

AMOP Lecture 2

Thur. 1.16 -Tue 1.21-Thur 1.23.2014

Relativity of wave-optics and Lorentz-Minkowski coordinates I.

Ch. 2 of Unit 8 CMwBang! and p.1-23 Relativity&QuantumTheory by Rule&Compass

1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames

Dueling lasers make lab frame space-time grid (CW or PW)

Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames with Review of Light

2. Applying Occam's razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms (Traditional: The "Roadrunner" Axiom)

CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear? ... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

3. Spectral theory of Einstein-Lorentz relativity

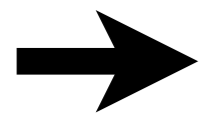
Applying Doppler Shifts to per-space-time (ck, ω) graph

CW Minkowski space-time coordinates (x, ct) and PW grids

Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ

Connection: Conventional approach to relativity and old-fashioned formulas

1. Optical wave coordinates and frames



Old-fashioned vs. New-fashioned spacetime frames

Dueling lasers make lab frame space-time grid (CW or PW)

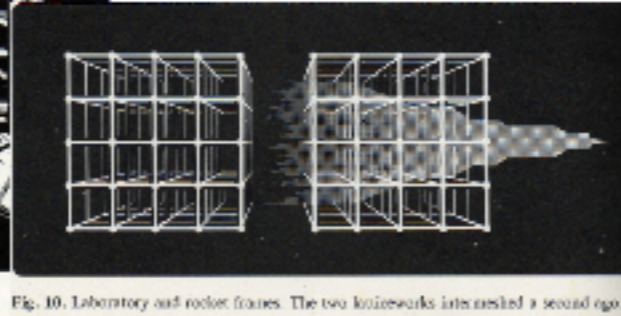
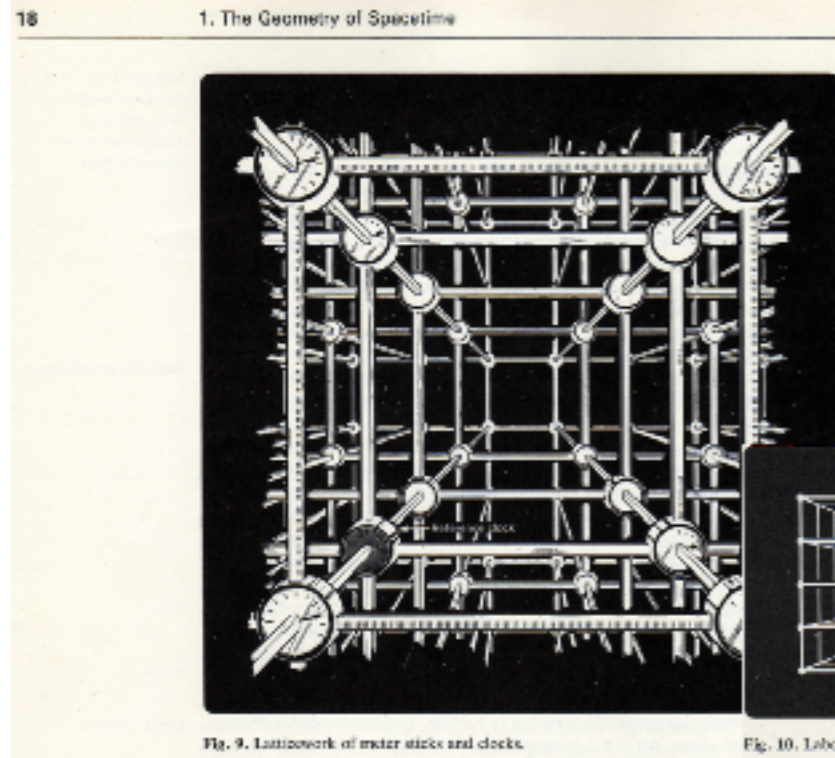
Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames

• Optical wave coordinate manifolds and frames

Shining some light on light using complex phasor analysis

Old-fashioned meter-stick-clock frames

E. F. Taylor and J. A. Wheeler Spacetime Physics (Freeman San Francisco 1966)



New-fashioned laser clocks & meter sticks

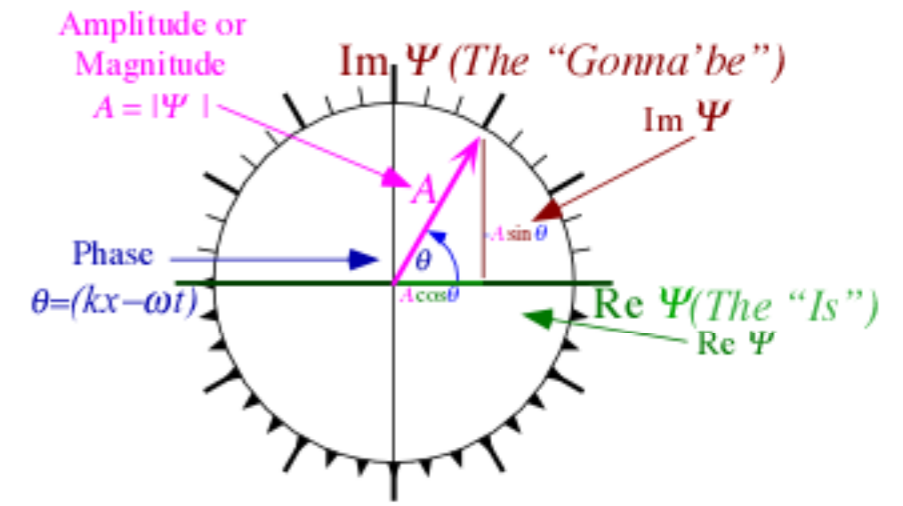
Complex Phasor Clocks : Tesla's AC "phasor"

Quantum Phasor Clock

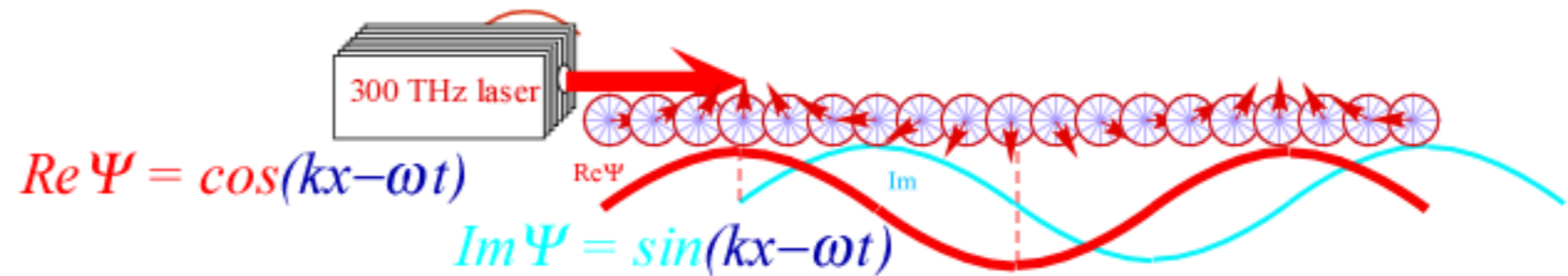
$$\Psi = Ae^{i(kx - \omega t)}$$

$$= A \cos(kx - \omega t) + i A \sin(kx - \omega t)$$

Phasor clocks turn clockwise in time for positive ω



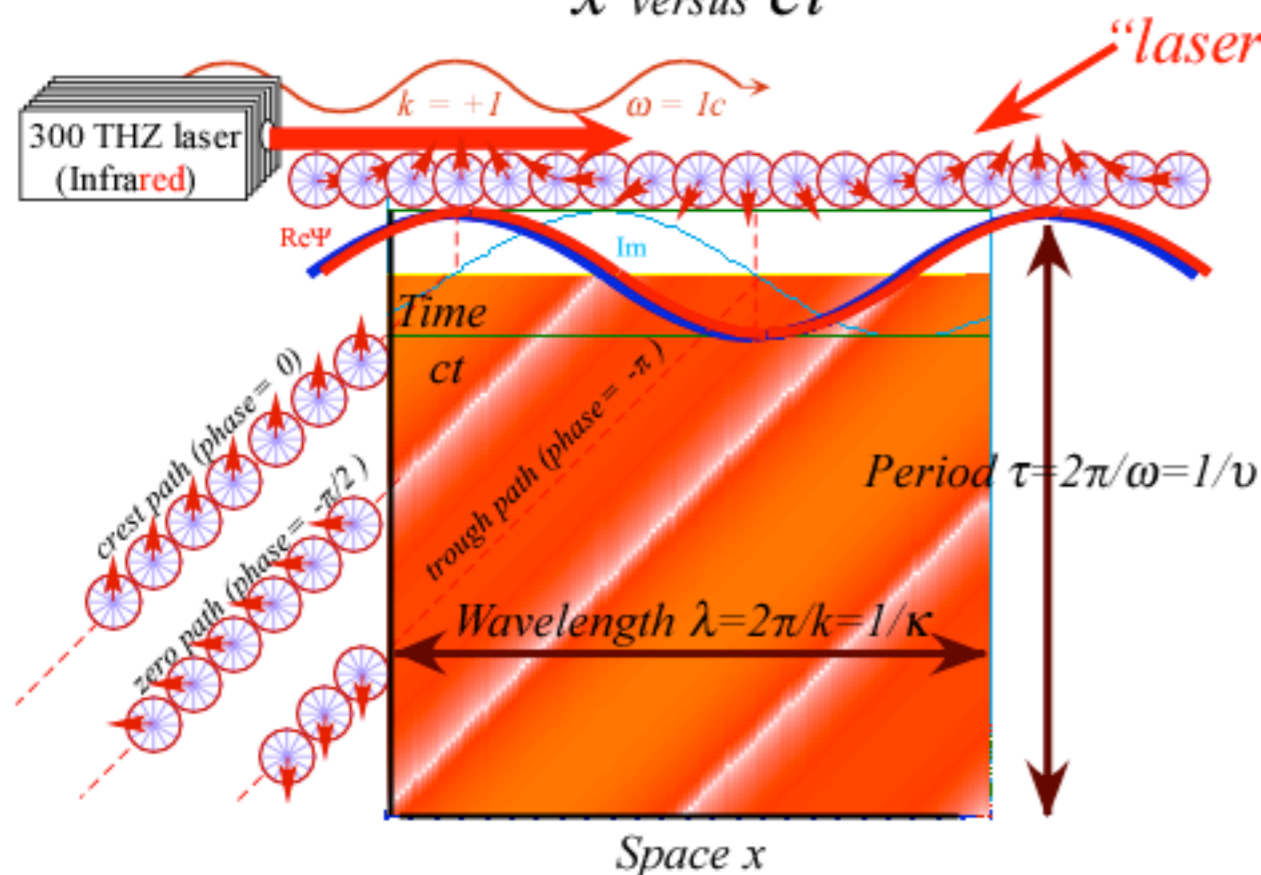
300THz Laser plane wave $\langle x, t | k, \omega \rangle = Ae^{i(kx - \omega t)}$



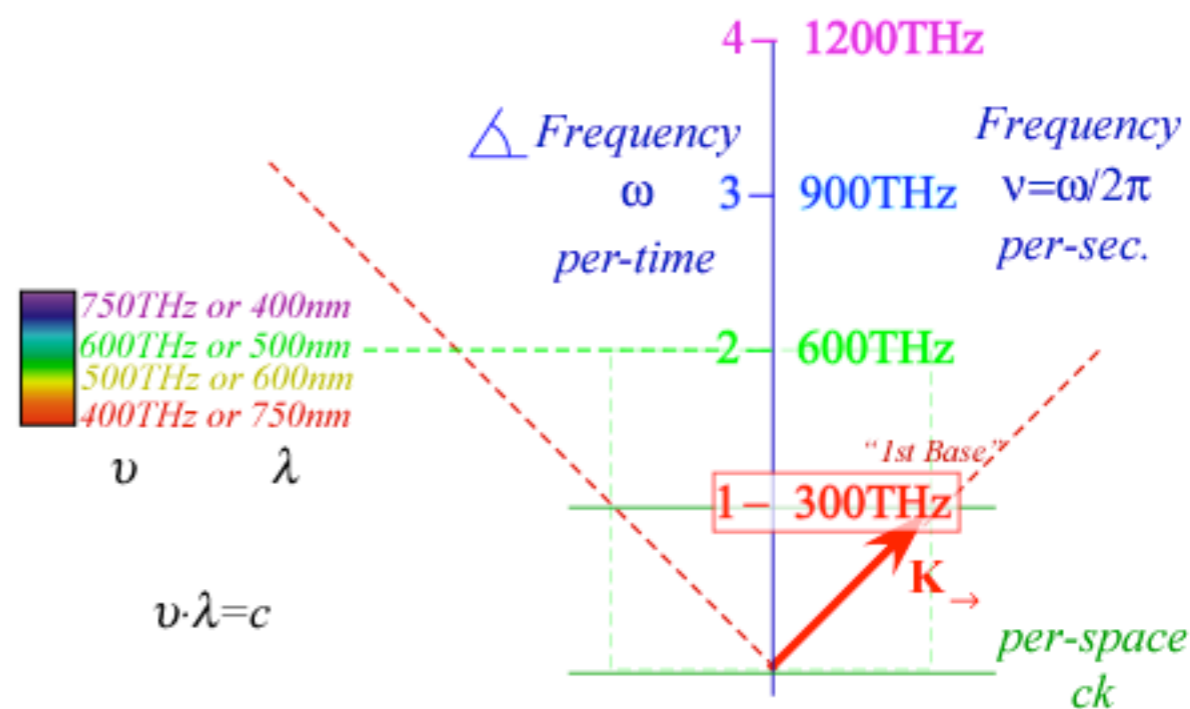
New-fashioned laser clocks & meter sticks (contd.)

Dual views:

(1.) Spacetime
 x versus ct



(2.) Per-Spacetime
 ω versus ck



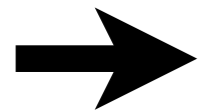
Single plane-wave meter-stick-clocks are too fast
 (can't catch 'em)

(...But at least this view is constant)

Interfering wave pairs needed
 to make rest frame coordinates...

1. Optical wave coordinates and frames

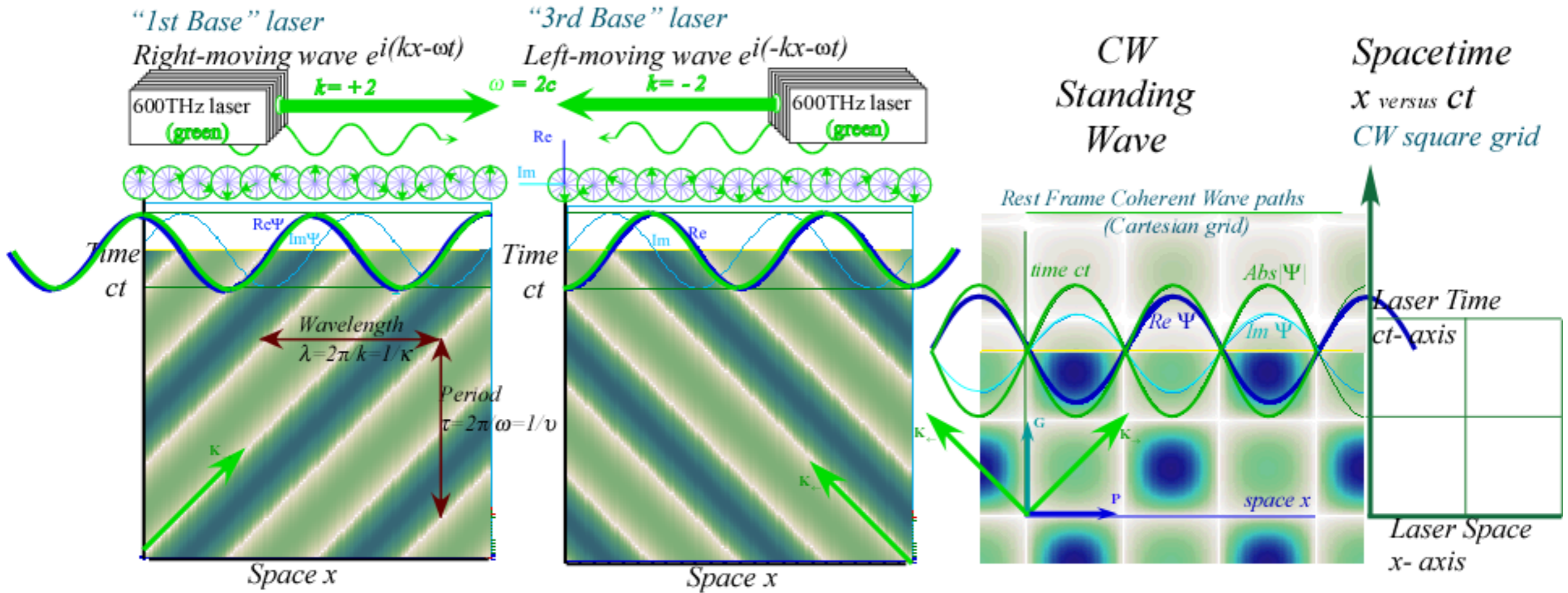
Old-fashioned vs. New-fashioned spacetime frames



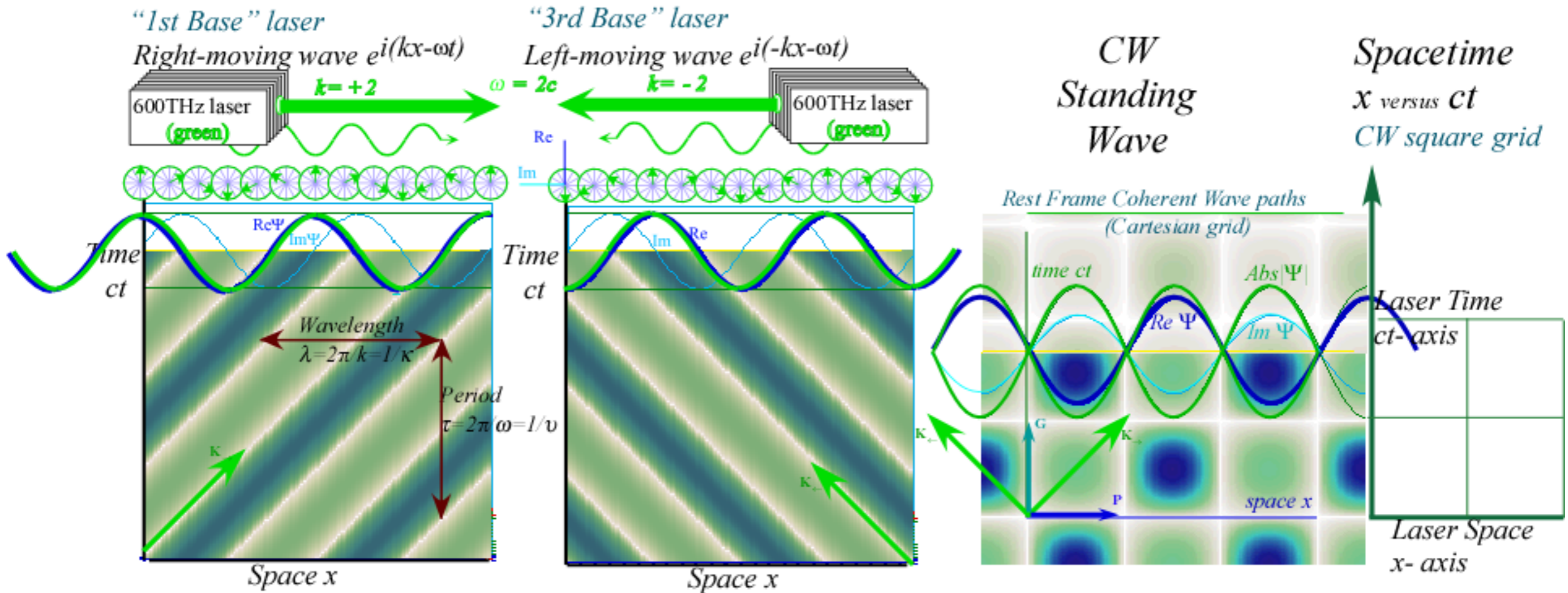
Dueling lasers make lab frame space-time grid (CW or PW)

Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames

Zeros of head-on CW sum gives (x, ct) -grid



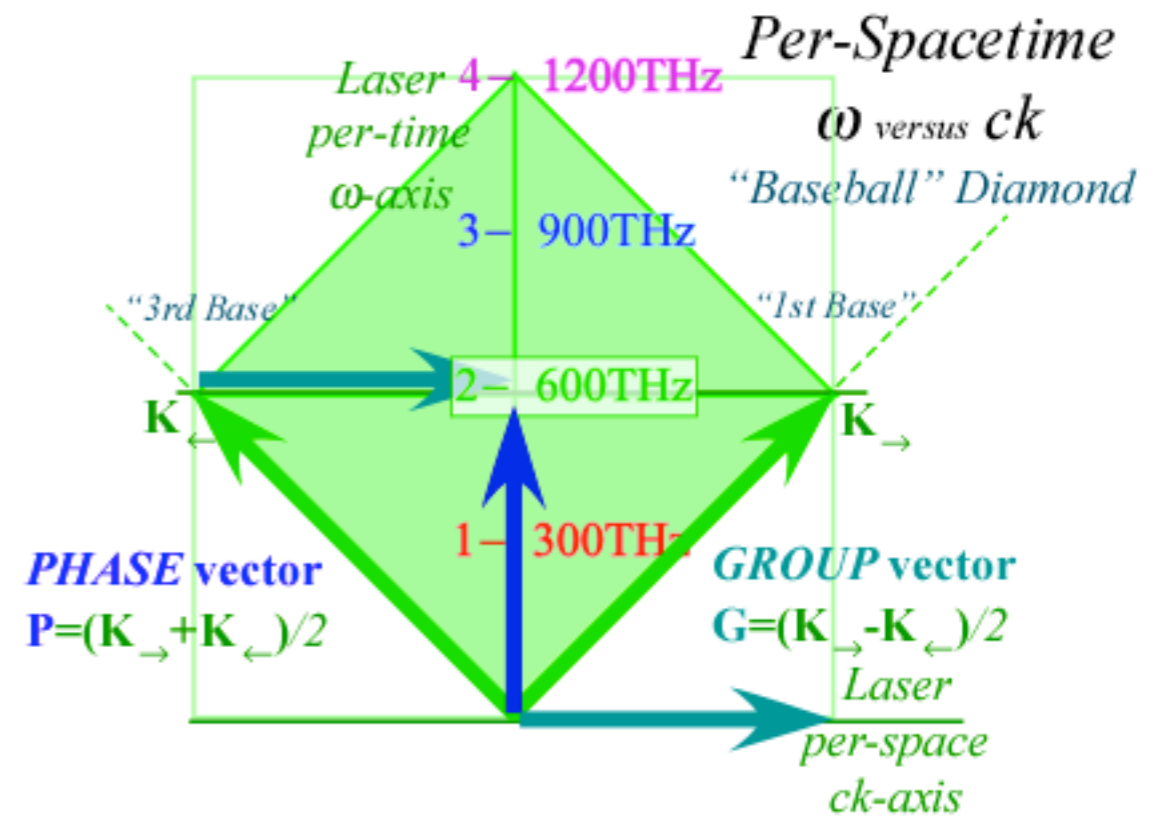
Zeros of head-on CW sum gives (x,ct)-grid



Find zeros by factoring sum:

$$\begin{aligned}
 \Psi &= e^{ia} + e^{ib} \\
 &= e^{i(a+b)/2} (e^{i(a-b)/2} + e^{-i(a-b)/2})
 \end{aligned}$$


Phase factor: $exp(i\frac{a+b}{2}) = e^{-i\omega t}$
 Group factor: $2\cos(\frac{a-b}{2}) = 2\cos(kx)$



1. Optical wave coordinates and frames

Old-fashioned vs. New-fashioned spacetime frames

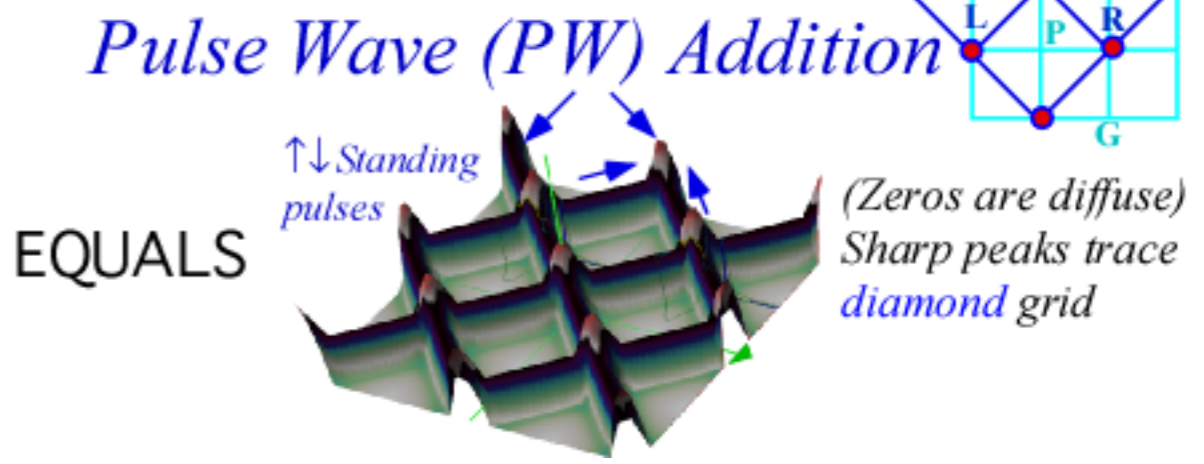
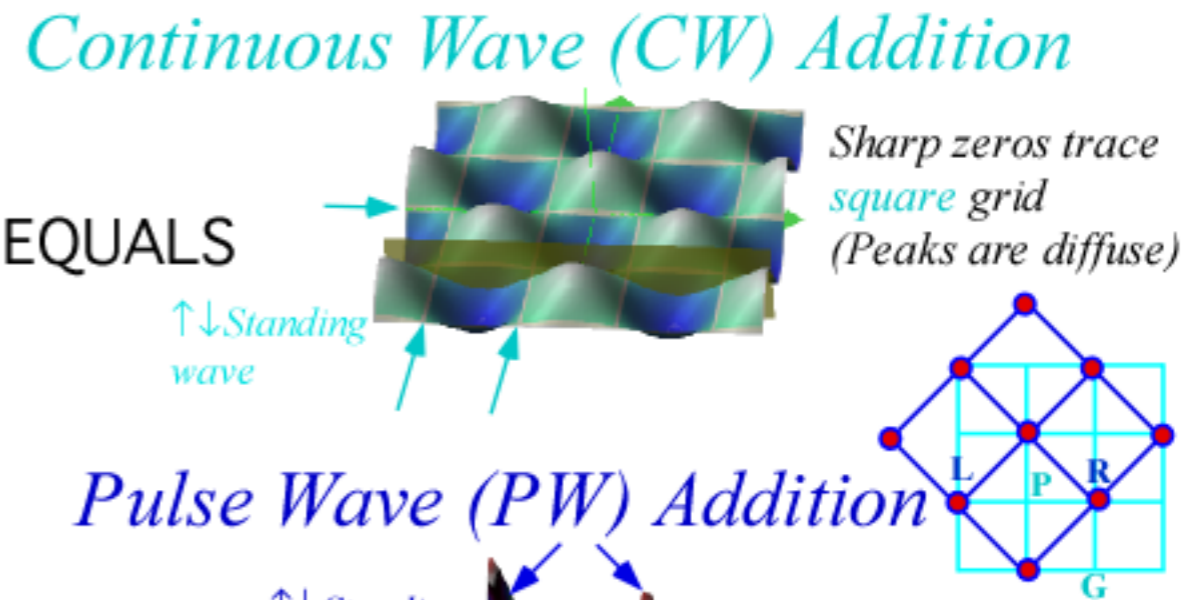
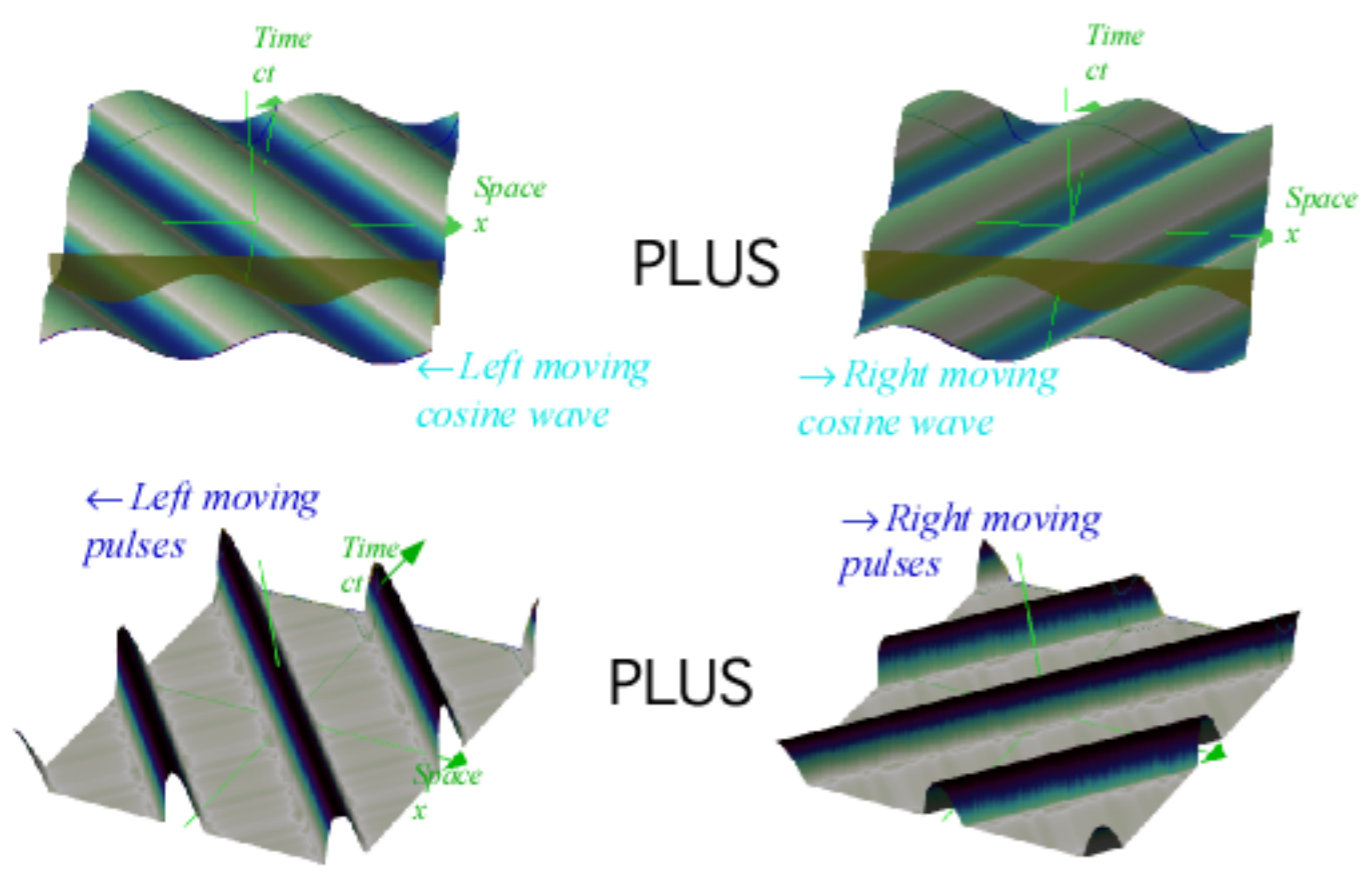
Dueling lasers make lab frame space-time grid (CW or PW)

 *Comparing Continuous-Wave (CW) vs. Pulse-Wave (PW) frames*

http://www.uark.edu/ua/pirelli/php/waves_interfering_montage.php

Newton's "Fits" in Optical Interference

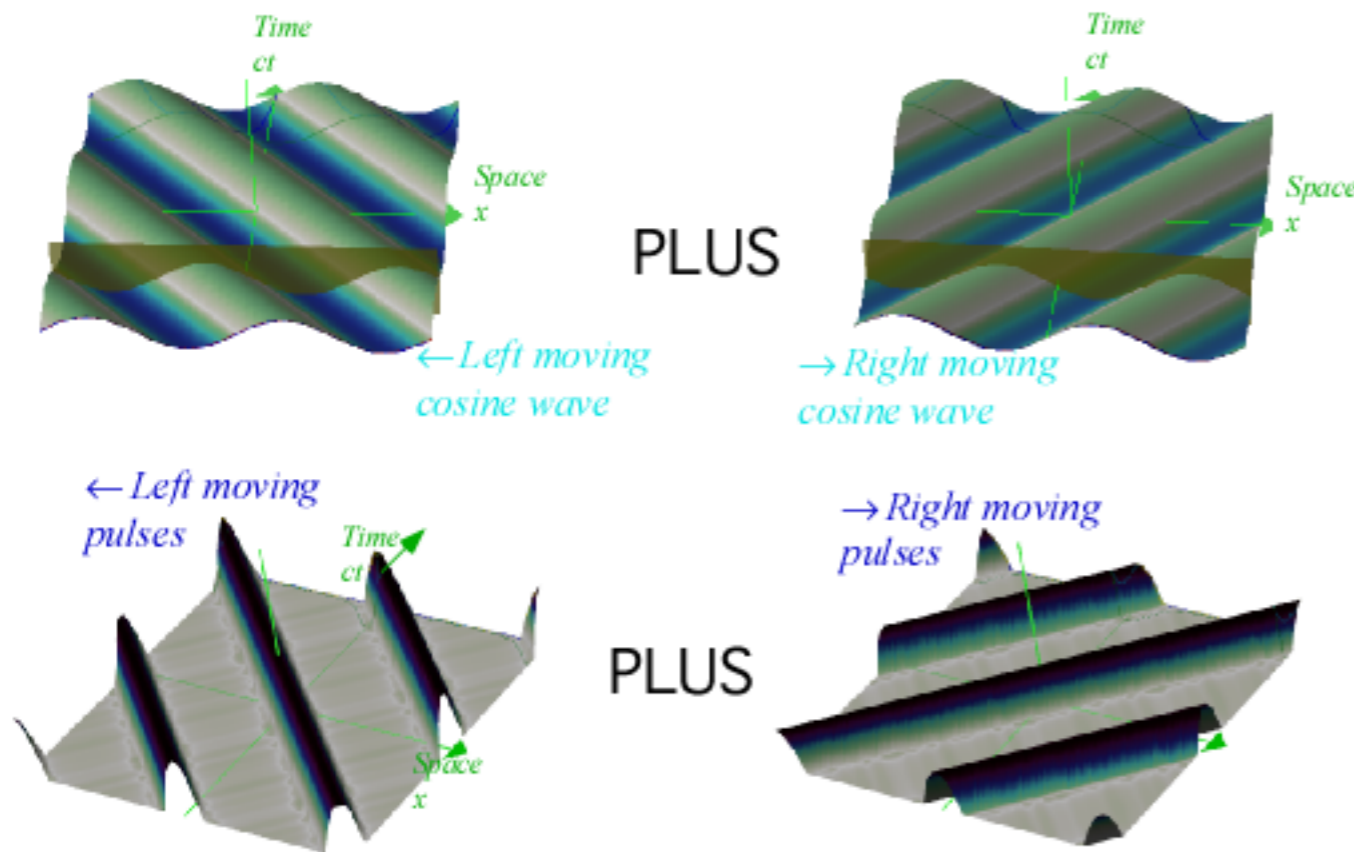
Newton complained that light waves have "fits" (what we now know as wave interference or resonance.)
 Examples of interference are head-on collision of two *Continuous Waves (2-CW)* or two *Pulse Waves (PW)*



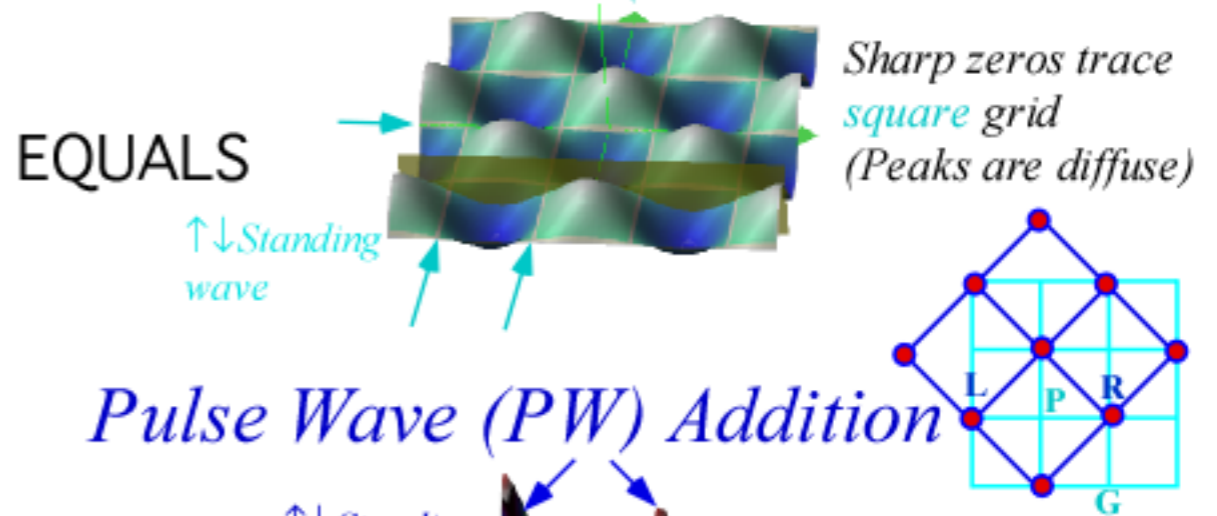
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Newton's "Fits" in Optical Interference

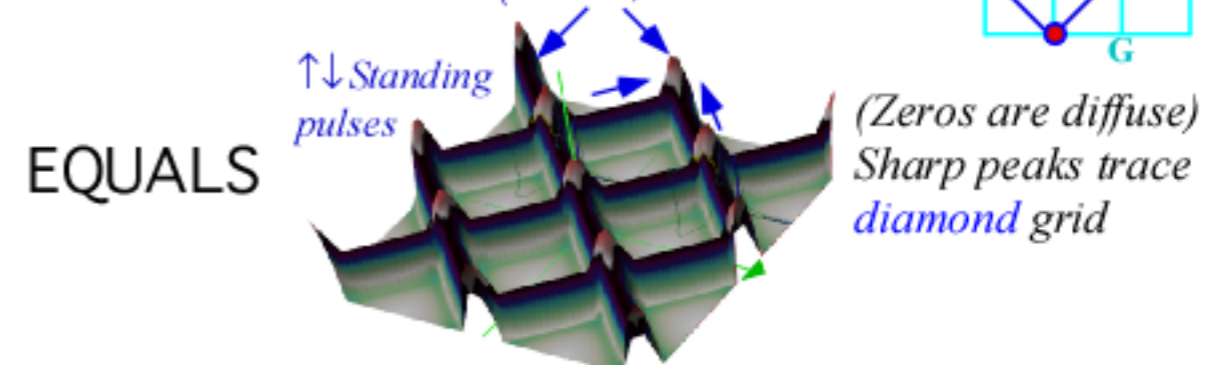
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Continuous Wave (CW) Addition



Pulse Wave (PW) Addition



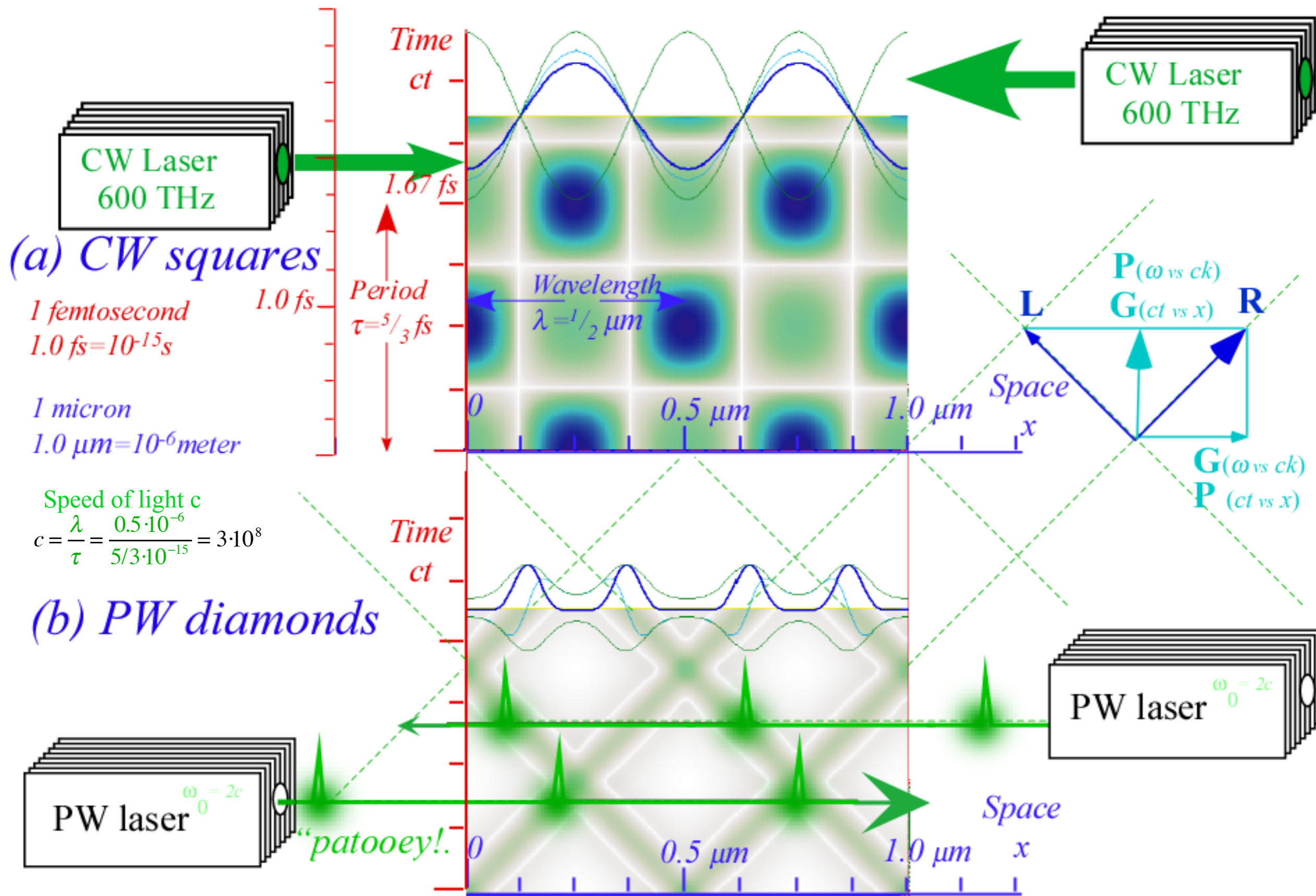
Pulse Wave (PW) sum compared with

- *PW* waves are OFF (0) or ON (1)
 - *PW* sum is Boolean $(0_L, 0_R), (0_L, 1_R), (1_L, 0_R), (1_L, 1_R)$.
 - *PW* time peak-diamond paths are wysiwyw. (What you see is what you expect!)
-

Continuous Wave (CW) sum

- *CW* waves range continuously from -1 to +1
 - *CW* sum is more subtle and nuanced interference.
 - *CW* time zero-square paths are subtle results of the half-sum **P**-rule and the half-difference **G**-rule of phase **P** and group **G** zeros.
-


http://www.uark.edu/ua/pirelli/php/waves_interfering_montage.php









Light waves are the lead actors in our portrayal of *relativity* and *quantum theory*.

