What's the Matter with Mass?: Getting relativity and quantum theory to come out of the closet

Bill Harter - University of Arkansas - Fayetteville

OAI
Ohio Aerospace Institute

- **What's the matter with Mass?**

  Shining some light on the elephant in the spacetime room

- **Optical wave coordinate manifolds and frames**

  Shining some light on light using complex phasor analysis

- **Applying Occam's razor to relativity axioms**

  Einstein Pulse-Wave (PW) axiom vs. Evenson Coherent-Wave (CW) axiom

- **Spectral development of relativistic mass mechanics**

  Doppler shifts and hyper-complex "phase-based" matter-wave mechanics

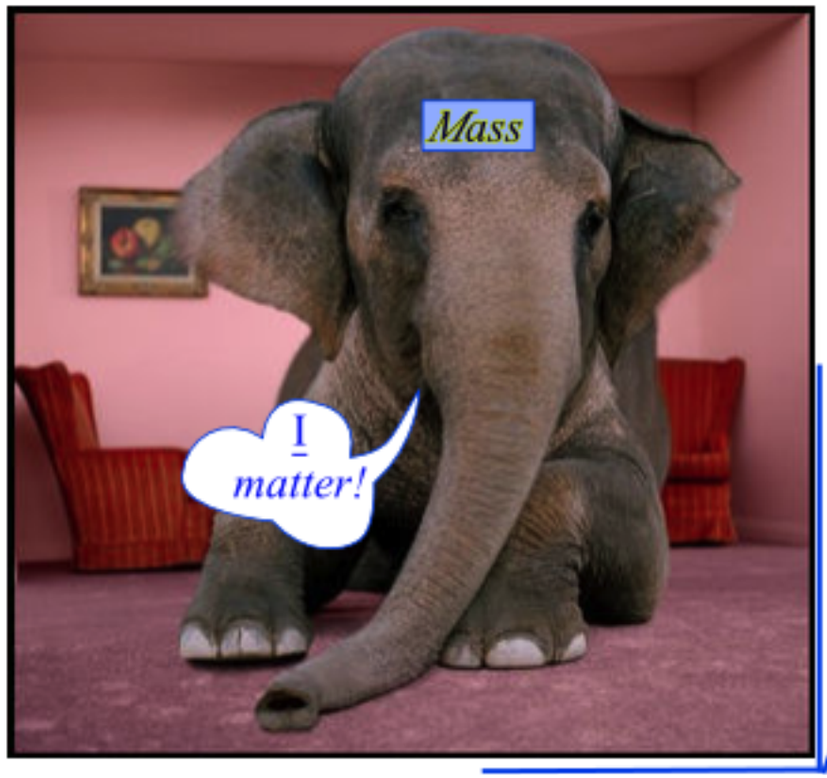
- **Geometry of relativistic Hamilton-Lagrange mechanics**

  Legendre contact transformations at extreme velocity

- *Wave frames of varying acceleration*

  Optical Einstein elevator, photon rockets, Compton acceleration

- ***What's the matter with Mass?***



A brief History of defining Mass M :

1590 Galileo's "impago"

$$M_{Galileo} = \frac{Momentum}{Velocity} = \frac{M \cdot v}{v}$$

1687 Newton's "inertia"

$$M_{Newton} = \frac{Change\ in\ Momentum}{Change\ in\ Velocity} = \frac{M \cdot a}{a}$$

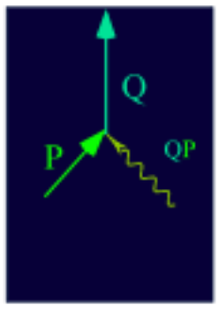
1905 Einstein's "rest mass"

$$M_{Einstein} = \frac{Energy}{(lightspeed)^2} = \frac{M \cdot c^2}{c^2}$$

Shining some light on the elephant in the spacetime room

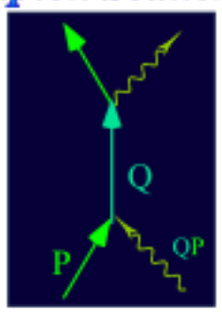
Fundamental light-matter processes:

Absorption A

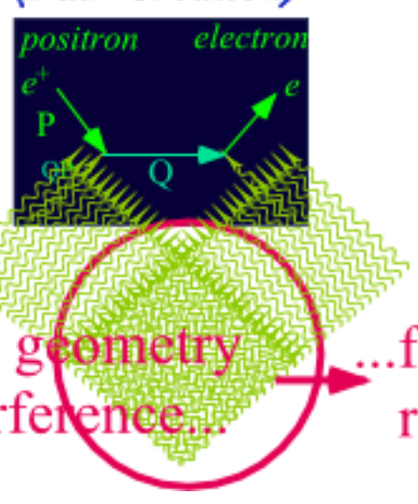


Emission E

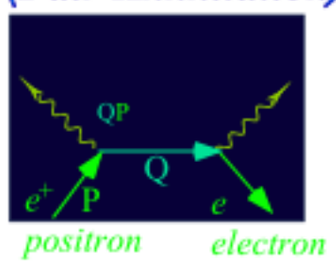
AE Together (Compton Scattering)



AA Together (Pair-Creation)



EE Together (Pair-Annihilation)



Does cavity QED have a future?

Hyper-complex geometry of optical interference... ...formulation of relativistic QM

○ *Optical wave coordinate manifolds and frames*

Shining some light on light using complex phasor analysis

• *Optical wave coordinate manifolds and frames*

Shining some light on light using complex phasor analysis

Old-fashioned meter-stick-clock frames

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)

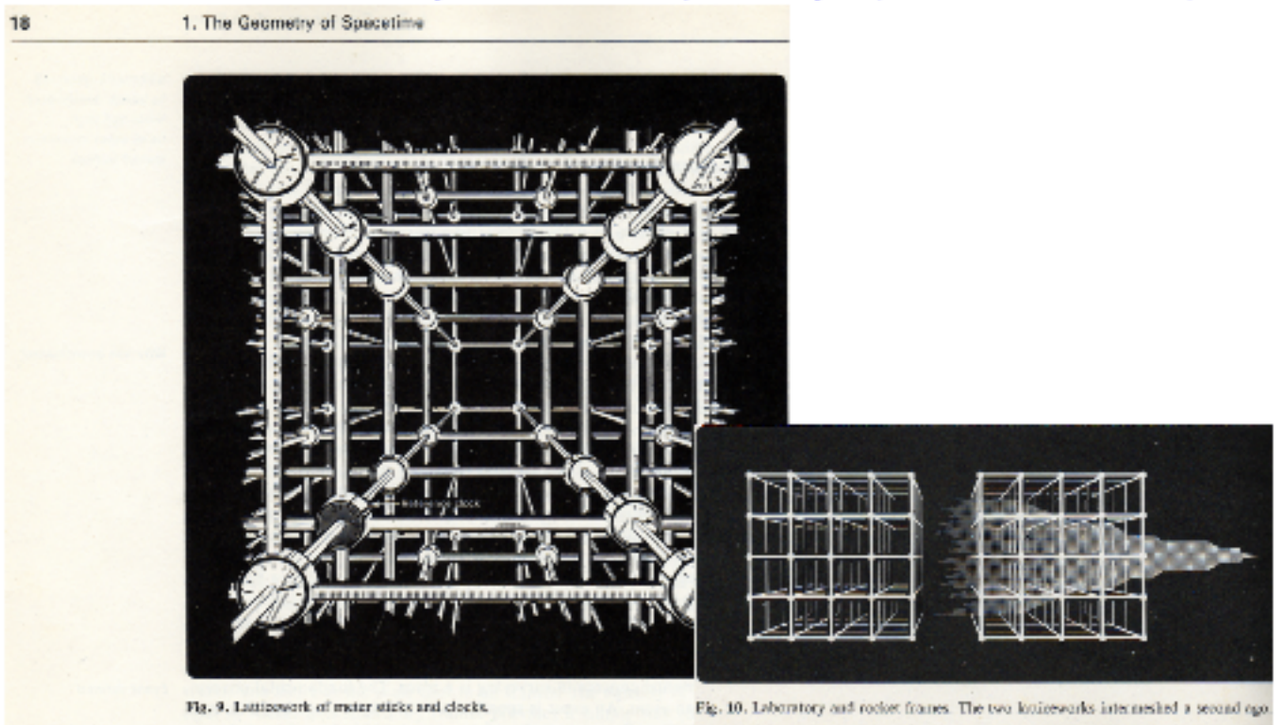


Fig. 9. Latticework of meter sticks and clocks.

Fig. 10. Laboratory and rocket frames. The two latticeworks interpenetrate a second ago.

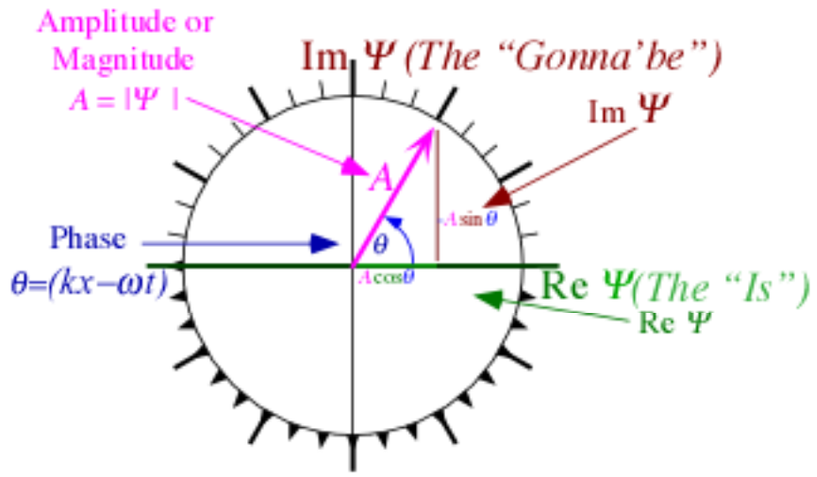
New-fashioned laser clocks & meter sticks

Complex Phasor Clocks : Tesla's AC "phasor"

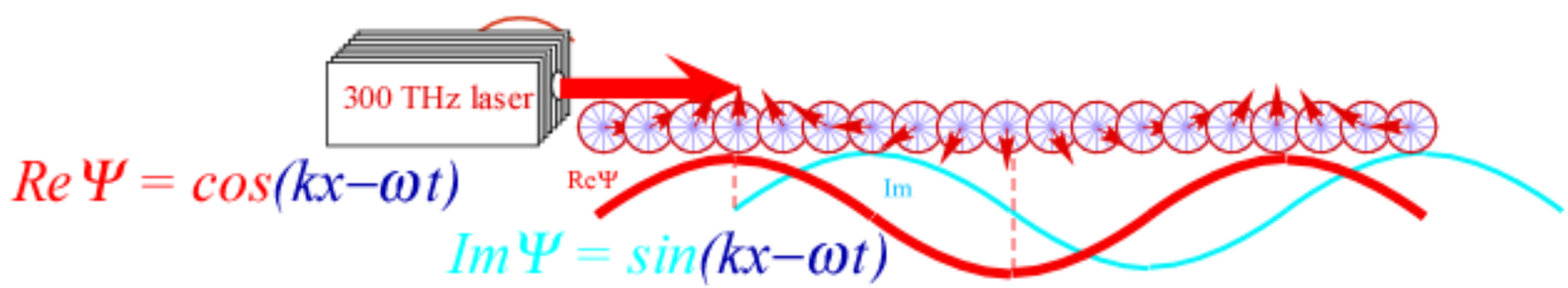
Quantum Phasor Clock

$$\Psi = Ae^{i(kx-\omega t)}$$
$$= A\cos(kx-\omega t)$$
$$+i\,A\sin(kx-\omega t)$$

Phasor clocks turn clockwise in time for positive ω



300THz Laser plane wave $\langle x,t \mid k,\omega \rangle = Ae^{i(kx - wt)}$



$$Re\,\Psi = \cos(kx-\omega t)$$

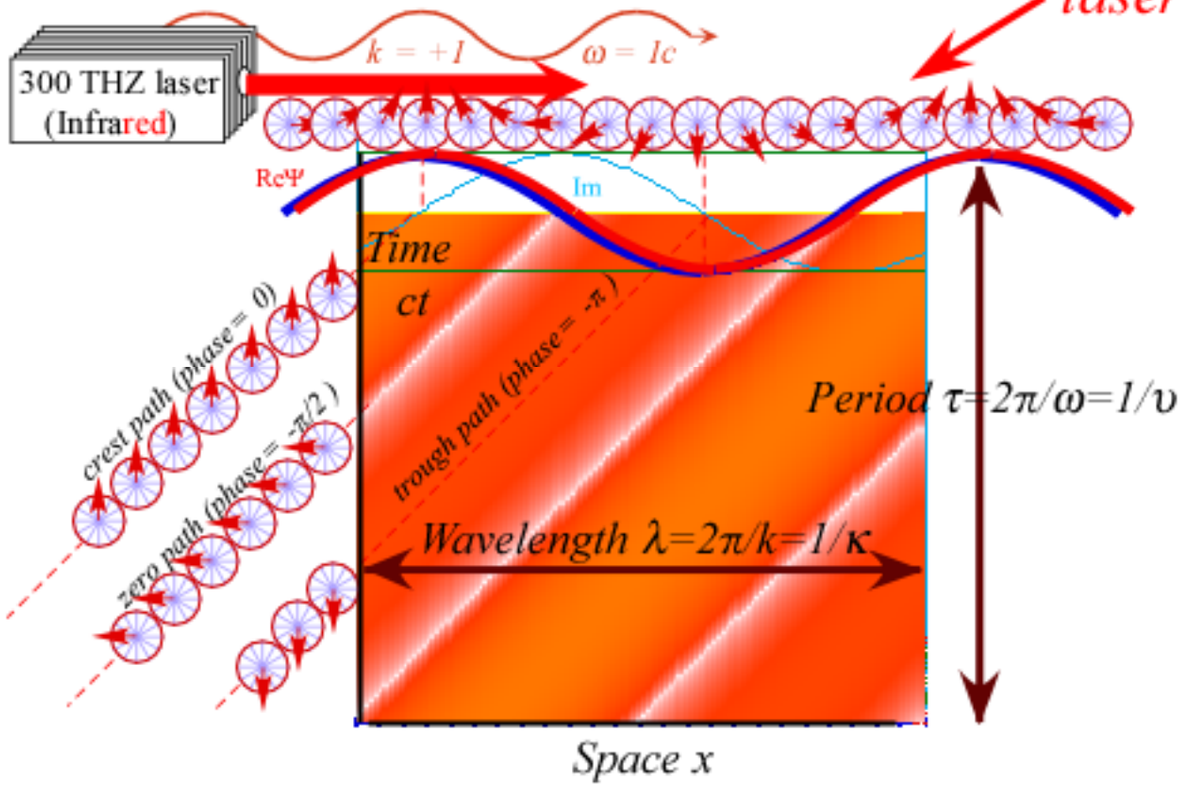
$$Im\,\Psi = \sin(kx-\omega t)$$

New-fashioned laser clocks & meter sticks (contd.)

Dual views:

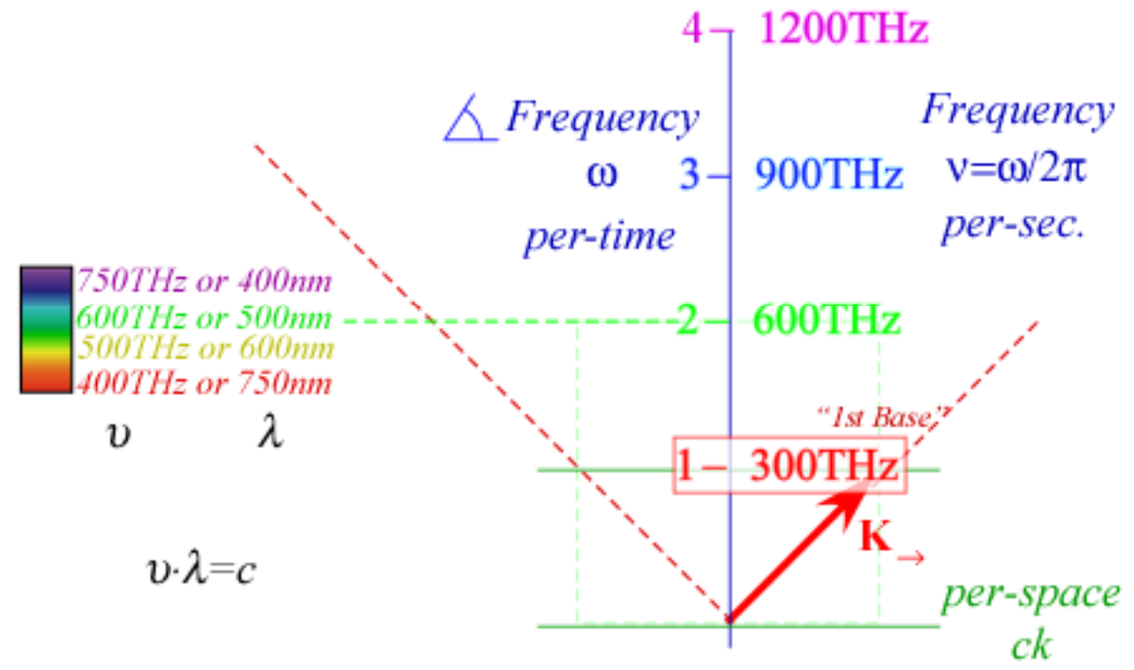
(1.) Spacetime

x versus ct



(2.) Per-Spacetime

ω versus ck

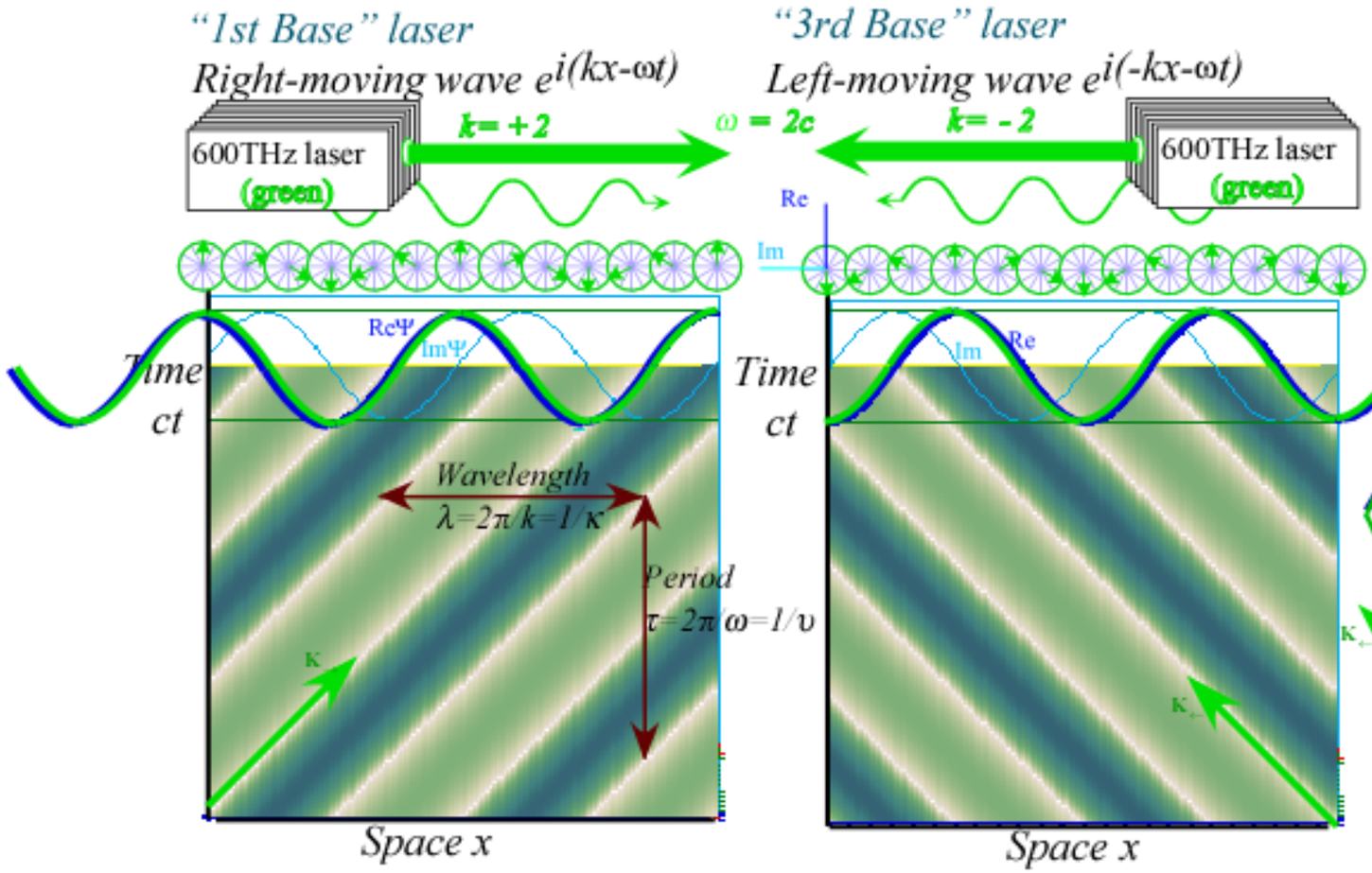


Single plane-wave meter-stick-clocks are too fast (can't catch'em)

(...But at least this view is constant)

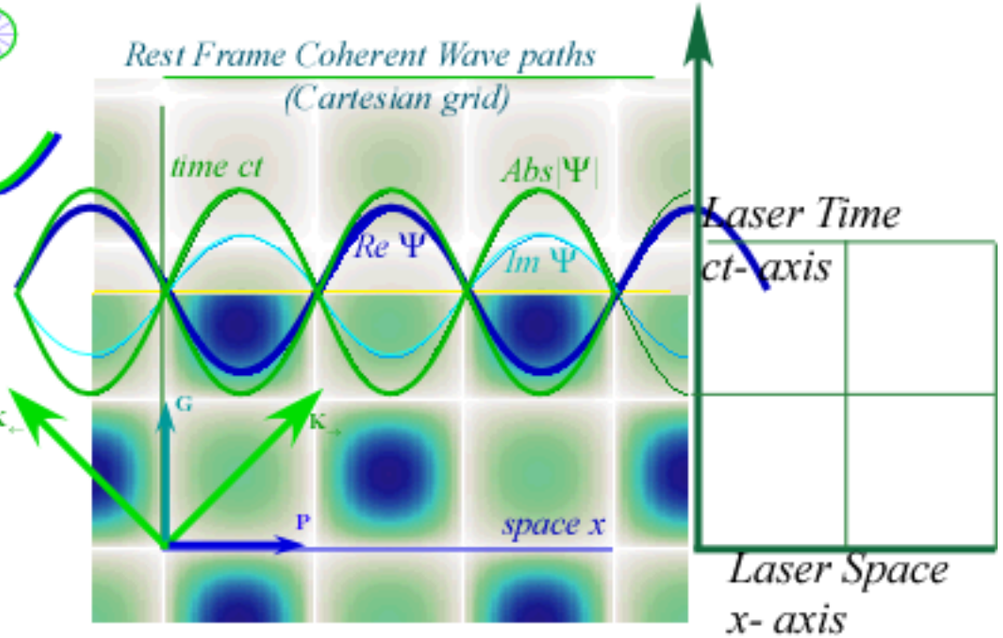
Interfering wave pairs needed to make rest frame coordinates...

