Lecture 7

Revised 12.21.12 from 9.11.2012

Geometry and Motion of Isotropic Harmonic Oscillators

(Ch. 9 and Ch. 11 of Unit 1)

Geometry of idealized "Sophomore-physics Earth"

Coulomb field outside *Isotropic Harmonic Oscillator (IHO) field inside*
Contact-geometry of potential curve(s)
"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"
Earth matter vs nuclear matter:
*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

Sinusoidal space-time dynamics derived by geometry
Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)
Constructing 2D Isotropic harmonic oscillator orbits using phasor plots
Examples with x-y phase lag : $\alpha_{x-y}=\alpha_x-\alpha_y=15°$, $30°$, *and* $\pm 75°$

Geometry of idealized “Sophomore-physics Earth”

→ *Coulomb field outside* *Isotropic Harmonic Oscillator (IHO) field inside*

Contact-geometry of potential curve(s)

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Earth matter vs nuclear matter:

*Introducing the “neutron starlet” and “***Black-Hole-Earth***”*

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

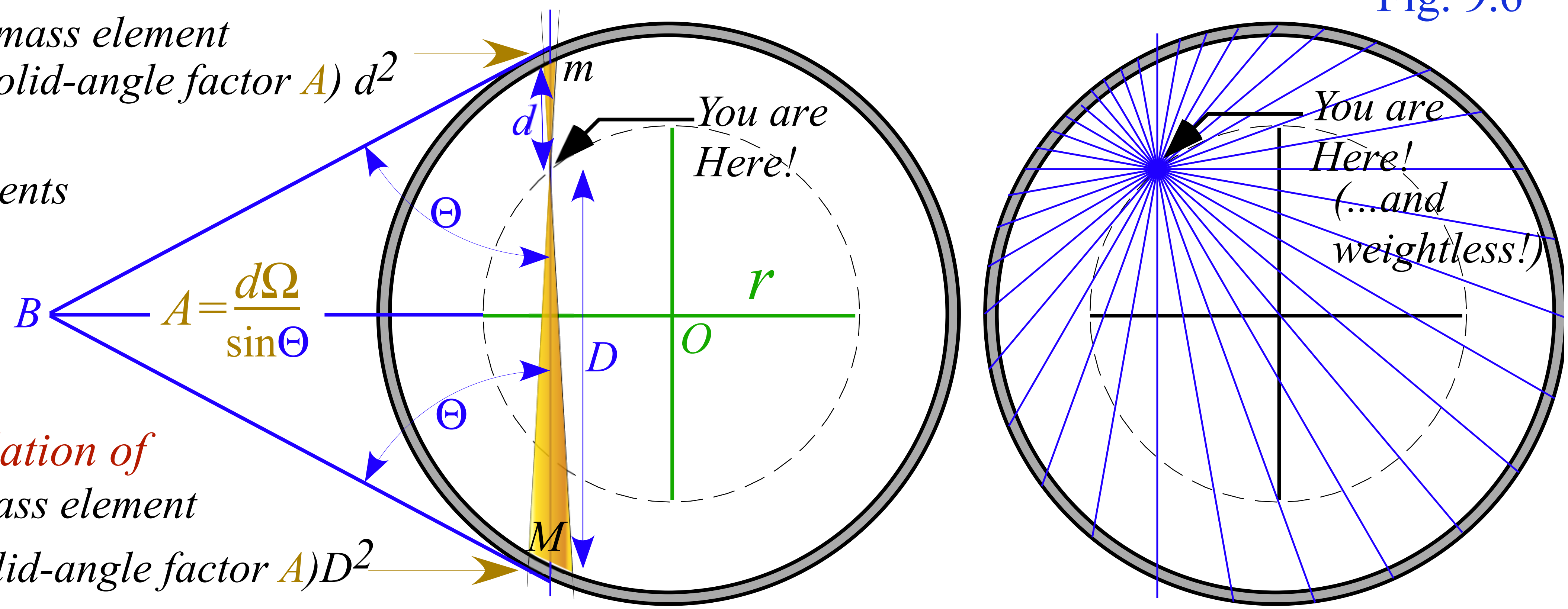
Shell mass element
m =(solid-angle factor A) d^2

Gravity at r
due to shell mass elements

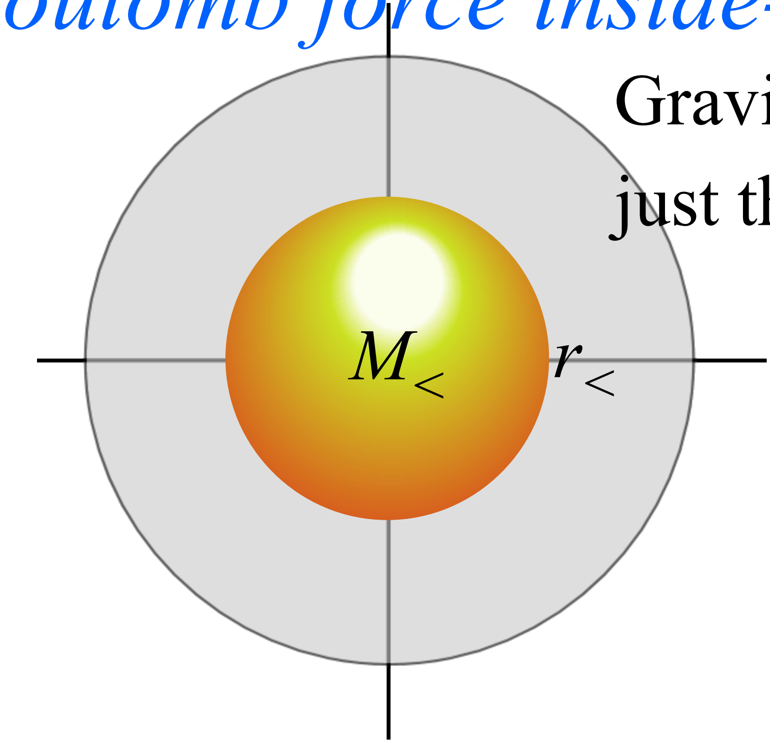
$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

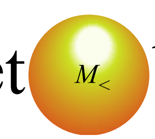
$$\left(\frac{D^2}{D^2} - \frac{d^2}{d^2}\right)A = 0$$

Cancellation of
Shell mass element
M =(solid-angle factor A)D^2



Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is
just that of planet  below $r_<$

Coulomb force vanishes inside-spherical shell (*Gauss-law*)

Unit 1
Fig. 9.6

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due to shell mass elements

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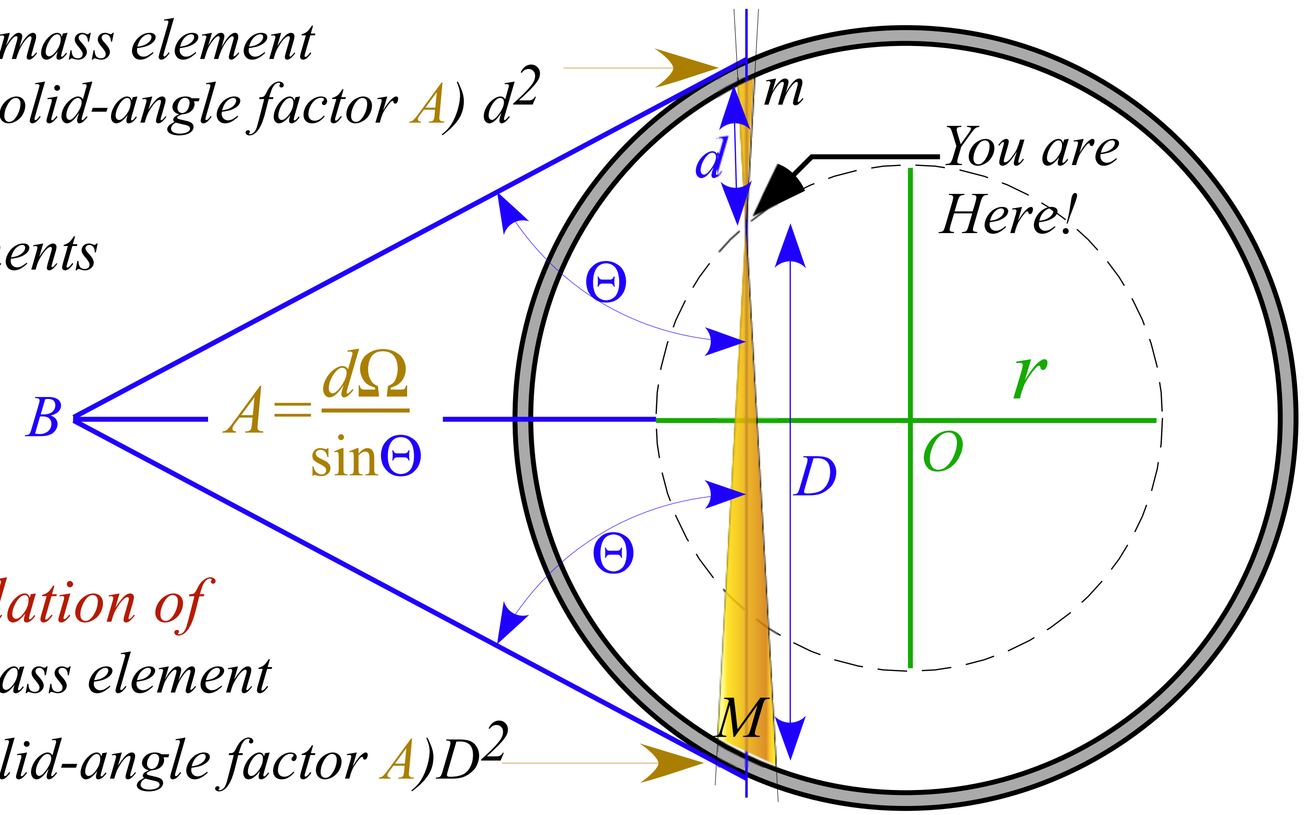
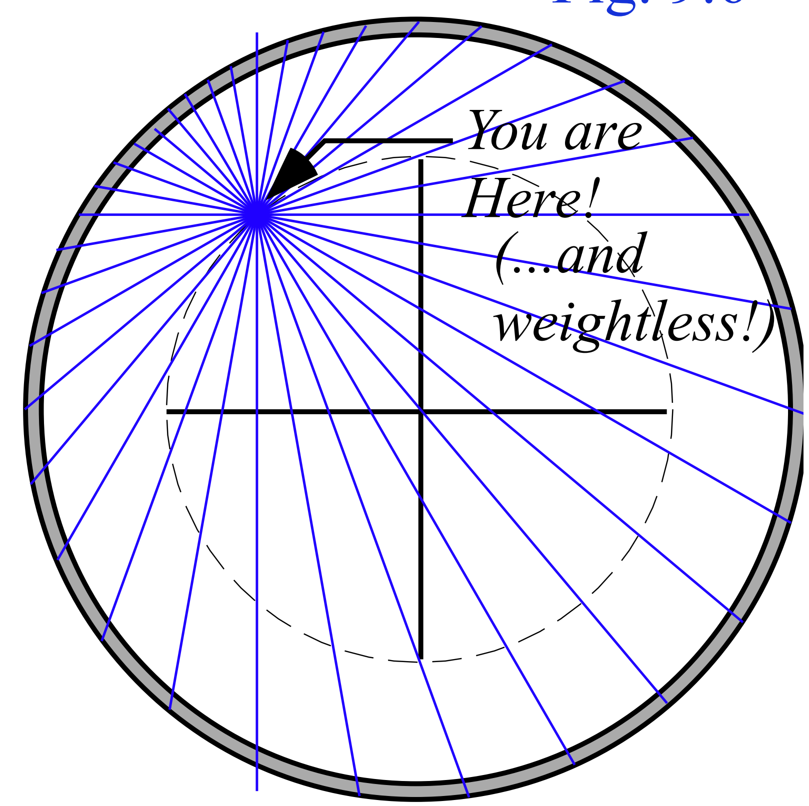


Cancellation of
Shell mass element
M =(solid-angle factor A)D^2



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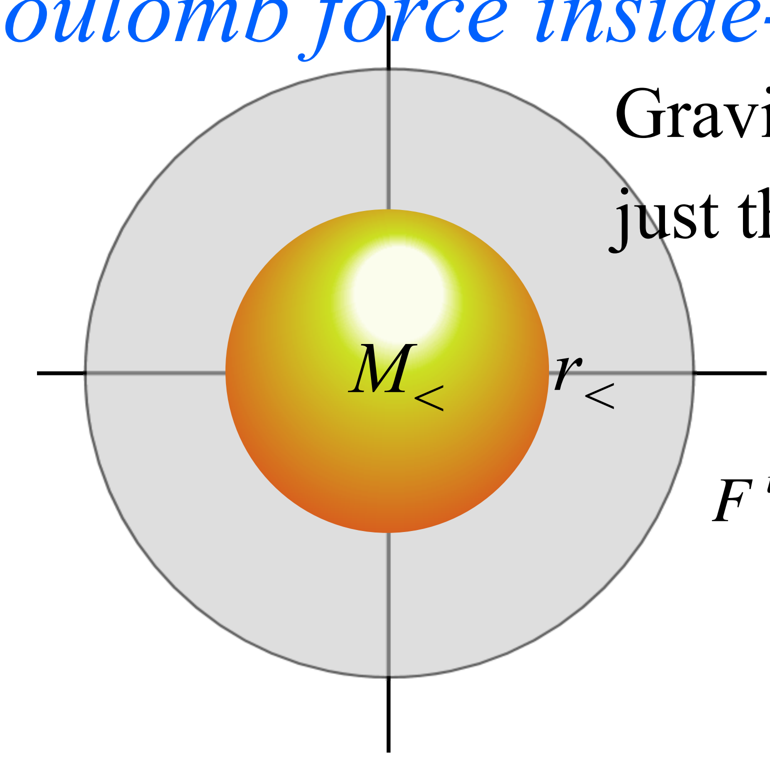

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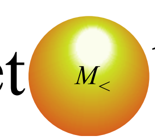
just that of planet  below $r_<$

Note:
Hooke's (linear) force law
for $r_<$ inside uniform body

$$F^{inside}(r_<) = G\frac{mM_<}{r_<^2} = Gm\frac{4\pi}{3}\frac{M_<}{\frac{4\pi}{3}r_<^3}r_< = Gm\frac{4\pi}{3}\rho_\oplus r_< = mg\frac{r_<}{R_\oplus} \equiv mg\cdot x$$

Coulomb force vanishes inside-spherical shell *(Gauss-law)*

Unit 1
Fig. 9.6

Shell mass element
m =(solid-angle factor A) d^2

Gravity at r
due to shell mass elements

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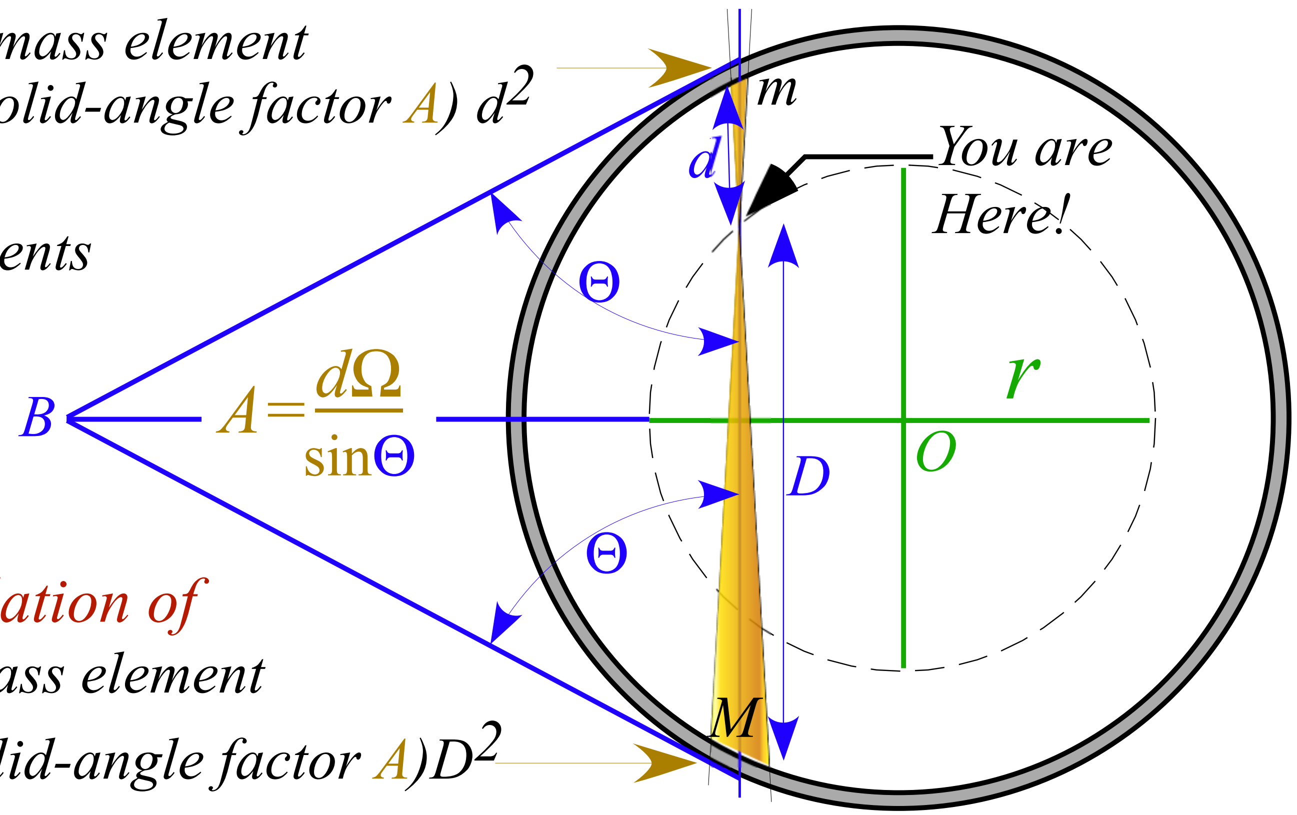

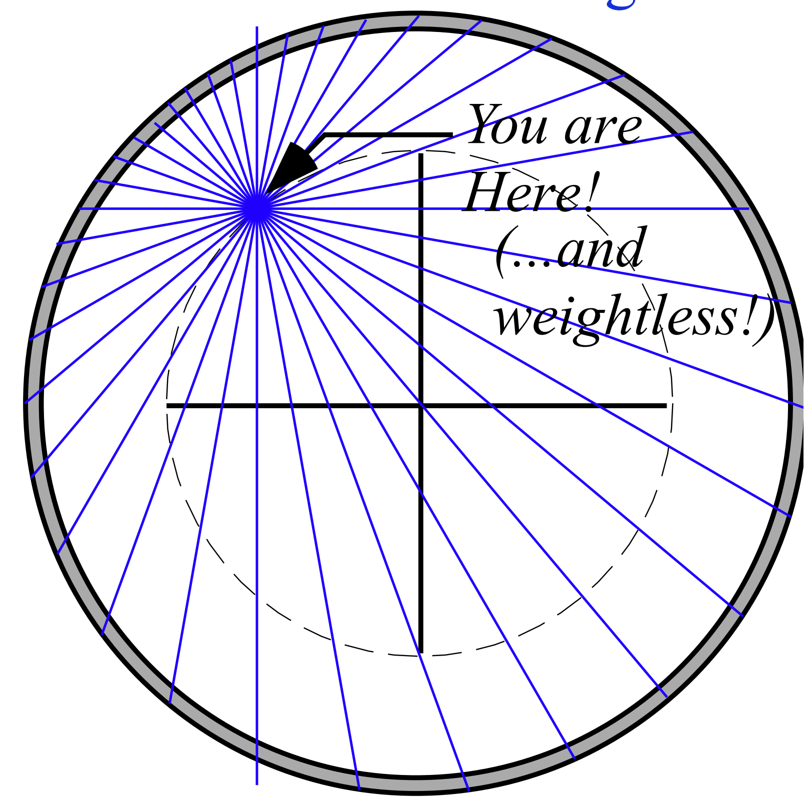

Cancellation of
Shell mass element
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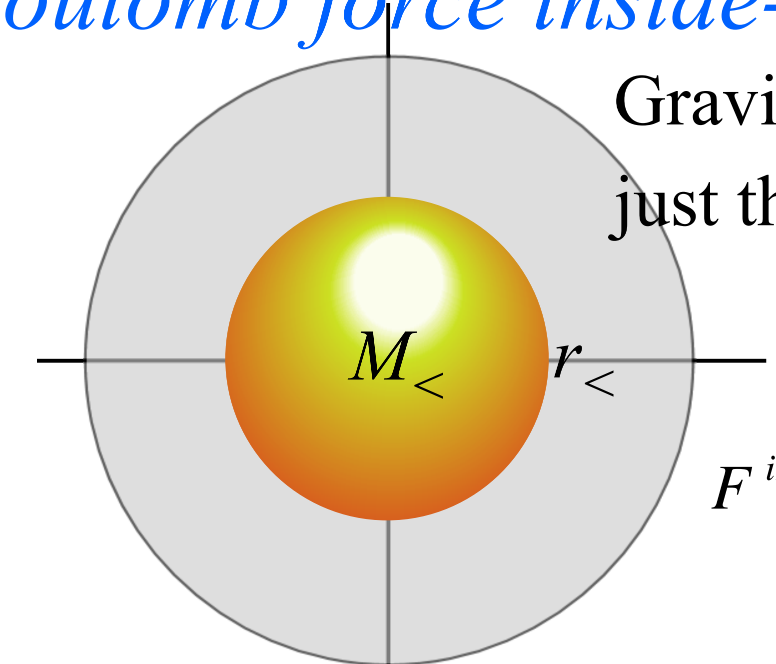
Coulomb force inside-spherical body due to stuff below you, only.



