Lecture 6

Revised 12.21.12 from 9.06.2012

Many-body 1D collisions

Elastic examples: Western buckboard, Bouncing column, Newton's cradle

Inelastic examples: "Zig-zag geometry" of freeway crashes

Super-elastic examples: This really **is** *"Rocket-Science"*

Geometry of common power-law potentials

Geometric (Power) series

- *"Zig-Zag" exponential geometry*
- *Projective or perspective geometry*

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

- *Compare mks units of Coulomb Electrostatic vs. Gravity*

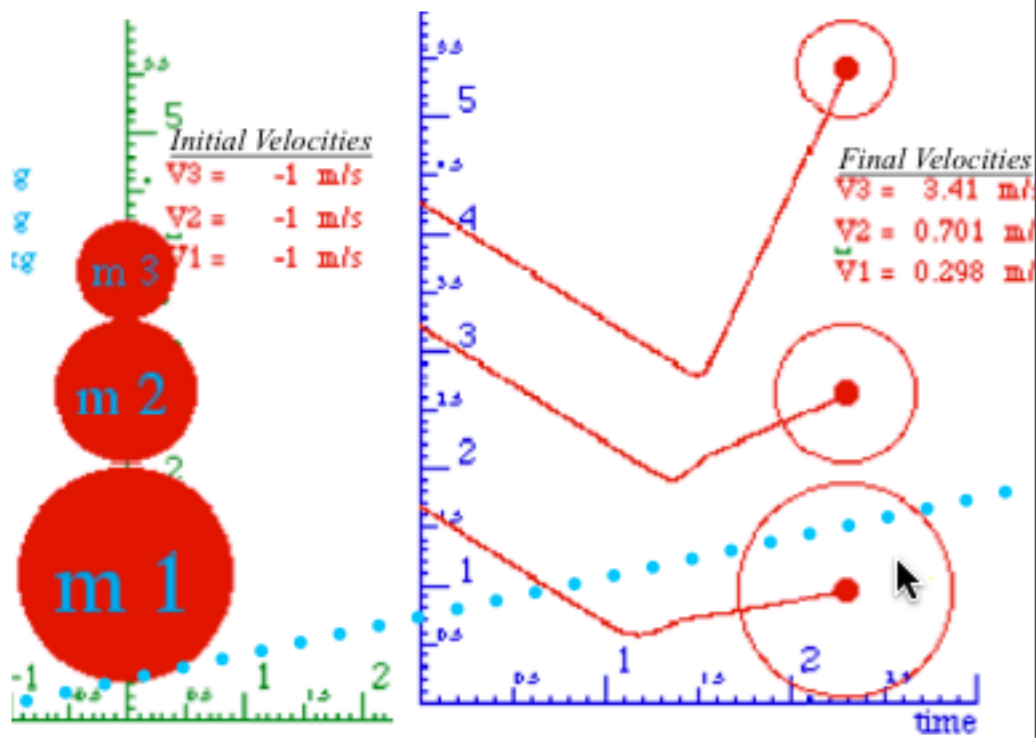
Many-body 1D collisions

→ *Elastic examples: Western buckboard, Bouncing column, Newton's cradle*

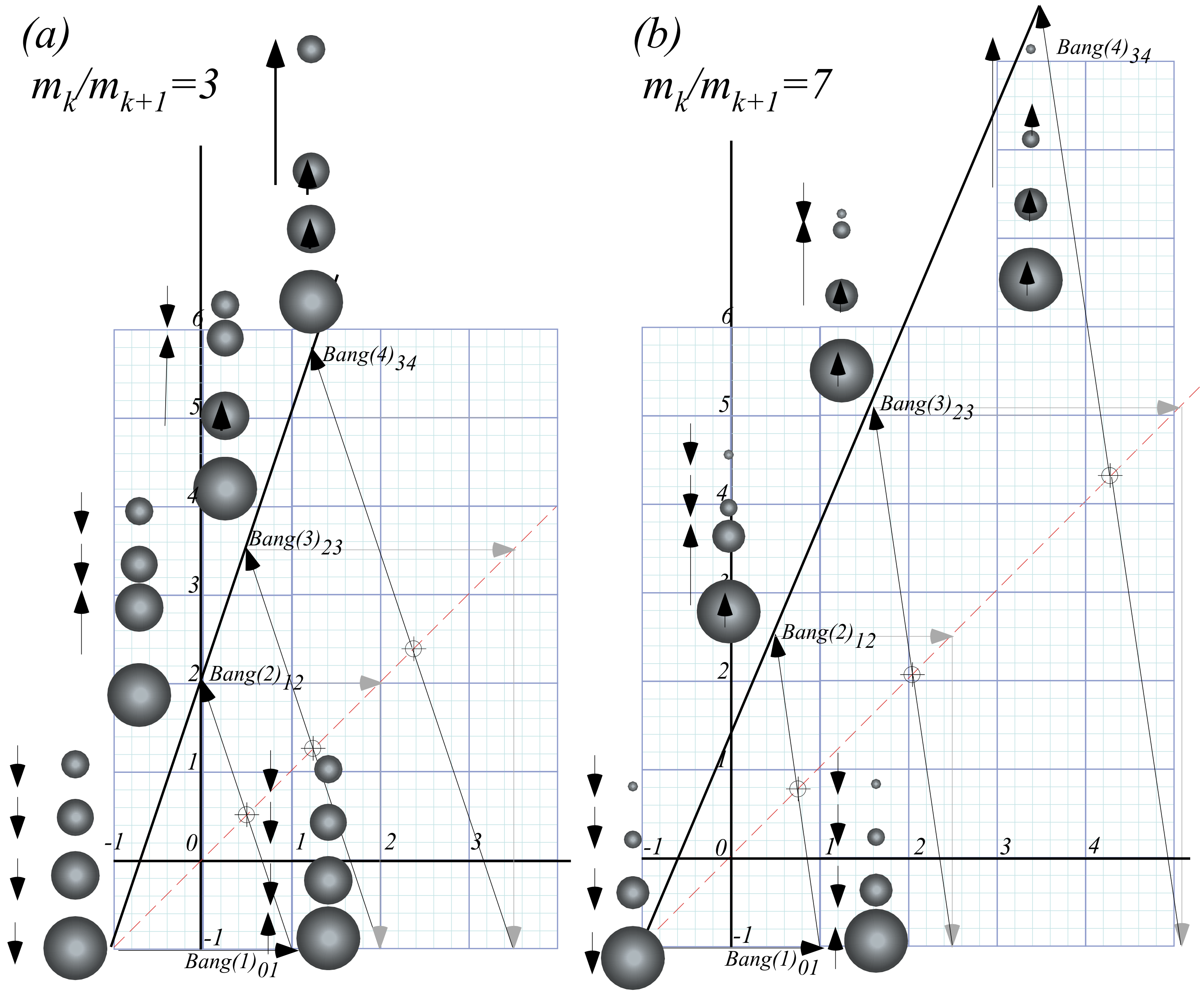
Inelastic examples: "Zig-zag geometry" of freeway crashes

Super-elastic examples: This really **is** *"Rocket-Science"*

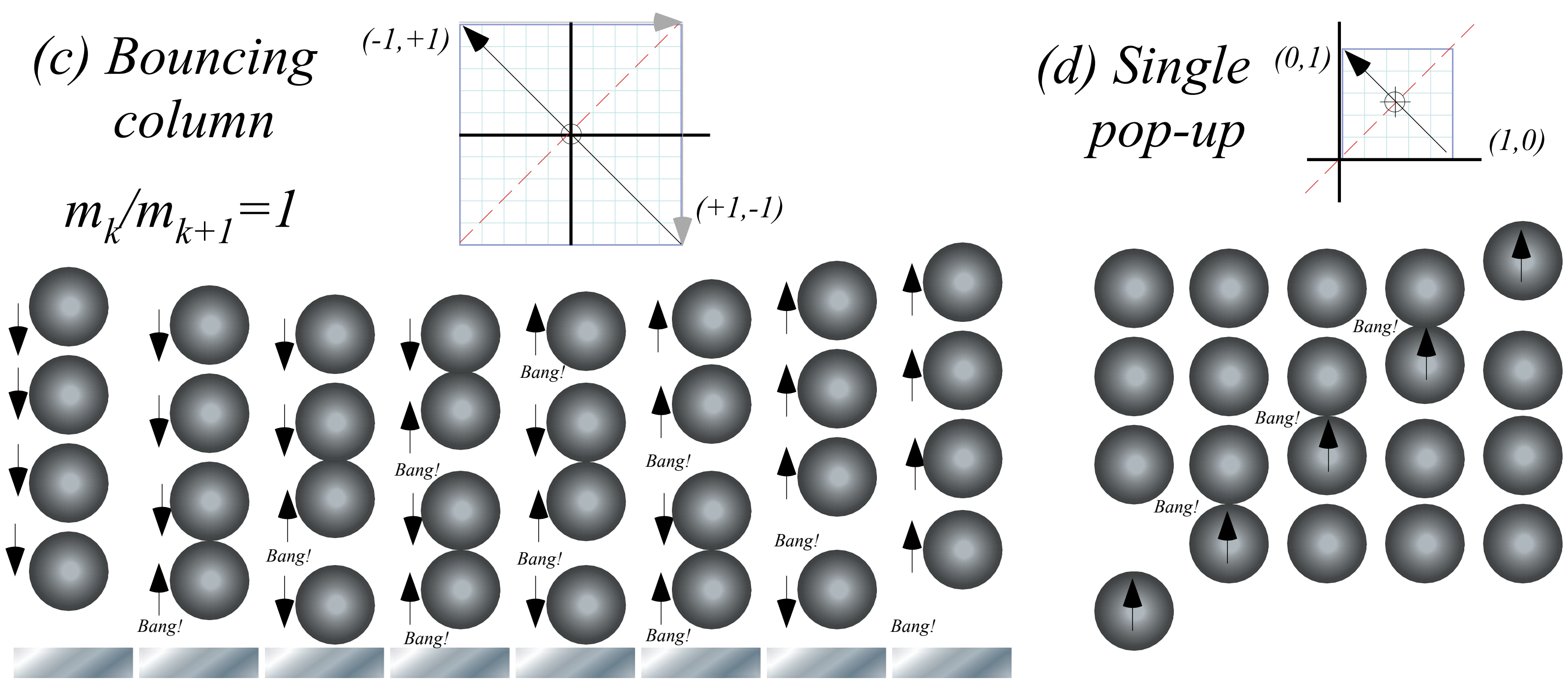
Western buckboard = *?????*



Western buckboard = Disaster!



Unit 1
Fig. 8.2a-b
4-Body IBM Geometry
Fig. 8.2c-d
4-Equal-Body Geometry



4-Equal-Body
"Shockwave" or pulse wave
Dynamics

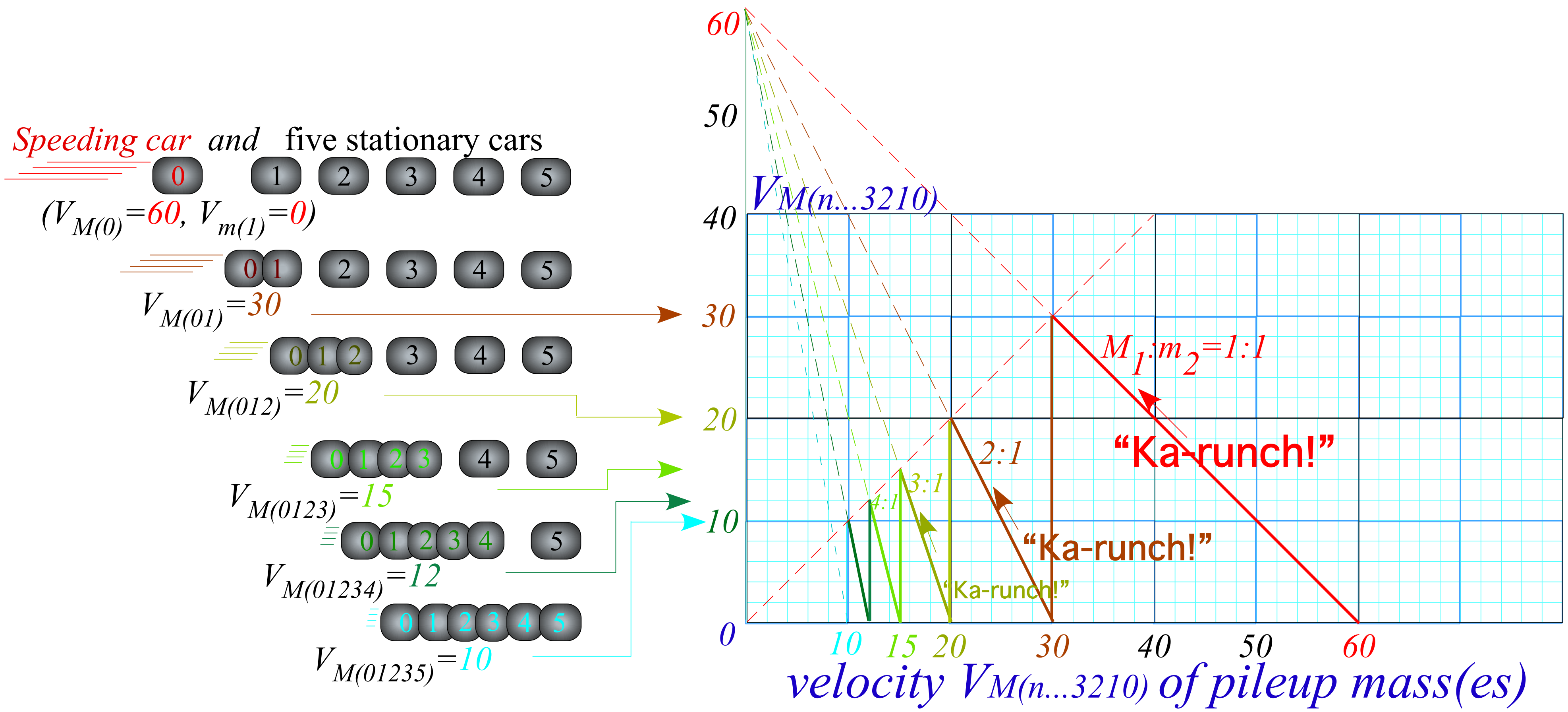
Opposite of continuous wave dynamics introduced in Unit 2

