Lecture 5

Revised 12.21.12 from 9.04.2012

Dynamics of Potentials and Force Fields

(Ch. 7 and Ch. 8 of Unit 1)

Potential energy geometry of Superballs and related things

Thales geometry and "Sagittal approximation"

Geometry and dynamics of single ball bounce

Examples: (a) Constant force (like kidee pool) (b) Linear force (like balloon)

Some physics of dare-devil-divers

Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and dynamics of 2-ball bounce (again with feeling)

The parable of RumpCo. vs CrapCorp.

The story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of 3-ball bounce

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

The story of USC pre-meds visiting Whammo Manufacturing Co.

Other bangings-on: The western buckboard and Newton's balls

Lecture 5 ends here

Crunch energy geometry of freeway crashes and related things

Crunch energy played backwards: This really ***is*** *"Rocket-Science"*

A Thales construction for momentum-energy

Potential energy geometry of Superballs and related things

→ *Thales geometry and "Sagittal approximation"*

Geometry and dynamics of single ball bounce

Examples: (a) Constant force (like kidee pool) (b) Linear force (like balloon)

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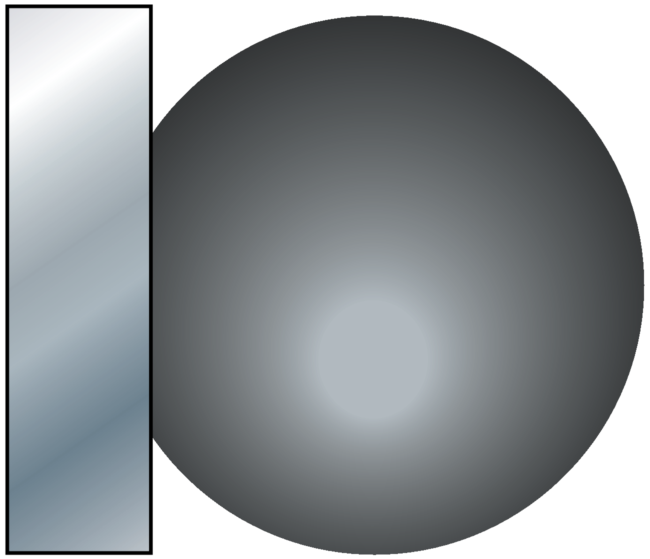
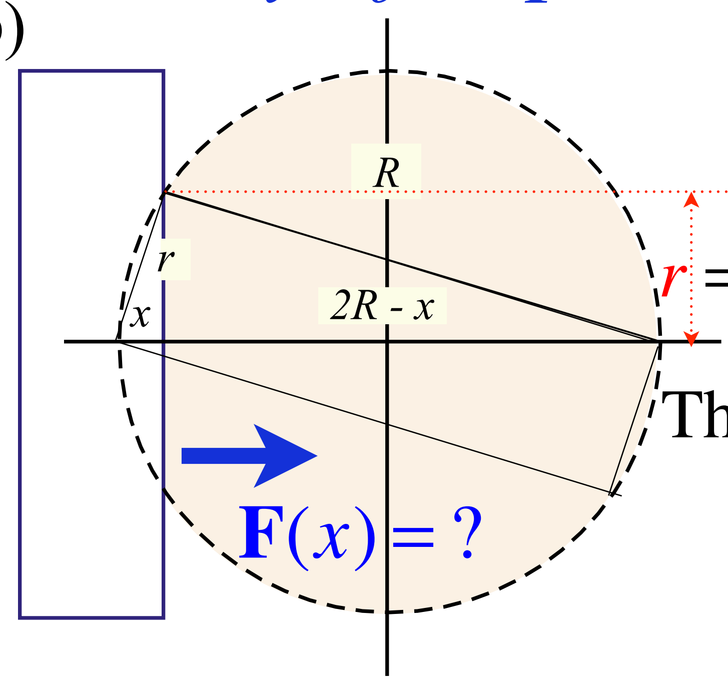
Potential Energy Geometry of Superballs and Related things

(a)



(b)



$r = \sqrt{x(2R-x)} \quad \left(\approx \sqrt{2Rx} \quad for: \; x << R\right)$

Thales' geometry and "Sagittal†" approx.

† "bow"

Unit 1
Fig. 7.1
(modified)

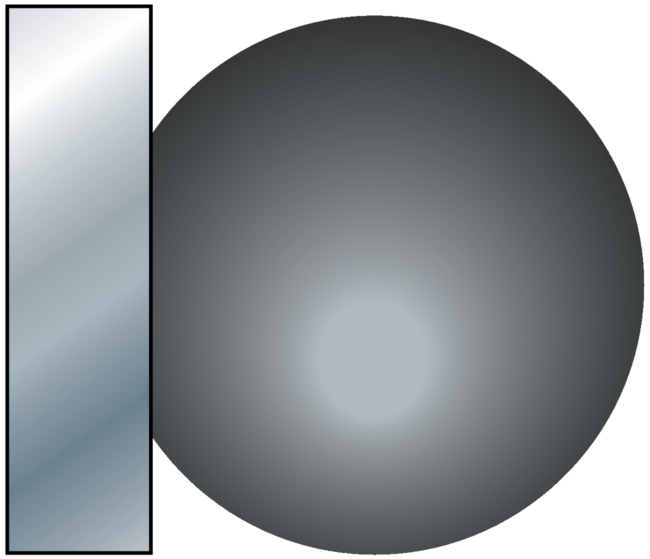
If superball was a balloon its bounce force law would be linear $F=-k\cdot x$ *(Hooke Law)*

(Pressure)

$$F_{balloon}(x) = P \cdot A = P \cdot \pi r^2 \approx 2\pi PRx$$

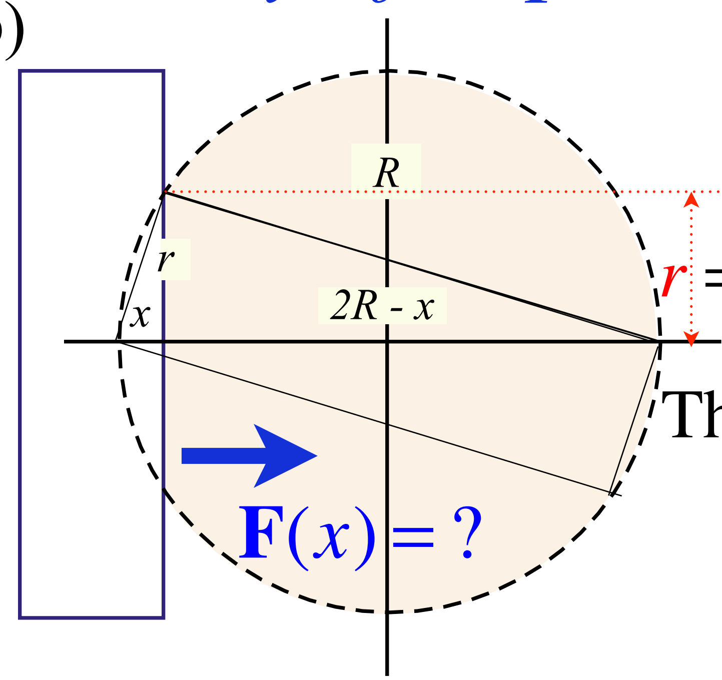
Potential Energy Geometry of Superballs and Related things

(a)



Unit 1
Fig. 7.1
(modified)

(b)



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Thales' geometry and "Sagittal†" approx.

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If superball was a balloon its bounce force law would be linear *F=-k·x* *(Hooke Law)*

$$F_{balloon}(x) = \overset{(Pressure)}{P} \cdot A = P \cdot \pi r^2 \approx 2\pi P R x$$

Instead superball force law depends on bulk *volume* modulus and is non-linear *F~ x^p? +?* *(Power Law?)*

$$Volume(X) = \int_0^X \pi r^2\, dx = \int_0^X \pi x(2R-x)\,dx = \int_0^X 2R\pi x\, dx - \int_0^X \pi x^2\, dx = R\pi X^2 - \frac{\pi X^3}{3} \approx \begin{cases} R\pi X^2 & (for: X << R) \\ \frac{4}{3}\pi R^3 & (for: X = 2R) \end{cases}$$

It also depends on velocity $\dot{x} = \frac{dx}{dt}$. *Adiabatic* differs from *Isothermal* as shown by *"Project-Ball*"*

* *Am. J. Phys.* **39**, 656 (1971)

Potential energy geometry of Superballs and related things

Thales geometry and "Sagittal approximation"

→ *Geometry and dynamics of single ball bounce* *(See Simulation)*

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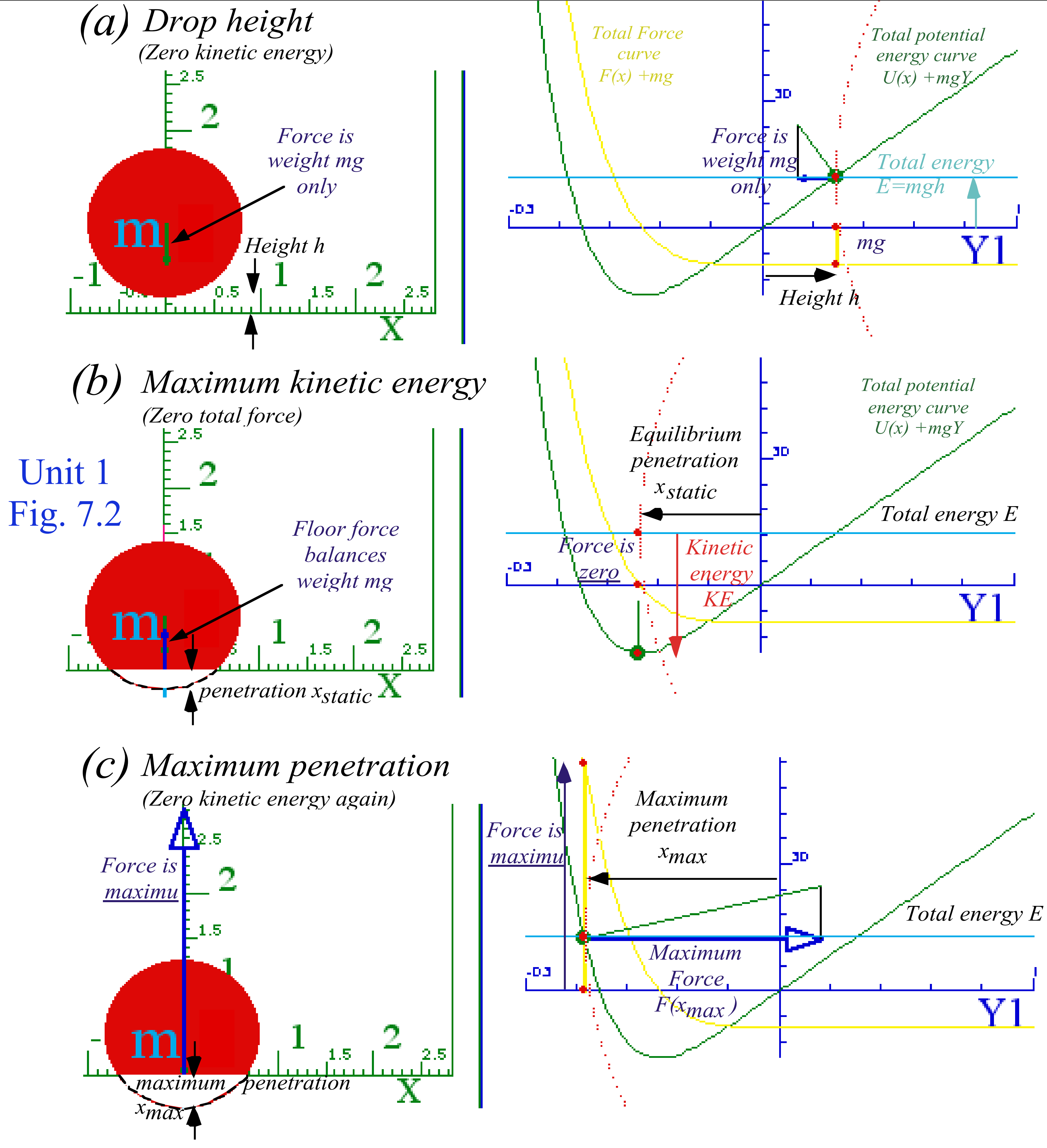

(a) Drop height
(Zero kinetic energy)
Force is weight mg only
Height h
Total Force curve F(x) +mg
Total potential energy curve U(x) +mgY
Force is weight mg only
Total energy E=mgh
mg
Y1
Height h
(b) Maximum kinetic energy
(Zero total force)
Floor force balances weight mg
penetration x_{static}
Equilibrium penetration x_{static}
Total potential energy curve U(x) +mgY
Total energy E
Force is zero
Kinetic energy KE
Y1
(c) Maximum penetration
(Zero kinetic energy again)
Force is maximu
maximum penetration
x_{max}
Force is maximu
Maximum penetration x_{max}
Total energy E
Maximum Force $F(x_{max})$
Y1
Unit 1 Fig. 7.2

(See Simulation)

- (•) Let mouse set: (x,y,Vx,Vy)
- ( ) Let mouse set force: F(t)

- ( ) Plot solid paths
- (•) Plot dotted paths
- ( ) Plot no paths

- ( ) Plot V1 vs. V2
- ( ) Plot Y1(t), Y2(t), ...
- (•) Plot PE of m1 vs. Y1
- ( ) Plot Y2 vs. Y1
- ( ) Plot user defined i.e - Y1 vs. Y2

- ( ) Balls initially falling
- (•) Balls initially fixed
- ( ) No preset initial values

Number of masses

1 | 1 Balls

Acceleration of gravity

1 | 1 100x{cm/s^2}

Collision friction (Viscosity)

0 | 0

- [x] Draw force vectors
- [x] Pause (once) at top
- [x] Constrain motion to Y-axis

Initial gap between balls

0 | 0 {cm}

Force power law exponent

4 | 4

Force Constant

50000 | 50000

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

1 | 1

Initial V =	0	0	y Max =	4	4
Initial x1 =	0.5	0.5	y Min =	-3	-3
Max x PE plot =	0.5	0.5	T Max =	6	6
F-Vector scale =	0.003	0.003	V2y Max =	3	3
Error step =	0.000001	0.000001	V2y Min =	-2	-2