Gravitational force at $r_<$ is just that of planet $M_<$ below $r_<$

Note:
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Earth surface gravity acceleration: $g = G\frac{M_\oplus}{R_\oplus^2} = G\frac{M_\oplus}{R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\frac{M_\oplus}{\frac{4\pi}{3}R_\oplus^3}R_\oplus = G\frac{4\pi}{3}\rho_\oplus R_\oplus = 9.8m/s$

$G=6.67384(80)\cdot 10^{-11}Nm^2/C^2 \sim (2/3)10^{-10}$

Coulomb force vanishes inside-spherical shell (Gauss-law)

Unit 1
Fig. 9.6

Gravity at r
due to shell mass elements

$$\frac{G\,M}{D^2} - \frac{G\,m}{d^2} =$$

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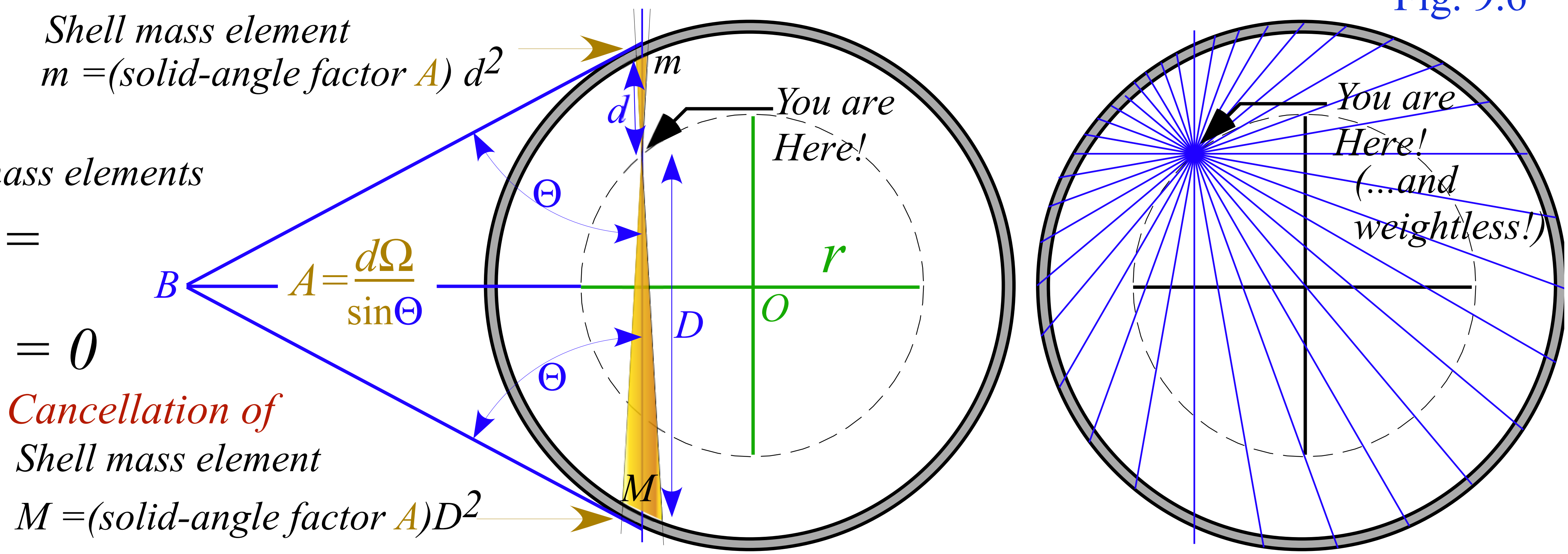

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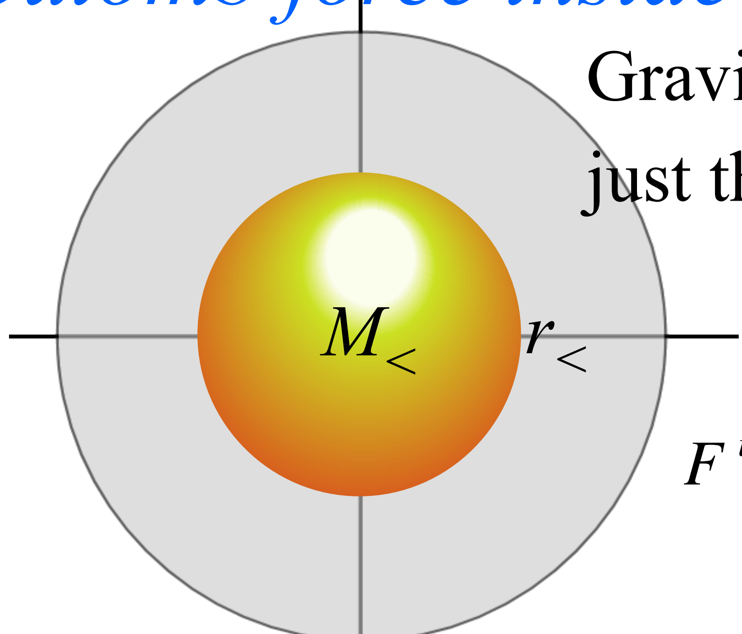

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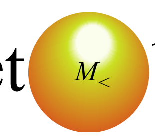
just that of planet  below $r_<$

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G=6.67384(80)·$10^{-11}Nm^2/C^2$ ~(2/3)10^{-10}

Earth radius: $R_\oplus = 6.371\cdot 10^6\,m \simeq 6.4\cdot 10^6\,m$

Earth mass: $M_\oplus = 5.9722 \times 10^{24}\,kg. \simeq 6.0\cdot 10^{24}\,kg.$

Solar radius: $R_\odot = 6.955 \times 10^8\,m. \simeq 7.0\cdot 10^8\,m.$

Solar mass: $M_\odot = 1.9889 \times 10^{30}\,kg. \simeq 2.0\cdot 10^{30}\,kg.$

Geometry of idealized "Sophomore-physics Earth"

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

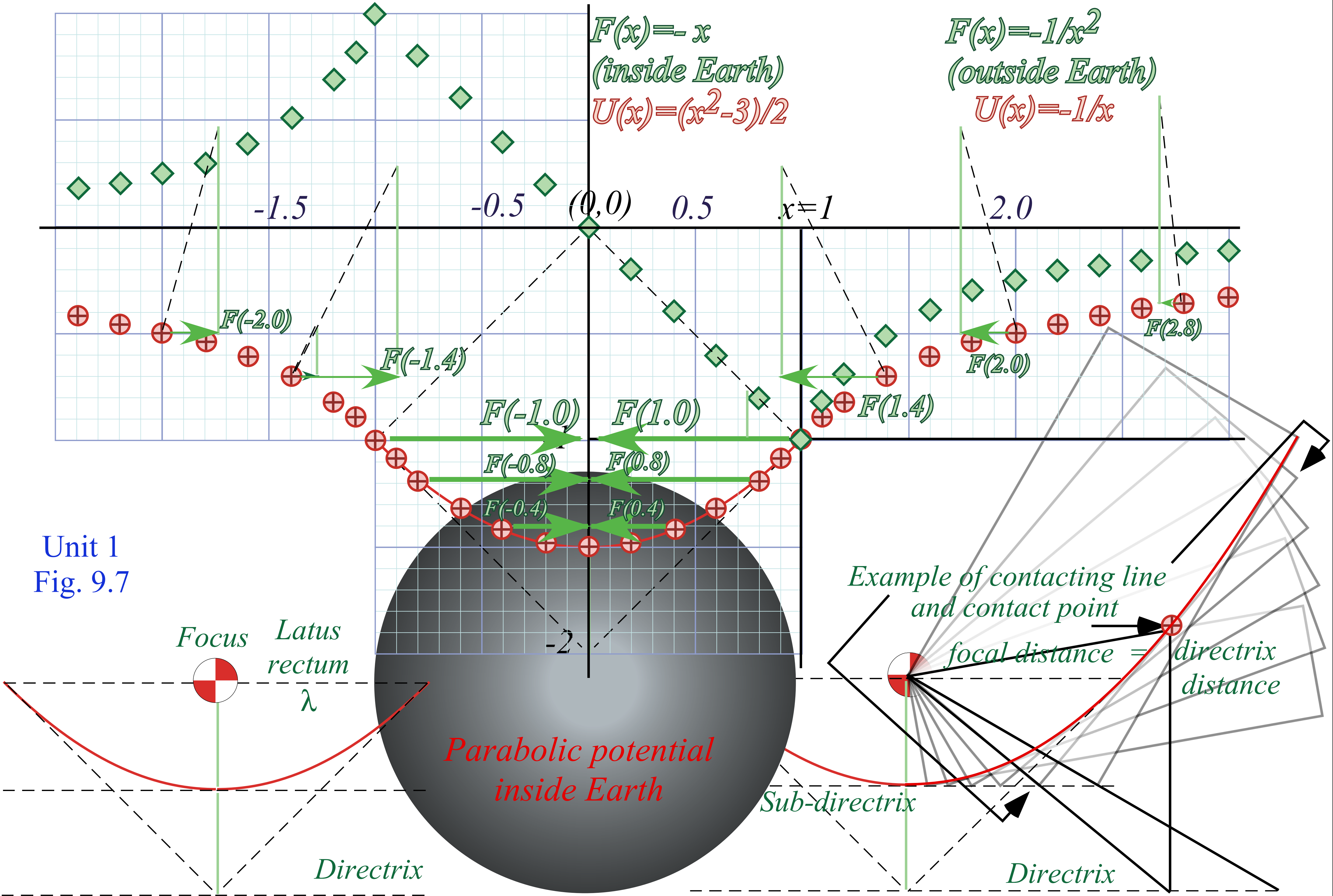
→ *Contact-geometry of potential curve(s)*

"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"

Earth matter vs nuclear matter:

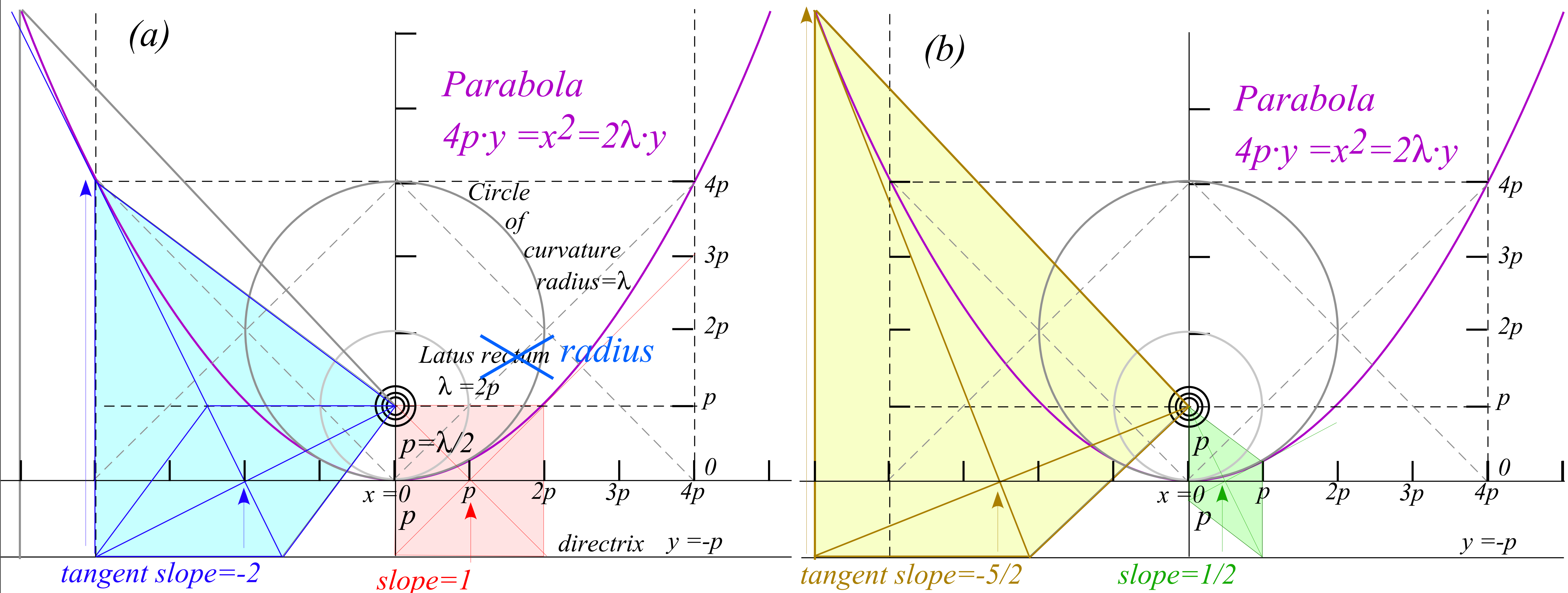
*Introducing the "neutron starlet" and "***Black-Hole-Earth***"*

The ideal "Sophomore-Physics-Earth" model of geo-gravity



...conventional parabolic geometry...carried to extremes...

(Review of Lect. 6 p.29)



Unit 1
Fig. 9.4

Geometry of idealized "Sophomore-physics Earth"

Coulomb field <u>outside</u> *Isotropic Harmonic Oscillator (IHO) field <u>inside</u>*

Contact-geometry of potential curve(s)

→ *"Crushed-Earth" models: 3 key energy "steps" and 4 key energy "levels"*

Earth matter vs nuclear matter:

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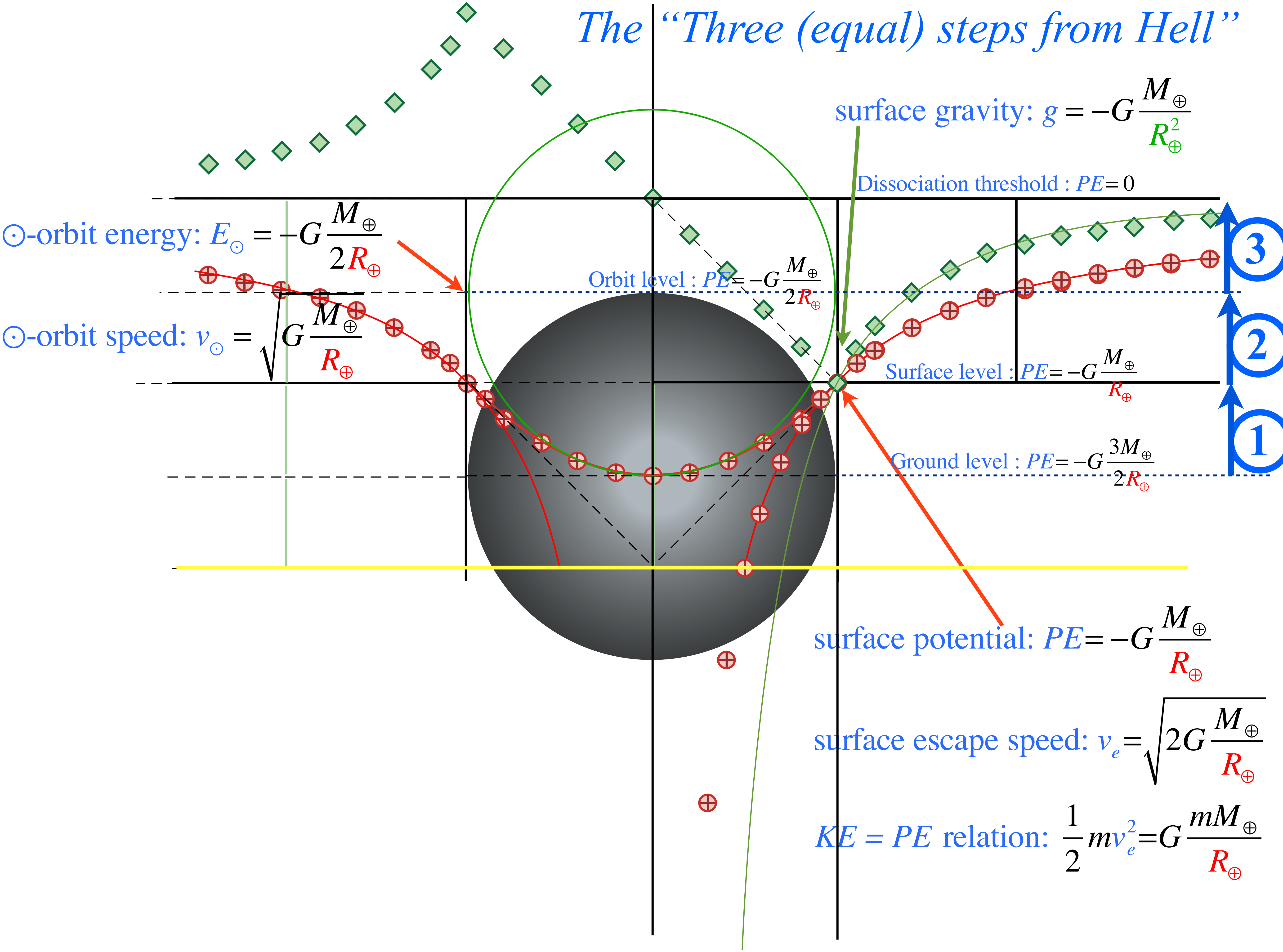
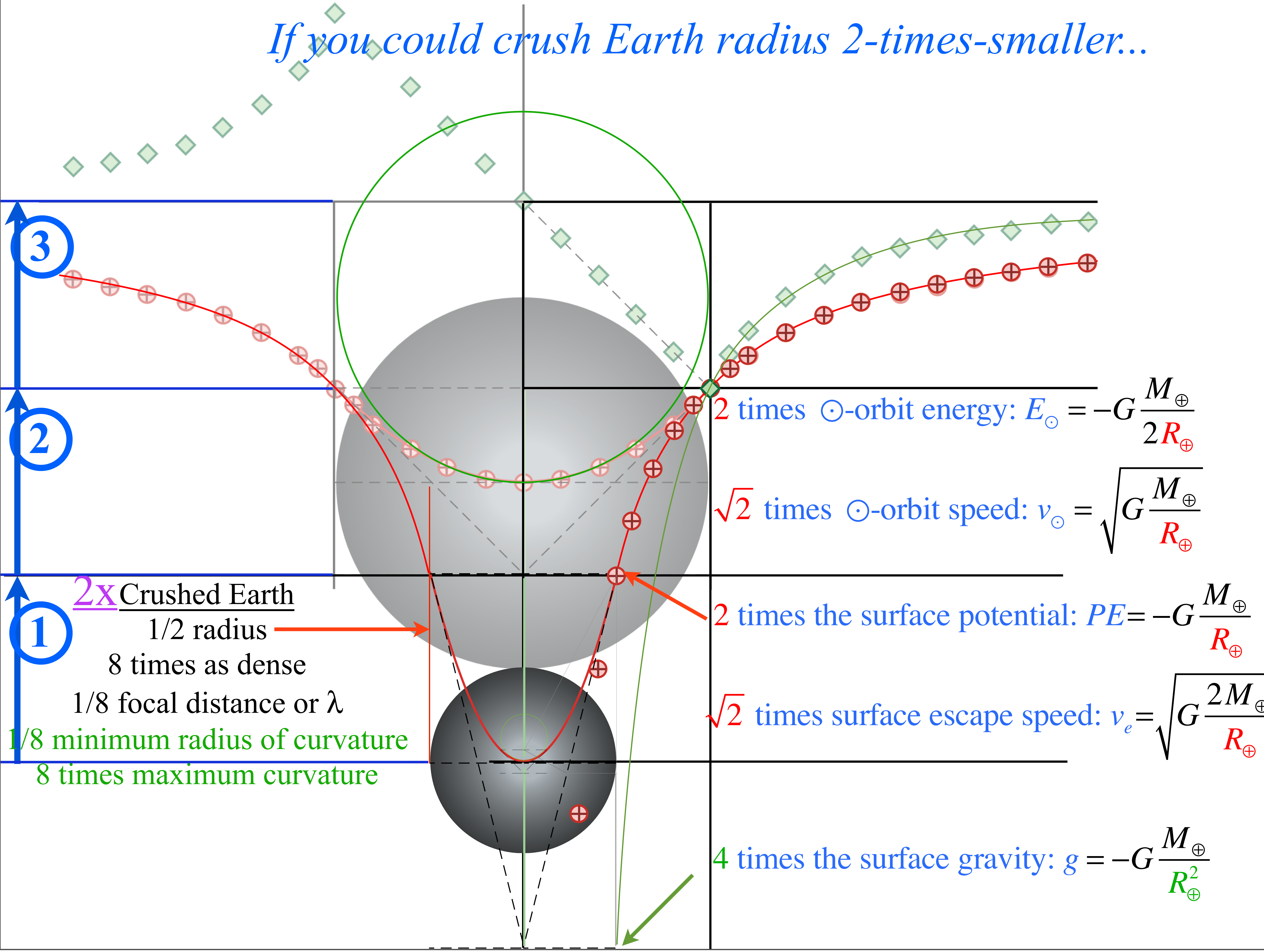
The "Three (equal) steps from Hell"



If you could crush Earth radius 2-times-smaller...