This differs from standard treatments following Einstein's original works that are based more on Newtonian notions of *particles*, *bodies*, and *rigid frames*.    

Light, which Newton also regarded as fundamentally *corpuscular* (particle-like), had by the late 1800's been shown to have a fundamental *wave nature* due to work of (among others) Young, Huygens, and most notably, Maxwell. 

Then one of Einstein's 1905 works, following Planck's 1900 hypothesis of *light quanta*, showed that light also had to have a *particle-like* nature.   

Wave-particle duality may be understood by looking at *interference properties* of *waves in general* and how that applies *in particular* to *light waves*. 

Relativity and *quantum mechanics* are practically the same subject when viewed in the light of wave *interference/resonance*, that is, *light wave addition*.   

It helps to introduce two *archetypes* of light waves and contrast them.

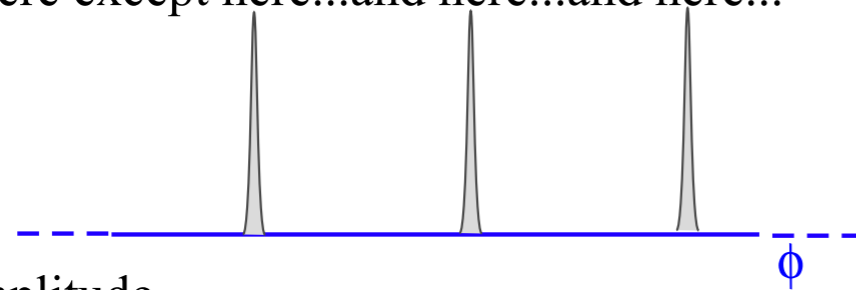
The first (*PW*) is a *Particle-like Wave* or part of a *Pulse-Wave* train.

The second (*CW*) is a *Coherent Wave* or part of a *Continuous-Wave* train.

(1) The *PW* archetype

PW amplitude is **ZERO**

everywhere except here...and here...and here...



PW amplitude...

...is mostly flat **ZEROS**.

...but has sharp **PEAKS**.

...is best defined by where it **IS**.

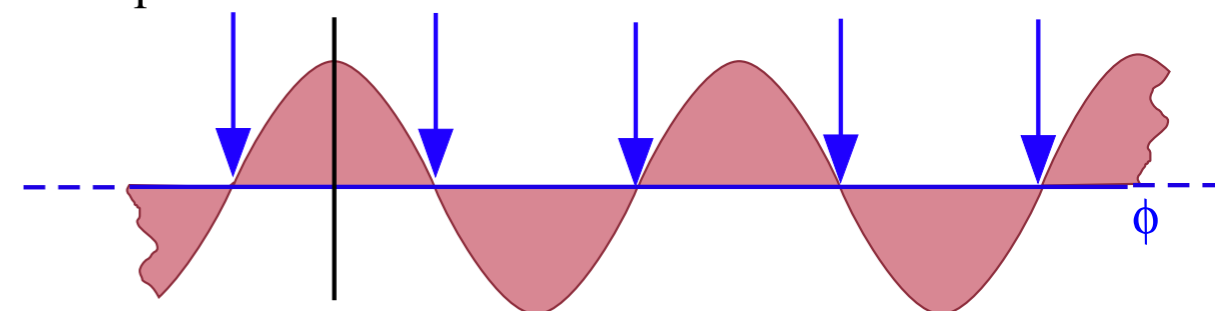
Ideal *PW* shape is a *Dirac Delta function*.

(Discussed on next page)

(2) The *CW* archetype

CW amplitude is **NON-zero**

everywhere except here...and here...and here...and here...



...is mostly **NON-zero** with rounded crests and troughs.

...but has sharp **ZEROS**.

...is best defined by where it **IS NOT**.

Ideal *CW* shape is a *cosine wave* ($\cos(\phi)$)

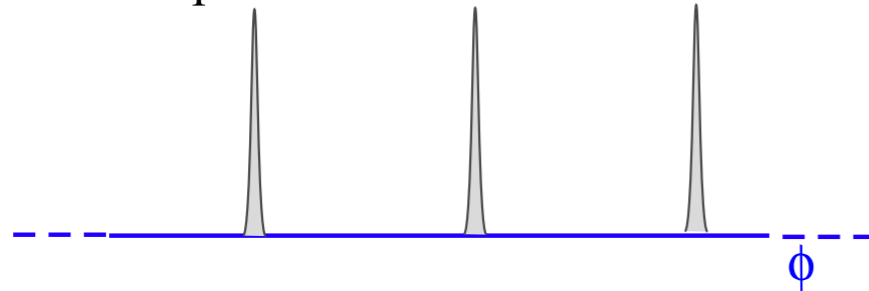
(Discussed on next page)

...continuing to contrast two light wave *archetypes*:

PW Pulse-Wave trains
 (1) *The PW archetype*

CW Continuous-Wave trains.
 (2) *The CW archetype*

PW amplitude is **ZERO**
 everywhere except here...and here...and here...



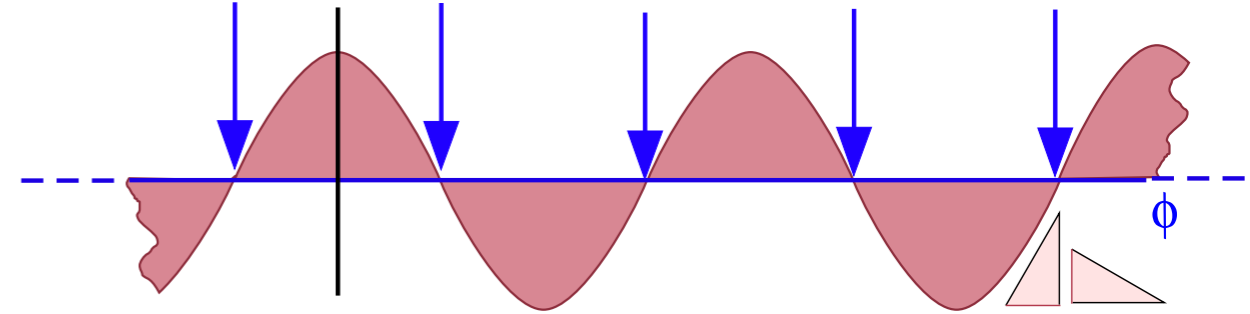
Ideal *PW* shape is the *Dirac Delta function* $\delta(\phi)$...
 infinitely high at one point and zero elsewhere!
 (Definition: $\delta(\phi)=\infty$ if $\phi=0$ else $\delta(\phi)=0$) Also,
 its area is one! ($\int d\phi \delta(\phi)=1$) This mathematical
 definition is not attainable in a laser lab.
 (An infinite pulse uses all the energy in the universe!)

Real laser lab *PW* shape varies a lot...
 ...Gaussian? ...sawtooth?...square?...etc.

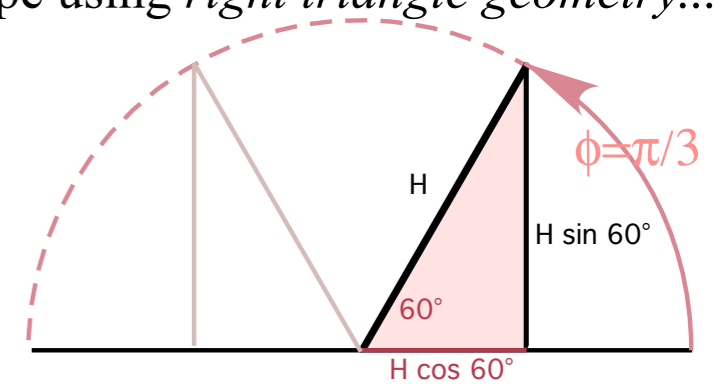
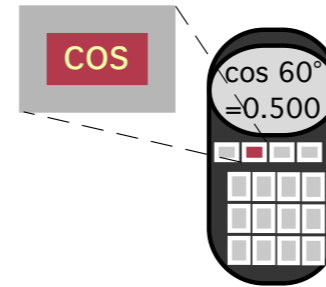


Quite a variety of shapes.
 (A jungle of possibilities!)

CW amplitude is **NON-zero**
 everywhere except here...and here...and here...and here...



Ideal *CW* *cosine wave* ($\cos(\phi)$) shape using *right triangle geometry*...
 is found in student-calculators,



Real laser lab *CW* shape is very nearly a *cosine wave* and can be
 tuned precisely to any *frequency* (or *color* if it's in *visible* spectrum)



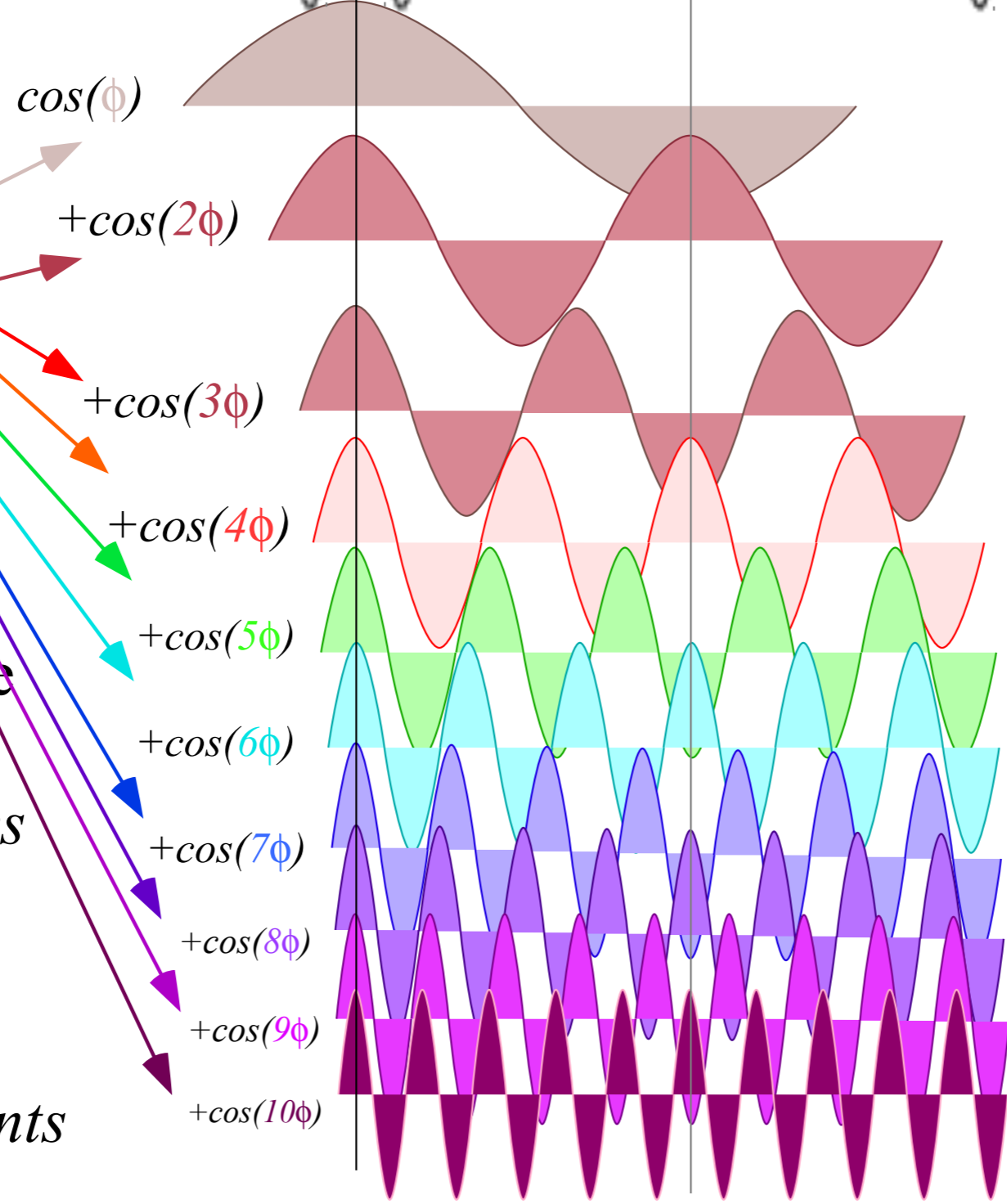
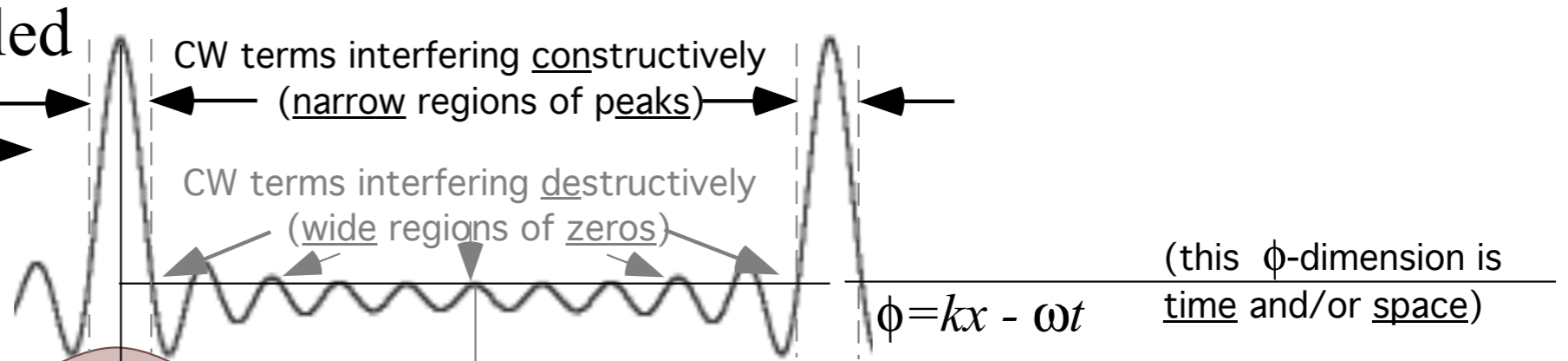
versus All the same shape.
 (Differing only in *frequency*, **amplitude**, and *phase*)

PW forms are also called *Wave Packets (WP)*

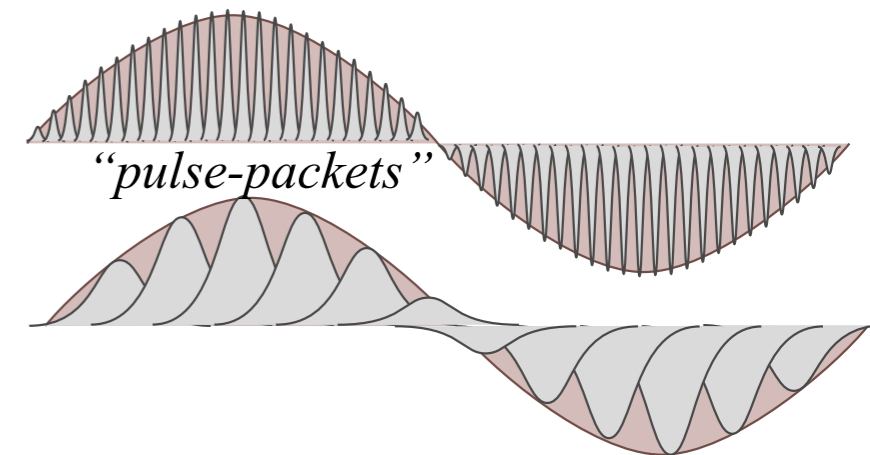
since they are interfering sums of many *CW* terms

(10-*Cosine Waves* make up this pulse)

CW terms are also called *Color Waves* or *Fourier Spectral Components*



... and *vice-versa* ... *CW* forms can be made *artificially* from *PW* sums ...



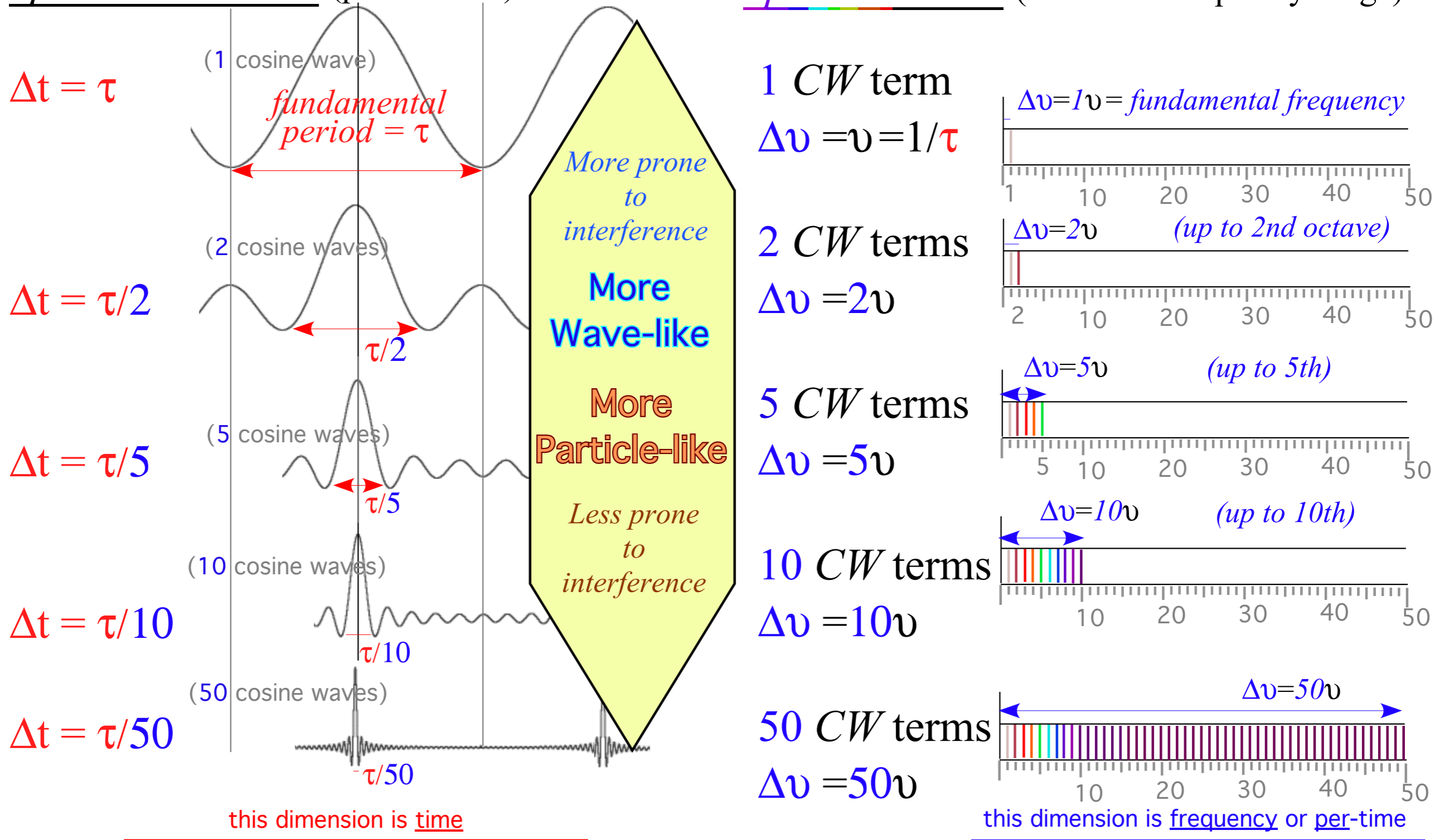
(this is digital *sampling* or *digital-to-analog synthesis*.)

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_anim.php

PW widths reduce proportionally with more *CW* terms (greater *Spectral* width)

Space-time width (pulse width)

Spectral width (harmonic frequency range)



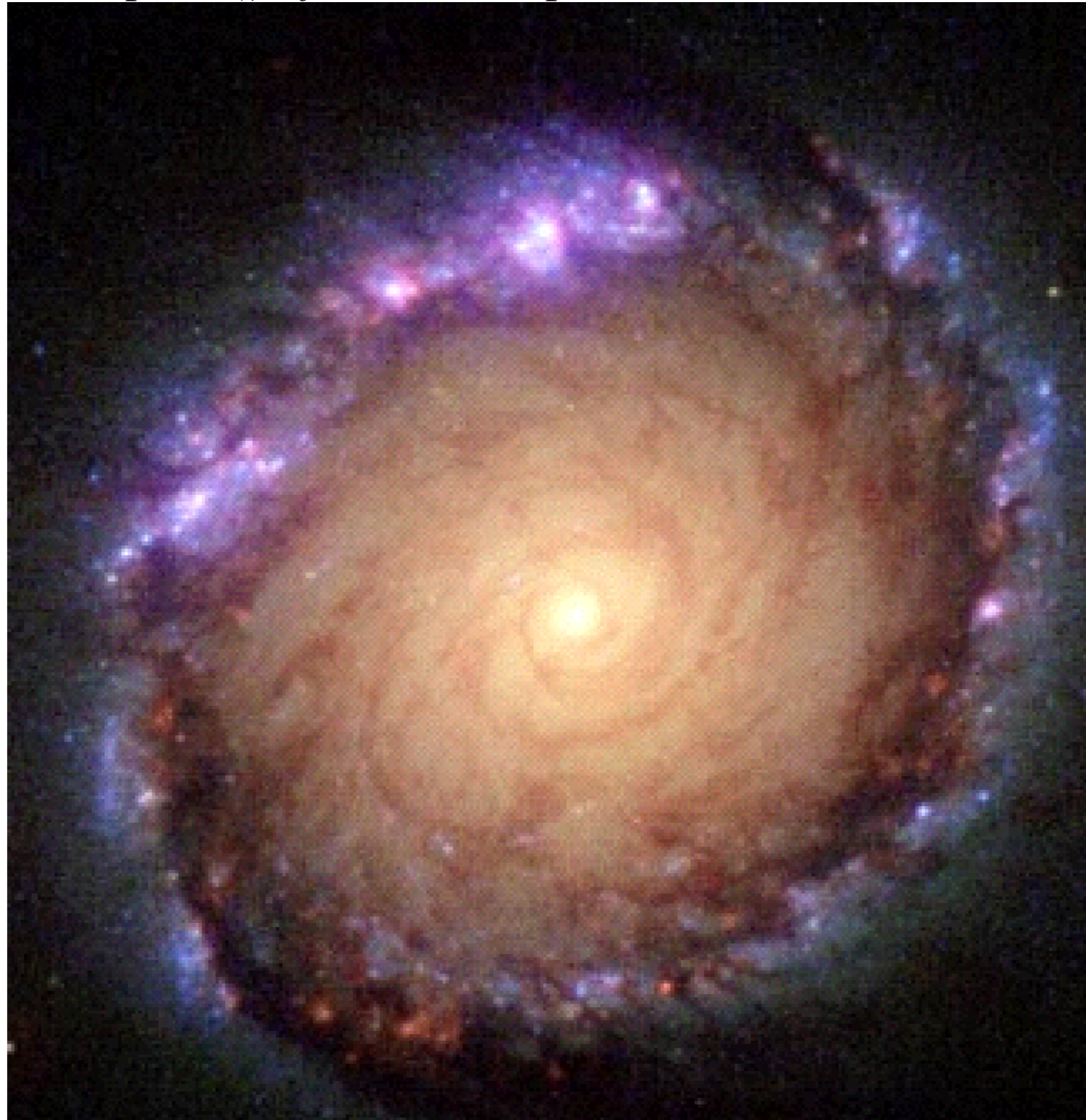
Fourier-Heisenberg product: $\Delta t \cdot \Delta \nu = 1$ (time-frequency uncertainty relation)

http://www.uark.edu/ua/pirelli/php/waves_pw_from_cw_width.php

How fast is light? Light goes one foot in a nano-second .

This may seem quite fast to us.

But, on a cosmic scale lightspeed is positively sub-glacial. In your lifetime light cannot cross one pixel (.) of the Hubble photo below.



Ways to recall (roughly) the speed c of light

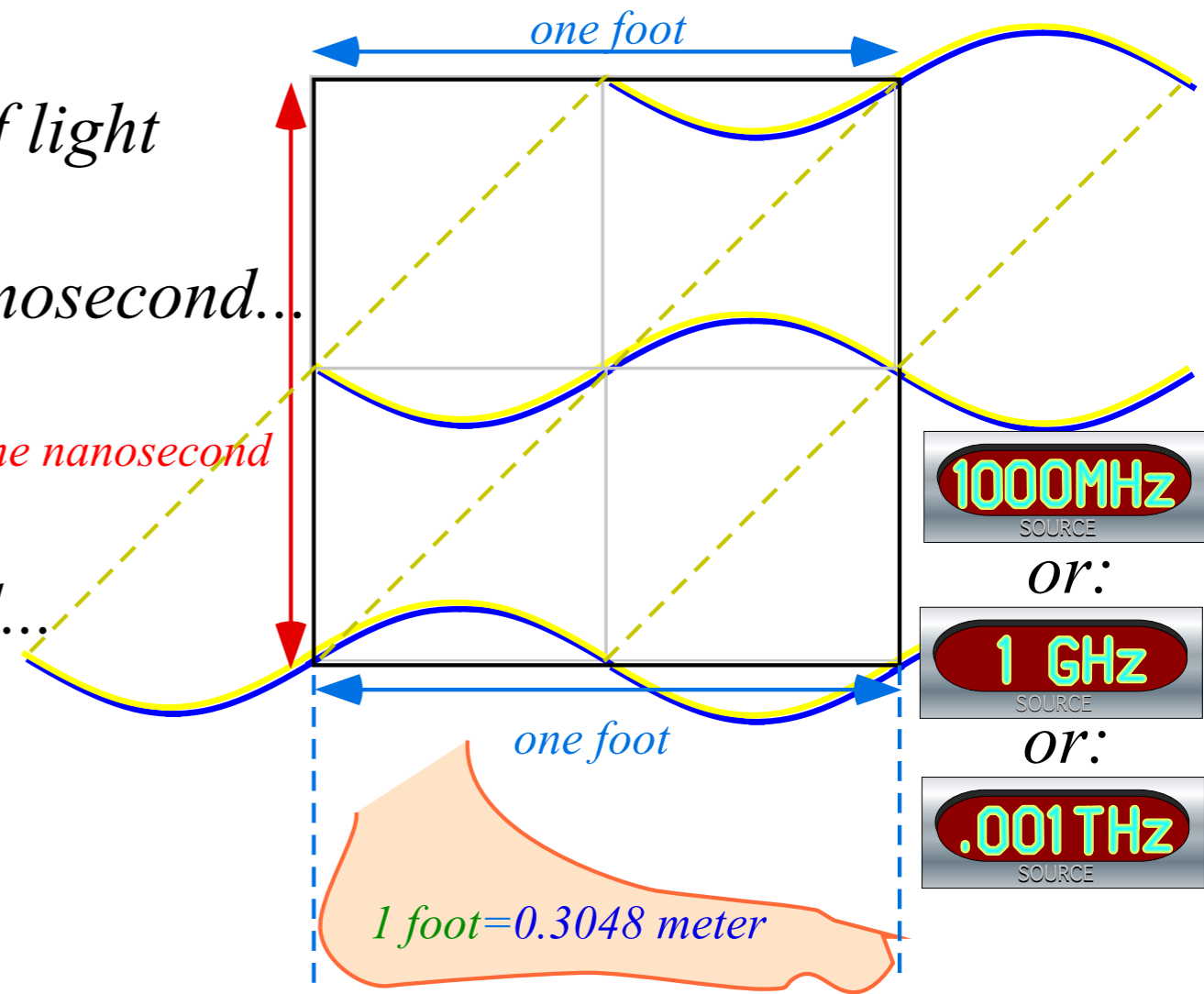
Light travels \sim one foot in one nanosecond...

1 foot \sim 1 light-nanosecond one nanosecond

US-English
units

...or \sim one billion feet in one second...

1 billion feet \sim 1 light-second



Light travels $\sim 3/10$ of a meter in one nanosecond...

$3/10$ meter \sim 1 light-nanosecond

Metric
units

...or \sim 3 hundred million meters in one second...

300,000,000 meters \sim 1 light-second

Current Standard c Value

K. M. Evenson - US NIST

$c = 299,792,458$ meters/second

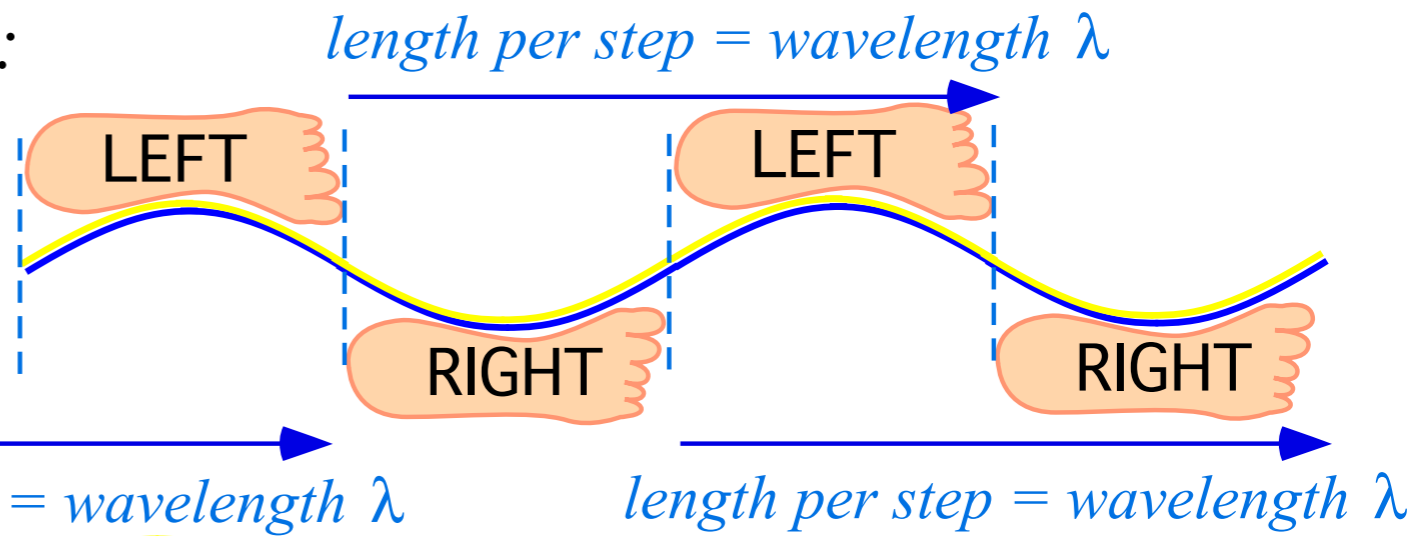
$c = 186,282.397$ miles/second

http://www.uark.edu/ua/pirelli/php/lightspeed_memmonic.php

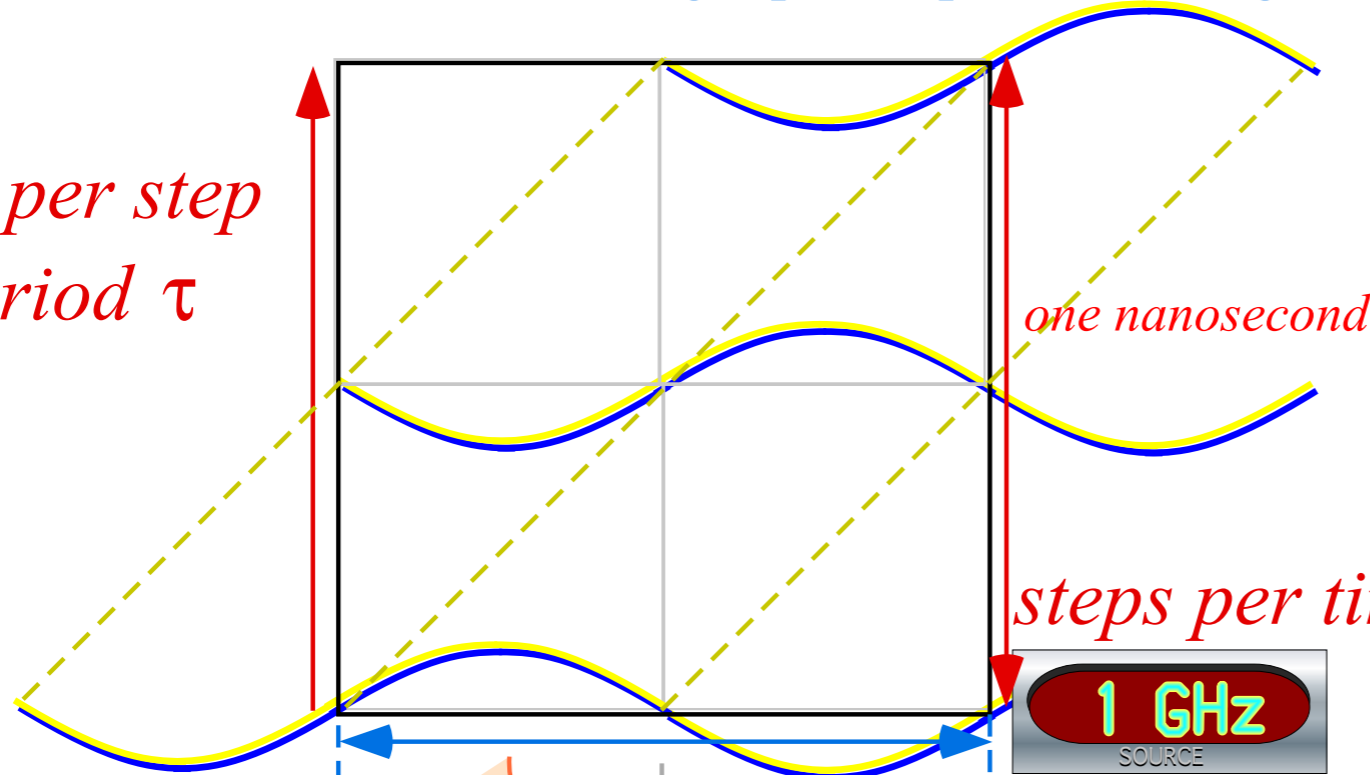
Think of light wave as pairs of steps:

Wave speed c is $\frac{\text{length per step}}{\text{time per step}}$

$$c = \frac{\lambda}{\tau}$$



time per step
= period τ



steps per time = Frequency $\nu = \frac{1}{\text{time per step}}$

$$\text{Frequency } \nu = \frac{1}{\tau}$$

one foot
 $1 \text{ foot} = 0.3048 \text{ meter}$

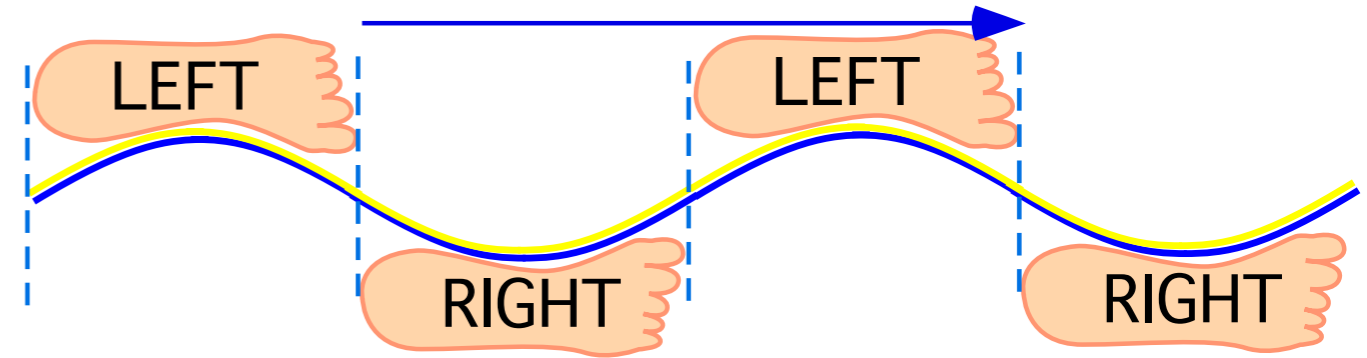
$1/2 \text{ foot} = 0.1524 \text{ meter}$

Light speed using *per-space* and *per-time*

meters per step = wavelength λ

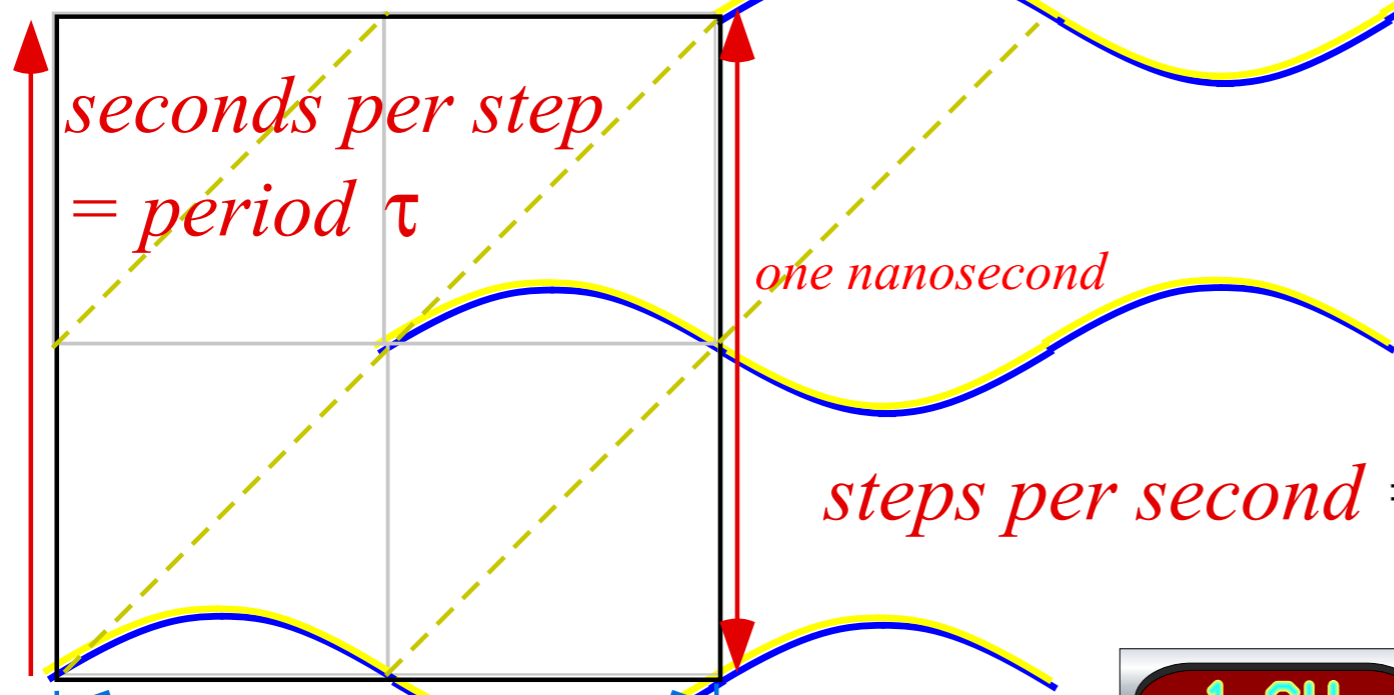
Wave speed c is $\frac{\text{steps per second}}{\text{steps per meter}}$