Many-body 1D collisions

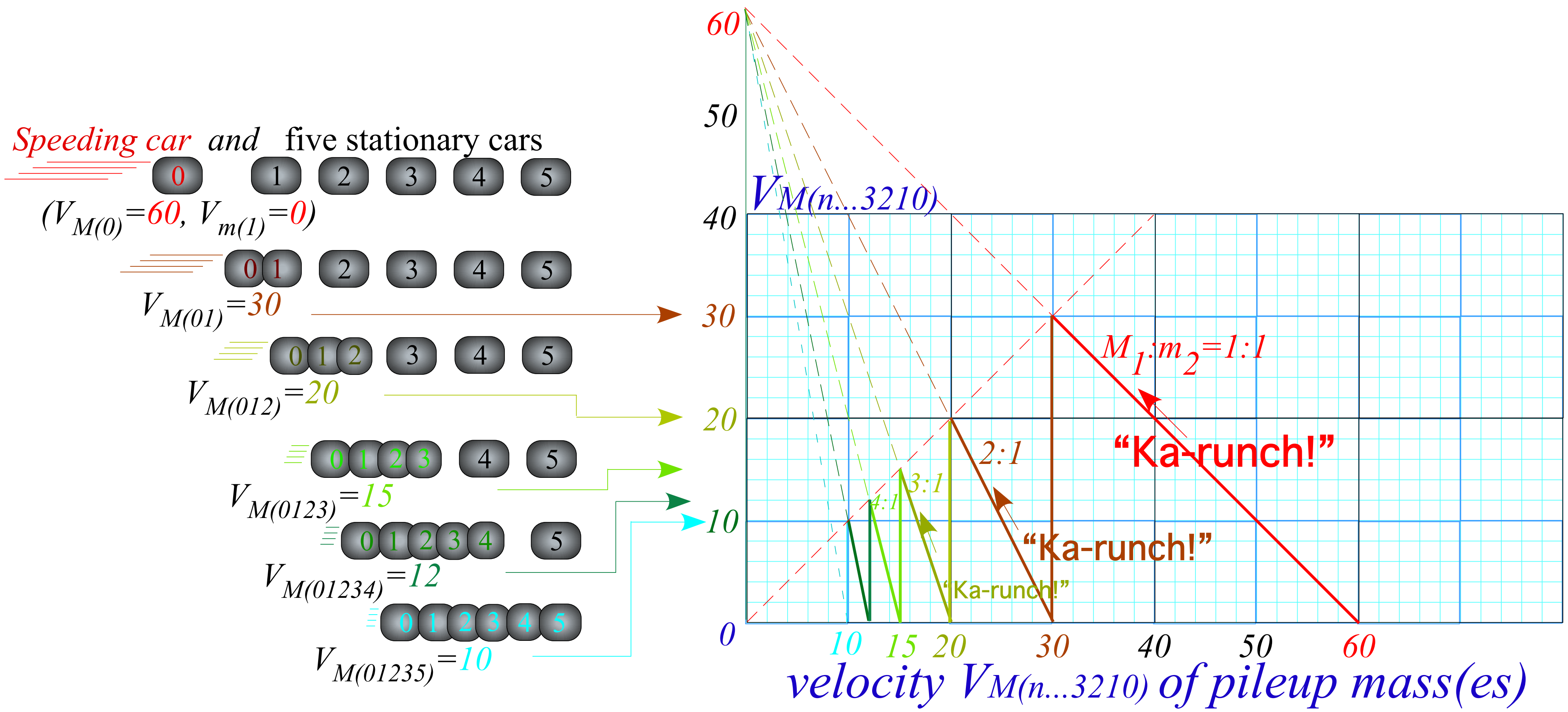
Elastic examples: Western buckboard, Bouncing column, Newton's cradle

→ *Inelastic examples: "Zig-zag geometry" of freeway crashes*

Super-elastic examples: This really **is** *"Rocket-Science"*



Unit 1
Fig. 8.5
Pile-up:
One *60mph* car
hits
five standing cars



Unit 1

Fig. 8.5

Pile-up:
One *60mph* car
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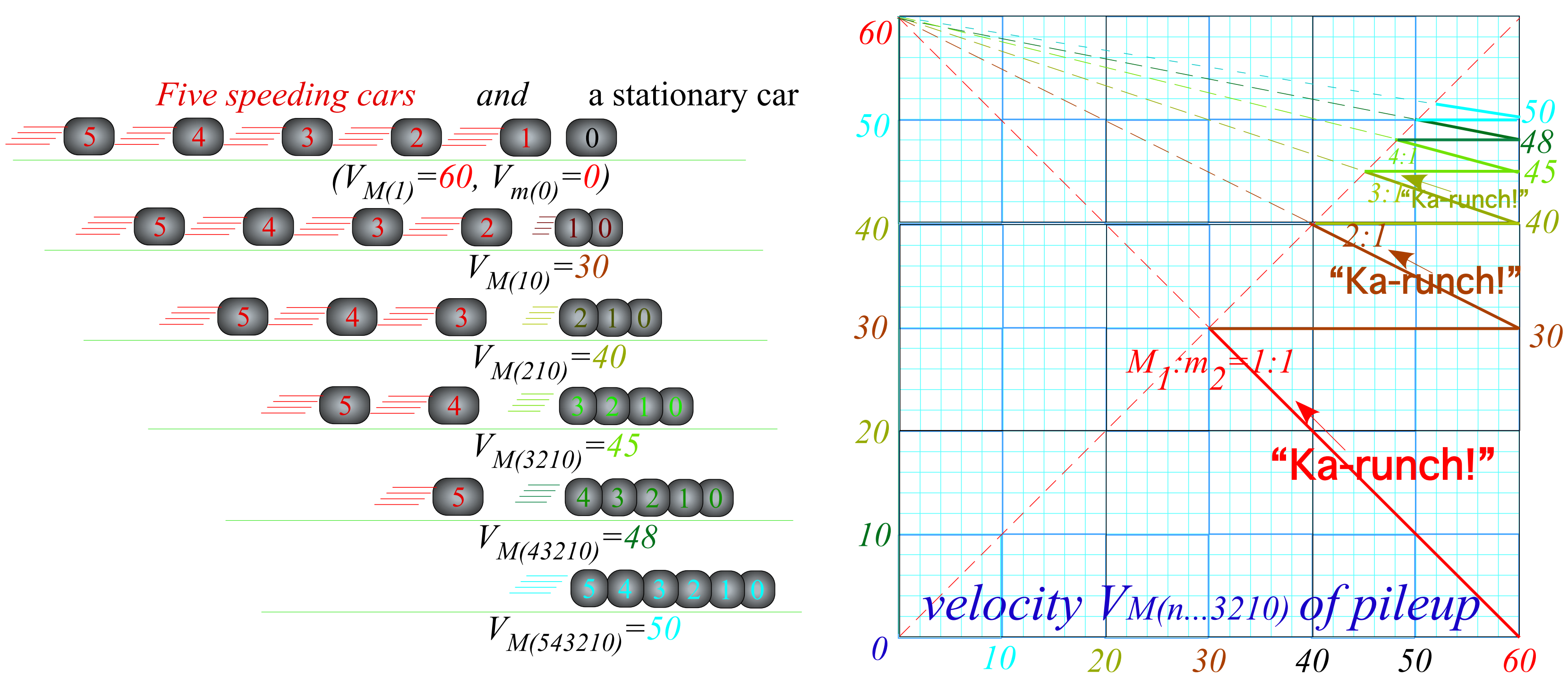

Fig. 8.6

Pile-up:
Five *60mph* cars
hit
one standing cars

Unit 1

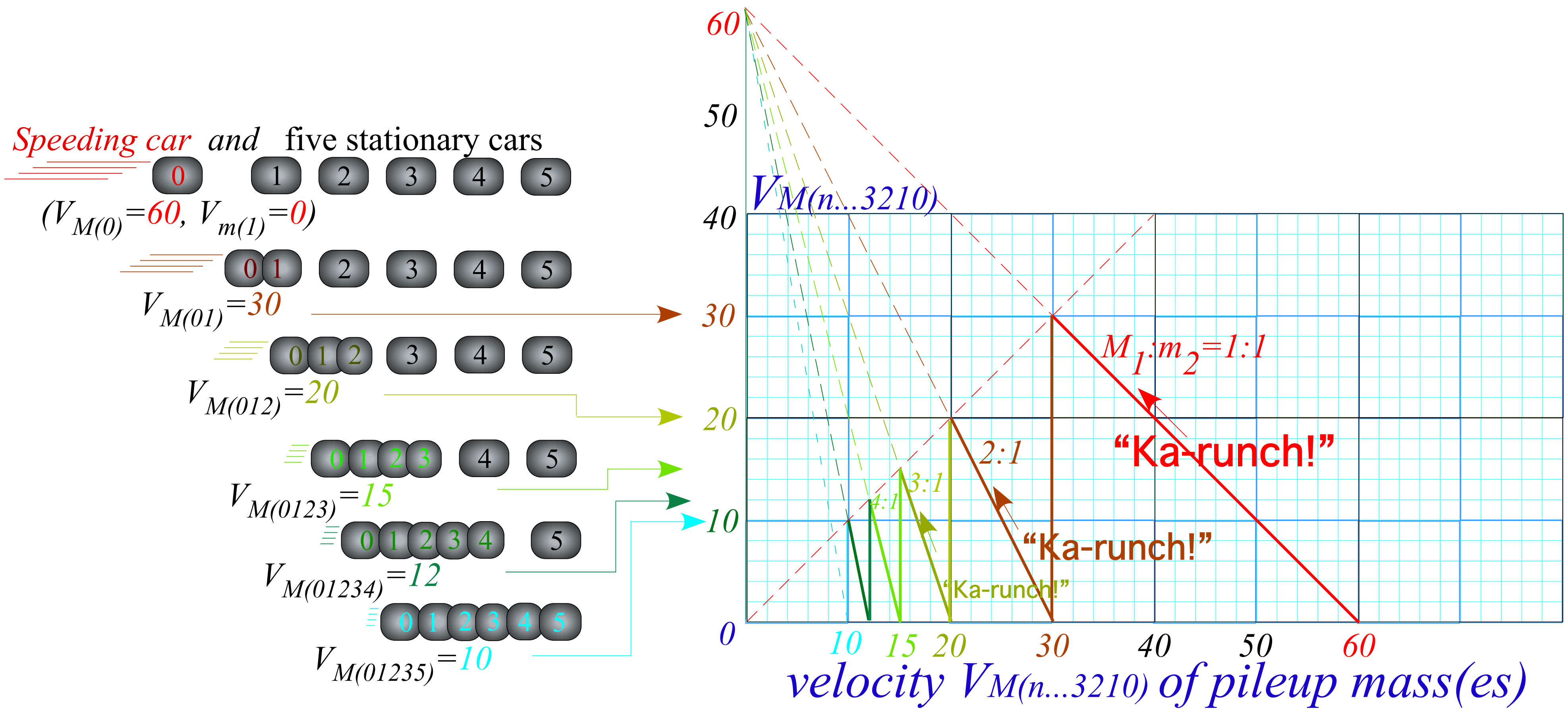

Fig. 8.5

Pile-up:
One *60mph* car
hits
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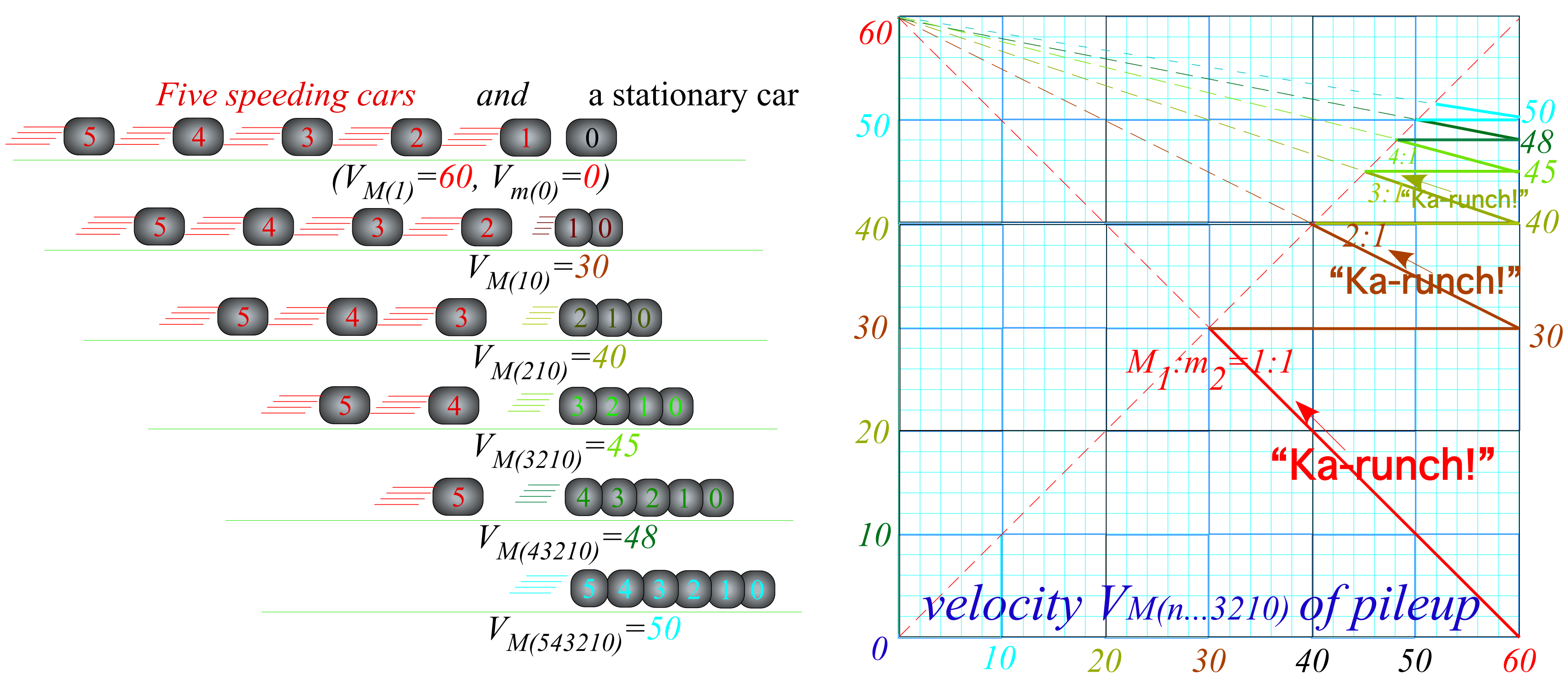
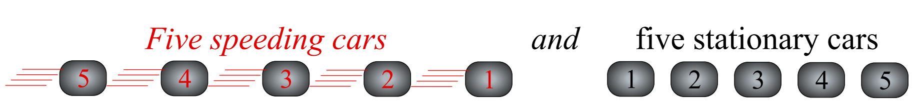


Fig. 8.6

Pile-up:
Five *60mph* cars
hit
one standing cars



(Fug-gedda-aboud-dit!!)