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Examples of "crushed" matter

Earth matter

$Earth\, mass: M_{\oplus} = 5.9722 \times 10^{24}\, kg. \simeq 6.0 \cdot 10^{24}\, kg.$ Density $\rho_{\oplus} \sim 6.0 \cdot 10^{24\text{-}21} \sim 6 \cdot 10^{3} kg/m^{3}$

$Earth\, radius: R_{\oplus} = 6.371 \cdot 10^{6}\, m \simeq 6.4 \cdot 10^{6}\, m \quad Earth\, volume: (4\pi/3) R_{\oplus}^{3} \simeq 4 \cdot 260 \cdot 10^{18} \sim 10^{21}\, m^{3}$

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Nuclear matter

Nucleon mass $=1.67 \cdot 10^{-27} kg. \sim 2 \cdot 10^{-27} kg.$

Say a nucleus of atomic weight *50* has a radius of *3 fm*, or *50* nucleons each with a mass $2 \cdot 10^{-27} kg$.

That's $100 \cdot 10^{-27} = 10^{-25}\,kg$ packed into a volume of ${}^{4\pi}/_{3} r^{3} = {}^{4\pi}/_{3}\,(3 \cdot 10^{-15})^{3}\,m^{3}$ or about $10^{-43}\,m^{3}$.

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Nuclear density is $10^{-25+43} = 10^{18} kg/m^{3}$ or a *trillion* (10^{12}) kilograms in the size of a fingertip.

Earth radius crushed by a factor of $0.5 \cdot 10^{-5}$ to $R_{crush\oplus} \simeq 300\,m$ would approach neutron-star density.

Examples of "crushed" matter

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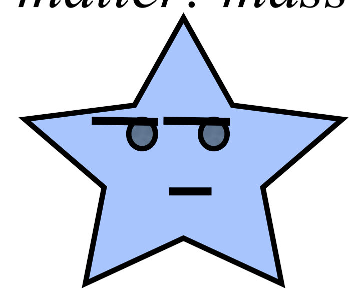
Introducing the "Neutron starlet" *1 cm³ of nuclear matter: mass= 10^{12} kg.*



Examples of "crushed" matter

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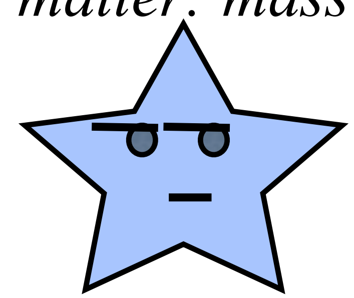
Introducing the "Neutron starlet"

1 cm^3 *of nuclear matter: mass*$= 10^{12}$ *kg.*



*Introducing the "***Black Hole Earth***"*

Suppose Earth is crushed so that its surface escape velocity is the speed of light $c = 3.0 \cdot 10^{12} m/s$.

$$c = \sqrt{(2GM/R_\otimes)}$$

$$R_\otimes = 2GM/c^2 = 9mm \sim 1cm \qquad \textit{(fingertip size!)}$$

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

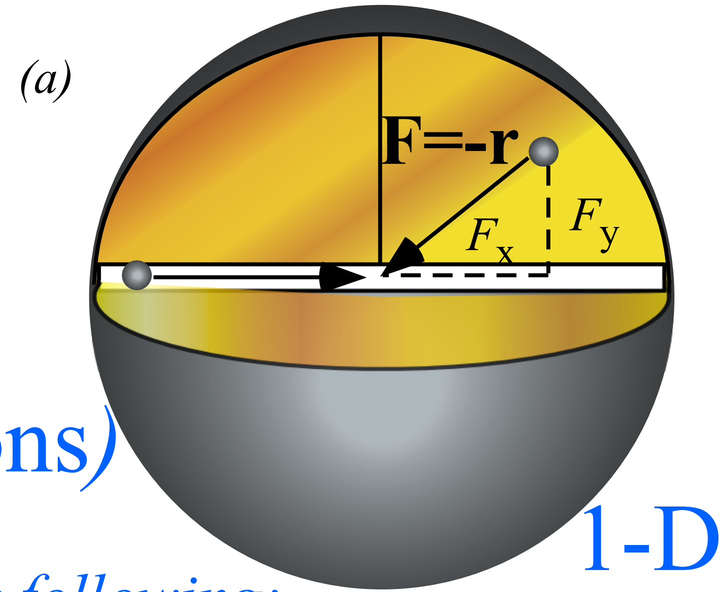
→ *Sinusoidal space-time dynamics derived by geometry*

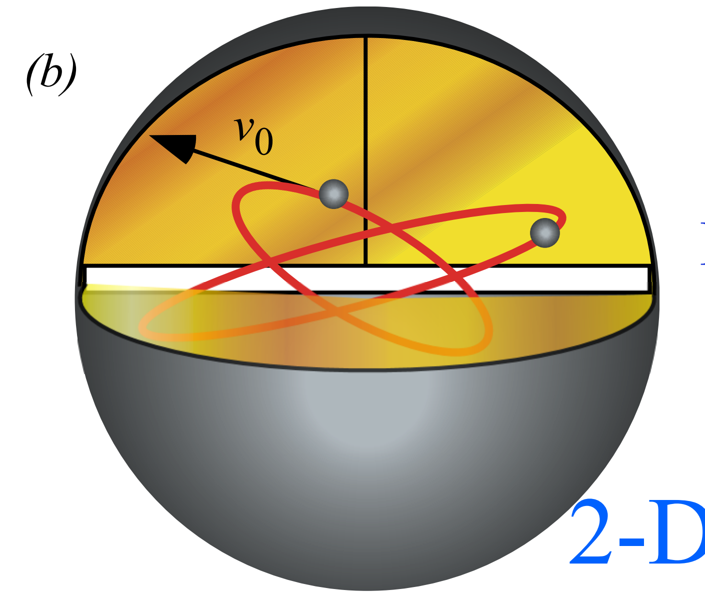
Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)

Constructing 2D Isotropic harmonic oscillator orbits using phasor plots

Examples with x-y phase lag : $\alpha_{x\text{-}y}=\alpha_x-\alpha_y=15°, 30°, \text{ and } \pm 75°$

Isotropic Harmonic Oscillator phase dynamics in uniform-body





Unit 1
Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

I.H.O. Force law

F =-x (1-Dimension)

F =-r (2 or 3-Dimensions)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

F =-x (1-Dimension)

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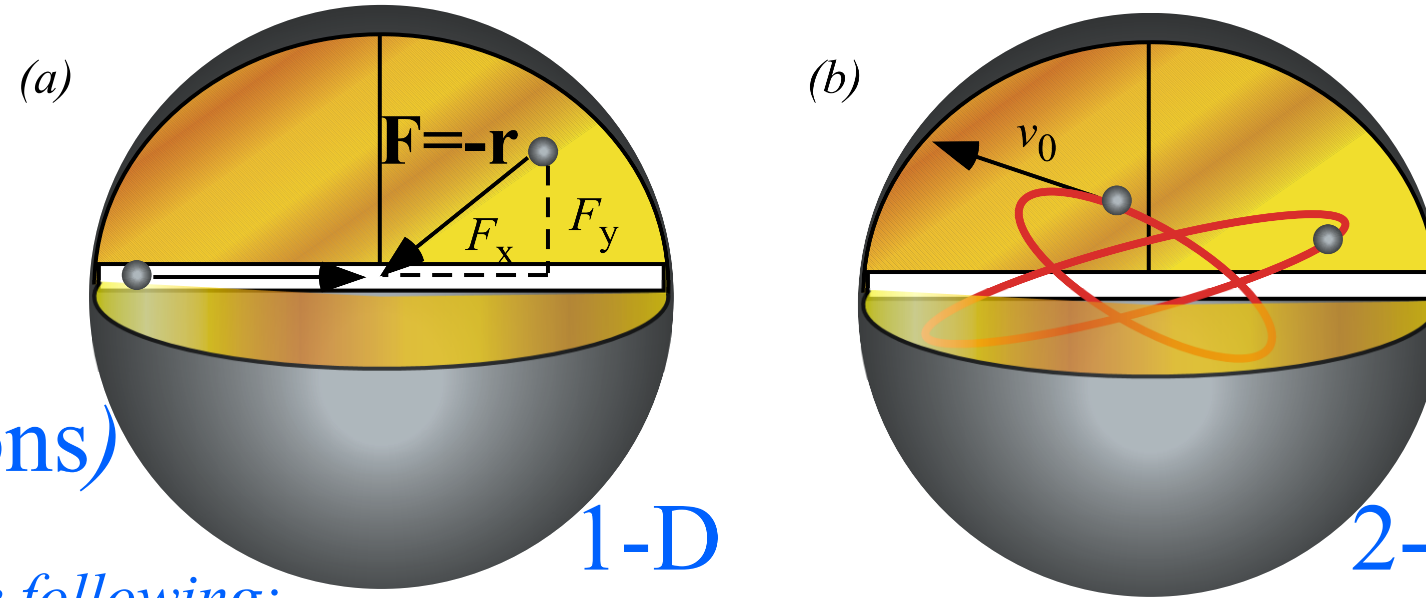

1-D

2-D

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Unit 1
Fig. 9.10

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$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta$$

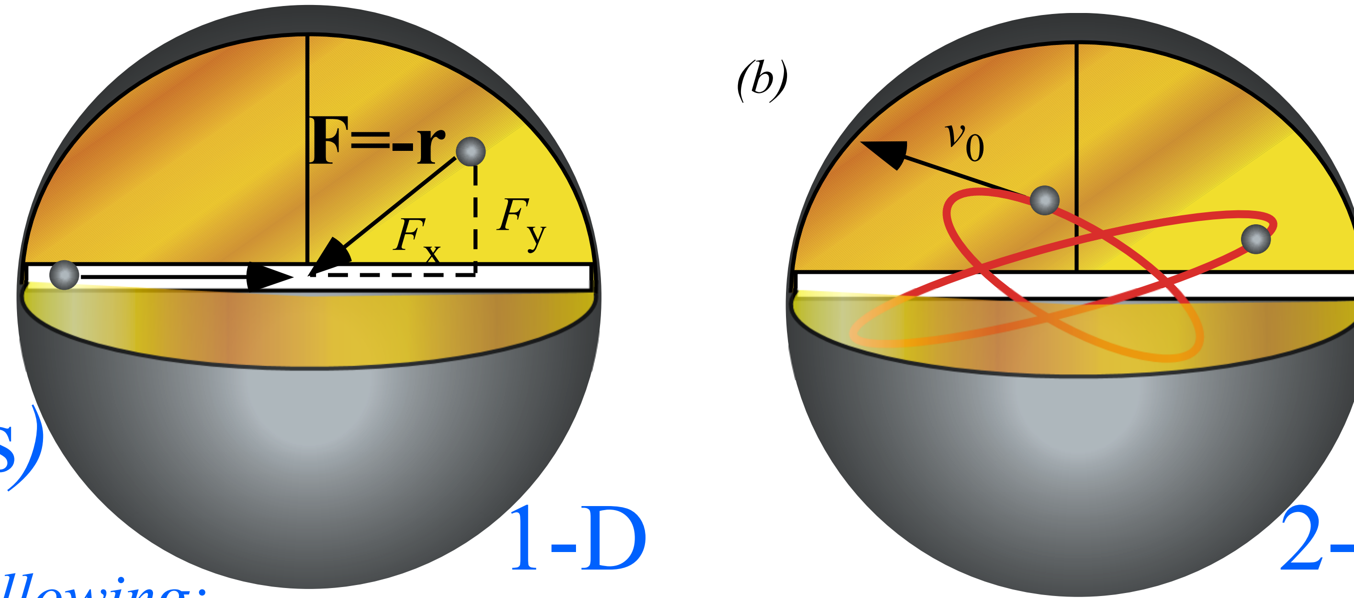
Isotropic Harmonic Oscillator phase dynamics in uniform-body

<u>*I.H.O. Force law*</u>

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)

(a) (b)



1-D 2-D

Unit 1
Fig. 9.10

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Each dimension x, y, or z obeys the following:

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$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta, \quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta$$

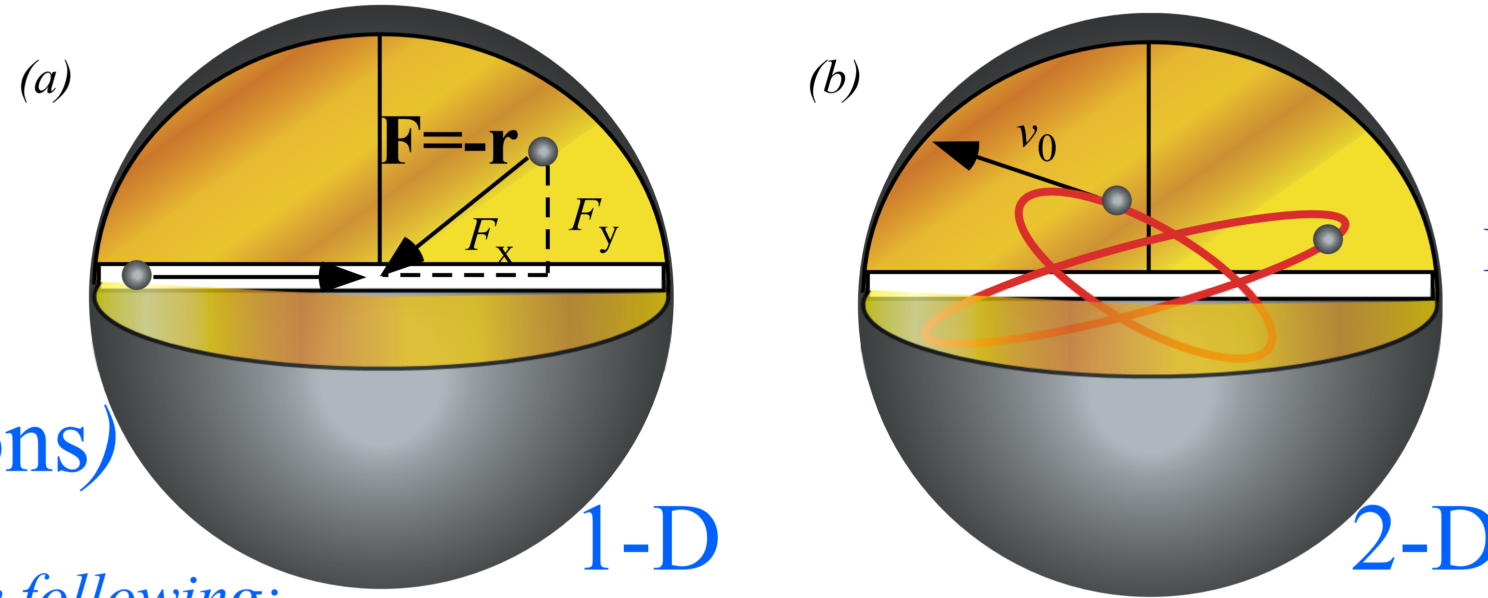
$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{by\ (1)} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta}}_{by\ def.\ (3)} = \underbrace{\omega\sqrt{\frac{2E}{k}}\cos\theta}_{by\ (2)}$$

Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

$F = -x$ (1-Dimension)

$\mathbf{F} = -\mathbf{r}$ (2 or 3-Dimensions)



Unit 1
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Another example of the old "scale-a-circle" trick...

$$Let:\ (1)\ v = \sqrt{2E/m}\cos\theta,\quad and:\ (2)\ x = \sqrt{2E/k}\sin\theta$$

$$\underbrace{\sqrt{\frac{2E}{m}}\cos\theta = v}_{\text{by (1)}} = \frac{dx}{dt} = \underbrace{\frac{d\theta}{dt}}_{\text{by def. (3)}}\frac{dx}{d\theta} = \omega\underbrace{\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta}_{\text{by (2)}}$$

by def. (3)

$$\omega = \underbrace{\frac{d\theta}{dt} = \sqrt{\frac{k}{m}}}_{\text{by (1)/(2)}}$$

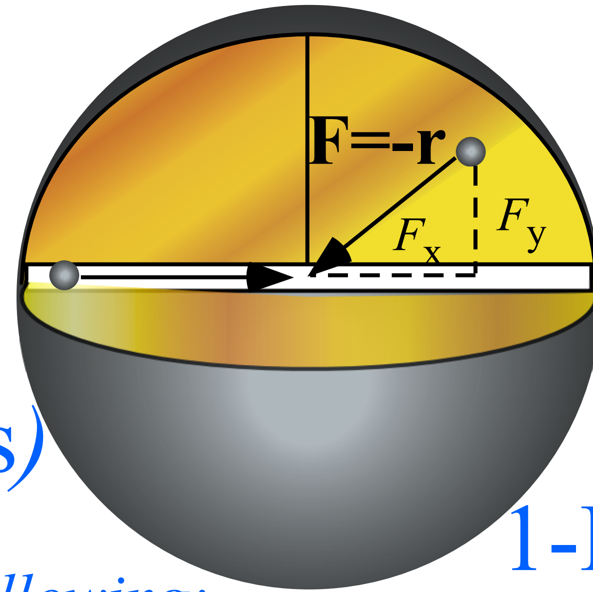
Isotropic Harmonic Oscillator phase dynamics in uniform-body

I.H.O. Force law

F =-x (1-Dimension)

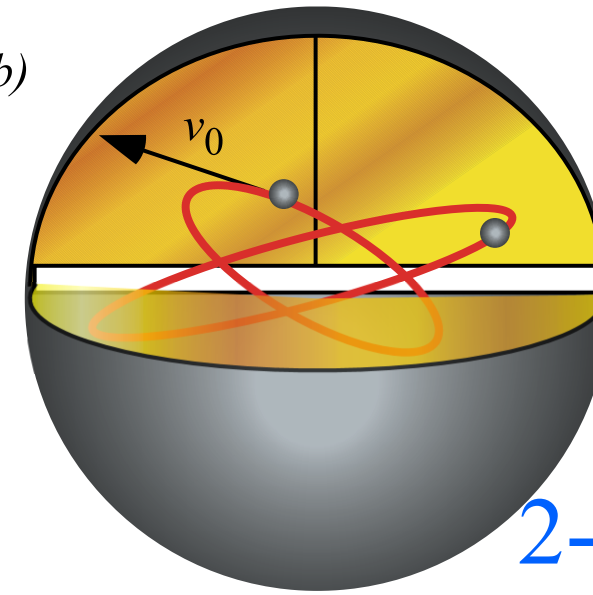
F =-**r** (2 or 3-Dimensions)

(a)



1-D

(b)



2-D

Unit 1 Fig. 9.10

(Paths are *always* 2-D ellipses if viewed right!)

Each dimension x, y, or z obeys the following:

$$Total\ E = KE + PE = \frac{1}{2}mv^2 + U(x) = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = const.$$

Equations for x-motion [x(t) and v_x=v(t)] are given first. They apply as well to dimensions [y(t) and v_y=v(t)] and [z(t) and v_z=v(t)] in the ideal isotropic case.

