Beware of the Ides of March!!

Lecture 21

Tue. 3.13-Thur 3.15.2012

2-Wave Interference: Phase and Group Velocity

(Ch. 0-1 of Unit 2)

1. Review of basic formulas for waves in space-time (x,t) or per-space-time (ω,k)

1-Plane-wave phase velocity

2-Plane-wave phase velocity and group velocity (1/2-sum &1/2-diff.)

2-Plane-wave real zero grid in (x,t) or (ω,k)

2. Geometric construction of wave-zero grids

Continuous Wave (CW) grid based on $\mathbf{K}_{phase}=(\mathbf{K}_a+\mathbf{K}_b)/2$ *and* $\mathbf{K}_{group}=(\mathbf{K}_a-\mathbf{K}_b)/2$ *vectors*

Pulse Wave (PW) grid based on primitive $\mathbf{K}_a=\mathbf{K}_{phase}+\mathbf{K}_{group}$ *and* $\mathbf{K}_b=\mathbf{K}_{phase}-\mathbf{K}_{group}$ *vectors*

When this doesn't work (When you don't need it!)

3. Beginning wave relativity

Dueling lasers make lab frame space-time grid

Einstein PW Axioms versus Evenson CW Axioms (Occam at Work)

Only CW light clearly shows Doppler shift

Dueling lasers make lab frame space-time grid

Fundamental wave dynamics based on Euler Expo-cosine Identity

$$(e^{ia} + e^{ib})/2 = e^{i(a+b)/2}(e^{i(a-b)/2}+e^{-i(a-b)/2})/2 = e^{i(a+b)/2} \bullet \cos(a-b)/2$$

Balanced (50-50) plane wave combination:

$\omega_p=(\omega_1+\omega_2)/2$ $\quad$ $\omega_g=(\omega_1-\omega_2)/2$

$k_p=(k_1+k_2)/2$ $\quad$ $k_g=(k_1-k_2)/2$

Overall or Mean phase $\quad$ *Relative or Group phase*

$$\Psi_{50_1\text{-}50_2}(x,t) = (1/2)\psi_{k_1}(x,t) + (1/2)\psi_{k_2}(x,t)$$

$$(1/2)e^{i(k_1x-\omega_1t)} + (1/2)e^{i(k_2x-\omega_2t)} = e^{i(k_px-\omega_pt)} \bullet \cos(k_gx-\omega_gt)$$

Velocity: meters/second or per-seconds/per-meter

1st plane phase velocity	*2nd plane phase velocity*	*Phase or Carrier velocity*	*Group or Envelope velocity*
$V_1 = \frac{\omega_1}{k_1}$	$V_2 = \frac{\omega_2}{k_2}$	$V_p = \frac{\omega_p}{k_p} = \frac{\omega_1+\omega_2}{k_1+k_2}$	$V_g = \frac{\omega_g}{k_g} = \frac{\omega_1-\omega_2}{k_1-k_2}$

Define **K**-*vectors in per-spacetime*

$\mathbf{K}_1=(\omega_1, k_1)$	$\mathbf{K}_2=(\omega_2, k_2)$	$\mathbf{K}_p=(\omega_p, k_p)$	$\mathbf{K}_g=(\omega_g, k_g)$
$=\mathbf{K}_p+\mathbf{K}_g$	$=\mathbf{K}_p-\mathbf{K}_g$	$=(\mathbf{K}_1+\mathbf{K}_2)/2$	$=(\mathbf{K}_1-\mathbf{K}_2)/2$

2. Geometric construction of wave-zero grids

Continuous Wave (CW) grid based on $\mathbf{K}_{phase}=(\mathbf{K}_a+\mathbf{K}_b)/2$ *and* $\mathbf{K}_{group}=(\mathbf{K}_a-\mathbf{K}_b)/2$ *vectors*

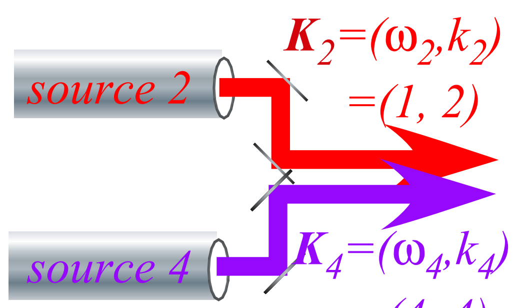
Pulse Wave (PW) grid based on primitive $\mathbf{K}_a=\mathbf{K}_{phase}+\mathbf{K}_{group}$ *and* $\mathbf{K}_b=\mathbf{K}_{phase}-\mathbf{K}_{group}$ *vectors*

When this doesn't work (When you don't need it!)

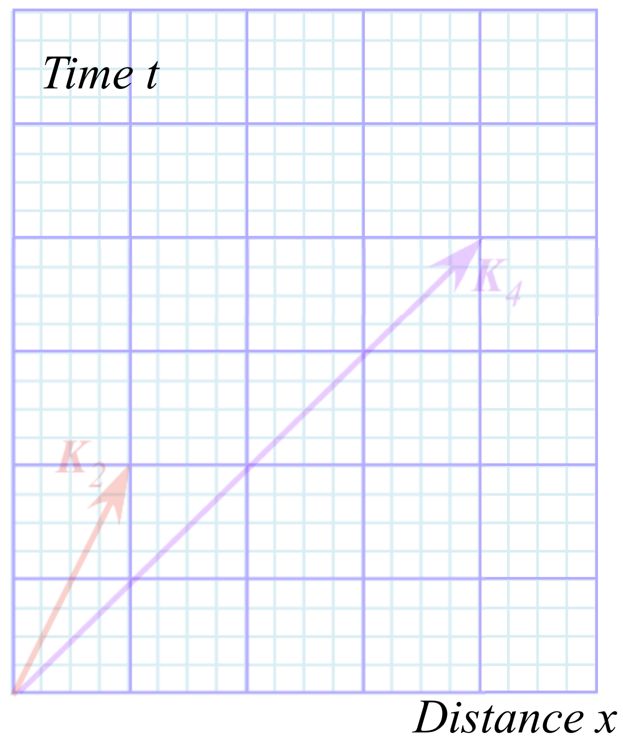
2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

$$\psi_+ = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2(\cos\frac{a+b}{2} + i\sin\frac{a+b}{2})\cos\frac{a-b}{2}$$

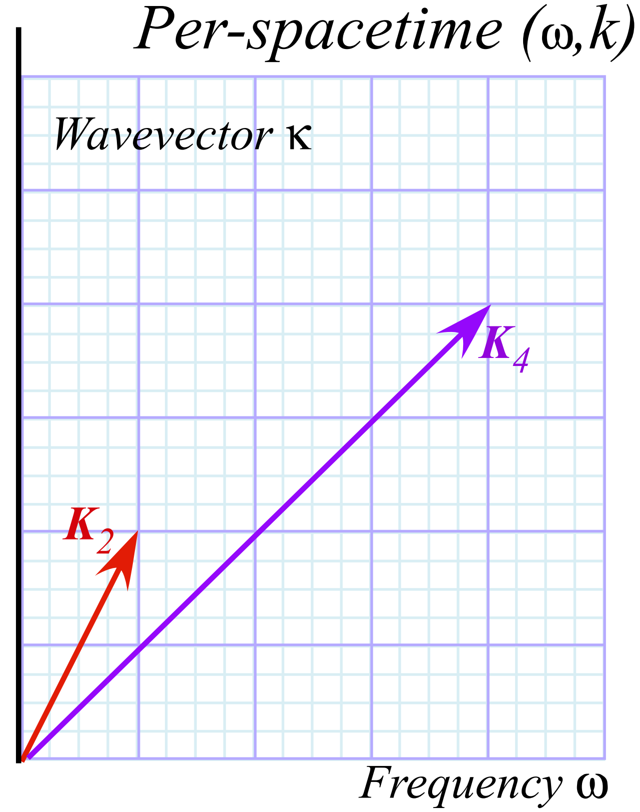
Suppose we are given two "mystery† sources"



Spacetime (x,t)



Per-spacetime (ω,k)



†*Schrodinger matter waves*

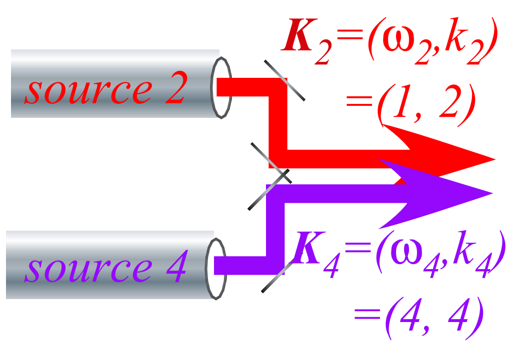
$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x - \frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x - \frac{\omega_a-\omega_b}{2}t\right)$$

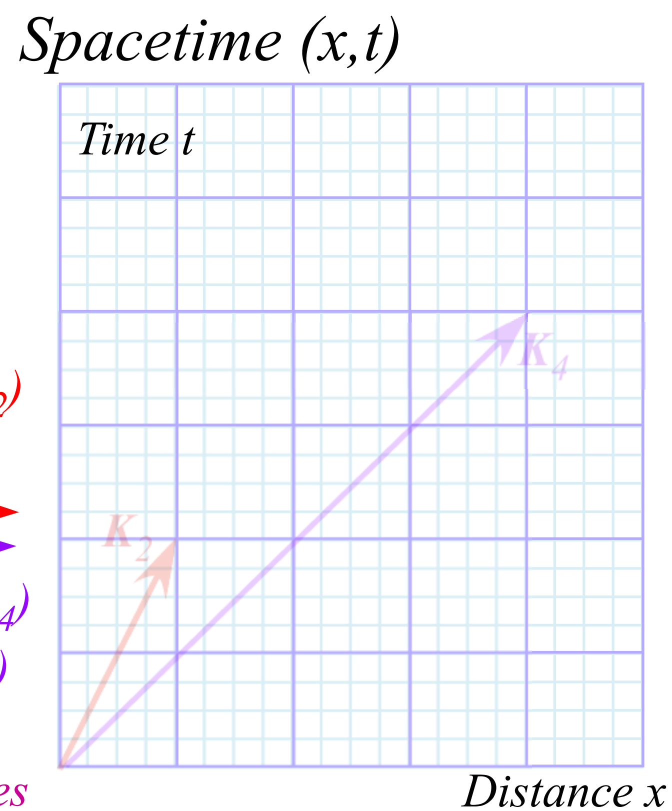
$$= \cos\left(k_{phase}x - \omega_{phase}t\right)\cos\left(k_{group}x - \omega_{group}t\right)$$

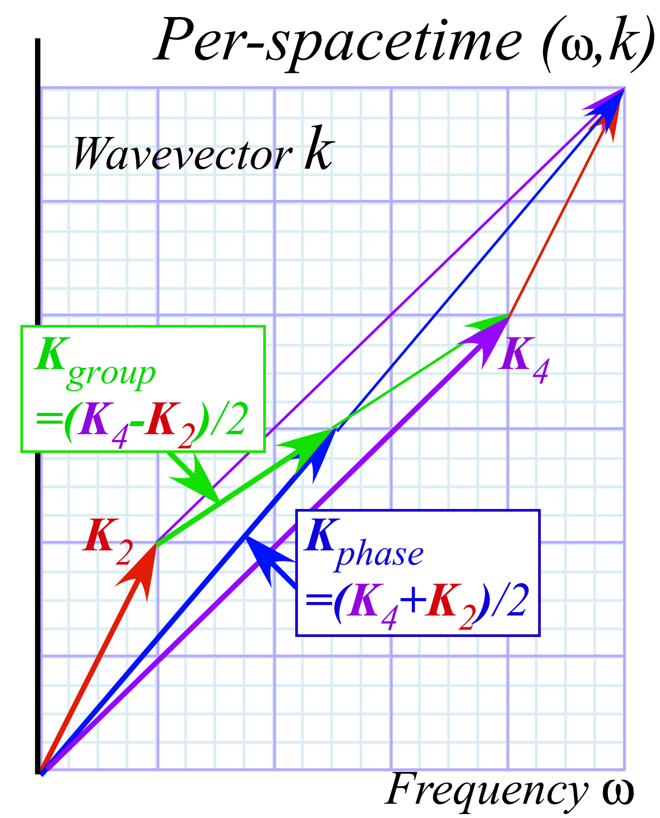
2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

$$\psi_+ = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2(\cos\frac{a+b}{2} + i\sin\frac{a+b}{2})\cos\frac{a-b}{2}$$

Suppose we are given two "mystery[†] sources"







[†] Schrodinger matter waves

$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x - \frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x - \frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x - \omega_{phase}t\right)\cos\left(k_{group}x - \omega_{group}t\right)$$

Space-time Reψ−zeros determined by:

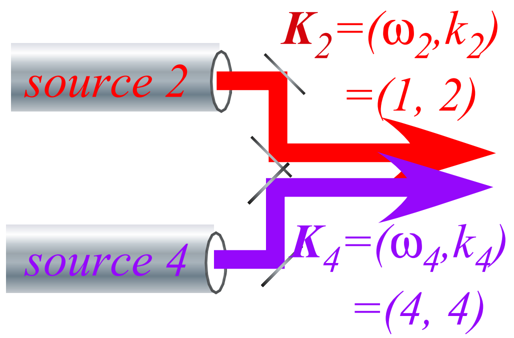
$k_{phase}x - \omega_{phase}t = m(\pi/2) \qquad m = \pm1, \pm3, \ldots$

$k_{group}x - \omega_{group}t = n(\pi/2) \qquad n = \pm1, \pm3, \ldots$

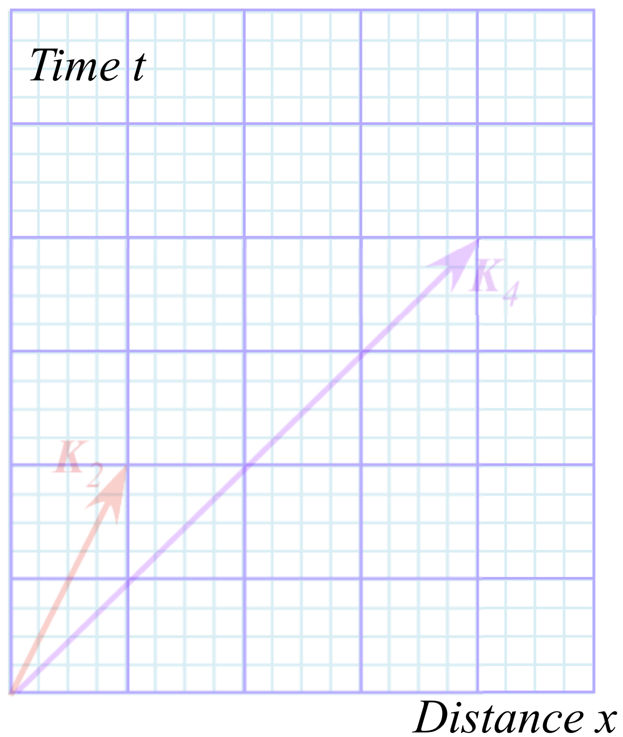
2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

$$\psi_+ = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2\left(\cos\frac{a+b}{2} + i\sin\frac{a+b}{2}\right)\cos\frac{a-b}{2}$$

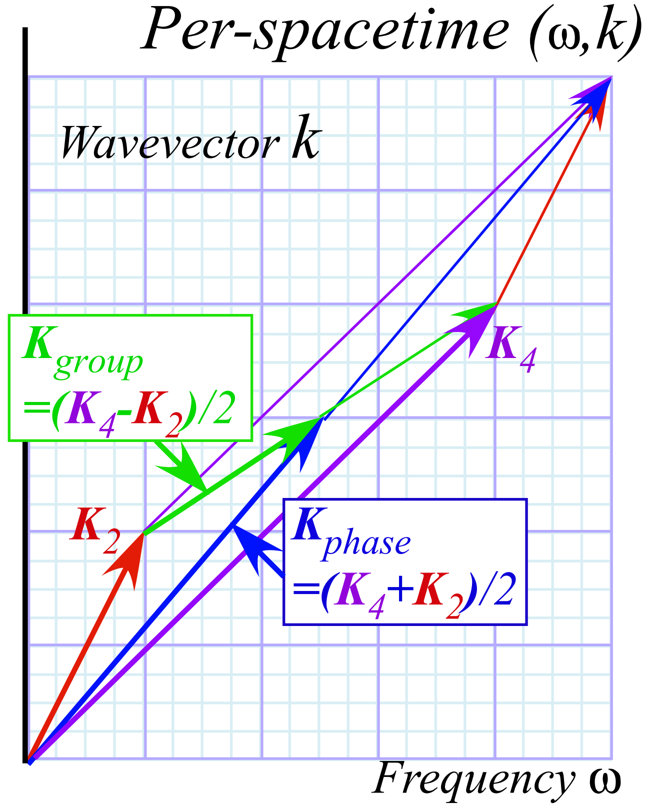
Suppose we are given two "mystery† sources"



Spacetime (x,t)



Per-spacetime (ω,k)



† Schrodinger matter waves

$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x - \frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x - \frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x - \omega_{phase}t\right)\cos\left(k_{group}x - \omega_{group}t\right)$$

Space-time Reψ−zeros determined by:

$$k_{phase}x - \omega_{phase}t = m(\pi/2) \qquad m = \pm1, \pm3, \ldots$$

$$k_{group}x - \omega_{group}t = n(\pi/2) \qquad n = \pm1, \pm3, \ldots$$

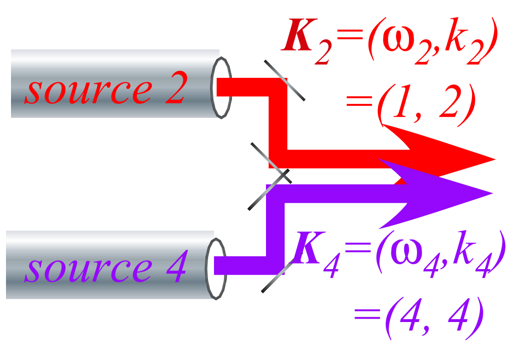
Matrix equation:

$$\begin{pmatrix} k_{phase} & -\omega_{phase} \\ k_{group} & -\omega_{group} \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

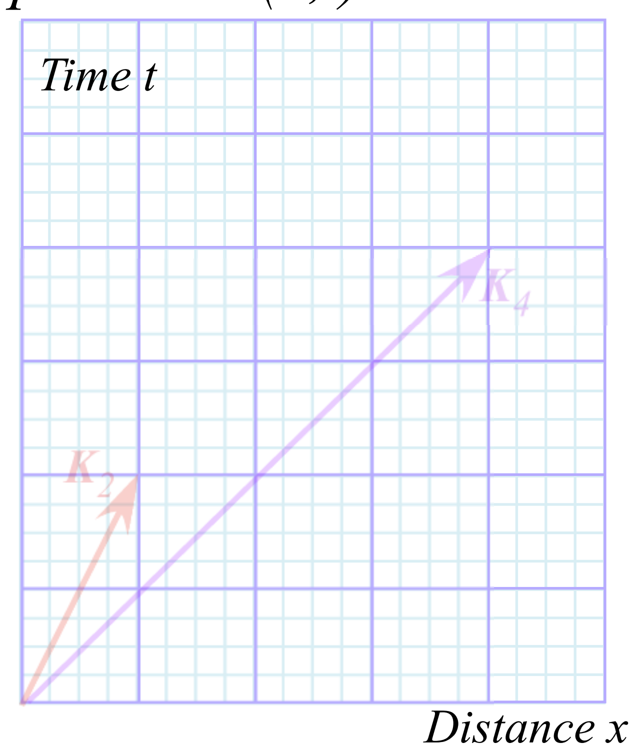
$$\psi_+ = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2\left(\cos\frac{a+b}{2} + i\sin\frac{a+b}{2}\right)\cos\frac{a-b}{2}$$

Suppose we are given two "mystery† sources"