Fig. 8.7

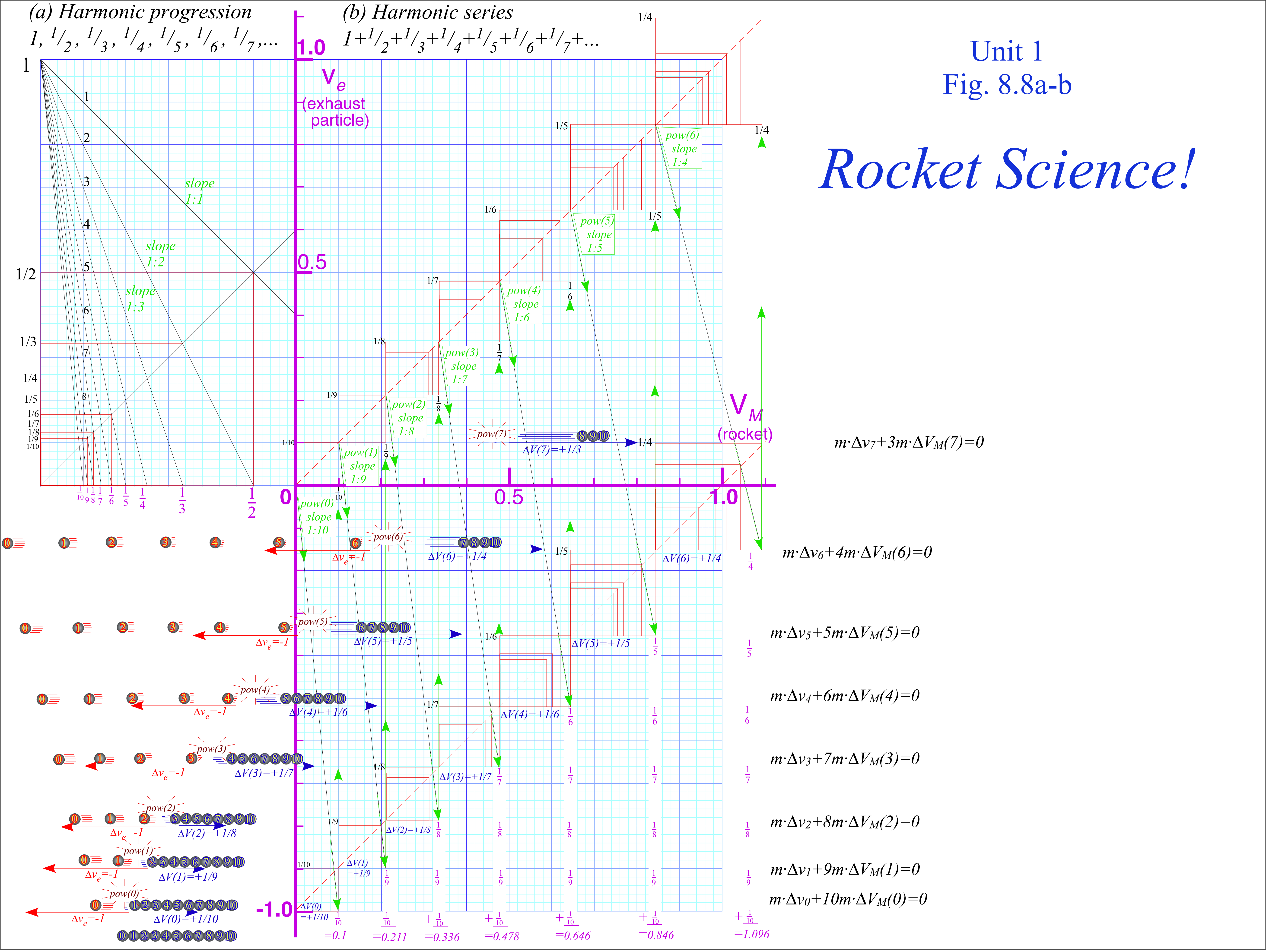
Pile-up:
Five *60mph* cars
hit
five standing cars

Many-body 1D collisions

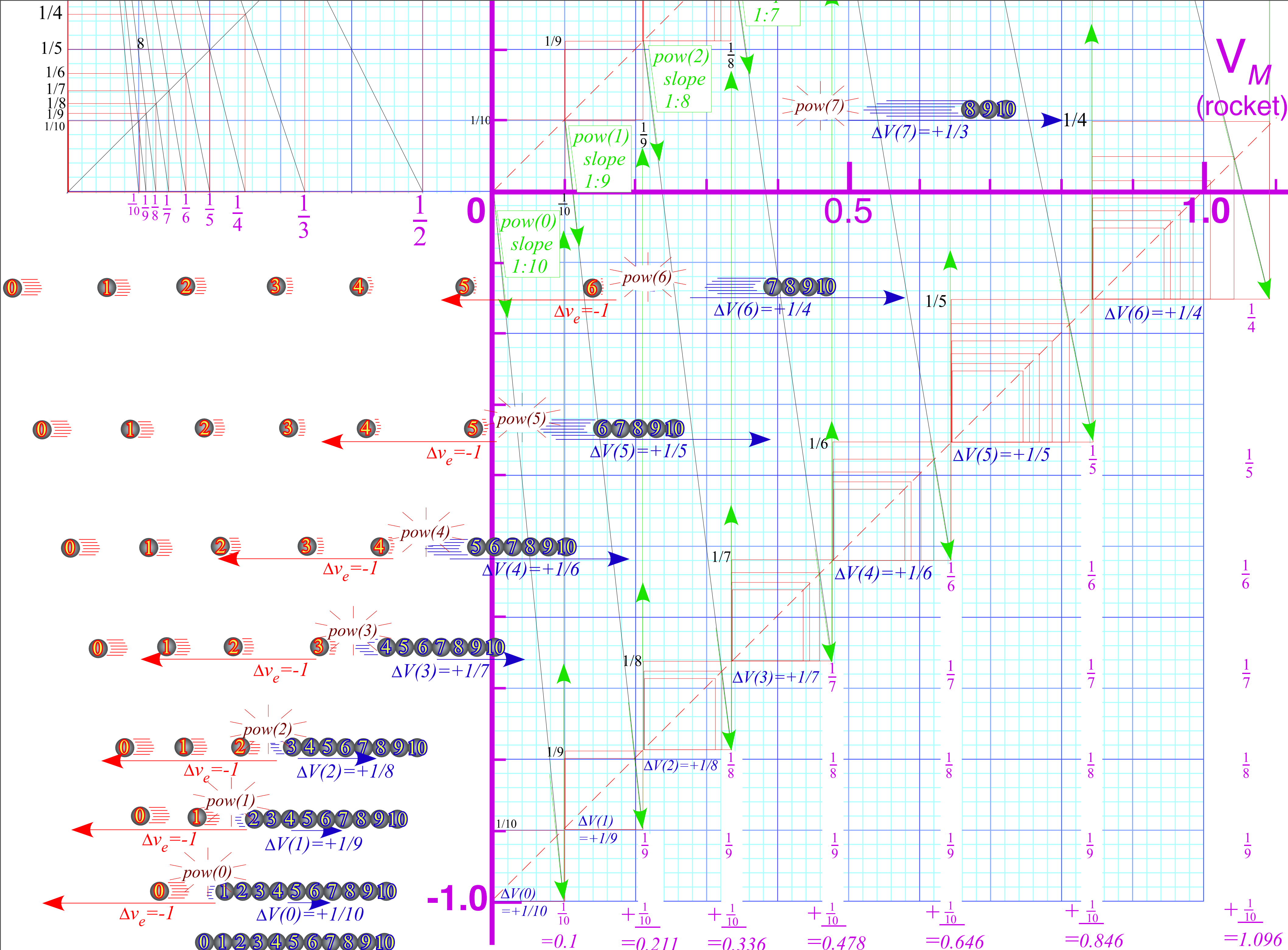
Elastic examples: Western buckboard, Bouncing column, Newton's cradle

Inelastic examples: "Zig-zag geometry" of freeway crashes

→ *Super-elastic examples: This really* **is** *"Rocket-Science"*



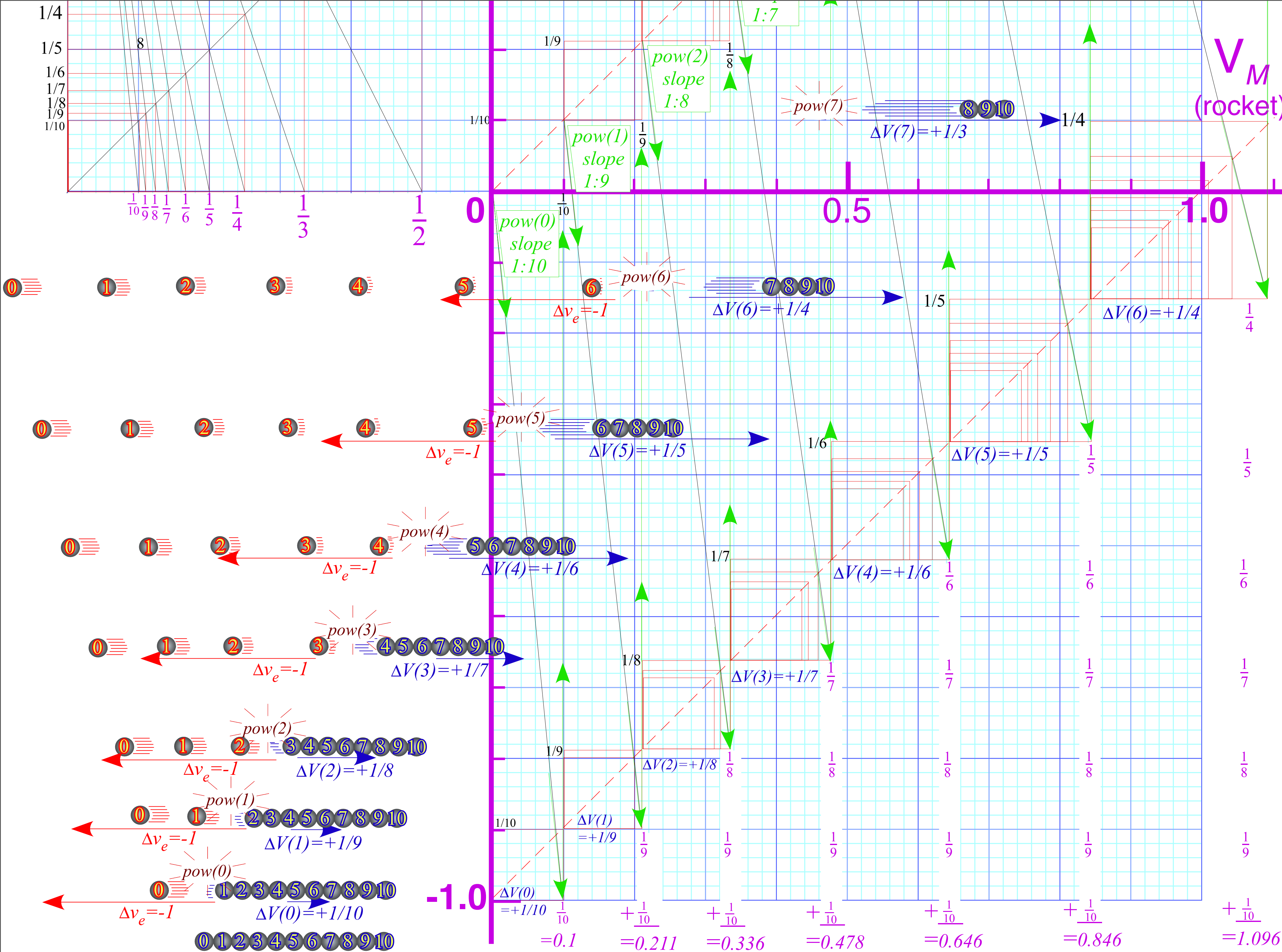
Unit 1
Fig. 8.8a-b
Rocket Science!
(a) Harmonic progression
1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/7,...
(b) Harmonic series
1+1/2+1/3+1/4+1/5+1/6+1/7+...
Ve
(exhaust particle)
VM
(rocket)
slope 1:1
slope 1:2
slope 1:3
pow(6) slope 1:4
pow(5) slope 1:5
pow(4) slope 1:6
pow(3) slope 1:7
pow(2) slope 1:8
pow(1) slope 1:9
pow(0) slope 1:10
ΔV(7)=+1/3
ΔV(6)=+1/4
ΔV(5)=+1/5
ΔV(4)=+1/6
ΔV(3)=+1/7
ΔV(2)=+1/8
ΔV(1)=+1/9
ΔV(0)=+1/10
Δve=-1
m·Δv7+3m·ΔVM(7)=0
m·Δv6+4m·ΔVM(6)=0
m·Δv5+5m·ΔVM(5)=0
m·Δv4+6m·ΔVM(4)=0
m·Δv3+7m·ΔVM(3)=0
m·Δv2+8m·ΔVM(2)=0
m·Δv1+9m·ΔVM(1)=0
m·Δv0+10m·ΔVM(0)=0
=0.1
=0.211
=0.336
=0.478
=0.646
=0.846
=1.096



0^{th}: $V(0)=1/10=0.1$ 1^{st}: $V(1)=1/10+1/9=0.211$ 2^{nd}: $V(2)=1/10+1/9+1/8=0.336$

3^{rd}: $V(3)=V(2)+1/7=0.478$ 4^{th}: $V(4)=V(3)+1/6=0.646$ 5^{th}: $V(5)=V(4)+1/5=0.846$

6^{th}: $V(6)=V(5)+1/4=1.096$ 7^{th}: $V(7)= V(6)+1/3=1.429$ 8^{th}: $V(8)=V(7)+1/2=1.929$



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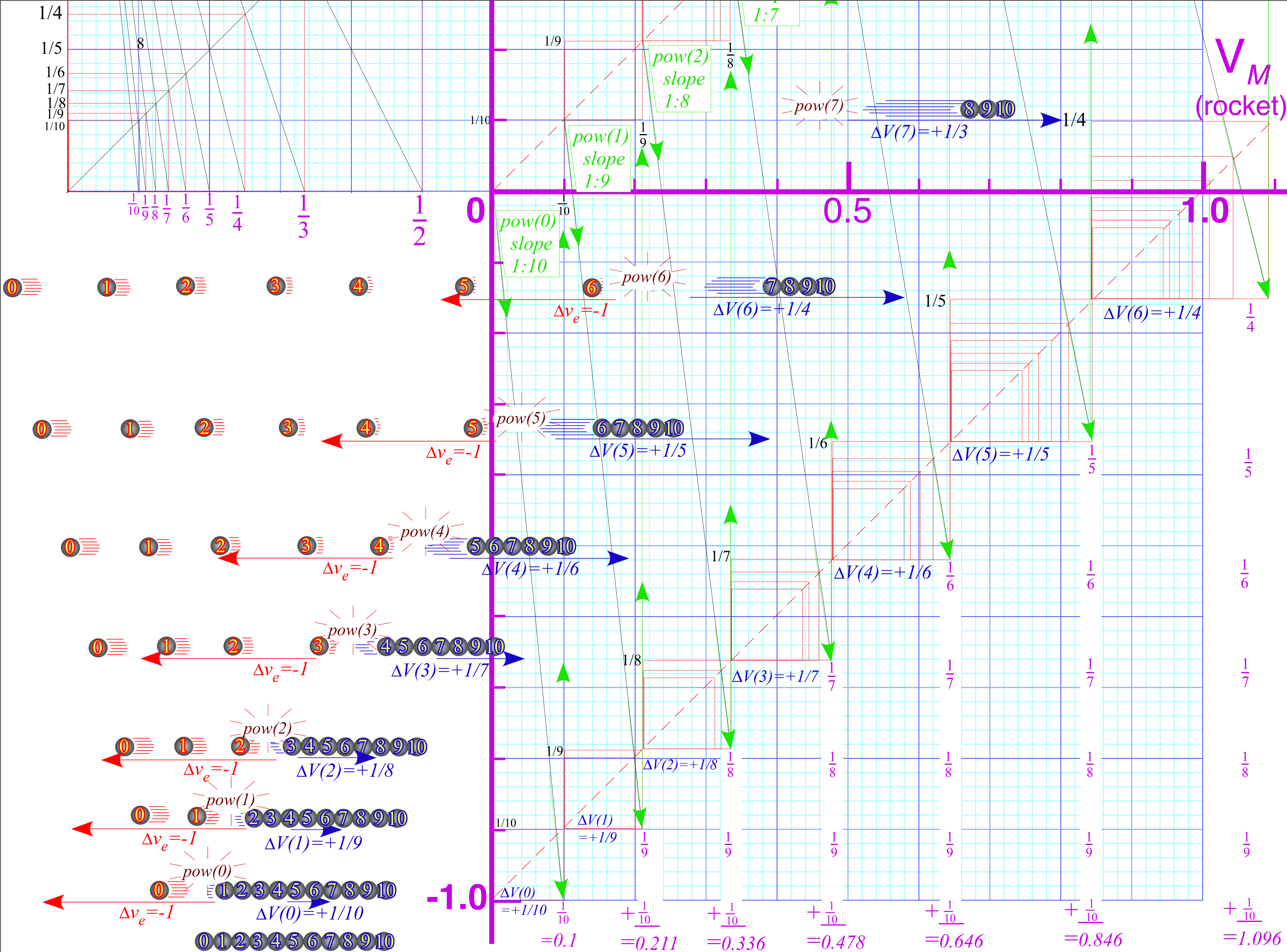
6^{th}: $V(6)=V(5)+1/4=1.096$ 7^{th}: $V(7)= V(6)+1/3=1.429$ 8^{th}: $V(8)=V(7)+1/2=1.929$

v_e *known as* **"Specific Impulse"**

By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\dfrac{dM}{M}$



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v_e known as

"Specific Impulse"

By calculus: $M\cdot\Delta V=-v_e\cdot\Delta M$ or: $dV=-v_e\dfrac{dM}{M}$ Integrate: $\int_{V_{IN}}^{V_{FIN}} dV=-v_e\int_{M_{IN}}^{M_{FIN}}\dfrac{dM}{M}$

