

V_1= -1.1 x10^ -2 {cm/s}



1990 BounceIt Mac simulations

Details of each case follows using newer *BounceIt Web simulations*

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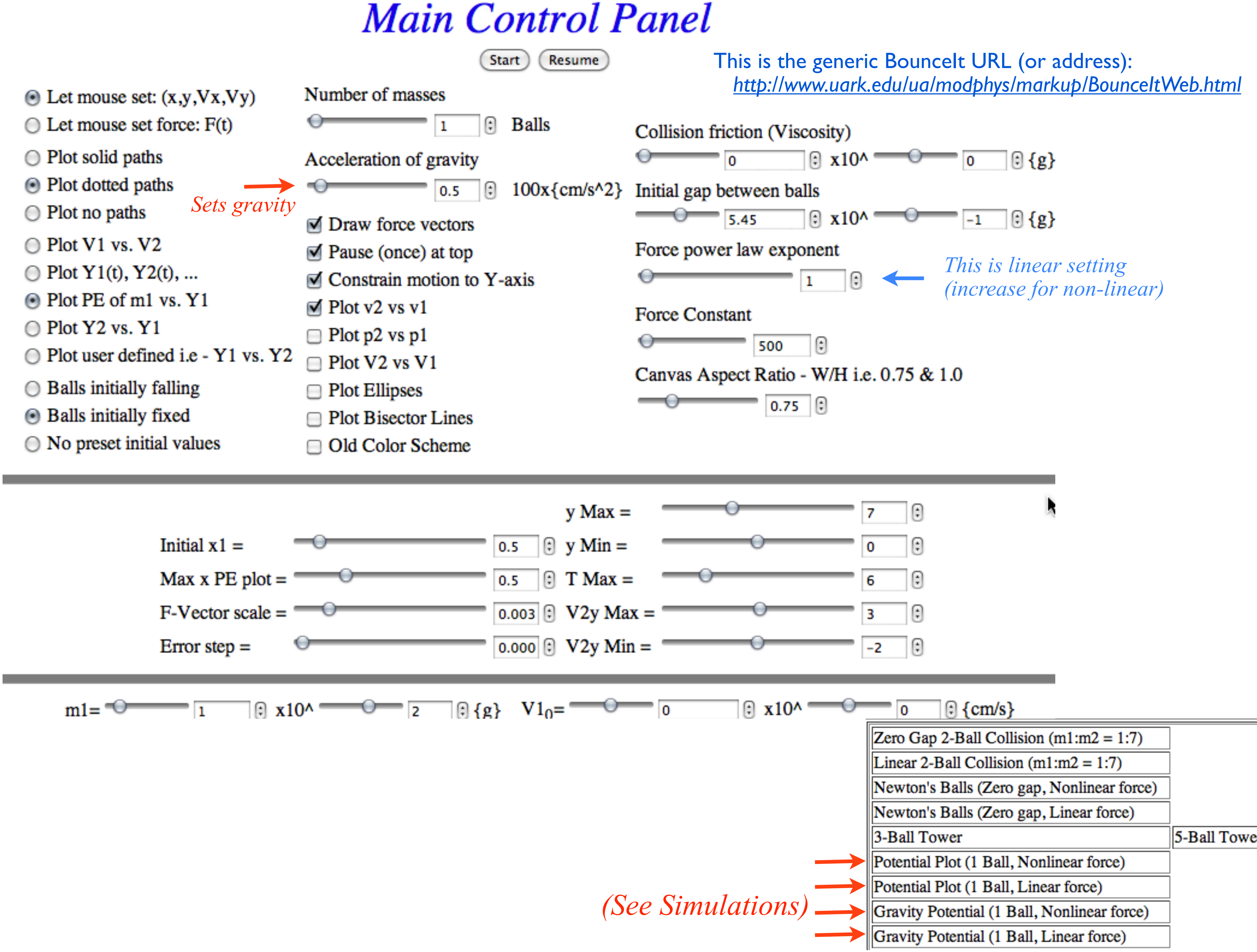
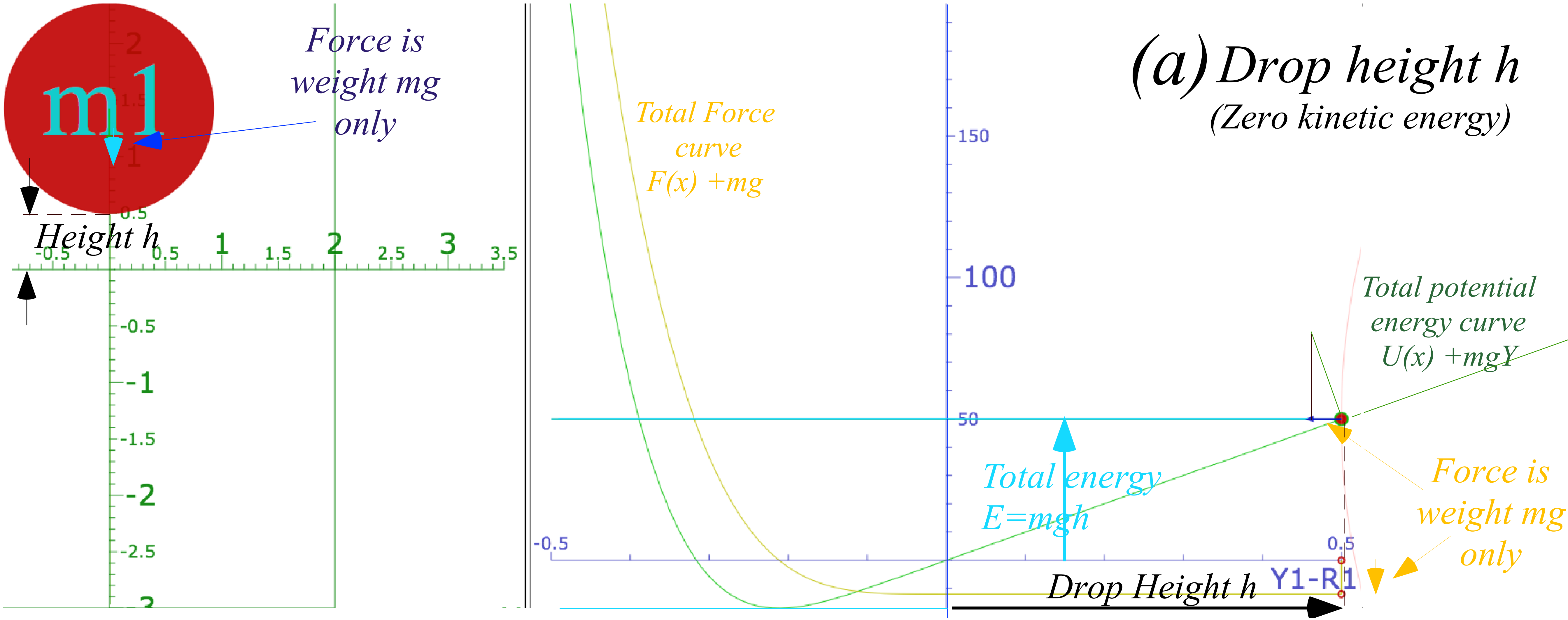


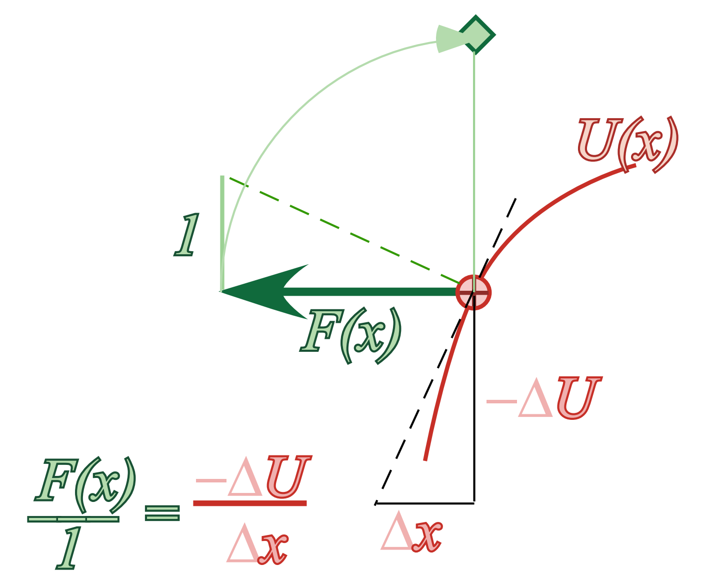
Sets gravity

This is linear setting (increase for non-linear)

(See Simulations)

(a) Drop height h
(Zero kinetic energy)

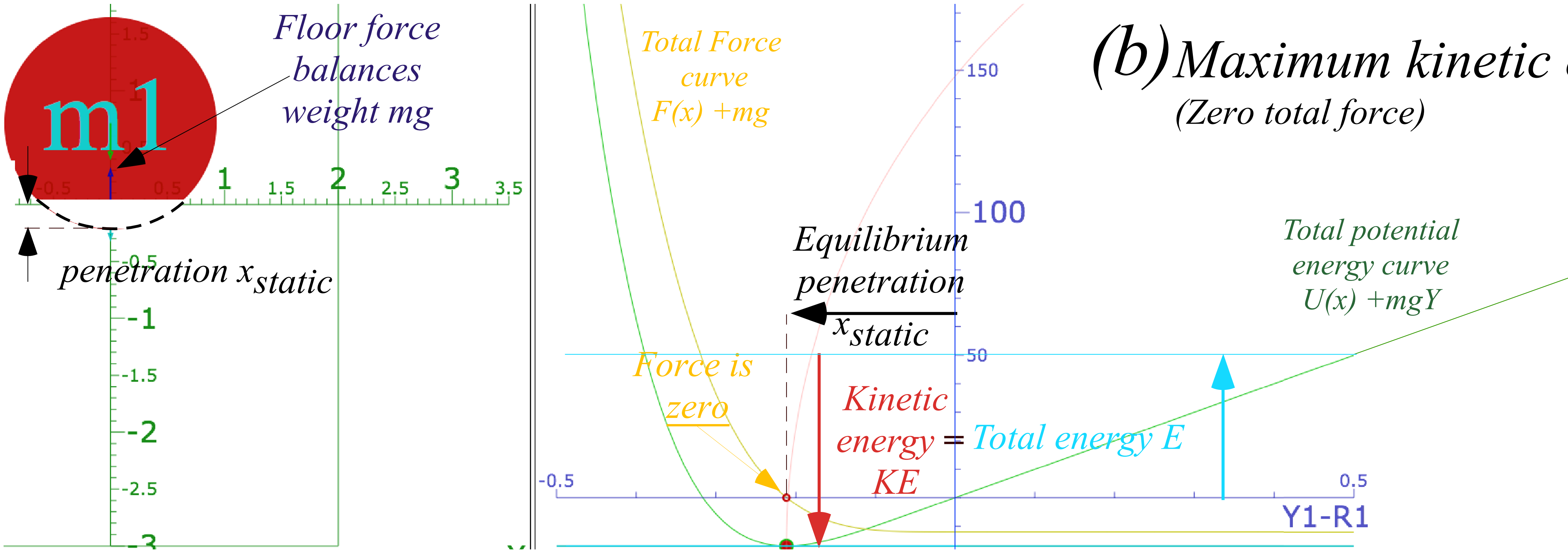


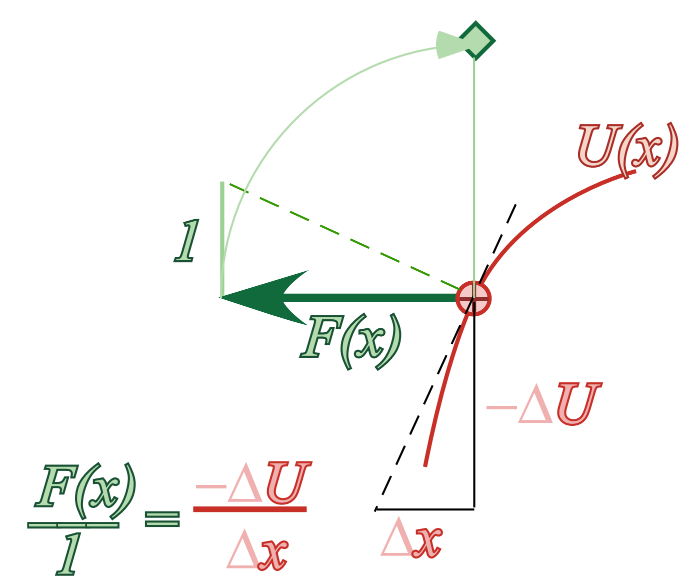


Display of Force vector using similar triangle constuction based on the slope of potential curve.

BounceIt Simulation: Force/Potential Plot
(Force power=4)

(b) Maximum kinetic energy
(Zero total force)

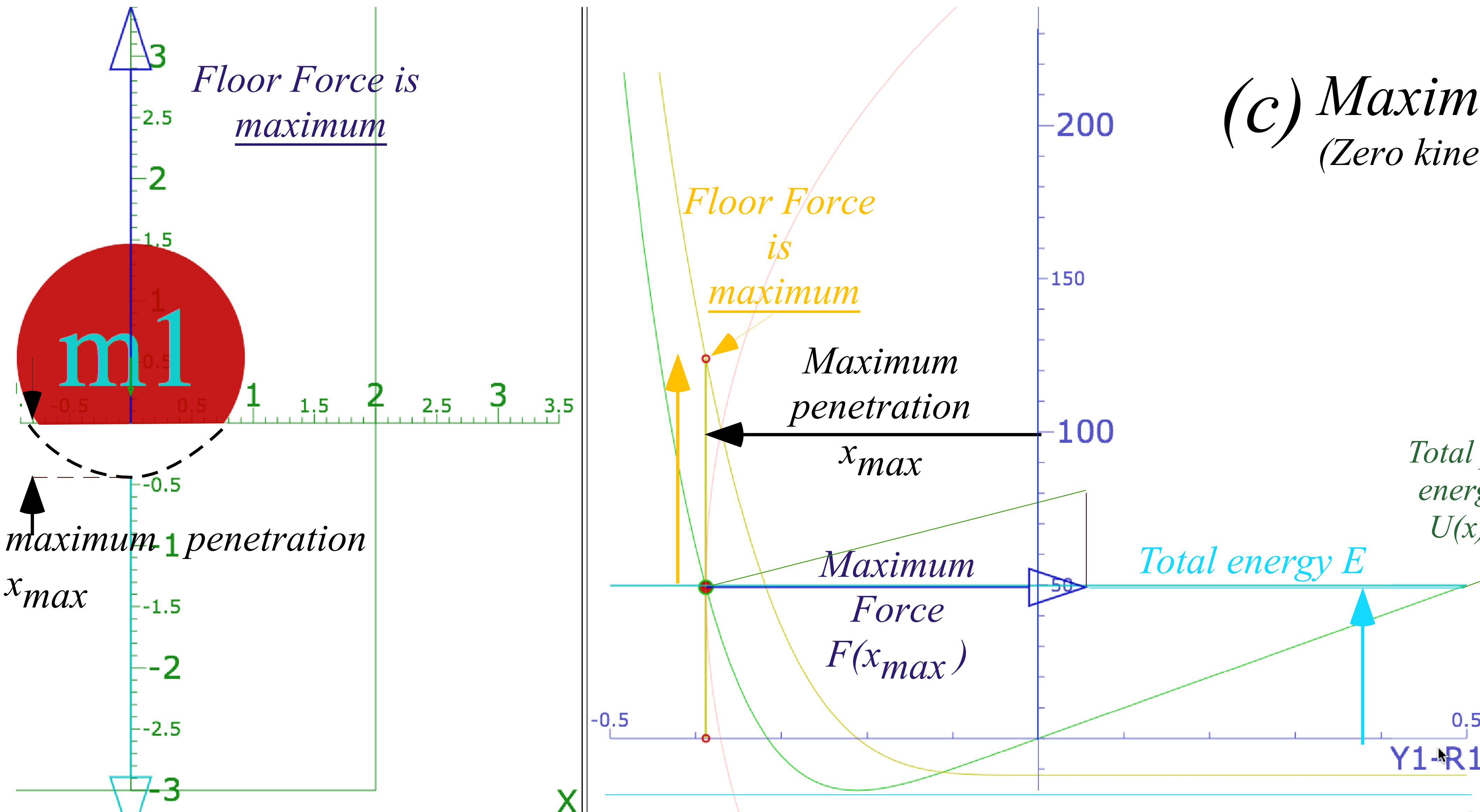


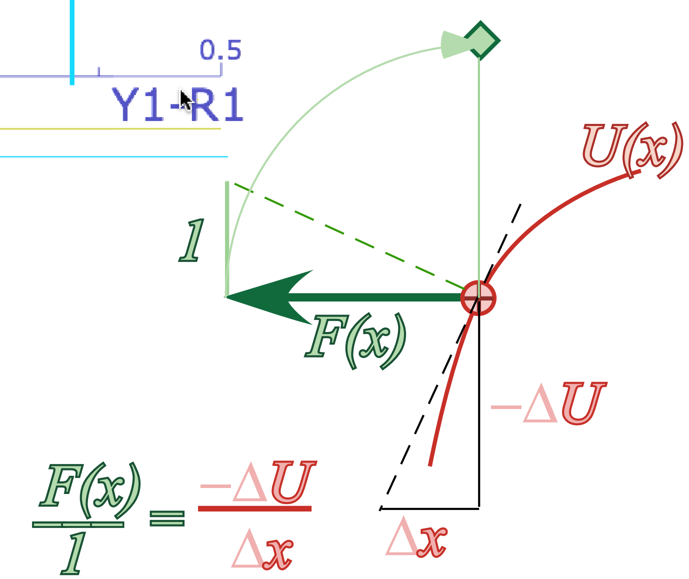


Display of Force vector using similar triangle constuction based on the slope of potential curve.

BounceIt Simulation: Force/Potential Plot (Force power=4)

(c) Maximum penetration
(Zero kinetic energy again)





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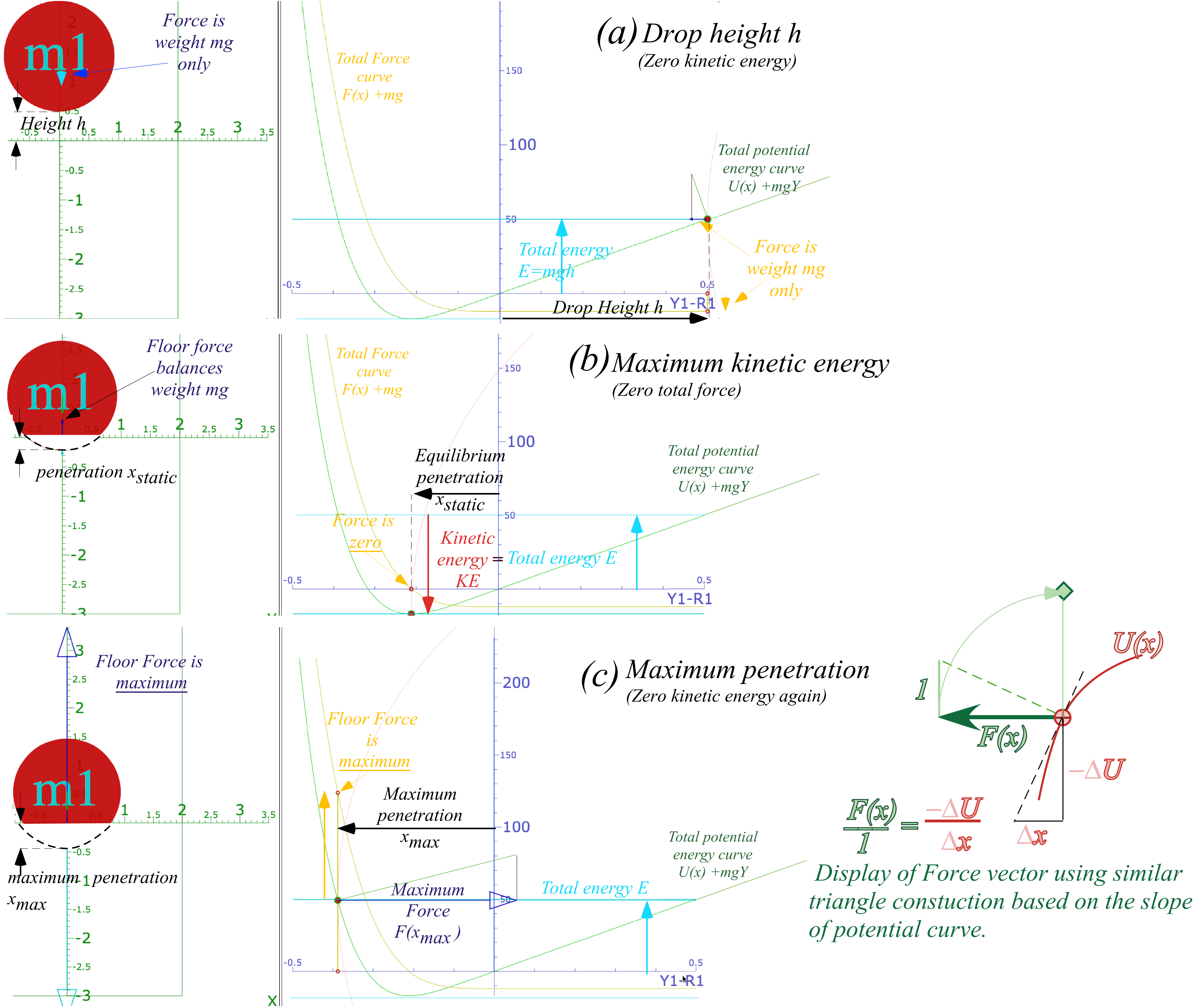
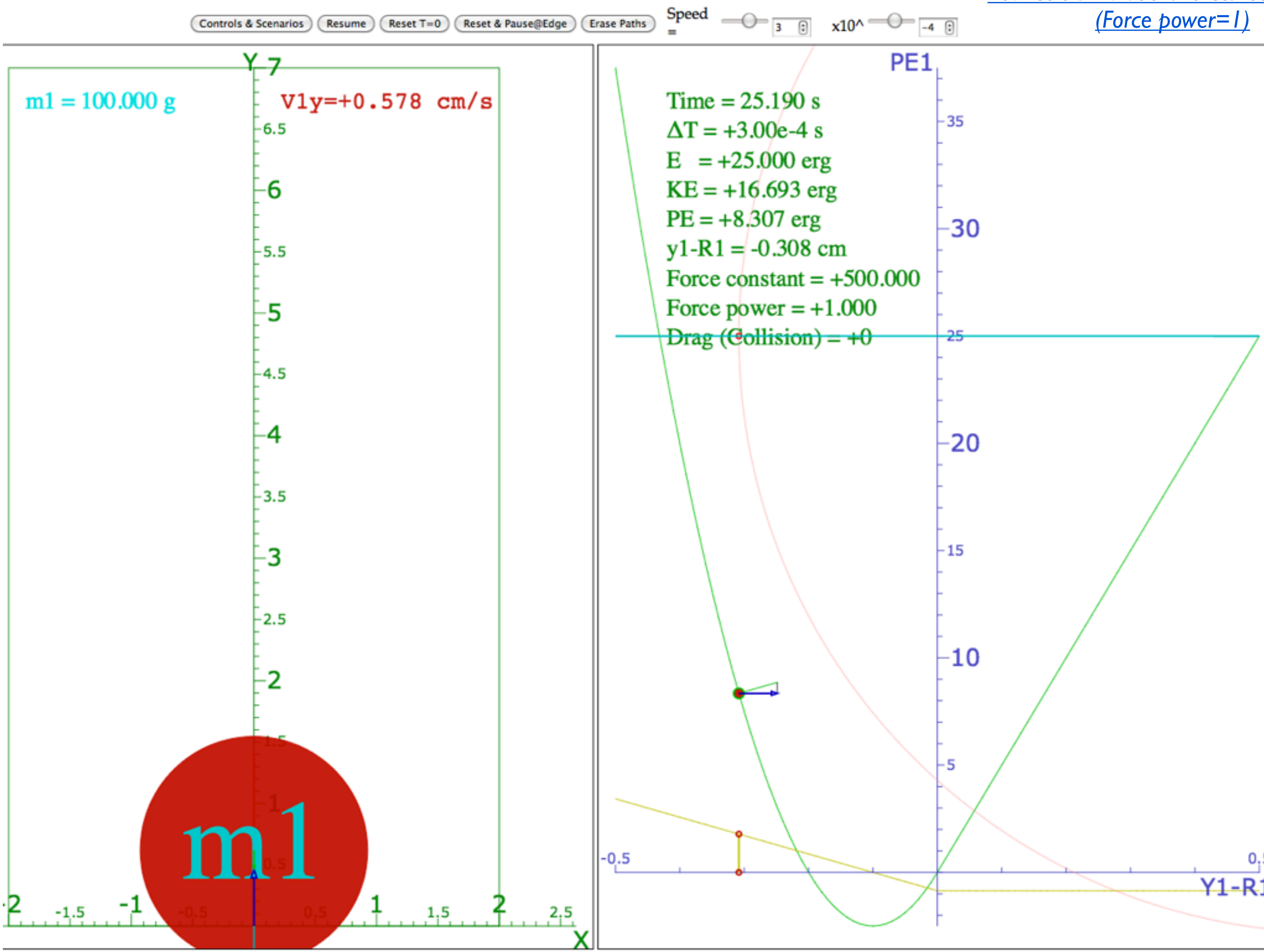

(a) Drop height h
(Zero kinetic energy)
Force is weight mg only
Height h
Total Force curve F(x) +mg
Total potential energy curve U(x) +mgY
Total energy E=mgh
Force is weight mg only
Drop Height h
Y1-R1
(b) Maximum kinetic energy
(Zero total force)
Floor force balances weight mg
penetration x static
Total Force curve F(x) +mg
Equilibrium penetration x static
Total potential energy curve U(x) +mgY
Force is zero
Kinetic energy KE
= Total energy E
Y1-R1
(c) Maximum penetration
(Zero kinetic energy again)
Floor Force is maximum
maximum penetration x max
Floor Force is maximum
Maximum penetration x max
Total potential energy curve U(x) +mgY
Total energy E
Maximum Force F(x max)
Y1-R1
U(x)
1
F(x)
−ΔU
Δx
F(x)/1 = −ΔU/Δx
Display of Force vector using similar triangle constuction based on the slope of potential curve.