Zeros of head-on CW sum gives (x,ct)-grid



CW Standing Wave

Spacetime
x versus ct
CW square grid



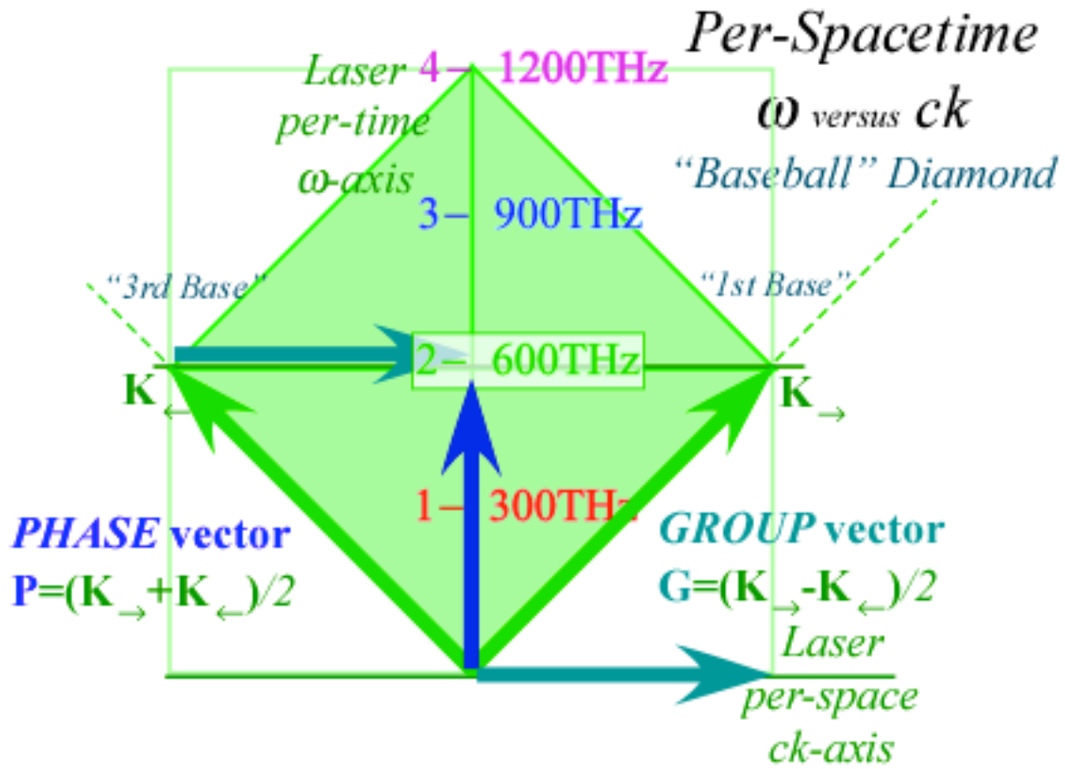
Find zeros by factoring sum:

$$\Psi = \underbrace{e^{ia}}_{(kx-\omega t)} + \underbrace{e^{ib}}_{(-kx-\omega t)}$$

$$= \underbrace{e^{i(a+b)/2}}_{\text{Phase factor:}} \underbrace{\left(e^{i(a-b)/2} + e^{-i(a-b)/2}\right)}_{\text{Group factor:}}$$

$$\exp i\left(\frac{a+b}{2}\right) = e^{-i\omega t} \qquad 2\cos\left(\frac{a-b}{2}\right) = 2\cos(kx)$$

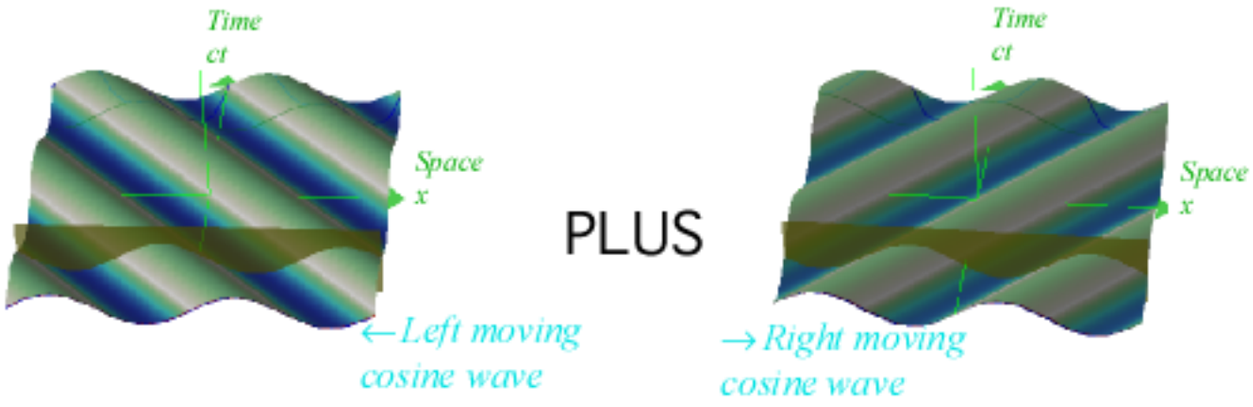
Per-Spacetime
ω versus ck
"Baseball" Diamond



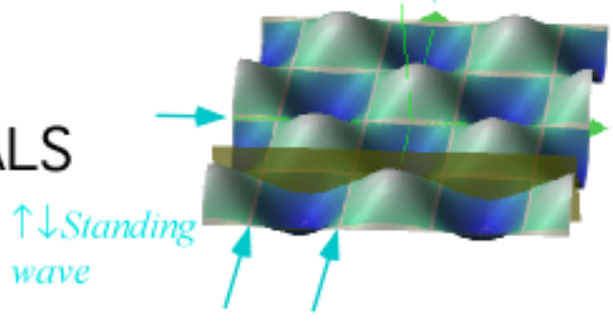
Newton's "Fits" in Optical Interference

Newton complained that light waves have *"fits"* (what we now know as wave *interference* or *resonance*.)
Examples of interference are head-on collision of two *Continuous Waves (2-CW)* or two *Pulse Waves (PW)*

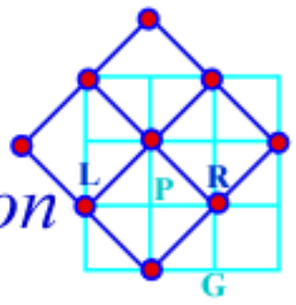
Continuous Wave (CW) Addition



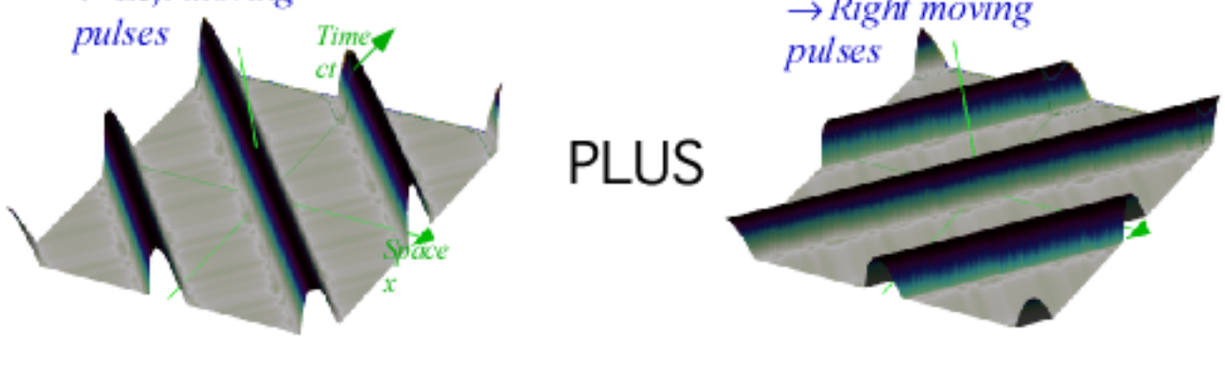
EQUALS



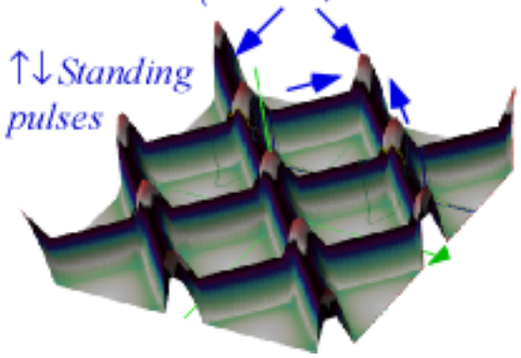
Sharp zeros trace square grid
(Peaks are diffuse)



Pulse Wave (PW) Addition



EQUALS

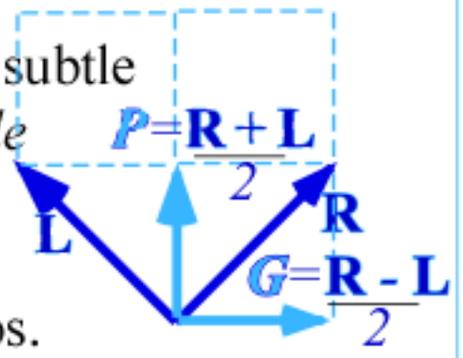


(Zeros are diffuse)
Sharp peaks trace diamond grid

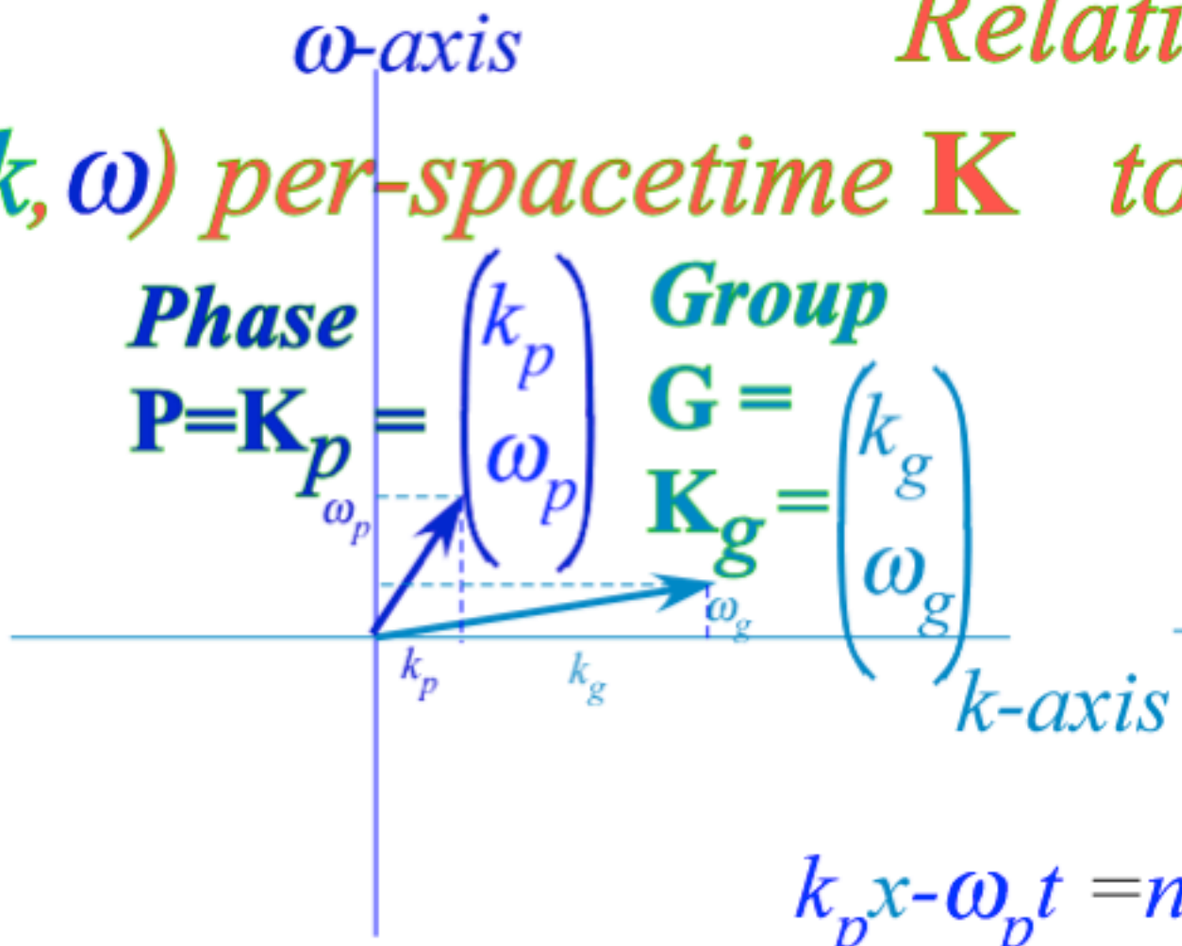
Pulse Wave (PW) sum compared with Continuous Wave (CW) sum

- *PW* waves are OFF (0) or ON (1)
- *PW* sum is Boolean $(0_L,0_R),(0_L,1_R),(1_L,0_R),(1_L,1_R)$.
- *PW* time *peak-diamond* paths are *wysiwye*.
  (What you see is what you expect!)

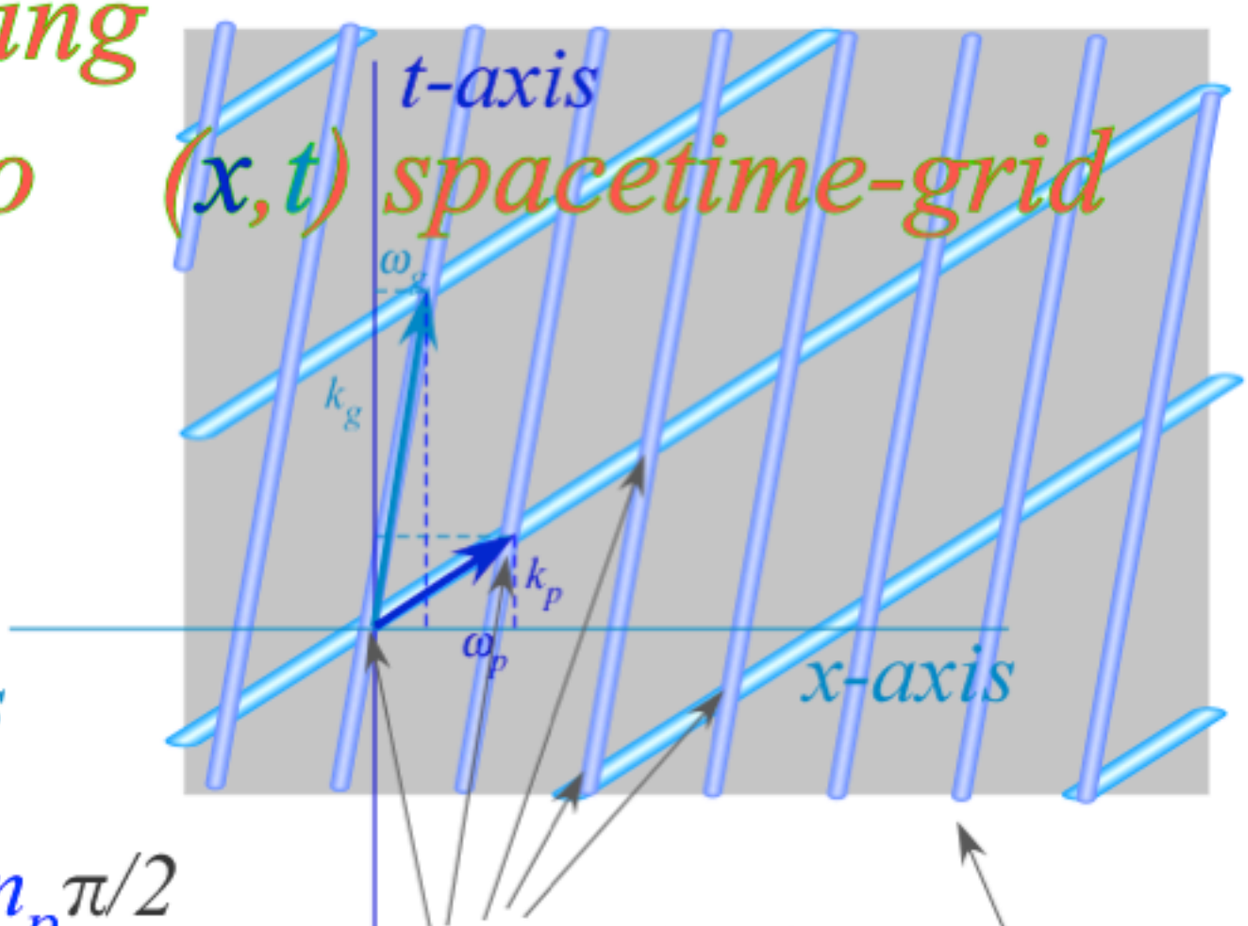
- *CW* waves range continously from -1 to +1
- *CW* sum is more subtle and nuanced *interference*.
- *CW* time *zero-square* paths are subtle results of the *half-sum* ***P****-rule* and the *half-difference* ***G****-rule* of *phase* ***P*** and *group* ***G*** zeros.



Relating (k,ω) per-spacetime **K** to (x,t) spacetime-grid



Phase $\mathbf{P}=\mathbf{K}_p=\begin{pmatrix} k_p \\ \omega_p \end{pmatrix}$ **Group** $\mathbf{G}=\mathbf{K}_g=\begin{pmatrix} k_g \\ \omega_g \end{pmatrix}$



$$k_p x-\omega_p t = n_p \pi/2$$

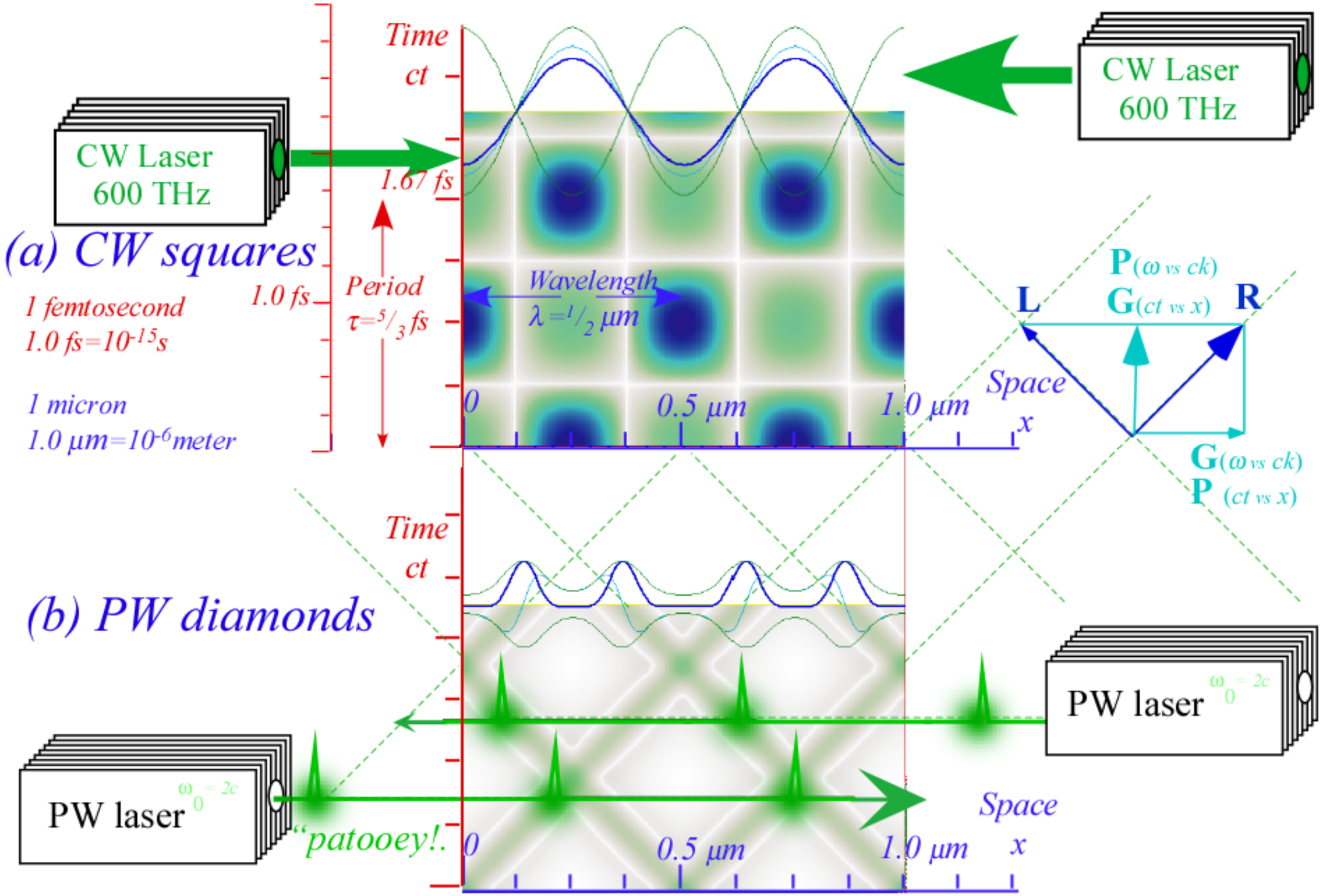
$$k_g x-\omega_g t = n_g \pi/2$$

lattice point equations for:
$n_p=\pm1,\pm2\ldots$ and $n_g=\pm1,\pm2\ldots$

$$\begin{pmatrix} k_p & -\omega_p \\ k_g & -\omega_g \end{pmatrix}\begin{pmatrix} x \\ t \end{pmatrix}=\begin{pmatrix} n_p \\ n_g \end{pmatrix}\pi/2$$

inverted

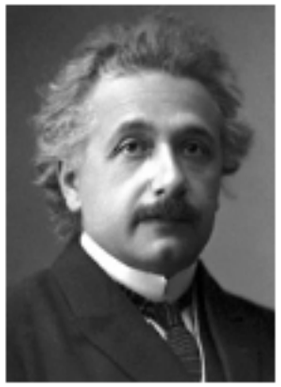
$$\begin{pmatrix} x \\ t \end{pmatrix}=\frac{\begin{pmatrix} -\omega_g & \omega_p \\ -k_g & k_p \end{pmatrix}\begin{pmatrix} n_p \\ n_g \end{pmatrix}}{det\left|\mathbf{K}_g \times \mathbf{K}_p\right|}\pi/2=\frac{-n_p\begin{pmatrix} \omega_g \\ k_g \end{pmatrix}+n_g\begin{pmatrix} \omega_p \\ k_p \end{pmatrix}}{det\left|\mathbf{K}_g \times \mathbf{K}_p\right|_{2/\pi}}$$


CW Laser
600 THz
CW Laser
600 THz
(a) CW squares
1 femtosecond
1.0 fs=10-15s
1 micron
1.0 μm=10-6meter
Time
ct
1.67 fs
1.0 fs
Period
τ=5/3 fs
Wavelength
λ=1/2 μm
0
0.5 μm
1.0 μm
Space
x
P(ω vs ck)
L
G(ct vs x)
R
G(ω vs ck)
P (ct vs x)
(b) PW diamonds
PW laser
ω0 = 2c
PW laser
ω0 = 2c
"patooey!.
Time
ct
0
0.5 μm
1.0 μm
Space
x

○ *Applying Occam's razor to relativity axioms*

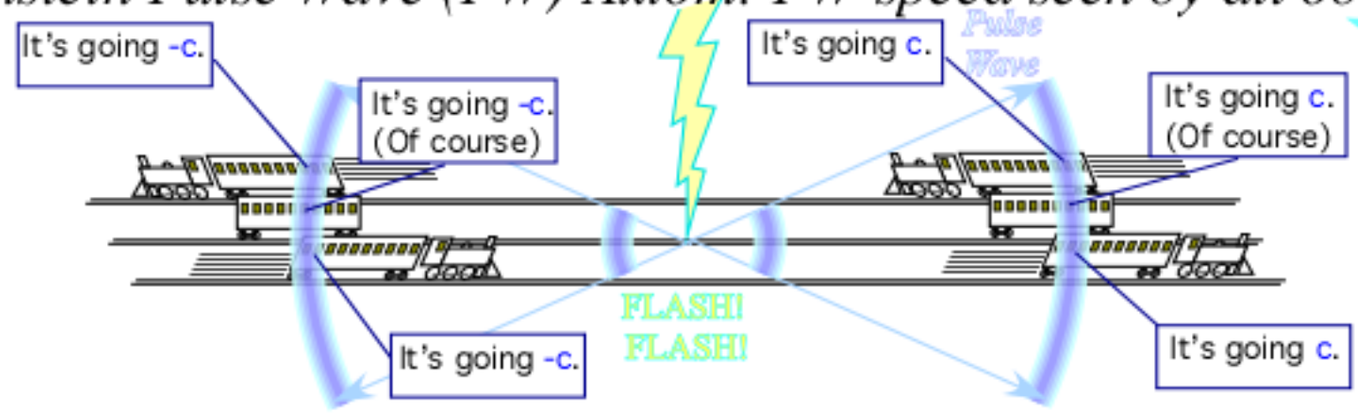
Einstein Pulse-Wave (PW) axiom vs. *Evenson Coherent-Wave (CW) axiom*

Albert Einstein



1879-1955

Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom is a "show-stopper"

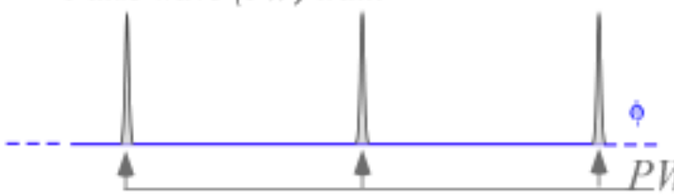
William of Ockham

1285-1349

Using Occam's Razor

(and Evenson's lasers)

Pulse wave (PW) train

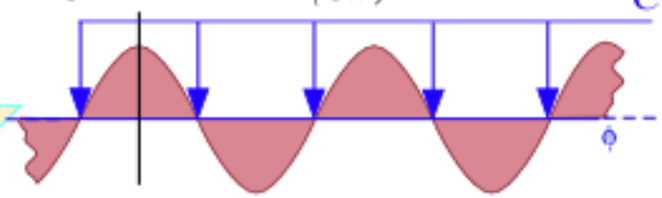


$A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \ldots$

Complicated

PW peaks precisely locate places where wave is.

Continuous wave (CW) train



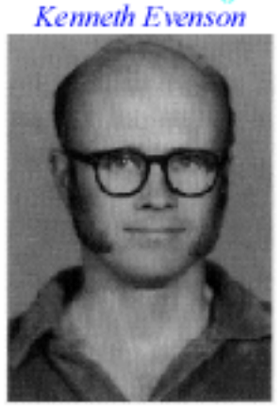
CW zeros precisely locate places where wave is not.

