AMOP Lecture 3
Tue 1.28-Thur 1.30.2014

Relativity of wave-optics and Lorentz-Minkowski coordinates II.

Ch. 3 of Unit 8 CMwBang! and p.1-28 Relativity&QuantumTheory by Rule&Compass

1. Spectral theory of Einstein-Lorentz relativity

Applying Doppler Shifts to per-space-time (ck,ω) graph

CW Minkowski space-time coordinates (x,ct) and PW grids

Relating Doppler Shifts b or r=1/b to velocity u/c or rapidity ρ

Connection: Conventional approach to relativity and old-fashioned formulas

Invariant hyperbolas and hyperbolic relations

2. Reciprocal dilation and contraction properties

The most old-fashioned form(ula) of all: Thales & Euclid means

Galileo wins one! (...in gauge space)

A point or two to review

*That old **Time-Reversal** meta-Axiom (that is so-oo-o neglected!)*

Inverse to Lorentz transformation T_{AB} *is* T_{BA}

..just as the arithmetic inverse of $\frac{\upsilon_A}{\upsilon_B}$ *is* $\frac{\upsilon_B}{\upsilon_A}$

..just as the arithmetic inver... of $e^{\rho_{AB}}$ *is* $e^{\rho_{BA}} = e^{-\rho_{AB}}$

..just as the arithmetic inver... of ρ_{AB} *is* $\rho_{BA} = -\rho_{AB}$

See animation: www.uark.edu/ua/pirelli/php/time_rev_sym.php

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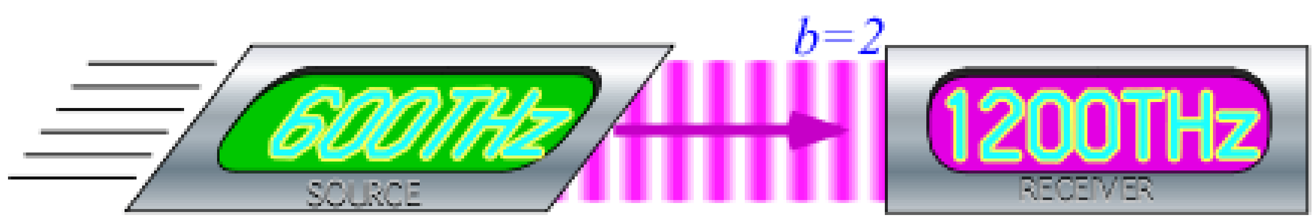
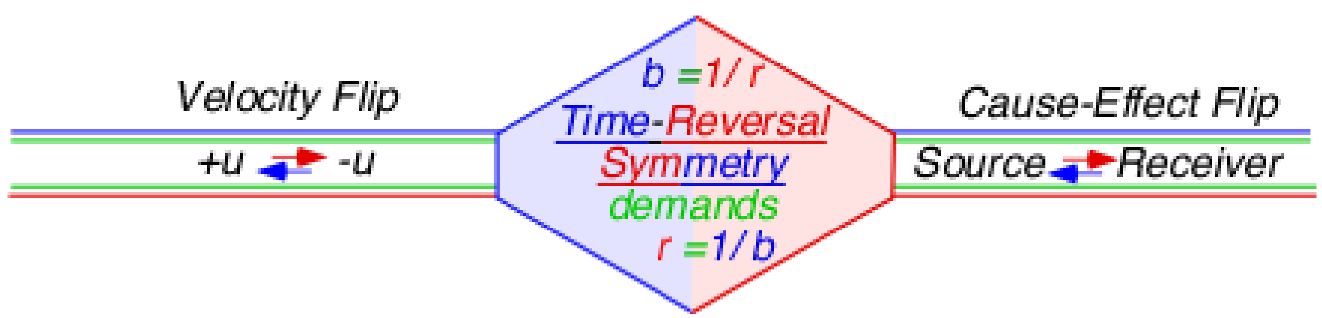
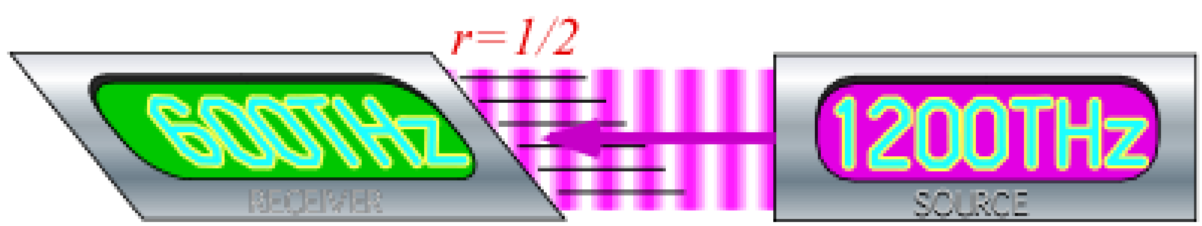
Detailed time reversal symmetry implies r=1/b.

Approaching source (600THz green)





Receding receiver sees Doppler red-shift of 1200THz source to 600THz (600THz)=r·(1200THz) with r=1/2



See animation: www.uark.edu/ua/pirelli/php/time_rev_sym.php

Review Spectral theory of Einstein-Lorentz Relativity

Applying Doppler Shifts to per-space-time (ck,ω) graph

CW Minkowski space-time coordinates (x,ct) and PW grids

Relating Doppler Shifts b or r=1/b to velocity u/c or rapidity ρ

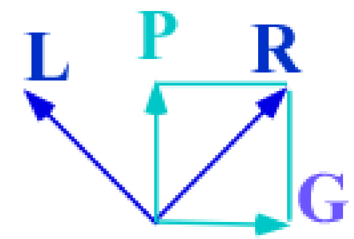
Lorentz transformation

Deriving Spacetime and per-spacetime coordinate geometry by:

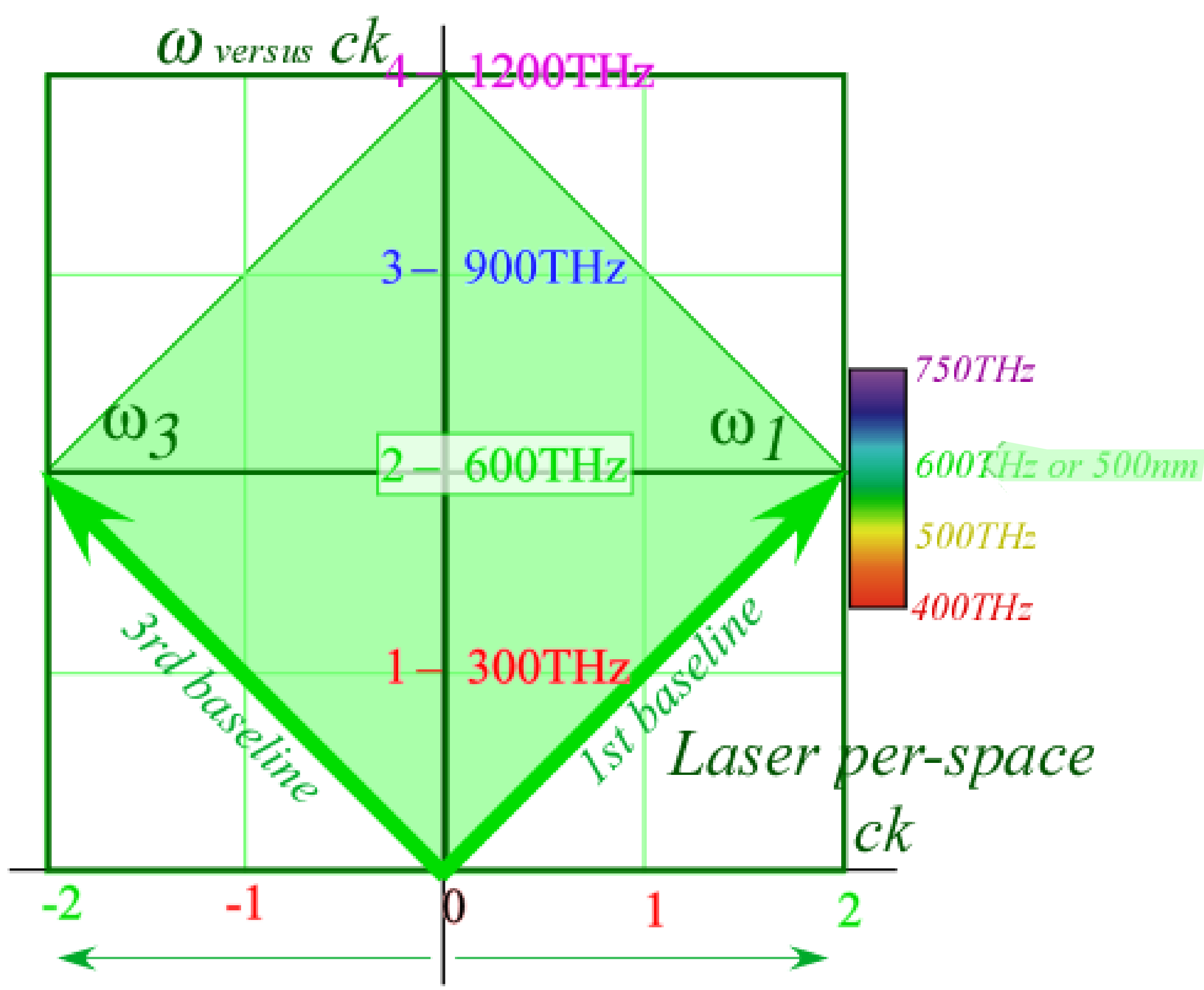
(1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$



LaserPer-Spacetime



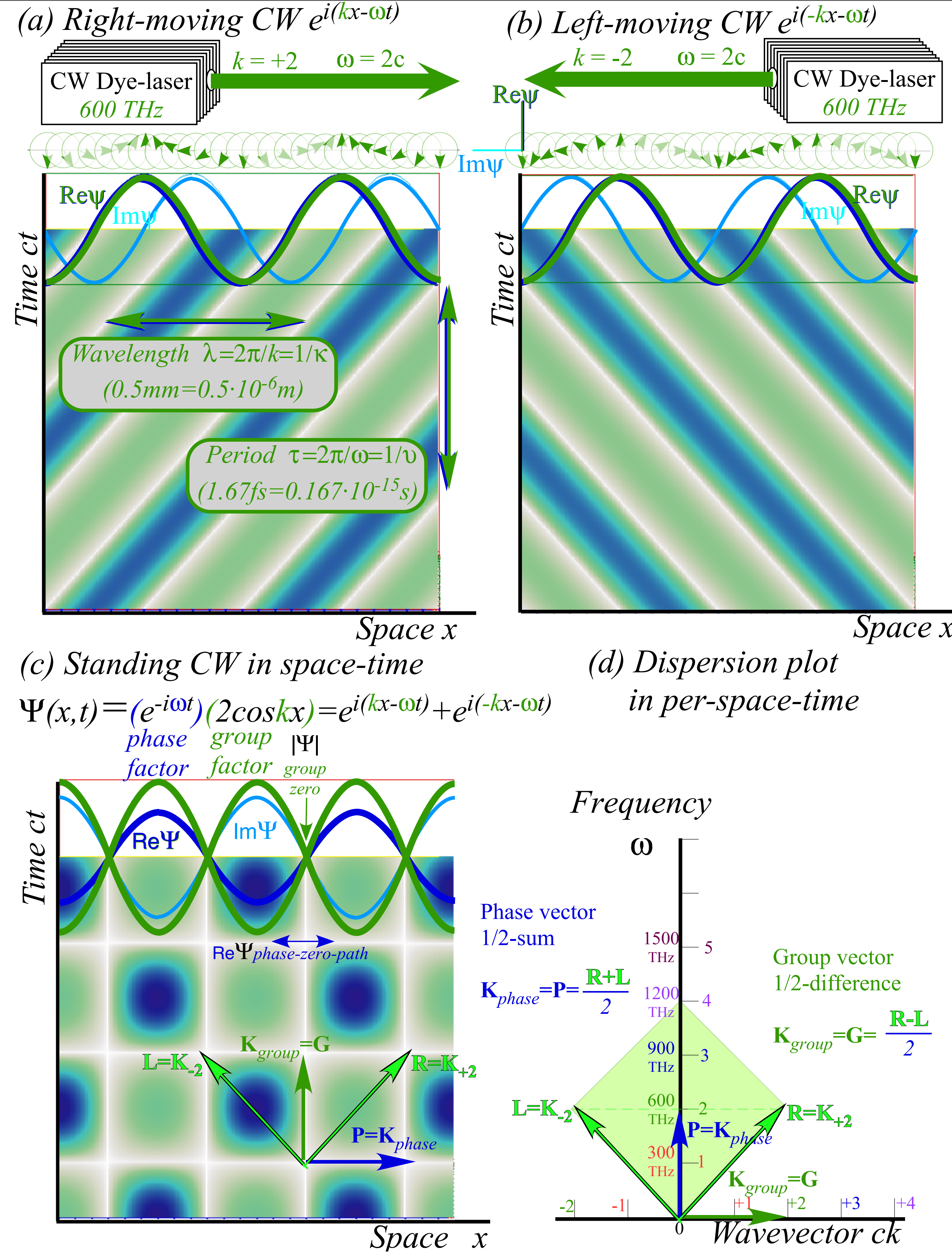


Fig. 5 in SR&QM

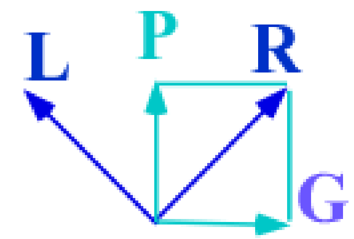
recall also:
p. 3-11 of Lect.1

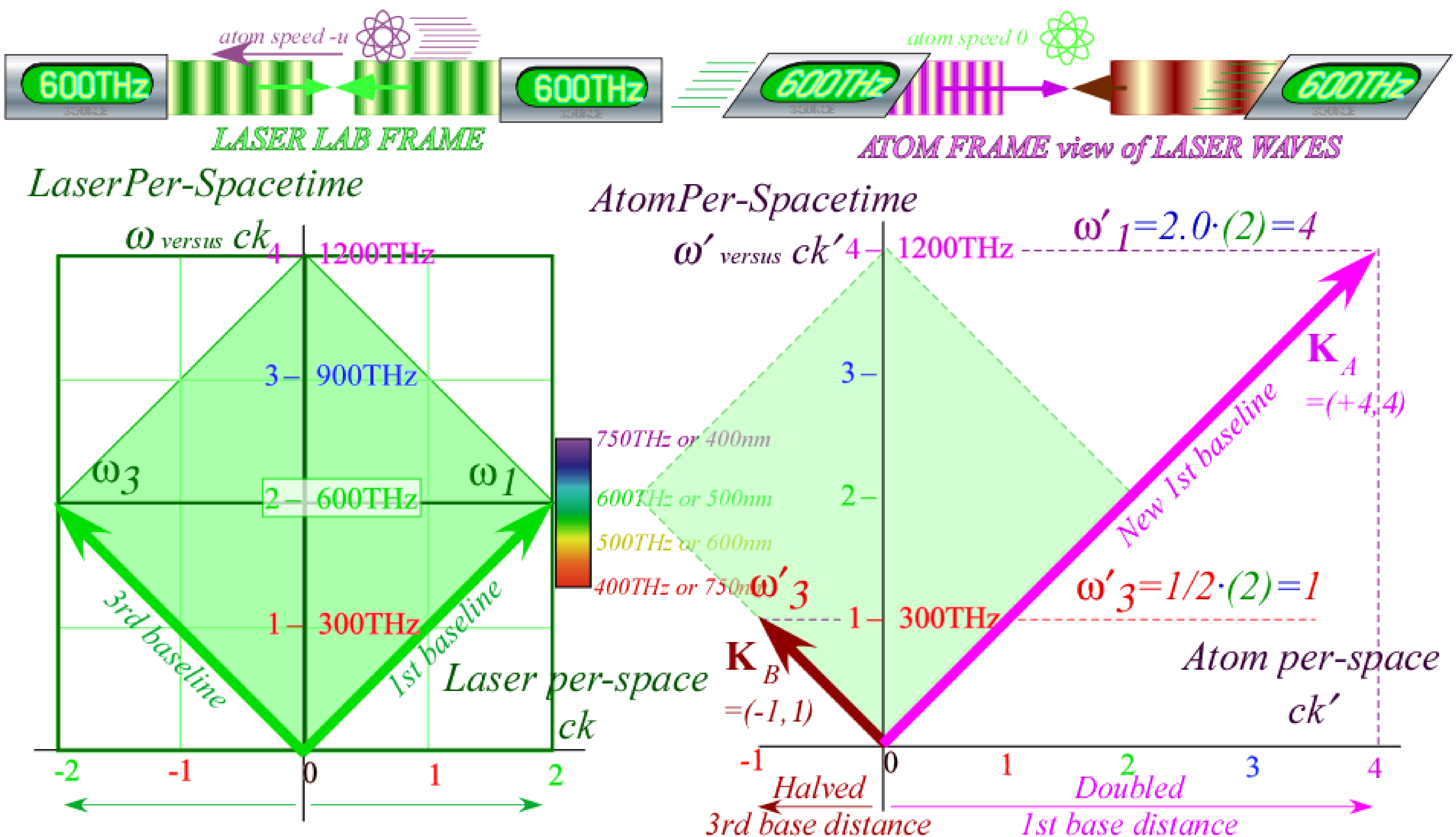
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom *"All colors go c"* keeps $\mathbf{K}_A$ and $\mathbf{K}_B$ on their baselines.

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase $\mathbf{P}=(\mathbf{R}+\mathbf{L})/2$ and Half-Difference Group $\mathbf{G}=(\mathbf{R}-\mathbf{L})/2$





Review Spectral theory of Einstein-Lorentz relativity

Applying Doppler Shifts to per-space-time (ck,ω) graph

→ *CW Minkowski space-time coordinates (x,ct) and PW grids*

Relating Doppler Shifts b or r=1/b to velocity u/c or rapidity ρ

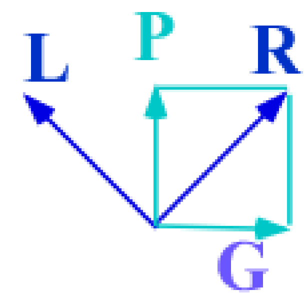
Lorentz transformation

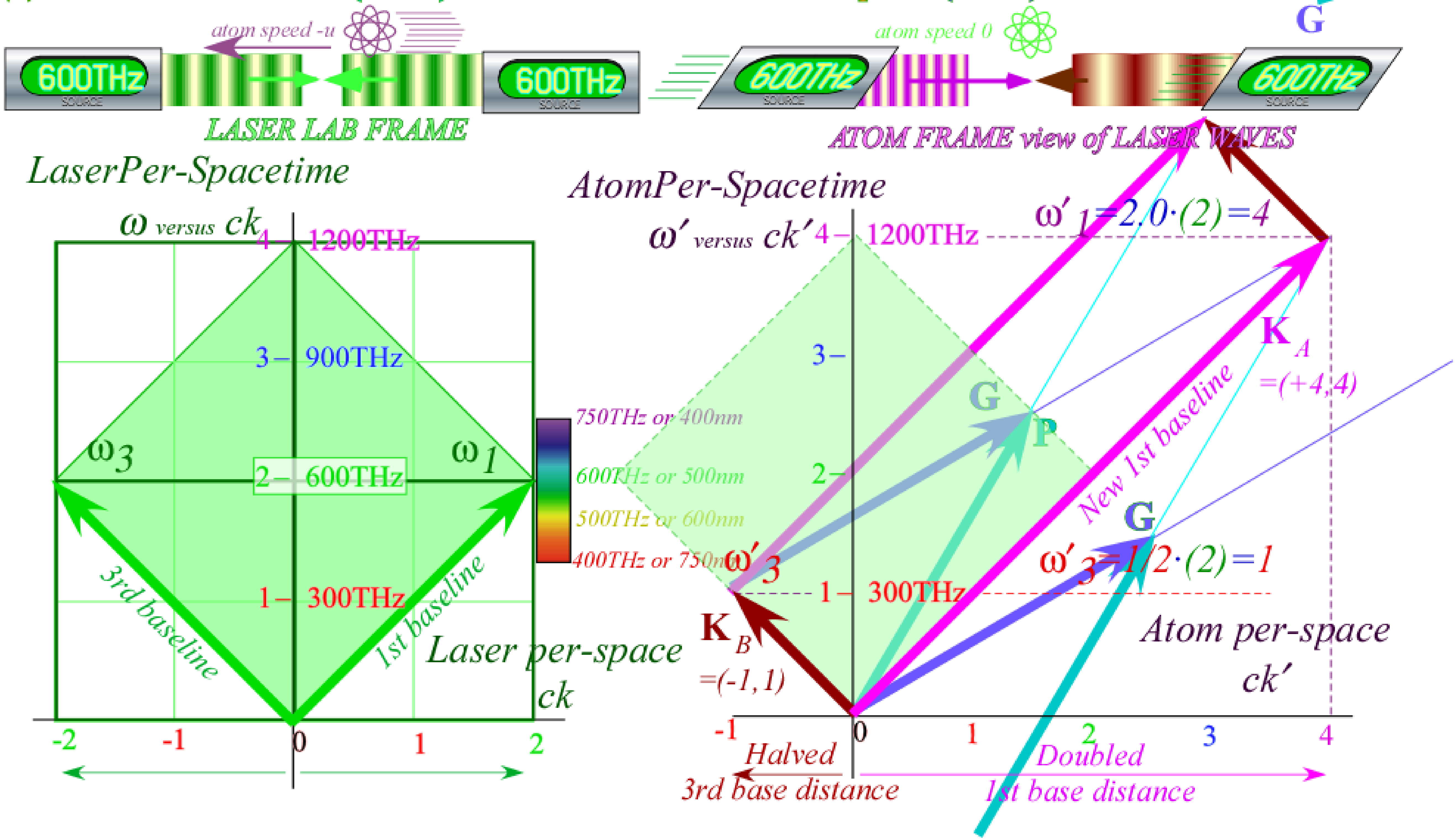
Deriving Spacetime and per-spacetime coordinate geometry by:

(1) *Evenson* CW axiom *"All colors go c"* keeps K_A and K_B *on their baselines.*

(2) Time-Reversal axiom: $r=1/b$

(3) Half-Sum Phase $P=(R+L)/2$ and Half-Difference Group $G=(R-L)/2$





Deriving Spacetime and per-spacetime coordinate geometry by:

(1) Evenson CW axiom *"All colors go c"* keeps K_A and K_B *on their baselines.*

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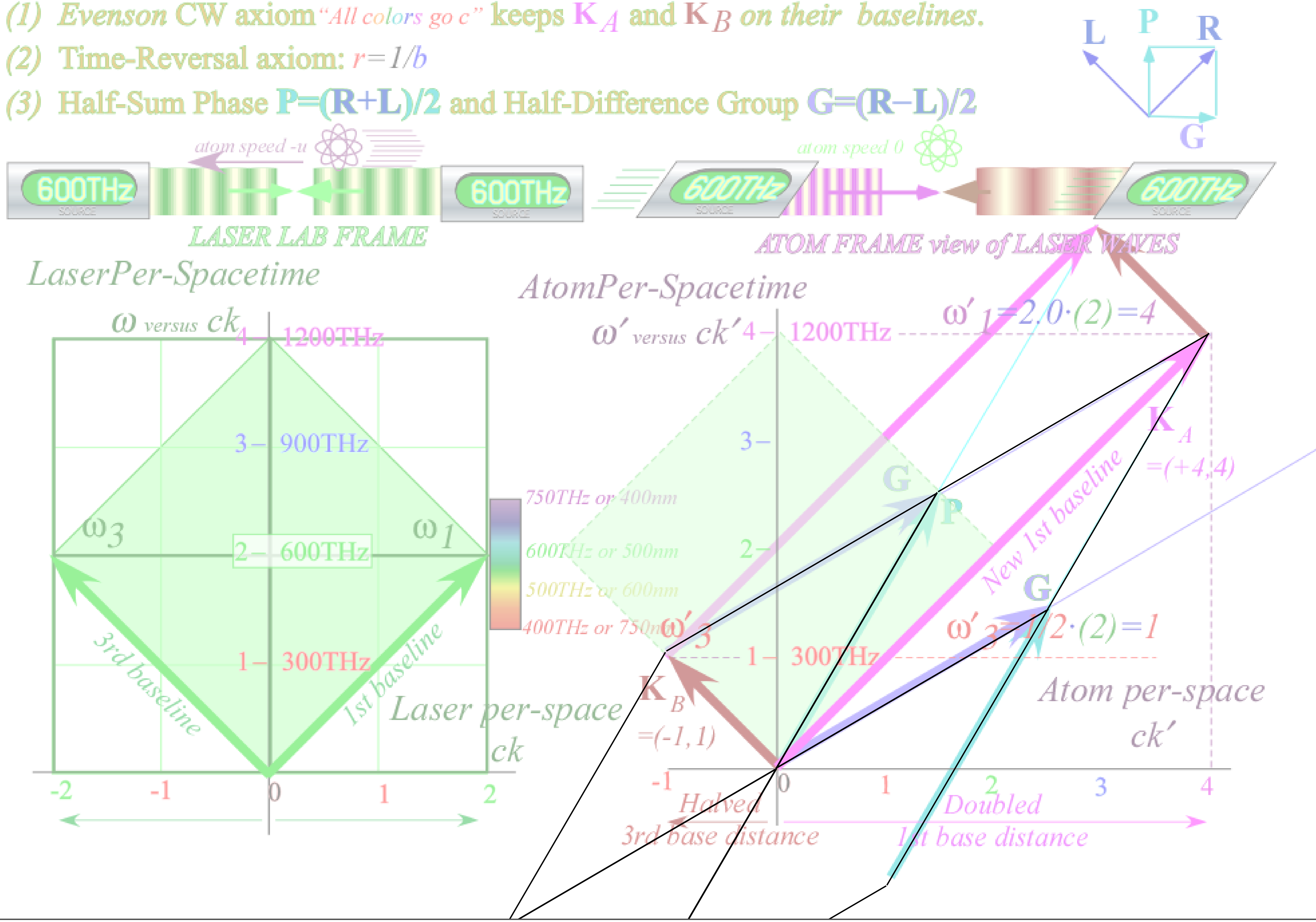