[BounceIt Simulation: Force/Potential Plot (Force power=1)]



Start | Resume

Number of masses: 1 Balls

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Acceleration of gravity: 0.5 100x{cm/s^2}

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Collision friction (Viscosity): 0 x10^ 0 {g}

Initial gap between balls: 5.5 x10^ -1 {g}

Force Constant: 5 x10^ 2 {g}

Usually need to increase k for p >1

Force power law exponent: 1

This is linear $F=-kx^1$ (increase p >1 for non-linear $F=-kx^p$)

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V_1= 0 x10^ 0 {cm/s}

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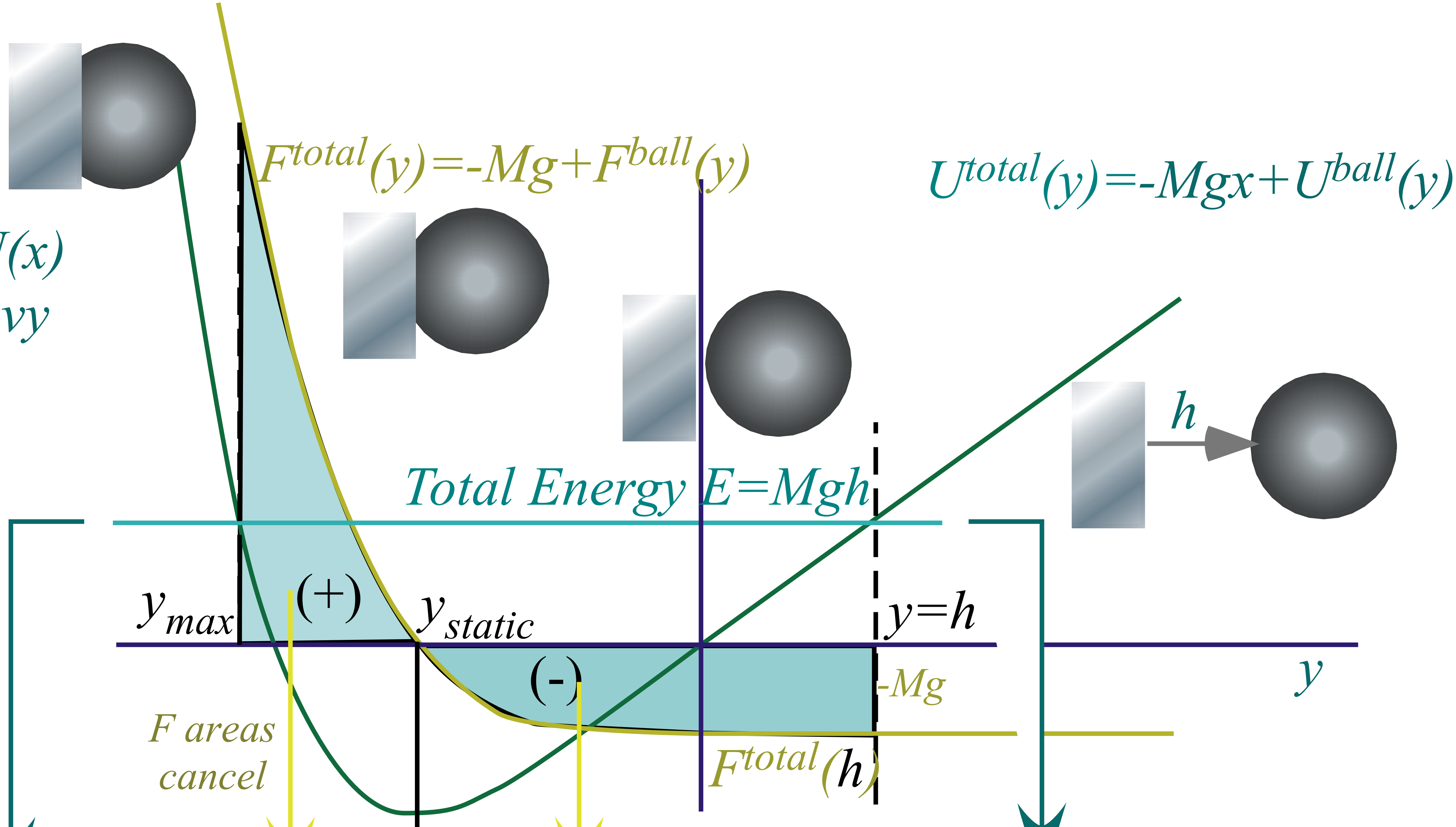
Force F(x) and Potential U(x) for soft heavy non-linear superball

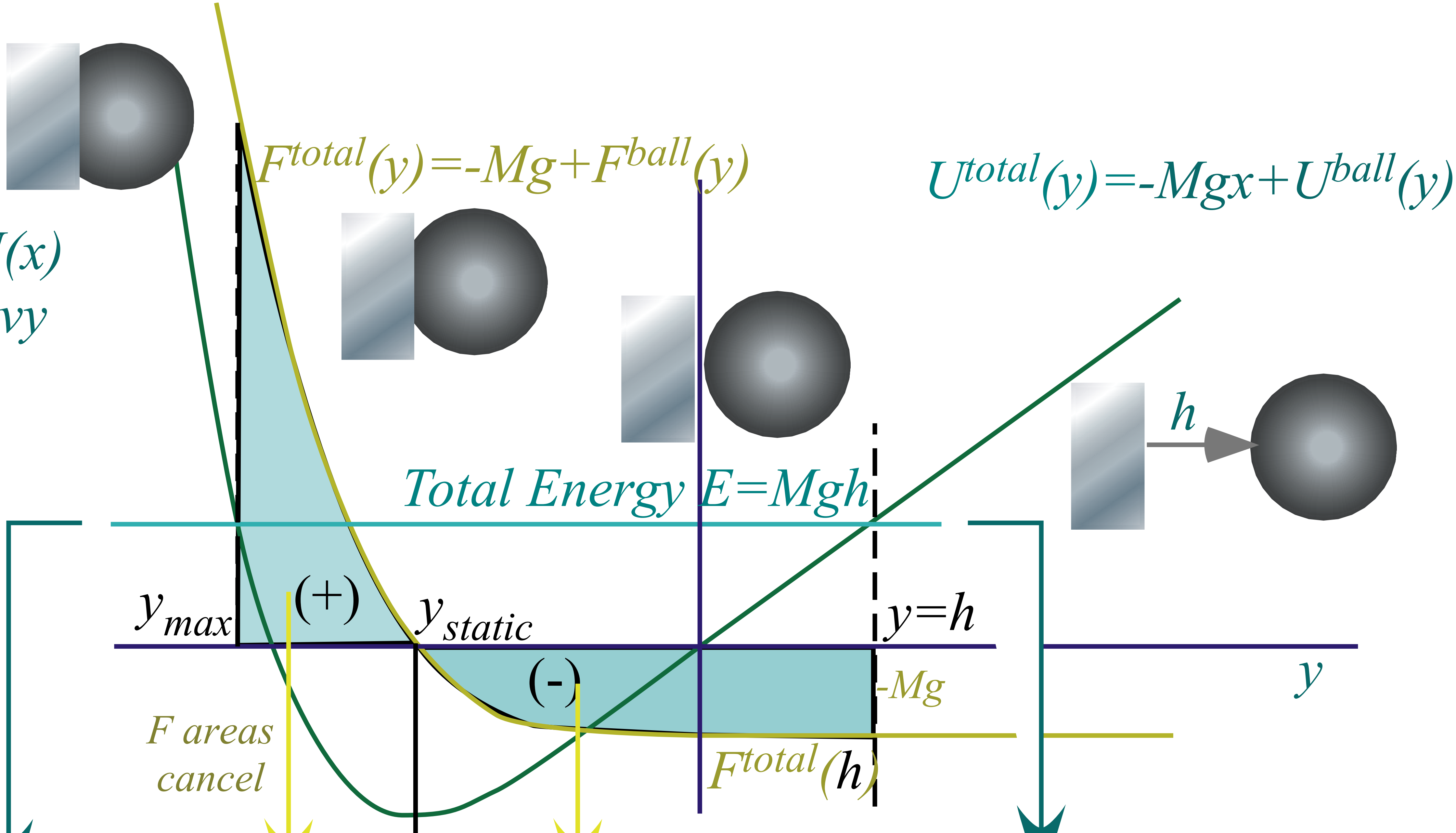
Unit 1
Fig. 6.5



$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y)\,dy + \int_{y=h}^{y_{static}} F^{total}(y)\,dy + U(h) \;=\; U(h) = E$$

$$F(x) = -\frac{dU(x)}{dx}$$

Force F(x)
and
Potential U(x)
for soft heavy
non-linear
superball



Unit 1
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$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

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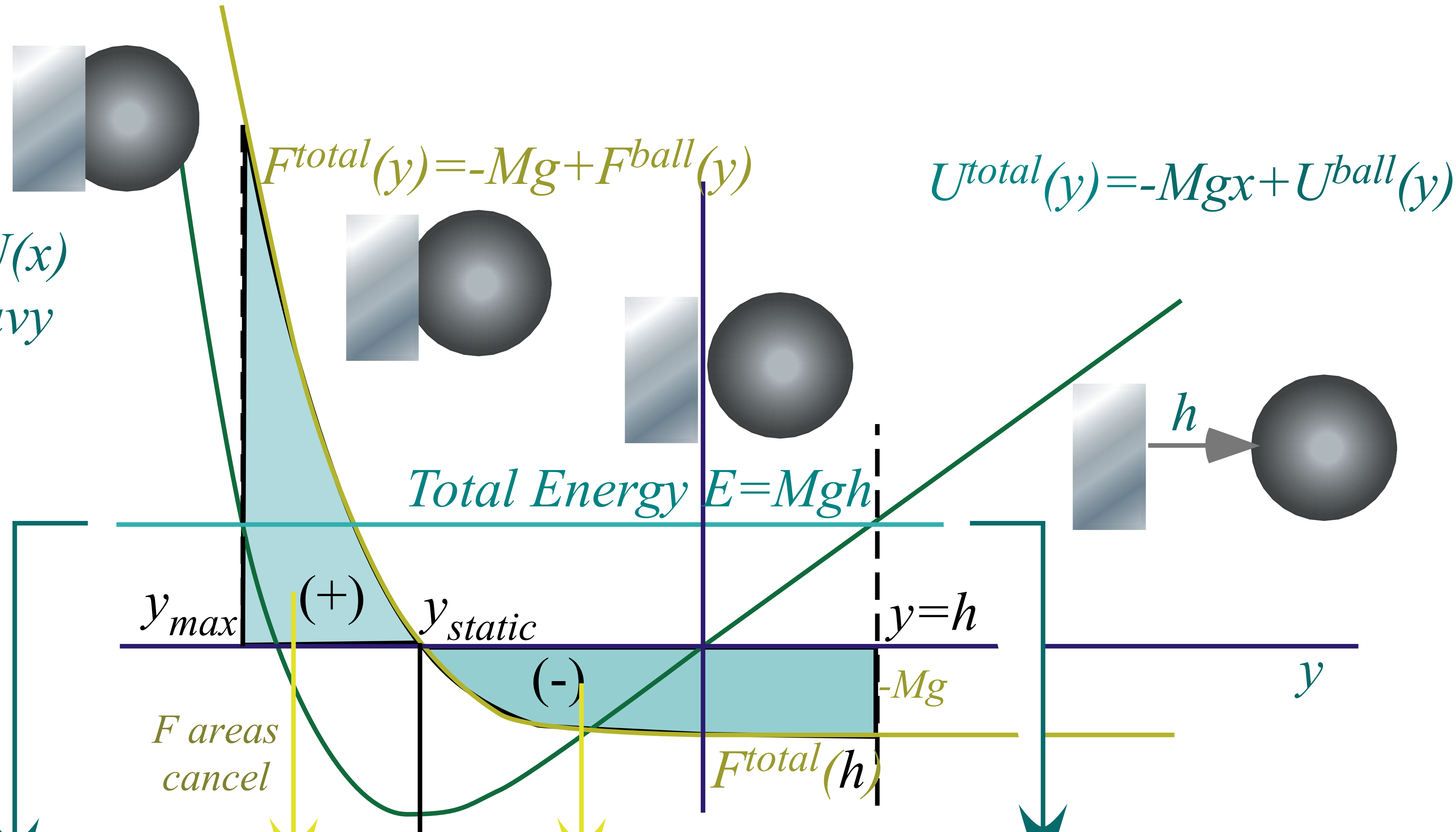

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$$Impulse = P = \int F(t)\,dt = Momentum\ acquired = Area\ of\ F(t) = P(t) \qquad F(t) = \frac{dP(t)}{dt}$$

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

General Non-linear force (like superball-floor or ball-bearing-anvil)

→ *Constant force F=-k (linear potential V=kx)* *(Simulations)*

Some physics of dare-devil-diving 80 ft. into kidee pool

Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

A story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of n-ball bounces

Analogy with shockwave and acoustical horn amplifier

Advantages of a geometric $m_1, m_2, m_3, \ldots$ series

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

Many-body 1D collisions

Elastic examples: Western buckboard

Bouncing columns and Newton's cradle

Inelastic examples: "Zig-zag geometry" of freeway crashes

*Super-elastic examples: This really **is** "Rocket-Science"*

Main Control Panel

Start | Resume

- (•) Let mouse set: (x,y,Vx,Vy)
- ( ) Let mouse set force: F(t)
- ( ) Plot solid paths
- (•) Plot dotted paths
- ( ) Plot no paths
- ( ) Plot V1 vs. V2
- ( ) Plot Y1(t), Y2(t), ...
- (•) Plot PE of m1 vs. Y1
- ( ) Plot Y2 vs. Y1
- ( ) Plot user defined i.e - Y1 vs. Y2
- ( ) Balls initially falling
- (•) Balls initially fixed
- ( ) No preset initial values

Sets gravity →

Number of masses: 1 Balls

Acceleration of gravity: 0.5 100x{cm/s^2}

- [x] Draw force vectors
- [x] Pause (once) at top
- [x] Constrain motion to Y-axis
- [x] Plot v2 vs v1
- [ ] Plot p2 vs p1
- [ ] Plot V2 vs V1
- [ ] Plot Ellipses
- [ ] Plot Bisector Lines
- [ ] Old Color Scheme

Collision friction (Viscosity): 0 x10^ 0 {g}

Initial gap between balls: 5.45 x10^ -1 {g}

Force power law exponent: 1 ← *This is linear setting (increase for non-linear)*

Force Constant: 500

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0: 0.75

---

		y Max =	7
Initial x1 =	0.5	y Min =	0
Max x PE plot =	0.5	T Max =	6
F-Vector scale =	0.003	V2y Max =	3
Error step =	0.000	V2y Min =	-2

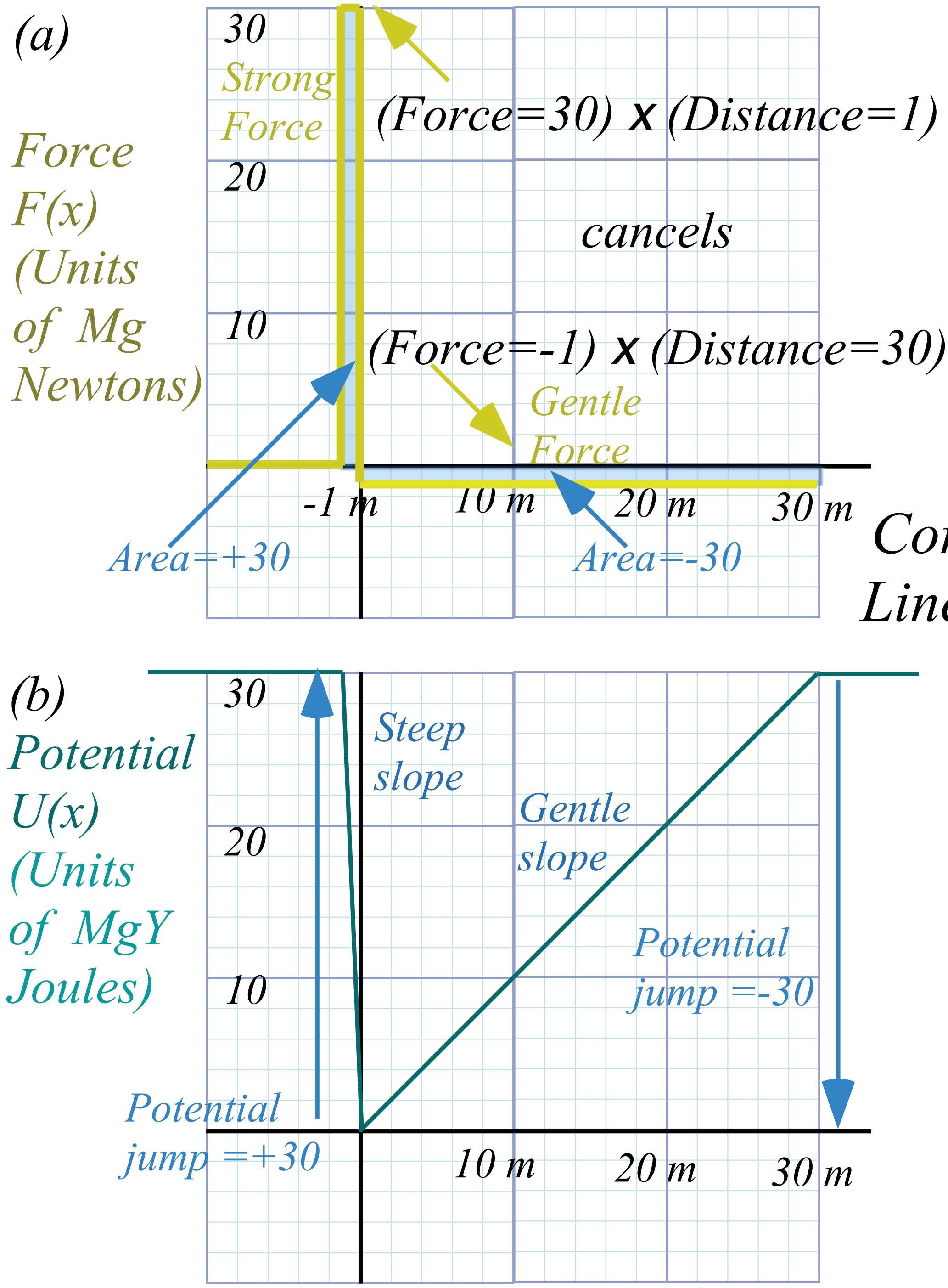
---

m1= 1 x10^ 2 {g} $V1_0$= 0 x10^ 0 {cm/s}

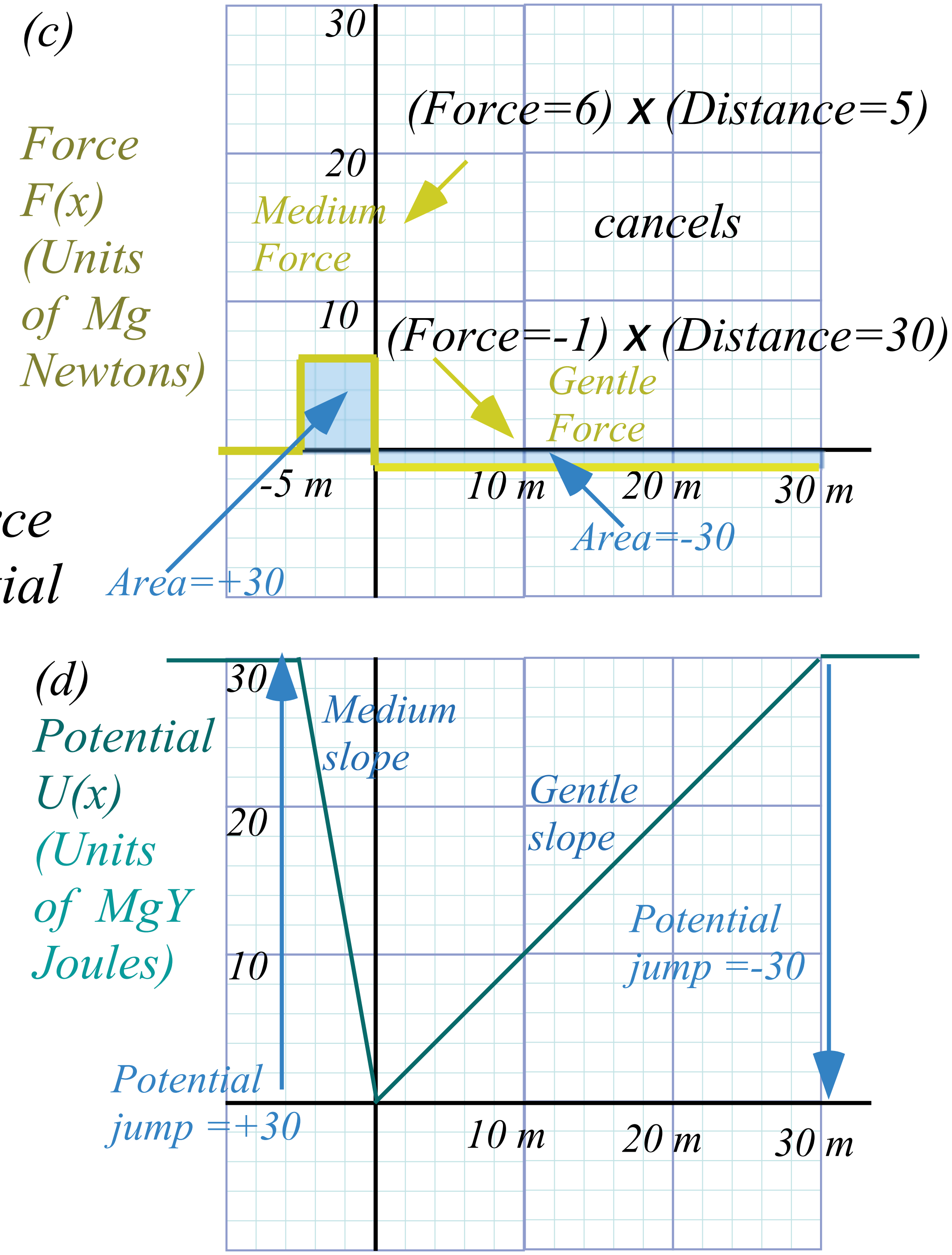
Zero Gap 2-Ball Collision (m1:m2 = 1:7)	
Linear 2-Ball Collision (m1:m2 = 1:7)	
Newton's Balls (Zero gap, Nonlinear force)	
Newton's Balls (Zero gap, Linear force)	
3-Ball Tower	5-Ball Towe
→ Potential Plot (1 Ball, Nonlinear force)	
→ Potential Plot (1 Ball, Linear force)	
→ Gravity Potential (1 Ball, Nonlinear force)	
→ Gravity Potential (1 Ball, Linear force)	

http://www.uark.edu/ua/modphys/markup/BounceItWeb.html

(See Simulations)



Constant Force Linear Potential Models: $F(x)=k$, $U(x)=-kx$



Unit 1 Fig. 6.3

$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

$$Impulse = P = \int F(t)\,dt = Momentum\ acquired = Area\ of\ F(t) = P(t) \qquad F(t) = \frac{dP(t)}{dt}$$

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General Non-linear force (like superball-floor or ball-bearing-anvil)

Constant force F=-k (linear potential V=kx) *(Simulations)*

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→ *Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))*

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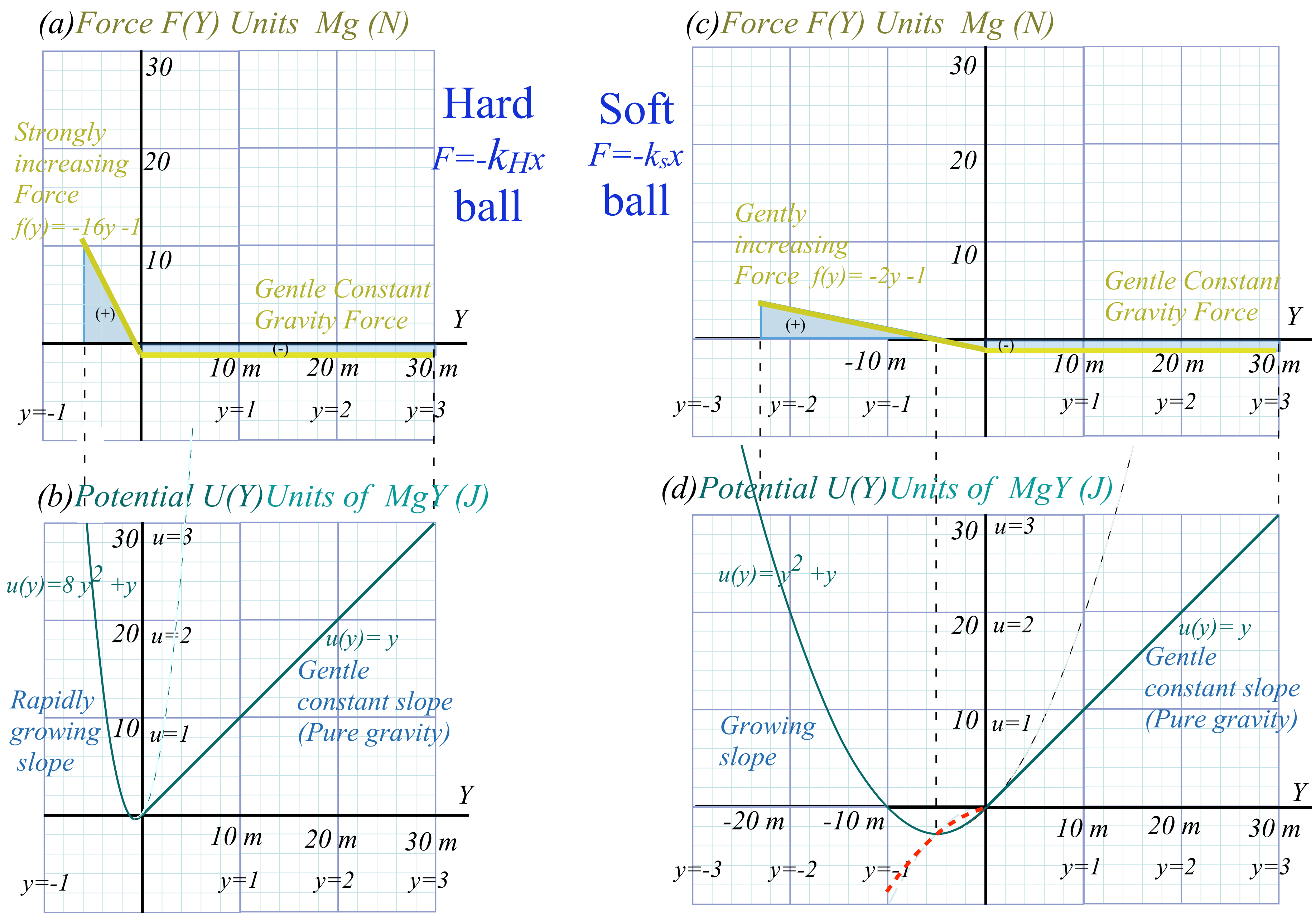
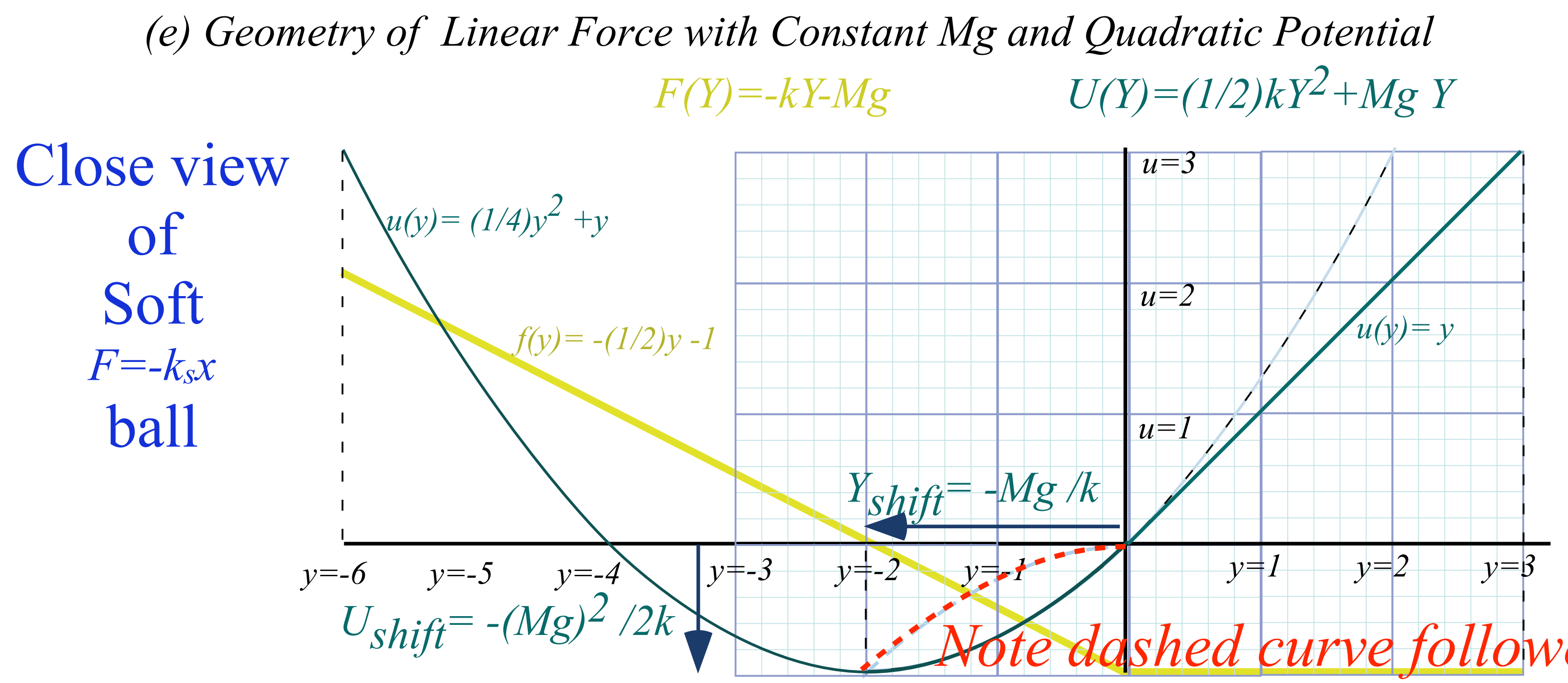


Unit 1
Fig. 6.4



$$F^{Total} = F^{grav} + F^{target} = \begin{cases} -Mg & (y \geq 0) \\ -Mg - ky & (y < 0) \end{cases}$$

$$U^{Total} = U^{grav} + U^{target} = \begin{cases} Mg\ y & (y \geq 0) \\ Mg\ y + \frac{1}{2}ky^2 & (y < 0) \end{cases}$$

Note dashed curve followed by PE minimum. Parabola? What?