$A \cos \omega t$

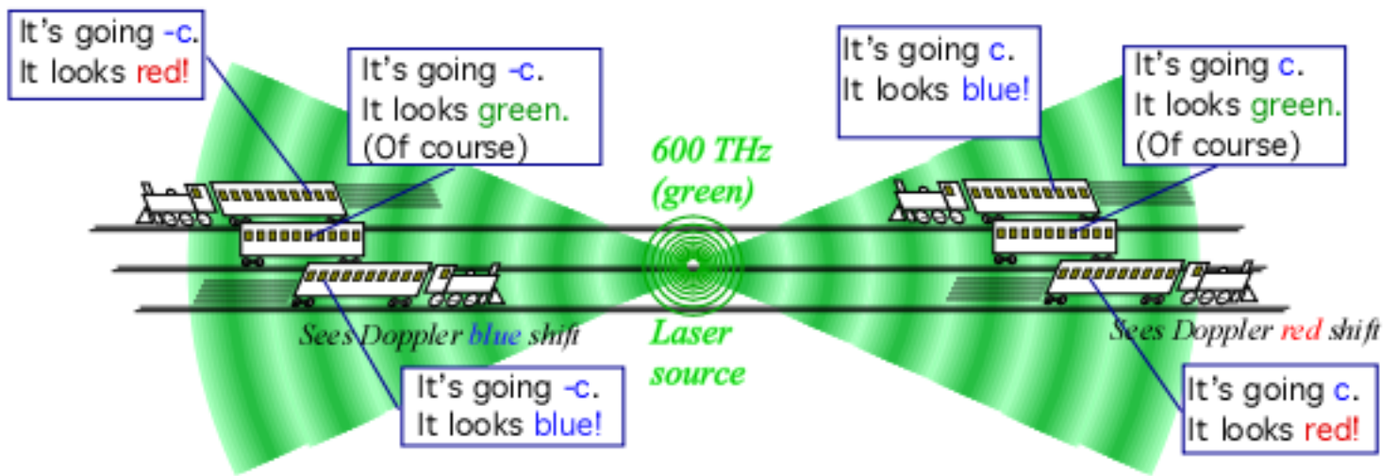
Simpler

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Kenneth Evenson



1929-2002
c=299,792,458 m/s

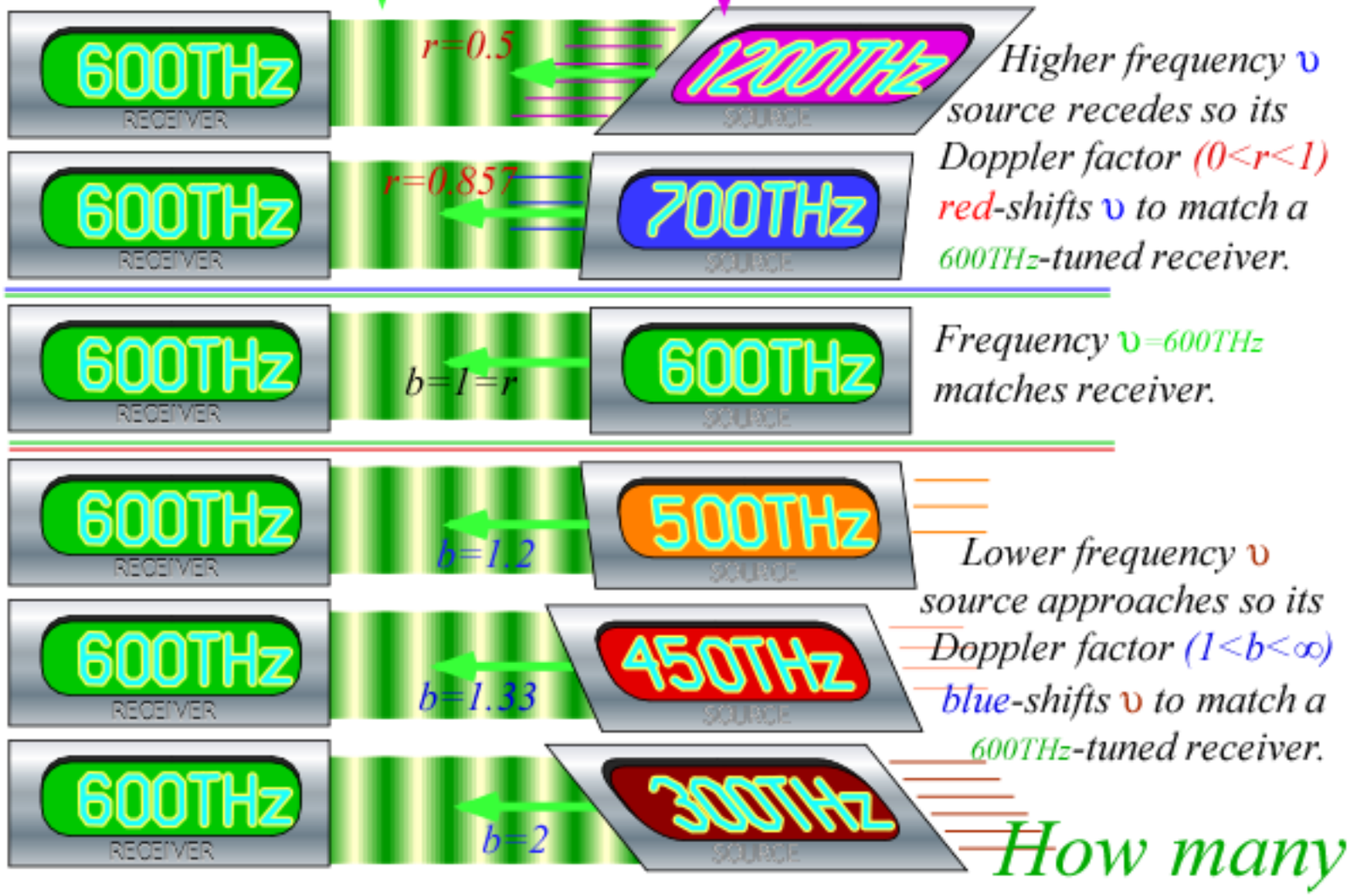


More self-evident "must-be" axiom

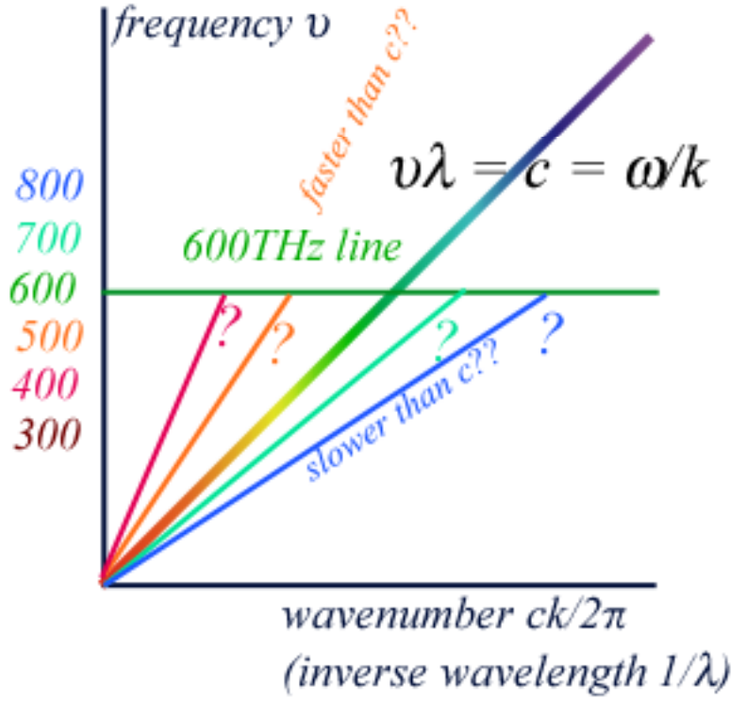
CW Axiom ("All colors go c.") based on Doppler effects

Showing that Green is Green is Green...(and all the same speed)...

Any color (like 600THz green) may be made by any other color source Doppler shifted by some speed u (less than c)



How many ways can you make 600THz green?



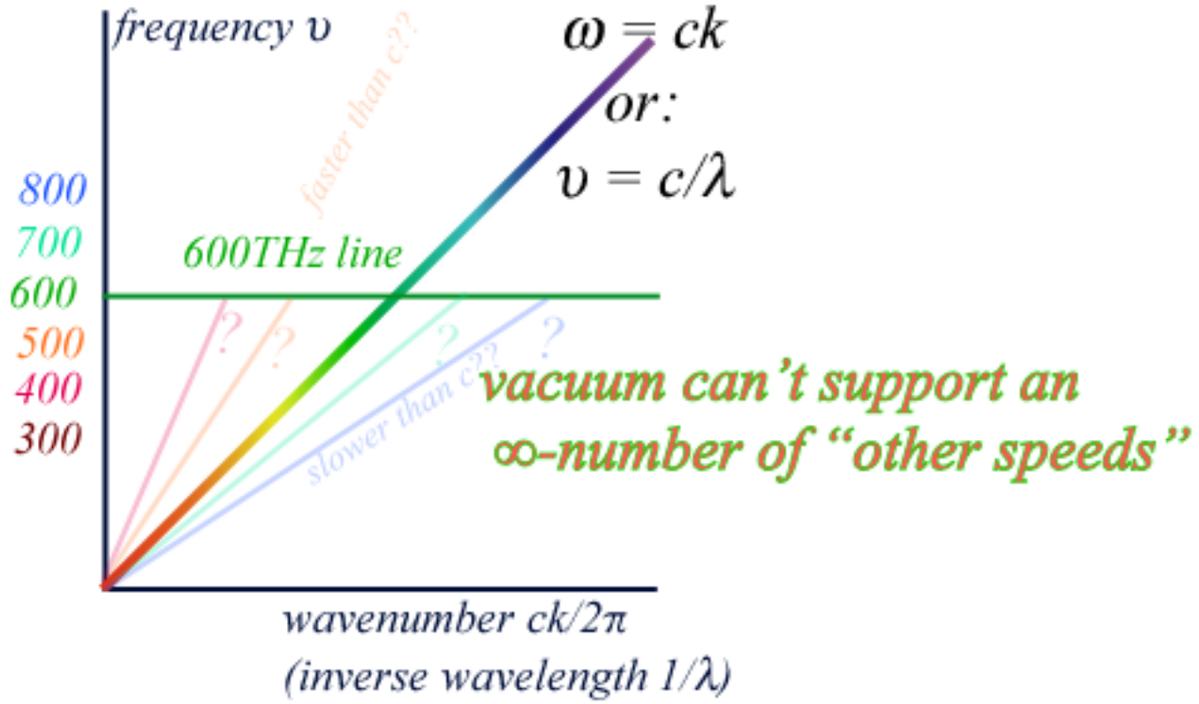
How many kinds of green exist? (It's either 1 or ∞.)

Evenson CW Axiom ("All colors go c.") is only reasonable conclusion:

Linear dispersion: $\omega = ck$

Linear dispersion means NO dispersion

Einstein PW is corollary of Evenson CW

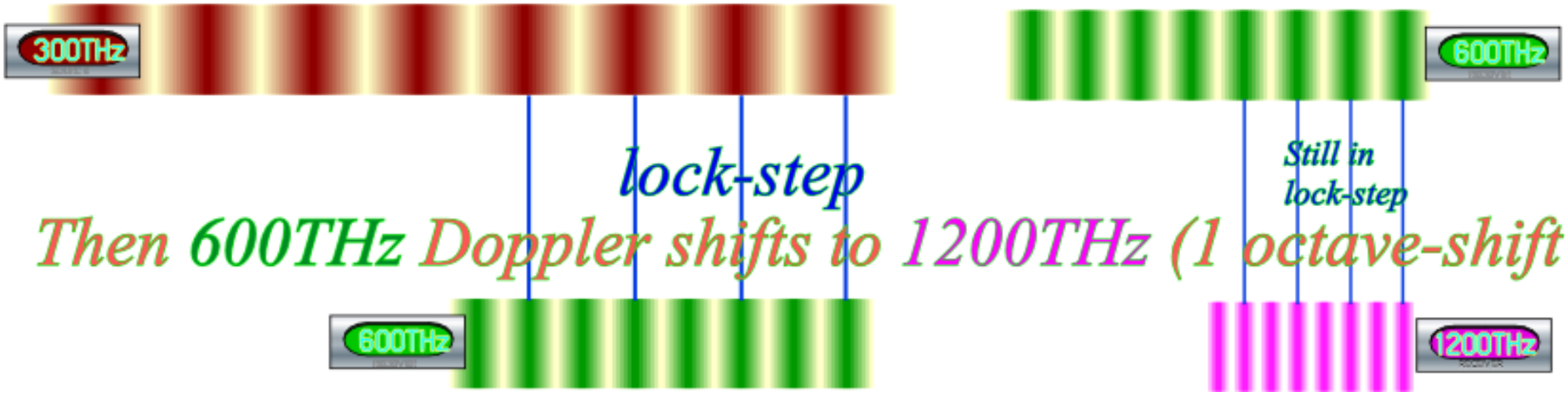


What if blue were to travel 0.001% slower than red from a galaxy 9 billion light years away? *(..and show up 10^5 years late)*

That would mean Good-Bye Hubble Astronomy!

If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)



Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)

Doppler shifts maintain frequency ratios (not differences)

1-D Doppler shifts {red=$e^{-\rho}$ ...blue=$e^{+\rho}$} *form a Lie Group*

3-D Doppler shifts are hypercomplex elements of Lorentz Group

Frequency blue shift b when Source-Receiver interval is >>CLOSING<<

$$\frac{\upsilon_{IN}}{\upsilon_{OUT}} = \frac{\upsilon_{Receiver}}{\upsilon_{Source}} = b = e^{+|\rho|} > 1$$

Frequency red shift r when Source-Receiver interval is <<OPENING>>

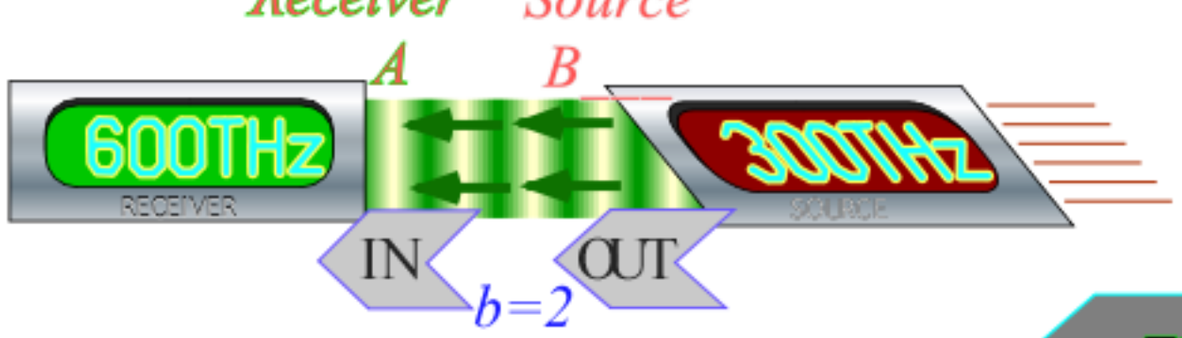
$$\frac{\upsilon_{Receiver}}{\upsilon_{Source}} = r = e^{-|\rho|} < 1$$

*Defining **Rapidity** ρ as logarithm of Doppler*

$$\rho = ln(b \text{ or } r)$$

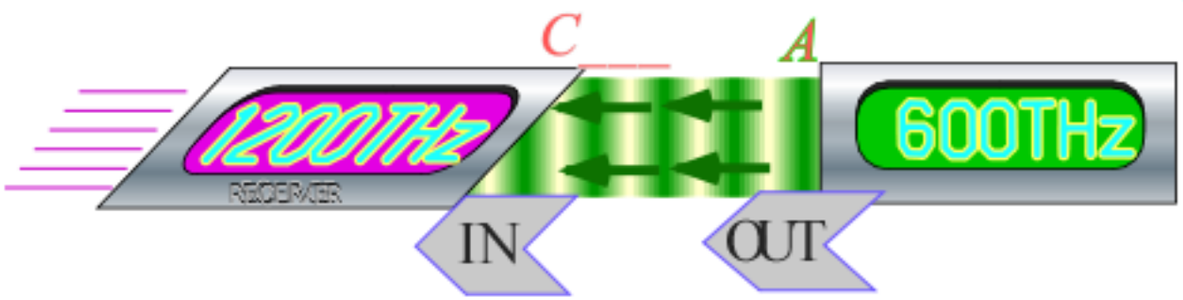
Examples:

Receiver *Source*



$$\rho = ln(2) = 0.69$$

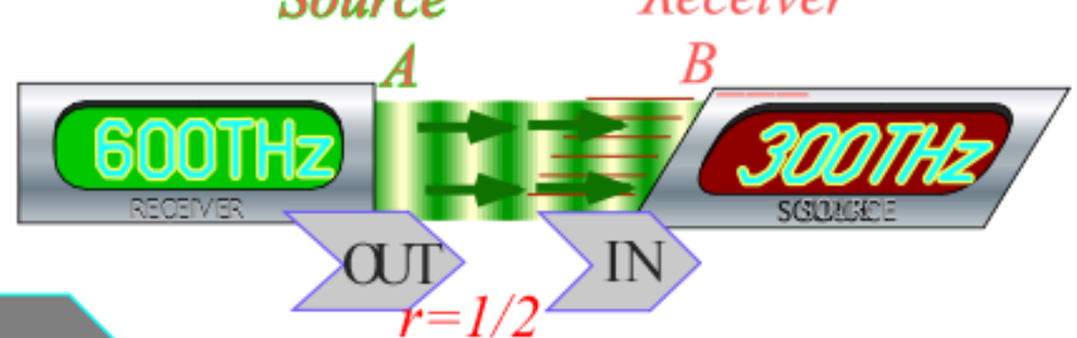
Receiver *Source*



Time Reversal

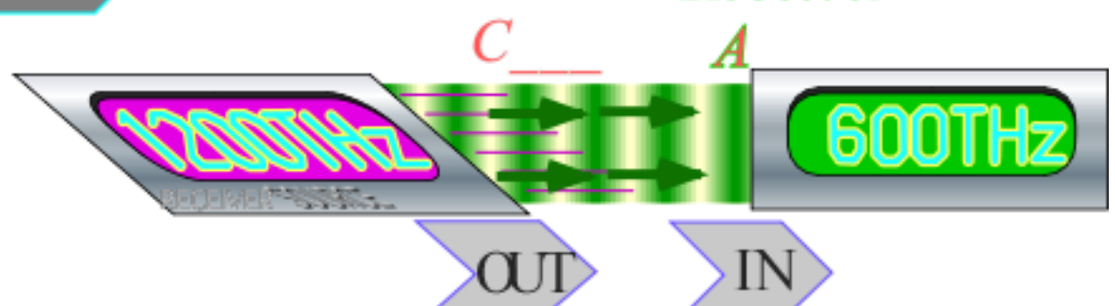
Examples:

Source *Receiver*

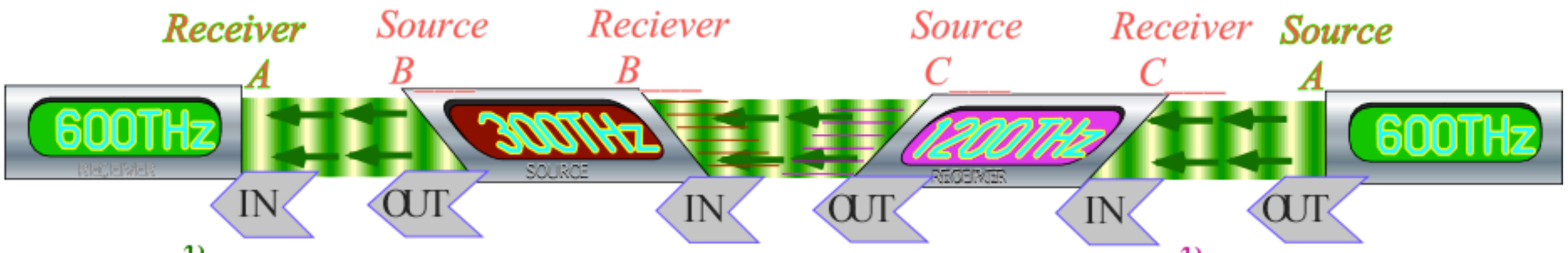


$$\rho = ln(1/2) = -0.69$$

Source *Receiver*



Each Doppler shift $\frac{\upsilon_A}{\upsilon_B}$ *maps to a Lorentz transformation* T_{AB}



$\frac{\upsilon_A}{\upsilon_B} = b_{AB} = e^{\rho_{AB}} = 2$

$\rho_{AB} = \ln(2) = 0.69$

$\frac{\upsilon_B}{\upsilon_C} = b_{BC} = e^{\rho_{BC}} = 1/4$

$\rho_{BC} = \ln(1/4) = -1.38$

$\frac{\upsilon_C}{\upsilon_A} = b_{CA} = e^{\rho_{CA}} = 2$

$\rho_{BC} = \ln(2) = 0.69$

Group product is represented: (*by IN-OUT "nematodes"*)

$$T_{AB} \cdot T_{BC} = T_{CA}$$

$$\frac{\upsilon_A}{\upsilon_B}\,\frac{\upsilon_B}{\upsilon_C} = \frac{\upsilon_A}{\upsilon_C}$$

$$e^{\rho_{AB}}\, e^{\rho_{BC}} = e^{\rho_{AC}} = e^{(\rho_{AB}+\rho_{BC})}$$

...and rapidity ρ_{AB} *is a Galilean (arithmetic) parameter*

To be shown: $\rho_{AB} = \operatorname{atanh}(u_{AB}/c)$ approaches (u_{AB}/c) for: $\rho_{AB} << 1$

Inverse to Lorentz transformation T_{AB} *is* T_{BA}

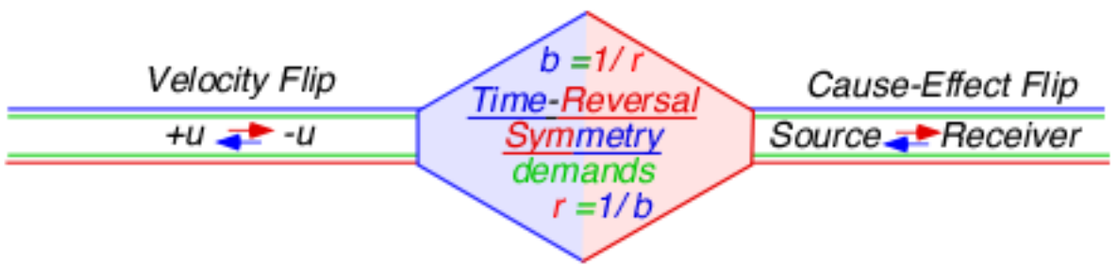
..just as the arithmetic inverse of $\frac{v_A}{v_B}$ *is* $\frac{v_B}{v_A}$

... of $e^{\rho_{AB}}$ *is* $e^{\rho_{BA}} = e^{-\rho_{AB}}$

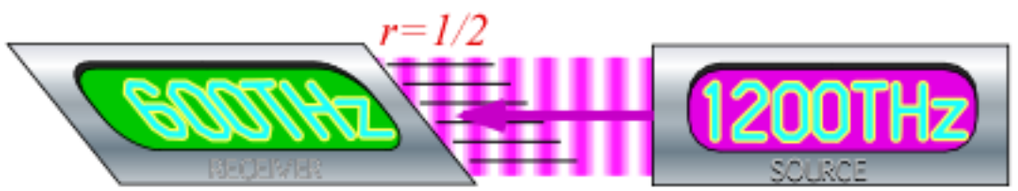
... of ρ_{AB} *is* $\rho_{BA} = -\rho_{AB}$

Detailed time reversal symmetry implies r=1/b.