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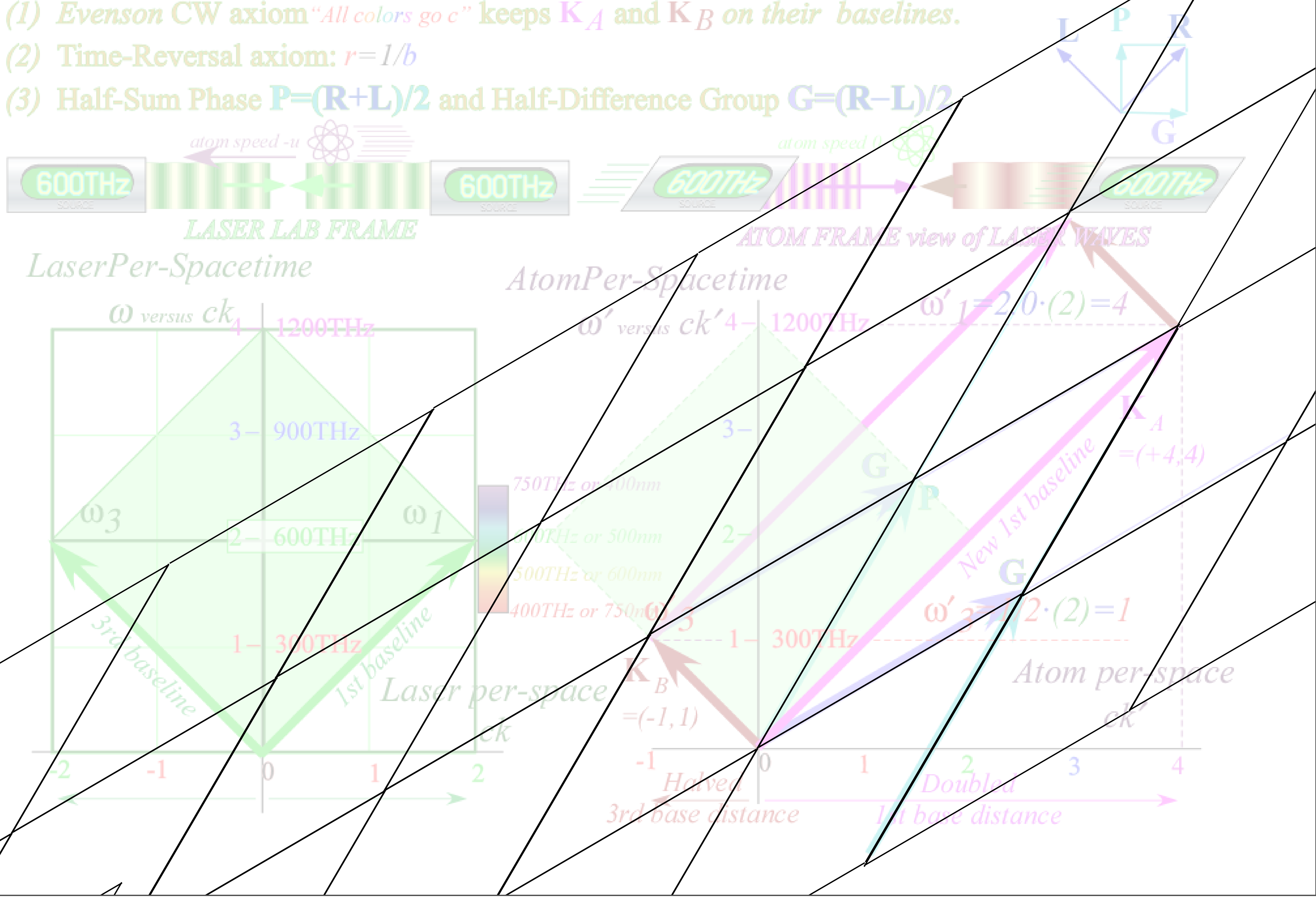
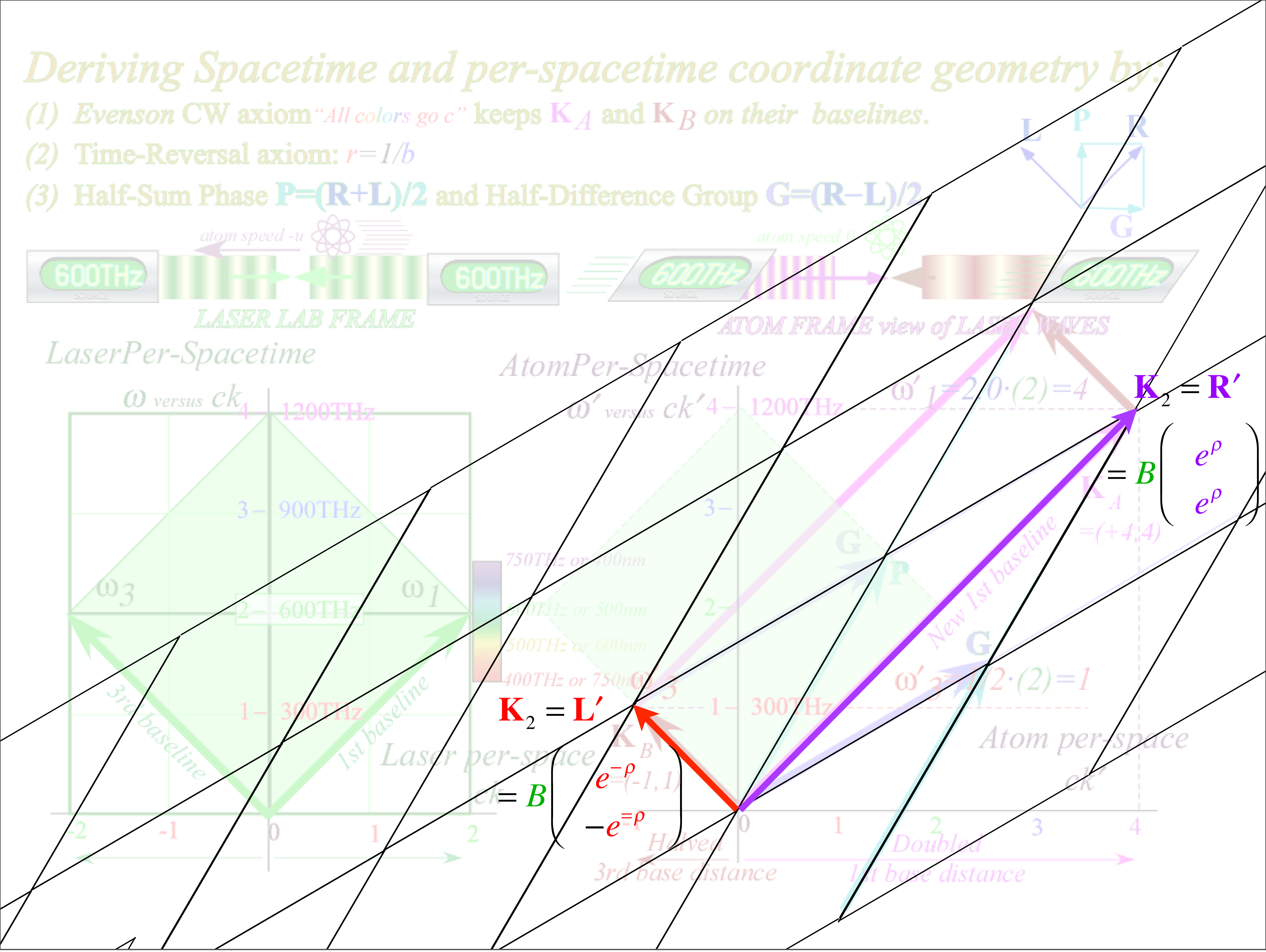
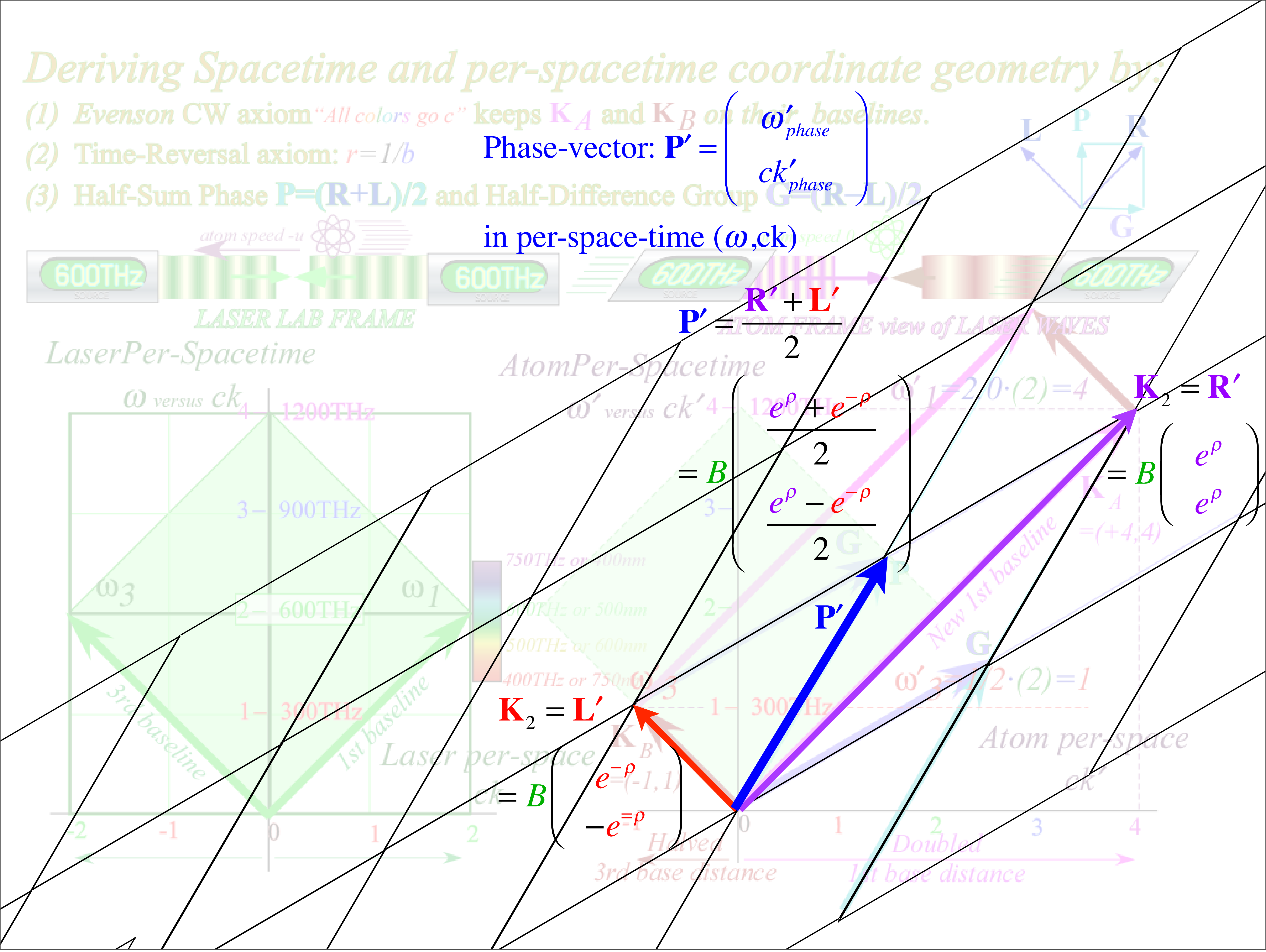



Deriving Spacetime and per-spacetime coordinate geometry by:
(1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.
(2) Time-Reversal axiom: r=1/b
(3) Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R−L)/2
L
P
R
G
atom speed -u
atom speed 0
600THz
LASER LAB FRAME
ATOM FRAME view of LASER WAVES
LaserPer-Spacetime
AtomPer-Spacetime
ω versus ck
ω′ versus ck′
1200THz
900THz
600THz
300THz
ω3
ω1
3rd baseline
1st baseline
Laser per-space
Atom per-space
New 1st baseline
K2 = R′
= B (e^ρ, e^ρ)
K2 = L′
= B (e^−ρ, −e^−ρ)
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
Halved 3rd base distance
Doubled 1st base distance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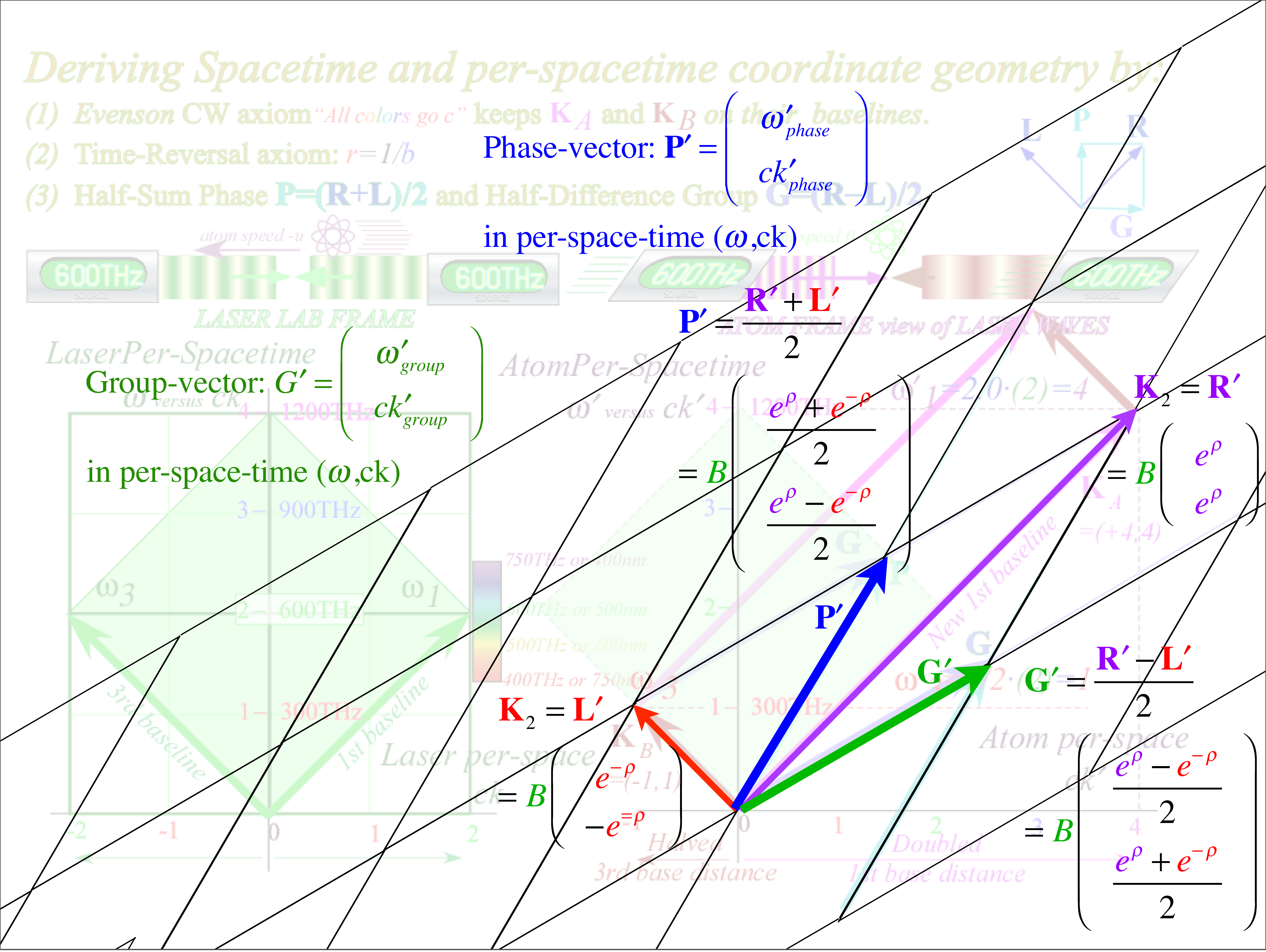
(1) Evenson CW axiom "All colors go c" keeps K_A and K_B on their baselines.
(2) Time-Reversal axiom: r=1/b
(3) Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R−L)/2
Phase-vector: $\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix}$
in per-space-time (ω,ck)
$\mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2}$
$= B \begin{pmatrix} \frac{e^{\rho} + e^{-\rho}}{2} \\ \frac{e^{\rho} - e^{-\rho}}{2} \end{pmatrix}$
$\mathbf{K}_2 = \mathbf{R}'$
$= B \begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix}$
$\mathbf{K}_2 = \mathbf{L}'$
$= B \begin{pmatrix} e^{-\rho} \\ -e^{=\rho} \end{pmatrix}$
P′
LASER LAB FRAME
ATOM FRAME view of LASER WAVES
LaserPer-Spacetime
AtomPer-Spacetime
600THz
1200THz
900THz
300THz
3rd baseline
1st baseline
New 1st baseline
Laser per-space
Atom per-space
Halved 3rd base distance
Doubled 1st base distance



Phase-vector: $\mathbf{P}' = \begin{pmatrix}\omega'_{phase}\\ ck'_{phase}\end{pmatrix}$

in per-space-time (ω,ck)

Group-vector: $G' = \begin{pmatrix}\omega'_{group}\\ ck'_{group}\end{pmatrix}$

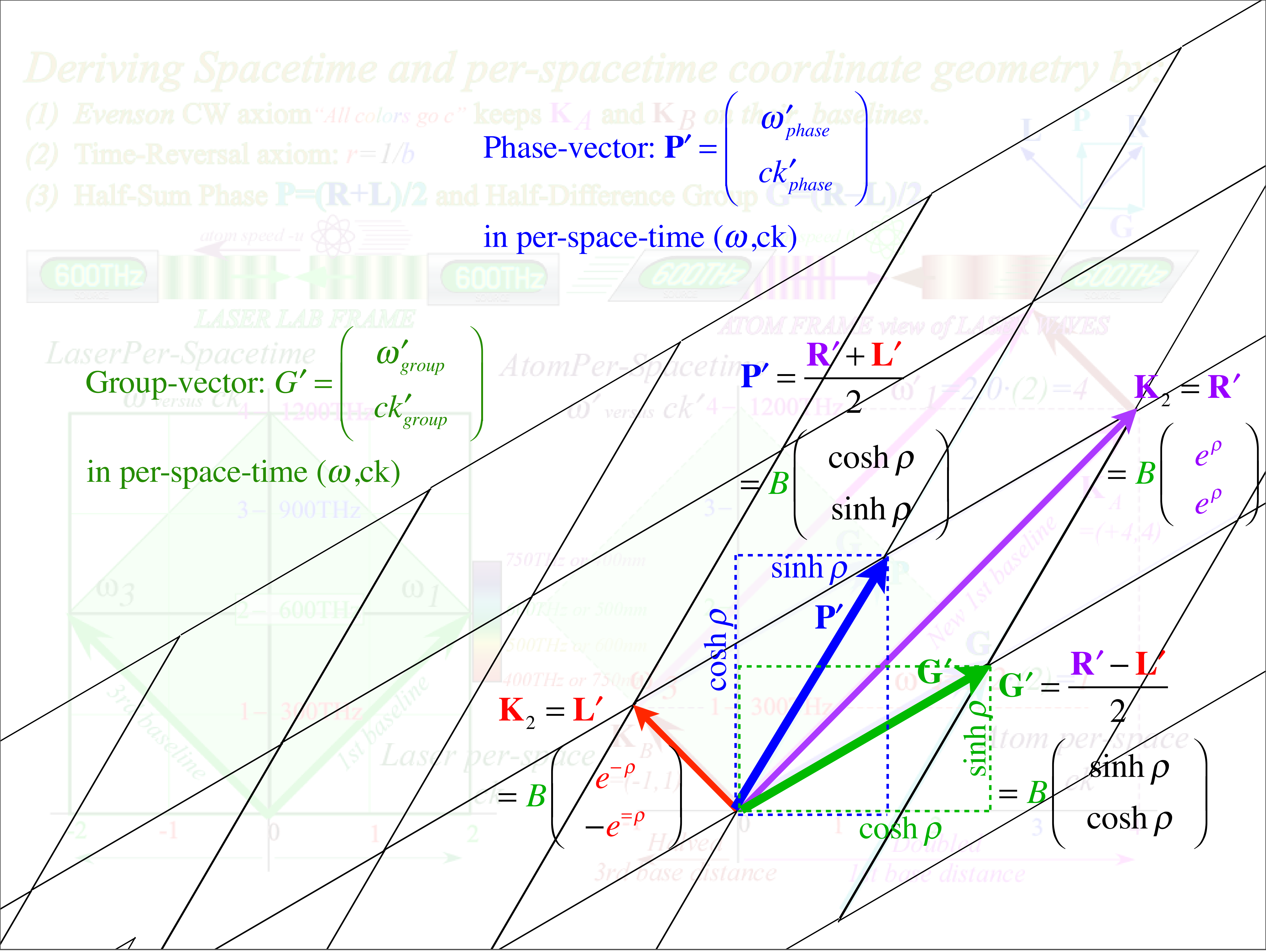
in per-space-time (ω,ck)

$$\mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = B\begin{pmatrix}\frac{e^{\rho}+e^{-\rho}}{2}\\ \frac{e^{\rho}-e^{-\rho}}{2}\end{pmatrix}$$

$$\mathbf{K}_2 = \mathbf{R}' = B\begin{pmatrix}e^{\rho}\\ e^{\rho}\end{pmatrix}$$

$$\mathbf{G}' = \frac{\mathbf{R}'-\mathbf{L}'}{2} = B\begin{pmatrix}\frac{e^{\rho}-e^{-\rho}}{2}\\ \frac{e^{\rho}+e^{-\rho}}{2}\end{pmatrix}$$

$$\mathbf{K}_2 = \mathbf{L}' = B\begin{pmatrix}e^{-\rho}\\ -e^{=\rho}\end{pmatrix}$$