$$c = \frac{\nu}{\kappa}$$



steps per meter = Wave-number $\kappa = \frac{1}{\text{meters per step}}$

$$\text{Wave-number } \kappa = \frac{1}{\lambda}$$



steps per second = Frequency $\nu = \frac{1}{\text{seconds per step}}$

$$\text{Frequency } \nu = \frac{1}{\tau}$$

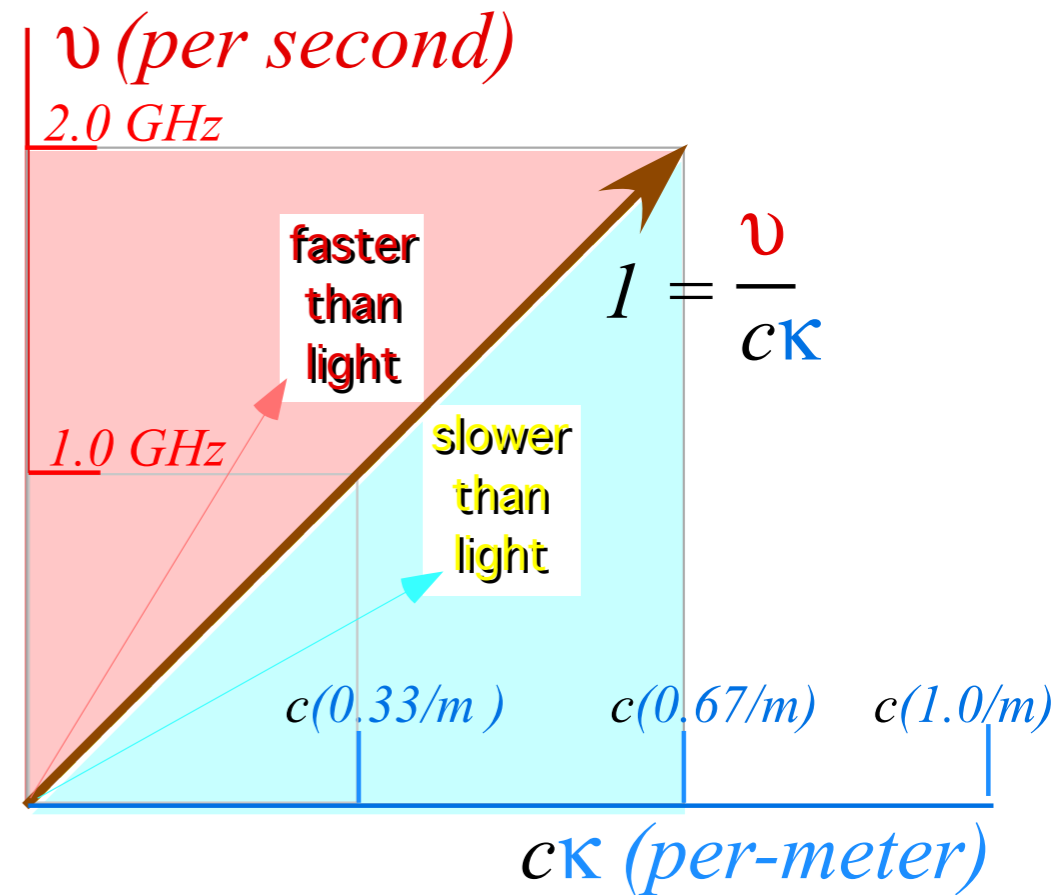
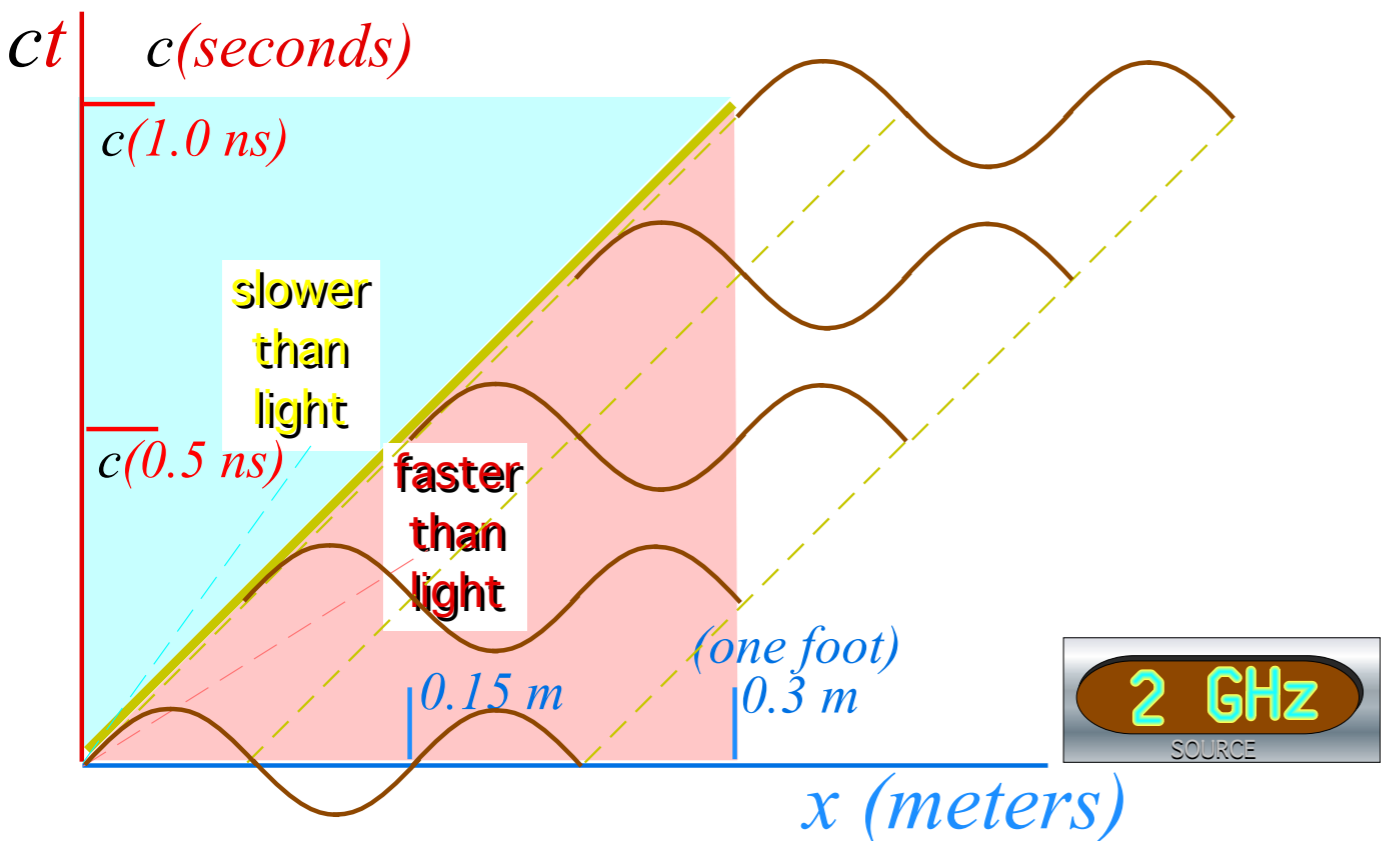
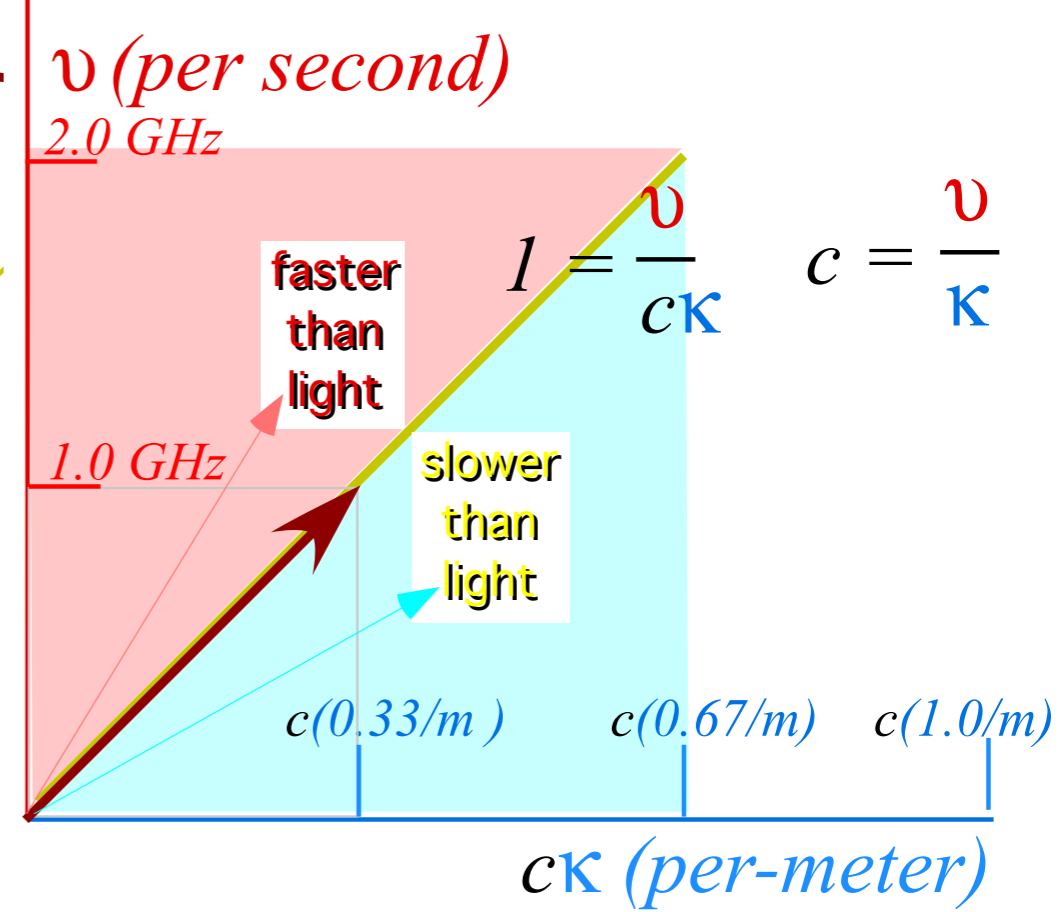
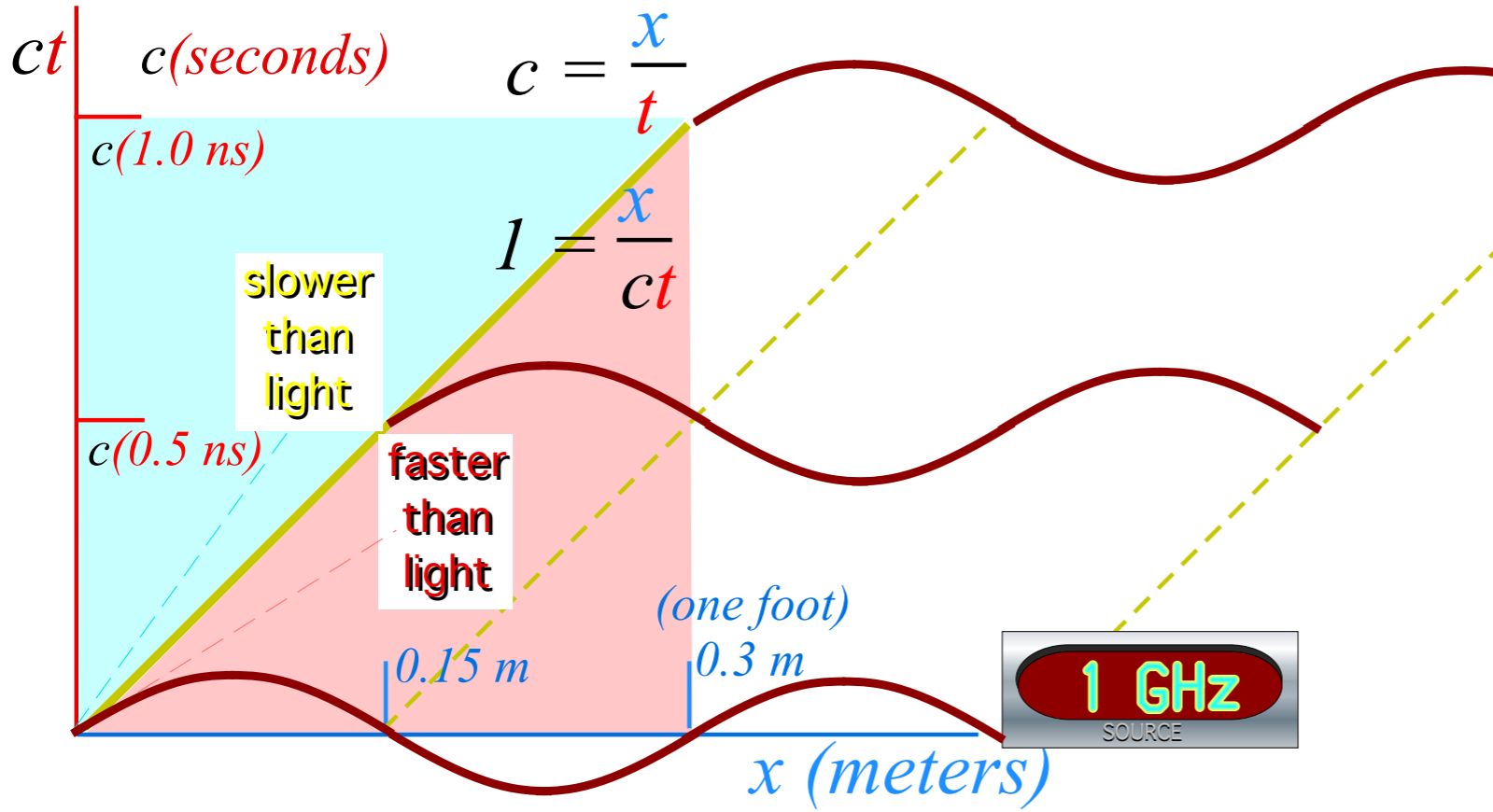


one foot
 1 foot = 0.3048 meter
 1/2 foot = 0.1524 meter

speed formulae:

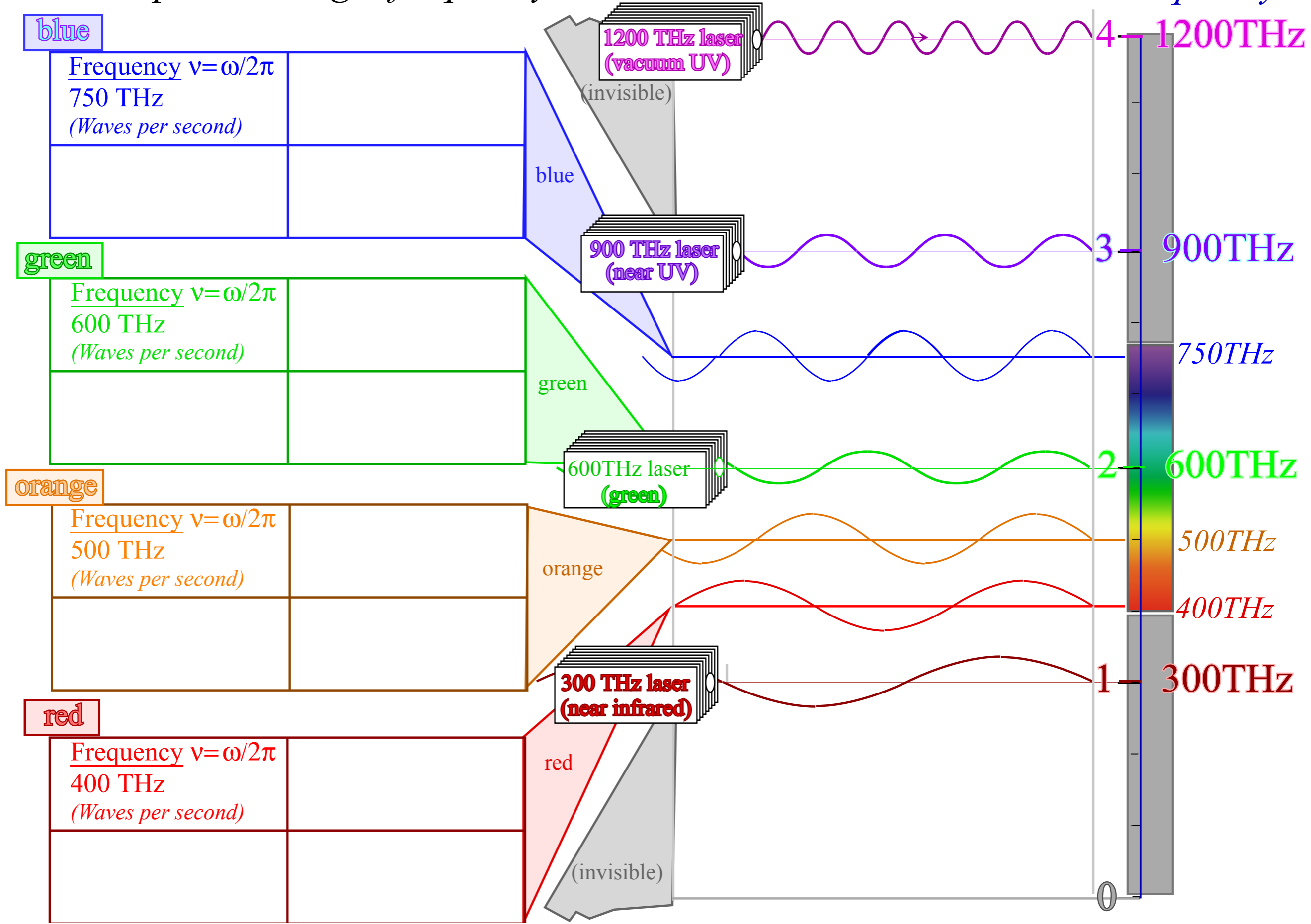
$c = \frac{\nu}{\kappa}$	$c = \lambda \cdot \nu$	$c = \frac{\lambda}{\tau}$
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Continuous Wave in Spacetime world defined by one vector in Per-Spacetime



Color depends on light frequency ν ...

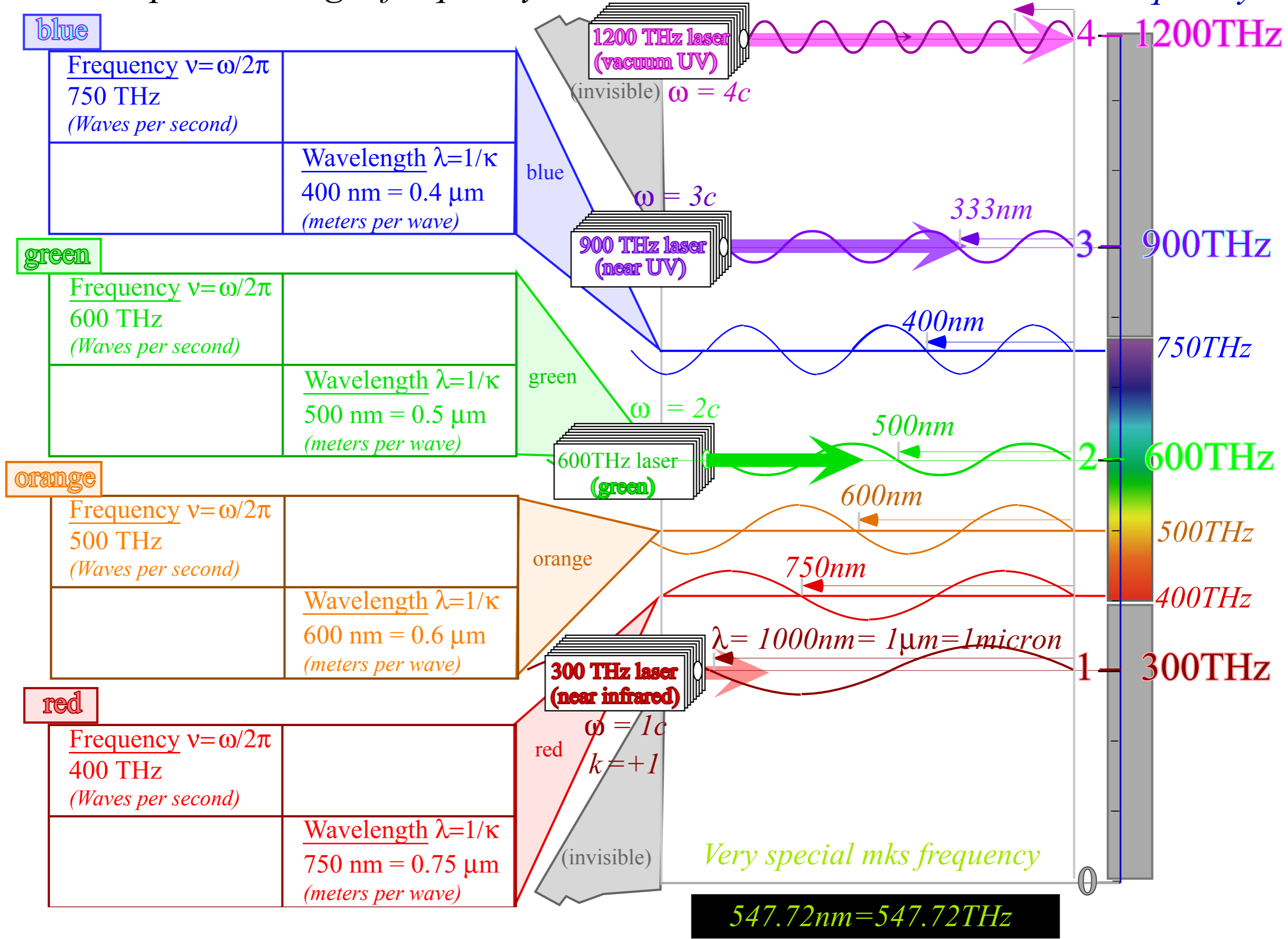
Frequency ν



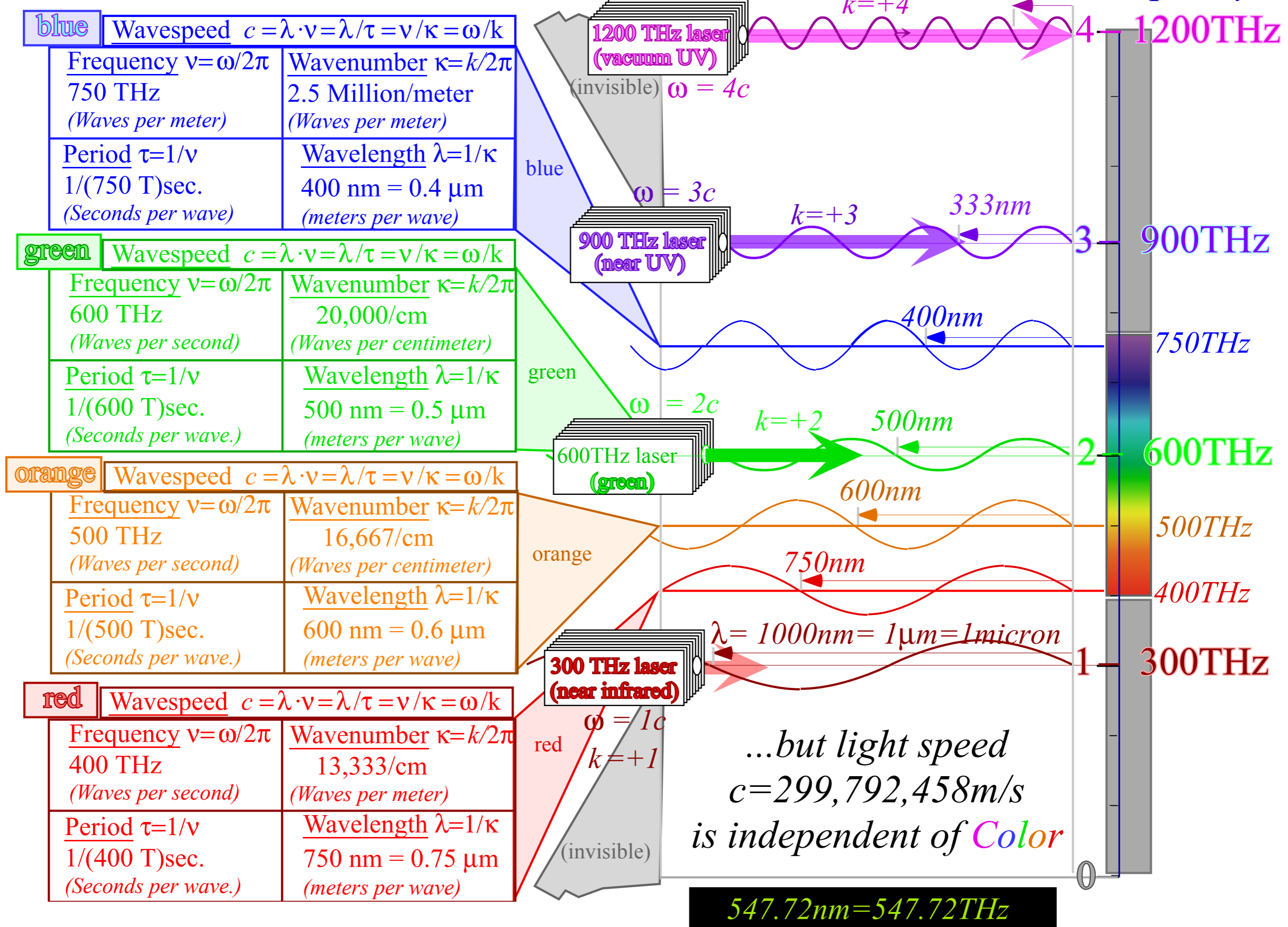
http://www.uark.edu/ua/pirelli/php/color_freq.php

Color depends on light frequency ν or wavelength λ ...

$\lambda=250\text{nm}$ Frequency ν

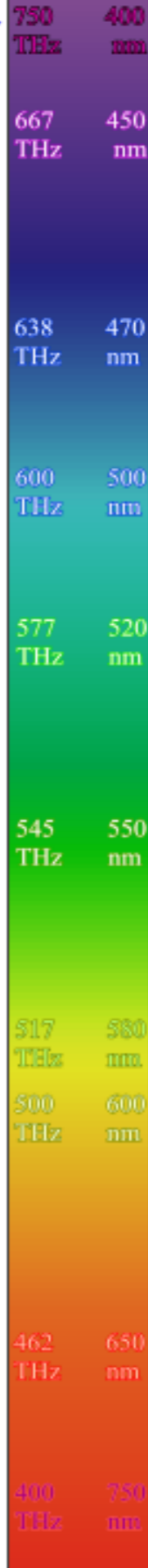


Color depends on light frequency ν or wavelength λ ...



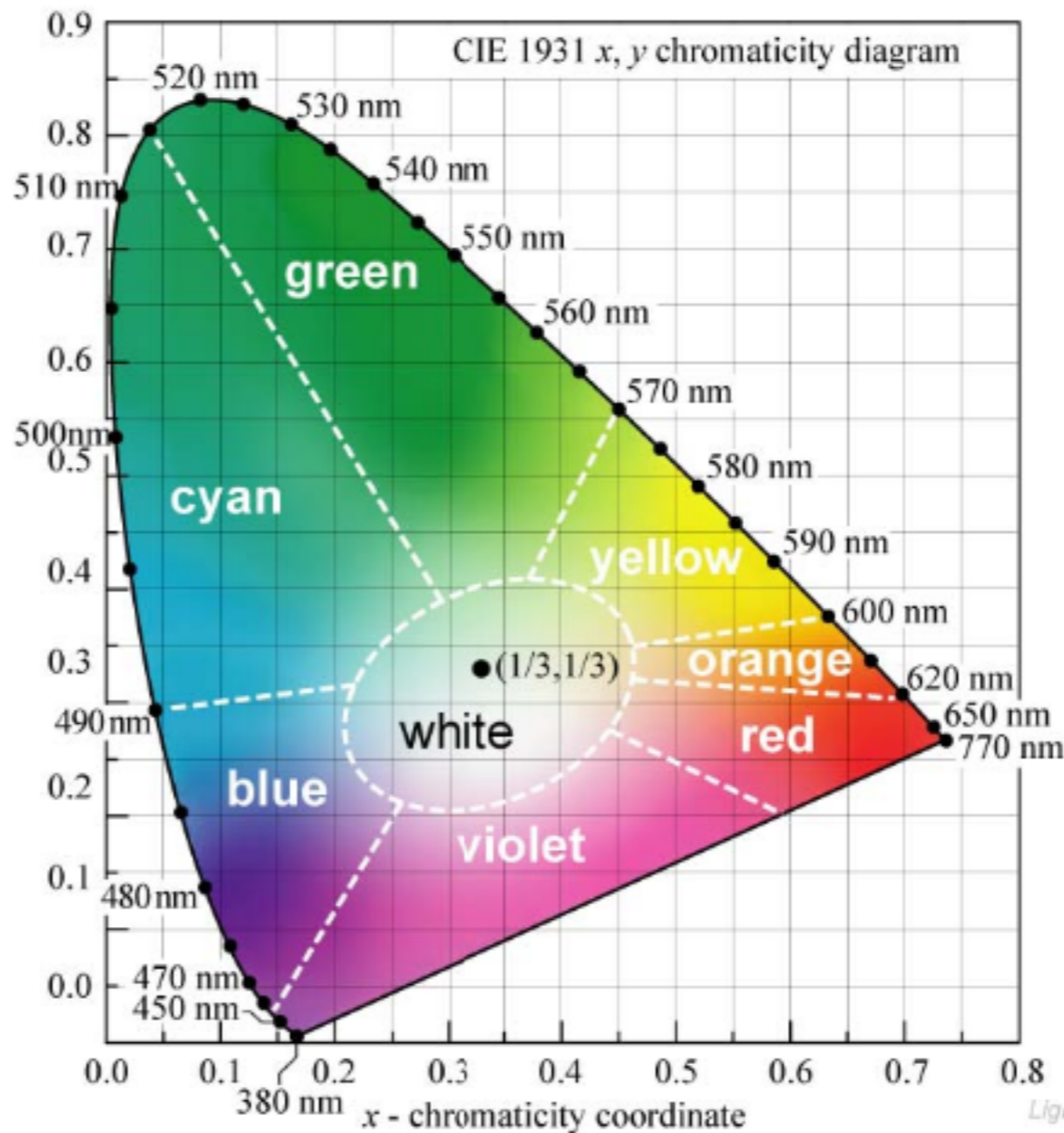
Frequency

Euclid to Einstein - A Colorful Approach to Relativity and Quantum Theory Harter 2006

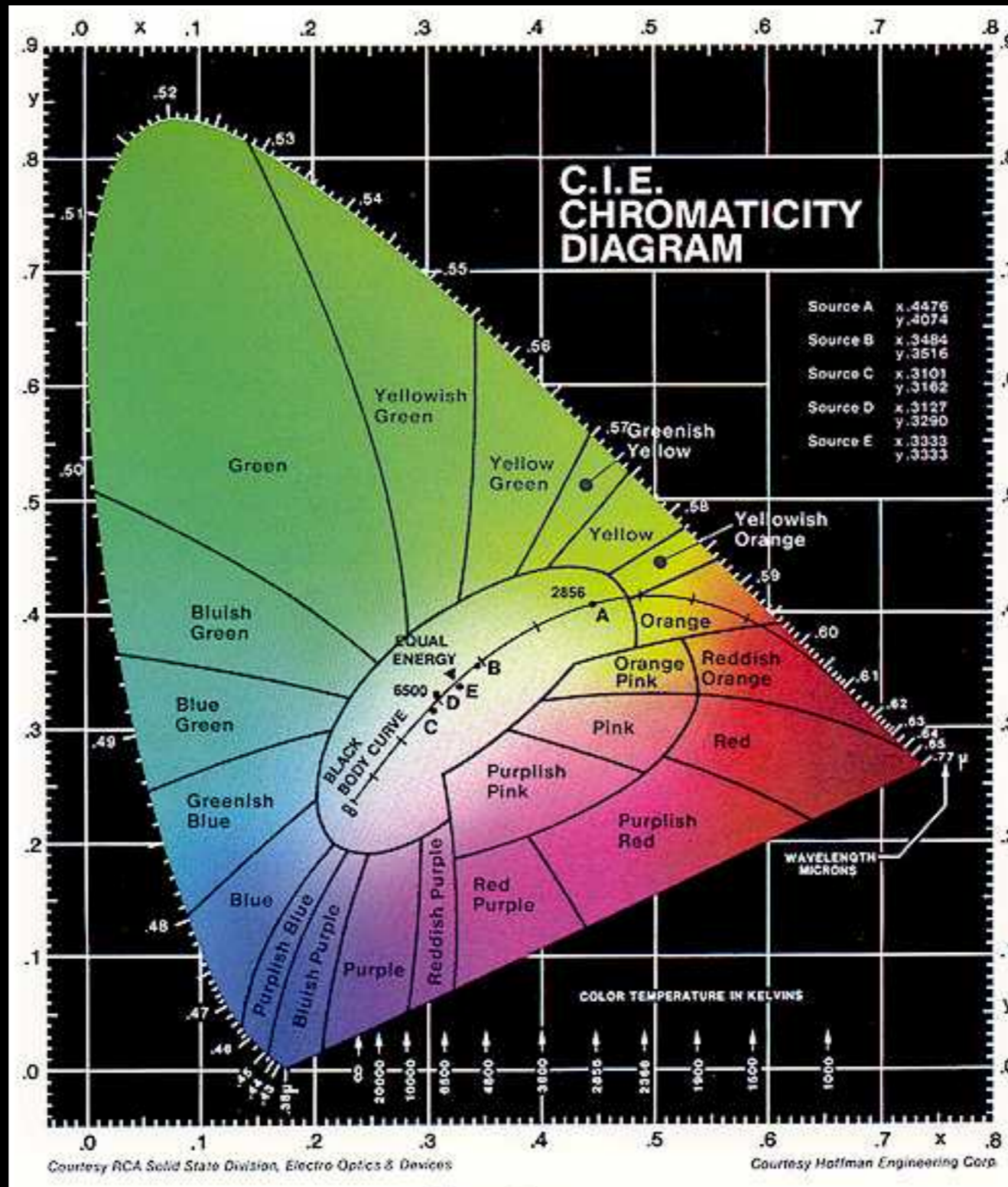


Wavelength vs. perceived color

1931 CIE "Slide Rule" for Human Perception



Lig



2. Applying Occam's razor to relativity axioms



→ *Einstein PW Axioms versus Evenson CW Axioms (Traditional: The "Roadrunner" Axiom)*

CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear? ... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

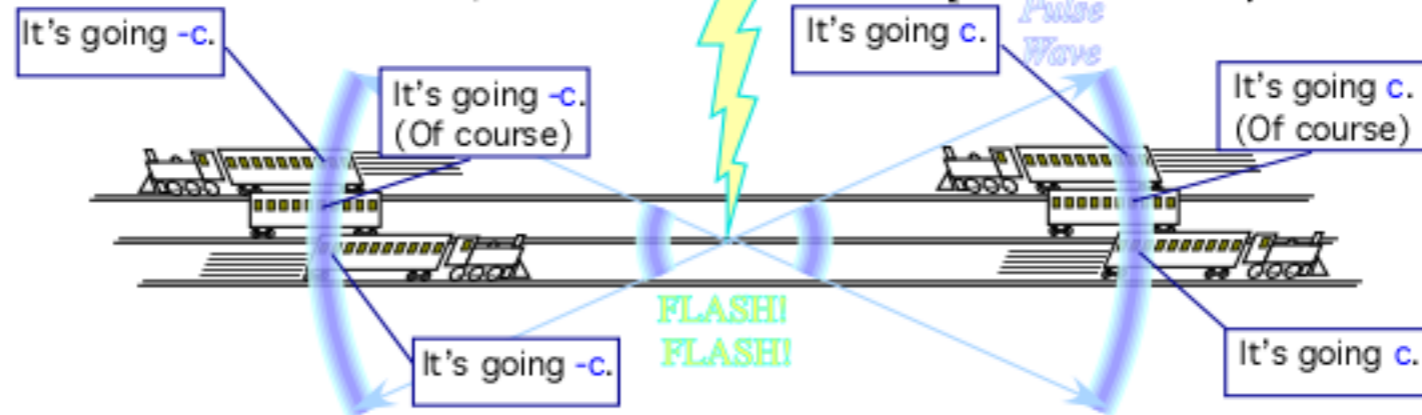
Tradition: Start with the "Roadrunner" Axiom!

Albert Einstein



1879-1955

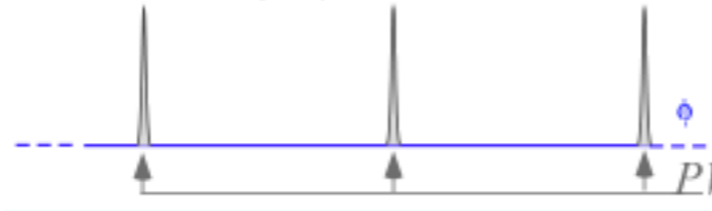
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom is a "show-stopper"



Pulse wave (PW) train



$$A_1 \cos \omega t + A_2 \cos 2\omega t + A_3 \cos 3\omega t + A_4 \cos 4\omega t + \dots$$

Complicated

PW peaks precisely locate places where wave is.

#	Release date	Title	Duration	Credits		Pseudo-Latin names given		Acme Corporation devices used	Books Studied
				Story/writing	Direction	For the Road Runner	For the Coyote		
1	1949-9-17	<i>Fast and Furry-ous</i>	6:55	Michael Maltese	Chuck Jones	Acceleratii incredibus	Carnivorous vulgaris	ACME Super Outfit	
2	1952-5-24	<i>Beep, Beep</i>	6:45	Michael Maltese	Chuck Jones	Accelerati incredibilus	Carnivorous vulgaris	Asprin, Matches, Rocket-Powered Roller Skates	None
3	1952-8-23	<i>Going! Going! Gosh!</i>	6:25	Michael Maltese	Chuck Jones	Acceleratti incredibilis	Carnivorous vulgaris	an anvil, a weather balloon, a street cleaner's bin, and a fan	
4	1953-9-19	<i>Ziping Along</i>	6:55	Michael Maltese	Chuck Jones	Velocitus tremenjus	Road-Runnerus digestus	Giant Kite Kit, Bomb, Detonator, Nitroglycerin	
5	1954-8-14	<i>Stop! Look! And Hasten!</i>	7:00	Michael Maltese	Chuck Jones	Hot-roddicus supersonicus	Eatibus anythingus	Bird Seed, Triple Strength Fortified Leg Muscle Vitamins	"How to Build a Burmese Tiger Trap"
6	1955-4-30	<i>Ready, Set, Zoom!</i>	6:55	Michael Maltese	Chuck Jones	Speedipus Rex	Famishus-Famishus	Glue	
7	1955-12-10	<i>Guided Muscle</i>	6:40	Michael Maltese	Chuck Jones	Velocitus delectibilus	Eatibus almost anythingus	ACME Grease	
8	1956-5-5	<i>Gee Whiz-z-z-z-z-z-z</i>	6:35	Michael Maltese	Chuck Jones	Delicious-delicious	Eatius birdius	ACME Triple Strength Battleship Steel Armor Plate, Rubber Band, Jet Bike	
9	1956-11-10	<i>There They Go-Go-Go!</i>	6:35	Michael Maltese	Chuck Jones	Dig-outius tid-bittius	Famishius fantasticus		
10	1957-1-26	<i>Scrambled Aches</i>	6:50	Michael Maltese	Chuck Jones	Tastyus supersonicus	Eternalii famishiis	ACME Dehydrated Boulders, Outboard Steam Roller	
11	1957-9-14	<i>Zoom and Bored</i>	6:15	Michael Maltese	Chuck Jones	Birdibus zippibus	Famishus vulgarus	ACME Bumblebees	
12	1958-4-12	<i>Whoa, Be-Gone!</i>	6:10	Michael Maltese	Chuck Jones	Birdius high-ballius	Famishius vulgaris ingeniusi	Tornado Seeds	
13	1958-10-11	<i>Hook, Line and Stinker</i>	5:55	Michael Maltese	Chuck Jones	Burnius-roadibus	Famishius-famishius		
14	1958-12-6	<i>Hip Hip-Hurry!</i>	6:13	Michael Maltese	Chuck Jones	digoutius-unbelieveablii	eatius-slobbius		
15	1959-5-9	<i>Hot-Rod and Reel!</i>	6:25	Michael Maltese	Chuck Jones	Super-sonicus-tastius	Famishius-famishius	Jet-Propelled Pogo Stick, Jet-Propelled Unicycle	None.
16	1959-10-10	<i>Wild About Hurry</i>	6:45	Michael Maltese	Chuck Jones	Batoutahelius	Hardheadipus oedipus	Giant Elastic Rubber Band, 5 Miles of Railroad Track, Rocket Sled, Bird Seed, Iron Pellets, Indestructo Steel Ball	None
17	1960-1-9	<i>Fastest with the Mostest</i>	7:20	None	Chuck Jones	Velocitus incalcublui	Carnivorous slobbius		

*List 1-17 of Roadrunner Episodes
Chuck Jones-Wikipedia-2012*

18	1960-10-8	<i>Hopalong Casualty</i>	6:05	Chuck Jones	Chuck Jones	speedipus-rex	Hard-headipus ravenus	Christmas Packaging Machine, Earthquake Pills
19	1961-1-21	<i>Zip 'N Snort</i>	5:50	Chuck Jones	Chuck Jones	digoutius-hot-rodus	evereadii eatibus	List 17-34 of Roadrunner Episodes Chuck Jones-Wikipedia-2012
20	1961-6-3	<i>Lickety-Splat</i>	6:20	Chuck Jones	Chuck Jones, Abe Levitow	Fastius tasty-us	Appetitius giganticus	
21	1961-11-11	<i>Beep Prepared</i>	6:00	John Dunn, Chuck Jones	Chuck Jones, Maurice Noble	Tid-bittius velocitus	Hungrii flea-bagius	ACME Iron Bird Seed
Film	1962-6-2	<i>Adventures of the Road Runner</i>	26:00	John Dunn, Chuck Jones, Michael Maltese	Chuck Jones	Super-Sonnicus Idioticus	Desertous-operativus Idioticus	
22	1962-6-30	<i>Zoom at the Top</i>	6:30	Chuck Jones	Chuck Jones, Maurice Noble	disappearialis quickius	overconfidentii vulgaris	Bird seed, instant icicle-maker, boomerang
23	1963-12-28	<i>To Beep or Not to Beep</i> ¹	6:35	John Dunn, Chuck Jones	Chuck Jones, Maurice Noble	None	None	
24	1964-6-6	<i>War and Pieces</i>	6:40	John Dunn	Chuck Jones, Maurice Noble	Burn-em upus asphaltus	Caninus nervous rex	Invisible Paint
25	1965-1-1	<i>Zip Zip Hooray</i> ²	6:15	John Dunn	Chuck Jones	Super-Sonnicus Idioticus	None	
26	1965-2-1	<i>Road Runner a Go-Go</i> ²	6:05	John Dunn	Chuck Jones	None	None	None
27	1965-2-27	<i>The Wild Chase</i>	6:30	None	Friz Freleng, Hawley Pratt	None	None	
28	1965-7-31	<i>Rushing Roulette</i>	6:20	David Detiege	Robert McKimson	None	None	
29	1965-8-21	<i>Run, Run, Sweet Road Runner</i>	6:00	Rudy Larriva	Rudy Larriva	None	None	
30	1965-9-18	<i>Tired and Feathered</i>	6:20	Rudy Larriva	Rudy Larriva	None	None	
31	1965-10-9	<i>Boulder Wham!</i>	6:30	Len Janson	Rudy Larriva	None	None	Deluxe Hi-bounce Trampoline Kit
32	1965-10-30	<i>Just Plane Beep</i>	6:45	Don Jurwich	Rudy Larriva	None	None	War Surplus Biplane
33	1965-11-13	<i>Haired and Hurried</i>	6:45	Nick Bennion	Rudy Larriva	None	None	Snow Machine, Magnetic Gun, Practice Bombs, Super Bomb, Kit
34	1965-12-11	<i>Highway Runnery</i>	6:45	Al Bertino	Rudy Larriva	None	None	

35	1965-12-25	<i>Chaser on the Rocks</i>	6:45	Tom Dagenais	Rudy Larriva	None	None	
36	1966-1-8	<i>Shot and Bothered</i>	6:30	Nick Bennion	Rudy Larriva	None	None	Suction Cups
37	1966-1-29	<i>Out and Out Rout</i>	6:00	Dale Hale	Rudy Larriva	None	None	No ACME labeled devices used.
38	1966-2-19	<i>The Solid Tin Coyote</i>	6:15	Don Jurwich	Rudy Larriva	None	None	
39	1966-3-12	<i>Clippety Clobbered</i>	6:15	Tom Dagenais	Rudy Larriva	None	None	
40	1966-11-5	<i>Sugar and Spies</i>	6:20	Tom Dagenais	Robert McKimson	None	None	Do-it-Yourself Kit Remote Control Missile-Bombs
41	1979-11-27	<i>Freeze Frame</i>	6:05	John W. Dunn Chuck Jones	Chuck Jones	Semper food-ellus	Grotesques appetitus	
42	1980-5-21	<i>Soup or Sonic</i>	9:10	Chuck Jones	Chuck Jones, Phil Monroe	Ultra-sonicus ad infinitum	Nemesis ridiculii	
43	1994-12-21	<i>Chariots of Fur</i> ³	7:00	Chuck Jones	Chuck Jones	Boulevardius-burnupius	Dogius ignoramii	
44	2000-12-30	<i>Little Go Beep</i>	7:55	Kathleen Helppie-Shipley, Earl Kress	Spike Brandt	Morselus babyfatus tastius	Poor schnookius	
45	2003-11-1	<i>The Whizzard of Ow</i>	7:00	Chris Kelly	Bret Haaland	<i>Geococcyx californianus</i> ⁴	<i>Canis latrans</i> ⁴	Book of Magic, Flying Broom, Bomb, Clear Paint
Film	2003-11-14	<i>Looney Tunes: Back in Action</i>	91:00	Larry Doyle	Joe Dante	None	Desertus operatus idioticus	
46	2010-7-30	<i>Coyote Falls</i> ³	2:59	Tom Sheppard ^[10]	Matthew O'Callaghan	None	None	Bird Seed, Bungee Cord
47	2010-9-24	<i>Fur of Flying</i> ³	3:03 ^[11]	Tom Sheppard	Matthew O'Callaghan ^[11]	None	None	Bonnie Bike, Mega-Motor, Football Helmet, Ceiling Fan
48	2010-12-17	<i>Rabid Rider</i> ³	3:07	Tom Sheppard	Matthew O'Callaghan	None	None	Hyper-Sonic Transport
49	TBA	<i>Untitled Wile E. Coyote and Road Runner Short Film</i>	5:38	Tom Sheppard	Matthew O'Callaghan	None	None	

2. Applying Occam's razor to relativity axioms

→ *Einstein PW Axioms versus Evenson CW Axioms (Traditional: The "Roadrunner" Axiom)*

CW light clearly shows Doppler shifts

Check that red is red is red, ... green is green is green, ... blue is blue is blue, ... etc.

Is dispersion linear? ... does astronomy work? ... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.