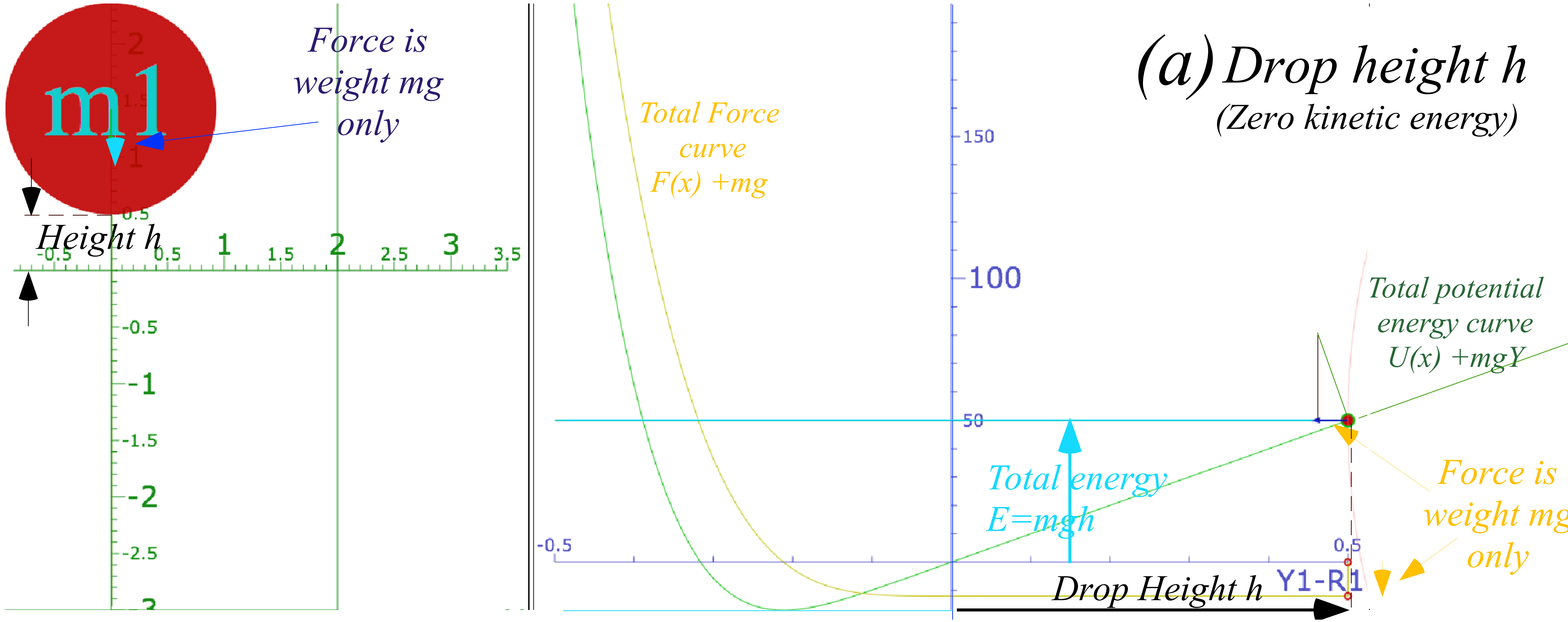
1st Mass m1 100 {g}

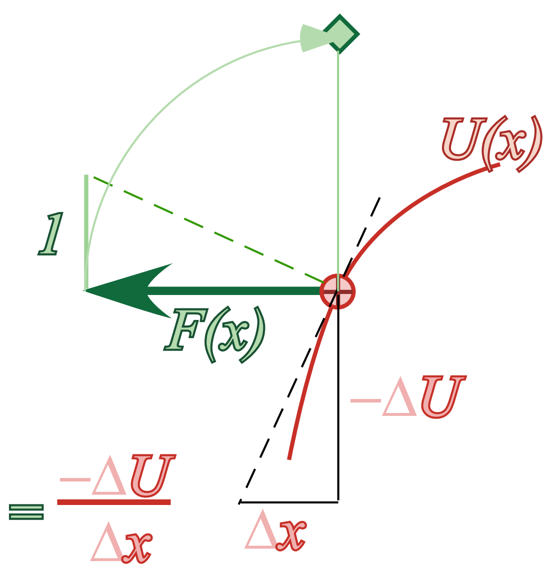
100

1st Mass V1 0 {cm/s}

0

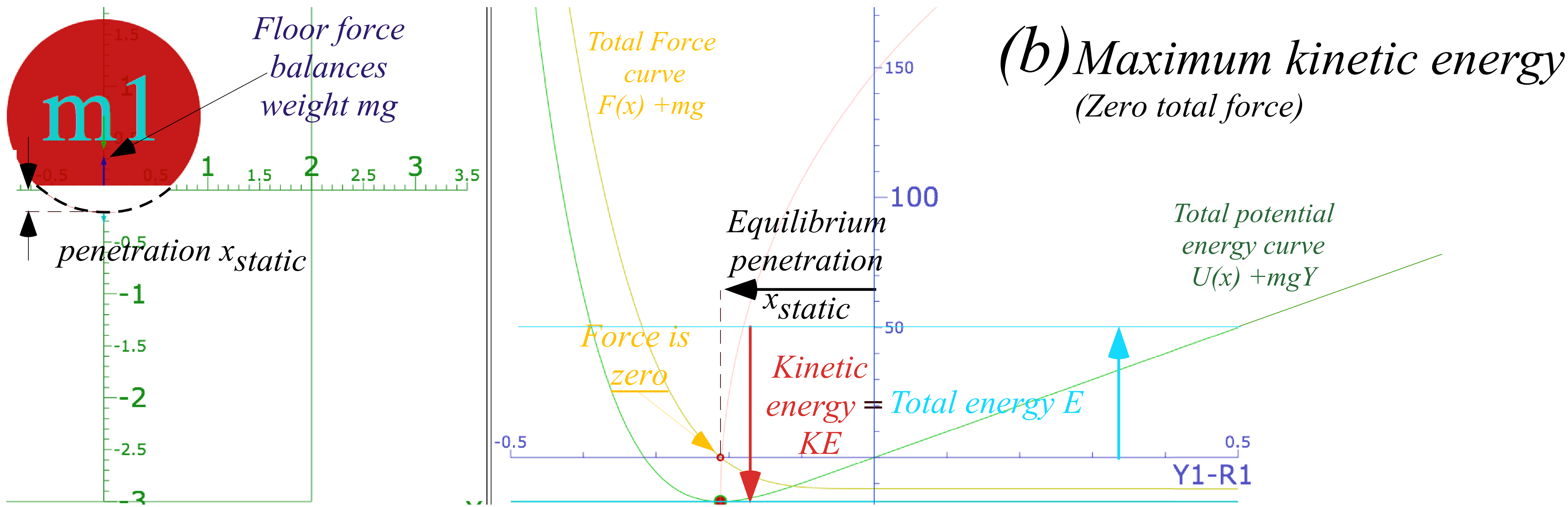
(a) Drop height h
(Zero kinetic energy)





$$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$$

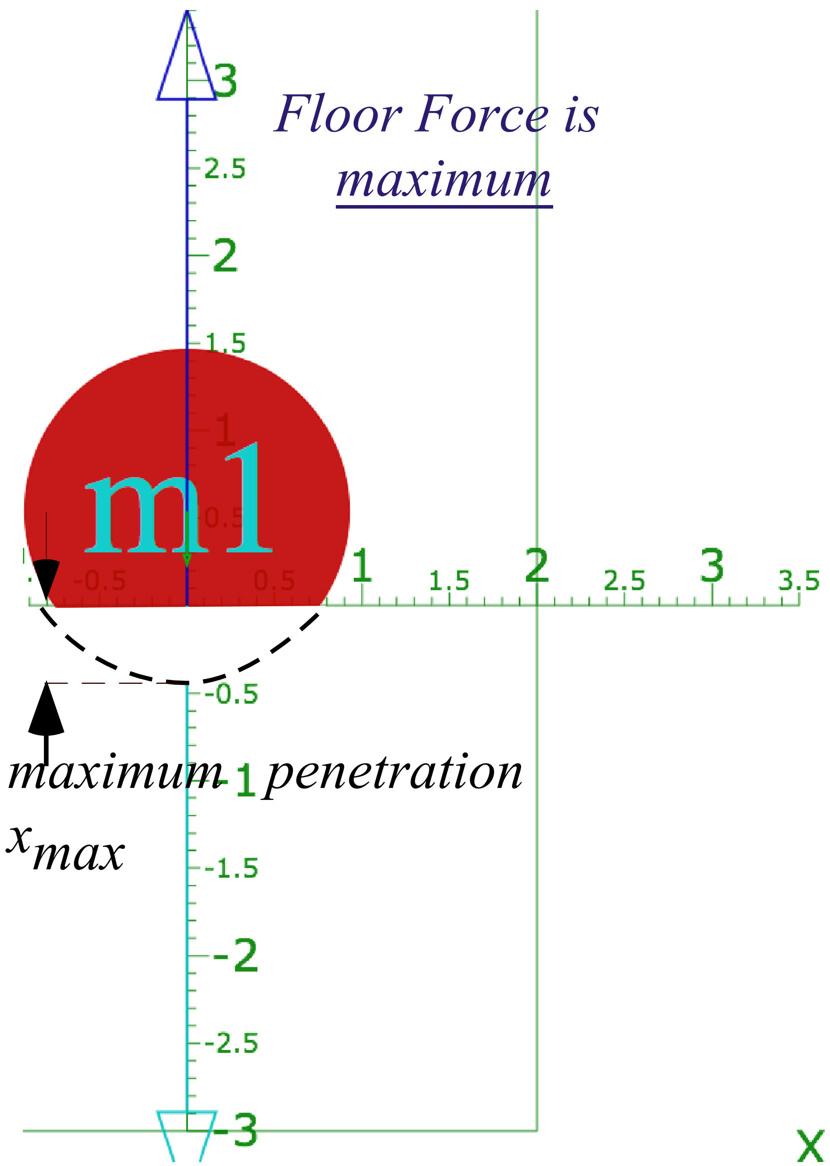
Display of Force vector using similar triangle constuction based on the slope of potential curve.

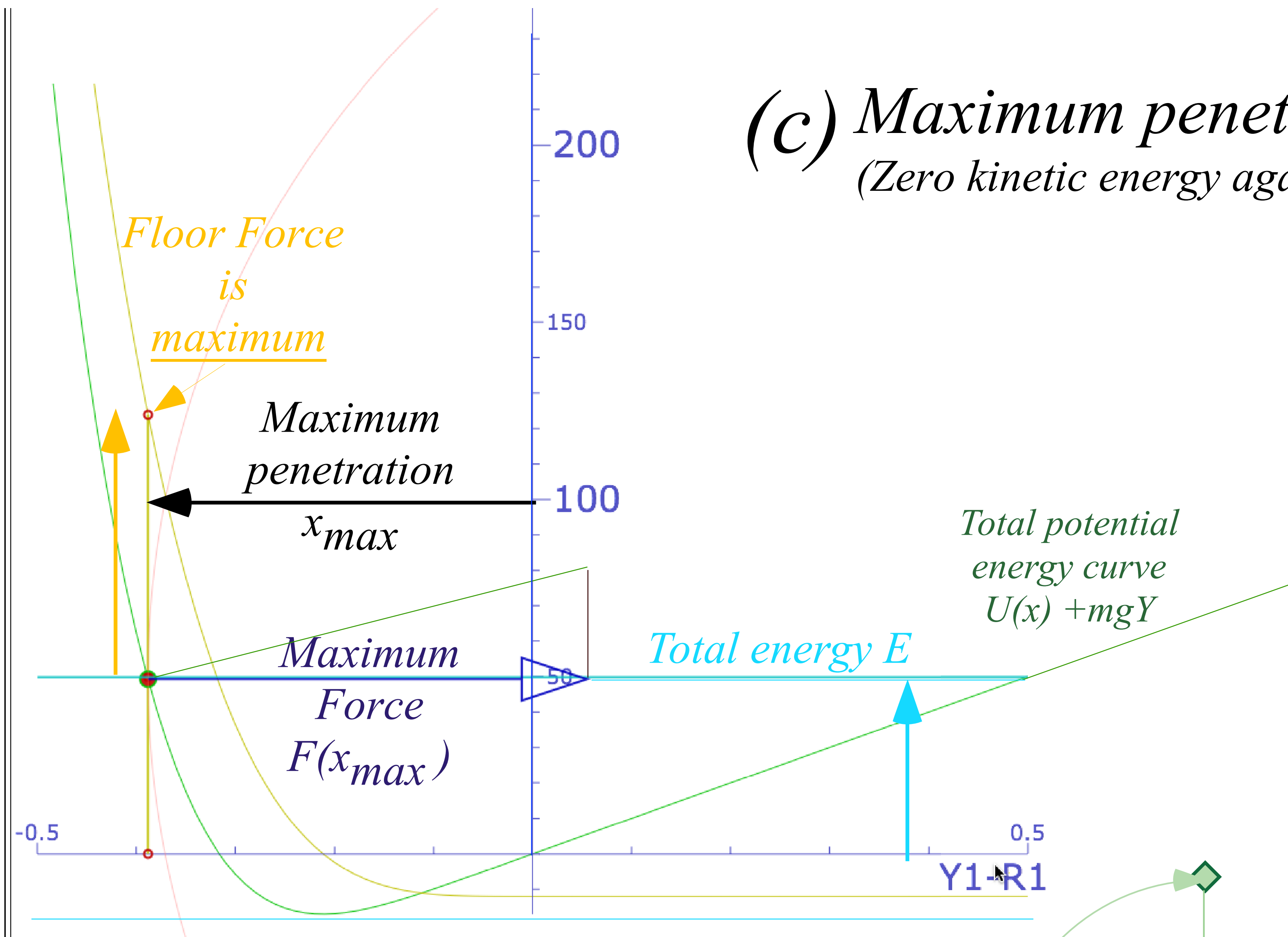


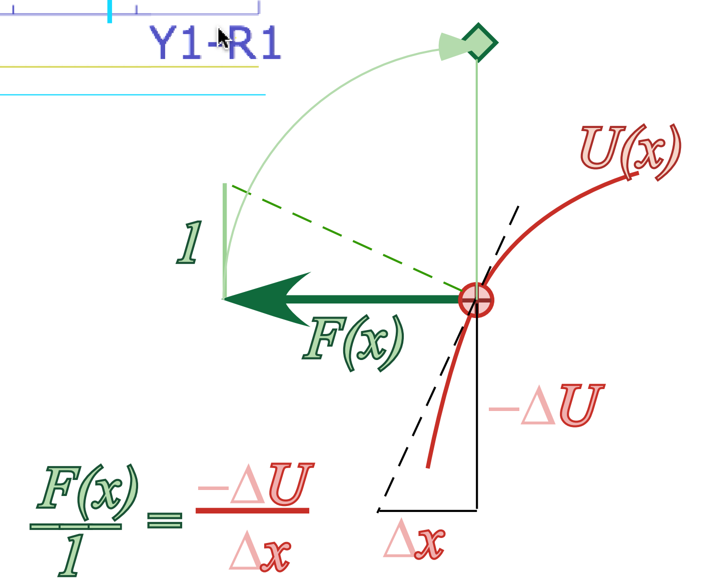
Floor force balances weight mg
m1
penetration x_{static}
(b) Maximum kinetic energy
(Zero total force)
Total Force curve F(x) +mg
Equilibrium penetration
x_{static}
Total potential energy curve U(x) +mgY
Force is zero
Kinetic energy KE
= Total energy E
Y1-R1

(c) Maximum penetration

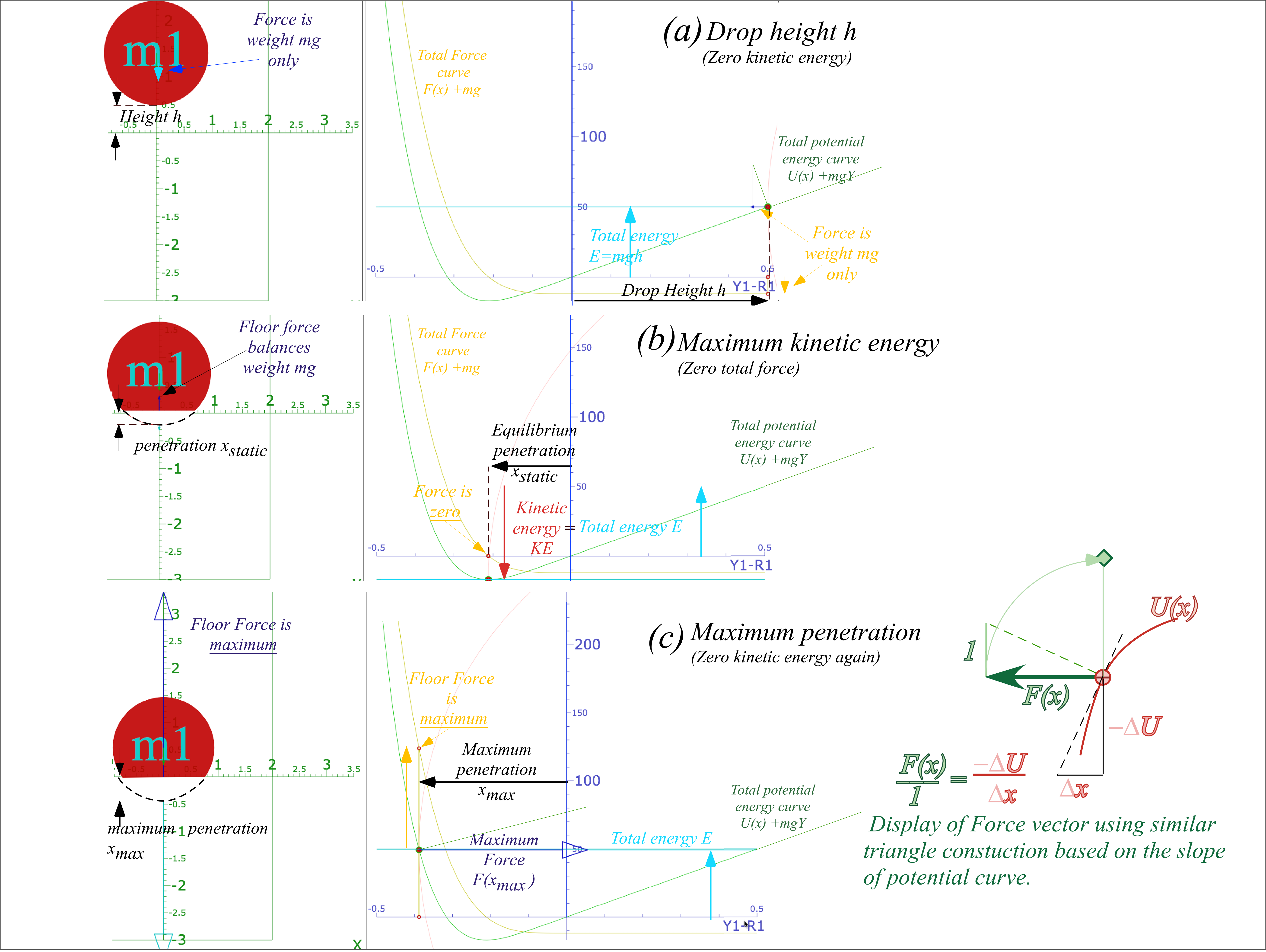
(Zero kinetic energy again)