Phase-vector: $\mathbf{P}' = \begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix}$
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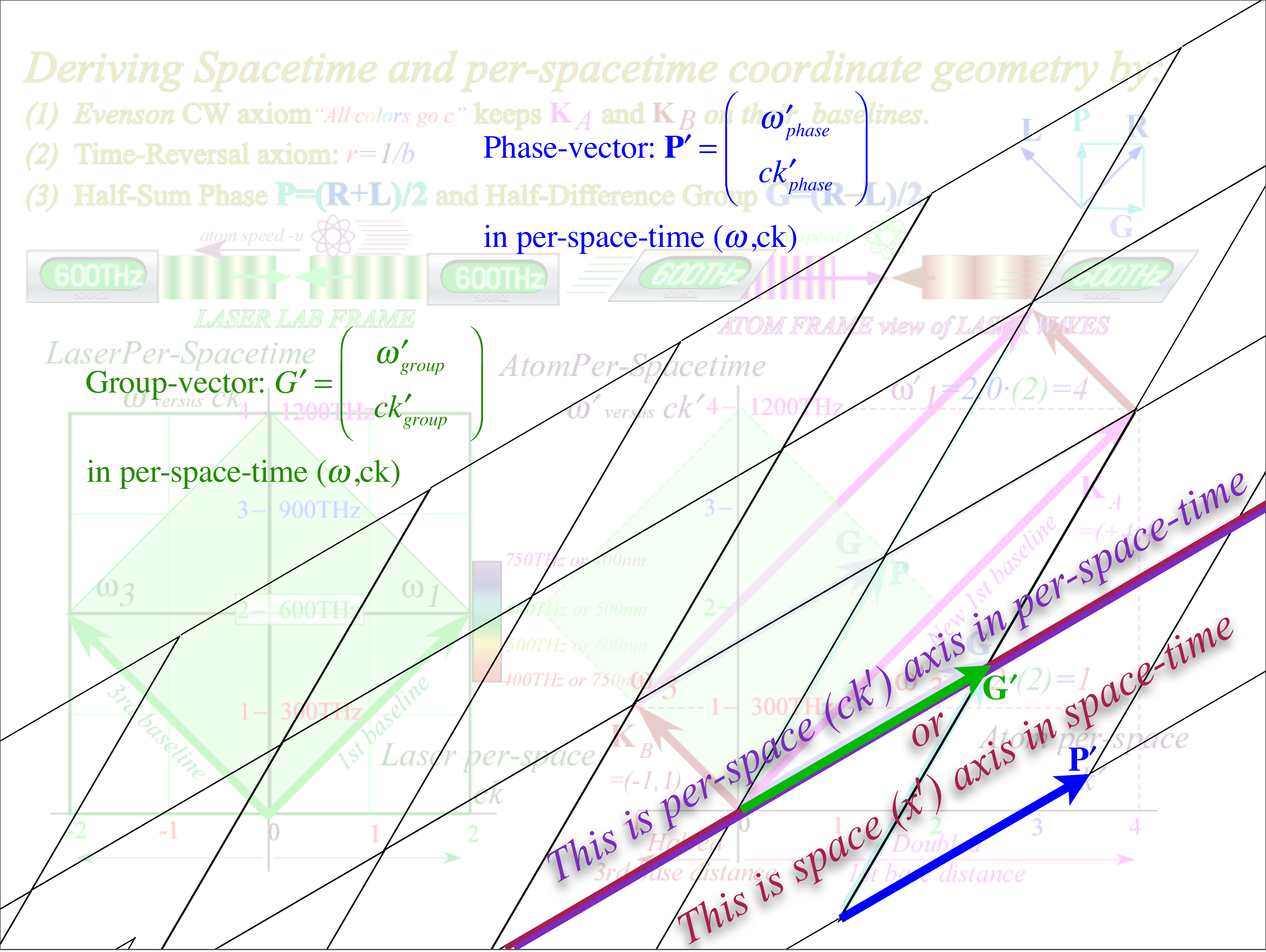
in per-space-time (ω,ck)
$\mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2}$
$= B \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix}$
$\mathbf{K}_2 = \mathbf{R}'$
$= B \begin{pmatrix} e^{\rho} \\ e^{\rho} \end{pmatrix}$
sinh ρ
cosh ρ
P′
G′
$\mathbf{G}' = \frac{\mathbf{R}' - \mathbf{L}'}{2}$
$= B \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix}$
sinh ρ
cosh ρ
$\mathbf{K}_2 = \mathbf{L}'$
$= B \begin{pmatrix} e^{-\rho} \\ -e^{=\rho} \end{pmatrix}$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in per-space-time (ω,ck)
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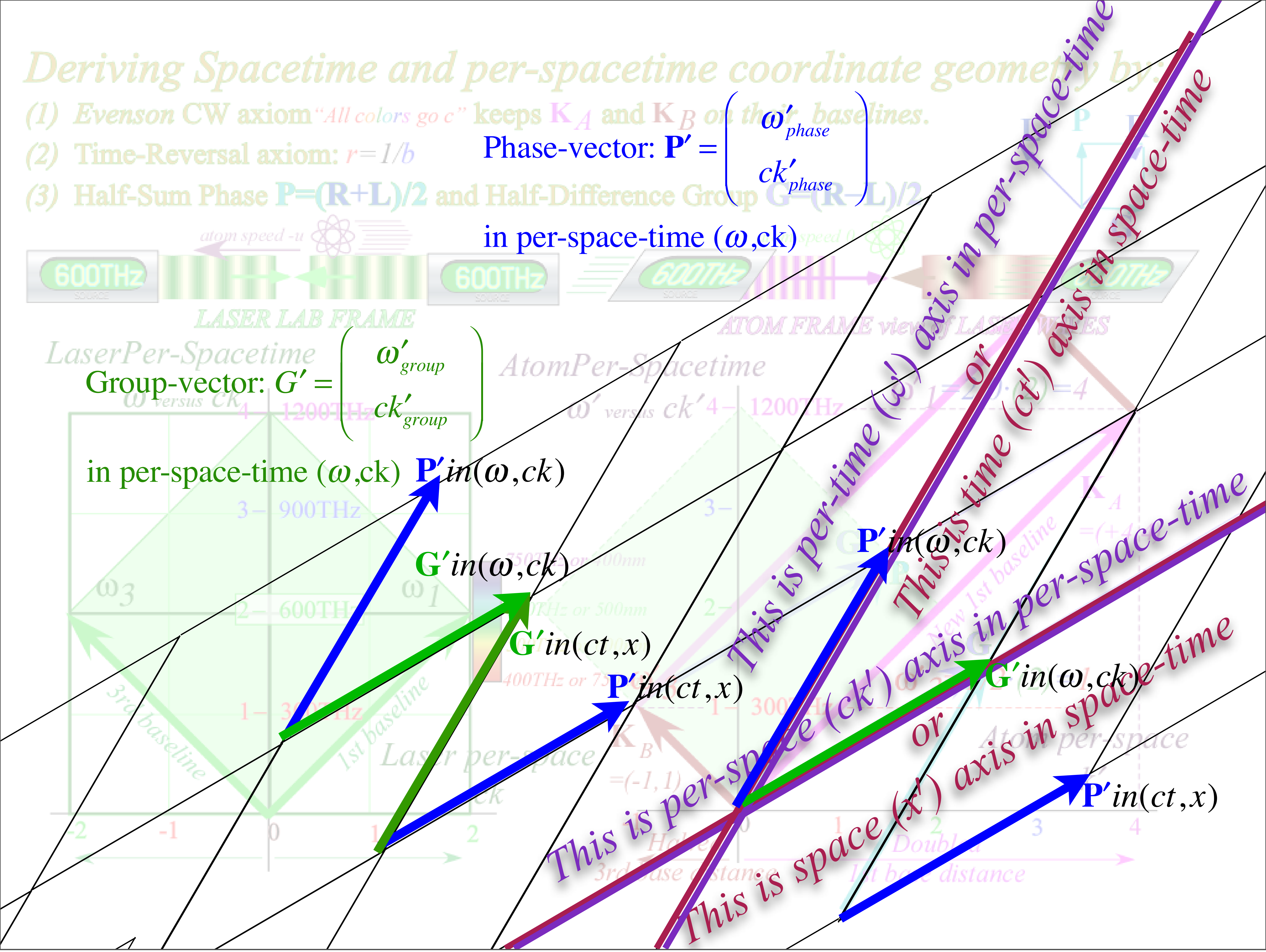
in per-space-time (ω,ck)
LASER LAB FRAME
ATOM FRAME view of LASER WAVES
LaserPer-Spacetime
AtomPer-Spacetime
600THz
1200THz
900THz
600THz
300THz
G′
P′
This is per-space (ck′) axis in per-space-time
or
This is space (x′) axis in space-time



Deriving Spacetime and per-spacetime coordinate geometry by:
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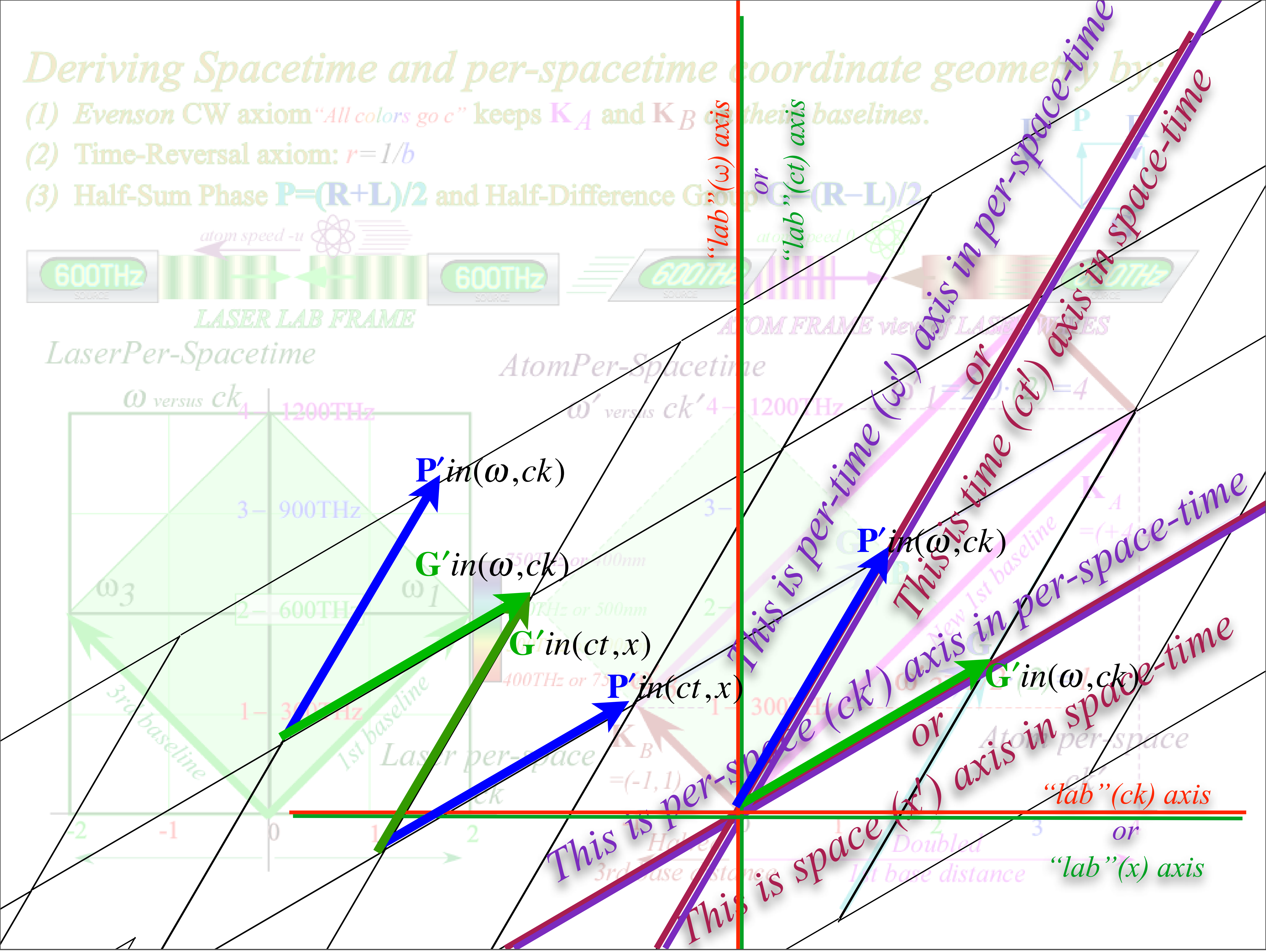
(2) Time-Reversal axiom: r=1/b
(3) Half-Sum Phase P=(R+L)/2 and Half-Difference Group G=(R-L)/2
Phase-vector: P′ = (ω′_phase, ck′_phase)
in per-space-time (ω,ck)
Group-vector: G′ = (ω′_group, ck′_group)
in per-space-time (ω,ck)
P′ in(ω,ck)
G′ in(ω,ck)
G′ in(ct,x)
P′ in(ct,x)
P′ in(ω,ck)
G′ in(ω,ck)
P′ in(ct,x)
This is per-time (ω′) axis in per-space-time
or
This is time (ct′) axis in space-time
This is per-space (ck′) axis in per-space-time
or
This is space (x′) axis in space-time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LASER LAB FRAME
ATOM FRAME view of LASER WAVES
LaserPer-Spacetime
AtomPer-Spacetime
"lab"(ω) axis
or
"lab"(ct) axis
This is per-time (ω′) axis in per-space-time
or
This is time (ct′) axis in space-time
This is per-space (ck′) axis in per-space-time
or
This is space (x′) axis in space-time
"lab"(ck) axis
or
"lab"(x) axis
P′in(ω,ck)
G′in(ω,ck)
G′in(ct,x)
P′in(ct,x)
P′in(ω,ck)
G′in(ω,ck)

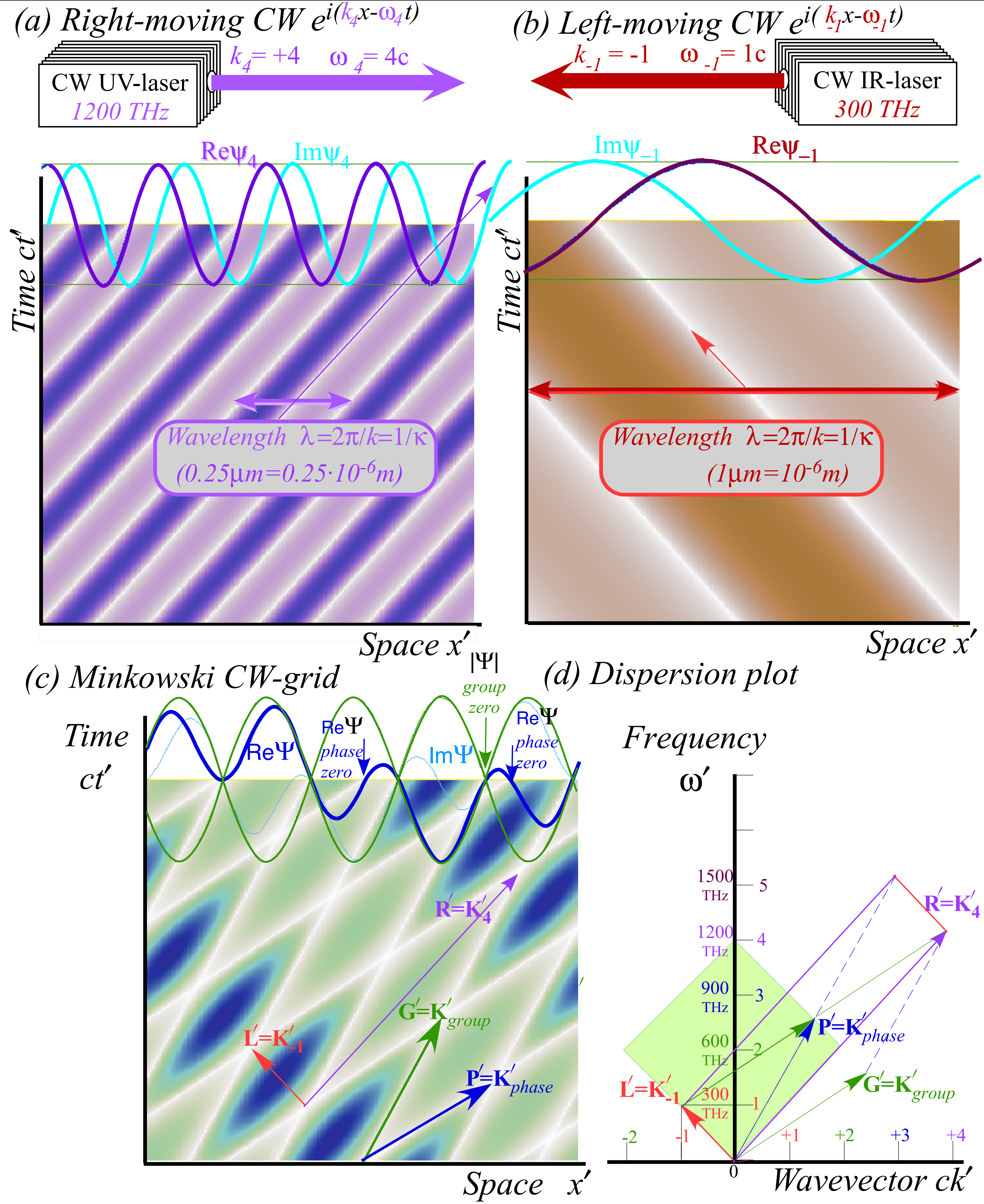
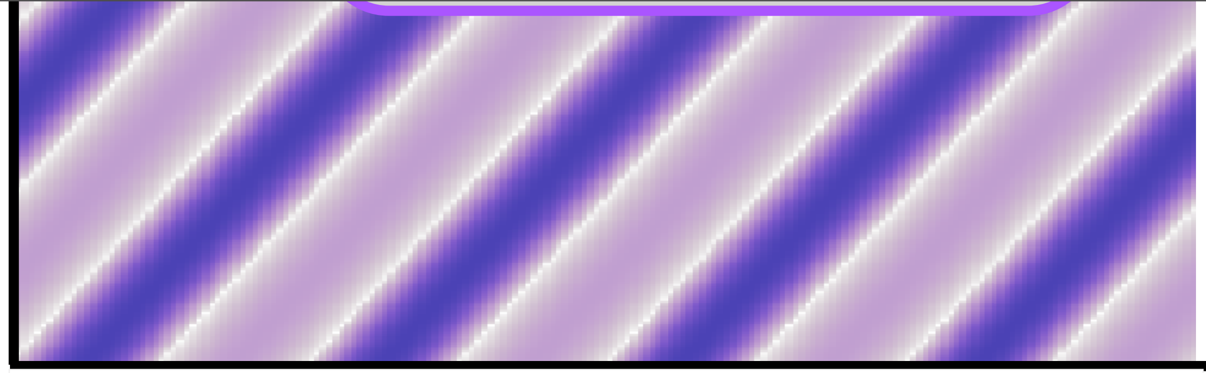
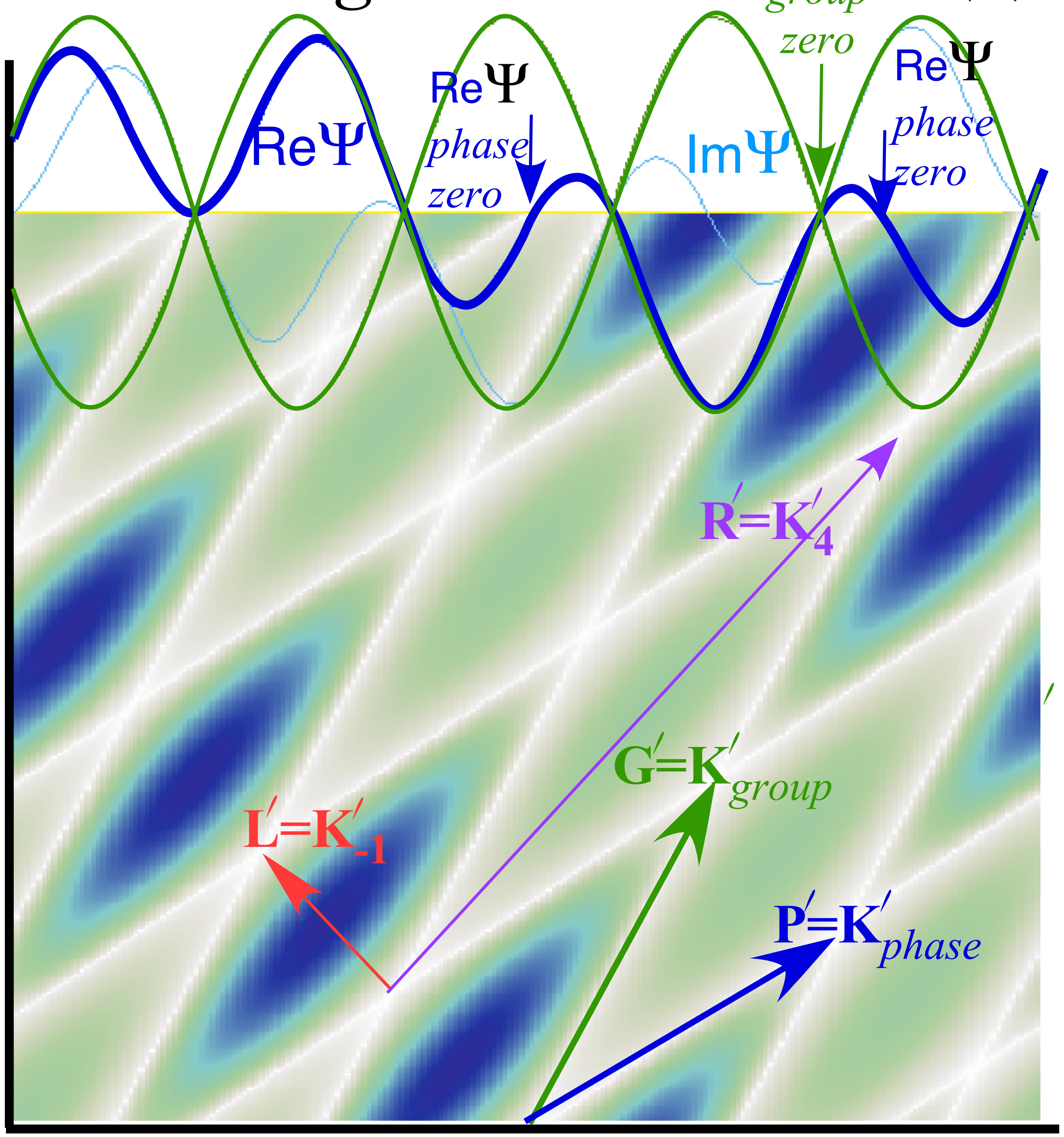


Fig. 9 in SR&QM

recall also:
p. 3-11 of Lect.1



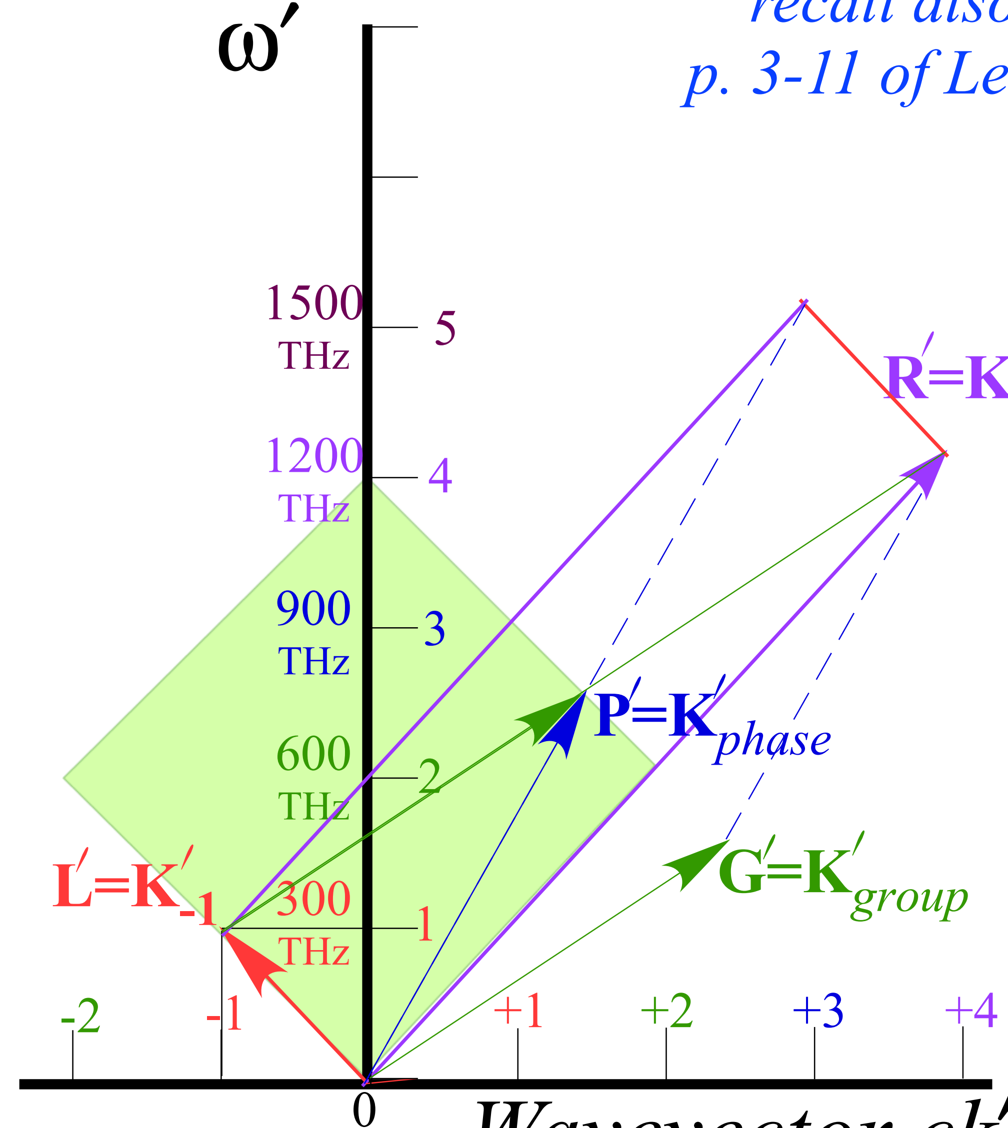
(c) Minkowski CW-grid

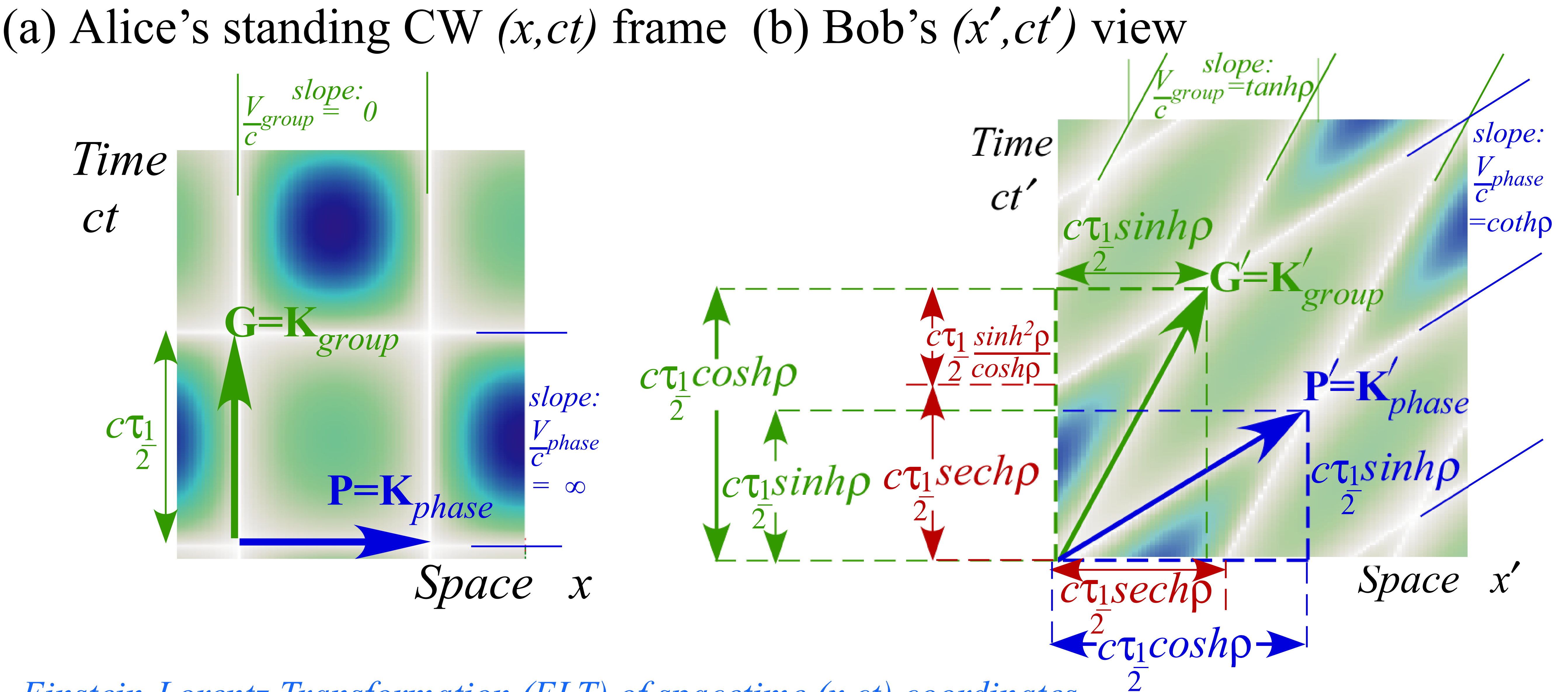


(d) Dispersion plot

Fig. 9 in SR&QM

recall also:
p. 3-11 of Lect.1





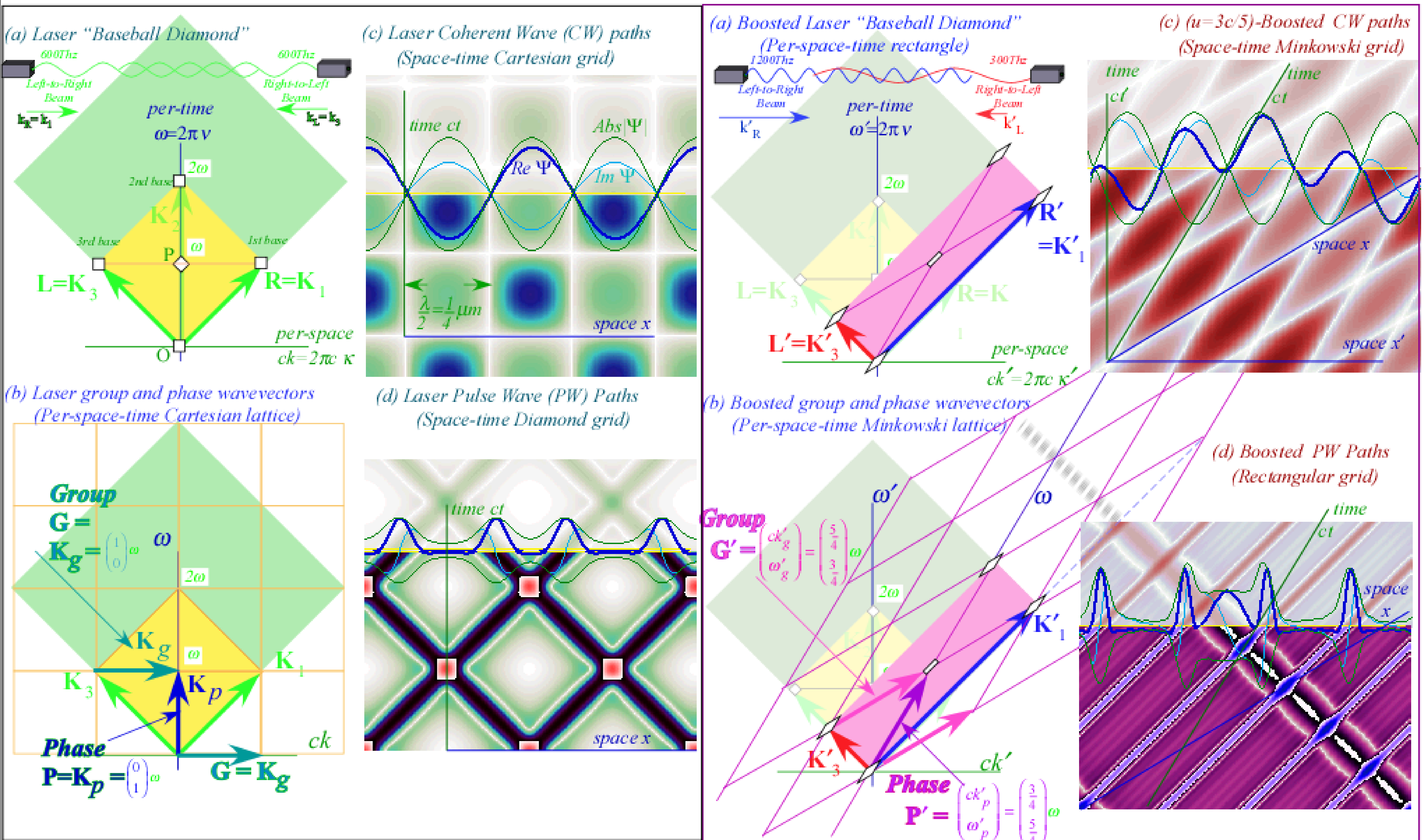
Einstein-Lorentz Transformation (ELT) of spacetime (x,ct) coordinates...