Approaching source (600THz green)



Receding receiver sees Doppler red-shift of 1200THz source to 600THz (600THz)=r·(1200THz) with r=1/2



○ *Spectral development of relativistic mass mechanics*

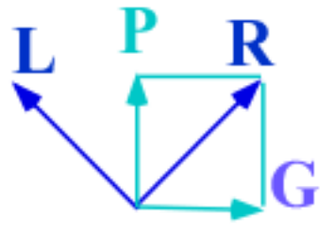
Doppler shifts and hyper-complex "phase-based" matter-wave mechanics

Deriving Spacetime and per-spacetime coordinate geometry by:

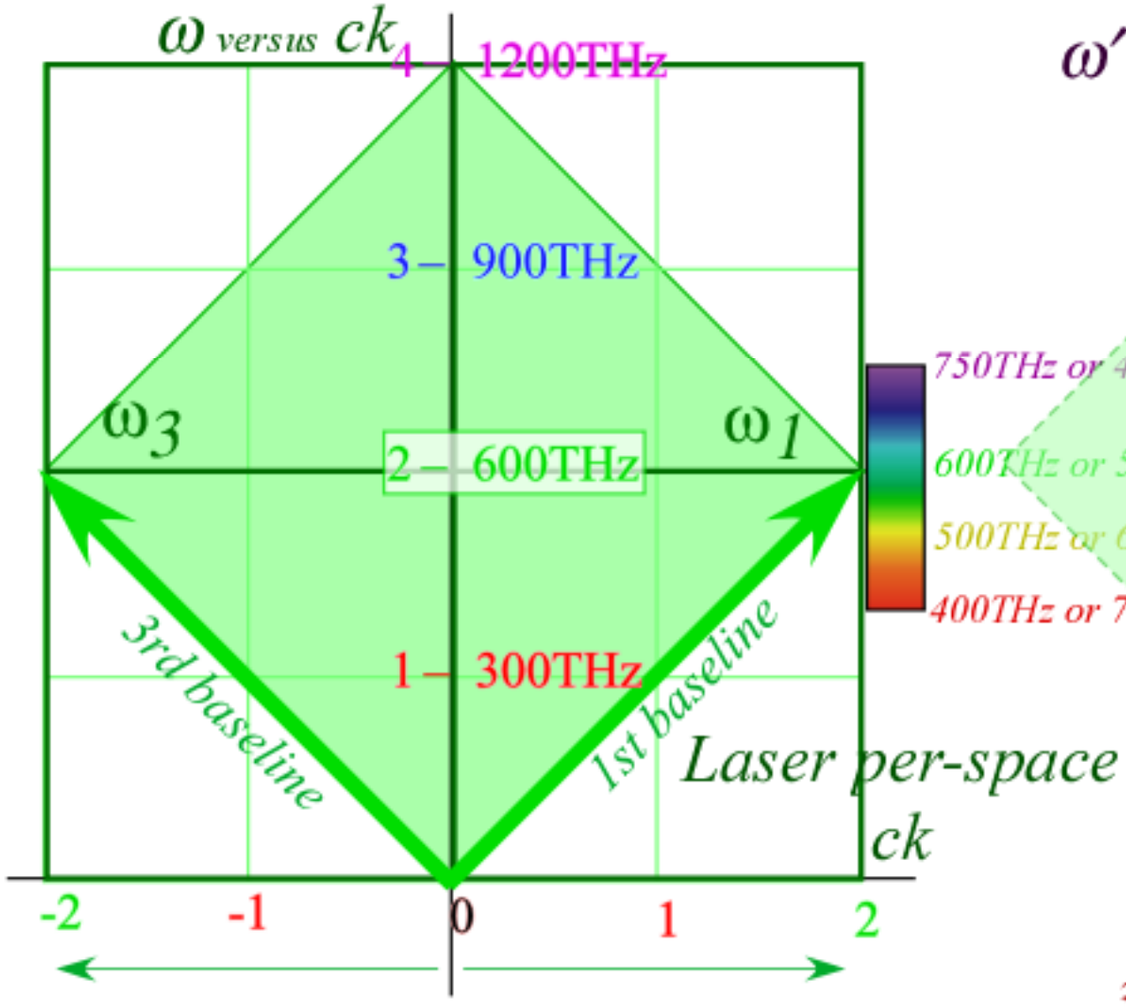
(1) Evenson **CW axiom** *"All colors go c"* **keeps** K_A **and** K_B ***on their baselines.***

(2) **Time-Reversal axiom:** $r=1/b$

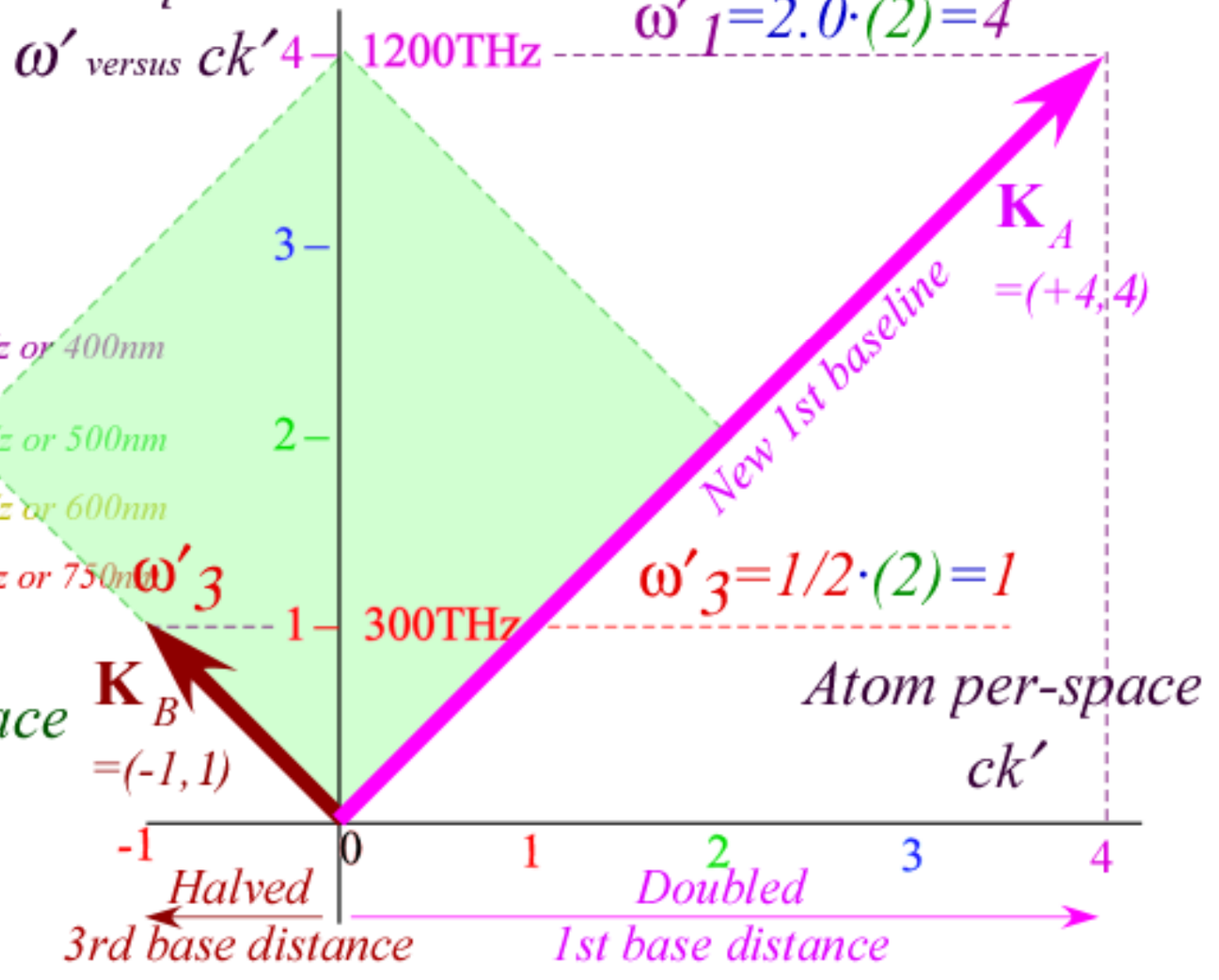
(3) **Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R–L)/2**



LaserPer-Spacetime



AtomPer-Spacetime

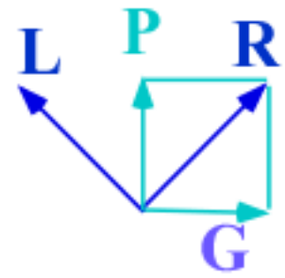


Deriving Spacetime and per-spacetime coordinate geometry by:

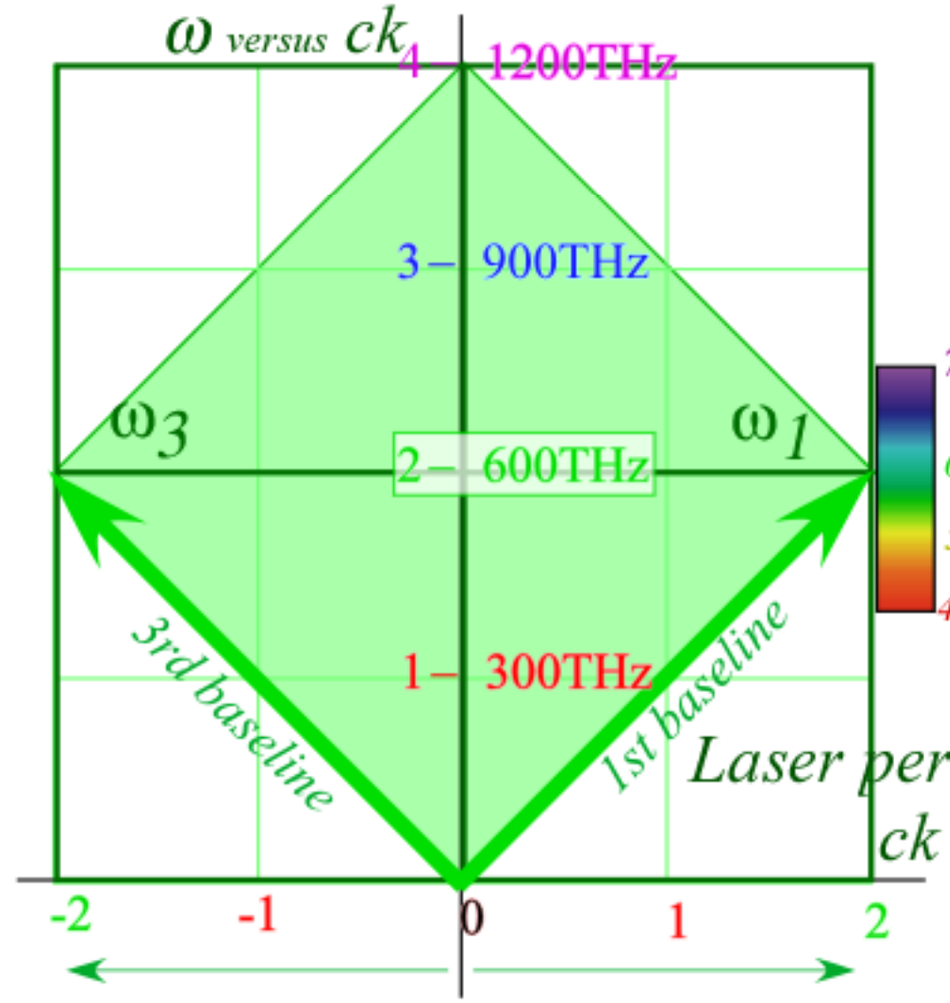
(1) Evenson CW axiom *"All colors go c"* keeps K_A and K_B *on their baselines.*

(2) Time-Reversal axiom: $r=1/b$

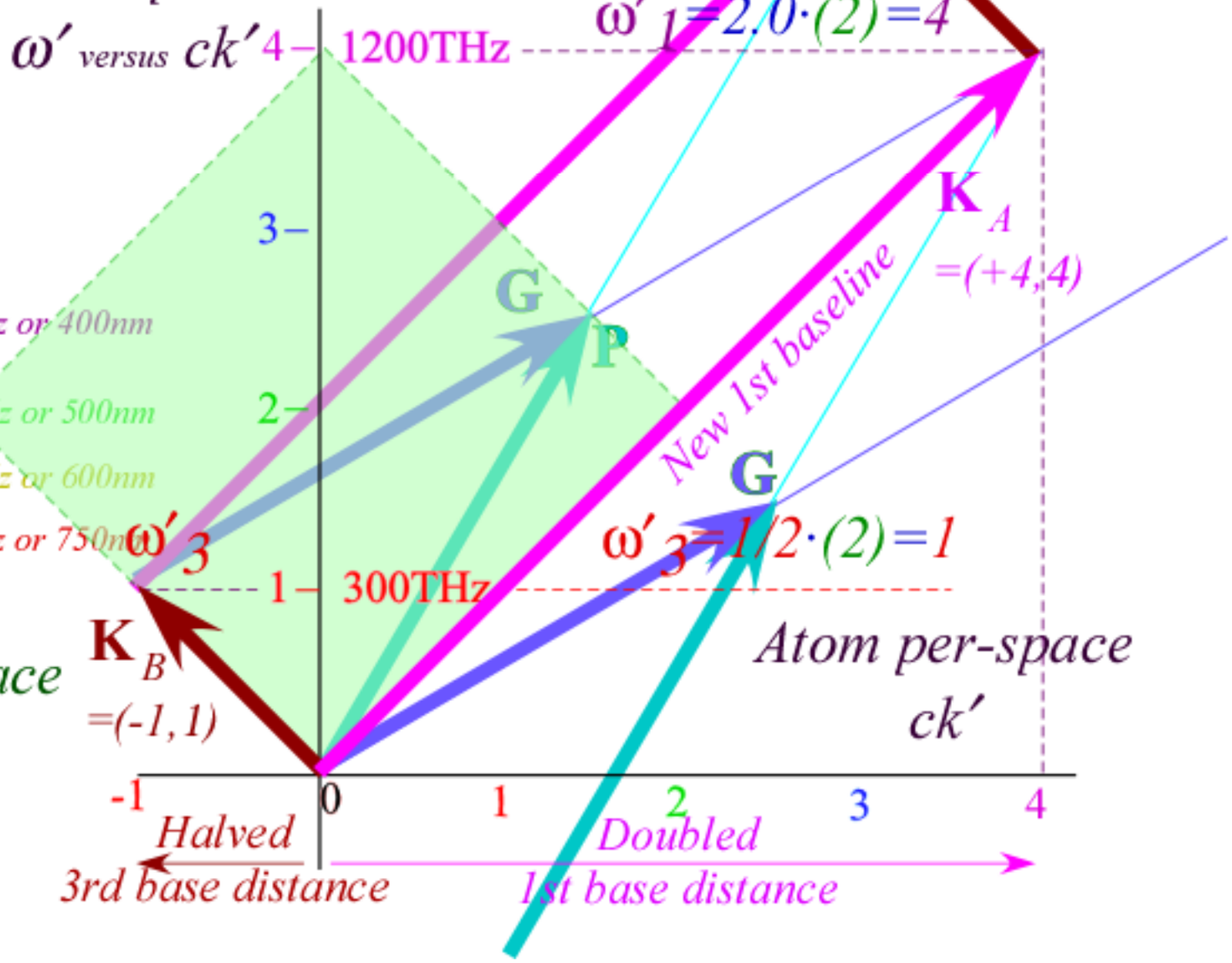
(3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$



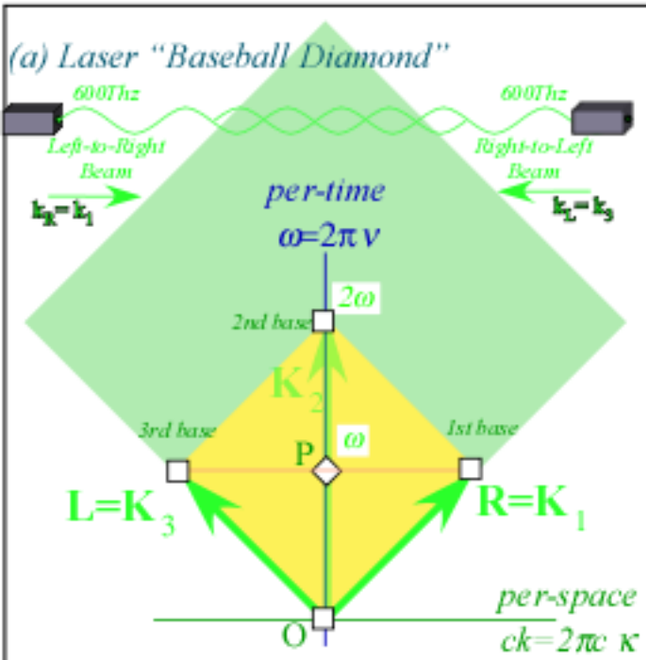
LaserPer-Spacetime



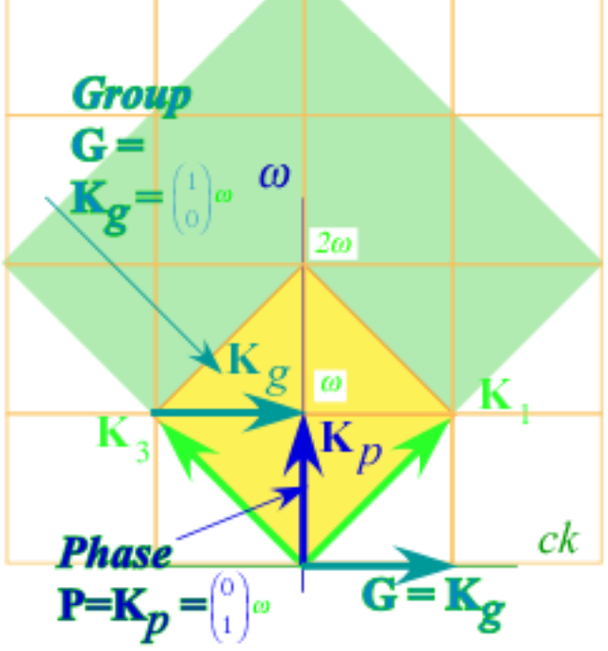
AtomPer-Spacetime



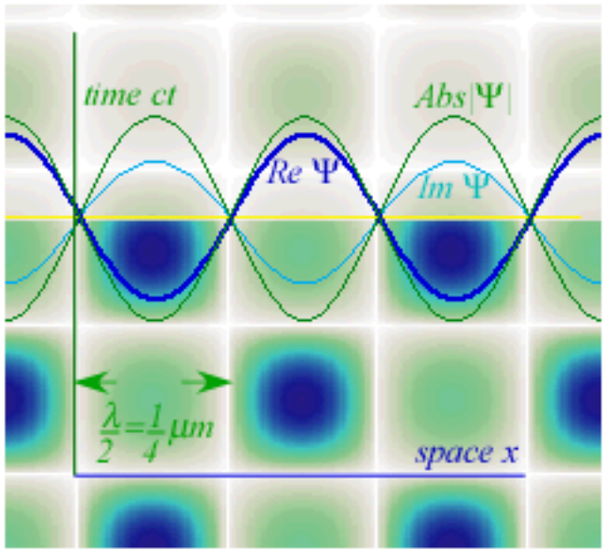
(a) Laser "Baseball Diamond"



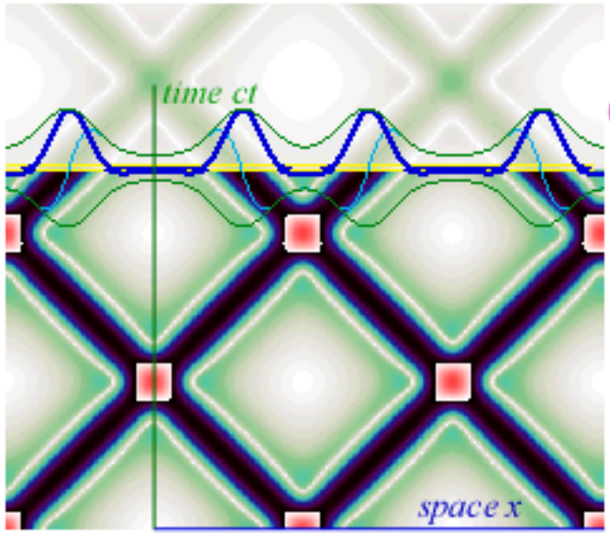
(b) Laser group and phase wavevectors
(Per-space-time Cartesian lattice)



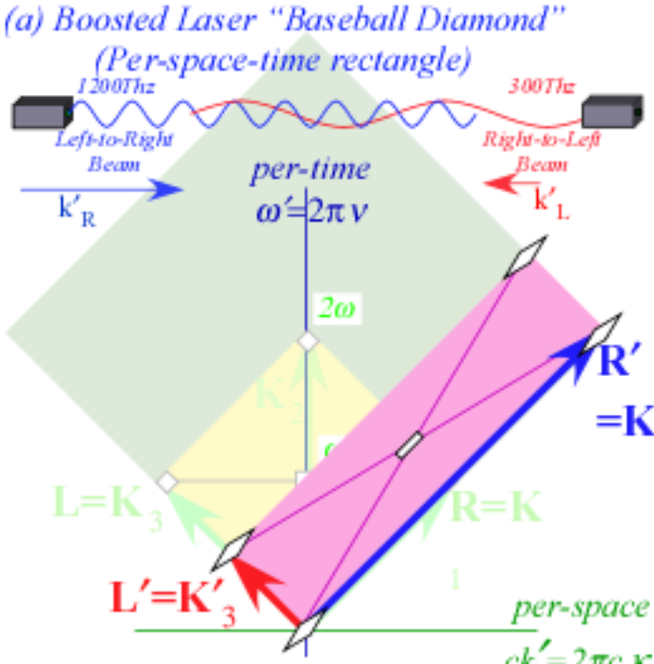
(c) Laser Coherent Wave (CW) paths
(Space-time Cartesian grid)



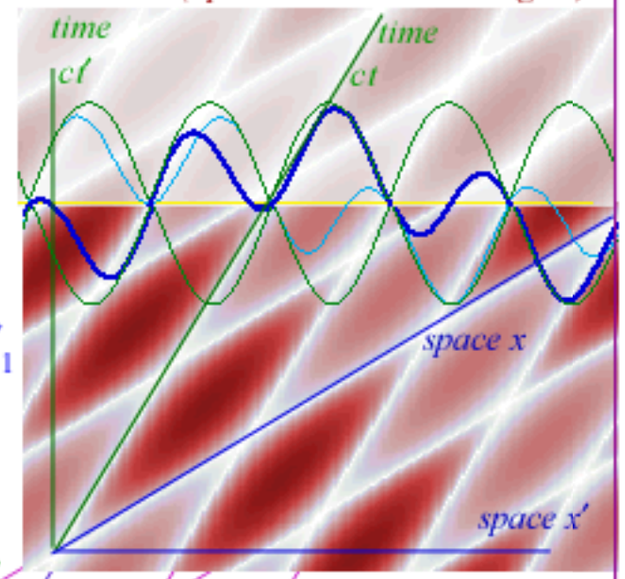
(d) Laser Pulse Wave (PW) Paths
(Space-time Diamond grid)



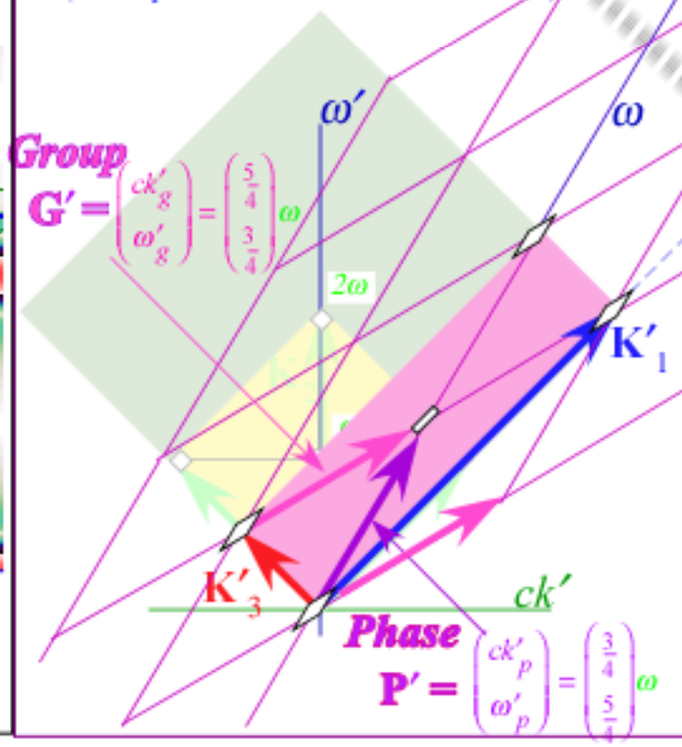
(a) Boosted Laser "Baseball Diamond"
(Per-space-time rectangle)



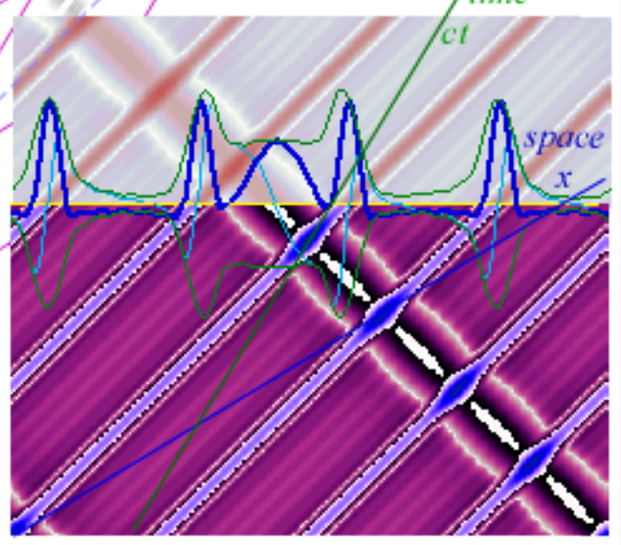
(c) (u=3c/5)-Boosted CW paths
(Space-time Minkowski grid)