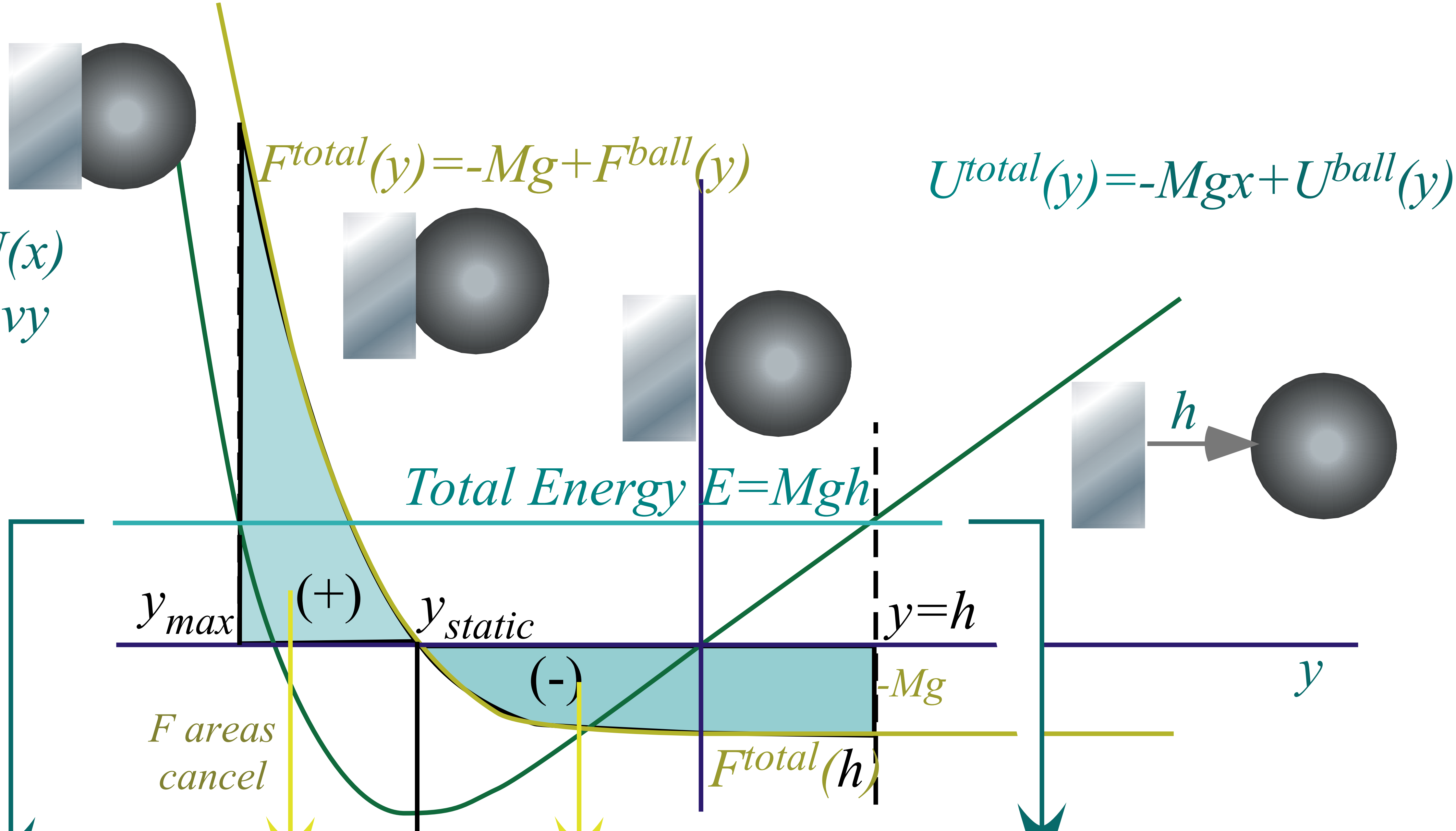


Display of Force vector using similar triangle constuction based on the slope


(a) Drop height h
(Zero kinetic energy)
Force is weight mg only
Height h
Total Force curve F(x) +mg
Total potential energy curve U(x) +mgY
Total energy E=mgh
Force is weight mg only
Drop Height h
Y1-R1
(b) Maximum kinetic energy
(Zero total force)
Floor force balances weight mg
penetration x_{static}
Total Force curve F(x) +mg
Equilibrium penetration x_{static}
Total potential energy curve U(x) +mgY
Force is zero
Kinetic energy KE = Total energy E
Y1-R1
(c) Maximum penetration
(Zero kinetic energy again)
Floor Force is maximum
maximum penetration x_{max}
Floor Force is maximum
Maximum penetration x_{max}
Total potential energy curve U(x) +mgY
Maximum Force $F(x_{max})$
Total energy E
Y1-R1
U(x)
1
F(x)
−ΔU
$\frac{F(x)}{1} = \frac{-\Delta U}{\Delta x}$
Δx
Display of Force vector using similar triangle constuction based on the slope of potential curve.

Force F(x) and Potential U(x) for soft heavy non-linear superball

Unit 1
Fig. 7.5

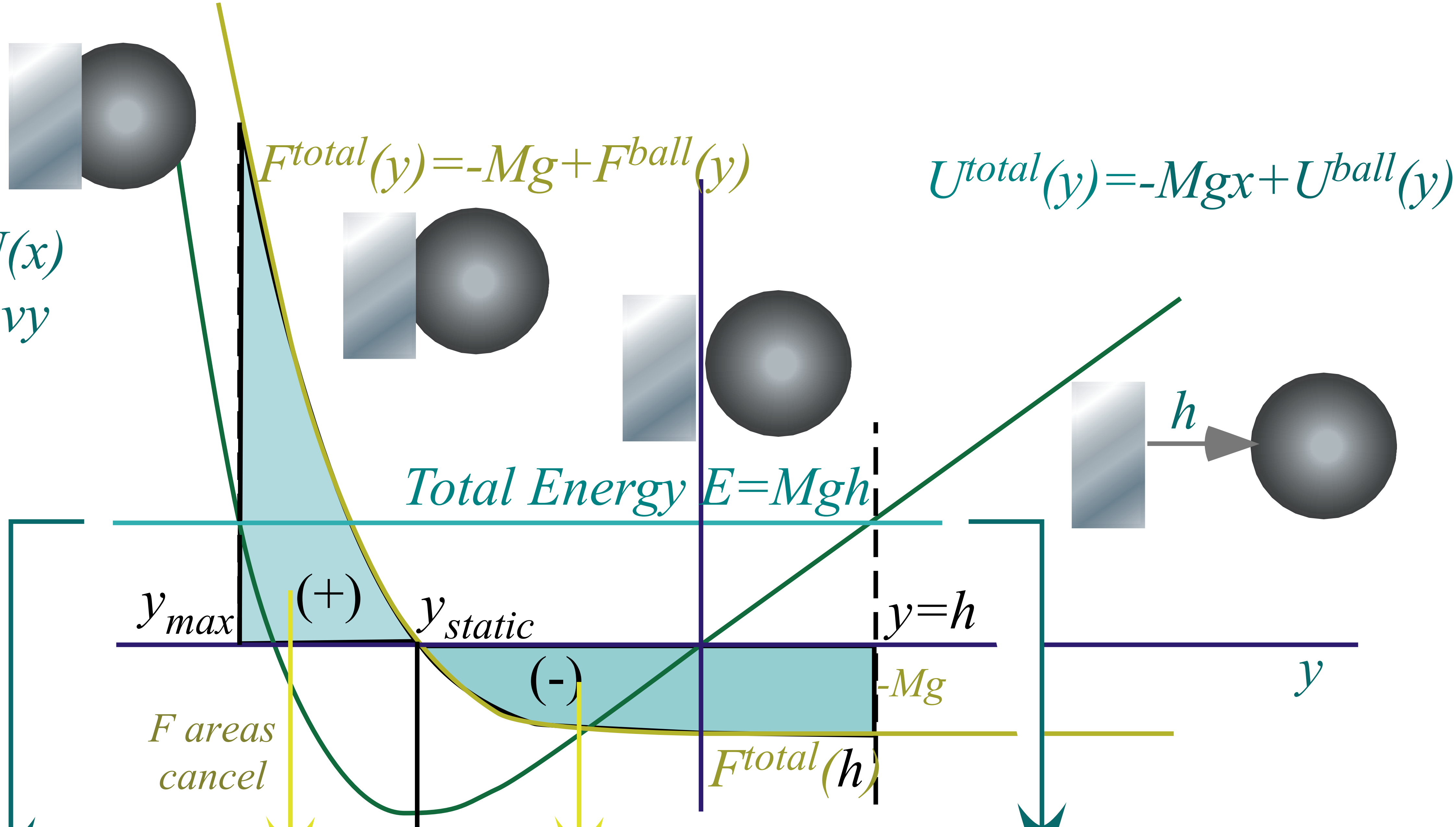


$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y)\,dy + \int_{y=h}^{y_{static}} F^{total}(y)\,dy + U(h) \;=\; U(h) = E$$

$$F(x) = -\frac{dU(x)}{dx}$$

Force F(x) and Potential U(x) for soft heavy <u>non</u>-linear superball

Unit 1
Fig. 7.5

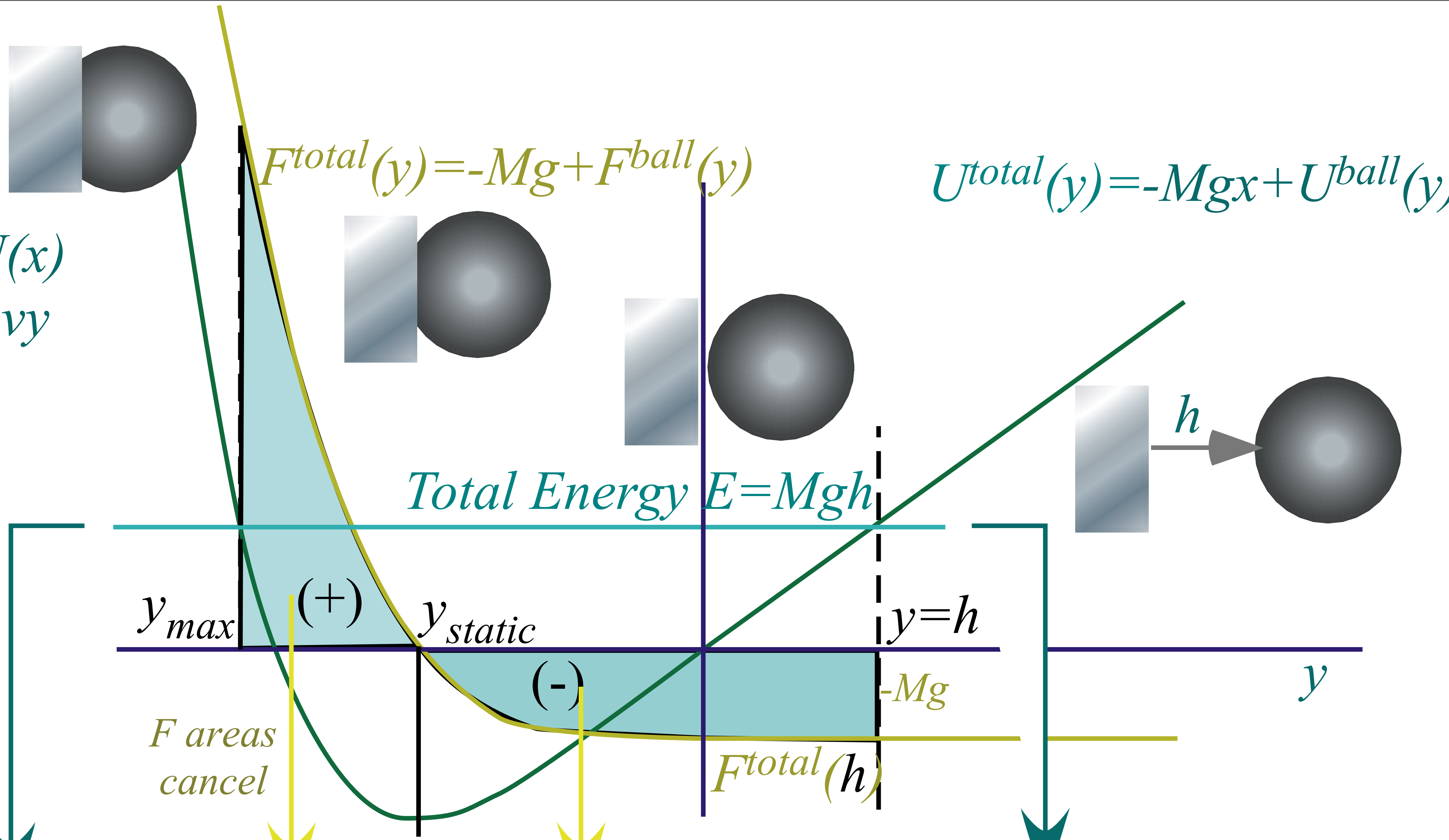


$$U^{total}(y_{max}) = \int_{y_{static}}^{y_{max}} F^{total}(y)\,dy + \int_{y=h}^{y_{static}} F^{total}(y)\,dy + U(h) = U(h) = E$$

$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

Force F(x) and Potential U(x) for soft heavy non-linear superball

Unit 1
Fig. 7.5



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$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

$$Impulse = P = \int F(t)\,dt = Momentum\ acquired = Area\ of\ F(t) = P(t) \qquad F(t) = \frac{dP(t)}{dt}$$

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Thales geometry and "Sagittal approximation"

Geometry and dynamics of single ball bounce

→ *Examples: (a) Constant force (like kidee pool) (b) Linear force (like balloon)*

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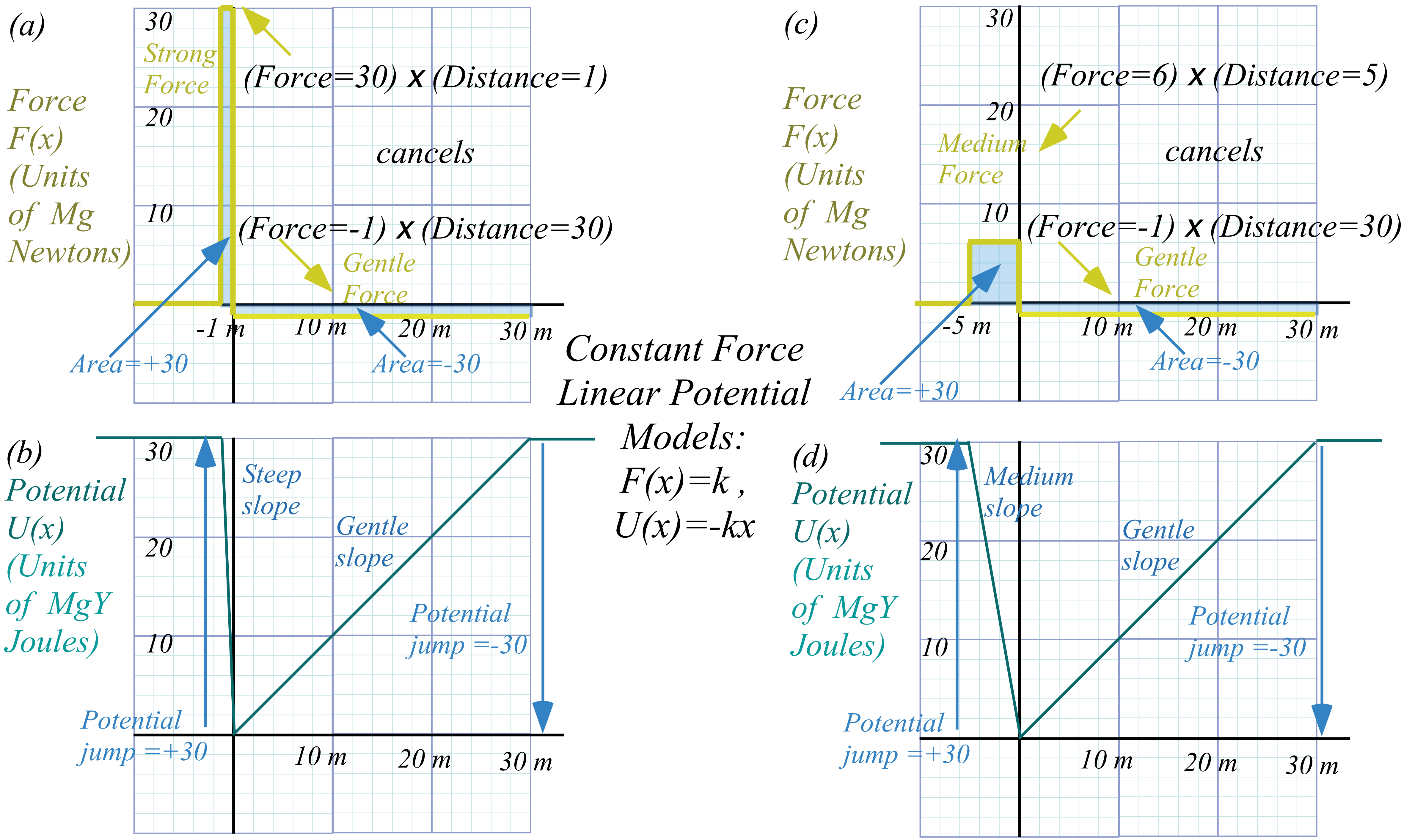

Constant Force Linear Potential Models:

$F(x)=k$, $U(x)=-kx$

Unit 1
Fig. 7.3

$$Work = W = \int F(x)\,dx = Energy\ acquired = Area\ of\ F(x) = -U(x) \qquad F(x) = -\frac{dU(x)}{dx}$$