$$\begin{pmatrix} x'_{(any)} \\ ct'_{(any)} \end{pmatrix}_B = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x_{(any)} \\ ct_{(any)} \end{pmatrix}_A \Leftrightarrow \begin{pmatrix} x_{(any)} \\ ct_{(any)} \end{pmatrix}_A = \begin{pmatrix} \cosh\rho & -\sinh\rho \\ -\sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} x'_{(any)} \\ ct'_{(any)} \end{pmatrix}_B$$

...is based upon the same ELT of per-spacetime (ck,ω) coordinates...

Old-fashioned notation (discussed below)

$$\begin{pmatrix} \omega'_{(any)} \\ ck'_{(any)} \end{pmatrix}_B = \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} \begin{pmatrix} \omega_{(any)} \\ ck_{(any)} \end{pmatrix}_A \quad \text{where:} \begin{pmatrix} \cosh\rho & \sinh\rho \\ \sinh\rho & \cosh\rho \end{pmatrix} = \begin{pmatrix} \gamma & \beta\cdot\gamma \\ \beta\cdot\gamma & \gamma \end{pmatrix}$$



Laser lab views

atom speed $-u=-\frac{3}{5}c$

Atom views (sees lab going $+u=\frac{3}{5}c$)

Review *Spectral theory of Einstein-Lorentz relativity*

Applying Doppler Shifts to per-space-time (ck,ω) graph

CW Minkowski space-time coordinates (x,ct) and PW grids

→ *Relating Doppler Shifts b or r=1/b to velocity u/c or rapidity ρ*

→ *Lorentz transformation*

→ *Connection: Conventional approach to relativity and old-fashioned formulas*

Invariant hyperbolas and hyperbolic relations

Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

$$\begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^{\rho} + e^{-\rho}}{2} \\ \frac{e^{\rho} - e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}' + \mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^{\rho} - e^{-\rho}}{2} \\ \frac{e^{\rho} + e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$

Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

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$$\begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^{\rho}-e^{-\rho}}{2} \\ \frac{e^{\rho}+e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$

Calculate phase velocity and group velocity of coordinate waves:

$$\frac{V_{phase}}{c} = \frac{\omega'_{phase}}{ck'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5}{3} = \coth\rho$$

$$\frac{V_{group}}{c} = \frac{\omega'_{group}}{ck'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3}{5} = \tanh\rho$$

$$\frac{V'_{group}}{c} = \frac{u}{c} = \tanh\rho = \frac{e^{\rho}-e^{-\rho}}{e^{\rho}+e^{-\rho}} = \frac{b-b^{-1}}{b+b^{-1}} = \frac{b^2-1}{b^2+1} \equiv \beta$$

old-fashioned relativity parameter $\beta = u/c$

Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

$$\begin{pmatrix} \omega'_{phase} \\ ck'_{phase} \end{pmatrix} = \mathbf{P}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^{\rho}+e^{-\rho}}{2} \\ \frac{e^{\rho}-e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \cosh\rho \\ \sinh\rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{5}{4} \\ \frac{3}{4} \end{pmatrix}$$

$$\begin{pmatrix} \omega'_{group} \\ ck'_{group} \end{pmatrix} = \mathbf{G}' = \frac{\mathbf{R}'+\mathbf{L}'}{2} = \omega_A \begin{pmatrix} \frac{e^{\rho}-e^{-\rho}}{2} \\ \frac{e^{\rho}+e^{-\rho}}{2} \end{pmatrix} = \omega_A \begin{pmatrix} \sinh\rho \\ \cosh\rho \end{pmatrix} = \omega_A \begin{pmatrix} \frac{3}{4} \\ \frac{5}{4} \end{pmatrix}$$

Calculate phase velocity and group velocity of coordinate waves:

$$\frac{V_{phase}}{c} = \frac{\omega'_{phase}}{ck'_{phase}} = \frac{\cosh\rho}{\sinh\rho} = \frac{5}{3} = \coth\rho$$

$$\frac{V_{group}}{c} = \frac{\omega'_{group}}{ck'_{group}} = \frac{\sinh\rho}{\cosh\rho} = \frac{3}{5} = \tanh\rho$$

$$\frac{V'_{group}}{c} = \frac{u}{c} = \tanh\rho = \frac{e^{\rho}-e^{-\rho}}{e^{\rho}+e^{-\rho}} = \frac{b-b^{-1}}{b+b^{-1}} = \frac{b^2-1}{b^2+1} \equiv \beta$$

old-fashioned relativity parameter
$\beta = u/c$

Solve $\frac{b^2-1}{b^2+1} \equiv \beta$ *for Doppler-blue factor:*

Connection to conventional approach to relativity and old-fashioned formulas

Given phase and group wave formulas:

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old-fashioned relativity parameter β=u/c

$$b^2-1=\beta b^2+\beta$$

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Solve $\frac{b^2-1}{b^2+1}\equiv\beta$ *for Doppler-blue factor:*

Connection to conventional approach to relativity and old-fashioned formulas

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$$b^2 - 1 = \beta b^2 + \beta$$

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old-fashioned asimultaneity coeff.
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Euclidian Geometry for Per-spacetime Relativity

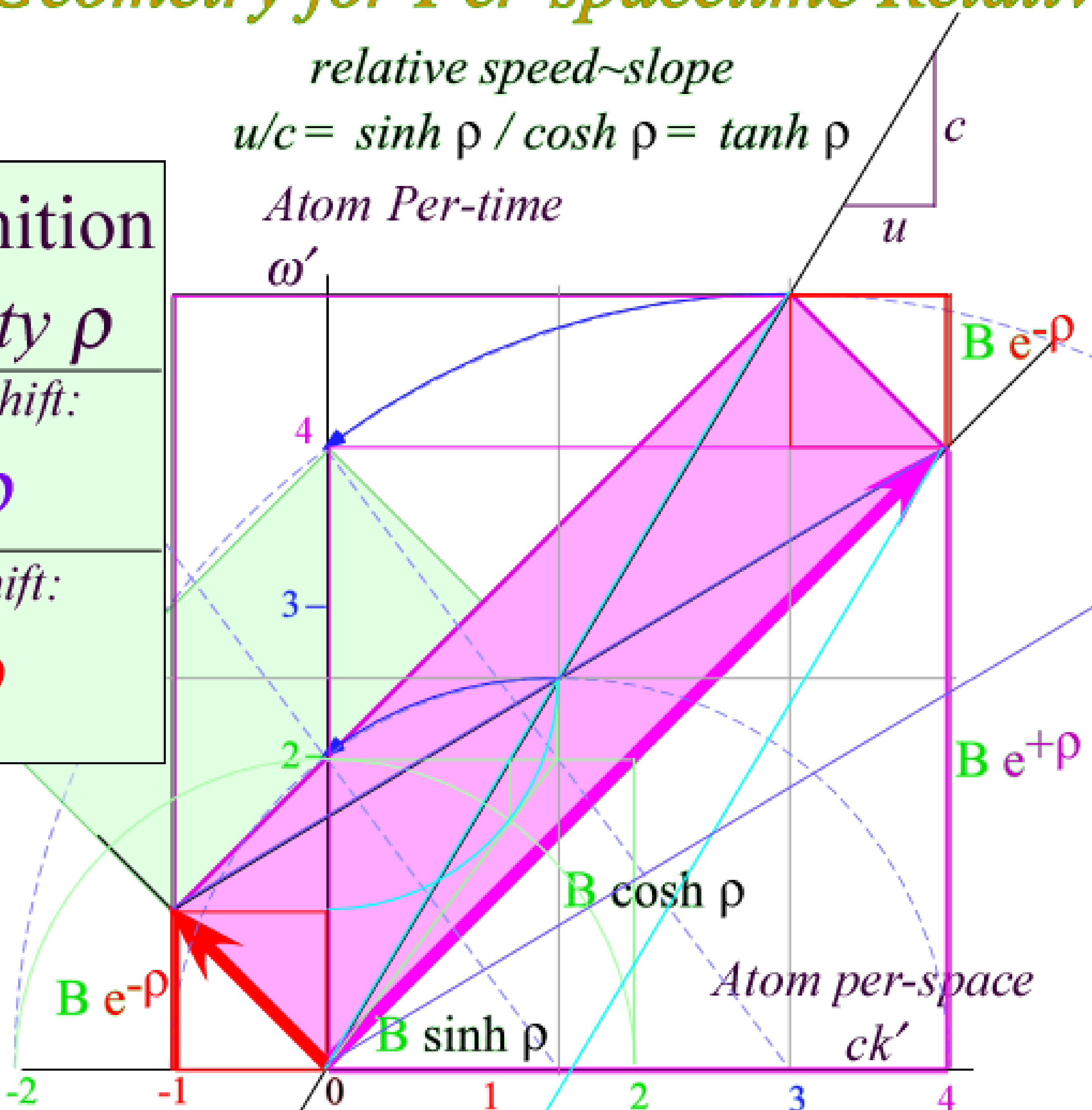
Key Definition of Rapidity ρ

Doppler blue shift:

$$Bb = Be^{+\rho}$$

Doppler red shift:

$$Br = Be^{-\rho}$$



Key Results:

ω *vs.* ck

"winks" vs. "kinks"

$$\omega = B\cosh\rho$$

$$ck = B\sinh\rho$$

group velocity:

$$\frac{\omega}{ck} = \frac{u}{c} = \tanh\rho$$

phase velocity:

$$\frac{ck}{\omega} = \frac{c}{u} = \coth\rho$$

$$B\sinh\rho = (Be^{+\rho} - Be^{-\rho})/2 \qquad B\cosh\rho = (Be^{+\rho} + Be^{-\rho})/2$$

Key Quantities

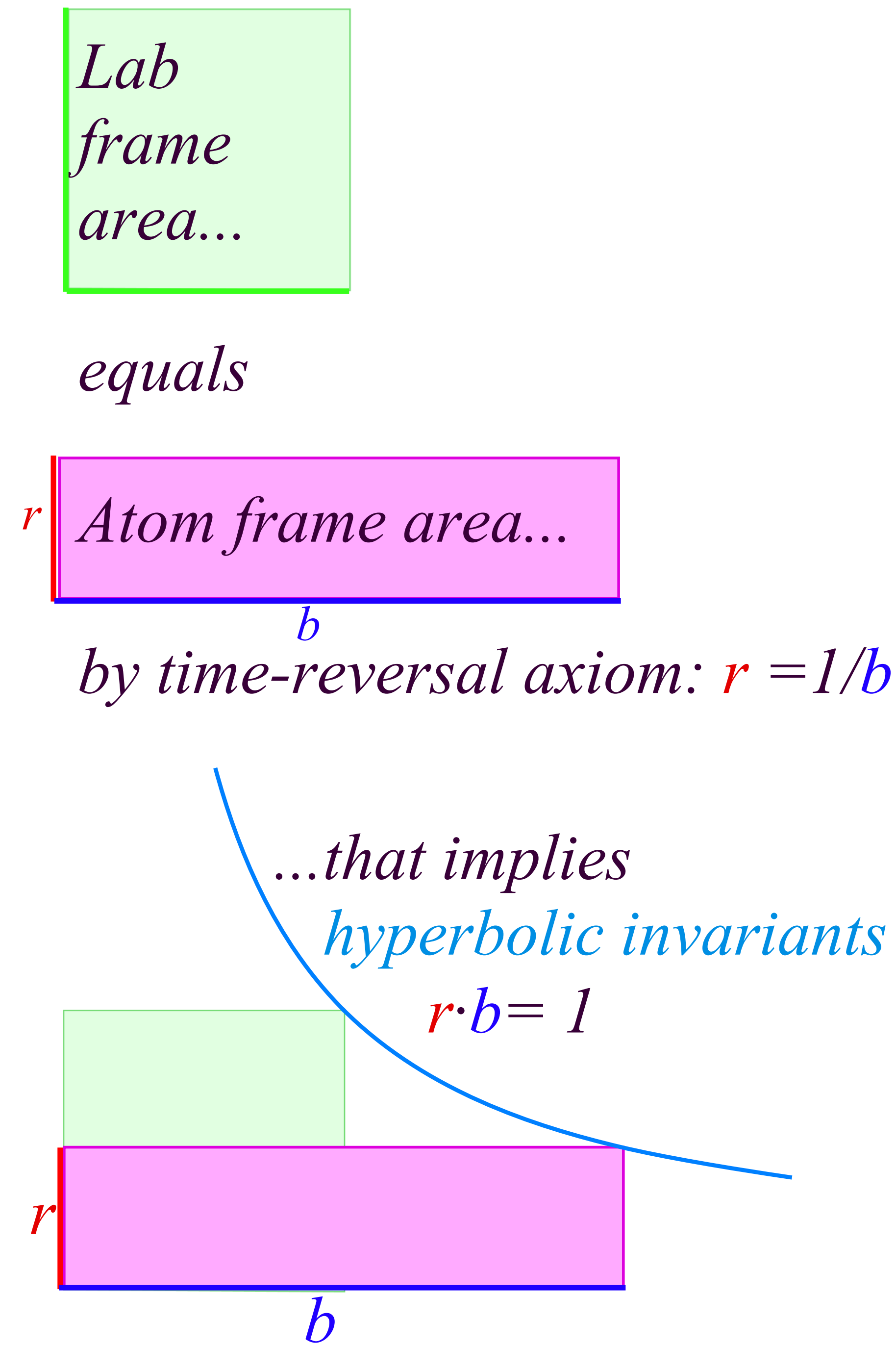
Lorentz-Einstein factors

$$\sinh\rho = \frac{\frac{u}{c}}{\sqrt{1-\frac{u^2}{c^2}}} \qquad \cosh\rho = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$$

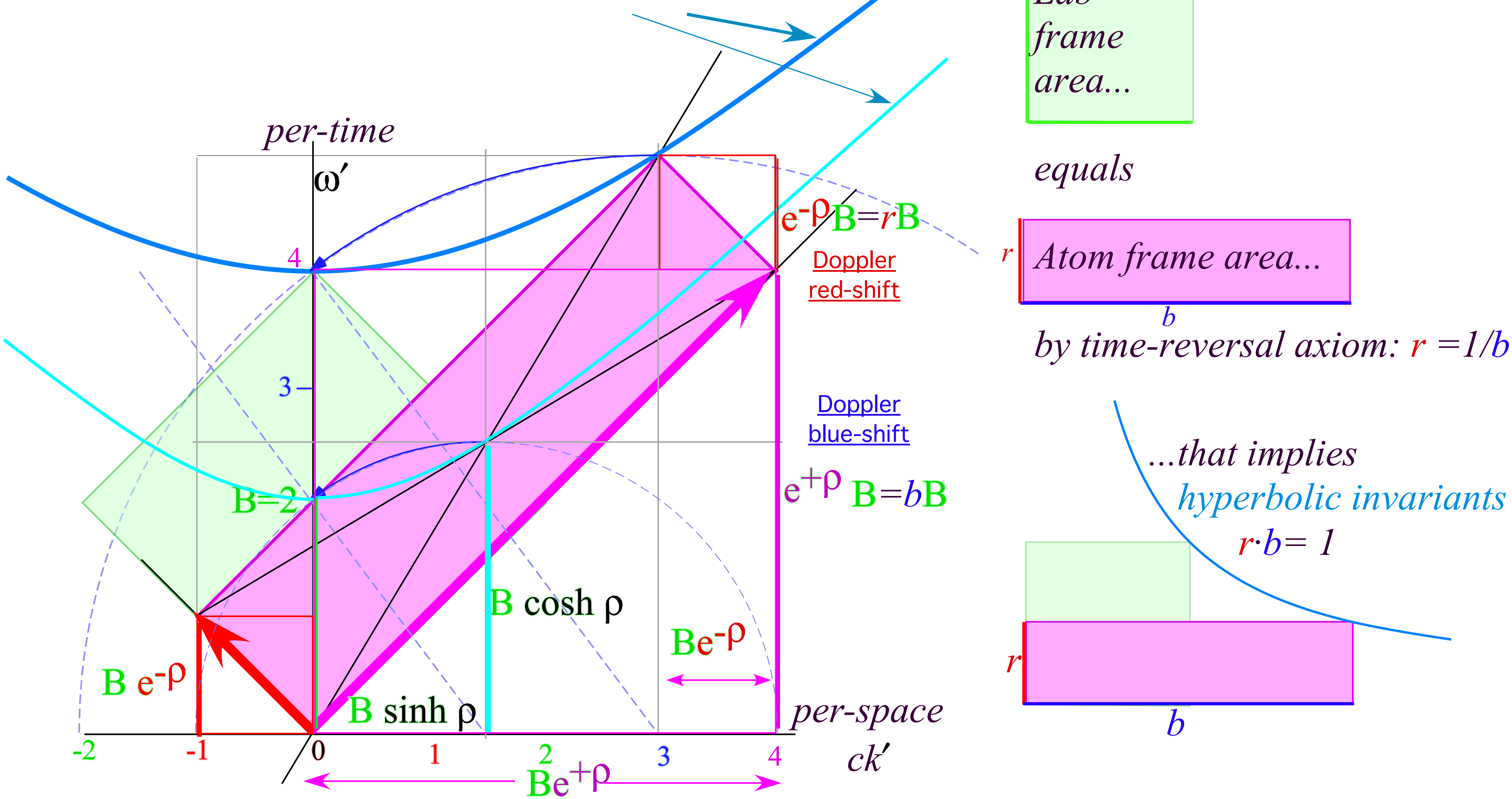
Related material at "per space-per-time" setting of: http://www.uark.edu/ua/modphys/testing/markup/RelaWavityWeb.html

Invariant hyperbolas and hyperbolic relations

Euclidian wave geometry with time-reversal symmetry imply

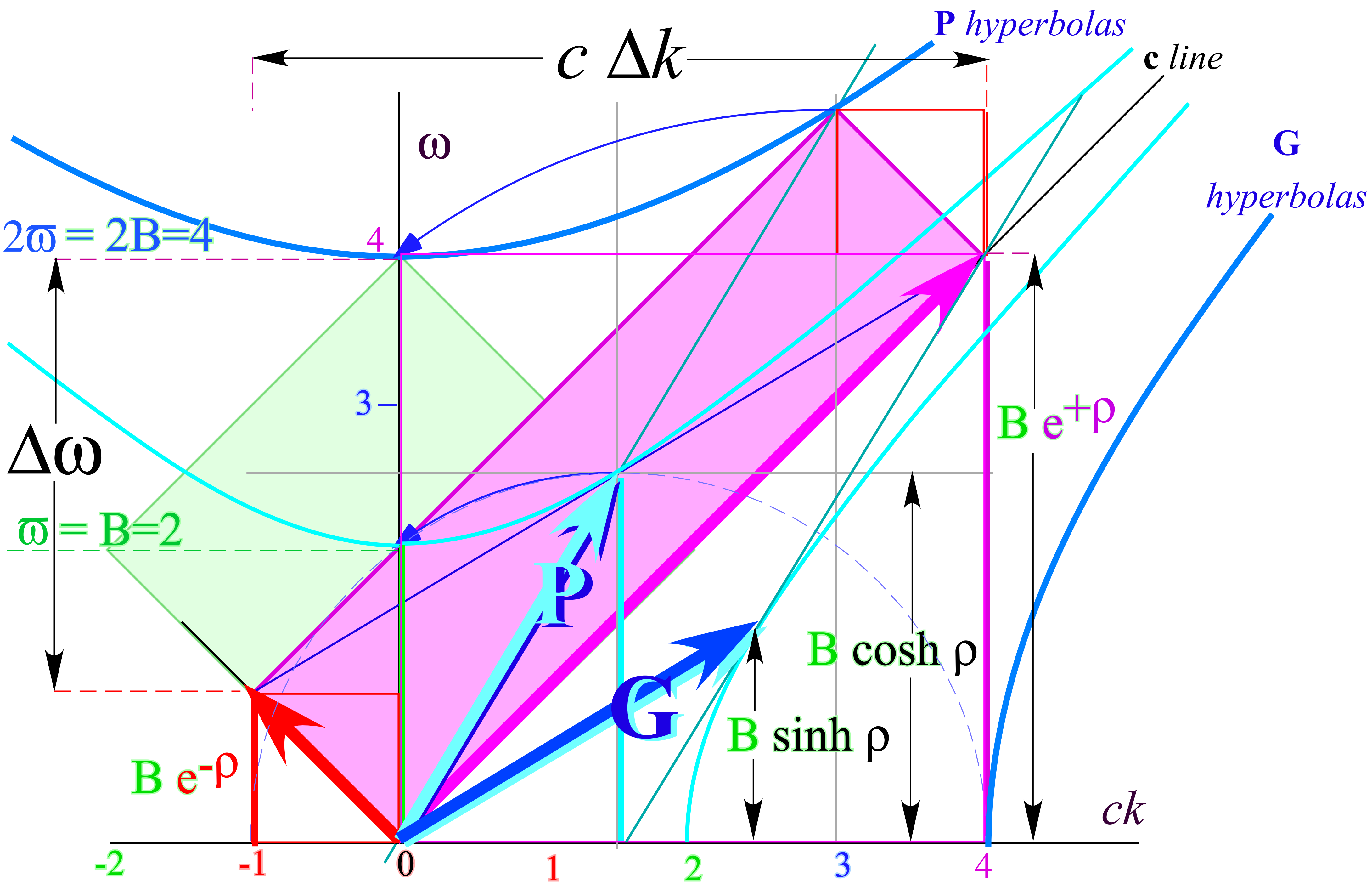


Euclidian wave geometry with time-reversal symmetry imply dispersion hyperbolas: $\omega = nB\cosh\rho$



$$B\sinh\rho = (B\,e^{+\rho} - B\,e^{-\rho})/2 \qquad B\cosh\rho = (B\,e^{+\rho} + B\,e^{-\rho})/2$$