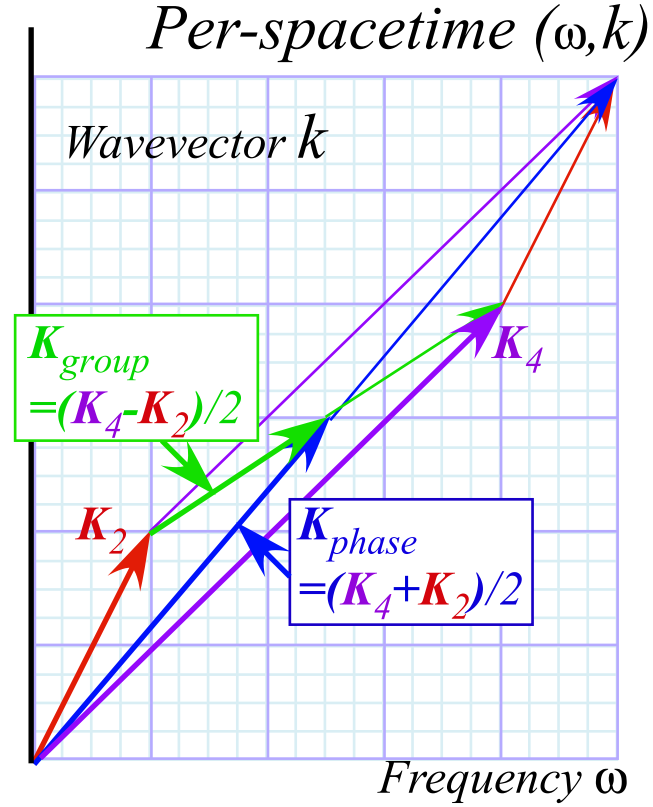


†Schrodinger matter waves

Spacetime (x,t)



Per-spacetime (ω,k)



$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x - \frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x - \frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x - \omega_{phase}t\right)\cos\left(k_{group}x - \omega_{group}t\right)$$

Space-time Reψ−zeros $\mathbf{X}_{m,n}$ determined by:

$$k_{phase}x - \omega_{phase}t = m(\pi/2) \qquad m = \pm 1, \pm 3, \ldots$$

$$k_{group}x - \omega_{group}t = n(\pi/2) \qquad n = \pm 1, \pm 3, \ldots$$

Matrix equation:

$$\begin{pmatrix} k_{phase} & -\omega_{phase} \\ k_{group} & -\omega_{group} \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

Inverse matrix equation:

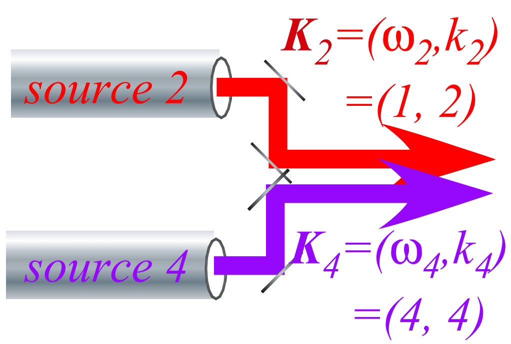
$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \frac{\begin{pmatrix} \omega_{group} & -\omega_{phase} \\ k_{group} & -k_{phase} \end{pmatrix}}{\left|\omega_{group}k_{phase} - \omega_{phase}k_{group}\right|}\begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \mathbf{X}_{m,n} = \left[m\mathbf{K}_{group} - n\mathbf{K}_{phase}\right]s_{gp}$$

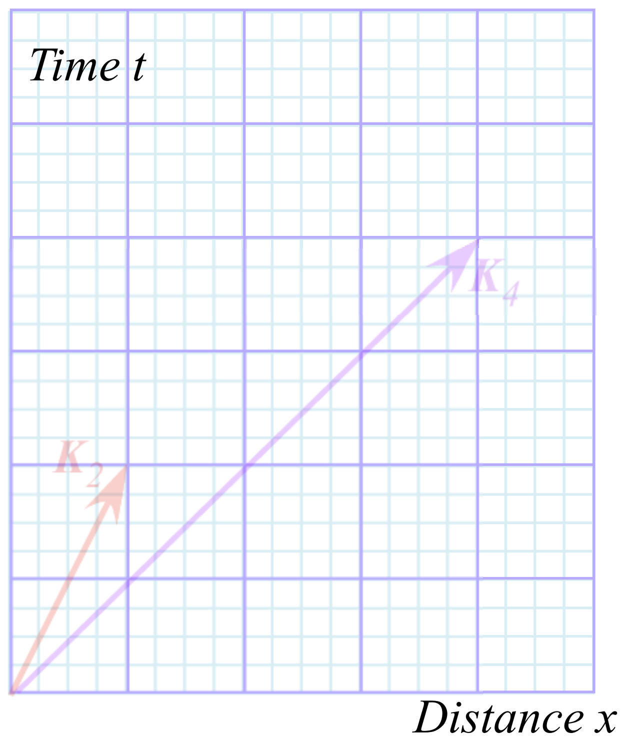
2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

$$\psi_+ = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2\left(\cos\frac{a+b}{2} + i\sin\frac{a+b}{2}\right)\cos\frac{a-b}{2}$$

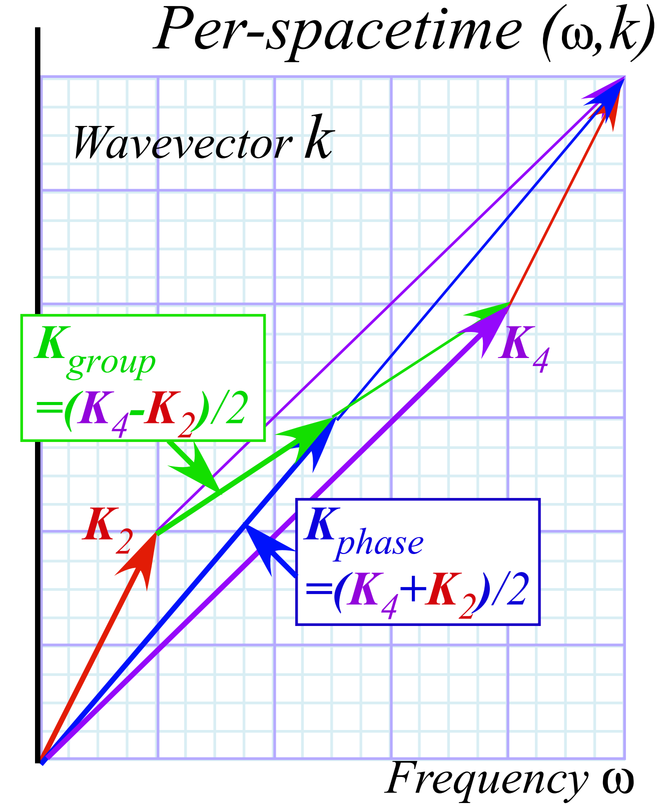
Suppose we are given two "mystery† sources"



Spacetime (x,t)



Per-spacetime (ω,k)



†Schrodinger matter waves

$$0 = \text{Re}\,\psi_+ = \text{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x - \frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x - \frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x - \omega_{phase}t\right)\cos\left(k_{group}x - \omega_{group}t\right)$$

Space-time Reψ–zeros $\mathbf{X}_{m,n}$ determined by:

$$k_{phase}x - \omega_{phase}t = m(\pi/2) \qquad m = \pm1,\pm3,\ldots$$

$$k_{group}x - \omega_{group}t = n(\pi/2) \qquad n = \pm1,\pm3,\ldots$$

Matrix equation:

$$\begin{pmatrix} k_{phase} & -\omega_{phase} \\ k_{group} & -\omega_{group} \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

Inverse matrix equation:

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \frac{\begin{pmatrix} \omega_{group} & -\omega_{phase} \\ k_{group} & -k_{phase} \end{pmatrix}}{\left|\omega_{group}k_{phase} - \omega_{phase}k_{group}\right|}\begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

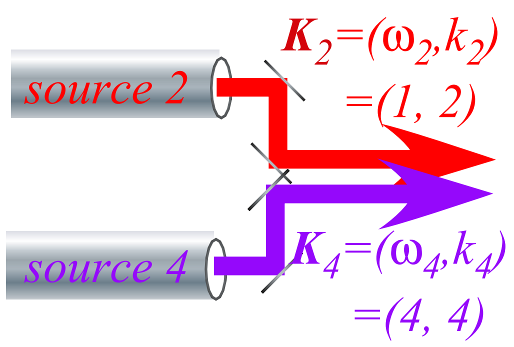
...and space-time scale factor: $s_{gp} = \dfrac{\pi}{2\left|\mathbf{K}_{group} \times \mathbf{K}_{phase}\right|}$

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \mathbf{X}_{m,n} = \left[m\mathbf{K}_{group} - n\mathbf{K}_{phase}\right]s_{gp}$$

2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

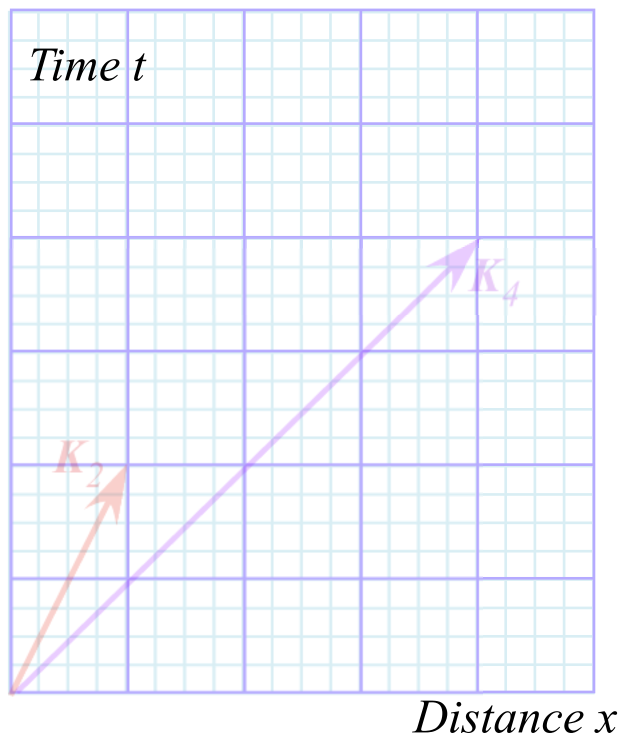
$$\psi_+ = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2(\cos\frac{a+b}{2} + i\sin\frac{a+b}{2})\cos\frac{a-b}{2}$$

Suppose we are given two "mystery† sources"

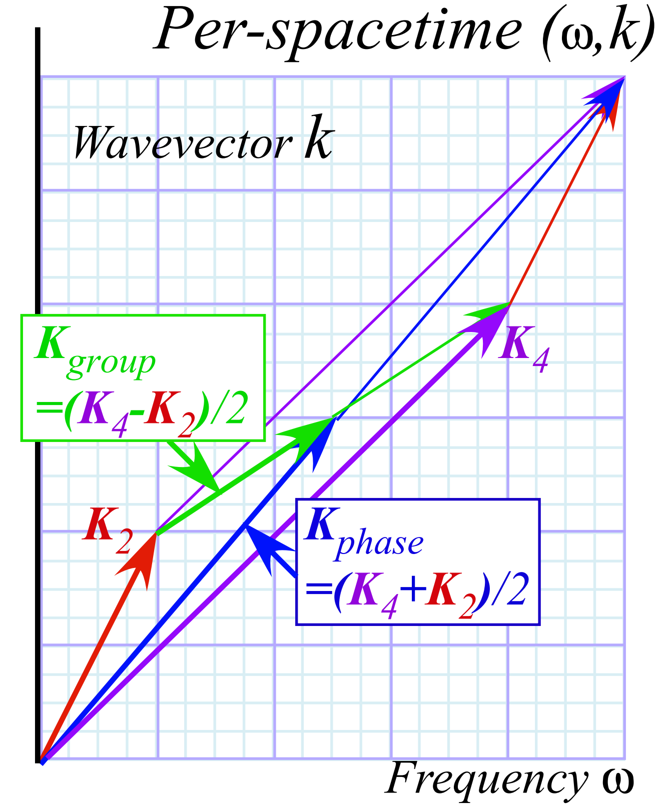


† Schrodinger matter waves

Spacetime (x,t)



Per-spacetime (ω,k)



$$0 = \text{Re}\,\psi_+ = \text{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x - \frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x - \frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x - \omega_{phase}t\right)\cos\left(k_{group}x - \omega_{group}t\right)$$

Space-time Reψ−zeros $\mathbf{X}_{m,n}$ determined by:

$$k_{phase}x - \omega_{phase}t = m(\pi/2) \qquad m = \pm1, \pm3, \ldots$$

$$k_{group}x - \omega_{group}t = n(\pi/2) \qquad n = \pm1, \pm3, \ldots$$

Matrix equation:

$$\begin{pmatrix} k_{phase} & -\omega_{phase} \\ k_{group} & -\omega_{group} \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

Inverse matrix equation:

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \frac{\begin{pmatrix} \omega_{group} & -\omega_{phase} \\ k_{group} & -k_{phase} \end{pmatrix}}{\left|\omega_{group}k_{phase} - \omega_{phase}k_{group}\right|}\begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

...and space-time scale factor:

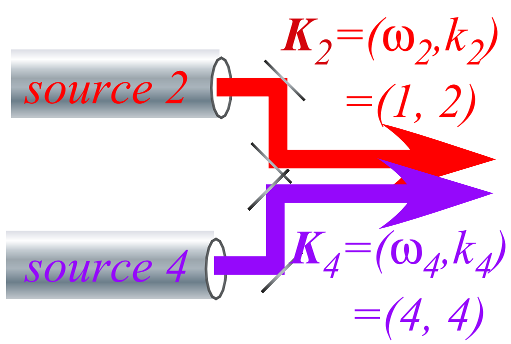
$$s_{gp} = \frac{\pi}{2\left|\mathbf{K}_{group} \times \mathbf{K}_{phase}\right|} = \frac{\pi}{2\left|1.5\cdot3.0 - 2.5\cdot1.0\right|} = \frac{\pi}{4}$$

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \mathbf{X}_{m,n} = \left[m\mathbf{K}_{group} - n\mathbf{K}_{phase}\right]s_{gp} \qquad m = \pm1, \pm3, \ldots \quad n = \pm1, \pm3, \ldots$$

2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

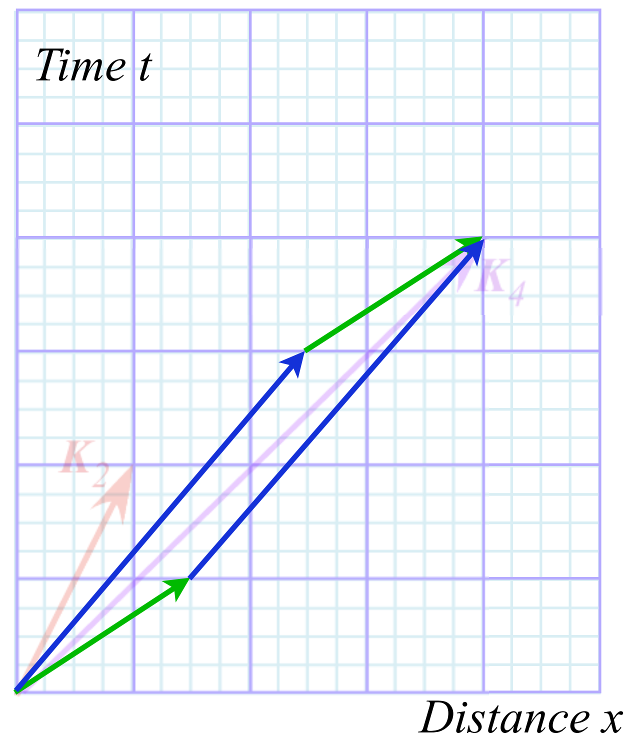
$$\psi_+ = e^{ia}+e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2\left(\cos\frac{a+b}{2}+i\sin\frac{a+b}{2}\right)\cos\frac{a-b}{2}$$

Suppose we are given two "mystery† sources"

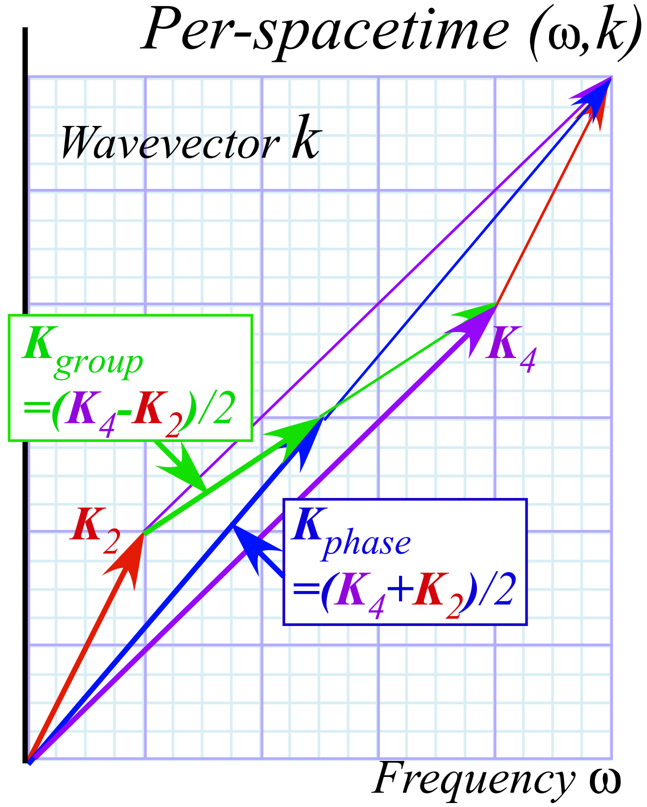


†Schrodinger matter waves

Spacetime (x,t)



Per-spacetime (ω,k)



$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x-\frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x-\frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x-\omega_{phase}t\right)\cos\left(k_{group}x-\omega_{group}t\right)$$

Space-time Reψ−zeros $\mathbf{X}_{m,n}$ *determined by:*

$$k_{phase}x-\omega_{phase}t = m(\pi/2) \qquad m=\pm1,\pm3,\ldots$$

$$k_{group}x-\omega_{group}t = n(\pi/2) \qquad n=\pm1,\pm3,\ldots$$

Matrix equation:

$$\begin{pmatrix} k_{phase} & -\omega_{phase} \\ k_{group} & -\omega_{group} \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

Inverse matrix equation:

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \frac{\begin{pmatrix} \omega_{group} & -\omega_{phase} \\ k_{group} & -k_{phase} \end{pmatrix}}{\left|\omega_{group}k_{phase}-\omega_{phase}k_{group}\right|}\begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

...and space-time scale factor:

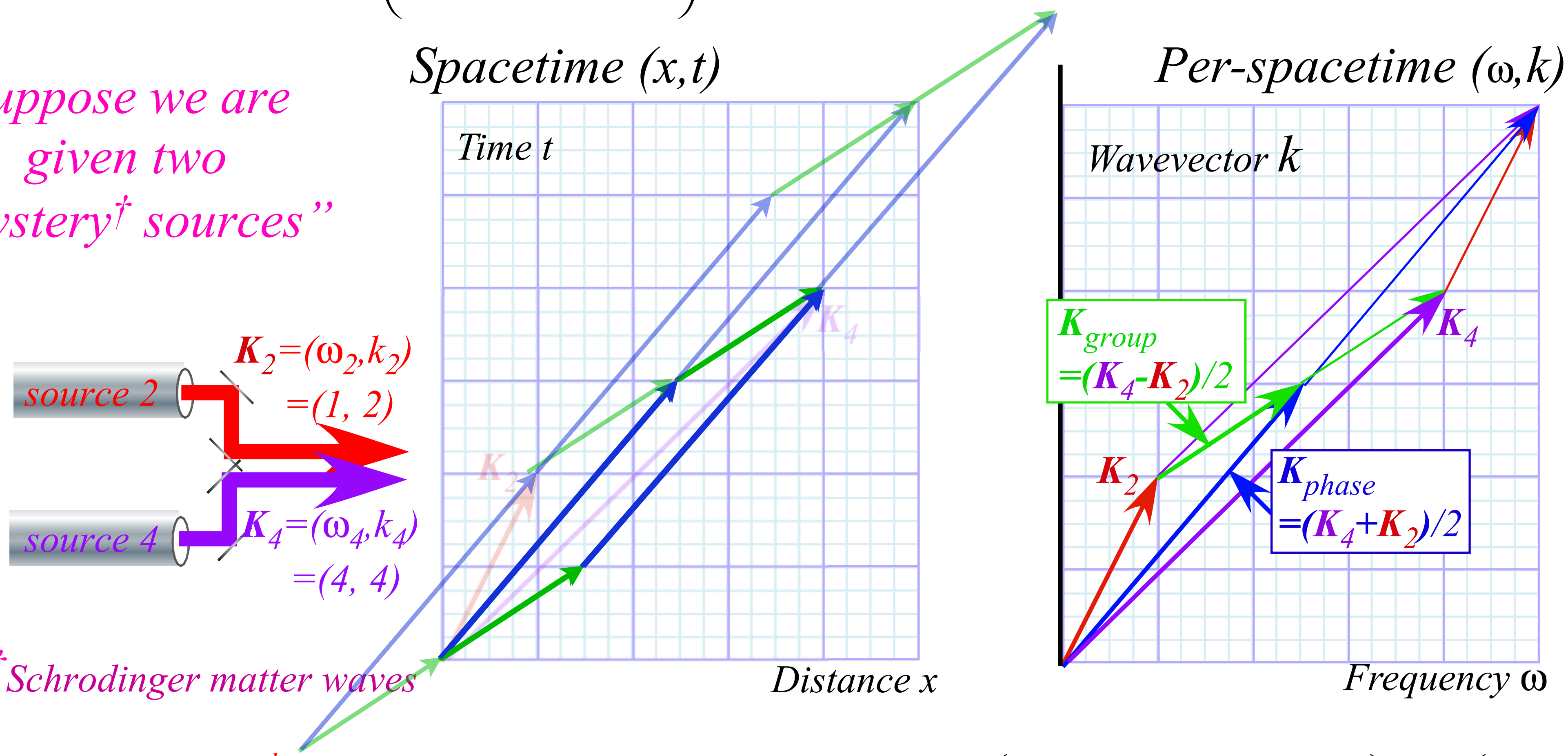
$$s_{gp} = \frac{\pi}{2\left|\mathbf{K}_{group}\times\mathbf{K}_{phase}\right|} = \frac{\pi}{2\left|1.5\cdot3.0-2.5\cdot1.0\right|} = \frac{\pi}{4}$$

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \mathbf{X}_{m,n} = \left[m\mathbf{K}_{group}-n\mathbf{K}_{phase}\right]s_{gp} \qquad m=\pm1,\pm3,\ldots \quad n=\pm1,\pm3,\ldots$$

2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

$$\psi_+ = e^{ia} + e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}} + e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2(\cos\frac{a+b}{2} + i\sin\frac{a+b}{2})\cos\frac{a-b}{2}$$

Suppose we are given two "mystery† sources"



†Schrodinger matter waves

$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x - \frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x - \frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x - \omega_{phase}t\right)\cos\left(k_{group}x - \omega_{group}t\right)$$

Space-time Reψ−zeros $\mathbf{X}_{m,n}$ determined by:

$$k_{phase}x - \omega_{phase}t = m(\pi/2) \qquad m = \pm1, \pm3, \ldots$$

$$k_{group}x - \omega_{group}t = n(\pi/2) \qquad n = \pm1, \pm3, \ldots$$

Matrix equation:

$$\begin{pmatrix} k_{phase} & -\omega_{phase} \\ k_{group} & -\omega_{group} \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

Inverse matrix equation:

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \frac{\begin{pmatrix} \omega_{group} & -\omega_{phase} \\ k_{group} & -k_{phase} \end{pmatrix}}{\left|\omega_{group}k_{phase} - \omega_{phase}k_{group}\right|}\begin{pmatrix} m \\ n \end{pmatrix}\frac{\pi}{2}$$

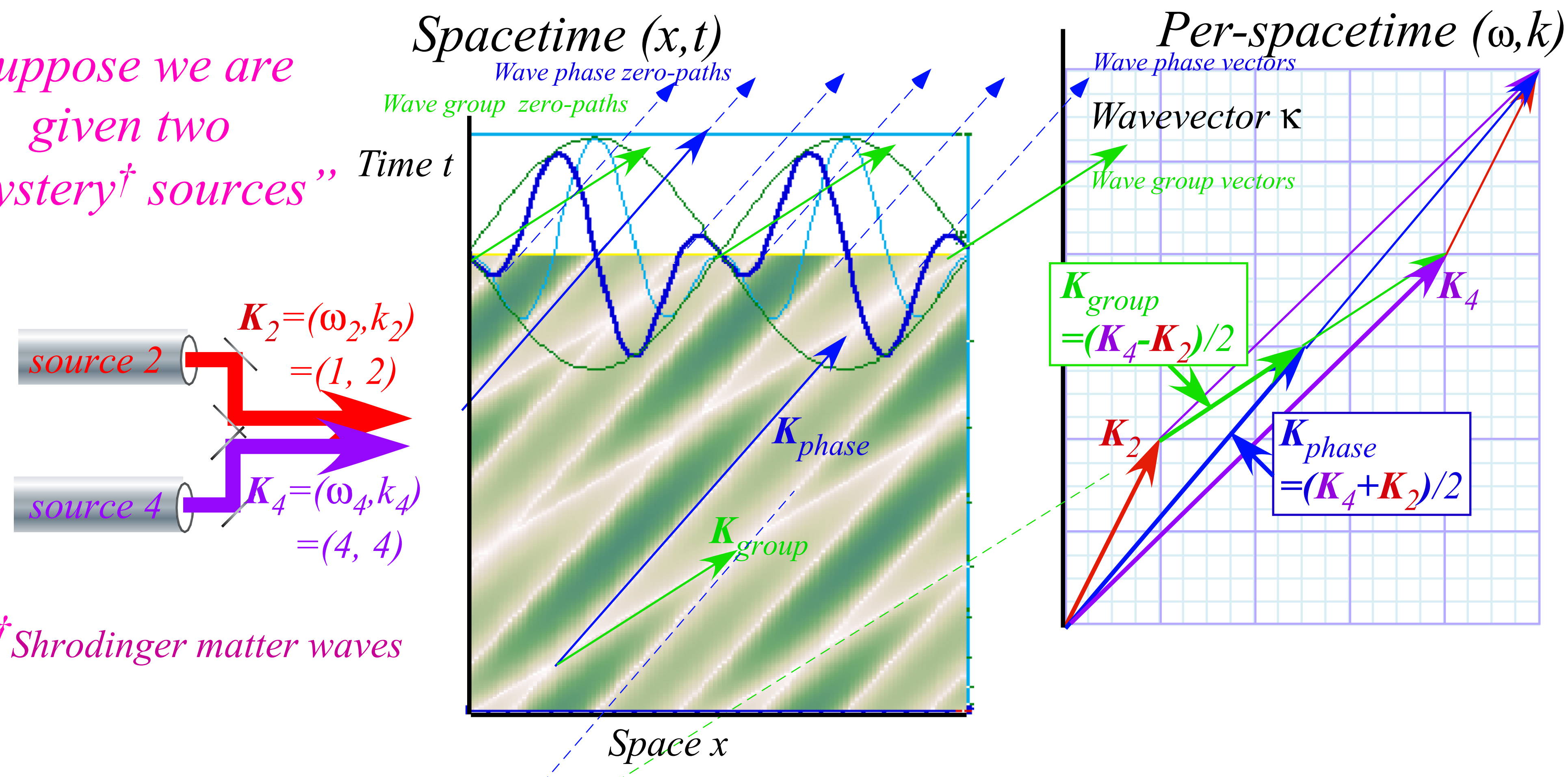
...and space-time scale factor:

$$s_{gp} = \frac{\pi}{2\left|\mathbf{K}_{group} \times \mathbf{K}_{phase}\right|} = \frac{\pi}{2\left|1.5\cdot3.0 - 2.5\cdot1.0\right|} = \frac{\pi}{4}$$

$$\begin{pmatrix} x_{m,n} \\ t_{m,n} \end{pmatrix} = \mathbf{X}_{m,n} = \left[m\mathbf{K}_{group} - n\mathbf{K}_{phase}\right]s_{gp} \qquad m = \pm1, \pm3, \ldots \quad n = \pm1, \pm3, \ldots$$

2-Source Case: Unifying Trajectory-Spacetime (x,t) and Fourier-Per-spacetime (ω,k)

Suppose we are given two "mystery† sources"



†Shrodinger matter waves

Wave("coherent")Lattice(Bases: $\mathbf{K}_{group}$ and $\mathbf{K}_{phase}$)

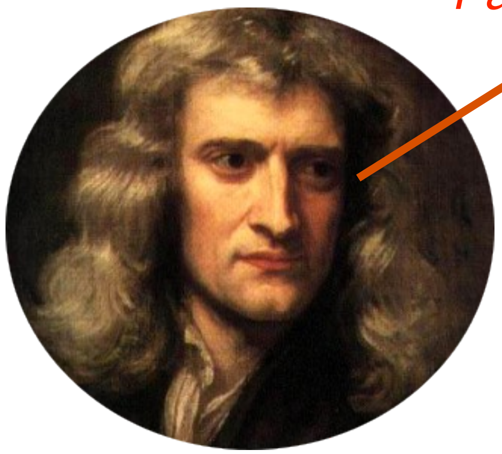
The wave-interference-zero paths given by K-vectors (ω_g, k_g) and (ω_p, k_p) .

2. Geometric construction of wave-zero grids

Continuous Wave (CW) grid based on $\mathbf{K}_{phase}=(\mathbf{K}_a+\mathbf{K}_b)/2$ *and* $\mathbf{K}_{group}=(\mathbf{K}_a-\mathbf{K}_b)/2$ *vectors*

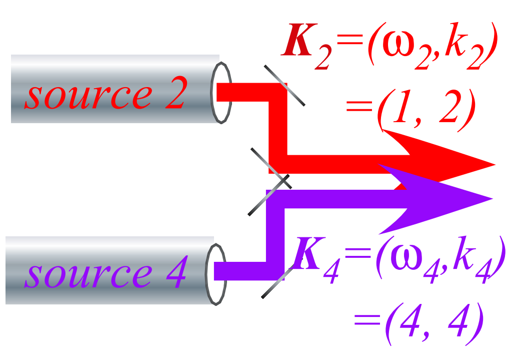
➡ *Pulse Wave (PW) grid based on primitive* $\mathbf{K}_a=\mathbf{K}_{phase}+\mathbf{K}_{group}$ *and* $\mathbf{K}_b=\mathbf{K}_{phase}-\mathbf{K}_{group}$ *vectors*

When this doesn't work *(When you don't need it!)*

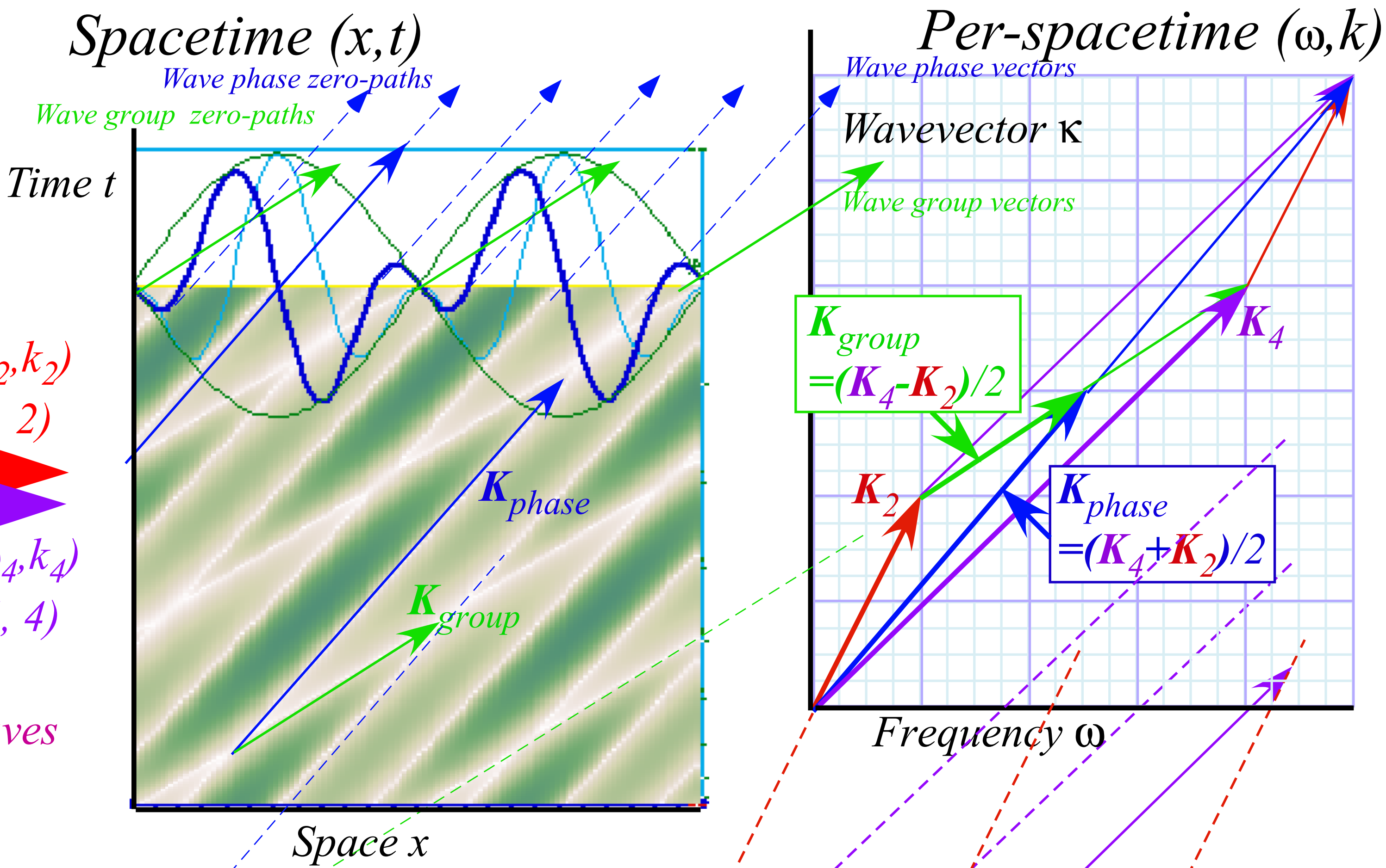


2-Source Case: Unifying Trajectory-Spacetime (x,t) and Fourier-Per-spacetime (ω,k)

Suppose we are given two "mystery† sources"

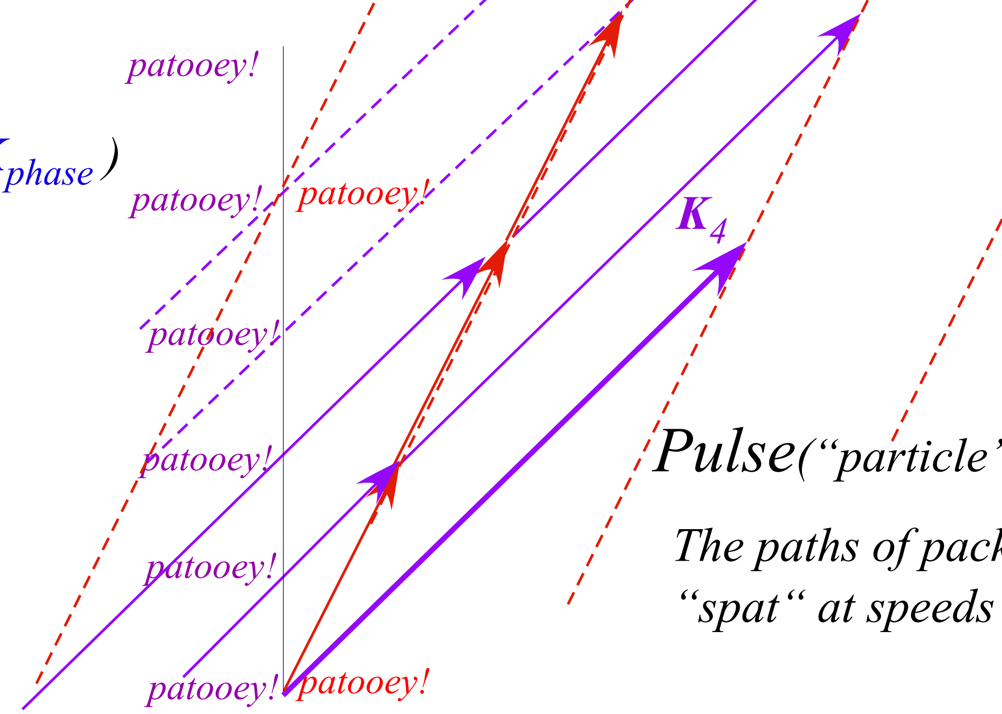


† Shrodinger matter waves

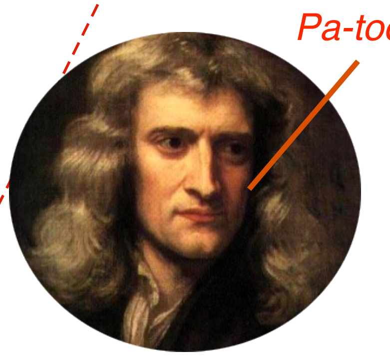


Wave("coherent")Lattice(Bases: K_{group} and K_{phase})

The wave-interference-zero paths given by K-vectors (ω_g,k_g) and (ω_p,k_p) .



"Waves are illusory!"
Corpuscles rule!
Pa-tooey!



Pulse("particle")Lattice(Bases: K_2 and K_4)

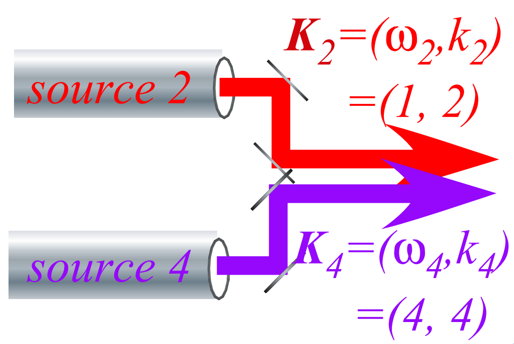
The paths of packets or Newtonian "corpuscles" "spat" at speeds V_2 and V_4 and rates ω_2 and ω_2

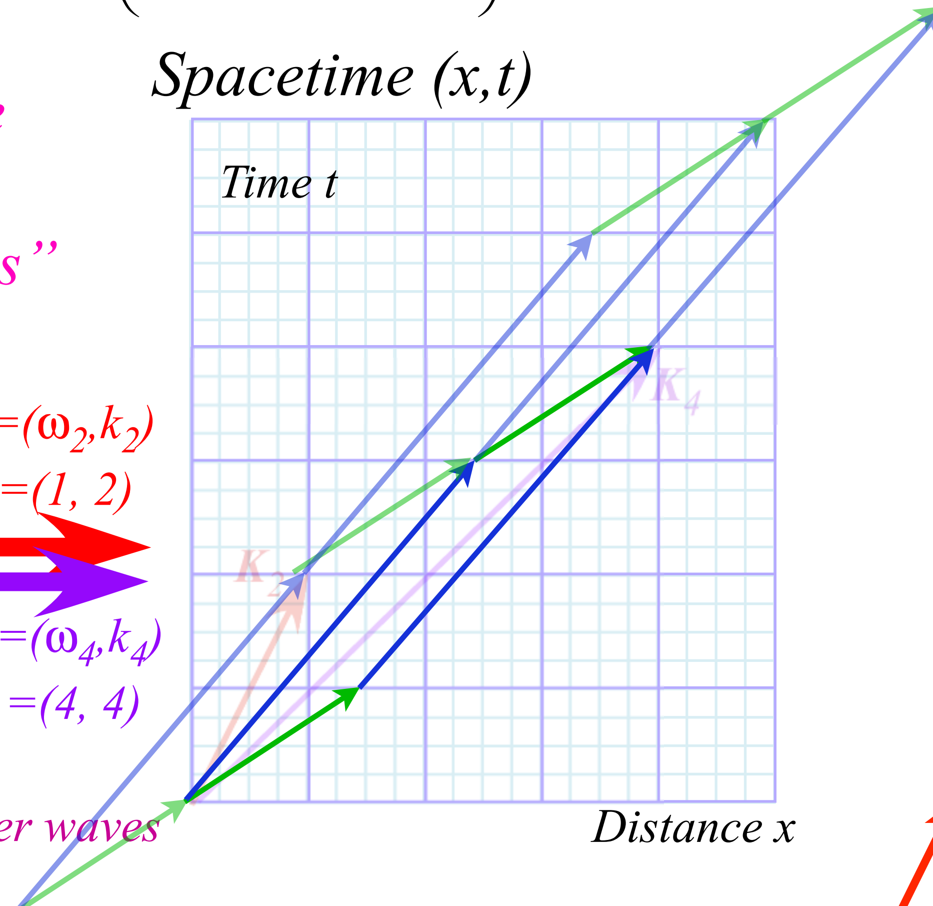
patooey! patooey! patooey! patooey! patooey! patooey! patooey!