The Rocket Equation: $V_{FIN}-V_{IN}=-v_e\left[\ln M_{FIN}-\ln M_{IN}\right]=v_e\left[\ln\dfrac{M_{IN}}{M_{FIN}}\right]$

Geometry of common power-law potentials

Geometric (Power) series

→ *"Zig-Zag" exponential geometry*

Projective or perspective geometry

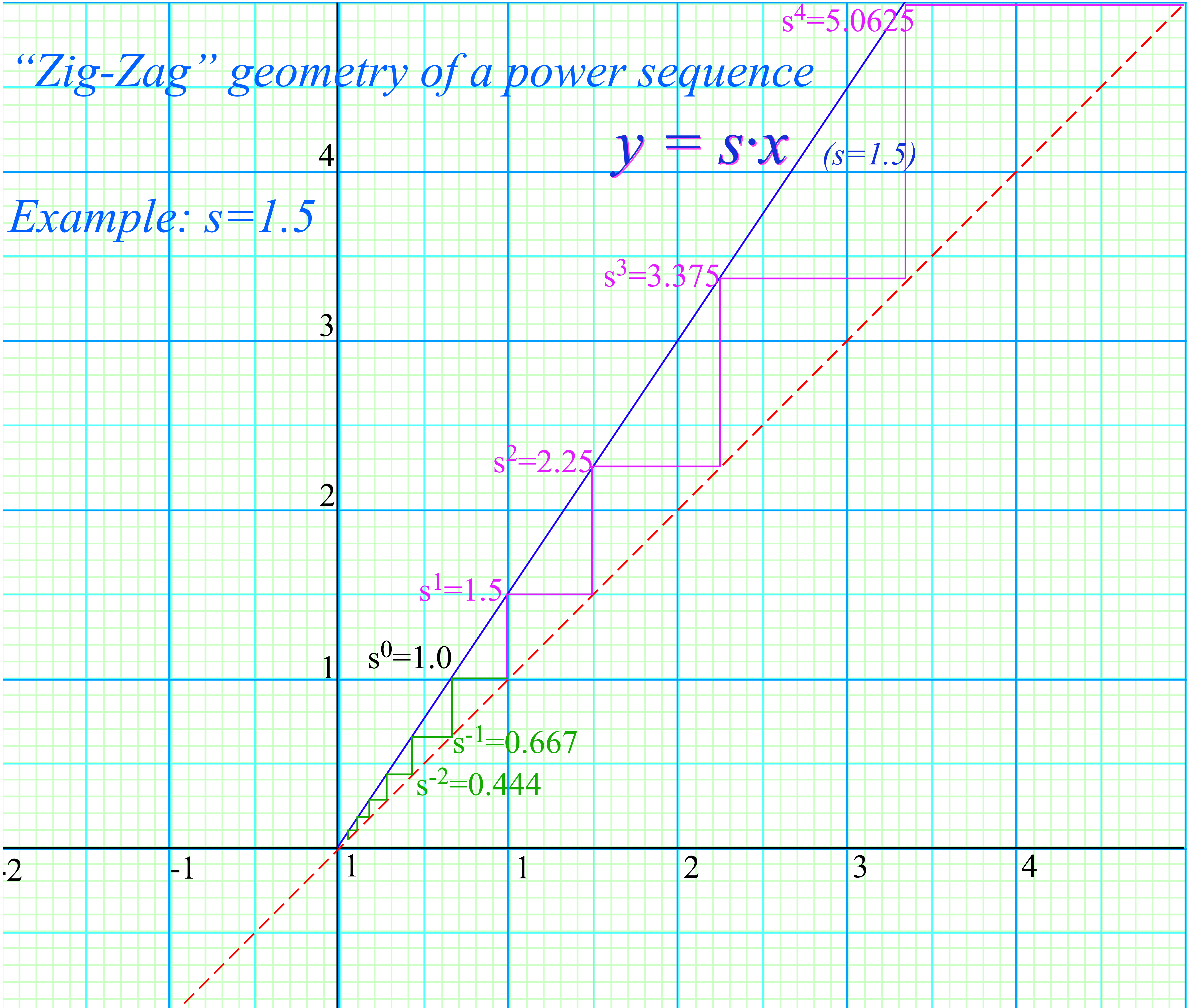
Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

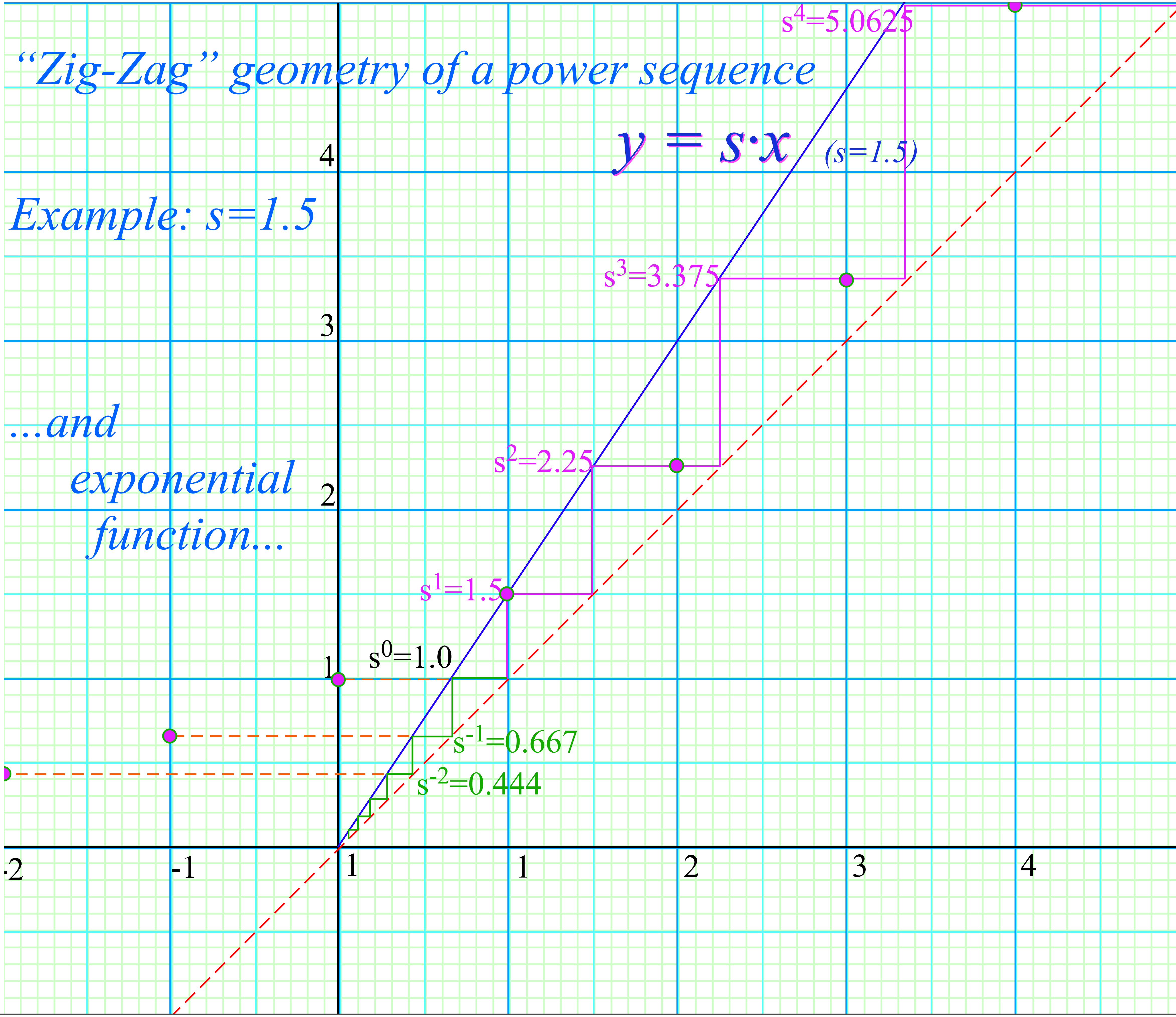
Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

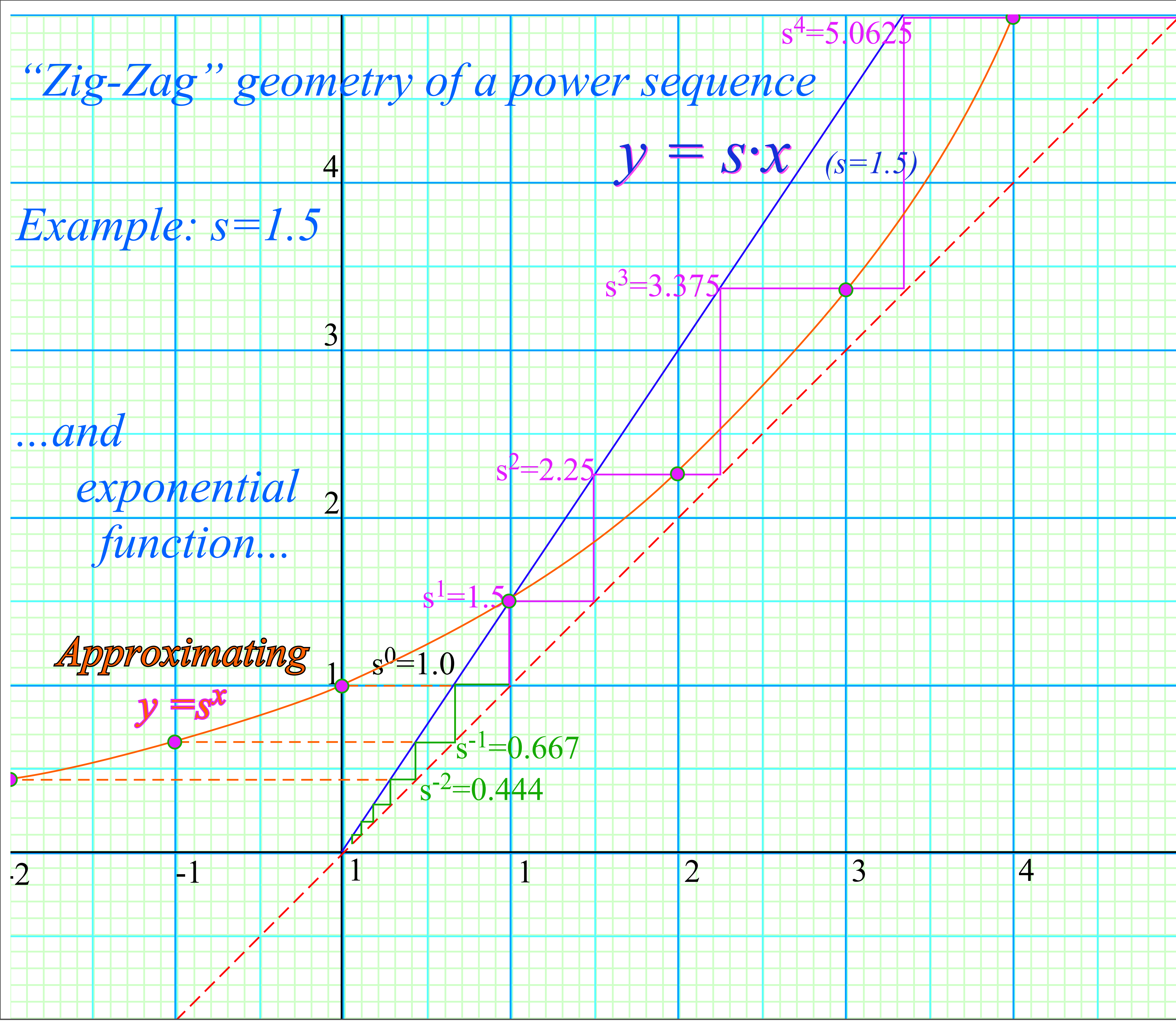
Compare mks units of Coulomb Electrostatic vs. Gravity

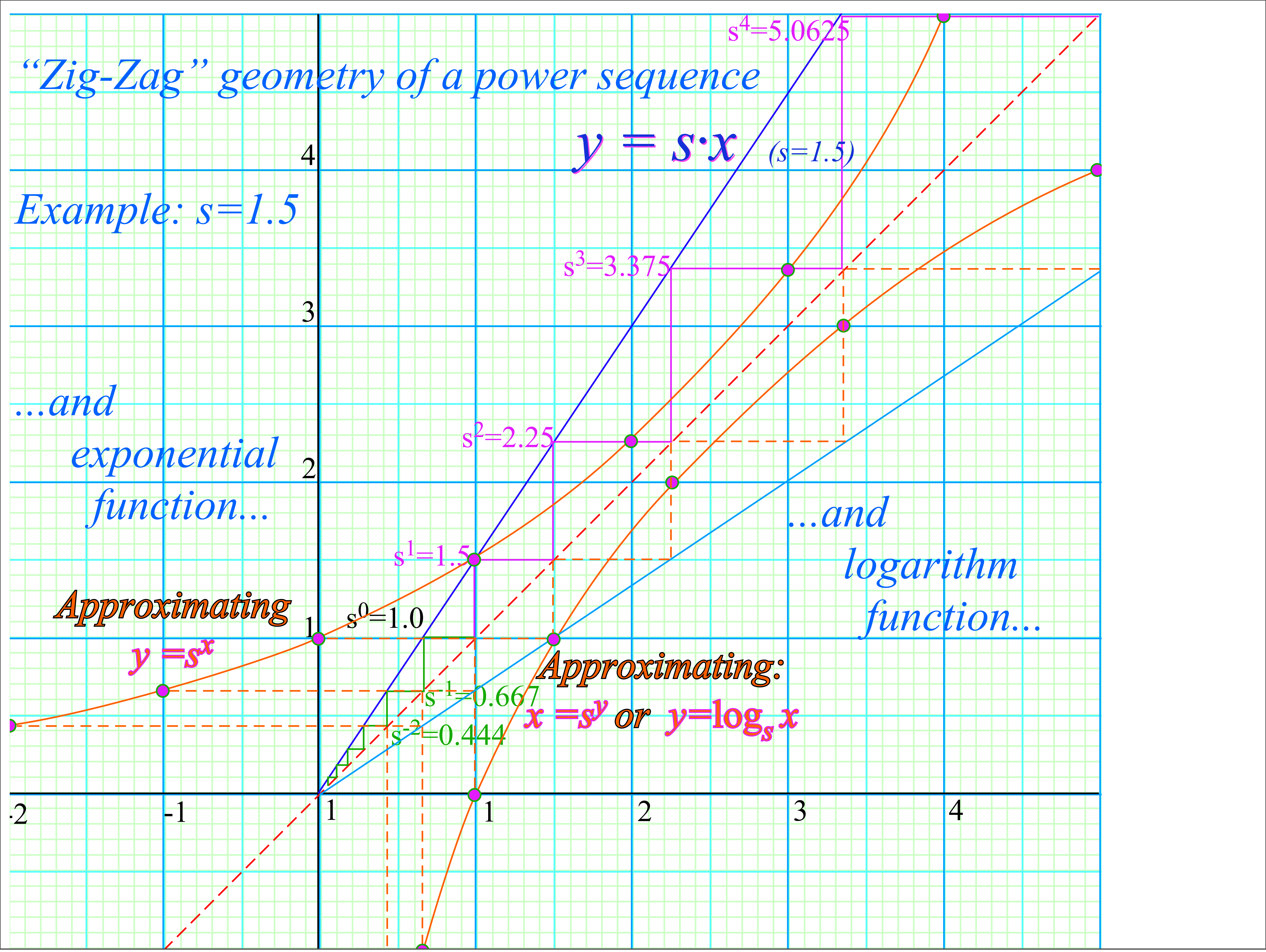
"Zig-Zag" geometry of a power sequence

Example: s=1.5




"Zig-Zag" geometry of a power sequence
Example: s=1.5
...and
exponential
function...
y = s·x
(s=1.5)
s4=5.0625
s3=3.375
s2=2.25
s1=1.5
s0=1.0
s-1=0.667
s-2=0.444
4
3
2
1
-2
-1
1
1
2
3
4