Group velocity u *and phase velocity* c^2/u *are hyperbolic tangent slopes*



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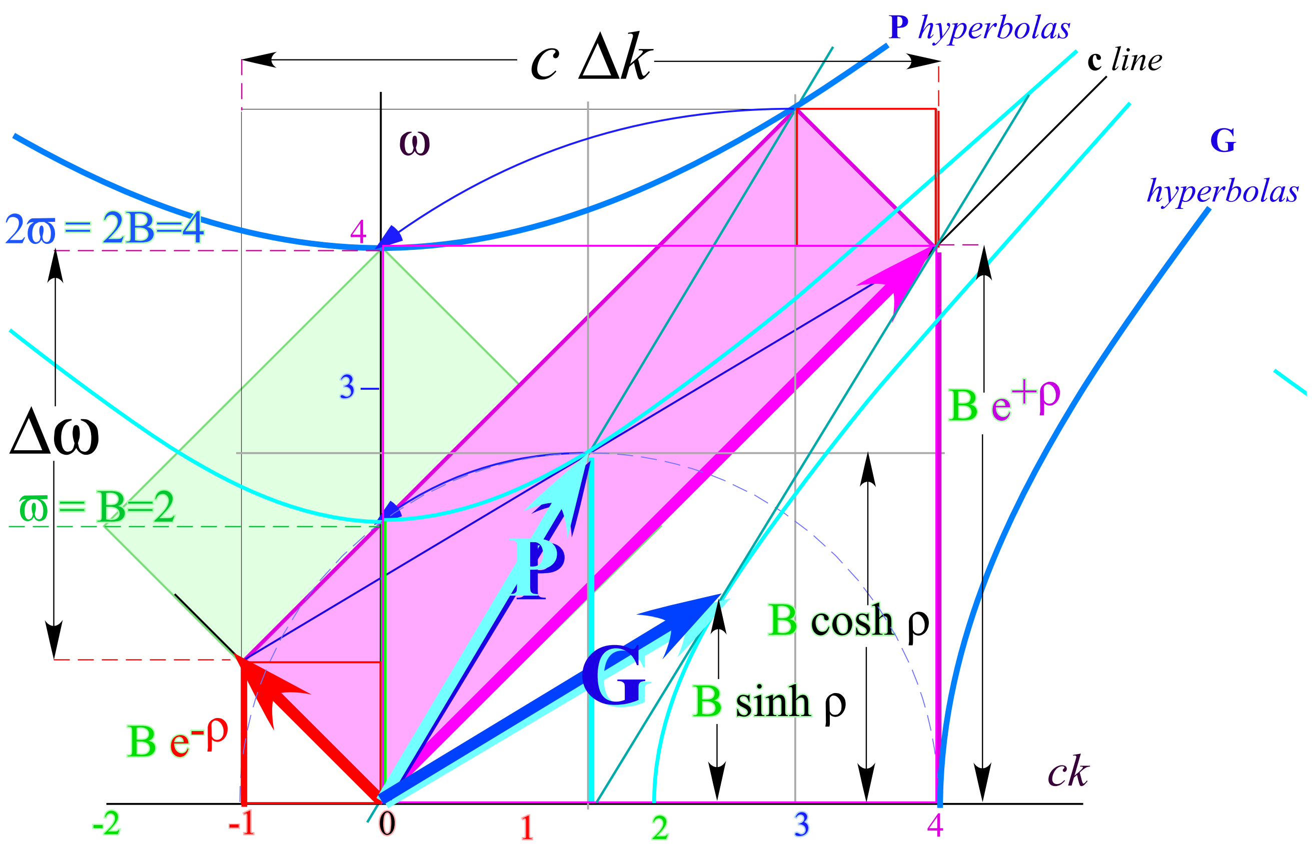

Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with LARGE Δk

(not infinitesimal)

Relativistic group wave speed u=c tanh ρ

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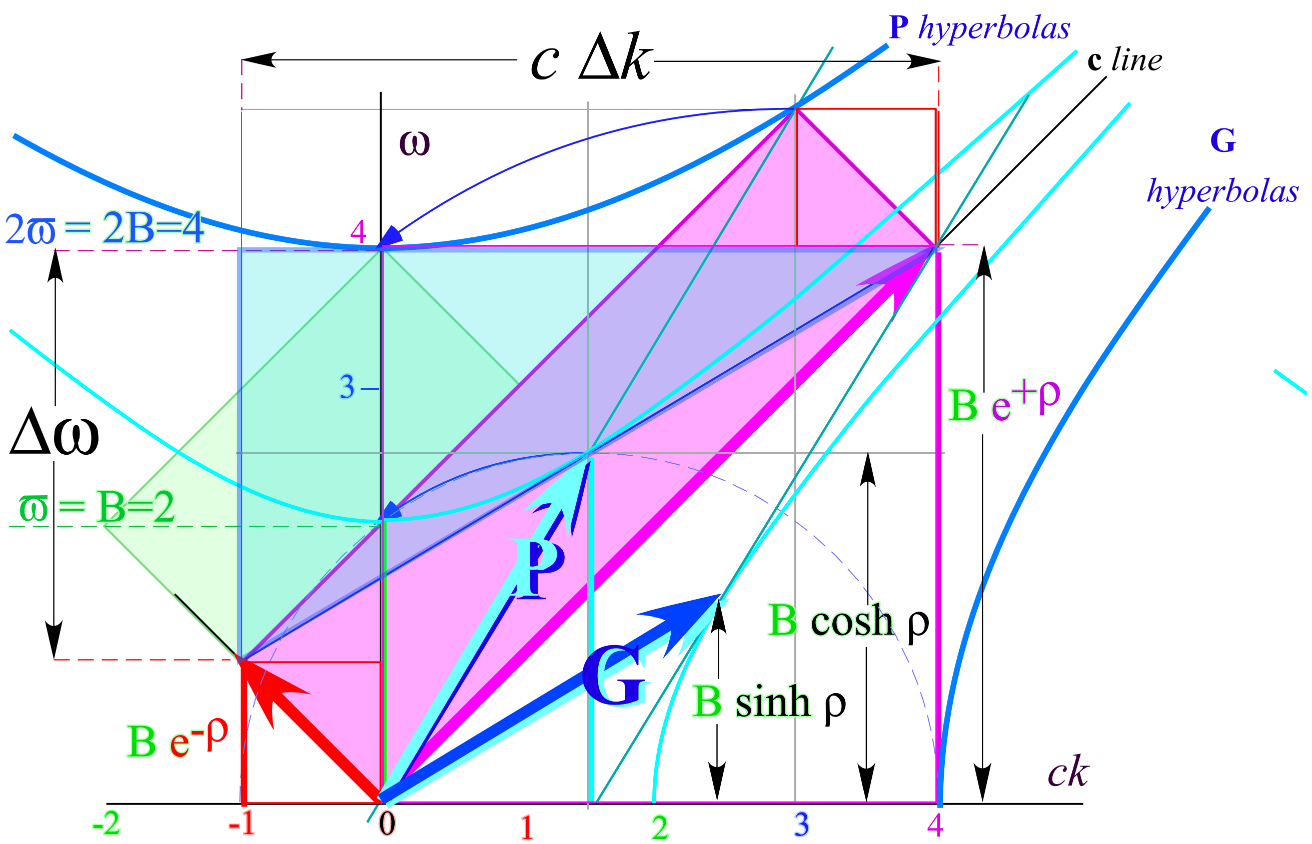

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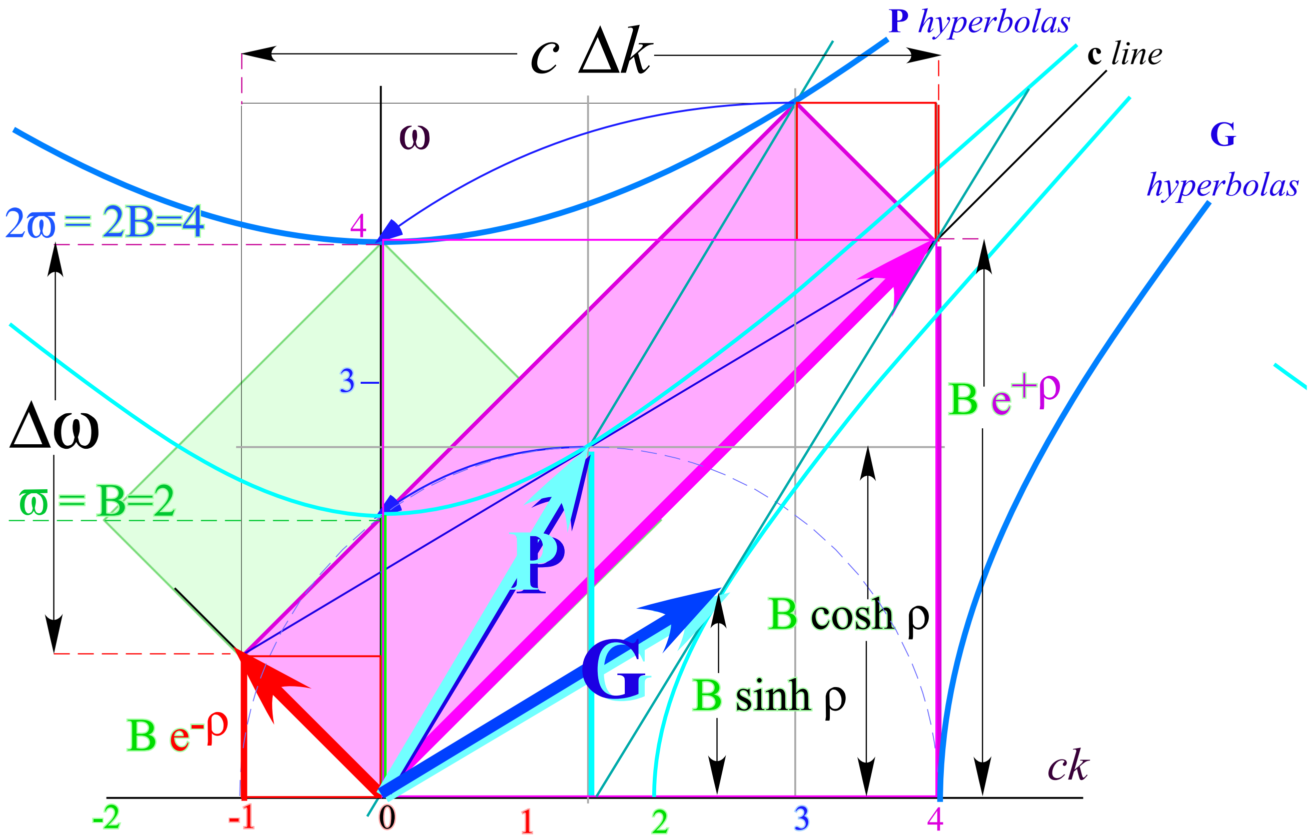
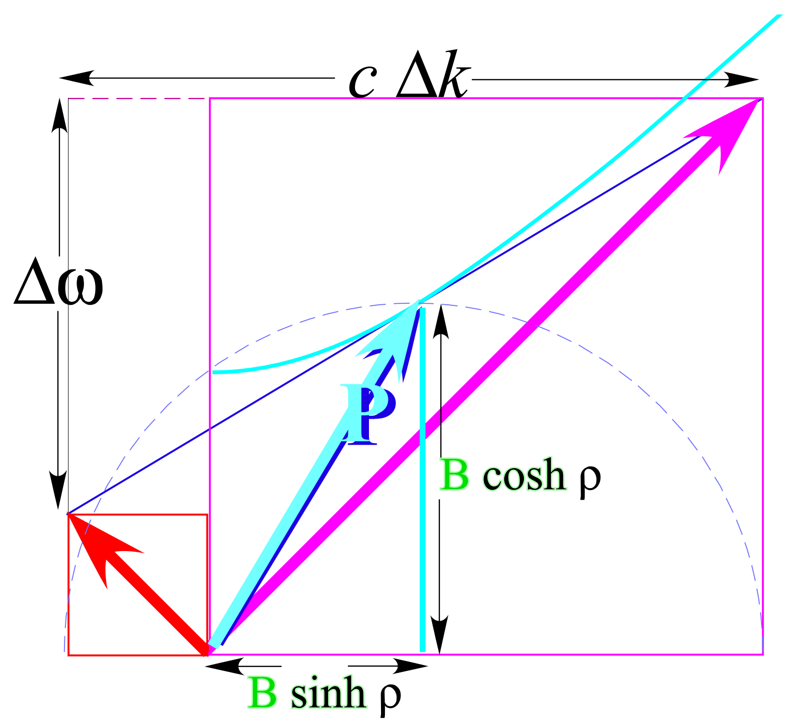


Rare but important case where

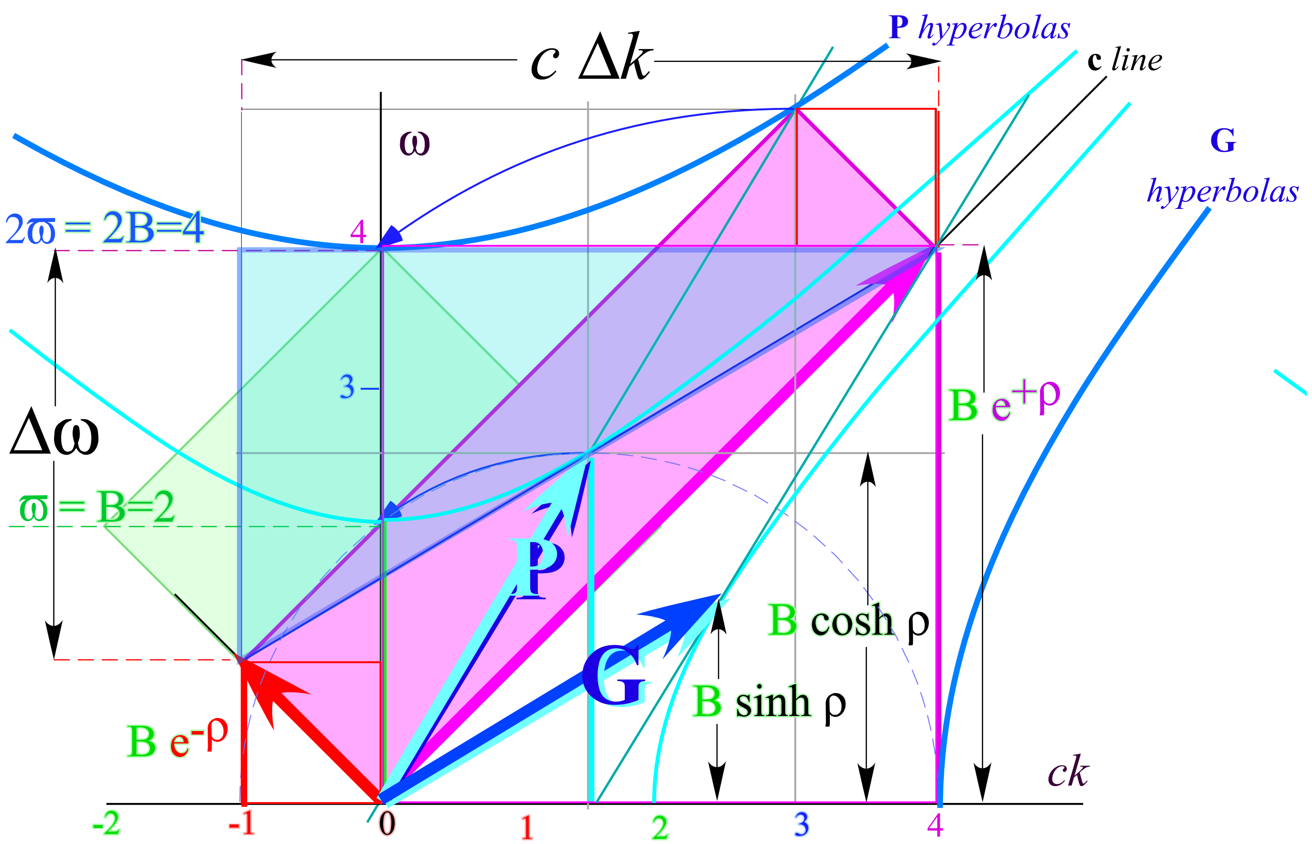
$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with LARGE Δk
(<u>not</u> infinitesimal)



Relativistic group wave speed $u=c \tanh\rho$

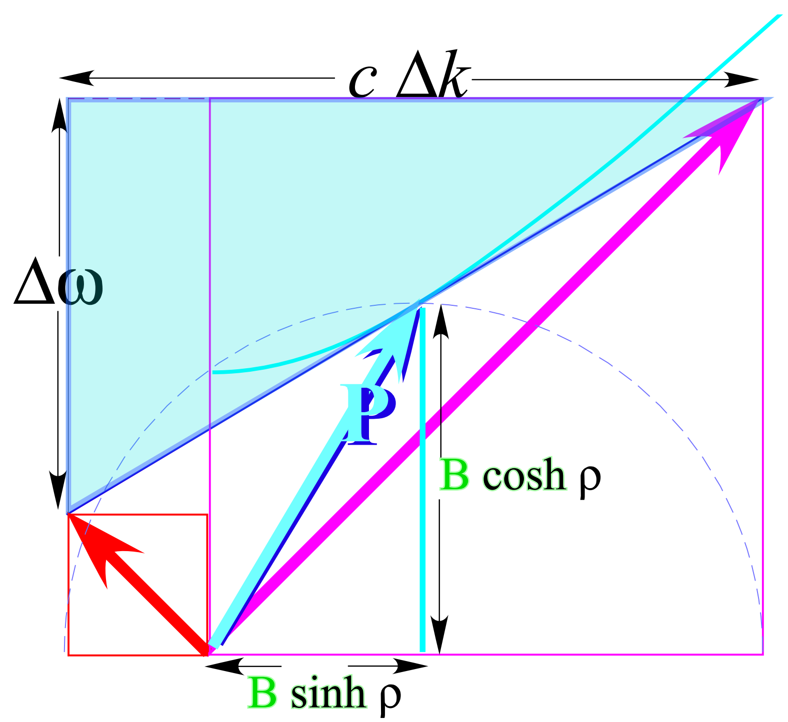
Group velocity u *and phase velocity* c^2/u *are hyperbolic tangent slopes*



Rare but important case where

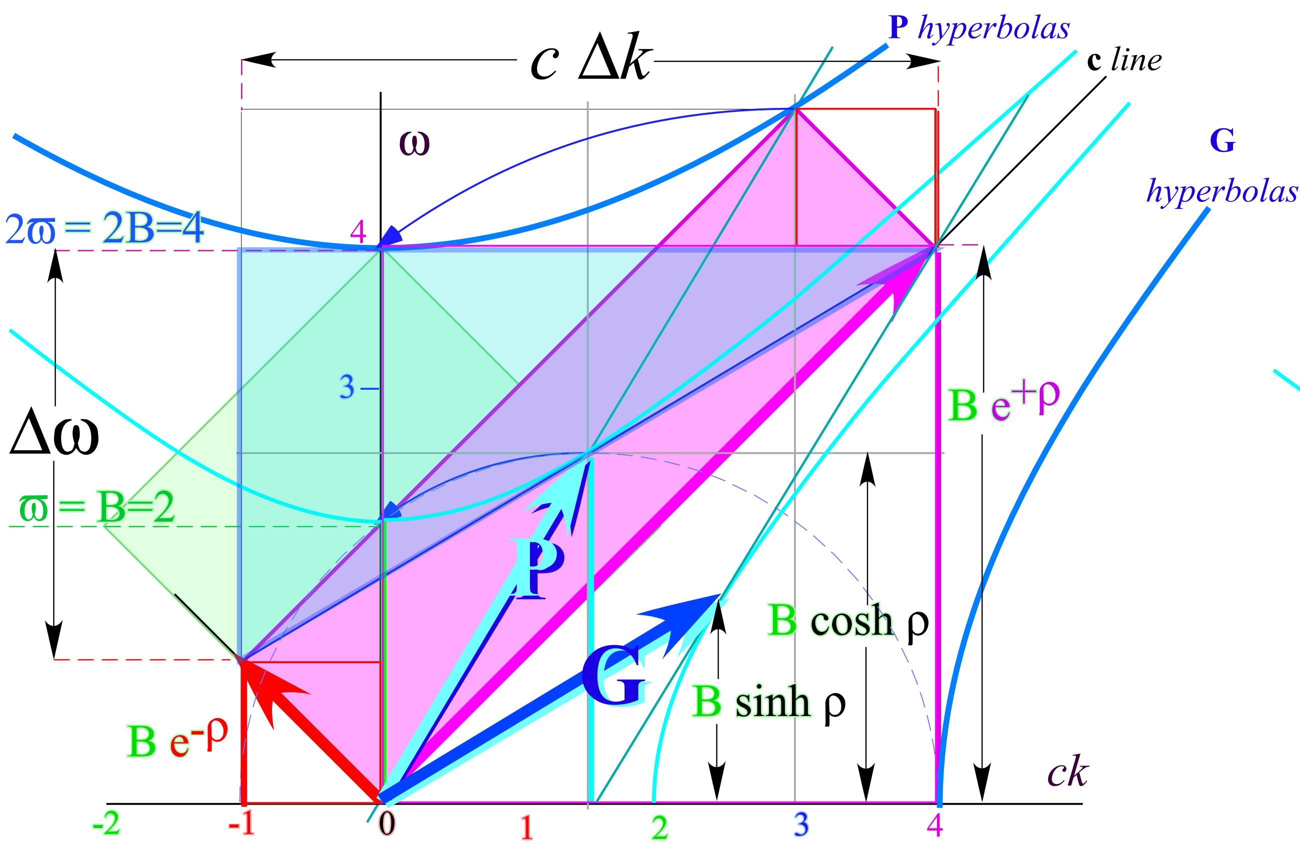
$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

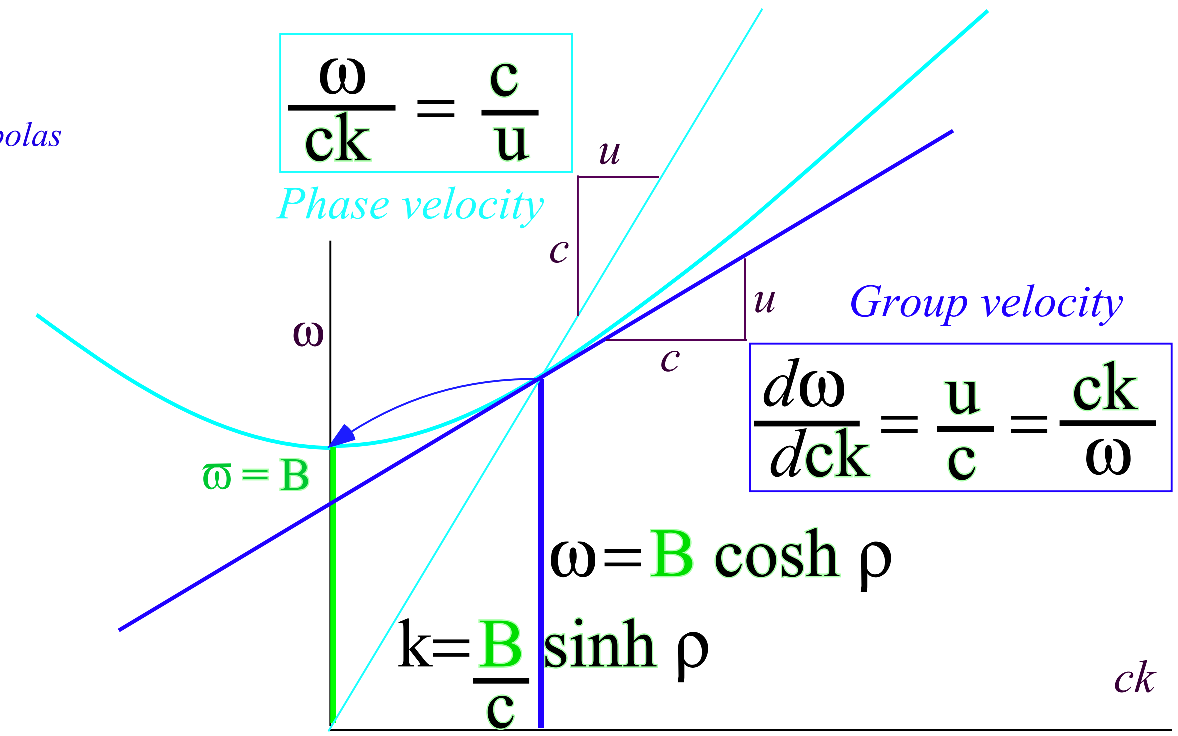
with LARGE Δk
(not infinitesimal)