Potential energy dynamics of Superballs and related things

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Geometry and dynamics of single ball bounce

→ *General Non-linear force (like superball-floor or ball-bearing-anvil)*

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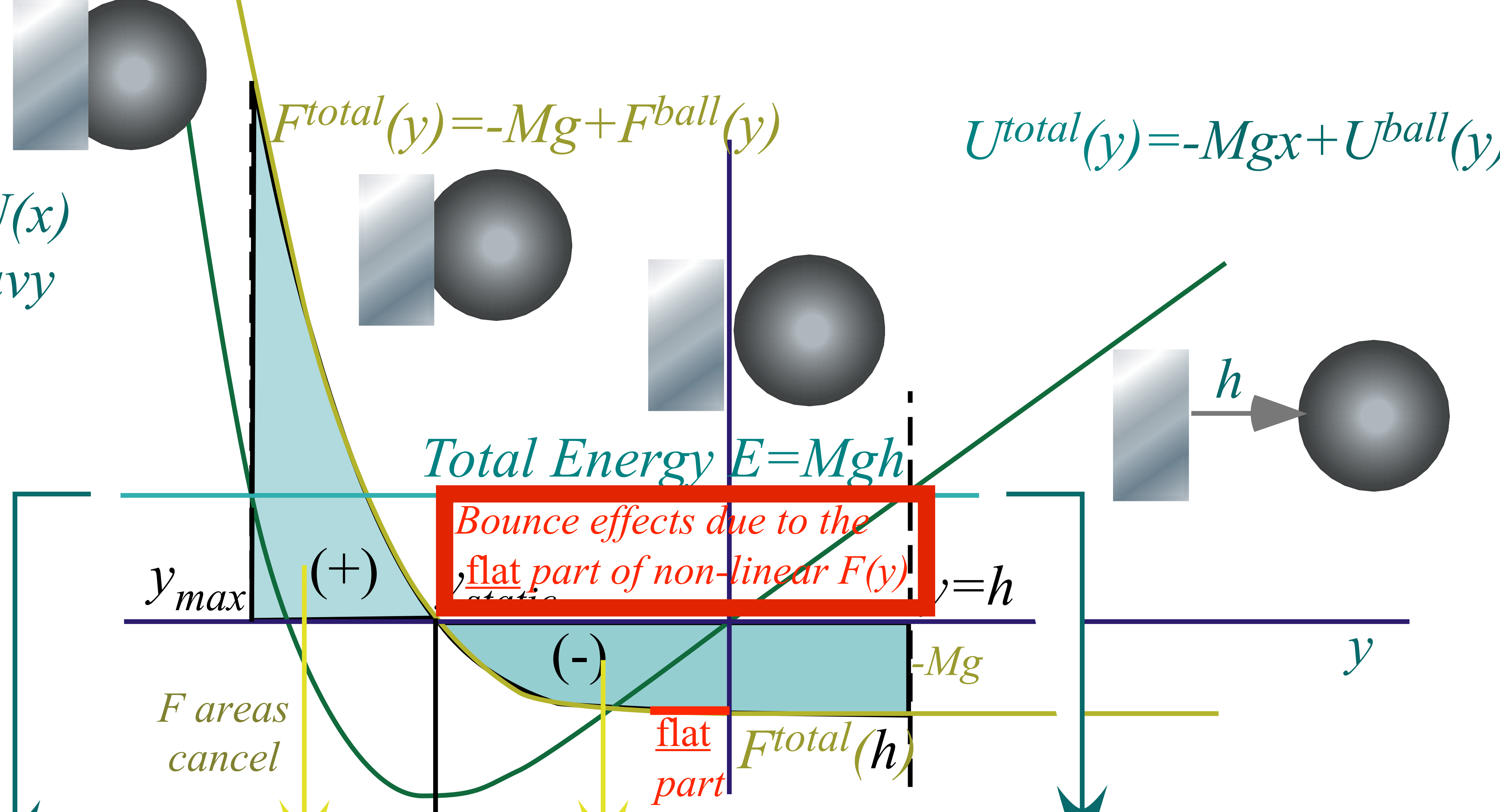
*Super-elastic examples: This really **is** "Rocket-Science"*

Force F(x) and Potential U(x) for soft heavy non-linear superball



Unit 1
Fig. 6.5

$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y)\,dy + \int_{y=h}^{y_{static}} F^{total}(y)\,dy + U(h) = U(h) = E$$

$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

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A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

Parable allegory for Los Alamos
Cheap&practical "seat-of-the pants" approach

Parable allegory for Livermore
Fancy&overpriced "political" approach

Advantages of a geometric m_1, m_2, m_3,... series

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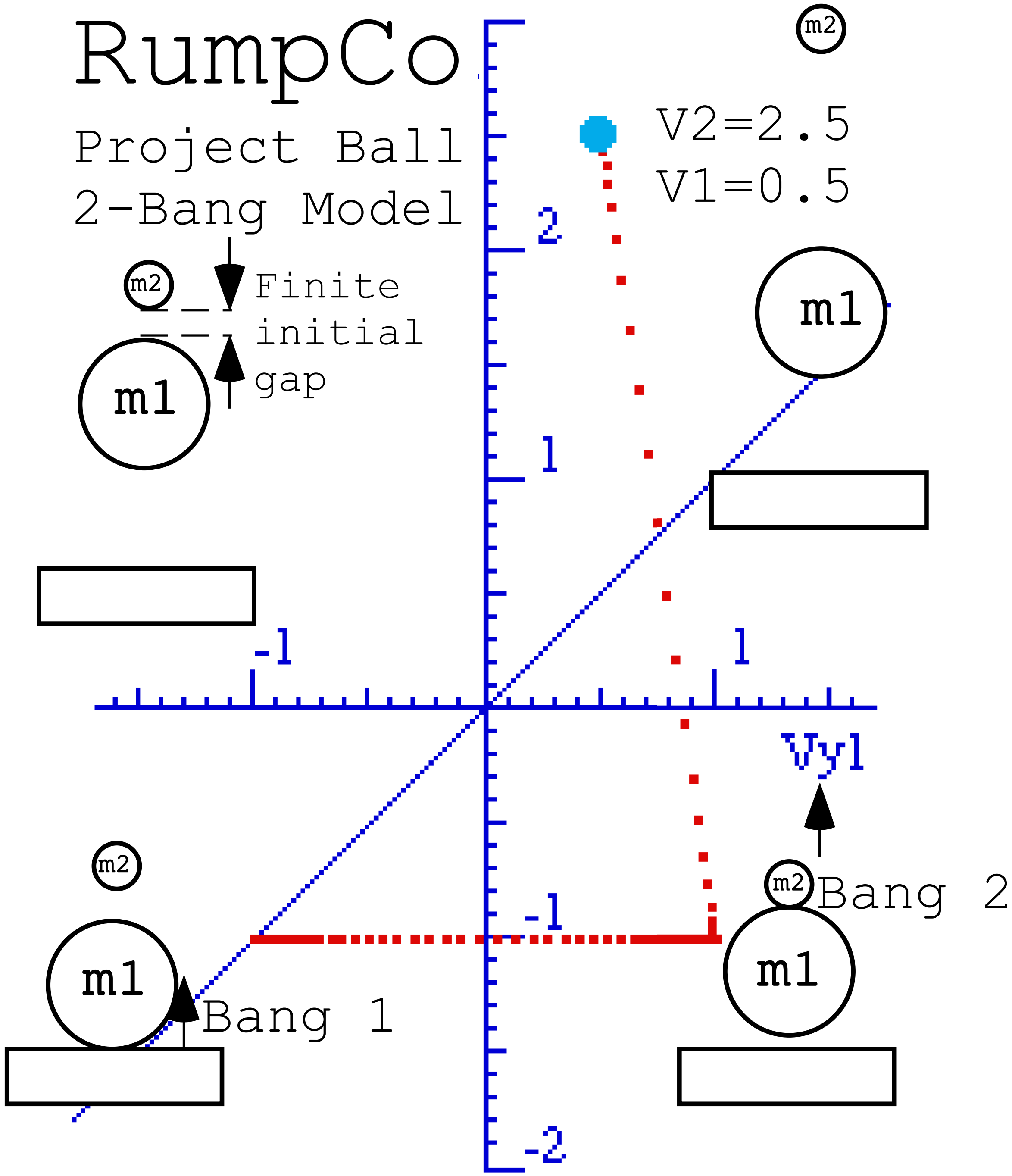
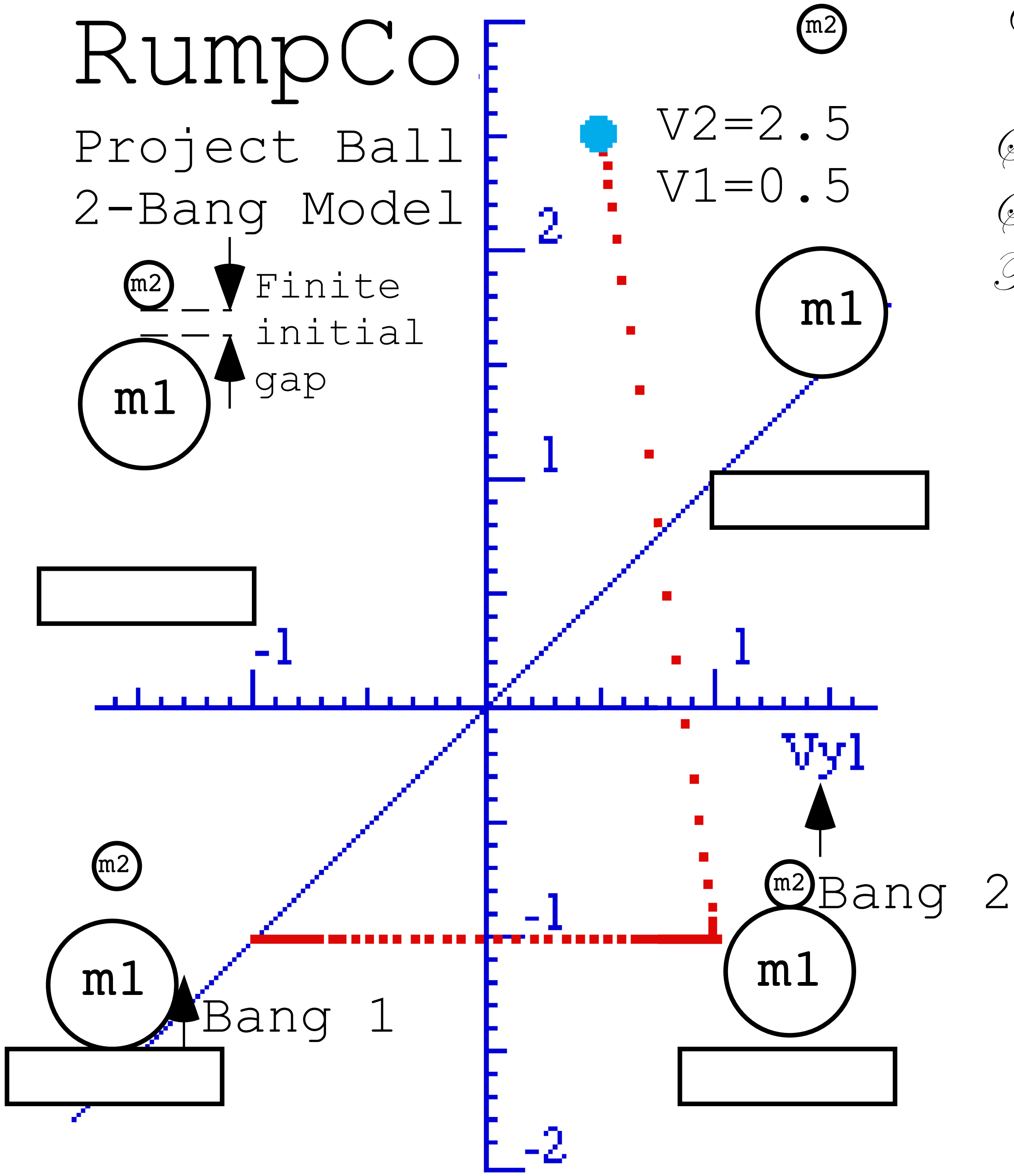


*Velocity amplification
or "throw" factor =2.5*

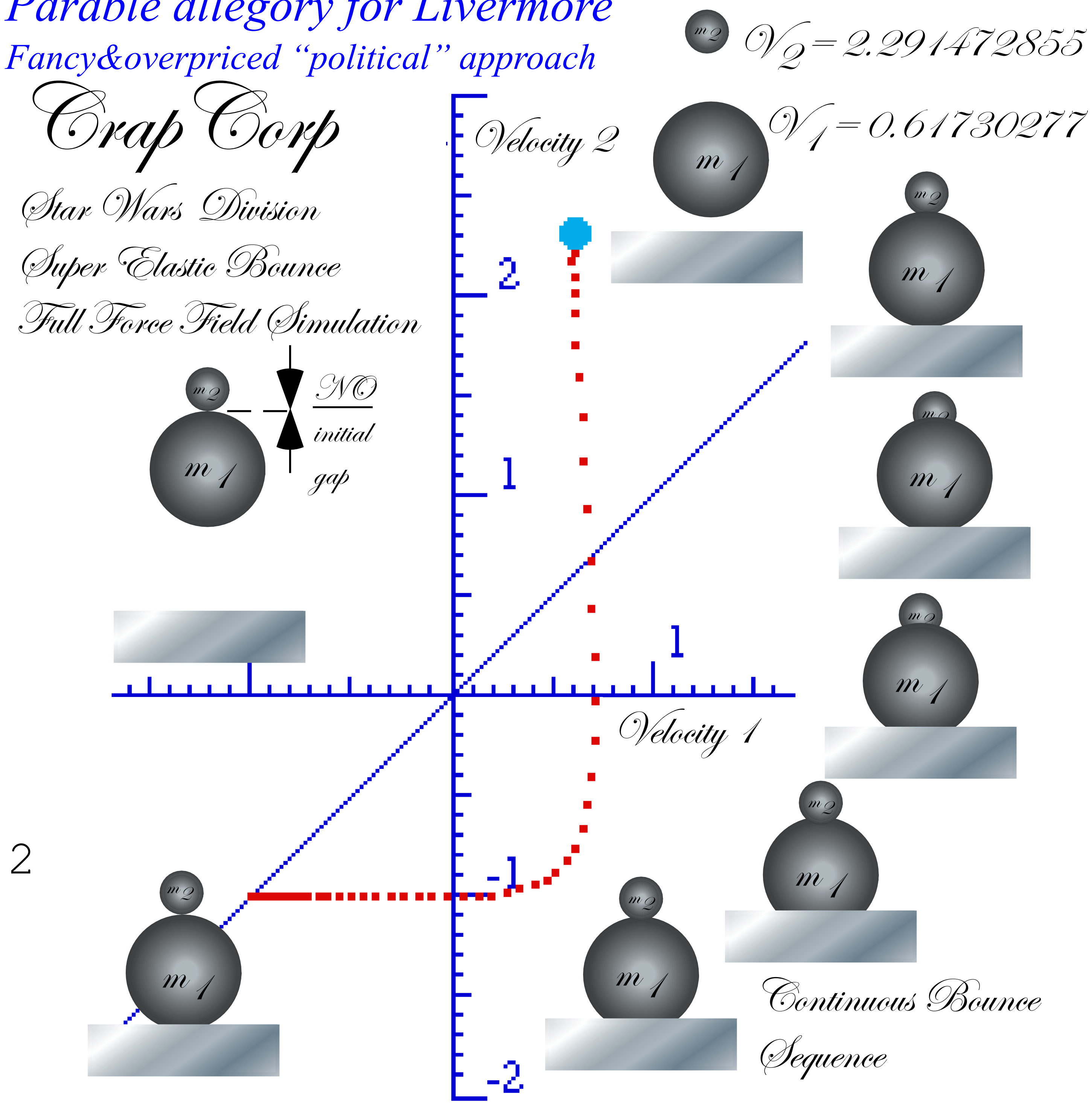
Unit 1
Fig. 6.6

Parable allegory for Los Alamos
Cheap&practical "seat-of-the pants" approach



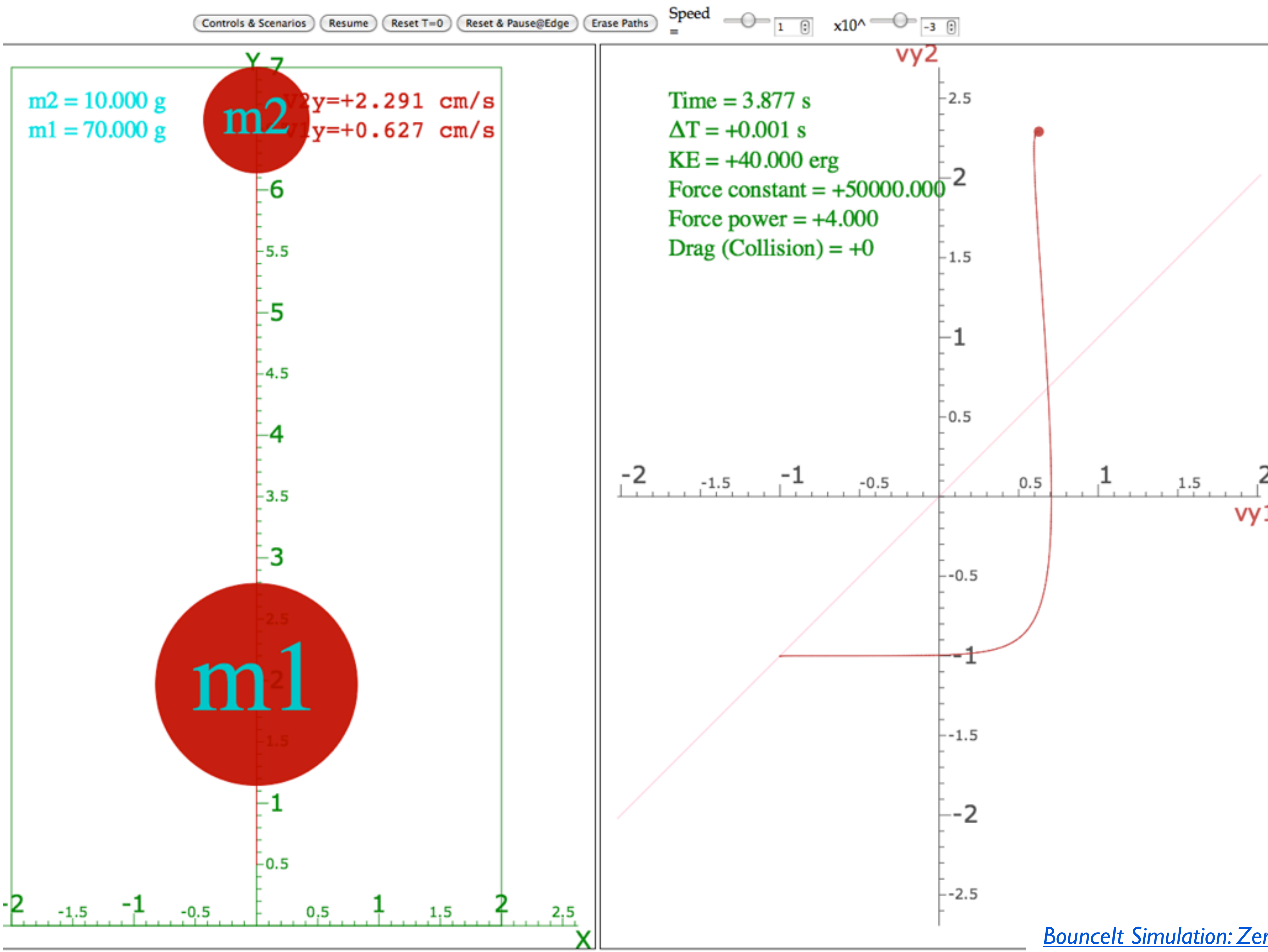
Velocity amplification
or "throw" factor =2.5

Parable allegory for Livermore
Fancy&overpriced "political" approach



Velocity amplification
or "throw" factor =2.3
(about equal to RumpCo finite gap experiment)

Unit 1
Fig. 6.6



[BounceIt Simulation: Zero Gap (Force power=4)]

BounceIt Simulation: Zero Gap (Force power=4)

Number of masses: 2 Balls

- Let mouse set: (x,y,Vx,Vy)
- Let mouse set force: F(t)
- Plot solid paths
- Plot dotted paths
- Plot no paths
- Plot V1 vs. V2
- Plot Y1(t), Y2(t), ...
- Plot PE of m1 vs. Y1
- Plot Y2 vs. Y1
- Plot user defined i.e - Y1 vs. Y2
- Balls initially falling
- Balls initially fixed
- No preset initial values

Acceleration of gravity: 0 100x{cm/s^2} — *Sets gravity*

- ☑ Draw force vectors
- ☐ Pause (once) at top
- ☑ Constrain motion to Y-axis
- ☑ Plot v2 vs v1
- ☐ Plot p2 vs p1
- ☐ Plot V2 vs V1
- ☐ Plot Ellipses
- ☑ Plot Bisector Lines
- ☐ Old Color Scheme
- ☑ Show right panel information
- ☑ Show left panel information
- ☐ Set Initial positions

Collision friction (Viscosity): 0 x10^ 0 {g}

Initial gap between balls: 0 x10^ 0 {g} — *zero-gap*

Force Constant: 5 x10^ 4 {g} — *Usually need to decrease k for p =1*

Force power law exponent: 4 — *This is non-linear $F=-kx^4$ (Set p =1 for linear $F=-kx^1$)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0: 0.75

		y Max =	7
Initial x1 =	1.09	y Min =	0
Max x PE plot =	0.5	T Max =	6
F-Vector scale =	0.003	V2y Max =	2.6944
Error step =	0.0000	V2y Min =	-2.694

Mass factor = 1

M_1= 7 x10^ 1 {g} V_1= -1 x10^ 0 {cm/s}

M_2= 1 x10^ 1 {g} V_2= -1 x10^ 0 {cm/s}

BounceIt Simulation: Zero Gap
(Force power=1)

Number of masses
2 Balls

Acceleration of gravity
0 100x{cm/s^2}

Collision friction (Viscosity)
0 x10^ 0 {g}

Initial gap between balls — *zero-gap*
0 x10^ 0 {g}

Force Constant *Usually need to increase k for p >1*
5 x10^ 2 {g}

Force power law exponent
1 ← *This is linear $F=-kx^1$ (increase $p>1$ for non-linear $F=-kx^p$)*

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0
0.75

- (•) Let mouse set: (x,y,Vx,Vy)
- ( ) Let mouse set force: F(t)
- ( ) Plot solid paths
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- [x] Draw force vectors
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- [x] Constrain motion to Y-axis
- [x] Plot v2 vs v1
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- [ ] Plot V2 vs V1
- [ ] Plot Ellipses
- [x] Plot Bisector Lines
- [ ] Old Color Scheme
- [x] Show right panel information
- [x] Show left panel information
- [ ] Set Initial positions

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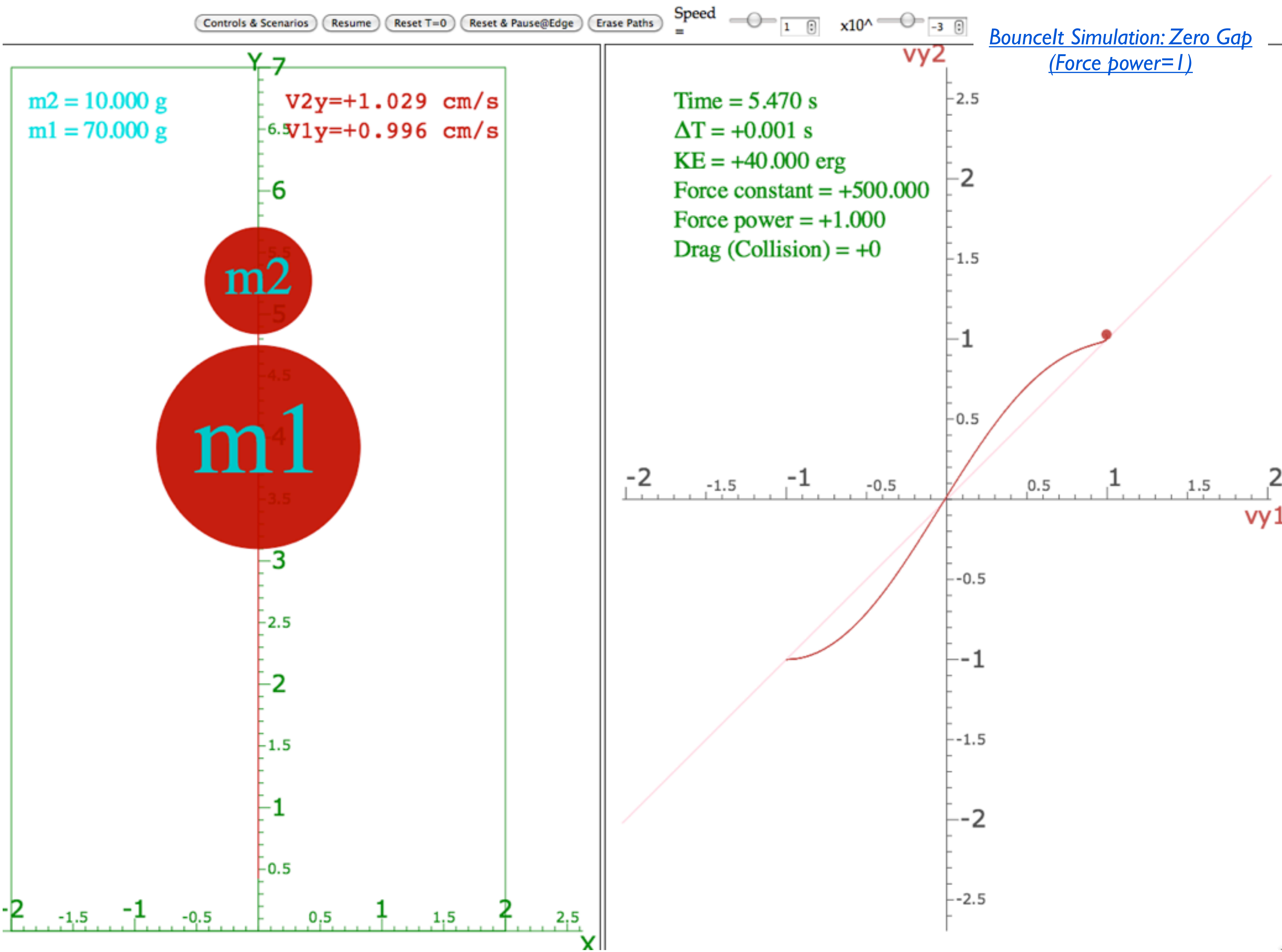
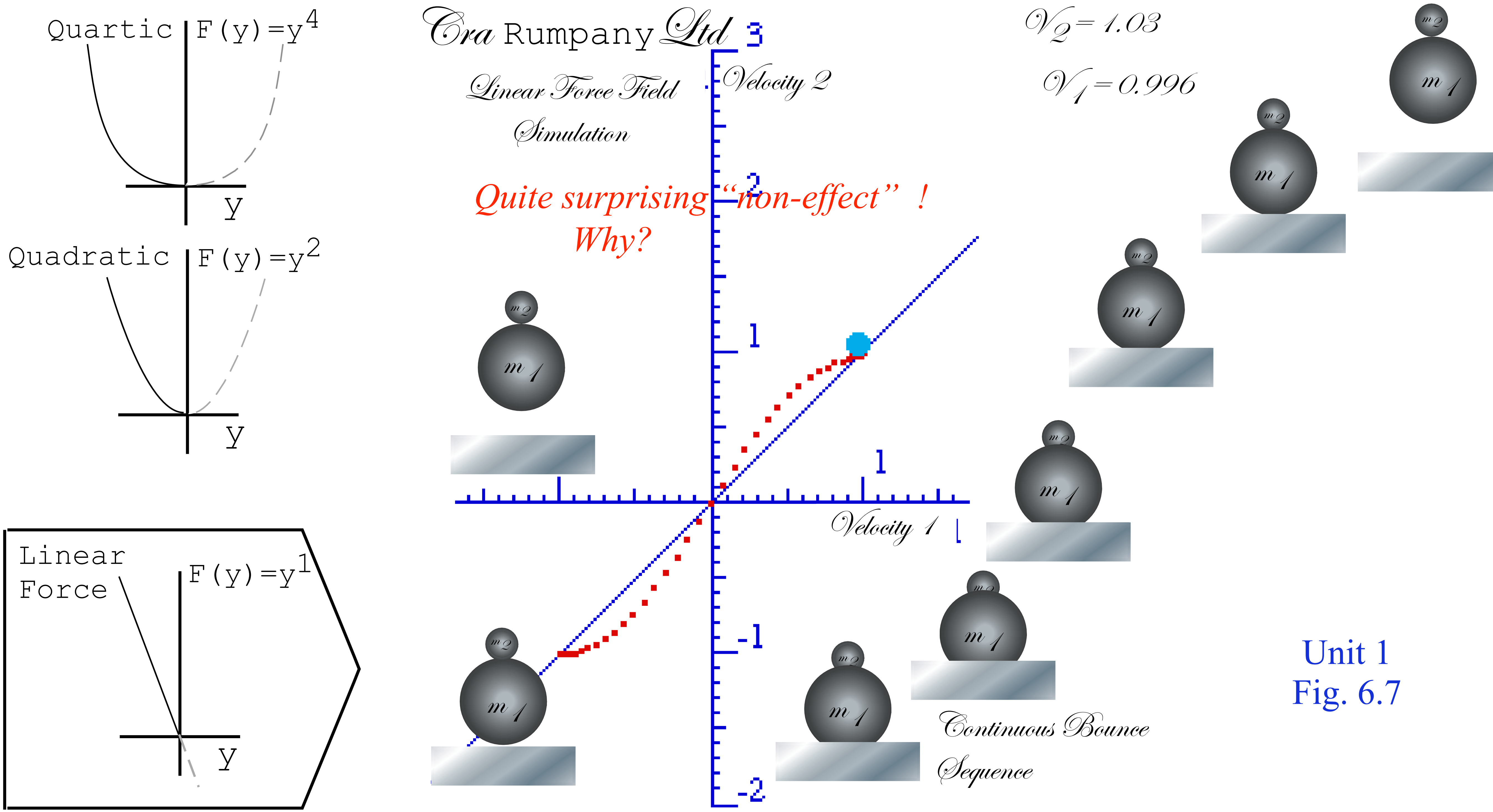
M_1= 7 x10^ 1 {g} V_1= -1 x10^ 0 {cm/s}

M_2= 1 x10^ 1 {g} V_2= -1 x10^ 0 {cm/s}

BounceIt Simulation: Zero Gap (Force power=1)



Cooperation between Los Alamos and Livermore yields insight to answer "What's going on?"