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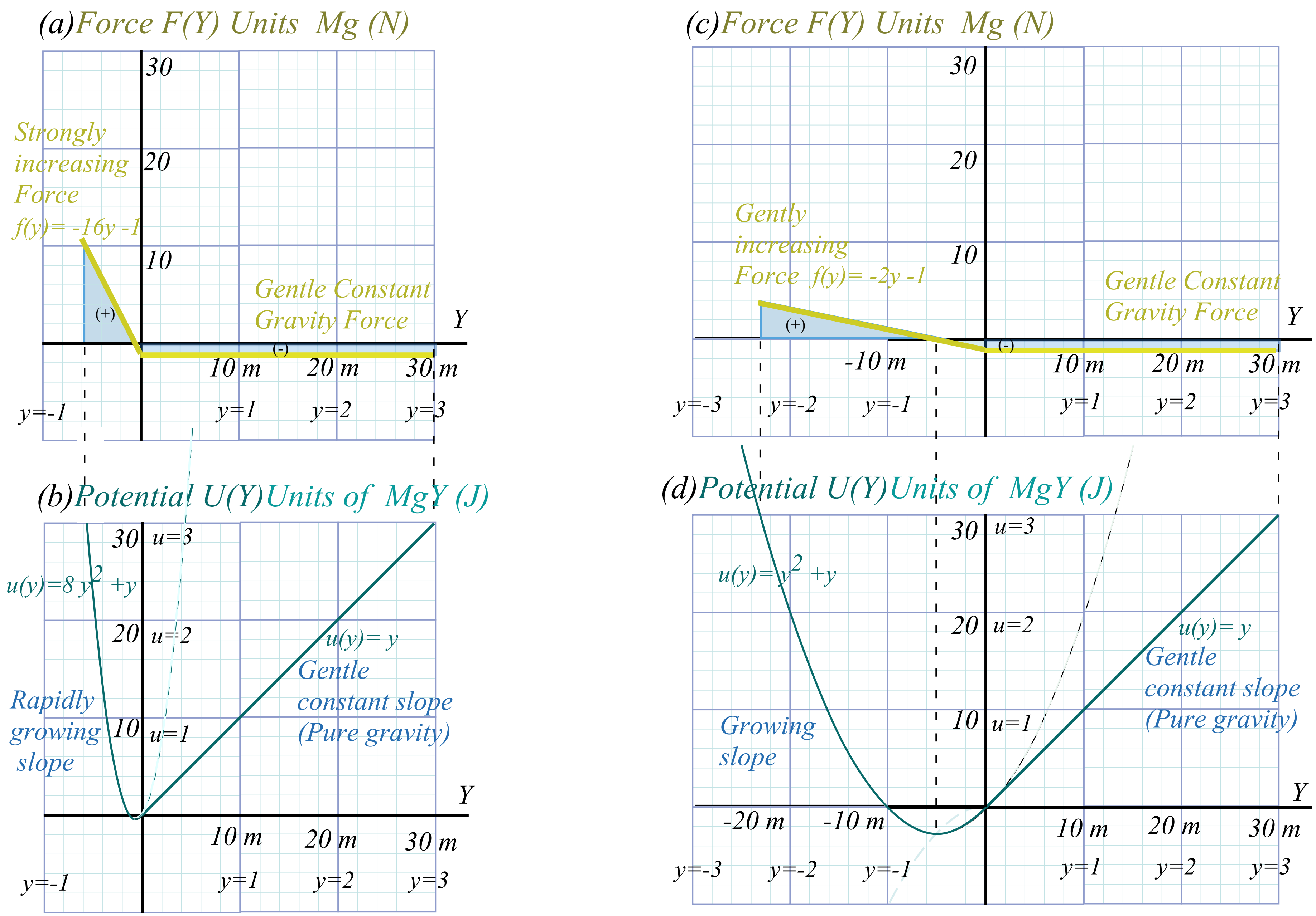

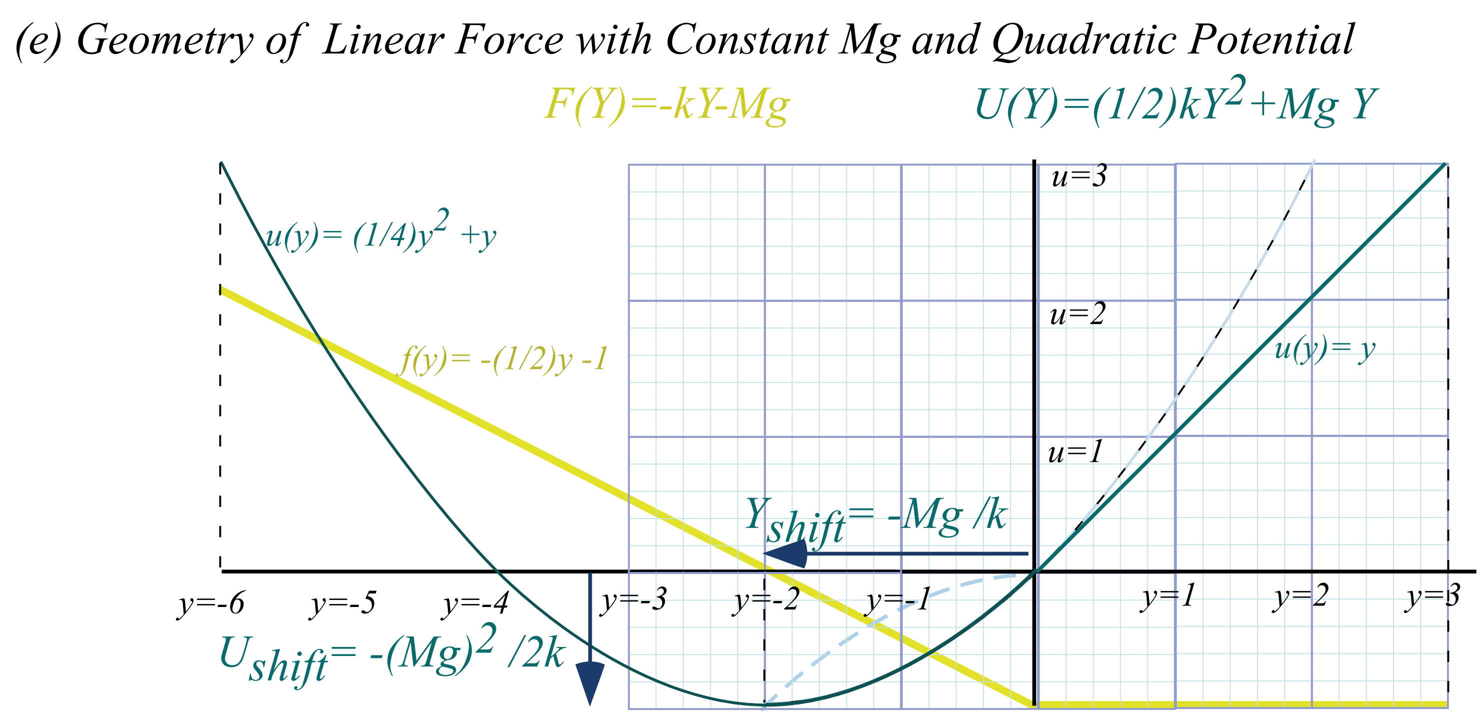

Unit 1
Fig. 7.4

$$F^{Total} = F^{grav} + F^{target} = \begin{cases} -Mg & (y \geq 0) \\ -Mg - ky & (y < 0) \end{cases}$$

$$U^{Total} = U^{grav} + U^{target} = \begin{cases} Mg\ y & (y \geq 0) \\ Mg\ y + \frac{1}{2}ky^2 & (y < 0) \end{cases}$$

- (•) Let mouse set: (x,y,Vx,Vy)
- ( ) Let mouse set force: F(t)

- ( ) Plot solid paths
- (•) Plot dotted paths
- ( ) Plot no paths

- ( ) Plot V1 vs. V2
- ( ) Plot Y1(t), Y2(t), ...
- (•) Plot PE of m1 vs. Y1
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- ( ) Plot user defined i.e - Y1 vs. Y2

- (•) Balls initially falling
- ( ) Balls initially fixed
- ( ) No preset initial values

Number of masses

1 | 1 Balls

Acceleration of gravity

2 | 2 100x{cm/s^2}

Collision friction (Viscosity)

0 | 0

- [x] Draw force vectors
- [x] Pause (once) at top
- [x] Constrain motion to Y-axis

Initial gap between balls

0 | 0 {cm}

Force power law exponent

1 | 1

Force Constant

2000 | 2000

Canvas Aspect Ratio - W/H i.e. 0.75 & 1.0

1 | 1

Initial V =	1	1	y Max =	7	7
Initial x1 =	0.01	0.01	y Min =	0	0
Max x PE plot =	0.5	0.5	T Max =	6	6
F-Vector scale =	0.003	0.003	V2y Max =	3	3
Error step =	0.000001	0.000001	V2y Min =	-2	-2

1st Mass m1 100 {g}

100

1st Mass V1 -1 {cm/s}

-1

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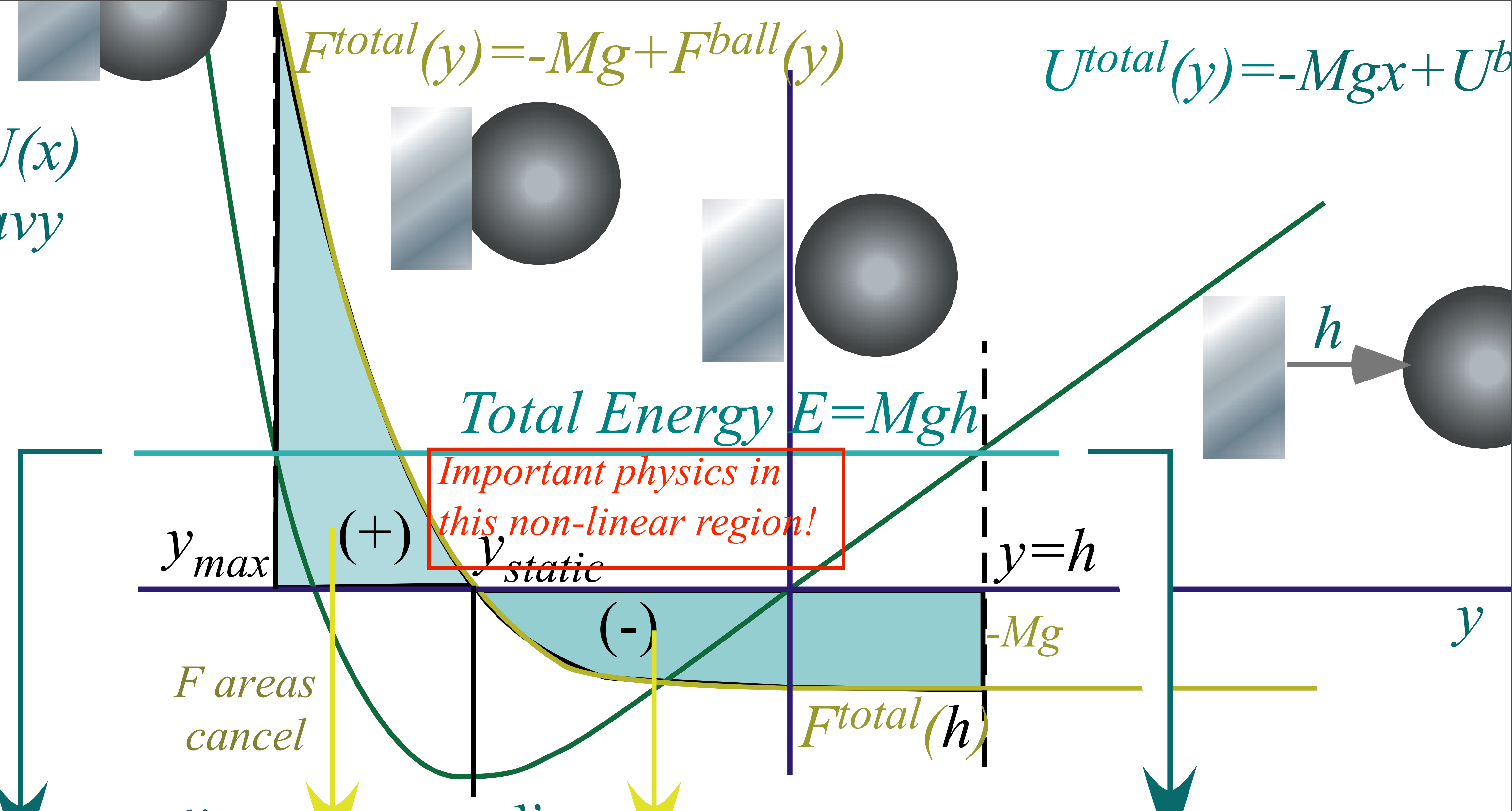
Force F(x) and Potential U(x) for soft heavy non-linear superball