"Zig-Zag" geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
...and
exponential
function...
Approximating
y = s^x
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
s^-1=0.667
s^-2=0.444
4
3
2
1
-2
-1
1
1
2
3
4


"Zig-Zag" geometry of a power sequence
y = s·x (s=1.5)
Example: s=1.5
...and
exponential
function...
...and
logarithm
function...
Approximating
y = s^x
Approximating:
x = s^y or y=log_s x
s^4=5.0625
s^3=3.375
s^2=2.25
s^1=1.5
s^0=1.0
s^-1=0.667
s^-2=0.444
4
3
2
1
-2
-1
1
1
2
3
4

Geometry of common power-law potentials

Geometric (Power) series

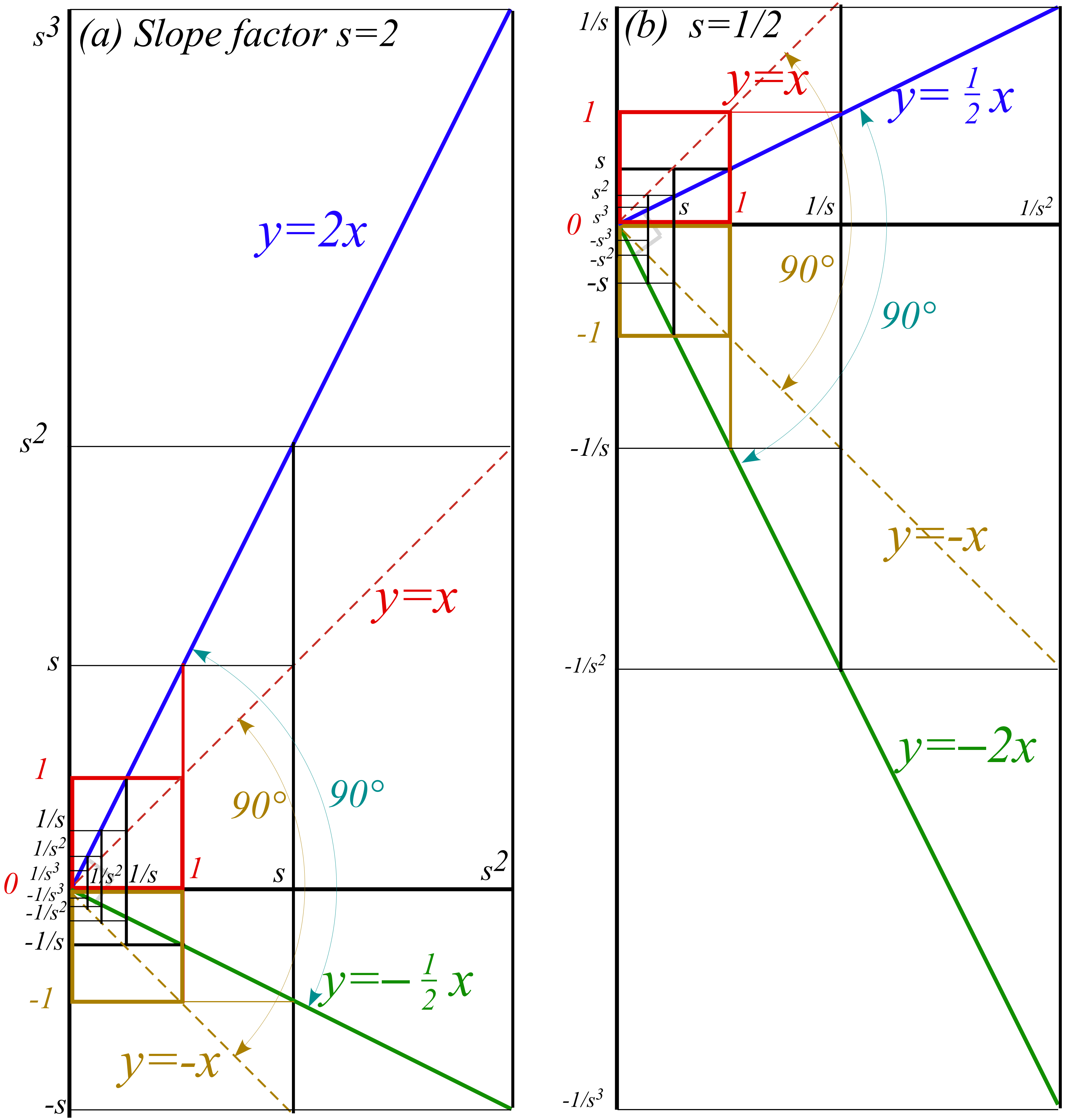
"Zig-Zag" exponential geometry

→ *Projective or perspective geometry*

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

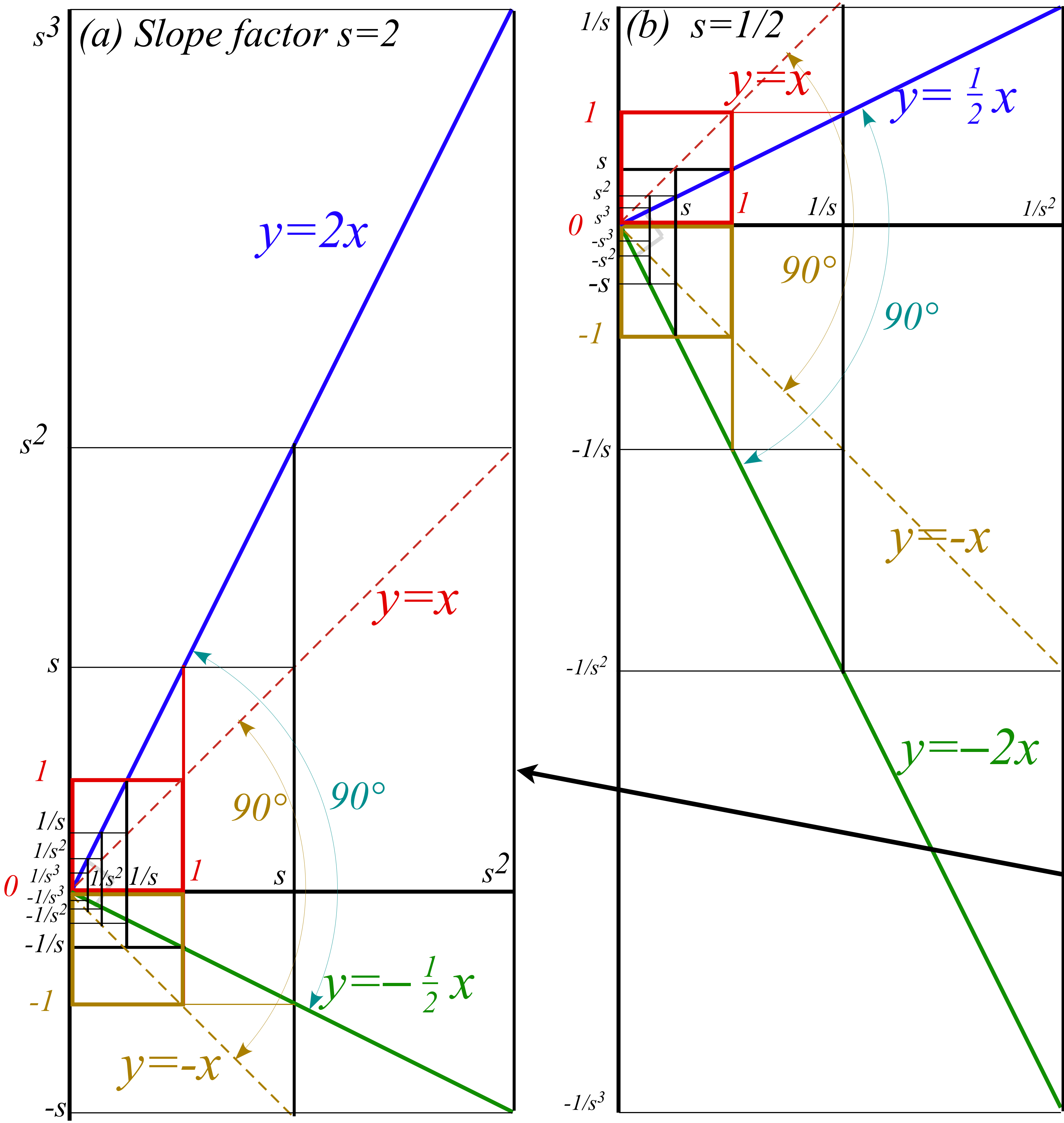
Compare mks units of Coulomb Electrostatic vs. Gravity



"Zig-Zags" give perspective geometry

(1D-vanishing point)

Unit 1
Fig. 9.2



"Zig-Zags" give perspective geometry

(1D-vanishing point)

Unit 1
Fig. 9.2

1st-day-of-school perspective of 12th-grader

1st-day-of-school perspective of 1st-grader

Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

→ *Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields*

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

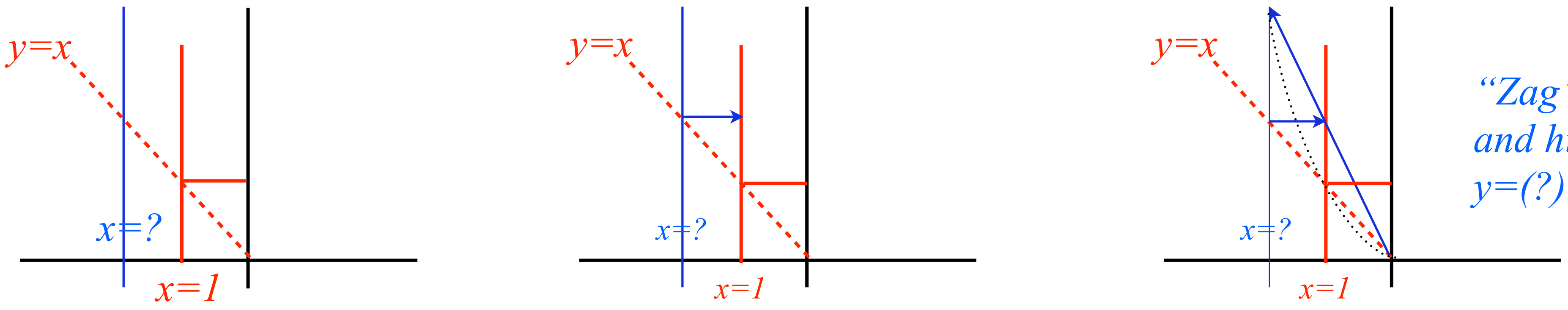
Compare mks units of Coulomb Electrostatic vs. Gravity

Each $y=x^2$ parabola point found by just one "Zig-Zag"

1. Pick an (x=?)-line

2. "Zig" from its y=x intersection to x=1 line

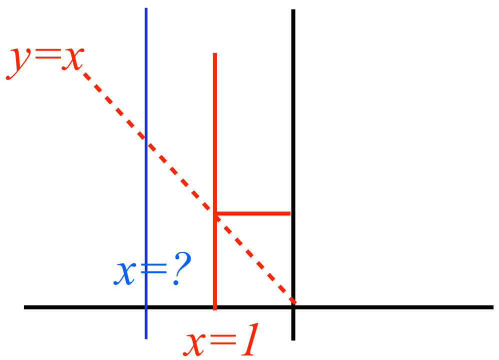
3. "Zag" from origin back to (x=?)-line



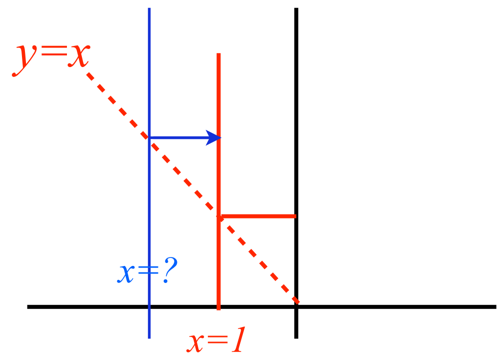
"Zag" line is y=(?)·x and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$

Each $y=x^2$ parabola point found by just one "Zig-Zag"

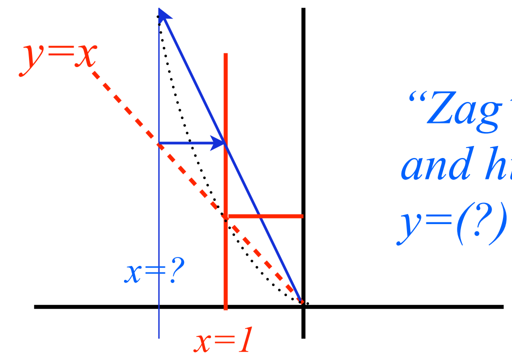
1. Pick an (x=?)-line



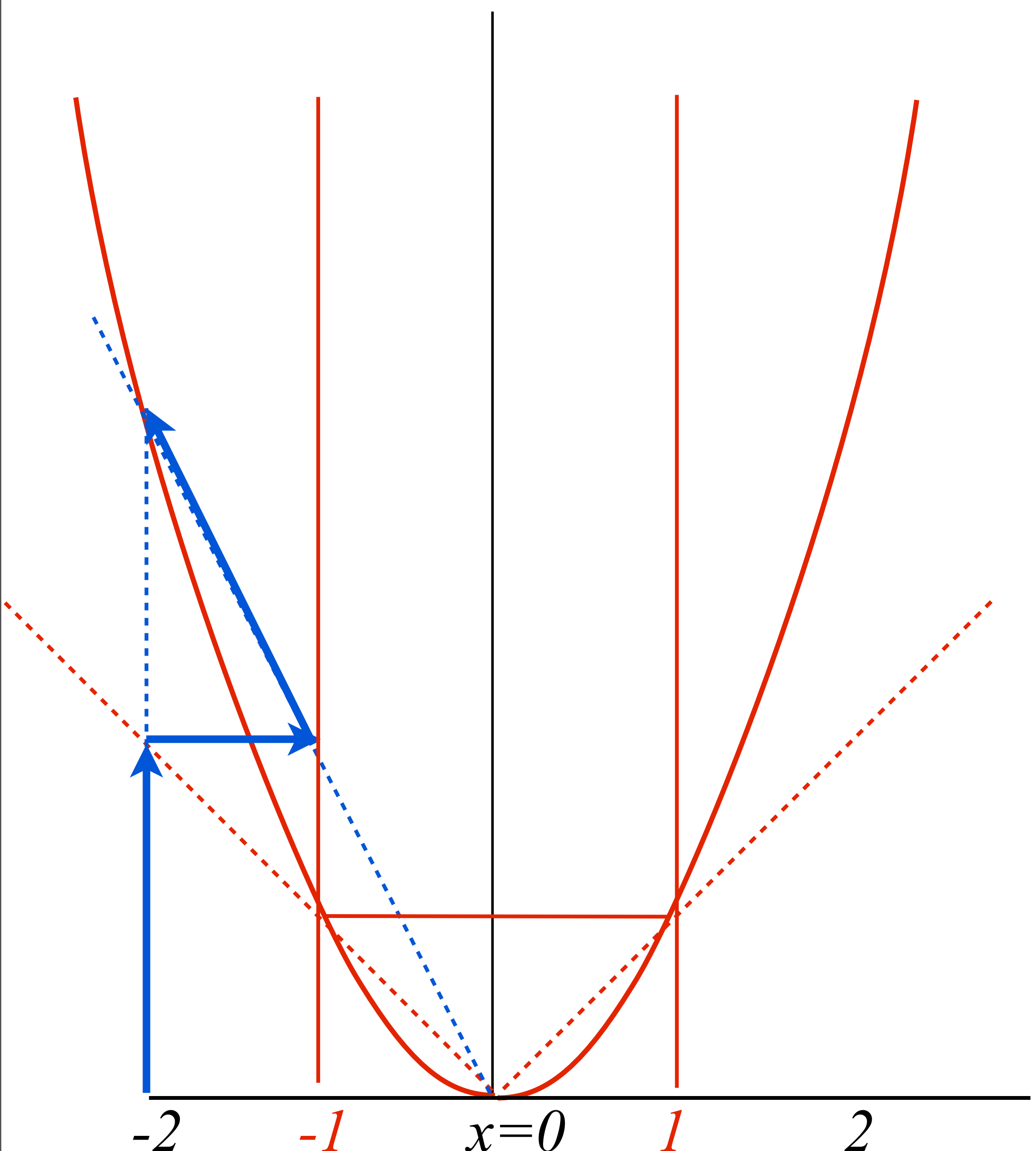
2. "Zig" from its y=x intersection to x=1 line