(b) Boosted group and phase wavevectors
(Per-space-time Minkowski lattice)

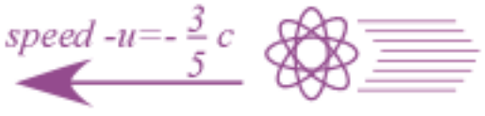


(d) Boosted PW Paths
(Rectangular grid)

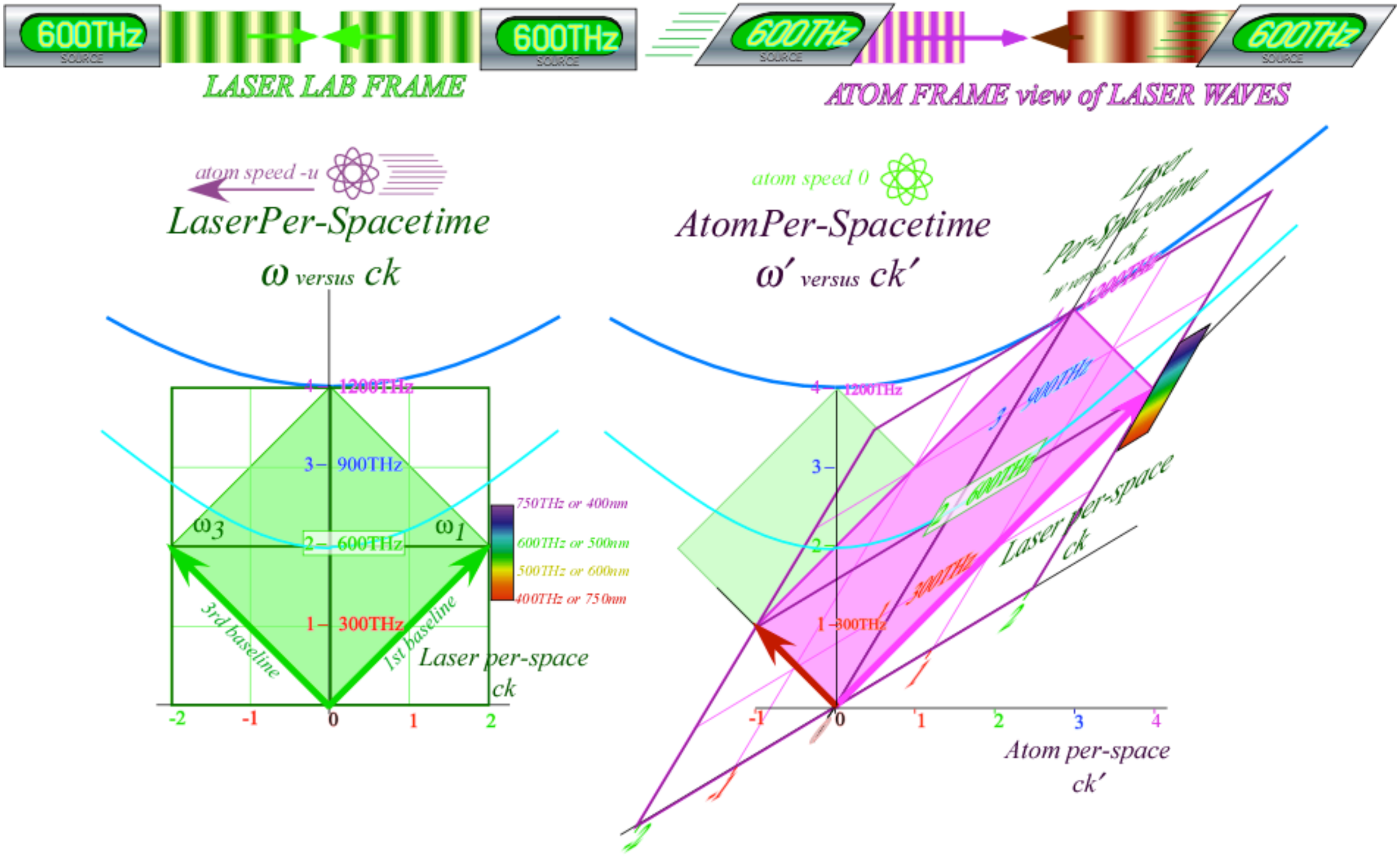


Laser lab views

atom speed $-u=-\frac{3}{5}c$



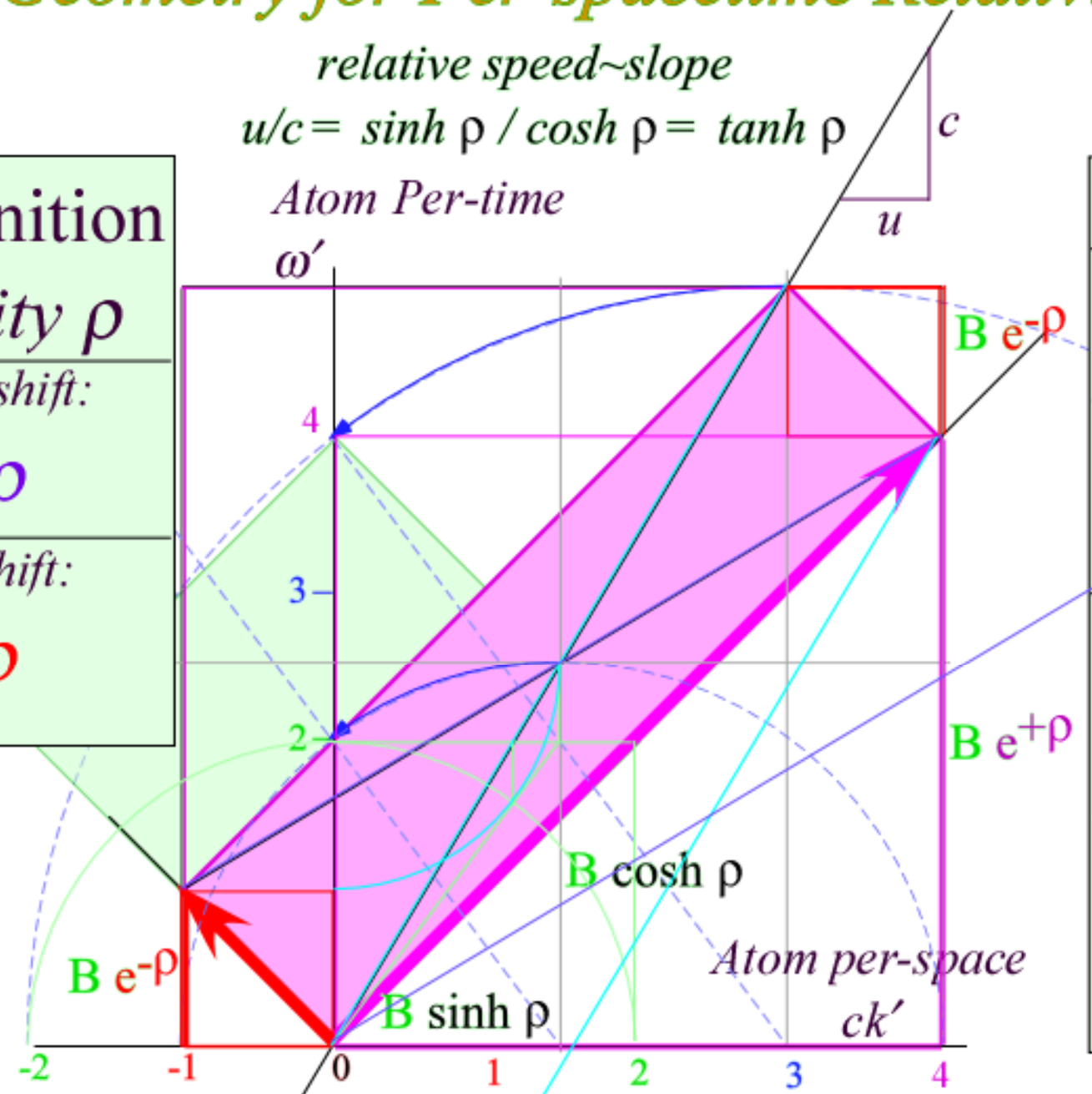
Atom views *(sees lab going* $+u=\frac{3}{5}c$*)*


600THz
SOURCE
600THz
SOURCE
LASER LAB FRAME
600THz
SOURCE
600THz
SOURCE
ATOM FRAME view of LASER WAVES
atom speed -u
LaserPer-Spacetime
ω versus ck
4 – 1200THz
3 – 900THz
2 – 600THz
1 – 300THz
ω3
ω1
3rd baseline
1st baseline
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
Laser per-space
ck
-2
-1
0
1
2
atom speed 0
AtomPer-Spacetime
ω′ versus ck′
Laser Per-Spacetime ω′ versus ck′
4 – 1200THz
3 –
2 –
1 – 300THz
900THz
600THz
300THz
Laser per-space ck
-1
0
1
2
3
4
Atom per-space
ck′

Euclidian Geometry for Per-spacetime Relativity

relative speed~slope

$u/c = \sinh\rho / \cosh\rho = \tanh\rho$



Key Definition of *Rapidity* ρ

Doppler blue shift:

$Bb = Be^{+\rho}$

Doppler red shift:

$Br = Be^{-\rho}$

Key Results:

ω *vs.* ck

"winks" vs. "kinks"

$\omega = B\cosh\rho$

$ck = B\sinh\rho$

group velocity:

$$\frac{\omega}{ck} = \frac{u}{c} = \tanh\rho$$

phase velocity:

$$\frac{ck}{\omega} = \frac{c}{u} = \coth\rho$$

$B\sinh\rho = (B e^{+\rho} - B e^{-\rho})/2$

$B\cosh\rho = (B e^{+\rho} + B e^{-\rho})/2$

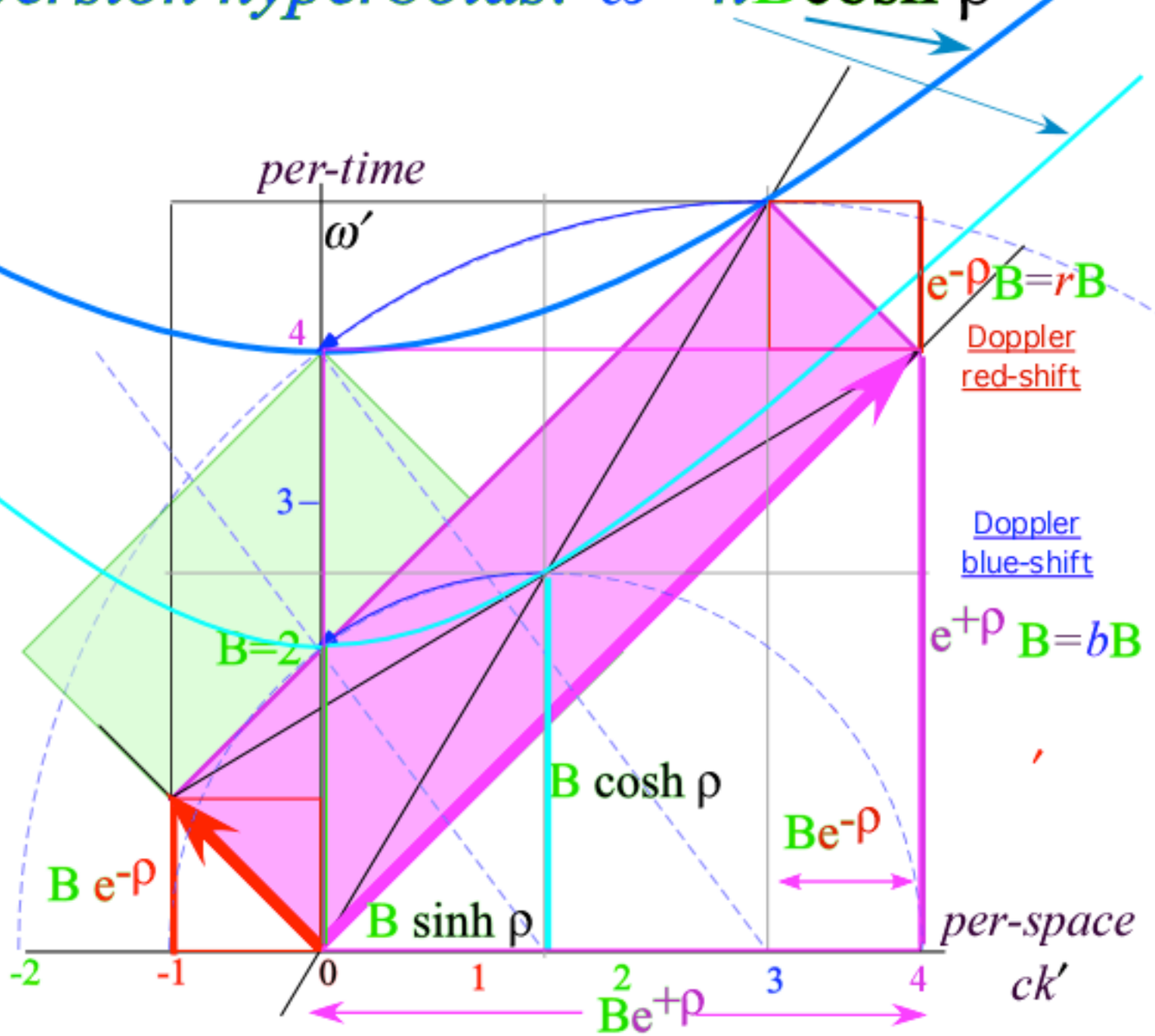
$$\sinh\rho = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}}$$

Key Quantities

Lorentz-Einstein factors

$$\cosh\rho = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas: $\omega = nB\cosh\rho$



Lab frame area...

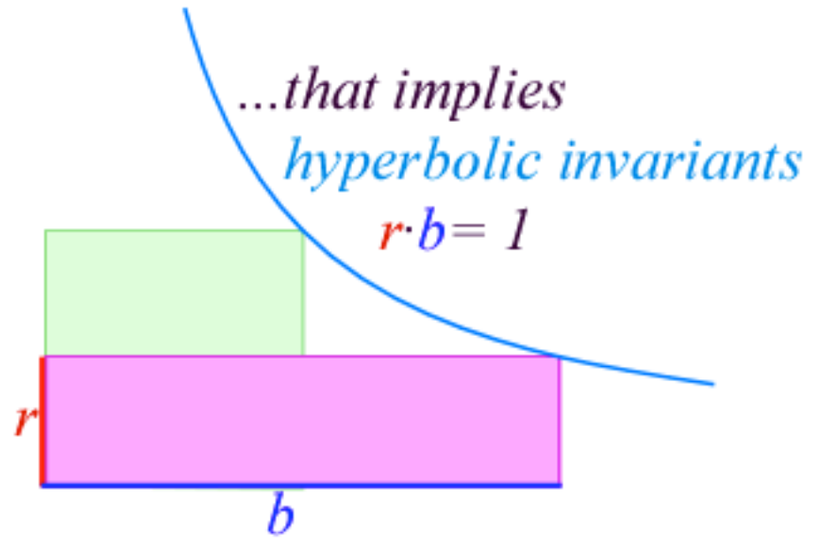
equals

Atom frame area...

r

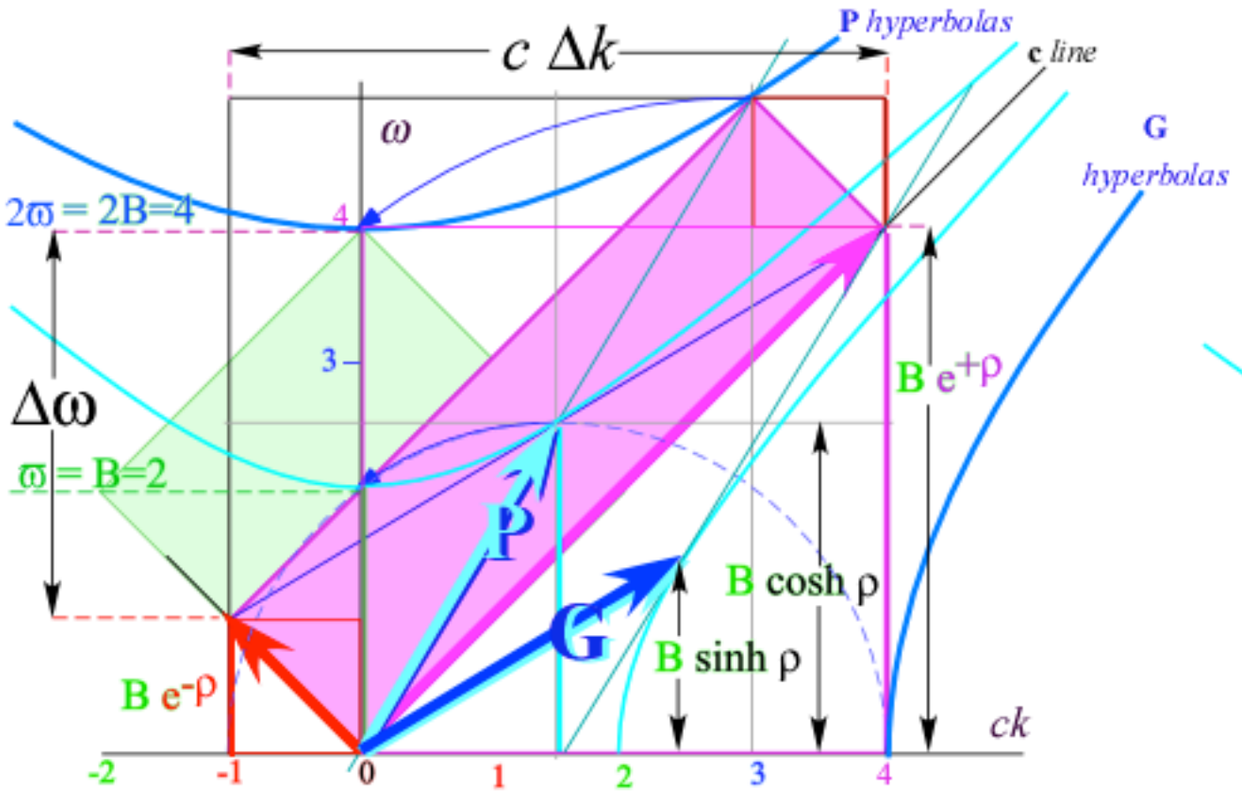
b

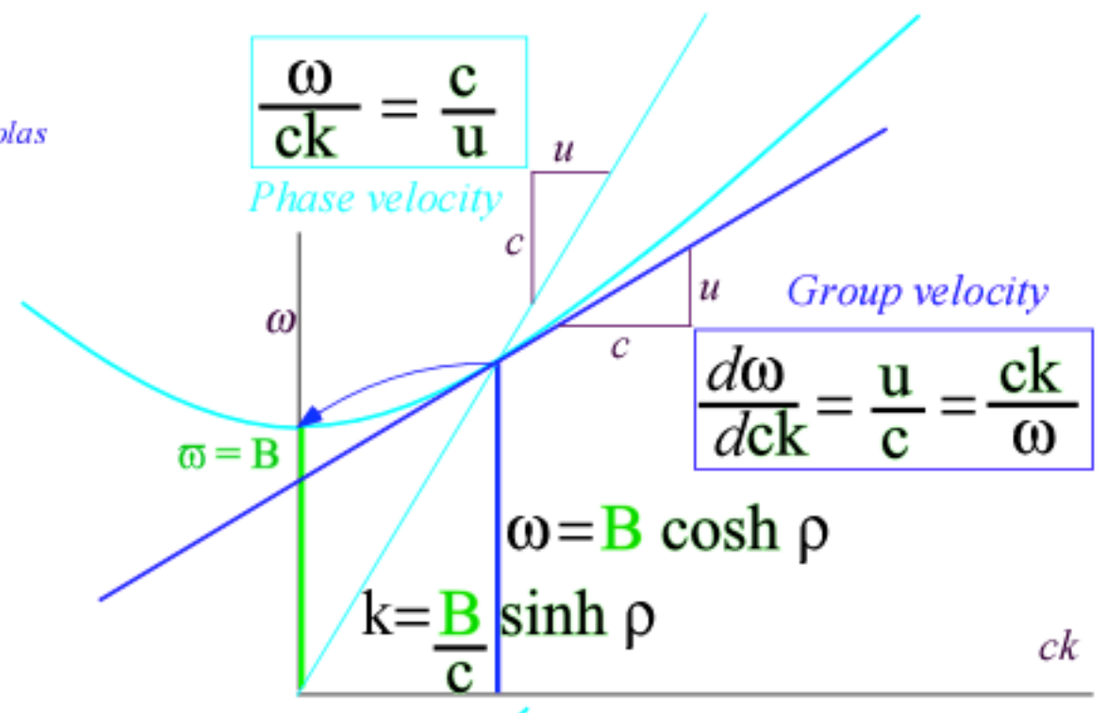
by time-reversal axiom: $r = 1/b$



$$B\sinh\rho = (B e^{+\rho} - B e^{-\rho})/2 \qquad B\cosh\rho = (B e^{+\rho} + B e^{-\rho})/2$$

Group velocity u and phase velocity c^2/u are hyperbolic tangent slopes



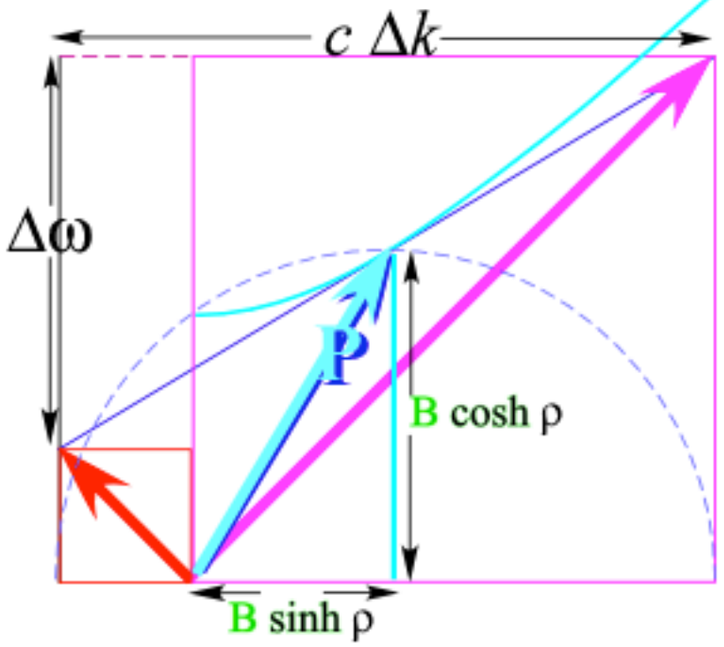


Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with LARGE Δk
(not infinitesimal)

Relativistic
group wave
speed u=c tanh ρ



Newtonian speed u~cρ

Low speed approximation

Rapidity ρ approaches u/c

CW Axioms ("All colors go c." and "r=1/b") imply hyperbolic dispersion then mechanics of matter

These follow from CW axioms

$$\omega = B\cosh\rho \cong B + \frac{1}{2}\frac{B}{c^2}u^2 \qquad k = \frac{B}{c}\sinh\rho \cong \frac{B}{c^2}u$$

$$E = \text{constant} + \frac{1}{2}Mu^2 \qquad p = Mu$$

(Newton's energy) *(Galileo's momentum)*

So *2-CW*-light *frequency* ω is like *energy* E while *k-number* is like *momentum* p, implies *Planck's* $E=s\cdot\omega$ scaling with factors: $s=\hbar=s$ *equal* to *DeBroglie's* $p=s\cdot k$.

$$E = s\omega = sB\cosh\rho \cong sB + \frac{1}{2}\frac{sB}{c^2}u^2 \qquad p = sk = \frac{sB}{c}\sinh\rho \cong \frac{sB}{c^2}u$$

Both relations imply: $M=\frac{sB}{c^2}$ giving a (famous) *rest energy constant.* : $sB=Mc^2$