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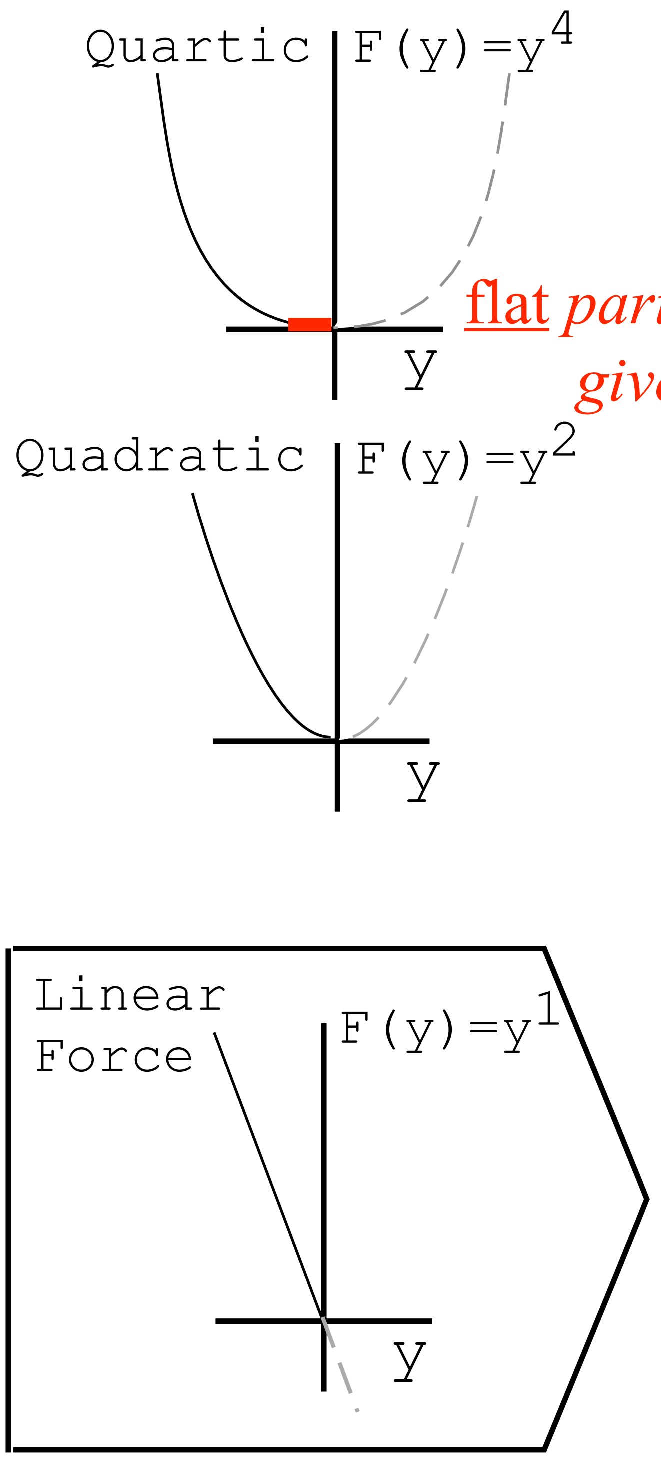
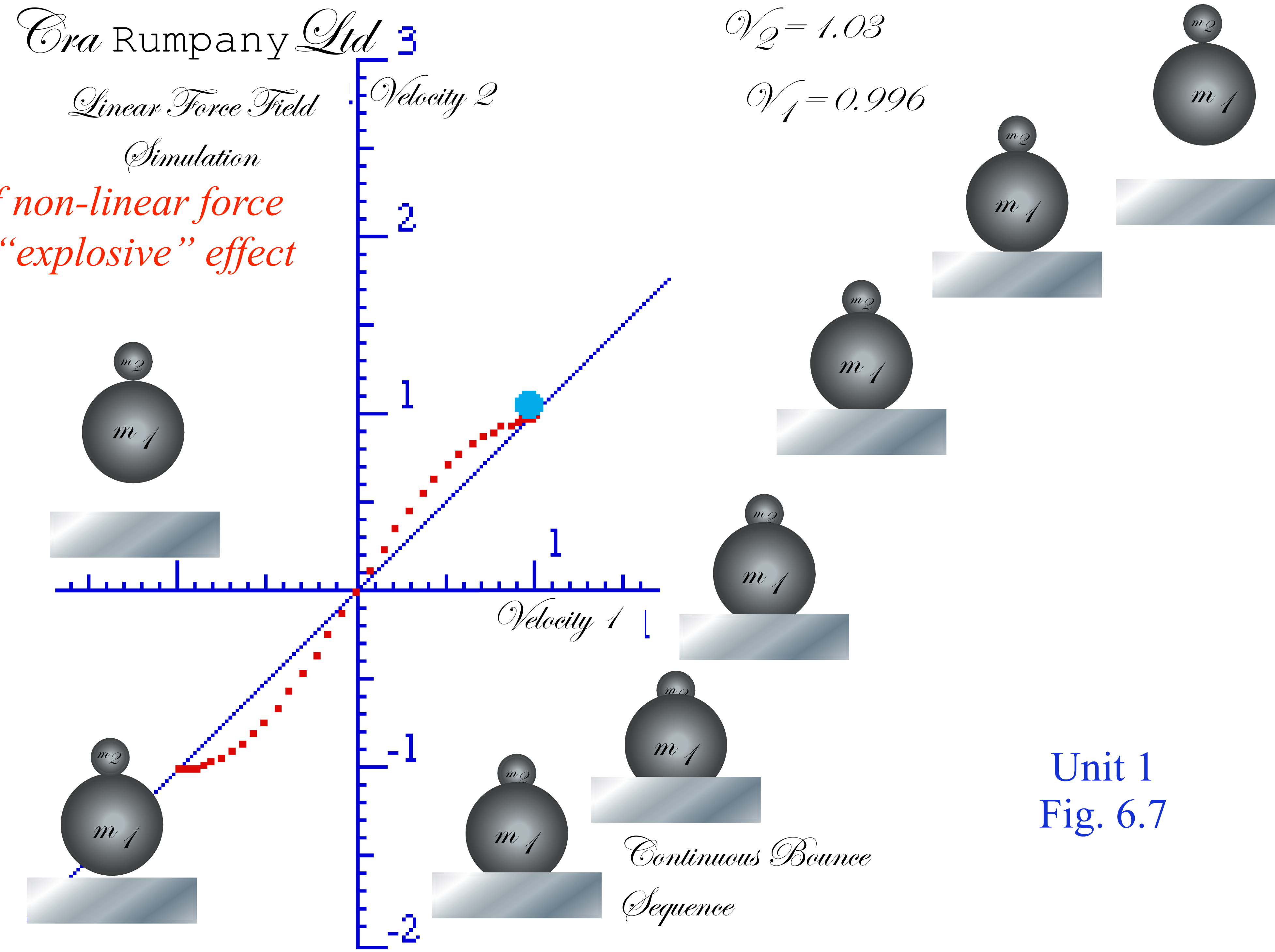
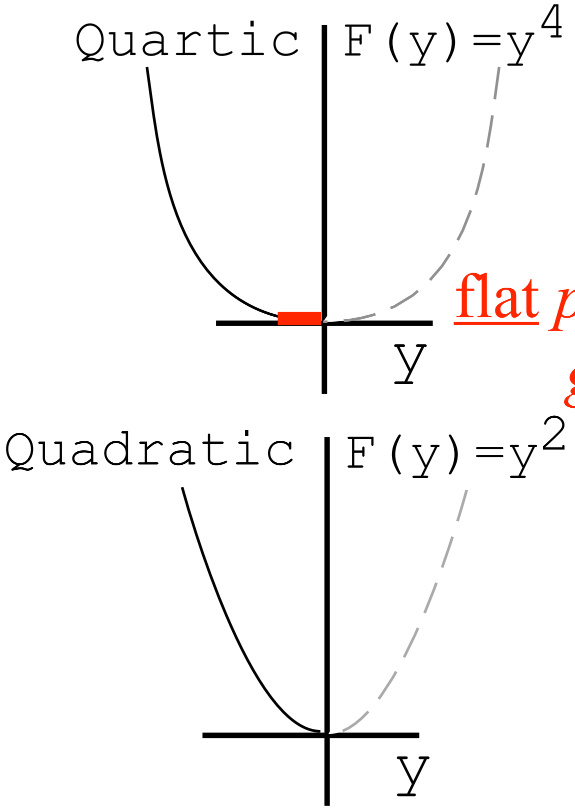


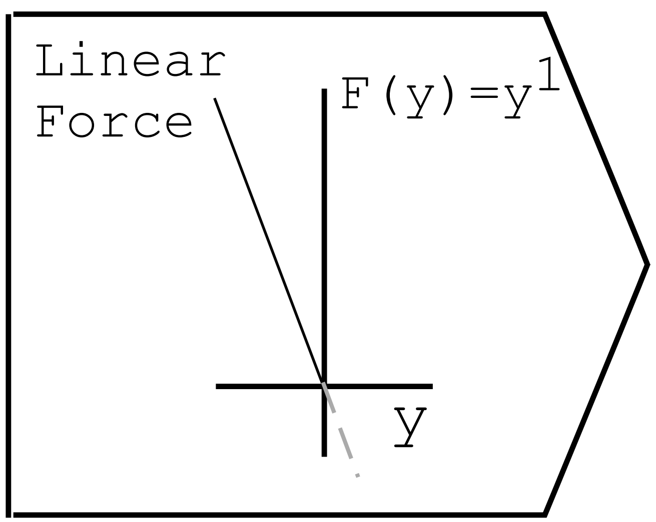


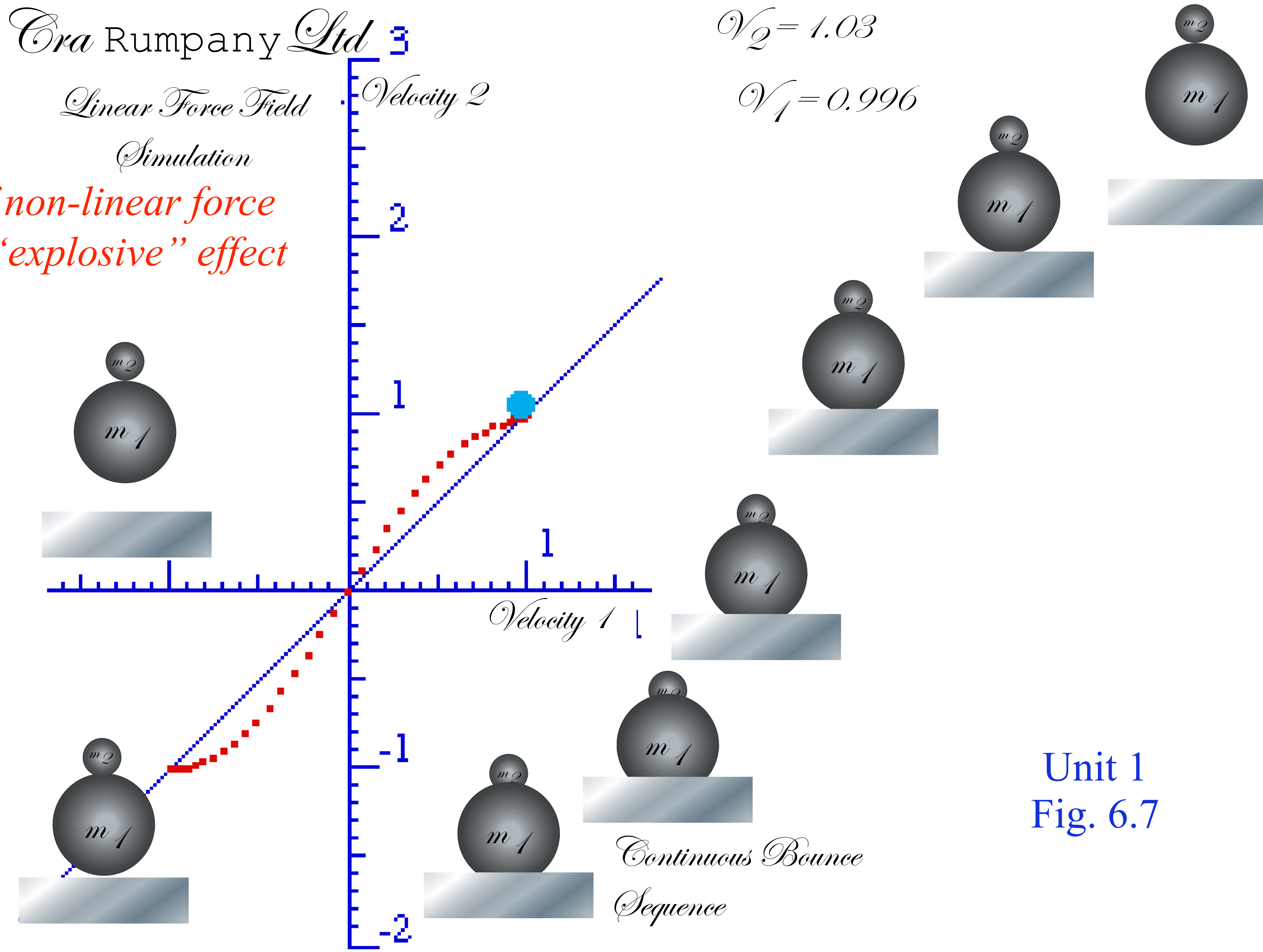
Unit 1
Fig. 6.7

Velocity amplification
or "throw" factor =1.03
(practically "no-throw")
for linear force F(y)= ky



flat part of non-linear force gives "explosive" effect





Unit 1
Fig. 6.7

Velocity amplification
or "throw" factor =1.03
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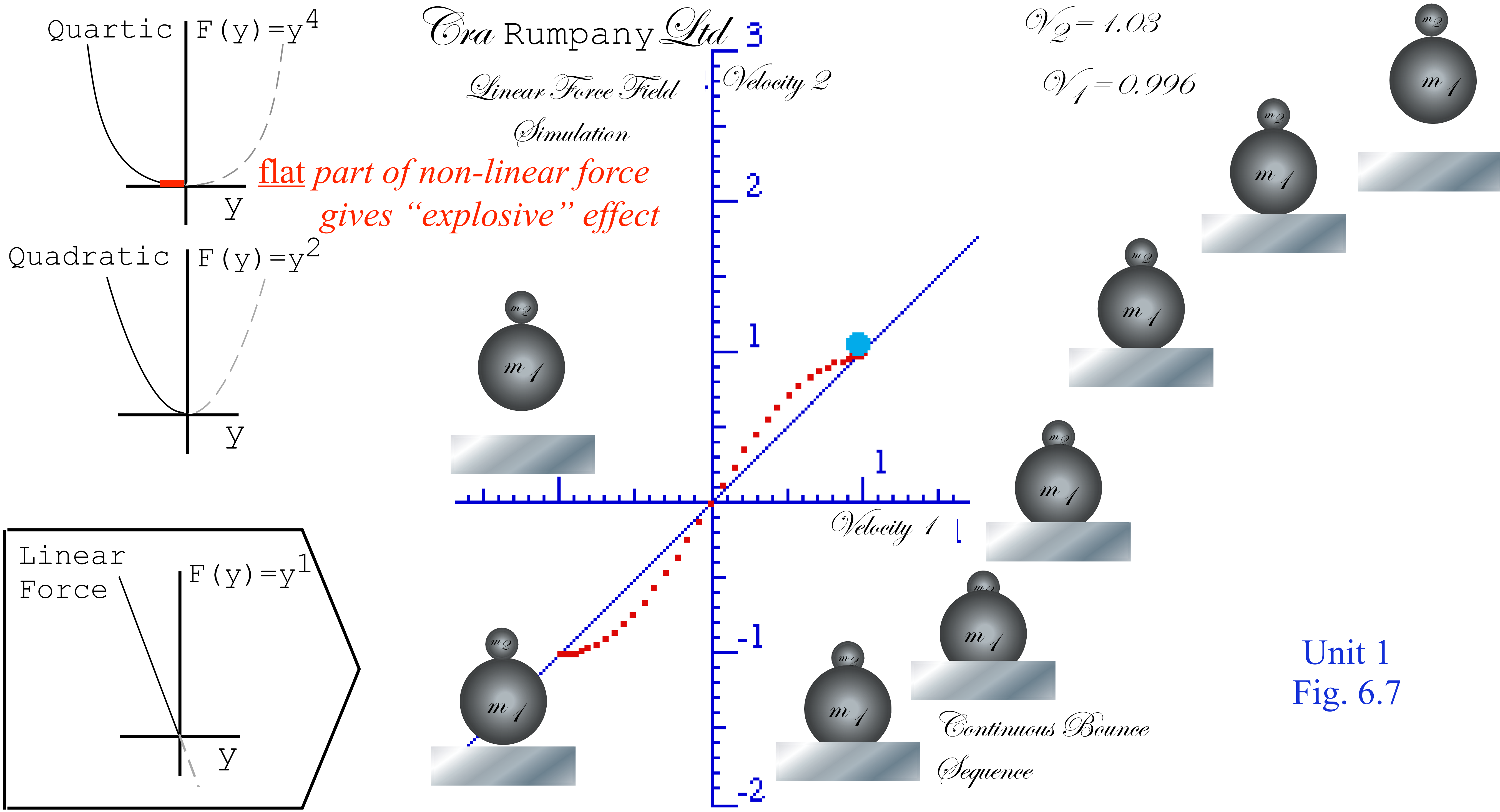
for linear force F(y)= ky

Lesson: Fasten your seatbelt

Cooperation between Los Alamos and Livermore yields insight to answer "What's going on?"



Unit 1
Fig. 6.7

Velocity amplification
or "throw" factor =1.03
(practically "no-throw")
for linear force F(y)= ky

Lesson: Fasten your seatbelt
TIGHTLY!

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

(a) Constant force F=-k (linear potential V=kx)

Some physics of dare-devil-diving 80 ft. into kidee pool *(Simulations)*

(b) Linear force F=-kx (quadratic potential $V=\frac{1}{2}kx^2$ (like balloon))

(c) Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and potential dynamics of 2-ball bounce

A parable of RumpCo. vs CrapCorp. (introducing 3-mass potential-driven dynamics)

→ *A story of USC pre-meds visiting Whammo Manufacturing Co.* *(Leads to Sagittal potential analysis of 2, 3, and 4 body towers)*

Geometry and dynamics of n-ball bounces

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A story of USC pre-meds visiting Whammo Manufacturing Co.

656 / June 1971

Velocity Amplification in Collision Experiments Involving Superballs

...and some results of "Project-Ball"

CLASS OF WILLIAM G. HARTER*
University of Southern California
Los Angeles, California 90007

(Received 25 September 1969; revised 25 September 1970)

If a pen is stuck in a hard rubber ball and dropped from a certain height, the pen may bounce to several times that height. The results of two such experiments, which can easily be duplicated in any undergraduate physics laboratory, are plotted for a range of mass ratios. A simple theoretical discussion which provides a qualitative understanding of the phenomenon is presented. A more complicated formulation which agrees very well with one of the experiments is also presented. The latter involves a simple analog computer program. Finally, an intriguing generalization of the phenomenon is considered.

* The members of the class of Dr. William G. Harter included: Calvin W. Gray, Jr., Robert C. Frickman, Brian P. Harney, Steven H. Hendrickson, Scott T. Jacks, David F. Judy, William D. Koltun, Sam C. Kaplan, Morton J. Kern, Edmund H. Kwan, Wayne E. Long, Michael E. Mason, William D. Moore, Willard W. Mosier, Gary P. Rudolf, Henry G. Rosenthal, William F. Skinner, Jay L. Stearn, Michael Weinberg, Mark Weiner, Frank J. Wilkinson, and David Willner.

INTRODUCTION

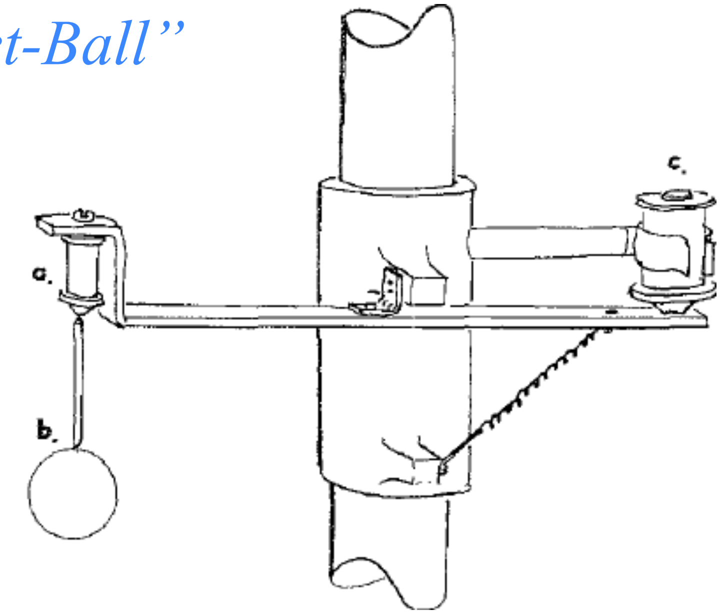
Shortly after the well-known Superball[1] appeared on the market, one of the authors quite accidentally discovered a surprising effect.[2] The point of a ball point pen is imbedded in the surface of a 3-in. diam Superball, and the pen and ball are dropped from a height of 4 or 5 ft so that the pen remains above the ball and perpendicular to a hard floor below. As the ball strikes the floor, the pen may be ejected so violently that it will strike the ceiling of the average room with considerable force. Furthermore, one can adjust the mass of the pen so that the ball remains completely at rest on the floor after ejecting the pen.

[1] Trade name of product by Whammo Manufacturing Co., San Gabriel, Calif.

ACKNOWLEDGMENT

We would like to thank John C. Fakan, John E. Heighway, and John H. Marburger for help during the initial and final stages of this project.

Class of W. G. Harter



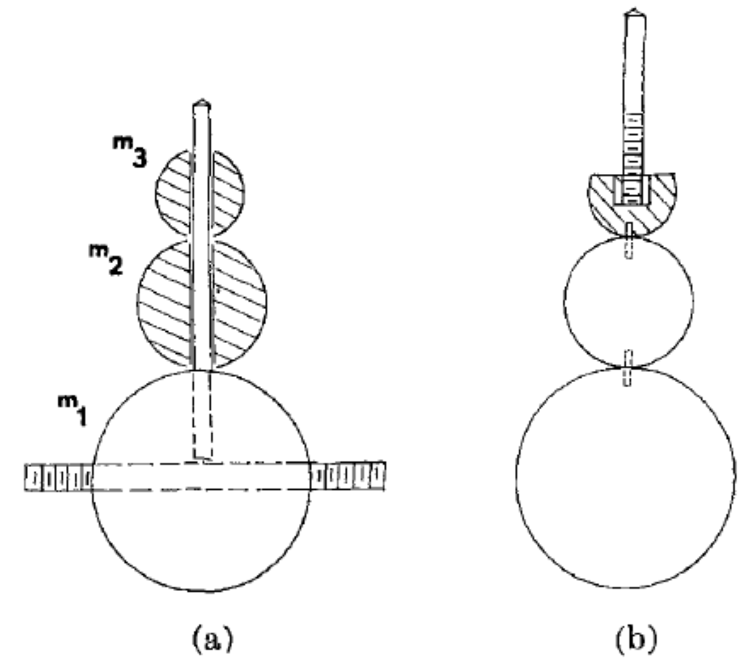


Fig. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

Much later....
Lots of profs try this out...
...including the unfortunate Harvard professor M. Tinkham...

(Still trying to find the video of the Tinkham incident...)

Basketball and Tennis Ball

Dropping a tennis ball on top of a basketball causes the tennis ball to bounce very high.

Source: 8.01 *Physics I: Classical Mechanics*, Fall 1999
Prof. Walter Lewin
http://ocw.mit.edu/high-school/physics/exam-prep/systems-of-particles-l

Course Material Related to This Topic:

- Watch video clip from Lecture 17 (21:30 - 24:08)
  http://videolectures.net/mit801f99_lewin_lec17/

A story of USC pre-meds visiting Whammo Manufacturing Co.

...and some results of "Project-Ball"

After initial big NBC splash (Ray Dunkin Reports) in Fall 1968, USC mechanical engineers kindly measured super-ball force curves F(y) with their precision tension meter and let us use their analog computer to calculate precise bounce heights.

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After this things began deteriorating in Old-Physics-Rm 69 (The Project-Ball-Room)

1. The fancy-pants computer theory did not jive with the fine drop-tower experiments.

2. USC B&G decided Rm 69 needed painting and kicked us out for a week.

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Days later, finally, got a car convoy together so we all could visit San Gabriel plant.

But, that was "Alpha-Wave" day for inventors at San Gabriel plant.
So we end up talking to Whammo lawyer/owner.

A story of USC pre-meds visiting Whammo Manufacturing Co.

...and some results of "Project-Ball"

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A call to Whammo Co. elicited interest in a big $$$$ product. Invited us to visit. Yay! $$$

Days later, finally, got a car convoy together so we all could visit San Gabriel plant.

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...and some results of "Project-Ball"

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Little paint spots on floor show what was wrong with our fancy-pants computer theory

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...and some results of "Project-Ball"

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Little paint spots on floor show what was wrong with our fancy-pants computer theory.

The engineering curves were <u>isothermal</u> not <u>adiabatic</u>.
Need latter. Can do latter by dropping dyed balls and measuring spot-size.

Measuring spot-size d *gives energy vs. height.*
Slope of E(x) gives force F(x) and G(x).

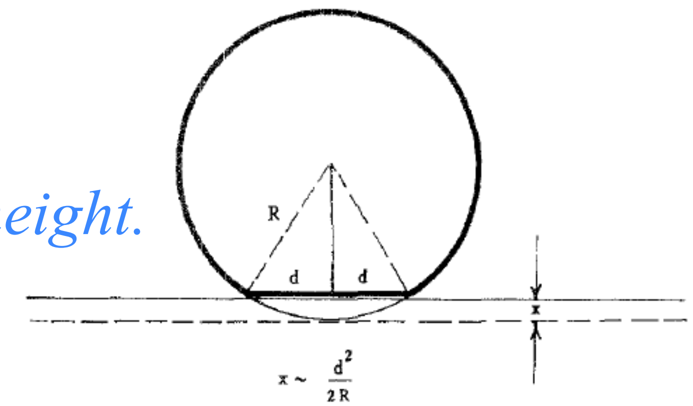
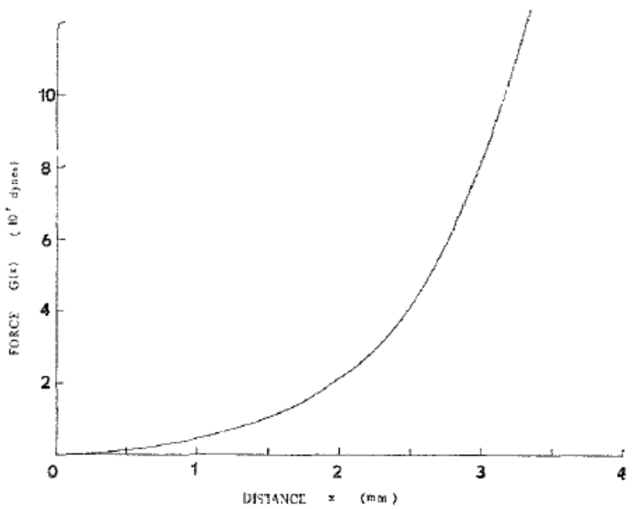


FIG. 10. Sagittal formula.

Collisions Involving Superballs



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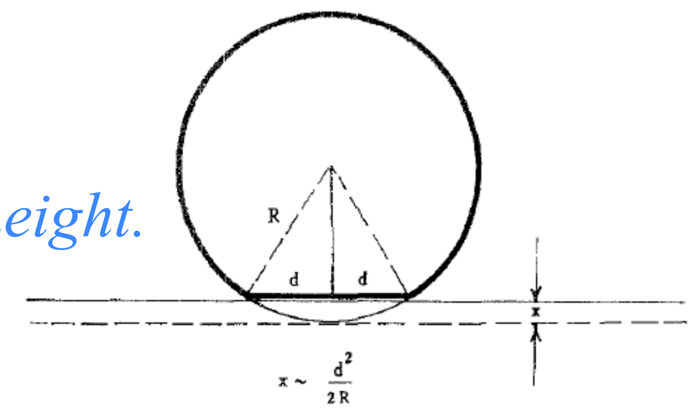

Fig. 10. Sagittal formula.

If $F(x)$ and $G(x)$ were linear for all x, then the

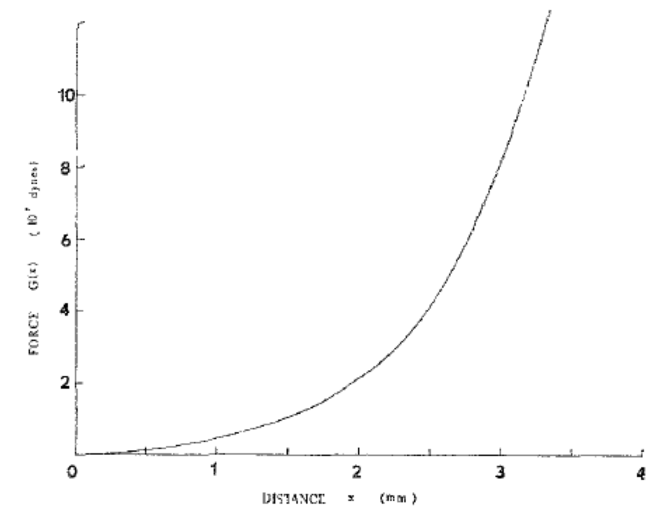

Fig. 12. Adiabatic force function $G(x)$.

Then fancy-pants computer theory can predict N-ball tower bounce

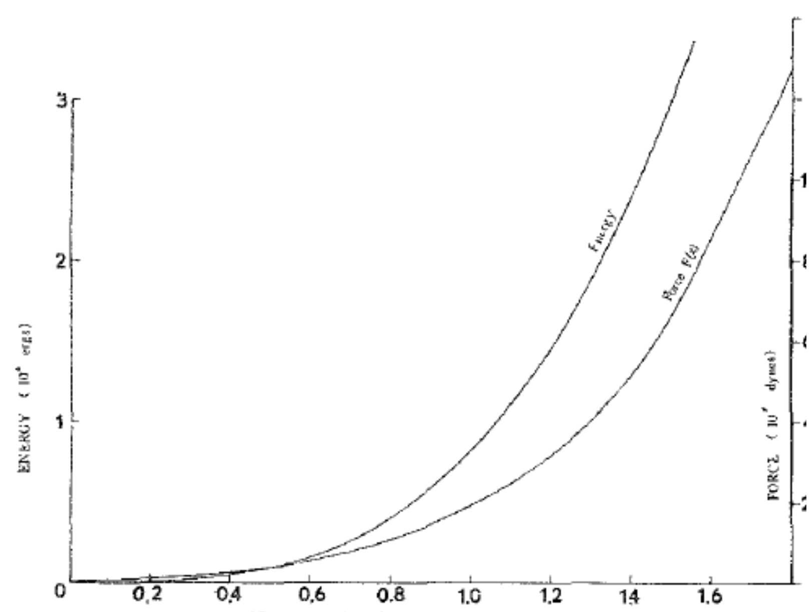

Fig. 11. Adiabatic force $F(x)$ and energy curves fo
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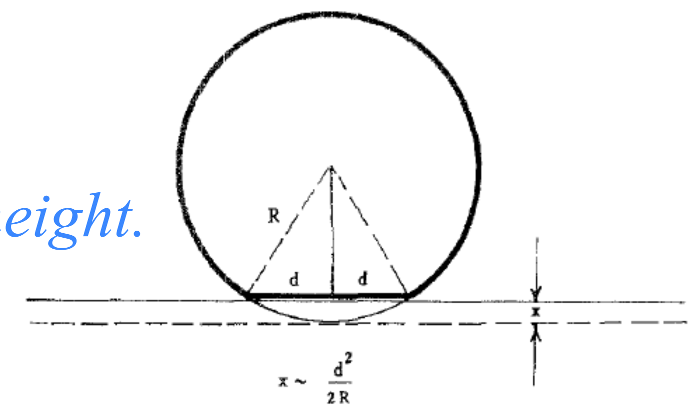

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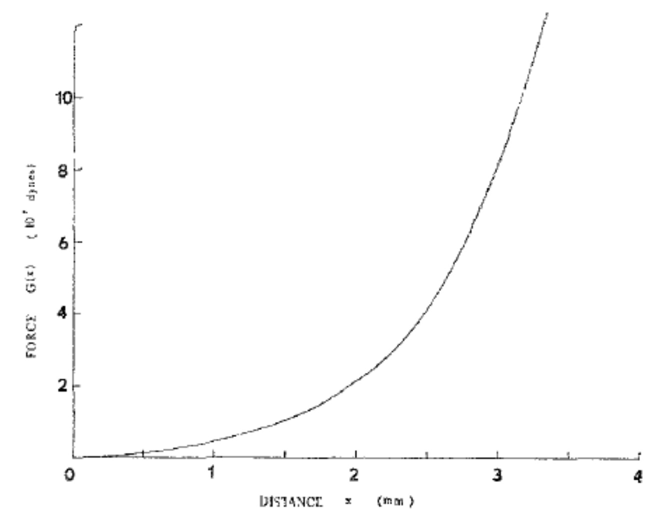

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If $F(x)$ and $G(x)$ were linear for all x, then the

Functions $F(x)$ and $G(x)$ were then placed on the function generators of the analog computer.