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = \left(\frac{v}{\sqrt{2E/m}}\right)^2 + \left(\frac{x}{\sqrt{2E/k}}\right)^2$$

$$1 = \frac{mv^2}{2E} + \frac{kx^2}{2E} = (\cos\theta)^2 + (\sin\theta)^2$$

Another example of the old "scale-a-circle" trick...

$$Let:\ \mathbf{(1)}\ v = \sqrt{2E/m}\cos\theta, \quad and:\ \mathbf{(2)}\ x = \sqrt{2E/k}\sin\theta$$

$$\sqrt{\frac{2E}{m}}\cos\theta = v = \frac{dx}{dt} = \frac{d\theta}{dt}\frac{dx}{d\theta} = \omega\frac{dx}{d\theta} = \omega\sqrt{\frac{2E}{k}}\cos\theta$$

by (1) *by def.* (3) *by* (2)

by def. (3)

$$\omega = \frac{d\theta}{dt} = \sqrt{\frac{k}{m}}$$

by (1)/(2)

by integration given constant ω:

$$\theta = \int \omega \cdot dt = \omega \cdot t + \alpha$$

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

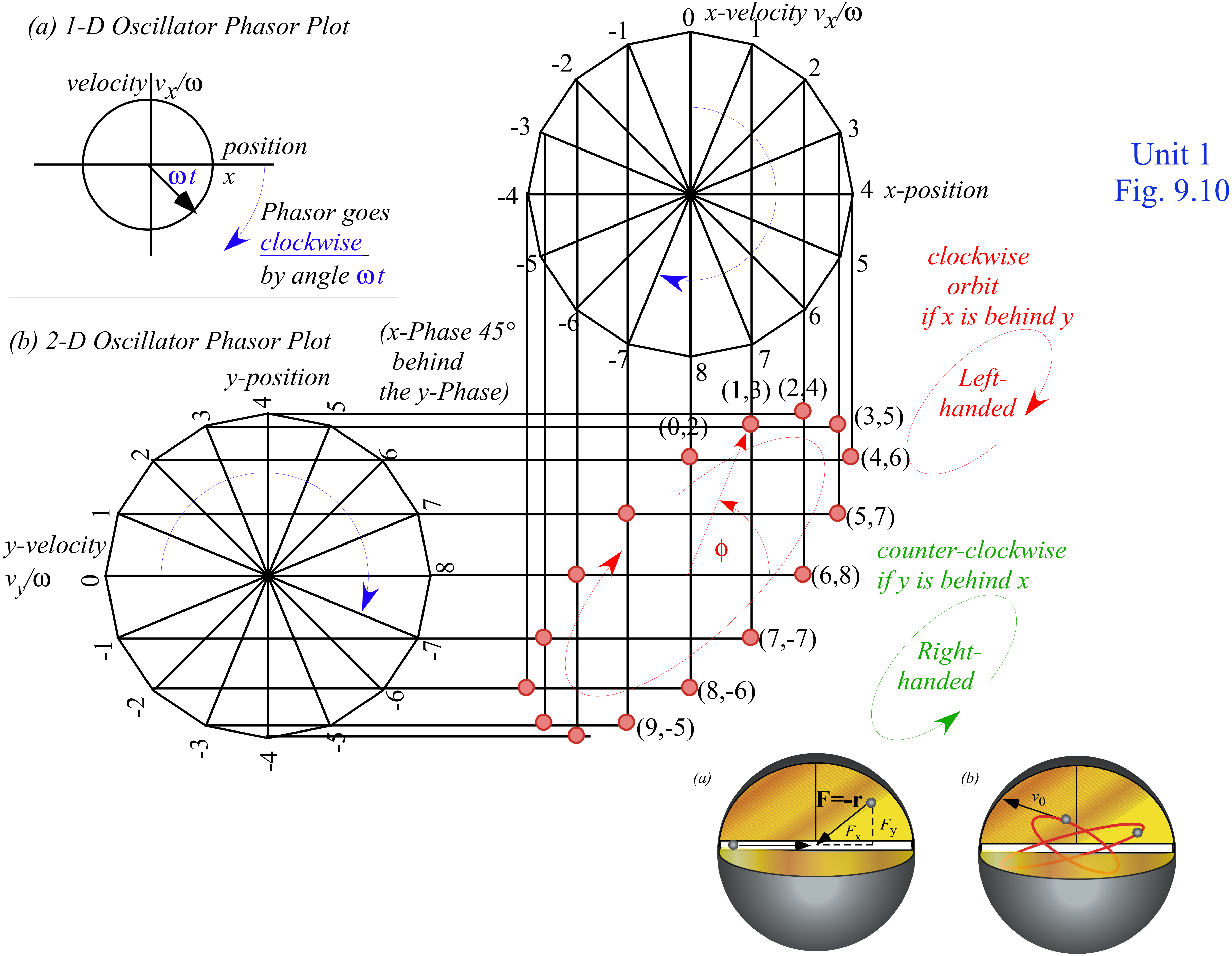
Sinusoidal space-time dynamics derived by geometry

Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)

→ *Constructing 2D Isotropic harmonic oscillator orbits using phasor plots*

Examples with x-y phase lag : $\alpha_{x\text{-}y}=\alpha_x-\alpha_y = 15°, 30°$, *and* $\pm 75°$

Isotropic Harmonic Oscillator phase dynamics in uniform-body



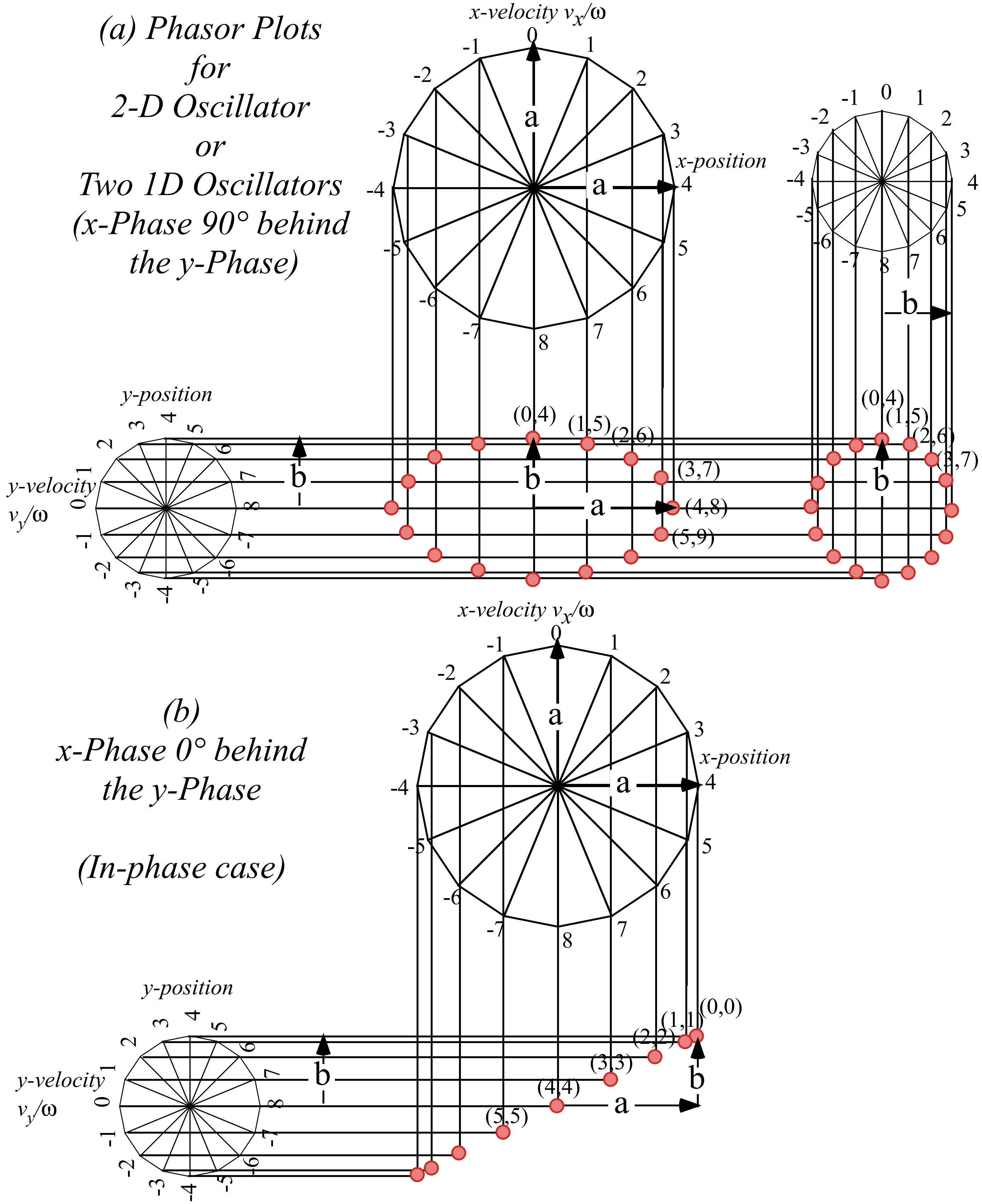
Unit 1
Fig. 9.10

Unit 1
Fig. 9.12

(a) Phasor Plots for 2-D Oscillator or Two 1D Oscillators (x-Phase 90° behind the y-Phase)

(b) x-Phase 0° behind the y-Phase

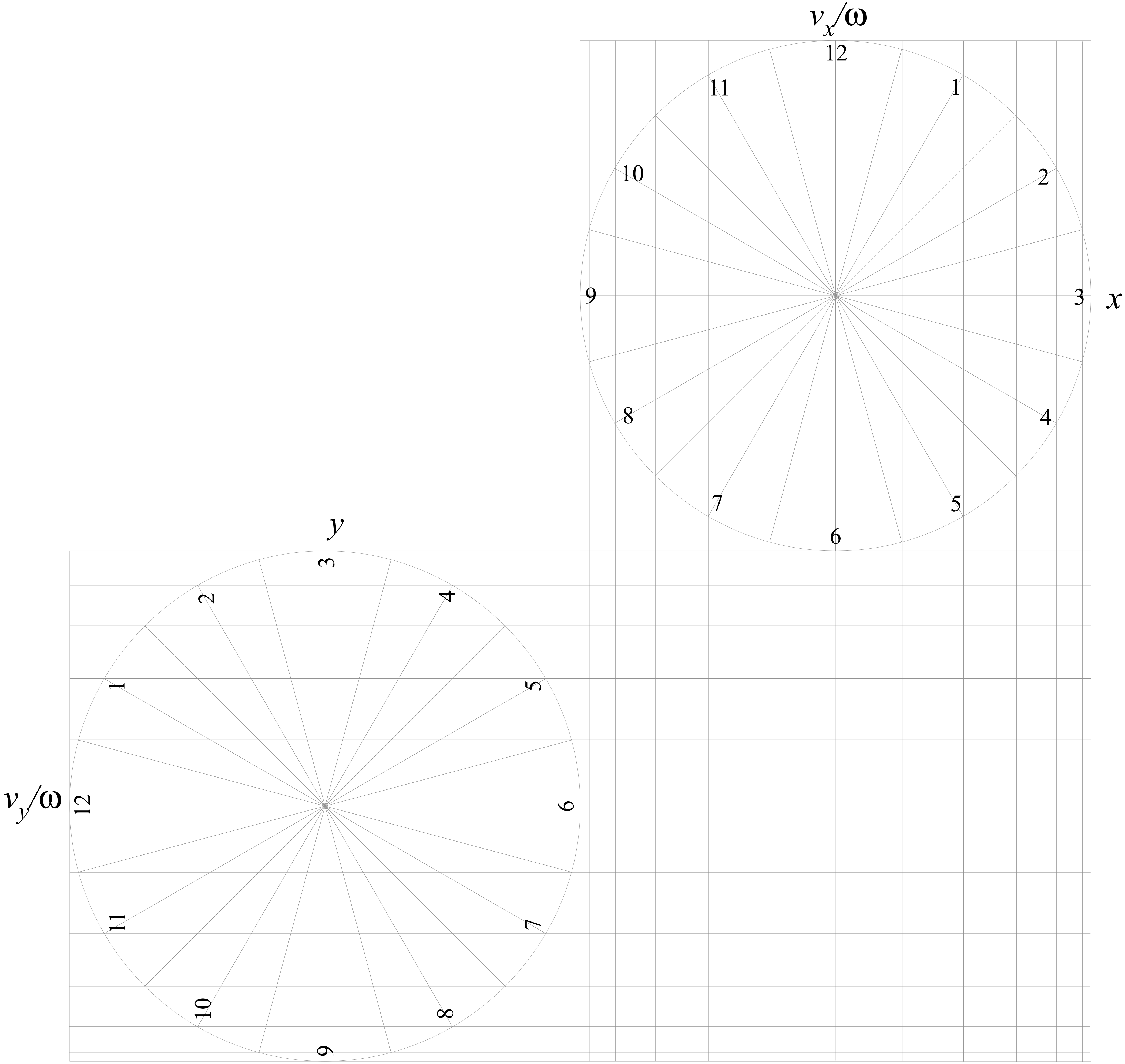
(In-phase case)



These are more generic examples with radius of x-phasor differing from that of the y-phasor.

Isotropic harmonic oscillator dynamics in 1D, 2D, and 3D

Sinusoidal space-time dynamics derived by geometry
Isotropic harmonic oscillator orbits in 1D and 2D (You get 3D for free!)
Constructing 2D Isotropic harmonic oscillator orbits using phasor plots
→ *Examples with x-y phase lag :* $\alpha_{x\text{-}y}=\alpha_x-\alpha_y = 15°, 30°$, *and* $\pm 75°$



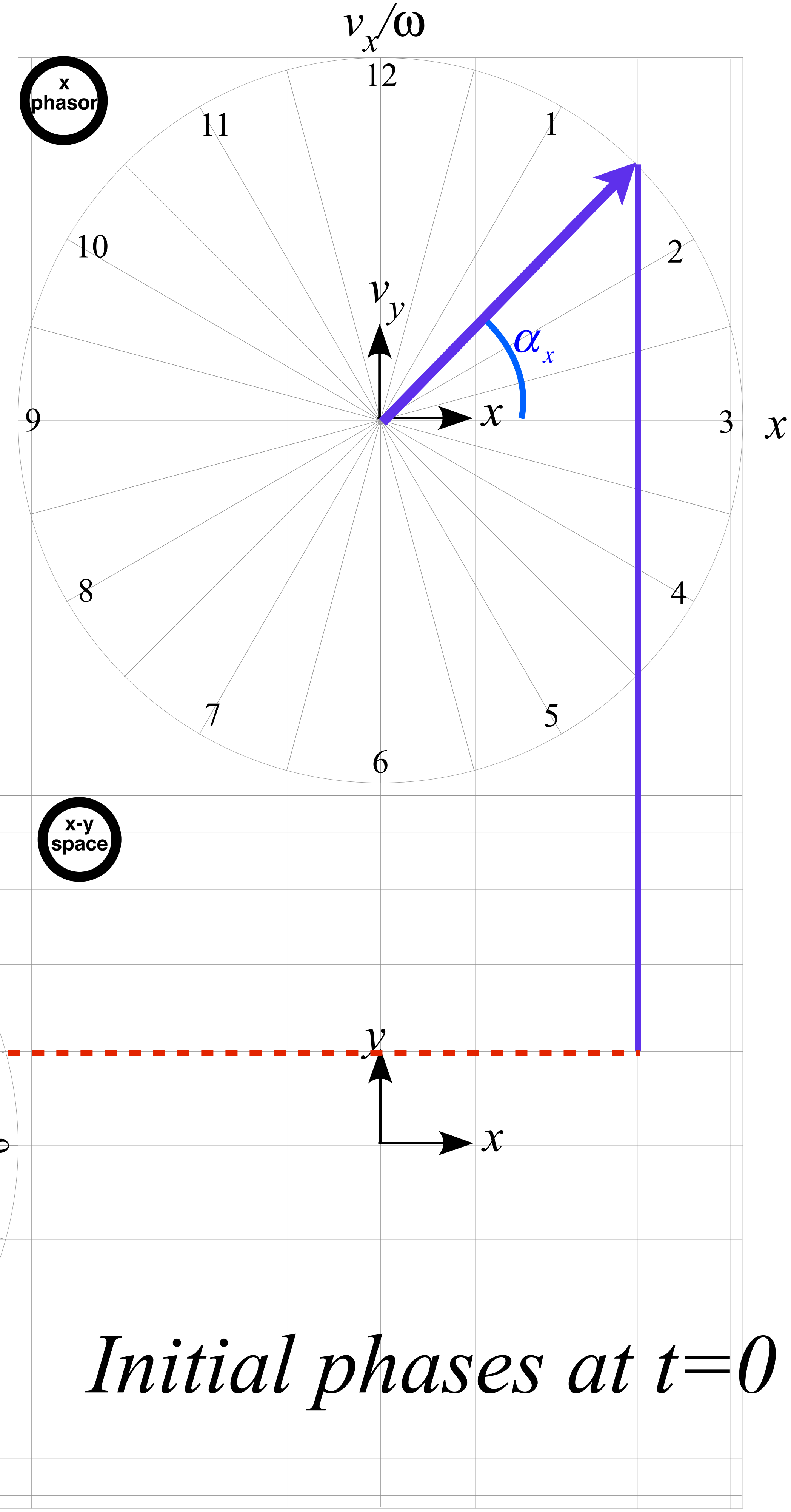
v_x/ω
12
11
1
10
2
9
3
x
8
4
7
5
6
y
3
2
4
1
5
v_y/ω
12
6
11
7
10
8
9

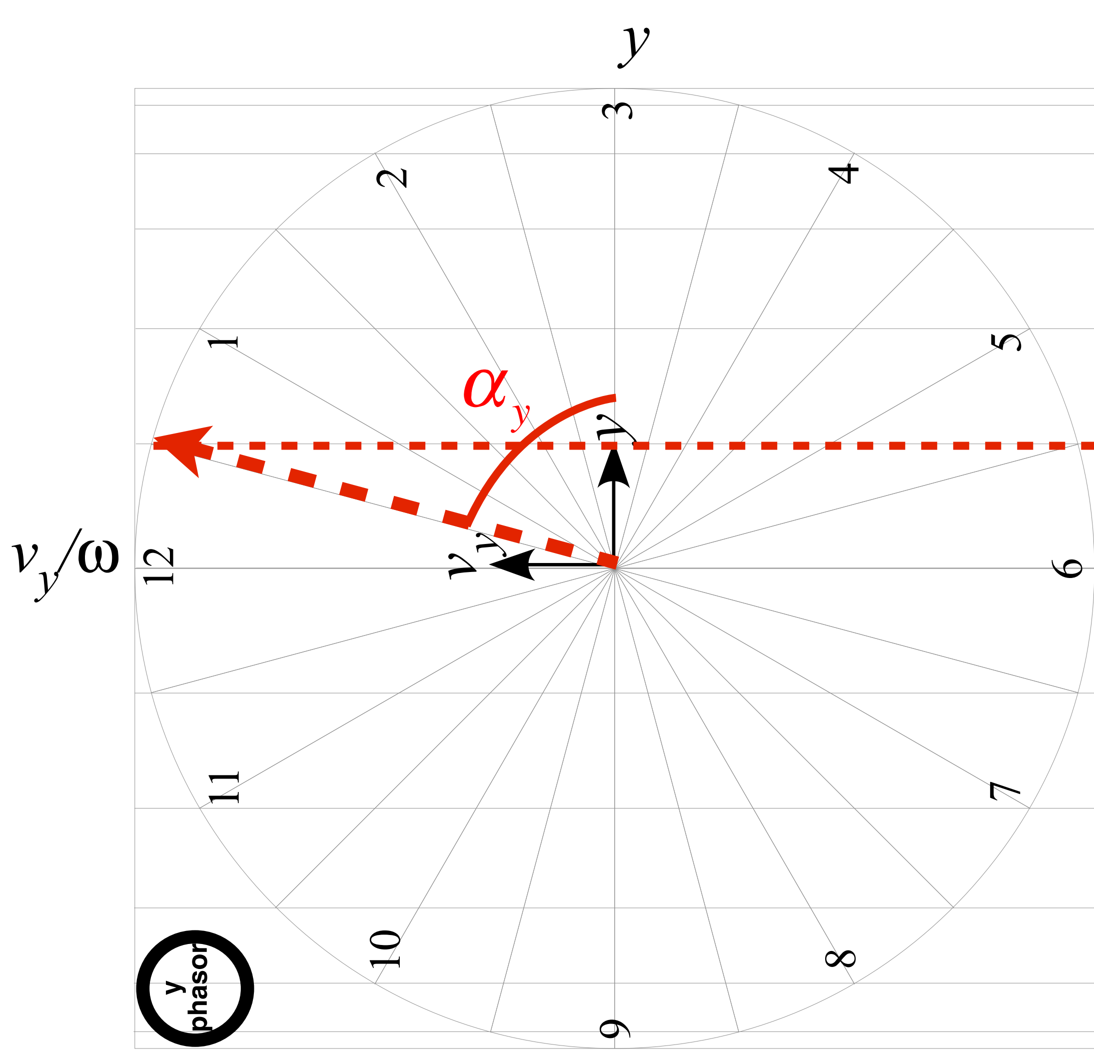
$$x(t) = A_x \cos(\omega \cdot t - \alpha_x) \qquad = A_x \cos(\alpha_x - \omega \cdot t)$$

$$\frac{v_x(t)}{\omega} = -A_x \sin(\omega \cdot t - \alpha_x) \quad = A_x \sin(\alpha_x - \omega \cdot t)$$

$$y(t) = A_y \cos(\omega \cdot t - \alpha_y) \qquad = A_y \cos(\alpha_y - \omega \cdot t)$$

$$\frac{v_y(t)}{\omega} = -A_y \sin(\omega \cdot t - \alpha_y) \quad = A_y \sin(\alpha_y - \omega \cdot t)$$