This then gives the famous *Einstein* *energy* E and also the *Einstein momentum* p

$$E = s\omega = Mc^2\cosh\rho \cong Mc^2 + \frac{1}{2}Mu^2 \qquad p = sk = Mc\sinh\rho \cong Mu$$

$$= \frac{Mc^2}{\sqrt{1-u^2/c^2}} \qquad = \frac{Mu}{\sqrt{1-u^2/c^2}}$$

Rest energy(u=0): ħB=Mc²

Rest momentum(u=0):p=0

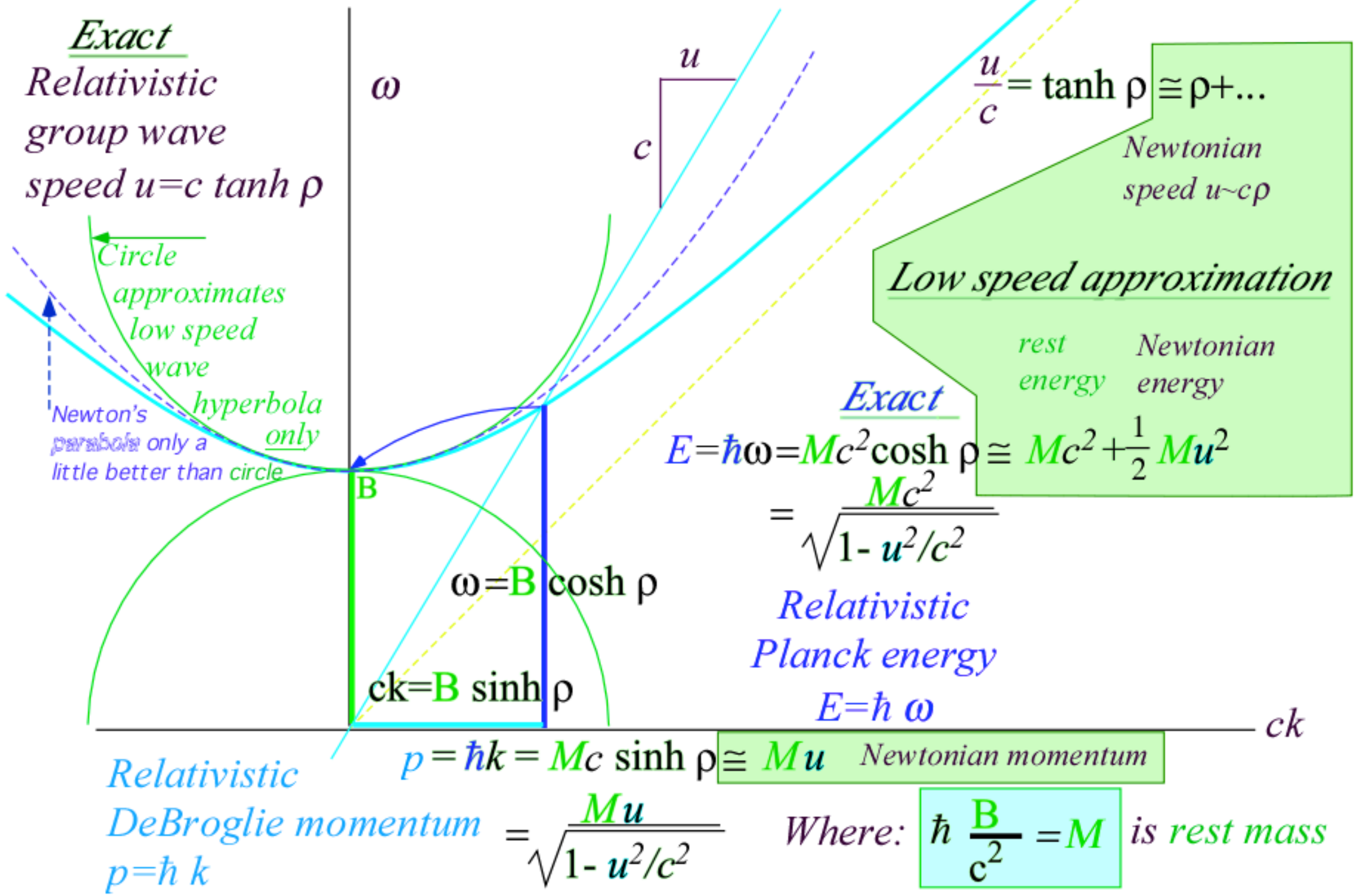
Scale factors determined by experiment

Planck's constant

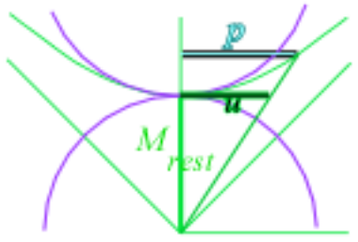
$s=\hbar=1.054572\cdot10^{-34}$ *Joule·s*

$h=6.626069\cdot10^{-34}$ *J·s* $=2\pi\hbar$

Summary of geometry ω-*vs*-*c*k or *E*-*vs*-*cp* relations with velocity *u* or rapidity ρ



What's the Matter With Light? *Three definitions of optical mass*



1. *Rest mass* $M_N=h\upsilon_N/c^2$ based on Planck's law $E=h\upsilon_N=Nh\upsilon_1$

Rest mass: $M_{rest}=E/c^2=h\upsilon_N/c^2$ *(Is invariant)*

2. *Momentum mass* is defined by Galileo's old formula $p=Mu$ with newer forms for momentum $p=M_{rest}u\cdot\cosh\rho=M_{rest}u\cdot/(1-u^2/c^2)^{1/2}$ and group velocity $u=d\omega/dk$. It is the ratio p/u of *momentum* to *velocity*.

Momentum mass: $M_{momentum}=p/u=M_{rest}\cosh\rho=M_{rest}/(1-u^2/c^2)^{1/2}$ *(Not invariant)*

3. *Effective mass* is defined by Newton's old formula $F=Ma$ with newer forms for $F=dp/dt=\hbar dk/dt$ *and* $a=du/dt=$ to give $F/a=(\hbar dk/dt)(dt/du)=\hbar dk/du=\hbar/(du/dk)$. It is the ratio F/a of *change of momentum* to the *change of velocity*,

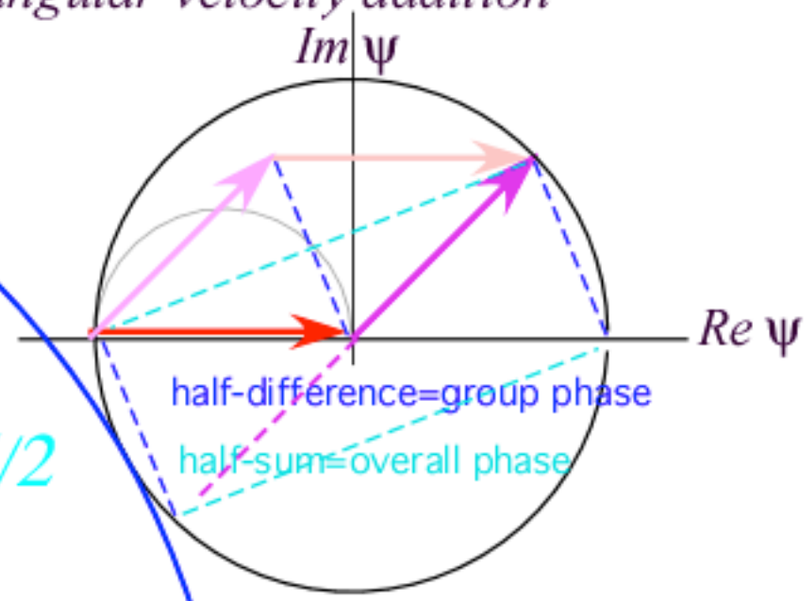
Effective mass: $M_{effective}=\hbar/(du/dk)=\hbar/(d^2\omega/dk^2)=M_{rest}\cosh^3\rho=M_{rest}/(1-u^2/c^2)^{3/2}$ *(Not invariant)*

Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition



geometric mean: $[1\cdot 4]^{1/2}=2$

difference mean: $\frac{1}{2}[4-1]=3/2$ *(HALF-DIFFERENCE)*

arithmetic mean: $\frac{1}{2}[4+1]=5/2=5/2$ *(HALF-SUM)*

ω

ck

-1 0 1 2 3 4

1 3/2 4 (*units of 300*THz)

Linear velocity $V_{group}/c=u/c$

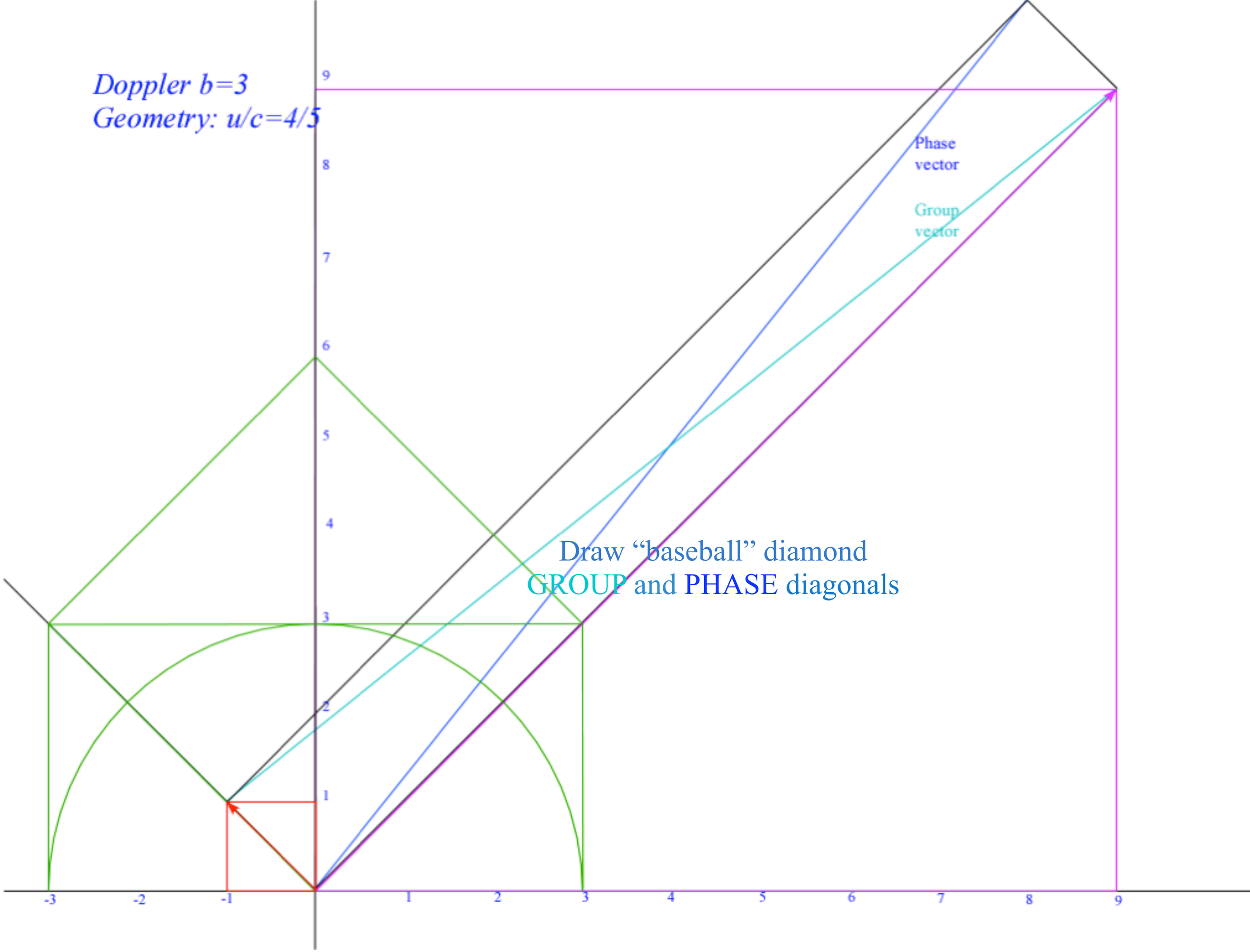
is (HALF-DIFF./HALF-SUM)=3/5

Summary:
Step-by-Step
Development of wave geometry
u/c=4/5

Doppler b=3
Geometry

Draw “baseball” diamonds
Original and Doppler shifted

Doppler b=3
Geometry: u/c=4/5



Doppler b=3
Geometry: u/c=4/5

Phase vector

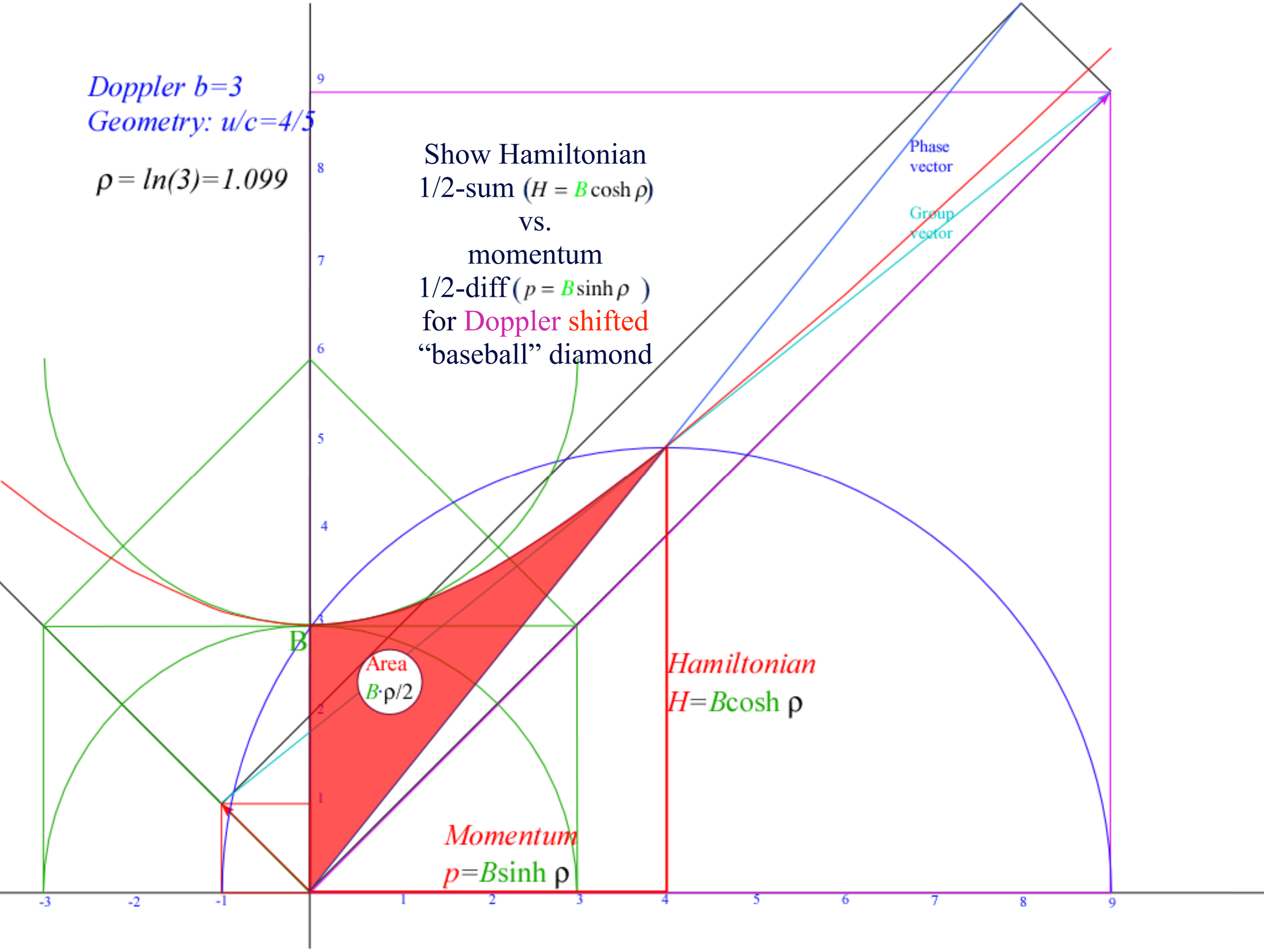
Group vector

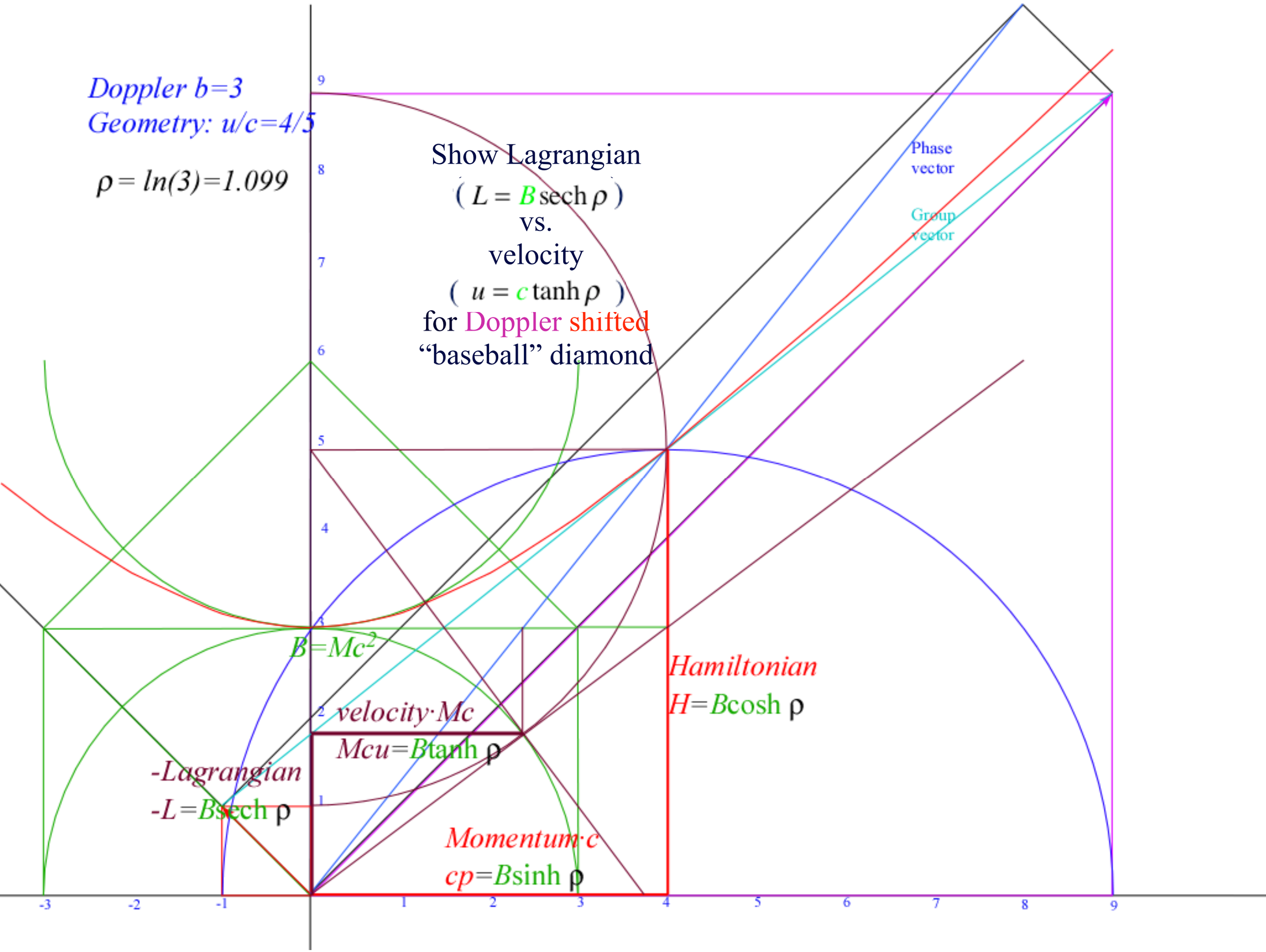
Given “baseball” diamonds
Draw Euclid Mean semi-circles
Original and Doppler shifted

Doppler b=3
Geometry: u/c=4/5

$\rho = ln(3) = 1.099$

Show Hamiltonian
1/2-sum $(H = B\cosh\rho)$
vs.
momentum
1/2-diff $(p = B\sinh\rho)$
for Doppler shifted
"baseball" diamond




Doppler b=3
Geometry: u/c=4/5
ρ = ln(3)=1.099
Show Lagrangian
(L = B sech ρ)
vs.
velocity
(u = c tanh ρ)
for Doppler shifted
“baseball” diamond
Phase vector
Group vector
B=Mc²
velocity·Mc
Mcu=Btanh ρ
-Lagrangian
-L=Bsech ρ
Momentum·c
cp=Bsinh ρ
Hamiltonian
H=Bcosh ρ

Note: Stellar Aberration Angle ϕ is a 3-Space object that arises naturally in wave geometry

Doppler b=3
Geometry: u/c=4/5

$\rho = ln(3) = 1.099$

$\tanh \rho = \sin \phi$

$\sinh \rho = \tan \phi$

$\phi = asin(4/5) = 0.92 = 53.1°$

Stellar Aberration Angle ϕ

Phase vector

Group vector

$\pi/2 - \phi$

$B = Mc^2$

velocity·Mc

$Mcu = B\tanh \rho$

-Lagrangian

$-L = B\,\mathrm{sech}\, \rho$

Momentum·c

$cp = B\sinh \rho$

Hamiltonian

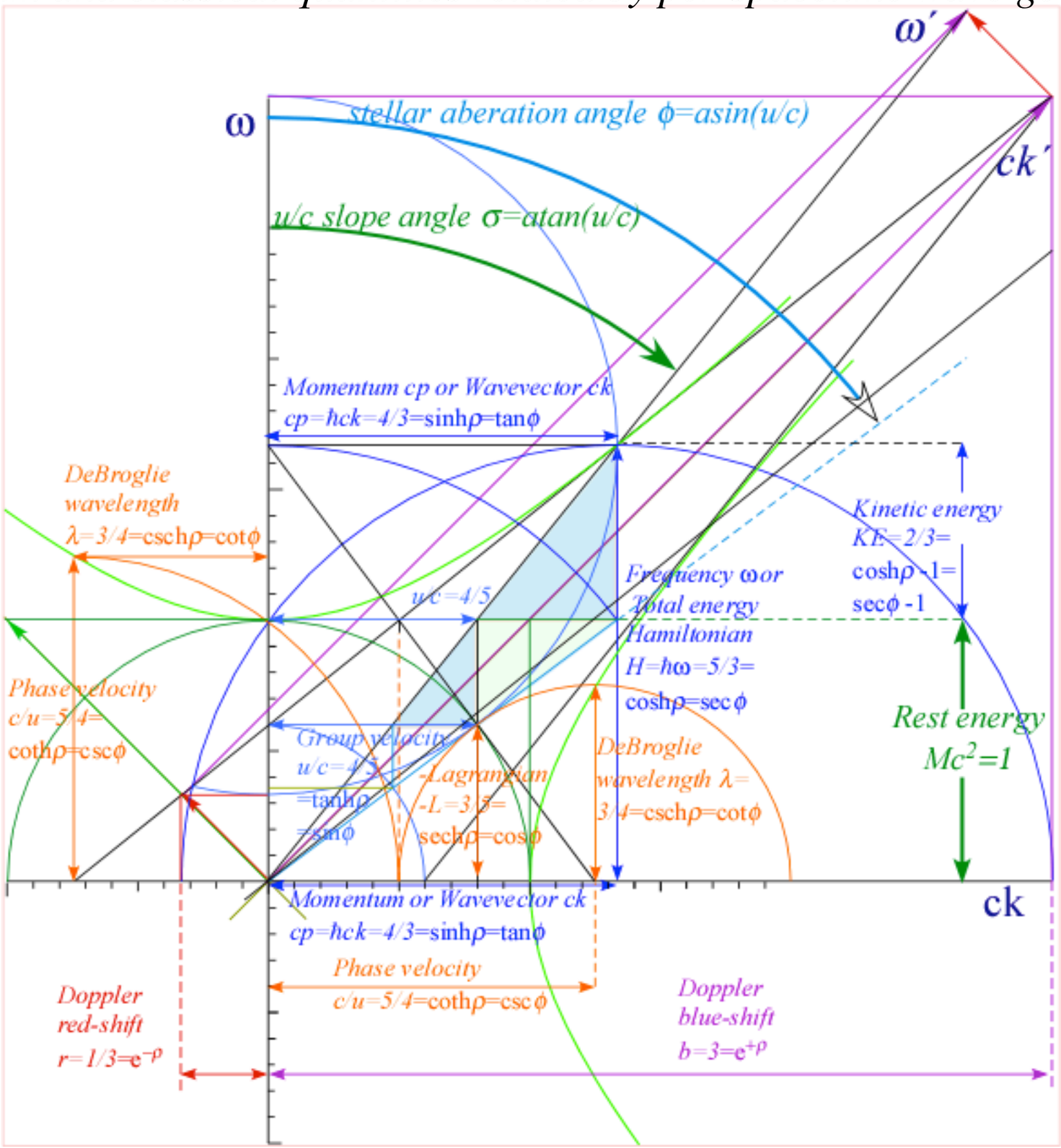
$H = B\cosh \rho$

Area $B \cdot \rho/2$

Area $B \cdot \phi/2$

-3 -2 -1 1 2 3 4 5 6 7 8 9

Key quantum and classical quantities related by per-spacetime wave geometry



These quantum and classical relations include
Poincare invariant action
Legendre contact H-to-L transformation