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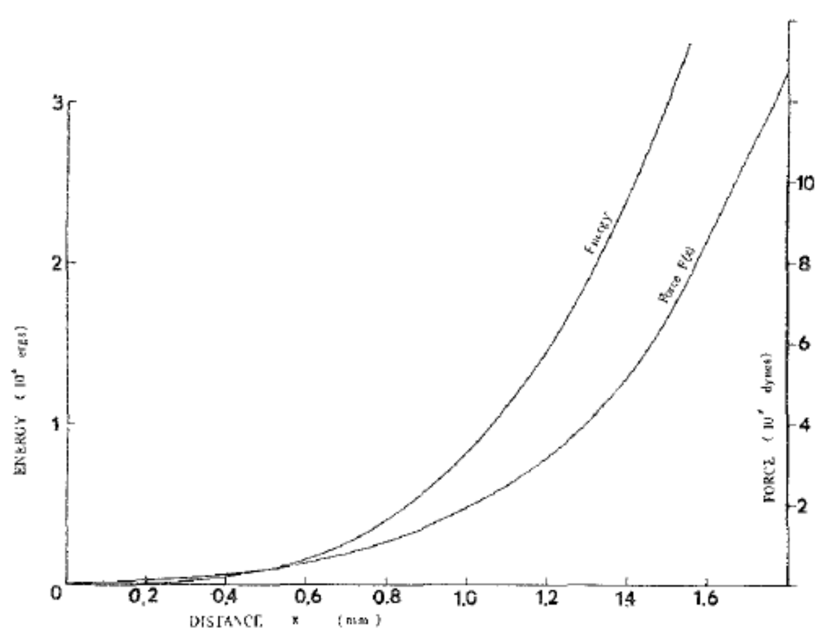

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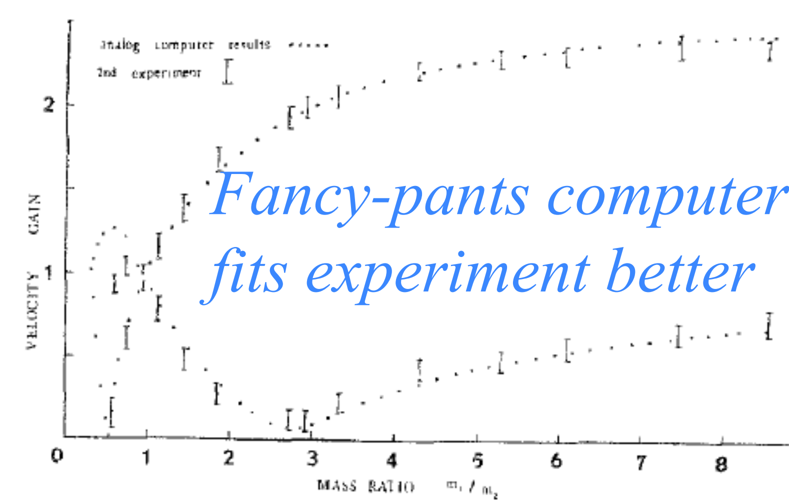
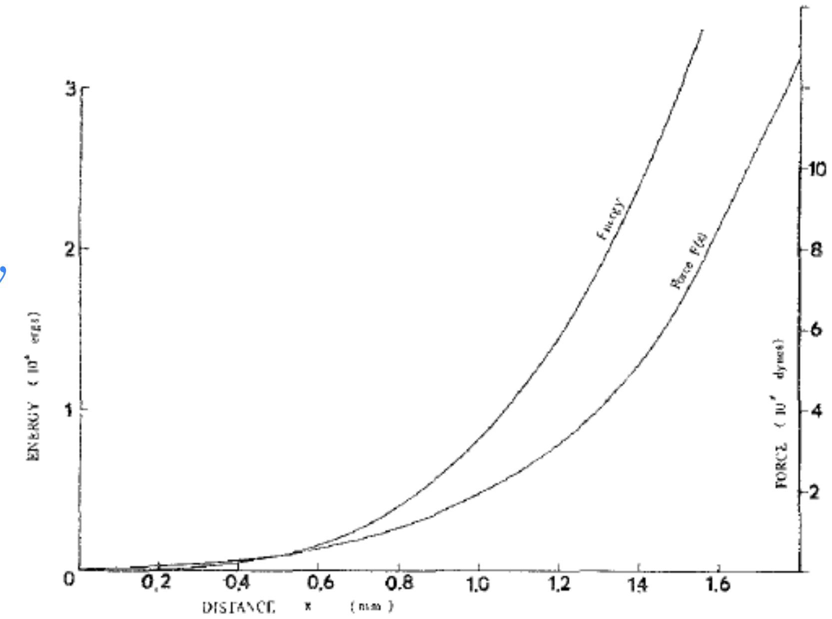


Fancy-pants computer theory fits experiment better

Fig. 13. Comparison between analog computer gain curves and second experiment.

Then fancy-pants computer theory can predict N-ball tower bounces



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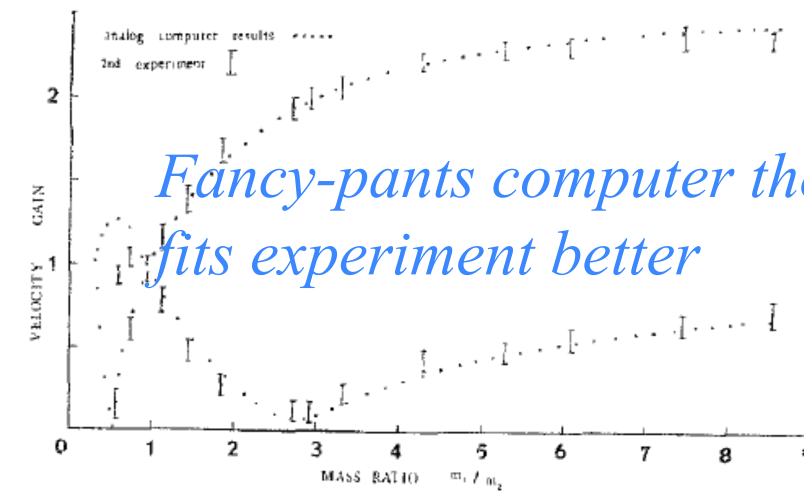

FIG. 13. Comparison between analog computer gain curves and second experiment.

Fancy-pants computer theory fits experiment better

Here are some 3-ball tower bounce predictions

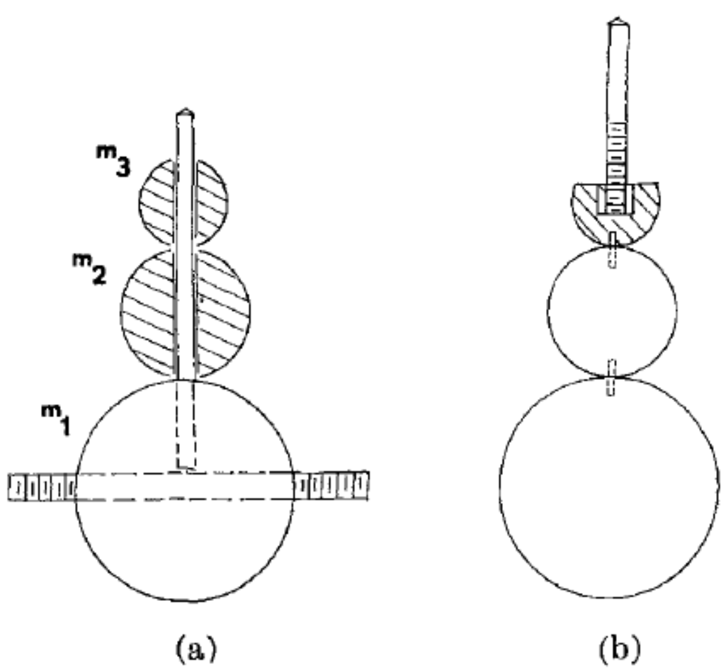

FIG. 14. Two designs for a multiple stage tower of balls. (a) Large number of balls can slide on a shaft. (b) Balls connected by small pins stand to lose appreciable amounts of binding energy.

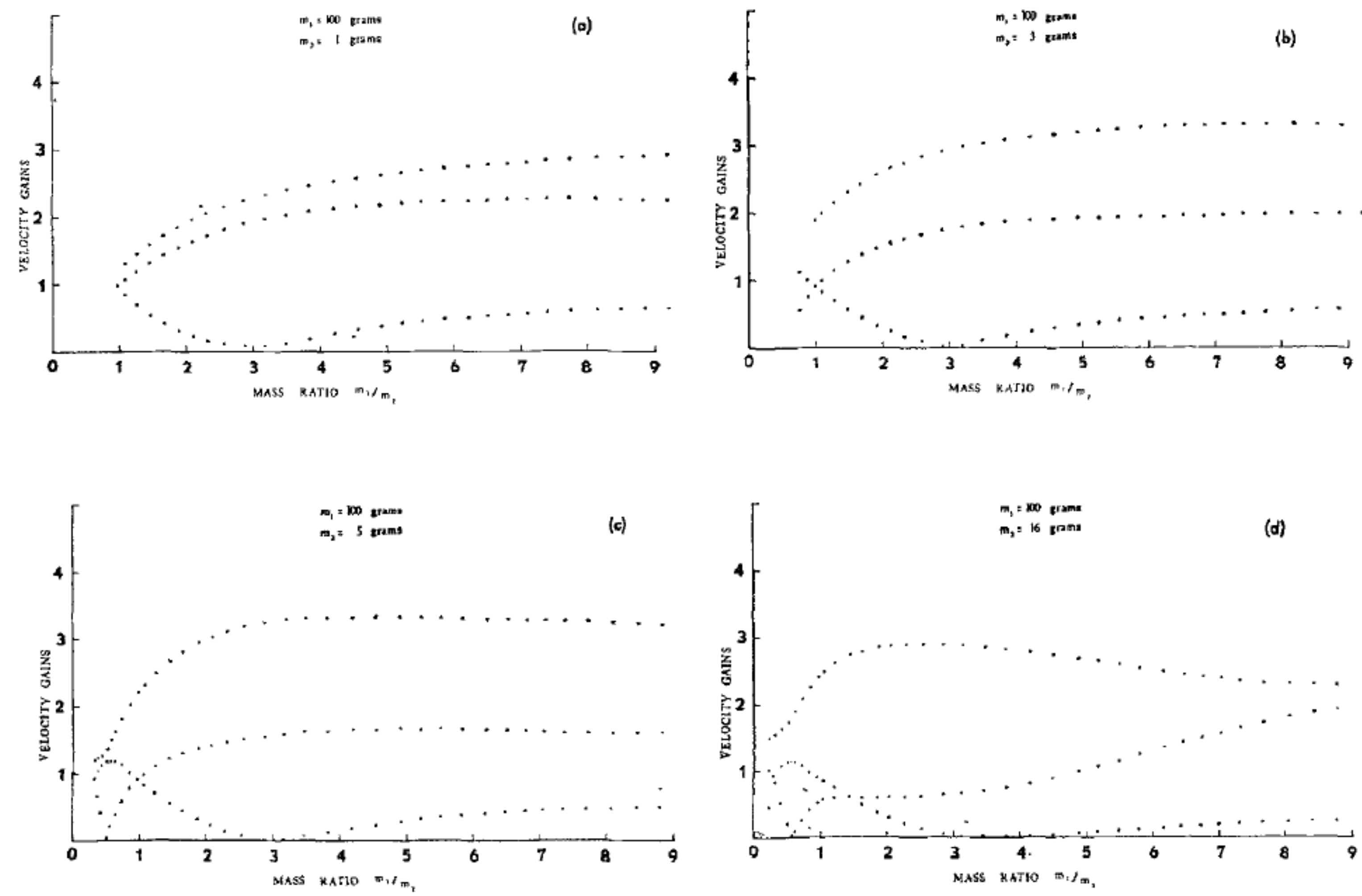

FIG. 15. (a)–(d) Analog computer output for velocity gains of three-ball system.

Potential energy dynamics of Superballs and related things

Thales geometry and "Sagittal approximation" to force law

Geometry and dynamics of single ball bounce

(a) Constant force F=-k (linear potential V=kx)

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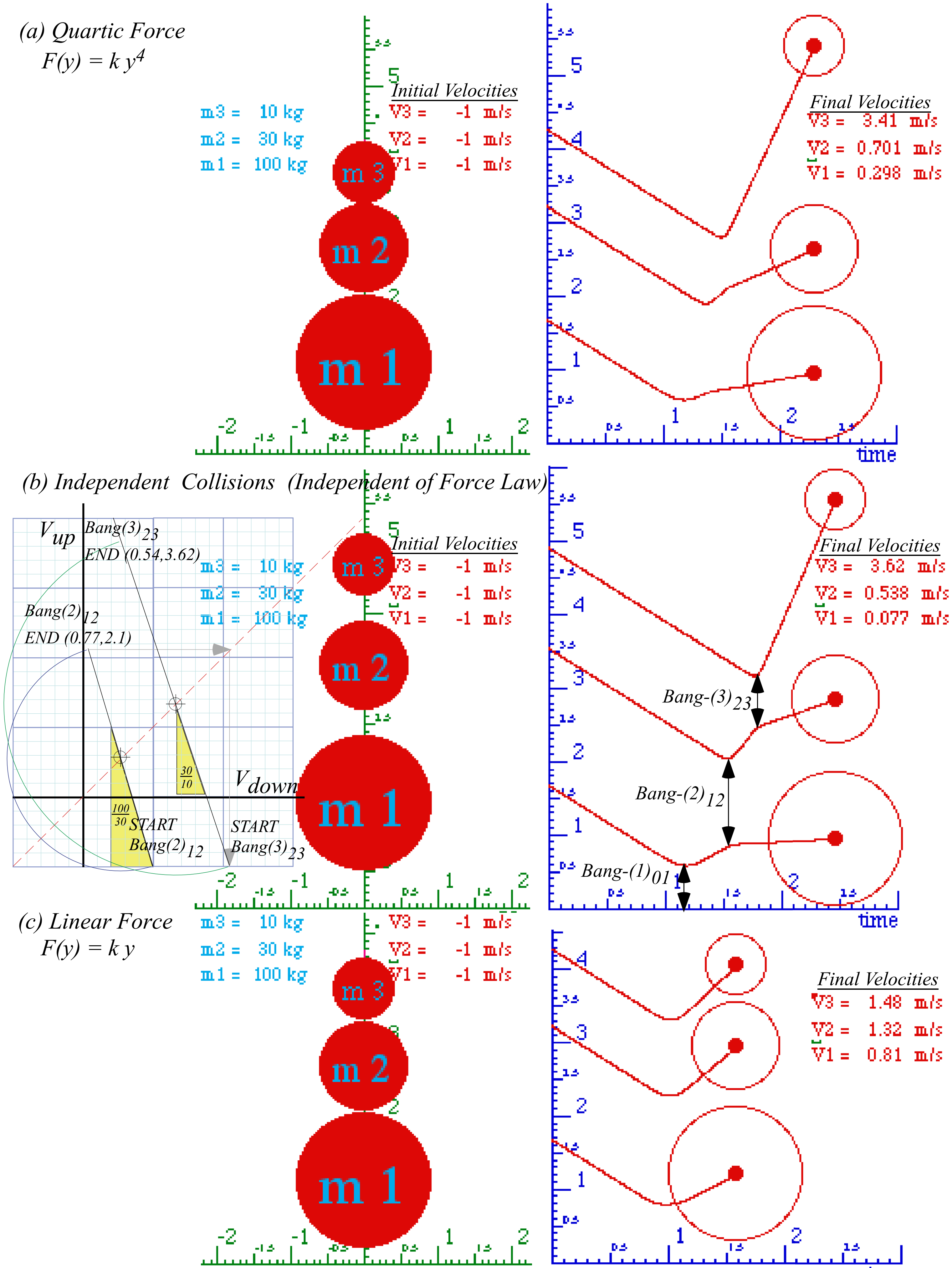
Inelastic examples: "Zig-zag geometry" of freeway crashes

*Super-elastic examples: This really **is** "Rocket-Science"*



Unit 1
Fig. 7.1a-c
Independent Bang Model (IBM)
3-Body Geometry

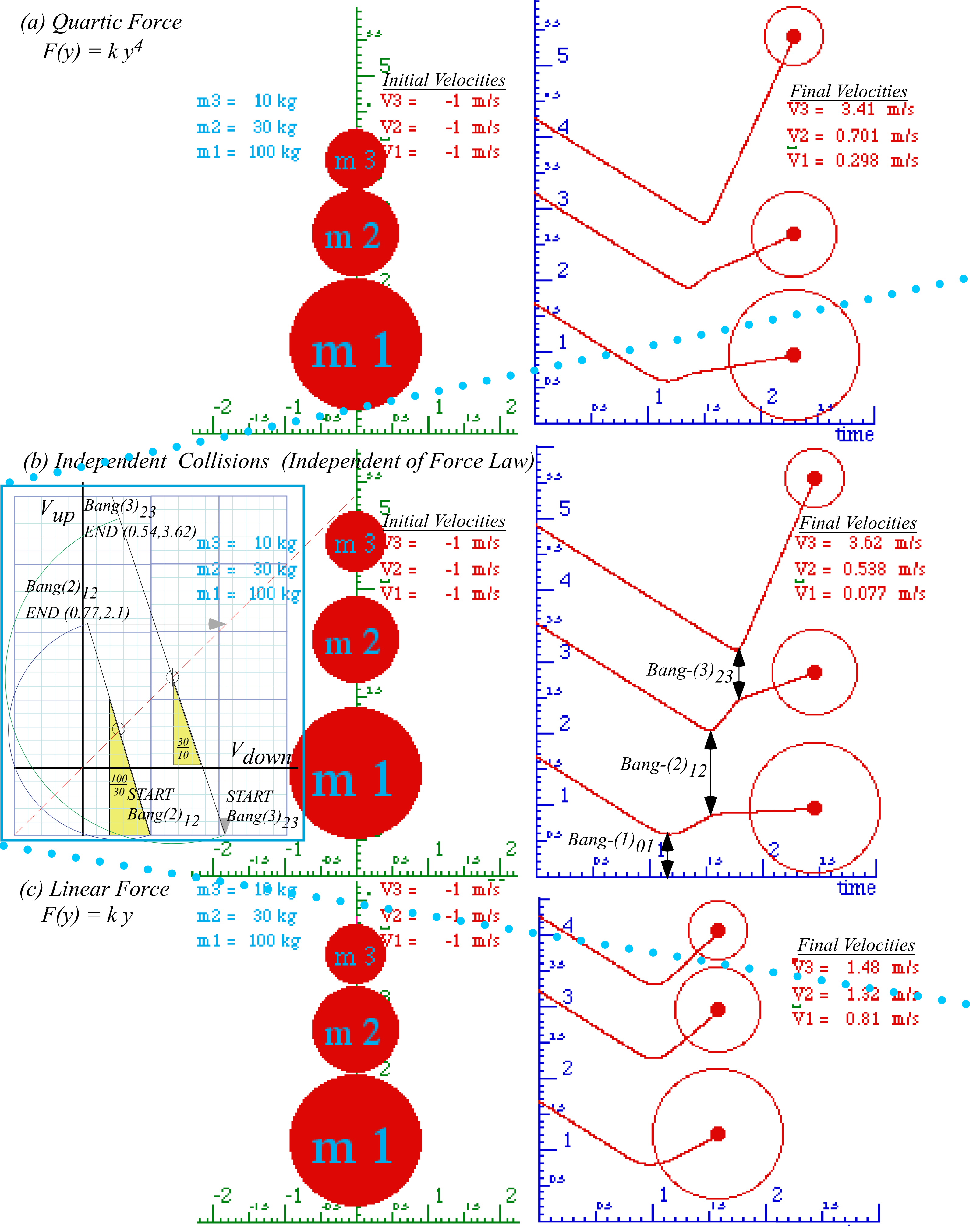
3-Ball Tower w/ Quartic Force

BounceIt Web

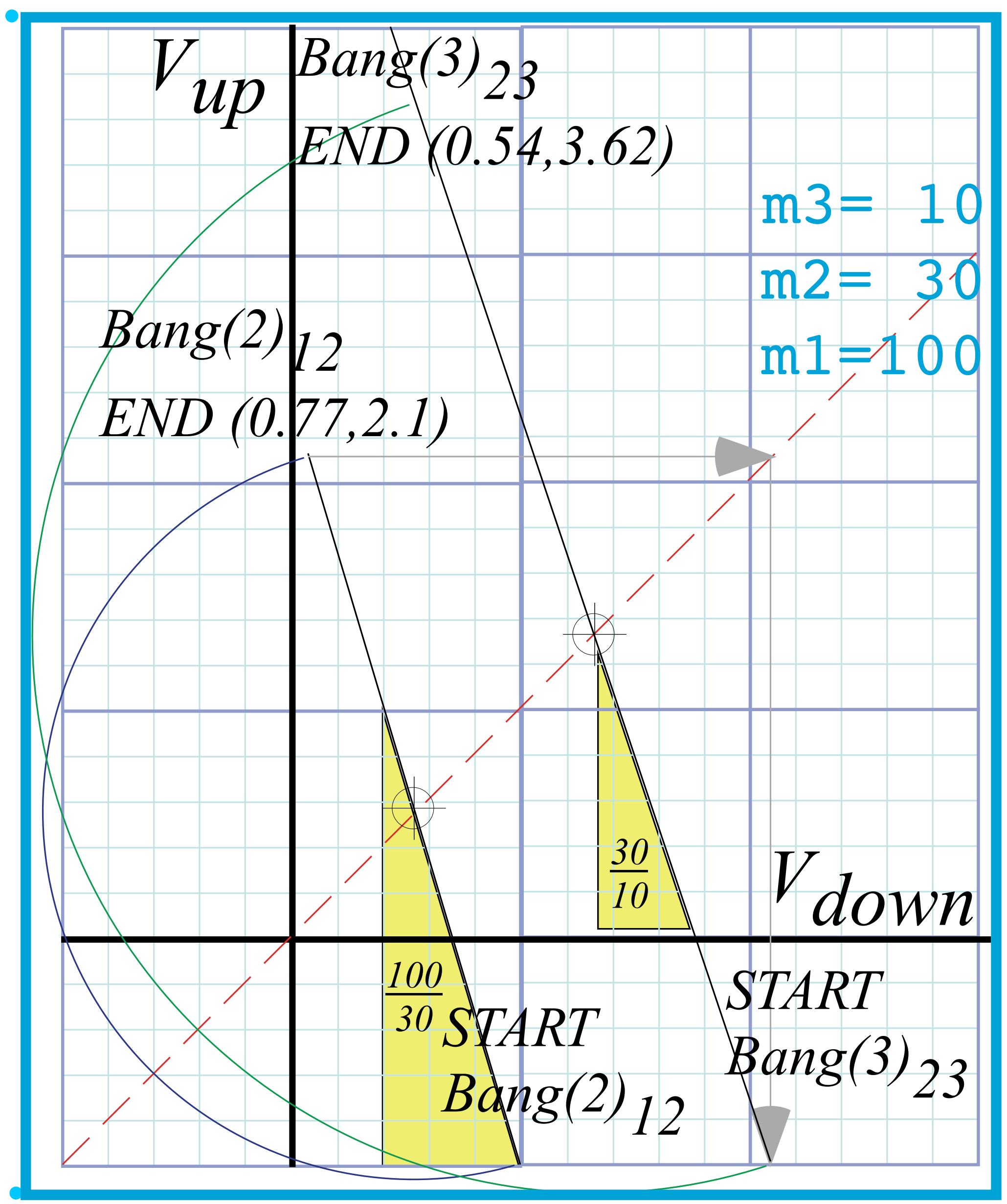
Newtonian Space-Time

Simulations

3-Ball Tower w/ Linear Force



Unit 1
Fig. 7.1b
Independent Bang Model (IBM)
3-Body Geometry

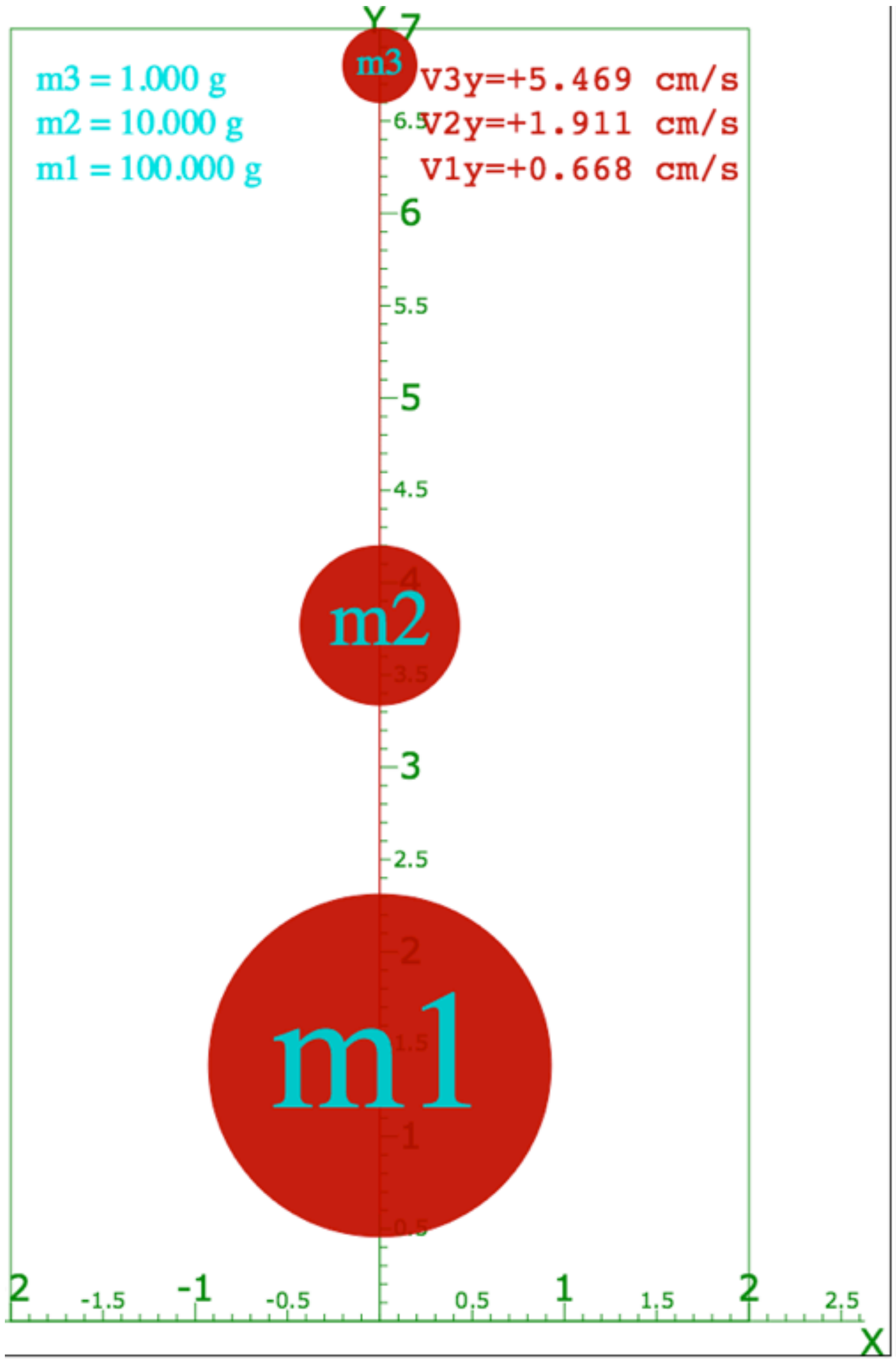


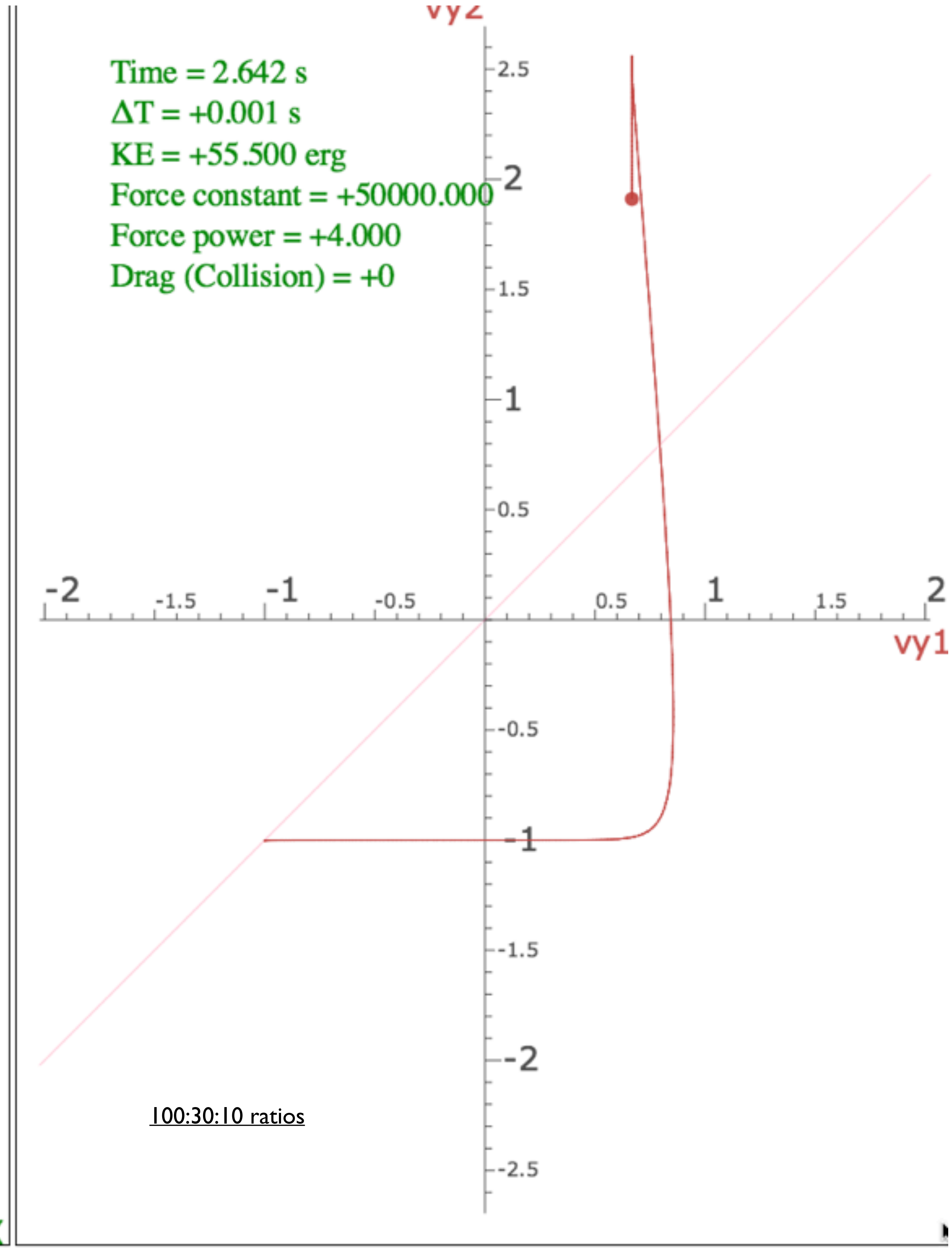
BounceIt Web
Newtonian Space-Time
Simulations