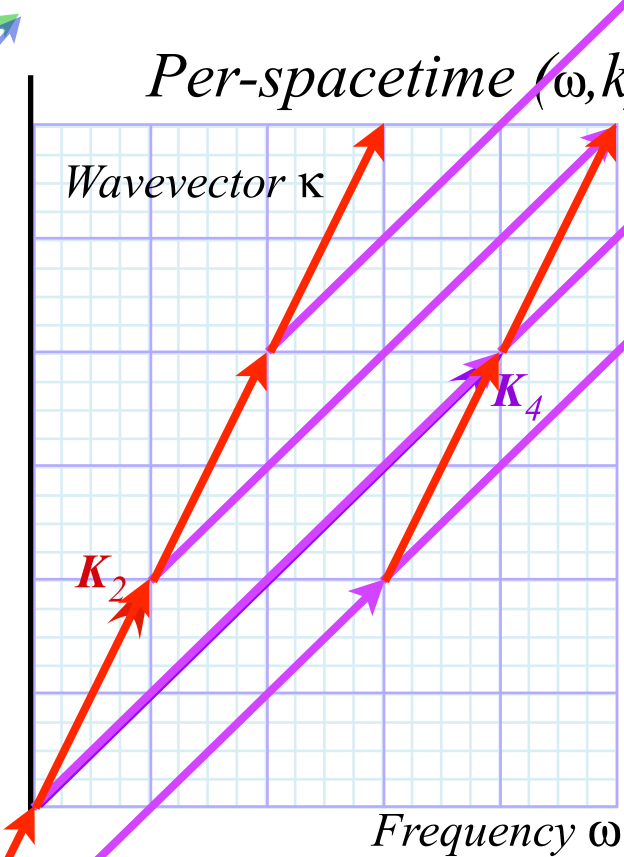
2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

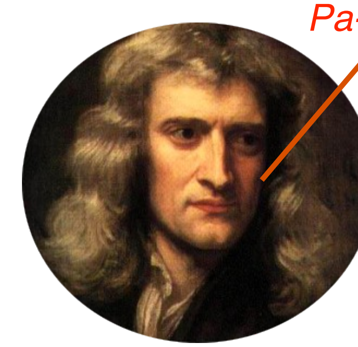
$$\psi_+ = e^{ia}+e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2\left(\cos\frac{a+b}{2}+i\sin\frac{a+b}{2}\right)\cos\frac{a-b}{2}$$

Suppose we are given two "mystery† sources"









†Schrodinger matter waves

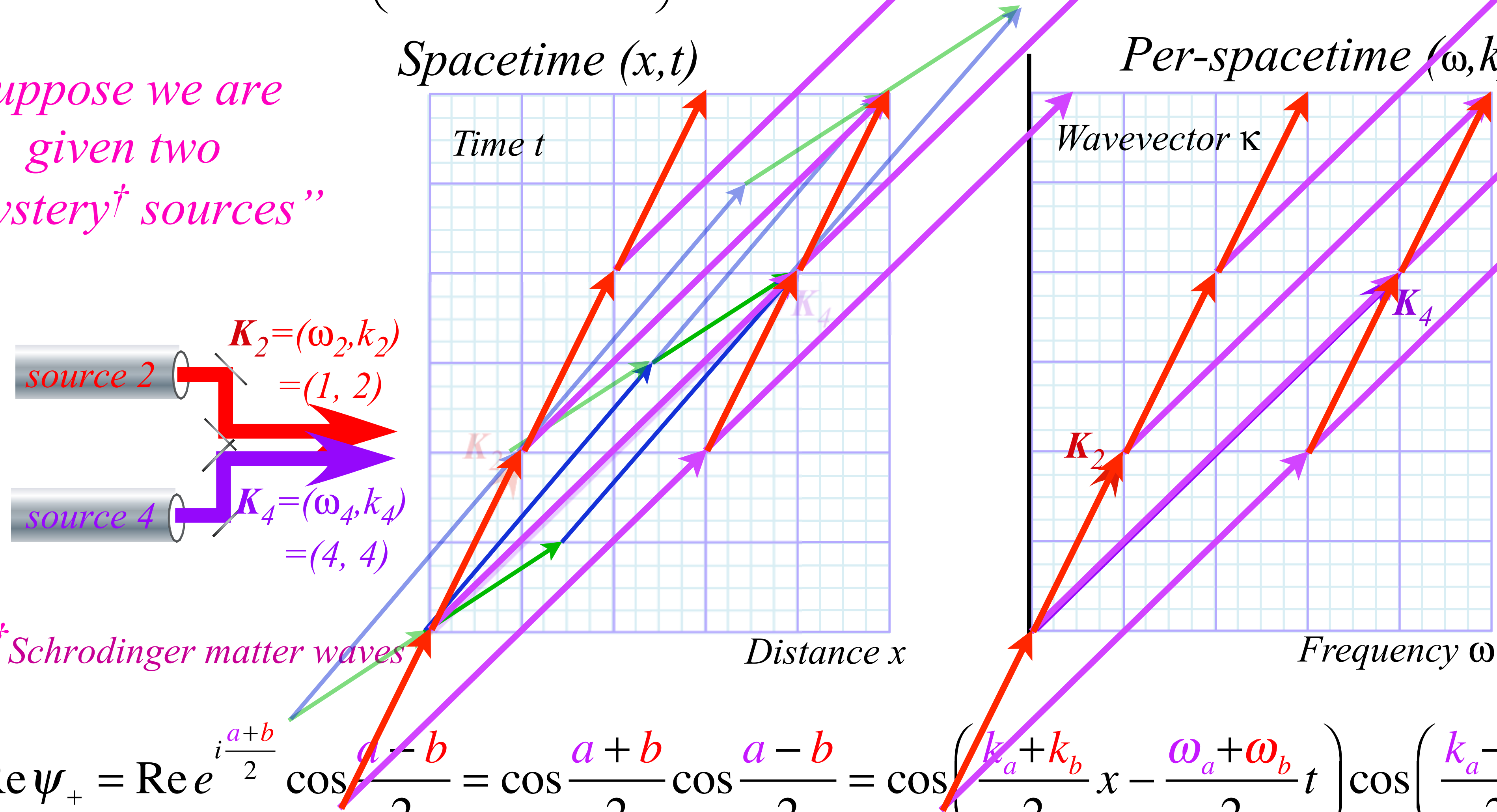
$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x-\frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x-\frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x-\omega_{phase}t\right)\cos\left(k_{group}x-\omega_{group}t\right)$$

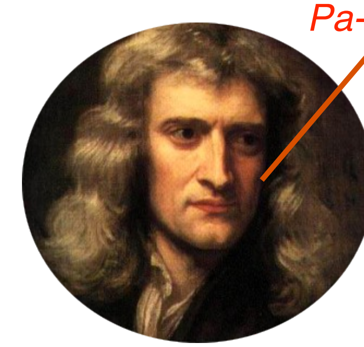
2-Wave Source: Unifying Trajectory-Space-time (x,t) and Fourier-Per-space-time (ω,k)

$$\psi_+ = e^{ia}+e^{ib} = e^{i\frac{a+b}{2}}\left(e^{i\frac{a-b}{2}}+e^{-i\frac{a-b}{2}}\right) = 2e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = 2\left(\cos\frac{a+b}{2}+i\sin\frac{a+b}{2}\right)\cos\frac{a-b}{2}$$

Suppose we are given two "mystery† sources"

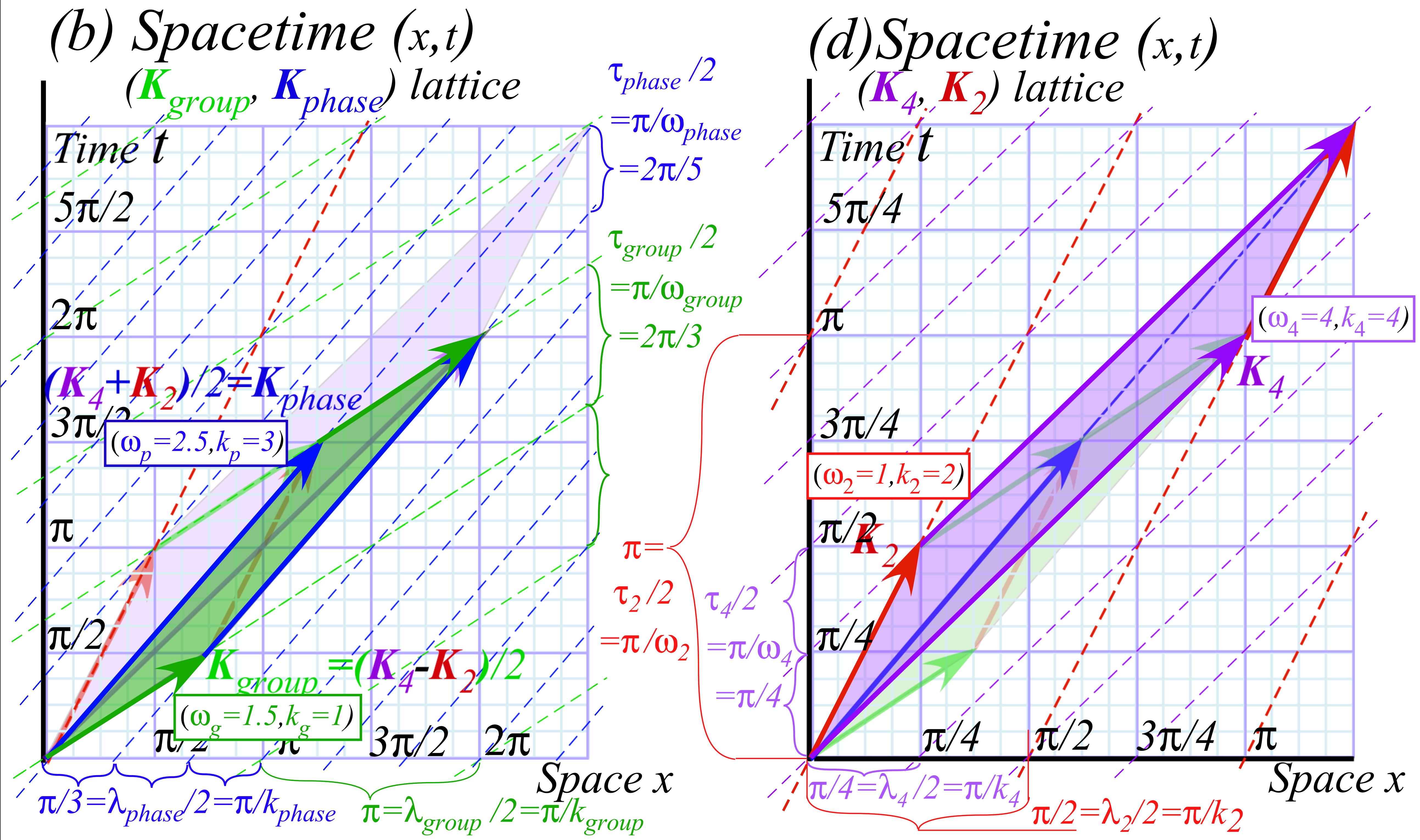


†Schrodinger matter waves



$$0 = \mathrm{Re}\,\psi_+ = \mathrm{Re}\,e^{i\frac{a+b}{2}}\cos\frac{a-b}{2} = \cos\frac{a+b}{2}\cos\frac{a-b}{2} = \cos\left(\frac{k_a+k_b}{2}x-\frac{\omega_a+\omega_b}{2}t\right)\cos\left(\frac{k_a-k_b}{2}x-\frac{\omega_a-\omega_b}{2}t\right)$$

$$= \cos\left(k_{phase}x-\omega_{phase}t\right)\cos\left(k_{group}x-\omega_{group}t\right)$$


(b) Spacetime (x,t)
(K_group, K_phase) lattice
Time t
5π/2
2π
3π/2
π
π/2
(K_4+K_2)/2=K_phase
(ω_p=2.5,k_p=3)
K_group =(K_4-K_2)/2
(ω_g=1.5,k_g=1)
π/2 π 3π/2 2π
Space x
τ_phase /2 =π/ω_phase =2π/5
τ_group /2 =π/ω_group =2π/3
π/3=λ_phase/2=π/k_phase
π=λ_group /2=π/k_group
π= τ_2 /2 =π /ω_2
(d)Spacetime (x,t)
(K_4, K_2) lattice
Time t
5π/4
π
3π/4
π/2
π/4
(ω_4=4,k_4=4)
K_4
(ω_2=1,k_2=2)
K_2
τ_4/2 =π/ω_4 =π/4
π/4 π/2 3π/4 π
Space x
π/4=λ_4/2=π/k_4
π/2=λ_2/2=π/k_2

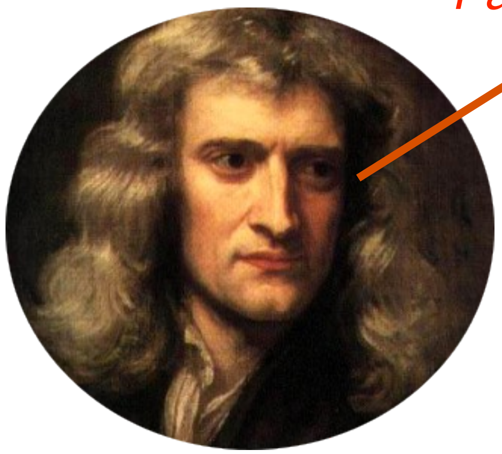
2. Geometric construction of wave-zero grids

Continuous Wave (CW) grid based on $\mathbf{K}_{phase}=(\mathbf{K}_a+\mathbf{K}_b)/2$ *and* $\mathbf{K}_{group}=(\mathbf{K}_a-\mathbf{K}_b)/2$ *vectors*

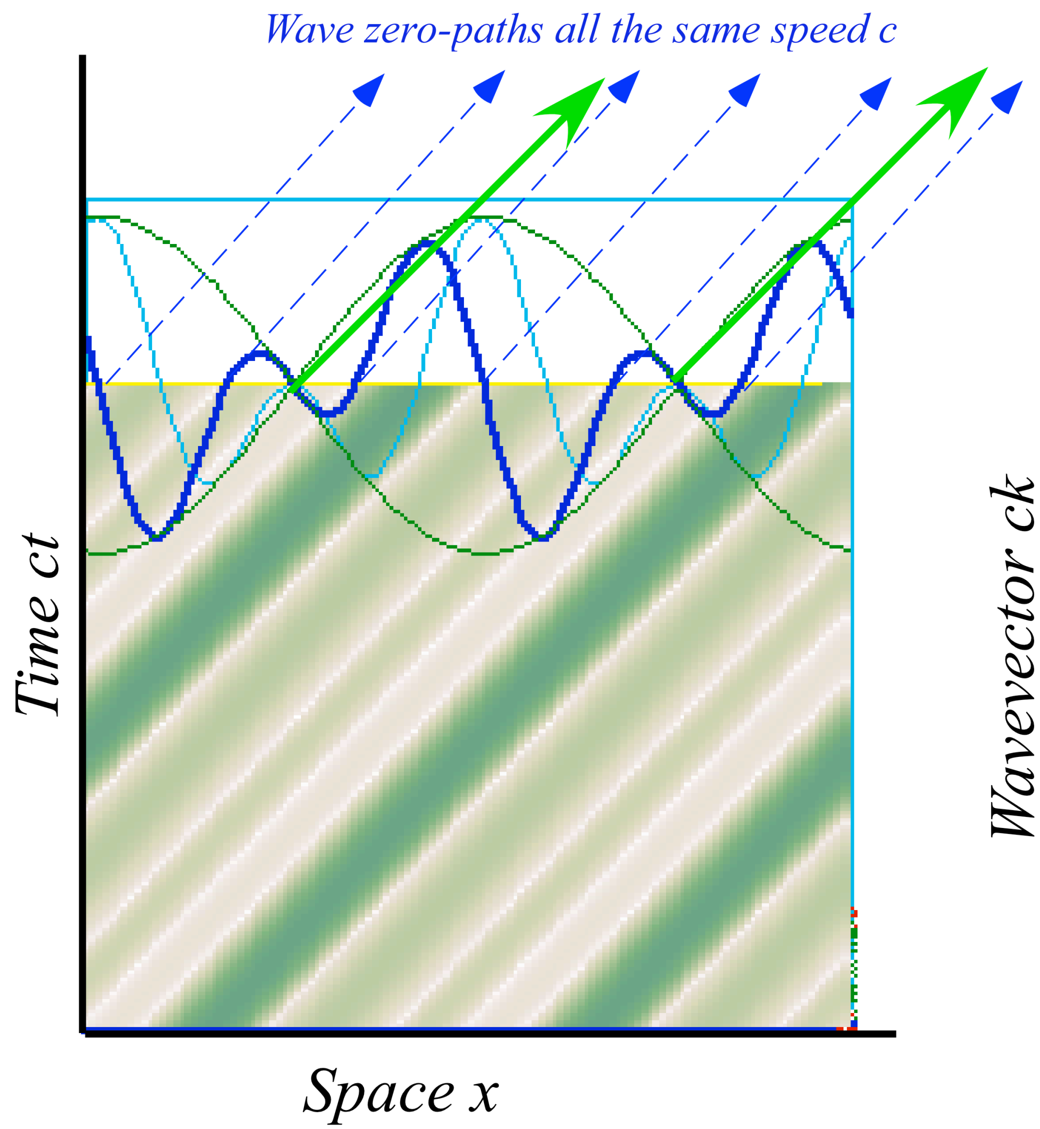
Pulse Wave (PW) grid based on primitive $\mathbf{K}_a=\mathbf{K}_{phase}+\mathbf{K}_{group}$ *and* $\mathbf{K}_b=\mathbf{K}_{phase}-\mathbf{K}_{group}$ *vectors*

→ *When this doesn't work* *(When you don't need it!)*

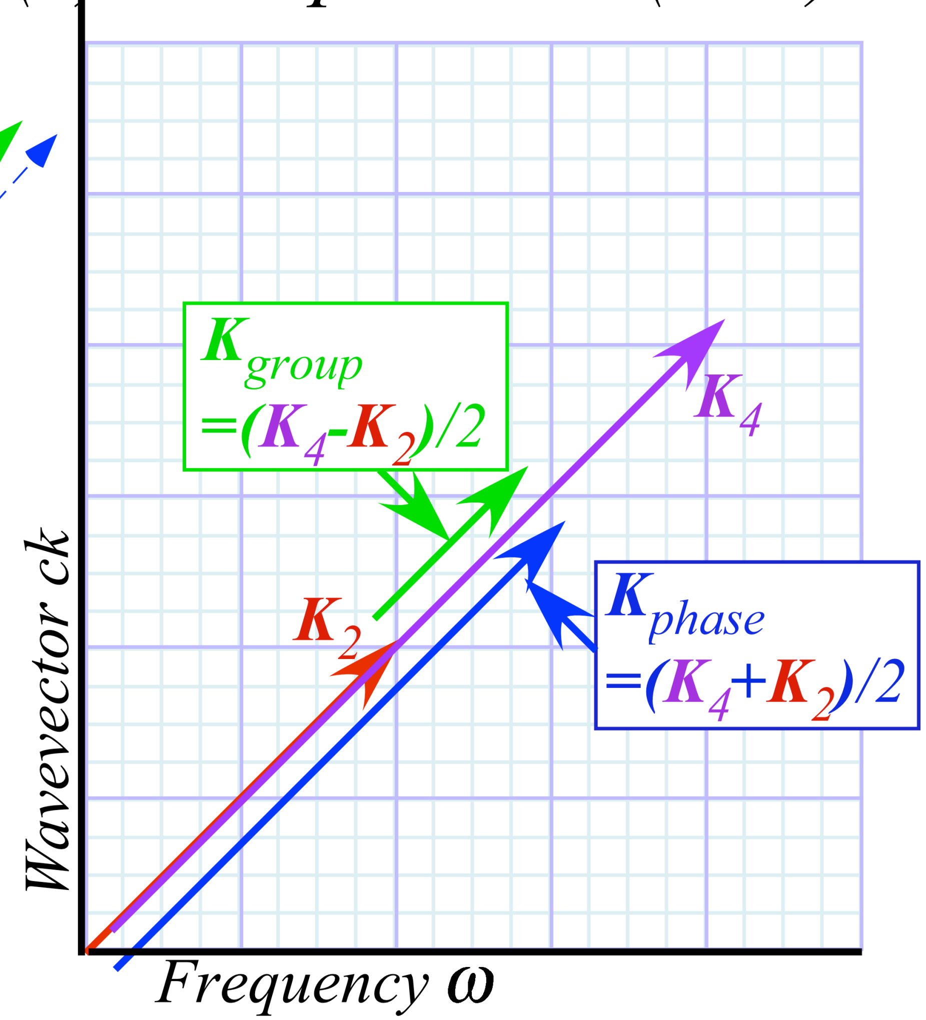
"Waves are illusory!"
Corpuscles rule!
Pa-tooey!