Relativistic group wave speed u*=c tanh* ρ

Group velocity u and phase velocity c^2/u are hyperbolic tangent slopes





Rare but important case where

$$\frac{d\omega}{dk} = \frac{\Delta\omega}{\Delta k}$$

with LARGE Δk
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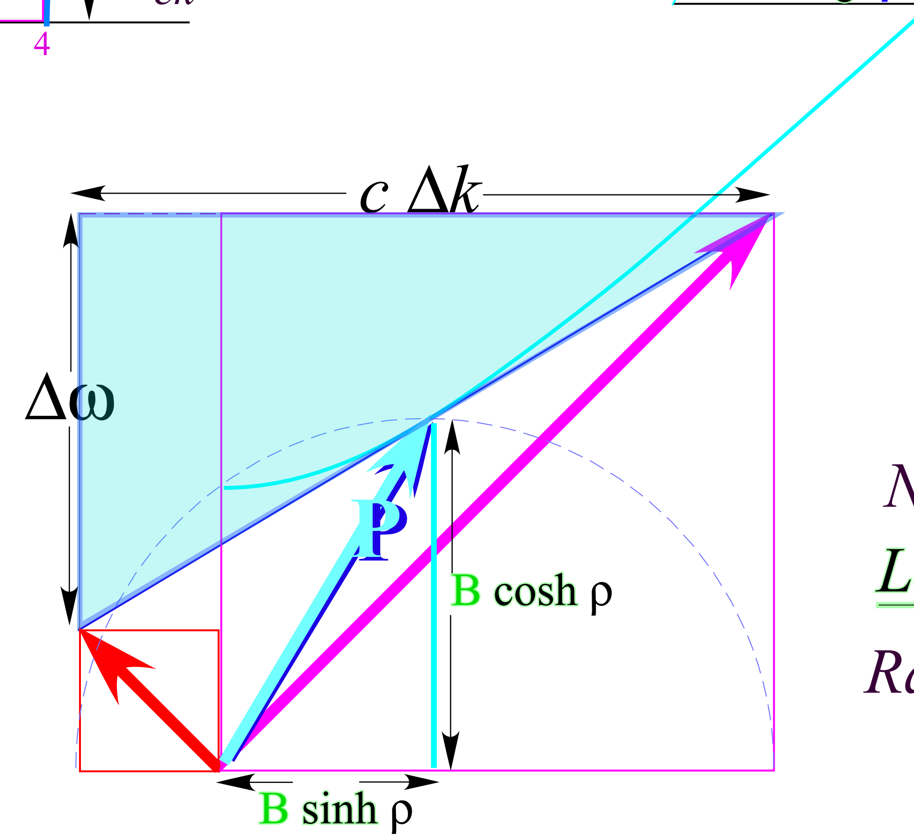
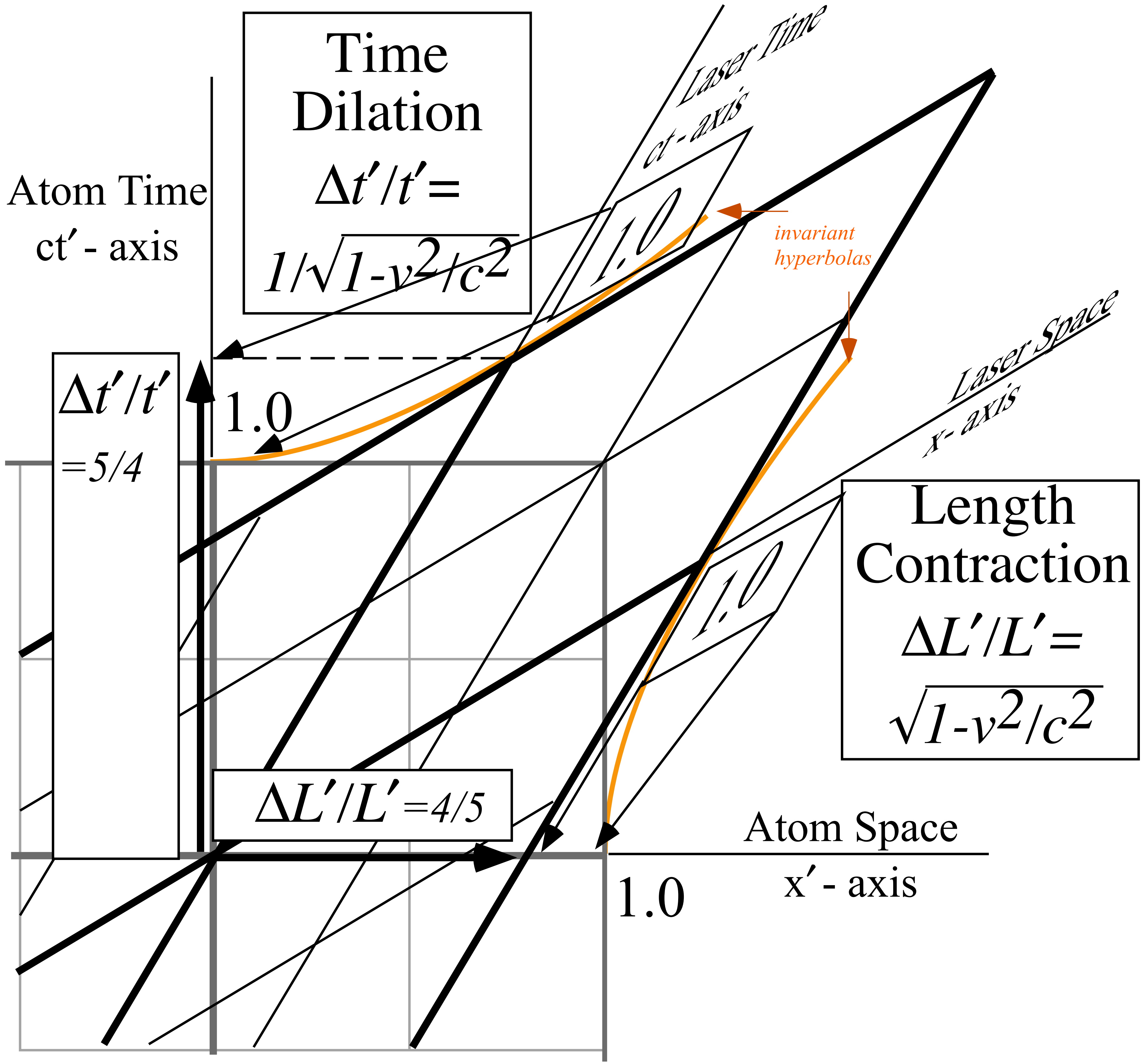


Relativistic group wave speed u=c tanh ρ

Newtonian speed u~cρ
Low speed approximation
Rapidity ρ approaches u/c

2. Reciprocal dilation and contraction properties

2. Reciprocal dilation and contraction properties



2. Reciprocal dilation and contraction properties

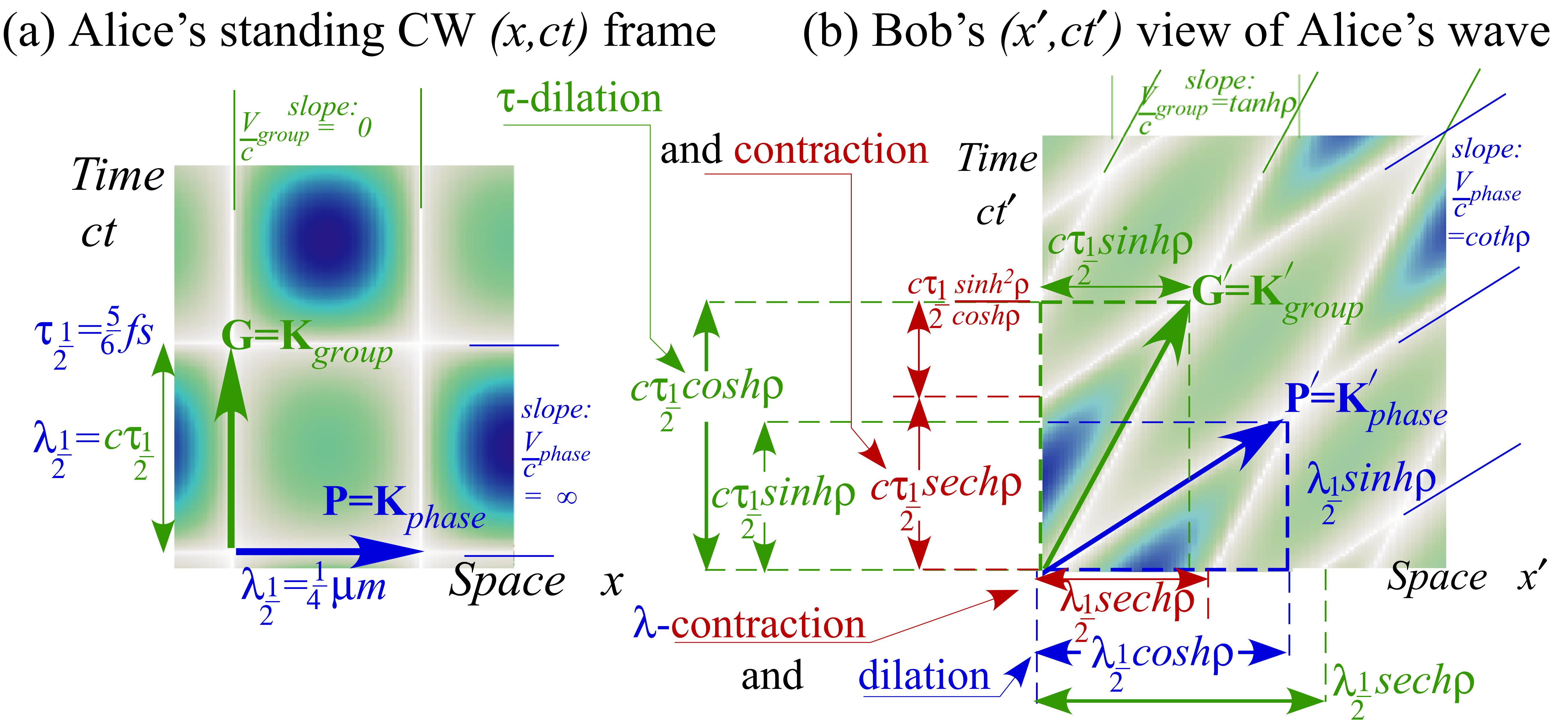
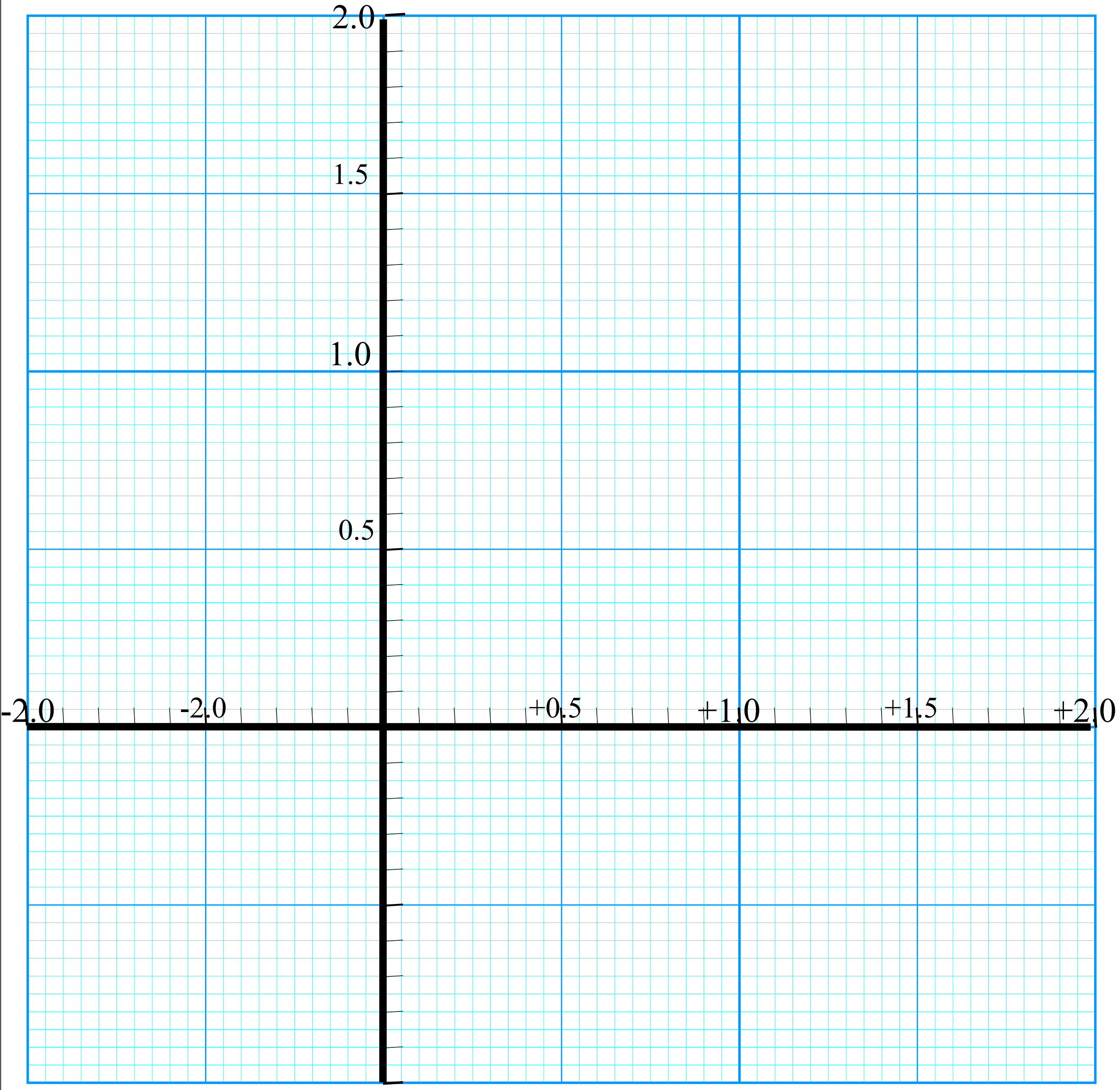
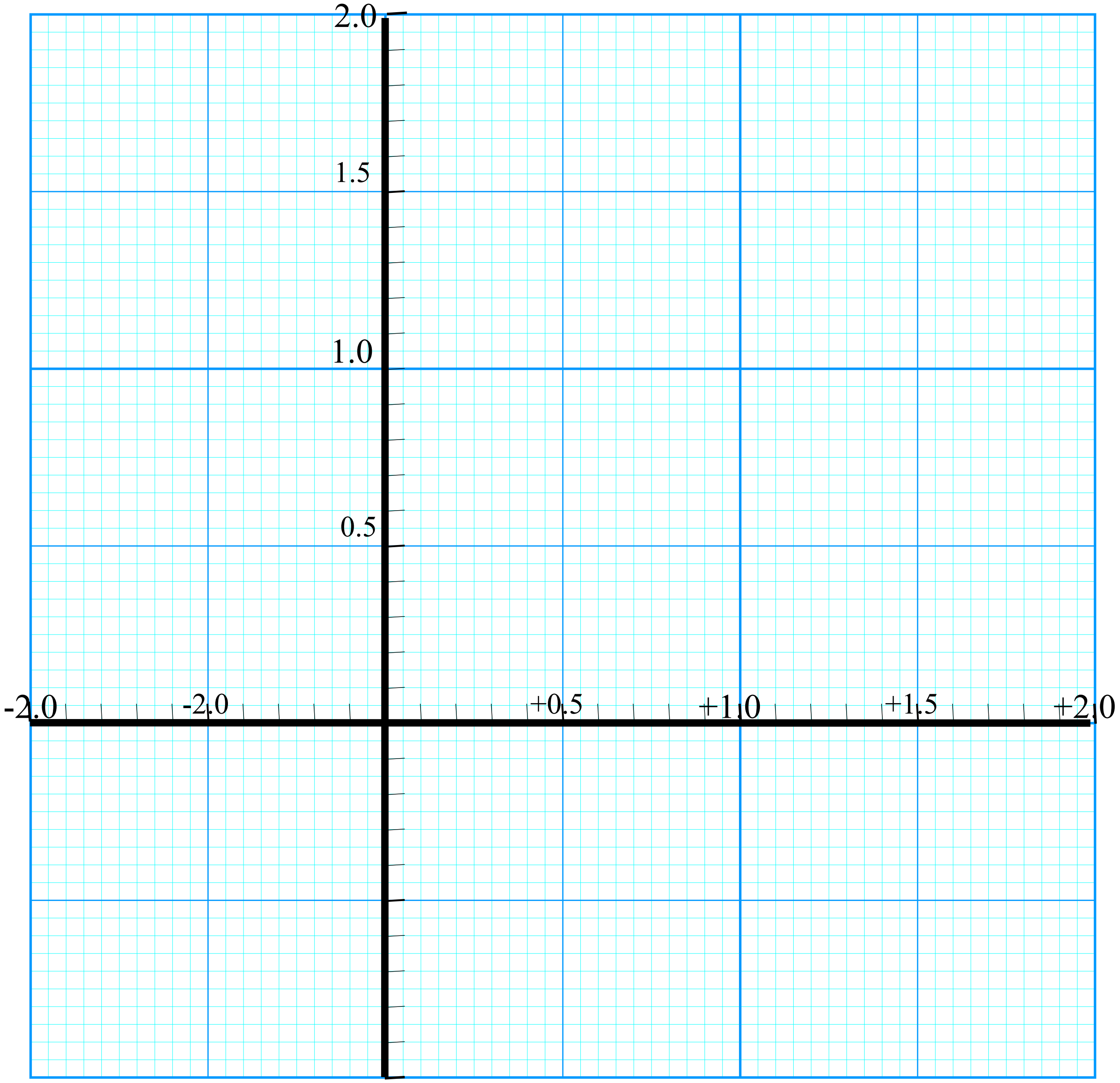


Fig. 10 SR&QMbyR&C



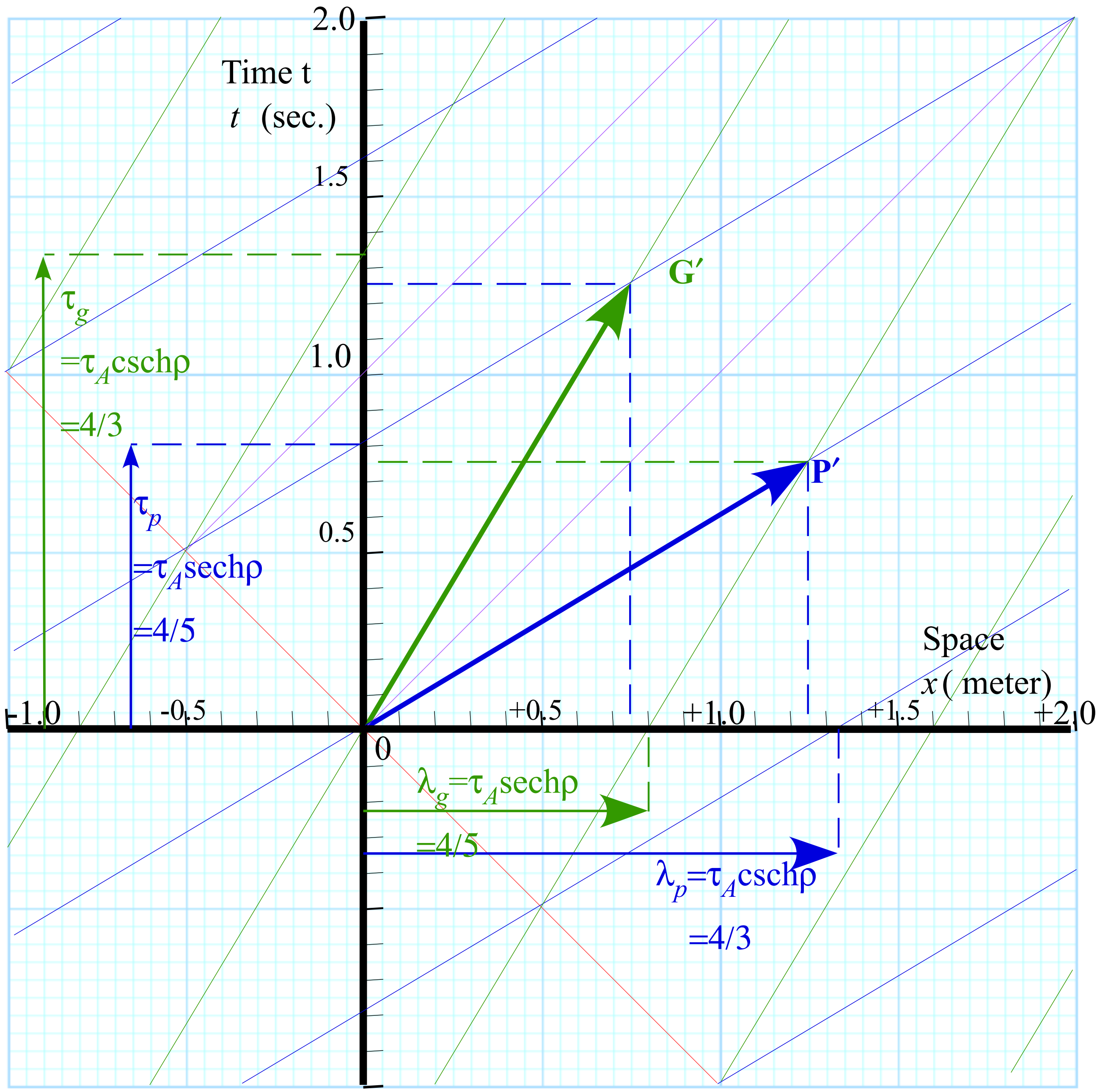
2.0
1.5
1.0
0.5
-2.0
-2.0
+0.5
+1.0
+1.5
+2.0



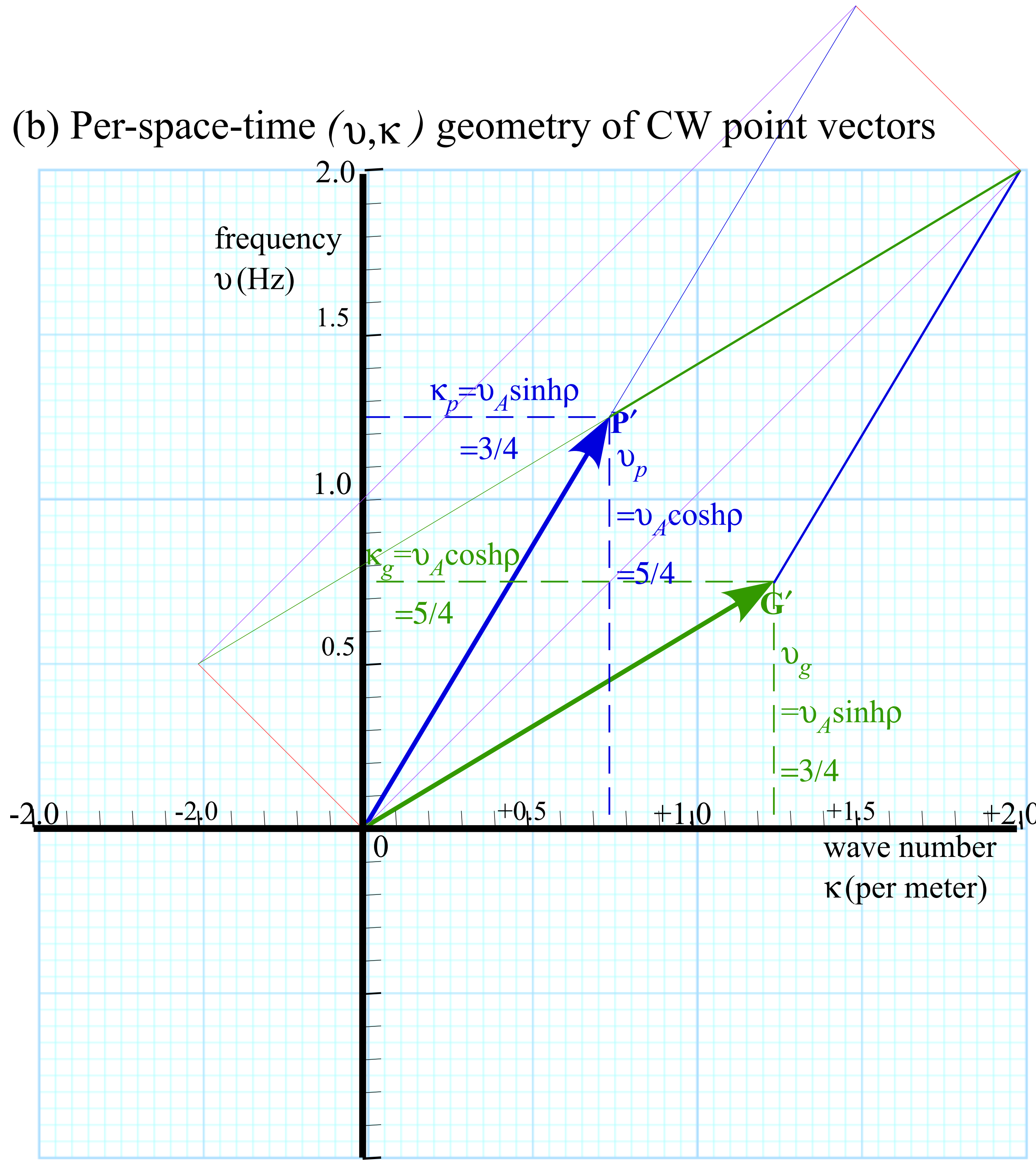
2.0
1.5
1.0
0.5
-2.0
-2.0
+0.5
+1.0
+1.5
+2.0

2. Reciprocal dilation and contraction properties

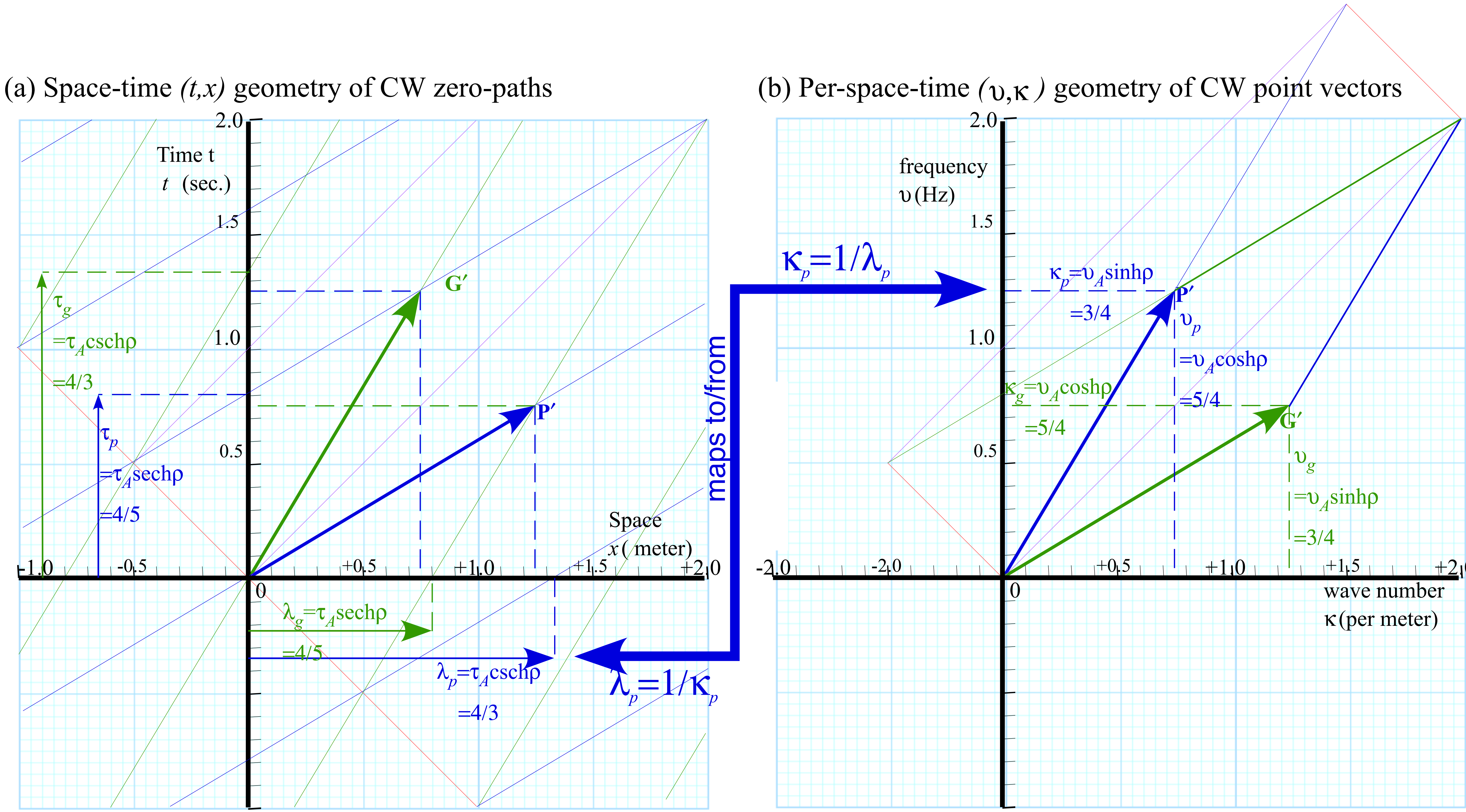
(a) Space-time *(t,x)* geometry of CW zero-paths