3-Ball Tower w/ Quartic Force

3-Ball Tower w/ Linear Force

BounceIt Web Simulation V_{y2} vs. V_{y1} Plot
3-Ball Tower w/ Quartic Force





[BounceIt Web Simulation V_{y2} vs. V_{y1} Plot](#)
[3-Ball Tower w/ Quartic Force](#)

Number of masses: 3 Balls

- (•) Let mouse set: (x,y,Vx,Vy)
- ( ) Let mouse set force: F(t)
- ( ) Plot solid paths
- (•) Plot dotted paths
- ( ) Plot no paths
- (•) Plot V1 vs. V2
- ( ) Plot Y1(t), Y2(t), ...
- ( ) Plot PE of m1 vs. Y1
- ( ) Plot Y2 vs. Y1
- ( ) Plot user defined i.e - Y1 vs. Y2
- ( ) Balls initially falling
- ( ) Balls initially fixed
- (•) No preset initial values

Acceleration of gravity: 0 100x{cm/s^2}

- [x] Draw force vectors
- [x] Pause (once) at top
- [x] Constrain motion to Y-axis
- [x] Plot v2 vs v1
- [ ] Plot p2 vs p1
- [ ] Plot V2 vs V1
- [ ] Plot Ellipses
- [x] Plot Bisector Lines
- [ ] Old Color Scheme
- [x] Show right panel information
- [x] Show left panel information
- [ ] Set Initial positions

Collision friction (Viscosity): 0 x10^ 0 {g}

Initial gap between balls: 0 x10^ 0 {g}

Force Constant: 5 x10^ 4 {g}

Force power law exponent: 4

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0: 0.75

---

y Max = 7

Initial x1 = 1.09 | y Min = 0

Max x PE plot = 0.5 | T Max = 6

F-Vector scale = 0.003 | V2y Max = 2.6944

Error step = 0.0000 | V2y Min = -2.694

Mass factor = 1

M_1= 1 x10^ 2 {g} | V_1= -1 x10^ 0 {cm/s}

M_2= 1 x10^ 1 {g} | V_2= -1 x10^ 0 {cm/s}

M_3= 1 x10^ 0 {g} | V_3= -1 x10^ 0 {cm/s}

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→ *Advantages of a geometric $m_1, m_2, m_3,$... series*

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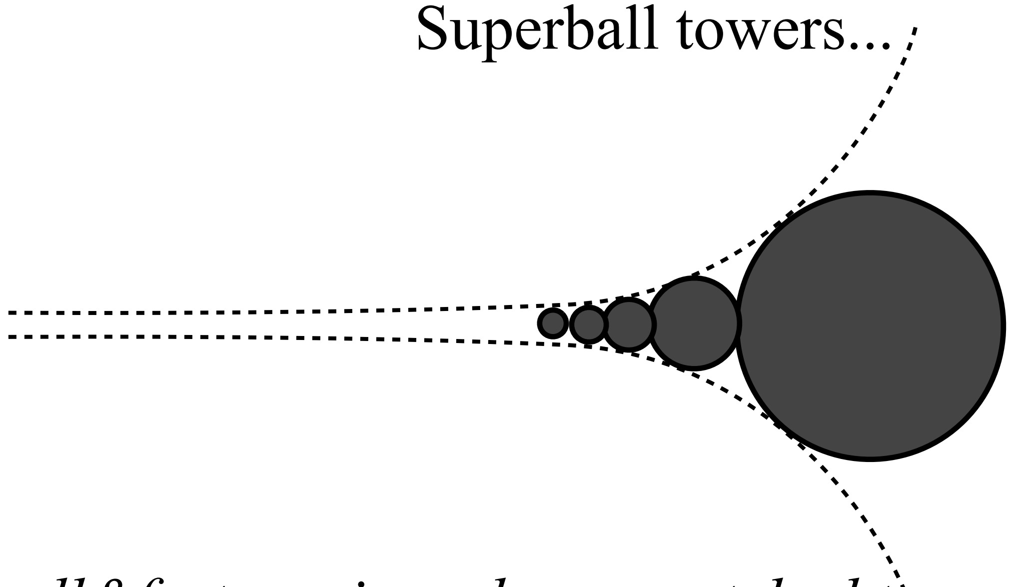
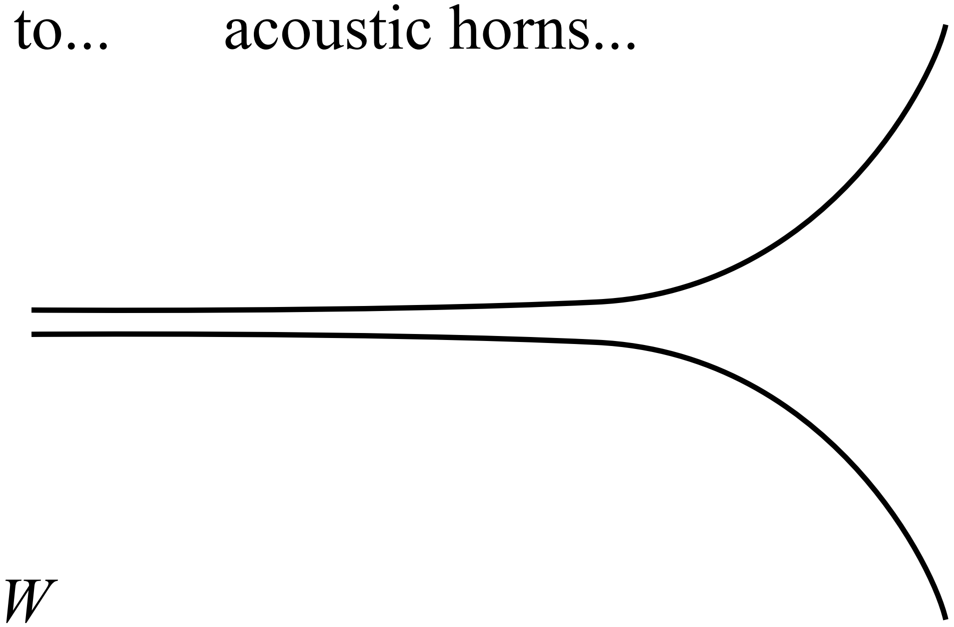
Bouncing columns and Newton's cradle

Inelastic examples: "Zig-zag geometry" of freeway crashes

*Super-elastic examples: This really **is** "Rocket-Science"*

Superball towers... analogous to... acoustic horns...





small&fast... impedance matched to... BIG&SLOW

[6] J. B. Hart and R. B. Herrmann, Amer. J. Phys. **36**, 46 (1968).

1.7.3 *The optimal idler (An algebra/calculus problem)*

To get highest final v_3 of mass m_3 find optimum mass m_2 in terms of masses m_1 and m_3 that does that.

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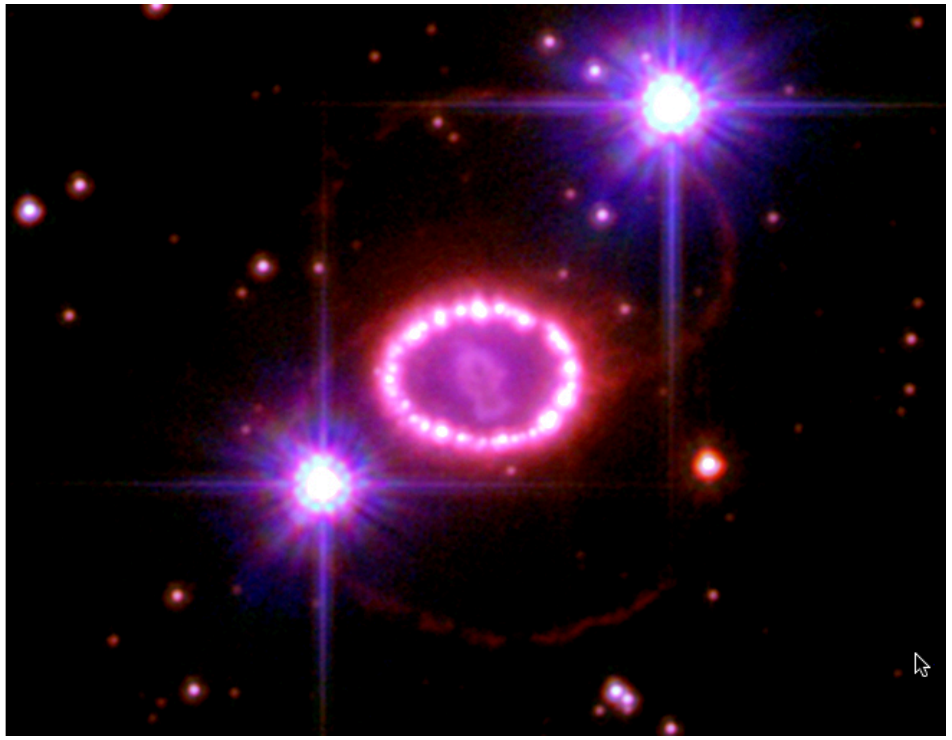
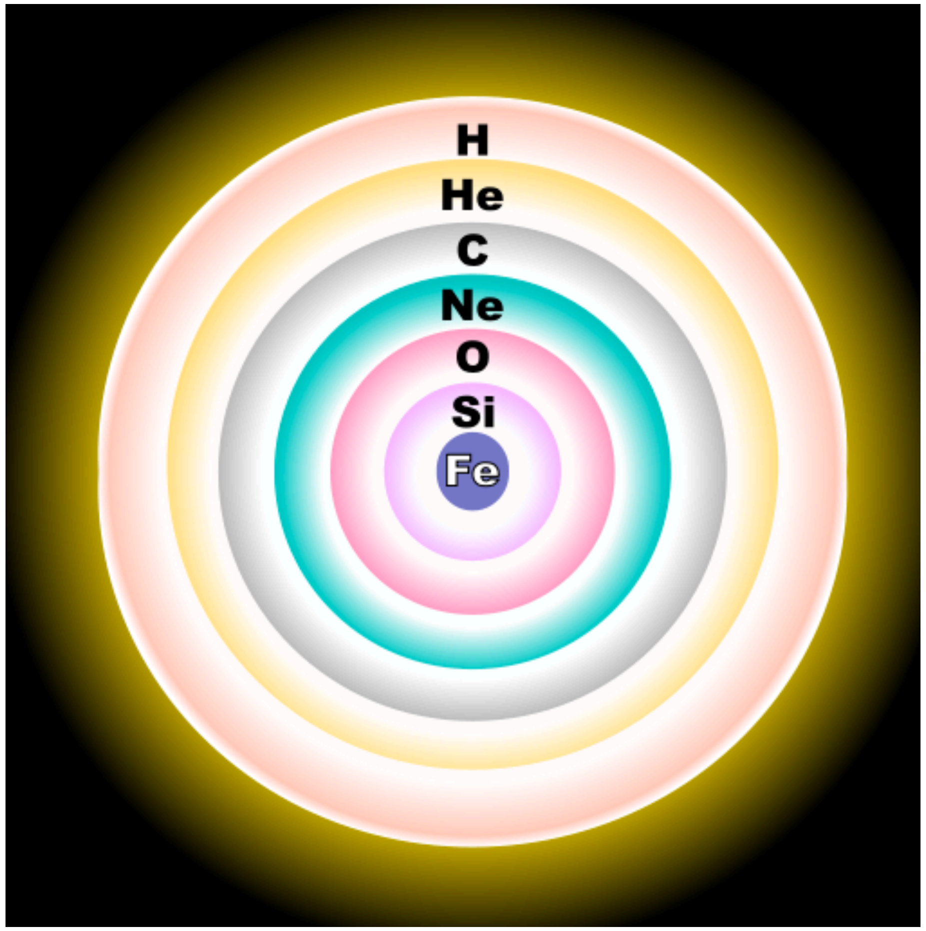


Source

http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

Author

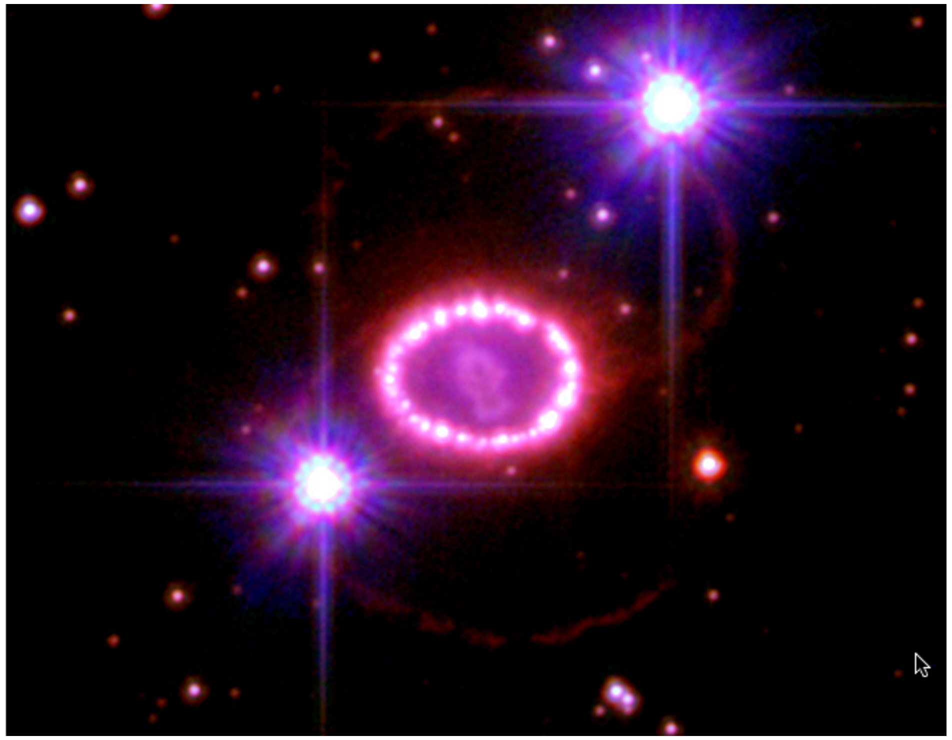
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)



Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 M⊙ star[6]		
			Temperature (Kelvin)	Density (g/cm^3)	Duration
hydrogen burning	hydrogen	helium	$7{\times}10^7$	10	10^7 years
triple-alpha process	helium	carbon, oxygen	$2{\times}10^8$	2000	10^6 years
carbon burning process	carbon	Ne, Na, Mg, Al	$8{\times}10^8$	10^6	10^3 years
neon burning process	neon	O, Mg	$1.6{\times}10^9$	10^7	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	$1.8{\times}10^9$	10^7	0.3 years
silicon burning process	silicon	nickel (decays into iron)	$2.5{\times}10^9$	10^8	5 days

A story of Stirling Colgate (Palmolive) and core-collapse supernovae



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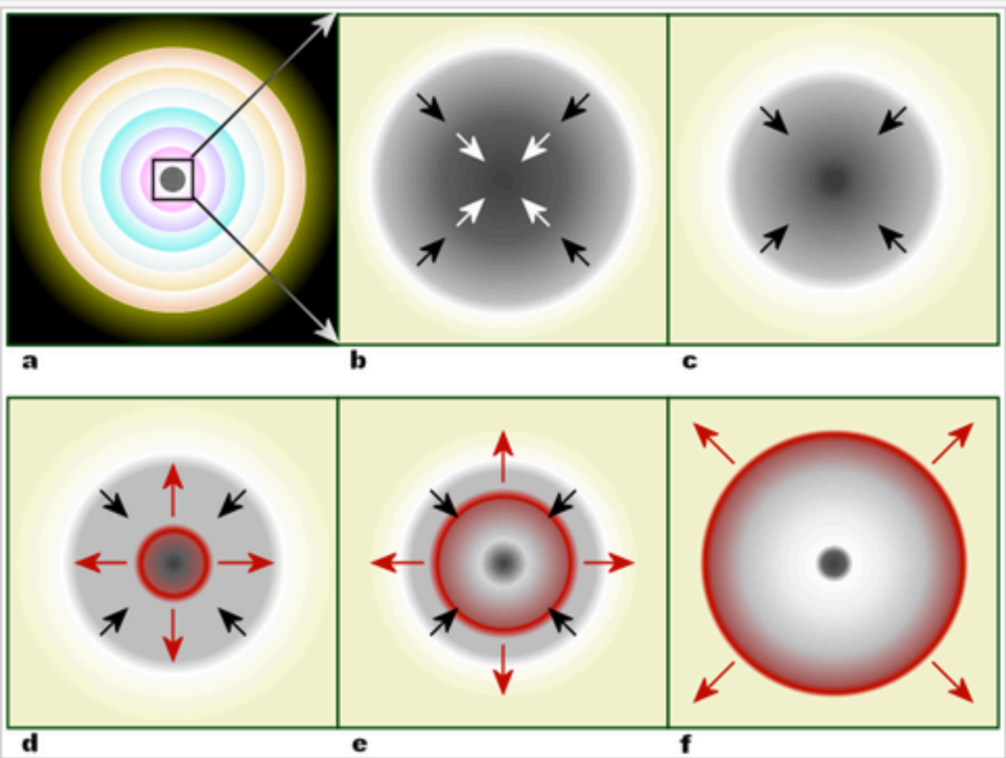
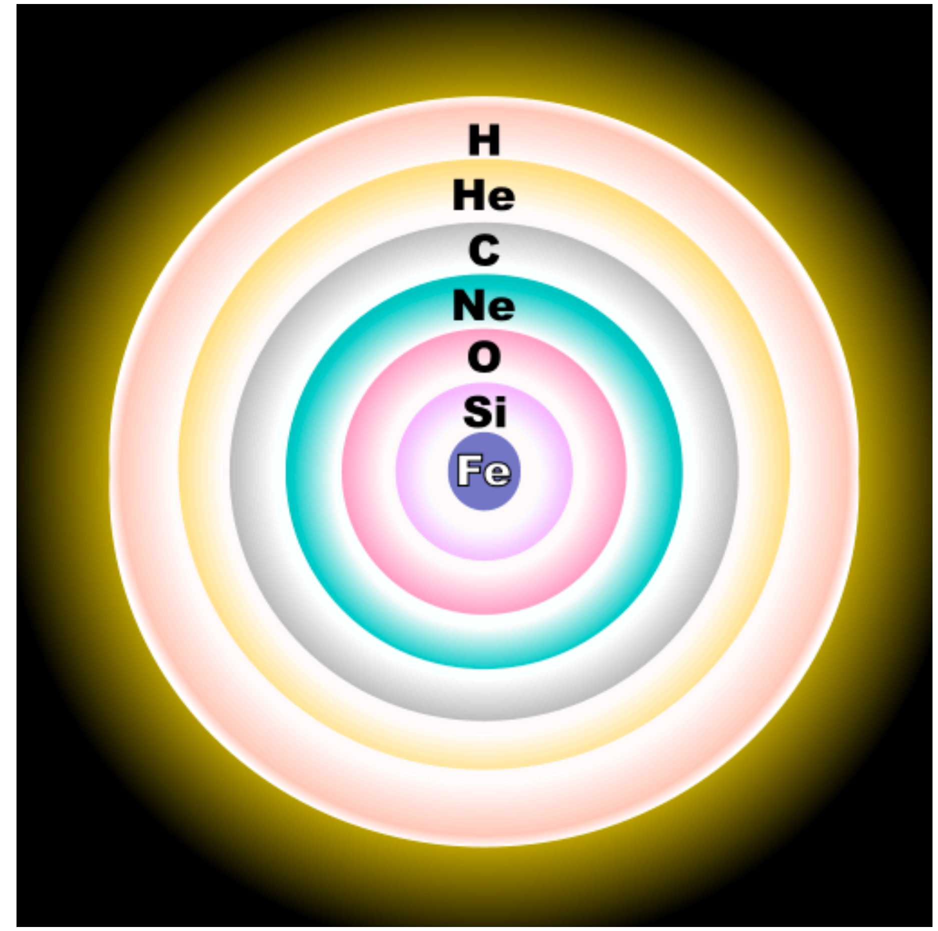


Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.



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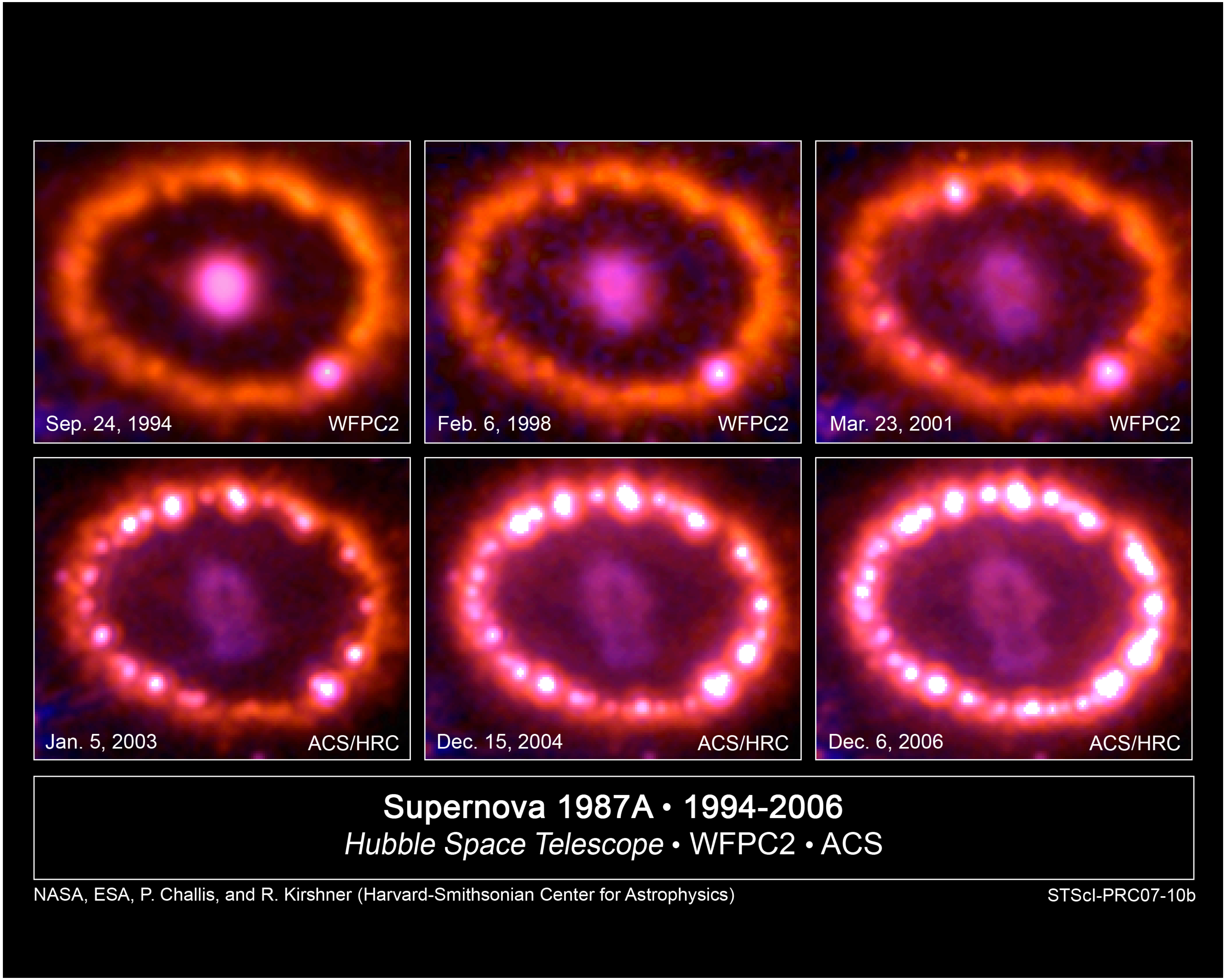
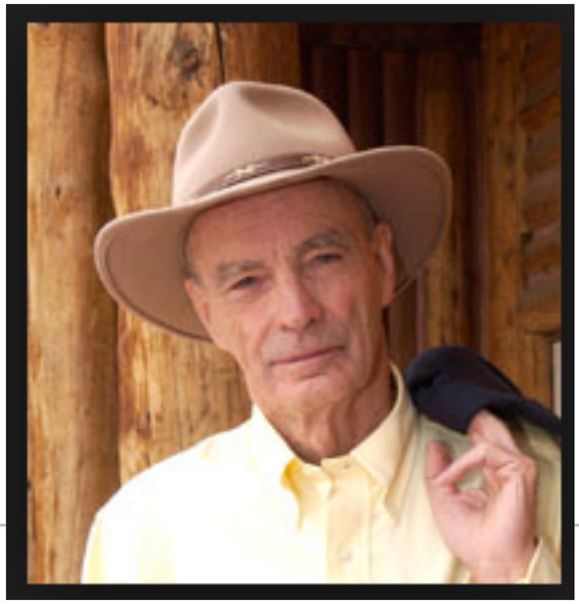

Sep. 24, 1994
WFPC2
Feb. 6, 1998
WFPC2
Mar. 23, 2001
WFPC2
Jan. 5, 2003
ACS/HRC
Dec. 15, 2004
ACS/HRC
Dec. 6, 2006
ACS/HRC
Supernova 1987A • 1994-2006
Hubble Space Telescope • WFPC2 • ACS
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)
STScI-PRC07-10b

Stirling Colgate



From Wikipedia, the free encyclopedia

Stirling Auchincloss Colgate (November 14, 1925 – December 1, 2013) was an American physicist at Los Alamos National Laboratory and a professor emeritus of physics, past president at the New Mexico Institute of Mining and Technology (New Mexico Tech),[1] and an heir to the Colgate toothpaste family fortune.[2] He was America's premier[citation needed] diagnostician of thermonuclear weapons during the early years at the Lawrence Livermore National Laboratory in California. While much of his involvement with physics is still highly classified, he made many contributions in the open literature including physics education and astrophysics.[3] He was born in New York City in 1925, to Henry Auchincloss and Jeanette Thurber (née Pruyn) Colgate.[4]

..an amusing off-color aside
story of Stirling Colgate's NMIMT resignation...

(Not told in Wikipedia!)

Quote

- "*I was always enamored with explosives, and eventually I graduated to dynamite and then nuclear bombs*."

Multiple-collision accelerator assembly

US 5256071 A

ABSTRACT

A device comprising several highly elastic objects is presented whose purpose is to demonstrate an unobvious consequence of fundamental laws of physics--the acceleration of an object to high speed by multiple collisions among a series of heavier objects moving at slower speed. The objects, each of different mass, are arrayed in close proximity in order of decreasing mass with their centers lying along a straight line. This arrangement of the assembly of objects is maintained by a constraining element which permits the assembly axis to be oriented in any desired direction and permits the assembly to be moved or manipulated as a unit in any desired way without destroying the arrangement of objects. In the preferred embodiment the elastic objects are polybutadiene balls (12), the constraining element is an interior guide-pin (10) fastened in the largest ball and extending radially therefrom, on which the remaining balls can slide freely because of diametrical holes formed in them. In use this multiple-collision accelerator assembly is suspended in vertical orientation, with the largest ball downward, by holding the tip-end of the guide-pin which extends beyond the littlest ball. The assembly is then dropped onto a solid surface (14), the striking of which produces a sharp impulse that is transmitted from the largest ball, through the assembly, causing the littlest ball to be projected to a height many times that from which the assembly was dropped.

Publication number	US5256071 A
Publication type	Grant
Application number	US 07/748,804
Publication date	Oct 26, 1993
Filing date	Aug 22, 1991
Priority date ⓘ	Aug 22, 1991
Fee status ⓘ	Paid
Inventors	Edward W. Hones, William G. Hones, Stirling A. Colgate
Original Assignee	Hones Edward W, Hones William G, Colgate Stirling A
Export Citation	BiBTeX, EndNote, RefMan

Patent Citations (3), Referenced by (4), Classifications (7), Legal Events (7)

External Links: USPTO, USPTO Assignment, Espacenet

(Point allowing patent over previous 1973 proposal (4))

1st publication describing theory and experiment of this device 20 years before.

Velocity Amplification in Collision Experiments Involving Superballs

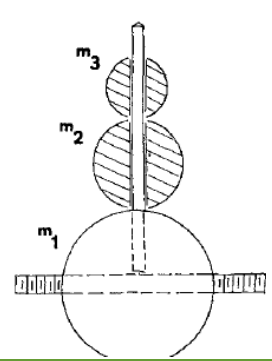
William G. Harter[1] (class of WGH)

— HIDE AFFILIATIONS

[1] University of Southern California, Los Angeles, California 90007

View the Scitation page for University of Southern California (USC).