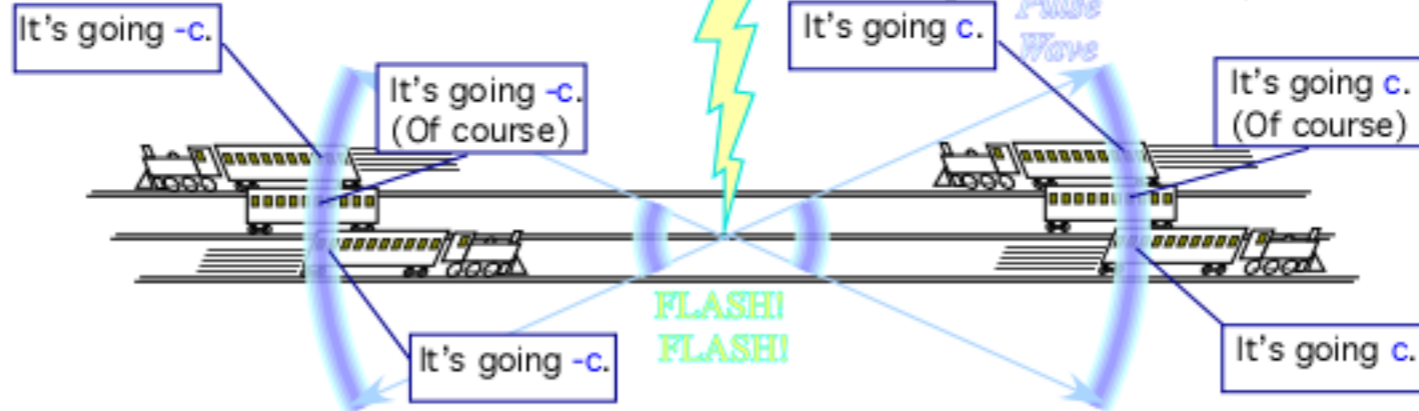
That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

Albert Einstein



1879-1955

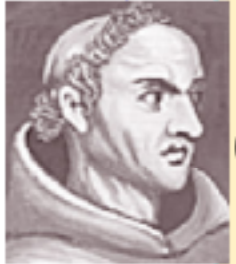
Einstein Pulse Wave (PW) Axiom: PW speed seen by all observers is c



A "road-runner" axiom is a "show-stopper"



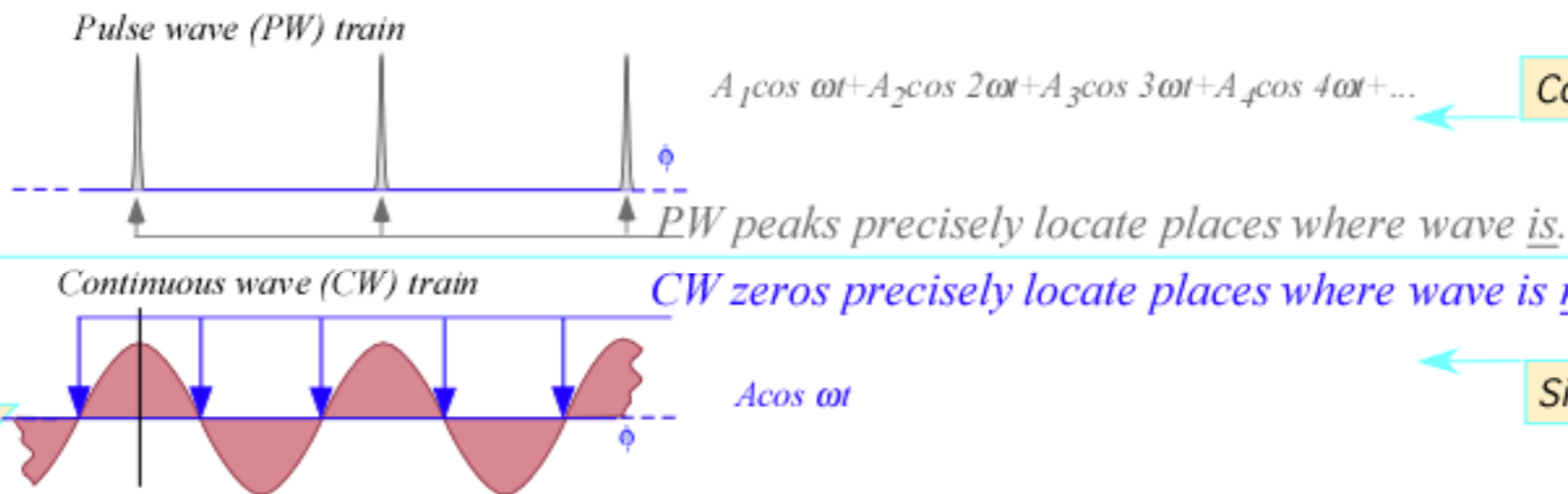
William of Ockham



1285-1349

Using Occam's Razor

(and Evenson's lasers)

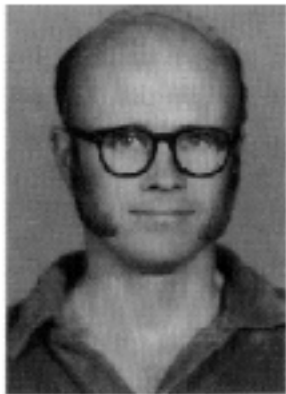


Complicated

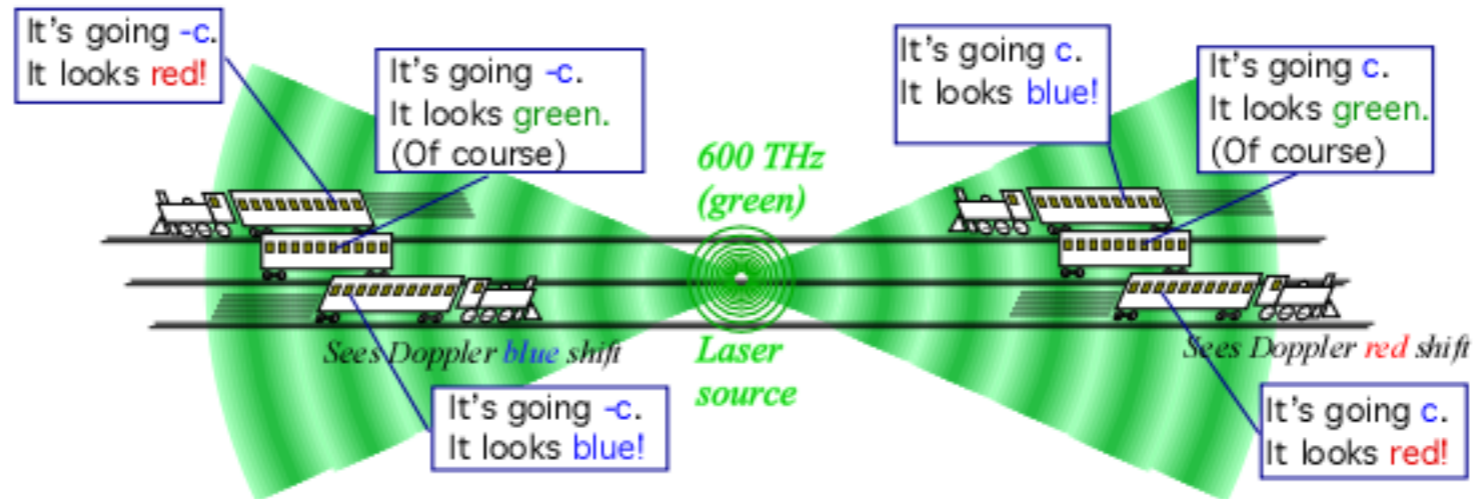
Simpler

Evenson Continuous Wave (CW) axiom: CW speed for all colors is c

Kenneth Evenson



1929-2002
 $c = 299,792,458 \text{ m/s}$



More self-evident "must-be" axiom

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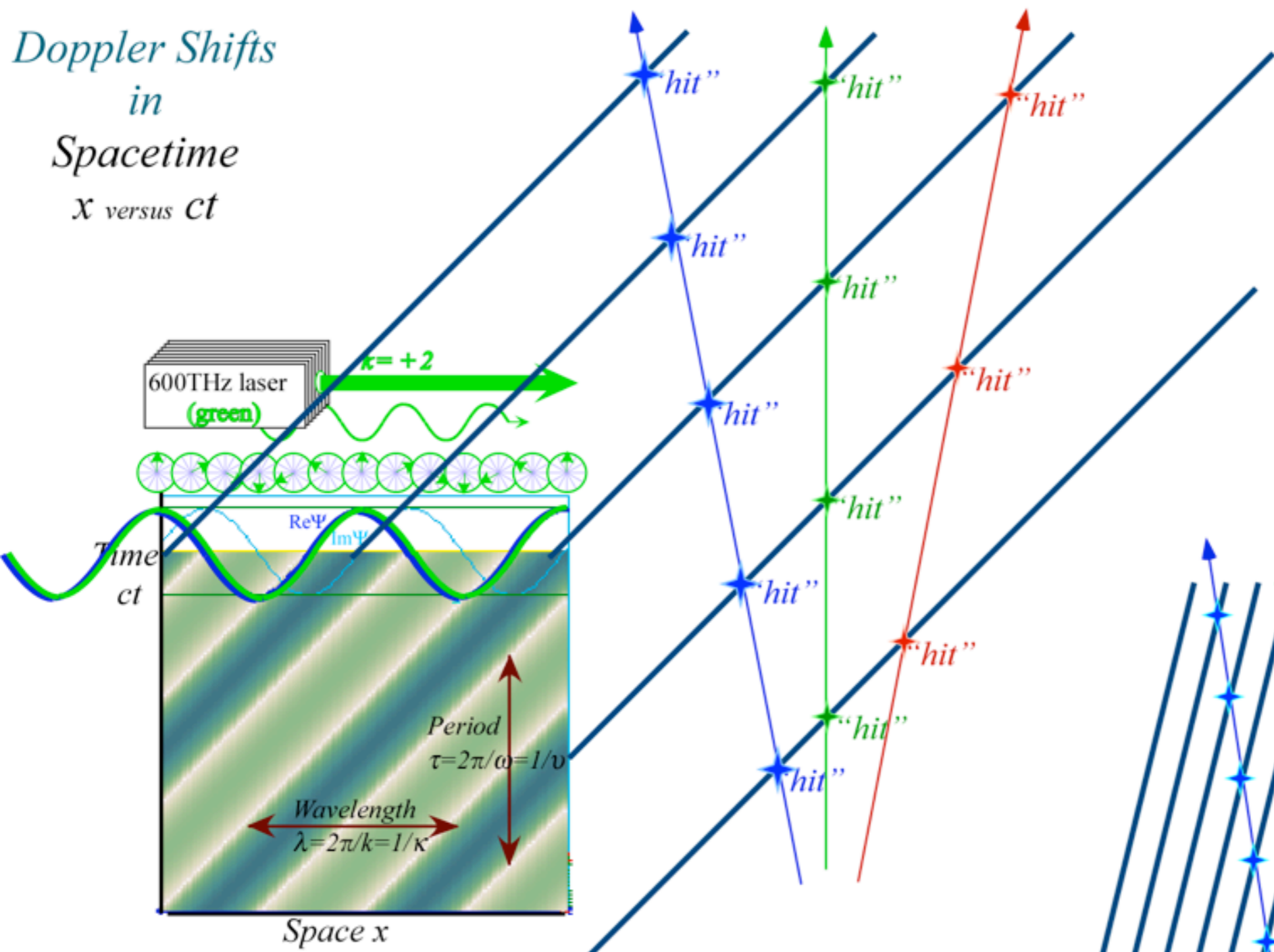
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Doppler Blueshift
 More "hits" per sec. if moving toward laser source

Doppler Redshift
 Fewer "hits" per sec. if moving away from laser source

Doppler Shifts in Spacetime
x versus *ct*



Doppler's picture needs revision for light whose period and wavelength both shift.

Why?

...So that all colors go the same speed!

$$v \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c$$

$$v \cdot \lambda = \frac{\lambda}{\tau} = \frac{\omega}{k} = c \quad \text{etc.}$$

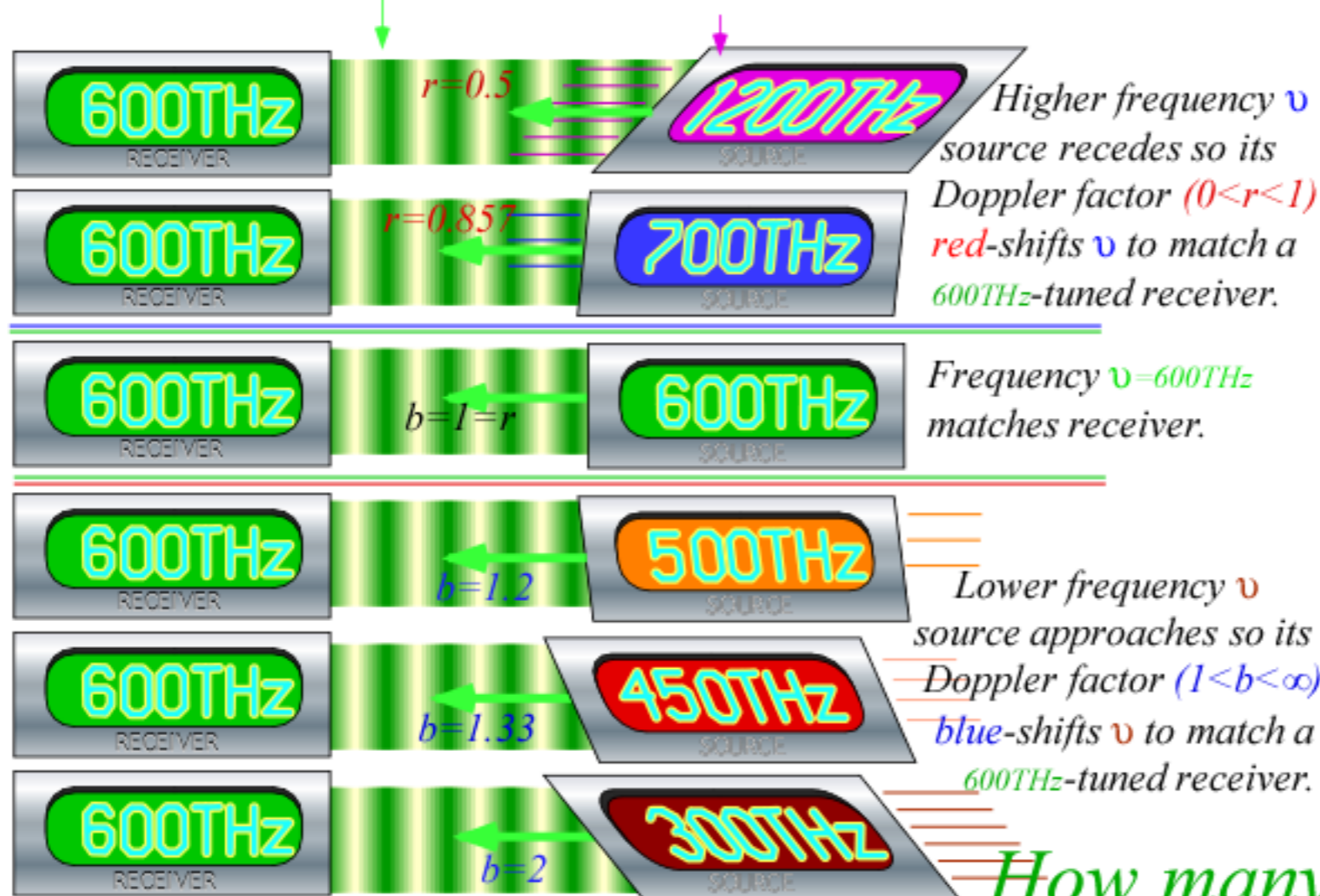
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Related subject matter at:
http://www.uark.edu/ua/pirelli/php/doppler_segue.php

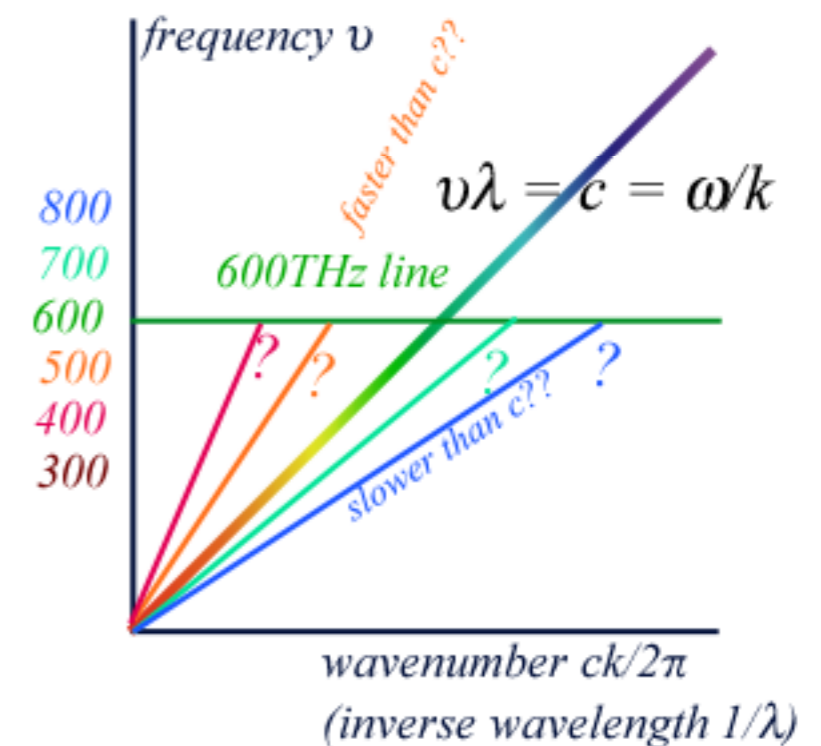
CW Axiom ("All colors go c.") based on Doppler effects

Showing that Green is Green is Green... (and all the same speed)...

Any color (like 600THz green) may be made by any other color source Doppler shifted by some speed u (less than c)



How many ways can you make 600THz green?



How many kinds of green exist?
(It's either 1 or ∞ .)

Related subject matter at:

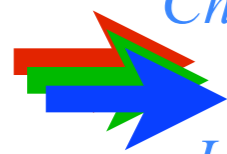
http://www.uark.edu/ua/pirelli/php/doppler_cw_logic_3.php

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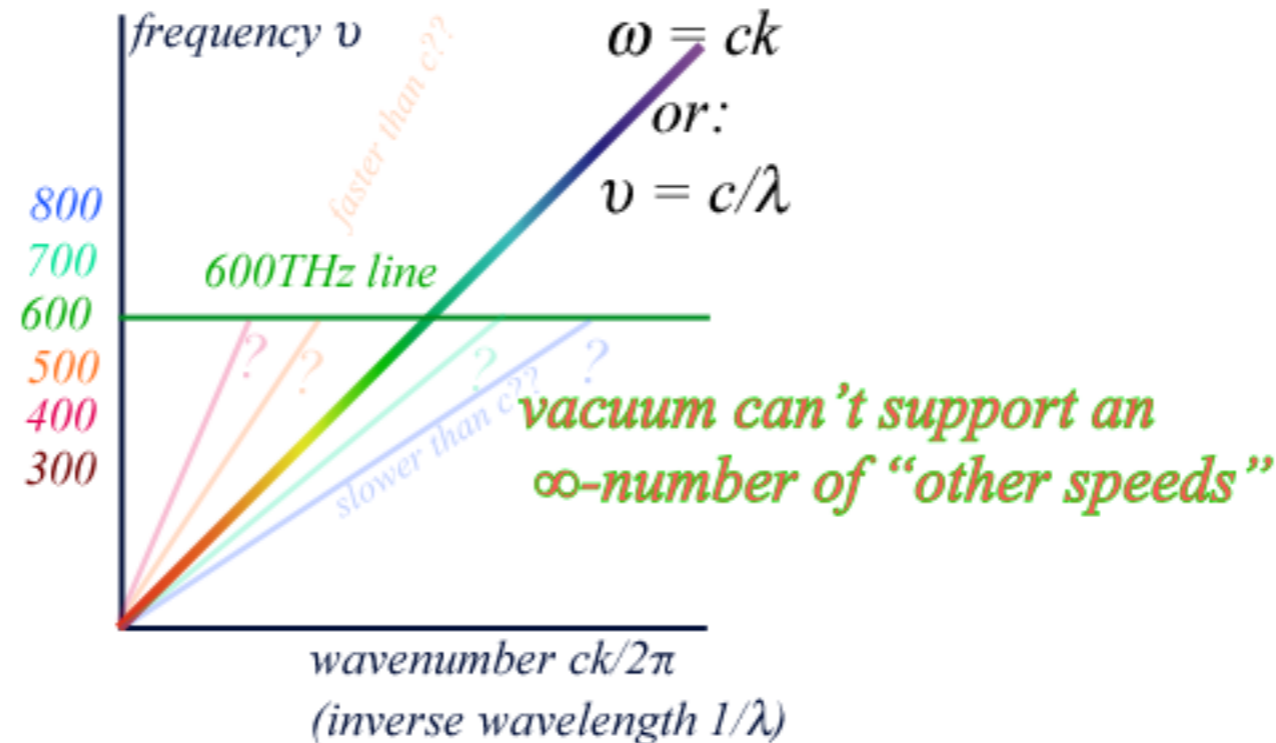
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Linear dispersion: $\omega = ck$

Linear dispersion means NO dispersion

Einstein PW is corollary of Evenson CW

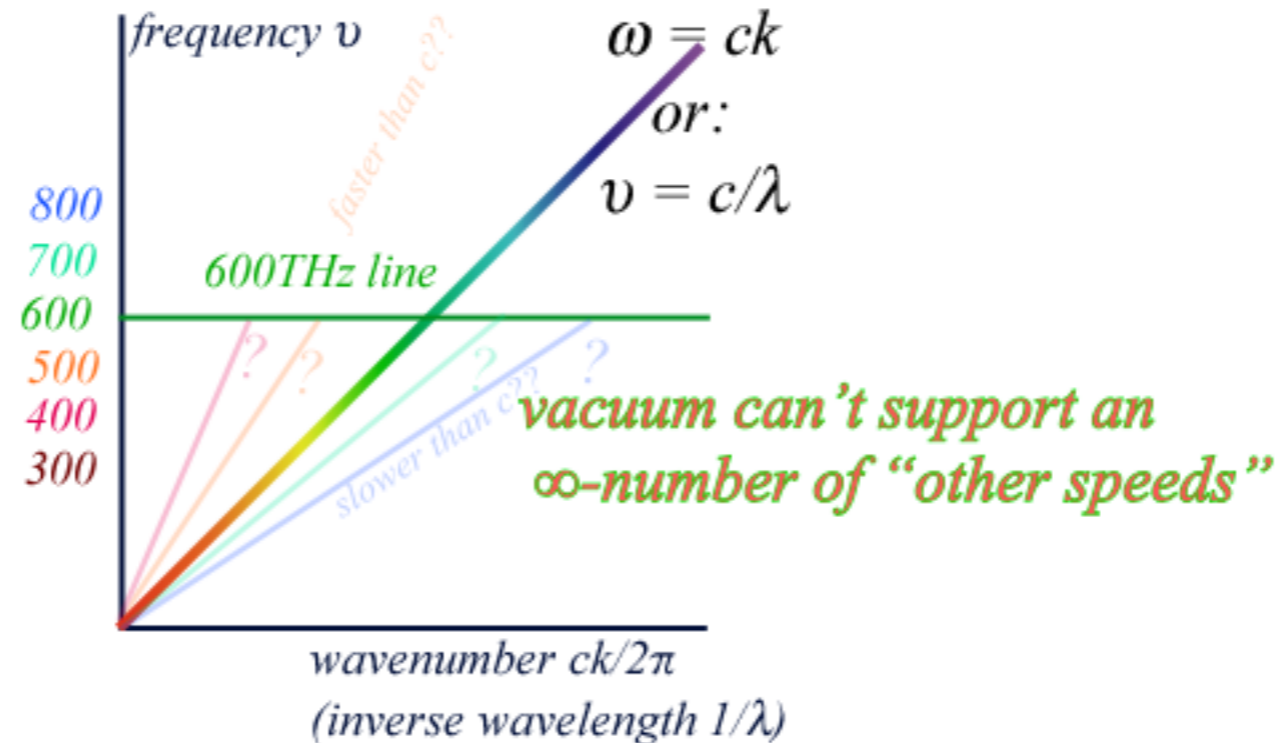


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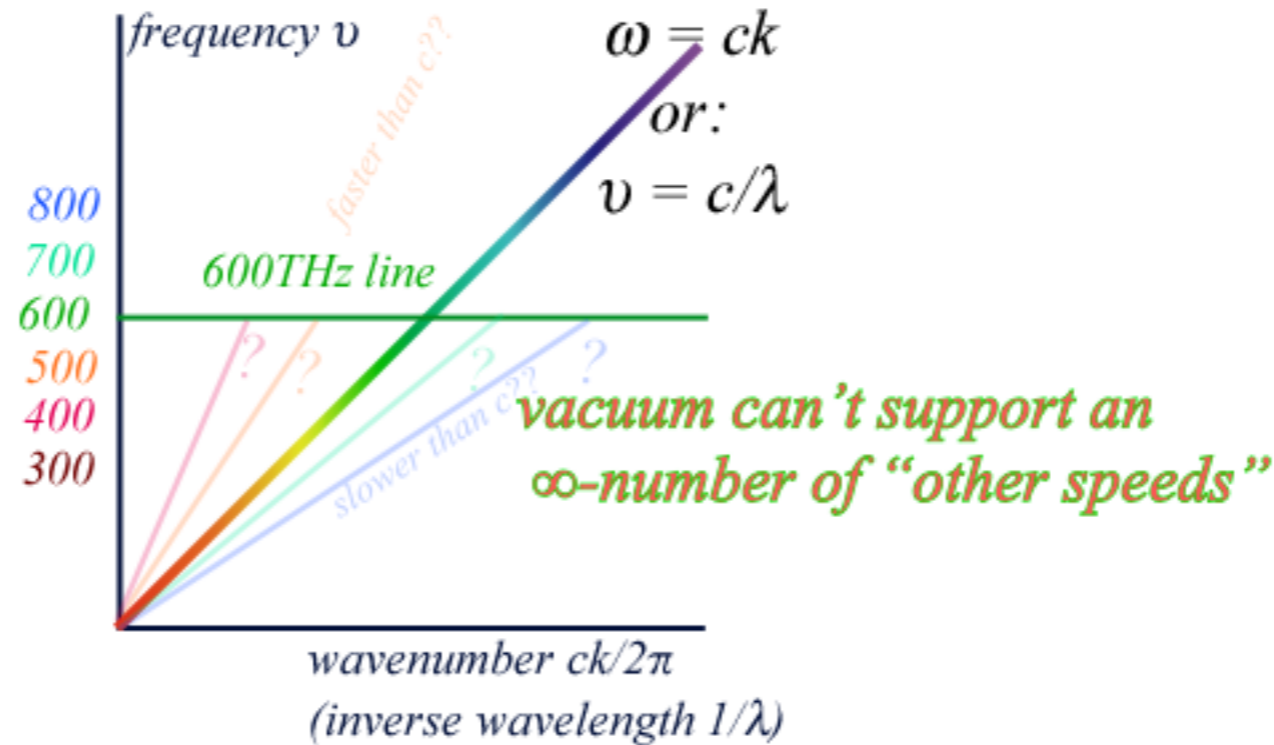


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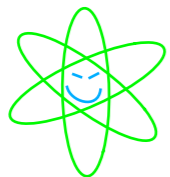
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*What if blue were to travel 0.001% slower than red
from a galaxy 9 billion light years away? (..and show up 10^5 years late)*

That would mean Good-Bye Hubble Astronomy!

*What if $\nu=600\text{THz}$ green excited an Ar atom but **NOT** a $\lambda=0.500\mu\text{m}$ optical cavity? (or vice-versa?)*



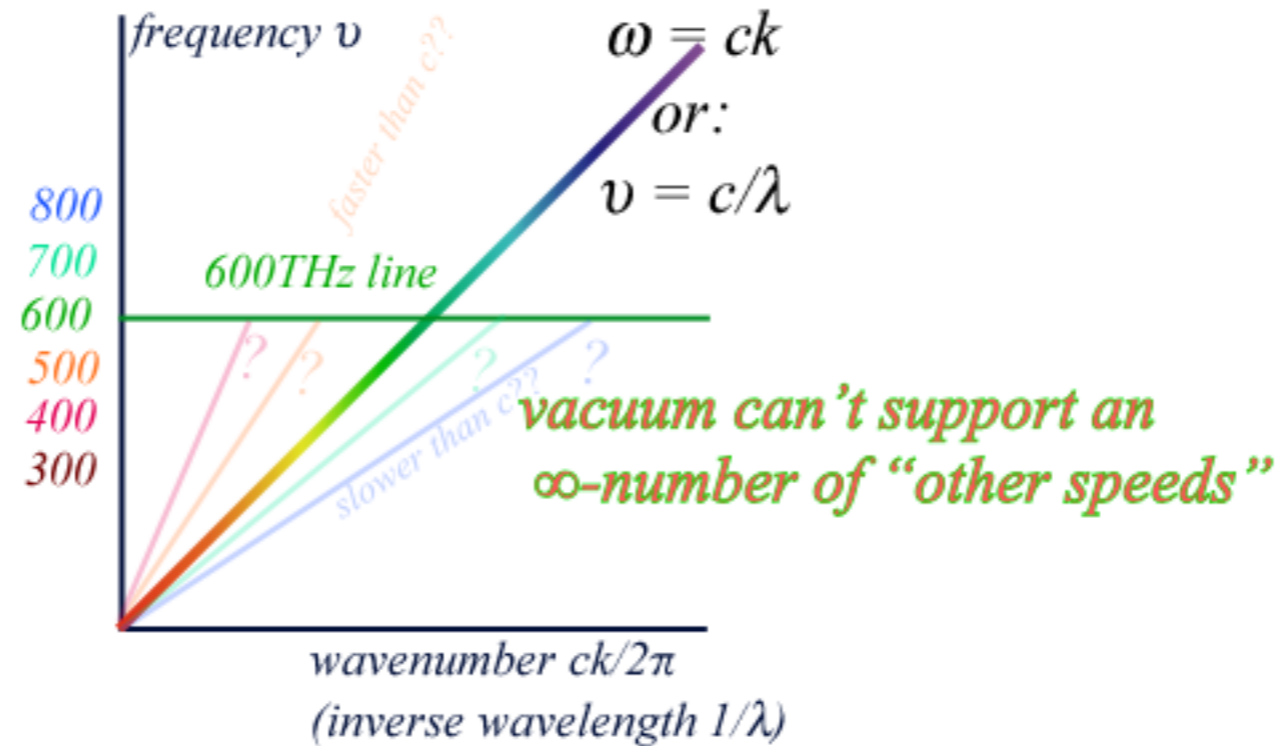
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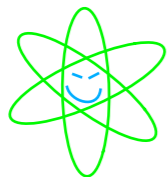
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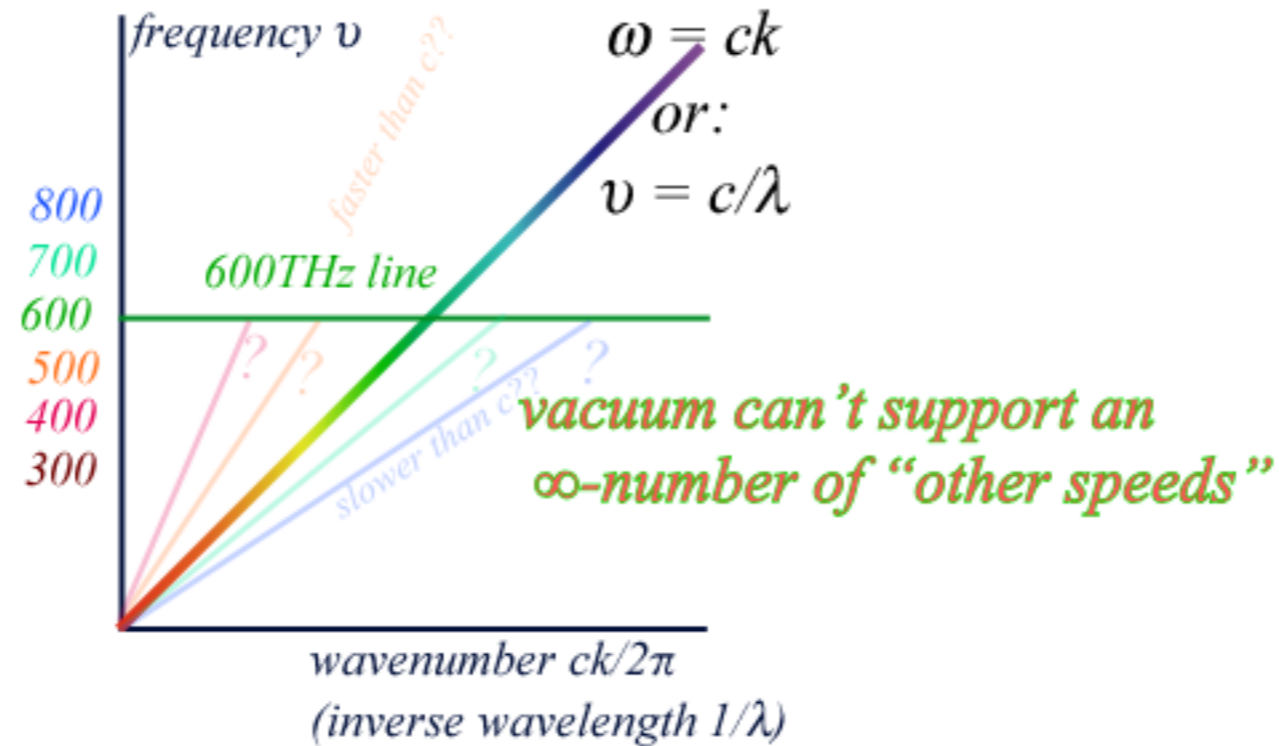


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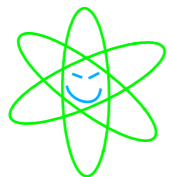
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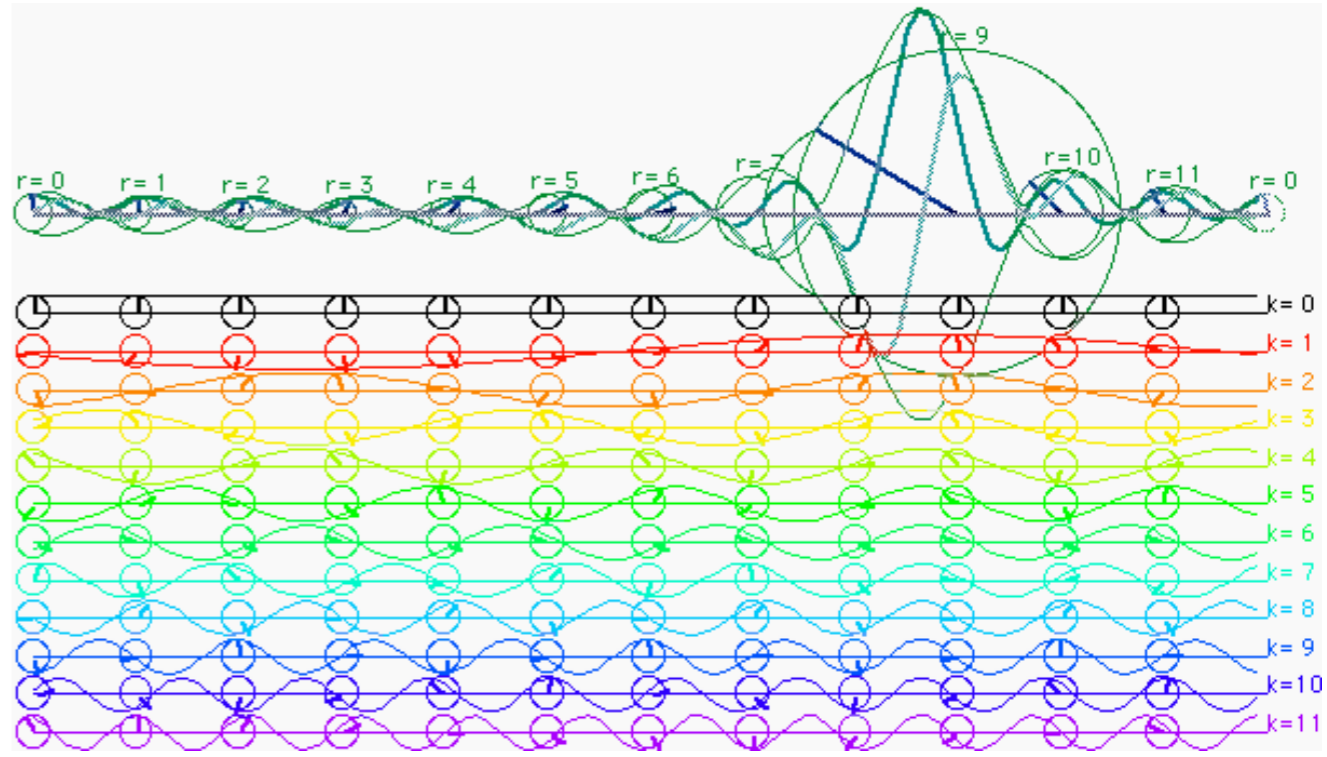
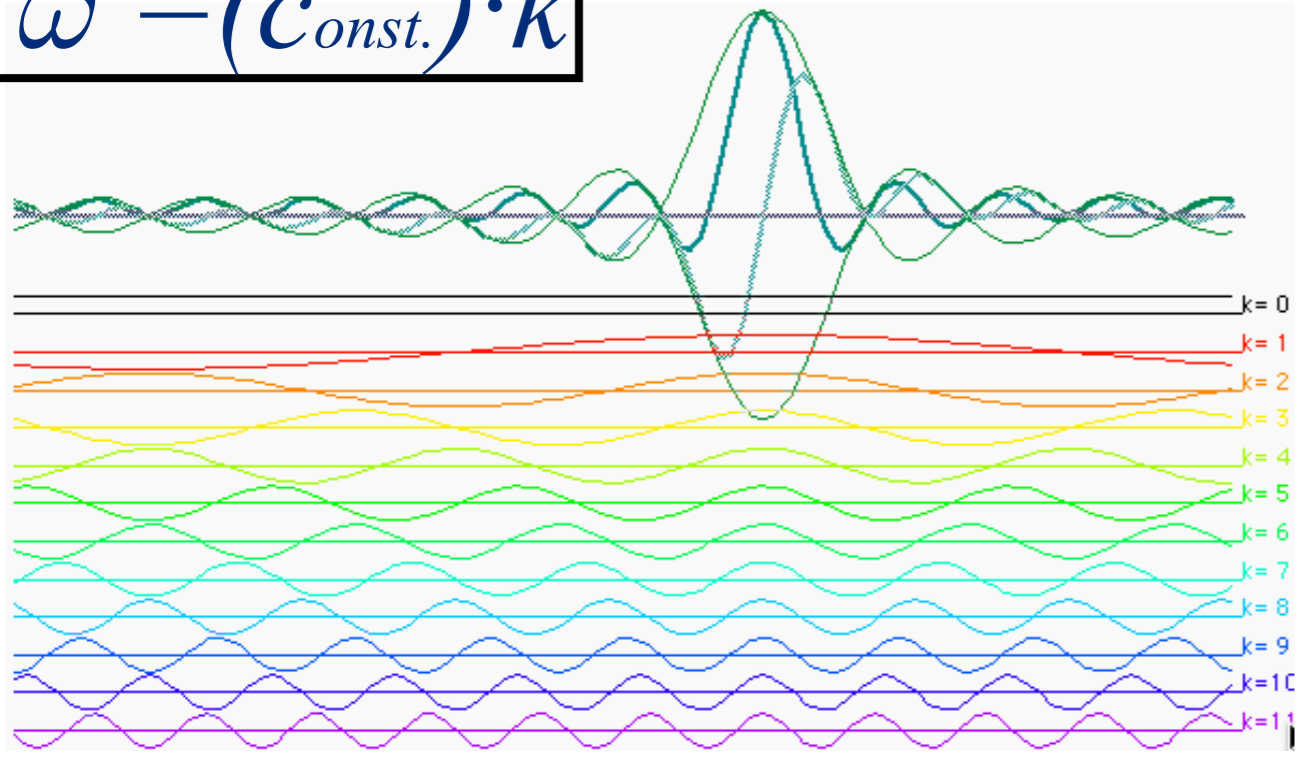
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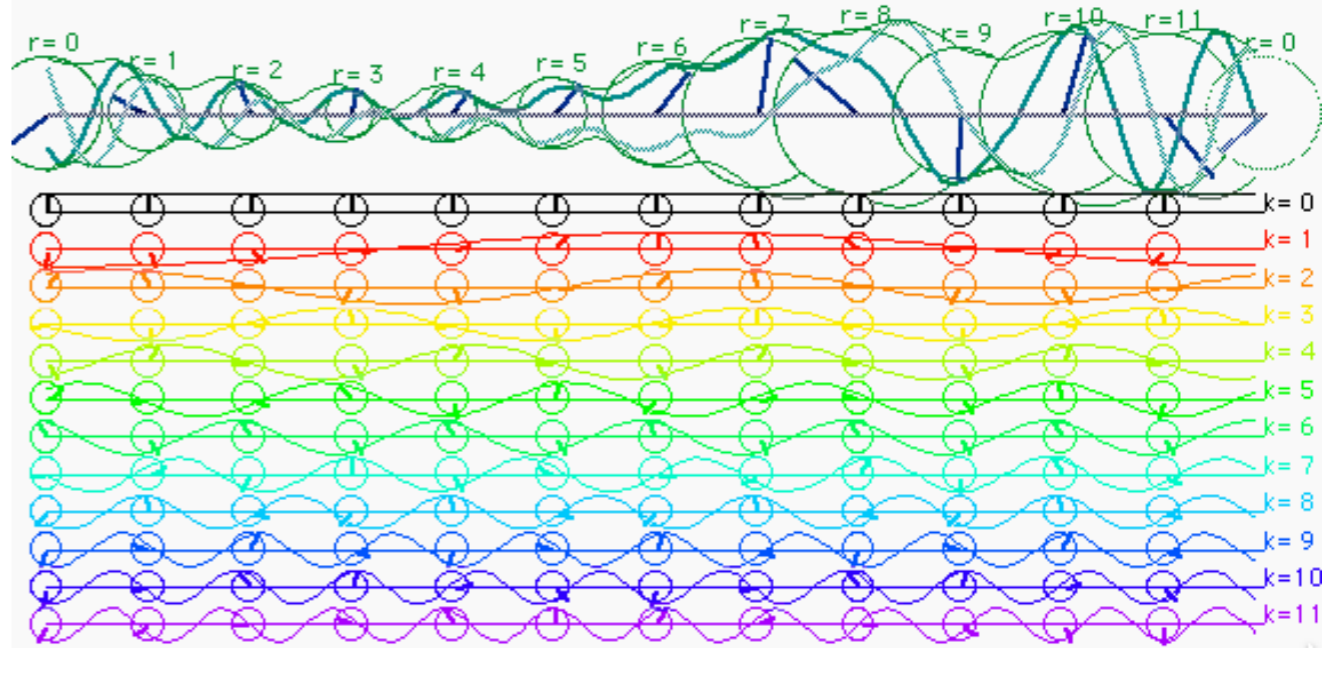
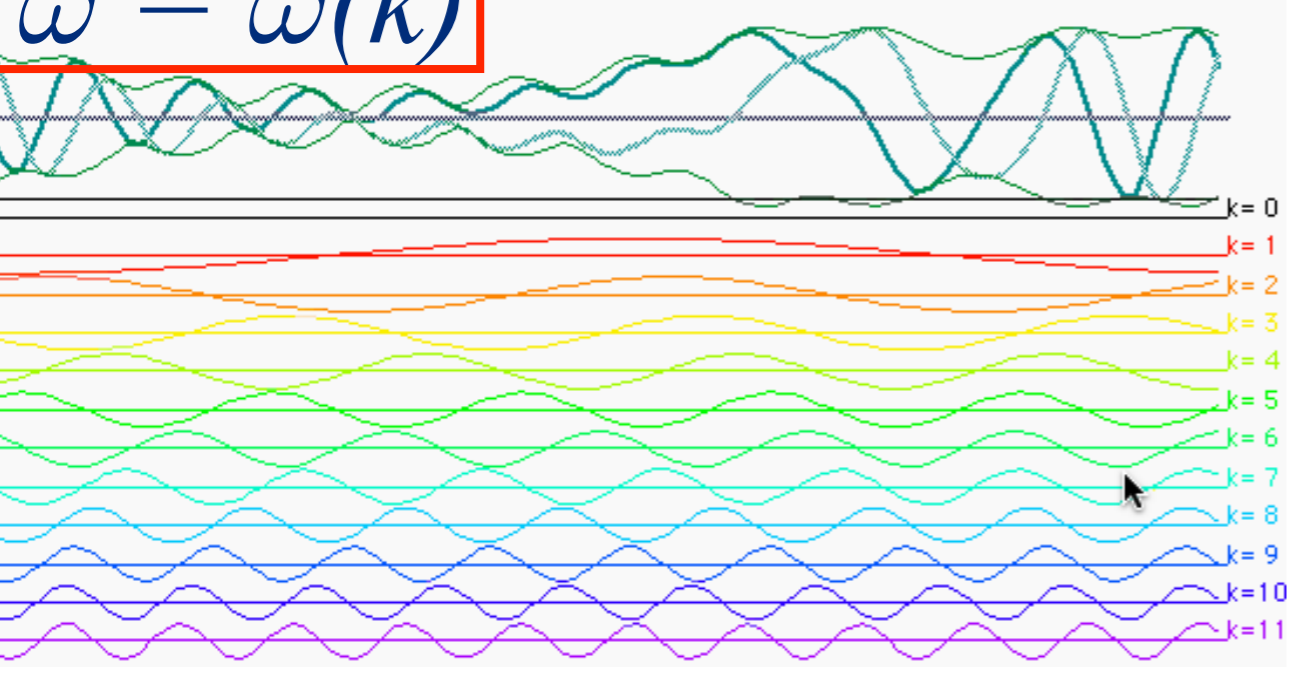
Linear Dispersion (means **NO** dispersion) has all colors (Fourier components) march in "lockstep"

$$\omega = (c_{const.}) \cdot k$$



NON-linear Dispersion (has dispersion) so different colors (Fourier components) go different speeds

$$\omega = \omega(k)$$



See animation: www.uark.edu/ua/pirelli/php/train_PW_Occum_Evenson.php

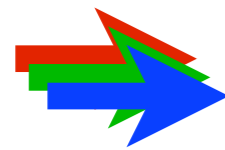
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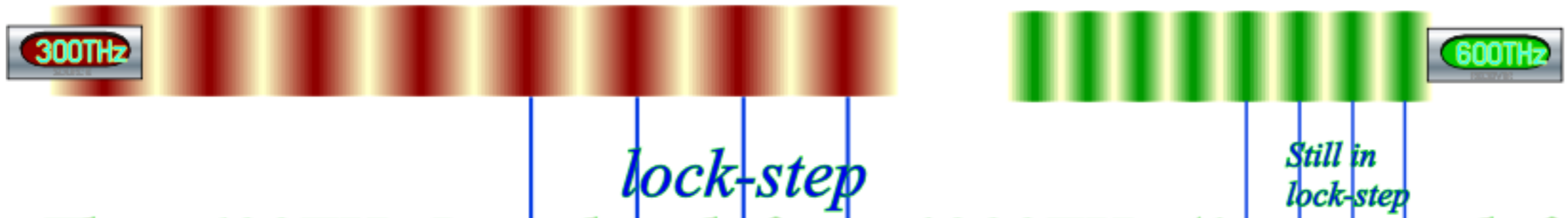
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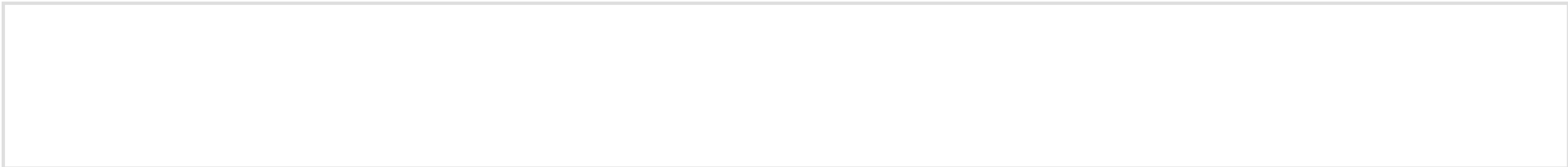
If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

If 300THz Doppler shifts to 600THz (1 octave-shift = 2.0)



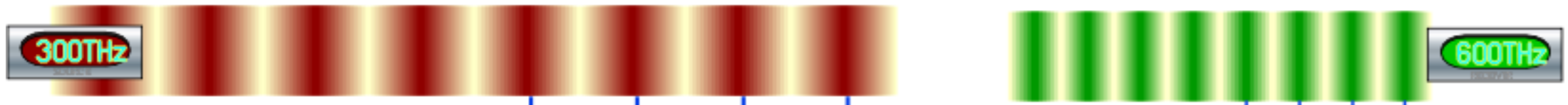
Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)





If all colors always march in lock-step then any Doppler shift must be geometric factor, that is, the same multiplier for all colors.