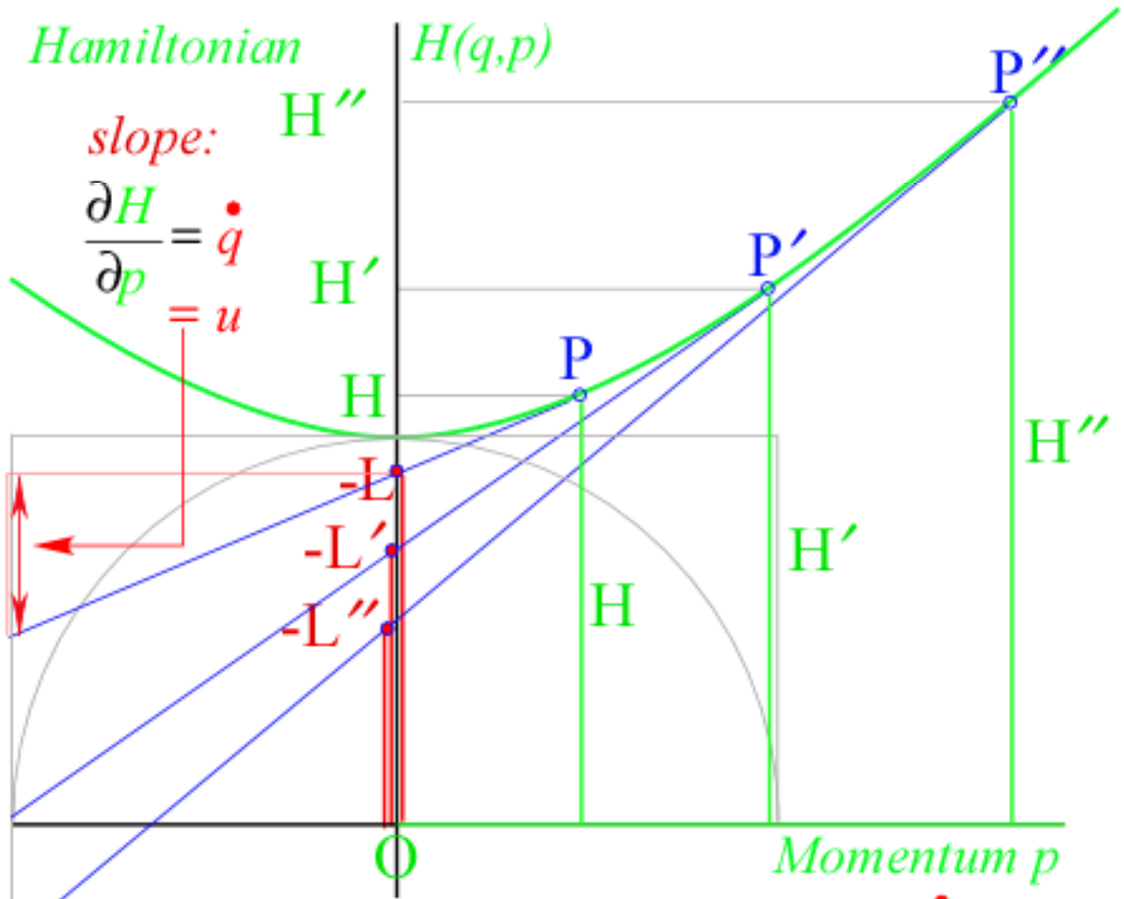
(These relate quantum and classical mechanics)

Poincare Invariant Action $dS=Ldt=p\,dq-H\,dt=\hbar d\Phi$ *(phase)*

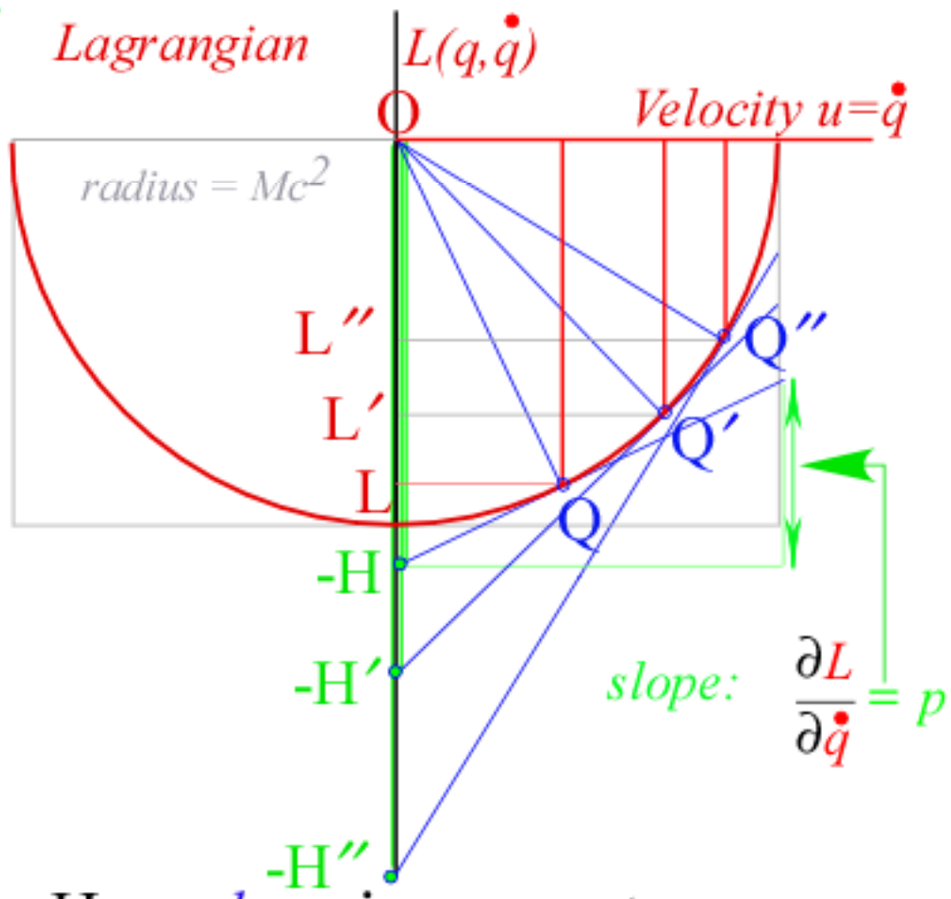
Hamiltonian $H(p,q)=p\dot{q}-L$ *vs.* *Lagrangian* $L(\dot{q}q)=p\dot{q}-H$

> *Contact transformation*: *(slope,-intercept)* of H (or L)
> *tangent* determines the *(X,Y coordinates)* of L (or H).

(Also, called a *Legendre contact transformation* which is a special case of a *Huygens transformation* that uses contacting tangent *curves* instead of *lines*.)



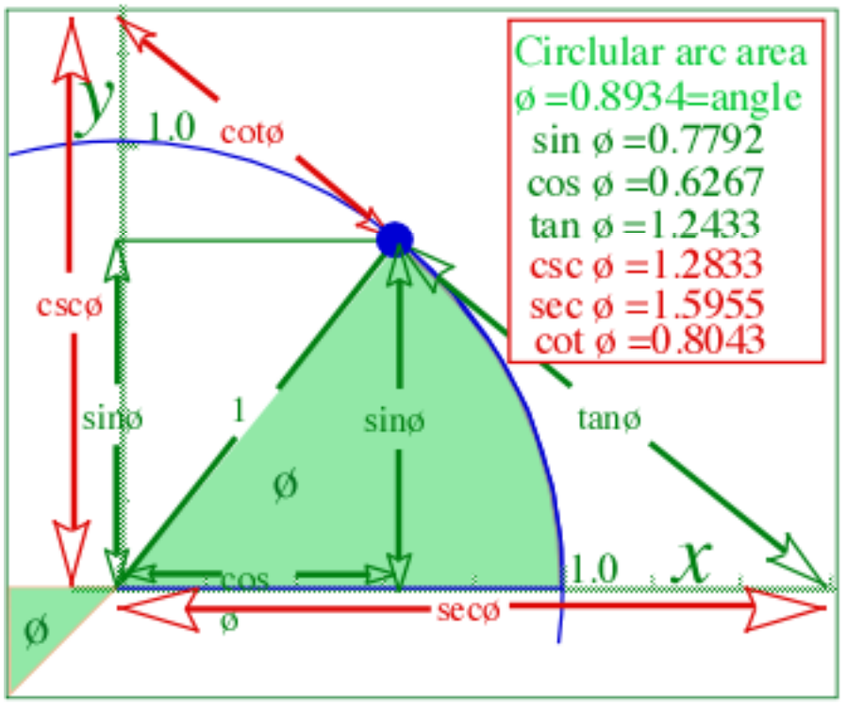
Here *slope* is group velocity $u=\dot{q}$
Y-coordinate is *energy* $H=\hbar\omega$



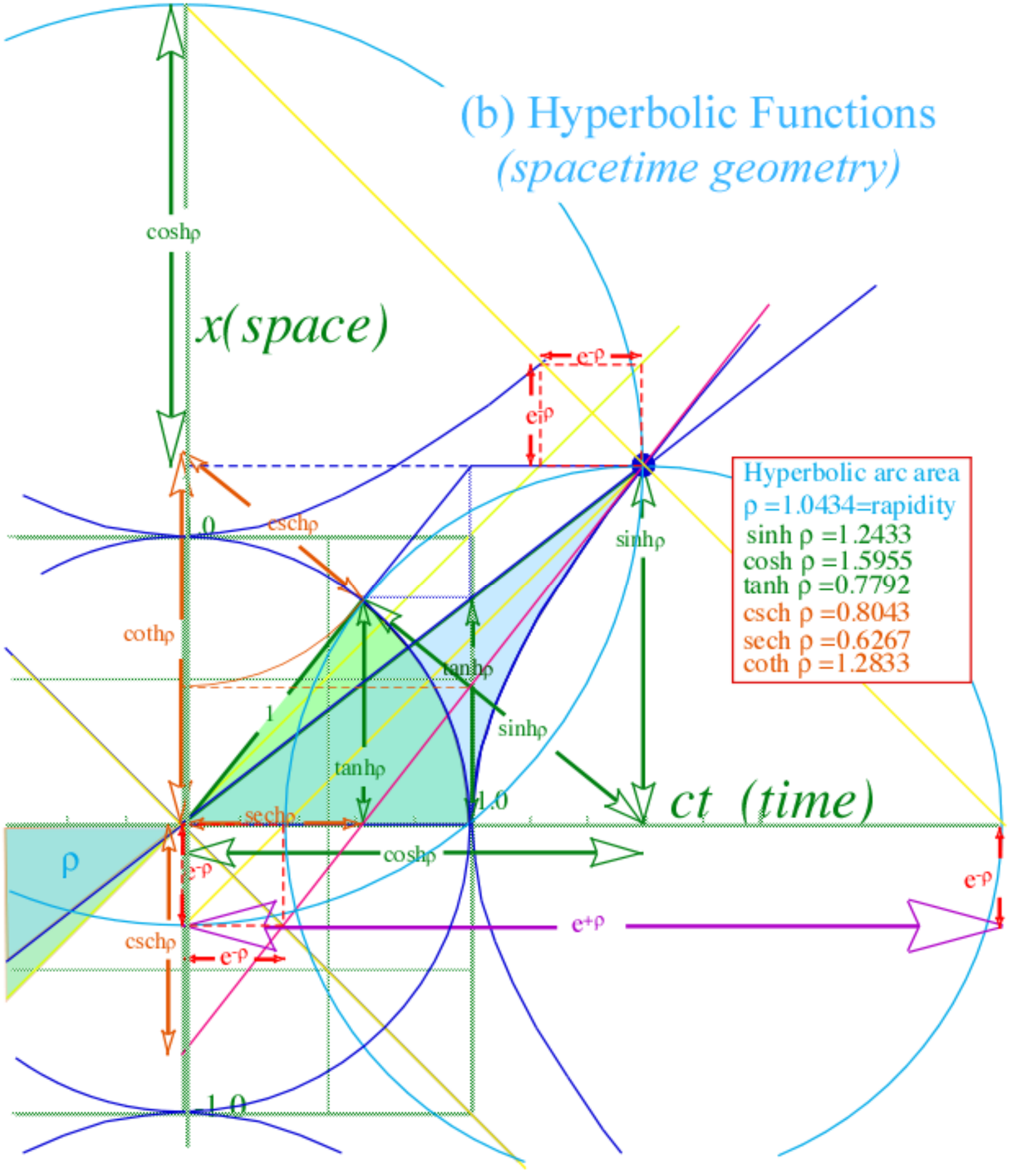
Here *slope* is momentum p
Y-coordinate is *phase rate* $L=\hbar\dot{\Phi}$

Underlying per-spacetime wave geometry
are the ancient relations between circle and hyperbola

(a) Circlular Functions
(plane geometry)



(b) Hyperbolic Functions
(spacetime geometry)



Wave frames of varying acceleration

Optical Einstein elevator, photon rockets, Compton acceleration

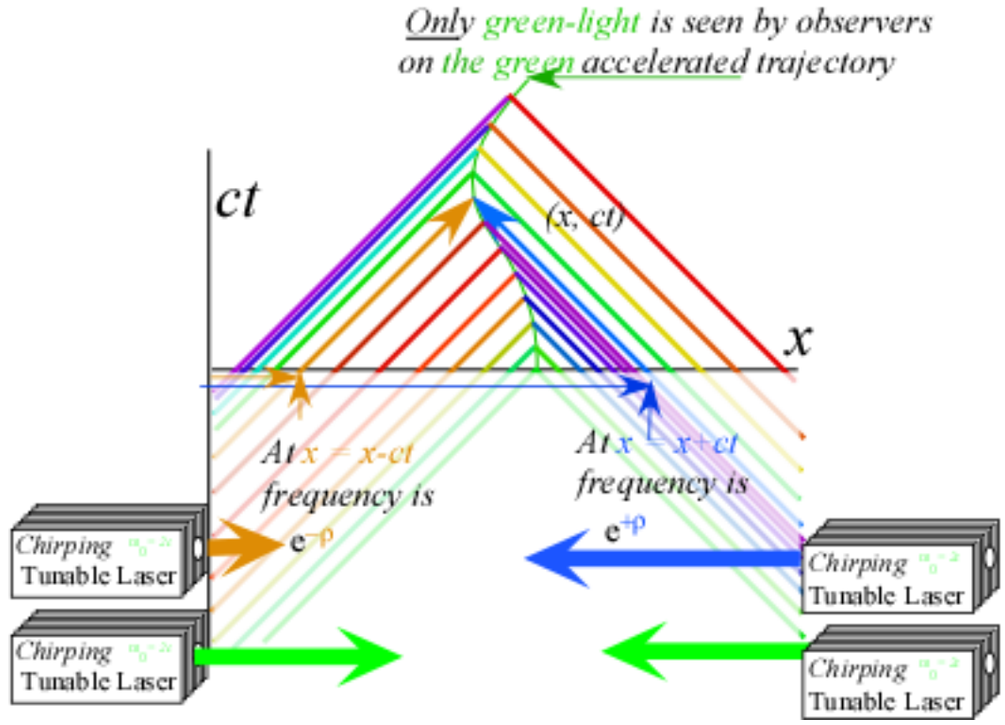
Wave frames of varying acceleration

Varying local acceleration $\rho = \rho(\tau)$

$$u = \frac{dx}{dt} = c\tanh(\tau)$$

$$\frac{dt}{d\tau} = \cosh\rho(\tau) \qquad \frac{dx}{d\tau} = \frac{dx}{dt}\frac{dt}{d\tau} = c\tanh\rho(\tau)\cosh\rho(\tau) = c\sinh\rho(\tau)$$

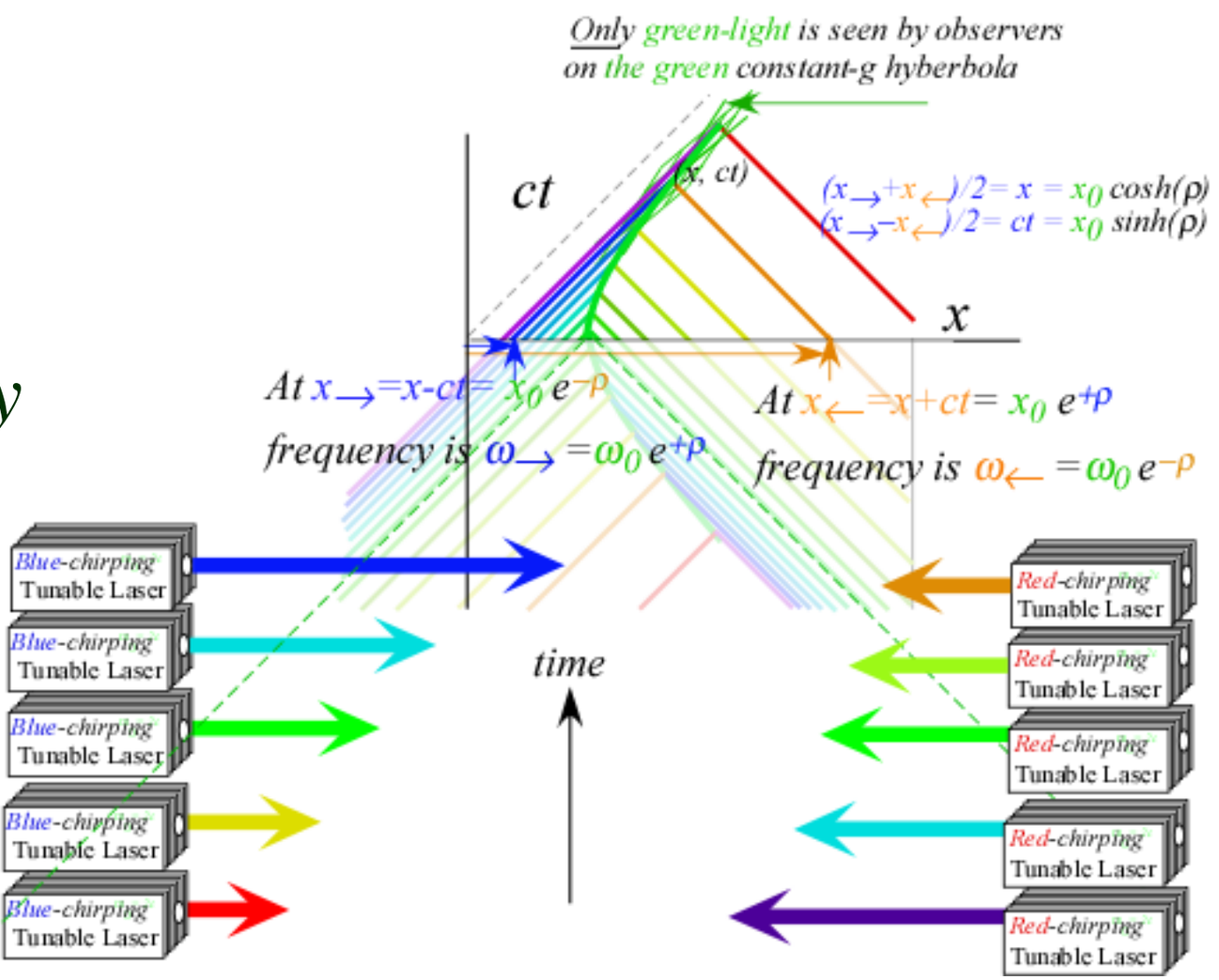
$$ct = c\int\cosh\rho(\tau)\,d\tau \qquad x = c\int\sinh\rho(\tau)\,d\tau$$



Wave frames of constant acceleration

Constant local acceleration $\rho = \frac{g\tau}{c}$ *"Einstein Elevator"*

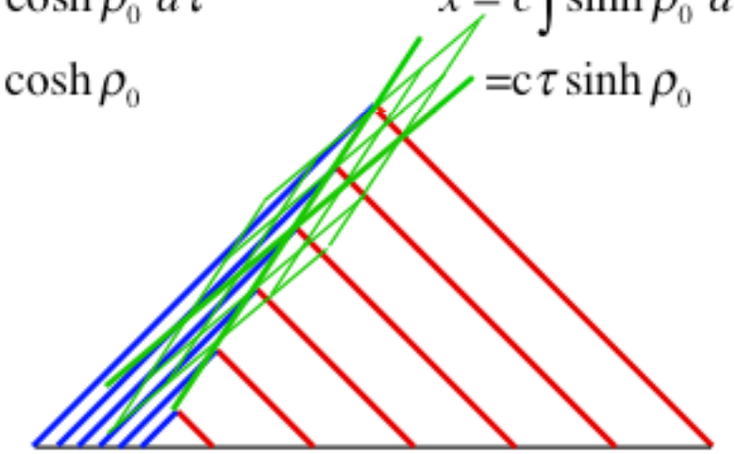
$$ct = c\int\cosh\frac{g\tau}{c}\,d\tau = \frac{c^2}{g}\sinh\frac{g\tau}{c} \qquad x = c\int\sinh\frac{g\tau}{c}\,d\tau = \frac{c^2}{g}\cosh\frac{g\tau}{c}$$

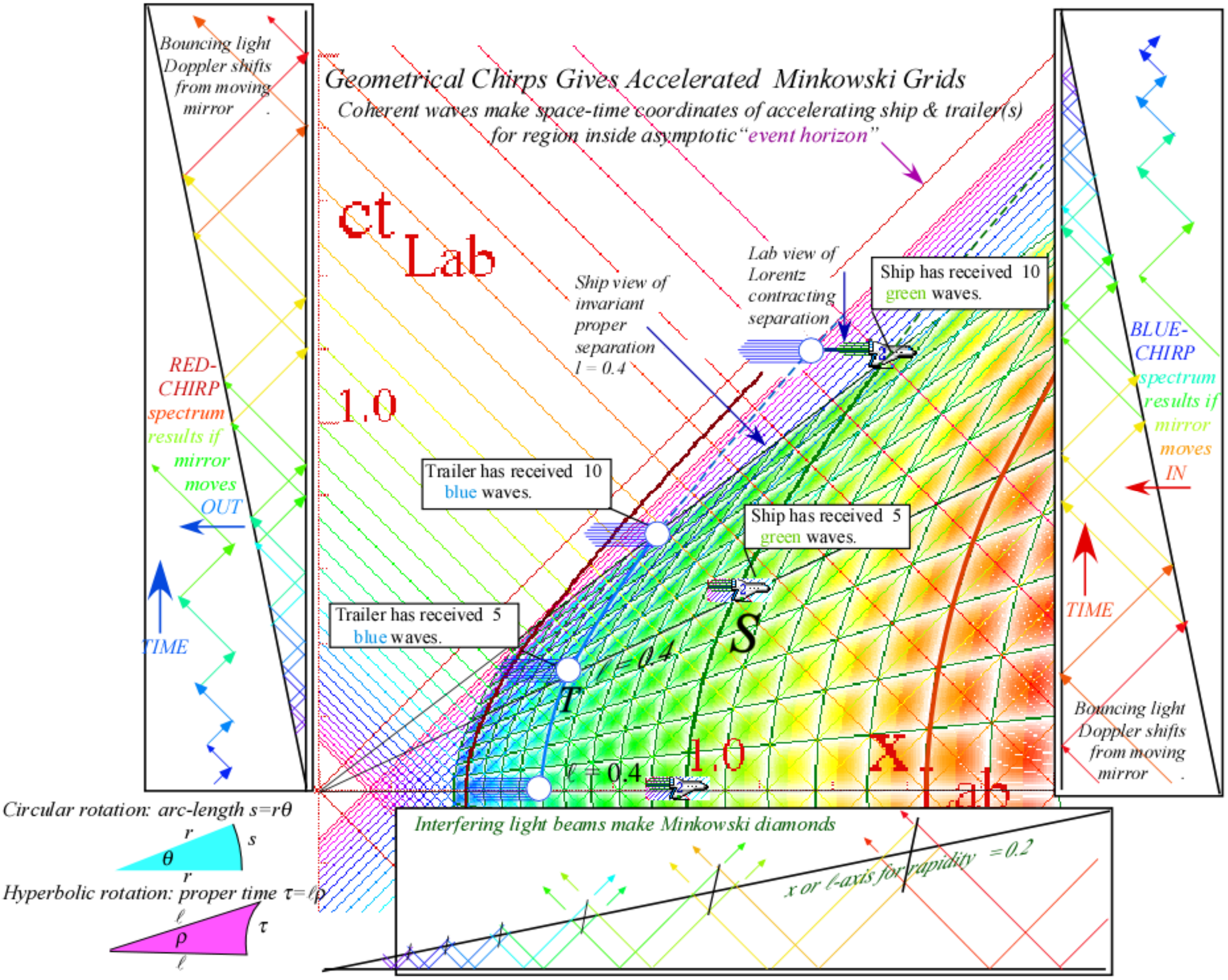


Wave frames of constant velocity

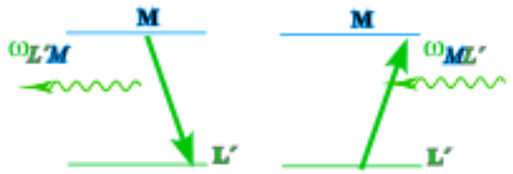
Constant velocity $\rho = \rho_0 = const.$ *"Lorentz transformation"*

$$ct = c\int\cosh\rho_0\,d\tau = c\tau\cosh\rho_0 \qquad x = c\int\sinh\rho_0\,d\tau = c\tau\sinh\rho_0$$