Am. J. Phys. **39**, 656 (1971); http://dx.doi.org/10.1119/1.1986253



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Potential energy dynamics of Superballs and related things

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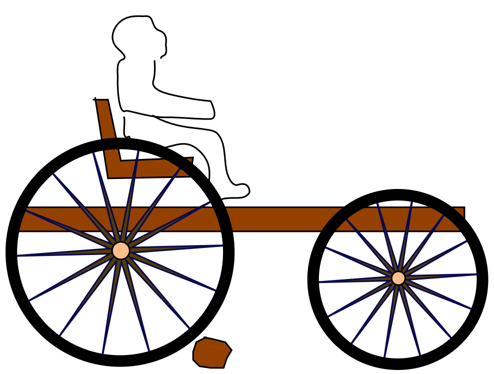
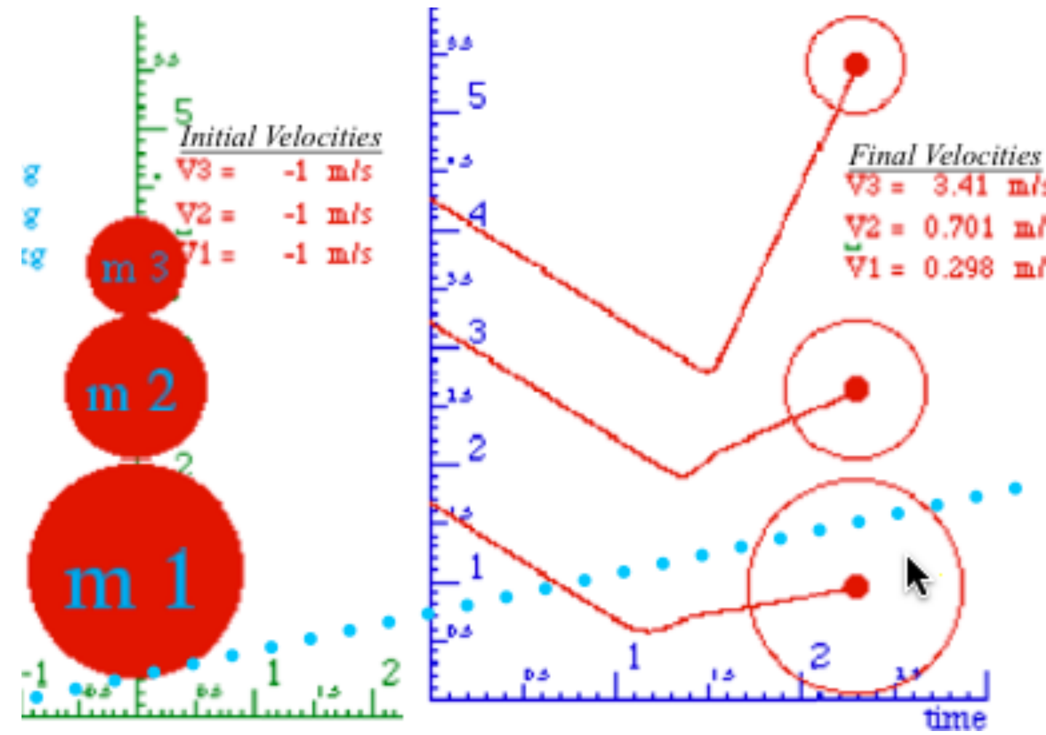
Inelastic examples: "Zig-zag geometry" of freeway crashes

Super-elastic examples: This really ***is*** *"Rocket-Science"*

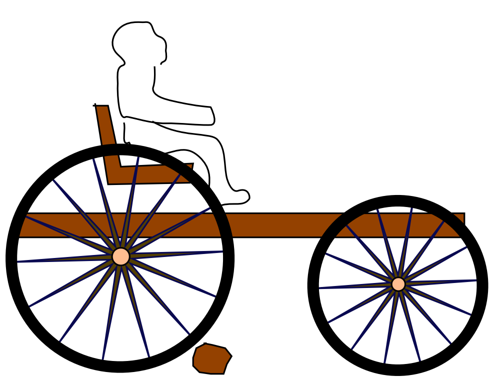
Western buckboard $=$ *?????*

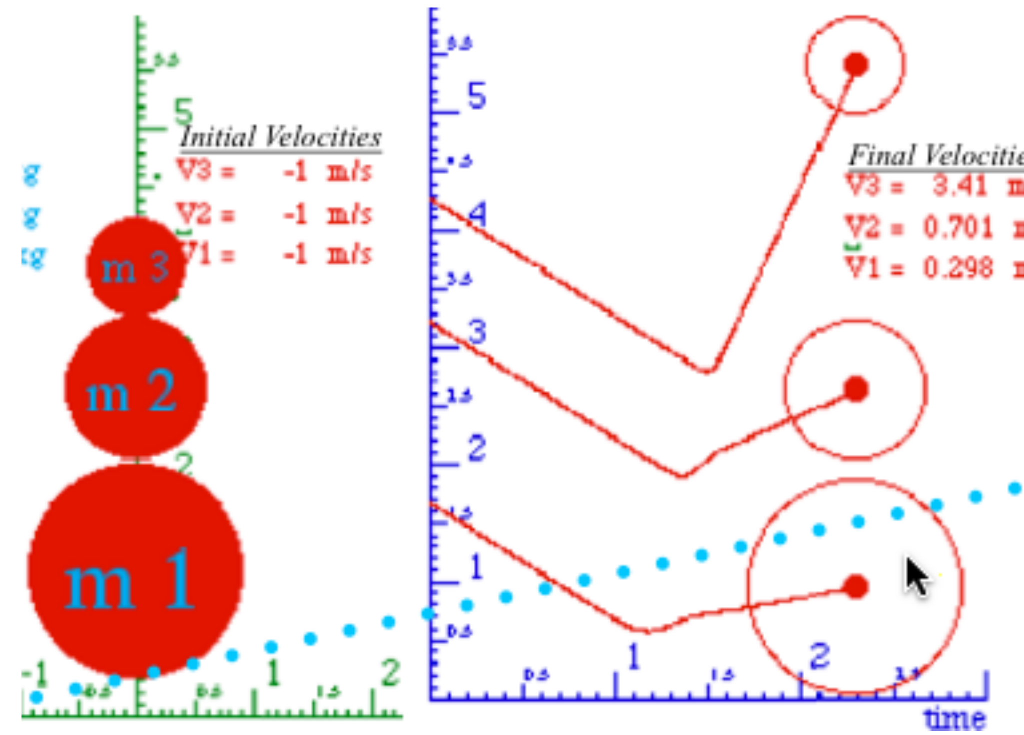
Western buckboard = ?????



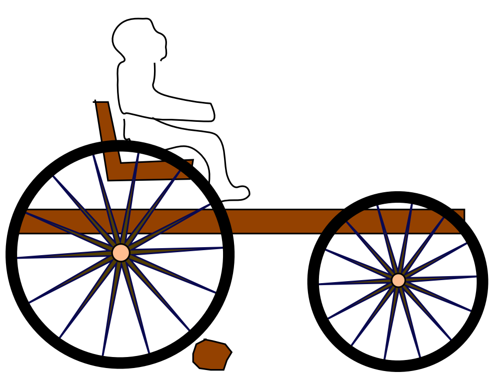


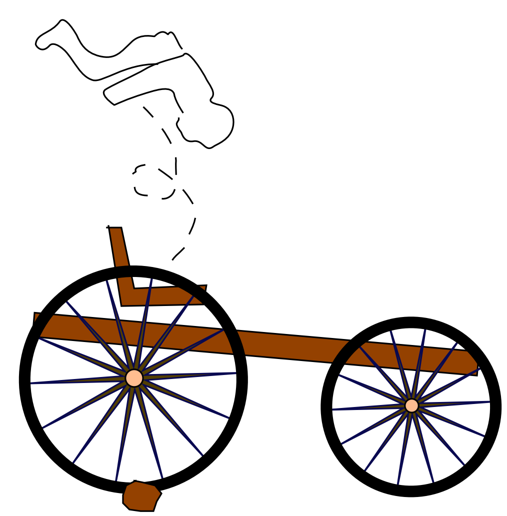
Western buckboard = *3-ball analogy*





Western buckboard = *3-ball analogy* *Disaster!*





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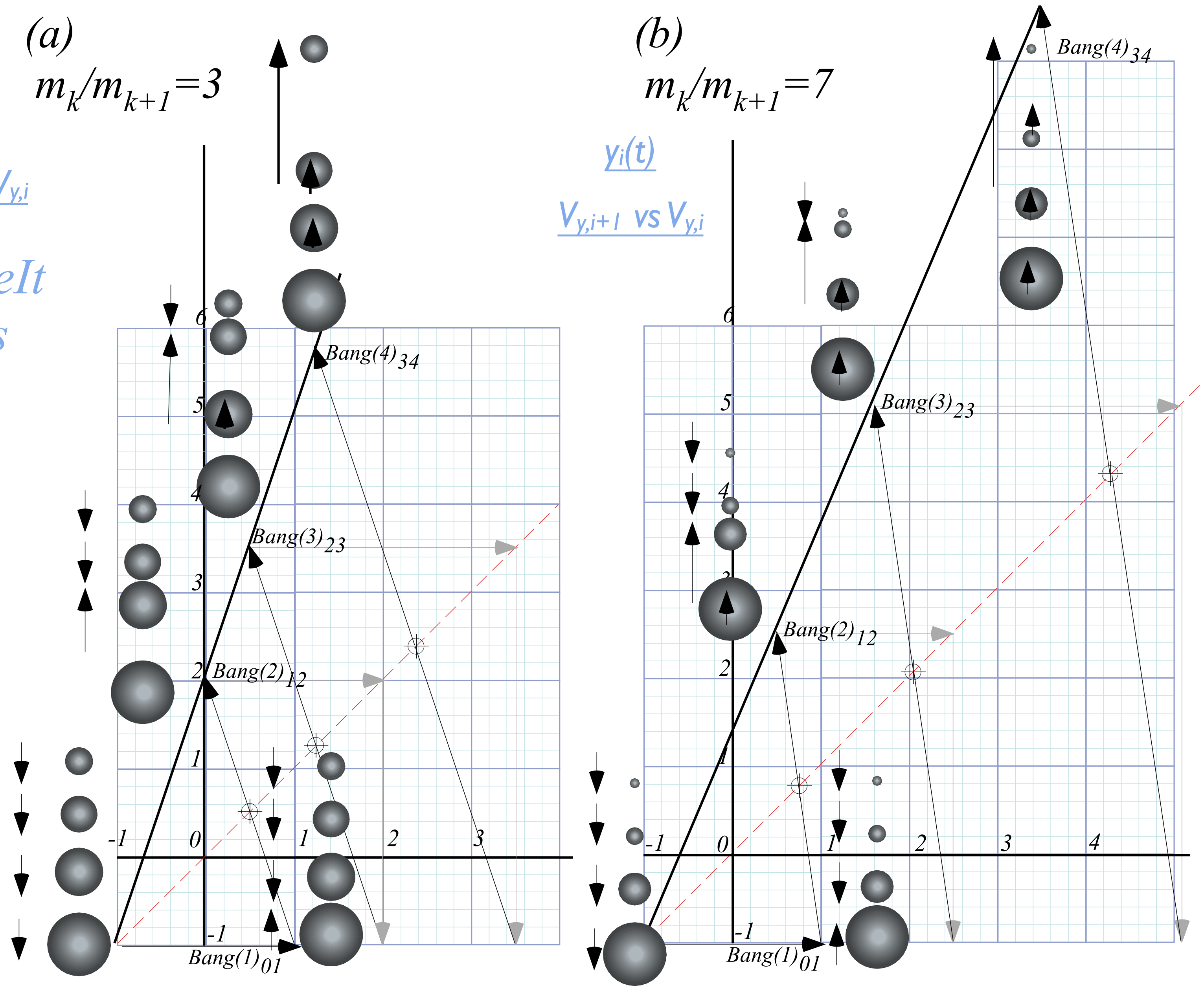
Unit 1
Fig. 7.2a-b
4-Body IBM Geometry
Fig. 7.2c-d
4-Equal-Body Geometry

[$y_i(t)$]

[$V_{y,i+1}$ vs $V_{y,i}$]

BounceIt Links



[$y_i(t)$]

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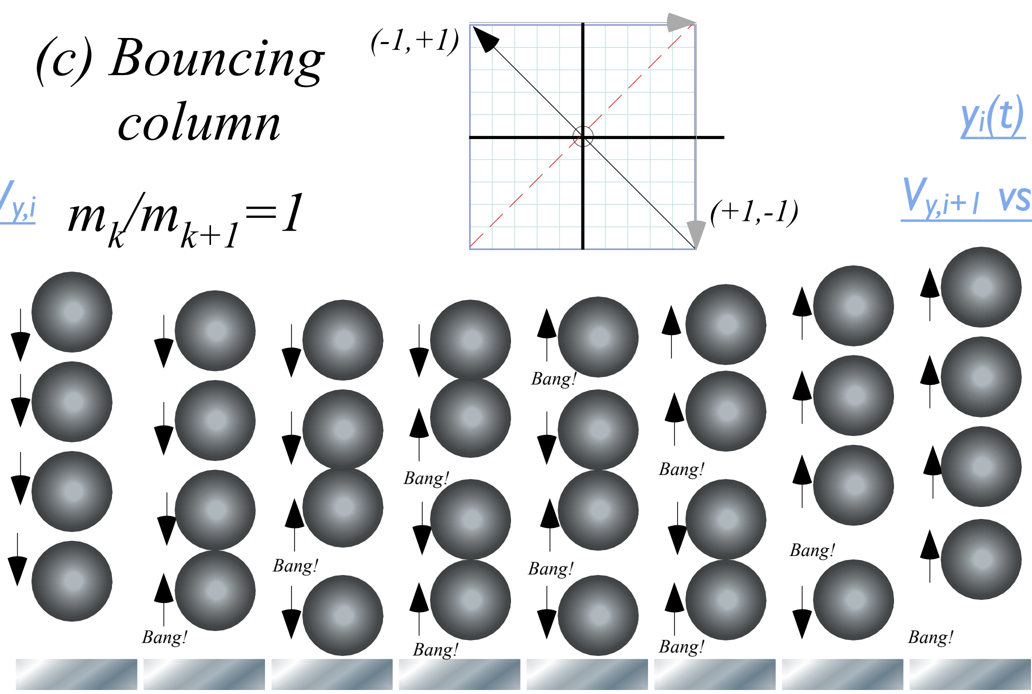

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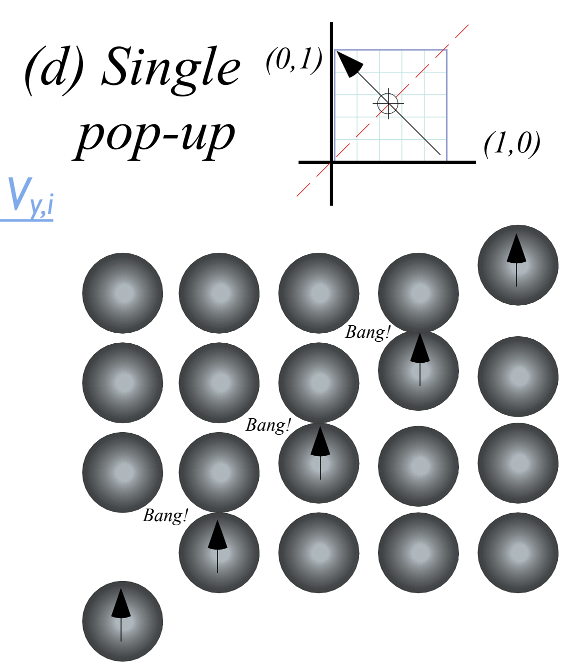

4-Equal-Body "Shockwave" or pulse wave Dynamics

Opposite of continuous wave dynamics introduced in Unit 2 or Lect. 6-9

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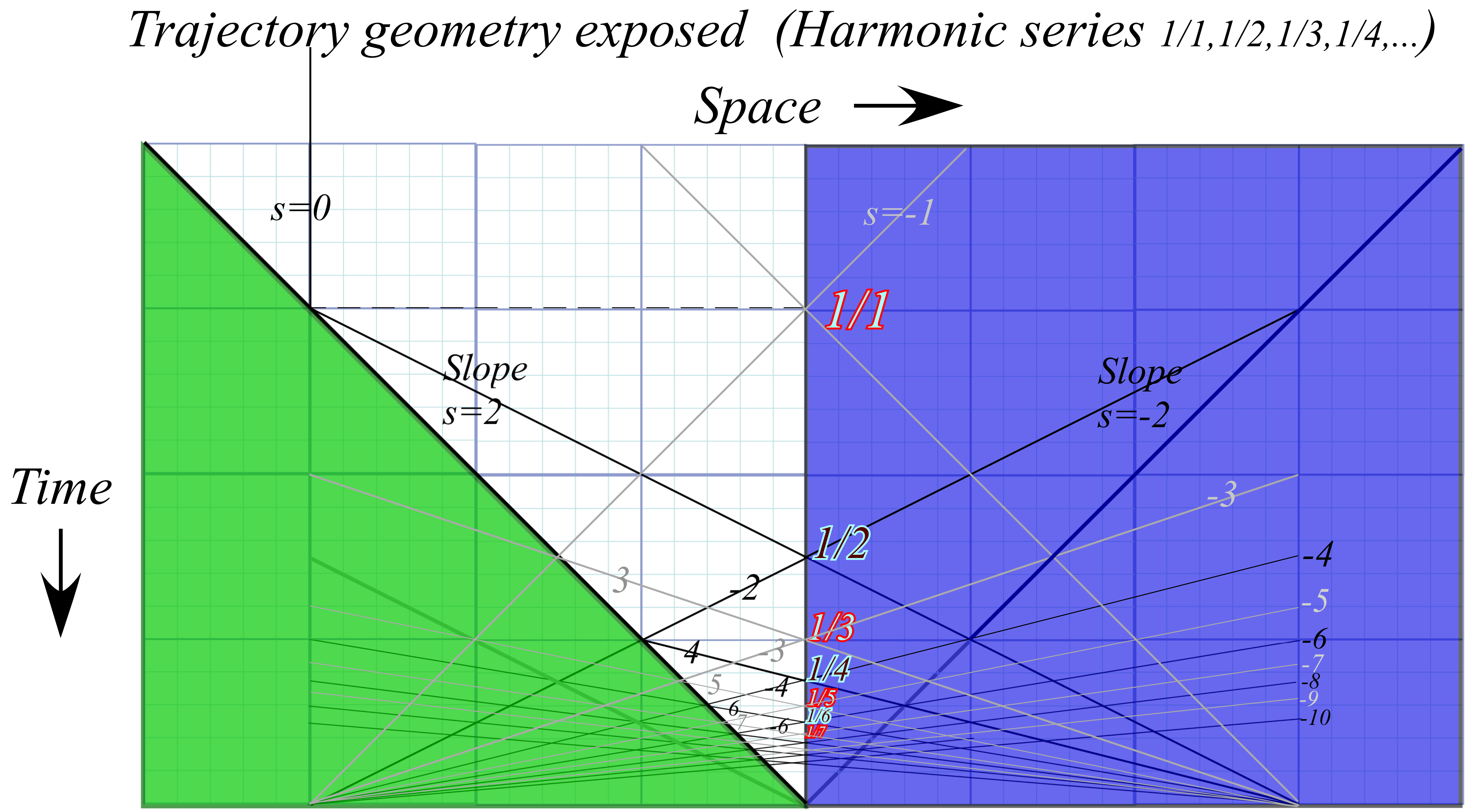
Inelastic examples: "Zig-zag geometry" of freeway crashes

First recall "zig-zag" fractions of "Monster Mash" in Lect. 4-5



Unit 1

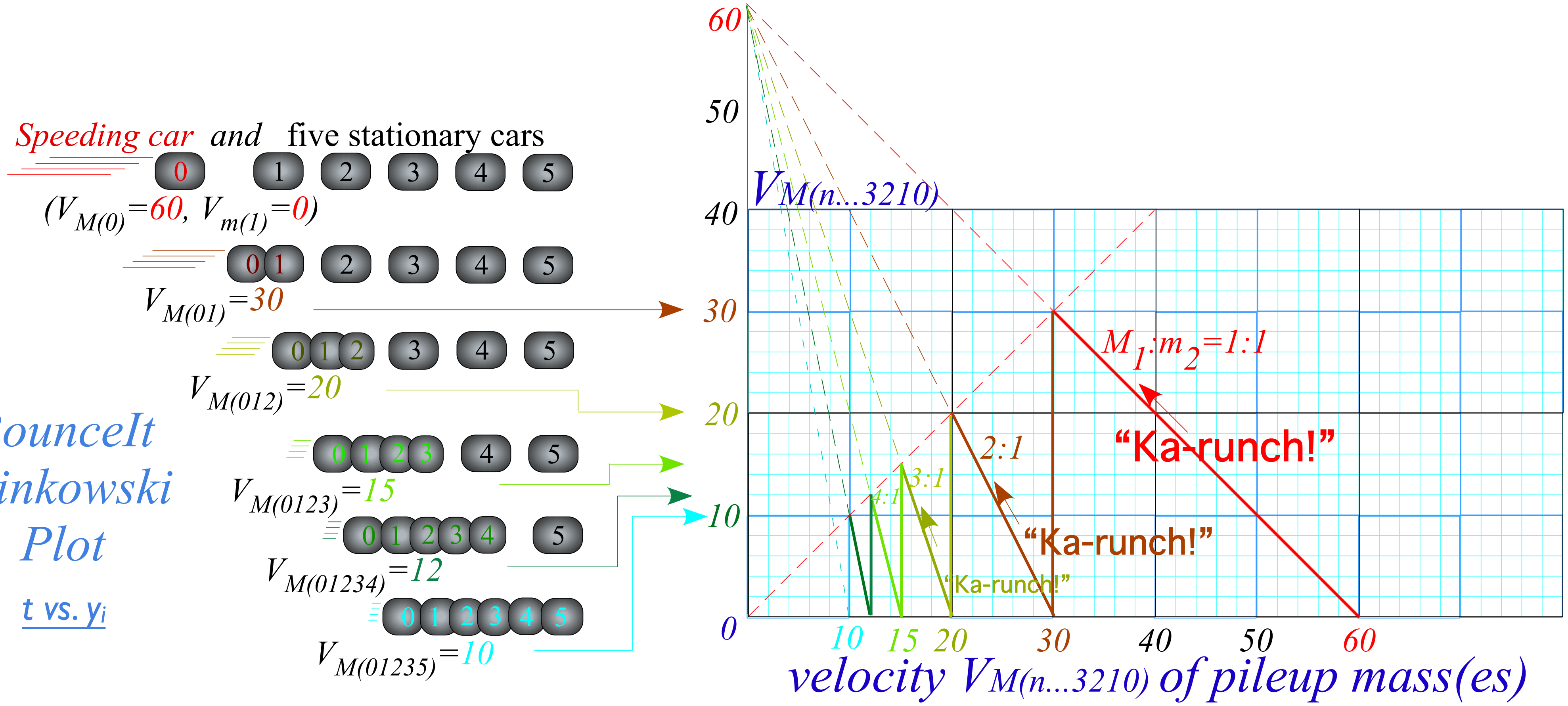


Fig. 7.5

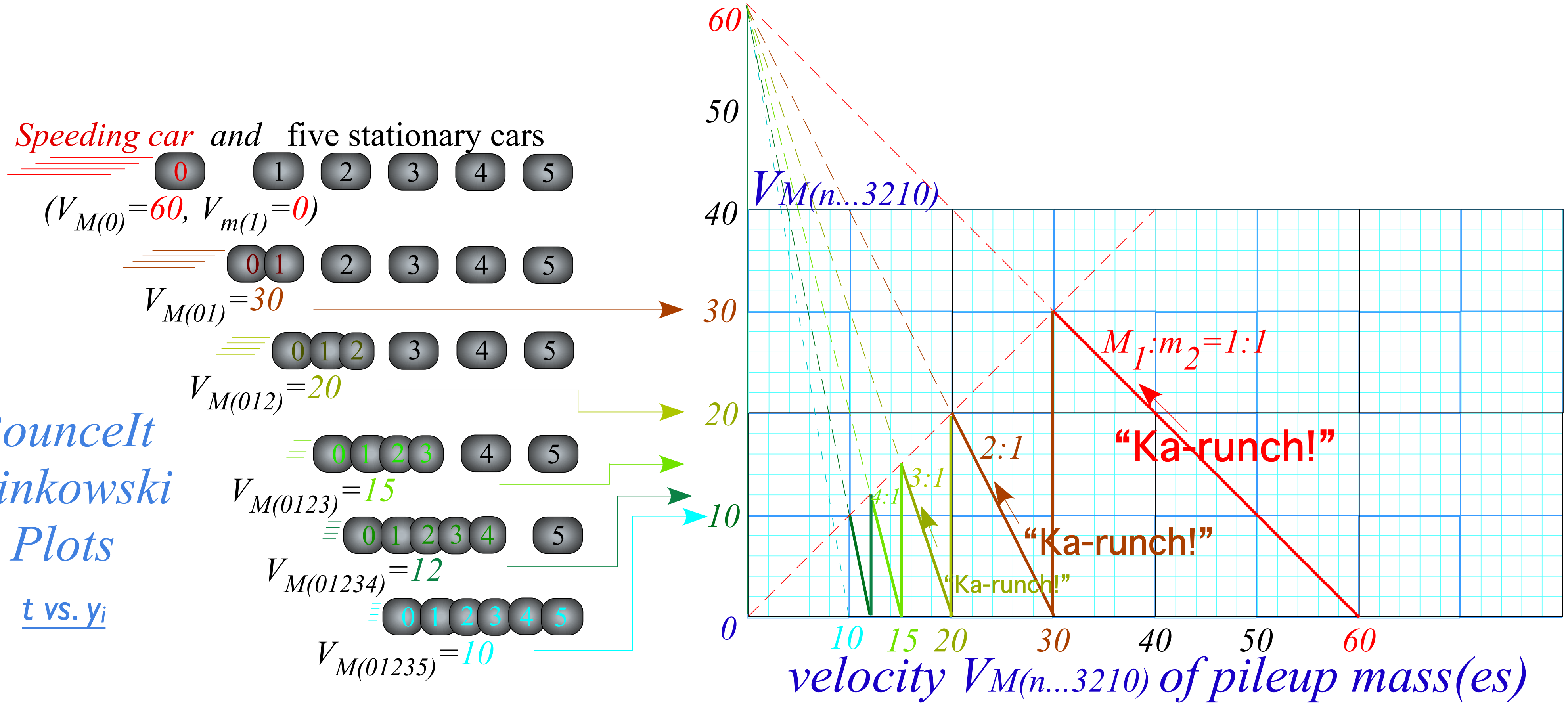
Pile-up:
One *60mph* car
hits
five standing cars

BounceIt Minkowski Plot

t vs. y_i

BounceIt Minkowski Plots

t vs. y_i



Unit 1

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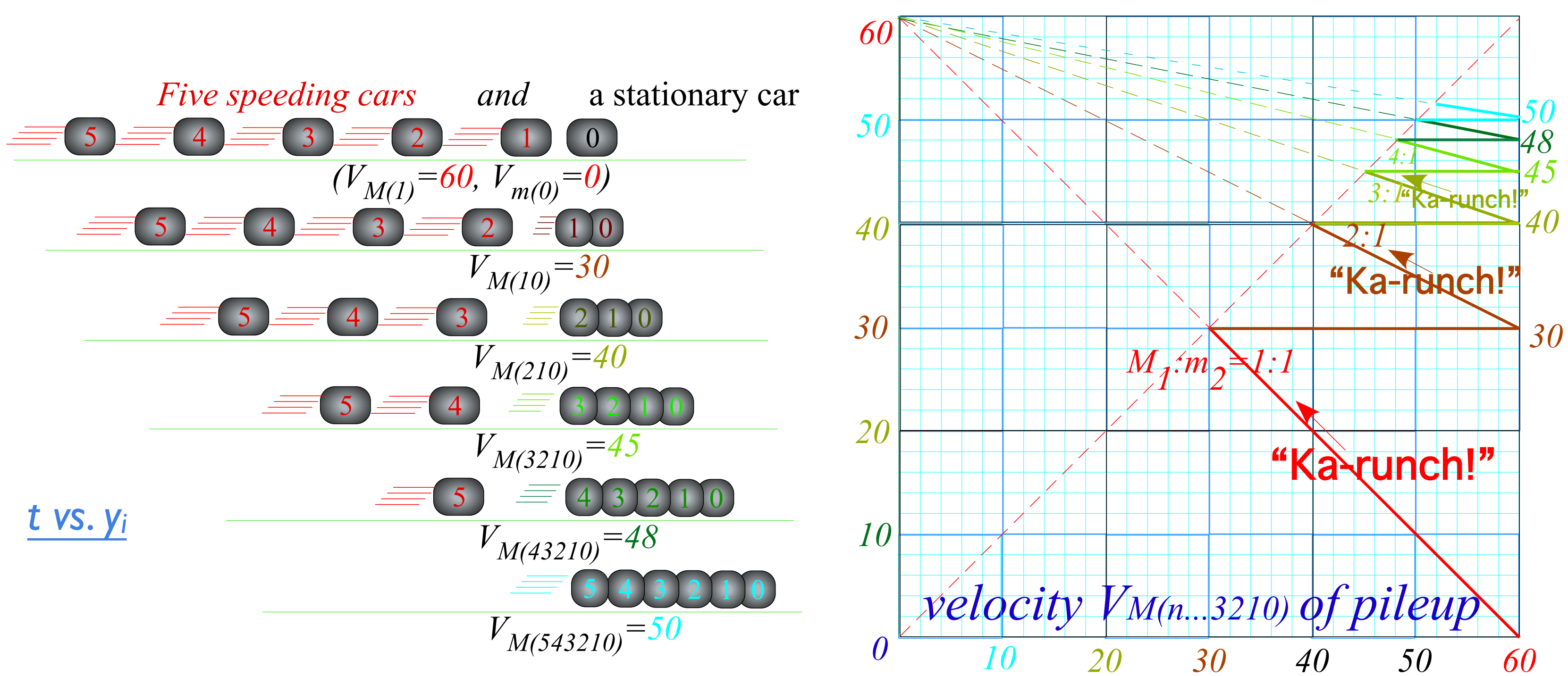
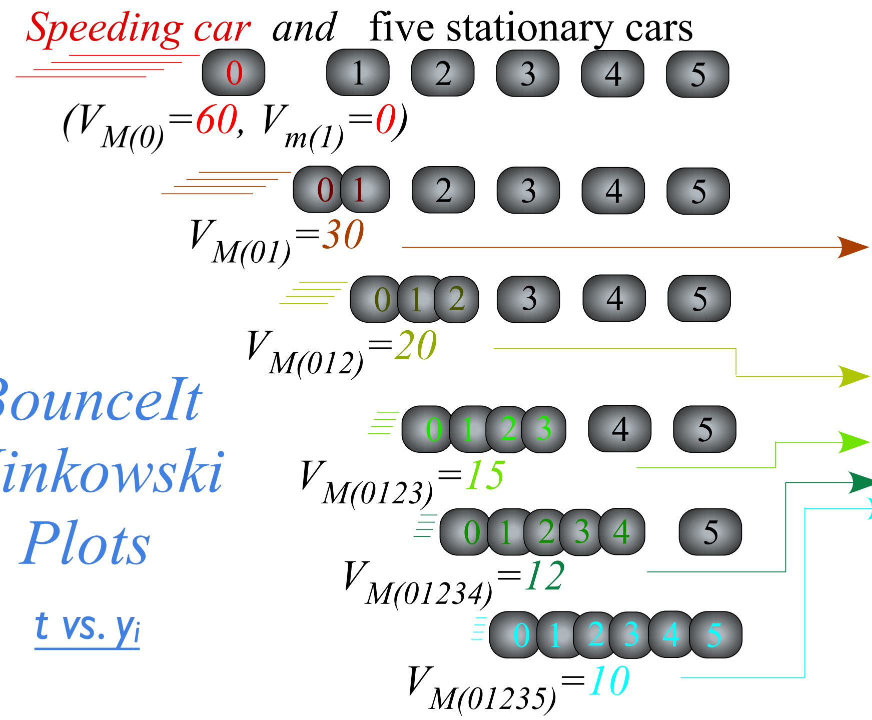