3. "Zag" from origin back to (x=?)-line



"Zag" line is y=(?)·x and hits (x=?)-line at $y=(?)\cdot(?)=(?)^2$



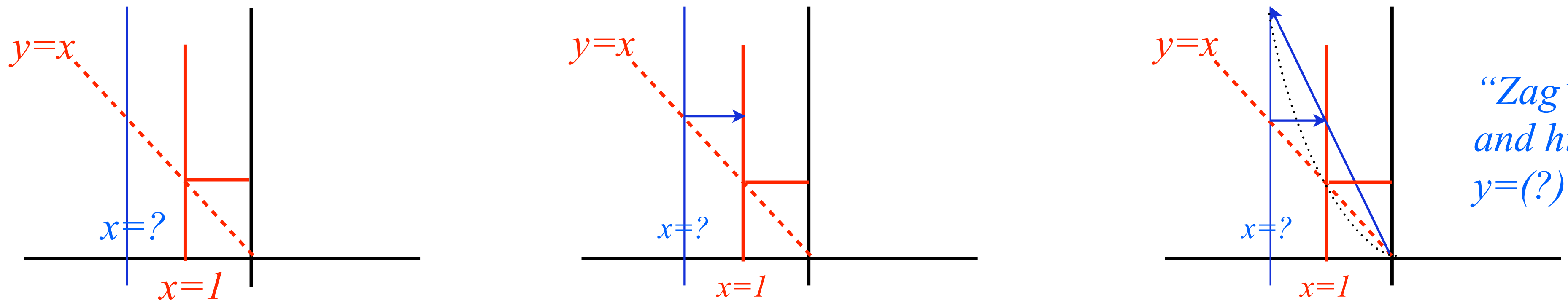
Unit 1
Fig. 9.1

Each $y=x^2$ parabola point found by just one "Zig-Zag"

1. Pick an (x=?)-line

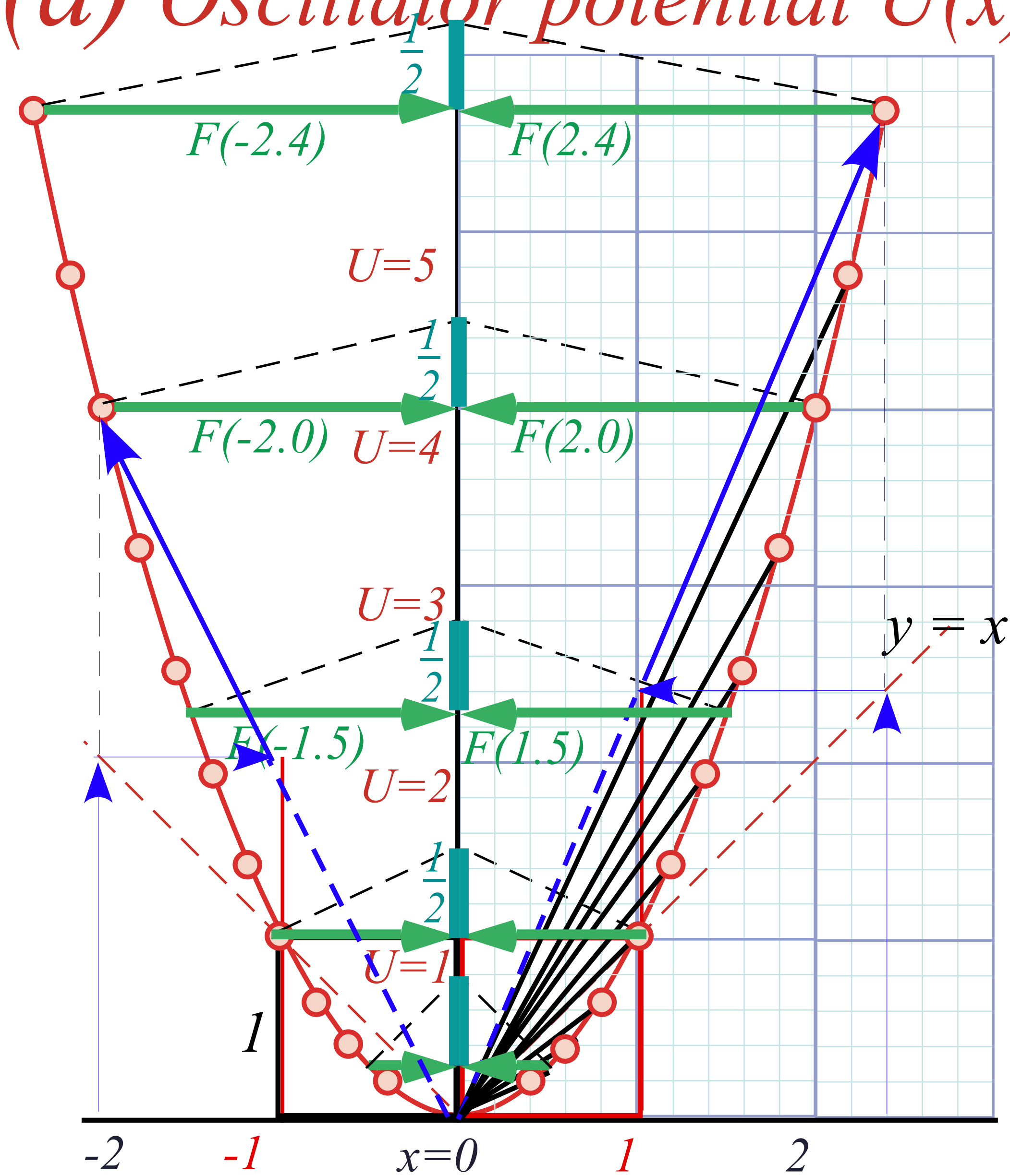
2. "Zig" from its y=x intersection to x=1 line

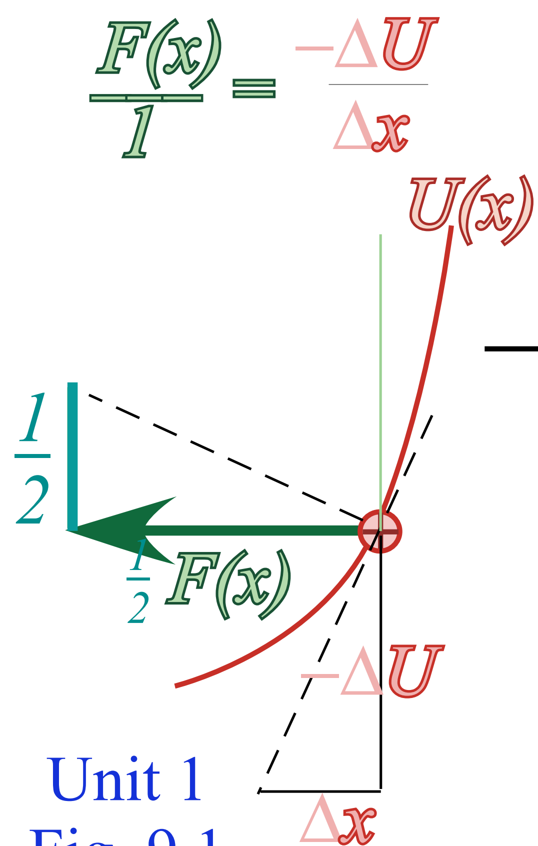
3. "Zag" from origin back to (x=?)-line



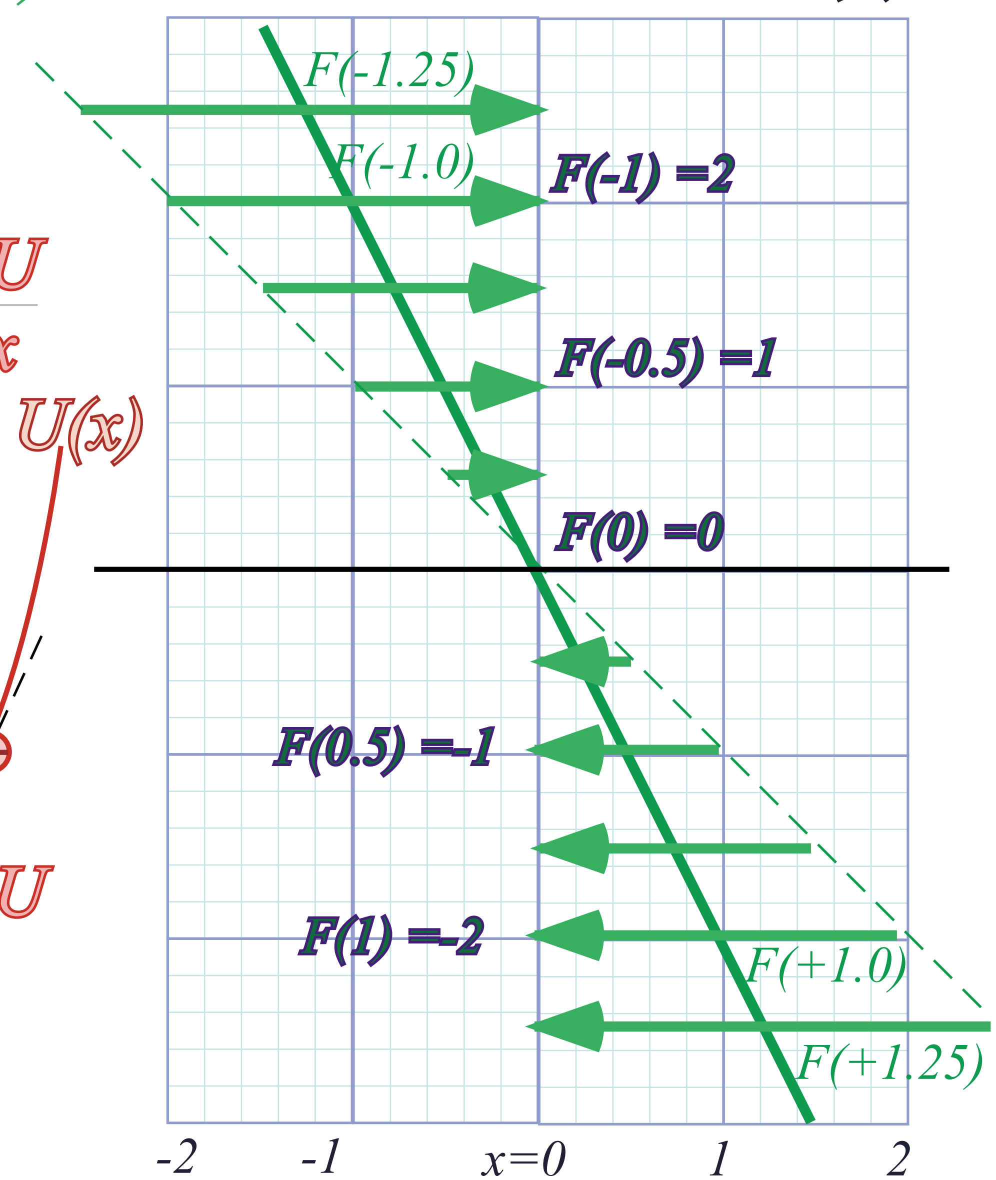
"Zag" line is y=(?)·x and hits (x=?)-line at y=(?)·(?)=(?)²