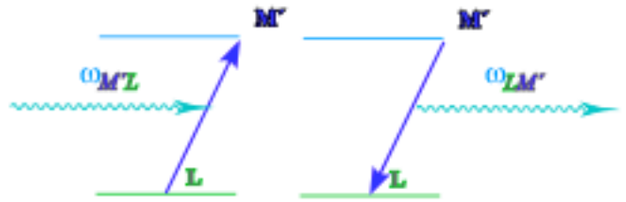


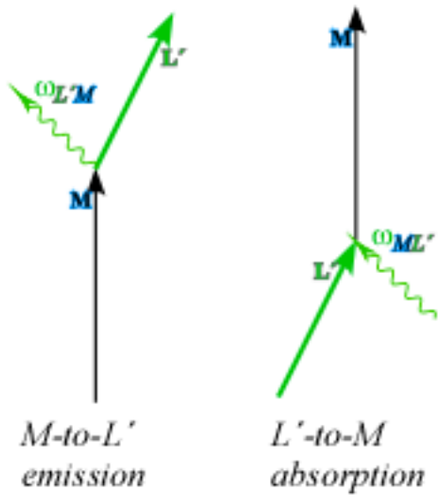

Geometrical Chirps Gives Accelerated Minkowski Grids
Coherent waves make space-time coordinates of accelerating ship & trailer(s)
for region inside asymptotic "event horizon"
Bouncing light Doppler shifts from moving mirror .
RED-CHIRP spectrum results if mirror moves OUT
TIME
ct Lab
1.0
Ship view of invariant proper separation l = 0.4
Lab view of Lorentz contracting separation
Ship has received 10 green waves.
Trailer has received 10 blue waves.
Ship has received 5 green waves.
Trailer has received 5 blue waves.
T
S
l = 0.4
l = 0.4
1.0
X Lab
BLUE-CHIRP spectrum results if mirror moves IN
TIME
Bouncing light Doppler shifts from moving mirror .
Circular rotation: arc-length s=rθ
r
θ
s
r
Hyperbolic rotation: proper time τ=ℓρ
ℓ
ρ
τ
ℓ
Interfering light beams make Minkowski diamonds
x or ℓ-axis for rapidity = 0.2

Wave geometry of 1-photon transitions and Compton recoil



Grotian 2-level diagrams

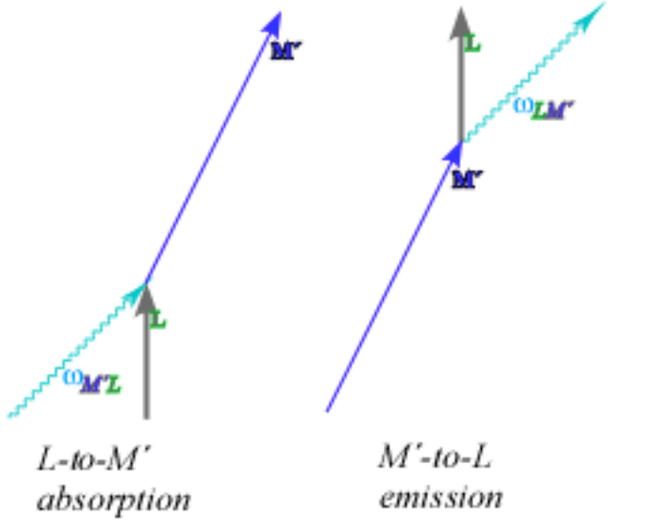




M-to-L′ emission

L′-to-M absorption

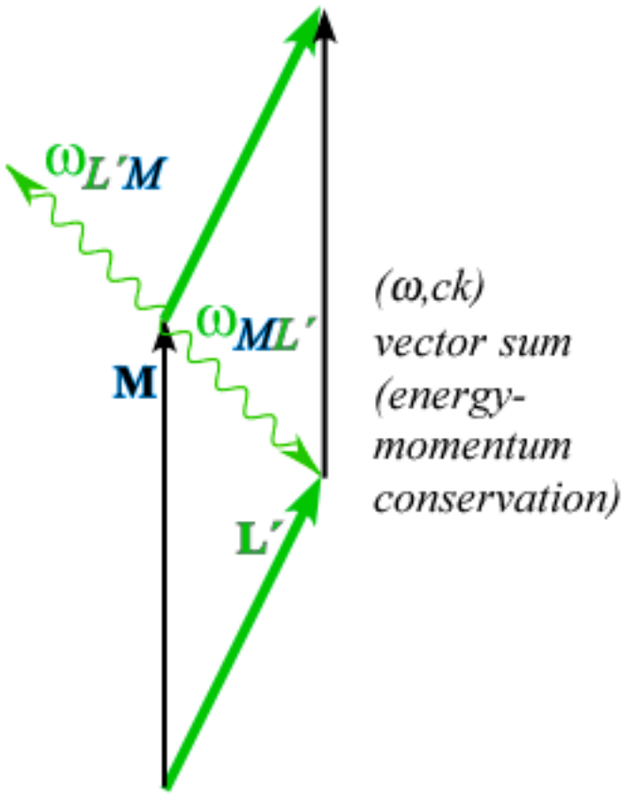
Feynman (ω,ck) diagrams (1-photon)



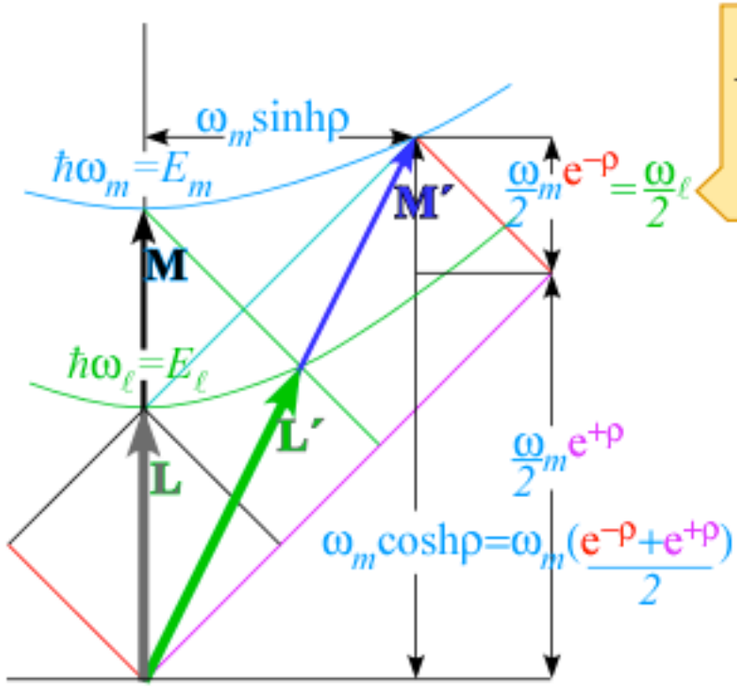
L-to-M′ absorption

M′-to-L emission

2-Level (ω,ck) "baseball" diamonds



(ω,ck) vector sum (energy-momentum conservation)



Key recoil relations:

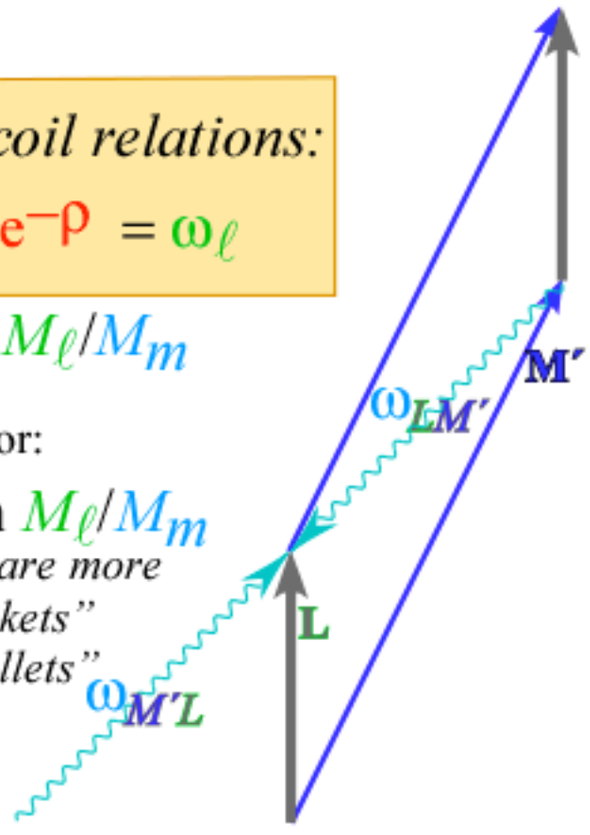
$$\omega_m e^{-\rho} = \omega_\ell$$

$$\rho = \ln M_\ell / M_m$$

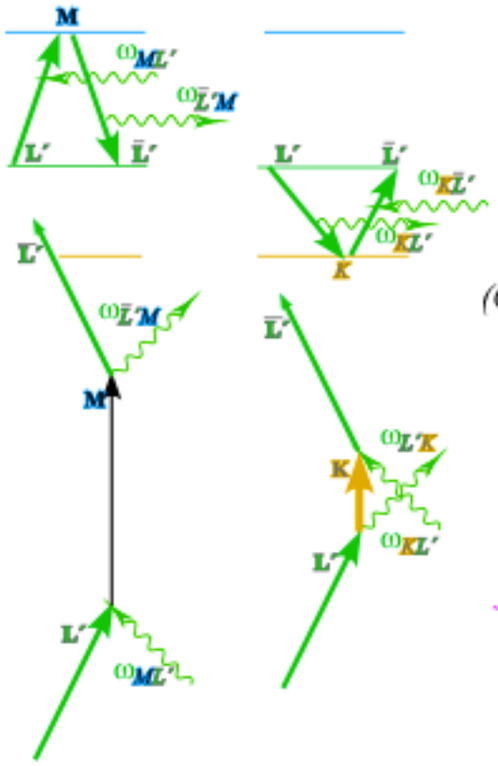
or:

$$u \sim c \ln M_\ell / M_m$$

Photons are more like "rockets" than "bullets"

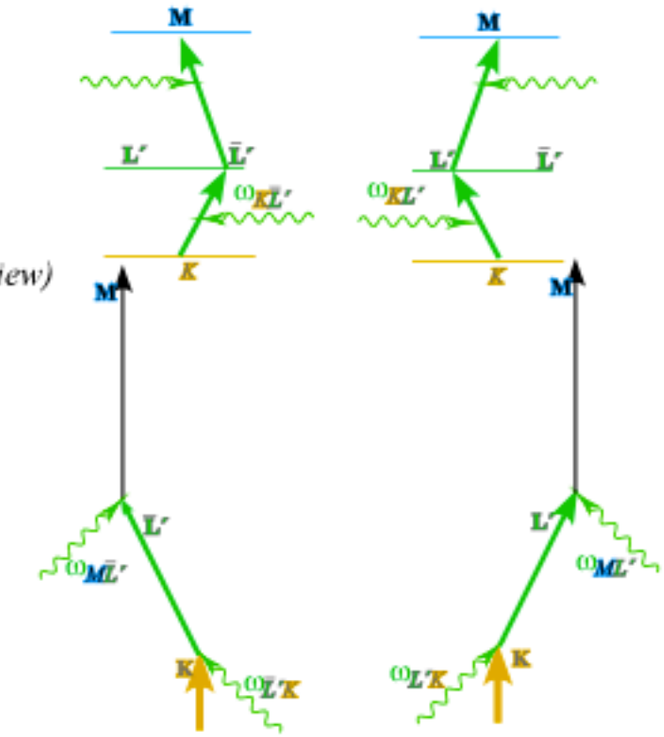


Wave geometry of 2-photon transitions and Compton scattering

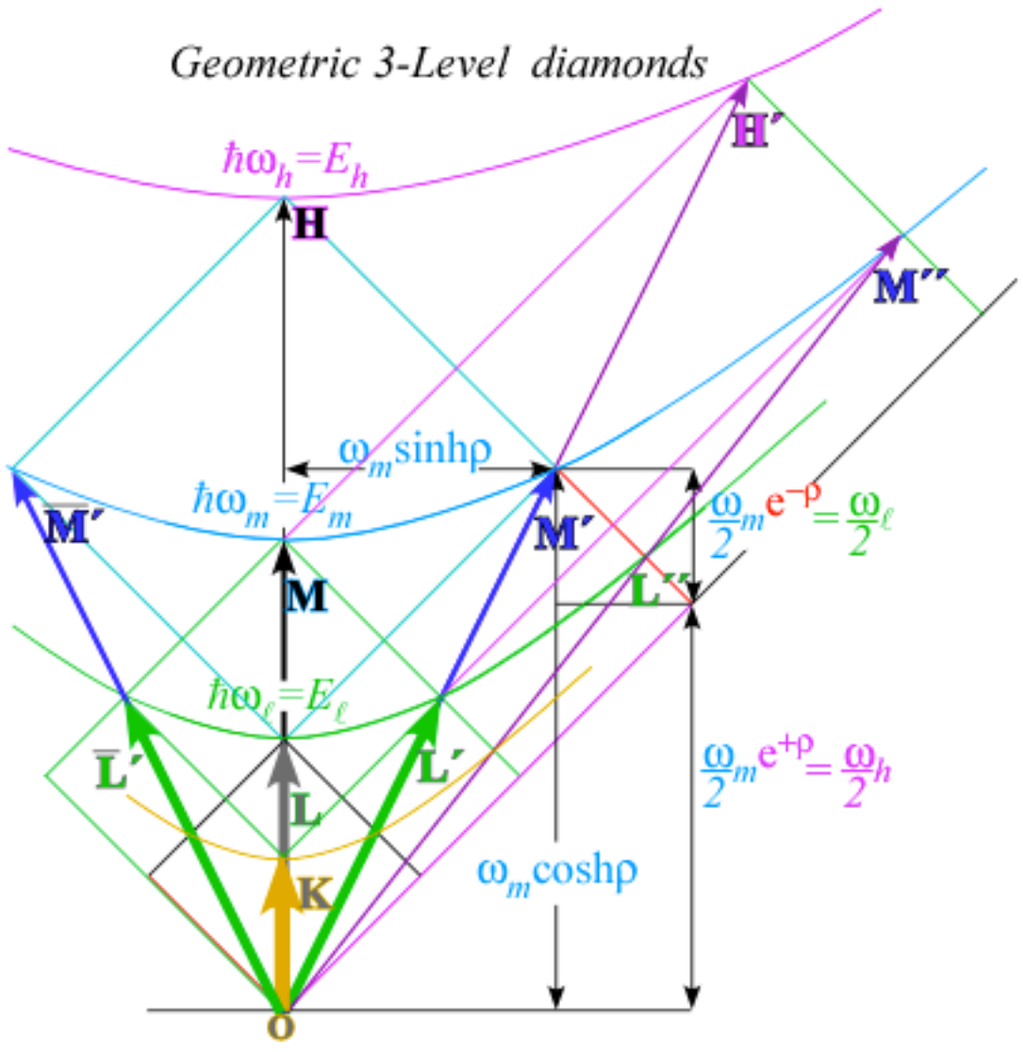


Compton scattering (Center of Momentum view)

2-photon absorption (Center of Momentum view)

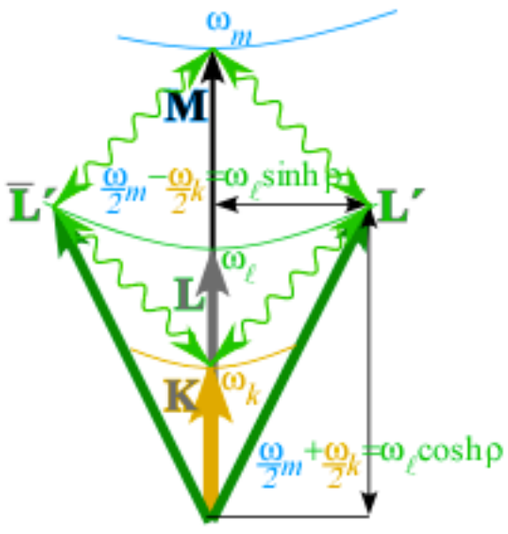


Geometric 3-Level diamonds

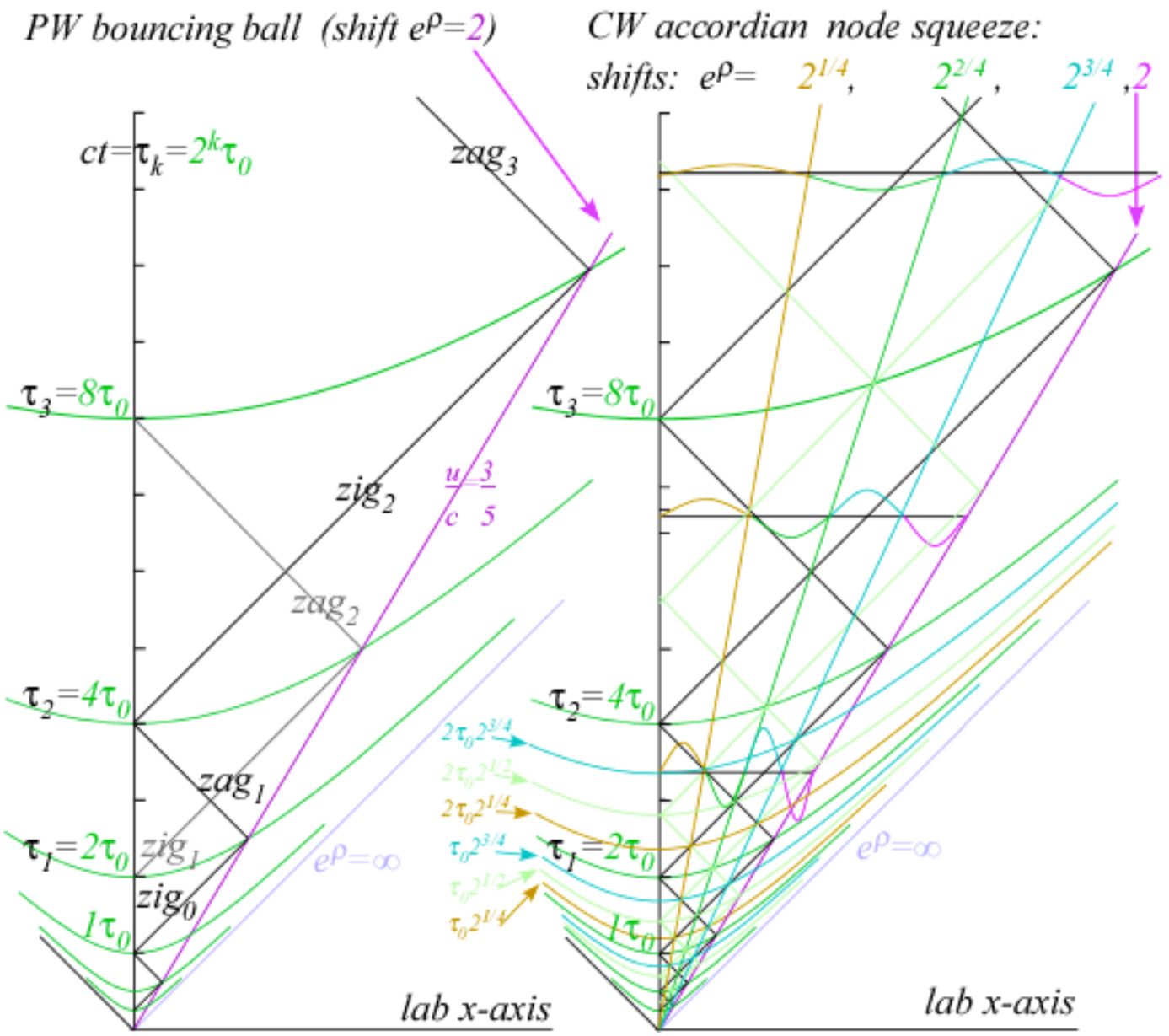


[ω_k, ω_ℓ, ω_m]

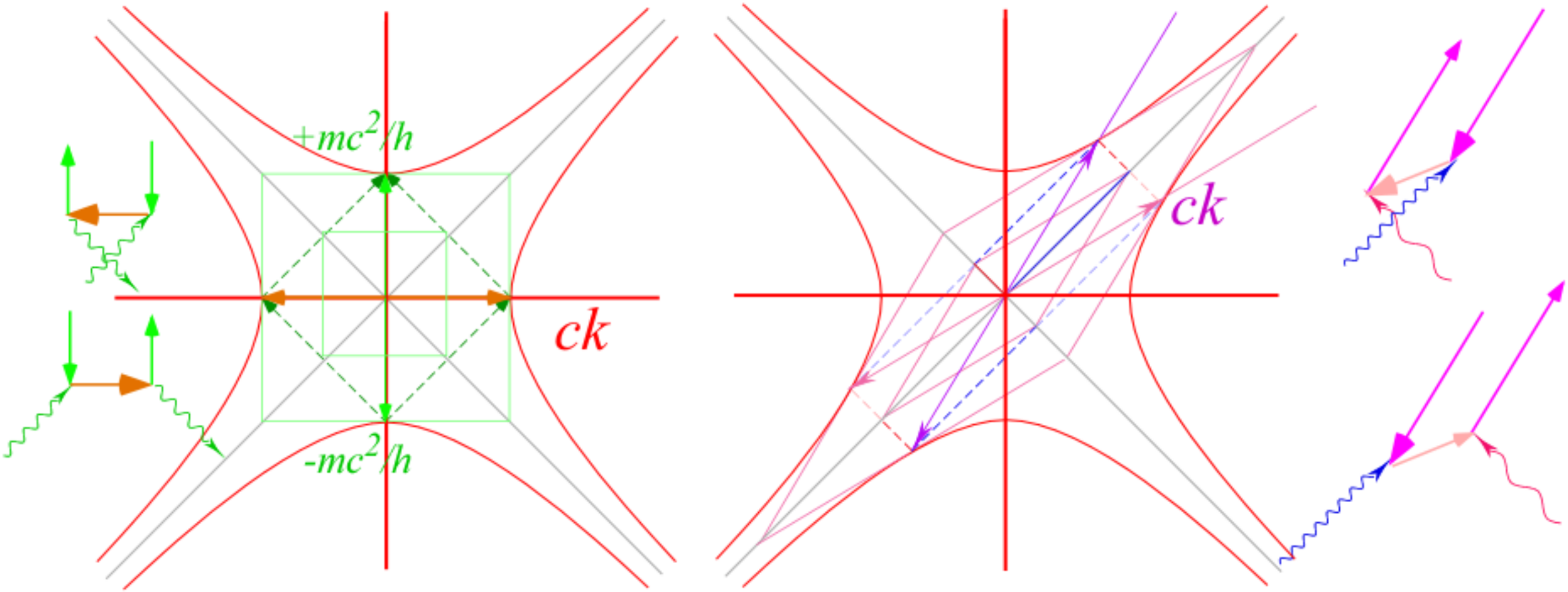
photon diamond



Spacetime view of Compton acceleration and wave chirp



Dirac Pair-Production Processes (A BIG mystery)

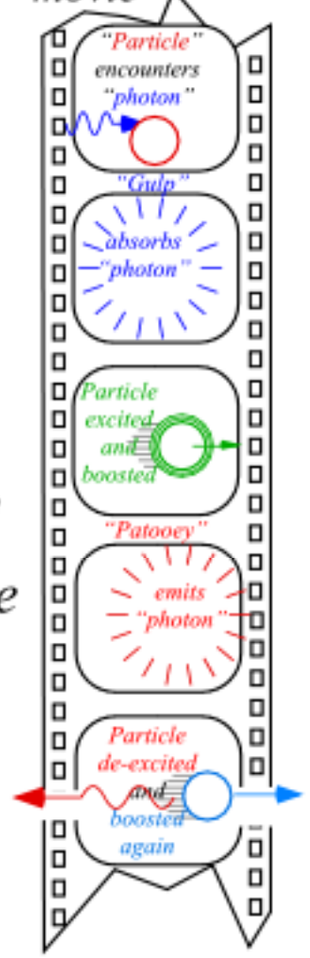


Conclusion: Wave geometry can simplify and clarify SR and QM
It's a wavy universe and one should think accordingly.

"Quantum Acceleration" in spacetime and per-spacetime

Spacetime:

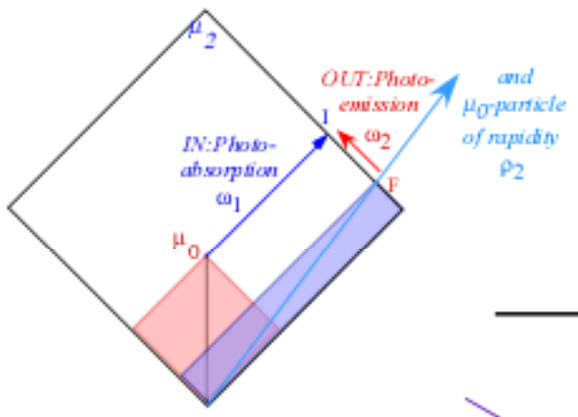
Classical "movie"



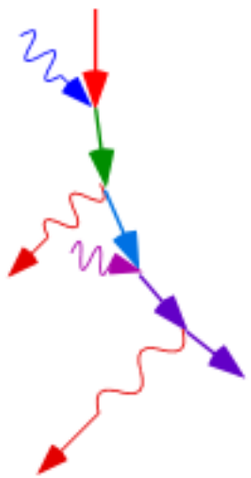
Per-spacetime:

(a) Elementary Compton process

Feynman diagram



(b) Two Compton processes



Etc.

...