Unit 1
Fig. 7.5

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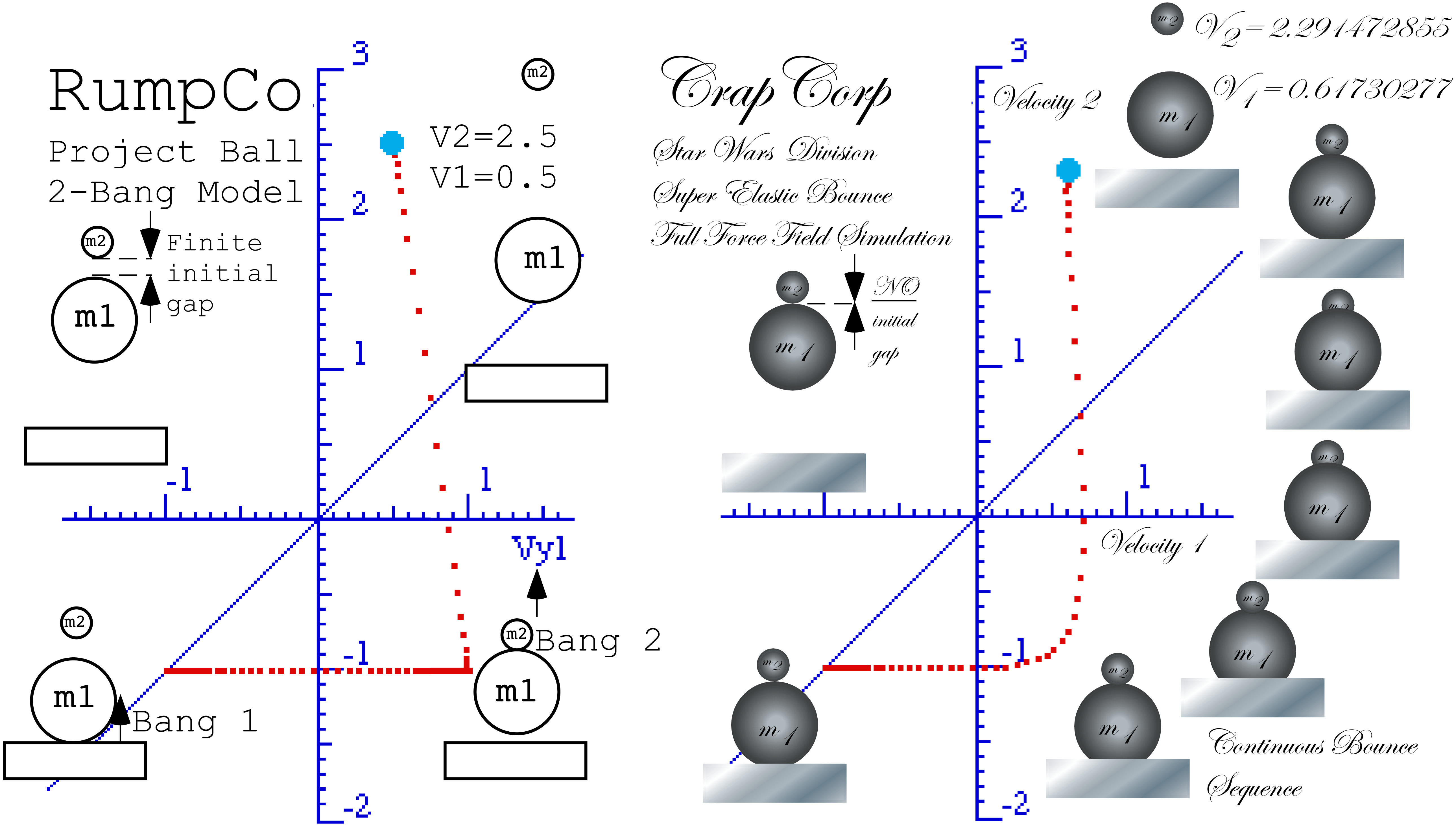
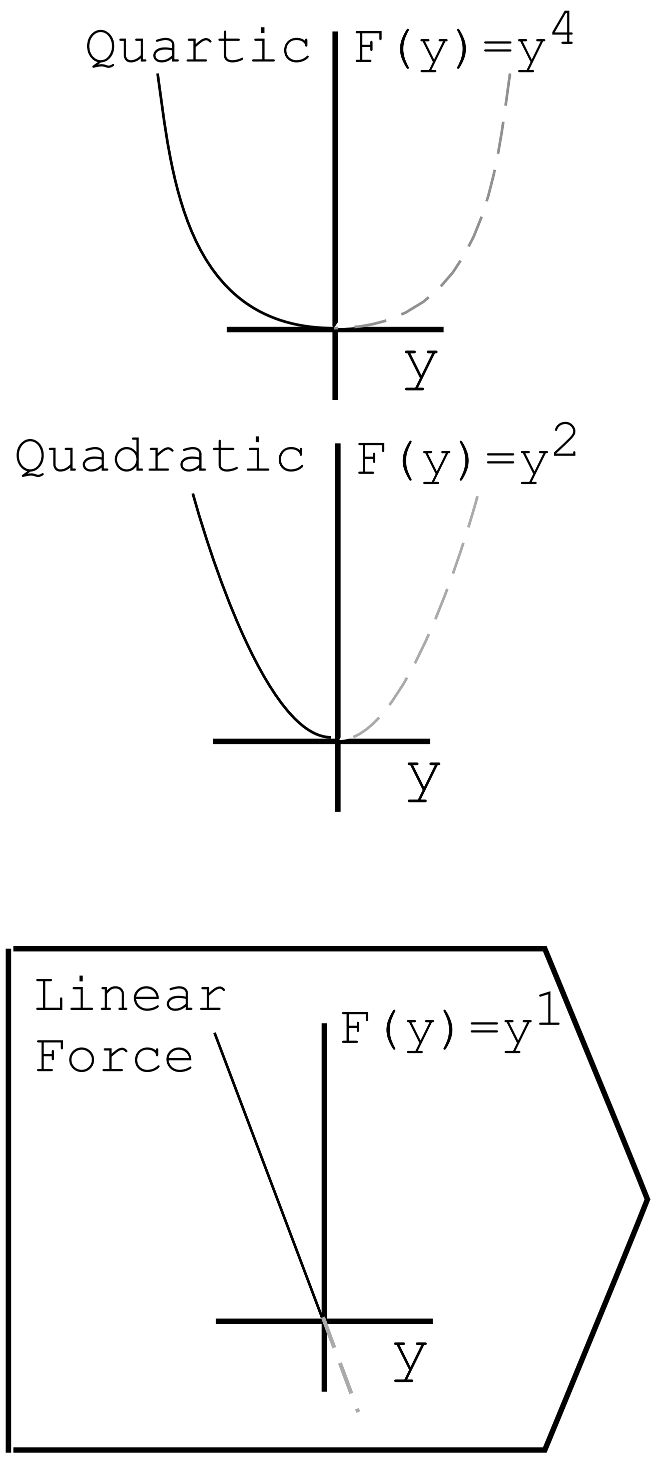
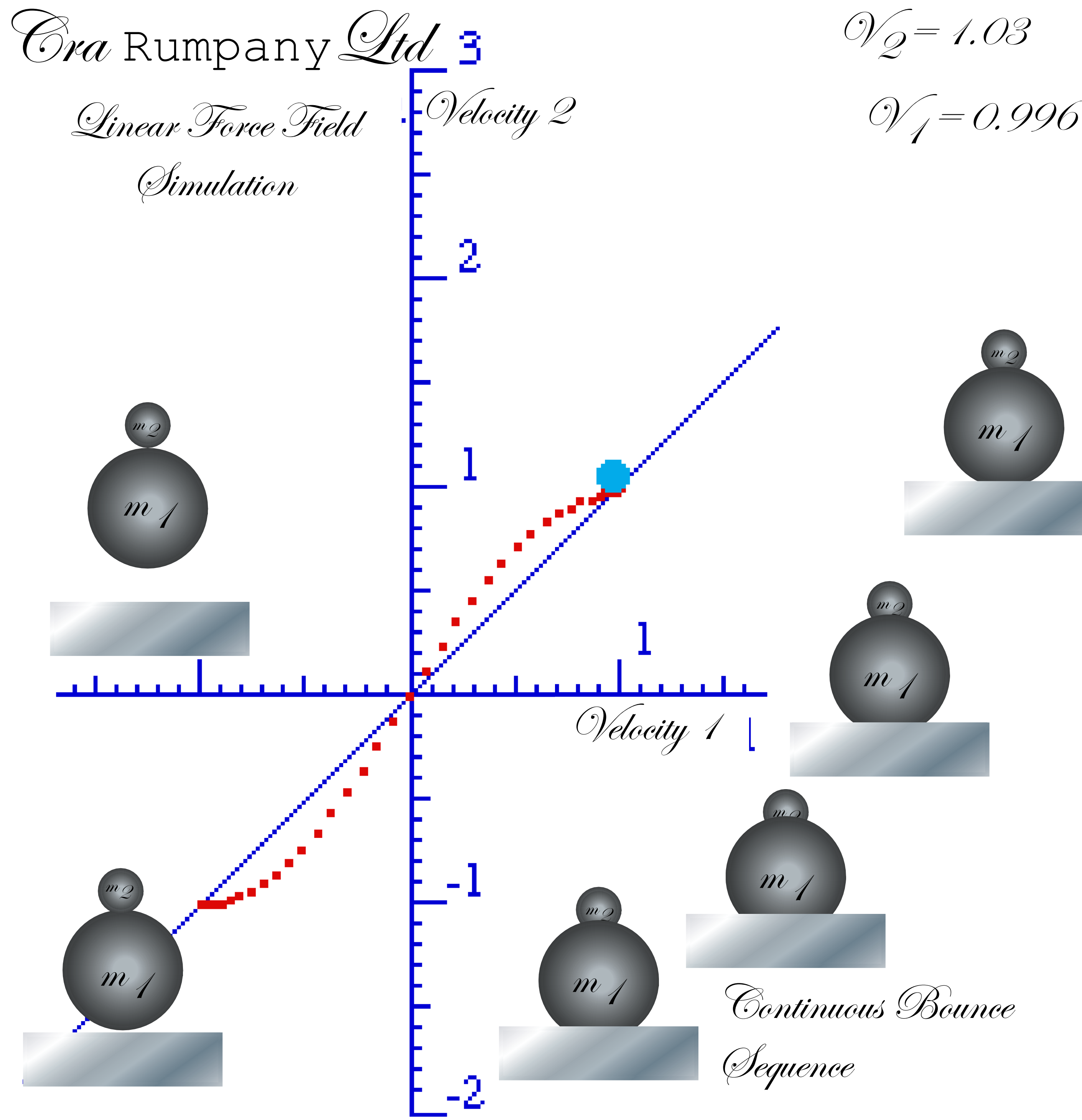
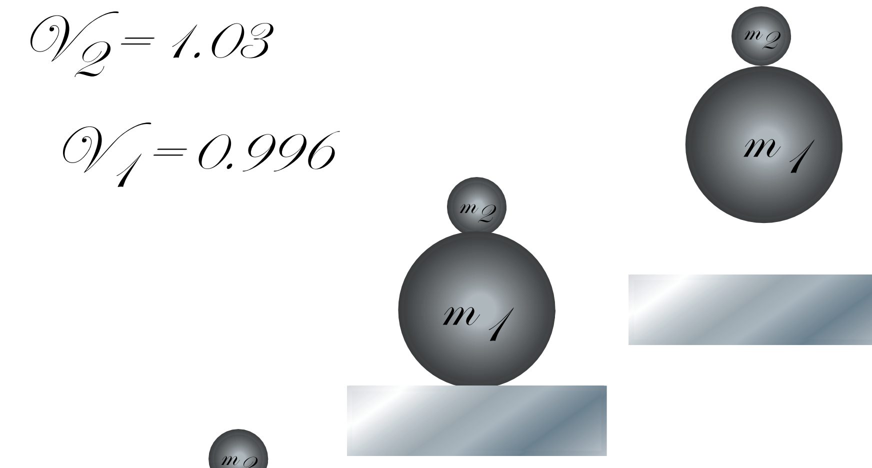


RumpCo
Project Ball
2-Bang Model
Finite
initial
gap
V2=2.5
V1=0.5
m2
m1
Vy1
Bang 1
Bang 2
3
2
1
-1
-2
Crap Corp
Star Wars Division
Super Elastic Bounce
Full Force Field Simulation
NO
initial
gap
Velocity 2
Velocity 1
V_2=2.291472855
V_1=0.61730277
Continuous Bounce
Sequence

Unit 1
Fig. 7.6


Quartic
F(y)=y4
y
Quadratic
F(y)=y2
y
Linear
Force
F(y)=y1
y


Cra Rumpany Ltd
Linear Force Field
Simulation
Velocity 2
3
2
1
-1
-2
1
Velocity 1
Continuous Bounce
Sequence


V2 = 1.03
V1 = 0.996

Unit 1
Fig. 7.7

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Thales geometry and "Sagittal approximation"

Geometry and dynamics of single ball bounce

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→ *Geometry and dynamics of 3-ball bounce*

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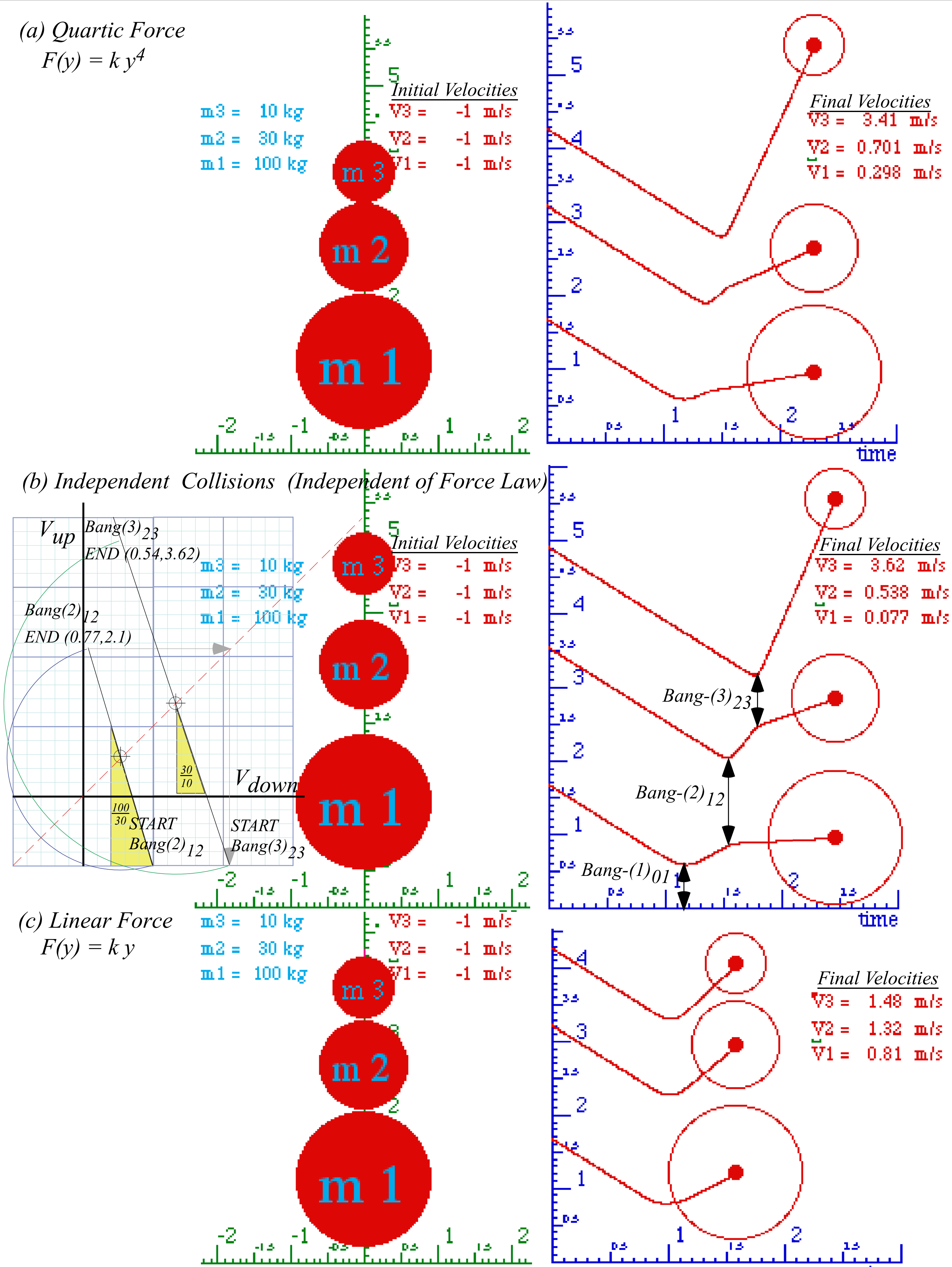
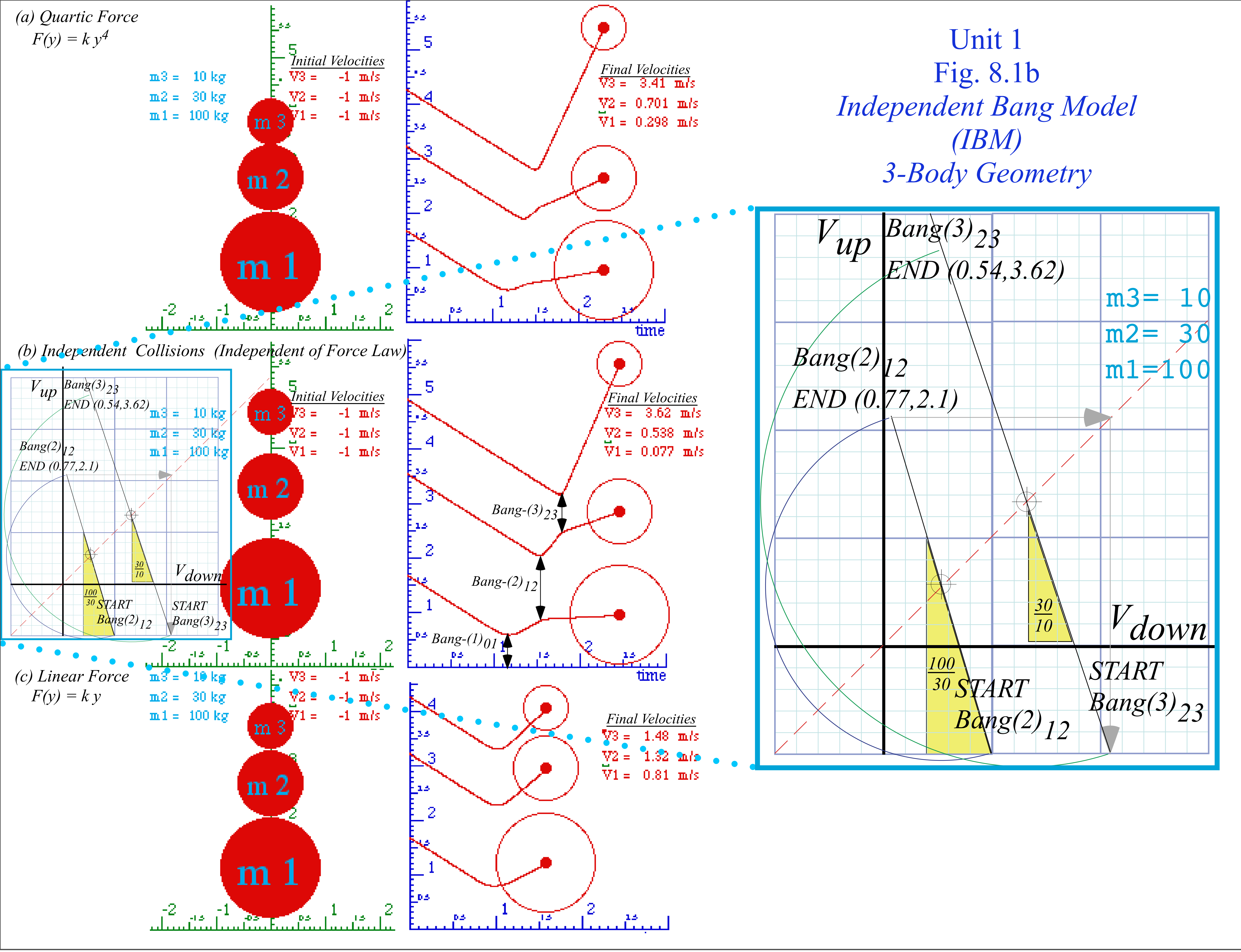
- *Other bangings-on: The western buckboard and Newton's balls*

Unit 1
Fig. 8.1a-c
Independent Bang Model (IBM)
3-Body Geometry



Unit 1
Fig. 8.1b
Independent Bang Model
(IBM)
3-Body Geometry



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Thales geometry and "Sagittal approximation"