(a) Oscillator potential $U(x)=x^2$ (b) Hooke-Law **Force F(x) =-2x**

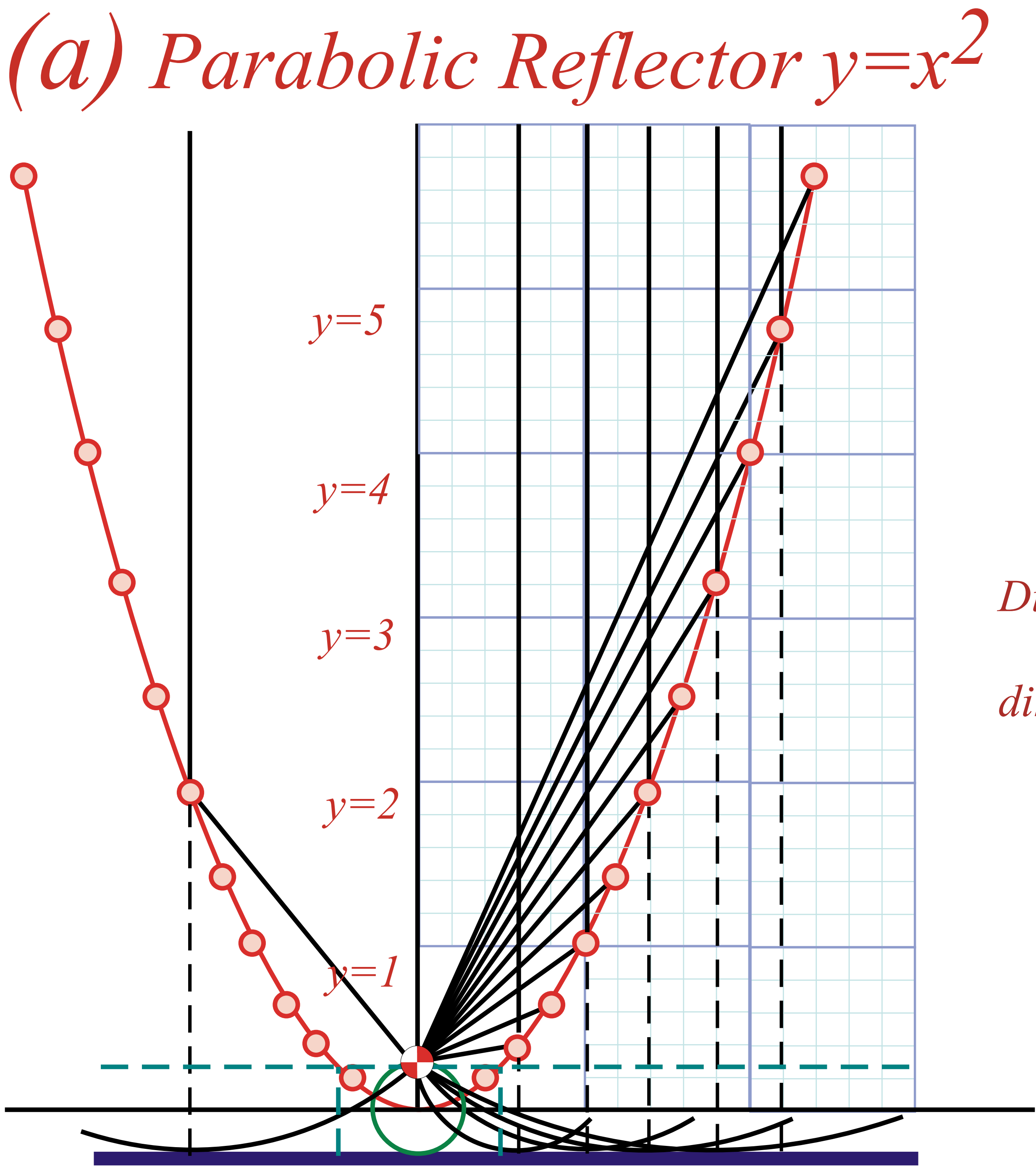


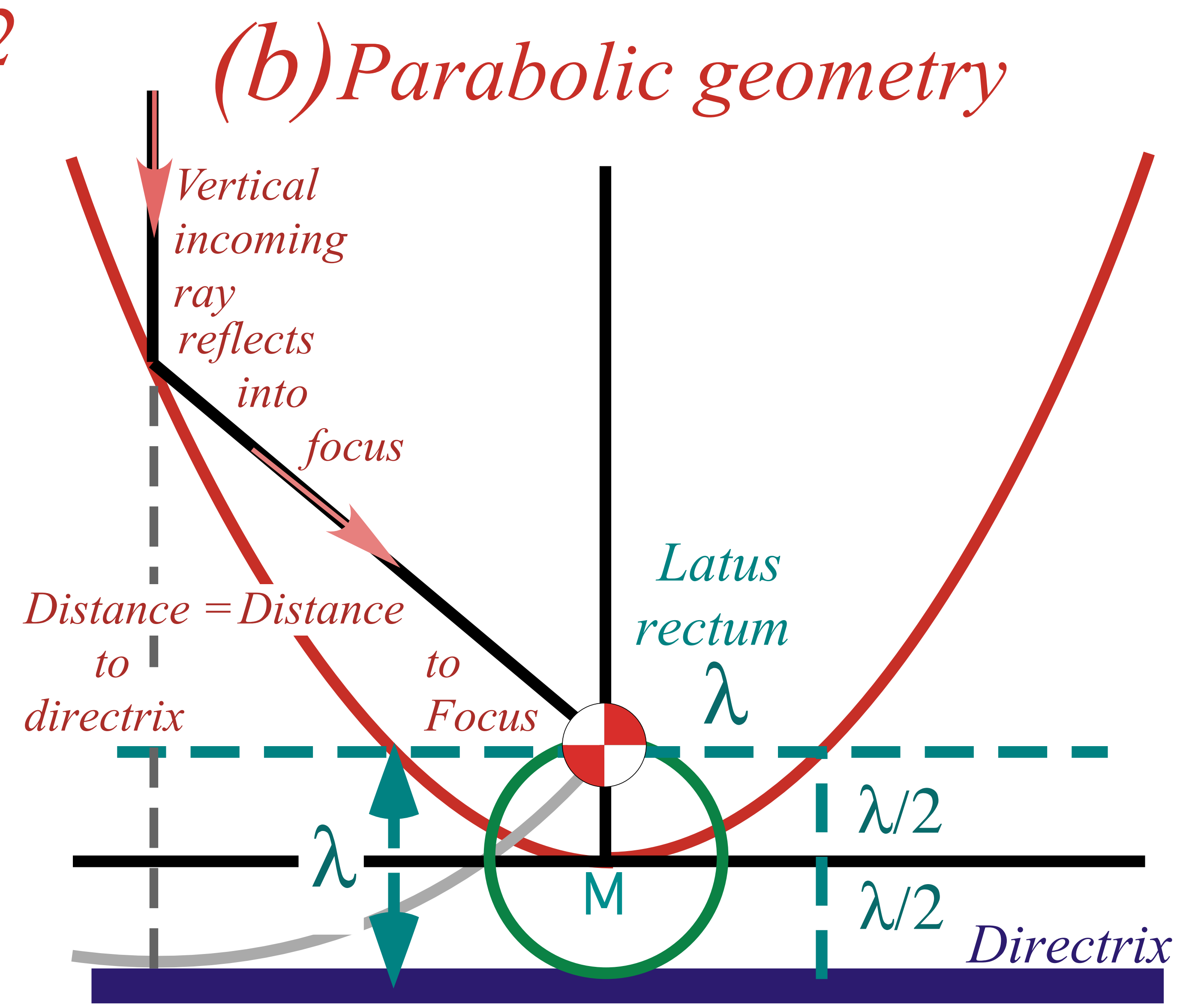


Unit 1
Fig. 9.1



A more conventional parabolic geometry…(uses focal point)

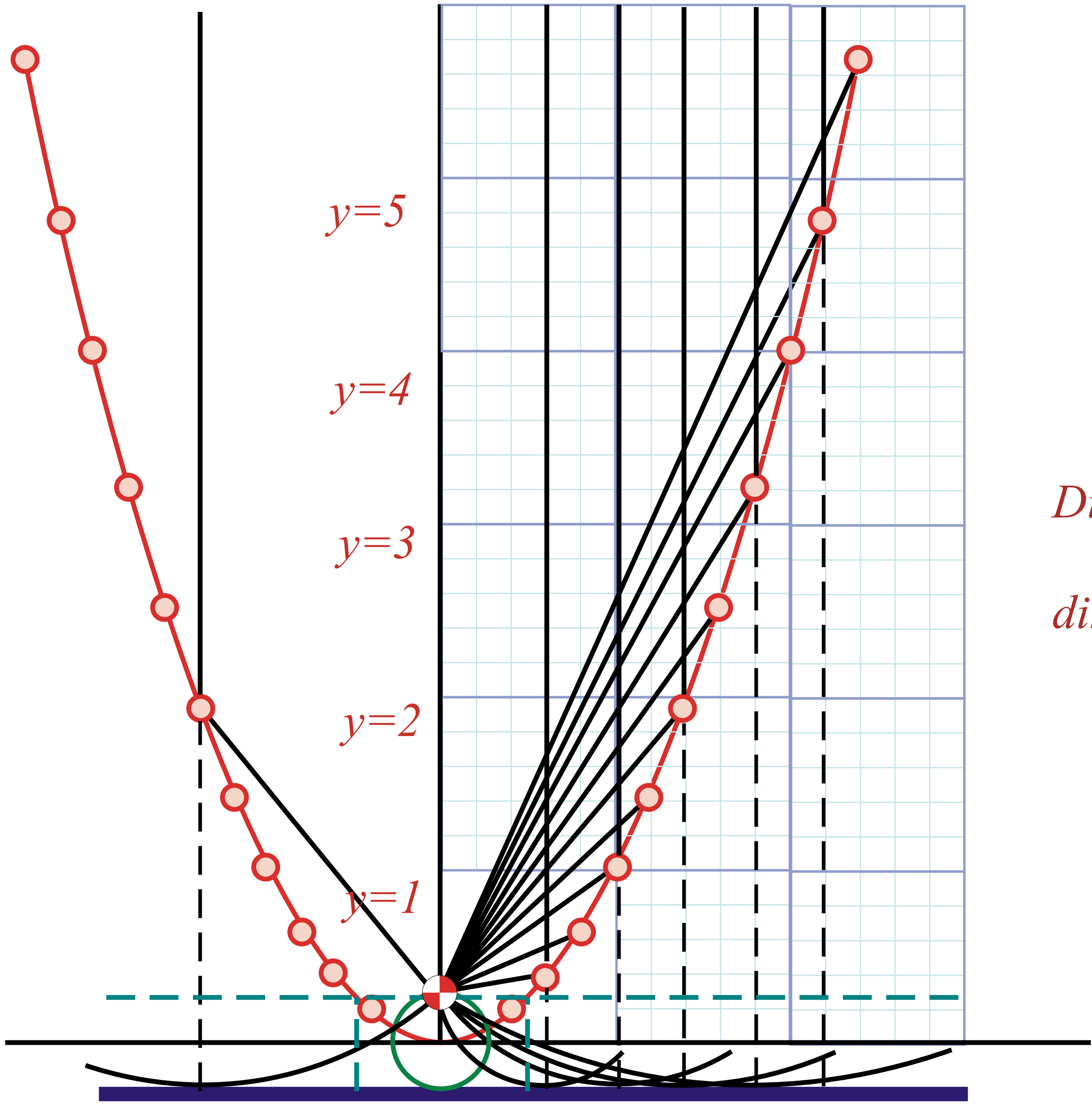




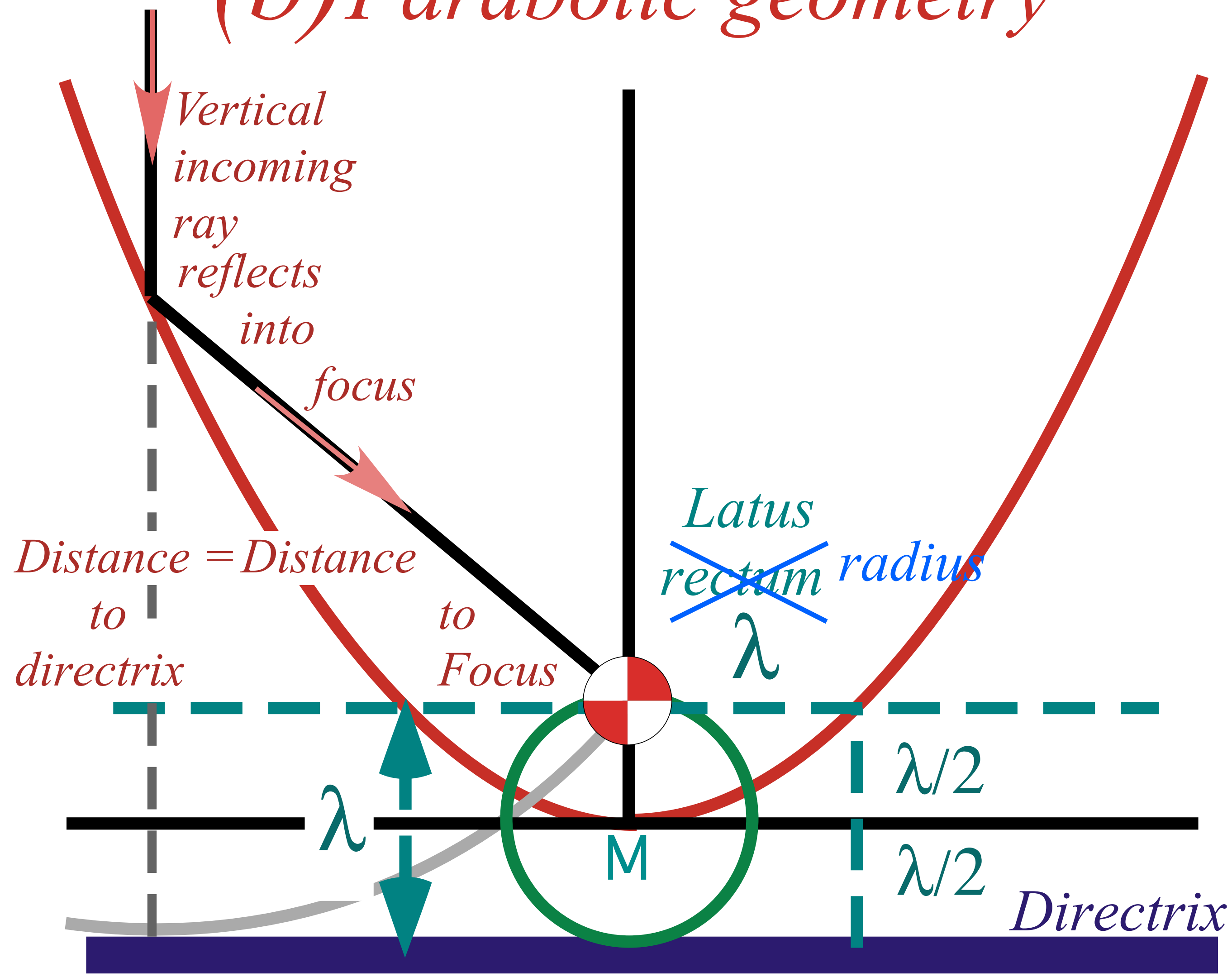
Unit 1
Fig. 9.3

A more conventional parabolic geometry...

(a) Parabolic Reflector $y=x^2$



(b) Parabolic geometry

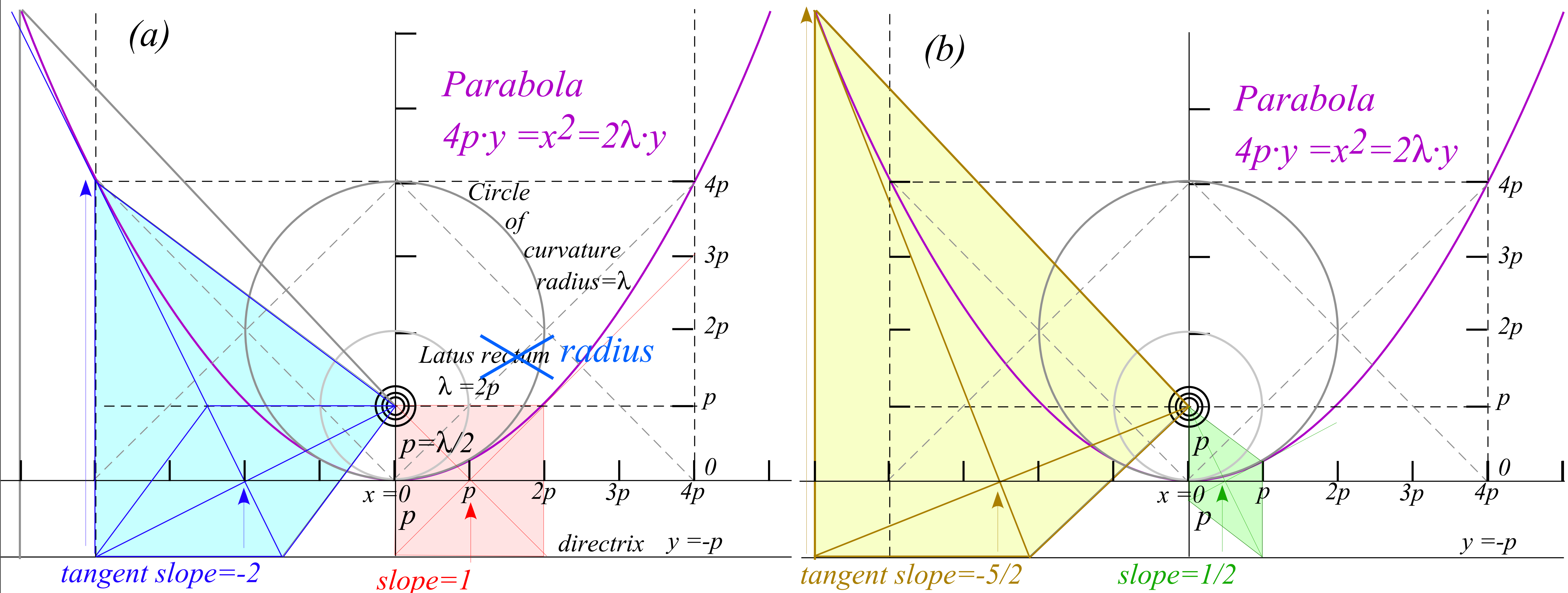


Better name† for λ : *latus radius*

† Old term *latus rectum* is exclusive copyright of
X-Treme Roidrage Gyms
Venice Beach, CA 90017

Unit 1
Fig. 9.3

...conventional parabolic geometry...carried to extremes...