(b) Per-space-time *(υ,κ)* geometry of CW point vectors



2. Reciprocal dilation and contraction properties



2. Reciprocal dilation and contraction properties

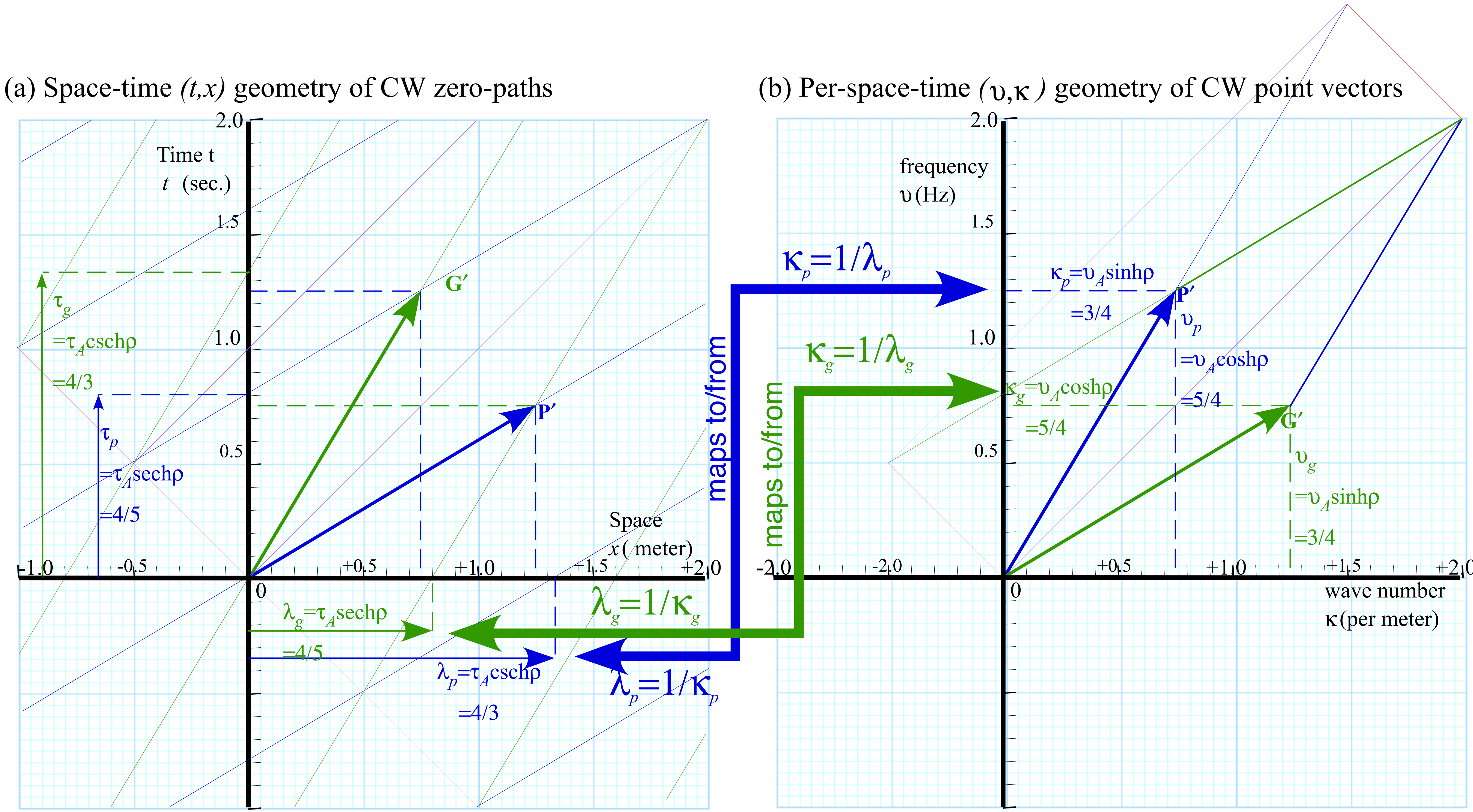


Fig. 11 SR&QMbyR&C

2. Reciprocal dilation and contraction properties

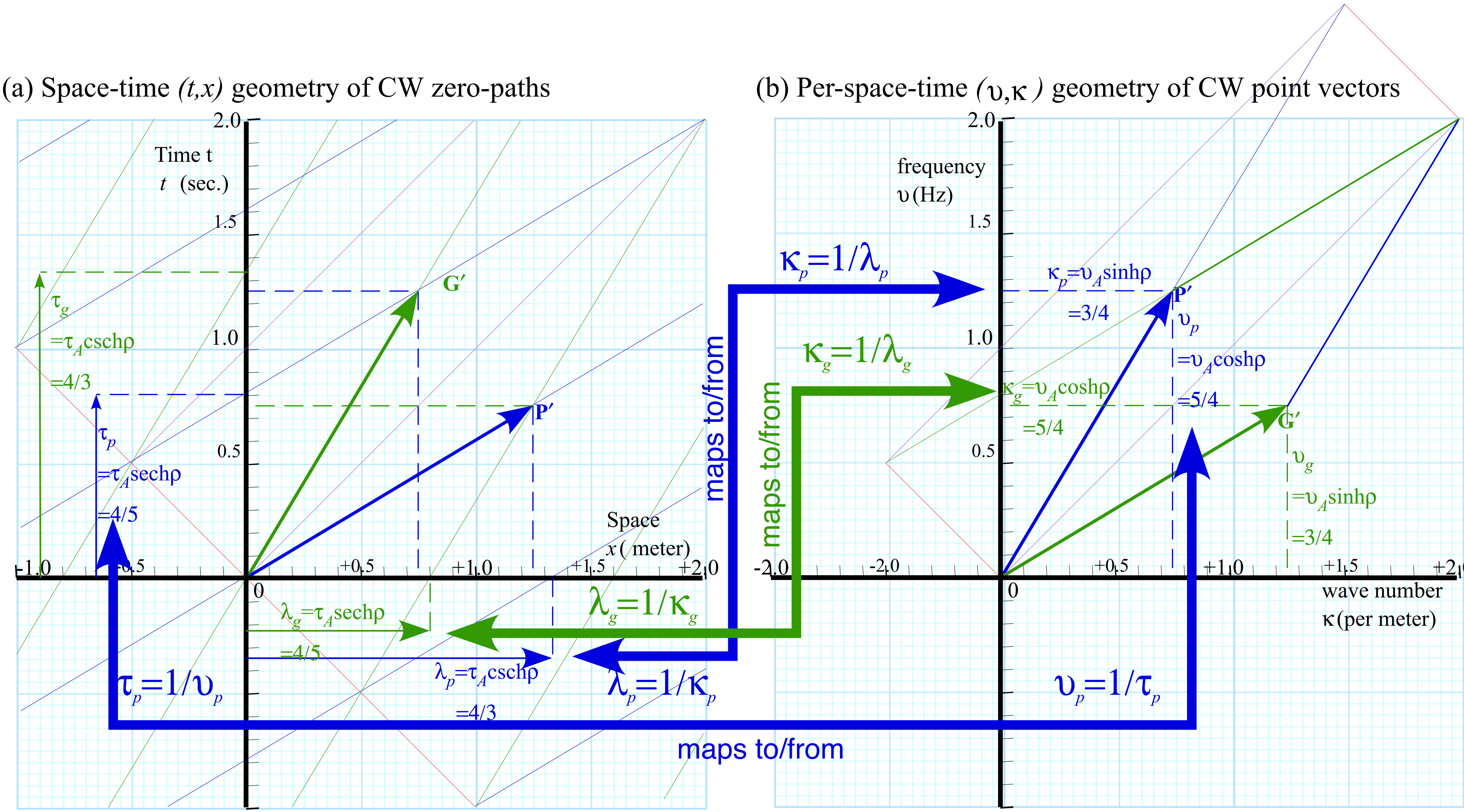


Fig. 11 SR&QMbyR&C

2. Reciprocal dilation and contraction properties

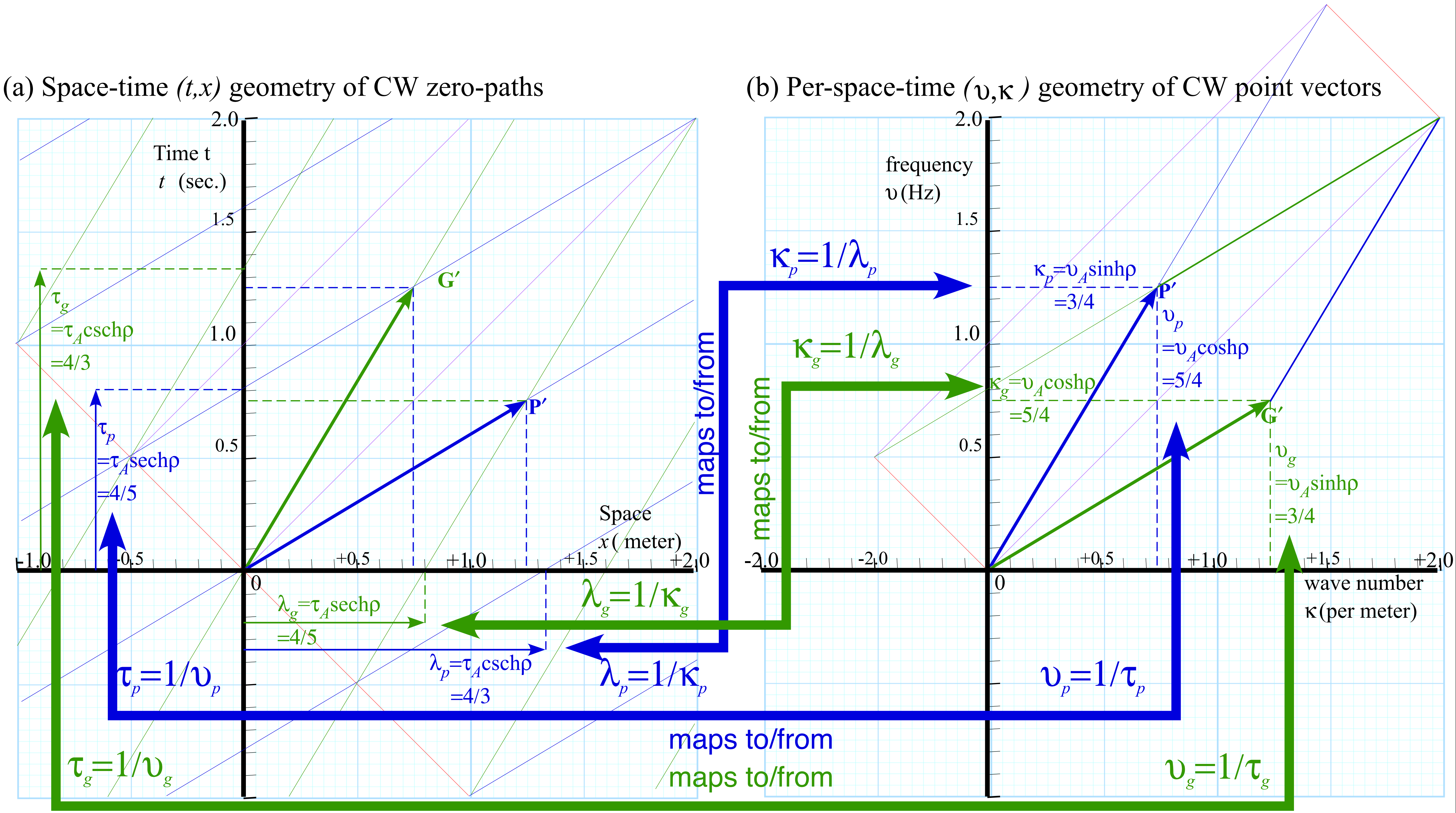


Fig. 11 SR&QMbyR&C

2. Reciprocal dilation and contraction properties

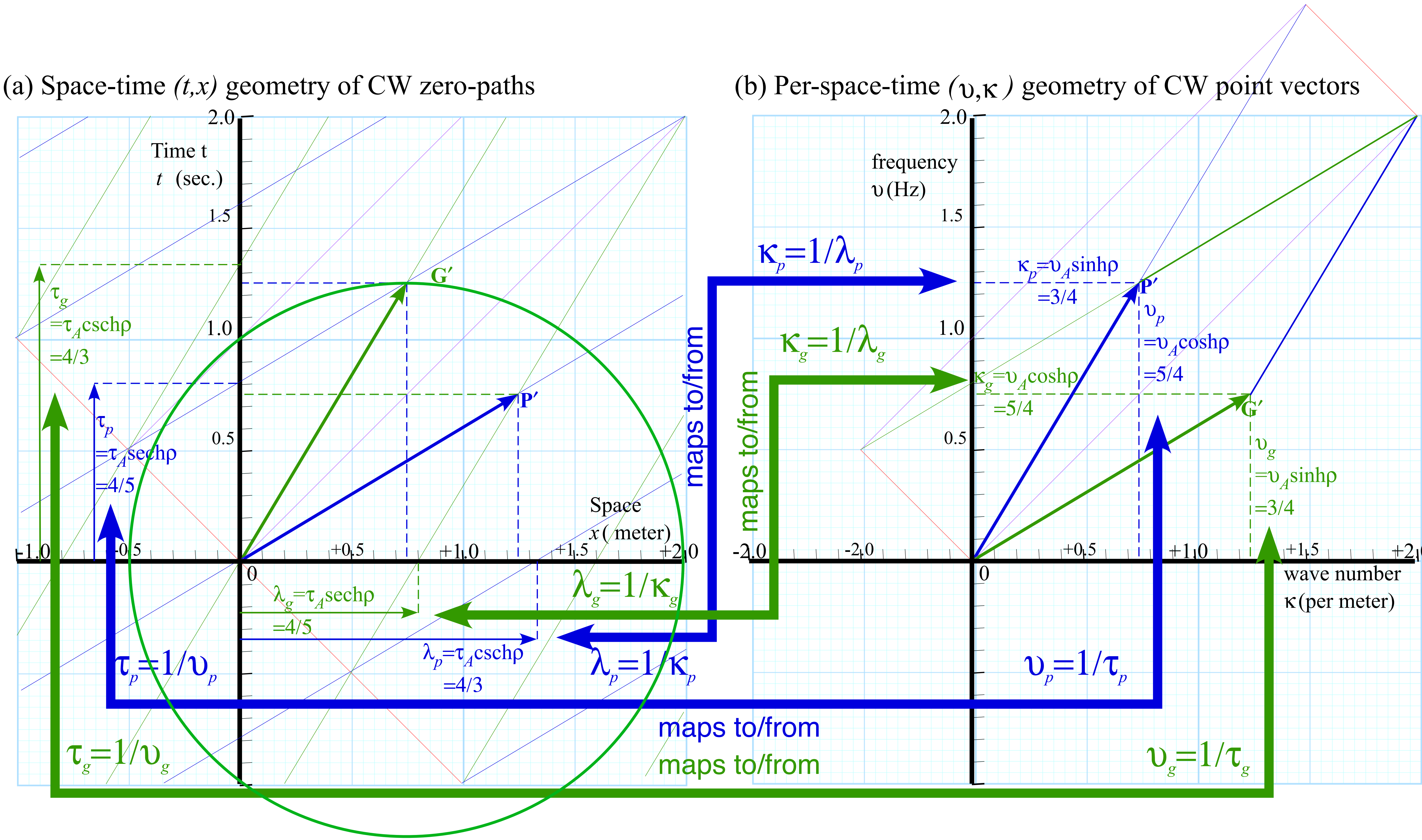


Fig. 11 SR&QMbyR&C

2. Reciprocal dilation and contraction properties

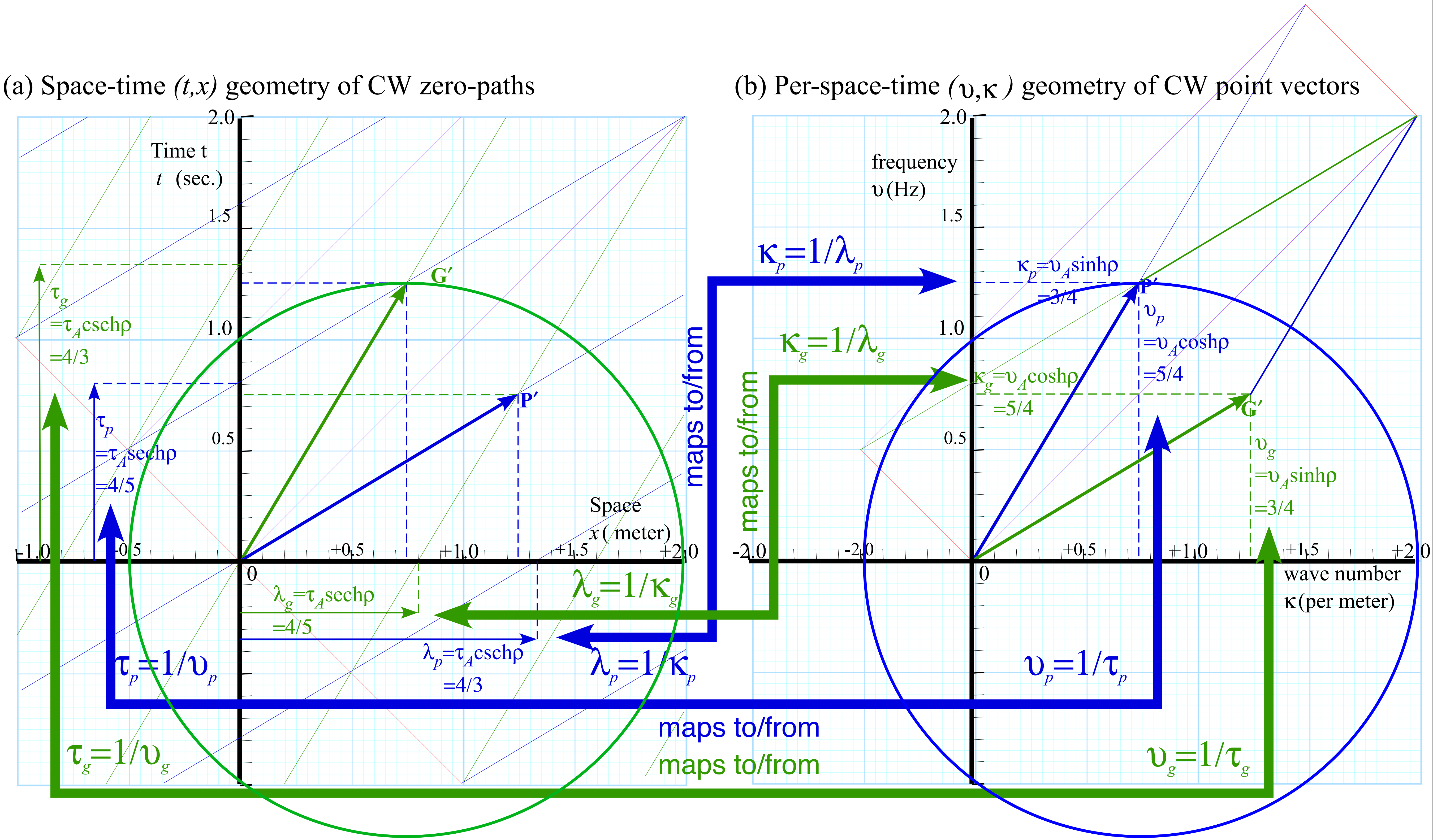


Fig. 11 SR&QMbyR&C

2. Reciprocal dilation and contraction properties

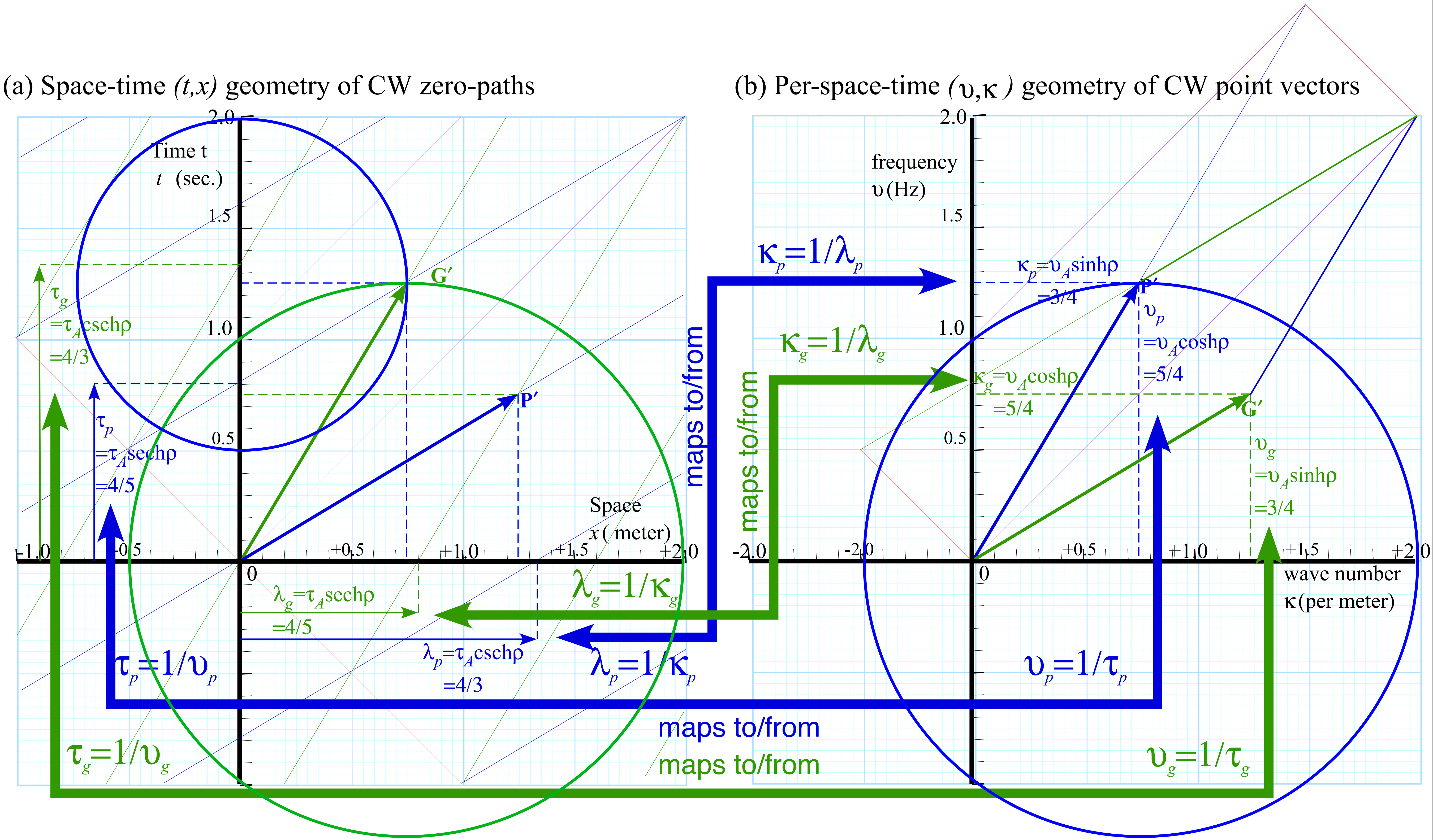


Fig. 11 SR&QMbyR&C

More connection to conventional approach to relativity and old-fashioned formulas

The most old-fashioned form(ula) of all: Thales & Euclid means

Galileo wins one! (...in gauge space)