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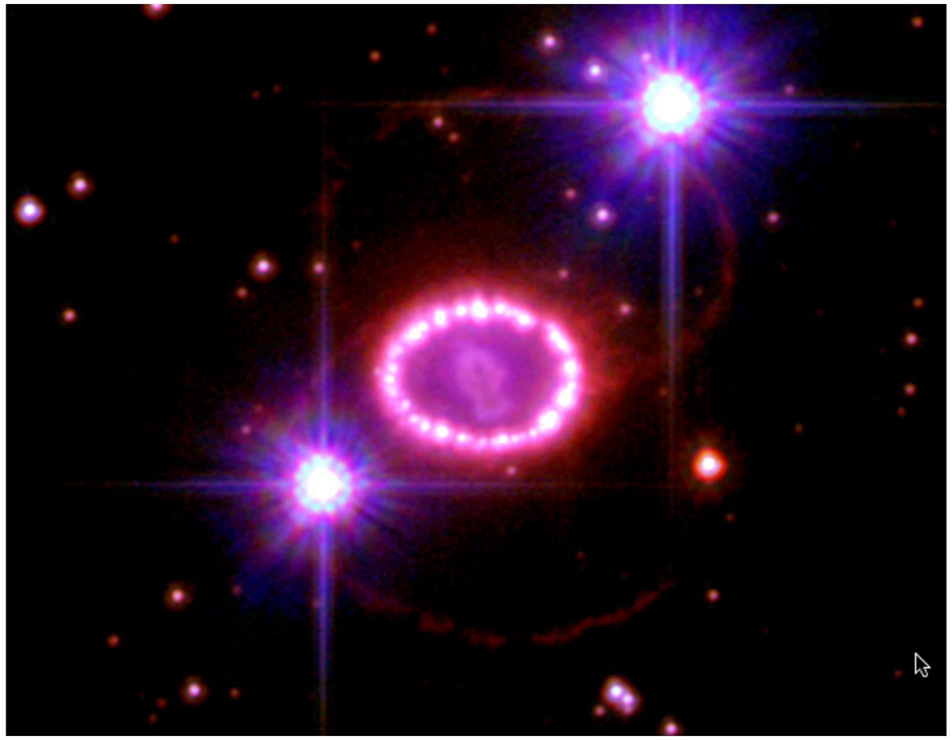
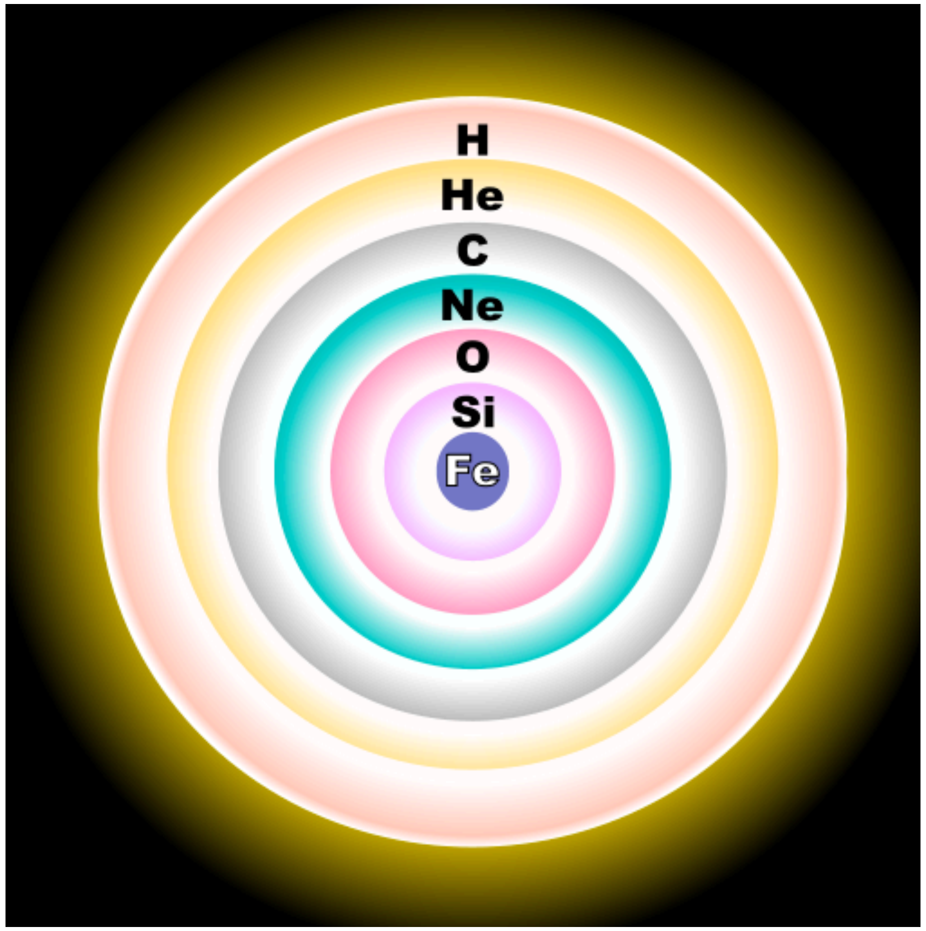


Source

http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

Author

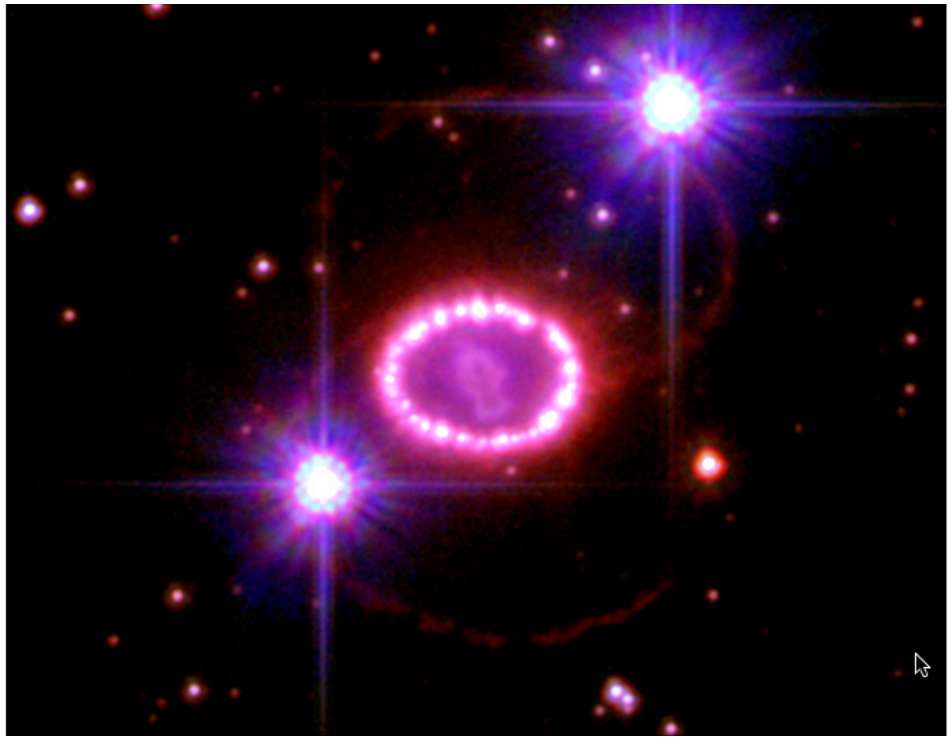
NASA, ESA, P. Challis, and R. Kirshner (Harvard-Smithsonian Center for Astrophysics)



Core-burning nuclear fusion stages for a 25-solar mass star

Process	Main fuel	Main products	25 $M_\odot$ star[6]		
			Temperature (Kelvin)	Density (g/cm^3)	Duration
hydrogen burning	hydrogen	helium	7×10^7	10	10^7 years
triple-alpha process	helium	carbon, oxygen	2×10^8	2000	10^6 years
carbon burning process	carbon	Ne, Na, Mg, Al	8×10^8	10^6	10^3 years
neon burning process	neon	O, Mg	1.6×10^9	10^7	3 years
oxygen burning process	oxygen	Si, S, Ar, Ca	1.8×10^9	10^7	0.3 years
silicon burning process	silicon	nickel (decays into iron)	2.5×10^9	10^8	5 days

A story of Stirling Colgate (Palmolive) and core-collapse supernovae



Source

http://hubblesite.org/newscenter/archive/releases/2007/10/image/a/

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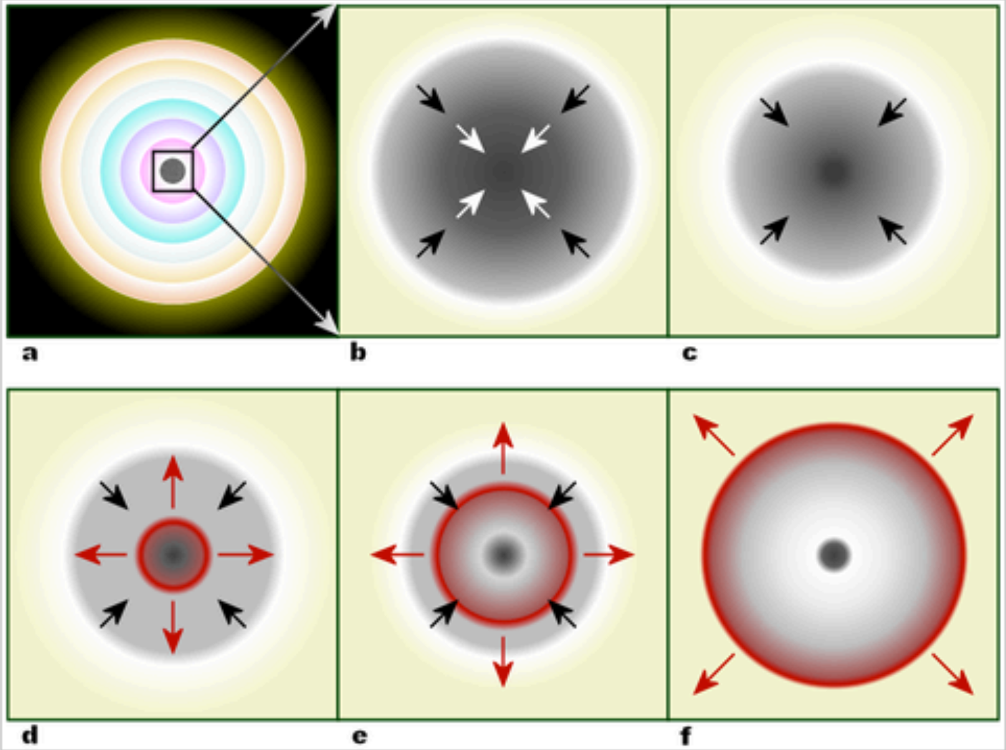
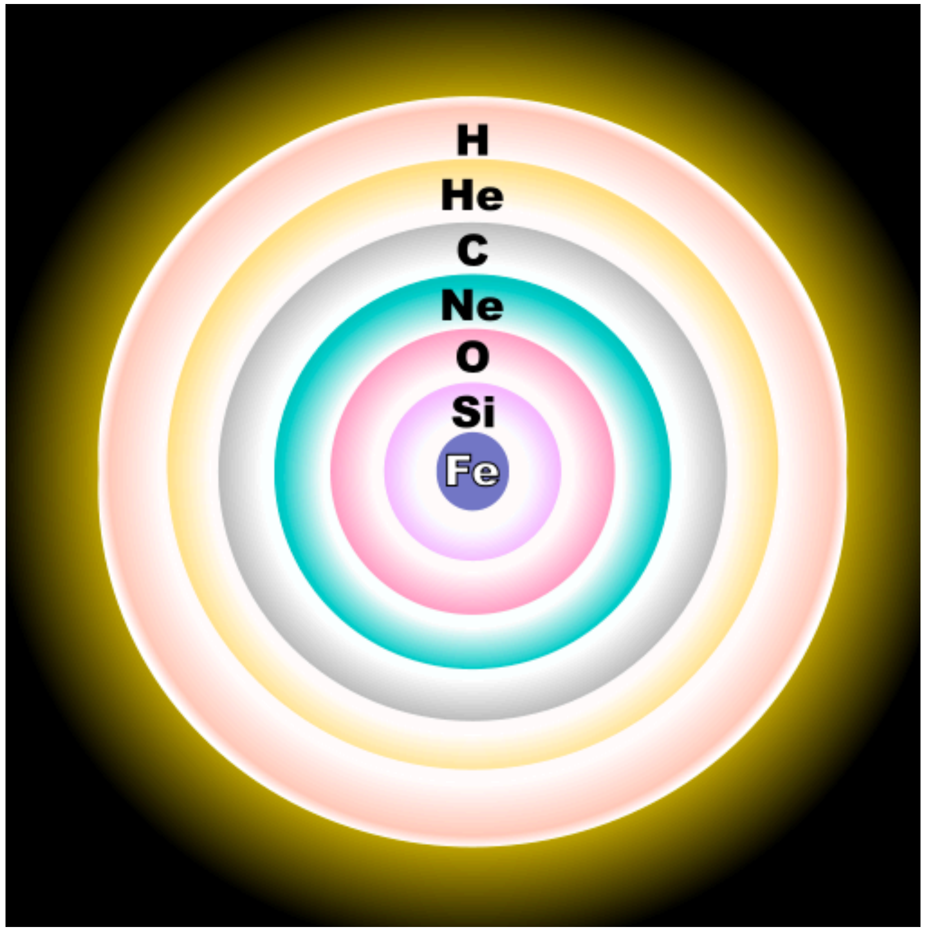


Within a massive, evolved star (a) the onion-layered shells of elements undergo fusion, forming a nickel-iron core (b) that reaches Chandrasekhar-mass and starts to collapse. The inner part of the core is compressed into neutrons (c), causing infalling material to bounce (d) and form an outward-propagating shock front (red). The shock starts to stall (e), but it is re-invigorated by neutrino interaction. The surrounding material is blasted away (f), leaving only a degenerate remnant.



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Potential energy geometry of Superballs and related things

Thales geometry and "Sagittal approximation"

Geometry and dynamics of single ball bounce

Examples: (a) Constant force (like kidee pool) (b) Linear force (like balloon)

Some physics of dare-devil-divers

Non-linear force (like superball-floor or ball-bearing-anvil)

Geometry and dynamics of 2-ball bounce (again with feeling)

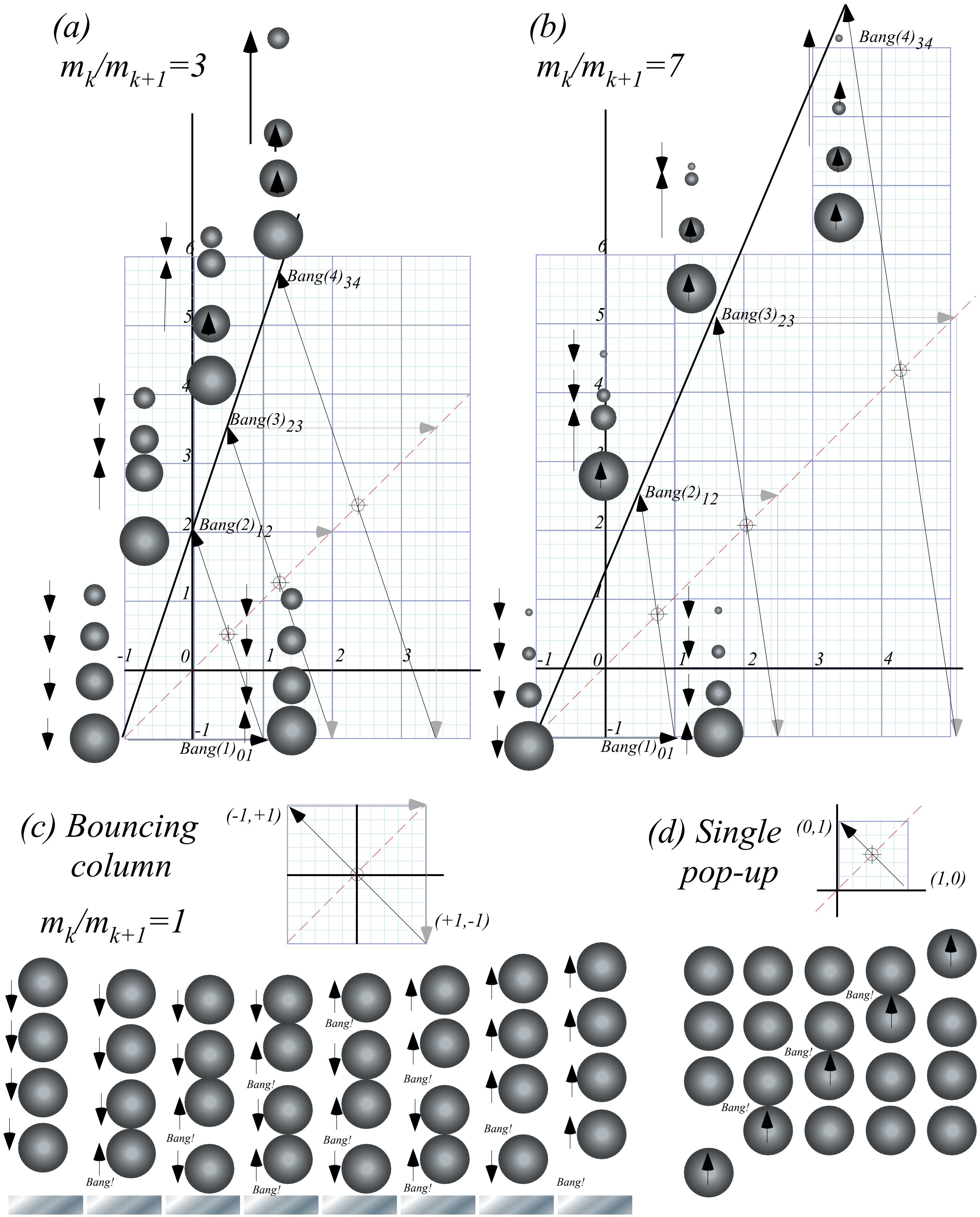
The parable of RumpCo. vs CrapCorp.