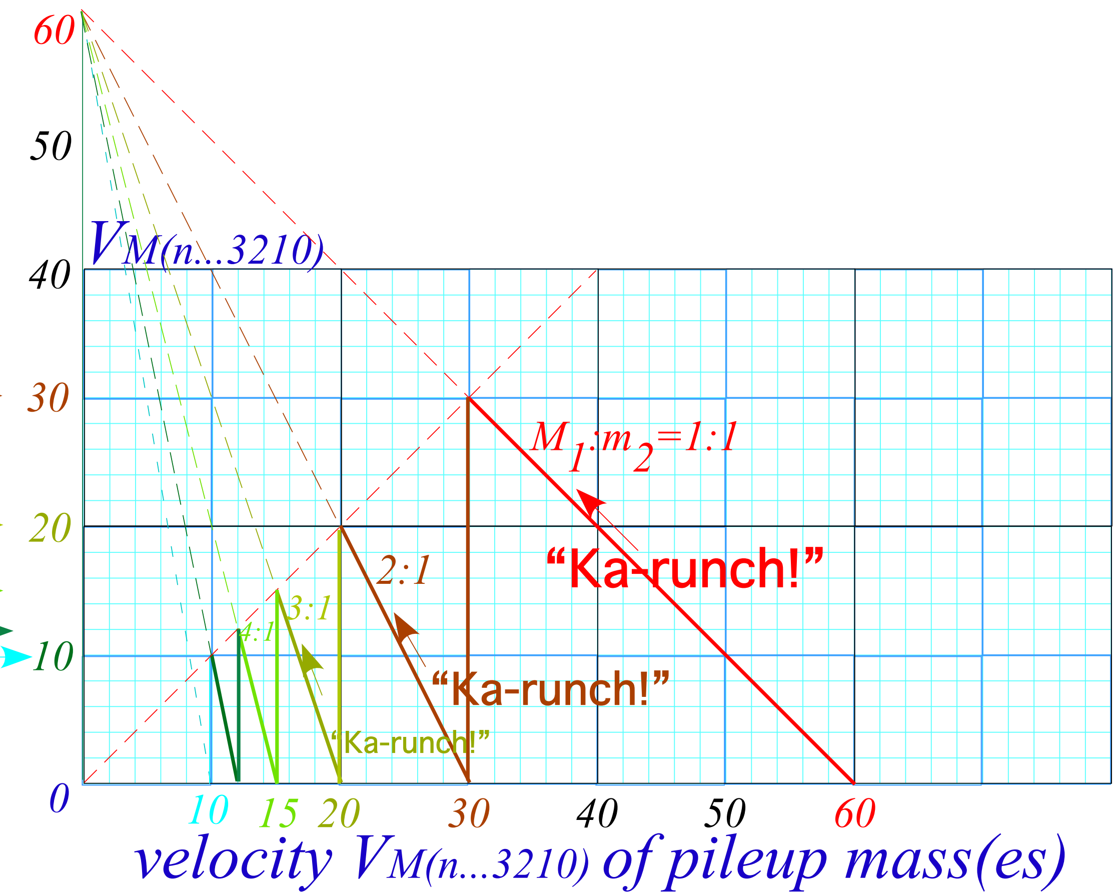
Fig. 7.6

Pile-up:
Five *60mph* cars
hit
one standing cars

Unit 1

BounceIt Minkowski Plots





t vs. y_i

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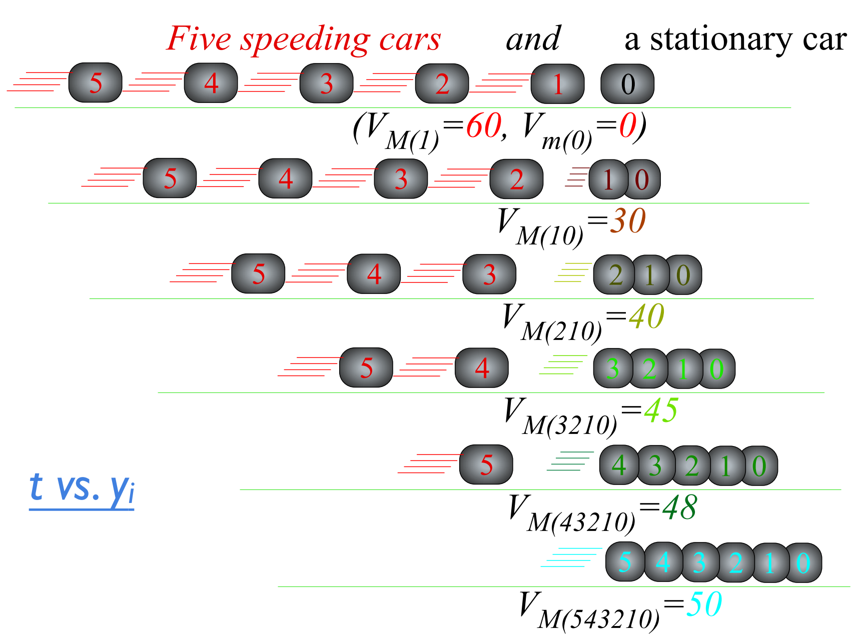
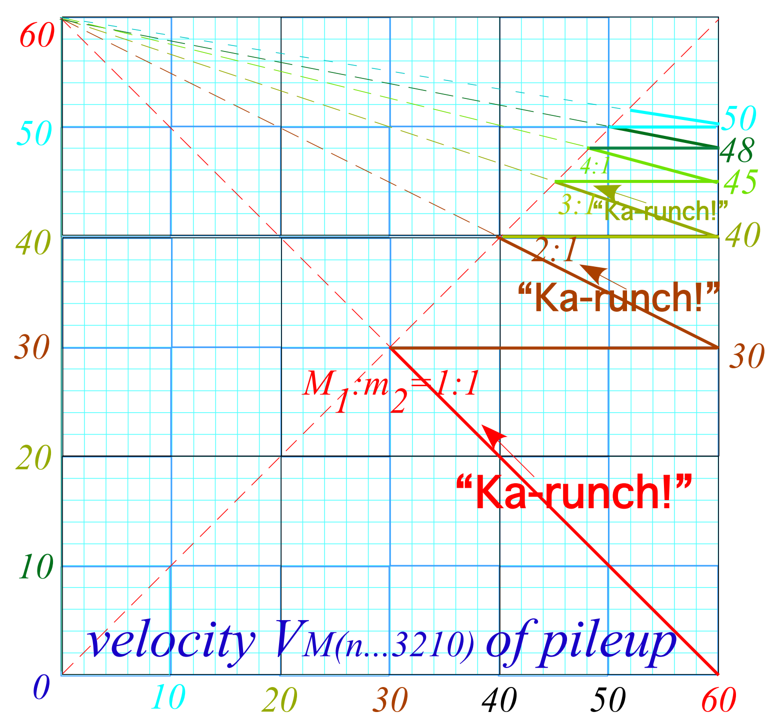
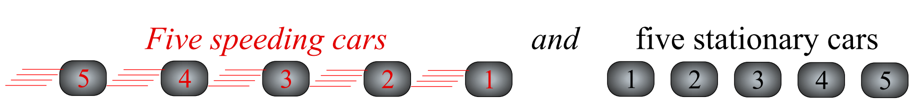




t vs. y_i

Fig. 7.6

Pile-up:
Five *60mph* cars
hit
one standing cars



t vs. y_i

(Fug-gedda-aboud-dit!!)

(Many possible scenarios depending on initial positions!)

Fig. 7.7

Pile-up:
Five *60mph* cars
hit
five standing cars

BounceIt: 5 into 5 Pile-up

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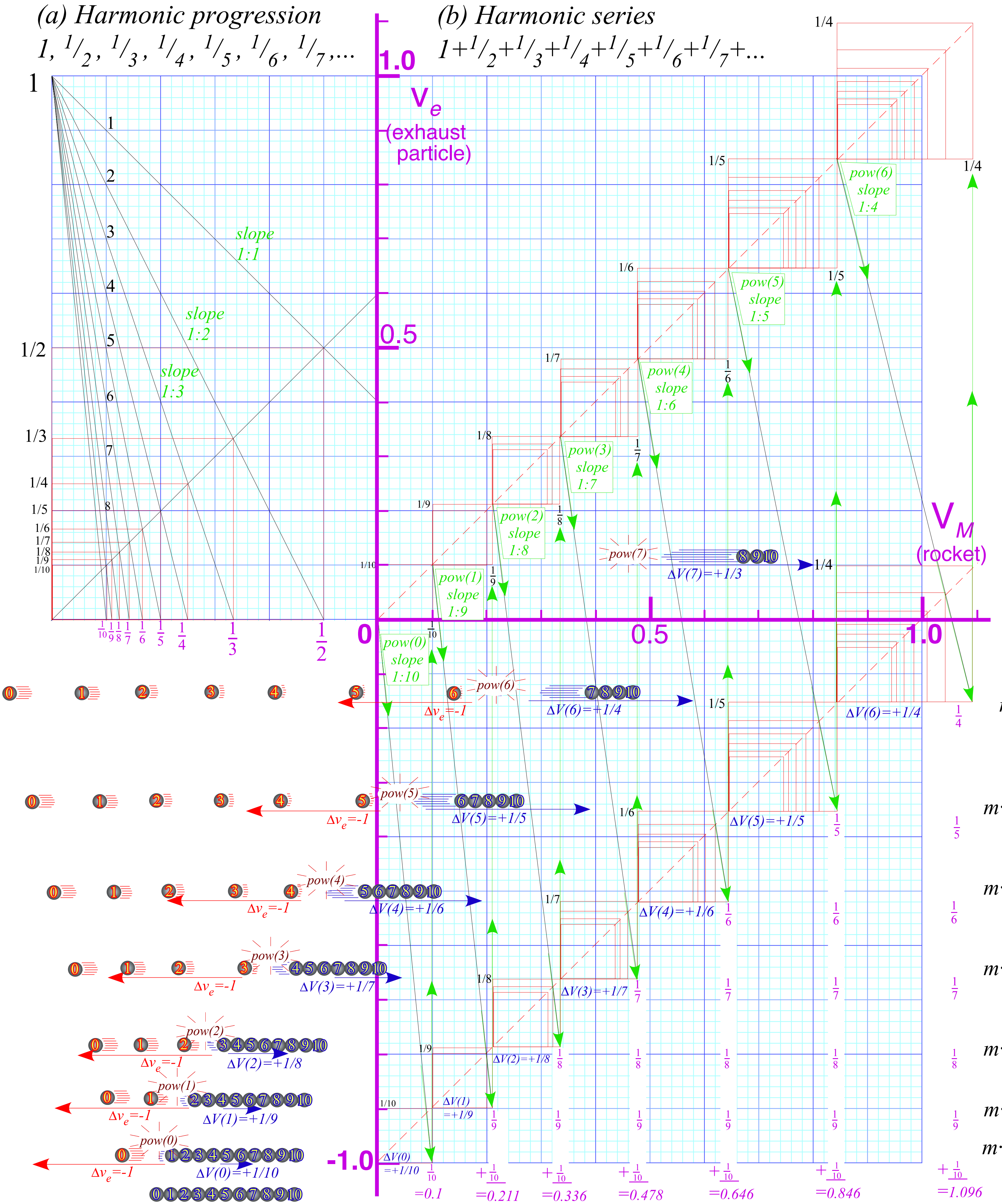


Unit 1
Fig. 7.8a-b

Rocket Science!

$$m\cdot\Delta v_7+3m\cdot\Delta V_M(7)=0$$

$$m\cdot\Delta v_6+4m\cdot\Delta V_M(6)=0$$

$$m\cdot\Delta v_5+5m\cdot\Delta V_M(5)=0$$

$$m\cdot\Delta v_4+6m\cdot\Delta V_M(4)=0$$

$$m\cdot\Delta v_3+7m\cdot\Delta V_M(3)=0$$

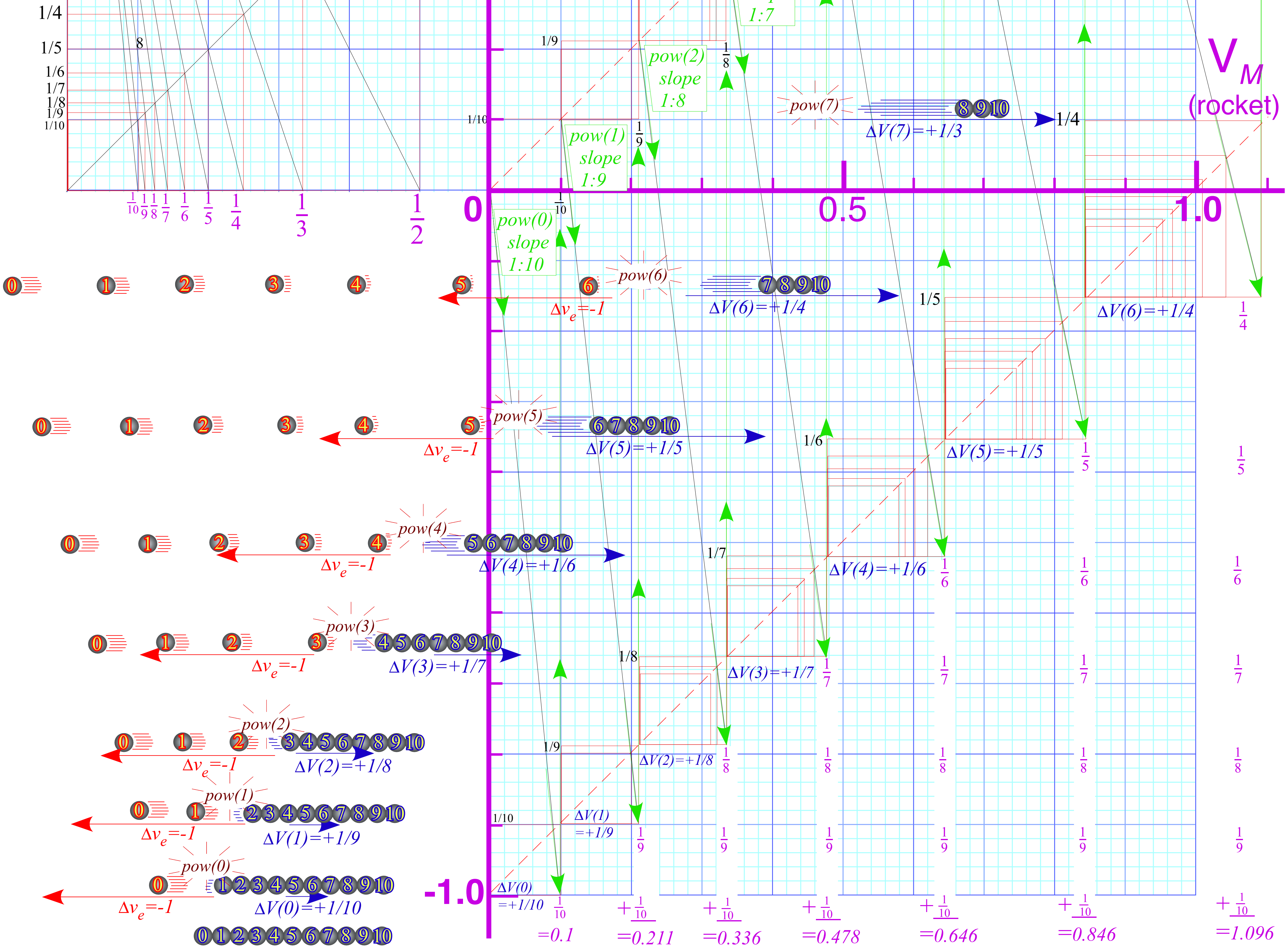
$$m\cdot\Delta v_2+8m\cdot\Delta V_M(2)=0$$

$$m\cdot\Delta v_1+9m\cdot\Delta V_M(1)=0$$

$$m\cdot\Delta v_0+10m\cdot\Delta V_M(0)=0$$

Unit 1
Fig. 7.8a-b

Rocket Science!



0^{th}: $V(0)=1/10=0.1$

1^{st}: $V(1)=1/10+1/9=0.211$

2^{nd}: $V(2)=1/10+1/9+1/8=0.336$

3^{rd}: $V(3)=V(2)+1/7=0.478$

4^{th}: $V(4)=V(3)+1/6=0.646$

5^{th}: $V(5)=V(4)+1/5=0.846$

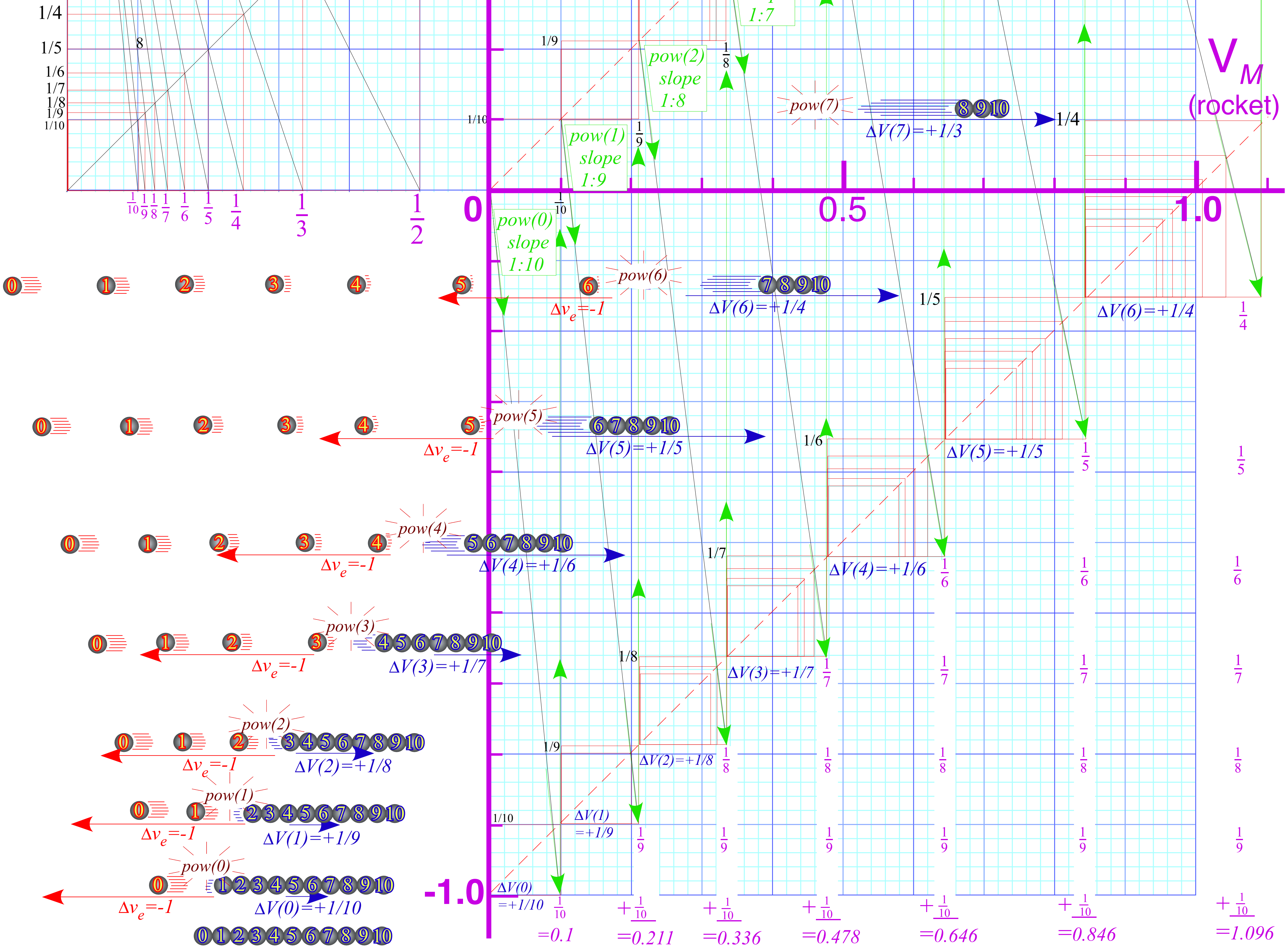
6^{th}: $V(6)=V(5)+1/4=1.096$

7^{th}: $V(7)= V(6)+1/3=1.429$

8^{th}: $V(8)=V(7)+1/2=1.929$

Unit 1
Fig. 7.8a-b

Rocket Science!



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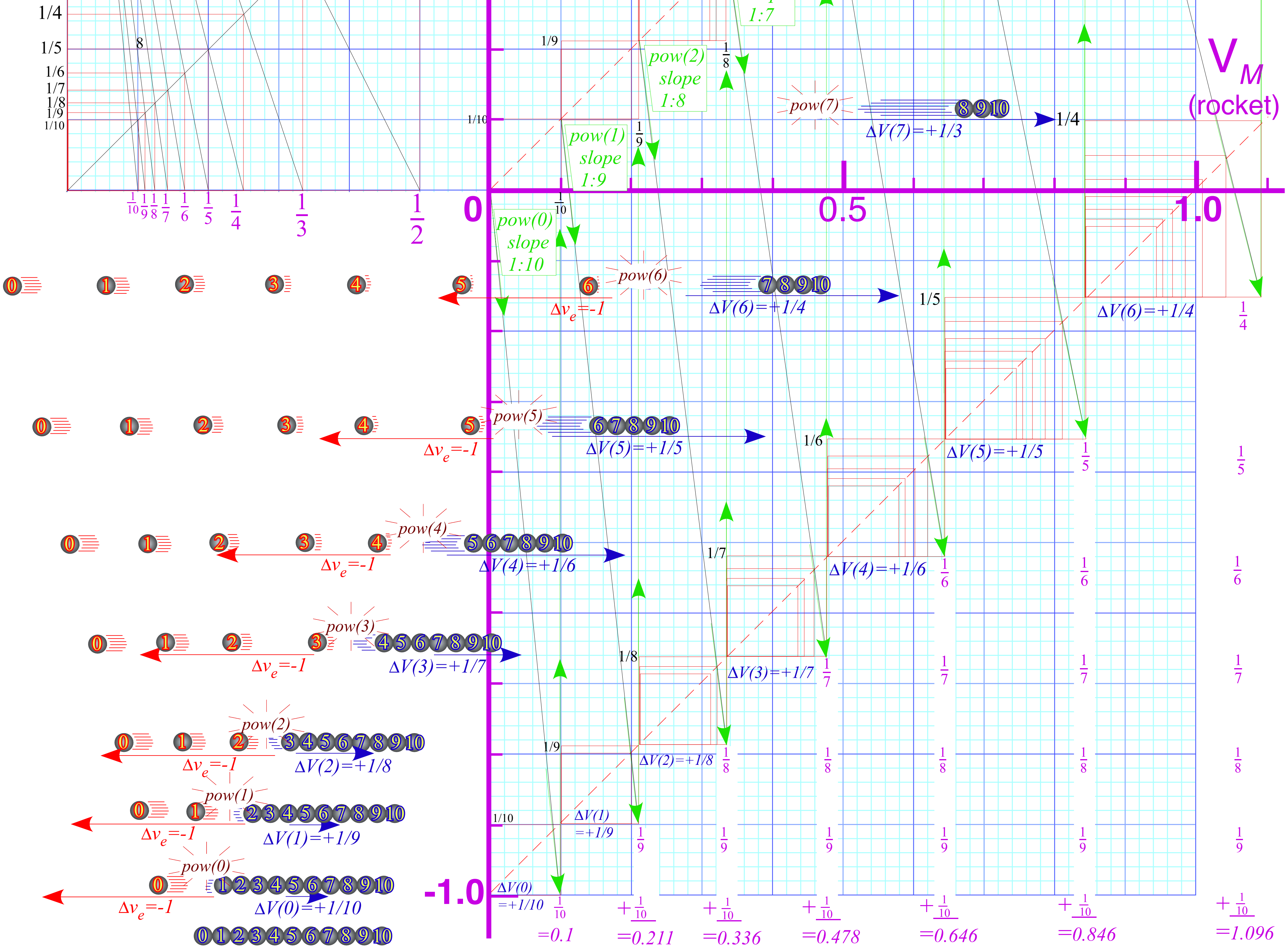
6^{th}: $V(6)=V(5)+1/4=1.096$ 7^{th}: $V(7)= V(6)+1/3=1.429$ 8^{th}: $V(8)=V(7)+1/2=1.929$

v_e known as "Specific Impulse"

By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\dfrac{dM}{M}$

Unit 1
Fig. 7.8a-b

Rocket Science!



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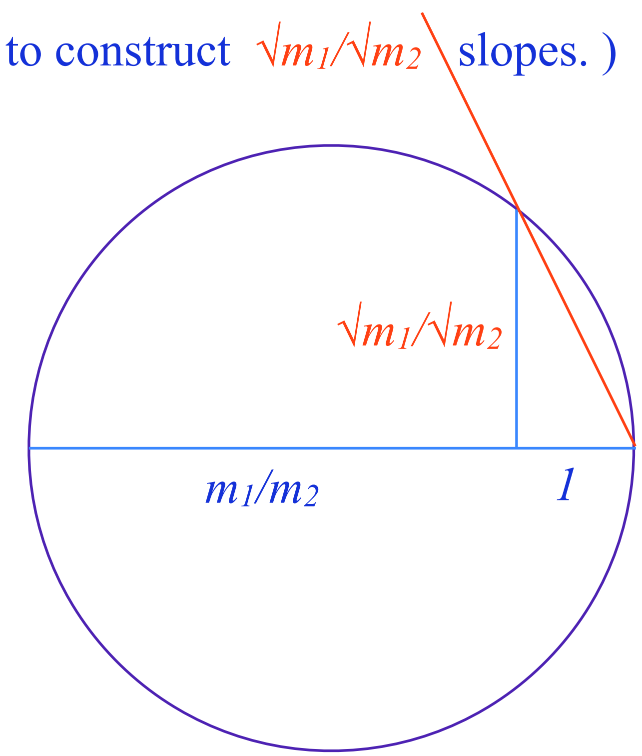
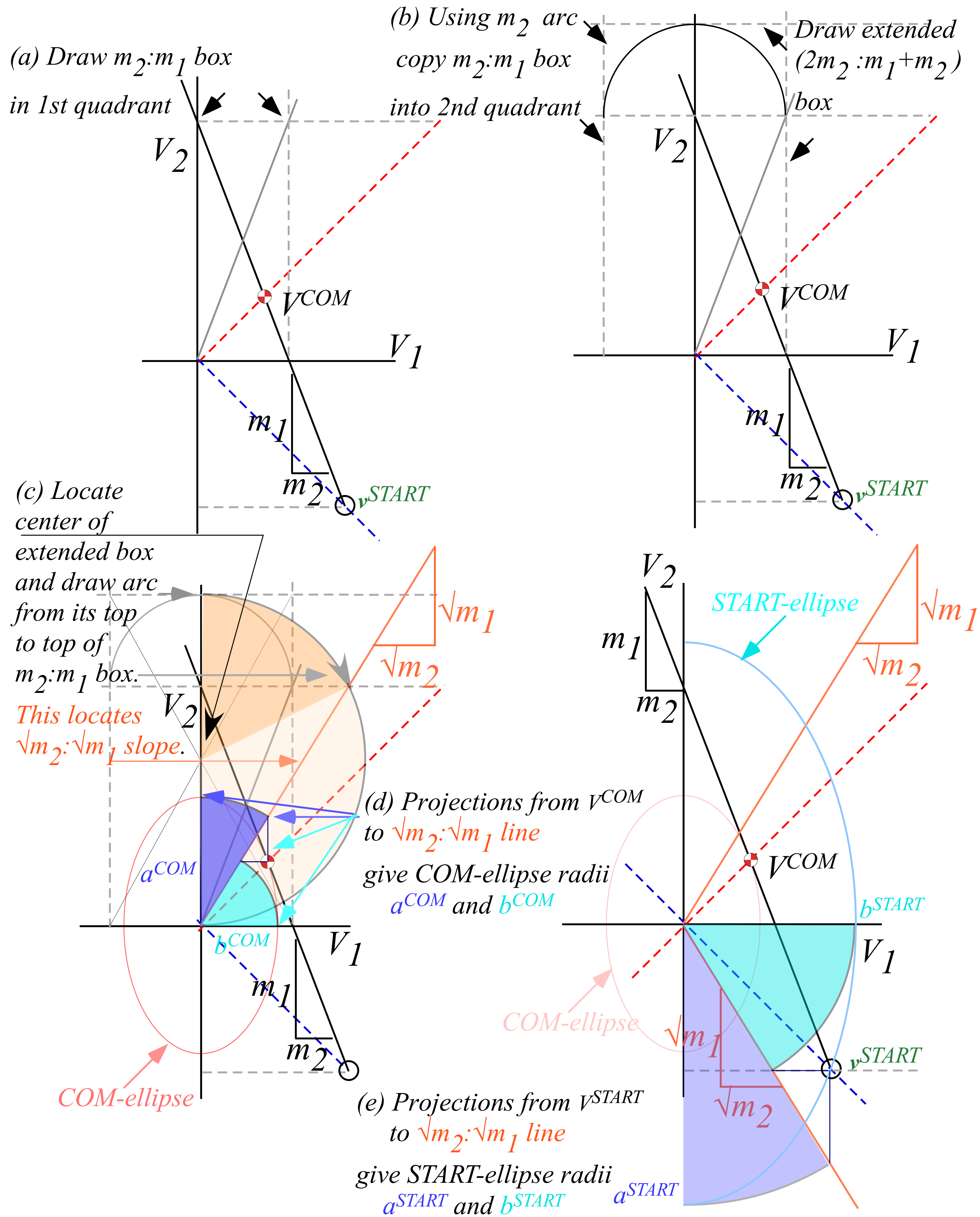
The Rocket Equation: $V_{FIN}-V_{IN}=-v_e\left[\ln M_{FIN}-\ln M_{IN}\right]=v_e\left[\ln\dfrac{M_{IN}}{M_{FIN}}\right]$

A Thales construction for momentum-energy

(Made obsolete by Estrangian scaling to circular (V_1,V_2) plots. Still, one has to construct $\sqrt{m_1}/\sqrt{m_2}$ slopes.)





Unit 1
Fig. 7.4a-d

This is a detailed construction of the energy ellipse in a Largangian (v_1,v_2) plot given the initial (v_1,v_2).

The Estrangian (V_1,V_2) plot makes the (v_1,v_2) plot and this construction obsolete.

(Easier to just draw circle through initial (V_1,V_2).)