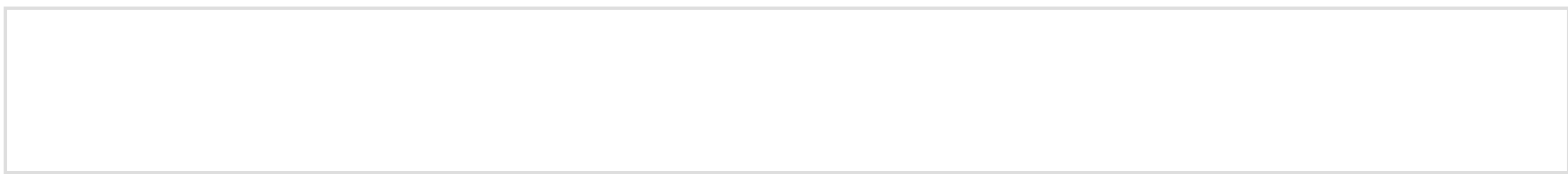
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lock-step

Still in lock-step

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lock-step

Still in lock-step

Then 600THz Doppler shifts to 1200THz (1 octave-shift = 2.0)



Doppler shifts maintain frequency ratios (not differences)

1-D Doppler shifts {red= $e^{-\rho}$... blue= $e^{+\rho}$ } form a Lie Group

3-D Doppler shifts are hypercomplex elements of Lorentz Group

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*Frequency blue shift b when
Source-Receiver interval is*

>>CLOSING<<

$$\frac{\nu_{IN}}{\nu_{OUT}} = \frac{\nu_{Receiver}}{\nu_{Source}} = b = e^{+|\rho|} > 1$$

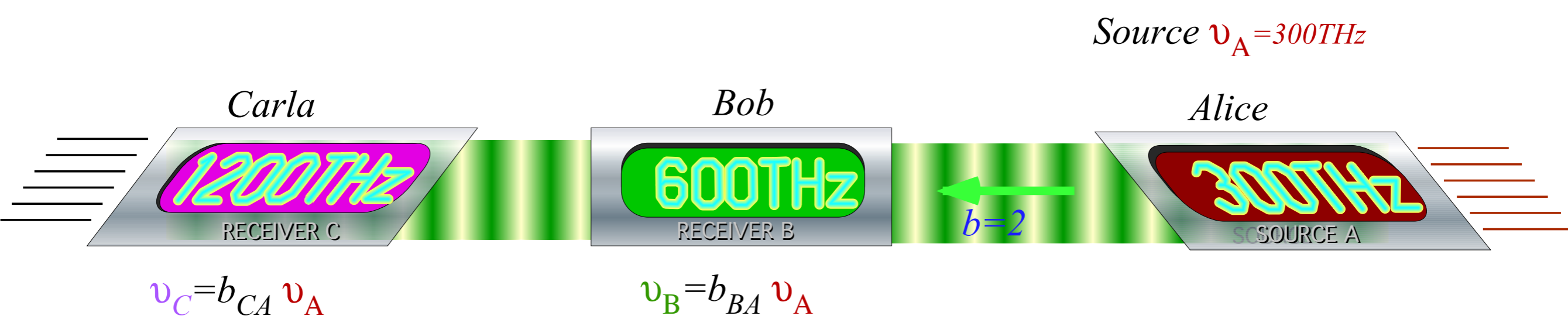
*Defining Rapidity ρ as
logarithm of Doppler*

$$\rho = \ln(b \text{ or } r)$$

*Frequency red shift r when
Source-Receiver interval is*

<<OPENING>>

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Source $\nu_A = 300\text{THz}$

Carla

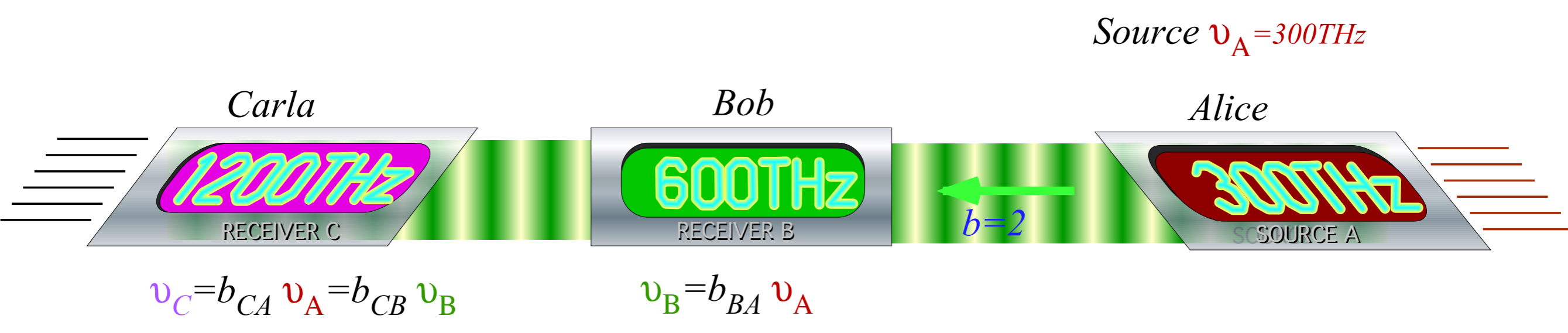
Bob

Alice



$$\nu_C = b_{CA} \nu_A$$

$$\nu_B = b_{BA} \nu_A$$



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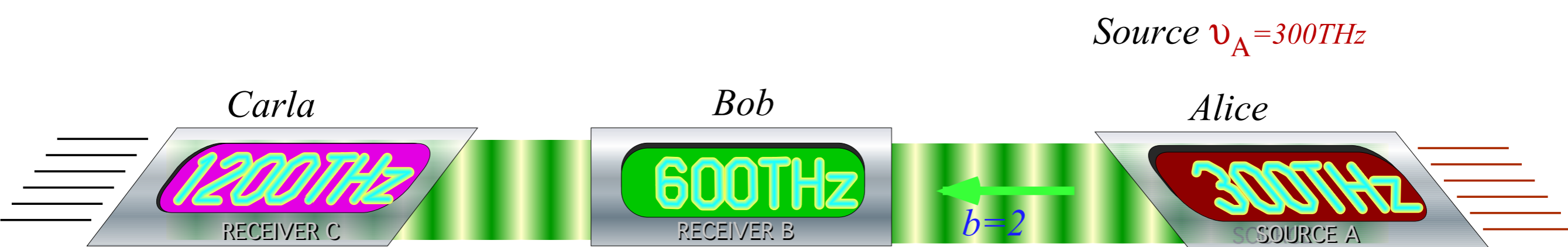
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$$\nu_C = b_{CA} \nu_A = b_{CB} \nu_B$$

$$\nu_B = b_{BA} \nu_A$$



$$\nu_C = b_{CA} \nu_A = b_{CB} \nu_B$$

$$= b_{CB} b_{BA} \nu_A$$

$$\nu_B = b_{BA} \nu_A$$

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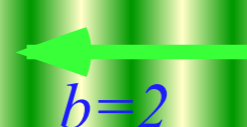
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Alice



$$\begin{aligned} \nu_C &= b_{CA} \nu_A = b_{CB} \nu_B \\ &= b_{CB} b_{BA} \nu_A \end{aligned}$$

$$\nu_B = b_{BA} \nu_A$$

2 times 2 = 4 Doppler arithmetic

Source $\nu_A = 300\text{THz}$



$$\begin{aligned} \nu_C &= b_{CA} \nu_A = b_{CB} \nu_B \\ &= b_{CB} b_{BA} \nu_A \end{aligned}$$

$$\nu_B = b_{BA} \nu_A$$

This implies:

$$b_{CA} = b_{CB} b_{BA}$$

b -Product rule

Frequency blue shift b when
Source-Receiver interval is
>>CLOSING<<

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<<OPENING>>

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b -Product rule

Frequency blue shift b when Source-Receiver interval is

>>CLOSING<<

$$\frac{v_{IN}}{v_{OUT}} = \frac{v_{Receiver}}{v_{Source}} = b = e^{+|\rho|} > 1$$

Defining Rapidity ρ as logarithm of Doppler

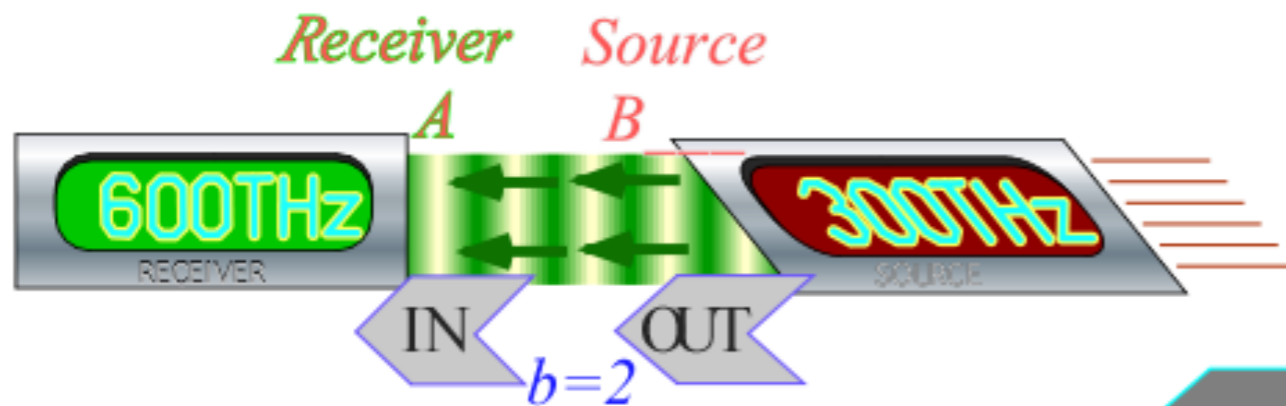
$$\rho = \ln(b \text{ or } r)$$

Frequency red shift r when Source-Receiver interval is

<<OPENING>>

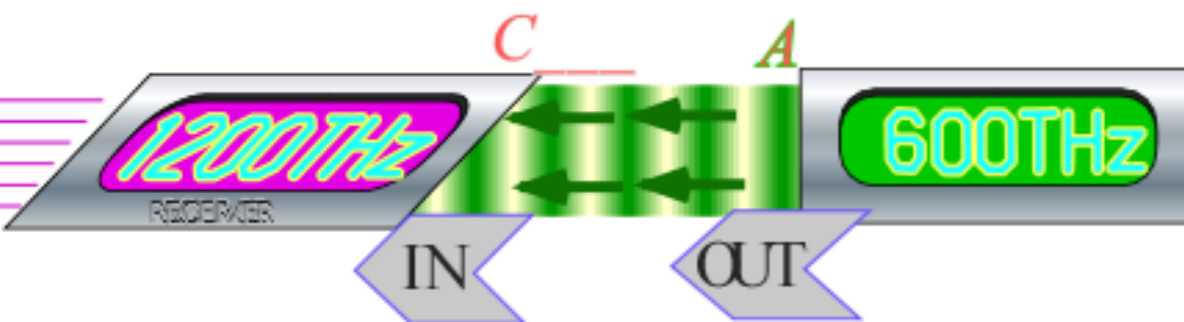
$$\frac{v_{Receiver}}{v_{Source}} = r = e^{-|\rho|} < 1$$

Examples:

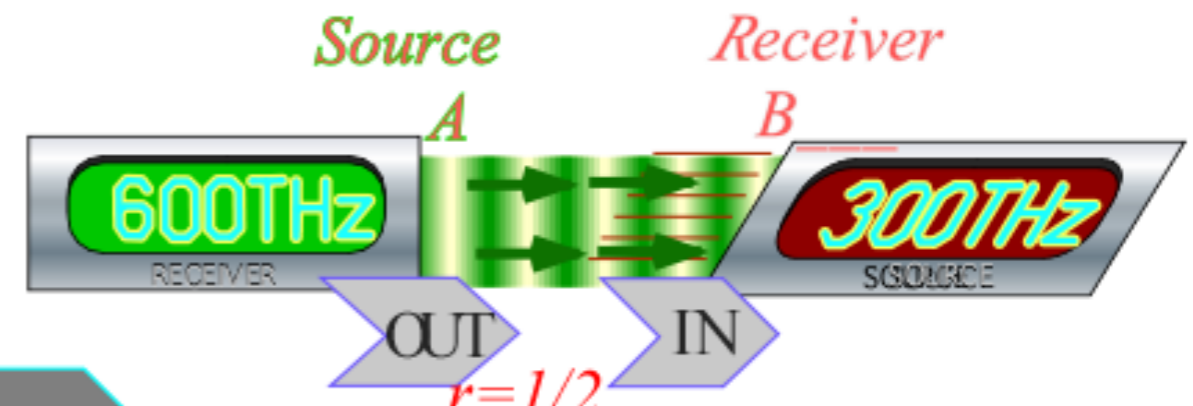


$$\rho = \ln(2) = 0.69$$

Receiver Source

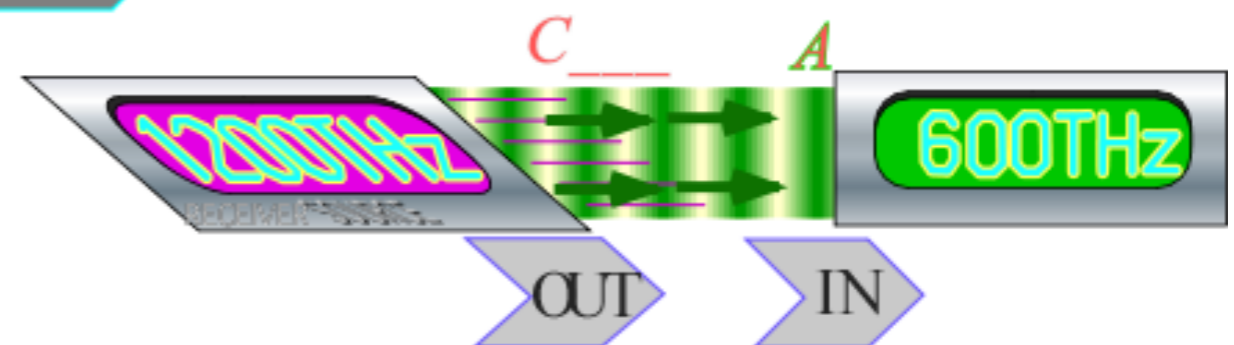


Examples:



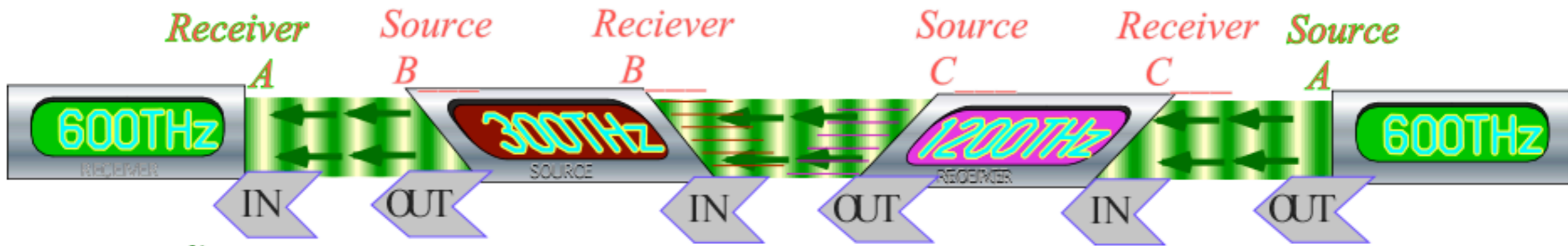
$$\rho = \ln(1/2) = -0.69$$

Source Receiver



Time Reversal

Each Doppler shift $\frac{\nu_A}{\nu_B}$ maps to a Lorentz transformation T_{AB}



$$\frac{\nu_A}{\nu_B} = b_{AB} = e^{\rho_{AB}} = 2$$

$$\rho_{AB} = \ln(2) = 0.69$$

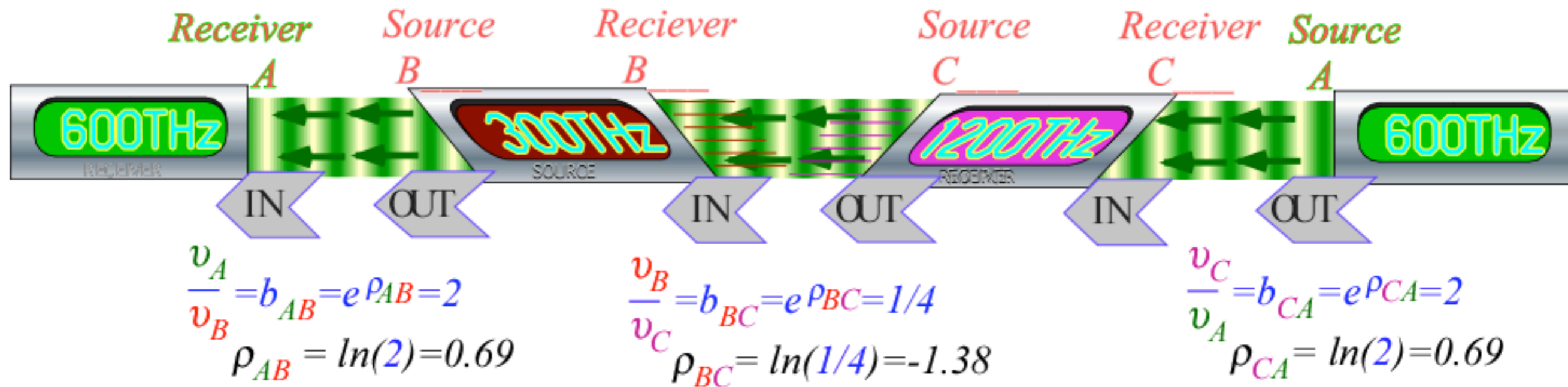
$$\frac{\nu_B}{\nu_C} = b_{BC} = e^{\rho_{BC}} = 1/4$$

$$\rho_{BC} = \ln(1/4) = -1.38$$

$$\frac{\nu_C}{\nu_A} = b_{CA} = e^{\rho_{CA}} = 2$$

$$\rho_{CA} = \ln(2) = 0.69$$

Each Doppler shift $\frac{v_A}{v_B}$ maps to a Lorentz transformation T_{AB}



Group product
is represented:
(by IN-OUT "nematodes")

$$T_{AB} \cdot T_{BC} = T_{CA}$$

$$\frac{v_A}{v_B} \frac{v_B}{v_C} = \frac{v_A}{v_C}$$

$$e^{\rho_{AB}} e^{\rho_{BC}} = e^{\rho_{AC}} = e^{(\rho_{AB} + \rho_{BC})}$$

...and rapidity ρ_{AB} is a Galilean (arithmetic) parameter

To be shown: $\rho_{AB} = \text{atanh}(u_{AB}/c)$ approaches (u_{AB}/c) for: $\rho_{AB} \ll 1$

2. Applying Occam's razor to relativity axioms

Einstein PW Axioms versus Evenson CW Axioms (Traditional: The "Roadrunner" Axiom)

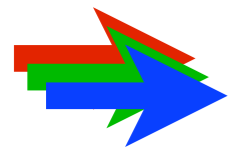
CW light clearly shows Doppler shifts

Check that red is red is red,...green is green is green,...blue is blue is blue,... etc.

Is dispersion linear? ... does astronomy work?... how about spectroscopy?

Is Doppler a geometric factor or arithmetic sum?

Introducing rapidity $\rho = \ln b$.



That old Time-Reversal meta-Axiom (that is so-oo-o neglected!)

*Inverse to Lorentz transformation T_{AB} is T_{BA}
..just as the arithmetic inverse of $\frac{v_A}{v_B}$ is $\frac{v_B}{v_A}$*

..just as the arithmetic inver... of $e^{\rho_{AB}}$ is $e^{\rho_{BA}} = e^{-\rho_{AB}}$

..just as the arithmetic inver... of ρ_{AB} is $\rho_{BA} = -\rho_{AB}$

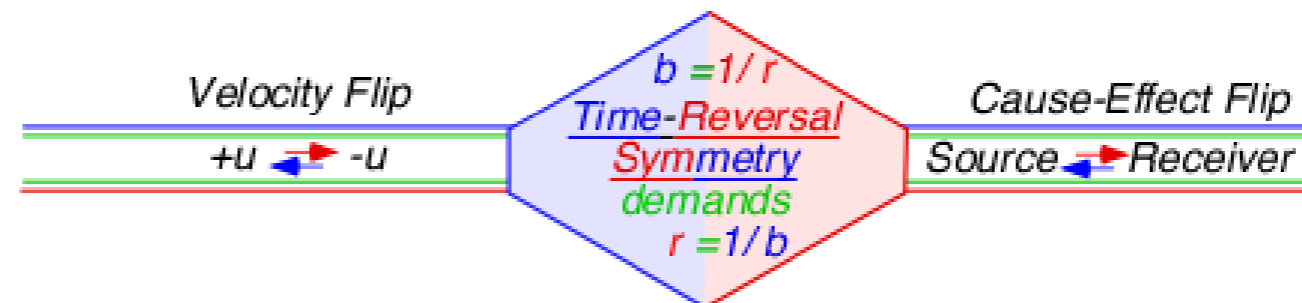
See animation: www.uark.edu/ua/pirelli/php/time_rev_sym.php

*Inverse to Lorentz transformation T_{AB} is T_{BA}
 ..just as the arithmetic inverse of $\frac{v_A}{v_B}$ is $\frac{v_B}{v_A}$*

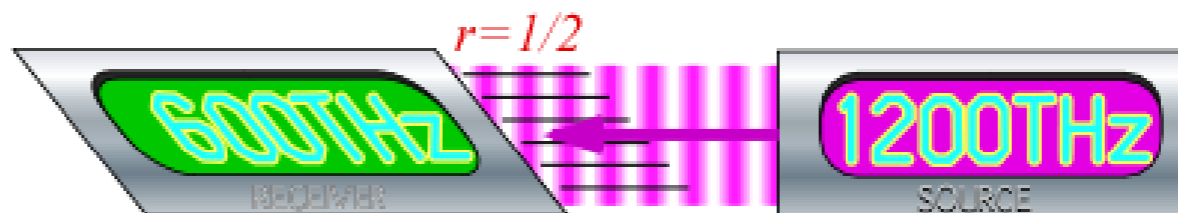
..just as the arithmetic inver... of $e^{\rho_{AB}}$ is $e^{\rho_{BA}} = e^{-\rho_{AB}}$

..just as the arithmetic inver... of ρ_{AB} is $\rho_{BA} = -\rho_{AB}$

*Detailed time reversal symmetry
 implies $r=1/b$.*



*Receding receiver sees
 Doppler red-shift of
 1200THz source to 600THz
 (600THz) = r · (1200THz)
 with $r=1/2$*



See animation: www.uark.edu/ua/pirelli/php/time_rev_sym.php

3. *Spectral theory of Einstein-Lorentz relativity*

 Applying Doppler Shifts to per-space-time (ck, ω) graph

CW Minkowski space-time coordinates (x, ct) and PW grids

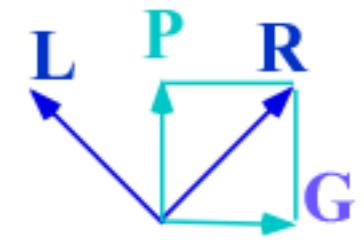
Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ

Lorentz transformation

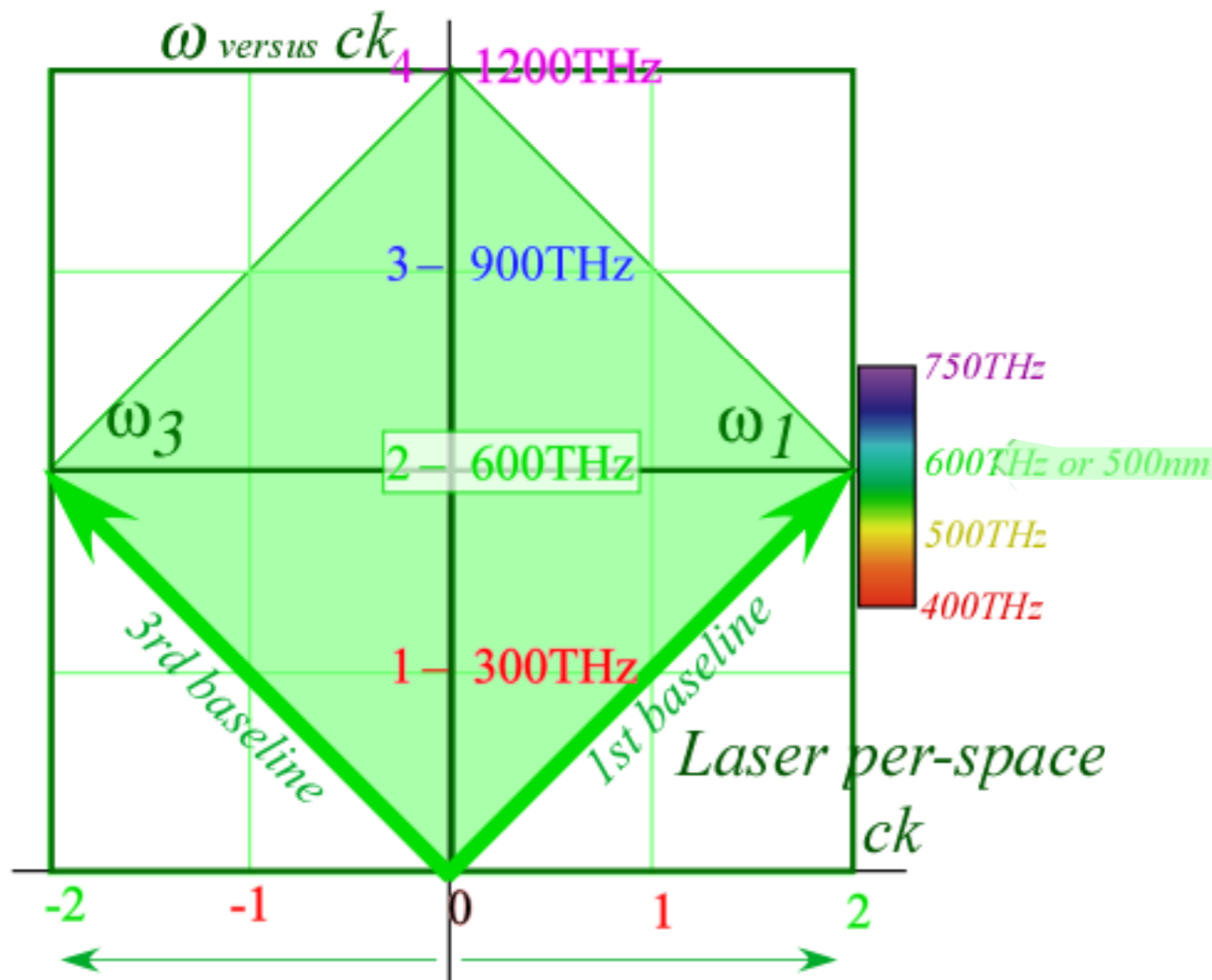
Connection: Conventional approach to relativity and old-fashioned formulas

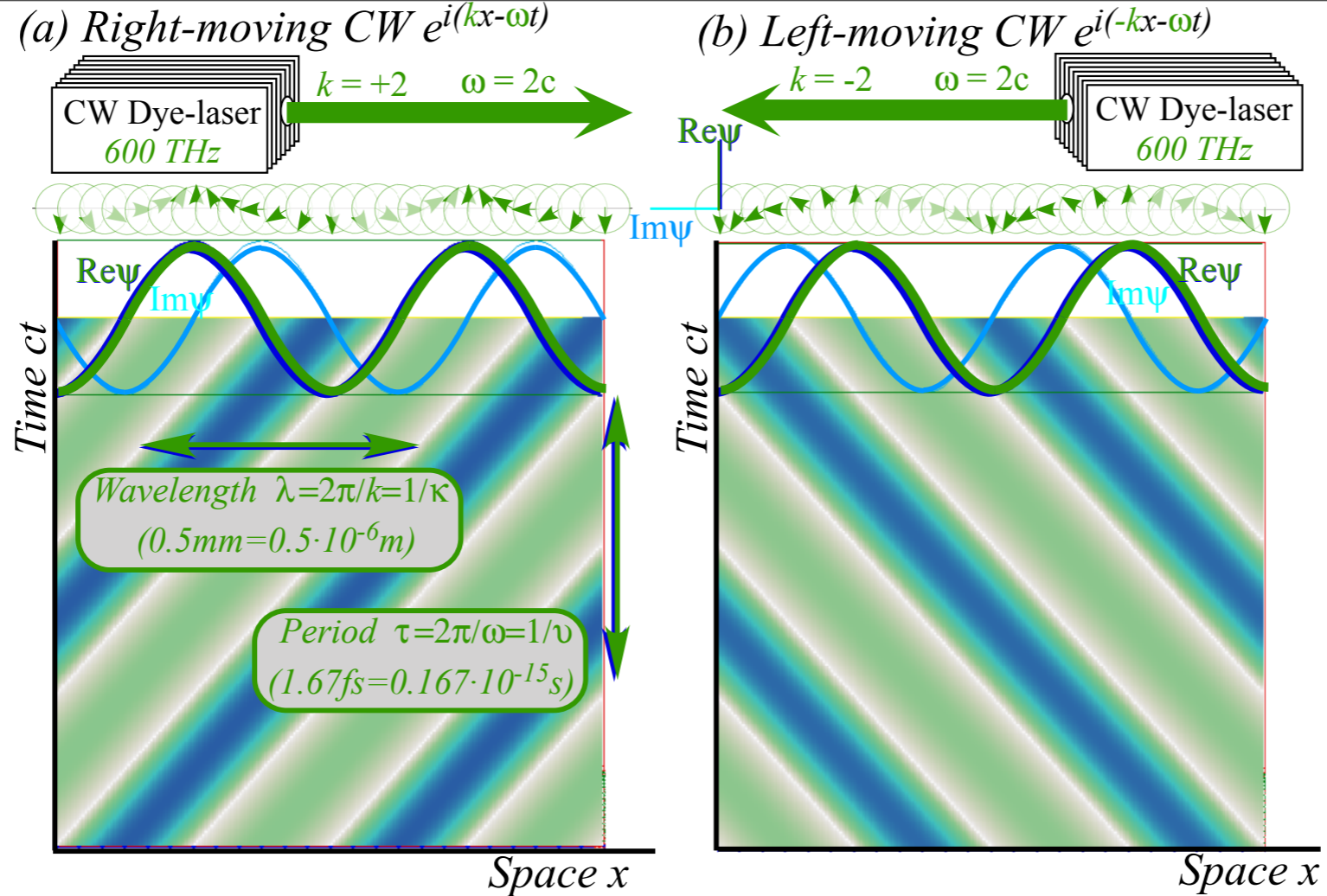
Deriving Spacetime and per-spacetime coordinate geometry by:

- (1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.
- (2) Time-Reversal axiom: $r=1/b$
- (3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$

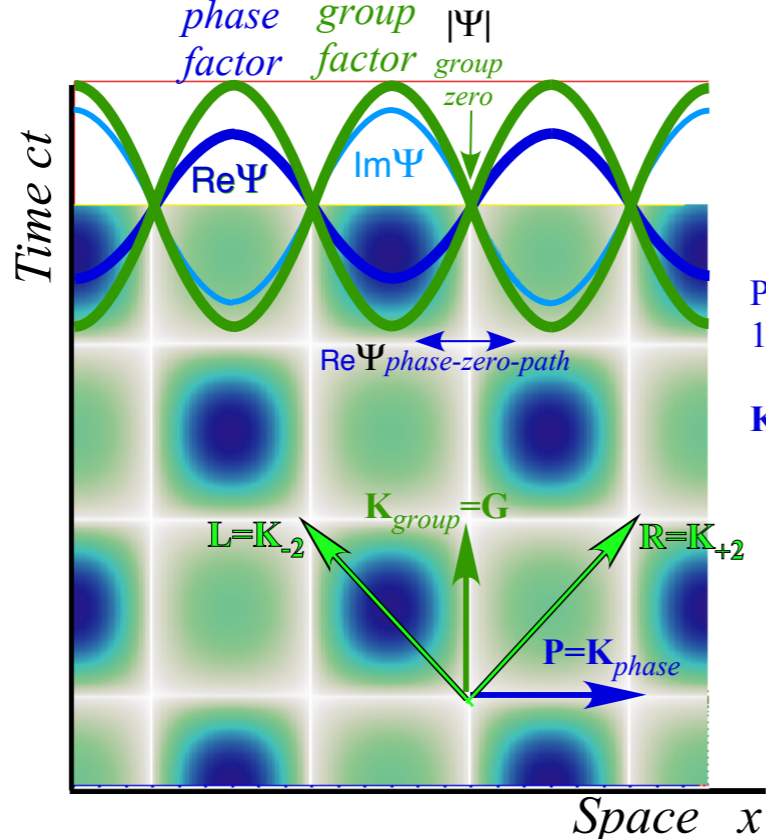


Laser Per-Spacetime





(c) Standing CW in space-time
 $\Psi(x,t) = (e^{-i\omega t}) (2\cos kx) = e^{i(kx-\omega t)} + e^{i(-kx-\omega t)}$



(d) Dispersion plot
 in per-space-time

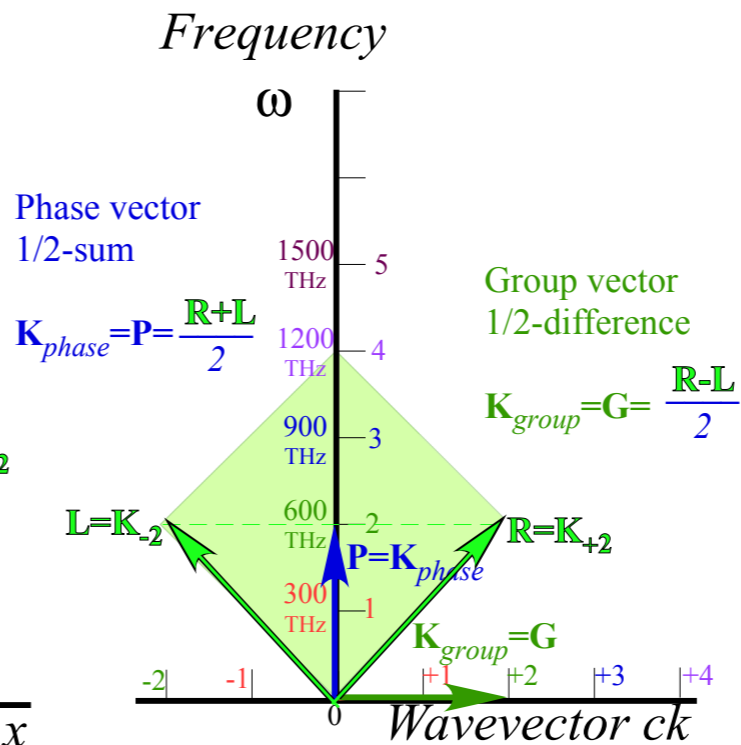
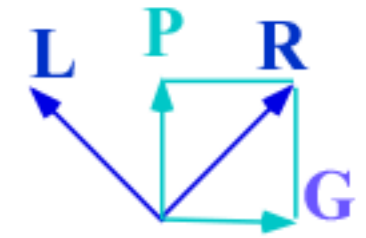