(a) Spacetime (x,ct)

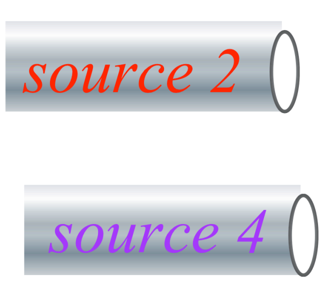


(b) Per-spacetime (ω,ck)

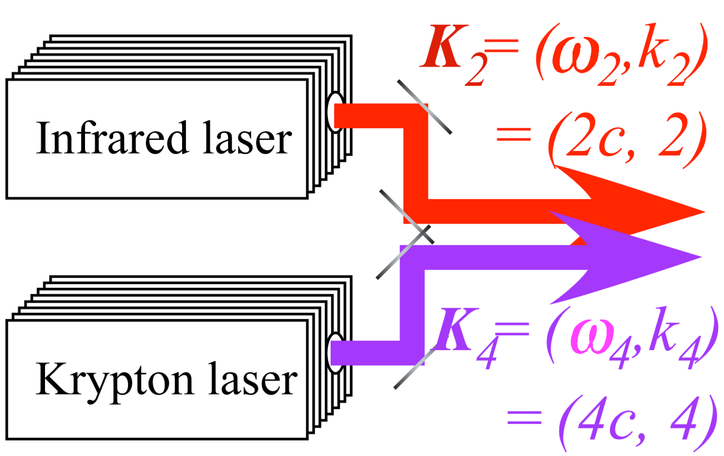


What happens when the grid area $\mathbf{K}_{group}\times\mathbf{K}_{phase}$ *is ZERO:*

$$s_{gp} = \frac{\pi}{2\left|\mathbf{K}_{group}\times\mathbf{K}_{phase}\right|} = \infty$$



Replaced by:



...But, if you collide the beams <u>Head-On</u>...

3. Beginning wave relativity

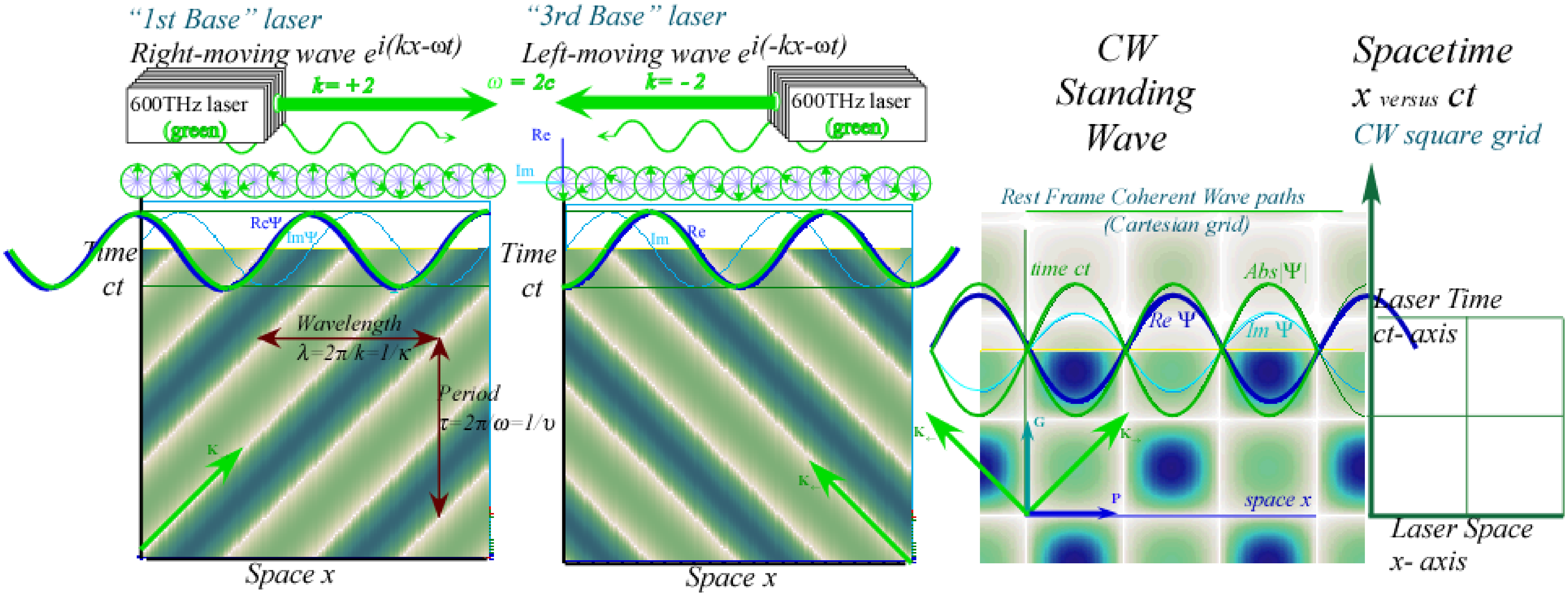
→ *Dueling lasers make lab frame space-time grid (CW or PW)*

Einstein PW Axioms versus Evenson CW Axioms (Occam at Work)

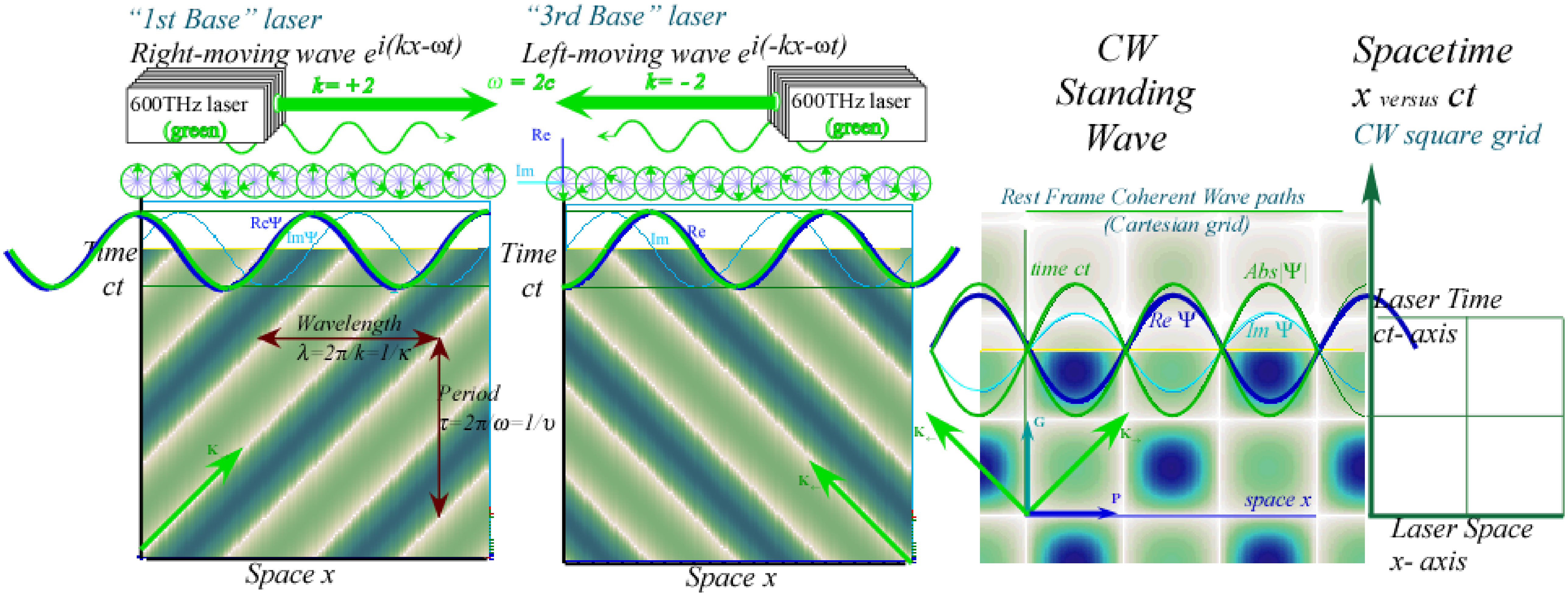
Only CW light clearly shows Doppler shift

Dueling lasers make lab frame space-time grid

Zeros of head-on CW sum gives (x,ct)-grid



Zeros of head-on CW sum gives (x,ct)-grid



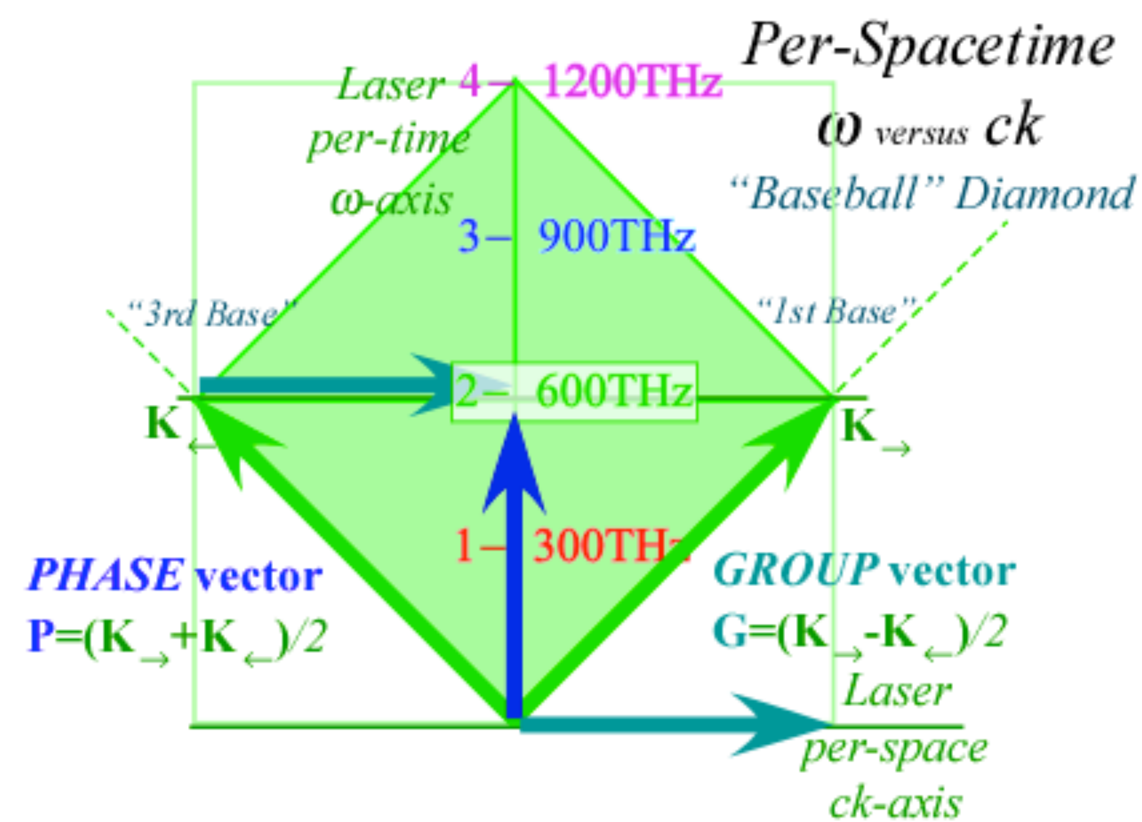
Find zeros by factoring sum:

$$\Psi = \overbrace{e^{ia}}^{(kx-\omega t)} + \overbrace{e^{ib}}^{(-kx-\omega t)}$$

$$= \underbrace{e^{i(a+b)/2}}_{\text{Phase factor:}} \underbrace{\left(e^{i(a-b)/2} + e^{-i(a-b)/2}\right)}_{\text{Group factor:}}$$

Phase factor: $\exp i\left(\frac{a+b}{2}\right) = e^{-i\omega t}$

Group factor: $2\cos\left(\frac{a-b}{2}\right) = 2\cos(kx)$



• *Optical wave coordinate manifolds and frames*

Shining some light on light using complex phasor analysis

Old-fashioned meter-stick-clock frames

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)

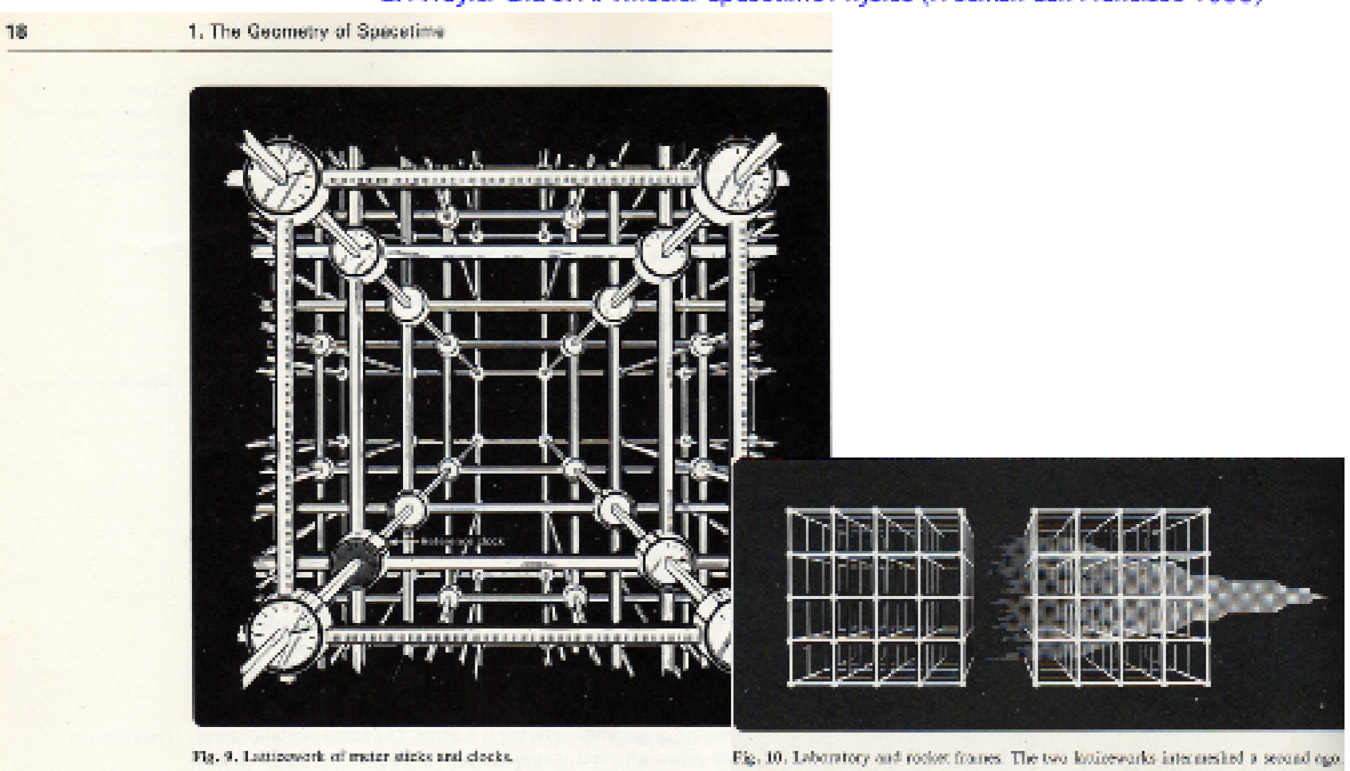


Fig. 9. Latticework of meter sticks and clocks.