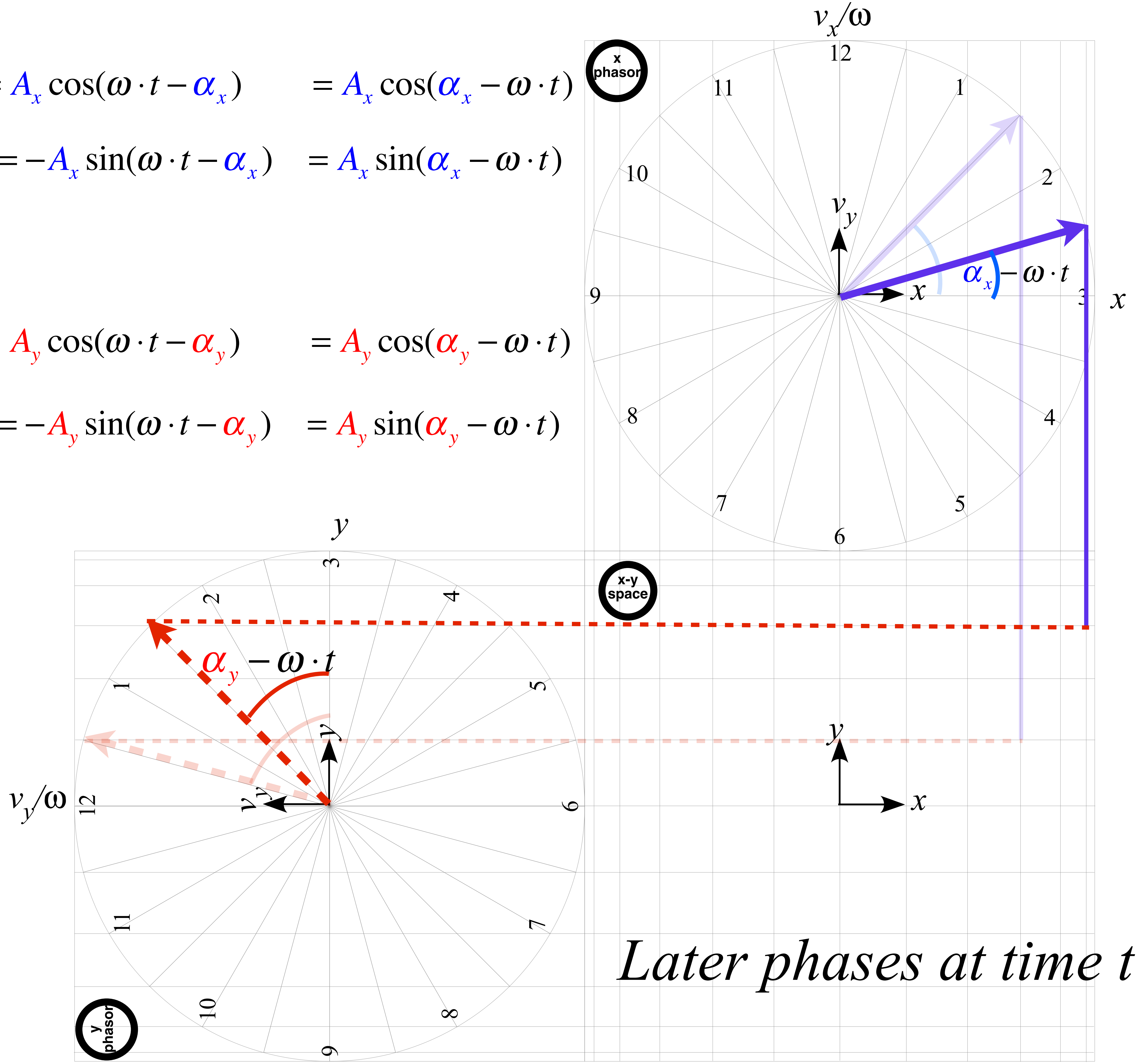
Initial phases at t=0

$$x(t) = A_x \cos(\omega \cdot t - \alpha_x) \qquad = A_x \cos(\alpha_x - \omega \cdot t)$$

$$\frac{v_x(t)}{\omega} = -A_x \sin(\omega \cdot t - \alpha_x) \quad = A_x \sin(\alpha_x - \omega \cdot t)$$

$$y(t) = A_y \cos(\omega \cdot t - \alpha_y) \qquad = A_y \cos(\alpha_y - \omega \cdot t)$$

$$\frac{v_y(t)}{\omega} = -A_y \sin(\omega \cdot t - \alpha_y) \quad = A_y \sin(\alpha_y - \omega \cdot t)$$



Later phases at time t

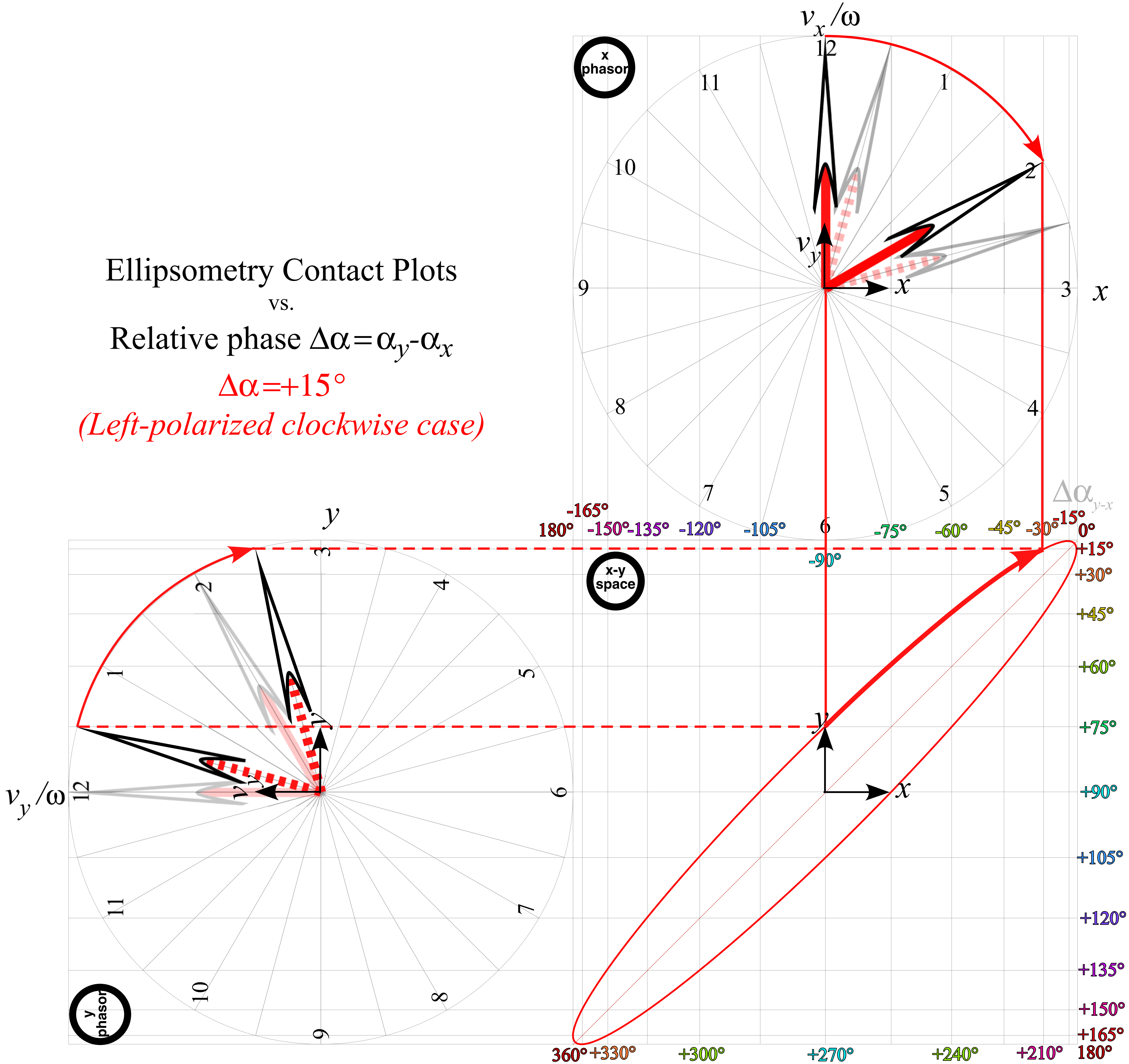
Ellipsometry Contact Plots

vs.

Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

$\Delta\alpha=+15°$

(Left-polarized clockwise case)



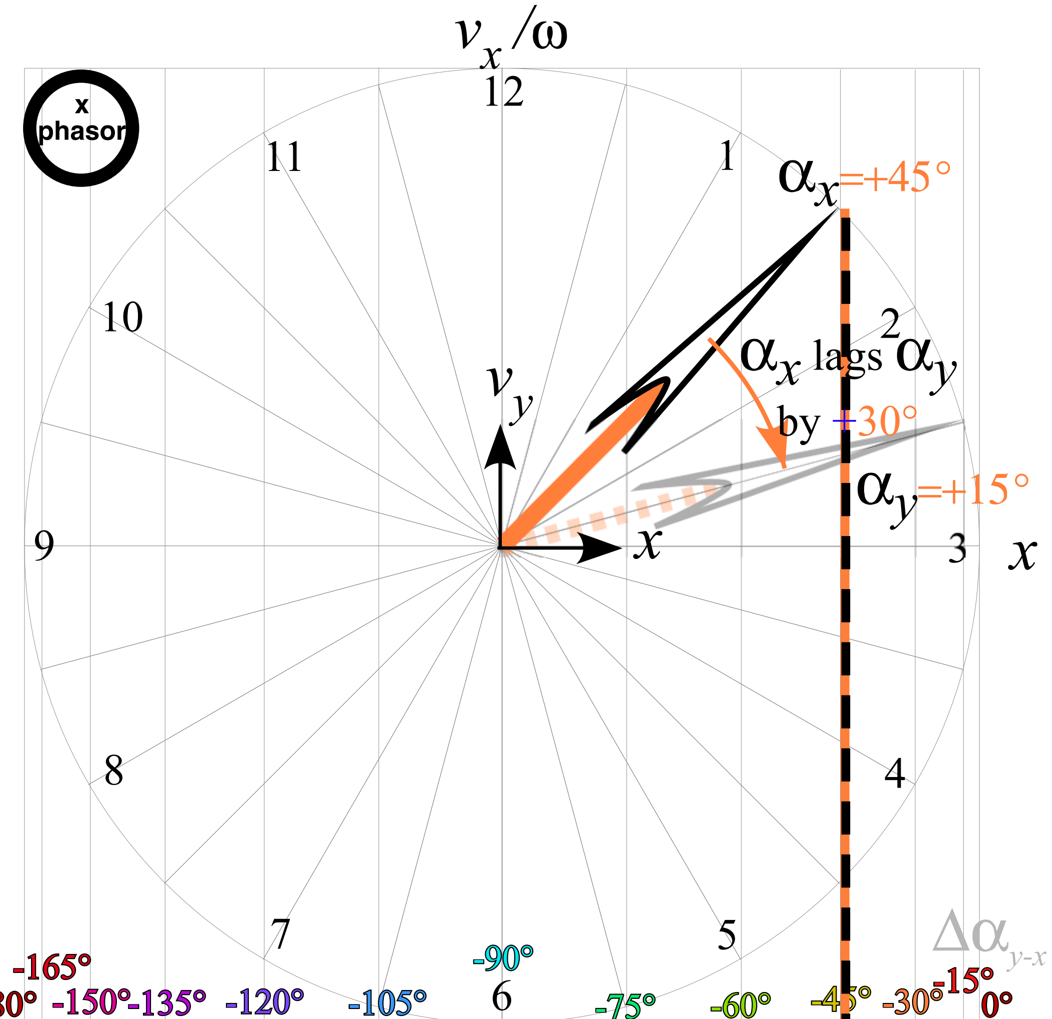
+30° case

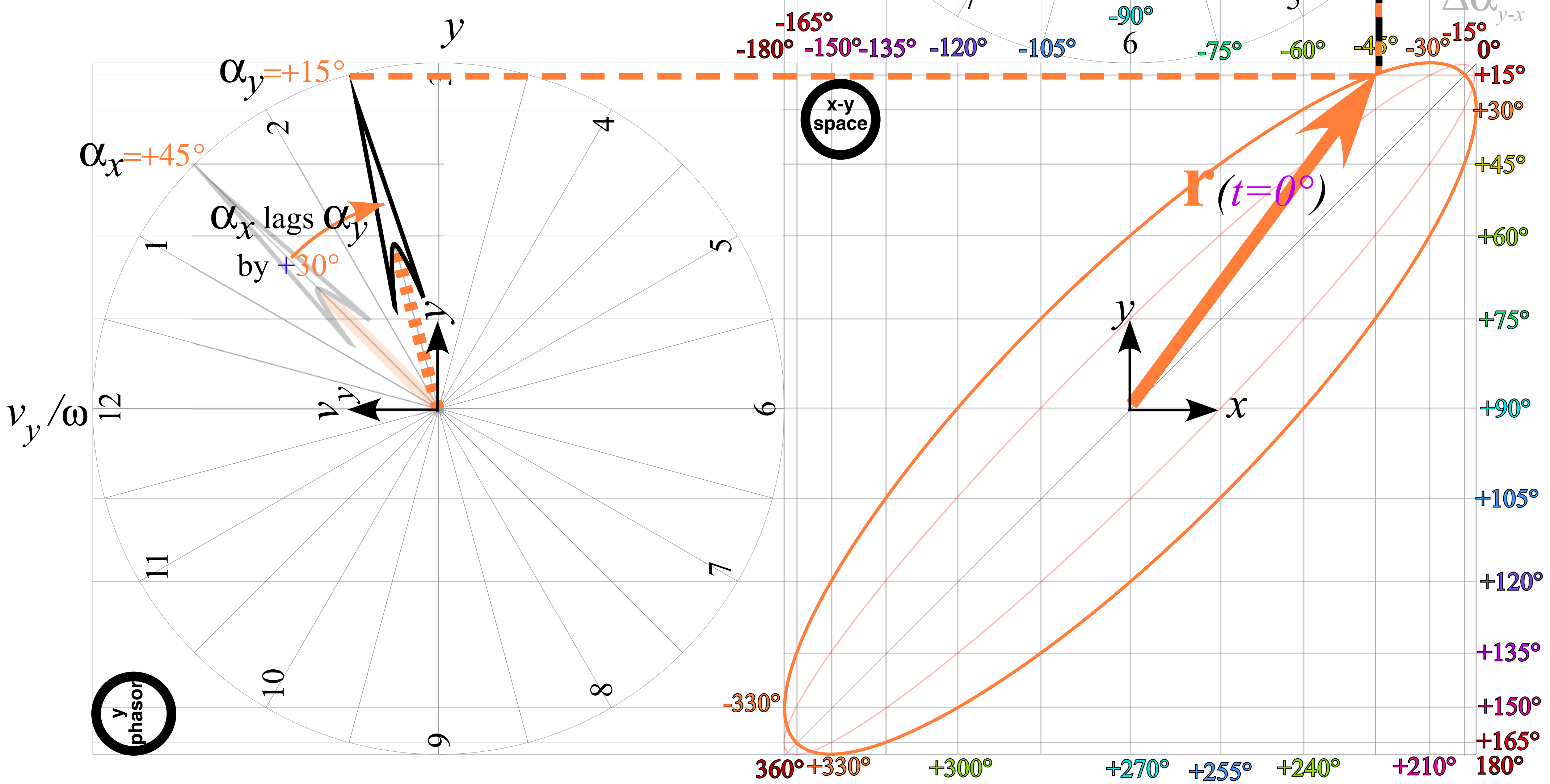
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha = \alpha_y - \alpha_x$

$\Delta\alpha = \alpha_x - \alpha_y = +30°$

Left-polarized (clockwise) orbit

Initial phase (α_x=45°, α_y=+15°)





+30° case

Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

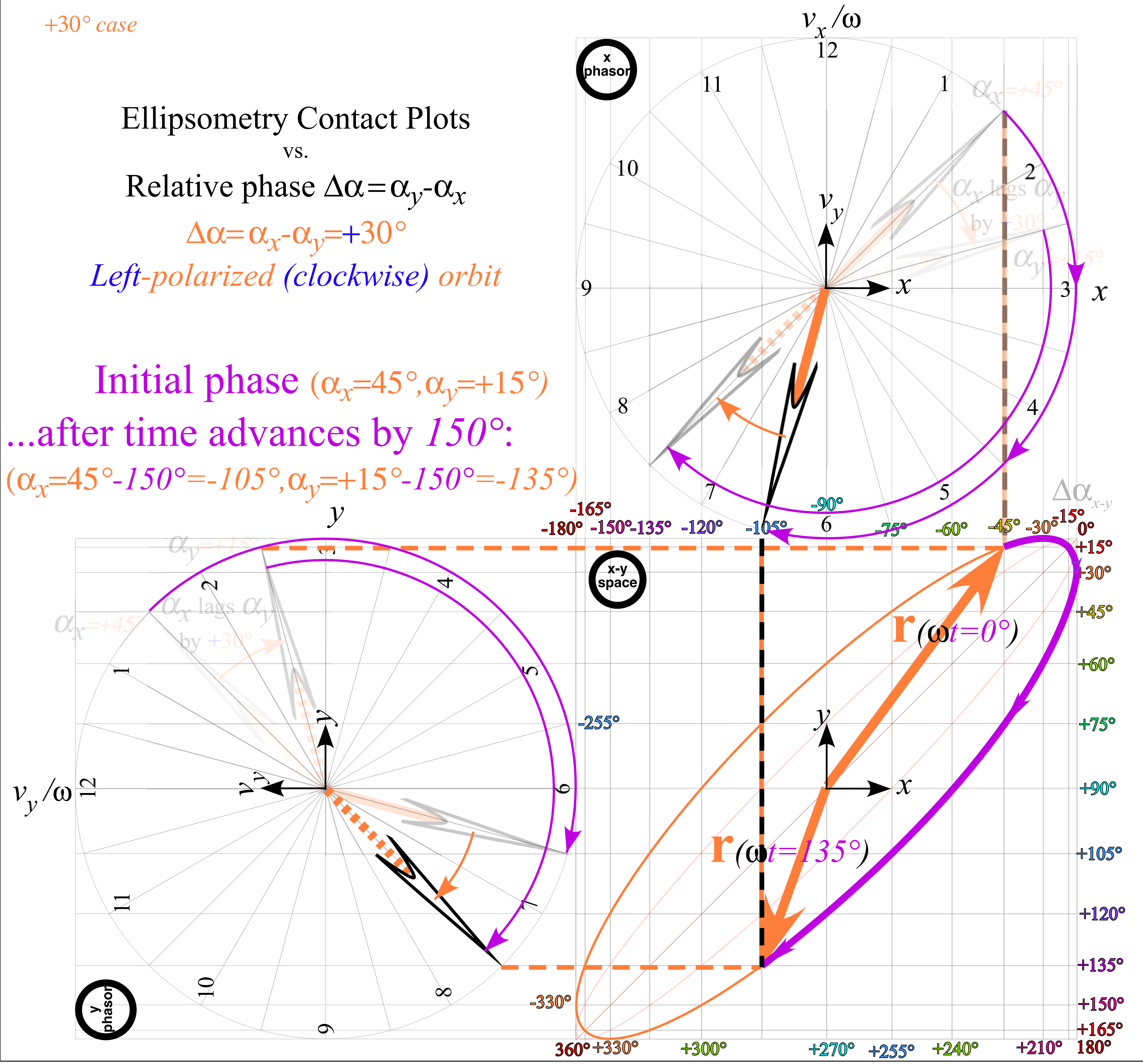
$\Delta\alpha=\alpha_x-\alpha_y=+30°$

Left-polarized (clockwise) orbit

Initial phase $(\alpha_x=45°, \alpha_y=+15°)$

...after time advances by *150°*:

$(\alpha_x=45°-150°=-105°, \alpha_y=+15°-150°=-135°)$



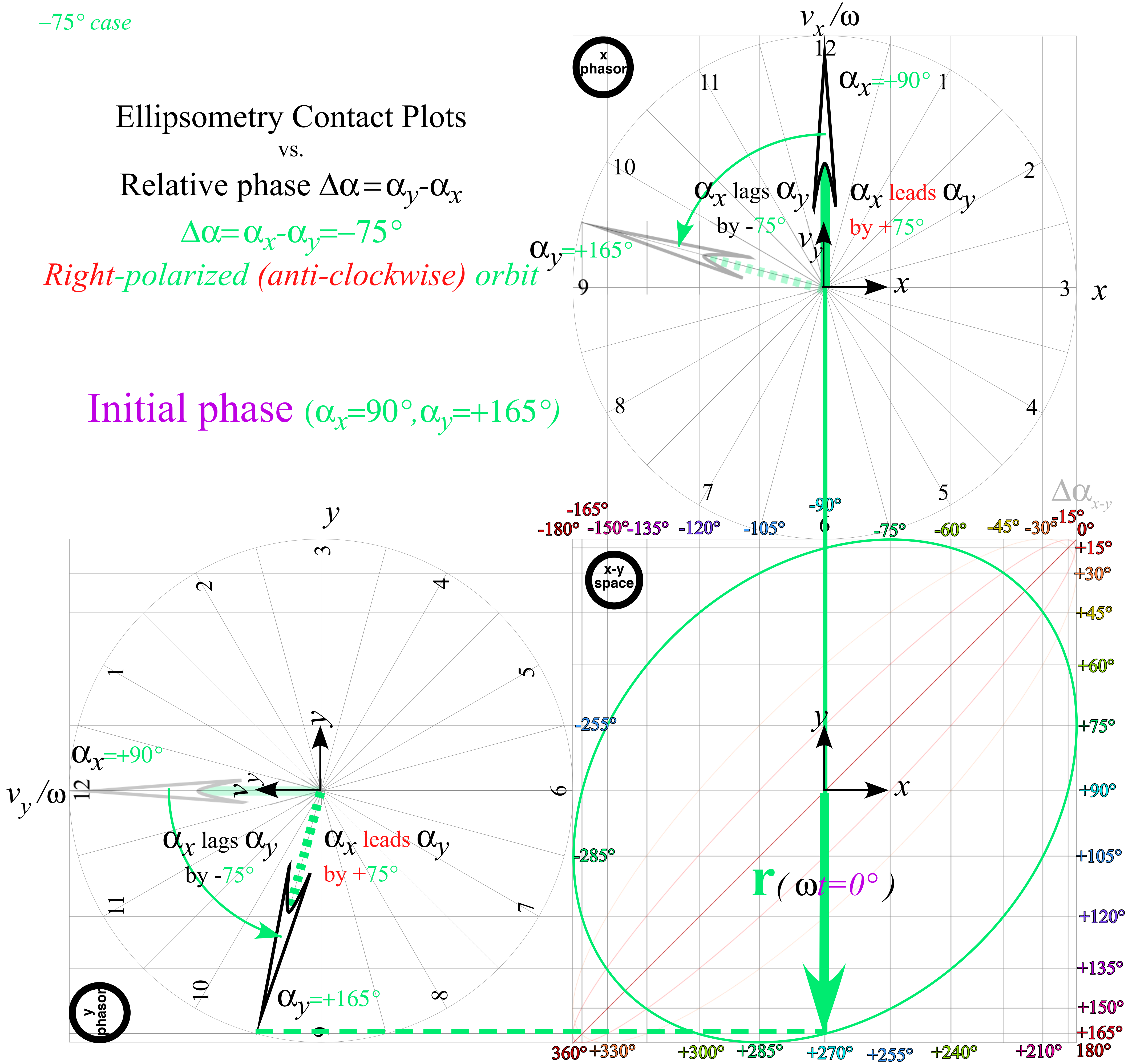
−75° case

Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

$\Delta\alpha=\alpha_x-\alpha_y=-75°$

Right-polarized (anti-clockwise) orbit

Initial phase $(\alpha_x=90°, \alpha_y=+165°)$



−75° case

Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_y-\alpha_x$

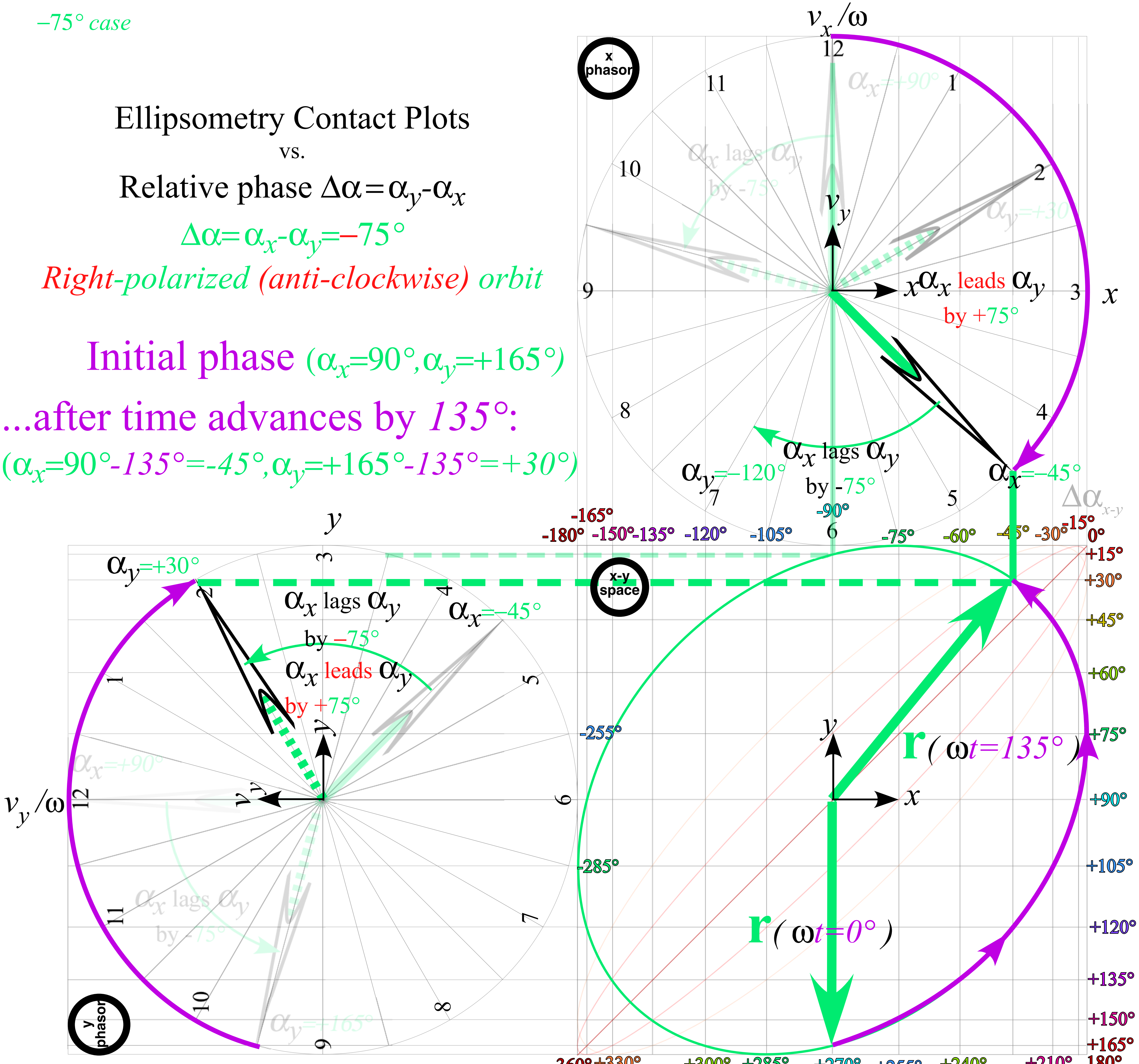
$\Delta\alpha=\alpha_x-\alpha_y=-75°$

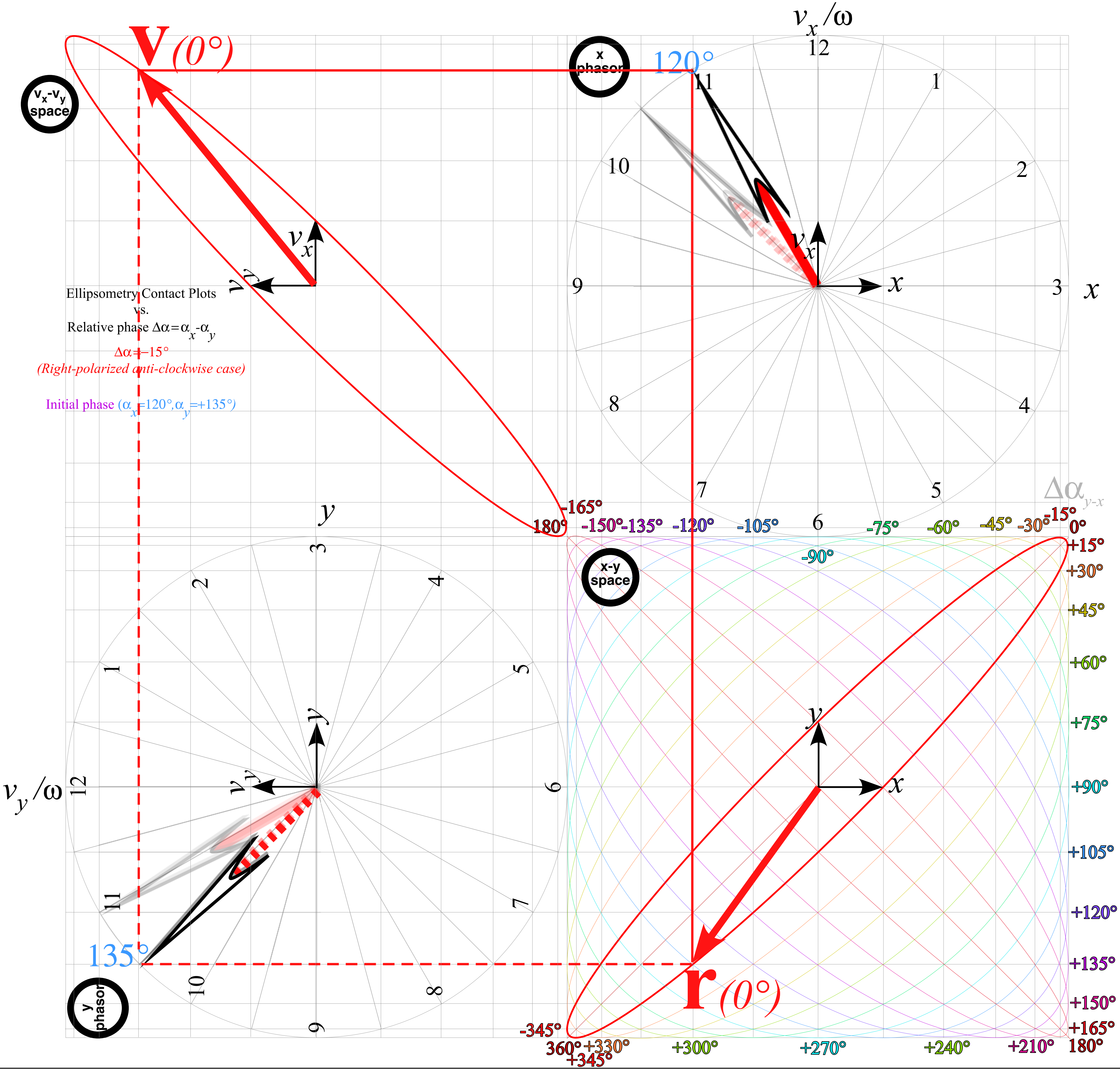
Right-polarized (anti-clockwise) orbit

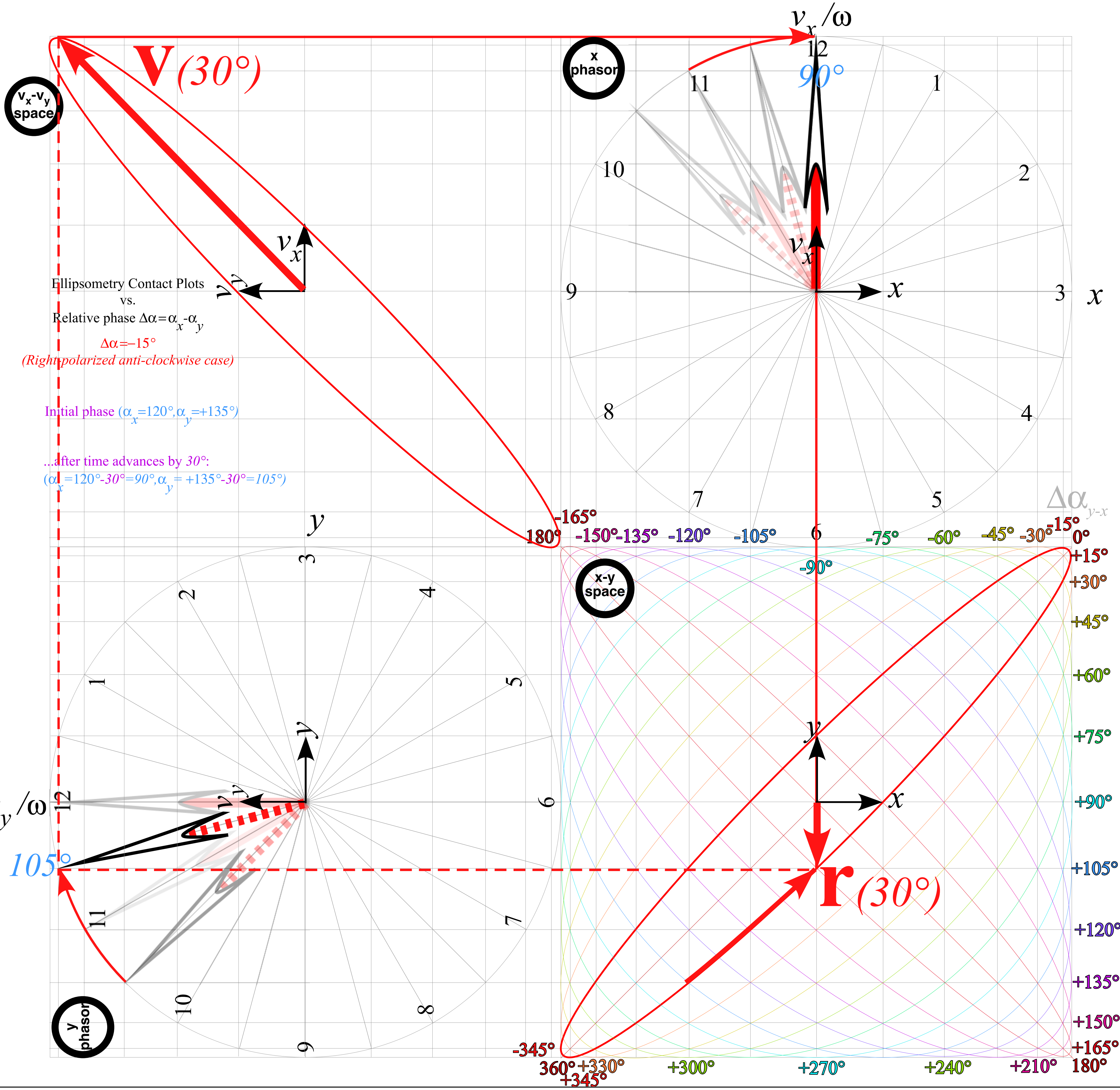
Initial phase $(\alpha_x=90°,\alpha_y=+165°)$

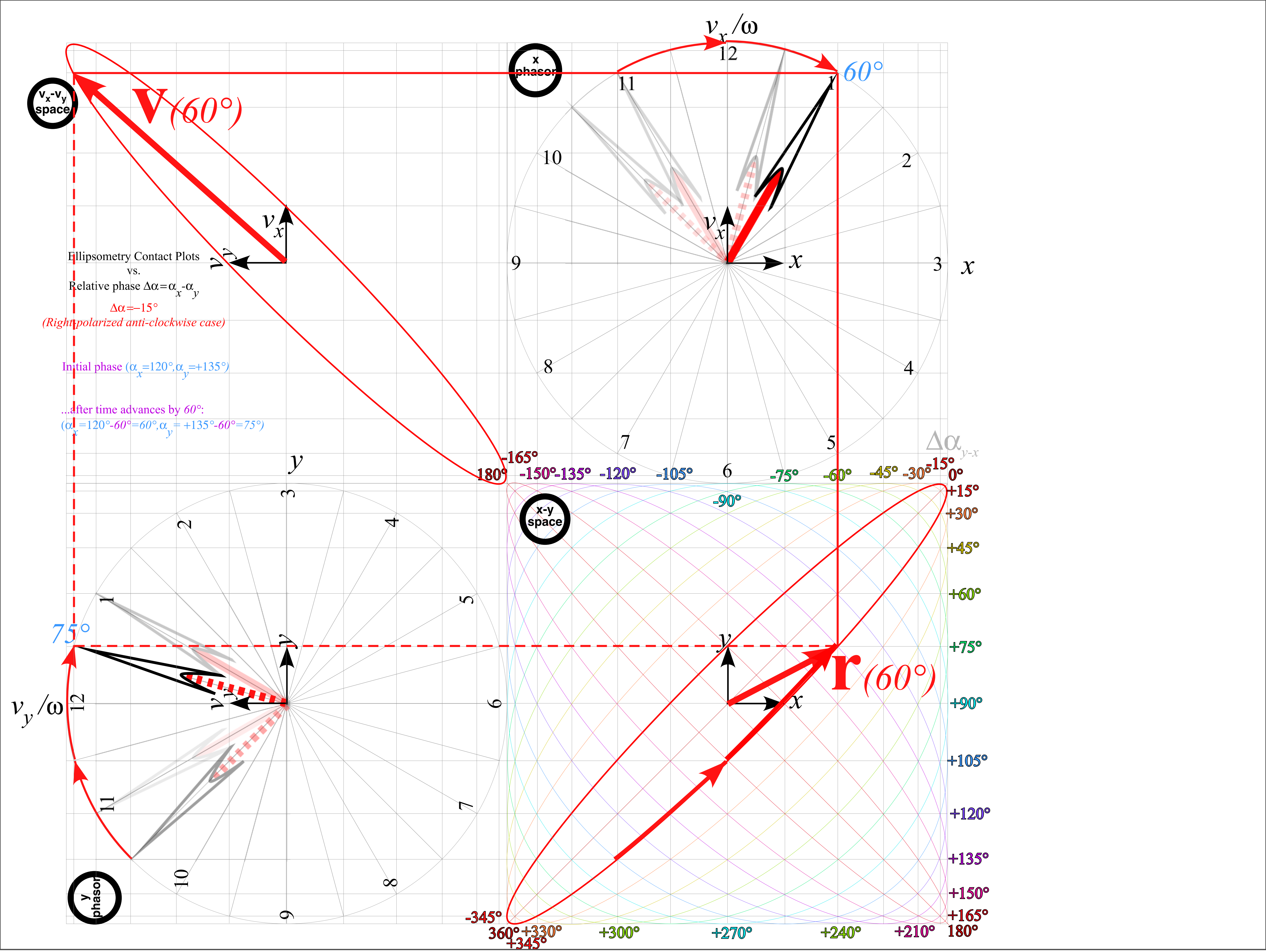
...after time advances by *135°*:

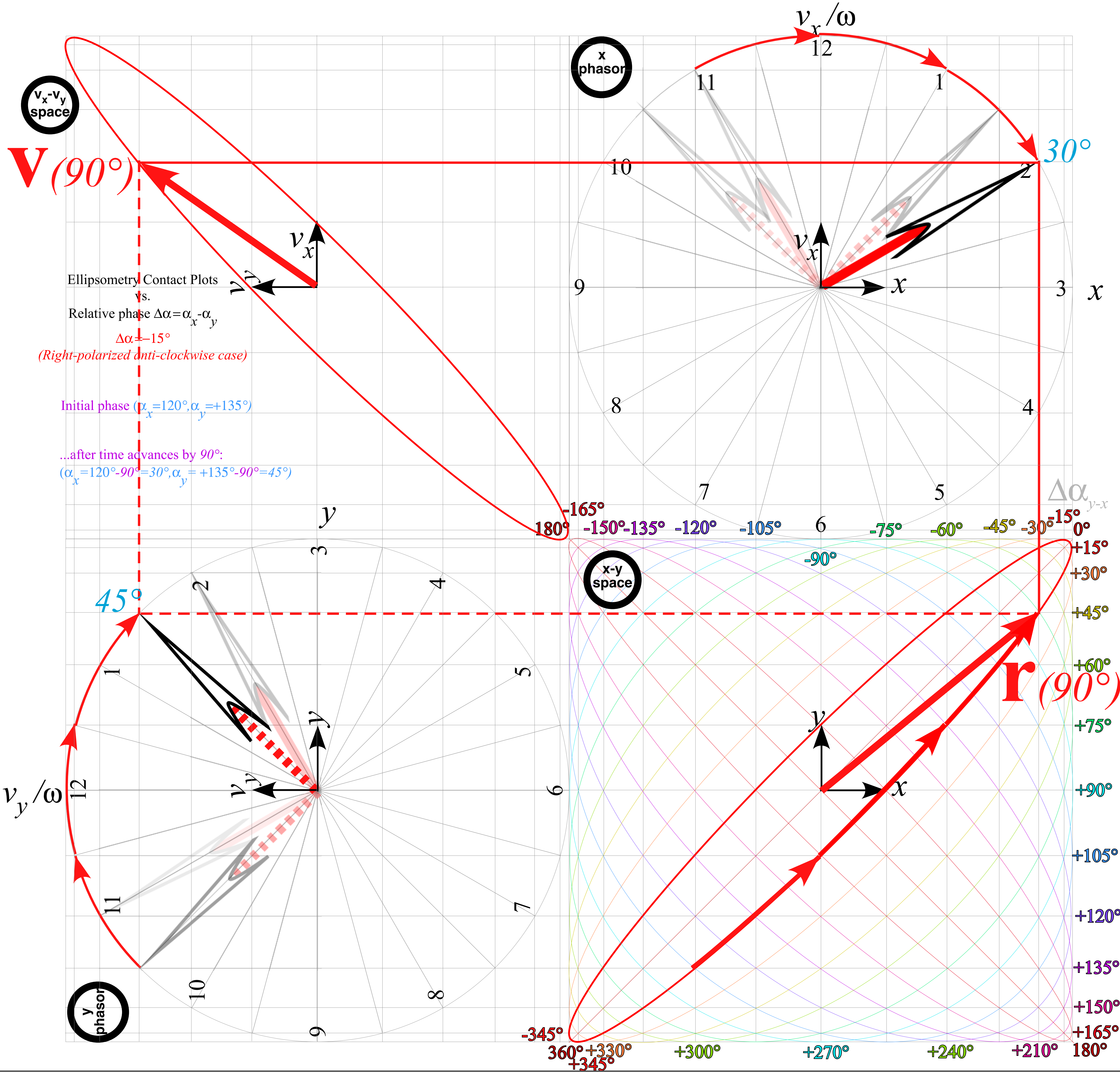
$(\alpha_x=90°-135°=-45°,\alpha_y=+165°-135°=+30°)$

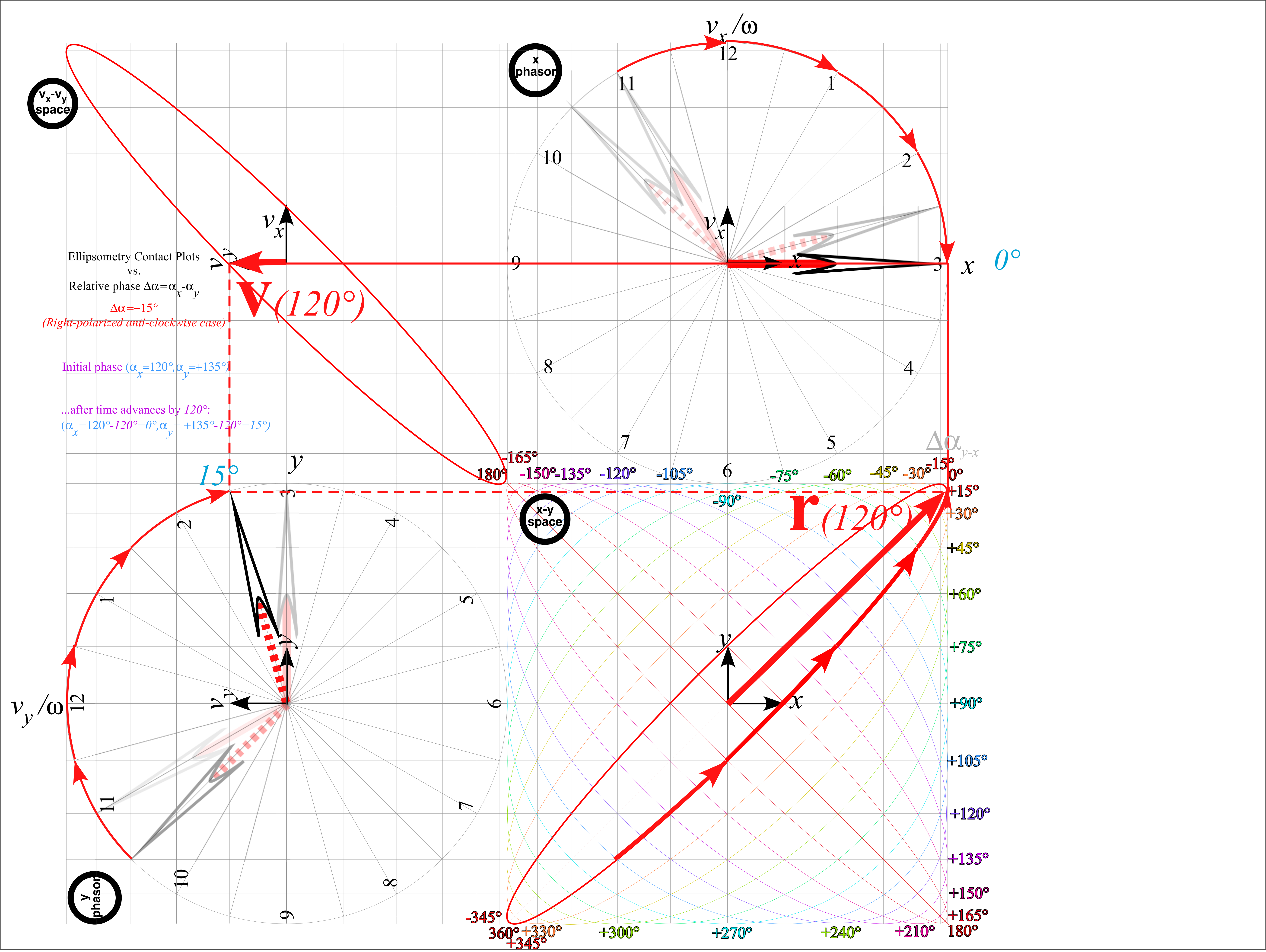



V(0°)
vx-vy space
x phasor
120°
vx/ω
x
vx
vy
Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
y
vy/ω
y phasor
135°
x-y space
r(0°)
Δαy-x
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°

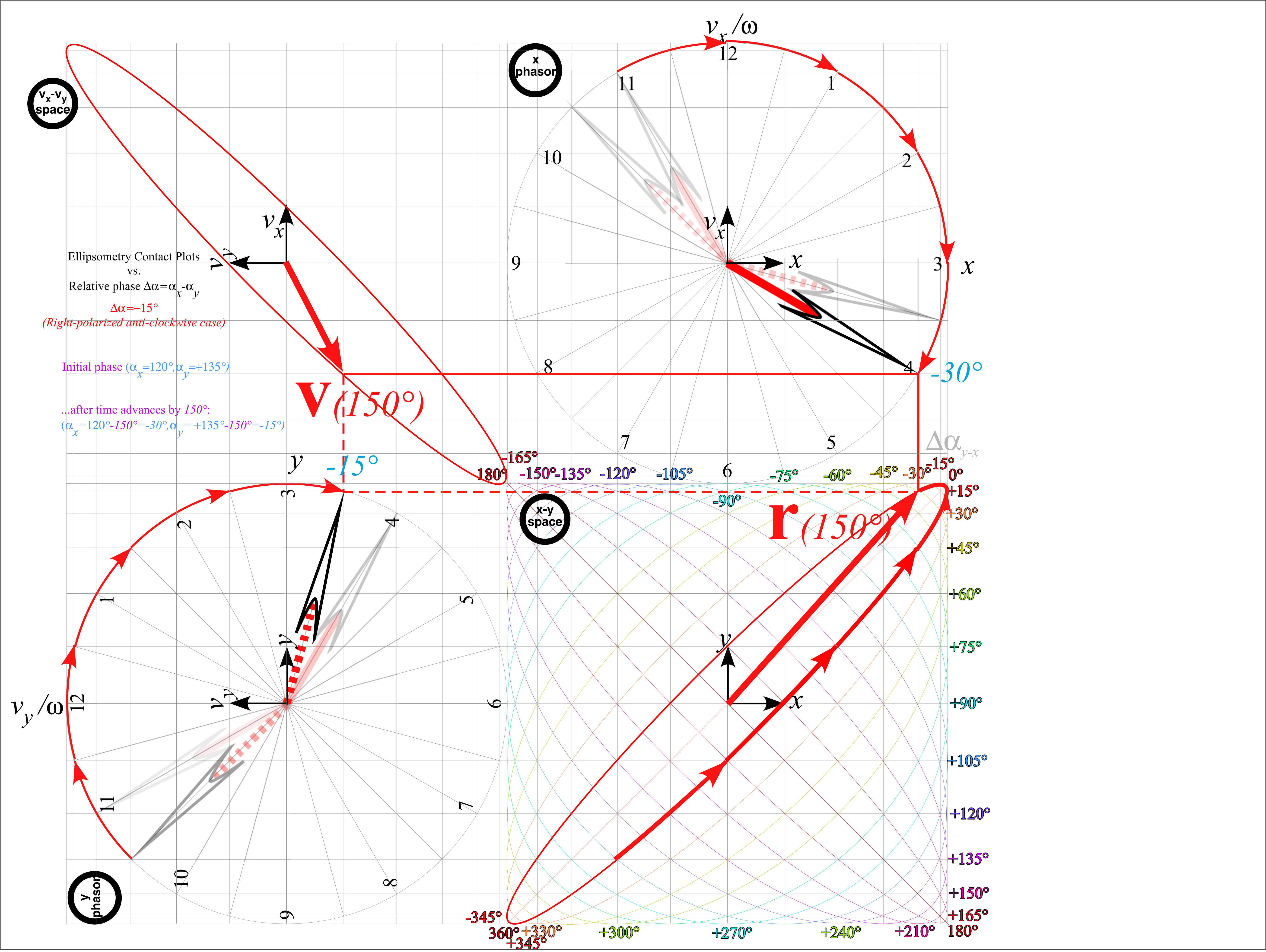

V(30°)
v_x-v_y space
x phasor
v_x/ω
90°
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right polarized anti-clockwise case)
Initial phase ($\alpha_x=120°$, $\alpha_y=+135°$)
...after time advances by 30°:
($\alpha_x=120°-30°=90°$, $\alpha_y=+135°-30°=105°$)
x-y space
y phasor
v_y/ω
105°
r(30°)
$\Delta\alpha_{y-x}$
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°

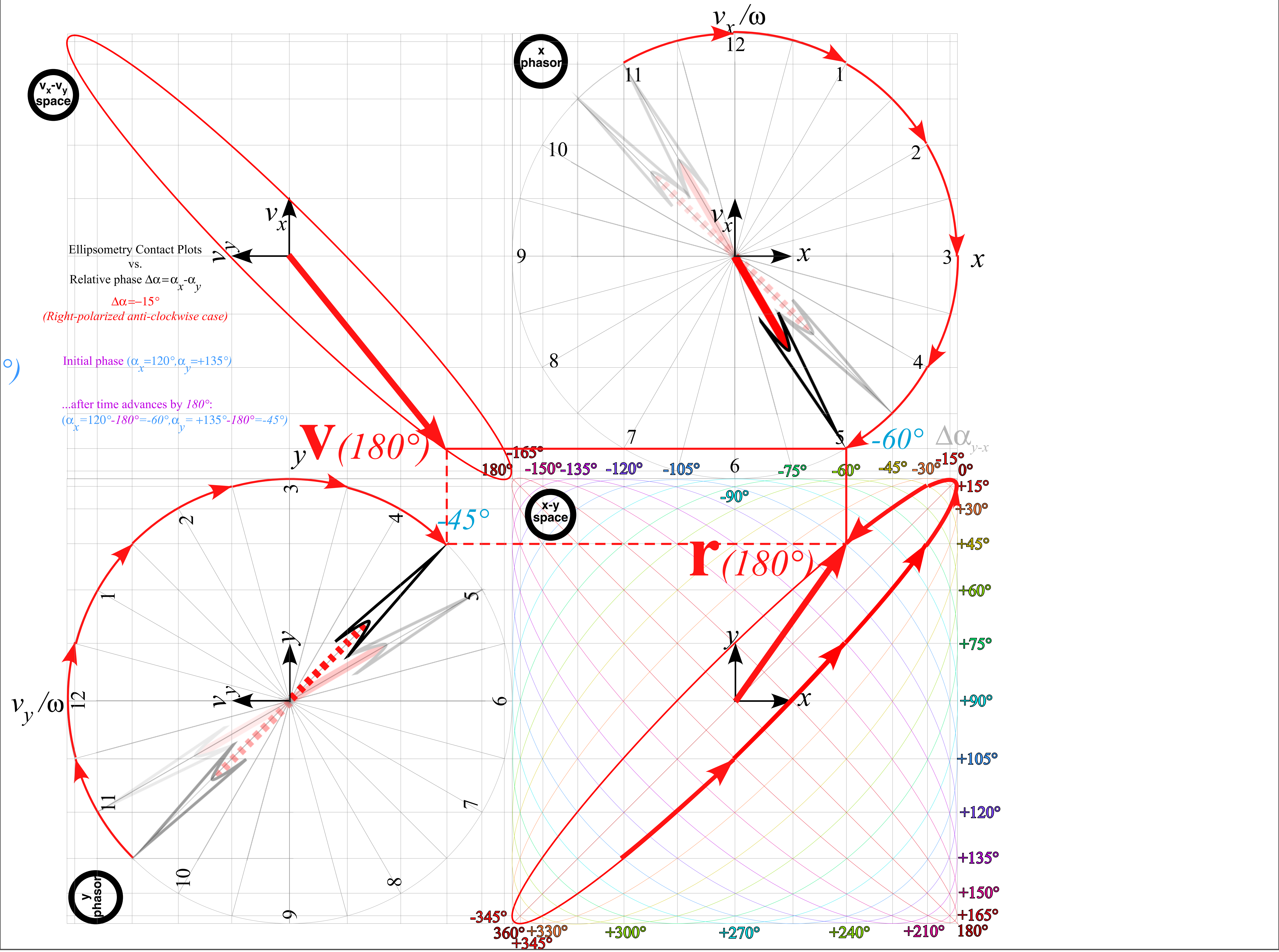

Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 60°:
(α_x=120°-60°=60°,α_y = +135°-60°=75°)
v_x-v_y space
x phasor
y phasor
x-y space
V(60°)
r(60°)
v_x/ω
v_y/ω
60°
75°
$\Delta\alpha_{y-x}$
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°


Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 90°:
(α_x=120°-90°=30°,α_y = +135°-90°=45°)
v_x-v_y space
x phasor
y phasor
x-y space
V (90°)
r (90°)
30°
45°
v_x/ω
v_y/ω
$\Delta\alpha_{y-x}$
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +345° +330° +300° +270° +240° +210°

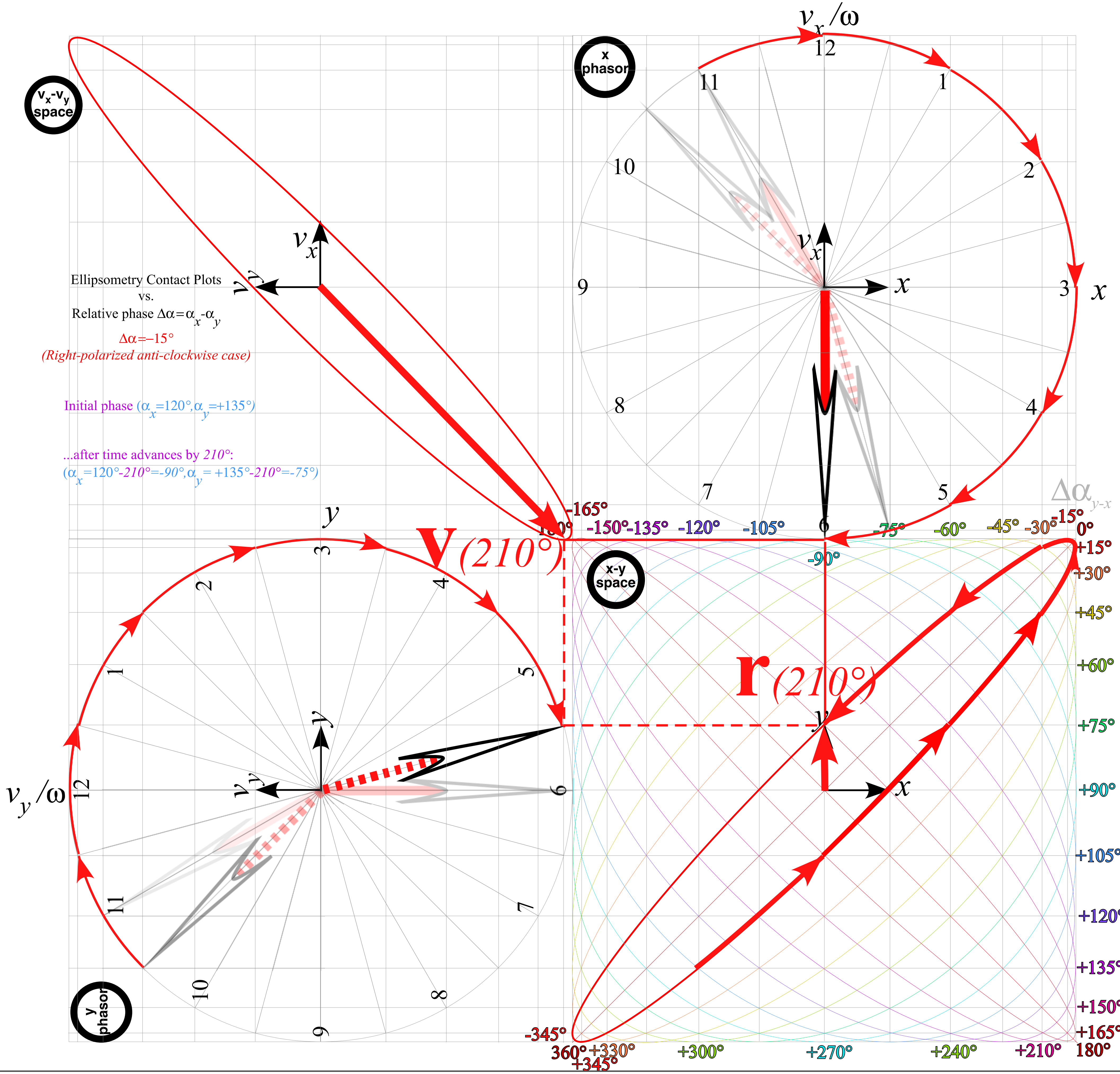


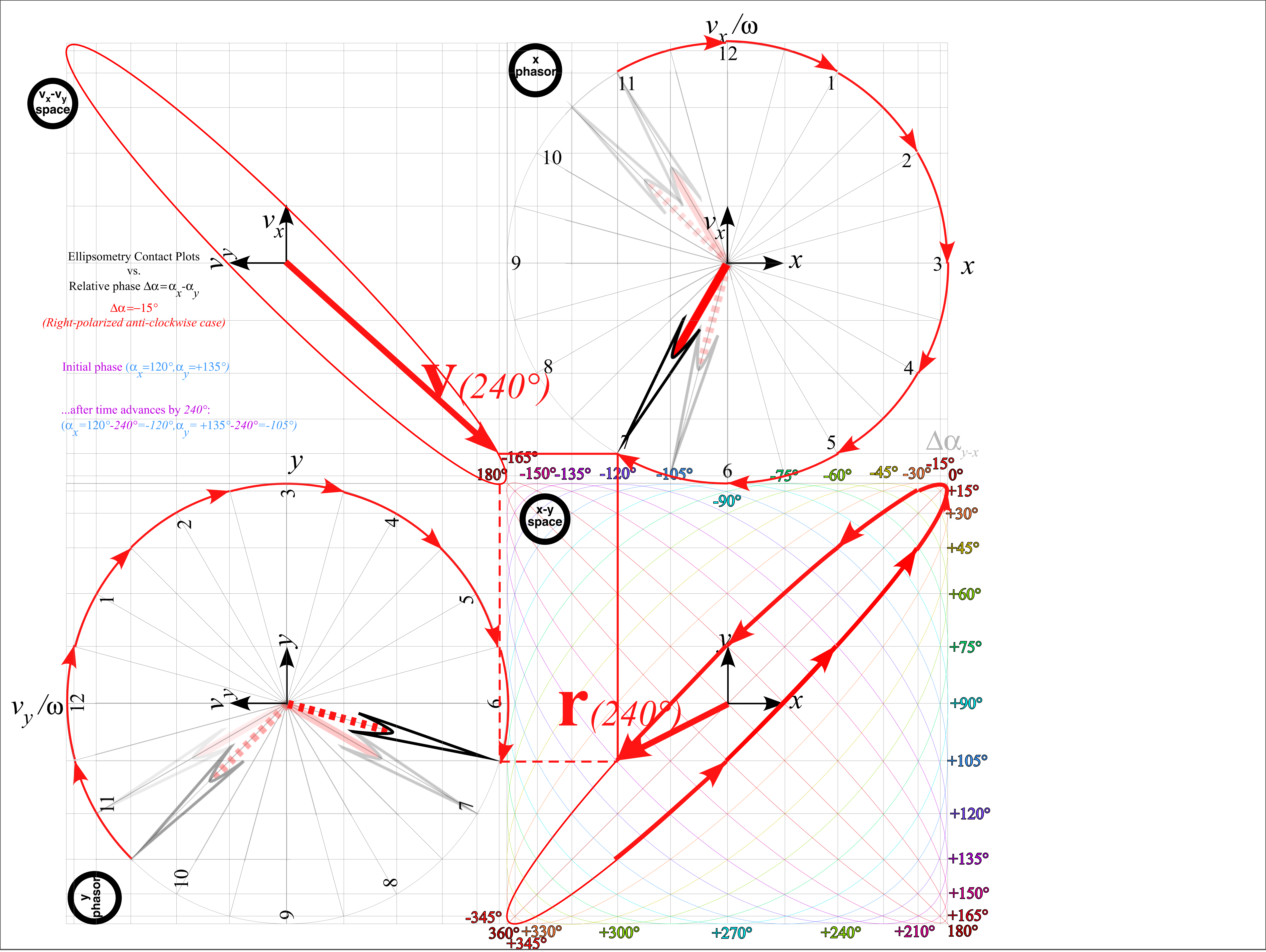
Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 120°:
(α_x=120°-120°=0°,α_y = +135°-120°=15°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(120°)
r(120°)
0°
15°
Δα_y-x