Fig. 5 in SR&QM

recall also:
 p. 3-11 of Lect.1

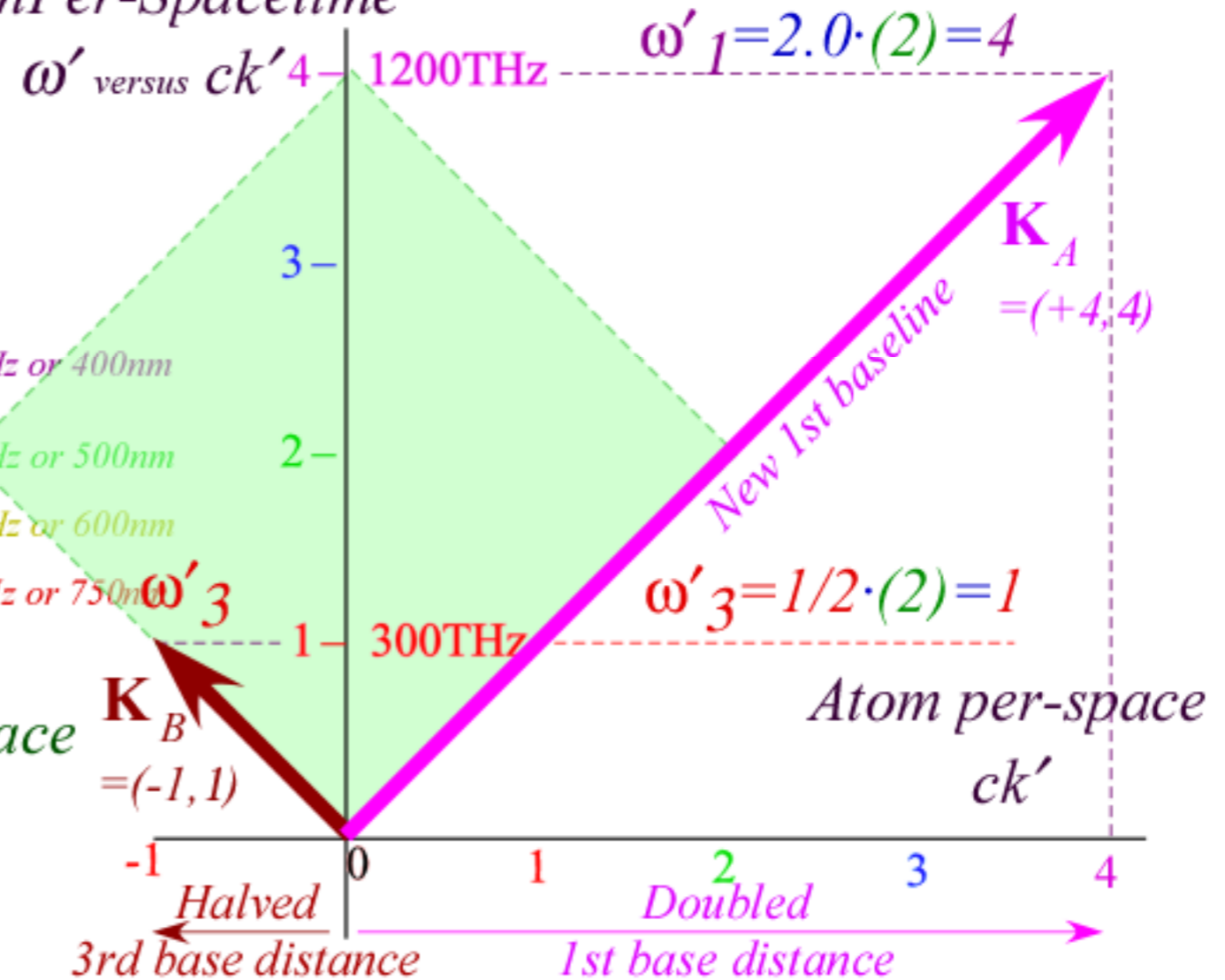
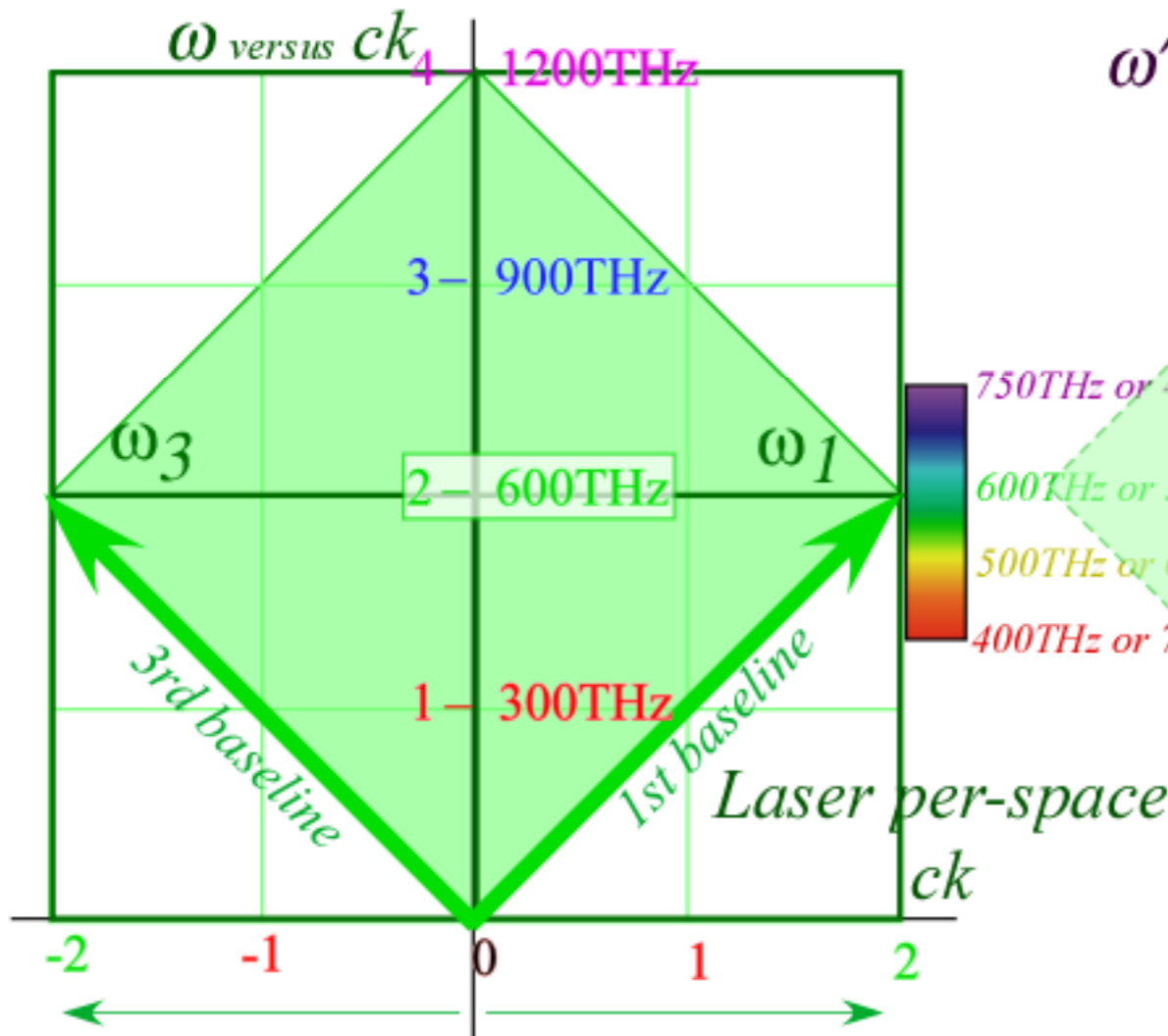
Deriving Spacetime and per-spacetime coordinate geometry by:

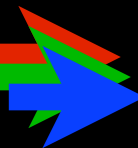
- (1) Evenson CW axiom "All colors go c " keeps K_A and K_B on their baselines.
- (2) Time-Reversal axiom: $r=1/b$
- (3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$



LaserPer-Spacetime

AtomPer-Spacetime

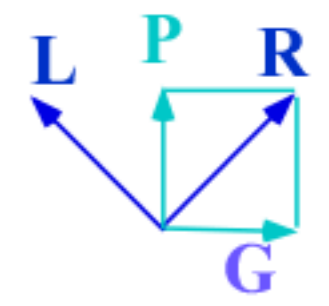


3. *Spectral theory of Einstein-Lorentz relativity*
Applying *Doppler Shifts* to per-space-time (ck, ω) graph
 CW Minkowski space-time coordinates (x, ct) and PW grids
Relating *Doppler Shifts* b or $r=1/b$ to velocity u/c or rapidity ρ
Lorentz transformation

Connection: Conventional approach to relativity and old-fashioned formulas

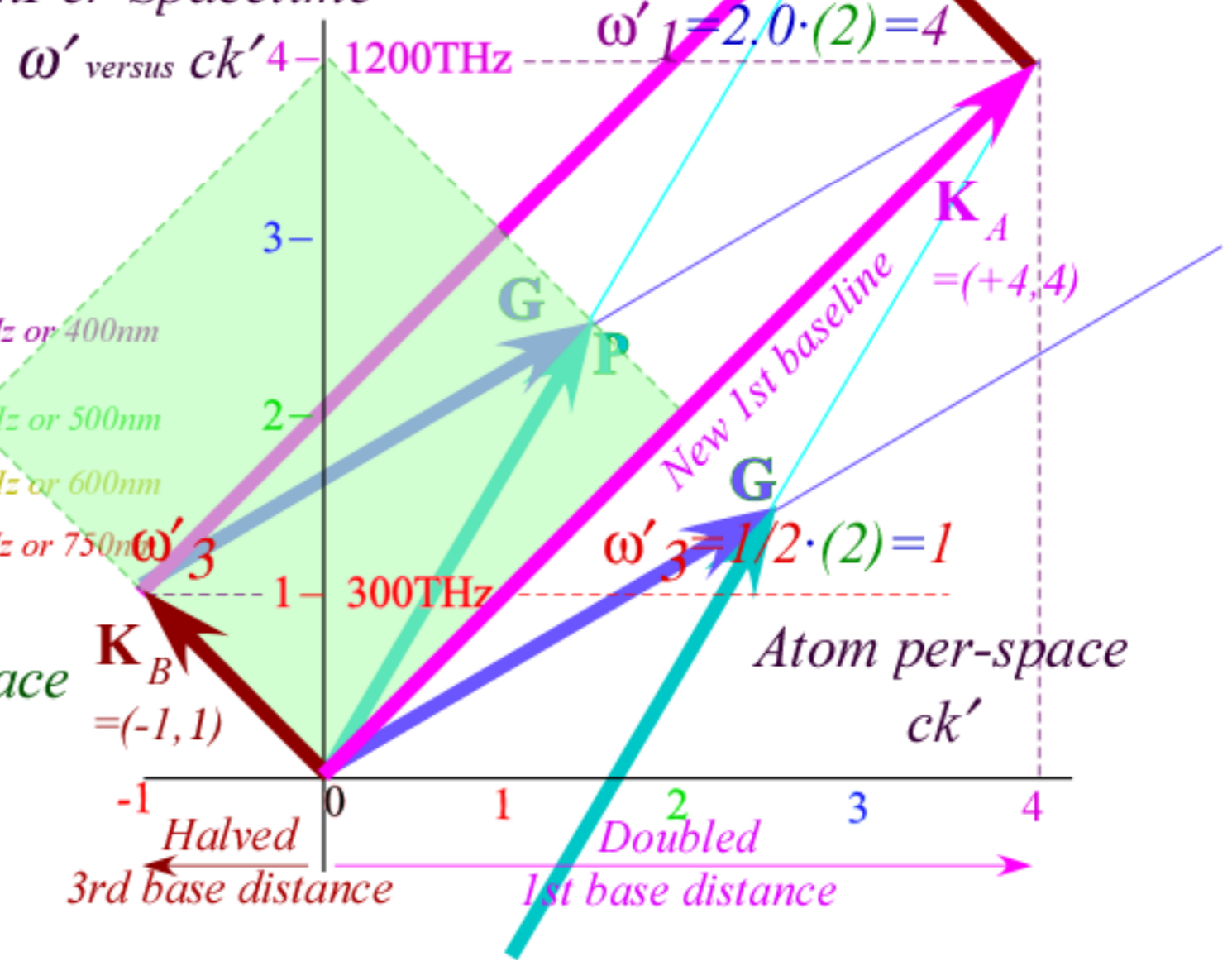
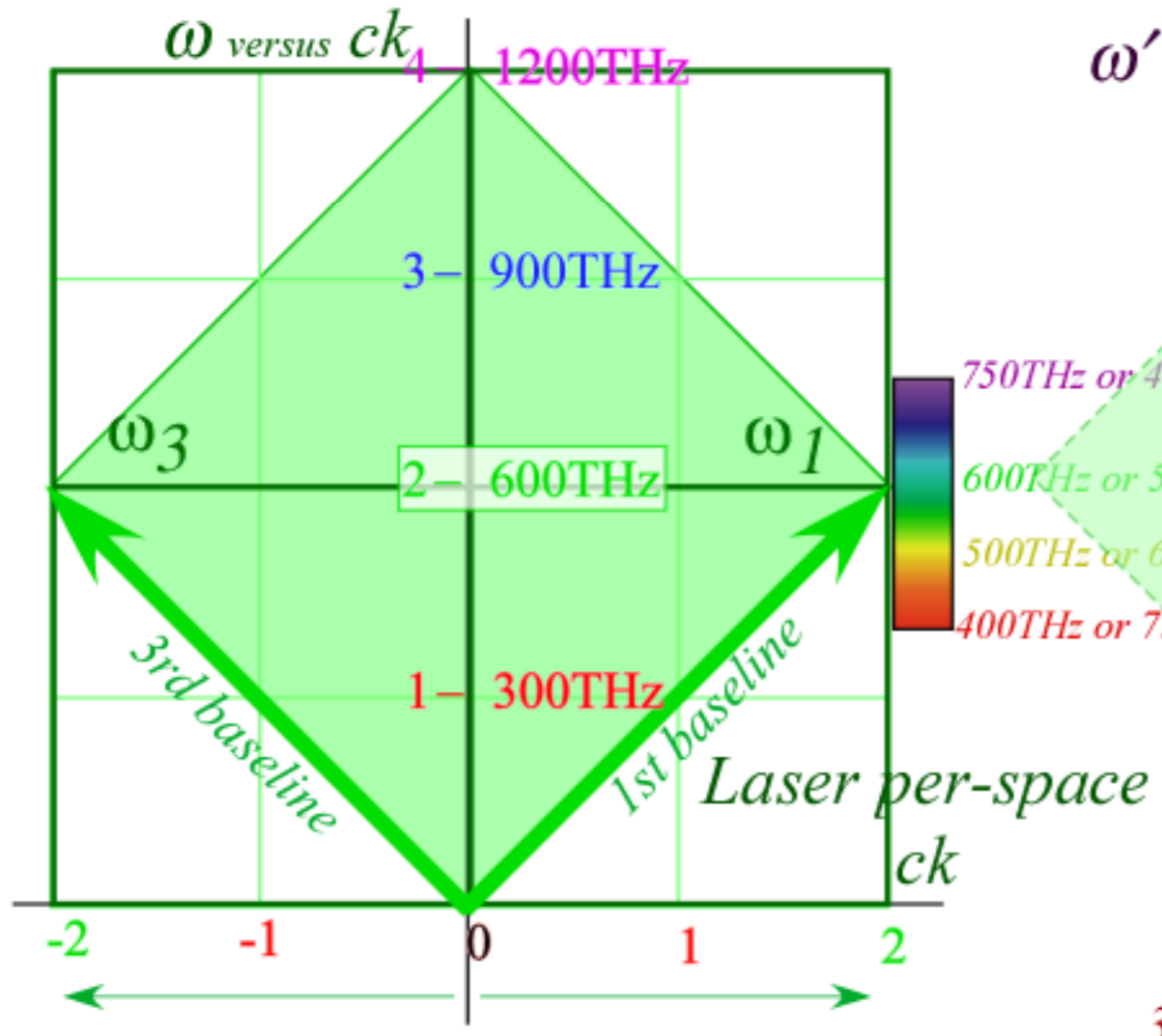
Deriving Spacetime and per-spacetime coordinate geometry by:

- (1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.
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LaserPer-Spacetime

AtomPer-Spacetime

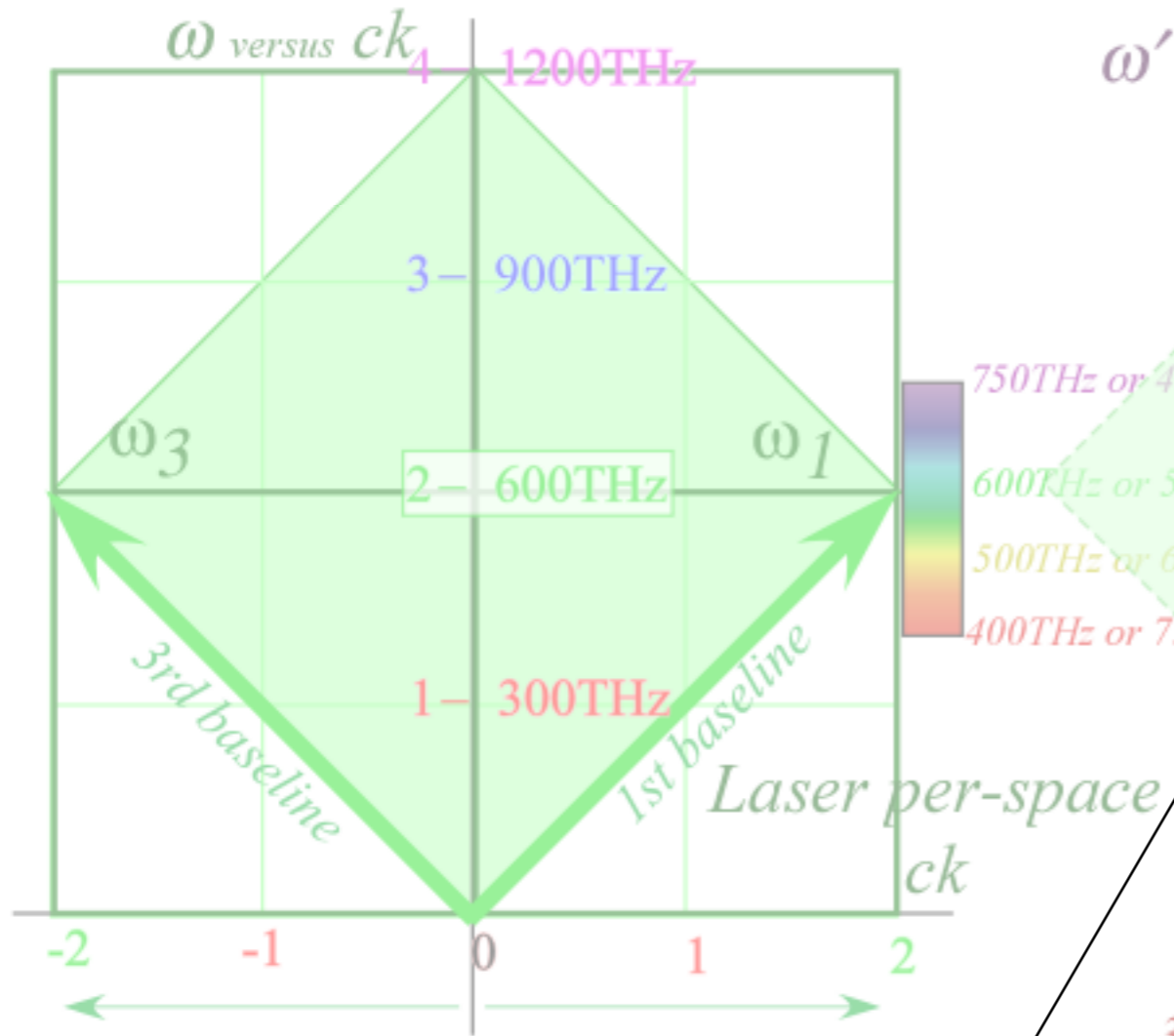


Deriving Spacetime and per-spacetime coordinate geometry by:

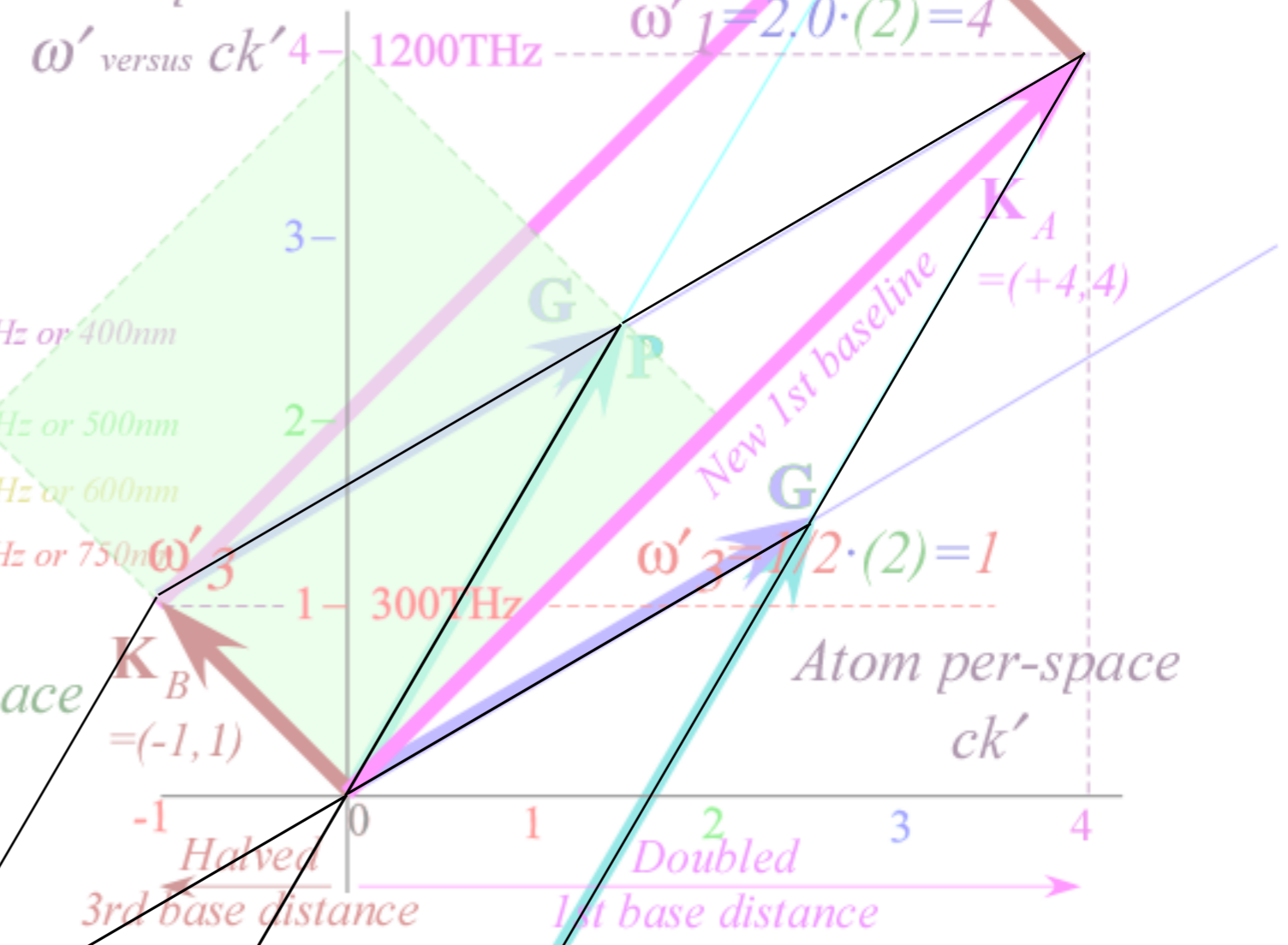
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Laser Per-Spacetime



Atom Per-Spacetime



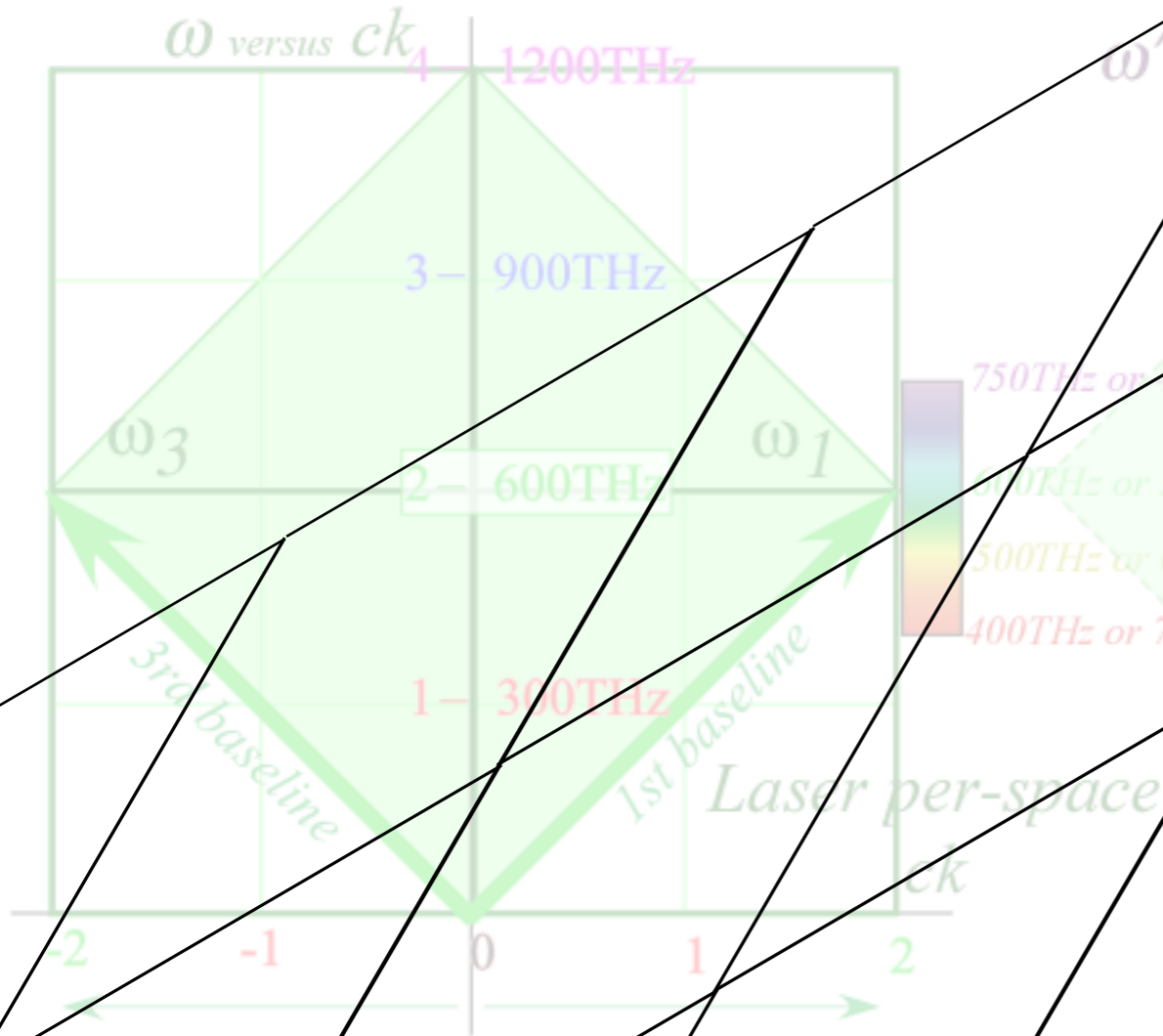
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LaserPer-Spacetime

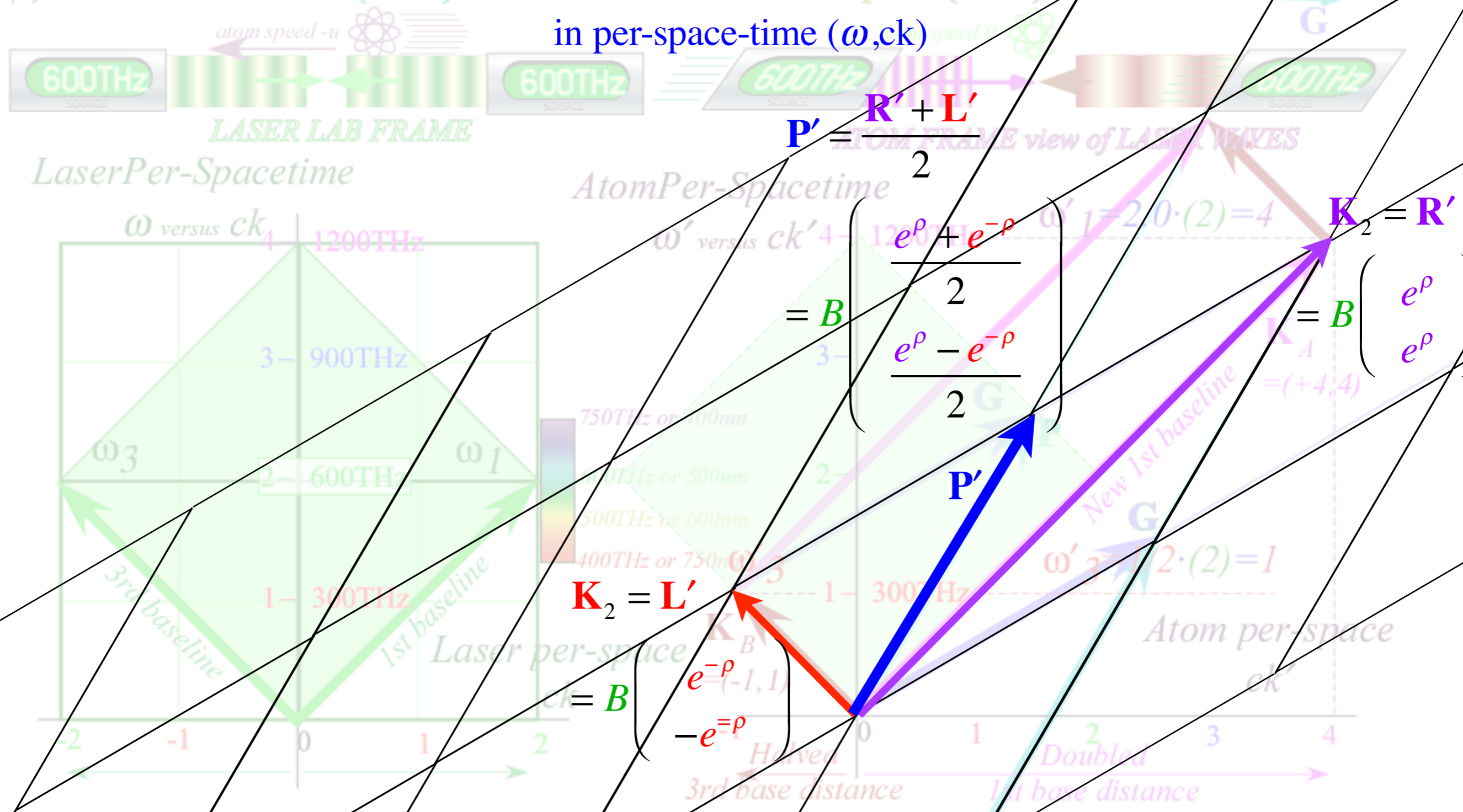
AtomPer-Spacetime



Deriving Spacetime and per-spacetime coordinate geometry by:

- (1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their ω' baselines.
- (2) Time-Reversal axiom: $r=1/b$
- (3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$

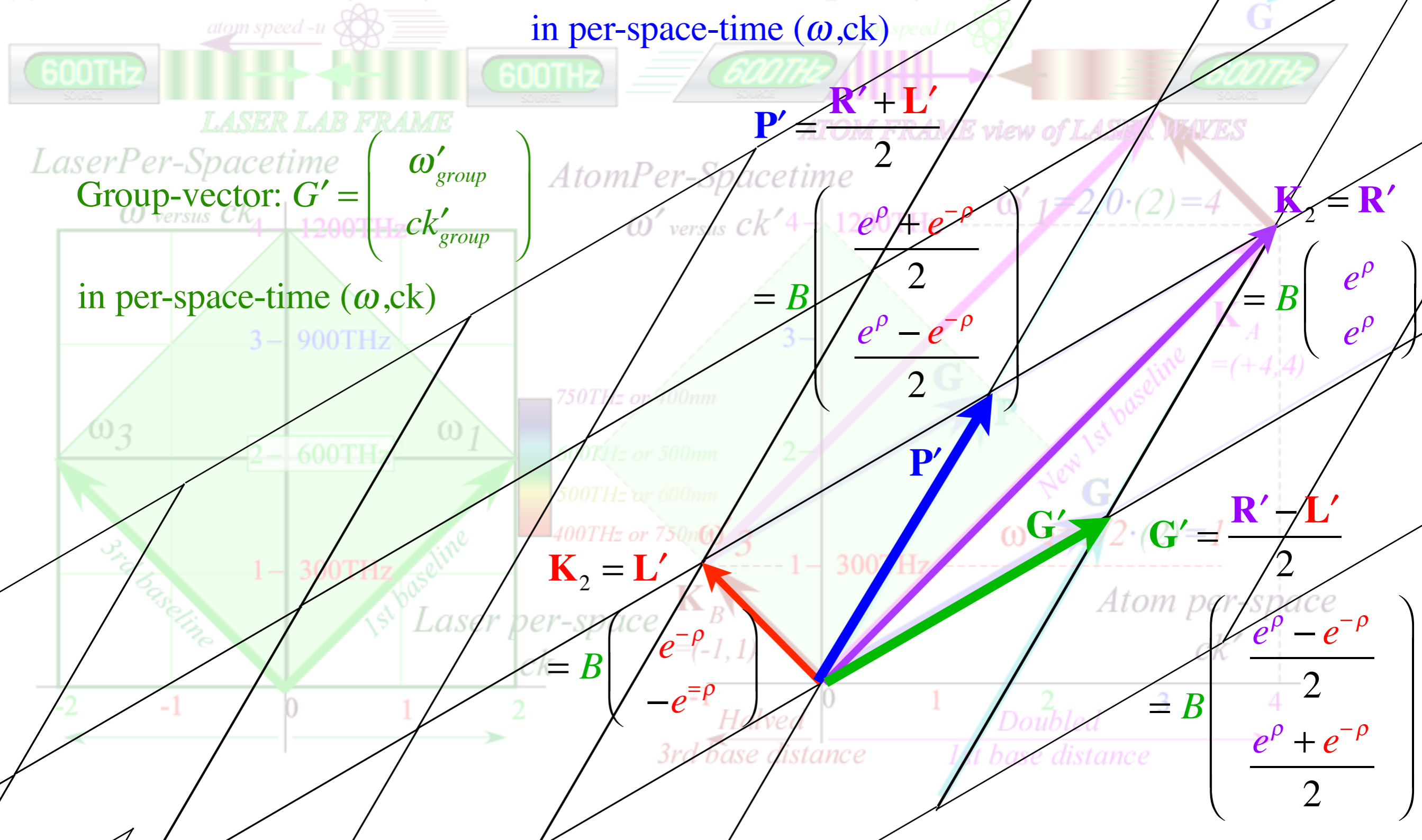
Phase-vector: $\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix}$
 in per-space-time (ω, ck)



Deriving Spacetime and per-spacetime coordinate geometry by:

- (1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their ω' baselines.
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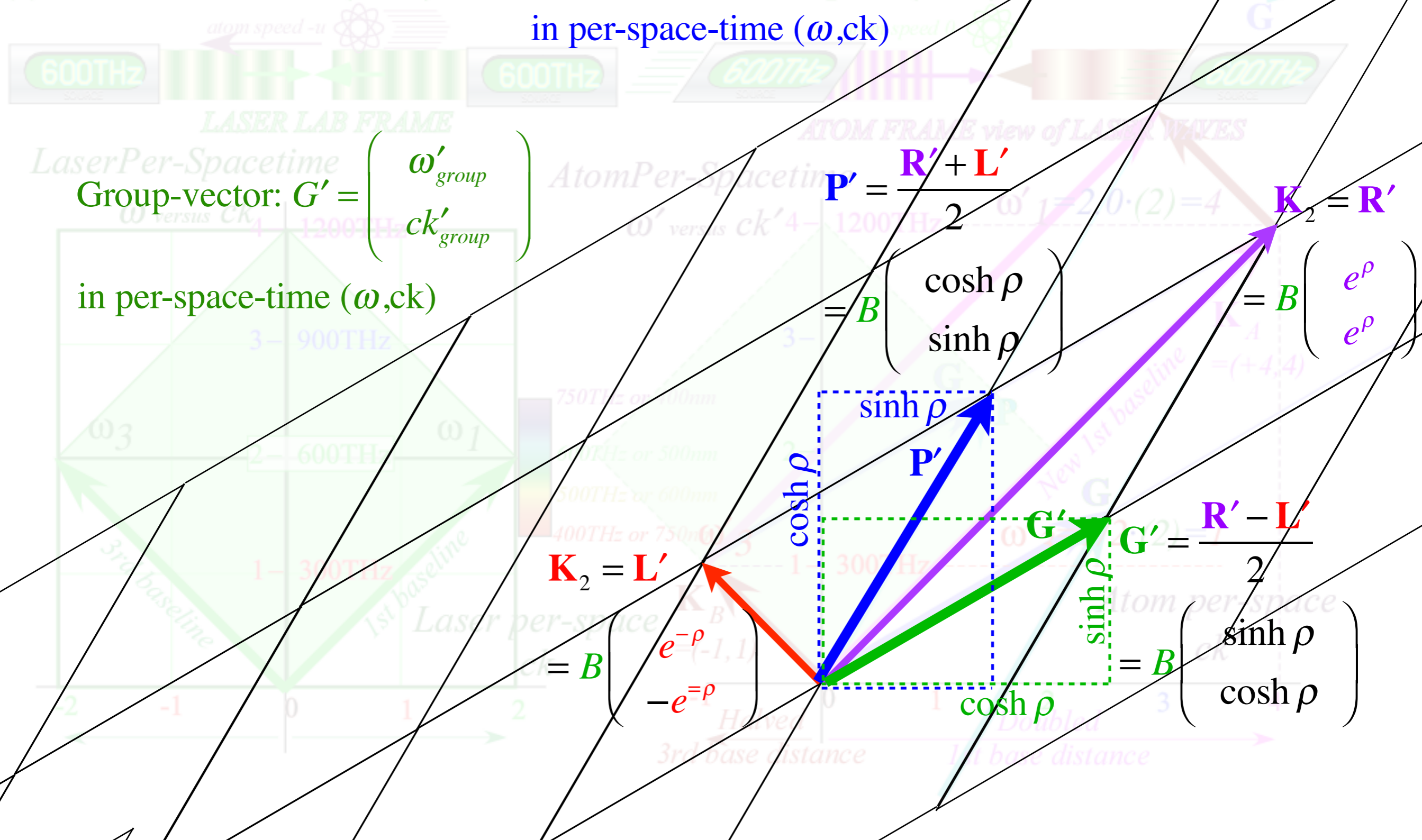
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Deriving Spacetime and per-spacetime coordinate geometry by:

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- (2) Time-Reversal axiom: $r=1/b$
- (3) Half-Sum Phase $\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$ and Half-Difference Group $\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$

Phase-vector: $\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix}$
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Deriving Spacetime and per-spacetime coordinate geometry by:

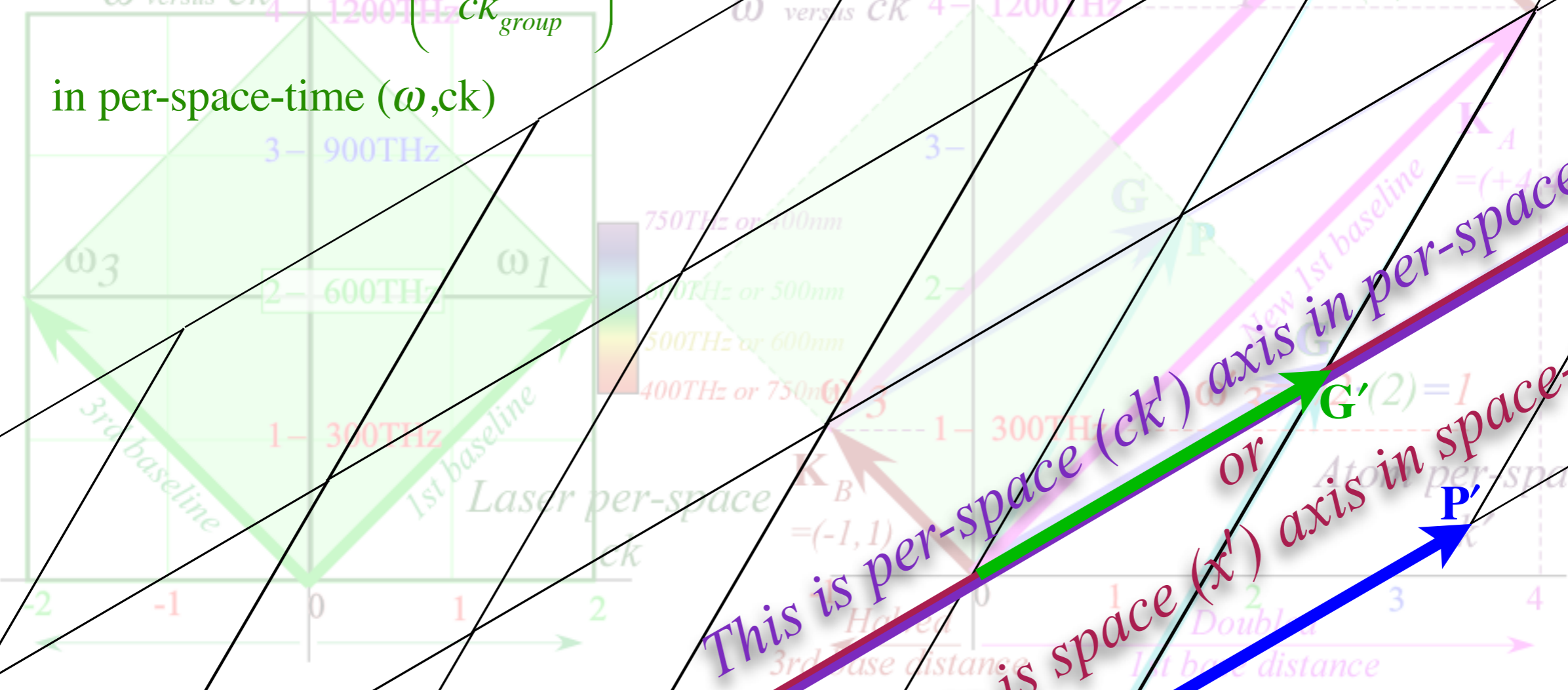
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- (3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$

Phase-vector: $\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix}$
 in per-space-time (ω, ck)



Laser Per-Spacetime
 Group-vector: $\mathbf{G}' = \begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix}$
 in per-space-time (ω, ck)

Atom Per-Spacetime
 ω' versus ck'



This is per-space (ck') axis in per-space-time

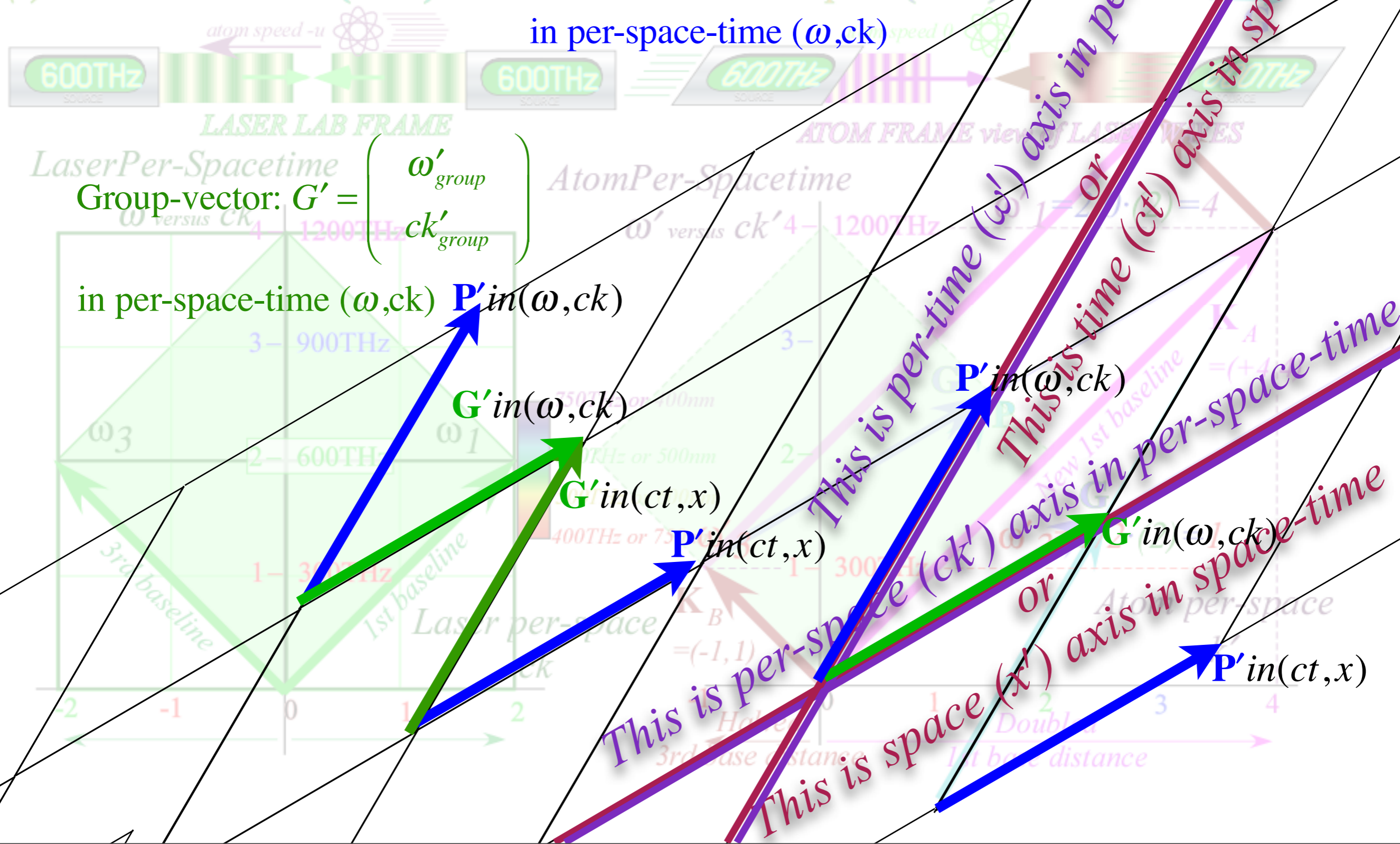
This is space (x') axis in space-time

Deriving Spacetime and per-spacetime coordinate geometry by:

- (1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their ω' baselines.
- (2) Time-Reversal axiom: $r=1/b$
- (3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$

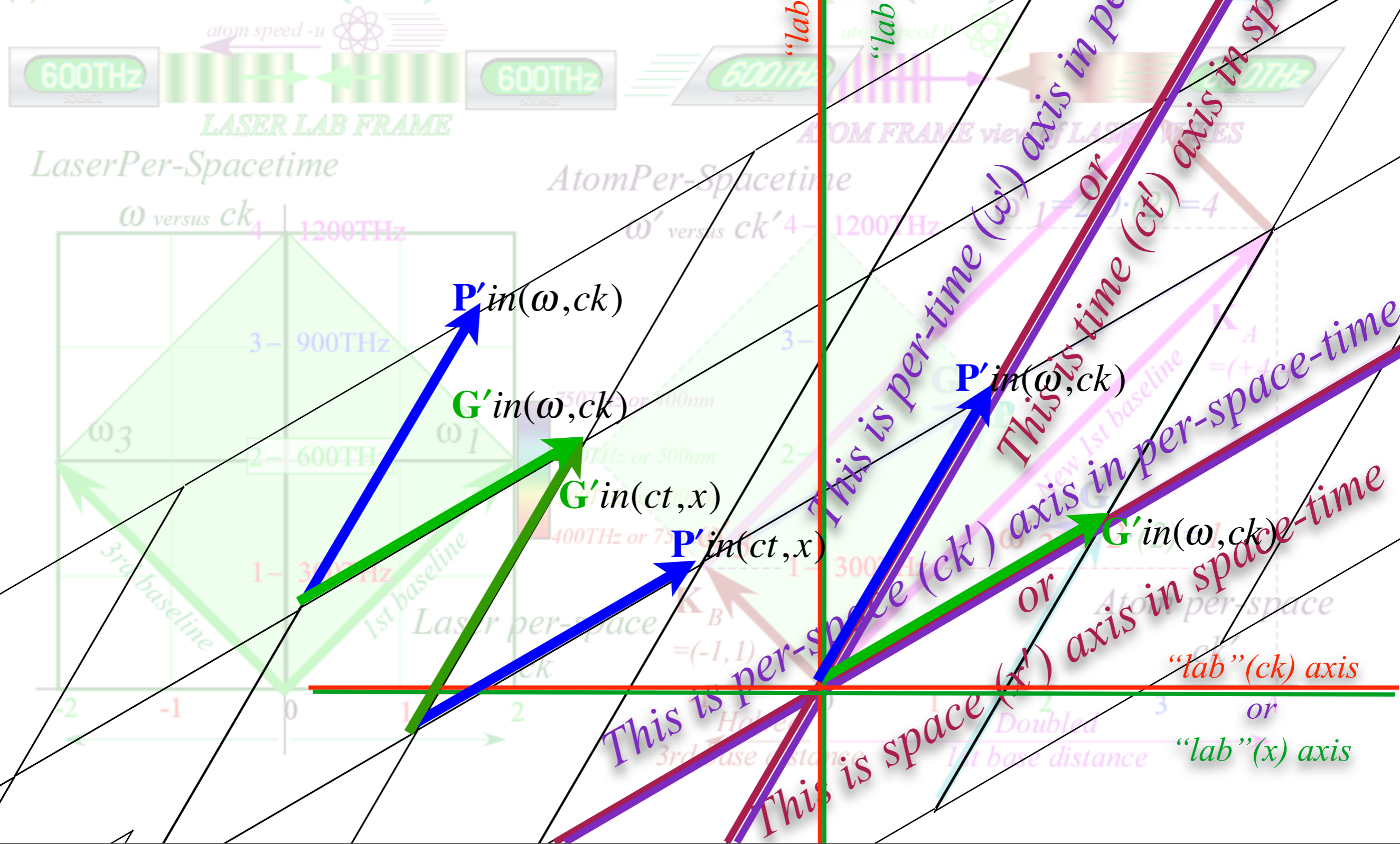
Phase-vector: $P' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix}$

in per-space-time (ω, ck)



Deriving Spacetime and per-spacetime coordinate geometry by:

- (1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.
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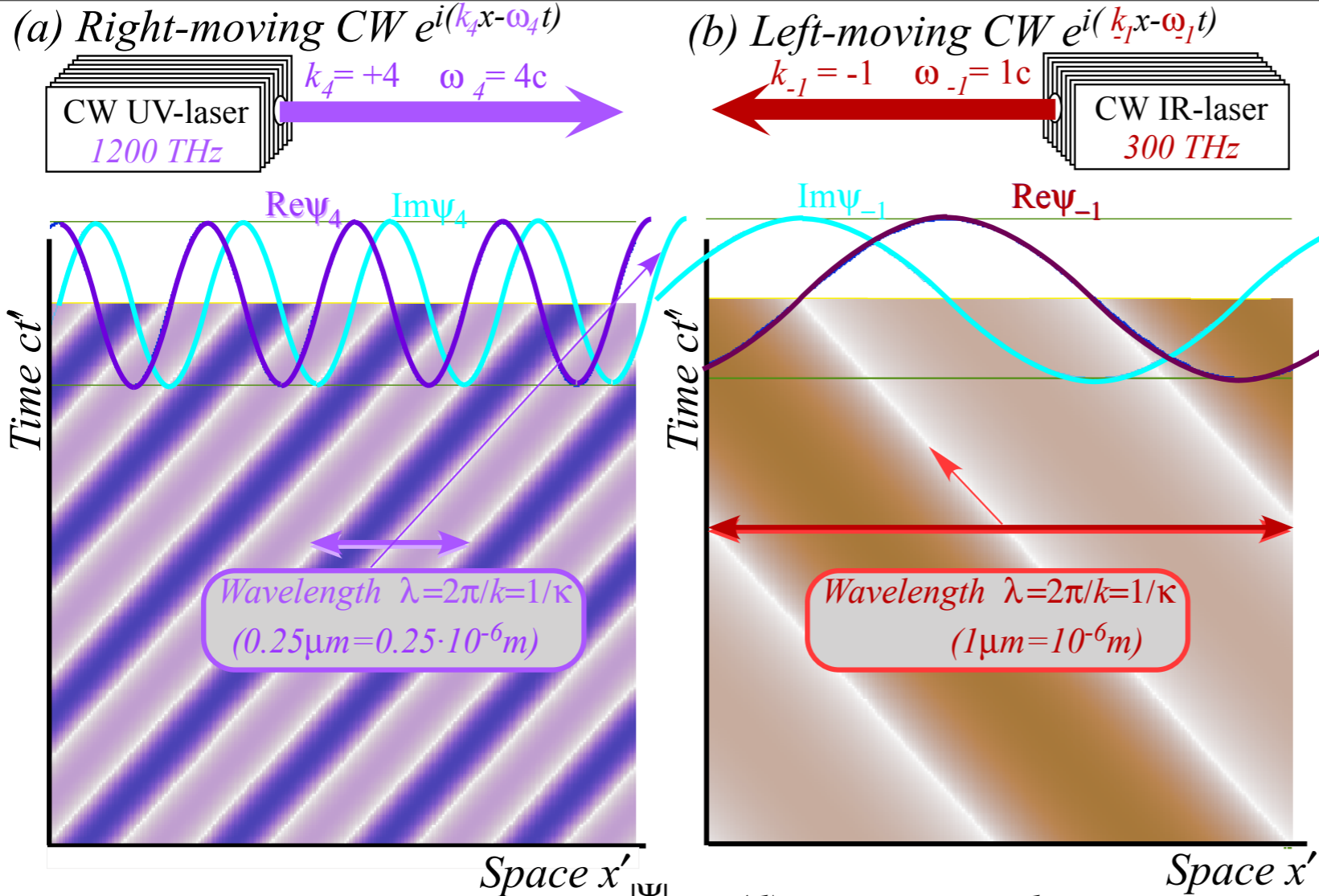
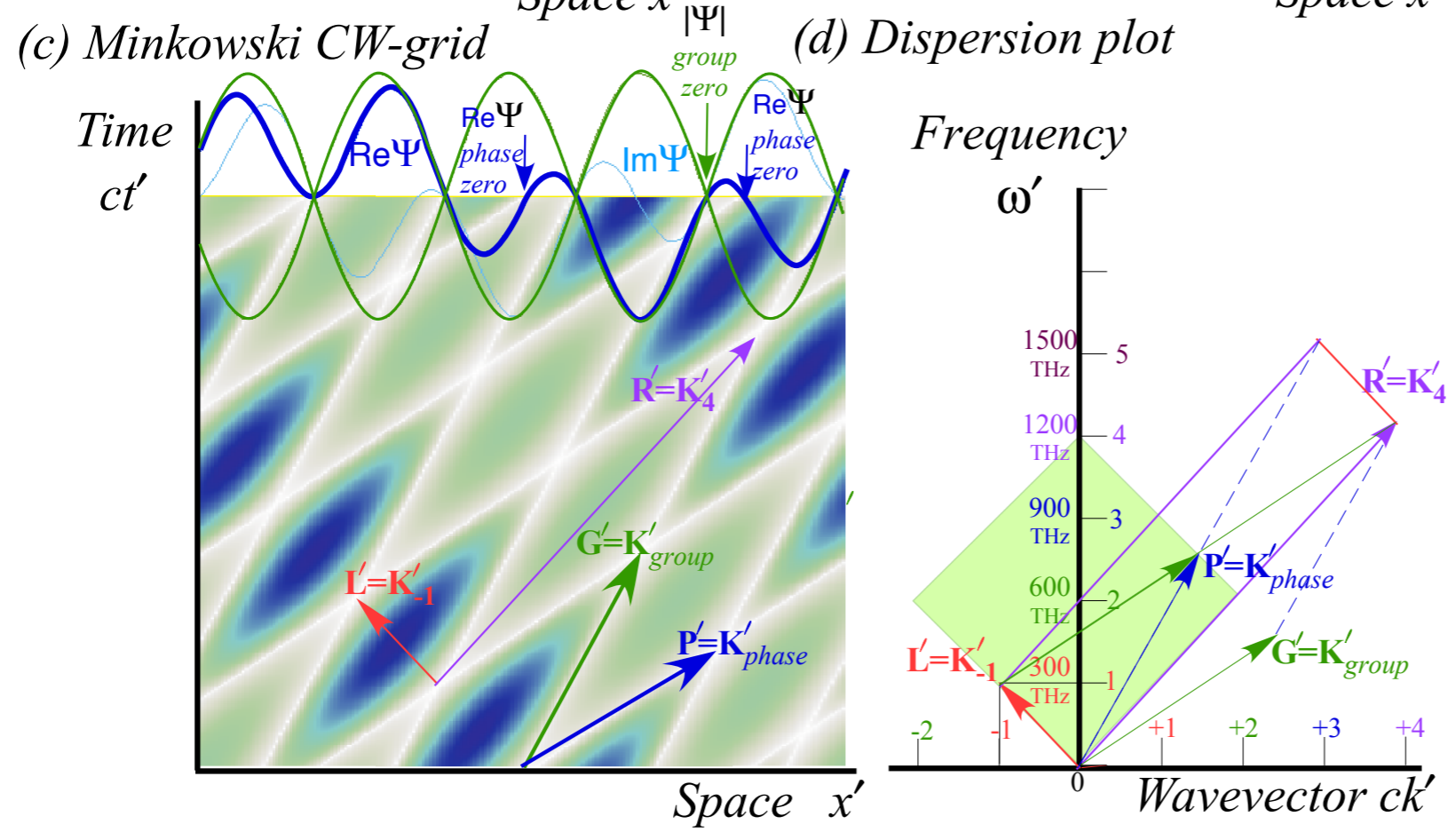
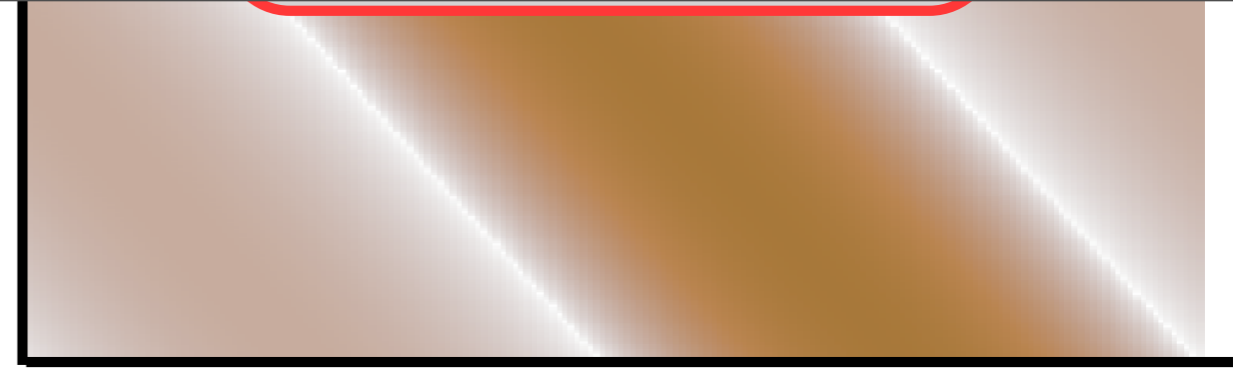
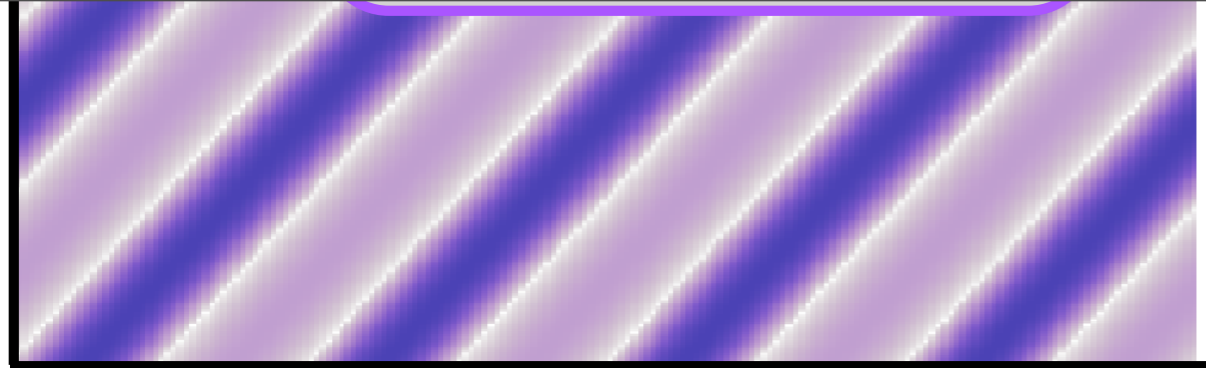


Fig. 9 in SR&QM

recall also:
 p. 3-11 of Lect.1





Space x'

Space x'

(c) Minkowski CW-grid

(d) Dispersion plot

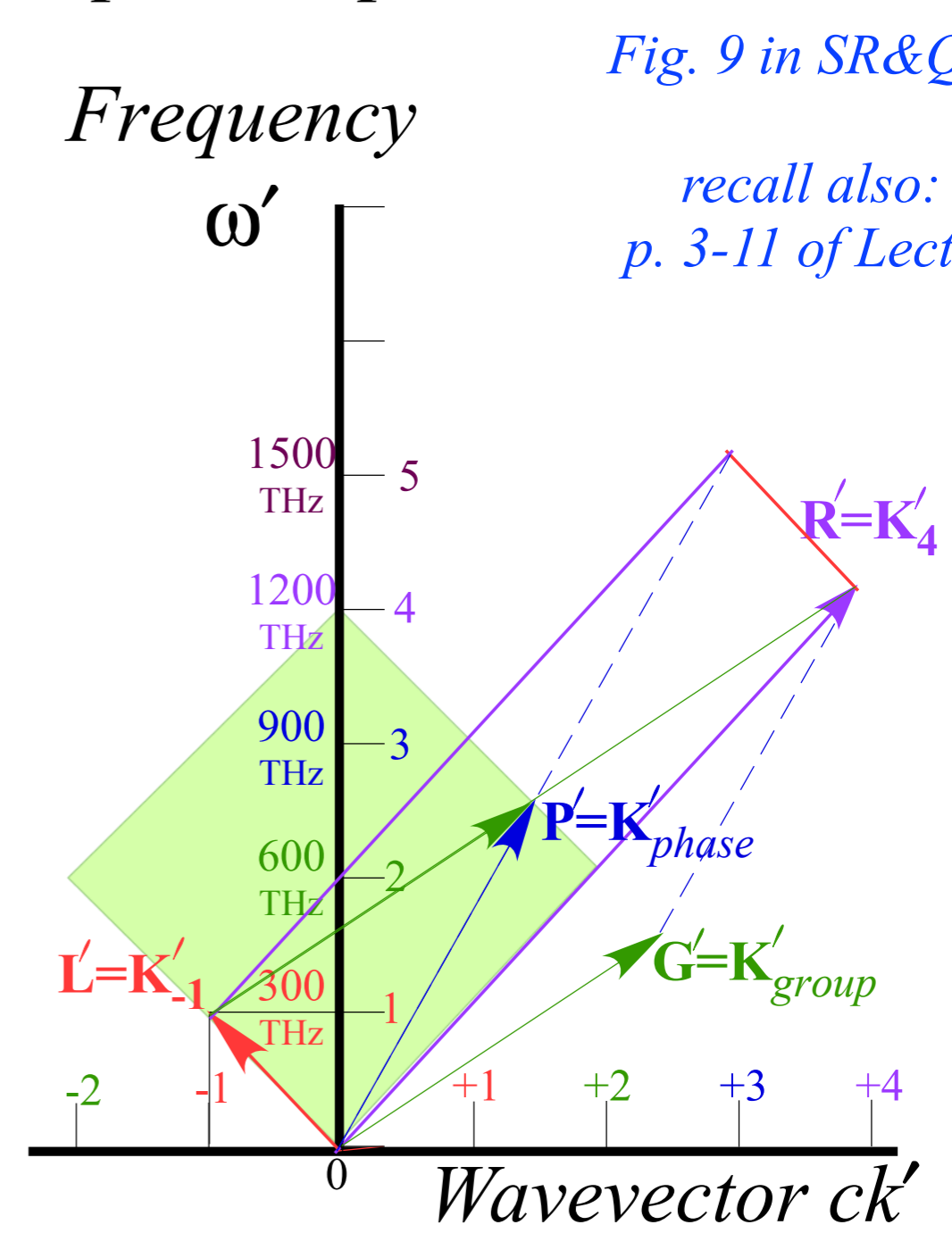
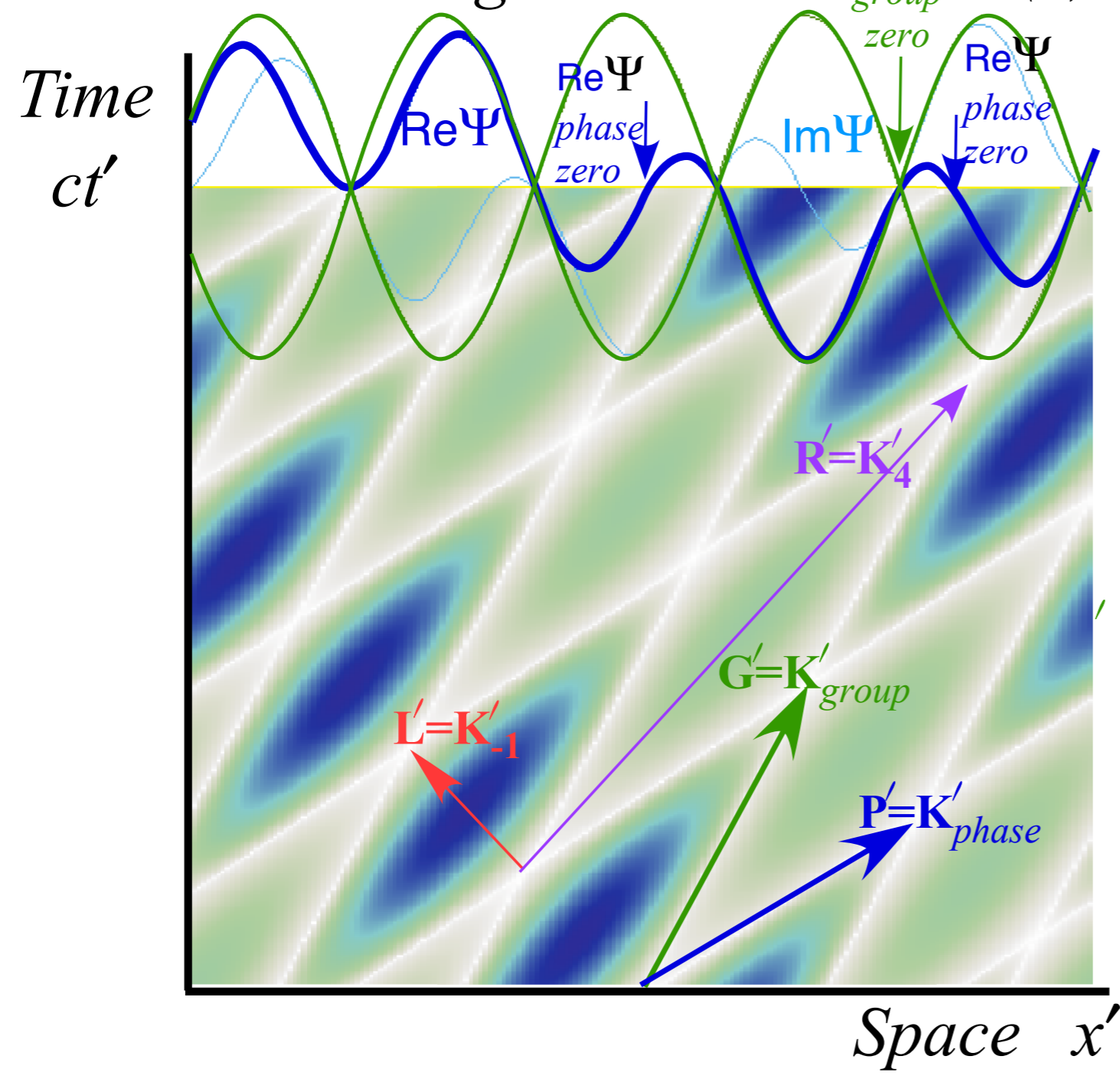
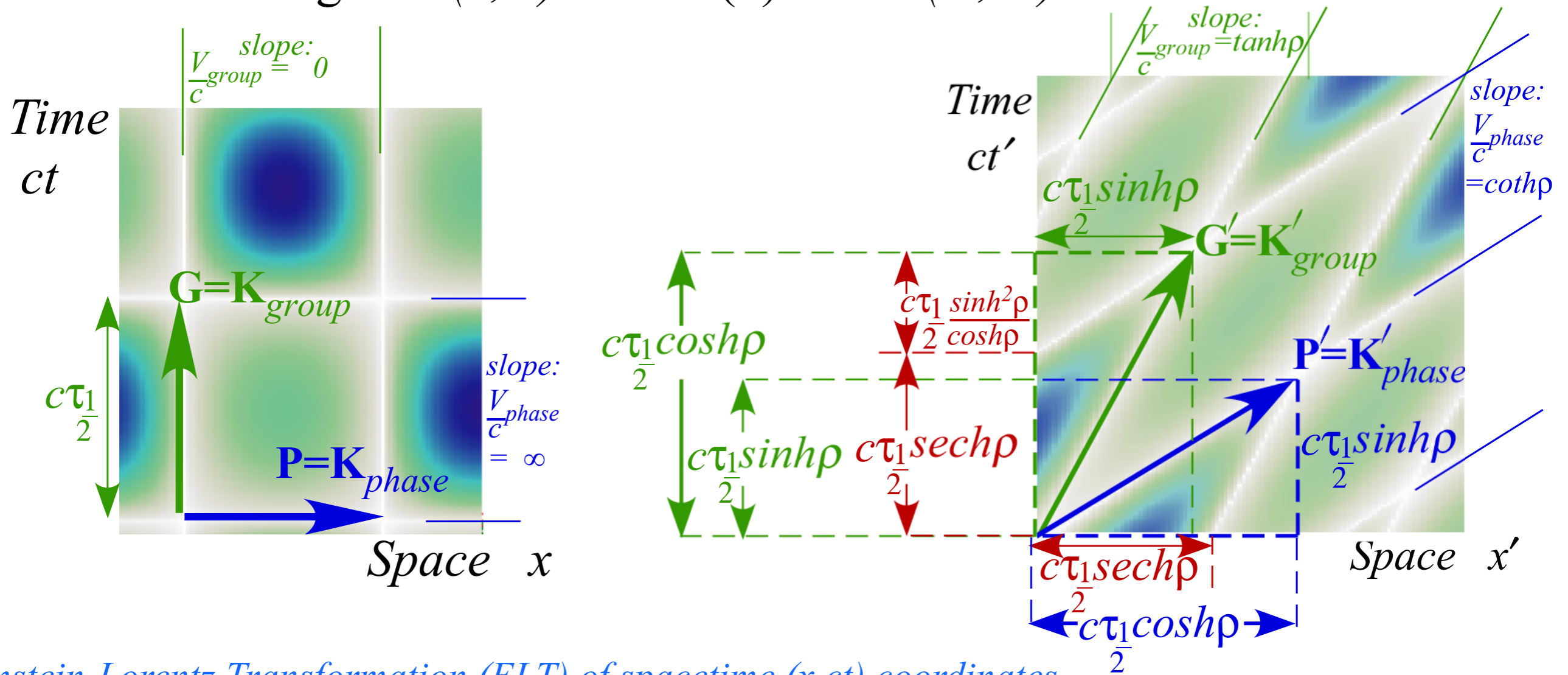


Fig. 9 in SR&QM

recall also:
p. 3-11 of Lect.1

(a) Alice's standing CW (x, ct) frame (b) Bob's (x', ct') view



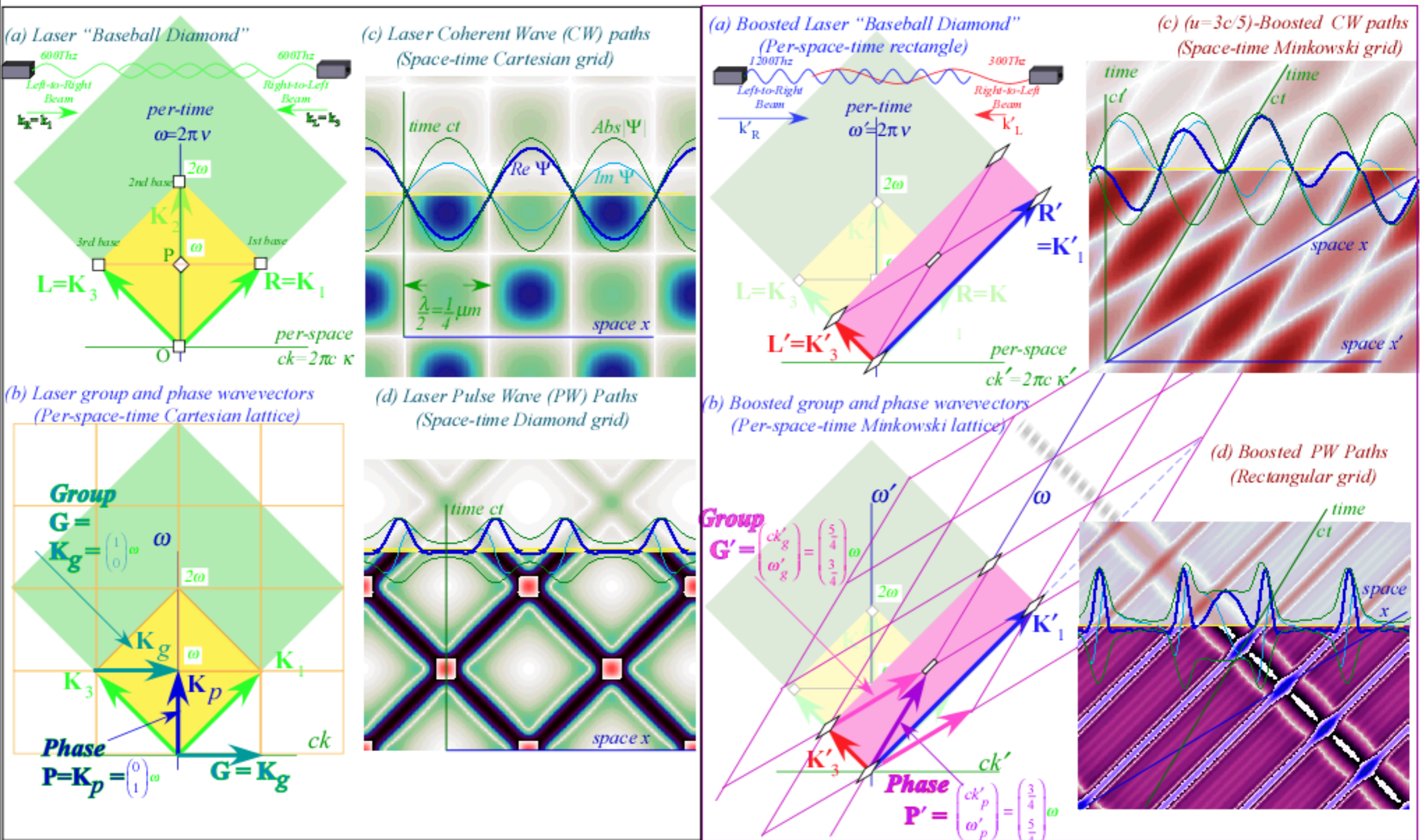
Einstein-Lorentz Transformation (ELT) of spacetime (x, ct) coordinates...

$$\begin{pmatrix} x'_{(any)} \\ ct'_{(any)} \end{pmatrix}_B = \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} x_{(any)} \\ ct_{(any)} \end{pmatrix}_A \Leftrightarrow \begin{pmatrix} x_{(any)} \\ ct_{(any)} \end{pmatrix}_A = \begin{pmatrix} \cosh \rho & -\sinh \rho \\ -\sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} x'_{(any)} \\ ct'_{(any)} \end{pmatrix}_B$$

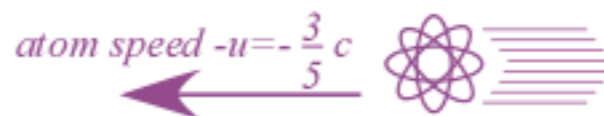
...is based upon the same ELT of per-spacetime (ck, ω) coordinates...

Old-fashioned notation (discussed below)

$$\begin{pmatrix} \omega'_{(any)} \\ ck'_{(any)} \end{pmatrix}_B = \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} \begin{pmatrix} \omega_{(any)} \\ ck_{(any)} \end{pmatrix}_A \text{ where: } \begin{pmatrix} \cosh \rho & \sinh \rho \\ \sinh \rho & \cosh \rho \end{pmatrix} = \begin{pmatrix} \gamma & \beta \cdot \gamma \\ \beta \cdot \gamma & \gamma \end{pmatrix}$$



Laser lab views




Atom views (sees lab going $+u = \frac{3}{5}c$)

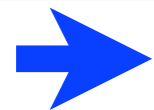
3. *Spectral theory of Einstein-Lorentz relativity*

Applying Doppler Shifts to per-space-time (ck, ω) graph

CW Minkowski space-time coordinates (x, ct) and PW grids

 *Relating Doppler Shifts b or $r=1/b$ to velocity u/c or rapidity ρ*

 *Lorentz transformation*



Connection: Conventional approach to relativity and old-fashioned formulas

Connection to conventional approach to relativity and old-fashioned formulas

Given *phase* and *group* wave formulas:

$$\begin{pmatrix} \omega'_{\text{phase}} \\ ck'_{\text{phase}} \end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^\rho + e^{-\rho}}{2} \\ \frac{e^\rho - e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh \rho \\ \sinh \rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} \omega'_{\text{group}} \\ ck'_{\text{group}} \end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^\rho - e^{-\rho}}{2} \\ \frac{e^\rho + e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$

Connection to conventional approach to relativity and old-fashioned formulas

Given *phase* and *group* wave formulas:

$$\begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^\rho + e^{-\rho}}{2} \\ \frac{e^\rho - e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh \rho \\ \sinh \rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^\rho - e^{-\rho}}{2} \\ \frac{e^\rho + e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$

Calculate *phase velocity*

and

group velocity of coordinate waves:

$$\frac{V_{phase}}{c} = \frac{\omega'_{phase}}{ck'_{phase}} = \frac{\cosh \rho}{\sinh \rho} = \frac{5}{3} = \coth \rho$$

$$\frac{V_{group}}{c} = \frac{\omega'_{group}}{ck'_{group}} = \frac{\sinh \rho}{\cosh \rho} = \frac{3}{5} = \tanh \rho$$

$$\frac{V'_{group}}{c} = \frac{u}{c} = \tanh \rho = \frac{e^\rho - e^{-\rho}}{e^\rho + e^{-\rho}} = \frac{b - b^{-1}}{b + b^{-1}} = \frac{b^2 - 1}{b^2 + 1} \equiv \beta$$

*old-fashioned
relativity
parameter*
 $\beta = u/c$

Connection to conventional approach to relativity and old-fashioned formulas

Given *phase* and *group* wave formulas:

$$\begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^\rho + e^{-\rho}}{2} \\ \frac{e^\rho - e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh \rho \\ \sinh \rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^\rho - e^{-\rho}}{2} \\ \frac{e^\rho + e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh \rho \\ \cosh \rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$

Calculate *phase velocity*

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group velocity of coordinate waves:

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*old-fashioned
relativity
parameter*
 $\beta = u/c$

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old-fashioned
relativity
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old-fashioned
Lorentz x -contraction
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old-fashioned
Lorentz *x*-contraction
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old-fashioned
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old-fashioned asimultaneity coeff.

$$\frac{u/c}{\sqrt{1-u^2/c^2}} = \sinh \rho$$

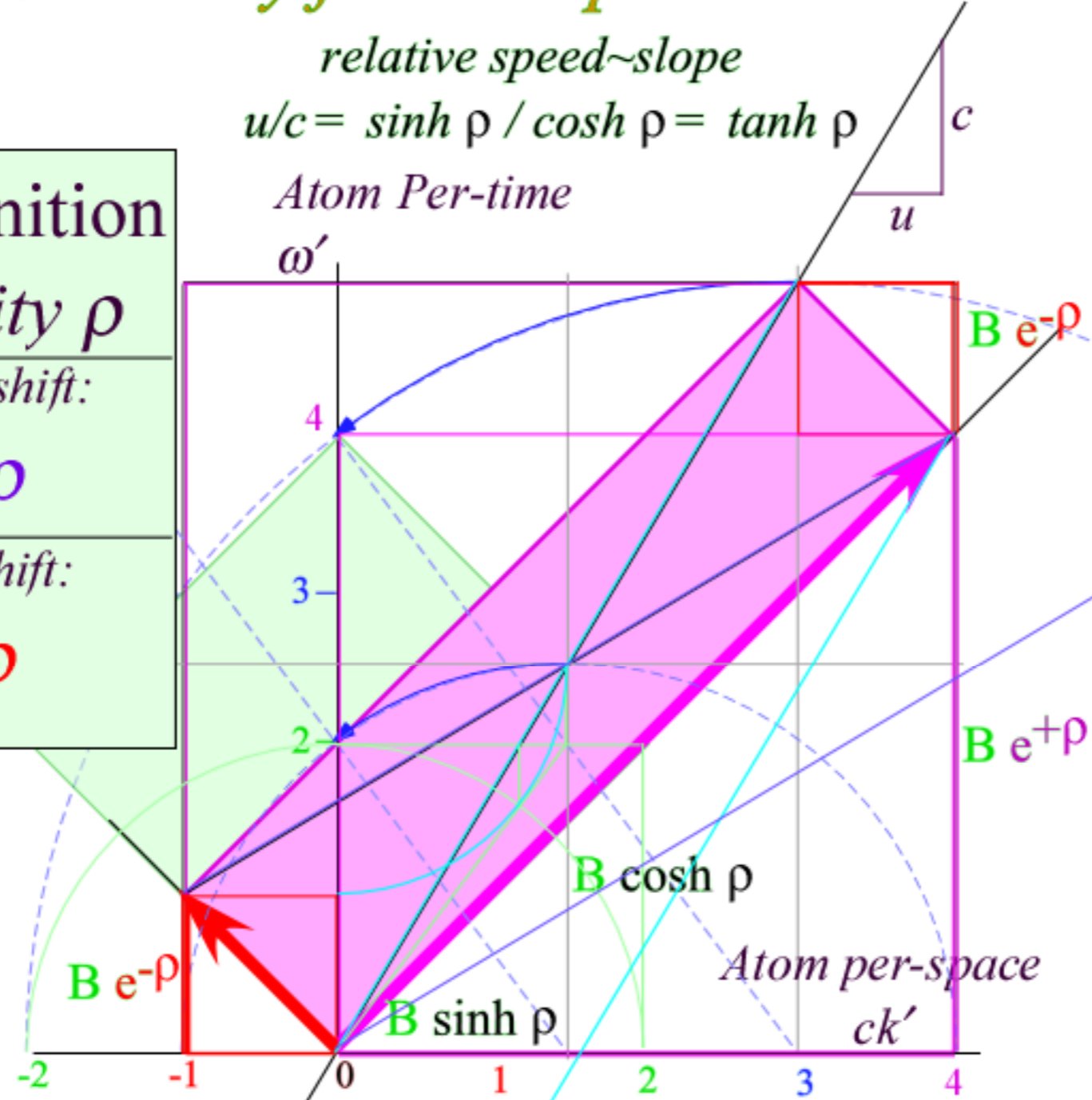
old-fashioned Einstein t-dilation parameter

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Euclidian Geometry for Per-spacetime Relativity

Key Definition of Rapidity ρ
 Doppler blue shift:
 $Bb = B e^{+\rho}$
 Doppler red shift:
 $Br = B e^{-\rho}$

relative speed~slope
 $u/c = \sinh \rho / \cosh \rho = \tanh \rho$



Key Results:

ω vs. ck
 “winks” vs. “kinks”

$\omega = B \cosh \rho$
 $ck = B \sinh \rho$

group velocity:
 $\frac{\omega}{ck} = \frac{u}{c} = \tanh \rho$

phase velocity:
 $\frac{ck}{\omega} = \frac{c}{u} = \coth \rho$

$$B \sinh \rho = (B e^{+\rho} - B e^{-\rho}) / 2$$

$$B \cosh \rho = (B e^{+\rho} + B e^{-\rho}) / 2$$

$\sinh \rho = \sqrt{1 - \frac{u^2}{c^2}}$	Key Quantities Lorentz-Einstein factors	$\cosh \rho = \sqrt{1 + \frac{u^2}{c^2}}$
-------------------------------------------	---------------------------------------------------	-------------------------------------------

Related material at
 “per space-per-time”
 setting of:

<http://www.uark.edu/ua/modphys/testing/markup/RelaVvavityWeb.html>

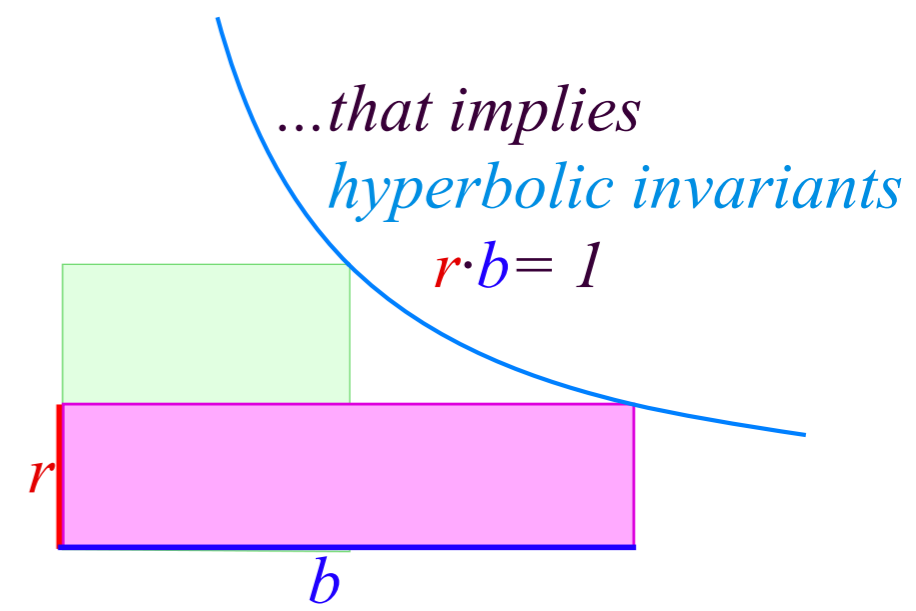
Euclidian wave geometry with time-reversal symmetry imply

*Lab
frame
area...*

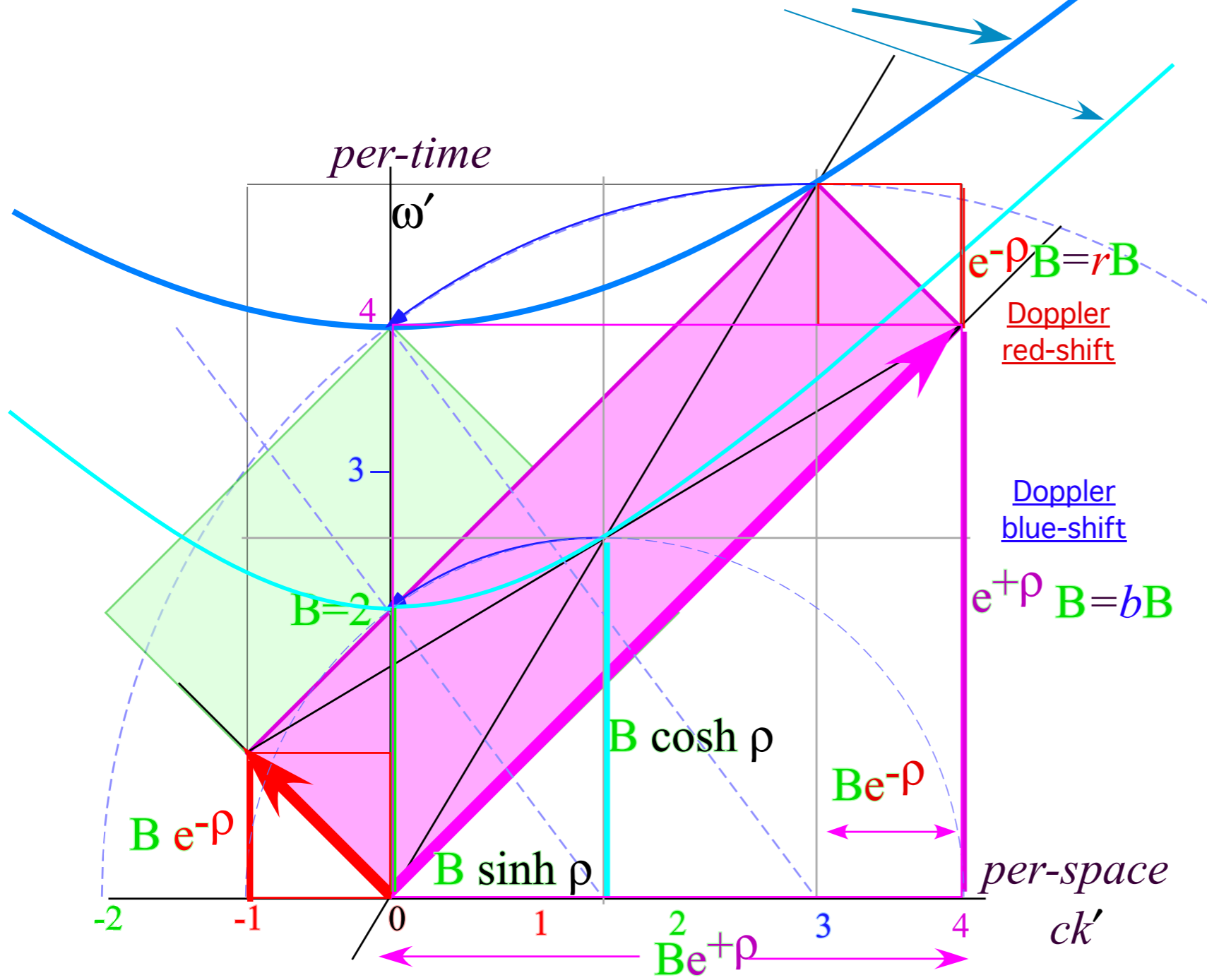
equals

r *Atom frame area...*

b
by time-reversal axiom: $r = 1/b$



Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas: $\omega = nB \cosh \rho$