Fig. 10. Laboratory and rocket frames. The two latticeworks intermeshed a second ago

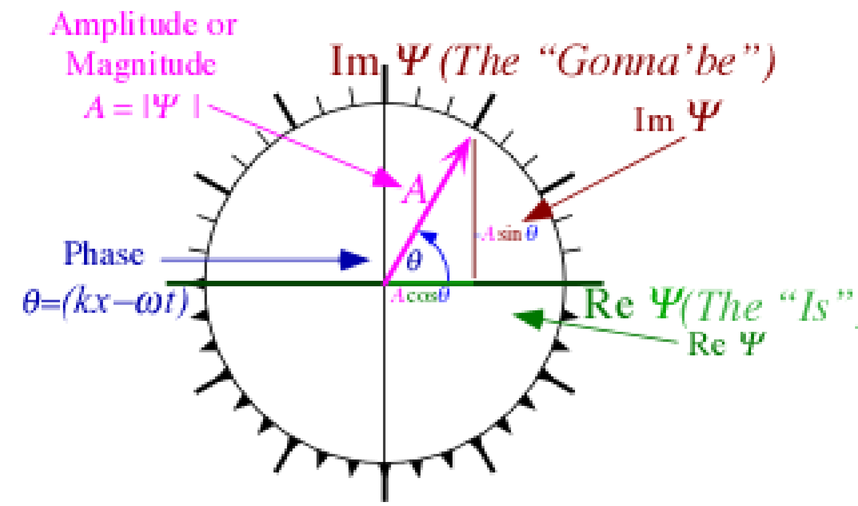
New-fashioned laser clocks & meter sticks

Complex Phasor Clocks : Tesla's AC "phasor"

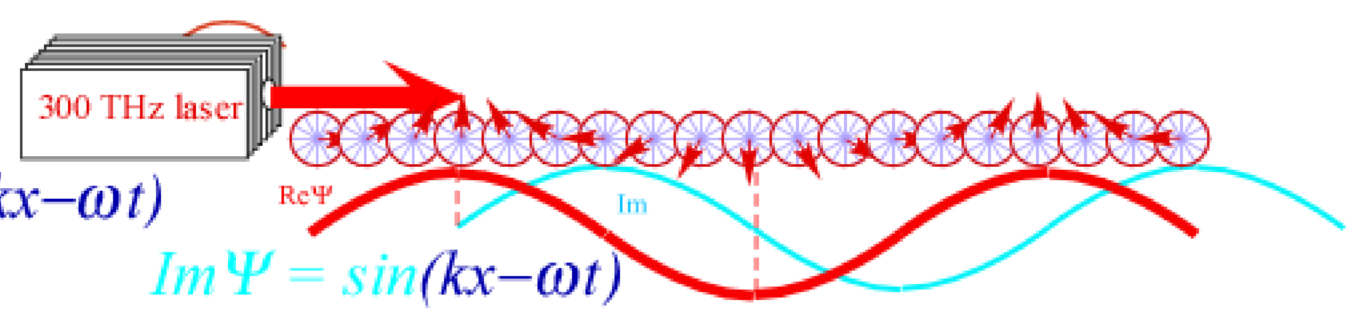
Quantum Phasor Clock

$$\Psi = Ae^{i(kx-\omega t)} = A\cos(kx-\omega t) + i\,A\sin(kx-\omega t)$$

Phasor clocks turn clockwise in time for positive ω



300THz Laser plane wave $\langle x,t\,|\,k,\omega\rangle = Ae^{i(kx\text{ - }wt)}$



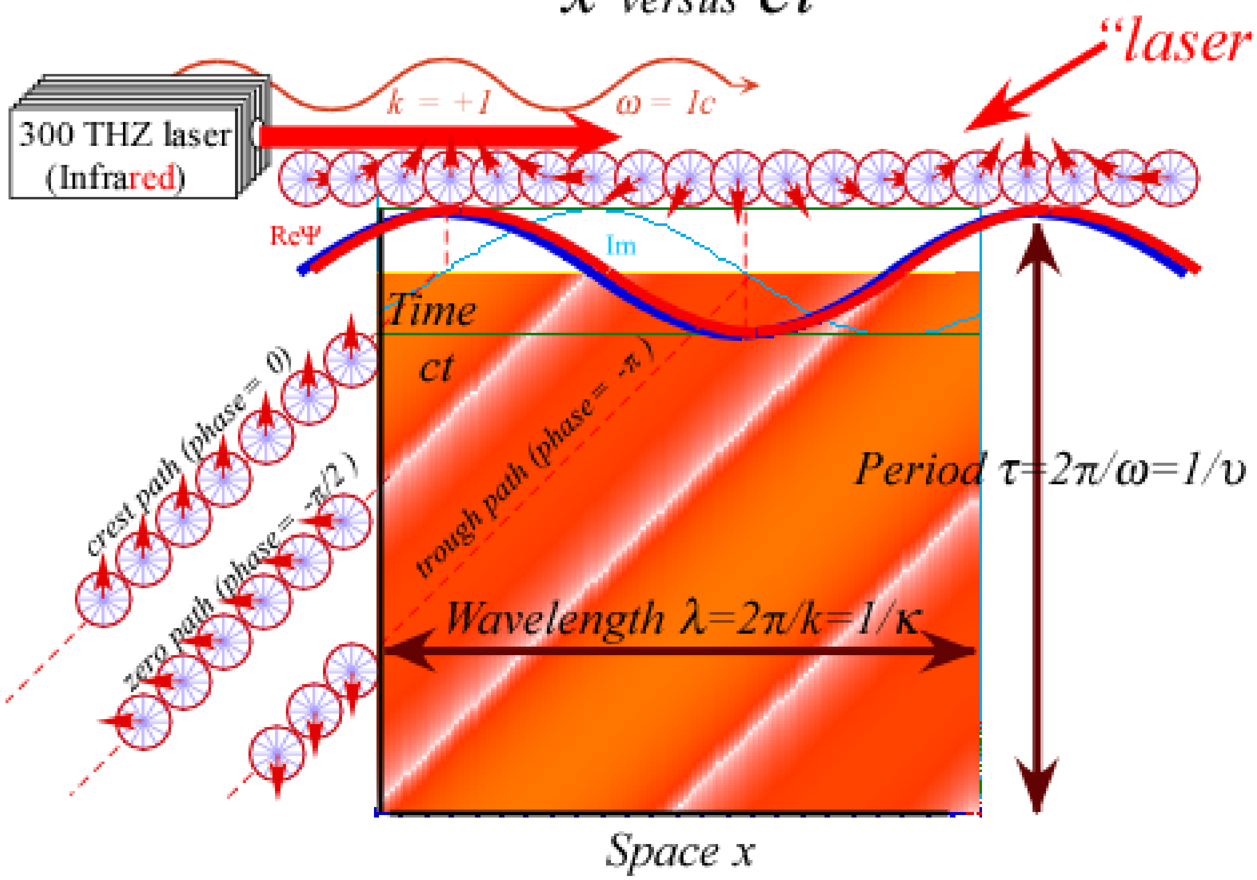
$Re\,\Psi = \cos(kx-\omega t)$

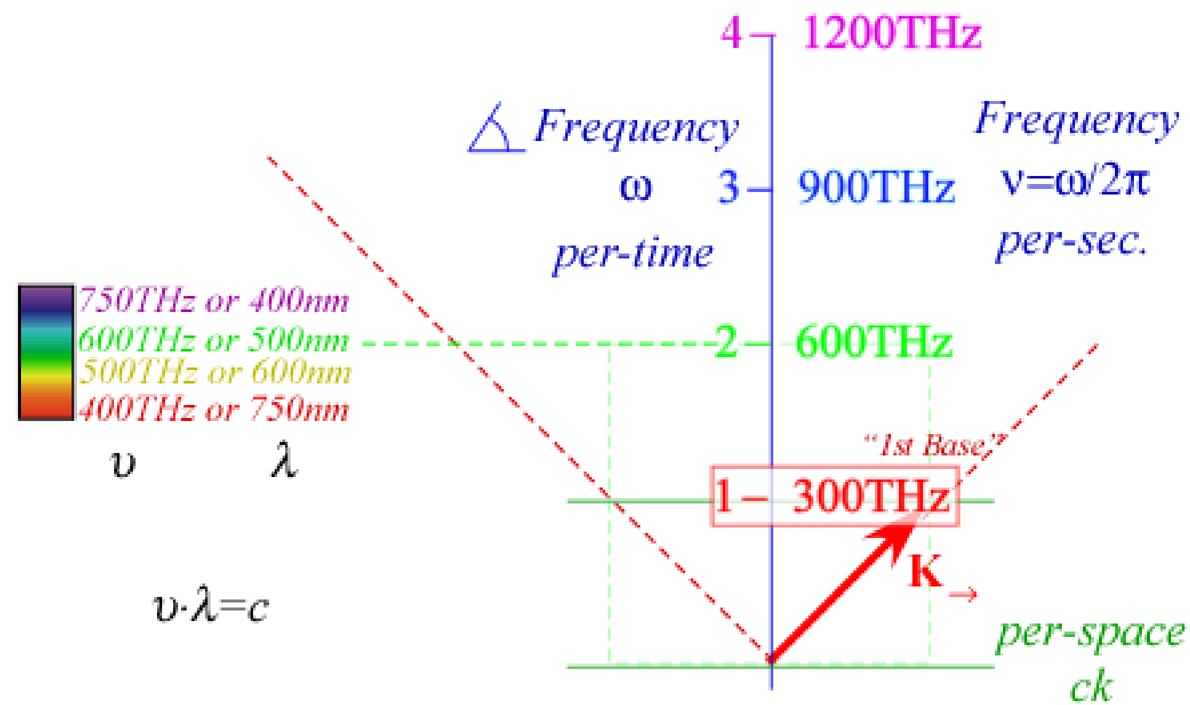
$Im\,\Psi = \sin(kx-\omega t)$

New-fashioned laser clocks & meter sticks (contd.)
Dual views:

(1.) Spacetime
x versus ct

(2.) Per-Spacetime
ω versus ck

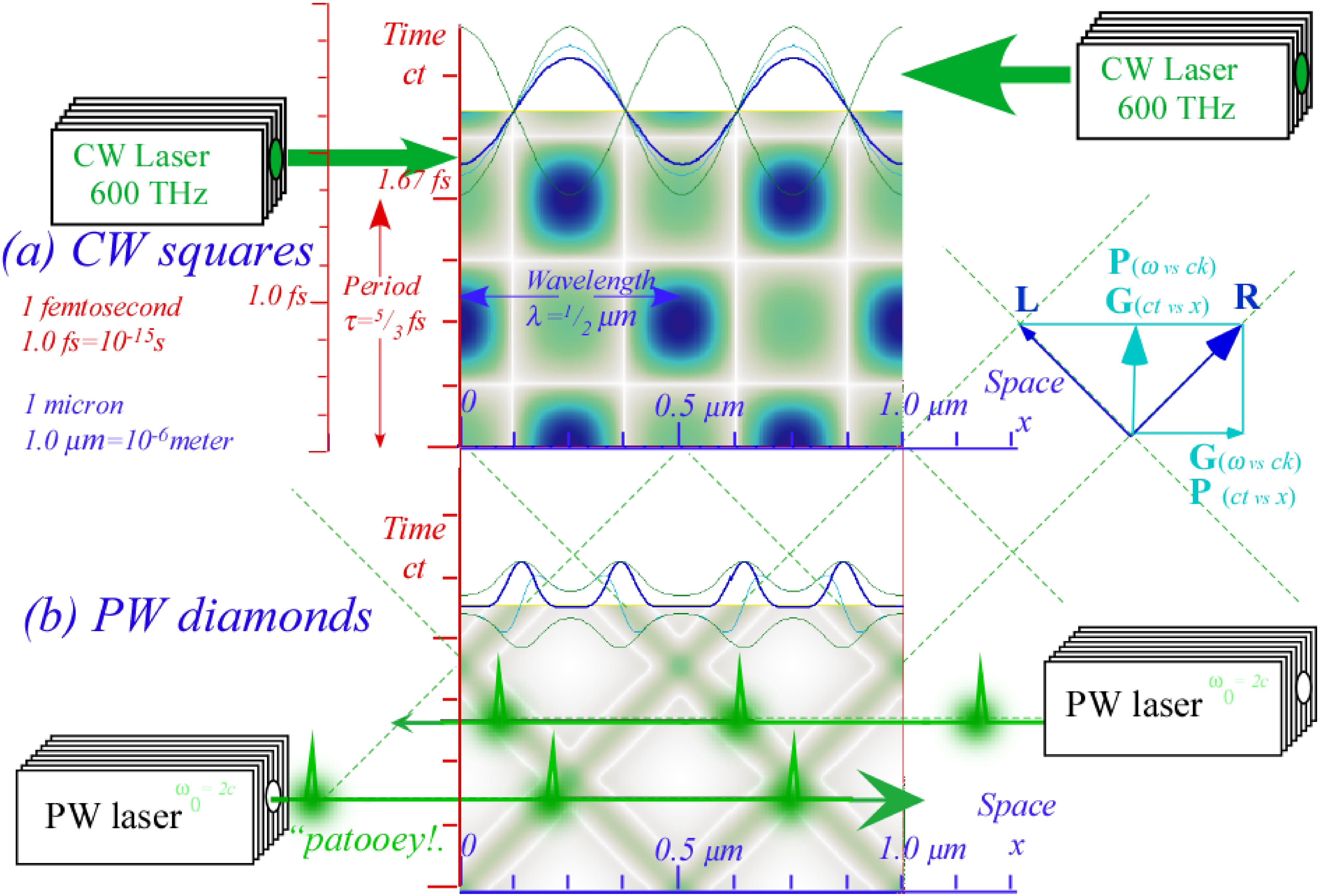




Single plane-wave meter-stick-clocks are too fast (can't catch'em)

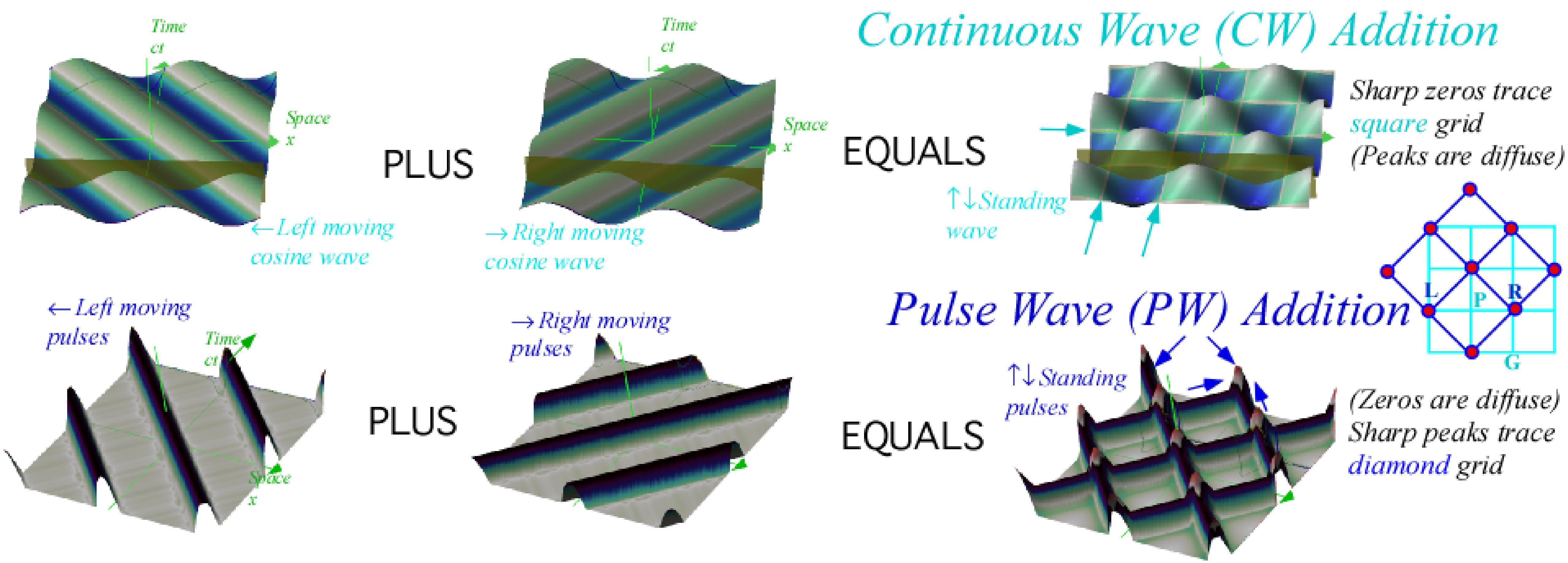
(...But at least this view is constant)

Interfering wave pairs needed to make rest frame coordinates...


CW Laser
600 THz
CW Laser
600 THz
Time
ct
1.67 fs
1.0 fs
Period
τ=5/3 fs
Wavelength
λ=1/2 μm
(a) CW squares
1 femtosecond
1.0 fs=10-15s
1 micron
1.0 μm=10-6meter
0
0.5 μm
1.0 μm
Space
x
P(ω vs ck)
G(ct vs x)
L
R
G(ω vs ck)
P (ct vs x)
(b) PW diamonds
Time
ct
PW laser
PW laser
"patooey!.
0
0.5 μm
1.0 μm
Space
x

Newton's "Fits" in Optical Interference

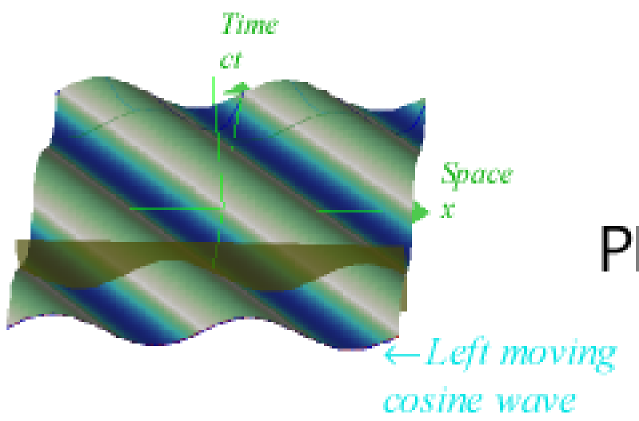
Newton complained that light waves have *"fits"* (what we now know as wave *interference* or *resonance*.)
Examples of interference are head-on collision of two *Continuous Waves (2-CW)* or two *Pulse Waves (PW)*



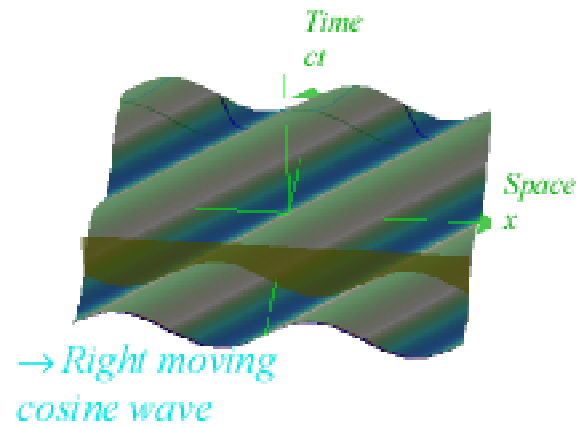
Newton's "Fits" in Optical Interference

Newton complained that light waves have *"fits"* (what we now know as wave *interference* or *resonance*.)
Examples of interference are head-on collision of two *Continuous Waves (2-CW)* or two *Pulse Waves (PW)*

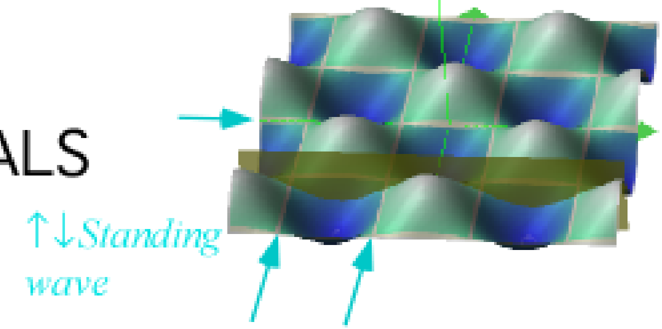
Continuous Wave (CW) Addition



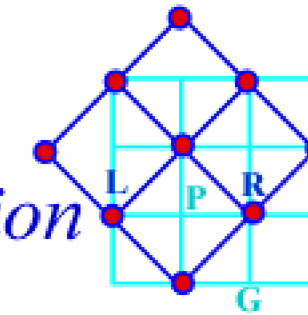
PLUS



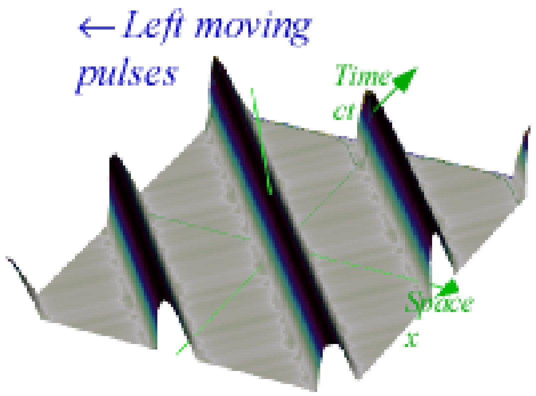
EQUALS



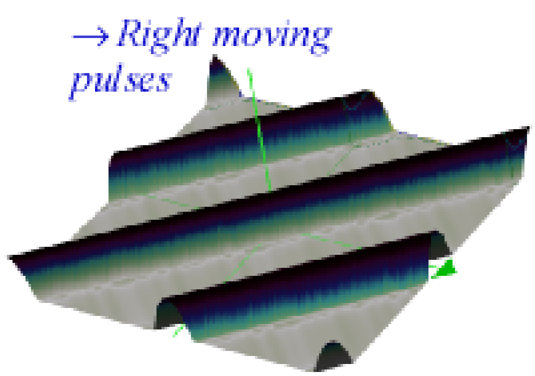
Sharp zeros trace
square grid
(Peaks are diffuse)



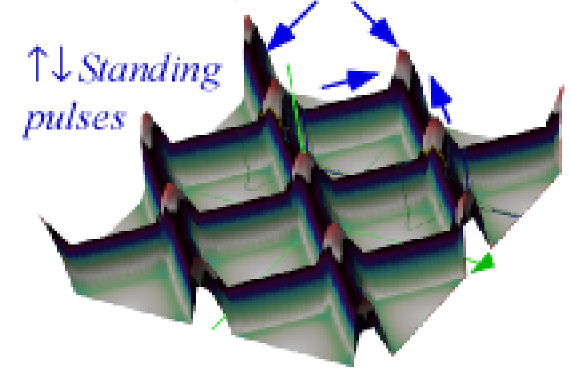
Pulse Wave (PW) Addition



PLUS



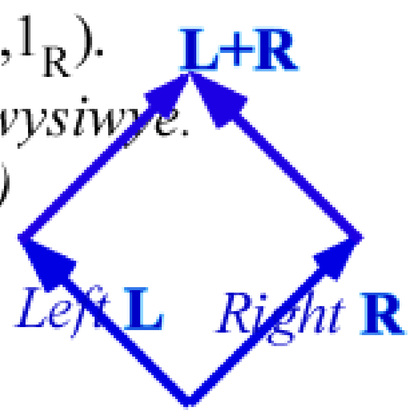
EQUALS



(Zeros are diffuse)
Sharp peaks trace
diamond grid

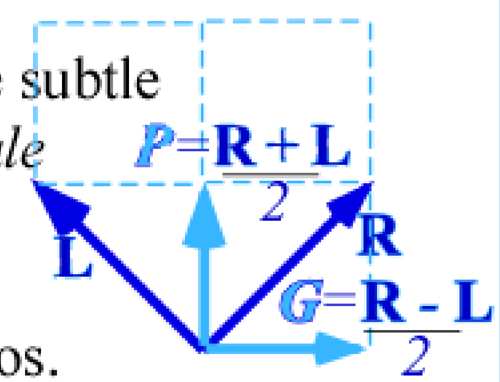
Pulse Wave (PW) *sum compared with*

- *PW* waves are OFF (0) or ON (1)
- *PW* sum is Boolean $(0_L,0_R),(0_L,1_R),$
  $(1_L,0_R),(1_L,1_R).$
- *PW* time *peak-diamond* paths are *wysiwye.*
  (What you see is what you expect!)