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 150°:
(α_x=120°-150°=-30°,α_y=+135°-150°=-15°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(150°)
r(150°)
-30°
-15°
Δα_y-x

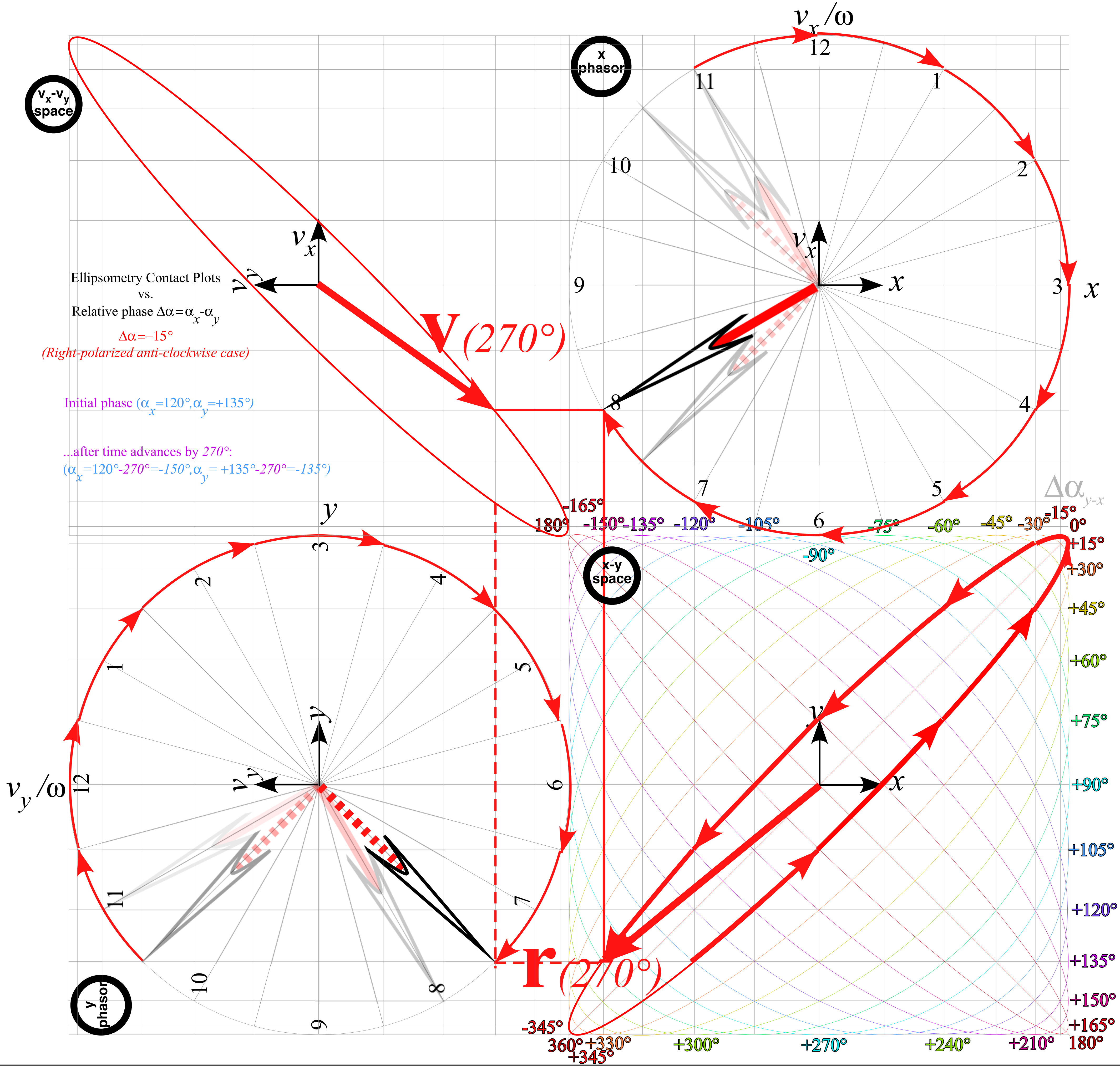


v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 180°:
(α_x=120°-180°=-60°,α_y= +135°-180°=-45°)
V(180°)
r(180°)
-60°
-45°
$\Delta\alpha_{y-x}$

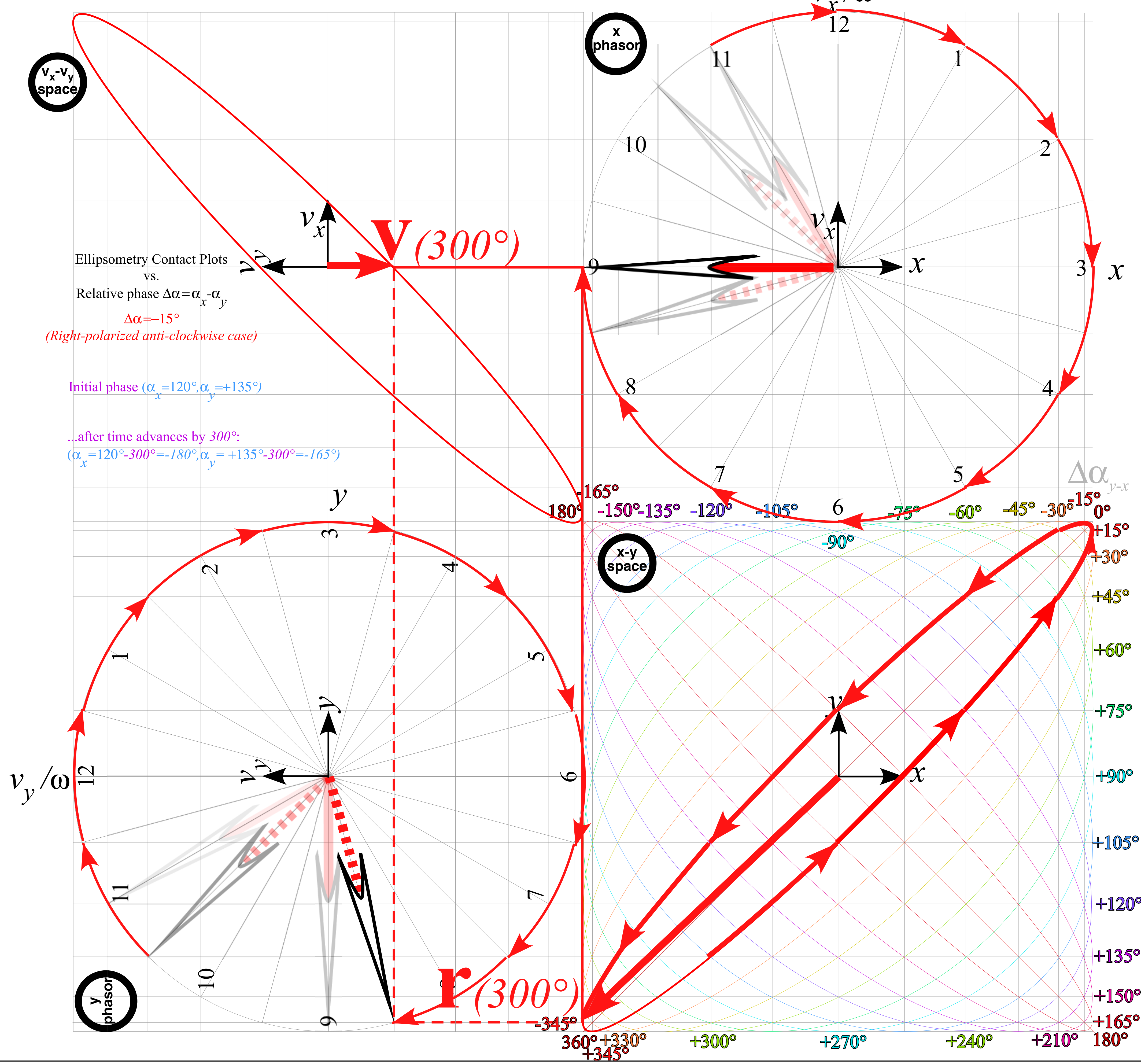

Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 210°:
(α_x=120°-210°=-90°,α_y = +135°-210°=-75°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(210°)
r(210°)
Δα_y-x

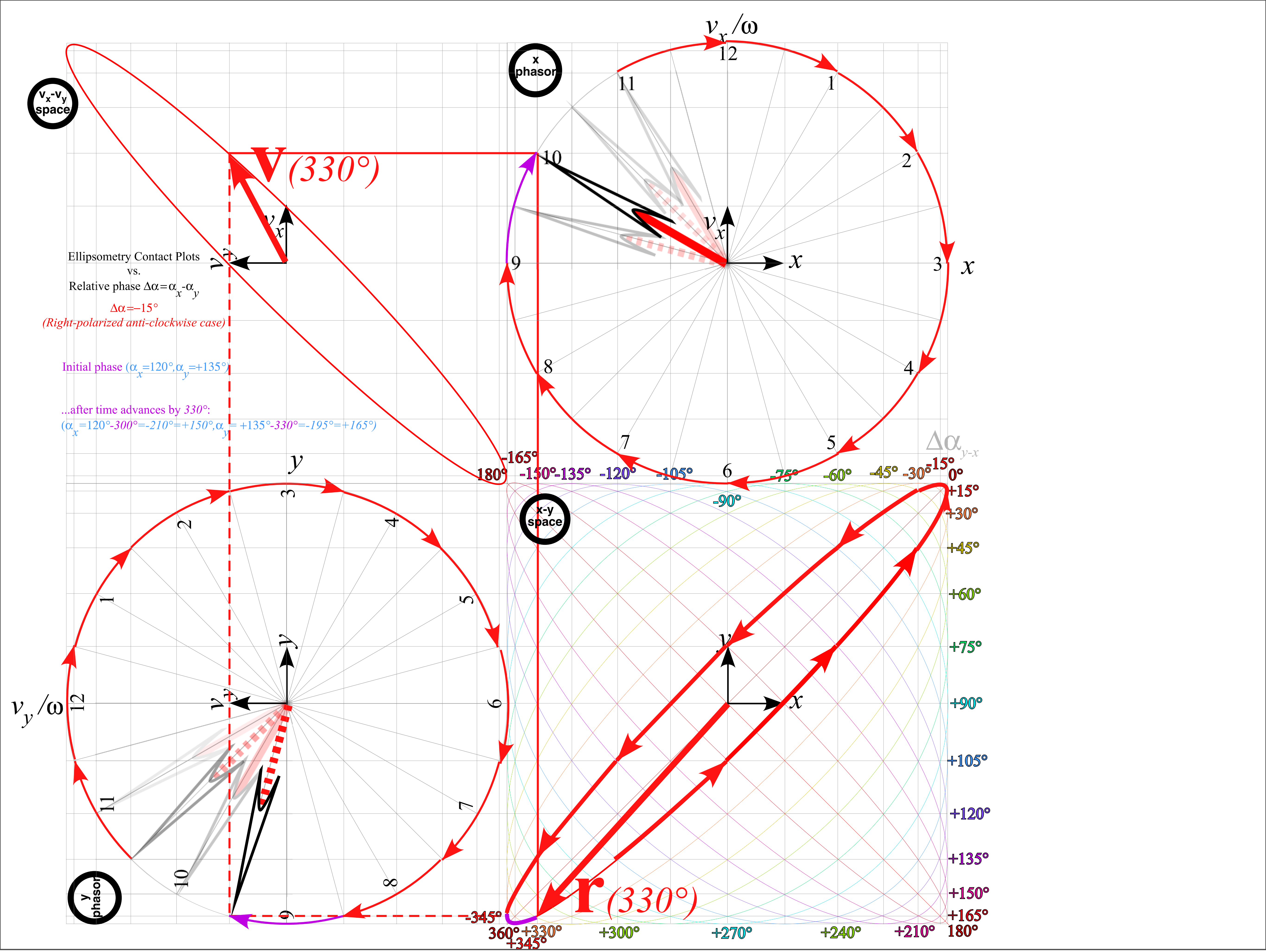


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 240°:
(α_x=120°-240°=-120°,α_y= +135°-240°=-105°)
v_x-v_y space
x phasor
y phasor
x-y space
v_x/ω
v_y/ω
V(240°)
r(240°)
Δα_y-x

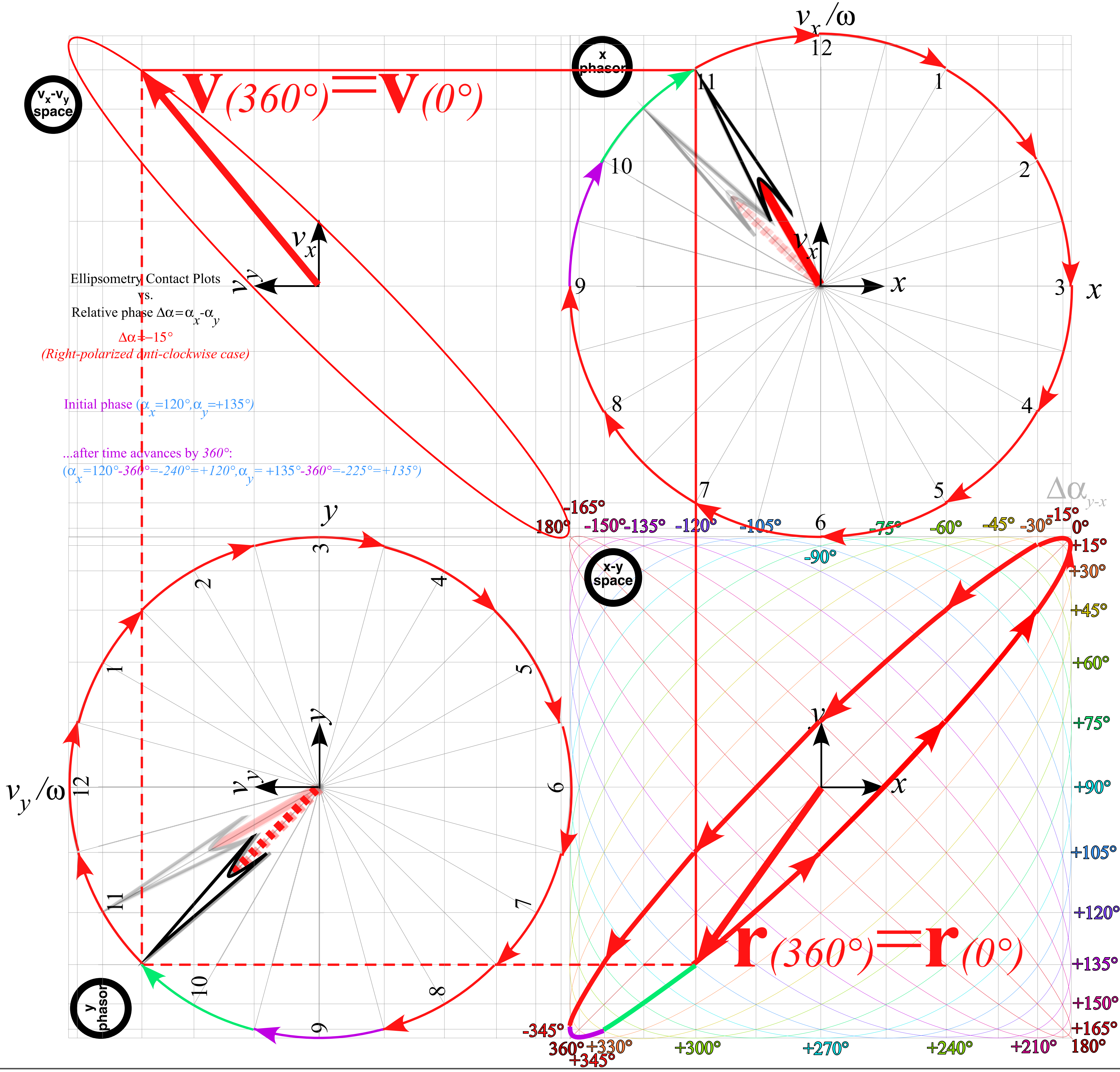


Ellipsometry Contact Plots
vs.
Relative phase Δα=αx-αy
Δα=−15°
(Right-polarized anti-clockwise case)
Initial phase (αx=120°,αy=+135°)
...after time advances by 270°:
(αx=120°-270°=-150°,αy = +135°-270°=-135°)
vx-vy space
x phasor
y phasor
x-y space
V(270°)
r(270°)
vx/ω
vy/ω
Δαy-x


Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=-15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 300°:
(α_x=120°-300°=-180°,α_y = +135°-300°=-165°)
v_x-v_y space
x phasor
y phasor
x-y space
V(300°)
r(300°)
v_x/ω
v_y/ω
Δα_y-x



Ellipsometry Contact Plots
vs.
Relative phase Δα=α_x-α_y
Δα=–15°
(Right-polarized anti-clockwise case)
Initial phase (α_x=120°,α_y=+135°)
...after time advances by 330°:
(α_x=120°-300°=-210°=+150°,α_y = +135°-330°=-195°=+165°)
v_x-v_y space
x phasor
y phasor
x-y space
V(330°)
r(330°)
v_x/ω
v_y/ω
Δα_y-x


v_x-v_y space
x phasor
y phasor
x-y space
$\mathbf{V}(360°)=\mathbf{V}(0°)$
$\mathbf{r}(360°)=\mathbf{r}(0°)$
Ellipsometry Contact Plots
vs.
Relative phase $\Delta\alpha=\alpha_x-\alpha_y$
$\Delta\alpha=-15°$
(Right-polarized anti-clockwise case)
Initial phase ($\alpha_x=120°, \alpha_y=+135°$)
...after time advances by 360°:
($\alpha_x=120°-360°=-240°=+120°, \alpha_y=+135°-360°=-225°=+135°$)
v_x/ω
v_y/ω
$\Delta\alpha_{y-x}$
180° -165° -150° -135° -120° -105° -90° -75° -60° -45° -30° -15° 0°
+15° +30° +45° +60° +75° +90° +105° +120° +135° +150° +165° 180°
-345° 360° +330° +345° +300° +270° +240° +210°