The story of USC pre-meds visiting Whammo Manufacturing Co.

Geometry and dynamics of 3-ball bounce

A story of Stirling Colgate (Palmolive) and core-collapse supernovae

→ *Other bangings-on: The western buckboard and Newton's balls*



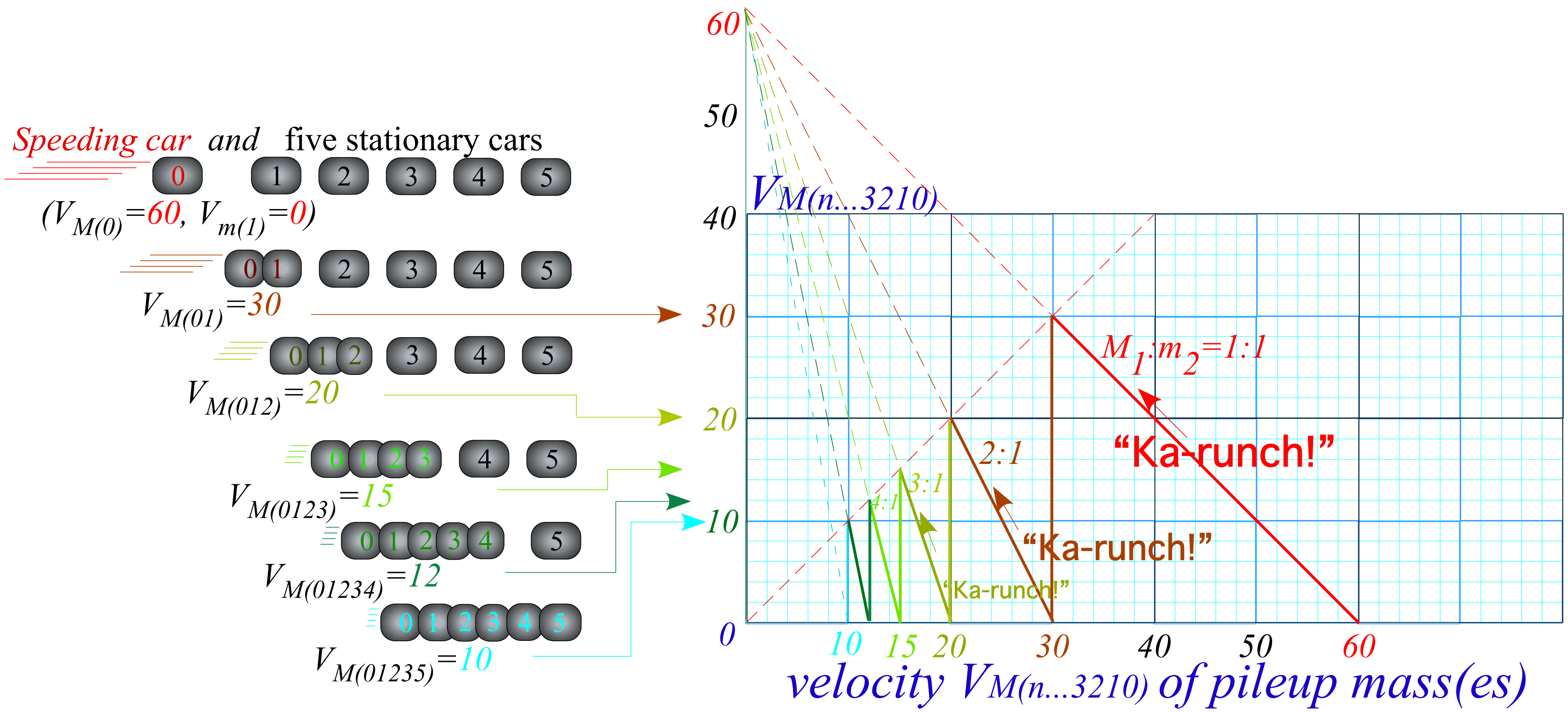
Unit 1
Fig. 8.2a-b
4-Body IBM Geometry
Fig. 8.2c-d
4-Equal-Body Geometry

4-Equal-Body
"Shockwave" or pulse wave
Dynamics

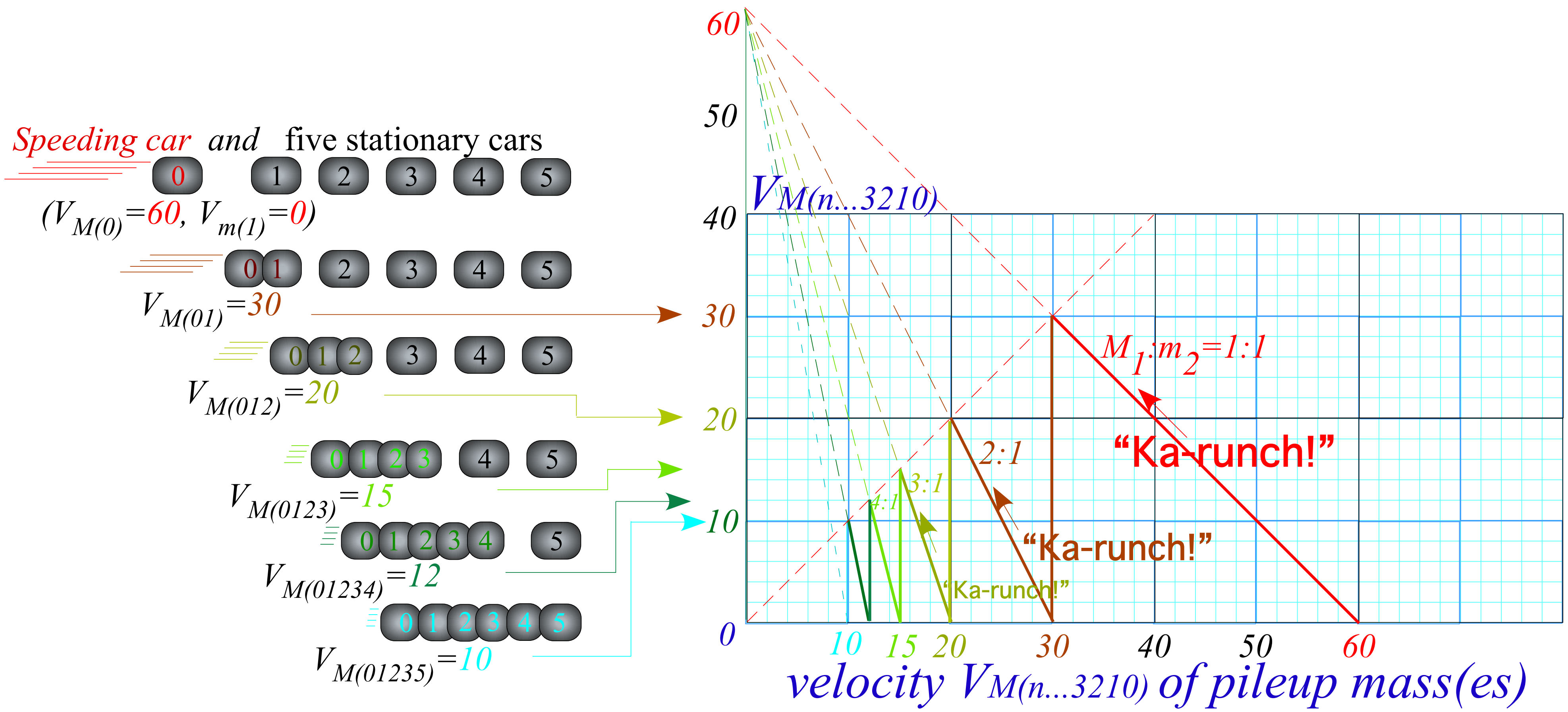
Opposite of continuous wave dynamics introduced in Unit 2

Crunch energy geometry of freeway crashes and related things

Crunch energy played backwards: This really **<u>is</u>** *"Rocket-Science"*



Unit 1
Fig. 8.5
Pile-up:
One *60mph* car
hits
five standing cars



Unit 1

Fig. 8.5

Pile-up:
One *60mph* car
hits
five standing cars

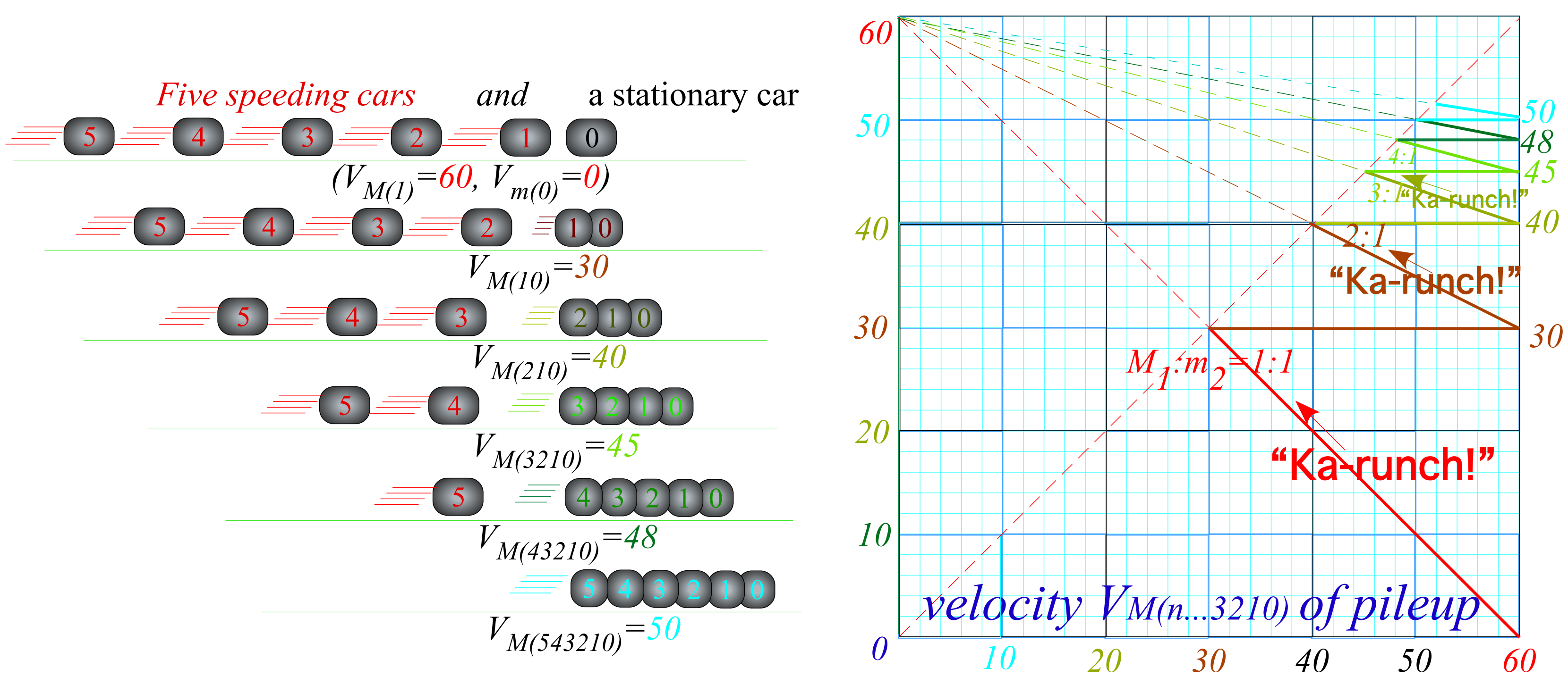


Fig. 8.6

Pile-up:
Five *60mph* cars
hit
one standing cars

Unit 1

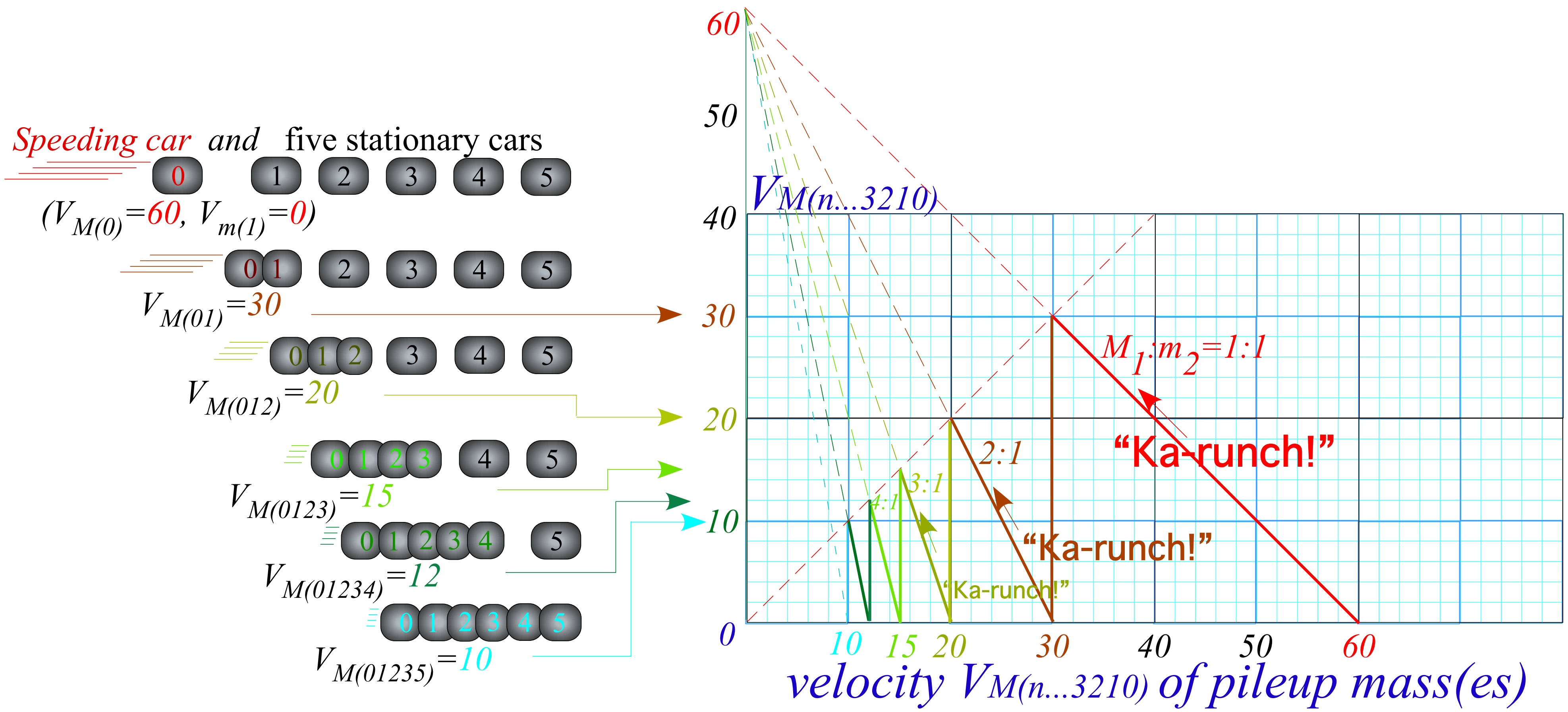


Fig. 8.5
Pile-up:
One *60mph* car
hits
five standing cars

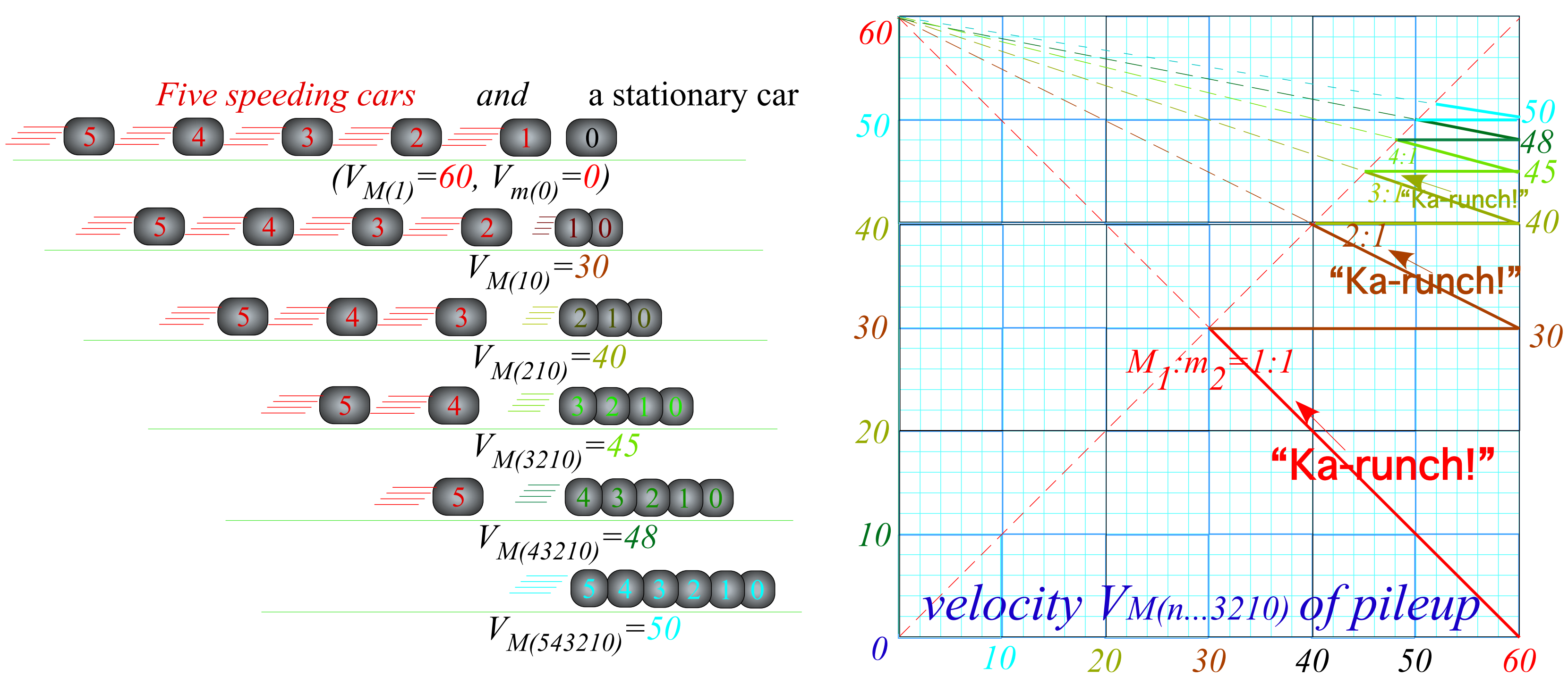
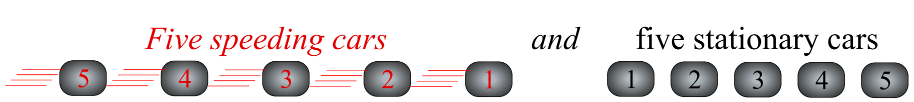