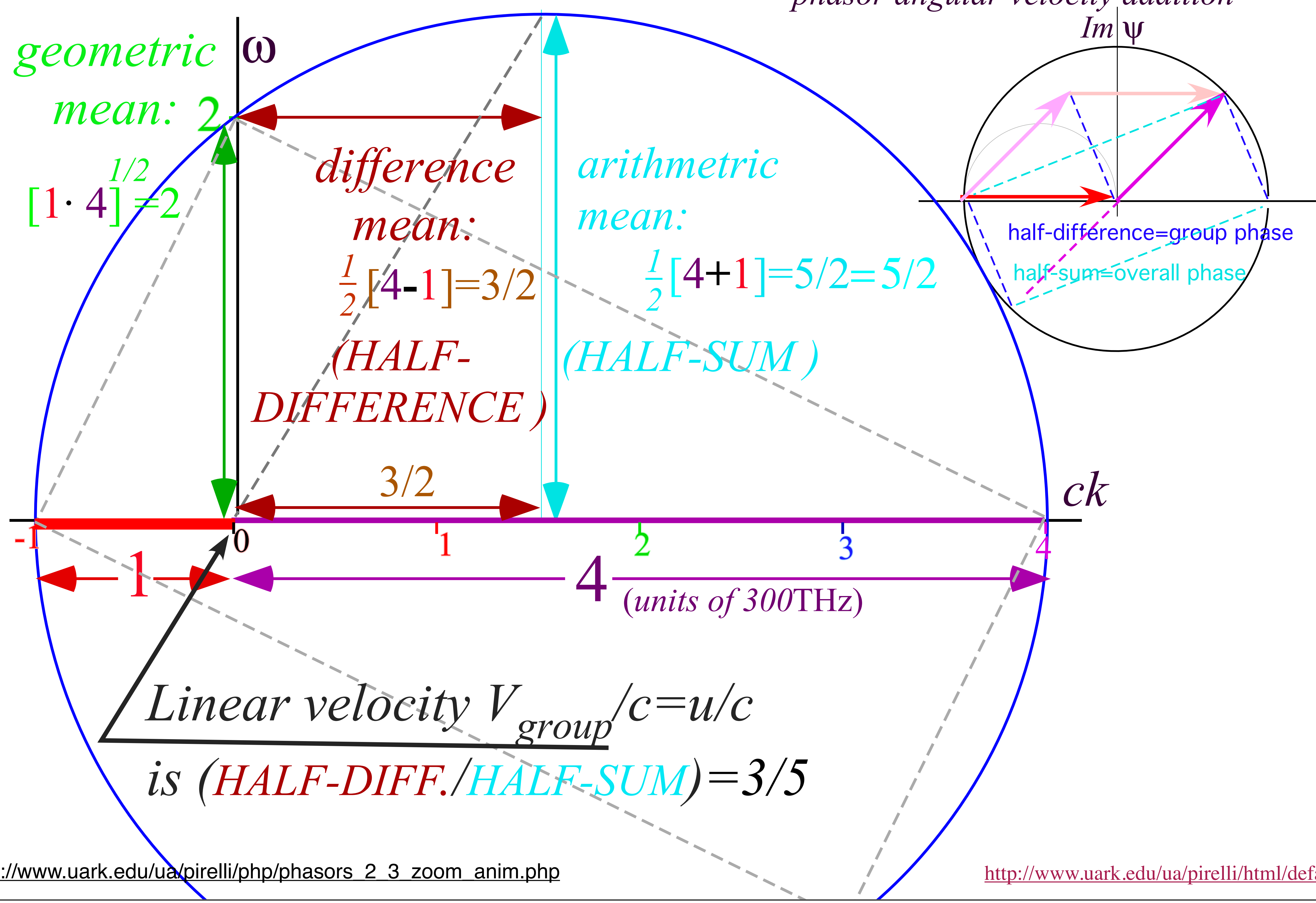
Connection to conventional approach to relativity and old-fashioned formulas
The most old-fashioned form(ula) of all: Thales & Euclid means

Euclid's 3-means (300 BC)

Geometric "heart" of wave mechanics

Thales (580BC) rectangle-in-circle

Relates to wave interference by (Galilean) phasor angular velocity addition



http://www.uark.edu/ua/pirelli/php/phasors_2_3_zoom_anim.php

http://www.uark.edu/ua/pirelli/html/default.html

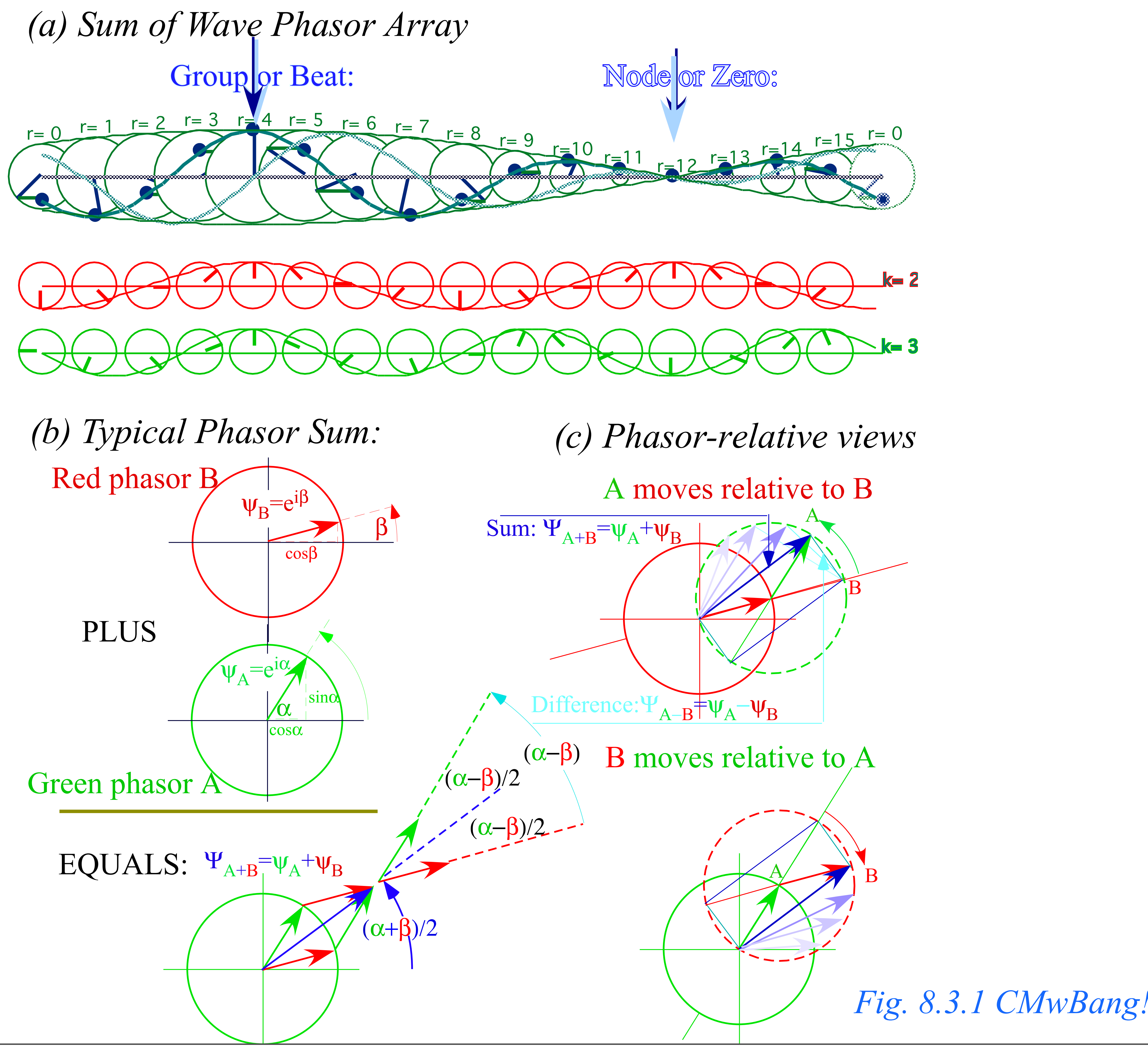
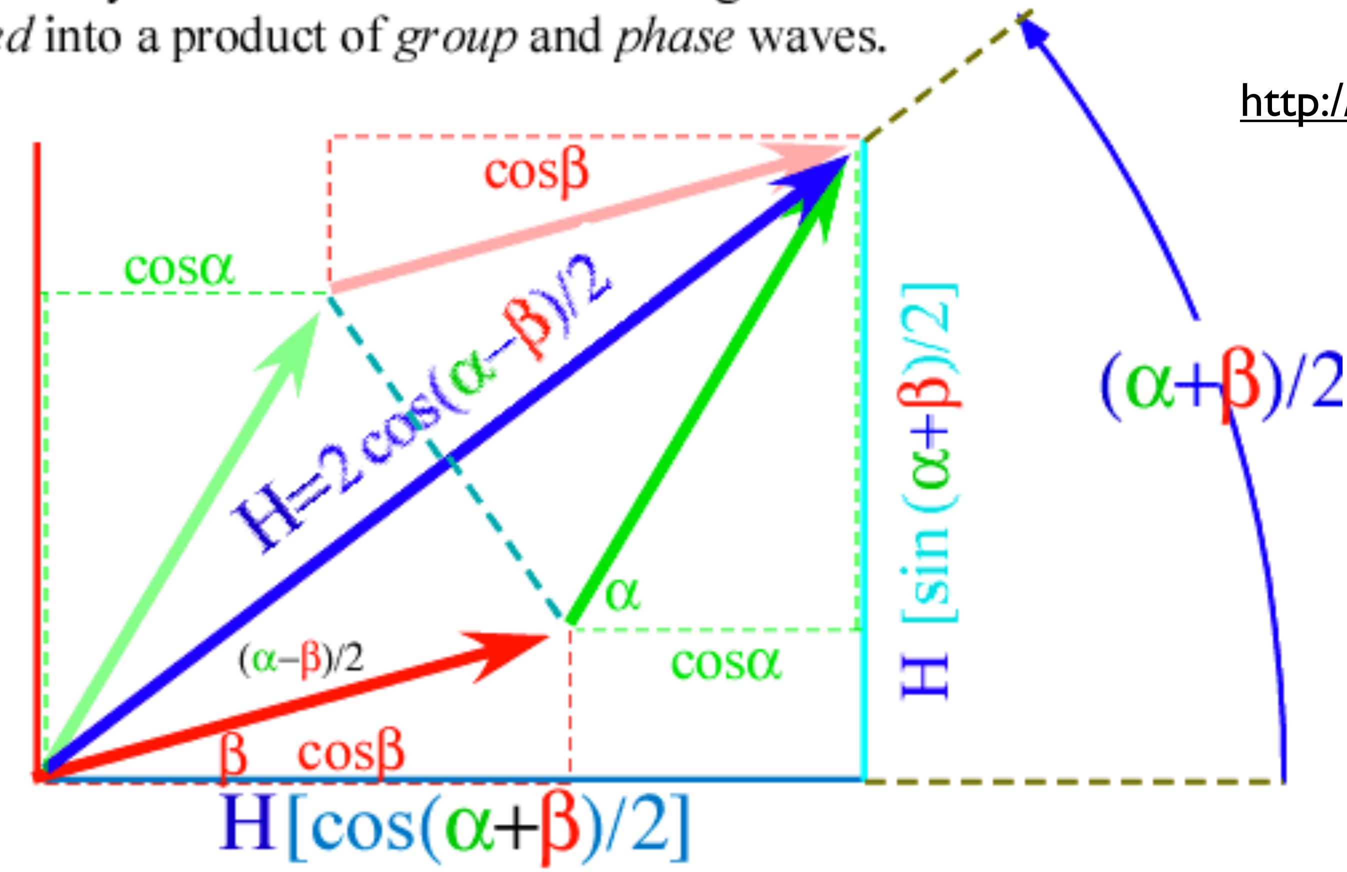


Fig. 8.3.1 CMwBang!

The detailed trigonometry of half-sum & difference angles is shown below.
The wave is *factored* into a product of *group* and *phase* waves.

http://www.uark.edu/ua/pirelli/php/half_sum_5.php



Main Result: Factoring algebraic sums helps to locate *wave zeros.*

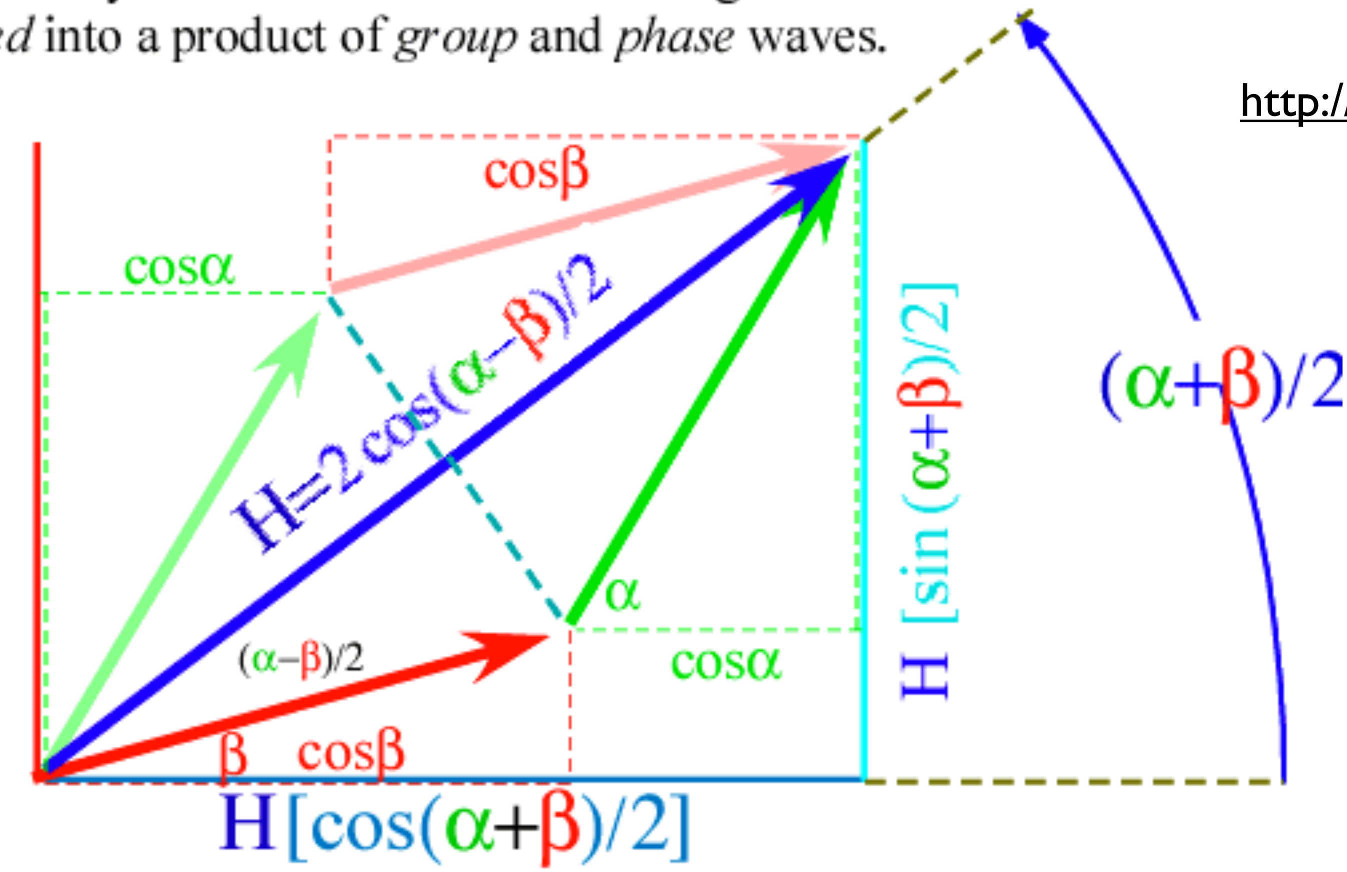
$\cos\alpha + \cos\beta \;=\; 2\cos(\alpha-\beta)/2 \;\bullet\; [\cos(\alpha+\beta)/2]$

$\sin\alpha + \sin\beta \;=\; 2\cos(\alpha-\beta)/2 \;\bullet\; [\sin(\alpha+\beta)/2]$

Sum is zeroed by *either* factor. Each factor's zero line is a *spacetime coordinate line*.

The detailed trigonometry of half-sum & difference angles is shown below.
The wave is *factored* into a product of *group* and *phase* waves.

http://www.uark.edu/ua/pirelli/php/half_sum_5.php

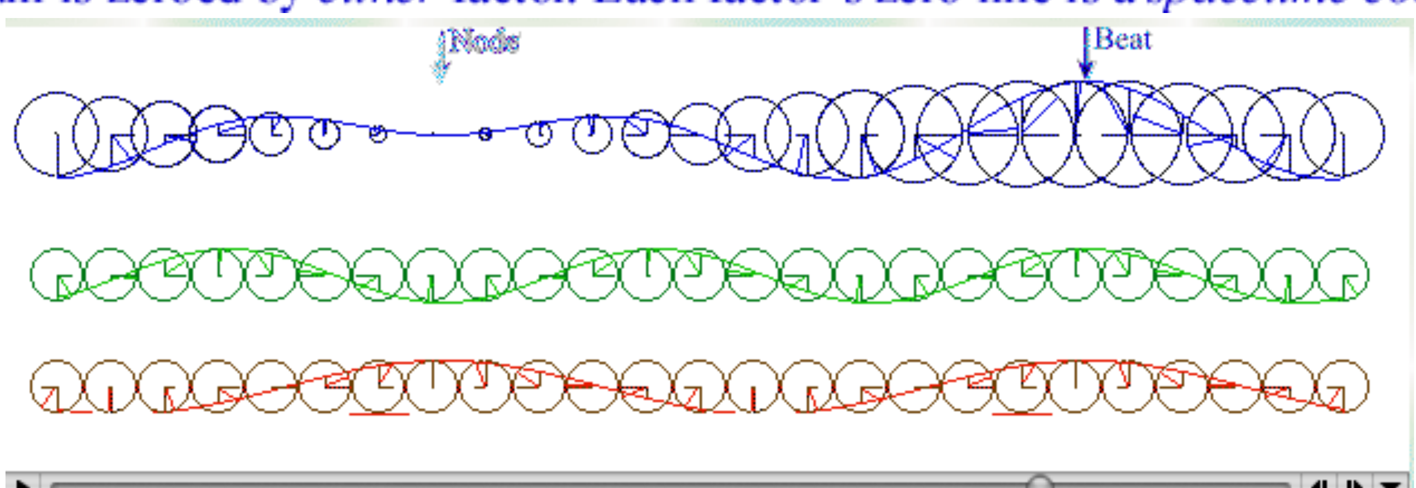


Main Result: Factoring algebraic sums helps to locate *wave zeros.*

$$\cos\alpha + \cos\beta = 2\cos(\alpha-\beta)/2 \cdot [\cos(\alpha+\beta)/2]$$

$$\sin\alpha + \sin\beta = 2\cos(\alpha-\beta)/2 \cdot [\sin(\alpha+\beta)/2]$$

Sum is zeroed by *either* factor. Each factor's zero line is a *spacetime coordinate line*.



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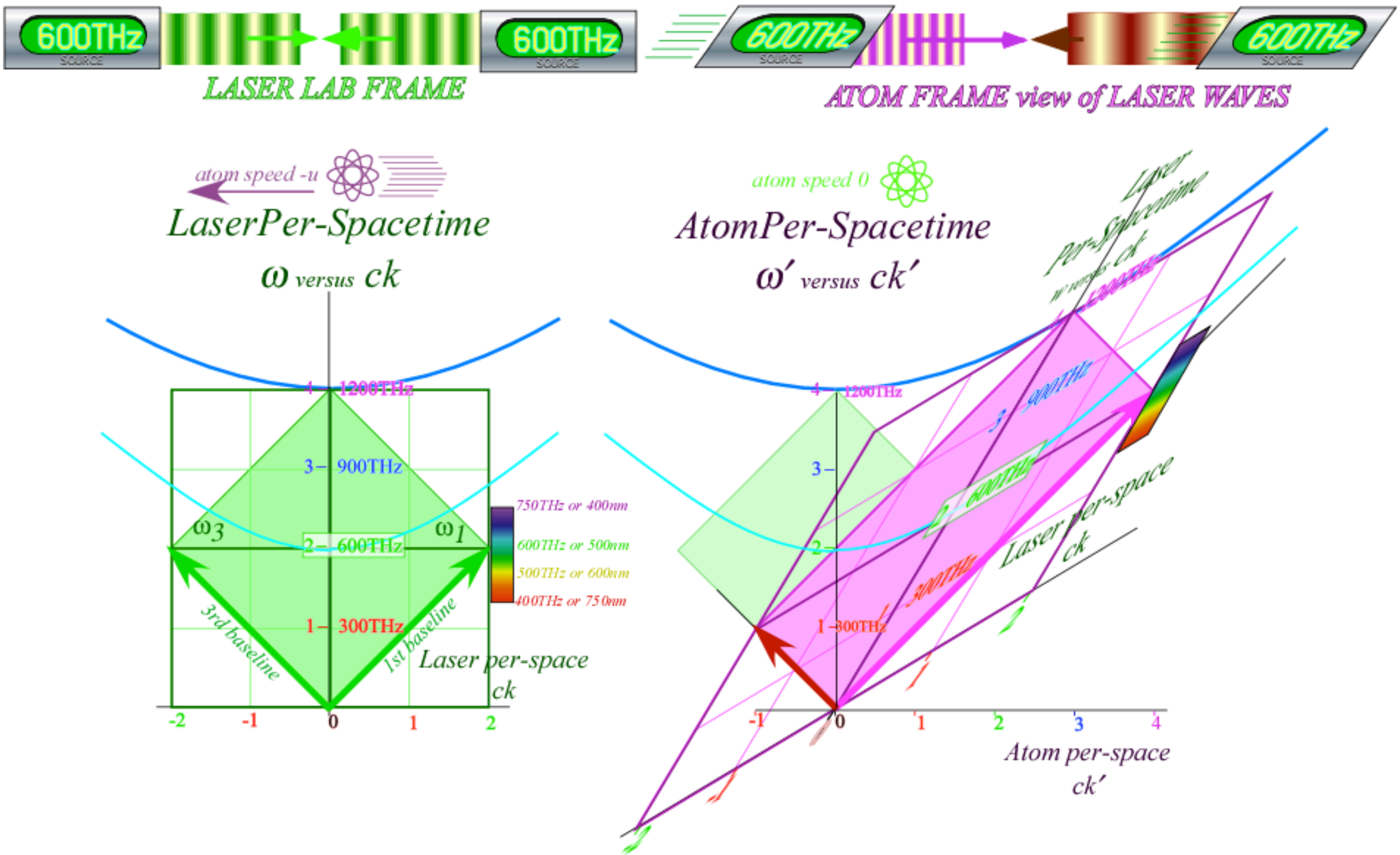
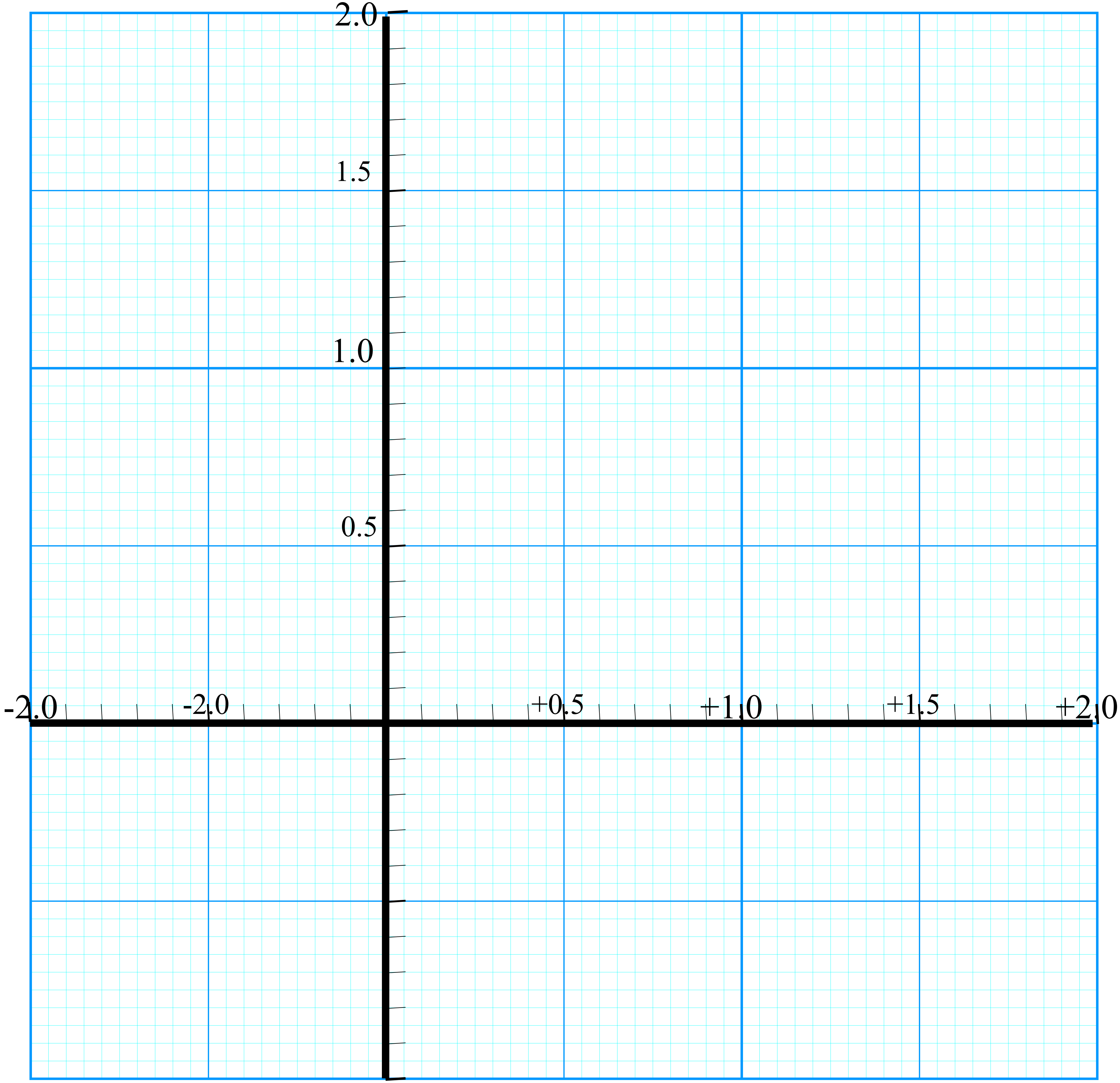


600THz
SOURCE
600THz
SOURCE
LASER LAB FRAME
600THz
SOURCE
600THz
SOURCE
ATOM FRAME view of LASER WAVES
atom speed -u
LaserPer-Spacetime
ω versus ck
atom speed 0
AtomPer-Spacetime
ω′ versus ck′
Laser Per-Spacetime ω versus ck
4 1200THz
3 900THz
2 600THz
1 300THz
ω3
ω1
3rd baseline
1st baseline
Laser per-space ck
-2 -1 0 1 2
750THz or 400nm
600THz or 500nm
500THz or 600nm
400THz or 750nm
4 1200THz
3
2
1 300THz
3 900THz
2 600THz
1 300THz
Laser per-space ck
-1 0 1 2 3 4
Atom per-space ck′


2.0
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