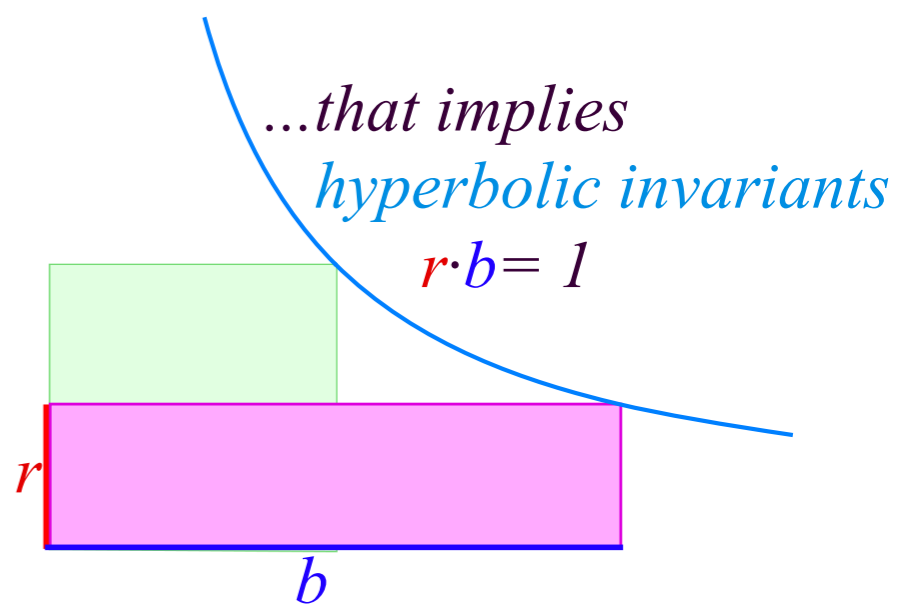


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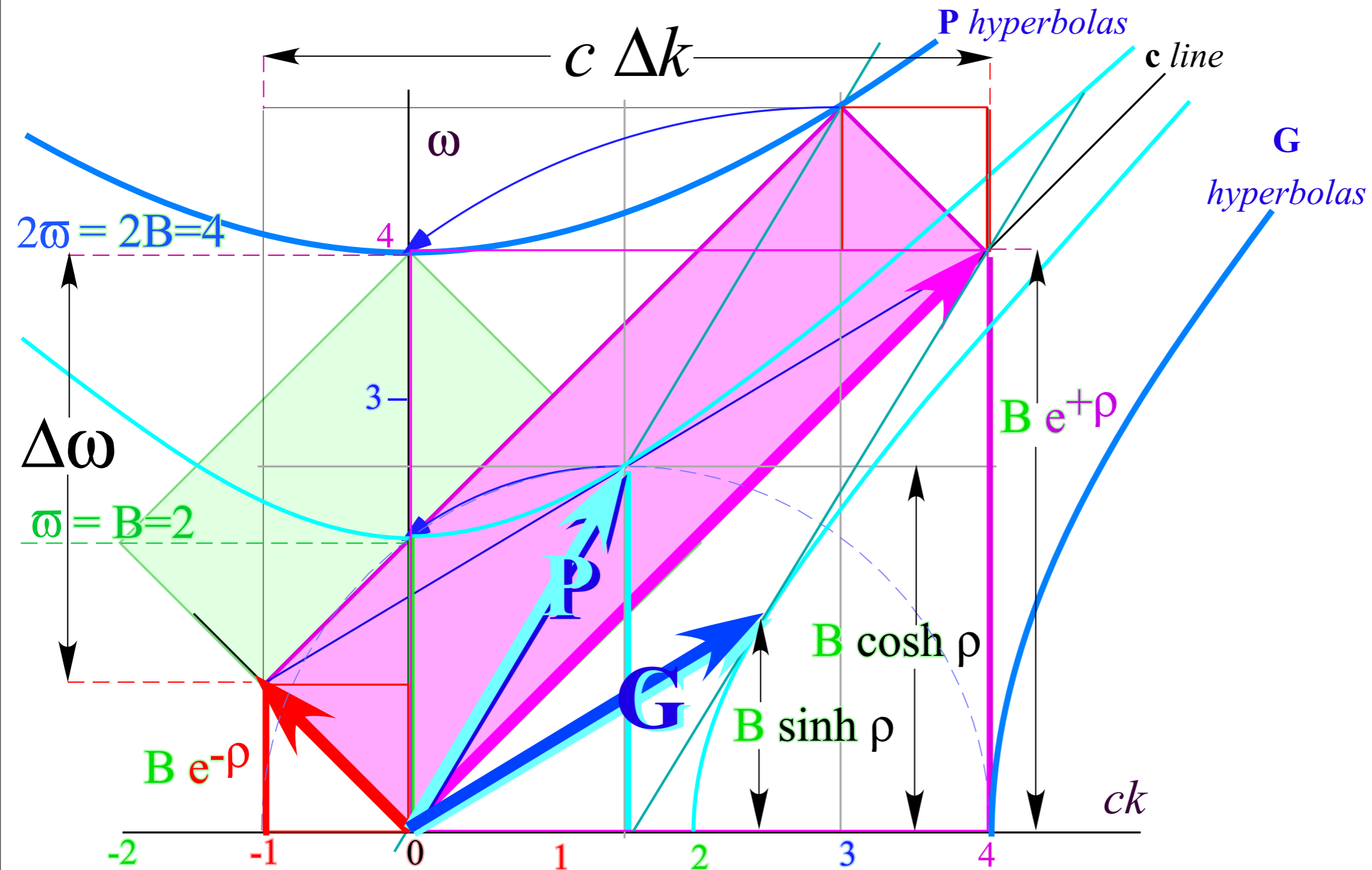
by time-reversal axiom: $r = 1/b$



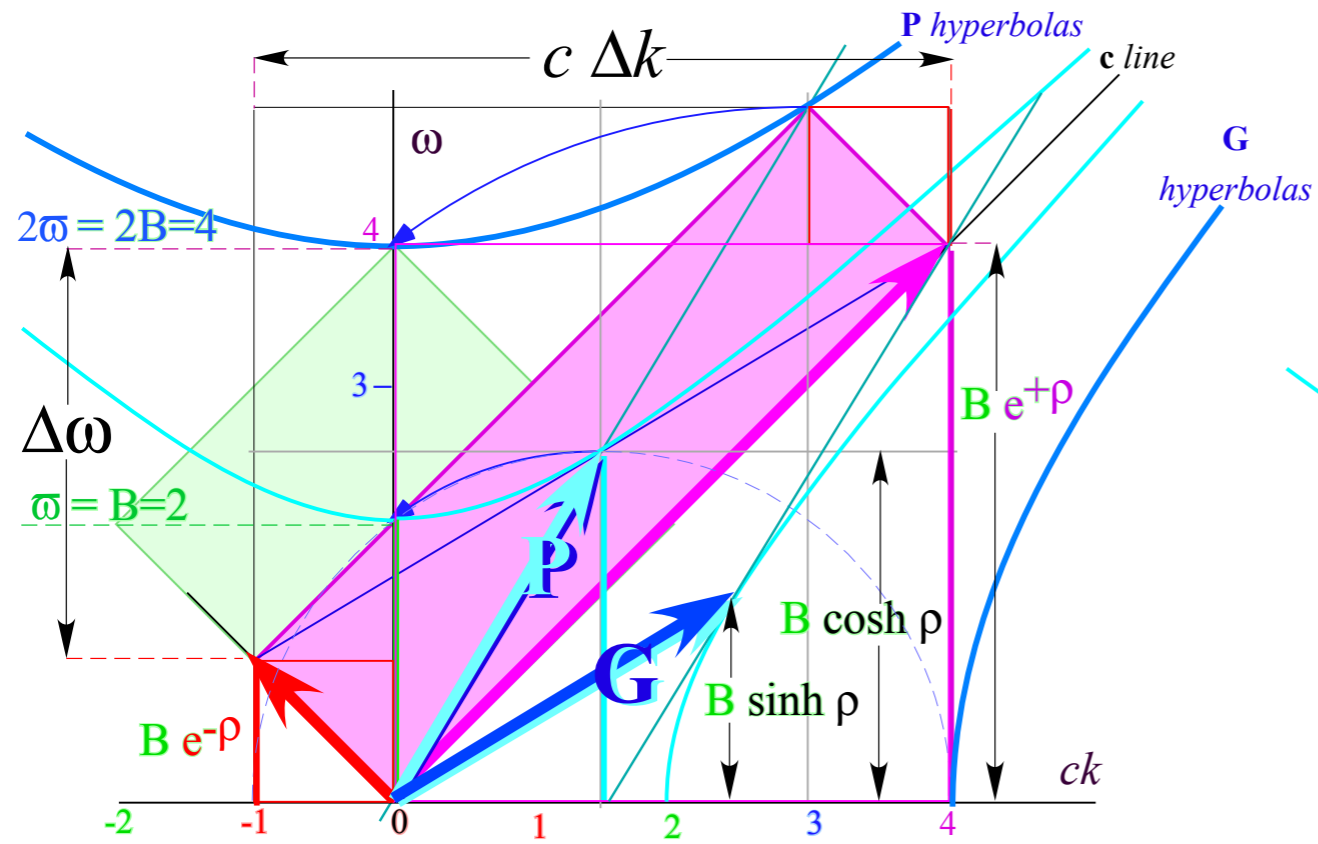
$$B \sinh \rho = (B e^{+\rho} - B e^{-\rho}) / 2$$

$$B \cosh \rho = (B e^{+\rho} + B e^{-\rho}) / 2$$

Group velocity u and phase velocity c^2/u are hyperbolic tangent slopes



Group velocity u and phase velocity c^2/u are hyperbolic tangent slopes



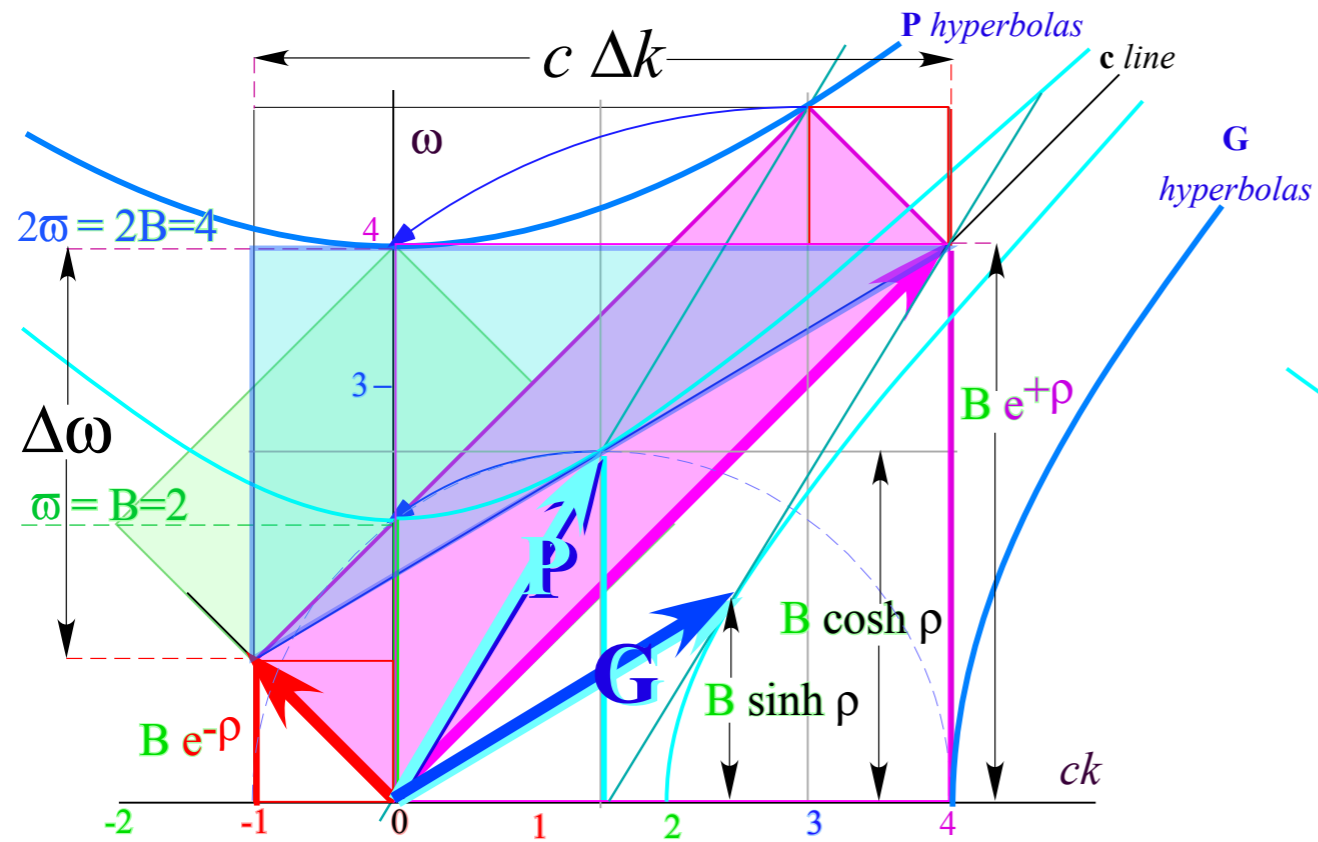
Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with *LARGE* Δk
(not infinitesimal)

Relativistic
group wave
speed $u = c \tanh \rho$

Group velocity u and phase velocity c^2/u are hyperbolic tangent slopes



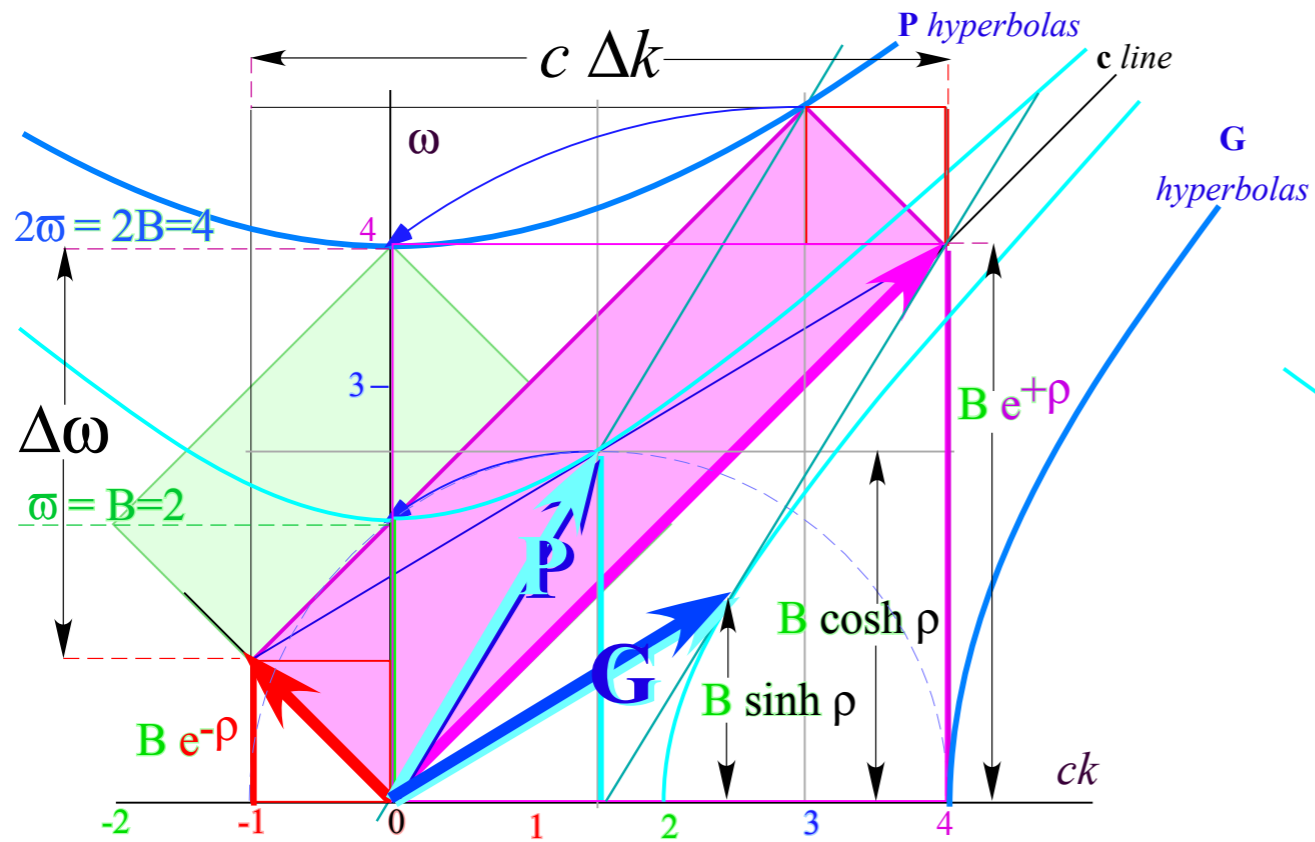
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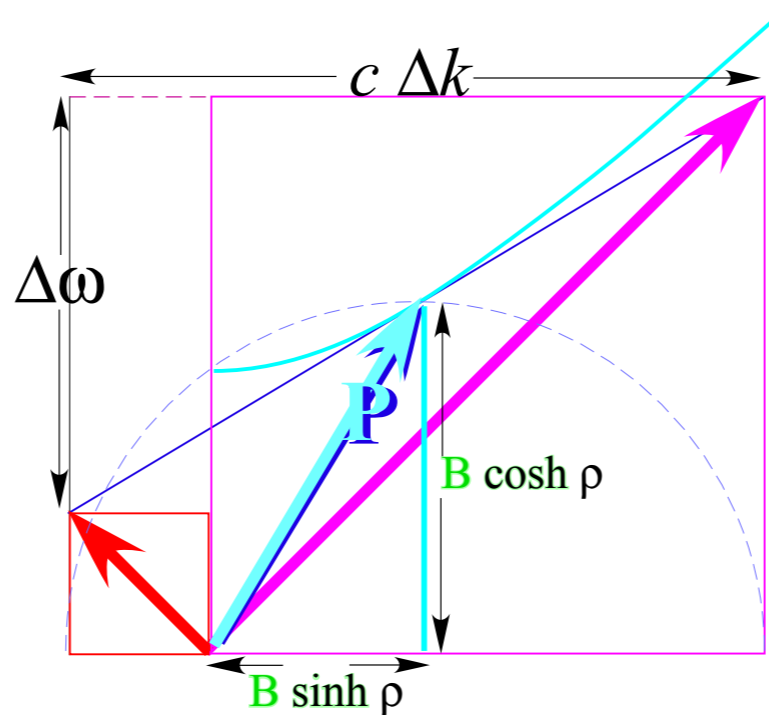


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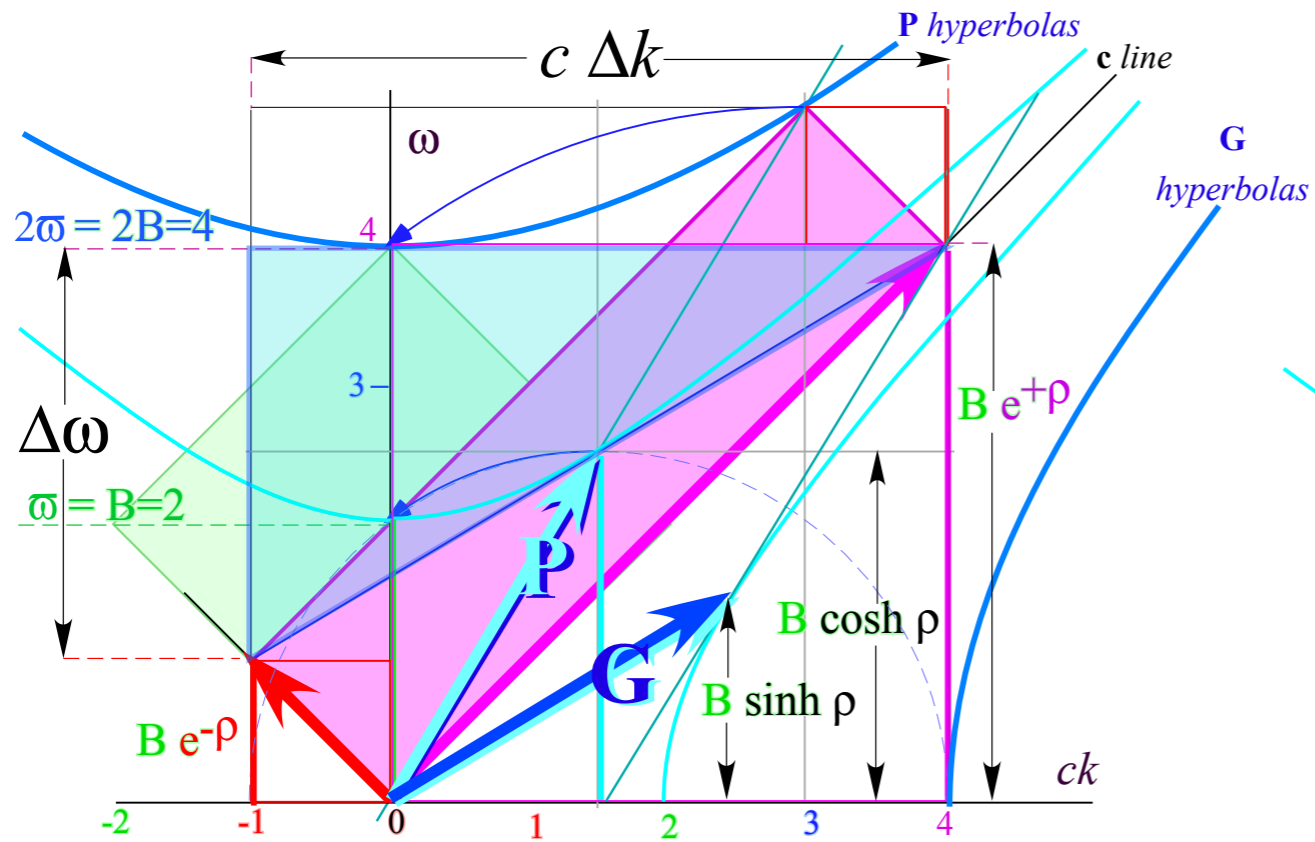
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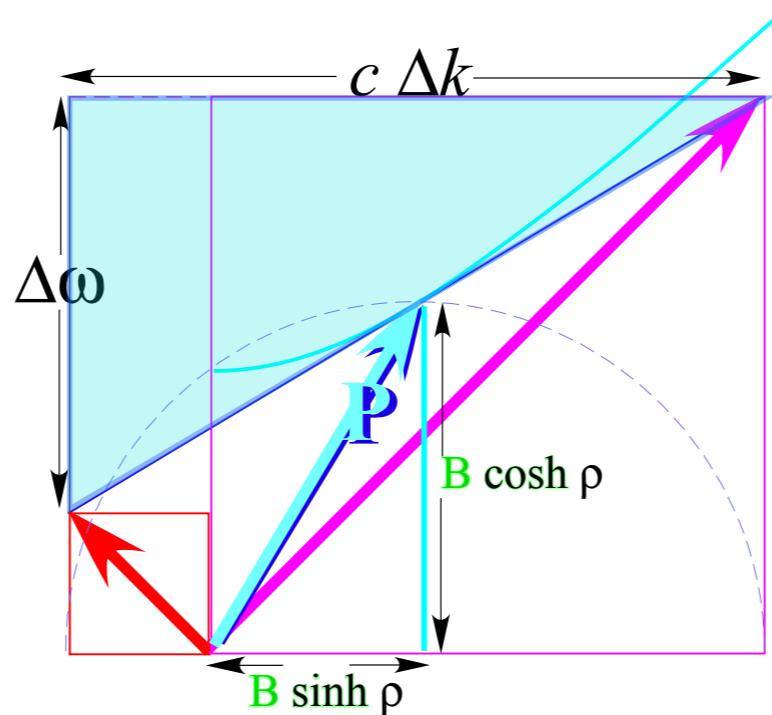


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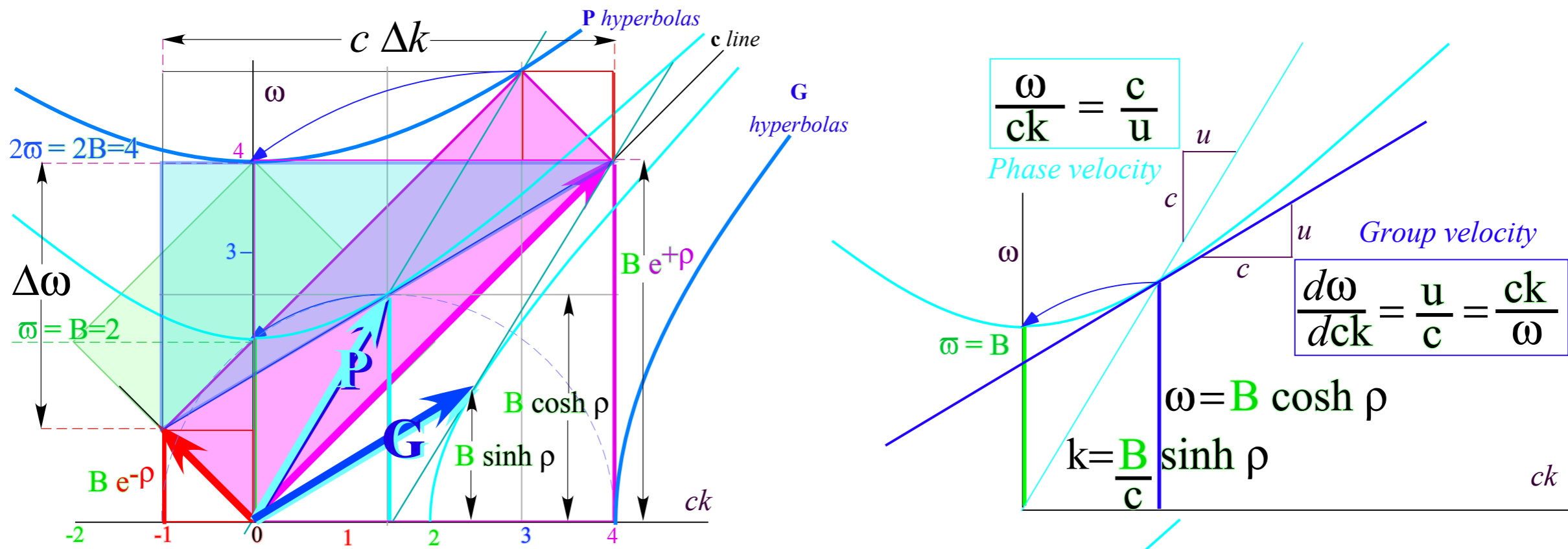
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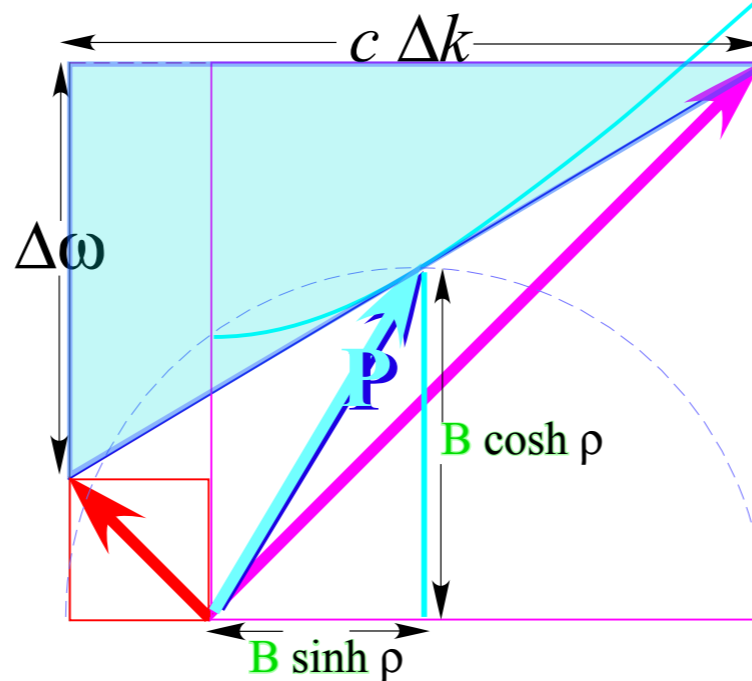


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Newtonian speed $u \sim c\rho$
Low speed approximation
Rapidity ρ approaches u/c

Connection to conventional approach to relativity and old-fashioned formulas

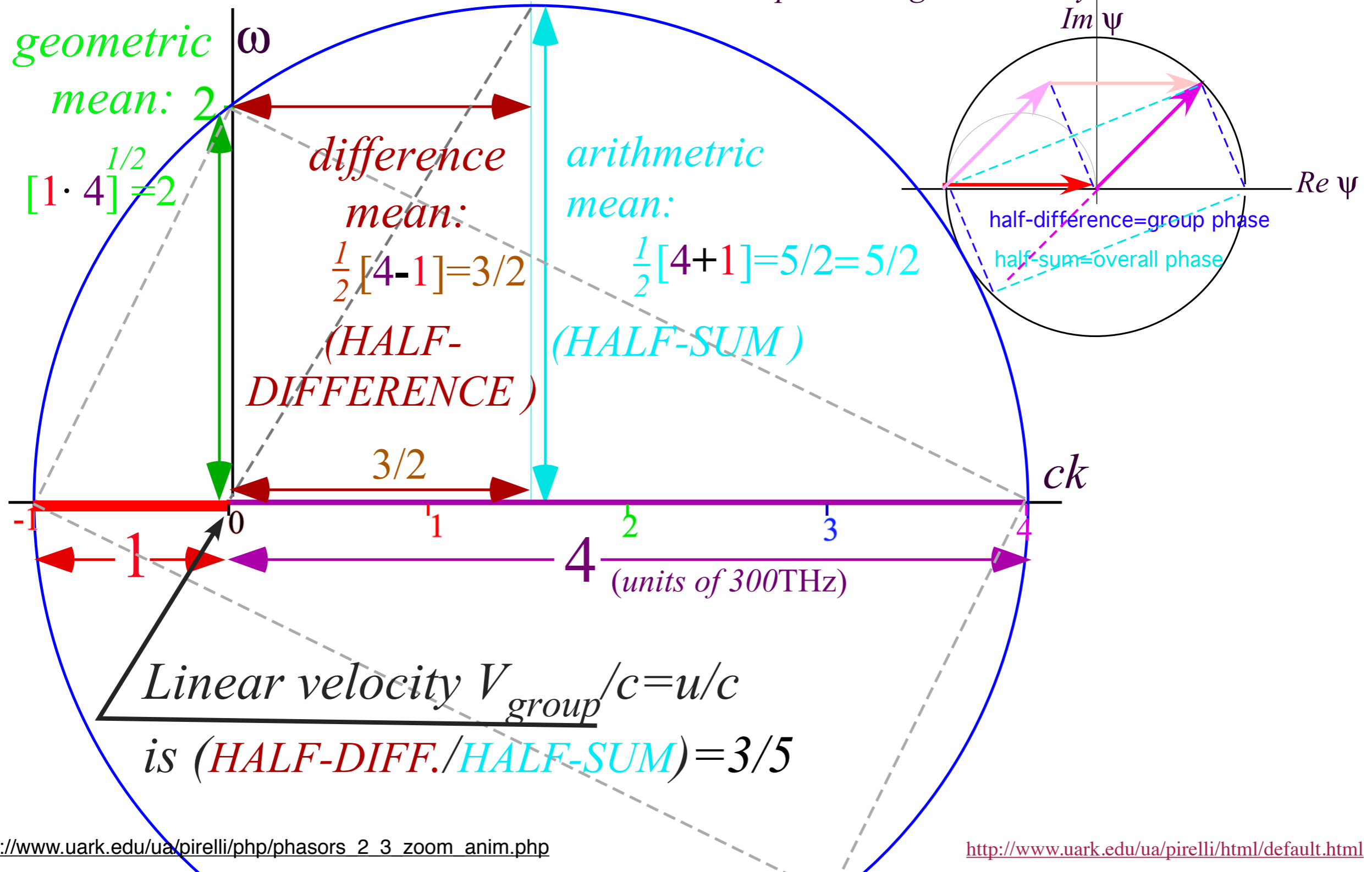
The most old-fashioned form(ula) of all: Thales & Euclid means

Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition

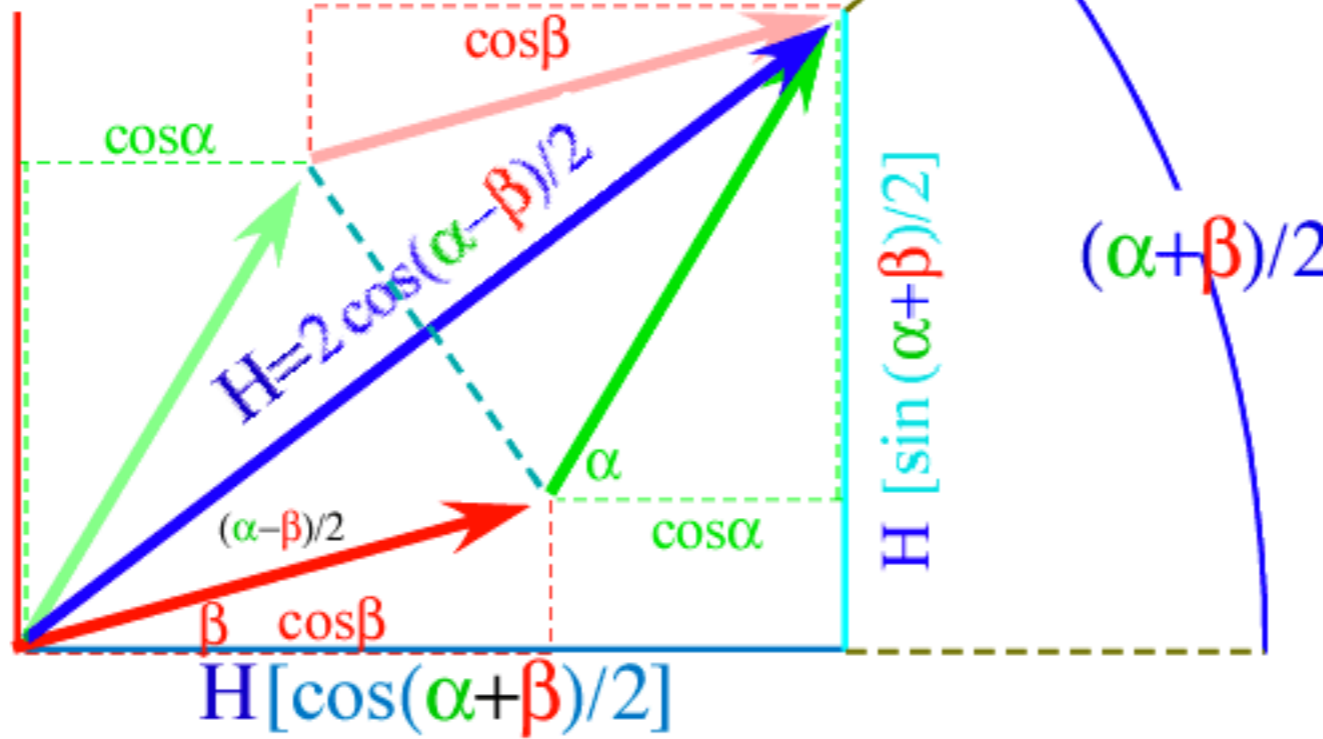


Connection to conventional approach to relativity and old-fashioned formulas

The most old-fashioned form(ula) of all: Thales & Euclid means

The detailed trigonometry of half-sum & difference angles is shown below.
The wave is *factored* into a product of *group* and *phase* waves.

http://www.uark.edu/ua/pirelli/php/half_sum_5.php



Main Result: Factoring algebraic sums helps to locate *wave zeros*.

$$\begin{aligned} \cos\alpha + \cos\beta &= 2 \cos(\alpha-\beta)/2 \cdot [\cos(\alpha+\beta)/2] \\ \sin\alpha + \sin\beta &= 2 \sin(\alpha-\beta)/2 \cdot [\sin(\alpha+\beta)/2] \end{aligned}$$



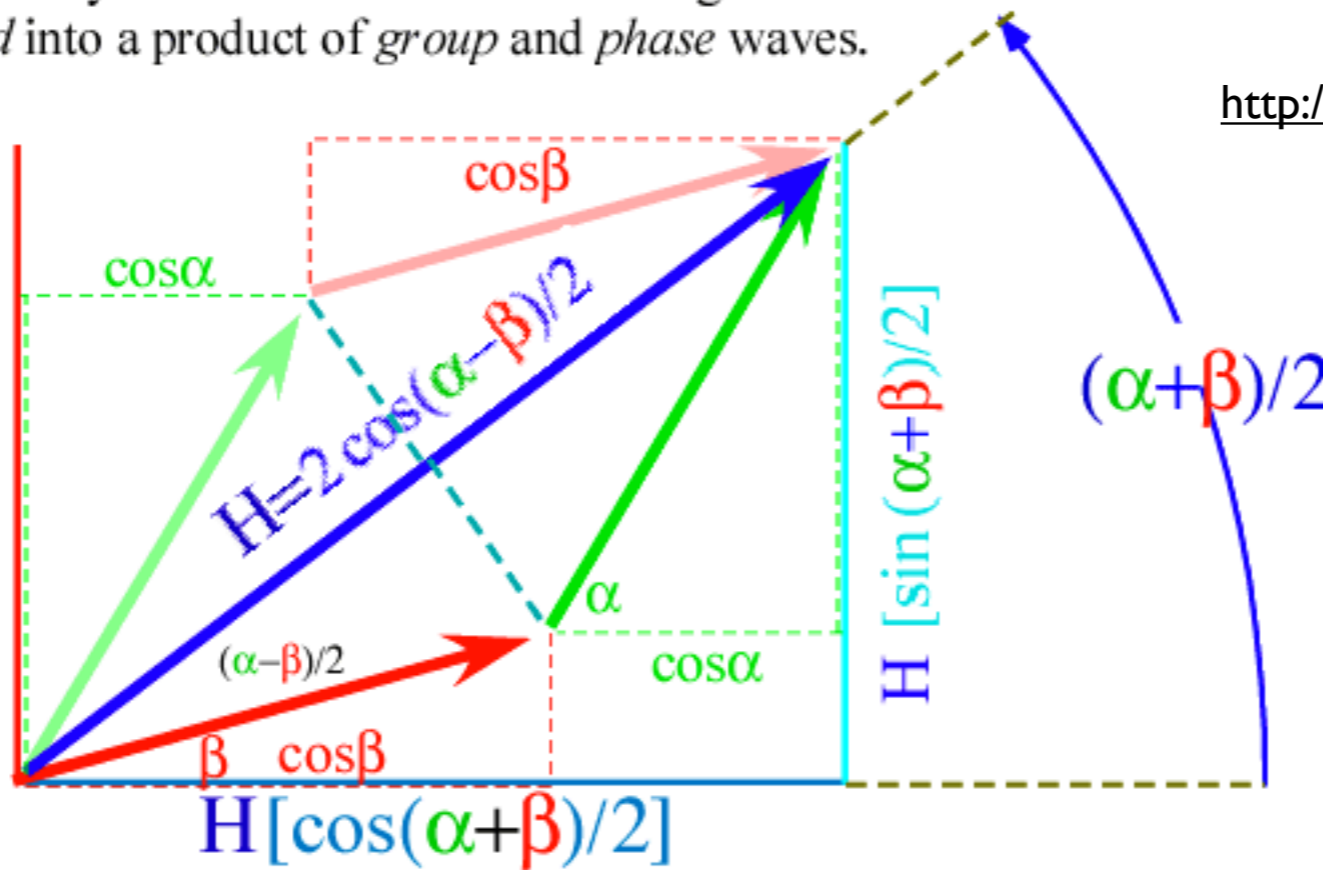
Sum is zeroed by *either* factor. Each factor's zero line is a *spacetime coordinate line*.

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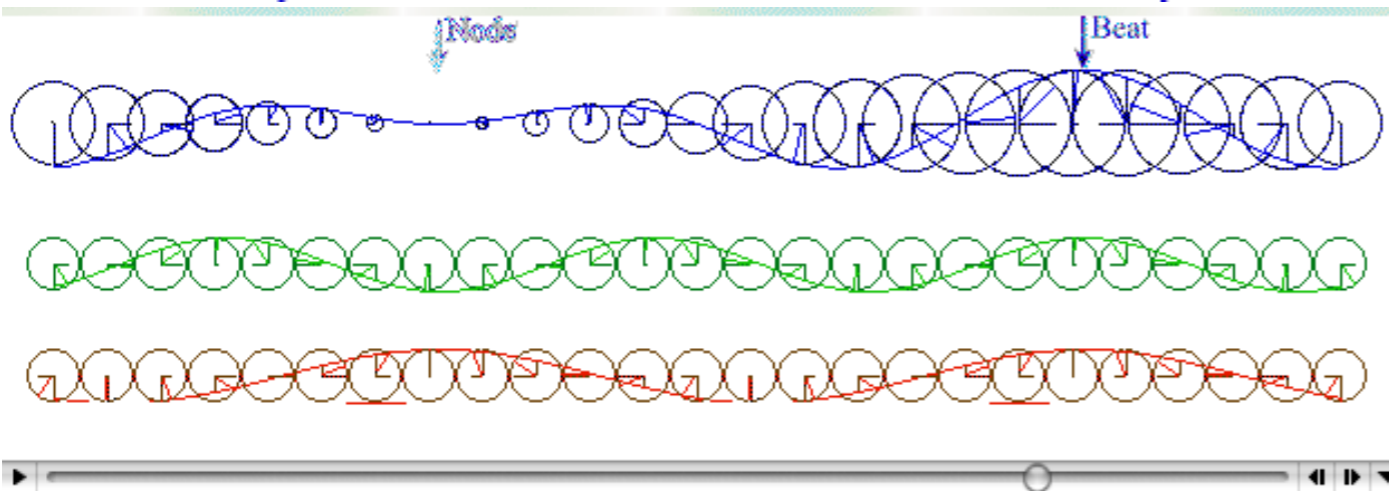


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http://www.uark.edu/ua/pirelli/php/phasors_2_3_anim.php



atom speed $-u$ 
LaserPer-Spacetime

atom speed 0 
AtomPer-Spacetime