Continuous Wave (CW) sum

- *CW* waves range continously from -1 to +1
- *CW* sum is more subtle and nuanced *interference*.
- *CW* time *zero-square* paths are subtle results of the *half-sum* ***P****-rule* and the *half-difference* ***G****-rule* of *phase* ***P*** and *group* ***G*** zeros.



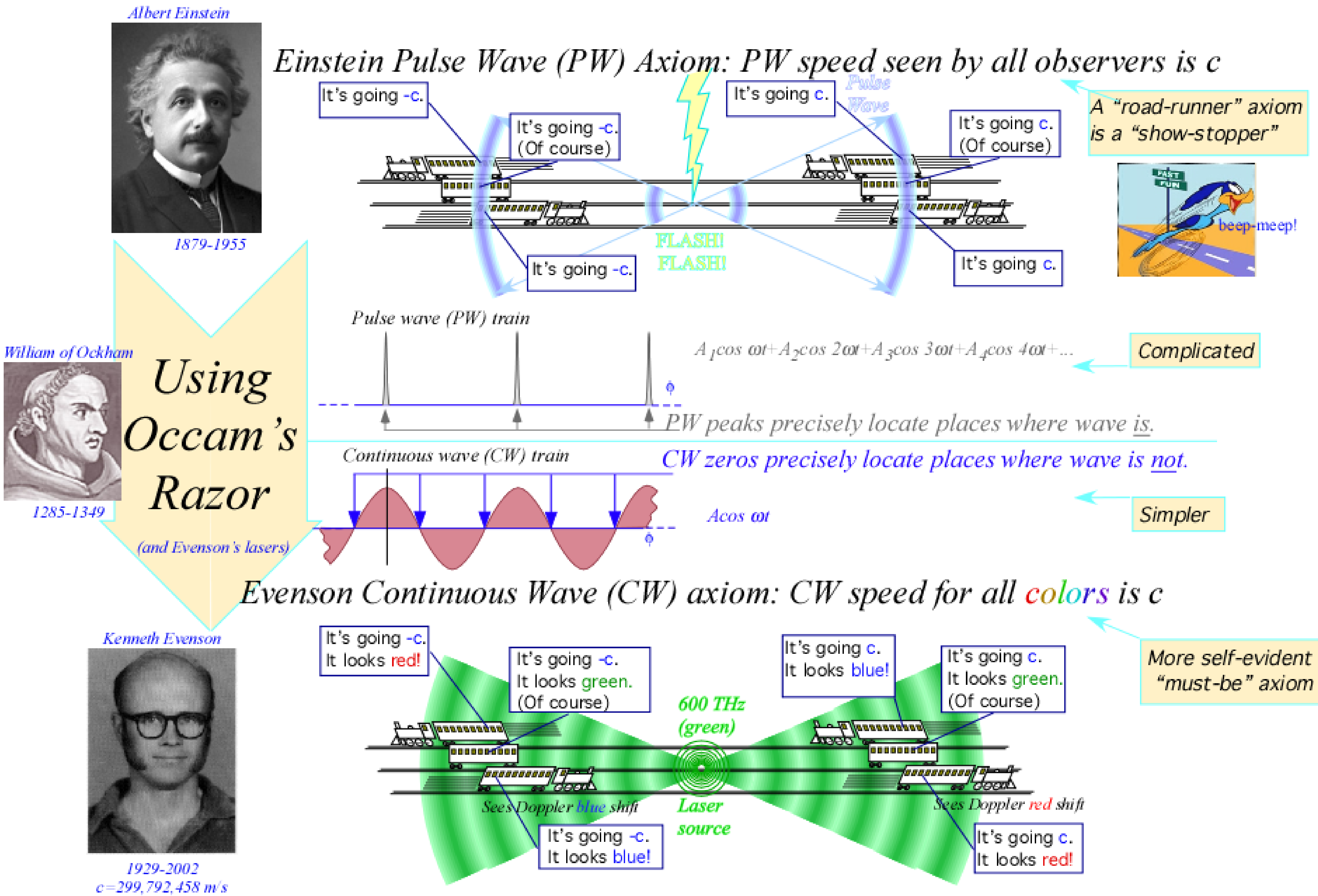
3. Beginning wave relativity

Dueling lasers make lab frame space-time grid (CW or PW)

→ *Einstein PW Axioms versus Evenson CW Axioms (Occam at Work)*

Only CW light clearly shows Doppler shift

Dueling lasers make lab frame space-time grid

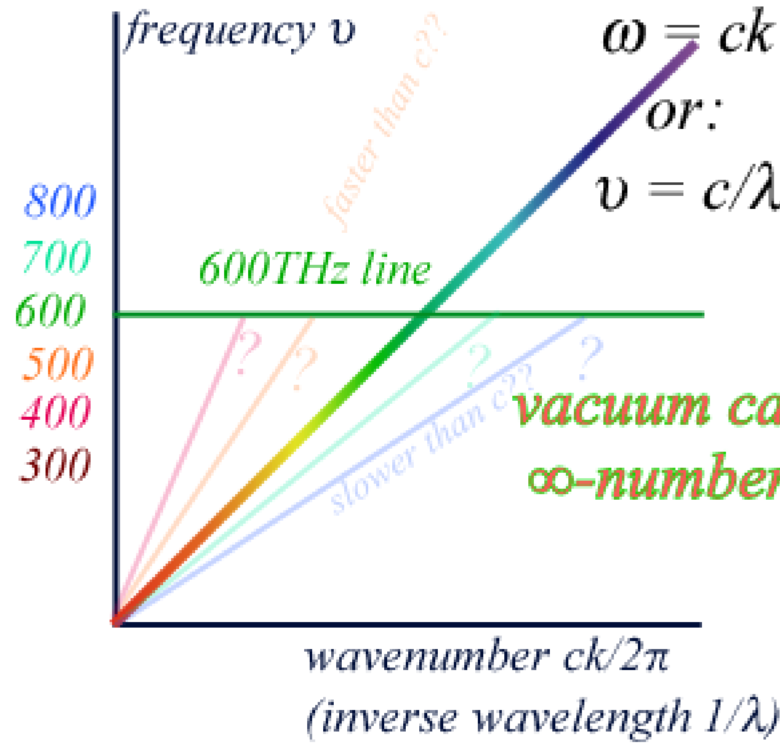

Albert Einstein
1879-1955
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c
It's going -c.
It's going -c.
(Of course)
It's going c.
Pulse Wave
It's going c.
(Of course)
A "road-runner" axiom is a "show-stopper"
beep-meep!
FLASH!
FLASH!
It's going -c.
It's going c.
Pulse wave (PW) train
$A_1\cos\omega t+A_2\cos 2\omega t+A_3\cos 3\omega t+A_4\cos 4\omega t+...$
Complicated
William of Ockham
1285-1349
Using Occam's Razor
(and Evenson's lasers)
PW peaks precisely locate places where wave is.
Continuous wave (CW) train
CW zeros precisely locate places where wave is not.
$A\cos\omega t$
Simpler
Evenson Continuous Wave (CW) axiom: CW speed for all colors is c
More self-evident "must-be" axiom
Kenneth Evenson
1929-2002
c=299,792,458 m/s
It's going -c.
It looks red!
It's going -c.
It looks green.
(Of course)
600 THz (green)
It's going c.
It looks blue!
It's going c.
It looks green.
(Of course)
Sees Doppler blue shift
Laser source
Sees Doppler red shift
It's going -c.
It looks blue!
It's going c.
It looks red!

Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

Linear dispersion: $\omega = ck$

Linear dispersion means NO dispersion

Einstein PW is corollary of Evenson CW



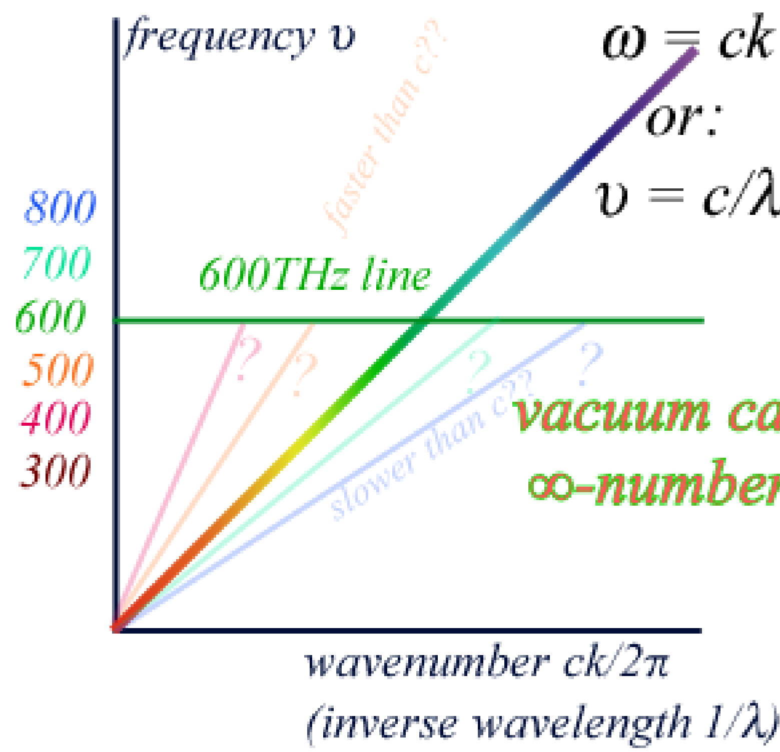
vacuum can't support an
∞-number of "other speeds"

Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

Linear dispersion: $\omega = ck$

Linear dispersion means NO dispersion

Einstein PW is corollary of Evenson CW



vacuum can't support an ∞-number of "other speeds"

What if blue were to travel 0.001% slower than red from a galaxy 9 billion light years away? (..and show up 10^5 years late)

That would mean Good-Bye Hubble Astronomy!

3. Beginning wave relativity

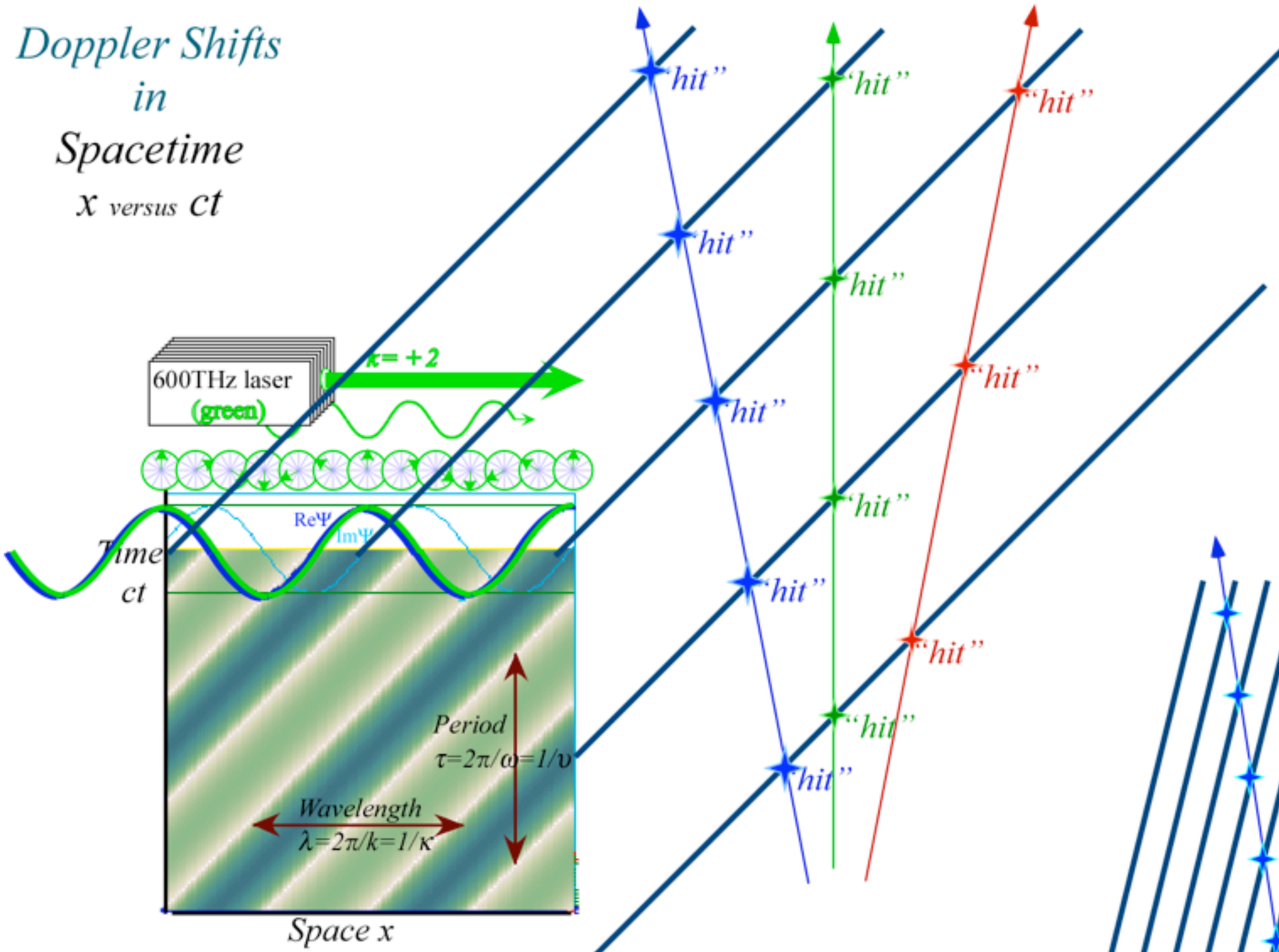
Dueling lasers make lab frame space-time grid (CW or PW)

Einstein PW Axioms versus Evenson CW Axioms (Occam at Work)

→ *Only CW light clearly shows Doppler shift*

Dueling lasers make lab frame space-time grid

Doppler Shifts in Spacetime
x versus ct

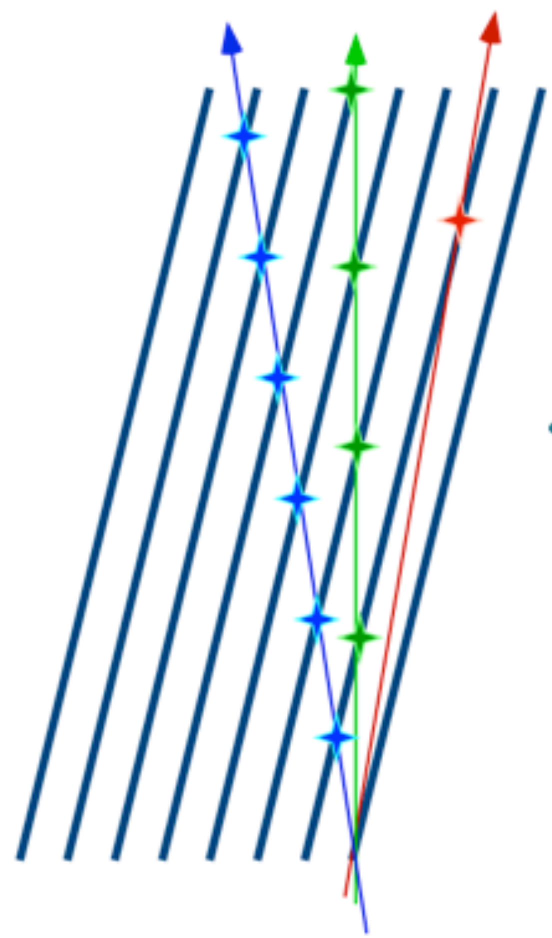


Doppler Blueshift
More "hits" per sec. if moving toward laser source

Doppler Redshift
Fewer "hits" per sec. if moving away from laser source

Doppler's picture needs revision for light whose period and wavelength both shift.

Why?



...So that all colors go the same speed!

$$\upsilon \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$$

$$\upsilon \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$$

$$\upsilon \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$$

etc.