Unit 1
Fig. 9.4

Geometry of common power-law potentials

Geometric (Power) series

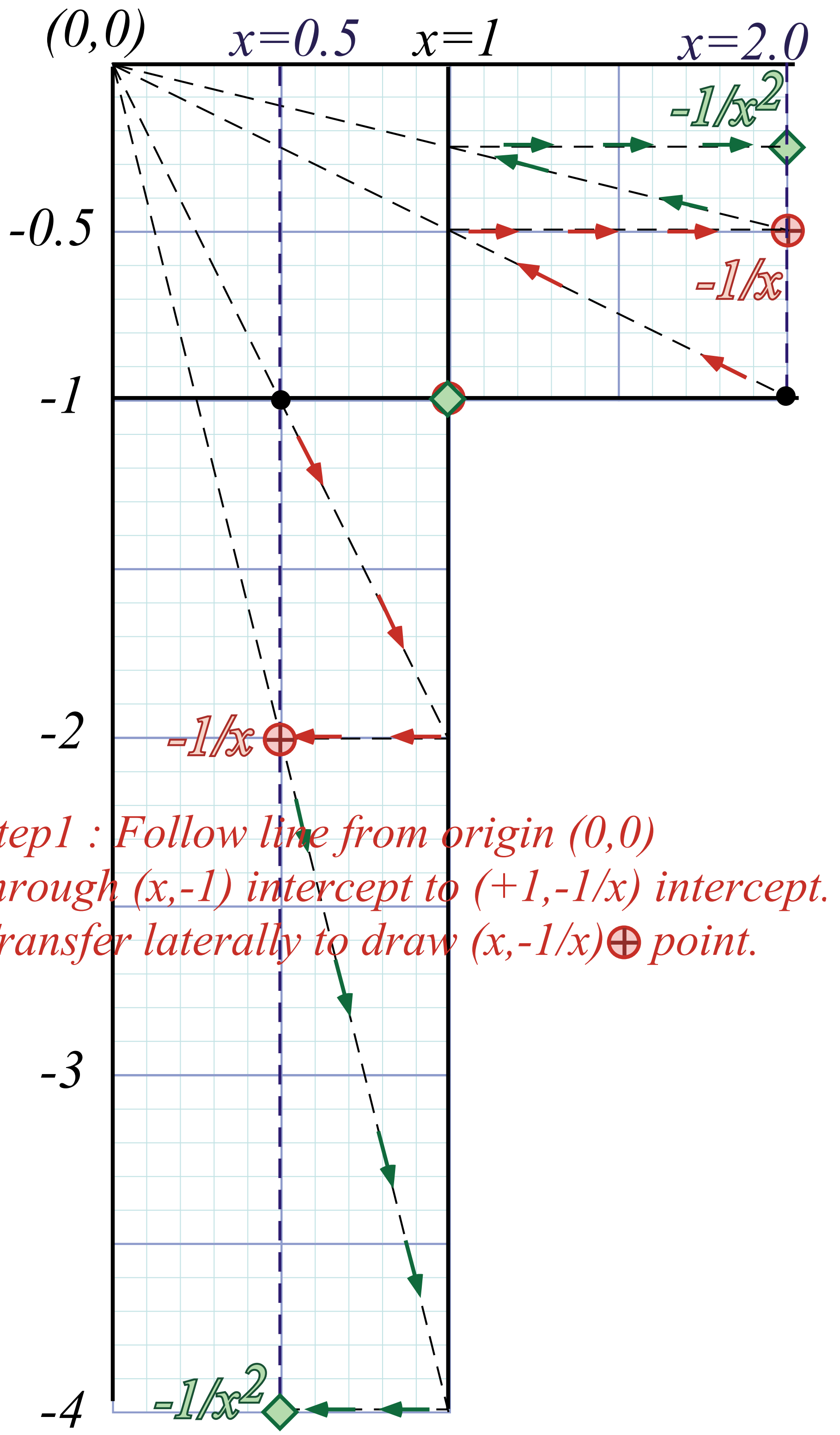
"Zig-Zag" exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

→ *Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields*

Compare mks units of Coulomb Electrostatic vs. Gravity

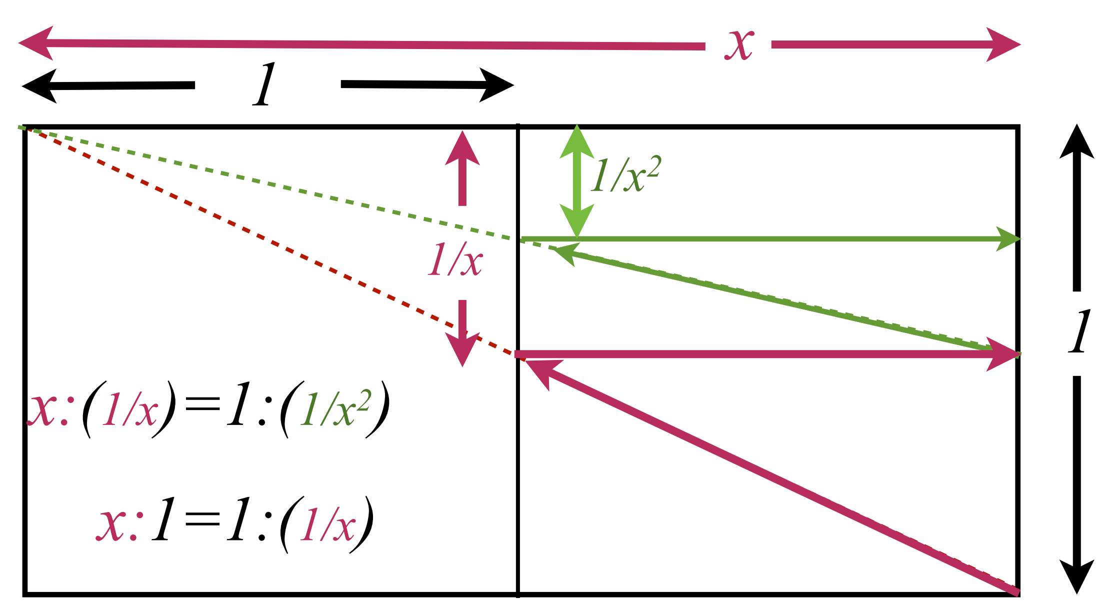


Unit 1
Fig. 9.4

Coulomb geometry
Force and Potential
$F(x)=-1/r^2$ $U(x)=-1/r$

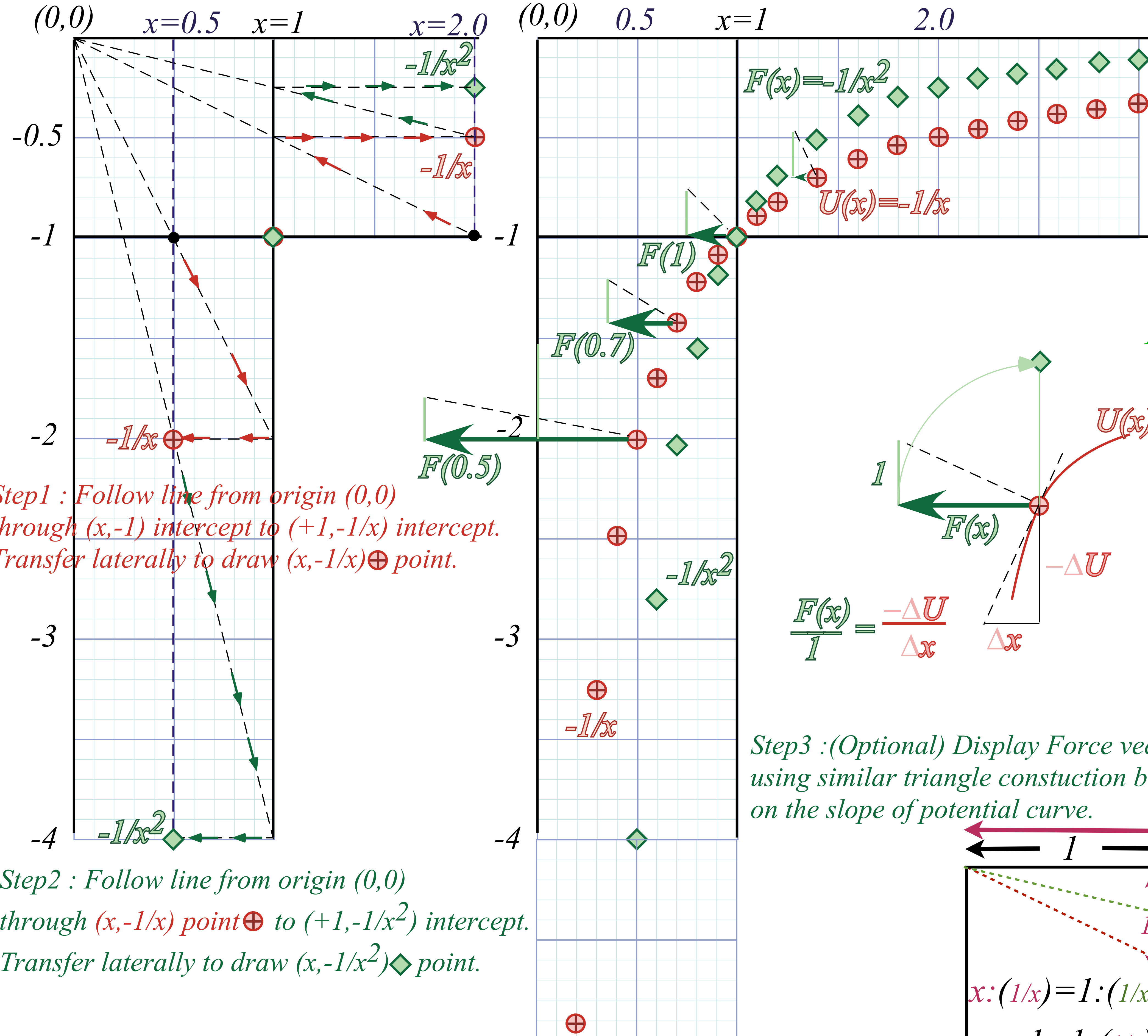
Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.

Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x²) intercept. Transfer laterally to draw (x,-1/x²)◇ point.



Unit 1
Fig. 9.4

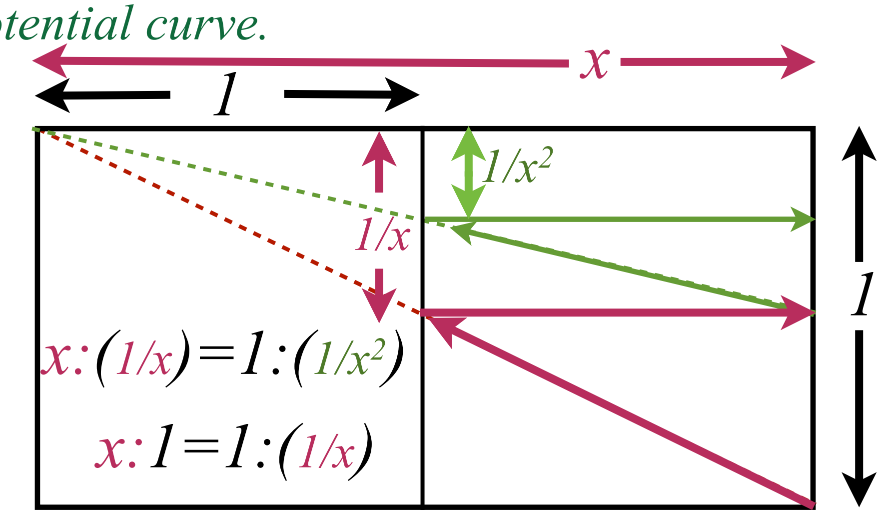
Coulomb geometry
Force and Potential
$F(x)=-1/r^2$ $U(x)=-1/r$



Step1 : Follow line from origin (0,0) through (x,-1) intercept to (+1,-1/x) intercept. Transfer laterally to draw (x,-1/x)⊕ point.

Step2 : Follow line from origin (0,0) through (x,-1/x) point⊕ to (+1,-1/x²) intercept. Transfer laterally to draw (x,-1/x²)◇ point.

Step3 :(Optional) Display Force vector using similar triangle constuction based on the slope of potential curve.



Geometry of common power-law potentials

Geometric (Power) series

"Zig-Zag" exponential geometry

Projective or perspective geometry

Parabolic geometry of harmonic oscillator $kr^2/2$ potential and $-kr^1$ force fields

Coulomb geometry of $-1/r$-potential and $-1/r^2$-force fields

→ *Compare mks units of Coulomb Electrostatic vs. Gravity*

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ *Coulomb*

↑
Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

...but 1 Ampere = 1 Coulomb/sec.

Compare mks units for Coulomb fields

1. Electrostatic force between q(Coulombs) and Q(C.)

$$F^{elec.}(r) = \pm \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9{,}000{,}000{,}000 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

More precise value for electrostatic constant : $1/4\pi\varepsilon_0 = 8.987{,}551 \cdot 10^9 Nm^2/C^2 \sim 9 \cdot 10^9 \sim 10^{10}$

quantum of charge: $|e| = 1.6022 \cdot 10^{-19}$ *Coulomb*

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

...but 1 Ampere = 1 Coulomb/sec.

"Fingertip Physics" of Ch. 9 notes that 1 (cm)^3 *of water (1/38 Mole) has (1/38)* $6 \cdot 10^{23}$ *molecules (about* $6 \cdot 10^{23}$ *electrons)*

That's about $6 \cdot 10^{23}\, 1.6022 \cdot 10^{-19}$ *Coulomb*
or about 10^{+5} *C or 10,000 Coulomb*

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↑

Repulsive (+)(+) or (-)(-)

Attractive (+)(-) or (-)(+)

vs

Always Attractive (so far)

↓

↕ COMPARE! ↕

BIG

vs

small

2. Gravitational force between m(kilograms) and M(kg.)

$$F^{grav.}(r) = -G\frac{mM}{r^2} \quad where: G = 0.000{,}000{,}000{,}067 \frac{Newtons \cdot meter \cdot square}{per\ square\ Coulomb}$$

More precise value for gravitational constant : $G = 6.67384(80) \cdot 10^{-11} Nm^2/C^2 \sim (2/3)10^{-10}$

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quantum of charge: |e|=1.6022·10^{-19} Coulomb

↑

Repulsive (+)(+) or (-)(-)
Attractive (+)(-) or (-)(+)

Discussion of repulsive force and PE in Ch. 9...

1(a). Electrostatic potential energy between q(Coulombs) and Q(C.)

$$U(r) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r} \quad where: \frac{1}{4\pi\varepsilon_0} \cong 9,000,000,000 \frac{Joule}{per\ square\ Coulomb}$$

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↑
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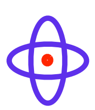
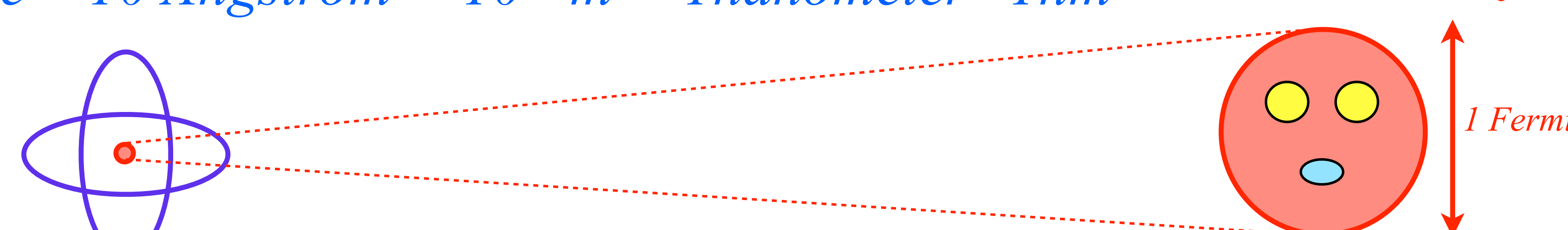


Atomic size ~ 1 Angstrom = 10^{-10} m

Nuclear size ~ 10^{-15} m = 1 femtometer = 1fm

Big molecule ~ 10 Angstrom = 10^{-9} m = 1nanometer=1nm

also: 1fm = 10^{-13} cm = 1Fermi
= 1Fm



atomic/chemical radii are 100,000 to 1,000,000 times smaller that of nuclei

...so nuclear energy 100,000 to 1,000,000 times **bigger** that of atomic/chemical...