Fig. 8.6
Pile-up:
Five *60mph* cars
hit
one standing cars



(Fug-gedda-aboud-dit!!)

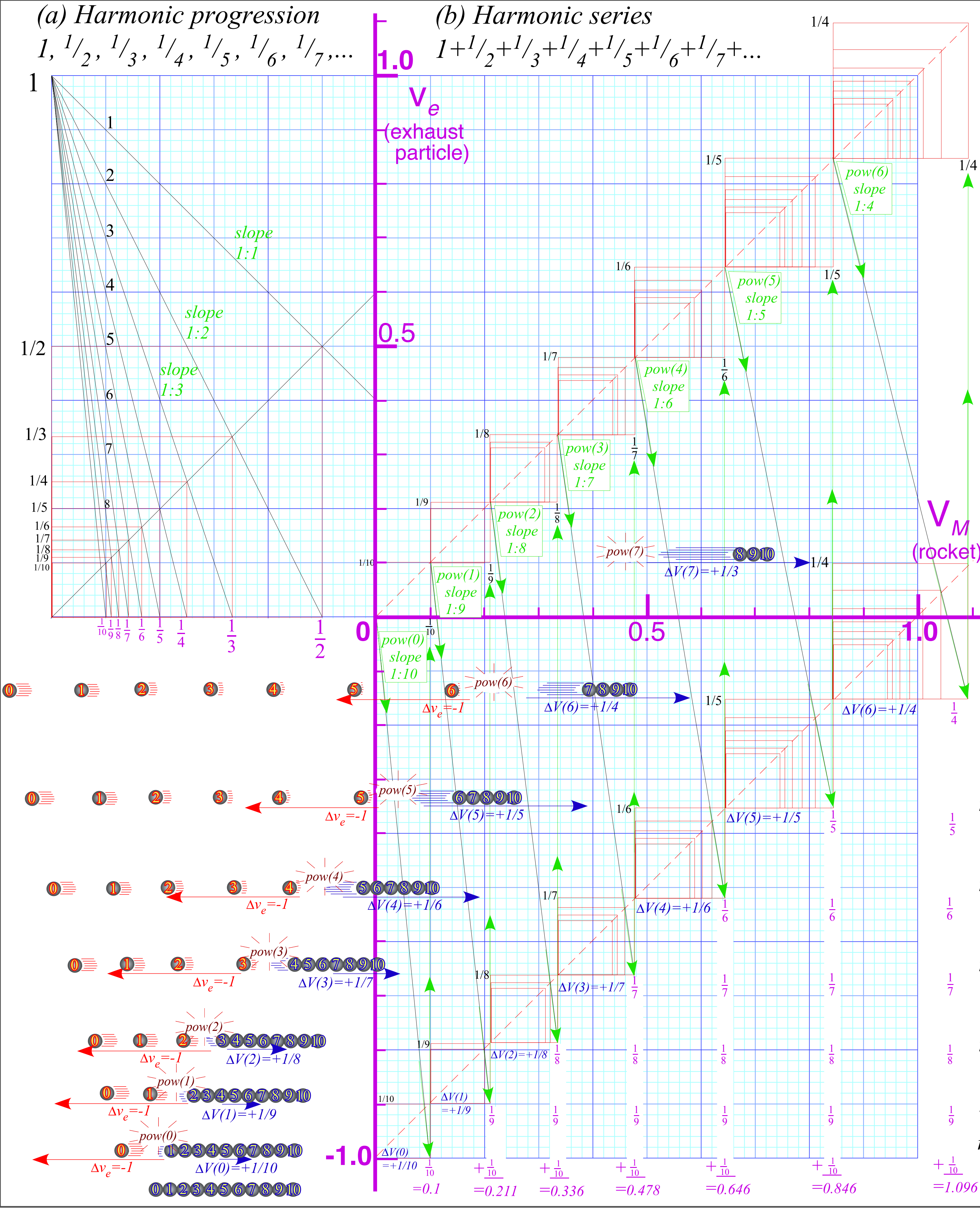
Fig. 8.7
Pile-up:
Five *60mph* cars
hit
five standing cars

Crunch energy geometry of freeway crashes and related things

→ *Crunch energy played backwards: This really **is** "Rocket-Science"*

Unit 1
Fig. 8.8a-b

Rocket Science!



$m{\cdot}\Delta v_7+3m{\cdot}\Delta V_M(7)=0$

$m{\cdot}\Delta v_6+4m{\cdot}\Delta V_M(6)=0$

$m{\cdot}\Delta v_5+5m{\cdot}\Delta V_M(5)=0$

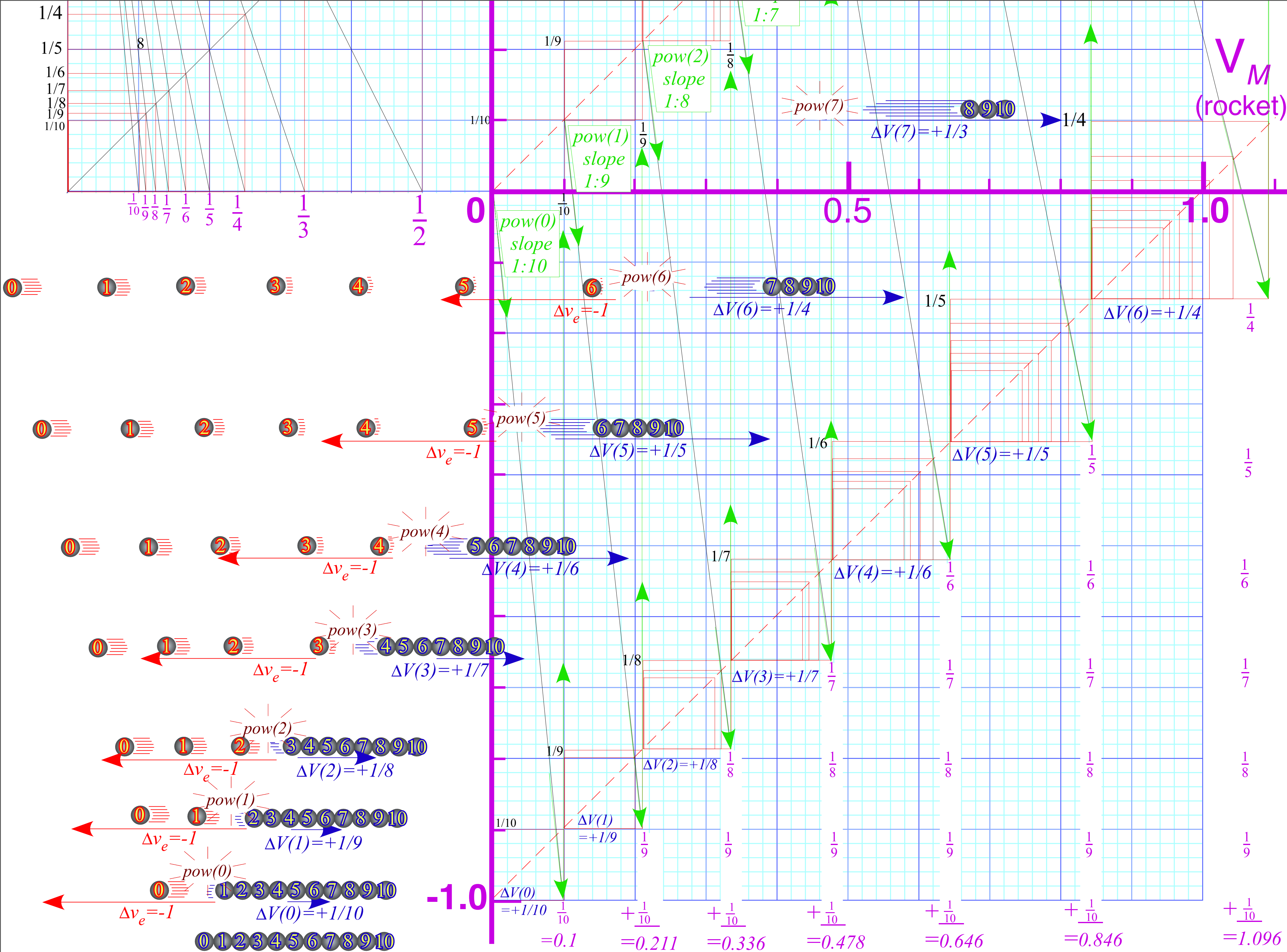
$m{\cdot}\Delta v_4+6m{\cdot}\Delta V_M(4)=0$

$m{\cdot}\Delta v_3+7m{\cdot}\Delta V_M(3)=0$

$m{\cdot}\Delta v_2+8m{\cdot}\Delta V_M(2)=0$

$m{\cdot}\Delta v_1+9m{\cdot}\Delta V_M(1)=0$

$m{\cdot}\Delta v_0+10m{\cdot}\Delta V_M(0)=0$



0^{th}: $V(0)=1/10=0.1$ 1^{st}: $V(1)=1/10+1/9=0.211$ 2^{nd}: $V(2)=1/10+1/9+1/8=0.336$

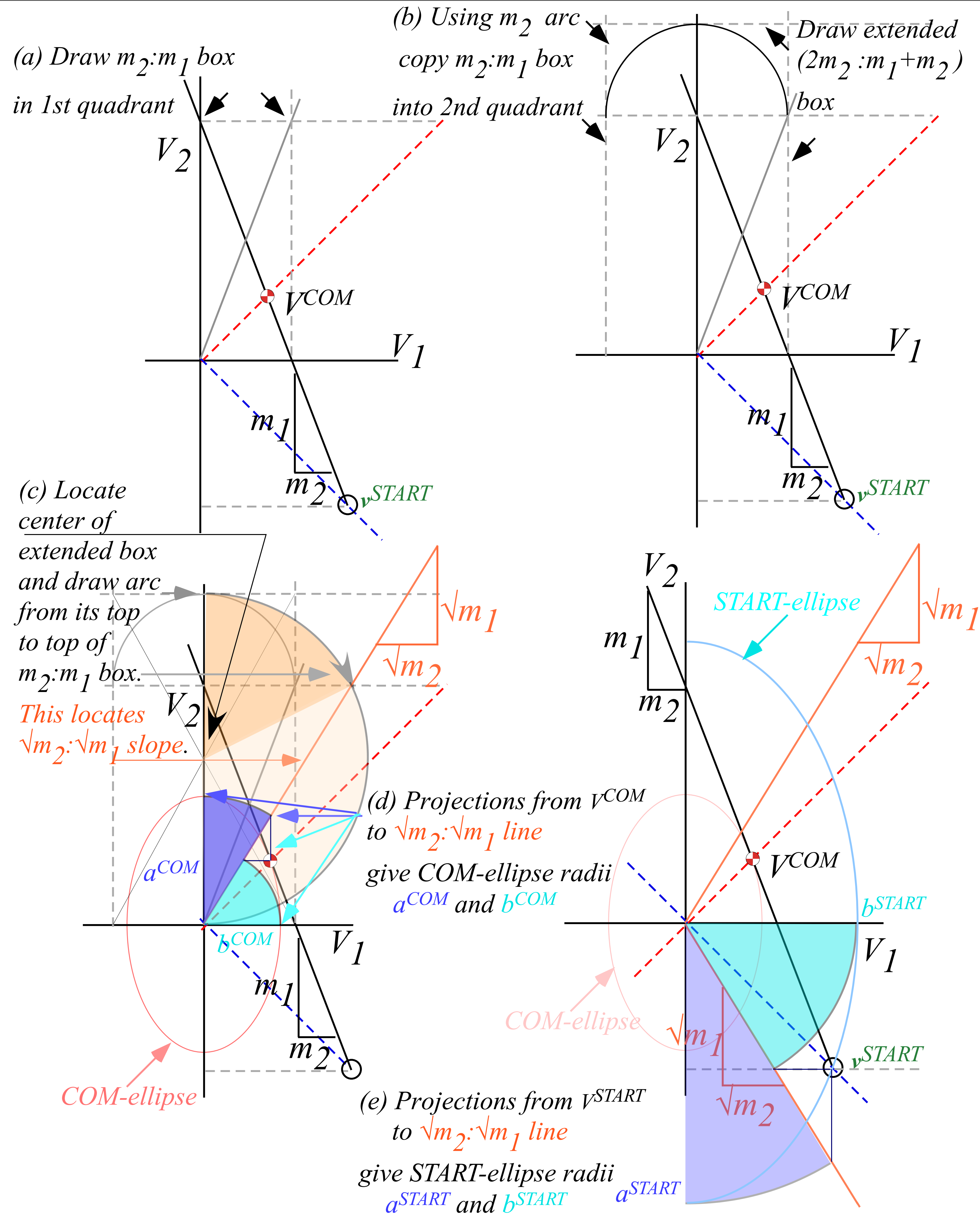
3^{rd}: $V(3)=V(2)+1/7=0.478$ 4^{th}: $V(4)=V(3)+1/6=0.646$ 5^{th}: $V(5)=V(4)+1/5=0.846$

6^{th}: $V(6)=V(5)+1/4=1.096$ 7^{th}: $V(7)= V(6)+1/3=1.429$ 8^{th}: $V(8)=V(7)+1/2=1.929$

By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\dfrac{dM}{M}$

The Rocket Equation: $V_{FIN}-V_{IN}=-v_e\left[\ln M_{FIN}-\ln M_{IN}\right]=v_e\left[\ln\dfrac{M_{IN}}{M_{FIN}}\right]$

A Thales construction for momentum-energy


(a) Draw m_2:m_1 box
in 1st quadrant
V_2
V^{COM}
V_1
m_1
m_2
v^{START}
(b) Using m_2 arc
copy m_2:m_1 box
into 2nd quadrant
Draw extended
($2m_2$:m_1+m_2)
box
V_2
V^{COM}
V_1
m_1
m_2
v^{START}
(c) Locate
center of
extended box
and draw arc
from its top
to top of
m_2:m_1 box.
This locates
$\sqrt{m_2}$:$\sqrt{m_1}$ slope.
V_2
$\sqrt{m_1}$
$\sqrt{m_2}$
a^{COM}
b^{COM}
V_1
m_1
m_2
COM-ellipse
(d) Projections from V^{COM}
to $\sqrt{m_2}$:$\sqrt{m_1}$ line
give COM-ellipse radii
a^{COM} and b^{COM}
V_2
m_1
m_2
START-ellipse
$\sqrt{m_1}$
$\sqrt{m_2}$
V^{COM}
b^{START}
V_1
COM-ellipse
$\sqrt{m_1}$
$\sqrt{m_2}$
v^{START}
(e) Projections from V^{START}
to $\sqrt{m_2}$:$\sqrt{m_1}$ line
give START-ellipse radii
a^{START} and b^{START}
a^{START}