3. Beginning wave relativity

Dueling lasers make lab frame space-time grid (CW or PW)

Einstein PW Axioms versus Evenson CW Axioms (Occam at Work)

Only CW light clearly shows Doppler shift

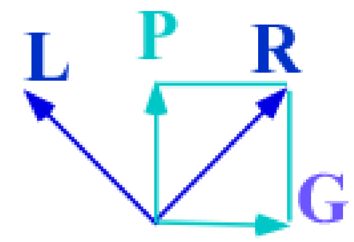
→ *Dueling lasers make lab frame space-time grid*

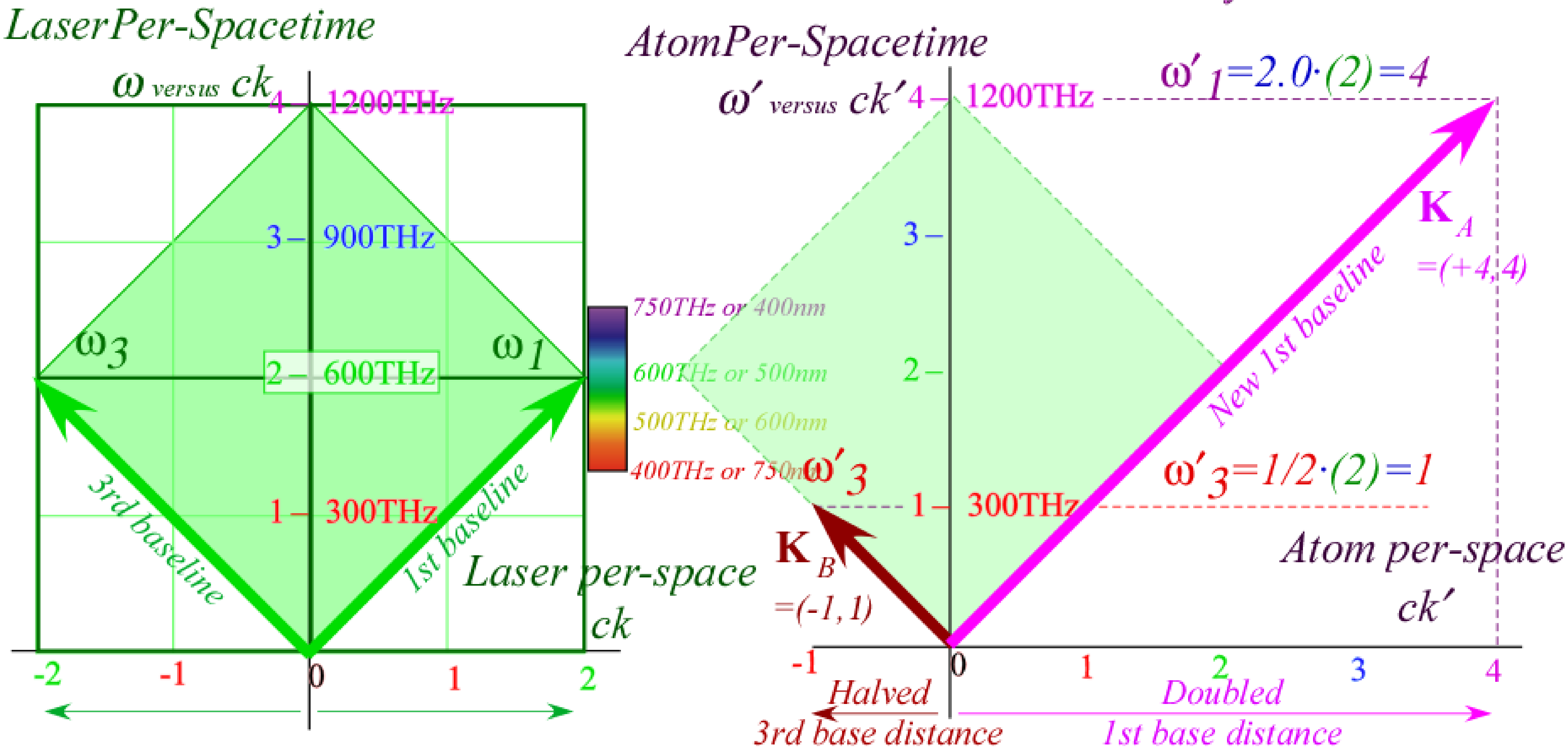
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom "All colors go c" keeps $\mathbf{K}_A$ and $\mathbf{K}_B$ on their baselines.

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase $\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$ and Half-Difference Group $\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$



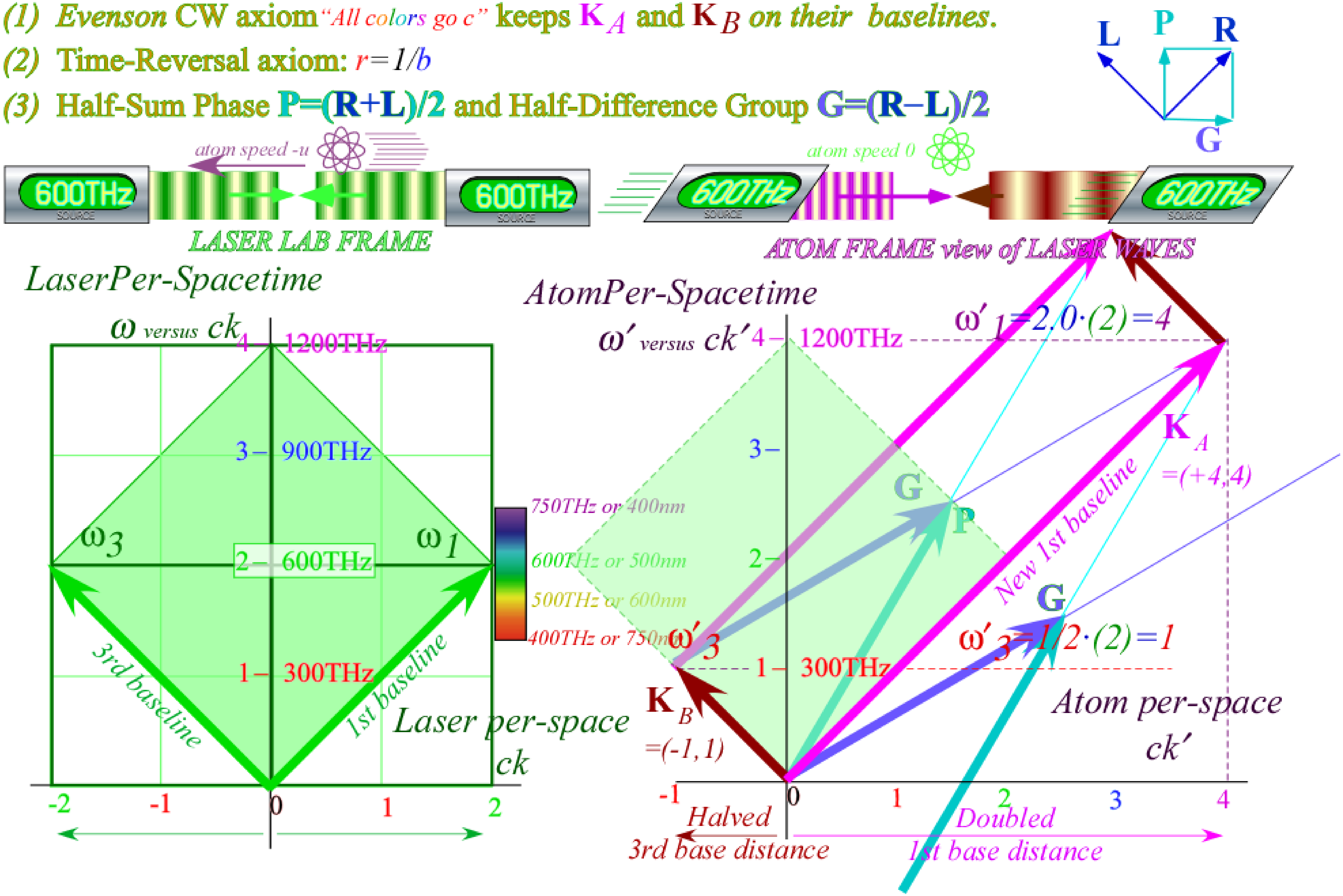


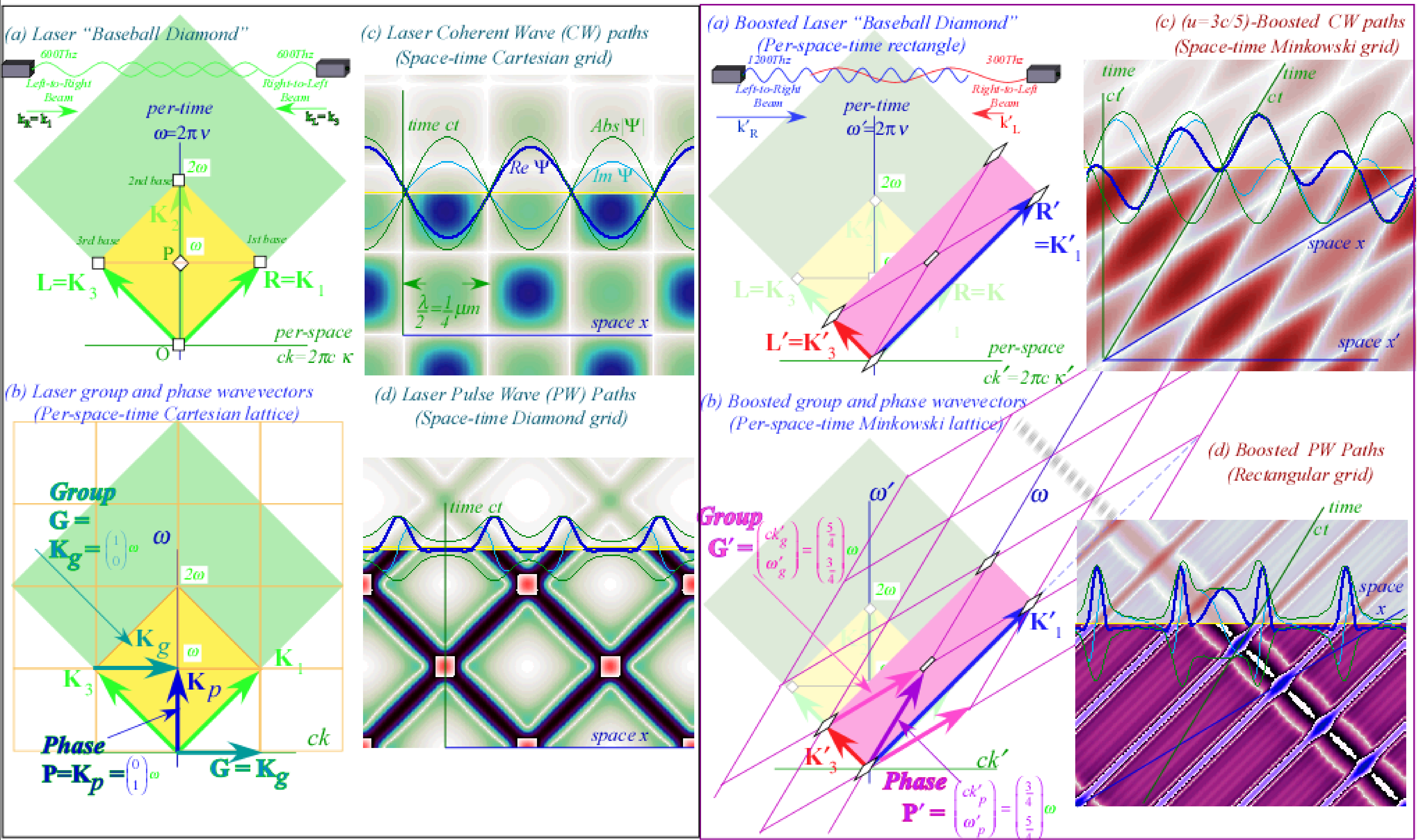
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom *"All colors go c"* keeps K_A and K_B *on their baselines.*

(2) Time-Reversal axiom: $r = 1/b$

(3) Half-Sum Phase **P=(R+L)/2** and Half-Difference Group **G=(R−L)/2**

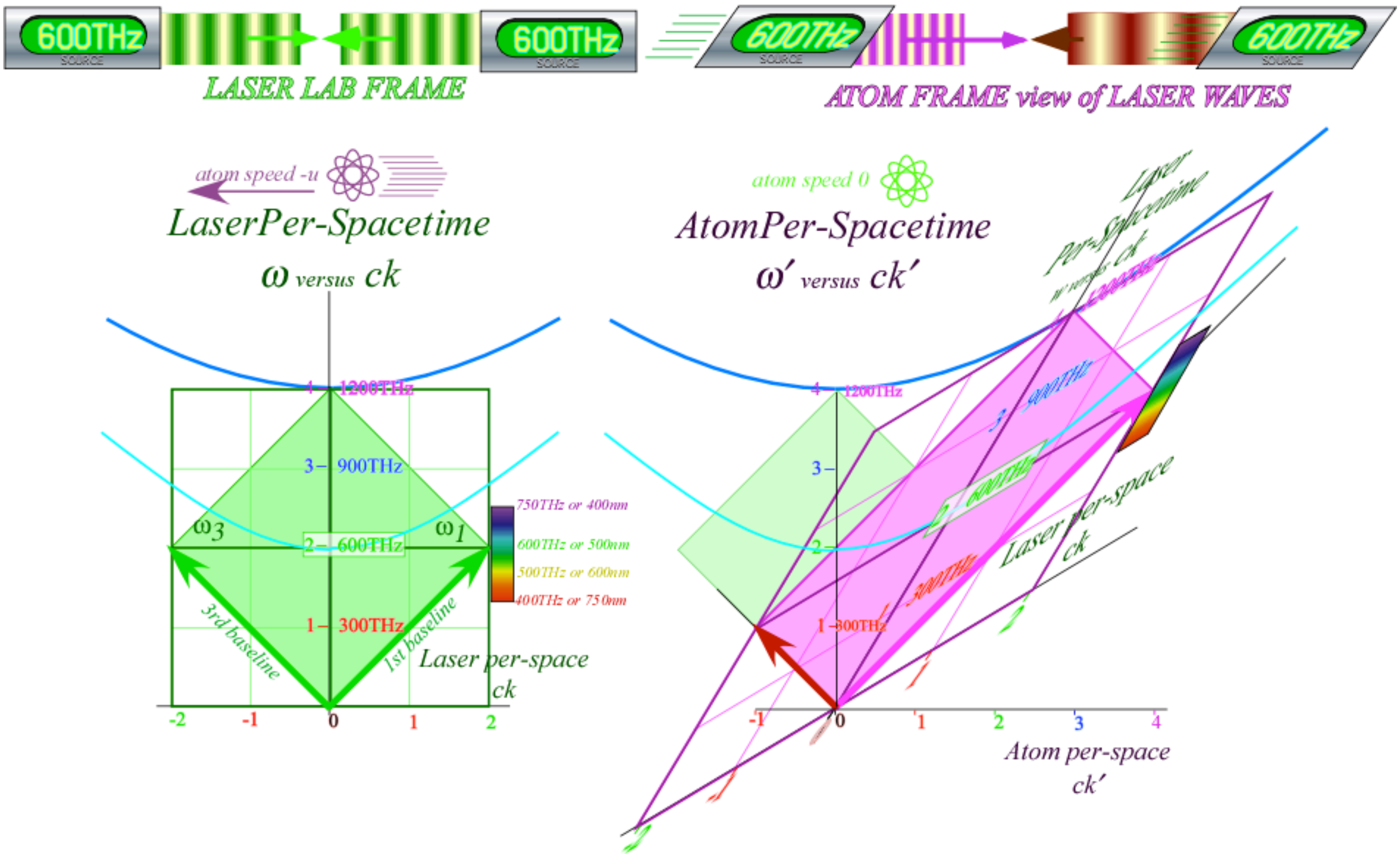




Laser lab views

atom speed $-u=-\frac{3}{5}c$

Atom views (sees lab going $+u=\frac{3}{5}c$)


600THz
SOURCE
LASER LAB FRAME
ATOM FRAME view of LASER WAVES
atom speed -u
LaserPer-Spacetime
ω versus ck
atom speed 0
AtomPer-Spacetime
ω′ versus ck′
1200THz
900THz
600THz
300THz
ω3
ω1
3rd baseline
1st baseline
Laser per-space
ck
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
Laser
Per-Spacetime
ω versus ck
Laser per-space
ck
Atom per-space
ck′

Euclidian Geometry for Per-spacetime Relativity

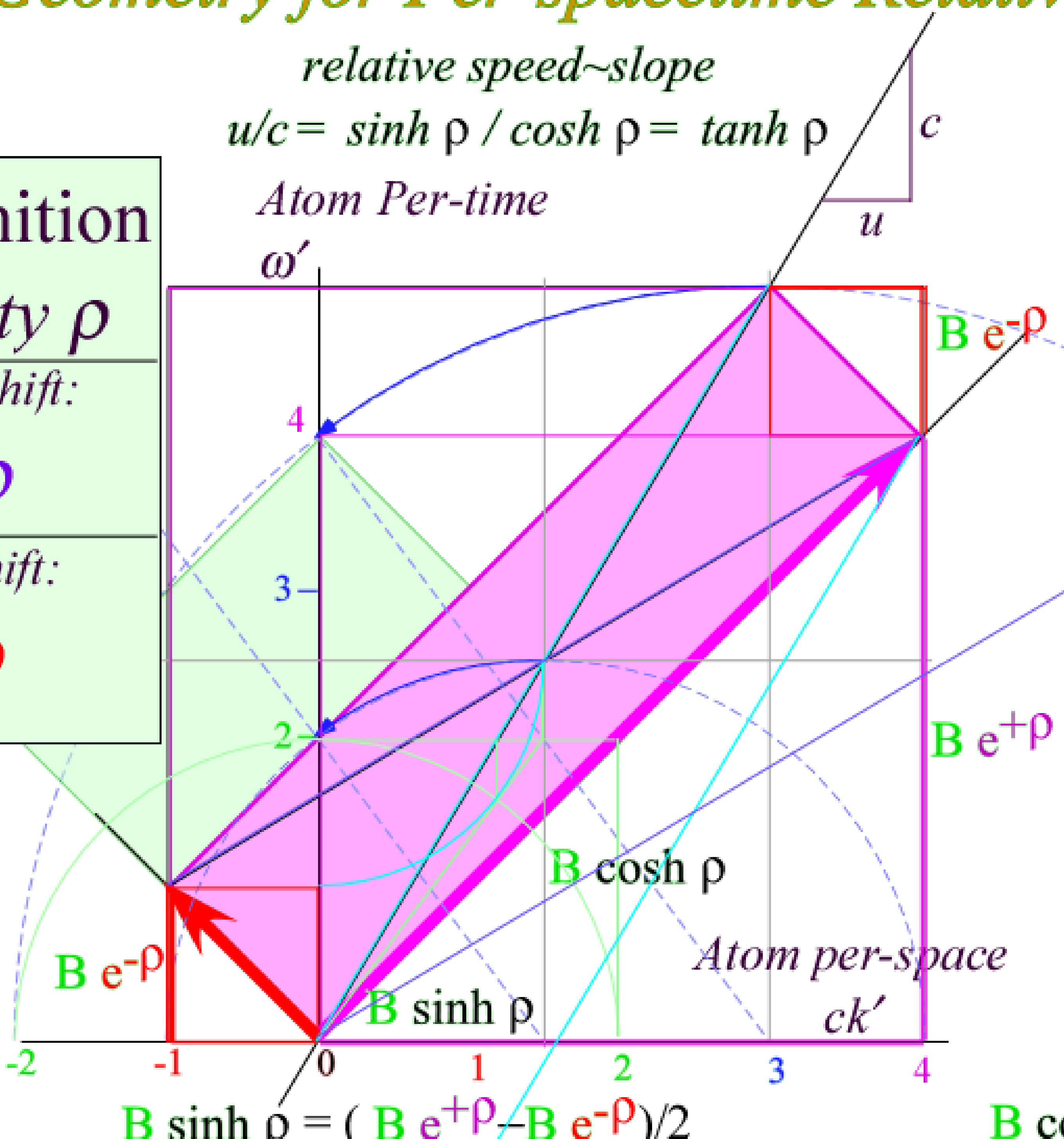
Key Definition of Rapidity ρ

Doppler blue shift:

$Bb = Be^{+\rho}$

Doppler red shift:

$Br = Be^{-\rho}$



Key Results:

ω *vs.* ck

"winks" vs. "kinks"

$\omega = B \cosh \rho$

$ck = B \sinh \rho$

group velocity:

$$\frac{\omega}{ck} = \frac{u}{c} = \tanh \rho$$

phase velocity:

$$\frac{ck}{\omega} = \frac{c}{u} = \coth \rho$$

Key Quantities

Lorentz-Einstein factors

$$\sinh \rho = \frac{\frac{u}{c}}{\sqrt{1 - \frac{u^2}{c^2}}}$$

$$\cosh \rho = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$$